# ENHANCING PPE SUPPLY CHAIN RESILIENCE DURING THE COVID-19 PANDEMIC USING MULTI-OBJECTIVE OPTIMIZATION UNDER UNCERTAINTY 

by

Cecil Ash

Submitted in partial fulfillment of the requirements for the degree of Master of Applied Science
at

Dalhousie University
Halifax, Nova Scotia
May 2021
© Copyright by Cecil Ash, 2021

## Table of Contents

List of Tables ..... iv
List of Figures ..... V
Abstract ..... vi
List of Abbreviations ..... vii
Acknowledgments ..... viii
1 Introduction ..... 1
1.1 Thesis Motivation ..... 3
1.2 Thesis Contributions ..... 3
1.3 Thesis Outline ..... 4
2 Literature Review ..... 6
2.1 Agility vs. Resilience ..... 6
2.2 Conceptual Drivers of SC Agility and Resilience ..... 7
2.3 Pandemic Disruptions ..... 8
2.4 Risk Mitigation Strategies ..... 9
2.5 Mathematical Optimization ..... 10
2.5.1 Objective Functions in SC Resilience Models ..... 11
2.5.2 Risk Measures ..... 12
2.5.3 Optimization Under Uncertainty ..... 14
2.6 Simulation ..... 16
2.7 Machine Learning ..... 16
2.8 Hybrid Approaches ..... 16
2.9 Research Gaps ..... 17
3 Problem Definition ..... 19
4 Optimization Framework ..... 22
4.1 Deterministic Formulation ..... 22
4.2 Solution Approach ..... 27
4.3 Optimization Under Uncertainty ..... 28
4.3.1 Robust Optimization Formulation ..... 29
4.3.2 Stochastic Programming Formulation ..... 31
4.3.3 Distributionally Robust Optimization Formulation ..... 32
5 Numerical Experiments ..... 37
5.1 Cost and Service Level Trade-Off ..... 40
5.2 Risk Mitigation Strategy Analysis ..... 43
5.2.1 Starting Inventory Quantity ..... 43
5.2.2 External Emergency Stockpile ..... 45
5.3 Inventory Level Behavior ..... 48
5.4 RO Model Conservatism ..... 49
5.4.1 RO Contract Utilization ..... 50
5.5 DRO Model Conservatism ..... 50
5.5.1 DRO Contract Utilization ..... 52
5.5.2 Relative Standard Deviation in Cost ..... 53
5.6 Value of Information ..... 54
6 Discussion ..... 56
6.1 Managerial Implications ..... 56
6.2 Theoretical Contributions ..... 59
6.3 Limitations ..... 60
6.4 Recommended Future Work ..... 60
7 Conclusion ..... 62
Bibliography ..... 64
A Matlab Code for Deterministic Model ..... 71
B Matlab Code for Two-Stage RO Model with SAA ..... 79
C Matlab Code for Multi-Stage RO Model with SAA ..... 88
D Matlab Code for Two-Stage DRO Model ..... 90
E Matlab Code for Multi-Stage DRO Model ..... 99
F Electronic Supplements ..... 101
G Copyright Permission Letter ..... 102

## List of Tables

5.1 Multi-stage dataset parameter value ranges ..... 40
5.2 Value of information experiment results ..... 55

## List of Figures

3.1 Representational diagram of the case study SC ..... 21
5.1 Pandemic severity factors by time period in two-stage dataset scenarios ..... 38
5.2 Plot of potential pandemic severity factor values in multi-stage dataset ..... 39
5.3 Deterministic model Pareto fronts using two-stage dataset ..... 42
5.4 Pareto fronts of the two-stage and multi-stage RO $(\alpha=0), \mathrm{SP}$, and DRO ( $\rho=0.8$ ) models ..... 43
5.5 Deterministic model costs by scenario and average quantity of starting inventory ..... 45
5.6 Expected cost and cost RSD for two-stage (left) and multi-stage (right) SP models versus size of emergency stockpile ..... 46
5.7 Plot of cost objective versus size of emergency stockpile ..... 47
5.8 Expected cost and cost RSD for two-stage (left) and multi-stage (right) SP models versus price of emergency stockpile ..... 48
5.9 Plot of RO minimax cost objective against values of $\alpha$ ..... 49
5.10 Plot of RO contract utilization against values of $\alpha$ ..... 50
5.11 Plot of DRO cost objective against values of $\rho$ ..... 51
5.12 Plot of DRO expected cost if estimated probabilities are correct against $\rho$ ..... 52
5.13 Plot of DRO contract utilization against values of $\rho$ ..... 53
5.14 Plot of DRO cost RSD against values of $\rho$ ..... 54


#### Abstract

This study presents a multi-period bi-objective mixed-integer supply planning model and applies it to a case study inspired by the operational challenges of a Canadian provincial healthcare provider's PPE supply chain during the COVID-19 pandemic. Uncertainties in the supply, price, and demand of PPE are considered. The cost minimization objective function is formulated using stochastic, robust, and distributionally robust optimization. The service-level objective function follows minimax robustness by minimizing the maximum shortage of any product in any time period and scenario. The $\epsilon$-constraint method is used to generate Pareto-optimal solutions and analyze the trade-off between the two competing objectives. Numerical experiments analyze model behaviour and the efficacy of emergency inventory and increased inventory levels as risk mitigation strategies. The distributionally robust optimization model is recommended with its ambiguity set size determined by the decision makers' relative preferences for average cost performance, worst-case cost performance, or cost variance.


## List of Abbreviations Used

| CVaR | Conditional Value at Risk |
| :--- | :--- |
| DRO | Distributionally Robust Optimization |
| PPE | Personal Protective Equipment (N95 Masks, Gloves, Gowns) |
| RO | Robust Optimization |
| SAA | Sample Average Approximation |
| SC | Supply Chain |
| SP | Stochastic Programming |
| SCND | Supply Chain Network Design |
| SCRM | Supply Chain Risk Management |
| VaR | Value at Risk |

## Acknowledgements

I would like to thank our research partner, the Supply Operations division of the Nova Scotia Health Authority, for guiding this research. Thank you to my supervisors, Dr. Venkatadri and Dr. Diallo, for being excellent leaders and colleagues. Thank you to Dr. Saif and Dr. VanBerkel for encouraging me to enter the M.A.Sc program. Thank you to Mariah, my mother, and my brother for all your support.

## Chapter 1

## Introduction

In recent decades, the prevailing strategy of supply chain (SC) design has been to reduce redundancies such as inventory in the name of short-term efficiency. The COVID-19 pandemic, and the ensuing disruption to SCs of all types, has exposed the risks of relying on a handful of low-cost, often foreign, producers. Medical SCs have received particular attention during the COVID-19 pandemic, as their disruption threatens the quality of healthcare services and containment of diseases.

Health service providers around the world have suffered shortages of medical equipment needed to combat the COVID-19 virus. Worldwide shortages of ventilators and personal protective equipment (PPE) paired with increased demand have pushed prices significantly above their norm. In April 2020, an independent non-profit entity called the Society for Healthcare Organization Procurement Professionals (2020) reported that the cost of N95 masks had risen 6,136\% during the COVID-19 pandemic compared to their pre-COVID prices. Surging demand and prices make it difficult for health organizations to secure the medical supplies that they need. This problem is exacerbated by a globally constrained supply of PPE. On April 13, 2020, a Canadian news agency reported that Canada had received around $6 \%$ of 293 million surgical masks ordered, around $0.5 \%$ of 130 million N95 masks ordered, and less than $0.5 \%$ of 900 million pairs of gloves ordered (Zimonjic, 2020). The severity of SC disruptions reported during the COVID-19 pandemic highlights the potential value of a resilienceoriented approach to SC design.

Even prior to COVID-19, the amount of research on SC resilience had increased significantly in the preceding years (Hosseini et al., 2019a). Resilience is a property
of SCs that describes their capacity to plan for, absorb, recover from, and adapt to unfavorable events (National Research Council, 2012). Resilience differs from other SC properties such as sustainability or robustness in its emphasis on recovering from disruptions and returning to the SC's original state (Golan et al., 2020). Enhancing SC resilience often coincides with decreasing SC risk which Jajja et al. (2018) define as both the likelihood and impact of disruption in SC sourcing, transportation, or operations; however, resilience is a property of SCs and risk is an environmental factor.

Motivated by the COVID-19 pandemic, Ivanov and Dolgui (2020b) introduced a new concept in SC research called SC viability. SC viability contains all the qualities of SC resilience in addition to the ability to survive long-term disruptions that scale unpredictably. While SC resilience typically studies singular disruptions during a fixed time window, SC viability studies the continuous evolution of SCs over longer, even infinite, time windows (Ivanov \& Dolgui, 2020b).

This thesis explores novel applications of mathematical optimization under uncertainty to improve the resilience and viability of a Canadian provincial healthcare provider's PPE SC. The presented models select suppliers and allocate forecasted demand among those suppliers during multiple time periods. Multi-period modelling is required since pandemics can disrupt SCs for multiple years and to varying degrees of intensity. In total, this research considers five extensions of the basic supplier selection problem:

1) Multi-periodicity to consider the long-term disruption and unpredictable spread of pandemics.
2) Multi-objective programming to analyze the trade-off between satisfying demand and cost performance.
3) Robust, stochastic, and distributionally robust optimization to consider uncertainty in PPE prices, supply, and demand.
4) Sample average approximation (SAA) to adjust model conservatism through chance-constraining the robust cost objective.
5) All-unit volume-based discounted pricing.

The case study to which this research is applied was inspired by the Nova Scotia Health Authority (NSHA), which is the largest health services provider in the Canadian province of Nova Scotia. The NSHA Supply Operations Department procures medical equipment and distributes it to hospitals located throughout Nova Scotia using numerous warehouses.

### 1.1 Thesis Motivation

This study is motivated by the operational challenges experienced by Canadian healthcare providers during the COVID-19 pandemic. Its goal is to develop mathematical models that can support procurement decision making prior to and during long-term SC disruptions with the specific aims of discovering generally applicable managerial insights and demonstrating the process by which any healthcare provider can implement the presented models. The efficacy of additional starting inventory and federal emergency PPE stockpiles as risk mitigation strategies is analyzed in the case study using the proposed optimization framework. The contents of this thesis are partially based on manuscripts (Ash et al., 2021a; Ash et al., 2021b) on the application of stochastic programming (SP) and robust optimization (RO) models and distributionally-robust optimization (DRO) models to healthcare SC optimization during pandemics.

### 1.2 Thesis Contributions

The contributions of this thesis are categorized as either managerial implications, referring to the insights and tools offered to SC managers, or theoretical contributions to academia. The managerial and theoretical contributions of this study are briefly
summarized in this section and discussed in further detail in Section 6.

This research analyzes the unique challenges of pandemics as SC disruptions. The presented optimization framework demonstrates how SC managers can use multiobjective optimization to assess the trade-off between two competing goals and multiperiod modelling to optimize SCs during long-term disruptions. The SP, RO, and DRO models serve as decision support tools with varied levels of risk tolerance and confidence in input data. The case study is based on a Canadian Healthcare Provider and considers three different types of sourcing. Multiple operational insights are uncovered regarding pre-pandemic inventory levels, emergency stockpile size and prices, sourcing mix, and model conservatism.

This study contributes to SC literature by augmenting the limited number of studies on multi-objective optimization under uncertainty. Although the theory behind multiobjective RO has been presented numerous times, including by Kuroiwa and Lee (2012), few research studies have applied it to real-world SCs. A realistic case study is analyzed with not only multi-objective RO, but also multi-objective DRO which is a novel approach in supplier selection research. This research presents a unique comparison between the value of information in two-stage and multi-stage recourse models. Finally, this thesis evaluates the effectiveness of risk mitigation strategies in the context of pandemics that spread unpredictably and disrupt SCs over multiple time periods.

### 1.3 Thesis Outline

The remainder of this paper is structured as follows. Relevant literature is discussed next followed by presentation of the case study and optimization framework used in this research in Section 3. Section 4 formulates a deterministic multi-objective supply planning model along with SP, RO, and DRO extensions to consider uncertainty.

Numerical experiments are conducted in Section 5. Section 6 discusses managerial and theoretical contributions, limitations, and avenues of future study. Conclusions are drawn in Section 7.

## Chapter 2

## Literature Review

Golan et al. (2020) recommend a comprehensive approach to designing resilient SCs that includes definitions, models, metrics, and disruption analysis. This literature review thus begins by presenting definitions and conceptual drivers of important SC properties. It then discusses the unique nature of pandemics as disruptions, summarizes common SC risk mitigation strategies, and reviews quantitative analysis methods.

This research was influenced by existing literature surveys on SC network design (SCND) under uncertainty (Govindan et al., 2017), SC risk management (SCRM) (Dolgui et al., 2018; Baryannis et al., 2018; Heckmann et al., 2015), and SC resilience (Golan et al., 2020; Hosseini et al., 2019a; Ivanov et al., 2017; Kamalahmadi \& Parast, 2016). SC viability had limited research at the time of writing due to its recent conception from the field of SC resilience. SC viability is thus framed as SC resilience in the context of longer-term disruptions.

### 2.1 Agility vs. Resilience

It is important to distinguish between SC agility and SC resilience, as both terms are properties of a SC related to SC disruptions. SC agility is typically defined as the ability to sense changes in the SC or external environment and then efficiently respond to them (Altay et al., 2018). Agility extends the definition of flexibility, which is simply the ability to rapidly respond to changes, by including the requirement to sense changes (Eckstein et al., 2014). Flexibility can thus be viewed as an antecedent of agility. Characteristics of agile SCs are fast delivery, reliability, flexibility in product
volume and mix, and the ability to customize products (Jajja et al., 2018).

SC agility and resilience share some similar capabilities such as short lead-times and flexible sourcing (Carvalho et al., 2012), but they should be viewed as separate strategies for improving SC performance. Wieland and Marcus Wallenburg (2013) categorize SC agility as a reactive strategy for achieving SC resilience. SC agility enables rapid response to new conditions, which increases SC resilience by quickly recovering to normal operations following a disruption (Carvalho et al., 2012).

### 2.2 Conceptual Drivers of SC Agility and Resilience

Using a dynamic capabilities approach, Brusset \& Teller (2017) present evidence that flexibility, which is defined as responsiveness to customer stimuli, and integration, which is defined as collaborative SC management and decision making with SC partners, both significantly improve SC resilience. Using similar methods, Jajja et al. (2018) conclude that awareness of risk motivates companies to improve their SC agility through supplier and customer integration.

Minimizing SCs' vulnerability to disruptions is a key driver of SC resilience. SCs that rely upon a few individual nodes to supply a disproportionately large amount of product are vulnerable to major shortages if those key nodes are disrupted. LimCamacho et al. (2017) propose a resilience index that is highest when product flows are more evenly distributed to all nodes in the SC. SC resilience is also enhanced by the geographical separation of suppliers which reduces the probability that regional disruptions will affect multiple suppliers (Kamalahmadi \& Parast, 2017; Hosseini et al., 2019b).

Communication and collaboration between SC members has been proven through qualitative and quantitative studies to enhance SC resilience (Hosseini et al., 2019a). Li et al. (2017) illustrate how inter-echelon information sharing can enhance SC
resilience by curtailing the propagation of disruptions to other echelons of the SC.

### 2.3 Pandemic Disruptions

SC disruptions are defined as events with low probabilities of occurrence and severe negative consequences on SC performance (Torabi et al., 2015). They have greater impact, less frequent occurrence, and potentially longer-term effects than operational risks, which include events like equipment failure, power outages, and personnel absence (Torabi et al., 2015; Jabbarzadeh et al., 2018).

While some recent studies (Golan et al., 2020; Mehrotra et al., 2020; Ivanov \& Dolgui, 2020b; Ivanov, 2020a; Ivanov, 2020b) discuss SC disruptions specifically in the context of the COVID-19 pandemic, the vast majority of research on SC disruptions study localized disruptions such as building fires, natural disasters, or political unrest. SC disruptions caused by pandemics differ from localized disruptions due to the potentially longer duration and unforeseeable propagation of pandemics (Ivanov, 2020a). The significance of these characteristics is demonstrated by Ivanov's (2020a) simulationbased analysis of the COVID-19 pandemic in which the greatest determinants of a pandemic's impact on SC performance were pandemic propagation speed, facility disruption duration, and the time at which facilities are opened or closed.

Another difference between pandemics and localized disruptions is that while the latter impedes sections of a SC, pandemics can simultaneously affect multiple geographic regions and echelons of the SC (Sheffi, 2015). Uncertainty in a SC's supply or demand side alone can seriously impede its efficiency, while volatility in multiple areas of a SC, as has been created by the COVID-19 pandemic, presents even greater challenges (Choi et al., 2019). Pandemics can also generate panic in the general public resulting in unstable pricing and demand (Sheffi, 2015).

Further complicating the predictability of pandemic disruptions is the variation in
epidemiological and pathological features among viruses. SARS-CoV-2, the virus behind the COVID-19 pandemic, spreads significantly faster and has higher infectivity than SARS-CoV and MERS-CoV (Goh et al., 2020) despite their shared ancestry (Hu et al., 2020).

### 2.4 Risk Mitigation Strategies

Risk mitigation strategies minimize the likelihood or impact of disruptions' adverse effects. The most common SC risk mitigation strategies are i) multiple sourcing, ii) backup sourcing, iii) emergency inventory, and iv) facility fortification (Govindan et al., 2017).

Multiple sourcing is achieved by sourcing products from multiple primary suppliers. This strategy provides greater flexibility when adapting to disruptions in the supply base (Costantino \& Pellegrino, 2010) and is more effective than single sourcing at mitigating the risks of high operating costs and low service levels (Sawik, 2014). Meena and Sarmah (2013) present a non-linear model that selects the optimal number of suppliers based on the trade-off between the volume-based discounts offered by each supplier and their risk of disruption. Tomlin (2006) uses a Markovian inventory model to show that multiple sourcing becomes a more effective risk mitigation strategy than emergency inventories as disruptions occur more frequently and for longer durations. Another risk mitigation strategy, backup sourcing, contracts the option to buy finished products or future production capacity from a supplier (Torabi et al., 2015). Typically, there is a fixed cost to reserve backup production capacity and then a per unit purchase price if it is utilized. Backup suppliers may also be vulnerable to disruptions. A third strategy, emergency inventories, pre-purchases units and stores them at locations throughout the SC to help meet demand if primary sources are disrupted. A fourth risk mitigation strategy is facility fortification which reduces the likelihood of facility
disruption. Installing handwashing stations and teaching employees to self-monitor their health are pandemic-related examples of this strategy.

Torabi et al. (2015) present a resilient supplier selection model which uses the four strategies mentioned above and fifth strategy that is called supplier continuity planning and entails collaborating with suppliers to develop their own disruption recovery plans. Another risk mitigation strategy found in literature is substituting products or raw materials with an alternative when primary sources are unavailable (Hosseini et al., 2019a). An example of this strategy during pandemics is the substitution of medical face shields with those produced using plastic sheets and 3D printing.

Dolgui et al. (2018) categorize all risk mitigation strategies as either proactive or reactive. Multiple sourcing, emergency inventory, and facility fortification are proactive strategies, as they defend the SC against disruptions with little attention paid to recovery. Back-up sourcing, supplier continuity planning, and product substitution are reactive strategies, as they focus on modifying a SC after disruptions occur to recover as quickly as possible.

Organizations may pursue more than one risk mitigation strategy. Evidence from Yoon et al. (2018) suggests that implementing multiple sourcing, back-up sourcing, and emergency inventories can improve SC performance more than utilizing only one of these approaches. Ultimately, risk mitigation approaches must be specifically tailored to the organization based on contextual variables and their propensity for cost or service-level focused performance (Yoon et al., 2018).

### 2.5 Mathematical Optimization

Mathematical optimization is a commonly applied technique for SCND under disruption risks (Dolgui et al., 2018; Hosseini et al., 2019a). Linear programming is a type of optimization that maximizes or minimizes a linear equation called the objective
function by changing the values of decision variables subject to constraints.

### 2.5.1 Objective Functions in SC Resilience Models

This section presents six categories of objective functions used in SC resilience and SCRM models. The first category is monetary objective functions such as minimizing cost or maximizing profit. Penalty costs for undesirable events, such as unmet demand, can be included in monetary objectives. Jeong et al. (2013) design emergency SCs using a multi-objective mixed-integer linear program (MILP) that minimizes operating cost and penalty costs incurred during disruptions. Rottkemper et al. (2012) minimize both cost and the amount of unsatisfied demand when solving a transshipment model for humanitarian relief SCs.

Another objective function approach is to measure performance level. One of the first quantitative resilience metrics proposed in literature was the area under the operating level curve during a disruption (Bruneau et al., 2003). Other examples include minimizing lost production (Simchi-Levi et al., 2015), minimizing the percent of demand that cannot be satisfied (Chen \& Miller-Hooks, 2012), and minimizing the total travel time of customers whose demand is satisfied by secondary sources (Azaron et al., 2020). In a COVID-19 motivated model, Mehrotra et al. (2020) minimize the maximum number of ventilators that any hospital is shorted in any time period. Khalili et al. (2016) simultaneously maximize three performance metrics: production capacity, transportation capacity, and emergency inventory availability as a percentage of the nominal capacity.

A third type of objective function is minimization of recovery time after a disruption. Torabi et al. (2015) perform SCND by minimizing the unit weighted time between the start of a disruption and when unmet demand is satisfied. Sahebjamnia et al. (2018) present a multi-objective MILP that minimizes profit-weighted time to recover to full
capacity as well as operating level loss following disruptions.

Minimizing the probability of occurence of undesirable events, such as costs exceeding budget, is another modeling approach (Azaron et al., 2008; Guillen et al., 2005).

A fifth objective function category is to optimize strategies that improve SC resilience. Hosseini et al. (2019b) select a resilient supply base by maximizing the sum of geographic distances between suppliers and minimizing total costs using a multi-object MILP. Yoon et al. (2018) select suppliers by maximizing the demand-weighted sum of supplier reliability scores. Wang et al. (2009) design a medical SC network by minimizing the maximum transport time between any warehouse and demand point. Margolis et al. (2018) propose a MILP that maximizes demand weighted connectivity, which is defined as the number of unique paths from suppliers to a demand point where all nodes in the path are only used in that path. Cardoso et al. (2015) list four resilience indicators that can be used in model constraints and objective functions: the number of nodes in the SC, the number of flows in the SC, the ratio of the number of flows to the number of potential flows, and the number of critical nodes which are those that have net flows above a certain threshold.

A sixth objective function category is the optimization of risk terms such as standard deviation, regret, or value-at-risk (VaR) (Heckmann et al., 2015). Examples of this in literature and a broader discussion of risk terms are presented in the following section.

### 2.5.2 Risk Measures

SC research has adopted numerous risk metrics from finance and insurance industries with the purpose of quantifying the likelihood and severity of variations in objective function value (Govindan et al., 2017). Heckmann et al. (2015) surveyed SCND literature and found that most quantitative risk measures fit into one of two categories. The first category is deviation-based metrics such as standard deviation, expected
deviation, regret, and semi-deviation from target. Deviation-based metrics describe the width of the objective function value distribution. The second category is downside risk metrics such as VaR and conditional value-at-risk (CVaR). Downside risk metrics describe the objective function value in worst-case scenarios beyond some probability of occurrence. Many risk metrics can only be computed if scenario probabilities are known.

Decision makers can assess the trade-off between performance and risk exposure using a weighted mean-risk term in the objective function (Govindan et al., 2017). Noyan (2012) proposes a SP model that optimizes the weighted sum of the expected cost and the CVaR for a given risk tolerance level. Jabbarzadeh et al. (2018) design a SC network under disruption risk using a stochastic-robust model that minimizes a weighted sum of the expected value and maximum regret, which is the difference between the objective values of a solution and the best possible solution for that scenario. Model preference for optimizing expected value or the risk-term is adjusted by changing the value of the weight parameter.

Risk-performance trade-offs can be assessed using multi-objective models where at least one of the objective functions is a risk-term (Govindan et al., 2017). Azaron et al. (2008) develop a tri-objective SP model that minimizes cost variance along with expected cost and the probability that costs exceed budget. Sabio et al. (2010) propose a multi-objective model that minimizes both expected cost and the worst-case cost.

Risk tolerance can also be modelled using risk constraints and chance constraints. In risk constraints, a risk-term is constrained by a parameter. In chance constrained programming, some constraints are only required to be satisfied a set portion of the time. In a resilient supplier selection model, Hosseini et al. (2019b) chance constrain the number of disruptions that each supplier can experience.

### 2.5.3 Optimization Under Uncertainty

RO, SP, and DRO were considered for optimization under uncertainty. SP optimizes the expected value of an objective function based on either the probability distribution function or discrete probabilities of unknown parameters realizing certain values. SP is commonly applied to SCND under disruption risks with the objective functions typically minimizing the expected total cost across the pre-disruption and postdisruption stages (Torabi et al., 2015; Chen \& Miller-Hooks, 2012; Ni et al., 2018). SP provides risk-neutral solutions, while RO provides risk-averse solutions by optimizing the performance of the worst-case scenario.

Model conservatism can protect decision makers' against common human biases. Jain et al. (2018) found that an overconfidence bias during sourcing causes buyers and suppliers to underestimate demand variability resulting in less reserve capacity and fewer suppliers in the supply base. Tang (2006) found that ignoring probabilities of occurrence protects decision makers from underestimating disruption risks and being ill-prepared for them. This characteristic of RO offers additional utility during unprecedented events like the COVID-19 pandemic, as historical data is often limited. DRO is a technique that unifies the RO and SP frameworks, making it less prone to the weaknesses of each individual approach (Shang \& You, 2018). While RO optimizes the worst-case outcome and SP optimizes the expected outcome, DRO optimizes the worst-case expected outcome among a set of possible probability distributions called the ambiguity set. DRO offers less conservative solutions than RO while still performing risk-averse decision making, and it counteracts the tendency to over-fit SP models by considering a family of probability distributions rather than just one. The strength of a DRO approach stems from utilizing available data to estimate the probability distribution without assuming the expected value is correct as SP does (Shang \& You, 2018).

Jia et al. (2020) propose a distributionally robust goal programming model to select sustainable suppliers and allocate the orders of a steel company under uncertainty in costs, emissions, and demand. They optimize the trade-off between four objective functions related to costs, carbon dioxide emissions, social impact, and suppliers' comprehensive value. Jia et al. (2020) found that sustainable supplier selection and order allocation models were commonly solved using fuzzy multi-objective or stochastic multi-objective programming and rarely using DRO. DRO has still been applied to other areas of SC optimization. Shang and You (2018) present a multi-stage DRO framework for industrial-scale process network planning and batch production scheduling with demand uncertainty. Gao et al. (2019) develop a two-stage DRO MILP to design shale gas SCs under uncertainty in supply and demand. Their DRO model is more tractable and performs better than its SP counterpart when given imperfect data inputs.

Wang et al. (2020) compare the out-of-sample performance of DRO, RO, and SP on facility location problems with uncertain demand and shipping costs. In their experiments, optimal solutions obtained from nominal data and out-of-sample data were closer in DRO models than SP models. They also found lower expected values, values-at-risk, and conditional-values-at-risk in DRO models compared to RO models. These results demonstrate the potentially superior performance of DRO to RO or SP when unexpectedly optimizing with out-of-sample data. Encountering out-of-sample data on pandemics is a likely occurrence, as pandemic severity can vary by geographic region and throughout its own progression due to virus mutation. The adjustable conservatism of DRO models and their protection against out-of-sample data makes them flexible for use by multiple different healthcare providers.

### 2.6 Simulation

Simulation is a powerful tool for predicting SC performance over time (Ivanov, 2020a). Although simulation does not guarantee optimality, it can provide additional capacities that optimization does not easily handle such as event randomness and complex inventory, sourcing, and shipping policies (Ivanov \& Dolgui, 2020a). Simulation allows decision makers to quickly estimate recovery time and other key performance indicators for various recovery plans and "what-if" scenarios.

### 2.7 Machine Learning

Where sufficient historical data is available, machine learning can be a useful tool for risk assessment, disruption identification, and automated decision making (Ivanov et al., 2018; Baryannis et al., 2018). The learning and prediction capabilities of machine learning can improve traditional SC modelling techniques by more accurately predicting probability distributions and future SC performance (Cavalante et al., 2019). Hosseini et al. (2019b) and Zhao \& You (2019) apply statistical learning techniques to estimate scenario probabilities and the probability density functions of uncertain parameters respectively for resilient SC design models.

### 2.8 Hybrid Approaches

Each quantitative technique discussed above offers unique capabilities and limitations. Mathematical optimization offers the ability to optimize highly complex systems, but it is limited in the quantity of data that it can handle (Baryannis et al., 2018). Dolgui et al. (2018) suggest that optimization models should concentrate on minimizing risk during SC design rather than modelling SC performance over time. Simulation is quite capable of time-dependent analysis, so it should be applied to contingency
planning and SC recovery after disruptions (Dolgui et al. 2018; Ivanov \& Dolgui, 2020a). Machine learning can automate decision making and learn from large amounts of data but has limitations when modelling complex systems (Baryannis et al., 2018). Hybrid applications of optimization, simulation, and machine learning techniques can provide even more powerful and holistic approaches to SCRM. Ke and Zhao (2008) solve a medical supplies distribution problem using simulation to model epidemic spread in the populace and optimization to design the SC network. Cavalcante et al. (2019) merge simulation and machine learning, referred to as a digital SC twin, to perform data-driven supplier selection. Their digital SC twin promotes SC agility and resilience through quick decision making based on data-driven experimentation of different scenarios. Ivanov and Dolgui (2020a) develop a decision-support system that combines simulation, optimization, and data analytics. Simulation-optimization tools optimize the SC network and simulate SC performance during various disruption scenarios. Data analytics are used to identify disruptions based on live process data and generate realistic disruption scenarios for contingency planning.

### 2.9 Research Gaps

The papers reviewed above, excluding Mehrotra et al. (2020) and Ivanov (2020a), analyze SC decision making in non-pandemic situations, so contextual parameters differ from this study. During pandemics, the PPE SC is fraught with complications as global demand sky rockets due to increased PPE usage, panic purchasing, and hoarding. This pressure on the SC can lead to contracts being unfulfilled, volatile market prices, and other foreseen and unforeseen consequences during long periods of disruption. According to Govindan et al. (2017), multi-period modelling is crucial for long-term disruptions that evolve unpredictably, yet a significant portion of SC disruption research studies two-stage models rather than multi-stage models.

SP is the most common approach to optimization under uncertainty in SC resilience (Hosseini et al., 2019a) and SCRM research (Baryannis et al., 2018). The problem with this trend is that SP assumes absolute correctness of its uncertainty scenario probability distribution. This may be unrealistic in the context of pandemics due to their infrequent occurrence and variance in epidemiological and pathological features. The literature is also lacking studies that combine RO and DRO with multi-objective optimization models.

## Chapter 3

## Problem Definition

This thesis studies the critically important PPE supply planning problem during the COVID-19 pandemic. Supply planning is defined as the selection of suppliers from multiple types of sources over multiple time periods. This is consistent with key decisions in SC disruption management identified by Hosseini et al. (2019b).

The models in this study incorporate five procurement factors that were identified in the case study of a Canadian healthcare provider. The first factor is the timing of purchases. The proposed models have multiple time periods to help SC managers decide when to make purchases. Modelling with multiple time periods is essential to capturing the long-term nature and unpredictable spread of pandemics.

The second procurement factor is competing strategic objectives. Health authorities consider various decision criteria when procuring medical supplies. Two common criteria, and those used by the healthcare provider in this case study, are to maximize the portion of PPE demand that is satisfied while also minimizing operating cost. Multi-objective optimization is used to optimize the trade-off between these two competing goals. A strength of this approach is its ability to optimize both cost and service level without a defined penalty cost for unmet demand which is also called shorted demand. Unmet demand can have long-lasting performance impacts making its cost to an organization unclear and difficult to accurately estimate (Simchi-Levi et al., 2018).

The third element of this framework is the use of RO, SP, and DRO to consider uncertainty in future PPE prices, supply, and demand. Recourse modelling techniques like these provide an intuitive approach to optimizing SCs under disruption-related
uncertainties (Govindan et al., 2017). Strategic decisions are made in the pre-disruption stage under uncertainty of future outcomes. These decisions cannot be easily adjusted in the short-term, so they have the same value regardless of the uncertainty scenario that is realized. Operational decisions are made in post-disruption modelling stages. These decisions can easily be adjusted in response to the realization of uncertain parameter values. The service-level objective function follows minimax robustness by minimizing the maximum shortage of any product in any time period and scenario. This approach facilitates health authorities' risk aversion regarding demand satisfaction. Costs are optimized using SP, RO, and DRO. Sample average approximation (SAA) is applied to the RO model to chance-constrain the minimax cost objective and adjust solution conservatism.

The fourth procurement factor is quantity-based pricing. Sourcing greater quantities from a single supplier often decreases unit cost through quantity-based discounts, but it exposes the SC to greater risk if that supplier is disrupted. The models in this study include all-unit volume-based discounted pricing. This differs from incremental discounts, which only apply to units in excess of the price break quantity, and businessvolume discounts which discount the price of all products bought from a supplier (Bohner \& Minner, 2017).

The fifth procurement factor is multiple types of sources: long-term contracts, onetime purchases on the open market, and federal emergency stockpiles. Long-term contracts deliver fixed quantities of PPE in each time period at fixed prices. One-off purchases from the open market vary in both quantity and price. Canada maintains a federal emergency supplies stockpile and allocates PPE to the provinces upon request (Government of Canada, 2021). This is modelled by parameterizing the total supply of emergency PPE available to the healthcare provider, while purchase quantities in each time period are decision variables. The price of emergency stockpile PPE remains
constant.

The models incorporate multiple sourcing and emergency inventory stockpiles, as these were identified to be promising risk mitigation strategies for risk-averse organizations in Tomlin (2006). The potential benefits of carrying larger inventory quantities prior to disruption are also analyzed.

To reduce SC complexity, warehouses are represented by individual decision variables for the net inventory and the net shipment quantities of each product in each time period. Similarly, the demand at all destinations is aggregated into a single parameter for each product and time period. The flow of PPE in the modelled SC is depicted in Figure 3.1.


Figure 3.1: Representational diagram of the case study SC

## Chapter 4

## Optimization Framework

Section 4.1 formulates a deterministic multi-objective model for resilient supply planning. The $\epsilon$-constraint solution method is outlined in Section 4.2. In Section 4.3, uncertainty is considered using both two-stage and multi-stage recourse models with RO, SP, and DRO cost objectives.

### 4.1 Deterministic Formulation

The following notation is used to formulate the supply planning models:

Sets
$\mathbb{P} \quad$ Products
II Suppliers
$\mathbb{K} \quad$ Warehouse capacities (sqft.)
$\mathbb{B} \quad$ Quantity-based price breaks
$\mathbb{T} \quad$ Time periods $\mathrm{t} \in 1 \ldots T$

## Parameters

$A_{i t}^{c} \quad$ Availability of supplier i to meet contractual obligations in time t
$A_{i t}^{o} \quad$ Availability of supplier i's nominal open market capacity in time t
$D_{p t} \quad$ Total hospital demand of product p in time t
$p_{p i t}^{o} \quad$ Open market price per unit of product p for supplier i in time t
$p_{p i}^{c} \quad$ Base contract price per unit of product p from supplier i
$p_{p}^{e} \quad$ Price per unit of product p in emergency stockpile
$C_{p i}^{1} \quad$ Cost to ship product p from supplier i to warehouses
$C_{p}^{2} \quad$ Cost to ship product p from emergency stockpile to warehouses
$C_{p}^{3} \quad$ Cost to ship product p from warehouses to hospitals
$C_{k}^{4} \quad$ Cost to have net warehouse capacity k
$C_{p}^{5} \quad$ Holding cost per unit of product p in inventory
$C^{6} \quad$ Administrative cost to contract each supplier
$F_{p i}^{1} \quad$ Fraction of product p from supplier i that is usable (not defective)
$F_{p i b}^{2} \quad$ Fraction of base contract price of product p for supplier i and discount b
$K_{k}^{1} \quad$ Square footage of net warehouse capacity k
$K_{p}^{2} \quad$ Square feet required to store one unit of product p
$Q_{i b}^{1} \quad$ Contract quantity where supplier i offers all-unit discount b
$Q_{p i}^{2} \quad$ Contract quantity maximum for product p from supplier i
$Q_{p i}^{3} \quad$ Contract quantity minimum for product p from supplier i
$Q_{p i}^{4} \quad$ Average quantity of product p that supplier i sells on open market per period
$Q_{p}^{5} \quad$ Net supply of product p in federal stockpile allocated to the healthcare provider
$V_{p}^{0} \quad$ Inventory of product p at start of disruption
M Very large number
$\epsilon \quad$ Parameter of the $\epsilon$-constraint approach to multi-objective optimization

## Decision Variables

$q_{p i b}^{c} \quad$ Periodic quantity of product p contracted from supplier i with discount b
$q_{p i t}^{o} \quad$ Quantity of product p procured from supplier i in time t on open market
$q_{p t}^{e} \quad$ Quantity of product p procured from federal stockpile in time t
$q_{p t}^{h} \quad$ Quantity of product p delivered from warehouses to hospitals in time t
$s_{p t} \quad$ Portion of demand for product p that is shorted in time t
$v_{p t} \quad$ Inventory level of product p at beginning of time t
$w_{k} \begin{cases}1 & \text { if net warehouse capacity is size } k \\ 0 & \text { otherwise }\end{cases}$
$y_{p i b} \quad \begin{cases}1 & \text { if product } p \text { is contracted from supplier } i \text { at price discount } b \\ 0 & \text { otherwise }\end{cases}$

Warehouse capacity $\left(w_{k}\right)$, and long-term contract selection $\left(q_{p i b}^{c}, y_{p i b}\right)$ are strategic decisions, so their values do not change during the modelled disruptions. Open market purchase quantities ( $q_{p i t}^{o}$ ), emergency stockpile consumption $\left(q_{p t}^{e}\right)$, transport quantities $\left(q_{p t}^{h}\right)$, PPE shortages $\left(s_{p t}\right)$, and inventory levels $\left(v_{p t}\right)$ are operational decisions that can change in each period of the modelled disruptions.

The models minimize the net costs in objective function $Z_{1}$ in Eq. 1, and they minimize the maximum portion of demand that is shorted in objective function $Z_{2}$ in Eq. 2 .

$$
\begin{align*}
& \text { Minimize } Z_{1}= \\
& \sum_{p} \sum_{i} \sum_{b} \sum_{t} A_{i t}^{c} F_{p i b}^{2} p_{p i}^{c} q_{p i b}^{c}+\sum_{p} \sum_{i} \sum_{t} p_{p i t}^{o} q_{p i t}^{o}+\sum_{p} \sum_{t} p_{p}^{e} q_{p t}^{e} \\
& +\sum_{p} \sum_{i} \sum_{b} \sum_{t} A_{i t}^{c} C_{p i}^{1} q_{p i b}^{c}+\sum_{p} \sum_{i} \sum_{t} C_{p i}^{1} q_{p i t}^{o}+\sum_{p} \sum_{t} C_{p}^{2} q_{p t}^{e}  \tag{1}\\
& +\sum_{p} \sum_{t} C_{p}^{3} q_{p t}^{h}+\sum_{k} C_{k}^{4} w_{k}+\sum_{p} \sum_{t} C_{p}^{5} v_{p t}+\sum_{p} \sum_{i} \sum_{b} C^{6} y_{p i b}
\end{align*}
$$

$$
\begin{equation*}
\text { Minimize } Z_{2}=\max _{p \in \mathbb{P}, t \in \mathbb{T}} s_{p t} \tag{2}
\end{equation*}
$$

Objective function $Z_{1}$ in Eq. 1 has 10 terms which account for costs associated with procurement, shipping, warehousing, inventory, and administration. Specifically, they are the long-term contract procurement costs, open market procurement costs, emergency stockpile procurement costs, shipping costs from contracted suppliers to warehouses, shipping costs from open market suppliers to warehouses, shipping costs from the emergency stockpile to warehouses, shipping costs from warehouses
to destinations, overhead cost of having net warehouse capacity $k$, inventory holding costs, and administration costs incurred by each long-term contract including the cost of negotiating with, documenting, and inspecting suppliers.

Objective function $Z_{2}$ in Eq. 2 minimizes the maximum portion of demand that is unsatisfied for any product $p$ and time period $t$. Optimizing the minimax term instead of a sum of shortages prevents the model from concentrating its product shortages at one hospital or in one time-period (Mak \& Shen, 2012; Mehrotra et al., 2020). Most health providers would prefer smaller shortages in several time periods over one large shortage.

The deterministic model is bound by constraints $3-20$.

$$
\begin{gather*}
q_{p i t}^{o} \leq A_{i t}^{o} Q_{p i}^{4} \quad \forall p \in \mathbb{P}, i \in \mathbb{I}, t \in \mathbb{T}  \tag{3}\\
q_{p i b}^{c} \leq Q_{p i}^{2} y_{p i b} \quad \forall p \in \mathbb{P}, i \in \mathbb{I}, b \in \mathbb{B}  \tag{4}\\
q_{p i b}^{c} \geq Q_{p i}^{3} y_{p i b} \quad \forall p \in \mathbb{P}, i \in \mathbb{I}, b \in \mathbb{B}  \tag{5}\\
q_{p i b}^{c} \geq Q_{i b}^{1} y_{p i b} \quad \forall p \in \mathbb{P}, i \in \mathbb{I}, b \in \mathbb{B}  \tag{6}\\
q_{p i b}^{c} \leq Q_{i, b+1}^{1} y_{p i b} \quad \forall p \in \mathbb{P}, i \in \mathbb{I}, b \in \mathbb{B}  \tag{7}\\
\sum_{p} K_{p}^{2} v_{p t} \leq \sum_{k} K_{k}^{1} w_{k} \quad \forall t \in \mathbb{T}  \tag{8}\\
\sum_{i} \sum_{b} A_{i t}^{c} F_{p i}^{1} q_{p i b}^{c}+\sum_{i} F_{p i}^{1} q_{p i t}^{o}+q_{p t}^{e}-q_{p t}^{h}+v_{p t}=v_{p, t+1} \quad \forall p \in \mathbb{P}, t \in \mathbb{T}  \tag{9}\\
v_{p, t=1}=V_{p}^{0} \quad \forall p \in \mathbb{P}  \tag{10}\\
v_{p, t=T+1} \geq V_{p}^{0} \quad \forall p \in \mathbb{P}  \tag{11}\\
q_{p t}^{h}-\left(1-s_{p t}\right) D_{p t}=0 \quad \forall p \in \mathbb{P}, t \in \mathbb{T}  \tag{12}\\
\sum_{k} w_{k}=1  \tag{13}\\
\sum_{b} y_{p i b} \leq 1 \tag{14}
\end{gather*} \forall p \in \mathbb{P}, i \in \mathbb{I} \quad l
$$

$$
\begin{gather*}
\sum_{t} q_{p t}^{e} \leq Q_{p}^{5} \quad \forall p \in \mathbb{P}  \tag{15}\\
q_{p i b}^{c} \geq 0 \quad \forall p \in \mathbb{P}, i \in \mathbb{I}, b \in \mathbb{B}  \tag{16}\\
q_{p i t}^{o} \geq 0 \quad \forall p \in \mathbb{P}, i \in \mathbb{I}, t \in \mathbb{T}  \tag{17}\\
q_{p t}^{e}, q_{p t}^{h}, s_{p t}, v_{p t} \geq 0 \quad \forall p \in \mathbb{P}, t \in \mathbb{T}  \tag{18}\\
w_{k} \in\{0,1\} \quad \forall k \in \mathbb{K}  \tag{19}\\
y_{p i b} \in\{0,1\} \quad \forall p \in \mathbb{P}, i \in \mathbb{I}, b \in \mathbb{B} \tag{20}
\end{gather*}
$$

Constraint 3 ensures that open market order quantities are less than each supplier's capacity in each time period. Constraints 4 and 5 enforce maximum and minimum contract sizes. The price discount binary variables are set by constraints 6 and 7 so that contract quantities are within their quantity-based discount bracket. Constraint 8 limits inventory space to the net warehouse capacity. Constraint 9 equates net inventory in the following time period to its current value plus or minus PPE shipments during the current period for each product. Constraint 10 sets the initial inventory level for each product. Inventory levels in the final time period $(T+1)$ must be equal to or greater than their starting inventory levels, which is enforced by constraint 11. Constraint 12 ensures that PPE quantities shipped to destinations exceed the service level for each product $p$ and period $t$. Constraint 13 ensures that only one net warehouse capacity is selected. Selecting the smallest warehouse capacity has zero cost, as the smallest option is the current capacity. Constraint 14 ensures that at most one discount price is used for each product-supplier combination. Constraint 15 guarantees that the sum of shipments from the federal emergency stockpile over all periods does not exceed the emergency stockpile fraction allocated to the healthcare provider. Constraints 16 - 20 are non-negativity and binary constraints.

This formulation makes the following assumptions. Contracted units that are never delivered are not paid for and there is no recourse if suppliers cannot produce the
contracted amount. Only PPE that passes quality inspection when it arrives at warehouses is stored in inventory. The cost of defective products is not refunded. Suppliers can offer all-unit volume-based price discounts on contracts but not on open market purchases. Lastly, demand must be satisfied in its respective time period, as back-ordering PPE is not an option in pandemic settings.

### 4.2 Solution Approach

The $\epsilon$-constraint approach was chosen to solve the multi-objective models in this study. This approach designates one objective function as the primary objective, while the remaining objective functions are added to the model constraints and bound by parameters $\epsilon$ (Ehrgott, 2005). The proposed model has two objectives, so it only requires one additional $\epsilon$-constraint.

Efficient solutions, also referred to as Pareto-optimal solutions, in multi-objective programming can not enhance the value of any objective function without degrading the value of another objective (Ehrgott et al., 2014). Efficient solutions are realized by optimizing (21) and then setting the value of $\epsilon$ to its objective value and optimizing (22). A set of efficient solutions obtained from different values of $\epsilon$ are typically referred to as a Pareto front.

$$
\begin{equation*}
\text { Min. } \quad Z_{1} \tag{21}
\end{equation*}
$$

s.t Eqs. $3-20$
$s_{p t} \leq \epsilon \quad \forall p \in \mathbb{P}, t \in \mathbb{T}$

$$
\begin{array}{ll}
\text { Min. } & \theta \\
\text { s.t } & s_{p t} \leq \theta \quad \forall p \in \mathbb{P}, t \in \mathbb{T}  \tag{22}\\
& \text { Eqs. } 3-20 \\
& Z_{1} \leq \epsilon
\end{array}
$$

### 4.3 Optimization Under Uncertainty

The SP, RO, and DRO models presented in this section incorporate scenario-based uncertainty in PPE supply, prices, and demand. This requires an additional notation set $\mathbb{S}$ for uncertainty scenarios $(\mathbb{S}=1 \ldots S)$. Parameters $A_{i t}^{c}, A_{i t}^{o}, D_{p t}$, and $p_{p i t}^{o}$ and recourse decision variables $q_{p i t}^{o}, q_{p t}^{e}, q_{p t}^{h}, s_{p t}$, and $v_{p t}$ receive an additional index to indicate their value in scenario $s$. Constraints $3,8,9,10,11,12,15,17,18$ and the $\epsilon$-constraint now exist $\forall s \in \mathbb{S}$, as they contain at least one uncertain parameters or recourse decision variable.

In two-stage recourse models, the value of uncertain parameters in all future time periods becomes known at the start of a disruption. Strategic decisions are made under uncertainty, but operational decisions are made with certainty. For example, warehouse capacity must be decided under uncertainty of the severity of future disruptions, but once a disruption begins, the open market purchase quantities are decided with knowledge of future demands, prices, and supplier availability. Conversely, multi-stage models make all decisions under uncertainty of future outcomes. In this context, the difference between strategic and operational decisions is that strategic decisions must remain fixed during all time periods, while operational decisions can assume different values in each period. The greater uncertainty in multi-stage recourse models causes their objective values to be worse than those from their two-stage counterparts.

In multi-stage uncertainty, scenarios with the same uncertain parameter realizations in the current and preceding time periods must have identical decision variable values.

Such scenarios do not exist in the two-stage dataset, so a multi-stage dataset was created in which there are three potential outcomes of uncertainty parameter values in each time period. Constraints 23-26 are added to the multi-stage recourse model to enforce the equality of decision variables $q_{p t s}^{e}, q_{p t s}^{h}, s_{p t s}$, and $q_{p i t s}^{o}$ in scenarios 1 to 3,4 to $6, \cdots, 3^{T}-2$ to $3^{T}$ in time period $T-1$, scenarios 1 to 9,10 to $18, \cdots, 3^{T}-8$ to $3^{T}$ in time period $T-2$, etc. Constraint 27 performs the same function for decision variable $v_{p t s}$ one time period ahead since $v_{p t s}$ represents the inventory at the beginning of a period rather than its end. Constraints 23-27 do not apply to scenarios numbered as a multiple of $3^{T-t}$ due to the three potential outcomes of uncertainty parameter values in each time period.

$$
\begin{align*}
& q_{p t s}^{e}=q_{p t, s+1}^{e} \forall p \in \mathbb{P}, t \in\{1 \ldots T-1\}, s \in \mathbb{S} \backslash\left\{3^{T-t}, 2 \times 3^{T-t} \ldots 3^{T}\right\}  \tag{23}\\
& q_{p t s}^{h}=q_{p t, s+1}^{h} \forall p \in \mathbb{P}, t \in\{1 \ldots T-1\}, s \in \mathbb{S} \backslash\left\{3^{T-t}, 2 \times 3^{T-t} \ldots 3^{T}\right\}  \tag{24}\\
& s_{p t s}=s_{p t, s+1} \forall p \in \mathbb{P}, t \in\{1 \ldots T-1\}, s \in \mathbb{S} \backslash\left\{3^{T-t}, 2 \times 3^{T-t} \ldots 3^{T}\right\}  \tag{25}\\
& q_{p i t s}^{o}=q_{p i t, s+1}^{o} \quad \forall p \in \mathbb{P}, i \in \mathbb{I}, t \in\{1 \ldots T-1\}, s \in \mathbb{S} \backslash\left\{3^{T-t}, 2 \times 3^{T-t} \ldots 3^{T}\right\}  \tag{26}\\
& v_{p, t+1, s}=v_{p, t+1, s+1} \forall p \in \mathbb{P}, t \in\{1 \ldots T-1\}, s \in \mathbb{S} \backslash\left\{3^{T-t}, 2 \times 3^{T-t} \ldots 3^{T}\right\} \tag{27}
\end{align*}
$$

### 4.3.1 Robust Optimization Formulation

The RO formulation incorporates what Soyster (1973) initially introduced as minimax or strict robustness. It requires each scenario to satisfy all constraints. The RO cost objective minimizes the maximum cost incurred in any scenario, while the service level objective minimizes the maximum portion of demand that is shorted for any product, time period, and scenario. Optimizing these minimax terms necessitates a new non-negative decision variable $\theta$. If cost is the main objective, $\theta$ must exceed $Z_{3}$ in Eq. $28 \forall s \in \mathbb{S}$. If service level is the main objective, $\theta$ must exceed $s_{p t s}$ for each
product $p$, time period $t$, and scenario $s$.

$$
\begin{align*}
& \text { Minimize } Z_{3}= \\
& \sum_{p} \sum_{i} \sum_{b} \sum_{t} A_{i t s}^{c} F_{p i b}^{2} p_{p i}^{c} q_{p i b}^{c}+\sum_{p} \sum_{i} \sum_{t} p_{p i t s}^{o} q_{p i t s}^{o}+\sum_{p} \sum_{t} p_{p}^{e} q_{p t s}^{e} \\
& +\sum_{p} \sum_{i} \sum_{b} \sum_{t} A_{i t s}^{c} C_{p i}^{1} q_{p i b}^{c}+\sum_{p} \sum_{i} \sum_{t} C_{p i}^{1} q_{p i t s}^{o}+\sum_{p} \sum_{t} C_{p}^{2} q_{p t s}^{e}  \tag{28}\\
& +\sum_{p} \sum_{t} C_{p}^{3} q_{p t s}^{h}+\sum_{k} C_{k}^{4} w_{k}+\sum_{p} \sum_{t} C_{p}^{5} v_{p t s}+\sum_{p} \sum_{i} \sum_{b} C^{6} y_{p i b}
\end{align*}
$$

This approach mimics that proposed by Kuroiwa and Lee (2012), where the concept of minimax robustness is extended to multi-objective models by optimizing objective functions' worst-case values across all uncertainty scenarios. This formulates the multi-objective RO model as a deterministic problem to which the $\epsilon$-constraint method can be applied. The efficient solutions of this approach are referred to as robust solutions.

RO model conservatism is made adjustable by applying chance constrained programming which is a technique that only requires some constraints to be satisfied a set portion of the time. An example of this in literature is Hosseini et al. (2019b) chance constraining the number of disruptions that each supplier can experience. Sample average approximation (SAA) is an approach proposed by Ahmed and Shapiro (2008) to approximate chance-constraints. In this model, the minimax cost constraint is chance constrained, allowing the cost of some scenarios to exceed the value of $\theta$. This is achieved by adding an arbitrarily large number $M$ to the right-hand side of the minimax cost constraint in a select number of scenarios. Binary decision variables $z_{s}$ select those scenarios. The number of scenarios where cost can exceed $\theta$ is bound by the product of parameter $\alpha$ and the total number of scenarios $S$.

Robust Pareto-optimal solutions with SAA of the minimax cost objective are obtained by optimizing (29) and then optimizing (30) with the value of $\epsilon$ set to the objective
value of the former model.

$$
\begin{array}{ll}
\text { Min. } & \theta \\
\text { s.t } & Z_{3} \leq \theta+M z_{s} \quad \forall s \in \mathbb{S} \\
& \sum_{s} z_{s} \leq \alpha S \tag{29}
\end{array}
$$

Eqs. 3-20, 23-27

$$
s_{p t s} \leq \epsilon \quad \forall p \in \mathbb{P}, t \in \mathbb{T}, s \in \mathbb{S}
$$

$$
\begin{aligned}
\text { Min. } & \theta \\
\text { s.t } & s_{p t s} \leq \theta \quad \forall p \in \mathbb{P}, t \in \mathbb{T}, s \in \mathbb{S}
\end{aligned}
$$

Eqs. 3-20, 23-27

Increasing the value of parameter $\alpha$ makes the model less risk-averse with regards to total cost while remaining completely risk-averse with regards to PPE shortages.

### 4.3.2 Stochastic Programming Formulation

This formulation replaces the RO cost objective with an expected cost function that is presented in Eq. 31. The first two terms represent the cost of strategic decisions regarding net warehouse capacity and contract quantities. These long-term decisions apply to all uncertainty scenarios, as they are not easily reversible. The third term represents the expected cost of operational decisions which are also referred to as recourse decisions since they can be changed depending on the scenario that is realized. The costs of recourse decisions are multiplied by their respective scenario's probability of occurrence $f_{s}$ resulting in an expected cost.

$$
\begin{align*}
& \text { Minimize } Z_{4}= \\
& \sum_{k} C_{k}^{4} w_{k}+\sum_{p} \sum_{i} \sum_{b} C^{6} y_{p i b}+\sum_{s} f_{s}\left[\sum_{p} \sum_{i} \sum_{b} \sum_{t} A_{i t s}^{c} F_{p i b}^{2} p_{p i}^{c} q_{p i b}^{c}\right. \\
& +\sum_{p} \sum_{i} \sum_{t} p_{p i t s}^{o} q_{p i t s}^{o}+\sum_{p} \sum_{t} p_{p}^{e} q_{p t s}^{e}+\sum_{p} \sum_{i} \sum_{b} \sum_{t} A_{i t s}^{c} C_{p i}^{1} q_{p i b}^{c}  \tag{31}\\
& \left.+\sum_{p} \sum_{i} \sum_{t} C_{p i}^{1} q_{p i t s}^{o}+\sum_{p} \sum_{t} C_{p}^{2} q_{p t s}^{e}+\sum_{p} \sum_{t} C_{p}^{3} q_{p t s}^{h}+\sum_{p} \sum_{t} C_{p}^{5} v_{p t s}\right]
\end{align*}
$$

Efficient solutions are obtained for the SP formulation by optimizing (32) and then optimizing (33) with the value of $\epsilon$ set to the objective value of the former model.

$$
\begin{aligned}
\text { Min. } & Z_{4} \\
\text { s.t } & \text { Eqs. 3-20, 23-27 } \\
& s_{p t s} \leq \epsilon \quad \forall p \in \mathbb{P}, t \in \mathbb{T}, s \in \mathbb{S} \\
\text { Min. } & \theta \\
\text { s.t } & s_{p t s} \leq \theta \quad \forall p \in \mathbb{P}, t \in \mathbb{T}, s \in \mathbb{S}
\end{aligned}
$$

Eqs. 3-20, 23-27

$$
Z_{4} \leq \epsilon
$$

### 4.3.3 Distributionally Robust Optimization Formulation

While RO optimizes the worst-case cost and SP optimizes the expected cost, DRO optimizes the worst-case expected cost among a set of possible probability distributions called the ambiguity set and denoted by $D$. Thus, DRO problems can be formally represented as $\inf _{x} \sup _{f \in D} \mathbb{E}_{f}[g(x, \xi)]$, where $f$ is the probability distribution that results in the worst-case expected value, and $\xi$ represents the uncertain parameters of future scenarios.

Different types of ambiguity sets exist in DRO. This project uses a statistical-distancebased ambiguity set which bounds a spherical region in the space of feasible probability distributions using a $\phi$-divergence function.
$\phi$-divergence, referred to as the distance between the estimated probability distribution $\hat{f}_{s}$ and any distribution $f_{s}$ in the ambiguity set $D$, is mathematically defined as:

$$
\begin{equation*}
I_{\phi}(f, \hat{f})=\sum_{s} \hat{f}_{s} \phi\left(\frac{f_{s}}{\hat{f}_{s}}\right) \tag{34}
\end{equation*}
$$

where $\phi(\mathrm{t})$ is some convex function for $\mathrm{t} \geq 0, \phi(1)=0,0 \phi\left(\frac{a}{0}\right):=\mathrm{a} \lim _{a \rightarrow \infty} \frac{\phi(t)}{t}$ for $\mathrm{a}>$ 0 , and $0 \phi\left(\frac{0}{0}\right):=0($ Ben-Tal et al., 2013).

The variation distance $\phi$-divergence function shown in Eq. 35 is used in this study to maintain model linearity. The reader is referred to Ben-Tal et al. (2013) for a summary of other $\phi$-divergence functions commonly used in the literature.

$$
\begin{equation*}
I_{\phi}(f, \hat{f})=\sum_{s}\left|f_{s}-\hat{f}_{s}\right| \tag{35}
\end{equation*}
$$

This DRO model constrains the $\phi$-divergence function by parameter $\rho$. The ambiguity set $D$, which is bound by the $\phi$-divergence function, is therefore defined as:

$$
\begin{equation*}
D(\hat{f}, \rho)=\left\{f \in \Lambda\left|\sum_{s}\right| f_{s}-\hat{f}_{s} \mid \leq \rho\right\} \tag{36}
\end{equation*}
$$

where $\Lambda$ represents the space containing all feasible probability distributions.
When $\rho$ is set to 0 , the only distribution in the ambiguity set $D$ is $\hat{f}$, so the problem reduces to SP. Increasing the value of $\rho$ increases the size of the ambiguity set thus making the DRO model more conservative. As $\rho \rightarrow \infty$, the ambiguity set $D$ contains all possible distributions, and the problem becomes RO.

An initial DRO formulation can be created by optimizing objective function 37 subject to constraints 3-20, 23-27, and 38-42. Decision variable $d_{s}$ is introduced to enact the absolute value operation in the variation distance $\phi$ function. Parameter $\hat{f}_{s}$ is set to $\frac{1}{S}$, as equally likely scenarios are assumed.

$$
\begin{align*}
& \min \sum_{k} C_{k}^{4} w_{k}+\sum_{p} \sum_{i} \sum_{b} C^{6} y_{p i b}+\max _{f_{s}, d_{s}} \sum_{s} f_{s}\left[\sum_{p} \sum_{i} \sum_{b} \sum_{t} A_{i t s}^{c} F_{p i b}^{2} p_{p i}^{c} q_{p i b}^{c}\right. \\
& +\sum_{p} \sum_{i} \sum_{t} p_{p i t s}^{o} q_{p i t s}^{o}+\sum_{p} \sum_{t} p_{p}^{e} q_{p t s}^{e}+\sum_{p} \sum_{i} \sum_{b} \sum_{t} A_{i t s}^{c} C_{p i}^{1} q_{p i b}^{c}  \tag{37}\\
& \left.+\sum_{p} \sum_{i} \sum_{t} C_{p i}^{1} q_{p i t s}^{o}+\sum_{p} \sum_{t} C_{p}^{2} q_{p t s}^{e}+\sum_{p} \sum_{t} C_{p}^{3} q_{p t s}^{h}+\sum_{p} \sum_{t} C_{p}^{5} v_{p t s}\right]  \tag{38}\\
& \sum_{s} f_{s}=1 \quad(\gamma) \\
& \sum_{s} d_{s} \leq \rho \quad(\pi)  \tag{39}\\
& -f_{s}-d_{s} \leq-\hat{f}_{s} \quad \forall s \in \mathbb{S} \quad\left(\psi_{s}^{+}\right)  \tag{40}\\
& f_{s}-d_{s} \leq \hat{f}_{s} \quad \forall s \in \mathbb{S}  \tag{41}\\
& f_{s}, d_{s} \geq 0 \quad \forall s \in \mathbb{S} \tag{42}
\end{align*}
$$

The third term of objective function 37 and constraints 38-42 form an inner maximization problem within the outer minimization problem. The inner and outer problems can be rectified using primal-duality theory. The standard formulation of a primal LP problem is:

$$
\text { (Primal) } \max \{\langle c, x\rangle: A x=b, x \geq 0\}
$$

where $A \in \mathbb{R}^{m \times n}, \operatorname{rank}(A)=m, b \in \mathbb{R}^{m}$, and $c \in \mathbb{R}^{n}$. The corresponding dual problem is:

$$
\text { (Dual) } \quad \min \left\{\langle b, y\rangle: A^{T} y+s=c, s \geq 0\right\}
$$

Dualizing the inner problem converts it from a maximization problem to a minimization problem and eliminates decision variables $f_{s}$ and $d_{s}$ in the process. $\gamma, \pi, \psi^{+}$, and $\psi^{-}$
are the dual variables attached to constraints 38-42 respectively. The dualized inner problem is formulated in equations 43-48.

$$
\begin{gather*}
\min _{\gamma, \pi, \psi_{s}^{+}, \psi_{s}^{-}} \gamma+\rho \pi-\sum_{s} \hat{f}_{s} \psi_{s}^{+}+\sum_{s} \hat{f}_{s} \psi_{s}^{-}  \tag{43}\\
\pi-\psi_{s}^{+}-\psi_{s}^{-} \geq 0 \quad \forall s \in \mathbb{S}  \tag{44}\\
\gamma-\psi_{s}^{+}+\psi_{s}^{-} \geq \sum_{p} \sum_{i} \sum_{b} \sum_{t} A_{i t s}^{c} F_{p i b}^{2} p_{p i}^{c} q_{p i b}^{c}+\sum_{p} \sum_{i} \sum_{t} p_{p i t s}^{o} q_{p i t s}^{o} \\
+\sum_{p} \sum_{t} p_{p}^{e} q_{p t s}^{e}+\sum_{p} \sum_{i} \sum_{b} \sum_{t} A_{i t s}^{c} C_{p i}^{1} q_{p i b}^{c}+\sum_{p} \sum_{i} \sum_{t} C_{p i}^{1} q_{p i t s}^{o}  \tag{45}\\
+\sum_{p} \sum_{t} C_{p}^{2} q_{p t s}^{e}+\sum_{p} \sum_{t} C_{p}^{3} q_{p t s}^{h}+\sum_{p} \sum_{t} C_{p}^{5} v_{p t s} \quad \forall s \in \mathbb{S} \\
\gamma \quad u . r .  \tag{46}\\
\pi \geq 0  \tag{47}\\
\psi_{s}^{+}, \psi_{s}^{-} \geq 0 \quad \forall s \in \mathbb{S} \tag{48}
\end{gather*}
$$

The dual of the inner problem is then reintroduced to the outer problem, and the DRO objective function becomes Eq. 49.

$$
\begin{equation*}
\text { Min. } Z_{5}=\sum_{k} C_{k}^{4} w_{k}+\sum_{p} \sum_{i} \sum_{b} C^{6} y_{p i b}+\gamma+\rho \pi-\sum_{s} \hat{f}_{s} \psi_{s}^{+}+\sum_{s} \hat{f}_{s} \psi_{s}^{-} \tag{49}
\end{equation*}
$$

Efficient DRO solutions are found by solving (50), then setting the value of $\epsilon$ to its objective value and optimizing (51).

Min. $\quad Z_{5}$
s.t Eqs. 3-20 (Nominal Cst.s)

Eqs. 23-27 (Multi-Stage Recourse Cst.s)
Eqs. 44-48 (DRO Cst.s)
$s_{p t s} \leq \epsilon \quad \forall p \in \mathbb{P}, t \in \mathbb{T}, s \in \mathbb{S}$

Min. $\quad \theta$
s.t $\quad s_{p t s} \leq \theta \quad \forall p \in \mathbb{P}, t \in \mathbb{T}, s \in \mathbb{S}$

Eqs. 3-20 (Nominal Cst.s)
Eqs. 23-27 (Multi-Stage Recourse Cst.s)
Eqs. 44-48 (DRO Cst.s)
$Z_{5} \leq \epsilon$

## Chapter 5

## Numerical Experiments

Experiments inspired by the NSHA case were run with the presented models using realistic data. Open market PPE prices vary between the average market prices prior to and during the COVID-19 pandemic as reported by the Society for Healthcare Organization Procurement Professionals (2020). Pandemic scenarios were generated based on plausible and actual pandemic trajectories seen around the world during the COVID-19 pandemic including single-wave, two-wave, and exponential growth.

The severity of the pandemic, which is denoted by a numerical factor, sets the values of the four uncertain parameters: demand $\left(D_{p t}\right)$, open market price $\left(p_{p i t}^{o}\right)$, open market supply $\left(A_{i t}^{o}\right)$, and contract fulfillment $\left(A_{i t}^{c}\right)$. PPE prices and demands have a positive correlation to pandemic severity, while open market and contract supply have a negative correlation to it. Open market prices $\left(p_{p i t}^{o}\right)$, contract fulfillment $\left(A_{i t}^{c}\right)$, and open market supplier availability $\left(A_{i t}^{o}\right)$ have a linear relationship with pandemic severity factors, so an X\% change in the pandemic severity factor value corresponds with an X\% change in these parameter values. Total hospital demand $\left(D_{p t}\right)$ increases by a factor of ( $1+$ pandemic severity factor $)$, so increases in the pandemic severity factor correspond with less than proportional increases in demand for PPE.

The two-stage recourse dataset contains ten scenarios named A through J. Their pandemic severity factors are plotted in Figure 5.1. Each scenario has eight time periods of a two month duration.


Figure 5.1: Pandemic severity factors by time period in two-stage dataset scenarios

The multi-stage recourse dataset contains four, four month time periods rather than eight, two month time periods to reduce model run-time. Each of the four time periods in the multi-stage dataset has three potential levels of pandemic severity resulting in $4^{3}=81$ scenarios in total. The values of pandemic severity factors in the multi-stage dataset are plotted in Figure 5.2.


Figure 5.2: Plot of potential pandemic severity factor values in multi-stage dataset

Three products are sourced by the model: 3-ply mask, isolation gown, and N95 mask. The purchase prices of the three PPE sources relative to one another changes depending on the time period and scenario. Emergency stockpile PPE costs more than all contract prices. Open market PPE is the cheapest source in some time periods and scenarios and the most expensive source in other time periods and scenarios.

There are seven potential suppliers. The single domestic supplier has higher prices and lower magnitude disruptions than the six foreign suppliers. Further, contract availability is disrupted less by pandemics than open market availability, as it is assumed that suppliers satisfy their contractual obligations before accepting open market orders. In the multi-stage dataset, the average availability of contracted suppliers $\left(A_{i t s}^{c}\right)$ is $71 \%$, while it is only $63 \%$ for open market suppliers $\left(A_{i t s}^{o}\right)$.

Due to the large number of parameters and uncertainty scenarios, only the upper and lower bounds of parameters in the multi-stage dataset are shown in Table 5.1. The entire two-stage and multi-stage datasets are available as electronic supplements. Modelling was performed in MATLAB with Gurobi 9.0.0 on a Dual-Core Intel © i5 CPU running at 1.6 GHz with 8.00 GB RAM.

Table 5.1: Multi-stage dataset parameter value ranges

| Parameter | Minimum Value | Maximum Value |
| :---: | ---: | ---: |
| $A_{i t s}^{c}$ | $0 \%$ | $99 \%$ |
| $A_{i t s}^{o}$ | $0 \%$ | $99 \%$ |
| $D_{p t s}$ | 110,000 | 525,000 |
| $p_{p i t s}^{o}$ | $\$ 0.05$ | $\$ 10.80$ |
| $p_{p i}^{c}$ | $\$ 0.10$ | $\$ 1.60$ |
| $p_{p}^{e}$ | $\$ 0.20$ | $\$ 1.60$ |
| $C_{p i}^{1}$ | $\$ 0.0025$ | $\$ 0.050$ |
| $C_{p}^{2}$ | $\$ 0.0030$ | $\$ 0.030$ |
| $C_{p}^{3}$ | $\$ 0.0038$ | $\$ 0.038$ |
| $C_{k}^{4}$ | $\$ 0$ | $\$ 20,000$ |
| $C_{p}^{5}$ | $\$ 0.0040$ | $\$ 0.032$ |
| $C^{6}$ | - | $\$ 1,000$ |
| $F_{p i}^{1}$ | $99 \%$ | $99 \%$ |
| $F_{p i b}^{2}$ | $94 \%$ | $100 \%$ |
| $K_{k}^{1}$ | $16,000 ~ s q f t$. | 24,000 sqft. |
| $K_{p}^{2}$ | $0.010 \frac{s q f t .}{\text { unit }}$ | $0.025 \frac{s q f t .}{u n i t}$ |
| $Q_{i b}^{1}$ | 0 | $1.0 \mathrm{E}+09$ |
| $Q_{p i}^{2}$ | 14,285 | 128,571 |
| $Q_{p i}^{3}$ | 1,000 | 3,000 |
| $Q_{p i}^{4}$ | 10,000 | 150,000 |
| $Q_{p}^{5}$ | 120,000 | 360,000 |
| $V_{p}^{0}$ | 50,000 | 150,000 |

### 5.1 Cost and Service Level Trade-Off

In this first experiment, Pareto fronts are generated and plotted with net cost as a function of the portion of demand satisfied. Efficient solutions are sampled by varying parameter $\epsilon$ between $0 \%$ and $20 \%$ at step sizes of $1 \%$. Cost is the main objective, so $\epsilon$ bounds the maximum shortage or product $p$ that can occur in any period $t$ and
scenario $s$.

The deterministic model was run for scenarios A through J. The resulting Pareto fronts are plotted in Figure 5.3. Scenarios B, A, and C are the respective first, second, and third highest-cost scenarios in the deterministic model across all sampled service levels. These three scenarios also have the highest cost in the two-stage SP model and are the first three scenarios removed by the SAA constraint in the two-stage RO model.

Scenarios A and C have the two lowest standard deviations in pandemic severity factor across time periods which may correspond with less opportunities to buy PPE at low costs resulting in higher overall costs. This hypothesis was tested by computing the correlation between scenario costs and their standard deviation in pandemic severity factors across time periods. This hypothesis is supported by a resulting correlation of -0.46 , but scenario B , which has a relatively high standard deviation, is still the highest cost scenario. One potential explanation of scenario B's high costs is that its maximum severity is in the first period, so the model has no opportunity to acquire inventory in preparation for the worst time periods.


Figure 5.3: Deterministic model Pareto fronts using two-stage dataset

Figure 5.4 plots the Pareto fronts of the two-stage and multi-stage RO without SAA, SP , and DRO with $\rho=0.8$ models using the multi-stage dataset. In this case, the two-stage and multi-stage RO models have identical Pareto fronts due to both models optimizing the same worst-case scenario regardless of two-stage or multi-stage recourse.

The slope of the Pareto fronts increases slightly as the service level increases. When cheaper suppliers have no product to sell, the model must purchase PPE from highercost suppliers. This increases the marginal cost of PPE and by extension the slope of the Pareto fronts.

## Pareto Fronts with Multi-Stage Dataset



Figure 5.4: Pareto fronts of the two-stage and multi-stage RO $(\alpha=0)$, SP, and DRO ( $\rho=0.8$ ) models

Figure 5.4 reflects that DRO is more conservative than SP but less conservative than RO. The DRO model has similar run-times to the SP and RO formulations despite the additional constraints and continuous decision variables in the DRO model.

### 5.2 Risk Mitigation Strategy Analysis

### 5.2.1 Starting Inventory Quantity

The value of parameter $V_{p}^{0}$, which is the inventory level for product $p$ in the first period, was varied to observe its impact on optimal costs in the deterministic model. The costs for scenarios A through J at four different starting inventory levels, which are represented by different bar colors, are plotted in Figure 5.5. These results show
that some scenarios decrease in cost given greater quantities of starting inventory, while other scenarios increase or experience little to no change in cost. This behavior is correlated with the maximum pandemic severity during the first three time periods. Scenarios B, F, H, and J, which clearly benefited from more starting inventory, have maximum pandemic severity factors between 0.21 and 0.25 during the first three time periods. Scenario B, whose severity starts at a maximum of 0.25 in period 1 and linearly decreases with the progression of time, experiences the greatest cost improvement. This supports the hypothesis proposed earlier that scenario B has high cost due to its lack of opportunity to gather inventory in preparation for maximum demand in period 1. Scenarios that experience little to no change in cost (A, C, G) have a maximum pandemic severity factor between 0.13 and 0.15 during the first three time periods. Scenarios whose cost clearly increased with greater quantities of starting inventory $(\mathrm{D}, \mathrm{E})$ have a maximum pandemic severity factor between 0.08 and 0.10 during the first three time periods. Scenario I seemed to fit into both the decreasing cost and constant cost groups, so it was not included in any group. The two-stage and multi-stage SP, RO, and DRO models all experienced decreases in their cost-objective value given more starting inventory.


Figure 5.5: Deterministic model costs by scenario and average quantity of starting inventory

### 5.2.2 External Emergency Stockpile

The following two experiments analyze how the size and prices of the emergency PPE stockpile impacts expected cost. The two-stage and multi-stage SP models at a $99 \%$ service level were run with different values for parameters $Q_{p}^{5}$ and $p_{p}^{e}$ which indicate the total supply of product $p$ in the emergency stockpile allocated to the Health Provider and the cost to consume emergency stockpile PPE respectively. Figure 5.6 plots the expected costs as blue bars and the cost standard deviation divided by the mean cost, also known as relative standard deviation (RSD), as red lines against the percent of total demand that could be satisfied by the emergency stockpile alone. Larger emergency stockpiles reduce cost RSD, but they have little effect on expected cost.

This experiment was then repeated with the RO and DRO models. Figure 5.7 plots the cost objectives of all five models at a $99 \%$ service level against the percent of total demand that could be satisfied by the emergency stockpile alone. Figure 5.7 shows
that the RO and DRO models benefit more from larger emergency stockpiles than the SP models. This indicates that larger emergency stockpiles cause a greater cost reduction for more severe scenarios, which are over-weighted in the RO and DRO models, than for less severe scenarios. Figure 5.7 illustrates a diminishing return on cost savings as the emergency stockpile size increases. With this data, increasing the quantity of emergency stockpile allocated to the Health Provider beyond $25 \%$ of their total expected demand seems uneconomical.


Figure 5.6: Expected cost and cost RSD for two-stage (left) and multi-stage (right) SP models versus size of emergency stockpile

## Cost Objective vs. Size of Emergency Stockpile



Figure 5.7: Plot of cost objective versus size of emergency stockpile

The quantity of allocated emergency stock was then held constant while the the cost of using it varied between $30 \%$ and $180 \%$ of the average contract price. The federal government could decide to heavily subsidize the costs of emergency stockpile PPE which is reflected in the case where emergency stock costs only $30 \%$ of the average contract price. Figure 5.8 plots the expected cost and cost RSD against various prices of emergency stock. As anticipated, cheaper emergency stockpile PPE significantly reduces expected cost. Cost RSD is minimized when the emergency PPE prices are roughly equal to the average contract price, but the improvements in cost RSD are smaller than those achieved by increasing the size of the emergency stockpile. The utilization of allocated emergency stock in the multi-stage model varies from $18 \%$ when its prices are highest to $94 \%$ when they are lowest.

Figures 5.6 and 5.8 indicate that cost variance is more effectively reduced by increasing the quantities of PPE allocated through the emergency stockpile rather than lowering the price of emergency stockpile PPE, while the latter strategy is more effective than the former at reducing expected cost.


Figure 5.8: Expected cost and cost RSD for two-stage (left) and multi-stage (right) SP models versus price of emergency stockpile

### 5.3 Inventory Level Behavior

In periods of severe pandemic spread, increased demand and prices paired with decreased supply causes inventory levels to drop. Both two-stage and multi-stage SP models exhibit this behaviour as shown by the negative correlation between pandemic severity factors and change in inventory levels that ranges in value between (-) 0.4 and (-) 0.9. The RO and DRO models exhibited weaker negative correlations than the SP model. This is likely caused by the former models' tendency to optimize the worst-case scenarios and find feasible but sometimes erratic solutions for scenarios with zero probabilities of occurrence.

Decreasing inventory holding cost or the storage space required per unit of PPE strengthens the negative correlation between inventory change and pandemic severity. A potential explanation of this behaviour is that stronger negative correlations can be thought of as more price sensitive purchasing. Low holding costs and space requirements make it cheaper for the model to purchase PPE in time periods with low prices and store it in inventory until use in more severe time periods.

Increasing the emergency stockpile size or decreasing emergency stockpile prices
weakens the negative correlation between inventory change and pandemic severity. This indicates that abundant and cheap emergency stock corresponds with less open market price sensitivity. The average inventory level also decreases as greater amounts of emergency stock become available at lower prices.

### 5.4 RO Model Conservatism

The RO model at a $99 \%$ service level was run with various values of $\alpha$ which relaxes or tightens the SAA chance-constraint on the cost objective. Larger values of $\alpha$ allow the cost of more scenarios to exceed the minimax cost objective. As shown in Figure 5.9, the minimax cost objective decreases as the value of $\alpha$ increases. The cost objective at $\alpha=0$ is equivalent to the optimal solution of the RO model without SAA.

$$
\text { RO Model Cost Objective vs. } \alpha
$$



Figure 5.9: Plot of RO minimax cost objective against values of $\alpha$

### 5.4.1 RO Contract Utilization

Figure 5.10 plots the percent of available contracts selected by the model at a $90 \%$ service level for various values of $\alpha$. Contract utilization was hypothesized to decrease as $\alpha$ increases and relaxes RO model conservatism. This behaviour is reflected in Figure 5.10.

RO Model Contract Utilization vs. $\alpha$


Figure 5.10: Plot of RO contract utilization against values of $\alpha$

### 5.5 DRO Model Conservatism

Increasing the value of $\rho$ increases the size of the ambiguity set which makes the DRO model more conservative. The worst-case expected cost values of the DRO model at a $99 \%$ service level are plotted for various values of $\rho$ in Figure 5.11. The worst-case expected cost increases along with the value of $\rho$. The cost objective at $\rho=0$ is equivalent to the optimal solution of a SP model with scenario probabilities $\hat{f}_{s}$. The objective function value of the DRO model converges on the objective value of the

RO model as $\rho$ increases and equals the robust solution for all instances of $\rho$ greater than 2.

Low, moderate, and high DRO conservatism ranges are defined by setting the value of $\rho$ within the intervals $[0,1],(1,1.6)$, and $[1.6, \infty)$ which are shaded in Figures 5.11 to 5.14 with darker shades indicating higher conservatism.


Figure 5.11: Plot of DRO cost objective against values of $\rho$

Higher conservatism reduces the risk of high expected costs associated with incorrect probability estimates, but it can also increase overall expected costs.

Figure 5.12 plots the expected cost of the DRO model at a $99 \%$ service level given that the scenario probability estimates $\hat{f}_{s}$ are correct against the value of $\rho$. Figure 5.12 confirms that increasing model conservatism generally decreases performance under the estimated probability distribution.

## Expected Cost with Estimated Probabilities vs. $\rho$



Figure 5.12: Plot of DRO expected cost if estimated probabilities are correct against $\rho$

### 5.5.1 DRO Contract Utilization

Figure 5.13 plots the percent of available contract units selected by the model at a $99 \%$ service level for various values of $\rho$. Contract prices can be higher or lower than open market prices depending on the scenario and time period, so it was anticipated that less conservative models would sign fewer long-term contracts than more conservative models. Figure 5.13 indicates that contract utilization is $82 \%$ with SP compared to $90 \%$ with RO. The DRO model contracts more units as the value of $\rho$ increases and has the same contract utilization for all values of $\rho$ greater than 1 .


Figure 5.13: Plot of DRO contract utilization against values of $\rho$

### 5.5.2 Relative Standard Deviation in Cost

Cost RSD was calculated for the DRO model at various values of $\rho$. Figure 5.14 shows that the DRO model with a $99 \%$ service level experiences a decrease in cost RSD as the value of $\rho$ increases. Interestingly, cost RSD increases again as $\rho$ approaches 2 , which is the point at which the DRO model is equivalent to a pure RO model. One potential explanation of this behaviour is that as the size of the ambiguity set increases, the DRO model optimizes fewer scenarios as the probability of many scenarios approaches 0. Sub-optimal solutions for scenarios with probabilities of occurrence equal to zero causes cost RSD to increase again when $\rho$ equals 2 , so it cannot be said definitively whether DRO or RO reduces cost RSD more. The DRO model does provide solutions with lower cost RSD than the SP model.

The DRO model with the value of $\rho$ set between 1 and 1.6 is recommended for decision makers that prioritize low cost variance. Decision makers in favour of average cost
performance can still benefit from the lower cost variance of DRO models with small values of $\rho$. For example, $\rho=0.6$ reduces cost RSD by $20 \%$ compared to $\rho=0$.

Relative Standard Deviation in Scenario Cost vs. $\rho$


Figure 5.14: Plot of DRO cost RSD against values of $\rho$

### 5.6 Value of Information

Studies that implement SP often calculate the value of information which compares the expected value obtained through SP to the best possible expected value that could be achieved if a deterministic model had perfect information about future states. Having presented two-stage and multi-stage models, this paper has the unique opportunity to explore what is rarely seen in other studies, a comparison of not two but three levels of uncertainty in value of information analysis. The two-stage and multi-stage SP models were run with the multi-stage dataset. The value of perfect information is the expected cost of running the deterministic model in each scenario with the same scenario probabilities as the SP model. The results of these experiments are presented in Table 5.2.

Table 5.2: Value of information experiment results

|  | Perfect <br> Information | Two-Stage SP | Multi-Stage SP |
| :---: | :---: | :---: | :---: |
| Expected Cost <br> (\$ in millions) | 1.856 | 1.859 | 1.868 |
| Cost of Uncertainty <br> (\% Above Perfect <br> Information Case | - | $0.2 \%$ | $0.6 \%$ |

The two-stage SP model has a lower expected cost than its multi-stage counterpart. This was anticipated as the two-stage model makes strategic decisions under uncertainty and operational decisions with certainty, while the multi-stage model makes both strategic and operational decisions under uncertainty. This allows the two-stage model to make more educated purchasing decisions, resulting in a cost that is closer to the deterministic model's expected cost with perfect information.

Realistically, many health authorities do not have the ability to predict the state of future time periods as is required for the two-stage models. They must use the multi-stage model, which has an expected cost that is only $0.6 \%$ greater than that with perfect information. The multi-stage model thus provides low-cost solutions while incorporating realistic levels of future uncertainty inherent in supply planning during a pandemic.

## Chapter 6

## Discussion

### 6.1 Managerial Implications

Implementing optimization models in real systems is often complicated by uncertainty about the future and multiple decision making criteria (Ide \& Schöbel 2015). The multi-objective SP, RO, and DRO models developed in this study incorporate both future uncertainties and multiple goals. They allow decision makers to select a solution along the trade-off between cost and service level that best serves their organizational strategy. Another benefit of multi-objective optimization is its independence of shortage penalty costs which have the undesirable traits of impacting optimal decisions and being difficult to accurately estimate.

The case study uncovered numerous operational insights for SCs disrupted by pandemics. Higher inventory levels may or may not improve worst-case or expected costs depending on the potential disruption scenarios. If rapid escalation of pandemic severity is probable, then SC managers should increase pre-pandemic inventory levels as they will have few opportunities once the pandemic starts to stockpile inventory. If pandemic severity is expected to increase slowly, then ample opportunities will exist during the pandemic to stockpile inventory and high levels of pre-pandemic inventory will be a cost burden. Another operational insight is that scenarios with early peaks and low variation in pandemic severity have the highest cost since inventory cannot be stockpiled prior to early peaks in severity and low variation in severity results in fewer opportunities to buy cheap PPE. Larger emergency PPE stockpiles reduce cost RSD, while cheaper emergency stockpiles reduce expected costs. Provincial health
authorities can use this insight to make request of the federal government for specific emergency stockpile allocations and prices. This insight supports findings of Kamalahmadi and Parast (2017) that emergency inventory effectively mitigates the negative impacts of disruptions. Finally, increasing the portion of fixed contracts in the PPE supply base improves the worst-case cost but increases expected cost performance. Signing more long-term contracts is beneficial in worst-case scenarios where open market PPE is limited and expensive, but it can be a hindrance in less-severe scenarios where cheaper PPE is available on the open market.

There are numerous points for managers to consider when selecting a suitable optimization approach. One factor is the availability of historical data. The certainty regarding future states and probability distribution estimates required for deterministic optimization and SP is generally not possible during pandemics when historical data is scarce or out-of-sample. DRO offers protection against over-fitting SP models by assuming that estimated probabilities can be incorrect. The protection against worst-case expected cost provided by more conservative DRO models comes at the expense of increasing expected cost when estimated probabilities are in fact correct. Greater skepticism in scenario probability estimates leads to RO which completely disregards them. SP and RO solutions can also be achieved with a DRO model by setting parameter $\rho$ to zero and 2 respectively.

Decision makers must select an appropriate value for DRO model hyperparameter $\rho$. Three intervals of $\rho$ values are proposed to prioritize average cost performance, cost variance, or worst-case cost performance. DRO with $\rho$ less than 1 is recommended for decision makers with higher confidence in scenario probability estimates and a preference for average cost performance. Even when decision makers are confident in the accuracy of their scenario probability estimates, the DRO model with small values of $\rho$ offers some advantages over SP models by decreasing cost variance and the risk of
out-of-sample predictions. DRO with $\rho$ greater than 1.6 should be used when decision makers prioritize worst-case cost performance over expected cost performance or have limited data with which to estimate the scenario probability distribution. Values of $\rho$ between 1 and 1.6 are ideal when decision makers prefer to minimize cost-variance or suspect that some historical data could be out-of-sample data. This moderate conservatism will best suit health authorities that have strict budget constraints. Requesting additional funds can be an arduous process, so minimizing cost variance and thus the chance of exceeding the planned budget is desirable. The values of $\rho$ that define these three intervals may vary for other datasets, so the DRO model should first be tested with all values of $\rho$ between 0 and 2 .

The DRO model selects more long-term contracts as the value of $\rho$ increases. This demonstrates that long-term contracts are an effective insurance policy against high costs in extreme disruption scenarios but over-weighting them in the supply base can decrease average cost performance. The added cost of some strategic decisions, such as sourcing from contracts rather than the open market, may be unsustainable indefinitely. For this reason, healthcare providers should develop systems that detect pandemics as soon as possible, perhaps through an internal business intelligence unit or frequent communication with the World Health Organization. Early warning notifications regarding a potential pandemic should result in immediate action to scale-up certain resilience strategies like contracting more suppliers. Conversely, less responsive strategic decisions, such as warehouse location, would remain constant in normal as well as disrupted time periods.

The definition of resilience adopted by this paper has four components: planning for, absorbing, recovering from, and adapting to disruptions. The first two components are delivered by consulting the presented models during the SC design phase. The latter two components are addressed post-disruption by reoptimizing the supply base
using newly available data. Decision variables $q_{p i b}^{c}$ for existing contracts would be constrained to the contracted amounts, while the unconstrained $q_{p i b}^{c}$ variables would represent contracts that are currently available. The duration of time periods in the post-disruption optimization model could be shortened from months to weeks to increase the complexity of pandemic scenarios considered by the model.

### 6.2 Theoretical Contributions

This paper contributes to SC literature through its focus on the rarely studied topic of pandemics as disruptions. A survey by Govindan et al. (2017) on SCND under uncertainty literature from 2000 to 2015 finds that less than $20 \%$ of studies focus on disruption risks while the rest focus on operational risk. Further, the majority of research on SC disruptions study localized events like natural disasters rather than wide-spread long-term disruptions like pandemics. Govindan et al. (2017) suggest further study of disruptions that impact the SC over multiple periods such as the one developed here. The presented modelling approach for long-term disruptions that evolve unpredictably is fundamental to SC viability research.

Multiple surveys of SC resilience literature have recommended the empirical study of multi-objective models that analyze the trade-off between costs and resilienceenhancing strategies, which this paper provides (Hosseini et al., 2019a; Kamalahmadi \& Parast, 2016). This study also adds to the limited amount of research on multiobjective optimization under uncertainty. Although some multi-objective optimization studies use SP to deal with uncertainty, far fewer studies incorporate RO and none were found in supplier selection research combining DRO with multi-objective optimization. While most studies adopt a single approach to modelling under uncertainty such as RO, DRO, or SP with either two-stage or multi-stage recourse, this study utilizes all three approaches as well as two-stage and multi-stage recourse. This facilitated a
comparison of the value of information in both two-stage and multi-stage uncertainty which has, to the best of my knowledge, not previously been performed in the literature. This study also provides insightful analysis on the relative variance achieved with DRO, RO, and SP. The results confirm that DRO achieves lower relative variance in the objective function than SP.

The use of RO, DRO, non-monetary objective functions, and multi-period models are all unconventional in SC resilience and SCRM (Hosseini et al. 2019a; Baryannis et al., 2018; Heckmann et al., 2015). Finally, this study answers the call of Govindan et al. (2017) for more SCND research to present models based on real-world applications with a case study of a Canadian provincial healthcare provider during the COVID-19 pandemic.

### 6.3 Limitations

This study contains limitations including its disregard for lead times on open market purchases. PPE is ordered and delivered in the same period which may be unrealistic for some open market suppliers. Another limitation is that the cost of strategic decisions are not evaluated on a long-term horizon. The cost to maintain additional warehouse capacity or source through premium-priced contracts in the disruption free periods between pandemics may be sizeable. This study might be improved by allowing the emergency stockpile supply to increase during longer pandemics to reflect the federal government's continual efforts to purchase and allocate PPE to provinces.

### 6.4 Recommended Future Work

The first potential avenue of future research is large scale optimization to handle the large number of scenarios required for multi-stage recourse models. The ability to model more time periods creates opportunities to improve model precision using shorter
period durations and incorporate multiple strategic decision periods. Ambulkar, Blackhurst, and Grawe (2014) find that SC resilience to high-impact disruptions requires active reconfiguration of resources, so allowing the model to adjust warehouse capacities and the portfolio of supplier contracts during a pandemic may improve performance.

Future work could also consider other types of sourcing such as contracts that guarantee the price but have a variable order quantity. Another potentially valuable modelling approach could be to group suppliers by geographic regions. Each region would experience different levels of pandemic severity rather than a single severity factor that applies globally as was the case in this study. Such a model would facilitate building on Hosseini et al.'s (2019b) research on supplier geographic dispersion as a resilience-enhancement strategy.

Effective supply planning is crucial to SC resilience, although another important component is satisfying volatile demand levels at points of usage. Agile SCs, inventory centralization, and transshipment models may be valuable tools in achieving SC resilience on the demand side.

Future studies could develop a more holistic SCRM approach that integrates the presented models with simulation or data analytics. Simulation could model pandemic spread in the populace while the models presented in this study periodically optimize the supply base. Data analytics provide the opportunity to use the abundance of data collected during the COVID-19 pandemic for more accurate parameter estimation and scenario generation for the models presented in this study.

## Chapter 7

## Conclusion

The COVID-19 pandemic has challenged healthcare SC managers with unprecedented volatility in the supply, demand, and price of PPE. In this paper, multi-period multiobjective optimization models perform resilient supply planning under pandemic induced uncertainty for a Canadian provincial healthcare provider. The $\epsilon$-constraint method produces sets of Pareto-optimal solutions along the trade-off between cost minimization and service level maximization.

Scenario-based uncertainty is incorporated using two-stage and multi-stage recourse models with SP, RO, and DRO objective functions. Multi-stage models make strategic and operational decisions under uncertainty, while two-stage models only make strategic decisions under uncertainty.

SAA of the minimax cost objective is introduced allowing decision makers to relax the RO model's conservatism regarding cost. The SP formulation provides riskneutral solutions for operating cost while remaining risk-averse regarding service level. SP carries the risk of performing poorly on out-of-sample data (Smith \& Winkler, 2006; Wang et al., 2020). This is pertinent in the context of pandemics as they are unpredictable and occur infrequently. DRO models minimize this risk by assuming the true distribution is contained within an ambiguity set whose size can be adjusted by changing the value of parameter $\rho$. Three intervals for the value of $\rho$ are recommended depending on the decision makers' preference for average cost performance, worst-case cost performance, or cost variance. This is important in the healthcare context, as many regional differences exist thus creating the need to model different risk-tolerances and budget restrictions. The presented models can enhance

SC resilience to and viability during pandemics by optimizing the supply base prior to disruption and re-optimizing it during a disruption as new information becomes available.

Healthcare providers can assess the effectiveness of new risk mitigation and procurement strategies using the proposed optimization framework. For example, the case study analysis uncovered generally applicable insights regarding emergency PPE stockpiles, pre-pandemic inventory levels, long-term contracts, and cost conservatism.

Value of information experiments revealed that the knowledge of future time periods possessed by the two-stage SP model helps it perform closer to the deterministic model with perfect information than the multi-stage SP model. It is more likely that healthcare providers will use multi-stage models, as the ability to predict pandemic severity several months into the future is somewhat unrealistic. The mere $0.6 \%$ difference in expected cost between the multi-stage model and deterministic model with perfect information demonstrates the potential of the multi-stage models to provide near-optimal supply planning solutions amidst realistic levels of pandemic induced uncertainty.

## Funding

This research was partially funded through grant NSERC ALLRP 550319-2020 from the Canada Natural Sciences and Engineering Research Council (NSERC).

## Data Disclaimer

The data in this paper is in no way reflective of the NSHA's actual operations during the COVID-19 pandemic. The results should not be used to pass judgement on the NSHA.

## Bibliography

Ahmed, S., \& Shapiro, A. (2008). Solving chance-constrained stochastic programs via sampling and integer programming. State-of-the-Art Decision-Making Tools in the Information-Intensive Age, 261-269. doi:10.1287/educ.1080.0048

Altay, N., Gunasekaran, A., Dubey, R., \& Childe, S. J. (2018). Agility and resilience as antecedents of supply chain performance under moderating effects of organizational culture within the humanitarian setting: A dynamic capability view. Production Planning \& Control, 29(14), 1158-1174. doi:10.1080/09537287.2018.1542174

Ambulkar, S., Blackhurst, J., \& Grawe, S. (2014). Firm's resilience to supply chain disruptions: Scale development and empirical examination. Journal of Operations Management, 33-34(1), 111-122. doi:10.1016/j.jom.2014.11.002

Azaron, A., Brown, K., Tarim, S., \& Modarres, M. (2008). A multi-objective stochastic programming approach for supply chain design considering risk. International Journal of Production Economics, 116(1), 129-138. doi:10.1016/j.ijpe.2008.08.002

Ash, C., Diallo, C., Venkatadri, U., \& VanBerkel, P. (2021a). Distributionally robust optimization of a Canadian healthcare supply chain to enhance resilience during the COVID-19 pandemic [Manuscript submitted for publication]. Department of Industrial Engineering, Dalhousie University.

Ash, C., Venkatadri, U., Diallo, C., \& VanBerkel, P. (2021b). Stochastic and robust PPE supply optimization under risks of disruption from the COVID-19 pandemic: A Canadian provincial healthcare perspective [Manuscript submitted for publication]. Department of Industrial Engineering, Dalhousie University.

Azaron, A., Venkatadri, U., \& Doost, A. F. (2020). Designing profitable and responsive supply chains under uncertainty. International Journal of Production Research. doi: 10.1080/00207543.2020.1785036

Baryannis, G., Validi, S., Dani, S., \& Antoniou, G. (2018). Supply chain risk management and artificial intelligence: State of the art and future research directions. International Journal of Production Research, 57(7), 2179-2202. doi:10.1080/00207543.2018.1530476

Ben-Tal, A., Hertog, D., Waegenaere, A., Melenberg, B., \& Rennen, G. (2013). Robust solutions of optimization problems affected by uncertain probabilities. Management Science, 59 (2), 341-357.

Bohner, C., \& Minner, S. (2017). Supplier selection under failure risk, quantity and business volume discounts. Computers \& Industrial Engineering, 104, 145-155. https://doi.org/10.1016/j.cie.2016.11.028

Bruneau, M., Chang, S. E., Eguchi, R. T., Lee, G. C., O’Rourke, T. D., Reinhorn, A. M., Shinozuka, M., Tierney, K., Wallace W. A., Winterfeldt, D. (2003). A framework to quantitatively assess and enhance the seismic resilience of communities. Earthquake Spectra, 19(4), 733-752. https://doi.org/10.1193/1.1623497

Brusset, X., \& Teller, C. (2017). Supply chain capabilities, risks, and resilience. International Journal of Production Economics, 184, 59-68. doi:10.1016/j.ijpe.2016.09.008

Cardoso, S. R., Barbosa-Póvoa, A. P., Relvas, S., \& Novais, A. Q. (2015). Resilience metrics in the assessment of complex supply-chains performance operating under demand uncertainty. Omega, 56, 53-73. doi:10.1016/j.omega.2015.03.008

Carvalho, H., Azevedo, S. G., \& Cruz-Machado, V. (2012). Agile and resilient approaches to supply chain management: Influence on performance and competitiveness. Logistics Research, 4(1), 49-62.

Cavalcante, I. M., Frazzon, E. M., Forcellini, F. A., \& Ivanov, D. (2019). A supervised machine learning approach to data-driven simulation of resilient supplier selection in digital manufacturing. International Journal of Information Management, 49, 86-97. doi:10.1016/j.ijinfomgt.2019.03.004

Chen, L., \& Miller-Hooks, E. (2012). Resilience: An indicator of recovery capability in intermodal freight transport. Transportation Science, 46(1), 109-123. doi:10.1287/trsc. 1110.0376

Choi, T.-M., Wen, X., Sun, X., \& Chung, S. (2019). The mean-variance approach for global supply chain risk analysis with air logistics in the blockchain technology era. Transportation Research Part E: Logistics and Transportation Review, 127, 178-191. doi:10.1016/j.tre.2019.05.007

Costantino, N., \& Pellegrino, R. (2010). Choosing between single and multiple sourcing based on supplier default risk: A real options approach. Journal of Purchasing and Supply Management, 16(1), 27-40. doi:10.1016/j.pursup.2009.08.001

Dolgui, A., Ivanov, D., \& Sokolov, B. (2018) Ripple effect in the supply chain: an analysis and recent literature. International Journal of Production Research, $56(1-2), 414-430$. doi: 10.1080/00207543.2017.1387680

Eckstein, D., Goellner, M., Blome, C., \& Henke, M. (2014). The performance impact of supply chain agility and supply chain adaptability: The moderating effect of product complexity. International Journal of Production Research, 53(10), 3028-3046. doi:10.1080/00207543.2014.970707

Ehrgott, M. (2005). Multicriteria optimization, second ed. Springer Berlin, Heidelberg, Germany.

Ehrgott, M., Ide, J., \& Schöbel, A. (2014). Minimax robustness for multi-objective optimization problems. European Journal of Operational Research, 239(1), 17-31. doi:10.1016/j.ejor.2014.03.013

Gao, J., Ning, C., \& You, F. (2019). Data-driven distributionally robust optimization of shale gas supply chains under uncertainty. AIChE Journal, 65 (3), 947-963.
Goh, G., Dunker, A., Foster, J., \& Uversky, V. (2020). Rigidity of the outer shell predicted by a protein intrinsic disorder model sheds light on the COVID-19 (wuhan-2019-ncov) infectivity. Biomolecules (Basel, Switzerland), $10(2), 331$.
Golan, M. S., Jernegan, L. H., \& Linkov, I. (2020). Trends and applications of resilience analytics in supply chain modeling: Systematic literature review in the context of the COVID-19 pandemic. Environment Systems and Decisions, 40(2), 222-243. doi:10.1007/s10669-020-09777-w

Government of Canada. (2021). National emergency strategic stockpile. Retrieved from: https://www.canada.ca/en/public-health/services/emergency-preparedness-response/national-emergency-strategic-stockpile.html

Govindan, K., Fattahi, M., \& Keyvanshokooh, E. (2017). Supply chain network design under uncertainty: A comprehensive review and future research directions. European Journal of Operational Research, 263(1), 108-141. doi:10.1016/j.ejor.2017.04.009

Guillén, G., Mele, F., Bagajewicz, M., Espuña, A., \& Puigjaner, L. (2005). Multiobjective supply chain design under uncertainty. Chemical Engineering Science, $60(6), 1535-1553$. doi:10.1016/j.ces.2004.10.023

Heckmann, I., Comes, T., \& Nickel, S. (2015). A critical review on supply chain risk - Definition, measure and modeling. Omega, 52, 119-132. doi:10.1016/j.omega.2014.10.004

Hosseini, S., Ivanov, D., \& Dolgui, A. (2019a). Review of quantitative methods for supply chain resilience analysis. Transportation Research Part E: Logistics and Transportation Review, 125, 285-307. doi:10.1016/j.tre.2019.03.001

Hosseini, S., Morshedlou, N., Ivanov, D., Sarder, M., Barker, K., \& Khaled, A. A. (2019b). Resilient supplier selection and optimal order allocation under disruption risks. International Journal of Production Economics, 213, 124-137. doi:10.1016/j.ijpe.2019.03.018

Hu, T., Liu, Y., Zhao, M., Zhuang, Q., Xu, L., \& He, Q. (2020). A comparison of COVID-19, SARS and MERS. PeerJ (San Francisco, CA), 8, E9725.

Ide, J., \& Schöbel, A. (2015). Robustness for uncertain multi-objective optimization: A survey and analysis of different concepts. OR Spectrum, 38(1), 235-271. doi:10.1007/s00291-015-0418-7

Ivanov, D. (2020a). Predicting the impacts of epidemic outbreaks on global supply chains: A simulation-based analysis on the coronavirus outbreak (COVID-19/SARS-CoV-2) case. Transportation Research Part E: Logistics and Transportation Review, 136, 101922. doi:10.1016/j.tre.2020.101922

Ivanov, D. (2020b). Viable supply chain model: Integrating agility, resilience and sustainability perspectives-lessons from and thinking beyond the COVID-19 pandemic. Annals of Operations Research. doi:10.1007/s10479-020-03640-6

Ivanov, D., \& Dolgui, A. (2020a). A digital supply chain twin for managing the disruption risks and resilience in the era of Industry 4.0. Production Planning and Control. Retrieved from https://doi.org/10.1080/09537287.2020.1768450.

Ivanov, D., \& Dolgui, A. (2020b). Viability of intertwined supply networks: Extending the supply chain resilience angles towards survivability. A position paper motivated by COVID-19 outbreak. International Journal of Production Research, 58 (10), 2904-2915.

Ivanov, D., Dolgui, A., Ivanova, M., \& Sokolov, B. (2018). Simulation Vs. Optimization Approaches to Ripple Effect Modelling in the Supply Chain. Dynamics in Logistics Lecture Notes in Logistics, 34-39. doi:10.1007/978-3-319-74225-0_5

Ivanov, D., Dolgui, A., Sokolov, B., \& Ivanova, M. (2017). Literature review on disruption recovery in the supply chain. International Journal of Production Research, 55(20), 6158-6174. doi:10.1080/00207543.2017.1330572

Jabbarzadeh, A., Haughton, M., \& Khosrojerdi, A. (2018). Closed-loop supply chain network design under disruption risks: A robust approach with real world application. Computers E Industrial Engineering, 116, 178-191. doi:10.1016/j.cie.2017.12.025

Jain, T., Hazra, J., \& Cheng, T.C.E. (2018). Sourcing under overconfident buyer and suppliers. International Journal of Production Economics, 206, 93-109.

Jajja, M. S., Chatha, K. A., \& Farooq, S. (2018). Impact of supply chain risk on agility performance: Mediating role of supply chain integration. International Journal of Production Economics, 205, 118-138. doi:10.1016/j.ijpe.2018.08.032

Jeong, K., Hong, J., \& Xie, Y. (2013). Design of emergency logistics networks, taking efficiency, risk and robustness into consideration. International Journal of Logistics Research and Applications, 17(1), 1-22. doi:10.1080/13675567.2013.833598

Jia, R., Liu, Y., \& Bai, X. (2020). Sustainable supplier selection and order allocation: Distributionally robust goal programming model and tractable approximation. Computers \& Industrial Engineering, 140, 106267.

Kamalahmadi, M., \& Parast, M. M. (2016). A review of the literature on the principles of enterprise and supply chain resilience: Major findings and directions for future research. International Journal of Production Economics, 171, 116-133. doi:10.1016/j.ijpe.2015.10.023

Kamalahmadi, M., \& Parast, M. M. (2017). An assessment of supply chain disruption mitigation strategies. International Journal of Production Economics, 184, 210230. doi:10.1016/j.ijpe.2016.12.011

Ke, Y., \& Zhao, L. (2008). Optimization of emergency logistics delivery model based on anti-bioterrorism. 2008 IEEE International Conference on Industrial Engineering and Engineering Management. doi:10.1109/ieem.2008.4738237

Khalili, S. M., Jolai, F., \& Torabi, S. A. (2016). Integrated production-distribution planning in two-echelon systems: A resilience view. International Journal of Production Research, 55(4), 1040-1064. doi:10.1080/00207543.2016.1213446

Kuroiwa D., \& Lee G.M. (2012) On robust multiobjective optimization. Vietnam J Math 40 (2833), 305-317.

Li, H., Pedrielli, G., Lee, L. H., \& Chew, E. P. (2017). Enhancement of supply chain resilience through inter-echelon information sharing. Flexible Services and Manufacturing Journal, 29(2), 260-285. doi:10.1007/s10696-016-9249-3

LLC Gurobi Optimization. (2020). Gurobi optimizer reference manual.
Lim-Camacho, L., Plagányi, É. E., Crimp, S., Hodgkinson, J. H., Hobday, A. J., Howden, S. M., \& Loechel, B. (2017). Complex resource supply chains display higher resilience to simulated climate shocks. Global Environmental Change, 46, 126-138. doi:10.1016/j.gloenvcha.2017.08.011

Mak, H., \& Shen, Z. (2012). Risk diversification and risk pooling in supply chain design. IIE Transactions, 44 (8), 603-621. doi:10.1080/0740817x.2011.635178

Margolis, J. T., Sullivan, K. M., Mason, S. J., \& Magagnotti, M. (2018). A multi-objective optimization model for designing resilient supply chain networks. International Journal of Production Economics, 204, 174-185. doi:10.1016/j.ijpe.2018.06.008

Meena, P., \& Sarmah, S. (2013). Multiple sourcing under supplier failure risk and quantity discount: A genetic algorithm approach. Transportation Research Part E: Logistics and Transportation Review, 50, 84-97. doi:10.1016/j.tre.2012.10.001

Mehrotra, S., Rahimian, H., Barah, M., Luo, F., \& Schantz, K. (2020). A model for supply-chain decisions for resource sharing with an application to ventilator allocation to combat COVID-19. doi:10.1101/2020.04.02.20051078

National Research Council. (2012). Disaster resilience: a national imperative. Washington DC: The National Academies Press. Retrieved from: https://doi.org/10.17226/13457.

Ni, N., Howell, B. J., \& Sharkey, T. C. (2018). Modeling the impact of unmet demand in supply chain resiliency planning. Omega, 81, 1-16. doi:10.1016/j.omega.2017.08.019

Noyan, N. (2012). Risk-averse two-stage stochastic programming with an application to disaster management. Computers $8 \mathcal{3}$ Operations Research, 39(3), 541-559. doi:10.1016/j.cor.2011.03.017

Rottkemper, B., Fischer, K., \& Blecken, A. (2012). A transshipment model for distribution and inventory relocation under uncertainty in humanitarian operations. Socio-Economic Planning Sciences, 46(1), 98-109. doi:10.1016/j.seps.2011.09.003

Sabio, N., Gadalla, M., Guillén-Gosálbez, G., \& Jiménez, L. (2010). Strategic planning with risk control of hydrogen supply chains for vehicle use under uncertainty in operating costs: A case study of Spain. International Journal of Hydrogen Energy, 35(13), 6836-6852. doi:10.1016/j.ijhydene.2010.04.010

Sahebjamnia, N., Torabi, S. A., \& Mansouri, S. A. (2018). Building organizational resilience in the face of multiple disruptions. International Journal of Production Economics, 197, 63-83. https://doi.org/10.1016/j.ijpe.2017.12.009

Sawik, T. (2014). Optimization of cost and service level in the presence of supply chain disruption risks: Single vs. multiple sourcing. Computers $8 \mathcal{F}$ Operations Research, 51, 11-20. doi:10.1016/j.cor.2014.04.006

Shang, C., \& You, F. (2018). Distributionally robust optimization for planning and scheduling under uncertainty. Computers © Chemical Engineering, 110, 53-68.

Sheffi, Y. (2015). A classification of catastrophes. The Power of Resilience, 27-52. doi:10.7551/mitpress/9780262029797.003.0002

Simchi-Levi, D., Schmidt, W., Wei, Y., Zhang, P. Y., Combs, K., Ge, Y, Gusikhin, O., Sanders, M., \& Zhang, D. (2015). Identifying risks and mitigating disruptions in the automotive supply chain. Interfaces, 45(5), 375-390. doi:10.1287/inte.2015.0804

Simchi-Levi, D., Wang, H., \& Wei, Y. (2018). Increasing supply chain robustness through process flexibility and inventory. Production and Operations Management, $27(8), 1476-1491$. doi:10.1111/poms. 12887

Smith, J. \& Winkler, R. (2006). The optimizer's curse: skepticism and post decision surprise in decision analysis. Management Science, 52 (3), 311-322.

Society for Healthcare Organization Procurement Professionals. (2020). COVID patient per day cost analysis. Retrieved from http://cdn.cnn.com /cnn/2020/images/04/16/shopp.covid.ppd.costs.analysis_.pdf

Soyster, A. L. (1973). Convex programming with set-inclusive constraints and applications to inexact linear programming. Operations Research, 21(5), 1154-1157. doi:10.1287/opre.21.5.1154

Tang, C. S. (2006). Robust strategies for mitigating supply chain disruptions. International Journal of Logistics Research and Applications, 9(1), 33-45. doi:10.1080/13675560500405584

Tomlin, B. (2006). On the value of mitigation and contingency strategies for managing supply chain disruption risks. Management Science, 52(5), 639-657. doi:10.1287/mnsc.1060.0515

Torabi, S., Baghersad, M., \& Mansouri, S. (2015). Resilient supplier selection and order allocation under operational and disruption risks. Transportation Research Part E: Logistics and Transportation Review, 79, 22-48. doi:10.1016/j.tre.2015.03.005

Wang, S., Chen, Z., \& Liu, T. (2020). Distributionally robust hub location. Transportation Science, 54 (5), 1189-1210.

Wang, H., Wang, X., \& Zeng, A. Z. (2009). Optimal material distribution decisions based on epidemic diffusion rule and stochastic latent period for emergency rescue. International Journal of Mathematics in Operational Research, 1(1/2), 76. doi:10.1504/ijmor.2009.022876

Wieland, A., \& Marcus Wallenburg, C. (2013). The influence of relational competencies on supply chain resilience: A relational view. International Journal of Physical Distribution ${ }^{\circ}$ Logistics Management, 43(4), 300-320.

Yoon, J., Talluri, S., Yildiz, H., \& Ho, W. (2018). Models for supplier selection and risk mitigation: A holistic approach. International Journal of Production Research, 56(10), 3636-3661. doi:10.1080/00207543.2017.1403056

Zhao, S., \& You, F. (2019). Resilient supply chain design and operations with decisiondependent uncertainty using a data-driven robust optimization approach. AIChE Journal, 65(3), 1006-1021. doi:10.1002/aic. 16513

Zimonjic, P. (2020). Canadian officials working around the clock to secure medical supplies, deputy minister says. CBC News. Retrieved from https://www.cbc.ca/news/politics/medical-supplies-coronavirus-purchases1.5533915

## Appendix A

## Matlab Code for Deterministic Model

```
%% File Names and Scenario Selection
clear
SheetName='Sheet1 ';
DataFileName = 'DataFile_twoStage.xlsx';
ResultsFileName = 'Results.xlsx';
scenario_number =6;
%% Index Initialization
sets=readmatrix(DataFileName, 'Sheet', 'Sets');
P=sets(1, 2); %number of products
I=sets(2, 2); %number of suppliers
K=3; %number of warehouse capacities available
B=sets(3, 2); %number of price breaks offered by suppliers, 1 = no price breaks
T=sets(4, 2); %number of time periods;
S=sets(5, 2); %number of scenarios
%% Sensitivity Analysis Variable
var = readmatrix(DataFileName, 'Sheet', 'V0_data');
sensitivityVariable = [var; 2*var; 3*var];
count_sensitivity = size(sensitivityVariable, 1);
%% Model Hyperparameters Initialization
epsilon_start = 0.20; %service level
step = -0.01;
count_eps = floor(abs(epsilon_start/step))+1;
qc_vars=[];
qo_vars = [];
w_vars=[];
inv_by_s=[];
qc_by_i = [];
qo_by_i = [];
OMcap_last_ep = [];
cnt_correl = 1;
%% Model Parameter Initialization
Ac_data=readmatrix(DataFileName, 'Sheet', 'Ac_data'); %Portion of supplier i's
    contract that is actually received in t
Ac_its=[];
for ct = 1:T
    for ci = 1:I
        Ac_its = [Ac_its, Ac_data(:,(ci-1)*T+ct)];
    end
end
Ac_its=reshape(Ac_its', 1, I*T*S);
Ac_it = Ac_its(I*T*(scenario_number - 1) +1:I*T*scenario_number);
Ao_data=readmatrix(DataFileName, 'Sheet', 'Ao_data'); %Portion of supplier i's open
    market capacity available in t
Ao_its=[];
for ct = 1:T
    for ci = 1:I
        Ao_its = [Ao_its, Ao_data(:,(ci-1)*T+ct)];
    end
end
Ao_its=reshape(Ao_its', 1, I*T*S);
Ao_it = Ao_its(I*T*(scenario_number -1)+1:I*T*scenario_number);
D_data=readmatrix(DataFileName, 'Sheet', 'D_data'); % open market purchase price for
```

```
        product p, supplier i, time period t, scenario s
D_pts=[];
for ct = 1:T
    for cp = 1:P
        D_pts = [D_pts, D_data (:, (cp-1)*T+ct)];
    end
end
D_pts=reshape(D_pts', 1, P*T*S);
D_pt = D_pts (P*T*(scenario_number - 1) +1:P*T*scenario_number );
po_data=readmatrix(DataFileName, 'Sheet', 'po_data'); % open market purchase price
        for product p, supplier i, time period t, scenario s
po_pits=[];
for ct = 1:T
    for ci = 1:I
            for cp = 1:P
                po_pits = [po_pits, po_data(:, (cp-1)*T+ct+(ci-1)*P*T)];
            end
    end
end
po_pits=reshape(po_pits ', 1, P*I *T*S);
po_pit = po_pits (P*I*T*(scenario_number -1)+1:P*I*T*scenario_number);
pc_pi = readmatrix(DataFileName, 'Sheet', 'pc_data'); % base contract price for
    product p, supplier i
pe_p = readmatrix(DataFileName, 'Sheet', 'pe_data'); %Emergency stock purchase price
C1_pi = readmatrix(DataFileName, 'Sheet', 'C1_data''); % cost to ship product p from
    supplier to WH
C2_p = 1.2*C1_pi(1:P); % cost to ship product p from emergency stock to WH
C3_p = readmatrix(DataFileName, 'Sheet', 'C3_data'); % cost to ship product p
    from_by_sc WH to hospital
C4_k = [0, 10000*(1:K-1)]; %cost to have capacity k at warehouse j
C5_p = readmatrix(DataFileName, 'Sheet', 'C5_data'); %holding cost per product p at
        warehouse j
C6 = 1000; %cost to establish supplier relationship
F1_pi = readmatrix(DataFileName, 'Sheet', 'F1_data'); %Reliability of supplier i (
        portion of product that passes QC and is usable)
F2_pib = readmatrix(DataFileName, 'Sheet', 'F2_data'); %Fraction of normal price
        charged by supplier i for product p with discount d
F2_pib= [ones (1,P*I), reshape(F2_pib', 1, P*I*(B-1))];
Q1_ib_1 = readmatrix(DataFileName, 'Sheet', 'Q1_data');% Quantity of any product
        where supplier i offers discount d - has dimensions I * (B+1)
Q1_ib_1 = [zeros(1,I), Q1_ib_1, 10^ 9*ones(1,I)];
Q2_pi = readmatrix(DataFileName, 'Sheet', 'Q2_data') ; %contract max
Q3_pi = readmatrix(DataFileName, 'Sheet', 'Q3_data'); %contract min
Q4_pi = readmatrix(DataFileName, 'Sheet', 'Q4_data''); %Nominal capacity of supplier i
    in units of product p
Q5_p = readmatrix(DataFileName, 'Sheet', 'Q5_data'); %Supply of product p in
        emergency stockpile
K1_k = 16000 + 4000*(0:K-1); %warehouse inventory capacities in square feet
K2_p = readmatrix(DataFileName, 'Sheet', 'K2_data'); %square feet required to store
        one unit of product p
V0_p = readmatrix(DataFileName, 'Sheet', 'V0_data'); %starting inventory
M=10^9; %very large number
%% Decision Variables
%qc: number product p purchased from_by_sc supplier i to warehouses in time period t
zero_qc = zeros(1, P*I*B);
%qo: number product p purchased from_by_sc backup supplier i to warehouses in time
        period t
zero_qo = zeros(1, P*I*T);
%qh: number product p shipped from_by_sc warehouses to hospitals in time period t
zero_qh = zeros(1, P*T);
%s: number of shorted units of product p at hospitals in time period t
zero_s = zeros(1, P*T);
```

```
%v: inventory level of product p at warehouses in time period t
zero_v = zeros(1, P*(T+1));
%w: 1 if warehouse j inventory capacity is size k and 0 otherwise
zero_w = zeros(1, K);
%y:1 if supplier i is selected as a primary supplier of product p and 0 otherwise
zero-y = zeros(1,P*I*B);
%qe: number of units of product p sent from_by_sc emergency stock to warehouses in t
    and s
zero_qe = zeros(1, P*T);
%theta is an auxiliary variable
zero_theta=0;
%% Locations of end of DVs
loc_qc=P
loc_qo=P*I *B+P}* I I TT
loc_qh=P*I*B+P*I *T +P*T;
loc_s}=\textrm{P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{I}*\textrm{T}+\textrm{P}*\textrm{T}+\textrm{P}*\textrm{T}
loc_v}=\textrm{P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{I}*\textrm{T}+\textrm{P}*\textrm{T}+\textrm{P}*\textrm{T}+\textrm{P}*(\textrm{T}+1)
loc}_\textrm{w}=\textrm{P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{I}*\textrm{T}+\textrm{P}*\textrm{T}+\textrm{P}*\textrm{T}+\textrm{P}*(\textrm{T}+1)+\textrm{K}
loc_y = P*I * B}+\textrm{P}*\textrm{I}*\textrm{T}+\textrm{P}*\textrm{T}+\textrm{P}*\textrm{T}+\textrm{P}*(\textrm{T}+1)+\textrm{K}+\textrm{P}*\textrm{I}*\textrm{B}
loc_qe=P
loc_theta=P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{I}*\textrm{T}+\textrm{P}*\textrm{T}+\textrm{P}*\textrm{T}+\textrm{P}*(\textrm{T}+1)+\textrm{K}+\textrm{P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{T}+1
%% Service Objective Function
serviceOF_pts = [repmat([zero_qc, zero_qo, zero_qh], P*T,1), eye(P*T) , repmat ([
    zero_v, zero_w, zero_y, zero_qe, zero_theta], P*T,1)];
%%***Sensitivity Analysis Loop***
for iteration_sensitivity = 1:count_sensitivity
    V0_p = sensitivityVariable(iteration_sensitivity, :);
    epsilon = epsilon_start;
    %% Theta Objective
    OF_theta = [zeros(1, loc_qe), 1];
    %% Cost Objective
    %qc
        qc_coeff=Ac_it * repmat(eye(I),1,T)';
        qc_coeff=repmat(kron(qc_coeff, ones(1,P)),1,B);
        qccst = qc_coeff.*[(repmat (pc_pi,1,B).*F2_pib) + repmat(C1_pi,1,B)];
        %qo
        qocst = [po_pit + repmat(C1_pi,1,T)];
        %qh
        qhcst = repmat(C3_p , 1 , T);
        %s
        scst=zero_s;
        %v
        vcst = repmat(C5_p , 1, T+1);
        %w
        wcst = C4_k;
        %y
        ycst = C 6*ones (1,P*I*B);
        %qe
        qecst = repmat ([C2_p + pe_p] , 1 , T);
        %theta
        thetacst = 0;
        costOF=[qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
        %% CST 3 - Open market supply supply CST - p,i,t,s
        %qc
        qccst=repmat(zero_qc, P*I*T, 1);
```

```
%qo
qocst=eye(P*I*T);
%qh
qhcst=repmat(zero_qh, P*I*T, 1);
%s
scst=repmat(zero_s, P*I*T, 1);
%v
vcst=repmat(zero_v, P*I*T, 1);
%w
wcst=repmat(zero_w, P*I*T, 1);
%y
ycst=repmat(zero_y, P*I*T, 1);
%qe
qecst=repmat(zero_qe, P*I*T,1);
%theta
thetacst = zeros( P*I*T , 1);
Acst3=[qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
bcst 3 = [kron(Ao_it, ones (1,P)).*repmat(Q4_pi,1,T)]';
%% CST 4 - Contract maximum CSTs - p,i,b
%qc
qccst=eye(P*I*B);
%qo
qocst=repmat(zero_qo, P*I*B, 1);
%qh
qhcst=repmat(zero_qh , P*I*B, 1);
%s
scst=repmat(zero_s, P*I*B, 1);
%v
vcst=repmat(zero_v, P*I*B, 1);
%w
wcst=repmat(zero_w, P*I*B, 1);
%y
ycst=-repmat(Q2_pi,1,B).*eye(P*I*B);
%qe
qecst=repmat(zero_qe, P*I*B,1);
%theta
thetacst = zeros( P*I*B , 1);
Acst4=[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
bcst4=zeros(P*I*B, 1);
%% CST 5 - Contract minimum CSTs - p,i,b
%qc
qccst=eye(P*I*B);
%qo
qocst=repmat(zero_qo, P*I*B, 1);
%qh
qhcst=repmat(zero_qh, P}*\textrm{I}*\textrm{B},1)
%s
scst=repmat(zero_s, P*I*B, 1);
%v
vcst=repmat(zero_v, P*I*B, 1);
%w
wcst=repmat(zero_w, P*I*B, 1);
%y
ycst=-repmat(Q3_pi,1,B).*eye(P*I*B);
%qe
qecst=repmat(zero_qe, P*I*B,1);
%theta
thetacst = zeros( P*I*B , 1);
Acst5=-[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
bcst5=-zeros(P*I*B, 1);
%% CST 6 - Contract order quantity exceeds price break min - p, i, b
%qc
```

```
qccst=eye(P*I*B);
%qo
qocst=repmat(zero_qo , P*I*B, 1);
%qh
qhcst=repmat(zero_qh , P*I*B, 1);
%s
scst=repmat(zero_s , P*I*B, 1);
%v
vcst=repmat(zero_v , P*I*B, 1);
%w
wcst=repmat(zero_w, P*I*B, 1);
%y
ycst=-kron(Q1_ib_1(1:I *B), ones(1,P)).*eye(P*I*B);
%qe
qecst=repmat(zero_qe, P*I*B,1);
%theta
thetacst = zeros( P*I*B, 1);
bcst6 = -zeros(P*I*B, 1);
Acst6 = - [qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
%% CST 7 - Contract order quantity is less than next price break - p, i, b
%qc
qccst=}=\operatorname{eye}(P*I*B)
%qo
qocst=repmat(zero_qo , P*I*B, 1);
%qh
qhcst=repmat(zero_qh , P*I*B , 1);
%s
scst=repmat(zero_s , P*I*B, 1);
%v
vcst=repmat(zero_v , P*I*B, 1);
%w
wcst=repmat(zero_w , P*I*B, 1);
%y
ycst=-kron(Q1_ib_1(I+1:I*(B+1)), ones (1,P)).*eye(P*I*B);
%qe
qecst=repmat(zero_qe, P*I*B,1);
%theta
thetacst = zeros( P*I*B, 1);
bcst7 = zeros(P*I*B, 1);
Acst7 = [qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
%% CST 8 - WH inventory capacity CST's - t+1,s
%qc
qccst=repmat(zero_qc, (T+1), 1);
%qo
qocst=repmat(zero_qo , (T+1), 1);
%qh
qhcst=repmat(zero_qh , (T+1), 1);
%s
scst=repmat(zero_s , (T+1), 1);
%v
vcst=[repmat(K2_p, 1, (T+1)).* kron(eye(T+1), ones (1,P))];
%w
wcst=-repmat (K1_k, (T+1), 1);
%y
ycst=repmat(zero_y, (T+1), 1);
%qe
qecst=repmat(zero_qe , (T+1),1);
%theta
thetacst = zeros(( (T+1) , 1);
Acst8=[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
bcst8=zeros ((T+1), 1);
%% CST 9 - WH inventory / flow balance CST - p,t,s
```

```
%qc
    qccst=repmat (kron(reshape(Ac_it, I,T)',ones (P,P)), 1, B).*(repmat (F1_pi, 1, B).*
        repmat( eye(P),T,I *B));
    %qo
    qocst=repmat(F1_pi, 1, T).*kron(eye(T), repmat(eye(P),1,I));
    %qh
    qhcst=-eye(P*T) ;
    %s
    scst=repmat(zero_s, P*T, 1);
    %v
    vcst = [[eye(P*T), zeros(P*T, P)]-[zeros(P*T, P), eye(P*T) ]];
    %w
    wcst=repmat(zero_w, P*T, 1);
    %y
    ycst=repmat(zero_y, P*T, 1);
    %qe
    qecst = eye(P*T);
    %theta
    thetacst = zeros( P*T, 1);
    Acst9=[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
    bcst9=zeros(P*T,1);
    %% CST 10 - Inventory period 1 equality constraints across scenarios - p,(s - 1)
    %qc
    qccst=repmat(zero_qc , P , 1);
    %qo
    qocst=repmat(zero_qo , P , 1);
    %qh
    qhcst=repmat(zero_qh , P , 1);
    %s
    scst=repmat(zero_s , P , 1);
    %v
    vcst= [eye(P), zeros(P, P*T)];
    %w
    wcst=repmat(zero_w , P , 1);
    %y
    ycst=repmat(zero-y , P , 1);
    %qe
    qecst=repmat(zero_qe, P, 1);
    %theta
    thetacst = zeros( P , 1);
    bcst10 = V0_p';
    Acst10 = [qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
    %% CST 11 - WH inventory in period 1 and T+1 are equal - p,s
    %qc
    qccst=repmat(zero_qc , P , 1);
    %qo
    qocst=repmat(zero_qo , P , 1);
    %qh
    qhcst=repmat(zero_qh , P , 1);
    %s
    scst=repmat(zero_s , P , 1);
    %v
    vcst= [zeros(P, P*T), eye(P)];
    %w
    wcst=repmat(zero_w , P , 1);
    %y
    ycst=repmat(zero_y , P , 1);
    %qe
    qecst=repmat(zero_qe, P, 1);
    %theta
    thetacst = zeros( P , 1);
    bcst11 = -V0_p';
    Acst11 = - [qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
```

```
384
385
386
387
388
389
390
392
393
394
395
```

%% CST 12 - Hospital demand satisfaction constraints - p,t,s

```
%% CST 12 - Hospital demand satisfaction constraints - p,t,s
%qc
%qc
qccst=repmat(zero_qc , P*T , 1);
qccst=repmat(zero_qc , P*T , 1);
%qo
%qo
qocst=repmat(zero_qo , P*T , 1);
qocst=repmat(zero_qo , P*T , 1);
%qh
%qh
qhcst=eye(P*T);
qhcst=eye(P*T);
%s
%s
scst=eye(P*T).*D_pt;
scst=eye(P*T).*D_pt;
%v
%v
vcst=repmat(zero_v , P*T , 1);
vcst=repmat(zero_v , P*T , 1);
%w
%w
wcst=repmat(zero_w , P*T , 1);
wcst=repmat(zero_w , P*T , 1);
%y
%y
ycst=repmat(zero-y , P*T , 1);
ycst=repmat(zero-y , P*T , 1);
%qe
%qe
qecst=repmat(zero_qe , P*T, 1);
qecst=repmat(zero_qe , P*T, 1);
%theta
%theta
thetacst = zeros( P*T , 1);
thetacst = zeros( P*T , 1);
Acst12 = [qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
Acst12 = [qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
bcst12 = D_pt';
bcst12 = D_pt';
%% CST 13 - Singular WH capacity is selected - singular
%% CST 13 - Singular WH capacity is selected - singular
bcst13 = 1;
bcst13 = 1;
Acst13 = [zero_qc, zero_qo, zero_qh, zero_s, zero_v, ones(1,K), zero_y, zero_qe,
Acst13 = [zero_qc, zero_qo, zero_qh, zero_s, zero_v, ones(1,K), zero_y, zero_qe,
    0];
    0];
%% CST 14 - At most one discount is applied - p,i
%% CST 14 - At most one discount is applied - p,i
%qc
%qc
qccst=repmat(zero_qc , P*I, 1);
qccst=repmat(zero_qc , P*I, 1);
%qo
%qo
qocst=repmat(zero_qo, P*I, 1);
qocst=repmat(zero_qo, P*I, 1);
%qh
%qh
qhcst=repmat(zero_qh, P*I, 1);
qhcst=repmat(zero_qh, P*I, 1);
%s
%s
scst=repmat(zero_s, P*I, 1);
scst=repmat(zero_s, P*I, 1);
%v
%v
vcst=repmat(zero_v, P*I, 1);
vcst=repmat(zero_v, P*I, 1);
%w
%w
wcst=repmat(zero_w, P*I, 1);
wcst=repmat(zero_w, P*I, 1);
%y
%y
ycst=repmat ( eye (P*I),1,B);
ycst=repmat ( eye (P*I),1,B);
%qe
%qe
qecst=repmat(zero_qe, P*I,1);
qecst=repmat(zero_qe, P*I,1);
%theta
%theta
thetacst = zeros( P*I, 1);
thetacst = zeros( P*I, 1);
bcst14 = ones(P*I, 1);
bcst14 = ones(P*I, 1);
Acst14 = [qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
Acst14 = [qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
%% CST 15 - Emergency stock supply constraint
%% CST 15 - Emergency stock supply constraint
%qc
%qc
qccst=repmat(zero_qc , P, 1);
qccst=repmat(zero_qc , P, 1);
%qo
%qo
qocst=repmat(zero_qo, P, 1);
qocst=repmat(zero_qo, P, 1);
%qh
%qh
qhcst=repmat(zero_qh , P, 1);
qhcst=repmat(zero_qh , P, 1);
%s
%s
scst=repmat(zero_s, P, 1);
scst=repmat(zero_s, P, 1);
%v
%v
vcst=repmat(zero_v, P, 1);
vcst=repmat(zero_v, P, 1);
%w
%w
wcst=repmat(zero_w, P, 1);
wcst=repmat(zero_w, P, 1);
%y
%y
ycst=repmat(zero-y, P, 1);
ycst=repmat(zero-y, P, 1);
%qe
```

%qe

```
```

    qecst= repmat(eye(P), 1, T);
    %theta
    thetacst = zeros( P , 1);
    bcst15 = Q5_p';
    Acst15 = [qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst];
    %% ***Epsilon FOR LOOP***
    for iter_eps = 1:count_eps
        %% Preparing First Model
        Acst_eps = serviceOF_pts;
        bcst_eps = epsilon*ones (P*T,1);
        %% Solving First Model - Cost Main OF
        Acst = [Acst3;Acst4;Acst5;Acst6;Acst7;Acst8;Acst11;Acst14;Acst15; Acst_eps];
        % A matrix non-equalities
    bcst = [bcst 3;bcst4;bcst5;bcst6;bcst7;bcst8;bcst11;bcst14;bcst15; bcst_eps];
            % b matrix non-equalities
        Acst_eq = [Acst9;Acst10;Acst12; Acst13]; % A matrix equalities
        bcst_eq = [bcst9;bcst10;bcst12;bcst13]; % b matrix equalities
        intcon = [loc_v +1:loc_y]; %setting D.V.s to integers
        LB = zeros(1, loc_theta); %LB is zeros for all variables
        UB=[inf(1, loc_v ), ones(1, loc_y -loc_v), inf(1, loc_theta-loc_y)]; %UB is ones
            for w,y,o, B.V.s, inf for remaining
        [sol1,val1] = intlinprog(costOF,intcon, Acst,bcst, Acst_eq, bcst_eq, LB,UB);
        %% Preparing Second Model
        fprintf('Part 2');
        x0(iter_eps, :)=sol1 ';
        x0(iter_eps, loc_theta)=epsilon;
        epsilon_two = val1;
        Acst_eps = costOF;
        bcst_eps = epsilon_two;
        Acst_theta = serviceOF_pts + [zeros(P*T, loc_qe), -ones(P*T, 1)];
        bcst_theta}=z\operatorname{zeros}(P*T,1)
        %% Solving Second Model - Shortage Main OF
        Acst = [Acst_theta; Acst3;Acst4;Acst5;Acst6;Acst7;Acst8;Acst11;Acst14;Acst15;
                Acst_eps]; % A matrix non-equalities
        bcst = [bcst_theta; bcst 3 ; bcst4;bcst5 ; bcst6;bcst7;bcst8;bcst11;bcst14;bcst15;
                bcst_eps]; % b matrix non-equalities
        Acst_eq = [Acst9;Acst10;Acst12;Acst13]; % A matrix equalities
        bcst_eq = [bcst9;bcst10;bcst12;bcst13]; % b matrix equalities
        intcon = [loc_v +1:loc_y]; %setting D.V.s to integers
        LB = zeros(1, loc_theta); %LB is zeros for all variables
        UB=[inf(1, loc_v), ones(1, loc_y-loc_v), inf(1, loc_theta-loc_y)]; %UB is ones
                for w,y,o, B.V.s, inf for remaining
        [sol2,val2] = intlinprog(OF_theta, intcon, Acst,bcst, Acst_eq, bcst_eq,LB,UB, x0(
            iter_eps,:));
        %% Increment Epsilon
        epsilon=epsilon+step;
    end
    end
% Output Results to Excel

```

\section*{Appendix B}

\section*{Matlab Code for Two-Stage RO Model with SAA}
```

%% File Names and Output Selection
clear
SheetName='Sheet1';
DataFileName = 'DataFile_twoStage.xlsx';
ResultsFileName = 'Results.xlsx';
%% Index Initialization
sets=readmatrix(DataFileName, 'Sheet', 'Sets');
P}=\mathrm{ sets (1, 2); %number of products
I=sets (2, 2); %number of suppliers
K=3; %number of warehouse capacities available
B=sets (3, 2); %number of price breaks offered by suppliers, 1 = no price breaks
T=sets (4, 2); %number of time periods;
S=sets (5, 2); %number of scenarios
alpha = 1; %level of risk in cost via SAA constraint
%% Sensitivity Analysis Variable
var = readmatrix(DataFileName, 'Sheet', 'Q5_data');
svtyVariable = [0.15*var; 0.2*var; 0.25*var; 0.3*var; 0.35*var; 0.40*var];
count_svty = size(svtyVariable, 1);
%% Model Hyperparameters Initialization
epsilon_start = 0.20; %service level
step = -0.01;
count_eps = floor(abs(epsilon_start/step)) +1;
rho = 0.8;
qc_vars=[];
qo_vars=[];
w_vars=[];
inv_by_s=[];
qc_by_i=[];
qo_by_i=[];
OMcap_last_ep = [];
cnt_correl = 1;
%% Model Parameter Initialization
Ac_data=readmatrix(DataFileName, 'Sheet', 'Ac_data'); %Portion of supplier i's
contract that is actually received in t
Ac_its=[];
for ct = 1:T
for ci=1:I
Ac_its = [Ac_its, Ac_data(:,(ci-1)*T+ct)];
end
end
Ac_its=reshape(Ac_its ', 1, I*T*S);
Ao_data=readmatrix(DataFileName, 'Sheet', 'Ao_data'); %Portion of supplier i's open
market capacity available in t
Ao_its=[];
for ct = 1:T
for ci = 1:I
Ao_its = [Ao_its, Ao_data(:,(ci - 1)*T+ct)];
end
end
Ao_its=reshape(Ao_its ', 1, I*T*S);

```
D_data=readmatrix (DataFileName, 'Sheet', 'D_data') ; \% open market purchase price for
        product \(p\), supplier i, time period \(t\), scenario \(s\)
D_pts \(=[]\);
for \(\mathrm{ct}=1: \mathrm{T}\)
        for \(\mathrm{cp}=1: \mathrm{P}\)
        D_pts \(=[\) D_pts, \(\quad\) _data \((:,(c p-1) * T+c t)] ;\)
        end
end
\(\mathrm{D}_{-} \mathrm{pts}=\) reshape \(\left(\mathrm{D}_{-} \mathrm{pts}{ }^{\prime}, \quad 1, \mathrm{P} * \mathrm{~T} * \mathrm{~S}\right) ;\)
po_data=readmatrix (DataFileName, 'Sheet', 'po_data'); \% open market purchase price
        for product p, supplier i, time period t, scenario s
po_pits \(=[]\);
for \(\mathrm{ct}=1: \mathrm{T}\)
        for \(\mathrm{ci}=1: \mathrm{I}\)
            for \(\mathrm{cp}=1: \mathrm{P}\)
                po_pits \(=[\) po_pits, po_data \((:,(\mathrm{cp}-1) * \mathrm{~T}+\mathrm{ct}+(\mathrm{ci}-1) * \mathrm{P} * \mathrm{~T})] ;\)
            end
        end
end
po_pits=reshape (po_pits ', \(1, ~ P * I * T * S) ;\)
pc_pi \(=\) readmatrix (DataFileName, 'Sheet', 'pc_data'); \% base contract price for
        product p, supplier i
pe_p \(=\) readmatrix (DataFileName, 'Sheet', 'pe_data'); \%ES purchase price
C1_pi \(=\) readmatrix (DataFileName, 'Sheet', 'C1_data') ; \% cost to ship product p from
    supplier to WH
C 2 _p \(=1.2 * \mathrm{C} 1\) _pi \((1: \mathrm{P}) ; \%\) cost to ship product prom emergency stock to WH
C3_p \(=\) readmatrix (DataFileName, 'Sheet', 'C3_data') ; \% cost to ship product p
    from_by_sc WH to hospital
\(\mathrm{C} 4 \_\mathrm{k}=[0,10000 *(1: \mathrm{K}-1)] ;\) \%cost to have capacity k at warehouse \(j\)
C5_p \(=\) readmatrix (DataFileName, 'Sheet', 'C5_data'); \%holding cost per product p at
        warehouse j
C6 \(=1000 ; \%\) cost to establish supplier relationship
F1_pi = readmatrix (DataFileName, 'Sheet', 'F1_data'); \%Reliability of supplier i (
    portion of product that passes QC and is usable)
    F2_pid \(=\) readmatrix (DataFileName, 'Sheet', 'F2_data') ; \%Fraction of normal price
        charged by supplier \(i\) for product \(p\) with discount d
    F 2 _pid \(=[\) ones \((1, \mathrm{P} * \mathrm{I})\), reshape ( F 2 _pid ', \(1, \mathrm{P} * \mathrm{I} *(\mathrm{~B}-1))]\);
    Q1_ib_1 = readmatrix (DataFileName, 'Sheet', 'Q1_data') ; \% Quantity of any product
        where supplier i offers discount \(d-h a s\) dimensions \(I *(B+1)\)
    Q1_ib_1 \(=\left[\operatorname{zeros}(1, I)\right.\), Q1_ib_1, \(\left.10^{\wedge} 9 * \operatorname{ones}(1, I)\right] ;\)
    Q2_pi \(=\) readmatrix (DataFileName, 'Sheet', 'Q2_data') ; \%contract max
    Q3_pi \(=\) readmatrix (DataFileName, 'Sheet', 'Q3_data') ; \%contract min
    Q4_pi \(=\) readmatrix (DataFileName, 'Sheet', 'Q4_data'); \%Nominal capacity of supplier i
        in units of product \(p\)
    Q5_p \(=\) readmatrix (DataFileName, 'Sheet', 'Q5_data') ; \%Supply of product p in
        emergency stockpile
    \(\mathrm{K} 1 \_\mathrm{k}=16000+4000 *(0: K-1) ; \%\) warehouse inventory capacities in square feet
K2_p \(=\) readmatrix (DataFileName, 'Sheet', 'K2_data'); \%square feet required to store
        one unit of product \(p\)
    V0_p \(=\) readmatrix (DataFileName, 'Sheet', 'V0_data'); \%starting inventory
\(\mathrm{M}=10^{\wedge} 9\); \%very large number
TS_data \(=\) readmatrix (DataFileName, 'Sheet', 'TS_data');
\(\% \%\) Decision Variables
\%qc: number product p purchased from_by_sc supplier i to warehouses in time period t
zero_qc \(=z e r o s(1, P * I * B) ;\)
\%qo: number product p purchased from_by_sc backup supplier i to warehouses in time
    period \(t\)
zero_qo \(=\) zeros \((1, \mathrm{P} * \mathrm{I} * \mathrm{~T} * \mathrm{~S}) ;\)
\%qh: number product p shipped from_by_sc warehouses to hospitals in time period t
zero_qh \(=z \operatorname{zeros}(1, P * T * S) ;\)
\(\%\) : number of shorted units of product \(p\) at hospitals in time period \(t\)
zero_s \(=\operatorname{zeros}(1, \mathrm{P} * \mathrm{~T} * \mathrm{~S})\);
```

%v: inventory level of product p at warehouses in time period t
zero_v = zeros(1, P*(T+1)*S);
%w:1 if warehouse j inventory capacity is size k and 0 otherwise
zero_w = zeros(1, K);
%y:1 if supplier i is selected as a primary supplier of product p and 0 otherwise
zero-y = zeros(1,P*I*B);
%pe: number of units of product p sent from_by_sc emergency stock to warehouses in t
and s
zero_qe = zeros(1, P*T*S);
%z is an auxiliary variable to enforce SAA constraint
zero_z = zeros(1, S);
%% Locations of end of DVs
loc_qc=P*I *B;
loc_qo=P*I *B+P*I*T*S;
loc_qh =P*I *B}+\textrm{P}*\textrm{I}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}
loc_s=P}* I * B+P * I *T *S+P*T*S+P*T*S
loc_v}=\textrm{P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{I}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*(\textrm{T}+1)*\textrm{S}
loc}_\textrm{w}=\textrm{P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{I}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*(\textrm{T}+1)*\textrm{S}+\textrm{K}
loc_y=P
loc_qe=P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{I}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*(\textrm{T}+1)*\textrm{S}+\textrm{K}+\textrm{P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{T}*\textrm{S}

```

```

loc_theta=P
%theta is an auxiliary variable to determine the max. demand shorted of any product
at any hospital in any time period
%% Objective Function - Theta
OF_theta = [zeros(1, loc_z), 1];
%% Service Objective Function
serviceOF_pts = [repmat([zero_qc, zero_qo, zero_qh], P*T*S,1), eye(P*T*S) , repmat([
zero_v, zero_w, zero_y, zero_qe, zero_z, 0], P*T*S,1)];
%%***Sensitivity Analysis Loop***
for iter_svty = 1:count_svty
%Sensitivity Analysis Variable
Q5_p = svtyVariable(iter_svty, :);
epsilon = epsilon_start;
%% CST 1 - Cost OF
%qc
qc_coeff=[];
for cntOF=1:S
qc_coeff=[qc_coeff; Ac_its ((cntOF-1)*I*T+1:cntOF*I*T) * repmat (eye(I), 1,T)'];
end
qc_coeff=repmat(kron(qc_coeff, ones(1,P)), 1,B);
qccst = qc_coeff.*repmat ([(repmat (pc_pi,1,B).*F2_pid) + repmat (C1_pi,1,B)], S, 1)
;
%qo
qocst = [po_pits + repmat(C1_pi,1,T*S)].* kron(eye(S), ones(1,P*I*T));
%qh
qhcst = kron(eye(S), repmat(C3_p , 1 , T));
%s
scst=repmat(zero_s , S , 1);
%v
vcst = kron(eye(S), repmat(C5_p , 1, T+1));
%w
wcst = repmat (C4_k, S, 1);
%y
ycst = C6*ones(S,P*I*B);
%z
qecst = kron(eye(S), repmat([C2_p + pe_p], 1, T));

```
```

%z
zcst = repmat(zero_z, S,1);
%theta
thetacst=zeros(S, 1);
costOF_s=[qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
%% CST 2 - SAA constraint - singular
bcst_SAA = alpha }*\mathrm{ S;
Acst_SAA =[zeros (1, loc_qe), ones(1, S), 0];
%% CST 3 - Open market supply supply CST - p,i,t,s
%qc
qccst=repmat(zero_qc, P*I*T*S, 1);
%qo
qocst=eye(P*I*T*S);
%qh
qhcst=repmat(zero_qh , P*I*T*S, 1);
%s
scst=repmat(zero_s, P*I*T*S, 1);
%v
vcst=repmat(zero_v, P*I*T*S, 1);
%w
wcst=repmat(zero_w, P*I*T*S, 1);
%y
ycst=repmat(zero_y, P*I*T*S, 1);
%z
qecst=repmat(zero_qe, P*I*T*S,1);
%z
zcst=repmat(zero_z, P*I*T*S, 1);
%theta
thetacst=zeros(P*I*T*S, 1);
Acst3=[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
bcst 3 = [kron(Ao_its, ones(1,P)).*repmat(Q4_pi,1,T*S)]';
%% CST 4 - Contract maximum CSTs - p,i,d
%qc
qccst=eye(P*I*B);
%qo
qocst=repmat(zero_qo, P*I *B, 1);
%qh
qhcst=repmat(zero_qh, P*I*B, 1);
%s
scst=repmat(zero_s, P*I*B, 1);
%v
vcst=repmat(zero_v, P*I*B, 1);
%w
wcst=repmat(zero_w, P*I*B, 1);
%y
ycst=-repmat(Q2_pi,1,B).*eye(P*I*B);
%z
qecst=repmat(zero_qe, P
%z
zcst=repmat(zero_z, P*I*B, 1);
%theta
thetacst=zeros(P*I*B, 1);
Acst4=[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
bcst4=zeros(P*I*B, 1);
%% CST 5 - Contract minimum CSTs - p,i,d
%qc
qccst=eye(P*I*B);
%qo
qocst=repmat(zero_qo, P*I*B, 1);
%qh
qhcst=repmat(zero_qh, P*I*B, 1);

```
```

244
245
246
247
248
249
249
250
251
251
253
254
256
256
257
258
259
259 260 261 262 263 264 265 266 267 268
269

```
```

271
272

```
%s
```

%s
scst=repmat(zero_s, P*I*B, 1);
scst=repmat(zero_s, P*I*B, 1);
%v
%v
vcst=repmat(zero_v, P*I*B, 1);
vcst=repmat(zero_v, P*I*B, 1);
%w
%w
wcst=repmat(zero_w, P*I*B, 1);
wcst=repmat(zero_w, P*I*B, 1);
%y
%y
ycst=-repmat(Q3_pi,1,B).*eye(P*I*B);
ycst=-repmat(Q3_pi,1,B).*eye(P*I*B);
%z
%z
qecst=repmat(zero_qe, P
qecst=repmat(zero_qe, P
%z
%z
zcst=repmat(zero_z, P*I*B, 1);
zcst=repmat(zero_z, P*I*B, 1);
%theta
%theta
thetacst=zeros(P*I*B, 1);
thetacst=zeros(P*I*B, 1);
Acst5=-[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
Acst5=-[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
bcst5=-zeros(P*I*B, 1);
bcst5=-zeros(P*I*B, 1);
%% CST 6 - Contract order quantity exceeds price break min - p,i,d
%% CST 6 - Contract order quantity exceeds price break min - p,i,d
%qc
%qc
qccst=eye(P*I*B);
qccst=eye(P*I*B);
%qo
%qo
qocst=repmat(zero_qo , P*I*B , 1);
qocst=repmat(zero_qo , P*I*B , 1);
%qh
%qh
qhcst=repmat(zero_qh , P*I*B , 1);
qhcst=repmat(zero_qh , P*I*B , 1);
%s
%s
scst=repmat(zero_s , P*I*B, 1);
scst=repmat(zero_s , P*I*B, 1);
%v
%v
vcst=repmat(zero_v , P*I*B , 1);
vcst=repmat(zero_v , P*I*B , 1);
%w
%w
wcst=repmat(zero_w , P*I*B , 1);
wcst=repmat(zero_w , P*I*B , 1);
%y
%y
ycst=-kron(Q1_ib_1(1:I*B), ones (1,P)).* eye(P*I*B);
ycst=-kron(Q1_ib_1(1:I*B), ones (1,P)).* eye(P*I*B);
%z
%z
qecst=repmat(zero_qe, P*I*B,1);
qecst=repmat(zero_qe, P*I*B,1);
%z
%z
zcst=repmat(zero_z, P*I*B, 1);
zcst=repmat(zero_z, P*I*B, 1);
%theta
%theta
thetacst=zeros(P*I*B, 1);
thetacst=zeros(P*I*B, 1);
bcst6 = -zeros(P*I*B, 1);
bcst6 = -zeros(P*I*B, 1);
Acst6 = -[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
Acst6 = -[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
%% CST 7 - Contract order quantity is less than next price break - p,i,d
%% CST 7 - Contract order quantity is less than next price break - p,i,d
%qc
%qc
qccst=eye(P*I*B) ;
qccst=eye(P*I*B) ;
%qo
%qo
qocst=repmat(zero_qo , P*I *B, 1);
qocst=repmat(zero_qo , P*I *B, 1);
%qh
%qh
qhcst=repmat(zero_qh , P*I*B , 1);
qhcst=repmat(zero_qh , P*I*B , 1);
%s
%s
scst=repmat(zero_s , P*I*B, 1);
scst=repmat(zero_s , P*I*B, 1);
%v
%v
vcst=repmat(zero_v , P*I*B, 1);
vcst=repmat(zero_v , P*I*B, 1);
%w
%w
wcst=repmat(zero_w , P*I*B, 1);
wcst=repmat(zero_w , P*I*B, 1);
%y
%y
ycst=-kron(Q1_ib_1(I+1:I*(B+1)), ones (1,P)).* eye(P*I*B);
ycst=-kron(Q1_ib_1(I+1:I*(B+1)), ones (1,P)).* eye(P*I*B);
%z
%z
qecst=repmat(zero_qe, P
qecst=repmat(zero_qe, P
%z
%z
zcst=repmat(zero_z, P*I*B, 1);
zcst=repmat(zero_z, P*I*B, 1);
%theta
%theta
thetacst=zeros(P*I*B, 1);
thetacst=zeros(P*I*B, 1);
bcst7 = zeros(P*I*B, 1);
bcst7 = zeros(P*I*B, 1);
Acst7 = [qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];

```
Acst7 = [qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
```

```
%% CST 8 - WH inventory capacity CST's - t+1,s
%qc
qccst=repmat(zero_qc , (T+1)*S, 1);
%qo
qocst=repmat(zero_qo , (T+1)*S, 1);
%qh
qhcst=repmat(zero_qh , (T+1)*S, 1);
%s
scst=repmat(zero_s, (T+1)*S, 1);
%v
vcst=kron(eye(S),[repmat(K2_p, 1, (T+1)).*kron(eye(T+1), ones (1,P))]);
%w
wcst=-repmat(K1_k, (T+1)*S, 1);
%y
ycst=repmat(zero_y, (T+1)*S, 1);
%z
qecst=repmat(zero_qe, (T+1)*S,1);
%z
zcst=repmat(zero_z, (T+1)*S, 1);
%theta
thetacst=zeros((T+1)*S, 1);
Acst8=[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
bcst8=zeros ((T+1)*S, 1);
%% CST 9 - WH inventory / flow balance CST - p,t,s
%qc
qccst=repmat(kron(reshape(Ac_its,I ,T*S)',ones (P,P)), 1, B).*(repmat(F1_pi, 1, B)
    .* repmat ( eye(P),T*S,I *B));
%qo
qocst=repmat(F1_pi, 1, T*S).* kron(eye(T*S), repmat(eye(P),1,I));
%qh
qhcst=-eye(P*T*S);
%s
scst=repmat(zero_s, P*T*S, 1);
%v
vcst=kron(eye(S), [[ eye(P*T), zeros(P*T, P)]-[zeros(P*T, P), eye(P*T)]]);
%w
wcst=repmat(zero_w, P*T*S, 1);
%y
ycst=repmat(zero_y, P*T*S, 1);
%z
qecst = eye(P*T*S);
%z
zcst=repmat(zero_z, P*T*S, 1);
%theta
thetacst=zeros(P*T*S, 1);
Acst9=[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
bcst9=zeros(P*T*S,1);
%% CST 10 - Inventory period 1 equality constraints across scenarios - p,(s-1)
%qc
qccst=repmat(zero_qc , P*S , 1);
%qo
qocst=repmat(zero_qo , P*S , 1);
%qh
qhcst=repmat(zero_qh , P*S , 1);
%s
scst=repmat(zero_s , P*S , 1);
%v
vcst= kron(eye(S), [eye(P), zeros(P, P*T)]);
%w
wcst=repmat(zero_w , P*S , 1);
%y
ycst=repmat(zero_y , P*S , 1);
%z
qecst=repmat(zero_qe, P*S, 1);
```

```
%z
    zcst=repmat(zero_z, P*S, 1);
    %theta
    thetacst=zeros( P*S ,1);
    bcst10 = repmat(V0_p, 1, S)';
    Acst10 = [qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
    %% CST 11 - WH inventory in period 1 and T+1 are equal - p,s
    %qc
    qccst=repmat(zero_qc , P*S, 1);
    %qo
    qocst=repmat(zero_qo , P*S, 1);
    %qh
    qhcst=repmat(zero_qh , P*S , 1);
    %s
    scst=repmat(zero_s , P*S, 1);
    %v
    vcst= kron(eye(S), [zeros(P, P*T), eye(P)]);
    %w
    wcst=repmat(zero_w, P*S, 1);
    %y
    ycst=repmat(zero_y , P*S , 1);
    %z
    qecst=repmat(zero_qe, P*S, 1);
    %z
    zcst=repmat(zero_z, P*S, 1);
    %theta
    thetacst=zeros( P*S ,1);
    bcst11 = -repmat(V0_p, 1, S)';
    Acst11 = - [qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
    %% CST 12 - Hospital demand satisfaction constraints - p,t,s
    %qc
    qccst=repmat(zero_qc , P*T*S, 1);
    %qo
    qocst=repmat(zero_qo , P*T*S, 1);
    %qh
    qhcst=eye(P*T*S) ;
    %s
    scst=eye(P*T*S).*D_pts;
    %v
    vcst=repmat(zero_v , P*T*S , 1);
    %w
    wcst=repmat(zero_w , P*T*S , 1);
    %y
    ycst=repmat(zero_y , P*T*S , 1);
    %z
    qecst=repmat(zero_qe , P*T*S , 1);
    %z
    zcst=repmat(zero_z, P*T*S, 1);
    %theta
    thetacst=zeros(P*T*S, 1);
    Acst12 = [qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
    bcst12 = D_pts';
    %% CST 13 - Singular WH capacity is selected - singular
    bcst13 = 1;
    Acst13 =[zero_qc, zero_qo, zero_qh, zero_s, zero_v, ones(1,K), zero_y, zero_qe,
    zero-z, 0];
    %% CST 14 - At most one discount is applied - p,i
    %qc
    qccst=repmat(zero_qc , P*I, 1);
    %qo
    qocst=repmat(zero_qo, P*I, 1);
```

```
%qh
qhcst=repmat(zero_qh, P*I, 1);
%s
scst=repmat(zero_s, P*I, 1);
%v
vcst=repmat(zero_v, P*I, 1);
%w
wcst=repmat(zero_w, P*I, 1);
%y
ycst=repmat(eye(P*I),1,B);
%z
qecst=repmat(zero_qe, P*I,1);
%z
zcst=repmat(zero_z, P*I, 1);
%theta
thetacst=zeros(P*I, 1);
bcst14 = ones (P*I, 1);
Acst14 = [qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
%% CST 15 - Emergency stock supply constraint
%qc
qccst=repmat(zero_qc , P*S, 1);
%qo
qocst=repmat(zero_qo, P*S, 1);
%qh
qhcst=repmat(zero_qh , P*S, 1);
%s
scst=repmat(zero_s, P*S, 1);
%v
vcst=repmat(zero_v, P*S, 1);
%w
wcst=repmat(zero_w, P*S, 1);
%y
ycst=repmat(zero_y, P*S, 1);
%z
qecst= kron(eye(S), repmat(eye(P), 1, T));
%z
zcst=repmat(zero_z, P*S, 1);
%theta
thetacst=zeros(P*S, 1);
bcst15 = repmat(Q5_p, 1, S)';
Acst15 = [qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, zcst, thetacst];
%%***Epsilon FOR LOOP***
for iter_eps = 1:count_eps
    %% Preparing First Model
        Acst_eps = serviceOF_pts;
        bcst_eps = epsilon*ones(P*T*S,1);
        Acst_theta = costOF_s + [zeros(S,loc_qe), -M*eye(S), -ones(S,1)];
        bcst_theta= zeros(S,1);
        %% Solving First-Cost Model
        Acst = [Acst_theta; Acst_SAA; Acst3;Acst4;Acst5;Acst6;Acst7;Acst8;Acst11;
            Acst14;Acst15; Acst_eps]; % A matrix non-equalities
        bcst = [bcst_theta; bcst_SAA; bcst3;bcst4;bcst5;bcst6;bcst7;bcst8;bcst11;
            bcst14;bcst15; bcst_eps]; % b matrix non-equalities
        Acst_eq = [Acst9;Acst10;Acst12;Acst13]; % A matrix equalities
        bcst_eq = [bcst9;bcst10;bcst12;bcst13]; % b matrix equalities
        intcon = [loc_v +1:loc_y, loc_qe+1:loc_z]; %setting D.V.s to integers
        LB}=\mathrm{ zeros(1, loc_theta); %LB is zeros for all variables
        UB = [inf(1, loc_v), ones(1, loc_y-loc_v), inf(1, loc_qe-loc_y), ones(1, loc_z-
            loc_qe), inf]; %UB is ones for w,y,o, B.V.s, inf for remaining
        [sol1,val1] = intlinprog(OF_theta,intcon, Acst,bcst, Acst_eq, bcst_eq, LB,UB);
        %% Preparing Second Model
```

```
            fprintf('Part 2');
            x0(iter_eps, :)=sol1';
            x0(iter_eps, loc_theta)=epsilon;
            epsilon_two = val1;
            Acst_eps = costOF_s + [zeros(S,loc_qe), -M*eye(S), zeros(S,1)];
            bcst_eps = epsilon_two*ones(S,1);
            Acst_theta = serviceOF_pts + [zeros (P*T*S, loc_z ), -ones (P*T*S,1) ];
            bcst_theta = zeros (P*T*S,1);
            %% Solving Second-Shortage Model
            Acst = [Acst_theta; Acst_SAA; Acst3;Acst4;Acst5;Acst6;Acst7;Acst8;Acst11;
                Acst14;Acst15; Acst_eps]; % A matrix non-equalities
            bcst = [bcst_theta; bcst_SAA; bcst3;bcst4;bcst5;bcst6;bcst7;bcst8;bcst11;
                bcst14;bcst15; bcst_eps]; % b matrix non-equalities
            Acst_eq = [Acst9;Acst10;Acst12;Acst13]; % A matrix equalities
            bcst_eq = [bcst9;bcst10;bcst12;bcst13]; % b matrix equalities
            intcon = [loc_v + 1: loc_y, loc_qe+1: loc_z]; %setting D.V.s to integers
            LB = zeros(1, loc_theta); %LB is zeros for all variables
            UB=[inf(1, loc_v ), ones(1, loc_y -loc_v), inf(1, loc_qe-loc_y), ones(1, loc_z-
                loc_qe), inf]; %UB is ones for w,y,o, B.V.s, inf for remaining
            [sol2,val2] = intlinprog(OF_theta,intcon, Acst,bcst, Acst_eq, bcst_eq,LB,UB, x0(
                iter_eps, :));
            %% Increment Epsilon
            epsilon=epsilon+step;
        end
end
% Output Results to Excel
```


## Appendix C

## Matlab Code for Multi-Stage RO Model with SAA

```
%% RO Two-Stage Model Code Lines (1) - (489)
%% CST 16 - Multi-stage equality cst - qe - P*(S*(T-1)-3-9-27)
PTS = [];
PTS_v = [];
PITS = [];
for ii = 1:T-1
    %S
    %kron(eye(3^ii), [ones(3^(4-ii)-1, 1), -eye(3^(4-ii)-1)]);
    %TS
    %kron(kron(eye(3^ii), [ones(3^(4-ii) - 1, 1), -eye(3^(4-ii)-1)]), [zeros(1,ii
                -1), 1, zeros(1,T-ii)]);
    %PTS
    PTS = [PTS; kron(kron(kron(eye(3^ii), [ones(3^(4-ii) - 1, 1), - eye(3^(4-ii) - 1)
        ]), [zeros(1,ii-1), 1, zeros(1,T-ii)]), eye(P))];
    %PTS_v
    PTS_v = [PTS_v; kron(kron(kron(eye(3^ii), [ones(3^(4-ii)-1, 1), -eye(3^(4-ii)
        -1)]), [zeros(1,ii), 1, zeros(1,T-ii)]), eye(P))];
    %PITS
    PITS = [PITS; kron(kron(kron(eye(3^ii), [ones(3^(4-ii) - 1, 1), -eye(3^(4-ii )
        -1)]), [zeros(1,ii-1), 1, zeros(1,T-ii)]), eye(P*I))];
end
bcst16 = zeros(P*(S*(T-1)-3-9-27), 1);
Acst16 = [repmat([zero_qc, zero_qo, zero_qh, zero_s, zero_v, zero_w, zero_y], P*(
    S*(T-1)-3-9-27), 1), PTS, repmat([zero_z, 0], P*(S*(T-1)-3-9-27), 1)];
%% CST 17 - Multi-stage equality cst - qh - P*(S*(T-1)-3-9-27)
bcst17 = zeros(P*(S*(T-1)-3-9-27), 1);
Acst17 = [repmat([zero_qc, zero_qo], P*(S*(T-1)-3-9-27), 1), PTS, repmat([zero_s,
    zero_v, zero_w, zero_y, zero_qe, zero_z, 0], P*(S*(T-1)-3-9-27), 1)];
%% CST 18 - Multi-stage equality cst - s - P*(S*(T-1)-3-9-27)
bcst18 = zeros(P*(S*(T-1)-3-9-27), 1);
Acst18 = [repmat([zero_qc, zero_qo, zero_qh], P*(S*(T-1)-3-9-27), 1), PTS, repmat
    ([zero-v, zero_w, zero-y, zero-qe, zero_z, 0], P*(S*(T-1)-3-9-27), 1)];
%% CST 19 - Multi-stage equality cst - v - P*(S*(T-1)-3-9-27)
bcst19 = zeros(P*(S*(T-1)-3-9-27), 1);
Acst19 = [repmat([zero_qc, zero_qo, zero_qh, zero_s], P*(S*(T-1)-3-9-27), 1),
    PTS_v, repmat([zero-w, zero-y, zero_qe, zero_z, 0], P*(S*(T-1)-3-9-27), 1)];
%% CST 20 - Multi-stage equality cst - qo - P*I*(S*(T-1)-3-9-27)
bcst20 = zeros(P*I*(S*(T-1)-3-9-27), 1);
Acst20 = [repmat([zero_qc], P*I*(S*(T-1)-3-9-27), 1), PITS, repmat([zero_qh,
    zero_s, zero_v, zero_w, zero_y, zero_qe, zero_z, 0], P*I*(S*(T-1)-3-9-27), 1)
    ];
%% ***Epsilon FOR LOOP***
for iter_eps = 1:count_eps
    %% Preparing First Model
    Acst_eps = serviceOF_pts;
    bcst_eps = epsilon*ones (P*T*S,1);
    Acst_theta = costOF_s + [zeros(S,loc_qe), -M*eye(S), -ones(S,1)];
    bcst_theta = zeros(S,1);
    %% Solving First-Cost Model
```


## Appendix D

## Matlab Code for Two-Stage DRO Model

## Stochastic-robust models are created by running the DRO models with $\rho=0$.

```
%% File Names and Output Selection
clear
SheetName='Sheet1';
DataFileName = 'DataFile_twoStage.xlsx';
ResultsFileName = 'Results.xlsx';
%% Index Initialization
sets=readmatrix(DataFileName, 'Sheet', 'Sets');
P=sets(1, 2); %number of products
I=sets (2, 2); %number of suppliers
K=3; %number of warehouse capacities available
B=sets(3, 2); %number of price breaks offered by suppliers, 1 = no price breaks
T=sets (4, 2); %number of time periods;
S=sets (5, 2); %number of scenarios
%% Sensitivity Analysis Variable
var = readmatrix(DataFileName, 'Sheet', 'Q5_data');
svtyVariable = [0.15*var; 0.2*var; 0.25*var; 0.3*var; 0.35*var; 0.40*var];
count_svty = size(svtyVariable, 1);
%% Model Hyperparameters Initialization
epsilon_start = 0.20; %service level
step = -0.01;
count_eps = floor(abs(epsilon_start/step)) +1;
rho = 0.8;
qc_vars=[];
qo_vars=[];
w_vars=[];
inv_by_s=[];
qc_by_i = [];
qo_by_i=[];
OMcap_last_ep = [];
cnt_correl = 1;
%% Model Parameter Initialization
Ac_data=readmatrix(DataFileName, 'Sheet', 'Ac_data'); %Portion of supplier i's
    contract that is actually received in t
Ac_its=[];
for ct = 1:T
    for ci = 1:I
        Ac_its = [Ac_its, Ac_data(:,(ci-1)*T+ct)];
    end
end
Ac_its=reshape(Ac_its ', 1, I*T*S);
Ao_data=readmatrix(DataFileName, 'Sheet', 'Ao_data'); %Portion of supplier i's open
    market capacity available in t
Ao_its=[];
for ct = 1:T
    for ci = 1:I
        Ao_its = [Ao_its, Ao_data(:,(ci - 1)*T+ct)];
    end
end
Ao_its=reshape(Ao_its ', 1, I*T*S);
D_data=readmatrix(DataFileName, 'Sheet', 'D_data'); % open market purchase price for
    product p, supplier i, time period t, scenario s
D_pts=[];
```

```
for ct = 1:T
    for cp = 1:P
        D_pts = [D_pts, D_data (:, (cp-1)*T+ct)];
    end
end
D_pts=reshape(D_pts', 1, P*T*S);
po_data=readmatrix(DataFileName, 'Sheet', 'po_data'); % open market purchase price
        for product p, supplier i, time period t, scenario s
po-pits=[];
for ct = 1:T
        for ci = 1:I
            for cp = 1:P
                po_pits = [po_pits, po_data(:,(cp-1)*T+ct+(ci-1)*P*T)];
            end
        end
end
po_pits=reshape(po_pits ', 1, P*I*T*S);
pc_pi = readmatrix(DataFileName, 'Sheet', 'pc_data'); % base contract price for
    product p, supplier i
pe_p = readmatrix(DataFileName, 'Sheet', 'pe_data'); %Emergency stock purchase price
C1_pi = readmatrix(DataFileName, 'Sheet', 'C1_data'); % cost to ship product p from
        supplier to WH
C2_p = 1.2*C1_pi(1:P); % cost to ship product p from emergency stock to WH
C3_p = readmatrix(DataFileName, 'Sheet', 'C3_data'); % cost to ship product p
        from_by_sc WH to hospital
C4_k = [0, 10000*(1:K-1)]; %cost to have capacity k at warehouse j
C5_p = readmatrix(DataFileName, 'Sheet', 'C5_data'); %holding cost per product p at
        warehouse j
C6 = 1000; %cost to maintain supplier relationship
f_hat_s = ones(1,S)/S; %Scenario probabilities
F1_pi = readmatrix(DataFileName, 'Sheet', 'F1_data'); %Reliability of supplier i (
        portion of product that passes QC and is usable)
F2_pid = readmatrix(DataFileName, 'Sheet', 'F2_data'); %Fraction of normal price
        charged by supplier i for product p with discount d
F2_pid= [ones(1,P*I), reshape(F2_pid', 1, P*I*(B-1))];
Q1_ib_1 = readmatrix(DataFileName, 'Sheet', 'Q1_data');% Quantity of any product
        where supplier i offers discount d - has dimensions I * (B+1)
Q1_ib_1 = [zeros(1,I), Q1_ib_1, 10^9*ones(1,I)];
Q2_pi = readmatrix(DataFileName, 'Sheet', 'Q2_data'); %contract max
Q3_pi = readmatrix(DataFileName, 'Sheet', 'Q3_data'); %contract min
Q4_pi = readmatrix(DataFileName, 'Sheet','Q4_data'); %Nominal capacity of supplier i
    in units of product p
Q5_p = readmatrix(DataFileName, 'Sheet', 'Q5_data'); %Supply of product p in
        emergency stockpile
K1_k = 16000 + 4000*(0:K-1); %warehouse inventory capacities in square feet
K2_p = readmatrix(DataFileName, 'Sheet', 'K2_data'); %square feet required to store
        one unit of product p
V0_p = readmatrix(DataFileName, 'Sheet', 'V0_data'); %starting inventory
M=10^9; %very large number
TS_data = readmatrix(DataFileName, 'Sheet', 'TS_data');
%% Decision Variables
%qc: number product p purchased from_by_sc supplier i to warehouses in time period t
zero_qc = zeros(1, P*I*B);
%qo: number product p purchased from_by_sc backup supplier i to warehouses in time
    period t
zero_qo = zeros(1, P*I*T*S);
%qh: number product p shipped from_by_sc warehouses to hospitals in time period t
zero_qh = zeros(1, P*T*S);
%s: number of shorted units of product p at hospitals in time period t
zero_s = zeros(1, P*T*S);
%v: inventory level of product p at warehouses in time period t
zero-v = zeros(1, P*(T+1)*S);
```

```
%w:1 if warehouse j inventory capacity is size k and 0 otherwise
zero_w = zeros(1, K);
%y:1 if supplier i is selected as a primary supplier of product p and 0 otherwise
zero-y = zeros(1,P*I*B);
%qe: number of units of product p sent from_by_sc emergency stock to warehouses in t
    and s
zero_qe = zeros(1, P*T*S);
%theta is an auxiliary variable for max shortage
zero_theta = 0;
%omega is a dual variable
zero_omega = 0;
%pi is a dual variable
zero-pi = 0;
%psi_pos is a dual variable
zero_psi_pos = zeros(1,S);
%phi_neg is a dual variable
zero_psi_neg = zeros(1,S);
%duals combined
zero_duals = [zero_omega, zero_pi, zero_psi_pos, zero_psi_neg];
%% Locations of end of DVs
loc_qc=P*I*B;
loc_qo=P*I *B+P*I *T*S;
loc_qh=P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{I}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}
loc_s}=\textrm{P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{I}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}
loc_v}=\textrm{P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{I}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*(\textrm{T}+1)*\textrm{S}
loc_w}=\textrm{P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{I}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*(\textrm{T}+1)*\textrm{S}+\textrm{K}
loc_y = P*I }*\textrm{B}+\textrm{P}*\textrm{I}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*(\textrm{T}+1)*\textrm{S}+\textrm{K}+\textrm{P}*\textrm{I}*\textrm{B}
loc_qe =P*I*B+P*I *T*S+P}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*(\textrm{T}+1)*\textrm{S}+\textrm{K}+\textrm{P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{T}*\textrm{S}
loc_theta=P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{I}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*(\textrm{T}+1)*\textrm{S}+\textrm{K}+\textrm{P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{T}*\textrm{S}+1
loc_omega=P
loc_pi=P
loc_psi_pos=P
loc_psi_neg=P*I*B+P*I*T*S+P*T}*\textrm{S}+\textrm{P}*\textrm{T}*\textrm{S}+\textrm{P}*(\textrm{T}+1)*\textrm{S}+\textrm{K}+\textrm{P}*\textrm{I}*\textrm{B}+\textrm{P}*\textrm{T}*\textrm{S}+1+1+1+\textrm{S}+\textrm{S}
%% Objective Function - Theta
OF_theta = [zeros (1, loc_theta - 1), 1, zero_duals ];
OF_dual_main = [zeros (1, loc_v ), C4_k, C6*ones(1,P*I*B), zeros(1, loc_theta-loc_y ), 1,
    rho, -f_hat_s, f_hat_s];
%% Service Objective Function
serviceOF_pts = [repmat([zero_qc, zero_qo, zero_qh], P*T*S,1), eye(P*T*S) , repmat([
    zero_v, zero_w, zero_y, zero_qe, zero_theta, zero_duals], P*T*S,1)];
%%***Sensitivity Analysis Loop***
for iter_svty = 1:count_svty
    %Sensitivity Analysis Variable
    Q5_p = svtyVariable(iter_svty, :);
    epsilon = epsilon_start;
    %% CST 1 - dual constraint 1 - S
    Acst1= - [zeros(S, loc_omega), ones(S,1), - eye(S), -eye(S)];
    bcst1= - zeros(S,1);
    %% CST 2 - dual constraint 2 - S
    %qc
        qc_coeff=[];
        for cntOF = 1:S
```

```
end
qc_coeff=repmat(kron(qc_coeff, ones (1,P)),1,B);
qccst = qc_coeff.*repmat ([(repmat (pc_pi,1,B).*F2_pid) + repmat (C1_pi,1,B)], S, 1)
        ;
%qo
qocst = [po_pits + repmat(C1_pi, 1,T*S)].* kron(eye(S), ones (1,P*I*T));
%qh
qhcst = kron(eye(S), repmat(C3_p , 1, T));
%s
scst=repmat(zero_s , S , 1);
%v
vcst = kron(eye(S), repmat(C5_p , 1, T+1));
%w
wcst = repmat(zero_w , S , 1);
%y
ycst = repmat(zero_y , S , 1);
%qe
qecst = kron(eye(S), repmat ([C2_p + pe_p], 1, T));
%theta
thetacst=zeros(S,1);
%dual
dualscst =[-ones(S,1), zeros(S,1), eye(S), -eye(S)];
Acst2=[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst, dualscst];
bcst2=zeros(S,1);
%% CST 3 - Open market supply supply CST - p,i,t,s
%qc
qccst=repmat(zero_qc, P}*\textrm{I}*\textrm{T}*\textrm{S}, 1)
%qo
qocst=eye(P*I*T*S);
%qh
qhcst=repmat(zero_qh, P*I*T*S, 1);
%s
scst=repmat(zero_s, P}*\textrm{I}*\textrm{T}*\textrm{S}, 1)
%v
vcst=repmat(zero_v, P*I*T*S, 1);
%w
wcst=repmat (zero_w, P*I*T*S, 1);
%y
ycst=repmat(zero_y, P*I*T*S, 1);
%qe
qecst=repmat(zero_qe, P*I *T*S,1);
%theta
thetacst=zeros (P*I*T*S,1);
%duals
dualscst=repmat(zero_duals, P}*\textrm{I}*\textrm{T}*\textrm{S},\quad1)
Acst3=[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst, dualscst];
bcst3=[kron(Ao_its, ones(1,P)).*repmat(Q4_pi,1,T*S)]';
%% CST 4 - Contract maximum CSTs - p,i,d
%qc
qccst=eye(P*I*B) ;
%qo
qocst=repmat(zero_qo, P}*\textrm{I}*\textrm{B},1)
%qh
qhcst=repmat(zero_qh, P*I*B, 1);
%s
scst=repmat(zero_s, P*I*B, 1);
%v
vcst=repmat(zero_v, P*I*B, 1);
%w
wcst=repmat (zero_w, P*I*B, 1);
%y
ycst=-repmat (Q2_pi,1,B).*eye (P*I*B);
%qe
```

```
qecst=repmat(zero_qe, P
%theta
thetacst=zeros( P*I*B,1);
%duals
dualscst=repmat(zero_duals, P}*\textrm{I}*\textrm{B}, 1)
Acst4=[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst, dualscst];
bcst4=zeros(P*I*B, 1);
%% CST 5 - Contract minimum CSTs - p,i,b
%qc
qccst=eye(P*I*B);
%qo
qocst=repmat(zero_qo, P*I*B, 1);
%qh
qhcst=repmat(zero_qh, P*I*B, 1);
%s
scst=repmat (zero_s, P*I*B, 1);
%v
vcst=repmat(zero_v, P*I*B, 1);
%w
wcst=repmat(zero_w, P*I *B, 1);
%y
ycst=-repmat(Q3_pi,1,B).*eye(P*I*B);
%qe
qecst=repmat(zero_qe, P*I*B,1);
%theta
thetacst=zeros( P*I*B,1);
%duals
dualscst=repmat(zero_duals ,P*I*B, 1);
Acst5=-[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst, dualscst];
bcst5=-zeros(P*I*B, 1);
%% CST 6 - Contract order quantity exceeds price break min - p, i, d
%qc
qccst=eye(P*I*B);
%qo
qocst=repmat(zero_qo , P*I*B , 1);
%qh
qhcst=repmat(zero_qh , P*I*B , 1);
%s
scst=repmat(zero_s , P*I*B, 1);
%v
vcst=repmat(zero_v , P*I*B, 1);
%w
wcst=repmat(zero_w , P*I*B , 1);
%y
ycst=-kron(Q1_ib_1(1:I*B), ones (1,P)).*eye (P*I*B);
%qe
qecst=repmat(zero_qe, P}*\textrm{I}*\textrm{B},1)
%theta
thetacst=zeros( P*I*B,1);
%duals
dualscst=repmat(zero_duals, P*I*B, 1);
bcst6 = -zeros(P*I*B, 1);
Acst6 = - [qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst, dualscst
            ];
%% CST 7 - Contract order quantity is less than next price break - p,i,d
%qc
qccst=eye(P*I*B);
%qo
qocst=repmat(zero_qo , P*I*B, 1);
%qh
qhcst=repmat(zero_qh , P*I *B , 1);
%s
```

```
scst=repmat(zero_s , P*I*B, 1);
%v
vcst=repmat(zero_v , P*I*B, 1);
%w
wcst=repmat(zero_w , P*I*B, 1);
%y
ycst=-kron(Q1_ib_1 (I+1:I*(B+1)), ones (1,P)) .* eye(P*I*B);
%qe
qecst=repmat(zero_qe, P*I*B,1);
%theta
thetacst=zeros( P*I*B,1);
%duals
dualscst=repmat(zero_duals , P}*\textrm{I}*\textrm{B}, 1)
bcst7 = zeros(P*I*B, 1);
Acst7 = [qcest, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst, dualscst];
%% CST 8 - WH inventory capacity CST's - t+1,s
%qc
qccst=repmat (zero_qc , (T+1)*S, 1);
%qo
qocst=repmat(zero_qo , (T+1)*S, 1);
%qh
qhcst=repmat(zero_qh , (T+1)*S, 1);
%s
scst=repmat(zero_s, (T+1)*S, 1);
%v
vcst=kron(eye(S),[repmat(K2_p, 1, (T+1)).* kron(eye(T+1), ones(1,P))]);
%w
wcst=-repmat(K1_k, (T+1)*S, 1);
%y
ycst=repmat(zero_y, (T+1)*S, 1);
%qe
qecst=repmat(zero_qe, (T+1)*S,1);
%theta
thetacst=zeros( (T+1)*S , 1);
%duals
dualscst=repmat(zero_duals , (T+1)*S, 1);
Acst8 = [qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst, dualscst];
bcst8=zeros ((T+1)*S, 1);
%% CST 9 - WH inventory / flow balance CST - p,t,s
%qc
qcost=repmat (kron(reshape(Ac_its,I,T*S)',ones (P,P)), 1, B).*(repmat(F1_pi, 1, B)
    .* repmat ( eye (P),T*S,I *B));
%qo
qocst=repmat(F1_pi, 1, T*S).* kron(eye(T*S), repmat (eye(P),1,I));
%qh
qhcst=-eye(P*T*S);
%s
scst=repmat(zero_s, P*T*S, 1);
%v
vcst=kron(eye(S), [[eye(P*T), zeros(P*T, P)]-[zeros(P*T, P), eye(P*T) ]]);
%w
wcst=repmat (zero_w, P*T*S, 1);
%y
ycst=repmat(zero_y, P*T*S, 1);
%qe
qecst = eye(P*T*S);
%theta
thetacst=zeros( P*T*S,1);
%duals
dualscst=repmat(zero_duals, P*T*S, 1);
Acst9=[qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst, dualscst];
bcst9=zeros(P*T*S,1);
```

```
%% CST 10 - Inventory period 1 equality constraints across scenarios - p,(s - 1)
%qc
qccst=repmat(zero_qc , P*S , 1);
%qo
qocst=repmat(zero_qo , P*S , 1);
%qh
qhcst=repmat(zero_qh , P*S , 1);
%s
scst=repmat(zero_s , P*S , 1);
%v
vcst= kron(eye(S), [eye(P), zeros(P, P*T)]);
%w
wcst=repmat(zero_w, P*S , 1);
%y
ycst=repmat(zero_y , P*S , 1);
%qe
qecst=repmat(zero_qe, P*S, 1);
%theta
thetacst=zeros( P*S,1);
%duals
dualscst=repmat(zero_duals, P*S, 1);
bcst10 = repmat(V0_p, 1, S)';
Acst10 = [qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst, dualscst
    ];
%% CST 11 - WH inventory in period 1 and T+1 are equal - p,s
%qc
qccst=repmat(zero_qc , P*S , 1);
%qo
qocst=repmat(zero_qo , P*S , 1);
%qh
qhcst=repmat(zero_qh , P*S , 1);
%s
scst=repmat(zero_s , P*S , 1);
%v
vcst= kron(eye(S), [zeros(P, P*T), eye(P)]);
%w
wcst=repmat(zero_w , P*S , 1);
%y
ycst=repmat(zero_y , P*S , 1);
%qe
qecst=repmat(zero_qe, P*S, 1);
%theta
thetacst=zeros( P*S,1);
%duals
dualscst=repmat(zero_duals, P*S, 1);
bcst11 = -repmat(V0_p, 1, S)';
Acst11 = - [qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst, dualscst
];
%% CST 12 - Hospital demand satisfaction constraints - p,t,s
%qc
qccst=repmat(zero_qc , P*T*S, 1);
%qo
qocst=repmat(zero_qo , P*T*S, 1);
%qh
qhcst=eye(P*T*S);
%s
scst=eye(P*T*S).*D_pts;
%v
vcst=repmat(zero_v , P*T*S , 1);
%w
wcst=repmat(zero_w , P*T*S , 1);
%y
ycst=repmat(zero_y , P*T*S, 1);
%qe
```

```
qecst=repmat(zero_qe , P*T*S , 1);
%theta
thetacst=zeros( P*T*S,1);
%duals
dualscst=repmat(zero_duals, P*T*S, 1);
Acst12 = [qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst, dualscst
        ];
bcst12 = D_pts ';
%% CST 13 - Singular WH capacity is selected - singular
bcst13 = 1;
Acst13 =[zero_qc, zero_qo, zero_qh, zero_s, zero_v, ones(1,K), zero_y, zero_qe,
    zero_theta, zero_duals];
%% CST 14 - At most one discount is applied - p,i
%qc
qccst=repmat(zero_qc , P*I, 1);
%qo
qocst=repmat(zero_qo, P*I, 1);
%qh
qhcst=repmat(zero_qh , P*I, 1);
%s
scst=repmat(zero_s, P*I, 1);
%v
vcst=repmat(zero_v, P*I, 1);
%w
wcst=repmat (zero_w, P*I, 1);
%y
ycst=repmat ( eye (P*I),1,B);
%qe
qecst=repmat(zero_qe, P*I,1);
%theta
thetacst=zeros( P*I,1);
%duals
dualscst=repmat(zero_duals, P*I, 1);
bcst14 = ones(P*I, 1);
Acst14 = [qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst, dualscst
        ];
%% CST 15 - Emergency stock supply constraint
%qc
qccst=repmat(zero_qc , P}*\mathrm{ S, 1);
%qo
qocst=repmat(zero_qo, P*S, 1);
%qh
qhcst=repmat(zero_qh, P*S, 1);
%s
scst=repmat(zero_s, P*S, 1);
%v
vcst=repmat(zero_v, P*S, 1);
%w
wcst=repmat(zero_w, P*S, 1);
%y
ycst=repmat(zero_y, P*S, 1);
%qe
qecst= kron(eye(S), repmat(eye(P), 1, T));
%theta
thetacst=zeros( P*S,1);
%duals
dualscst=repmat(zero_duals, P*S, 1);
bcst15 = repmat(Q5_p, 1, S)';
Acst15 = [qccst, qocst, qhcst, scst, vcst, wcst, ycst, qecst, thetacst, dualscst
    ];
%% ***Epsilon FOR LOOP***
```

```
    for iter_eps = 1:count_eps
    %% Preparing First Model
    Acst_eps = serviceOF_pts;
    bcst_eps = epsilon *ones(P*T*S,1);
    %% Solving First Model-Cost
    Acst = [Acst1; Acst2; Acst3;Acst4;Acst5;Acst6;Acst7;Acst8;Acst11; Acst14;
        Acst15; Acst_eps]; % A matrix non-equalities
    bcst = [bcst1; bcst2; bcst3;bcst4;bcst5;bcst6;bcst7;bcst8;bcst11;bcst14;
        bcst15; bcst_eps]; % b matrix non-equalities
    Acst_eq = [Acst9;Acst10;Acst12; Acst13]; % A matrix equalities
    bcst_eq = [bcst9;bcst10;bcst12;bcst13]; % b matrix equalities
    intcon = [loc_v +1:loc_y]; %setting B.V.s to integers
    LB = [zeros(1, loc_theta), -inf, zeros(1, loc_psi_neg-loc_omega)]; %LB is zeros
        for all variables
    UB=[inf(1, loc_v), ones(1, loc_y - loc_v), inf(1, loc_psi_neg-loc_y)]; %UB is
        ones for w,y,o, B.V.s, inf for remaining
    [sol1, val1] = intlinprog(OF_dual_main, intcon, Acst,bcst, Acst_eq, bcst_eq, LB,UB)
        ;
        %% Preparing Second Model
        fprintf('Part 2');
        x0(iter_eps, :)=sol1 ';
        x0(iter_eps, loc_theta)=epsilon;
        epsilon_two = val1;
        Acst_eps = OF_dual_main;
        bcst_eps = epsilon_two;
        Acst_theta = serviceOF_pts + [zeros(P*T*S, loc_theta - 1), -ones (P*T*S,1),
        zeros(P*T*S, loc_psi_neg-loc_theta)];
    bcst_theta = zeros(P*T*S,1);
    %% Solving Second Model-Shortage
    Acst = [Acst_theta; Acst1; Acst2; Acst3;Acst4;Acst5;Acst6;Acst7;Acst8;Acst11;
        Acst14;Acst15; Acst_eps]; % A matrix non-equalities
    bcst = [bcst_theta; bcst1; bcst2; bcst 3;bcst4;bcst5;bcst6;bcst7;bcst8;bcst11;
        bcst14;bcst15; bcst_eps]; % b matrix non-equalities
    Acst_eq = [Acst9;Acst10;Acst12;Acst13]; % A matrix equalities
    bcst_eq = [bcst9;bcst10;bcst12;bcst13]; % b matrix equalities
    intcon = [loc_v +1:loc_y]; %setting B.V.s to integers
    LB = [zeros(1, loc_theta), -inf, zeros(1, loc_psi_neg-loc_omega)]; %LB is zeros
        for all variables
    UB=[inf(1, loc_v), ones(1, loc_y - loc_v), inf(1, loc_psi_neg-loc_y)]; %UB is
        ones for w,y,o, B.V.s, inf for remaining
    [sol2,val2] = intlinprog(OF_theta,intcon, Acst,bcst, Acst_eq, bcst_eq, LB,UB, x0(
        iter_eps, :));
        %% Increment Epsilon
        epsilon=epsilon+step;
        end
end
% Output Results to Excel
```


## Appendix E

## Matlab Code for Multi-Stage DRO Model

Stochastic-robust models are created by running the DRO models with $\rho=0$.

```
%% DRO Two-Stage Model Code Lines (1) - (506)
%% CST 16 - Multi-stage equality cst - qe - P*(S*(T-1) - 3-9-27)
PTS = [];
PTS_v = [];
PITS = [];
for ii = 1:T-1
    %S
    %kron(eye(3^ii ), [ones(3^(4-ii ) -1, 1), -eye(3^(4-ii) - 1)]);
    %TS
    %kron(kron(eye(3^ii), [ones(3^(4-ii) - 1, 1), -eye(3^(4-ii)-1)]), [zeros(1, ii
                -1), 1, zeros(1,T-ii)]);
    %PTS
    PTS = [PTS; kron(kron(kron(eye(3^ii), [ones(3^(T-ii) - 1, 1), -eye(3^(T-ii ) - 1)
        ]), [zeros(1,ii-1), 1, zeros(1,T-ii)]), eye(P))];
    %PTS_v
    PTS_v = [PTS_v; kron(kron(kron(eye(3^ii ), [ones(3^(T-ii ) - 1, 1), -eye(3^(T-ii )
        -1)]), [zeros(1, ii), 1, zeros(1,T-ii)]), eye(P))];
    %PITS
    PITS = [PITS; kron(kron(kron(eye(3^ii), [ones(3^(T-ii) - 1, 1), - eye(3^(T-ii )
        -1)]), [zeros(1, ii - 1), 1, zeros(1,T-ii)]), eye(P*I))];
end
bcst16 = zeros(P*(S*(T-1)-3-9-27), 1);
Acst16 = [repmat([zero_qc, zero_qo, zero_qh, zero_s, zero_v, zero_w, zero_y], P*(
    S*(T-1)-3-9-27), 1), PTS, repmat([zero_theta, zero_duals], P*(S*(T-1)-3-9-27)
    , 1)];
%% CST 17 - Multi-stage equality cst - qh - P*(S*(T-1) - 3-9-27)
bcst17 = zeros(P*(S*(T-1)-3-9-27), 1);
Acst17 = [repmat([zero_qc, zero_qo], P*(S*(T-1)-3-9-27), 1), PTS, repmat([zero_s,
    zero_v, zero_w, zero_y, zero_qe, zero_theta, zero_duals], P*(S*(T-1)-3-9-27)
        1)];
%% CST 18- Multi-stage equality cst - s - P*(S*(T-1)-3-9-27)
bcst18 = zeros(P*(S*(T-1)-3-9-27), 1);
Acst18 = [repmat([zero_qc, zero_qo, zero_qh], P*(S*(T-1)-3-9-27), 1), PTS, repmat
    ([zero_v, zero_w, zero_y, zero_qe, zero_theta, zero_duals], P* (S*(T-1)
    -3-9-27), 1)];
%% CST 19 - Multi-stage equality cst - v - P*(S*(T-1)-3-9-27)
bcst19 = zeros(P*(S*(T-1)-3-9-27), 1);
Acst19 = [repmat([zero_qc, zero_qo, zero_qh, zero_s], P*(S*(T-1)-3-9-27), 1),
    PTS_v, repmat([zero_w, zero_y, zero_qe, zero_theta, zero_duals], P*(S*(T-1)
    -3-9-27), 1)];
%% CST 20 - Multi-stage equality cst - qo - P*I*(S*(T-1)-3-9-27)
```



```
Acst20 = [repmat ([zero_qc], P*I*(S*(T-1)-3-9-27), 1), PITS, repmat ([zero_qh,
    zero_s, zero_v, zero_w, zero_y, zero_qe, zero_theta, zero_duals], P*I*(S*(T
    -1)-3-9-27), 1)];
%% ***Epsilon FOR LOOP***
for iter_eps = 1:count_eps
    %% Preparing First Model
    Acst_eps = serviceOF_pts;
    bcst_eps = epsilon *ones(P*T*S,1);
    %% Solving First Model-Cost
```

            Acst \(=[\) Acst1; Acst2 ; Acst3;Acst4;Acst5;Acst6;Acst7;Acst8;Acst11;Acst14;
                Acst15; Acst_eps]; \% A matrix non-equalities
            \(\mathrm{bcst}=[\mathrm{bcst} 1 ; \mathrm{bcst} 2 ; \mathrm{bcst} 3 ; \mathrm{bcst} 4 ; \mathrm{bcst} 5 ; \mathrm{bcst} 6 ; \operatorname{bcst} 7 ; \mathrm{bcst} 8 ; \mathrm{bcst} 11 ; \mathrm{bcst} 14 ;\)
                bcst15; bcst_eps]; \% b matrix non-equalities
            Acst_eq \(=[\) Acst9; Acst10; Acst12; Acst13; Acst16; Acst17; Acst18; Acst19;
                Acst20]; \% A matrix equalities
            bcst_eq \(=[\operatorname{bcst} 9 ; b \operatorname{cst} 10 ; b \operatorname{cst} 12 ; \mathrm{bcst} 13 ; \mathrm{bcst} 16 ; \operatorname{bcst} 17 ; \operatorname{bcst} 18 ; \operatorname{bcst} 19\);
                bcst20]; \% b matrix equalities
    

for all variables

ones for $w, y, o, B . V . s, i n f$ for remaining
$[$ sol1, val1] $=$ intlinprog (OF_dual_main, intcon, Acst, bcst, Acst_eq, bcst_eq, LB, UB)
;
$\%$ Preparing Second Model
fprintf('Part 2');
$\mathrm{x} 0($ iter_eps, $:)=$ soll $^{\prime}$;
$x 0($ iter_eps, loc_theta) $=$ epsilon;
epsilon_two $=$ vall;
Acst_eps $=$ OF_dual_main;
bcst_eps $=$ epsilon_two;
Acst_theta $=$ serviceOF_pts + zeros $(\mathrm{P} * \mathrm{~T} * \mathrm{~S}, \quad$ loc_theta -1$), \quad-\operatorname{ones}(\mathrm{P} * \mathrm{~T} * \mathrm{~S}, 1)$,
zeros $(\mathrm{P} * \mathrm{~T} * \mathrm{~S}, \quad$ loc_psi_neg-loc_theta) ];
bcst_theta $=$ zeros $(\mathrm{P} * \mathrm{~T} * \mathrm{~S}, 1)$;
$\%$ Solving Second Model-Shortage
Acst $=$ [Acst_theta; Acst1; Acst2; Acst3;Acst4;Acst5;Acst6;Acst7;Acst8;Acst11;
Acst14; Acst15; Acst_eps]; \% A matrix non-equalities
bcst $=$ [bcst_theta; bcst1; bcst $2 ; \operatorname{bcst} 3 ; b \operatorname{cst} 4 ; b \operatorname{cst} 5 ; b \operatorname{cst} 6 ; b \operatorname{cst} 7 ; b \operatorname{cst} 8 ; b \operatorname{cst} 11$;
bcst14;bcst15; bcst_eps]; $\%$ b matrix non-equalities
Acst_eq $=$ Acst9; Acst10; Acst12; Acst13; Acst16; Acst17; Acst18; Acst19;
Acst20]; \% A matrix equalities
bcst_eq $=[\operatorname{bcst} 9 ; b \operatorname{cst} 10 ; b \operatorname{cst} 12 ; \operatorname{bcst} 13 ; \operatorname{bcst} 16 ; \operatorname{bcst} 17 ;$ bcst18; bcst19;
bcst20]; \% b matrix equalities
intcon $=$ [loc_v +1 : loc_y]; \%setting B.V.s to integers
$L B=\left[\operatorname{zeros}\left(1, \operatorname{loc}\right.\right.$ _theta) $,-\inf , \quad \operatorname{zeros}\left(1, \operatorname{loc}_{-}\right.$psi_neg-loc_omega)$] ; \%$ LB is zeros
for all variables

ones for $w, y, o, B . V . s, ~ i n f ~ f o r ~ r e m a i n i n g ~$
[sol2, val2] $=$ intlinprog (OF_theta, intcon, Acst, bcst, Acst_eq, bcst_eq, LB, UB, x0 (
iter_eps, :) ;
$\%$ Increment Epsilon
epsilon=epsilon+step;
end
end
\% Output Results to Excel

## Appendix F Electronic Supplements

The complete two-stage and multi-stage datasets are attached to this submission as electronic supplements. They are also available at the DalSpace website. The two-stage dataset file is named 'CecilAsh2021_TwoStageData.csv', and the multi-stage dataset file is named 'CecilAsh2021_MultiStageData.csv'. The indexing convention is described in the CSV file. Parameters are identified using the parameter notation presented in Section 4.1.

## Appendix G

## Copyright Permission Letter

April 16 ${ }^{\text {th }}, 2021$
I am preparing my M.A.Sc thesis for submission to the Faculty of Graduate Studies at Dalhousie University, Halifax, Nova Scotia, Canada. I am seeking your permission to include a manuscript version of the following paper(s) as a chapter in the thesis:

1) Distributionally robust optimization of a Canadian healthcare supply chain to enhance resilience during the COVID-19 pandemic. Cecil Ash, Claver Diallo, Uday Venkatadri, Peter VanBerkel. Submitted for publication to Information Systems and Operational Research (acceptance pending).
2) Stochastic and robust PPE supply optimization under risks of disruption from the COVID-19 pandemic: A Canadian provincial healthcare perspective. Cecil Ash, Uday Venkatadri, Claver Diallo, Peter VanBerkel. Submitted for publication to Computers \& Industrial Engineering (acceptance pending).

Canadian graduate theses are collected and stored online by the Library and Archives of Canada. I am also seeking your permission for the material described above to be stored online with the LAC. Further details about the LAC thesis program are available on the LAC website (www.bac-lac.gc.ca).

Full publication details and a copy of this permission letter will be included in the thesis.
Yours sincerely,
Cecil Ash

Permission is granted for:
a) the inclusion of the material described above in your thesis.
b) for the material described above to be included in the copy of your thesis that is sent to the Library and Archives of Canada (formerly National Library of Canada) for online storage.

| Name: | Claver Diallo | Title: | Professor |
| :--- | :--- | :--- | :--- |
| Date: |  |  |  |
| Signature: |  |  |  |
| Name: | Uday Venkatadri | Title: | Professor |
| Signature: | Date: |  |  |
| Name: | Peter VanBerkel |  |  |
| Signature: |  | Title: | Associate Professor |

