# OPTIMIZATION MODELS FOR COORDINATING FLOCK PROCUREMENT IN A POULTRY COMPANY

by

Kenneth Reay Dolson

Submitted in partial fulfillment of the requirements
for the degree of Master of Applied Science

at

Dalhousie University
Halifax, Nova Scotia
March 2021

# Table of Contents

# List of Tables

# List of Figures

## Abstract

A poultry company must coordinate the collection of flocks of chickens from many farms to meet the demand of its customers while fulfilling the facility's operational constraints. The profits of the farmers, each of whom have production goals, must also be considered due to supply management. A modified version of the multi-period lot sizing with supplier selection problem framework is used to formulate three models for the problem: an integer program, a weighted goal program, and a minmax goal program. A two-stage stochastic variation of the problem is then considered, in which forecasts of the average weight of each flock are uncertain. The three proposed models are adapted to solve the stochastic variation of the problem. Soft robust optimization is used with the integer program. Time series analysis is used to identify the growth rate of each flock as a normally distributed random variable and weight predictions are made based on this, then the deterministic and stochastic models are tested. The experiments reveal that in both the deterministic and stochastic problems, the minmax goal programming approach significantly reduces the maximum expected deviation from optimality without significantly impacting quota fulfillment, reducing risk of a large deviation from optimality for all parties. The value of stochastic solution is also observed to be quite large, reducing the maximum expected deviation from optimality by between 40.3% and 86.6% of its original value when a stochastic formulation is implemented for five test weeks.

## Acknowledgements

Thank you to Dr. Ahmed Saif, Dr. Hassan Sarhadi, Dr. M. Ali Ülkü and Dr. Uday Venkatadri as the members of my supervisory committee for helping push me to write the best thesis I could. In addition, thank you to the Mitacs Accelerate program and an anonymous industry partner for funding this research.

# Chapter 1

# Introduction

The poultry industry is important in Canada. Over the past decade, Canadians' chicken consumption has continually increased while their consumption of beef and pork products has decreased. Using availability as a proxy for consumption, this is noted by a 2019 Statistics Canada study [75], which observes a 7.6% increase in chicken availability in Canada between 2009 and 2017 compared to 9.4% and 11.8% decreases in the availability of pork and beef, respectively. This shift may be economically motivated. While its 19.1% price increase over that period of time was similar to the 26.3% increase for pork, it was much smaller than the 63.1% increase for beef. In more absolute terms, chicken is a popular protein source for many Canadians due to its affordability, carrying an average price of $7.94/kg as of October 2020 in comparison to $11.38/kg for ground beef (typically the least expensive type of beef) or $11.65/kg for pork chops [74].

Affordable food is particularly important in Nova Scotia, where as of 2018 the median household income for single-parent families was 15% lower than the national median and 9% lower for two-parent families [72]. Despite these income discrepancies, the average Nova Scotian household only spent $20/month less than the national average on groceries in 2017 [73]. In fact, according to 2020 studies by the Government of Nova Scotia [31] and Statistics Canada [76], 13.3% of Nova Scotians lived below the poverty line in 2018 and 15.3% experienced food insecurity, both the highest proportions of anywhere in Canada. These statistics demonstrate an above-average need for economical, nutritious food in Nova Scotian communities. As the most cost-effective meat, it is important to keep chicken production stable and efficient to avoid the adverse impact of unnecessary price increases on food insecurity.

This stability is facilitated by a supply management system [77], allowing Canada to limit the production of chicken (as well as other poultry, eggs and dairy) to what its citizens will actually consume, ensuring predictable prices and consistent availability.

A supply management system has three components: production quotas, minimum prices, and import tariffs. The first two components are managed by provincial boards of farmers, while the third is federally administrated. A production quota establishes the maximum total weight of chicken that a farmer can produce without being forced to pay a fine per unit weight. The minimum price is the lowest price per unit weight at which farmers may sell their flocks to processors. Finally, high import tariffs keep most foreign chicken products out of the Canadian market. Collectively, the three components of the supply management system create an uncompetitive market designed to meet public need. The tariffs ensure the chicken sold in Canada is primarily domestic, while minimum prices and production quotas discourage farmers from trying to undercut their competitors.

The uncompetitive environment encouraged by the supply management system homogenizes the experience of a processing company purchasing from different farmers because the prices are typically the same and the farmers have production quotas that encourage them to grow chickens of a similar average weight. Accordingly, the system encourages teamwork between the farmers and the processing companies. Each farmer wants to produce their full quota amount to maximize profit and they will be strongly disincentivized from working with a processing company again if the company makes decisions purely in its own interest rather than considering the farmer's quota.

Before chicken can be sold to consumers, it must be prepared and packaged by a poultry processing company. The company purchases flocks of chickens from a variety of farmers and retains their business by determining an optimal procurement schedule to help all of them meet their production quotas. Procurement scheduling is important because a flock of chickens must be processed in a relatively narrow window of time. Once the chickens have reached the minimum acceptable size, it typically takes only three to four days before they grow too large to produce a product customers will purchase, meaning that a farmer cannot simply produce the amount requested by the decision maker and set it aside to await pickup. The farmer is effectively producing continuously until the flock has been processed.

Because not all chickens grow at the same rate, their exact weight on a given day in the future cannot be known. Thus, to help the farmers meet their targets, forecasts must be made about the weights of the flocks before scheduling them to get all farmers as close to their targets as possible. The schedule is additionally limited by the company's operating constraints, which dictate the amount of chicken to be processed each day as well as restricting the average flock sizes and locations that can be scheduled for collection on the same day. The uncertainty of the weight forecasts and how that can affect scheduling practices should also be considered.

While this thesis focuses on a supply management problem in the context of agriculture, its model may be adapted for applications elsewhere. For example, independent power producers (IPPs) are used as a source of electricity in some countries such as Taiwan [41], although the state continues to distribute power to its citizens. If the country wishes to reduce its carbon footprint, the power distributor might offer the IPPs guarantees of a minimum share of the market in exchange for producing more renewable energy. The distributor would then be responsible for minimizing cost by planning how much power would be purchased from each IPP as demand fluctuates over the course of the day while considering that some IPPs have power generation targets to meet.

## 1.1 Overview

In Chapter 2, a review of literature relevant to this problem is provided. This includes literature about the poultry industry, the multi-period lot sizing with supplier selection problem (MLSSP), the stochastic lot sizing problem and multiobjective formulations used to solve it, and common scenario generation techniques. Opportunities for contribution are identified concerning the modelling of a system under supply management, production planning with competing interests, the stochastic lot sizing problem under uncertain supply, and comparison of different solution methods.

In Chapter 3, a detailed description of the Deterministic Flock Procurement Problem (DFPP) is given and the problem is compared to the MLSSP. DFPP is identified as deterministic because its weight forecasts are assumed to be certain knowledge of the future. An integer program (IP) is formulated to solve the problem, then its limitations are evaluated. Two goal programming (GP) formulations are then considered, a weighted GP (WGP) and a minmax GP (MGP). The WGP is identified as presenting a higher risk of loss in a single week to each competing interest, although it notes that this risk may be offset by higher expected quota fulfillment levels throughout the sixteen-week quota period. A need to compare the relative risk of loss in a single week against the possibility of gain across the quota period is identified for the IP, WGP, and MGP formulations of the DFPP, which are identified as [DIP], [DWGP], and [DMGP] respectively.

In Chapter 4, DFPP is modified to consider the Stochastic Flock Procurement Problem (SFPP), in which weight forecasts are considered uncertain and their stochasticity can be represented by a scenario set. Soft robust optimization (SRO) [60, 61] is used to reformulate the IP from Chapter 3, then stochastic versions of the WGP and MGP are also generated. The increased risks of large losses by a single party posed by the SRO and stochastic WGP frameworks are identified and theoretically compared, then the need to compare the relative risk of loss in a single week against the possibility of gain across the quota period is identified for the SRO, WGP, and MGP formulations of SFPP, which are identified as [RP], [SWGP], and [SMGP] respectively. A visual map of the formulations for DFPP and SFPP can be observed in Figure 1.1 demonstrating how the different solution methods relate to each other.

In Chapter 5, a weight forecasting model is developed based on a case study. It finds that the growth rate of each flock can modelled as a normally distributed random variable and estimated by an 8-point moving average. The process of determining parameters and generating scenario sets from the case data is described. Lastly, a computational experiment finds that the MGP formulation causes a notable reduction in risk of a large deviation without significantly impacting expected gains in quota fulfillment for both DFPP and SFPP.

Finally, Chapter 6 summarizes the findings of this thesis and proposes considerations for future work.

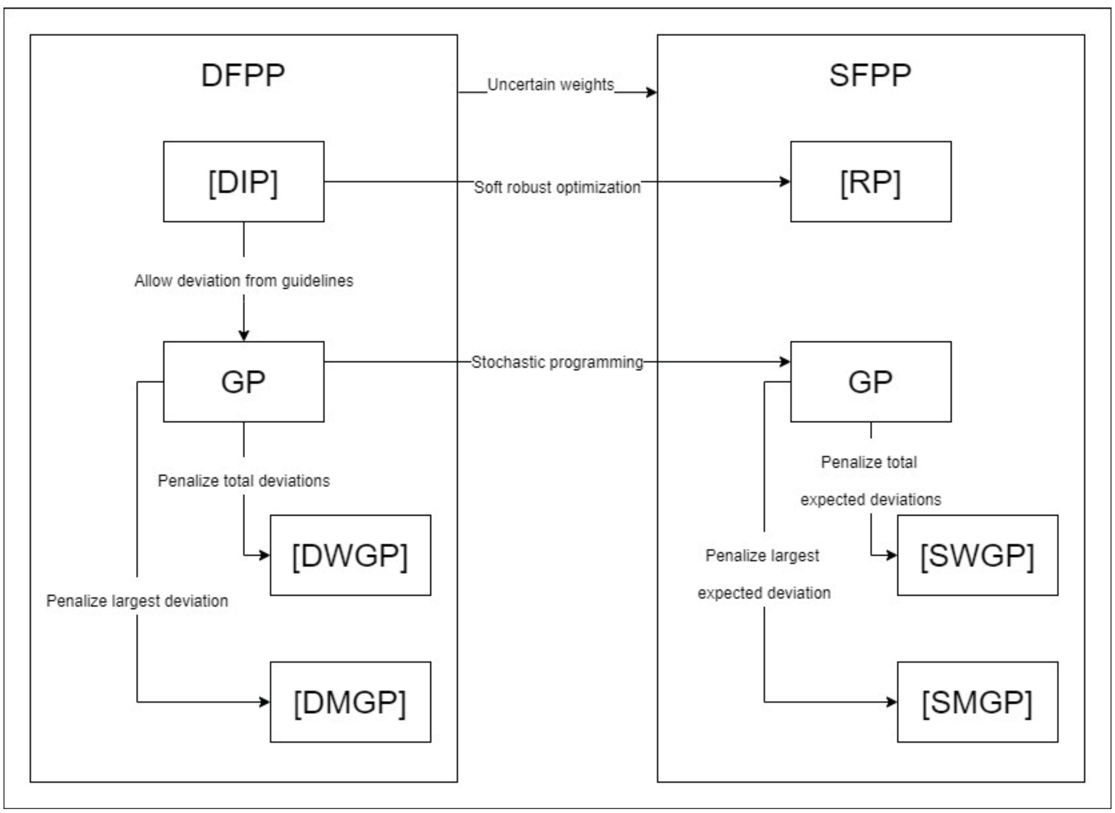


Figure 1.1: A map of the formulations presented in this thesis and how they relate to each other.

# Chapter 2

# Literature Review

This chapter contains an overview of literature relevant to DFPP and SFPP. First, it reviews papers concerning poultry production planning, noting that not many exist. Next, it defines the multi-period lot sizing problem and surveys the techniques currently used to solve variants of this problem. Third, it examines papers that have solved the multi-period lot sizing problem under at least one uncertain parameter, specifically identifying stochastic goal programming as a technique used in these situations and briefly regarding common scenario generation techniques one might use for these stochastic problems. Finally, it identifies opportunities for contribution in the literature, most notably a lack of production planning problems considering competing financial interests and of studies of the stochastic lot sizing problem under uncertain supply.

## 2.1 Poultry Industry

Several applications of schedule optimization for the poultry industry have been published, so it is prudent to review these first. Taube-Netto [78] is among the first to recognize the potential in poultry production planning and provides a detailed breakdown of the process before presenting software designed to save Sadia Concordia SA more than $50 million over a three-year period. It notably breaks this complex process into individual modules such as Site Planning and Chick Planning. This thesis primarily focuses on what is identified as the Flock Planning & Control module, the step of the process when flocks are identified for slaughter on a particular day. While the procurement process is described in great detail, Taube-Netto does not formulate a scheduling model at any point, preferring simply to state that the software developed is capable of scheduling flocks.

Oliveira and Lindau [1] present a framework for poultry scheduling when pickup days are already fixed and the collection times must be scheduled to minimize loss of mass. While the details of the solution are not of interest because DFPP and SFPP do not consider scheduling practices on a more granular level than choosing a day for each flock, its characterization of the problems inherent in the choosing a slaughter by age or slaughter by weight methodology is useful. Slaughter by age is distinguished by a simple solution and volatile final result due to the variance of average flock weights, whereas slaughter by weight is noted as requiring additional equipment in the barns and requiring additional time-sensitive information which might narrow the window inside which a schedule can actually be made.

Brevik *et al.* [16] and Solano-Blanco *et al.* [71] both propose a mixed-integer linear program for production planning and scheduling decisions for a poultry manufacturer seeking to minimize cost. While minimization of cost is desirable, the cases studied are of companies with a higher level of vertical integration than what is considered in DFPP or SFPP, appearing to possess more agency over the quantities and timing of chicks placed in hatcheries.

Khamjan *et al.* [48] establish a swine procurement model considering pig growth and size distribution. While this is not explicitly about poultry, it is still similar enough to merit consideration. It treats different size ranges as different products, then uses a two-phase heuristic to predict which combinations of products herds will contain at different periods of time and create a schedule which minimizes cost based on that. It is worth noting that this approach assumes deterministic knowledge of the growth rates of the herd.

Aparicio *et al.* [9] propose a weighted additive model to assess the technical efficiency of dairy farmers in Canada. Like the Canadian poultry industry, the Canadian dairy industry is subject to supply management. The proposed model evaluates the average deviation from optimal production in each province, noting that this deviation from optimality becomes smaller when the production quotas assigned to each dairy farmer are taken into account. The proposed model offers high-level insights without being able to provide specific operational recommendations, as well as failing to account for other production constraints when evaluating optimality.

Focusing more on flock weight forecasting and less on scheduling, Johansen *et al.* [43, 44] and Johansen *et al.* [45] use dynamic neural network models to effectively lower the root mean squared forecasting error in comparison to existing poultry weight models. Reducing forecasting error is crucial in providing a higher degree of confidence in the optimality of one's chosen scheduling methodology. Huang *et al.* [39] and Xiao *et al.* [88] both have similar levels of success in forecasting poultry growth rates when comparing the performance of back-propagation neural networks and multiple linear regressions. All five sources consider detailed information about environmental conditions throughout the flock's life cycle, but nothing in the literature defines a forecasting model for growth rates when the farmers do not keep a log of these conditions.

## 2.2 Lot Sizing Problem

The multi-period lot sizing problem was first defined in 1958 by Wagner and Whitin [85]. It identifies a forecast of product demand $d_t$ over a time horizon $t \in T$ and aims to determine how many units $x_t$ to order in each time period $t$ to minimize cost. In 2005, Basnet and Leung [14] were the first to consider the multi-period lot sizing problem with supplier selection. This extension of the model considers a set of products $i \in I$ and a set of suppliers $j \in J$. Each product $i$ has demand $D_{it}$ in each time period $t$ and each supplier $j$ has a purchase price $P_{ij}$ for each product $i$. The model aims to decide how many units $x_{ijt}$ of each product $i$ must be ordered from each supplier $j$ in each time period $t$ to minimize cost.

Once forecasts have been made for each flock's growth rate, the core of DFPP becomes a variant of the multi-period lot sizing problem with supplier selection because it is a situation in which a purchaser must efficiently acquire a product from a set of suppliers in a given set of time periods. In the case of DFPP and SFPP, it is notable that the decision maker must purchase each product offered by each supplier exactly once, and the question is to determine when each purchase can be made to satisfy demand. By examining the different ways other multi-period lot sizing problems with supplier selection have been approached, ideas for the best solution to DFPP and SFPP may be generated.

The multi-period lot sizing problem is commonly modeled as a mixed integer program (MIP) [5, 7, 8, 14, 21, 24, 49, 50, 51, 59, 65, 84]. While these are all unique variations on the basic multi-period lot sizing problem with supplier selection (for example, only some discuss variants of the problem with multiple products [5, 14, 21, 49, 50, 59, 65]), they all have an objective function which seeks exclusively to minimize cost, or similarly maximize profit in the case of Mohammadi *et al.* [59]. While maximizing profit is important to any business, this is insufficient for DFPP and SFPP because it does not account for how a decision in one period could affect the decision maker's relationship with a supplier in a future period. Supplier satisfaction has not previously been considered as a complication of the multi-period lot sizing problem. The methodology must consider the profits of many farmers at once, differentiating between solutions that are good for every farmer and those that are very good for most of them and bad for a few. Thus, we begin to look toward a multiobjective programming approach in the literature.

Assadipour and Razmi [11] consider the problem of inventory lot sizing and supplier selection for an assembly system. Like the aforementioned lot sizing problems, the paper seeks strictly to maximize profit, but it diverges from these problems in its multiobjective formulation. It is formulated as a multiobjective problem due to the imprecise nature of several coefficients and thus the fuzzy nature of the expected profit for a given solution, seeking to maximize the expected profit while minimizing the risk of lower profit. These objectives are then combined into a single objective function to create a mixed integer program which is solved by particle swarm optimization. Awareness of this approach may be valuable because, while it does not consider satisfaction of multiple parties, it considers a solution for multiple objective functions associated with the lot sizing problem at once.

Azadnia *et al.* [12] propose a multi-period lot sizing model in which sustainable supplier selection is considered, a form of supplier selection accounting not only for economic considerations but also for environmental and social criteria by calculating sustainability scores for each supplier. Maximization of economic, environmental and social scores is considered alongside minimization of cost in a multiobjective model which is successfully solved with both a weighted sum method and an augmented $\epsilon$-constraint method. Both of these methods are effective, but it is worth noting that the augmented $\epsilon$-constraint method produced a number of Pareto-optimal solutions rather than just one solution, and when the solutions were brought to the case study company's experts they found it difficult to select the best solution from among those presented. An approach like this could cause undue loss of productivity while its similarly efficient solutions are analyzed if new problems are solved on a regular basis. The weighted sum method produces comparable solutions to the augmented $\epsilon$-constraint method. It can also be noted that the multiobjective formulations incur less than a 2% cost increase in exchange for a significant sustainability increase. The tradeoff between cost and sustainability demonstrates the potential efficacy of multiobjective programming in remaining cost-efficient while accounting for social factors.

Liao and Rittscher [53] examine a multi-period procurement lot sizing problem considering both supplier and carrier selection, developing a multiobjective programming model to minimize total cost, defects, and late deliveries. It combines them via a similar weighted sum method as proposed in Azadnia *et al.* [12], but experiments with how different weights affect solution quality. The proposed model generates solutions using a genetic algorithm customized to suit the problem. While it does not provide a baseline against which the quality of its solutions can be evaluated, its use of sensitivity analysis to establish a set of weights which lead to effective solutions is an analytical tool worthy of consideration.

Rezaei and Davoodi [66] present a multiobjective formulation of the multi-period, multi-product lot sizing problem with supplier selection. This formulation attempts to minimize cost while maximizing both the quality and service levels. Like Liao and Rittscher [53], the authors develop a solution with a genetic algorithm, using the NSGA-II algorithm outlined by Deb *et al.* [22] as a base and introducing a novel, problem-specific operator to achieve convergence toward a solution more quickly. The formulation attempts to identify overall trends in quality from suppliers using an exponential time-dependent function to prioritize suppliers whose products are on average of higher quality. The total service level objective function is an appealing idea because it helps address the issue of uncertain supply by not assuming the amount requested from a supplier is always equivalent to the amount delivered.

Razmi and Maghool [63] propose a fuzzy multiobjective model for the multi-period, multi-product lot sizing problem with supplier selection further complicated by discount price schemes. The authors develop solutions using both the augmented $\epsilon$-constraint method and the reservation-level Tchebycheff procedure (RLTP) to balance the minimization of cost and maximization of total value of purchasing, a metric which accounts for qualitative performance criteria. Application of both methods to a numerical example allows the authors to draw useful conclusions about optimal purchasing habits, such as when delay of payment is useful with different suppliers in the event of a limited budget. It is also noted that while the methods find good solutions - RLTP generally finding better solutions than augmented $\epsilon$-constraint - the solver struggles as the problem size grows and a metaheuristic method might be needed to handle problems with more than ten suppliers and five time periods. In the case of a tool being designed for practical use, this is an important note because computational efficiency leading to a reasonable runtime is important to many users. They may not want to wait for hours; particularly when there is a chance they may need to update some data and run the program again. Nonetheless, a Tchebycheff method should be considered as a solution if the issue of computational efficiency can be resolved.

Ustun and Demirtas [82, 83] experiment with Tchebycheff methods to address a multiobjective take on the multi-period lot sizing problem with supplier selection with a two-stage methodology. First, they use an analytic network process to evaluate intangible qualities and develop weights to attach to suppliers based on expert opinions of 14 criteria pertaining to each supplier. Second, they use Tchebycheff methods to balance budget, quality, and purchasing value goals. One problem [82] is solved using a hybrid of weighted and Tchebycheff GP, while the other [83] uses RLTP much like Razmi and Maghool [63]. The latter [83] also tries $\epsilon$-constraint and preemptive GP methods, but finds them less effective than the Tchebycheff approach. The authors claim that RLTP has a significant advantage over goal programming due to the decision maker's participation in an interactive decision process, but this may once again come up as an issue when presented as a tool to users expected to regularly solve new problems without technical knowledge of the program or access to technical guidance. In this sense, a less interactive approach which will consistently produce an acceptable solution is preferable, an experience which is afforded by the weighted Tchebycheff goal programming [82]. A similar model is later used by Demirtas and Ustun [23] and solved purely with weighted goal programming, ignoring the Tchebycheff metric entirely. It achieves a solution, but does not find the quality of its solution noteworthy.

Choudhary and Shankar [19] also propose a goal programming model for joint decisions pertaining to lot sizing and supplier selection. It compares three GP techniques – preemptive, non-preemptive, and weighted Tchebycheff fuzzy GP – using a value path approach to analyze the tradeoffs offered by making improvements on any one of the multiple objective functions. This is an attractive form of analysis for this problem because it enables scenario-based analysis according to user-defined factors, working well in tandem with the weight sensitivity analysis of Choudhary and Shankar [18] and Liao *et al.* [53].

## 2.3 Stochastic Optimization

Stochastic programming is a branch of methodologies commonly used to deal with uncertainty in mathematical programs. Specifically, the stochastic lot sizing problem is a well-studied variant of the lot sizing problem featuring uncertain demand. It is typically a multi-period problem which does not feature supplier selection, formulated as a MIP seeking to minimize cost. Existing literature has scrutinized the efficacy of many existing heuristics [13, 17, 25, 46, 80] or proposed new ones [42, 52, 67], used Monte Carlo simulations [4, 20], or even found ways to modify the problem to create tractable formulations [13, 37, 79]. However, very little work has been done to understand the lot sizing problem with known demand and uncertain supply. While this type of problem is certainly less common, it is relevant to the agricultural industry, where goods are often marketed by weight and the exact weight of goods to be marketed is unknown until their arrival at the facility.

While it still deals with uncertain demand rather than supply, Kang and Lee [46] propose a solution to the stochastic lot sizing problem which accounts for supplier selection, an uncommon consideration in the literature. It formulates a multiobjective model seeking to both minimize total cost and maximize service level, solving it by using the $\epsilon$-constraint method to transform the multiobjective formulation into a mixed integer program. It then finds that this MIP can be efficiently solved by a modified version of the heuristic dynamic programming model originally presented as a solution to the lot sizing problem by Wagner and Whitin [85]. The model handles the stochasticity of the demand with a chance constraint, assuming normal distribution and placing orders such that the demand will be satisfied with at least $\alpha$

certainty according to $z$-values defined for the problem. This form of problem-solving may be useful because a modified version of the service level chance constraint could be relevant to DFPP and SFPP, in which flocks must be purchased from farmers at the right weights to help them meet their quotas.

Stochastic lot sizing problems sometimes incorporate robust optimization techniques as a tradeoff between optimization of expected value and of worst-case scenarios, providing a more moderate form of risk aversion. Different authors have done this in different ways. For example, Hu and Hu [38] and Keyvanshokooh *et al.* [47] identify multiple sources of uncertainty in a two-stage stochastic program and solve one stage's variables stochastically while defining an uncertainty set for the other stage's variables and solving it with robust optimization. Curcio *et al.* [20] optimize the expected value of a multistage stochastic problem while using a budget polyhedral uncertainty set to ensure model robustness.

Azizi *et al.* [13] and Hu and Hu [37] both approach the stochastic lot sizing problem with a scenario approach rather than using chance constraints. While solving the problem for an exhaustive list of scenarios quickly becomes computationally unrealistic when a different set of scenarios is available for each time period, they use scenario generation based on the mean, variance, kurtosis and skewness of the data to discretize the demand's continuous distribution before using fast-forward scenario reduction to choose a subset of scenarios that is collectively most representative of all possibilities, in one instance reducing 3125 scenarios to just 15 while losing very little information. In particular, Hu and Hu [37] utilize a stability test to see how far they can lower the number of scenarios in the interest of efficiency while keeping enough of them to develop a useful solution.

### 2.3.1 Stochastic Goal Programming

Stochastic goal programming is an effective way of balancing multiple objectives under uncertainty. This uncertainty may be captured by any of several methods, including fuzzy programming [26, 54, 57, 69], chance constraints [2, 56, 62, 64, 68, 69, 91] or scenario sets [10, 56, 92]. Scenario sets become useful for assessing the expected value of an objective function when decisions are made in multiple stages rather than all at once, and are thus appealing for SFPP.

Arabi *et al.* [10] consider the design of an algal biofuel supply chain network, building a goal program which seeks to simultaneously maximize profit and minimize greenhouse gas emissions. The alternative fuel price is considered under uncertainty, and the weighted goal program constructed penalizes deviations from the optimal value for each objective function in each scenario proportional to the probability of that scenario. Monte Carlo simulation is used to generate scenario data in the case study and the conclusion is drawn that the stochastic model is more profitable than the deterministic model.

Mahmoodirad and Niroomand [56] similarly consider another supply chain network design problem which seeks to maximize profit and minimize environmental impact. It considers five different cost parameters associated with transportation under uncertainty, using a belief degree-based system to construct scenarios in the absence of historical data. A goal programming approach is attempted with a scenario-based expected value formulation as well as with a chance constraint-based formulation. While the study does not declare the quality of one method's solution to be superior to the other, it can be noted that the computational time of the chance constrained method grows more quickly than the scenario-based formulation.

Zhou and Erdogan [92] introduce a two-stage resource allocation model for wildfire suppression and resident evacuation, creating a preemptive goal program which minimizes the number of people in high-risk areas, then the costs incurred by the fire. Population density, percent of population evacuated, and property value are all considered as uncertain parameters affected by the fire's intensity and speed. The resulting plan is deemed comprehensive and realistic, although the authors do not compare it with any other resource allocation models to indicate improvement.

Although it is not common, some stochastic GP techniques have been mixed with soft robust optimization [60, 61]. In SRO, much like two-stage stochastic GP, uncertain parameters are represented by a set of scenarios while decision variables are divided into here-and-now and wait-and-see variables. Penalty variables are then defined to allow some of the constraints to be violated, and a new objective function is introduced including a penalty function. It is uncommon because SRO and a stochastic GP both penalize deviations, but the approach can be useful in programs where deviations from a goal value and deviations from feasibility are two separate things. Yu and Li [89] first highlighted the efficacy of this approach in a GP context, applying a slightly modified version of SRO to a stochastic GP to improve computational efficiency. Zahedi-Seresht *et al.* [90] also investigate the utility of SRO in solving stochastic GPs, formulating a set of scenarios for a data envelopment analysis (DEA) model. It creates a weighted GP using scenario probabilities as weights and finds its robust counterpart, solving the DEA problem $S$ times more efficiently than existing methods and obtaining comparable results, where $S$ is the number of scenarios. While these papers do not attempt to quantify the robustness of their solution in comparison to other methods, they do demonstrate the conditions under which SRO can be effectively applied to stochastic GP problems.

### 2.3.2 Scenario Generation

The stochasticity of a program is often represented by scenarios and their corresponding probabilities. These scenarios are by definition discrete, which makes stochastic programming quite easy for a system which has a finite and predictable set of states, but more difficult for a system whose uncertainty is dictated by a continuous distribution. SFPP is a two-stage problem, so it represents uncertainty with scenarios. The marginal distributions of each source of uncertainty in the problem can be discretized by generating a set of scenarios that are representative enough of them to help the program produce an effective solution.

Several methods of scenario generation for continuous distributions have been used in the literature, although one of the most common is the moment matching method proposed by Høyland and Wallace [40]. It provides a generalized model which can be adjusted and solved to produce a multi-period scenario tree based on the relevant statistical properties of a distribution. Several researchers [27, 33, 36, 81] use it effectively to generate large decision trees across long planning horizons, the latter taking advantage of measurements such as skewness and kurtosis to set the problem up effectively despite an asymmetrical, non-Gaussian distribution. Unfortunately, this method can result in an intractably large scenario set when considering many independent sources of uncertainty.

Random sampling is another scenario generation method frequently used by the literature [15, 30, 32, 35, 70, 86]. Unlike moment matching, it does not seek to generate a set of scenarios that is as evenly spaced out as possible; rather, it sets a finite number of realizations and draws randomly from each distribution to create a scenario set. Multiple scenario sets are generated and solved to ensure a stable solution has been found. For example, Homem-de-Mello [35] solves 25 iterations of each problem configuration to ensure stability.

While moment matching and random sampling are the two most prevalent methods, other techniques for scenario generation exist. Before proposing a different scenario generation method, Hochreiter and Pflug [34] note that optimal scenario sets result simply from minimization of a probability metric or distance between the sets of points, leaving the number of conceivable generators at least as large as the number of statistical distance measurement methods. It goes on to consider cases in which moment matching may produce strange results by illustrating examples in which two very different distributions both have the same mean, variance, skewness and kurtosis and solves these problems by minimizing Wasserstein distances rather than using the traditional moment matching method. Hochreiter and Pflug claim that this is a particularly useful method if the system is very sensitive to the tails of the distribution because it offers more weight to the tails.

## 2.4 Opportunities for Contribution

Several opportunities for contributions can be identified in the different areas of focus studied in this literature review. This thesis aims to make contributions in these areas. According to the current literature, no published work could be found doing any of the following:

- Modelling an agricultural system under supply management to create a prescriptive optimization model.
- Considering the multi-period lot sizing problem under the influence of multiple stakeholders with competing financial interests.
- Considering a multiobjective formulation of the multi-period lot sizing problem with uncertain supply.
- Comparing the performance of SRO and stochastic GP techniques.

The body of work studying production planning in the poultry industry or in agricultural systems under supply management is quite small. It lacks material in which multiple farmers' interests must be considered in parallel, despite the fact that consistently developing solutions that ensure each farmer turns a good profit on their flock is an essential component of ensuring the ability to do repeat business with them. The only published work that could be found concerning agricultural supply management makes high-level observations about the productivity of the farmers it studies, but cannot provide operational recommendations. A useful contribution in this area would be to formulate a model to optimize farmer productivity subject to supply management and operational constraints. This contribution would create a model that would remain useful with some adjustments when applied to poultry production without supply management or another supply managed industry with different operational constraints.

The primary objective function of the lot sizing problem has historically been profit. While multiobjective functions have been established to enable the decision maker to make what they feel is an appropriate tradeoff on profit in exchange for another criterion, such as quality or environmental sustainability, they have all considered a single party that benefits from the decisions suggested by the model. This can be observed in Table 2.1, which identifies several criteria relevant to DFPP or SFPP and shows every paper reviewed which addresses a multi-period lot sizing problem with more than one of these criteria.

| Reference | Multi-Supplier | Multiobjective | Multi-Party | Stochastic |
|---|---|---|---|---|
| [11], [12], [23], [53], [63], [82], [83] | ✓ | ✓ | | |
| [46] | ✓ | ✓ | | ✓ |
| [67] | | ✓ | | ✓ |
| This thesis | ✓ | ✓ | ✓ | ✓ |

Table 2.1: Comparison of references.

The profit of multiple parties must be considered separately from other goals because those parties have competing interests, and so a significant loss to one goal in exchange for marginal improvements to several others may not be an acceptable solution. Whereas Table 2.1 shows that the current body of work focuses on the profit of the manufacturer, a useful contribution in this area would be to develop a formulation of the lot sizing problem attempting to maximize the benefit of multiple parties at once while continuing to work within the manufacturer's constraints. In addition to its application to the system defined in this thesis, this contribution would enable sustainable models such as minimizing the ecological impact of harvesting a commodity in several locations at once.

Stochastic versions of the lot sizing problem have been studied extensively under uncertain demand, but not under uncertain supply. This observation is consistent with the typical lot sizing problem in practice: customer demands are often unknown and must be predicted but it is assumed that the supplier will be able to deliver exactly the requested number of units. However, suppliers in the agricultural sector deal with living, growing products. They cannot be certain of exactly how many units they will produce until their product has been harvested. Accordingly, a model for a lot sizing problem in this sector should be capable of accounting for supplier uncertainty. A useful contribution in this area would be to propose a lot sizing model which considers certain demand and uncertain supply. In addition to its applications in agriculture, uncertain supply is a useful consideration if a supplier may experience production shortages after orders have been placed.

While SRO and stochastic GP techniques have been explored in the literature, no direct comparisons have been made of their efficacy. Both can be used when the multi-period lot sizing problem has manufacturing guidelines which may need to be violated to develop a feasible solution. A useful contribution would be to compare SRO and stochastic GP to evaluate any performance tradeoffs each may offer over the other. Beyond the multi-period lot sizing problem, this contribution would be useful to any optimization problem seeking to minimize cost or maximize profit while satisficing other criteria.

It can be noted that the only published work studying poultry weight forecasting also uses detailed environmental data to make predictions. Other work has assumed a uniform growth rate across every flock. A practical managerial contribution in this area would be to develop a method of scheduling that is applicable to poultry processors with multiple suppliers whose operations they do not control. This method would be able to balance the interests of each supplier while using a technique requiring minimal information from the supplier to make predictions about each flock's growth rate.

# Chapter 3

# Deterministic Flock Procurement Problem

This chapter discusses several formulations that can be used to solve DFPP. First, the problem is defined and its similarity to the MLSSP is described. Next, the notation used in each formulation is collected and formally stated. Third, an IP is developed based on the work done by Basnet and Leung [14] and its limitations are evaluated. Finally, a WGP formulation is defined based on work by Ustun and Demirtas [82, 83], its limitations are similarly evaluated, and an alternative MGP formulation is proposed to mitigate the drawbacks of the IP and WGP methods.

## 3.1 Notation

The following notations are used to formulate the IP, WGP, and MGP presented in this chapter.

*Sets:*

$I = \{1, 2, ..., |I|\}$ The set of eligible flocks, indexed by $i$ and $i'$.

$T = \{1, 2, ..., |T|\}$ The set of days, indexed by $t$.

$J = \{1, 2, ..., |J|\}$ The set of farmers, indexed by $j$.

$G = \{1, 2, ..., |G|\}$ The set of geographic areas, indexed by $g$ and $g'$.

*Decision Variables:*

$x_{it}$ A binary variable, 1 if flock $i$ is picked up on day $t$, 0 otherwise.

$y_{gt}$ A binary variable, 1 if area $g$ is visited on day $t$, 0 otherwise.

$r_{i'it}$ A binary variable, 1 if flocks $i'$ and $i$ are picked up on the same day $t$, 0 otherwise.

*Parameters:*

$e_{ij}$ 1 if flock $i$ belongs to farmer $j$, 0 otherwise.

$w_{it}$ The expected average weight per chicken of flock $i$ on day $t$ in kg.

$n_i$ The number of chickens in flock $i$.

$q_j$ The desired portion of the weight quota of farmer $j$.

$f_{gi}$ 1 if flock $i$ is located in area $g$, 0 otherwise.

$c_{g'g}$ 1 if pickups can be made from areas $g'$ and $g$ in the same day, 0 otherwise.

$\epsilon$ The allowable deviation from $\frac{1}{|T|}$ for the proportion of total weight produced on day $t$ to total weight produced across all days $t \in T$.

$\lambda_d$ The weight of goal $d$ in the WGP for $d \in \{1, 2, 3\}$.

$k_d$ The normalization constant of goal $d$ in the WGP and MGP for $d \in \{1, 2, 3\}$.

*Intermediate Variables:*

$\phi$ The maximum deviation of the MGP.

$\theta_t^1$ Goal 1 on day $t$; the total amount of chicken, in kg, produced on day $t$.

$\theta_{i'it}^2$ Goal 2 for flocks $i'$ and $i$ on day $t$; the difference between the average weights of flocks $i'$ and $i$ if they are both scheduled on day $t$ and 0 otherwise.

$\theta_j^3$ Goal 3 for farmer $j$; the proportion of $q_j$ farmer $j$ expects to produce in total.

$z_t^{1+}$ The amount by which goal 1 exceeds target amount 1 on day $t$.

$z_t^{1-}$ The amount by which target amount 1 exceeds goal 1 on day $t$.

$z_{i'it}^{2+}$ The amount by which goal 2 exceeds target amount 2 between flock $i'$ and flock $i$ on day $t$. A corresponding $z_{i'it}^{2-}$ is not relevant to the problem.

$z_j^{3+}$ The amount by which goal 3 exceeds target amount 3 for farmer $j$.

$z_j^{3-}$ The amount by which target amount 3 exceeds goal 3 for farmer $j$.

## 3.2 Problem Description

A poultry company purchases flocks of chickens from many barns in their region. Due to industry standards, each barn follows a similar pattern for raising their flocks. When the flocks are newly hatched, they are delivered from a hatchery to a farmer. Hatcheries are separate businesses from farms, owned and operated by different people. Farms are economic entities, each controlled by a single owner who might have multiple barns. When the flocks are between three and four weeks old, the farmers will weigh some of the chickens and send an average weight to the decision maker at the poultry company. The measurement sent to the poultry company is referred to as an interim weight. When the flocks are approximately five weeks old, the poultry company sends trucks to collect them to be taken to the plant for processing. This process is depicted in Figure 3.1.

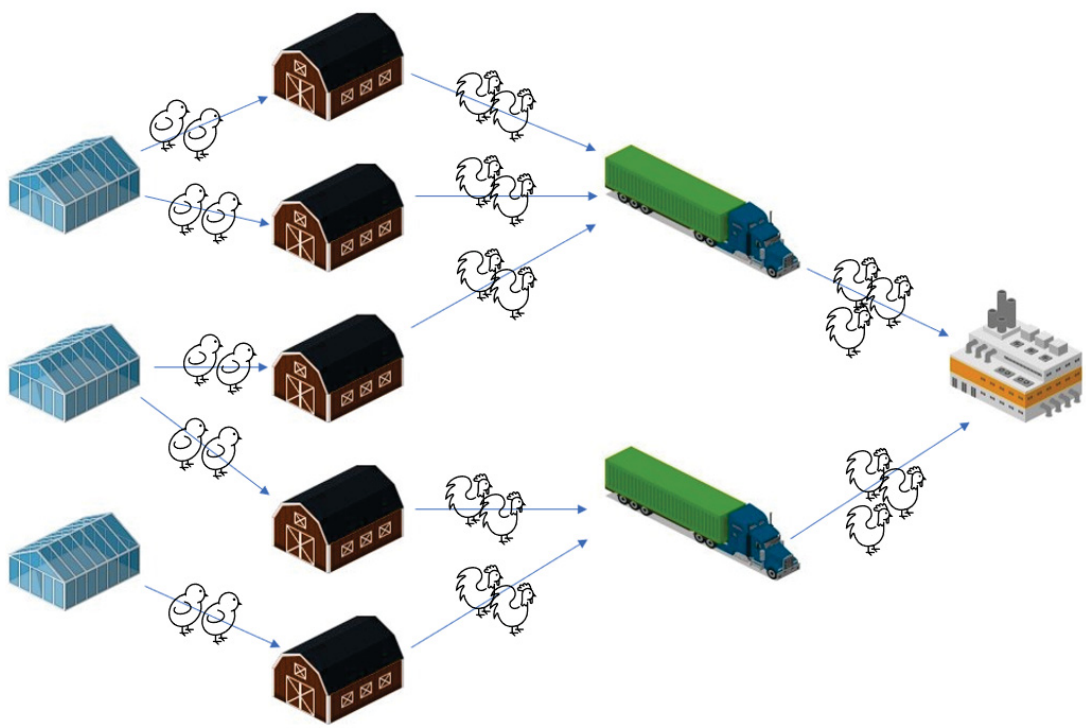

Figure 3.1: Flocks from hatching to processing

The barns take between one and three weeks to be cleaned, depending on the farmer, then a new flock is placed. The cleaning time is usually three weeks, so a typical flock cycle lasts for eight weeks. The division of responsibilities among the hatcheries, farmers, and the poultry company is illustrated in Figure 3.2. While each party may offer input on any step in the process, the decision is ultimately made by the party to whom the step has been assigned. The diagram also denotes the timeframe relative to flock collection in which each stakeholder has responsibilities.

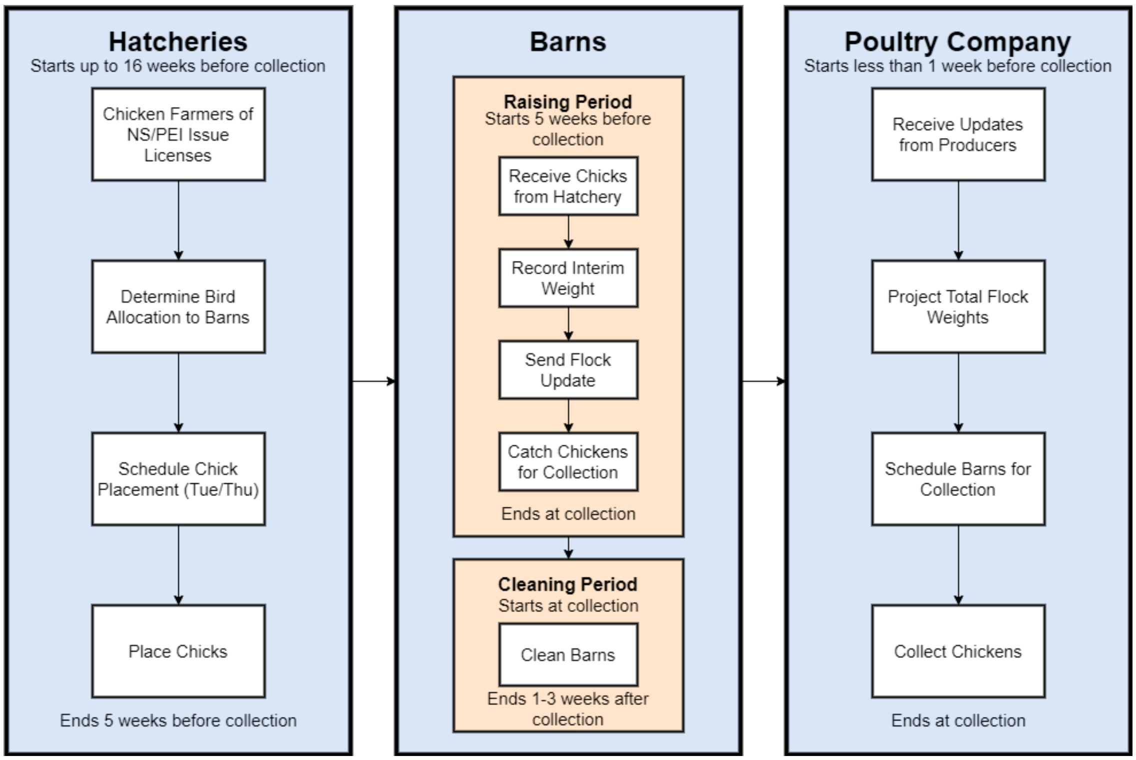


Figure 3.2: Diagram illustrating the responsibilities of each stakeholder in the procurement process.

It can be observed from Figure 3.2 that the poultry company begins making decisions less than one week before flocks are collected, once they have received interim weights from these farmers. Although the decision maker may offer input earlier in the process, they do not hold decision-making power until this point. They must make forecasts to determine when flocks will be ready for collection and what their average weight will be at that time. The window of time in which a flock is appropriate for collection is typically about three days before its expected value in sales sharply declines, so it is important for forecasts to be accurate enough to ensure each flock is scheduled inside that window. Flocks are typically processed between 34 and 39 days old. The procurement manager's professional judgment is used to determine the final eligible set of flocks to be processed before the schedule is created. Flocks are collected between day 1 and day $|T|$. After $|T|$ days of flock collection and processing, $|T|$ days of downtime occur in which no flocks can be collected or processed. Forecasts can be made about the average weight of a flock $2|T|$ days before it is collected. Scheduling decisions must be made at least $|T| + 1$ days in advance to give the transportation department time to arrange collection of the flocks. The scheduling process occurs once every $2|T|$ days. A visualization of the timeline can be observed in Figure 3.3.

<table>
<tr><td>Day</td><td>-|T|</td><td>...</td><td>0</td><td>1</td><td>...</td><td>|T|</td></tr>
<tr><td>Activity</td><td>Make scheduling decisions</td><td colspan="2"></td><td colspan="3">Collect flocks</td></tr>
</table>

Figure 3.3: The timeline of the DFPP planning process.

It can be noted that each week can be treated as a discrete problem rather than considering a rolling horizon model because knowledge is not gained about the next scheduling period until the current scheduling period can no longer be adjusted. For example, consider the timeline presented in Figure 3.3. Flocks are collected in the current scheduling period between day 1 and day $|T|$. The next set of flocks will be collected in the next scheduling period between day $2|T|+1$ and day $3|T|$. The latest a decision can be made about any part of the current scheduling period is day -1, at which point forecasts can only be made until day $2|T| - 1$. Because no forecasts can be made about the next scheduling period during the planning of the current scheduling period, each scheduling period is considered a separate problem.

Procurement schedules must account for operational constraints in three areas: the processing plant, transportation, and farmers. The workload for the employees at the plant should remain balanced each day to allow them to work at a consistent pace rather than overworking them one day and underworking them the next, so similar total weights of chicken should be processed each day because quantities of chicken are measured in kilograms (kg). The difference between average flock weights collected in a given day also cannot be too large. The machinery is set at the beginning of each day based on the average size of the chickens that will be processed that day. The defect rate has been observed to increase if flocks with more than a 0.2 kg average weight difference are processed on the same day. For example, scheduling 2.15 and 2.35 kg flocks on the same day is optimal, scheduling 2.10 and 2.40 kg flocks together is not.

Transportation constraints must also be considered when modelling the problem. The distance between some of the barns supplying the poultry company may be quite large. It is assumed that the company's transportation department recommends that collection from two distant areas cannot be scheduled on the same day because this distance might increase the number of trucks required to collect the same number of flocks. Geographically, clusters of barns can be identified. The decision maker may decide that certain areas may not be scheduled on the same day.

Chicken production in Canada is regulated by provincial boards. These boards define sixteen-week periods and a quota for each farmer. A farmer's quota is the total amount of chicken, in kg, they can produce in a quota period without paying a penalty fee per unit weight. Because farmers sell chicken by weight, producing less than their quota makes their flocks in that period less profitable than they could be. Thus, each farmer has a goal amount and producing under or over that amount is undesirable because it is less profitable. It is advisable for the poultry company to use their schedule to help the farmers meet their quotas as closely as they can because it incentivizes the farmers to continue to work with the poultry company rather than with their competitors. A farmer considers their quota fulfilled if they have produced between 99% and 102% of the quota amount.

Note that DFPP makes the following assumptions:

- The trucks collecting the flocks do not have a maximum capacity. If a non-local cluster is scheduled to be visited on a given day, it is feasible to collect any number of flocks from that cluster.

- The processing facility of the poultry company does not have a production capacity. It is feasible to schedule any number of flocks on the same day as long as none are from competing non-local clusters.

- Demand is sufficient to ensure that all chickens are sold on the day they are processed. Considerations for holding cost or perishable goods are not necessary.

- Geographic clusters are defined such that $|G| \leq |T|$

### 3.2.1 Lot Sizing Problem

The formulation of the MLSSP can be adapted to suit DFPP. The MLSSP was first addressed in 2005 by Basnet and Leung [14]. Defining a set of products $I$, a set of suppliers $J$, and a set of time periods $T$, the authors construct a MIP to decide how many units of each product $i$ to order from each supplier $j$ in each time period $t$. An objective function is used to minimize costs under the constraint of meeting demand in each time period. Its decision variables are defined as $X_{ijt}$, the number of product $i$ ordered from supplier $j$ in period $t$, and $Y_{jt}$, a binary variable which is 1 if an order is placed from supplier $j$ in period $t$ and 0 otherwise. Its parameters are defined as $D_{it}$, the demand for product $i$ in period $t$, $P_{ij}$, the purchase price per unit of product $i$ from supplier $j$, $H_i$, the holding cost of product $i$ per period, and $O_j$, the transaction cost for supplier $j$. An analogue to each of these variables can be identified in DFPP with the exception of the costs $H_i$ and $O_j$, which can be set to 0 because holding is assumed to not occur and transaction costs are fixed based on which flocks have been selected for collection before actually creating the procurement schedule. While additional constraints must be formulated, Basnet and Leung [14] provide a good framework to which these constraints can be added.

First, consider the sets $I$, $J$, and $T$. $I$ is the set of products to be scheduled in the original formulation. In DFPP, each flock waiting to be scheduled will have a unique impact on the problem, contributing a different amount to the production quota of a different farmer. The flocks cannot be treated interchangeably, and thus can each be considered a unique product $i \in I$, making $I$ the set of flocks in DFPP. $J$ is the set of suppliers in the original formulation. The flocks are sold by the farmers, so $J$ is the set of farmers in DFPP. Note that each flock $i \in I$ is exclusively available from a single farmer $j \in J$. $T$ is the set of time periods in the original formulation. DFPP decides on which day each flock will be collected, making each day $t \in T$ a time period.

Next, consider the decision variables $X_{ijt}$ and $Y_{jt}$. In the original formulation, $X_{ijt}$ is defined as the number of product $i$ ordered from supplier $j$ in period $t$. In DFPP, the decision to be made is whether a flock $i$ is collected from farmer $j$ in period $t$. The decision is binary because only collecting part of a flock is not possible. Additionally, because only one farmer $j$ can supply each flock $i$, the the more compact decision variable $x_{it}$ can be used. In the original formulation, $Y_{jt}$ is a binary variable determining whether an order is placed from supplier $j$ in period $t$ and it is used in the objective function to total the transaction costs when minimizing cost. Because transaction cost itself is not a concern, $Y_{jt}$ is unnecessary. However, as previously mentioned, there is a constraint on which geographic areas can be visited in the same day. Defining $G$ as the set of geographic areas and $g \in G$ as an area in this set, this can be captured by the parameter $c_{g'g}$, a binary parameter which is 1 if areas $g'$ and $g$ can be visited in the same day and 0 otherwise. The decision variable $y_{gt}$ can now also be established, a binary variable which is 1 when area $g$ is visited in period $t$ and 0 otherwise. To ensure areas $g'$ and $g$ are not visited unless $c_{g'g} = 1$, the constraint

$$y_{g't} + y_{gt} \leq c_{g'g} + 1 \quad \forall g' \in G, g \in G, t \in T \tag{3.1}$$

can be established. Like the transaction cost, this constraint complicates ordering certain products depending on where they come from. To complete the geographic constraints, $f_{gi}$ can be defined as a binary parameter which is 1 if flock $i$ is located in area $g$ and 0 otherwise. The decision variable $x_{it}$ must be connected to this parameter, so a constraint is used to ensure $y_{gt}$ is 1 if any flock collected in period $t$ belongs to area $g$.

$$\sum_{i \in I} f_{gi} x_{it} \leq |I| y_{gt} \quad \forall g \in G, t \in T \tag{3.2}$$

It can be noted that when $f_{gi} = 0$, $x_{it}$ may assume a value of 1 regardless of the value of $y_{gt}$: this is acceptable because $y_{gt}$ should not constrain $x_{it}$ unless flock $i$ belongs to area $g$.

The parameters of the original MLSSP formulation must now be considered. The demand in the original formulation is $D_{it}$, the demand in units for product $i$ in period $t$. However, demand must be considered differently in the case of DFPP. When processed on day $t$, flock $i$ produces a total weight of chicken based on the number of chickens in the flock, $n_i$, and the average weight per chicken in flock $i$ on day $t$, $w_{it}$. Thus, the total weight produced by harvesting flock $i$ on day $t$ is $n_i w_{it}$. The units of this measurement are kg. The expected total weight to be harvested across all days is

$$\frac{\sum_{t\in T}\sum_{i\in I} n_i w_{it}}{|T|}, \tag{3.3}$$

because the expected value of $w_{it}$ for flock $i$ across all days $t \in T$ is $\sum_{t\in T} \frac{w_{it}}{|T|}$. Note that deterministic knowledge of $w_{it}$ is assumed in DFPP. Later, SFPP will be considered, in which a distribution of possible $w_{it}$ values are used rather than assuming the single set will always be correct. If the poultry company produces a similar overall amount of chicken in kg each day, they will satisfy the demand of their customers. Thus, the demand for each day $t \in T$ is

$$D_t = \frac{\sum_{k\in T}\sum_{i\in I} n_i w_{ik}}{|T|^2}, \tag{3.4}$$

which can be balanced by a constraint.

$$\sum_{i\in I} n_i w_{it} x_{it} \geq D_t \quad \forall t \in T \tag{3.5}$$

It can be noted that this demand, unlike the demand of the original MLSSP, is not indexed by flock $i$ because chicken is the same product once it reaches the consumer. However, production requirements must also be considered for the farmers. Each farmer has a production quota to meet, and it is in the best interest of the decision maker to help the farmer meet their quota as closely as possible so they will continue to be incentivized to work with the poultry company. This quota can be viewed as each farmer creating demand for their own flocks. Let $e_{ij}$ be a binary parameter taking the value of 1 if flock $i$ is owned by farmer $j$ and 0 otherwise. The total amount of chicken produced by farmer $j$ across all time periods $t \in T$ can be calculated by $\sum_{i\in I}\sum_{t\in T} e_{ij} n_i w_{it} x_{it}$. Letting the demand created by farmer $j$ be a production goal $q_j$, considering that farmers are satisfied producing at 99-102% of their quota, this demand can be fulfilled by the constraints

$$\sum_{i\in I}\sum_{t\in T} e_{ij} n_i w_{it} x_{it} \geq 0.99 q_j \qquad \forall j \in J \tag{3.6}$$

$$\sum_{i\in I}\sum_{t\in T} e_{ij} n_i w_{it} x_{it} \leq 1.02 q_j \qquad \forall j \in J. \tag{3.7}$$

These constraints ensure that each farmer $j$ produces a total weight between 99-102% of $q_j$.

The original objective function is to minimize cost, however, it can be noted that the cost of materials is not directly an issue for the decision maker because the company expects to spend the same amount per kg regardless of supplier or time period. The primary concern when scheduling procurement is to ensure each farmer meets their production quota without exceeding it and being forced to pay a penalty. Because deviation from a farmer's maximum production quota results in a proportional loss of profits for that farmer, this deviation can be considered a cost. Thus, an objective function must be formulated to maximize production for each farmer.

$$\max \sum_{i \in I} \sum_{t \in T} \sum_{j \in J} \frac{e_{ij} n_i w_{it} x_{it}}{q_j} \tag{3.8}$$

This objective function scales the production by weight of each farmer $j$ according to their production goal $q_j$ to ensure that farmers with lower production goals are not forced to disproportionately underproduce: for example, a deviation of 1000 kg for a farmer with production goal $q$ has a similar profit margin impact to a deviation of 5000 kg for a farmer with production goal $5q$. Using suboptimal production as an analogue to profit in this way also allows the final comparison to the MLSSP formulation to be drawn: $P_{ij}$, the price of product $i$ from supplier $j$, is analogous to $n_i w_{it}$ in DFPP because they both define the scale of the contribution of the decision variable $X_{ijt}$ or $x_{it}$ to the objective function.

Finally, additional constraints must be considered to finish adapting this framework to DFPP. Production is constrained by the average size per chicken of flocks scheduled on the same day, which should not have a difference of greater than 0.20 kg. Defining a binary variable $r_{i'it}$, which indicates whether flock $i'$ and flock $i$ have both been scheduled on day $t$, constraints can be defined to ensure no two flocks scheduled on the same day have average weights which differ by more than 0.20 kg.

$$r_{i'it}|w_{i't} - w_{it}| \leq 0.20 \quad \forall i' \in I, i \in I, t \in T \tag{3.9}$$

$$r_{i'it} \leq x_{i't} \quad \forall i' \in I, i \in I, t \in T \tag{3.10}$$

$$r_{i'it} \leq x_{it} \quad \forall i' \in I, i \in I, t \in T \tag{3.11}$$

$$r_{i'it} \geq x_{i't} + x_{it} - 1 \quad \forall i' \in I, i \in I, t \in T \tag{3.12}$$

Constraint (3.9) allows flocks $i'$ and $i$ to be scheduled on the same day only if their average weight difference is less than 0.20 kg, while Constraints (3.10) - (3.11) force $r_{i'i}$ to 0 if $x_{i't}$ or $x_{it}$ is 0 and Constraint (3.12) forces $r_{i'it}$ to 1 if $x_{i't}$ and $x_{it}$ are both 1. A constraint can be defined to ensure that each flock is scheduled for collection exactly once.

$$\sum_{t \in T} x_{it} = 1 \tag{3.13}$$

Because $x_{it}$ is a binary variable, this constraint forces it to adopt the value of 1 for exactly one day $t \in T$.

Having defined a framework to be used for DFPP, IP and GP formulations to solve the problem can now be considered.

## 3.3 Integer Programming

The literature review notes that a MIP is a common method of modelling the MLSSP. It was first used when the problem was introduced by Basnet and Leung [14] and it is an effective approach when the sole objective of the decision maker is to minimize cost or maximize profit. Having defined the objective of DFPP to be maximizing production for each farmer, using quota fulfillment as an analogue for profit, an IP formulation is a natural first step for DFPP. Note that DFPP generates an IP rather than a MIP because all of its decision variables are binary. This formulation is presented below and henceforth referred to as [DIP].

$$
\begin{aligned}
\text{[DIP]: max} \quad & \sum_{i\in I}\sum_{t\in T}\sum_{j\in J}\frac{e_{ij}n_i w_{it}x_{it}}{q_j} && && (3.14)\\
\text{s.t.} \quad & \frac{|T|\sum_{i\in I} n_i w_{it}x_{it}}{\sum_{t\in T}\sum_{i\in I} n_i w_{it}} \leq \frac{1}{|T|}+\epsilon && \forall t\in T && (3.15)\\
& \frac{|T|\sum_{i\in I} n_i w_{it}x_{it}}{\sum_{t\in T}\sum_{i\in I} n_i w_{it}} \geq \frac{1}{|T|}-\epsilon && \forall t\in T && (3.16)\\
& \sum_{i\in I}\sum_{t\in T} e_{ij}n_i w_{it}x_{it} \leq 1.02q_j && \forall j\in J && (3.17)\\
& r_{i'it}|w_{i't}-w_{it}| \leq 0.20 && \forall i'\in I, i\in I, t\in T && (3.18)\\
& r_{i'it} \leq x_{i't} && \forall i'\in I, i\in I, t\in T && (3.19)\\
& r_{i'it} \leq x_{it} && \forall i'\in I, i\in I, t\in T && (3.20)\\
& r_{i'it} \geq x_{i't}+x_{it}-1 && \forall i'\in I, i\in I, t\in T && (3.21)\\
& \sum_{t\in T} x_{it} = 1 && \forall i\in I && (3.22)\\
& \sum_{i\in I} f_{gi}x_{it} \leq |I|y_{gt} && \forall g\in G, t\in T && (3.23)\\
& y_{g't}+y_{gt} \leq c_{g'g}+1 && \forall g'\in G, g\in G, t\in T && (3.24)\\
& x_{it}, y_{gt}, r_{i'it} \in \{0,1\} && \forall g\in G, i'\in I, i\in I, t\in T && (3.25)
\end{aligned}
$$

Many of the constraints are presented as they were initially formulated, although some have been adjusted to fit [DIP]. Equation (3.14) is as it was initially formulated, an objective function seeking to maximize the sum of the proportions of the production goals $q_j$ produced by each farmer $j$. Constraints (3.15) - (3.16) have been reformulated from the demand constraint initially presented in Constraint (3.5), notably by separating part of $D_t$ onto the left hand side to leave only $\frac{1}{|T|}$ on the right hand side, allowing a tolerance $\epsilon$ to be added. When $\epsilon = 0$, the company is required to produce exactly $\frac{1}{|T|}$ of its total weight of chicken each day. Due to the varying sizes of the flocks, it is unlikely that a feasible solution will exist in which exactly the same amount of chicken can be produced every day, so a tolerance is added. The tolerance must be two-sided to prevent slightly underproducing on all days except one and severely overproducing on one day: for example, if Constraint (3.15) was not used in a 4-day week with tolerance $\epsilon = 0.03$, three days could each produce 22% of the total weight and the fourth day could produce 34% of the total weight. Constraint (3.17) sets a production limit for farmer $j$ at $1.02q_j$ kg. Because the objective function already seeks to maximize production, the minimum production to satisfy a farmer of $0.99q_j$ is not used. Constraint (3.18) ensures that no two flocks $i'$ and $i$ scheduled on the same day have average weights that differ by more than 0.20 kg. This constraint and Constraints (3.19) - (3.21) that follow it to constrain $r_{i'it}$ are exactly as they were previously defined and so do not require a more detailed explanation. The same is true of Constraint (3.22) ensuring each flock is picked only once in the set of days $T$, Constraint (3.23) ensuring geographic areas are scheduled in the same day only when this is allowed, and Constraint (3.24) ensuring that flocks located in a geographic area can be picked only when this area is visited. Finally, Constraint (3.25) sets $x_{it}$, $y_{gt}$, and $r_{i'it}$ as binary variables.

This formulation is idealistic. While it reflects the constraints of DFPP in a perfect world, the parameters of the problem may not always be such that a feasible solution exists because there is no guarantee that a schedule can be created that ensures the proportion of total weight produced each day is within the range of $\frac{1}{|T|} \pm \epsilon$. Even if such a schedule exists, there is also no guarantee that it will respect the maximum daily weight spread of 0.20 kg.

It is important for a formulation solving DFPP to always have a feasible solution. The poultry company must continue to schedule flocks for collection every week regardless of whether a schedule can be created that follows all of their guidelines. Therefore, a method should be used which attempts to follow all of these guidelines, but allows deviations if no feasible solution would otherwise exist. This can be done with a multiobjective program.

## 3.4 Goal Programming

Multiobjective programming is another approach often used to optimize the MLSSP when the problem becomes more complex than simply maximizing expected profit subject to dynamic demand. Common objectives added to the problem include minimizing defect rate or maximizing an environmentally sustainable criterion. A general procedure for developing a multiobjective framework is demonstrated by Ustun and

Demirtas [82, 83], in which the authors define a set of objective functions and their constraints, then choose a solution method that is anticipated to be effective. This section takes the same approach to DFPP, then uses the framework developed to create WGP and MGP formulations.

### 3.4.1 Formulating Objectives

Anything the formulation must achieve should be defined by either an objective function or a constraint. While a constraint must be met for a solution to be considered feasible, an objective function simply takes the best value it can. Because the priority of the decision maker is to always develop a solution, even if it does not follow all of their scheduling guidelines, any constraint which poses a risk of causing the problem to have no feasible solution should be converted to an objective function. Two requirements can be identified which fit this description: the weight balancing requirement indicated by Constraints (3.15) - (3.16) and the maximum average weight difference per day indicated by Constraint (3.18).

The first objective function to formulate is the weight balancing function. It is defined by

$$\min \quad \sum_{t\in T}(z_t^{1+} + z_t^{1-}) \tag{3.26}$$

$$\text{s.t.} \quad \theta_t^1 = \frac{|T|\sum_{i\in I} n_i w_{it} x_{it}}{\sum_{t\in T}\sum_{i\in I} n_i w_{it}} \qquad \forall t \in T \tag{3.27}$$

$$\theta_t^1 - z_t^{1+} + z_t^{1-} = \frac{1}{|T|} \qquad \forall t \in T. \tag{3.28}$$

Each instance of $\theta_t^1$ for $t \in T$ has a target value of $\frac{1}{|T|}$ rather than requiring maximization or minimization, and its deviations from the target value are measured by $z_t^{1+}$ and $z_t^{1-}$. Minimizing these deviations will get $\theta_t^1$ as close to the target value as possible. $\theta_t^1$ quantifies the proportion of weight produced on day $t$ to the total weight produced across all days $t \in T$. Because this objective function aims to guide $\theta_t^1$ to a target value, GP is an effective form of multiobjective optimization. Thus, further objective functions defined in this section should be defined in a way that allows them to be added to a GP formulation.

The next objective function to formulate is the maximum average weight difference function. Preparing to put it into a GP formulation, it can be defined by

$$\min \quad \sum_{i' \in I} \sum_{i \in I} \sum_{t \in T} z^{2+}_{i'it} \tag{3.29}$$

$$\text{s.t.} \quad \theta^2_{i'it} = r_{i'it} |w_{i't} - w_{it}| \qquad \forall i' \in I, i \in I, t \in T \tag{3.30}$$

$$\theta^2_{i'it} - z^{2+}_{i'it} \leq 0.20 \qquad \forall i' \in I, i \in I, t \in T. \tag{3.31}$$

Each instance of $\theta^2_{i'it}$ should be penalized if it exceeds 0.20. $\theta^2_{i'it}$ quantifies the difference in average weight per chicken between flock $i'$ and flock $i$ on day $t$ if they are both collected that day, and takes a value of 0 otherwise. A negative deviation term is not included in the objective function because the solution should not be penalized for pairing flocks with average weights per chicken that differ by less than 0.20 kg.

Finally, the third objective function to consider is maximizing the proportion of the production goal $q_j$ produced by each farmer $j$. The goal can be modified slightly to attempt to satisfy as many farmers as possible rather than directly trying to maximize their production. As previously noted, farmers are satisfied when producing between 99-102% of their production quota. It is also technically possible for farmers to produce more than 102% of their quota, but it is not profitable to do this because they are penalized for the total weight produced past 102%. Considering that producing under 99% or over 102% is undesirable, an objective function can be formulated.

$$\min \quad \sum_{j \in J} (z^{3+}_j + z^{3-}_j) \tag{3.32}$$

$$\text{s.t.} \quad \theta^3_j = \sum_{i \in I} \sum_{t \in T} \frac{e_{ij} n_i w_{it} x_{it}}{q_j} \qquad \forall j \in J \tag{3.33}$$

$$\theta^3_j - z^{3+}_j \leq 1.02 \qquad \forall j \in J \tag{3.34}$$

$$\theta^3_j + z^{3-}_j \geq 0.99 \qquad \forall j \in J \tag{3.35}$$

Each instance of $\theta^3_j$ is penalized if the farmer's total production is under $0.99q_j$ or if it exceeds $1.02q_j$. $\theta^3_j$ quantifies the anticipated total weight of chicken produced by farmer $j$ across all days $t \in T$.

It can be noted that while $q_j$ is linked to the quota amount for farmer $j$, it can be changed each week according to how closely the farmer's goals were met earlier in the quota period. For example, consider a farmer who possesses two identical barns and is assigned an 80 000 kg quota over 16 weeks. Each barn can produce a flock every 8 weeks and their cycles are offset by 4 weeks: one produces flocks in weeks 4 and 12, the other produces flocks in weeks 8 and 16. Table 3.1 displays how the farmer's production target is adjusted over time if the schedule forces the farmer to produce only 90% of their production target each time. Initial Quota refers to the quota amount, in kg, that has not yet been produced at the start of the week. Amount Produced refers to the amount of chicken, in kg, produced that week. End Quota refers to the quota amount, in kg, that has not yet been produced at the end of the week.

| Week | $q_j$ (kg) | Initial Quota (kg) | Amount Produced (kg) | End Quota (kg) |
|---|---|---|---|---|
| 4 | 20 000 | 80 000 | 18 000 | 62 000 |
| 8 | 20 667 | 62 000 | 18 600 | 43 400 |
| 12 | 21 700 | 43 400 | 19 530 | 23 870 |
| 16 | 23 870 | 23 870 | 21 483 | 2387 |

Table 3.1: A theoretical farmer produces 90% of their production target each week.

Table 3.1 shows that if the farmer produces 90% of their production target in each of the four weeks they are scheduled to produce, they will still produce 97% of their quota. A similar exercise, producing 110% per week instead of 90% per week, would show that the farmer would produce 103% of their quota. A farmer experiencing a deviation from optimality in one week does not require preferential treatment in subsequent weeks to ensure they meet their quota. It is still important to attempt to meet production targets in the early weeks of a quota period to ensure production targets will be attainable in the later weeks of the quota period. In a practical application, the extent to which production targets can be adjusted may depend on company guidelines regarding the range of average weights in which a flock should be purchased.

Having defined the objective functions which must be balanced, and considering the constraints previously defined in [DIP], a goal program can now be created.

### 3.4.2 Weighted Goal Program

When a GP formulation is implemented for the MLSSP in the literature, it can be observed that it is commonly formulated as a WGP. It allows the decision maker to designate a weight for each of the objective functions according to the perceived importance of achieving the target value of that function. The WGP formulation is provided below and henceforth referred to as [DWGP].

$$\text{[DWGP]: min} \quad \lambda_1 \sum_{t \in T} \frac{z_t^{1+} + z_t^{1-}}{k_1} + \lambda_2 \sum_{i' \in I} \sum_{i \in I} \sum_{t \in T} \frac{z_{2i'it}^{+}}{k_2} + \lambda_3 \sum_{j \in J} \frac{z_{3j}^{+} + z_{3j}^{-}}{k_3} \tag{3.36}$$

$$\begin{aligned} \text{s.t.} \quad & \text{Constraints (3.19) - (3.25)} \\ & \text{Constraints (3.27) - (3.28)} \\ & \text{Constraints (3.30) - (3.31)} \\ & \text{Constraints (3.33) - (3.35)} \end{aligned}$$

$$z_t^{1+}, z_t^{1-}, z_{i'it}^{2+}, z_j^{3+}, z_j^{3-} \geq 0 \quad \forall i' \in I, i \in I, j \in J, t \in T \tag{3.37}$$

The [DWGP] formulation is composed primarily of constraints that have previously been established. The objective function presented in Equation (3.36) is a sum of the deviations weighted by $\lambda$. Each deviation is scaled by a normalization constant $k$ to allow their summation despite having different units. Constraints (3.19) - (3.25) carry over from the [DIP] formulation with no modification, while the constraints identified from (3.27) to (3.35) can be used because they have been designed specifically to fit into a goal program. Constraint (3.37) provides a non-negativity constraint for the deviations from each goal.

[DWGP] improves on [DIP] in an important way by allowing the scheduling guidelines to be violated and attempting to minimize the violations. This practice ensures that even if no schedule exists which perfectly adheres to the guidelines, a schedule can always be generated that almost adheres to them. Importantly, recalling that farmers can produce more than 102% of their quota if they pay a penalty which makes the additional weight produced no longer profitable, [DWGP] also allows a farmer to be scheduled to produce slightly over $1.02q_j$ if it is preferable to underproducing. For example, consider a farmer who has one flock. The total weight of the flock grows by approximately $0.05q_j$ per day. If location constraints prevent the flock from being scheduled for collection on the day its weight will be $0.98q_j$, it can be scheduled for the following day when its weight will be $1.03q_j$. If the [DIP] constraint is in place, this decision cannot be made and the flock must be scheduled for collection a day earlier rather than later, when its weight will be $0.93q_j$. Better solutions can be reached when overproducing and underproducing are both considered undesirable but possible.

It can be noted that the location constraints have been left as constraints rather than being converted to another objective function; this decision is made based on the assumption that geographic clusters have been defined such that $|G| \leq |T|$. The location constraints only present the possibility of infeasibility if $|G| > |T|$, because a worst-case-scenario schedule would no longer exist in which each location is assigned a single day on which all of its flocks will be collected. Because $|G| \leq |T|$ for each instance the decision maker will solve, an additional goal concerning the locations visited in the same day is not necessary.

[DWGP] is not flawless, however. By only considering the weighted total of the deviations, the program fails to consider how the magnitude of a single deviation might affect the practicality of implementing its associated solution. For example, consider an instance of DFPP with 15 farmers, then consider two solutions to this problem each with an identical amount of deviation in $\theta^1_t$ and $\theta^2_{i'it}$. In the first solution, 14 of the 15 farmers produce at 99% of their production goal, while one farmer produces at 85% of their production goal. The total deviation for this solution is 14 percentage points. In the second solution, all 15 farmers produce at 98% of their production goal. The total deviation for this solution is 15 percentage points. Although [DWGP] would choose the first solution, it is likely that the farmers, and therefore the decision maker, would prefer the second. [DWGP] does not adequately consider the interests of multiple parties simultaneously if at least one of those parties is averse to a loss in a single week in exchange for the possibility of a gain across multiple weeks.

DFPP is solved every week for a similar set of farmers. Two approaches to the probability of loss in a single week can be defined for these farmers: individualistic and collectivistic. In the individualistic approach, each party wishes to minimize the deviation that affects them each week. A party in the context of DFPP is any entity with a unique interest in optimizing part of the objective function, specifically a farmer or the poultry company itself. Each farmer is interested in minimizing the deviation from optimality of their own production. The poultry company is interested in balancing production across each day and minimizing the maximum average weight difference between two flocks collected in the same day. This means that DFPP has $|J| + 1$ parties with conflicting interests which must be balanced. An individualistic party seeks to minimize their loss in a single week, so they do not find individual solutions which cause them to occasionally sustain a much larger loss than the other parties acceptable, even if doing so increases their expected value across a long time horizon during which many iterations of DFPP are solved. Conversely, a collectivistic party seeks to minimize the total loss of all parties in a single week. By accepting that they may occasionally sustain a much larger loss than the other parties, they may increase their expected value across a long time horizon during which many iterations of DFPP are solved.

The [DIP] and [DWGP] formulations both take the collectivistic approach. While the benefits of this approach should certainly be considered, in reality, farmers may prefer an individualistic approach. Accordingly, a formulation should be designed to suit this preference and enable a quantitative comparison of the approaches.

### 3.4.3 Minmax Goal Program

Minmax, or Chebyshev, goal programming (MGP) was first introduced by Flavell [28] in 1976. It defines an achievement function for a GP framework which seeks to minimize the maximum deviation, scaled by the normalization constant $k$, from any goal. This is a useful GP formulation because it prioritizes optimizing the worst-case scenario. MGP is ideal for the farmers who prefer an individualistic approach because it ensures that whichever party must accept the largest deviation from optimality could not be assigned a better solution without forcing another party to accept an even larger deviation from optimality. In the example case with 15 farmers provided earlier, a MGP formulation would choose to schedule 15 farmers to each produce at 98% rather than scheduling 14 of them to produce at 99% and the last at 85%. The MGP formulation of DFPP is presented below and henceforth referred to as [DMGP].

$$
\begin{aligned}
\text{[DMGP]: min} \quad & \phi && && (3.38)\\
\text{s.t.} \quad & \frac{z_t^{1+} + z_t^{1-}}{k_1} \leq \phi && \forall t \in T && (3.39)\\
& \frac{z_{i'it}^{2+}}{k_2} \leq \phi && \forall i' \in I, i \in I, t \in T && (3.40)\\
& \frac{z_j^{3+} + z_j^{3-}}{k_3} \leq \phi && \forall j \in J && (3.41)\\
& \text{Constraints (3.19) - (3.25)} && &&\\
& \text{Constraints (3.27) - (3.28)} && &&\\
& \text{Constraints (3.30) - (3.31)} && &&\\
& \text{Constraints (3.33) - (3.35)} && &&\\
& z_t^{1+}, z_t^{1-}, z_{i'it}^{2+}, z_j^{3+}, z_j^{3-} \geq 0 && \forall i' \in I, i \in I, j \in J, t \in T && (3.42)
\end{aligned}
$$

The objective function of [DMGP] is presented in Equation (3.38) alongside the additional constraints it brings to the problem in Constraints (3.39) - (3.41). These constraints ensure that $\phi$ assumes the value of the greatest deviation. As in [DWGP], a normalization constant $k$ has been used to allow the deviations to be compared quantitatively. The rest of the formulation is identical to the constraints presented in [DWGP].

Generating solutions with [DMGP] solves the problems presented by [DIP] and [DWGP] solutions. By using a GP formulation to allow some constraints to be violated, [DMGP] avoids the possibility of having no feasible solution that is presented by [DIP]. By specifically using a MGP formulation to minimize the maximum deviation, [DMGP] avoids assuming the collectivistic nature of [DIP] and [DWGP], lowering risk for all parties involved.

While an inability to generate a feasible solution is not a desirable trait in a formulation for DFPP, a collectivistic approach should not automatically eliminate a formulation from consideration. If the increase in expected value is sufficient, it may justify the increased risk assumed by each party. The risk and reward associated with this approach must be analyzed, but first, the problem should be considered under uncertainty. The expected value of $w_{it}$ is used to generate solutions in [DIP], [DWGP], and [DMGP], but it is a prediction which carries an inherent degree of uncertainty rather than being a known value. Adaptations to the formulations presented in this chapter should be considered to account for this uncertainty.

# Chapter 4

# Stochastic Flock Procurement Problem

This chapter discusses SFPP, a modified version of DFPP which acknowledges the uncertainty inherent in the weight predictions. First, the modified problem is described as a two-stage stochastic optimization problem. Next, changes or additions to notation are listed. Finally, modified versions of [DIP], [DWGP], and [DMGP] are proposed which address SFPP.

## 4.1 Notation

The following notations are used to model the stochastic problem. Only notations that have changed from or been added to the deterministic problem are mentioned.

*Sets:*

$S = \{1, 2, ..., |S|\}$ The set of scenarios, indexed by $s$.

*Decision Variables:*

$x_{its}$ A binary variable, 1 if flock $i$ is picked up on day $t$ in scenario $s$, 0 otherwise.

$r_{i'its}$ A binary variable, 1 if flocks $i'$ and $i$ are picked up on the same day $t$ in scenario $s$, 0 otherwise.

*Parameters:*

$w_{its}$ The expected average weight of flock $i$ on day $t$ of scenario $s$ in kg.

$p_s$ The probability of scenario $s$.

$\psi$ The weight of the feasibility penalty function used in SRO.

*Intermediate Variables:*

$\phi_s$ — The maximum deviation in scenario $s$ for the stochastic MGP.

$\theta^1_{ts}$ — Goal 1 on day $t$; the total amount of chicken, in kg, produced on day $t$ in scenario $s$.

$\theta^2_{i'its}$ — Goal 2 for flocks $i'$ and $i$ on day $t$ in scenario $s$; the difference between the average weights of flocks $i'$ and $i$ if they are both scheduled on day $t$ and 0 otherwise.

$\theta^3_{js}$ — Goal 3 for farmer $j$ in scenario $s$; the proportion of $q_j$ farmer $j$ expects to produce in total.

$z^{1+}_{ts}$ — The amount by which goal 1 exceeds target amount 1 on day $t$ in scenario $s$.

$z^{1-}_{ts}$ — The amount by which target amount 1 exceeds goal 1 on day $t$ in scenario $s$.

$z^{2+}_{i'its}$ — The amount by which goal 2 exceeds target amount 2 between flock $i'$ and flock $i$ on day $t$ in scenario $s$.

$z^{2-}_{i'its}$ — The amount by which target amount 3 exceeds goal 2 between flock $i'$ and flock $i$ on day $t$ in scenario $s$.

$z^{3+}_{js}$ — The amount by which goal 3 exceeds target amount 3 for farmer $j$ in scenario $s$.

$z^{3-}_{js}$ — The amount by which target amount 3 exceeds goal 3 for farmer $j$ in scenario $s$.

$z^1_{ts}$ — SRO penalty amount for weight balancing on day $t$ in scenario $s$.

$z^2_{i'its}$ — SRO penalty amount for exceeding the maximum average weight difference between flock $i'$ and flock $i$ on day $t$ in scenario $s$.

$z^3_{js}$ — SRO penalty amount for deviating from production goal $q_j$ in scenario $s$.

## 4.2 Problem Description

DFPP was previously described in Chapter 3. While the proposed solution methods are useful given the assumptions of DFPP, the quality of their solutions may degrade if the assumption of known future weights is relaxed. This relaxation acknowledges the inherent variability present in predicted weight values. DFPP is optimized only for the expected scenario, so it might not remain optimal if a scenario other than the average one is realized. The quality of the solution can be improved by reducing the number of decisions made under uncertainty.

Two sets of decision variables are currently calculated under uncertainty: $x_{it}$, a binary variable which dictates when each flock is picked up, and $y_{gt}$, a binary variable which dictates when collection teams are sent to different geographic areas for pickup. From a practical perspective, $x_{it}$ can be determined any time before the trucks are sent out to collect the flocks, but $y_{gt}$ must be determined far enough in advance for the transportation department to make arrangements for drivers and catchers to work that day.

Assume that average flock weight predictions can be made with certainty no more than $|T|$ days in advance, but predictions with uncertainty can be made up to $2|T|$ days in advance. Also assume that geographic clusters must be chosen $|T| + 1$ days before the start of the week to give the transportation department time to coordinate drivers and catchers. Consider Figure 4.1, a timeline of the SFPP planning process.

| Day | -\|T\| | ... | -1 | 0 | 1 | ... | \|T\| |
|---|---|---|---|---|---|---|---|
| Activity | Determine geographic clusters | | | Determine flock collection dates | Collect flocks | | |

Figure 4.1: A timeline of the planning process for SFPP.

Figure 4.1 shows that the decision maker must determine $y_{gt}$, the geographic clusters to be visited each day, on day $-|T|$. The day each flock must be collected, $x_{it}$, does not need to be determined until day 0. Because day 0 is $|T|$ days before the last day, the average flock weights can be known with certainty when $x_{it}$ is determined.

SFPP can be modeled as a two-stage stochastic program. In the first stage, the geographic areas which need to be visited each day are determined, still bound by the restrictions that certain areas cannot be scheduled on the same day. $y_{gt}$ is determined while the flock weights $w_{its}$ can be predicted but are still uncertain. Uncertainty in the flock weight is captured through a finite number of scenarios, indexed by $s \in S$, so the subscript $s$ is added to $w_{it}$ to indicate a range of possible outcomes. The second stage chooses which barn floors will be collected each day, deciding $x_{its}$ for each scenario $s$. Once the scenario has been identified on day 0 based on which set of weights $w_{its}$ most closely matches the week's predicted weights, the appropriate set of $x_{its}$ will determine which flocks are picked up each day.

## 4.3 Robust Optimization

The first formulation to adapt is [DIP]. However, some additional modifications must be made for this formulation to be worthy of consideration. A prominent concern when using [DIP] to address DFPP is that its constraints limit it too much, often leaving no feasible solution to be found as a result. If [DIP] is reformulated as a stochastic program with no other modifications, this concern is not addressed. SRO can be used to reformulate it and allow some of its constraints to be violated at a penalty cost.

The essential part of SRO to establish is the composition of its objective function, which takes the form

$$\sigma(y, x_1, x_2, ..., x_s) + \psi\rho(z_1, z_2, ..., z_s). \tag{4.1}$$

Appropriate $\sigma(.)$ and $\rho(.)$ functions must be chosen. The feasibility penalty weight $\psi$ can be decided when numerical analysis begins. Considering $\xi_s$ as the objective function of [DIP] under scenario $s$, a good choice is its expected value $\sigma(.) = \sum_{s \in S} p_s \xi_s$. This function is consistent with the collectivistic approach [DIP] is originally described as taking. A common and effective choice for the feasibility penalty function is $\rho(.) = \sum_{s \in S} |z_s|$. Considering these functions, [DIP] can be reformulated using SRO. The reformulation is presented below and henceforth referred to as [RP].

$$\begin{aligned}\text{[RP]: max} \quad & \sum_{i\in I}\sum_{t\in T}\sum_{j\in J}\sum_{s\in S}\frac{e_{ij}n_i w_{its}x_{its}}{q_j}p_s - \\ & \psi\sum_{s\in S}p_s(\sum_{t\in T}|\frac{z^1_{ts}}{k_1}| + \sum_{i'\in I}\sum_{i\in I}\sum_{t\in T}|\frac{z^2_{i'its}}{k_2}| + \sum_{j\in J}|\frac{z^3_{js}}{k_3}|)\end{aligned} \tag{4.2}$$

$$\text{s.t.}\frac{|T|\sum_{i\in I}n_i w_{its}x_{its}}{\sum_{t\in T}\sum_{i\in I}n_i w_{its}} + z^1_{ts} = \frac{1}{|T|} \qquad \forall t\in T, s\in S \tag{4.3}$$

$$r_{i'its}|w_{i'ts} - w_{its}| + z^2_{i'its} \leq 0.20 \qquad \forall i'\in I, i\in I, t\in T, s\in S \tag{4.4}$$

$$\sum_{i\in I}\sum_{t\in T}\frac{e_{ij}n_i w_{its}x_{its}}{q_j} + z^3_{js} = 1.02 \qquad \forall j\in J, s\in S \tag{4.5}$$

$$r_{i'its} \leq x_{i'ts} \qquad \forall i'\in I, i\in I, t\in T, s\in S \tag{4.6}$$

$$r_{i'its} \leq x_{its} \qquad \forall i'\in I, i\in I, t\in T, s\in S \tag{4.7}$$

$$r_{i'its} \geq x_{i'ts} + x_{its} - 1 \qquad \forall i'\in I, i\in I, t\in T, s\in S \tag{4.8}$$

$$\sum_{t\in T}x_{its} = 1 \qquad \forall i\in I, s\in S \tag{4.9}$$

$$\sum_{i\in I}x_{its}f_{gi} \leq |I|y_{gt} \qquad \forall g\in G, t\in T, s\in S \tag{4.10}$$

$$y_{g't} + y_{gt} \leq c_{g'g} + 1 \qquad \forall g'\in G, g\in G, t\in T \tag{4.11}$$

$$x_{its}, y_{gt}, r_{i'its} \in \{0,1\} \qquad \forall g\in G, i'\in I, i\in I, t\in T, s\in S. \tag{4.12}$$

Equation (4.2) is the objective function of [RP]. It calculates the expected value of farmer quota fulfillment across all scenarios and penalizes feasibility violations according to their absolute value. These violations are weighted according to their expected probabilities and additionally by the term $\psi$, whose magnitude determines the tradeoff between solution robustness and model robustness. Solution robustness ensures the solution is nearly optimal according to $\sigma(.)$ for any scenario, whereas model robustness ensures the solution is nearly feasible according to $\rho(.)$ for any scenario. In the case of [RP] specifically, more solution robustness means the optimal quota fulfillment of the farmers has been considered more important, whereas more model robustness means that more weight has been placed on minimizing the deviations from the scheduling guidelines. An increase in $\psi$ implies an increase in model robustness and a corresponding decrease in solution robustness. The reverse is also true. Constraint (4.3) adds a penalty term $z$ to its [DIP] counterpart, Constraints (3.15) - (3.16), and simplifies them as one constraint by assuming that the tolerance $\epsilon = 0$ because the constraint can now be violated at a penalty cost. Constraint (4.4) adds a penalty term $z$ to its [DIP] counterpart, Constraint (3.18). Constraints (4.6) - (4.8) constrain $r_{i'its}$ and are exactly as they were previously defined in Constraints (3.19) - (3.21), except the decision variable $r_{i'it}$ is now additionally indexed by $s$. Similarly, the decision variable $x_{it}$ becomes $x_{its}$, modifying Constraint (3.22) to Constraint (4.9) to ensure each flock is picked only once in the set of days $T$ in each scenario $s \in S$. Constraint (3.23) is also modified to Constraint (4.10) to ensure that geographic areas are scheduled in the same day only when this is allowed. Constraint (4.11) is the same as Constraint (3.24), ensuring that flocks located in a geographic area can be picked only when this area is visited. Finally, Constraint (4.12) sets $x_{its}$, $y_{gt}$, and $r_{i'its}$ as binary variables.

While Constraint (4.5) also adds a penalty term to its counterpart, Constraint (3.17), it can be noted that it additionally changes the sign of the constraint from $\leq$ to $=$. This change is necessary because $\rho(.)$ has been added to the objective function. If the sign of the constraint is left as $\leq$, $z^3_{js}$ will remain 0 when a farmer underproduces but will increase when that farmer overproduces. Even accounting for the fact that overproducing naturally increases $\sigma(.)$ and underproducing naturally decreases $\sigma(.)$, the imbalance forces overproduction to penalize the objective function $\psi - 1$ times more harshly than underproduction despite the fact that they are equally undesirable. For example, consider a farmer $j$ who is currently producing at 102% of their production goal $q_j$. If this farmer instead produces at 103% of $q_j$, the objective function will increase by $0.01p_s$ in $\sigma(.)$ and decrease by $0.01p_s\psi$ in $\rho(.)$ whether the sign used is $\leq$ or $=$. The net impact of this 1% increase on the objective function is that it decreases by $0.01p_s(\psi - 1)$. Conversely, consider if this farmer produces at 101% of $q_j$. If the sign in the constraint is $\leq$, $z^3_{js} = 0$. The impact on $\sigma(.)$ is that it will decrease by $0.01p_s$ and the impact on $\rho(.)$ is 0. The net impact of the 1% decrease on the objective function is that it decreases by $0.01p_s$. However, if the sign in the constraint is $=$, $z^3_{js} = 0.01p_s$, so the impact is that the objective function will decrease by $0.01p_s$ in $\sigma(.)$ and decrease by $0.01p_s\psi$ in $\rho(.)$. The net impact of the 1% decrease on the objective function is that it decreases by $0.01p_s(\psi + 1)$. As long as $\psi$

does not assume a value close to 1, changing the sign in Constraint (4.5) from $\leq$ to $=$ causes overproduction and underproduction to be penalized more evenly.

The [RP] reformulation effectively addresses the most major concern with [DIP], namely the risk of not finding a feasible solution. Because [RP] is a formulation which seeks primarily to maximize the sum of expected farmer quota fulfillment, it is a collectivistic formulation. Its performance must be evaluated to test the risk it poses of a large maximum deviation and the corresponding reward it may carry of higher expected farmer quota fulfillment. Before this assessment can be done, however, stochastic reformulations of [DWGP] and [DMGP] must still be defined.

## 4.4 Goal Programming

The multiobjective optimization framework provided by Ustun and Demirtas [82, 83] can be easily adapted to suit a two-stage stochastic program. Adaptations of the GPs defined for DFPP in Chapter 3 are presented in this section. The purpose and value of WGP and MGP formulations for DFPP has already been stated in Chapter 3, and these GP methods remain useful when the problem is stochastic.

The first objective function to reformulate is the weight balancing function. It is defined by:

$$\min \quad \sum_{s\in S}\sum_{t\in T} p_s(z_{ts}^{1+} + z_{ts}^{1-}) \tag{4.13}$$

$$\text{s.t.} \quad \theta_{ts}^1 = \frac{|T| \sum_{i\in I} n_i w_{its} x_{its}}{\sum_{t\in T}\sum_{i\in I} n_i w_{its}} \qquad \forall t \in T, s \in S \tag{4.14}$$

$$\theta_{ts}^1 - z_{ts}^{1+} + z_{ts}^{1-} = \frac{1}{|T|} \qquad \forall t \in T, s \in S. \tag{4.15}$$

Minimizing the deviations $z_{ts}^{1+}$ and $z_{ts}^{1-}$ will get $\theta_{ts}^1$ as close to its target value $\frac{1}{|T|}$ as possible. The objective function balances the deviations from each scenario $s$ by weighting it according to the probability of occurrence of that scenario, $p_s$. $\theta_{ts}^1$ quantifies the proportion of weight produced on day $t$ to the total weight produced across all days $t \in T$ in scenario $s$.

The next objective function to reformulate is the maximum average weight difference function. It is defined by:

$$\min \quad \sum_{s\in S}\sum_{i'\in I}\sum_{i\in I}\sum_{t\in T} p_s z^{2+}_{i'its} \tag{4.16}$$

$$\text{s.t.} \quad \theta^2_{i'its} = r_{i'its}|w_{i'ts} - w_{its}| \qquad \forall i' \in I, i \in I, t \in T, s \in S \tag{4.17}$$

$$\theta^2_{i'its} - z^{2+}_{i'its} \leq 0.20 \qquad \forall i' \in I, i \in I, t \in T, s \in S. \tag{4.18}$$

Each instance of $\theta^2_{i'its}$ should be penalized if it exceeds 0.20. $\theta^2_{i'its}$ quantifies the difference in average weight per chicken between flock $i'$ and flock $i$ on day $t$ in scenario $s$ if they are both collected that day, and takes a value of 0 otherwise. A negative deviation term is not included in the objective function because the solution should not be penalized for pairing flocks with average weights per chicken that differ by less than 0.20 kg. The objective function balances the deviations from each scenario $s$ by weighting it according to the probability of occurrence of that scenario, $p_s$.

Finally, the third objective function to consider is maximizing the proportion of the production goal $q_j$ produced by each farmer $j$ across all scenarios $s \in S$. Similar to how it is done in the deterministic formulation, the goal is modified slightly from the [RP] to attempt to satisfy as many farmers as possible rather than directly trying to maximize their production. As previously noted, farmers are satisfied when producing between 99-102% of their production quota, so this objective function can be defined by:

$$\min \quad \sum_{s\in S}\sum_{j\in J} p_s(z^{3+}_{js} + z^{3-}_{js}) \tag{4.19}$$

$$\text{s.t.} \quad \theta^3_{js} = \sum_{i\in I}\sum_{t\in T} \frac{e_{ij} n_i w_{its} x_{its}}{q_j} \qquad \forall j \in J, s \in S \tag{4.20}$$

$$\theta^3_{js} - z^{3+}_{js} \leq 1.02 \qquad \forall j \in J, s \in S \tag{4.21}$$

$$\theta^3_{js} + z^{3-}_{js} \geq 0.99 \qquad \forall j \in J, s \in S. \tag{4.22}$$

Each instance of $\theta^3_{js}$ is penalized if the farmer's total production is under $0.99q_j$ or if it exceeds $1.02q_j$. $\theta^3_{js}$ quantifies the anticipated total weight of chicken produced by farmer $j$ across all days $t \in T$ for each scenario $s \in S$. Having defined the objective functions which must be balanced, and considering the constraints previously defined in [RP], goal programs can now be created.

### 4.4.1 Weighted Goal Program

It is straightforward to convert [DWGP] to a two-stage stochastic formulation, simply changing the variables which must now be indexed by scenario $s \in S$ and taking the expected value of the objective function across all scenarios. The formulation is presented below and henceforth referred to as [SWGP].

$$\text{[SWGP]: max} \quad \sum_{s \in S} p_s \left( \lambda_1 \sum_{t \in T} \frac{z_{ts}^{1+} + z_{ts}^{1-}}{k_1} + \lambda_2 \sum_{i' \in I} \sum_{i \in I} \sum_{t \in T} \frac{z_{i'its}^{2+}}{k_2} + \lambda_3 \sum_{j \in J} \frac{z_{js}^{3+} + z_{js}^{3-}}{k_3} \right) \tag{4.23}$$

s.t. Constraints (4.6) - (4.12)

Constraints (4.14) - (4.15)

Constraints (4.17) - (4.18)

Constraints (4.20) - (4.22)

$$z_{ts}^{1+}, z_{ts}^{1-}, z_{i'its}^{2+}, z_{js}^{3+}, z_{js}^{3-} \geq 0 \quad \forall i' \in I, i \in I, j \in J, t \in T, s \in S \tag{4.24}$$

The [SWGP] formulation is composed primarily of constraints that have previously been established. The objective function presented in Equation (4.23) is the expected value of the sum of the deviations weighted by $\lambda$ across all scenarios $s \in S$. Each deviation is scaled by a normalization constant $k$ to allow their summation despite having different units. Constraints (4.6) - (4.12) carry over from the [RP] formulation with no modification, while the constraints identified from (4.14) to (4.22) can be used because they have been designed specifically to fit into a goal program. Constraint (4.24) provides a non-negativity constraint for the deviations from each goal.

The [RP] formulation focuses specifically on maximizing production. It allows violation of the constraints which might otherwise prevent it from generating a feasible solution and penalizes the objective function according to these violations, but its objective function is still focused directly on maximizing production. [SWGP] takes a more balanced approach by trying to minimize any deviations from the characteristics that make a solution desirable. While [SWGP] is still a collectivistic approach because its objective function evaluates deviations from the goals according to the total of those deviations rather than considering their size individually, it generates a solution which treats each goal with equal importance rather than constructing an objective function focused on maximizing quota fulfillment and only incidentally penalizing it for deviations from the other goals.

[SWGP] is expected to create a more balanced solution than [RP] because it places as much importance on $\theta^1_{ts}$ and $\theta^2_{i'its}$ as it does on $\theta^3_{js}$, which is anticipated to result in a solution with less deviation from the weight balancing or maximum average weight difference goals than [RP] would produce. If both are determined to be acceptable collectivistic approaches and the farmers wish to take a collectivistic approach, the decision maker determines how much risk of a large deviation they feel is acceptable to assume for their goals while continuing to efficiently collect and process every flock. The risk and reward associated with these formulations can be observed through numerical analysis, but a MGP formulation must be constructed first.

### 4.4.2 Minmax Goal Program

Just as [DWGP] can be converted to [SWGP], so too can [DMGP] be converted to a stochastic MGP by appropriately indexing scenario-dependent parameters and variables by $s \in S$ and taking the expected value of the maximum deviation in each scenario as the objective function. The formulation is presented below and henceforth referred to as [SMGP].

$$\text{[SMGP]: min} \quad \sum_{s \in S} p_s \phi_s \tag{4.25}$$

$$\text{s.t.} \quad \frac{z^{1+}_{ts} + z^{1-}_{ts}}{k_1} \leq \phi_s \qquad \forall t \in T, s \in S \tag{4.26}$$

$$\frac{z^{+}_{2i'its}}{k_2} \leq \phi_s \qquad \forall i' \in I, i \in I, t \in T, s \in S \tag{4.27}$$

$$\frac{z^{+}_{3js} + z^{-}_{3js}}{k_3} \leq \phi_s \qquad \forall j \in J, s \in S \tag{4.28}$$

Constraints (4.6) - (4.12)

Constraints (4.14) - (4.15)

Constraints (4.17) - (4.18)

Constraints (4.20) - (4.22)

$$z^{1+}_{ts}, z^{1-}_{ts}, z^{2+}_{i'its}, z^{3+}_{js}, z^{3-}_{js} \geq 0 \quad \forall i' \in I, i \in I, j \in J, t \in T, s \in S. \tag{4.29}$$

The objective function of [SMGP] is presented in Equation (4.25) alongside the additional constraints it brings to the problem in Constraints (4.26) - (4.28). These constraints ensure that $\phi_s$ assumes the value of the greatest deviation in scenario $s$, and the objective function is the expected value of this deviation across all scenarios. It can be noted that this problem can be robustified by minimizing the maximum deviation over all scenarios, in which case $\phi$ would not be subscripted by $s$. It is likely that such an approach would produce an unnecessarily conservative solution. While each party affected by the solution wishes to minimize their own risk of a large deviation, one outlier scenario $s$ for which a good solution could not be generated could make the whole solution much worse despite only having a small probability of occurrence. As in [SWGP], a normalization constant $k$ is used to allow the deviations to be compared quantitatively. The rest of the formulation is identical to the constraints presented in [SWGP], carrying Constraints (4.6) - (4.12) over from the [RP] formulation with no modification and using the constraints identified between Constraints (4.14) and (4.22) because they have been designed specifically to fit into a goal program.

[SMGP] and its deterministic counterpart [DMGP] are individualistic formulations. [SMGP] seeks to minimize the maximum expected deviation across all scenarios $s \in S$, and it is expected to be preferable to farmers who are averse to loss in a single week even if it leads to gain across the quota period. This individualistic approach must now be compared against the proposed collectivistic approaches [RP] and [SWGP]. They should be tested to observe if an individualistic approach can be expected to produce a significantly lower maximum risk, if a collectivistic approach can be expected to produce significantly higher expected quota fulfillment, and what the magnitude of the exchange is if one exists between maximum risk and optimal quota fulfillment.

# Chapter 5

# Case Study

With several formulations now developed to solve the deterministic and stochastic forms of this problem, their efficacy should be compared by attempting to quantify the tradeoffs between the individualistic and collectivistic formulations. Before this can be done, the models must be tested with real data. The company that provided the data is kept anonymous by referring to them as ABC Poultry. The weight forecasting process is discussed in detail, eventually deciding to model growth rates as a normally distributed random variable and predict them with an 8-point moving average. The process of generating the other parameters that were not directly provided by ABC Poultry data is also discussed, including location clusters, farmer weight goals, normalization constants, SRO penalty weights, and GP weights. Methods of scenario generation are considered for the stochastic problem, deciding to use the random sampling method to create sets of 20 scenarios and run multiple trials to ensure stability. Finally, the deterministic and stochastic problems are tested with their respective formulations, finding that in both instances the collectivistic approach creates a risk of suboptimal production in one week without offering the benefit of a noticeable increase in expected production optimality. The experiment concludes that [DMGP] should be used for the deterministic problem and [SMGP] should be used for the stochastic problem.

## 5.1 Case Study Details

This research is sponsored by a Mitacs Accelerate grant. It contains a problem that was solved in partnership with an industry client, ABC Poultry, as part of a design project. As is typical of the design process in industry, some work had to be done to define the problem more clearly before a solution could be proposed and agreed upon by all stakeholders. Several techniques were used to define the problem, including a series of interviews, a precedence diagram, supplier-input-process-output-customer

(SIPOC) analysis, and finally a project charter.

The first step was to have a series of conversations with the ABC Poultry procurement manager, who is responsible for the process, to gain a better understanding of the scheduling procedures and their decision-making role in them. A list of questions was prepared in advance for each interview to create a structure for the conversation and ensure certain things were discussed, but it was not intended to be an exhaustive list of the questions to be asked. After three of these sessions, enough information had been gathered to create a precedence diagram.

A precedence diagram was the next logical step because it would encapsulate on a single page exactly what had been gleaned from the interviews about the procurement process. This was helpful for two reasons: it could be provided to ABC Poultry for verification so any changes could be made, thus lowering the risk of miscommunication affecting the design process, and it was an efficient way to convey information about the process to stakeholders from outside the company. It identified three major decision makers in a flock's life cycle - a hatchery, a farmer, and ABC Poultry - and showed the order of events that would happen to the flock from beginning to end while denoting which party had control over each event and when each decision had to be made. Once all parties had reviewed and agreed upon the precedence diagram, a SIPOC analysis was conducted.

SIPOC analysis was used to examine the processes driven by ABC Poultry's decisions. The supplier and input sections were used to identify parties who provided data used for decision-making and what that data was. The process section identified which decisions were being made with those data sets. Finally, the output and customer sections identified the information produced by the processes and where the information had to go. This information was also reviewed with ABC Poultry's procurement manager. Once the SIPOC diagram had been adjusted, it provided a valuable framework showing which data would be necessary for completion of the project.

Finally, a project charter was created in consultation with all of the project's stakeholders. By defining a business case, problem statement, project scope, goals, and a contact plan, the project's purpose and boundaries were very clearly defined. Regular reference to this document ensured that the project accomplished everything it set out to do without wasting any time or resources by adding features or solving problems outside the project scope. By clearly defining which things the project would and would not do, the project charter was influential on the generation of a complete, detailed problem description.

The tool developed has successfully been integrated into ABC Poultry's procurement planning process. The tool serves three major purposes: centralizing procurement data, tracking hatchery placement cycles, and scheduling flocks for collection. Centralizing the procurement data is an important feature because different sets of data were previously stored in many different files, requiring tedious cross-referencing to regularly generate reports. Tracking hatchery placement cycles is important because it allows ABC Poultry to see which barns are expected to produce two flocks in a sixteen-week quota period and which are expected to produce three. This allows the user to ensure a similar number of flocks can be scheduled each week, potentially recommending that the hatcheries move barns forward or back a week in their placement schedule as needed. While ABC Poultry does not have the authority to decide exactly when the barns will receive their flocks, their requests regarding moving barns forward or backward by a week are generally respected by the hatcheries. Due to the previously decentralized nature of their data, this was a time-consuming process for ABC Poultry before the tool centralized the data and automated the process of identifying which barns would be ready for new flocks each week. Finally, a feature to schedule flocks for collection is included with the tool. It allows the user to define a date range and minimum age for flock collection, then see weight forecasts for every flock that will be eligible for collection in the date range. The user confirms which flocks will be collected in the date range and the target average weight for each flock, then the program generates a schedule. This scheduling feature solves the problem at the core of this thesis. Before implementing the tool, ABC Poultry would retrieve the collection of flocks and develop a solution to the scheduling problem manually, first making a manual prediction about the average weight per chicken of each flock.

The purpose of the tool, defined in consultation with ABC Poultry, was to formalize the procurement planning process and minimize the number of sources to be consulted before making procurement decisions. Automation was encouraged where possible when formalizing aspects of the process. Improvement of scheduling performance was intentionally not identified as a goal because it would be too difficult to measure given the records that had been kept. A direct measurement of the savings provided by the tool could not be performed because ABC Poultry did not provide financial data, but because the tool eliminates several previously time-consuming tasks through automation and centralization of data, making these tasks trivial or allowing them to be delegated, ABC Poultry considers the tool a success.

## 5.2 Weight Forecasting

While it is straightforward to consult the data to determine many of the parameters in the goal program, weight predictions do not come readily available with the interim weights. These predictions must be made based on the characteristics of the data. ABC Poultry's existing weight prediction procedure is manual: their procurement manager will look at the last two to three average flock sizes and make an estimate as a multiple of 0.05. The success of this method has been tracked over several years and shown to be approximately 85% according to the company's success metric, which considers a prediction useful if it is within 100 grams of the true value and not useful otherwise. The goal of this section is to match the efficacy of the existing forecasting method while taking a more systematic approach, allowing the weight forecasting process to be automated rather than relying on intuition.

### 5.2.1 Growth Model Shape

The first step to predicting weights is to understand how the chickens are expected to grow. ABC Poultry uses two breeds of chickens. Both companies provide an expected growth chart for their chickens based on extensive observation, noting their expected weight day-by-day. For the purposes of this analysis, these growth charts are treated as the true population mean. The expected weight of the chickens every day for the first six weeks of their lives is graphed in Figure 5.1.

Farmers typically send ABC Poultry an update on the average weight of their flock when the flock is 25-30 days old, with most flocks slaughtered at 35-40 days old. The growth of the flock must be predicted from the provided interim weight of 25-30

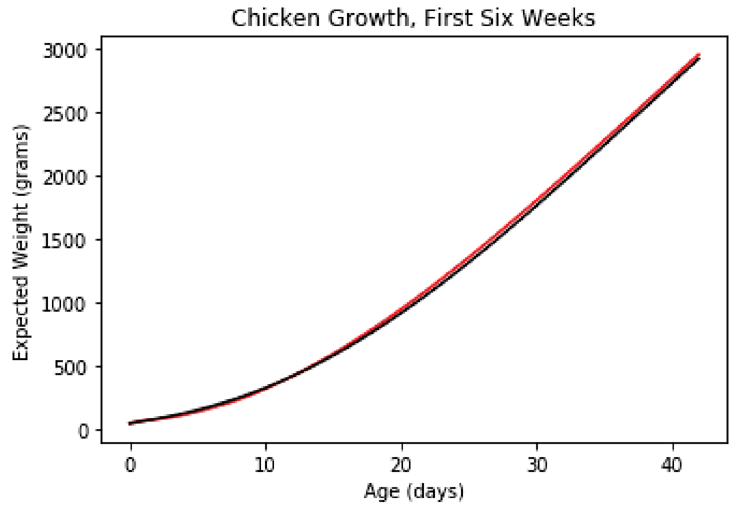


Figure 5.1: Daily expected weights of the two chicken breeds from 0 to 42 days old. One breed is shown in red, the other in black.

days. The expected weights of the two breeds between three and six weeks old are shown in Figure 5.2.

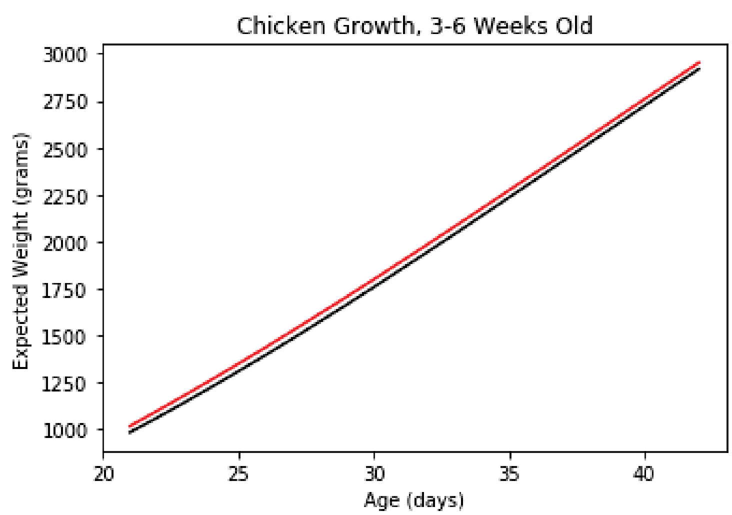


Figure 5.2: Daily weights of the two chicken breeds from 21 to 42 days old. One breed is shown in red, the other in black.

It can be observed that while one breed is expected on average to be slightly heavier at three weeks old, both breeds appear to grow at very similar rates between three and six weeks old. It can also be observed that linear regression is an effective approximation of the function. A linear regression based on ABC Poultry data between 2011 and 2017 can be observed in Figure 5.3.

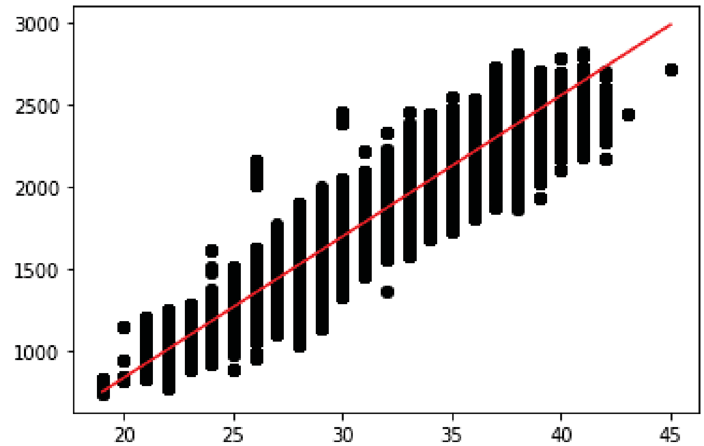


Figure 5.3: Linear regression representing average flock growth between 2011 and 2017.

The linear function yielded by this regression, $\hat{y} = 86x - 876$, is shown in red and the 120270 points used for the regression are shown in black. At first inspection, the function appears to fit well. This is confirmed by an R-squared value of $R^2 = 0.89$, showing that the linear regression explains approximately 89% of the model's variance. The efficacy of this model can be tested by using it to make predictions about the future. Because the model was trained on data from 2011-2017, it can be tested with data from 2018 forward.

ABC Poultry has a simple binary metric to measure proportion of useful predictions based on what they find is relevant to their business practices. Each prediction of a flock's average weight is considered useful if it is within 100 grams of the true value (either higher or lower is acceptable) and not useful otherwise. In the case of the linear regression $\hat{y} = 86x - 876$, 26.62% of predictions made for 2742 flocks processed between 2018 and 2020 are considered useful. This is an underwhelming statistic. It can be improved by remembering the aforementioned interim measurement, which is received before a prediction must be made about a flock. When a data point is provided to associate with the flock, the regression's $b_0$, or intercept, value becomes less necessary. The purpose of the intercept value is to anchor the function somewhere vertically on the graph, whereas the $b_1$, or slope, value determines how fast the chickens are anticipated to grow. When an actual data point is provided, $b_1$ can be used on its own to predict a second $y$-value given a second $x$-value. When this is done (assuming a growth rate of 86 g/day for each flock from the interim weight), 40.59%

of the predictions made for the same 2742 flocks tested earlier are considered useful. This statistic is a noticeable improvement on 26.62%, but should still be improved to achieve the 85% accuracy of the manual forecasting method. To further improve the predictions, the different growth rates of each flock must be analyzed and accounted for in the model.

### 5.2.2 Predicting Growth Rates

Analyzing growth rates is important because, unlike analyzing flock weights directly, it allows the way growth rates change over time to be studied so any patterns can be identified. While weights are influenced by demand and when ABC Poultry decides to slaughter each flock, growth rates are not. These growth rates can be considered as a time series, which provides several opportunities for analysis. The graph of the time series can be observed in Figure 5.4.

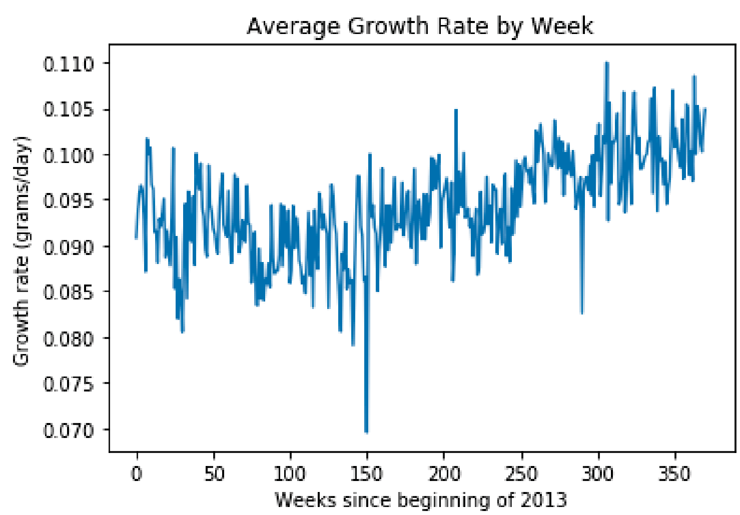


Figure 5.4: Time series representation of average flock growth between 2013 and 2020.

The time series begins in 2013 because data was not logged consistently enough prior to then. Each point in the time series corresponds to the average across one week of all growth rates observed from flocks slaughtered that week. It does appear that growth rates trend in different directions over time, most notably that they have generally trended upward over the last 3-4 years. How these growth rates are related to future growth rates can be represented by graphing their autocorrelation. A graph displaying autocorrelation from a lag of -52 to +52 weeks (a ±1 year window) can be observed in Figure 5.5.

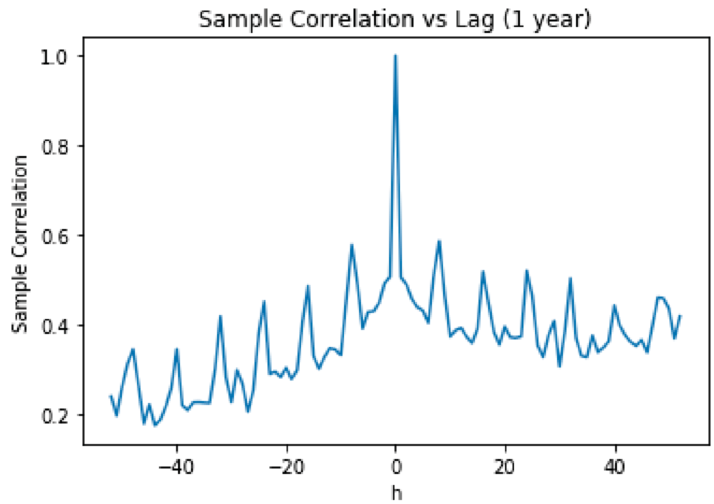


Figure 5.5: Autocorrelation between lags of ±52 weeks.

Some amount of periodicity is immediately evident upon inspection of the graph. It can be observed that the autocorrelation experiences spikes every 8 weeks and does not show any other interesting patterns. While a theoretical autocorrelation graph should be symmetrical, it is understandable for a ±1 year graph to be as asymmetrical as this one with a relatively small sample size of 7 years. Periodicity can be further examined by using a power spectrum, as shown in Figure 5.6.

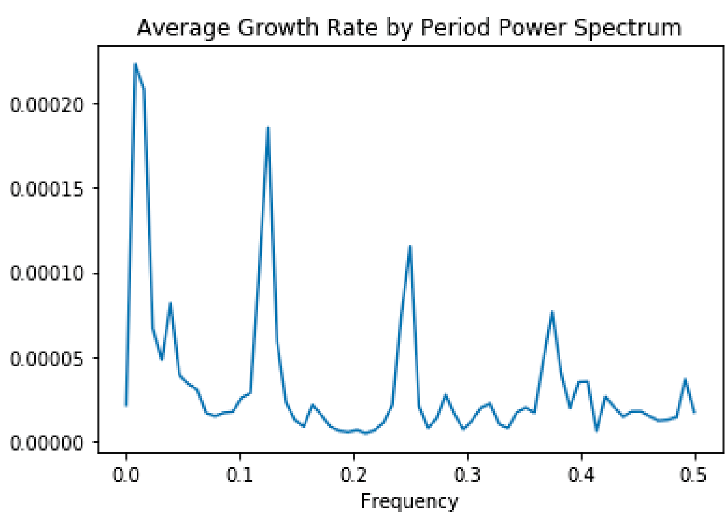


Figure 5.6: Power spectrum showing frequency composition of the growth rate time series.

Ignoring the spike immediately following 0 on the frequency axis, which typically exists on a power spectrum as a product of its resolution (e.g. because the power spectrum has 128 points, it spikes at $f = \frac{1}{128}$), the power spectrum is dominated by an impulse at approximately $f = 0.125$, corresponding to a period of $T = \frac{1}{0.125} = 8$ weeks as observed in the autocorrelation function. Smaller spikes also exist near $f = 0.25$ and $f = 0.375$: as these are multiples of $f = 0.125$, it is likely that they are harmonics of it rather than being indicative of anything meaningful about the time series itself.

Inspection of the farmers' practices shows what might introduce an 8 week period to the growth rates. The farmers partnered with ABC Poultry keep each flock on a different barn floor, and the vast majority of barn floors exist on 8-week cycles. Most flocks spend 5 weeks growing on the barn floor, then are removed and the next 3 weeks are spent cleaning that barn floor before the next flock arrives. Introducing the barn floor as a factor would account for a number of smaller, potentially relevant factors at once: a non-exhaustive list of examples could include feed type or amount, environmental conditions, average stress levels, and any farmer behaviours that could affect growth rates.

Because the barn floor appears to be the only factor to account for in the growth rates, the next logical step is to separate the data according to barn floor. Once sorted, an autocorrelation function can be graphed for each barn floor. Points on these autocorrelations will generally be spaced 8 weeks apart, as there are typically 8 weeks between flocks, so graphing them in the lag window $\pm 7$ is expected to provide a roughly $\pm 1$ year window through which to view the function. The average autocorrelation function across a sample of 203 barn floors is shown in Figure 5.7.

It can be observed that the autocorrelation function quickly drops to 0 and stays close to it. The red lines represent a confidence interval of $\pm \frac{1.96}{\sqrt{N}}$, inside which 95% of non-zero autocorrelations are expected to exist if the process is independent and identically distributed, or i.i.d. Consider $X_t$, the growth rate on a barn floor in period $t$. Its autocovariance at lag $h$ is $\gamma(h) = E(X_{t+h} - \mu)(X_t - \mu)$, which quantifies the relationship between any growth rate $X_t$ and a later growth rate $X_{t+h}$. The autocorrelation at lag $h$ is $\rho(h) = \gamma(h)/\gamma(0)$, quantifying the relationship between $X_t$ and $X_{t+h}$ . Because all of the non-zero-lag autocorrelations from $\rho(-7)$ to $\rho(7)$ are inside the confidence interval, it is assumed that the process is i.i.d.

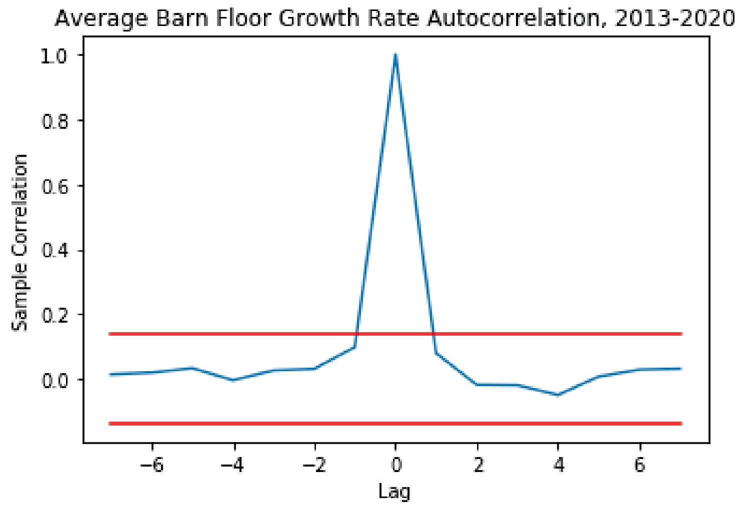


Figure 5.7: Average autocorrelation function of all barn floors from 2013 to 2020.

The next step is to determine the distribution of the process. A histogram illustrates the shape of the data, as shown in Figure 5.8.

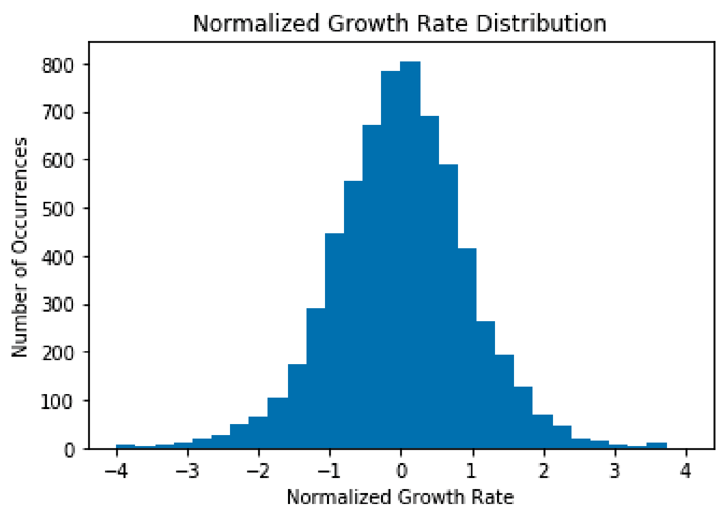


Figure 5.8: Histogram of growth rates observed from each flock on all barn floors standardized by that barn floor's mean and variance, 2013-2020.

The histogram is constructed by standardizing each flock's growth rate according to the mean and variance in that flock's barn floor. Working under the assumption that all barn floors will assume a distribution with the same shape, just having different parameters, standardizing them makes it possible to combine them and judge the shape of their distribution more accurately. As one might anticipate, the growth rates appear to follow a normal distribution. This can be confirmed with a QQ plot, shown in Figure 5.9.

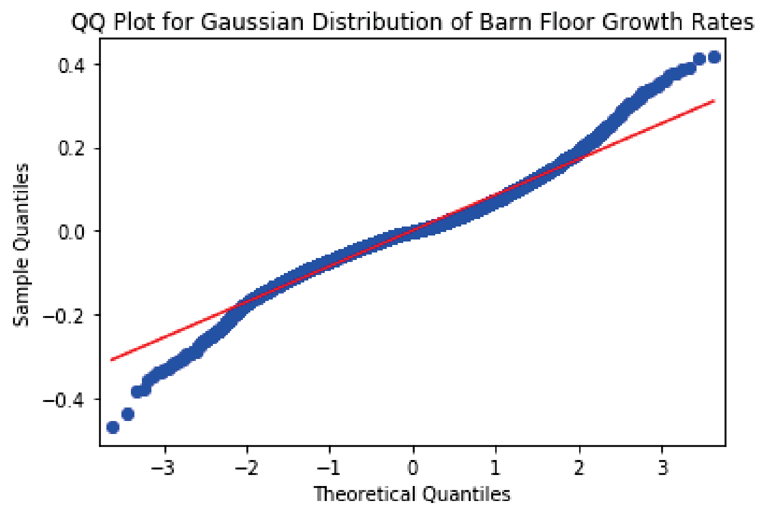


Figure 5.9: QQ plot comparing sample vs theoretical quantiles of growth rate data for Gaussian distribution.

A QQ, or quantile-quantile, plot functions by comparing the theoretical quantiles of a distribution against the sample quantiles of the data set provided. It is an efficient way of visualizing how well a data set fits the normal distribution because the points will form a straight line if the data is normally distributed. While the tails diverge from the expected values as they approach 3 standard deviations from the mean, the rest of the sample and theoretical values match up closely enough that normality is a reasonable assumption. This means that while each barn floor may have different parameters $\mu$ and $\sigma^2$, the sequence of growth rates for each of them is ultimately represented by a random variable $\{X_t\} \sim N(\mu, \sigma^2)$. It is reasonable to make a prediction about a given barn floor with this information as $E(X_t) = \mu$. There is no autocorrelation, so no predictions can be made with respect to how the growth rates will fluctuate about $\mu$ for each future $t$.

The true average growth rate for a barn floor, $\mu$, cannot be known in implementation, so a substitute average $\bar{x}$ must be used as an approximation instead. While it would be ideal to be able to use $\bar{x}$ determined from across the barn floor's history, this is not reasonable in practice. If anything changed on the barn floor that would significantly affect the parameters, it would take years for this change to be meaningfully reflected in $\bar{x}$. Thus, the model must remain responsive to change while still using enough data points that it reasonably attempts to accurately estimate the mean. To that end, the mean is estimated with an 8-point moving average. The number 8 was chosen because it was the largest number that worked with the information about the barns' conditions provided by ABC Poultry. Too many factors affecting the health of the chickens would have changed further back than 8 cycles in the same barn, as things such as food and medication are often changed in unsynchronized sixteen-month rotations. The model can be represented as:

$$\{X_t\} = \frac{\sum_{n=1}^{8} X_{t-n}}{8} \tag{5.1}$$

Thus, predictions can be made for a flock based on growth rates with the following formula:

$$W_{final} = W_{interim} + X_t n_{days} \tag{5.2}$$

Where $W_{final}$ is the final average weight of the flock, $W_{interim}$ is the average weight of the flock when the farmer provides an interim weight at 25-30 days, and $n_{days}$ is the number of days anticipated between the interim weight and the flock's slaughter. When the model is tested for its proportion of useful predictions, just as the variations on the linear regression predictions were tested earlier to find 26.62% and 40.59%, it estimates 84.12% of its predictions are useful. A 99% confidence interval estimates the true proportion of useful predictions this model will produce exists in the range [82.29%, 85.96%]. This is close enough to satisfy the goal of 85% accuracy.

### 5.2.3 Inspecting Residuals

Once a time series model has been chosen, it is important to examine its residuals when compared to the time series itself. If its residuals demonstrate any patterns, this may be a sign that the time series model should account for another factor. While many time series models are technically examined in this section, using the assumption about standardizing their data from Figure 5.8, a histogram of their standardized residuals can also be graphed in Figure 5.10.

Based on the normal distribution provided for reference on top of the histogram, it can be observed that the residuals are also normally distributed with a mean of 0.

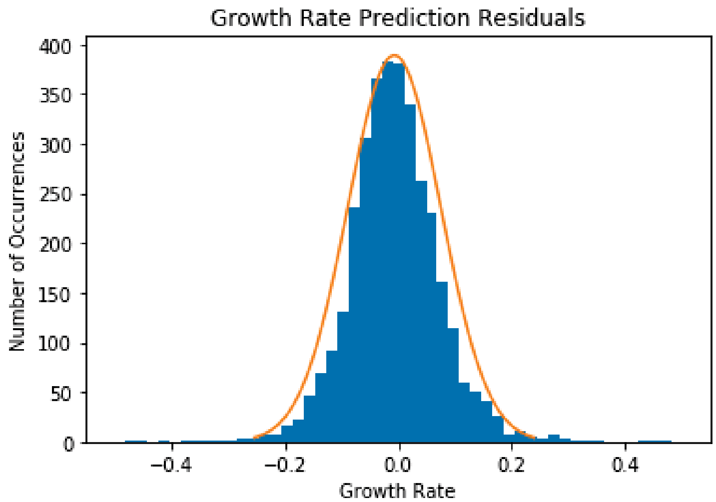


Figure 5.10: Histogram of standardized residuals from all growth rate predictions, 2013-2020.

Their autocorrelation can also be checked to ensure their independence, as shown in Figure 5.11.

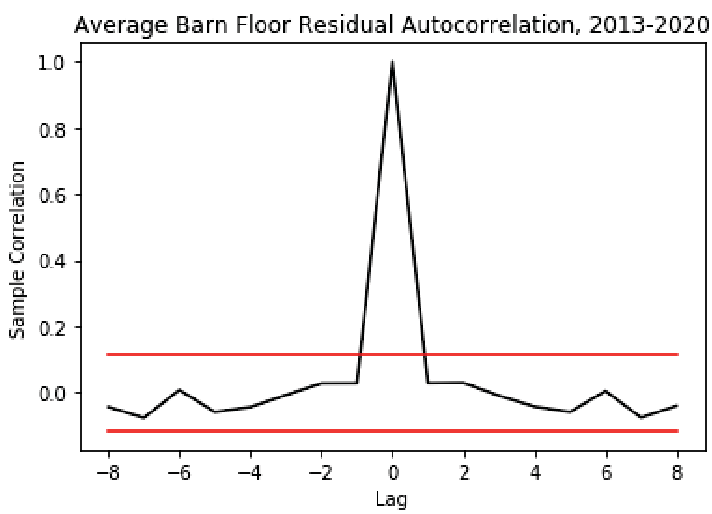


Figure 5.11: Autocorrelation of standardized residuals from all growth rate predictions, 2013-2020.

Just as in the case of the growth rates themselves, it can observed that there is no autocorrelation present in the residuals. It is assumed that they are random white noise, and thus that modelling the growth rates as a normally distributed random variable is acceptable without requiring any additional terms.

## 5.3 Determining Parameters

Many of the problem's parameters are objectively defined by data, although several had to be generated based on other parameters or defined in consultation with ABC Poultry based on their current scheduling practices. Determining geographic clusters and their relationships, weight goals, normalization constants and penalty weights required effort beyond simply reading data. While weight predictions technically fall under this category as well, they have already been discussed in detail.

The first thing which must be discussed is the geographic clustering used to simplify the problem's location constraints. A less formal version of this process was initially described by the company, but it was historically done subjectively rather than by using codified relationships between barns. Fortunately, the natures of the locations lent themselves well to clustering. The location of each barn was recorded on a map of the area surrounding the ABC Poultry processing facility. Figure 5.12 depicts a map of clusters similar to the one used in this case study. The real map is not shown to maintain the anonymity of ABC Poultry.

Figure 5.12: Clusters of ABC Poultry barns.

ABC Poultry accepted the clusters without modification, allowing the set $G$ to be defined for the problem. At the company's request, members of two different non-local clusters cannot be scheduled on the same day. This information allows $c_{g'g}$ to be defined for the problem. For other applications of this model, $c_{g'g}$ could assume a value of 1 for a pair of non-local clusters that could be scheduled on the same day.

Values for the weight goals $q_j$ must be generated for experimental purposes because records were not kept of each farmer's goals. It is important to note that the value $q_j$ is more granular than the corresponding quota for that farmer. While a quota is defined for a sixteen-week period, the weight goal is defined differently for each week based on the flocks to be sold. The weight goal may also be set higher or lower for the same number of chickens based on the farmer's production to date in the quota period. Accordingly, each farmer's weight goal in a week is assumed to be a random number from a uniform distribution bounded by $\sum_{i \in I} e_{ij} n_i w_{i,1}$ and $\sum_{i \in I} e_{ij} n_i w_{i,|T|}$, where $j$ identifies the farmer. These limits are chosen for the uniform distribution because they represent the lower and upper limits of what the farmers are expected to be able to produce. Farmer $j$ would produce $\sum_{i \in I} e_{ij} n_i w_{i,0}$ kg if all of their flocks were collected on day 1 and $\sum_{i \in I} e_{ij} n_i w_{i,|T|}$ kg if all of their flocks were collected on day $|T|$. The values are determined more methodically in practice: farmers are generally instructed to grow their flocks with a target of 2.25 kg/chicken, so $q_j$ is set according to this and adjusted if it will help the farmer meet their quota. Setting targets like this is possible because of the opportunity for manual review before the problem is solved. The scheduling tool makes the weight goals $q_j$ available for review and editing by ABC Poultry's procurement manager prior to creating a schedule in case they have additional information that might cause them to be higher or lower than otherwise expected, such as a flock's unique target weight. A lack of opportunity for manual review coupled with the fact that general production targets have often changed in the company's history means that the $q_j$ generated from the random uniform distribution is more likely to provide a problem that can be appropriately solved.

Normalization constants $k$ also had to be defined in the interest of comparing the deviations from each goal value and accurately identifying the maximum deviation. The process began by setting each farmer's normalization constant $k_3$ to 1, scaling their $\frac{z}{k}$ values so a 1% increase in deviation would result in a 0.01 increase in the term. Once $k_3$ had been selected, it could be taken as a fixed value relative to which $k_1$ and $k_2$ would be decided. This was done through sensitivity testing of a sample week in the Excel tool, which uses [DMGP], and confirmed across several more sample weeks.

First, $k_2$ was set to a large number $M$. This would temporarily ensure that weight spreads over 0.20 kg were not penalized, effectively making the only two goals of the program to help all farmers meet their goals and to evenly spread the total weight produced across each day. Then $k_1$ was set to 0.01 and an iteration of the problem was solved. If $k_1$ is too low, it will penalize the total weight spread too harshly and only optimize the problem based on which schedule gets the weight per day closest to even rather than giving any consideration to the farmers' quotas. If $k_1$ is too high, it will fail to penalize uneven days in a meaningful way and could potentially allow too many flocks to be scheduled for collection on the same day. The amount was slowly increased until arriving at $k_1 = 0.33$, which created a schedule that appeared to strike an appropriate balance between balanced days and quota management. This value was then used to generate four sample week schedules, each of which also balanced the requirements appropriately. With the value of $k_1$ now fixed, the same process was applied to $k_2$, yielding a value of $k_2 = 0.5$.

The penalty weight $\psi$ for [RP] and the goal programming weights $[\lambda_1, \lambda_2, \lambda_3]$ for [DWGP] and [SWGP] were also selected through a similar process, although they were tested in Python rather than in Excel. [RP] was tested with $\psi = 0.1, 1, 5, 10,$ and 100, and found that appropriately balanced solutions were achieved with $\psi = 10$. The goal programming weights were set inversely proportional to the size of the set determining how many deviation variables they had, setting $\lambda_1$ and $\lambda_2$ to $\frac{1}{|T|}$ and $\lambda_3$ to $\frac{1}{|J|}$. No further adjustments were made because these weights produced appropriately balanced solutions in the test weeks.

## 5.4 Scenario Generation

Each barn floor has an associated random variable used to model its growth rate, which is normally distributed with mean $\mu$ and standard deviation $\sigma$. For any set of days $T$, each member of the set of flocks $I$ is associated with one of these random variables. A scenario set with realizations of $|I|$ random variables must be generated for that week. The literature identifies two dominant methods of scenario generation: moment matching, first used by Høyland and Wallace [40] and random sampling, such as Homem-de-Mello [35] uses. Both methods seek to generate a scenario set which retains all marginal distributions, meaning that the set of realizations for each variable should have the same mean $\mu$ and standard deviation $\sigma$ as the distribution of that variable.

The advantage of the moment matching method is that it allows the decision maker to generate a complete multi-period scenario tree that respects the marginal distributions of each random variable in each time period even if a low number of scenarios has been chosen for each time period. This is helpful because it allows the decision maker to keep the complete decision tree to a manageable size as the number of time periods increases. The most significant disadvantage of moment matching is its long computational time as a non-convex problem, which is exacerbated by the large number of independent random variables presented by SFPP. The program must find a solution matching the mean and standard deviation of each of these variables simultaneously.

The advantage of moment matching over random sampling becomes diminished as the number of scenarios per time period increases. As this number increases, the set of randomly sampled scenarios naturally begins to approximate the marginal distributions more closely. This can be observed through experimentation. If $n$ random observations are drawn from a normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$, and the sample mean $\bar{x}$ and standard deviation $s$ are recorded as unbiased estimators of the distribution's parameters, $N$ trials of this experiment can be performed to develop confidence intervals for $\bar{x}$ and $s$. Table 5.1 shows the results of this experiment with $N = 1000$ sets of samples drawn and 95% confidence intervals constructed for various sample sizes $n$.

| $n$ | $\bar{x}$ | $s$ |
|---|---|---|
| 5 | [-0.88, 0.87] | [0.28, 1.61] |
| 10 | [-0.62, 0.60] | [0.53, 1.42] |
| 20 | [-0.44, 0.45] | [0.68, 1.30] |
| 40 | [-0.30, 0.31] | [0.76, 1.21] |
| 80 | [-0.22, 0.22] | [0.84, 1.15] |
| 160 | [-0.16, 0.15] | [0.89, 1.11] |

Table 5.1: 95% confidence intervals for $N = 1000$ sample sets of size $n$.

Predictably, the estimates become more accurate as the sample size increases. The confidence intervals in Table 5.1 have a relationship with the sample size $n$. If the lower bound of the confidence interval for $\bar{x}$ at sample size $n$ is defined as $LB_n$ and its upper bound is defined as $UB_n$, a relationship of $\frac{UB_{4n}-LB_{4n}}{UB_n-LB_n} \approx 0.5$ can be observed, which is consistent with the Central Limit Theorem: as the sample size quadruples, the width of the confidence interval is reduced by approximately 50%. The same relationship holds for $s$, so as $n$ continues to increase, the accuracy of the estimators $\bar{x}$ and $s$ will improve proportionately to this relationship.

Using a larger scenario set is not trivial. As Florian *et al.* [29] note, the multi-period lot sizing problem is NP-hard, so its transformations must also be NP-hard. As transformations of the multi-period lot sizing problem, DFPP and SFPP are both NP-hard. Each formulation of SFPP has a large number of variables and constraints, and will have to be tested multiple times to ensure consistency. The case study, for example, has average set sizes of $|I| = 30$, $|J| = 15$, $|T| = 4$, and $|G| = 4$. The number of decision variables and constraints these parameters create can be observed in Table 5.2.

| Model | Decision Variables | Constraints |
|---|---|---|
| DIP | 3736 | 18269 |
| DWGP | 7374 | 25507 |
| DMGP | 7375 | 29126 |
| RP | $3720\|S\| + 16$ | $18249\|S\| + 16$ |
| SWGP | $7358\|S\| + 16$ | $25521\|S\| + 16$ |
| SMGP | $7359\|S\| + 16$ | $29140\|S\| + 16$ |

Table 5.2: The number of decision variables and constraint when each model is applied to the case study.

Considering the NP-hardness of the problems and the large problem sizes shown in Table 5.2, it is clear that the size of SFPP must be carefully managed to obtain a solution in a reasonable time. Based on the options tested in Table 5.1, $n = 20$ scenarios will be used for the numerical analysis because it presents substantial improvements over $n = 5$ and $n = 10$ in both $\bar{x}$ and $s$ while presenting a confidence interval for its estimate of the mean that is less than one standard deviation wide. If creating a histogram of a standard normal distribution, a middle bin of $[-0.5, 0.5]$ would be reasonable because it is a good measure of the middle of the distribution. It can be claimed with 95% confidence that $\bar{x}$ would belong in this range for $n = 20$, whereas this claim cannot be made for $n = 10$. While it would be recommended in practice to use the largest possible scenario set that could be solved in the amount of computational time allotted to the program, a smaller number of scenarios will suffice for the purpose of efficiently comparing the models. When considering a larger scenario set, the L-shaped method [55] could be implemented to reduce computational time.

## 5.5 Experiment Results

When designing a method to evaluate the relative efficacy of each formulation, it is important to remember that any of them will look like the best option if its objective function is the sole criterion in the analysis. Accordingly, several criteria should be used for experimentation reflecting the benefits of the individualistic and collectivistic approaches presented to the farmers. Solution time will also be measured to determine if one approach is significantly easier or more difficult to compute than the others, information which could in practice inform the number of scenarios used to create a schedule.

The criterion to be used to evaluate the individualistic dimension of the problem is the objective function of the minmax goal program. This quantifies the expected maximum allowable deviation from optimality for any one party, which could be one of the farmers or ABC Poultry. When it is measured for a collectivistic approach and compared with the individualistic approach, it demonstrates the magnitude of the risk of a large deviation from optimality assumed by each party.

Several criteria must be used to evaluate the collectivistic dimension of the problem effectively. While the objective function of the minmax goal program can be used as the only criterion for the individualistic dimension because it identifies the worst-case deviation, which is the single deviation most likely to make a solution undesirable regardless of which goal it affects, the collectivistic dimension must examine the impact on each goal to ensure solutions are balanced. When this is measured for an individualistic approach and compared with the collectivistic approach, it demonstrates the magnitude of the gains that could be made by the farmers by assuming increased risk of a large deviation from optimality.

The following is a list of the five criteria to be recorded:

- Time: the average amount of time the solver took to solve a trial. Its units are seconds or minutes and its optimal value is 0.

- Objective function value (OFV): the average value assumed by the minmax objective function for the solution of a trial. It has no units and its optimal value is 0.

- Deviation from quota (DFQ): how close to the optimal range of 99-102% of a farmer's production goal the average solution comes for the average farmer.

- Weight balance (WB): a metric referring to how close to evenly balanced the weights are on an average day. Its units are percentage points and its optimal value is 0.

- Weight spread (WS): the maximum difference in average weight per chicken of two flocks scheduled on the same day in a trial. Its units are kg and its optimal value is any number between 0 and 0.20, inclusive.

It is unlikely that one method will be superior in every category, although this method should be recommended if this is the case. This analysis seeks to observe whether an inverse relationship exists between OFV and DFQ, suggesting that as the value of OFV becomes less desirable, the value of DFQ becomes more desirable. The goal of this analysis is to characterize how much individual risk the farmers take on by adopting a collectivistic approach, and how much expected benefit that approach offers. As long as the average values of WB and WS are acceptable for a formulation, detailed analysis of their exact values is not necessary. Determining which values are acceptable must be done situationally: while a WS value of 0.30 is generally undesirable, for example, it may be unavoidable in some weeks. This situational evaluation should be done in relation to the results from the other formulations. Consequently, a decision rule must be developed to define whether a formulation is acceptably balanced. In this experiment, any formulation presenting OFV more than 0.100 above the minimum OFV calculated by a MGP should be rejected. An OFV more than 0.100 above the minimum implies that an improvement is possible comparable to reducing DFQ by 10 percentage points, improving WB by 3.3 percentage points, or reducing a WS of at least 0.25 kg by 0.05 kg, all without introducing another deviation of that size elsewhere. However, this decision rule is just the first step. If a solution is not rejected by the decision rule, it does not mean that it has equal merit to other solutions, only that the solutions are balanced enough that the comparison is worth making.

All experiments described in this section were run on a Windows 10 PC with a Ryzen 5 3600X processor at 3.8 GHz and 8 GB of RAM. The solver CPLEX was used via the Python package docplex. The solver was given two stopping conditions: a 15-minute time limit and a 1% optimality gap.

### 5.5.1 Deterministic Models

The first step is to test the deterministic models. 53 weeks were selected from between 2018 and 2020 in which a similar amount of chicken was processed during each day the facility was open. A tolerance of 5% of the week's total weight was used to decide whether two amounts were similar. The [DMGP], [DWGP], and [DIP] formulations were solved for each week. Based on the results from [DMGP] and [DWGP], the weight balancing tolerance parameter $\epsilon = 0.03$ was used for [DIP]. The results are recorded in Table 5.3 and subsequently visualized in Figure 5.13. Note that the units of time are seconds.

| Trial | Time (s) | OFV | DFQ | WB | WS |
|---|---|---|---|---|---|
| DMGP | 3.48 | 0.136 | 6.93 | 2.65 | 0.26 |
| DWGP | 5.85 | 0.202 | 5.73 | 2.75 | 0.22 |
| DIP | 1.40 | 0.083 | 3.68 | 1.70 | 0.20 |

Table 5.3: Data from testing 53 weeks of the deterministic formulations.

Figure 5.13 scales each criterion according to the maximum value of that criterion in Table 5.3. For example, the maximum Time value in the table is 5.85 for [DWGP], so the value for [DMGP] is scaled to $3.48/5.85 = 0.59$. [DIP] initially looks superior to [DMGP] and [DWGP] because a low value is ideal for each solution, but it should be noted that these statistics were only collected from trials in which the solver obtained a solution. Whereas [DMGP] and [DWGP] each produce a solution for all 53 weeks, [DIP] produces a solution for only 4 of the 53. [DIP] demonstrates that concerns about its inability to produce feasible solutions are well-founded and justifies the GP and SRO approaches taken in the subsequent formulations of DFPP and SFPP. Because these subsequent formulations attempt to minimize violation of scheduling guidelines rather than setting a maximum value the violations can take, they ensure feasibility

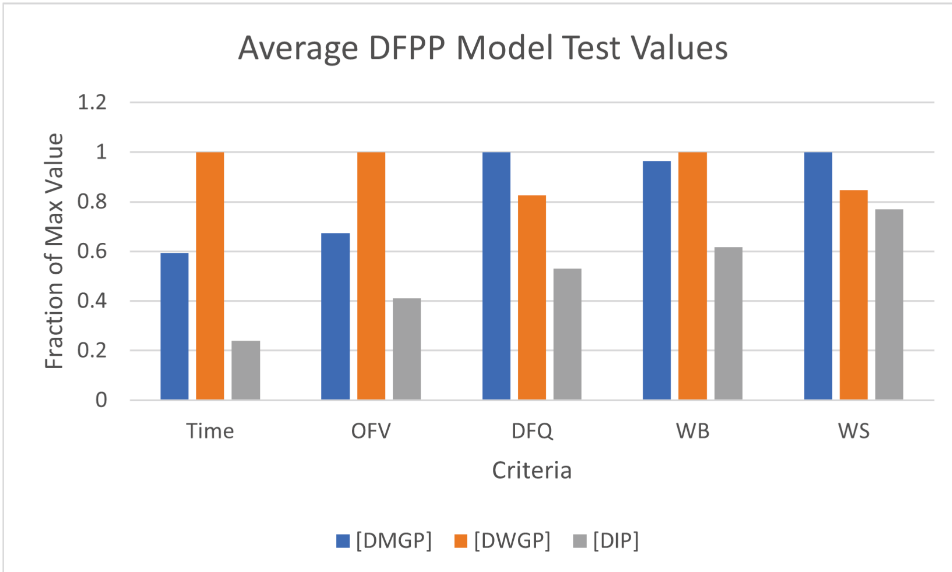


Figure 5.13: Data from testing 53 weeks of the deterministic formulations.

regardless of the parameters of the problem. Thus, [DIP] should be removed from consideration entirely.

[DMGP] and [DWGP] can both be solved in a reasonable time and both achieve an acceptable WB and WS, so their OFV and DFQ can be compared. A paired t-test cannot be used to compare the OFV or DFQ values of [DMGP] and [DWGP] because their differences are not normally distributed. While its non-parametric equivalent, the Wilcoxon signed-rank test [87] might be considered because it does not require an assumption of normal distribution, this test only determines whether the mean difference of the paired observations is 0. The test cannot quantify the magnitude of the difference, which is an essential component of evaluating the tradeoff between [DWGP] and [DMGP]. The data can still be inspected by visualizing it with a histogram and observing the shape of its distribution. [DMGP] offers large improvements on the maximum deviation of several of the solutions calculated for [DWGP]. These improvements can be observed in Figure 5.14.

In 8 of 53 cases, the difference is 0. In the other 45, [DMGP] offers the preferable OFV, in 9 such cases offering an improvement of more than 0.10, which would cause the [DWGP] solutions for these cases to be rejected based on the previously defined decision rule. This difference illustrates the stability of [DMGP] as well as the volatility of [DWGP]: in 6 of the 9 cases, the difference of the maximum deviations is at

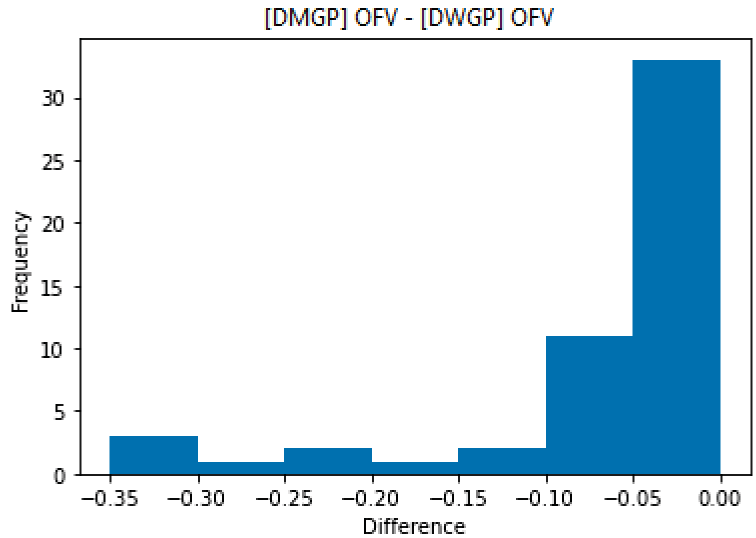


Figure 5.14: The difference between [DMGP] OFV and [DWGP] OFV.

least 0.20, more than double the threshold for rejection.

The large improvement [DMGP] provides in OFV is contrasted by a small improvement the averages presented in Table 5.3 suggest [DWGP] might offer in DFQ over [DMGP]. Their differences can be inspected in a histogram, presented in Figure 5.15.

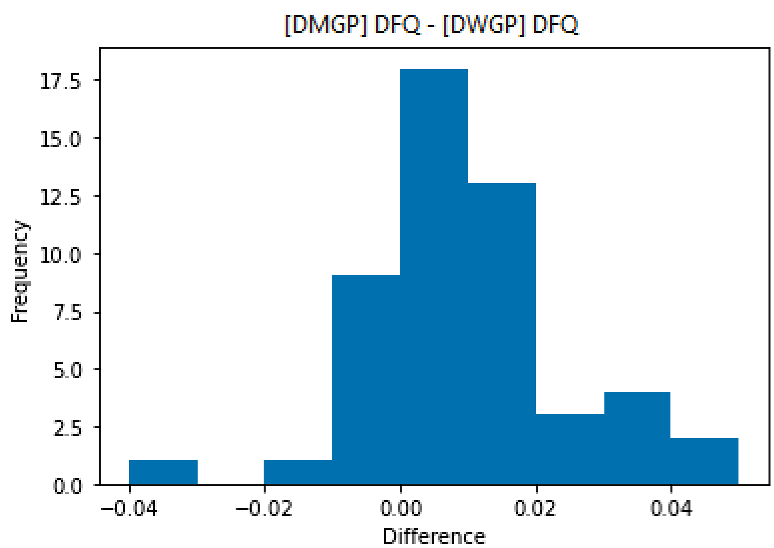


Figure 5.15: The difference between [DMGP] DFQ and [DWGP] DFQ.

It is not immediately evident from Figure 5.15 whether [DWGP] reliably offers the better DFQ value. The most likely outcome appears to be a modest improvement in DFQ of less than 2 percentage points, consistent with the average difference of 1.2, but improvement is not guaranteed. In 11 of 53 cases, the DFQ of [DWGP] is worse than that of [DMGP].

Based on this analysis, it can be concluded that choosing [DWGP] over [DMGP] poses a significant risk in exchange for an insignificant reward. 9 of 53 [DWGP] solutions in the test data contain a deviation large enough to cause the decision maker to reject the solution, something which is never a concern for a [DMGP] solution. Regardless of whether the farmers prefer an individualistic or collectivistic approach, the [DMGP] model should be used when solving the deterministic problem to ensure the solution will not be rejected.

### 5.5.2 Stochastic Models

Having compared the deterministic models, the stochastic models can similarly be tested on 5 randomly selected weeks from the 53 weeks used to test the deterministic problem. Based on the parameters defined for the problem, each week can take up to 20 hours to test. 25 scenario sets are generated for each week, each containing 20 scenarios, and the average of each evaluation criterion is displayed in Table 5.4. The data from the table is subsequently visualized in Figure 5.16, which shows the average value across all 5 test weeks for each criterion. Each criterion has been scaled based on its maximum observed value in the same manner as Figure 5.13. Time is measured in seconds. Having previously concluded that [DMGP] is the best solution to the deterministic problem, it is solved for the stochastic problem in addition to [SMGP], [RP], and [SWGP] to provide a baseline from which to calculate the value of stochastic solution (VSS).

The first thing that can be noted from Table 5.4 is the inferiority of the [DMGP] solution in comparison to the solutions from the other formulations. While it produces a solution in a matter of seconds rather than taking minutes and its average DFQ and WB are comparable to those of the stochastic solutions, the average WS forces OFV to assume a much higher value than it does for the other formulations. It is likely that its higher average WS comes from an inability to account for multiple scenarios. While the values of DFQ and WB are insulated by the effect of risk pooling through multiple flocks because underestimation of one growth rate may be

| Trial | Time (s) | OFV | DFQ | WB | WS |
|---|---|---|---|---|---|
| DMGP-1 | 5.4 | 0.325 | 3.72 | 0.78 | 0.36 |
| SMGP-1 | 720.0 | 0.099 | 3.92 | 1.52 | 0.25 |
| RP-1 | 586.2 | 0.165 | 4.03 | 2.18 | 0.22 |
| SWGP-1 | 910.8 | 0.139 | 4.38 | 1.21 | 0.24 |
| DMGP-2 | 3.0 | 0.356 | 5.20 | 3.62 | 0.38 |
| SMGP-2 | 408.0 | 0.165 | 4.86 | 3.45 | 0.28 |
| RP-2 | 384.6 | 0.349 | 4.45 | 5.98 | 0.23 |
| SWGP-2 | 760.8 | 0.230 | 4.51 | 3.32 | 0.26 |
| DMGP-3 | 3.0 | 0.322 | 3.43 | 0.54 | 0.36 |
| SMGP-3 | 352.8 | 0.059 | 3.65 | 1.00 | 0.22 |
| RP-3 | 316.8 | 0.134 | 3.61 | 1.65 | 0.21 |
| SWGP-3 | 892.2 | 0.094 | 3.49 | 0.85 | 0.22 |
| DMGP-4 | 4.8 | 0.360 | 3.64 | 0.76 | 0.38 |
| SMGP-4 | 577.2 | 0.069 | 3.79 | 1.27 | 0.23 |
| RP-4 | 475.8 | 0.148 | 3.38 | 2.07 | 0.21 |
| SWGP-4 | 916.8 | 0.120 | 3.90 | 1.02 | 0.23 |
| DMGP-5 | 1.8 | 0.404 | 6.23 | 3.12 | 0.40 |
| SMGP-5 | 386.4 | 0.221 | 6.01 | 4.33 | 0.31 |
| RP-5 | 221.4 | 0.306 | 5.73 | 4.43 | 0.31 |
| SWGP-5 | 357.0 | 0.279 | 5.99 | 2.98 | 0.32 |

Table 5.4: Average data from 25 trials for each of 5 test weeks for each stochastic formulation and [DMGP].

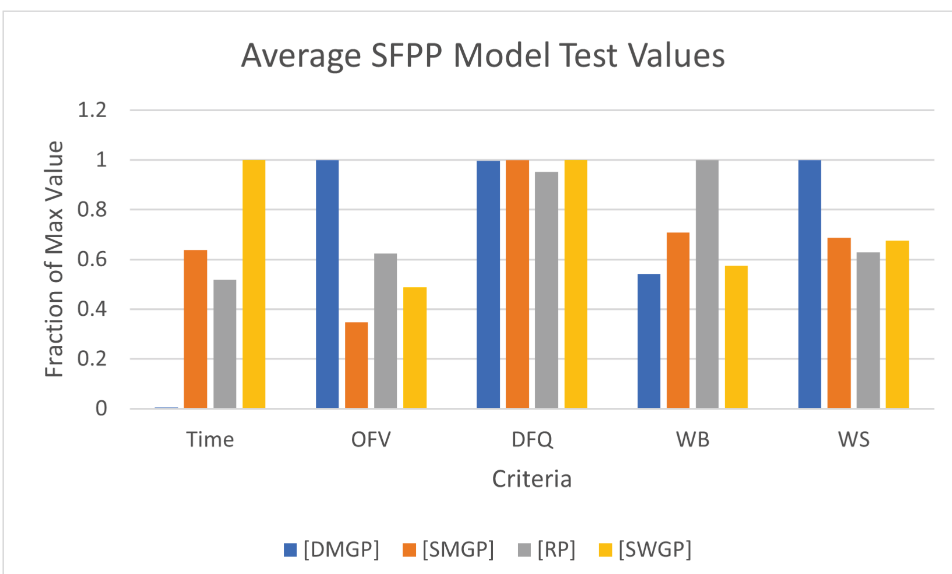


Figure 5.16: Average values of criteria across 5 test weeks for each stochastic formulation and [DMGP].

offset by overestimation of another, WS experiences the opposite effect. Consider a simple version of SFPP with one day and one farmer who must meet a 3000 kg quota. Additionally, assume that each flock will produce 80%, 100%, or 120% of its predicted weight, each scenario with $\frac{1}{3}$ probability. The farmer can raise one 3000 kg flock, two 1500 kg flocks, or three 1000 kg flocks. Each flock will contain the same number of chickens. The probabilities of the worst-case scenarios for DFQ and WS can be observed in Table 5.5 as the number of flocks changes. WB is omitted because the problem only concerns one day, so balancing weight across each day is not relevant.

| Number of Flocks | DFQ Worst-Case Probability | WS Worst-Case Probability |
|---|---|---|
| 1 | 66.7% | 0% |
| 2 | 22.2% | 22.2% |
| 3 | 7.4% | 44.4% |

Table 5.5: The probability of maximum deviation from the goal occurring in DFQ and WS as the number of flocks changes.

A worst-case scenario for DFQ is for the farmer to produce either 2400 or 3600 kg of chicken, both a 600 kg deviation from their 3000 kg quota. A worst-case scenario for WS is for any flock to produce 80% of its predicted weight while any other flock produces 120% of its predicted weight. Table 5.5 shows that as the number of flocks increases, the probability of DFQ realizing one of its worst-case scenarios decreases while the probability of WS realizing one of its worst-case scenarios increases. If a second day is added to the problem and 3000 kg are also scheduled for that day, the WB worst-case scenario is that all of the flocks on one day produce 2400 kg while all of the flocks on the other day produce 3600 kg. WB will be similar to DFQ by decreasing its probability of realizing a worst-case scenario as the number of flocks increases.

In the context of the case study, approximately 8 flocks are scheduled per day. When DFPP is solved, average flock weights are still uncertain. DFQ and WB are both insulated from the negative effects of this uncertainty because of the number of flocks scheduled each day. This effect can be observed in Table 5.4, where [DMGP] exhibits DFQ and WB values that are as good as, or sometimes better than, those of the formulations designed to handle SFPP. Conversely, enough flocks are scheduled each day to make WS more likely to assume a higher, and therefore worse, value.

When a DFPP solution is applied to SFPP, risk pooling shields DFQ and WB from assuming worse values due to variance, but it does not protect WS. Because WS still becomes worse when a DFPP solution is applied to SFPP, the average OFV can also be expected to be higher. In the case study, the OFV of [DMGP] is high enough that it crosses the threshold for rejection for SFPP in each test week.

[SMGP] produces a high VSS when compared to [DMGP]. Because OFV for SFPP is determined by the objective function of [SMGP], and [DMGP] is the deterministic formulation of [SMGP], the OFV difference of the two formulations is the VSS. A paired t-test can be conducted between the [DMGP] OFV and [SMGP] OFV to determine 95% confidence intervals for the VSS in each week, presented in Table 5.6. It displays the lower (LB) and upper (UB) bounds of the confidence interval for the expression $[DMGP]_{OFV} - [SMGP]_{OFV}$ in the unitless terms of OFV, as well as a percentage improvement of the expected OFV for [DMGP] that week.

| Week | LB | UB | LB (%) | UB (%) |
|---|---|---|---|---|
| 1 | 0.194 | 0.257 | 59.6 | 79.1 |
| 2 | 0.175 | 0.208 | 49.2 | 58.4 |
| 3 | 0.248 | 0.279 | 77.0 | 86.6 |
| 4 | 0.265 | 0.290 | 73.6 | 80.6 |
| 5 | 0.163 | 0.203 | 40.3 | 50.2 |

Table 5.6: 95% confidence intervals for the VSS reformulating [DMGP] as [SMGP].

The significance of the differences presented in Table 5.6 is that they illustrate the reduction in maximum expected deviation presented by using a stochastic formulation of the MGP, [SMGP], rather than using a deterministic formulation, [DMGP]. For example, a 95% confidence interval for the difference in week 4 is [0.265, 0.290], which corresponds to one of the following improvements without introducing a similarly-sized devation elsewhere: a DFQ improvement of 26.5-29.0 percentage points, a WB improvement of 8.8-9.7 percentage points, or a WS reduction of 0.13-0.15 kg from an initial solution of at least 0.33-0.35 kg. Even in the week with the least improvement, the stochastic solution still improves OFV by more than 40%, and in some cases may offer improvements upward of 80%. This illustrates the importance of using the stochastic formulation for scheduling if possible. Because the lower bound of the difference is also greater than 0.100 in every case, this confirms with 97.5%

certainty that [DMGP] must be rejected, having $\frac{\alpha}{2}$ certainty rather than $\alpha$ because the comparison is one-tailed.

The VSS presented by using [SMGP] instead of [DMGP] becomes plain when considered in a practical context. Consider the SFPP timeline presented in Figure 4.1, in which geographic clusters are chosen for each day on day $-|T|$, flock collection days are determined on day 0, and flocks are collected between days 1 and $|T|$. The difference between DFPP and SFPP is that DFPP also requires flocks to be scheduled for collection on day $-|T|$. While the flock weights are still uncertain on day $-|T|$, their expected value is used because DFPP presents no opportunity for recourse when the flock weights become known $|T|$ days later. The VSS represents the improvement in performance that can be achieved by delaying the decision to schedule flocks for collection by $|T|$ days. It has been observed that [DMGP] presents a much less desirable WS than the other formulations tested for SFPP, despite displaying a comparable DFQ and WB. It can be concluded that the primary effect of delaying the decision to schedule flocks for collection by $|T|$ days is a significant WS improvement. Because WS is monitored in the interest of minimizing defect rates, lowering it significantly also lowers defect rates. Therefore, implementing [SMGP] instead of [DMGP] lowers defect rates.

Knowing now that [DMGP] can be ignored, [SMGP], [RP], and [SWGP] can be examined. In each week, their WB and WS are all at comparable levels which allows their OFV and DFQ to be compared, although making this claim about weeks 2 and 5 does merit brief discussion. In week 2, [RP] has a noticeably lower WS than the other two formulations. However, this solution comes at the cost of a much worse WB than the other solutions achieve. Because each formulation has been forced to take on one somewhat undesirable value, it can be concluded that a solution with good WB and WS cannot be constructed without significantly impacting DFQ. Thus, the OFV and DFQ of each non-[DMGP] solution in Week 2 can be compared. Week 5 experiences a similar issue: a solution with a desirable WS is not found. Even [SMGP], designed specifically to minimize the worst deviation, cannot on average find a solution with a lower WS than 0.31 without severely impacting another criterion. While a WS of 0.30 or greater is generally undesirable, there is no evidence of a better option given the flocks that must be scheduled that week. Thus, the OFV and DFQ of each non-[DMGP] solution in Week 5 can be compared.

It may be noted that [RP] is compared to [SMGP] and [SWGP] on the basis of expected performance despite the fact that it is a robust formulation and the other two are stochastic formulations. Any consideration of this approach must account for the fact that $\sigma(.)$, the non-penalty function part of the [RP] objective function, is singularly focused on maximizing the expected quota fulfillment of the farmers. While $\sigma(.)$ is penalized by $\rho(.)$, the expected value of deviations from the scheduling guidelines, this metric is similarly evaluated in [SMGP] and [SWGP]. This leaves $\sigma(.)$ as the unique element of the objective function in [RP], granting the formulation a stronger focus on maximizing the expected quota fulfillment of the farmers, which therefore minimizes the deviation from quota, DFQ. Accordingly, it is acceptable to compare [RP] to [SMGP] and [SWGP] on the basis of expected performance because the only unique element of its objective function still helps to minimize expected DFQ.

OFV can first be considered to evaluate the magnitude of the deviation risked by using each formulation. Paired t-tests can be used to develop confidence intervals of the difference in OFV for each pair of formulations in each week. The results of these tests, using 95% confidence intervals, are presented in Table 5.7 and visualized in Figure 5.17. Percentages are provided based on the first number in the calculation, i.e. if the calculation is [A]-[B] then the appropriate LB (%) is calculated by LB/[A]. The same is true of UB (%) and UB. While these percentages are not referenced in the analysis, they are intended to provide the reader with a point of reference illustrating the significance of the differences presented.

Interpreting Table 5.7 requires considering the calculation provided in the first column. For example, examine the first row. The calculation $[RP]_{OFV} - [SMGP]_{OFV}$ finds the difference between the OFV of [RP] and the OFV of [SMGP]. If the difference is a positive number, OFV is higher for [RP]; if the difference is a negative number, OFV is higher for [SMGP]. LB and UB show that a 95% confidence interval for the difference is [0.034, 0.097], demonstrating that the OFV of [RP] is expected to be higher, and therefore worse, with at least 95% confidence. Because $[RP]_{OFV}$ is the first term in the Calculation column, LB (%) and UB (%) present LB and UB respectively as percentages of the OFV of [RP], suggesting that [SMGP] improves the

| Calculation | Week | LB | UB | LB (%) | UB(%) |
|---|---|---|---|---|---|
| $[RP]_{OFV} - [SMGP]_{OFV}$ | 1 | 0.034 | 0.097 | 20.5 | 59.0 |
| $[RP]_{OFV} - [SWGP]_{OFV}$ | 1 | 0.019 | 0.034 | 11.5 | 20.6 |
| $[SWGP]_{OFV} - [SMGP]_{OFV}$ | 1 | 0.008 | 0.070 | 5.8 | 50.0 |
| $[RP]_{OFV} - [SMGP]_{OFV}$ | 2 | 0.169 | 0.200 | 48.4 | 57.3 |
| $[RP]_{OFV} - [SWGP]_{OFV}$ | 2 | 0.101 | 0.136 | 28.9 | 39.0 |
| $[SWGP]_{OFV} - [SMGP]_{OFV}$ | 2 | 0.057 | 0.075 | 24.8 | 32.6 |
| $[RP]_{OFV} - [SMGP]_{OFV}$ | 3 | 0.068 | 0.083 | 50.7 | 61.9 |
| $[RP]_{OFV} - [SWGP]_{OFV}$ | 3 | 0.033 | 0.047 | 24.6 | 35.1 |
| $[SWGP]_{OFV} - [SMGP]_{OFV}$ | 3 | 0.032 | 0.039 | 34.0 | 41.5 |
| $[RP]_{OFV} - [SMGP]_{OFV}$ | 4 | 0.072 | 0.083 | 48.6 | 56.1 |
| $[RP]_{OFV} - [SWGP]_{OFV}$ | 4 | 0.028 | 0.040 | 18.9 | 27.0 |
| $[SWGP]_{OFV} - [SMGP]_{OFV}$ | 4 | 0.039 | 0.048 | 32.5 | 40.0 |
| $[RP]_{OFV} - [SMGP]_{OFV}$ | 5 | 0.074 | 0.096 | 24.2 | 31.4 |
| $[RP]_{OFV} - [SWGP]_{OFV}$ | 5 | -0.001 | 0.060 | -0.3 | 19.6 |
| $[SWGP]_{OFV} - [SMGP]_{OFV}$ | 5 | 0.028 | 0.084 | 10.0 | 30.1 |

Table 5.7: Paired t-test comparisons of the difference of mean OFV values for each pairing of [SMGP], [RP], and [SWGP] for each week.

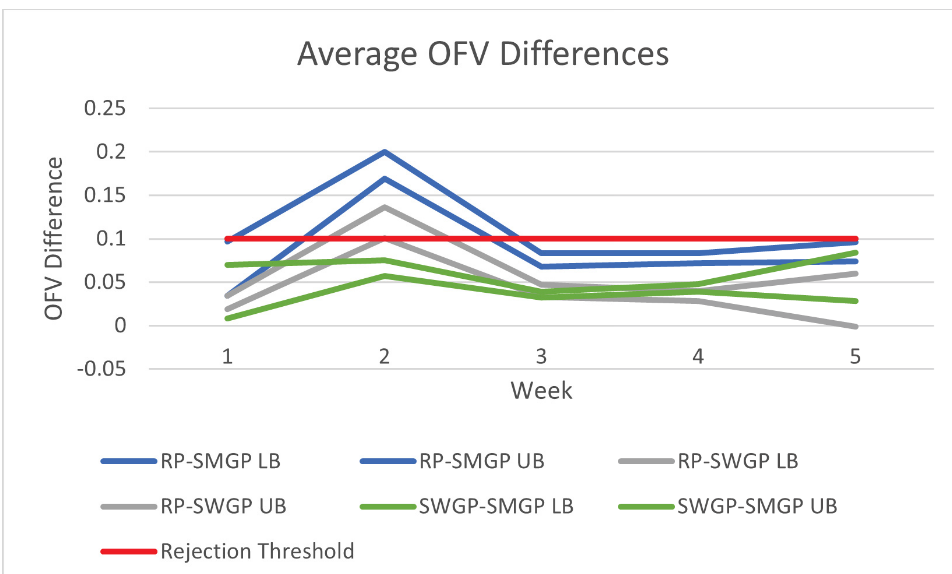


Figure 5.17: Paired t-test comparisons of the difference of mean OFV values for each pairing of [SMGP], [RP], ad [SWGP] for each week.

OFV of [RP] by between 20.5% and 59.0%.

Upon observing Table 5.7, a consistent pattern emerges: [SMGP] will produce the best OFV and [RP] will produce the worst OFV. The only time a result suggests otherwise is in the comparison of [RP] and [SWGP] in week 5, where the 95% confidence interval contains 0. If a 93% confidence interval is constructed to make the same comparison, it does not contain 0. [SWGP] is still heavily favoured to provide a better OFV. While [SMGP] is optimized according to OFV, and thus it is predictable that it will be the best by this metric, it is still useful to be able to quantify the improvement.

On average, [RP] has an OFV 0.098 higher than [SMGP]. If the decision maker could be guaranteed this would be the OFV difference with very little variance, [RP] might be an acceptable model, but it is very close to the threshold for rejection of 0.100. Its observed worst-case scenario should be identified to consider whether the model might produce a solution that would be rejected by ABC Poultry. According to the week 2 confidence interval in Table 5.7, it can be claimed with 95% confidence that [SMGP] offers an OFV improvement over [RP] in the range [0.169, 0.200]. Week 2 presents the largest OFV improvement between any pair of the stochastic formulations in any week, an improvement which greatly exceeds the threshold for rejection. The magnitude of the corresponding improvements is comparable to those referenced in the analysis of the deterministic solutions. A 0.200 OFV reduction corresponds to bringing the worst DFQ level of a farmer 20 percentage points closer to the optimal value, improving the WB of the worst day by 6.7 percentage points, or reducing a WS of at least 0.30 kg by 0.10 kg. It is useful to observe this because it makes the decision maker aware of a chance that [RP] will produce a solution imbalanced enough that it cannot be used.

[SWGP] offers more moderate OFV results: it is consistently better than [RP], but worse than [SMGP]. It does not produce any solutions with a significant enough improvement to be made in OFV to merit rejection, averaging an OFV increase of 0.048. According to the confidence intervals presented in Table 5.7, the largest improvement [SMGP] may offer over it in any of the test weeks is 0.084, corresponding to bringing the worst DFQ level of a farmer 8.4 percentage points closer to the optimal value, improving the WB of the worst day by 2.8 percentage points, or or reducing a WS of at least 0.242 kg by 0.042 kg. While these improvements are desirable, none of them are indicative of a solution imbalanced enough to be rejected.

Data indicates that moving from using [SMGP] to [RP] risks a much larger maximum deviation, possibly leading to rejection of the solution. Moving from using [SMGP] to [SWGP] risks a more moderate increase in maximum deviation the farmers may find acceptable in exchange for a DFQ improvement if they agree to a collectivistic approach, although this exchange cannot be properly evaluated until its reward is quantified. This reward can be examined in a similar manner to OFV. The results of the paired t-test comparisons of DFQ, using 95% confidence intervals for each pairing of [SMGP], [RP], and [SWGP], are presented in Table 5.8 and subsequently visualized in Figure 5.18. As in Table 5.7, percentages are provided to give the reader a point of reference, LB (%) is calculated by LB/[A] for [A]-[B], and the same is true of UB (%) and UB.

| Calculation | Week | LB | UB | LB (%) | UB(%) |
|---|---|---|---|---|---|
| $[SMGP]_{DFQ} - [RP]_{DFQ}$ | 1 | -0.262 | 0.044 | -6.7 | 1.1 |
| $[SWGP]_{DFQ} - [RP]_{DFQ}$ | 1 | 0.153 | 0.547 | 3.5 | 12.5 |
| $[SMGP]_{DFQ} - [SWGP]_{DFQ}$ | 1 | -0.597 | -0.320 | -15.2 | -8.2 |
| $[SMGP]_{DFQ} - [RP]_{DFQ}$ | 2 | 0.226 | 0.591 | 4.7 | 12.2 |
| $[SWGP]_{DFQ} - [RP]_{DFQ}$ | 2 | -0.106 | 0.223 | -2.4 | 4.9 |
| $[SMGP]_{DFQ} - [SWGP]_{DFQ}$ | 2 | 0.216 | 0.484 | 4.4 | 10.0 |
| $[SMGP]_{DFQ} - [RP]_{DFQ}$ | 3 | -0.114 | 0.195 | -3.1 | 5.3 |
| $[SWGP]_{DFQ} - [RP]_{DFQ}$ | 3 | -0.263 | 0.012 | -7.5 | 0.3 |
| $[SMGP]_{DFQ} - [SWGP]_{DFQ}$ | 3 | 0.062 | 0.271 | 1.7 | 7.42 |
| $[SMGP]_{DFQ} - [RP]_{DFQ}$ | 4 | 0.120 | 0.319 | 3.2 | 8.4 |
| $[SWGP]_{DFQ} - [RP]_{DFQ}$ | 4 | 0.081 | 0.329 | 2.1 | 8.4 |
| $[SMGP]_{DFQ} - [SWGP]_{DFQ}$ | 4 | -0.066 | 0.096 | -1.7 | 2.5 |
| $[SMGP]_{DFQ} - [RP]_{DFQ}$ | 5 | 0.122 | 0.429 | 2.0 | 7.1 |
| $[SWGP]_{DFQ} - [RP]_{DFQ}$ | 5 | 0.140 | 0.385 | 2.3 | 6.4 |
| $[SMGP]_{DFQ} - [SWGP]_{DFQ}$ | 5 | -0.137 | 0.163 | -2.3 | 2.7 |

Table 5.8: Paired t-test comparisons of the difference of mean DFQ values for each pairing of [SMGP], [RP], and [SWGP] for each week.

Table 5.8 can be interpreted similarly to Table 5.7, with the one difference being that it examines DFQ differences rather than OFV differences. While a clear hierarchy emerges when OFV is examined in Table 5.7, the same is not true when DFQ is examined in Table 5.8. For each pairing, the paired t-test is inconclusive for two of the five weeks: weeks 1 and 3 for [SMGP] and [RP], weeks 2 and 3 for [SWGP] and [RP], and weeks 4 and 5 for [SMGP] and [SWGP]. It can be observed that in the pairings

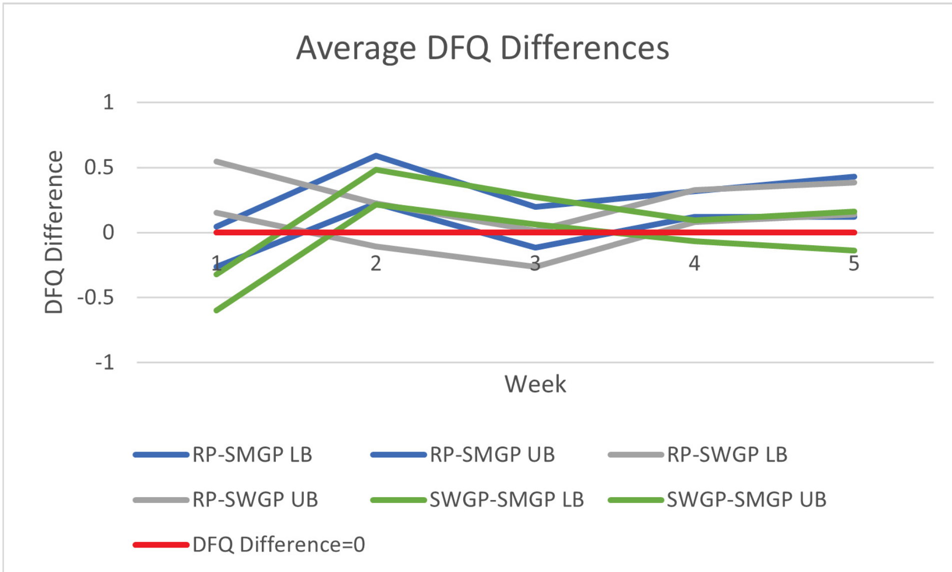


Figure 5.18: Paired t-test comparisons of the difference of mean DFQ values for each pairing of [SMGP], [RP], and [SWGP] for each week.

where the tests are not inconclusive, [RP] shows an average of a 0.301 percentage point improvement over [SMGP] and a 0.273 percentage point improvement over [SWGP]. Because [SMGP] is better than [SWGP] with 95% confidence in week 1 and worse with 95% confidence in weeks 2 and 3, no assertion of an improvement in either direction should be made.

Ultimately, the test weeks give no evidence that any formulation will consistently yield a better DFQ value than another formulation. This makes the collectivistic approach unappealing, asking the farmers to adopt an increased risk of suboptimal profit in a single week without providing an incentive of increased expected profit. Accordingly, [SMGP] should be used to solve the stochastic problem rather than [RP] or [SWGP] regardless of whether the farmers prefer an individualistic or collectivistic approach.

### 5.5.3 Managerial Insights

Even if a decision maker chooses to continue scheduling by hand rather than implementing [DMGP] or [SMGP], the analysis provided by this chapter still offers several contributions. The insights offered may be particularly useful to those working in the poultry industry. The following observations can be made:

- Useful predictions can be made about the average growth rate of a flock of chickens based on the growth rates of previous flocks raised in the same barn.

- Risk pooling offers an improvement in performance for meeting quotas and balancing weight across days in exchange for a reduction in performance in managing the maximum average weight difference between flocks, potentially leading to a higher defect rate.

- Delaying the scheduling of flocks for procurement until their average weights are known results in schedules which adhere significantly better to production guidelines.

A weight forecasting model is established which has comparable accuracy to the existing ABC Poultry method without requiring additional human input. Having the ability to make forecasts without requiring detailed knowledge of each farmer is useful because it enables automation of the process and allows the responsibility to be assumed by new people more easily. Identifying that average flock growth rates can be estimated by the average of previous growth rates in the same barn creates a forecasting model which is less reliant on the data recorded by farmers. Previous models explored in the literature [39, 43, 44, 45, 88] have required daily records of average flock weights and environmental factors such as temperature and humidity. These factors must either be recorded automatically by expensive state-of-the-art equipment some farmers may not possess, or manually, which expects a farmer to make a time-consuming daily change to their routine. This forecasting model only relies on a single piece of information from the farmer, which can be recorded manually with inexpensive equipment. The weight forecasting model developed in this thesis is a useful contribution to poultry companies purchasing from farmers who lack state-of-the-art equipment.

The effect of risk pooling on DFQ, WB, and WS is observed, concluding that using a greater number of flocks meets quotas and balances total weight across days more reliably, but risks a higher defect rate. By evaluating the best DFPP solution method, [DMGP], in the context of SFPP, it can be observed that WS is adversely affected by uncertainty when an average of approximately 8 flocks per day are scheduled. Based on the reasoning presented earlier in this chapter, it can be concluded that reducing the number of flocks per day will help reduce WS, and therefore defect rate. Consideration should be given to how the severely the magnitude of the reduction will negatively impact DFQ and WB. If a poultry company decides an acceptable trade may be made to improve WS at the expense of the other two metrics, this observation is a useful contribution by suggesting that the company should take steps to ensure their suppliers produce fewer, larger flocks.

The large VSS presented between [DMGP] and [SMGP] illustrates the power of delaying schedule decisions until flock weights are known. In terms of the case study, [DMGP] and [SMGP] are the closest to ABC Poultry's manual scheduling approach because they are all individualistic approaches. Farmers have different methods of providing average flock weights. Some weigh their chickens manually, creating the need for the deterministic formulations, but others have outfitted their barns with more modern equipment. This equipment includes scales built into parts of the barn itself and provides daily digital updates on the average flock weights. In a week where every barn to be scheduled has this technology, last-minute schedule adjustments with certain knowledge of the flock weights are possible. By offering a significant reduction in maximum deviation if the exact schedule is not finalized until the day before collection begins, the two-stage stochastic approach offers a valuable way for a poultry company to reduce WS, and thus average defect rates, in these weeks. It can be noted that this contribution is also valuable to poultry companies in areas not subject to supply management. While the model would need to be adjusted by replacing the goal of meeting farmer quotas with a goal of maximizing profit, reducing WS to reduce defect rates is still appealing. This information incentivizes any poultry company to delay their flock scheduling until their weight forecasts become certain, even if trucks must be scheduled to visit geographic clusters earlier than that.

# Chapter 6

# Conclusion

This thesis considers a production planning problem in which forecasts must be made about an uncertain supply, then this supply must be scheduled across a time horizon subject to a set of operating constraints while considering the competing interests of multiple parties simultaneously in the solution.

After reviewing the relevant literature, Chapter 2 identifies an opportunity for contribution by considering a production planning problem which must balance competing interests and consider the stochastic multi-period lot sizing problem under uncertain supply. By creating and testing three formulations each for the deterministic and stochastic problem, useful models are created to accomplish this goal in the context of DFPP and SFPP.

Chapter 3 applies two common approaches to the multi-period lot sizing problem to solve DFPP by developing an integer program [DIP] and a weighted goal program [DWGP], then proposes a less frequently considered approach in the minmax goal program [DMGP]. It qualifies [DIP] and [DWGP] as collectivistic and [DMGP] as individualistic, then considers that the collectivistic solutions present an increased risk of loss in a single week in exchange for the potential of expected gain. It concludes that testing should be performed to quantify this relationship.

Chapter 4 considers SFPP, a modified version of DFPP with uncertainty incorporated into the weight predictions by having farmers send a second set of average flock weights as late as possible. It uses soft robust optimization to adapt [DIP] to the problem as [RP], then stochastic versions of [DWGP] and [DMGP] are proposed as [SWGP] and [SMGP], respectively. It is noted that [RP] and [SWGP] are still collectivistic approaches while [SMGP] is individualistic, concluding that testing should be done to quantify the tradeoff in single-week risk vs expected reward in case it differs from that of the deterministic problem.

Chapter 5 analyzes flock history from ABC Poultry to model the growth rate of each flock as a normally distributed random variable, then approximates the expected value of this variable with an 8-point moving average to make predictions. When the models of the deterministic problem [DIP], [DWGP], and [DMGP] are tested, [DMGP] is observed as the superior solution because it provides a significant reduction in the risk of a large maximum deviation in exchange for a statistically insignificant change in production optimality for the farmers. When the models of the stochastic solution [RP], [SWGP], and [SMGP] are tested, [SMGP] is observed as the superior solution because it provides statistically significant improvements on test data, yielding reductions in expected maximum deviation ranging from 5.8% to 61.9% while demonstrating no evidence of a significant reduction in production optimality.

This thesis makes the following contributions:

- An agricultural system is modelled under supply management to create prescriptive optimization models: DFPP and SFPP both define agricultural systems under supply management. After creating three models to solve each problem, the MGP approach is observed to produce the superior optimization model for both problems.

- The multi-period lot sizing problem is considered under the influence of multiple stakeholders with competing financial interests: DFPP and SFPP both require the optimization of the production of competing stakeholders. The best solution methods [DMGP] and [SMGP] consistently reduce the maximum expected deviation from optimality, thereby lowering the risk of a solution disproportionately impacting a farmer, without reducing the expected quota fulfillment of that farmer.

- A multiobjective formulation of the multi-period lot sizing problem with uncertain supply is created: [SMGP] and [SWGP], the stochastic GP formulations of SFPP, both provide a multiobjective formulation of the multi-period lot sizing problem with uncertain supply.

- The performance of SRO and stochastic GP techniques is compared: [RP] is compared to [SWGP] and [SMGP]. SRO may provide a small average improvement in the criterion its objective function optimizes (DFQ) at the cost of larger maximum deviations from optimality compared to stochastic GP.

In addition, the following managerial insights are provided:

- Useful predictions can be made about the average growth rate of a flock of chickens based on the growth rates of previous flocks raised in the same barn. In the case study, a forecasting model is established using a moving average of the growth rates for the 8 flocks most recently raised in the same barn.
- Risk pooling offers an improvement in performance for meeting quotas and balancing weight across days in exchange for a reduction in performance in managing the maximum average weight difference between flocks, potentially leading to a higher defect rate. In the case study, it is observed that scheduling approximately 8 flocks per day provides good performance in DFQ and WB but unacceptable performance in WS.
- Delaying the scheduling of flocks for procurement until their average weights are known results in schedules which adhere significantly better to production guidelines. In the case study, it is observed across five test weeks that delaying this decision reduces maximum deviation by between 40.3% and 86.6%.

The case study shows that ABC Poultry should implement a minmax goal programming approach to procurement scheduling moving forward, and additionally that the company should encourage farmers to adopt the practice of sending a second set of weights the day before collection begins for the week.

## 6.1 Future Work

SFPP assumes that farmers providing average flock weights on Sunday enable predictions accurate enough to be considered certain. If this is no longer considered to be the case, and it is assumed that a flock's average weight cannot be known with certainty until the day before collection, SFPP could be considered as a multi-stage stochastic problem. Once the week of collection arrives, farmers provide daily average weights until their flocks are collected. Each day is its own stage, so the problem has $n + 1$ stages for $n$ days. The geographic areas to be visited would still be selected in the first stage the week before collection, but each subsequent stage would only determine the flocks to be collected that day.

While the problem of uncertain supply is solved for SFPP using scenario generation, alternative methods of handling uncertainty could also be considered for the problem. Alternative formulations could be considered in which the uncertainty of the weight predictions is captured by chance constraints or fuzzy programming.

Weather conditions might be considered. A bridge could become closed for a day due to inclement weather, causing some of the barns the decision maker purchases from to become inaccessible that day. A multi-stage stochastic programming approach could be considered by assessing the probability of bridge closure for each day during planning, then making an updated set of decisions each day when the 24-hour forecast reveals whether the bridge will be closed the next day.

Another variant of DFPP or SFPP might be considered where farmers receive priority if they have produced an unsatisfactory overall amount across a recent sixteen-week quota period. This extension would take the form of a modification to the GP framework assigning higher weights to deviations by farmers who had recently under- or over-produced across an entire quota period. In addition to their applications in the WGP, the weights could still be used in the MGP by modifying what constitutes two deviations of the same magnitude. While this thesis shows in Chapter 3 that adjusting production targets each week helps produce quota amounts more accurately, it does not consider how to balance who is impacted by large deviations over time. Preemptive GP could also be used. A higher priority than any other goal could be assigned to minimizing deviations for farmers who have recently been forced to deviate from their quotas or a lower priority than any other goal could be assigned to minimizing deviations for farmers who have not. The extension would be a valuable contribution because it is helpful to farmers to reduce their chances of suboptimal production in consecutive quota periods.

An additional dimension of stochasticity could also be considered for SFPP by assuming an uncertain flock size based on a variable mortality rate. Not every egg placed with a farmer becomes a chicken that is processed because some of them die due to sickness or environmental factors. A model predicting flock size at collection and a formulation considering the uncertainty inherent in that prediction would be a useful contribution. A model could also be created for mortality rate based on the weather. As climate change creates more extreme temperatures, it becomes harder for chickens to survive in the back of a truck, particularly in areas of the world that are already especially hot or cold. Their mortality rates could be monitored and corrective action taken if an increase was observed over time, ensuring more sustainable production.

A poultry company can also consider sustainability by ensuring they purchase from suppliers with sustainable practices. Azadnia *et al.* [12] present a multi-period lot sizing model with supplier selection in which each supplier is assigned a sustainability score based on environmental, social, and economic qualitative criteria. While the decision maker for DFPP and SFPP cannot choose to collect flocks from some farmers and not others, sustainability scores could be used to determine which farmers to prioritize as suppliers when requesting flocks from farmers. Alternatively, a multi-stage stochastic problem could be considered in which the first stage selects which farmers will receive flocks from the hatcheries each day and the second and third stages are the two stages of SFPP. A problem like this requires more decision making power than is assumed in DFPP or SFPP because the decision maker also controls the hatcheries.

A larger poultry company might be considered without downtime between each week of production, creating a modified version of SFPP. A rolling horizon model is useful when production occurs every day. Remembering that geographic clusters must be selected for collection $2|T|$ days in advance, flock weight predictions can be made $2|T|$ days in advance and average flock weights can be known with certainty $|T|$ days in advance, a stochastic dynamic programming model can be established. Each day $t$ is a new stage requiring $y_{g,t+2|T|}$ and $x_{i,t+1}$ to be decided and sent to the transportation department, influenced by the decision variables $x_{i,t+2,s}$ through $x_{i,t+2|T|,s}$. It can be noted that while the average flock weights would be known for each day until $t + |T|$, $x_{i,t+2,s}$ through $x_{i,t+|T|,s}$ would still be indexed by scenario because more weight predictions become certain every day and the decision must only be made a day in advance.

Vehicle routing decisions might be added to the problem. Oliveira and Lindau [1] propose a routing problem which seeks to minimize loss of mass between flock collection and processing, an approach which could be taken to DFPP or SFPP. Observations could then be made about whether implementing this approach leads to an increase in prediction accuracy, or if the weight forecasting model could be modified to predict the average weight of a flock based on its collection time.

Decisions on behalf of the transportation department might also be considered in a bilevel program that treats the transportation department as the leader and the procurement department scheduling flocks for collection as the follower. A variant of DFPP or SFPP could be considered in which the transportation department at the poultry company can set their policy each week about which geographic clusters can be visited in the same day. The department would seek to minimize cost by properly utilizing its drivers and equipment, anticipating which policy would result in the most efficient schedule each week.

Bilevel programming can also be used to assist decision making in hatcheries. One hatchery supplies many farms, and those farms may supply multiple poultry companies. Assume each poultry company can make predictions based on the age of a flock and the barn it is raised in, rather than requiring an interim weight. The poultry companies can solve DFPP or SFPP and use the objective function of their formulation to evaluate their satisfaction with the flock placement schedule a hatchery generates. As the leader, the hatchery would place flocks with farms to maximize the satisfaction of the followers, who are the poultry companies. Different formulations could use stochastic and robust optimization and observe the differences in their solutions.

A supply management system is not a constraint everywhere in the world: while it is used across Canada, countries as close as the United States do not have a similar system [77]. If Canada abandoned the supply management system, DFPP and SFPP would change. Because farmers would no longer have quotas to meet, the decision maker would be free to consider flock purchases based on the most profitable schedule. SFPP would become a multi-product MLSSP with stochastic supply and demand, considering chickens in different weight ranges as different products. Bilevel programming can also be considered as a solution to capture the competitive nature of the free market, viewing the decision maker at the poultry company as the leader and the decision maker at each farm as a follower. The poultry company would schedule flocks for collection and offer to pay a premium for chickens of a certain size because they are easier to sell, incentivizing farmers to make a decision more complex than raising the flock to be as heavy as possible.

If the supply management system is abolished, the assumption that holding cost is unnecessary might be relaxed. Canada initially adopted the supply management system to control a production surplus [3] and it is possible that the surplus could return if the system disappeared. If the demand of the market is less than the supply of chicken, the poultry company must hold the surplus until it can be sold. Consideration must also be given to the perishability of the chicken, which can only be held for a finite amount of time before becoming unsellable and incurring a waste cost. The multi-period lot sizing problem with perishable goods has been considered in the literature [6, 58], but not with supplier selection.

Perishability can also be considered in the context of a major disruption to the market such as a pandemic. While demand for any amount of supply can be a reasonable assumption under supply management, the assumption does not hold if the market changes quickly enough. Consider the effects of the COVID-19 pandemic. Many restaurants temporarily closed when Canada went into lockdown in 2020 [3], causing demand for food products including poultry to decrease rapidly and for some of those perishable products to be wasted. An extension of this thesis could consider a version of DFPP or SFPP which can be implemented when the decision maker is at a heightened risk of lockdown. The model would use the probability of lockdown to penalize the objective function based on the proportion of sales that could be lost if restaurants closed again. Assuming that restaurants are more likely to purchase smaller chickens because they pay a price per unit weight but sell their pieces for a flat rate, the proportion of each flock expected to be purchased by restaurants would vary based on the flock's average size.

# Bibliography


[1] Gilson Adamczuk Oliveira and Luis Antonio Lindau. A framework for delivery scheduling in the poultry industry. *Journal of Scheduling*, 15(6):757–772, 2012.

[2] Ivo Adan, Jos Bekkers, Nico Dellaert, Jully Jeunet, and Jan Vissers. Improving operational effectiveness of tactical master plans for emergency and elective patients under stochastic demand and capacitated resources. *European Journal of Operational Research*, 213(1):290–308, 2011.

[3] Aleks, ra Sagan · The Canadian Press · Posted: Apr 09, 2020 3:56 PM ET | Last Updated: April 27, and 2020. Why canada's dairy farmers are dumping milk despite food supply issues in COVID-19 | CBC news, 2020.

[4] Douglas Alem, Eduardo Curcio, Pedro Amorim, and Bernardo Almada-Lobo. A computational study of the general lot-sizing and scheduling model under demand uncertainty via robust and stochastic approaches. *Computers & Operations Research*, 90:125–141, 2018.

[5] Hesham K. Alfares and Rio Turnadi. Lot sizing and supplier selection with multiple items, multiple periods, quantity discounts, and backordering. *Computers & Industrial Engineering*, 116:59–71, 2018.

[6] Zohreh Alipour, Fariborz Jolai, Ehsan Monabbati, this link will open in a new window Link to external site, and Nima Zaerpour. General lot-sizing and scheduling for perishable food products. *RAIRO: Recherche Opérationnelle*, 54, 2020. Place: Paris, France Publisher: EDP Sciences.

[7] S. Aouadni and A. Rebai. Inventory management with supplier selection problem: Supply uncertainty. In *2013 5th International Conference on Modeling, Simulation and Applied Optimization (ICMSAO)*, pages 1–5, 2013.

[8] S. Aouadni and A. Rebai. Lot-sizing problem with supplier selection considering safety stock. In *2013 International Conference on Control, Decision and Information Technologies (CoDIT)*, pages 766–769, 2013.

[9] Juan Aparicio, Magdalena Kapelko, Lidia Ortiz, and Jesus T. Pastor. Production under a quota system: an extension of the weighted additive model to assess technical efficiency. *INFOR: Information Systems and Operational Research*, 55(3):227–242, 2017. Publisher: Taylor & Francis _eprint: https://doi.org/10.1080/03155986.2016.1272961.

[10] Mahsa Arabi, Saeed Yaghoubi, and Javad Tajik. Algal biofuel supply chain network design with variable demand under alternative fuel price uncertainty: A case study. *Computers & Chemical Engineering*, 130:106528, 2019.

[11] Ghazal Assadipour and Jafar Razmi. Possibilistic inventory and supplier selection model for an assembly system. *The International Journal of Advanced Manufacturing Technology*, 67(1):575–587, 2013.

[12] Amir Hossein Azadnia, Muhamad Zameri Mat Saman, and Kuan Yew Wong. Sustainable supplier selection and order lot-sizing: an integrated multi-objective decision-making process. *International Journal of Production Research*, 53(2):383–408, 2015. Publisher: Taylor & Francis _eprint: https://doi.org/10.1080/00207543.2014.935827.

[13] Vahid Azizi, Guiping Hu, and Mahsa Mokari. A two-stage stochastic programming model for multi-period reverse logistics network design with lot-sizing. *Computers & Industrial Engineering*, 143:106397, 2020.

[14] Chuda Basnet and Janny M. Y. Leung. Inventory lot-sizing with supplier selection. *Computers & Operations Research*, 32(1):1–14, 2005.

[15] Paolo Brandimarte. Multi-item capacitated lot-sizing with demand uncertainty. *International Journal of Production Research*, 44(15):2997–3022, 2006. Publisher: Taylor & Francis _eprint: https://doi.org/10.1080/00207540500435116.

[16] Elisabeth Brevik, Anna Ø Lauen, Maria C. B. Rolke, Kjetil Fagerholt, and Jone R. Hansen. Optimisation of the broiler production supply chain. *International Journal of Production Research*, 58(17):5218–5237, 2020. Publisher: Taylor & Francis _eprint: https://doi.org/10.1080/00207543.2020.1713415.

[17] A. Cem Randa, Mustafa K. Doğru, Cem Iyigun, and Ulaş Özen. Heuristic methods for the capacitated stochastic lot-sizing problem under the static-dynamic uncertainty strategy. *Computers & Operations Research*, 109:89–101, 2019.

[18] Devendra Choudhary and Ravi Shankar. Modeling and analysis of single item multi-period procurement lot-sizing problem considering rejections and late deliveries. *Computers & Industrial Engineering*, 61(4):1318–1323, 2011.

[19] Devendra Choudhary and Ravi Shankar. A goal programming model for joint decision making of inventory lot-size, supplier selection and carrier selection. *Computers & Industrial Engineering*, 71:1–9, 2014.

[20] Eduardo Curcio, Pedro Amorim, Qi Zhang, and Bernardo Almada-Lobo. Adaptation and approximate strategies for solving the lot-sizing and scheduling problem under multistage demand uncertainty. *International Journal of Production Economics*, 202:81–96, 2018.

[21] Leopoldo Eduardo Cárdenas-Barrón, José Luis González-Velarde, and Gerardo Treviño-Garza. A new approach to solve the multi-product multi-period inventory lot sizing with supplier selection problem. *Computers & Operations Research*, 64:225–232, 2015.

[22] K. Deb, A. Pratap, and T. Meyarivan. A fast elitist non-dominated sorting genetic algorithm for multi-objective: NSGA-II. In *Proceedings of the Parallel Problem Solving from Nature VI Conference*, pages 846–858. Springer-Verlag, 2000.

[23] Ezgi Aktar Demirtas and Ozden Ustun. Analytic network process and multi-period goal programming integration in purchasing decisions. *Computers & Industrial Engineering*, 56(2):677–690, 2009.

[24] Lisha Duan and José A. Ventura. A dynamic supplier selection and inventory management model for a serial supply chain with a novel supplier price break scheme and flexible time periods. *European Journal of Operational Research*, 272(3):979–998, 2019.

[25] David Escuín, Lorena Polo, and David Ciprés. On the comparison of inventory replenishment policies with time-varying stochastic demand for the paper industry. *Journal of Computational and Applied Mathematics*, 309:424–434, 2017.

[26] Hamed Fazlollahtabar and Ermia Aghasi. A two-stage mechanism as a decision support for marketing via customer satisfaction measures. *Journal of the Chinese Institute of Industrial Engineers*, 29(4):270–281, 2012. Publisher: Taylor & Francis _eprint: https://doi.org/10.1080/10170669.2012.691118.

[27] Yonghan Feng and Sarah M. Ryan. Scenario construction and reduction applied to stochastic power generation expansion planning. *Computers & Operations Research*, 40(1):9–23, 2013.

[28] R. B. Flavell. A new goal programming formulation. *Omega*, 4(6):731–732, 1976.

[29] M. Florian, J. K. Lenstra, and A. H. G. Rinnooy Kan. Deterministic production planning: Algorithms and complexity. *Management Science*, 26(7):669–679, 1980. Publisher: INFORMS.

[30] Steven A. Gabriel, Jifang Zhuang, and Ruud Egging. Solving stochastic complementarity problems in energy market modeling using scenario reduction. *European Journal of Operational Research*, 197(3):1028–1040, 2009.

[31] Government of Nova Scotia. Indicators of prosperity, 2020.

[32] C. A. Hans, P. Sopasakis, J. Raisch, C. Reincke-Collon, and P. Patrinos. Risk-averse model predictive operation control of islanded microgrids. *IEEE Transactions on Control Systems Technology*, 28(6):2136–2151, 2020. Conference Name: IEEE Transactions on Control Systems Technology.

[33] Holger Heitsch and Werner Römisch. Scenario tree modeling for multistage stochastic programs. *Mathematical Programming*, 118(2):371–406, 2009.

[34] Ronald Hochreiter and Georg Ch. Pflug. Financial scenario generation for stochastic multi-stage decision processes as facility location problems. *Annals of Operations Research*, 152(1):257–272, 2007.

[35] Tito Homem-de Mello. On rates of convergence for stochastic optimization problems under non-independent and identically distributed sampling. *Society for Industrial and Applied Mathematics Journal on Optimization*, 19(2):524–551, 2008.

[36] Zhengyang Hu and Guiping Hu. A two-stage stochastic programming model for lot-sizing and scheduling under uncertainty. *International Journal of Production Economics*, 180:198–207, 2016.

[37] Zhengyang Hu and Guiping Hu. A multi-stage stochastic programming for lot-sizing and scheduling under demand uncertainty. *Computers & Industrial Engineering*, 119:157–166, 2018.

[38] Zhengyang Hu and Guiping Hu. Hybrid stochastic and robust optimization model for lot-sizing and scheduling problems under uncertainties. *European Journal of Operational Research*, 284(2):485–497, 2020.

[39] Peijie Huang, Piyuan Lin, Shangwei Yan, and Meiyan Xiao. Seasonal broiler growth performance prediction based on observational study. *Journal of Computers*, 7(8):1895–1902, 2012.

[40] Kjetil Høyland and Stein W. Wallace. Generating scenario trees for multistage decision problems. *Management Science*, 47(2):295–307, 2001. Publisher: INFORMS.

[41] Industrial Technology Research Institute. Taiwan overview, 2013.

[42] Wen Jiao, Ju-Liang Zhang, and Hong Yan. The stochastic lot-sizing problem with quantity discounts. *Computers & Operations Research*, 80:1–10, 2017.

[43] S. V. Johansen, J. D. Bendtsen, and J. Mogensen. Broiler growth optimization using optimal iterative learning control. In *2019 American Control Conference (ACC)*, pages 2203–2208, 2019. ISSN: 2378-5861.

[44] S. V. Johansen, J. D. Bendtsen, and J. Mogensen. Broiler slaughter weight forecasting using dynamic neural network models. In *2019 IEEE 6th International Conference on Industrial Engineering and Applications (ICIEA)*, pages 463–470, 2019.

[45] S. V. Johansen, M. R. Jensen, B. Chu, J. D. Bcndtsen, J. Mogensen, and E. Rogers. Broiler growth optimization using norm optimal terminal iterative learning control. In *2018 IEEE Conference on Control Technology and Applications (CCTA)*, pages 1258–1264, 2018.

[46] He-Yau Kang and Amy H. I. Lee. A stochastic lot-sizing model with multi-supplier and quantity discounts. *International Journal of Production Research*, 51(1):245–263, 2013. Publisher: Taylor & Francis _eprint: https://doi.org/10.1080/00207543.2012.674649.

[47] Esmaeil Keyvanshokooh, Sarah M. Ryan, and Elnaz Kabir. Hybrid robust and stochastic optimization for closed-loop supply chain network design using accelerated benders decomposition. *European Journal of Operational Research*, 249(1):76–92, 2016.

[48] S. Khamjan, K. Piewthongngam, and S. Pathumnakul. Pig procurement plan considering pig growth and size distribution. *Computers & Industrial Engineering*, 64(4):886–894, 2013.

[49] Thomas Kirschstein and Frank Meisel. A multi-period multi-commodity lot-sizing problem with supplier selection, storage selection and discounts for the process industry. *European Journal of Operational Research*, 279(2):393–406, 2019.

[50] Kuldeep Lamba, Surya Prakash Singh, and Nishikant Mishra. Integrated decisions for supplier selection and lot-sizing considering different carbon emission regulations in big data environment. *Computers & Industrial Engineering*, 128:1052–1062, 2019.

[51] Amy H. I. Lee, He-Yau Kang, Chun-Mei Lai, and Wan-Yu Hong. An integrated model for lot sizing with supplier selection and quantity discounts. *Applied Mathematical Modelling*, 37(7):4733–4746, 2013.

[52] Hongyan Li and Anders Thorstenson. A multi-phase algorithm for a joint lot-sizing and pricing problem with stochastic demands. *International Journal of Production Research*, 52(8):2345–2362, 2014. Publisher: Taylor & Francis _eprint: https://doi.org/10.1080/00207543.2013.864053.

[53] Zhiying Liao and Jens Rittscher. Integration of supplier selection, procurement lot sizing and carrier selection under dynamic demand conditions. *International Journal of Production Economics*, 107(2):502–510, 2007.

[54] Shu-Cheng Lin, Han-Wen Tuan, and Peterson Julian. An improvement for fuzzy stochastic goal programming problems. *Mathematical Problems in Engineering*, 2017. ISSN: 1024-123X Pages: e8605652 Publisher: Hindawi Volume: 2017.

[55] Francois Louveaux and John R. Birge. L-shaped method for two-stage stochastic programs with recourseL-shaped method for two-stage stochastic programs with recourse. In Christodoulos A. Floudas and Panos M. Pardalos, editors, *Encyclopedia of Optimization*, pages 1943–1945. Springer US, 2009.

[56] Ali Mahmoodirad and Sadegh Niroomand. A belief degree-based uncertain scheme for a bi-objective two-stage green supply chain network design problem with direct shipment. *Soft Computing*, 24(24):18499–18519, 2020.

[57] M. Masoud, H. A. Khalifa, S. Q. Liu, M. Elhenawy, and P. Wu. A fuzzy goal programming approach for solving fuzzy multi-objective stochastic linear programming problem. In *2019 International Conference on Industrial Engineering and Systems Management (IESM)*, pages 1–6, 2019.

[58] S.M.J. Mirzapour Al-E-Hashem, Z. Sazvar, A. Baboli, Y. Rekik, and M. Akbari Jokar. A capacitated multi-product dynamic lot-sizing problem by considering expiration dates; a new approach. *IFAC Proceedings Volumes*, 46(9):152–157, 2013. Publisher: Elsevier.

[59] Milad Mohammadi, Majid Esmaelian, and Arezoo Atighehchian. Design of mathematical models for the integration of purchase and production lot-sizing and scheduling problems under demand uncertainty. *Applied Mathematical Modelling*, 84:1–18, 2020.

[60] John M. Mulvey and Andrzej Ruszczynski. A new scenario decomposition method for large-scale stochastic optimization. *Operations Research*, 43(3):477, 1995. Num Pages: 14 Place: Linthicum, United States Publisher: Institute for Operations Research and the Management Sciences.

[61] John M. Mulvey, Robert J. Vanderbei, and Stavros A. Zenios. Robust optimization of large-scale systems. *Operations Research*, 43(2):264, 1995. Num Pages: 18 Place: Linthicum, United States Publisher: Institute for Operations Research and the Management Sciences.

[62] B. B. Pal, S. Gupta, and D. Chakraborti. A genetic algorithm based stochastic simulation approach to chance constrained interval valued multiobjective decision making problems. In *2010 Second International conference on Computing, Communication and Networking Technologies*, pages 1–7, 2010.

[63] Jafar Razmi and Elham Maghool. Multi-item supplier selection and lot-sizing planning under multiple price discounts using augmented $\epsilon$-constraint and tchebycheff method. *The International Journal of Advanced Manufacturing Technology*, 49(1):379–392, 2010.

[64] Chong Feng Ren, Rui Huan Li, Liu Dong Zhang, and Ping Guo. Multiobjective stochastic fractional goal programming model for water resources optimal allocation among industries. *Journal of Water Resources Planning and Management*, 142(10):04016036, 2016. Publisher: American Society of Civil Engineers.

[65] Jafar Rezaei and Mansoor Davoodi. Genetic algorithm for inventory lot-sizing with supplier selection under fuzzy demand and costs. In Moonis Ali and Richard Dapoigny, editors, *Advances in Applied Artificial Intelligence*, Lecture Notes in Computer Science, pages 1100–1110. Springer, 2006.

[66] Jafar Rezaei and Mansoor Davoodi. Multi-objective models for lot-sizing with supplier selection. *International Journal of Production Economics*, 130(1):77–86, 2011.

[67] Navid Sahebjamnia, F. Jolai, S. A. Torabi, and Mohsen Aghabeiglo. A novel fuzzy stochastic multi-objective linear programming for multi-level capacitated lot-sizing problem: a real case study of a furniture company. *The International Journal of Advanced Manufacturing Technology*, 84(1):749–767, 2016.

[68] Francisco Salas-Molina, Juan A. Rodriguez-Aguilar, and David Pla-Santamaria. A stochastic goal programming model to derive stable cash management policies. *Journal of Global Optimization*, 76(2):333–346, 2020.

[69] Ali Shakourloo. A multi-objective stochastic goal programming model for more efficient remanufacturing process. *The International Journal of Advanced Manufacturing Technology*, 91(1):1007–1021, 2017.

[70] Kailash Chand Sharma, Prerna Jain, and Rohit Bhakar. Wind power scenario generation and reduction in stochastic programming framework. *Electric Power Components and Systems*, 41(3):271–285, 2013. Publisher: Taylor & Francis _eprint: https://doi.org/10.1080/15325008.2012.742942.

[71] A. L. Solano-Blanco, J. E. Gonzalez, L. O. Gomez-Rueda, J. J. Vargas-Sanchez, and A. L. Medaglia. Integrated planning decisions in the broiler chicken supply chain. *International Transactions in Operational Research*, 00:1–24, 2020.

[72] Statistics Canada. Table 11-10-0012-01 distribution of total income by census family type and age of older partner, parent or individual.

[73] Statistics Canada. Table 11-10-0125-01 detailed food spending, canada, regions and provinces.

[74] Statistics Canada. Table 18-10-0002-01 monthly average retail prices for food and other selected products.

[75] Statistics Canada. Poultry and egg statistics, may 2019 and annual 2018, 2019.

[76] Statistics Canada. Table 13-10-0385-01 household food security by living arrangement, 2020.

[77] John Paul Tasker. Supply management in Canada — a primer, 2018.

[78] Miguel Taube-Netto. Integrated planning for poultry production at sadia. *Interfaces*, 26(1):38–53, 1996.

[79] Dariush Tavaghof-Gigloo and Stefan Minner. Planning approaches for stochastic capacitated lot-sizing with service level constraints. *International Journal of Production Research*, 0(0):1–21, 2020. Publisher: Taylor & Francis _eprint: https://doi.org/10.1080/00207543.2020.1773003.

[80] Horst Tempelmeier and Timo Hilger. Linear programming models for a stochastic dynamic capacitated lot sizing problem. *Computers & Operations Research*, 59:119–125, 2015.

[81] Nikolas Topaloglou, Hercules Vladimirou, and Stavros A. Zenios. A dynamic stochastic programming model for international portfolio management. *European Journal of Operational Research*, 185(3):1501–1524, 2008.

[82] Ozden Ustun and Ezgi Aktar Demirtas. Multi-period lot-sizing with supplier selection using achievement scalarizing functions. *Computers & Industrial Engineering*, 54(4):918–931, 2008.

[83] Ozden Ustun and Ezgi Aktar Demı˙rtas. An integrated multi-objective decision-making process for multi-period lot-sizing with supplier selection. *Omega*, 36(4):509–521, 2008.

[84] Alejandro Vital Soto, Nusrat T. Chowdhury, Maral Z. Allahyari, Ahmed Azab, and Mohammed F. Baki. Mathematical modeling and hybridized evolutionary LP local search method for lot-sizing with supplier selection, inventory shortage, and quantity discounts. *Computers & Industrial Engineering*, 109:96–112, 2017.

[85] H. M. Wagner and T. M. Whitin. Dynamic version of the economic lot size model. *Management Science*, 5(1):89–96, 1958.

[86] Wolfram Wiesemann, Daniel Kuhn, and Berç Rustem. Maximizing the net present value of a project under uncertainty. *European Journal of Operational Research*, 202(2):356–367, 2010.

[87] Wilcoxon, F. Individual comparisons by ranking methods. *Biometrics Bulletin*, 1(6):80–83, 1945.

[88] M. Xiao, P. Huang, P. Lin, and S. Yan. Broiler growth performance analysis: From correlation analysis, multiple linear regression, to neural network. In *2010 4th International Conference on Bioinformatics and Biomedical Engineering*, pages 1–4, 2010. ISSN: 2151-7622.

[89] Chian-Son Yu and Han-Lin Li. A robust optimization model for stochastic logistic problems. *International Journal of Production Economics*, 64(1):385–397, 2000.

[90] Mazyar Zahedi-Seresht, Gholam-Reza Jahanshahloo, and Josef Jablonsky. A robust data envelopment analysis model with different scenarios. *Applied Mathematical Modelling*, 52:306–319, 2017.

[91] Yahia Zare Mehrjerdi. A multiple objective stochastic approach to vehicle routing problem. *The International Journal of Advanced Manufacturing Technology*, 74(5):1149–1158, 2014.

[92] Siqiong Zhou and Ayca Erdogan. A spatial optimization model for resource allocation for wildfire suppression and resident evacuation. *Computers & Industrial Engineering*, 138:106101, 2019.