International Comparisons of Poverty Intensity: Index Decomposition and Bootstrap Inference

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1. Introduction

The purpose of this study is: (i) to propose a modified index of poverty intensity which is suitable for survey data with sampling weights; (ii) to introduce a bootstrap-based statistical inference of this index and the Gini index of inequality; (iii) to decompose the index of poverty intensity into three meaningful and familiar poverty measures, the poverty rate (sometimes called the headcount ratio), the average poverty gap ratio among the poor, and the overall Gini index of poverty gap ratios; and (iv) to apply the above measures to actual data to provide an international comparison of poverty intensity and contributing factors across major industrialized countries and over time.

In this paper, we analyze changes of poverty intensity over time for the following countries: Australia (1981, 1985, and 1989), Belgium (1985, 1988, and 1992), Canada (1971, 1975, 1981, 1987, 1991 and 1994), Denmark (1987 and 1992), Finland (1987 and 1991), France (1979, 1981, and 1984), Germany (1981, 1983, and 1984), Israel (1979, 1986, and 1992), Italy (1986 and 1991), the Netherlands (1983, 1987, and 1991), Norway (1979, 1986, and 1991), Sweden (1975, 1981, 1987, and 1992), United Kingdom (1979, and 1986), and United States (1974, 1979, 1986, 1991, and 1994).

We assume that within all the sampled countries, at all dates: (i) household income (after tax) is equally shared among all household members , (ii) the OECD equivalence scale adequately accounts for economies of scale in family consumption; and (iii) the poverty line is represented by half the median equivalent income.

2. The Sen-Shorrocks Index: Decomposition and Inference

Since Sen (1976) proposed a poverty index and a set of desirable criteria for evaluating a poverty index in his seminal paper, research on poverty indices has received considerable attention. As the Sen index is not replication invariant, not continuous in individual incomes, and fails to satisfy the transfer axiom, Shorrocks (1995) has recently proposed a modified Sen index (the Sen- Shorrocks index or the S-S index hereafter) for measuring the intensity of poverty.

The S-S index is proposed assuming that all the income data of a population are known and nonstochastic. Let ith-person's income of the population size N be Y_i, such that $Y_1 < Y_2 < \ldots < Y_N$. and the poverty line be $z > 0$. Let Q ($< N$) be the number of individuals whose income is less than z. For the ith poor person, the poverty gap is $z-Y_i$, and the poverty gap ratio (X_i) is $(z-Y_i)/z$. The S-S index is defined as [see Shorrocks (1995)]:

$$P(Y;z)=\frac{1}{N^2}\sum_{i=1}^{Q}(2N-2i+1)\frac{z-Y_i}{z}. \tag{1}$$

It can be regarded as a weighted "average" of individual poverty gap ratios of the poor. The S-S index is desirable because (i) it is symmetric, replication invariant, monotonic, homogeneous of degree zero in individual incomes and the corresponding poverty line, and normalized to take values in the range [0,1]; (ii) it is continuous in individual incomes and consistent with the transfer axiom; and (iii) it admits a geometric interpretation. $P(Y;z)$ can be computed based on Equation (1) if the individual incomes of all members of the population are available.

The decomposition of the Sen-Shorrocks index of poverty intensity [as shown in Shorrocks (1995, p.1228)] is given by:

$$P(Y;z) = \mu(X)[1+G(X)], \tag{2}$$

where $\mu(X)$ and $G(X)$ are the average poverty gap ratio and Gini coefficient of poverty gap ratios (among all people), with the non-poor population's X_i being set to zero.

A further decomposition can be based on the fact that $\mu(X)$ is simply the weighted average of the average poverty gap ratio among the poor [GAP] and the poverty gap ratio among the non-poor (i.e. zero), where the weights are the population proportions (i.e., the poverty rate [RATE] and one minus the poverty rate, respectively).

It is easy to see that: $\mu(X) = (RATE)\,(GAP) + (1\text{-}RATE)\,(0) = (RATE)(GAP)$.

Hence,

$$P(Y;z) = (RATE)\,(GAP)\,(1+G(X)). \tag{3}$$

The overall percentage rate of change in poverty intensity can then be expressed as the sum of the percentage changes in the poverty rate, average poverty gap ratio (among the poor), and Gini index of inequality in the poverty gap ratios (among all people):

$$\Delta \ln P(Y;z) = \Delta \ln(RATE) + \Delta \ln(GAP) + \Delta \ln(1+G(X)). \tag{4}$$

The data that economists normally use contain the sample incomes of households with sampling weights. Let m households in the sample be ordered by their equivalent incomes in an ascending order and be indexed by i. Let the total number of households whose equivalent income is below the poverty line z be q ($< m$). Let the sample household equivalent income of household i, that is shared by all the members of that family, be y_i. Let the number of family members of the ith household be n_i, and the sampling weight of the ith household w_i. Thus the total number of individuals is $\sum_{i=1}^{m} n_i w_i$. To accommodate complex survey data, the following formulation for the S-S index for survey data with sampling weights is proposed below:

$$P(y;z) = \frac{1}{[\sum_{l=1}^{m} n_l w_l]^2} \sum_{i=1}^{q} \sum_{j=1}^{n_i w_i} [2(\sum_{l=1}^{m} n_l w_l) - 2(j + \sum_{k=1}^{i} n_{k-1} w_{k-1}) + 1] \frac{z - y_i}{z}, \tag{5}$$

where $n_0 = 0$, and $w_0 = 0$.

To compute the bootstrap standard deviation of the modified S-S index estimator, we resample both equivalent incomes and the sampling weights associated with them. We generate a random integer t, from a uniform distribution defined over the support from zero to the total number of the households m'. Then we use this random integer to draw the tth household equivalent income, the number of members of the tth household, and the sampling weight. The new sample of size m' is denoted by $\{y_i^*, w_i^*, n_i^*\}_{i=1}^{m'}$. The new sample can then be used to compute a new S-S index denoted as $P(y^{*t}, z^{*t})$. Repeating this process T times (e.g. T=200) gives $P(y^{*1}, z^{*1}), P(y^{*2}, z^{*2}), \ldots, P(y^{*T}, z^{*T})$. The bootstrap variance is computed as the sample variance from the large number of the standard S-S index estimates from the resampling. We denote the sampling variance of $P(y;z)$ as $\sigma^2(P(y;z))$, see Efron (1982, Chapter 8) for details. $P(y;z)$ has asymptotic standard normal distribution; hence we construct a 95% confidence interval of ±2 standard deviation surrounding the S-S index estimate in ranking the examined countries.

3. Results

The 1970's LIS data show a fairly clear dichotomy between a "European" and a "North American" level of poverty intensity. Norway (1979) and Sweden (1979) have least poverty intensity, while the UK (1979) is very close. In the early 1970's, Canada's point estimate of poverty intensity exceeds that of the U.S., but the statistical uncertainty surrounding the estimates of Canada (1971) and the U.S. (1974) indicates that no clear judgement is possible. However, by 1986 the U.S. had moved into a class by itself. Canada, like Australia, had moved into the high end of a continuum of "European-style" poverty intensity. Austria, Germany and the Benelux and Scandinavian countries rank low in poverty intensity, depending somewhat on the year of observation for their precise ranking (Notably, Sweden is not particularly exemplary among this group). France, Italy, Israel, Australia and Canada are at the top end of European style poverty intensity - but there is enough statistical uncertainty to caution against being more exact in country rankings.

Table 1 decomposes the level of the Sen-Shorrocks index as per Equation (3) and the changes observed in poverty intensity as per Equation (4). It is noteworthy that, in practice, percentage changes in $\ln(1+G(x))$ are always an order of magnitude smaller than percentage changes in the poverty rate and the average poverty gap ratio. Since inequality in the poverty gap ratios among all people (i.e. $1+G(x)$) does not change much, changes in poverty intensity are dominated by changes in the poverty rate and the average poverty gap ratio. To a first approximation, the percentage change in poverty intensity is the sum of the percentage change in the poverty rate and the percentage change in the average poverty gap ratio of the poor.

We use our estimates of the bootstrap variance reported in Table 1 to indicate (with an asterisk) the changes in poverty intensity within countries that are statistically significant at a 95% level of confidence (i.e., differ by more than two standard deviations from the prior years's estimate). Only ten of 26 observed changes in poverty intensity pass this test - negative shifts and positive shifts are exactly matched.

In some countries, there have been quite large increases in poverty intensity, albeit with differing underlying causes. The 31% increase in Dutch poverty intensity between 1987 and 1991 was driven entirely by an increase in the average poverty gap ratio among the poor - but although Sweden experienced a similar increase in poverty intensity between 1981 and 1987, two-thirds of that increase was due to an increase in the poverty rate.The 39% increase in poverty intensity in the United Kingdom between 1979 and 1986 was almost equally due to an increase in the poverty rate and increase in the average poverty gap ratio among the poverty. Countries experiencing significant declines in poverty intensity include Norway (between 1979 and 1986) France (between 1981 and 1984) and Israel (between 1986 and 1992).

The largest changes in the intensity of poverty in Canada came in the 1970's, particularly between 1971 and 1975. Reductions in poverty intensity in the 1980's and 1990's were much more modest and not statistically significant at a 95% level of confidence. In both the 1970's and the 1980's, declines in Canadian poverty intensity were split fairly evenly between declines in the poverty rate and decreases in the average poverty gap ratio among the poor.

The U.S. data on poverty intensity indicates that the big change in U.S. poverty - an increase of 20% in poverty intensity - occurred in the early 1980's, between 1979 and 1986. About seven tenths of that increase in poverty intensity is ascribable to an increase in the poverty rate, with the remainder being due to an increase in the average poverty gap ratio among the poor. The recession of the early 1980's, coinciding with cuts in welfare benefits, evidently hit the poor of the U.S. rather hard.

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SUMMARY of RESUME

This paper proposes an alternative formulation for the Sen-Shorrocks index of poverty intensity for survey data with sampling weights and decomposes the Sen-Shorrocks index into the poverty rate, average poverty gap ratio, and Gini index of poverty gap ratios. This decomposition allows the percentage change in poverty intensity to be approximated as the sum of the percentage changes in the poverty rate and average poverty gap ratio. To account for sampling variation, this paper also uses the bootstrap method to compute the confidence interval surrounding the Sen-Shorrocks index estimate in international comparisons using Luxemburg Income Study data. Cross-sectional and longitudinal analyses indicate that in the early 1970's poverty intensity in Canada and the U.S. was almost indistinguishable, but in the 1970's Canadian poverty intensity decreased. Large increases in poverty intensity occurred in the 1980's in the United States, the United Kingdom, and Sweden.

Table 1 - Sampling Variance and Decompostition of the Sen-Shorrocks Index

		S-S Index (P)	Standard Deviation of 200 Bootstraps	Decomposition of Level			$\Delta \ln(P)$	Decomposition of Change		
				RATE	*GAP*	$(1+G(x))$		$\Delta \ln$ (*RATE*)	$\Delta \ln$ (*GAP*)	$\Delta \ln$ $(1+G(x))$
Finland	**87**	0.0195	0.00126	0.041	0.243	1.978				
	91	0.0190	0.00118	0.041	0.234	1.979	-0.03	0.010	-0.036	0.001
Belgium	**85**	0.0206	0.00177	0.044	0.237	1.979				
	88	0.0208	0.00237	0.047	0.224	1.977	0.008	0.067	-0.058	-0.001
	92	0.0195	0.00220	0.045	0.221	1.978	-0.07	-0.051	-0.015	0.001
Norway	**79**	0.0249	0.00175	0.041	0.307	1.978				
	86	0.0207	0.00214	0.037	0.285	1.980	-0.185*	-0.112	-0.074	0.001
	91	0.0211	0.00231	0.035	0.303	1.982	0.02	-0.043	0.063	0.001
Netherlands	**83**	0.0235	0.00274	0.040	0.299	1.978				
	87	0.0259	0.00335	0.048	0.271	1.976	0.098	0.200	-0.101	-0.001
	91	0.0353	0.00390	0.047	0.380	1.975	0.309*	-0.030	0.339	0.000
Denmark	**87**	0.0382	0.00186	0.064	0.302	1.969				
	92	0.0355	0.00174	0.053	0.340	1.972	-0.07	-0.194	0.119	0.000
Sweden	**75**	0.0257	0.00180	0.043	0.303	1.977				
	81	0.0286	0.00216	0.047	0.309	1.972	0.107	0.089	0.021	-0.003
	87	0.0390	0.00165	0.058	0.344	1.966	0.309*	0.205	0.107	-0.003
	92	0.0372	0.00198	0.052	0.363	1.969	-0.05	-0.104	0.054	0.002
France	**79**	0.0482	0.00266	0.081	0.305	1.954				
	81	0.0597	0.00488	0.096	0.318	1.948	0.214*	0.176	0.041	-0.003
	84	0.0431	0.00221	0.080	0.276	1.957	-0.326*	-0.190	-0.140	0.005
UK	**79**	0.0324	0.00348	0.067	0.245	1.966				
	86	0.0479	0.00265	0.081	0.300	1.960	0.390*	0.190	0.204	-0.003
Israel	**79**	0.0543	0.00553	0.138	0.205	1.919				
	86	0.0596	0.00401	0.130	0.238	1.921	0.092	-0.060	0.151	0.001
	92	0.0480	0.00369	0.117	0.212	1.931	-0.215*	-0.106	-0.115	0.005
Canada	**71**	0.1020	0.00246	0.149	0.359	1.914				
	75	0.0757	0.00179	0.123	0.318	1.929	-0.299*	-0.186	-0.120	0.008
	81	0.0634	0.00246	0.113	0.290	1.935	-0.177*	-0.089	-0.091	0.003
	87	0.0595	0.00300	0.109	0.281	1.937	-0.06	-0.031	-0.033	0.001
	91	0.0561	0.00248	0.107	0.271	1.938	-0.06	-0.022	-0.037	0.000
Australia	**81**	0.0620	0.00249	0.100	0.319	1.947				
	85	0.0586	0.00338	0.091	0.329	1.951	-0.06	-0.089	0.030	0.002
	89	0.0648	0.00259	0.102	0.329	1.943	0.101	0.106	-0.001	-0.004
US	**74**	0.0990	0.00330	0.146	0.355	1.913				
	79	0.0972	0.00291	0.155	0.328	1.909	-0.02	0.065	-0.081	-0.002
	86	0.1185	0.00344	0.180	0.349	1.888	0.198*	0.145	0.063	-0.011
	91	0.1162	0.00288	0.177	0.346	1.892	-0.02	-0.013	-0.009	0.002
	94	0.1246	0.00154	0.183	0.360	1.889	0.07	0.031	0.040	-0.001