# ITERATIVE ESTIMATION AND DEMODULATION FOR IMPROVED JOINT RANDOM ACCESS SATELLITE COMMUNICATIONS

by

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Submitted in partial fulfillment of the requirements for the degree of Master of Applied Science

at

Dalhousie University Halifax, Nova Scotia December 2015

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#### Abstract

This thesis investigates high throughput random access communications in the satellite up-link environment. In this environment it is typical for a large number of un-coordinated, mobile transmitters scattered over a large geographic area to connect intermittently to a single receiver over the satellite channel. This configuration fundamentally results in a random access scenario, where a large number of potential users contend for access to limited channel resources. Complexity is increased due to the geographic spread and localized weather effects which make it possible for each user have its own independent channel that must be estimated to ensure a high level of system performance.

To this point research efforts have been focused on the development of access protocols capable of approaching the single user capacity, and have succeeded in asymptotically approaching it, however previous efforts have shown that the application of joint detection and multiple packet reception (MPR) techniques make it is possible to surpass the capacity of the current single-user random access channel. The majority of this previous work is focused on improving ALOHA protocols which show a common behavior of collapsing when the number of users is the system exceeds the number of signaling dimensions. While previous results have excelled at reducing the packet loss ratio and achievable throughputs at low SNR with low normalized system loads, little has been done to address the high-SNR case where the expected number of transmissions per signaling dimension is greater than one. In this thesis it is proven that it is possible to operate in an uncoordinated multi-user environment with system loads up to 1.96 in the high-SNR region under realistic conditions.

# List of Abbreviations and Symbols Used

ESA European Space Agency

LMS Land Mobile Satellite

GEO Geosynchronous Orbit

LEO Low Earth Orbit

ETSI European Telecommunications Standard Institute

RCS Return Channel via Satellite
ACM Adaptive Coded Modulation

SNR Signal to Noise Ratio

QPSK Quadrature Phase Shift Keying

QAM Quadrature Amplitude Modulation

dB Decibels

TDMA Time Division Multiple Access

FMDA Frequency Division Multiple Access

CDMA Code Division Multiple Access

AWGN Additive White Gaussian Noise

CRDSA Contention Resolution Diversity Slotted ALOHA

SIC Successive Interference Cancellation IRSA Irregular Repetition Slotted ALOHA

CSA Coded Slotted ALOHA

CRA Contention Resolution ALOHA

FEC Forward Error Correction

ECRA Enhanced Contention Resolution ALOHA

E-SSA Enhanced Spread Spectrum ALOHA

PLR Packet Loss Ratio

MuSCA Multi-slot Coded ALOHA
CSI Channel State Information

MSE Mean Square Error

LS Least Squares

MMSE Minimum Mean Square Error

MAP Maximum a posteriori

ISI Inter-Symbol Interference

ECC Error Control Coding

GMAC Gaussian Multiple Access Channel

PS Partitioned Signaling

MAI Multiple Access Interference

DS-CDMA Direct-Sequence Code Division Multiple Access

FIR Finite Impulse Response
APP A Posteriori Probability

LLR Log Likelihood Ratio

r(t) Received Signal

 $u_k(t)$  Transmitted Signal

K Number of Users

z(t) Additive Gaussian Noise

R Rate

C Capacity

W Bandwidth

 $P_k$  Power of the  $k_{th}$  User

K Number of Users

r Received Signal

H Channel Impulse Response

x Transmitted Symbol

z Additive Gaussian Noise Term

p Pilot Symbol

 $\hat{H}$  Estimate of the Channel Impulse Response

 $E(\dot{)}$  Expectation Function

 $C_h$  Channel Covariance Matrix

 $C_n$  Noise Covariance Matrix

 $\alpha$  System Load

Number of Available Signaling Dimensions

 $d_k$  Information Symbols from the  $k_{th}$  user

M Repetition Factor

 $d_{k,m}$  Repeated Information Symbols

 $d_{k,m}^{\prime}$  Repeated and Interleaved Information Symbols

 $s_k$  16-QAM Signals Formed by Superposition

y Multi-user Received Signal

n Noise Term

 $\sigma$  Noise Variance p(t) Shaping Pulse

 $\Delta f_k$  Frequency Offset

g(x) Error Variance Function

 $\varepsilon_r$  Reconstruction Error

 $c_k$  Pilot Sequence

 $\gamma^2$  Energy Normalization Term

Q Number of Filter Taps

 $F_k$  Filter Noise Reduction Factor

 $P_p$  Energy of the Pilot Signal

 $P_d$  Energy of the Data Signal

# Acknowledgements

I would like to thank my friends and family for their support during the months spent preparing this thesis. I would especially like to acknowledge my supervisor, Dr. Christian Schlegel, for his support and the interesting conversations we have had regarding the material presented herein.

This work has been published in part, and presented at the 8th International Symposium on Turbo Coding & Iterative Information Processing in Bremen, Germany.

# Chapter 1

#### Introduction

In 2014 the global satellite industry had a reported value of over \$203 billion, a value which has more than doubled in the past decade [1]. Of the approximately 1,261 satellites currently operating, 52% of them are dedicated to communications, with the majority of those being deployed in the commercial sector. Of all the services provided by these communications satellites the demand for mobile services is growing at a staggering rate; experiencing 25% growth in the past year alone [1]. The nature of this demand will lead to a noticeable increase in mobile traffic in the near future and the type of services being offered will result in this traffic appearing random and highly bursty at the satellite receiver. An unavoidable consequence of this type of traffic is that packet collisions will occur at the receiver. With current receivers, these collisions are often seen as unresolvable. Due to the increasing probability of collision that comes with an increase in traffic, and the long round-trip propagation delays (approximately 240ms for geosynchronous satellites), the retransmission of the collided packets should be avoided whenever possible.

The scenario discussed above describes what is known as uncoordinated random access. This is a system where a number of uncoordinated users attempt to access communication resources in a completely random fashion. This problem has received much attention in the past, which will be discussed in detail in the following sections. Despite this, the need for improved random access protocols continues to grow. This scenario becomes vastly important as we begin to consider the future of wireless communications, with things such as sensor networks and the highly touted Internet of Things set to become an everyday part of the average persons life. To get the most out of these services we must first solve the problem of unresolvable collisions at the receiver, the following work strives to do just that.

In an attempt to solve the problems associated with uncoordinated random access communications this work will focus on the problem of channel estimation in a system employing iterative decoding and demodulation. Channel estimation is an important aspect of any wireless communication system because it allows the signal to be accurately reconstructed at the receiver regardless of any distortions introduced by the channel which exists between the transmitter and receiver. Exact estimation of these distortions becomes a critical component when considering multi-user communications. The cancellation based receivers discussed in detail herein rely heavily on accurate channel information to reconstruct the transmitted signals to enable the cancellation process. Any error present in the estimation process will translate into reconstruction errors which negatively impact the receiver performance.

Cancellation based receivers rely on resolving packet collisions by decoding users iteratively while treating the interference from every other user transmitting concurrently as an additional source of noise. Once a single user is decoded their signal is subtracted from the received data stream and decoding of the next signal can begin. This process continues until either all users are decoded or the cancellation process has resulted in degradation of the received signal to such an extent that no more user data can be recovered. The latter case is typically caused by inaccurate channel estimation which results in an inaccurate reconstruction of the original signal. The most important artifact of this inaccurate reconstruction is that when the reconstructed signal is removed from the received data the signal mismatch results in incomplete cancellation which can be modeled as residual noise. This residual noise will grow with each user involved in the collision providing a functional limit on the number of users which can be received concurrently.

The following work focuses on improving throughput in the dense multi-user random access satellite up-link environment by exploiting the concepts of generalized modulation and improved low complexity iterative estimation and cancellation techniques. The bulk of this work is performed with a focus on the satellite channel, however it should be noted that the results can be easily extended to a number of communication environments. The focus lies on the satellite environment because it has been traditionally utilized for system testing and presents a relatively benign channel for testing compared to other environments such as underwater, or indoor mobile. Additionally, due to the geographic spread of users and long propagation delays the uncoordinated multi-user satellite environment presents itself as the classic problem of multiple user random access, with it's own set of complications which will be discussed in detail. Finally, Advancement in this field of research has been indicated as a priority by both the European Space Agency (ESA) and Industry Canada.

The rest of the thesis is organized as follows: Chapter 2 provides the reader with the necessary background in satellite communications, multiple access, estimation theory and iterative processing. In Chapter 3 the design of the system is outlined, including the pertinent information regarding the satellite environment and the receiver structure. Finally, results and performance analysis of the proposed receiver are provided in Chapter 4.

# Chapter 2

# Background

#### 2.1 Satellite Communications

The satellite industry is rapidly advancing to meet the demand for wide-spread interactive wireless services and offer a competitive alternative to terrestrial based communication systems. This demand is shifting away from broadcast style communications such as television and radio towards short interactive applications such as tracking of ships and planes which leads to a dense multi-user scenario on the satellite up-link. The industrial effects of this shift can be seen with the launch of ALPHASAT in 2013 by the European Space Agency; with the ability to handle more than 750 communications channels, it represents the largest communications satellite launched to date. Modern communication satellites typically operate in a geosynchronous orbit, which means the satellite remains stationary above a single point on the earth as it spins. This orbit allows for a single satellite to continually service approximately 40% of the earth's surface in the extreme case. This area includes difficult terrains where terrestrial solutions may not be effective, or even possible, such as marine or mountainous areas. Covering such a large area of land means there is a very large number of users which could potentially access the satellite at any given time.

High end communications satellites mitigate the effects of servicing such a large area by using a multi-spot beam approach. That is, they utilize an array of receivers and tuned antennas which employ frequency reuse and polarization as a means of segmenting the service area into smaller, more manageable sub-regions. This approach leads to an increase in capacity across the entire region by reducing the area and number of users which must be serviced by each receiver. For the remainder of this thesis the focus will be on the performance of a single spot beam, which corresponds to a single receiver in the antenna array. Although this spot beam is much smaller than the total coverage area of the satellite it still encompasses a very large number of potential users which may access the satellite at any given time.

Much work has been done in developing a comprehensive model of the land mobile satellite (LMS) channel. The satellite community generally accepts the statistical channel model proposed by Fontán et al. in 2001 [2]. In this model the channel is represented by a three state Markov model; where each of the three states represents a different state of shadowing: line of sight (state 1), moderate shadowing (state 2), and deep shadowing (state 3) conditions as illustrated for a single user in Figure 2.1. This channel illustration is based on the results of a measurement campaign carried out by the German Aerospace Center [3] which provides the Markov transition probabilities and underlying Loo parameters required for the model in [2] for each of the states for a wide range of realistic scenarios. In practice, these states are dictated by the motion of the mobile terminal as it moves through the environment.

For the purpose of this thesis, analysis will be focused on a single fading state, line-of-sight, where high SNR with low levels of power fluctuation are expected at the receiver. The fluctuations in each Markov state are determined by the parameters of the underlying Loo distribution [4] which dictates the fading characteristic of the channel within each of the various states. Fading parameters in this model can be decomposed into: very slow, represented by the Markov states; slow, represented by a log-normal power distribution which represents the small scale changes in attenuation within each of the three fading environments; and fast variations introduced by the random fluctuations experienced during transmission through the atmosphere.

The model presented in [2] provides a statistical method for generating a discrete time series representing realistic amplitude and phase variations of the transmitted signal for a number of relevant, real-world scenarios. While this model is generally accepted it is important to note that other authors have opted for a simplified two state Markov process [5] which separates the transmission into good and bad conditions with an underlying fading profile.

Although the satellite channel has been thoroughly investigated it does not come without it's own host of issues which must be overcome to ensure efficient and reliable high speed communications for future wireless applications. There are a number of atmospheric effects which must be considered: most notably high levels of attenuation and absorption due to the presence of water vapour in the atmosphere, along with atmospheric scintillation which causes random rapid phase and amplitude variations

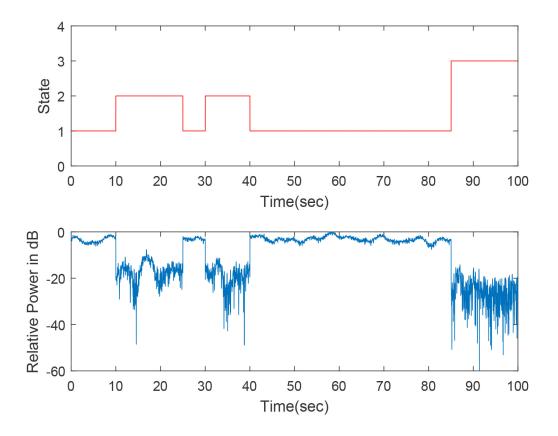


Figure 2.1: LMS channel example

as the signal passes through the tropo- and ionospheres [5]. Due to the geographic spread of users and the localized weather and atmospheric effects discussed above, each user may have a unique channel realization, making accurate channel estimation in a multi-user environment a complex task. Channel estimation is further complicated when considering the geosynchronous (GEO) orbits popular for communications satellites; the long propagation delay resulting from a  $72,000 \, \mathrm{km} \, (\sim 240 \, \mathrm{ms})$  round trip makes closed-loop processing impractical. This may be circumvented by the use of low earth orbit (LEO) satellites which maintain a much lower elevation, however their highly dynamic orbits create a sizable Doppler which must be tracked.

There are a number of relevant standards published by the European Telecommunications Standard Institute (ETSI) which govern transmission in the satellite environment. One very important aspect of the return channel via satellite (RCS-2) standard is the inclusion of adaptive coded modulation (ACM). The inherent fluctuations in the received power leads to a wide range of operating signal to noise ratios

(SNR) which leads to different requirements on the signaling. An operating range of 0 dB to 14 dB  $E_s/N_0$  is typical in practice. To take advantage of this, the standard allows for rate 1/3 QPSK during poor conditions to improve reliability and up to rate 5/6 16-QAM to increase throughput during good operating conditions. The reader is directed to [6, 7, 8] for additional details regarding the relevant standards.

#### 2.2 Multiple Access

Since the introduction of wireless communications scientists and engineers have been searching for techniques which allow multiple users to share the communication resources without reducing the throughput of the channel. The techniques developed have become known as multiple access techniques and have taken on many forms over the years. The simplest of these techniques involves slicing the channel into time slots and assigning them to users in the system, this technique is known as time division multiple access (TDMA). The resource may also be separated by assigning a subset of the available bandwidth to each user, known as frequency division multiple access (FDMA). Another alternative, where is user is assigned a unique spreading sequence, code division multiple access (CDMA), has seen extensive use as an access technology in wireless networks. These techniques are highly effective but they also rely on a high degree of coordination and synchronization between the users, this is not always practical or even possible in real life situations.

A perfect example of the multiple access scenario is the satellite return channel, or up-link, where multiple ground based users contend for a single satellite receiver (see Figure 2.2). Due to the geographic spread of users, the long propagation delays, and the nature of the services offered, a high level of coordination between the users and a user and the receiver is infeasible. This scenario perfectly encompasses what has become known as an uncoordinated random access scenario.

In this scenario, and in the absence of fading, the received signal r(t) can be described as the sum of the transmitted signals  $u_k(t)$  from each of the K users with an additive Gaussian noise (AWGN) term z(t).

$$r(t) = \sum_{k=1}^{K} u_k(t) + z(t)$$
(2.1)

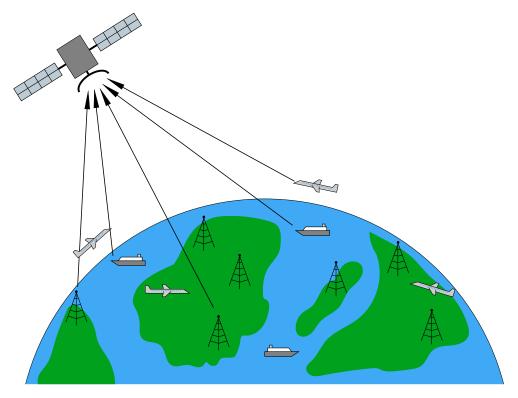


Figure 2.2: Multiple access satellite up-link scenario

Random access has been studied extensively in the past for a wide array of applications. The overarching goal of this research has been to maximize throughput while supporting a high number of concurrent users. The predominant protocols studied are the slotted ALOHA protocols developed by Abramson in 1970 [9]. These protocol effectively doubled the potential throughput of the original un-slotted ALOHA protocol whose throughput f(x) is given by

$$f(x) = x * e^{-2x} (2.2)$$

while allowing for a higher system load (x) before collapsing as illustrated in Figure 2.3. For the TDMA case, the system load may be defined as the number of active users in the system per time slot. By organizing transmission into pre-defined time slots without scheduling, the throughput of ALOHA was increased to

$$f(x) = x * e^{-x} \tag{2.3}$$

which made it capable of achieving throughput as high as 0.37 packets/time slot for a system load equivalent to one transmission (user) per time slot. This throughput is

equivalent to 37% of the single user TDMA throughput of one packet per time slot. If two or more users transmitted in the same slot a collision occurred and the data transmitted in the slot was lost, users involved in a collision would wait some time and try again. The actual throughput in bits/s/Hz is dependent upon the modulation and coding used. The bulk of the recent research has focused on developing protocols capable of resolving these collisions without suffering a reduction in throughput with much of the work aimed at applications in the satellite up-link environment; many of this are advancements or variations on the original ALOHA protocols.

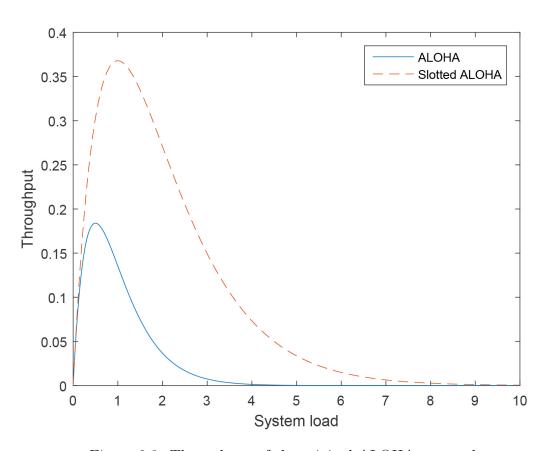


Figure 2.3: Throughput of the original ALOHA protocols

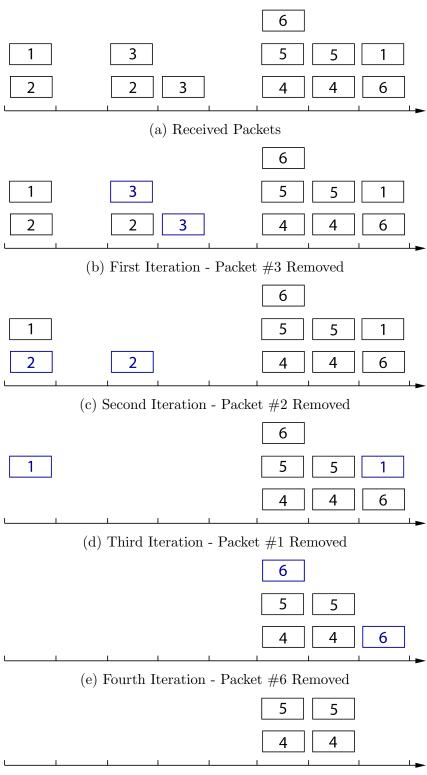
The first notable improvement was the introduction of Contention Resolution Diversity Slotted ALOHA (CRDSA) [10] in 2007. CRDSA improves upon traditional slotted ALOHA by introducing an additional redundant copy of each packet into the transmission frame and using successive interference cancellation (SIC) at the receiver to increase throughput to a maximum of 0.52 packets/time slot under a normalized system load of 0.65. The system load is normalized with respect to the number of

packet copies transmitted. This was accomplished through the principal that if a packet is received (i.e it isn't involved in a collision) then it can be decoded and its copies can be removed from other time slots which may lead to more recoverable packets by removing the colliding packet from the time slot. An example of this protocol can be seen in Figure 2.4.

This protocol has been included in the most recent DVB-RCS standards and thus represents the state of the art in terms of what has been applied to a real system. However, this process is not perfect and there are still instances where unresolvable collisions occur. To reduce the number of unresolvable collisions a new protocol was developed where a variable number of packet copies were transmitted. By allowing for more than two copies of each packet to be transmitted the throughput was further increased to 0.8 packets/slot at a normalized system load of 0.85 [11].

At this point a variable number of packet copies were allowed to be transmitted and researchers quickly realized that the cancellation structure resembled that of a bipartite or Tanner graph. This realization allowed for the theories developed for coding on graphs to be applied to the iterative cancellation process [12, 13] which lead to a new protocol dubbed Irregular Repetition Slotted ALOHA (IRSA) which was capable of achieving throughput approaching the single user channel capacity (0.97 packets/slot) in the asymptotic case at normalized loads approaching 1 transmission/slot. It was however, still limited to 0.8 packets/slot under realistic delay and frame length constraints. The ability to impose a statistical distribution on the number of packet repetitions lead to the development of Coded Slotted ALOHA (CSA) [14] which relied on encoding each packet prior to transmission to achieve the throughput of IRSA while using higher rate codes on average. This scheme was further improved in [15, 16] by taking it one step further and breaking each packet into sub slots and applying the concepts of CSA on a per slot basis. With a properly chosen distribution these protocols were capable of achieving throughputs asymptotically close to the single user channel capacity of 1 packet/slot utilizing low rate codes, but suffered the drawback of requiring a complex statistical distribution to determine the access probability of each user in the system.

The advancements described above provided a great framework for approaching the single user capacity however it was quickly realized that they relied on fairly



(f) Final Iteration - Unresolvable Collision Between Packets 4 and 5  $\,$ 

Figure 2.4: Example of Contention Resolution ALOHA

accurate network synchronization which may be prohibitive in realistic scenarios. To combat this issue Contention Resolution ALOHA (CRA) [17] removed the need for network wide synchronization and added Forward Error Correction (FEC). Removing the slotted (TDMA) transmission framework present in the various protocols outlined above increases the probability that a collision will occur due to the presence of partial collisions. The inclusion of FEC however means that some of these partial collisions can be correctly recovered and removed during the SIC process. These changes allow CRA to surpass CRDSA in the high SNR region. CRA shows linear performance over the entire range of normalized system loads for in the high SNR region with 2 repetitions and rate 1/2 FEC. As the number of repetitions is increased the throughput is reduced with respect to the normalized load but robustness is increased in the low SNR region.

This attempt at improving random access in the satellite environment was followed up in [18] with the introduction of Enhanced Contention Resolution ALOHA (ECRA). ECRA improved upon CRA by allowing for the cleanly received portions of partially collidied packets to be combined in an effort to create complete, or nearly complete, packets which could be decoded with the assistance of FEC. This protocol outperformed the original CRA in every way, excelling especially in scenarios where the system load is very high (i.e the number of active users is greater than the number of available slots) while maintaining good performance in low SNR however, it still falls short of CRDSA in the very low SNR region (<2dB). Another attempt to relax the network wide synchronization requirements was made in [19] with the introduction of Enhanced Spread Spectrum ALOHA (E-SSA) which combines asynchronous spread spectrum techniques with low rate FEC and SIC to provide very low packet loss ratios (PLR) at throughputs approaching the single user capacity of the channel.

In 2013 a novel extension to the CRDSA protocol was proposed by introducing the concept of decoding rounds. The new scheme was dubbed Frameless ALOHA [20] and operates with variable length frames and decoding rounds. Each decoding round represents a number of iterations of the SIC; where this protocol diverges from CRDSA is that associated with each round is an end condition. In CRDSA the iterative process continues until there are no resolvable collisions or a set number of iterations have been reached, in Frameless ALOHA the end condition for each round

of the iterative process depends on either decoding a pre-defined portion of the active users or reaching a set number of iterations. Once this end condition has been reached a new round is started with only the users who were not successfully decoded in the previous round contending for channel resources. The length of the transmission frame and the value of the end condition are both dependent upon an estimation of the number of active users and vary with each round in an attempt to maximize the throughput. In [21] the end condition is analyzed and the optimum is found to exist when instantaneous throughput is maximized, that is once the throughput begins to drop the current round will end and a new round begins. This protocol excels when the number of active users is comparable to the number of transmission slots, referred to as a high system load, in this region CRDSA and CSA begin to collapse and their throughput is greatly diminished. Frameless ALOHA is shown to be capable of achieving throughputs up to 0.88 packets/slot with a system load of 0.9 users/slot under the assumption of an optimal end condition for the processing rounds.

CRDSA was again revisited last in 2014 when some of the original researchers attempted to make the protocols truly asynchronous [22], to achieve this the concept of a virtual frame is introduced. In this new methodology the transmission frame is known locally at each user without coordination which inherently creates an asynchronous transmission environment. This asynchronicity leads to the presence of partial collisions which may be decoded as in ECRA increasing performance while at the same time greatly decreasing the delay experienced by each user as they no longer need to wait until the beginning of a potentially long global frame to transmit their data. Additionally, the number of packet repetitions required to achieve a high level of performance is greatly reduced with simulations showing as few as 2 packet repetitions being sufficient to achieve performance greater than CRDSA, CSA, or E-SSA.

However, these schemes all suffer the same fundamental drawback; they treat the channel as a single user environment, when it is inherently a multiple access channel [23]. The capacity of which actually increases with the aggregate received power (2.4); this indicates that throughput of greater than one packet per time slot are possible.

$$R \le C = W \log_2 \left( 1 + \frac{\sum_{k=1}^K P_k}{N} \right) [bits/s]$$
 (2.4)

This potential for surpassing the single user channel capacity was shown in 2012 by [24] with the development of Multi-slot Coded ALOHA (MuSCA). This protocol exploited the multi-user nature of the channel to achieve throughput up to 1.4 packets/slot while the system is heavily loaded, up to 1.3 transmission/slot. This impressive throughput was further increased in [24] by employing irregular degree distributions inspired by IRSA to the cancellation process. This lead to the throughput being increased to a peak of 1.43 packets/slot at the cost of increased complexity. Both of these schemes utilize iterative processing and joint detection, two concepts which form the basis for the work presented herein. However, these and the protocols presented above rely on the assumption of perfect channel estimation which is a known issue in any realistic environment. Accurate channel state information is crucial to the iterative cancellation processes which allow all of the protocols presented in this section to achieve desired performance but channel estimation in this environment is largely ignored in the literature. Acquisition of reliable channel state information for the cancellation process represents a key component of the research presented in this thesis.

It should be noted that protocols resembling those discussed above have been developed and implements in various fields outside of satellite communications. Some notable examples are for use in RFID tags [26], and mobile networks with hidden nodes [27, 28].

#### 2.3 Channel Estimation

In wireless communications all information must pass through what a medium known as the channel which introduces some nature of signal distortion and noise to the transmitted signal. The goal of channel estimation is to quantify these effects in such a way that the original signal can be accurately reconstructed at the receiver. To investigate the basics of estimation theory we will focus on the case of a single user. Under this assumption we can use vector representation to describe the received

signal 
$$\mathbf{r} = [r_0, r_1, ..., r_{k-1}]^T$$
 as 
$$\mathbf{r} = \mathbf{H} * \mathbf{x} + \mathbf{z} \tag{2.5}$$

The goal of channel estimation in this case is to determine the unknown channel vector  $\mathbf{H}$  in the presence of noise  $\mathbf{z}$ . The contents of the channel vector,  $\mathbf{H} = [H(0), H(1), ..., H(Q-1)]$ , are commonly referred to as the Channel State Information (CSI) and represents the combined effects of a number of channel impairments such as: scattering, fading, and attenuation. This information is crucial to reconstruct the transmitted symbols,  $\mathbf{x} = [x_0, x_1, ..., x_{k-1}]^T$ , from the received signal  $\mathbf{r}$ . For the remainder of the development we will model the channel time variations as a tapped delay line, or Finite Impulse Response(FIR) filter [29] (Figure: 2.5) with Q taps.

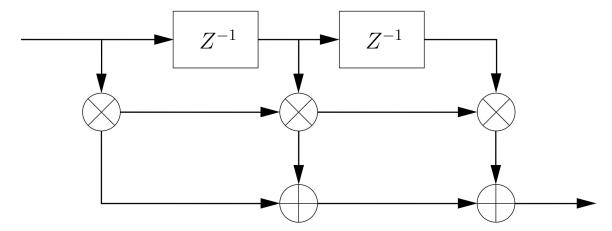


Figure 2.5: Tapped delay line representation of the channel

The performance of an estimator can be quantified by how closely it approximates the real signal, this metric is known as the mean square error (MSE).

$$MSE = E[|\hat{\boldsymbol{H}} - \boldsymbol{H}|^2] \tag{2.6}$$

where  $E[\cdot]$  is the statistical expected value of the input and  $\hat{\boldsymbol{H}}$  represents the estimate of  $\boldsymbol{H}$ . The random access protocols described in section 2.2 rely heavily on accurate signal reconstruction to drive the Successive Interference Cancellation Process, imperfect reconstruction leads to residual noise which limits the performance of the system; this concept will be discussed in more detail in the following chapters.

Development of techniques for determining the unknown vector  $\mathbf{H}$  has been the focus of much research and several good methods have been developed. The methods of interest in the context of this thesis rely on a training, or pilot sequence to be

transmitted across the channel along with the data. This pilot sequence consists of a set of known data which is used to estimate the instantaneous CSI. In the simplest case a number of pilot symbols are appended at the beginning of the data frame. We denote the pilot as a vector  $\mathbf{p} = [p_0, p_1, ..., p_{n-1}]$  and focus on how it effected by the channel, such that the received sequence becomes:

$$\boldsymbol{r} = \boldsymbol{H} * \boldsymbol{p} + \boldsymbol{z} \tag{2.7}$$

One popular method which relies only on knowledge of the received signal y, and the transmitted signal p to generate an estimate of the unknown channel vector H is known as least squares (LS) estimation. Which attempts to satisfy the criteria:

$$\hat{\boldsymbol{H}}_{LS} = \arg\min_{\boldsymbol{H}} (\boldsymbol{r} - \boldsymbol{H} \boldsymbol{p})^{\boldsymbol{H}} (\boldsymbol{r} - \boldsymbol{H} \boldsymbol{p})$$
(2.8)

Based on this the following closed form expression can be obtained:

$$\hat{\boldsymbol{H}}_{LS} = (\boldsymbol{p}^H \boldsymbol{p})^{-1} \boldsymbol{p}^H \boldsymbol{r} \tag{2.9}$$

where  $(\cdot)^H$  represents the Hermitian, or conjugate transpose of the vector. It is important to note that the performance of the Least Squares algorithm is highly dependent upon the signal to noise ratio, and the relative pilot length as clearly illustrated in figure: 2.6.

In certain instances we can improve the estimation by incorporating some a priori knowledge of the channel or the noise; this covers a class of algorithms known as Bayesian estimation, we will explore the minimum mean square error (MMSE) estimator in more detail. The MMSE estimator relies on knowledge of the covariance of both the channel and noise vectors to produce an estimate based on the maximum a posteriori (MAP) criteria under the assumption that the channel coefficients are zero mean Gaussian distributed [30]. This assumption is valid for the generic tapped-delay line channel model used herein because the tap values are assumed to be the result of summing numerous random channel effects.

$$\hat{\boldsymbol{H}}_{MAP} = \arg\max_{\boldsymbol{H}} \frac{p(\boldsymbol{r}|\boldsymbol{H})p(\boldsymbol{H})}{p\boldsymbol{r}}$$
(2.10)

which, under some mild assumptions leads to the solution of the MMSE estimator given by:

$$\hat{\mathbf{H}}_{MMSE} = C_H \boldsymbol{p}^H (\boldsymbol{p} C_H \boldsymbol{p}^T + C_N)^{-1} \boldsymbol{r}$$
(2.11)

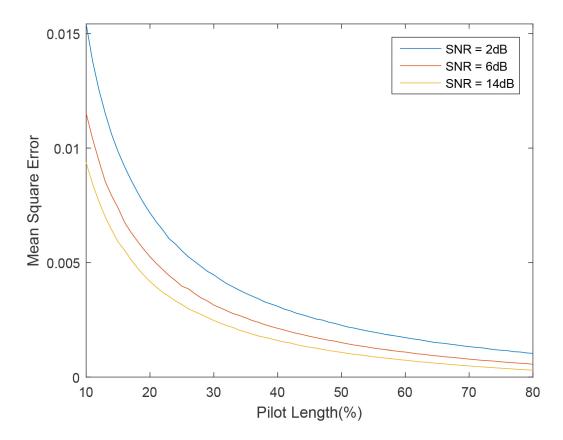


Figure 2.6: Performance of the least squares estimation algorithm with respect to SNR and relative pilot length

where  $C_H$  and  $C_N$  are the channel and noise covariance matrices respectively and may be computed as follows

$$cov(\boldsymbol{H}) = C_H = \boldsymbol{E}[\boldsymbol{H}\boldsymbol{H}^T] \tag{2.12}$$

An important class of channel estimation algorithms known as data-aided estimation have been developed to compliment, or even replace the pilot-aided techniques discussed above. The main purpose behind these structures is to reduce the required signaling overhead by reducing the amount of training sequence required to accurately estimate and track the channel variations. This is accomplished in large by utilizing the decoded data from the previous frame to update the channel estimate of the current frame [31, 32, 33, 34, 35]. This approach is only valid for slowly varying channels where users are continuously active, they are also prone to error propagation due to

incorrect decisions being fed back to the estimator. A potentially more interesting advancement in data-aided estimation techniques came about with the advent of Turbo Coding [36] and the popularization of iterative receiver structures. This advancement was based on taking advantage of the soft information generated by the receiver to iteratively improve the estimation; this avoids the potential for error propagation inherent in decision-directed systems which rely on the hard decision output.

One of the first attempts at using the soft information to improve estimation was seen in [37] where the reliability information from the soft output Viterbi algorithm was used to construct a virtual pilot from the most reliable data, this was shown to outperform the hard output data-aided estimation techniques over a wide range of signal to noise ratios. In [38] the prohibitive nature of utilizing hard feedback estimation is noted and a soft input MMSE estimator is developed and shown to outperform traditional MMSE algorithms, specifically results are shown for a multiuser environment where each channel must be independently estimated. It is also of note that the complexity of joint MMSE in a multi-user environment is prohibitive for most applications, this problem may be addressed by separating each user into K data streams and employing a sub-optimal MMSE algorithm with reduced complexity. These results are supported by [39] where soft-iterative estimation is shown to out perform hard feedback over flat-fading channels and [40] show the performance of soft-iterative least squares algorithms in time-varying frequency selective channels. Similar results are seen in [41, 42] where soft-iterative estimation algorithms outperform their hard decision, and classical counter-parts in a number of scenarios. More recently [43] combines classical turbo estimation with soft-iterative estimation by combining estimates from the training sequences and decoded data based on the MMSE criteria to create a hybrid system. Furthermore, complexity is reduced by removing the interleaving, channel decoding, and de-interleaving blocks from the iterative estimation process. The soft estimation techniques shown in these papers rely heavily on the iterative receiver structures ushered in with by turbo coding to outperform their classical counterparts in nearly all situations of interest.

#### 2.4 Iterative Information Processing

In 1993 a new class of convolution codes was introduced which utilized parallel concatenation of two component codes and introduced an iterative decoding methodology where decoding is performed in by passing the soft information back and forth between soft-input, soft-output decoders. Thanks to the iterative process this new receiver was capable of achieving performance close to the Shannon limit using relatively simple component codes, a feat never before achieved in the coding community. These new codes were deemed "turbo codes", due to the feedback loop being analogous to the mechanical feedback used in turbochargers. Hagenauer argued that the iterative turbo principle could be applied to many problems other than decoding, such as: equalization, multi-user detection, and channel coding to name a few [44]. In the relatively short history of the turbo principle this has proven to be correct; iterative structures have found their way into a number of important applications, providing performance well beyond that achievable with classical single-path algorithms. A very important application of these so-called, turbo principles, appears in the task of channel equalization where inter-symbol interference (ISI) must be corrected prior to forward error correction, these equalizers are aptly named turbo equalizers [45] and are prevalent in many modern communications systems.

Turbo Equalization is based on separating the decoding and error control coding (ECC) sections of an iterative receiver, a key component in the dual stage receiver (see Figure 2.7) proposed in [46]. In this paper the receiver is shown to perform within 1 bit/dimension of the Shannon capacity for any SNR. While other capacity approaching techniques exist for the Gaussian Multiple Access Channel (GMAC) (2.4) exist [47] they are generally quite complex to implement. By separating the detection and error control decoding operations capacity can be approached in a simple and robust fashion with negligible decrease in performance. The method presented relies on the concepts of Partitioned Signalling (PS) [48] where signature sequences are partitioned and interleaved prior to transmission; this operation effectively suppresses interference in the multi-user environment allowing for significant increase the concurrent number of users accessing the system with only a marginal increase in complexity. Additionally, this offers protection against the variations in received power which plague many mutli-user receivers. The system also relies on the assumption of a random matrix

channel which is created inherently by the random nature of PS. It is shown that for this channel the combination of PS and iterative demodulation always performs at least as well as MMSE filtering with reduced complexity.

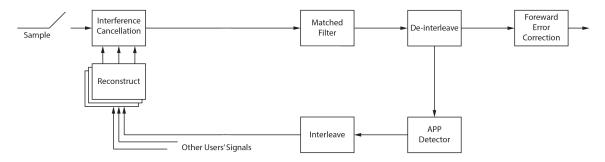


Figure 2.7: Two stage capacity achieving receiver

An important metric for the performance of the two-stage decoder is the system load  $(\alpha)$  which is defined as

$$\alpha = \frac{K}{N} \tag{2.13}$$

where K represents the number of users and N is the number of available signalling dimensions. The effects of power distributions on the capability of the receiver to support high system loads is analyzed and found to go infinity under the assumption of exponential power distributions and 2.08 in the worst case where all users are received at the same power level. Additionally, it is shown that the two-stage receiver with partitioned signalling described above is capable of achieving the capacity of the GMAC with the application of spatial coupling with a sufficiently large coupling window [49]. It is of importance to note that capacity is achieved with complexity that is linear in the number of users and the achievable system load goes to infinity if the SNR is allowed to go to infinity, effectively overcoming the interference limitation present in all multiple access communication systems.

#### 2.5 Iterative Interference Cancellation

Interference plays a major role in multi-user communications systems; to the extent that the performance of many multiple access systems are inherently limited by the multiple access interference (MAI) caused by concurrent transmission of other users. Much research has been performed with the goal of combating this limitation by removing the interference through iterative methods known as iterative, or successive interference cancellation. The multiple-access protocols discussed in section 2.2 all feature SIC as a main component of the proposed algorithms. In this section we will look into the cancellation process in greater detail.

It didn't take long for multi-user or joint detection to be recognized as a necessary advancement to improve throughput of multiple-access communications systems. However, the optimal multi-user detection algorithms were seen as being too complex for practical implementation as their complexity is exponential with the number of users  $\mathcal{O}(2^k)$  [50]. In [51] a multi-stage receiver based on SIC was proposed which exhibited complexity which was linear in the number of users. The SIC based receiver was further investigated for direct-sequence code division multiple cccess (DS-CDMA) in [52]. To properly understand the successive interference cancellation process it is imperative to recall the multiple-access equation (2.1) where the transmitted signal of each active user combine at the receiver. The proposed protocol acts by decoding and subtracting users successively beginning with the user with the highest received power and continuing iteratively until the lowest power user has been decoded. The benefits of proceeding with cancellation in this way are twofold; first, the highest power user will suffer the least interference from other users therefore being the easiest to decode, and second, removing this user from the received data stream will have the biggest impact of reducing the interference affecting the other users. Subtraction is performed by regenerating the decoded signal using the known modulation sequence and the channel estimate.

$$r_k^m = r_k^{m-1} - h_k d_k^m + z^m (2.14)$$

where  $r_k^m$  is the received signal at iteration m, calculated by subtracting the remodulated signal of the  $k^{th}$  user from the received signal at the previous iteration and  $z^m$  is the noise at the  $m^{th}$  iteration. It is noted that through cancellation the MAI decreases with each iteration, however the noise due to imperfect cancellation increases with each iteration; this residual noise effectively limits to extent to which SIC can be used to remove MAI. Turbo principals were quickly applied to the cancellation process to produce a soft interference cancellation system [53]. By taking advantage of the soft information the problem of error propagation can be avoided and the residual error minimized.

In [54] a simple iterative cancellation based receiver is proposed that is capable of achieving the Shannon capacity of the AWGN channel with optimal rate and power control. Furthermore, it is shown that under the worst case scenario, a simple linear cancellation receiver and repetition coding performs just as well as an optimal joint decoding system with uncoded transmission. The dynamics of this cancellation process are studied extensively in [55] and compared with the MMSE receiver. Additionally, interference Cancellation has also been suggested to improve channel estimation in heavily populated multi-user environments [56] [57]. The interference is reduced with each iteration which allows for a better estimate to be generated on the cleaner received signal.

# Chapter 3

# System Design

#### 3.1 Signalling Methodology

A multi-user communication system is proposed for the uncoordinated random access satellite up-link environment by coupling the capacity-achieving receiver discussed above with an iterative estimation loop. This approach has been previously studied in [59] and [60] but will be revisited with a focus on performance in a realistic satellite communications environment.

To keep the signalling methodology both generic and scalable generalized modulation will be utilized to construct  $2^{2B}$ -QAM constellations from the superposition of B constituent QPSK signals. To achieve this the information symbols  $\mathbf{d}_k$  from each of the K users are separated into multiple power streams according to

$$P_k = 4^{k-1}P_0; \quad 1 \le k \le B$$
 (3.1)

where  $P_0$  is the power level of the lowest power data stream. In the case of 16-QAM this translates into a high power and a low power data stream. With B=2 the high power data stream will have 4 times the power of the low power data stream. Repetition coding is applied to each data stream, that is, each symbol  $d_k$  is repeated M times, giving the symbols  $d_{k,m}$  which are passed through an interleaver  $(\pi)$  to generate  $d'_{k,m}$  which are randomly distributed through the transmission frame of length L. This is similar to the packet repetition used in CRDSA, except instead of repeated packets being distributed randomly throughput a frame, the repetition and random interleaving is performed on a per symbol basis. The interleaved symbols  $d'_{k,m}$  are finally utilized to form the 16-QAM symbols  $(s_k)$  based on the superposition principal discussed above. That is

$$\boldsymbol{s}_k = \sum_{m=1}^{M} \boldsymbol{d}'_{k,m} \tag{3.2}$$

as shown in Figure 3.1.

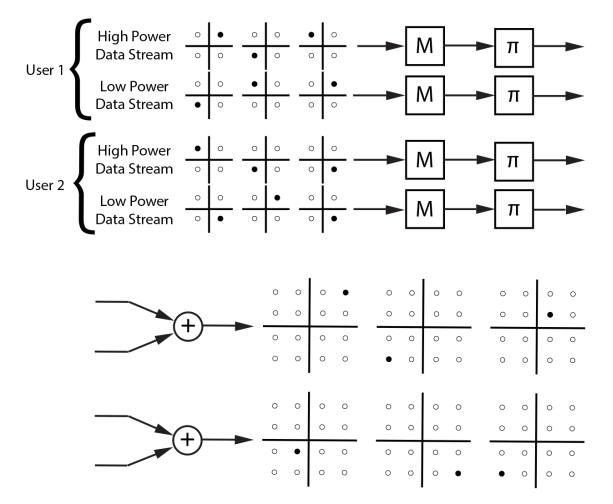


Figure 3.1: Example of the proposed signaling mechanism with two users.

It is important to note that the proposed system utilizes time division training sequences for channel estimation, however the ideas presented herein are readily extendable to other pilot schemes without loss of generality. For this specific case, the symbols  $d_k$  consist of both data symbols and finite-length training (pilot) sequences  $c_k$ . The pilot sequence may be inserted at any point in the data stream such that it does not interfere with the data symbols. For the sake of convenience it is assumed that each user is coded with a unique signature sequence which allows them to be identified and separated at the receiver. This assumption allows us to focus on the signal processing at the receiver without worrying about higher layer functionality and will not be addressed further in this work.

With this formulation of  $s_k$  the system can now be described as

$$y = \sum_{k=1}^{K} \frac{1}{\sqrt{M}} s_k * h_k + \sigma n$$
(3.3)

where  $h_k$  is the discrete, time-varying, channel impulse response which may be unique for each user. In all comparisons we normalize performance to energy per information bit and the spectral efficiency to bits per unit bandwidth, and n has unit variance, so that  $\sigma^2$  is the variance of the additive noise. The scaling factor  $1/\sqrt{M}$  is required to normalize the power of the transmitted symbols due to the repetition coding used, that is, the repetition does not increase the power of the transmitted signal. The sequence of symbols y is length  $L \times N$  samples, where L is the number of transmitted symbols and N is the number of samples per symbol. This particular signalling is required to take advantage of the receiver introduced in [46].

In a realistic system each symbol  $s_k(t)$  will be generated as a sequence of sample pulses for each user, that is the signal  $s_k(t)$  is formally generated as the interpolation

$$s_k(t) = \sum_{n=1}^{LN} s_k[n]p(t - nTc)$$
 (3.4)

where p(t) is some shaping pulse, for example a root-Nyquist pulse.

This definition of  $s_k(t)$  leads to the received signal being formally written as

$$y(t) = \sum_{k=1}^{K} e^{j2\pi\Delta f_k t} s_k(t - \tau_k) + n(t)$$
 (3.5)

where  $\tau_k$  is a random delay of the k-th signal, and  $\Delta f_k$  is the transmitter-receiver carrier mismatch for signal k. These affects take into account both the channel

distortions  $h_k$  and inaccuracies in the processing chain at the transmitter. Due to the low-cost nature of the mobile user terminals, oscillator drifts can result in significant frequency offsets. This is assumed to be the primary source of carrier mismatch in the system. It is also assumed that the frequency offset is constant for the duration of a single transmission and does not need to be tracked, although this extension could be easily made.

For notational simplicity we introduce the term

$$r_k(t) = e^{j2\pi\Delta f_k t} s_k(t - \tau_k) \tag{3.6}$$

such that (3.5) can be rewritten as

$$y(t) = \sum_{k=1}^{K} r_k(t) + n(t)$$
(3.7)

The signal y(t) arrives at the receiver after transmission through the channel.

#### 3.1.1 Receiver Design

Sample-based processing is intended at the receiver, and not necessarily Nyquist-based processing, a filter matched to p(t) is used at the receiver although matched filtering is not strictly required. After receiver-matched filtering the sample-based signal of the  $k^{th}$  user is given by

$$r_k(t) = e^{j2\pi\Delta f_k t} \sum_{v=1}^{NL} s_k[v] q(t - vT_c - \tau_k)$$
(3.8)

where q(x) = p(x) \* p(-x). This model is relevant in the satellite environment where channel distortion and complex fading can be modeled by the channel response h(t); then the overall pulse is given by q(x) = p(x) \* h(t) \* p(-x) instead. The received signal is then sampled asynchronously at rate  $t = rT_c$  to obtain a sampled version of the received signal  $r_k(t)$  as

$$r_k(rT_c) = r[r] = e^{j2\pi\Delta f_k r T_c} \sum_{v=1}^{NL} s_k[v] q((r-v)T_c - \tau_k)$$
(3.9)

which can be rewritten as a discrete finite-impulse response (FIR) form as

$$r_k[r] = \sum_{v=1}^{NL} s_k[v] f_k[r-v] = \sum_{v:\text{filter span}} s_k[r-v] f_k[v]$$
 (3.10)

where the taps of the filter that model the asynchronous sampling process are given by

$$f_k[v] = e^{2\pi j \Delta f_k r T_c} q \left( v T_c - \tau_k \right) \tag{3.11}$$

The time variation of these taps is not explicitly expressed in the notation, but is caused primarily by  $\Delta f_k$  and the drift of  $\tau_k$ . After sampling and filtering the received signal passes through a de-interleaver  $(\pi^{-1})$  to prepare it for the *a posteriori* probability (APP) decoder which Log-Likelihood Ratios (LLR) according to

$$LLR_{d_k}(\lambda) = \log\left(\frac{p(d_k = 1)}{p(d_k = 0)}\right)$$
(3.12)

which leads to the soft-bit estimate

$$\tilde{d}_k = \tanh\left(\frac{\lambda}{2}\right) \tag{3.13}$$

where  $(\tilde{\cdot})$  is used to represent a soft estimate of the true value.

The LLR values are passed through another interleaver  $(\pi)$  before being used along with the channel estimate  $\tilde{\boldsymbol{h}}_{k'}$  to reconstruct an estimate of the original signal  $\tilde{\boldsymbol{x}}_{k'}$  for cancellation.

$$\tilde{\boldsymbol{x}}_{k'} = 4\tilde{\boldsymbol{d}}_{k',hp}\tilde{\boldsymbol{h}}_{k',hp} + \tilde{\boldsymbol{d}}_{k',lp}\tilde{\boldsymbol{h}}_{k',lp}$$
(3.14)

The canceled input for the k-th user/data stream is now generated as

$$\boldsymbol{y}_k = \boldsymbol{y} - \sum_{k' \neq k} \tilde{\boldsymbol{x}_{k'}} + n \tag{3.15}$$

for each user in the system.

For large systems, the iterative demodulator is well described by a single-parameter iterative equation. In the large-systems limit the residual noise variance of the iterative process is governed by the iteration equation [54, Equation 11]. To understand the iterative equations we need to introduce the function g(x)

$$g(x) = E[(1 - \tanh(x + \sqrt{x}\xi))^{2}]$$
(3.16)

which is the error variance of a binary soft-bit estimate of a bipolar signal observed in Gaussian noise with variance x which is analyzed in detail in [61]. The variance

of the iterative demodulator which dictates the system performance is now described by

$$\sigma_{i}^{2} = \frac{\alpha}{2} \left( 4 E_{hp} \left[ (d - \tilde{d})^{2} \right] + E_{lp} \left[ (d - \tilde{d})^{2} \right] \right) + \sigma^{2}$$

$$= \frac{\alpha}{2} \left( 4 g(\sigma_{i-1}^{2}/4) + g(\sigma_{i-1}^{2}) \right) + \sigma^{2}$$
(3.17)

where the subscripts hp and lp represent the low and high power data stream respectively.  $\sigma_i^2$  is the effective noise variance, which depends on the iteration i, that is, on the level of cancellation. The generation of the channel estimates  $\tilde{h}_k$  will be discussed in detail in the next section.

#### 3.2 Estimator Development

By representing the channel as a tapped delay line with Q taps the estimator can be viewed as a Finite Impulse Response (FIR) filter. This estimator is a crucial addition to the iterative receiver shown in Figure 3.2, where an estimation block which has been added to take advantage of iterative estimation techniques to produce a highly reliable channel estimate  $\tilde{h}_k$  of each user in the densely populated multi-user environment. The goal of the estimator is to minimize the square error between the estimate  $\tilde{h}$  and the true value h. Any errors in the channel estimation will lead to enhanced linear inference at the decoder; that is, the residual variance of iteration i, ( $\sigma_i^2$ ) is increased with respect to the predictions of (3.17).

By separating the estimated channel response for each user into a known term and an error term  $(\Delta \mathbf{h}_k)$  we can write the estimate of  $\mathbf{h}_k$  as

$$\tilde{\boldsymbol{h}}_k = (\boldsymbol{h}_k + \Delta \boldsymbol{h}_k) \tag{3.18}$$

The reconstruction error  $\varepsilon_r$  can then be quantified as

$$\varepsilon_r = d_k \mathbf{h}_k - \tilde{d}_k \tilde{\mathbf{h}}_k$$

$$\varepsilon_r = (d_k - \tilde{d}_k) \mathbf{h}_k - \tilde{d}_k \Delta \mathbf{h}_k$$
(3.19)

Making the assumption that the reconstruction error  $\Delta h_k$  is white with variance  $\sigma_{i,h}^2$  and recalling that our data streams consist of two power levels as in (3.14), we

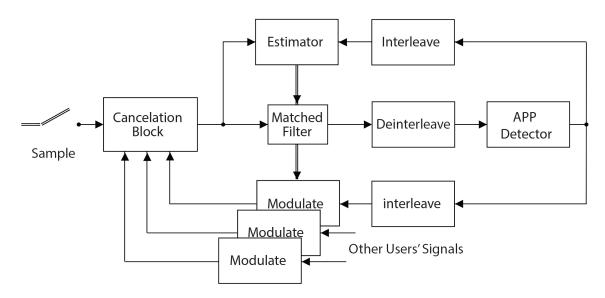


Figure 3.2: Proposed two stage receiver with integrated estimator

can modify (3.17) to include the effects of the iterative estimator and compute the performance of the iterative estimation loop as a function of the estimation error as

$$\sigma_{i}^{2} = \frac{\alpha}{2} \left( 4g(\sigma_{i-1}^{2}/4) + g(\sigma_{i-1}^{2}) \right) + \frac{\alpha}{2} \left( 4\left( 1 - g(\sigma_{i-1}^{2}/4) \right) + \left( 1 - g(\sigma_{i-1}^{2}) \right) \right) \sigma_{i,h}^{2} + \sigma^{2}$$
(3.20)

where the variance of the estimation process is a function of the variance of the previous iteration

$$\sigma_{i,h}^2 = f(\sigma_{i-1}^2) \tag{3.21}$$

For notational convenience we introduce the terms

$$g_1(\sigma_{i-1}^2) = 4g(\sigma_{i-1}^2/4) + g(\sigma_{i-1}^2)$$
(3.22)

and

$$g_2(\sigma_{i-1}^2) = 4\left(1 - g(\sigma_{i-1}^2/4)\right) + \left(1 - g(\sigma_{i-1}^2)\right) \tag{3.23}$$

such that (3.20) can be rewritten as

$$\sigma_{i-1}^2 = \frac{\alpha}{2} \left( g_1(\sigma_{i-1}^2) + g_2(\sigma_{i-1}^2) \sigma_{i,h}^2 \right) + \sigma^2$$
 (3.24)

The task then becomes selecting an estimator for use in the iterative loop. To this end, we explore the relatively simple least squares estimator due to its ease of implementation and low complexity which is very attractive for an iterative implementation. Other estimators may be considered which are capable of producing a more accurate estimation of the channel taps by utilizing more available information at the cost of complexity. As it will be shown, the simple LS estimator is sufficient for the envisioned application.

It is important to recall the transmitted sequence  $d_k$  contains both data and the pilot sequence  $c_k$  which are used to generate an initial channel estimate.

$$\tilde{\boldsymbol{h}}_{k,0} = (\boldsymbol{c}_k^H \boldsymbol{c}_k)^{-1} \boldsymbol{c}_k^H \boldsymbol{y}_k \tag{3.25}$$

The accuracy of the estimation is dependent upon the length of the training sequence, and the signal-to-noise and interference ratio in the channel. In subsequent iterations the soft symbol estimates  $\tilde{\boldsymbol{d}}_k$  from the APP decoder are used in conjunction with the pilot signal  $\boldsymbol{c}_k$ . Therefore, we introduce the term  $\boldsymbol{s}_k$  which contains the known pilot signal and the partially decoded data which is utilized as soft pilots in the later iterations.

$$\tilde{\boldsymbol{h}}_{k,i} = (\boldsymbol{s}_k^H \boldsymbol{s}_k)^{-1} \boldsymbol{s}_k^H \boldsymbol{y}_k \qquad i < 0 \tag{3.26}$$

Which, with properly designed pilot sequences collapses to

$$\tilde{\boldsymbol{h}}_{k,i} = \mathrm{E}\left[\boldsymbol{s}_k^H\right] \boldsymbol{y}_k \qquad i < 0 \tag{3.27}$$

The estimator in (3.27) is basically a rake-type matched filter, that is, each tap of the equivalent discrete channel model is correlated against the pilot sequence, so the v-th tap is estimated as

$$\tilde{h}[v] = h_k[v] + \gamma^2 w[v] \tag{3.28}$$

where w[v] is the residual noise left over after integration. Assessing the performance of the estimator condenses now into computing the variance of that noise term. The energy normalization term  $\gamma^2 = 1/\left(E_p + 4E_{shp}[\tilde{d}_k^2] + E_{slp}[\tilde{d}_k^2]\right)$  is determined by the *effective* pilot signal power, and changes with each iteration.

It is important to note that in subsequent iterations (i > 0) it is possible to generate an estimate of the channel tap correlations  $\mathrm{E}\left[\tilde{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^H\right]$ . This leads to the possibility that the minimum mean-square error (MMSE) could be utilized to improve

the estimation in later iterations at the cost of increased complexity, thus creating a hybrid estimation scheme. This MMSE estimate is given as

$$\tilde{\boldsymbol{h}}_{\text{mmse}} = E\left[\boldsymbol{h}_{k}\boldsymbol{h}_{k}^{H}\right] \mathbf{s}^{H} \left[\mathbf{s} E\left[\boldsymbol{h}_{k}\boldsymbol{h}_{k}^{H}\right] \mathbf{s}^{H} + \sigma^{2}\mathbf{I}\right]^{-1} \boldsymbol{y}_{k}.$$
 (3.29)

# Chapter 4

# Results and Analysis

### 4.1 Convergence

The system given by (3.20), (3.21) can be used to predict the performance of an adequately large iterative receiver given a specific estimator performance. Since each of the signals  $s_k(t)$  is distorted by its own equivalent discrete filter (model), and there is no direct correlation between the filter taps of different signals, the received signals  $k' \neq k$  will contribute very little to the estimation of the Q taps  $h_k[v]$ . The iterative demodulator, on the other hand, will subsequently remove the interference and thus provide the main contribution to the estimation of  $h_k[v]$ .

The estimation error  $\sigma_{i,h}^2$  is computed as the variance of the residual noise  $w_p$  as

$$\sigma_{i,h}^{2} = E\left[|\boldsymbol{h} - \tilde{\boldsymbol{h}}|^{2}\right]$$

$$= \gamma^{2} \sigma_{i}^{2} \frac{Q\Delta f_{k}}{f_{s}}$$

$$= \gamma^{2} \sigma_{i}^{2} F_{k}$$
(4.1)

where the filter noise reduction factor  $F_k = \frac{Q\Delta f_k}{f_s}$  depends on the number of taps that need to be estimated, and the ratio of symbol frequency to filter cutoff frequency. Clearly  $F_k \ll 1$  for the estimator to work. In reality, the energy normalization term is determined by both the energy of pilot signal which is fixed, and the energy of the soft pilot signal which will change with each iteration. Therefore it is convenient to write  $\gamma^2$  as

$$\gamma^2 = \frac{1}{P_p + P_d \left( g_2(\sigma_{i-1}^2) \right)} \tag{4.2}$$

which leads to (4.1) being rewritten as

$$\sigma_{i,h}^2 = \frac{\sigma_i^2}{P_n + P_d\left(q_2(\sigma_{i-1}^2)\right)} F_k \tag{4.3}$$

With (4.3) substituted into (3.20) and (3.21), we obtain

$$\sigma_i^2 = \frac{\alpha}{2} \left( g_1(\sigma_i^2) + g_2(\sigma_{i-1}^2) \frac{\sigma_i^2}{P_p + P_d \left( g_2(\sigma_{i-1}^2) \right)} F_k \right) + \sigma^2$$
 (4.4)

To interpret this result, we use  $P_d \gg P_p$ , and consider the latter iterations, i.e.,  $g(\sigma_{i-1}^2) \not\approx 0$ . Then the following simplification can be used

$$\sigma_i^2 \approx \frac{\alpha}{2} \left( g_1(\sigma_{i-1}^2) + \frac{\sigma_i^2}{P_d} F_k \right) + \sigma^2 \tag{4.5}$$

which simply means that the variance transform function of the APP decoder is altered by the addition of a linear term, whose magnitude is completely defined by the filter requirements needed to track the channel changes.

The convergence behavior of iterative systems is often characterized by singleparameter iterative equations, such as (3.17), here rewritten in the standard convergence form

$$x - \frac{\alpha}{2}g_1(x) - \sigma^2 > 0; \quad \text{where } x = \sigma_i^2$$
 (4.6)

For an iterative estimator of the form (4.4), where the estimator uses *only* the pilot signal in its repeated estimation the modified convergence equation is given by

$$x - \frac{\alpha'}{2}g(x) - \sigma^2 - \frac{\alpha x g_2(x)}{2} \frac{F_k}{P_p} > 0$$
 (4.7)

and, finally, an iterated estimator which also uses the emerging data information to drive the estimation obeys the convergence equation

$$x - \frac{\alpha}{2}g_1(x) - \sigma^2 - \frac{\alpha x g_2(x)}{2} \frac{F_k}{P_p + P_d(g_2(x))} > 0$$
(4.8)

The convergence equation describes the dynamical behavior of the system in the large-system limit. It typically has either one or three fixed points, with the right most fixed point being the one reached by the system during operation. If three fixed points exist, the final signal-to-noise ratio is typically too small to allow low-BER operation, and the system load  $\alpha$  has to be lowered until only a single fixed point exists. This limiting system load is the maximum load supportable by a given  $E_s/\alpha$ . With these convergence equations it is now possible to study receiver performance in the system of interest.

The system of interest is a multiuser satellite up-link in the Ka band (27.5GHz), in this band the doppler is dominated by the oscillator drift inherent in low-cost transmitters. Assuming a symbol rate  $f_s$  of  $10^6$  Hz and 0.1ppm oscillators the frequency drift  $\Delta f_k/f_s = 2.75 \times 10^{-3}$ , estimating 10 filter taps leads to a filter noise suppression of  $F_k = 2.75 \times 10^{-2}$  being required for proper operation. Following the DVB-SC standard of 36 pilot symbols for every 1440 data symbols, the energy of the pilot can be viewed as 2.55% of the total energy used in transmission. At this low of a pilot power, simple single-pass estimation with iterative demodulation results in very low achievable system loads based on this receiver. For illustrative purposes we assumed that 10 times as much power is utilized for the pilot symbols than dictated in the standard, which comes with the obvious drawbacks of higher transmit power, or decreased symbol rate. This scenario is shown in Figure 4.1 for an SNR of 10dB and a load of  $\alpha = 1.44$ , where it can be seen that even increasing the pilot power by a factor of 10 in a high-SNR environment the single pass estimation system is limited.

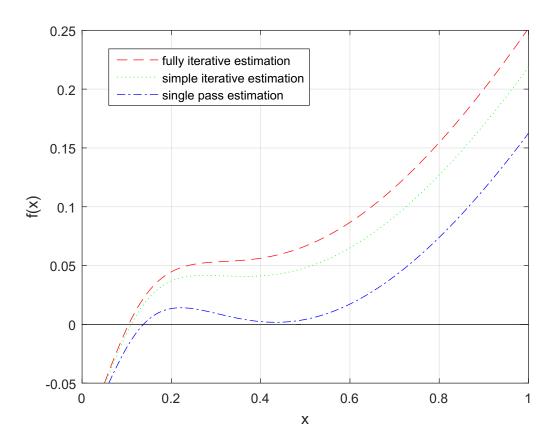


Figure 4.1: Convergence plot - 10dB

Under the same conditions the iterative system using just the pilot data achieves a system load of  $\alpha = 1.64$  and the full iterative system discussed herein is capable of achieving a load of  $\alpha = 1.73$  in the high-SNR environment.

The proposed system excels in the high SNR region where a high number of users can be supported concurrently. As shown in Figure 4.2 the achievable system load is highly dependent on the SNR of the system. At 14 dB SNR the single pass estimator is only capable of achieving  $\alpha = 1.66$  while the simple iterative systems and fully iterative systems are limited to  $\alpha = 1.86$  and  $\alpha = 1.96$  respectively.

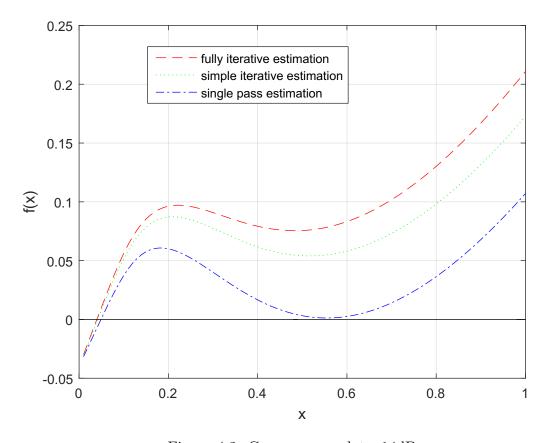


Figure 4.2: Convergence plot - 14dB

The potential for increased system load is apparent in the high-SNR region which exists when the satellite system is operating under good, or line-of-sight conditions. In more adverse shadowing states the increased attenuation will reduce the achievable increase, and the system will collapse to single user performance.

## Chapter 5

#### Conclusion

In this thesis it has been demonstrated that it is possible to support system loads up to 1.96 users per signaling dimension in an uncoordinated multi-user satellite environment with a system operating in the high-SNR region under realistic conditions. This has been achieved by integrating a low complexity receiver into an already established two stage iterative decoder structure. These results represents a significant improvement in an area where research has been mostly limited to approaching the single user TDMA capacity of the channel when this is known to not be a limiting factor. The receiver presented herein is capable of surpassing previous attempts in the high SNR reason when the ability to support a high number of users is required, however it may not be the best choice applications requiring for low SNR operation.

The system presented in this thesis represents a baseline system with room to be expanded by investigating higher order modulation and different methods of coding such as LDPC to improve operation in the low SNR region. Additionally, the performance demonstrated represents a worst case scenario where the received power of the users is equal, when in fact the expected log-normal power distribution will result in increased system performance which has yet to be studied in detail. Additionally, the estimator studied for this purpose was a simple Least Squares Estimator which has the benefit of being low complexity, but does not achieve the same performance of a more complex estimator. The impacts of various estimator structures should be studied to ensure the top level of system performance for a given application. Furthermore, the topic of channel estimation has been largely ignored in the literature regarding access protocols, which has been shown to significantly decrease system performance. It is possible that this method of estimation could be applied to some existing iterative systems to ensure their theoretical performance is attainable under realistic channel conditions.

Finally, it should be noted that although the focus of this thesis was on operation

in the satellite environment but it is not limited and may find application in any system where closed loop processing is infeasible or a high system load is a required feature. This includes networks with coordination where the number of active users may surpass the number of signaling dimensions, such as a slotted TDMA system with more users than available time slots.

# **Bibliography**

- [1] "2015 State of the Satellite Industry Report," Satellite Industry Association, Washington, DC, Ind. Rep. SSIR-2015, Sept. 2015.
- [2] F. Perez Fontán et al., "Statistical Modeling of the LMS Channel," *IEEE Trans. Veh. Technol.*, vol. 50, no. 6, pp. 1549-1567, Nov. 2001.
- [3] S. Scalisem, H. Ernst, and H. Harles, "Measurement and Modeling of the Land Mobile Satellite Channel at Ku-Band," *IEEE Trans. Veh. Technol.*, vol. 57, no. 2, pp. 693-703, Mar. 2008.
- [4] C. Loo, "A Statistical Model for a Land Mobile Satellite Link," *IEEE Trans. Veh. Technol.*, vol. 34, no. 3, pp. 122-127, Aug. 1985.
- [5] E. Lutz, "Modelling of the Land Mobile Satellite Communications Channel," in *IEEE-APS Topical Conf. Antennas and Propagation in Wireless Communications.*, Torino, Sept 2013, pp. 199-202.
- [6] Digital Video Broadcasting (DVB); Second Generation DVB Interactive Satellite System (DVB-RCS2); Part 1: Overview and System Level Specification, ETSI Standard TS 101 545-1 V1.2.1, 2014.
- [7] Digital Video Broadcasting (DVB); Second Generation DVB Interactive Satellite System (DVB-RCS2); Part 2: Lower Layers for Satellite standard, ETSI Standard EN 301 545-2 V1.1.1, 2012.
- [8] Digital Video Broadcasting (DVB); Second Generation DVB Interactive Satellite System (DVB-RCS2); Part 3:Higher Layers Satellite Specification, ETSI Standard TS 101 545-3 V1.1.1, 2012.
- [9] N. Abramson, "The ALOHA System Another Alternative for Computer Communications", in *Proc. AFIPS Computer Conf.*, Nov, 1970, vol. 37, pp. 281-285.
- [10] E. Casini, R. De Gaudenzi, and O. del Rio Herrero, "Contention Resolution Diversity Slotted ALOHA(CRDSA): An Enhanced Random Access Scheme for Satellite Access Packet Networks," *IEEE Trans. Wireless Commun.*, vol. 6, no.4, pp. 1408-1419, Apr. 2007.
- [11] G. Liva, "Contention Resolution Diversity Slotted ALOHA with Variable Rate Burst Repetitions", in *IEEE Global Telecommunications Conf.*, Miami, FL, Dec. 2010, pp. 1-6.
- [12] G. Liva, "A Slotted ALOHA Scheme Based on Bipartite Graph Optimization", in *Int. ITG Conf. Source and Channel Coding*, Siegen, Jan. 2010, pp. 1-6.

- [13] G. Liva, "Graph-Based Analysis and Optimization of Contention Resolution Diversity Slotted ALOHA," *IEEE Trans. Commun.*, vol. 59, no. 2, pp. 477-487, Deb. 2011.
- [14] E. Paolini, G. Liva, and M. Chiani, "High Throughput Random Access via Codes on Graphs: Coded Slotted ALOHA," in *IEEE Int. Conf. Communications*, Kyoto, Jun. 2011, pp. 1-6.
- [15] E. Paolini, G. Liva, and M. Chiani, "Graph-Based Random Access for the Collision Channel without Feedback: Capacity Bound," in *IEEE Global Telecommunications Conf.*, Houston, TX, Dec. 2011, pp. 1-5.
- [16] M. Chiani, G. Liva, and E. Paolini, "The Marriage Between Random Access and Codes on Graphs: Coded Slotted ALOHA," in 1st Annu. IEEE European Conf. Satellite Telecommunications, Rome, Oct. 2012, pp. 1-6
- [17] C. Kissling, "Performance Enhancements for Asynchronous Random Access Protocols over Satellite," in *IEEE Int. Conf. Communications*, Kyoto, Jun 2011, pp.1-6.
- [18] F. Clazzer, and C. Kissling, "Enhanced Contention Resolution ALOHA -ECRA" in *Proc. 9th Int. Conf. Systems, Communication and Coding*, Munich, Jan. 2013, pp. 1-6.
- [19] O. del Rio Herrero, and R. De Gaudenzi, "High Efficiency Satellite Multiple Access Scheme for Machine-to-Machine Communications," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 48, pp. 2961-2989, Oct. 2012.
- [20] C. Stefanovic, K.F. Trilingsgaard, N.K. Pratas, and P. Popovski, "Joint Estimation and Contention Resolution Protocol for Wireless Random Access," in *IEEE Int. Conf. Communications*, Budapest, Jun. 2013, pp. 3382-3387.
- [21] C. Stefanovic, and P. Popovski, "ALOHA Random Access that Operates as a Rateless Code," *IEEE Trans. Commun.*, vol. 61, no. 11, pp. 4653-4662, Nov. 2013.
- [22] R. De Gaudenzi, O. del Rio Herrero, G. Acar, and E. Garrido Barrabes, "Asynchronous Contention Resolution Diversity Slotted ALOHA: Making CRDSA Truly Asynchronous," *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 6193-6206, Nov. 2014.
- [23] C. Schlegel, and A. Grant, *Coordinated Multiuser Communications*, Dordrecht, Netherlands: Springer Publishing, 2006.
- [24] B. Huyen-Chi, J. Lacan, and M.L. Boucheret, "An Enhanced Multiple Random Access Scheme for Satellite Communications," in *Wireless Communications Symposium*, London, Apr. 2012, pp. 1-6.

- [25] B. Huyen-Chi, J. Lacan, and M.L. Boucheret, "Multi-Slot Coded ALOHA with Irregular Degree Distribution," in 1st Annu. IEEE European Conf. Satellite Telecommunications, Rome, Oct. 2012, pp. 1-6.
- [26] F. Ricciato, and P. Castiglione, "Pseudo-Random ALOHA for Enhanced Collision-Recovery in RFID," *IEEE Commun. Lett.*, vol. 17, no. 3, pp. 608-611, Mar. 2013.
- [27] A.S. Tehrani, A.G. Dimakis, and M.J. Neely, "SigSag: Iterative Detection Through Soft Message-Passing," *IEEE J. Sel. Areas Commun.*, vol. 5, no. 8, pp. 1512-1523, Dec. 2011.
- [28] S. Arabi, E. Sabir, and M. Sadik, "ZigZag Decoding-Enabled Muli-Hop Ad Hoc Networks: Taking Benefit from Collisions," 5th Int. Conf. Next Generation Networks and Services, Casablanca, May 2014, pp. 96-100.
- [29] R. Ziegler, and J. Cioffi, "Estimation of Time-Varying Digital Radio Channels," *IEEE Tras. Veh. Tech.*, vol. 41, no.2, pp. 134-151, May 1992.
- [30] H. Arslan, and G. Bottomley, "Review: Channel Estimation in Narrowband Wireless Communication Systems," Wirel. Commun. Mob. Comput., vol.1, pp. 201-219, 2001.
- [31] M. Juntti, and M. Latva-aho, "Multiuser Receivers for CDMA Systems in Rayleigh Fading Channels," *IEEE Trans. Veh. Tech.*, vol. 49, no. 3, pp. 885-899, May 2000.
- [32] V. Lottici, A. D'Andrea, and U. Mengali, "Channel Estimation for Ultra-Wideband Communications," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 9, pp. 1638-1645, Dec. 2002.
- [33] M. Khalighi, J. Boutros, and J-F. Hérald, "Data-aided Channel Estimation for Turbo-PIC MIMO Detectors," *IEEE Commun. Lett.*, vol. 10, no. 5, pp.350-352, May 2006.
- [34] P. Banelli, R.C. Cannizzaro, and L. Rugini, "Data-Aided Kalman Tracking for Channel Estimation in Doppler-Affected OFDM Systems," *IEEE Int. Conf.* Acoustics Speech and Signal Processing, Honolulu, HI., Apr. 2007, vol. 3, pp.133-136.
- [35] J. Ylioinas, and M. Juntti, "Iterative Joint Detection, Decoding, and Channel Estimation in Turbo-Coded MIMO-OFDM," *IEEE Trans. Veh. Tech.*, vol. 58, no. 4, pp.1784-1796, May 2009.
- [36] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon Limit Error-Correcting Coding and Decoding: Turbo-Codes," *IEEE Int. Conf. Communications*, Geneca, May 1993, pp.1064-1070.

- [37] L. Jarbot, "Combined Decoding and Channel Estimation of OFDM Systems in Mobile Radio Networks," *IEEE. Conf. Veh. Tech.*, Phoenix, AZ., May 1997, vol. 3, pp. 1601-1604.
- [38] H. El Gamal, and E. Geraniotis, "Iterative Multiuser Detection for Coded CDMA Signals in AWGN and Fading Channels," *IEEE J. Sel. Areas Commun.*, vol. 18, no. 1, pp. 30-41, Jan. 2000.
- [39] M. Valenti, and B. Woerner "Iterative Channel Estimation and Decoding of Pilot Assisted Turbo Codes Over Flat-Fading Channels," *IEEE J. Sel. Areas Commun.*, vol. 19, no. 9, pp. 1697-1705, Sep. 2001.
- [40] R. Otnes, and M. Tüchler, "Iterative Channel Estimation for Turbo Equalization of Time-Varying Frequency-Selective Channels," *IEEE Trans. Wireless Com*mun., vol. 3, no. 6, pp. 1918-1923, Nov. 2004.
- [41] S. Song, A.C. Singer, and K.M. Sung, "Soft Input Channel Estimation for Turbo Equalization," *IEEE Trans. Signal Process.*, vol. 52, no. 10, pp. 2885-2893, Oct. 2004.
- [42] M. Zhao, Z. Shi, and M.C. Reed, "Iterative Turbo Channel Estimation for OFDM System over Rapid Dispersive Fading Channel," *IEEE Trans. Wireless Com*mun., vol. 7, no. 8, pp. 3174-3184, Aug. 2008.
- [43] M. Liu, M. Crussière, and J.F. Hélard, "A Novel Data-Aided Channel Estimation with Reduced Complexity for TSD-OFDM Systems," *IEEE Trans. Broadcast.*, vol. 58, no. 2, pp. 247-260, Jun. 2012.
- [44] J. Hagenauer, "The Turbo Principle: Tutorial Introduction and State of the Art," *Proc. Int. Symp. Turbo Codes and Related Topics*, Brest, France, Sept. 1997, pp. 1-11.
- [45] C. Douillard, M. Jezequel, C. Berrou, A. Picart, P. Didier, and A. Glavieux, "Iterative Correction of Intersymbol Interference: Turbo-Equalization," *European Trans. Telecomm.*, vol. 6, pp. 507-511, Oct. 1995.
- [46] D. Truhachev, C. Schlegel, and L. Kryzmien, "A Two-Stage Capacity-Achieving Demodulation/Decoding Method for Random Matrix Channels," *IEEE Trans. Inf. Theory*, vol. 55, no. 1, pp. 136-146, Jan. 2009.
- [47] G. Caire, S. Guemgar, A. Roumy, and S. Verdu, "Maximizing the Spectral Efficiency of Coded CDMA Under Successive Decoding," *IEEE Trans. Inf. Theory*, vol. 50, no. 1, pp. 152-164, Jan. 2004.
- [48] C. Schlegel, "CDMA with Partitioned Spreading," *IEEE Commun. Lett.*, vol. 11, no. 12, pp. 913-915, Dec. 2007.

- [49] C. Schlegel, and D. Truhachev, "Multiple Access Demodulation in the Lifted Signal Graph with Spatial Coupling," *IEEE Trans. Inf. Theory*, vol. 59, no. 4, pp. 2459-2470, Apr. 2013.
- [50] S. Verdú, "Minimum Probability of Error for Asynchronous Gaussian Multiple-Access Channels," *IEEE Trans. Inf. Theory*, vol. 32, no. 1, pp. 85-96, Jan. 1986.
- [51] M.K. Varanasi, and B. Aazhang, "Multistage Detection in Asynchronous Code-Division Multiple-Access Communications," *IEEE. Trans. Commun.*, vol. 18, no. 4, pp. 509-519, Apr. 1990.
- [52] J.M. Holtzman, "DS/CDMA Successive Interference Cancellation," IEEE 3rd Int. Symp. Spread Spectrum Techniques and Applicat., Oulu, Jul. 1994, vol. 1, pp. 69-78.
- [53] A. Lampe and J. Huber, "On improved multiuser detection with soft decision interference cancellation, in *Proc. Int. Conf. Commun.*, Vancouver, BC, Canada, June 1999, pp. 172176..
- [54] C. Schlegel, Z. Shi, and M. Burnashev, "Optimal Power/Rate Allocation and Code Selection for Iterative Joint Detection of Coded Random CDMA," *IEEE Trans. Inform. Theory*, vol. 52, no. 9, pp. 4286-4294, Sept. 2006.
- [55] M. Burnashev, C. Schlegel, W. Krzymien, and Z. Shi, "Analysis of the Dynamics of Iterative Interference Cancellation in Iterative Decoding," *Problems of Inform. Transmission*, vol. 40, no. 4, pp. 297-317, May 2004.
- [56] M. Kobayashi, J. Boutros, and G. Caire, "Successive Interference Cancellation with SISO Decoding and EM Channel Estimation," *IEEE J. Sel. Areas Com*mun., vol. 19, no. 8, pp. 1450-1460, Aug. 2001.
- [57] A. Lampe, "Iterative Multiuser Detection with Integrated Channel Estimation for Coded DS-CDMA," *IEEE Trans. Commun.*, vol. 50, no. 8, pp. 1217-1223, Aug. 2002.
- [58] C. Schlegel, D. Truhachev, "Generalized Modulation and Iterative Demodulation," *IEEE Int. Zurich Seminar Commun.*, Zurich, Mar. 2008, pp. 76-69.
- [59] P. Dickson, and C. Schlegel, "Iterative Demodulation and Channel Estimation for Joint Random Access Satellite Communications," in *Turbo Codes and Iterative* Information Processing (ISTC), 2014 8th International Symposium on, Bremen, Aug. 2014, pp. 188-192.
- [60] C. Schlegel, and M. Jar, "Iterative Demodulation and Channel Estimation for Asynchronous Joint Multiple Access Reception," *IEEE Trans. Signal Process*. Aug. 2014.

[61] M. Burnashev, C. Schlegel, and W. Krzymien, "Analysis of the Dynamics of Iterative Cancellation Decoding," *Problems of Information Transmission* vol 40. no. 4, pp. 297-317, Nov. 2004.