STATISTICAL SAMPLE SIZE FOR QUALITY CONTROL PROGRAMS OF CEMENT-BASED SOLIDIFICATION/STABILIZATION

by

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To my father, S. M. A. Hannan

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ABSTRACT

Sampling requirements for the quality control (QC) of cement-based solidification/stabilization (S/S) construction cells do not currently consider the reliability of the hydraulic conductivity sample nor the explicit risk associated with actual flow through the cells. This thesis addresses the issues associated with sampling requirements of a cement-based S/S construction cell during a QC program via probabilistic simulation and via theory taking into account the spatial variability associated with hydraulic conductivity of the entire cement-based S/S system. The sampling requirements are determined by considering a hypothesis test, having null that the constructed material is unacceptable, and targeting acceptable probabilities of making erroneous decisions. Two types of errors that may result in the hypothesis test are: 1) a Type I error where the sample data rejects the null hypothesis even though the null is correct, and 2) a Type II error where the sample data fails to reject the null hypothesis even though it is false. Probabilistic simulations are performed to examine the influence of a soil-cement material's mean, variance, and correlation length on sampling requirements for a QC program of cement-based S/S construction cells. It is found that to achieve target Type I and Type II error probabilities, samples should be collected at higher frequencies when the mean hydraulic conductivity is close to the regulatory value, coefficient of variation is 1.0 or less and the correlation length is at an intermediate value. An example is presented to illustrate how the results can be used in practice. An analytical approach is also presented for selecting the sample size for cement-based S/S construction cell's QC program. Analytical solutions are developed to compute the probabilities of Type I and Type II errors as a function of the number of samples taken and the statistics of the hydraulic conductivity field. The solutions are verified by probabilistic simulations. A set of hydraulic conductivity field data of an existing cement-based S/S system is statistically analyzed to assess its spatial variability. A lognormal distribution is found to be a reasonable fit to the data. Recommendations are provided for conservative QC sampling requirements for that system.

LIST OF ABBREVIATIONS AND SYMBOLS USED

A = Construction cell area perpendicular to flow

D = Effective dimension of the construction cell

G = Standard normal random field

G_i = Local average of G over the i th element

H_0 = Null hypothesis

H_a Alternative hypothesis

i = Hydraulic gradient across the construction cell

k_{crit} = Regulatory hydraulic conductivity

k_{eff} = Effective hydraulic conductivity

k'_{eff} = Normalized effective hydraulic conductivity

k_G = Sample geometric average

k_i = Hydraulic conductivity of the i th element

k_j = Hydraulic conductivity of the j th sample

$\ln k$ = Log-hydraulic conductivity field

l = Number of samples in each of the x and y directions

m = Number of elements

n = Number of samples

n_1 = Number of realizations where $k_G < k_{crit}$ while actual $k_{eff} > k_{crit}$

n_2 = Number of realizations with $k_G > k_{crit}$ while actual $k_{eff} < k_{crit}$

p_1 = Probability of a Type I error

p_2 = Probability of a Type II error

Q = Total flow through the construction cell

T_i = Dimension of the random field in the i th direction

X = Planar dimension of the construction cell in the x direction

Y = Planar dimension of the construction cell in the y direction

X/Y = Aspect ratio of the construction cell

θ_i = Correlation length in the i th direction of the $\ln k$ random field, $i = 1, 2$

θ_k = Random field correlation length for hydraulic conductivity

$\theta_{\ln k}$ = Correlation length of the $\ln k$ random field

$\theta'_{\ln k}$ = Normalized correlation length of the $\ln k$ random field

μ_k = Mean of the hydraulic conductivity field

μ'_k = Normalized mean of the hydraulic conductivity field

$\mu_{\ln k}$ = Mean of the log-hydraulic conductivity field, $\ln k$

$\sigma_{\ln k}$ = Standard deviation of the log-hydraulic conductivity field, $\ln k$

σ_k = Standard deviation of the hydraulic conductivity field

v_k = Coefficient of variation of the hydraulic conductivity field

$\rho_{\ln k}$ = Correlation coefficient between points in the $\ln k$ random field

$\gamma_{\ln k}$ = Variance reduction function when $\ln k$ is averaged over some volume

γ = Same as $\gamma_{\ln k}$

τ_i = Distance between points in the i th direction of the random field, $i = 1, 2$

Δx = Planar dimension of the element in the x direction

Δy = Planar dimension of the element in the y direction

$\mu_{\ln k_G}$ = Mean of the log-sample geometric average, $\ln k_G$

$\mu_{\ln k_{eff}}$ = Mean of the log- effective hydraulic conductivity, $\ln k_{eff}$

$\sigma_{\ln k_G}$ = Standard deviation of the log-sample geometric average, $\ln k_G$

$\sigma_{\ln k_{eff}}$ = Standard deviation of the log-effective hydraulic conductivity, $\ln k_{eff}$

ρ = Correlation coefficient between $\ln k_{eff}$ and $\ln k_G$

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CHAPTER 1

INTRODUCTION

1.1 GENERAL

Cement-based solidification/stabilization (S/S) is a source-controlled remediation technology in which cement is mixed with contaminated media such as soil, sediment, sludge, or industrial waste to minimize the migration of the contaminants and thereby to limit the contamination of groundwater and/or surface water. For a contaminated site, S/S may be performed by dividing the entire site into a number of smaller cells (which will be referred to as construction cells for this study) for quality control (QC) purposes. QC programs normally involve testing of individual construction cells for strength, hydraulic conductivity and leachability; each construction cell being approved if it is determined to have "passed" pre-established performance limits by a regulatory body.

In terms of hydraulic conductivity, the objective of the performance criteria is to ensure, with some certainty, a sufficiently small hydraulic flow. For one dimensional flow, the deterministic equation governing the total advective flow, Q, through a saturated S/S construction cell is given by Darcy's law as follows,

$$Q = k_{eff} iA \tag{1.1}$$

where k_{eff} is the effective hydraulic conductivity of the construction cell, i is the hydraulic gradient across the cell and A is the area perpendicular to the direction of flow. The effective hydraulic conductivity, k_{eff}, is defined as the single value of hydraulic conductivity which yields the same total flow through the cell as does the actual spatially varying hydraulic conductivity field (see Fenton and Griffiths, 1993). To ensure that the construction cell will perform effectively in restricting contaminant migration via advection, samples are collected and tested during construction in a QC program to establish adequate “performance”. In essence, the purpose is to ensure that the effective hydraulic conductivity meets the given performance specification. If the estimated effective hydraulic conductivity is less than or equal to the regulatory hydraulic conductivity, k_{crit}, then the construction cell is considered to be acceptable. Otherwise it is deemed unacceptable and must be repaired or replaced.

The question is: How many samples should be taken in order to reliably make the decision regarding the construction cell being acceptable or unacceptable? In practice, samples are collected based on the sample density method (USACE, 2000), which requires a certain number of samples per unit volume. The required number of samples is not affected by the statistics of the sampled field. Since the randomness of a system increases as the variability of the material composing the system increases, it is logical to believe that sampling at the same frequencies over construction cells having different variability will result in different levels of reliability of the effective hydraulic conductivity estimate. This thesis aims to address the issues associated with sampling requirements of a cement-based S/S construction cell during a QC program to achieve a certain confidence in the decision (acceptable or unacceptable) regarding this cell based on the estimated effective hydraulic conductivity and the spatial variability associated with hydraulic conductivity of the entire cement-based S/S system. The purpose of this chapter is to provide some background information about cement-based S/S systems and their QC sampling requirements and reliability prior to presenting the research work in the following chapters.

1.2 CEMENT-BASED S/S

Solidification/stabilization is a chemical treatment technology for contaminated material. USEPA (2000) defines solidification and stabilization separately, as follows:

Solidification involves the processes by which contaminants are entrapped within a solid cementitious matrix. Solidification may or may not accompany chemical processes (processes that involve chemical reactions). Contaminant migration is restricted by decreasing the surface area exposed to leaching and/or by encapsulating contaminated material within a low-permeability material.

Stabilization involves the processes by which contaminants are transformed into less soluble, mobile, or toxic forms. During stabilization, chemical reactions occur between contaminants and the stabilizing agent.

S/S was employed in 24% of the source-controlled remediation at Superfund projects in the United States (USEPA, 2004). The performance of S/S treated material depend on many factors: waste type, water content, reagent type, reagent mix-ratio, curing time, and temperature (Conner and Hoeffner, 1998). Out of the different remediation technologies, Portland cement-based (which will be referred to as simply cement-based in this thesis) S/S has been widely applied since 1950. It was selected as one of the remediation technologies in the Sydney Tar Ponds project, a $400 million CAD clean-up project in Canada (http://en.wikipedia.org/wiki/Sydney_Tar_Ponds).

Cement-based S/S has the following advantages over other remediation technologies (Conner and Hoeffner, 1998; Shi and Spence, 2004; Paria and Yuet, 2006):

- Low cost and ease of use,
- Good physical and chemical stability,

- Low permeability,
- Well known hydration reactions (reactions between cement and water in setting and hardening),
- Availability and non-toxicity of chemical ingredients,
- Good compressive strength,
- Low leachability of some constituents,
- High resistance to biodegradation.

A brief overview of cement-based S/S process (ITRC, 2011) is presented below:

Cement-based S/S involves mixing cement, additives, and water with contaminated material using mechanical equipment. Additives with sorptive properties can be used to improve the performance of S/S processes to treat organic contaminants, since organic contaminants do not bond with the cementitious minerals formed by the hydration of cement. There are two methods to add cement to the contaminated material: dry and wet. In the dry method, cement is applied "dry" onto the contaminated material and then mixed. In the wet addition method, cement is mixed with water to form a grout or paste which is then mixed with contaminated material. Dry addition is more common and is feasible for shallow S/S applications. Cement-based S/S treatment process can be either in-situ (i.e., material remains in place when treatment is performed) or ex-situ (i.e., treatment is performed on excavated material). A variety of equipment is available for both in-situ and ex-situ S/S treatment processes. The choice of equipment depends on the characterstic of the contaminants and the geometry of the contaminated area.

Common ex-situ mixing methods are pugmill and batch mixing. In pugmill mixing, contaminated material and cement are mixed in a trough-like mixing chamber which has two horizontal shafts with paddles attached to counter-rotating shafts. In batch mixing, a variety of vessels are used to mix batches of contaminated material with cement.

Common in-situ mixing methods are rotary mixing, excavator mixing, and auger mixing. In rotary mixing, cement is spread on top of the material, and a rotary mixer mixes the contaminated material and cement. While mixing, water is added either in front or behind the mixer as necessary. Excavator mixing is suitable for shallow depth (less than 6 m) and is performed by dividing the entire contaminated area into grid cells. The contaminated material and cement are mixed by the excavator bucket. Mixing homogeneity in excavator mixing depends on the expertise of the excavator operator and the amount of time spent mixing each grid cell.

Auger mixing is the most commonly used method for in-situ cement-based S/S process. In auger mixing, contaminated material is mixed with cement either by a single auger or array of augers. An auger treats a column of soil 0.6-3.7 m in diameter and up to 20 m in depth. Both dry and wet methods are used for auger mixing. Dry mixing is suitable for low-strength soils with high moisture content, whereas wet mixing is suitable for soils with moisture content less than 60%. The treatment process is completed by executing a series of overlapping columns over the project area.

After mixing cement and/or additives to the contaminated material, the treated material is left to cure. In the in-situ process, the treated material is left in place whereas, in the ex-situ process, the treated material is spread on a waste consolidation cell in lifts of uniform thickness, compacted and allowed to cure. The ex-situ treated material can be returned either to the original location or another location on the project area.

1.3 Quality control programs and reliability

The sample density method (USACE, 2000) that is commonly used to specify the sampling frequency for the QC of S/S construction cells, requires a certain number of samples per unit volume (i.e., 1 hydraulic conductivity sample per 500 m^3 of a cement-based S/S construction cell). A construction cell is considered to be acceptable if the average of sampled hydraulic conductivities is less than or equal to 1×10^{-8} m/s, no individual sampled hydraulic conductivity is greater than 1×10^{-7} m/s, and no more than 20% of the sampled hydraulic conductivities exceed 1×10^{-8} m/s (ITRC, 2011).

Unfortunately, the sample density method does not adjust for site variability. Performing simulations, Benson et al. (1994) found that samples should be collected at higher frequency for highly variable soils and for soils having the mean hydraulic conductivity at or close to the regulatory value. The sample density method may result in either "undersampling" or "oversampling" (Benson et al., 1994). Although Bensons' findings are for compated clay liners which have variability in their composition due to the inherent and induced (due to construction) variability, they can also be used for similar constructed systems such as cement-based S/S construction cells.

The precision-of-estimator methods (the error of sampling and the sequential sampling methods (Richardson, 1992)), which are basically confidence interval methods, are used to select the sample size for a QC program of waste-containment systems. In the error of sampling method , the sample size is determined prior to data collection, whereas, the predetermined sample size is adjusted based on the collected data in the sequential sampling method. The goal of the precision-of-estimator methods is to select a sample size such that the estimate of the property of interest (e.g., hydraulic conductivity) will lie within certain bounds with a specified probability. For example, the precision-of-estimator methods can be used to specify the sample size such that the estimate of the mean hydraulic conductivity does not vary by more than 5×10^{-9} m/s from the true mean (m/s) with a 95% probability. The advantage of the precision-of-estimator methods over the sample density method is that instead of considering the volume of the system, the precision of the estimate is considered in selecting the sample size.

Menzies (2008) proposed a hypothesis test based method to determine QC sample sizes for soil liner systems. The work presented in this thesis for assessing QC sample sizes of cement-based S/S construction cells is also a hypothesis test based method, which is described in more detail in the following chapters. Random fields are used to model the hydraulic conductivity fields in the hypothesis tests.

1.4 Random Fields

Random fields are commonly used to model spatially variable engineering properties (Fenton and Griffiths, 2008). Relatively simple, Gaussian-based, random fields are described by three parameters, the mean, μ, the standard deviation, σ, and the correlation length, θ. Frequently, the standard deviation is expressed as a coefficient of variation, v, which is the ratio between the standard deviation, σ, and the mean, μ, $v = \sigma/\mu$. The correlation length is the distance over which the property of interest is significantly correlated and beyond which is largely uncorrelated. Mathematically, θ can be defined as the area under the correlation function (Vanmarcke, 1984);

$$\theta = \int_{-\infty}^{+\infty} \rho(\tau) d\tau \qquad (1.2)$$

where $\rho(\tau)$ is the correlation function which characterizes the spatial dependence between two points in the random field separated by a distance τ.

Smaller values of θ imply a rapidly varying field, while larger values of θ imply a slowly varying field. Figure 1.1 shows two random field (one-dimensional) realizations for two different values of θ. The figure on the left, having small θ (i.e., 0.04), displays rapid variation in the field, while the figure on the right, having large θ (i.e., 2.0), shows slow variation in the field (Fenton and Griffiths, 2008).

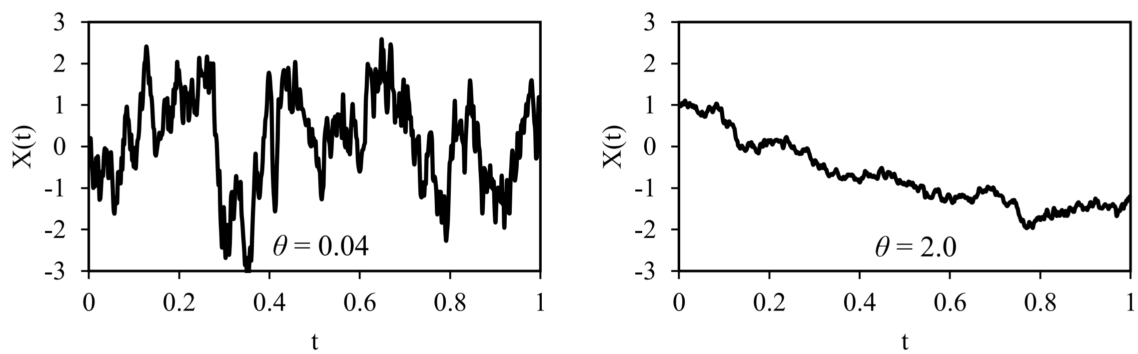

Figure 1.1 Sample realizations of $X(t)$ for two different correlation lengths

In this research, the hydraulic conductivity field is represented using a two-dimensional spatially variable random field. The hydraulic conductivity is assumed to be the local average over some volume. Local averaging reduces the variance of the random field. The final variance depends on the volume selected for local averaging, decreasing as the local averaging volume increases (Fenton and Griffiths, 2008). The variance reduction function, $\gamma(T)$, is used to express the amount of variance reduction when averaged over some length, T.

1.5 Background on Sampling Theory

The overall objective of QC sampling of cement-based S/S construction cells is to ensure that each cell will be acceptable (i.e., that its effective hydraulic conductivity, k_{eff} will be less than the regulatory hydraulic conductivity, k_{crit}). The decision about whether a construction cell is acceptable or not is made on the basis of a set of samples taken from the cell. This decision making process can be formulated as a hypothesis test where the null hypothesis (H_0) is that the cell is unacceptable, so that the burden of proof is on showing that the alternative hypothesis (H_a) is true, at an appropriate level of confidence.

$$\begin{aligned} H_o &: k_{eff} \geq k_{crit} \\ H_a &: k_{eff} < k_{crit} \end{aligned} \tag{1.3}$$

Two types of errors may result in making this decision about the acceptability of a cell: 1) concluding that the S/S construction cell is acceptable when it is not (Type I error), or, 2) failing to conclude that the S/S construction cell is acceptable when it actually is (Type II error). The challenge is to determine how many samples should be collected in order to ensure that the probability of making either type of error will be acceptably small.

Taking an infinite number of samples from the construction cell will eliminate any chances of making a decision error, but this is neither physically nor economically feasible. This means that some chance of error will always exist and so it is necessary to relate the error probabilities with the number of samples taken in order to determine the number of samples required.

Analytical solutions exist to determine the sample size required to ensure that the probabilities of Type I and II errors are sufficiently small (see, e.g., chapter 8 of Devore, 2008). These solutions, however, assume that the samples are independent. Since the construction cell hydraulic conductivity values are generally correlated, the existing analytical solutions cannot be used to determine required sample sizes for the quality control of construction cells. The goal of this study is to investigate how the probabilities of Type I and Type II errors change as a function of the number of samples taken within a construction cell.

1.6 RESEARCH OBJECTIVES

This thesis has three distinct objectives:

1) to perform probabilistic simulations to examine the influence of the correlation length, hydraulic conductivity mean and coefficient of variation on sampling requirements to achieve target probabilities for Type I and Type II errors for the QC program of cement-based S/S construction cells;
2) to develop analytical solutions to compute the probabilities of Type I and Type II errors as a function of the number of samples taken and the statistics of the hydraulic conductivity field. These solutions will allow the estimation of the sample size required for the QC program of cement-based S/S construction cells to achieve target Type I and Type II error probabilities; and

3) to statistically analyze the hydraulic conductivity field data of an existing cement-based S/S system to assess its spatial variability and to provide recommendations for conservative sampling requirements for the QC program of that S/S system.

1.7 THESIS ORGANIZATION

This thesis is organized as follows:

Chapter 2 reports on a parametric study performed to examine the influence of correlation length, hydraulic conductivity mean and coefficient of variation on sampling requirements for the QC program of cement-based S/S construction cells using probabilistic simulations. The simulation employs a modified version of the two-dimensional random finite element method (RFEM) program, mrflow2d (Fenton and Griffiths, 1993). Modifications made to mrflow2d for this study were to enable the sampling of the random field at prescribed locations. For a specific number of samples, the influence of correlation length, hydraulic conductivity mean and coefficient of variation on the probabilities of Type I and Type II errors are examined in Chapter 2. Plots are provided which can be used to estimate required number of samples and an example is presented to illustrate how the results can be used in practice.

Chapter 3 presents an analytical approach to estimate QC sample sizes required to achieve target Type I and Type II error probabilities in cement-based S/S consruction cells. Analytical solutions are developed to compute the probabilities of Type I and Type II errors as functions of the number of samples and the statistics of the hydraulic conductivity field. In order to validate the proposed analytical solutions, the analytically computed Type I and Type II error probabilities are compared to those estimated via probabilistic simulations in Chapter 2. An example is presented to illustrate how the proposed analytical approach can be used in practice to assess required sample size for the QC program of cement-based S/S construction cells.

In **Chapter 4**, a set of hydraulic conductivity field data of an existing cement-based S/S system is statistically analyzed to assess its spatial variability (described by a distribution and a correlation function). The spatial variability associated with the hydraulic conductivity data is then used to compute the Type I and Type II error probabilities associated with a specific number of samples using the analytical solutions presented in Chapter 3. Recommendations are provided in Chapter 4 for conservative sampling requirements for the QC program of that S/S system.

Chapter 5 summarizes the results presented in Chapters 2, 3, and 4 and draws conclusions from these results. Chapter 5 also presents recommendations for further study.

CHAPTER 2

A PARAMETRIC STUDY ON QUALITY CONTROL SAMPLE SIZES OF CEMENT-BASED SOLIDIFICATION/STABILIZATION

2.1 GENERAL

This chapter aims to examine the influence of the correlation length and hydraulic conductivity mean and coefficient of variation on sampling requirements during a QC program of cement-based S/S of a construction cell via simulation. The goal is to determine the number of samples required to achieve a certain confidence in the decision (acceptable or not) about the cell based on the estimated effective hydraulic conductivity.

Random fields will be used here to model the hydraulic conductivity field. The sampling problem will be investigated by simulating possible realizations of the two-dimensional hydraulic conductivity field, virtually sampling each realization at selected locations and then deciding whether the realization is acceptable or not on the basis of the sample results. An error in the decision is made if either the cell is deemed to be acceptable, when it is not (Type I error), or if the cell is deemed to be unacceptable, when it is actually acceptable (Type II error). As will be shown, the probability of making a decision error reduces as the number of samples increases, not surprisingly, and the task is to determine just how many samples are required to reduce the error probabilities to acceptable levels.

The random conductivity field realizations will be simulated using a method called Local Average Subdivision (LAS) (Fenton and Vanmarcke, 1990). The LAS algorithm preserves the spatial correlation, over the ensemble, between local averages of the property. LAS directly simulates realizations of 'local' averages. As mentioned previously, local averaging reduces the variance of the random field. In the two-dimensional model considered here, the final variance depends on the area selected for local averaging, decreasing as the local averaging area increases (Fenton and Griffiths, 2008). Further details regarding the correlation structure and variance reduction used in the random field model can be found in Section 2.3.

Research relating to the sampling requirements for a QC program of cement-based S/S construction cells is not available in literature, so far as the author is aware. Some research has been conducted on the sampling requirements for soil liner systems, which is similar to the requirements for cement-based S/S construction cells, as discussed next.

Benson et al. (1994) presented a method to select the number of samples that should be collected and tested during the construction of compacted soil liners in order to ensure reliable liners at some confidence level. Not surprisingly, they found that the accuracy of the estimate increases as the sample size increases and also showed that samples should be collected at higher frequency for soils having highly variable hydraulic properties as well as for soils with mean hydraulic conductivity close to the regulatory value. In their investigation, simulations were performed using a three-dimensional stochastic model with varying hydraulic conductivity mean, variance, and liner thickness. However, they did not explicitly consider the random field nature of the liner, that is independence between adjacent elements in their model was assumed for simplicity, i.e., they ignored the correlation between hydraulic conductivity values.

Menzies (2008) examined the influence of the correlation length on sampling requirements of soil liner systems in order to achieve target reliability against excessive flow through the liner. Influences of the hydraulic conductivity mean and variance on sampling requirements were investigated using a two-dimensional stochastic model to perform simulations. In Menzies' study, two types of hypothesis test errors were considered, i.e., Type I where the sample data led to the conclusion that the liner was acceptable when it was not and Type II where the sample data suggested that the liner was unacceptable when it actually was acceptable. It was found that a "worst case" correlation length existed, which was about 5%-10% and 2%-3% of the liner size in any direction, that maximized the probabilities of Type I and Type II errors, respectively. Menzies (2008) also found that for a particular sample size, both types of error probabilities reached a maximum value when the mean hydraulic conductivity of the liner was close to the regulatory value, requiring more samples in this case to achieve the same reliability as obtained when the mean hydraulic conductivity is farther away from the regulatory value. In his stochastic model, Menzies used the arithmetic average of the hydraulic conductivity field to be the effective hydraulic conductivity, since for soil liners, the dimension parallel to the flow is thin relative to the dimension perpendicular to the flow. He also assumed the correlation structure to be isotropic. This work extends that of Menzies' to a case where the flow is in-plane so that geometric averaging is required.

As mentioned above, the author has been unable to find literature dealing specifically with the sampling requirements for QC programs of cement-based S/S construction cells. This study aims to investigate this problem through the use of random field simulations.

2.2 PROBABILISTIC SIMULATIONS

The construction cells investigated in this study are designed to provide a barrier against horizontal flow and are thin (vertically) relative to their planar dimension, as shown in Figure 2.1. Because the cell is relatively thin, the flow is largely in the plane and a two-dimensional flow model is acceptably accurate. Since a two-dimensional flow model is also much faster, computationally, than a three-dimensional model, the two-dimensional model will be used here. The hypothesis test problem is thus studied here using Monte Carlo simulations employing a modified version of the two-dimensional random finite element method (RFEM) program, mrflow2d (Fenton and Griffiths, 2008). The original program was designed to analyze stochastic fluid flow problems and is described in Fenton and Griffiths (1993). The program is modified in this study to enable the sampling of the random field at prescribed locations. Also in the modified program, finite element method is not used to obtain the flow through the field, instead geometric avergage of the field is used to represent the flow, which is justified by Fenton and Griffiths (1993), Dagan (1982) and Gutjher et al. (1978). The mesh discretization used in the simulations is as shown in Figure 2.1.

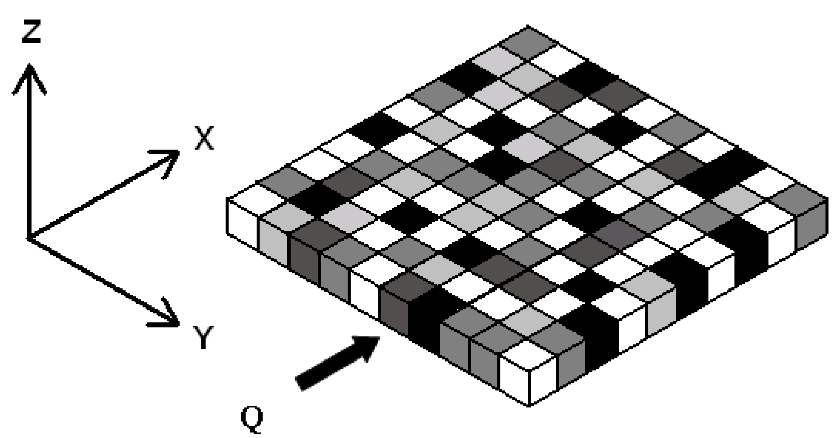

Figure 2.1: Illustration of mesh discretization used in the simulations

The flow regime assumes that an impervious boundary exists on the top and bottom, and on the left and right, faces of Figure 2.1. A uniform unit pressure head was applied on the front face which directs the flow, *Q*, in the x direction. The inputs to the model are the mean and standard deviation of point-scale hydraulic conductivity, correlation lengths (assumed isotropic), the number of elements in each direction, the element size, and the number and locations of the samples to be taken. Given these inputs, the RFEM model generates a random field of lognormally distributed hydraulic conductivity. The steps followed in the simulations are as follows:

1. Given the mean, standard deviation and correlation length of the hydraulic conductivity at the point-scale, generate a realization of the local averages, G_i, for $i = 1,2,......, m$, where m is the specified number of elements in the model, using the Local Average Subdivision (LAS) algorithm (Fenton and Vanmarcke, 1990). Each local average, G_i, is the arithmetic average of a standard Gaussian field, G over the i th element.

2. The lognormally distributed hydraulic conductivity value, k_i, is assigned to the i th element through the transformation $k_i = \exp\{\mu_{\ln k} + \sigma_{\ln k} G_i\}$, where $\mu_{\ln k}$ and $\sigma_{\ln k}$ are the mean and standard deviation of the logarithm of k obtained from the specified mean and standard deviation μ_k and σ_k via the transformations:

$$\sigma^2_{\ln k} = \ln\left(1 + \nu^2_k\right) \tag{2.1a}$$

$$\mu_{\ln k} = \ln \mu_k - \frac{1}{2}\sigma^2_{\ln k} \tag{2.1b}$$

where $\nu_k = \sigma_k / \mu_k$ is the coefficient of variation.

3. Sample the field at the specified element locations. This is done simply by recording the value of k_j generated for the jth sampled element. Measurement error is assumed to be zero.

4. Compute the geometric average, k_G, of the sample and the effective hydraulic conductivity of the entire conductivity field, k_{eff} as follows,

$$k_G = \exp\left\{\frac{1}{n}\sum_{j=1}^{n}\ln k_j\right\} \tag{2.2}$$

$$k_{eff} = \exp\left\{\frac{1}{m}\sum_{i=1}^{m}\ln k_i\right\} \tag{2.3}$$

where

$n =$ number of samples taken from the random field,

$k_j =$ hydraulic conductivity of the j th sampled element of the random field,

$m =$ number of elements of the random field, and

$k_i =$ hydraulic conductivity of the i th element of the random field.

Fenton and Griffiths (1993) demonstrated that the geometric average was the best estimate of the effective hydraulic conductivity for relatively square flow regimes, where the effective hydraulic conductivity was defined by them to be the single value of hydraulic conductivity which yields the same total flow through the cell as does the actual spatially varying hydraulic conductivity field. Hence, geometric averages of the element hydraulic conductivities and the samples are used to obtain the actual and the predicted effective hydraulic conductivity of the random field, respectively. In other words, the effective hydraulic conductivity, k_{eff}, used in this study, closely approximates the uniform (spatially constant) hydraulic conductivity value which yields the same total flow as computed through the actual spatially random hydraulic conductivity field. If $k_{eff} > k_{crit}$, then the total flow through the cell exceeds the regulatory limit and the cell is unacceptable.

The geometric average, k_G, is the sample estimate of the effective hydraulic conductivity, k_{eff}. If $k_G < k_{crit}$, then the cell is *deemed* to be acceptable, even though it may not be (Type I error). Alternatively, if $k_G > k_{crit}$, then the cell is deemed to be unacceptable, even though it may actually be acceptable (Type II error). For each realization, the sample geometric average, k_G, and the effective hydraulic conductivity, k_{eff} are compared to the regulatory hydraulic conductivity, k_{crit}. This comparison results in one of the following four outcomes being recorded for each realization:

- The effective hydraulic conductivity of the random field is below the regulatory value and the sample data agrees with this $(k_G < k_{crit} \cap k_{eff} < k_{crit})$. This is a favorable outcome.
- Both k_G and the actual effective hydraulic conductivity of the random field are above the regulatory value $(k_G > k_{crit} \cap k_{eff} > k_{crit})$. This outcome will result in the cell being deemed to be unacceptable but is a favorable outcome since it is predicted by the sample.
- k_G is less than the regulatory hydraulic conductivity, while the actual effective hydraulic conductivity of the field exceeds the regulatory value $(k_G < k_{crit} \cap k_{eff} > k_{crit})$. This is an unfavorable Type I error (cell is assumed

acceptable when it is not) resulting in the worst outcome of this hypothesis test, where an unsafe cell is deemed to be safe.

- k_G is greater than the regulatory value, while the actual effective hydraulic conductivity of the field is less than the regulatory value $(k_G > k_{crit} \cap k_{eff} < k_{crit})$. This is an unfavorable Type II error (cell is assumed unacceptable when it is actually acceptable) which would require some unnecessary work, such as excavating the treated material and reapplication of the S/S process for the construction cell, resulting in a higher project cost.

Of the two types of errors, the Type I error is the worst from an environmental protection standpoint since it results in an unacceptable cell being accepted. The above steps are repeated over n_{sim} realizations for each parameter set (as discussed in the next section) to estimate the probabilities of Type I (p_1) and Type II (p_2) errors, according to:

$$p_1 = \frac{n_1}{n_{sim}} \tag{2.4}$$

$$p_2 = \frac{n_2}{n_{sim}} \tag{2.5}$$

where n_1 is the number of realizations where $k_G < k_{crit}$ while $k_{eff} > k_{crit}$, n_2 is the number of realizations where $k_G > k_{crit}$ while $k_{eff} < k_{crit}$, and n_{sim} is the total number of realizations considered.

2.3 PARAMETRIC STUDY

In order to enable the results to be scaled to any desired regulatory hydraulic conductivity, k_{crit}, the mean of the point-scale hydraulic conductivity of the input distribution, μ_k, and the effective hydraulic conductivity, k_{eff}, can be normalized by the regulatory hydraulic conductivity, k_{crit}.

$$\mu'_k = \frac{\mu_k}{k_{crit}} \tag{2.6}$$

$$k'_{eff} = \frac{k_{eff}}{k_{crit}} \tag{2.7}$$

where μ'_k is the normalized mean hydraulic conductivity and k'_{eff} is the normalized effective hydraulic conductivity.

The correlation length, $\theta_{\ln k}$, can also be non-dimensionalized by dividing by the effective dimension of the construction cell, D, where $D = \sqrt{XY}$ and X and Y are the planar dimensions of the construction cell;

$$\theta'_{\ln k} = \frac{\theta_{\ln k}}{D} \tag{2.8}$$

Non-dimensionalizing the correlation length allows the results to be scaled to any construction cell size so long as it has same (or similar) aspect ratio (X/Y) as used in this study, which is $X/Y = 1$. If the region under consideration is not square, it is approximated by a square region of size $D \times D$.

Parametric variations considered in the simulations were as follows:

- Normalized mean hydraulic conductivity, μ'_k = 0.01, 0.1, 0.9, 1.0, 1.1, 1.2, 1.4, 1.6, 1.8, 2.0, 3.0, and 10.0,
- Coefficient of variation, v_k = 0.1, 1.0, 2.0, and 5.0,
- Normalized correlation length, $\theta'_{\ln k}$ = 0.01, 0.05, 0.1, 0.5, 1.0, 5.0, and 10.0,
- Number of samples, $n = 1$, 4, 9, 16, 25, and 49 (i.e., in increments of l^2, where l is the number of samples in each of the x and y directions). For each value of n, the field is sampled at equispaced locations, as illustrated in Figure 2.2.

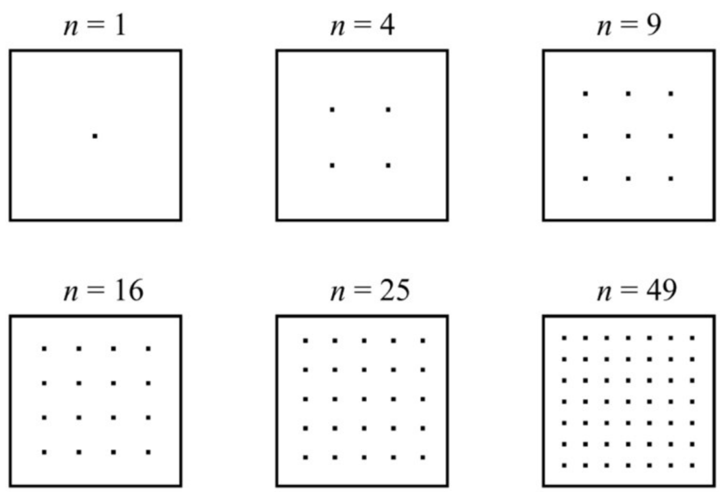

Figure 2.2: Sampling locations shown as small black squares

The lognormally distributed random hydraulic conductivity field is fully specified by its mean, its variance, and its correlation structure. In this study, the correlation between pairs of $\ln k$ values is assumed to be Markovian having the following separable correlation function (which is a product of two directional correlation functions – see, e.g., Vanmarcke, 1984, for more details.),

$$\rho_{\ln k}(\tau_1, \tau_2) = \exp\left(-2|\tau_1|/\theta_1\right)\exp\left(-2|\tau_2|/\theta_2\right) \tag{2.9}$$

in which τ_i is the distance between points in each coordinate direction, $i=1$ and 2. The decay rate parameters θ_i, for $i=1$ and 2, are the directional correlation lengths. In this study, the correlation lengths are assumed to be equal; $\theta_1 = \theta_2 = \theta_{\ln k}$.

Since the correlation function is separable, its corresponding variance reduction function (see Vanmarcke, 1984) is also separable and can be written explicitly as the product:

$$\gamma_{\ln k}(X,Y) = \gamma(X)\gamma(Y) \tag{2.10}$$

where

$$\gamma(X) = \frac{\theta_{\ln k}^2}{2X^2}\left[\frac{2|X|}{\theta_{\ln k}} + \exp\left\{\frac{-2|X|}{\theta_{\ln k}}\right\} - 1\right] \tag{2.11}$$

and similarly for $\gamma(Y)$.

Regarding the finite element model, a sensitivity analysis was performed in order to examine the influence of the element size on the output quantities of interest (i.e., the probabilities of Type I and Type II errors). A domain of size (1×1) was discretized into 32×32, 64×64, 72×72, 80×80, 104×104, 128×128, and 256×256 elements. All mesh resolutions gave similar results (see Appendix A). Based also on reasonable computing time, a (64×64) element density was selected for all simulations. The number of realizations selected was $n_{sim} = 25000$ for all parameter sets considered. This means that the standard deviation of any probability estimate is $\sqrt{\hat{p}(1-\hat{p})/n_{sim}}$, where $\hat{p}$ is the estimated probability, which, for small $\hat{p}$ is approximately $0.0063\sqrt{\hat{p}}$. In other words, the Monte Carlo simulation can reasonably accurately estimate $\hat{p}$ down to about 1/10000.

2.4 RESULTS

2.4.1 INFLUENCE OF CORRELATION LENGTH ON ERROR PROBABILITIES

It is instructive to first consider the probabilities of Type I and II errors at the limiting values of $\theta_{\ln k}$. At the lower limit, when $\theta_{\ln k}$ is equal to 0, points within the field will have no correlation with each other, which means that the $\ln k$ field is white noise (Fenton and Griffiths, 2008). In this case, any local average of $\ln k$ will consist of an infinite number of independent values whose average is a non-random constant (equal to the median) so that one (local average) sample is sufficient to completely specify the effective hydraulic conductivity of the field. That is, the probability of making any type of error (i.e., either Type I or Type II) will be zero on the basis of one or more samples if $\theta_{\ln k} = 0$. At the other extreme, when $\theta_{\ln k} \to \infty$, points within the random field are perfectly correlated with each other which means that they are all equal if the field is stationary, as assumed here. In this case, the field can be represented by a single (random) hydraulic conductivity value so that one sample is sufficient to predict the actual effective hydraulic conductivity of the entire field, resulting in error probabilities again being equal to 0. At intermediate correlation lengths (i.e., between zero and infinity), the probabilities of Type I and II errors are non-zero and will be affected by the number of samples taken – fewer samples will result in larger error probabilities. Figure 2.3 shows the influence of the normalized correlation length on the probability of a Type I error for different numbers of samples ($n = 1$, 4, 9, 16, 25, and 49) for $\mu_k' = 1.0$ and $v_k = 1.0$. Each point on the plot is obtained using 25000 realizations and indicates that, for given number of samples, as the correlation length increases the probability of a Type I error at first increases and then decreases, as expected. For example, when $\mu_k' = 1.0$, $v_k = 1.0$, and

$n = 4$, the probability of a Type I error increases from close to 0 at a normalized correlation length of 0.1 to a maximum value of 0.0362 at a normalized correlation length of 1.0, and then decreases to 0.0190 when the normalized correlation length reaches 10.0. The probability continues to decrease thereafter to 0 as $\theta'_{\ln k} \rightarrow \infty$ (not shown). The highest error probability occurs at a "worst case" correlation length, in this case at about $\theta'_{\ln k} = 1.0$. Since the actual correlation length is rarely, if ever, known at any site, the practical importance of the existence of a "worst case" correlation length is that it can be used to produce sampling plans which are conservative, that is, guaranteed to have error probabilities no higher than specified in the sampling design.

Figure 2.3 also shows that for given correlation length, the probability of a Type I error decreases as the number of samples increases. For example, when $\mu'_k = 1.0$, v_k = 1.0, and $\theta'_{\ln k} = 0.5$, the probability of a Type I error decreases from 0.0322 when $n = 4$ to

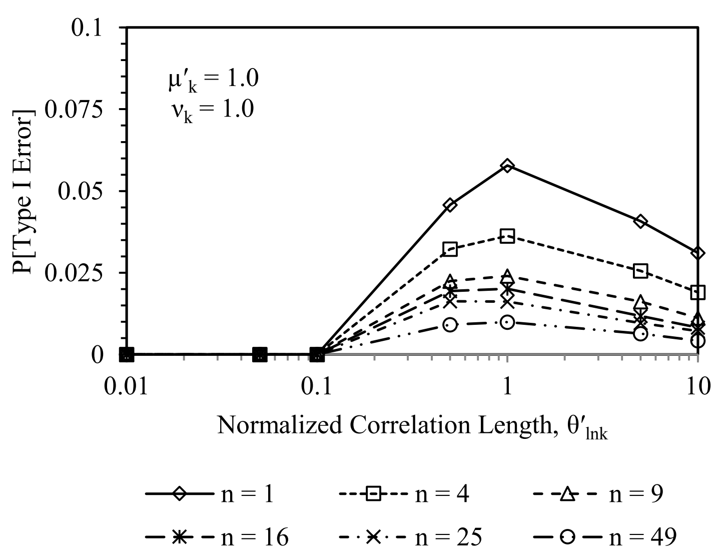

Figure 2.3: Influence of correlation length on the probability of a Type I error for mean and coefficient of variation of 1.0

0.0092 when $n = 49$. When $\theta'_{\ln k} = 0.01$ and 0.1, the probability of a Type I error of close to 0 is obtained for all number of samples.

Figure 2.3 also indicates that the probability of a Type I error will decrease towards 0 as correlation length increases beyond the worst case for one or more samples, as expected.

Figure 2.4 illustrates the influence of the normalized correlation length on the probability of a Type II error for various numbers of samples ($n = 1$, 4, 9, 16, 25, and 49) for $\mu'_k = 1.0$ and $\nu_k = 1.0$. Similar to Figure 2.3, a "worst case" correlation length occurs at an intermediate correlation length, in this case at around 10% to 50% of the field dimension. For example, when $\mu'_k = 1.0$, $\nu_k = 1.0$, and $n = 4$, the probability of a Type II error starts at 0.0296, increases to 0.1818, and then drops back down to 0.0232 for $\theta'_{\ln k}$ = 0.01, 0.1, and 10.0, respectively.

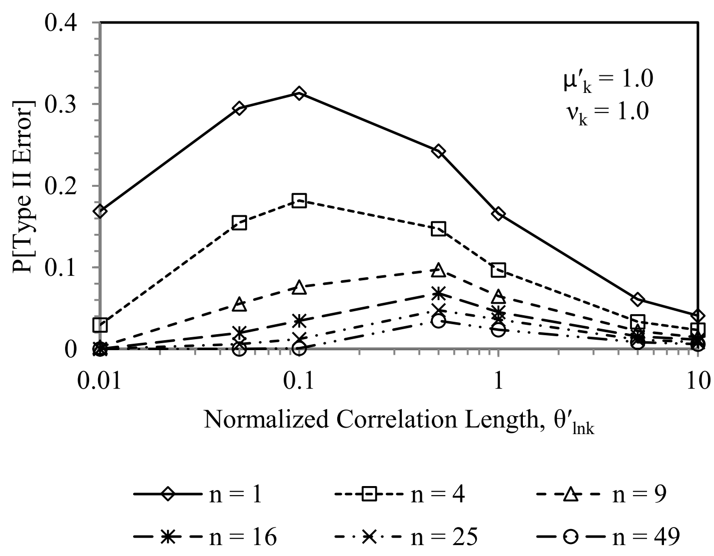

Figure 2.4: Influence of correlation length on the probability of a Type II error for mean and coefficient of variation of 1.0

Figure 2.4 also shows that an increase in the number of samples decreases the probability of a Type II error. For example, when $\mu'_k = 1.0$, $v_k = 1.0$, and $\theta'_{\ln k} = 0.5$, the probability of a Type II error decreases from 0.1471 when $n = 4$ to 0.0346 when $n = 49$. The converging nature of the plots on both sides of the worst case indicates that at very low and high correlation lengths, the probability of a Type II error tends to 0, which is as expected.

Similar trends to those shown in Figures 2.3 and 2.4 are seen for all other parameter set combinations considered and the results are included in Appendix B. The "worst case" correlation lengths occur at about 0.1 to 5 times the field dimension for Type I errors and at about 0.01 to 10 times the field dimension for Type II errors. In general, the "worst case" correlation length is somewhere between 0.01 and 1.0 times the field dimension. For most of the following comparisons, an intermediate worst case correlation length of $\theta'_{\ln k} = 0.5$ has been selected.

2.4.2 Influence of Mean on Error Probabilities

When the mean hydraulic conductivity of the random field is much less than the regulatory hydraulic conductivity, both the effective hydraulic conductivity and the sample geometric average will almost always be less than the regulatory value so that the probabilities of Type I and II errors will be small. Similarly, when the mean hydraulic conductivity is much higher than the regulatory value, both the effective hydraulic conductivity and the sample geometric average will almost always be higher than the regulatory value so that, again, the probabilities of Type I and II errors will be small. The highest decision error probabilities will occur when the mean hydraulic conductivity is close to the regulatory value. Figures 2.5 and 2.6 illustrate the influence of the mean on the probabilities of Type I and Type II errors, respectively, for $\nu_k = 1.0$, $\theta'_{\ln k} = 0.5$, and $n = 4$, 16, and 49. For given number of samples, the highest probability of a Type I error

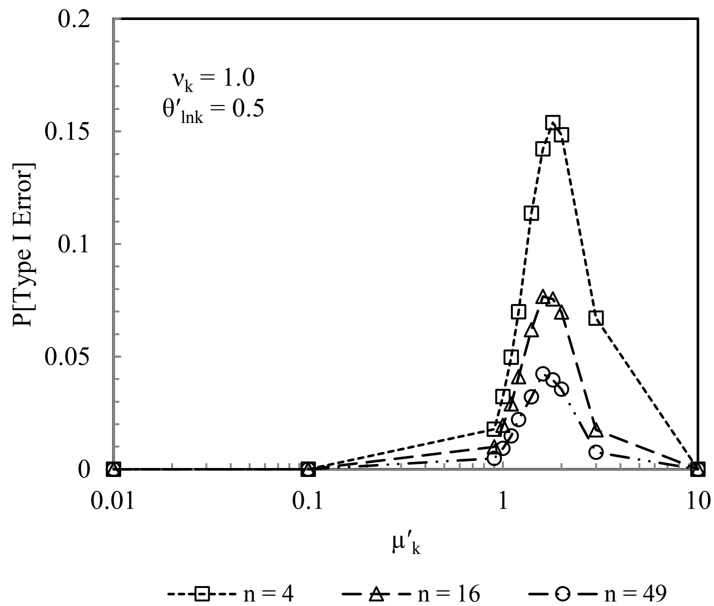

Figure 2.5: Influence of mean on the probability of a Type I error

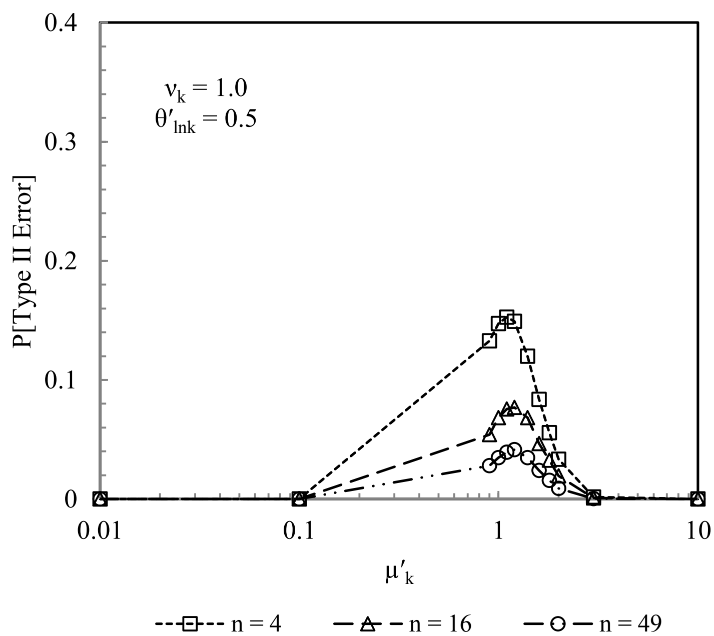

Figure 2.6: Influence of mean on the probability of a Type II error

in Figure 2.5 occurs when the mean hydraulic conductivities are about 1.7 times the regulatory value. For example, in the case where $v_k = 1.0$, $\theta'_{\ln k} = 0.5$, and $n = 4$, the probability of a Type I error reaches a maximum of about 0.15 when $\mu'_k = 1.7$.

Similarly the highest probabilities of a Type II error (Figure 2.6) are observed when $\mu'_k = 1.1$. For example, for $v_k = 1.0$, $\theta'_{\ln k} = 0.5$, and $n = 4$, the probability of a Type II error reaches a maximum of about 0.15 when $\mu'_k = 1.1$.

2.4.3 Influence of Coefficient of Variation on Error Probabilities

Figures 2.7 and 2.8 illustrate the influence of the coefficient of variation on the probabilities of Type I and II errors, respectively, for $\mu'_k = 1.0$, $\theta'_{\ln k} = 0.5$, and varying *n*. Points on the plots are obtained using 25000 realizations. The figures show that both Type I and Type II error probabilities (mostly) decrease with increasing coefficients of variation. For example, for $\mu'_k = 1.0$, $\theta'_{\ln k} = 0.5$, and $n = 4$, probabilities of Type I and Type II errors decrease from 0.0322 to 0.0106 and from 0.1472 to 0.1181, respectively, when the coefficient of variation increases from 1 to 2. However, the probability of Type II errors does tend to show a maximum at around a coefficient of variation of 1.0, so that this value of v_k seems to be a "worst case" for the probability of Type II errors. The probability of a Type I error drops back down to 0 as $v_k \to 0$ (not shown), and it is found that the probability of a Type I error begins to decrease below $v_k = 0.1$, so that the coefficient of variation of 0.1 is the "worst case" for the probability of Type I errors.

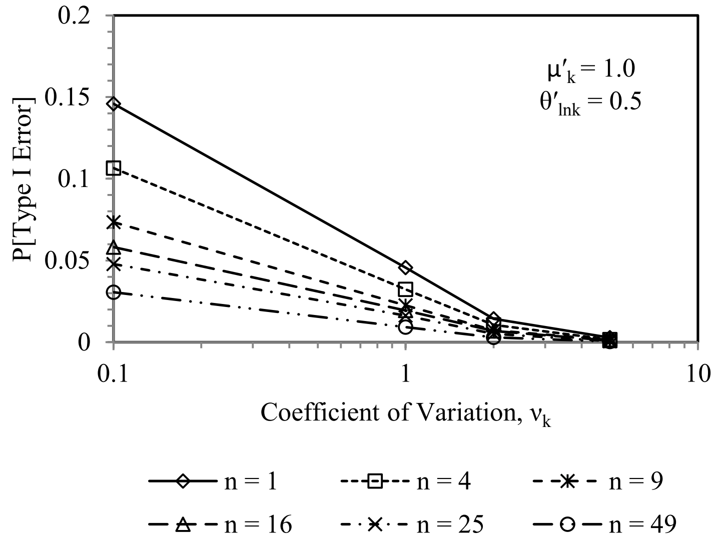

Figure 2.7: Influence of coefficient of variation on the probability of a Type I error

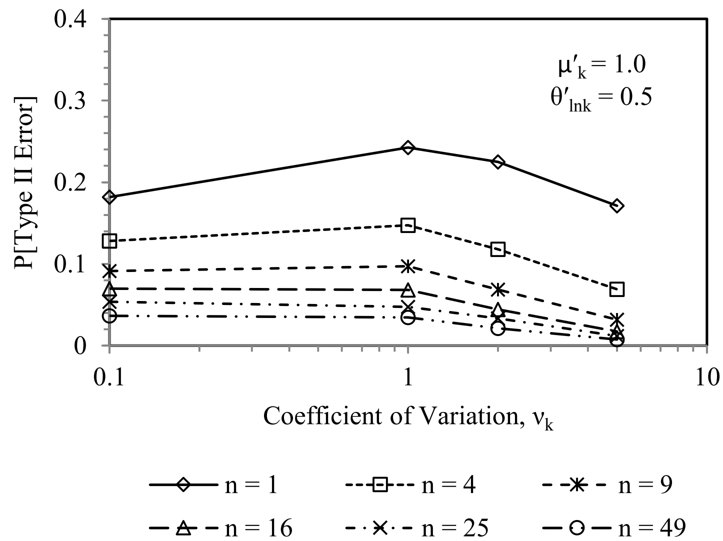

Figure 2.8: Influence of coefficient of variation on the probability of a Type II error

2.4.4 INFLUENCE OF NUMBER OF SAMPLES ON ERROR PROBABILITIES

It is expected that in a QC program of a cement-based S/S construction cell, increasing n decreases the chances of making an error in the decision about the approval of the construction cell. When the entire cell is sampled at every point, the probability of making a decision error will be zero. Figures 2.9 and 2.10 show the influence of the number of samples on the probabilities of making a Type I and a Type II error, respectively for different normalized means (i.e., μ'_k = 0.01, 0.1, 0.9, 1.0, 1.1, and 10.0), $\nu_k = 1.0$ and $\theta'_{\ln k} = 0.5$. These figures indicate that as the number of samples increase, the probabilities of Type I and Type II errors decrease as expected. Also as expected, the probabilities of both types of errors are very close to zero when the normalized mean is far from 1.0.

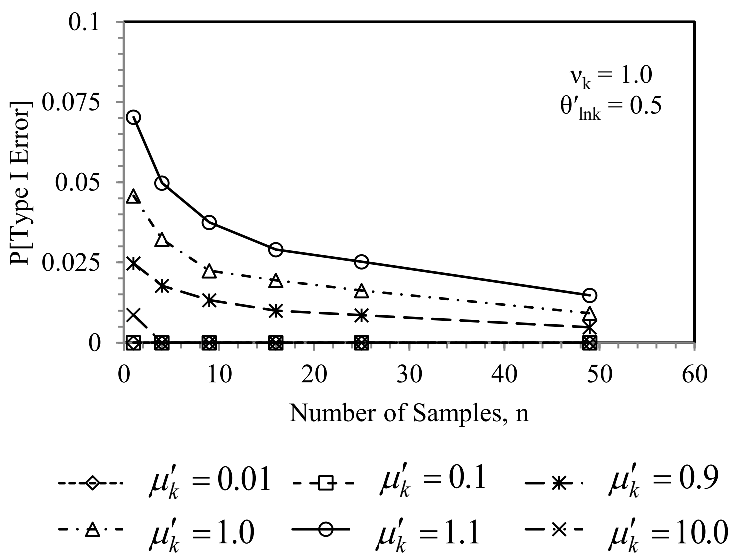

Figure 2.9: Influence of number of samples on the probability of a Type I error

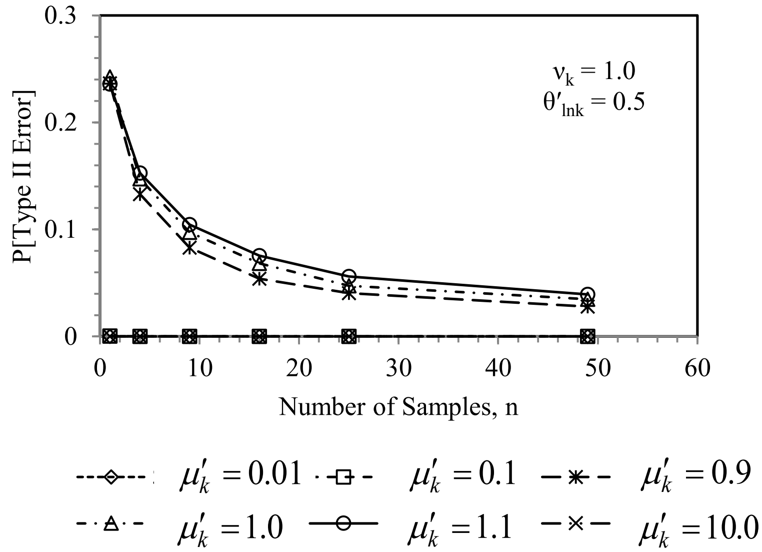

Figure 2.10: Influence of number of samples on the probability of a Type II error

2.5 Illustrating the Scalability of the Simulation Results

Simulations are performed for an example to illustrate the scalability of the simulation results presented in the previous section (which considered a 1×1 cell). The example construction cell size is 10 m×10 m which is modeled using 160×160 elements each of size 0.0625 m×0.0625 m. The normalized mean and coefficient of variation of point-scale hydraulic conductivity specified in the simulations are both 1.0, normalized correlation lengths considered are 0.01, 0.05, 0.1, 1.0, and 10.0, and the number of samples used are 1, 4, 9, 16, and 25. Figures 2.11 and 2.12 present the comparison between the simulation results for the probabilities of a Type I and a Type II error for the example problem and the case considered in this chapter. Good agreements are obtained for both error probabilities between the two cases for all normalized correlation lengths, which justify the scalability of the simulation results presented in this chapter.

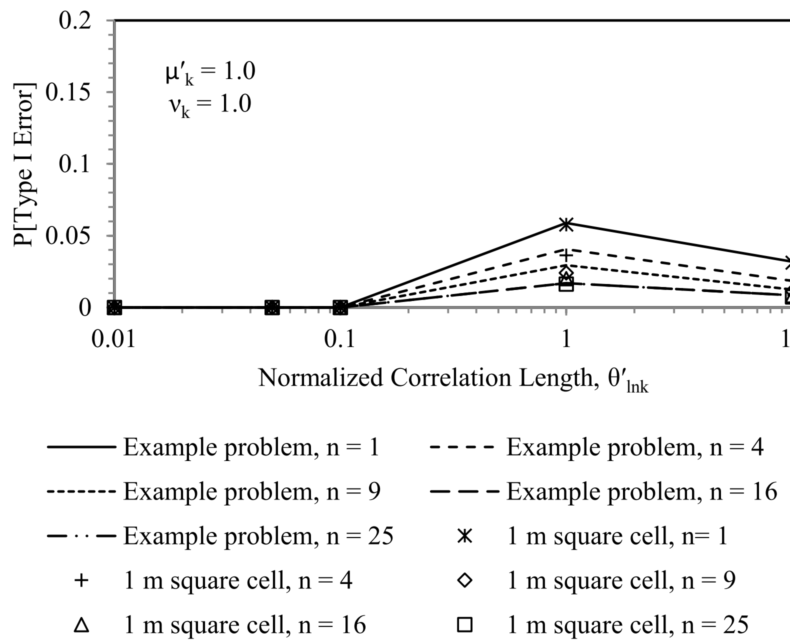

Figure 2.11: Comparison of the simulation results for the probability of a Type I error between a (10 m×10 m) and a 1×1 cell

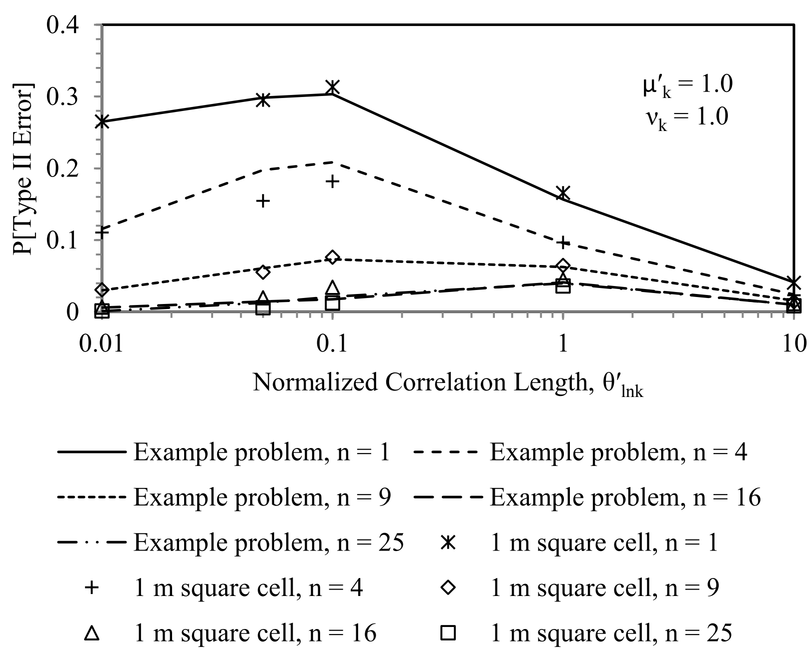

Figure 2.12: Comparison of the simulation results for the probability of a Type II error between a (10 m×10 m) and a 1×1 cell

2.6 SUMMARY AND CONCLUSIONS

In this chapter, Monte Carlo simulations are performed using a modified version of the two-dimensional random finite element method (RFEM) program, mrflow2d, to examine the influence of the correlation length, hydraulic conductivity mean and coefficient of variation on sampling requirements for a QC program of cement-based S/S construction cells. The modification made to the program enables the sampling of the random field at prescribed locations. Also in the modified program, instead of using the finite element method, geometric average of the field is used to obtain the flow through the field.

Based on the results obtained in this study, the following conclusions can be drawn:

- For a specific number of samples in the QC program, the greatest probability of making an error in the hypothesis test occurs at a "worst case" correlation length, indicating that more samples are required at this correlation length. The "worst case" correlation lengths are found to be 0.1 to 5 times the effective construction cell dimension (square root of the construction cell area) for the probability of a Type I error and 0.01 to 10 times the effective construction cell dimension for the probability of a Type II error. In general, the "worst case" correlation length is somewhere between 0.01 and 1.0 times the field dimension. The worst case correlation length leads to conservative sampling requirements to achieve target hypothesis error probabilities.
- For a specific number of samples, the greatest error probabilities occur when μ_k' is approximately 1.7 for Type I errors and 1.1 for Type II errors. This suggests

that more samples are required when the normalized mean hydraulic conductivity is in the range 1.1 to 1.7 in order to ensure that cells are properly identified as being unacceptable or acceptable (note that although the population mean μ_k may be above k_{crit}, individual cells may very well have $k_{eff} < k_{crit}$). For a constant number of samples, the probabilities of Type I and Type II errors rapidly approach zero when the mean hydraulic conductivity deviates significantly from the regulatory value (e.g. μ_k' = 0.01, 0.1, and 10.0). This, of course, implies that targeting the mean hydraulic conductivity well below the regulatory value is desirable, although possibly more expensive. Note that targeting a lower mean hydraulic conductivity may have no benefits with respect to the required number of QC samples, since the worst case must always be assumed prior to sampling.

- Increasing the number of samples is effective in decreasing both Type I and Type II error probabilities, which, of course, agrees with statistical theory.
- For a specific number of samples, an increase in the hydraulic conductivity coefficient of variation, v_k, generally results in a decrease in probabilities of Type I and Type II errors, at least when $\mu_k' = 1.0$ and $v_k > 1$. This reduction in error probability is largely because the resistance to flow increases as v_k increases, due to downstream blockages, so that the value of k_{eff} decreases with increasing v_k. The general implication is that when μ_k' is approximately 1.0, more samples will be required to achieve acceptably small error probabilities when v_k is 1 or less.

- The good agreement obtained between the simulation results for both Type I and Type II error probabilities for a (10 m×10 m) construction cell and a 1×1 construction cell indicates the scalability of the simulation results presented in this chapter.

CHAPTER 3

AN ANALYTICAL APPROACH TO ASSESS QUALITY CONTROL SAMPLE SIZES OF CEMENT-BASED SOLIDIFICATION/STABILIZATION

3.1 GENERAL

In Chapter 2, probabilistic simulations were performed to examine the influence of correlation length, hydraulic conductivity mean and coefficient of variation on the probabilities of a Type I and a Type II error for a specific number of samples taken from cement-based S/S construction cells during a QC program. Plots provided in Chapter 2 can be used to assess the sampling requirements for the QC program of cement-based S/S construction cells to achieve target error probabilities (i.e., Type I and Type II) about the decision regarding the acceptance or rejection of the cells. This chapter aims to develop an analytical approach to assess the required sample size for the QC programs of cement-based S/S construction cells. Analytical solutions will be developed to compute the probabilities of Type I and Type II errors as a function of the number of samples taken and the statistics of the hydraulic conductivity field. The soutions will be verified by the probabilistic simulations of Chapter 2. An example will be presented to illustrate how the proposed method can be used in practice to estimate the required sample size for the QC program of cement-based S/S construction cell.

The analytical approximations to the probabilities of Type I and Type II errors will be presented in the next section.

3.2 ANALYTICAL SOLUTIONS FOR THE PROBABILITIES OF TYPE I AND TYPE II ERRORS

As mentioned previously, random fields are commonly used to model spatially variable engineering properties (Fenton and Griffiths, 2008) and will be used here to model the soil hydraulic conductivity field. The resulting random field can be conditioned on the samples taken from the field. The resulting conditional distributions of k_{eff} and k_G can in turn be determined analytically and used to estimate the probabilities of making Type I or II errors in the approval decision process, leading to the determination of the number of samples required to achieve target error probabilities.

In this study, the random hydraulic conductivity field is assumed to be two-dimensional. This two-dimensional assumption is reasonable, as discussed in Chapter 2, if the conditioned layer is thin relative to its planar area. In the site modelling, the field is broken up into m elements and each element hydraulic conductivity is assumed to be the geometric average of point-scale hydraulic conductivity over that element and is assumed to be lognormally distributed (as assumed in the simulations). The lognormal assumption of hydraulic conductivity is reasonable for the hydraulic conductivity of S/S sites, as will be shown in Chapter 4 through a statistical analysis of a real site.

The probabilities of a Type I and a Type II error will be mathematically formulated in the next sub-section.

3.2.1 MATHEMATICAL FORMULATIONS OF THE PROBABILITIES OF A TYPE I AND A TYPE II ERROR

For the hypothesis test considered in this thesis, the probability of a Type I error, p_1, can be defined as,

$$p_1 = \mathrm{P}\left[k_G < k_{crit} \cap k_{eff} > k_{crit}\right] \tag{3.1}$$

Similarly, the probability of a Type II error, p_2, can be defined as,

$$p_2 = \mathrm{P}\left[k_G > k_{crit} \cap k_{eff} < k_{crit}\right] \tag{3.2}$$

where k_G and k_{eff} are as defined by Eq's (2.2) and (2.3), respectively.

As assumed in the simulations, both the predicted, k_G and the actual effective hydraulic conductivity, k_{eff}, of the random field are assumed here to be the geometric averages of the sample and element hydraulic conductivities, respectively.

In Eq's (3.1) and (3.2), both k_G and k_{eff} are assumed to be lognormally distributed, which means that the logarithms of the sample geometric average, $\ln k_G$, and the actual effective hydraulic conductivity, $\ln k_{eff}$, are normally distributed with means of $\mu_{\ln k_G}$ and $\mu_{\ln k_{eff}}$, respectively, and standard deviations of $\sigma_{\ln k_G}$ and $\sigma_{\ln k_{eff}}$, respectively, and covariance $Cov\left[\ln k_G, \ln k_{eff}\right]$. The lognormal assumption is reasonable for both predicted and the actual effective hydraulic conductivities, since they are approximated by the geometric averages of the lognormally distributed sample and element hydraulic conductivities, respectively (Fenton and Griffiths, 2008).

Assuming $\ln k_G$ and $\ln k_{eff}$ follow a bivariate normal distribution, $f_{\ln k_{eff} \ln k_G}(u,v)$, the probabilities of Type I and Type II errors can be expressed as,

$$p_1 = \int_{-\infty}^{\ln k_{crit}} \int_{\ln k_{crit}}^{+\infty} f_{\ln k_{eff} \ln k_G}(u,v)\,du\,dv \tag{3.3}$$

$$p_2 = \int_{\ln k_{crit}}^{+\infty} \int_{-\infty}^{\ln k_{crit}} f_{\ln k_{eff} \ln k_G}(u,v)\,du\,dv \tag{3.4}$$

where,

$$f_{\ln k_{eff} \ln k_G}(u,v) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left(u^2 - 2\rho uv + v^2\right)\right\} \tag{3.5}$$

$u = \left(r - \mu_{\ln k_{eff}}\right)/\sigma_{\ln k_{eff}}$, $v = \left(s - \mu_{\ln k_G}\right)/\sigma_{\ln k_G}$, and ρ is the correlation coefficient between $\ln k_{eff}$ and $\ln k_G$, and all other terms are as defined previously. The means and standard deviations of $\ln k_{eff}$ and $\ln k_G$ and ρ are defined in Appendix C.

There is no closed form solution for the integral of the bivariate normal distribution. An approximate method proposed by Owen (1959) to obtain the bivariate normal probability is used in this study to obtain the probabilitites of a Type I error and a Type II error defined by Eq's (3.3) and (3.4), respectively.

3.2.2 SOLUTION METHOD

Owen (1959) proposed an approximate method for obtaining the bivariate normal probability. The solutions obtained by Owen (1959) were for the bivariate normal probability,

$$B(h,w;\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{h}\int_{-\infty}^{w} \exp\left[-\frac{1}{2(1-\rho^2)}\left(u^2 - 2\rho uv + v^2\right)\right] du\,dv \tag{3.6}$$

where u and v are defined above and are standard normal random variables and ρ is the correlation coefficient between the normally distributed random variables corresponding to u and v (i.e., $\ln k_{eff}$ and $\ln k_G$). The approximation to $B(h,w;\rho)$ is as follows

$$B(h,w;\rho) = \frac{1}{2}\Phi(h) + \frac{1}{2}\Phi(w) - T(h,a_h) - T(w,a_w) \tag{3.7}$$

if $hw > 0$ or if $hw = 0, h$ or $w \geq 0$, and

$$B(h, w; \rho) = \frac{1}{2}\Phi(h) + \frac{1}{2}\Phi(w) - T(h, a_h) - T(w, a_w) - \frac{1}{2} \tag{3.8}$$

if $hw < 0$ or if $hw = 0, h$ or $w < 0$,

where

$$a_h = \frac{w}{h\sqrt{1-\rho^2}} - \frac{\rho}{\sqrt{1-\rho^2}} \tag{3.9a}$$

$$a_w = \frac{h}{w\sqrt{1-\rho^2}} - \frac{\rho}{\sqrt{1-\rho^2}} \tag{3.9b}$$

$$T(h, a_h) = \frac{1}{2\pi}\int_0^{a_h} \frac{\exp\left[-\frac{1}{2}h^2\left(1+u^2\right)\right]}{1+u^2} du \tag{3.10a}$$

$$T(w, a_w) = \frac{1}{2\pi}\int_0^{a_w} \frac{\exp\left[-\frac{1}{2}w^2\left(1+v^2\right)\right]}{1+v^2} dv \tag{3.10b}$$

and Φ is the standard normal cumulative distribution function.

Eq's (3.10a) and (3.10b) are valid for $a_h < 1$ and $a_w < 1$, respectively.

When $a_h > 1$, $T(h, a_h)$ is defined by Eq. (3.10c).

$$T(h, a_h) = \frac{1}{2}\Phi(h) + \frac{1}{2}\Phi(ha_h) - \Phi(h)\Phi(ha_h) - T\left(ha_h, \frac{1}{a_h}\right) \tag{3.10c}$$

Similarly, when $a_w > 1$, $T(w, a_w)$ is defined by Eq. (3.10d).

$$T(w, a_w) = \frac{1}{2}\Phi(w) + \frac{1}{2}\Phi(wa_w) - \Phi(w)\Phi(wa_w) - T\left(wa_w, \frac{1}{a_w}\right) \tag{3.10d}$$

$T\left(ha_h, \frac{1}{a_h}\right)$ in Eq. (3.10c) can be defined by Eq. (3.10a) replacing h by ha_h and a_h by $\frac{1}{a_h}$ in Eq. (3.10a).

Similarly, $T\left(wa_w, \frac{1}{a_w}\right)$ in Eq. (3.10d) can be defined by Eq. (3.10b) replacing w by wa_w and a_w by $\frac{1}{a_w}$ in Eq. (3.10b).

Using Owen (1959)'s method for obtaining the bivariate normal probability, as presented above, the probability of a Type I error, is

$$p_1 = \frac{1}{2}\Phi(h) - \frac{1}{2}\Phi(w) + T(h, a_h) + T(w, a_w) \tag{3.11a}$$

if $hw > 0$ or if $hw = 0, h$ or $w \geq 0$, and

$$p_1 = \frac{1}{2}\Phi(h) - \frac{1}{2}\Phi(w) + T(h, a_h) + T(w, a_w) + \frac{1}{2} \tag{3.11b}$$

if $hw < 0$ or if $hw = 0, h$ or $w < 0$

The probability of a Type II error is

$$p_2 = \frac{1}{2}\Phi(w) - \frac{1}{2}\Phi(h) + T(h, a_h) + T(w, a_w) \tag{3.12a}$$

if $hw > 0$ or if $hw = 0, h$ or $w \geq 0$, and

$$p_2 = \frac{1}{2}\Phi(w) - \frac{1}{2}\Phi(h) + T(h, a_h) + T(w, a_w) + \frac{1}{2} \tag{3.12b}$$

if $hw < 0$ or if $hw = 0, h$ or $w < 0$

where

$$h = \frac{\ln k_{crit} - \mu_{\ln k_G}}{\sigma_{\ln k_G}} \tag{3.13a}$$

$$w = \frac{\ln k_{crit} - \mu_{\ln k_{eff}}}{\sigma_{\ln k_{eff}}} \quad (3.13b)$$

a_h, a_w, $T(h, a_h)$, and $T(w, a_w)$ have the same meanings as Eq's (3.9a), (3.9b), (3.10a), and (3.10b), respectively, and $u = \frac{r - \mu_{\ln k_{eff}}}{\sigma_{\ln k_{eff}}}$ and $v = \frac{s - \mu_{\ln k_G}}{\sigma_{\ln k_G}}$.

Derivations of Eq's (3.11) and (3.12) are presented in Appendix D.

3.3 VERIFICATION

The type of probabilistic analyses presented in the previous section could be performed using simulation programs such as the modified version of a two-dimensional random finite element method (RFEM) program, mrflow2d, as presented in Chapter 2. It requires significant time and expertise to complete a simulation for a set of statistical parameters. The advantage of the analytical solutions presented in this chapter is that they enable one to quickly compute the probabilities of Type I and Type II errors for a specific number of samples and the statistics of the random field. However, the developed analytical solutions given by Eq's (3.11) and (3.12) need to be verified, which is done in this chapter by comparing to probabilistic simulations.

Simulations are performed using a modified version of the two-dimensional random finite element method (RFEM) program, mrflow2d, following the method described in Section 2.2.

For a 20 m×20 m random field, discretized into 256×256 elements, parametric variations considered in the simulations were:

Normalized point-mean hydraulic conductivity, μ'_k =0.5, 1.2, and 1.5, Coefficient of variation, $v_k = 0.5$, 1.0, and 2.0,

Correlation length, $\theta_{\ln k} = 1$ m, 3 m, and 10 m,

Number of samples, $n = 1$, 4, 25, and 100 (i.e., in increments of l^2, where l is as defined in Chapter 2).

For all number of samples, the field is sampled at equispaced locations. Specifically, for the jth sample in the x direction, where, $j = 1,2,.....,l$, the sampled element number in the x direction, i_{xs}, is given by:

$$i_{xs} = \left\lfloor \frac{m_x}{l+1} \right\rfloor j \tag{3.14}$$

where m_x is the number of elements in the x direction. The sampled element number in the y direction is computed similarly.

A separable Markovian correlation function, having an associated variance reduction function is assumed in the simulations here, as was assumed in Chapter 2.

For all parameter sets considered, the probabilities of Type I and Type II errors estimated via simulation are compared to those computed analytically using Eq's (3.11) and (3.12), respectively, as illustrated in Figures 3.1 and 3.2, respectively. Excellent agreements are obtained between the theory and the simulation for both probabilities of a Type I and a Type II error for all parameter sets considered, indicating that the proposed analytical solutions can be used to compute the probabilities of Type I and Type II errors with reasonable confidence. The small discrepancies seen in Figures 3.1 and 3.2 are likely due to natural sampling variation.

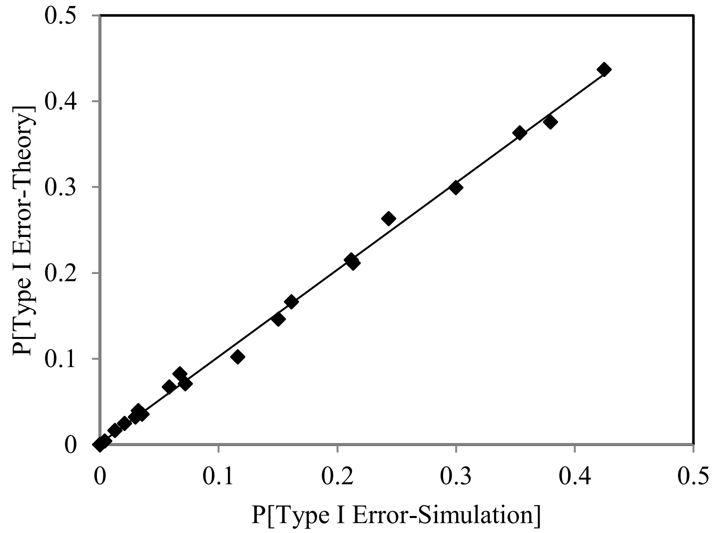

Figure 3.1: Comparison between the theory and simulation for the probability of a Type I error

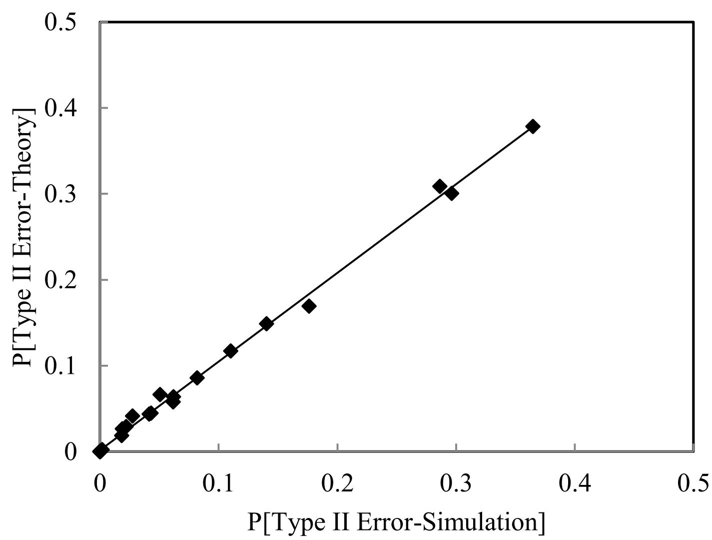

Figure 3.2: Comparison between the theory and simulation for the probability of a Type II error

3.4 PROCEDURE TO SELECT SAMPLE SIZE

The analytical solutions presented in Section 3.2 to compute the probabilities of Type I and Type II errors (i.e., Eq's (3.11) and (3.12), respectively) can be used to estimate the sample size required for the QC program of cement-based S/S construction cell to achieve target Type I and Type II error probabilities.

The following steps can be taken to select the sample size, given the desired probabilities of Type I and Type II errors and the statistics of the random hydraulic conductivity field.

1. For the specified μ_k and v_k, compute $\sigma^2_{\ln k}$ and $\mu_{\ln k}$ using Eq's (2.1a) and (2.1b), respectively.

2. Computing $\gamma_{\ln k}(X,Y) = \gamma(X)\gamma(Y)$, where $\gamma(X)$ and $\gamma(Y)$ can be computed using Eq. (2.11), compute $\mu_{k_{eff}}$ and $\sigma_{k_{eff}}$ using Eq's (C.1) and (C.2), respectively.

3. Compute $\sigma_{\ln k_{eff}}$ and $\mu_{\ln k_{eff}}$ using Eq's (C.4) and (C.3), respectively.

4. Choose a specific sample size and compute $\mu_{\ln k_G}$ and $\sigma_{\ln k_G}$ using Eq's (C.5) and (C.6), respectively. Computation of $\sigma_{\ln k_G}$ requires computations of the variance reduction function over the element, $\gamma_{\ln k}(\Delta x, \Delta y) = \gamma(\Delta x)\gamma(\Delta y)$, where Δx and Δy are the dimensions of the element in the x and y directions, respectively, and $\frac{1}{n^2}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\rho_{\ln k}(\mathbf{x}_i - \mathbf{x}_j)$, where $\rho_{\ln k}(\mathbf{x}_i - \mathbf{x}_j) = \exp\{-2(x_{ix} - x_{jx})/\theta_{\ln k}\}\exp\{-2(x_{iy} - x_{jy})/\theta_{\ln k}\}$. Compute $\gamma(\Delta x)$

using Eq. (2.11), replacing X in Eq. (2.11) by Δx. Compute $\gamma(\Delta y)$ in a similar manner.

5. Compute ρ using Eq. (C.7).

6. Compute h, w, a_h, and a_w using Eq's (3.13a), (3.13b), (3.9a), and (3.9b), respectively. When $a_h < 1$, compute $T(h, a_h)$ using Eq. (3.10a), and when $a_h > 1$, compute $T(h, a_h)$ using Eq. (3.10c). Similarly, compute $T(w, a_w)$ using Eq's (3.10b) and (3.10d), when $a_w < 1$, and $a_w > 1$, respectively.

7. Compute the probabilities of a Type I (p_1) and a Type II (p_2) error using Eq's (3.11) and (3.12), respectively.

8. If the computed probabilities of both Type I and Type II errors reach the targeted values, then the chosen sample size can be considered as the required one, otherwise, choose another sample size and repeat steps 1-7 until target values are reached for both probabilities of a Type I and a Type II error.

3.5 Using the Proposed Method to Obtain QC Sample Size: An Example

An example is provided in this section to clarify the method presented in the previous section to assess QC sample size of cement-based S/S construction cell for achieving target Type I and Type II error probabilities.

Consider a cement-based S/S construction cell that has a plan area of 10 m×10 m. The mean hydraulic conductivity of the proposed cell is to be less than 1×10^{-8} m/s with a coefficient of variation of 1.0. An upper bound of $\mu_k = 1\times10^{-8}$ m/s will be assumed here.

The correlation length is assumed to be 3 m in both planar directions. The regulatory requirement for the hydraulic conductivity of the cell is 1×10^{-8} m/s. It is necessary to determine the number of samples required to achieve a 5% probability for both Type I and Type II errors.

For statistical purposes, dividing the 10 m×10 m cell into 160×160 elements, each of size 0.0625 m×0.0625 m, and assuming only one sample from the centre of the 10 m×10 m cell, the following computations are performed.

Given the mean and coefficient of variation of the point-scale hydraulic conductivity, the variance and mean of log-k are as follows:

$$\sigma^2_{\ln k} = \ln\left(1+v_k^2\right)$$

$$= \ln(1+1)$$

$$= 0.6931$$

$$\mu_{\ln k} = \ln \mu_k - \frac{1}{2}\sigma^2_{\ln k}$$

$$= \ln\left(1\times10^{-8}\right) - \frac{1}{2}(0.6931)$$

$$= -18.7672$$

Using $\gamma_{\ln k}(X,Y) = \gamma(X)\gamma(Y)$, where $X = Y = 10\,\text{m}$, $\gamma(X) = \dfrac{\theta^2_{\ln k}}{2X^2}\left[\dfrac{2|X|}{\theta_{\ln k}} + \exp\left\{\dfrac{-2|X|}{\theta_{\ln k}}\right\} - 1\right]$, and similarly for $\gamma(Y)$, the variance reduction function over the cell is computed as 0.0650. Similarly, the variance reduction function over the element, $\gamma_{\ln k}(\Delta x, \Delta y)$, where $\Delta x = \Delta y = 0.0625$ m, is computed as 0.9727.

The mean and standard deviation of the actual effective hydraulic conductivity, k_{eff} of the field can be computed to be,

$$\mu_{k_{eff}} = \exp\left\{\mu_{\ln k} + \frac{1}{2}\gamma_{\ln k}(X,Y)\sigma^2_{\ln k}\right\}$$

$$= \exp\left\{-18.7672 + \frac{1}{2}(0.0650)(0.6931)\right\}$$

$$= 7.2323\times 10^{-9} \text{ m/s}$$

$$\sigma_{k_{eff}} = \sqrt{\mu^2_{k_{eff}}\left[\exp\left\{\sigma^2_{\ln k}\gamma_{\ln k}(X,Y)\right\}-1\right]}$$

$$= \sqrt{\left\{\left(7.2323\times 10^{-9}\right)^2\right\}\left[\exp\left\{(0.6931)(0.0650)\right\}-1\right]}$$

$$= 1.5532\times 10^{-9}$$

The standard deviation and mean of log-k_{eff} can be computed as,

$$\sigma_{\ln k_{eff}} = \sqrt{\ln\left(1+\left(\frac{\sigma_{k_{eff}}}{\mu_{k_{eff}}}\right)^2\right)}$$

$$= \sqrt{\ln\left(1+\left(\frac{1.5532\times 10^{-9}}{7.2323\times 10^{-9}}\right)^2\right)}$$

$$= 0.2123$$

$$\mu_{\ln k_{eff}} = \ln\left(\mu_{k_{eff}}\right) - \frac{1}{2}\sigma^2_{\ln k_{eff}}$$

$$= \ln\left(7.2323\times 10^{-9}\right) - \frac{1}{2}(0.2123)^2$$

$$= -18.7372$$

Using $\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\rho_{\ln k}\left(\mathbf{x}_i-\mathbf{x}_j\right)=\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\exp\left\{-2\left(x_{ix}-x_{jx}\right)/\theta_{\ln k}\right\}\exp\left\{-2\left(x_{iy}-x_{jy}\right)/\theta_{\ln k}\right\}$,

the mean and standard deviation of log-k_G can be computed as follows:

$$\mu_{\ln k_G}=\mu_{\ln k}=-18.7672$$

$$\sigma_{\ln k_G}\cong\sqrt{\frac{1}{n^2}\left[n\left(\sigma_{\ln k}^2\gamma_{\ln k}(\Delta x,\Delta y)\right)+\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\sigma_{\ln k}^2\rho_{\ln k}\left(\mathbf{x}_i-\mathbf{x}_j\right)\right]}$$

$$=\sqrt{\frac{1}{1^2}\left[(1)(0.6931)(0.9660)+(0.6931)(1)\right]}$$

$$=0.8211$$

Using

$$\rho=\frac{Cov\left[\ln k_G,\ln k_{eff}\right]}{\sigma_{\ln k_G}\sigma_{\ln k_{eff}}}\cong\frac{1}{\sigma_{\ln k_G}\sigma_{\ln k_{eff}}nm_xm_y}\left[n\sigma_{\ln k}^2\gamma_{\ln k}(\Delta x,\Delta y)+\sum_{k=1}^{n}\sum_{\substack{i=1\\ i\neq k}}^{m_x}\sum_{\substack{j=1\\ j\neq k}}^{m_y}\sigma_{\ln k}^2\rho_{\ln k}\left(\mathbf{x}_k-\mathbf{x}_{ij}\right)\right],$$

the correlation coefficient between log-k_{eff} and log-k_G can be computed as 0.3328.

h, w, a_h, and a_w can be computed as follows:

$$h=\frac{\ln\left(1\times10^{-8}\right)-\mu_{\ln k_G}}{\sigma_{\ln k_G}}$$

$$=0.4220$$

$$w=\frac{\ln\left(1\times10^{-8}\right)-\mu_{\ln k_{eff}}}{\sigma_{\ln k_{eff}}}$$

$$=1.6320$$

$$a_h = \frac{w}{h\sqrt{1-\rho^2}} - \frac{\rho}{\sqrt{1-\rho^2}}$$

$$= 3.7476$$

$$a_w = \frac{h}{w\sqrt{1-\rho^2}} - \frac{\rho}{\sqrt{1-\rho^2}}$$

$$= -0.0786$$

Since $a_h > 1$, $T(h, a_h)$ can be computed using

$T(h, a_h) = \frac{1}{2}\Phi(h) + \frac{1}{2}\Phi(ha_h) - \Phi(h)\Phi(ha_h) - T\left(ha_h, \frac{1}{a_h}\right)$ as 0.1659, where

$T\left(ha_h, \frac{1}{a_h}\right) = \frac{1}{2\pi}\int_0^{1/a_h} \frac{\exp\left[-\frac{1}{2}(ha_h)^2(1+u^2)\right]}{1+u^2}du$, is computed using 16-point Gauss quadrature as 0.0115.

Using 16-point Gauss quadrature, $T(w, a_w) = \frac{1}{2\pi}\int_0^{a_w} \frac{\exp\left[-\frac{1}{2}w^2(1+v^2)\right]}{1+v^2}dv$ is computed as -0.0033.

The probabilities of a Type I (p_1) and a Type II (p_2) error are computed as,

$$p_1 = \frac{1}{2}\Phi(h) - \frac{1}{2}\Phi(w) + T(h, a_h) + T(w, a_w)$$

$$= \frac{1}{2}\Phi(0.4220) - \frac{1}{2}\Phi(1.6320) + 0.1659 - 0.0033$$

$$= 0.0201$$

$$p_2 = \frac{1}{2}\Phi(w) - \frac{1}{2}\Phi(h) + T(h, a_h) + T(w, a_w)$$

$$= \frac{1}{2}\Phi(1.6320) - \frac{1}{2}\Phi(0.4220) + 0.1659 - 0.0033$$

$$= 0.3052$$

Since the computed probability of a Type II error (30.52%) is greater than the target value (5%), the probabilities of Type I and Type II errors are further computed for the number of samples of 4, 9, 16, 25, and 49, locating the samples at equal spacing in both of the x and y directions of the cell. Table 3.1 presents computed Type I and Type II error probabilities for all number of samples (i.e., 1, 4, 9, 16, 25, and 49) and shows that both Type I and Type II error probabilities are less than 5% when the number of samples is 49. This suggests that 49 is the required number of samples for this example case. Table 3.1 also presents the Type I and Type II error probabilities assuming the correlation length to be the "worst case" i.e., 10 m for this example.

Table 3.1 : The probabilities of Type I (p_1) and Type II (p_2) errors for $\mu_k = 1\times10^{-8}$ m/s, $v_k = 1.0$, $\theta_k = 3$ and 10 m, and varying n

n	$\theta_k = 3$ m		$\theta_k = 10$ m	
	p_1	p_2	p_1	p_2
1	0.0201	0.3052	0.0601	0.1664
4	0.0162	0.1889	0.0402	0.0996
9	0.0136	0.1240	0.0303	0.0700
16	0.0119	0.0883	0.0243	0.0539
25	0.0095	0.0640	0.0210	0.0465
49	0.0080	0.0437	0.0194	0.0377

The results presented in Table 3.1 shows that for a specific number of samples, the Type I error probability increases significantly, whereas, the Type II error probability decreases when the correlation length increases from 3 to 10 m, suggesting that for this particular case, the correlation length of 3 m corresponds to the overall “worst case”. When the correlation length is 10 m, the number of samples of 25 can be suggested for this example case to achieve a 5% error probability for both Type I and II errors.

3.6 SUMMARY

In this chapter, an analytical approach is proposed to estimate the sample size for the QC program of a cement-based S/S construction cell to achieve target Type I and Type II error probabilities for the hypothesis test considered in this study. Analytical solutions are developed to compute the probabilities of Type I and Type II errors. The developed analytical solutions are functions of the number of samples taken and the statistics of the hydraulic conductivity field. For a range of parameter sets, the analytically computed probabilities of a Type I and a Type II error are compared to those estimated via probabilistic simulations and the comparison results in excellent agreement, allowing the probabilities of a Type I and a Type II error to be computed analytically with reasonable confidence. An example is presented to illustrate how the proposed method can be used in practice to assess QC sample size of cement-based S/S construction cell.

CHAPTER 4

SPATIAL VARIABILITY ASSOCIATED WITH HYDRAULIC CONDUCTIVITY OF CEMENT-BASED SOLIDIFICATION/STABILIZATION: A CASE STUDY

4.1 GENERAL

Statistical methods are used to analyze the probability of excessive hydraulic flow through systems and/or the risk associated with quality control (QC) of systems. In such analyses, the hydraulic conductivity is treated as a random field, which is described by a distribution and a correlation function (Vanmarcke, 1977). The correlation function is parameterized by the correlation length, $\theta_{\ln k}$, which is a measure of the degree of persistence between hydraulic conductivity values over space. In the reliability analyses, hydraulic conductivity is described probabilistically, since hydraulic conductivity is spatially variable both for natural soil (Byers and Stephens, 1983; Freeze and Cherry, 1979) and compacted soil liners (Rogowaski et al., 1985; Benson, 1993). The distribution of hydraulic conductivity at a point is often found to be lognormal for both natural and compacted soils (Freeze, 1975; Krapac et al., 1989; Johnson et al., 1990; Benson et al., 1993) and the correlation length is found to be 1 to 3 m for compacted soil liners (Benson, 1991). Since no study is found in the literature which attempts to find the distribution and correlation length describing the spatial variability of hydraulic conductivity of cement-based S/S, a set of hydraulic conductivity data from an existing cement-based S/S system is statistically analyzed for this purpose in this study. The spatial variability associated with hydraulic conductivity is then utilized to assess sampling requirements for the QC program of this case study.

4.2 SITE

A site having an area of 31 hectares and an average depth of 3.9 m, was contaminated by 700,000 tonnes of coal-based contaminants generated from steel production over the past 100 years. The contaminated site has been treated using cement-based S/S. During treatment, the site was divided into 2160 construction cells, of different areas, in order to keep the volume of each cell approximately constant (since the contaminated depth was different for different cells). During the QC program, multiple samples were collected from each cell and tested for hydraulic conductivity following ASTM D 5078. Each cell was approved individually if it was determined that the average of hydraulic conductivity measurements over each cell was at or below the regulatory value (i.e., 1×10^{-8} m/s). As with most cement-based S/S projects, the number of samples taken from each cell to make this decision about the acceptance or rejection of the cell did not consider the risk of an erroneous decision associated with finite QC sampling. Figure 4.1 shows locations of centres of cells, which were considered as sampling locations of average hydraulic conductivity values over cells for this study.

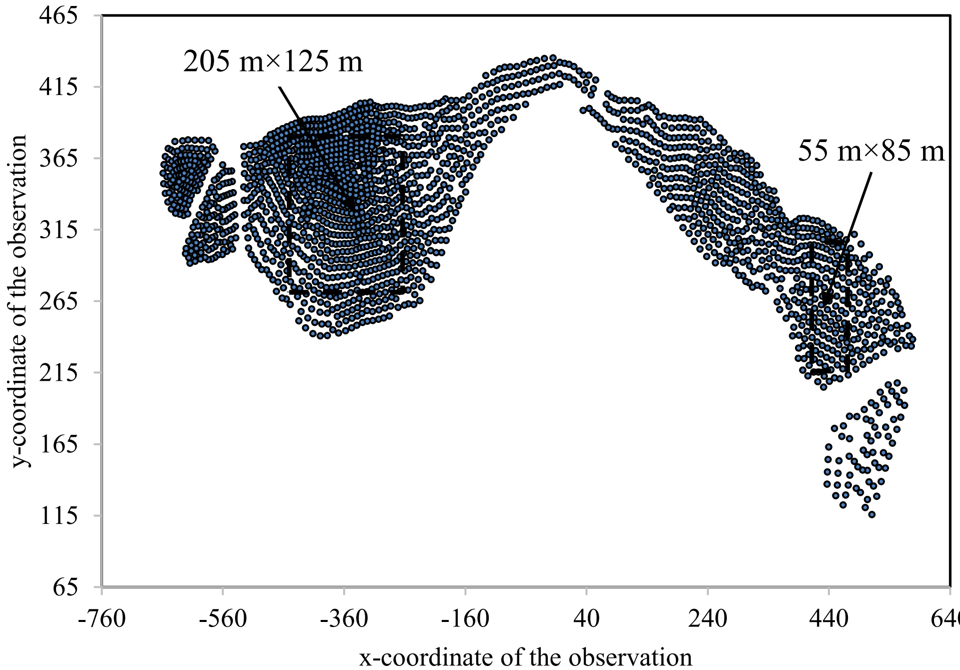

Figure 4.1: Sampling locations of hydraulic conductivity data set shown as small black squares

4.3 STATISTICAL ANALYSES

Out of the 2160 hydraulic conductivity values (each is the average of multiple hydraulic conductivity values over a cell), 2086 hydraulic conductivity values were reported. The statistical analyses performed in this study were based on the available set of 2086 hydraulic conductivity data in which the mean, variance and distribution were assumed stationary. The available hydraulic conductivity data was in the normalized form (i.e., hydraulic conductivity measured in m/s was normalized by the regulatory hydraulic conductivity, 1×10^{-8} m/s).

Statistical analyses performed in this study are presented in the following sub-sections:

4.3.1 DISTRIBUTION

For the data set considered in this study, the hydraulic conductivity (K) was hypothesized as being lognormally distributed, which has the probability density function (PDF) as given below:

$$f_K(k) = \frac{1}{\sqrt{2\pi}\sigma_{\ln K} k} \exp\left\{-\frac{1}{2}\left(\frac{\ln k - \mu_{\ln K}}{\sigma_{\ln K}}\right)^2\right\} \qquad k > 0 \tag{4.1a}$$

$$f_K(k) = 0 \qquad k \leq 0 \tag{4.1b}$$

where $\mu_{\ln K}$ and $\sigma_{\ln K}$ are the mean and standard deviation of $\ln k$.

The goodness-of-fit was tested using the Chi-square and the Anderson-Darling (A-D) goodness-of-fit tests. The A-D test statistic is calculated as

$$A_n^2 = \left(-\frac{1}{2}\left\{\sum_{i=1}^{n}(2i-1)\left[\ln Z_i + \ln(1 - Z_{n+1-i})\right]\right\}\right) - n \tag{4.2}$$

where $Z_i = \hat{F}(X_{(i)})$ is the fitted CDF of $X_{(i)}$ for $i = 1,2,........, n$, and n is the sample size. If the test statistic falls outside the critical region, the null hypothesis of lognormality is rejected.

A lognormal distribution with $\mu_{\ln K'} = -1.30$ and $\sigma_{\ln K'} = 1.02$, where K' is the normalized hydraulic conductivity, is a reasonable fit to the data set (Figure 4.2), although the p- value of the Chi-square goodness-of-fit test for this fit is 0.0. The A-D test statistic for this fit is 8.20.

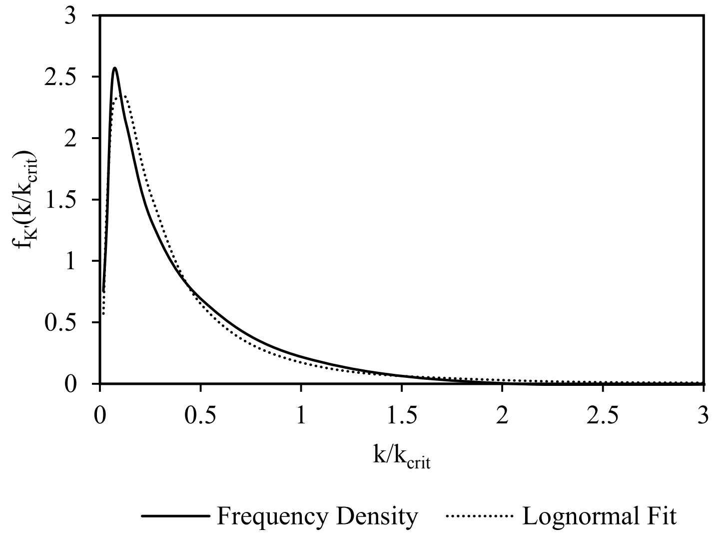

Figure 4.2: Frequency-density plot of hydraulic conductivity, with fitted lognormal distribution

4.3.2 CORRELATION LENGTH

Figure 4.1 indicates that hydraulic conductivity data set used in this study to perform statistical analyses was irregularly scattered. Since the classical estimators for the correlation structure require equispaced data, the scattered hydraulic conductivity data set was transformed into a 5 m spaced data set, in both x and y directions, using linear interpolation method (see the MATLAB class "TriscatteredInterp").

For a set of values, V, and corresponding locations, X, in two-dimensional space, the "TriscatteredInterp" first creates the Delaunay triangles at locations, X. The Delaunay triangles are such triangles that are formed by a set of points, P, in such a way that no point in P lies within the circumcicle of any Delaunay triangle. An interpolant is then created which fits a surface of the form $V = f(X)$, called the convex hull (the convex hull of a set Y of points in the Euclidean plane is the smallest convex set that contains Y). The interpolant always goes through the data points specified by the sample. The interpolant can be evaluated at any query location that falls within the hull. In this study, the interpolant created using the scattered hydraulic conductivity data set was evaluated at 5 m grid in two-dimensional space. The MATLAB code is presented in Appendix E. Two sub-sites having 5 m spaced hydraulic conductivity values, one on the left and the other on the right sides of the entire S/S site (areas enclosed by dashed lines in Figure 4.1), of sizes 205 m×125 m and 55 m×85 m, respectively, were chosen to estimate the directional and isotropic correlation lengths.

Different methods are available in the literature to estimate the correlation length. In one of the methods, the correlation length was estimated by best fitting the theoretical correlation model to the sample correlation function (Degroot and Beacher, 1993; Fenton, 1999; Jaksa et al., 1999; Fenton and Griffiths, 2008; Wackernagel, H., 2003; Zhang et al., 2008; Lloret et al., 2013). Vanmarcke (1977) proposed a method based on the variance reduction function. The variance reduction function based-method proposed by Wickremensinghe and Campanella (1993) was used in many studies to estimate the correlation length of Cone Penetration Test (CPT) data (Lloret et al., 2012; Lloret et al., 2013). Jaksa et al. (1993) used the technique of the semi-variogram to estimate the correlation length of CPT data of stiff, over-consolidated clay in the city of Adelaide. In Dasaka and Zhang (2012)'s study, random field theory was combined with the conventional estimation methods of correlation length. Phoon and Fenton (2004) used the bootstrap approach to estimate the sample correlation function.

In this study, using a set of hydraulic conductivity data obtained from an existing cement-based S/S system, the correlation length was estimated by best fitting the theoretical correlation model to the sample correlation function. The exponentially decaying correlation function was used as the theoretical correlation model.

The exponentially decaying correlation model which was used to fit the sample correlation function, is given below:

$$\rho(j\Delta\tau) = \exp\left\{\frac{-2|j\Delta\tau|}{\theta}\right\} \tag{4.3}$$

The method of moments was used to estimate the sample correlation function. The directional moment estimators of the correlation function between two hydraulic conductivity values of an existing cement-based S/S system, separated by distances $j\Delta x$ and $j\Delta y$ in the x and y directions, respectively, where $j = 0,1,........, n_x - 1$ and $j = 0,1,........, n_y - 1$, in the x and y directions , respectively, are given by Eq's (4.4) and (4.5) , respectively, and the isotropic moment estimator of the correlation function between two hydraulic conductivity values separated by a distance $j\Delta x$ (assuming $\Delta x = \Delta y$) in both x and y directions, where $j = 0,1,........, \max(n_x, n_y) - 1$, is given by Eq. (4.6).

$$\hat{\rho}(j\Delta x) = \frac{1}{\hat{\sigma}_K^2 (n_y (n_x - j) - 1)} \sum_{l=1}^{n_y} \sum_{i=1}^{n_x - j} (X'_{il})(X'_{i+j,l}) \tag{4.4}$$

$$\hat{\rho}(j\Delta y) = \frac{1}{\hat{\sigma}_K^2 (n_x (n_y - j) - 1)} \sum_{l=1}^{n_x} \sum_{i=1}^{n_y - j} (X'_{li})(X'_{l,i+j}) \tag{4.5}$$

$$\hat{\rho}(j\Delta x) = \frac{1}{\hat{\sigma}_K^2 (n_y (n_x - j) + n_x (n_y - j) - 1)} \left\{ \sum_{l=1}^{n_y} \sum_{i=1}^{n_x - j} (X'_{il})(X'_{i+j,l}) + \sum_{l=1}^{n_x} \sum_{i=1}^{n_y - j} (X'_{li})(X'_{l,i+j}) \right\} \tag{4.6}$$

where $X'_{il} = k_{il} - \mu_K$ is the deviation in hydraulic conductivity about the mean, k_{il} is the conductivity value interpreted at coordinates $((i-1)\Delta x, (j-1)\Delta y)$, n_x and n_y are the number of samples in the x and y directions, respectively, and $\Delta x = \Delta y = 5$ m in this case. The subscripts on X' index first the x direction and second the y direction.

For the 205 m×125 m sub-site having 5 m spaced hydraulic conductivity values, estimated x and y directions and isotropic correlation lengths are 16.0 m, 10.2 m and 12.3 m, respectively. For the 55 m×85 m sub-site having 5 m spaced hydraulic conductivity values, estimated x and y directions and isotropic correlation lengths are 14.6 m, 8.9 m and 11.1 m, respectively. The isotropic correlation length is between x and y direction correlation lengths, as expected, because it is obtained by averaging over all data pairs in either direction. An average of estimated isotropic correlation lengths is 11.7 m, or approximately 12 m. Figures 4.3 and 4.4 show directional and isotropic correlation functions, estimated using 205 m×125 m and 55 m×85 m sub-sites, respectively, at different lags. Figures 4.3 and 4.4 show that the curves become quite erratic at higher lags. This is typical since they are based on fewer sample pairs as the lag increases.

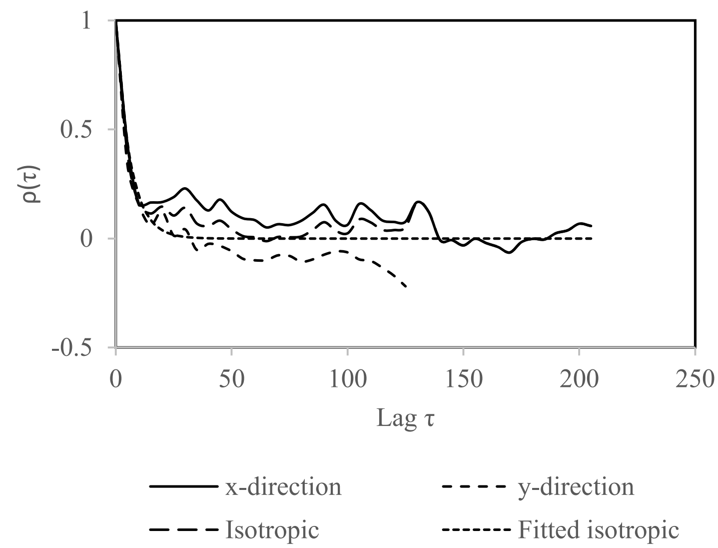

Figure 4.3: Directional and isotropic correlation functions at different lags, estimated using a 205 m×125 m sub-site having 5 m spaced hydraulic conductivity values

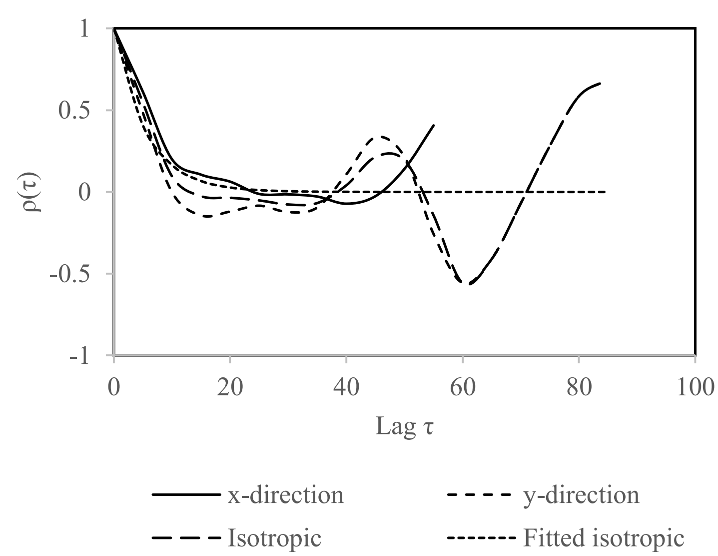

Figure 4.4: Directional and isotropic correlation functions at different lags, estimated using a 55 m×85 m sub-site having 5 m spaced hydraulic conductivity values

4.4 ERROR PROBABILITIES

The probabilities of Type I and Type II errors were computed for different numbers of samples taken from a sub-site of the entire S/S site of size 55 m×85 m discretized into 2048×2048 elements each of size $\frac{55}{2048}$ m× $\frac{85}{2048}$ m. For computing the error probabilities for varying number of samples, for each number of samples, the samples were located at equal spacing in both of the x and y directions of the sub-site. For each number of samples, the sampled element numbers in the x and y directions were obtained using Eq. (3.14). The probabilities of Type I (p_1) and Type II (p_2) errors were computed using Eq's (3.11) and (3.12), respectively.

The random field representing the hydraulic conductivity of a cement-based S/S system can be used to assess the reliability associated with QC sampling. The parameters of the random hydraulic conductivity field (i.e., $\mu_k' = 0.47$, $v_k = 1.7$, and $\theta_k = 12$ m) derived from the studied cement-based S/S system are used to compute the Type I and Type I error probabilities for the number of samples of 1, 4, 9, and 16, taken from the 55 m×85 m sub-site of the existing S/S site. The results are presented in Table 4.1.

The results presented in Table 4.1 indicate that when the number of samples is 16, both Type I and Type II error probabilities are very very small when μ_k' is known ahead of time to be 0.47. According to the current sampling requirements specified by the USACE (2000) for the QC program of cement-based S/S of 1 sample/500 m^3, this 55 m×85 m sub-site requires 36 samples (= (1/500) ×55×85×3.9). Thus, the current QC sampling regulation of cement-based S/S seems to be conservative for this particular S/S system (see Table 4.1), again if the value of μ_k' is known to be much smaller than 1.0.

Table 4.1: The probabilities of Type I (p_1) and Type II (p_2) errors for μ'_k = 0.47, v_k = 1.7, θ_k = 12 m and different n over 55 m×85 m sub-site

n over 55 m×85 m sub-site	p_1	p_2
1	less than 0.0001	0.1003
4	less than 0.0001	0.0064
9	less than 0.0001	0.0002
16	less than 0.0001	less than 0.0001

However, the Type I and Type II error probabilities presented in Table 4.1 can not be obtained prior to the QC program, since the hydraulic conductivity mean, coefficient of variation and correlation length are unknown prior to QC sampling. In order to assess QC sampling requirements, the "worst case" hydraulic conductivity mean, coefficient of variation and correlation length need to be used in the determination of error probabilities. According to the results presented in Chapter 2, the "worst case" normalized mean and coefficient of variation of hydraulic conductivity are approximately 1.5 and 1.0, respectively. As well as the "worst case" mean and coefficient of variation, a normalized mean of 1.0 is also considered to assess QC sampling requirements over the 55 m×85 m sub-site. The "worst case" correlation lengths are 12 m and 68 $\left(=\sqrt{(55\times 85)}\right)$ m (which correspond to the normalized correlation lengths of 0.17 and 1.0, respectively). The Type I and Type II error probabilities are computed for the number of samples of 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 225, 400, 625, and 900. The results are presented in Tables 4.2 and 4.3.

Table 4.2: The probabilities of Type I (p_1) and Type II (p_2) errors for $\mu'_k = 1.0$, $v_k = 1.0$, $\theta_k = 12$ and 68 m, and varying n

n over 55 m×85 m sub-site	$\theta_k = 12$ m		$\theta_k = 68$ m	
	p_1	p_2	p_1	p_2
1	0.0020	0.3357	0.0607	0.1692
4	0.0017	0.2058	0.0408	0.1014
9	0.0016	0.1224	0.0310	0.0716
16	0.0015	0.0752	0.0250	0.0553
25	0.0014	0.0488	0.0212	0.0452
36	0.0013	0.0339	0.0179	0.0378
49	0.0010	0.0244	0.0157	0.0325
64	0.0009	0.0188	0.0140	0.0288
81	0.0009	0.0151	0.0127	0.0259
100	0.0009	0.0122	0.0115	0.0231
225	0.0007	0.0059	0.0079	0.0154
400	0.0005	0.0039	0.0062	0.0123
625	0.0004	0.0031	0.0052	0.0107
900	0.0004	0.0022	0.0041	0.0078

The results presented in Table 4.2 indicate that when $\mu'_k = 1.0$ and $v_k = 1.0$, $\theta_k =$ 68 m is the "worst case" for all number of samples for the probability of a Type I error,

Table 4.3: The probabilities of Type I (p_1) and Type II (p_2) errors for $\mu'_k = 1.5$, $v_k = 1.0$, $\theta_k = 12$ and 68 m, and varying n

n over 55 m×85 m sub-site	$\theta_k = 12$ m		$\theta_k = 68$ m	
	p_1	p_2	p_1	p_2
1	0.3165	0.1737	0.1376	0.1158
4	0.2449	0.1287	0.0888	0.0762
9	0.2073	0.1136	0.0653	0.0567
16	0.1769	0.1009	0.0516	0.0451
25	0.1527	0.0904	0.0430	0.0378
36	0.1336	0.0817	0.0363	0.0320
49	0.1173	0.0736	0.0315	0.0278
64	0.1051	0.0675	0.0280	0.0248
81	0.0951	0.0622	0.0253	0.0224
100	0.0862	0.0575	0.0227	0.0202
225	0.0587	0.0418	0.0154	0.0137
400	0.0447	0.0326	0.0122	0.0109
625	0.0376	0.0276	0.0106	0.0094
900	0.0298	0.0228	0.0079	0.0071

and $\theta_k = 12$ m is the "worst case" for the number of samples of 1 to 25, and for the rest of the number of samples (i.e., 36 to 900), $\theta_k = 68$ is the "worst case" for the probability of a Type II error. Sampling requirement to achieve target 5% probability for both Type I and Type II errors for $\mu_k' = 1.0$ and $\nu_k = 1.0$ is 25 for both $\theta_k = 12$ and 68 m.

The results presented in Table 4.3 indicate that when $\mu_k' = 1.5$ and $\nu_k = 1.0$, for both Type I and Type II error probabilities, $\theta_k = 12$ m is the "worst case" for all number of samples. Sampling requirements to achieve target 5% probability for both Type I and Type II errors for $\mu_k' = 1.5$ and $\nu_k = 1.0$ are 25 and 400 when $\theta_k = 68$ and 12 m, respectively, suggesting the number of samples of 400 over a 55 m×85 m sub-site of the entire S/S site to be conservative to achieve target 5% probability for both Type I and II errors. According to USACE (2000), sampling requirement over 55 m×85 m sub-site of 36 seems to be unconservative to achieve 5% probabilitiy for both Type I and Type II errors for this assumed mean and coefficient of variation.

4.5 SUMMARY AND CONCLUSIONS

In this chapter, a set of hydraulic conductivity data with corresponding locations obtained from an existing cement-based S/S system, is statistically analyzed to assess its spatial variability. The spatial variability of hydraulic conductivity is described by a random field with a certain distribution and correlation length. In order to make use of the classical estimators for the correlation structure (which are based on equispaced data), irregularly scattered hydraulic conductivity data set is interpolated onto a two-dimensional 5 m grid using the linear interpolation method available in MATLAB under the class "TriscatteredInterp". Two sub-sites having 5 m spaced hydraulic conductivity values of sizes 205 m×125 m and 55 m×85 m are used to estimate directional and isotropic correlation lengths. In order to assess QC sampling requirements, the spatial variability of hydraulic conductivity of the system is then used to compute the error probabilities (i.e., Type I and Type II) for different numbers of samples taken from a 55 m×85 m sub-site of the entire cement-based S/S site. The Type I and Type II error probabilities are also computed for the "worst case" conditions of hydraulic conductivity mean, coefficient of variation and correlation length, and varying number of samples to provide recommendations for conservative QC sampling requirements over 55 m×85 m sub-site.

The following conclusions can be drawn from this study:

- A lognormal distribution with the mean and standard deviation of the logarithm of the normalized hydraulic conductivity of -1.30 and 1.02, respectively, is found to be a reasonable fit to the hydraulic conductivity data.
- The x and y directions and isotropic correlation lengths are estimated to be 16.0 m, 10.2 m, and 12.3 m, respectively, considering a 205 m×125 m sub-site having 5 m spaced hydraulic conductivity values, and the x and y directions and isotropic correlation lengths are estimated to be 14.6 m, 8.9 m, and 11.1 m, respectively, considering a 55 m×85 m sub-site having 5 m spaced hydraulic conductivity values. An average isotropic correlation length is found to be 11.7 m, or approximately 12 m.
- For the spatial variability of hydraulic conductivity of the S/S system, i.e., for $\mu'_k = 0.47$, $v_k = 1.7$, and $\theta_k = 12$ m, the Type I error probabilities for any of the number of samples of 1, 4, 9, and 16 are found to be less than 0.0001, whereas, the Type II error probabilities for the number of samples of 1, 4, 9, and 16 are found to be 0.1003, 0.0064, 0.0002, and less than 0.0001, respectively. The

USACE (2000) sampling recommendation for the QC program of cement-based S/S found to be conservative for this particular case.

- The probabilities of Type I and Type II errors computed for various number of samples and the "worst case" conditions of hydraulic conductivity mean (i.e., 1.0 and 1.5 times the regulatory value), coefficient of variation (i.e., 1.0), and correlation lengths (i.e., 12 and 68 m) suggest the number of samples of 400 to be conservative to achieve 5% probability for both Type I and Type II errors. The USACE (2000) sampling recommendation for the QC program of cement-based S/S would be unconservative for this particular case.

CHAPTER 5

CONCLUSIONS

5.1 SUMMARY AND CONCLUSIONS

Sampling requirements for the quality control (QC) of cement-based solidification/stabilization (S/S) construction cells do not currently specify the sample size with a consideration of the accuracy of the estimated effective hydraulic conductivity of the cells from the samples, nor by considering the risk associated with drawing the wrong conclusions about the acceptability of the cells. Research related to the sampling requirements for the QC program of cement-based S/S construction cell and the spatial variability associated with the hydraulic conductivity of cement-based S/S systems is not available in literature. This thesis aims to address the issues associated with sampling requirements of a cement-based S/S construction cell during a QC program to achieve a certain confidence in the decision (acceptable or unacceptable) regarding each cell via simulation and via theory taking into account the spatial variability associated with hydraulic conductivity of the entire cement-based S/S system. In order to address the sampling issue, this study considers a hypothesis test, where the null hypothesis was that the S/S construction cell had an unacceptable flow rate. Two types of errors that resulted in the hypothesis test were: 1) a Type I error where the sample data rejected the null hypothesis even though the null was correct. This error results in the cell being deemed acceptable when it is actually not, and 2) a Type II error where the sample data failed to reject the null hypothesis even though it was false. This results in the cell being assumed unacceptable when it is actually acceptable. The purpose of this study is to determine the number of samples required to achieve target probabilities for both Type I and Type II errors. Probabilistic simulations performed in this thesis to assess sampling requirements for the QC programs of cement-based S/S construction cells to achieve target hypothesis test errors is an extension of Menzies (2008)' work. In order to determine sampling requirements for QC programs of soil liner systems to achieve target Type I and Type II errors, Menzies (2008) considered the arithmetic average of the hydraulic conductivity field to be the effective hydraulic conductivity. The work presented in this thesis considers the geometric average of the hydraulic conductivity field to be the effective hydraulic conductivity, since flow was in-plane.

A summary of conclusions drawn in this study is presented below:

The objective of Chapter 2 was to present a parametric study to examine the influence of hydraulic conductivity mean, coefficient of variation, and correlation length on sampling requirements during the QC program of a cement-based S/S construction cell by performing Monte Carlo simulations. The simulation employed a modified version of the two-dimensional random finite element method (RFEM) program, mrflow2d. The modification made to the program for this study enables the sampling of the random field at prescribed locations. Also in the modified version, finite element method is not used to obtain the flow through the field, instead geometric average of the field is used to represent the flow through the field. In order to perform a parametric study, a two-dimensional cement-based S/S construction cell was simulated. The influence of hydraulic conductivity mean, coefficient of variation and correltion length on both Type I and Type II errors was examined. It was found that, for a specific number of samples, the greatest Type I and Type II error probabilities occurred at some "worst case" correlation length, which was found to be 0.1 to 5 times the effective field dimension for the probability of Type I errors and 0.01 to 10 times the effective field dimension for the probability of Type II errors. In general, the "worst case" correlation length is somewhere between 0.01 and 1.0 times the field dimension. This "worst case" correlation length would be conservative in designing sampling requirements to achieve a target reliability about the decision regarding approval of the cell.

Chapter 2 also showed that for a specific number of samples, the greatest error probabilities occurred at the normalized mean of point-scale hydraulic conductivity of about 1.7 for a Type I error and 1.1 for a Type II error, indicating more sample requirements at these mean hydraulic conductivities would be required. It was shown in Chapter 2 that both Type I and Type II error probabilities approached zero when the mean hydraulic conductivity was far below or far above the regulatory value (i.e., $\mu_k' =$ 0.01, 0.1, and 10.0), suggesting that in general the mean hydraulic conductivity should be targeted well below the regulatory value. As expected, increasing the number of samples was found to be effective in decreasing both Type I and Type II error probabilities.

For a specific number of samples, an increase in the hydraulic conductivity coefficient of variation resulted in a decrease in both Type I and Type II error probabilities when the normalized mean of point-scale hydraulic conductivity was 1.0. This suggests that when μ_k' is approximately 1.0, more samples will be required to achieve acceptably small error probabilities when ν_k is 1.0 or less.

Simulations were performed considering a construction cell of dimension 1×1 in order to make the results scalable. An example (considering a 10 m×10 m cell) was presented to illustrate the scalability of the results presented in Chapter 2. The good agreement obtained between the simulation results for both Type I and Type II error probabilities for a (10 m×10 m) construction cell and a 1×1 construction cell indicates the scalability of the simulation results presented in this chapter.

The goal of Chapter 3 was to develop an analytical approach for selecting the sample size required for a cement-based S/S construction cell's QC program. In order to meet this objective, analytical solutions were developed for computing the probabilities of Type I and Type II errors as a function of the number of samples taken and the statistics of the hydraulic conductivity field. In order to validate the proposed analytical solutions, the analytically computed Type I and Type II error probabilities were compared to those estimated via probabilistic simulations for a range of parameter sets and were found to have excellent agreement, allowing the Type I and Type II error probabilities to be computed analytically with reasonable confidence. An example was presented in Chapter 3 to illustrate how the proposed method can be used in practice to assess the required sample size for the QC program of cement-based S/S construction cells.

In order to address the deficiency in the literature about the spatial variability associated with hydraulic conductivity of cement-based S/S systems, Chapter 4 aimed to perform statistical analyses on a set of hydraulic conductivity data obtained from a real cement-based S/S system to assess its spatial variability. A lognormal distribution was found to be a reasonable fit to the data. The goodness-of-fit was tested using the Chi-square and the Anderson-Darling tests. In order to estimate directional and isotropic ccorrelation lengths, irregularly scattered hydraulic conductivity data set was transformed into 5 m spaced data set in two-dimensions using the linear interpolation method available in MATLAB under the class "TriscatteredInterp". The method followed by the

“TriscatteredInerp” to obtain interpolated hydraulic conductivity values is described in Chapter 3. Using two sub-sites having 5 m spaced hydraulic conductivity values of sizes 205 m×125 m and 55 m×85 m, the x and y directions and isotropic correlation lengths were estimated by fitting an exponentially decaying correlation model to the sample correlation functions. For the 205 m×125 m sub-site, estimated x and y directions and isotropic correlation lengths were 16.0 m, 10.2 m, and 12.3 m, respectively, and for a 55 m×85 m sub-site, estimated x and y directions and isotropic correlation lengths were 14.6 m, 8.9 m, and 11.1 m, respectively. An average isotropic correlation length was 11.7 m or approximately 12 m. The spatial variability derived for the hydraulic conductivity of the existing cement-based S/S system was used to assess the sampling requirements over a 55 m×85 m sub-site of the entire S/S site. The computed probabilities of Type I and Type II errors for various sample sizes considering the “worst case” conditions of hydraulic conductivity mean, coefficient of variation and correlation length, presented in Chapter 4, can be used to assess conservative sampling requirements for the QC program of the 55 m×85 m sub-site of the existing S/S site.

5.2 FUTURE WORK

Research related to the reliability of cement-based S/S systems is not available in literature. Although the work presented in this thesis addressed the issue associated with sampling requirements for the QC program of cement-based S/S considering the reliabilitity associated with the decision regarding the acceptance or rejection of the system, there are still some issues that should be included into future research, such as,

- This study assumes equal correlation length in both planar directions. Depending on the type of contaminated material, cement-based S/S may have anisotropic correlation length. Consideration of this anisotropy in the correlation length in future research may be more rational.

- The sampling issue should be addressed by performing three-dimensional analyses.
- The "worst case" correlation length renders the results size independent – this issue needs more study in future research. There must actually be a trade-off between cell size and potential replacement cost, i.e. if the construction cell is taken to be too large, then it is very expensive to replace if the test dictates that it should be replaced.
- In addition to advection, uncertainty in diffusion and sorption could be considered to investigate the sampling issue associated with the flow though cement-based S/S systems.

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APPENDIX A

INFLUENCE OF MESH RESOLUTION ON ERROR PROBABILITIES

Table A.1: Sensitivity Analysis, $\mu'_k = 1.0$, $\nu_k = 1.0$, $\theta'_{\ln k} = 0.5$, and $n = 9$

Mesh Resolution	p_1	p_2	Time (sec)
32×32	0.02388	0.09472	15
64×64	0.02244	0.09736	34
72×72	0.02316	0.09648	41
80×80	0.02248	0.09628	48
104×104	0.02476	0.09696	85
128×128	0.02304	0.09468	120
256×256	0.02388	0.09644	430

APPENDIX B

INFLUENCE OF CORRELATION LENGTH ON TYPE I AND TYPE II ERROR PROBABILITIES

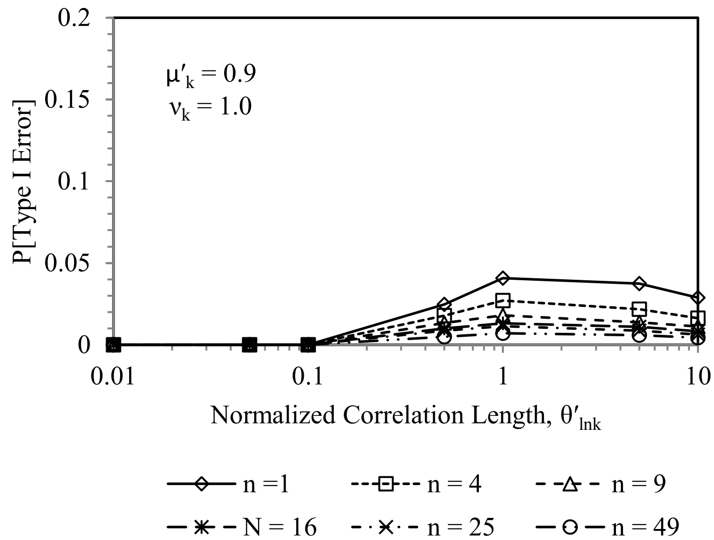

Figure B.1: Influence of correlation length on the probability of a Type I error for mean of 0.9 and coefficient of variation of 1.0

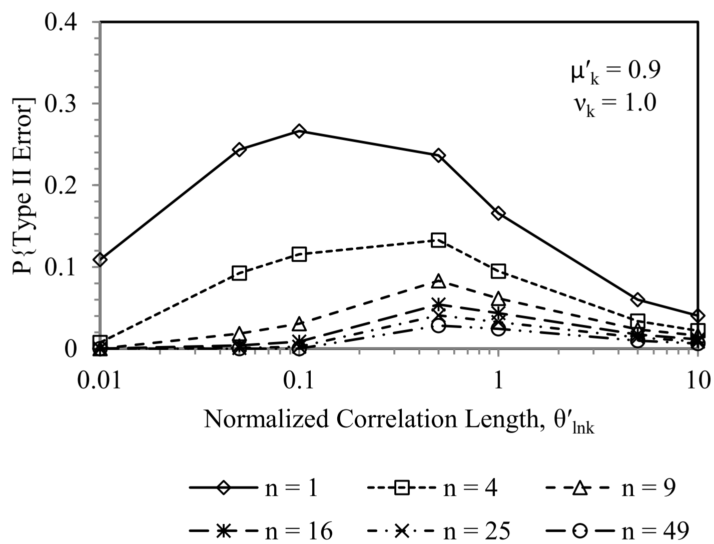

Figure B.2: Influence of correlation length on the probability of a Type II error for mean of 0.9 and coefficient of variation of 1.0

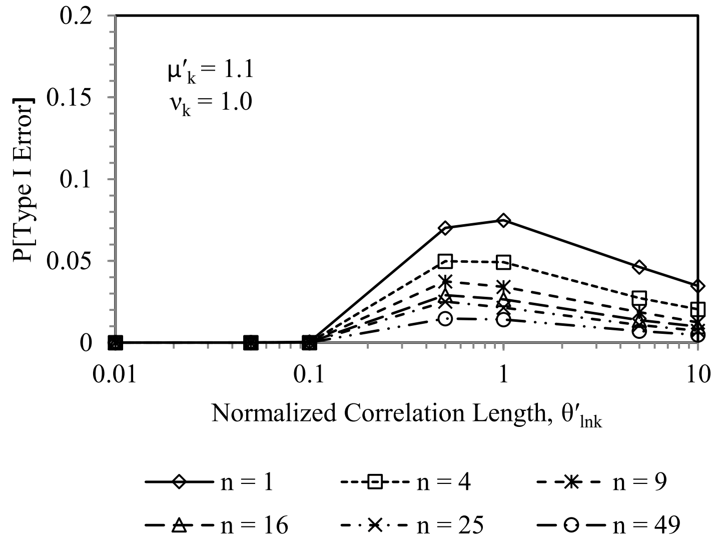

Figure B.3: Influence of correlation length on the probability of a Type I error for mean of 1.1 and coefficient of variation of 1.0

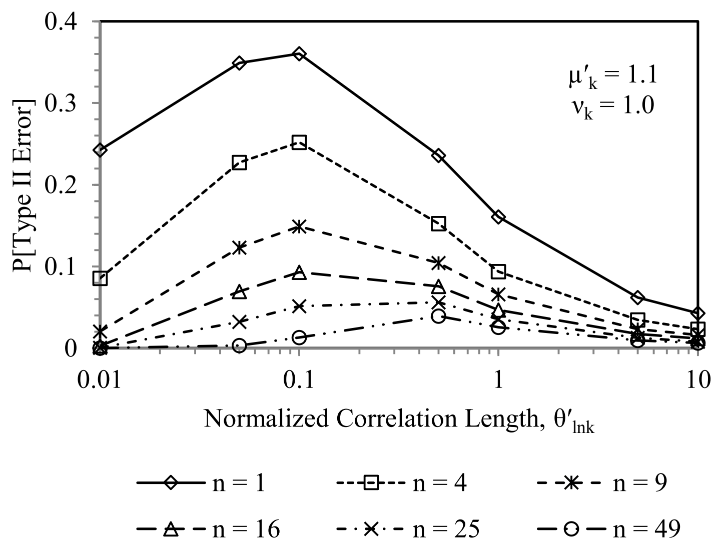

Figure B.4: Influence of correlation length on the probability of a Type II error for mean of 1.1 and coefficient of variation of 1.0

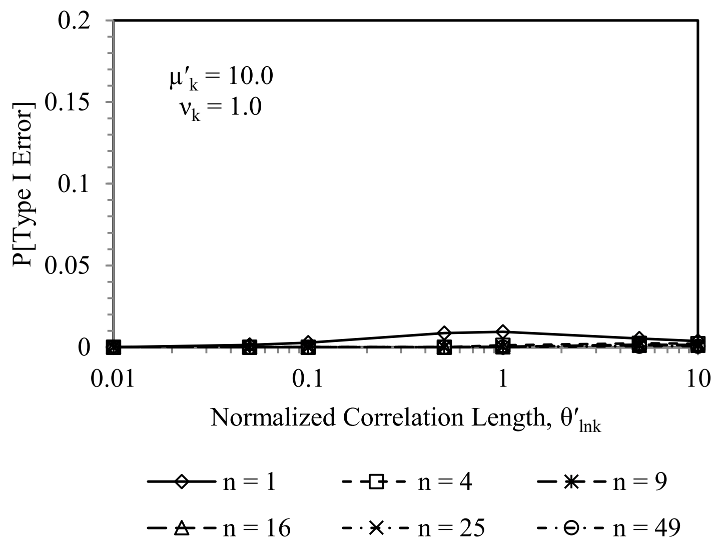

Figure B.5: Influence of correlation length on the probability of a Type I error for mean of 10.0 and coefficient of variation of 1.0

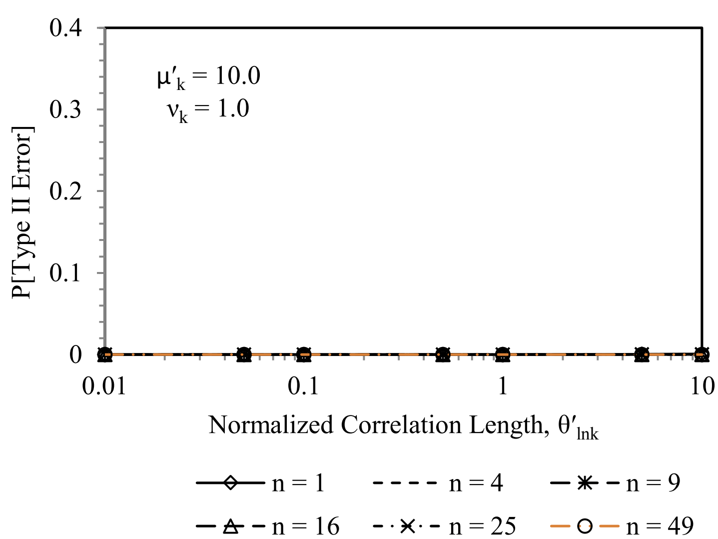

Figure B.6: Influence of correlation length on the probability of a Type II error for mean of 10.0 and coefficient of variation of 1.0

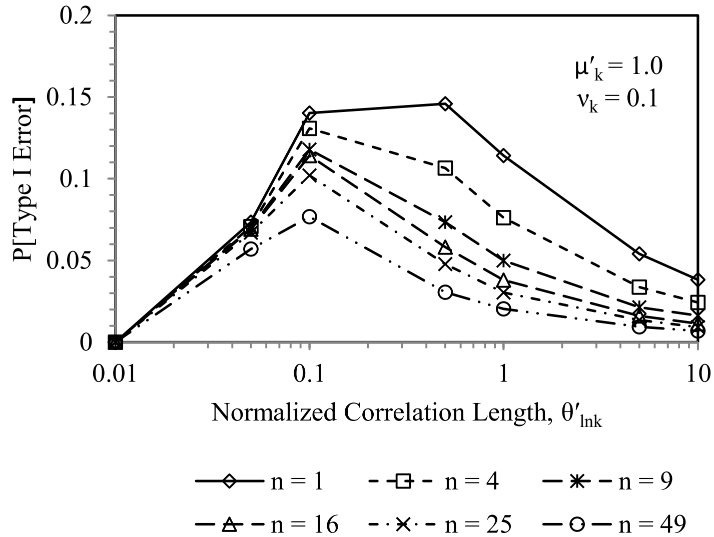

Figure B.7: Influence of correlation length on the probability of a Type I error for mean of 1.0 and coefficient of variation of 0.1

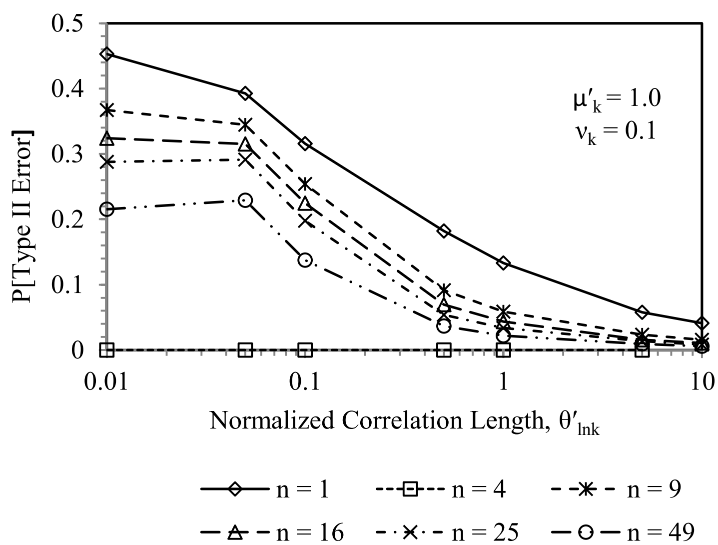

Figure B.8: Influence of correlation length on the probability of a Type II error for mean of 1.0 and coefficient of variation of 0.1

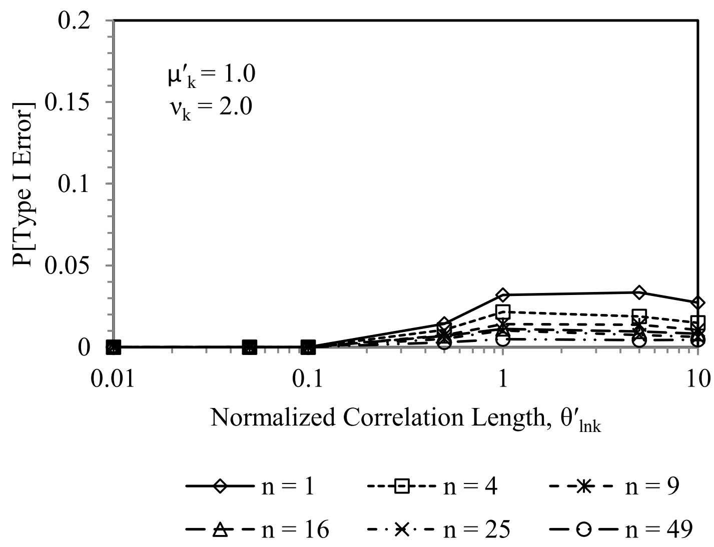

Figure B.9: Influence of correlation length on the probability of a Type I error for mean of 1.0 and coefficient of variation of 2.0

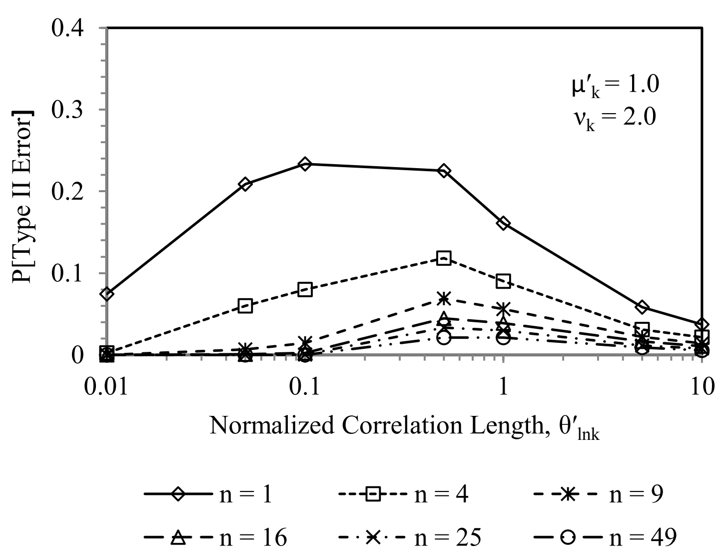

Figure B.10: Influence of correlation length on the probability of a Type II error for mean of 1.0 and coefficient of variation of 2.0

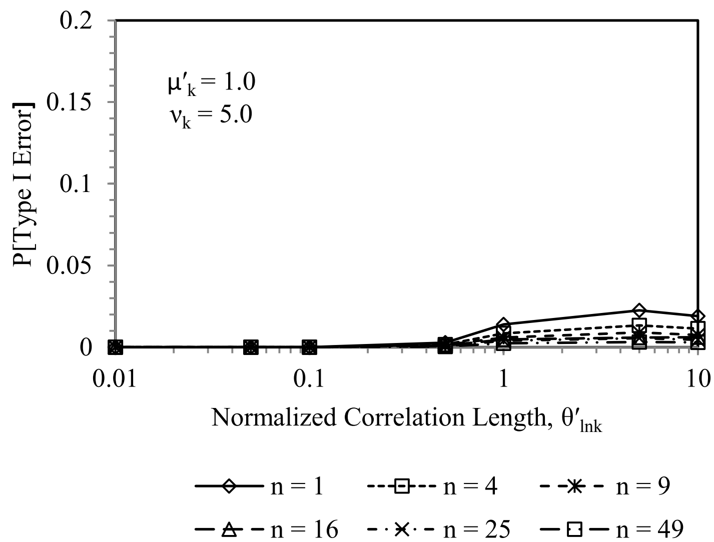

Figure B.11: Influence of correlation length on the probability of a Type I error for mean of 1.0 and coefficient of variation of 5.0

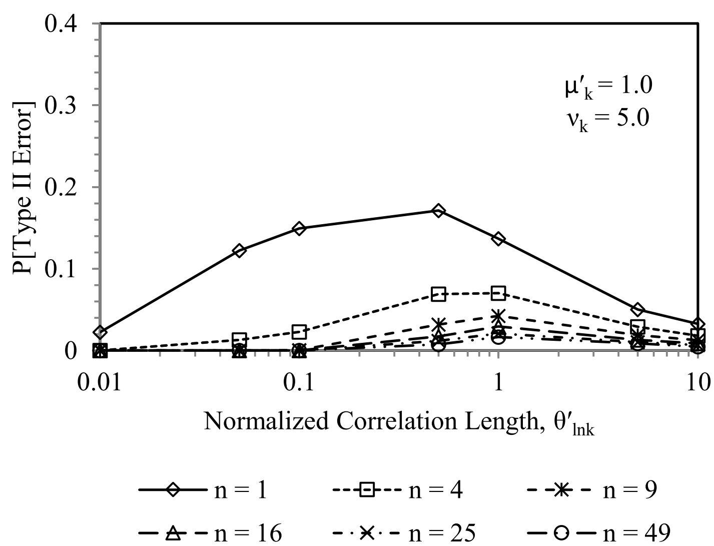

Figure B.12: Influence of correlation length on the probability of a Type II error for mean of 1.0 and coefficient of variation of 5.0

APPENDIX C
STATISTICS OF GEOMETRIC AVERAGE

Assuming k_{eff} to be the geometric average of m element hydraulic conductivities and a Markovian correlation structure (Vanmarcke, 1984) with a separable correlation function (which is a product of directional correlation functions) and correspondingly a separable variance reduction function, i.e., see Equations 2.10 and 2.11, the mean and standard deviation of the actual effective hydraulic conductivity of the S/S construction cell , k_{eff} , can be calculated as,

$$\mu_{k_{eff}} = \exp\left\{ \mu_{\ln k} + \frac{1}{2}\gamma_{\ln k}(X,Y)\sigma_{\ln k}^2 \right\} \tag{C.1}$$

$$\sigma_{k_{eff}} = \sqrt{\mu_{k_{eff}}^2 \left[\exp\left\{ \sigma_{\ln k}^2 \gamma_{\ln k}(X,Y) \right\} - 1 \right]} \tag{C.2}$$

where $\mu_{\ln k} = \ln \mu_k - \frac{1}{2}\sigma_{\ln k}^2$, $\sigma_{\ln k}^2 = \ln\left(1 + \nu_k^2\right)$, $\nu_k = \frac{\sigma_k}{\mu_k}$ is the coefficient of variation of point-scale hydraulic conductivity.

The mean and standard deviation of log-k_{eff} can be computed as,

$$\mu_{\ln k_{eff}} = \ln\left(\mu_{k_{eff}}\right) - \frac{1}{2}\sigma_{\ln k_{eff}}^2 \tag{C.3}$$

$$\sigma_{\ln k_{eff}} = \sqrt{\ln\left(1 + \nu_{k_{eff}}^2\right)} \tag{C.4}$$

where $\nu_{k_{eff}} = \frac{\sigma_{k_{eff}}}{\mu_{k_{eff}}}$ is the coefficient of variation of the actual effective hydraulic conductivity.

Assuming k_G to be the geometric average of n sample hydraulic conductivities, the mean and standard deviation of the logarithm of sample geometric average, log-k_G, can be calculated as,

$$\mu_{\ln k_G} = \mu_{\ln k} \tag{C.5}$$

and

$$\sigma_{\ln k_G} \cong \sqrt{\frac{1}{n^2}\left[n\left(\sigma^2_{\ln k}\gamma_{\ln k}(\Delta x, \Delta y)\right) + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\sigma^2_{\ln k}\rho_{\ln k}\left(\mathbf{x}_i - \mathbf{x}_j\right)\right]} \tag{C.6}$$

where $\mathbf{x}_i = \{x_{ix}, x_{iy}\}$ are the spatial coordinate of the centre of the ith sample and assuming a Markovian correlation structure with a separable correlation function (which is a product of directional correlation functions) and isotropic correlation lengths,

$$\rho_{\ln k}\left(\mathbf{x}_i - \mathbf{x}_j\right) = \exp\left\{-2\left|x_{ix} - x_{jx}\right|/\theta_{\ln k}\right\}\exp\left\{-2\left|x_{iy} - x_{jy}\right|/\theta_{\ln k}\right\}$$

The correlation coefficient between $\ln k_{eff}$ and $\ln k_G$, ρ is given by,

$$\begin{aligned}\rho &= \frac{Cov\left[\ln k_G, \ln k_{eff}\right]}{\sigma_{\ln k_G}\sigma_{\ln k_{eff}}} \\ &\cong \frac{1}{\sigma_{\ln k_G}\sigma_{\ln k_{eff}} n m_x m_y}\left[n\sigma^2_{\ln k}\gamma_{\ln k}(\Delta x, \Delta y) + \sum_{k=1}^{n}\sum_{\substack{i=1\\ i\neq k}}^{m_x}\sum_{\substack{j=1\\ j\neq k}}^{m_y}\sigma^2_{\ln k}\rho_{\ln k}\left(\mathbf{x}_k - \mathbf{x}_{ij}\right)\right]\end{aligned} \tag{C.7}$$

where m_x and m_y are the number of elements in the x and y directions, respectively, such that $m_x \times m_y = m$.

APPENDIX D

DERIVATIONS OF ERROR PROBABILITIES USING OWEN (1959)'S METHOD

Let $f_{\ln k_{eff} \ln k_G}(u,v)$ be the bivariate normal probability density function of random variables $\ln k_{eff}$ and $\ln k_G$, $f_{\ln k_G}(v)$ be the marginal probability density function of $\ln k_G$ and $B(h,w;\rho)$, where $h = \dfrac{\ln k_{crit} - \mu_{\ln k_G}}{\sigma_{\ln k_G}}$, $w = \dfrac{\ln k_{crit} - \mu_{\ln k_{eff}}}{\sigma_{\ln k_{eff}}}$, be Owen(1959)'s solution for the bivariate normal probability.

Then, the probability of a Type I error, $p_1 = \mathrm{P}\left[\ln k_G < \ln k_{crit} \cap \ln k_{eff} > \ln k_{crit}\right]$, is,

$$\int_{-\infty}^{\ln k_{crit}} \int_{\ln k_{crit}}^{+\infty} f_{\ln k_{eff} \ln k_G}(u,v)\,du\,dv$$

$$= \int_{-\infty}^{\ln k_{crit}} \left\{ f_{\ln k_G}(v) - \int_{-\infty}^{\ln k_{crit}} f_{\ln k_{eff} \ln k_G}(u,v)\,du \right\} dv$$

$$= \Phi\left(\frac{\ln k_{crit} - \mu_{\ln k_G}}{\sigma_{\ln k_G}}\right) - \int_{-\infty}^{\ln k_{crit}} \int_{-\infty}^{\ln k_{crit}} f_{\ln k_{eff} \ln k_G}(u,v)\,du\,dv$$

$$= \Phi\left(\frac{\ln k_{crit} - \mu_{\ln k_G}}{\sigma_{\ln k_G}}\right) - B(h,w;\rho)$$

$$= \Phi\left(\frac{\ln k_{crit} - \mu_{\ln k_G}}{\sigma_{\ln k_G}}\right) - B\left(\frac{\ln k_{crit} - \mu_{\ln k_G}}{\sigma_{\ln k_G}}, \frac{\ln k_{crit} - \mu_{\ln k_{eff}}}{\sigma_{\ln k_{eff}}}; \rho\right)$$

$$= \Phi\left(\frac{\ln k_{crit} - \mu_{\ln k_G}}{\sigma_{\ln k_G}}\right) - \left[\frac{1}{2}\Phi\left(\frac{\ln k_{crit} - \mu_{\ln k_G}}{\sigma_{\ln k_G}}\right) + \frac{1}{2}\Phi\left(\frac{\ln k_{crit} - \mu_{\ln k_{eff}}}{\sigma_{\ln k_{eff}}}\right) - T(h,a_h) - T(w,a_w)\right]$$

$$= \frac{1}{2}\Phi(h) - \frac{1}{2}\Phi(w) + T(h, a_h) + T(w, a_w)$$

That is, the probability of a Type I, p_1 can be derived to:

$$p_1 = \frac{1}{2}\Phi(h) - \frac{1}{2}\Phi(w) + T(h, a_h) + T(w, a_w) \tag{D.1}$$

In a similar manner, the probability of a Type II error, $p_2 = \mathrm{P}\lfloor \ln k_G > \ln k_{crit} \cap \ln k_{eff} < \ln k_{crit} \rfloor$ can be derived to:

$$p_2 = \frac{1}{2}\Phi(w) - \frac{1}{2}\Phi(h) + T(h, a_h) + T(w, a_w) \tag{D.2}$$

The above expressions for the probabilities of Type I and Type II errors are valid if $hw > 0$ or if $hw = 0, h$ or $w \geq 0$. If $hw < 0$ or if $hw = 0, h$ or $w < 0$, the probabilities of Type I and Type II errors can be derived to Eq's (D.3) and (D.4), respectively.

$$p_1 = \frac{1}{2}\Phi(h) - \frac{1}{2}\Phi(w) + T(h, a_h) + T(w, a_w) + \frac{1}{2} \tag{D.3}$$

$$p_2 = \frac{1}{2}\Phi(w) - \frac{1}{2}\Phi(h) + T(h, a_h) + T(w, a_w) + \frac{1}{2} \tag{D.4}$$

APPENDIX E

THE MATLAB CODE THAT GENERATES INTERPOLATED HYDRAULIC CONDUCTIVITY VALUES ON A 2-D, 5 m GRID

A spreadsheet containing three columns of information: x, y and z, where (x, y) is the position of each observation in m and z is the corresponding normalized hydraulic conductivity value, is used to obtain the interpolated values on a 5 m grid in 2-D space. Using the scattered dataset, the "TriscatteredInterp" first creates a function, which fits a convex hull. A grid is then created with x positions ranging from -660 to 580 and y positions ranging from 115 to 440, with 5 m spacing in both x and y directions. The function is then evaluated at each query location (i.e., at each grid point). The MATLAB code that generated the interpolated 5 m spaced hydraulic conductivity values is as given below:

```
n=2086;
x=xlsread('correlation_length.xls','c4:c2089');
y=xlsread('correlation_length.xls','d4:d2089');
z=xlsread('correlation_length.xls','e4:e2089');
F = TriScatteredInterp(x,y,z);
min_x = -660;
delta_x = 5;
max_x = 580;
grid_x = min_x:delta_x:max_x;
min_y = 115;
delta_y = 5;
max_y = 440;
grid_y = min_y:delta_y:max_y;
[qx,qy] = meshgrid(grid_x,grid_y)
qz = F(qx,qy)
```

APPENDIX F

HYDRAULIC CONDUCTIVITY DATA WITH LOCATIONS

x - cord	*y* - cord	*k*	*x* - cord	*y* - cord	*k*	*x* - cord	*y* - cord	*k*
-15.651	435.336	0.19	-132.36	408.071	0.27	-368.29	396.195	0.59
-29.969	434.96	0.14	-95.475	414.631	0.16	-376.68	394.981	0.24
-42.554	433.609	0.15	-4.2236	416.864	0.16	-384.71	393.894	1
-53.231	432.585	0.18	-14.586	415.677	0.099	-391.97	392.847	0.89
-63.424	430.766	0.17	-25.52	414.636	0.12	-97.994	407.593	0.38
-75.474	429.277	0.4	-35.8	412.9	0.14	-107.48	406.586	0.035
-86.411	429.143	0.13	-46.004	411.554	0.19	-116.8	404.676	0.32
-95.07	428.841	0.43	-55.993	410.548	0.15	-126.94	401.714	0.24
-103.65	427.754	0.61	-65.409	410.013	0.23	-137.98	398.798	0.14
-113.03	425.479	0.61	-75.877	409.07	0.09	-60.79	402.817	0.22
-126.45	423.024	0.48	-86.926	408.247	0.16	-74.659	401.406	0.24
-4.2236	432.22	0.35	-388.03	300.875	0.42	-85.555	400.549	0.59
-13.872	429.977	0.018	-202.48	403.48	0.041	-471.99	272.008	0.088
-23.747	429.62	0.086	-198.55	397.396	0.34	-477.96	280.134	0.2
-33.371	428.459	0.05	-195.3	392.28	0.54	-488.47	286.961	0.3
-43.041	427.058	0.23	-191.43	386.169	0.053	-217.86	394.403	6.1
-52.606	425.754	0.11	-187.19	379.636	0.56	-211.61	388.012	0.76
-62.838	424.512	0.76	-174.9	401.179	0.025	-206.22	379.142	0.34
-72.727	423.174	0.2	-183.09	373.257	0.98	-202.11	370.581	1
-82.731	422.39	0.42	-178.87	366.594	0.041	-197.09	361.596	0.55
-91.788	421.672	0.96	-174.49	360.76	0.15	-192.03	352.906	0.66
-99.766	420.942	0.44	-170.18	354.018	0.21	-186.32	344.156	0.75
-107.51	419.65	0.32	-165.74	346.799	0.51	-180.87	335.392	0.31
-115.09	418.112	0.66	-219.71	401.254	0.66	-176.13	327.211	0.88
-123.99	416.892	0.26	-227.23	401.694	3.1	-154.41	408.626	0.05
-133.81	415.721	0.2	-235.08	400.974	0.6	-164.03	405.928	0.12
-144.22	412.683	0.53	-243.27	400.331	0.71	-175.08	405.589	0.067
-5.5493	424.415	0.041	-251.31	399.752	6.1	-185.43	400.47	0.066
-17.345	423.204	0.29	-258.68	400.191	0.75	-193.08	406.759	0.012
-28.35	421.838	0.063	-266.25	400.89	0.077	-197.52	404.774	0.029
-39.144	420.371	0.014	-274.3	401.535	0.65	-226.4	395.156	1
-107.96	412.888	0.1	-317.35	404.515	0.96	-233.79	394.255	0.82
-49.504	419.238	0.29	-326.24	403.865	1	-241.42	393.662	0.91
-60.681	417.937	0.53	-333.59	403.457	0.21	-248.67	393.2	0.98

x-cord	*y*-cord	*k*	*x*-cord	*y*-cord	*k*	*x*-cord	*y*-cord	*k*
-72.401	416.668	0.24	-341.76	401.709	0.68	-255.65	393.254	1
-120.12	410.39	0.27	-351.04	399.497	1.3	-263.08	394.343	0.57
-83.871	415.46	0.33	-359.76	397.782	3.6	-159.26	350.312	0.16
-163.73	357.919	0.72	-146.82	372.826	0.068	-246.92	374.012	0.82
-168.26	365.011	0.18	-148.47	381.068	0.042	-422.91	270.154	0.72
-172.88	372.008	0.033	-137.12	382.638	0.04	-410.73	266.512	0.35
-177.41	379.224	0.14	-128.1	383.186	0.016	-398.97	264.307	0.12
-181.75	386.051	0.39	-121.12	377.965	0.033	-387.88	263.164	0.18
-163.67	398.811	0.59	-134.87	367.735	0.69	-270.61	395.294	0.92
-154.01	401.039	0.07	-139.42	375.246	0.29	-277.64	396.063	5.7
-143.98	404.964	0.06	-129.51	373.281	0.14	-284.94	395.525	1
-106.91	398.891	0.56	-408.49	307.1	0.82	-292.45	394.696	4.3
-96.214	399.911	0.73	-167.63	336.69	0.087	-300.23	394.849	0.82
-160.02	341.503	0.45	-183.32	405.662	0.15	-398.86	392.452	0.29
-152.75	352.84	0.89	-180.64	357.426	0.68	-406.01	392.462	0.018
-157.7	361.899	0.05	-186.29	366.277	0.15	-472.73	326.06	0.041
-162.85	370.396	0.94	-192.34	375.077	0.37	-412.52	392.524	0.17
-167.68	377.944	0.084	-198.44	385.805	0.016	-419.28	392.392	0.18
-172.3	386.019	0.048	-204.54	395.162	0.07	-426.31	391.956	0.26
-178.57	392.775	0.07	-208.95	403.491	0.25	-497.17	375.196	0.23
-449.23	262.899	0.8	-214.52	401.735	0.27	-433.56	391	0.19
-81.049	393.664	0.037	-208.72	392.483	0.35	-440.53	389.721	0.29
-94.962	392.27	0.24	-202.22	382.616	0.08	-447.29	388.488	0.48
-106.66	391.59	0.039	-196.15	372.116	0.038	-454.48	387.163	0.94
-117.45	397.058	1	-189.89	361.332	0.53	-480.59	321.965	0.55
-147.45	359.035	0.16	-183.13	351.12	0.22	-462.22	385.773	0.053
-152.45	367.825	0.34	-171.6	332.043	0.081	-469.46	384.183	0.12
-156.88	375.447	0.05	-249.96	281.869	0.22	-476.05	382.569	0.051
-158.9	381.546	0.048	-242.55	286.501	0.36	-482.19	381.491	0.35
-186.84	392.436	0.057	-235.54	291.866	0.12	-488.97	378.505	0.062
-191.04	398.302	0.037	-228.74	298.924	0.35	-468.7	322.282	0.051
-426.89	296.517	0.049	-221.64	305.986	0.097	-455.24	325.957	0.12
-159.99	386.848	0.51	-217.96	312.245	0.46	-508.04	294.122	0.42
-146.6	388.766	0.079	-222.05	319.87	0.28	-497.89	304.933	0.59
-136.18	391.201	6.1	-225.94	327.838	0.34	-488.2	314.704	0.24
-117.92	388.812	0.03	-230.26	335.942	0.47	-513.56	363.852	0.32
-112.86	382.377	0.08	-235.01	344.678	0.99	-524.17	360.939	0.022

x - cord	*y* - cord	*k*	*x* - cord	*y* - cord	*k*	*x* - cord	*y* - cord	*k*
-100.36	385.185	0.12	-240.26	352.761	0.97	-310.48	402.022	0.27
-126.89	393.342	0.024	-245	360.082	0.78	-305	401.263	0.17
-141.21	363.652	0.037	-248.17	367.244	1	-300.07	400.314	0.99
-294.51	400.666	0.28	-206.76	350.601	0.31	-207.83	339.308	0.032
-289.69	401.401	0.51	-201.46	342.544	0.24	-212.56	347.357	0.89
-281.74	401.581	0.86	-196.58	334.458	0.69	-217.29	356.6	1
-226.47	388.429	0.99	-191.32	325.764	0.57	-222.23	367.215	0.94
-244.17	385.873	7.9	-186.51	316.147	0.11	-230.93	367.746	1
-252.27	385.481	0.68	-307.28	396.143	0.19	-448.54	278.697	0.8
-259.55	386.467	0.071	-493.35	318.857	0.038	-261.39	374.825	0.061
-181.35	321.437	0.57	-314.95	396.485	0.3	-244.04	269.691	0.83
-185.53	329.746	0.35	-324.38	395.956	0.1	-203.3	314.838	0.75
-190.58	338.171	0.92	-459.55	328.366	0.05	-206.74	322.311	0.092
-195.78	346.627	0.31	-334.54	394.885	0.16	-210.58	329.506	0.32
-202.4	356	0.31	-502.54	333.892	0.19	-214.68	336.718	0.96
-240.25	367.255	0.14	-497.82	336.458	0.23	-218.92	344.829	0.31
-207.92	366.392	0.99	-344.65	393.45	0.74	-223.34	353.337	0.78
-211.31	375.187	0.28	-492.06	341.335	0.83	-227.5	361.563	0.91
-214.74	382.299	6.3	-191.74	307.693	0.08	-455.58	266.659	0.78
-219.98	386.615	0.38	-199.59	299.051	0.18	-434.94	251.485	0.19
-149.87	395.228	0.021	-207.8	290.15	0.32	-423.87	246.461	0.14
-162.16	391.902	0.26	-216.58	280.384	0.037	-412.01	242.423	0.14
-171.49	394.458	0.11	-235.59	265.621	0.14	-399.77	240.975	0.037
-264.87	385.291	0.68	-245.57	262.914	0.14	-388.06	240.988	0.012
-316.55	400.492	0.75	-270.34	389.032	0.75	-378.93	245.201	0.39
-323.73	399.901	0.71	-278.37	390.579	0.64	-369.59	242.788	0.29
-330.79	398.978	0.36	-286.62	390.109	0.3	-357.77	244.526	0.16
-336.81	398.575	0.34	-499.26	365.438	0.13	-254.29	263.386	0.19
-342.98	397.631	0.12	-295.36	389.563	0.17	-245.4	275.721	0.66
-349.43	396.099	0.29	-302.47	390.242	0.13	-237.84	280.273	0.38
-356.28	394.718	0.6	-227.73	375.483	0.77	-230.81	286.429	0.56
-363.06	393.376	0.57	-237.22	373.528	0.92	-224.27	293.48	0.57
-369.82	392.083	0.76	-232.74	274.84	0.74	-217.45	300.387	0.83
-505.86	374.11	0.17	-224.44	282.219	0.71	-209.97	307.657	0.87
-257.46	379.836	0.93	-216.78	290.734	7.2	-212.13	315.341	0.3
-248.33	379.489	0.88	-208.51	299.216	0.87	-215.86	323.041	0.52
-238.42	379.812	0.74	-200.61	307.777	7.6	-219.66	330.36	0.27

x-cord	y-cord	k	x-cord	y-cord	k	x-cord	y-cord	k
-229.51	381.471	0.73	-194.32	315.369	0.92	-223.51	337.755	0.18
-220.13	378.483	0.23	-198.85	324.043	0.76	-231.57	353.142	0.053
-216.81	369.406	5.9	-203.24	331.911	0.56	-236.31	360.436	0.075
-436.98	268.439	0.6	-236.14	333.248	0.54	-248.96	328.723	0.49
-426.31	264.009	0.43	-240.27	340.679	0.14	-252.71	335.807	0.4
-431.4	258.005	0.15	-244.96	347.795	0.049	-257.42	343.208	0.052
-420.41	253.526	0.083	-249.93	355.32	0.9	-262.3	351.393	0.84
-408.54	250.338	0.15	-253.97	364.082	0.28	-266.21	359.642	0.26
-397.23	249.148	0.15	-254.13	373.221	0.9	-268.23	367.856	1
-386.66	249.197	0.14	-417.34	282.489	0.14	-267.2	376.846	0.79
-369.23	249.204	0.25	-407.74	278.759	0.18	-429.03	279.247	0.72
-413.71	259.729	0.25	-398.79	276.259	0.81	-404.69	285.679	0.15
-400.35	257.516	0.42	-389.64	274.751	0.23	-396.07	282.846	0.31
-387.96	256.883	0.21	-383.01	268.529	0.19	-380.86	274.81	0.11
-379.54	259.342	0.11	-347.1	253.176	0.13	-373.46	267.44	0.15
-442.98	274.995	0.82	-338.33	254.761	0.22	-371.65	260.652	0.85
-433.64	274.026	0.27	-263	272.861	0.96	-362.29	262.372	0.31
-419.36	275.875	0.45	-257.66	279.527	0.083	-353.32	263.965	0.067
-410.23	272.917	0.22	-253.6	288.024	0.05	-344.7	265.698	0.086
-401.79	270.661	0.1	-246.83	292.042	0.1	-335.81	267.04	0.08
-393.26	269.245	0.26	-245.17	302.011	0.11	-268.7	271.936	0.63
-322.67	258.473	0.081	-239.43	307.412	0.14	-260.31	286.413	0.053
-371.64	254.489	0.11	-233.78	312.612	0.11	-257.91	292.894	0.14
-357.18	251.33	0.12	-237	318.931	0.048	-256.18	301.793	0.2
-347.91	246.418	0.19	-240.27	326.096	0.15	-254.47	311.579	0.092
-339.2	248.216	0.42	-243.72	333.167	0.34	-249.87	317.02	0.24
-330.02	250.19	0.14	-247.81	340.206	0.11	-253.85	324.172	0.035
-321.87	251.188	0.062	-252.41	347.353	0.55	-257.39	330.48	0.064
-313.19	251.612	0.32	-256.24	353.324	0.059	-260.95	336.682	0.58
-303.63	252.437	0.07	-259.24	360.186	0.53	-264.55	342.826	0.61
-294.97	253.406	0.37	-261.23	367.596	0.63	-268.01	349.79	0.2
-285.74	254.657	0.11	-363.03	256.038	0.65	-271.77	357.981	0.03
-271.47	260.742	0.16	-355.04	257.524	0.12	-274.19	367.289	0.085
-260.8	262.464	0.45	-346.88	258.951	0.1	-273.8	376.226	0.4
-254.71	272.903	0.14	-338.3	260.564	0.12	-272.36	382.938	0.96
-240.18	297.137	0.44	-330.88	256.501	0.085	-462.71	267.248	0.077
-233.42	303.352	0.12	-251.25	297.007	0.24	-435.14	285.365	0.26

x - cord	y - cord	k	x - cord	y - cord	k	x - cord	y - cord	k
-226.74	310.452	0.26	-249.81	306.918	0.12	-426.82	285.645	0.35
-228.81	318.075	0.13	-243.12	312.783	0.071	-414.48	289.297	0.85
-232.36	326.049	0.083	-245.16	320.6	0.25	-387.52	281.358	0.36
-372.66	275.24	0.096	-313.24	263.725	0.069	-278.88	384.505	5.6
-366.59	268.612	0.092	-303.21	264.756	0.96	-312.33	268.933	0.1
-358.99	269.434	0.24	-292.44	266.214	0.11	-302.04	270.135	0.88
-349.54	271.058	0.049	-282.03	267.886	0.22	-292.13	271.387	0.076
-338.7	272.939	0.086	-274.15	276.723	0.17	-273.8	293.403	0.046
-496.13	287.146	0.12	-265.75	287.956	0.38	-275.37	300.875	0.074
-329.64	263.165	0.035	-268.31	296.287	0.24	-274.16	309.597	0.5
-321.88	265.429	0.012	-267.74	304.372	0.22	-277.46	313.736	0.095
-313.49	257.957	0.31	-262.89	314.734	0.51	-381.82	293.396	0.44
-303.17	259.082	0.22	-267.49	322.624	0.19	-367.97	288.864	0.17
-292.95	260.331	0.2	-271.65	329.558	0.57	-360.69	288.894	0.13
-283.37	261.64	0.12	-275.12	335.958	0.22	-352.4	288.8	0.98
-275.13	266.927	0.11	-278.07	342.621	0.15	-342.53	289.608	0.15
-266.25	280.118	0.041	-276.11	353.645	0.77	-397.78	304.352	0.44
-262.35	298.436	0.43	-409.73	301.431	0.39	-286.79	347.844	0.68
-260.21	306.803	0.032	-399.49	298.656	0.53	-469.13	278.489	0.062
-257.35	318.166	0.19	-390.67	295.514	0.11	-460.54	274.462	0.15
-261.43	324.649	0.047	-376.24	288.26	0.7	-454.48	286.432	0.11
-265.42	331.092	0.44	-361.08	282.764	0.49	-439.83	287.978	0.78
-269.14	337.841	0.7	-281.75	273.202	0.24	-314.74	273.675	0.36
-272.53	345.412	0.21	-273.35	285.64	0.76	-306.41	275.195	0.18
-278.97	362.31	0.44	-268.14	312.163	0.04	-297.92	276.362	0.25
-280.18	370.192	0.84	-272.7	319.687	0.66	-289.65	277.511	0.15
-280.29	377.376	3.6	-276.89	326.637	0.82	-281.44	278.812	0.17
-421.47	294.121	0.23	-280.5	333.596	0.12	-280.87	284.935	0.37
-411.36	295.366	0.21	-283.86	340.694	0.041	-280	291.641	0.31
-401.24	292.446	0.16	-280.86	348.927	0.14	-280.98	298.827	0.2
-392.69	289.398	0.15	-282.87	355.362	0.18	-280.63	307.159	0.44
-384.8	287.971	0.23	-284.8	362.089	0.52	-279.27	320.12	0.054
-379.04	281.691	0.19	-285.75	369.841	0.58	-283	327.784	0.07
-370.48	282.204	0.27	-285.91	377.494	0.53	-287.17	335.64	0.2
-363.54	276.011	0.15	-284.63	384.743	0.05	-290.74	343.583	0.58
-354.72	276.352	0.33	-288.72	355.037	0.082	-293.33	351.248	5.2
-346.86	277.384	0.27	-290.03	361.543	0.26	-456.65	280.369	0.54

x - cord	*y* - cord	*k*	*x* - cord	*y* - cord	*k*	*x* - cord	*y* - cord	*k*
-338.33	278.421	0.92	-350.53	283.388	0.042	-432.25	298.366	0.39
-329.85	271.376	0.82	-340.96	284.413	0.15	-420.49	304.274	0.41
-322.56	271.737	0.13	-333.87	282.212	0.087	-405.98	312.019	0.24
-396.67	309.701	0.34	-289.02	316.616	0.41	-302.6	330.291	0.13
-387.77	306.879	0.51	-292.62	322.843	0.8	-306.24	336.848	0.18
-372.25	294.169	0.074	-296.16	329.339	0.38	-303.81	349.261	0.18
-327.5	278.416	0.89	-302.77	342.585	0.66	-304.39	356.366	0.1
-320.16	279.285	0.51	-299.5	357.594	0.11	-477.94	293.123	0.068
-312.61	280.57	0.59	-295.3	371.115	0.36	-460.03	300.563	0.099
-304.39	281.81	0.092	-380.4	374.482	0.061	-443.25	311.298	0.096
-296.45	282.984	0.27	-580.3	364.519	0.2	-438.19	309.858	0.19
-288.58	284.071	0.2	-497.32	348.501	0.49	-427.19	312.909	0.12
-363.9	294.529	0.088	-503.4	343.006	0.33	-333.47	302.542	0.16
-356.32	294.019	0.091	-467.96	284.667	0.048	-328.35	296.408	0.094
-348.67	294.444	0.81	-443.31	291.341	0.078	-321.81	297.528	0.086
-341.45	295.357	0.18	-437.16	301.558	0.055	-315.37	298.436	0.88
-329.43	285.244	0.12	-483.46	283.63	0.11	-308.66	299.175	0.22
-323.07	285.635	0.057	-452.04	292.67	0.14	-302.25	299.866	0.17
-315.94	286.679	0.14	-449.05	298.619	0.078	-297.23	301.853	0.066
-308.67	287.547	0.1	-418.3	309.62	0.65	-295.2	317.309	0.11
-301.38	288.333	0.11	-379.6	298.74	0.54	-476.75	300.66	0.14
-293.93	289.409	0.12	-371.04	299.309	0.074	-471.45	297.876	0.075
-286.71	290.548	0.079	-362.62	299.798	0.45	-423.51	317.677	0.24
-286.74	296.091	0.07	-354.78	299.855	0.4	-415.81	317.612	0.47
-286.7	302.561	0.12	-347.24	300.289	0.79	-380.28	305.31	0.2
-285.61	310.443	0.042	-339.71	301.298	0.22	-374	304.935	0.17
-283.32	317.247	0.18	-334.87	295.689	0.19	-367.16	305.105	0.93
-287.18	324.246	0.37	-332.83	290.786	0.24	-360.27	305.314	0.62
-290.5	330.402	0.13	-464.68	290.623	0.17	-353.42	305.443	0.68
-293.62	336.364	0.079	-462.41	295.918	0.095	-346.32	305.773	0.24
-296.37	342.271	0.17	-445.38	303.581	0.093	-339.61	306.832	0.11
-298	349.233	0.73	-430.01	307.965	0.078	-332.97	308.764	0.65
-294.81	359.392	0.3	-383.72	312.128	0.21	-328.17	303.006	0.17
-290.8	368.458	0.046	-324.14	291.376	0.48	-322.28	303.473	0.21
-590.46	356.017	0.04	-316.64	292.405	0.76	-315.75	304.266	0.2
-290.76	376.129	0.18	-309.24	293.392	0.56	-309.08	305.043	0.3
-289.98	383.774	0.14	-301.74	293.994	0.24	-302.53	305.775	0.18

x-cord	*y*-cord	*k*	*x*-cord	*y*-cord	*k*	*x*-cord	*y*-cord	*k*
-585.69	360.426	0.39	-294.18	294.696	0.39	-489.44	294.631	0.3
-292.45	301.212	0.42	-299.96	336.125	0.19	-457.24	304.958	0.57
-290.81	309.664	0.096	-299.12	323.879	0.2	-377.1	311.631	1
-321.91	333.678	0.12	-436.08	382.573	0.084	-434.12	335.696	0.73
-328.19	323.272	0.72	-443.11	381.062	0.075	-441	328.446	0.062
-322.11	324.517	0.22	-449.87	379.835	0.057	-420.74	339.77	0.69
-451.39	320.079	0.074	-456.85	378.58	0.064	-411.41	339.017	0.089
-447.23	323.525	0.1	-463.83	377.361	0.042	-403.55	338.968	0.44
-419.07	326.854	0.12	-471.03	375.571	0.026	-393.86	348.248	0.073
-410.65	324.582	0.17	-478.36	373.73	0.11	-386.93	347.269	0.73
-428.92	326.521	0.53	-486.15	371.208	0.27	-380.55	345.59	0.16
-478.8	312.307	0.21	-375.09	386.747	0.79	-373.7	344.124	0.2
-464.02	319.239	0.069	-473.45	315.713	4.5	-366.44	342.563	0.41
-463.31	381.361	0.097	-424.24	335.333	0.22	-358.7	340.843	0.87
-471.72	379.482	0.071	-414.78	335.049	0.16	-350.28	339.852	0.36
-480.63	377.466	0.47	-406.5	333.449	0.46	-341.08	339.308	0.47
-434.97	328.903	0.49	-396.53	338.422	0.1	-332.23	340.35	0.38
-426.3	330.576	0.37	-389.33	337.108	0.048	-325.91	349.789	0.76
-416.66	330.893	2.7	-382.7	335.439	0.78	-319.13	357.601	0.28
-408.57	328.748	0.16	-375.85	334.343	0.22	-353.43	385.519	0.37
-398.11	333.242	0.25	-368.53	333.659	0.059	-366.31	383.97	3.9
-390.92	331.975	0.11	-361.4	332.447	0.054	-373.73	382.993	0.79
-383.98	330.192	5.8	-354.18	331.439	1.4	-381.52	381.916	0.16
-376.7	329.099	0.091	-346.97	330.965	0.33	-389.23	380.855	0.18
-369.46	328.462	0.1	-339.57	330.505	0.19	-397.05	379.971	0.19
-362.74	327.591	0.22	-331.49	331.084	0.15	-404.73	379.616	0.47
-355.52	326.843	0.082	-322.59	342.997	0.14	-412.04	379.278	5.5
-347.66	326.465	0.41	-303.6	386.052	0.43	-418.69	379.346	0.29
-340.12	326.082	0.11	-395.41	343.457	0.62	-402.02	344.737	0.037
-331.02	327.039	0.083	-388.8	342.467	1	-410	343.488	0.39
-325.69	332.011	0.14	-382.33	340.786	0.19	-417.73	344.61	0.52
-489.46	373.507	4.3	-375.49	339.369	0.23	-428.34	341.514	0.99
-499.03	370.203	0.83	-367.96	338.37	0.91	-438.87	338.456	5
-508.14	369.521	0.025	-360.11	336.681	0.11	-332.32	344.909	0.063
-516.83	367.87	0.07	-352.21	335.704	0.55	-326.85	354.381	2.8
-525.44	366.55	0.065	-344.47	335.306	0.62	-311.14	361.212	0.31
-407.59	383.373	0.1	-336.71	334.927	0.12	-425.05	379.549	0.048

x-cord	*y*-cord	*k*	*x*-cord	*y*-cord	*k*	*x*-cord	*y*-cord	*k*
-415.08	383.329	0.46	-329.38	337.238	0.063	-431.62	379.274	0.023
-422.02	383.562	0.91	-323.43	346.369	0.32	-438.08	378.027	0.61
-428.83	383.496	0.66	-445.3	331.351	0.24	-444.24	376.768	0.11
-451.01	375.214	0.097	-444.96	344	0.58	-373.06	375.12	0.24
-457.39	373.691	0.031	-436.04	346.897	1	-365.93	375.669	0.54
-463.59	372.336	0.045	-626.9	338.344	0.11	-518.68	351.237	0.095
-469.87	371.668	0.87	-502.92	308.691	0.56	-511.15	355.812	0.73
-477.21	369.652	0.76	-502.03	324.883	0.16	-520.04	343.734	0.62
-484.74	367.822	0.59	-489	327.624	0.63	-519.39	335.555	0.25
-448.84	334.339	0.017	-480.82	332.404	0.15	-519.03	327.009	0.46
-442.51	340.88	0.068	-464.59	334.276	0.51	-517.58	317.77	0.29
-433.02	343.938	0.059	-484.3	335.638	0.43	-542.12	332.865	0.14
-424.8	346.383	3.3	-468.31	336.825	0.74	-543.6	340.955	0.1
-636.57	337.519	0.12	-459.7	342.227	0.49	-545.02	348.931	0.12
-411.22	349.774	0.95	-410.83	375.172	0.051	-546.24	356.216	0.18
-401.01	348.873	6.5	-417.34	374.872	0.096	-547.15	331.852	0.4
-372.48	348.838	0.36	-423.74	375.029	0.13	-548.56	340.384	0.25
-365.63	347.015	0.046	-430.04	375.157	0.15	-550.34	348.242	0.8
-358.33	345.266	0.1	-644.49	342.787	0.54	-501.9	355.809	0.1
-349.91	343.578	0.73	-436.76	374.015	0.084	-509.27	351.275	0.34
-340.61	342.854	0.51	-465.84	367.863	0.11	-515.23	345.951	0.35
-332.73	351.886	0.47	-472.95	366.703	0.031	-322.1	384.25	0.73
-328.34	358.075	0.082	-485.8	361.765	0.74	-331.09	383.163	0.11
-320.37	361.668	0.7	-491.21	361.08	0.71	-339.83	382.234	0.33
-311.85	364.853	4.3	-503.6	359.57	3.1	-348.31	380.326	0.75
-497.42	322.518	0.43	-515.24	358.165	0.46	-513.82	337.836	0.39
-476.4	330.162	0.2	-621.56	337.008	0.19	-513.34	330.232	0.37
-462.29	331.299	0.72	-524.83	354.502	0.18	-512.86	322.495	0.43
-452.25	337.188	0.31	-525.93	345.127	0.46	-509.37	315.074	0.16
-491.3	369.165	0.089	-525.55	335.709	0.28	-540.63	325.48	0.057
-496.23	359.876	0.43	-524.81	325.619	0.26	-539.71	317.59	0.13
-505.61	364.144	0.51	-523.16	316.135	0.1	-539.25	309.347	0.16
-298.04	364.538	0.23	-488.97	352.314	1	-547.16	308.641	0.044
-366.11	379.708	0.34	-435.93	370.248	0.056	-544.5	315.5	0.32
-373.36	379.015	0.084	-428.77	370.769	0.078	-545.63	324.158	0.3
-380.86	378.11	0.064	-421.94	370.76	0.063	-550.96	355.988	0.21
-388.28	377.166	0.058	-414.34	370.976	0.049	-555.94	356.051	0.22

x-cord	y-cord	k	x-cord	y-cord	k	x-cord	y-cord	k
-395.81	376.316	0.097	-406.51	371.541	0.41	-555.62	347.71	0.62
-403.51	375.735	0.069	-398.44	372.281	3.8	-553.35	339.451	0.81
-454.9	340.137	0.14	-389.05	373.475	1.4	-552.23	330.647	0.12
-550.42	322.563	0.12	-561.03	346.994	0.67	-358.25	372.304	0.17
-549.54	314.595	0.14	-562.01	338.159	1.5	-365.76	367.939	0.18
-615.39	335.723	0.23	-561.46	329.172	0.21	-373.34	367.626	0.95
-500.09	351.911	0.68	-555.52	321.138	0.26	-381.45	367.017	0.54
-508.04	347.177	0.97	-554.84	313.422	0.063	-389.65	366.105	0.79
-508.58	340.72	0.79	-462.89	345.661	0.45	-434.62	354.561	0.82
-507.68	332.259	0.92	-452.32	347.587	0.42	-426.14	353.036	0.87
-507.19	324.109	1	-441.08	352.954	0.15	-420.76	350.056	3.8
-557.92	338.779	0.18	-566.1	352.881	0.16	-575.02	333.335	0.93
-556.83	329.42	1.3	-566.46	345.178	0.26	-564.88	306.358	0.25
-495.26	331.661	1	-566.67	337.044	0.25	-574.26	349.653	0.23
-486.8	339.741	0.95	-560.51	320.681	0.18	-574.79	341.383	1
-356.72	379.065	0.2	-560.1	312.211	0.058	-445.92	370.368	6.5
-339.7	378.942	0.28	-466.24	348.593	0.47	-451.49	370.462	1.4
-330.85	379.738	0.71	-455.33	350.744	0.29	-458.58	368.605	2.6
-365.65	371.755	0.13	-358.14	375.881	0.18	-482.12	347.947	1.7
-373.03	371.297	0.39	-348.84	377.02	0.095	-411.45	347.195	2.1
-380.52	370.762	0.13	-339.6	375.534	0.24	-391.43	351.676	0.88
-388.53	370.072	0.52	-330.82	376.238	0.34	-381.35	349.888	4.2
-396.51	368.973	0.33	-321.9	380.656	0.71	-397.4	365.131	0.66
-404.62	367.951	0.3	-570.24	350.573	0.15	-405.3	364.178	0.18
-412.27	366.944	0.29	-553.65	300.01	0.22	-413.39	362.952	0.6
-418.72	366.42	0.098	-570.9	338.281	0.59	-421.7	362.281	0.33
-424.55	366.329	0.18	-568.23	330.425	6.8	-431.49	362.62	0.26
-430.42	366.235	0.21	-565.45	324.385	0.47	-441.02	364.478	0.16
-437.73	366.361	0.14	-564.93	315.558	0.1	-559.24	301.331	0.13
-442.16	371.37	0.18	-557.49	306.464	0.22	-561.31	297.691	0.036
-447.64	346.633	1	-547.34	304.314	0.089	-328.87	361.583	0.36
-438.74	349.876	1	-544.47	301.633	0.074	-319.93	365.499	0.98
-429.72	349.785	0.36	-507.07	312.068	0.65	-309.88	368.878	0.79
-474.62	342.944	0.85	-478.11	345.689	1	-299.23	370.735	0
-471.55	340.014	0.83	-482.82	356.186	0.18	-302.59	370.088	0.39
-518.97	309.47	0.23	-484.44	350.176	0.76	-480.15	365.157	0.06
-495.6	344.435	0.42	-469.52	351.397	11	-313.05	384.127	0.78

x-cord	y-cord	k	x-cord	y-cord	k	x-cord	y-cord	k
-491.77	355.728	0.79	-458.47	353.844	0.95	-471.86	354.24	1.8
-560.73	360.506	0.13	-448.54	353.247	3.1	-461.26	356.812	0.39
-560.82	354.835	0.17	-348.82	374.171	0.68	-568.49	297.646	0.059
-569.09	306.303	0.27	-314.64	373.364	0.093	-390.98	354.382	0.9
-569.13	315.267	0.036	-305.72	375.534	0.045	-380.76	352.427	0.36
-569.75	323.868	0.63	-357.85	365.694	0.15	-368.44	351.808	0.99
-579.21	333.862	0.085	-349.35	367.945	0.62	-359.23	349.368	0.95
-578.95	344.583	0.67	-339.98	369.109	0.39	-349.97	346.848	0.69
-431.71	357.405	0.37	-323.44	373.733	0.18	-340.98	346.251	0.87
-440.06	360.934	6.4	-472.1	360.269	0.5	-335.93	350.645	0.43
-447.89	365.088	1	-454.11	362.512	0.8	-338.38	359.754	0.88
-457.41	365.222	6.1	-444.07	359.409	0.48	-586.47	336.862	0.67
-467.89	363.644	0.81	-405.47	360.704	0.22	-586.76	327.307	0.25
-477.64	362.038	0.39	-397.51	361.453	0.34	-587.34	318.466	0.64
-474.45	356.465	0.31	-389.71	362.485	0.9	-587.97	310.108	0.24
-463.57	359.427	0.79	-381.88	360.137	5.5	-588.77	302.674	0.075
-453.7	358.752	0.48	-373.79	360.641	0.72	-589.69	295.711	0.05
-445.39	356.054	0.31	-365.81	361.013	0.42	-597.11	294.597	0.17
-330.1	364.423	0.17	-357.61	362.568	0.25	-606.08	292.981	0.38
-321.01	368.251	0.27	-348.38	365.153	0.98	-614.88	291.674	0.13
-310.61	371.467	0.82	-339.03	366.394	0.69	-417.22	357.464	0.66
-294.85	379.105	0.12	-582.76	340.993	0.29	-397.56	357.877	5
-305.29	379.146	0.28	-582.8	331.736	4.9	-390.22	358.83	0.96
-313.27	380.431	0.6	-578.87	324.966	0.21	-415.28	355.258	0.054
-321.89	377.135	0.69	-578.51	317.351	0.16	-406.78	354.684	0.2
-331.85	371.797	0.83	-578.38	309.582	0.28	-399.61	354.8	0.25
-339.71	372.219	0.66	-578.57	301.956	0.05	-341.48	359.951	0.44
-349.09	371.061	0.52	-578.72	294.632	0.19	-339.61	351.767	0.78
-357.92	369.021	0.32	-335.33	360.184	6.2	-345.42	349.815	0.25
-365.88	364.412	0.37	-582.9	323.549	0.87	-589.96	333.21	0.47
-373.77	364.058	0.12	-582.89	315.417	0.27	-590.91	323.42	0.85
-381.67	363.56	0.32	-583.13	308.071	0.1	-592.13	314.723	0.1
-574.54	325.531	0.47	-583.76	300.967	0.1	-593.19	305.776	0.23
-573.91	317.914	0.23	-584.41	294.204	0.064	-596.86	299.093	0.51
-573.46	310.639	0.24	-427.32	359.667	5.2	-606.88	297.317	0.8
-573.58	303.481	0.067	-415.06	359.895	3.1	-616.87	295.641	0.33
-573.47	295.82	0.11	-405.71	357.404	6.3	-387.76	356.477	0.57

x-cord	*y*-cord	*k*	*x*-cord	*y*-cord	*k*	*x*-cord	*y*-cord	*k*
-313.87	376.74	0.15	-423.65	355.281	0.85	-378.04	354.195	0.98
-331.12	367.708	0.87	-412.86	352.547	0.62	-377.53	356.623	0.7
-323.66	370.584	0.06	-401.88	352.176	0.53	-368.45	355.022	0.73
-360.86	353.232	0.95	-612.38	304.271	1.8	-598.88	354.045	0.38
-353.68	351.096	0.32	-620.78	302.148	0.39	-631.64	339.374	0.92
-349.02	353.771	0.1	-599.24	325.568	0.85	-635.47	370.203	0.3
-344.07	355.023	0.88	-622.08	329.501	0.47	-640.72	345.097	0.89
-345.59	361.119	0.25	-628.84	330.346	0.23	-612.3	339.295	0.92
-625.03	325.093	0.99	-635.14	330.996	0.098	-617.78	340.506	0.88
-631.67	325.98	0.06	-641.06	332.483	0.12	-622.46	341.5	0.6
-638.49	326.639	0.17	-645.38	336.329	0.15	-627.1	342.711	0.59
-645.39	329.49	0.09	-649.04	341.084	0.2	-632.21	343.659	0.19
-651.02	334.612	0.23	-651.22	346.515	0.25	-636.24	345.123	0.49
-655.4	340.543	0.062	-652.9	352.183	0.24	-637.67	349.043	0.083
-604.16	301.948	0.21	-652.17	358.409	0.17	-642.72	350.08	0.21
-612.34	300.207	0.51	-649.81	364.222	0.14	-642.34	355.365	0.21
-621.81	298.492	0.096	-645.1	370.928	0.15	-640.66	359.973	0.75
-371.9	358.018	0.089	-639.37	376.947	0.44	-638.32	364.588	0.61
-363.3	357.49	0.49	-626.91	377.313	0.036	-627.66	371.862	0.5
-618.17	326.186	4	-617.89	377.956	0.4	-619.14	372.466	0.55
-513.8	297.773	0.33	-606.99	377.684	0.31	-611.58	372.234	0.52
-518.19	302.241	0.4	-595.78	377.661	0.12	-604.08	371.65	0.36
-348.96	359.662	0.45	-617.39	357.451	0.053	-595.54	371.628	0.073
-355.57	360.545	0.86	-618.82	353.823	0.37	-584.16	377.45	0.055
-354.98	358.241	0.11	-601.48	317.63	0.59	-586.45	371.13	0.12
-354.98	355.147	0.067	-602.6	311.458	0.66	-577.08	372.989	0.39
-593.98	330.947	0.057	-608.57	309.405	0.59	-593.39	365.657	0.39
-657.96	347.315	0.079	-616.69	307.201	0.13	-598.6	360.622	0.65
-658.88	354.681	0.12	-640.92	369.581	1.6	-602.22	356.5	0.93
-602.67	342.508	0.73	-644.74	362.998	0.13	-602.28	349.939	0.78
-598.28	347.08	0.9	-646.9	357.894	0.1	-605.93	346.287	0.59
-594.42	351.628	0.31	-647.67	352.782	0.25	-608.96	342.647	0.28
-657.55	360.736	0.2	-646.74	347.641	0.079	-614.25	344.183	0.91
-655.33	366.311	0.16	-604.98	320.537	0.64	-619.41	345.506	0.75
-650.59	371.859	0.13	-611.6	311.778	0.12	-624.54	346.537	0.74
-595.31	323.906	0.8	-609.15	316.877	0.4	-629.22	347.373	0.34
-596.43	317.759	0.21	-585.21	366.966	0.37	-632.58	348.584	0.96

x-cord	*y*-cord	*k*	*x*-cord	*y*-cord	*k*	*x*-cord	*y*-cord	*k*
-597.44	311.37	0.22	-640.38	339.28	0.35	-638.04	352.803	0.11
-597.86	304.579	0.027	-590.89	362.455	0.036	-636.88	357.119	0.051
-604.49	306.126	0.94	-595.28	358.268	0.74	-634.41	362.408	0.14
-630.05	367.32	0.9	420.586	321.279	0.35	209.76	392.519	0.82
-621.96	367.52	0.96	404.551	322.95	0.56	202.523	393.372	0.84
-614.43	367.443	0.94	391.098	322.77	0.24	193.863	393.337	0.55
-606.64	366.692	0.37	379.888	319.24	0.09	184.892	392.903	0.11
-597.89	366.894	0.14	367.795	321.43	0.12	176.132	393.411	0.83
-601.19	362.285	0.81	362.41	325.069	0.058	167.177	394.619	0.17
-605.35	358.594	0.57	357.284	329.467	0.75	159.257	393.997	0.49
-605.84	352.28	0.82	352.64	335.955	0.54	154.018	397.924	0.33
-609.6	347.866	0.87	348.829	342.15	0.095	148.237	400.729	0.38
-614.25	348.657	0.55	340.548	343.06	0.13	136.41	401.016	0.12
-633.29	353.348	0.37	331.179	346.214	0.12	123.634	405.323	0.46
-631.79	357.987	0.93	326.239	353.625	0.25	113.138	405.644	0.32
-629.78	362.689	0.54	320.2	360.135	0.13	102.176	405.67	0.047
-625.09	363.044	0.8	311.344	363.728	0.31	389.184	225.274	0.37
-628.96	353.429	0.57	303.904	366.593	0.31	109.028	364.291	0.12
-626.87	358.491	0.49	294.77	368.6	0.26	120.821	360.938	0.081
-618.92	363.739	0.59	283.794	368.613	0.4	396.588	216.911	0.13
-612.44	363.563	0.24	274.138	371.505	0.09	405.154	212.97	0.086
-606.04	363.208	0.77	266.728	376.273	0.076	421.517	208.918	0.23
-608.55	359.885	0.64	556.24	194.422	0.076	149.463	351.189	0.085
-608.63	354.619	0.19	546.759	182.414	1	158.103	346.832	0.12
-612.57	352.053	0.26	537.699	170.568	0.39	166.648	336.569	0.094
-619.59	349.743	0.41	529.477	159.189	0.9	510.438	115.874	0.054
-625.56	350.838	0.063	513.437	135.721	0.78	551.114	198.695	0.4
-624.21	354.732	0.46	503.231	126.307	0.4	533.598	177.083	0.79
-622.22	359.556	0.85	447.458	314.712	0.63	515.317	151.986	1
-615.35	360.504	0.42	428.578	319.504	0.99	506.041	138.677	0.93
-611.93	356.218	0.88	412.965	321.835	0.46	456.433	307.503	0.6
269.35	295.319	0.58	397.49	323.202	0.085	446.338	309.954	0.58
284.94	287.05	0.1	385.172	320.969	0.062	437.202	312.198	0.55
550.05	268.61	0.76	373.691	318.701	0.12	427.895	313.943	0.36
534.21	276.304	0.27	260.244	379.238	0.086	419.216	315.176	0.32
398.4	225.065	0.048	249.049	383.636	0.11	411.352	316.041	0.81
407.17	220.225	0.33	242.349	386.222	0.12	403.969	316.266	0.66

x-cord	*y*-cord	*k*	*x*-cord	*y*-cord	*k*	*x*-cord	*y*-cord	*k*
414.244	215.643	0.13	235.834	388.749	0.14	396.944	316.288	0.77
458.019	311.688	0.24	227.236	392.249	0.11	389.288	316.307	0.34
438.312	317.005	0.39	218.044	391.805	0.11	381.022	313.534	0.088
372.816	313.73	0.085	384.76	255.568	0.41	176.457	379.429	0.14
365.248	315.674	0.11	315.067	285.919	0.71	563.707	202.684	0.6
359.662	317.931	0.16	229.951	326.036	0.34	543.33	202.151	0.99
355.044	321.848	0.14	224.187	329.452	0.32	536.334	192.717	0.82
351.098	327.086	0.23	218.383	333.29	0.29	520.807	171.459	0.47
347.087	332.179	0.12	381.184	306.544	0.15	512.353	159.315	0.039
343.596	336.841	0.16	374.834	307.04	0.056	503.143	146.701	0.6
336.76	337.91	0.23	368.5	307.792	0.26	491.546	131.203	0.25
329.496	339.599	0.47	361.588	308.797	0.55	497.886	231.367	0.73
324.566	345.159	0.64	355.441	312.049	0.37	552.629	207.911	0.4
319.873	353.581	0.13	350.058	317.08	0.25	543.76	271.529	0.094
312.703	357.418	0.31	345.278	322.27	0.067	462.468	303.146	0.68
304.488	360.416	0.12	341.593	327.65	0.059	518.391	128.724	0.13
296.482	362.996	0.3	336.301	331.378	0.75	526.187	138.9	0.1
287.751	363.91	0.45	480.167	222.661	0.74	532.832	148.318	0.055
276.622	364.946	0.86	329.11	332.856	0.38	538.638	156.719	0.51
266.348	368.97	0.66	322.793	334.943	0.65	544.534	165.792	0.24
257.577	373.117	0.38	319.558	341.17	0.16	551.669	175.366	0.2
248.454	372.923	0.51	317.72	347.639	0.79	558.413	183.855	0.059
240.657	377.207	0.85	311.553	351.777	0.081	564.317	192.298	0.07
232.279	382.968	0.22	304.669	353.89	0.045	537.487	280.742	0.062
222.364	386.363	0.25	298.731	355.814	0.31	454.625	302.535	0.86
211.857	386.32	0.11	292.294	357.87	0.099	445.468	304.917	0.22
201.723	386.746	0.15	286.33	358.229	0.48	437.201	307.024	0.38
191.897	386.652	0.012	277.632	358.373	0.98	429.657	308.409	0.23
180.378	386.38	0.31	269.183	360.757	1	421.724	309.881	0.059
168.196	387.808	1	261.561	363.901	0.67	413.154	310.685	0.26
499.299	120.856	0.41	254.254	366.692	0.74	404.569	310.226	1
470.128	222.795	0.17	245.297	366.165	0.87	396.517	309.978	0.48
471.499	136.453	0.98	238.002	370.898	0.41	388.568	310.456	0.089
454.904	144.739	0.098	229.738	376.984	0.28	381.078	299.305	0.46
438.38	154.385	0.24	220.066	379.697	0.056	373.381	299.747	0.46
423.101	250.565	0.98	209.986	379.692	0.23	365.939	300.622	0.43
415.151	253.753	0.3	200.614	379.819	0.41	358.596	302.338	0.82

x - cord	*y* - cord	*k*	*x* - cord	*y* - cord	*k*	*x* - cord	*y* - cord	*k*
407.234	257.66	0.38	191.619	379.478	0.42	351.509	305.501	0.11
398.001	260.491	0.23	183.813	379.349	0.31	467.19	129.558	0.47
389.112	264.196	0.31	169.795	380.348	0.12	345.697	310.48	0.26
340.51	316.308	0.51	414.972	304.53	0.17	440.862	294.045	0.44
335.774	322.71	0.3	408.498	304.399	0.27	296.21	341.34	1.6
453.847	136.458	0.44	400.914	303.719	0.69	288.252	343.029	0.13
327.794	325.672	0.28	393.646	303.675	0.81	280.23	344.626	0.06
318.895	329.725	0.12	387.482	301.994	0.06	272.081	346.741	1
537.741	206.566	0.51	380.963	292.681	0.29	264.267	348.962	0.47
530.303	196.831	0.28	372.633	292.968	0.68	256.132	351.89	0.46
522.654	187.209	1	364.176	293.731	0.32	248.738	355.281	0.77
515.597	177.016	0.11	355.808	295.439	1	242.152	360.213	0.73
509.326	167.325	0.17	347.702	299.074	0.44	235.249	365.511	0.055
502.839	157.847	0.13	340.293	304.872	0.27	227.594	371.366	0.41
495.288	146.921	0.61	334.673	311.345	0.61	560.299	255.729	0.059
484.798	134.177	0.76	330.01	315.908	0.23	551.184	248.168	0.094
530.522	271.514	0.44	326.571	317.139	0.11	533.805	250.516	0.11
475.342	302.106	0.6	319.808	320.921	0.19	526.23	257.234	0.4
468.397	307.986	0.042	312.704	325.702	1	518.148	263.35	0.21
452.803	296.993	0.28	312.897	335.61	0.084	476.808	295.432	0.17
443.563	299.492	0.44	311.355	344.911	0.068	472.438	291.042	0.23
435.383	301.487	0.24	304.369	346.547	0.45	461.211	288.535	0.42
290.969	349.887	0.17	297.8	348.357	0.36	569.095	247.967	0.2
283.403	350.695	0.28	463.151	122.53	0.46	560.87	246.08	0.055
275.388	352.202	0.17	451.093	129.058	0.28	547.718	236.595	0.45
266.993	354.924	0.31	436.815	137.063	0.79	232.559	316.113	0.54
259.177	357.647	0.15	307.636	278.104	0.74	306.197	331.46	0.65
251.696	360.795	0.47	437.465	145.569	0.22	305.268	339.926	0.047
163.024	382.46	0.27	257.675	304.927	0.051	285.694	335.966	0.54
553.461	264.194	0.047	248.613	308.304	0.18	277.578	338.084	0.43
542.078	262.819	0.39	240.099	311.913	0.094	269.048	340.116	0.48
482.746	251.825	0.18	524.323	201.501	0.51	260.868	342.85	0.26
527.286	265.069	0.62	516.851	192.05	0.2	253.153	345.854	0.55
518.247	272.088	0.059	509.314	181.401	1	245.704	349.374	0.28
525.175	278.981	0.42	502.745	171.256	0.54	238.622	354.457	0.11
483.311	295.308	0.32	496.344	160.817	0.47	231.881	360.55	0.075
468.581	299.174	0.52	432.187	212.579	0.3	225.354	365.664	0.31

x - cord	*y* - cord	*k*	*x* - cord	*y* - cord	*k*	*x* - cord	*y* - cord	*k*
462.009	294.468	0.51	557.007	259.802	0.26	217.869	363.913	0.23
427.975	302.998	0.13	549.388	255.126	0.36	155.814	384.924	0.41
421.327	303.836	0.23	450.545	291.444	0.25	218.157	372.968	0.18
208.877	373.086	0.03	307.926	320.681	0.38	273.377	331.036	0.43
200.915	373.26	0.19	541.731	246.436	0.5	281.881	329.396	0.73
192.914	372.798	0.38	522.406	252.062	0.54	485.946	184.481	0.44
185.33	372.393	0.32	513.947	258.548	0.31	473.43	168.981	0.45
177.385	372.299	0.25	510.693	268.879	0.19	473.091	156.708	0.92
169.16	372.955	0.19	472.517	284.894	0.42	201.642	330.704	0.18
160.583	375.41	0.4	462.561	281.446	0.13	346.009	282.796	0.39
509.766	193.986	0.17	453.455	276.281	0.18	352.936	280.104	0.93
501.067	182.727	0.44	443.961	278.801	0.11	572.21	240.774	0.23
481.41	311.921	0.057	434.314	281.446	0.27	551.805	242.756	0.43
204.123	337.649	0.098	432.651	295.009	0.06	530.896	244.573	0.46
195.99	337.735	0.18	425.887	295.889	0.22	518.376	246.858	0.42
187.968	337.876	0.31	419.599	296.832	0.15	510.412	252.607	0.36
177.624	338.591	0.71	413.488	297.912	0.4	442.179	271.801	0.72
442.95	217.108	0.21	406.684	297.635	0.2	431.871	274.777	0.46
156.586	340.999	0.78	399.487	296.852	0.44	428.311	287.054	0.75
144.24	346.495	0.17	393.374	294.231	0.51	388.335	293.55	0.89
453.345	220.465	0.28	299.264	333.222	0.48	142.36	402.323	0.17
462.502	217.699	0.14	291.951	333.531	0.15	123.85	398.619	0.32
431.062	205.046	0.18	149.939	388.321	0.18	112.926	398.706	0.27
453.285	284.039	0.31	152.157	376.582	0.35	101.31	398.871	0.12
444.774	286.173	0.094	157.533	368.544	0.64	144.404	390.752	0.31
436.418	288.247	0.88	167.809	365.499	0.26	145.316	379.281	0.32
487.501	149.162	0.12	211.002	329.763	0.49	147.454	369.748	0.43
473.95	143.454	0.13	177.555	364.759	0.48	154.745	362.263	0.41
456.136	153.408	0.46	185.395	365.225	0.8	460.841	170.479	0.93
217.798	324.517	0.68	192.481	365.662	0.29	448.601	176.019	0.33
439.36	163.049	0.9	199.693	366.096	0.46	473.682	184.608	1
497.719	189.25	0.78	206.26	365.799	0.31	463.5	177.94	0.7
481.463	153.064	0.74	211.46	364.506	0.062	472.84	278.278	0.38
491.189	243.509	0.34	222.641	358.862	0.5	463.159	273.686	0.17
358.403	287.512	0.79	228.273	354.109	0.45	377.504	286.535	0.75
349.243	290.298	0.13	233.924	348.629	0.26	368.046	286.571	0.59
341.266	294.954	0.41	241.707	344.314	0.44	361.575	280.736	0.03

x-cord	*y*-cord	*k*	*x*-cord	*y*-cord	*k*	*x*-cord	*y*-cord	*k*
334.628	299.927	0.55	249.749	339.866	0.26	359.093	273.993	0.84
328.952	306.1	0.23	258.026	336.293	0.091	354.118	264.624	0.45
317.255	314.744	0.53	265.718	333.505	0.15	335.059	272.239	0.096
337.931	281.908	0.29	216.336	355.157	0.15	427.332	262.521	0.42
337.282	289.304	0.44	209.848	356.397	0.091	416.78	266.584	0.51
329.266	293.651	0.23	203.846	358.381	0.33	417.741	273.595	0.4
323.995	300.071	0.44	196.676	358.517	0.43	408.804	273.467	0.47
319.727	306.139	0.31	188.8	358.267	0.61	401.618	273.453	0.16
492.36	226.406	0.52	181.109	357.909	0.18	395.484	276.961	0.33
131.461	400.309	0.35	173.119	357.633	0.42	369.86	265.555	1
123.779	391.322	0.58	144.13	362.921	0.25	320.808	276.184	0.31
112.99	391.573	0.079	138.991	371.305	0.092	316.06	297.152	0.47
100.695	392.427	0.38	138.939	381.356	0.29	264.411	325.74	0.26
537.338	257.193	0.5	139.042	392.898	0.33	257.919	328.115	0.15
448.874	211.598	0.39	133.034	389.05	0.28	251.664	331.294	0.36
440.274	208.965	0.58	122.256	383.968	0.22	245.182	334.256	0.28
399.11	289.846	0.43	112.29	384.52	0.18	238.805	337.803	0.36
406.502	290.513	0.64	101.246	385.301	0.061	232.723	341.532	0.14
553.317	231.819	0.31	90.802	385.48	0.21	134.987	364.356	0.27
414.02	290.578	0.32	482.531	276.678	0.36	130.703	382.18	0.29
421.957	289.381	0.65	472.601	270.906	0.43	121.157	376.27	0.13
424.631	277.617	0.4	463.075	265.809	1	110.867	377.127	0.23
428.973	268.598	0.55	451.192	261.804	1	100.364	378.342	0.17
439.893	265.331	0.4	437.255	259.192	0.38	87.3948	374.79	0.1
452.785	268.681	0.28	522.482	148.807	0.065	131.774	373.602	0.069
165.239	358.876	0.58	328.698	275.278	1	452.025	248.219	0.45
577.595	238.318	0.41	325.653	287.629	0.57	440.763	245.495	0.85
559.559	237.722	1	271.877	323.68	0.25	429.686	241.275	1
528.264	240.018	0.3	566.914	234.06	0.28	457.656	210.954	0.76
514.622	241.417	0.19	525.289	235.828	0.84	471.36	213.554	0.74
505.131	264.685	0.44	515.09	234.678	0.41	484.367	217.524	0.59
420.016	281.708	0.08	505.612	248.224	1	495.944	220.884	0.31
412.655	282.921	0.36	502.516	259.696	0.3	506.473	223.737	1
405.165	282.775	0.48	500.056	269.679	0.53	517.041	226.485	0.4
397.132	283.84	0.36	491.313	271.486	0.25	530.27	229.241	0.7
387.223	286.483	0.11	481.114	267.816	0.59	540.571	234.956	0.42
364.593	268.314	0.85	471.03	263.184	0.36	536.3	237.063	0.71

x - cord	*y* - cord	*k*	*x* - cord	*y* - cord	*k*	*x* - cord	*y* - cord	*k*
358.748	259.268	0.42	460.408	258.623	0.93	575.622	233.735	0.27
225.67	346.316	0.12	446.748	256.252	0.49	503.605	241.79	0.5
220.577	350.322	0.4	433.75	253.544	0.22	496.702	255.931	0.31
489.687	263.953	0.54	56.7523	394.939	0.42	406.596	250.523	0.33
263.397	318.309	0.24	62.758	392.446	0.25	398.234	254.175	0.56
255.777	320.524	0.38	69.0404	389.6	0.87	389.773	252.894	0.63
249.432	323.635	0.25	75.662	386.656	0.08	312.559	309.016	0.47
242.934	327.223	0.35	33.8495	398.615	0.19	259.64	296.39	0.34
465.313	249.901	0.35	43.9349	400.032	0.12	250.75	299.203	0.1
216.267	344.059	0.39	167.969	343.943	0.18	241.813	302.891	0.22
210.887	349.054	0.093	177.915	344.272	0.38	232.537	307.475	0.16
203.375	350.633	0.055	187.515	344.95	0.17	223.793	312.458	0.45
195.76	351.424	0.091	196.526	344.691	0.19	487.035	230.728	0.75
188.086	351.445	0.12	206.55	343.941	0.35	487.607	238.917	1.1
179.163	350.924	0.27	212.719	337.213	0.18	480.528	245.786	0.85
169.656	350.846	0.39	236.935	322.155	0.26	469.854	242.066	0.74
82.7713	385.635	0.18	243.815	318.278	0.055	460.639	239.398	0.27
98.1018	371.658	0.32	251.07	314.635	0.3	447.751	237.279	0.26
108.739	370.302	0.068	259.25	311.552	0.26	437.075	232.273	0.72
119.871	369.415	0.25	308.864	289.734	0.45	420.057	236.522	0.44
373.697	261.461	0.29	313.505	274.943	0.093	411.974	239.675	0.81
379.69	272.506	0.31	377.834	255.999	0.3	394.95	248.341	0.77
386.366	281.216	0.13	382.849	266.265	0.3	381.953	246.923	0.82
478.598	259.973	0.44	399.732	265.423	0.85	302.295	278.933	0.76
467.17	256.523	0.34	151.996	356.792	0.12	238.248	296.422	0.5
455.865	253.305	0.51	160.945	352.824	0.36	227.093	301.554	0.85
443.19	251.048	0.56	494.079	248.824	0.5	215.395	317.499	0.42
431.648	247.744	0.33	487.735	256.582	0.76	208.57	322.596	0.69
424.881	256.746	0.36	508.353	231.575	0.87	199.638	323.694	0.68
416.209	260.138	0.23	499.662	236.339	0.21	190.167	323.375	0.33
408.517	264.286	0.55	191.765	330.319	0.84	177.454	329.202	0.84
392.401	271.092	0.19	182.055	331.445	0.32	266.11	291.635	0.11
387.451	275.22	0.072	472.345	247.671	0.46	217.186	306.945	0.32
235.97	331.293	0.097	460.807	245.253	0.54	208.936	312.664	0.1
228.707	335.341	0.43	450.647	242.874	0.31	403.519	244.112	0.65
222.894	339.763	0.17	439.495	239.014	0.19	426.287	228.035	0.24
140.945	355.795	0.45	428.604	234.163	0.46	456.29	234.585	0.38

x - cord	*y* - cord	*k*	*x* - cord	*y* - cord	*k*	*x* - cord	*y* - cord	*k*
130.978	357.162	0.25	225.078	319.785	0.32	469.42	236.09	0.69
126.79	366.147	0.097	420.635	243.826	0.48	198.801	316.527	0.55
50.532	396.989	0.44	413.71	246.724	0.51	252.343	291.921	0.29
297.508	282.949	0.052	301.144	324.544	0.13	499.751	286.853	0.39
306.08	293.719	0.087	303.81	315.911	0.11	506.683	297.623	0.06
390.049	241.249	0.19	306.86	307.275	0.36	523.037	289.273	0.074
408.977	233.901	0.39	271.856	300.984	0.21	93.1214	405.604	0.25
417.512	230.55	0.26	281.159	322.363	0.25	91.3005	396.697	0.44
480.462	238.213	0.86	297.862	299.463	0.9	86.6273	406.851	0.65
477.748	228.518	0.1	285.889	293.862	0.082	84.5016	397.497	0.24
475.002	232.65	0.16	288.678	321.428	0.23	80.8103	408.076	0.58
465.136	231.049	0.19	270.944	309.814	0.11	77.6011	399.229	0.79
454.024	228.338	0.19	276.221	317.023	0.056	74.5966	410.161	0.013
445.421	230.715	0.24	297.981	313.931	0.056	72.0646	402.866	0.021
435.474	225.874	0.12	300.166	306.915	0.073	36.3408	412.928	0.46
424.654	222.048	0.39	293.427	303.813	0.19	47.3216	409.278	0.013
399.863	238.852	0.44	286.466	300.215	0.069	39.282	418.111	0.026
369.056	253.173	0.35	279.262	298.097	0.13	50.9275	413.774	0.017
290.776	283.149	0.14	275.949	306.155	0.069	43.8061	423.209	0.021
297.736	292.354	0.11	281.05	313.085	0.15	54.6995	419.906	0.017
415.504	224.465	0.29	289.279	314.613	0.082	3.4706	431.433	0.37
406.526	227.798	0.36	290.508	308.978	0.082	3.95358	422.315	0.77
397.363	233.99	0.45	284.146	305.899	0.096	11.0518	430.824	0.44
389.463	232.337	0.33	482.248	285.543	0.38	10.9455	421.801	0.66
379.637	240.769	0.073	488.407	291.813	0.25	19.0434	429.194	0.76
492.642	305.166	0.096	497.973	294.759	0.37	18.7018	419.891	0.6
461.878	224.79	0.25	514.508	285.303	0.039	26.4528	416.771	0.51
444.432	224.158	0.19	495.707	277.595	0.2	27.572	426.263	0.41
433.726	219.572	0.22	488.615	280.444	0.11	33.8874	423.563	0.87
422.462	215.656	0.33	492.086	288.543	0.26			
292.405	327.644	0.19	512.329	277.421	0.19			
305.492	300.773	0.11	504.087	275.037	0.44			
276.066	291.689	0.15	506.001	281.327	0.23			
268.087	314.078	0.76	496.982	282.941	0.39			