THE ROOT OF ALL DIMENSIONS

by

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This thesis is dedicated, first and foremost, to the Demiurge, the prime algorithm from whence originate the formulae and schematics of the universe, the apprehension of which is the highest purpose of our existence. I dedicate this work also to Plato and Aristotle, and to all those great minds throughout the history of humanity who have advanced our knowledge of the cosmos to which we belong, and who continue, against all of the challenges of our present age, to guide us toward the most noble achievement of governing ourselves in proper accord with the laws of nature.

Table of Contents

Abstract

This thesis aims to resolve the disjunction of ontology and epistemology in Platonist mathematical philosophy. This disharmony results from the assumption of the non-spatiotemporal existence of abstract mathematical entities, a principle which fails to account both for our knowledge of mathematics, and for the operation of mathematical principles in the tangible universe. In order to address the problem, we will examine the definition and function of mathematical principles as expressed in Plato's philosophy, referring also to Aristotle in order to identify the logical restrictions governing the Platonist position. Our investigation of the nature and role of mathematical principles within the Platonist cosmos will lead us to the consideration of the geometric structures of matter as described in the *Timaeus*, and of the role of mathematics in the essential connection of the human intellect to the schematics and algorithms of the natural world.

List of Abreviations and Symbols Used

Arist.	Aristotle
Pl.	Plato
CTK	Causal Theory of Knowledge
Nicom.	Nicomachus Gerasanus
Plot.	Plotinus
Procl.	Proclus
Ar.	*Introduction to Arithmetic*
Metaph.	*Metaphysics*
EN.	*Nicomachean Ethics*
Ti.	*Timaeus*
Tht.	*Theaetetus*
Phlb.	*Philebus*
Phdr.	*Phaedrus*
Chrm.	*Charmides*
Sph.	*Sophist*
Plt.	*Statesman*
R.	*Republic*
Lg.	*Laws*
Grg.	*Gorgias*
Phd.	*Phaedo*
LI.	*On Indivisible Lines*
Ph.	*Physics*
Prm.	*Parmenides*
Men.	*Meno*
In Alc.	*Commentary on Plato's Alcibiades*
In Euc.	*Commentary on the First Book of Euclid's Elements*
In Prm.	*Commentary on Plato's Parmenides*
In Ti.	*Commentary on Plato's Timaeus*
A Po.	*Posterior Analytics*
GC.	*On Generation and Corruption*
Σ	Summation
Sin	Sine
Cos	Cosine
√	Square Root

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Chapter 1: Introduction

As beings defined by our faculty for rational articulation of reality, it belongs to us to apprehend the code of laws governing the system to which we belong, that we ourselves might function within the best possible manner within that structure. In seeking to grasp the truths of the cosmos, our approach to the explanation of the structure and activity of the universe is to describe it in terms of the rules of mathematics. In what manner, however, are we to define these mathematical principles? Are they mere constructs of the human imagination, or do they rather represent a system of regulations belonging properly to reality? If, moreover, the laws of mathematics constitute operative patterns within reality, does our use of mathematical principles in the explanation of the cosmos reflect an essential connection between the human intellect and the structure of the natural world as a whole? In addressing such questions as these, we advance toward a greater capacity for inquiry regarding the structure of the cosmos, as well as the proper place of our species within that system.

We must first, however, address the problems with the thesis that mathematics is a contrivance of the human imagination. As explained by Shapiro (1997), this position is asserted by the mathematical philosophy of intuitionism, a branch of the anti-realist school which rejects the law of excluded middle[1] (and thus rejects the binary truth or falsehood of all mathematical statements) on the basis that "These methodological principles are symptomatic of faith in the transcendental existence of mathematical objects or the transcendental truth of mathematical statements."[2] The intuitionist argument therefore places our knowledge of the universe, and thus our role within it, in a particularly vulnerable position. If the mathematical foundations of scientific theories are trivialized, then it becomes largely impossible for us to construct any meaningful articulation of the universe.

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[1] Arist. *Metaph*. IV. 1011b23-24. Aristotle describes in this passage the law of excluded middle by stating that "No truth can be allowed between two opposite statements, but rather it is necessary to affirm one of the possibilities and deny the other." ἀλλὰ μὴν οὐδὲ μεταξὺ ἀντιφάσεως ἐνδέχεται εἶναι οὐθέν, ἀλλ᾽ ἀνάγκη ἢ φάναι ἢ ἀποφάναι ἓν καθ᾽ ἑνὸς ὁτιοῦν

[2] Stewart Shapiro. *Philosophy of Mathematics: Structure and Ontology*. (New York and Oxford: Oxford University Press, 1997), 23.

Even without the rejection of the law of excluded middle, the treatment of mathematical principles as human constructions proves to be problematic in several regards. It devalues our activity of scientific investigation to a task of imposing our vision of order upon a world that we perceive as being otherwise chaotic and devoid of rational structure. This approach places us in the uncomfortable position of mastery over that which we examine, as opposed to one in which we recognize our place as self-aware components, or cognizant cogs, within the system to which we belong. It also fails to account for the natural properties of the human intellect that incline it towards inquiry into the nature of reality; for it treats mathematics, and by extension, all of science, as nothing more than a product of the human imagination. In this respect, the anti-realist position creates an inconsistency within itself; for in attempting to argue that scientific principles are mere projections of the human mind upon the observed natural world, we necessarily assume the givenness of human thought; for otherwise we must even treat the anti-realist position as a mere contrivance of imagination.

We have, furthermore, shown ourselves to be ill-suited for the mastery of the natural world that would be bestowed upon us by the anti-realist argument, as we have all too frequently failed to grasp the ineluctable truth of the finite magnitude and multiplicity which belongs necessarily to tangible objects – a truth which ought to indicate to us that the laws of mathematics constitute a real governing force within the cosmos, and that it therefore behooves us to apprehend the laws, such that we may understand their relevance to us, and use them to properly guide the direction of our existence.

To be sure, however, these arguments against the anti-realist position of mathematical philosophy are not intended to show that anti-realism is incorrect, but rather that it may be inadvisable for us to assume that it is correct. Similarly, our consideration of mathematical realism will not demonstrate that the realist position is correct, but rather will present a possible explanation for how it might be correct. Throughout the course of our investigation, any statements that we present as truth are to be understood as such only within the context of our proposed solution to the disjunction of ontology and epistemology. It may indeed be impossible for us to determine conclusively which of these two positions is correct; for as long as the sensible world as we observe it is assumed to be real, the correctness of either argument is possible; and due to our

confidence in sensible reality over the authority of rational argumentation, there is no argument that can speak to the truth of sensible reality.

The consideration of mathematical principles as operative within reality is central to Platonist mathematical philosophy, which, as we shall later observe, may be considered erroneous in terms of its faithfulness to Plato's philosophy. As Balaguer (1998) explains, mathematical Platonism maintains that mathematical objects, including numbers, "…are non-spatiotemporal and exist independently of us and our mathematical theorizing…"[3] Balaguer also notes, however, that according to Benacerraf's argument from the causal theory of knowledge, or CTK, the truth of mathematical Platonism makes it impossible for us to attain knowledge of mathematics. CTK maintains that in order for a particular person to possess knowledge of a certain object or principle, the former and the latter must be "causally related" to one another "in an appropriate way." Benacerraf concludes that if mathematical objects exist outside of the spatiotemporal realm, they are not causally related to humans, and that if, therefore, mathematical Platonism holds true, it is impossible for us to possess mathematical knowledge.[4] To be sure, if mathematical objects are not causally related to the spatiotemporal realm in any regard, then it follows that they have no bearing on the structure and motion of tangible entities, and since they would, in this case, have no relevance to our understanding of mathematics, they would be utterly without purpose.

It is possible for us to solve this problem, while still maintaining the reality of mathematical principles, if we are able to explain some manner in which the separation of mathematical objects from the spatiotemporal world need not preclude their causal relation to tangible beings. A variation of this solution is proposed by Gödel, who

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[3] Mark Balaguer. *Platonism and Anti-Platonism in Mathematics*. (Oxford: Oxford University Press, 1998), 5.

[4] Balaguer. *Platonism and Anti-Platonism in Mathematics*. 22. See also Kenneth Dorter. *Form and Good in Plato's Eleatic Dialogues: The* Parmenides*,* Sophist*,* Theaetetus*, and* Statesman. (Berkeley and Los Angeles: University of California Press, 1994), 39, 41. Op. Cit. Pl. *Prm*. 133a-c,124b-c.. In the passage quoted by Dorter, the problem presented is that if the Forms are entirely separate from sensible reality in their existence, it would be exceedingly difficult to demonstrate that the separate existence of the Forms does not preclude our knowledge of them. Indeed, one of the arguments against our knowledge of the Forms is that they are not within us in any respect (Pl. *Prm*. 134b-c). As we shall observe later, however, the nature of the presence of the Forms within the human intellect is of critical importance in explaining our knowledge of them.

suggests that the human intellect attains knowledge of the objects of mathematics by means of "mathematical intuition." The problem with Gödel's position, according to Balaguer, is that it does not account for the assumed lack of causal relation between the objects of mathematics and the spatiotemporal realm. A possible Platonist counter-argument, Balaguer explains, is that the intellect is non-spatiotemporal. He rejects this postulation, however, despite giving little support for doing so, and states that even the identification of the intellect as non-spatiotemporal does not necessarily imply that the mind communicates with the objects of mathematics.[5] There is also, however, no reason to assume that the separation of an object from the spatiotemporal realm must necessarily prevent the object from being causally related to the latter. Indeed, it may be the case that such an unjustified assumption must be put aside in order for a solution to the ontological-epistemological conflict of mathematical realism to be possible.

We shall consider such a solution through our examination of the true definition and function of mathematical principles within the philosophical teachings of Plato. Our inquiry shall draw primarily on the *Timaeus*, supplemented by other Platonic texts such as the *Republic* and the *Phaedo*. We will also consult Aristotle's reflections on mathematics, particularly as set forth in the *Metaphysics*, in order to interpret Plato with greater clarity, and to determine the restrictions on the realist position based on Aristotle's arguments against perceived errors in Plato's thought. Proceeding from the doctrine of an eternal, unchanging model as the origin of all knowledge and existence and the image of perfection to which all things seek to return, we observe that the principles of mathematics, from the foundational unit concept, to the relations of geometric structure, to functions of vast complexity, constitute nothing less than the language of reality itself. Such laws, as we shall see, are dictated by the requirements defined within the schematics of the cosmos, while also governing the structure of these schematics. Amongst themselves, the laws of mathematics must also serve a mutually defining role towards one another, with rudimentary operations constituting the foundations of complex operations, which in turn dictate the functionality of rudimentary functions. In

[5] Balaguer. *Platonism and Anti-Platonism in Mathematics*. 25-26.Op. Cit. J. Katz. *Language and Other Abstract Objects*. (Totowa, New Jersey: Rowman and Littlefield, 1981), 201. Balaguer maintains that the communication of information between two objects is an activity belonging purely to the physical realm. On a general note, one of the primary weaknesses in his argument against mathematical Platonism is that it rests on the assumption of a purely tangible reality.

this regard, the laws of mathematics function as governing principles within the intelligible, as the intelligible is with respect to the tangible; for in their capacity as the patterns by which the intelligible is the highest perfection of all existence, they constitute the foundation by which all order within the intelligible is defined.

Chapter 2: Εἶδος

2.1 Upholding Non-Contradiction: The Necessary and the Impossible in the Philosophical Language of Mathematics

In order to discover the precise definition of mathematical principles within Plato's philosophy, we must first establish the specifications that define all of the objects of knowledge, including the principles of mathematics. These rules are as follows: (1) The Demiurge (the divine craftsman discussed in the *Timaeus*) is understood to be good, and the cosmos to be beautiful.[6] From this requirement, it follows (2) that the cosmos is constructed according to an eternal principle.[7] From this rule, it follows, in turn, that (3) the model that constitutes this eternal principle is entirely unchanging, or "always in the same way."[8] This requirement dictates (4) that the model described can never be in an incomplete state; and from this law, it follows (5) that within the cosmic model there can no temporal succession; for the immediate and eternal completion and perfection of the cosmic model precludes the possibility that it is subject to any process of construction. All cases of dependency within the model must then be mutual, such that any definitions contained within the model are completely simultaneous insofar as there is no priority or posterity in their relation to one another.

It seems also (6) that there must indeed be schematic definitions of some sort contained within the cosmic model, as Plato characterizes the inquiry occurring in the *Timaeus* a consideration of "how the framework of models brought itself to perfection…"[9] In mentioning a framework of models, Plato may be referring to the model of the cosmos, or to the sensible cosmos itself (or perhaps to both), yet in any case, it is reasonable to suggest that the schematic models of which we have spoken are defined within the great cosmic model; for if the objects of the sensible are models, then each one must be a model of something. It seems also to be the case (7), according to Plato's

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[6] Pl. *Ti*. 29a2-3. εἰ μὲν δὴ καλός ἐστιν ὅδε ὁ κόσμος ὅ τε δημιουργὸς ἀγαθός

[7] Pl. *Ti*. 29a3-6. δῆλον ὡς πρὸς τὸ ἀίδιον ἔβλεπεν: εἰ δὲ ὃ μηδ᾽ εἰπεῖν τινι θέμις, πρὸς γεγονός. παντὶ δὴ σαφὲς ὅτι πρὸς τὸ ἀίδιον: ὁ μὲν γὰρ κάλλιστος τῶν γεγονότων, ὁ δ᾽ ἄριστος τῶν αἰτίων

[8] Thomas A. Blackson. *Inquiry, Forms, and Substances: Studies in Plato's Metaphysics and Epistemology*. Philosophical Studies Series 62. (Dordrecht, The Netherlands: Kluwer Academic Publishers, 1995), 133. Op. Cit. Pl. *Ti*. 27d5-28a4. ἀεὶ κατὰ ταὐτὰ ὄν

[9] Pl. *Ti*. 28c5-29a1. πρὸς πότερον τῶν παραδειγμάτων ὁ τεκταινόμενος αὐτὸν ἀπηργάζετο

position, that all knowledge belongs to a single structure; for Socrates states in the *Theaetetus*, to which Crombie (1963) directs our attention, that knowledge is "... *not many things, but one.*" [10] The statement cited by Crombie need not necessarily imply that all knowledge belongs to a single structure, and may merely indicate that there is only one definition of knowledge. Crombie appears, however, to interpret the statement as an indication of the essential unity of all knowledge, as he explains that Socrates is, in this case, refuting Theaetetus' assumption of knowledge having some sort of range. In part of the passage of the *Theaetetus* that is cited by Crombie, Theaetetus characterizes such disciplines as geometry, as well as various fields of craftsmanship, as being encompassed within knowledge, using such terms as "all" and "each" to identify each discipline individually.[11] Socrates responds to Theaetetus by stating that the latter is "asking one thing to be given as many," and for a "patchwork" or "quilt" (ποικίλα) instead of a "single stretch of fabric" (ἁπλοῦ).[12] The language used by Socrates in this case suggests not only a unified structure for all knowledge, but a framework that is unified in a simple manner, insomuch as it is not composed of disparate pieces brought together, but rather is properly one, with each portion of it being essentially connected to all others.

It also stands to reason (8) that the single system of all knowledge is the cosmic model according to which the Demiurge constructs the sensible universe, as the belonging of all knowledge to a single system dictates that this system must contain knowledge regarding the architecture of the universe at all levels; and as we have observed before, the sensible cosmos (and presumably, the cosmic model as well, as there is no clear justification for the cosmos possessing non-accidental qualities absent from the model on which it is based) constitutes a framework of models, or τῶν παραδειγμάτων ὁ τεκταινόμενος; and as such, it would seem that any non-accidental truth pertaining to the tangible world ought to originate in the schematic according to which the tangible world is constructed. From (7) and (8), it follows that the objects of mathematical knowledge belong to the same foundation as all other objects of

[10] I. M. Crombie. *An Examination of Plato's Doctrines*. Volume II: *Plato On Knowledge and Reality*. (London: Routledge and Kegan Paul Ltd.., 1963), 368. Op. Cit. Pl. *Tht*. 146-7.

[11] Pl. *Tht*. 146c8-d2. γεωμετρία τε καὶ ἃς νυνδὴ σὺ διῆλθες, καὶ αὖ σκυτοτομική τε καὶ αἱ τῶν ἄλλων δημιουργῶν τέχναι, πᾶσαί τε καὶ ἑκάστη τούτων, οὐκ ἄλλο τι ἢ ἐπιστήμη εἶναι

[12] Pl. *Tht*. 146d3-4. ἓν αἰτηθεὶς πολλὰ δίδως καὶ ποικίλα ἀντὶ ἁπλοῦ

knowledge, and that the schematic to which they belong is the eternal model of the cosmos.

The belonging of the objects of mathematical knowledge to the same system as that which contains all intelligible principles seems to contradict the notion of non-spatiotemporal, independently existing mathematical objects as discussed by Balaguer.[13] McLarty (2005) demonstrates, furthermore, that such a concept is in fact foreign to Plato's thought concerning mathematical objects. McLarty draws our attention to the fact that it is Glaucon, one of the participants in the dialogue of the *Republic*, who suggests that the objects of geometry are eternal substances.[14] Socrates, as McLarty reminds us, corrects Glaucon's error by explaining that the objects of mathematics are 'mere preludes to the song itself that we must learn.'[15] The immediate result of McLarty's observation is that Plato's position concerning the station of mathematical objects in the ordering of reality is restored to potential viability in terms of its capacity to harmonize its ontology with mathematical epistemology. More still is revealed in considering the logical impossibilities that follow from the characterization of abstract geometric structures as subsistent entities.

For the purpose of determining what is contrary to the nature and function of mathematical objects, and thereby attaining a more precise apprehension of their relation to the sensible world, the guidance of Aristotle is of particular benefit to our task. In Book M of the *Metaphysics*, Aristotle dismantles the arguments for the subsistence of mathematical objects. He identifies two possibilities, or more precisely, impossibilities, for the subsistence of the objects of mathematics. The first such suggestion is that numbers, points, lines, planes, and solids are present as distinct entities within sensible beings. Aristotle indicates the problem with this position by stating,

> The argument that these objects are indeed within, and together with, tangible beings, is fictitious and it has been determined that in these troubled questions that it

[13] See n. 3

[14] Colin McLarty. "'Mathematical Platonism' *Versus* Gathering the Dead: What Socrates teaches Glaucon", *Philosophia Mathematica*(III) 13, no. 2 (2005):116. Op. Cit. Pl. *Rep*. 527b.

[15] McLarty. "'Mathematical Platonism' *Versus* Gathering the Dead: What Socrates teaches Glaucon", 116. Op. Cit. Pl. *Rep*. 531d.

> is impossible for two solids to be together, and indeed it is also determined from the same argument that other faculties and nature are within sensible objects and are in no way separate; thus it has been found that these things are prior, though concerning these things it is manifestly impossible for them to divide corporeal objects in any way; for they would be divided by planes, and these would be divided by lines, which would, in turn, be divided according to points, so if it is impossible to divide the point, it is likewise impossible to divide the line and so on.[16]

In this passage, Aristotle seeks to demonstrate that the objects of geometry, that is to say, points, lines, planes, and solids, do not exist within sensible beings as a distinct self-sufficient presence, but rather as components accounting for the structural and functional attributes of the tangible objects to which they belong. Concerning geometric solids specifically, Aristotle states that these objects cannot exist as tangible solids distinctly within sensible beings, since it is assumed to be impossible for two or more tangible solid objects to occupy the same physical space simultaneously. He indicates that even if the objects of mathematics exist in sensible beings as an intangible subsistent presence, they are indivisible based on the indivisibility of the points that comprise them, such that the tangible objects within which they reside are likewise indivisible. If mathematical objects cannot be subsistent, either tangibly or intangibly, within sensible entities, then they must necessarily be present as components of the objects that contain them. If the presence of mathematical objects within sensible entities is that of components, it then seems to follow that the objects of mathematics cannot be present within tangible beings in a non-active manner. That is to say, they may only be present within tangible entities insofar as they function as the specifications of structure and movement belonging to the object in question. It is therefore by no means difficult for us to anticipate Aristotle's specification that it is impossible to remove mathematical objects from the sensible substance to which

[16] Arist. *Metaph*. XIII. 2. 1076a39-1076b9. ὅτι μὲν τοίνυν ἔν γε τοῖς αἰσθητοῖς ἀδύνατον εἶναι καὶ ἅμα πλασματίας ὁ λόγος, εἴρηται μὲν καὶ ἐν τοῖς διαπορήμασιν ὅτι δύο ἅμα στερεὰ εἶναι ἀδύνατον, ἔτι δὲ καὶ ὅτι τοῦ αὐτοῦ λόγου καὶ τὰς ἄλλας δυνάμεις καὶ φύσεις ἐν τοῖς αἰσθητοῖς εἶναι καὶ μηδεμίαν κεχωρισμένην: ταῦτα μὲν οὖν εἴρηται πρότερον, ἀλλὰ πρὸς τούτοις φανερὸν ὅτι ἀδύνατον διαιρεθῆναι ὁτιοῦν σῶμα: κατ᾽ ἐπίπεδον γὰρ διαιρεθήσεται, καὶ τοῦτο κατὰ γραμμὴν καὶ αὕτη κατὰ στιγμήν, ὥστ᾽ εἰ τὴν στιγμὴν διελεῖν ἀδύνατον, καὶ τὴν γραμμήν, εἰ δὲ ταύτην, καὶ τἆλλα
It is worthy of note that the inability of solid objects to simultaneously occupy the same physical space may be treated as natural adherence to the Law of Non-Contradiction. The understanding of this impossibility as such presupposes that the fullness or emptiness of the physical space is indeed a binary variable.

they belong.[17] Aristotle thus emphasizes what ought already to be self-evident, which is the fact that while the objects of geometry are present in sensible beings, they constitute the parameters of structure and movement that properly belong to the entities in which they are manifest. If these geometrical objects are removed from the beings in which they are present, the aforesaid entities would therefore cease to adhere to their proper definition, as they would be entirely amorphous and motionless.

Based on the fact that sensible beings are dependent upon abstract, geometrically expressed variables for adherence to the specifications belonging to their intelligible definitions, it is apparent to us that tangible entities must, in some respect, be subordinate to mathematical objects. Aristotle elaborates on the exact nature of this dependency in his characterization of the objects of incomplete, indefinite, or undefined magnitudes [ἀτελὲς μέγεθος] as being "prior [to sensible objects] with respect to generation [γενέσει μὲν πρότερόν], yet posterior [to sensible objects] in substance [τῇ οὐσίᾳ δ' ὕστερον]."[18] From this explanation, it is possible for us to identify a process in which the objects of geometry, including numbers, points, lines, planes, and solids, are responsible for defining the essential characteristics of dimension and activity for specific sensible beings. Inasmuch as they constitute the quantitative attributes of a tangible entity, the objects of geometry must be defined prior to sensible beings, though tangible οὐσίαι and mathematical objects are nevertheless mutually co-dependent, such that neither may survive the destruction of the other; for the structural integrity of a sensible being cannot survive the removal of the parameters of spatial magnitude dictated by its definition; and since those parameters represent a dependent presence within sensible objects, any damage to a sensible object will prevent the mathematical objects belonging to it from remaining intact; and if a sensible being is somehow obliterated completely, the mathematical objects belonging to it will similarly be destroyed. Since tangible entities depend upon mathematical objects to maintain their structure, it stands to reason that the

[17] Arist. *Metaph*. XIII. 2. 1077a13-14.

[18] Arist. *Metaph*. XIII. 2. 1077a18-20. Aristotle uses the term ἀτελὲς μέγεθος to refer to a magnitude still in the process of generation that has not yet been incorporated into a tangible entity. He provides no explanation for this term, although one possible interpretation is that it implies a magnitude that is incomplete insofar as it has not been fully expressed in substance. It may also be understood as an undefined magnitude, if specific magnitudes are defined by their activity in substance.

objects of mathematics are always functioning for this purpose within the entities to which they belong, and must therefore constitute a non-idle presence therein.

If one treats geometric objects purely with respect to their priority to sensible substances, without taking into account the precise nature of their agency in the generation of tangible beings, then it seems absurd for a superior class of objects to be contingent while their subordinates are subsistent. In *Metaphysics* A, Aristotle himself argues against the notion of that which is relative to some other determining factor being prior to that which is independent, or "according to itself."[19] Ironically, however, it is due to their role as regulators of structure and motion that the objects of geometry are unable to subsist as an idle presence within sensible beings; in order to abide by the mathematical laws specifically pertaining to the finite tangible level of existence, they must be present within αἰσθητοί as active components.

In characterizing geometric objects, it may be most accurate to say that they are not independent entities, but are rather patterns that account for the dimensional specifications according to which sensible beings adhere to the structural properties belonging to their intelligible definitions. If, however, the function of numbers, points, lines, planes, and solids at the sensible level is to serve as the "bones" of αἰσθητοί, as Aristotle's argument seems to dictate, one must determine the exact capacity in which they fulfill this purpose. It is not plausible for the geometric objects present within sensible beings to occupy this role, since they would perish with the destruction of the αἰσθητοί of which they are components. A possible explanation for the true nature and activity of mathematical objects is that they belong more properly to the intelligible principles to which sensible beings adhere, and that αἰσθητοί possess mathematical objects only by obedience to the eternal schematics that govern them. This postulation is supported by Pedersen (1974), who identifies geometric structure as a property of Form.[20] That is to say, as Pedersen explains, the Form contains the geometric structure that belongs essentially to the type of sensible object that it governs, along with other properties such as "gravity and lightness" as well as all characteristics pertaining to

[19] Arist. *Metaph*. I. 9. 990b20-22. τὸ πρός τι τοῦ καθ'αὑτό

[20] Olaf Pedersen. *Early Physics and Astronomy: A Historical Introduction*, Rev. Ed. (1974, 1993; repr., Cambridge: Cambridge University Press, 1996), 29.

interaction with other types of entities. In the event that the essential geometric structure of a sensible object is not defined within the Form that governs it, then the only conceivable explanation for the manner in which sensible beings possess distinct geometric structure is that their geometric structure originates from themselves. Such a result would be absurd, as it would dictate that the essential geometric properties of sensible beings, which are also determining factors for characteristics of movement, have the same origin as the accidental qualities of sensible beings; and as such there would be no clear distinction between the structural properties that belong essentially to tangible entities, and those that are merely accidental qualities. The only other possible solution would be for the objects of geometry to exist prior to sensible beings and independently from Forms, though as we shall soon observe, this conclusion also proves to be unacceptable. As Aristotle indicates in *Metaphysics* M, the χώρησις of geometric objects proves to be untenable, as it results in the following structure,

> There are therefore, once again, lines belonging to these planes, prior to which, by the same argument, there will necessarily be other lines and points, and from these there will be other prior points for the prior lines, to which nothing else will be prior. The result is then absurd (for one additional set of solids corresponds to sensible beings, as well as three sets of planes: those above sensible beings, those in mathematical solids, and those above the planes in mathematical solids. There will also be four sets of lines, and five sets of points. To which mathematical objects will science then pertain? It will not pertain to the planes, lines, and points in the unchanging solid, for science always treats that which is prior.) The argument is also the same concerning numbers; for each of the other points there will be different units, and for each sensible being, subordinate to the intelligible, is thus a type of mathematical number.[21]

[21] Arist. *Metaph*. XIII. 2. 1076b25-37. πάλιν τοίνυν τούτων τῶν ἐπιπέδων ἔσονται γραμμαί, ὧν πρότερον δεήσει ἑτέρας γραμμὰς καὶ στιγμὰς εἶναι διὰ τὸν αὐτὸν λόγον: καὶ τούτων τῶν ἐκ ταῖς προτέραις γραμμαῖς ἑτέρας προτέρας στιγμάς, ὧν οὐκέτι πρότεραι ἔτεραι. ἄτοπός τε δὴ γίγνεται ἡ σώρευσις （συμβαίνει γὰρ στερεὰ μὲν μοναχὰ παρὰ τὰ αἰσθητά, ἐπίπεδα δὲ τριττὰ παρὰ τὰ αἰσθητά—τά τε παρὰ τὰ αἰσθητὰ καὶ τὰ ἐν τοῖς μαθηματικοῖς στερεοῖς καὶ τὰ παρὰ τὰ ἐν τούτοις—γραμμαὶ δὲ τετραξαί, στιγμαὶ δὲ πενταξαί: ὥστε περὶ ποῖα αἱ ἐπιστῆμαι ἔσονται αἱ μαθηματικαὶ τούτων; οὐ γὰρ δὴ περὶ τὰ ἐν τῷ στερεῷ τῷ ἀκινήτῳ ἐπίπεδα καὶ γραμμὰς καὶ στιγμάς: ἀεὶ γὰρ περὶ τὰ πρότερα ἡ ἐπιστήμη）: ὁ δ᾽ αὐτὸς λόγος καὶ περὶ τῶν ἀριθμῶν: παρ᾽ ἑκάστας γὰρ τὰς στιγμὰς ἕτεραι ἔσονται μονάδες, καὶ παρ᾽ ἕκαστα τὰ ὄντα, τὰ αἰσθητά, εἶτα τὰ νοητά, ὥστ᾽ ἔσται γένη τῶν μαθηματικῶν ἀριθμῶν

In the hierarchy described by Aristotle, the bottom level is that of sensible solids. The sensible solids are composed of sensible planes, which are, in turn, composed of sensible lines, which are themselves composed of sensible points. The sensible points, firstly, are governed by a set of intelligible points. The sensible lines are governed by a set of intelligible lines, which are, in turn, composed of another set of intelligible points. The sensible planes adhere to a set of intelligible planes, which are composed of another set of intelligible points. Presiding over sensible planes is a set of intelligible planes, which are in turn composed of intelligible lines and points. Finally, the sensible solids would be governed by intelligible solids, which would, in turn, be composed of intelligible planes, points, and lines. Altogether, there would be two sets of solids, three sets of planes, four sets of lines, and five sets of points. The viability of the hierarchy described by Aristotle would necessitate that all sets of geometric objects adhere equally to the mathematical laws pertaining to their class. It would be absurd for those numbers, points, lines, planes, and solids which are posterior to sensible beings, to be mathematically functional if the objects prior to them are entirely inert, or are less versatile in their functionality. In his examination of Aristotle's treatment of mathematical objects, Hussey (1991) posits that the hierarchy described by Aristotle constitutes an infinite regress.[22] Hussey's postulation indeed proves to be correct upon our examination of the hierarchy in terms of mathematical functionality. Within this analysis, the first intelligible point must have at least three dimensional parameters if we are to ascribe any essential mathematical characteristics to posterior points. This property, as we shall now observe, leads to an infinite succession of causes and effects.

According to these parameters, each primary point would possess a minimum of three axis variables, each of which would correspond to one of the dimensions of height, width, and depth, and possibly others as well. Assuming the existence of even one such axis, it follows that primary points are defined according to a linear progression, which constitutes a line prior to primary points. In other words, unless each point is defined within the context of at least one linear continuum, it is mathematically inoperative, and thus utterly meaningless. With the presence of two axes, there is also a plane prior to

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[22] Edward Hussey. "Aristotle on Mathematical Objects", *Apeiron: A Journal For Ancient Philosophy and Science* 24, no. 4 (December 1991), 107.

these points, and with three axes, a solid as well. Since we are not able to identify any demonstrable limit to the length of these axes, it must be assumed that they are of infinite length, for any designated endpoint, if it is a real absolute endpoint and not merely relative to a particular finite geometric structure, would constitute a point possessing only partial geometric operability; yet we are unable to explain how a point of limited geometric functionality might be possible, unless we choose to describe it arbitrarily as such. Thus, since we may assume the axes to be infinite, we are also unable to recognize a beginning point for any of them, and so it would seem that the axes are infinite in multiplicity as well as in length. Each axis would then be composed of an infinite number of points, and there would also be an infinite abundance of lines, planes, and solids. Each line, furthermore, would be infinitely divisible, for Aristotle indicates in his treatise on indivisible lines, that if there existed indivisible lines, then not only would it be impossible to measure any lines, but all lines would be devoid of a midpoint.[23] As such, there would not only be an infinite progression of points according to the unlimited length of the axes, but an infinite continuum of points as well, since each point introduced to the structure would include an infinite number of points within the range of its division. Here we reach an ἀπορία, for on the one hand, as Aristotle has explained, there cannot be a minimum interval beyond which intelligible points are indivisible, for this property would then extend to sensible beings, thereby rendering movement impossible. If, however, we attempt to resolve this problem by making intelligible lines and points infinitely divisible, then we must admit to the presence of the infinite multitude of resulting points, for it would otherwise be necessary to designate the same limit to division as that which we have only now observed to be impossible. We would thus find ourselves returned to the conclusion of Zeno, according to which there can be no real motion,[24] for in order to progress from one point to another, it would be necessary to traverse an infinite expanse of numerical values. Based upon this impasse, the existence of geometric objects as self-subsisting entities, whether within sensible beings or separate from them, is utterly impossible.

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[23] Arist. *LI*. IV. 969b34-5. See also Hett, note a, p 426 for the problem of movement in conjunction with indivisible lines.

[24] Leigh Atkinson. "Where Do Functions Come from?", *The College Mathematics Journal* 33, no. 2 (March 2002), 108.

Nevertheless, if all knowledge belongs to a single structure, and if that structure is complete and immutable, then each dimension must adhere to a code of unchanged, universally applicable laws and formulae. Concerning the relation of these patterns and specifications to the particular objects that they govern, Hussey directs our attention to Aristotle's characterization of mathematical objects as 'representative objects,' such that "…they possess just those properties which (i) are shared by all (actual or possible) individual members of the class that they represent, and (ii) are *representative* properties, i.e., belong to individuals qua members of that class."[25] Although this definition is sufficient for the classification of mathematical objects according to rudimentary genera, a certain amount of elaboration is necessary in order to explain geometric structures with respect to their essential properties and their relation to the cosmic model. One might envision, for instance, a universal model of geometric lines, such that all individual lines are constructed in accordance with the essential attributes of that model, although this model would not necessarily account for all origins that dictate the length of a line. That is to say, the principle governing the lines that constitute spatial magnitudes will not necessarily apply under all circumstances to those lines that are representative of trigonometric ratios such as sine or cosine values, or to those lines corresponding to any geometrically expressible non-spatiotemporal value.

To be sure, all principles pertaining to lines, whether they are spatial magnitudes or some other property, are defined at the level of the intelligible. They are differentiated by the manner in which operations translate into the characteristics of sensible beings. We might also suggest that there is some distinction between the geometric objects representing the structural characteristics of sensible οὐσίαι, and those representing the patterns of motion that are proper to the same. For instance, the arcs that constitute the path of motion of a rock as it skips across a lake might be treated from a purely geometrical standpoint, in which case the only formulae that would have bearing upon the size and structure of the arcs would be those belonging specifically to the laws of geometric magnitude and proportion. If these arcs of motion are considered within their natural context, however, they must be functions of several other factors, such as the weight and shape of the stone, and the force and trajectory with which the rock is thrown.

[25] Hussey. "Aristotle on Mathematical Objects", 118.

This example also serves to illustrate the distinction between geometric structures considered as such in an abstract sense and geometric structures examined according to the formulae by which they occur in the natural world. As we shall observe at later points within our inquiry, this differentiation will prove to be of vital importance in explaining the epistemological and teleological importance of mathematical knowledge. Based upon the distinctions that we have just considered regarding the intelligible articulation of geometric structures, it would likely be most accurate to describe mathematical objects not as specific instances of universal abstract models, but as instances of functions by which the parameters of structure and movement for sensible substances are calculated in accordance with their intelligible definition.

2.2 Dimension, Angle, Structure, and Motion: The Foundations of Mathematics and the True Nature of Numbers

Concerning these primary functions[26], it seems that they cannot be prior to ἀριθμός, for the principles of magnitude and multiplicity are contained within them. Here, we may therefore observe the true nature of ἀριθμοί, their operations, and the manner of their succession, that is to say, whether they always exist and operate simultaneously with one another, or whether instead they represent a progression from the simple to the composite. In order to articulate in their entirety the agency of ἀριθμοί within the structure and operation of the cosmos, we shall look to the principles of number theory that have been credited to the Pythagoreans, for although we face considerable difficulty in identifying them conclusively with their supposed contributions of mathematics, their observations regarding the properties of ἀριθμοί may provide us with more profound insight into the intellectual foundations of Plato's mathematical thought.

[26] To give but a few examples, we might count the arithmetic addition and subtraction operations among the most basic of the primary functions. We might also include those of multiplication and division, though whether we ought to consider these latter functions to be as simple as addition and subtraction is a question of the essential nature of mechanical processes implied within multiplication and division. We might, for instance, interpret multiplication as $\sum_{i=1}^{y} x$, such that the multiplication of the two factors x and y is equivalent to the summation of x over a number of iterations equal to y. Similarly, we might describe the division operation as $x - (\sum_{i=1}^{x-1} y)$, or the summation of the divisor y over a number of iterations one fewer than the dividend x, subtracted from x. In this case, the definitions of multiplication and division would both be dependent upon that of addition, while that of division would also depend upon the definition of subtraction, and possibly that of multiplication as well. In this case, the definitions of multiplication and division would be more complex than those of addition and subtraction, and thus would not be counted among the most basic primary functions. See also Atkinson . "Where Do Functions Come from?" , 111-112 which traces the earliest instances of the modern representation of mathematical functions to Galileo's studies of motion, as well as the principles of analytic geometry first described by Descartes and Fermat.

These teachings, furthermore, may represent our best hope for tracing the mathematical philosophy of Plato to its origins, for a multitude of sources Ancient and contemporary suggest connections between Plato and the first Pythagorean order. Plato is widely regarded to have received some of his teachings under the instruction of the Pythagoreans. In the *Metaphysics*, Aristotle notes that Plato's principle of participation in Forms is almost identical to the Pythagorean concept of imitation of numbers, differing only in terminology.[27] While Socrates' correction of Glaucon's mistake in the *Republic* may cast a measure of doubt on Aristotle's remark,[28] Irwin (1992), furthermore, recognizes the mathematical teachings of the Pythagoreans among the myriad of sources which appear to serve as the foundations of Plato's philosophy.[29] In order to avoid the errors of treating mathematical objects as executive causes when they are better understood as schematic attributes, it will be necessary to take into account Aristotle's arguments against the separation of the objects of mathematics from Form and sensible substance. Throughout our inquiry, we will thereby seek to develop a possible explanation for the operation of mathematical principles as schematic specifications for intelligible paradigms and their expression within sensible reality.

In the investigation of ἀριθμοί as the defining principles of Form and the regulating factors for adherence to Form on the tier of the sensible, we arrive at the examination of numbers as the root of all dimensions. For this purpose, it would be absurd to presume that only abstract quantitative values that are not assigned to any multiplicity or magnitude are counted among ἀριθμοί, and as Ridgeway (1896) suggests, the concept of ἀριθμός was understood by the Pythagoreans to encompass not only arithmetic values but all objects of mathematics. Ridgeway relates as examples such principles as those of ἐπίπεδοι ἀριθμοί, which he interprets as 'superficial numbers,' and

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[27] Arist. *Metaph*. I. 6. 987b11-13. τὴν δὲ μέθεξιν τοὔνομα μόνον μετέβαλεν: οἱ μὲν γὰρ Πυθαγόρειοι μιμήσει τὰ ὄντα φασὶν εἶναι τῶν ἀριθμῶν, Πλάτων δὲ μεθέξει, τοὔνομα μεταβαλών

[28] See n. 14, 15.

[29] T. H. Irwin, "Plato: The intellectual background," in *The Cambridge Companion to Plato*, ed. Richard Kraut. (Cambridge: Cambridge University Press, 1992; 22nd printing 2010), 51. Op. Cit. On Pythagorean mathematics and metaphysics, see D. J. Furley, *The Greek Cosmologists,* vol. 1 (Cambridge, 1987), 57-60; C. H. Kahn, "Pythagorean Philosophy Before Plato," in *The Presocratics*, ed. Alexander P. D. Mourelatos (Garden City, N.Y., 1974), CHAP 6. On the importance of mathematics see *Grg*. 507e6-508a8; *Rep*. 522c-525c. On astronomy and cosmology see esp. G. Vlastos, *Plato's Universe* (Seattle, 1975).

στερεοὶ ἀριθμοί, which he identifies as 'solid numbers.'[30] These geometrical number concepts are discussed in detail by Nicomachus of Gerasa, a Platonist Neo-Pythagorean of the second century AD[31], regarded by Iamblichus as a "true Pythagorean."[32] In his *Introduction to Arithmetic*, Nichomachus discusses geometric numbers as follows:

> A point is the foundation of a dimension, though it is not a dimension, and a dimension is itself the foundation of a line, though not a line itself, while a line is the foundation of a plane, yet is not a plane itself, and is also the foundation of the second dimension, though it is not itself the second dimension. It is then reasonable that the plane is the foundation of the solid, though not a solid itself, and the foundation of the third dimension, though it is not itself the third dimension. Similarly, among numbers, the unit is the foundation of every number, as the line is one according to a unit of progression, while the linear number is the foundation of a planar number, as a subsequent dimension is the foundation of a surface in the manner of a plane, while the planar number is the foundation of a solid number, as is the third dimension from the origin towards the depth of such a structure;[33]

In this consideration of figured numbers, Nichomachus' concept for the solid number is that of a tetrahedron through the emanation of three lines from a single point of origin. Tubbs (2009), references this solid as a geometric representation of 4 as the principle of

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[30] William Ridgeway. "What Led Pythagoras to the Doctrine That the World Was Built of Numbers?", *The Classical Review* 10, no. 2 (March 1896): 92. Ridgeway gives the Pythagoreans little credit for mathematical sophistication. In particular, he is of the position that the Pythagoreans regarded the world to be composed of geometric solids, in contrast to the modern position. Ridgeway draws this conclusion from the notion of sensible objects as "imitations of numbers," and from the classification of numbers according to geometric properties, which he interprets as a difficulty on the part of both Plato and of Pythagoras before him in separating numbers from geometric figures on a conceptual level. It be the case however, that the principle of geometric numbers does not reflect any type of difficulty. He fails, however, to give suitable attention to Aristotle's account of the scientific methodology of mathematics, in which Aristotle describes the approach of mathematicians as one in which they treat physical magnitudes as separate from the sensible characteristics associated with them. (D. W .K. Modrak, "Aristotle on the Difference between Mathematics and Physics and First Philosophy," Nature Knowledge and Virtue: Essays in memory of Joan Kung *Apeiron: A Journal for Ancient Philosophy and Science* 22, no. 4 (December 1989), 122. Op. Cit. Arist. *Metaph*. 1077b27-31; cf. 193b24-6.).

[31] Leonid Zhmud. *Pythagoras and the Early Pythagoreans*. (Oxford: Oxford University Press, 2012), 6.

[32] Dominic J. O'Meara. *Pythagoras Revived: Mathematics and Philosophy in Late Antiquity*. (Oxford: Oxford University Press, 1989; reprint 2002), 15.

[33] Nicom. *Ar.* ii. 7. 1-3, ed. Hoche 86. 9-87. 6. Translation partly after Thomas (2006)

three-dimensional structure. He describes 1, 2, and 3 as the principles of point, dimension, and plane respectively, and explains that the Pythagorean doctrine according to which 10 is associated with the divine, is a function of 10 being the sum of the dimensional root numbers 1, 2, 3, and 4, and therefore representative of completion and perfection.[34] These numbers would then function as dimensional principles, accounting for the axes of spatial magnitude necessary for the construction of three-dimensional objects.

Rather than treating these numbers as causes, however, it would be more accurate to treat them as pre-requisite operations that must be active within reality in order to define the structure of space and time. That is to say, the intelligible operations of dimensional ἀριθμοί must be active in order for the existence of sensible beings to be possible. If these dimensional principles belong to the cosmic model (which seems almost certain, given the need for their ubiquitous operation at the sensible level, and, as we shall observe later, the pre-determined need for three-dimensional structure), then it follows, according to (5), that the order of their generation must be simultaneous and instantaneous. The unit principle, the most basic of the dimensional principles, seems the most likely to be correctly identified with the origin of this generation, for as explained by Hersh (1997), citing Boyer and Merzbach (1991), 'one' was held by the Pythagoreans to be "the generator of numbers, and the number of reason."[35] We might interpret this precept as indicating that the intelligible mathematical 1, treated geometrically as the point principle, has contained within it (according to (5), in which we determined the impossibility of sequential succession at the level of the intelligible) the principles of all other numerical values and mathematical operations. This possibility is merely conjecture, however, since we cannot be certain that our interpretation is consistent with the connotation intended by the Pythagoreans. As indicated by Nikulin (2002), the teachings of the Pythagoreans treat oneness as an "active limiting principle" and the dyad

[34] Robert Tubbs. *What Is A Number?*. (Baltimore: The John Hopkins University Press, 2009), 12.

[35] Reuben Hersh. *What is Mathematics, Really?*. (Oxford: Oxford University Press, 1997), 93. Op. Cit. C. Boyer. *A History of Mathematics*, 2nd ed., revised by Uta C. Merzbach. (New York: Willey, 1991), 53.

as a "passive limited principle."[36] Based on this explanation, along with the identification of the point with the unit, we may say that the first activity of the geometric point marks the beginning of its function as the foundation of all dimensions that follow.

Compounding this observation with Nicomachus' identification of the point as the foundation of the dimension, which, in turn, is distinguished from the line as the foundation of the latter, it follows that the second point, which limits the line is also the foundation of another dimension, which is the foundation of the plane. The limit of the plane dimension constitutes the foundation of the third dimension, the limitations of which produces a solid structure. At the intelligible level, these dimensions and limitations ought to be understood not as segments of a sequential process, but rather as an immediate and simultaneous activity; for unless the emanation of the dimension from the point begins at an arbitrary instant, prior to which the point remains completely idle insofar as it is mathematically inoperative, we must assume that the emanation of the dimension belongs to the geometric point operation immediately such that it is fully realized and in no regard potential. In other words, according to (4), which states that the cosmic model must be immediately and eternally complete, it is necessarily the case that the principle of the mathematical unit-point as defined at the intelligible level must always be fully mathematically operative. Also, with respect to the mathematical points that may be isolated by human intellect through abstraction from sensible beings, it is possible to conceive of an infinite range of angular directions for any dimension originating from such a point. Since we understand the characteristics of the abstracted point according to the mathematical principles of space, angle, and finitude as they are operative within the sensible object from which the point is abstracted, we may say that according to these laws, the point operation must allow for infinite directions of linear dimensional progression.

Nicomachus, explains, moreover, that the dimension, although the foundation of the line, is not the line itself. Since the line is associated with the geometric 2, it may be identified with the recognition of a second point, delineating a finite portion of an

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[36] Dmitri Nikulin. *Matter, Imagination, and Geometry:Ontology, natural philosophy and mathematics in Plotinus, Proclus and Descartes*. Ashgate New Critical Thinking in Philosophy. (Aldershot: Ashgate Publishing Ltd., 2002), 26. Cp. Kallikratidos, 103.11 Thesleff; Iamblichus. Theolog. Arithm. 7,19; 9,6; Pythagoras. Heir. log. 164.24 Thesleff; Anon. Alexandri 234.18-20 Thesleff. On the role and dialectic of *hen* and *aoristos dyas* see: Hösle 1984, 459 sqq.

ostensibly infinite dimension. Thus do we observe one of the ways in which the one might serve as an active limiting operation to the passive dyad. It would, however, be absurd to assume that the dimension remains indivisible prior to a particular moment for the occurrence of the line, so we must assume that the unit of progression according to which the line is defined, is immediately present upon the dimensional continuum. It may therefore be inferred that the activity of the line begins simultaneously with that of the point. Since the line, as Nicomachus indicates, is the foundation of both the plane and of the second dimension, the activity of the second dimension ought to then occur immediately and simultaneously with the operation of the initial point; for in the definition of a finite geometric structure among an infinite multitude of possible dimensions, the second dimension must emanate from the point that designates the limit of the line, and as in the case of the initial point, the emanation must be immediate, for it would otherwise be potential and occur at an arbitrary moment. The activity of the plane would therefore begin in the same instant, as would that of the third dimension and the solid, and thus the activity of all dimensional operations would begin at the same ontological moment.

Within this infinite network of intelligibly active dimensions, made unequivocally actual through their participation in the ordering of sensible beings, the importance of the unit as a mathematical principle is immediately apparent. This special significance is, as one might expect, a function of the unit being understood as the origin from which numbers derive. As Tubbs (2009) states, this position was held by Pythagoras, who, furthermore, did not regard 1 as a number for exactly this reason.[37] In Book Λ of the *Metaphysics*, Aristotle seems to ascribe similar significance to the unit, stating 1 to be "the foundation of number qua number."[38] As indicated, however, by Apostle (1952), the unit itself is understood to be indivisible,[39] and Apostle additionally attributes this

[37] Tubbs. *What is a Number?* 12.

[38] Arist. *Metaph*. X. 1. 1052b.23-24 διὸ τὸ ἓν ἀριθμοῦ ἀρχὴ ᾗ ἀριθμός

[39] Hippocrates George Apostle. *Aristotle's Philosophy of Mathematics.* (Chicago: The University of Chicago Press, 1952), 84. Op. Cit. Arist. *Metaph*. V. 6. 1016b17-25, X. 1. 1052b15-24, X. 6. 1056b32-34.

apprehension to Plato as well as the Pythagoreans.[40] As is now easily apparent to us, the indivisibility of the unit is indefensible in the face of the incommensurability of the diagonal of a square with it sides, along with the applicability of the Pythagorean theorem to a right isosceles triangle. In the case of both the square and the right isosceles triangle, the length of the diagonal will be equal to the length of one of the sides multiplied by √2. Thus, if the unit is understood to be indivisible, the value √2, by means of the fact that it is a non-integer real number, will be impossible. If, furthermore, we assume that there is some minimum fraction, such that the divisibility of all numbers is finite, then it must still follow that √2 is impossible; for √2, as an irrational number that contains an infinite multiplicity of non-repeating decimal places, must allow for infinitely minute base-10 divisions of the unit value, including tenths, hundredths, thousandths, ten-thousandths, and so on. As Tubbs (2009) explains, the discovery of the incommensurability of the diagonal of the square with its sides resulted in the refutation of the Pythagorean commensurability assumption, according to which the ratio of the lengths of two line segments would always be expressible as a ratio of two positive whole numbers. He states also, however, that following this observation, the Greeks did not accommodate irrational values to the study of arithmetic, but rather removed numbers from their approach to geometry, though he cites no source to substantiate this claim.[41] Wedberg (1955) states, by contrast, that incommensurable proportions were merely confined to the study of geometry.[42] More significantly, however, he suggests that Plato's view of ideal units is reminiscent of geometric points as understood by the Pythagoreans.[43] In all these cases, it is in accordance with the single, undivided (though perhaps not entirely indivisible) units.

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[40] Hippocrates George Apostle. *Aristotle's Philosophy of Mathematics.* 84. Op. Cit Arist. *Metaph*. 1001a9-12, X. 1-2. 1053b9-15.

[41] Tubbs. *What is a Number?*. 19-22.

[42] Anders Wedberg. *Plato's Philosophy of Mathematics*. (Stockholm: Almqvist and Wiksell, 1955), 24. Op. Cit. Arist. *A Po*. 76b9.

[43] Wedberg. *Plato's Philosophy of Mathematics*. 25. Op. Cit. Cf. 76. In the earlier instance, Wedberg interprets ideal units according to Plato as being indivisible. This understanding is based upon Aristotle's distinction of points and lines, according to which both are stated to be entirely indivisible (Op. Cit. Arist. *Metaph*. 1016b29-31.). It is more appropriate, however, if they are indivisible, to treat them as being so in a certain qualified sense rather than absolutely; for the assumption that they are indivisible in the absolute

If indeed magnitudes and multiplicities were to be measured by some means other than that of a consistent unit, then we would be at a loss to determine a method that would be viable for such a purpose. If the unit is merely assigned a value other than 1, then all other multiplicities will be different absolutely, yet will remain functionally identical relative to the new value of the unit. In other words, even if a number other than 1 is chosen to represent the base unit, the value of the unit and the relative values of all numbers in proportion to that unit must remain the same if any mathematically meaningful statement is to be made. If no consistent value is assumed for the unit of multiplicity and magnitude, then the principles of mathematics as they are currently understood are overturned, since two instances of what is supposedly the same multiplicity or magnitude may differ from one another due to disparities in the units that comprise them. Since it seems that the consistent unit must then be the principle according to which all multiplicities and magnitudes are determined, it ought to be the case that, in accordance with our observations concerning the ordering of geometric principles, the activity of numbers does not occur by succession, but rather belongs to the same ontological moment; for otherwise, the full extent of mathematical principles would not be immediately realized with the activity of the mathematical unit, and the mathematical unit would lack full functionality prior to the activity of multiplicity, and the prior would thus be affected by the posterior. Thus would all mathematical principles be realized in the first geometric point, and therefore in the first mathematical unit. Having determined that the unit/point principle belongs to the same ontological moment as operations that follow from it, it follows, according to (5) that the principles of the circle and angle must be similarly simultaneous in their ordering.

In *Metaphysics* H, Aristotle describes the circle to be σχῆμα ἐπίπεδον.[44] One possible interpretation of this term is that of a "foundational figure," since it is discussed

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sense would contradict Aristotle's demonstration of the non-existence of indivisible lines (Op. Cit. 11), as well as his premise for the necessary divisibility of sensible οὐσίαι, which is itself a supporting argument for the non-subsistence of geometric objects within tangible beings. The term used by Aristotle in *Metaphysics* Book V is μηδαμῇ διαιρετὸν which Wedberg interprets to mean "in no way divisible," although this expression might be interpreted as meaning, "in no way divided," such that division of the unit is possible, though it is only potential. Given, however, the necessary mathematical and physical divisibility of tangible entities, this expression would be more correctly interpreted to mean, "in no way divided," such that division of the unit is possible, yet unnecessary.

[44] Arist. *Metaph*. VIII. 6. 1045a35.

as a simile to Aristotle's characterization of matter as the essential aspect of objects perceptible through the senses. This interpretation also seems to be supported by Aristotle's remark in *Metaphysics Z* stating that unlike the definition of the syllable, in which all of its elements are specified, the definition of the circle does not include the partitions of the circle.[45] The term σχῆμα ἐπίπεδον might also refer to a planar figure, which would indeed be compatible with a two-dimensional circle. Apostle states that although the circle is often defined according to the straight line, that is, by the equidistance of all lines emanating from the centre of the circle to the circumference, it is more correct to understand the circle and the straight line as being "simultaneous by nature."[46] This statement may be interpreted as indicating that principles of structure and proportion governing the circle and those governing the straight line are interdependent. Indeed, since we have demonstrated that the operations of the line and the plane ought to be similarly simultaneous, it seems to follow that the principles governing angles as defined in the ratios and divisions of the circle are necessary for the full functionality of the dimensional operations, just as the dimensional principles are necessary for the definition of the circle.

In the *Timaeus*, it appears at first glance as though the circle is to be understood as posterior to the principles of dimension. Through observation of Timaeus' account, however, it is possible to recognize the addition of the circle and the sphere to the World-Soul according to a pre-determined objective, as he states that the Demiurge rounded off the structure of the World-Soul into a circle for the purpose of producing a sphere, based on the fact that it is the "most perfect of all shapes and having the greatest likeness himself."[47] Our observation of the circular structure of the cosmos as intelligibly pre-determined is confirmed by Runia (1986) in his analysis of the reading of the *Timaeus* by Philo of Alexandria, in which he draws attention to the necessity of the cosmos being

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[45] Arist. *Metaph*. VII. 9. 1034b. τοῦ μὲν γὰρ κύκλου ὁ λόγος οὐκ ἔχει τὸν τῶν τμημάτων, ὁ δὲ τῆς συλλαβῆς ἔχει τὸν τῶν στοιχείων

[46] Apostle. *Aristotle's Philosophy of Mathematics.*116. Arist. 92b19-22, 1407b26-28, 14b33-15a1, 142b7-10.

[47] Pl. *Ti*. 33b6. πάντων τελεώτατον ὁμοιότατόν τε αὐτὸ ἑαυτῷ σχημάτων

constructed according to an eternal model in order to be κάλλιστος, the most beautiful of all things coming into being.[48] Thus, although the process described by Timaeus appears to be sequential, the model according to which it is carried out is already complete, and therefore it seems that the dimensional operations and the principles of circular geometry are intelligibly and ontologically simultaneous.

The intelligible and ontological simultaneity of the circular and dimensional principles, as required by (5) is also demonstrable by examining the operational connections of mathematical principles. We will use the sine and cosine functions for the purpose of familiarity, even though these functions were unknown to Pythagoras, Plato, and Aristotle.[49] We are, in fact, able to observe this simultaneity within a single trigonometric identity, according to which, as Van Brumemelen (2013) explains, for any two acute angles α and β, $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.[50] This identity alone suggests that some articulation of the cosine function is contained within the definition of the sine function. Both of these principles are contained within the Pythagorean Theorem, as well as within the laws of circular geometry. Within a right triangle, for instance, the sine of each acute angle is equal to the ratio of the side opposite to the angle to the hypotenuse of the triangle, while the cosine is equal to the ratio of the adjacent line to the hypotenuse. On a right triangle for which the hypotenuse γ is assigned the value of 1, and for which there is a certain angle α, the value of the hypotenuse may be expressed as $\sqrt{(\sin\alpha)^2 + (\cos\alpha)^2}$. These identities translate into the unit circle, that is, a circle in which a value of 1 is assigned to the radius, and the angle in question is treated as a fraction of the circle, that is to say, it is equal to *x*/360 where *x* is equal to the measure of the angle in degrees. It is crucial to note that trigonometric ratios such as the sine and cosine belong essentially to the proportional properties of the circle by means of the divisibility of the circle into angles; and based on the immediate completion and absence of sequential succession characterizing the definition of mathematical principles within

[48] David T. Runia. *Philo of Alexandria and the* Timaeus *of Plato*. (Leiden: E.J. Brill, 1986), 113. Op. Cit. Pl. *Ti*. 28c5-29b1.

[49] Glen van Brummelen. *The Mathematics of the Heavens and Earth: The Early History of Astronomy*. (Princeton: Princeton University Press, 2009), 95-96.

[50] Glen van Brummelen. *Heavenly Mathematics: The Forgotten Art of Spherical Trigonometry*. (Princeton and Oxford: Princeton University Press, 2013), 10.

the cosmic model, it would also be the case that all formulae and identities pertaining to angles are immediately defined alongside those belonging to the circle. The circle, it would seem, must therefore be defined alongside the dimensional principle of the plane, which, according to Nicomachus of Gerasa, is defined within 2 as a dimensional root number. Owing, once again, to the completeness and absence of sequential succession within the intelligible, the value of 2 must in turn be defined within the dimensional root number 1, and thus all principles pertaining to circular geometry and trigonometry ought to be defined within 1 as the first number and unit principle.

2.3 Functions and Forms: Determining the Proper Place of Mathematical Principles in the Universe Described by Plato

Though the guidance of Aristotle and the Pythagoreans is invaluable for determining the activity of mathematical laws and operations, it is not sufficient for the execution of this task. In treating mathematics according to the Pythagorean doctrine alone, there is the risk of regressing to the notion of numbers as the primary causes of existence, perhaps in part due to the inclination of the Pythagoreans to treat the symbolic identities of numbers as schematically defined mandates for the inclusion of certain properties in the construction of the cosmos; for instance, as Horky (2013) explains, citing Aristotle, certain "so-called Pythagoreans" based the addition of the Counter-Earth to their planetary model on the basis that 10, being the number associated with perfection, was also assumed to be the correct number of heavenly spheres.[51] The explanation attributed to the Pythagoreans for what they perceived as the structure of the cosmos therefore leaves a great deal to be desired. While it treats numbers as rational patterns for the ordering of the cosmos, it does not, at least in this instance, give any indication of the logical structure to which numbers belong. Additionally, the approach described by Aristotle in the passage cited by Horky does not include any explanation for the precise manner in which mathematical principles are translated into the structure of the sensible world.

[51] Phillip Sidney Horky. *Plato and Pythagoreanism*. (Oxford: Oxford University Press, 2013), 23. Op. Cit. Arist. *Metaph*. I. 5. 985b23-986b21. Translation after M. Schofield. "Pythagoreanism: Emerging from the Presocratic Fog (*Metaphysics* A 5)", In *Aristotle's Metaphysics Alpha: Symposium Aristotelicum*. ed. C. Steel. (Oxford: Oxford University Press, 2012).

Plato's approach, however, places mathematical principles into a larger teleological context within the cosmic model, allowing dimensional root numbers to be unified within a single structure (for as we shall later observe, Aristotle finds it to be impossible for ideal numbers to exist as discrete objects, and refutes the presence of such a concept within Plato's thought). Plato does not dispense with the dimensional root numbers, rather presenting a more comprehensive explanation of their nature, according to which they are defined within principles pertaining to more than geometric structure alone. As Mugler (1948) explains, the definitions of the four elements account for the dimensional root numbers, with fire and earth accounting for the two "extremes," that is, the point-1 and the solid-4 respectively, while air and water account for the line-2 and the plane-3.[52] This approach, as stated by Mugler, is derived from the Pythagorean doctrine, as he states that the Demiurge's plan is "inspired by the ancient dream of the Pythagoreans to explain the universe according to number…"[53] It might, furthermore, be observed, that the dimensional principles seem to translate into certain aspects of Plato's epistemology, as the description of the various levels of mathematical knowledge in the *Republic* might be said to correspond, to a certain extent, to the dimensional principles. This education begins with simple arithmetic calculation, which is stated to be "refined and useful for many things that we desire."[54] Alone, however, it is insufficient for the examination of visible and tangible corporeal objects.[55] Geometry is therefore necessary as well,[56] yet as we have observed previously from Socrates' correction of Glaucon's error regarding the nature of geometric objects, geometry alone is not sufficient to

[52] Charles Mugler. *Platon et la Recherche Mathématique de son Époque*. (Strasbourg and Zurich: Éditions P. H. Heitz, 1948), 82. Op. Cit Pl. *Ti*. 32b5-7. "…le Démiurge établit entre ces deux éléments extrême deux autres éléments comme moyens, l'air et l'eau, de manière que le rapport de feu à l'air soit égal à celui de l'air à l'eau, et ce dernier égal au rapport de l'eau à la terre…"

[53] Mugler. *Platon et la Recherche Mathématique de son Époque.* 82. "…dans ce plan inspiré par le vieux rêve des Pythagoriciens d'expliquer l'univers par le nombre…"

[54] Pl. *R*. VII. 525d1-2. ὡς κομψόν ἐστι καὶ πολλαχῇ χρήσιμον ἡμῖν πρὸς ὃ βουλόμεθα

[55] Pl. *R*. VII. 525d7-8. οὐδαμῇ ἀποδεχόμενον ἐάν τις αὐτῇ ὁρατὰ ἢ ἁπτὰ σώματα ἔχοντας ἀριθμοὺς προτεινόμενος διαλέγηται

[56] Pl. *R*. VII. 526c10-11. τὸ ποῖον; ἢ γεωμετρίαν, ἔφη, λέγεις;αὐτὸ τοῦτο, ἦν δ᾽ ἐγώ

articulate the principles of mathematics in terms of their ontological context.[57] Astronomy is thus required as well,[58] and we may regard each of these mathematical sciences as a parallel to one of the dimensional principles; for since, first of all, arithmetic calculation involves operations involving numbers belonging to an ordered continuum, it may be associated with the linear dimension. Geometry, despite its consideration of three-dimensional figures, may be likened more closely to the planar dimension, since it is inclined to treat visible shapes diagrammatically, as representations derived from nature, rather than as objects within nature. It is, then, the treatment of the planetary orbits as they are in nature which identifies astronomy as a higher science than geometry, and since astronomy builds upon calculation and geometry and adds the element of motion, it may be considered the most complete of the mathematical sciences and may therefore be associated with the solid dimension.

The concept of ideal numbers, that is to say, numbers as intelligible principles pertaining to certain specific mathematical values and operation, is treated more overtly in the *Phaedo*, for which Wedberg refers to Socrates' explanation that each specific instance of a certain multiplicity adheres to a particular intelligible mathematical principle, with 2, for example, being an instance of an eternal model of duality.[59] Within the *Phaedo*, Cebes, one of the Pythagoreans with whom Socrates converses as he awaits his execution, interprets Socrates' position thus,

> And that you should say to be great that which you do not know except by each particular instance manifesting according to participation in the idea of being for each thing that participates, and you hold the cause of two to be none other than that of participation in duality, and that it is necessary for two to be a participation in such a thing, and that it is necessary for one to be a participation in singularity, and

[57] See n. 14, 15.

[58] Pl. *R*. VII. 527d1.

[59] Wedberg. *Plato's Philosophy of Mathematics*. 77. Op. Cit. Cf. 135.

that there are separations and applications of these, and there are compositions of others if you allow them to separate…[60]

A literal interpretation of this passage would suggest that for each positive integer value, there exists an intelligible prototype to which all specific instances of that value must adhere. Moravcsik (1992) understands the passage in this way, stating that according to the *Phaedo*, numbers are indeed to be understood as Forms such that they are objects of knowledge, "and of which other entities can participate in ways to be explicated."[61] In this reading of the *Phaedo*, it appears counterintuitive that specific multiplicities and magnitudes should adhere to a principle that is only the model of the corresponding value rather than following a single schematic governing all mathematical laws and functions.

If we assume each magnitude and multiplicity to participate in a distinctive model according to its value, then it is clear that each specific multiplicity is more versatile in its mathematical functionality than the model in which it participates; for if each ideal number is to remain distinct, it cannot be added, subtracted, divided, multiplied, or combined in any other sort of operation with another ideal number to produce any other value, whether ideal or specific. If another ideal number is produced from such an operation, then that ideal number will in that instance be an effect of those ideal numbers that were the operands, while in another operation, it may be effects of those numbers if such an operation constitutes an opposite process. For example, an ideal number that is the sum of two other ideal numbers might also produce any one of the same values that functions as operands in the operation of addition. The succession of causes and effects among mathematical principles would be tangled, except in the case that all ideal numbers are assumed to be ontologically simultaneous, an explanation similar to the structure that has previously been proposed.[62] Aristotle suggests, moreover, that ideal numbers of the type that we presently examine, are entirely absent from Plato's thought,

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[60] Pl. *Phd*. 101c2-9. καὶ μέγα ἂν βοφῃς ὅτι οὐκ οἶσθα ἄλλως πως ἕκαστον γιγνόμενον ἢ μετασχὸν τῆς ἰδίας οὐσίας ἑκάστου οὗ ἂν μετάσχῃ, καὶ ἐν τούτοις οὐκ ἔχεις ἄλλην τινὰ αἰτίαν τοῦ δύο γενέσθαι ἀλλ᾽ ἢ τὴν τῆς δυάδος μετάσχεσιν, καὶ δεῖν τούτου μετασχεῖν τὰ μέλλοντα δύο ἔσεσθαι, καὶ μονάδος ὃ ἂν μέλλῃ ἓν ἔσεσθαι, τὰς δὲ σχίσεις ταύτας καὶ προσθέσεις καὶ τὰς ἄλλας τὰς τοιαύτας κομψείας ἐᾷς ἂν χαίρειν, παρεὶς ἀποκρίνασθαι

[61] Julius Moravcsik. *Plato and Platonism*. (Cambridge, Mass: Blackwell, 1992), 63.

[62] See pages 12-15.

stating in the *Nicomachean Ethics* that the Platonists of Plato's lifetime rejected the possibility of succession among Ideas, and therefore refuted the existence of ideal numbers.[63] Thus, a possible alternative interpretation of the ideal numbers discussed in the *Phaedo* is that they are to be understood not as models for entities, but rather as intelligible instructions according to which actions occur on the level of specific beings. Prior (1983) proposes such an interpretation, stating that there is an alternate connotation for the term παράδειγμα, which, although it may be defined as "example," or "instance," may also be understood as "pattern." He regards this connotation as the most appropriate in referring to the Forms associated with predicated characteristics. For this sense of the term he cites *Protagoras* 326c8, in which life according to laws is likened to behaviour according to a certain pattern.[64]

Gerson (2004) presents a similar argument, using the examples of the terms "like," and "similar," and explaining that these are not "well-formed or perspicuous metaphysical concepts, and that the Forms are best understood as "natures" rather than as "really distinct entities."[65] The understanding of Forms in this respect appears to contain the implication not of mere templates, but rather of schematics that contain not only specifications of structure for sensible beings, but also of pre-programmed patterns of activity according to the definition of the entity in question. If these patterns of activity are indeed to be understood as patterns and not as arbitrary motions free from prediction according to natural reason, then it seems that they ought to follow a series of regulations that must necessarily be included within their natures, and must therefore be prior to those things that are termed as Forms. Such a dependency would suggest that prior to the Forms to which sensible beings adhere, there ought to be at least one other class of

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[63] Arist. *EN*. I. 6. 1096a17-19. οἱ δὴ κομίσαντες τὴν δόξαν ταύτην οὐκ ἐποίουν ἰδέας ἐν οἷς τὸ πρότερον καὶ ὕστερον ἔλεγον, διόπερ οὐδὲ τῶν ἀριθμῶν ἰδέαν κατεσκεύαζον

[64] William J. Prior. "The Concept of Παράδειγμα in Plato's Theory of Forms", *Apeiron: A Journal For Ancient Philosophy and Science* 17, no. 1 (June 1983), 36. Op. Cit. *Webster's Seventh New Collegiate Dictionary* (Springfield, G.C. Merriam Co., 1972) p.610, *The American Heritage Dictionary of the English Language* (Boston, Houghton Mifflin co., 1978), p.950, Liddell, Scott, and Jones, *A Greek-English Lexicon* (Oxford, clarendon, 1940), p.1307, Pl. *Prt*. 326c8.

[65] Lloyd P. Gerson. "Plato on Identity, Sameness, and Difference", *The Review of Metaphysics* 58, no. 2 (Dec., 2004), 305, 329.

intelligibles which constitute the principles of structure and activity regulating the generation and motion of sensible beings.

The concept of a system comprised of two classes of intelligible principles has been subject to consideration in contemporary scholarship, with one example having been proposed by Thesleff (1999). Thesleff distinguishes the more general category of Forms as referring to the entirety of the objects of knowledge, including the schematics of sensible beings, as well as the intelligible principles pertaining to intangible concepts.[66] He indicates, however, that Ideas are to be understood as a particular subset of Forms, particularly those which may be termed 'Value Forms,' of which he gives such examples as 'good,' 'fine,' 'strong,', and 'healthy,' which are distinguished as abstract concepts that constitute attributes rather than schematics governing sensible entities..[67] Thesleff also includes ideal numbers, such as those of the μονάς and δυάς mentioned in the *Phaedo*, among the class of intelligible principles to be understood as Ideas.[68] If Ideas constitute a proper subset of Forms in reality as well as in abstraction, then they ought to belong to the same ontological moment as all other Forms, since it would be absurd for them to either be prior to a genus to which they belong, or to be posterior to those Forms that are dependent upon them for their essential characteristics. In order for Thesleff's theory to adhere to (5), that is, to avoid sequential succession, the activity of all Forms therefore ought to be recognized as simultaneous and interdependent, so that those Forms outside of the subset of Ideas are anticipated in the conception of Ideas. There is no reason to assume, however, that the distinction posited by Thesleff necessarily represents a real dichotomy at the intelligible level.

In conjunction with our consideration of ideal numbers, we must consider additionally the generation and functionality of the dimensional principles of geometry as they are articulated by Plato. This task will lead to the examination of the *Timaeus*,

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[66] Holger Thesleff. "Studies in Plato's Two-Level Model", (Commentationes Humanarum Litterarum 113), Societas Scientiarum Fennica, Helsinki: 1999, in *Platonic Patterns: A Collection of Studies*, ed. Holger Thesleff. (Athens, Las Vegas, and Zurich: Parmenides Publishing, 2009), 439.

[67] Thesleff. "Studies in Plato's Two-Level Model", 441. Op. Cit. Santas 1983, Dorter 1994.

[68] Thesleff. "Studies in Plato's Two-Level Model", 447. Op. Cit. Pl. *Phaedo*. 101b.

wherein the eponymous philosopher describes thus the initial activity of these laws through the work of the Demiurge,

> It needed to be exactly corporeal, visible, and tangible, so he separated fire, and yet even then it was not visible, nor was it tangible without any solid, and it was not solid without earth; so did the god, being the first of all things, make the corporeal by mixing together earth and fire. It was impossible for the first and the second to be combined beautifully with a third being separate; for it was necessary that in the centre there be some connection binding both of these together. And so to bind it, he, the most beautiful, made it so that in binding it would be supremely one, which brought to completion the most beautiful mathematical proportion. For whenever the middle is raised by either third numbers or powers, so that it goes first toward the same side, and then towards the far end, and then back again, and the end point towards the middle, and the middle towards the first end, and the middle and the beginning and the end becoming such that the end point and the beginning both go towards the middle, so it follows that on all sides out of necessity, that each will become one with all others. So if a plane, not having any depth, must become the body of all, the same thing then rises up from the centre, such that it is bound in the same way, and its solid nature will therefore then be manifest, the solids will never be one, for two centres always fit together, and between fire and earth the god placed water and air, and towards one another in such a way that it was possible for to bring that calculation upward towards completion, with fire adjacent to air, air adjacent to water, and as with air adjacent to water, water adjacent to earth, he bound and combined the universe to be visible and tangible. And through these ways, and the number of four units, and out of such things he begat the body of the cosmos through a harmony of proportion and he held love for these things, such that they would be bound towards him alone, inseparable by one another except to be bound by him.[69]

[69] Pl. *Ti*. 31b4-32c4. σωματοειδὲς δὲ δὴ καὶ ὁρατὸν ἁπτόν τε δεῖ τὸ γενόμενον εἶναι, χωρισθὲν δὲ πυρὸς οὐδὲν ἄν ποτε ὁρατὸν γένοιτο, οὐδὲ ἁπτὸν ἄνευ τινὸς στερεοῦ, στερεὸν δὲ οὐκ ἄνευ γῆς: ὅθεν ἐκ πυρὸς καὶ γῆς τὸ τοῦ παντὸς ἀρχόμενος συνιστάναι σῶμα ὁ θεὸς ἐποίει. δύο δὲ μόνω καλῶς συνίστασθαι τρίτου χωρὶς οὐ δυνατόν: δεσμὸν γὰρ ἐν μέσῳ δεῖ τινα ἀμφοῖν συναγωγὸν γίγνεσθαι. δεσμῶν δὲ κάλλιστος ὃς ἂν αὐτὸν καὶ τὰ συνδούμενα ὅτι μάλιστα ἓν ποιῇ, τοῦτο δὲ πέφυκεν ἀναλογία κάλλιστα ἀποτελεῖν. ὁπόταν γὰρ ἀριθμῶν τριῶν εἴτε ὄγκων εἴτε δυνάμεων ὠντινωνοῦν ᾖ τὸ μέσον, ὅτιπερ τὸ πρῶτον πρὸς αὐτό, τοῦτο αὐτὸ πρὸς τὸ ἔσχατον, καὶ πάλιν αὖθις, ὅτι τὸ ἔσχατον πρὸς τὸ μέσον, τὸ μέσον πρὸς τὸ πρῶτον, τότε τὸ μέσον

At the beginning of this passage, Plato indicates a predetermined need for the existence of tangible, stereometric entities. This requirement agrees with the suggestion that intelligible definition of the point ought to account immediately for all basic dimensional levels leading up to the sterometric level and perhaps even beyond it. The simultaneous definition and distinction of all dimension levels might explain the designation of 10 as the perfect number as opposed to 4, which, according to the Pythagorean doctrine, is considered representative of perfection; for 10, as the summation of all representative numbers associated with the dimensional levels implies a model that accounts for all dimensional levels, both together and as considered distinctly. Within this particular passage, Plato appears to equate each of the elements with one of the dimensional levels, suggesting that even beyond the significance of elemental solids (which we shall examine later), each of the elements can be regarded as what might be called a rational symbol, such that it serves, in a sense, as a metaphor for one of the four basic dimensional levels, except that in this instance, the symbolic connection is not merely incidental, but rather constitutes a real relation that is defined intelligibly.

One of the distinctions readily apparent in Plato's account of geometric dimensional principles is the recurring theme of predetermined perfection and of the objective of optimizing the beauty of the structure that is described. We observe also that the process described occurs with the purpose of allowing for the existence of solid objects on the tangible level. Mohr (2005), in interpreting the *Timaeus*, argues that the primary bodies, the elements of earth, air, fire, and water, are to be understood as "geometrical particles" even in the initial chaotic state of existence. The Demiurge, Mohr states, does not impose intelligible properties upon these elements, but rather regulates them with respect to proportion, such that they adhere to the intelligible patterns that are

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μὲν πρῶτον καὶ ἔσχατον γιγνόμενον, τὸ δ᾽ ἔσχατον καὶ τὸ πρῶτον αὖ μέσα ἀμφότερα, πάνθ᾽ οὕτως ἐξ ἀνάγκης τὰ αὐτὰ εἶναι συμβήσεται, τὰ αὐτὰ δὲ γενόμενα ἀλλήλοις ἓν πάντα ἔσται. εἰ μὲν οὖν ἐπίπεδον μέν, βάθος δὲ μηδὲν ἔχον ἔδει γίγνεσθαι τὸ τοῦ παντὸς σῶμα, μία μεσότης ἂν ἐξήρκει τά τε μεθ᾽ αὑτῆς συνδεῖν καὶ ἑαυτήν, νῦν δὲ στερεοειδῆ γὰρ αὐτὸν προσῆκεν εἶναι, τὰ δὲ στερεὰ μία μὲν οὐδέποτε, δύο δὲ ἀεὶ μεσότητες συναρμόττουσιν: οὕτω δὴ πυρός τε καὶ γῆς ὕδωρ ἀέρα τε ὁ θεὸς ἐν μέσῳ θείς, καὶ πρὸς ἄλληλα καθ᾽ ὅσον ἦν δυνατὸν ἀνὰ τὸν αὐτὸν λόγον ἀπεργασάμενος, ὅτιπερ πῦρ πρὸς ἀέρα, τοῦτο ἀέρα πρὸς ὕδωρ, καὶ ὅτι ἀὴρ πρὸς ὕδωρ, ὕδωρ πρὸς γῆν, συνέδησεν καὶ συνεστήσατο οὐρανὸν ὁρατὸν καὶ ἁπτόν. καὶ διὰ ταῦτα ἔκ τε δὴ τούτων τοιούτων καὶ τὸν ἀριθμὸν τεττάρων τὸ τοῦ κόσμου σῶμα ἐγεννήθη δι᾽ ἀναλογίας ὁμολογῆσαν, φιλίαν τε ἔσχεν ἐκ τούτων, ὥστε εἰς ταὐτὸν αὑτῷ συνελθὸν ἄλυτον ὑπό του ἄλλου πλὴν ὑπὸ τοῦ συνδήσαντος γενέσθαι

already proper to them. In this argument, Mohr specifies that the Demiurge is bringing these elements into accord with paradigmatic Forms.[70] Mohr's argument appears to agree with the manner in which the Demiurge activates the principle of stereometric structure within the World-Soul, for Timaeus describes this process as one that involves the separation of the primary corporeal elements into their four distinct structures, and the cyclical connection of the elements. The language used by Plato to describe this action therefore suggests that the primary corporeal structures possess certain immediate intelligible characteristics, or that the elements described at the earlier portion of the text are to be understood as patterns added to the World-Soul by the Demiurge to function as schematics of structure and motion to which tangible matter must adhere. In either case, we will doubtless observe that matter is subordinated to mathematical principles in preparation for the generation of specific sensible οὐσίαι.

[70] Richard D. Mohr. *God and Forms in Plato*. (USA: Parmenides Publishing, 2005; Originally published as *The Platonic Cosmology*. Leiden: E.J. Brill, 1985) 111-112. Op. Cit. Pl. *Ti*. 53c ff., 55d7, 53a7-c3, 69b4, cf. μέτρον, 68b6.

Chapter 3: Ὕλη

3.1 Clarifying Confused Mass: Geometry, Physics, and the Nature of Matter

The relation of motion to matter, and therefore the essential nature of matter, is closely connected with the significance of necessity as a factor in the construction of the universe. In the account of the cosmogony as set forth in the *Timaeus*, the Demiurge is stated on several occasions to construct the cosmos ἐξ ἀνάγκης, or "from necessity."[71] Vlastos (1941) describes necessity, or ἀνάγκη, as "the "secondary" cause, which is "necessary," irrational, fortuitous, and disorderly."[72] As an irrational force, necessity must, in some regard, be at odds with the rational governance of the cosmic model, yet in a different sense, it must collaborate with reason. Timaeus indicates, for instance, that even after the application of precise geometric structure to matter, the difference in movement speed on the part of the elemental solids based on size, with the solids possessing the smallest sides being the most agile, is a function of necessity.[73] Since necessity continues to hold sway over matter even after its ordering, it is clear that the Demiurge has not removed necessity from the cosmos; rather, it would seem that matter has been patterned so as to take advantage of the natural inclinations of necessity for the purpose of directing matter to conform to intelligible paradigmatic specifications. From this relation between reason and necessity, it would follow that the paradigms that govern the sensible cosmos must account for the operations of necessity, and that the role of matter in the construction of the universe is defined within the intelligible.

The greatest difficulty that we will encounter in addressing the intelligibility of matter is the characteristic of movement and flux according to which it is defined. The conflict that we seek to address is between the impermanence associated with matter, and the geometric properties ascribed to them by Plato. Our task is to explain the nature of matter in terms of its role in the operation of intelligible mathematical principles as expressed in the Forms, on the level of sensible beings. Part of this inquiry will involve the consideration of whether the geometric structures of matter are particles in

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[71] Pl. *Ti*. 28a9, 32a5, 37c2.

[72] Gregory Vlastos. "Morals, Politics, Metaphysics", in *Platonic Studies*. ed. Gregory Vlastos. (Princeton, New Jersey: Princeton University Press, 1973), 155. Op. Cit. Pl. 48a, 56c, 68e, 47e, Cf. Pl. *Phlb*. 26d6, 7.

[73] Pl. *Ti*. 56a6-7. ταῦτ᾽ οὖν δὴ πάντα, τὸ μὲν ἔχον ὀλιγίστας βάσεις εὐκινητότατον ἀνάγκη πεφυκέναι

themselves, or whether they are patterns of motion followed by simple base particles of a different sort. In addressing this question, we will also determine whether, in explaining matter from a mathematical standpoint, we are stating what matter is, or rather, how matter behaves.

For this purpose, we will chiefly examine the account of the elements as given in the *Timaeus*, considered in conjunction with Ancient criticisms, particularly those of Aristotle, as well as the responses of contemporary scholars, so as to give a mathematically robust explanation of the structure and activity of tangible matter. The intricacy of the geometric theory of matter appears somewhat ironic given Plato's rejection of sensible entities as objects, a position which, as explained by Lloyd (1968), has been interpreted as an 'anti-empirical bias' according to which Plato's ontology has been criticised as anti-scientific by certain scientists of the last century.[74] Lloyd opposes this argument, however, and explains that Plato, along with Pythagoras, has also been considered to "stand nearer to modern physical science than does Aristotle."[75] This suggestion appears to be at least somewhat correct with regard to Aristotle's own argument concerning matter. As stated by Byrne (2001), Aristotle maintains that the mathematical sciences do not examine matter, and argues that they are more precise for this reason.[76] Similar to Plato, however, Aristotle is observed to hold the four elements of earth, air, fire and water to be primary bodies;[77] yet since Aristotle's position rejects the mathematical analysis of the primary bodies, it would seem that his position concerning matter is somewhat more rudimentary than that which is presented by Plato. It may therefore be argued that he fails to acknowledge the need for a complete articulation of mathematical principles to turn its inquiry towards the distinguishing properties of matter as defined by their precise characteristics of structure and movement.

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[74] G. E. R. Lloyd. "Plato as a Natural Scientist", *The Journal of Hellenic Studies* 88 (1986), 79. Op. Cit. T. L. Heath, *Aristarchus of Samos* (Oxford, 1913) 138-9, commenting on *Republic* 529a-530b.

[75] Lloyd. "Plato as a Natural Scientist", 78. Op Op. Cit. *Science and the Modern World* (Cambridge, 1926).

[76] Christopher Byrne. "Matter and Aristotle's Material Cause", *Canadian Journal of Philosophy* 32, no. 1 (March 2001), 94. Op. Cit. Arist. *Metaph*. II. 3. 995a14-18.

[77] Kurt Smith. *Matter Matters*. (Oxford: Oxford University Press, 2010), 45. Op. Cit. Sir David Ross. *Aristotle* London: Methuen & Co Ltd., 1923), 66, and A. E. Taylor. *Aristotle*. (New York: Dover Publications, 1955), 71.

The need for these distinctions is indicated in Aristotle's treatise *On Generation and Corruption*, for Aristotle states therein that fire and earth are inclined to move toward the "limits" of space whereas earth and water move towards the "centre."[78] The ascription of limits and a centre to the space in question would imply finite magnitude, and would indicate that this space is subject to mathematical measurement. With regard, moreover, to the nature of motion, Aristotle explains in the *Physics* that motion is classified among those things that are understood to be continuous, and that place, void, and time are among those conditions that are considered to be necessary in order for motion to occur.[79] It would seem that any meaningful examination of time and place would require some sort of mathematical expression. As far as void is concerned, assuming that it is an enclosed space relative to two or more solid surfaces and not absolute void (a consideration that we shall address at a later point), then it stands to reason that any questions pertaining to the magnitude of such a space would belong to geometric analysis. Aristotle must then acknowledge that any attempt to articulate the nature of motion in an entirely non-mathematical manner would be incomplete and imprecise. The problem presented, therefore, by Aristotle's position, is not that the ontological significance of mathematical principles is refuted, but rather that the manner in which the human mind understands mathematical principles may be insufficient for the task of treating them ontologically.

As explained by Lear (1982), Aristotle's assertion that mathematicians examine non-spatiotemporal objects of thought is not regarded by commentators as a sufficient epistemological solution to the ontological problems associated with the conventional interpretation of Plato.[80] Lear makes specific reference to a passage of the *Physics* in

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[78] Arist. *GC*. II. 3. 330b31-33. trans. Harold H. Joachim. in *The Basic Works of Aristotle*. ed. Richard McKeon. Modern Library Paperback Edition. (New York: The Modern Library, 2001) 51.

[79] Arist. *Ph*. 200b15-21. trans. R.K. Gaye and R.P. Hardie. in *The Basic Works of Aristotle*, ed. Richard McKeon. Modern Paperback Library Edition. (New York: The Modern Library, 2001), 253.

[80] Jonathan Lear. "Aristotle's Philosophy of Mathematics", *The Philosophical Review* 91, no. 2 (April 1982), 161. Op. Cit. Ian Mueller, "Aristotle on Geometrical Objects," *Archiv für Geschichte der Philosophie*, 157 (1970). Lear specifies here that Aristotle is frequently interpreted as arguing that sensible οὐσίαι "do not perfectly instantiate mathematical properties." Op. Cit. Ian Mueller, "Aristotle on Geometrical Objects," *Archiv für Geschichte der Philosophie*, 52 (1970); Julia Annas, *Aristotle's Metaphysics* M *and* N (Oxford: Clarendon Press, 1976), p. 29. Since the imperfections in the adherence of tangible οὐσίαι to their essential geometric structures is a function of the inclusion within the definition of

which Aristotle states that the mathematician separates mathematical objects from sensible beings in thought due to the fact that they are then "separable from motion (kinesis)." If the mathematician does not treat the objects of physics with respect to motion, then we are at a loss to explain how it is possible to construct an ontologically relevant λόγος of mathematics; for Aristotle's argument against separate Forms in *Metaphysics* B is supported in part by the absurdity of the notion that the object of astronomical science is a separate sky which remains unmoving while movement occurs in the sensible sky.[81] The critical aspect of Aristotle's premise is that the Form cannot be bereft of any essential characteristic that is present in the objects that participate in it, thus dictating that if movement belongs properly to a certain sensible being, that movement is also defined within the paradigm to which that being adheres.

Consequently, if the mathematical sciences treat magnitudes as separate from movement, then not only are they unable to produce complete apprehension of intelligible paradigms, but they would also likely fail to account for the laws of mathematics specifically pertaining to spatial motion and change. The analysis of motion and change is of critical importance in the capacity of mathematical science to articulate the structure and activity of the cosmos, and thus, to advance toward the consideration of the principles of mathematics in terms of their operation within reality. The inclusion of motion within mathematical investigation is a defining characteristic of astronomy, which Plato indicates in the *Republic* to be of a higher tier of inquiry than arithmetic and geometry in terms of its capacity "…to lead us toward the most beautiful things and to possess those that are most accurate…"[82] While astronomy includes the principles of both geometry and arithmetic, it also adds the element of motion to its analysis, as well as

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these substances of their necessary application to matter, it is necessary that the parameters under which these deviations occur are represented on the level of the intelligible. According to Aristotle, however, mathematicians examine only the geometrically perfected abstractions of sensible beings. As such, the epistemological inadequacy of abstraction might be said to at least partially be characterized by its inability to account for variation and deviation in the structure and activity of sensible beings.

[81] Arist. *Metaph*. II. 2. 997b15-22. ὥστ᾽ ἐπείπερ ἡ ἀστρολογία μία τούτων ἐστίν, ἔσται τις καὶ οὐρανὸς παρὰ τὸν αἰσθητὸν οὐρανὸν καὶ ἥλιός τε καὶ σελήνη καὶ τἆλλα ὁμοίως τὰ κατὰ τὸν οὐρανόν. καίτοι πῶς δεῖ πιστεῦσαι τούτοις; οὐδὲ γὰρ ἀκίνητον εὔλογον εἶναι, κινούμενον δὲ καὶ παντελῶς ἀδύνατον

[82] Pl. *R*. VII. 529c7-d1. ταῦτα μὲν τὰ ἐν τῷ οὐρανῷ ποικίλματα, ἐπείπερ ἐν ὁρατῷ πεποίκιλται, κάλλιστα μὲν ἡγεῖσθαι καὶ ἀκριβέστατα τῶν τοιούτων ἔχειν

language pertaining specifically to objects residing in tangible space. Since geometry and arithmetic fall short of this measure of accuracy, they would also be insufficient for explaining with full precision the subordination of matter to Form. Thus, it would appear that, contrary to Aristotle's position, the mathematical sciences are in fact less precise if they do not examine sensible matter, for they consequently fail at the task of articulating the role of geometric principles in imparting Form to matter.

Plato, however, undertakes this task in the *Timaeus* wherein the eponymous philosopher relates an account of the geometric structures ascribed to each of the elements. Though this account is not necessarily without error, and though Timaeus himself states that it is merely likely, it provides us with a potentially viable foundation on which to develop λόγος of the mathematical properties of matter that agrees with the principles of first philosophy and the ontological simultaneity of mathematical functions. Our inquiry shall therefore concentrate on the mutable nature of matter in connection with the theory of elemental solids as detailed in the *Timaeus*. This flux is partly explained by Gill (1987), who draws our attention to one of Plato's characterizations of matter in the *Timaeus*. In this instance it is appropriate to refer to fire and water not as "this," but rather as "what is such."[83] This description suggests that the elemental properties of matter are not to be understood as non-predicated objects, but rather as states that may be ascribed to tangible matter which would be otherwise indescribable. It seems then to follow that this definition should belong to the geometric patterns of the elements. The foundations of these elemental solids are from triangular structures, which Timaeus describes thus,

> So he made two triangular solids for fire and for the other elements, the isosceles and the scalene. Those which were once indistinct in character were now distinguished. For he differentiated the four classes of the elements, not being correctly visible, from one another in order to encompass all of nature; for from the triangles of which we have proposed four classes, three of them have sides which are not all equal to one another, while the fourth alone comes together from the isosceles triangle. It is thus not possible for the smallest triangles to be broken into smaller triangles, nor for

[83] Mary Louise Gill. "Matter and Flux in Plato's *Timaeus*", *Phronesis* 34 (October 1986), 34-35. Op. Cit. Pl. *Ti*. 49c7-50a4.

> the largest to be combined into larger triangles, and the same is true of the other three types of triangles; for they would all be produced from one of the triangles that had been broken loose, and from the larger ones, many smaller triangles would follow, receiving their proper structures from them, until the smaller triangles are scattered into many other triangles, such that a single number having come into being from one great multitude produces one different form. I have explained how these transform into one another; in this way each shape would come into being, and would be considered to follow from these numbers being combined. Thus would the first shape originate, and the smallest component, an element of this, would have a hypotenuse that is double the length; a pair of these triangles are added together according to the diagonal, and three triangles of the type produced are joined at the centre along their short sides and diagonals, such that one equilateral triangle comes into being from those which are six in number.[84]

The rules governing the triangles that constitute the foundation of the elemental solids seem to disrupt the mathematical integrity of this theory, for Timaeus states that they cannot be divided beyond their smallest size, nor can they be combined beyond their largest size. This restriction leads to the same problems described by Aristotle concerning indivisible lines,[85] for if the triangles are indivisible at their smallest size, then it stands to reason that the lines of which they are comprised are similarly indivisible, and are therefore mathematically inoperable. Indeed, given the impossibility of spatiotemporal

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[84] Pl. *Ti*. 54b2-e3. προῃρήσθω δὴ δύο τρίγωνα ἐξ ὧν τό τε τοῦ πυρὸς καὶ τὰ τῶν ἄλλων σώματα μεμηχάνηται, τὸ μὲν ἰσοσκελές, τὸ δὲ τριπλῆν κατὰ δύναμιν ἔχον τῆς ἐλάττονος τὴν μείζω πλευρὰν ἀεί. τὸ δὴ πρόσθεν ἀσαφῶς ῥηθὲν νῦν μᾶλλον διοριστέον. τὰ γὰρ τέτταρα γένη δι᾽ ἀλλήλων εἰς ἄλληλα ἐφαίνετο πάντα γένεσιν ἔχειν, οὐκ ὀρθῶς φανταζόμενα: γίγνεται μὲν γὰρ ἐκ τῶν τριγώνων ὧν προῃρήμεθα γένη τέτταρα, τρία μὲν ἐξ ἑνὸς τοῦ τὰς πλευρὰς ἀνίσους ἔχοντος, τὸ δὲ τέταρτον ἓν μόνον ἐκ τοῦ ἰσοσκελοῦς τριγώνου συναρμοσθέν. οὔκουν δυνατὰ πάντα εἰς ἄλληλα διαλυόμενα ἐκ πολλῶν σμικρῶν ὀλίγα μεγάλα καὶ τοὐναντίον γίγνεσθαι, τὰ δὲ τρία οἷόν τε: ἐκ γὰρ ἑνὸς ἅπαντα πεφυκότα λυθέντων τε τῶν μειζόνων πολλὰ σμικρὰ ἐκ τῶν αὐτῶν συστήσεται, δεχόμενα τὰ προσήκοντα ἑαυτοῖς σχήματα, καὶ σμικρὰ ὅταν αὖ πολλὰ κατὰ τὰ τρίγωνα διασπαρῇ, γενόμενος εἷς ἀριθμὸς ἑνὸς ὄγκου μέγα ἀποτελέσειεν ἂν ἄλλο εἶδος ἕν. ταῦτα μὲν οὖν λελέχθω περὶ τῆς εἰς ἄλληλα γενέσεως: οἷον δὲ ἕκαστον αὐτῶν γέγονεν εἶδος καὶ ἐξ ὅσων συμπεσόντων ἀριθμῶν, λέγειν ἂν ἑπόμενον εἴη. ἄρξει δὴ τό τε πρῶτον εἶδος καὶ σμικρότατον συνιστάμενον, στοιχεῖον δ᾽ αὐτοῦ τὸ τὴν ὑποτείνουσαν τῆς ἐλάττονος πλευρᾶς διπλασίαν ἔχον μήκει: σύνδυο δὲ τοιούτων κατὰ διάμετρον συντιθεμένων καὶ τρὶς τούτου γενομένου, τὰς διαμέτρους καὶ τὰς βραχείας πλευρὰς εἰς ταὐτὸν ὡς κέντρον ἐρεισάντων, ἓν ἰσόπλευρον τρίγωνον ἐξ ἓξ τὸν ἀριθμὸν ὄντων γέγονεν

[85] See n. 23.

motion which Aristotle demonstrates to follow from the indivisibility of lines and points, it seems quite clear to us at this point that the possibility of movement within tangible space presupposes the infinite divisibility of all finite magnitudes.

It is therefore impossible for the indivisibility of the stoichiometric triangles at their smallest size to imply indivisibility in the mathematical sense. Lloyd (1952) suggests, however, that there are two senses in which division may be understood. He indicates that Plato acknowledged the infinite divisibility of magnitudes on the level of abstract mathematics, but distinguished this system from the 'philosophical' mathematics discussed in the *Republic* and the *Philebus*.[86] In the *Republic*, Glaucon uses this differentiation in order to argue against the identification of sensible beings with the objects of knowledge, making reference to those who hold these entities to be indivisible mathematically on the basis that they are essentially indivisible.[87] In the *Philebus*, Socrates speaks of those things which are "unequal units" insofar as they are entities belonging to a particular genus or to a designated group. He states that within this group, the smallest objects are also the largest, yet explains that this rule does not apply to the "μονάδα μονάδος," or the unit as unit, such that abstract values are infinitely divisible according to their definition, whereas tangible οὐσίαι are essentially largest and smallest in terms of their divisibility.[88] Regardless of whether or not the stoichiometric triangles are to be understood as particles, it is readily evident that their indivisibility cannot be mathematical; for if they are particles, then their mathematical indivisibility would translate into the sides of the triangles, such that they would be subject to the limitations associated with indivisible lines. If, however, they are not particles, but are rather

[86] A. C. Lloyd. "Plato's Description of Division", *The Classical Quarterly* 2, ½ (January –April 1952), 106. Op. Cit. Arist. *Metaph*. 992a22; cf. 1084b1-2, Pl. *Prm.* 164c-165d; Cf. 142c7-145a2, Pl. *Phlb*. 56d-e, Pl. *R*. 525d-526a, Pl. *Sph.* 256e-257a.

[87] Pl. *R*. VII. 525d8-e4. οἶσθα γάρ που τοὺς περὶ ταῦτα δεινοὺς αὖ ὡς, ἐάν τις αὐτὸ τὸ ἓν ἐπιχειρῇ τῷ λόγῳ τέμνειν, καταγελῶσί τε καὶ οὐκ ἀποδέχονται, ἀλλ᾽ ἐὰν σὺ κερματίζῃς αὐτό, ἐκεῖνοι πολλαπλασιοῦσιν, εὐλαβούμενοι μή ποτε φανῇ τὸ ἓν μὴ ἓν ἀλλὰ πολλὰ μόρια

[88] Pl. *Phlb*. 56d9-e3. οἱ μὲν γάρ που μονάδας ἀνίσους καταριθμοῦνται τῶν περὶ ἀριθμόν, οἷον στρατόπεδα δύο καὶ βοῦς δύο καὶ δύο τὰ σμικρότατα ἢ καὶ τὰ πάντων μέγιστα: οἱ δ᾽ οὐκ ἄν ποτε αὐτοῖς συνακολουθήσειαν, εἰ μὴ μονάδα μονάδος ἑκάστης τῶν μυρίων μηδεμίαν ἄλλην ἄλλης διαφέρουσάν τις θήσει

patterns, then the problem that results from their mathematical indivisibility is greater still, for these paths of motion will be non-traversable as a result.

In both of these cases, it would be impossible to determine any proportional relation between the side lengths of an isosceles stoichiometric triangle and those of the scalene, thus prohibiting the assembly of the elemental solids, which Timaeus thus describes,

> Combining four equilateral triangles at three conjoined planar angles he made a single solid angle, coming from the collinear joining of the planar angles; and thus he assembled the first solid among those four that are the most perfect, scattered throughout the entire orbit, toward equal proportion and uniformity. The second he made out of these triangles, which were assembled to be from eight equilateral triangles, and so completed a single solid angle from four planes; and coming into being from these the second body thus attained perfection. When third was made from twice sixty of these put together with twelve solid angles, based on five planar equilateral triangles arranged adjacently around the centre, he also put together twenty sides from equilateral triangles. It was necessary to create a solid other than these elements, and so from arranging four isosceles triangles around a central point, he completed a square; six such shapes were combined to produce eight solid angles, from each being put together on three straight planes; and the structure of the assembled body became a cube, having six square sides.[89]

It would seem that the construction of the elemental solids also includes the application of their visible properties, as Timaeus states that the Demiurge "commanded the solids to

[89] Pl. *Ti*.54e3-55c4. τρίγωνα δὲ ἰσόπλευρα συνιστάμενα τέτταρα κατὰ σύντρεις ἐπιπέδους γωνίας μίαν στερεὰν γωνίαν ποιεῖ, τῆς ἀμβλυτάτης τῶν ἐπιπέδων γωνιῶν ἐφεξῆς γεγονυῖαν: τοιούτων δὲ ἀποτελεσθεισῶν τεττάρων πρῶτον εἶδος στερεόν, ὅλου περιφεροῦς διανεμητικὸν εἰς ἴσα μέρη καὶ ὅμοια, συνίσταται. δεύτερον δὲ ἐκ μὲν τῶν αὐτῶν τριγώνων, κατὰ δὲ ἰσόπλευρα τρίγωνα ὀκτὼ συστάντων, μίαν ἀπεργασαμένων στερεὰν γωνίαν ἐκ τεττάρων ἐπιπέδων: καὶ γενομένων ἓξ τοιούτων τὸ δεύτερον αὖ σῶμα οὕτως ἔσχεν τέλος. τὸ δὲ τρίτον ἐκ δὶς ἑξήκοντα τῶν στοιχείων συμπαγέντων, στερεῶν δὲ γωνιῶν δώδεκα, ὑπὸ πέντε ἐπιπέδων τριγώνων ἰσοπλεύρων περιεχομένης ἑκάστης, εἴκοσι βάσεις ἔχον ἰσοπλεύρους τριγώνους γέγονεν. καὶ τὸ μὲν ἕτερον ἀπήλλακτο τῶν στοιχείων ταῦτα γεννῆσαν, τὸ δὲ ἰσοσκελὲς τρίγωνον ἐγέννα τὴν τοῦ τετάρτου φύσιν, κατὰ τέτταρα συνιστάμενον, εἰς τὸ κέντρον τὰς ὀρθὰς γωνίας συνάγον, ἓν ἰσόπλευρον τετράγωνον ἀπεργασάμενον: ἓξ δὲ τοιαῦτα συμπαγέντα γωνίας ὀκτὼ στερεὰς ἀπετέλεσεν, κατὰ τρεῖς ἐπιπέδους ὀρθὰς συναρμοσθείσης ἑκάστης: τὸ δὲ σχῆμα τοῦ συστάντος σώματος γέγονεν κυβικόν, ἓξ ἐπιπέδους τετραγώνους ἰσοπλεύρους βάσεις ἔχον

be painted in diverse colours."[90] It is possible that these colours refer to the various types of minerals and other types of chemicals associated with the various types of elements, for Plato associates with gold the properties of water in terms of the manner in which it is poured and hardened into a particular shape through the use of some sort of mould. Such would imply that the visual and chemical properties are defined in the calculations of the elemental structures rather than being imparted directly to the particles themselves; and as we will recall, Plato suggests that even without the calculation of the solids, the distinction of the elements is already given according to their order in proximity to one another.[91] The language that Plato uses to describe the application of geometric structure to the elements suggests, as follows, that they are assigned to the elements as fully calculated patterns,

> To earth he gave the shape of a cube; for among the four types of elements earth is unmoving, and among the moulded bodies it is necessary that the one that becomes earth have sides that are entirely unmovable; a side that is constructed according to the foundations of the triangles described will be unmovable according to the nature of equal sides in conjunction with the unequal, and from each equilateral triangle the square was built according to proportion, and from the motionless whole it began movement out of necessity. We therefore maintain that we ought to assign this as the most likely theory for earth, and of those remaining we assign to water the slightly moving shape, the greatly moving shape to fire, and the middling shape to air; and the smallest shape belongs to fire, the largest belongs to water, and the medium shape belongs to air; and the sharpest belongs to fire, the second sharpest belongs to air, and the least sharp belongs to water. And so in order to understand all of these, one must thereby understand the small, as each one of each type is not seen by us on account of its smallness, but we perceive the entirety of all such things assembled; and furthermore, from the mathematical proportions pertaining to quantity and movement and the other potentialities everywhere, we consider the god, to whom

[90] Pl. *Ti*.55c5-6. ἐπὶ τὸ πᾶν ὁ θεὸς αὐτῇ κατεχρήσατο ἐκεῖνο διαζωγραφῶν

[91] Pl. *Ti*. 32b3-8. οὕτω δὴ πυρός τε καὶ γῆς ὕδωρ ἀέρα τε ὁ θεὸς ἐν μέσῳ θείς, καὶ πρὸς ἄλληλα καθ᾽ ὅσον ἦν δυνατὸν ἀνὰ τὸν αὐτὸν λόγον ἀπεργασάμενος, ὅτιπερ πῦρ πρὸς ἀέρα, τοῦτο ἀέρα πρὸς ὕδωρ, καὶ ὅτι ἀὴρ πρὸς ὕδωρ, ὕδωρ πρὸς γῆν, συνέδησεν καὶ συνεστήσατο οὐρανὸν ὁρατὸν καὶ ἁπτόν

> nature, out of necessity, having been commanded, readily complied, to assemble these by proportion according to him for all of the elements through precisions of the completed solids.[92]

Considering Plato's earlier characterization of the different types of primary matter as "what is such," it is reasonable to suggest that the geometric structures ascribed to the elements are not to be understood as corporeal shapes in the most literal sense, but rather as patterns of movement defined by the necessary properties of each element. This position is further supported by Timaeus' account of the manner in which completely formulated geometric patterns are imparted to matter that is delineated in type according to its relative position.

As to the specific character of these patterns, they may be interpreted as spatial patterns, or as geometric number patterns associated with the activity of the elements, yet to understand them as corporeal entities in the same sense as composite sensible objects would suggest a measure of permanence contrary to Plato's earlier characterization of matter. Mortley (1967) nevertheless refers to the elemental solids as particles, yet still recognizes within them a quality that remains compatible with them if they are recognized as patterns of movement, and proves in that case to be of far greater importance; that is, the properties by which they are distinguished in sensory observation are not "secondary qualities," but are rather a function of the shape of the solids.[93] This

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[92] Pl. *Ti*. 55d8-56c7. γῇ μὲν δὴ τὸ κυβικὸν εἶδος δῶμεν: ἀκινητοτάτη γὰρ τῶν τεττάρων γενῶν γῆ καὶ τῶν σωμάτων πλαστικωτάτη, μάλιστα δὲ ἀνάγκη γεγονέναι τοιοῦτον τὸ τὰς βάσεις ἀσφαλεστάτας ἔχον: βάσις δὲ ἥ τε τῶν κατ᾽ ἀρχὰς τριγώνων ὑποτεθέντων ἀσφαλεστέρα κατὰ φύσιν ἡ τῶν ἴσων πλευρῶν τῆς τῶν ἀνίσων, τό τε ἐξ ἑκατέρου συντεθὲν ἐπίπεδον ἰσόπλευρον ἰσοπλεύρου τετράγωνον τριγώνου κατά τε μέρη καὶ καθ᾽ ὅλον στασιμωτέρως ἐξ ἀνάγκης βέβηκεν. διὸ γῇ μὲν τοῦτο ἀπονέμοντες τὸν εἰκότα λόγον διασῴζομεν, ὕδατι δ᾽ αὖ τῶν λοιπῶν τὸ δυσκινητότατον εἶδος, τὸ δ᾽ εὐκινητότατον πυρί, τὸ δὲ μέσον ἀέρι: καὶ τὸ μὲν σμικρότατον σῶμα πυρί, τὸ δ᾽ αὖ μέγιστον ὕδατι, τὸ δὲ μέσον ἀέρι: καὶ τὸ μὲν ὀξύτατον αὖ πυρί, τὸ δὲ δεύτερον ἀέρι, τὸ δὲ τρίτον ὕδατι. ταῦτ᾽ οὖν δὴ πάντα, τὸ μὲν ἔχον ὀλιγίστας βάσεις εὐκινητότατον ἀνάγκη πεφυκέναι, τμητικώτατόν τε καὶ ὀξύτατον ὂν πάντῃ πάντων, ἔτι τε ἐλαφρότατον, ἐξ ὀλιγίστων συνεστὸς τῶν αὐτῶν μερῶν: τὸ δὲ δεύτερον δευτέρως τὰ αὐτὰ ταῦτ᾽ ἔχειν, τρίτως δὲ τὸ τρίτον. ἔστω δὴ κατὰ τὸν ὀρθὸν λόγον καὶ κατὰ τὸν εἰκότα τὸ μὲν τῆς πυραμίδος στερεὸν γεγονὸς εἶδος πυρὸς στοιχεῖον καὶ σπέρμα: τὸ δὲ δεύτερον κατὰ γένεσιν εἴπωμεν ἀέρος, τὸ δὲ τρίτον ὕδατος. πάντα οὖν δὴ ταῦτα δεῖ διανοεῖσθαι σμικρὰ οὕτως, ὡς καθ᾽ ἓν ἕκαστον μὲν τοῦ γένους ἑκάστου διὰ σμικρότητα οὐδὲν ὁρώμενον ὑφ᾽ ἡμῶν, συναθροισθέντων δὲ πολλῶν τοὺς ὄγκους αὐτῶν ὁρᾶσθαι: καὶ δὴ καὶ τὸ τῶν ἀναλογιῶν περί τε τὰ πλήθη καὶ τὰς κινήσεις καὶ τὰς ἄλλας δυνάμεις πανταχῇ τὸν θεόν, ὅπῃπερ ἡ τῆς ἀνάγκης ἑκοῦσα πεισθεῖσά τε φύσις ὑπεῖκεν, ταύτῃ πάντῃ δι᾽ ἀκριβείας ἀποτελεσθεισῶν ὑπ᾽ αὐτοῦ συνηρμόσθαι ταῦτα ἀνὰ λόγον

[93] Raoul J. Mortley. "Primary Particles and Secondary Qualities in Plato's Timaeus", *Apeiron: A Journal for Ancient Philosophy and Science* 2, no. 1 (November 1967), 17.

characterization is certainly appropriate to spatial or abstract patterns of motion, for it seems absurd to suggest that the non-accidental effects of these patterns would be a function of a secondary non-intelligible affectation.

They may alternatively be regarded as calculated patterns to be applied to the elemental mass that the Demiurge is stated to separate into its components at an earlier portion of the *Timaeus*.[94] This interpretation seems self-evident given the language of Timaeus's account, according to which the Demiurge appears to assign the appropriate elemental solids to elements that have already been distinguished from one another. According to the Matrix hypothesis examined by Ostenfeld (1982), these solids are understood as belonging to a "universal Matrix." Ostenfeld presents the Matrix hypothesis as a possible interpretation of the Receptacle concept, inasmuch as the elemental solids are imprinted into the Matrix, within which their size and shape translate into properties of weight and speed. Ostenfeld expresses this type of receptacle as something of a mould, and describes it as a Matrix according to Plato's reference to the ἐκμαγεῖον, which is stated to be "shaped by the things pressed into it."[95] The elemental solids, as well as the tangible entities constructed from them, would therefore constitute patterns imprinted within an otherwise indistinct mass.

Ostenfeld identifies the elemental solids as atoms, and states that their tangible properties are partly a function of their belonging to the Matrix. He explains also that the primary characteristics of cosmic matter (i.e. ordered matter) are those of shape, size, and location, with attributes such as weight, motion, sharpness, and hardness being derived. Ostenfeld thereby determines that the elemental solids are not to be understood as fully

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[94] See n. 69.

[95] Erik Nis Ostenfeld. *Form, Matter, and Mind*, Martinus Nijhoff Philosophy Library Volume 10 (The Hague: Martinus Nijhoff Publishers, 1982), 125. Op. Cit. Pl. *Ti*. 50c2-6. In the text of the *Timaeus*, the use of the expression διασχηματιζόμενον ὑπὸ τῶν εἰσιόντων carries the implication of shapes produced as impressions within some sort of medium, and therefore seems to suggest that Ostenfeld's universal Matrix theory is congruous with Plato's explanation of matter as a receptacle of Form. Although Ostenfeld does not explicitly specify the relation of the universal Matrix to the cosmic model, it stands to reason that, given the eternity of the cosmic model, the Matrix cannot be understood to have prior existence to the cosmic model. Since we have determined (4) that the model can never be in an incomplete state, and, as we shall later observe, it belongs essentially to the cosmic model to be instantiated at the sensible level, it also stands to reason that the instantiation of the cosmic model must be immediate, for the cosmic model would otherwise be incomplete. It then follows that the universal Matrix must immediately have instantiations of Form imprinted within it.

geometric or corporeal, but rather as intermediate with respect to these classes. He describes the universal Matrix, furthermore, as being full on the basis that Plato refutes the presence of void within the universe described in the *Timaeus*. Ostenfeld specifies also that the solids alone do not constitute atoms, but are only understood as such in conjunction with the Matrix, such that they might be understood as imprints upon the Matrix. He also significantly identifies the geometric characteristics as kinematic, lending credence to the identification of the elemental solids as patterns of motion.[96] According to Ostenfeld's observation, the absence of void in the cosmogony as described in the *Timaeus* appears to be unqualified, though Keyt (1961) indicates that in Aristotle's analysis, the principle of void, or κένον, is present in Plato's λόγος of the natural world. Keyt explains that place and void may be understood in several different senses. One of the proposed conceptions of place, which, according to Keyt, Aristotle associates with Plato, is that of διάστημά τι, which refers to an extension between "the boundaries of the vessel," or τὰ πέρατα τοῦ ἀγγείου. This extension is understood to be a constantly present distance in addition to the distance defined by the movement of a sensible object. Aristotle also refutes the existence of such an extension, on the basis that it would constitute a subject to which magnitude would be ascribed, thus demonstrating absolute emptiness to be impossible.[97]

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[96] Ostenfeld. *Form, Matter, and Mind*. 125-7. Concerning the sharpness of the atoms, Ostenfeld refers to *Timaeus* 61d-e. In this passage, Timaeus explains that the apparent heat of fire atoms is due to their sharpness, a characterization which might be seen to suggest that the atoms are in fact corporeal. The term used to imply sharpness, ὀξύ, may also be understood to imply swiftness, which in this case may be appropriate to suggest faster motion on the part of fire atoms in contrast to surrounding atoms which move at a slower speed. Ostenfeld references Plato's refutation of void at *Timaeus* 52e and 58a. At the former of these two passages, it is suggested that the presence of space between the elemental layers would prevent their movement. In this passage it is also indicated that the elements should not be equidistant from one another, nor should they be balanced in stasis, for they would consequently be incapable of balanced movement.

[97] David Keyt. "Aristotle on Plato's Receptacle", *The American Journal of Philology* 82, no. 3 (July 1961), 291. Aristotle discusses void in *Physics* at 214a13 (Trans. R.P. Hardie and R.K. Gaye.). One of the interpretations of void, as he explains, is that it is an entirely empty space entirely devoid of any corporeal substance. One of the arguments that has developed from this interpretation is that "void is the matter of the body," a position which Aristotle rejects on the basis that it appears to imply the existence of non-predicated matter. The compatibility of the rejection of non-predicated matter with the refutation of absolute void appears to require a model similar to that which is described in the *Timaeus* with respect to matter; for if the physical space surrounding solid objects does not constitute absolute void, then it must follow that this space is entirely filled with matter of some sort. If there may be no predicated matter, then this matter must be structured according to a precise pattern, such as the shape that Timaeus ascribes to air. In identifying place with the "boundaries of the vessel" at *Physics* 212a13, Aristotle expresses the vessel as

Since the boundaries of the vessel do not constitute the limits of the receptacle itself (which we shall soon consider in further detail), but rather the limits of specific sensible objects, the identification of unoccupied space with absolute void will present difficulties regardless of whether Ostenfeld's Matrix principle is understood to be a correct explanation of the physics of the *Timaeus*. If we reject Ostenfeld's Matrix, then the elemental solids must be fully corporeal, as they would not, consequently, be unified in a geometric network. In both cases, the characterization of unoccupied physical space as absolute void bereft of all matter would render certain natural phenomena inexplicable. We would, for instance be unable to account for the fact that a flame, which ought to be identified with the fire element, is extinguished upon being enclosed on all sides; for if the space surrounding the flame prior to its enclosure is absolute void, then it should add nothing to the flame, such that there is no functional difference between the space within the enclosure and the space outside of the enclosure. Such, however, is not the case, and it therefore seems that unoccupied space cannot accurately be regarded as absolute void.

Within Ostenfeld's Matrix, the presence of absolute void would imply that there are areas of interruption within the Matrix, such that it is not perfectly continuous, even (assuming that it is finite) within its designated boundaries. In this instance, the void would render atoms unable to function properly within the Matrix; for the void would be such that they would be unable to traverse it. Ostenfeld indicates that atoms as described in the physics of the *Timaeus* are not truly corporeal, and that their tangible properties are characterized by the manner in which their geometric structure behaves within the context of the Matrix.[98] If, therefore, unoccupied space is to be identified as absolute

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a sensible object rather than as a limit to the receptacle itself. He explains the διάστημά τι at *Physics* 211b7-8 to be an extension between the boundaries of a corporeal object, and argues that the space between the boundaries does not exist independently beyond the displacement of the corporeal object in question (211b13-17), since there would otherwise be an infinite multitude of places within the same physical space according to the infinite number of possible arrangements for the objects occupying the space (211b19-26). Keyt interprets Aristotle as attributing the principle of διάστημά τι to Plato based on the remarks of the former at *Physics* 209b11, at which Aristotle explains that Plato identified matter with space. Aristotle regards Plato's position as a function of the fact that place is contained by the boundaries of the corporeal object, and is therefore understood to constitute matter (209b5-10). The only circumstance, however, under which this characterization of space would imply absolute void, would be that in which the matter detailed in the *Timaeus* is understood to be non-predicated in such a manner as to be non-existent altogether. The identification of matter with space otherwise suggests a perspective on matter similar to the Matrix discussed by Ostenfeld.

[98] See n. 96.

void, then sensible entities will be incapable of movement; for the motion of tangible objects is contingent upon physical properties such as weight and hardness, which are properties of a sensible object's interaction with the Universal Matrix, such that they are not inherent characteristics of the atoms comprising the entity. Atoms would consequently be capable of moving only within insular matrices defined by the boundaries, and sensible οὐσίαι would be unable to carry out the activities proper to the paradigms to which they must adhere.

It is self-evident to us that the geometric patterns ascribed to the elements are necessitated by the paradigms according to which sensible οὐσίαι are constructed. Since the universe is understood to be produced according to necessity as determined by an eternal model within which the intelligible principles for sensible beings must be defined according to their properties of structure and movement, it must therefore also contain the parameters of their material composition. In Mueller's treatment of the theory of matter discussed in the *Timaeus* (2005), he explains that matter, identified with the receptacle, is understood by Plato to be devoid of any inherent shape, but instead "receives the likeness of the eternal forms although in itself it has no particular character."[99] This explication of matter indicates that the ordering of the elements according to the stoichiometric solids implies an application of Form to the previously amorphous mass, not only insofar as it is subordinated to mathematical principles, but also inasmuch as the ordering of matter according to these patterns anticipates the paradigms of sensible οὐσίαι. Broadie (2012) provides a further indication of the nature of the ordering of pre-cosmic matter, explaining that the elements initially moved in a random, disorderly manner.[100] The ordering to the elements would then seem to imply the reconfiguration of their

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[99] Ian Mueller. "Mathematics and the Divine in Plato", in *Mathematics and the Divine: A Historical Study*, ed. L. Bergmans and T. Koetsier (Amsterdam: Elsevier, 2005), 108.

[100] Sarah Broadie. *Nature and Divinity in Plato's* Timaeus. (Cambridge: Cambridge University Press, 2012), 182. Op. Cit. Pl. *Ti*. 48a5-7; cf.30a2-6. In the earlier of these passages, the Demiurge is described as having arrayed the elements into τάξις, a term which may also identified with the combat formation of soldiers (Liddell and Scott, 1996), suggesting the imposition of order upon the elements implies bringing them into a structured pattern of motion. In the later passage, Timaeus describes the elements as the "wandering cause," a term possibly intended to emphasize the connection of the principles that will ultimately govern their movement with the mathematical laws that guide the heavenly spheres, since these are also termed as "wanderers." (Pl. *Ti*. 38c6.)

movements according to a precise pattern, an interpretation further emphasized by Plato's identification of their ordering with a term used to describe military formations,[101] which may also be a reference to the importance of mathematical knowledge in military strategy as detailed in the *Republic*.[102] We may thus identify two different juxtapositions of motion and passivity; for under pre-cosmic conditions we observe that matter moves in a random and incoherent fashion, and therefore remains indolent inasmuch as it is unable to become anything or otherwise serve any purpose; while cosmic matter is in motion inasmuch as it fulfills the task adhering to the parameters of intelligible paradigms, yet is also at rest insofar as it has settled into regular patterns and abandoned its previously unstable disposition.

In addressing the connection between the role of matter as a Receptacle and its geometric properties, Broadie states that the Receptacle is considered to be that in which the event of becoming occurs,[103] as opposed to an alteration transpiring merely within the spatial position of the object in question, an explanation which Broadie characterizes as nearer to the Aristotelian position. She states that under this representation, the elements are not to be understood as objects unto themselves, but rather as qualities predicated of sensible entities. Quite significantly, moreover, geometric properties such as triangular shape are classified in this manner as well.[104] This characterization would be applicable

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[101] See n. 100.

[102] Pl. *R*. VII. 527d2-6. ἐμοὶ γοῦν, ἔφη: τὸ γὰρ περὶ ὥρας εὐαισθητοτέρως ἔχειν καὶ μηνῶν καὶ ἐνιαυτῶν οὐ μόνον γεωργίᾳ οὐδὲ ναυτιλίᾳ προσήκει, ἀλλὰ καὶ στρατηγίᾳ οὐχ ἧττον. ἡδὺς εἶ, ἦν δ᾽ ἐγώ, ὅτι ἔοικας δεδιότι τοὺς πολλούς, μὴ δοκῇς ἄχρηστα μαθήματα προστάττειν

[103] Broadie. *Nature and Divinity in Plato's* Timaeus. 188. Op. Cit. Pl. *Ti*. 49e7-8. In this passage, the use of the term περιφερόμενον suggests not only containment, by also movement around the shape of the object being contained.

[104] Broadie. *Nature and Divinity in Plato's* Timaeus. 188-9. Op. Cit. Pl. *Ti*. 50a6, b2, cf. c3. In the second of these passage, the shapes such as triangles are stated not to be ὡς ὄντα, but rather to be τ ὸ τοιο ῦτον, while in c3 it is stated that "an imprint is made upon all nature (ἐκμαγεῖον γὰρ φύσει παντὶ κεῖται)." This description is consistent with Ostenfeld's Matrix theory, suggesting the Universal Matrix as a suitable explanation for the application of intelligible paradigms to matter. If the characterization of geometric shapes as predicated qualities rather than as discrete objects is assumed to be correct, it would appear to be compatible with the Matrix argument presented by Ostenfeld. It seems, furthermore, to be consistent with the definition of the objects of mathematics not as specific numbers or geometric entities, but rather as functions, formulae, and relational laws. The difficulty with this definition, as Broadie indicates, is that Plato does not provide any viable argument to support it. The lack of substantiation for this position may simply be a function of the fact that the cosmogony described in the *Timaeus* is considered to be probable but not certain. A possible alternative explanation is that the function of matter as a vessel for movement

to the elemental solids as well, and is consistent with Ostenfeld's specification that the elemental solids are not tangible in themselves.[105] The concept of shape as a predicated property is also of significance in establishing the relation between the elemental solids and the models according to which their structure is determined; for in identifying the four elements as τὸ τοιοῦτον,[106] Plato gives credence to the characterization of the elemental solids not as stable, self-subsistent entities, but rather as calculated patterns of motion, to which the otherwise indescribable particles of matter must adhere. Broadie specifies, however, that identification of the elements as predicated qualities is not explicitly stated in the geometric description of matter. She indicates, however, that the geometric account of the elements is necessary in order to provide an articulation of the types of motion associated with the elements, as such an explanation is absent from the Receptacle account.[107] These representations of matter, taken together, provide clear indication that the Receptacle must be prepared before it is able to receive intelligible paradigms, and this preparation is defined by the ordering of the elements according to proportion.[108] Since the specifications of this ordering are dictated by the properties defined in the Forms, it may be easily inferred that the stoichiometric properties of sensible οὐσίαι must be defined within the paradigms to which these entities adhere. It may be said also that these paradigms must include the laws of stoichiometry in order to fully account for the properties of movement belonging to tangible beings.

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and change is assumed to be self-evident; for if the movement of sensible beings are considered to be dictated by the paradigms to which the entities in question adhere, it would be absurd for specific alterations and motions to occur within the paradigms, assuming that they are regarded as the objects of knowledge.

[105] See n. 94.

[106] See n. 83.

[107] Broadie. *Nature and Divinity in Plato's* Timaeus. 195. Op. Cit. Pl. *Ti*. 55d8-56b3. According to this account, it is evident that among the mobile solids, those of greater mobility possess fewer vertices. One possible explanation for this rule is that the solids represent geometric numbers rather than spatial particles, with each vertex representing a dependent variable, and each triangle signifying a relation of height, width, and depth. This theory will be considered further in section 2.2 in relation to the possible association of elemental solids with vector geometry.

[108] Broadie. *Nature and Divinity in Plato's* Timaeus. 191. Cit Pl. *Ti*. 69b2-c7.

We have previously determined that the paradigms of sensible beings are dependent upon ontologically operative mathematical formulae and functions for their properties of structure and motion. This necessity is demonstrated with particular clarity through the parameters of movement and alteration as specified within the Forms; for we are at a loss to explain the mobility and mutability of sensible οὔσίαι, except through consideration of these properties at the quantum level. Though the stoichiometric model set forth is presented as speculation, and is crude in comparison with present-day knowledge of motion at the atomic and subatomic level, it represents a significant attempt to explain distinctions among the essential characteristics of motion as they belong to tangible objects. It also reflects awareness of the importance of the relative properties of objects in motion; for in the absence of such relations, there is great difficulty in explaining such phenomena as the difference in water displaced by two distinct objects of identical spatial dimensions. In addressing the functions and formulae of motion as distinct properties, we shall examine two possible explanations for the precise functionality of the elemental solids as principles of motion.

3.2 Solid Vectors and Dimensional Relations: Two Theories Concerning the Structure and Motion of Matter

In our consideration of matter within the Platonist cosmic system, the identification of matter as τὸ τοιοῦτον, or "what is such,"[109] appears to preclude the identification of the elemental solids as self-subsistent, fully tangible structures. As Timaeus elaborates on the patterns of motion associated with these solids, he distinguishes them from earth, air, fire, and water, but treats them rather as qualities, such that a certain material may be considered hydromorphic, geomorphic, pyromorphic, or aeromorphic, a characterization that he relates as follows,

> From all of these we have thus introduced what is most likely. Earth packed together is pulled apart by fire, if it should experience the sharpness of fire, and it will be disassembled in the sharpness of fire, air, and water, and as far as the parts of earth are pushed together in some place, they will be drawn back together toward one another to become earth – for they cannot, under any circumstances come together

[109] See n. 83.

into another shape – and water is divided under fire and under air, which allows it to become one assembled solid of fire and two of air; The two pieces of air are formed from a single disassembled solid [of water] if solids of fire are produced. Contrariwise, whenever fire is encompassed by air, water, and earth, with few among many, being moved in orbits, being attacked and defeated it breaks apart, two solids of fire into one combined shape of air; and from air having been overpowered and broken apart as well as two and a half water solids a single whole shape will be joined together. Thus we ought to have inferred these things once again, as whenever there is containment of other elements in fire there will be some type of angle beneath it, and it would be cut along the sides by the sharpness, while coming together according to the nature of each, it will cease to be cut; for each type will be one and the same to it, and it will not be possible for it to produce any change, and it will not be similarly affected through containment by these solids – until it sets one that is stronger against one that is of lesser resilience, it will not cease in being unbound. And once again whenever the smaller solids are encompassed by the many larger solids, the smaller solids, having been broken apart, will be quenched, and aiming to come together toward the prevailing form they will cease to be quenched, and air will take shape out of fire, and water out of air; and if it moves against combinations of other types, it will not stop them from coming apart, or until they have either been entirely driven and released so that they fly according to their innate dispositions, or have been overpowered, one of many will become the same as the most powerful, so that it remains together with that solid. And furthermore, according to these effects, they exchange places entirely; for he [the god] separated the multitude of genera according to the movement given to them, the differences between them, and the commonalities between them, and they were carried by shock, toward the place for each one, to which they ought to adapt.[110]

[110] Pl. *Ti*. 56c1-57c6. ἐκ δὴ πάντων ὧνπερ τὰ γένη προειρήκαμεν ὧδ᾽ ἂν κατὰ τὸ εἰκὸς μάλιστ᾽ ἂν ἔχοι. γῆ μὲν συντυγχάνουσα πυρὶ διαλυθεῖσά τε ὑπὸ τῆς ὀξύτητος αὐτοῦ φέροιτ᾽ ἄν, εἴτ᾽ ἐν αὐτῷ πυρὶ λυθεῖσα εἴτ᾽ ἐν ἀέρος εἴτ᾽ ἐν ὕδατος ὄγκῳ τύχοι, μέχριπερ ἂν αὐτῆς πῃ συντυχόντα τὰ μέρη, πάλιν συναρμοσθέντα αὐτὰ αὐτοῖς, γῆ γένοιτο—οὐ γὰρ εἰς ἄλλο γε εἶδος ἔλθοι ποτ᾽ ἄν—ὕδωρ δὲ ὑπὸ πυρὸς μερισθέν, εἴτε καὶ ὑπ᾽ ἀέρος, ἐγχωρεῖ γίγνεσθαι συστάντα ἓν μὲν πυρὸς σῶμα, δύο δὲ ἀέρος: τὰ δὲ ἀέρος τμήματα ἐξ ἑνὸς μέρους διαλυθέντος δύ᾽ ἂν γενοίσθην σώματα πυρός. καὶ πάλιν, ὅταν ἀέρι πῦρ ὕδασίν τε ἤ τινι γῇ περιλαμβανόμενον ἐν πολλοῖς ὀλίγον, κινούμενον ἐν φερομένοις, μαχόμενον καὶ νικηθὲν καταθραυσθῇ,

Upon initial examination, the ascription of planar angles[111] and sharpness to the elemental solids appears, in spite of our previous observations, to carry indications of fully tangible and self-subsisting solid structures. As we have noted before, however, the identification of matter as τὸ τοιοῦτον, such that its structures are more accurately understood as states rather than as entities, precludes this interpretation of the elemental solids. A further problem that results from the interpretation of the elemental solids as fully tangible and subsistent shapes is that which results from the inclusion of planar angles among their components; for if each elemental solid constitutes a stable, tangible assembly, then the elemental solids will consequently be hollow. As we have observed before, the empty space that would be thereby contained by the planes comprising the solids could not be absolute void, as the presence of this type of unoccupied space on the level of matter is indicated in the *Timaeus* to render movement impossible.[112] While the containment of absolute void within the solids may not pose this problem if the solids remain static, Plato explains that the triangular planes comprising each solid disassemble and reconstitute themselves into the structures of various elemental solids, and therefore we are at a loss to explain the interplay of the solids and the void; for it must remain the case that the solids cannot be situated in an insular fashion surrounded by void, since, as explained before, they would be incapable of motion.

They would thus be adjacent to one another, and since they would contain absolute void, the spatial regions occupied by each solid would be mutually exclusive of one another. In this scenario there would not be available space to allow the solids to

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δύο πυρὸς σώματα εἰς ἓν συνίστασθον εἶδος ἀέρος: καὶ κρατηθέντος ἀέρος κερματισθέντος τε ἐκ δυοῖν ὅλοιν καὶ ἡμίσεος ὕδατος εἶδος ἓν ὅλον ἔσται συμπαγές. ὧδε γὰρ δὴ λογισώμεθα αὐτὰ πάλιν, ὡς ὅταν ἐν πυρὶ λαμβανόμενον τῶν ἄλλων ὑπ᾽ αὐτοῦ τι γένος τῇ τῶν γωνιῶν καὶ κατὰ τὰς πλευρὰς ὀξύτητι τέμνηται, συστὰν μὲν εἰς τὴν ἐκείνου φύσιν πέπαυται τεμνόμενον—τὸ γὰρ ὅμοιον καὶ ταὐτὸν αὑτῷ γένος ἕκαστον οὔτε τινὰ μεταβολὴν ἐμποιῆσαι δυνατὸν οὔτε τι παθεῖν ὑπὸ τοῦ κατὰ ταὐτὰ ὁμοίως τε ἔχοντος—ἕως δ᾽ ἂν εἰς ἄλλο τι γιγνόμενον ἧττον ὂν κρείττονι μάχηται, λυόμενον οὐ παύεται. τά τε αὖ σμικρότερα ὅταν ἐν τοῖς μείζοσιν πολλοῖς περιλαμβανόμενα ὀλίγα διαθραυόμενα κατασβεννύηται, συνίστασθαι μὲν ἐθέλοντα εἰς τὴν τοῦ κρατοῦντος ἰδέαν πέπαυται κατασβεννύμενα γίγνεταί τε ἐκ πυρὸς ἀήρ, ἐξ ἀέρος ὕδωρ: ἐὰν δ᾽ εἰς ταὐτὰ ἴῃ καὶ τῶν ἄλλων τι συνιὸν γενῶν μάχηται, λυόμενα οὐ παύεται, πρὶν ἢ παντάπασιν ὠθούμενα καὶ διαλυθέντα ἐκφύγῃ πρὸς τὸ συγγενές, ἢ νικηθέντα, ἓν ἐκ πολλῶν ὅμοιον τῷ κρατήσαντι γενόμενον, αὐτοῦ σύνοικον μείνῃ. καὶ δὴ καὶ κατὰ ταῦτα τὰ παθήματα διαμείβεται τὰς χώρας ἅπαντα: διέστηκεν μὲν γὰρ τοῦ γένους ἑκάστου τὰ πλήθη κατὰ τόπον ἴδιον διὰ τὴν τῆς δεχομένης κίνησιν, τὰ δὲ ἀνομοιούμενα ἑκάστοτε ἑαυτοῖς, ἄλλοις δὲ ὁμοιούμενα, φέρεται διὰ τὸν σεισμὸν πρὸς τὸν ἐκείνων οἷς ἂν ὁμοιωθῇ τόπον

[111] See n. 89.

[112] See n. 97.

reconfigure themselves, and thus it would be impossible for them to encompass absolute void. The only other viable alternative is for the solids to contain some measure of another element, and as in the previous circumstance, it would be impossible for the spaces occupied by the elements to be mutually exclusive of one another; for otherwise, each solid is filled with smaller solids, which would in turn be filled with smaller solids themselves, and the same would be true of each set of contained solids, such that the elemental solids would be nested within one another in an infinitely recursive pattern, thus resulting in an instance of infinite regress, which is ontologically impossible. It would therefore appear that the only viable alternative is a system in which the elemental solids are not only capable of sharing spatial regions with one another, but are also connected in a vast geometric network.

Within this network, the absence of the void is a critical factor in allowing compatibility of the mutability of matter with the articulation of a geometric principle to explain its behaviour. Without absolute void, we may reasonably assume that the elemental solids do not move as particles within unoccupied space, and that their constituent triangular planes therefore need not be regarded as subsistent and fully tangible entities, and therefore do not present a contradiction to the flux of matter. There appears to be little possibility of identifying the same extent of compatibility between flux and structure in the geometric theory of matter that has been attributed to the Pre-Socratic Democritus. Democritus' system, as described by Aaronson (2013), consists of atoms of variable size, weight, and shape moving constantly within the void according to mathematical laws.[113] This system, however, is by no means without error, as indicated by Seide (1981), who cites Plutarch in his identification of ναστός as a potential problem in the functionality of Democritus' geometric atomism.[114] Seide notes that there is a possibility of ναστός being interpreted as absolute indivisibility (absolute Unteilbarkeit)

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[113] Scott Aaronson. *Quantum Computing Since Democritus*. (Cambridge: Cambridge University Press, 2013), 1.

[114] Reinhard Seide. "Zum Problem des Geometrische Atomismus bei Demokrit", *Hermes* 109, Bd., H. 3 (1981), 265-6."Nur wenige Fragmente sind es, auf die wir uns bie der Untersuchung dieser Probleme stützen können, in erster Linie das in Plutarchs Schrift »De communibus notitiis adversus Stoicos« Überlieferte über Demokrits Versuch der Berechnung des Kegelvolumens und die dabei aufgetretenen Schwierigkeiten (Teil II dieser Arbeit). Soddann die Schrift περὶ ἀλόγων γραμμῶν καὶ ναστῶν (Über irrationale Linien und Kontinuum - D. L. 9, 47)."

and states that there must be a way of ensuring infinite divisibility.[115] While he explains that no such certainty is indicated by Democritus directly,[116] he observes that Aristotle defines the concept of ναστός or "Kontinuum" in the *Physics* as unity,[117] and considers it to be similarly unlikely for the term ἀδιαίρετος to imply absolute indivisibility according to Parmenides' position.[118] Atomists such as Democritus and Leucippus, as he explains, employed the Eleatic approach to the "foundations and permanence of true existence,"[119] and that according to this approach, atoms are understood to be indivisible *qua* ontological unity rather than in the absolute sense.[120] In particular, it would be impossible for the atoms described in the *Timaeus* to be mathematically indivisible, as they would consequently be unable to function as patterns of motion; for if they were mathematically indivisible, it would follow that the linear pathways of the solids possessed no midpoints.[121] As Hett (1936) indicates in the commentary accompanying Aristotle's discussion of indivisible lines, the absence of midpoints on the linear pathways of the solids would dictate that any traversal of the pathways would have to occur without passing through any intermediate point, a manner of movement which would be impossible.[122]

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[115] Seide. "Zum Problem des Geometrischen Atomismus bei Demokrit", 266-7. "...welche die Möglichkeit unendlicher Teilbarkeit geometrischer Größen sichern soll, gegen die Demokriteer (Δημοκρίτειοι) gerichtet sie."

[116] Ibid., 267. "Damit ist aber doch über Demokrits Standpunte selbst nichts gesagt;"

[117] Ibid., 268. "Aristoteles beseichnet dort das Kontinuum al seines (- also in Gegensatz zu seiner Behauptung in der Physik -)..."

[118] Ibid. "Ebensowenig wie bei Parmenides kann ἀδιαίρετος hier die absolute Unteilbarkeit bedeuten."

[119] Ibid., 269. "So übernahmen die Atomisten zwar die eleatische Grundposition der Unveränderlichkeit des wahrhaft Seienden..."

[120] Ibid. "Dieses Atom hat nun sämtliche Eigenschaften des parmenideischen ὅν: es ist εἷς (d. H. Jedes Atom ist ein in sich geschlossenes Ganzes), ἀδιαίρετος und συνεχής."

[121] See n. 23.

[122] Arist. *LI*., in *Aristotle Minor Works*. Loeb Classical Library 307. ed. and trans. W. S. Hett. (Cambridge, Massachusetts and London, England: Harvard University Press), 1936), 426.

These solids are, however, vaguely defined in terms of their activity as patterns of motion, and therefore allow for a multitude of possible explanations concerning their functionality. The first of the two theories to be treated in our investigation is that the elemental solids represent triangular number structures in accordance with the principles of triangular numbers described by Nicomachus of Gerasa. In the *Introduction to Arithmetic*, Nicomachus expresses two models for the principles of triangular numbers, and the first, which he identifies as the Pythagorean model, is explained thus,

> The Pythagorean approach is based on odd numbers; for it places a given odd number that is less than the others at the edge and taking the square from that number, and subtracting 1 from the result of this operation, places half of this value as the larger edge; while adding one to the result of this operation produces the hypotenuse; for instance, 3 is squared to become 9, from which 1 is then subtracted to produce 8, which is then divided in half to produce 4 and thus a right triangle is discovered with side lengths of 3, 4, and 5.[123]

The same triangle may also be produced through a similar method, which Nicomachus describes as the Platonic approach, and details as follows,

> The Platonic approach proceeds from even numbers; for it takes a given even number and places it as the length of the first side, and then, dividing it by 2, and then squaring the half and adding 1 to the square it produces the hypotenuse, while subtracting 1 from the square produces the other edge. For example, it might take 4, and divide it in half and square it, once again producing 4. In subtracting 1 it produces 3, and then adding 1 it produces 5, and so it has the same triangle, which has been completed according to a different method. Thus is assembled the same triangle from sides of 5, 3, and 4.[124]

The triangles described in these passages each constitute one half of an isosceles triangle, yet in order for it to be a potentially viable formula for the triangles constituting the elemental solids of air, water, and fire, it must be capable of producing a scalene right triangle equivalent to precisely half of an equilateral triangle. The larger of the two acute

[123] Procl. *In Eucl*. I. ed. Friedlein 428. 7-429. 8. Translated partly after Thomas (2006).

[124] Procl. *In Eucl*. I. ed. Friedlein 428. 7-429. 8. Translated partly after Thomas (2006).

angles in this triangle must therefore have a measurement of 60°, and the length of the smallest edge must be exactly equal to half of the length of the hypotenuse. The length of the middle side must be equal to precisely (√3)/2, or approximately 0.8660,[125] of the hypotenuse. Since √3 constitutes an irrational value, it cannot be divided or multiplied by an integer to produce a rational number, and thus if the value of the hypotenuse is an integer, then the value of the length of the middle edge cannot be a rational number according to the same unit of measurement. The two formulae for triangular numbers described by Nicomachus are therefore not suitable for the purpose of producing a complete explanation of the construction of the elemental solids; for though they may be used in the construction of the isosceles triangles that comprise earth solids, they are not applicable for the other types of elemental solids.

Another, perhaps more viable triangular number theory for the elemental solids would be to treat each of the right triangles comprising the solids as a triple tuple which consists of a value corresponding to each of the *x*, *y*, and *z* axes, such that these are variables corresponding to a location in three-dimensions space, and are predicated of the most basic unit of matter (about which no determinate statement may be made, except for its role as that which is moved according to cosmic patterns), henceforth termed as a prime particle. Each solid would signify a relation of these prime particles, with each relation consisting of a number of prime particles equal to the number of right triangles comprising the solid, such that a fire solid is comprised of 8 prime particles, with 16 for an air solid, 40 for a water solid, and 48 for an earth solid. The solids would be unable to function properly if the variables corresponded to the edges of the triangles, as their values would then remain constant, and they would therefore be incapable of movement. The variables must then correspond to the vertices of the right triangle, such that each vertex shared by two triangles constitutes an instance of a spatial variable that is equal for two different prime particles; the edges of the triangles, meanwhile, would signify the proportion of values to one another. The distances, furthermore, might reasonably be measured according to the size of a prime particle, if they are assumed to be of the same

[125] H. A. Elliot, K. D. Fryer, J. C. Gardner, and Norman J. Hill. *Vectors, Matrices, and Algebraic Structures*. (Canada: Holt, Rinehart and Winston of Canada, Limited, 1972), 421.

size.[126] Throughout the movement of the prime particles, the variable proportions would remain, as would the equalities of the spatial variables between particles. The disintegration and reintegration of particles might be explained by possible disruptions in proportion based on proximity to other solids, since absolute void is absent from the cosmos described in the *Timaeus*. There is, however, little evidence for this theory apart from its apparent functional plausibility, as the *Timaeus* provides no conclusive indication of its correctness, beyond its possible accuracy according to the negative statements which may be made about matter based on identity as a state rather than an entity.

The second theory is that the elemental solids are correctly interpreted as assemblies of geometric vectors that represent the patterns of movement followed by the most basic units of matter. There is difficulty in determining with certainty whether a formal concept of vectors is present among the mathematical knowledge of the Classical world. In translating *Physics* III 202b17-20, Gaye and Hardie (1941) suggest at least a rudimentary understanding of vector mathematics on the part of Aristotle, as the passage in question as interpreted by Gaye and Hardie speaks of two vectors traversing the same distance, AB and BA, and acknowledges that they are not equal to one another.[127] The term used by Aristotle in the Greek text of the *Physics* to refer to vectors is τὸ διίστασθαι,[128] a use of language that suggests a geometric variable that simultaneously represents an activity of motion. In this respect, we might identify each line that comprises the elemental solids as a vector, such that each solid is an interconnection of vectors arranged end to end according to the geometric pattern of each element. In order for the dismantling and reconstitution of solids to occur, as in the previous theory, it

[126] It is reasonable to suggest that the primary particles in a cosmic context should be identical in size, firstly in accordance with the ordering of the elements according to proportion, and also based on the fact that particles of varying sizes moving within the same geometric relation might prevent the inertia necessary for the movement of matter according to the dynamics of the elemental solids.

[127] Arist. *Ph*. III. 3. 202b17-20. trans. R. K. Gaye and R. P. Hardie. in *The Basic Works of Aristotle*, ed. Richard McKeon. Modern Paperback Library Edition. (New York: The Modern Library, 2001), 257.

[128] Arist. *Ph*. III. 202b19. Aristotle's use of a verbal noun to signify distance and separation (Liddell and Scott. *Greek-English Lexicon*, 428.) also gives a possible indication as to Aristotle's affirmative position concerning the objects of mathematics, suggesting that he regards them as actions inasmuch as they constitute executions of the formulae from which they are derived. Aristotle's use of language will be examined further in discussing the epistemology of mathematics.

seems likely to be the case that each solid also implies a relation of several prime particles, for there would otherwise be some difficulty in explaining the divergence of solids according to which a solid of water may be broken into solids of air and fire. In order for the solids to be broken and reconstituted according to their differing mass and sharpness, it is necessary for the vector system of each solid to be capable of motion among other such systems, and as in the previous theory, the disassembly and reassembly of solids may be said to occur when a sharp solid disrupts the patterns of a dull solid and forces the prime particles that comprise it to adopt different trajectories.

The movement of elemental solids therefore demands the execution of certain operations upon the vectors belonging to elemental solids, such that they are altered in angle though not in length. This type of operation proves to be problematic for the representation of the dynamics of elemental solids within the context of Plato's ontology; for Marsden and Tromba (2012) explain that the principles associated with vectors were fully formalized in modern mathematical scholarship under William Rowan Hamilton, who provided a complete definition for vectors following their initial identification by Newton in the *Principia*.[129] Thus, while the vector theory of solids may be demonstrable as an explanation, it would require the admission of principles inappropriate to the context of mathematical knowledge in the Classical world. The triangular number theory therefore seems a more plausible characterization for elemental solids, not only because it is more compatible with the mathematic principles of Platonic thought, but also through its apparently greater departure from the identification of the elemental solids as static structures. If it is assumed to be correct, this theory is particularly representative of the identification of mathematical functions and relations with geometric structures according to intelligible patterns.

3.3 Organized Chaos and Necessary Evil: Explaining the Mathematical Origins of Accidents, Privation, and Chance

In considering the mathematical operation of matter in Plato's ontology, we must account for the imperfections that are manifest on the level of tangible entities. Although the objects of sensation, as Aristotle explains, do not constitute perfect representations of

[129] Jerrold E. Marsden and Anthony Tromba. *Vector Calculus*. Sixth ed. (New York: W. H. Freeman and Company, 1976, 1981, 1988, 1996, 2003, 2012), xxii.

geometric abstracts,[130] the accidental variations that occur at a sensible level, since they are confined to the spatiotemporal liminations of the corporeal realm, must still be restricted by mathematical parameters. It may, furthermore, be the case that if the elemental solids were restricted in such a way as to adhere seamlessly to the structural parameters of intelligible paradigms, they would lack the capacity to carry out the patterns of motion defined within the paradigms. Mason (2006) explains that according to Plato's ontology, objects on the level of αἴσθησις function according to necessity, which, when ungoverned by a rational pattern, acts in a disorderly manner, but which is also capable of being exploited by intellect for the purpose of cosmic structure.[131] Given the requirement, however, that sensible entities must differ in their activity in accordance with specific conditions, it seems to follow that sensible objects are necessarily capable of acting in a manner different from that which is suitable to the circumstance in question; for in order for sensible objects to act in accordance with a rational pattern, it stands to reason that their behaviour must be dictated by the parameters of specific scenarios, and that the actions of which they are capable, as well as the conditions under which these actions are to be performed, are defined on the level of paradigm.

Thus, in order for sensible beings to correctly follow the paradigm to which they adhere, they must retain their capacity to act contrariwise to intellect; for if, in each possible scenario, they are incapable of acting in a manner contrary to intellect, then not only are their capabilities mutable, but they are also strictly limited in their instantiation of paradigm, as at any given moment, they will possess only a fraction of the capabilities represented on the level of paradigm.

Consequently, if sensible beings are incapable of acting in a manner contrary to their perfection as it is defined paradigmatically, then their nature must alter in a manner appropriate to a specific circumstance, which presents difficulty in ascribing to them a precise identity. They will, moreover, be ironically unable to attain perfection, inasmuch as perfection implies complete adherence to paradigm, as they will be possessed of only a

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[130] See n 80.

[131] Andrew S. Mason. "Plato on Necessity and Chaos", *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition* 127, no. 2, Selected Papers from the American Philosophical Association, Pacific Division, 2004 Meeting (January 2006), 284.

fraction of the functions defined as proper to them on the level of the intelligible. The necessity that we have described may be explained as follows:

$Let\ T\ represent\ the\ set\ of\ all\ tangible\ objects.$

$Let\ A\ represent\ the\ set\ of\ all\ actions\ that\ may\ occur\ on\ the\ tangible\ level.$

$Let\ C(a)\ indicate\ that\ a\ certain\ action\ a\ is\ permitted\ by\ the\ scenario\ in\ question.$

$Let\ P(o,a)\ indicate\ that\ the\ object\ o\ is\ capable\ of\ the\ action\ a\ in\ the\ given\ scenario.$

$\forall t | t \in T\ let\ A_t\ represent\ the\ set\ of\ all\ actions\ included\ in\ the\ paradigm\ of\ t.$

$\forall t(t \in T \rightarrow \forall a(a \in A \rightarrow ((a \rightarrow A_t \wedge C(a)) \rightarrow P(t,a))))$

The final statement indicates that for all tangible objects, it is the case that for all actions possible at the tangible level, assuming that a certain action belongs to the paradigm of a particular tangible object, and that the given action is possible in the scenario in question, it then follows that the object is capable of that action under the stated conditions. Unless all actions possible within these parameters are in accord with intellect, it must be that some such functions are either irrelevant or contrary to the requirements of reason. Since it is on the level of matter that disorderly motion is understood to occur, it seems to follow that the possibility of irrational variance must also extend to qualities in addition to actions. From the multitude of possible variations and deviations in the structure and activity of sensible οὐσίαι, we might find ourselves led toward the consideration of a potentially infinite multitude of alternate permutations for the universe. The *Timaeus* nonetheless refutes the notion of a plurality or infinite multitude of κόσμοι when Timaeus addresses the question, "Is it correct to say that there is one firmament, or rather that there are many and infinite firmaments?"[132] He determines that there must be one firmament only, stating, "There will be one firmament, if it is to be constructed according to paradigm; for that which encompasses all cannot in any way be posterior to something else; for it is contradictory [to say] that there must be an alternate image of each thing, that they should not be part of one another, and it is not correct to claim that they are likenesses of each other and that they encompass one another."[133] Timaeus indicates in

[132] Pl. *Ti.* 31a2-3. πότερον οὖν ὀρθῶς ἕνα οὐρανὸν προσειρήκαμεν, ἢ πολλοὺς καὶ ἀπείρους λέγειν ἦν ὀρθότερον;

[133] Pl. *Ti.* 31a4-8. τὸ γὰρ περιέχον πάντα ὁπόσα νοητὰ ζῷα μεθ᾽ ἑτέρου δεύτερον οὐκ ἄν ποτ᾽ εἴη: πάλιν γὰρ ἂν ἕτερον εἶναι τὸ περὶ ἐκείνω δέοι ζῷον, οὗ μέρος ἂν εἴτην ἐκείνω, καὶ οὐκ ἂν ἔτι ἐκείνοιν ἀλλ᾽ ἐκείνῳ τῷ περιέχοντι τόδ᾽ ἂν ἀφωμοιωμένον λέγοιτο ὀρθότερον

this passage that if there exists a plurality or infinite multiplicity of tangible universes, these universes will encompass one another, and will be defined according to one another. This relation might be interpreted as indicating that alternate permutations of the sensible universe would be differentiated from one another intelligibly, such that variations in structure and activity would occur not only in accordance with paradigm, but also on the basis of the particulars of each permutation.

According to the system described in the *Timaeus*, the existence of multiple permutations of each specific tangible entity is also unnecessary; for if the sensible cosmos is to be understood to be constructed according to such a model as to conform to the ἀναλογία κάλλιστα,[134] it would be superfluous to construct multiple versions of the same instantiation of a particular paradigm entirely for the purpose of accounting for non-essential variations and deviations on the part of sensible beings. To be precise, it would be extraneous for privations and deviations to be distinctly defined if they are determined as violations of the specifications belonging to paradigm. The absence of this rendundancy from Plato's thought is compatible with the argument presented by Meldrum (1950), which rejects the concept of a distinct principle pertaining particularly to evil, and thereby supports the identification of evil as deviation from the paradigmatic specifications of the cosmic model. Meldrum attacks Cornford's position, which maintains that evil originates on the level of the intelligible. Cornford's thesis is based partially on his interpretation of the *Timaeus* in conjunction with the *Laws* and the *Phaedrus*, according to which ψυχή is to be understood as the ἀρχή κινήσεως, or the origin of motion. Meldrum, however, refutes this argument on the identification of the irrational force of necessity as the source of evil within Plato's ontology.[135]

Within his counter-position, Meldrum also notes that the connotation of κίνησις presented within the *Timaeus* differs from that of the *Laws*, as the former treats κίνησις as spatial motion which is included within γένεσις, whereas the *Laws* indicates that γένεσις

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[134] Pl. *Ti.* 31c3-4.

[135] M. Meldrum. "Plato and the ἈΡΧΗ ΚΑΚΩΝ", *The Journal of Hellenic Studies* 70 (1950), 60. Op. Cit. Pl. *Ti*. 52d.

belongs to κίνησις.[136] If κίνησις is to be similarly interpreted within both texts, then the definition of γένεσις must differ in each case, with one possible distinction being the identification of γένεσις with construction from pre-cosmic matter in the *Timaeus*, and with generation as confined to tangible being in the *Laws*.

This differentiation appears to be confirmed in *Laws* X, in which κίνησις is stated by one of the speakers, Athenaeus, to be capable of moving other objects but incapable of self-motion,[137] and to be responsible for moving these objects "toward union, separation, generation, and destruction."[138] The absence of self-motion on the part of the type of κίνησις discussed in the *Laws* suggests that it signifies non-intelligent motion, such that it requires guidance by an external agent to function in an orderly manner. It is possible that this agent is to be identified as ψυχή, which is stated at an earlier passage of *Laws* X to be prior to φύσις, in opposition to the position of the Physicists according to which elements such as fire and air are considered to be primary causes,[139] and is stated by Athenaeus' companion Cleinias to be the ἀρχη κινήσεως in such a way as to distinguish it from the cause of motion.[140] Since ψυχή in this context is differentiated from the primary cause of existence, we may presume that Cleinias speaks not of the Demiurge, but perhaps of the World-Soul. The distinction between ψυχή and the Demiurge is, however, not the relevant differentiation in this case, for neither ψυχή nor the Demiurge is the source of disorderly motion. As we have determined before, matter is stated in the *Timaeus* to have moved in a random and unruly manner prior to the imposition of order,[141] which requires

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[136] Meldrum. "Plato and the ἈΡΧΗ ΚΑΚΩΝ", 60. Concerning the disparate connotations of γένεσις and κίνησις, see Meldrum, n. 8, also with reference to Pl. *Prm*. 155e-156b and Cornford, *Plato and Parmenides*, 197. See also Meldrum, n. 10, which makes reference to the identification of τὸ σωματοειδές, or corporeal being as the origin of necessity, citing Pl. *Ti.* 46e. For the exact description of Cornford's identification of ψυχή as the ἀρχή κινέσεως, see Meldrum, Op. Cit. 13, in reference to Cornford, *Plato's Cosmology*. 205.

[137] Pl. *Lg*. X. 894b8-9. Ἔστω τοίνυν ἡ μν ἕτερα δυναμένη κινε ῖν κίνησις, ἑαυτὴν δὲ ἀδυνατο ῦσα

[138] Pl. *Lg*. X. 894b10-11. κατά τε συγκρίσεις ἔν τε διακρίσεσιν αὔξαις τε καὶ τῷ ἐναντίῳ καὶ γενέσεσι καὶ φθοραῖς

[139] Pl. *Lg*. X. 892c2-5. φύσιν βούλονται λέγειν γένεσιν τὴν περὶ τὰ πρῶτα: εἰ δὲ φανήσεται ψυχὴ πρῶτον, οὐ πῦρ οὐδὲ ἀήρ, ψυχὴ δ᾽ ἐν πρώτοις γεγενημένη, σχεδὸν ὀρθότατα λέγοιτ᾽ ἂν εἶναι διαφερόντως φύσει

[140] Pl. *Lg*. X. 896b2-3. οὔκ, ἀλλὰ ἱκανώτατα δέδεικται ψυχὴ τῶν πάντων πρεσβυτάτη, γενομένη γε ἀρχὴ κινήσεως

[141] See n. 100.

the agency of the Demiurge as well as the operation of ψυχή. Thus, since disorderly motion belongs to the realm of necessity, and is brought into precisely determined patterns through intellect, it follows that potential variations in structure and motion do not require definition through alternative permutations of the same tangible instantiation of paradigm, since these variations ought to require no definition except in comparison to the patterns belonging to paradigm. More significantly, however, the continued presence of disorderly motion following the ordering of matter leads us to the consideration of the manner in which these movements may be altered by the imposition of order while remaining erratic.

We may suggest by inference that the disorderly motion that persists following the introduction of structure to the universe constitutes a type of movement on the part of the elemental solids that is within the capacity characterized by their structure yet contrary to the actions dictated by paradigm. It would seem that in the strictest sense, the chaotic movements of elemental solids occur without the direction of intellect, while in another respect, they are subordinate to intellect in an indirect manner insofar as their potential range is contingent upon variables such as size and density, acting in conjunction with mathematical laws defined according intellect. The position presented by Clegg (1976) appears to agree with this understanding of disorderly motion, as he states that while the type of movement originating with soul is "teleologically directed," or guided toward a purpose defined by intellect, he identifies the type of motion belonging to matter as being derived from such characteristics as texture and weight.[142] Derived motion represents an instance in which mathematical laws, particularly those that pertain by definition to tangible entities and matter, may come into conflict with the teleological objectives determined according to intellect; for although these laws and variables are required for the structure and motion of sensible beings, they are also operative in the disposition of cosmic matter, which would seem to be capable of functioning at cross-purposes with paradigm.

In this regard, even disorderly motion on the part of cosmic matter adheres to intelligible principles on the basis of the patterns imparted to it by the Demiurge, and in

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[142] Jerry S. Clegg. "Plato's Vision of Chaos", *The Classics Quarterly, New Series* 26, no. 1 (1976), 53.

doing so is capable of violating the specifications defined on the level of paradigm. Mohr (1981) argues, however, that the account of disorderly motion presented in Plato's *Statesman* maintains that movement of that type is not guided by intellect in any manner, whether direct or indirect. Mohr identifies and attacks four arguments in favour of intellect as a cause of disorderly movement, but the position most relevant to our current investigation is that which treats disorderly motion as an "inadvertent but inevitable" result of the agency of the World-Soul. As he explains, however, this argument requires that the mechanical model described in *Statesman* 270a6-8 be interpreted as the efficient cause of the reverse circuit of the universe rather than merely an explanation for its duration.[143] According to this passage it is evident that even through necessity, without the agency of the Demiurge or of ψυχή, the reverse circuit functions according to mathematical specifications, for even in reversed movement, it is propelled in κατὰ καιρὸν, or in accordance with proportion.[144]

The most likely explanation for this phenomenon seems to be that there are two degrees of mathematical order at work on the level of sensible entities in Plato's ontology. The first of these degrees constitutes direct adherence to mathematical laws on the part of matter in a manner that does not necessarily conform to the teleological specifications defined on the level of paradigm. The second implies compliance with mathematical principles according to teleological objectives as defined according to paradigm. As explained by Miller (2003), the first degree of mathematical order, which we would identify with necessity, is considered by certain scholars, such as Archer-Hind, to encompass the laws governing the physical cosmos.[145] Miller appears to concur with this position, as she considers it probable for necessity as it is discussed in the *Timaeus* to

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[143] Richard D. Mohr. "Disorderly Motion in Plato's "Statesman" ", *Phoenix* 35, no. 3 (Autumn 1981), 201. Op. Cit. Pl. *Plt*. 270a6-8; Harold Cherniss. *Aristotle's Criticism of Plato and the Academy* 1 (Baltimore 1944) 444-450; rev. of A.-J. Festugière, *La Révélation d'Hermès Trismégiste* 2: *Le Dieu cosmique in Gnomon* 22 (1950) 207-210; "The Sources of Evil according to Plato" *ProcPhilSoc*. 98 (1954) 23-30, reprinted in G. Vlastos, ed., *Plato*2 (Garden City, N.Y. 1971) 244-258 (on the *Statesman* myth in particular see nn. 21, 44); and Leonardo Tarán, "The Creation Myth in Plato's *Timaeus*, in J.P. Anton and G.L. Kustas, eds., *Essays in Ancient Greek Philosophy* (Albany, N.Y. 1971) 386-388.

[144] Pl. *Plt*. 270a6.

[145] Dana Miller. *The Third Kind in Plato's Timaeus*. Hypoknemata 145. (Göttingen, Germany: Vandenhoeck & Ruprecht, 2003), 63.

belong to the "auxiliary causes," citing *Timaeus* 46e1-2.[146] In this passage Plato makes reference to objects that come into being according to necessity as a secondary cause,[147] continuing from his explanation that necessity "follows the love of intellect and knowledge from the intelligible nature of the first cause."[148] It is reasonable to suggest, based on Plato's discussion of the manner in which the Demiurge orders the motion of matter,[149] that disorderly motion within the context of cosmic reality represents an adherence of matter to its initial ordering in a manner that is simultaneously contrary to the teleological specifications of paradigm. This deviation is evidently necessary, for in its absence, matter would only be capable of moving in response to direct control by νοῦς. The physical cosmos would then fall short of the best model, as sensible beings would then be entirely incapable of self-motion; for in the absence of necessity, reason would be the only cause of motion among tangible objects, and as such, sensible beings would remain immobile without direct compulsion by the Demiurge. If however, necessity is operative in tangible entities, then the agency of reason does not impart motion to these beings, but rather directs their irrational kinetic inclinations toward their teleological objective as defined in the cosmic model.

Necessity therefore proves to be especially significant for living sensible beings, as it is crucial in accounting for the connection between their intellectual faculties and their physical actions. Without necessity, there would be no principle of motion operative at the tangible level apart from rational sovereignty of the Demiurge, and thus living entities, being incapable of moving by their own accord, would functionally be little more than marionettes guided by a rational control external to themselves. This scenario would pose a particular problem for the human intellect, for Plato explains in the *Timaeus* that it

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[146] Miller. *The Third Kind in Plato's Timaeus*. 67. Op. Cit. Pl. *Ti*. 46e1-2.; Cf. Easterling 1967, 28: "In fact νοῦς and ἀνάγκη appear not as two opposing forces in direct confrontation; rather they form a partnership (though the cooperation between them is limited) in which νοῦς is the senior and more important partner."

[147] Pl. *Ti*. 46e1-2. ἕτερα δὲ κατὰ ἀνάγκης κινούντων γίγνονται, δευτέρας

[148] Pl. *Ti*. 46d7-e1. τὸν δὲ νοῦ καὶ ἐπιστήμης ἐραστὴν ἀνάγκη τὰς τῆς ἔμφρονος φύσεως αἰτίας πρώτας μεταδιώκειν

[149] See n. 100.

is necessity that guides us to turn our minds toward the pursuit of knowledge.[150] Thus, the same force that is responsible for chaos, chance, and error is necessary for sensible beings to strive toward the schematics of their perfection, and for us to pursue justice through knowledge of ourselves within the context of the universe.

[150] Pl. *Ti*. 46d7-e1. τὸν δὲ νοῦ καὶ ἐπιστήμης ἐραστὴν ἀνάγκη τὰς τῆς ἔμφρονος φύσεως αἰτίας πρώτας μεταδιώκειν

Chapter 4: Ἐπιστήμη

4.1 The Schematics of Thought: On the Principles of Geometry and Number Theory as the Foundations of Knowledge

The functionality of mathematical principles within the cosmos described by Plato may be further articulated through examination of Plato's thought regarding both the manner in which mathematical principles are known by the human intellect, as well as the importance of mathematical knowledge in the entirety of wisdom. Plato's position according to the *Timaeus* and the *Meno* suggests that the formulae and paradigms of mathematics are already present within the human intellect. According to the account presented in the *Timaeus*, these laws and functions might be said to belong to the model of the cosmos that is stated to constitute the human intellect, varying in its accuracy according to the degree of wisdom that the mind has attained. Timaeus discusses the perfection of intellect through mathematical inquiry as follows,

> Now, morning, night, and months were observed, and the cycles of these periods, equinoxes, and solstices of this produced number and the measurement of time, and gave us the investigation of the entirety of nature. From these we developed the division of philosophy, from which the greatest good did not come, nor was it present when it was presented to the mortal kind by the gods. I say that this is the highest good of sensible things, but of what other, lesser goods should we sing praises, as the philosopher should not be so blind that he sings laments and dirges? But from this it must be said that such is, with respect to ourselves, beyond those other causes, and it has been determined that god granted us sight so that, gazing upon the orbits of the heavens through our minds, we might guide the orbital paths by way of reason, innate to all beings, learning thoroughly and calculating according adherence to the nature of precision, imitating in all respects the unchanging nature of gods, and stabilize the wandering patterns within ourselves.[151]

[151] Pl. *Ti*. 47a4-c6. νῦν δ᾽ ἡμέρα τε καὶ νὺξ ὀφθεῖσαι μῆνές τε καὶ ἐνιαυτῶν περίοδοι καὶ ἰσημερίαι καὶ τροπαὶ μεμηχάνηνται μὲν ἀριθμόν, χρόνου δὲ ἔννοιαν περί τε τῆς τοῦ παντὸς φύσεως ζήτησιν ἔδοσαν: ἐξ ὧν ἐπορισάμεθα φιλοσοφίας γένος, οὗ μεῖζον ἀγαθὸν οὔτ᾽ ἦλθεν οὔτε ἥξει ποτὲ τῷ θνητῷ γένει δωρηθὲν ἐκ θεῶν. λέγω δὴ τοῦτο ὀμμάτων μέγιστον ἀγαθόν: τἆλλα δὲ ὅσα ἐλάττω τί ἂν ὑμνοῖμεν, ὧν ὁ μὴ φιλόσοφος τυφλωθεὶς ὀδυρόμενος ἂν θρηνοῖ μάτην; ἀλλὰ τούτου λεγέσθω παρ᾽ ἡμῶν αὕτη ἐπὶ ταῦτα αἰτία, θεὸν ἡμῖν ἀνευρεῖν δωρήσασθαί τε ὄψιν, ἵνα τὰς ἐν οὐρανῷ τοῦ νοῦ κατιδόντες περιόδους χρησαίμεθα ἐπὶ τὰς περιφορὰς τὰς τῆς παρ᾽ ἡμῖν διανοήσεως, συγγενεῖς ἐκείναις οὔσας, ἀταράκτοις

The most plausible explanation for this structure appears to be that the human intellect consists of a model of the entire cosmos, the completeness and accuracy of which is the measure of our knowledge. If it is understood thus, such a model would encompass not only the orbits of the planets, but all intelligible principles governing the structure and activity of the sensible cosmos. In the second book of his commentary on the *Timaeus*, Proclus lends credence to this characterization of the intellect by arguing for the essential connection of all intelligible principles, such that they are present in one another.[152] A similar principle of connection is suggested in the explanation of collection and division as these concepts are discussed in the *Philebus* and in the *Phaedrus* (which we shall consider at a later point in relation to dialectic). Collection is understood as the gathering of objects of various kinds into a single class according to an essential common property, while division involves the delineation of different subtypes within collections according to essential points of distinction.[153] If these processes are carried out correctly, then we ought to articulate the classifications that are defined at the level of the intelligible, for instance, those of plants and animals, and the various subordinate classifications belonging to them.

Since there can be no sequential succession of causes and effects at the intelligible level, it follows that just as each classification contains within it all those definitions belonging to it, each definition contains within it the classification to which it belongs; each species is connected, indirectly through the genus to which it belongs, to all other species belonging to that genus, and thus it seems that if collection and division are performed properly, we may observe the manner in which Proclus' position on the relation of Forms is, to a certain degree, consistent with that of Plato. If such unity belongs essentially to intelligible principles, it then stands to reason that their complete apprehension by human intellect requires that they be understood in the context of the paradigmatic system to which they belong, and thus, that knowledge of the objects of

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τεταραγμένας, ἐκμαθόντες δὲ καὶ λογισμῶν κατὰ φύσιν ὀρθότητος μετασχόντες, μιμούμενοι τὰς τοῦ θεοῦ πάντως ἀπλανεῖς οὔσας, τὰς ἐν ἡμῖν πεπλανημένας καταστησαίμεθα

[152] Procl. *In Ti*. 430.30-431.5. In *Proclus, Commentary on Plato's* Timaeus *Volume II*. Ed. and Trans. David T. Runia and Michael Share. (Cambridge: Cambridge University Press, 2008), 317.

[153] See n. 182.

science may only reach its full perfection through the articulation of all paradigms within a single intelligible model.

It would be absurd for the paradigm of the planetary orbits to be absent from this model, so it seems to follow that the orbits stated to belong to the soul are contained within an approximation of the unchanging model of the cosmos. Strange (1999), citing *Timaeus* 30b, explains that the intellect serves to "mediate soul and body,"[154] and in this passage, such mediation is described thus,

> So by calculation he devised then a task so that the things belonging to the sensible would not, according to nature, be unintelligent, since they would possess completely the entirety of the good, yet it was impossible for them to possess intellect apart from soul. And through calculation and intellect within soul, he contrived the entire unification of soul within a corporeal vessel, thereby bringing his work to perfection so that it would be most beautiful and best according to nature.[155]

This passage clearly indicates that ψυχή is nothing less than that by which sensible entities are capable of acting in accordance with the intelligible. It must then, it would seem, be within ψυχή that our faculties of knowledge reside. At any rate, the role of mathematical understanding in the ordering of human ψυχή suggests that such principles belong inherently, in some respects, to the structure of our intellect.[156]

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[154] Steven Strange, "The Double Explanation in the *Timaeus*", in *Plato 1: Metaphysics and Epistemology*. Volume 1. Oxford Readings in Philosophy. Ed. Gail Fine. (Oxford: Oxford University Press, 1999), 402. Op. Cit. Pl. *Ti*. 30b.

[155] Pl. *Ti*. 30b1-6. λογισάμενος οὖν ηὕρισκεν ἐκ τῶν κατὰ φύσιν ὁρατῶν οὐδὲν ἀνόητον τοῦ νοῦν ἔχοντος ὅλον ὅλου κάλλιον ἔσεσθαί ποτε ἔργον, νοῦν δ᾽ αὖ χωρὶς ψυχῆς ἀδύνατον παραγενέσθαι τῳ. διὰ δὴ τὸν λογισμὸν τόνδε νοῦν μὲν ἐν ψυχῇ, ψυχὴν δ᾽ ἐν σώματι συνιστὰς τὸ πᾶν συνετεκταίνετο, ὅπως ὅτι κάλλιστον εἴη κατὰ φύσιν ἄριστόν τε ἔργον ἀπειργασμένος

[156] Stephen Menn. "Aristotle and Plato on God as Nous and as the Good", *The Review of Metaphysics* 45, no. 3 (March 1992), 546, 556-7. Op. Cit. Pl. *Phlb*. 28c6-8, 30c9-10, *Ti*. 30a2-c1, 30b3, 46d5-6 *Sph*. 249a4-8. In this article, Menn shows a distinction between the divine νοῦς, which, as he explains, is described in *Philebus* 28c6-8 as "the king for us of heaven and earth," and which he identifies as "the Demiurge of the *Timaeus*," and instantiated νοῦς, whose operation at the sensible level requires ψυχή. As Menn states in reference to *Philebus* 30c9-10, *Timaeus* 30b3, and *Sophist* 249a4-8, "…Plato says that *nous* cannot come-to-be or be present in anything without soul…" Menn clarifies this dependency by stating that this requirement does not preclude the independent existence of the divine νοῦς, nor does it imply that instantiated νοῦς is the same as ψυχή. As Menn informs us, this relation rather implies that "nothing except a soul can participate in *sophia* or *nous*. Instantiated νοῦς, it seems, must therefore be contained within ψυχή insofar as it constitutes an activity of the latter, and yet must also contain the image of ψυχή as it is

This interpretation is further supported by the geometric experiment of the *Meno*, whereby Socrates demonstrates, using a young slave boy as his subject, the argument for the attainment of knowledge through recollection. Following his demonstration, Socrates speaks thus with his eponymous companion Meno regarding the observed method of apprehension,

> '…Is this how he was taught everything? For if it is correct in any way to say that he knows these things, they would otherwise come into being and develop within the household.'
>
> 'But I know for a fact that no one has taught them to him.'
>
> 'And yet he has these postulations, does he not?'
>
> 'It would certainly seem so, Socrates.'
>
> …
>
> 'If, therefore, the truth of things is, for us, always within the soul, and the soul is immortal, is it not then necessary that in knowing you do not chance upon assumptions – for this does not constitute recollection of such things – but that you attempt to seek and remember?'
>
> 'You seem to speak correctly, Socrates, though I am not sure.'[157]

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apprehended by νοῦς. Although the divine νοῦς contains the perfect image of ψυχή according to which ψυχή is instantiated, it must also belong to the νοῦς to have its instantiation contained within ψυχή in order to be present within sensible entities.

[157] Pl. *Men*. 85e3-8, 86b1-4. δεδίδαχέν τις τοῦτον γεωμετρεῖν; οὗτος γὰρ ποιήσει περὶ πάσης γεωμετρίας ταὐτὰ ταῦτα, καὶ τῶν ἄλλων μαθημάτων ἁπάντων. ἔστιν οὖν ὅστις τοῦτον πάντα δεδίδαχεν; δίκαιος γάρ που εἶ εἰδέναι, ἄλλως τε ἐπειδὴ ἐν τῇ σῇ οἰκίᾳ γέγονεν καὶ τέθραπται.

Μένων
ἀλλ᾽ οἶδα ἔγωγε ὅτι οὐδεὶς πώποτε ἐδίδαξεν.

Σωκράτης
ἔχει δὲ ταύτας τὰς δόξας, ἢ οὐχί;

Μένων
ἀνάγκη, ὦ Σώκρατες, φαίνεται.

…

Σωκράτης
οὐκοῦν εἰ ἀεὶ ἡ ἀλήθεια ἡμῖν τῶν ὄντων ἐστὶν ἐν τῇ ψυχῇ, ἀθάνατος ἂν ἡ ψυχὴ εἴη, ὥστε θαρροῦντα χρὴ ὃ μὴ τυγχάνεις ἐπιστάμενος νῦν—τοῦτο δ᾽ ἐστὶν ὃ μὴ μεμνημένος—ἐπιχειρεῖν ζητεῖν καὶ ἀναμιμνῄσκεσθαι;

Plato thereby indicates clearly that the principles of mathematics constitute an intrinsic component of ψυχή, an attribute that is also readily evident in the description of ψυχή presented in the *Timaeus*.

Taken in conjunction with Timaeus' account, the argument in the *Meno* suggests that not only are mathematical laws, formulae and definitions inherent to ψυχή, but they are also the foundations of its essential architecture. In this regard, the attainment of mathematical understanding also advances us toward a greater measure of self-knowledge. Since, as Socrates explains to Meno, the truth of existing things, or the truth of existences, with respect us,[158] is always within ψυχή, we furthermore cannot but assume that the paradigms of the cosmos are contained, in some respect, within the human intellect. Scott (1995) appears to concur with this interpretation of knowledge through recollection, but understands the universal principles contained within ψυχή to be separate entities beyond particulars.[159] While these universals may be considered to be separate in a qualified sense insofar as they are distinct from the particulars that adhere to them, the suggestion that they are unequivocally separate seems incompatible with the operation of ψυχή as the intelligible activity of paradigm within the realm of the sensible.[160] If, moreover, all intelligible principles are contained within one another, as Proclus suggests,[161] then through their accordance with the specifications of the paradigm by which they are governed, specific entities must also account for their essential relation to all other objects. Based upon our observations of the natural world, for example, the fact, that certain animals require particular types of plants for sustenance, and more generally, the overall interdependence of various species toward one another within an a particular ecosystem, it seems that Proclus' position is correct; for unless the connections that bind that natural world together are purely accidental, they must also be represented within the intelligible. There is also some indication in the *Sophist* that Plato might hold a

[158] Pl. *Men*. 86b1. ἡ ἀλήθεια ἡμῖν τῶν ὄντων

[159] Dominic Scott. "Platonic Recollection", in *Plato 1: Metaphysics and Epistemology*. Volume 1. Oxford Readings in Philosophy. (Oxford: Oxford University Press, 1999), 95.

[160] See n.136.

[161] See n. 152.

position similar to that of Proclus. Additionally, as we shall observe later (see n. 177), the processes of collection and division as discussed in the *Philebus* and the *Phaedrus* enable the articulations of the essential connections between distinct paradigms according to the properties held in common between them. At any rate, it seems to follow that since knowledge of the paradigms is contained within ψυχή, the structure of which consists of mathematical parameters, mathematical knowledge ought to have special significance as the foundation for the full articulation of the intelligible.

In addition to the direct conclusion of Socrates' mathematical demonstration, which shows that geometric truths are contained within ψυχή and are apprehended through recollection, the original objective of the *Meno* is also of crucial significance to our inquiry. Socrates and Meno initially seek to determine the manner in which ἀρετή is acquired.[162] If ἀρετή is apprehended through recollection, then its definition must be contained already within ψυχή. Thus, if the excellence implied by ἀρετή is defined within the Good (for otherwise we are at a loss to explain how it is to be measured), it then follows that knowledge of the Good is to be understood as inherently present within ψυχή. As Santas (1980) explains, the Good is described in the *Republic* as "'the cause' of truth and knowledge',"[163] and that even mathematical understanding, according to the position expressed by Socrates, is considered to be uncertain and "hypothetical" without apprehension of the Good, and that full apprehension of mathematics requires that we then turn our knowledge of the Good upon the level of hypothesis.[164] To be sure, while apprehension of mathematical principles is indicated to be necessary for our attainment of the Good, it is not sufficient for this purpose. As indicated in Socrates' criticism of the methods commonly employed by geometers, it is possible to articulate mathematical principles entirely in themselves, without reference to their activity in nature, but in

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[162] Pl. *Men*. 70a1-2.

[163] Gerasimos Santas. "The Form of the Good in Plato's *Republic*", in *Plato 1: Metaphysics and Epistemology*. Volume 1. Oxford Readings in Philosophy. (Oxford: Oxford University Press, 1999), 247. Op. Cit. Op. Cit. Pl. *R*. VI. 509a-b.

[164] Santas. "The Form of the Good in Plato's *Republic*", 247. Op. Cit. Pl. *R*.VI. 509b-511e. We do not assume, however, that apprehension of mathematical principles must necessarily lead to ascension toward the Good. In Plato's argument against the traditional approach to geometry, which we shall examine shortly, Socrates notes that mathematicians treat the principles of arithmetic and geometry purely with respect to themselves without considering them in the ontological and epistemological context.

taking this approach, one fails to produce a complete articulation of these laws and formulae.

Based on Socrates' argument, it is plausible that there are two main elements to the hypothetical understanding of mathematical principles. The first aspect is that prior to apprehension of the Good, we are able to understand mathematical laws, formulae, and functions with respect to themselves, but are incapable of recognizing their activity within reality. The second element is that prior to apprehension of mathematical truths as belonging to the Good, we are only able to articulate them as observed facts, and are not yet capable of demonstrating by rational means that they must necessarily be true, and, by consequence, are unable to explain their significance in the context of mathematical systems as a whole. In the passage cited by Santas concerning hypothetical and precise knowledge of mathematics, the two levels of knowledge are differentiated as follows,

> 'Insofar as this is the case, the soul is forced to investigate from hypothesis, deriving not from the origin, but from the result, which is something different – since that which is based on the origin is non-hypothetical – going from hypothesis and without the thing upon which the likenesses are based, and through these carries out pursuit of the models themselves.'
>
> …
>
> 'And therefore geometers furnish themselves with visible shapes and produce articulations of these, not of the objects of thought, but each according to itself alone, producing articulations of the square itself and the diameter itself, but does not draw the shape of the thing itself, and they shape and draw the same things, and of these there are shadows and images in the water, and indeed, consulting these images, they seek to observe the shapes alone and not the objects of thought beyond them.'
>
> 'You speak truthfully,' he said.
>
> 'They claim that the shape is the thing known, and that the soul, in seeking it, is compelled to consult hypotheses, not beginning from first principles, and so it is unable to move beyond hypotheses, and will consult the images themselves as copies

of the first principles for each thing as visible objects of thought worthy of reverence.'[165]

In this passage, Socrates describes the error of geometers as their inclination to regard abstract geometric figures as the sources of mathematical first principles, when in fact these shapes are mere explanatory illustrations that serve to facilitate the articulation of the principles that they instantiate. In identifying these diagrams with paradigms, geometers remove the principles of mathematics from their context within reality, and thus do not examine them in a way that advances their apprehension of the true objects of knowledge.

Nevertheless, the argument of the divided line indicates that this imperfect understanding of mathematical concepts is necessary in order for the human intellect to progress to the realm of the intelligible. Based on this requirement, it is reasonable to presume that the laws and formulae belonging to geometry and the other mathematical sciences must be apprehended with respect to themselves before it is possible to articulate their operations in the context of the cosmos.

According to Smith (1981), the abstract objects of geometric investigation are most commonly identified with the third segment of the Divided Line which represents διάνοια, since these objects are understood to constitute perceptible representations of intelligible models, which he describes as "visible images of Forms, and the Forms

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[165] Pl. *R*. VI. 510b4-8, 510d5-511a8. ἦι τὸ μὲν αὐτοῦ τοῖς τότε μιμηθεῖσιν ὡς εἰκόσιν χρωμένη ψυχὴ ζητεῖν ἀναγκάζεται ἐξ ὑποθέσεων, οὐκ ἐπ᾽ ἀρχὴν πορευομένη ἀλλ᾽ ἐπὶ τελευτήν, τὸ δ᾽ αὖ ἕτερον—τὸ ἐπ᾽ ἀρχὴν ἀνυπόθετον—ἐξ ὑποθέσεως ἰοῦσα καὶ ἄνευ τῶν περὶ ἐκεῖνο εἰκόνων, αὐτοῖς εἴδεσι δι᾽ αὐτῶν τὴν μέθοδον ποιουμένη.

...

οὐκοῦν καὶ ὅτι τοῖς ὁρωμένοις εἴδεσι προσχρῶνται καὶ τοὺς λόγους περὶ αὐτῶν ποιοῦνται, οὐ περὶ τούτων διανοούμενοι, ἀλλ᾽ ἐκείνων πέρι οἷς ταῦτα ἔοικε, τοῦ τετραγώνου αὐτοῦ ἕνεκα τοὺς λόγους ποιούμενοι καὶ διαμέτρου αὐτῆς, ἀλλ᾽ οὐ ταύτης ἣν γράφουσιν, καὶ τἆλλα οὕτως, αὐτὰ μὲν ταῦτα ἃ πλάττουσίν τε καὶ γράφουσιν, ὧν καὶ σκιαὶ καὶ ἐν ὕδασιν εἰκόνες εἰσίν, τούτοις μὲν ὡς εἰκόσιν αὖ χρώμενοι, ζητοῦντες δὲ αὐτὰ ἐκεῖνα ἰδεῖν ἃ οὐκ ἂν ἄλλως ἴδοι τις ἢ τῇ διανοίᾳ.

ἀληθῆ, ἔφη, λέγεις.

τοῦτο τοίνυν νοητὸν μὲν τὸ εἶδος ἔλεγον, ὑποθέσεσι δ᾽ ἀναγκαζομένην ψυχὴν χρῆσθαι περὶ τὴν ζήτησιν αὐτοῦ, οὐκ ἐπ᾽ ἀρχὴν ἰοῦσαν, ὡς οὐ δυναμένην τῶν ὑποθέσεων ἀνωτέρω ἐκβαίνειν, εἰκόσι δὲ χρωμένην αὐτοῖς τοῖς ὑπὸ τῶν κάτω ἀπεικασθεῖσιν καὶ ἐκείνοις πρὸς ἐκεῖνα ὡς ἐναργέσι δεδοξασμένοις τε καὶ τετιμημένοις.

imaged."[166] The problem, however, in identifying geometric abstracts with the objects of διάνοια, as Smith explains, is that according to the structure of the divided line, the objects of διάνοια and νόησις must belong to the intelligible, whereas the models through which geometers articulate the laws and formulae of figures and spatial proportions are visible. Smith presents the arguments of several scholars concerning the exact nature of the objects of διάνοια, one of whom, J. Adam, states that the objects of διάνοια represent perfect intelligible models of the perceptible structures examined by geometers. While this position, as Smith indicates, agrees with Aristotle's interpretation of Plato, it fails to properly address the necessary distinction between the objects of διάνοια and those of νόησις, since the perfection of the objects of διάνοια would, in Adam's argument, suggest, rather incorrectly, that they are functionally identical to the Forms, which belong to νόησις. Smith also describes the argument presented by D. Gallop, according to whom the objects of διάνοια are most accurately understood as λόγοι of the objects of νόησις, a position which, as Smith indicates, does not contain the same problem as that which is proposed by Adam.[167] The λόγοι proposed in Gallop's theory must, however, articulate the objects of νόησις in a strictly qualified sense, according to Plato's characterization of the mathematical knowledge of geometers as hypothetical;[168] for it seems to be the case that the apprehension of geometric principles on the level of διάνοια, if it is correct in its explanation of the objects of νόησις, cannot begin from these uppermost intelligibles, but rather must approach them through accurate abstraction from sensible models which instantiate the first principles examined in the hypotheses at the level of νόησις.

We might further argue that it is not sufficient for the λόγοι posited by Gallop to simply constitute accurately abstracted inferences concerning the objects of νόησις; for based on Socrates' description of the practice of geometry in the *Republic*,[169] we may

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[166] Nicholas D. Smith. "The Objects of "Dianoia" in Plato's Divided Line", *Apeiron: A Journal for Ancient Philosophy and Science* 15, no. 2 (December 1981), 129. Op. Cit. Pl. *R*. VI. 510b, 510d-e, 511a; 510c-511d.

[167] Smith. "The Objects of "Dianoia in Plato's Divided Line", 130-131. Op. Cit. Pl. *R*. VI. 509d; J. Adam. *The Republic of Plato*. (Cambridge: Cambridge University Press, 1902).; David Gallop. "Image and Reality in Plato's *Republic*", *Archiv für Geschichte der Philosophie* (1965), pp. 113-131.

[168] See n. 165.

[169] See n. 165.

infer that it is possible for geometers to draw correct conclusions based on abstraction from sensible models, while nonetheless failing to develop any explanation concerning the ontological operation of the laws and formulae abstracted. We may therefore postulate that the objects of διάνοια are to be identified not only with accurate observationally derived explanations, but specifically with accurate abstractions leading to correct conclusions regarding the operations of the cosmos as defined on the level of paradigm. This interpretation of διάνοια appears particularly likely based on the fact that in the description cited by Smith,[170] the primary division in the divided line is stated, as follows, to be the division of the sensible and intelligible,

> 'So therefore, we partition the divided line into unequal parts, cutting, in turn, each part according to the same ratio, into the classifications of the sensible and the intelligible, and the things that are clear and ambiguous to you will be next to one another, while among the sensible, the second partition belongs to images – I am saying that the images are prior to the shadows, with the shadows afterwards being as reflections in the water, with the light and the flat surface coming together such that this is the entirety, if you understand my meaning.'
>
> 'Indeed I do understand it.'
>
> 'You must then, it would seem, place the other such segment, for the entirety of all living things in addition to ourselves, as well as the entire class of things planted and crafted.'
>
> 'I am placing it,' he said.
>
> 'Would you say,' I said, 'that to separate what is true from what is not, as the opinion from the known, we would thus separate the thing itself from its likeness?'
>
> 'Certainly,' he said, 'and very much so.'
>
> 'It seems then that the portion pertaining to knowing must be divided as well.'[171]

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[170] See n. 166.

[171] Pl. *R*. VI. 509d6-510a10. ὥσπερ τοίνυν γραμμὴν δίχα τετμημένην λαβὼν ἄνισα τμήματα, πάλιν τέμνε ἑκάτερον τὸ τμῆμα ἀνὰ τὸν αὐτὸν λόγον, τό τε τοῦ ὁρωμένου γένους καὶ τὸ τοῦ νοουμένου, καί σοι ἔσται σαφηνείᾳ καὶ ἀσαφείᾳ πρὸς ἄλληλα ἐν μὲν τῷ ὁρωμένῳ τὸ μὲν ἕτερον τμῆμα εἰκόνες—λέγω δὲ τὰς εἰκόνας πρῶτον τὸ μὲν ἕτερον τμῆμα εἰκόνες—λέγω δὲ τὰς εἰκόνας πρῶτον μὲν τὰς σκιάς, ἔπειτα τὰ ἐν τοῖς ὕδασι φαντάσματα καὶ ἐν τοῖς ὅσα πυκνά τε καὶ λεῖα καὶ φανὰ συνέστηκεν, καὶ πᾶν τὸ τοιοῦτον, εἰ κατανοεῖς.

Since the structure of the Divided Line places the illusory likenesses of actual sensible objects within the lowest tier of apprehension, we may infer that only accurate mathematical conclusions derived from perceptible models ought to be counted among the objects of διάνοια; for it would seem that incorrect conclusions should be regarded as false perceptions of the same class as shadows. In order, however, for the objects of διάνοια to lead the intellect toward apprehension of the intelligible principles at the level of νόησις, it is not sufficient for us to consider them only with respect to themselves, lest we fail to account for their operation in complete reality. Mohr (1981) uses Plato's theory of recollection to explain the operation of the third level of the Divided Line. He suggests that the images according to which geometers develop hypotheses are "images of the Ideas in the seat of the consciousness," and that connection of hypotheses on the part of the scientist is a function of the recovery of pre-existing connections as defined within ψυχή according to the origin of such connections within the intelligible.[172]

Given the special significance for ψυχή attributed to mathematical knowledge in the *Timaeus*, it would seem that hypotheses pertaining to geometry and number theory are not only subject to the connections that occur by means of recollection, but are instrumental in facilitating them. Mohr suggests that the progression from διάνοια to

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ἀλλὰ κατανοῦ.

τὸ τοίνυν ἕτερον τίθει ᾧ τοῦτο ἔοικεν, τά τε περὶ ἡμᾶς ζῷς καὶ πᾶν τὸ φυτευτὸν καὶ τὸ σκευαστὸν ὅλον γένος.

τίθημι, ἔφη.

ἦ καὶ ἐθέλοις ἂν αὐτὸ φάναι, ἦν δ' ἐγώ, διῃρῆσθαι ἀληθείᾳ τε καὶ μή, ὡς τὸ δοξαστὸν πρὸς τὸ γνωστόν, οὕτω τὸ ὁμοιωθὲν πρὸς τὸ ᾧ ὡμοιώθη;

For an explanation of the mathematical error within the structure of the Divided Line, see Robert S. Brumbaugh. "Plato's Divided Line", *The Review of the Metaphysics* 5, no. 4 (June 1953), 530, where Brumbaugh indicates that the ratio ascribed to the divided line is not possible, for if a line is partitioned according to a certain ratio m/n, and both sections are, in turn, divided according to the same ratio, the entire structure will not adhere to the same ratio consistently, as the two middle sections of the line will be of equal length. Brumbaugh suggests, however, (p532-3, Op. Cit. Pl. *R*. IX. 580d2, VII. 525b, X. 602e, 534d5, *Plt*. 257a.) that Plato's apparent miscalculation in the construction of the Divided Line is conscious and intentional, and is meant to serve as a warning regarding the dangers of accepting a completely literal interpretation of visible mathematical models.

[172] Richard Mohr. "The Divided Line and the Doctrine of Recollection In Plato", *Apeiron:A Journal for Ancient Philosophy and Science* 18, no. 1 (June 1984), 37-8. Op. Cit. Pl. *R*.VI. 510c; Pl. *R*.VI. 510c, 500c2-4 "τεταγμένα ... κόσμῳ πάντα καὶ κατὰ λόγον ἔχοντα", 508b, 509b.

νόησις in the Divided Line is enabled not only by hypotheses, but occurs also in the observation of sensible entities which function as "paradigms (παραδείγματα) in the study of Ideas (529d7-8)."[173] It would therefore appear as though the process that occurs in the ascent from διάνοια to νόησις is a direction of the connections produced through hypothesis downward toward sensible entities treated as paradigms, after which a similar return may occur from the level of νόησις to the lower tiers of the Divided Line. In this manner, it would seem, the model of the cosmos innately present within ψυχή, as detailed in the *Timaeus*, attains closer adherence to the intelligible paradigm according to which it is defined. Benson (2010) explains that the first stage of the process associated with the dianoetic level of the Divided Line, the proof stage, begins with the development of a hypothesis in an attempt to determine the answer to a particular question. This stage is followed by the confirmation of the hypothesis, in which case, as Benson states, "one seeks to confirm the truth of the hypothesis, [2a] first by identifying a further hypothesis from which the original hypothesis can be derived and showing how this derivation goes until one reaches "something adequate," and then [2b] by testing the consequences of the hypotheses to see whether they agree with one another."[174] Benson's explanation appears compatible with Mohr's argument, and taking both positions in conjunction with one another we may posit that the connection and demonstration of hypotheses as described by Benson, assuming compatible conclusions follow from the truth of the hypotheses in question, constitute the recovery of paradigmatic connections within the consciousness as presented by Mohr.

Beyond the account related in the *Timaeus* concerning the repair of the orbits of ψυχή, there is strong evidence in the *Republic* that the apprehension of geometry and number theory is of special significance in the process of the recovery of connections

[173] Mohr. "The Divided Line and the Doctrine of Recollection in Plato", 39. Op. Cit. Pl. *R*.VI. 511a, 510d5-8; Pl. *Phd*. 103b5, 8, e5; Pl. *Phdr*. 266a2-3; Pl. *Phlb*. 64a1-2. Mohr's characterization of sensible entities as "paradigms" must be interpreted in a qualified sense, since the unqualified use of the term would refer to paradigms as defined in the intelligible. It would be more accurate to describe tangible objects as referential examples which are instrumental in the testing of hypotheses, although they are not the final objects of knowledge toward which this verification is directed.

[174] Hugh H. Benson. "Plato's Philosophical Method in the *Republic*", in *Plato's 'Republic'*, ed. Mark L. McPherran. (Cambridge: Cambridge University Press, 2010), 191. Op. Cit. Bailey 2006, p. 102; Benson, 2003; Benson, 2006.

within the intellect. Socrates gives clear indication that mathematical knowledge represents the foundation of the processes through which we attain apprehension of reality, explaining as follows the correlation between mathematical skill and mastery of all scholarly disciplines,

> 'Why? You have also said before that just as those who are skilled by nature in calculation are said to similarly excel in all sciences, while those who are slow, when they are educated and given to practice, and are not helped by another, will they not, just like those who are keen, become more capable in all respects?'[175]

It thus appears to be the case that in Plato's position, the mathematical sciences constitute the language of rational articulation within intellect, and that the principles of these sciences are therefore not merely recollected, but also aid in the process of recollection. The recollection of mathematical principles, and therefore the reconfiguration of the orbits of ψυχή, must then, it seems, constitute a reconnection of the logical structures that are innately present within intellect. Plato thus indicates that the principles of mathematics constitute nothing less than the foundation of all knowledge, for although they do not account for a full articulation of reality in themselves, they nonetheless function as the thread by which the fabric of the intelligible is fully unified within the activity of reason. In examining the process through which our mathematical knowledge develops, we therefore advance not only the measure of our self-knowledge, but also our apprehension of the logical ordering of existence.

4.2 The Language of Truth and Being: Explaining the Nature of Mathematical Laws and Formulae through the Methods and Results of Human Knowing

Through our observations of the structure of the Divided Line, as well as Plato's Doctrine of Recollection, it is shown quite clearly to us that our apprehension of the full reality of mathematical laws and formulae occurs in the articulation of their operation as the patterns of precision within the intelligible, and the activity of the intelligible within the realm of the sensible. On a general level, there can be little doubt that the logical pattern

[175] Pl. *R*. VII. 526b5-9. τί δέ; τόδε ἤδη ἐπεσκέψω, ὡς οἵ τε φύσει λογιστικοὶ εἰς πάντα τὰ μαθήματα ὡς ἔπος εἰπεῖν ὀξεῖς φύονται, οἵ τε βραδεῖς, ἂν ἐν τούτῳ παιδευθῶσιν καὶ γυμνάσωνται, κἂν μηδὲν ἄλλο ὠφεληθῶσιν, ὅμως εἴς γε τὸ ὀξύτεροι αὐτοὶ αὑτῶν γίγνεσθαι πάντες ἐπιδιδόασιν;

of mathematical demonstration also belongs essentially to the conditionally structured laws governing the activity of the cosmos. Mueller (1992) explains the procedure of mathematical demonstration through *diorismos*, whereby one seeks to prove a certain hypothesis *P* "by searching for propositions that imply *P*, propositions that imply those, and so on until one reaches propositions already established;"[176] This method of inference is virtually identical to that which is described by Benson,[177] and in those instances in which the process of demonstration ultimately leads to a necessary truth belonging to the structure of reality, it is reasonable to suggest that the intelligible succession of conditions and results connecting the axiom to the demonstrated hypothesis ought to be identical to the stepwise pattern of proof carried out by the scientist. This procedure of inquiry would then coincide with that which is described by Mohr, whereby the images of essential logical connections belonging to the objects of knowledge are restored within ψυχή as part of the process of recollection.[178] Cherniss, furthermore, whom Mueller cites in reference to the method of inference through *diorismos*, describes a process of reasoning in which the proposition to be investigated is assumed to be true, with all consequences of its truth being considered, until the succession of results leads to one that is known with certainty to be either true or false.[179] If this process of inference may be said to lead to true knowledge pertaining to the objects of science as defined within the intelligible, then it must follow that, assuming the hypothesis in question is demonstrated to be true, the logical progression by which it is proven must be identical with the intelligible order of conditions and results according to which this proposition is true according to the relations belonging essentially to paradigms; for if the rational structures of reality are defined within ψυχή, then any accurate inference made concerning the objects of knowledge, assuming, at least, that it is true in a non-trivial sense, constitutes a parallel

[176] Ian Mueller. "Mathematical Method, Philosophical Truth", in *The Cambridge Companion to Plato*, ed. Richard Kraut. (New York: Cambridge University Press, 1992, 22nd reprint 2010), 175. Op. Cit. Harold Cherniss, "Plato as Mathematician," *Review of Metaphysics* 4 (1951): 395-426, reprinted in Harold Cherniss. *Selected Papers*, ed. Leonardo Tarán (Leiden, 1977).

[177] See n. 174.

[178] See n. 172.

[179] Harold Cherniss. "Plato as Mathematician", *The Review of Metaphysics* 4, no. 3 (March 1951), 414.

with the essential logical structures governing the definitions and relations that guide the activities of the universe.

It is indeed difficult, if not impossible, for us to explain the manner in which the essential unifying connections of the cosmos are knowable, unless correct, non-trivial relations according to inference on the part of human intellect also constitute real relations on the level of paradigm. Within the positions of both Plato and Plotinus, Martin (1982) likens the process of dialectical reasoning to the mathematical structure of formal logic. Martin cites Taylor (1926), who describes the dialectic discussed in the *Republic* as "reduction of mathematics to rigorous deduction from expressly formulated logical premises," as well as Plotinus, according to whom the human intellect "is nourished on the proposition of logic, is skilled in following discussions, works by reasonings, examines links of demonstration, and comes to know the world of Being also by the steps of logical process…"[180] This passage, as Martin interprets it, suggests the process of proof from proposition is one in which paths of reasoning pertaining to reality, that is to say, the "links of demonstration," are not produced, but are rather revealed; at any rate, if the results from this process constitute knowledge rather than simply accurate opinion, then the logical connections that are apprehended must, it seems, be essential relations within paradigm rather than mere constructs of the human imagination. It ought to be noted that Martin appears to be misquoting Plotinus, who, in the passage in question, explores the definition of evil rather than discussing dialectic. In his remarks of dialectic in I. 3. 4, however, Plotinus states,

> What is dialectic, and what must it give to the things that are prior? It is the faculty for expressing the articulation of each thing, as well as distinguishing each thing from the others, as well as their points of commonality; it is within them, and is that by which each thing is whatever it is, and by which such things may be said to exist or not to exist, being apart from existence.[181]

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[180] R. M. Martin. "On Logical Structure and the Plotinic Cosmos", in *The Structure of Being: A Neoplatonic Approach*, ed. R. Blaine Harris. Studies in Neoplatonism: Ancient and Modern Volume IV. (Albany: State University of New York Press, 1982), 11, 12. Op. Cit. A. E. Taylor, *Plato, The Man and His Work* (London: Methuen, 1963, first published in 1926), p. 293; Plot. I. 8. 1.

[181] Plot. I. 3. 4. 1-6.

According to Plotinus, dialectic is therefore not simply the articulation of definitions and connections on the part of the human intellect, but does in fact constitute the logical structure of sameness, likeness, difference and multiplicity.[182] Since dialectic belongs to a higher tier of the Divided Line than that of hypothesis, its pattern of reasoning would necessarily treat the definitions and relations contained within paradigm as unified and simultaneous; for, as Nikulin explains, the discursive method of reasoning belonging to διάνοια "cannot embrace the whole of thinking in the structure of all-unity," and that "discursive thinking always presupposes a fundamental duality of indemonstrable first principles (e.g., axioms) and formal rules of deduction…"[183]

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[182] Crombie. *An Examination of Plato's Doctrines*. Volume II: *Plato On Knowledge and Reality*. 368-74. The articulation of the essential connections belonging to various intelligible paradigms may also be identified with the principle of collection and division as discussed in the *Philebus* and the *Phaedrus*. As explained by Crombie, the concept of collection, or *sunagôgê*, is associated with two definitions. The definition presented in the *Philebus* seems to treat collection as "giving a selection of specimens chosen to illustrate the range over which the kind extends." The other definition, detailed in the *Phaedrus*, implies the unification of disparate genera into a single class according to an essential common property. Crombie states that in the passage cited , Pl. *Phlb*. 25d5-9, *Socrates* states that although he and Protarchos have collected the *apeiron*, that is, the indefinite or the unlimited, or, to be sure, that which belongs to the *apeiron* (τὴν τοῦ ἀπείρου), they have failed (or rather simply not yet succeeded), at collecting that which belongs to the *peras*, or the limited (τὴν τοῦ περατοειδοῦς). In this passage, Socrates then proceeds to state that the latter type of collection should also be possible (ἀλλ᾽ ἴσως καὶ νῦν ταὐτὸν δράσει). Crombie comments that the giving of specimens seems a less likely definition for collection, given that it does not appear sufficient for the role of collection alongside division within dialectic. The specimen-focused definition of collection does indeed appear to be weaker insofar as it defines genera based upon the specific species that comprise them, rather than according to the essential characteristics that would identify a certain species as a member of the genus in question. Crombie paraphrases the later definition from Pl. *Phdr*. 249b6 as ""to unify in thought what is presented in manifold sense-experiences"." In, Pl. *Phdr*. 249b-c, Plato states, "It is necessary for a human to unify that which is discussed according to image, going from many sensible things towards grasping one thing by reasoning…" δεῖ γὰρ ἄνθρωπον συνιέναι κατ᾽ εἶδος λεγόμενον, ἐκ πολλῶν ἰὸν αἰσθήσεων εἰς ἓν λογισμῷ συναιρούμενον The principle of division, or *diairesis*, as Crombie indicates, is understood as the precise partitioning of collections according to essential distinctions between those things that belong to them. This division, Crombie states, must occur ""at a joint"," an expression presented in Pl. *Phdr*. 265e as "κατ᾽ ἄρθρα." Thus, rather than dividing a collection haphazardly and in an uncalculated fashion, the correct method of division differentiates the various members of the collection by isolating them from the defining common property of the collection. Through division, one might also identify other collections, and identify the properties of each member of the initial collection which necessitate its possession of the common characteristic defining this collection. The processes of collection and division do not restrict our classification of species to genera alone, for we might also suggest connections between various species according to similar adherence to a specific mathematical principle. We must also recognize that dialectic is itself subject to collection and division through its partition into two processes, which are unified as the highest level of inquiry. In this regard, the activity of dialectic is also directed toward itself.

[183] Nikulin. *Matter, Imagination and Geometry*, 151.

These rules of deduction must therefore, in reality, be unified with the first principles that connect them on the hypothetical level, such that, while the succession from premise to conclusion occurs in a stepwise manner within διάνοια, it must be simultaneous and instantaneous at the level of νόησις. Nikulin cites the *Theaetetus* in describing the discursive process of reasoning,[184] and in the passage in question, the exchange between Socrates and Theaetetus proceeds as follows,

> 'Whatever is different according to your observation is different in that respect and not as belonging to thought.'
>
> 'Indeed.'
>
> 'So whenever thought does such a thing, is it not also necessary for it to consider both that which is identical to it and that which is different?'
>
> 'It is necessarily so; for both types of things are shares of the whole.'
>
> 'Most beautiful. Do you judge me to know?'
>
> 'What am I judging?'
>
> 'We are considering the principle by which the soul passes through itself if it perceives things. For it seems to me as thought contemplation is nothing other than discussion, enquiring about itself and separating from itself, both affirming and denying. Whenever it enquires, whether it progresses at the slowest or the swiftest speed, and whenever it affirms something without question, we observe it in the same way. '[185]

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[184] Nikulin. *Matter, Imagination and Geometry*, 150-151. Op. Cit. Pl. *Tht*. 189d-190a, *R*. VII. 533e-534c, cf. *Sph*. 260e, 263d-e, Arist. *Metaph*. 1025b 3sqq.; de An. 431a 1 sqq.

[185] Pl. *Tht*. 189d7-190a4.
Σωκράτης
ἔστιν ἄρα κατὰ τὴν σὴν δόξαν ἕτερόν τι ὡς ἕτερον καὶ μὴ ὡς ἐκεῖνο τῇ διανοίᾳ τίθεσθαι.

Θεαίτητος
ἔστι μέντοι.

Σωκράτης
ὅταν οὖν τοῦθ᾽ ἡ διάνοιά του δρᾷ, οὐ καὶ ἀνάγκη αὐτὴν ἤτοι ἀμφότερα ἢ τὸ ἕτερον διανοεῖσθαι;

Θεαίτητος
ἀνάγκη μὲν οὖν: ἤτοι ἅμα γε ἢ ἐν μέρει.

Σωκράτης
κάλλιστα. τὸ δὲ διανοεῖσθαι ἆρ᾽ ὅπερ ἐγὼ καλεῖς;

If the object being treated hypothetically is understood to be part of the same intelligible system, as Socrates and Theaetetus agree, then it cannot be sufficient to articulate such an object in a stepwise manner; for each stage in the process of διάνοια is, in reality, simultaneously predefined with all others according to the Good, and to each stage belongs a distinct system of rational connections for which we do not properly account in using this progression to answer a single question alone. The same must be true of mathematical questions treated in this manner, such that on the level of the intelligible, the logic governing the procession of premises and conclusions is not only entirely simultaneous and unified, but that this procession is unified by virtue of an overall structure that is circular rather than linear; for the beginning of this structure must also be its end in regard to the objectives that determine the functionality of mathematical laws and formulae. We cannot, furthermore, treat any mathematical hypothesis within its full ontological context without also giving comprehensive consideration to each stage of the process by which it is affirmed or refuted. The differentiation between the inferences of διάνοια and those of νοήσις therefore suggests that the entirety of mathematical principles must belong to a single, ontologically simultaneous system, the precise structure of which is determined by the unified intelligible model of the entire cosmos.

For the articulation of the true objects of science, it is therefore not sufficient to treat observations of number and shape purely with respect to themselves; they must, on the contrary, be examined in regard to the manner in which they are unchanging, as well as that in which they are constantly in motion inasmuch as they are at work constantly as a universal pattern of life and motion. It therefore comes as no surprise to us that Plato regards some mathematical sciences superior to others in terms of the extent to which they are able to articulate reality, and that he therefore holds astronomy at the highest tier

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Θεαίτητος
τί καλῶν;

Σωκράτης
λόγον ὃν αὐτὴ πρὸς αὑτὴν ἡ ψυχὴ διεξέρχεται περὶ ὧν ἂν σκοπῇ. ὥς γε μὴ εἰδώς σοι ἀποφαίνομαι. τοῦτο γάρ μοι ἐνδάλλεται διανοουμένη οὐκ ἄλλο τι ἢ διαλέγεσθαι, αὐτὴ ἑαυτὴν ἐρωτῶσα καὶ ἀποκρινομένη, καὶ φάσκουσα καὶ οὐ φάσκουσα. ὅταν δὲ ὁρίσασα, εἴτε βραδύτερον εἴτε καὶ ὀξύτερον ἐπᾴξασα, τὸ αὐτὸ ἤδη φῇ καὶ μὴ διστάζῃ, δόξαν ταύτην τίθεμεν αὐτῆς.

of the mathematical sciences, and the last to be taught within the mathematical education doctrine of the Callipolis, which Socrates explains as follows,

> 'I was in a hurry then,' I said, 'to treat things which must be taken slowly. For following the inquiry concerning the dimension of depth, which we considered laughable, proceeding past geometry, I spoke of astronomy, which pertains to motion according to depth.'
>
> 'You are correct,' he said.
>
> 'We must therefore place astronomy as the fourth science,' I said, as though the beginning was otherwise incomplete, lest the *polis* neglect it.'
>
> 'That is likely. And it seems to me, Socrates, that before you scorned the notion of astronomy as something worthy to be pursued, whereas now you heap praise upon it. For to me it seems entirely to be something visible which leads the soul toward that which is beyond sight and away from the things within it.'
>
> 'It is so,' I said, 'except for the matter of the visible, for it does not seem so to me.'
>
> 'But how?' he said.
>
> 'As these things lead the soul towards philosophy, they cause it to see downwards.'
>
> 'How do you mean?' he said.
>
> 'You do not seem low-minded to me,' I said, 'toward that which is the science of things beyond sight; for you are daring, and if one observes by lifting their head towards the things embroidered on the ceiling, such would not lead them towards the intelligible but towards the visible; but you hold them to be equally beautiful,' I said kindly. '[186]

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[186] Pl. *R*. VII. 528d7-529b3. σπεύδων γάρ, ἔφην, ταχὺ πάντα διεξελθεῖν μᾶλλον βραδύνω: ἐξῆς γὰρ οὖσαν τὴν βάθους αὔξης μέθοδον, ὅτι τῇ ζητήσει γελοίως ἔχει, ὑπερβὰς αὐτὴν μετὰ γεωμετρίαν ἀστρονομίαν ἔλεγον, φορὰν οὖσαν βάθους.
ὀρθῶς, ἔφη, λέγεις.

τέταρτον τοίνυν, ἦν δ᾽ ἐγώ, τιθῶμεν μάθημα ἀστρονομίαν, ὡς ὑπαρχούσης τῆς νῦν παραλειπομένης, ἐὰν αὐτὴν πόλις μετίῃ.

εἰκός, ἦ δ᾽ ὅς. καὶ ὅ γε νυνδή μοι, ὦ Σώκρατες, ἐπέπληξας περὶ ἀστρονομίας ὡς φορτικῶς ἐπαινοῦντι, νῦν ᾖ σὺ μετέρχῃ ἐπαινῶ: παντὶ γάρ μοι δοκεῖ δῆλον ὅτι αὕτη γε ἀναγκάζει ψυχὴν εἰς τὸ ἄνω ὁρᾶν καὶ ἀπὸ τῶν ἐνθένδε ἐκεῖσε ἄγει.
ἴσως, ἦν δ᾽ ἐγώ, παντὶ δῆλον πλὴν ἐμοί: ἐμοὶ γὰρ οὐ δοκεῖ οὕτως.

ἀλλὰ πῶς; ἔφη.

Since Socrates holds astronomy to be a science pertaining to motion according to depth, it would seem that while geometry pertains to the principles of dimension only with respect to themselves, astronomy examines these principles as active within the cosmos. In order for astronomy to be relevant to philosophy, it cannot truly pertain specifically to the visible heavenly objects themselves, for it has already been established, as Glaucon notes in the same passage, that astronomy cannot be the highest of the mathematical sciences if it only treats the firmament as it is perceptible by sensory faculties. It would also appear that astronomy cannot simply examine the general geometric laws pertaining to the observed patterns of movement of the planets, but must also consider the mathematical laws pertaining, by definition, to the properties according to which the planets follow the observed patterns of orbit. The former of these postulations, that is, that astronomy must not, in the strictest sense, examine the planets themselves, is confirmed when Socrates clarifies the proper approach to the science, stating,

> 'It is necessary in this respect,' I said. 'While it falls short of many truths to hold the fabric in the sky as the most beautiful and precise of all things, since it belongs to the sensible, the speed and slowness of these things are among the numbers pertaining to truth and within the true patterns pertaining to activity, these are moved by one another and move those things that are within them, and the things that are apprehended by articulation and contemplation, you will not see. Do you think so?' 'Not at all,' he said.[187]

In this passage, our later postulation, which states that astronomy must examine mathematical principles specifically dedicated to the motion of tangible objects in

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ὡς μὲν νῦν αὐτὴν μεταχειρίζονται οἱ εἰς φιλοσοφίαν ἀνάγοντες, πάνυ ποιεῖν κάτω βλέπειν.

πῶς, ἔφη, λέγεις;

οὐκ ἀγεννῶς μοι δοκεῖς, ἦν δ᾽ ἐγώ, τὴν περὶ τὰ ἄνω μάθησιν λαμβάνειν παρὰ σαυτῷ ἥ ἐστι: κινδυνεύεις γὰρ καὶ εἴ τις ἐν ὀροφῇ ποικίλματα θεώμενος ἀνακύπτων καταμανθάνοι τι, ἡγεῖσθαι ἂν αὐτὸν νοήσει ἀλλ᾽ οὐκ ὄμμασι θεωρεῖν. ἴσως οὖν καλῶς ἡγῇ, ἐγὼ δ᾽ εὐηθικῶς.

[187] Pl. *R.* VII. 529c7-d6. ὦδε, ἦν δ᾽ ἐγώ. ταῦτα μὲν τὰ ἐν τῷ οὐρανῷ ποικίλματα, ἐπείπερ ἐν ὁρατῷ πεποίκιλται, κάλλιστα μὲν ἡγεῖσθαι καὶ ἀκριβέστατα τῶν τοιούτων ἔχειν, τῶν δὲ ἀληθινῶν πολὺ ἐνδεῖν, ἃς τὸ ὂν τάχος καὶ ἡ οὖσα βραδυτὴς ἐν τῷ ἀληθινῷ ἀριθμῷ καὶ πᾶσι τοῖς ἀληθέσι σχήμασι φοράς τε πρὸς ἄλληλα φέρεται καὶ τὰ ἐνόντα φέρει, ἃ δὴ λόγῳ μὲν καὶ διανοίᾳ ληπτά, ὄψει δ᾽ οὔ: ἢ σὺ οἴει;
οὐδαμῶς γε, ἔφη

relation to one another, is shown to be correct. As noted previously, this manner of inquiry does not study the planets in themselves specifically, but rather considers the pattern of their motion as a model for the totality of mathematical activity within the cosmos, taking into account the laws pertaining to mass, speed, and relative distance, as well as accounting for the essential connection by which, according to Proclus,[188] the paradigms to which tangible entities adhere are contained within one another by way of essential connection. Indeed, part of the explanation for the identification of astronomy as the highest of the mathematical sciences is that it articulates the essential connection of numbers, points, lines planes, and solids.

Within the paradigmatic laws dedicated to the architecture of the heavens, the four basic dimension levels of geometry are expressed according to their proper possession of one another, such that the point contains the definition of the line, plane, and solid, and each of the other dimensional levels is defined within the others.[189] At the pre-dialectical level, however, the study of astronomy ought to be considered to belong to διάνοια rather than to νόησις; for it has not yet abandoned its need for the examination of sensible models, albeit natural, active models, for its approach toward intelligible laws and functions. As Socrates explains to Glaucon, the apprehension of the intelligible on the philosophical level leads to a downward examination of tangible beings,[190] such that specific objects are not known through themselves, but rather are completely articulated according to the intelligible models that govern them. Hopkins (2011) notes that according to the consideration of the Platonic and Neoplatonic division of the mathematical sciences by Klein (1934), astronomy is categorized among both geometry and arithmetic, which are themselves classified among the sciences pertaining to the

[188] See n. 152.

[189] We might even suggest that the relation of the geometric dimensional levels is a parallel to the structure of the Divided Line itself. Each of the four tiers of the Divided Line is comparable to one of the geometric dimensional levels insofar each tier beyond the lowest constitutes a development upon those that precede it. The comparison is particularly apt if the Divided Line constitutes a pattern of apprehension which is defined among the objects of knowledge; for in that case, all levels of the Divided Line are defined at the highest level of the Divided Line, such that in attaining the measure of apprehension that constitutes true knowledge, we come to contemplate the process of our own thinking.

[190] See n. 186.

mathematical objects of thought, with the other mathematical sciences, including those of calculation or logistic, geodetics, music, optics, and mechanics.

This division, according to Klein, "goes back to Plato."[191] Arithmetic and geometry thus ought to be recognized among those mathematical sciences which diverge to the least extent from the true objects of knowledge. Astronomy, being a union of both geometry and arithmetic which treats the principles of both sciences within the context of nature, approaches, out of all fields of inquiry belonging to the tier of διάνοια, the nearest to the mathematical algorithmic paradigms of existence as they are at the level of the intelligible and as images within ψυχή. While it therefore seems to be the case that astronomy represents the study of mathematical principles in a manner that closely approaches the nature of their ontological origin, it nevertheless requires the prior apprehension of these laws and formulae in respect to themselves. Such is indicated by Klein (1968) in his explanation of the distinction between the arithmetic and logistic sciences. He refers initially to the *Gorgias*, wherein Plato characterizes arithmetic as it pertains particularly to the study of ""the even and the odd," and elaborates by citing the *Theaetetus*, according to which arithmetic is that by which "…one is oneself master of the sciences of number, and is able as a teacher to pass them on to another.""[192] Concerning the distinction of logistic from arithmetic, Klein refers to the *Gorgias* and the *Charmides*, both of which present similar definitions for logistic; for the former states that logistic ""…studies the even and the odd with respect to the multitude which they [the single even and odd] make both with themselves and with each other."" The latter definition likewise states that logistic treats the even and the odd in terms of ""what multitude they make with themselves and with one another.""[193] Based upon Plato's

[191] Burt C. Hopkins. *The Origin of the Logic of Symbolic Mathematics: Edmund Husserl and Jacob Klein*. (Bloomington and Indianapolis: Indiana University Press, 2011), 154, 155.

[192] Jacob Klein. *Greek Mathematical Thought and the Origins of Algebra*. Trans. Eva Brann. (Dover edition: New York: Dover Publications Inc., 1992; Massachusetts Institute of Technology edition: Cambridge: The M.I.T Press, 1968) 17, 19. Op. Cit. Pl. *Grg*. 451a-c, *Tht*. 198a-b. Originally published as "Die griechische Logistik und die Entstehung der Algebra", in *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abteilung B: *Studien*, Vol. 3, fasc. 1 (Berlin, 1934), pp. 18-105 (Part I); fasc. 2 (1936), pp. 122-235 (Part II).

[193] Klein. *Greek Mathematical Thought and the Origin of Algebra*. 17. Op. Cit. Pl. *Grg*. 451a-c, *Chrm*. 165e-166a-b.

characterization of arithmetic of the foundation of all human knowledge concerning number, Klein also suggests that the former is also distinguished based on its dependence upon the latter.[194] This relation comes as no surprise to us, for it stands to reason that through apprehension of the universal laws concerning mathematical operations, one would attain greater aptitude in the activity of calculation as it pertains to all manner of tasks.

It is similarly reasonable to suggest, however, that the relation between the mathematical sciences belonging to νόησις and those of διάνοια is such that those of one level are capable of complementing those of the other in either direction; for just as one who is well-versed in both arithmetic and geometry will be more likely to excel in mechanics, so too is it possible that accurate observations made from the correct application of geometry and arithmetic may lead to the development of hypotheses the investigation of which may further advance one's apprehension of both sciences. This postulation is supported by the passage in the *Timaeus* in which Plato confirms the importance of proficiency in logistic (λογισμός) for the proper configuration of the image of the World-Soul.[195] We may therefore reasonably infer that when the intellect hones its aptitude in logistic, it also improves its apprehension of the universal principles constituting both arithmetic and geometry, and thereby further increases its capability in logistic, mechanics, and the other mathematical sciences belonging to διάνοια.[196]

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[194] Klein. *Greek Mathematical Thought and the Origin of Algebra*. 19. Op. Cit. Pl. *Tht*. 198a-b, Cf. *Ion*. 537e: …ὅτι πέντε εἰσὶν οὗτοι οἱ δάκτυλοι…τῇ αὐτῇ τέχνῃ γιγνώσκομεν τῇ ἀριθμητικῇ… -- "…that these fingers amount to five…we know by this same arithmetical art…."

[195] Pl. *Ti*. 47c2.

[196] Michael McCloskey. "Cognitive Mechanisms in Numerical Processing: Evidence from Acquired Dyscalcula", In *Numerical Cognition*, ed. Stanislas Dehaene. A Cognition Special Issue. (Previous edition: Amsterdam: Elsevier Science Publishers, B. V.; Current edition: Cambridge, Massachusetts and Oxford, United Kingdom: Blackwell Publishers, 1993), 113 (See Figure 1, McCloskey et al. (1985)), 115. Originally published as McCloskey, M., 1992. Cognitive mechanisms in numerical processing: evidence from acquired dyscalcula. Cognition, 44: 107-157. . Somewhat curiously, though perhaps accidentally, McCloskey notes a similar relation of the arithmetical faculties to the process of calculation based on an observation of acquired dyscalculia. He determines, in particular, that the activity of calculation is subordinate to the arithmetical cognition, and in the model that he examines, the cognitive mechanism pertaining to arithmetic processes the operation symbol(s) of the equation in question (for which McCloskey gives the example of the multiplication sign), for which the calculation mechanism then carries out the relevant procedure. This model thus suggests that the capacity to identify the definition of a particular mathematical operation belongs to the arithmetical mechanism. Thus, in order to perform the operation in question, the intellect must possess arithmetical apprehension of the definition of that

It appears, furthermore, as though music ought to be of similar importance among the mathematical sciences belonging to διάνοια in terms of its role in the perfection of the image of the World-Soul within ψυχή. He states that the orbits of the soul are ordered according to ratio in a manner akin to that of musical sound,[197] an ordering which Kytzler (1959) interprets as a system of categorically organized ratios. He identifies the ordering of the World-Soul as a structure of different operations delineated in a stepwise manner in increasing order.[198] In particular, he identifies the delineations as number-rows (Zahlenreihen), which consist of various ordering of notes corresponding to certain mathematical ratios, with the unit being associated with e´´´.[199] This interpretation of the structure of the World-Soul, if the ratios contained therein are able to account, either directly or indirectly, for all possible integer ratios, would also agree with the notion of all numerical values being ontologically simultaneous with the unit value. This position is, in turn, compatible with the ontological simultaneity of all mathematical principles within the World-Soul.

4.3 Ἐπιστήμη and Τέχνη: Understanding the Laws of Nature and Translating them into the Ordering of Human Activity

Within the ordering of human life toward the Good on the political and individual level, the apprehension of mathematical principles, both in themselves and in their cosmic operations, ought to enable precise articulation of the systems of the natural world to a measure that we are able to emulate them. As we have previously observed, there appear to be parallels between the ordering of the elements described in the *Timaeus* and the precise organization of human activity within the political community. We have noted,

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operation; for otherwise, although the calculation mechanism may possess the functional processes necessary to perform the operation, it will be uncertain of which processes it ought to use, and of the order in which it ought to use them, and will therefore be unable to construct a suitable algorithm to solve the equation.

[197] Pl. *Ti*. 47c6-d1. λόγος τε γὰρ ἐπ᾽ αὐτὰ ταῦτα τέτακται, τὴν μεγίστην συμβαλλόμενος εἰς αὐτὰ μοῖραν, ὅσον τ᾽ αὖ μουσικῆς φωνῇ

[198] Bernhard Kytzler. “Die Weltseele und musikalische Raum”, *Hermes* 87. Bd., H. 4 (December 1959), 393. “Dieser metaphysischen Komposition der Weltseele folgt die im letzten Satz angekündigte neue Eiteilung, eine Einteilung in Zahlen, die in drei sich stufenweise ergänzenden Operationen vorgenommen wird Note: e´´´ corresponds to musical notation conventions used by Kytzler.”

[199] Kytzler. “Die Weltseele und musikalische Raum”, 393, 406.

for instance, Socrates' example of the practical use for mathematics in the ordering of military formations.[200] It therefore appears to be the case that the role of mathematical principles in the ordering of human activity may be identified on two levels. The higher, and perhaps, to a certain extent, less direct, of these levels is that by which mathematical knowledge leads to apprehension of the Good by means of the perfection of the image of the Good within ψυχή. The lower, yet perhaps more direct of these two tiers constitutes the role of mathematical knowledge in the ordering of human activity towards it highest possible degree of excellence. Plato illustrates in the *Republic* the manner in which mathematical principles are required for the correct execution of the multitude of practical tasks necessary for the livelihood of the *polis* through his explanation of the need for the mathematical science of astronomy, which is stated thus,

> 'But why? Should we place astronomy third among these? That does not seem correct.'
>
> 'It seems so to me,' he said, for it is necessary for the accurate perception of hours, months, and yearly cycles, not only for farming and sailing, but also just as much so for military strategy.'[201]

Since the mathematical analysis of the motion of heavenly objects is understood to be of vital importance to necessities of agrarian, nautical, and military pursuits, it then stands to reason that skill in calculation of this sort is similarly crucial for the leaders of the political community, that they may order the livelihood of society to function in unison with the cycles of the natural world.

The governance of society according to the laws of proportion is, furthermore, necessary in order to ensure that the *polis* is able to maintain the resources needed for their survival. In a warning that proves to be frighteningly prescient, Plato speaks in an earlier passage of the *Republic* about the plight of the luxurious society, which, as a

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[200] See n. 102.

[201] Pl. *R*. VII. 527d1-4. τί δέ; τρίτον θῶμεν ἀστρονομίαν; ἢ οὐ δοκεῖ;

ἐμοὶ γοῦν, ἔφη: τὸ γὰρ περὶ ὥρας εὐαισθητοτέρως ἔχειν καὶ μηνῶν καὶ ἐνιαυτῶν οὐ μόνον γεωργίᾳ οὐδὲ ναυτιλίᾳ προσήκει, ἀλλὰ καὶ στρατηγίᾳ οὐχ ἧττον.

consequence of its excesses, suffers from ill health, and is plagued with resource shortages that drive it into armed conflict with its neighbours, as explained thus,

> ‘Will we not, furthermore, be in need of physicians, since we will need to be treated for eating so much more than before?’
>
> ‘Certainly.’
>
> ‘And we will need more space as well, since that would suit a growing city, for it will be beyond the capacity of a small area of land. Do we not agree?’
>
> ‘As you say,’ he said.
>
> ‘Then more space must therefore be claimed, if we are to distribute and plow a suitable amount of land, and if, furthermore, so that there should be some for each of us, and if each person has surpassed their need for the sake of infinite wealth, exceeding the boundaries of necessity?’
>
> ‘It will be quite necessary, Socrates,’ he said.’
>
> ‘We would have to make war after that, Glaucon. How could it be otherwise?’
>
> ‘It is as you say,’ he said.
>
> ‘And it is not necessary for us to say, whether war would be responsible for good or ill, but this alone, that we have already witnessed the waging of war, from which great ill befalls the *polis*, the individual, and the general public, whenever war occurs.’[202]

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[202] Pl. *R*. II. 373d1-e7.
οὐκοῦν καὶ ἰατρῶν ἐν χρείαις ἐσόμεθα πολὺ μᾶλλον οὕτω διαιτώμενοι ἢ ὡς τὸ πρότερον;

πολύ γε.

καὶ ἡ χώρα γέ που, ἡ τότε ἱκανὴ τρέφειν τοὺς τότε, σμικρὰ δὴ ἐξ ἱκανῆς ἔσται. ἢ πῶς λέγομεν;

οὕτως, ἔφη.

οὐκοῦν τῆς τῶν πλησίον χώρας ἡμῖν ἀποτμητέον, εἰ μέλλομεν ἱκανὴν ἕξειν νέμειν τε καὶ ἀροῦν, καὶ ἐκείνοις αὖ τῆς ἡμετέρας, ἐὰν καὶ ἐκεῖνοι ἀφῶσιν αὐτοὺς ἐπὶ χρημάτων κτῆσιν ἄπειρον, ὑπερβάντες τὸν τῶν ἀναγκαίων ὅρον;

πολλὴ ἀνάγκη, ἔφη, ὦ Σώκρατες.

πολεμήσομεν δὴ τὸ μετὰ τοῦτο, ὦ Γλαύκων; ἢ πῶς ἔσται;

οὕτως, ἔφη.

We may therefore characterize the failure of the luxurious city most accurately as a fundamentally mathematical error; for the desire for infinite wealth on the part of its citizens is indicative of their inability to apprehend the arithmetic and geometric finitude of the realm of the sensible.

The just city may then be classified as a precise proportional structure, which itself belongs to a grand system of proportions encompassing the entirety of the natural world. This explanation is one possible interpretation for the association of justice with ratio by several philosophers, including Plato, Archytas, and Iamblicus. This concept is detailed by Ausland (2006) who states that according to Plato's position, rational, spirited, and appetitive portions of ψυχή are likened to first, fourth, and octave portions of the concord respectively, while Iamblichus identifies justice with the mean of 9, which is equal to 5.[203] Perhaps the most likely explanation behind the use of ratios as symbolic concepts for justice, is that relations of that type may be said to represent a certain dynamic of governance and exchange within the political community. This interpretation is readily apparent in Iamblichus' argument, part of which, as Ausland tells us, is the fact that 5 is the "middle number" between 1 and 9, with the numbers before it having "less than is fitting, and the numbers following it having "more than is fitting." This value, according to Iamblichus, is also "fitting to each in accordance with the ratio of equality" based on the fact that it is equivalent to one ninth of the sum of 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9. Iamblichus also likens this summation to a balance scale, with elements on either side of the mean being equal in terms of excess and deficiency, such that 1, for instance, is of equal deficiency from 5 to the amount by which 9 is in excess of 5.[204] Leaving aside all considerations of this mean in terms of its numerological significance, we may regard its association with justice as a "thought symbol," such that it implies a universal abstract

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καὶ μηδέν γέ πω λέγωμεν, ἦν δ᾽ ἐγώ, μήτ᾽ εἴ τι κακὸν μήτ᾽ εἰ ἀγαθὸν ὁ πόλεμος ἐργάζεται, ἀλλὰ τοσοῦτον μόνον, ὅτι πολέμου αὖ γένεσιν ηὑρήκαμεν, ἐξ ὧν μάλιστα ταῖς πόλεσιν καὶ ἰδίᾳ καὶ δημοσίᾳ κακὰ γίγνεται, ὅταν γίγνηται.

[203] Hayden. W. Ausland. "The Mathematics of Justice", in *Reading Plato In Antiquity*, eds. Harold Tarrant and Dirk Baltzly. (London: Gerald Duckworth & Co., Ltd., 2006), 112, 113-114. Op. Cit. Pl. *R*. IV. 373d.; Iamb. *In Nicom*. 18.2-21., *Theol. Arithm*. 37.10ff.

[204] Ausland. "The Mathematics of Justice", 113-114. Op. Cit. Op. Cit. Pl. *R*. IV. 373d.; Iamb. *In Nicom*. 18.2-21., *Theol. Arithm*. 37.10ff.

representation for a pattern of activity that occurs within the natural world, and which must be emulated within socio-political systems. It is analogous to several examples of balance within the dynamics of human activity, such as impartiality in legal disputes, and moderate use of natural resources in a manner that is carefully calculated in order to ensure that consumption does not outpace renewal.

If these "justice ratios" are, as we have suggested, to be understood as thought symbols, then it seems, if they are pure and underived representations of the intelligible principle of justice, then the object of knowledge pertaining to justice is also a mathematical relation of some sort. This postulation receives credence from Plato's characterization of the just soul, which, as Ausland explains, is thus described by Plato as a musical harmony,

> 'And that which I said is true, as it seems, that justice does not belong to what is external to us, but is rather internal, and thus in truth concerns itself and those things belonging to it, and does not allow everything outside itself within it, nor does the class of objects within the soul allow them to interfere with one another, but, bringing the household into being, which is originated and ordered from it, and, becoming loved by the household, and fitting together on three levels, as three boundaries of harmony joined in a non-contrived manner, the lowest, the highest, and the middle, and if there should be any other between them, all of these are bound together, with one coming entirely out of many, self-controlled and harmonized, so that it is active only if it governs the acquisition of property, bodily health, matters of the state, around its own image, leading in all these things and being called the just and beautiful approach, and one preserves this habit and hones it to perfection only when one engages in attention to justice and in science, and leaves aside all unjust behaviour, along with ignorance and incorrect opinion.'[205]

[205] Pl. *R*. IV. 443c9-444a2. τὸ δέ γε ἀληθές, τοιοῦτόν τι ἦν, ὡς ἔοικεν, ἡ δικαιοσύνη ἀλλ᾽ οὐ περὶ τὴν ἔξω πρᾶξιν τῶν αὑτοῦ, ἀλλὰ περὶ τὴν ἐντός, ὡς ἀληθῶς περὶ ἑαυτὸν καὶ τὰ ἑαυτοῦ, μὴ ἐάσαντα τἀλλότρια πράττειν ἕκαστον ἐν αὑτῷ μηδὲ πολυπραγμονεῖν πρὸς ἄλληλα τὰ ἐν τῇ ψυχῇ γένη, ἀλλὰ τῷ ὄντι τὰ οἰκεῖα εὖ θέμενον καὶ ἄρξαντα αὐτὸν αὑτοῦ καὶ κοσμήσαντα καὶ φίλον γενόμενον ἑαυτῷ καὶ συναρμόσαντα τρία ὄντα, ὥσπερ ὅρους τρεῖς ἁρμονίας ἀτεχνῶς, νεάτης τε καὶ ὑπάτης καὶ μέσης, καὶ εἰ ἄλλα ἄττα μεταξὺ τυγχάνει ὄντα, πάντα ταῦτα συνδήσαντα καὶ παντάπασιν ἕνα γενόμενον ἐκ πολλῶν, σώφρονα καὶ ἡρμοσμένον, οὕτω δὴ πράττειν ἤδη, ἐάν τι πράττῃ ἢ περὶ χρημάτων κτῆσιν ἢ περὶ σώματος θεραπείαν ἢ καὶ πολιτικόν τι ἢ περὶ τὰ ἴδια συμβόλαια, ἐν πᾶσι τούτοις ἡγούμενον καὶ ὀνομάζοντα δικαίαν μὲν καὶ

This passage suggests that the image of justice within ψυχή is to be understood as a pattern of proportion comprised of three levels, within which the rational element presides over the emotional and appetitive elements, and not only holds these lower elements in proper proportion to itself, but also maintains within them the proper balance on non-rational influences.

Since this entire structure ought to be defined within the intelligible, the entire hierarchy may be likened to the Divided Line, with three levels instantiated below the tier of paradigm, while they are defined and known according to the intelligible principle in which they are all fully articulated. We may say with certainty that this proportional structure must be contained within the cosmic model, as justice must be governed according to such a pattern as defined within the intelligible; for we have already determined, as stated in the *Meno*, that knowledge pertaining to reality must be innately present within us,[206] and that the Good is the origin of knowledge,[207] and have thereby concluded that the image of the Good is present within the human intellect according to the intelligible. In order for the paradigm of justice to govern the entirety of the cosmos, it is reasonable to suggest that it must be an intricate operation consisting of a vast multitude of interconnected ratios, along with several regulatory formulae. If it is to have sovereignty over all objects of the sensible world, then all of the paradigms to which tangible entities adhere must contain not only the portion of the "justice algorithm" that pertains directly to their own definition, but the entire formula for their correct pattern of activity in conjunction with all other sensible beings.

Thus, through recollection of the image of justice contained within the human intellect, we are able to attain knowledge of the proper functionality of all entities within the structure of the cosmos. It is likely for this reason that, as Burnyeat (2000) explains, Plato treats harmonics as a particularly important realm of mathematical science. Burnyeat explains that Plato's position, in contrast to the doctrine associated with Pythagoras, emphasized the examination of harmonious ratios at an abstract level for the

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καλὴν πρᾶξιν ἣ ἂν ταύτην τὴν ἕξιν σῴζῃ τε καὶ συναπεργάζηται, σοφίαν δὲ τὴν ἐπιστατοῦσαν ταύτῃ τῇ πράξει ἐπιστήμην, ἄδικον δὲ πρᾶξιν ἣ ἂν ἀεὶ ταύτην λύῃ, ἀμαθίαν δὲ τὴν ταύτῃ αὖ ἐπιστατοῦσαν δόξαν

[206] See n.157.

[207] See n.163.

purpose of attaining apprehension of beauty and goodness, and refuted the notion of the concord of numbers being a function of the pleasantness of the sounds associated with them.[208] We may thus assume that the pleasantness of the sounds produced by concordant notes is in fact a result of the mathematical concord to which they adhere, as opposed to its cause.

Since, as we have determined, the image of justice, and of harmony as well, must be present within the human consciousness in order for us to apprehend justice and harmony within the intelligible, it stands to reason that concordant sounds are experienced pleasantly by us according to the intelligible image of harmony and justice as it is contained within ψυχή. As Tubbs explains, the ratio of harmonious sounds according to Pythagoras' supposed observations is always a ratio of two integers,[209] a description that is consistent with the Euclidean definitions, which, as Burnyeat explains, included not only double, triple, and quadrupal ratios, but also epimoric ratios, which are expressed as $n + 1 : n$ for a certain number n.[210] As Brumbaugh explains, however, Plato cautions us against the interpretation of mathematical examples in an overly literal manner.[211] It might therefore be more accurate to interpret concordant numbers not as actual representations of goodness and beauty, but rather as examples of the operations that imitate the abstract pattern pertaining to the universal activity of justice.

The various classifications of ratio discussed by Burnyeat would nonetheless be necessary for the intricate intelligible structure of justice accounting for the entire cosmos. This system would likely be a closed network of ratio relations, with each paradigm being connected, either directly or indirectly to all others according to a delicate mathematical balance. On the level of sensible οὐσίαι, this type of operation may be most clearly represented through population dynamics of animals and plants within the natural world, as well as the daily life of the citizens of a political community. Szpiro

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[208] M. F. Burnyeat. "Plato on Why Mathematics is Good for the Soul", in *Mathematics and Necessity*, ed. Timothy Smiley. Proceedings of the British Academy 103. (Oxford: Oxford University Press, 2000), 47.

[209] Tubbs. *What is a Number?*. 2.

[210] Burnyeat. "Plato on Why Mathematics is Good for the Soul", 47-8. Op. Cit. Euc. VII. 37.

[211] See n. 151.

(2010) supports this interpretation in describing the mathematical structure which Plato identifies in the *Laws* as the best proportion for the governance of a city. He states the ideal population of the city to be 50,000, with a total of 5,040 households. This number, as Szpiro explains, is stated by the Athenian stranger (assumed to be either Socrates or Plato) to be a "convenient number," which, as Szpiro suggests, is based on the fact that it has 59 divisors in total, including the first 10 positive integers, which makes it suitable for the proportional division of labour and wealth.[212] This aspect of Plato's definition of a just city is consistent with our earlier hypotheses for the role of mathematical principles in the activity of statecraft. The requirement for such precise calculation in the ordering and preservation of the political community receives sufficient clarification in the *Republic*, wherein Plato indicates that one of the primary concerns of rulers must be the task of ensuring that a city's consumption of resources does not outpace the natural cycles by which these resources are able to replenish themselves;[213] a just ruler must therefore hold the appetites of the populace in check, and ensure that the multiplicity of the population does not grow to such an extent that necessities of its livelihood exceed the capacity of the natural world; for in such eventualities, as Socrates explains, armed conflict with neighbouring cities will ensue, and the city that initiates this conflict in order to sate its growing hunger will either fall to the might of its enemies, or, having claimed all available resources through conflict, will ultimately exhaust them and perish through starvation.

A just ruler must therefore have precise apprehension of proportion, as well as an accurate understanding of the cyclical patterns of renewal that are operative within the natural world. It is of equal importance for the ruler to remain constantly aware of the mathematical limitations of the sensible world as defined by its stereometric construction, so as to distinguish these limitations from the potentially infinite cycles of renewal by which nature is able to restore that of which it has been deprived. A philosophical ruler must therefore not only possess the aptitude to make accurate calculations, but must be habituated toward mathematical analysis to such an extent as to ensure constant vigilance

[212] George G. Szpiro. *Numbers Rule: The Vexing Mathematics of Democracy, from Plato to the Present*. (Oxford and Princeton: Princeton University Press, 2010), 5.

[213] See n. 197.

regarding the proportional ordering of the city. The structure of Magnesia, the political community described in the *Laws*, suggests an example of the specifications that must be calculated in order to address the concerns set forth in the *Republic*. Saunders (1992) argues that the socio-political structure detailed in the later dialogues, such as the *Laws* and the *Statesman*, is continuous from the program introduced in the *Republic*. In both texts, there is a strong emphasis on the importance of division, which he interprets as an indication of Plato's prevailing adherence to the theory of the Forms. The significance of this study, as identified by Saunders in examining the *Statesmen*, is the apprehension of the intelligible patterns governing the divisions that occur among sensible entities.[214] Within the task of governance, a science of this sort would furnish the principles according to which the distinct elements of the natural world are able to carry out activity that is proper to their intelligible definition by functioning in cooperative harmony. The ruler is thereby able to structure the political community in a manner that not only emulates the patterns of division governing nature, but also participates in the patterns on which it is based, such that it does not damage or diminish them in any way, but is rather unified seamlessly with them. In this respect, the work of the ruler is comparable to the division of the elements by the Demiurge as described in the *Timaeus*. It is, furthermore, possible for us to identify both the penultimate and uppermost principles of the Divided Line within the mathematical ordering of the city; for at the rank of the hypothetical, we observe the direct application of mathematical proportion to the structure and activity of the political community; simultaneously, at the tier of the intelligible, we may recognize the governance of the city in harmony with the natural world in accordance with our apprehension of the essential relations between tangible beings as defined within paradigm.

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[214] Trevor J. Saunders. "Plato's Later Political Thought", in *The Cambridge Companion To Plato*, ed. Richard Kraut. (New York: Cambridge University Press, 1992, 22nd reprint 2010), 468. Op. Cit. Pl. *Plt*. 866d7*ff*, 867b3, 861b, 932e*ff* (cited by Saunders in note 28), cf. R. F. Stanley, *An Introduction to Plato's Laws*, (Oxford, 1983) 136, on 630e-631a; V. Brochard, "Les Lois de Platon et la théorie des Idées," in *Etudes de philosophie ancienne et de philosophie moderne*, ed. V. Delbos (Paris, 1926), 151-68, Guthrie, *History of Greek Philosophy*, 5:378-81, Stalley, *Introduction to Plato's Laws*, 133-6, Pl. *Lg*. 967e, Saunders, "Notes on the *Laws* of Plato," *Bulletin of the Institute of Classical Studies*, supp 28 [London, 1972], n. 10; Pl. *Pol*. 262a*ff*, 285a-b, cf. *Phdr*. 265e, Sauders, note 28.

The mathematical ordering of the political community by means of proportion, in accordance with the dictates of the intelligible, has the effect of prolonging the survival of the city, such that if is ruled at such a level of excellence indefinitely, it might conceivably flourish throughout the existence of the universe. In the *Timaeus*, Plato explains that the supreme beauty of the cosmos requires that it be constructed according to an eternal principle.[215] If the cosmos is supremely beautiful in accordance with the eternity of the model to which it adheres, then it stands to reason that a civilization which, through sustainable governance informed by precise calculation, remains eternally prosperous and wise, is therefore supremely beautiful through its likeness to the model of the cosmos. Thus, through study of the orbits of the World-Soul as mandated in the *Timaeus*,[216] the human intellect becomes better suited to the task of leadership, for not only does it thereby avail itself of the faculties of calculation necessary for the proper ordering of the state, but is also guided by the essential structures of relation in nature as defined intelligibly, and by the model of perfection toward which this ordering must be directed. The just city must therefore have its highest measure of excellence articulated in its foundation, just as the perfection of the entire cosmos is defined from its inception according to the schematic by which it is governed.

[215] Pl. *Ti*. 29a. εἰ μὲν δὴ καλός ἐστιν ὅδε ὁ κόσμος ὅ τε δημιουργὸς ἀγαθός, δῆλον ὡς πρὸς τὸ ἀίδιον ἔβλεπεν

[216] See n. 151

Chapter 5: Τέλος

5.1 Beginning at the End: The Mathematical Definition of Teleological Objectives within Intelligible Principles

Plato's description of the intelligible model of the universe indicates that this schematic represents the predefined completion and perfection of the entirety of existence. That is to say, the objectives towards which all entities strive may be correctly identified with the specifications according to which they originate. This predetermined completion may be inferred from Timaeus' explanation that "if the cosmos is beautiful and the Demiurge good, then he conceived of it according to an eternal image."[217] While the eternity of this image necessarily implies that it is imperishable, it also appears that the model cannot be subject to change in any respect; for, as Blackson states, the intelligible Forms that Socrates describes to Cebes in the *Phaedo* are unchangeable.[218] If the models belonging to this paradigm are entirely unchanging and eternal, then it is impossible for them to be brought to completion from an unfinished state, so they must be instantly and eternally complete and perfect.

The mathematical laws and formulae governing structure and motion within the sensible world must therefore be fully defined according to the specifications of a complete reality. The entire system must be ordered in such a way that just as the predetermined structure of the cosmos dictates the operation of mathematical principles, these same principles condition the structure and activity of the Forms governing sensible beings. As explained by Proclus, each Form is also contained within all others according to its relation to them.[219] Gonzalez (2002) does not identify this particular type of connection in his examination of Plato's dialogues, though he explains that "If the forms are universals, then their communion with each other would necessarily be radically

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[217] Pl. *Ti*. 29a2-3. εἰ μὲν δὴ καλός ἐστιν ὅδε ὁ κόσμος ὅ τε δημιουργὸς ἀγαθός, δῆλον ὡς πρὸς τὸ ἀίδιον ἔβλεπεν

[218]. Blackson. *Inquiry, Forms, and Substances: Studies in Plato's Metaphysics and Epistemology*. 133-34. Cit. Pl. *Phd*. 78e2, Ti. 27d5-28a4. ἀεὶ κατὰ ταύτὰ ὅν

[219] See n. 152.

unlike their communion with their exemplifications."[220] A type of connection similar to that which is described by Proclus seems, however, to follow from the principles of collection and division presented in the *Phaedrus* and the *Philebus*.[221] Collection and division are defined, respectively, as the grouping of various types of objects according to common essential properties, and the delineation of classifications of objects according to distinguishing traits. These processes reveal a variety of classifications among various types of beings, some of which may in fact be essential classifications, such as that of animals, which might, in turn, be divided into vertebrates and invertebrates, those with lungs and those with gills, and so on. Each type, it seems, must relate back to the classifications to which it essentially belongs, based on the characteristics which designate it as a member of that classification. Thus, just as each classification contains those types of objects that belong to that classification, so too does each type of object contain the definitions for those classifications to which it belongs. It would therefore, by extension, contain the definitions of all other types of objects belonging to the same classification. Based, furthermore, on the absence of sequential succession within the cosmic model according to (5), it follows that since there is no succession between larger classifications and the subtypes that comprise them, the relation between all classifications and the types contained within them is one of reciprocal definition. Additionally, based on (6), according to which the cosmic model is understood as a "framework of models,"[222] it seems that it would be absurd for these classifications and the relations binding them to be separate from the cosmic model.

It is therefore evident to us, according to both the plurality of paradigms at the level of the intelligible and the need for the ontologically immediate full operation of number, that the unit value must also be completely operational, such that it is able to account for all multiplicities and divisions of itself. This requirement, as Thesleff indicates, is addressed in the Eleatic dialogues, through Plato's treatment of the problem of plurality

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[220] Francisco J. Gonzalez. "Plato's Dialectic of Forms", in *Plato's Forms: Varieties of Interpretation*, ed. William A. Welton. (Lanham, Maryland: Lexington Books, 2002), 45.

[221] See n. 153, 182.

[222] See n. 9

within the singular entirety of the One.[223] The passages cited by Thesleff consider not only the problem of multiplicity within the One, but also the manner of relation that defines the connection between the Forms. In the earlier of these passages, Socrates draws attention to the problem of plurality and otherness within the unity of beings, as follows,

> He told us that Socrates, having listened, tried once more to understand the first hypothesis of the initial argument, and once he was certain of it, he said, 'How can you say such a thing, Zeno? If many things exist, then what you suggest would be impossible if many things existed, since some things would need to be identical to one another, while others would need to be different from one another; for things that are different from one another obviously cannot be identical, just as things that are identical cannot be different from one another. Have I understood you correctly?'
>
> 'You have,' said Zeno.
>
> 'If it is impossible for different things to be identical to each other and identical things to be different from each other, would it not then be impossible for many things to exist? For if there are many things, they would be unable to coexist. Is this what your arguments are meant to indicate, that, if many things did not exist, they would not be in conflict with each other? Considering the way in which you urge us to accept these proofs as you present them, you seem rather confident that they will produce the results that you expect, that is, that there cannot be a multitude of things. Is that what you are saying, or have I misunderstood you?'[224]

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[223] Thesleff. "Studies in Plato's Two-Level Model," 458. Cit. cf. Jatakari, 1990, for Thesleff's identification of young Socrates with Plato, Pl. *Prm*. 127d-e, 128e, 129a-b, e2, 130a1.

[224] Pl. *Prm*. 127d6-128a1. τὸν οὖν Σωκράτη ἀκούσαντα πάλιν τε κελεῦσαι τὴν πρώτην ὑπόθεσιν τοῦ πρώτου λόγου ἀναγνῶναι, καὶ ἀναγνωσθείσης, πῶς, φάναι, ὦ Ζήνων, τοῦτο λέγεις; εἰ πολλά ἐστι τὰ ὄντα, ὡς ἄρα δεῖ αὐτὰ ὅμοιά τε εἶναι καὶ ἀνόμοια, τοῦτο δὲ δὴ ἀδύνατον: οὔτε γὰρ τὰ ἀνόμοια ὅμοια οὔτε τὰ ὅμοια ἀνόμοια οἷόν τε εἶναι; οὐχ οὕτω λέγεις;

οὕτω, φάναι τὸν Ζήνωνα.

οὐκοῦν εἰ ἀδύνατον τά τε ἀνόμοια ὅμοια εἶναι καὶ τὰ ὅμοια ἀνόμοια, ἀδύνατον δὴ καὶ πολλὰ εἶναι; εἰ γὰρ πολλὰ εἴη, πάσχοι ἂν τὰ ἀδύνατα. ἆρα τοῦτό ἐστιν ὃ βούλονταί σου οἱ λόγοι, οὐκ ἄλλο τι ἢ διαμάχεσθαι παρὰ πάντα τὰ λεγόμενα ὡς οὐ πολλά ἐστι; καὶ τούτου αὐτοῦ οἴει σοι τεκμήριον εἶναι ἕκαστον τῶν λόγων, ὥστε καὶ ἡγῇ τοσαῦτα τεκμήρια παρέχεσθαι, ὅσουσπερ λόγους γέγραφας, ὡς οὐκ ἔστι πολλά; οὕτω λέγεις, ἢ ἐγὼ οὐκ ὀρθῶς καταμανθάνω;

The presence of all Forms within one another, as suggested by Proclus, and the classification-based connection system coinciding with collection and division, are both possible resolutions for the unity-multiplicity discrepancy encompassing the entirety of the intelligible. As is readily evident to us, full operability of mathematics within a unified structure of existence requires its compatibility with multiplicity; for absence of succession within the intelligible dictates that the unit principle must itself be capable of accounting for the unit value as the basis for all other values as well as all possible fractions of itself, and must also be able to account for inequality, lest all mathematical operations be impossible.

The inclusion of some principle of multiplicity within the intelligible is perhaps necessary for the operation of the fractions and products of the unit value in both a direct and indirect capacity; for, according to (7), which states that all knowledge belongs to a single structure,[225] then within that structure, there must be a law of some sort to account for the variety of operations that are bound together as part of such a system. The multiplicity of paradigms would thus be crucial not only for the diversity and multiplicity of sensible objects, but also for the plurality of laws and formulae pertaining to the activity of number. Any viable theory concerning the structure of the intelligible must therefore account for the plurality not only of the paradigms directly governing tangible entities, but also for the paradigms pertaining to relation, proportion, and other algorithmic principles. Thesleff[226] draws our attention to Socrates' response to the apparent problem in Zeno's argument, and in the cited passage, the solution proceeds as follows,

> 'But I accept it,' said Socrates, 'and I suppose that it holds as you say.' And then he said to me, 'surely you hold that the Form should be distinct from its likeness, and likewise, that the same should be the case for its opposite, which is different; and with two of these things, being two in number, do you agree that we hold me, you, and all other such things to share in the Many? Would you say also that those things possessing likeness become similar to that which they resemble according to that

[225] See n. 10.

[226] See n. 223.

particular thing, and unlike that from which they differ also according to that thing, such that either one becomes as such towards the other? And if all things of either type have opposites, and if it belongs to both of these things to partake of both similarities and differences, such a thing would be surprising? For if the same thing which proves to be similar to something else then differs from that thing, or if something different from it should become similar to it, I said that I would consider such a thing amazing; and if either of the things participating in one of these principles appeared to be affected, it would not seem at all strange to me, Zeno, nor would it seem strange if a particular one were therefore to participate altogether in the One, or if many things participate in the multitude. If that which is one proves to be many, and if the Many likewise proves to be one, I will also be surprised at that.'[227]

If certain objects are called similar or different from one another according to that to which they are compared, and are understood also to participate in similarity and in difference, then there must be an essential relation of some sort between all objects at the level of the intelligible; for if two specific entities differ from one another in a manner that is not accidental, then surely the comparison must be essential and intelligible, since any proper comparison must be made according to paradigm as opposed to a sensible, particular object. This *essential* comparison is undoubtedly necessary for the proper operation of mathematical principles, for it would otherwise be impossible, both within the Good and within human apprehension of the Good, to account for the equalities and inequalities belonging to the foundation of mathematical reasoning; for without some principle for determining identity and distinction, it is impossible to formulate any definitive mathematical statement. Within this passage, Socrates does not allude to any

[227] Pl. *Prm*. 128e5-129c1. Passage interpreted with assistance of Dorter, 1994. ἀλλ᾽ ἀποδέχομαι, φάναι τὸν Σωκράτη, καὶ ἡγοῦμαι ὡς λέγεις ἔχειν. τόδε δέ μοι εἰπέ: οὐ νομίζεις εἶναι αὐτὸ καθ᾽ αὑτὸ εἶδός τι ὁμοιότητος, καὶ τῷ τοιούτῳ αὖ ἄλλο τι ἐναντίον, ὃ ἔστιν ἀνόμοιον: τούτοιν δὲ δυοῖν ὄντοιν καὶ ἐμὲ καὶ σὲ καὶ τἆλλα ἃ δὴ πολλὰ καλοῦμεν μεταλαμβάνειν; καὶ τὰ μὲν τῆς ὁμοιότητος μεταλαμβάνοντα ὅμοια γίγνεσθαι ταύτῃ τε καὶ κατὰ τοσοῦτον ὅσον ἂν μεταλαμβάνῃ, τὰ δὲ τῆς ἀνομοιότητος ἀνόμοια, τὰ δὲ ἀμφοτέρων ἀμφότερα; εἰ δὲ καὶ πάντα ἐναντίων ὄντων ἀμφοτέρων μεταλαμβάνει, καὶ ἔστι τῷ μετέχειν ἀμφοῖν ὅμοιά τε καὶ ἀνόμοια αὐτὰ αὑτοῖς, τί θαυμαστόν; εἰ μὲν γὰρ αὐτὰ τὰ ὅμοιά τις ἀπέφαινεν ἀνόμοια γιγνόμενα ἢ τὰ ἀνόμοια ὅμοια, τέρας ἂν οἶμαι ἦν: εἰ δὲ τὰ τούτων μετέχοντα ἀμφοτέρων ἀμφότερα ἀποφαίνει πεπονθότα, οὐδὲν ἔμοιγε, ὦ Ζήνων, ἄτοπον δοκεῖ, οὐδέ γε εἰ ἓν ἅπαντα ἀποφαίνει τις τῷ μετέχειν τοῦ ἑνὸς καὶ ταὐτὰ ταῦτα πολλὰ τῷ πλήθους αὖ μετέχειν. ἀλλ᾽ εἰ ὃ ἔστιν ἕν, αὐτὸ τοῦτο πολλὰ ἀποδείξει καὶ αὖ τὰ πολλὰ δὴ ἕν, τοῦτο ἤδη θαυμάσομαι

system of interconnection analogous to that which is presented by Proclus, although, as stated before, we may observe a similar sort of structure based on the manner in which processes of collection and division reveal logical connections between paradigms. Within this structure, even paradigms that are not contained directly within one another might be contained indirectly accorded to a nested hierarchy. Alternatively, they may be bound by some other type of rational connection, such as the overlap of common essential properties across a variety of disparate classifications, by belonging to the same type of ecosystem as defined by their essential position in the natural world, or even by necessarily similar use of a common mathematical principle.

The fact that Socrates does not initially account for such a system may be a function of the relatively rudimentary level of philosophical development by which he is characterized in the *Parmenides.* As explained by Dorter (1994), Parmenides suggests that due to Socrates' youth, his philosophical wisdom has not yet reached its greatest extent (Pl. *Prm*. 131e.); for Socrates is unable to explain the reality of the multiplicity of entities, since these entities must, by consequence, be simultaneously similar and dissimilar. Socrates, as Dorter notes, finds similarity and dissimilarity impossible to reconcile, and thus overlooks the fact that two objects that are governed by the same Form must be simultaneously similar and dissimilar simply by means of their common classification in conjunction with their individual distinction.[228] Socrates is also in error in his apparent identification of the One and the Many with two different intelligible principles; for such distinction dictates that the One must already allow for multiplicity in order to contain multiple intelligible definitions, in which case a definition pertaining specifically to multiplicity would be superfluous. That is to say, if the One and the Many were distinct and disparate principles, the existence of the Many would necessitate that the One also function as a principle of multiplicity in order to allow for a principle other than itself; yet if the One also accounts for multiplicity, the existence of the Many as a distinct principle is unnecessary.

[228] Kenneth Dorter. *Form and Good in Plato's Eleatic Dialogues: The* Parmenides*,* Sophist*,* Theaetetus*, and* Statesman. (Berkeley, Los Angeles, and London: University of California Press, 1994), 23, 24. Op. Cit. Pl. *Prm*. 127e, 129c, 131e, R. E. Allen, *Plato's Parmenides* (Oxford: Blackwell, 1983), 78.

The argument that Socrates rejects in this instance is likely the more accurate, as it suggests that the One and the Many are in fact the same principle. That is to say, the singular structure constituting all of reality also accounts for multiplicity, both as the mathematical unit principle, and as the structure in which all paradigms reside. As we have previously observed, the model according to which the Demiurge constructs the cosmos in the *Timaeus* is understood to be unchanging,[229] thus dictating that all principles belonging to it must belong to it immediately and eternally, such that there is no need or possibility for anything to be added to it. The unit value, as we have already established, would therefore be fully operative mathematically, such that all possible divisions and multiplicities of the unit would be intelligibly realized from the outset. Plato addresses this property at a slightly later point in the *Parmenides*, suggesting, as follows, that the unity of being implies the belonging of the arithmetical concepts of 2 and 3 to the same singular structure,

> 'And therefore if both the One and the Other exist, should I then say that they both belong to one another?'
>
> 'Yes.'
>
> 'And if they have been described correctly, will they both be the same in that regard, and not two?'
>
> 'Both cannot be true.'
>
> 'And if they are two, is there some device by which each of these is not itself one?'
>
> 'There is no such device.'
>
> 'In that case, each of these things combined together are a pair, and each of them is one.
>
> 'So it is said.'
>
> 'And if each of these things is one, does combining one with these things not produce three altogether?'
>
> 'Yes.'
>
> 'Would three not be odd and two even?'
>
> 'How could it be otherwise?'

[229] See n. 8.

> ‘And at any rate, is it not necessary for two to be a pair of things, and for three to be a triple of things, if it happens to be the case that two is the double of one and three is the triple of one?’[230]

This passage might be interpreted as an indication that numerical values beyond the unit value do not require that there exist a distinct intelligible principle pertaining to each specific value beyond the unit value. It suggests that each such value is in fact derived directly from the unit value, and thus the participation of specific instances of values higher than 1 in Forms that are specific to the value in question, appears to be unnecessary; for if each numerical value greater than 1 is derived from the unit value, then the unit value is the intelligible principle for all numerical values, such that any principle pertaining to a specific value other than 1 would be redundant. At any rate, in order for the unit value to be fully, immediately capable of mathematical operations, it must be infinitely divisible, and in order to be infinitely divisible, it must have multiplicity as a proper attribute in order for 1 as an integer value to be divided into fractions.

The full articulation of the One with respect to its mathematical operation, due to its immediate intelligible completeness, must also be considered in the context of the

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[230] Pl. *Prm*. 143c8-e2. καὶ ἐὰν οὐσία τε καὶ ἕτερον ἢ ἕτερόν τε καὶ ἕν, καὶ οὕτω πανταχῶς ἐφ᾽ ἑκάστου ἄμφω λέγω;

ναί.

ὧ δ᾽ ἂν ἄμφω ὀρθῶς προσαγορεύησθον, ἆρα οἷόν τε ἄμφω μὲν αὐτὼ εἶναι, δύο δὲ μή;

οὐχ οἷόν τε. ὧ δ᾽ ἂν δύο ἦτον, ἔστι τις μηχανὴ μὴ οὐχ ἑκάτερον αὐτοῖν ἓν εἶναι; οὐδεμία. τούτων ἄρα ἐπείπερ σύνδυο ἕκαστα συμβαίνει εἶναι, καὶ ἓν ἂν εἴη ἕκαστον.

φαίνεται.

εἰ δὲ ἓν ἕκαστον αὐτῶν ἐστι, συντεθέντος ἑνὸς ὁποιουοῦν ἡτινιοῦν συζυγίᾳ οὐ τρία γίγνεται τὰ πάντα;

ναί.

τρία δὲ οὐ περιττὰ καὶ δύο ἄρτια;

πῶς δ᾽ οὔ;

τί δέ; δυοῖν ὄντοιν οὐκ ἀνάγκη εἶναι καὶ δίς, καὶ τριῶν ὄντων τρίς, εἴπερ ὑπάρχει τῷ τε δύο τὸ δὶς ἓν καὶ τῷ τρία τὸ τρὶς ἕν;

paradox that is ascribed to the One in the *Parmenides*. This paradox is described by Dorter in his account of the first hypothesis that follows from the assumption that there is a One such that this One constitutes the entirety of existence. This hypothesis states that since the One cannot be comprised of parts, and cannot be many, it must be ἄπειρον, or indefinite. According to this hypothesis, however, it is also impossible for the One to be subject to definition, for it cannot even be the same as itself due to the fact that such a designation, as Dorter observes, would imply multiplicity.[231] In *Parmenides* 139b-c, Plato specifies that even if the One is considered to be the same as itself, such would imply that it must be different from something else, which necessitates multiplicity.[232] If the One is treated in the capacity of the single structure of all mathematical principles, then it must, at least in a certain sense, be regarded as indefinite; for it is the intelligible definition not only of the unit, but of all multiplicity, from the infinitely minute to the infinitely vast. The definition of multiplicity accounts not only for the distinction within myriad operations, rules, and formulae contained within the grand structure, but also for the correct functionality of these operations. The paradigms governing all types of entities within the cosmos must also belong to the One, and just as these paradigms are subject to mathematical rules and functions, they also constitute the objectives towards which these rules and functions are formulated.

Thus, since the principles simultaneously define one another, it would not be accurate to say that they follow a linear pattern of succession; rather, the pattern to which they adhere might, as we have noted before, be better described as a closed network of paradigms wherein each is essentially connected to all others, such that the entirety of all is within each; or, at the very least, all paradigms are bound together directly or indirectly, with some being contained within one another, while others are bound together by a variety of other types of rational connections. To be sure, this structure may correspond more closely to the model that coincides with collection and division as discussed in the *Philebus* and the *Phaedrus*. Proclus, moreover, explains in his commentary on the *Parmenides* that being must simultaneously be one and many, and

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[231] Dorter. *Form and Good in Plato's Eleatic Dialogues: The* Parmenides*,* Sophist*,* Theaetetus*, and* Statesman. 54-5. Cit. Pl. *Prm*. 137c-d, 139b-e, 141e.

[232] Pl. *Prm*. 139b5-c2.

that "every monad has a plurality correlative with it, and every plurality is comprehended under some appropriate monad."[233] Proclus, it would appear, is indicating that every monad (i.e. every instance of a single discrete object), at least in respect to its mathematical properties, has the essential characteristic of being multiplied into an infinite number of pluralities of itself, and that every multiplicity, whether it is a multiplicity of specific objects or an abstract arithmetic multiplicity, represents a plurality of the type of monad in question. In stating that "every plurality is comprehended under some appropriate monad," Proclus' intended interpretation may be twofold; for in addition to explaining that each multiplicity is defined by a specific monad of the type in question, he might also be indicating that every specific plurality is governed by the universal paradigm of the monad, which, it would seem, represents one aspect of the activity of the ontological One, that is to say, the One in the capacity of the totality of existence, as opposed to the mathematical unit principle.

The ontological One must thus be distinguished from a mere mathematical monad, of which infinitely large multiplicities are possible, and which is divisible into infinitely miniscule fractions of itself. The ontological One, in contrast, cannot be understood in a purely mathematical sense, and therefore cannot be multiplied into a plurality of instances in the same sense as a purely mathematical monad, though it may be considered to be a multiplicity in a certain qualified sense; for Proclus states in his commentary on the *Alcibiades*, to which Dillon and Morrow make reference, that "As we have said elsewhere about the dialogues, each one must possess what the whole cosmos possesses…"[234] This plurality of the One within itself is a possible resolution for the problem of the first hypothesis, according to which the One is indefinite, and therefore indivisible and unable to admit of multiplicity within itself. The plurality of the One must be within the One, as the multiplicity of the One outside of itself would be untenable; for in such a case it would still be necessary for all such realities to belong to a single

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[233] Procl. *In Prm*. 620. In *Proclus*' *Commentary on Plato's* Parmenides. ed. and trans. John M. Dillon and Glenn R. Morrow. (Princeton, New Jersey: Princeton University Press, 1987), 21.

[234] Procl. *In Alc*.10.3ff. In *Proclus' Commentary on Plato's* Parmenides. ed. and trans. John M. Dillon and Glenn. (Princeton, New Jersey: Princeton University Press, 1987), 3. Cf. Procl. In Ti. 430.30-431.5. In Proclus, Commentary on Plato's Timaeus Volume II. Ed. and Trans. David T. Runia and Michael Share. (Cambridge: Cambridge University Press, 2008), 317.

structure of being, and the multiplicity of all such realties would necessarily account for the intelligible principle of plurality. In this scenario, there would be, at best, no functional difference between the existence of multiple realities and that of a single reality alone. The multitude of realities would thus represent a system containing unnecessary redundancies, which would add nothing to the solution to the question of the manner in which the One might account for multiplicity. The multiplicity of the ontological One must therefore be a plurality of instances of the One within itself.

The structure of this intelligible first plurality also accounts for the indivisibility of the ontological One; for since each Form must be related, either directly or indirectly, and either through containment or some other type of rational connection, to all others, each tangible entity therefore also participates, however imperfectly, in the entirety of the One. For the One to be divisible in the unequivocal sense would also imply that the One is composite, since as Sayre (1983) indicates, the characterization of a certain object according to a whole or parts implies composition, a quality which is contrary to the accurate articulation of the Forms.[235] If the One is understood to be composite, then it follows that each of the individual Forms must exist prior to their integration into a single cosmic model. Such a result would be absurd; for if each of the Forms is already connected in some respect to all others, as Proclus argues, and as the explanations of collection and division suggest, then the singular structure of the intelligible is already predefined within its parts, such that the essential connections by which the Forms are inextricably bound together are already in place. In this scenario, if the One is stated to be composed, then it is impossible to explain how it might be considered as such, since the Forms are already complete in their connections to one another.

Thus, if the One is understood to be composed, it must be assumed that prior to the integration of the Forms into the One, they would not contain within them the singular structure of the intelligible. We may thus rule out the possibility of the composite one; for both the assembly of the Forms into a composite One, and the addition of the totality of the intelligible to the Forms are contrary to the definition of the

[235] Kenneth M. Sayre. *Plato's Late Ontology: A Riddle Resolved*. (Princeton, New Jersey: Princeton University Press, 1983), 23. Cit. Pl. *Prm*. 131a5, 137c7-d1, *Sph*. 245a1-2, *Tht*. 205a4-5, *Phd*. 78c6, *R.* V. 476a6-9.

Forms, for they violate the specification in the *Timaeus* that the intelligible must be "always in the same way."[236] Thus, to state that each sensible entity participates in a specific Form does not imply that it participates in a particular part of the One, but rather that it participates, albeit imperfectly, in the One altogether. Such adherence occurs in several senses simultaneously; for each specific tangible being is not only numerically one, but must also be in accord with the entirety of the One through the essential connections of the paradigm to which it adheres.

The paradigm that governs the entity in question must, in turn, obey the mathematical rules that dictate the patterns of the structure and activity proper to it, even as it also partakes in the activity of defining them; for if the model of the cosmos is to be entirely unchanging, as Plato has indicated to be the case, then it must be immediately complete, such that the paradigms that constitute the objectives of mathematical principles must dictate the parameters of these laws and formulae even as they depend upon these same rules for their own essential characteristics. Through these aspects of their adherence to the intelligible model of the cosmos, sensible beings, though they cannot be called fully identical to the One, nonetheless belong to it and may be likened to it. It is in this respect that, as Dorter explains, "The One must be both other than other (by definition) and the same as the others…"[237] It stands to reason that only by their simultaneous difference and sameness with respect to the One are specific entities capable of being properly distinct from one another within the One. It is therefore in this respect also that mathematical operations are possible, for the One must account for numerical equality and inequality. The structure of the One is therefore such that the same principle that allows for distinction amongst the paradigms belonging to the intelligible is also that which is the foundation of the laws and formulae governing the specifications of shape and movement defined within these paradigms; and though its definition is not solely confined to that of the mathematical unit, it represents the first instance of any object to be numerically one. This first instance of the unit value simultaneously encompasses the entire multiplicity of all entities, and thus the Many

[236] See n. 8.

[237] Dorter. *Form and Good in Plato's Eleatic Dialogues: The* Parmenides*,* Sophist*,* Theaetetus*, and* Statesman. 56. Cit. Pl. *Prm*. 145e-147b

belongs essentially to the One, and those things counted among the Many, in that they are different from the One while being within it, must strive for the One as the pattern from whence they originate. Since it is therefore both the point of origin and the final point of convergence, the One is therefore, as Dorter interprets from the first and second hypotheses, both a limit as well as unlimited.[238] It is a limit insofar as it represents the highest extent of excellence for all of existence, and is the source of the parameters of order within the cosmos; yet it is simultaneously unlimited inasmuch as it is the foundation of infinite mathematical multiplicity and magnitude.

5.2 The Undying Draftsman: Mathematical Truth in the Intellect of the Demiurge, and Its Relation to Knowledge in the Human Mind

The plurality that abides within the One is defined not only in the multiplicity of paradigms within the intelligible, but also in the instantiation of the eternal model of the cosmos on the level of the tangible. Plato explains in the *Timaeus* that it belongs to the Demiurge to desire other things to be like himself, based on the fact that he is good, and therefore cannot be jealous.[239] Since the process by which the Demiurge makes other things to be like himself is that by which he constructs the sensible cosmos according to its intelligible model, it must follow that this model not only belongs properly to the Demiurge, but also that the model *is* the Demiurge. In the *Timaeus*, one of the two types of causes described by Plato, is that by which entities are "made distinct out of thought, and each time, bring random chance to completion."[240] Since the process by which the Demiurge constructs the sensible cosmos is stated to involve the proportional ordering of

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[238] Dorter. *Form and Good in Plato's Eleatic Dialogues: The Parmenides, Sophist, Theaetetus, and Statesman*. 60. Cit. Pl. *Phlb*. 23c-25b, *Sph*. 246a-c. Dorter states that the understanding of the One as a limit relates to its definition as "beyond divisibility." This explanation might imply that, despite being the origin to the unit principle that constitutes the foundation of mathematical operations, or perhaps because of its significance in this capacity, the One cannot be subject to the mathematical principles belonging to it in the same manner as a tangible, spatially limited object; for within the intelligible, the laws of mathematics and the paradigms governing the sensible are dependent upon one another in a reciprocal manner, as opposed to the somewhat passive subservience of the tangible to these laws. Dorter notes that this differentiation is an anticipation of the discourse concerning the distinction between the limit and the unlimited in *Philebus* 23c-25b and *Sophist* 246a-c.

[239] Pl. *Ti*. 29e1-3. ἀγαθὸς ἦν, ἀγαθῷ δὲ οὐδεὶς περὶ οὐδενὸς οὐδέποτε ἐγγίγνεται φθόνος: τούτου δ᾽ ἐκτὸς ὢν πάντα ὅτι μάλιστα ἐβουλήθη γενέσθαι παραπλήσια ἑαυτῷ

[240] Pl. *Ti*. 46e5-6. ὅσαι μονωθεῖσαι φρονήσεως τὸ τυχὸν ἄτακτον ἑκάστοτε ἐξεργάζονται

the elements,[241] it must follow that the type of construction is that of primary ordering out of indistinct chaos, as opposed to the other type of creation, which comes not only from intellect but also from "beautiful and good things."[242] For the Demiurge to construct the cosmos in a derived manner such as this rather than that of the primary ordering out of chaos would suggest that the intelligible model of the cosmos is outside of the Demiurge. Such a manner of construction indicates that the thing which is created is made in imitation of a model which is outside of the craftsman; yet since the construction of the sensible cosmos according to the model described in the *Timaeus* is characterized by the Demiurge's will to make his work to be like himself, it must follow that the intelligible model is indeed the Demiurge. The separation of the Demiurge from the cosmic model would, moreover, contravene the requirement stated in the *Timaeus* according to which the cosmic model is not subject to potency;[243] for if the cosmic model is separate from that which communicates it to matter in the construction of the sensible cosmos, then it requires an external agent to carry out this task, and is thus subject to potency; yet the cosmic model cannot be subject to potency, and thus it cannot be separate from the Demiurge.

We may therefore easily infer that the model containing the mathematical foundations of the cosmos must also actively perform calculations according to these formulae, and given that the model wills all things to be like itself, it must share with the sensible world the laws and formulae contained within it, and order them in such a way as to allow them to function in a continuously active manner as the patterns of structure and motion within reality. Livio (2009) appears to concur with this interpretation of the Demiurge as described by Plato, as he states that "…for him, the mathematical character of the world is simply a consequence of the fact that "God always geometrizes.""[244] The type of geometric activity in which the Demiurge is constantly engaged certainly cannot

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[241] See n. 88, 91, 108.

[242] Pl. *Ti*. 46e4. μετὰ νοῦ καλῶν καὶ ἀγαθῶν

[243] Pl. *Ti*. 30a3. μηδὲν εἶναι κατὰ δύναμιν

[244] Mario Livio. *Is God a Mathematician?* (New York: Simon & Schuster, 2009), 36.

be of the same sort of which Socrates finds mortal geometers to be guilty;[245] for the Demiurge's apprehension of geometry pertains directly to the objects of knowledge, as opposed to the method attributed to mortal geometers, according to which contrived diagrams are mistakenly regarded as the sources of mathematical knowledge. Not only does the Demiurge's geometric apprehension therefore attain the Good unfailingly, but it also does so without the need for any type of sensible intermediary. It differs also from the manner in which the human intellect grasps at the intelligible; for the apprehension of mathematical principles on the part of the Demiurge begins and reaches its completion at the level of νόησις.

The human intellect, though it always contains the image of the intelligible, does not immediately possess it in a perfect manner, and so the mind must restore its apprehension of the objects of knowledge to a completed state. Socrates, as we have previously observed, demonstrates in the *Meno* that this restoration occurs within the human mind by means of recollection, whereby through inquiry, that which is already within us is made accessible to our faculty of articulation.[246] We have noted also that the process of this recollection seems to imply the re-assembly of the structure of the cosmos within the human intellect, since Plato makes reference in the *Timaeus* to the "wandering patterns," or πεπλανημέναι, within the soul, stating that we must stabilize these patterns in order for them to attain greater likeness to those of the Demiurge.[247] The special significance of mathematical knowledge is, furthermore, indicated by Plato's specification that the stabilization of the πεπλανημέναι occurs through the development of skill in the activity of calculation.[248] It may therefore be said that mathematical knowledge fulfills a crucial constructive function in the ordering of the rational framework of the objects of knowledge, both at the level of the intelligible and as apprehended by mortal faculties.

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[245] See n. 165.

[246] See n. 157

[247] Pl. *Ti*. 47c2-3. μιμούμενοι τὰς τοῦ θεοῦ πάντως ἀπλανεῖς οὔσας

[248] See n. 151.

Plato lends further credence to this postulation in the *Republic* by indicating that those who excel naturally in mathematics display similar aptitude in all areas of scholarly inquiry, while even those whose aptitude for mathematics is initially weak become more adept in other disciplines by honing their mathematical ability.[249] Taken in conjunction with Plato's explanation of the role of calculation, or λογισμός, in the reconfiguration of the orbits of the human ψυχή, it seems to follow that the role of mathematics in the attainment of knowledge is that of refining and ordering the image of the cosmos within the human intellect. The identification of the human ψυχή is demonstrated in Timaeus' account of the construction of the mortal soul by the lesser gods begotten by the Demiurge, in which he states,

> …emulating their own creator, they borrowed as many portions of fire, earth, water, and air from the cosmos as would be given again, and assembling those that had been taken into a single structure, they did not bind the loose pieces together as though with fetters, but rather they fused them into one structure by the smallest pieces using invisible bolts, completing all of them each one by one, they bound the orbits of the immortal soul into an ebbing and flowing body.[250]

There are therefore several possible explanations for the process of re-aligning the orbits of the soul through mathematical inquiry. We may suggest, as has been previously stated, that the attainment of excellence in the activity of calculation enables us to perfect the image of the cosmos within our minds in accordance with that of the Demiurge, an interpretation which, as we shall soon observe, may be, to some extent, compatible with the other explanations.

We might, for instance, posit that the alignment of the orbits of the soul implies the departure from false belief and the stabilization of correct belief toward true knowledge. As Fine (2003) explains, the superiority of knowledge to true belief, according to Plato, is a function of its connection to an *aitias logismos*, interpreted by

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[249] See n. 175.

[250] Pl. *Ti*. 42e8-43a6. μιμούμενοι τὸν σφέτερον δημιουργόν, πυρὸς καὶ γῆς ὕδατός τε καὶ ἀέρος ἀπὸ τοῦ κόσμου δανειζόμενοι μόρια ὡς ἀποδοθησόμενα πάλιν, εἰς ταὐτὸν τὰ λαμβανόμενα συνεκόλλων, οὐ τοῖς ἀλύτοις οἷς αὐτοὶ συνείχοντο δεσμοῖς, ἀλλὰ διὰ σμικρότητα ἀοράτοις πυκνοῖς γόμφοις συντήκοντες, ἓν ἐξ ἁπάντων ἀπεργαζόμενοι σῶμα ἕκαστον, τὰς τῆς ἀθανάτου ψυχῆς περιόδους ἐνέδουν εἰς ἐπίρρυτον σῶμα καὶ ἀπόρρυτον

Fine to signify an "explanatory account," which, as Plato indicates, is constructed by means of recollection.[251] True belief might then be described as accidental, such that, if it is identified with the orbits of the soul, it might be likened to a motion whose speed and direction are only correct by chance, and might therefore be diverted from its appropriate path by some external influence which misleads the mind toward false belief. The movement associated with true knowledge, however, would imply a correct pattern of movement that is formulaically defined in such a way that it is not prone to being distorted in its trajectory by the influence of misconceptions.

As with the previous theory, mathematical knowledge functions in this case as the rational foundation by which truth is possessed in a non-accidental capacity. Similarly to the previously stated postulation, λογισμός is instrumental in furnishing the foundations of the essential rational connections between the images of the Forms belonging to the human intellect; or rather it illuminates those connections that are already present, but have hitherto been unavailable to the faculties of articulation. We must also note that in the passage cited by Fine, *Meno* 98a, the turn of phrase used by Socrates does not explicitly indicate an explanatory account as Fine suggests; for Socrates rather states that "something must bind the causes themselves to reasoning."[252] This interpretation nevertheless largely agrees with Fine's explanation, providing the same argument

[251] Gail Fine. "Inquiry in the *Meno*", in *Plato on Knowledge and Forms: Selected Essays*. ed. Gail Fine. (Oxford: Clarendon Press, 2003), 58. At citation 19 (p 50) of this article, Fine explains that regardless of whether *Meno* 98a is interpreted to indicate that knowledge constitutes "justified true belief," it is certain that "Plato offers only one definition for knowledge." Beginning in the previous passage, Socrates tells Meno, "For true beliefs, which ought to remain for some time, accomplish beauty and all necessary and good things; yet they do not wish to remain for a great deal of time, but rather they flee from the human soul, and so they are not of much value, until something should bind the causes themselves to reasoning." (Pl. *Men*. 97e-98a.) According to this passage, the interpretation of Plato's definition of knowledge as "justified true belief" appears to be largely correct, though perhaps not sufficient. It might rather be said that such true belief is not simply justified, but also bound to the rational structure of the intellect, such that we do not merely accept them as truth on the basis of trust, but rather are able to explain the scientific causes behind them through independent inference. They are not only reliably anchored against influences that might otherwise push them astray, but are also capable of guiding our inquiry along further paths toward apprehension of reality. In citation 26 (p. 55), Fine also considers Plato's identification of the construction of the explanatory account with recollection (Pl. *Men*. 98a), and specifies that some explanatory accounts, such as that by which one might know the way to Larissa, need not necessarily occur through recollection. This observation is significant for the distinction between essential scientific knowledge and awareness of those propositions whose truth or falsehood is relative to opinion or convention.

[252] Pl. *Men*. 98a3-4. ἕως ἄν τις αὐτὰς δήσῃ αἰτίας λογισμῷ

regarding the differentiation of knowledge from true belief. It serves to demonstrate, moreover, the significance of λογισμός as a connective and constructive activity that is instrumental in binding scientific laws and paradigms together within the human intellect. It is also worthy of our consideration to note that λογισμός carries the connotations both of reasoning and of calculation.[253] It may be used in a seemingly non-mathematical capacity to imply the activity of reasoning in a more general sense; yet the previously stated significance of calculation as it pertains to the analysis of the motions of the heavenly spheres, and, in turn to the stabilising of the orbits of ψυχή may point to a simultaneous dual meaning of λογισμός in the contexts of the *Meno* and the *Timaeus*, such that λογισμός as it is discussed in Plato's thought identifies the activity of reasoning as a type of mathematical calculation.

Regardless of whether Plato understands all inference as an unequivocally mathematical activity, his identification of mathematics with the foundation of the activity of reason is largely unmistakable, considering both the importance of calculation in the ordering of the orbits of the human soul, and the role of mathematical reasoning in the attainment of knowledge in all scholarly disciplines.[254] It seems also to be the case that mathematical knowledge does not necessarily, in itself, propel the soul toward the Good, for Plato, as we have noted before, takes issue with the tendency of the mathematicians of his time, to treat abstract geometric structures as the objects of thought.[255] Shapiro (1997) elaborates on the implications of this error, explaining that later geometers, including Euclid, use language that suggests construction and motion in discussing abstract geometric objects, and thereby make the mistake of attributing these characteristics to those things which, according to Plato, ought to be regarded as "eternal and unchanging."[256] A further problem with the approach against which Plato argues, is that those things being treated as the objects of science are human contrivances as

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[253] Liddell and Scott. *Greek-English Lexicon with a Revised Supplement*..1056.

[254] See n. 175.

[255] See n. 165.

[256] Stewart Shapiro. *Philosophy of Mathematics: Structure and Ontology*. (New York: Oxford University Press, 1994), 21. Cit Pl. *R*. VII.

opposed to models belonging to the intelligible. To be sure, they are constructs of the human mind derived from the principles belonging to paradigms. The accurate construction of these contrived models would, furthermore, require correct apprehension of the laws of geometry as defined within the intelligible, and as such they prove to be unreliable; for since their accuracy is contingent upon the correct apprehension of geometric paradigms on the part of the geometer, one whose comprehension of geometry at the level of the intelligible is poor will be inclined to produce flawed diagrams; and since these diagrams are flawed, the inferences made from them will be similarly erroneous; and moreover, any student of geometry, regardless of innate talent, might be led to incorrect conclusions as a result of false premises if there are errors in the diagrams produced by their instructor.

Patterson (2007) states that according to Plato's position, the representation of intelligible principles by sensible means is admissible, and perhaps even necessary, for "the discovery and proof of geometrically interesting theorems…" He clarifies, however, that the geometer must account for the ontological and epistemological context of the theory being represented diagrammatically.[257] That is to say, the geometer must acknowledge, and, where necessary, specify to others, that the visual model is merely an articulation of a principle which cannot itself be perceived by means of sensory faculties. The geometer must also take into account the relation of the logical structure of the proof in question to the essential characteristics of the relevant principles, along with their context within the intelligible as it is considered altogether. It is therefore quite evident to us that although mathematical knowledge is necessary for the ascension of the soul towards the Good, it is nevertheless not sufficient; for rather than treating mathematical logic in a purely abstract manner, a philosophical mathematician must examine it in a dialectical manner, so as to consider the orbits of the World-Soul as specified in the *Timaeus.*

The approach of the philosophical mathematician is differentiated from the non-philosophical study of mathematics by several characteristics. Firstly and perhaps most importantly, the philosophical mathematician always recognizes that the stepwise

[257] Richard Patterson. "Diagrams, Dialectic, and Mathematical Foundations in Plato", *Apeiron: A Journal for Ancient Philosophy and Science* 40, no. 1 (March 2007), 2.

formulation of inferences is merely the attainment of eternal, ontologically simultaneous truths. Philosophical mathematics also does not assume diagrammatic representations of geometric principles to be true sources of knowledge, but rather recognizes them as derivations of human apprehension of mathematical laws. Accordingly, the philosophical mathematician does not place undue faith in diagrammatic representations, but rather seeks to verify the accuracy of the reasoning on which the diagrams are based, and considers, more generally, the activity of our mathematical reasoning in the construction of diagrammatic images. Philosophical mathematics also cannot restrict itself in the scope of its inquiry; that is to say, upon the confirmation or refutation of a particular theorem, it must then consider the implications of that conclusion within a larger scientific context, considering what other truths must follow from what has been determined, not only at a purely mathematical level, but also, where applicable, in terms of their significance for physical reality. As we shall soon observe, the full significance of mathematical inquiry in the movement of the soul towards the Good is dependent upon the structural relation of soul to intellect according to the model presented by Plato, and therefore upon the exact explanation of all that is implied in the alignment of the orbits of the human soul in such a way as to attain likeness to those of the Demiurge.

Within this model, Plato does not appear to equate intellect precisely with ψυχή, but rather suggests that the former belongs to the latter. The account presented in the *Timaeus* states that the Demiurge imbued the corporeal structure of the cosmos with ψυχή "by means of calculation and intellect in soul."[258] This passage appears to draw not only a distinction between νοῦς and ψυχή, but between νοῦς and λογισμός. It particularly suggests that νοῦς and λογισμός are contained within ψυχή. This interpretation appears to agree with that which is presented by Mohr (1982), who aims to demonstrate that the Demiurge, as pure reason, does not constitute ψυχή, but rather νοῦς independent of ψυχή. Mohr contends that, contrary to the argument presented by Cherniss, νοῦς need only be within ψυχή in those instances in which it is contained within an entity that does not, in itself, constitute pure reason.[259] According to Mohr's argument, it would seem that ψυχή

[258] Pl. *Ti*. 30b4. διὰ δὴ τὸν λογισμὸν τόνδε νοῦν μὲν ἐν ψυχῇ

[259] Richard D. Mohr. "The Relation of Reason to Soul", in *The Platonic Cosmology*. ed. Richard D. Mohr. Philosophia Antiqua XLII. (Leiden: E. J. Brill, 1985), 179-180, 181. Cit. H. F. Cherniss. *Aristotle's*

is to be understood as the means by which νοῦς bound within a corporeal vessel is able to communicate reason to that which contains it. This function is confirmed in *Timaeus* 43a-b, in which Plato states that the soul "carried life and was carried by it," and that it "moved the whole of the living thing."[260] Plato also indicates that ψυχή contains the emotional and possibly appetitive characteristics of the human consciousness, for he states that in constructing the human soul, the lesser gods created by the Demiurge added these characteristics in the following manner,

> "…secondly mixed in pleasure, pain, and love, and following these they also added fear and courage, and added others besides, with as many opposites; and if mortals are strong, they will live according to justice and prevail over injustice."[261]

Based upon this passage, we may observe, firstly, that action in accordance with justice requires dominion of the rational aspect of the soul over its more chaotic elements, and secondly, that the various characteristics of emotion contained within the soul, as pairs of opposites, constitute extremes between which the intellect must guide the soul toward a just balance.

The second observation agrees with an element of mathematical precision in the apprehension of justice, for each set of opposites might be seen as a range between which the intellect must calculate, in a certain sense, the noblest mean value between the two extremes. The concept of acting in a just manner by way of control, and thereby prevailing over injustice, is compatible with the explanation of the just soul as set forth in the *Phaedo* as well as the *Republic*. In the former of these texts, O'Connell (1997) notes a dualistic relation between the human soul and the corporeal vessel in which it is contained. This relation, as O'Connell states, is one of "ethical antagonism," within which the influences of the body and the soul are at cross-purposes with one another, for while the soul strives for wisdom, the body diverts the consciousness with appetitive

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Criticism of Plato and the Academy I. Baltimore: John Hopkins Press, 1944. Cit. Pl. *Sph*. 248e6-249d4, after Fowler.

[260] Pl. *Ti* 43a7-b1. ὥστε τὸ μὲν ὅλον κινεῖσθαι ζῷον

[261] Pl. *Ti*. 42a6-b2. δεύτερον δὲ ἡδονῇ καὶ λύπῃ μεμειγμένον ἔρωτα, πρὸς δὲ τούτοις φόβον καὶ θυμὸν ὅσα τε ἑπόμενα αὐτοῖς καὶ ὁπόσα ἐναντίως πέφυκε διεστηκότα: ὧν εἰ μὲν κρατήσοιεν, δίκῃ βιώσοιντο, κρατηθέντες δὲ ἀδικίᾳ

temptations and potentially misleading sense experiences.[262] It may not be by mere chance that in this relation, we are able to perceive a comparable distinction to that of the corporeal and intelligible on a larger cosmic level. Such a postulation, however, would be more compatible with the structure of the human soul as described in the *Republic*. Brennan (2012) suggests that the spirited portion of the tripartite structure of the soul fulfills a similar purpose for humans to that of the World-Soul within the cosmos as a whole, inasmuch as it functions as an intermediary between the intelligible and the corporeal, communicating with both and enabling the former to govern the latter.[263] The only plausible explanation for this interaction is that the rational soul must communicate its apprehension of the intelligible to the spirit; and it stands to reason that the more closely the rational soul approaches to the greatest possible extent of its perfection, the greater the similarity will be between the intelligible as it is apprehended within the human intellect and the intelligible as it truly is according to the Demiurge.

We must also consider Plato's account of the construction of the human soul by the lesser gods, in which he states that they emulated the work of the Demiurge.[264] We have also observed that the soul is described as possessing orbits, the perfection of which brings them to emulate those of the Demiurge, and that a clear distinction is drawn between intellect and soul. Thus, if the human soul is fashioned in the image of the World-Soul, and the perfection of the human intellect is the articulation of the intelligible according to the structure of the eternal cosmic model, then it would appear that the structure of the intellect within the soul reflects the duality of the cosmos, with the intellect as a parallel to the cosmic model.

Since νοῦς and λογισμός are instrumental in the task of ψυχή being imbued into a corporeal structure, it would seem that part of the significance of λογισμός in the alignment of the orbits of the soul is the task of understanding the divergence of the soul

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[262] Robert J. O'Connell, S.J. *Plato on the Human Paradox*. (New York: Fordham University Press: 1997), 114-115. Cit. Pl. *Phd*. 68b-81b.

[263] Tad Brennan. "The Nature of the Spirited Part of the Soul and its Object", in *Plato and the Divided Self*. Eds. Rachael Barney, Tad Brennan, and Charles Brittain. (Cambridge: Cambridge University Press, 2012), 123, Cit. Pl. *Ti*. 31c.

[264] Pl. *Ti*. 41c. τρέπεσθε κατὰ φύσιν ὑμεῖς ἐπὶ τὴν τῶν ζῴων δημιουργίαν, μιμούμενοι τὴν ἐμὴν δύναμιν περὶ τὴν ὑμετέραν γένεσιν

from its perfected state through its entry into a tangible vessel; for Plato explains in the *Timaeus* the result of the binding of the soul into a corporeal structure, describing this aspect of the imbuement of body with soul as follows,

> These things, pressed together in a stream were not strong, and did not hold their ground, and so they carried life, and were carried by it, and moved the whole of the living thing, falling randomly and irrationally into disarray, and had six types of motion altogether; for it moved back and forth between the right and the left, and mindlessly downward in all directions, wandering toward six areas.[265]

Since, as we have previously noted, it appears to us as though the soul is structured according to a duality that emulates that of the intelligible and the sensible, we may draw several important conclusions regarding what it means for the soul to behave according to the Good. We must state, firstly, that since this duality is merely an instantiation of the overall structure of the cosmic model and the World-Soul, it is given to imperfection. As a result, it may happen that the intellect either does not hold dominion over the spirited and appetitive aspects of the soul, or, if it does govern the lower realms of the soul, but has not attained sufficient apprehension of the intelligible, it will not be able to communicate the perfection of the cosmic model to the lower soul in such a manner that the latter emulates the structure and motion of the World-Soul.

In our earlier consideration of the connection between intelligible mathematical principles and epistemology, we posited the connection between human mathematical knowledge and justice to be such that through precise understanding of the principles of mathematics as they are operative in the natural world, we are able to determine the manner in which we ought to behave as components of that system. This observation appears to be expressed by Gadamer (1978), who suggests that the interplay of the rational and appetitive portions of the soul is "subordinated to the root principle of mass and measure..."[266] We have also suggested that the role of these laws in the governance of

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[265] Pl. *Ti*. 43a6-b2. αἱ δ' εἰς ποταμὸν ἐνδεθεῖσαι πολὺν οὔτ' ἐκράτουν οὔτ' ἐκρατοῦντο, βίᾳ δὲ ἐφέροντο καὶ ἔφερον, ὥστε τὸ μὲν ὅλον κινεῖσθαι ζῷον, ἀτάκτως μὴν ὅπῃ τύχοι προϊέναι καὶ ἀλόγως, τὰς ἓξ ἁπάσας κινήσεις ἔχον

[266] Hans-Georg Gadamer. *Die Idee des Guten zwischen Plato und Aristoteles.* Sitzungsberichte der Heidelberger Akademie der Wissenschaften Philosophisch-historische Klasse.3. (Heidelberg: Carl Winter Universitätsverlag, 1978), 22.

a political community would be to ensure that it would attain the greatest possible extent of ἀρετή by allowing it to function in such a manner as to survive eternally in the manner of the cosmic model. Planinc (1991) argues, however, that the use of the city as a parallel to the human soul is not meant to be interpreted literally, and even rejects the notion that the definition of justice presented in the *Republic* constitutes an explanation of political justice.[267] Even if we assume Planinc's argument to be correct, his position does little to diminish the clear relation of mathematical apprehension of natural laws on the part of the intellect, to action in accordance with objectives belonging to justice as defined within the intelligible. Indeed, the reasoning given by Socrates and his companions for the use of the structure of a city as a parallel for the individual human soul is that, as Planinc explains, Socrates "…would first describe justice in a bigger thing, a city, in order that justice in a smaller thing, a soul, might be seen more easily afterwards..."[268] While the image of the city described in the *Republic* may be intended only as a parallel to illustrate more easily the definition of justice on the level of soul, it nevertheless proves particularly effective in explaining the manner in which the intellectual properties of the just soul translate into the behaviour of the individual in the context of both the political community and the natural world as a whole.

Mathematical knowledge, as we have previously noted, informs just existence based on awareness of the finitude of tangible substance, as well as a comprehensive understanding of the cycles governing the functionality of nature. As such, the just person will be wise enough to eschew the excesses of the luxurious city,[269] whose inhabitants, through their folly, face eventual death, whether by starvation or by violent conflict with their neighbours. When a political community is governed by those with correct apprehension of the activity of nature as governed by temporal cycles,[270] and of the finitude and perishability of sensible substance, it has the capacity to be regulated in such

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[267] Zdravko Planinc. *Plato's Political Philosophy: Prudence in the* Republic *and in the* Laws. (Columbia and London: University of Missouri Press, 1991), 53.

[268] Planinc. *Plato's Political Philosophy: Prudence in the* Republic *and in the* Laws. 53

[269] See n. 202.

[270] Pl. *Ti*. 47a6-7. χρόνου δὲ ἔννοιαν περί τε τῆς τοῦ παντὸς φύσεως ζήτησιν ἔδοσαν

a way that its use of natural resources does not outpace the cycles according to which they are replenished. In the *Laws*, Plato appears to give more precise consideration to the manner in which such knowledge might be applied to the task of statecraft. The city of Magnesia, as noted before, is structured so that it has a population of 50, 000, with a total of 5,040 households, with the latter of the two numbers being considered particularly suitable for the division of wealth and labour due to the fact that it has 59 divisors, including the first 10 positive integers.[271] If there is a measure of continuity between the *Republic* and the *Laws*, Magnesia may similarly represent, albeit in a possibly more literal capacity, a parallel for the structure of the just soul. Planinc does indeed identify such a connection based on the comparison of the Nocturnal Council to the human νοῦς.[272] In the passage in question, Plato speaks of "…merging the image of the intellect in the head with the community…"[273] In this case, it may be that the direction of Plato's inquiry in the *Laws* is, to a certain extent, the opposite of that which occurs in the *Republic*; for in the *Republic*, the city was used as a model for the examination of justice within the soul, whereas in the *Laws*, the seat of reason, seemingly already established, is characterized as the cornerstone for the governance of the city. It would also appear to be the case that the *Laws* connects the relation of astronomy to the soul as discussed in the *Timaeus* with the principles of statecraft presented in the *Republic*. As Planinc explains, it is through the studies of arithmetic, geometry, and astronomy that the citizens of Magnesia are able to raise their souls toward the divine. Through the studies of geometry and astronomy, they also come to understand the manner in which the growth of their city follows a circular pattern.[274] Thus the work of the Nocturnal Council becomes akin to that of the Demiurge; for their leadership has the effect of raising the citizens of Magnesia towards the Good, while ordering them in a manner that might be likened to the Demiurge's ordering of matter towards a cosmic state.

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[271] See n. 212.

[272] Planinc. *Plato's Political Philosophy: Prudence in the* Republic *and in the* Laws. 215. Cit. Pl. *Lg*. XII 969b-c; Cf. Pl. *R*. VII. 536b.

[273] Pl. *Lg*. XII. 969b7. κεφαλῆς νοῦ τε κοινωνίας εἰκόνα τινά πως συμμείξαντες

[274] Planinc. *Plato's Political Philosophy: Prudence in the* Republic *and in the* Laws. 215. Cit. Pl. *Lg*. V. 747b, VII. 809c-e, 817e-18d, 821a-22c.

The role of mathematics in the tasks of statecraft and military leadership, and the possible parallel of these orderings to that of the cosmos, are consistent with the significance of mathematical knowledge as described in the *Republic*. In the *Republic*, Plato notes that geometric principles ought to be implemented in military strategy, stating their use in "…charging toward enemy encampments and other objectives, gathering and extending armies, assembling encampments, and formations of movement in encampments and in battle…"[275] As we have previously observed, Plato's language in describing the ordering of matter toward a cosmic state suggests a parallel between the structured gathering of atoms and the direction of military formations,[276] and may thus imply the application of mathematical principles in the regulation of human activity in accordance with natural principles. Plato demonstrates, furthermore, that astronomy is similarly significant in the governance of the political community, given that it proves invaluable in "…having effective observation of hours and the anniversaries of months, not only for farming and navigation, but also for military strategy…"[277] The application of astronomy to such purposes as these would imply the ordering of the political community in consonance with the heavenly orbits, and according to the *Timaeus*, it is in this manner as well that the individual soul ought to be ordered.[278] Thus, when the activity of the *polis* is conducted according to the heavenly orbits, the perfection of the individual soul may be translated into that of the entire political community.

5.3 Construction and Optimization: Investigating the Function of Geometric Matter in the Activity of Form and the Ascent of Sensible Being Toward its Perfection

As we have observed previously in examining the *Timaeus*, the Demiurge, being entirely good and without jealousy, wills all other things to be like himself. As Plato explains,

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[275] Pl. *R*. VII. 526d2-6. πρὸς γὰρ τὰς στρατοπεδεύσεις καὶ καταλήψεις χωρίων καὶ συναγωγὰς καὶ ἐκτάσεις στρατιᾶς καὶ ὅσα δὴ ἄλλα σχηματίζουσι τὰ στρατόπεδα ἐν αὐταῖς τε ταῖς μάχαις καὶ πορείαις διαφέροι ἂν αὐτὸς αὑτοῦ γεωμετρικός τε καὶ μὴ ὤν

[276] See n. 100, 102.

[277] Pl. *R*. VII. 527d2-4. τὸ γὰρ περὶ ὥρας εὐαισθητοτέρως ἔχειν καὶ μηνῶν καὶ ἐνιαυτῶν οὐ μόνον γεωργίᾳ οὐδὲ ναυτιλίᾳ προσήκει, ἀλλὰ καὶ στρατηγίᾳ

[278] See n. 151.

potency, or δύναμις, as with jealousy, does not belong to the Demiurge.[279] The Demiurge also explains to the lesser gods, whom he has placed within the sensible heavens, that the universe is incomplete without the generation of mortal entities.[280] It therefore stands to reason that all things willed by the Demiurge must be carried out. On this basis, it is clear that since it belongs essentially to the paradigms governing sensible entities to be instantiated on the level of matter, it stands to reason that the atomic specifications of sensible beings must, in some regard, be defined on the level of Form.

From this observation it is readily evident that the elemental solids as they are described in the *Timaeus* must be defined in some respect within the model of the cosmos. Just as Form must be brought to its completion through application to matter, it has been suggested that it belongs essentially to matter to be subordinated to Form. As Kutash (2011) explains, such a position is presented by Proclus, who maintains that "matter is not simply a passive *hupokeimenon*, a recipient of Form prior to its activation, but an active component and even opponent of Form."[281] Assuming that Proclus' position is correct, we must infer that matter must indeed be defined on the level of the intelligible, and that, as we have considered earlier,[282] the mathematical operability of matter requires that it allow for the possibility of conflicting with the Good. We are therefore faced with a strange paradox, for since matter must allow for all mathematical variations that are possible given the properties of its movement, it must also allow for deviations which are contrary to the schematics of the intelligible. We consider here the interplay of reason and necessity, and the manner in which the former essentially mandates this interaction.

In our previous consideration of mathematical principles in relation to Plato's explanation of matter, we postulated that the most likely theory for the correct definition of the elemental solids was that each of the triangles constitutes a triple tuple related to a

[279] Pl. *Ti*. 30a2-3. φλαῦρον δὲ μηδὲν εἶναι κατὰ δύναμιν

[280] Pl. *Ti*. 41b7-8. θνητὰ ἔτι γένη λοιπὰ τρία ἀγέννητα: τούτων δὲ μὴ γενομένων οὐρανὸς ἀτελὴς ἔσται

[281] Emilie Kutash. *Ten Gifts of the Demiurge: Proclus on Plato's* Timaeus. (New York: Bristol Classical Press, 2011), 44.

[282] See section 2.3.

three-dimensional coordinate system. According to this theory, each of the initial right triangles comprising the solids, which, in number, would be 8 for pyromorphic solids, 16 for aeromorphic solids, 40 for hydromorphic solids, and 48 for geomorphic solids, would represent a mathematical gathering of prime particles (i.e. a particle of matter about which no definitive statement may be made, save that it is the most basic unit of matter). Each vertex of each triangle would then represent one of the variables for the *x*, *y*, and *z* as the spatial location of a single particle. Some of the vertices of the right triangles comprising these solids will be shared between triangles, thereby implying axes of spatial alignment between two particles. These alignments of triangles would constitute clusters of prime particles moving together with one another in formation.

Under this system, the various measures of speed and sharpness[283] attributed to the solids would be defined according to the number of interdependent triple variable tuples corresponding to the spatial locations of the particles contained within the cluster. For instance, tetrahedral fire solids would contain only eight prime particles moving in formation according to the relations in distance defined by the differences in value between the *x*, *y*, and *z* variables corresponding to the location of each particle belonging to the solid. As such, whenever one particle of a pyromorphic cluster is propelled in a certain direction, there will be very few other particles that are required to "catch up" with the particle set in motion. The agility of a pyromorphic cluster is also a result of the fact that fewer particles are required to make course corrections in the event that one of the particles should have the direction of its movement altered. The larger particle clusters, being more ponderous in their movement due to the greater number of particles which must move in formation, will not have strong resistance against the smaller, more agile clusters, and will thus be easily dispersed by them. As we noted earlier, Ostenfeld's interpretation of the elemental solids appears to be compatible with this postulation, as he suggests that the solids are neither fully geometric nor fully tangible, but rather intermediate between both classes in some respect.[284] Friedländer's explanation of matter according to Plato's description of the elemental solids also appears to support the

[283] See n. 110.

[284] See n. 96.

identification of the elemental solids as patterns of motion followed by prime particles, for he describes matter as being "something like "space" and at the same time something like "stuff"…" Immediately before this description, he also states that matter can only be defined when it is in a certain state, providing the examples of stones and clouds.[285] Friedländer's characterization of matter as comprehensible in its various states rather than according to the prime particles themselves seems to lend further support to the notion of the elemental solids constituting patterns of motion as opposed to fully corporeal structures. His simultaneous characterization of matter as space and stuff is also compatible with Plato's rejection of the principle of void within his atomic theory,[286] since it accommodates the resulting requirement for elemental particles to be capable of passing through one another.

Proclus presents a similar position as well, referring to triangles as "ideas of natural science," and stating that the three types of triangles, equilateral, scalene, and isosceles, constitute a principle of unified plurality among things that are "in every way divided and changeable…"[287] Proclus' treatment of the elemental solids may add a second simultaneous level to the operation of mathematical principles in sensible being. In addition to constituting patterns pertaining directly to the motion of particles within cosmic matter, the triangular structure may be a thought-symbol to signify unification of multiplicity.

Concerning the position of chance and accident in the approach of sensible beings toward the Good, the predetermined requirement for intelligible paradigms to be expressed at the level of the sensible requires the possibility for tangible objects to deviate from their perfection as that perfection is defined in the intelligible. The main defining element within this type of order is the interaction of intellect with necessity. Friedländer explains necessity as one of the two causes that are operative in the natural

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[285] Paul Friedländer. *Plato: An Introduction*. Volume 1. Bollingen Series LIX. trans. Hans Meyerhoff. (New York: Bollingen Foundation, Inc., 1958), 250. Cit. Gaston Milhaud, *Les Philosophes géomètres de la Grèce* (Paris, 1934), p. 293. Originally published in German as *Platon: Seiswahrheit und Lebenswirklichkeit*. 2nd ed. (Berlin: W. de Gruyter & Co., Berlin, 1954), 250.

[286] See n. 94.

[287] Procl. *in Euc*. 166-67. In *A Commentary on the First Book of Euclid's Elements*. ed. and trans. Glenn R. Morrow. (Prince, New Jersey: Princeton University Press, 1970) 132.

world, though it is only a "concomitant or accessory cause."[288] From this description it would seem that necessity constitutes a force that must be subordinated to intellect in order for the construction of the cosmos to occur, though it may not be possible to bring necessity into total compliance with the intelligible. Necessity must remain active in at least some regard, a truth which is evident to us not only through the tendency of the sensible world towards imperfection, but also, as De Vogel (1986) demonstrates, based on the requirement for "the receptacle – as it were, the nurse – of all Becoming…"[289] As De Vogel notes, reason does not appear to suppress necessity altogether. Indeed it seems, based upon De Vogel's observations, that to suppress reason altogether would be impossible, for sensible beings, upon being constructed would then be incapable of motion, and thus unable to act according to the specifications by which they are defined within the intelligible.

Based on the need for movement, the crucial interaction of necessity with intellect may be partly, if not entirely, defined by transition and the operation of time. Based on *Timaeus* 49a-50a, Cornford (1971) confirms our previous postulation that the elemental solids should not be regarded as permanent, self-subsistent entities, but rather as impermanent states.[290] We are at a loss to explain how this mutability is possible without the inclusion of a temporal variable. Time, as Cornford explains, is stated in the *Timaeus* to be governed by the movements of the heavenly spheres, and is understood as a "moving likeness of eternity…"[291] The problem that must be addressed in this case is that of the infinite mathematical divisibility of time that might make all motion and activity impossible. This concern is raised by Mills and Strang (1974), wherein Strang identifies a significant difficulty of defining the relation between time and motion; for it seems that if a sensible being is constantly in flux, then nothing definitive may be said about it; yet if it at any point it is not in motion, then it may be altogether impossible for it to move at all;

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[288] Friedländer. *Plato: An Introduction*. 248.

[289] C. J. De Vogel. *Rethinking Plato and Platonism*. (Leiden: E. J. Brill, 1986) 207. Cit. Pl. *Ti*. 48a.

[290] Cornford. *Plato's Cosmology.* International Library of Psychology, Philosophy, and Scientific Method. (Initial Publication Great Britain: Kegan Paul, Trench, Trubner, & Co. Ltd., 1937, Reprints London: Routledge and Kegan Paul Ltd., 1948, 1952, 1956, 1966, and 1971), 178.

[291] Cornford. *Plato's Cosmology*. 97-98. Op. Cit. Pl. *Ti*. 37c-38c.

for if motion occurs over time, which is divided into an indivisible minimum unit, then based on our prior observations of Aristotle's treatise concerning indivisible lines, an object that stops moving altogether will be unable to resume its motion.[292] For this problem, one solution might be to suggest two different levels of motion for the object in question. One of these levels would constitute the particular motion or change occurring, while the other would be equated with the rate at which motion occurs, an essential property which would itself be in constant flux. As such, we are able to observe, with respect to motion and time, a multi-layered interaction of the limit and the unlimited; for the One, to which all being belongs, constitutes the limit insofar as it is the first point of origin for all things; yet by means of its eternity, it is also, in a certain regard, unlimited. Time thus constitutes a limitation of eternity, yet it is, in turn, infinitely divisible and continues indefinitely. It must, however, admit to limited divisions of itself in order for specific events to be possible, and yet in order to allow for these divisions, it must impose a limit on the stasis of the object by including constant flux in its rate of change. Such is necessary in order for sensible beings to approach the perfection of their nature as defined within the Good. This system, must, in itself, be defined within the intelligible, and so it must be that the One defines the Many, and that the Many ultimately returns to the One.

In this system, the Many is articulated within the One in three respects, or rather in four, if we count the duality of articulations to be a real articulation of multiplicity. The first of these articulations is that of the One as the mathematical unit principle, within which all multiplicities, fractions, and other operations must be immediately defined. The second is that of the multiplicity of paradigms defined within the intelligible, and the third is that of the application of those paradigms to matter for the construction of sensible substances, as mandated by the cosmic model's desire for all things to be like it. The first two articulations are mutually defining of one another, since the essential structures and activities defined within the paradigms are governed by the same mathematical principles that they themselves mandate, and it is according to the need for these principles that they are defined as they are within the unit principle. These two articulations are, in turn, defined by the requirement for the application of paradigm to

[292] K.W. Mills and Colin Strang. "Plato and the Instant", *Proceedings of the Aristotelian Society, Supplementary Volumes* 48 (1974), 65-7, 75. Cit. Pl. *Prm*. 183b4-5, 152b-c, 180b2-3, Arist. *Phys*. 187a1-3, cf. *LI*. 968a18-23.

matter, and they are similarly responsible for governing the manner in which paradigm is communicated to matter. The need for these three articulations of multiplicity might be recognized as an articulation of multiplicity as well, and just as the requirement for the inclusion of the other three articulations would necessitate an articulation encompassing all of them, the fourth articulation would enable and define the other three articulations. All of these articulations might be likened to certain dimensional principles. The first might be associated with both the point and line principles, since it accounts for both the single mathematical unit and the multiplicity of the same. The second might be likened to the plane principle, since it contains the schematics that must be brought to completion at the tangible level, while the third would be connected with the solid principle, since it accounts for the application of paradigm to matter. The fourth articulation as the sum of the others, might then constitute the symbolic 10 in its representation of perfection.

Chapter 6: Conclusion

From the role of mathematical principles as specifications of structure, relation, and motion as defined within the eternal model of the cosmos, it follows that the activity of these laws as a single system is one of self-motion. This self-motion expresses itself both within the structure of the cosmic model, as well as in the patterns of relation and dependency belonging to the definitions of mathematical principles. First of all, based upon the unchanging nature of the cosmic model, according to which it is, as Timaeus notes, "always in the same way," it follows that the model can never be in an incomplete state, but rather is always fully perfected. Timaeus also states that the Demiurge, whom we have previously identified with the cosmic model, "is not at all subject to potency." That is to say, nothing belongs to the cosmic model which must be brought to its final objective by an external agent. There is therefore, within this model, no chronological succession of cause and effect such as we would observe on a spatiotemporal level. All processes of generation and alteration that would occur in a sequential manner among tangible objects are represented simultaneously (inasmuch as the fact that intelligible paradigms are non-temporal dictates that there can be no chronological distinction between them), such that the final objectives of these processes define the initial phases, as well as all other stages leading up to their completion. The definition of physical causes and effects within the intelligible therefore translates into self-motion by means of the fact that it constitutes a pattern of succession that is circular rather than linear; for at the level of the cosmic model, the final results of all processes in nature (for instance, the adult stages in the life cycles of insects) constitute the causes of the initial steps, which, in turn, proceed towards the final objectives.

The same must also then be true of the processes of rational demonstration belonging to the articulation of scientific truth. At the level of the intelligible, the conclusion of any proof, whether a strictly mathematical proof or one pertaining to some consideration within physical science, must define the same premises from which it

follows, such that all steps in the demonstration are contained, in order yet also simultaneously, within the truth that is determined from them. In the cosmic model, such demonstrations thereby represent a logical structure that may be described as truth articulating itself. The imperfection of the mode of inference employed by the human intellect is therefore a function of its sequential structure, as well as its inclination to concentrate on a single specific question. Our method of mathematical demonstration, for instance, might be considered to be something of a dissection of the theorem that we are attempting to prove or disprove, since it treats all supporting or refuting arguments individually. Even as we use this method, we must nevertheless recognize that all of these arguments are contained within the conclusion of the hypothesis that they aim to verify. Additionally, in order to grasp the full significance of a tested hypothesis, both in terms of the conclusion itself and the process followed in reaching it, we must consider such a theorem within the larger context of the intelligible framework of reality. We ought to explore, for instance, the ramifications of the proof or disproof of certain mathematical theorems for the physical sciences. Questions of this sort would, in turn, require an examination of the extent to which those laws which are expressible purely in the language of mathematics, are instrumental in defining the structure and activity of the natural world. Particularly worthy of our inquiry is the relation between mathematical possibility and physical possibility, with respect to both the manner in which the former governs that latter, and the way in which mathematical possibility surpasses the limitations of physical reality.

Within the model of the cosmos, the principles of mathematics must follow the same circular pattern of connection as all intelligible paradigms, both within the system of mathematics and in relation to the objects of knowledge pertaining to tangible entities. We have observed, for instance, that as it is defined within an eternal, unchanging model, the unit principle must constitute the definition for all degrees of multiplicity and magnitude. Magnitude, in this case, must act alongside multiplicity and must be inseparable from it, such that all arithmetic values are also geometrically functional. The unit principle therefore contains the definition of the geometric point, which contains the definition of all lines, planes, and solids, as well as an infinite multiplicity of dimensional axes. There can be no complete definition of points, lines, planes, and solids without that

of the circle, for the laws that govern angles, which are articulated within the principles belonging to the circle, are necessary for the full operation of all geometric structures. All rules and concepts following from the definitions of points, lines, planes, solids, angles, and circles, must be defined along with them, since the model of the cosmos is unchanging and does not admit of potentiality, and thus no principle belonging to it can ever be in an incomplete state. The circular relation of mathematical principles ought to be a function of their connection to those paradigms pertaining specifically to the natural world; for these paradigms are responsible for dictating the same mathematical functions that are required to govern their essential specifications. These mathematical functions are dependent upon rudimentary laws and operations, the structure and activity of which is, conversely, defined according to the teleological objectives of the same functions of which they constitute the foundations. The overall structure is such that the paradigms governing tangible beings dictate the specifications of the same mathematical functions that constitute their foundations, while the relation of these functions to their foundational operations follows a similar circular structure. Thus, within the grand order of the cosmos, the laws of mathematics hold an exalted position as the articulations of the precision of the Good; for it is these specifications that act both as the foundations of reason within nature, and the measures of perfection to which all nature seeks to return.

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