A DYNAMIC SELECT SECTOR SPDRS ETFS PORTFOLIO OPTIMIZATION MODEL WITH REGIME-SWITCHING ECONOMIC INDICATORS

by

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To my beloved parents and grandparents

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ABSTRACT

This thesis studies a dynamic Select Sector SPDRs ETFs portfolio optimization problem. The objective of the optimization model is to maximize the risk-adjusted expected return of a portfolio similar to a logarithmic utility maximization. The conditional value-at-risk measure is chosen to be an additional risk exposure constraint. The vector auto-regression (1) regime-switching economic factor model estimated with the expectation-maximization algorithm is employed to identify different market regimes over time. The expected ETFs returns and their variance-covariance matrix used in the objective function of the optimization model are generated by a regime-switching asset pricing model. Both regime-switching models have proven to be superior to respective single-regime models due to their greater predictive ability. The optimized portfolio performance evaluated by Sharpe ratio, Treynor ratio and Jensen's alpha are all statistically significant compared to those of the equally weighted ETFs portfolio and S&P 500 stock index. This illustrates that incorporating the regime-switching technique, the portfolio optimization model is effective and successful under both bull and bear market conditions.

LIST OF ABBREVIATIONS USED

SPDR	Standard & Poor's Depositary Receipt
ETF	Exchange Traded Fund
S&P	Standard & Poor's
Amex	American Stock Exchange
CAPM	Capital Asset Pricing Model
APT	Asset Pricing Theory
FF	Fama-French
HMM	Hidden Markov Model
CPI	Consumer Price Index
VaR	Value-at-Risk
CVaR	Conditional Value-at-Risk
SPX	S&P 500 Stock Index
TYS	U.S. Treasury Yield Spread
UCS	U.S. Credit Spread
UNI	U.S. Unexpected Inflation
CCI	U.S. Comsumer Confidence Index
LEI	U.S. Chicago Board Leading Economic Indicators Index
LIBOR	London Interbank Offered Rate
XLY	Consumer Discretionary Sector ETF
XLP	Consumer Staples Sector ETF
XLE	Energy Sector ETF
XLF	Financial Services Sector ETF
XLB	Materials Sector ETF
XLI	Industrials Sector ETF
XLK	Technology Sector ETF
XLU	Utilities Sector ETF
XLV	Healthcare Sector ETF
EM	Expectation-Maximization
BIC	Bayesian Information Criterion
RMSE	Root Mean Square Error
DJIA	Dow Jones Industrial Average

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CHAPTER 1 INTRODUCTION

Shares of Exchange Traded Funds (ETFs) are purchased and sold in the secondary market, much like the stocks or shares of closed-end funds. They provide an efficient way to assemble and trade a portfolio of securities. Like traditional stocks and bonds, ETFs that mirror equity indices with full replication of the index in the portfolio can be traded intraday. ETFs can also be used for speculative trading strategies, such as short selling and trading on margin. Deville (2007) states that ETF-specific structure allows for more efficient and fairer pricing by consistently being traded very close to the value of the underlying portfolio in a contemporaneously priced market. Advantages that make ETFs extremely competitive include enhanced tax efficiency through in-kind redemption process and transparency. ETFs also have narrower spreads, lower management fees and lower transaction costs than conventional “open-end” mutual funds or private equities (Fabozzi, 2002).

Standard & Poor’s Depositary Receipts (SPDRs) are the shares of a unit trust that holds an S&P 500 portfolio, which were developed by the American Stock Exchange (Amex). Select Sector SPDRs are unique ETFs that divide S&P 500 into nine sectors and each stock in the S&P 500 is assigned to a Sector SPDR. These SPDRs are managed with the objective of matching the price and yield performance of their underlying sector indices. They provide diversification within a particular sector and are ideal tools to achieve varying levels of equity exposures within an asset allocation strategy. Therefore, we are especially interested in an investment portfolio constructed using Sector ETFs. As in a standard portfolio optimization model, expected asset returns and their variance-covariance matrix are derived from an asset pricing model. The Capital Asset Pricing Model (CAPM) (Treynor, 1961 and 1962; Sharpe, 1964; Lintner, 1965; Mossin, 1966), Asset Pricing Theory (APT) (Ross, 1976; Roll and Ross, 1980) and Fama-French (FF) model (Fama and French, 1989) are among the most celebrated asset pricing models, which are used for this purpose. With constant parameters and linear relationships between asset return and factors, these asset pricing models do work when the financial market operates normally. However, when the market crashes such as the Internet Bubble from 2000 to 2002 and the global financial crisis starting from 2007, the market regime changes substantially and unexpectedly. Overwhelming empirical evidence shows that asset return distributions are fat-tailed and skewed (Rachev et al., 2005). As a result, limitations of these traditional asset pricing models are exposed during financial crises, as they more or less fail to predict expected asset returns correctly since they depend on static parameters.

Consequently, the limitations of the classic asset pricing models cause a challenge to portfolio optimization. Portfolio separation mean-variance analysis (Markowitz, 1952) that ensures a portfolio resides on the efficient frontier, is one of the theoretical breakthroughs. The necessary inputs of this model, such as expected returns, standard deviations and correlations, are estimated using the traditional Sharpe-Lintner CAPM (Lummer et al., 1994). According to Chopra and Ziemba (1993), mean-variance optimization is very sensitive to errors in the estimates of inputs. Furthermore, the study of Ragunathan and Mitchell (1997) displays that financial crises are characterized by a significant increase in correlations of stock price movements, which may seriously degrade the benefits of diversification. Traditional portfolio optimization models have become unreliable, and we need a dynamic portfolio optimization model that is able to characterize different economic conditions. To obtain such a dynamic optimization model, we first need to improve the asset pricing model.

Economic condition is not static and alternates among various states over time. Given that the alternating market regimes over time is not always observable, the hidden Markov model (HMM) would be an extraordinarily useful tool in addressing this issue. Although the economic sentiment regimes are latent, information about them is found embedded in the economic factors. The hidden Markov model is composed of two stochastic processes: the economic indicator observation sequence and the latent regime sequence. One critical assumption of the hidden Markov model is that given a regime, the observed factor process is independent across periods. Utilizing the observed factor sequence, the hidden Markov model can capture different distributions of observed factor process across regimes, as well as to identify the hidden path of regimes. Accordingly, the hidden Markov model can help to obtain state-dependent relationships between asset returns and macroeconomic indicators, in addition to capturing market timing when regime changes.

In this thesis, we apply the hidden Markov model to construct a dynamic vector auto-regression (1) regime-switching factor model. Six economic indicators -- S&P 500 stock index, Treasury yield spread, credit spread, unexpected inflation, consumer confidence index and leading economic indicators index – are employed to characterize the potential market regimes. New market information in factor data is included through the structural Markov chain to obtain updated market conditions. Then, a dynamic regime-switching asset pricing model is established by incorporating predicted factor values derived from the regime-switching factor model as its independent variables. This model is thus ameliorated with state-dependent parameters and known path of market regimes.

Improved predictions of ETFs returns and variance-covariance matrix across time for the portfolio optimization model are therefore provided by this asset pricing model. Using conditional value-at-risk as the risk exposure constraint, we derive the dynamic optimal weights for Sector ETFs at each time point. Portfolio performance is evaluated using three classic performance measurements – Sharpe ratio, Treynor ratio and Jensen's alpha. As the regime-switching technique is utilized through a hidden Markov model across the three consecutive models, we expect that our ETFs portfolio optimization model outperforms its benchmarks under all economic regimes.

The remaining thesis will be constructed as follows. Section 2 reviews the literature of related methodology and measurement. Section 3 presents a data description on two data sets -- six macroeconomic indicators and nine Select Sector SPDRs ETFs, which we use to fit our models. The methodology of the regime-switching factor model, the regime-switching asset pricing model and the ETFs portfolio optimization model is discussed in Section 4. Section 5 shows the empirical results as well as their interpretations and the conclusions are given in Section 6.

CHAPTER 2 LITERATURE REVIEW

2.1 ECONOMIC FACTORS

Research on the interactions between stock returns and economic variables has been well established. These studies show that variations of stock returns can be explained by economic factors. For example, Flannery and Protopapakis (2002) highlight that economic indicators influence both stock market volatility and returns. Chang (2009) shows that S&P 500 stock returns and volatilities are based on macroeconomic factors, and the degree of influence changes conditional with stock market conditions.

S&P 500 index is one of the factors, as it reflects the equity market risk premium. Equity market risk premium was introduced in CAPM, a special case of APT model with only one factor exposed. Chen, Ross and Roll (1986) show that various macroeconomic shocks, such as inflation change, Treasury bond yield spread and corporate bond credit spread are important factors in the market model. Treasury bond yield spread is the difference between the yields on long- and short-term government securities and is a proxy of interest rate level. Corporate bond credit spread, as a measure of default risk, is the difference between yields on risky corporate and governments bonds. These factors can explain stock returns and can be quite useful in asset pricing (Tangjitprom, 2012)**.** Gilchrist and Zakrajsek (2010) also reveal that compared to default-risk and other financial indicators, credit spread is a robust predictor of future economic growth. In general, a wider credit spread usually indicates the upcoming market downturn. Fama and Schwert (1977), as well as Fama and Gibbons (1982) demonstrate that nominal equity returns are negatively correlated to unexpected inflation. Consumer confidence index and leading economic indicators index are proximal measurements of investor sentiment toward the economy. Charoenrook (2005) finds that changes in consumer sentiment are positively correlated to contemporaneously excess market returns.

Consequently, we adopt these six key economic factors -- S&P 500 stock index, U.S. Treasury yield spread, U.S. credit spread, U.S. unexpected inflation, U.S. consumer confidence index and U.S. Chicago Board leading economic indicators index. These indicators are used in the regime-switching factor model to recognize the concealed economic regimes.

2.2 EFFICIENCY OF MARKOV REGIME-SWITCHING TECHNIQUE

Research of Hamilton (1989) and Harvey (1989) first displays that a regime-switching model identifies various market conditions of the economy. It is an adoption of Markov switching approach with conditional covariance of market returns and does an excellent job in statistical inference. In the research of Alizadeh and Nomikos (2004), it displays that hedge ratios depending on Markov regime-switching models are superior to the ones using the other models. In addition, more reliable results are obtained when joint distributions can be switched stochastically between regimes. Works by Longin and Solnic (2001), and Ang, Chen and Xing (2006) show that correlations between stock returns vary over time and they increase substantially in a downside market. Such variation is explained by considering state dependence through regime-switching models. Schaller and van Norden (1997), and Perez-Quiros and Timmermann (2000) illustrate that regime-switching models considering different market regime conditions in a business cycle are quite successful. Papers of Ang and Bekaert (2002), and Guidolin and Timmermann (2003, 2005) also reveal some evidence that supports this viewpoint. As a result, we expect that our models depending on the regime-switching models would be more efficient since it captures the changing of the market condition.

2.3 CONDITIONAL VALUE-AT-RISK

Setting up the risk capital requirement for a period of time is extremely important for an investment portfolio. For our portfolio optimization problem, we need to apply some capital reserve measures. Value-at-risk (VaR) is one of the well-known risk measures that are used to determine the amount of assets to be kept in reserves. For a given portfolio, probability and time horizon, VaR is defined as a threshold value such that the probability that the mark-to-market loss on the portfolio over the given time horizon exceeds this value is the given probability level (Jorion, 2006). A risk measure should have certain properties such as normalization, transitivity and monotonicity (Artzner, et al., 1999). While VaR has been widely used instead of the traditional standard deviation, a deviation risk measure rather than a capital reserve measure, John Hull (2006) discusses the limitations of VaR. For example, VaR does not estimate losses in the tail of the return distribution, and is lack of sub-additivity.

In this thesis, we use conditional value-at-risk (CVaR) as our risk exposure constraint in the portfolio optimization model, since CVaR can overcome the limitations of VaR. CVaR estimates the loss in the left tail by calculating the conditional expectation of the loss after the VaR threshold has been breached. It is more sensitive to the left-tail shape of the return distribution. A portfolio with a low CVaR value necessarily has a low VaR value as well (Rockafellar and Uryasev, 2000).

2.4 PORTFOLIO PERFORMANCE EVALUATION MEASURES

Portfolio performance of Select Sector SPDRs ETFs needs to be evaluated using historical returns. Sharpe ratio (Sharpe, 1966), Treynor ratio (Treynor, 1966) and Jensen's alpha (Jensen, 1968) are three traditional and classic measurements.

Sharpe ratio describes a trade-off between the expected return and corresponding standard deviation of a portfolio, and Treynor ratio is directly related to CAPM. They both reflect a return-risk relationship by estimating excess return per unit of risk. The difference is that the total risk is employed in Sharpe ratio, while for Treynor ratio method, the overall portfolio is assumed to be efficiently diversified and only systematic risk is of importance. Expected returns and standard deviations of the portfolio are computed with the formula generated in Markowitz portfolio optimization (1952). Jensen's alpha also straight comes from CAPM, and it computes excess asset returns by using CAPM risk premium.

CHAPTER 3 DATA DESCRIPTION

3.1 ECONIMIC FACTOR DATA SET

Two data sets are employed in this study. One contains monthly macroeconomic factor data series such as the S&P 500 index (SPX), U.S. Treasury yield spread (TYS), U.S. credit spread (UCS), U.S. unexpected inflation (UNI), U.S. consumer confidence index (CCI) and U.S. Chicago Board leading economic indicators index (LEI). The last two factors are related to the market sentiment. The six factors are employed in the regime-switching factor model to recognize the number of the hidden market regimes.

Some raw data from January, 1978 to June, 2013 are collected from DataStream. They comprise of S&P 500 stock index, U.S. 10-year Treasury bond yield, U.S. interbank federal fund yield, 3-month LIBOR rate, U.S. 3-month T-bill rate, U.S. CPI change, U.S. consumer prices, U.S. consumer confidence index and U.S. Chicago Board leading economic indicators index. This raw data need to be transformed properly, as these series are usually not stationary and have a unit root problem. SPX and CCI are calculated as logarithmic changes multiplied by 100. TYS is the spread between the yields on the U.S. 10-year Treasury bond and the U.S. interbank federal fund. UCS is the spread between the 3-month LIBOR rate and U.S. 3-month T-Bill rate. UNI is the difference between U.S. CPI change, the realized inflation, and U.S. consumer prices, the expected inflation that reflects the future tendency. LEI is the difference of the values in previous and current periods. All the data observations are in percentage.

After the transformation, the available sample period of economic factor data is from February, 1978 to June, 2013. Data from February, 1978 to December, 2008 are used to do the in-sample test, and data from January, 2009 to June, 2013 are employed for the out-of-sample test.

3.2 PORTFOLIO PERFORMANCE EVALUATION MEASURES

The other data set is the observations on logarithmic asset returns of Select Sector SPDRs ETFs for nine sectors. Sector ETFs data are applied in the regime-switching asset pricing model and the portfolio optimization model. The Sector ETFs data contain Consumer Discretionary (XLY), Consumer Staples (XLP), Energy (XLE), Financial Services (XLF), Materials (XLB), Industrials (XLI), Technology (XLK), Utilities (XLU) and Healthcare (XLV). Monthly closing price data from January, 1999 to June, 2013 are obtained from Bloomberg. We utilize the logarithmic returns of these nine SPDRs by taking the difference between the logarithms of the closing prices in time t-1 to t and then multiplying 100. All the data observations are in percentage.

After the transformation, the available sample period of SPDRs ETFs data is from February, 1999 to June, 2013. Data from February 1999 to December 2008 are used to do the in-sample test, and data from January 2009 to June 2013 are adopted to do the out-of-sample test.

CHAPTER 4 METHODOLOGY

4.1 Vector Auto-Regression Regime-Switching Factor Model

Asset returns from different time periods are usually treated as random draws from the same statistical distribution, ignoring the economic situations associated with those periods. However, previous research has shown that returns and volatilities of assets can have different relationships with the same macroeconomic indicator under different market conditions of the economy. Empirical distribution of stock returns generated under different regimes are usually fat-tailed on the left-hand side of the normal density distribution. Therefore, a regime-switching model can help to better capture the characteristics of stock returns that reflect different market sentiments under unobserved economic and financial market regimes.

Vector auto-regression model is a multivariate time series model that has numerous advantages. It is a flexible model and can examine the lead-lag relationships among the variables in the model. It has proven to be especially useful for describing the dynamic behavior of economic and financial time series and for forecasting as well. Accordingly, we fit the economic factor data into a vector auto-regression model to identify the latent market states across periods.

4.1.1 Regime-Switching Factor Model

Transition Probability Matrix

Assume that the unobserved market regimes follow a Markov chain with a finite number of regimes K, $M_1, M_2, \ldots, M_t \in \{1, 2, \ldots, K\}$, which correspond to the observed economic factor data over time. One important property of the Markov chain is that given the information of present, the prediction of future is conditionally independent of the past information. Assume that the regime transition probability matrix is P

$$P = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1k} \\ P_{21} & P_{22} & \cdots & P_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ P_{k1} & P_{k2} & \cdots & P_{kk} \end{pmatrix} \quad (1)$$

where

$$P_{ij} = P_r\{M_t = j \mid M_{t-1} = i\}, i, j \in \{1, 2, \ldots, K\} \quad (2)$$

is the transition probability from regime i at time t-1 to regime j at time t.

Regime-Switching Regression Model

We first construct a vector auto-regression (1) regime-switching model. Observations of economic factors at time t-1 are the independent variables and those of factors at time t are the dependent variables in the regression. Both independent and dependent variables are in-sample factor data.

$$Y_t = \alpha_{Mt} + Y_{t-1}\beta_{Mt} + \varepsilon_{Mt}, \quad \varepsilon_{Mt} \sim NID(0, \Sigma_{Mt}), \quad (3)$$

where Y_t is the vector of the six economic factors (SPX, UNI, TYS, UCS, CCI and LEI) across time; M_t stands for the unobserved regime; α_{Mt} and β_{Mt} are regime dependent parameters, as for different regimes, estimations of α_{Mt} and β_{Mt} are different; ε_{Mt} follows an identical and independent multivariate normal distribution with zero mean and Σ_{Mt} variance-covariance matrix. All the information about the market regime including optimal number of regimes, α_{Mt} and β_{Mt} parameters and regime transition probability P_{ij} is embedded in the data series of factors. We can use model (3) to decide the switching points of the market regimes and to predict future factors. To demonstrate the advantage of the regime-switching model, a standard vector auto-regression model with single regime is also examined.

4.1.2 In-Sample and Out-of-Sample Predictions on Factors

Denote the posterior probability of regime k at time t by $q_t(k)$. The unconditional joint probability distribution of factor predictions is a multivariate mixture of normal distributions. Hence, the unconditional in-sample predicted factor values at time t is

$$E[Y_t] = E[E[Y_t|M_t]] = \sum_{k=1}^{K} (\alpha_k + Y_{t-1}\beta_k)\, q_t(k) \tag{4}$$

The prior probability of regime being k at time t+1 is

$$p_{t+1}(k) = P_{1k}\, q_t(1) + P_{2k}\, q_t(2) + \ldots + P_{Kk}\, q_t(K) \tag{5}$$

while P_{ik}, i = 1, 2,…, K is the regime transition probability from regime i at time t to regime k at time t+1.

We use the in-sample estimated regime-dependent parameters to conduct the out-of-sample test. Accordingly, the unconditional out-of-sample predicted value of economic factor at time t+1 is

$$E[Y_{t+1}] = E[E[Y_{t+1}|M_{t+1}]] = \sum_{k=1}^{K} (\alpha_k + Y_t\beta_k)\, p_{t+1}(k) \tag{6}$$

with a variance-covariance matrix

$$V[Y_{t+1}] = \sum_{k=1}^{K} (p_{t+1}(k)\, \Sigma_k) + \sum_{k=1}^{K} (E[Y_{t+1}|M_{t+1} = k] - E[Y_{t+1}])^2\, p_{t+1}(k). \tag{7}$$

4.1.3 Parameter Estimation Algorithm

Expectation-Maximization Algorithm for Parameter Estimation

In 4.1.1, the parameters we need to estimate are regime dependent parameters of the regression model, α_{M_t}, β_{M_t} and Σ_{M_t}, and transition probability of the market regime, P_{ij}.

Let Θ be the set of parameters $\{\alpha_{Mt}, \beta_{Mt}, \Sigma_{Mt}, P_{ij}\}$. Y is the sequence of in-sample historical data of the factors over time, and M is the sequence of in-sample market regimes across time. The optimal estimate of the set of parameters Θ is the one that maximizes the log-likelihood function of joint probabilities of Y and M, conditional on Θ

$$\max_{\Theta} \quad \{\ln P_r (Y, M|\Theta)\} \tag{8}$$

As documented in MacLean et al. (2011), the estimation process is an application of the expectation-maximization (EM) algorithm (Dempster et al., 1977). It is an iterative method for finding maximum likelihood estimates of model parameters. It consists of two steps, the E-Step and the M-Step. The iterative algorithm is described as follows:

1. **E-Step:** Set an initial value Θ^0 for the true parameter set Θ, and then find a tight lower bound to the true maximum log-likelihood. Calculate the conditional distribution function $Q^*(M) = P_r(M|Y, \Theta^0)$, and determine the expected log-likelihood, $E^{Q^*}[\ln P_r (Y, M|\Theta)]$.

2. **M-Step:** Maximize the expected log-likelihood with respect to the conditional distribution Q^* of the latent variable to obtain an improved estimate of Θ. The improved estimate is

$$\Theta^1 = \arg\max_{\Theta} \{E^{Q^*}[\ln P_r (Y, M|\Theta)]\} \tag{9}$$

With Θ^1 being the new initial value for Θ, we return to the E-Step.

The initialization of model parameters is generated by stochastic simulations. This EM algorithm guarantees to converge to a local optimal solution as iterations proceed and parameters are updated.

Posterior Probability and Hidden Path of Market Regime

Forward-backward algorithm is usually attributed to Baum et al. (1970) and Baum (1972). It is an inference algorithm for hidden Markov models and computes the posterior marginal of all latent state variables given a sequence of observed data. By employing this algorithm, we can calculate the probability of market regime given the observation sequence of economic indicators, $P_r(M_t|Y)$. In our model, it is defined as the posterior probability of regime at time t. The estimates generated by the forward-backward algorithm are key inputs to obtain parameter estimations in EM algorithm, which has been described previously.

After we acquire all the estimates of model parameters by utilizing EM algorithm, we are able to compute the out-of-sample posterior probability across time. $P_r(Y|M_t)$ is calculated using the probability density function

$$f(Y_t|M_t) = \frac{1}{\sqrt{|\Sigma_{M_t}|(2\pi)^n}} exp^{[-1/2\,((Y_t|M_t - E[Y_t|M_t])'\,(\Sigma M_t{}^{-1})(Y_t|M_t - E[Y_t|M_t])]},$$

$$\text{for } n = 1,2,\ldots 6 \qquad (10)$$

and $P_r(M_t|Y)$ is updated by Bayesian inference step, using Bayes' Theorem with the following formula

$$P_r(M_t|Y) = \frac{[P_r(Y|M_t) * P_r(M_t)]}{\Sigma M_t\,[P_r(Y|M_t) * P_r(M_t)]} \qquad (11)$$

where $P_r(M_t)$ is the prior probability of regime at time t.

In-sample hidden path of economic regimes is derived by the the Viterbi algorithm. It is a dynamic programming algorithm for finding the most likely sequence of hidden states given the sequence of factor observations, which is

$$M^* = \arg\max_{M} P_r(M|Y). \qquad (12)$$

4.1.4 Optimal Number of Regimes

Optimal number of market regimes inferred from the economic factor data is evaluated and determined based on Bayesian Information Criterion (BIC), using formula

$$\mathrm{BIC}\ (K) = -2\ln\ (L|K, \Theta(K)) + f\ (K, \Theta(K))\ \ln\ (T) \tag{13}$$

where $\Theta(K)$ is the set of parameters $\{\alpha_{Mt}, \beta_{Mt}, \Sigma_{Mt}, P_{ij}\}$; L is the likelihood function conditional on the number of regimes K and the set of parameters $\Theta(K)$; $f\ (K, \Theta(K))$ denotes the number of parameters and is a function of market regimes K; T is the in-sample size of factor data. $f\ (K, \Theta(K))\ \ln\ (T)$ is a penalty term for the number of parameters in the model, as when fitting models, it is possible to increase the likelihood by only adding parameters.

For K =1, 2, 3, 4 and 5, we run 1000 stochastic simulations for each K and select the simulation that minimizes the BIC (K). This gives us five minimized BIC (K) values. The optimal number of regimes K is the one with the smallest value of the five BIC (K) values.

4.1.5 Criteria for Predictability of Regime-Switching Factor Model

Since we have the factor predictions, we need to check how accurate the regime-switching factor model can predict factors. Predicting direction percentage is employed as a criterion of accuracy to check the predictability of regime-switching factor model. It is the percentage of the number of periods the observed factor and the predicted one have the same sign over the total periods. If this percentage is no less than 60%, we may consider our regime-switching factor model efficient in predicting factor values. This simple but direct indication can be quite helpful to investors. In-sample and out-of-sample predictions of the standard regression model with single regime are also calculated as a contrast.

Furthermore, root mean square error (RMSE), a measure of the differences between the predicted values by model and the observed values, can reveal the distinctions in the accuracy of predictions between regime-switching model and single-regime model.

4.2 REGIME-SWITCHING ASSET PRICING MODEL

The classic asset pricing models such as CAPM, APT and Fama-French factor models relate the asset prices to the important economic factors such as market risk premium, yield spread, credit spread and consumer confidence index. However, the critical assumption for these asset pricing models is that all the asset returns are generated under one single market regime. As a result, the relationships between equity returns and economic indicators are linear and constant over time. And they would fail to predict expected asset returns correctly by depending on static parameters when the economic market is in a regime-switching fashion.

Therefore, we accommodate the regime-switching process by taking the regime-switching factor predictions rather than the observed values as the independent variables, combined with the posterior probability of market regimes, to our asset pricing model. One key assumption for this accommodation is that all the information needed to estimate the posterior probability of market regimes has been fully reflected by historical factor values. Consequently, SPDRs ETFs observation performance does not affect the distribution of regime posterior probability.

4.2.1 Regression Model and Parameter Estimations

The regime-switching asset pricing model is built with predicted factors at time t as independent variables and observed Sector SPDRs logarithmic returns at time t as dependent variables in the regression model

$$R_t = A_{M_t} + \hat{Y}_{t, M_t} B_{M_t} + \Phi_{M_t}, \quad \Phi_{M_t} \sim \text{NID}(0, \Gamma_{M_t}) \tag{14}$$

R_t is the vector of the logarithmic returns of the nine Sector ETFs (XLY, XLP, XLE, XLF, XLB, XLI, XLK, XLU and XLV) from time t-1 to t; $\hat{Y}_{t, M_t}$ is the predicted factor values coming from the regime-switching factor model; A_{M_t} and B_{M_t} are regime dependent parameters while B_{M_t} is the sensitivity of economic factors; Φ_{M_t} is an N-dimensional multivariate normal distribution with zero mean and Γ_{M_t} variance-covariance matrix. Similarly, we use a linear asset pricing model with only one regime as a contrast.

4.2.2 In-Sample and Out-of-Sample Predictions on ETFs Returns

Likewise, the unconditional joint probability distribution of predicted ETFs returns is a multivariate mixture of normal distributions. Hence, the unconditional in-sample ETFs return predictions from time t-1 to t is

$$E[R_t] = E[E[R_t|M_t]] = \sum_{k=1}^{K} (A_k + \hat{Y}_{t,k} B_k)\, q_t(k) \tag{15}$$

and the unconditional out-of-sample return predictions from time t to t+1 is

$$E[R_{t+1}] = E[E[R_{t+1}|M_{t+1}]] = \sum_{k=1}^{K} (A_k + \hat{Y}_{t+1,k} B_k)\, p_{t+1}(k) \tag{16}$$

with a variance-covariance matrix

$$V[R_{t+1}] = \sum_{k=1}^{K} (p_{t+1}(k)\, \Gamma_k) + \sum_{k=1}^{K} (E[R_{t+1}|M_{t+1} = k] - E[R_{t+1}])^2\, p_{t+1}(k). \tag{17}$$

4.2.3 Criteria for Predictability of Regime-Switching Asset Pricing Model

Again, we need to check the accuracy of ETFs return predictions from this asset pricing model that incorporates the regime-switching technique by utilizing predicting direction percentage. There is a hypothesis for stock prices stating that they follow random walks. Hence, if the model correctly predicts the direction no less than half of the time (≥50%), we will say that this model has great goodness of fit. Replicating the methods used in the regime-switching factor model, in-sample and out-of-sample predictions of the linear regression model will also be reported, and root mean square errors of both asset pricing models will be compared.

4.3 PORTFOLIO OPTIMIZATION MODEL WITH CVAR CONSTRAINT

The goal of the investment is to maximize an investor's expected utility. Levy and Markowitz (1979) develop an approximation for the expected utility by a function of mean and variance based on a Taylor series expansion. On the grounds of this approximation, our objective here is to maximize the risk-adjusted expected return of a target portfolio while the risk exposure of the portfolio is constrained to some degree of tolerance. With all the nine predicted ETFs returns at hand from the regime-switching asset pricing model, we can construct an optimized portfolio by allocating different weights among the nine Sector SPDRs ETFs. Transaction costs for Sector ETFs can be very low with discount brokers and may be negligible when the investment amount is large. Thus to simplify, we assume that there are no transaction costs.

4.3.1 Objective Function of Model

Let W_t be the portfolio weight vector of the nine ETFs from time t-1 to t and let $(x_t|M_t)$ denote the portfolio return conditional on each regime $M_t = 1, 2\ldots K$ at time t, then

$$x_t|M_t = E[R_t|M_t]\, W_t \tag{18}$$

Also assume that every conditional probability distribution for the ETFs portfolio returns at each regime follows a normal distribution with mean μ_i and standard deviation σ_i, i = 1,2,…, K, hence

$$f(x_t|M_t=1) \sim N(\mu_1, \sigma_1), f(x_t|M_t=2) \sim N(\mu_2, \sigma_2)_{\ldots,} f(x_t|M_t=k) \sim N(\mu_k, \sigma_k) \tag{19}$$

The density of x_t conditional on the random variable M_t taking the value i is

$$f(x_t|M_t=i, \mu_i, \sigma_i) = \frac{1}{\sigma_i\sqrt{2\pi}} exp^{\{[-(x_t|M_t=i\ -\mu_i)^2]/2\sigma_i^2\}} \text{ for } i = 1,2,\ldots, K \tag{20}$$

We also have the prior probabilities of each regime at time t, which are $p_t(1)$, $p_t(2)$,…, $p_t(K)$ for regime 1,2,…, K, respectively. Let Ω be a vector of parameters that involves μ_1, μ_2,…, μ_K, σ_1, σ_2,…, σ_K as well as $p_t(1)$, $p_t(2)$,…, $p_t(K)$. Then the unconditional joint probability density function of the portfolio is a linear combination of these individual density functions, which is a multivariate mixture of normal distributions

$$f(x_t, \Omega) = p_t(1) * \frac{1}{\sigma_1\sqrt{2\pi}} exp^{\{[-(x_t-\mu_1)^2]/2\sigma_1^2\}} + p_t(2) * \frac{1}{\sigma_2\sqrt{2\pi}} exp^{\{[-(x_t-\mu_2)^2]/2\sigma_2^2\}} + \ldots + p_t(K) * \frac{1}{\sigma_K\sqrt{2\pi}} exp^{\{[-(x_t-\mu_K)^2]/2\sigma_K^2\}} \tag{21}$$

Portfolio return variance is a proper measure of risk because the difference between arithmetic average rate of return and geometric average rate of return is proportional to the variance (Messmore, 1995). As a result, we add a risk component in the form of portfolio return variance, which is a function to quantify financial risk. The variance of the portfolio, $V_t(w)$, is

$$V_t(w) = W_t' \, V[R_t] \, W_t \quad (22)$$

Hence, the objective function of the optimization problem is

$$E_t(w) = r_{ft} + [E(R_t) - r_{ft}]' \, W_t - \frac{1}{2} W_t' \, V[R_t] \, W_t \quad (23)$$

$E_t(w)$ is the expected return of a portfolio and r_{ft} is the risk free rate.

Regime-switching technique is incorporated here by providing unconditional ETFs return predictions $E(R_t)$ and variance-covariance matrix $V[R_t]$. They are obtained from the regime-switching asset pricing model and are inputs of the portfolio optimization problem.

4.3.2 Model Constraints

Conditional Value-at-Risk

Value-at-risk (VaR), a method in terms of percentiles of loss distributions, is one of the risk measures that satisfies the current regulations of risk management requirement in finance business. VaR can be quite efficiently estimated and managed when underlying risk factors are normally distributed. However, for non-normal distributions, VaR may have undesirable properties (Artzner, et al., 1999). VaR is not a coherent risk measure since it violates the sub-additivity property, which is

$$\text{If } X, Y \in L, \text{ then } \rho(X + Y) \leq \rho(X) + \rho(Y). \quad (24)$$

However, the conditional value-at-risk (CVaR), an alternative risk measure, is coherent. Therefore, it is suggested that CVaR is used instead of VaR (Rockafellar and Uryasev, 2000). CVaR is the conditional expected loss under the condition that it exceeds VaR.

Minimization of CVaR also leads to near optimal solutions in VaR terms because VaR never exceeds CVaR.

Let $\zeta_{\alpha}(x_t)$ denote VaR with the α significance level at time t, then we have

$$\int_{-\infty}^{VaR} f(x_t)\,dx_t = \int_{-\infty}^{\zeta\alpha(x_t)} f(x_t)\,dx_t = \alpha \tag{25}$$

Hence, our risk exposure constraint of CVaR is

$$CVaR = \frac{1}{\alpha}\int_{-\infty}^{VaR} x_t\, f(x_t) dx_t = \frac{1}{\alpha}\int_{-\infty}^{\zeta\alpha(x_t)} x_t\, f(x_t)\,dx_t \tag{26}$$

which should be larger than some negative tolerance δ.

Bounds for Long and Short Positions

In the standard Sharpe-Lintner CAPM, all the investors are assumed to be able to invest capital at a risk free rate and to borrow it at the same rate. Bounds of long and short positions for both portfolio and individual ETF need to be set up according to risk preference of investors as well as investment regulations of Sector ETFs. Let W_{it} denote individual ETF weight contributed to the portfolio at time t, i = 1,2,…, 9, then the limit condition of the total long and short positions for the portfolio is

$$L_p \leq \sum_{i=1}^{9} W_{it} \leq U_p \tag{27}$$

where L_p is the lower bound of portfolio investment weight and U_p is its upper bound. The limit condition of long and short positions for individual ETFs is

$$l \leq W_{it} \leq u \tag{28}$$

where l is the lower bound of individual ETF investment weight and u is its upper bound.

4.3.3 Mathematical Expression of Portfolio Optimization Model

Consequently, the portfolio optimization problem can be represented as follows:

$$\max_{w} \quad E_t(w) = r_{ft} + [E(R_t) - r_{ft}]' W_t - \frac{1}{2} W_t' V[R_t] W_t$$

$$\text{subject to} \quad \frac{1}{\alpha} \int_{-\infty}^{\zeta\alpha(x_t)} x_t \, f(x_t) \, dx_t \geq \delta$$

$$L_p \leq \sum_{i=1}^{9} W_{it} \leq U_p, \; l \leq W_{it} \leq u \tag{29}$$

4.3.4 Benchmarks and Measures for Portfolio Performance

Selecting benchmarks for the dynamic optimized portfolio is important. Benchmarks provide a frame of reference about portfolio return and risk, and help investors to monitor the investment progress of the portfolio. S&P 500 stock index, a passive but well-established index, is a proxy of market portfolio, and thus can be adopted as one of the benchmarks. In addition, a portfolio with a simple buy-and-hold strategy and equal weights to invest in the nine SPDRs ETFs across time can be used as another powerful benchmark.

After setting the benchmarks, we can evaluate our optimized portfolio. The three methods utilized here to assess the portfolio performance are Sharpe ratio, Treynor ratio and Jensen’s alpha.

Sharpe Ratio

$$E(r_p) = r_f + \frac{E(r_m) - r_f}{\sigma_m} \sigma_p \tag{30}$$

shows that there is always a trade-off between the expected return of a portfolio and its total risk. $E(r_p)$ is the expected return of portfolio; r_f is the risk-free rate; $E(r_m)$ is the expected return of market portfolio while σ_m is its corresponding standard deviation; σ_p stands for the portfolio standard deviation. $E(r_p)$ is simply the weighted average of expected returns of the Sector ETFs

$$E(r_p) = E(R_t)' \, W_t \tag{31}$$

The square of σ_p, the variance of portfolio can be derived as follows:

$$Var\,(E(r_p)) = W_t \,' \, V \, W_t \tag{32}$$

where V is the variance-covariance matrix of historical data of Sector ETFs. Thus, the Sharpe ratio of the portfolio is expressed as

$$S_p = \frac{E(r_p) - r_f}{\sigma_p}. \tag{33}$$

Higher Sharpe ratio means higher excess return per unit of total risk, indicating a better portfolio performance.

Treynor Ratio

To apply this measure, it is assumed that the whole portfolio is well diversified and therefore only systematic risk matters. The expected return of the portfolio under this condition is

$$E(r_p) = r_f + \beta_p \, [E(r_m) - r_f] \tag{34}$$

where β_p is the portfolio systematic risk, and $\beta_p = \frac{cov(p,m)}{Var(m)}$. The Treynor ratio is thus stated as

$$T_p = \frac{E(r_p) - r_f}{\beta_p}. \tag{35}$$

Likewise, higher Treynor ratio suggests greater excess return per unit of systematic risk, implying a preferable portfolio performance for an efficiently diversified portfolio.

Jensen's Alpha

Jensen's alpha is also related to CAPM. Excess return of a portfolio can be completely explained by the CAPM risk premium, and

$$\alpha_p = R_p - \{r_f + \beta_p [E(r_m) - r_f]\} = R_p - E(r_p) \tag{36}$$

where R_p denotes the actual portfolio return. Larger positive α signifies a greater abnormal positive return and therefore a better portfolio performance.

CHAPTER 5 EMPIRICAL RESULTS

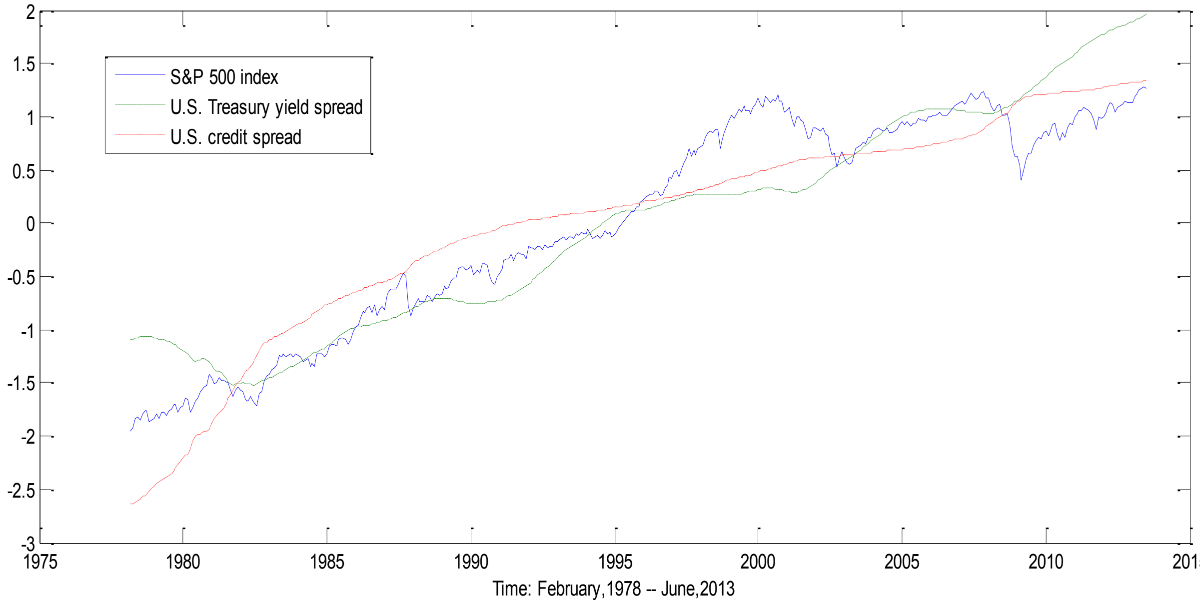
5.1 DYNAMIC REGIME-SWITCHING FACTOR MODEL

5.1.1 Statistical Summary of Economic Factor Data Set

As we have mentioned before, the macroeconomic indicator data set embeds all the information concerning economic conditions and market regimes.

Figure 1 shows the historical performances of the six indicators from February, 1978 to June, 2013. From the graphs, we can see that S&P 500 index (SPX), Treasury yield spread (TYS) and credit spread (UCS) have basically the same increasing trend over time, while unexpected inflation (UNI) is negatively correlated with leading economic indicators index (LEI).

Table 1 displays a statistical summary of the factor data set. We can see that SPX, TYS, CCI and LEI series exhibit negative skewness, which indicates that they have longer left tails, while UCS and UNI series show positive skewness. All the factors have large excess kurtosis, especially UCS, suggesting that it has a serious heavy-tail problem. The other factors have heavy tails to some degree. Average monthly S&P 500 stock index return is 0.6801% with a standard deviation of 4.4607%. The values of normality test for the six indicators stand for the p-values of the test. The p-values of the test for all these six factors are 0.001, which is far less than 0.05. Therefore, we should reject the null hypothesis that the factors are characterized by a multivariate normal distribution at a 5% significance level. The largest correlation value between each pair of factors is around 0.5. This implies that there would be no potential multicollinearity problem in the regression model.



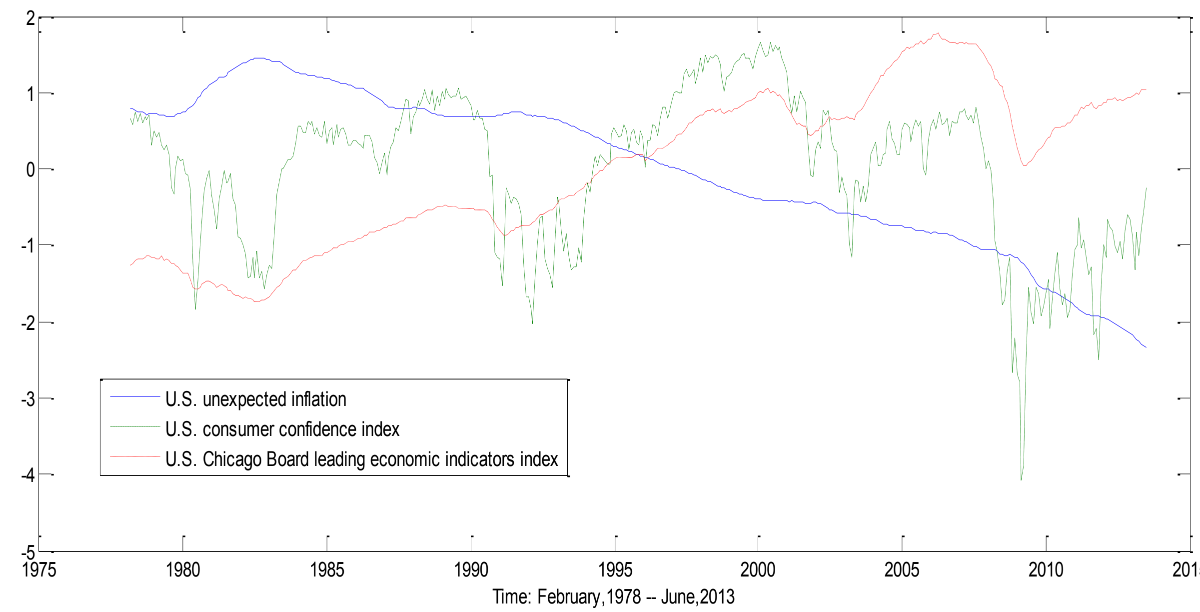


Figure 1 Historical Performances of Economic Factors from February, 1978 to June, 2013. The top panel shows the trends of S&P 500 index (SPX), Treasury yield spread (TYS) and credit spread (UCS), while the bottom panel shows those of unexpected inflation (UNI), consumer confidence index (CCI) and Chicago Board leading economic indicators index (LEI).

Table 1 Statistical Summary of Factor Sample Data Distributions from February, 1978 to June, 2013.

	SPX	TYS	UCS	UNI	CCI	LEI
Mean	0.6801	1.2268	0.8584	-0.7359	-0.0661	0.0936
Std. Dev.	4.4607	1.7030	0.9084	1.2996	8.4558	0.5892
Median	1.1004	1.4200	0.5300	-0.8200	-0.1097	0.2000
Skewness	-0.9121	-1.2060	2.6447	0.8080	-0.2419	-1.3969
Kurtosis	6.0681	5.7189	12.4291	6.5752	8.3906	6.9058
Normality Test						
P-Values	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
Correlations						
SPX	1.0000	0.0053	-0.1156	-0.0818	0.1782	0.2611
TYS	0.0053	1.0000	-0.5576	-0.4948	0.1311	0.3667
UCS	-0.1156	-0.5576	1.0000	0.5080	-0.1841	-0.3894
UNI	-0.0818	-0.4948	0.5080	1.0000	-0.0573	-0.2272
CCI	0.1782	0.1311	-0.1841	-0.0573	1.0000	0.3747
LEI	0.2611	0.3667	-0.3894	-0.2272	0.3747	1.0000

5.1.2 Optimal Number of Market Regimes and Parameter Estimations

Using Bayesian Information Criterion (BIC), we can identify the optimal number of market regimes. Table 2 displays the minimum values of BIC among the 1000 simulations for K=1, 2, 3, 4 and 5, as well as the corresponding maximum log-likelihood values. From Table 2, BIC is minimized when K=2, indicating that the market would be most likely to switch between two regimes given the in-sample factor data.

Table 2 Optimal Number of Market Regimes. It is obtained using in-sample factor data from February, 1978 to December, 2008. The minimized BIC value is achieved at two regimes.

Number of Regimes	1	2	3	4	5
Min BIC of 1000 Simulations for Each Regime	7188.0	6530.8	6652.7	6820.5	7074.2
Maximum Log-Likelihood	-3407.7	-2884.0	-2743.9	-2620.8	-2534.8

By employing the expectation-maximization algorithm, we could obtain the in-sample regime-dependent estimated parameters of α_{Mt}, β_{Mt} and Σ_{Mt}. Results of the regime-dependent estimated coefficients α_{Mt} and β_{Mt} are presented in Table 3, and Table 4 records the corresponding standard errors of these estimates. The estimated parameters and their standard errors of standard model with single regime are also presented for comparison. From Table 3, we can tell that both directions and magnitudes can be quite different for the regime-dependent estimates and for the ones of the standard model. In addition, most estimated parameters for Regime One are statistically significant at a 10% significance level, while those for the standard model are not. These results imply that the one-regime model probably not be capable to capture the real distribution of factors.

Table 3 Estimated Parameters for the Regime-Switching Factor Model. The vector auto-regression regime-switching factor model is:

$$Y_t = \alpha_{Mt} + Y_{t-1}\,\beta_{Mt} + \varepsilon_{Mt},\ \varepsilon_{Mt} \sim NID\,(0, \Sigma_{Mt})$$

As a comparison, the estimated parameters of the standard model are also reported. Estimated parameters are obtained using in-sample factor data from February, 1978 to December, 2008. Most estimated parameters for Regime One model are statistically significant at 10% significance level, while they are not for the standard model with single regime.

		α	β_{SPX}	β_{TYS}	β_{UCS}	β_{UNI}	β_{CCI}	β_{LEI}
SPX	Regime1	-0.5843	0.2154	0.0212	0.5180	-0.1539	-0.0566	1.8288
	Regime2	-0.0280	-0.1139	-0.0303	1.8010	0.0584	0.0319	0.0289
Standard Model		0.3997	0.0341	-0.1381	0.1925	-0.2103	-0.0024	0.7967
TYS	Regime1	-0.4025	0.0129	0.9713	0.2327	-0.0298	-0.0199	-0.0658
	Regime2	0.0686	0.0014	0.9836	-0.0710	0.0273	-0.0033	-0.0282
Standard Model		-0.1063	0.0027	0.9811	0.1271	-0.0129	-0.0121	-0.0191
UCS	Regime1	1.2421	-0.0341	-0.1379	0.3729	0.0989	0.0184	-0.1815
	Regime2	0.1147	-0.0113	-0.0282	0.8083	-0.0318	-0.0008	0.0231
Standard Model		0.4266	-0.0176	-0.0596	0.6822	0.0897	0.0113	-0.0674
UNI	Regime1	-0.1364	0.0253	-0.1467	0.1635	0.6772	-0.0065	0.1209
	Regime2	-0.2395	-0.0005	0.0144	0.0201	0.6952	0.0025	-0.2851
Standard Model		-0.2211	0.0076	-0.0640	0.1559	0.7305	-0.0016	-0.0528
CCI	Regime1	4.3266	0.4948	0.8452	-2.7322	1.8454	-0.3296	3.1514
	Regime2	-1.7496	0.3002	0.3052	0.3380	-0.5788	0.0018	1.3789
Standard Model		0.0024	0.4105	0.4823	-0.8456	0.7677	-0.0707	1.8382
LEI	Regime1	-0.2465	0.0386	0.0462	0.0773	0.0371	-0.0116	0.8605
	Regime2	-0.1546	0.0320	0.1505	-0.0156	-0.1346	0.0101	0.0297
Standard Model		-0.0553	0.0329	0.0658	-0.0041	-0.0099	0.0015	0.4696

Table 4 Standard Errors of the Estimated Parameters for the Regime-Switching Factor Model. They are obtained using in-sample factor data from February, 1978 to December, 2008.

		α	β_{SPX}	β_{TYS}	β_{UCS}	β_{UNI}	β_{CCI}	β_{LEI}
SPX	Regime1	0.6667	0.0545	0.1652	0.3127	0.2011	0.0377	0.4837
	Regime2	0.5336	0.0527	0.1760	0.5477	0.2974	0.0315	0.5167
Standard Model		0.4954	0.0544	0.1697	0.3214	0.2356	0.0341	0.4922
TYS	Regime1	0.1281	0.0105	0.0317	0.0601	0.0386	0.0072	0.0929
	Regime2	0.0369	0.0037	0.0122	0.0379	0.0206	0.0022	0.0358
Standard Model		0.0659	0.0072	0.0226	0.0427	0.0313	0.0045	0.0655
UCS	Regime1	0.1119	0.0092	0.0277	0.0525	0.0338	0.0063	0.0812
	Regime2	0.0228	0.0023	0.0075	0.0234	0.0127	0.0013	0.0221
Standard Model		0.0600	0.0066	0.0205	0.0389	0.0285	0.0041	0.0596
UNI	Regime1	0.1059	0.0087	0.0262	0.0497	0.0320	0.0060	0.0768
	Regime2	0.0632	0.0062	0.0208	0.0649	0.0352	0.0037	0.0612
Standard Model		0.0681	0.0075	0.0233	0.0442	0.0324	0.0047	0.0655
CCI	Regime1	1.0576	0.0865	0.2620	0.4961	0.3191	0.0598	0.7673
	Regime2	0.8373	0.0828	0.2761	0.8596	0.4667	0.0495	0.8108
Standard Model		0.7845	0.0861	0.2688	0.5089	0.3731	0.0540	0.7793
LEI	Regime1	0.0422	0.0035	0.0105	0.0198	0.0127	0.0024	0.0306
	Regime2	0.0528	0.0052	0.0174	0.0542	0.0294	0.0031	0.0511
Standard Model		0.0485	0.0053	0.0166	0.0315	0.0231	0.0033	0.0482

5.1.3 Predictability of Regime-Switching Factor Model

That how well the regime-switching factor model can predict the future factors needs to be examined. Both in-sample and out-of-sample predictions are computed using formula (4) and (6). We first plot the predicted values estimated by both regime-switching model and standard model, and the empirical factor values as well to get a visual intuition. Figure 2 documents the plots for S&P 500 index (SPX), Treasury yield spread (TYS) and credit spread (UCS), while factors of unexpected inflation (UNI), consumer confidence index (CCI) and leading economic indicators index (LEI) are recorded in Figure 3. Based on these two figures, the regime-switching model clearly does a much better job in predicting future values than the standard one for SPX, CCI and LEI. As for TYS, UCS and UNI, both models have pretty excellent goodness of fit. Therefore, we turn to the reports of the predicting direction percentage and root mean square error for both models.

From Table 5 and Table 6, we can tell that the predicting direction percentages of the regime-switching model for all the six factors are higher than those of the standard model. Moreover, root mean square errors of the regime-switching model are all smaller compared to those of the standard model. These results are sufficient to show that the regime-switching factor model surpasses the standard model with single regime in predicting future factors.

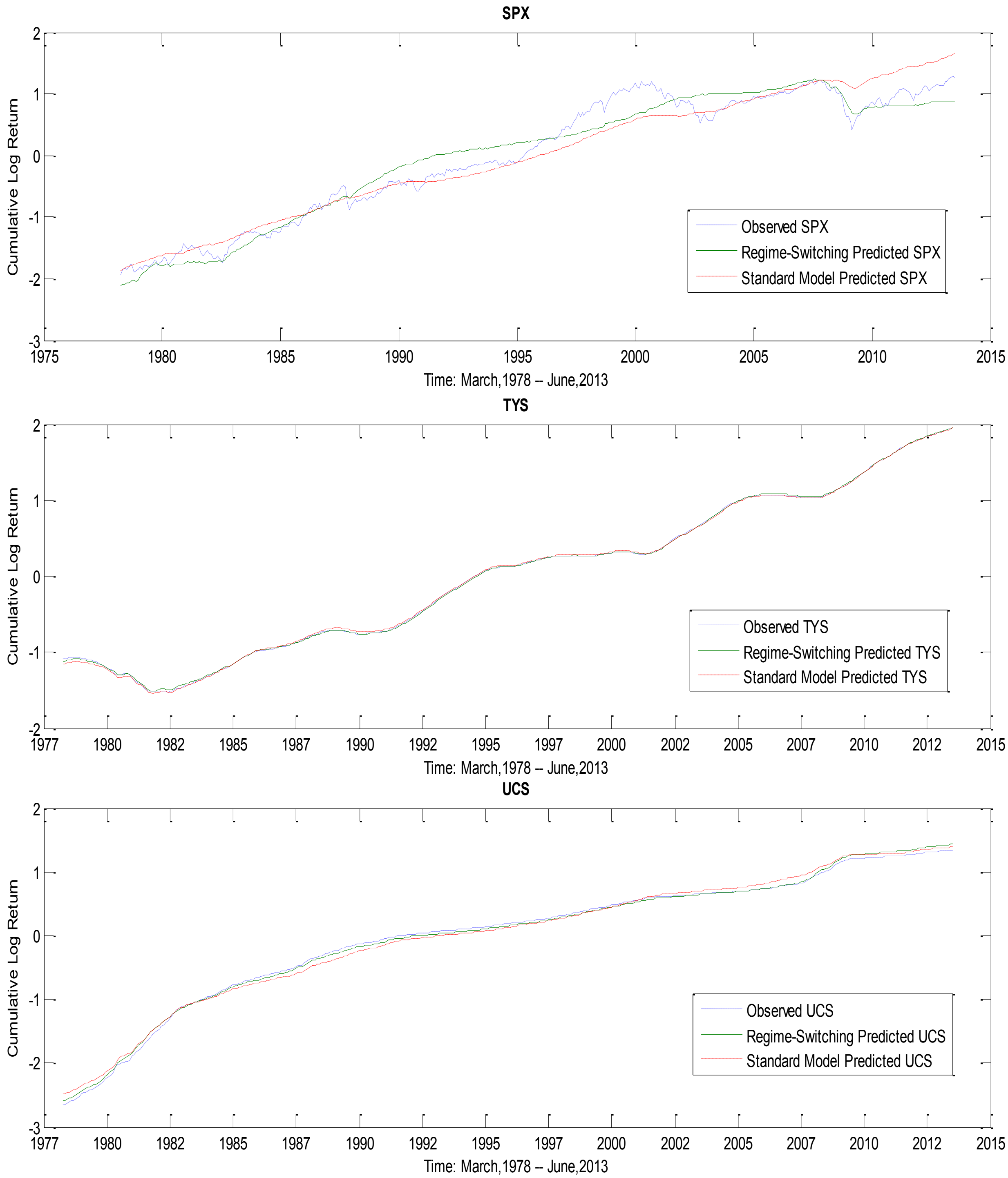


Figure 2 Predicted and Observed Economic Factors. This figure displays the predicted and observed values of three factors from March, 1978 to June, 2013. The top panel shows the predicted and historical values of S&P 500 index (SPX), the middle panel shows the ones of Treasury yield spread (TYS), while the bottom panel shows the ones of credit spread (UCS).

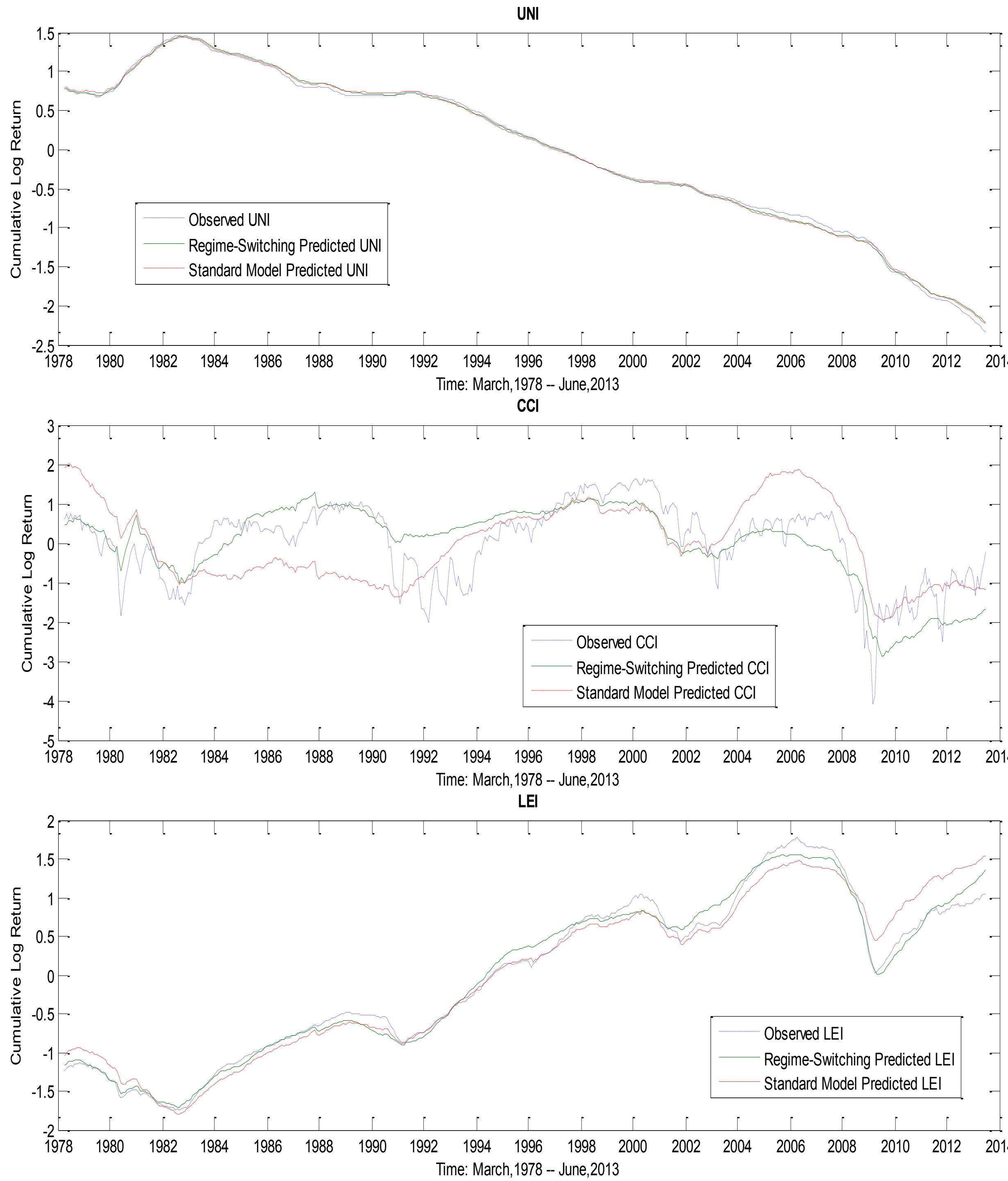


Figure 3 Predicted and Observed Economic Factors (ctd.). This figure displays the predicted and observed values of three factors from March, 1978 to June, 2013. The top panel shows the predicted and historical values of unexpected inflation (UNI), the middle panel shows the ones of consumer confidence index (CCI), while the bottom panel shows the ones of leading economic indicators index (LEI).

Table 5 Predictability of Factor Models. Predicting direction percentages of both models are calculated using in-sample plus out-of-sample predicted and observed data from March, 1978 to June, 2013. The performance of the regime- switching model is better than that of the single-regime model.

Model	SPX	TYS	UCS	UNI	CCI	LEI
Regime-Switching	0.6203	0.9552	0.9976	0.8844	0.5943	0.7382
Single-Regime	0.6038	0.9481	0.9811	0.8726	0.5708	0.7335

Table 6 Root Mean Square Errors for Factors. Root mean square errors of both models are calculated using in-sample plus out-of-sample predicted and observed data from March, 1978 to June, 2013.

Model	SPX	TYS	UCS	UNI	CCI	LEI
Regime-Switching	4.2854	0.5430	0.4475	0.5957	7.8357	0.3886
Single-Regime	4.4125	0.5573	0.5015	0.6126	7.9160	0.4449

5.1.4 Economic Interpretation of Market Regimes and Hidden Path

Given the in-sample factor data, there is most likely to be two market regimes and the corresponding estimated transition probability matrix P is

$$P = \begin{pmatrix} 0.9201 & 0.0799 \\ 0.0309 & 0.9691 \end{pmatrix}$$

It suggests that if the market is currently in Regime One, then in the next time point, the probability that it will still stay in Regime One is 0.9201 and that it will switch to Regime Two is 0.0799; If the market is currently in Regime Two, then in the next period, the probability that it will still remain in Regime Two is 0.9691, while the one it will move to Regime One is 0.0309. This estimated transition probability matrix signifies that the market will be quite stable in one regime for a while and would not jump back and forth between the two regimes in a very short time period.

The in-sample hidden path is estimated with the Viterbi algorithm. However, it is completely in accordance with the implication given by the in-sample posterior probability of market regime. That is, if the posterior probability of any regime excesses 0.5 at time t, the hidden path will show that market performs in keeping with that regime. We then distribute the in-sample factor data into two groups according to the posterior probability of regime over time. If the posterior probability of Regime One is larger than 0.5, we assign the corresponding factors at that time into Regime One group. If it goes the other way, we allocate them into Regime Two group. Table 7 documents the average performances of factors by each regime across time from March, 1978 to June, 2013. From the reported statistics, Regime One can be labeled as a bear market, since the average S&P 500 stock index (SPX) return is negative, together with a substantially positive credit spread (UCS). Furthermore, consumer confidence index (CCI) and leading economic indicators index (LEI) display negative signs. Unexpected inflation (UNI), on average, is positive for Regime One. In contrast, Regime Two appears to be a bull market, as the average S&P 500 return is a considerably positive number. It is

Table 7 Average Performances of Factors by Regime. They are obtained using in-sample factor data from February, 1978 to December, 2008. Regime One is labeled as a bear market and Regime Two is labeled as a bull market.

Regime	SPX	TYS	UCS	UNI	CCI	LEI
1	-0.0248	0.2546	2.0286	0.3865	-0.5650	-0.2398
2	0.8530	1.2983	0.5687	-0.8319	-0.1784	0.1758

also associated with a much smaller positive credit spread compared to that of Regime One, and we know that the credit spread is more likely to diverge in a downturn market. The leading economic indicators index is positive, which is a potential sign of a bull market, although consumer confidence index is slightly negative. The unexpected inflation factor is negative for Regime Two.

We already know that the in-sample hidden path exactly coincides with the implication obtained from the in-sample posterior probability. Accordingly, we have a good reason to believe that out-of-sample hidden path can be decided with out-of-sample posterior probability. Since we can estimate the posterior probability for out-of-sample data, we are able to decide the corresponding out-of-sample hidden path based on that. If the posterior probability of Regime One at time t is larger than 0.5, the market is most likely to behave under Regime One condition, thus the hidden regime at that time is Regime One; otherwise, it is most probable to perform under Regime Two and the corresponding latent regime is Regime Two. Based on this criterion, we calculate the respective frequencies by regime within a year. Table 8 presents the results of frequencies of each regime by year from March, 1978 to June, 2013, since the estimated posterior probabilities from model (3) are only available from March, 1978. Moreover, we plot the hidden path of the market regimes for the same period according to the corresponding posterior probability across time, which is given in Figure 4.

Table 8 Frequency of Regimes by Year. For year 1978, the posterior probabilities for regimes are only available for 10 months.

Regime	1978	1979	1980	1981	1982	1983	1984	1985	1986
1	6	4	12	12	12	6	12	3	1
2	4	8	0	0	0	6	0	9	11
Regime	1987	1988	1989	1990	1991	1992	1993	1994	1995
1	7	0	0	0	1	0	0	0	0
2	5	12	12	12	11	12	12	12	12
Regime	1996	1997	1998	1999	2000	2001	2002	2003	2004
1	0	0	0	0	0	0	0	1	0
2	12	12	12	12	12	12	12	11	12
Regime	2005	2006	2007	2008	2009	2010	2011	2012	2013
1	0	0	5	12	9	0	0	0	0
2	12	12	7	0	3	12	12	12	12

Combining together Table 8 and Figure 4, we find that from September, 1979 to June, 1983, economic factor data capture a bear market. Since then, bull market prevailed until October, 1987. The following bear regime did not last long, continued by a long-term prosperity of stock market from December, 1987 to July, 2007. Although during this period, the market regime occasionally jumped to bear market, it bounced back quickly. The turning point appeared again in August, 2007, and it lasted until September, 2009. Since then, another long-term bull market has begun and it still prevails at the end of the sample period.

This hidden path trail derived from the factor data highly coincides with what really happened in stock market from March, 1978 to June, 2013. The stock market from 1979 to 1983 was destroyed by the high inflation, and then it went through a period of booming with the Dow Jones Industiral Average (DJIA) peaking in August, 1987 until the Black Monday, October 19, 1987. Stock markets around the world crashed, leading to the largest one-day percentage decline in DJIA. After that, the United States stock market experienced a long time-frame flourish described as the secular bull market, with short upsets including the efficient market collapse between 2000 and 2002 triggered by the

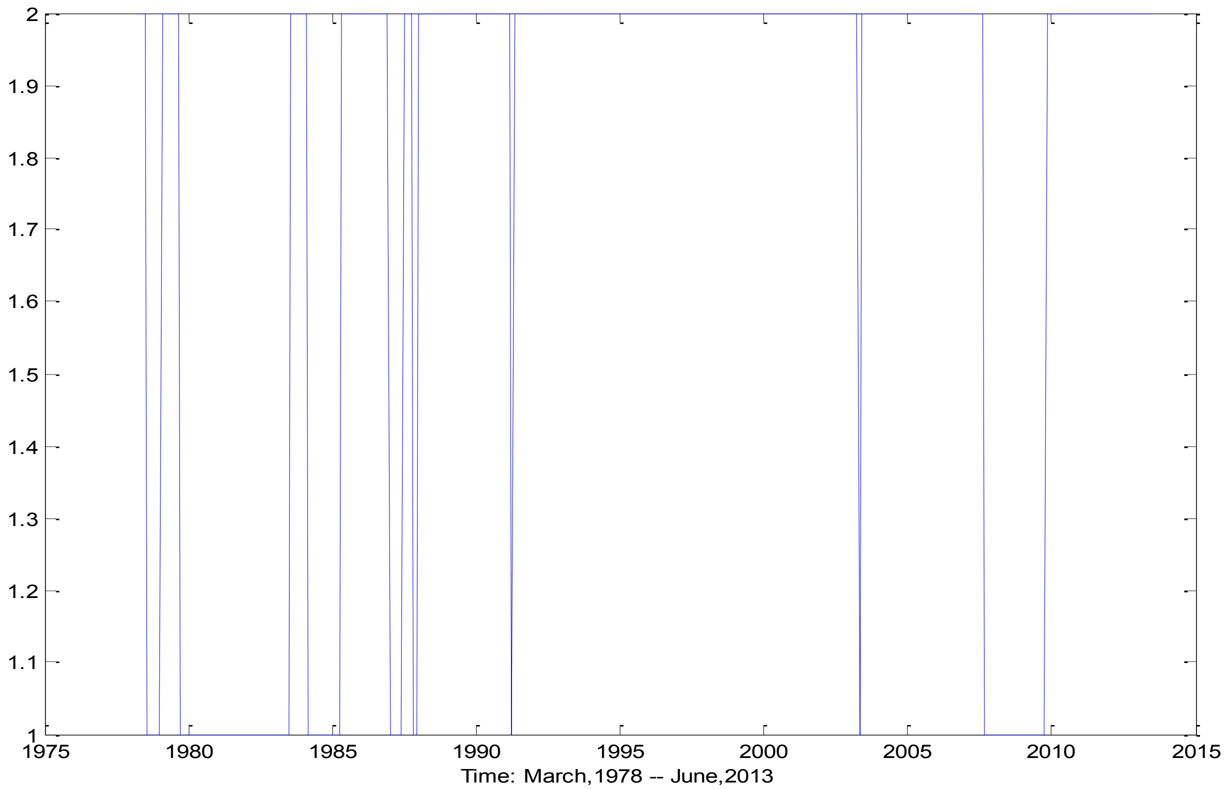


Figure 4 Hidden Path of Market Regimes. The out-of-sample hidden path of market regimes, from January, 2009 to June, 2013, is plotted according to the corresponding posterior probability for out-of-sample time period.

dot-com bubble. Watershed showed up by August, 2007 when the U.S. stock market fell amid credit fears. Caused by housing bubble and subprime, it is considered as the worst financial crisis since the Great Depression and it persisted until the middle of 2009. The stock market has been kept prosperous after then. The high degree of consistency between the estimated market regime and the empirical history of U.S. stock market regime strengthen the credibility of our regime-switching factor model.

5.2 Dynamic Regime-Switching Asset Pricing Model

5.2.1 Statistical Summary of Sector ETFs Data Set

As ETFs returns have different distributions under different economic conditions, the ETFs returns applied in our regime-switching asset pricing model need to be able to mirror both bull and bear market regimes. The time period for SPDRs ETFs data used in the regime-switching model is from February, 1999 to June, 2013, which includes enough transitions between the two market regimes, and therefore can be representative.

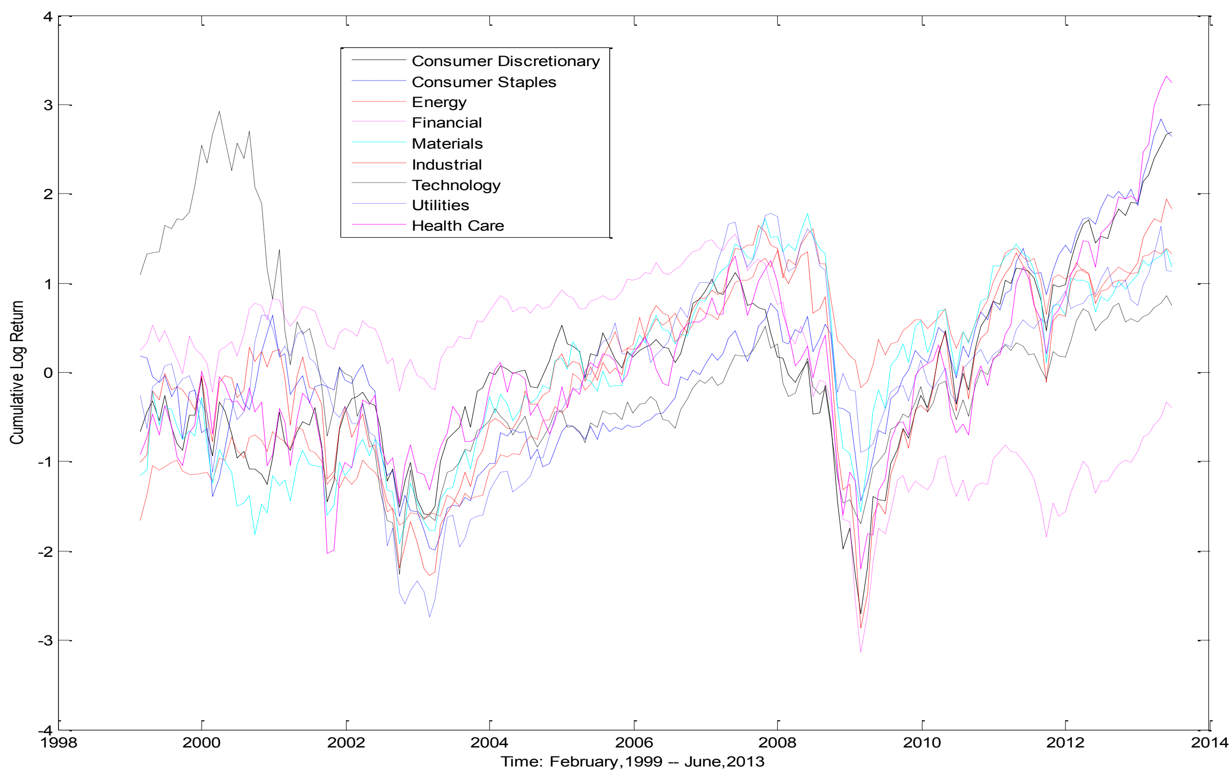


Figure 5 Historical Performances of Select Sector SPDRs ETFs. This figure presents nine Sector ETFs historical return data from February, 1999 to June, 2013.

From Figure 5, we can see that majority of the SPDRs ETFs have similar trends during the sample period, except that Financial Services (XLF) and Technology (XLK) have larger fluctuations, indicating higher risks along with these two ETFs.

Table 9 presents a statistical summary of Sector ETFs data set. In accordance with Figure 5, all the average values of these SPDRs ETFs are positive other than XLF and XLK, while these two ETFs are characterized with greater standard deviations among the nine ETFs. All the ETFs depict negative skewness, especially for Consumer Staples (XLP) and Utilities (XLU). Excess kurtosis also exists, and XLF has the largest one among the nine ETFs. This is superb evidence that the assumption for traditional asset pricing models that stock returns follow a normal distribution is misstated. In addition, the null hypothesis that ETFs returns are characterized by a multivariate normal distribution is rejected at a 5% significance level. All the correlations between each pair of ETFs are positive, indicating that long and short strategies can be employed alternately in keeping with the corresponding economic conditions. Estimated parameters of the regime-switching and linear models and their corresponding standard errors are documented in Table 10 and Table 11.

Table 9 Statistical Summary of the Sample Data Distributions of U.S. Sector SPDRs ETFs. These distributions are computed using nine Sector ETFs return data from February, 1999 to June, 2013.

	XLY	XLP	XLE	XLF	XLB	XLI	XLK	XLU	XLV
Mean	0.4158	0.2268	0.7388	-0.1179	0.3450	0.3251	-0.1226	0.1410	0.3225
Std. Dev.	5.6198	3.6237	6.4605	6.8503	6.5999	5.7197	7.5423	4.5221	4.1314
Median	0.5661	0.6443	1.0807	0.2714	0.2184	0.9433	0.7055	0.8453	0.9490
Skewness	-0.3564	-0.9925	-0.4239	-0.8304	-0.3085	-0.5561	-0.5931	-0.9148	-0.5926
Kurtosis	3.8258	4.7917	3.7819	6.2210	4.5227	4.5975	4.3889	4.7328	4.3841
Normality Test									
P-Values	0.0213	0.0010	0.0169	0.0010	0.0030	0.0011	0.0016	0.0010	0.0017
Correlations									
XLY	1.0000	0.4938	0.4713	0.7769	0.7678	0.8197	0.6731	0.3652	0.6919
XLP	0.4938	1.0000	0.3957	0.5730	0.4704	0.5375	0.2668	0.5434	0.5139
XLE	0.4713	0.3957	1.0000	0.4737	0.6630	0.6344	0.4257	0.5040	0.4440
XLF	0.7769	0.5730	0.4737	1.0000	0.6963	0.7879	0.4939	0.4100	0.6383
XLB	0.7678	0.4704	0.6630	0.6963	1.0000	0.8495	0.5606	0.4464	0.6350
XLI	0.8197	0.5375	0.6344	0.7879	0.8495	1.0000	0.6530	0.4710	0.7080
XLK	0.6731	0.2668	0.4257	0.4939	0.5606	0.6530	1.0000	0.2265	0.6138
XLU	0.3652	0.5434	0.5040	0.4100	0.4464	0.4710	0.2265	1.0000	0.4419
XLV	0.6919	0.5139	0.4440	0.6383	0.6350	0.7080	0.6138	0.4419	1.0000

Table 10 Estimated Parameters for the Regime-Switching Asset Pricing Model. The regime-switching asset pricing model is:

$$R_t = A_{Mt} + \hat{Y}_{t,\,Mt}\, B_{Mt} + \Phi_{Mt}, \qquad \Phi_{Mt} \sim NID\,(0, \Gamma_{Mt})$$

As a contrast, estimated parameters of the linear regression model are also reported. Estimated parameters are obtained using in-sample ETFs data from February, 1999 to December, 2008. Most estimated parameters for Regime One model are statistically significant at 10% significance level, while they are not for the linear regression model with single regime.

		A	BSPX	BTYS	BUCS	BUNI	BCCI	BLEI
XLY	Regime1	15.3465	-0.4754	-1.1353	-10.9767	0.4502	-0.2393	-1.3348
	Regime2	1.3911	-1.2050	-0.0892	0.0078	0.7538	-0.3236	-0.3362
Linear Regression		4.9538	-3.3057	-1.2301	-4.0090	-0.8287	0.3331	0.9424
XLP	Regime1	-5.0587	0.2891	-1.1521	2.0798	-3.0292	0.7281	-2.8397
	Regime2	3.2404	1.3192	-0.4683	-5.4033	0.8734	0.4679	-2.0708
Linear Regression		0.9694	1.8095	-0.0404	-1.7277	0.8343	0.0985	-3.2013
XLE	Regime1	-23.3588	6.4062	-3.4365	8.3853	-6.7039	1.3273	-20.2229
	Regime2	4.4042	1.9602	-1.6771	-8.2598	-0.5274	-0.0421	5.7583
Linear Regression		0.4053	1.9538	-0.6458	-1.0779	-1.3298	0.1930	-1.6475
XLF	Regime1	19.8580	-3.7440	-3.5175	-13.4143	-0.9366	0.2048	3.7840
	Regime2	3.2132	0.8780	-0.3807	-4.4661	1.1244	0.2465	-1.4247
Linear Regression		8.3420	-4.1888	-1.9728	-6.7779	0.5192	-0.3892	7.0121
XLB	Regime1	-10.5805	3.1512	-1.7627	4.7645	-5.4676	0.7848	-5.8048
	Regime2	0.7509	-0.9368	-0.4996	-1.0812	-1.5583	-0.3431	0.1983
Linear Regression		0.7986	4.9937	-0.0542	-4.8714	-0.7983	0.4712	-9.9307
XLI	Regime1	-1.0344	-0.7521	-1.9871	-1.0369	-3.2864	0.8177	-1.5591
	Regime2	2.0639	-0.8296	-0.4606	-1.3314	-0.4655	0.2536	-2.2036
Linear Regression		1.7361	3.3932	-0.1274	-4.0962	0.1900	0.4860	-7.5756
XLK	Regime3	-6.0024	1.6572	-1.2042	0.9896	-4.4588	0.7152	-5.0938
	Regime4	-6.7989	1.8335	1.4413	9.3382	-0.2466	0.3991	-2.2832
Linear Regression		-1.8349	4.8858	0.6621	-2.7397	0.1989	-0.1384	-6.0762
XLU	Regime1	-19.0924	3.7927	-0.3771	8.3713	-5.4564	1.2865	-10.4033
	Regime2	6.9190	-0.5851	-2.2449	-6.7305	2.7182	-0.1868	8.3379
Linear Regression		0.0980	-0.3311	-0.7360	0.7024	-1.6394	1.3782	-5.5822
XLV	Regime1	-2.7281	0.0833	-0.9339	0.3793	-4.3089	0.1066	0.7789
	Regime2	0.1100	0.3850	-0.1946	1.7013	1.3557	-0.0201	3.0342
Linear Regression		1.2133	-2.8601	-0.9735	0.0752	-1.7192	0.2272	3.3338

Table 11 Standard Errors of Estimated Parameters for the Regime-Switching Asset Pricing Model. They are obtained using in-sample ETFs data from February, 1999 to December, 2008.

		A	BSPX	BTYS	BUCS	BUNI	BCCI	BLEI
XLY	Regime1	7.3255	1.1825	1.0889	4.3803	2.1920	0.4181	3.4868
	Regime2	2.4962	1.8652	0.8028	5.5191	1.3136	0.8238	6.3509
Linear Regression		2.5282	4.9536	1.1854	2.7089	2.1480	0.5988	7.8577
XLP	Regime1	4.1442	0.6690	0.6160	2.4780	1.2401	0.2365	1.9726
	Regime2	1.7532	1.3100	0.5639	3.8764	0.9226	0.5786	4.4606
Linear Regression		1.7445	3.4180	0.8180	1.8692	1.4821	0.4132	5.4219
XLE	Regime1	7.8593	1.2687	1.1683	4.6995	2.3518	0.4486	3.7409
	Regime2	2.8010	2.0929	0.9009	6.1931	1.4740	0.9244	7.1263
Linear Regression		2.9816	5.8421	1.3981	3.1948	2.5332	0.7063	9.2672
XLF	Regime1	9.0230	1.4565	1.3413	5.3954	2.7000	0.5150	4.2948
	Regime2	2.3310	1.7417	0.7497	5.1539	1.2266	0.7693	5.9306
Linear Regression		2.5767	5.0487	1.2082	2.7609	2.1892	0.6103	8.0086
XLB	Regime1	7.1912	1.1608	1.0690	4.3000	2.1518	0.4105	3.4229
	Regime2	2.9540	2.2073	0.9501	6.5314	1.5545	0.9748	7.5156
Linear Regression		2.9257	5.7325	1.3718	3.1349	2.4857	0.6930	9.0933
XLI	Regime1	7.2523	1.1707	1.0781	4.3365	2.1701	0.4139	3.4519
	Regime2	2.3584	1.7622	0.7585	5.2144	1.2411	0.7783	6.0002
Linear Regression		2.4124	4.7268	1.1312	2.5849	2.0496	0.5714	7.4981
XLK	Regime1	7.4508	1.2027	1.1075	4.4553	2.2295	0.4253	3.5465
	Regime2	4.1205	3.0789	1.3253	9.1106	2.1683	1.3598	10.4835
Linear Regression		3.9856	7.8092	1.8688	4.2706	3.3862	0.9441	12.3876
XLU	Regime1	4.5900	0.7409	0.6823	2.7446	1.3735	0.2620	2.1848
	Regime2	2.0799	1.5541	0.6689	4.5987	1.0945	0.6864	5.2917
Linear Regression		2.1888	4.2887	1.0263	2.3453	1.8597	0.5185	6.8031
XLV	Regime1	4.4562	0.7193	0.6624	2.6646	1.3334	0.2544	2.1211
	Regime2	1.9800	1.4795	0.6368	4.3778	1.0419	0.6534	5.0375
Linear Regression		1.9647	3.8496	0.9212	2.1052	1.6693	0.4654	6.1065

5.2.2 Predictability of Regime-Switching Asset Pricing Model

Likewise, we calculate the predicting direction percentages and root mean square errors for both regime-switching and linear regression models. According to Table 12 and Table 13, we can see that most of the predicting direction percentages of the regime-switching model are better than those of the linear regression model. In addition, majority of the root mean square errors of the regime-switching model are smaller compared to the ones of the linear regression model. These results display that in general, the regime-switching asset pricing model outperforms the linear regression model with single regime in predicting ETFs returns. Figure 6, 7 and 8 depict predicted ETFs returns over time by the regime-switching asset pricing model, using predicted values by simple linear asset pricing model and observed data as comparisons.

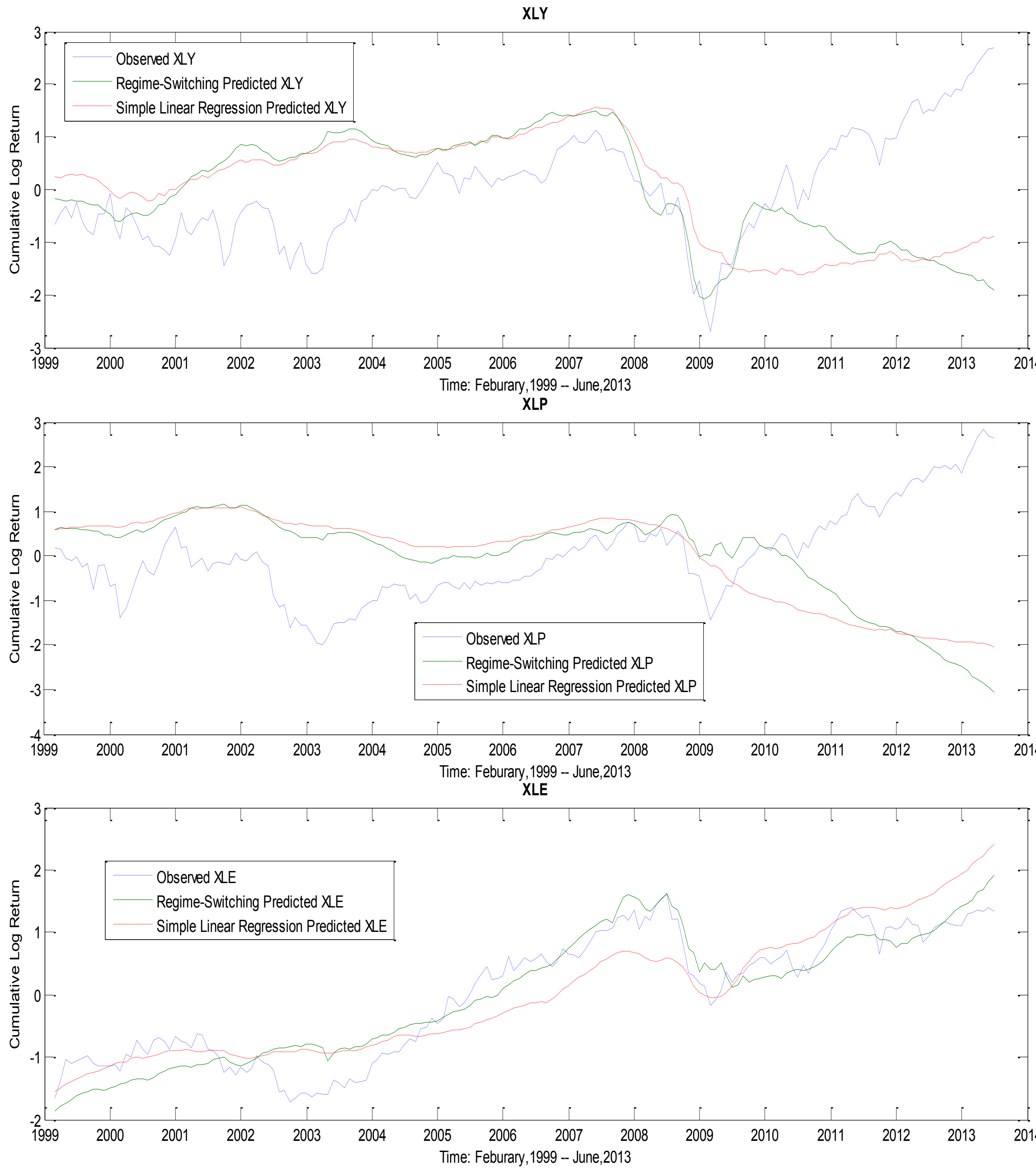


Figure 6 Predicted and Observed SPDRs ETFs Returns. This figure displays predicted and observed values of three Sector ETFs from February, 1999 to June, 2013. The top panel shows predicted and historical values of Consumer Discretionary (XLY), the middle panel shows the ones of Consumer Staples (XLP), while the bottom panel shows the ones of Energy (XLE).

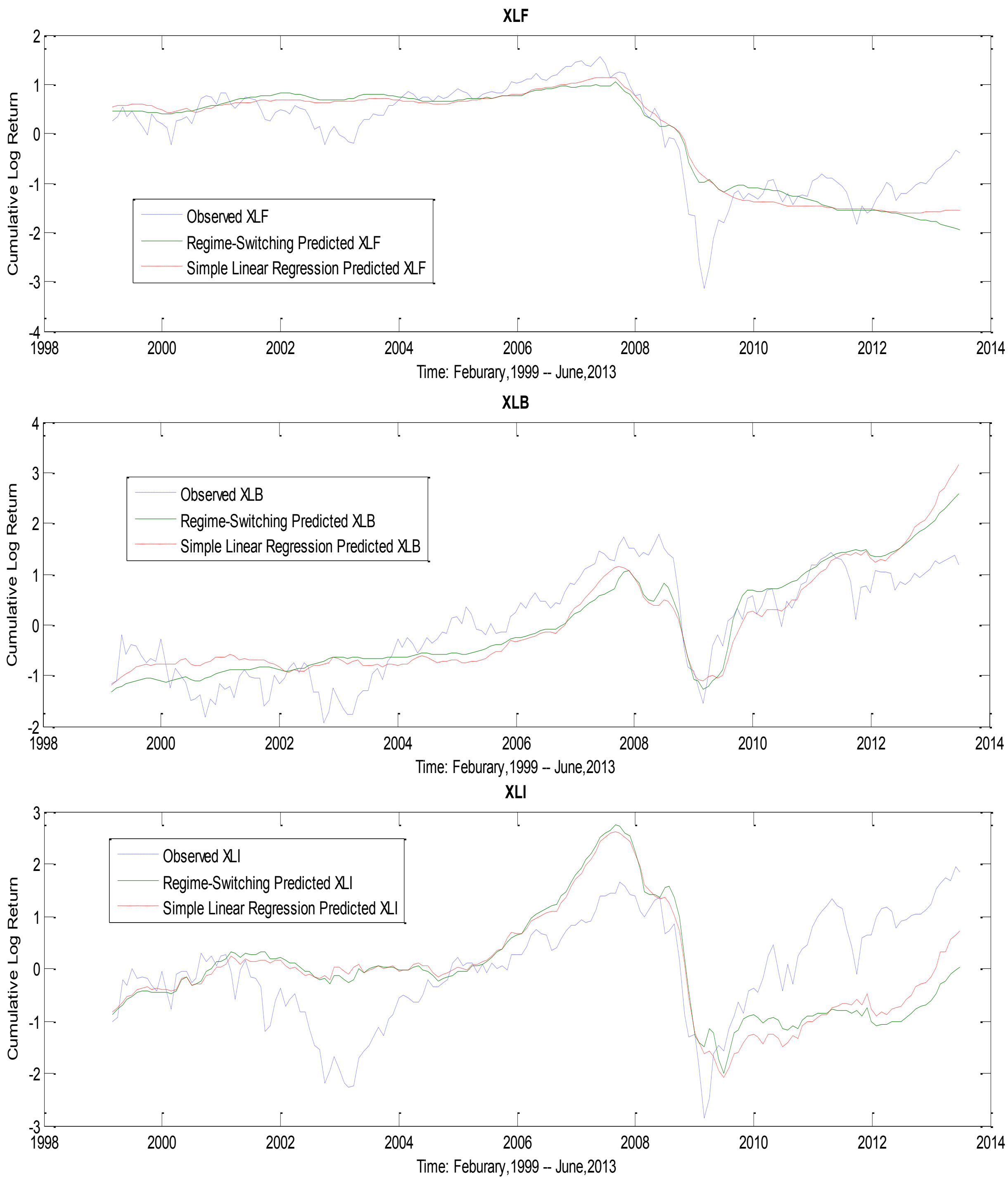


Figure 7 Predicted and Observed SPDRs ETFs Returns (ctd.). This figure displays predicted and observed values of three Sector ETFs from February, 1999 to June, 2013. The top panel shows predicted and historical values of Financial Services (XLF), the middle panel shows the ones of Materials (XLB), while the bottom panel shows the ones of Industrials (XLI).

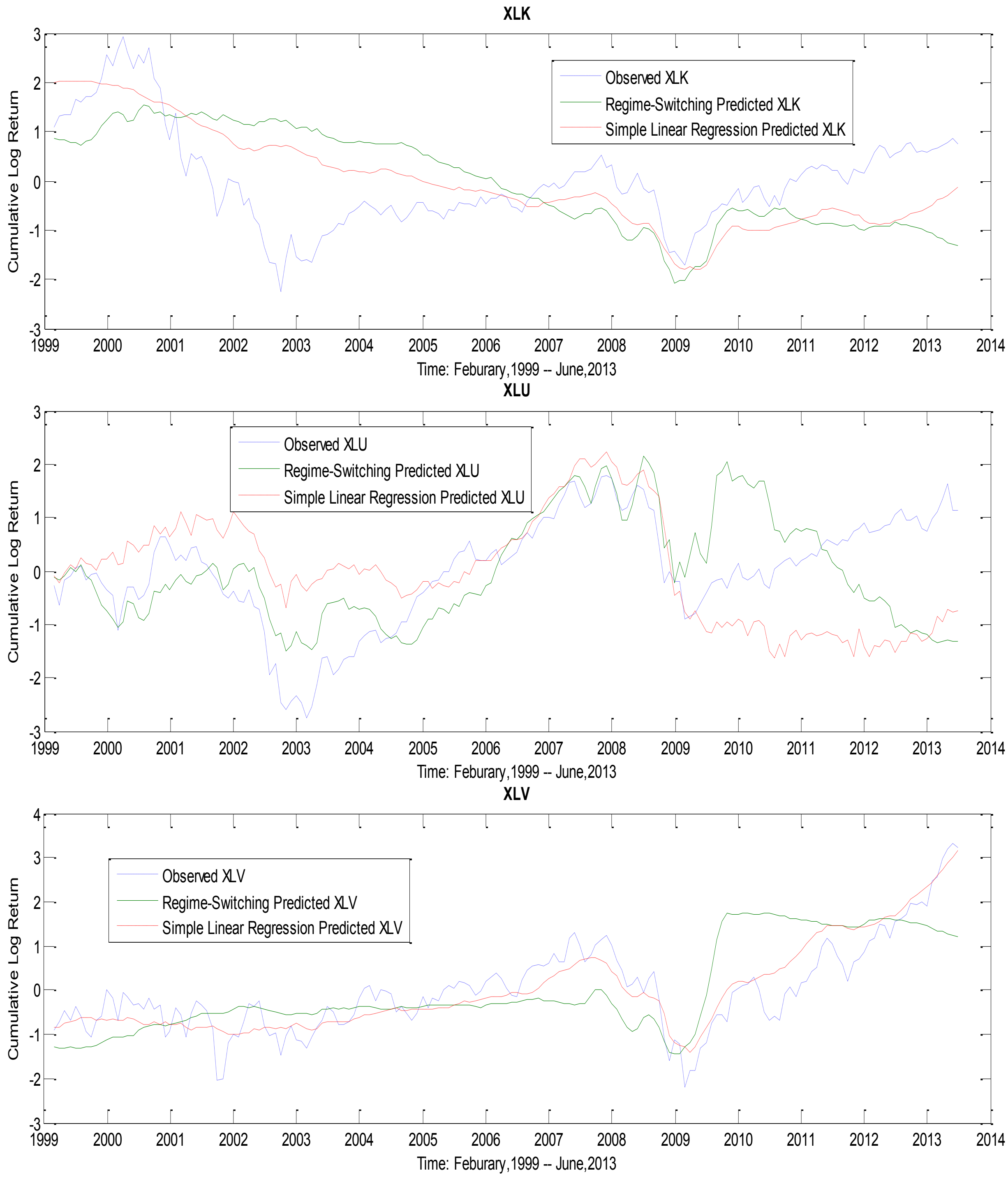


Figure 8 Predicted and Observed SPDRs ETFs Returns (ctd.). This figure displays predicted and observed values of three Sector ETFs from February, 1999 to June, 2013. The top panel shows predicted and historical values of Technology (XLK), the middle panel shows the ones of Utilities (XLU), while the bottom panel shows the ones of Healthcare (XLI).

Table 12 Predictability of Asset Pricing Models. Predicting direction percentages of both models are calculated using in-sample plus out-of-sample predicted and observed data from February, 1999 to June, 2013. The performance of regime-switching model is better than that of the linear model on the whole.

Model	XLY	XLP	XLE	XLF	XLB	XLI	XLK	XLU	XLV
Regime-Switching	0.5145	0.5665	0.6069	0.5260	0.5723	0.6358	0.5318	0.6243	0.5445
Linear Model	0.5434	0.5318	0.6067	0.5780	0.5491	0.6127	0.5723	0.5954	0.5376

Table 13 Root Mean Square Errors for Sector ETFs. Root mean square errors of both models are calculated using in-sample plus out-of-sample predicted and observed data from February, 1999 to June, 2013.

Model	XLY	XLP	XLE	XLF	XLB	XLI	XLK	XLU	XLV
Regime-Switching	5.4038	3.6886	6.0440	6.3985	6.2038	5.4711	7.2107	4.3029	4.2069
Linear Model	5.4576	3.6470	6.2398	6.4904	6.3189	5.4728	7.3883	4.3131	4.0142

5.3 DYNAMICS OF OPTIMIZED PORTFOLIO MODEL

5.3.1 Setting up Parameters for Portfolio Optimization

When we deal with the risk exposure, on the one hand, we would like to make the portfolio risk controllable while on the other hand, we need to understand that too much restriction on the risk exposure would hamper potential higher fund returns. Therefore, proper parameters need to be set to achieve the optimal result from the objective function.

According to the trading regulation, all the nine Sector SPDRs are allowed for short selling. And based on Table 9 of our study, all the correlations between each pair of ETFs are positive. Therefore, some short positions may hedge risk substantially in a downward market, whereas more long positions may greatly improve the portfolio return in a bull market.

We set the upper bound of portfolio position as 1.2, meaning that investors can borrow 20% of the total portfolio value at the risk free rate for the investment in SPDRs ETFs as a maximum. The lower bound of the portfolio investment weight is -0.3, implying that investors can short 30% of the total portfolio value at most. Similarly, we impose a limit of 0.8 on the long position and -0.2 on the short position for each ETF. The same weight limits are applied over the entire sample period and 3-month T-bill rate is used for the risk free rate.

A 5% significance level is employed here, indicating that there is a 0.05 probability that this portfolio will fall in value by more than the pre-specified VaR over a one-month period. While CVaR assesses the conditional expectation of the loss in the left tail after the VaR threshold is breached, we make the negative tolerance, δ to be -0.01 in the constraint. This suggests that the conditional expected loss of the portfolio after it exceeds VaR over a one-month period will be no more than 0.01.

5.3.2 Portfolio Performance with Empirical ETFs Data

As the new information on the unconditional predicted return values and variance-covariance matrix of ETFs is updated monthly over time, so will be the optimal weights allocated to each ETF. At each time period, the optimized portfolio performance is evaluated by computing portfolio return and risk with optimal ETF weights using actual ETFs returns and variance-covariance matrix. For the benchmarks utilized in the optimal portfolio performance – the equally weighted portfolio with the buy-and-hold strategy and the S&P 500 stock index – returns and risks are also calculated with historical data.

Figure 9 presents the trends of the cumulative wealth by investing the regime-switching optimized portfolio of Sector SPDRs, the equally weighted portfolio of Sector SPDRs and S&P 500 index respectively from February, 1999 to June, 2013. Assume that the initial cumulative wealth is equal to 1. From this figure, we can explicitly find that our regime-switching optimized portfolio has the most superior performance among the three investment strategies on the whole. It is followed by the performance of the equally weighted portfolio, while S&P 500 index behaves the worst. The minimum point of the cumulative wealth for the optimally weighted portfolio is 0.9388 and its maximum point is 1.9972. This implies that during the sample period, the invested wealth has gained a 99.72% cumulative growth and it has not much loss in its principal when undertaking its worst time. In contrast, S&P 500 index hits its rock bottom at 0.5745 and maximizes at 1.2744. This is a suggestion that the wealth invested in S&P 500 index would loss almost half of its principal with its poorest performance while it has only accumulated 27.44% growth during these thirteen and half years. The difference of profits generated by the two investment strategies can be extraordinarily tremendous if the principal is huge.

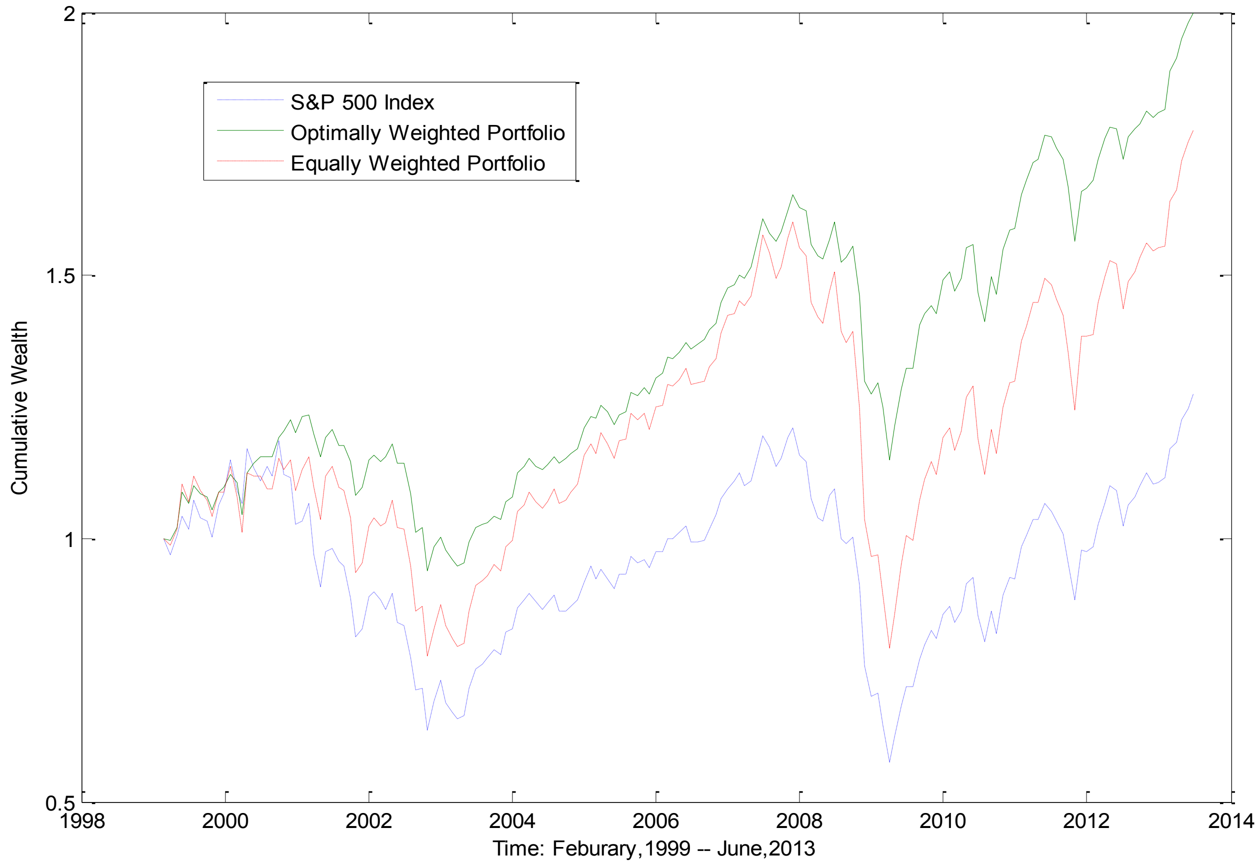


Figure 9 Portfolio Performance with Empirical Data. This figure shows cumulative wealth performances of regime-switching optimal weighted portfolio, equally weighted portfolio with a simple buy-and-hold strategy and S&P 500 index using real data from February, 1999 to June, 2013. The performance of optimized portfolio is superior to those of equally weighted portfolio and S&P 500 index.

The plot for cumulative wealth behaviors of the regime-switching optimized portfolio together with its two benchmark strategies does provide us an intuitive display of the portfolio performance. However, more than just considering the absolute growth of the investment as the only factor by evaluating the portfolio performance, we should take into account the risk with respect to the corresponding return and compare risk-adjusted returns among the three strategies. Consequently, we use three classic portfolio performance evaluation measures to make our results more convincing, which are Sharpe ratio, Treynor ratio and Jensen’s alpha.

Table 14 Portfolio Performance Measurements. The performances of optimized portfolio evaluated by three measures – Sharpe ratio, Treynor ratio and Jensen's alpha are all superior to those of equally weighted portfolio and S&P 500 index.

Investment Strategies	Regime-Switching Optimally Weighted	Equally Weighted	S&P 500 Index
Average Monthly Growth	0.33%	0.25%	0.13%
Average Monthly Total Risk	2.89%	4.47%	4.56%
Monthly Sharpe Ratio	0.0516	0.0154	-0.0115
Systematic Risk Beta	0.5714	0.9524	1.0000
Monthly Treynor Ratio	0.0026	7.1726E-04	-5.2456E-04
Monthly Jensen's Alpha	0.18%	0.12%	0.00%

Table 14 documents the portfolio performance measurement statistics for the three strategies. Average monthly growth for all the three strategies is calculated by taking the mean of monthly growth for optimized portfolio, equally weighted portfolio and S&P 500 index using historical data from February, 1999 to June, 2013. Average monthly total risk for optimized and equally weighted portfolios are computed by taking the average of their respective standard deviations. For the total risk of S&P 500 index, we just simply calculate its standard deviation as it is constant across time. In addition, we use the S&P 500 index as the proxy of the market portfolio and compute the systematic risk based on that. As a result, the systematic risk beta for S&P 500 index strategy is 1 and the monthly Jensen's alpha for it is 0. From Table 14, we can tell that the regime-switching optimally weighted portfolio has the largest average monthly growth among the three, and meanwhile its average monthly total risk is lowest. It also has much lower systematic risk with the market compared to that of the equally weighted portfolio. Furthermore, Sharpe ratio, Treynor ratio and Jensen's alpha methods for the regime-switching optimized portfolio all outperform those for the other two strategies. These statistics are adequate to exhibit that our Select Sector SPDRs portfolio optimization model accommodating regime-switching technique is highly efficient.

5.3.3 Economic Implication of Optimal Investment Weights

Different Sector ETFs tend to perform differently under the same market condition. Therefore, sector rotation strategies require investors to hold overweight positions in strong sectors and underweight positions in weaker sectors. In addition, some Sector ETFs are characterized by higher variations than the others. By examining carefully the weights allocated to the optimized portfolio for each time point, we may obtain some implications on how Sector ETFs should be chosen to construct an investment portfolio under different economic conditions.

The sample period we utilize to construct our ETFs portfolio optimization problem is from February, 1999 to June, 2013. This involves the latter part of the so-called secular bull market from 1988 to 2007, the recession caused by the 2007-08 financial crisis, and another market booming starting from the midst of 2009. Therefore, this selected sample period is sufficient for studying the ETFs weight allocations generated by the regime-switching portfolio optimization model.

The result of the optimal weight matrix from the portfolio optimization model depicts that short position strategy is indeed employed in the portfolio optimization process. However, this strategy is only adopted with rare occasions, which mostly happened during the 2007 financial crisis period, and short weights are mostly assigned to Financial Services (XLF) sector. This makes much sense because short positions may lower the portfolio risk in a downward market condition and financial industry was going through an extremely hard time during the 2007-2009 bear market.

Table 15 Statistics of Optimal Weight Allocated among ETFs. Long positions allocated to Financial Services (XLF) and Technology (XLK) are minimized under both market regimes while there are no positive weights assigned to Financial Services (XLF), Materials (XLB) and Industrials (XLI) in depression market condition.

Number of Positive Weight over 0.1	XLY	XLP	XLE	XLF	XLB	XLI	XLK	XLU	XLV
In Entire Time Periods	22	110	46	5	24	27	7	89	75
In Regime1	4	21	3	0	0	0	3	12	10
In Regime2	18	89	43	5	24	27	4	77	65

Since almost all the assigned weights in the optimal portfolio are long positions over time, we decide to inspect only the positive weights. Table 15 records the number of the weight allocated to a particular Sector ETF in all the time points if the weight is over 0.1, as well as in Regime One and Regime Two, respectively. On the basis of Table 15, we can see that the number that a relatively heavy weight is assigned to a Sector ETF is practically negatively correlated to its standard deviation. While Financial Services (XLF) and Technology (XLK) have the highest total risk, they possess the least frequencies of more than 0.1 positive positions allocated by our model during the entire sample period. Meanwhile, Consumer Staples (XLP) has the lowest standard deviation, whereas it owns the most times with more weight among the nine Sector ETFs over time. Apart from that, Utilities (XLU) and Healthcare (XLV) also have substantial number of occasions to get greater long positions. By checking the optimal weight vector in different market regimes carefully, we conclude that Energy (XLE), Financial Services (XLF), Materials (XLB) and Industrials (XLI) should not be assigned any positive weights in terrible market situations. This implies that these sectors behave weaker than the other sectors when the market is experiencing its difficult time.

CHAPTER 6 CONCLUSION

Previous studies have shown that macroeconomic factors and ETFs returns do not follow an independent and identical normal distribution, and our research verifies this by exploring the vector auto-regression regime-switching factor model and the regime-switching asset pricing model. Examining the regime-switching factor model by wielding the expectation-maximization algorithm, we identify that there are potentially two alternating market regimes. We also depict the most likely hidden path of market regimes revealed by the information of economic factors, and it highly coincides with the regime switches in the real stock market. By employing the regime-switching asset pricing model, we find that Sector ETFs returns do have different relationships with economic factors under different market sentiments, and obtain predicted ETFs returns and variances at each time point. Predictabilities of both regime-switching models are confirmed to be more significant than the ones of simple linear models with only one regime.

Predictions of ETFs returns and corresponding variance-covariance matrix across time are generated from the regime-switching asset pricing model. They are applied in the objective function of the dynamic ETFs portfolio optimization model. With the conditional value-at-risk and proper long and short positions limits as risk exposure restrictions, the optimal investment weights of ETFs are obtained from the optimization model. Portfolio performance is evaluated by Sharpe ratio, Treynor ratio and Jensen's alpha measures. An equally weighted ETFs portfolio with the buy-and-hold strategy and S&P 500 index are used as performance benchmarks. The significant results demonstrate that compared to the benchmarks, our dynamic optimized ETFs portfolio implemented with the regime-switching technique displays greater excellence and efficiency under both positive and negative market sentiments.

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