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Physics Letters B 638 (2006) 310–313
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Tilt and phantom cosmology

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Received 19 April 2006; accepted 18 May 2006
Available online 2 June 2006
Editor: M. Cvetič

Abstract

We show that in tilting perfect fluid cosmological models with an ultra-radiative equation of state, generically the tilt becomes extreme at late times and, as the tilt instability sets in, observers moving with the tilting fluid will experience singular behaviour in which infinite expansion is reached within a finite proper time, similar to that of phantom cosmology (but without the need for exotic forms of matter).

PACS: 98.80.Jk; 04.40.Nr

Cosmological data, including galaxy, CMB and supernovae observations [1], seem to be consistent with a cosmological constant or a dark phantom energy with effective equation of state parameter $\gamma < 0$ [2]. We shall study cosmological models with a tilting, but otherwise conventional, perfect fluid, and show that the models have dynamical behaviour similar to that of phantom cosmology, but without the need for any exotic forms of matter and consequently avoiding the pathologies in these models, such as the existence of ghosts.

For spatially homogeneous (SH) Bianchi cosmologies models, the universe is foliated into space-like hypersurfaces [3], and there are two naturally defined time-like vectors: the unit vector field, n^a, normal to the group orbits, and the four-velocity, u^a, of the perfect fluid. If u^a is not aligned with n^a, the model is called *tilted* (and non-tilted or orthogonal otherwise) [4]. Usually, the kinematical quantities associated with the normal congruence n^a of the spatial symmetry surfaces, rather than the fluid flow u^a, are used as variables. Following [4], a tilt variable v is introduced, so that in an orthonormal frame where $n^a = (1, 0, 0, 0)$, we have

$$u^a = \frac{1}{\sqrt{1 - v^2}}(1, v, 0, 0). \qquad (1)$$

We will assume a perfect-fluid matter source with $p = (\gamma - 1)\mu$ as equation of state, where μ is the energy density, p is the pressure, and γ is a constant. Causality then requires γ to be in the interval $0 \leqslant \gamma \leqslant 2$. A positive cosmological constant may also be included in the models. Such SH tilted cosmologies with a γ-law perfect fluid source have been studied by a number of authors [5–11]. It is known that the tilt can become extreme ($v^2 \to 1$) asymptotically to the future. In particular, let T be the proper time as measured along the fluid congruence, and let us define the quantity ΔT:

$$\Delta T \equiv \int_{\tau_0}^{\infty} \frac{1}{H}\sqrt{1 - v^2}\, \mathrm{d}\tau, \qquad (2)$$

where τ is a dynamical time variable defined in terms of the clock time t and the Hubble scalar H by $\frac{\mathrm{d}\tau}{\mathrm{d}t} = H$. If ΔT is finite, then the fluid congruence is future incomplete: the fluid observers will reach infinite expansion within finite proper time. Therefore, in spite of the fact that such a spacetime may be future geodesically complete [12], the worldlines defined by the fluid congruence u^a may become null with respect to the normal congruence n^a, sometimes so quickly that this occurs within finite fluid proper time. We note that for a SH cosmology for which $v^2 \to 1$ (which is a necessary condition for ΔT to be finite) the Hubble parameter of the fluid congruence, H_{fluid} also diverges.

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0370-2693
doi:10.1016/j.physletb.2006.05.048

The locally rotational symmetric (LRS) Bianchi type V perfect fluid models were studied in [7]. The global structure of Bianchi type V models was studied in more detail in [6]. It was shown that for models with $4/3 < \gamma \leqslant 2$, to the future the tilt can become extreme (in a finite time as measured along the fluid congruence) and, as T approaches a finite limiting value, the fluid worldlines become null with respect to n^a, and its length scale ℓ and some of its kinematic variables (i.e., the expansion, the shear, the acceleration, and the vorticity) diverge, but its matter density and curvature scalars tend to zero. Although this peculiarity only occurs for ultra-radiative fluids, which are unlikely to dominate the late universe, we will show that this behaviour can even occur for universes where ultra-radiative fluid is not dominant.

We shall first show that the fluid proper time is finite as the solution approaches its asymptotic state (the extremely tilted Milne equilibrium point M^-, defined by $\Sigma = 0$, $A = 1$, $v = -1$, $\Omega = 0$, $\Omega_\Lambda = 0$, and with deceleration parameter $q = 0$) for $4/3 < \gamma < 2$ in the absence of a cosmological constant [13]. The decay rates for the Hubble scalar $H = \theta/3$ and the quantity $\sqrt{1 - v^2}$ are $H \propto e^{-\tau}$, $\sqrt{1 - v^2} \propto \exp[-(5\gamma - 6)\tau/(2 - \gamma)]$, whence

$$\Delta T \propto \int_{\tau_0}^{\infty} \exp\left(\frac{-2(3\gamma - 4)}{2 - \gamma}\tau\right) d\tau, \tag{3}$$

which is finite for $4/3 < \gamma < 2$.

Spatially homogeneous cosmological models with a positive cosmological constant were investigated using dynamical systems methods in [13], extending the tilted LRS Bianchi type V analysis of [7] to the $\Lambda \neq 0$ case. A de Sitter point with extreme tilt is the future attractor for $\gamma > 4/3$. Therefore, in general, for Bianchi type V models with $\gamma > 4/3$, the tilt again becomes extreme at late times and the fluid motion is no longer orthogonal to the surfaces of homogeneity.

Although it is known that expanding non-type-IX Bianchi models with a positive cosmological constant isotropize to the future (cosmic no hair theorem) [14], and that this result applies to tilted models, the isotropization is with respect to the congruence normal to the homogeneous symmetry surfaces—not the fluid congruence [15]. Thus in the Bianchi type V models the spacetime generically becomes de Sitter-like, in accordance with the cosmic no hair theorem, but since the tilt does not die away, isotropization of the cosmology does not occur with respect to the fluid congruence.

This is a generic feature of spatially homogeneous models. In general, SH models are not asymptotically isotropic. The spatially homogeneous models do isotropize to the future in the presence of a positive cosmological constant [14]. By investigating the asymptotic behaviour of a SH model with a cosmological constant and a tilted perfect fluid with $p = (\gamma - 1)\mu$ as the future de Sitter model (with an extremely tilted perfect fluid) attractor is approached we obtain $H \propto H_0$ and: $\Delta T \propto \int_{\tau_0}^{\infty} \exp[-(3\gamma - 4)\tau/(2 - \gamma)]\, d\tau$, which is finite for $4/3 < \gamma < 2$. In the absence of a cosmological constant, in general SH models are not asymptotically isotropic. Nevertheless, these models can spend a long time close to a flat Friedmann model corresponding to a saddle point. Considering one non-tilting perfect fluid with $p_\perp = (\gamma_\perp - 1)\mu_\perp$ and one tilting fluid with $p = (\gamma - 1)\mu$ (with $\gamma_\perp < \gamma$), as the Friedmann equilibrium point is approached we obtain: $\Delta T \propto \int_{\tau_0}^{\infty} \exp[-(3\gamma - 4 - \frac{3}{2}\gamma_\perp(2 - \gamma))\tau/(2 - \gamma)]\, d\tau$. This means that for solutions spending a finite (but arbitrarily long) time close to the saddle F, exhibiting quasi-isotropic behaviour consistent with observations, the Hubble parameter as measured by the fluid, H_{fluid}, can become arbitrarily large.

There will be different anisotropic asymptotic end-states depending on Bianchi types. In the Bianchi type VII_0 model with a tilted γ-law perfect fluid [11,16], the future asymptotic state for $\gamma > 4/3$ was found to be anisotropic and extremely tilted with $H \propto e^{-2\tau}$ and $\Delta T \propto \int_{\tau_0}^{\infty} \exp[(8 - 5\gamma)\tau/(2 - \gamma)]\, d\tau$. For models with $\gamma > 8/5$, we find that this integral is finite. The Bianchi type VIII models are asymptotically extremely tilted for $1 < \gamma < 2$ and the asymptotic solution is an extremely Weyl-curvature dominated model [17,18], with $H \propto \tau^{\frac{1}{4}} \exp(-\frac{3}{2}\tau)$, and $\Delta T \propto \int_{\tau_0}^{\infty} \tau^{\frac{\gamma}{4(2-\gamma)}} \exp[-3(3\gamma - 4)\tau/2(2 - \gamma)]\, d\tau$, which is finite for $4/3 < \gamma < 2$.

It is known that for general spatially inhomogeneous perfect fluid models with a cosmological constant, the de Sitter solution with extreme tilt (where the tilt refers to the fluid tilt with respect to a congruence with an acceleration that tends to zero) is locally stable for $4/3 < \gamma < 2$ [12,19]. From Equations (3.43) and (3.28) of [19], we have that $H \propto H_0$ and

$$\Delta T \propto \int_{\tau_0}^{\infty} \exp\left(-\frac{3\gamma - 4}{2 - \gamma}\tau\right) d\tau, \tag{4}$$

which is finite for $4/3 < \gamma < 2$, as required. We note that for $4/3 < \gamma < 2$, the generic behaviour is $v \to 1$, which implies that inflation does not isotropize an ultra-radiative fluid. Even if inflation is turned off after a certain number of e-foldings, H_{fluid} can become arbitrary large. Nevertheless, as noted above, this result does not contradict the cosmic no-hair theorem [14].

Therefore, we have found that, for $\gamma > 4/3$, the fluid congruence becomes null with respect to the normal congruence in finite fluid proper time, and a 'kinematic singularity' develops for the fluid congruence. To fully understand the behaviour of these models and their physical properties, the dynamics need to be studied using a formulation adapted to the fluid (i.e., utilizing a fluid-comoving frame). By using the boost formulae relating the normal and fluid congruences this singular behaviour for the fluid congruence can be confirmed.

This mathematical instability might lead to some interesting physics. Expanding universes that come to a violent end after a finite proper time have arisen in a different context [2,20]. Models with a constant equation of state parameter $\gamma < 0$, dubbed "phantom energy", lead to a singularity commonly called the *big rip*. In this paradigm, during the cosmic evolution the scale factor grows more rapidly than the Hubble distance and consequently blows up in a finite proper time, and is typically characterized by a divergent pressure and acceleration. As the *big rip* singularity is approached, both the strong and weak energy conditions are violated. For the so-called *sudden future singularities* [20] the strong energy condition needs not be violated, yet a future singularity forms within finite time. In our examples, no energy conditions are violated; in fact, the energy density in the fluid frame, μ_{fluid}, tends to zero as the singularity is approached.

The Hubble scalar for the fluid frame can be computed from the boost formula for H, and is of the form $H_{\text{fluid}} = BH/\sqrt{1-v^2}$ [21]. The above examples lead to a diverging H_{fluid}, with $H_{\text{fluid}} \propto e^{-(q_{\text{fluid}}+1)\tau_{\text{fluid}}}$, and $\tau_{\text{fluid}} = B\tau$ in the limit. We then examine the value of the deceleration parameter q_{fluid} and compare with the critical value of -1. Equivalently, we can compare with a phantom fluid in a flat, isotropic model using the effective equation of state parameter γ_{eff}, give

$$\gamma_{\text{eff}} = \frac{2}{3}(1 + q_{\text{fluid}}), \tag{5}$$

with corresponding critical value of 0.

In the general case of inhomogeneous cosmological models with a cosmological constant, the de Sitter model with extreme tilt is the future asymptotic state for $\gamma > 4/3$, and we have

$$q_{\text{fluid}} = -\frac{3}{2}(3\gamma - 4) - 1 < -1. \tag{6}$$

Consequently, as the de Sitter asymptotic state is approached the dynamical effect of the ultra-radiative perfect fluid congurence behaves similarly to a phantom energy in an isotropic and spatially flat spacetime. Note that inflation does not stop the big rip from occurring.

Let us now briefly consider the SH models discussed above. For the LRS Bianchi type V models, on the approach to the extremely tilted Milne solution (for $\gamma > 4/3$) we find that

$$q_{\text{fluid}} = -\frac{3}{2}(3\gamma - 4) - 1 < -1.$$

For the Bianchi type VIII model with $\gamma > 4/3$ we again obtain

$$q_{\text{fluid}} = -\frac{3}{2}(3\gamma - 4) - 1 < -1.$$

For the Bianchi type VII_0 model with $\gamma > 8/5$ we obtain

$$q_{\text{fluid}} = -\frac{3}{2}(5\gamma - 8) - 1 < -1;$$

this difference arises due to the fact that the type VII_0 models are geometrically more special than Bianchi type VIII models. Similarly, for the two-fluid example, we have

$$q_{\text{fluid}} < -1,$$

for $\gamma > \frac{2(4+3\gamma_\perp)}{3(2+\gamma_\perp)}$. A similar behaviour also occurs for Bianchi type VII_h models (see [21]). In the generic SH models (such as, for example, the Bianchi type VIII model) the ultra-radiative perfect fluid effectively behaves dynamically like a phantom energy in the sense that the length scale and the Hubble scalar diverges as the future asymptotic state is approached. In the more special examples, the requirement is that the threshold equation of state must be equal to or higher than that of radiation.

Therefore, as the future asymptotic state is approached the ultra-radiative perfect fluid effectively behaves like a phantom energy in an isotropic and spatially flat spacetime. It is important to note that the energy conditions of the perfect fluid are nowhere violated.

Let us discuss the physical consequences of this dynamical behaviour in a little more detail. One can consider models that spend a period close to isotropy (i.e., close to a Friedmann saddle point), with a small tilt. Thereafter, the models begin to evolve away from isotropy. Since the tilt is non-zero, for $\gamma > 4/3$ the models generically evolve towards an asymptotic state with extreme tilt. As the tilt instability sets in during the transient regime, observers moving with the tilting fluid will experience a transition from a decelerating expansion to an accelerating expansion, and later, extremely accelerating expansion mimicking that of a phantom cosmology. Moreover, unlike in conventional phantom cosmology, in the models studied here there is no need for any exotic forms of matter; conventional matter which is tilting suffices. In a braneworld approach, accelerating universes can also result without a cosmological constant or other form of dark energy [22]. Indeed, other pathologies, such as the existence of ghosts, are avoided in the models described here. This is also the case in alternative models to phantom cosmology which result from alternative theories of gravity, theories with non-minimal couplings, and models in which the dark energy and quintessence field interact [23]. In addition, as noted above, due to the existence of future attractors with extreme tilt the dynamical behaviour described here is generic.

Although the dynamics in the comoving dark energy models and the tilting models are qualitatively similar, the actual quantitative (physical) predictions of the two different models may differ (due to the different physical transient time scales in each model). For example, the age of the universe as measured in the two models may differ. As an example, let us calculate numerically the age in a LRS Bianchi type V model which starts near a flat FL model at the time of decoupling. In a conventional comoving dust model (with no dark energy) with present energy density $\Omega_{\text{dust},0} \approx 0.2$, the maximum age is approximately 11.8 billion years (which, as is well known, is on the low side compared with the ages of the oldest astrophysical objects in the Universe). In a comoving dust model with present energy density $\Omega_{\text{dust},0} \approx 0.2$ and a cosmological constant $\Omega_{\Lambda,0} \approx 0.8$, the estimated age is about 15.0 billion years, which is about a 27% increase over the age for the dust model and consistent with current observational data. In a LRS Bianchi type V model with a comoving dust model with $\Omega_{\text{dust},0} \approx 0.2$ and a (second) tilting perfect fluid with a total effective energy density $\Omega_{\text{tilt},0} \approx 0.8$, then according to the tilted fluid observer with present energy density $\Omega_{\text{dust},0,\text{fluid frame}} = 0.2$, the maximum age as measured in the tilting fluid frame appears to be (depending on the initial conditions in the various numerical experiments) about 13.2 billion years (this particular value occurs for the initial conditions: $A_0 = 0.641$, $\Sigma_{+,0} = 0.0056$, $v = -0.443$; $\Omega_0 = 0.0111$). This is a 12% increase over the original age of 11.8 billion years (and marginally consistent with observations). The situation is expected to be similar in the more general Bianchi VIII models, although the quantitive predictions will depend on the precise initial conditions and it is possible more fine tuning may be necessary.

Therefore, although qualitatively the calculated age of the universe in both the dark energy models and the tilting fluid models are similar in that the age is greater than in the conventional models (without dark energy or a second tilting fluid), quantitatively the increased age appears to be greater in the dark energy models. However, the tilted fluid models are still physically viable. In future work we shall further study physical predictions of the tilting fluid models. In addition to the age problem, it is also of interest to investigate the effect of tilt on cosmic microwave background radiation observations and whether these models offer a possible explanation for the various anomalies on large angular scales found in the WMAP data [24,25].

Acknowledgements

This work was supported by a Killam Postdoctoral Fellowship (S.H.) and the Natural Sciences and Engineering Research Council of Canada (A.C.).

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