

# Deflection Sensors Utilizing Optical Multi-Stability

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**Abstract.** Deflection sensors have attracted significant attention due to their wide application in pressure and temperature measurements in practical systems. Several techniques have been proposed, studied, and tested to realize optical deflection sensor elements, including Mach-Zehnder (MZI), and Fabry-Pérot interferometers. In this work, a novel optical deflection sensor that is comprised of two cascaded optical resonators is proposed and analyzed. The proposed structure is designed to operate in the multi-stable (input to output) regime. As the first resonator is equipped with a movable mirror, which is connected to a diaphragm in order to sense changes in deflection, the second resonator is filled with non-linear material. It is demonstrated that such a structure has a novel memory property, aside from having the ability to yield instant deflection measurements. This novel property is attributed to the non-linear refractive index of the medium of the second resonator. Furthermore, the sensor sensitivity (which is the ratio of the change in the output light intensity to the change in the induced deflection) is enhanced due to the input-output multi-stable behavior of the proposed structure. This device possesses a promising potential for applications in future smart sensors.

**Keywords:** Fabry-Perot resonators, Intelligent sensors, Nonlinear optics, Optical bi-stability.

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## INTRODUCTION

The force of interaction between a manipulator and its environment can be measured by a number of technologies. As an external force is applied to a sensing element, a deformation or change in its shape is induced. This deformation can be detected either by using a measurement of the change of a physical property, such as resistance or capacitance, or directly through the use of an optical device. There are several types of micro force sensors, including strain gauge, piezoelectric, optical, and capacitive sensors. The optical sensors are recognized for their inherent advantages over other types of sensors. For example, optical sensors possess immunity to electromagnetic interference, which in turn improves the operability in a harsh environment, as well as a capability to accept more than one input. High miniaturization, flexibility, and light weight are further useful features offered when an optical route is chosen.

Usually, optical interferometric sensors are composed of a diaphragm that can be deformed and displaced by a certain distance from an optical fiber end or a mirror. This displacement results in the formation of a tuned or detuned resonant cavity [1].

The cavity length will change when a deflection of the diaphragm is applied. Once the change in the reflected or transmitted optical signal of a known wavelength is detected, the induced deflection can easily be measured [2,3]. Optical pressure sensors utilizing Mach-Zehnder interferometer (MZI), or Fabry-Perot interferometer, have been reported [4,5]. In a Fabry-Perot based sensor, the variation of the cavity length due to a displacement is read out as intensity changes. Similarly, in MZI based sensors, the phase change in the sensing arm of an MZI due to an applied pressure is read out also as intensity change.

On the other hand, non-linear optics has various applications based on different optical structures including optical resonators and MZI interferometers. A non-linear medium can be defined as the medium that has a dielectric polarization,  $P$ , responding in a non-linear fashion to the electric field,  $E$ , of the light. The non-linear behavior is usually employed in applications which involve high light intensities such as those typically produced by a laser.

Optical multi-stability is one known nonlinear optical phenomenon, which results in optical systems having multiple possible outputs for a given input. The studies of multi-stable optics have experienced significant attention, as the exploration of possible ways to control light by means of light is a potential towards all-optical processing. The principles of optical bi-stability can be utilized to produce a plethora of vital functions such as optical switching or limiting. In general, the bi-stable optics is crucial for fiber optic communications, optical logic circuits, and optical computers. The concept of a bi-stable optical device was originally reported in 1969. The idea was based on a saturable absorber in a Fabry-Perot etalon [6]. Intensive research has been focused on this area of study after the first experimental demonstration, articulated in 1975 [7]. In this particular work, the mechanism beyond the bi-stability is attributed to an optical field-induced refractive index change in a non-linear material filling a Fabry-Perot etalon [8]. This is known as the non-linear dispersion type of an intrinsic bi-stable optical device. Another type of optical bi-stable devices was demonstrated in 1977. This device type is known as using the hybrid bi-stability [9]. It is based on a second order non-linear electro-optical crystal placed inside a Fabry-Perot etalon. The refractive index change of the crystal is modulated by an external electric field which is proportional in strength to the optical feedback provided by the Fabry-Perot cavity.

In this work, a new modified structure is proposed. We suggest a sensor structure to consist of two cascaded optical resonators instead of a single resonant cavity. The role of the second resonator is to operate as an all-optical multi-stable element, while the first resonator transforms the deflection change into light change. As a multi-stable system, when the input light to the second resonator changes, the optical output follows a nontrivial function. That is because the properties of the nonlinear medium filling the second resonator are changing respectively with the light intensity. This results in unstable output ranges. Thus as the input of the second resonator is changing, the output will jump from one stable input to another, passing over the unstable ranges. In this work, we analyzed this behavior utilizing a linear stability criterion, and we present a phase diagram that matches stable outputs to their causing deflections. Thereafter, we found that such a deflection sensor is capable of recognizing previous deflection values, thanks to the optical multi-stability. Furthermore, the sensor sensitivity, which is the ratio of the change in the output light

intensity to the change in the induced deflection, is enhanced thanks to an intensity jump from one stable output value to another.

Although a material with a large third-order optical nonlinearity still represents a fundamental challenge to implementing efficient photonic devices based on nonlinear optics, a new development of techniques for fabrication and production of artificial materials has contributed substantially to the evolution of nonlinear optics, while the overall aim is to develop materials that possess large non-linear properties, and satisfy all of the technological requirements required in particular applications such as wide transparency range, fast response, and high damage threshold. In this work, our numerical estimations utilizing carefully chosen highly nonlinear materials show promising results.

The paper is organized as follows. In Section II the proposed structure is described and modeled. Section III presents the numerical simulations. Finally, section IV summarizes the results and conclusions.

## OPTICAL STRUCTURE AND MODELING

As depicted in Fig. 1, the first resonator has a movable mirror that converts the applied deflection into a change on the second resonator's input light. The second resonator is filled with a nonlinear material. A convenient nonlinear material that possesses a strong nonlinear refractive index and weak absorption is required. After conducting a material's survey, we found that castor oil is a good candidate [10]. A numerical example of the evolution of the proposed structure, utilizing the castor oil, will be provided.

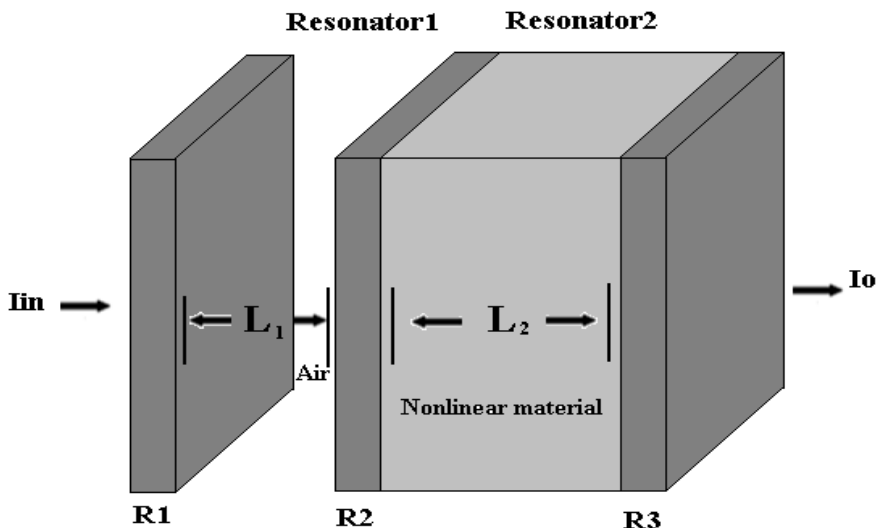


FIGURE 1. Optical structure.

The entire structure can be optically modeled by the light transmission. The overall light transmission can be given by [1]

$$I_o = T_T I_{in}, \quad (1)$$

$$T_T = 1 - R_T, \quad (2)$$

$$R_T = \frac{R_1 + R_{NL} - 2\sqrt{R_1 R_{NL}} \cos \phi_1}{1 + R_1 R_{NL} - 2\sqrt{R_1 R_{NL}} \cos \phi_1}, \quad (3)$$

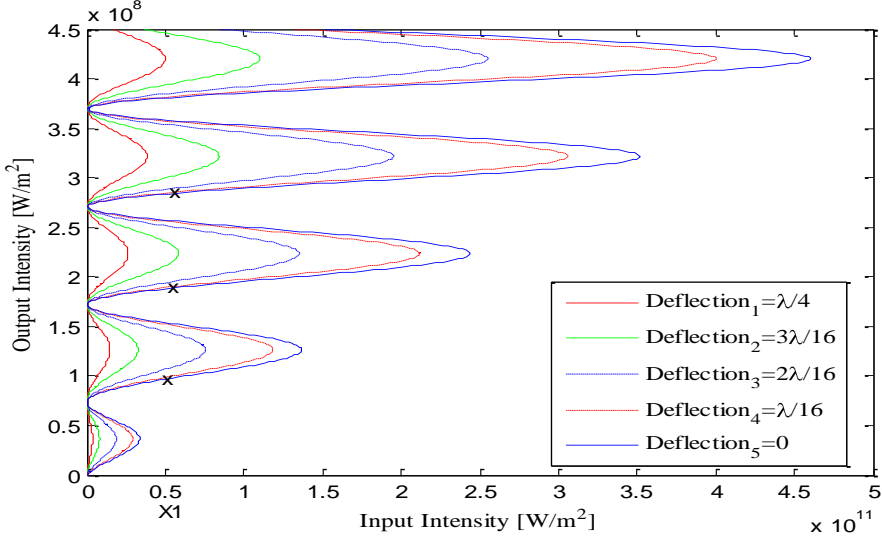
where  $\phi_1 = \frac{4\pi L_1 n_1}{\lambda}$ ,  $L_1$  is the air-gap length,  $n_1$  is the linear refractive index of resonator 1,  $\lambda$  is the wavelength of the light source,  $R_1$  is the reflectivity of mirror 1. The nonlinear reflectivity of the second resonator  $R_{NL}$  can be given by [11, 12]

$$R_{NL} = \frac{1 + R_o^2 e^{-2\alpha L_2} - 2R_o e^{-\alpha L_2} \cos \phi_2 - (1 - 2R_o + R_o^2) e^{-\alpha L_2}}{1 + R_o^2 e^{-2\alpha L_2} - 2R_o e^{-\alpha L_2} \cos \phi_2}, \quad (4)$$

where  $R_o = R_2 = R_3$ ,  $R_2$  is the reflectivity of mirror 2,  $R_3$  is the reflectivity of mirror 3,  $\phi_2 = \frac{4\pi L_2}{\lambda} (n + 2 \frac{I_o}{T_{R_o}} n_2)$ ,  $L_2$  is the nonlinear material length,  $n$  is the linear refractive index of resonator 2,  $n_2$  is the nonlinear refractive index of resonator 2, and  $T_{R_o} = 1 - R_o$ ,  $\alpha = \alpha_o + 2 \frac{I_o}{T_{R_o}} \beta$ ,  $\alpha_o$  is the linear absorption coefficient and  $\beta$  is the nonlinear absorption coefficient. Here, all the mirrors are assumed lossless.

## NUMERICAL SIMULATIONS AND RESULTS

As the second resonator is optically multi-stable, a given input value has multiple possible output values. In addition, the first resonator is designed to transform the applied deflection change into a change in the second resonator light input. Therefore, in the case of having a fixed light input to the first resonator, a given deflection has multiple transmitted output light values. The relationship between the input and output intensities at different deflections is shown in Fig. 2.



**FIGURE 2.** The input and output intensities at different deflections

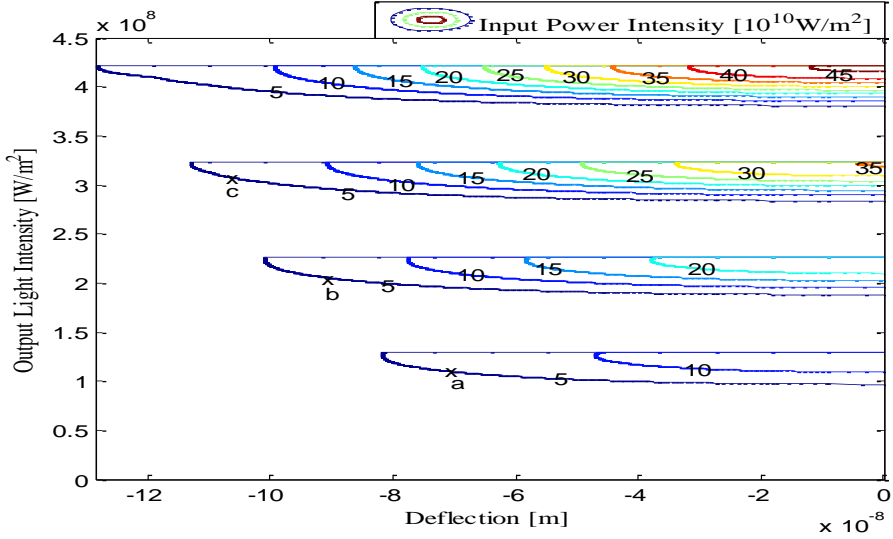
As can be concluded from Fig. 2, the output space cannot directly be calculated from presumed inputs. This is because there are no unique solutions, as the case in most nonlinear systems. For example, for a given input light value, let's say  $X_1$  in Fig. 2, there are many possible output values. We approached this numerical dilemma by calculating the output-input pairs in an inverse direction. We presumed reasonable outputs and then solved the corresponding input values.

However, in the case of any multi-stable system, some of the possible input-output pairs might not be stable. Thus, a stable criterion is needed to identify the stable input-output pairs. Applying the linear stability criteria, it can be shown that each stable input-output pair obey the following condition [13]

$$\left(\phi_{2NL} \frac{\partial T_{NL}}{\partial \phi_2}\right) \left\langle \frac{T_{NL}}{I_o} \right\rangle, \quad (5)$$

where  $\phi_{2NL} = \frac{4\pi d_2}{\lambda} \left(2 \frac{I_o}{T_{R_o}} n_2\right)$  and  $T_{NL} = 1 - R_{NL}$ .

Hereby, a phase diagram of all possible stable optical outputs for a given input, as a function of the induced deflection can be obtained. In Fig. 3, such a phase diagram is shown, utilizing the castor oil in the second resonator.



**FIGURE 3.** The phase diagram of all possible stable optical outputs.  $R_1=0.35$ ,  $R_2=0.96$ ,  $R_3=0.96$ ,  $L_1=10.15 \mu\text{m}$ ,  $L_2=40 \mu\text{m}$  and  $\lambda=514 \text{ nm}$ .

Observing Fig. 3, one can see that the system evolution shows a hysteresis-like behavior, on the transmitted output light versus the applied deflection. In other words, at a certain induced deflection, the value of the transmitted optical output depends on the history of that applied deflection, as well as on its absolute value. For example, referring to Fig. 3, if the input intensity is set to be  $5 \times 10^{10} \text{ (W/m}^2\text{)}$  and the optical output threshold value taken as  $0.977 \times 10^8 \text{ (W/m}^2\text{)}$  then as a deflection is applied, the output intensity will be correspondingly increased. If the applied deflection is released, after the output light reached point (a), the output light will consequently go back to  $0.977 \times 10^8 \text{ (W/m}^2\text{)}$ . However, if a further deflection is applied, the output will jump to point (b). Consequently, if the applied deflection is released, the output will be at another second threshold value of  $1.83 \times 10^8 \text{ (W/m}^2\text{)}$ . On the other hand, after reaching point (b), if a further deflection is applied, the output will be driven to point (c), where a third threshold value of  $2.85 \times 10^8 \text{ (W/m}^2\text{)}$  is obtained at zero applied deflection, and so on and so forth. Thus, it is evident that the system offers to memorize previous applied deflections.

Moreover, the sensor sensitivity is expected to be enhanced, thanks to abrupt optical output jumps due the presence of unstable output ranges. The nonlinear reflectivity of the second resonator  $R_{NL}$  will change because it depends on the light intensity inside the resonator 2 due to change in deflection, causing a change in total output intensity. The sensitivity of the whole structure can be derived as

$$I_{sen} = \left| \frac{\partial I_o}{\partial L_1} \right| = \left| \frac{\partial}{\partial L_1} \left[ \frac{I_{in} (1 + R_1 R_{NL} - R_1 - R_{NL})}{1 + R_1 R_{NL} - 2\sqrt{R_1 R_{NL}} \cos \phi_1} \right] \right|$$

$$\begin{aligned}
&= \frac{I_{in} \left[ \frac{\partial R_{NL}}{\partial L_1} (R_1 - 1)(1 + R_1 R_{NL} - 2\sqrt{R_1 R_{NL}} \cos \phi_1) \right]}{(1 + R_1 R_{NL} - 2\sqrt{R_1 R_{NL}} \cos \phi_1)^2} \\
&\quad \frac{\left[ R_1 \frac{\partial R_{NL}}{\partial L_1} - \left( \frac{1}{\sqrt{R_{NL}}} \frac{\partial R_{NL}}{\partial L_1} \cos \phi_1 - \frac{8\pi}{\lambda} \sqrt{R_1 R_{NL}} \cos \phi_1 \right) (1 + R_1 R_{NL} - R_1 - R_{NL}) \right]}{(1 + R_1 R_{NL} - 2\sqrt{R_1 R_{NL}} \cos \phi_1)^2}, \quad (6)
\end{aligned}$$

where  $\frac{\partial R_{NL}}{\partial L_1}$  is the ratio of the change in nonlinear reflectivity of the second resonator,  $R_{NL}$ , to the change in the induced deflection, which can be given as

$$\frac{\partial R_{NL}}{\partial L_1} = \frac{\partial R_{NL}}{\partial I_o} \frac{\partial I_o}{\partial L_1}, \quad (7)$$

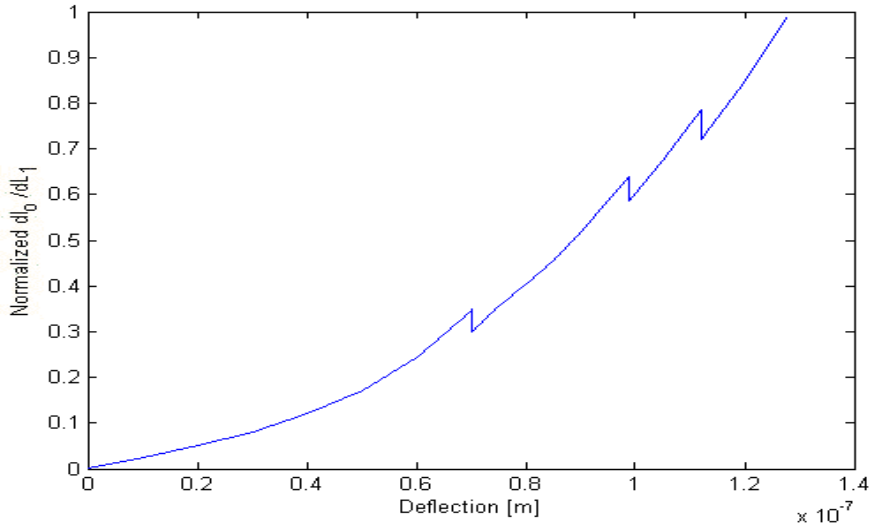
where  $\frac{\partial R_{NL}}{\partial I_o} = \frac{16R_o \pi L_2 n_2 e^{-2\alpha L_2} \sin \phi_2 (1 - 2R_o + R_o^2)}{\lambda T_{R_o} (1 + R_o^2 e^{-2\alpha L_2} - 2R_o e^{-\alpha L_2} \cos \phi_2)^2}$ .

Substitute Eq. (7) in Eq. (6), the sensitivity of the whole structure can be written as

$$I_{sen} = \frac{\left[ -\frac{8\pi}{\lambda} (1 + R_1 R_{NL} - R_1 - R_{NL}) \sqrt{R_1 R_{NL}} \sin \phi_1 \right]}{\frac{\delta^2}{I_{in}} - \frac{\partial R_{NL}}{\partial I_o} \left[ (R_1 - 1)\delta + (-R_1 + \left( \sqrt{\frac{R_1}{R_{NL}}} \right) \cos \phi_1)(1 + R_1 R_{NL} - R_1 - R_{NL}) \right]}, \quad (8)$$

where  $\delta = 1 + R_1 + R_{NL} - 2\sqrt{R_1 R_{NL}} \cos \phi_1$ .

Figure 4 shows the sensor sensitivity of the structure, utilizing the castor oil in the second resonator.



**FIGURE 4.** The system sensitivity.

As can be seen in Fig. 4, the sensitivity is enhanced for the wider range of the applied deflection. This is another novel property of the structure in addition to the memory-like property.

## CONCLUSIONS

An optical multi-stability resonator structure has been proposed to be used as a deflection sensing element. The structure consists of a linear and a nonlinear resonator connected in a cascaded fashion. The first resonator is filled with air and has a movable mirror that is coupled to a diaphragm. The second resonator is filled with a nonlinear optical material.

We found that the dependence of the system evolution on the induced deflection exhibits a hysteresis-like character. In other words, the value of the optical output, at a certain induced deflection, depends on the history of that induced deflection. In addition, the system sensitivity is found to be enhanced thanks to an intensity jump from one stable output value to another. We believe that these novel properties increase the controlling and the sensing functionality and flexibility of novel devices and systems.

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