

A REGIME SWITCHING MULTIFACTOR MODEL FOR THE STOCK AND
BOND RETURNS

by

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Submitted in partial fulfilment of the requirements
for the degree of Master of Arts

at

Dalhousie University
Halifax, Nova Scotia
August 2012

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DALHOUSIE UNIVERSITY
DEPARTMENT OF ECONOMICS

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Dated: August 24th, 2012

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DALHOUSIE UNIVERSITY

DATE: August 24th, 2012

AUTHOR: Shuichang Xie

TITLE: A REGIME SWITCHING MULTIFACTOR MODEL FOR THE STOCK
AND BOND RETURNS

DEPARTMENT OR SCHOOL: Department of Economics

DEGREE: MA CONVOCATION: October YEAR: 2012

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Table of Contents

List of Tables	vi
List of Figures	vii
Abstract	ix
List of Abbreviations and Symbols Used	x
Acknowledgments	xii
Chapter 1 Introduction	1
Chapter 2 Literature Reviews	6
2.1 CAPM and APT	6
2.2 Risk Factors	7
2.3 Regime-switching Model	8
Chapter 3 Data	11
3.1 Data Description	11
3.2 Descriptive Statistics	12
Chapter 4 Methodology	18
4.1 Regime-switching Model	18
4.2 Parameter Estimation	19
4.3 Number of Regime	20
4.4 Out-of-sample Performance	20
4.5 Means and Correlations under Different Regimes	22

4.6	Mean-variance Efficient Frontier	22
4.7	Portfolio Implication	25
Chapter 5 Empirical Analysis		27
5.1	Number of Regimes and Its Interpretation	27
5.2	Parameter Estimation in the RS Model	29
5.3	Out-of-sample Performance in the RS Model	34
5.4	Regime-dependent Mean-Variance Frontiers	34
5.5	Portfolio Performance	40
5.5.1	Regime Inferred Portfolio (RIP)	41
5.5.2	Probability Implied Portfolio (PIP)	46
Chapter 6 Conclusion and Discussion		52
6.1	Conclusion	52
6.2	Discussion	53
Bibliography		55
Appendix: EM Algorithm		60

List of Tables

3.1 Abbreviations, Measurements and Sources	13
3.2 Descriptive Statistics	15
5.1 BIC and Maximum Log Likelihood(MLL) for Different Regimes	27
5.2 Regime-dependent Descriptive Statistics	30
5.3 Estimated Parameters in the RS and SR Model	32
5.4 Variance-covariance of Residuals for the SR and RS Models	33
5.5 Weights of Stock and Bond for the Tangent Portfolios	39
5.6 Sharpe Ratios for RIP and the Compared Portfolios (In-sample)	44
5.7 Sharpe Ratios for RIP and the Compared Portfolios (Out-of-sample)	45
5.8 Sharpe Ratios for PIP and the Compared Portfolios (In-sample)	49
5.9 Sharpe Ratios for PIP and the Compared Portfolios (Out-of-sample)	51

List of Figures

3.2.1 Histograms of Monthly Stock and Bond Returns	14
3.2.2 Historical Performances of Standardized Stock and Bond Index . . .	16
3.2.3 Average Correlations between Stock and Bond Returns	17
5.1.1 Posterior Probability of Regimes and Histogram of Regimes	29
5.2.1 Comparison of Predicted Stock Return in the RS and SR Models (In-sample)	35
5.2.2 Comparison of Predicted Bond Return in the RS and SR Models (In-sample)	36
5.3.1 Comparison of Predicted Stock and Bond Returns in the SR and RS Models (Out-of-sample)	37
5.4.1 Regime-dependent Mean-variance Frontiers	39
5.5.1 Performance of Regime Inferred Portfolio (In-sample)	42
5.5.2 Weights of Stock and Bond in Regime Inferred Portfolio (In-sample)	43
5.5.3 Performance of <i>RIP</i> and the Comparison Portfolios under the Weight Constraints (In-sample)	44
5.5.4 Weights for Stock and Bond in Regime Inferred Portfolio under the Weights Constraint (In-sample)	45
5.5.5 Performance of <i>RIP</i> and the Comparison Portfolios and Weights of Bond and Stock in <i>RIP</i> (Out-of-sample)	46
5.5.6 Performance of Probability Implied Portfolio (In-sample)	47
5.5.7 Weights of Stock and Bond in Probability Implied Portfolio (In- sample)	48

5.5.8 Performance of <i>PIP</i> and the Comparison Portfolios under the Weight Constraints (In-sample)	48
5.5.9 Weights for Stock and Bond in Probability Implied Portfolio under the Weights Constraints (In-sample)	49
5.5.10 Performance of <i>PIP</i> and the Comparison Portfolios and the Weights of Bond and Stock in PIP (Out-of-sample)	50

Abstract

In contrast to the studies of constant or time-varying correlations between stock and bond returns, in this thesis, I explore the regime-dependent correlations between stock and bond returns. Specifically, I start with a comprehensive asset pricing model, i.e., a regime-switching multifactor model, and then investigate the regime-dependent correlations between stock and bond returns. Based on the BIC, the number of regimes in the regime-switching model is optimally determined to be two. For the two regimes, the directions of the regime-dependent correlations appear to be significantly different. Also, the magnitudes of the regime-dependent correlations are substantially larger in these two regimes than the correlation in the single regime.

With my findings in the regime-dependent correlations, I then examine the performance of portfolio strategies. Throughout the in-sample and out-of-sample tests, I find that the two portfolio strategies, regime inferred portfolio and probability implied portfolio, can outperform the benchmark, S&P 500.

List of Abbreviations and Symbols Used

Abbreviations:

- AIR: Actual Inflation Rate
- APT: Arbitrage Pricing Theory
- BIC: Bayesian Information Criterion
- CAPM: Capital Asset Pricing Model
- CBOE: Chicago Board Options Exchange
- COP: Change of Oil Price
- EDY: Equity Dividend Yield
- ETF: Exchanged Traded Fund
- JB-test: Jarque-Bera Test
- MLL: Maximized Log Likelihood
- SR: Single Regime
- RS: Regime-switching
- RIP: Regime Inferred Portfolio
- PIP: Probability Implied Portfolio
- UCS: US Credit Spread
- UIP: Growth of Industrial Output
- VIX: Change of CBOE Volatility Index

Symbols:

- r_s : Stock Return

r_b : Bond Return
 r_f : Riskfree Rate
R: Predicted Return Vector
F: Factors
 M : Regimes
 α_{M_t} : Regime-dependent Intercept
 β_{M_t} : Regime-dependent Beta/Coefficient
 ϵ_{M_t} : Regime-dependent Error Matrix
 Σ_{M_t} : Regime-dependent Variance-covariance Matrix
 p : Transition Probability
P: Regimes Transition Matrix
 K : Number of Regimes
 q : Posterior Probability
q: Vector of Posterior Probability
 V : Portfolio Return
W: Regime-dependent Weight Vector
 ω : Weight Vector
 λ : Lagrangian Multiplier

Acknowledgments

I would not have been able to complete this project without the help from a number of people. First and foremost, I would like to thank my supervisor, Dr. Yonggan Zhao, who has been abundantly generous with his time and efforts. Without his patience and consideration over the past several months, I would surely not have completed this research. I am also thankful for his financial support during this project.

I am grateful to my committee member, Dr. Leonard C. MacLean, for the invaluable time he gave to this project. Special thanks go to Dr. Kuan Xu, who has taught and inspired me both in and outside of the classroom. I truly appreciate his encouragement throughout my two-year graduate degree, especially the enthusiasm and support during the early stage of this project.

I would like to acknowledge the financial support from the Department of Economics, and the editorial assistance from the Writing Center. I would like to thank all of my family and friends for their support and encouragement throughout my life.

Chapter 1

Introduction

As the importance of diversification in portfolio management was recognized after the work of mean-variance analysis by Markowitz (1952), the correlations among asset returns in a portfolio have drawn substantial attention from both academics and practitioners. One of the most important asset returns correlations is the correlation between stock and bond returns provided that a typical portfolio is largely constituted by equities and bonds. In the current investment community, it is not surprising to see a financial market phenomenon, called “flight to quality”, which is an investor behavior in the financial market. That is, investors transfer their investment into safer financial assets, like treasury bonds, when investors experience a bearish equity market. Hence, understanding the co-movement between stock and bond returns might have a significant role to play for investors, particularly for money managers.

There is extensive academic literature exploring this area, and it can be grouped into two categories: the direction and magnitude of correlation. From the standpoint of discounted cash flows, it seems that stock returns should be positively correlated with bond returns. Shiller (1982) argues that the prices of bond, land and other assets, which can result in cash streams in the future, should co-vary with stock prices. Shiller and Beltratti (1990) point out that there is a positive correlation between excess stock and bond returns. In practice, however, it is not

uncommon to see stock returns negatively correlated with bond returns. Defining the excess stock and bond returns as the differences between monthly returns and the one-month T-bill rate, Campbell and Ammer (1991) study the factors that drive stock and bond prices. They find a small correlation between excess stock returns and excess bond returns, and a marginal effect of real interest rate on stock and bond returns. They claim that the excess stock return is mainly from news about the future excess stock returns, while excess bond returns are mainly from news regarding the future inflation. Connolly, Stivers and Sun (2005) explore how the time varying correlation between daily stock and Treasury bond returns can explain the uncertainty of the stock market, and conclude that the bond return is relatively higher than the stock return when the stock market is increasingly volatile.

In terms of the magnitude of the correlation between stock and bond returns, academics tend to agree that the correlation appears to be time-varying and also vary across different economic conditions. Scruggs and Glabadanidis (2003) strongly reject models imposing a constant correlation restriction on stock and bond returns. Barsky (1989) claims that the comovement between stock and bond returns relies on economic conditions. Specifically, he emphasizes that it is likely for companies to face lower profit and real interest rates when the economy is experiencing lower productivity and higher market risk. The decrease in profit and real interest rates would result in an ambiguous effect on both stock and bond returns.

Although there are a lot academics who have examined the correlation between stock and bond returns from both the perspective of direction and magnitude and pointed out that this correlation is time-varying, it seems only a few of them focus on this correlation under different economic states. Some studies employ regime-switching models to explore the correlation between asset returns. For example, Bansal, Connolly and Stivers (2009) adopt a bivariate regime-switching model to investigate the daily future contract returns for US stock and ten-year

Treasury notes, and then conclude that a higher stock volatility is associated with lower stock-bond correlation and higher mean return of bonds. A similar study is conducted by Hobbes, Lam and Loudon (2007). They use an autoregressive regime-switching model to reflect how stock returns are correlated with bond return under different economic states. The most recent study is from Baele, L., Bekart, G. and Inghelbrecht, K. (2010). They study the determinants of correlation between stock and bond returns, and find that the fundamental factors of macroeconomics contribute little to the correlation between bond and stock returns, but have impacts on the bond returns. Thus, in this thesis, I analyze the evolution of this correlation over different economic regimes under the framework of a regime-switching factor model.

The motivation to investigate this correlation is from several facts. First of all, the stock and bond returns tend to be positively correlated in the early 1990s, and then this followed by a seemingly random pattern between positive and negative correlation after the collapse of the dotcom bubble. Secondly, a number of historical studies focus on either the constant correlation versus the time-varying correlations between stock and bond returns. It is worthwhile to take a look at the “midpoint”, the regime-dependent correlations, between stock and bond returns. Pelletier (2006) studies the variance between multiple time series, and finds that the regime-dependent correlations¹ can fit the data better than the dynamic conditional correlations. Moreover, the importance of the correlations between stock and bond returns in portfolio construction also motivates the setup of this study. Finally, the historical studies (Guidolin and Timmerman (2005), Liu, Xu and Zhao (2011)) have successfully employed the RS factors model to price asset returns.

¹ According to Pelletier (2006), regime-dependent correlation is defined as that the correlation between two time series is constant within a regime, and is different across the regimes. This definition is applied to the entire thesis.

The first objective of this research is to adopt a regime switching multifactor model to estimate stock and bond returns, and then explore the regime-dependent correlations between stock and bond returns. The second objective of this research is to utilize the regime-dependent correlation, and compare the mean-variance frontiers for the bond-stock portfolio under different regimes. Finally, based on the previous findings, I examine the performance of portfolio strategies, which can potentially be implemented in the current investment community, under the framework of RS model.

To address the above objectives, I adopt a generic stock and bond pricing model, i.e., RS multifactor mode. Following the Bayesian information criterion (BIC), the number of regimes in the RS model is optimally determined to be two. Based on the characteristics of each regime, these two optimal regimes are interpreted as the negative regime in regime 1 and the positive regime in regime 2 respectively. Under the framework of the two-regime RS model, I find that the six utilized factors have predictive power for either the future stock or bond returns. To confirm the forecast ability in the adopted RS model, both the in-sample and out-of-sample tests are conducted. The in-sample test evidently shows the RS model forecasts the actual returns better than the single regime model, i.e., the multiple linear regression model without regimes. For the out-of-sample test, it is unclear, especially for the stock return, whether the RS model predict the returns better than the SR model, but the RS model does show a clearer up-and-down trend compared with the SR model.

Thereafter, by referring to the regime-dependent variance-covariances of the residuals in the RS and SR models, I find that the regime-dependent correlations are significantly different across the two regimes. Following these regime-dependent correlations, I investigate the regime-dependent mean-variance frontiers. The results are that the positive regime has the best trade-off between return and risk, and the negative regime has the smallest global minimum variance portfolio among the three frontiers. Moreover, for each regime, the correspond-

ing tangent portfolio is constructed, and it is interesting to see that the Sharpe ratio of the tangent portfolio in either regime 1 or regime 2 is greater than that in single regime. Finally, with the previous findings, I examine the performance of two portfolio strategies under the framework of RS model. The results suggest that both the portfolios, regime implied portfolio and posterior probability implied portfolio, can outperform the benchmark, S&P 500, in both the in-sample and out-of-sample tests.

The remaining of this thesis is organized as follows. Chapter 2 reviews the historical studies pertaining to this study. Chapter 3 describes the data being used in this thesis. Chapter 4 shows the methodology that this thesis adopts, and then followed by a comprehensive empirical analysis in Chapter 5. Lastly, I conclude the findings from the empirical analysis and discuss the possible future works in Chapter 6.

Chapter 2

Literature Reviews

It is widely known that the intrinsic value of an asset is the discounted value of its cash flows that this asset can generate in the future. That is, the discount rates and forecast cash flows of assets centralize the prices of assets. Nonetheless, the investigation of asset returns, i.e., the discounted rates, has captured much attentions from academics since the work of Harry Markowitz (1952), who pioneers the modern portfolio theory with his work of the mean-variance analysis.

In this chapter, I would review some current pricing models. In this thesis, I mainly focus on the review of factor pricing models, with an emphasize on CAPM and APT. Meanwhile, the review of risk factors and Markov regime-switching models are conducted.

2.1 CAPM and APT

In the modern finance, one of the most influential works that should be mentioned is the Capital asset pricing model (Sharpe (1964)), which measures the required rate return of an asset relative to the return of the market. This model is built on the mean-variance analysis of Markowitz (1952). A number of other financial professionals, such as Treynor (1961, 1962), Lintner (1965a,b) and Mossin (1966), have also all contributed to the development of this asset pricing model.

Admittedly, the CAPM paves the way of the modern finance. However, the

model has several limitations. For instance, one of the key assumptions is that the asset returns and market returns are jointly normally distributed. However, due to the extreme events, this assumption generally does not hold, especially for stock returns, which tends to have a fat left tail. Moreover, the empirical studies (Fama and French (1992) and Black (1993)) find that this model is not supported by the historical data. One of the main reasons why this model fails is because there are other factors that can contribute to assets returns. To some extent, this weakness in CAPM motivates the study of the arbitrage pricing theory from (Ross (1976)).

In contrast to the assumption of the CAPM that only the market return can measure the riskiness of assets, the arbitrage pricing theory (APT) states that there are several risk factors that should be taken into account when the risk of an asset is measured. In terms of what factors should be considered, a number of studies have been carried out since the presence of APT, and some selected studies are reviewed in the next section.

2.2 Risk Factors

Since the development of the APT model of Ross (1976), there has sprung up a number of literature that is trying to identify what factors, except the market return, can contribute asset returns.

Fama and French (1993) study the common risk factors for the stock and bond returns and five common risk factors are identified. Those factors are: an overall market factor, firm-size related factor, book to market equity, maturity and default risks. Chan, Karceki and Lakonishok (1998) comprehensively explore what factors can explain the stock and bond returns. Specifically, a set of different styles of factors, such as fundamental factors, technical factors, macroeconomic factors, statistical factors and the market factors, are used to examine the explanation power of the stock and bond returns in their work. They find that the

default premium and term premium can help explain the asset returns, while the macroeconomic factors show poor explanations for asset returns.

Some studies find that several macroeconomic factors, such as the inflation rate, the growth rate of industrial output, and the change of oil prices, provide forecast ability for future asset returns. For example, Chen, Roll, and Ross (1986) examine the impact of economic variables on the stock market. They claim that the industrial output and inflation rates have strong impacts on the stock return. A group of studies, which use different sets of macroeconomic variables to model asset returns, also show the importance of macroeconomic factors on asset pricing (Fama and French (1989), (1993), Ferson and Harvey(1991), Shanken and Weinstein(1990)).

In the paper of Chen, Roll, and Ross (1986), it is shown that yield spread and credit spread can significantly explain the expected stock return. Similar conclusions are drawn from several other studies (Keim and Stambaugh (1986), Campbell (1987), Clare and Thomas (1992)). Another important factor that has been examined frequently is the factor of dividend yield. A number of studies (Rozeff (1984), Harvey (1991), Ferson and Harvey (1991)) have shown that dividend yield can be used to predict bond and stock returns. Most recently, the factor of CBOE Volatility Index, which measures the volatility of stock return has been successfully added to predict asset returns. The proven empirical analysis can be found in the studies, such as Connolly, Stivers and Sun (2005) and Liu, Xu and Zhao(2011). Both of them point out that the factor of volatility has some explanation power for asset returns.

2.3 Regime-switching Model

Although the APT addresses some weaknesses of the CAPM, there are still some limitations with the APT model. One limitation is the linear relation between asset returns and risk factors, and the other one is the static multifactor model.

It is well known that the risk premiums for asset returns tend to be time-varying. For example, Liu, Zhao and Xu (2011) study the time-varying risk premium for sector selected ETFs and find that the sensitivities to the factors are regime-varying. Hence, a model, which is capable of capturing this characteristics, should be utilized.

Recently, Markov regime-switching models, which is pioneered by the work of Hamilton (1989), have been favorably adopted in several areas in financial economics, such as, asset pricing, the term structure of interest rate, the exchange rate and the joint distribution of bond and stock returns. For instance, as for asset pricing, it is generally known that the distribution of asset returns appears to be regime-dependent. To be specific, it is observable that the stock return in a bullish regime performs better than that in a bearish regime, which, in other words, means that the distribution of the stock return in these two regimes are different. Ang and Bekaert (2002a) explore the equity returns in US, Germany and UK and find the characteristics for the equity returns are different in the two identified regimes.

The advantage of a regime-switching model is that this model is capable to identify the unobserved regimes, and allow the estimated parameters to be regime-dependent. From the perspective of the predictive power of models, the regime-dependent parameters are usually more favorable and can result in better forecasting performance compared with the general linear models. For example, Guidolin, Hyde, McMillan and Ono (2010) investigate the predicted power of macroeconomic factors on UK asset returns. They compare the predictive performance of different models and find that the Markov regime switching model shows the strongest predictive power among the several asset pricing models.

Lastly, the work from Ang and Bekaert (2002b), in which they apply the RS model to the asset allocation, shows that portfolio constructed under the regime-switching model can perform better than the portfolios with static strategies. Afterthat, similar studies have also been explored. For example, Mulvey and

Zhao (2011) develop an investment model under the framework of RS model. They claim that both the in-sample and out-of-sample tests support the fact that the proposed investment model can significantly outperform the other portfolios.

Chapter 3

Data

This chapter provides a general data description, including the measurement of stock and bond returns and factors as well as the data sources, and the descriptive statistics of data.

3.1 Data Description

In this thesis, I analyze the data from the perspective of US investors. I assume that there are only two classes of risky assets, i.e., equity and bond, that are available for investment. I use the monthly data to analyze their returns. The S&P 500 Index and the US Benchmark 10 Year Datastream Government Index are respectively used to measure the stock and bond returns.

The full sample period¹ is from February 1990 to February 2012 with the in-sample period from February 1990 to February 2010, and the out-of-sample period from March 2010 to February 2012. The reason why this sample period is selected is because the financial market experiences various stages², such as the prosperity in early 1990s and the following dot-com bubble as well as the most recent financial crisis. Thus, the empirical results from this sample period would

¹As one of the objectives of this thesis is to use the risk latent factors to predict stock and bond returns, then the time period for the factors is from February 1990 to January 2012, while the time period for the stock and bond returns is from March 1990 to February 2012.

²For example, in stock markets, there are bullish stage, bearish stage.

be more convincing.

In terms of factors, as discussed in Chapter 2, there is extensive literature that has confirmed that there indeed exists a set of factors that can be applied to predict future asset returns. In this thesis, the factors that are utilized to forecast the stock and bond returns can be categorized as the macroeconomic factors and non-macroeconomic factors.

For the macroeconomic factors, as summarized in the literature review, historical studies have shown that the unexpected inflation rate, actual inflation, the growth rate of industrial output have strong predictive power for future asset returns. In this thesis, three fundamental macroeconomic indicators are used to capture the future bond and stock returns. These factors are: change of oil prices, growth rate of industrial output and actual inflation rate. The three non-macroeconomic indicators being considered in this thesis are: change of CBOE volatility index, change of equity dividend yield and US credit spread. Table 3.1 documents the abbreviation, measurement, and the data sources of each factor, plus the stock and bond returns, and the riskfree rate.

3.2 Descriptive Statistics

Table 3.2 presents the summary statistics of each variable for the full sample period. All the numbers are in percentage. As expected, the average monthly stock return is higher than the average monthly risk-free rate (0.2807%)³, while it is surprising to see that the average monthly bond return is lower than the average monthly risk-free rate. Interestingly, the coefficient of variation of stock return (8.2056) is lower than the coefficient of variation of bond return (15.0007), which means, for a certain level of investment return, investors in the 10-year government bond are required to take a relatively higher risk compared with investors for equity.

³The average monthly risk-free rate refers to the average of the 3-month T-bill rate divided by 12 during the full the sample period.

Table 3.1 Abbreviations, Measurements and Sources

Abbr.	Variables	Measurements	Sources
r_s	Stock Return	$100 * (\ln(P_t) - \ln(P_{t-1}))$	S&P 500 Index, <i>Datastream</i> .
r_b	Bond Return	$100 * (\ln(P_t) - \ln(P_{t-1}))$	US Benchmark 10 Year Datastream Government Index, <i>Datastream</i> .
r_f	Riskfree Rate	r_f	US Three-month T-bill Rate, <i>Datastream</i> .
VIX	CBOE Volatility Index	$100 * (\ln(P_t) - \ln(P_{t-1}))$	CBOE SPX Volatility VIX, <i>Datastream</i> .
EDY	Equity Dividend Yield	$DR_t - DR_{t-1}$	US S&P 500 Composite - Dividend Yield (DR), <i>Datastream</i> .
UCS	U.S. Credit Spread	$IR_t - BR_t$	US Monthly Interbank Rate (IR), US 3-Month T-bill Rate (BR), <i>Datastream</i> .
COP	Change of Oil Price	$100 * (\ln(P_t) - \ln(P_{t-1}))$	World Market Oil Price Index, <i>Datastream</i> .
UIP	Growth Rate of Industrial Output	$100 * (\ln(P_t) - \ln(P_{t-1}))$	US Industrial Production Index, <i>Datastream</i> .
AIR	Actual Inflation Rate	AR_t	US CPI All Urban Sample: All Items - Annual Inflation Rate (AR), <i>Datastream</i> .

Table 3.1 shows the abbreviations, measurements and data sources for the stock return, bond return, the riskfree rate and the factors.

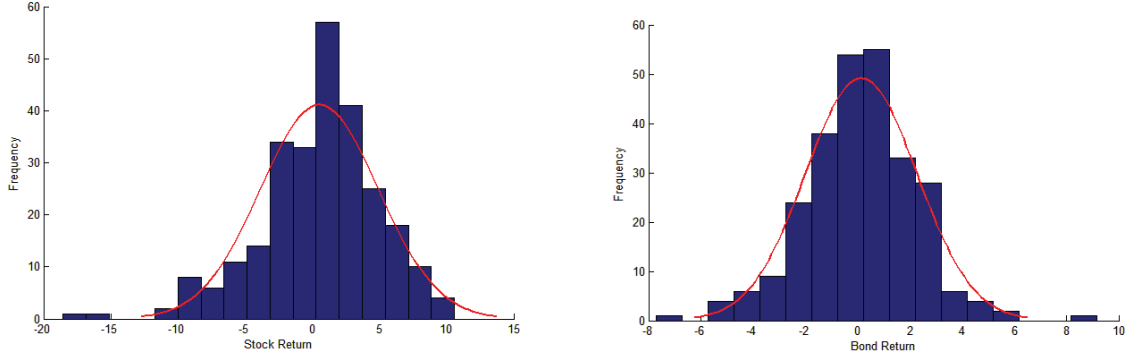


Figure 3.2.1: Histograms of Monthly Stock and Bond Returns

Figure 3.2.1: the left histogram shows the distribution of monthly stock returns, and also plots normal density function with the mean and standard deviation estimated from the in-sample monthly stock returns. The right histogram presents the distribution of monthly bond returns, and plots the normal density function with the mean and standard deviation estimated from the in-sample monthly bond returns.

This thesis adopts the RS model to price the stock and bond returns, so it is crucial to examine how the stock and bond returns are distributed. From the JB-test, it is clear that both null hypotheses that the stock and bond returns are normally distributed are rejected at 1% significant level. To visualize this result, the histograms of the monthly stock and bond returns during the full-sample period are displayed in Figure 3.2.1. It is noticeable that the distribution of stock returns appears to be skewed, which is consistent with several historical studies (*e.g.*, Cont (2001)). Hence, it is convincing that the general linear factors pricing model is biased, and a model that is able to better characterize this observed distribution should be studied.

This thesis explores the correlations between stock and bond returns, and constructs portfolios based on the characteristics of the regime-dependent correlations between stock and bond returns. Therefore, the historical performances of stock and bond returns for the full sample period are standardized and presented in Figure 3.2.2. From the figure, it seems that the correlations between stock and bond returns are positive before the dot-com bubble, and then the correlation seems to be negative during the period of dot-com bubble. Similarly, this correlation turns to be positive again during the economic recovery after the dot-com crisis, and

Table 3.2 Descriptive Statistics

	r_s	r_b	VIX	EDY	UCS	COP	UIP	AIR
<i>Panel A</i>								
Mean	0.5370	0.1421	-0.1205	-0.0054	0.4496	0.6413	0.1717	2.7453
Std	4.4064	2.1316	17.1358	0.0942	0.3863	8.6641	0.6734	1.2707
Median	1.0099	0.1897	-1.3575	-0.0100	0.3575	1.6052	0.2180	2.7628
Kurtosis	4.5385	4.4287	3.7357	9.8797	13.2124	6.3297	10.9429	5.1384
Skewness	-0.7716	-0.0172	0.5288	0.6816	2.6009	-0.1134	-1.6671	-0.3453
JB-test	0.0010	0.0015	0.0028	0.0010	0.0010	0.0010	0.0010	0.0010
<i>Panel B</i>								
Correlation	1.0000							
	-0.0937	1.0000						
	-0.6425	0.1091	1.0000					
	-0.7979	0.0289	0.4584	1.0000				
	-0.1478	0.1229	0.1187	0.1510	1.0000			
	-0.0344	-0.1439	0.0586	0.0876	-0.1959	1.0000		
	0.0093	-0.0833	0.0574	0.1068	-0.2832	0.1526	1.0000	
	-0.1144	0.0229	0.0883	0.1646	0.3258	-0.0431	-0.0800	1.0000

Table 3.2 shows the statistics of data for the full sample period with a total of 265 observations in this thesis. All numbers are in percentage, and the means and standard deviations of stock and bond returns are calculated monthly. In Panel A, the mean, standard deviation, median, kurtosis, and skewness of each variable are reported. Also, the p-values of normality test of Jarque-Bera test (JB-test), in which the null hypothesis is the data are normally distributed and alternative hypothesis is the data are not normally distributed, are presented. In Panel B, the correlations among variables are displayed.

again goes to be negative during the most recent financial crisis.

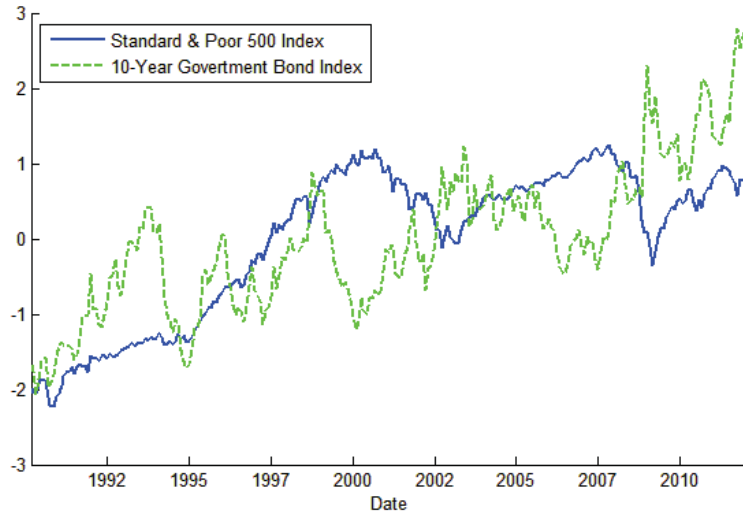


Figure 3.2.2: Historical Performances of Standardized Stock and Bond Index
 Figure 3.2.2 shows the trend of the historical stock and bond performances for the full sample period. That is, both the S&P 500 Index and the 10-year Government Bond Index for the full sample period are normalized by their corresponding means and standard deviations.

To be more specific, following the approach of Longin & Solnik (1995), another figure that shows the average time-varying correlations between stock and bond returns is provided in Figure 3.2.3. These average correlations are calculated in a rolling window of 12 months⁴ over the course of full sample period, and then reported and plotted at the end of each rolling window. Clearly, this correlation tends to be time-varying with a range from 0.9 to -0.9 , which, once again, enhances the significance to investigate the correlation between bond and stock returns. Meanwhile, from the perspective of investors, it also reminds the question that how investors can take advantage of the time-varying correlations when a bond-stock portfolio is being constructed.

Regarding the correlations among risky asset returns and the common factors, the magnitudes of correlations between stock return and non-macroeconomic factors are all higher than the correlations between bond return and those factors. However, for the macroeconomic factors, the correlations between bond return

⁴The reason why the rolling window is selected as 12 months is justified in the section of empirical analysis.

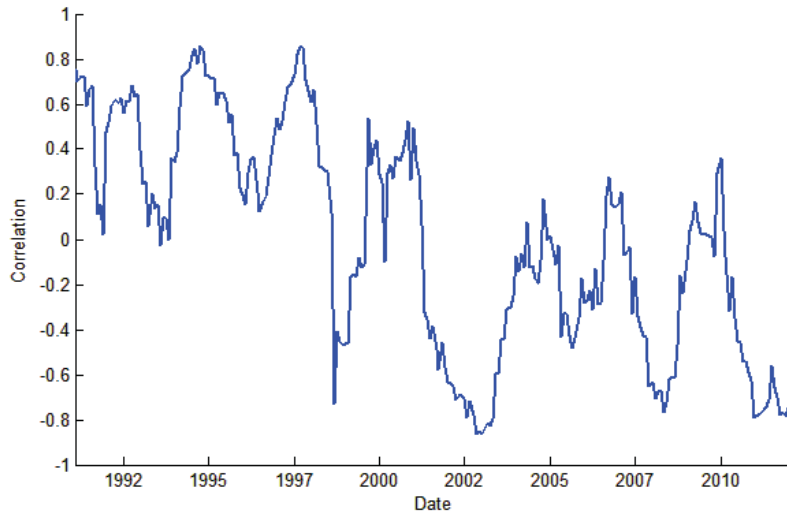


Figure 3.2.3: Average Correlations between Stock and Bond Returns

Figure 3.2.3 shows the average correlations between stock and bond returns for the full sample period. These average correlations are equally calculated in a rolling window of 12 months over the course of full sample period, and then this correlation is reported at the end of each rolling window. Thereafter, this correlation is plotted at the end of each rolling window date.

and those factors, except the actual inflation rate, are higher than the correlations between stock returns and those factors. In terms of the correlations among factors, their magnitudes range from 0.0431 to 0.4584, which avoids the potential problem of multicollinearity caused by including a set of independent variables in regression models.

Chapter 4

Methodology

In this chapter, I discuss the methodology being employed in this thesis. Starting with a generic asset pricing model, a RS model, I then show how the number of regimes is optimally determined and how an out-of-sample test can be conducted under the RS model. After that, with the regime-dependent correlations between stock and bond returns, the regime-dependent mean-variance frontiers are explored. Finally, to take the advantage of the regime-dependent correlations between stock and bond returns, the strategies of portfolio construction under the framework of the RS model are presented.

4.1 Regime-switching Model

Let the number of regimes in the financial market be K , and the transitions of the regimes are governed by the first order Markov chain. Also, assume the market is in regime i at current time period $t - 1$ and denote it as $M_{t-1} = i$, then the stock and bond returns at time t can be predicted through a set of common latent factors. This prediction can be expressed as follows:

$$\mathbf{R}_t = \alpha_{M_t} + \mathbf{F}_{t-1} \beta_{M_t} + \epsilon_{M_t},$$

where $\mathbf{R}_t = (r_{b,t}, r_{s,t})'$ is a 2×1 vector of predicted returns for the stock and bond at time t ; \mathbf{F}_{t-1} is a matrix of the common latent factors that drive bond and stock returns at time $t - 1$; ϵ_{M_t} is an error matrix that follows an identical and independent bivariate normal distribution with a zero mean vector, $E(\epsilon_{M_t}) = \mathbf{0}$, and variance-covariance matrix, Σ_{M_t} ; and α_{M_t} and β_{M_t} are regime-dependent parameters. The transitions of the regimes follow the Markov chain with an initial regime distribution q_0 and a constant probability matrix

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1K} \\ p_{21} & p_{22} & \cdots & p_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ p_{K1} & p_{K2} & \cdots & p_{KK} \end{bmatrix},$$

where $p_{ij} = Pr\{M_t = j \mid M_{t-1} = i\}$ refers to the transition probability that the market would transit from regime i at time $t - 1$ to regime j at time t .

4.2 Parameter Estimation

The EM algorithm, proposed by Dempster, Laird and Rubin (1977), is adopted to estimate the parameters in this thesis. The EM algorithm is a two-step parameter estimation with E-step, the estimation of missing data on regimes, and M-step, the maximization of the likelihood based on the E-step of the missing data estimation on regimes. The Appendix in the end of this thesis documents the details of this EM algorithm and the following summarizes these two steps:

1. **E-step:** Set initial value Φ^0 for the true parameter set Φ , calculate the arbitrary item, $Q^*(\Gamma) = P(\Gamma; \mathbf{X} \mid \Phi^0)$, and determine the expected log-likelihood, $E^{Q^*}[\ln P(\mathbf{X}, \Gamma \mid \Phi)]$.
2. **M-step:** Maximize the expected log-likelihood with respect to the arbitrary distribution Q^* of the hidden variable to obtain an improved estimate of Φ .

The improved estimated is

$$\Phi^* = \underset{\Phi}{\operatorname{arg\,max}} \{E^{Q^*}(\ln P(\mathbf{X}, \Gamma | \Phi))\},$$

where Φ^* is the new initial value for Φ .

Thereafter, reset the initial value from Φ^0 to Φ^* , and implement the E-step and M-step again. Following this iterative procedure, the parameters can be obtained when the log-likelihood is maximized.

4.3 Number of Regime

The number of parameters under the above RS model would be substantially increasing once one more regime is included. However, as we know, the principle to create a model is to determine the most parsimonious but accurate model among many plausible candidate models. In other words, it is crucial to balance the tradeoff between the number of regimes and the predictability of the model. In this thesis, therefore, the optimal number of regimes is determined based on the decision rule of Bayesian information criterion (BIC). That is, the number of regimes is determined when the value of BIC is minimized. Mathematically,

$$BIC = -2\ln(L | K, \Phi(K)) + f(K, \Phi(K))\ln(N)$$

where L is the likelihood function given the number of regimes K , $\Phi(K) = \{\alpha_{M_t}, \beta_{M_t}, \Sigma_{M_t}, \mathbf{P}\}$ is the set of parameters, $f(K, \Phi(K))$ indicates the number of parameters, and N is the sample size of the observed data.

4.4 Out-of-sample Performance

With the estimated parameters in the RS model, it is important to see how accurate this model can predict the future stock and bond returns. Hence an out-of-

sample test for the RS model should be conducted. Specifically, given the latent factors during the out-of-sample period and the in-sample estimated parameters under the RS model, then the regime-dependent stock and bond returns can be predicted.

As an illustrative example, here I show how to predict stock and bond returns at time t with the latent risk factors at time $t - 1$. Denote $\hat{\mathbf{R}}_{M_t,t}$ as the regime-dependent predicted stock and bond returns at time t , then

$$\hat{\mathbf{R}}_{M_t,t} = \hat{\alpha}_{M_t} + \mathbf{F}_{t-1} \hat{\beta}_{M_t}$$

where $\hat{\alpha}_{M_t}$ and $\hat{\beta}_{M_t}$ are the in-sample regime-dependent estimated parameters; \mathbf{F}_{t-1} is the latent risk factors matrix at time $t - 1$.

To find out the expected predicted returns for stock and bond at time t , I first illustrate prior probability for the market being in a specific regime at time period t . Under the framework of the regime switching model, the posterior probability for the market staying in each regime at time $t - 1$ is estimated and denoted as a vector, \mathbf{q}_{t-1} , where the i^{th} element, q_{t-1}^i , refers to the probability of the market being in regime i at time $t - 1$. Hence, the prior probability vector \mathbf{p}_t , where the j^{th} element indicates the probability of the market being in regime j at time t , is

$$\mathbf{p}_t = q_{t-1}^1 p_{1j} + q_{t-1}^2 p_{2j} + \cdots + q_{t-1}^K p_{Kj} = \mathbf{q}_{t-1} \mathbf{P}.$$

Thus, the expected predicted stock and bond return at time t is

$$E(\hat{\mathbf{R}}_t | \mathbf{F}_{t-1}) = \mathbf{q}'_{t-1} \mathbf{P} \hat{\mathbf{R}}_{M_t,t} = \mathbf{q}'_t \mathbf{P} (\hat{\alpha}_{M_t} + \mathbf{F}_{t-1} \hat{\beta}_{M_t}).$$

As a comparison for the predictive power of the RS model, the bivariate regression of the stock and bond returns are also estimated during the in-sample period. Thereafter, under this regression, the future stock and bond returns can be predicted during the out-of-sample period and compared with the correspond-

ing forecasting value under the RS model.

4.5 Means and Correlations under Different Regimes

Given the regime switching factor model,

$$\mathbf{R}_t = \alpha_{M_t} + \mathbf{F}_{t-1} \beta_{M_t} + \epsilon_{M_t},$$

the mean stock and bond returns in the k^{th} regime are

$$E(\mathbf{R}_t \mid \mathbf{F}_{t-1}, M_t = k) = \alpha_k + \mathbf{F}_{t-1} \beta_k$$

where $k = 1, 2, \dots, K$; and the variance-covariance matrix for the residuals of the stock and bond returns in the k^{th} regime are Σ_k .

By converting the regime-dependent variance-covariance matrices of the residuals, the regime-dependent correlation matrices can be obtainable. Thus, the elements in the off-diagonal of the regime-dependent correlation matrices represent the regime-dependent correlations between stock and bond returns.

4.6 Mean-variance Efficient Frontier

Building on the work of mean-variance analysis by Markowitz (1952), I discuss the regime-dependent mean-variance frontiers for the stock-bond portfolio in this section.

Assume there are only two risky financial assets, i.e., stock and bond, that are available for investors in the financial market and investors rebalance their portfolio at each time period. Denote \mathbf{W}_{t-1}^i as the regime-dependent weight vector for the risky assets at time $t-1$, then the regime-dependent risky portfolio return at time t is

$$V_{M_t, t} = \mathbf{W}_{t-1}^{i'} \mathbf{R}_{M_t, t}.$$

Thus, the regime-dependent expected return for this risky portfolio at time t is

$$E(V_t | M_{t-1} = i, \mathbf{F}_{t-1}) = \mathbf{W}_{t-1}^{i'} E(\mathbf{R}_t | M_{t-1} = i, \mathbf{F}_{t-1});$$

where $E(\mathbf{R}_t | M_{t-1} = i, \mathbf{F}_{t-1}) = \sum_{j=1}^K (\alpha_j + F_{t-1} \beta_j) p_{ij}$; The regime-dependent expected variance for this risky portfolio at time t is

$$\begin{aligned} Var(V_t | M_{t-1} = i, \mathbf{F}_{t-1}) &= \mathbf{W}_{t-1}^{i'} \{Var[\mathbf{R}_t | M_{t-1} = i, \mathbf{F}_{t-1}]\} \mathbf{W}_{t-1}^i \\ &= \mathbf{W}_{t-1}^{i'} \boldsymbol{\Omega}_i \mathbf{W}_{t-1}^i. \end{aligned}$$

To be specific,

$$\begin{aligned} \boldsymbol{\Omega}_i &= Var[\mathbf{R}_t | M_{t-1} = i, \mathbf{F}_{t-1}] \\ &= \sum_{j=1}^K \boldsymbol{\Sigma}_j p_{ij} + \sum_{j=1}^K \{[E(\mathbf{R}_t | M_t = j, \mathbf{F}_{t-1}) - E(\mathbf{R}_t | M_{t-1} = i, \mathbf{F}_{t-1})]^2\} p_{ij} \end{aligned}$$

where $\boldsymbol{\Sigma}_j$ is the variance-covariance matrix for the residuals of the stock and bond returns in the j^{th} regime and $E[\mathbf{R}_t | M_t = j, \mathbf{F}_{t-1}] = \alpha_j + \mathbf{F}_{t-1} \beta_j$.

As an investor, the objective is to maximize the portfolio return for a given level risk. That is, investors are seeking to construct the portfolios that result in the maximized Sharpe ratios. Mathematically, this maximization problem can be stated as follows:

$$\begin{aligned} Max \frac{E(V_t | M_{t-1} = i, \mathbf{F}_{t-1}) - r_f}{\mathbf{W}_{t-1}^i Var(V_t | M_{t-1} = i, \mathbf{F}_{t-1})^{\frac{1}{2}}} &= \frac{\mathbf{W}_{t-1}^{i'} E(\mathbf{R}_t | M_{t-1} = i, \mathbf{F}_{t-1}) - r_f}{(\mathbf{W}_{t-1}^{i'} \boldsymbol{\Omega}_i \mathbf{W}_{t-1}^i)^{\frac{1}{2}}} \\ s.t. : (1) \mathbf{W}_{t-1}^{i'} \mathbf{1} &= 1. \end{aligned}$$

where r_f refers to the risk-free rate. The solution for this optimization can be solved via the method of Lagrange multipliers. For simplification, I drop the subscript and superscript for the above maximization problem. Thus, the Lagrangian

function for this optimization problem is

$$L(\mathbf{W}, \lambda) = (\mathbf{W}'\mathbf{R} - r_f)(\mathbf{W}'\boldsymbol{\Omega}\mathbf{W})^{-\frac{1}{2}} + \lambda(1 - \mathbf{W}'\mathbf{1}). \quad (4.6.1)$$

Taking the first derivative of equation (4.6.1) with respect to \mathbf{W} and λ can result in:

$$\begin{aligned} \frac{\partial L}{\partial \mathbf{W}} &= \mathbf{R}(\mathbf{W}'\boldsymbol{\Omega}\mathbf{W})^{-\frac{1}{2}} - (\mathbf{W}'\mathbf{R} - r_f)(\mathbf{W}'\boldsymbol{\Omega}\mathbf{W})^{-\frac{3}{2}}\boldsymbol{\Omega}\mathbf{W} + \lambda\mathbf{1} = 0 \\ \frac{\partial L}{\partial \lambda} &= 1 - \mathbf{W}'\mathbf{1} = 0. \end{aligned}$$

By solving the above equation system, then the optimal weight vector for the risky portfolio is

$$\mathbf{W}^* = \frac{\boldsymbol{\Omega}^{-1}(\mathbf{R} - r_f\mathbf{1})}{\mathbf{1}'\boldsymbol{\Omega}^{-1}(\mathbf{R} - r_f\mathbf{1})}.$$

In order to make the notation consistent with the above description, this optimal regime-dependent weight vector can be written as :

$$\mathbf{W}_{t-1}^{i*} = \frac{\boldsymbol{\Omega}_i^{-1} [E(\mathbf{R}_t | M_{t-1} = i, \mathbf{F}_{t-1}) - r_f\mathbf{1}]}{\mathbf{1}'\boldsymbol{\Omega}_i^{-1} [E(\mathbf{R}_t | M_{t-1} = i, \mathbf{F}_{t-1}) - r_f\mathbf{1}]}.$$

From the solution, it is evident that the investor's decision at time $t-1$ depends on which regime the market is in at time period $t-1$. In other words, it means that investors can conduct the regime-dependent investment if they have the knowledge of the regimes of the market at time period $t-1$.

Following the above process, the regime-dependent means and variance-covariance matrices are also known. Hence, the tangent portfolio at each frontier and the mean-variance frontiers can be explored and compared across the regimes.

4.7 Portfolio Implication

The existence of regime-dependent investment might have a significant role to play for the investors who have the knowledge of the regimes at time period $t - 1$, but it might be not useful for the investors who do not have the knowledge of the regimes at time period $t - 1$. Therefore, I now show how to construct an efficient portfolio by incorporating the existence of the regimes, and the expected return and variance of the efficient portfolio are not conditioned in the regimes.

With the regime-dependent expected returns and the variances for the risky portfolio, thus the expected risky portfolio return without conditioning to the regimes is

$$E(V_t | \mathbf{F}_{t-1}) = \sum_{i=1}^K E(V_t | M_{t-1} = i, \mathbf{F}_{t-1}) q_{t-1}^i = \omega'_{t-1} \mathbf{u}'_t \mathbf{q}_{t-1}$$

where $\mathbf{u}_t = E[\mathbf{R}_t | M_{t-1} = i, \mathbf{F}_{t-1}]$ is a $K \times 2$ matrix and stands for the regime-dependent expected returns for the risky assets at time period t ; and ω_{t-1} refers to the weight vector for the risky assets at time period $t - 1$.

The expected variance of the risky portfolio without conditioning to the regimes is

$$\begin{aligned} Var(V_t | \mathbf{F}_{t-1}) &= \omega'_{t-1} [Var[\mathbf{R}_t | \mathbf{F}_{t-1}]] \omega_{t-1} \\ &= \omega'_{t-1} \{E[Var(\mathbf{R}_t | M_{t-1}, \mathbf{F}_{t-1})] + Var[E(\mathbf{R}_t | M_{t-1}, \mathbf{F}_{t-1})]\} \omega_{t-1} \end{aligned}$$

where

$$\begin{aligned} E[Var[\mathbf{R}_t | M_{t-1}, \mathbf{F}_{t-1}]] &= \sum_{i=1}^K q_{t-1}^i Var[\mathbf{R}_t | M_{t-1} = i, \mathbf{F}_{t-1}] \\ &= \sum_{i=1}^K q_{t-1}^i \Omega_i \end{aligned}$$

and

$$Var[E(\mathbf{R}_t|M_{t-1}, \mathbf{F}_{t-1})] = \sum_{i=1}^K q_{t-1}^i \{E(V_t | \mathbf{F}_{t-1}) - E[\mathbf{R}_t|M_{t-1} = i, \mathbf{F}_{t-1}]\}^2.$$

Again, the main goal for investors is to maximize the expected portfolio return for any targeted risk that investors are willing to take it. Hence, the efficient portfolio for them can be constructed by maximizing the Shape ratio, or

$$\begin{aligned} \underset{\omega_{t-1}}{Max} \quad & \frac{E(V_t | \mathbf{F}_{t-1}) - r_f}{Var(V_t | \mathbf{F}_{t-1})^{\frac{1}{2}}} & (4.7.1) \\ = \quad & \frac{\omega'_{t-1} \mathbf{u}'_t \mathbf{q}_{t-1} - r_f}{\{\omega'_{t-1} \{E[Var(\mathbf{R}_t|M_{t-1}, \mathbf{F}_{t-1})] + Var[E(\mathbf{R}_t|M_{t-1}, \mathbf{F}_{t-1})]\} \omega_{t-1}\}^{\frac{1}{2}}} \\ \text{s.t. :} \quad & \omega' \mathbf{1} = 1 \end{aligned}$$

Similarly, this optimization in equation (4.7.1) can be solved by employing the method of Lagrangian multipliers and the solution to this maximization can finally be derived, i.e.,

$$\omega_{t-1} = \frac{\Psi^{-1}(\mathbf{u}'_t \mathbf{q}_{t-1} - r_f \mathbf{1})}{\mathbf{1}' \Psi^{-1}(\mathbf{u}'_t \mathbf{q}_{t-1} - r_f \mathbf{1})}$$

where $\Psi = \{E[Var(\mathbf{R}_t|M_{t-1}, \mathbf{F}_{t-1})] + Var[E(\mathbf{R}_t|M_{t-1}, \mathbf{F}_{t-1})]\}$. Undoubtedly, the allocation of investment in risky asset at time $t - 1$ depends on the posterior probabilities for the regimes that the market is in at time $t - 1$. With the framework of the RS model, the posterior probabilities at time $t - 1$ are known. That is, the investor's investment decision can be determined at time $t - 1$.

Chapter 5

Empirical Analysis

Up to now, the methodology to address the objectives have been discussed and presented in Chapter 4. In this chapter, a comprehensive empirical analysis is conducted, starting from the asset pricing and ending up with portfolio constructions.

5.1 Number of Regimes and Its Interpretation

As discussed in Section 4.3, it is important to balance the tradeoff between the predictability and the number of regimes in the RS model, and then a criterion determining the number of regimes should be utilized. Table 5.1 shows the maximized log likelihood value (MLL) and the value of BIC under different regimes. Following the criterion of BIC that the model is optimal when the value of BIC is minimized, then it is clear that the model should be optimally selected when the number of regimes is 2.

Table 5.1 BIC and Maximum Log Likelihood (MLL) for Different Regimes

Regime	1	2	3	4	5	6
MLL	-1189.97	-1138.28	-1093.41	-1061.33	-1037.40	-989.26
BIC	2456.66	2440.99	2449.88	2495.35	2568.07	2603.31

Within the two regimes, the transition matrix, which shows how these two unobserved market regimes are transitioned between two time periods, is

$$\mathbf{P} = \begin{bmatrix} 0.9675 & 0.0325 \\ 0.0198 & 0.9802 \end{bmatrix}.$$

It is observable that the transition probabilities from one regime to the other regime are very low in both regimes, which means these two regimes are very stable. The stabilized regimes clearly play a significant role from the perspectives of investors. This is because the characteristics of stabilized regimes might help investors reduce the frequency of rebalance, and then save the cost resulting from portfolio rebalance.

To visualize the two regimes, Figure 4.1.1 shows the implied posterior probabilities of each regime, and the histogram of implied regimes in the RS model during the in-sample period. From the left graph, it is noticeable that regime 2 is relatively more stable than regime 1 because when the market transits to regime 2, it stays in this regime for a relatively longer time compared to that of regime 1. This phenomenon can be once again observed by referring to the right graph – histograms of regimes. The number of months in regime 1 (77) is greater than regime 2 (163), which is consistent with several historical studies, such as Ang and Chen (2002) and Guidolin and Ria (2010).

Under the RS model, the market can be inferred to a specific regime at each time period, then the regime-dependent sample statistics can be calculated by sorting the periods by regime. Table 5.2 provides the regime-dependent statistics for each variable during the in-sample period. It is no doubt that the characteristics for regime 1 are significantly different from that for regime 2. In regime 1, the mean stock return is negative, and is lower than the mean stock return shown in Table 3.1. However, the mean bond return increases from 0.1421, the mean bond return for the full sample period, to 0.4308 in regime 1. Furthermore, it is surprising to see that correlation between stock and bond returns in either

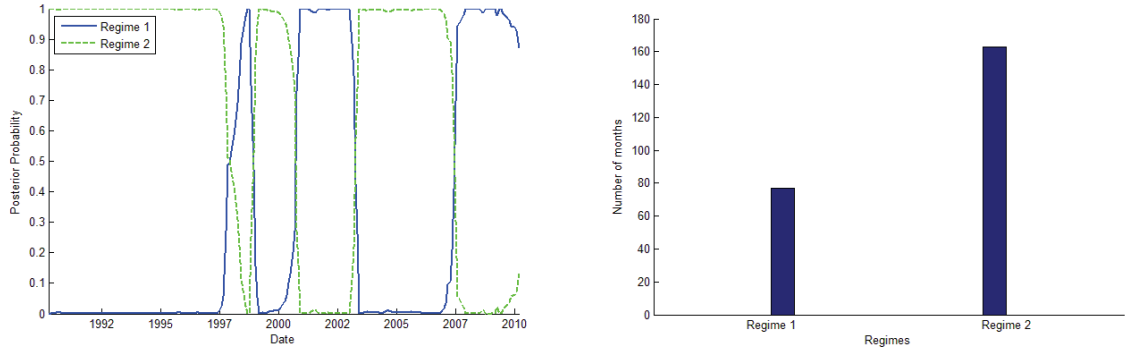


Figure 5.1.1: Posterior Probability of Regimes and Histogram of Regimes

Figure 5.1.1: the left figure shows the implied posterior probabilities for different regimes, and the right figure shows the histogram of the implied regimes in RS model during the in-sample period.

regime 1 or regime 2 changes substantially compared with the correlation between stock and bond returns without regime. To be specific, the correlation between stock and bond returns changes from -0.0937 in the full sample period to -0.3179 in regime 1 and 0.3501 in regime 2. In the meantime, it seems the correlations between either stock return or bond return and any of the risk factors have all experienced to some degree. The particular examples are the correlations between stock return and each macroeconomic factor, and the correlations between bond return and each non-macroeconomic factor.

According to those regime-dependent characteristics, regime 1 can be interpreted as the negative regime, while regime 2 can be perceived as the positive regime¹. Considering the fact that the mean stock return in regime 2 is significantly higher than that in regime 1, thus regime 1 can also be viewed as the bearish regime and regime 2 can also be seen as the bullish regime.

5.2 Parameter Estimation in the RS Model

Table 5.3 shows the parameter estimation of both stock and bond returns under the RS and single regime (SR) models. Starting with the parameters in the SR

¹In this thesis, the positive (negative) regime mainly indicates that the correlation between stock and bond returns is positive (negative) in this regime.

Table 5.2 Regime-dependent Descriptive Statistics

	r_s	r_b	VIX	EDY	UCS	COP	UIP	AIR
<i>Panel A (Regime 1)</i>								
Mean	-0.8265	0.4308	1.4040	0.0100	0.6219	-0.1145	-0.1297	2.1441
Standard Deviation	5.9217	2.4322	20.6747	0.1197	0.5593	10.0201	0.8989	1.6301
Correlation	1.0000							
	-0.3179	1.0000						
	-0.7459	0.2746	1.0000					
	-0.7798	0.2064	0.5113	1.0000				
	-0.2181	0.1921	0.2292	0.3565	1.0000			
	0.1123	-0.2293	0.0136	-0.2088	-0.2984	1.0000		
	0.0128	-0.0321	0.0301	0.0878	-0.2448	0.1842	1.0000	
	-0.2526	0.0254	0.1928	0.3657	0.5006	-0.0799	-0.1539	1.0000
<i>Panel B (Regime 2)</i>								
Mean	1.1160	-0.0673	-0.6814	-0.0143	0.4134	0.8639	0.2765	3.0776
Standard Deviation	3.2218	1.9525	14.4153	0.0779	0.2591	8.4253	0.5165	0.9715
Correlation	1.0000							
	0.3501	1.0000						
	-0.4800	-0.1622	1.0000					
	-0.8007	-0.2938	0.3531	1.0000				
	0.1051	0.0569	-0.0347	-0.1777	1.0000			
	-0.2784	-0.0488	0.1515	0.4186	-0.0758	1.0000		
	-0.1468	-0.1084	0.1348	0.2092	-0.1733	0.1400	1.0000	
	-0.1008	0.0801	0.0028	0.0350	0.3558	-0.0323	-0.2322	1.0000

Table 5.2 shows the regime-dependent statistics during the in-sample period. In Panel A, the mean and standard deviation for each variable in regime 1 is reported and both the correlations between stock and risk factors and the correlations between bond return and risk factors in this regime are presented. In Panel B, the mean and standard deviation for each variable in regime 2 is reported and both the correlations between stock and risk factors and the correlations between bond return and risk factors in this regime are presented.

model, as expected, only a few show predictive power for the stock and bond returns. More specifically, none of the nonmacroeconomic factors have a significant predictability for the stock return, and the only macroeconomic factor that has a significant predictive power for the stock return is *UIP*. For the bond return, the *COP* and *EDY* both exhibit forecast ability.

For the stock return in the RS model, it is noticeable that most of the non-macroeconomic factors are statistically significant at 1% level in regime 2, while none of them are statistically significant at 1% level in regime 1. Considering the characteristics of regime 1 and regime 2, this finding seems to be reasonable, because that investors would expect to receive a significant risk premium for the stock when the market is bullish (regime 1). By contrast, those risk premiums would not be rewarded when the market is bearish (regime 2). For the macroeconomic factors, it is interesting to see that if a factor is significant in one regime, then it would be insignificant in the other regime. One particular example is the *COP* factor, which shows a strong predictability in regime 2 for the stock return but this predictability disappears in regime 1.

Regarding the predictive power of the factors in the bond return, it tends to be quite different from that for the stock return. Most of the nonmacroeconomic and macroeconomic factors are significant in both regime 1 and regime 2. Compared with the regime-dependent pattern in the stock return, the risk factors in the bond return do not appear to be regime-dependent. That is, if a factor is significant in one regime, then it usually remains significant in the other regime. For example, the *COP* factor displays significant foreseeability in both regime 1 and regime 2 for bond return.

It is surprising to see that the non-macroeconomic factor, *VIX*, is neither significant in regime 1 nor in regime 2 for the stock return. This factor, therefore, does not have any significant predictive power. Nevertheless, this factor shows a significant predictive power for the bond return in both regime 1 and regime 2. Another factor that should be mentioned is *UIP*, which presents a significant

Table 5.3 Estimated Parameters in the Regime-switching (RS) and Single Regime (SR) Models

	Intercept	<i>VIX</i>	<i>EDY</i>	<i>UCS</i>	<i>COP</i>	<i>UIP</i>	<i>AIR</i>
<i>Panel A</i>							
Regime 1	0.7510	-0.0308	-5.3757	1.4562	0.0263	2.2329	-0.9847
Regime 2	1.5694	-0.0066	9.4743	3.3110	-0.1013	-0.2162	-0.5008
Single Regime (SR)	0.9127	-0.0198	-4.0882	0.3985	-0.0238	1.6547	-0.3091
<i>Panel B</i>							
Regime 1	0.6292	0.0201	3.6709	0.7649	0.0369	0.4090	0.2493
Regime 2	0.7142	0.0144	3.0553	0.8153	0.0261	0.4076	0.2159
Single Regime (SR)	0.7028	0.0183	3.3242	0.7851	0.0317	0.4191	0.2241
<i>Panel C</i>							
Regime 1	0.2971	0.0190	6.1600	0.3180	-0.0312	0.9114	-0.0103
Regime 2	-0.3937	-0.0247	4.8769	0.4364	-0.0334	-0.3353	0.1063
Single Regime (SR)	-0.1866	-0.0064	5.9762	0.5546	-0.0367	0.3112	0.0107
<i>Panel D</i>							
Regime 1	0.2401	0.0077	1.4011	0.2919	0.0141	0.1561	0.0952
Regime 2	0.4409	0.0089	1.8863	0.5033	0.0161	0.2517	0.1333
Single Regime (SR)	0.3391	0.0088	1.6039	0.3788	0.0153	0.2022	0.1081

Table 5.3 presents the parameter estimation for both stock and bond returns under the RS and SR models. Panel A shows the coefficient for each factor for the stock return in the RS and SR models and Panel B provides the corresponding standard error for each coefficient for the stock return. Panel C shows the coefficient for each factor for the RS and SR models, and Panel D provides the corresponding standard error for each coefficient for the bond return.

predictive power in regime 1 for both stock and bond returns, but this forecasting power does not exist in regime 2 for either stock or bond returns.

To sum up, the significance of parameters in the RS model for both the stock and bond returns are different from that in the SR model. Specifically, most of the parameters that are not significant in the SR model show predictive power for, either the stock or bond returns, in the RS model.

Besides the coefficients of the factors in the SR and RS models, Table 5.4 shows the variance-covariance of the residuals in the SR model and RS models. Hence, by transferring the regime-dependent variance-covariance matrices, the correlations between stock and bond returns implied by the SR and RS models can be obtained. These correlations are -0.0161 in single regime, -0.4579 in regime 1 and 0.3249 in regime 2.

Table 5.4 Variance-covariance of Residuals for the SR and RS Models

	Single Regime		Regime 1		Regime 2	
	r_s	r_b	r_s	r_b	r_s	r_b
r_s	17.3471	-0.1346	27.0665	-4.7306	9.1912	1.8435
r_b	-0.1346	4.0382	-4.7306	3.9428	1.8435	3.5032

Following the estimated parameters, it is worthwhile to see if the predicted stock and bond returns fit the actual returns during the in sample data. Figure 5.2.1 shows the comparison of the predicted stock return, under the RS and SR models, with the actual stock return. It can be seen that the cumulative log predicted stock return in the RS model is close to that the actual returns. In addition, the fit under the RS model is much better than that in the SR model. Similarly, Figure 5.2.2 compares the predicted bond return, under the RS and SR models, with the actual bond return for the in-sample period. From the figure, it is unclear whether the predicted bond return in the RS model is better than that in the SR model, especially during the most recent period. Hence, to quantify the comparison, the root mean square errors (RMSE) in the prediction of bond returns in RS and SR models are calculated. For the RS model, the RMSE is

1.8962% against 2.0053% in the SR model, which confirms that the prediction of the bond return in the RS is also better than that in the SR model.

5.3 Out-of-sample Performance in the RS Model

From the in-sample test, it suggests that the performance of the predictions in the RS model for either the stock or bond returns are indeed better than that in the SR model. To further investigate the robustness of this result, an out-of-sample test for the RS and SR models is conducted.

Figure 5.3.1 shows the cumulative log predicted stock and bond returns, under both the RS and SR models, during the out-of-sample period. From the left graph, it seems that the predicted stock return in the RS does not fit the actual stock return as good as the predicted return in the SR model. However, compared with the predicted stock return in the SR model, the trend for the predicted stock return shown in the RS model tend to be more consistent with the trend of the actual stock return. Regarding the bond return, it is visualized from the right graph that the predicted bond return in the RS model does fit the actual bond return better than that in the SR model.

5.4 Regime-dependent Mean-Variance Frontiers

From the estimated parameters in the RS model, it is known that the correlation between stock and bond returns is different across the regimes, and the direction of this correlation in regime 1 is contrast to that in regime 2. Therefore, following Ang and Bekaert (2002a), I discuss the regime-dependent mean-variance frontiers and the corresponding tangent portfolio in each regime in this section. As a comparison, the single regime mean-variance frontier and its according tangent portfolio are constructed as well.

To derive the mean-variance frontier and the tangent portfolio in each regime, the question centralizes on the determination of the risk-free rate. In the thesis,

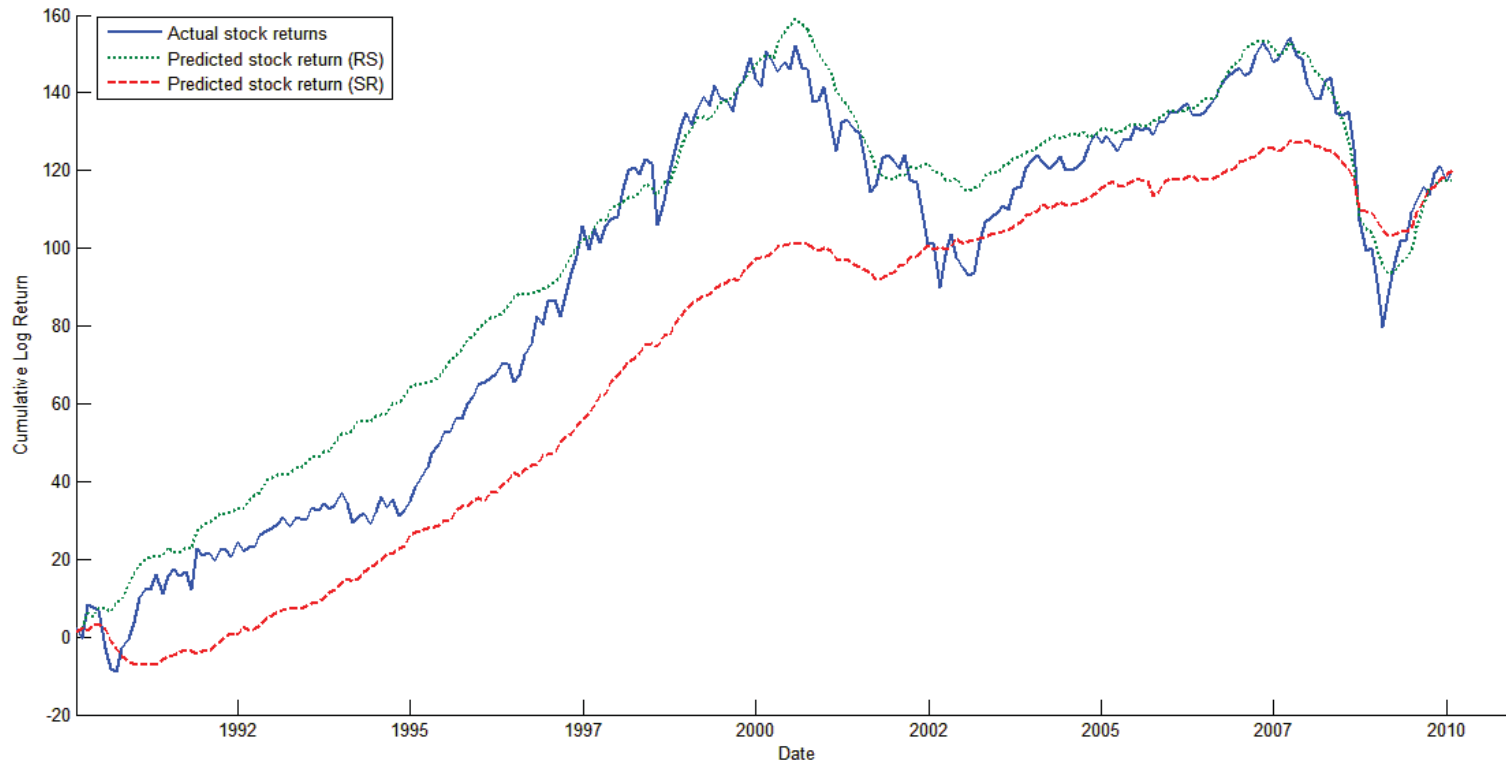


Figure 5.2.1: Comparison of Predicted Stock Return in the RS and SR Models (In-sample)

Figure 5.2.1 compares the predicted stock return, under both the RS and SR models, with the actual stock return during the in-sample period. The dash line represents the predicted cumulative log stock return under the single regime model (SR), the solid line is the actual cumulative stock return and the dotted line refers the predicted cumulative stock return under the regime-switching (RS) model.

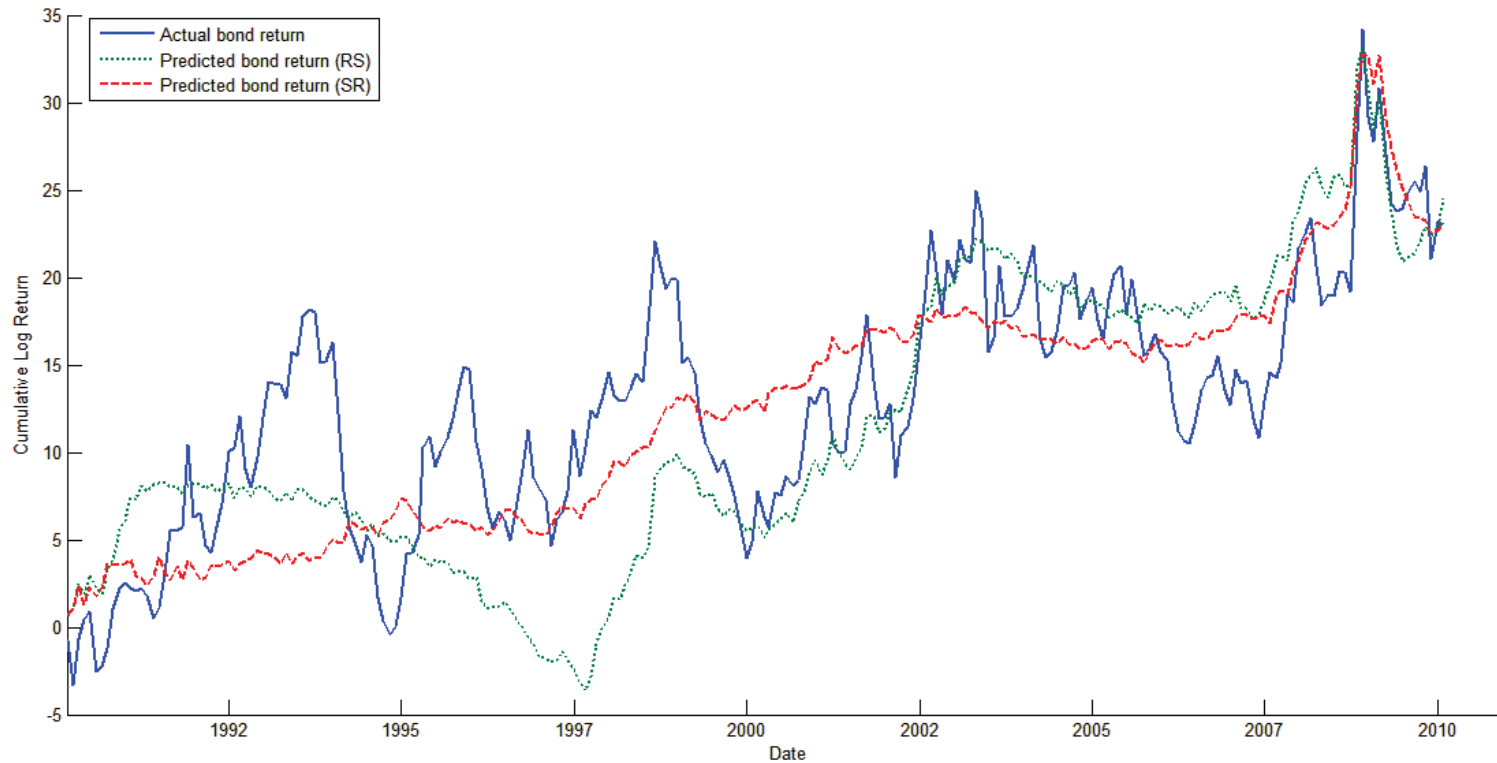


Figure 5.2.2: Comparison of Predicted Bond Return in the RS and SR Models (In-sample)

Figure 5.2.1 compares the predicted bond return, under both the RS and SR models, with the actual stock return during the in-sample period. The dash line represents the predicted cumulative log bond return under the single regime model (SR), the solid line is the actual cumulative bond and the dotted line refers the predicted cumulative bond return under the regime-switching (RS) model.

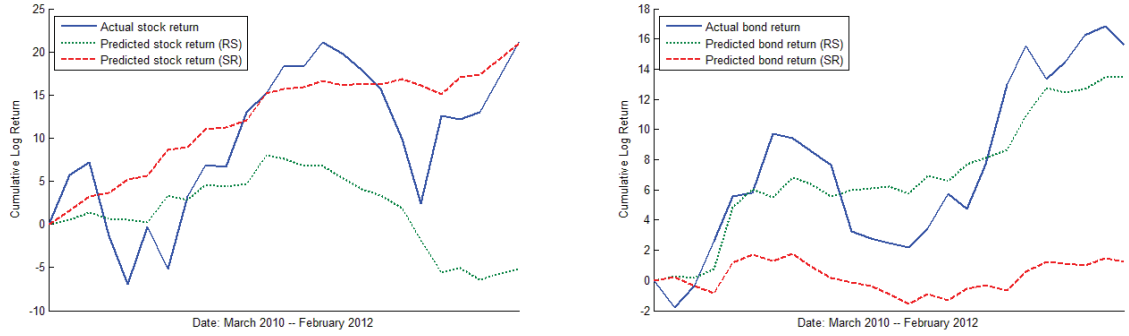


Figure 5.3.1: Comparison of Predicted Stock and Bond Returns in the SR and RS Models (Out-of-sample)

Figure 5.3.1 provides the cumulative log predicted and actual bond and stock returns during the out-of-sample period. The out-of-sample period is from March 2010 to February 2012. The left (right) graph compares the predicted stock (bond) return, under both the RS and SR models, with the actual stock (bond) return during the out-of-sample period. The dash line represents the predicted cumulative log stock (bond) return under the SR model, the solid line is the actual cumulative stock (bond) and the dotted line refers the predicted cumulative bond return under the RS model.

the riskfree rate is assumed to be regime-dependent. Specifically, as the implied regime² is known for each time period under the framework RS model, then it is possible to classify the riskfree rate into a specific regime at each time period. Hence, by sorting the regimes, the average regime-dependent riskfree rate is calculated as the average of the riskfree rates within the regime during the in-sample period. With this process, the average riskfree rate in regime 1 is 2.83%, 4.21% in regime 2 and 3.76% in single regime.

Figure 5.4.1 shows the mean-variance frontier for each regime. As expected, the frontier in regime 2 comes to the top, followed by the frontier for regime 1 in the middle and the frontier for single regime in the bottom. The intuition behind that is that regime 2 has the best tradeoff between return and risk in the risky portfolio among these two regimes and single regime, while the tradeoff in single regime is the worst. That is to say, the investment feasible sets in the framework of RS model dominate that in the SR model, which is consistent with the findings

²With the RS model, the posterior probability for each regime is estimated at each time point. If the posterior probability of market being in regime 1 is greater or equal to 50%, then the regime in the market at that time period is inferred to regime 1.

in the literature (Ang and Bakert (2002b) and Clarke and de Silva (1998)).

As an example to illustrate this point, suppose the level of risk³ that an investor is willing to take is one (or 100%) per year, then the investor can expect an annualized portfolio return of 12.54% in regime 2, 7.89% in regime 1 and only 4.46% in single regime. That is, given a certain level of risk, the payoff is the worst in single regime, and the best in regime 2. The differences in the characteristics of the regimes can be taken into account of this striking result. To be specific, the stock market tends to be bullish in regime 2, as a result, investors will expect a relatively higher return. When the market is in regime 1, even though the stock market is bearish in this regime, investors can transfer their investment into bond market as the bond return is relatively higher than the stock return in this regime. That is also the possible reason why the phenomenon of “flight to quality” occurs when stock markets perform poorly.

Meanwhile, Figure 5.4.1 also shows the tangent portfolio for each frontier. To make the tangent portfolio feasible, the weights of stock and bond are constrained to be within zero and one, and the sum of each weight is equal to one. Following these constraints, Table 5.5 reports the weights of stock and bond for the tangent portfolio in different regimes. From the table, it is interesting to see the weights for stock and bond differ among the three tangent portfolios. For the tangent portfolios in regime 1 and regime 2, both of them weight heavily on the better-perform asset in their corresponding regime. For example, in regime 1, the bond market performs relatively better than the stock market, correspondingly, the tangent portfolio in this regime requests total investment on the bond market. In regime 2, as the bond and stock returns are positively correlated and the stock market outperform the bond market, not surprisingly, a rational investor would put the total investment on the stock market.

Furthermore, it is not surprising to see that the Sharpe ratio in regime 2 (0.2525) is the highest, and followed by regime 1 (0.0984) and single regime

³The level of risk is defined as the standard deviation of the risky portfolio.

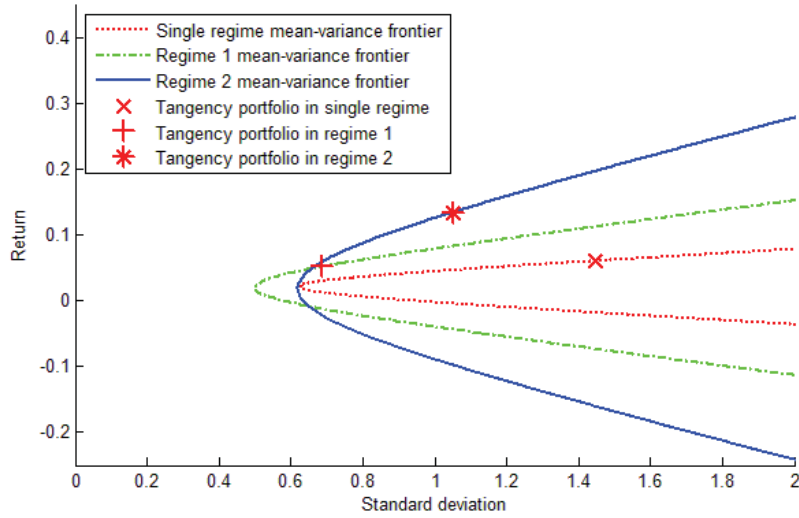


Figure 5.4.1: Regime-dependent Mean-variance Frontiers

Figure 5.4.1 shows the regime-dependent mean-variance frontiers. The risk-free rate that is used to calculate the regime-dependent frontier is the average of regime-dependent 3-month T-bill rate during the in sample period. That is, the average T-bill rate is 3.76% in the single regime, 2.83% in regime 1 and 4.21% in regime 2. The risky portfolio return is annualized by multiplying 12 and the standard deviation is annualized by multiplying $\sqrt{12}$. All numbers are in decimal. The dashdot line represents the mean-variance frontier in regime 1, the dotted line shows the mean-variance frontier in single regime and the solid line displays the mean-variance frontier in regime 2. The plus-mark is the tangent portfolio in regime 1, the x-mark is the tangent portfolio in the single regime, and the star-mark is the tangent portfolio in regime 2.

Table 5.5 Weights of Stock and Bond for the Tangent Portfolios

	Single Regime	Regime 1	Regime 2
w_s	1.0000	0.0000	1.0000
w_b	0.0000	1.0000	0.0000

Table 5.5 reports the weights of bond and stock for the tangent portfolios in the different regimes. The riskfree rate being employed to calculate the regime-dependent tangent portfolio is also regime-dependent, with 3.76% in the single regime, 2.83% in regime 1 and 4.21% in regime 2.

(0.0449). As an investor, the main goal is to maximize the Sharpe ratio of his investment given the certain constraints. Under the framework of RS model, the Sharp ratio in either regime 1 or regime 2 is improved, which means that investors can achieve a better tradeoff between return and risk if they have knowledge of regimes and can conduct the regime-dependent investments.

Lastly, it is noticeable that regime 1 has the smallest global minimum variance risky portfolio among all three global minimum variance risky portfolios. The reason being that can be attributed to the differences in the correlations between stock and bond returns across the regimes. In regime 1, the correlation between stock and bond returns is negative, consequently, the stock-bond portfolio in this regime might have a relatively better diversified effect on the portfolio. This diversified effect in regime 1 can also be seen, when the annualized standard deviations of the regime-dependent portfolios is roughly between 0.5 and 0.7. More specifically, within this range of the portfolio risks, the payoff in regime 1 is better than the payoff for the same level of portfolio risk in regime 2. However, as pointed out in Section 5.1, the stock market in regime 2 is bullish, which seems that the payoff in this regime should have been better than that in regime 1. For the portfolio risks within 0.5 and 0.7, the reason why this does not occur is because the diversified effect dominates the portfolio performance during this range of portfolio risks.

5.5 Portfolio Performance

In this section, I examine the portfolio strategies under the framework of the RS model. As pointed out in Chapter 4, investors can conduct regime-dependent investment if they have the knowledge of which regime the market is in at time period $t - 1$. For example, if investors know the market at time period $t - 1$ is in regime 1, they would make the investment decision based on the characteristics of regime 1 at time period $t - 1$. To distinguish this strategy with other strategies,

I define this portfolio, constructed under this strategy, as the regime inferred portfolio (*RIP*), which means investors construct the portfolios from the knowledge of the regimes inferred by the RS model.

Alternatively, investors can build their portfolios with the posterior probability implied by the RS model. That is to say, investors are uncertain about the market regimes at time period $t - 1$, but they know the probability of the market being in a certain regime at time period $t - 1$. In other words, investment decision is based on the posterior probability of regimes implied by the RS model, and the portfolio constructed under this strategy is defined as probability implied portfolio (*PIP*).

It is worthwhile to take a look into the performance of these two proposed portfolio strategies over the alternative portfolios.

5.5.1 Regime Inferred Portfolio (RIP)

In Chapter 4, I obtain the regime-dependent weights for the risky assets via the maximization of the Sharpe ratio. At the same time, the inferred regime for a certain time point is known⁴ under the framework of the RS model. Hence, at each time period, the investment decision can be made by referring to which regime the market is in, and then accordingly choose the regime-dependent weights of stock and bond at that time period.

To illustrate the process of this portfolio construction, I assume that, on March 30th, 1990, an investor was making the investment decision about how to allocate the weights of the stock and bond in his portfolio. Under the RS model, the posterior probabilities for the market being in regime 1 and regime 2 on March 30th, 1990, are estimated with 0.04% in regime 1 and 99.96% in regime 2. Since the probability of the market being in regime 2 is greater than 50%, then the regime for the market on March 30th, 1990, is inferred to be regime 2. Also, the investor has the knowledge of asset allocation for each regime. That is, if the

⁴Within the RS model, the posterior probability for the market being in a specific regime is estimated. If the posterior probability is greater than or equal to 50%, then that regime is the inferred regime under the RS model.

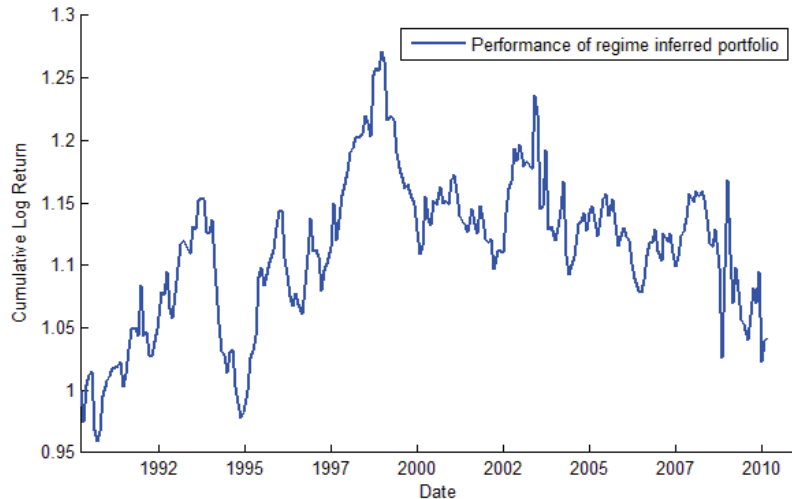


Figure 5.5.1: Performance of Regime Inferred Portfolio (In-sample)

market is in regime 1, the weights he would allocate in the bond and stock in the next period is 78.75% in the stock and 21.25% in the bond. If the market is in regime 2, then the weights in the stock and bond become 105.51% and -5.51% ,⁵ respectively. Together with the fact that the RS model infers the market to be in regime 2 on March 30th, 1990, this suggests that, under the strategy of regime inferred portfolio, the investor should long 105.51% of his asset in the stock and short -5.51% of his asset in the bond on March 30th, 1990.

Following the above process, the weights of the stock and bond for *RIP* at each time period can be estimated during the in-sample period. For \$1 investment on the *RIP*, the performance of this portfolio is plotted on Figure 5.5.1, and the corresponding weights of the stock and bond at each rebalanced time is displayed on Figure 5.5.2. As we can see from Figure 5.5.2, the weights of the stock and bond at some time periods are extremely high or low. As pointed out in the studies (Black and Litterman (1992)), this is due to the strong sensitivity of sample moments of portfolios, and is also one of the weaknesses of the mean-variance analysis. Consequently, it is not feasible to implement this strategy without some appropriate weight constraints.

⁵The negative weight means that the investor would short 5.51% of the bond and use this financing to invest the stock.

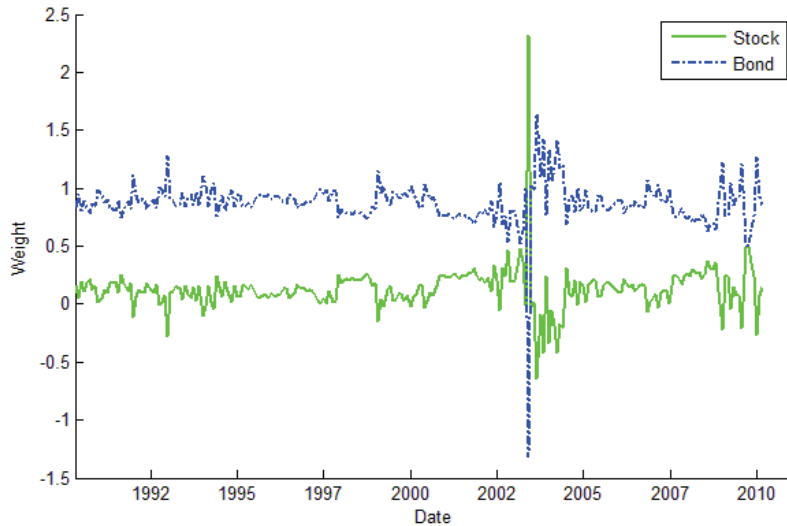


Figure 5.5.2: Weights of Stock and Bond in Regime Inferred Portfolio (In-sample)

To make the *RIP* feasible for practitioners, the weights of bond and stock should be constrained to a certain level. As an investor, the main objective is to maximize the portfolio return for a given level of risk, alternatively, the investor is seeking a portfolio that results in a maximized Sharpe ratio. Therefore, in thesis, one of the main constraints on the weights is that the Sharpe ratio should be subjected to be positive. However, the positive constraint on Sharpe ratio might result in some extreme weights for stock and bond, so another constraint on the individual weights of stock and bond should be imposed. As the riskfree rate in the early 1990s is relatively high, this high riskfree rate results in a big challenge for the constraint of positive Sharpe ratio. To tackle this issue, the individual weight for stock and bond is constrained to be a relatively large value, i.e, within -7 and 7.

As a comparison to the performance of *RIP*, an equally-weighted portfolio, in which the stock and bond are equally invested, is constructed. Figure 5.5.3 shows the performance of \$1 investment in *RIP* and the comparison portfolios, the equally-weight portfolio and the benchmark-S&P 500, during the in-sample period. Figure 5.5.4 presents the weights of stock and bond for *RIP* at each time period during the in-sample period. It is noticeable that there are two time

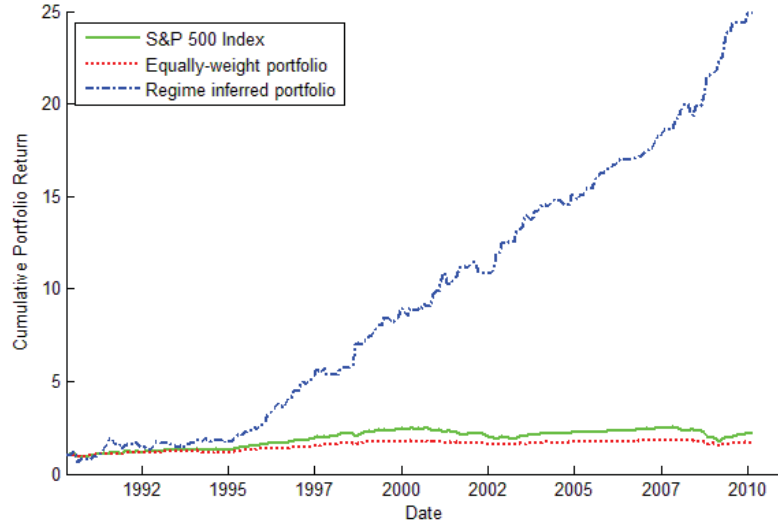


Figure 5.5.3: Performance of *RIP* and the Comparison Portfolios under the Weight Constraints (In-sample)

Figure 5.5.3 compares the performance of \$1 investment in *RIP* and the other portfolios under the weight constraints during the in-sample period. There are two extra weight constraints on *RIP*, one of which is the constraint of positive Sharpe ratio and the other one is the individual weight constraint, -7 and 7.

Table 5.6 Sharpe Ratios for *RIP* and the Compared Portfolios (In-sample)

	S&P 500	Equal-weight	<i>RIP</i>
Mean	0.5010	0.2984	9.9899
Standard Deviation	4.3628	2.4083	23.7335
Sharpe Ratio	0.0439	-0.0046	0.4079

Table 5.6 shows the means, standard deviations and Sharpe ratios for *RIP* and the comparison portfolios during the in-sample period. The risk-free rate that is used to calculate the Sharpe ratios is the average of the 3-month T-bill rate (3.71%) during the in-sample period. The means and standard deviations are expressed in the percentage.

periods in which the individual weight for stock is greater than the individual weight constraint. The reason might be due to the expected returns of stock and bond returns. Specifically, the expected stock and bond returns depend on the latent risk factors, as a result of the factor model, the noise of these latent risk factors are also being taken into account in the expected returns of stock and bond. This issue can be addressed by smoothing the noise of the factors, but this is not the interest of this thesis.

It is visualized that the performance of *RIP* is significantly better than that of the other portfolios. To quantify the comparisons, the Sharpe ratio for each

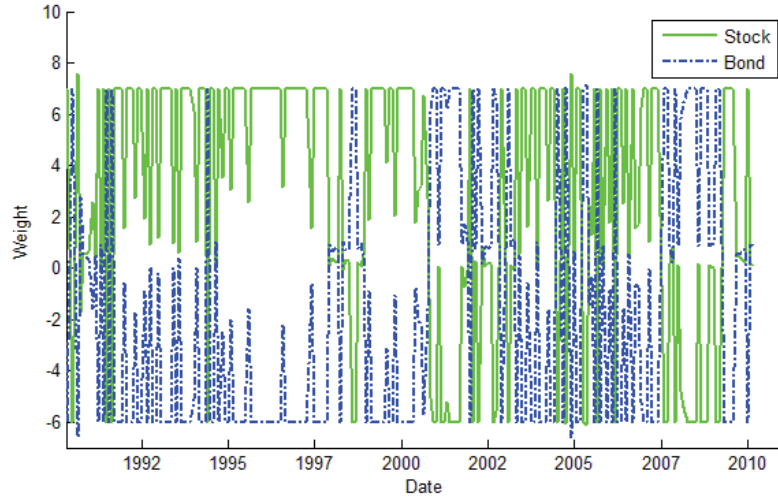


Figure 5.5.4: Weights for Stock and Bond in Regime Inferred Portfolio under the Weights Constraint (In-sample)

Table 5.7 Sharpe Ratios for RIP and the Compared Portfolios (Out-of-sample)

	S&P 500	Equal-weight	RIP
Mean	0.8845	0.7669	1.0709
Standard Deviation	4.9931	1.7957	4.0530
Sharpe Ratio	0.1755	0.4224	0.2622

Table 5.7 shows the means, standard deviations and Sharpe ratios for *RIP* and the comparison portfolios during the out-of-sample period. The risk-free rate that is used to calculate the Sharpe ratio is the average of the 3-month T-bill rate (0.1004%) during the out-of-sample period. The means and standard deviations are expressed in the percentage.

portfolio during the in-sample period is calculated and reported in Table 5.6. From the comparisons of Sharpe ratios, it is clear that the *RIP* indeed performs better than the other portfolios during the in-sample period.

In order to check the robustness of this result, an out-of-sample test is conducted. In Figure 5.5.5, the left graph compares the performance of *RIP* and the other portfolios, and the right graph shows the weights of bond and stock for *RIP* during the out-of-sample period. Also, the Sharpe ratio for each portfolio during the out-of-sample period is reported in Table 5.7. The Sharpe ratio in *RIP* is lower than the Sharpe ratio in equally-weight portfolio, but higher than the Sharpe ratio of S&P 500 Index during the out-of-sample period. This suggests that the *RIP* outperforms the benchmark.

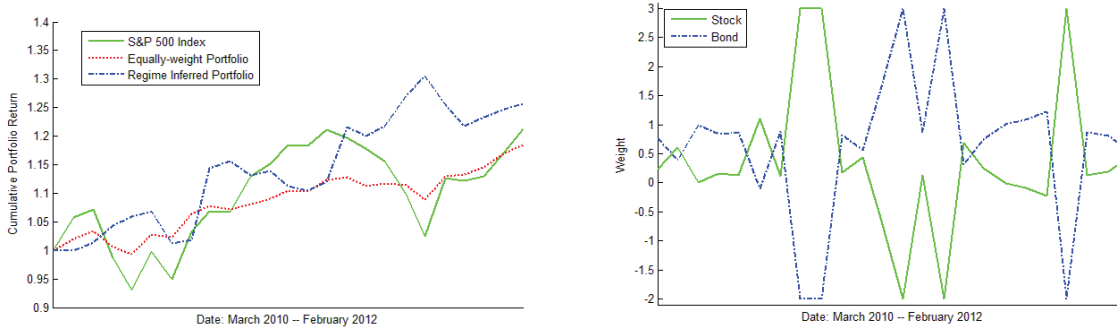


Figure 5.5.5: Performance of RIP and the Comparison Portfolios and Weights of Bond and Stock in RIP (Out-of-sample)

Figure 5.5.5: the left graph compares the performance of *RIP* and the other alternative portfolios during the out-of-sample period, and the right graph shows the weights of stock and bond for the *RIP* during the out-of-sample period. Similarly, there are two extra constraints on the weights of *RIP*, one of which is the positive Sharpe ratio and the other one is the individual weight. The constrain on the individual weight in the out-of-sample test is between -3 and 3, and is different from the individual weights constraints during the in-sample period. This is because the constrain of the positive Sharpe ratio is valid at any time periods, when the individual weight subjects to be -3 and 3.

5.5.2 Probability Implied Portfolio (PIP)

The probability implied portfolio is the portfolio constructed by incorporating the posterior probability estimated in the RS model. The basic idea for this strategy is that the investor has the knowledge of the probability of which regime the market is in, when he faces the investment decision. Similarly, the following shows the mechanism about how an investor can build its portfolio under this strategy.

Again, I assume that the investor was facing an investment decision on March 30th, 1990. He knew the probabilities of the market being in regime 1 and regime 2 are 0.04% and 99.96% respectively on March 30th, 1990. In contrast with the investor for *RIP*, this investor would establish a portfolio by incorporating the fact that the market in a specific regime is only probabilistic. For example, even though the investor knew the probability for the market staying in regime 1 on March 30th, 1990 is extremely low, he did not want to ignore the potential risks resulting from ignoring the fact that the market was indeed in regime 1 on March 30th, 1990. Hence, he constructed the portfolio based on the possible regimes

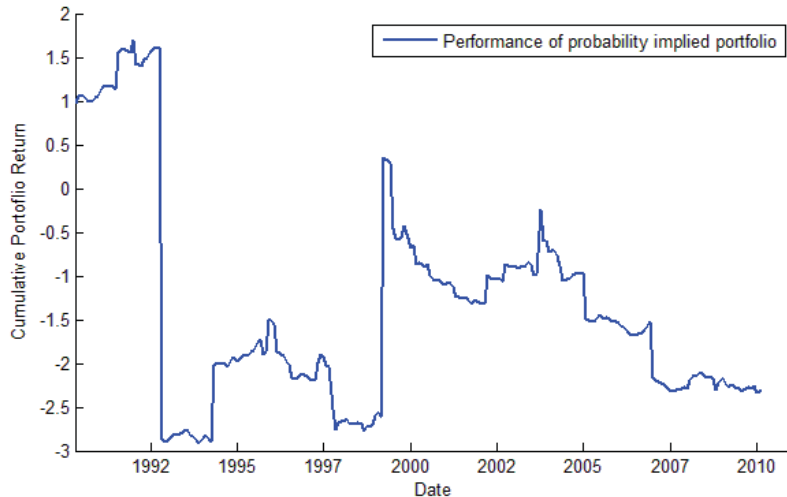


Figure 5.5.6: Performance of Probability Implied Portfolio (In-sample)

instead of a certain regime on March 30th, 1990. The portfolio constructed under this strategy is the so-called probability implied portfolio (*PIP*) in this thesis.

Figure 5.5.6 shows the performance of \$1 investment on *PIP* during the in-sample period and Figure 5.5.7 shows the weights of stock and bond for *PIP* during the in-sample period. Evidently, the performance of *PIP* has a strong volatility and the weights for bond and stock are also extremely high or low at some time points during the in-sample period. That is to say, the weights of *PIP* should also be constraint at some degree. To do so, following the same constraints as that in *RIP*, the weights of stock and bond in *PIP* are also subjected to the two extra constraints. The first one is the positive Sharpe ratio and the other one is the individual weight constraint, -7 and 7.

With the above portfolio construction process and the weight constraints, Figure 5.5.8 presents the performance of each portfolio during the in-sample period, and Figure 5.5.9 shows the weights of bond and stock at each time period for *PIP* during the in-sample period. As expected, there are also two time periods, in which the weight of stock is greater than the individual weight constraint. Similarly, this phenomenon can also be explained by the noise of the factors included in the RS model.

Table 5.8 reports the Sharpe ratios for *PIP* and the comparison portfolios

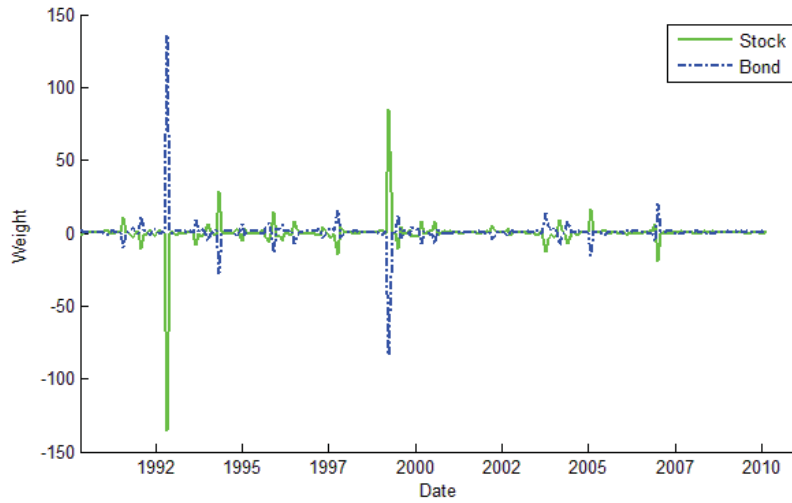


Figure 5.5.7: Weights of Stock and Bond in Probability Implied Portfolio (In-sample)

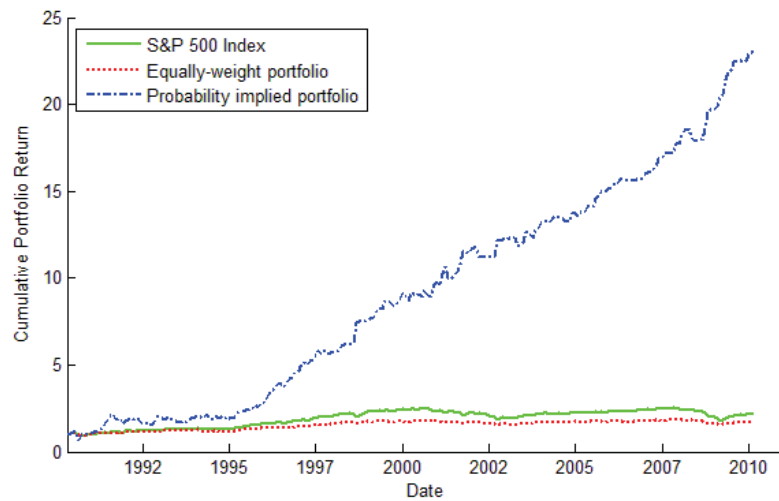


Figure 5.5.8: Performance of *PIP* and the Comparison Portfolios under the Weight Constraints (In-sample)

Figure 5.5.8 compares the performance of \$1 investment in *PIP* and the other portfolios under the weight constraints during the in-sample period. There are two extra weight constraints on *PIP*, one of which is the constraint of the positive Sharpe ratio and the other one is the individual weight constraint, -7 and 7.

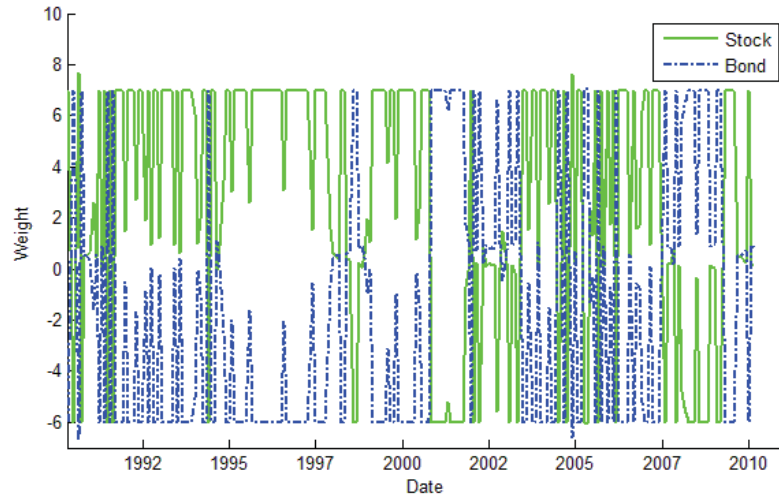


Figure 5.5.9: Weights for Stock and Bond in Probability Implied Portfolio under the Weights Constraints (In-sample)

Table 5.8 Sharpe Ratios for *PIP* and the Compared Portfolios (In-sample)

	S&P 500	Equal-weight	PIP
Mean	0.5010	0.2984	9.1896
Standard Deviation	4.3628	2.4083	23.8959
Sharpe Ratio	0.0439	-0.0046	0.4051

Table 5.8 shows the means, standard deviations and Sharpe ratios for *PIP* and the comparison portfolios during the in-sample period. The risk-free rate that is used to calculate the Sharpe ratios is the average of the 3-month T-bill rate (3.71%) during the in-sample period. The means and standard deviations are expressed in the percentage.

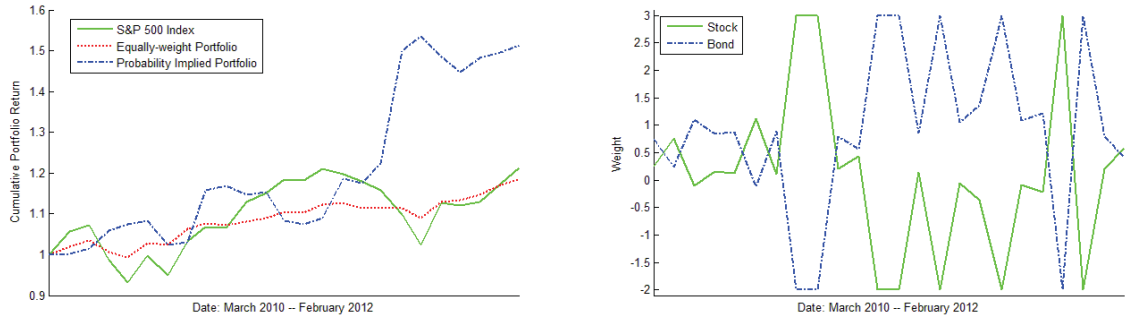


Figure 5.5.10: Performance of *PIP* and the Comparison Portfolios and the Weights of Bond and Stock in *PIP* (Out-of-sample)

Figure 5.5.10: the left graph compares the performance of *PIP* and the other alternative portfolios during the out-of-sample period, and the right graph shows the weights of stock and bond for the *PIP* during the out-of-sample period. Similarly, there are two extra constraints on the weights of *PIP*, one of which is the positive Sharpe ratio and the other one is the individual weight. The constrain on the individual weight in the out-of-sample test is between -3 and 3, and is different from the individual weights constraints during the in-sample period. This is because the constrain of the positive Sharpe ratio is valid at any time periods, when the individual weight subjects to be -3 and 3.

during the in-sample period. As the Sharpe ratio of *PIP* is the highest among the three portfolios, it suggests that *PIP* performs better than the other portfolios during the in-sample period.

For the out-of-sample period, Figure 5.5.10 shows the performance of \$ 1 investment in *PIP* and the comparison portfolios, and the weights of bond and stock for *PIP*. Although the cumulative portfolio return of *PIP* is greater than the other portfolios, the risk of this portfolio might also be greater. Hence, Table 5.9 shows the Sharpe ratio for each portfolio during the out-of-sample test. From the Sharpe ratios in Table 5.9, we can see that *PIP* only outperforms the benchmark, S&P 500, during the out-of-sample test.

To conclude, within the framework of regime-switching model, the regime inferred portfolio (*RIP*) and the probability implied portfolio (*PIP*) are investigated. The performances of the two portfolios are compared with the other alternative portfolios. The in-sample test suggests that these two portfolios perform better than other portfolios, while the out-of-sample test indicates that these two portfolios only outperform the benchmark, S&P 500. In terms of the comparison of the

Table 5.9 Sharpe Ratios for *PIP* and the Comprised Portfolios (Out-of-sample)

	S&P 500	Equal-weight	PIP
Mean	0.8845	0.7669	1.0709
Standard Deviation	4.9931	1.7957	4.0530
Sharpe Ratio	0.1755	0.4224	0.2622

Table 5.9 shows the means, standard deviations and Sharpe ratios for *PIP* and the comparison portfolios during the out-of-sample period. The risk-free rate that is used to calculate the Sharpe ratio is the average of the 3-month T-bill rate (0.1004%) during the out-of-sample period. The means and standard deviations are expressed in the percentage.

performance of *RIP* and *PIP*, it is interesting to see that the Sharpe ratios for *PIP* during both the in-sample period and the out-sample period are very close to that of *RIP*.

Chapter 6

Conclusion and Discussion

In this Chapter, I will conclude the findings from the empirical analysis, and also discuss the future works relating to this thesis.

6.1 Conclusion

In this thesis, I study the stock and bond returns under the framework of regime switching multifactor model. Starting with the asset pricing model, i.e, regime-switching multifactor model, I use three nonmacroeconomic factors and three macroeconomic factors to capture the bond and stock returns. As the determination of the number of regimes in the RS model centralizes the accuracy of estimated parameters, the BIC is selected to determine the number of regimes in this thesis. From the estimated parameters in the RS and SR models, there are evidences that most of the factors show the predictive power for the future stock and bond returns in the RS model, while only a few factors present the foreseeability in single regime model, i.e., the multiple linear regression model without regimes. Meanwhile, from the in-sample test, it is evident that the RS model forecasts the actual returns better than the single regime model. To further examine the forecast ability in the adopted RS model, an out-of-sample test is conducted. However, it is unclear whether the RS model predict the returns, especially for the stock return, better than the SR model, but the RS model does show a clearer

up-and-down trend compared with the SR model.

Thereafter, based on the regime-dependent correlations, the regime-dependent mean-variance frontiers are investigated for the different regimes. The mean-variance frontier in regime 2 has the best tradeoff between return and risk, while the tradeoff in single regime is the worst. Also, I find that regime 1 has the lowest global minimum variance portfolio among all three frontiers. One possible reason is that the correlation between stock and bond returns is negative in regime 1, as a result, the portfolio in regime 1 shows a better diversification compared with the others. In terms of the tangent portfolios in these regimes, it is surprising to see that the Sharpe ratio for either regime 1 or regime 2 is greater than that in single regime.

Finally, I examine two portfolio strategies implied by the RS model. One is the regime inferred portfolio (*RIP*) and the other one is the posterior probability implied portfolio (*PIP*). Both the in-sample and out-of-sample tests suggest that these two portfolios perform better than the benchmark, S&P 500. In terms of the comparison of the performance between *RIP* and *PIP*, it is interesting to see that the Sharpe ratios for *PIP* during both the in-sample period and the out-sample period are very close to that of *RIP*.

6.2 Discussion

In this thesis, I study the regime-dependent correlations of stock and bond returns, and examine the performance of the two portfolio strategies under the framework of RS model. As an extension of the two asset model in this study, the other classes of assets can also be considered to examine whether the included assets can help construct a better diversified portfolio. Due to the globalization, the traditional diversification between stock and bond may not be able to adequately diversify the system risk in the current sophisticated financial market. Therefore, the other classes of assets, such as commodity and currency, can be included as

the investment tools for investors to construct a sufficiently diversified portfolio.

Moreover, as pointed out in Chapter 5, within the RS Multifactor model, the noise of the factors is included in the expected asset returns. The expected returns with the noise of factors might result in some extreme investment weights . To tackle this problem, the noise of these factors can be ruled out by beginning with an autoregressive regime-switching factor model, and then use the predicted factors to forecast the asset returns. By doing so, the noise of the factors would be smoothing out in the expected returns of assets. Another approach to addressing this issue is to use the regime switching autoregressive model to price the asset returns, instead of the regime-switching factor model.

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Appendix : EM Algorithm

This Appendix intends to show the method of parameter estimation under the RS model. Essentially, the EM algorithm is a two-step parameter estimation with E-step, the estimation of missing data on regimes, and M-step, the maximization of the likelihood based on the E-step of the missing data estimation on regimes.

Throughout this section, for notational simplicity, the subscriptions of the parameters are temporary to be ignored in the following derivation of EM algorithm. Let the set of parameter be Φ , the sequence of observations of the bond and stock returns and common factors be \mathbf{X} , the sequence of unobserved regimes over time be $\mathbf{\Gamma}$. Therefore, the parameters can be estimated by maximizing the likelihood function $P(\mathbf{X}, \mathbf{\Gamma}|\Phi)$, which is the same as maximizing the log likelihood function

$$\max_{\Phi} \left\{ \ln \sum P(\mathbf{X}, \mathbf{\Gamma}|\Phi) \right\}, \quad (6.2.1)$$

where $P(\mathbf{X}, \mathbf{\Gamma}|\Phi)$ is the joint probability distribution function of \mathbf{X} and $\mathbf{\Gamma}$. Using the Jensen's inequality, then the log likelihood function

$$\ln \sum P(\mathbf{X}, \mathbf{\Gamma}|\Phi) \geq \sum Q(\mathbf{\Gamma}) \ln \frac{P(\mathbf{X}, \mathbf{\Gamma}|\Phi)}{Q(\mathbf{\Gamma})}, \quad (6.2.2)$$

where $Q(\mathbf{\Gamma})$ is an arbitrary item.

Beginning with a set of initial values for the parameters in Φ , then the objective is to find a tight lower bound to the true maximum log-likelihood, as shown on the right hand side of equation (6.2.2). The optimal distribution function Q is obtained by maximizing the right hand of equation (6.2.2) for the initial value of

parameter, denoted Φ^0 .

Associating with the observed data and initial value of parameter, as well as the maximum of the right hand side function of equation(6.2.2), then the arbitrary item is

$$\mathbf{Q}(\mathbf{\Gamma}) = \mathbf{P}(\mathbf{\Gamma}; \mathbf{X} | \Phi^0). \quad (6.2.3)$$

Substituting equation (6.2.3) into equation (6.2.2), then lower bound function achieves the log-likelihood $\ln P(\mathbf{X} | \Phi^0)$ of the observed data for the initial parameter estimate Φ^0 . This is called the E-Step.

Suppose that Φ is true maximum likelihood estimate, then we have $\ln P(\mathbf{X}, \mathbf{\Gamma} | \Phi) \geq P(\mathbf{X}, \mathbf{\Gamma} | \Phi^0)$ for any approximation Φ^0 and

$$\ln P(\mathbf{X} | \Phi) \geq \sum P(\mathbf{\Gamma}; \mathbf{X} | \Phi^0) \ln \left(\frac{P(\mathbf{X}, \mathbf{\Gamma} | \Phi^0)}{P(\mathbf{\Gamma}; \mathbf{X} | \Phi^0)} \right) \geq \ln P(\mathbf{X} | \Phi^0) \quad (6.2.4)$$

In order to improve the initial estimate Φ^0 , M-step is implemented by maximizing the middle term in the inequality equation (6.2.4). That is to maximize the expected log-likelihood of the join data of \mathbf{X} and $\mathbf{\Gamma}$ with respect to Φ

$$E^{Q^*}(\ln P(\mathbf{X}, \mathbf{\Gamma} | \Phi)) = \sum P(\mathbf{\Gamma}; \mathbf{X} | \Phi^0) \ln (P(\mathbf{X}, \mathbf{\Gamma} | \Phi)).$$

Hence, an improved estimated for the parameter set Φ is

$$\Phi^* = \arg \max_{\Phi} \{E^{Q^*}(\ln P(\mathbf{X}, \mathbf{\Gamma} | \Phi))\}.$$

With each iteration, it is clear that log-likelihood is increasing under this algorithm. When the increase is approaching to a very small value that is close to zero, then the true parameter Φ is achieved. An iterative algorithm can be designed as follows:

1. **E-step:** Set initial value Φ^0 for the true parameter set Φ , calculate the

arbitrary function, $Q^*(\Gamma) = P(\Gamma; \mathbf{X} | \Phi^0)$, and determine the expected log-likelihood, $E^{Q^*}[\ln P(\mathbf{X}, \Gamma | \Phi)]$.

2. **M-step:** Maximize the expected log-likelihood with respect to the arbitrary distribution Q^* of the hidden variable to obtain an improved estimate of Φ .

The improved estimated is

$$\Phi^* = \underset{\Phi}{\operatorname{arg\,max}} \{E^{Q^*}(\ln P(\mathbf{X}, \Gamma | \Phi))\},$$

where Φ^* is the new initial value for Φ .

Under the EM framework, the parameters are obtainable by estimating the missing variables and maximizing the likelihood function.