

A FREILP Approach for Long-Term Planning of
MSW Management System in HRM, Canada

by

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DALHOUSIE UNIVERSITY

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Abstract

The municipal solid waste (MSW) management system is consisted of planning, development, execution of capital works, and so on. Too many factors in the system make the decision making process plagued with uncertainties, vagueness and complication. Interval-parameter Linear Programming (ILP) is widely used to deal with uncertainties existed in the MSW system and to assist optimal decision making. However, the existing ILP solution algorithms, i.e., best-worst case algorithm and 2-step algorithm, are found to be ineffective through a validity checking process. Moreover, the results from ILP cannot reflect the linkage between decision risks and the system return.

In this study, a fuzzy risk explicit interval-parameter linear programming (FREILP) model is developed and applied to the long-term planning of the MSW management system in Halifax Regional Municipality (HRM). This method is specifically designed to deal with extensive uncertainties existed in the MSW management system and to provide decision supports to HRM planners. In the model, ILP is used to reflect uncertainties existed in both objective function and constraints. Based on the basic ILP, a risk function is defined to assist in finding solutions with minimum system cost while minimizing the system risk, under certain aspiration levels. The aspiration level could be conservative, medium or aggressive, and can thus be presented as a fuzzy set to reflect the preference of decision makers. Three sets of solutions are obtained accordingly. Besides, the model was also solved under the aspiration level from 0 to 1, with a step of 0.1, for providing a comprehensive decision support.

This approach can effectively reflect dynamic, interactive, uncertain characteristics, as well as the interactions between overall cost and risk level of the MSW management system, thus provide valuable information to support the decision-making process, such as waste allocation pattern, timing and expansion capacities of the municipal solid waste management activities. The result can directly reflect the tradeoff between decision risks and the system return.

List of Abbreviations and Symbols Used

BWC	Best-worst case algorithm
C&D	Construction and demolition
CCP	Chance-constrained programming
CDF	Cumulative distribution Function
DF	Discount factor
DP	Dynamic programming
FEP	Front end processor
FFP	Fuzzy flexibility programming
FLP	Fuzzy linear programming
FMP	Fuzzy mathematical programming
FPP	Fuzzy possibility programming
FREILP	Fuzzy risk-explicit interval linear programming
HHW	Household hazardous waste
HRM	Halifax regional municipality
ILP	Interval-parameter linear programming
IPMP	Interval-parameter mathematical programming
IMSW	Integrated municipal solid waste
LP	Linear programming
MILP	Mix-integer linear programming
MLP	Multi-objective mathematical programming
MSW	Municipal solid waste
NRL	Normalized risk level
PDF	Probability distributions function
RDF	Residual disposal facility
REILP	Risk explicit interval-parameter linear programming
SMP	Stochastic mathematical programming

A^{\pm}	A set of interval parameters of ILP constraints
B^{\pm}	A set of right-hand side of parameters of ILP
C^{\pm}	A set of interval parameters of ILP objective function
CA_{ik}	Capital cost (expansion cost) for facility i under expansion option k (\$)
CP_i	Current capacity of facility i (tonnes/year when $i=1$ and 2 ; tonnes when $i=3$)
DF_t	Discount factor in period t
EP_{ikt}	The expanding capacity of option k (tonnes/year when $i=1$ and 2 ; tonnes when $i=3$)
HWG_t	Wastes disposed by HRM (tonnes/5 years)
P	Feasible decision space of ILP solved by 2-step algorithm
P^l	Smallest decision space of ILP solved by 2-step algorithm
P^u	Largest decision space of ILP solved by 2-step algorithm
Q	Decision space of ILP solved by BWC algorithm
Q^l	Smallest decision space of ILP solved by BWC algorithm
Q^u	Largest decision space of ILP solved by BWC algorithm
RR	Residue rate from both recycling and composting facilities
T	A particular time period
TWG_t	Total wastes generated in HRM (tonnes/ 5 years)
UC_i	The unit collection and transportation cost for facility i

UO_i	Unit operating cost for the facility i (\$/tonne)
UR_i	Unit revenue from facility i (\$/tonne)
X^\pm	A set of decision variables of ILP
X_{opt}^\pm	Optimum decision variable values
X_{opt}^-	Lower bound of optimum decision variable values
X_{opt}^+	Upper bound of optimum decision variable values
X_{it}	Waste flow allocated to the facility i in period t (tonnes/5 years)
	Binary variable:
Y_{ikt}	$Y_{ikt} = 1$ when facility i (with capacity option k) needs to be developed in period t $Y_{ikt} = 0$ when there is no expansion
a_i	Interval parameters of ILP constraints
b_i	Right-hand side of parameters of ILP
c_j	Interval parameters of ILP objective function
f^\pm	Value of objective function
f_{jopt}^\pm	Optimum objective function interval
f_{jopt}^+	Upper bound of optimum objective function
f_{jopt}^-	Lower bound of optimum objective function
$f_{2-stepopt}$	Optimum objective function solved by 2-step algorithm
f_{BWCopt}	Optimum objective function solved by BWC algorithm
k_1	Number of positive c_j parameters

λ_0	Aspiration level
λ_{ij}	Risk level variables
ξ	Risk function
	Different waste processing facilities:
I	$i=1$ for recycling facility
	$i=2$ for composting facility
	$i=3$ for landfill
T	The planning time period, $t=1,2,\dots,6$
x_j	Decision variables of ILP
x_{jopt}^\pm	Optimum decision variable intervals of x_j
x_{jopt}^+	Upper bound of optimum decision variable x_j
x_{jopt}^-	Lower bound of optimum decision variable x_j
-	“-” superscript represents the lower bound of an interval-parameter or variable
+	“+” superscripts represents the upper bound of an interval-parameter or variable

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CHAPTER 1

INTRODUCTION

1.1. Statement of the Problem

1.1.1. Municipal Solid Waste (MSW) Management System

Municipal solid waste (MSW) includes “garbage, refuse, sludge, rubbish, tailings, debris, litter and other discarded materials resulting from residential, commercial, institutional and industrial activities which are commonly accepted at a municipal solid waste management facility”. (Nova Scotia Environment and Labour, 2004)

Generally, three major components exist in a MSW management system. They include waste generation districts, waste processing facilities, and landfill. Transfer stations are also included in the system in some big cities like New York City (USEPA, 2000). In 1996, Environment Canada defined the Integrated Municipal Solid Waste (IMSW) system as a management system that contains programs which can “reduce or reuse the wastes produced, and/or divert wastes from traditional disposal facilities to recycling, composting, and/or incineration processes” (Environment Canada, 1996). Based on this definition, a MSW management system must be substantially modified to perform the far more complex tasks to ensure that wastes are initially reduced, certain products are reused, and wastes are separated at their sources from recyclables and compostables which can then be processed, stored, marketed and transported to buyers (Baetz et al., 1989).

As a consequence, many factors must be considered in planning such an integrated management system. Examples include collection techniques to be adopted, how to allocate the trash, when to expand the existing facilities, where to develop a new facility, and how to control the total system cost (Wilson, 1985). Interactions among these factors make the management system more complicated. Besides, conflicts may exist among different decision-makers and interest groups, such as local government officials, facility

owners and operators, consultants, regulatory agency specialists, and residents. Multiple options may lead to different levels of satisfaction for each of the stakeholders (Barlshen, 1996). This complexity makes it difficult to identify the optimal MSW management options among decision makers. Thus, optimization techniques that could consider and incorporate factors within a general framework, rather than examine them in isolation, would be desirable for providing a holistic analysis of the factors, and for a comprehensive evaluation of the related activities and policy responses (Baetz, 1990b; Huang et al., 2001).

1.1.2. Optimization Model Development for MSW Planning

Optimization techniques for waste management were first proposed by Anderson in 1968. Since then, a number of waste-related planning models have been developed. They include the deterministic optimization models, such as linear, mixed-integer linear, dynamic, and multi-objective programming (Anderson, 1968; Jenkins, 1982; Baetz et al., 1989; Baetz, 1990a&1990b; Thomas and Baetz, 1990). However, it has been recognized that deterministic optimization techniques are not sufficient to model such a complex problem, particular its uncertain features. To better reflect uncertainties in waste management systems, several optimization techniques were developed. They include fuzzy mathematical programming (FMP) (Zimmermann, 1985; Huang, 1994; Leimbach, 1996; Chang, 1997; Lee and Wen, 1997; Chanas, 2000; Seongwon et al., 2003), stochastic mathematical programming (SMP) (Zare and Daneshmand, 1995; Schwarm, 1999; Schultz, 2003), interval-parameter mathematical programming (IPMP) (Moore, 1979; Zou et al., 1999; Rocha and Kreinovich, 2003) and some hybrid or integrated programming methods (Huang et al., 1993b; Huang et al., 1995; Infanger and Morton, 1996; Xia et al., 1997; Huang et al., 2001; Huang et al., 2002; Li et al., 2009a; Sun et al., 2009; Nie et al., 2009; Guo and Huang, 2009a&2009b; Li and Huang, 2010a;). Mixed outcomes have been obtained when different methods were used to reflect system uncertainties in hypothetical or real-world case studies. Compared to fuzzy and stochastic programming, in terms of data quality and requirements, IPMP does not need the information of membership functions or distribution of parameters which may be hard to

obtain in practical applications. Moreover, fuzzy and stochastic methods often lead to more complicated sub-models and may not be practical for many real life situations. Interval-parameter linear programming (ILP) is one of the IPMP which can effectively deal with uncertainties without leading to more complicated sub-models. Because of this characteristic, ILP has been widely used in many environmental and civil modeling fields (Ben-Israel and Robers, 1970; Rommelfanger et al., 1989; Huang and Moore, 1993; Tong, 1994; Hansen and Walster, 2004; Maqsood et al., 2004; Li et al, 2008&2009b; Guo et al, 2010; Wu et al., 2010; Dong et al., 2011), including MSW system planning and management issues. Three solution algorithms have developed to facilitate the use of ILP, including Monte-Carlo simulation, best-worst case analysis (BWC) and 2-step interactive algorithm.

The Monte-Carlo simulation method randomly sets values for each parameter within their interval range to form a classic Linear Programming (LP) model. This method could be most reliable in simulating a real model situation if the modeler sets values for parameters and runs the model millions of times. However, it is not always realistic to run the models millions of times, especially when facing a real environmental systems planning problem with hundreds or thousands of decision variables and constraints. As more realistic solution methods, BWC analysis and 2-step algorithm were developed (Huang and Moore. 1993; Tong, 1994; Chinneck and Ramadan, 2000). Both approaches reformulate the original model using extreme constraints to represent the most conservative and the most aggressive conditions. The main difference between the two algorithms is that the 2-step algorithm differentiates the selection of extreme parameter values with different signs after the objective function is fixed, while the BWC treats all the parameters without discrimination. These two algorithms have been widely used in ILP applications. Both algorithms provide an interval solution space, and each point in the interval solution space becomes a potential solution to form a decision alternative for implementation. However, in practical decision making process, when the decision makers need to pick a point from the interval solution space for an implementation scheme, there will be problems of feasibility and optimality, which might be resulted from the flaws in the algorithm development. Therefore, validity checking seems to be

absolutely desired for the ILP algorithms in order to improve the applicability of the ILP modeling results.

Moreover, the result of the ILP lacks of a linkage between decision risks and system return. Different decision makers may expect divergent system returns under various acceptable risk levels. Thus, a modeling framework which could not only be effectively and efficiently solved but also can reflect the linkage between system return and decision risks became desirable. This modeling framework was preliminarily examined by Zou et al. (2010) through developing a risk explicit interval-parameter linear programming model (REILP). However, several deficiencies still exist in the REILP, including the infeasibility problem, the risk function issue, and the difficulty of the aspiration level selection. These deficiencies will be addressed in this study as an evolution of the proposed REILP approach.

1.1.3. MSW Management System in HRM

Nova Scotia is the first province in Canada that has achieved the target of a 50% solid waste diversion rate, according to the criteria established by the Canadian Council of Ministers of the Environment (Walker et al., 2004). The Halifax Regional Municipality (HRM), as the capital of Nova Scotia, provided an outstanding contribution to this high diversion rate achievement. HRM reported in January 2011 that the current diversion rate was right on target at 60% (CMC, 2010 Newsletter). Along with the diversion target, HRM developed several long-term goals for effectively managing the MSW management system of HRM, including maximizing the 3Rs (Reduction, Reuse and Recycling), maximizing environmental sustainability and minimizing costs, and fostering stewardship and values of a conserver society (FCM, 2000).

To reach the diversion rate target and the long-term environmental goals, HRM has strived to establish a new MSW management system since 1999, along with many programs including source separation program, new landfill site construction, and materials recovery facility expansion program (Walker et al., 2004). During the

implementation of these programs and continuous improvement of the MSW diversion rate, cost is a key factor that must be considered by decision makers. This new MSW management system has a gross annual budget of 32 million Canadian dollars. Capital costs include \$8.5 million for the carts, mini bins and tracking system; \$24 million for the mixed waste processing system and the back-end stabilization plant; \$20 million for the new landfill (which opened in January 1999) construction and access roads; and \$500,000 for the upgrade to the Materials Recovery Facilities (MRF). The mixed waste processing and stabilization operation employed 85 people (Goldstein and Gray, 1999). This is a significant cost for a city like the size of HRM. Furthermore, if a higher diversion rate is desired, or if the facilities need to be expanded or relocated, additional investment would be needed.

There is always a tradeoff between the system investment and environment effects. Economic growth is usually sacrificed for the good of the environment. The ideal and best solution for environmental protection is usually unaffordable. It is more realistic to find an optimal solution which is economically feasible and environmentally acceptable. In HRM, a problem that decision makers have to consider and find the answer is what the least-cost strategy for achieving the waste diversion rate target as well as the long-term environmental goals is. Previously, very few studies have reported a comprehensive MSW management system study for HRM. Ghadiri (2004) developed a fuzzy-stochastic mixed integer LP model for the long-term planning and management of the MSW system of HRM. This study focused on addressing system uncertainties associated with the MSW management processes and it has provided a very valuable decision support tool for local decision makers and been appreciated by them. However, the diversion rate target was not included in the study and the model was solved by the 2-step algorithm. No validity checking for the ILP method was conducted and the connections between decision risks and system benefits were not examined as well. All the relevant issues will be addressed in our study for providing better and more reliable decision support information for the HRM MSW managers.

1.2. Objective

As an extension of previous efforts on MSW management system modeling, this study will focus on the development of a fuzzy risk explicit interval linear programming model and its application to the HRM case study. This study entails the following objectives:

- Validity checking of two ILP solution algorithms, i.e., Best-Worse Case algorithm and the 2-step interactive algorithm. A numerical example will be formulated and solved by the Monte-Carlo simulation, BWC and 2-Step algorithm respectively for investigating the validities of BWC and 2-Step algorithm, and the focus will be on the feasibility and optimality of the interval solutions.
- Development of a Fuzzy Risk Explicit Interval Linear Programming (FREILP) model for reflecting complex connections between system return and decision risk. The proposed FREILP model is designed to minimize the decision risks while the total system cost is maintained at a minimum level and the aspiration level is preferably selected by the decision makers. In addition, problems of model infeasibility and risk function formulation will also be discussed.
- Application of the developed FREILP model to the long-term planning of the MSW management system in HRM. The modeling results could provide HRM managers scientific basis for generating practical MSW management implementation schemes. In this application, three options based on the different aspiration levels of decision makers will be provided, including aggressive schemes, medium schemes, and conservative schemes.

1.3. Structure of the Thesis

The structure of this thesis is organized as follows:

Chapter 1 introduces the MSW system and the application of mathematical programming to the planning and management of the MSW system. Problems associated with the current modeling studies are discussed, leading to the need of a new approach that could provide decision support for effective and efficient MSW planning and management.

Chapter 2 provides a comprehensive literature review with respect to previous efforts in using optimization techniques to solve MSW planning and management issues. The focuses have been placed on discussing uncertainty-handling optimization techniques and their applications to MSW management system. A summary has been provided for identifying a few knowledge gaps that need to be addressed in future studies.

Chapter 3 uses a numerical example to conduct the validity checking of existing algorithms to ILP models with focuses on checking the feasibility and optimality of the model solutions.

Chapter 4 discusses the details of FREILP model formulation and development. The solution process for solving the FREILP model is also provided.

Chapter 5 presents a detailed description of the MSW management system of HRM and the FREILP-based model formulation.

Chapter 6 provides the modeling results obtained from the REILP model and FREILP model. The implications of the modeling results to the planning and management of the HRM waste system is also discussed.

Chapter 7 is a summary of this study along with some conclusions. Recommendation for further work is also provided in this chapter.

CHAPTER 2

LITERATURE REVIEW

2.1. Conventional MSW Management and Planning Methods

Since the 1960's, much efforts have been devoted to develop mathematical programming models to support the decision makers of MSW management system and to evaluate the relevant operational and investment policies. Anderson firstly proposed an economic optimization method for the planning of MSW in 1968 (Anderson, 1968). After that, a number of waste-related deterministic optimization models have been developed.

Chiampi et al.(1982) applied the linear programming (LP) techniques to investigate the relative costs of hazardous waste management schemes. Hsieh and Ho (1993) also used the LP to optimize the solid waste disposal and recycling system. Jenkins developed a mix-integer linear programming (MILP) method and applied it to the planning of the municipal waste reclamation system for Southeastern Ontario (Jenkins, 1982). Huang et al. applied the MILP in the MSW management system in China (Huang et al., 1997). Baetz et al. applied the dynamic programming (DP) model to determine the optimal sizing and timing of facility expansions for Mecklenburg County, North Carolina, US, the Long Island Community in New York, US, and the Regional Municipality of Hamilton, Ontario, Canada (Beatz et al., 1989); Chang and Wang (1995) applied a multi-objective linear programming model (MLP) to a MSW system. They considered both economic and environmental objectives while establishing and selecting the long term optimal management alternatives. Solano et al. (2002) used LP and developed an integrated solid waste-management model to assist in identifying desired solid waste management strategies that could meet cost, energy and environmental emission objectives.

As environmental policies have become more integrated and strict, it has been recognized that the above deterministic optimization techniques are not sufficient to model complex

MSW management problems. In MSW management system, many processes need to be considered by decision makers, including waste collection, allocation, transportation, treatment and disposal (Wilson, 1985). These processes contain many factors that interact with each other with multi-period, multi-layer, and multi-objective features (Thomas et al., 1990). The spatial and temporal variations of many system components may further multiply the uncertainties in the system (Thompson and Tanapat, 2005). Therefore, these factors are associated with uncertainties and hard to be evaluated in precise terms.

Since the deterministic optimization techniques require deterministic data and crisp model constraints, optimization techniques that can reflect uncertainties became desirable. Various approaches that can deal with uncertainties have been developed and applied to MSW management study. When parameters in the model are fuzzy sets, the optimal problem becomes the fuzzy mathematical programming (FMP). Comparing to FMP problems, stochastic mathematical programming (SMP) models are similar in style but probability distributions are governing the data. When the parameters are known only within certain bounds, the approach to tackling such problems is called interval-parameter mathematical programming (IPMP) or robust optimization.

2.2. Optimization Approaches that Deal with Uncertainties

2.2.1. Fuzzy Mathematical Programming

Fuzzy mathematical programming (FMP) is based on the fuzzy set theory formalized by Professor Lofti Zadeh in 1965. Comparing to the classical sets (membership can only be 0 or 1), the member of fuzzy sets could be described with the aid of a membership function valued in the real unit interval $[0, 1]$. Parameters in both the objective function and the constraints could be fuzzy sets.

FMP has been applied to many optimization problems, waste management included. Koo et al. (1991) accomplished the location planning of a regional hazardous waste treatment

center by using FMP. In the same year, Lee et al. (1991) applied FMP to a dredged material management problem, using the environmental risk and cost as fuzzy parameters. Chang and Wang (1997) applied a fuzzy goal programming approach for the optimal planning of solid waste management system in a metropolitan region in Taiwan. It demonstrated how fuzzy objectives of the decision maker can be quantified under various types of solid waste management alternatives. More recently, Fan et al. (2009) developed a fuzzy linear programming (FLP) model for the long term planning of MSW management system. It can deal with uncertainties expressed as fuzzy sets that exist in the constraint's left-hand and right-hand sides and objective function.

FMP can be categorized into two major streams: fuzzy flexibility programming (FFP) and fuzzy possibility programming (FPP) (Inuiguchi and Sakawa, 1994). In FFP, the flexibility in the constraints and fuzziness in the objective are represented by fuzzy sets and denoted as “fuzzy constraints” and “fuzzy goal” respectively, which can be expressed as membership grades. However, FFP could hardly tackle uncertainties expressed as ambiguous coefficients in the objective function and constraints (Inuiguchi and Tanino, 2000; Inuiguchi and Ramik, 2000; Inuiguchi et al., 2003). In FPP, fuzzy parameters are introduced into the modeling framework, and these parameters are presented as fuzzy sets with possibility distributions. The limitation of FPP is that when many uncertain parameters are expressed as fuzzy sets in a model, interactions among these uncertainties may lead to serious complexities, particularly for large-scale practical problems (Huang et al., 1993a). Also, a key factor of FMP, which is also the central concept of fuzzy set theory, is the membership function of fuzzy sets, numerically representing the degree to which an element belongs to a set. But the membership functions of parameters are not so easy to define. An inaccurate membership function may lead to undesirable results.

2.2.2. Stochastic Mathematical Programming

Stochastic mathematical programming (SMP) is derived from probability theory. In SMP, random elements are introduced to account for probabilistic uncertainty in the coefficients. Wilson and Baetz (2001) developed a derived probability model for curbside

waste collection activities that allowed for analyzing stochastic information in the MSW management. The inherent uncertainties in a model can be expressed as stochastic elements in the constraint matrix, the right-hand side stipulations, or the objective functions (Sengupta, 1972). However, if all parameters are expressed as random variables, the model would be extremely hard to solve and often leads to infeasibility problems.

Chance-constrained programming (CCP) is one of the SMP methods that contain only the right-hand side parameters (b_i) as random distributions. The CCP approaches do not require all the constraints to be satisfied. Instead, it allows a certain level of violation of constraints with random b_i under some circumstance (Loucks et al., 1981). As described in the following equation $P[g_i(\bar{x}) \geq b_i] \geq p$, the probability that the i^{th} constraint is satisfied is p , where p is greater than 0 and less than 1. The random b_i can be determined by its distribution and the possibility p .

As many factors other than the right-hand side stipulations exist as uncertainties in the system, CCP is often combined with other uncertainty reflecting methods like FMP and IPMP. For example, Maqsood et al. (2004) developed an inexact two-stage stochastic mixed-integer linear programming model for waste management. Li and Huang (2006) applied the inexact two-stage stochastic mixed-integer linear programming to solid waste management in the City of Regina. Guo and Huang developed an inexact fuzzy-stochastic mixed-integer programming approach and applied the approach to the long term planning of MSW management system in the city of Regina, Canada (Guo and Huang, 2009a& 2009b). Li et al. (2009a) developed an inexact fuzzy-stochastic constraint-softened programming method for waste-management systems planning by introducing FFP into an inexact multistage stochastic programming framework, where a number of violation variables for the constraints are introduced, allowing in-depth analyses of tradeoffs among economic objective, satisfaction degree, and constraint-violation risk.

The major strength of SMP method is that it does not simply reduce the complexity of the problems; it allows decision makers to have a complete view of the effects of

uncertainties and the relationships between uncertain inputs and resulting solutions (Huang, 1994). The major problem of SMP is that no sufficient data are available to obtain the probability distribution functions (PDF) for random parameters. In addition, even if these functions are available, it is extremely hard to solve a large scale stochastic management system planning problem with all uncertain data being expressed as PDFs (Birge and Louveaux, 1997; Luo et al., 2007).

2.2.3. Interval-Parameter Mathematical Programming

The FMP and SMP can effectively reflect uncertainties in the model system. However, in many practical problems, the available data is often not enough to be presented as distribution functions or membership functions. Instead, the obtained data may often show a robust distribution or no distribution information at all. It is more difficult for planners or decision makers to specify distributions than to define a fluctuation interval for uncertainties (Huang, et al., 1992). For example, the daily waste generation rate often fluctuates within a certain interval, but it may be difficult to state a reliable probability distribution for this variation (Li and Huang, 2010b). Interval-parameter mathematical programming (IPMP) is an alternative for handling uncertainties in the model's constraints and objectives (Huang et al., 1992). Unlike FMP and SMP, IPMP does not need the membership functions or distributions of inexact parameters. It is based on the interval analysis, and needs only the lower and upper bounds of uncertain parameters. The interval analysis was proposed by Moore (1979) and then extended into IPMP. The IPMP method emphasizes the intrinsic vagueness of its informational characteristics during parameter estimation (Change et al., 1997).

Due to the simplicity of IPMP, the last three decades have seen a wide application of it to deal with uncertainties in many fields (Ben-Israel and Robers, 1970; Rommelfanger et al., 1989; Huang and Moore, 1993; Tong, 1994; Hansen and Walster, 2004; Maqsood et al., 2004; Li et al, 2008&2009b; Guo et al, 2010; Wu et al., 2010; Dong et al., 2011). However, similar to FPP, when many uncertain parameters are expressed as intervals, interactions among these uncertainties may lead to serious complicated problems that are

hard to solve, particularly for large-scale practical problems (Huang et al., 1993). Interval-parameter linear programming (ILP) is a key member of the IPMP family and is a method that can deal with uncertainties expressed as intervals without any distributional information and without leading to more complicated problems. The ILP allows the interval information to be directly communicated into the optimization process and resulting solution. Early applications of ILP incorporated interval numbers into the objective function (Ishibuchi and Tanaka, 1990), constraint matrix (Huang and Moore, 1993; Tong, 1994), right-hand sides of constraints, and all of the above (Huang, 1996; Huang et al., 1992& 1995). Case studies of ILP include hypothetical examples for solid waste management (Huang et al., 1992&1995; Maqsood and Huang, 2003; Maqsood et al., 2004; Li and Huang, 2006; Li et al., 2006a), water resources allocation (Huang and Loucks, 2000; Li et al., 2006b; Maqsood et al., 2005), and flood diversion planning (Li et al., 2007). Practical examples include water quality management in China (Huang, 1998), solid waste management for the city of Regina (Li and Huang, 2006), and water system planning in Amman, Jordan (Rosenberg and Lund, 2009).

The reason that ILP has been widely used in the past decades is that it can effectively reflect the uncertainties in the modeling system, the information requirement is low, and the solution algorithms are easy to use. Three algorithms have been used to solve ILP models, including Monte Carlo simulation algorithm, two-step interactive algorithm, and best-worst case (BWC) algorithm. Monte Carlo simulation method relies on repeated random sampling to compute the results (Rubinstein, 1981). It has a very high computational requirement, so it is unrealistic to solve complicated models with a large number of uncertain parameters and variables. Two-step interactive algorithm and BWC algorithm were developed by Huang and Moor (1992) and Tong (1994) respectively. Both algorithms solve the model by generating two deterministic sub-models that correspond to the lower bound and the upper bound of the objective functions. With regards to the two algorithms, Rosenberg (2009) reviewed several ILP models and summarized that the two-step algorithm sometimes cannot provide a good performance. No validity checking for the two algorithms has been conducted previously in term of the feasibility and optimality of the obtained solutions.

2.2.4. Hybrid and Mixed Mathematical Programming

Combining advantages of the above optimization methods, many hybrid and mixed programming approaches have been developed. Due to the flexibility of interval numbers, most approaches combine ILP with other optimization methods. For example, Huang et al. (1993b) proposed a grey fuzzy flexible programming method and applied it to MSW management systems to tackle uncertainties that presented as fuzzy sets and intervals. Chang et al. (1997) proposed a fuzzy interval multi-objective mixed integer programming model to evaluate solid waste management strategies. The study demonstrated how uncertain factors could be quantified by specific membership functions and interval numbers in a multi-objective model. Maqsood I. et al. (2004) developed a hybrid optimization approach, inexact two-stage mixed integer linear programming model for MSW management. The model improved upon the mixed integer, two-stage stochastic and ILP approaches by allowing uncertain parameters presented as random distributions and discrete intervals. Li and Huang (2010b) developed an interval-based possibilistic programming method for the planning of waste management, with minimized system cost and environmental impact. This model was a combination of FMP and SMP

These hybrid and mixed approaches can combine the advantages and avoid the limitation of single methods, and flexibly use the availability of input data. However, all these interval-based methods need to be solved by ILP algorithms eventually. As a result, if the existing ILP algorithms prove to have fundamental flaws in its solution process, the validity of these previous ILP-based hybrid studies will be problematic. It is then in a pressing need of re-examining the existing ILP algorithms to overcome their limitations. This has been proven to be very difficult (Zhou et al., 2009). Otherwise, risks of the decisions generated by these flawed optimization processes need to be evaluated if they have to be implemented.

2.3. Previous Studies on Solid Waste Management in HRM

The Halifax Regional Municipality (HRM) was founded in 1996 with the combination of four communities: Halifax, Dartmouth, Bedford, and Halifax County (HRM, 2010a). It is the largest population center of the Canadian east coast and the capital of the province of Nova Scotia. HRM is also a municipality that committed to environmental sustainability. It was the first winner of FCM-CH2M HILL Sustainable Community Awards for the community-based waste resource management strategy (FCM, 2000). By implementing many environmental programs (Walker et al., 2004), HRM now has a very high waste diversion rate, which is 60% (HRM, 2010c).

Ghadiri (2004) developed an interval-parameter fuzzy-stochastic mixed integer linear programming model for the long term planning of MSW management system in HRM. The modeling results can help to answer questions related to types, timing and sites of the MSW management activities, as well as reflect dynamic, interactive, and uncertain characteristics of the MSW system in HRM. However, the diversion rate target was not considered in the developed model and also the model was solved by the 2-step algorithm. Both facts lead to obvious limitations in Ghadiri's study. The method proposed in this thesis attempts to address these issues in a desirable fashion.

CHAPTER 3

VALIDITY CHECKING FOR ILP ALGORITHMS

3.1. Existing ILP Solution Algorithms

Definition 3.1.1: An *interval-parameter linear programming (ILP)* model (for maximized problems) is defined as (Huang et al. 1992):

$$\text{Max } f^\pm = C^\pm X^\pm \quad (3.1.1)$$

$$\text{s.t. } A^\pm X^\pm \leq B^\pm \quad (3.1.2)$$

$$X^\pm \geq 0 \quad (3.1.3)$$

where,

$$C^\pm = [c_1^\pm, c_2^\pm, \dots, c_n^\pm]$$

$$X^\pm = [x_1^\pm, x_2^\pm, \dots, x_n^\pm]^T$$

$$B^\pm = [b_1^\pm, b_2^\pm, \dots, b_m^\pm]^T$$

$$A^\pm = \{a_{ij}^\pm\}, \quad i=1, \dots, m; \quad j=1, 2, \dots, n.$$

For minimize problems, the ILP model is as the following:

$$\text{Min } f^\pm = C^\pm X^\pm \quad (3.1.4)$$

$$\text{s.t. } A^\pm X^\pm \geq B^\pm \quad (3.1.5)$$

$$X^\pm \geq 0 \quad (3.1.6)$$

Where the “-” and “+” superscripts represent lower and upper bounds of an interval-parameter or variable respectively.

Since interval parameters exist in the objective function and constraints to reflect uncertainties in the model, the optimal solutions of the ILP model are also intervals:

$$f_{opt}^{\pm} = [f_{opt}^{-}, f_{opt}^{+}] \quad (3.1.7)$$

$$X_{opt}^{\pm} = [x_{1opt}^{\pm}, x_{2opt}^{\pm}, \dots, x_{nopt}^{\pm}] \quad (3.1.8)$$

$$x_{jopt}^{\pm} = [x_{jopt}^{-}, x_{jopt}^{+}], \quad j = 1, 2, \dots, n \quad (3.1.9)$$

As intervals exist in ILP models, and no software can directly solve the model with intervals in it, the ILP models need to be converted into some formats that software can recognize and solve. Presently, three algorithms have been developed and used for solving ILP problems, including Monte Carlo simulation algorithm, 2-step algorithm, and best-worst case (BWC) algorithm.

3.1.1. Monte Carlo Simulation Algorithm

Monte Carlo simulation method is a computational algorithm that relies on repeated random sampling to compute the results (Rubinstein, 1981). It is useful for modeling phenomena with significant uncertainty in inputs. To solve an ILP problem, it randomly sets values for parameters within their ranges to form a classic Linear Programming model (Sabelfeld, 1991). The process of Monte Carlo simulation algorithm (Robert and Casella, 2004.) for ILP models is described in the following steps:

[Step 1] Assign random feature to each interval parameter (i.e., a_{ij} , b_i , and c_j) in the ILP model with probability distribution functions (PDFs), and then convert each PDF into its cumulative distribution function (CDF). Figure 3.1 gives an example of the PDF of a normally distributed parameter and it could be converted into its CDF as shown in Figure 3.2. Normal distribution is the most commonly used distribution of interval parameters, although other distributions like uniform distribution and exponential distribution could also describe the distributing characteristic of parameters.

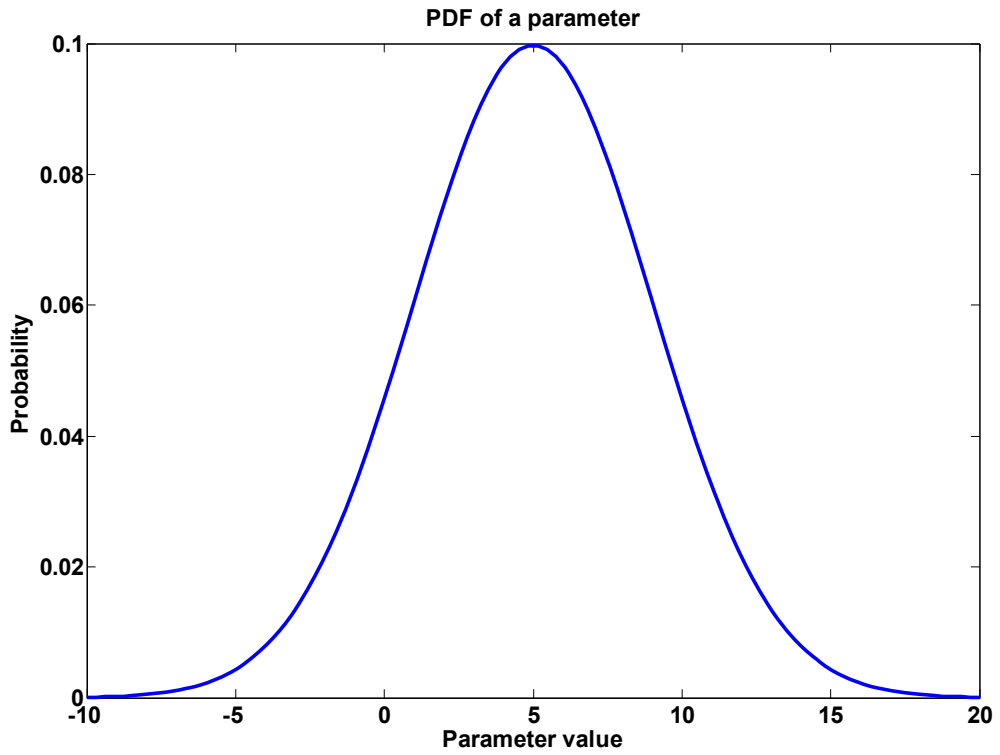


Figure 3.1 The PDF curve of a parameter

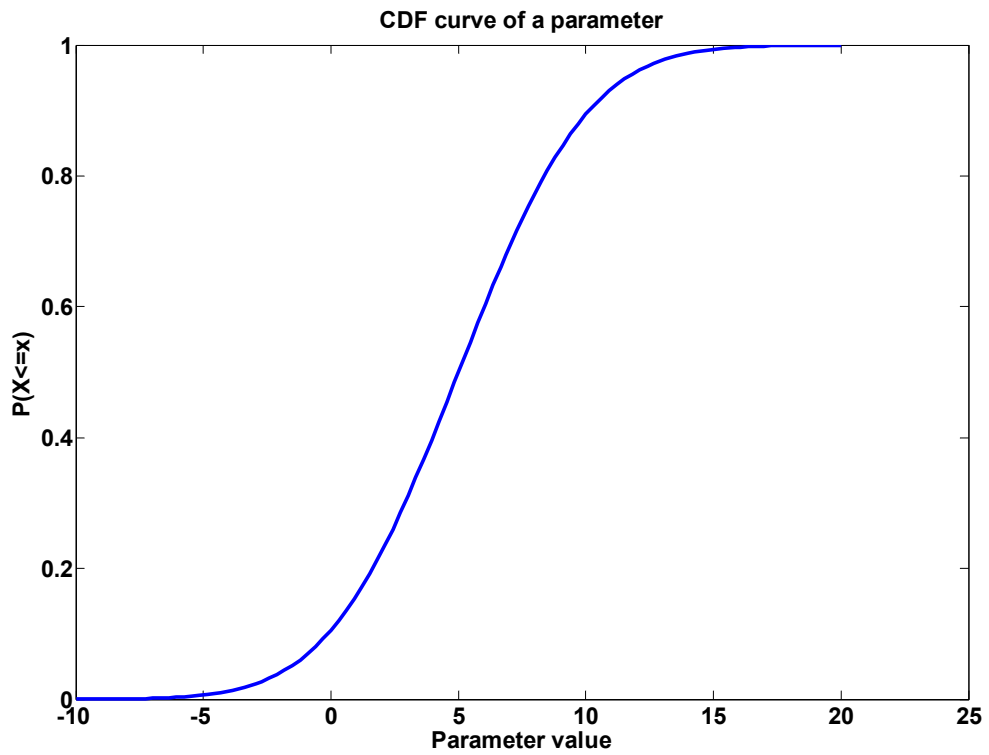


Figure 3.2 The CDF curve of a parameter converted from PDF

[Step 2] Use random number generator (computed code available in most programming software) to generate a random number between 0 and 1, denoted as r .

[Step 3] Relate the generated random number r to the CDF curve for each parameter, and then get a set of deterministic values for parameters in a_{ij} , b_i , c_j . An example of how to randomly set a value for a parameter is shown in Figure 3.3, in which the parameter is found to be 5.5.

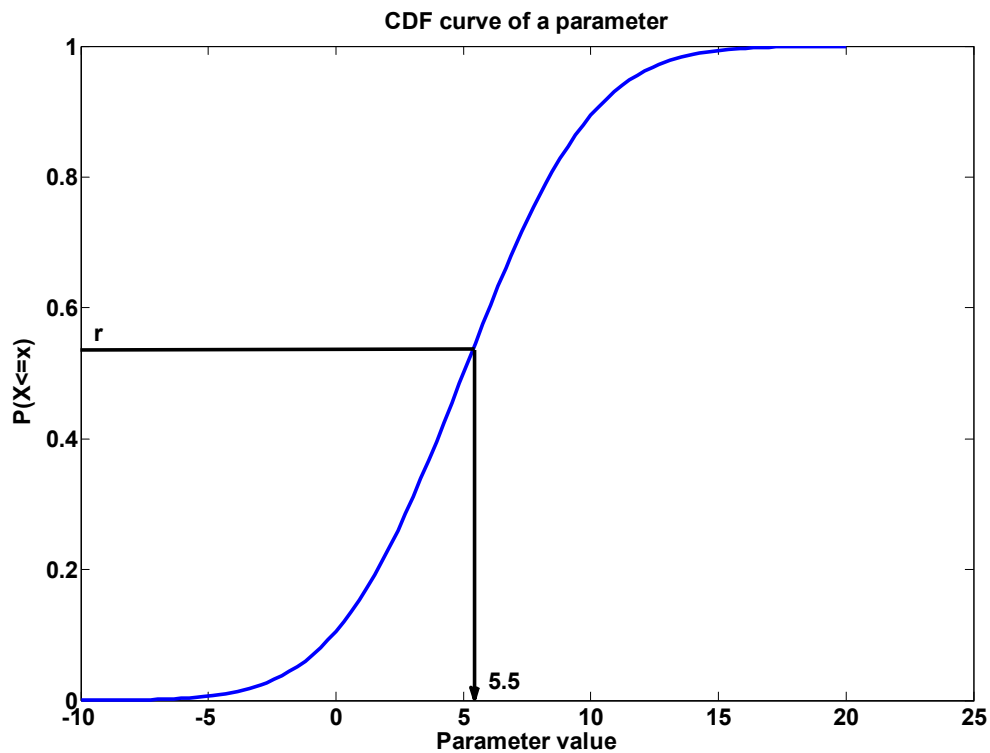


Figure 3.3 An example of randomly setting a value for a parameter

[Step 4] Use the set of deterministic parameter values to replace the interval a_{ij} , b_i , c_j and thus form a classic deterministic linear programming model. Then solve the classic deterministic LP model and produce a set of deterministic solution.

[Step 5] Repeat Steps 2, 3 and 4 for a sufficient number of runs until a distribution of the solution for each specific decision variable is obtained.

Monte Carlo simulation method could be a very successful algorithm for ILP problems (Jahanshahloo et al, 2008) because it can generate the sound results through simulating the real-world situation. However, because of its reliance on large numbers of repeated computation of random or pseudo-random numbers, Monte Carlo simulation method has a very high computational requirement. Millions of times, or even billions of simulation need to be conducted in order to obtain a distribution for a specific solution. This makes the method unrealistic to solve complicated practical models with a large number of uncertain parameters and variables. In addition, in real-world problems, it is almost impossible to obtain sufficient data to formulate distributions for interval parameters (Nawrocki, 2001). Therefore, the best-worst case algorithm and the 2-step algorithm were developed as alternatives to solve the ILP models.

3.1.2. Two-Step Algorithm

Two-step algorithm was introduced by Huang and Moore in 1993. It is a 2-step interactive method, which at first formulates a sub-model to solve for the upper bound of objective function when it is a maximization problem, and then solve the sub-model corresponding to the lower bound of objective function (Huang and Moore, 1993).

For the n interval coefficients c_j^\pm ($j = 1, 2, \dots, n$) in the original objective function

$f^\pm = C^\pm X^\pm$ in the ILP model, if k_1 of these coefficients are non-negative, and the rest are negative, let the n coefficients be rearranged such that $c_j^\pm \geq 0$ ($j = 1, 2, \dots, k_1$) and $c_j^\pm < 0$ ($j = k_1 + 1, k_1 + 2, \dots, n$). The sub-model corresponding to the upper bound of the objective function (when the objective is to be maximized) is formulated as:

$$\text{Max} \quad f^+ = \sum_{j=1}^{k_1} c_j^+ x_j^+ + \sum_{j=k_1+1}^n c_j^+ x_j^- \quad (3.1.10)$$

$$\text{s.t.} \quad \sum_{j=1}^{k_1} |a_{ij}^-| \text{Sign}(a_{ij}^-) x_j^+ + \sum_{j=k_1+1}^n |a_{ij}^+| \text{Sign}(a_{ij}^+) x_j^- \leq b_i^+, \quad \forall i \quad (3.1.11)$$

$$x_j^\pm \geq 0, \quad \forall j \quad (3.1.12)$$

The sub-model (3.1.10)-(3.1.12) is a classic linear programming model, which can be solved with any existing algorithm such as the Simplex method. After solving the above sub-model corresponding to the upper bound of the objective function, we can then formulate the sub-model corresponding to the lower bound:

$$\text{Max} \quad f^- = \sum_{j=1}^{k_1} c_j^- x_j^- + \sum_{j=k_1+1}^n c_j^- x_j^+ \quad (3.1.13)$$

$$\text{s.t.} \quad \sum_{j=1}^{k_1} |a_{ij}|^+ \text{Sign}(a_{ij}^+) x_j^- + \sum_{j=k_1+1}^n |a_{ij}|^- \text{Sign}(a_{ij}^-) x_j^+ \leq b_i^-, \quad \forall i \quad (3.1.14)$$

$$x_j^\pm \geq 0, \quad \forall j \quad (3.1.15)$$

$$x_j^- \leq x_{jopt}^+, \quad j = 1, 2, \dots, k_1 \quad (3.1.16)$$

$$x_j^+ \geq x_{jopt}^-, \quad j = k_1 + 1, k_1 + 2, \dots, n \quad (3.1.17)$$

where x_{jopt}^+ ($j = 1, 2, \dots, k_1$) and x_{jopt}^- ($j = k_1 + 1, k_1 + 2, \dots, n$) are the optimal solutions generated from sub-model (3.1.10)-(3.1.12).

The sub-model (3.1.13)-(3.1.17) is also a classic linear programming model that can be easily solved to obtain the optimal solutions. Thus, after implementing the above two-step method, we can obtain optimal interval solutions of the ILP model as $x_{jopt}^\pm = [x_{jopt}^-, x_{jopt}^+]$ and $f_{opt}^\pm = [f_{opt}^-, f_{opt}^+]$.

For minimization problems, the sequence of solving the sub-models is opposite. The first sub-model could be formulated as (3.1.18) to (3.1.20):

$$\text{Min} \quad f^- = \sum_{j=1}^{k_1} c_j^- x_j^- + \sum_{j=k_1+1}^n c_j^- x_j^+ \quad (3.1.18)$$

$$\text{s.t.} \quad \sum_{j=1}^{k_1} |a_{ij}|^+ \text{Sign}(a_{ij}^+) x_j^- + \sum_{j=k_1+1}^n |a_{ij}|^- \text{Sign}(a_{ij}^-) x_j^+ \leq b_i^+, \quad \forall i \quad (3.1.19)$$

$$x_j^\pm \geq 0, \quad \forall j \quad (3.1.20)$$

The second sub-model could then be formulated as equations (3.1.21) to (3.1.25).

$$\text{Min} \quad f^+ = \sum_{j=1}^{k_1} c_j^+ x_j^+ + \sum_{j=k_1+1}^n c_j^+ x_j^- \quad (3.1.21)$$

$$\text{s.t.} \quad \sum_{j=1}^{k_1} |a_{ij}^-| \text{Sign}(a_{ij}^-) x_j^+ + \sum_{j=k_1+1}^n |a_{ij}^+| \text{Sign}(a_{ij}^+) x_j^- \leq b_i^-, \quad \forall i \quad (3.1.22)$$

$$x_j^+ \geq x_{jopt}^-, \quad j = 1, 2, \dots, k_1 \quad (3.1.23)$$

$$x_j^- \leq x_{jopt}^+, \quad j = k_1 + 1, k_1 + 2, \dots, n \quad (3.1.24)$$

$$x_j^\pm \geq 0, \quad \forall j \quad (3.1.25)$$

Similar as maximization models, after solving the above two sub-models, we can obtain optimal interval solutions of the original ILP model.

3.1.3. Best-Worst Case Algorithm

The best-worst case algorithm (BWC) (Tong, 1994; Chinneck and Ramadan, 2000) is similar as the 2-step algorithm in solving two sub-models. The major difference between them lies in that the 2-step algorithm differentiates the selection of extreme parameter values (i.e., lower or upper bounds of coefficients) for decision variables in the objective function based on their different signs (i.e., negative or positive coefficients in c_j), while the BWC treats all the parameters without discrimination. The solution process of the BWC algorithm is provided below for a maximization problem:

The first step of the BWC algorithm is to formulate the Best-Case sub-model corresponding to the upper bound of the objective function as follows:

$$\text{Max} \quad f^+ = c_j^+ x_j \quad (3.1.26)$$

$$\text{s.t.} \quad a_{ij}^- x_j \leq b_i^+, \quad \forall i \quad (3.1.27)$$

$$x_j \geq 0, \quad \forall j \quad (3.1.28)$$

The second step is to formulate and solve the sub-model corresponding to the worst-case situation:

$$\text{Max} \quad f^- = c_j^- x_j \quad (3.1.29)$$

$$\text{s.t.} \quad a_{ij}^+ x_j \leq b_i^-, \quad \forall i \quad (3.1.30)$$

$$x_j \geq 0, \quad \forall j \quad (3.1.31)$$

For minimization problems, the BWC algorithm reformulates the original ILP model into following two sub-models corresponding to best- and worst-case, respectively.

Best-Case Sub-model:

$$\text{Min} \quad f^- = c_j^- x_j \quad (3.1.32)$$

$$\text{s.t.} \quad a_{ij}^+ x_j \geq b_i^-, \quad \forall i \quad (3.1.33)$$

$$x_j \geq 0, \quad \forall j \quad (3.1.34)$$

Worst case Sub-model:

$$\text{Min} \quad f^+ = c_j^+ x_j \quad (3.1.35)$$

$$\text{s.t.} \quad a_{ij}^- x_j \geq b_i^+, \quad \forall i \quad (3.1.36)$$

$$x_j \geq 0, \quad \forall j \quad (3.1.37)$$

In the BWC algorithm, “best case” or “worst case” refers to a situation that both objective function and constraints of the best case or worst case sub-model represent the “best” or “worst” extremities, respectively, of the original ILP model. Specifically, for a maximization problem as indicated in the model (3.1.26-3.1.31), the objective function of the best-case sub-model represents the upper bound of the original ILP model (model 3.1.26), and its constraints delimit a largest decision space for the optimal solution to be searched from it (model 3.1.27); while the objective function of the worst-case sub-model

gives the lower bound of the original ILP model (model 3.1.29) and its constraints bound a smallest decision space. However, for a minimization problem as indicated in the model (3.1.32-3.1.37), the objective function of the best case sub-model represents the lower bound of the original ILP model (model 3.1.32), and its constraints delimit a smallest decision space for the optimal solution to be searched from it (model 3.1.33); while the objective function of the worst-case sub-model gives the upper bound of the original ILP model (model 3.1.35) and its constraints bound a largest decision space.

3.1.4. Comparison of Two-Step and BWC Algorithms

(1) Model Equivalence

Theorem 3.1.1: The 2-step algorithm is equivalent to the BWC algorithm for an ILP problem if and only if $Sign(a_{ij}^{\pm}) = Sign(c_j^{\pm})$, $\forall i, j$:

$$Max \quad f^{\pm} = \sum_{j=1}^{k_1} c_j^{\pm} x_j^{\pm} + \sum_{j=k_1+1}^n c_j^{\pm} x_j^{\pm} \quad (3.1.38)$$

$$s.t. \quad \sum_{j=1}^{k_1} a_{ij}^{\pm} x_j^{\pm} + \sum_{j=k_1+1}^n a_{ij}^{\pm} x_j^{\pm} \leq b_i^{\pm}, \quad \forall i \quad (3.1.39)$$

$$x_j^{\pm} \geq 0, \quad \forall j \quad (3.1.40)$$

Proof: Model (3.1.38-3.1.40) gives a general ILP model which is used as an illustrative example in the proof. Assume $c_j^{\pm} > 0$ for $j=1, 2, \dots, k_1$ and $c_j^{\pm} < 0$ for $j=k_1+1, k_1+2, \dots, n$. Also, let $Sign(c_{ij}^{\pm}) = 1$ for $j=1, 2, \dots, k_1$ and $Sign(c_{ij}^{\pm}) = -1$ for $j=k_1+1, k_1+2, \dots, n$. Thus, according to the condition of Theorem 3.1.1, we have $Sign(a_{ij}^{\pm}) = Sign(c_{ij}^{\pm}) = 1$ for $j=1, 2, \dots, k_1$ and $Sign(a_{ij}^{\pm}) = Sign(c_{ij}^{\pm}) = -1$ for $j=k_1+1, k_1+2, \dots, n$. The proof of Theorem 3.1.1 is provided as follows:

The first step we need to prove is that the objectives functions of two sub-models reformulated by the two algorithms are equivalent to each other. The 2-step algorithm reformulates the objective function of the original ILP model in a way that the decision variables are regrouped according to the signs of the corresponding coefficients (c_j^\pm), as indicated in model (3.1.41 and 3.1.42). It is quite obvious that the upper-bound objective function (i.e., f^+) reformulated by the 2-step algorithm is equivalent to the objective function of the best-case sub-model from the BWC algorithm, as indicated in model (3.1.41), and the lower-bound objective function (i.e., f^-) from the 2-step algorithm is equivalent to that of the worst-case sub-model as seen in model (3.1.42). The only treatment is to ignore the upper- or lower-bound sign associated with the decision variables and the sign could be ignored mathematically without changing the model formulation.

Sub-model #1(best-case sub-model objective function):

$$\begin{aligned}
 \text{Max} \quad f^+ &= \sum_{j=1}^{k_1} c_j^+ x_j^+ + \sum_{j=k_1+1}^n c_j^+ x_j^- & (3.1.41) \\
 &= \sum_{j=1}^{k_1} c_j^+ x_j + \sum_{j=k_1+1}^n c_j^+ x_j \\
 &= \sum_{j=1}^n c_j^+ x_j
 \end{aligned}$$

Sub-model #2(worst-case sub-model objective function):

$$\begin{aligned}
 \text{Max} \quad f^- &= \sum_{j=1}^{k_1} c_j^- x_j^- + \sum_{j=k_1+1}^n c_j^- x_j^+ & (3.1.42) \\
 &= \sum_{j=1}^{k_1} c_j^- x_j + \sum_{j=k_1+1}^n c_j^- x_j \\
 &= \sum_{j=1}^n c_j^- x_j
 \end{aligned}$$

With equivalent objective functions being proved, we also need to prove that the corresponding decision spaces delimited by the their own constraints are also identical to each other under the condition of $Sign(a_{ij}^\pm) = Sign(c_j^\pm)$.

For 2-step algorithm, the feasible decision space for the sub-model #1 (f^+) is defined as:

$$P^u = \left\{ X^\pm \left| \sum_{j=1}^{k_1} |a_{ij}^-| \text{Sign}(a_{ij}^\pm) x_j^+ + \sum_{j=k_1+1}^n |a_{ij}^+| \text{Sign}(a_{ij}^\pm) x_j^- \leq b_i^+, x_j^-, x_j^+ \in X^\pm, X^\pm \geq 0, \forall i \right. \right\} \quad (3.1.43)$$

For BWC algorithm, the feasible decision space for the best-case sub-model is defined as:

$$Q^u = \left\{ X^\pm \left| \sum_{j=1}^{k_1} a_{ij}^- x_j + \sum_{j=k_1+1}^n a_{ij}^- x_j \leq b_i^+, x_j^-, x_j^+ \in X^\pm, X^\pm \geq 0, \forall i \right. \right\} \quad (3.1.44)$$

Since $\text{Sign}(a_{ij}^\pm) = \text{Sign}(c_j^\pm)$, we have $\text{Sign}(a_{ij}^\pm) = 1$ for $j=1, 2, \dots, k_1$ and $\text{Sign}(a_{ij}^\pm) = -1$

for $j=k_1+1, k_1+2, \dots, n$. In addition, $|a_{ij}^-| = a_{ij}^-$ for $j=1, 2, \dots, k_1$ because they are all

positive coefficients, and $|a_{ij}^+| = -a_{ij}^-$ for $j=k_1+1, k_1+2, \dots, n$ because they are all negative

coefficients. Therefore, we have:

$$\sum_{j=1}^{k_1} |a_{ij}^-| \text{Sign}(a_{ij}^\pm) x_j^+ = \sum_{j=1}^{k_1} (a_{ij}^-)(1) x_j^+ = \sum_{j=1}^{k_1} a_{ij}^- x_j^+, \quad \forall i \quad (3.1.45)$$

$$\sum_{j=k_1+1}^n |a_{ij}^+| \text{Sign}(a_{ij}^\pm) x_j^- = \sum_{j=k_1+1}^n (-a_{ij}^-)(-1) x_j^- = \sum_{j=k_1+1}^n a_{ij}^- x_j^-, \quad \forall i \quad (3.1.46)$$

Combine (3.1.45) and (3.1.46), we have:

$$P^u = \left\{ X^\pm \left| \sum_{j=1}^{k_1} a_{ij}^- x_j^+ + \sum_{j=k_1+1}^n a_{ij}^- x_j^- \leq b_i^+, x_j^-, x_j^+ \in X^\pm, X^\pm \geq 0, \forall i \right. \right\} \quad (3.1.47)$$

If we ignore the upper- or lower-bound sign associated with the decision variables, it is apparent that $P^u = Q^u$. It indicates that the feasible decision spaces delimited respectively by two algorithms for sub-model #1 and best-case sub-model are identical to each other. It can then be concluded that the 2-step algorithm is equivalent to the BWC algorithm if the condition $\text{Sign}(a_{ij}^\pm) = \text{Sign}(c_j^\pm)$ exists.

Now we need to prove that, only if $Sign(a_{ij}^{\pm}) = Sign(c_j^{\pm})$, this theorem still holds. Let's assume that there exists an a_{ij}^{\pm} (where $i=p, j = q$), $Sign(a_{ij}^{\pm}) \neq Sign(c_j^{\pm})$.

If $p \leq k_1$, it is obvious that $Sign(a_{pq}^{\pm}) = -1$ and $|a_{pq}|^- = -a_{pq}^+$. Following the same procedure as shown in equations (3.1.45) to (3.1.47), it is easy to see that $P^u \neq Q^u$, indicating that the 2-step algorithm is not equivalent to the BWC algorithm. Similarly, it is straightforward to see that this condition also holds for $p > k_1$. Therefore, it has been proved that only when $Sign(a_{ij}^{\pm}) = Sign(c_j^{\pm})$, $\forall i, j$, the 2-step algorithm is equivalent to the BWC algorithm.

Theorem 3.1.1 gives the conditions for 2-step algorithm to be equivalent to the BWC algorithm. If an ILP problem satisfies the condition of $Sign(a_{ij}^{\pm}) = Sign(c_j^{\pm})$, $\forall i, j$, the solutions obtained by these two algorithms will be same; otherwise the solutions will differ from each other.

(2) Feasible Decision Space

Theorem 3.1.2: Suppose an ILP problem has interval inequalities as

$$Q = \left\{ X^{\pm} \left| \sum_{j=1}^n a_{ij}^{\pm} x_j \leq b_i^{\pm}, x_j \in X^{\pm}, X^{\pm} \geq 0, \forall i \right. \right\} \quad (3.1.48)$$

the largest and smallest feasible decision space corresponding to the upper bound and lower bound of the objective function solution can be presented as:

$$Q^u = \left\{ X^{\pm} \left| \sum_{j=1}^n a_{ij}^- x_j \leq b_i^+, x_j \in X^{\pm}, X^{\pm} \geq 0, \forall i \right. \right\} \quad (3.1.49)$$

$$Q^l = \left\{ X^\pm \left| \sum_{j=1}^n a_{ij}^+ x_j \leq b_i^-, x_j \in X^\pm, X^\pm \geq 0, \forall i \right. \right\} \quad (3.1.50)$$

This theorem was firstly stated by Tong (1994) without providing a proof, and was then proved by Chinneck and Ramadan (2000) for the form of minimization problems.

Theorem 3.1.2 gives the largest and smallest feasible decision space of an ILP model. A practical interpretation for an ILP model is that the interval parameters can potentially take any value between its prescribed lower and upper bound. When each parameter takes a value within its range, the ILP becomes a classic LP problem, and the feasible decision space of this classic LP is found to be located between the smallest and largest feasible decision space of the original ILP problem, and we define this classic LP as an event model of the ILP problem.

Definition 3.1.2: An *event model* of an ILP is defined as a classic LP model where the interval parameters in A^\pm, B^\pm and C^\pm take a specific set of crisp values within their respective lower and upper bounds.

Based on this definition, apparently, two sub-models reformulated by the 2-step algorithm and BWC algorithm are two specific event models representing two opposite extreme conditions of the original model, respectively. As the matter of fact, Q^u in model 3.1.40 represents the constraints of the best-case sub-model by BWC and Q^l in model 3.1.41 represents the constraints of the worst-case sub-model by BWC. In other words, the feasible decision spaces bounded by the BWC algorithm represent the largest and smallest feasible spaces of the original ILP model. From this definition, it becomes understandable that the feasible decision spaces provided by the 2-step algorithm only represent two general event model situations, and are enclosed by the BWC feasible spaces. It can then be concluded that the feasible decision space of an ILP delimited by BWC is larger than or equal to (when $Sign(a_{ij}^\pm) = Sign(c_j^\pm), \forall i, j$ according to Theorem 3.1.1) the feasible space bounded by 2-step algorithm.

(3) Optimal Solution

Theorem 3.1.3: The optimal solutions obtained by the 2-step algorithm can be different from that obtained by the BWC algorithm in that:

$$f_{2\text{-stepopt}}^+ \leq f_{BWCopt}^+ \quad (3.1.51)$$

$$f_{2\text{-stepopt}}^- \geq f_{BWCopt}^- \quad (3.1.52)$$

Proof: In Theorem 3.1.1, it has been proved that when $Sign(a_{ij}^\pm) = Sign(c_j^\pm), \forall i, j$, two sub-models reformulated by the 2-step algorithm and BWC algorithm are equivalent, and thus the solutions obtained from two algorithms will be same. Therefore, we have $f_{2\text{-stepopt}}^+ = f_{BWCopt}^+$ and $f_{2\text{-stepopt}}^- = f_{BWCopt}^-$.

For a general situation when $\exists i, j, Sign(a_{ij}^\pm) \neq Sign(c_j^\pm)$, this theorem also holds. Theorem 3.1.2 tells that the feasible decision space for solving the upper bound objective function of BWC algorithm (i.e. best-case sub-model) is given as Q^u in model 3.1.49. According to Definition 3.1.2, the feasible decision space for solving the sub-model #1 from the 2-step algorithm is smaller than Q^u and is enclosed by Q^u when $\exists i, j, Sign(a_{ij}^\pm) \neq Sign(c_j^\pm)$. Based on the fundamental linear programming theory, the maximum objective function value obtained by the BWC algorithm (i.e., f_{BWCopt}^+) is equal to or greater than that obtained by the 2-step algorithm. Thus, we have $f_{2\text{-stepopt}}^+ \leq f_{BWCopt}^+$. Similarly, the minimum objective function value obtained by the BWC algorithm is equal to or less than that from the 2-step algorithm, i.e., $f_{2\text{-stepopt}}^- \geq f_{BWCopt}^-$.

Remark 3.1.1: Both 2-step and BWC algorithms were proposed to account for system uncertainties in an ILP problem, according to Theorems 3.1.2 and 3.1.3, the 2-step algorithm searches for the optimal solutions in a smaller feasible decision space, and moreover, the decrease of feasible decision space is caused simply by the way the left-hand

sides of the sub-model constraints are formulated. Due to this space decrease, the 2-step algorithm arbitrarily ignores some system uncertainties to a certain degree and this ignorance was not theoretically or mathematically justified. In this sense, the BWC algorithm seems to be a better method in handling the uncertainties presented as intervals.

Remark 3.1.2: In terms of the optimal solutions, both algorithms use the optimal solutions generated from each sub-model to form an interval optimal solution for each decision variable as well as an interval objective function value. For example, when the 2-step algorithm is used for solving a maximization ILP problem, its upper bound sub-model is solved first to obtain the upper bound solutions for decision variables x_{jopt}^+ , where $j = 1, 2, \dots, k_1$, and the lower bound solution for decision variables x_{jopt}^- , where $j = k_1 + 1, k_1 + 2, \dots, n$. Then, its lower bound sub-model is solved to obtain the lower bound solutions for decision variables x_{jopt}^- , where $j = 1, 2, \dots, k_1$, and the upper bound solution for decision variables x_{jopt}^+ , where $j = k_1 + 1, k_1 + 2, \dots, n$. The obtained upper bound and lower bound solutions will then be combined to form the final interval optimal solution for the original ILP problem, i.e., $x_{jopt}^\pm = [x_{jopt}^-, x_{jopt}^+]$, $\forall j, j = 1, 2, \dots, n$. The optimal value of the objective function of the original ILP model is also an interval: $f_{opt}^\pm = [f_{opt}^-, f_{opt}^+]$, where f_{opt}^+ and f_{opt}^- represent the upper- and lower- bound values of the original objective function, respectively, as given in model (3.1.44) and (3.1.45).

$$f_{opt}^+ = f(x_{1opt}^+, x_{2opt}^+, \dots, x_{k_1opt}^+, x_{(k_1+1)opt}^-, x_{(k_1+2)opt}^-, \dots, x_{(n)opt}^-) \quad (3.1.53)$$

$$f_{opt}^- = f(x_{1opt}^-, x_{2opt}^-, \dots, x_{k_1opt}^-, x_{(k_1+1)opt}^+, x_{(k_1+2)opt}^+, \dots, x_{(n)opt}^+) \quad (3.1.54)$$

In the practical decision-making process, the decision makers can choose any values within the interval ranges for each decision variable to develop a practical implementation scheme, depending on their preferences to different policies and their interpretation of system risk and economic return (Huang et al, 1993; Zou et al, 2000; Yeh and Tung, 2003).

3.2. Validity Checking for Two-Step and BWC Algorithms

As ILP models are commonly formulated for decision problems in many fields (Ben-Israel and Robers, 1970; Rommelfanger et al. 1989; Huang and Moore 1993; Tong 1994; Hansen and Walster 2004), the 2-step and BWC algorithms are also widely employed to solve these models (Huang and Moore 1993; Oliveira and Antunes 2007; Qin et al. 2007). As explained in Remark 3.1.1, 2-step algorithm ignores some of the system uncertainties when reformulating the sub-model constraints and this treatment could be a potential flaw of this algorithm and could very possibly lead to feasibility and optimality concerns towards the generated interval optimal solutions. This concern has triggered off a desire to check the validity of both algorithms which has not been conducted previously. In this study, a numerical example is designed to illustrate the validity checking process for both algorithms, and the focus of the validity checking is on the investigation of any infeasible solutions existed in the generated interval optimal solution and any optimal solutions missing from it.

3.2.1. A Numerical Example for Validity Checking

In order to perform the validity checking for both algorithms, a minimized ILP model with two decision variables and two interval constraints was designed as follows and used as the numerical example:

$$\text{Min} \quad f = [2,3]x_1 + x_2 \quad (3.2.1)$$

$$\text{s.t.} \quad x_1 - [1.2,1.4]x_2 \geq [3,4] \quad (3.2.2)$$

$$x_1 + [1.5,2.0]x_2 \geq [5,6] \quad (3.2.3)$$

$$x_1, x_2 \geq 0 \quad (3.2.4)$$

Before solving this ILP model by 2-step and BWC algorithms, the first step is to generate a large number of event models using the Monte-Carlo Simulation method as described in Section 3.1.1. Each event model is a classic deterministic LP model which can be easily solved. By solving these event models, a large number of solution sets for decision

variables can be produced and the solution ranges of the objective function and decision variables can then be obtained. The larger the numbers of the event models are solved, the better the solution resolution and accuracy could be. In this study, 50 million event models were generated and solved by the Monte-Carlo simulation method, and the obtained interval solutions are: $f = [8.18, 15.50]$, $x_1 = [3.76, 4.95]$, $x_2 = [0.30, 1.11]$.

(1) 2-Step Algorithm Solution

Based on the 2-step interactive algorithm described in Section 3.1.2, two sub-models corresponding to f^- and f^+ could be formulated as follows:

Sub-model #1:

$$\text{Min} \quad f^- = 2x_1^- + x_2^- \quad (3.2.5)$$

$$\text{s.t.} \quad x_1^- - 1.4x_2^- \geq 3 \quad (3.2.6)$$

$$x_1^- + 2x_2^- \geq 5 \quad (3.2.7)$$

$$x_1^-, x_2^- \geq 0 \quad (3.2.8)$$

Sub-model #2:

$$\text{Min} \quad f = 3x_1^+ + x_2^+ \quad (3.2.9)$$

$$\text{s.t.} \quad x_1^+ - 1.2x_2^+ \geq 4 \quad (3.2.10)$$

$$x_1^+ + 1.5x_2^+ \geq 6 \quad (3.2.11)$$

$$x_1^+, x_2^+ \geq 0 \quad (3.2.12)$$

$$x_1^+ \geq x_{1opt}^- \quad (3.2.13)$$

$$x_2^+ \geq x_{2opt}^- \quad (3.2.14)$$

In Sub-model #2, x_{1opt}^- in constraint (3.2.13) and x_{2opt}^- in constraint (3.2.14) are the optimal solutions of decision variables from Sub-model #1. Both sub-models are classic

deterministic LP models and could be solved easily. The optimal interval solutions obtained by using 2-step algorithm are $f = [8.24, 15.41]$, $x_1 = [3.82, 4.89]$, $x_2 = [0.59, 0.74]$.

(2) BWC Algorithm Solution

According to the BWC algorithm explained in Section 3.1.3, two sub-models corresponding to the best-case and worst-case situations can be formulated as follows:

Best-Case Sub-model:

$$\text{Min} \quad f^- = 2x_1^- + x_2^- \quad (3.2.15)$$

$$\text{s.t.} \quad x_1^- - 1.2x_2^- \geq 3 \quad (3.2.16)$$

$$x_1^- + 2x_2^- \geq 5 \quad (3.2.17)$$

$$x_1^-, x_2^- \geq 0 \quad (3.2.18)$$

Worst-Case Sub-model:

$$\text{Min} \quad f^+ = 3x_1^+ + x_2^+ \quad (3.2.19)$$

$$\text{s.t.} \quad x_1^+ - 1.4x_2^+ \geq 4 \quad (3.2.20)$$

$$x_1^+ + 1.5x_2^+ \geq 6 \quad (3.2.21)$$

$$x_1^+, x_2^+ \geq 0 \quad (3.2.22)$$

Both sub-models are classic deterministic LP models and could be solved easily. The optimal interval solutions obtained by the BWC algorithm are $f = [8.13, 15.58]$, $x_1 = [3.75, 4.97]$, $x_2 = [0.63, 0.69]$.

3.2.2. Result Interpretation and Validity Checking

(1) Optimality Checking

According to the principle of Monte Carlo simulation algorithm, although a large number of event model runs (50 million times) were implemented, the optimal solution space provided by the Monte-Carlo simulation should be narrower than the real solution space of the original example model, at most get very close to it. Before checking the validity of the optimal interval solutions obtained by 2-step and BWC algorithms, two facts should be noted: (1) every optimal solution generated by solving Monte-Carlo simulation even model represents a subset of true optimal solution sets of the original model, and solution infeasibility is not an issue; (2) the optimal solution spaces provided by both 2-step and BWC algorithm should completely include the optimal solution space from the Monte-Carlo simulation method. Mathematically, it yields:

$$x_{1opt}^- \leq 3.76 \leq 4.95 \leq x_{1opt}^+, x_{2opt}^- \leq 0.30 \leq 1.11 \leq x_{2opt}^+, \text{ and } f_{opt}^- \leq 8.18 \leq 165.50 \leq f_{opt}^+$$

Based on these two facts, if the optimal solution space provided by 2-step or BWC algorithm does not cover the interval ranges of Monte-Carlo simulation results, i.e., the above relationship cannot be satisfied, this could lead to two significant consequences: (1) some optimal solution pairs are missing from the 2-step algorithm or BWC algorithm; (2) the optimal solutions produced by both algorithms might include some pair points which are infeasible.

The results obtained from three algorithms are summarized in Table 3.1 for solution comparison purpose. From Table 3.1, it can be seen that interval ranges of the optimal solutions provided by 2-step algorithm for both decision variable x_1 , x_2 and objective function f are all smaller than that provided by the Monte-Carlo simulation. It is obvious that some optimal solution pairs (x_1, x_2) are missing from the 2-step algorithm, and this algorithm fails the validity checking in terms of optimality validity. This result is also in line with the previous observation that the sub-models reformulated by 2-step algorithm do not represent the actual extreme situations.

Comparing to the 2-step algorithm, BWC has a better performance on decision variable x_1 which covers its full interval range produced by the Monte-Carlo simulation; however, its performance on decision variable x_2 is worse with an even smaller interval range than 2-step algorithm. It is obvious that, similar as the 2-step algorithm, some optimal solution pairs (x_1, x_2) are missing from the BWC algorithm as well, and this algorithm fails the checking in terms of optimality validity. According to the fundamental principles of BWC algorithm, reformulated best-case and worst-case sub-models represent two extreme situations of the original model, and the missing optimal solutions are not supposed to occur. This needs further investigation and is out of the scope of this study. As to the objective function, the BWC provides the largest ranges among three algorithms to cover the most optimistic and most pessimistic decision spaces, which allows the system to have a more flexible performance.

Table 3.1 Results obtained from three ILP algorithms

Algorithms	x_1		x_2		Objective function	
	x_1^-	x_1^+	x_2^-	x_2^+	f^-	f^+
Monte Carlo simulation	3.76	4.95	0.30	1.11	8.18	15.50
2-step algorithm	3.82	4.89	0.59	0.74	8.24	15.41
BWC	3.75	4.97	0.63	0.69	8.13	15.58

(2) Feasibility Checking

Figure 3.4 gives the feasible decision space and optimal solution space for the numerical example provided by the 2-step algorithm. The lines BJIG and CKLF represent the boundaries of the feasible decision space delimited by two sub-models' constraints (3.2.6) and (3.2.10), respectively; while the dotted blue lines beside each of them represent two BWC sub-models' constraints reformulated by the same original constraints. The lines AJKD and HILE represent the boundaries of the feasible decision space delimited by two sub-models' constraints (3.2.7) and (3.2.11), respectively; these two lines are in

coincidence with BWC sub-models' constraints (3.2.17) and (3.2.21) since all their coefficients are positive. Figure 3.4 gives the feasible decision space delimited by the 2-step algorithm which is under the lines of BJIG and CKLF and above the lines of AJKD and HILE. We can basically divide the entire feasible decision space into several regions and categories: the triangle CDK represents the absolute feasible region which satisfies all the constraints; the big triangle above line BJIG, and the big triangle below line HILE represent infeasible region as they violate at least one original constraint; the space bounded by points in sequence B, J, I, E, D, K, C represents softly feasible region which means the solution pairs (x_1, x_2) are not guaranteed to satisfy all the constraints; Quadrangle IJKL is the feasible optimal solution space provided by 2-step algorithm.

The rectangular grey area in Figure 3.4 is the optimal solution region for decision variable pair (x_1, x_2) obtained by the 2-step algorithm. This plot can help explain the infeasibility checking results. This figure shows that most of the 2-step optimal solutions are located in the softly feasible region. There is a small triangular area (MOP) right above the dotted blue line which is located in an infeasible region. Any optimal solution pairs obtained by the 2-step algorithm and located in this small triangular area are infeasible solutions for the original example model. For example, when $x_1=3.82$ and $x_2=0.74$, which is the left upper point of the solution rectangle (point M), the constraint (3.2.2) would be violated. It is obvious that infeasible solutions have been generated by the 2-step algorithm. In addition, the little triangular area (RNQ) under the blue dotted blue line is the non-optimal solution space. Solutions in this area are valid but not optimal.

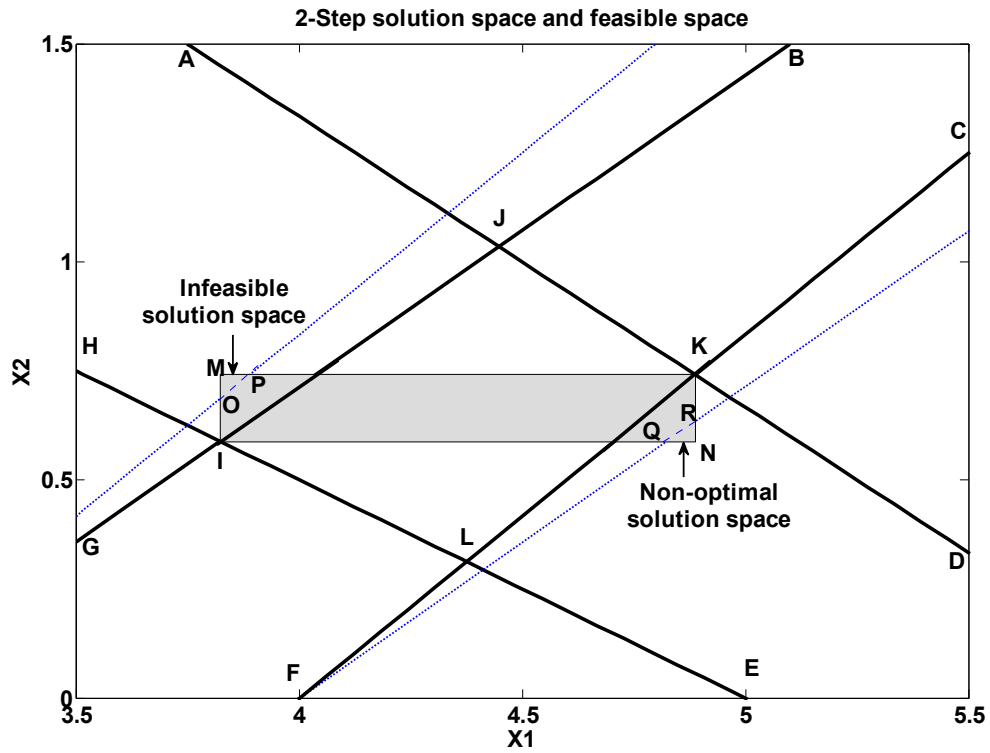


Figure 3.4 Two-step solution space and feasible space

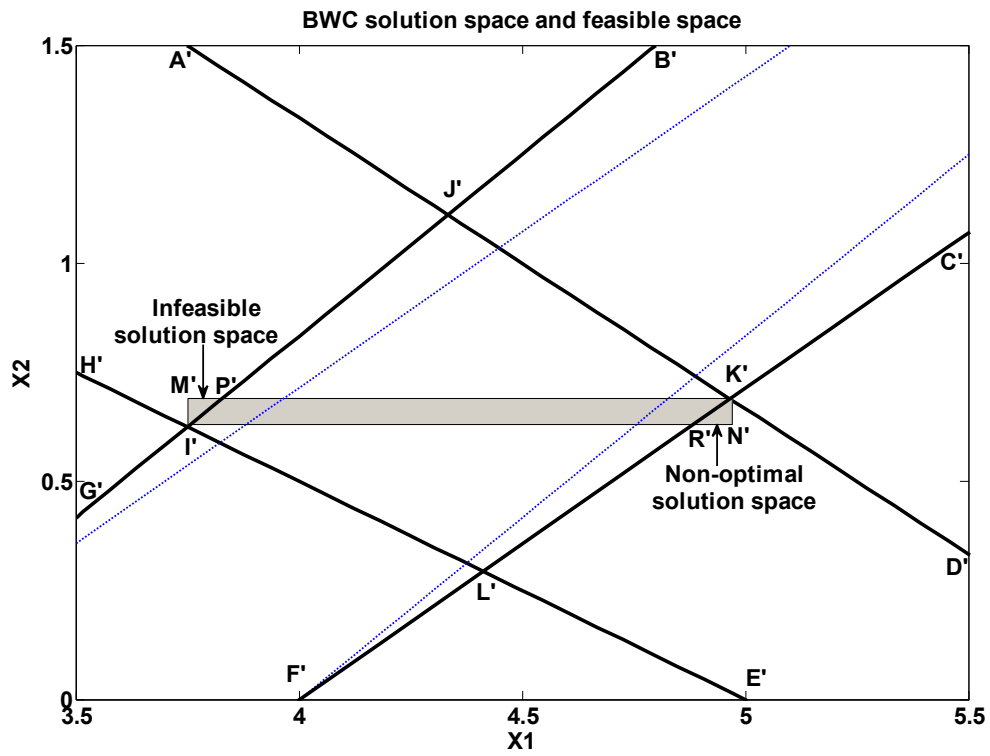


Figure 3.5 BWC solution space and feasible space

Figure 3.5 presents the feasible decision space and optimal solution space provided by the BWC algorithm. Line B'J'I'G' represents the constraint of (3.2.16); Line C'K'L'F' represents the constraint of (3.2.20); Line A'J'K'D' represents the constraint of (3.2.17); and Line H'I'L'E' represents the constraint of (3.2.21). The two dotted blue lines represent the two corresponding constraints from the 2-step algorithm. The triangle C'D'K' is an absolutely feasible region as it satisfies all the constraints; the big triangle above Line B'J'I'G', and the big triangle below Line H'I'L'E' are the infeasible regions as they violate at least one constraint; the space bounded by points in sequence B', J', I', E', D', K', C' represents softly feasible region which means the solution pairs (x_1, x_2) are not guaranteed to satisfy all the constraints; the quadrangle I'J'K'L' is the feasible optimal solution space. The rectangular grey area in Figure 3.5 represents the optimal interval solution obtained by the BWC algorithm. Once again, it shows that most of the BWC optimal solutions are located in the softly feasible region. The small triangular grey area located right above Line B'J'I'G' includes all the infeasible solution pairs which are generated by the BWC algorithm.

The observations from Figures 3.4 and 3.5 indicate that both 2-step and BWC algorithm fails the solution feasibility checking, and the optimal solutions provided by them are not always valid. From the decision-making standpoint, the generation of infeasible solutions might lead to a risky even failed decision.

Figures 3.4 and 3.5 can also help explain the missing optimal solutions from both 2-step and BWC algorithms. For example, for the minimized problem (3.2.15), the optimization solution exists when both x_1 and x_2 take the values as small as the constraints permit. In Figure 3.5, a solution at point L' is obviously a feasible solution and can provide a lower objective function than point K', since both x_1 and x_2 take smaller values at point L' than point K'. However, point K' is included in the optimal solution generated by the BWC algorithm while point L' is missing from the solution.

In addition, the solution spaces from both algorithms are mostly located within the softly feasible regions. It indicates that the solution provided cannot be guaranteed to be feasible in all situations. There exists the risk of violating some constraints in the softly feasible region. If we have to use the optimal solutions generated by two algorithms to develop practical implementation schemes, the risk level of violating the constraints needs to be considered. However, the existing ILP models cannot incorporate this risk into its decision-making process.

The results from this numerical example can help get the following conclusions in terms of validity checking for both 2-step and BWC algorithms: (a) the optimal solutions are not always valid and part of the results might be infeasible; (b) some optimal solutions are missing and not included in the obtained interval solutions. Two possible solutions may be helpful for dealing with this dilemma: (1) improving or redeveloping both algorithms, and (2) incorporating the decision risks into the decision-making process for helping develop practical implementation policies.

CHAPTER 4

FREILP MODEL DEVELOPMENT

4.1. Risk Explicit Interval Linear Programming (REILP)

4.1.1. REILP Modeling Approach

From the ILP validity checking conducted in Chapter 3, it has proved that the optimal solutions provided by both 2-step and BWC algorithms are not always valid. The ILP solutions are mostly located in the softly-feasible decision region, indicating that some solutions would have risks of violating some of the constraints. Moreover, both algorithms tend to generate infeasible or suboptimal implementation schemes. If the solutions are used for actual decision making, the decision makers need to acknowledge the potential risks associated with the generated decisions for making good use of them. However, the existing ILP solution algorithms are incapable of reflecting the linkage between decision risks and system performance. It is desired to develop new approaches.

To overcome the limitations of ILP algorithms while maintaining the strengths of ILP, a Risk Explicit Interval Linear Programming (REILP) was recently proposed. The development of a risk explicit ILP (REILP) model is presented as following (Zou et al., 2010).

Based on Definition 3.1.1, an event model of a general ILP model can be formulated as:

$$\text{Max} \quad f = \sum_{j=1}^n [c_j^- + \lambda_0 (c_j^+ - c_j^-)] x_j \quad (4.1.1)$$

$$\text{s.t.} \quad \sum_{j=1}^n [a_{ij}^+ - \lambda_{ij} (a_{ij}^+ - a_{ij}^-)] x_j - [b_i^- + \eta_i (b_i^+ - b_i^-)] \leq 0, \forall i \quad (4.1.2)$$

$$x_j \geq 0, \forall j \quad (4.1.3)$$

$$0 \leq \lambda_0 \leq 1 \quad (4.1.4)$$

$$0 \leq \lambda_{ij} \leq 1, \forall i, j \quad (4.1.5)$$

$$0 \leq \eta_i \leq 1, \forall i \quad (4.1.6)$$

Apparently, the model represented by equations (4.1.1) to (4.1.6) is a classic LP model, which corresponds to a specific set of crisp value of each coefficient given λ_0, λ_{ij} and η_i .

By re-arranging terms in equations (4.1.1) to (4.1.6), the model becomes:

$$\text{Max} \quad f = \sum_{j=1}^n [c_j^- x_j + \lambda_0 (c_j^+ - c_j^-) x_j] \quad (4.1.7)$$

$$\text{s.t.} \quad \sum_{j=1}^n a_{ij}^+ x_j - b_i^- \leq \sum_{j=1}^n \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-), \forall i \quad (4.1.8)$$

$$x_j \geq 0, \forall j \quad (4.1.9)$$

$$0 \leq \lambda_0 \leq 1 \quad (4.1.10)$$

$$0 \leq \lambda_{ij} \leq 1, \forall i, j \quad (4.1.11)$$

$$0 \leq \eta_i \leq 1, \forall i \quad (4.1.12)$$

Let $\mu = \lambda_0 (c_j^+ - c_j^-) x_j$, and $\xi_i = \sum_{j=1}^n \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-)$, where $i = 1, 2, \dots, m$,

the model can be reformatted as:

$$\text{Max} \quad f = \sum_{j=1}^n (c_j^- x_j + \mu) \quad (4.1.13)$$

$$\text{s.t.} \quad \sum_{j=1}^n a_{ij}^+ x_j - b_i^- \leq \xi_i, \forall i \quad (4.1.14)$$

$$x_j \geq 0, \forall j \quad (4.1.15)$$

When μ and ξ_i equal to 0, the model (4.1.13) to (4.1.15) becomes the worst-case sub-model of the BWC algorithm and the worst-case sub-model represents a most pessimistic situation. In an interval decision environment, the solution obtained for the most pessimistic sub-model would have no risk of violating the constraints since the formulation has guaranteed satisfying the tightest constraints (i.e., risk = 0 when μ and ξ_i

equal to 0). In cases where ξ_i takes values greater than 0, the constraints are relaxed by a level of ξ_i to obtain optimal solutions for achieving higher system return; in the meantime, the solution itself would be subjected to certain level of risk of violating the constraints. Obviously, the larger the ξ_i , the higher the risk would be associated with the solutions until ξ_i reaches its maximum values when both $\lambda_{ij} = 1 (\forall i, j)$ and $\eta_i = 1 (\forall i)$, which represents the most optimistic situation. Therefore, $\xi_i (\forall i)$ is qualified to evaluate the risk level of a decision, representing the possibility of a decision violating the constraints.

Definition 4.1.1: Function $\xi_i = \sum_{j=1}^n \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-)$ is defined as the *risk function* for constraint i in an ILP problem.

From equations (4.1.13) to (4.1.15), we know that (a) when $\xi_i = 0$, the decision based on the optimal solution has no risk of violating the corresponding constraint; and (b) when $\xi_i > 0$, the decision based on the optimal solution will have a level of risk of violating the corresponding constraint in proportion to the value of ξ_i .

The original ILP model is to maximize the objective function (i.e., system return). Since the system return and decision risk represent two conflicting factors in practical decision making process, a sound and satisfactory decision can be obtained only through minimizing the risk function while maximizing the system return. This leads to a multi-objective optimization problem:

$$\text{Max} \quad f = \sum_{j=1}^n c_j^- x_j + u \quad (4.1.16)$$

$$\text{Min} \quad \xi = \oplus_i \left[\sum_{j=1}^n \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-) \right] \quad (4.1.17)$$

$$s.t. \quad \sum_{j=1}^n a_{ij}^+ x_j - b_i^- \leq \xi_i, \forall i \quad (4.1.18)$$

$$x_j \geq 0, \forall j \quad (4.1.19)$$

Where \oplus is a general arithmetic operator which can be a simple addition, a weighted addition, simple arithmetic mean, weighted arithmetic mean, or a max operator. The subscript for \oplus_i , i , suggests that the operator be applied across constraints to obtain a unified risk function for the entire optimization problem. For each individual constraint, the constraint-wise risk function $\sum_{j=1}^n \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-)$ can differ from that of another constraint by order of magnitude due to different categories of b_i as well as the incorporation of interactions among λ_{ij} , a_{ij}^+ , a_{ij}^- , x_j , η_i , b_i^+ and b_i^- in the function. Therefore, it is necessary to convert the constraint-wise risk function into comparable magnitude. A simple method through scaling each constraint-wise risk function by $\frac{1}{b_i^-}$ can be a feasible choice, which essentially represents a fractional risk factor from the most pessimistic case. In application, more refined approaches can be developed to better reflect the decision environment for the specific case.

To solve the multi-objective programming problem, the model (add the model numbering here) can be re-formulated as:

$$Min \quad \xi = \oplus_i \left[\sum_{j=1}^n \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-) \right] \quad (4.1.20)$$

$$s.t. \quad \sum_{j=1}^n (c_j^- x_j + \mu) \geq f_{opt}^- + \lambda_0 (f_{opt}^+ - f_{opt}^-) \quad (4.1.21)$$

$$\sum_{j=1}^n a_{ij}^+ x_j - b_i^- \leq \xi_i, \forall i \quad (4.1.22)$$

$$\lambda_0 = \lambda_{pre} \quad (4.1.23)$$

$$0 \leq \lambda_{ij} \leq 1 \quad (4.1.24)$$

$$x_j \geq 0, \forall j \quad (4.1.25)$$

$$0 \leq \eta_i \leq 1, \forall i \quad (4.1.26)$$

Definition 4.1.2: The optimization model (model numbering) is derived from the original ILP model and includes a risk-minimization objective function. This model is defined as a *Risk Explicit ILP (REILP)* model.

Here, λ_0 is pre-defined by the decision makers, representing the degree of aggressiveness, or alternatively, the aspiration level of decision makers given the uncertainties in the optimization model. When $\lambda_0 = 0$, the model is corresponding to the least aggressive case where the most conservative and safe solution is expected. On the other hand, when $\lambda_0 = 1$, the model is corresponding to the most aggressive case where the most optimistic solutions but risky solutions will be generated. Apparently, in most real-world situations, decision makers would prefer balanced solutions with $0 < \lambda_0 < 1$ to the extreme solutions represented by $\lambda_0 = 0$ or $\lambda_0 = 1$ as the basis of practical decision making. Therefore, the task is to find the optimal solutions with least risk level for a desired degree of aggressiveness.

The risk optimization model REILP (4.1.20) to (4.1.26) is a nonlinear model. The non-linearity is generated by the introduction of risk level variables (i.e., λ_0 and λ_{ij}) to represent the complex non-linear interactions of uncertainties between different variables and terms in a constraint. It is apparent that for a specific constraint, if a large λ_{ij} is associated with a small x_j the large λ_{ij} would have small contribution to the risk in the decision. On the other hand, if the λ_{ij} is associated with a large x_j , it would result in significant contribution to the overall risk of decision making.

4.1.2. Discussion of REILP

(1) Aspiration Level λ_0

For responding to the issues associated with the ILP solution, the REILP approach was developed attempting to provide decision-makers more satisfactory and practical implementation schemes through minimizing the decision risks while maximizing the system return. The improvement over the existing 2-step and BWC solutions is that the risks associated with the possible optimal solutions and decisions derived from them could be explicitly incorporated into the decision-making process. However, one potential problem associated with the aspiration level λ_0 needs to be further discussed.

In the formulation of a REILP model, its original objective was converted into a constraint with the following format (Zou et al., 2010):

$$\sum_{j=1}^n (c_j^- x_j + \mu) \geq f_{opt}^- + \lambda_0 (f_{opt}^+ - f_{opt}^-),$$

where,

$$\mu = \lambda_0 (c_j^+ - c_j^-) x_j,$$

So we get:

$$\sum_{j=1}^n [(c_j^- + \lambda_0 (c_j^+ - c_j^-)) x_j] \geq f_{opt}^- + \lambda_0 (f_{opt}^+ - f_{opt}^-) \quad (4.1.27)$$

Where, λ_0 appears in both right-hand side and left-hand side of this constraint, representing the aspiration level that reflects the decision-maker's preference and needs to be preset. A major assumption used behind this formulation is that the system return coefficient c_j has the same changing rate (i.e., λ_0) from its lower bound as f_{opt}^- . However, this is not always true in a real-world decision making problem, and c_j and f_{opt}^- might take

different rates changing with their own intervals. A better formulation for the inequality (4.1.27) should be:

$$\sum_{j=1}^n [(c_j^- + \lambda_j(c_j^+ - c_j^-))x_j] \geq f_{opt}^- + \lambda_0(f_{opt}^+ - f_{opt}^-) \quad (4.1.28)$$

Where, λ_j is the changing rate for $c_j, j = 1, \dots, n$, and it is not necessarily equal to the aspiration level λ_0 in most cases.

(2) Risk Function

In the REILP formulation, the risk function was defined as (Zou et al., 2011):

$$\xi_i = \sum_{j=1}^n \lambda_{ij}(a_{ij}^+ - a_{ij}^-)x_j + \eta_i(b_i^+ - b_i^-)$$

By this equation, the risk function can reflect the risk of violating the corresponding constraints a_{ij} and b_j , but cannot reflect the risk of violating the constraints of c_j , and it cannot directly reflect the relation between the risk function and the aspiration level. When the original objective function is converted into a constraint as (4.1.28), the risk of violating this secondary constraint should also be considered. This fits the common sense that a higher aspiration level, which means an aggressive decision and a higher system return, will affect the risk level of the whole system. The risk function without considering the risk level of parameters c_j and aspiration level λ_0 is not sufficient.

(3) Preset of Aspiration Level

It might be simple for some decision makers to preset an aspiration level and then obtain a series of results. However, in some situations, decision makers may not have clear understanding of the aspiration level and cannot select a sound aspiration level that

represents their professional judgment of the situation. Blindly selecting an aspiration level may lead to undesirable results.

4.1.3. An Illustrative Example of REILP

(1) The Example and Results

In the development of the REILP approach, an example was given to illustrate the applicability of the approach (Zou et al., 2010). The example was a land-use management problem for nutrient loading control and maximum profits gain. In this hypothetical case, there is 1,200 acre of lands in a watershed that is available for two types of crop production. It is known that crop 1 can reach unit productivity of 4,326-4,920 kg/acre, with a net profit of \$0.26 to 0.3/kg, and the production of crop 2 can reach 3,480 – 4,120 kg/acre, which can realize a net profit of 0.22 to 0.29 dollar/kg. To produce crop 1, the unit area nitrogen and phosphorus loading discharged to a lake in the watershed is 4.3 to 5.2 kg/acre/yr and 0.42 to 0.48 kg/acre/yr, respectively. For crop 2, the loading rates are 3.2 to 3.6 kg/acre/yr for nitrogen and 0.27 to 0.32 kg/acre/yr for phosphorus. It is known from a Total Maximum Daily Load (TMDL) study that the total loading of nitrogen and phosphorus discharged into the lake cannot be greater than 4,144 and 379 kg/yr, respectively, without considering an explicit margin of safety. However, when a 10% margin of safety is imposed, the maximum allowable loading for nitrogen and phosphorus are 3,730 and 341 kg/yr, respectively. The watershed authorities need a land-use planning scheme to optimally allocate lands to different crops in order to maximize the crop production profit while satisfying the environmental requirements in terms of nitrogen and phosphorus discharges. This land-use ILP model was firstly formulated as follows:

$$\text{Max} \quad f = [0.26,0.3] * [4326,4920] * X_1 + [0.22,0.29] * [3480,4120] * X_2 \quad (4.1.29)$$

$$\Rightarrow \text{Max} \quad f = [1125,1476] * X_1 + [765,1194.8] * X_2$$

$$\text{s.t.} \quad X_1 + X_2 \leq 1200$$

$$[4.3,5.2]* X_1 + [3.2,3.6]* X_2 \leq [3730,4144]$$

$$[0.42,0.48]* X_1 + [0.27,0.32]* X_2 \leq [341,379]$$

$$X_1, X_2 \geq 0$$

Through the BWC algorithm, an interval maximized system benefit could be obtained: $f = [803250, 1511470]$. According to the REILP approach, this original land-use ILP model can be converted into a risk explicit ILP model:

$$\begin{aligned} \text{Min} \quad \xi = & r_3(5.2 - 4.3)X_1 / 3730 + r_4(3.6 - 3.2)X_2 / 3730 + r_5(4144 - 3730) / 3730 \\ & + r_6(0.48 - 0.42)X_1 / 341 + r_7(0.32 - 0.27)X_2 / 341 + r_8(379 - 341) / 341 \quad (4.1.30) \end{aligned}$$

s.t.

$$(1125 + r_0(1476 - 1125))X_1 + (765 + r_0(1194.8 - 765))X_2 \geq 803250 + r_0(1511470 - 803250)$$

$$X_1 + X_2 \leq 1200$$

$$5.2X_1 + 3.6X_2 - 3730 \leq r_3(5.2 - 4.3)X_1 + r_4(3.6 - 3.2)X_2 + r_5(4144 - 3730)$$

$$0.48X_1 + 0.32X_2 - 341 \leq r_6(0.48 - 0.42)X_1 + r_7(0.32 - 0.27)X_2 + r_8(379 - 341)$$

$$X_1, X_2 \geq 0$$

$$0 \leq r_0, r_3, r_4, r_5, r_6, r_7, r_8 \leq 1$$

Where, r_0 is the aspiration level that needs to be preset by decision makers. The solutions of the example problem are given in Table 4.1 (Zou et al., 2010). In the table, NRL refers to the Normalized Risk Level, which is calculated by multiplying the risk function value by a number to make the smallest NRL value close to 0 and the greatest close to 1.

Table 4.1 The optimal solutions of the example problem

r_0	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Profit (10^5 \$)	8.03	8.74	9.45	10.20	10.90	11.60	12.30	13.00	13.70	14.40	15.10
X_1 (acre)	531	0	0	0	0	0	2	66	132	202	276
X_2 (acre)	277	1082	1110	1136	1160	1181	1198	1134	1068	998	924
NRL	0.00	0.10	0.20	0.28	0.36	0.43	0.50	0.60	0.70	0.81	0.93

Source: Zou et al., 2010

(2) Revisit of the Example Model and Results

As discussed in the previous section, when the original objective function is transformed

into a new constraint (4.1.27), $\sum_{j=1}^n [(c_j^- + \lambda_0(c_j^+ - c_j^-))x_j] \geq f_{opt}^- + \lambda_0(f_{opt}^+ - f_{opt}^-)$, λ_0

appears in both right-hand side and left-hand side of this constraint. It is assumed that the system return coefficient c_j has the same changing rate (i.e., λ_0) from its lower bound as f_{opt} . However, in a real-world decision making problem, the changing rate for c_j (λ_j) is not necessarily same as the changing rate for f_{opt} (i.e., λ_0). Using λ_0 in both hand sides may lead to infeasible problems. A better formulation for the inequality (4.1.27) should be:

$$\sum_{j=1}^n [(c_j^- + \lambda_j(c_j^+ - c_j^-))x_j] \geq f_{opt}^- + \lambda_0(f_{opt}^+ - f_{opt}^-)$$

Under this circumstance, we use r_1 for X_1 , r_2 for X_2 , and still λ_0 for f_{opt} . The example model (4.1.29) could then be reformulated as (4.1.31), which is different from the model (4.1.30):

$$\begin{aligned} \text{Min} \quad \text{RISK} = & r_3(5.2 - 4.3)X_1 / 3730 + r_4(3.6 - 3.2)X_2 / 3730 + r_5(4144 - 3730) / 3730 \\ & + r_6(0.48 - 0.42)X_1 / 341 + r_7(0.32 - 0.27)X_2 / 342 + r_8(379 - 341) / 341 \end{aligned} \quad (4.1.31)$$

s.t.

$$(1125 + r_1(1476 - 1125))X_1 + (765 + r_2(1194.8 - 765))X_2 \geq 803250 + r_0(1511470 - 803250)$$

$$X_1 + X_2 \leq 1200$$

$$5.2X_1 + 3.6X_2 - 3730 \leq r_3(5.2 - 4.3)X_1 + r_4(3.6 - 3.2)X_2 + r_5(4144 - 3730)$$

$$0.48X_1 + 0.32X_2 - 341 \leq r_6(0.48 - 0.42)X_1 + r_7(0.32 - 0.27)X_2 + r_8(379 - 341)$$

$$X_1, X_2 \geq 0$$

$$0 \leq r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, \leq 1$$

where r_0 is the aspiration level given the decision-makers.

Table 4.2 The optimal solutions of the model (4.1.31)

Aspiration level	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
r_1	1	1	1	1	1	1	1	1	1	1	1
r_2	1	1	1	1	1	1	1	1	1	1	1
r_3	0	0	0	0	0	0	0	1	0.484	0.484	0.999
r_4	0.004	0.004	0.04	0	0	0	0	0.420	0.864	1	0.999
r_5	0	0	0	0	0	0	0	0.003	0.003	0.358	0.999
r_6	0	0	0	0	0	0	0	0.347	0.271	0.271	0.271
r_7	0	0	0	0	0	0	0	0.127	0.451	0.792	1
r_8	0	0	0	0	0	0	0	0	0	0	0.962
X_1 (acre)	543	592	639	686	590	323	39	0	0	24	276
X_2 (acre)	2	2	2	2	180	570	980	1087	1146	1176	924
Profit (10^3 \$)	803	874	945	1016	1087	1157	1228	1299	1370	1441	1511
Risk function	0	0	0	0	0	0	0	0.070	0.182	0.306	0.532

The model (4.1.31) was solved by LINGO, and the solutions was obtained and presented in Table 4.2. The results indicate that $r_1, r_2, r_3, r_4, r_5, r_6, r_7$ and r_8 all satisfy the range of $[0, 1]$, and also a reasonable increasing trend between system benefit and aspiration level is observed. However, the values of risk function all take 0 for r from r_1 to r_6 , indicating that there is no linkage between the decision risks (in terms of the aspiration levels) and system return.

As discussed in section 4.1.2, the risk function defined by Zou et al. (2010) can reflect the risk of violating the corresponding constraints a_{ij} and b_j , but cannot reflect the risk of violating the constraints of c_j , and it cannot directly reflect the relation between the risk function and the aspiration level. When the original objective function is converted into a constraint as (4.1.28), the risk of violating this secondary constraint should also be considered.

Thus, Based on the definition and formulation of the risk function:

$$\xi_i = \sum_{j=1}^n \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-)$$

$$\xi = \oplus_i \left[\sum_{j=1}^n \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-) \right]$$

And the secondary constraints that:

$$\sum_{j=1}^n (c_j^- + \lambda_j (c_j^+ - c_j^-) x_j) \geq f_{opt}^- + \lambda_0 (f_{opt}^+ - f_{opt}^-)$$

A comprehensive risk function could be formulated as:

$$\xi = \oplus_i \left[\sum_{j=1}^n \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-) \right] + \oplus_k \left[\sum_{j=1}^n \lambda_j (c_j^+ - c_j^-) x_j + \lambda_0 (f_{opt}^+ - f_{opt}^-) \right] \quad (4.1.32)$$

By choosing the general operator \oplus_i and \oplus_k as $\frac{2}{b_i^+ + b_i^-}$ and $\frac{2}{f_{opt}^+ + f_{opt}^-}$, the risk

function then becomes:

$$\xi = \sum_{i=1}^m \frac{2}{b_i^+ + b_i^-} [\sum_{j=1}^n \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-)] + \frac{2}{f_{opt}^+ + f_{opt}^-} [\sum_{j=1}^n \lambda_j (c_j^+ - c_j^-) x_j + \lambda_0 (f_{opt}^+ - f_{opt}^-)] \quad (4.1.33)$$

The equation (4.1.33) is the risk function that will be used in the model (4.1.31) to replace its objective function. Solving this new model and the obtained results are given in Table 4.3. The results indicate that the risk function is directly related with the aspiration level, and the system benefit increases with the increasing risk function.

Table 4.3 The optimal solutions when the risk function is (4.1.33)

Aspiration level	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Risk function	0	0.176	0.353	0.529	0.705	0.886	1.068	1.269	1.501	1.739	2.029
r_1	0	0.122	0.140	0.520	0.900	1	1	1	1	1	1
r_2	0	0.416	1	1	1	1	1	1	1	1	1
r_3	0	0	0	0	0	0	0.114	1	1	1	1
r_4	0	0	0	0	0	0.160	0.182	0.265	0.358	0.411	1
r_5	0	0	0	0	0	0	0.003	0.166	0.563	1	0.999
r_6	0	0	0	0	0	0	0	0	0	0	1
r_7	0	0	0	0	0	0	0	0.127	0	0.152	0.707
r_8	0	0	0	0	0	0	0	0	0.681	1	1
X_1 (acre)	531	531	531	531	531	366	142	0	0	24	276
X_2 (acre)	269	269	269	269	269	516	852	1087	1146	1176	924
Profit (10^3 \$)	803	874	945	1015	1087	1157	1228	1299	1370	1441	1511

4.2. FREILP Model Development

In the REILP, the aspiration level, λ_0 , represents the degree of aggressiveness and the aspiration level of decision makers under an uncertain decision-making environment. When $\lambda_0 = 0$, the model corresponds to the least aggressive case and the most conservative and safe solutions will be obtained; while when $\lambda_0 = 1$, the model corresponds to the most aggressive case and the most optimistic but risky solutions will be obtained. The aspiration level needs to be given and preset by the decision makers before running the model. However, when facing many practical decision-making problems, the decision makers may not be able to determine exactly their specific aspiration levels; and more importantly, different decision makers and stakeholders may have different opinions about their preferences to different decisions and policies in terms of their aggressiveness or conservativeness. In this sense, the aspiration levels exist in the human thinking, being fuzzy in nature. As an extension to previous efforts, this study attempts to develop a fuzzy based REILP approach to account for the fuzziness associated with the aspiration levels, and thus improve the applicability of the proposed method.

4.2.1. Fuzzy Set Theory

Fuzzy Set Theory (Zadeh, 1965; Zadeh, 1968) was formalized as an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition — an element either belongs or does not belong to the set. By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the real unit interval $[0, 1]$. Fuzzy sets generalize classical sets, since the indicator functions of classical sets are special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. The fuzzy set theory can be used in a wide range of domains in which information is incomplete or imprecise.

Definition 4.2.1: If X is a collection of objects denoted generically by x , then a *fuzzy set* \tilde{A} in X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$$

$\mu_{\tilde{A}}(x)$ is called the **membership function** or grade of membership of x in \tilde{A} , which maps X to the membership space M . When M contains only the two points, 0 and 1, \tilde{A} is non-fuzzy. The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed.

For example, a fuzzy set of “real numbers close to 10” (Zimmermann, 1991) could be expressed as the following expression and the corresponding curve is given in Figure 4.1:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | \mu_{\tilde{A}}(x) = (1 + (x - 10)^2)^{-1}\}$$

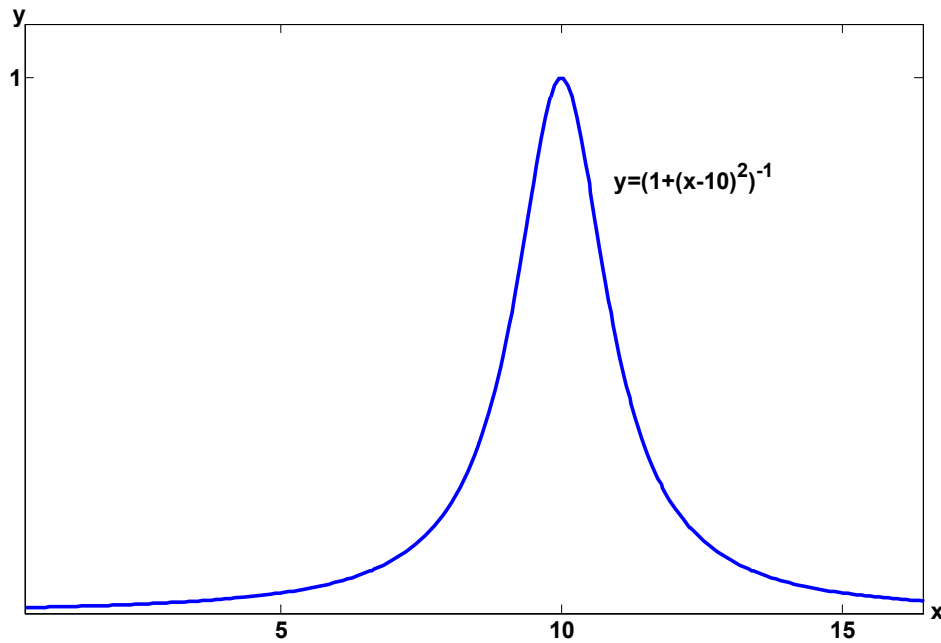


Figure 4.1 An example of the membership function of the fuzzy set “real number close to 10”

For any given number, x , the membership grade could be calculated by the above expression $\mu_{\tilde{A}}(x)$. For example, when $x = 7$, the membership grade of “number 7 close to 10” is $\mu_{\tilde{A}}(7) = (1 + (7 - 10)^2)^{-1} = 0.1$; and when $x = 10$, the membership grade is

$\mu_{\tilde{A}}(7) = (1 + (10 - 10)^2)^{-1} = 1$. The membership grade indicates the possibility or likelihood of a real number belonging to this fuzzy set.

Definition 4.2.2: The crisp set of elements that belong to the fuzzy set \tilde{A} at least to the degree α is called the *α -level set* (Zadeh, 1975; Zimmermann, 1991).

$$A_\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha\}$$

Given the α -level, the α -level set could be obtained simply by drawing a line of α -level in the figure of the membership function. The elements above the line would be the α -level set. So the degree α is also called α -cut.

For the fuzzy set “real numbers close to 10”, when α -cut = 0.5, the line of α -cut = 0.5 cuts the membership function into two parts (above and below the line) and two intersection points were obtained with $x = 9$ and $x = 11$, respectively (see Figure 4.2). So when α -cut = 0.5, the elements of the α -level set include all the real numbers between 9 and 11 with their membership grades all being greater than 0.5. The α -cut concept has been widely used in the practical decision-making problems to deal with system fuzziness.

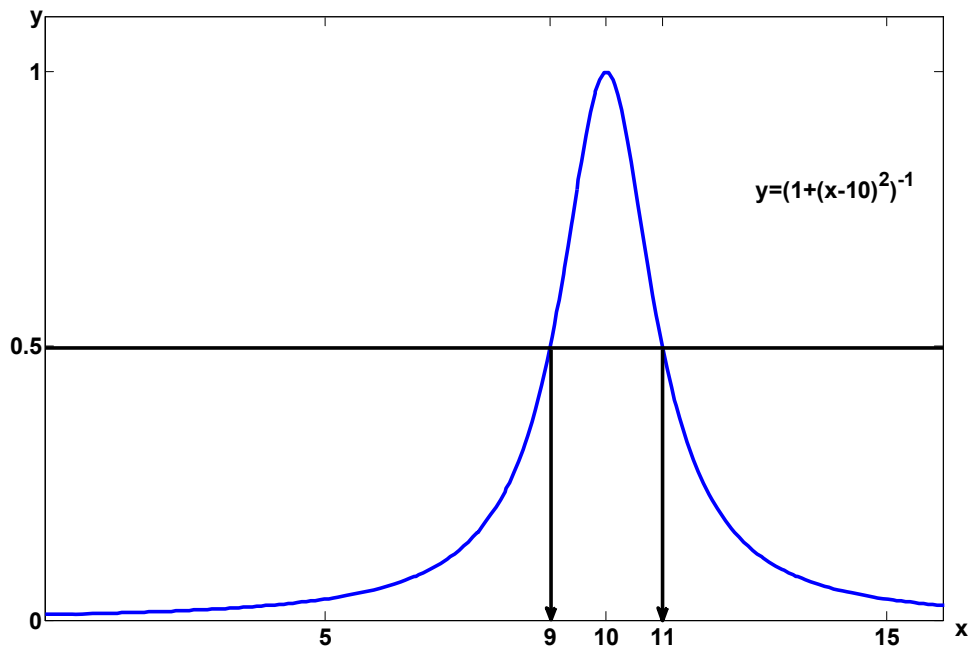


Figure 4.2 The membership function of “real numbers close to 10” under α -cut = 0.5

Lofti Zadeh also developed a membership function for “young people” (Zadeh, 1972) as following:

$$\tilde{A}(x) = \begin{cases} 1 & 0 \leq x \leq 25 \\ \left[1 + \left(\frac{x-50}{5}\right)^{-2}\right]^{-1} & 25 \leq x \leq 100 \end{cases}$$

This is based on the assumption that people’s ages are from 0 to 100. The plot of membership function of “young people” is shown in Figure 4.3. Under α -cut of 0.7, the α -cut subset of the fuzzy set “young people” would include all people aged from 0 to 28.

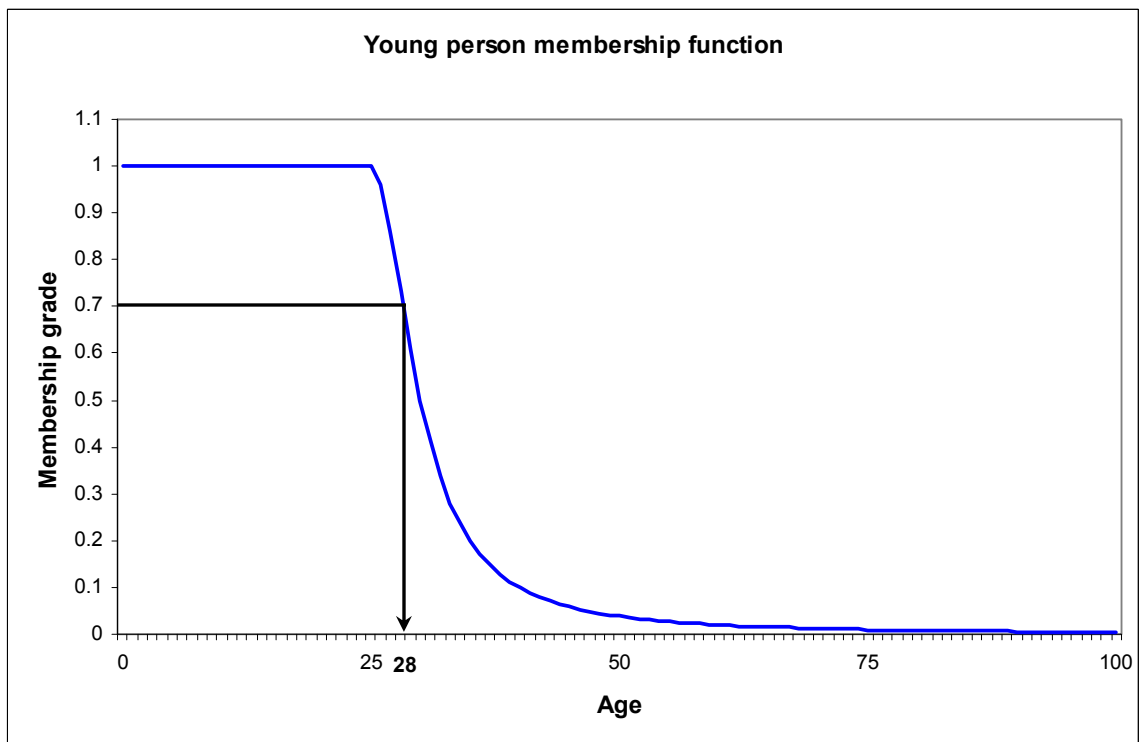


Figure 4.3 The membership function of the fuzzy set “young people” and the subset of $\alpha=0.7$

4.2.2. Fuzzy Nature of the Aspiration Level (λ_0)

As explained earlier, the aspiration level, λ_0 , has fuzzy characteristics in nature. Different values of aspiration levels represent the degree of aggressiveness or conservativeness of decision makers. In this study, three levels of λ_0 are considered, including aggressive aspiration level, medium aspiration level, and conservative aspiration level. Their membership functions are developed based on the fuzzy set theory of Zadeh and his “young person” membership function. Figure 4.4 gives the membership function curve of the fuzzy conservative aspiration level. Its x-axis represents the aspiration level instead of age (Figure 4.3) and its ranges change from $[0, 100]$ of age to $[0, 1]$ of the aspiration level. The membership function of the aggressive aspiration level is the opposite of the conservative one, as shown in Figure 4.6. The membership function of the medium aspiration level is a combination of conservative and aspiration level functions, as given in Figure 4.5. The equations and figures of the membership functions for three situations are provided below.

(1) Conservative aspiration level membership function:

$$\tilde{A} = \begin{cases} 1 & 0 \leq \lambda_0 \leq 0.25; \\ [1 + (4\lambda_0 - 1)^2]^{-1} & 0.25 < \lambda_0 \leq 1; \end{cases} \quad (4.2.1)$$

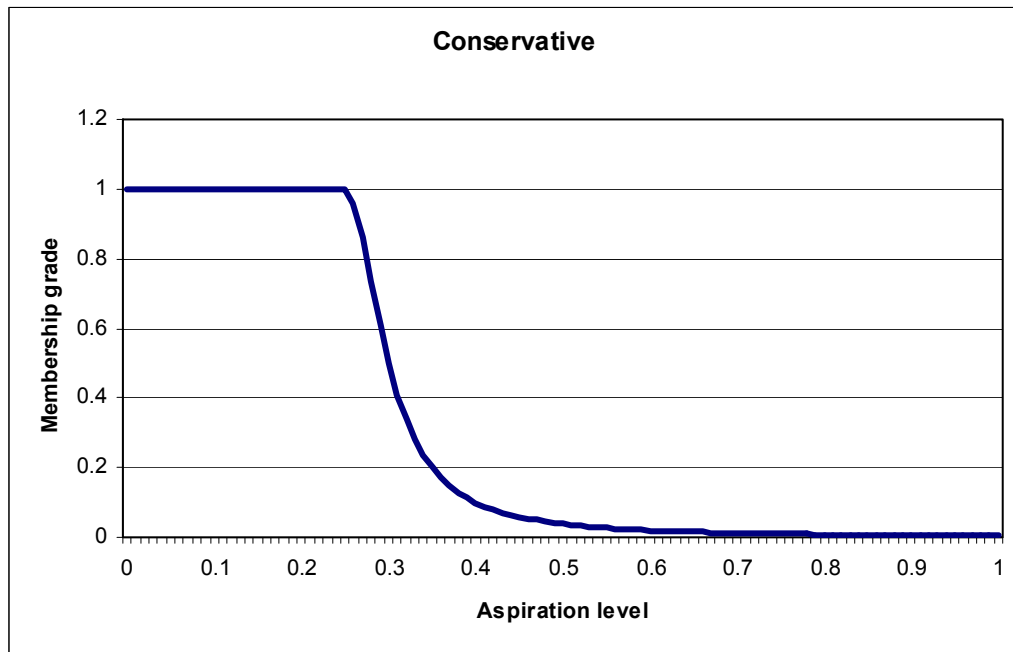


Figure 4.4. The membership function of conservative aspiration level

(2) Medium aspiration level membership function

$$\tilde{M} = \begin{cases} [1 + (15 - 40\lambda_0)^2]^{-1} & 0 \leq \lambda_0 < 0.375; \\ 1 & 0.375 \leq \lambda_0 \leq 0.625; \\ [1 + (40\lambda_0 - 15)^2]^{-1} & 0.625 < \lambda_0 \leq 1; \end{cases} \quad (4.2.2)$$

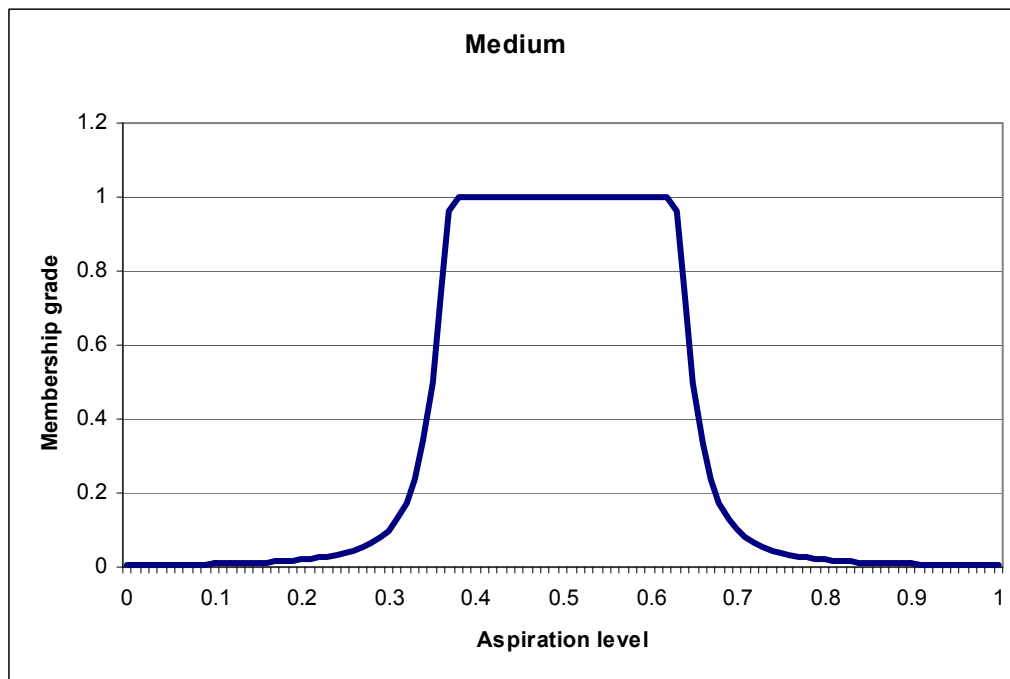


Figure 4.5 The membership function of medium aspiration level

(3) Aggressive aspiration level membership function

$$\tilde{B} = \begin{cases} [1 + (15 - 20\lambda_0)^2]^{-1} & 0 \leq \lambda_0 \leq 0.75; \\ 1 & 0.75 \leq \lambda_0 \leq 1; \end{cases} \quad (4.2.3)$$

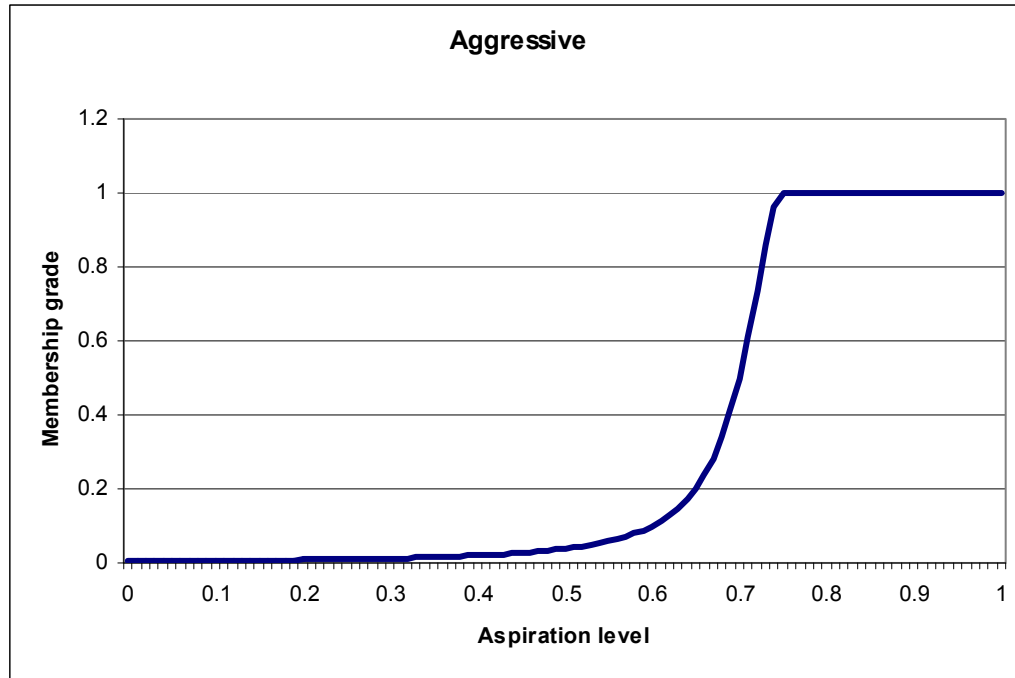


Figure 4.6 The membership function of aggressive aspiration level

4.2.3. Fuzzy Risk Explicit Interval Linear Programming (FREILP)

With the aspiration level being handled as the fuzzy numbers, a Fuzzy Risk Explicit Interval Linear Programming (FREILP) can then be formulated as follows.

For a maximized ILP problem, we have:

$$Max \quad f^{\pm} = \sum_{j=1}^n c_j^{\pm} x_j^{\pm} \quad (4.2.4)$$

$$s.t. \quad \sum_{j=1}^n a_{ij}^{\pm} x_j^{\pm} \leq b_i^{\pm}, \quad \forall i$$

$$x_j^{\pm} \geq 0, \forall j$$

The objective function of its FREILP formulation is to minimize the risk function:

$$\xi = \sum_{i=1}^m \frac{2}{b_i^+ + b_i^-} \left[\sum_{j=1}^n \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-) \right] + \frac{2}{f_{opt}^+ + f_{opt}^-} \left[\sum_{j=1}^n \lambda_j (c_j^+ - c_j^-) x_j + \lambda_0 (f_{opt}^+ - f_{opt}^-) \right] \quad (4.2.5)$$

Constraints include:

$$\sum_{j=1}^n (c_j^- + \lambda_j (c_j^+ - c_j^-) x_j) \geq f_{opt}^- + \lambda_0 (f_{opt}^+ - f_{opt}^-) \quad (4.2.6)$$

$$\sum_{j=1}^n a_{ij}^+ x_j - b_i^- \leq \sum_{j=1}^n \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-), \forall i \quad (4.2.7)$$

$$0 \leq \lambda_{ij}, \lambda_j, \eta_i \leq 1 \quad (4.2.8)$$

$$x_j \geq 0, \forall j \quad (4.2.9)$$

In the above formulation from (4.2.5) to (4.2.6), the aspiration level λ_0 is treated as a fuzzy set.

For a minimized ILP problem, we have:

$$Min \quad f^{\pm} = \sum_{j=1}^n c_j^{\pm} x_j^{\pm} \quad (4.2.10)$$

$$s.t. \quad \sum_{j=1}^n a_{ij}^{\pm} x_j^{\pm} \geq b_i^{\pm}, \quad \forall i$$

$$x_j^{\pm} \geq 0, \forall j$$

The objective function of its FREILP is also to minimize the risk function, which is the same as the model (4.2.5).

Constraints include

$$\sum_{j=1}^n (c_j^+ - \lambda_j (c_j^+ - c_j^-) x_j) \geq f_{opt}^+ - \lambda_0 (f_{opt}^+ - f_{opt}^-) \quad (4.2.11)$$

$$b_i^+ - \sum_{j=1}^n a_{ij}^- x_j \geq \sum_{j=1}^n \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-), \forall i \quad (4.2.12)$$

$$0 \leq \lambda_{ij}, \lambda_j, \eta_i \leq 1 \quad (4.2.13)$$

$$x_j \geq 0, \forall j \quad (4.2.14)$$

Similarly, the aspiration level λ_0 is also treated as the fuzzy set.

4.2.4. Solution Process for the FREILP Model

For solving the models formulated in Section 4.2.3, the solution process includes the following steps:

[Step 1] Use the BWC algorithm to convert the original ILP model into two sub-models and solve both sub-models to find the solutions of the lower bound and the upper bound of the objective function of the original ILP model.

[Step 2] Use the solutions of objective function obtained in Step 1 to formulate a fuzzy REILP model as given in equations (4.2.4) to (4.2.14).

[Step 3] According to the preference of the decision makers, define the aspiration level as conservative, medium, or aggressive, accordingly, and also choose an α -cut level to cut the membership function curve. Two α -cut crisp values obtained would be used as the aspiration level input for solving the formulated FREILP model.

[Step 4] Run the FREILP model by LINGO and find out the solutions.

CHAPTER 5

FREILP MODEL APPLICATION TO MSW SYSTEM IN HRM

In this study, the developed approach is applied to the Municipal Solid Waste (MSW) management system in Halifax Regional Municipality (HRM), Canada, not only for testing its applicability to real-world problems, but also for providing the municipal waste managers with a more practical decision support tool. A FREILP model is developed for the long-term planning for the MSW management system in HRM. The planning horizon is 30 years starting from the year of 2011, and is divided into 6 planning periods with 5 years in each period. The latest data recorded for modeling purposes include a period from April 2009 to March of 2010 which is used to represent the input data for the year of 2010. All the monetary values will be converted to the value of 2010 Canadian dollar considering the annual inflation factor in the coming years.

5.1. Overview of MSW System in HRM

The Halifax Regional Municipality (HRM) is the largest population center of Canadian East Coast and the capital city of the province of Nova Scotia. The municipality was founded in 1996 through combining four communities: Halifax, Dartmouth, Bedford, and Halifax County (HRM, 2010a). It is the economic and cultural centre of Canada's East Coast, accounting for 40% of Nova Scotia's population, and is one of Canada's prime tourist locations (HRM, 2010b).

The HRM is committed to environmental sustainability. It is one of the highest waste diversion rate municipalities in Canada (Walker, et al., 2004). By August 2009, the waste diversion rate in HRM has reached 59%, which is in the 4th place across Canada following the Regional District of NaNaimo, British Columbia (64%), the city of Victoriavill, Quebec (64%), and Charlottetown, Prince Edward Island (60%) (FCM, 2009).

In 1995, the HRM Waste Resource Management Strategy was developed to meet three long-term goals (CSC, 1995):

- To maximize the 3 Rs (reduction, reuse and recycling) of MSW.
- To maximize environmental sustainability and minimize costs.
- To foster stewardship and values of a conserver society.

The research toward this strategy began in the early 1990's, when the local raw waste landfill was reaching its designed capacity and began causing odour and other environmental problems (Goldstein and Gray, 1999; HRM 2007a). To achieve these goals, the HRM, along with public and private partners, has implemented many programs since then, as shown in Table 5.1 below. In 2000, HRM became the first winner of FCM-CH2M HILL Sustainable Community Awards for the community-based waste resource management strategy (FCM, 2000).

Currently, the MSW system in HRM has 3 components (HRM, 2002): landfill, recycling, and composting. HRM operates one landfill without relying on incineration facilities. All the refuses are delivered to the Otter Lake Waste Processing and Disposal Facilities. The designed capacity is 150,000 tonnes per year for 25 years (from 1998 to 2023) (Goldstein and Gray, 1999). HRM also has one recycling facility in operation, located at 50 Chain Lake Drive, with the capacity of 28,000 tonnes per year (Goldstein and Gray, 1999). Two composting facilities are in use in HRM for compostable materials: New Era Farms facility, and Miller Composting facility. The nominal capacity of each facility is 25,000 tonnes per year (Friesen, 1999; Friesen 2000). There is no solid waste transfer station being in use in HRM. Only one type of vehicle, large garbage truck, is being used in HRM.

Table 5.1 MSW management programs implemented by HRM

Program name	Description
Source separation	Waste of organics, recyclables and trash were separated at source, with biweekly collection of organics and trash, weekly collection of recyclables in most areas and biweekly recyclables collection in the rural areas of the county.
Collection zones creation	Creation of eight collection zones (from 25 before amalgamation) with six haulers.
Aerated carts	Use of aerated carts for organics collection.
New landfill site construction	One site that includes a mixed waste processing facility designed to handle 119,000 tonnes/year of MSW; a 13 channel agitated bed composting system to process the mixed waste after recyclables are removed; and a landfill for stabilized waste. HRM owns these facilities. However, the design, building and operation of these facilities are the responsibility of Mirror Nova Scotia.
Composting facilities improvement	Two separate composting facilities with total processing capacity of 50,000 tonnes/year. Both facilities are privately owned and operated, each with put or pay guarantees by HRM of 20,000 tonnes/year.
Materials recovery facility expansion	Expansion of an existing materials recovery facility.

Source: Goldstein and Gray, 1999

5.1.1. Waste Generation

In 2008, Canadians produced over 1,031 kilograms of residential waste per person, virtually the same per capita production as in 2006 (Statistics Canada, 2008). In April 2006, Natural Resources Canada and Environment Canada summarized the waste generation data for year 2002 and part of the data is shown in Table 5.2. Comparing to other provinces, Nova Scotia has the lowest waste generation rate across Canada.

Table 5.2 Waste generation in Canada, 2002

Province/Territory	Population	Residential generation (tonnes)	Generation rate (kg/capita)
Newfoundland and Labrador	519,270	231,291	445
Prince Edward Island	136,998	--	--
Nova Scotia	934,392	252,012	270
New Brunswick	750,183	256,190	342
Quebec	7,443,491	3,471,000	466
Ontario	12,096,627	4,388,239	363
Manitoba	1,155,492	494,535	428
Saskatchewan	995,490	321,069	323
Alberta	3,114,390	1,159,697	372
British Columbia	4,114,981	1,354,177	329
Yukon Territory, Northwest Territories and Nunavut	111,297	--	--
Canada Total	31,361,611	12,008,338	382

Source: Natural Resources Canada and Environment Canada, 2006.

As HRM is the largest and the most developed city in Nova Scotia, the municipal waste generation in HRM would be higher than the average rate in Nova Scotia. Here for HRM, the MSW includes both residential and commercial solid wastes. By the latest approachable statistic data provided by Solid Waste Resources of HRM, the solid waste generation and collection data are shown in Table 5.3. A total of 373,989 tonnes of solid waste were generated from April 1st 2009 to March 31st 2010, in which 34.7% were from residential sources and 65.3% were from commercial sources. Among all the wastes, refuse, organics, recycling and drop-off materials accounts for 59.9% of total wastes and were collected and disposed by HRM. All the other wastes including fibers private recycling, backyard composting, construction and demolition (C&D) waste, and household hazardous waste (HHW) were collected and disposed by in-person or private companies (HRM By-Law S600, 2007; HRM By-Law L200, 2002), and these wastes are not included

in the planning. As for the ratio of 59.9%, in this study, the ratio of total wastes being disposed by HRM is estimated to be a little lower with a range of [57%, 58.5%] due to the existence of waste residues which are not collected and disposed by HRM.

Table 5.3 Yearly waste generation in HRM from April 2009 to March 2010

Waste category	Residential waste (tonne)	Commercial waste (tonne)	Total waste (tonne)	Residential Ratio of the total
Refuse	63,703	85,537	149,240	39.9%
Organics	35,806	16,291	52,097	13.9%
Recycling	17,420	5,312	22,732	6.1%
Fibers private recycling (Est)	N/A	43,000	43,000	11.5%
Backyard composting (Est)	5,000	N/A	5,000	1.3%
Drop-off materials (Est)	7,500	N/A	7,500	2.0%
C&D	N/A	93,920	93,920	25.1%
HHW(Est)	500	N/A	500	0.1%
Totals	129,929	244,060	373,989	100.0%
Diversion rate	50.97%	64.95%	60%	--

Source: HRM Solid Waste Resources, 2010c

Total waste generation is directly related to the population of a city. In 2006, the population of HRM has reached 372,679 (2006 census, Statistics Canada) and spreads 2,224 square miles and ranges from high-density urban settings to rural communities (HRM, 2010a). Comparing to the population in 2001, which was 359,111, the population in HRM has increases 3.8% in 5 years. If HRM keeps this population growth rate, the population will be around 386,760 in 2010. According to the waste generation data (373,989 tonnes in 2010), the waste generation rate would be 0.967 tonne per capita per year. In this study, an interval waste generation rate with a range of [0.95, 0.98] tonne per capita per year was used to reflect its uncertain feature. Based on available information, the population and waste

generation data in HRM for the next 30 years can be estimated and are provided in Table 5.4.

Table 5.4 Estimate of population and waste generation in HRM for the planning horizon

Year	Population	Waste generation(tonne/year)		Waste generation(tonne/5years)	
		lower level	upper level	lower level	upper level
2001	359111	341155	351929	--	--
2006	372679	354045	365225	--	--
2011	386760	367422	379024		
2012	389682	370198	381888		
2013	392626	372995	384774	1865081	1923978
2014	395593	375813	387681		
2015	398582	378653	390611		
2016	401594	381514	393562		
2017	404628	384397	396536		
2018	407686	387301	399532	1936617	1997773
2019	410766	390228	402551		
2020	413870	393176	405593		
2021	416997	396147	408657		
2022	420148	399141	411745		
2023	423323	402156	414856	2010896	2074398
2024	426521	405195	417991		
2025	429744	408257	421149		
2026	432991	411342	424331		
2027	436263	414450	427538		
2028	439559	417581	430768	2088025	2153962
2029	442881	420737	434023		
2030	446227	423916	437302		
2031	449599	427119	440607		
2032	452996	430346	443936		
2033	456419	433598	447290	2168111	2236578
2034	459867	436874	450670		
2035	463342	440175	454075		
2036	466843	443501	457506		
2037	470371	446852	460963		
2038	473925	450228	464446	2251270	2322363
2039	477506	453630	467956		
2040	481114	457058	471491		

5.1.2. Recycling

Recycling turns materials that would otherwise become waste into valuable resources.

Many benefits (USEPA, 2008) can be expected from recycling: it saves energy; it reduces the need for landfilling and incineration; it prevents pollution caused by the manufacturing of products from virgin materials; it decreases emissions of greenhouse gases; it conserves natural resources such as timber, water, and minerals; it helps sustain the environment for future generations; it can also protect and expand employment opportunities.

Many recycling programs (HRM, 2007b; HRM, 2011b) are being implemented in HRM. They include Blue Bag Program, Paper Recycling Program, Corrugated Cardboard Program, and some other provincial programs like Paint Recycling Program, Household Hazardous Waste (HHW) program, and Used Tire Management Program, Derelict Vehicle Program, and Safe Sharps Bring-Back Program.

Blue Bag program requests residents to put recyclables into a clear blue bag for curbside collection. The recyclables in the blue bag include plastic bottles and containers, all plastic bags used for grocery, retail, bread, dry cleaning and frozen food, and bubble wrap, glass bottles and jars, steel and aluminum cans, clean aluminum foil and plates, paper milk cartons, mini sips and tetra juice packages.

Paper recyclables include dry and clean paper, newspapers and flyers, glossy magazines and catalogues, envelopes, paper egg cartons, paperbacks and phone books. These paper recyclables should be placed in a grocery bag, retail carry-out bag or a clear bag, and be kept separately from Blue Bag recyclables.

Corrugated cardboard program encourages people to fold boxes flat and tie them in bundles no more than 2 ft x 3 ft x 8 inches in size. Corrugated cardboard is “waffled” between the layers such as appliance boxes and pizza boxes. And the bundles should be placed beside the blue bag.

Nova Scotians purchase more than 3 million containers of paint every year, and up to 25 % of this paint is not used after the purchase (NS Environment and Labour, 2003). In the past, most of this paint was either burned in dumps or buried in landfills. The recent paint-recycling program recovers thousands of liters of paint and paint cans. It allows consumers to return surplus paint to any one of the province's 85 recycling depots at no charge. It applies to all latex, oil and solvent-based paints, including aerosol paint cans, but does not apply to specially formulate industrial, automotive or marine coatings. New recycled paints will be manufactured from the recovered waste paint.

By February 2009, the provincial government has banned most common electronic equipment from landfills. The banned electronic equipments include televisions, computers, audio and video playback and recording systems, telephones (corded and cordless), fax and answering machines, computer scanners, cell Phones and other wireless devices. Since the ban, HRM has encouraged its residents to bring their old electronic equipment to their local Atlantic Canada Electronics Stewardship (ACES) recycling centre (HRM, 2010d).

Currently, there is one recycling facility in operation in HRM, located at 50 Chain Lake Drive, with a capacity of 28,000 tonnes per year. The Materials Recycling Facility (MRF) of HRM processes all blue bag and fiber (paper and cardboard) collected in residential curbside program and the small amount of material deposited in the onsite public drop off bins (HRM, 2011a). Recyclable materials include newspapers, fiber, cardboard, glass and steel containers, aluminum etc (see Table 5.4). Besides that, 22 recycling centers are separately located around HRM opening 7 days a week to collect recyclables (RRFB, 2011). Residents can send their used bottles or cans to the collection center and get the deposit.

As given in Table 5.2, 6.1% of generated wastes were recycled by the recycling facilities in HRM and 11.5% were recycled by private recycling companies. Two recycling streams contributed a 20% of recycle rate in HRM. According to the documented report of USEPA (USEPA, 2008), the availability of recyclables could be as high as 35% of the total solid

wastes generated. So in this study, the ratio of recyclables is bounded between 6% and 35%.

5.1.3. Composting

Composting is the natural breakdown of organic materials by living organisms, including bacteria, fungi, worms and small insects. Any material from a living source, plant or animal, is called "organic". The end product is a dark, earthy, soil-like substance called compost. Compost has its market value because it can be used as a soil amendment or as a medium to grow plants. It serves as a marketable commodity and is a low-cost alternative to standard landfill cover and artificial soil amendments. Composting also extends municipal landfill life by diverting organic materials from landfills and provides a less costly alternative to conventional methods of remediating (cleaning) contaminated soil (USEPA, 1994).

Presently, two composting facilities are serving HRM: the New Era Farm composting facility and the Miller composting facility, with a total capacity of 50,000 tonnes/year. The New Era Farms compost facility (Friesen, 1999; Friesen, 2000), located in 61 Evergreen Place, Ragged Lake, has a capacity of 25,000 tonnes/year. It was sized to allow expansion by an additional 10,000 tonnes. It has three main areas: a receiving and preprocessing building, a composting pad and a curing structure. The Miller Composting (Friesen, 1999; Friesen, 2000) utilizes the Ebara technology to compost waste organics. Same as the New Era, the Miller site has a capacity of 25,000 tonnes/year. It is located at 80 Gloria McClusky Avenue, Burnside, Nova Scotia's largest business park. The building footprint is 55,000 sq. ft., set on a 20 acre lot. The property could accommodate a second plant directly beside the original one if needed in the future. The main building has three areas: receiving and preprocessing, composting and curing.

From April 1st 2009 to March 31st 2010, 52,097 tonnes of organics were composted by the two composting facilities in HRM, in which, 35,806 tonnes were from residential sources and 16,291 tonnes were from commercial sources. It diverted 13.9% of total waste

generated from landfill. This percentage could be improved to about 26%. As documented in US Environmental Protection Agency (USEPA, 1994), yard trimmings and food residuals together constitute 26 percent of municipal solid waste stream. The composting wastes are bounded as 12% to 26% of total generated wastes in this study.

Backyard composting (HRM, 2011) is also promoted in HRM. Backyard or onsite composting can be conducted by residents and other small-quantity generators of organic waste on their own property. By composting these materials onsite, select businesses can significantly reduce the amount of waste that needs to be disposed of and thereby save money from avoiding disposal costs, and home owners can generate natural fertilizer that can be applied to lawns and gardens to help condition the soil and replenish nutrients. The household practicing backyard composting diverts approximately 5,000 to 7,5000 tonnes per year of wastes from landfill and it contributed 12.25% of total organic material generated in HRM in 2010 (see Table 5.2).

5.1.4. Landfill

Currently, the Otter Lake Landfill is the only landfill being operated in HRM. It opened for full operations in 1999 to replace the closed Sackville Landfill, with a nominal capacity of 150,000 tonnes/year and a designed life time of 25 years from year 1998 to 2023. The site is 200 acres in size and employs over 100 workers. This facility includes three major components:

(1) Front End Processor (FEP) – This is the first stage of Otter Lake Facility where garbage arrives and bags are opened and inspected. It consists of a system of conveyors, bag breaker, sorting platforms and mechanical screening operations. The FEP allows for identification and removal of material that should not be going to landfill. Clean recyclable paper, metals and containers are removed during the sorting process. Scrap metal (i.e., appliances) is separated for recycling. By FEP, the Otter Lake landfill makes some avenue from recyclable wastes. In the 2010 budget of solid waste resources in HRM, approximately \$80,000 revenue was received from the FEP.

(2) Waste Stabilization Facility (WSF) - Mixed wastes containing organic material leftover from the FEP is brought by conveyor and undergoes processing in the waste stabilization facility. The process is much the same as composting, where after a period of 18 to 21 days, the materials leaving this process is a dry-like fluff which then goes to the landfill. This process could significantly reduce not only the volume of wastes entering the landfill cells (about 40%), but also the strength of the landfill leachate since a large amount of the organics have been degraded in this process.

(3) Residual Disposal Facility (RDF) - This is the place to landfill or bury the residues from the WSF process. The landfill contains a liner system, a cover system, a leak detection system, a leachate and gas management system, surface drainage control and environmental monitoring controls. In the Otter Lake landfill, 9 cells were designed 4 of them have been filled before 2009. Table 5.5 presents the capacities of unfilled cells. Cell 5 was started to use from January 2009. The original capacity is 546,000 tonnes and it is estimated that an approximate 167,500 tonnes of garbage has been placed in Cell 5 by March 31st, 2010. The total capacity of Otter Lake Landfill by March 31st 2010 is 2,421,500 tonnes.

Table 5.5 Capacity of the Otter Lake Landfill by March 31, 2010.

Cell #	Capacity unfilled
Cell 5	378,500
Cell 6	503,000
Cell 7	520,000
Cell 8	520,000
Cell 9	500,000
Total	2,589,000

5.1.5. Incinerator

Incineration, as a type of thermal treatment, is recognized as an effective and environmentally sound disposal method for a wide range of wastes. In the early 1990s when the Sackville Landfill was constructed and still in operation was causing severe environmental damages to its nearby communities. The Municipality was then planning to build an incinerator close to the urban center of the region. However, the plan was eventually rejected due to various environmental and economic concerns by the Nova Scotia Ministry of Environment. There is no incinerator facility serving HRM.

5.1.6. Transfer Station

Waste transfer stations are facilities where municipal solid wastes are unloaded from the collection vehicles and briefly held before they are reloaded onto larger long-distance transport vehicles for shipment to landfills or other treatment or disposal facilities (USEPA, 2002). The HRM used to own a solid waste transfer station in Lady Hammond Road for the past few decades. But it has been closed since May 2000 (HRM, 2000). According to HRM's Integrated Solid Waste/Resource Management System, HRM no longer needs a solid waste transfer station. Waste materials could be collected for a direct shipment to the Otter Lake landfill and be separated, sorted and processed there as well (HRM, 1999).

5.2. Model Input Data

5.2.1. Discount Factor

Since the planning problem under consideration includes long multiple planning periods, discount factors have to be considered for each planning period to obtain the total present value for the objective function. The discount factor (DF) is defined as the coefficient which a future dollar should multiply by to be converted to the present value. The discount factor could be calculated through the following equation:

$$DF = \left(\frac{1}{(1+r)} \right)^t$$

Where r is a fixed discount rate and is determined by the interest rate and inflation rate. It is a fixed discount rate and is determined by the interest rate and inflation rate. t is the time factor. In this study, a discount rate of 3.18% was used for the discount factor calculation, based on the predictions of Canada's inflation and interest rates in the coming years (IndexMundi, 2010; Scotia Bank, 2011).

In this study, the 30-year long planning horizon is divided into 6 planning period, with 5 years in each period. All the costs and revenues in the coming planning years are calculated as 2011 dollars. The discount factors for each year and period average are calculated using the above equation and are provided in Table 5.6. For the net present value of operation and transportation costs, an average discount factor is chosen for each time period. For the capital costs used for facility development and expansion, it is assumed that, if the MSW management system requires additional capacity at the beginning of a particular period, the development or expansion of that facility has to be completed by the end of the previous period. Thus, the discount factor for facility development and expansion would be the regular discount factor from the previous period. In addition, it is assumed that all garbage streams can be handled by the existing facilities in the first planning period, and no new facilities are needed.

Table 5.6 Discount factors

Period	Year	Discount factor	period average DF
1	2011	0.969	0.911
	2012	0.939	
	2013	0.910	
	2014	0.882	
	2015	0.855	
2	2016	0.829	0.779
	2017	0.803	
	2018	0.778	
	2019	0.754	
	2020	0.731	
3	2021	0.709	0.666
	2022	0.687	
	2023	0.666	
	2024	0.645	
	2025	0.625	
4	2026	0.606	0.570
	2027	0.587	
	2028	0.569	
	2029	0.552	
	2030	0.535	
5	2031	0.518	0.487
	2032	0.502	
	2033	0.487	
	2034	0.472	
	2035	0.457	
6	2036	0.443	0.417
	2037	0.429	
	2038	0.416	
	2039	0.403	
	2040	0.391	

5.2.2. Waste Collection and Transportation Cost

HRM serves single family households, row houses, duplexes, semi-detached, small apartments (up to six units) with curbside collection, to collect the waste. MSW collection

in HRM is equipped to serve about 120,500 households, including 5,500 condominiums (HRM, 2002).

According to HRM Solid Waste Resource Collection and Disposal By-law (By-law No. S600, 2007), residents in HRM are required to “source-separate all collectible waste generated from eligible premises at the point of generation so as to comply with the provincial disposal bans and to facilitate their recycling, composting or disposal in accordance with the Municipality’s waste resource management system” (HRM, 2007c).

After the waste source separation, organics and trash are collected biweekly and recyclables are collected weekly (biweekly in the rural areas of the county). Aerated carts are used for organics collection. Eight collection zones were created for the purpose of waste collection (from 25 before amalgamation). The zones and approximate number of the served households are presented in Table 5.7. Figure 5.1 shows the spatial distribution of the 8 collection areas in HRM.

Table 5.7 HRM waste collection areas and the number of the households served.

Area No.	Area	No. of the served households	
		(1998)	(2002)
1	Halifax	27,750	28,861
2	Dartmouth	19,806	20,815
3	Bedford, Hammond Plains, Pockwock & Area	7,355	9,230
4	Beechville, Lakeside, Timberlea, Prospect & West	12,104	13,668
5	Sackville, Shubenacadie Lakes & Area	17,712	19,387
6	Cole Harbour, Westphal, Eastern Passage & Area	11,361	12,637
7	North and East Preston, Lake Major, Lake Loon, Cherrybrook, Lawrencetown & Area	6,507	7,227
8	Elderbank, Musquodoboit & all Eastern Shore	6,758	7,163
	Total	109,353	188,988

Source: HRM solid waste resources, 2002

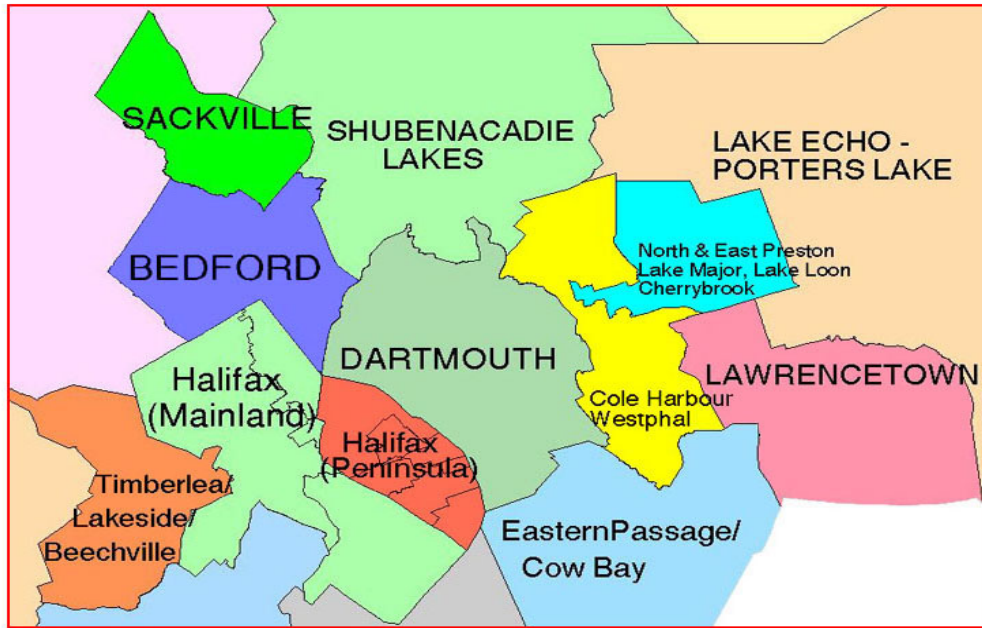


Figure 5.1 HRM waste collection areas

Waste collection and transportation cost is usually estimated based on the vehicle type and the worker's salary. Mulholland has calculated the cost in 1997 as shown in Table 5.8. In HRM, since most wastes are collected by automated truck with cart lifter and packer, the collection and transportation cost would be around [21, 37] dollar per tonne in 1997, which is around [32, 56] dollar per tonne currently.

Table 5.8 Waste collection and transportation data

Waste collection type	Automated
Salary per hour (\$/h)	24.87
Cost of vehicle per hour (\$/h)	27
Volumetric capacity (m ³)	22
length of working day (h)	8
Daily loads (tonnes)	[12 20]
Total collection and transportation costs (\$/tonne)	[21 37]

Based on the 2010 Solid Waste Resources budget provided by HRM, the yearly collection and transportation cost was about 13 million dollars and the breakdown of the collection cost for garbage, organics and recyclables are provided in Table 5.9. The average collection cost for garbage, organics and recyclables could then be calculated, as presented in Table 5.9. Since the collection and transportation cost is an uncertain parameter affected by many factors, the intervals are assigned to them based on the calculated average for reflecting the uncertainties. In this study, the collection and transportation costs for garbage, organics, and recyclables are [32, 35], [85, 90], and [170, 180] (\$ per tonne), respectively.

Table 5.9 Waste collection and transportation costs for different types of wastes

Cost breakdown	Annual cost (\$)	Annual amount (tonnes)	Average cost (\$/tonne)
Garbage	4981455.24	149240	33.38
Organics	4498828.87	52097	86.35
Recyclables	3977536.78	22732	174.98

Source: 2010 HRM solid waste resources budget

5.2.3. Waste Facility Operating Cost

In 2010, HRM had a total of operating cost of 31 million dollars for running MSW management facilities, in which 67.7% were allocated to for landfill, 24.3% for organic composting, and 8.1% for recycling facilities. The detailed breakdown operating costs for different facilities were provided in Table 5.10. As indicated in the table, the average operating cost for each facility could be calculated as \$141.25, \$145.08, and \$110.98 per tonne for landfill, composting and recycling facilities, respectively. In this study, interval operating costs are used to account for their uncertainties, and they are [140, 144], [142, 147], and [108, 113] (\$/tonne) for landfill, composting and recycling facilities, respectively.

Table 5.10 Operating cost of different facilities in HRM

Facility	Annual total (\$)	Annual amount (tonnes)	Average cost (\$/tonne)
Landfill	21,080,700	149240	141.25
Composting	7,558,000	52097	145.08
Recycling	2,522,700	22732	110.98

Source: 2010 HRM solid waste resources budget

5.2.4. Revenues

Generally speaking, recycling and composting collections can generate substantial revenues. Recyclables could be sold directly to some companies and compost has the market value because it could be used for gardening. In HRM, not only the recyclable and organics, but also the garbage being trucked to landfill, could generate revenues, simply because the Front End Processor (FEP) of the Otter Lake landfill sort out clean recyclable paper, metals and containers which can be sold to the market. In 2010, there were \$8,000 generated the landfill FEP recyclables, and over 1.2 million dollars generated by composting and recycling facilities (Table 5.11). As indicated in Table 5.11, the average revenues from different facility are 0.05 \$/tonne, 23.09 \$/tonne, and 54.80 \$/tonne for landfill, composting and recycling facilities, respectively. In this study, the revenue coefficients were also assigned as intervals, with the ranges of [0, 0.1], [22, 25], and [52, 57] \$/tonne for landfill, composting and recycling facilities, respectively. Table 5.12 presents the relevant cost and revenue coefficients used in the model. Table 5.13 presents the estimated costs and revenues for the 6 planning periods. Costs are getting less with periods since the discount factors are taken into account.

Table 5.11 Revenues estimation for waste management facilities

Revenue breakdown	Yearly revenue (\$)	Yearly tonnage (tonne)	Average revenue (\$/tonne)
Garbage	8,000	149240	0.05
Organics	1,203,000	52097	23.09
Recyclables	1,245,769	22732	54.80

Source: 2010 HRM solid waste resources budget

Table 5.12 Costs and revenues estimation

Cost(\$/tonne)	Collection	Operation	Revenue
Landfill	[32,35]	[140,144]	[0.03,0.07]
Composting	[85,90]	[142,147]	[22,25]
Recycling	[170,180]	[108,113]	[52,57]

Table 5.13 Transportation cost, operation cost, and revenue for waste management facilities

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Collection and transportation costs (\$/tonne)						
Landfill	[32, 35]	[31, 33.9]	[30.1, 32.9]	[29.1, 31.8]	[28.2, 30.9]	[27.3, 29.9]
Composting	[85, 90]	[82.4, 87.2]	[79.8, 84.5]	[77.3, 81.9]	[74.9, 79.3]	[72.6, 76.9]
Recycling	[170, 180]	[164.7, 174.4]	[159.6, 169.0]	[154.7, 163.8]	[149.9, 158.7]	[145.2, 153.8]
Operating costs (\$/tonne)						
Landfill	[140, 144]	[135.7, 139.5]	[131.5, 135.2]	[127.4, 131.0]	[123.4, 127.0]	[119.6, 123.0]
Composting	[142, 147]	[137.6, 142.4]	[133.3, 138.0]	[129.2, 133.7]	[125.2, 129.6]	[121.3, 125.6]
Recycling	[108, 113]	[104.7, 109.5]	[101.4, 106.1]	[98.3, 102.8]	[95.2, 99.6]	[92.3, 96.5]
Revenues (\$/tonne)						
Landfill	[0.03, 0.07]	[0.03, 0.07]	[0.03, 0.07]	[0.03, 0.06]	[0.03, 0.06]	[0.03, 0.06]
Composting	[22, 25]	[21.3, 24.2]	[20.7, 23.5]	[20.0, 22.7]	[19.4, 22.0]	[18.8, 21.4]
Recycling	[52, 57]	[50.4, 55.2]	[48.8, 53.5]	[47.3, 51.9]	[45.8, 50.3]	[44.4, 48.7]

5.2.5. Capital Costs for Facility Expansion and Development

Along with the growth of population and economy in HRM, the waste generation will keep increasing in the coming years. Before the total amount of wastes generated reaches beyond the capacity of existing facilities, HRM needs the capital investment to expand the existing facilities or develop new facilities.

The total capital cost for the Otter Lake landfill was 44 million Canadian dollars in 1998, which equals 66.1 million 2010 dollars. The landfill was designed with a nominal capacity of 150,000 tonnes/year till 2023 (that equals a total capacity of 3,750,000 tonnes). In this study, the landfill is allowed to expand only once for the entire 30-year planning horizon, with a same capital cost of 66.1 million dollars and a same capacity of 3,750,000 tonnes.

A survey conducted by Chi and Huang (1998) lists the number of different capacity composting facilities in operation in each province of Canada, and the capital costs for building them were also estimated (see Table 5.14). A small capacity facility has a composting capacity less than 5000 tonne/year, and it costs from 595,000 dollars to 980,000 dollars; a medium capacity facility has a composting capacity between 5000-25000 tonne/year and costs from 1 to 6 million dollars; and a large capacity facility has its composting capacity larger than 25,000 tonne/year and costs over 15 million dollars. In this study, three capacity options were selected for possible composting expansion, including 10,000 tonnes/year, 15,000 tonnes/year, and 25,000 tonnes/year. The estimated costs for their expansions are presented in Table 5.15, and they are also converted into 2010 dollars.

Table 5.14 The number of composting facilities in each province of Canada and estimated capital costs

Province	Total composting facilities	Number of facilities falling in the range of		
		0-5,000 tonne/yr	5,001-25,000 tonne/yr	25,001+ tonne/yr
Newfoundland	1	1		
PEI	2	2		
Nova Scotia	2	1	1	
New Brunswick	1	1		
Quebec	19	18	1	
Ontario	37	26		2
Manitoba	6	6		
Saskatchewan	2	2		
Alberta	8	7	1	
British Columbia	7	4	3	
NWT	-	-	-	-
Yukon	1	1		
Total	86	69	15	2
Low estimated capital costs		\$595,000	\$1,000,000	\$15,800,000
Medium estimated capital costs		\$787,500	\$2,000,000	\$18,850,000
High estimated capital costs		\$980,000	\$6,000,000	\$21,900,000

Source: Chi and Huang, 1998.

Table 5.15 Composting facility expansion options and the estimated costs

Options	Capacity (tonnes/yr)	Cost in 1998 (M\$)		Cost in 2010(M\$)	
		Lower bound	Upper bound	Lower bound	Upper bound
Recent	50000	-	-	-	-
Option1	10000	2	2.4	3.0	3.7
Option2	15000	3.5	4	5.3	6.1
Option3	25000	5	6	7.5	9.2

Chi and Huang (1998) also gave the number of recycling facilities which were in operation in each province of Canada and their treatment capacities as well as an estimated capital cost for building them in terms of per tonne. These data are presented in Table 5.16. In this study, three possible options were considered for recycling facility expansion. They are 10,000 tonnes/year, 20,000 tonnes/year, and 30,000 tonnes/year. The interval capital costs are [3.6, 4.4], [7.2, 8.8], [10.8, 13.1] million dollars, respectively.

Table 5.16 MRF facilities in each province of Canada and their annual capacities

Province	Number of MRF facilities	Amount of recyclables processed annually (tonne/year)		
		Residential	IC&I	C&D
Newfoundland	2	2,872	25,655	330
PEI	1	1,896	11,490	0
Nova Scotia	11	9,074	48,829	0
New Brunswick	7	3,876	30,913	0
Quebec	15	227,806	1,350,194	0
Ontario	51	478,890	1,366,441	705,791
Manitoba	9	3,514	63,568	0
Saskatchewan	3	21,400	79,034	0
Alberta	5	17,597	150,630	56,190
British Columbia	39	100,798	411,836	508,188
NWT	1	51	2,211	991
Yukon	1	213	1,311	485
Total	145	867,987	3,542,112	1,271,975
Low estimated capital costs(\$/tonne)		\$238.00	\$238.00	\$238.00
Medium estimated capital costs(\$/tonne)		\$264	\$264	\$264
High estimated capital costs(\$/tonne)		\$291	\$291	\$291

Source: Chi and Huang, 1998.

Table 5.17 Recycling facility expansion options and the estimated costs

Options	Capacity (tonnes/yr)	Capital cost estimated (\$)	
		Lower Bound	Upper Bound
Recent	28000	-	-
Option 1	10000	3584000	4382000
Option 2	20000	7168000	8764000
Option 3	30000	10752000	13146000

Based on the collected data, Table 5.18 presents the details of waste management facility expansion options considered for HRM, their capacities, and changes of their cost ranges

with planning period. It is noted that the capital costs become gradually less with period since the discount factors are taken into consideration.

Table 5.18 Capital cost for waste management facilities

Facility	Option	Capacity	Capital cost in different periods (10 ⁶ \$)					
			Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Landfill	Option 1	3,750,000 tonnes	[66, 70]	[64, 67.8]	[62.0, 65.7]	[60.0, 63.7]	[58.2, 61.7]	[56.4, 59.8]
	Option 1	10,000 tonnes/yr	[3, 3.7]	[2.9, 3.6]	[2.8, 3.5]	[2.7, 3.4]	[2.6, 3.3]	[2.6, 3.2]
Composting	Option 2	15,000 tonnes/yr	[5.3, 6.1]	[5.1, 5.9]	[5.0, 5.7]	[4.8, 5.6]	[4.7, 5.4]	[4.5, 5.2]
	Option 3	25,000 tonnes/yr	[7.5, 9.2]	[7.3, 8.9]	[7.0, 8.6]	[6.8, 8.4]	[6.6, 8.1]	[6.4, 7.9]
Recycling	Option 1	10,000 tonnes/yr	[3.6, 4.4]	[3.5, 4.3]	[3.4, 4.1]	[3.3, 4.0]	[3.2, 3.9]	[3.1, 3.8]
	Option 2	20,000 tonnes/yr	[7.2, 8.8]	[7.0, 8.5]	[6.8, 8.3]	[6.6, 8.0]	[6.3, 7.8]	[6.2, 7.5]
	Option 3	30,000 tonnes/yr	[10.8, 13.1]	[10.5, 12.7]	[10.1, 12.3]	[9.8, 11.9]	[9.5, 11.5]	[9.2, 11.2]

5.2.6. Residue Rate

In HRM, the approximate residue rates for the recycling and composting facilities are estimated to be 8% of the incoming tonnage of the wastes, and the residues from recycling and composting facilities need to be landfilled.

5.3. Model Development for MSW System in HRM

As discussed above, three components exist in the MSW management system in HRM. They are waste generation district, waste processing facilities, and landfill. The waste allocation flow chart is shown in Figure 5.2. Wastes are generated from each district, and collected and transported to waste processing facilities and landfill. Recyclable and organic wastes will be recovered or composed and sold to the market. Waste residues will be sent to landfill. The HRM-MSW model uses the waste allocation flows as the decision variables, with an objective of minimizing total system costs.

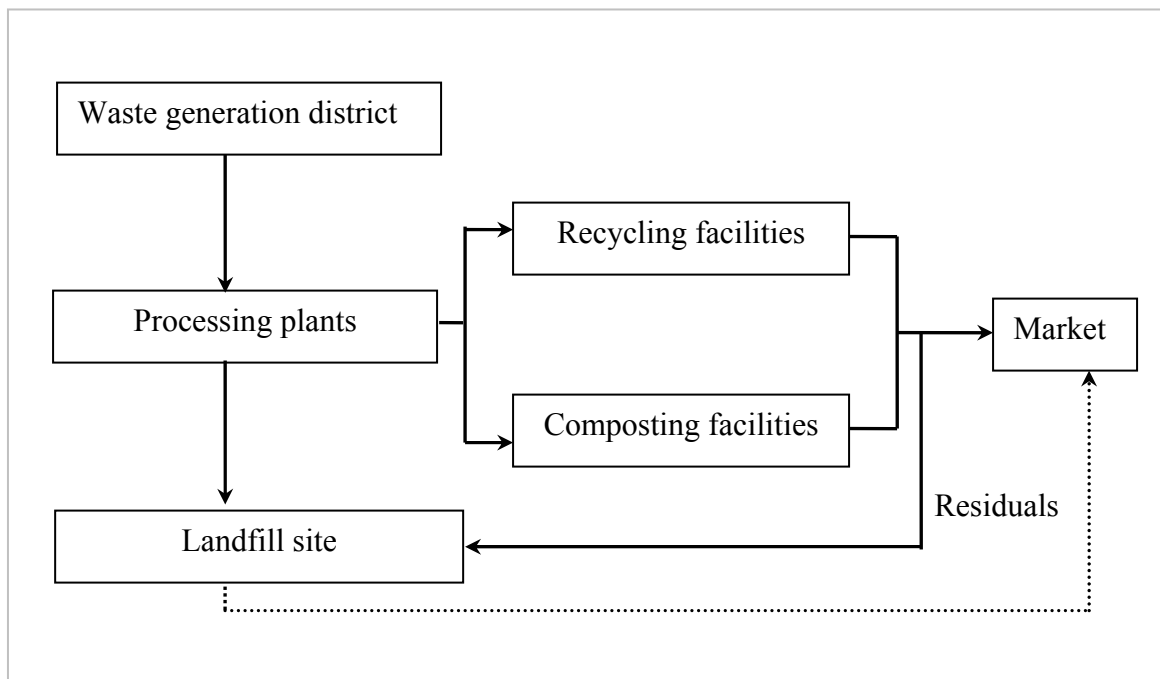


Figure 5.2 The waste allocation flow chart

5.3.1. Objective Function

The original objective of this model is to minimize the total system cost. Total costs are determined by collection and transportation cost, waste facilities operating cost, capital cost and the revenues. The residual market values of facilities are not considered in this study. The decision variables, denoted as X_{it} , represent the waste flow pattern from HRM to different facility i in different planning period t . The objective function is given as follows:

$$\text{Minimize Total Cost} = \text{(I)} + \text{(II)} + \text{(III)} - \text{(IV)} \quad (5.3.1)$$

Where, (I) is the waste collection and transportation cost;

(II) is the facility operating cost;

(III) is the facility expansion cost; and

(IV) is the revenues from three waste disposal facilities.

(I) Collection and Transportation Cost

$$\sum_{t=1}^6 \sum_{i=1}^3 X_{it} DF_t UC_i \quad (5.3.2)$$

$$+ UC_3 RR * \sum_{t=1}^6 \sum_{i=1}^2 X_{it} DF_t \quad (5.3.3)$$

Where, (5.3.2) is the total collection and transportation cost for wastes allocated to facility i in the entire planning horizon. (5.3.3) is the collection and transportation cost for the residues generated in recycling facility and composting facility to landfill.

X_{it} is the waste flow allocated to the facility i in period t (tonnes/5 years). i represents different waste processing facilities: $i=1$ for recycling facility, $i=2$ for composting facility, and $i=3$ for landfill. t is the time period, $t=1,2,\dots,6$. DF_t is the discount factor in period t . UC_i is the unit collection and transportation cost for facility i (\$/tonne). RR is the residue generation rate from both recycling and composting facilities, which is estimated as 8% in this study.

(II) Operating Cost

$$\sum_{t=1}^6 \sum_{i=1}^3 X_{it} DF_t UO_i \quad (5.3.4)$$

$$+ UO_3 RR * \sum_{t=1}^6 \sum_{i=1}^2 X_{it} DF_t \quad (5.3.5)$$

Where, (5.3.4) is the total operating cost at all three facilities. Equation (5.3.5) is the operating cost at landfill for the residues from recycling and composting facilities. UO_i is the unit operating cost for the facility i (\$/tonne).

(III) Capital Cost

$$\sum_{t=1}^6 \sum_{i=1}^3 \sum_{k=1}^3 Y_{ikt} DF_{t-1} CA_{ik} \quad (5.3.6)$$

If a facility needs to be expanded to satisfy the wastes disposal needs in period t , it is assumed that the expansion would be completed in the previous period, which is period $t-1$. Thus the discount factor DF_{t-1} is used in (5.3.6). CA_{ik} is the capital cost (expansion cost) for facility i under expansion option k (\$). Three options ($k=1, 2, 3$) are available for either recycling or composting facility, and only one option is considered for landfill expansion ($CA_{32} = 0, CA_{33} = 0$). Y_{ikt} is a binary variable; $Y_{ikt} = 1$ when facility i (with capacity option k) needs to be developed in period t , and $Y_{ikt} = 0$ when there is no expansion.

(IV) Revenue

$$\sum_{t=1}^6 \sum_{i=1}^3 X_{it} DF_t UR_i \quad (5.3.7)$$

Where, UR_i is the unit revenue from facility i (\$/tonne).

5.3.2. Constraints

(1) Mass Balance Constraints

$$\sum_{i=1}^3 X_{it} \geq HWG_t, \quad \forall t \quad (5.3.8)$$

For any period t , the wastes allocated to all facilities should be equal to or greater than the wastes disposed by HRM (denoted as HWG_t , tonnes/5 years).

(2) Diversion Rate Constraints

$$\frac{[RR*(X_{1t} + X_{2t}) + X_{3t}]}{TWG_t} \leq 40\%, \quad \forall t \quad (5.3.9)$$

For any period t , in order to ensure the waste diversion rate no less than 60%, the wastes allocated to landfill should be less than 40% of total wastes generated (TWG_t , tonnes/5 years).

(3) Capacity Limitation Constraints

$$X_{iT} \leq 5year * [CP_i + \sum_{k=1}^3 \sum_{t=2}^T Y_{ikt} EP_{ikt}], \quad \forall T; i = 1, 2. \quad (5.3.10)$$

$$RR*(X_{1T} + X_{2T}) + X_{3T} \leq CP_3 + \sum_{t=2}^T Y_{ikt} EP_{31t} - \sum_{t=1}^T [RR*(X_{1t} + X_{2t}) + X_{3(t-1)}], \quad \forall T. \quad (5.3.11)$$

Equation (5.3.10) is the capacity limitation constraints for recycling and composting facilities ($i=1, 2$). For a particular time period T ($T=1, 2, \dots, 6$), the wastes allocated to the recycling or composting facilities should be less than their respective capacity. CP_i is the current capacity of facility i (tonnes/year when $i=1$ and 2 ; tonnes when $i=3$); EP_{ikt} is the expanding capacity of option k (tonnes/year when $i=1$ and 2 ; tonnes when $i=3$). Since no

expansion will occur for period 1, t is thus from 2 to T . The value of $\sum_{k=1}^3 \sum_{t=2}^T Y_{ikt} EP_{ikt}$ will be 0 if $T=1$.

Constraint (5.3.11) represents the capacity constraint for landfill. For a particular period T , all the wastes being delivered to landfill, including the residues from other facilities, should be less than its available capacity. CP_3 is the existing capacity of landfill. $\sum_{t=2}^T Y_{ikt} EP_{31t}$ represents the expanded capacity of landfill from period 2 to period T , and equals to 0 when $T=1$. $\sum_{t=1}^T [RR^*(X_{1t} + X_{2t}) + X_{3(t-1)}]$ is the total wastes being delivered to landfill for final disposal from period 1 to period T .

(4) Technical and Other Constraints

$$X_{it} \geq 0, \quad \forall i, t. \quad (5.3.12)$$

$$\frac{X_{1t}}{TWG_t} = [6\%, 35\%], \quad \forall t \quad (5.3.13)$$

$$\frac{X_{2t}}{TWG_t} = [12\%, 26\%], \quad \forall t \quad (5.3.14)$$

$$\sum_{k=1}^3 Y_{ikt} \leq 1, \quad \forall i, t. \quad (5.3.15)$$

Equation (5.3.12) is the technical constraint. All the decision variables must be equal to or greater than 0. Equation (5.3.13) is the availability constraint for recyclables. (5.3.14) is the availability constraint for organic wastes. The last constraint (5.3.15) indicates that no more than one expansion will be allowed for one facility in one period.

CHAPTER 6

RESULTS AND DISCUSSIONS

Based on the modeling approach developed in Chapter 4, the HRM-WSM model was firstly solved by the REILP approach for 11 pre-assigned aspiration levels from 0 to 1 with a step of 0.1. The REILP solutions could provide valuable information for decision makers who understand the modeling approach and have specific aspiration levels for making their decisions. The model was then solved by the proposed FREILP approach. Aspiration levels representing the conservative, medium, and aggressive decision makers will be provided with three α -cuts, and the solutions under these aspiration levels will be generated and discussed. With the solution of FREILP, decision makers do not need to set a specific aspiration level for the model. They can simply from one based on their either conservative, medium, or aggressive preference.

6.1. REILP Results

For the REILP model, a set of solutions can be obtained under a specific aspiration level. Table 6.1 gives the optimal solution for the objective function under the aspiration level from 0 to 1, with a step of 0.1. It includes the total system cost as well as its breakdown among collection/transportation, operating and capital costs. Table 6.2 presents, under different aspiration levels, the values of risk function, total wastes generated, and the total waste flow allocated to different facilities with the 30-year planning horizon. When the aspiration level = 0, the risk function is equal to 0 with a highest total system cost of 978.58 million dollars. On the other end, when the aspiration level = 1, the risk function value is also the highest with a lowest total system cost of 873.02 million dollars.

Table 6.1 REILP solution for the objective function under different aspiration levels

Aspiration level	Total system cost (10 ⁶ \$)	Total collection cost (10 ⁶ \$)	Total operation cost (10 ⁶ \$)	Total capital cost (10 ⁶ \$)	Total revenue (10 ⁶ \$)
0	978.58	300.70	676.40	52.02	50.53
0.1	968.03	285.88	661.27	70.77	49.89
0.2	957.47	290.17	666.15	50.79	49.64
0.3	946.91	287.43	660.96	49.44	50.90
0.4	936.36	280.64	655.66	51.83	51.78
0.5	925.80	276.43	652.63	48.40	51.65
0.6	915.25	273.18	644.72	48.40	51.05
0.7	904.69	270.49	636.76	48.12	50.68
0.8	894.14	267.49	628.72	48.12	50.20
0.9	883.58	265.67	621.36	46.69	50.14
1	873.02	259.24	618.50	45.95	50.66

Table 6.2 REILP wastes flow allocation and the values of risk function under different aspiration levels

Aspiration level	Risk function	Total waste generated (10 ⁶ tonne)	To landfill (tonne)	To recycling facility (tonne)	To composting facilities (tonne)
0	0.00	12.70	4875997	791001	1767737
0.1	0.06	12.70	4877258	771351	1771633
0.2	0.06	12.70	4877294	770894	1771633
0.3	0.07	12.70	4875997	791001	1767737
0.4	0.10	12.70	4875997	791001	1767737
0.5	0.14	12.70	4878411	766001	1762566
0.6	0.21	12.70	4849761	766001	1724359
0.7	0.29	12.70	4804006	767910	1689759
0.8	0.38	12.70	4745086	762104	1656749
0.9	0.48	12.68	4644856	760763	1653832
1	0.63	12.32	4745878	740125	1536398

Based on the solutions provided in Table 6.1 and 6.2, the relationship between the aspiration level, risk function and total system cost can be plotted in Figure 6.1. With the increase of aspiration level, the risk function is on the rise, while the total system cost decreases. It indicates that a lower system cost comes along with a higher decision risk which might violate the system constraints, while a safer decision needs a higher cost for investing facilities and management. Figure 6.1 also shows that the system cost and the aspiration level have a linear correlation. This is because that in FREILP, the constraint converted from the original objective function (minimized system cost) is proportionally subject to the aspiration level, as given in the equation: $f_{opt} = f_{opt}^- + \lambda_0(f_{opt}^+ - f_{opt}^-)$. The risk function is not linearly correlated with the aspiration level, as given by the following equations:

$$\xi = \sum_{i=1}^m \frac{2}{b_i^+ + b_i^-} [\sum_{j=1}^n \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-)] + \frac{2}{f_{opt}^+ + f_{opt}^-} [\sum_{j=1}^n \lambda_j (c_j^+ - c_j^-) x_j + \lambda_0 (f_{opt}^+ - f_{opt}^-)]$$

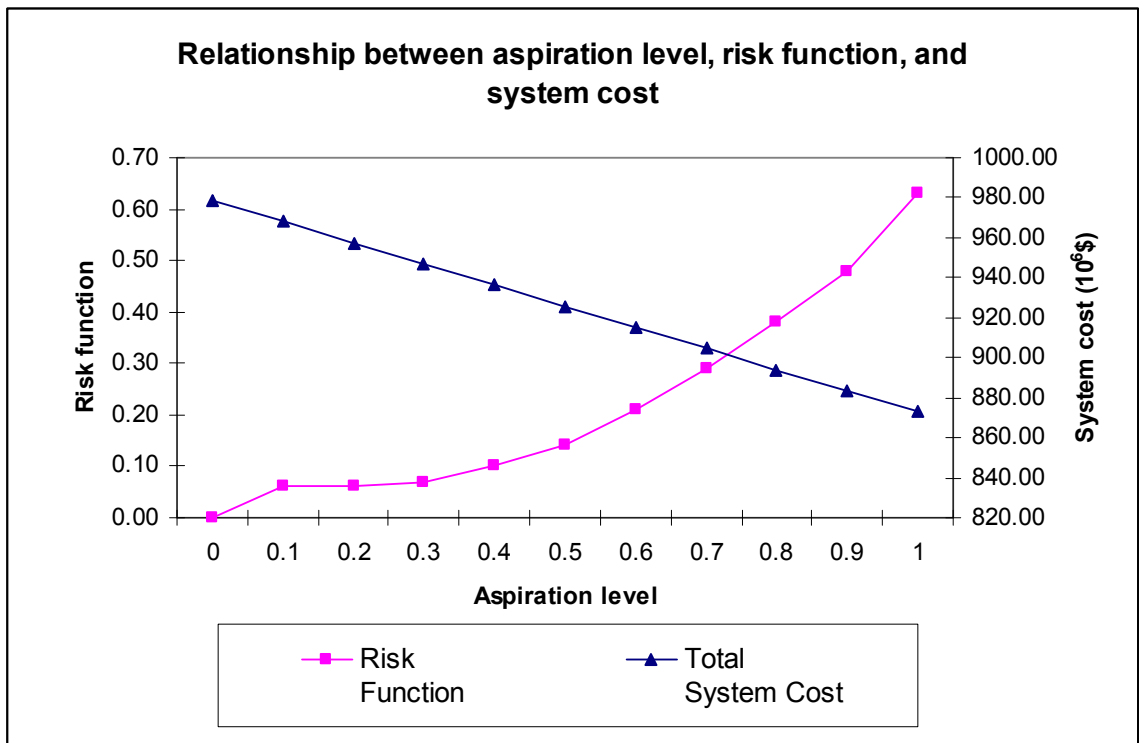


Figure 6.1 Relationship between aspiration level with risk function and system cost

Table 6. 3 Distribution of total system cost from the REILP model

	Expenditure or income (10 ⁶ \$)	Percentage (%)
Collection and transportation costs		
Landfill	[98.72, 111.38]	[9.6, 10.69]
Recycling	[78.77, 89.84]	[7.66, 8.53]
Composting	[136.57, 161.80]	[7.95, 8.85]
Operation costs		
Landfill	[431.89, 458.26]	[41.89, 46.76]
Recycling	[50.04, 56.40]	[4.86, 5.42]
Composting	[136.57, 161.80]	[13.27, 14.78]
Capital costs		
Landfill	[43.96, 46.62]	[4.27, 4.76]
Recycling	[0, 0]	[0, 0]
Composting	[2.00, 5.48]	[0.19, 0.22]
Gross system costs	[923.69, 1028.84]	[100, 100]
Revenue		
Landfill	[0.1, 0.2]	[0.18, 0.41]
Recycling	[25.95, 26.41]	[51.82, 51.96]
Composting	[24.04, 24.21]	[47.64, 48.01]
Gross system benefits	[50.08, 50.83]	[100, 100]
Net system costs	[873.02, 978.58]	

Table 6.3 shows the distribution of system costs among different facilities in intervals. They were formed based on the biggest and smallest values of each parameter obtained from all 11 event models under 11 aspiration levels. Among all the costs, the operating cost contributes the most percentage to the total system cost, and the operating cost of landfilling wastes accounts for over 40 percent of total system costs and is the highest one among all the costs. This is because the waste flow allocated to landfill is much more than

that allocated to the other facilities. Meanwhile, the revenues generated by the landfill are much lower than revenues of recycling and composting facilities. The capital cost for the recycling facility is 0, indicating that the capacity of the existing recycling facility in HRM can satisfy all the recyclable needs, and the facility does not need to be expanded in the next 30 years.

Wastes flow allocated to landfill in each period is shown in Figure 6.2. Apparently the waste flow increases in each period. This is because the municipal population is increasing and the total waste generated will increase accordingly. As the landfill is the main facility to dispose of wastes, the waste flow allocated to the landfill will increase accordingly.

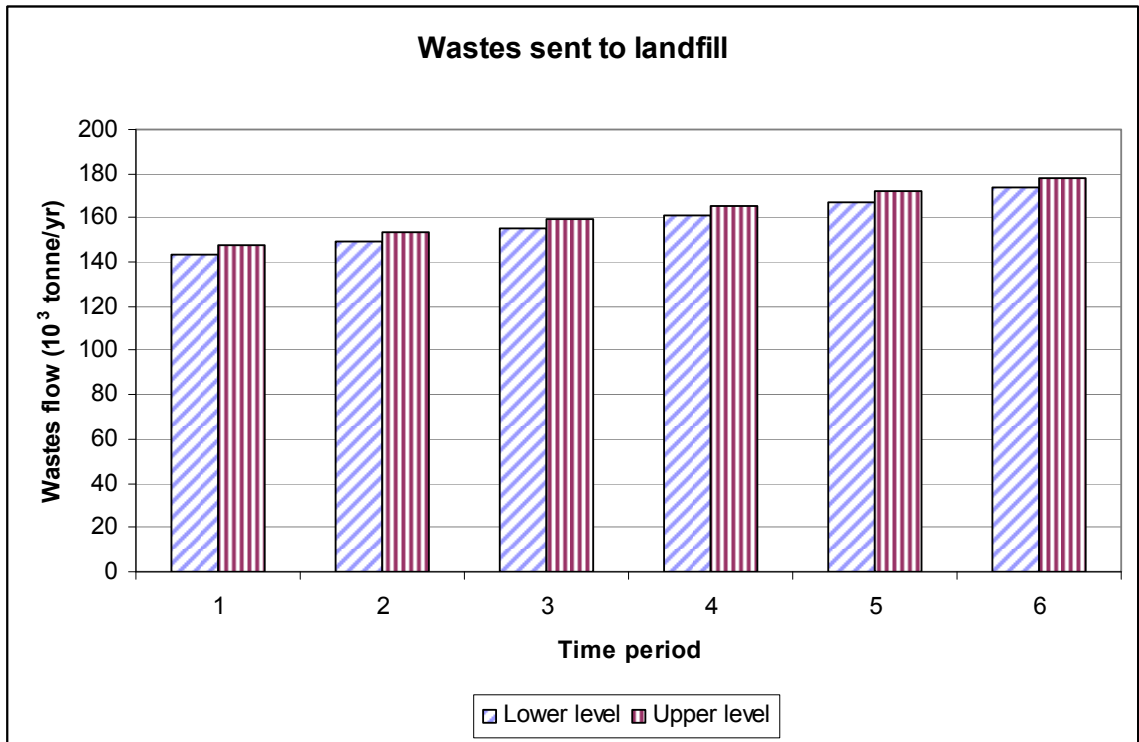


Figure 6.2 Waste flow allocated to the landfill (REILP)

Figure 6.3 illustrates the waste flow allocated to the recycling facility in each period of the planning horizon. Although the waste flow increases gradually in each period, it is lower than the existing capacity (which is 28,000 tonnes per year) for the entire planning horizon. Therefore, there will be no need for the recycling facilities to be expanded in the coming 30 years.

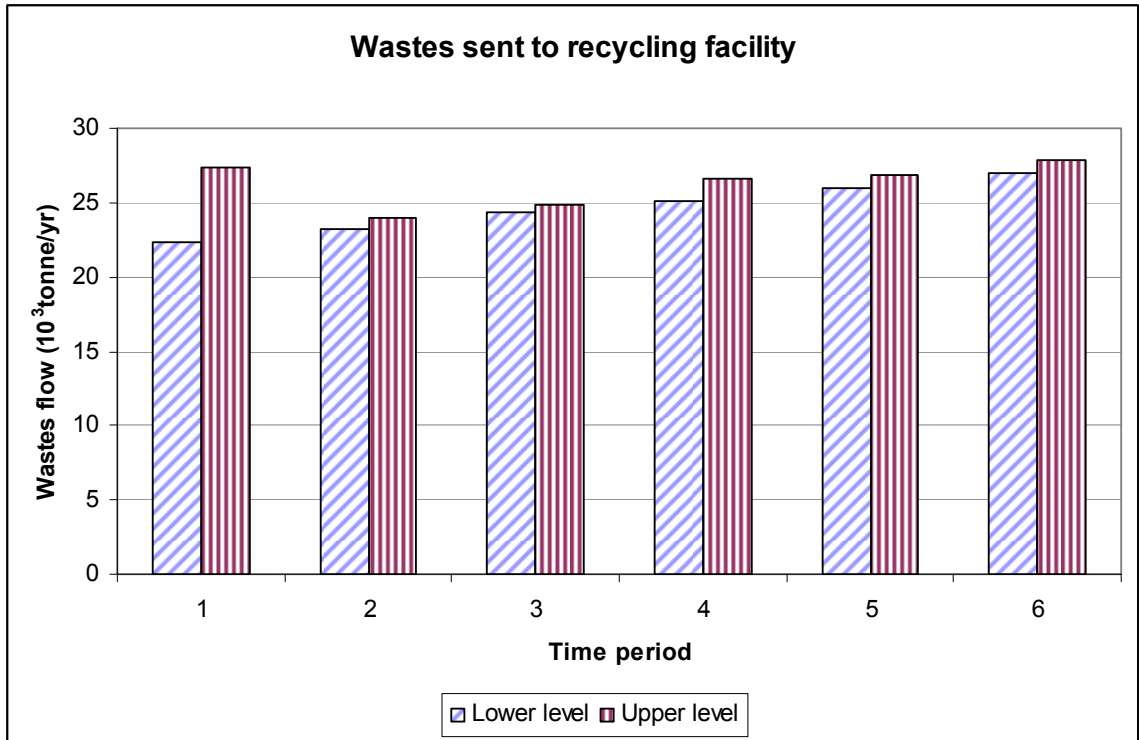


Figure 6.3 Waste flow allocated to the recycling facilities (REILP)

Waste flow allocated to the composting facilities is illustrated in Figure 6.4. In the Figure, the left bar represents the lower bound of waste flow, and the right bar is the upper bound of waste flow. It can be observed that the waste flow allocated to the composting facilities keeps increasing. Starting from period 2, the upper level of waste flow goes beyond the existing composting capacity of HRM (i.e., 50,000 tonnes per year). Starting from period 5, the upper level of waste flow goes over 60,000 tonnes per year, but is less than 70,000 tonnes per year. This indicates that the composting facility needs to be expanded for 10,000 tonnes per year (Option 1 of composting facility expansion plan) by the end of period 1, and it needs to be expanded for 10,000 tonnes per year again by the end of period 4 to satisfy the needs of disposing compostable wastes.



Figure 6.4 Waste flow allocated to the composting facilities (REILP)

6.2. FREILP Results under Different α -cuts

Before running the FREILP model, the aspiration levels under conservative, medium, or aggressive decision cases need to be determined by selecting different α -cut levels, as described in Chapter 4. Figure 6.5 gives an example of finding interval conservative aspiration levels under different α -cut levels. For example, when α -cut = 0.6, the interval aspiration level is [0, 0.29]. Table 6.7 gives the interval aspiration levels for all three decision cases under α -cut levels of 0.5, 0.6, and 0.7.

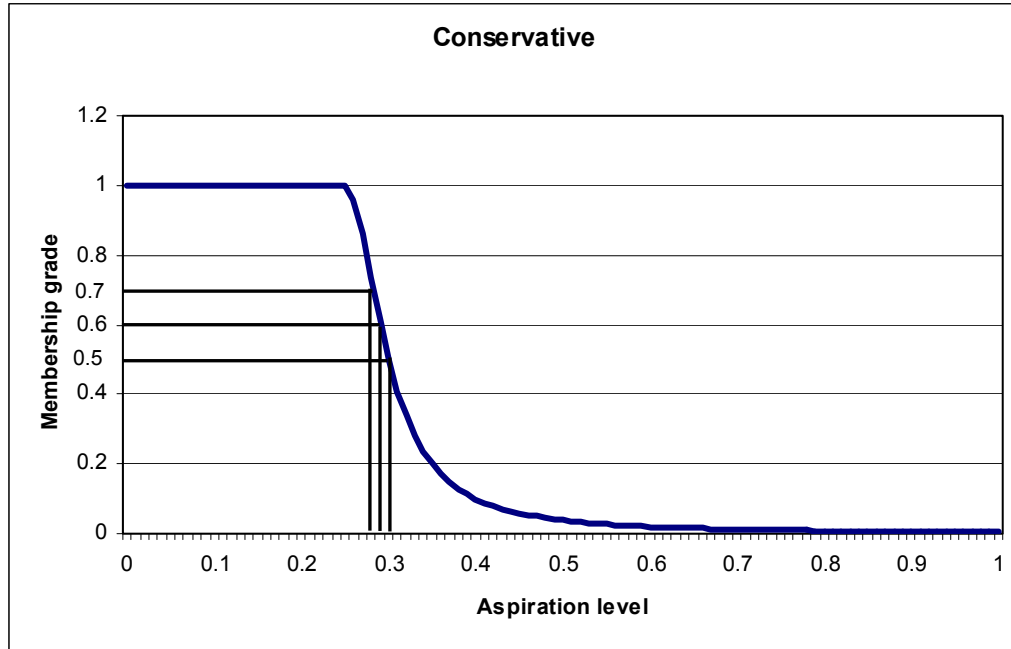


Figure 6.5 Determination of the interval aspiration levels under α -cut levels of 0.5, 0.6 and 0.7 for conservative scenarios

6.2.1. Scenario 1: α -cut = 0.5

Three groups of solutions, including conservative, medium and aggressive solutions, under the α -cut of 0.5, are provided in Table 6.4. As the aspiration level increases, the risk function increases from $[0, 0.07]$ for conservative case to $[0.08, 0.24]$ for medium case and $[0.29, 0.63]$ for aggressive case; while the total system cost decreases from $[946.91, 978.58]$ to $[909.97, 941.64]$, and to $[873.02, 904.69]$ million dollars for the entire planning horizon, respectively. The components included in the total system cost, i.e., waste transportation and collection cost, facility operating cost, and capital cost, also decreases. Table 6.4 also presents the optimal solutions for total waste generation and allocation to different facilities. As shown in Table 6.4, most solutions are obtained as intervals, representing the uncertainties in the system. However, waste generation and waste allocation solutions for the conservative decisions are crisp values, since the upper bounds and the lower bounds happen to be same with the aspiration level being 0 and 0.3.

Table 6.4 FREILP solution when α -cut = 0.5

0.5-cut	Conservative	Medium	Aggressive
Aspiration level	[0.00, 0.30]	[0.35, 0.65]	[0.70, 1]
Risk function	[0.00, 0.07]	[0.08, 0.24]	[0.29, 0.63]
Total cost (10 ⁶ \$)	[946.91, 978.58]	[909.97, 941.64]	[873.02, 904.69]
Collection cost (10 ⁶ \$)	[287.43, 300.70]	[271.54, 284.72]	[259.23, 270.49]
Operation cost (10 ⁶ \$)	[660.96, 676.40]	[642.33, 659.67]	[618.50, 636.76]
Capital cost (10 ⁶ \$)	[49.44, 52.02]	[48.69, 48.76]	[45.95, 48.12]
Revenue (10 ⁶ \$)	[50.90, 50.53]	[50.59, 51.52]	[50.66, 50.68]
Waste generation (10 ⁶ tonne)	12.70	[12.68, 12.70]	[12.32, 12.70]
Waste to landfill (10 ⁶ tonne)	4.88	[4.87, 4.88]	[4.75, 4.80]
Waste to recycling (10 ⁶ tonne)	0.79	[0.77, 0.79]	[0.74, 0.77]
Waste to composting (10 ⁶ tonne)	1.77	[1.68, 1.77]	[1.54, 1.69]

6.2.2. Scenario 2: α -cut = 0.6

The optimal solutions of FREILP model under the α -cut of 0.6 are given in Table 6.5.

Under the α -cut of 0.6, the aspiration levels are equal to [0, 0.29], [0.36, 0.65], [0.71, 1] for conservative, medium, and aggressive decisions, respectively. As indicated in the Table, the optimal risk function for these decisions are [0, 0.07], [0.09, 0.24], and [0.29, 0.63], respectively. The total system cost and each cost component decrease with the increase of the aspiration levels.

Table 6.5 FREILP solution when α -cut = 0.6

0.6-cut	Conservative	Medium	Aggressive
Aspiration level	[0.00, 0.29]	[0.36, 0.65]	[0.71, 1]
Risk function	[0.00, 0.07]	[0.09, 0.24]	[0.29, 0.63]
Total cost (10 ⁶ \$)	[947.97, 978.58]	[909.97, 940.58]	[873.02, 903.64]
Collection cost (10 ⁶ \$)	[289.32, 300.70]	[271.54, 285.34]	[259.23, 269.94]
Operation cost (10 ⁶ \$)	[659.07, 676.40]	[642.33, 655.80]	[618.50, 637.40]
Capital cost (10 ⁶ \$)	[50.11, 52.02]	[48.69, 51.34]	[45.95, 46.69]
Revenue (10 ⁶ \$)	50.53	[50.59, 51.91]	[50.39, 50.66]
Waste generation (10 ⁶ tonne)	12.70	[12.68, 12.70]	[12.32, 12.68]
Waste to landfill (10 ⁶ tonne)	4.88	[4.87, 4.88]	[4.75, 4.84]
Waste to recycling (10 ⁶ tonne)	0.79	[0.77, 0.79]	[0.74, 0.77]
Waste to composting (10 ⁶ tonne)	1.77	[1.68, 1.77]	[1.54, 1.66]

6.2.3. Scenario 3: α -cut = 0.7

When α -cut = 0.7, the interval aspiration level for conservative, medium, and aggressive decisions are [0, 0.28], [0.35, 0.64], and [0.72, 1], and the minimized risk function values are [0, 0.07], [0.08, 0.24], and [0.3, 0.63] respectively. Similar trends can be observed for the solutions of the total system cost, waste generation, and waste allocation to different facilities, as indicated in Table 6.6.

Table 6.6 FREILP solution when α -cut = 0.7

0.7-cut	Conservative	Medium	Aggressive
Aspiration level	[0.00, 0.28]	[0.35, 0.64]	[0.72, 1]
Risk function	[0.00, 0.07]	[0.08, 0.24]	[0.30, 0.63]
Total cost (10 ⁶ \$)	[949.03, 978.58]	[911.02, 941.64]	[873.02, 902.58]
Collection cost (10 ⁶ \$)	[288.12, 300.70]	[271.87, 284.72]	[259.23, 269.55]
Operation cost (10 ⁶ \$)	[663.55, 676.40]	[641.81, 659.67]	[618.50, 635.34]
Capital cost (10 ⁶ \$)	[48.80, 52.02]	[48.12, 48.76]	[45.95, 48.12]
Revenue (10 ⁶ \$)	[50.53, 51.45]	[50.77, 51.52]	[50.43, 50.66]
Waste generation (10 ⁶ tonne)	12.70	12.70	[12.32, 12.70]
Waste to landfill (10 ⁶ tonne)	4.88	[4.84, 4.88]	[4.75, 4.80]
Waste to recycling (10 ⁶ tonne)	0.79	[0.77, 0.79]	[0.74, 0.76]
Waste to composting (10 ⁶ tonne)	1.77	[1.70, 1.77]	[1.54, 1.68]

6.3. Discussion

The optimal solutions under the α -cut of 0.6 are used for further result analysis and discussion. Table 6.7 gives the optimal waste flows allocated to each facility in different planning periods. Generally, the waste flows allocated to all the facilities are increasing over time in the planning horizon.

The expansion plans for waste disposal facilities are shown in Table 6.8. It is indicated that the landfill capacity needs to be expanded before the period 4 across the conservative, medium, or aggressive decisions. In other words, the landfill must be expanded by the end of period 3 (year 2025) to meet the waste disposal needs. The current designed lifetime of the Otter Lake landfill is from 1999 to 2023. However, by implementing the optimal MSW allocation pattern obtained from this study, the lifetime of existing landfill would be extended for another two years. For recycling facilities, no expansion is needed, which

means the existing recycling facilities have enough capacity to treat the recyclables in the coming 30 years, and no capital cost will be needed for expanding the recycling facilities. As for the composting facilities, it should be expanded for 10,000 tonnes per year by the end of the first and third period for conservative and medium planning. For the aggressive planning, they need to be expanded only once during the 30-year planning horizon.

Table 6.7 Waste allocation solutions

0.6-cut	Conservative	Medium	Aggressive
Aspiration level	[0, 0.29]	[0.36, 0.65]	[0.71, 1]
Risk function	[0, 0.07]	[0.09, 0.24]	[0.29, 0.63]
Waste sent to landfill (tonnes)			
Period 1	[735467, 738640]	[698096, 735467]	[698096, 718462]
Period 2	766971	[766971, 768674]	[735087, 746019]
Period 3	796388	[796388, 798156]	[774632, 798156]
Period 4	826934	[826934, 827513]	[804344, 828770]
Period 5	858652	[858652, 859431]	[835194, 859431]
Period 6	891586	[891586, 920582]	[867228, 920582]
Total	[4875997, 4879171]	[4875997, 4872452]	[4840122, 4745878]
Waste sent to recycling (tonnes)			
Period 1	[136887, 140000]	[115000, 140000]	[111905, 115000]
Period 2	119866	119866	[116197, 119866]
Period 3	124464	124464	[121579, 124464]
Period 4	133134	[129238, 133134]	[125282, 129238]
Period 5	134195	[134195, 140000]	[130087, 140000]
Period 6	139342	[138000, 139342]	[135076, 138000]
Total	[787887, 791001]	[766568, 791001]	[740125, 766568]
Waste sent to composting (tonnes)			
Period 1	250000	250000	[232729.5, 250000]
Period 2	281860	[260579, 281860]	[241656.5, 260579.1]
Period 3	292671	[270574, 292671]	[250000, 270573.7]
Period 4	300000	[296656, 300000]	[260548.9, 280951.6]
Period 5	315552	[300000, 315552]	[270543.3, 300000]
Period 6	327655	[300000, 327655]	[280919.5, 300000]
Total	1767737	[1677809, 1767737]	[1536398, 1662104]

Table 6.8 Facility expansion solutions

0.6-cut	Conservative		Medium		Aggressive	
Aspiration Level	0	0.29	0.36	0.65	0.71	1
Landfill expansion						
Period 1	0	0	0	0	0	0
Period 2	0	0	0	0	0	0
Period 3	0	0	0	0	0	0
Period 4	1	1	1	1	1	1
Period 5	0	0	0	0	0	0
Period 6	0	0	0	0	0	0
Recycling expansion options						
Period 1	0	0	0	0	0	0
Period 2	0	0	0	0	0	0
Period 3	0	0	0	0	0	0
Period 4	0	0	0	0	0	0
Period 5	0	0	0	0	0	0
Period 6	0	0	0	0	0	0
Composting expansion option 1						
Period 1	0	0	0	0	0	0
Period 2	1	1	1	1	1	0
Period 3	0	0	0	0	0	0
Period 4	0	0	0	0	0	1
Period 5	1	1	1	0	0	0
Period 6	0	0	0	0	0	0
Composting expansion option 2						
Period 1	0	0	0	0	0	0
Period 2	0	0	0	0	0	0
Period 3	0	0	0	0	0	0
Period 4	0	0	0	0	0	0
Period 5	0	0	0	0	0	0
Period 6	0	0	0	0	0	0
Composting expansion option 3						
Period 1	0	0	0	0	0	0
Period 2	0	0	0	0	0	0
Period 3	0	0	0	0	0	0
Period 4	0	0	0	0	0	0
Period 5	0	0	0	0	0	0
Period 6	0	0	0	0	0	0

6.3.1. Conservative-Risk Decision Support

If the decision makers choose conservative decisions, the waste allocation plans for recycling, composting, and landfill facilities are shown in Figures 6.6, 6.7, and 6.8. As illustrated in Figure 6.6, the wastes allocated to the recycling facilities are relatively more in the first period, and become much less in the second period. Starting from period 2, the wastes allocation for recycling facilities gradually increases over time. This is due to the expansion of composting facilities. In Figure 6.7, it is shown that the capacity of composting facilities would be 60,000 tonnes per year for period 2. Therefore, more wastes could be sent to composting facilities for a lower treatment cost. The increase of waste flow allocated to the recycling facilities after period 2 is due to the increase of total waste generation. All the waste allocation flows are under the above line in Figure 6.6, which indicates the current capacity (28,000 tonnes per year) is sufficient to meet the disposal needs for recyclable wastes, and no expansion is needed for recycling facilities in 30-year planning horizon.

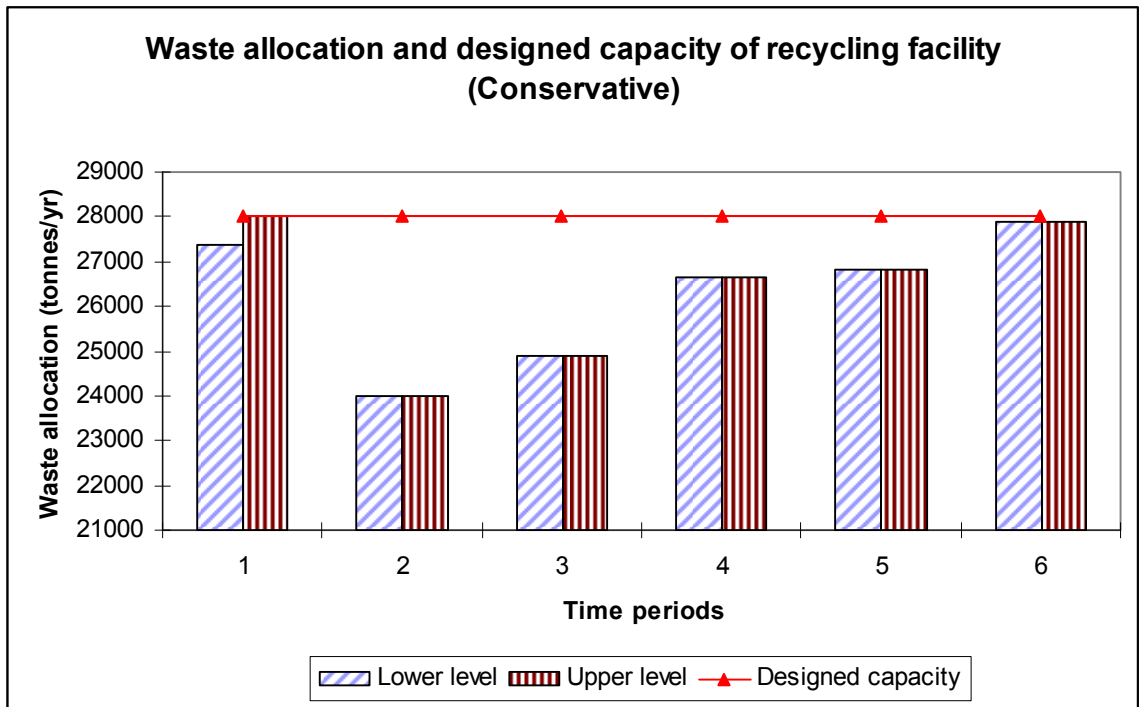


Figure 6.6 Waste allocation and design capacity of recycling facilities under conservative-risk decisions

Figure 6.7 illustrates the waste allocation and the designed capacity of composting facilities under the conservative decisions. The capacity needs to be expanded to 60,000 tonnes per year for period 2 and 70,000 tonnes per year for period 5. In other words, the option 1 for composting facility expansion will be used in periods 1 and 4 to ensure all the compostable wastes be treated in the entire planning horizon.

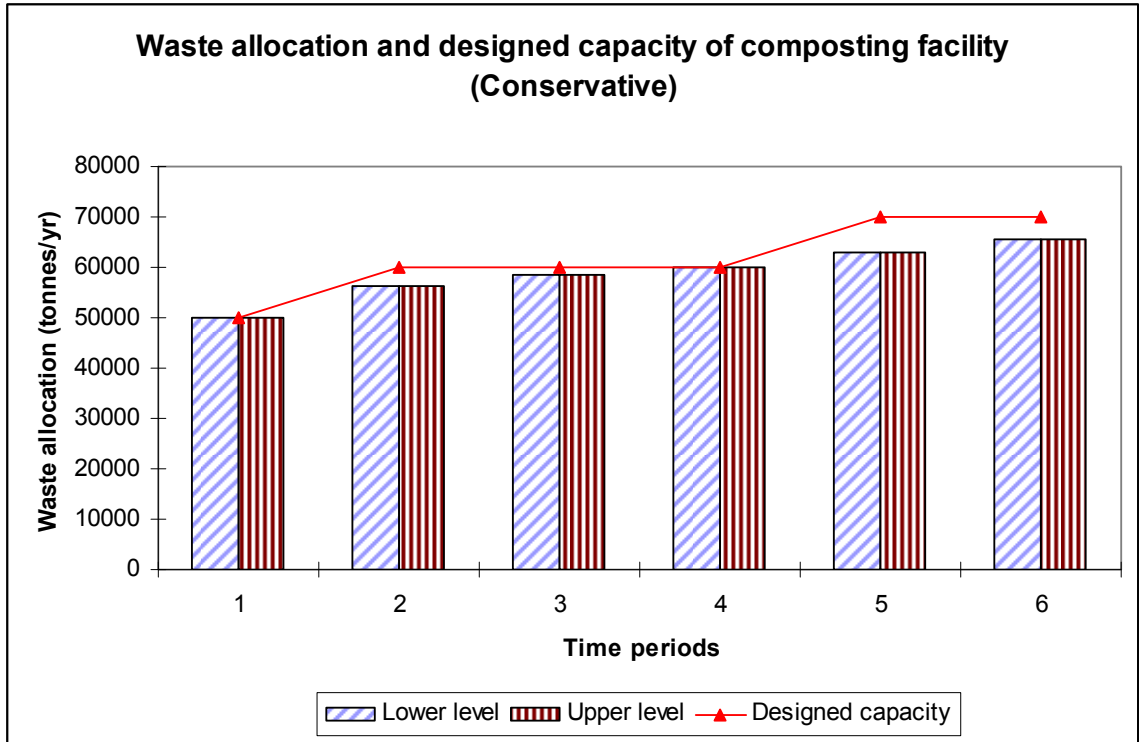


Figure 6.7 Waste allocation and designed capacity of composting facilities under conservative-risk decisions

The waste flows allocated to the landfill are illustrated in Figure 6.8. The waste flow increases steadily over time. On the one hand, in order to achieve the diversion rate of over 60%, the wastes allocated to the landfill have to be less than 40% of the total waste generated. On the other hand, the waste flow to landfill should be kept at a high level simply because the operating costs of landfilling wastes are much lower than that of recycling and composting facilities. This makes the optimal waste flows of landfill around 40% of the total waste generated, and has a steady increase as the total waste generation increases over the years.

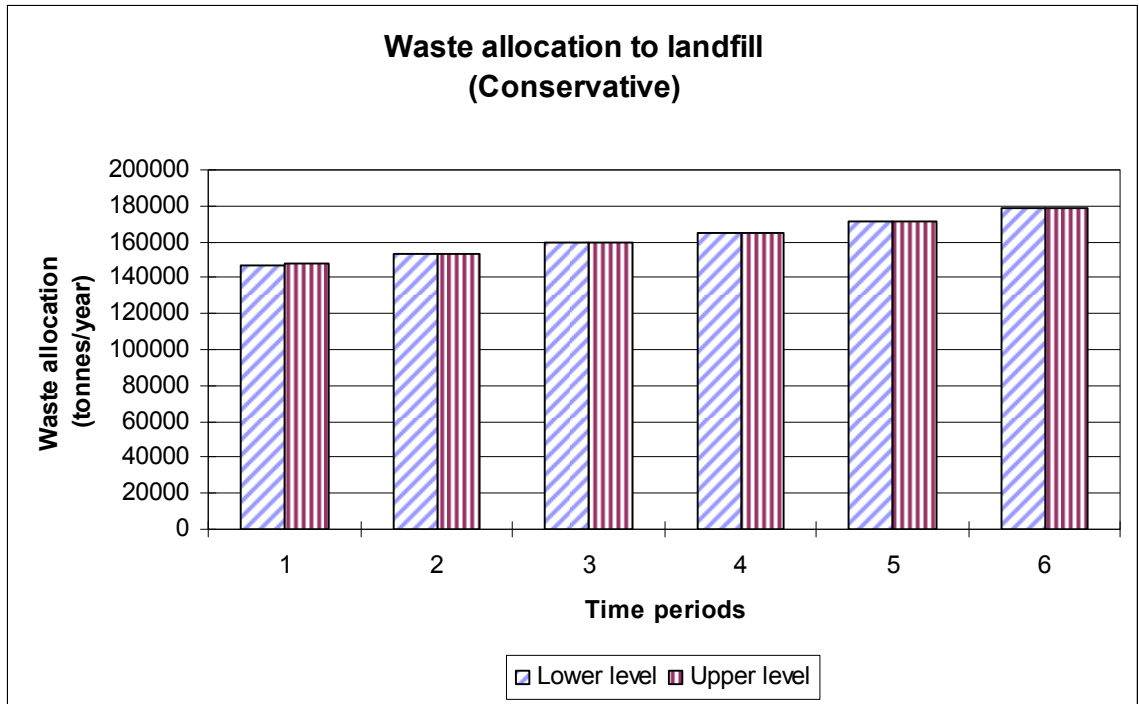


Figure 6.8 Waste allocation to landfill under conservative-risk decisions

Unlike the capacity of recycling or composting facilities, the landfill is accumulating the wastes in its cell and thus its capacity is consumed up and reduced on a daily basis. In this study, the capacity of Otter lake landfill gradually reduces, as shown in Figure 6.9, and will be mostly used up in period 3. The remaining capacity would be [119500, 122674] tonnes by the end of period 3, which is not enough for the coming waste flow of 826934 tonnes in period 4 (see Table 6.7). Consequently, the landfill will need an expansion in period 3 to make sure the capacity will be enough for period 4 and thereafter. With the expansion, the available landfill capacity by the end of period 4 will become [3042566, 3045740] tonnes. The available capacity after the whole planning horizon would be [1292329, 1295503] tonnes.

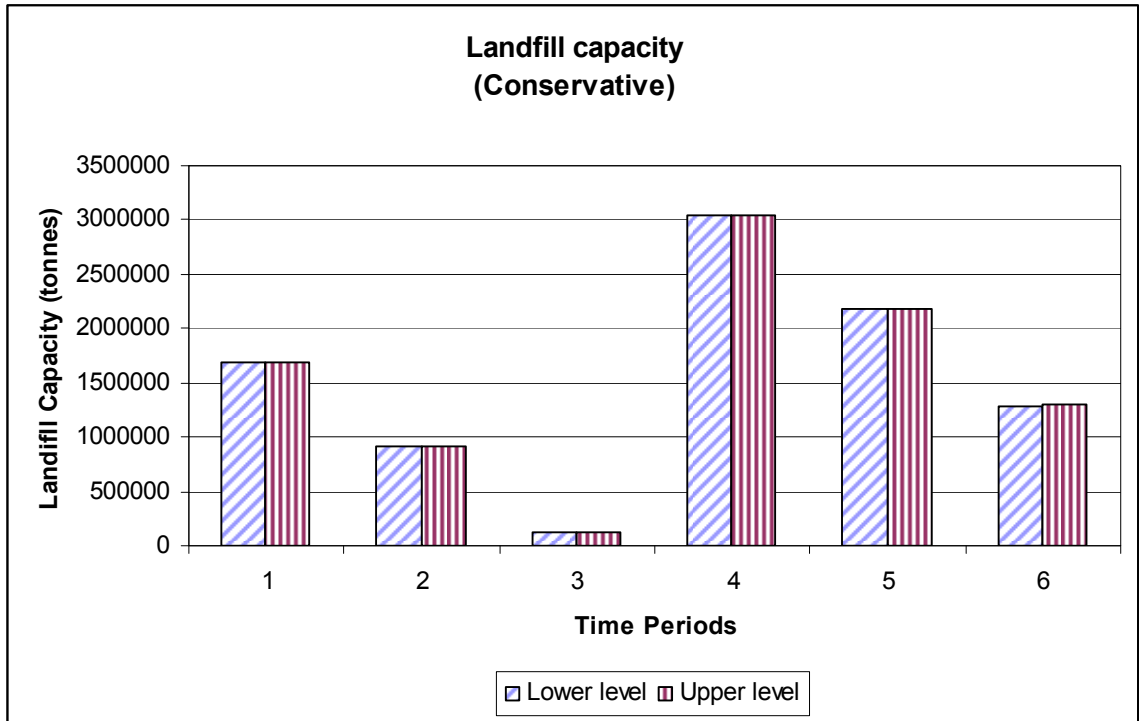


Figure 6.9 Available landfill capacity under conservative-risk decisions

6.3.2. Medium-Risk Decision Support

If the decision makers choose medium decisionst, the waste allocation plans for recycling, composting, and landfill facilities are shown in Figures 6.10, 6.11, and 6.12. As shown in Figure 6.10, the recycling facility does not need to be expanded in the coming 30 years. The optimal waste flow for medium risk decisions would be [23000, 28000], 23973, 24893, [25848, 26627], [26839, 28000], and [27600, 27868] tonnes per year in the coming 6 periods. The capacity of the existing recycling facilities of 280,000 tonnes per year is capable of disposing all these recyclables in the entire planning horizon.

The waste flows allocated to composting capacities are 50000, [52116, 56372], [54115, 58534], [59331, 60000], [60000, 63110], and [60000, 65531] tonnes per year in the coming 6 periods, as illustrated in Figure 6.11. To compost all these wastes, the composting facilities need to be expanded to 60000 tonnes per year by the end of period 1 and to 70000 tonnes per year by the end of period 4.

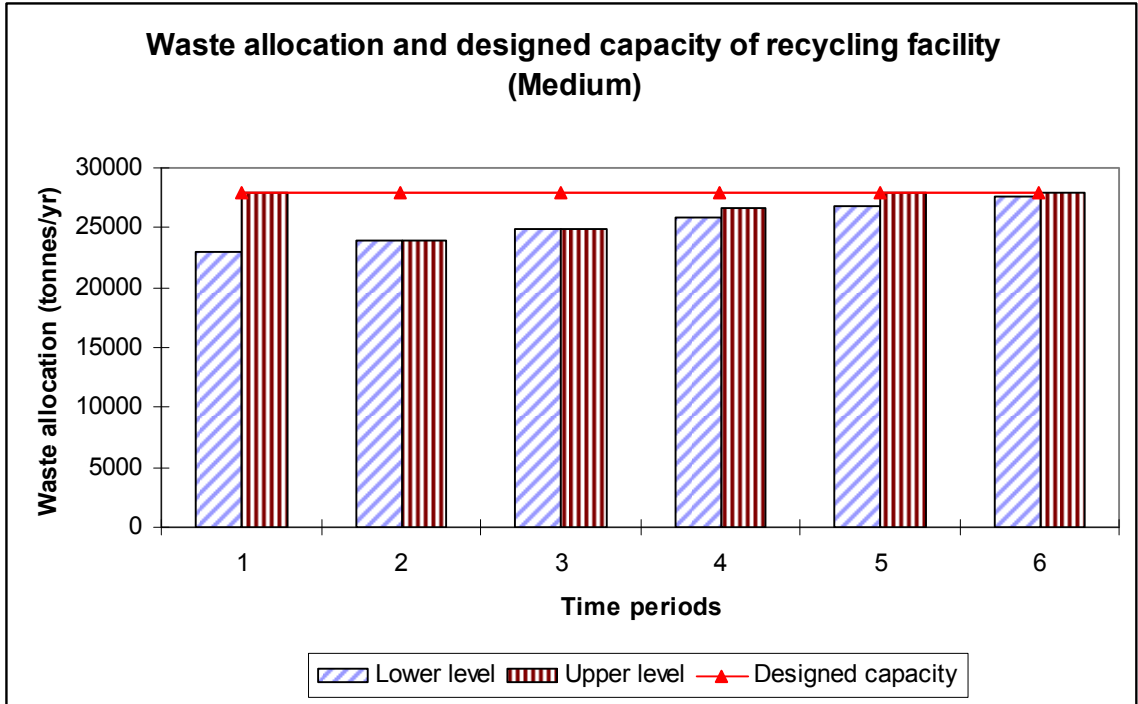


Figure 6.10 Waste allocation and designed capacity of recycling facilities under medium-risk decisions

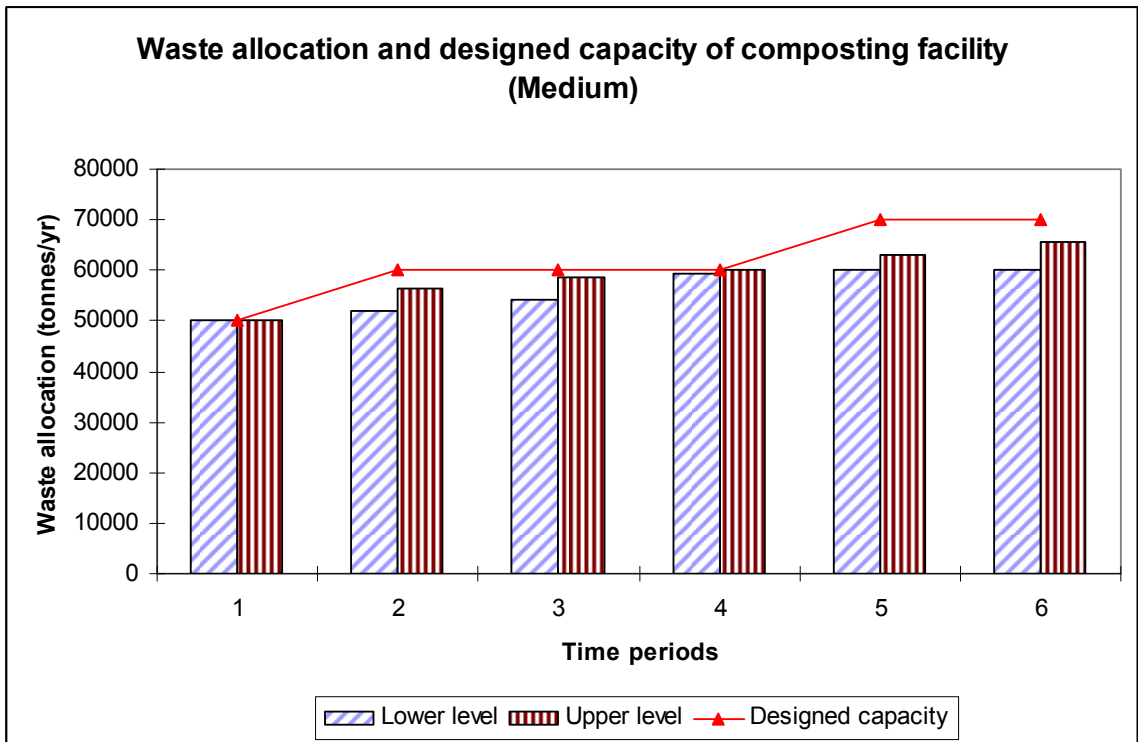


Figure 6.11 Waste allocation and designed capacity of composting facilities under medium-risk decisions

The optimal waste flows allocated to the landfill for the medium risk level decisions are [139619, 143692], [147018, 149204], [154926, 159631], [160869, 165754], [167039, 171886], and [173446, 184116] tonnes per year, as illustrated in Figure 6.12. As the wastes accumulate in the landfill, the capacity would be reduced to [126734, 156574] by the end of period 3. However, the waste flow for the coming period 4 would be [826934, 827513] tonnes, which is much more than the remaining capacity, so the landfill needs to be expanded by the end of period 3, as illustrated in Figure 6.13. After the expansion, the total available capacity by the end of period 4 would be [3045740, 3079061] tonnes and will be consumed gradually as time passes. The final remaining capacity of the landfill by the end of the entire planning horizon would be [1295503, 1299048] tonnes.

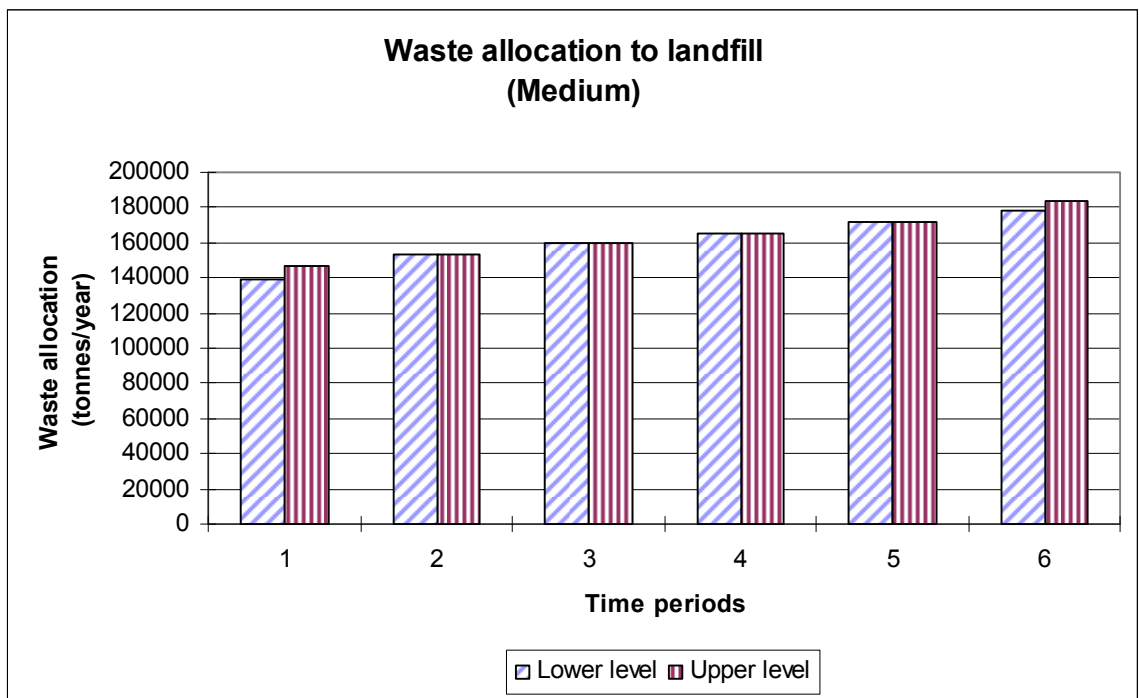


Figure 6.12 Waste flow allocation to landfill under medium-risk decisions

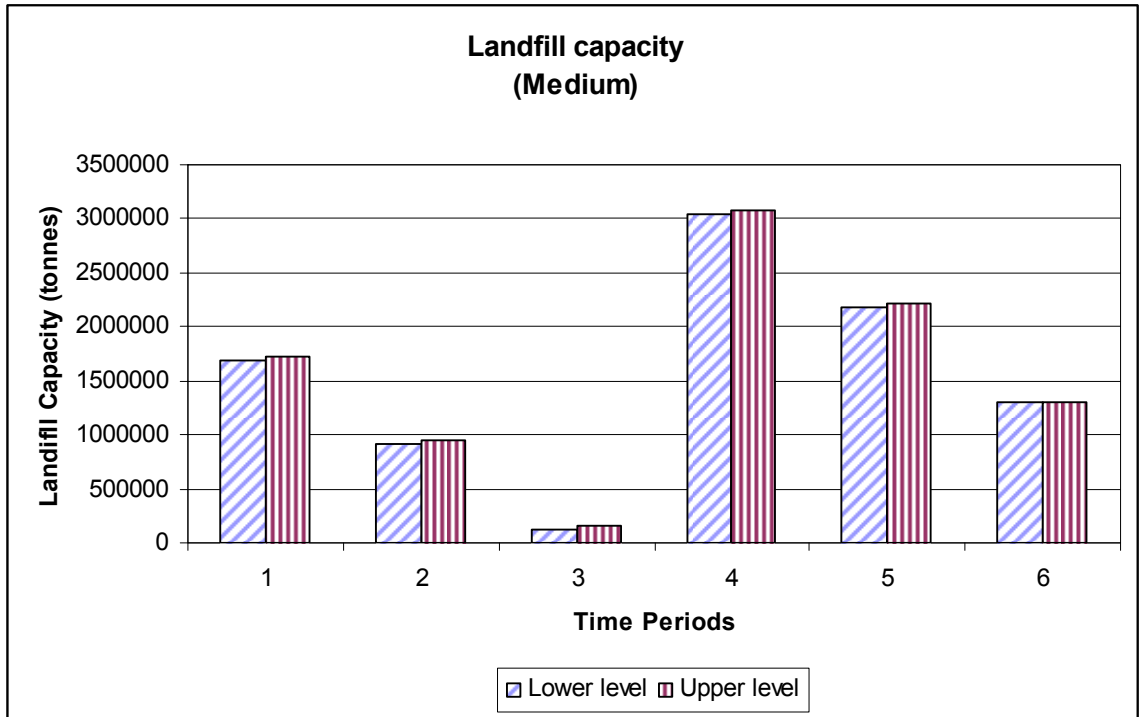


Figure 6.13 Available capacity of landfill at the end of each period under medium-risk decisions

6.3.3. Aggressive-Risk Decision Support

If the decision makers choose aggressive decisions, the waste flows allocated to different MSW facilities are provided in Figures 6.14, 6.15, and 6.16. As illustrated in Figure 6.14, even under the most aggressive situation, the capacity of the existing recycling facilities i.e., 280,000 tonnes per year, is sufficient for the treatment of all the recyclable wastes in the coming 30 years. The optimal allocation for recyclables are [22381, 23000], [23239, 23973], [24316, 24893], [25056, 25848], [26017, 28000], and [27015, 27600] tonnes per year in each period.

The composting facilities need to be expanded for only once, under the aggressive decisions, as shown in Figure 6.15. The option 1 expansion (i.e., 60,000 tonnes per year) by the end of period 1 will be able to handle the compostable wastes flows of [46546, 50000], [48331, 52115], [50000, 54114.74], [52110, 56190], [54109, 60000] and [56184, 60000] tonnes per year for the coming 6 periods.

The wastes allocated to the landfill are [139619, 143692], [147018, 149204], [154926, 159631], [160869, 165754], [167039, 171886], and [173446, 184116] tonnes per year in the coming 6 periods, as shown in Figure 6.16. Due to waste accumulation and capacity consumption, the remaining landfill capacity by the end of each period would be [1703038, 1723404], [957020, 988317], [182388, 190160], [3111391, 3128044], [2251960, 2292850], and [1331378, 1425622] tonnes. It is apparent that the expansion of landfill needs to be conducted before period 4 (Figure 6.17). The remaining landfill capacity of the aggressive decisions is larger than that of the conservative decisions.

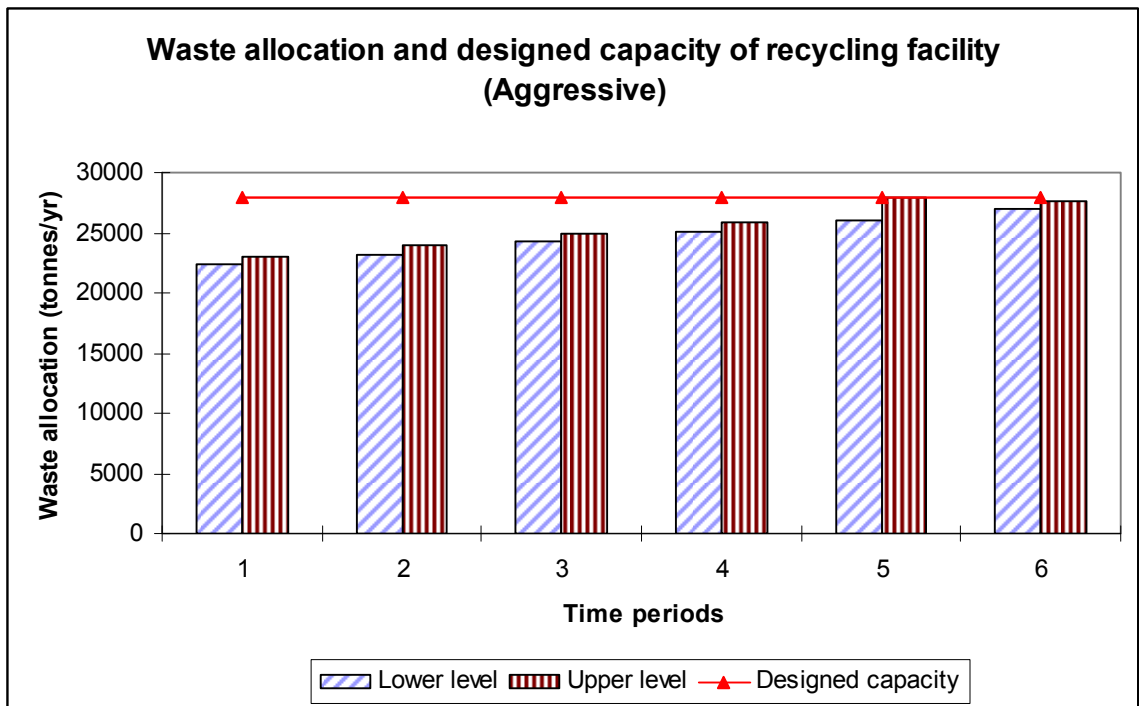


Figure 6.14 Waste allocation and designed capacity of recycling facilities under aggressive-risk decisions

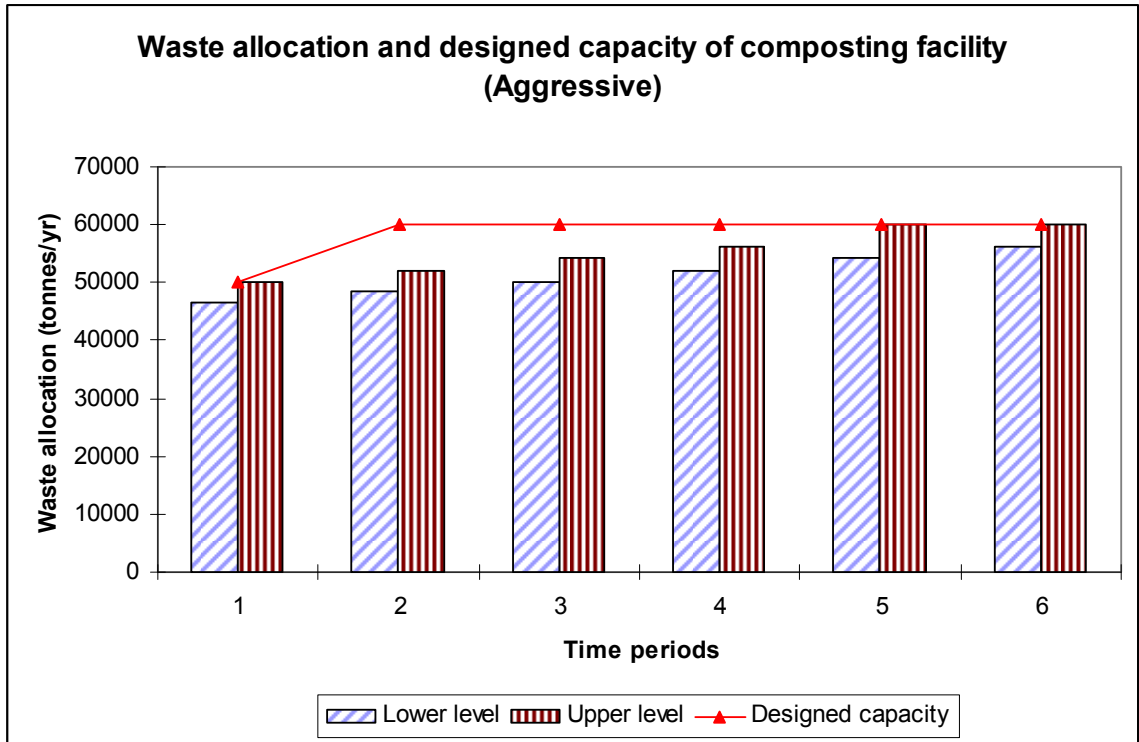


Figure 6.15 Waste allocation and designed capacity of composting facilities under aggressive-risk decisions

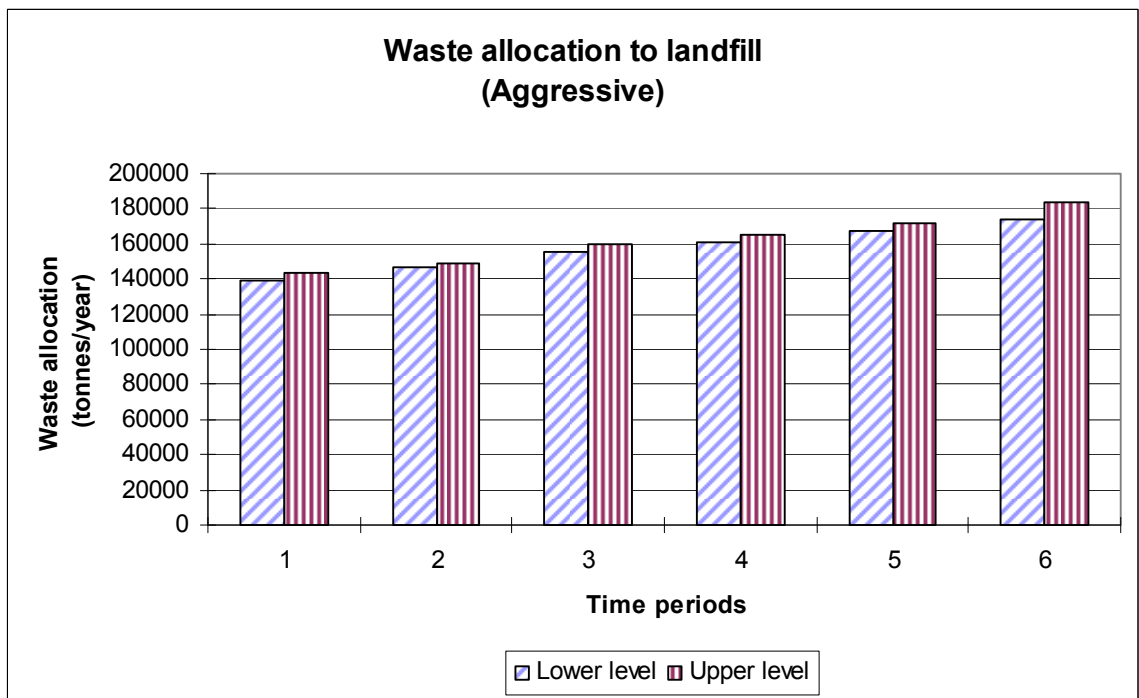


Figure 6.16 Waste flow allocation to the landfill under aggressive-risk decisions

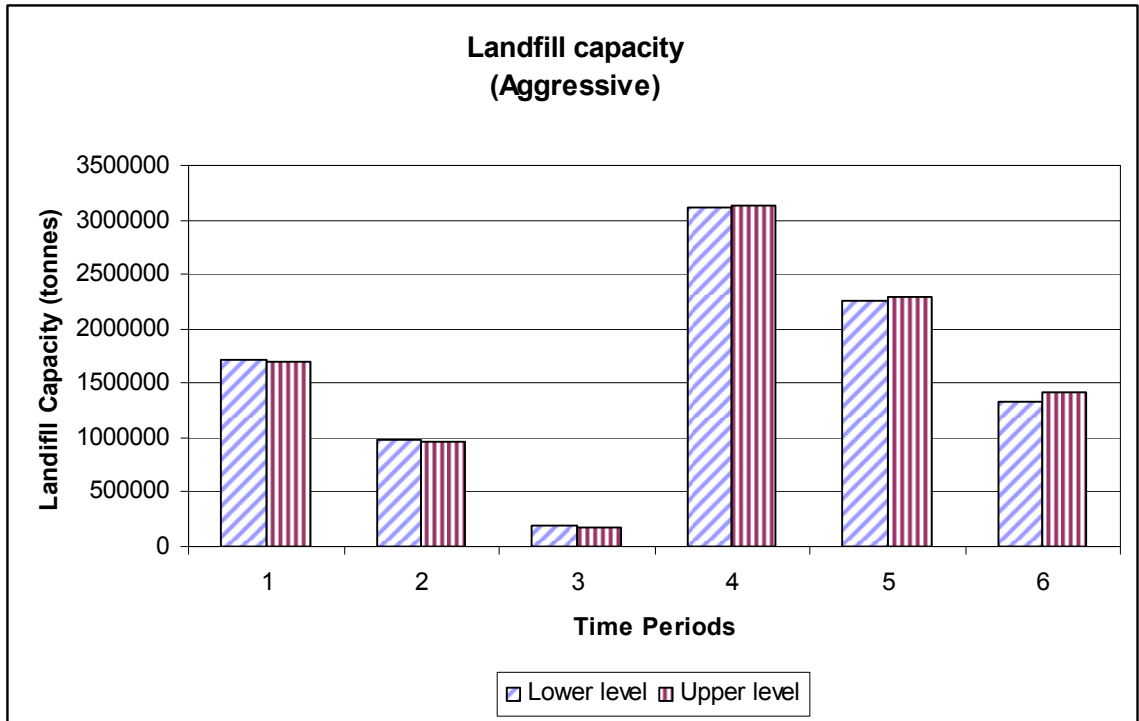


Figure 6.17 Available landfill capacity at the end of each period under aggressive-risk decisions

6.3.4. Comparison of Three Decision Supports

As most of the solutions are obtained as intervals, the mean values of the upper level and the lower level of each of the parameters and variables are calculated in this section for the purpose of a simple comparison. For example, the risk function values for conservative, medium and aggressive functions are $[0, 0.07]$, $[0.09, 0.24]$, and $[0.29, 0.63]$, so the mean values of them, 0.035, 0.165 and 0.46 are used for comparison.

Within the same time period, the waste flow sent to a facility would be higher for a conservative decision than for an aggressive decision. This is because the aggressive decision assumes the waste generation amount to be in a lower level, so that the lower system cost could be achieved. This assumption is certainly helpful for the economic benefit, but the risk of violating the constraints improves because it is not guaranteed that the waste generation amount is within the lower level. If the waste generation happens to be high, the solution for the aggressive decision will not satisfy the real need of waste

disposal. Therefore, the risk function for aggressive decision is relatively high. On the contrary, the risk function for conservative decision is low, but the expected total system cost is high, as illustrated in Figure 6.18.

The waste generation values for conservative, medium and aggressive decision supports are shown in Figure 6.19. Since the total wastes generated are anticipated to be less for the aggressive solution, the wastes sent to different disposal facilities would also be less in the aggressive decision, and the wastes flows under the conservative decision are higher, as illustrated in Figure 6.20, Figure 6.21 and Figure 6.22.

The tradeoff between the risk function and total system cost is illustrated in Figure 6.23. Generally, the aggressive decision support will result in a lower system cost and lower waste generation estimation, with a higher risk level of violating the constraints; the conservative decision support will result in a higher system cost and higher waste generation estimation, with a lower risk level of violating the constraints; and the medium decision support will lead to a result in between. No single decision is superior to others in every aspect, so no one option is strongly recommended. It would be planning makers' responsibility to select one among the three options, based on their evaluation of MSW system in HRM and their preferences.

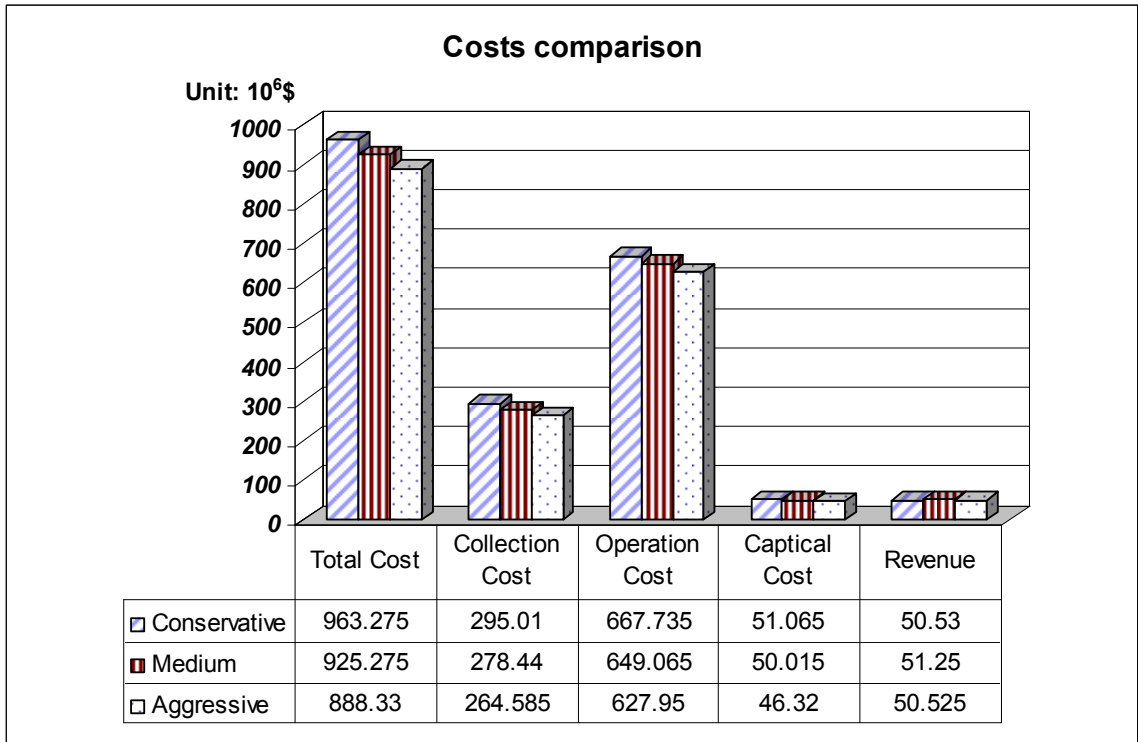


Figure 6.18 Costs comparison

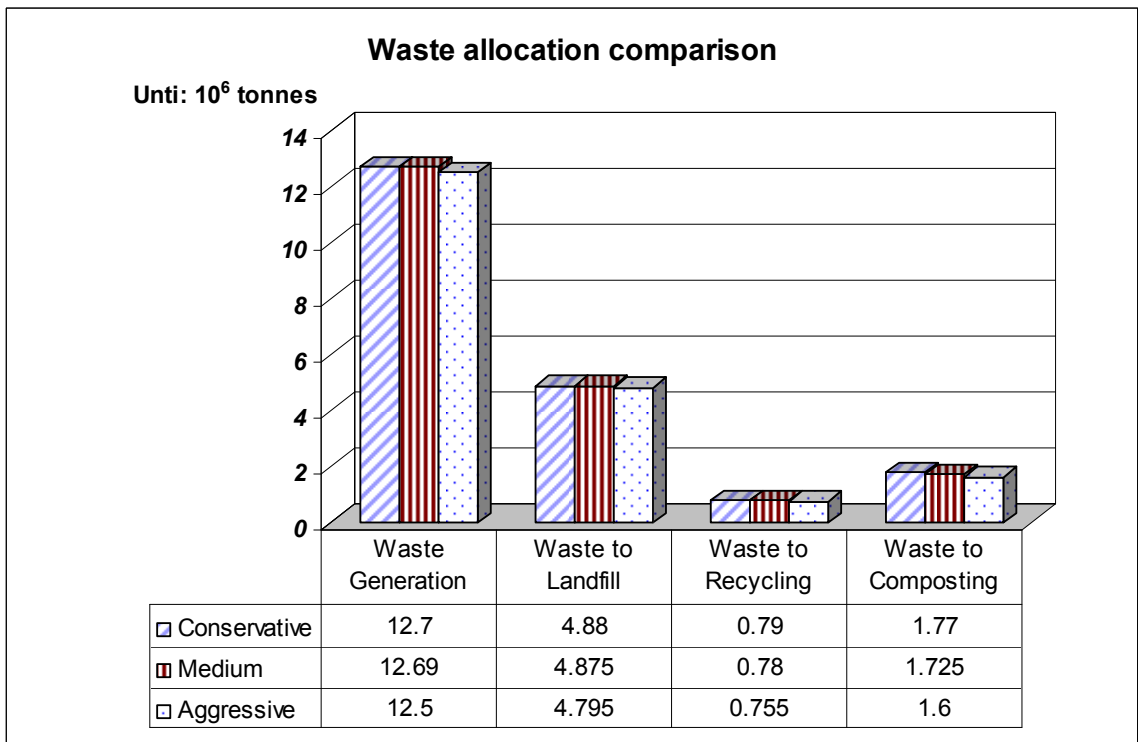


Figure 6.19 Waste allocation comparison

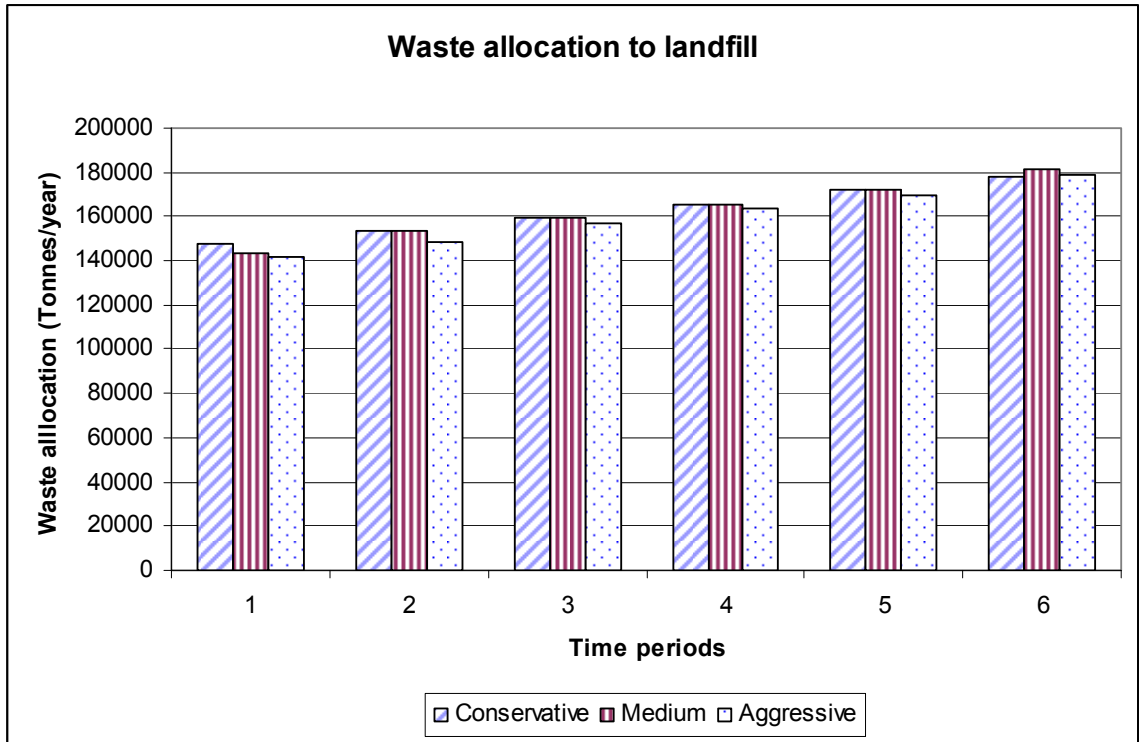


Figure 6.20 Landfill waste flows comparison

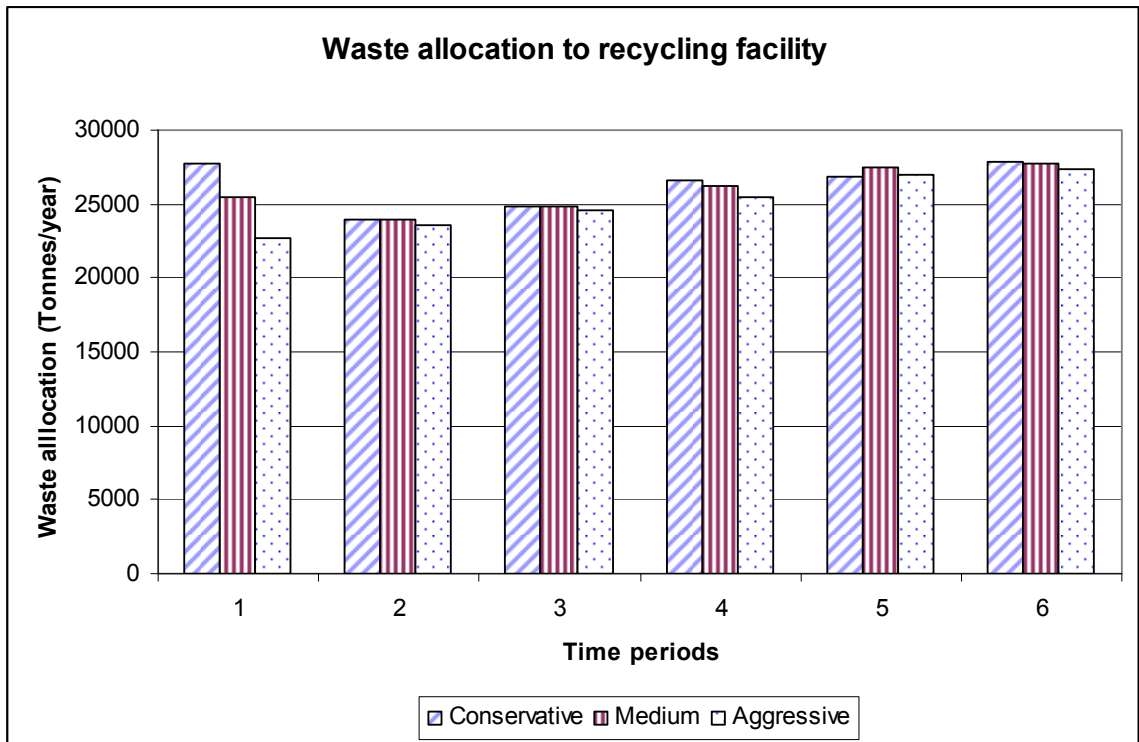


Figure 6.21 Recycling waste flows comparison

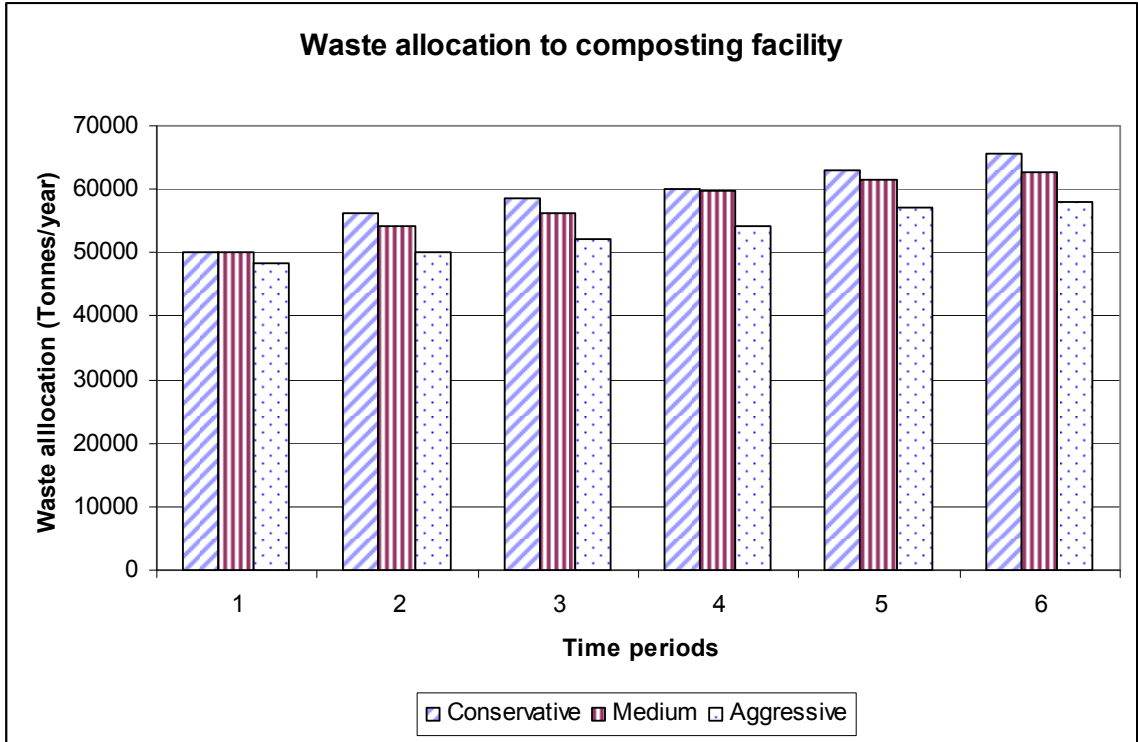


Figure 6.22 Composting waste flows comparison

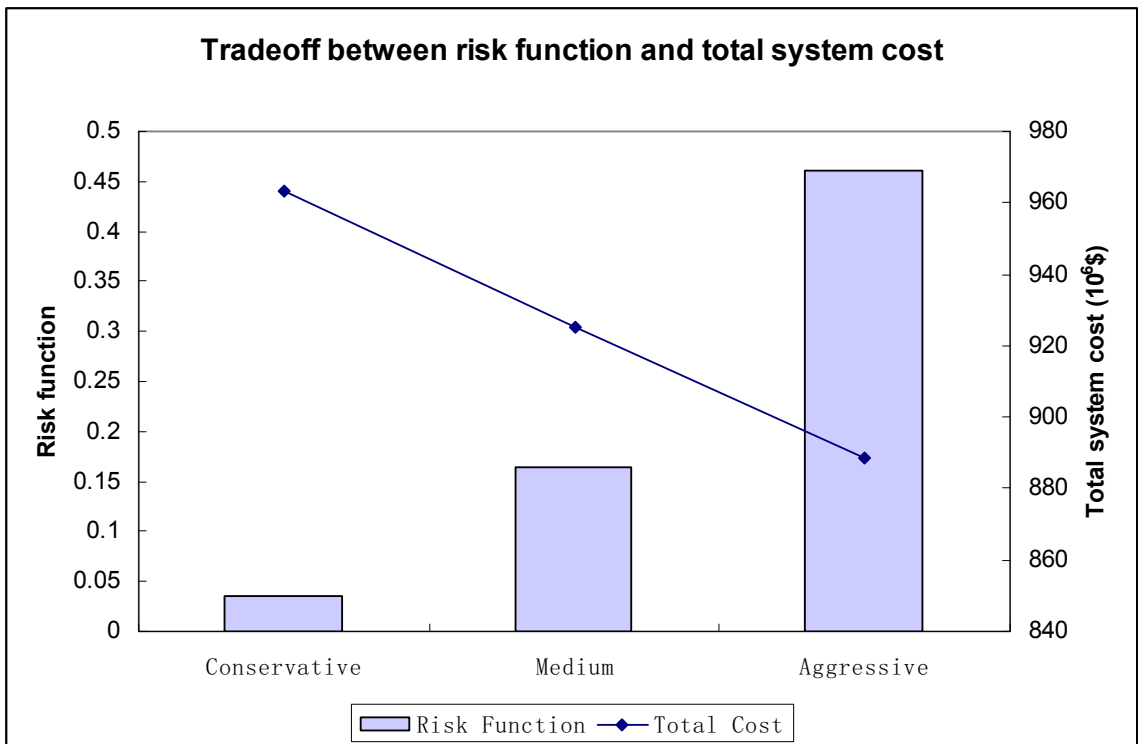


Figure 6.23 Composting waste flows comparison

CHAPTER 7

SUMMARY AND CONCLUSION

7.1. Summary

In this study, a fuzzy risk explicit interval linear programming method is proposed and applied to the long-term planning of the MSW management system in HRM, Canada. The approach can provide a practical decision support through reflecting the tradeoff between system benefits and decision risks. From this study, some conclusions could be summarized as follows:

(1) Three existing algorithms that are being used to solve ILP problems include Monte Carlo simulation method, 2-step algorithm, and BWC algorithm. Monte Carlo simulation method needs extensive computing efforts and is not applicable for complicated real-world problems. Validity checking for 2-step algorithm and BWC algorithm has been conducted through an illustrative numerical example, and the results indicate that both algorithms could produce pseudo-optimal solutions which contain infeasible optimal solutions as well as non-optimal solutions. More in-depth efforts should be placed on either algorithm itself or decision-support process.

(2) Problems of the existing risk explicit ILP method are examined. REILP method can assist decision makers to make a crisp decision by offering the tradeoff between the decision risk and the system return. The risk function was defined to enable finding solutions with the minimum decision risk. The original ILP objective function, which is usually to maximize the system return or to minimize the system cost, was transformed into a constraint to keep the system return in a desired level. Due to the potential problems associated with the REILP, a fuzzy REILP method is proposed in this study to improve upon existing approach.

(3) An optimization approach FREILP is proposed as the further development of REILP. By finding and correcting the root of the infeasibility problem, restructuring the risk

function formulation, and introducing fuzzy set theory in selecting aspiration levels, FREILP can provide more reliable and more practical supports to decision makers. The improvement of FREILP over the existing 2-step and BWC solutions is that the risks associated with the possible optimal solutions and decisions derived from them could be incorporated into the decision-making process.

(4) The developed FREILP model is applied to the long-term planning of the MSW management system in HRM, Canada. A traditional ILP model is firstly developed to minimize the total MSW management system cost over a 30 years period. This is achieved by selecting the waste flows allocated to different waste disposal facilities as decision variables, with the constraints of mass balance, diversion rate, and capacity limitations. Then, the ILP is transformed into a REILP model, having the risk function as the new minimized objective function. Aspiration level is the key factor of the transformed REILP model. To provide a comprehensive decision support, the model is firstly solved as REILP. Eleven groups of solutions are obtained with 11 preset aspiration levels from 0 to 1, with the step of 0.1. These solutions can provide specific decision support for those decision makers who are familiar with the modeling and have a crisp aspiration level in mind. For those decision makers who are not sure about the aspiration levels, FREILP is also solved and three groups of solutions are provided and compared, including aggressive schemes, medium schemes, and conservative schemes.

(5) While minimizing the system risk and the total system costs, FREILP can provide the waste allocation flows and expansion plans for three decision levels: conservative, medium, and aggressive. For the conservative solution schemes, the total system cost of MSW management system in HRM is estimated as [947.97, 978.58] million dollars and the wastes generation is estimated as 12.7 million tonnes over the 30 year period. The landfill should be expanded by the end of period 3 and the composting capacity should be expanded to 60,000 tonnes per year in period 1 and to 70,000 tonnes per year in period 4. For the medium solution schemes, the total system cost is estimated as [909.97, 940.58] million dollars and the waste generation would be [12.68, 12.70] million tonnes. The facility expansion plan would be the same as conservative outcome. For the aggressive

solution schemes, the total system cost is evaluated as [873.02, 903.64] million dollars and the waste generation is [12.32, 12.68] million tonnes. Only one expansion (expanding to 60,000 tonnes per year) is expected by the end of period 1 in the aggressive situation. All these solutions can ensure the waste diversion rate of 60% is achievable.

(6) The model results show that the FREILP approach is able to efficiently explore the interval uncertainty space and generate optimal decision schemes that directly reflect the tradeoff between decision risks and system return, allowing decision makers to make effective and practical decisions, based on the risk-reward information generated by the FREILP modeling analysis.

7.2. Research Achievements

This study represents a new contribution to ILP and its solution algorithms. It checked the validity of ILP algorithms through a numerical example, which is not conducted by previous studies. The result of validity checking shows that both algorithms could produce infeasible optimal solution schemes as well as non-optimal solution schemes.

This study is the first attempt at developing a FREILP, and more importantly, applying it to a real and advanced MSW planning and management problem. The knowledge gained in this study would provide valuable support for research and real life applications. The solutions for the case study can provide an effective decision support for MSW management system planning makers in HRM.

7.3. Recommendation for Future Research

The original objective function of the ILP model in this study is to minimize the total system cost while achieving the environmental goal, which is a waste diversion rate of 60%. In the model construction, the total system cost consists of transportation and collection cost, operation cost, capital cost, and revenue. Besides these factors, the residual

market value of facilities by the end of the planning horizon should also be considered as a component of the total system cost. For example, the landfill with a capacity of 1,291,329 tonnes and the one with a capacity of 1,425,622 tonnes by the end of the planning horizon will have different market values. This residual value will affect the ongoing planning of MSW management. However, due to no residual market value data was obtainable during the conduction of this study is conducted, the residual market value of waste disposal facilities in HRM is not considered in this study. More comprehensive and updated data will improve the accuracy of the model results, so that more helpful and efficient decision supports could be provided.

After the validity check of ILP algorithms were conducted, this study focuses on developing an approach that can minimize system risk as well as minimize the system cost. The solutions provided by either REILP or FREILP are practical and effective. However, they cannot provide the entire solution space for the original ILP model. This means part of the feasible and optimal solutions could have been missed. Therefore, further study in developing a method that is effective, practical, and computationally affordable to solve ILP is recommended.

From the standpoint of practical applications, the developed methodology framework, FREILP, could be further employed to other engineering decision-making problems, such as resource management problems, and the regional air or water pollution control planning problems.

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