THE ELECTRICAL RESISTANCE AND TEMPERATURE COEFFICIENT OF ICE.—BY J. H. L. JOHNSTONE, B. SC., Dalhousie University, Halifax, N. S.*

Read 13th May, 1912.

The following investigation was first begun in January, 1911, with the object of determining the resistence of ice.

A great many difficulties were subsequently met with, which resulted, as will be seen, in a modification of the original methods of experiment; and several other problems appeared, closely connected with the one treated of in this paper, the chief one of which is the effect of polarization, and its nature as related to ice. The latter problem is to be investigated fully at a later time.

Dr. H. L. Bronson, when working in the Physical laboratory at McGill University, noticed the peculiarities connected with this problem and as a result this work was undertaken, with his guidance, by the writer.

The only measurements of the resistance of ice, that could be found after a diligent search, were obtained from a paper by Ayrton and Perry.[1] As these measurements appear to be the only ones published, a brief summary, together with the results of their work, is given here.

Aryton and Perry measured the resistance of ice as follows:

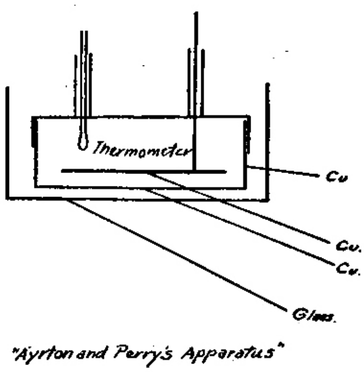

FIG. 1.

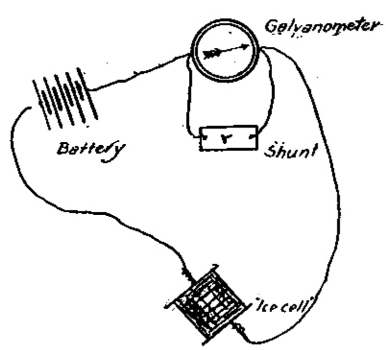

FIG. 2.

* Contributions from the Science Laboratories of Dalhousie University—[Physics]

1 Ayrton, M. E., and Perry: Proc. London Phys. Soc., Vol. II, 178, March, 1877.

Ice was frozen from distilled water in a copper vessel, like that shown in Fig. 1. Connections were then made as shown in Fig. 2, the current passing through the ice being measured by a galvanometer. Knowing the E. M. F. of the cells in the

TABLE I.

AYRTON AND PERRY'S
VALUES.
FOR THE RESISTIVITY OF ICE AND H_2O.

TEMPERATURE °C.	RESISTIVITY.
−12.4	22.4×10^8
−6.2	10.23×10^8
−5.02	9.486×10^8
−3.5	6.42×10^8
−3.0	5.693×10^8
−2.46	4.844×10^8
−1.50	3.876×10^8
−0.2	2.84×10^8
+0.75	1.188×10^8
+2.2	2.48×10^7
+4.0	9.10×10^6
+7.75	5.4×10^5
+11.02	3.4×10^5

Resistance in ohms. ("BA").

TABLE II.

Specimen.
Ayrton and Perry's Results.

Voltage	Time	Gal. Deflection
2.61	0	30.1
4.25	1	39.5
	3	36.5
	4½	50.5
8.7	5½	49.1
17 4	6½	72.7
	7½	69.0
	12½	59.5
28.7	13½	76.5
	17½	67.2
Ice short	circuited for 4	minutes
87.	—— great swing	off scale.
	23½	212
	26½	145
	83	66.2
	129	66.2

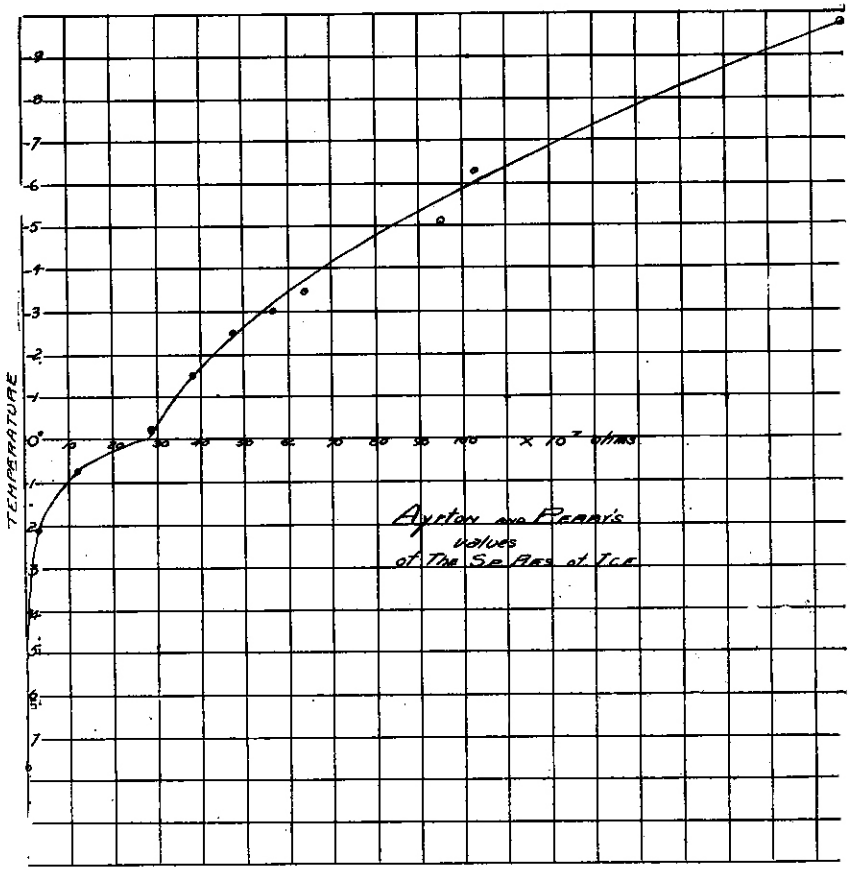

FIG. 3.

circuit, the resistance of the sample of ice was calculated from these data, and from this, the specific re.istance of the ice. This was done for several temperatures.

A great difficulty was encountered in determining the actual value of the current passing through the ice. Ayrton and Perry were troubled greatly by polarization effects which, at that time, they were unable to determine the nature of. As they could not eliminate this effect, which will be shown to be very considerable at times with the method of experimenting, their results do not appear to be very reliable.

The method just outlined was first used and investigated, with results similar to those obtained by Ayrton and Perry[1] and to those obtained by Dr. Bronson.[2]

A D'Arsonval galvanometer, manufactured by Leeds and Northrup, was used as a current measurer, its sensitivity and resistance being first determined.

The resistance was determined by several methods, the mean of these several values being taken and found equal to 1930 ohms, at 17°C.

The sensitivity, or current which produces a deflection of one scale division was determined as follows:

The galvanometer was connected in a circuit as shown in Fig. 4.

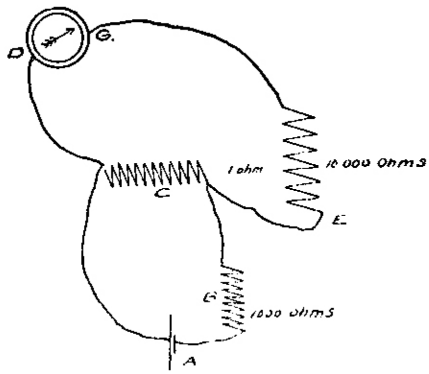

FIG. 4.

E is a 10,000 ohm resistance coil,
C is a standard one, (1), ohm coil,
A is a storage cell,
B is a 1000 ohm coil.

The E. M. F. of the storage cell, as determined by a Weston voltmeter was 2.00 volts, so the current passing through the galvanameter,

$$i = \frac{\frac{2}{1000}}{10000 + 1930} \text{ amperes.}$$

1. Loc. cit.
2. Loc. cit.

It was observed that this current produced a deflection of 51 scale divisions. Therefore the current necessary to produce a deflection of one scale division, will be

$$\frac{\frac{2}{1000}}{(10001 + 1930)\,51} = 3.29 \times 10^{-9} \text{ amp.}$$

The specimen of ice was prepared as follows: Two brass electrodes,—circular discs, were made; a copper rod was soldered to one of them and a copper wire was soldered to the edge of the other one. A cylinder of ice, 3 cm. in height, was cut from ice, obtained from the Dartmouth Lakes. The electrodes were then frozen to this cylinder of ice by warming them slightly and then pressing them to the upper and lower surfaces of the ice. This conductivity cell, so to speak, was placed on a plate of parafine wax, and the whole thing was placed in a box, which was kept in the open air, shaded from the sun. Of course experiments could only be performed when the air was below the temperature of 0°C, which was quite frequent at this period of the year. Several sets of readings are given below, together with a set of readings from Ayrton and Perry's papers.[1] The apparatus was connected up as in Fig. 2.

In the actual experiment, the current was made to pass through the ice for a considerable period of time, in some cases the circuit being unbroken for 48 hours.

When the current was suddenly reversed after flowing for quite a length of time in one direction, a very much greater deflection of the galvanometer was obtained than at first. This deflection decreased somewhat with time. Thus for instance, the deflection changes from 9 divisions, on one side of the zero, to 13 divisions on the other side when the current is reversed through the ice. This is an increase of 40% of the current, which passed through the ice in the initial case. If this is all due to

1. Loc. cit.

TABLE III.

JAN. 24, 25, 26, 1911.

Time		Temp.	Shunt	Voltage	Deflection	
					Right	Left
Jan. 24—	4.15		1 ohm.	20.9	9	13
	4.45		"	"	6	15
	4.50		"	40.7		27.5
	8.0		"	20.9		3.5
	10.00		40	1 "	102.0	106.0
" 25—	10.45	−.5°C	30	"	140*-65	
" "	12.00	"	"	"	†73-23	83-37
" "	12.35	"	"	"	80.-29	
	1.00	"	"	"	16	
	2.00	+ 1°	"	"	175-158	
" 26—	10.25	− 6°	"	"	17-15	19-15
	10.30	"	"	20.1 cells	25	
	10.45	"	"	40.7	102-23	
	12.50	− 5°	"	10 cells	15-16½	

* The current took 10 seconds to fall from "140-65" in value.
† The current took 3 seconds to fall from 73-23 in value.
In the 2nd column we have the temp. of air beside the ice, recorded.
In the last column the deflections of the gal., and the deflections when the current is reversed, are given.

polarization, the phenomena we have here to treat of, are quite different from the so-called electrolytic polarization effects. Ayrton and Perry[1] noticed similar effects on reversal, and on short-circuting their "cell," (see page 128). In one case, it will be seen that on short-circuiting their cell, the current increased about 175% of its original value. Similar results were consistently obtained by the writer.

1. Loc. cit.

As resistances calculated from such values of the current as the above would have little or no value, a method was then sought, whereby the effects just indicated could be eliminated.

A great deal of time was spent in attempting to find out the nature of this polarization and measure its value. The "Tuning Fork"[1] method of measuring electrolytic polarization was used first of all, with water as the electrolyte, and correct values were obtained. But when the ice cell was substituted for the water cell, this method would give no results on account, principally, of capacity effects. A "commutator" method was also tried with similar results.

By using Kohlrausch's method for measuring the resistance of electrolytes, the polarization effect would probably be eliminated. However, the maximum resistance that can be measured by this method is of the order of 10^6 ohms. As ice has a specific resistance of more than 10^8 ohms, this method is not practicable. It might be possible however, by taking thin sections of a block of ice, to measure its resistance by means of Nernst's conductivity apparatus.[2]

Now one of the methods of measuring the resistance of a solid conductor, is to determine the drop in potential between two sections of the substance, when a steady current is flowing through these sections. Knowing the values of i and e, the resistance can be calculated.

A method very similar to the above was adopted, and as will be shown, the effects of polarization, etc., will be eliminated as far as the measurement of the resistance is concerned.

The apparatus was set up, as shown in Fig. 5. B, is a "U" tube, 12 cm. in height, with a bore of about 2 cm. a and a_1 are glass tubes of 2½ mm. bore, with platinum points sealed at the ends c and c_1. b and b_1 are glass tubes of 4 or 5 mm. bore, with

1 Watson, M. :—Text book of Physics, page 790.

2 Nernst, W. :—Zeitschr. f. phys. Chem., 14, 622. (1894); also Maltby, M. E. : *ibid.* 18, 133, (1895).

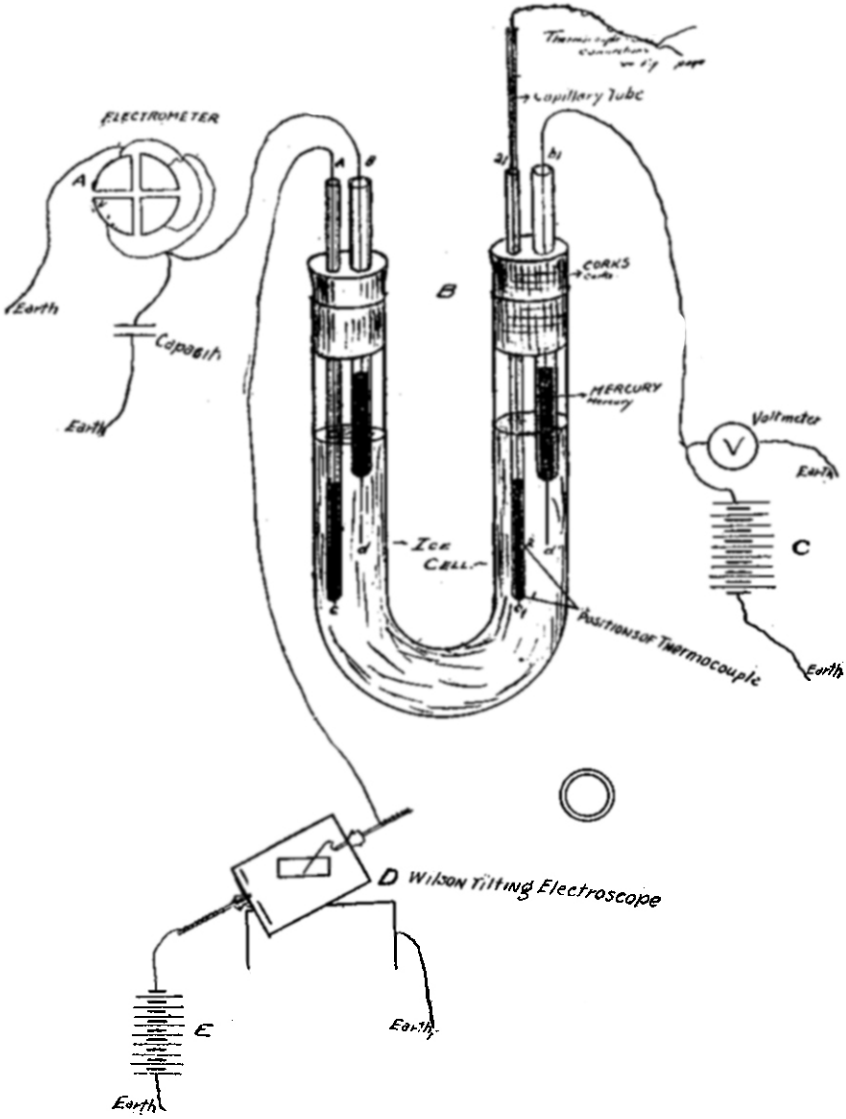
ELECTROMETER
A
Earth
Earth
B
Capillary Tube
CORKS
MERCURY
Ice
Cell
Positions of Thermocouple
Voltmeter
Earth
C
Earth
D Wilson Tilting Electroscope
E
Earth
Earth

FIG. 5.

platinum wires, d and d_1 sealed in the end as shown in the figure. These four tubes are fitted carefully in corks, and then in the "U" tube as shown.

Now suppose we fill the tube with an electrolyte and pass a current through it, by way of the electrodes b and b_1. There will result a polarization at these electrodes and the value of the current will vary somewhat with the time. If this current is measured by a sensitive galvanometer and if the difference in potential of the two electrodes is measured by a voltmeter, a value of the resistance of the electrolyte could be calculated at any particular time; but this is a varying quantity, and not the true resistance of the electrolyte.

Now if two other electrodes a and a (see Fig. 5), are placed in the position shown in the figure, their ends c and c_1 being lower somewhat than the extremities of the platinum wires d and d_1, and then a current is passed through the electrolyte by way of the electrodes d and d_1; then if we measured the difference of potential between the points c and c_1, by some electrostatic instrument and knew the value of current, we could calculate the value of the resistance of the electrolyte between the points c and c_1, and the resistance so calculated would be constant in value and unaffected by polarization. This would be so, because when there is a variation in the current due to polarization or any other causes, there will be a proportional change in the potential difference between the two potential electrodes, so that the ratio of the potential difference to the current will be constant (with a const. temp.). Therefore the resistance determined in this way will have a constant value. Thus, polarization effects will be eliminated.

The current passing through the electrolyte, (which was ice in this case), was measured by a *Dolezalek* electrometer, A, (see Fig. 5). The potential of each potential electrode was measured by a *Wilson Tilting* electroscope, D, the plate of the electroscope being kept at a potential of 320 volts, from small storage cells. A calibration curve for this instrument, as it was used in these experiments is shown in Fig. 6. It

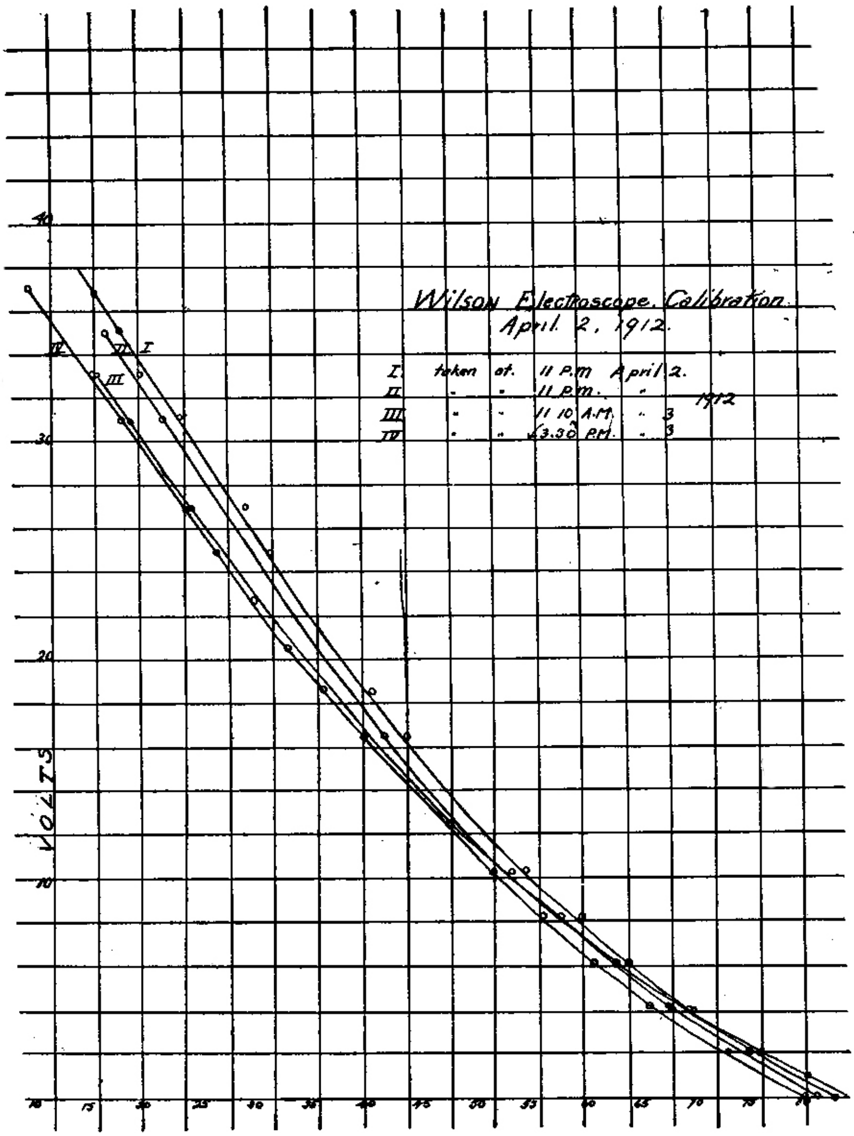
Wilson Electroscope. Calibration.
April 2, 1912.
I. taken at. 11 P.M April 2.
II " " 11 P.M. "
III " " 11 10 A.M. " 3
IV " " 3.30 P.M. " 3
1912
VOLTS
40
30
20
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80

FIG. 6.

will be seen that this method of measuring high resistances of any kind, insulators for example, is very advantageous, for currents as small as 10^{-14} amperes, can be measured with ease by the *Dolezalek* electrometer, while the *Wilton Tilting* electroscope can be made very sensitive. For electrolytes with low resistances, the current could be measured by a galvanometer and the potential by the electrometer, since the latter as a current measure would be too sensitive for use in this case.

Preparation of the Ice.

Pure water was obtained with a resistivity of about 1×10^6 ohms.

After cleaning the "U" tube very carefully—first, in a solution of potassium bichromate and sulphuric acid, then in an alcohol-ether solution and then washing several times in distilled water, both hot and cold—the pure water was put in the tube, its resistance carefully determined and temperature noted. The specific resistance of a sample of this water was then determined by the *Kohlrausch* method, the temperatures being the same in the two cases. From these results the "cell constant" of the apparatus can be calculated.

The "U" tube was then placed inside a cylindrical glass vessel, about 15 cm. in diameter and 45 cm. in height. This was then placed in an earthenware-jar, which in turn was surrounded by an ice-salt mixture, contained in a bucket. The while apparatus was kept in a refrigerator.

A thermo-couple (Fig 7) consisting of a German-silver-iron junction, was used to measure the temperature of the ice, which was formed in the "U" tube. The junction was enclosed in a capillary tube, which could be slipped in and out of one of the electrode tubes, "a ", (Fig. 5). For a diagram of the connections of the thermo-couple see Fig. 7. A very careful calibration of this instrument was made over the range of

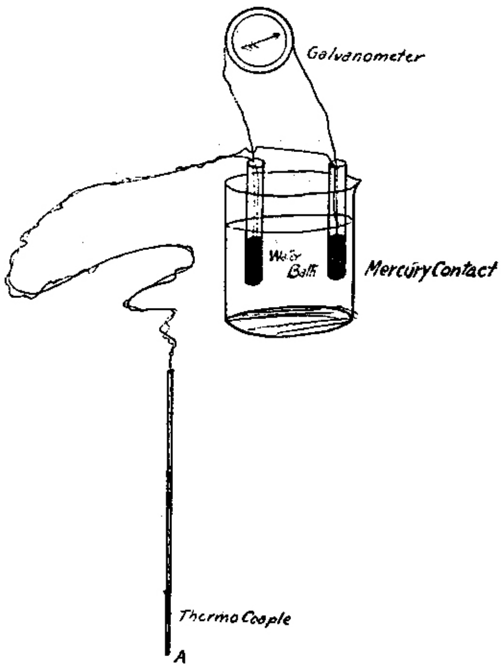

FIG. 7.

temperatures through which it was to be used in the experiment, viz.—from 5° to —12°C.

It was found very difficult to freeze the water in the "U" tube in the method described, without the tube being broken by the expansion of the ice. To obviate this difficulty a piece of rubber tubing, about .5 cm. in diameter and 15 cm. in length, and very carefully cleaned by boiling, etc., was closed at one end, by means of a glass stopper. The other end was also closed with the exception of a small hole, the size of a pin-head. This tubing was placed inside the "U" tube, care being taken to prevent any quantity of water entering through the opening in the one end. When the water expands on freezing, it can be seen that a certain amount of freedom is allowed it, by its being able to push in the lateral surface of the rubber tubing, the air inside escaping through the pin-hole. It was found that the water could be frozen with ease in this way without the glass tube being broken.

The electrometer was then calibrated. In this experiment, the needle was changed to a potential of about 200 volts. The electroscope was then calibrated over the range at which it was to be used.[1]

To obtain a set of readings at different temperatures, the "U" tube was connected up with the electrometer and electroscope and source of current, (Fig. 5). In most of the experiments the current was obtained from 10 storage cells, which gave an E. M. F. of about 20 volts. The "U" tube was very carefully packed in the glass vessel with "felt" and so temperature changes were slow. The time, in seconds, for the electrometer needle to pass over 100 scale divisions, was recorded on a stop-watch. The temperature of the ice was then read from the thermo-couple. The potential difference of the two electrodes, c and c_1 (Fig. 5), was then determined from the electroscope readings at these points.

If $d \equiv$ the scale divisions passed over per second by the electrometer needle;

D $\equiv$ the number of divisions per volt, and C $\equiv$ the capacity of the system in microfarads, then the current, i, passing through the ice will be,

$$\frac{C\,d}{10^6\,D} \text{ amperes.}$$

If $V \equiv$ the potential difference of the two electrodes c and c_1, then R, the resistance of the electrolyte between c and c_1 will be,

$$\frac{V \times 10^6 \times D}{C\,d} \text{ ohms.}$$

If k denote the cell constant of the apparatus then the specific resistance of the ice will be,

$$\frac{V \times 10^6 \times D \times k}{C\,d} \text{ ohms.}$$

1. See Fig. 6.

In the following table column I gives the time at which the readings were taken; column II the readings of the electroscope, when the gold leaf was connected with each one of the potential electrodes, c and c_1; column III, the time taken for the electrometer needle to pass over 100 scale divisions; column IV, the temperature of the water bath in Fig. 7, the reading of the galanometer, and the calculated temperature of the ice; column V, the electromotive force used in the experiment; column VI, the capacity of the system; column VII, the calculated resistance and column VIII, the specific resistance of the ice. In the experiments performed, readings were taken as the temperature of the ice decreased to the minimum temperature for the particular salt and ice mixture in the outer vessel, and then as the temperature rose up to zero.

TABLE IV.

RESISTANCE OF ICE

CELL. CONSTANT = .315 April 2, 1912

TIME	ELECTROSCOPE I.	ELECTROSCOPE II.	ELECTRO-METER	TEMPERATURE I. Th. Cp	TEMPERATURE II. Th. Cp	Cal.	Temp	E. M. F.	Cap.	SPECIFIC RES. (Ohms.)
4.10	15-83	61.1-83	1m. 24 s.	126	125	16.9	19.3°	41.2	0.05	3 86x10^9
4 30	"	"	1m. 25 s.	127.8	125.5	17.0	19.7	"	"	3.90x10^9
4.40	"	"	1m. 28 s.	128	125.5	16.9	19.9	"	"	4.04x10^9
5.55	"	64.5-83	1m. 30 s	127	122	16.9	19.6	"	"	4.13x10^9
7 40	14 5-80	62.0-80	1m. 17 s.	121½	"	16.3	18.6	"	"	3.56x10^9
8 00	82-15	62.5-82	1m. 13 s.	120.2	118.8	16.4		"	"	3.47x10^9
9.00	15-81.5	59.0-81	1m. 0 s.	117.0	117.0	16.4	17.2	"	"	2.69x10^9
10 00	16-81 3	58-81-5	45 s	112.6	112.0	"	15 9	"	"	1.98x10^9
10.04	81 5-16	"	46 "	111.1	111.0	"		"	"	
10 15	"	"	44 "	112.1	111.6	"	15.8	"	"	1.94x10^9
10.20	81.3-16.5	58-81.3	45 "	111.0		16.4		"	"	
10.40	"	"	40 "	109.5	110.0	"		"	"	
11.00	14.5-81	55.5-81	22 "	95.0	93	"	10.8	"	"	
"	"	"	89 "	"	"	"	"	"	0.2	0.947x10^9
11.10	"	"	90 "	94.5	"	"	10.6	"	"	0.958x10^9
11.17	15 81.5	55-81.5	88 "	93.0	91.5	16.3	10.3	"	"	0.936x10^9
11.40	"	"	88 "	90		"	9.4	"	"	0.852x10^9
12.00	16-81.5	53.8-81.5	41½"	75	73.5	16.8	4.5	"	"	0.457x10^9
12.15	"	"	39 "	72.5	72.0	16.75	3.85	"	"	0.430x10^9
12.25	15-81.5	54.2-81.5	38½"	71.5	70.0	16.8	3.5	"	"	0.410x10^9
A. 3.										
10.15	23-80.5	67 80.5	48 "	77.3	76.0	14.8	7.1	"	"	0.464x10^9
10.25	"	"	" "	77.1	76.0	"		"	"	
11.10	20-82.5	64-82 5	79 "	62.1	61.0	15.2	2.3	"	1.00	0.169x10^9
11.15	"	66-82.5	85 "	63.0	61.3	15.2	2.6	"	"	0.181x10^9
11.33	"	"	36 "	77.5	77.0	15.3	6.8	"	0.20	0.390x10^9
11.37	"	"	39 "	81.0	79.1	"	7.8	"	"	0.423x10^9
11.47	21.6-82.6	62-82.6	21 "	56.8	55.6	15.4	0.6	"	1.00	0 0386x10^9
11.55	"	"	19½"	55.0		"	0.1	"	"	0.0349x10^9
12.00	"	"	19 "	"		"		"	"	
12.10	20.0-82.4	58-82.4	21 "	55.1	57.0	"	0°	"	"	0.0367x10^9

It was found that the rubber tubing in the "U" tube affected the conductivity of the contents to quite an extent, and in this particular experiment the water was frozen without the presence of the rubber, and also in the suceeding experiments. However, the general shape of the temperature resistance curve was found to be the same in every experiment performed.

The water, which was put in the cell originally, had a specific resistance of about 1.4×10^6 ohms at 17.9°C.

In the measurement of the temperature of the ice between the "potential" terminals, by the thermo-couple, it was found that there was a considerable temperature-gradient in the "U" tube in the direction of its length, although the tube was enclosed in three separate vessels, in a refrigerator and closely packed with felt. The junction of the thermo-couple was enclosed in a capillary-tube, (see Fig. 7), so that it could be placed inside the "potential-terminal-tube," c_1 (see Fig. 5). A reading of the galvanometer was taken when the end of the capillary tube reached the end of the "potential-terminal-tube." The capillary tube was then drawn up about 3 centimeters or so and another galvanometer reading was made. The average of these two readings was taken as the average temperature of the ice between the potential terminals.

When the water was frozen, cracks were observed through the ice in the "U" tube, and it was found impossible to obtain ice at low temperatures, by freezing in the tube, without the cracks

These cracks may have a considerable effect on the resistance of the ice. So the accuracy of the values of the specific resistance as given in this paper, is limited by this uncertainty.

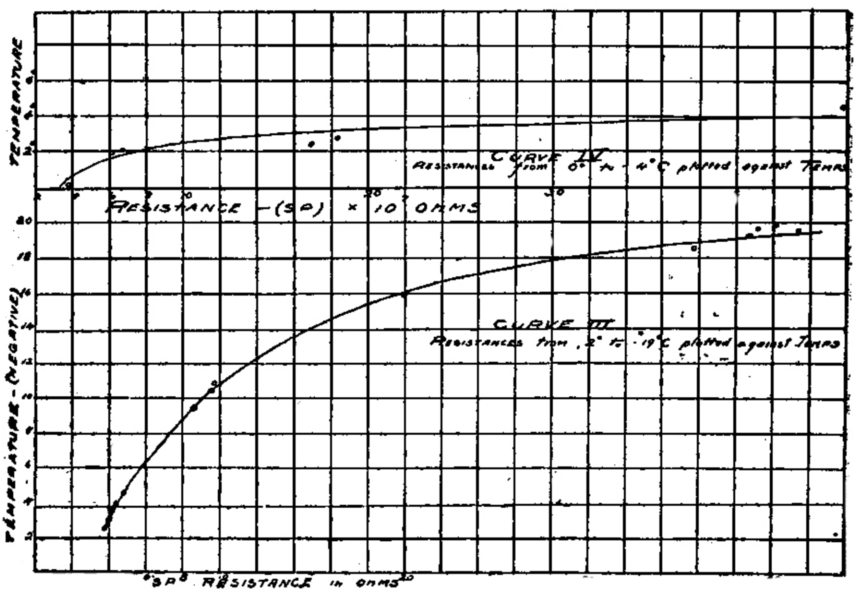

FIG. 8.

A temperature-resistance curve is shown in Fig. 8, and it will be observed that it is nearly *expotential.* This was found to be the case in every curve of six, plotted.

The specific resistance of the ice was found by multiplying the resistance of the ice between the "potential-terminals" by the "cell-constant"[1] of the tube.

To determine the temperature co-efficient of the resistance at different temperatures, the *cotangent* of the temperature-resistance curve, (Fig. 9), was determined graphically at different temperatures, and this was divided by the resistance of the ice at this point. Thus if R_t is the specific resistance at temperature t then the temperature co-efficient at this temperature will be $\frac{1}{R_t} \times \frac{d\,R_t}{d\,t}$. For the temperature co-efficient curve, see Fig. 9.

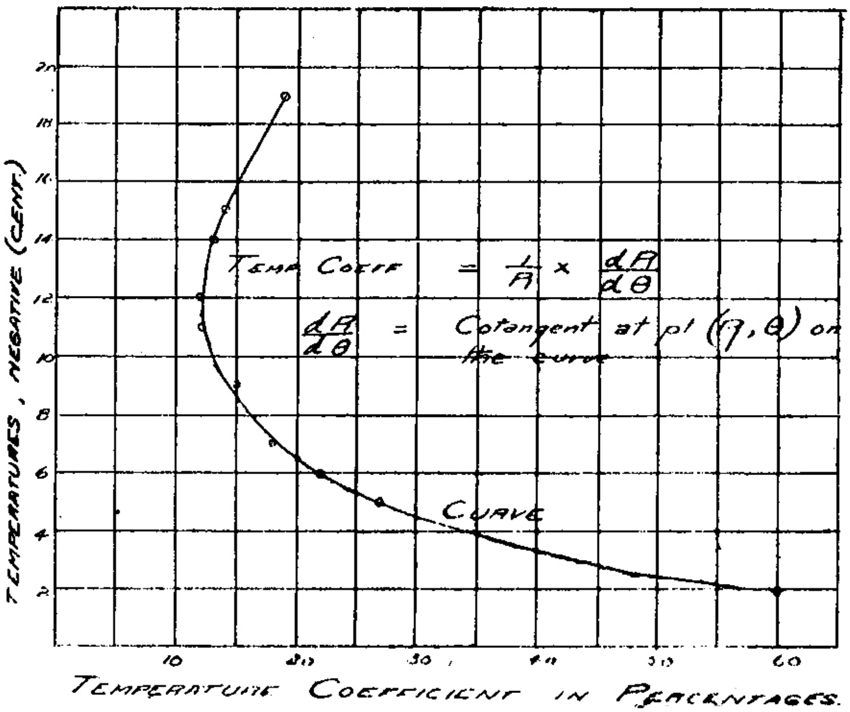

FIG. 9.

1. Kohlrausch, F., Physico-Chem. Measurements.

Summary.

1. The specific resistance of ice has been determined at temperatures ranging from 0° to — 19° C.

2. The effects of electrolytic-polarization have been eliminated by the method used.

3. *The value of the temperature-co-efficient of the resistance of ice has been determined at different temperatures and its value has been found to be very much higher than the temperature-co-efficient of ordinary electrolytes. It decreases in value as the temperature decreases from zero.*

The values obtained for the specific resistance of ice compare fairly well with those obtained by Ayrton and Perry, using a different method.

In conclusion I wish to thank Dr. H. L. Bronson, who suggested this work, and without whose kind supervision and assistance, this research could not have been undertaken.

Dalhousie University, Halifax, N. S.
April 20th, 1912.