OPTIMIZATION OF THE SELECTIVE MAINTENANCE PROBLEM FOR LARGE-SCALE SYSTEMS UNDER UNCERTAINTY

by

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To my parents.

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Abstract

As complex engineering systems have become more prevalent in recent years, the importance of reliability and maintenance has become increasingly apparent. Maintenance is essential to keep systems functioning properly and ensure that they perform their intended functions. However, limited resources such as budget, time, and repairperson availability can make it challenging to perform all necessary maintenance. In these situations, it is crucial to optimally allocate available maintenance resources to carefully selected components within a system and perform the necessary maintenance actions in order to ensure satisfactory performance after maintenance. Such a maintenance policy is called selective maintenance (SM). When tasks are assigned to multiple repairpersons, potentially with different skill levels and costs, it is referred to as the joint selective maintenance and repairperson assignment problem (JSM–RAP).

This dissertation explores four themes dealing with the optimization of JSM–RAP for large-scale systems under uncertainty. The dissertation starts with the first theme which provides a critical review of SM literature, identifying challenges and potential areas for future research. The second theme introduces four column-generation-based algorithms to effectively address the JSM–RAP for large-scale systems. The third theme presents a piecewise-linear-approximation-based approach (PLA) and a distributionally robust chance-constrained program with a Wasserstein ambiguity set (DRC-W) to handle uncertain maintenance duration in large-scale instances of the JSM–RAP. The fourth theme reformulates the JSM–RAP as a mixed-integer exponential conic program before a robust optimization framework is used to capture the maintenance quality uncertainty through non-symmetric budget uncertainty sets allowing the level of decision-maker conservatism to be controlled.

The proposed JSM–RAP models are applied to several illustrative examples. The results demonstrate the effectiveness and advantages of the proposed models.

List of Abbreviations Used

BIP	Binary Integer Programming
BP–PLA	CG–PLA embedded into a Branch-and-Price-procedure
BP–ECO	CG–ECO embedded into a Branch-and-Price-procedure
CC	Chance Constraint
CG	Column Generation
CG–PLA	CG method using PLA to solve nonlinear subproblems
CG–ECO	CG method using ECO to solve nonlinear subproblems
CG-SMP	Column Generation algorithm for the SMP
CM	Corrective Maintenance
CR	Corrective Replacement
CV	Coefficient of Variation
CVaR	Conditional Value at Risk
DAS	Distributional Ambiguity Set
DN	Do Nothing
DRCC	Distributionally Robust Chance-constraint
DRC-W	DRCC Program with a Wasserstein Ambiguity Set
DRO	Distributionally Robust Optimization
ECO	Exponential Conic Optimization
IM	Imperfect Maintenance
JSM–RAP	Joint Selective Maintenance and Repairperson Assignment Problem
KP	Knapsack Problem
LP	Linear Program
MdMCKP	Multidimensional Multiple-choice Knapsack Problem
MINLP	Mixed Integer Nonlinear Programming
MIECP	Mixed Integer Exponential Conic Program
MR	Minimal Repair
PCC	Partial Concave Conjugate
PLA	Piecewise Linear Approximation
PM	Preventive Maintenance
PR	Preventive Replacement
RMP	Restricted Master Problem
RO	Robust Optimization
SM	Selective Maintenance
SMP	Selective Maintenance Problem
SP	Sub-problem
VaR	Value at Risk

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Chapter 1

Introduction

In today's fast-paced and highly competitive business environment, maintenance has become a central element for ensuring the efficient and reliable operation of production-distribution assets. The ability for firms to compete successfully depends on the ability of production systems and equipment to perform at high levels of quantity and quality. The increasing demands for improved product quality, reduced throughput time and enhanced operating effectiveness within a rapidly changing customer demand environment continue to require a high level of maintenance performance (Ben-Daya et al., 2009).

Maintenance is defined as the combination of all technical and administrative actions including supervision actions intended to retain an item in, or restore it to, a state in which it can perform its required function. This can include activities such as inspection, cleaning, testing, adjustment, and replacement of worn or defective parts, and the overhaul or rebuilding of equipment (Saraswat and Yadava, 2008). Coordinating maintenance activities with production schedules to maintain high performance can be challenging as maintenance often needs to be done while equipment is not in use, potentially disrupting production. To address this challenge, many modern systems in manufacturing, production, and service industries use alternating sequences of scheduled missions and break periods for maintenance (Cao et al., 2018a).

During scheduled breaks, maintenance actions are performed on components to improve the system's ability to successfully perform its subsequent missions. However, the limited maintenance time and budget, spare parts and repair crews availability during these breaks can restrict the number and levels of maintenance activities that can be performed. Without effective maintenance planning, these systems are at risk of unexpected failures that can lead to not only increased downtime, decreased reliability, costly repairs and replacements, and lost revenue, but also injury or loss of human life. The decision problem that aims to prevent these risks by selecting the components to maintain and the level of maintenance actions to carry out within the scheduled maintenance break is known as the *selective maintenance problem* (SMP) (Rice et al., 1998).

The SMP has its origins in a collaborative project between C. Richard Cassady, a professor of Industrial Engineering at the University of Arkansas, and the US Air Force Research Laboratory. The project, which took place in the late 1990s, aimed to develop a modelling methodology for managing SM decisions and integrating maintenance planning and sortie scheduling for the Air Force (Cassady et al., 2003, 2004, 2005a,b). In 1998, Wanda Faye Rice, a graduate student working on the project, investigated the SMP in her thesis titled “Optimal Selective Maintenance Decisions for Series Systems" (Rice, 1999). This is assumed to be the first known *academic publication* on the topic of the SMP as currently known and modelled. An empirical definition of selective maintenance can be traced back to Fisher (1965) where a conceptual framework of the SM is proposed at a high level without any formal modelling.

Rice’s study (Rice, 1999), along with other early studies in the field, considered a basic version of the SMP in which a small-scale system made up of identical components operating in alternating sequences of scheduled missions and break periods for maintenance, with a single repair channel that can only perform one type of maintenance action (replacement) with the aim of maximizing the system reliability. These assumptions were necessary for a foundational understanding of the concept, but they do not accurately reflect the complexities and uncertainties of real-world problems. As the field progressed, over the last two decades researchers began to incorporate more realistic assumptions and consider a broader range of factors and provide extensions to system structures, maintenance policies, resource limitations, modeling methods, and solution algorithms to make the models more applicable in real-world situations such as wind turbines (O’Neil et al., 2022a), coal conveyor systems (Liu et al., 2009), machining lines in an engine shop (Zhu et al., 2011), army tanks (Sharma et al., 2017), nuclear fuel production systems (Zhao et al., 2019b), aircraft turbine engine systems (Wang et al., 2019), and flow transmission systems (Liu et al., 2020).

Despite recent advancements in formulating and solving the SMP, several limitations that restrict practitioners’ ability to implement it remain. Limitations such as difficulty in quickly solving large-scale problems as encountered in many industrial and military settings and the lack of models dealing with uncertainty in key maintenance parameters are some of the most pressing ones.

Recently, a substantial extension of the SMP that addresses several complexities of real-world systems was proposed by Diallo et al. (2019b). This extension called the *Joint Selective Maintenance and Repairperson Assignment Problem* (JSM-RAP) allows maintenance actions to be performed by multiple repairpersons with varying skill levels and costs. These repairpersons can perform a spectrum of maintenance actions ranging from minimal repair (MR) to replacement. The JSM-RAP jointly determines the selection of maintenance actions and the assignment of tasks to repairpersons, providing a more comprehensive solution for real-world systems. A novel approach called the two-phase method was developed by Diallo et al. (2018) to transform the non-linear JSM-RAP into a multi-dimensional multiple-choice knapsack problem (MdMCKP) by generating all feasible combinations of components, maintenance levels and repairpersons. This method optimally selects a subset of patterns to minimize the total maintenance cost or maximize the reliability for the next mission. However, the proposed model (JSM-RAP) still shares similar limitations with other models in the SM literature, specifically their inability to handle large-scale systems and address uncertainty in maintenance parameters. These limitations mainly stem from the lack of tractable formulations for the SMP.

With limited failure events and/or short usage periods of the components, it is challenging to accurately estimate the probability distributions of key parameters with only a limited amount of maintenance records. To that extent, risk-neutral stochastic or fuzzy models, which are the current approaches used in literature to address the uncertainty in the SMP, may not be suitable as they tend to have poor out-of-sample performance when the training sample size is small (Smith and Winkler, 2006), are generally intractable when evaluating expected loss functions (Hanasusanto et al., 2016), and do not take potential negative scenarios or provide any performance guarantees (Birge and Louveaux, 2011), all of which are imperative for mission-based SMP. Alternatively, Robust Optimization (RO) is a promising approach for efficiently dealing with uncertainty whilst assuring worst-case system reliability without the need for large sample sizes, yet tractable formulations remain an essential requisite to make RO a successful tool for large-scale instances of the SMP which has been proved to be NP-hard (Rice, 1999). To address these challenges, this dissertation proposes six novel formulations that can handle the JSM–RAP for large-scale systems and addresses the uncertain nature of the duration and quality of maintenance actions using RO-based techniques.

1.1 Research themes

This dissertation explores four themes dealing with the optimization of the JSM–RAP for large-scale systems under uncertainty. Each theme is developed in a dedicated chapter. Theme 1 introduces a comprehensive critical review of the literature on the SMP and identifies challenges and potential opportunities for future research. Theme 2 presents four algorithms based on column generation (CG) to solve the JSM–RAP in large-scale multicomponent systems. Themes 3 and 4 explore methods to optimize solutions for large-scale instances of JSM–RAP when the maintenance duration and the quality of maintenance actions are uncertain, respectively.

1.1.1 Theme 1: A critical review of selective maintenance for mission-oriented systems: Challenges and a roadmap for novel contributions

Despite many extensions to the SMP having been proposed in recent years, only two literature reviews on the subject have been conducted. The first by Xu et al. (2015) covered papers published between 1998 and 2014, and the second by Cao et al. (2018a) covered papers up to 2017. However, there has been a significant surge in research in this field in recent years along with new advancements in robust optimization and machine learning. This necessitates conducting an up-to-date critical review of the literature and creating a roadmap for future developments in SMP models to make them more relevant and effective in addressing industry-scale problems.

Theme 1 provides a comprehensive critical review of the literature in the field of SM, with a review of a total of 119 research articles related to SM optimization. These references are reviewed using a systematic classification and analysis framework. A selection of notable models is discussed in depth. Additionally, the challenges and limitations of current methods are identified and opportunities for future research are presented. We address the following research questions.

1. How has the literature on the SMP developed over the last two decades?
2. How can the characteristics of the SMP be categorized?
3. What are the main contributions and limitations of the existing literature?

4. What are the notable models in the literature of SM?

5. What are the challenges, gaps and opportunities for future research that can improve the academic and industrial contributions of the SMP?

1.1.2 Theme 2: Branch-and-price algorithms for large-scale mission-oriented maintenance planning problems

The main challenge encountered in most SMP models is that their formulations are difficult to solve optimally, particularly for industrial-size problems that would allow practitioners to implement and use the SMP for their production and service assets. (Rice, 1999) proved that the basic SMP is NP-hard, and so are all its extensions, meaning that computational efforts increase exponentially with problem size. The large-scale instances of the problem, and in particular its JSM–RAP extension, are still challenging to solve due to their combinatorial and nonlinear nature. Therefore, novel reformulations, approximations, and solution methods that can handle real-life systems consisting of hundreds of multicomponent assets are still needed (Cao et al., 2018a; Diallo et al., 2019b).

Under Theme 2, four algorithms based on CG are developed to address the JSM–RAP for large-scale systems. The approach involves breaking down the JSM–RAP into a master problem and multiple subproblems that are solved to generate maintenance patterns, also known as columns. Two methods are developed to handle the mixed-integer nonlinear subproblems: a piecewise-linear approximation (PLA) and an exact reformulation into mixed-integer exponential conic programs (MIECP). Branch-and-price (B&P) algorithms are developed by embedding the CG method into a branch-and-bound tree to restore solution integrality and guarantee its optimality. Accordingly, the four proposed CG-based algorithms are: the CG method using a PLA to solve the nonlinear subproblems (CG–PLA), the CG method that utilizes exponential conic optimization (ECO) to solve the nonlinear subproblems (CG–ECO), the CG-PLA approach integrated into a branch-and-bound (B&B) process (BP–PLA), and the CG-ECO approach integrated into a B&B process (BP–ECO). A heuristic procedure is developed to quickly provide a feasible solution to initiate the column-generation algorithm. Furthermore, a stabilization scheme is used to accelerate convergence. Numerical experiments validate the proposed approach and demonstrate its added value in terms of computation time and solution quality.

The main goal the chapter answers is to develop new methods to deal with the JSM–RAP for large-scale systems. The following questions are considered:

1. How to efficiently decompose the JSM–RAP into a master problem and multiple subproblems and generate maintenance patterns (columns)?

2. How to handle the mixed-integer nonlinear subproblems obtained when applying CG to the JSM–RAP?

3. How to embed the CG method into a branch-and-bound tree to restore solution integrality and guarantee its optimality?

4. How to use a stabilization scheme to accelerate the convergence of the proposed CG approach?

1.1.3 Theme 3: Distributionally-robust chance-constrained optimization of selective maintenance under uncertain repair duration

Most SMP models assume that the duration of maintenance actions is known in advance, but this assumption is unrealistic due to factors such as variability of component conditions and repairperson skill levels, which can lead to low system reliability or long overtime (Khatab et al., 2017a). To address this uncertainty, most papers in the literature use stochastic programming. Despite its intuitive risk-neutral attitude and favorable convergence properties that make it a popular modelling approach when dealing with random factors in many application areas (Birge and Louveaux, 2011), this approach has limitations such as poor out-of-sample performance, especially when the size of the training sample is small (Smith and Winkler, 2006), and intractability due to the need to compute multivariate integrals (Hanasusanto et al., 2016). These limitations are particularly relevant in the maintenance industry where failure events can be infrequent and there may not be enough data to accurately fit distribution functions.

Theme 3 introduces the PLA-based approach and a distributionally robust chance-constrained program with a Wasserstein ambiguity set (DRC-W) to deal with uncertain maintenance duration in large-scale instances of JSM–RAP. The proposed PLA-based model linearizes the objective function of JSM–RAP and ensures effective maintenance plans can be determined with a high probability of completion. Numerical experiments show the added value of the proposed approach in terms of computation time and the overall quality of solutions.

The main goal of the chapter is to propose a new methodology to address uncertain maintenance duration for large-scale instances of JSM–RAP with limited maintenance records. The following questions are addressed:

1. How can the nonlinear JSM-RSP be tightly approximated as a mixed-integer linear program (MILP) that can effectively handle large-scale instances of the JSM–RAP?
2. How can probabilistic guarantees for maintenance plan completion be achieved in the JSM-RSP?
3. How to capture uncertain maintenance duration through a Wasserstein ambiguity set in JSM-RSP?
4. What advantages does the DRC-W provide over the deterministic case?

1.1.4 Theme 4: Robust selective maintenance optimization under maintenance quality uncertainty

The majority of SM models assume that the post-maintenance reliability of a component is fully determined by the maintenance level selected for it (*i.e.*, the maintenance quality is deterministic). However, this hypothesis turns out to be inaccurate since the quality of maintenance can be swayed by a range of aspects not part of the planner's responsibility, including the proficiency of the technician performing the maintenance, the instruments and methods utilized, operating conditions, and other uncontrollable variability-inducing factors.

Theme 4 introduces the optimization of the JSM–RAP when the quality of maintenance actions is uncertain, thus leading to uncertain post-maintenance reliability of system components. The robust optimization (RO) framework is used to capture the maintenance quality uncertainty via non-symmetric budget uncertainty sets, which allow the level of decision-maker conservatism to be controlled. Both the nominal (deterministic) and robust problems are reformulated as mixed-integer exponential conic programs that can be solved using off-the-shelf solvers. Numerical experiments on benchmark instances demonstrate the favorable computational performance of the proposed reformulations and the importance of considering maintenance quality uncertainty when creating SM plans.

The main goal of this chapter is to propose a formulation to handle uncertain maintenance quality in large-scale instances of the JSM–RAP. Three research questions are considered.

1. How can JSM-RSP be exactly reformulated as a MIECP that can be effectively handled by modern off-the-shelf solvers?
2. How the uncertainty of the post-maintenance reliability of the components could be addressed in JSM–RAP?
3. How can the robust version of JSM–RAP be tractably reformulated as a MIECP problem?

1.2 Dissertation outline

This dissertation is a thesis by articles and is comprised of four manuscripts (one is published, one is resubmitted after revisions and two are submitted for publication). After the introduction in Chapter 1, Chapter 2 presents a comprehensive critical review of the literature on the SMP. As shown in Figure 1.1, models in the SM literature are identified, classified, and analyzed using a systematic classification and analysis framework. A selection of notable models is discussed in depth. Additionally, the chapter identifies the challenges and limitations of current methods and presents opportunities for future research. Results of this chapter have been submitted for publication in a peer-reviewed journal. A preliminary condensed version of this review work was presented at the 10$^{\text{th}}$ IFAC Conference on Manufacturing Modelling Management and Control 2022 and published in the proceedings of the conference as Al-Jabouri et al. (2022).

Chapter 3 presents four CG-based algorithms to tackle the JSM–RAP for large-scale systems as shown in Figure 1.1. A manuscript resulting from this chapter has been published in *Computers and Operations Research* (Al-Jabouri et al., 2023).

Chapter 4 introduces the PLA-based approach and a distributionally robust chance-constrained program with a Wasserstein ambiguity set to deal with uncertain maintenance duration in large-scale instances of JSM–RAP. A preliminary version of this chapter was presented at the ECSO-CMS 2022, Joint European Conference on Stochastic Optimization – Computational Management Science Conference. An extended version was submitted for publication in a peer-reviewed journal.

Chapter 5 introduces the optimization of the JSM–RAP when the quality of maintenance actions is uncertain, thus leading to uncertain post-maintenance reliability of system components. The robust optimization framework is used to capture the maintenance quality uncertainty via non-symmetric budget uncertainty sets, which allows the level of decision-maker conservatism to be controlled. Both the nominal and robust problems are reformulated as mixed-integer exponential conic programs that can be solved using off-the-shelf solvers. A manuscript resulting from this chapter has been submitted for publication in a peer-reviewed journal.

Chapter 6 draws the conclusions of this work by summarizing important observations and introducing directions for future work based on the outcomes of the research.

Figure 1.1 below gives an overview of the organization of this dissertation and the links between the four research contributions.

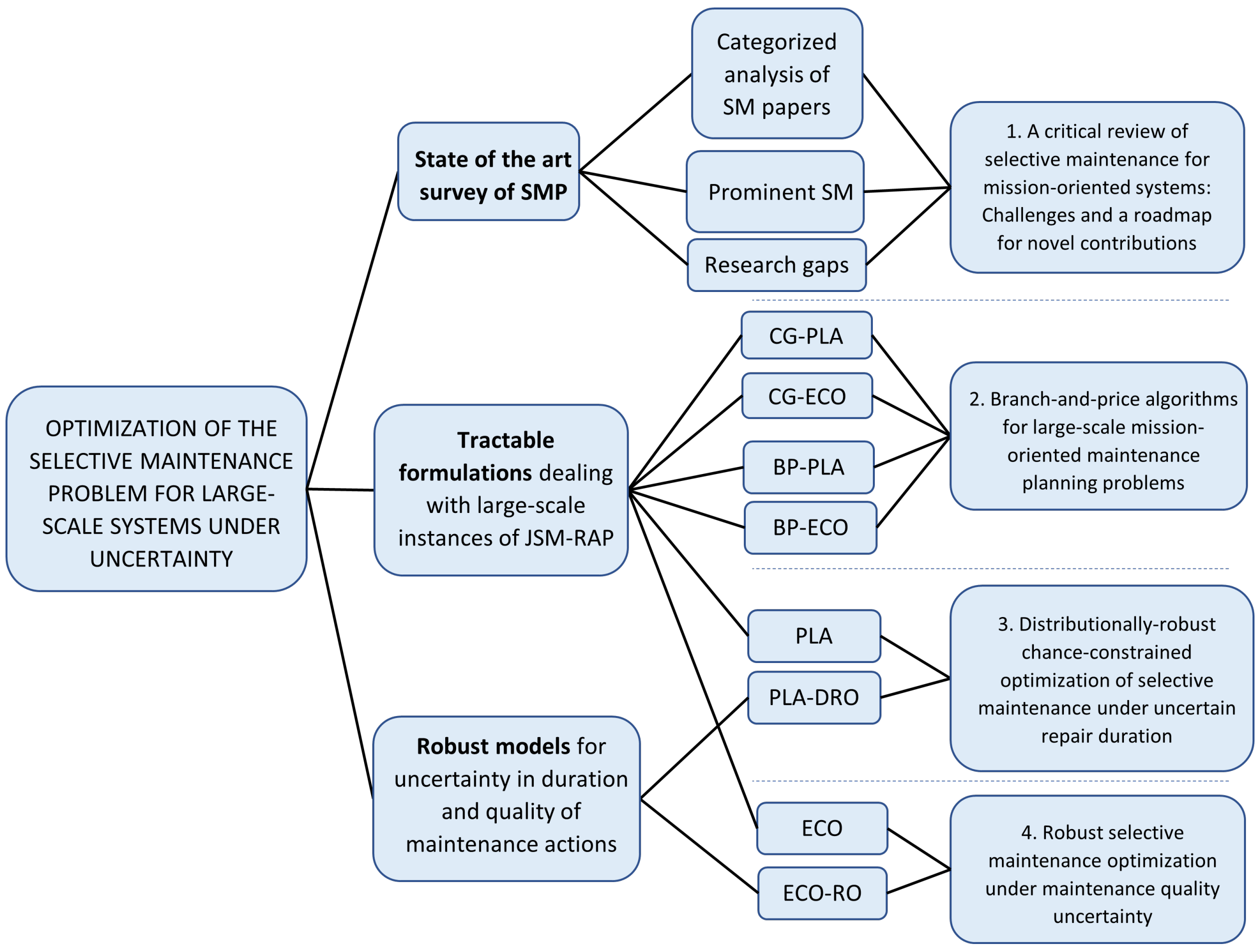

Figure 1.1: Dissertation outline

Chapter 2

A critical review of selective maintenance for mission-oriented systems: Challenges and a roadmap for novel contributions

2.1 Introduction

Maintenance is an essential operation for industrial and economic assets as its costs can account for a significant percentage of a firm's production costs (Bevilacqua and Braglia, 2000). Adequate maintenance strategies can prevent unexpected production interruptions, lower spare parts inventory and operational costs, and even increase the lifespan of industrial machines. Therefore, maintenance engineering and optimisation have been widely studied and many interesting and significant results have been produced for a wide range of maintenance optimisation models. Over the last 20 years, a growing number of mathematical models have been developed in the literature for the design of optimal maintenance policies for repairable systems.

Many modern systems operate in an alternating sequence of missions and break periods during which maintenance actions are performed. These mission-oriented multi-component systems are commonly found in manufacturing, production, and service industries such as production lines, aircraft, ships, and trucks that operate continuously until their missions are interrupted to undergo maintenance. Similarly, new asset types such as unmanned autonomous vehicles and advanced combat/defensive systems also exhibit such patterns. The selective maintenance (SM) strategy is particularly suited for these mission-oriented systems. To enhance the ability of such systems to successfully complete their subsequent missions, maintenance actions are carried out on components during scheduled breaks. However, limited maintenance resources such as time, budget, spare parts, and repair crews restrict the number and levels of maintenance activities that can be performed before the next mission. The optimal selection of components to maintain and the level of maintenance actions to perform is known as the selective maintenance problem (SMP) and can be traced back to (Rice et al., 1998) and (Rice, 1999). The

SMP is a prevalent problem in many industrial systems such as wind turbines (O'Neil et al., 2022a), coal conveyor systems (Liu et al., 2009), machining lines in an engine shop (Zhu et al., 2011), army tanks (Sharma et al., 2017), nuclear fuel production systems (Zhao et al., 2019b), aircraft turbine engine systems (Wang et al., 2019), and flow transmission systems (Liu et al., 2020).

The purpose of this paper is to provide an updated and comprehensive review of the literature on the SMP up to the year 2022. A preliminary version of this review, which appeared in (Al-Jabouri et al., 2022), has been significantly extended by adding many more references and expanding the discussion. The review makes the following contributions:

1. A comprehensive and categorized analysis of SM papers based on a systematic classification and analysis framework.

2. A mapping of key characteristics of the SM papers.

3. A review of prominent SM maintenance models along with the necessary assumptions and solving methodologies.

4. An examination of research gaps and suggestions for future research avenues.

Despite the many extensions proposed for SMP in the past two decades, only two limited literature reviews have been published, one by Xu et al. (2015) covering papers published between 1998 and 2014, and another by Cao et al. (2018a) covering papers published up to 2017. However, since then there has been a significant growth of research in the field of SM as the number of publications between the years 2018 and 2022 (54 out of 119) is nearly equivalent to the total number of research articles published before 2018. In addition, new advances in robust optimisation approaches and machine learning have emerged in the SM context, thus motivating the authors to undertake an updated critical review of the literature and propose a road-map for future developments of the SMP models to make them more relevant and address industry-scale problems.

The remainder of this paper is organized into four sections. In Section 2.2, the scope, research procedure, and review framework are introduced. In Section 2.3, the identified

SMP papers are classified according to two SMP feature categories: *formulation characteristics* which is composed of three sub-groups of characteristics related to system, maintenance and mathematical model characteristics, and the *solution approaches* which are categorized depending on whether they are exact or approximate methods. Prominent SM maintenance models along with the necessary assumptions and solution methodology are addressed in section 2.4. Finally, in Section 2.5, research gaps identified are highlighted and discussed, and a structured list of future research directions is outlined.

2.2 Scope and review methodology

This paper conducts a comprehensive review of research articles pertaining to the SMP published up to 2022. The review includes both a categorization of the articles based on SMP features and a detailed examination of notable models that encompass a wide range of SMP characteristics. The reviewed literature includes books, peer-reviewed conference papers, and journal articles, which were sourced from various academic research databases such as Proquest, Google Scholar, Engineering Village (Compendex), and Web of Science using the keywords such as “maintenance”, “maintenance planning”, and “maintenance policy” in combination with “selective”, “problem” and/or “models”. Filtering was then applied to the more than 6,000 results returned to exclude articles from unrelated fields such as medicine, psychology, etc. A second layer of filtering was applied to remove articles dealing with general maintenance but not addressing selective maintenance as defined above. The list was thus reduced to 119 references (78 journal publications, 37 conference papers, 3 reports, and 1 thesis) published up to 2022. Figure 2.1 illustrates the distribution of papers by publication year. It can be observed that 45% of the papers (54 out of 119) were published between 2018 and 2022, indicating the significant growth of research in this field in recent years and the importance of the proposed literature review. The 3-year moving average trend plotted in Figure 2.1 also shows a more than doubled publication rate between 2016 and 2022. Figure 2.2 presents the distribution of papers by publishing journal, with only venues having two or more SMP papers included. The journal with the most SMP publications is Reliability Engineering and System Safety with 17 papers out of 119.

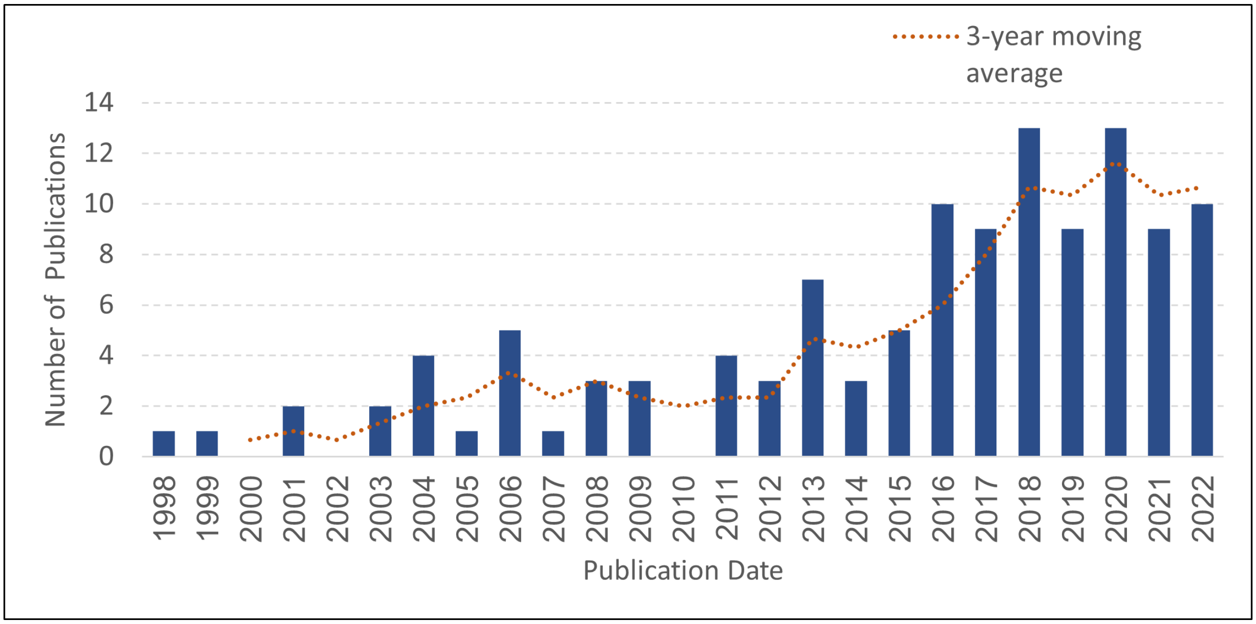

Figure 2.1: Publication year distribution of referenced papers

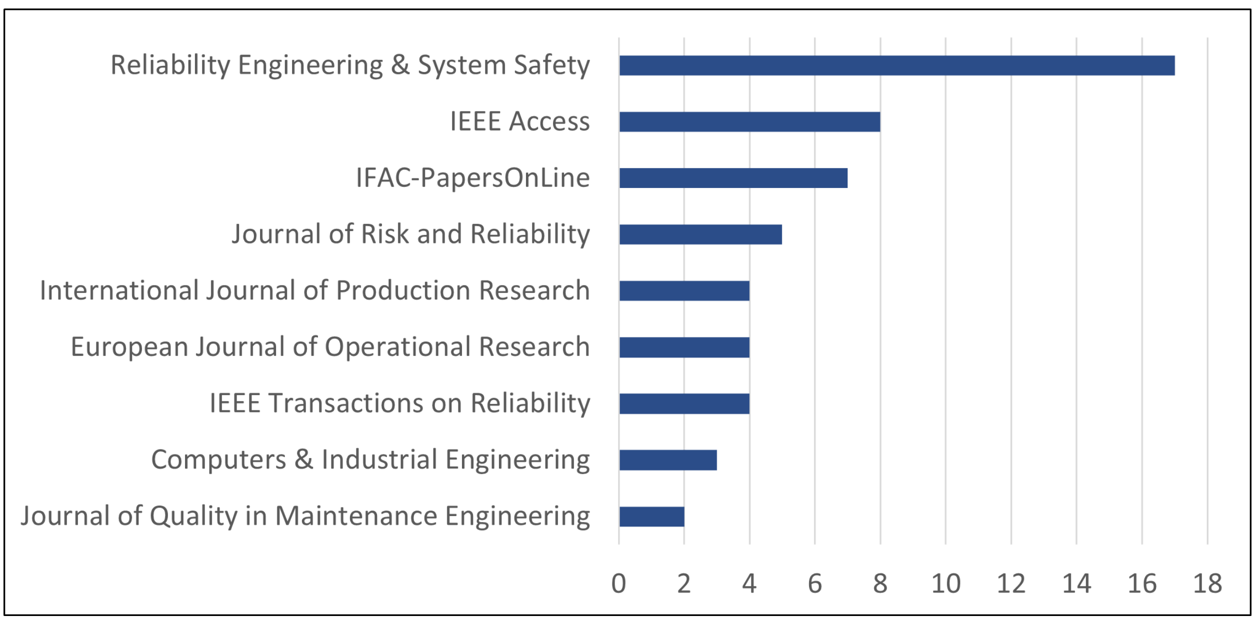

Figure 2.2: Distribution of referenced papers by journal

Table 2.1 presents the most frequently cited articles from the literature review, separated into two sections: the first section lists the overall most cited articles, while the second section lists the most cited articles that have been published in 2016 or later.

Table 2.1: Most cited articles (Google Scholar as of December 15, 2022)

	All-time most cited articles	Citations		Recent most cited articles	Citations
1	Cassady et al. (2001a)	234	1	Liu et al. (2020)	82
2	Duan et al. (2018)	230	2	Diallo et al. (2018)	58
3	Cassady et al. (2001b)	214	3	Shahraki et al. (2020)	53
4	Rice et al. (1998)	184	4	Khatab et al. (2018c)	48
5	Pandey et al. (2013b)	143	5	Jiang and Liu (2020b)	29
6	Dao et al. (2014)	130	6	Chaabane et al. (2020a)	28
7	Lust et al. (2009)	106	7	Yang et al. (2018)	27
8	Dao and Zuo (2017b)	102	8	Diallo et al. (2019b)	27
9	Liu et al. (2018)	100	9	Khatab et al. (2018b)	26
10	Pandey et al. (2013a)	97	10	Cao et al. (2018b)	20

2.3 Categorisation of SMP modelling and solution methods

To provide a comprehensive structure for the existing literature, this section is framed according to two SMP feature categories: *formulation characteristics* and *solutions approaches*. The proposed review framework is depicted in Figure 2.3. The *formulation characteristics* category is composed of three groups of characteristics related to system, maintenance and mathematical model characteristics. The *solution approaches* are discussed depending on whether they are exact methods or approximate algorithms. Table 2.3 provides a two-dimensional mapping of all papers reviewed, highlighting their key characteristics. The abbreviations used in Table 2.2 are explained in Table 2.2.

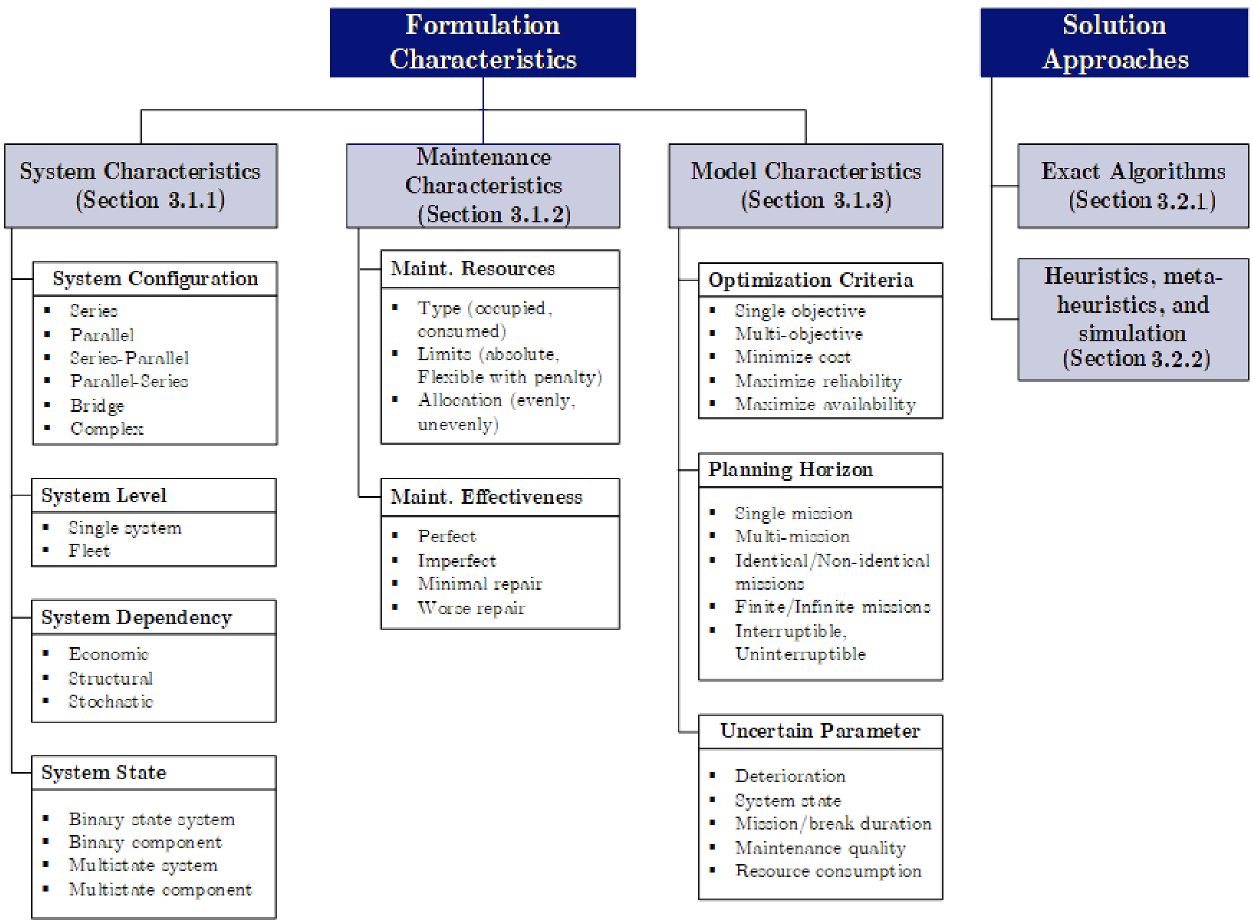

Figure 2.3: Framework of the proposed categorization of SMP modelling and solution approaches (Al-Jabouri et al., 2022)

Table 2.2: Abbreviation of terms used in the mapping table

System configuration/level		**System & components states**	
S	Series	BSS	Binary state system
P	Parallel	BSC	Binary state components
SP	Series-Parallel	MSS	Multistate system
CP	Complex	MSC	Multi state components
F	Fleet-level		
System dependency		**Maintenance degree**	
Ind	Independent	Pf	Perfect
Econ	Economic dependence	Impf	Imperfect
Stoch	Stochastic dependence	W	Worse
Struct	Structural dependence	M	Minimal repair
Planning horizon			
Id.M	Identical missions	Un.M	Unidentical missions
optimisation criteria			
R	System reliability	Av	System availability
#SM	Number of missions	#MB	Number of breaks
TP	Total system profit	UTC	Unit time cost
MC	Maintenance Cost	MT	Maintenance time
GT	Gap time	RV	Reliability variance
DT	Down Time	PC	Performance capacity
EE	Energy efficiency	ME	Maintenance energy
Exact algorithm			
TE	Total enumeration	B&B	Branch-and-Bound
2-ph	Two-phase approach	Mx&Mn	Max min approach
ECM	ϵ-constraint method		
(Meta)-Heuristics			
CH	Construction heuristic	ABC	Artificial bee colony
DE	Differential evolution	DGSA	Hybrid DE & gravitation search
EGD	Extended great deluge	EA	Exhaust algorithm
SGA	Sequential game algorithm	HA	Heuristic algorithm
GA	Genetic algorithm	NSGA-II	Non-dominated sorting GA
PSO	Particle swarm optimisation	ACO	Ant colony optimisation
TS	Tabu search	SA	Simulated annealing
CG	Column generation		
Deep learning			
DM	Data mining technique	DL	Deep learning technique
DQN	Deep Q-Network method		
Stochastic parameters			
DEv	Dynamic environment	Dtr	Deterioration
RC	Recourse Consumption	MBD	Maintenance Break Duration
MD	Mission Duration	MQ	Maintenance Quality
R.Av	Repairpersons Availability		
Notes			
Fzy prg.	Fuzzy programming	Oc. res.	Occupied resources
Size	Problem size	Inf. mis.	Infinite mission
Int. mis.	Interruptible mission	Comm. solver	Commercial solver

Table 2.3: Mapping and classification based on main characteristics

#	Article	System config.	System states	Sys. dependency	Maint. degree	Planing horizon	Optim. criteria	Stoch. param.	Notes	Solution Approaches				
										Exact alg.	Heuristic	Simulation	Solver	Deep learning
1	Rice et al. (1998)	SP	BSS,BSC	Ind.	Pf	Id.M	Max R	–	size ≥ 20	TE	HA			
2	Meng et al. (1999)	SP	MSS,MSC	Ind.	Impf, M	Id.M	Min MC	–		TE				
3	Cassady et al. (2001a)	SP	BSS,BSC	Ind.	Pf, M	Un.M	Max R	Dtr.		TE		x		
4	Cassady et al. (2001b)	CP	BSS,BSC	Ind.	Pf	Id.M	Max R Min MC Min MT	–		TE				
5	Cassady et al. (2003)	SP(F)	BSS,BSC	Ind.	Pf	Un.M	Max R	-	size ≥ 20		GA			
6	Yu and Schneider (2003)	S	BSS,BSC	Ind.	Pf	Id.M	Max Av	Dtr. MBD MD DEv	Oc. res. Inf. mis.			x		
7	Schneider and Cassady (2004)	SP (F)	BSS,BSC	Ind.	Pf	Id.M	Max R	-		TE		x		
8	Cassady et al. (2004)	SP	BSS,BSC	Ind.	Pf	Id.M	Max #SM	-		TE				
9	Rainwater et al. (2004)	SP (F)	BSS,BSC	Ind.	Pf	Id.M	Max R	-		TE				
10	Rajagopalan and Cassady (2004)	SP	BSS,BSC	Ind.	Pf	Id.M	Max R	-		TE				
11	Cassady et al. (2005b)	SP (F)	MSS,BSC	Ind.	Pf	Un.M	Min MT	-	Oc. res. size ≥ 20	TE	HA			
12	Rajagopalan and Cassady (2006)	SP	BSS,BSC	Ind.	Pf	Id.M	Max R	-		TE				
13	Schneider (2006)	SP (F)	BSS,BSC	Ind.	Pf	Id.M	Max R Max #SM	-		TE				
14	Iyoob et al. (2006)	SP	BSS,BSC	Ind.	Pf	Id.M	Max R Min MC	-		TE				
15	Thibaut and Jacques (2006)	SP	BSS,BSC	Ind.	Pf, M	Id.M	Min MC Max R	Dtr.			HA			
16	Hoai and Luong (2006)	S	BSS,BSC	Ind.	Pf, M	Un.M	Min MC Max Av	Dtr.					x	

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Table 2.3: Mapping and classification based on main characteristics

#	Article	System config.	System states	Sys. dependency	Maint. degree	Planing horizon	Optim. criteria	Stoch. param.	Notes	Solution Approaches: Exact alg.	Solution Approaches: Heuristic	Solution Approaches: Simulation	Solution Approaches: Solver	Solution Approaches: Deep learning
17	Khatab et al. (2007)	SP	BSS,BSC	Ind.	Pf	Id.M	Max R	Dtr.	size ≥ 20		HA		MATLAB	
18	Khatab et al. (2008a)	SP	BSS,BSC	Ind.	Pf, M	Id.M	Min MC	Dtr.			EGD			
19	Khatab et al. (2008b)	SP	BSS,BSC	Ind.	Impf, M	Un.M	Min MC	Dtr.			SA			
20	Khatab and Ait-Kadi (2008)	SP	MSS,BSC	Ind.	Pf, M	Un.M	Min MC	Dtr.			EGD			
21	Lust et al. (2009)	SP	BSS,BSC	Ind.	Pf, M	Id.M	Max R	Dtr.	size ≥ 20	B&B	CH&TS			
22	Liu et al. (2009)	SP	MSS,BSC	Ind.	Impf, M	Un.M	Max R	Dtr.			GA			
23	Maillart et al. (2009)	SP	BSS,BSC	Ind.	Pf	Id.M	Max #SM	-	Inf. mis.	TE				
24	Zhu et al. (2011)	SP	BSS,BSC	Ind.	Impf, M	Id.M	Min UTC	Dtr.	size ≥ 20		CH&TS			
25	Ali et al. (2011b)	SP	BSS,BSC	Ind.	Pf	Id.M	Max R	-					Lingo	
26	Ali et al. (2011a)	SP	BSS,BSC	Ind.	Pf	Id.M	Max R	RC					Lingo	
27	Lv et al. (2011)	SP (SF)	BSS,BSC	Ind.	Pf	Id.M	Max R	Dtr. RC			PSO			
28	Chen et al. (2012)	SP	MSS,BSC	Stoch.	Impf, M	Un.M	Max R	Dtr. DEv			GA			
29	Pandey et al. (2012)	SP	BSS,BSC	Ind.	Impf, M	Id.M	Max R	Dtr.			PSO&DE			
30	Maaroufi et al. (2012)	CP	BSS,BSC	Econ. Stoch.	Pf	Un.M	Min MC	Dtr.		TE				
31	Maaroufi et al. (2013b)	SP	BSS,BSC	Econ.	Impf	Un.M	Min MC	Dtr.	Oc. res.			x		
32	Gupta et al. (2013)	SP	BSS,BSC	Ind.	Pf	Id.M	Max R Min MC Min MT	-	size ≥ 20 Fzy prg.				Lingo	
33	Maaroufi et al. (2013a)	CP	BSS,BSC	Econ. Stoch.	Pf	Un.M	Min MC	Dtr.		TE		x		
34	Pandey et al. (2013b)	SP	BSS,BSC	Ind.	Impf, M	Id.M	Max R	Dtr.			DE			

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Table 2.3: Mapping and classification based on main characteristics

#	Article	System config.	System states	Sys. dependency	Maint. degree	Planing horizon	Optim. criteria	Stoch. param.	Notes	Solution Approaches				
										Exact alg.	Heuristic	Simulation	Solver	Deep learning
35	Ali et al. (2013)	SP	BSS,BSC	Ind.	Pf	Id.M	Max R Min MC	RC					Lingo	
36	Pandey et al. (2013a)	SP	MSS,MSC	Econ.	Impf	Id.M	Max R	Dtr.			DE			
37	Pandey and Zuo (2013)	SP	BSS,BSC	Ind.	Impf, M	Un.M	Min MC Min #MB	Dtr.			DE			
38	Gupta et al. (2014)	SP	BSS,BSC	Econ.	Pf	Id.M	Min MC Min MT	RC	size ≥ 20				Lingo	
39	Pandey and Zuo (2014)	SP	MSS,MSC	Stoch.	Impf, M	Id.M	Max R	Dtr.			DE			
40	Dao et al. (2014)	SP	MSS,MSC	Econ.	Impf	Id.M	Max R	Dtr.			DE			
41	Haseen et al. (2015)	SP	BSS,BSC	Ind.	Pf	Id.M	Max R	Dtr. RC	size ≥ 20 Fzy prg.				Lingo	
42	Schneider and Cassady (2015)	SP (F)	BSS,BSC	Ind.	Pf	Id.M	Max R Min MC	-	size ≥ 20	Mx& Mn				
43	Djelloul et al. (2015)	SP	BSS,BSC	Ind.	Impf, M	Id.M	Max R	Dtr. MD					x	
44	Hou and Qian (2015)	SP	BSS,BSC	Ind.	Pf, M	Un.M	Max R	Dtr.					MATLAB	
45	Dao and Zuo (2015)	S	MSS,MSC	Stoch.	Impf	Id.M	Max TP	Dtr.			GA			
46	Cao et al. (2016b)	SP	BSS,BSC	Ind.	Impf, M	Un.M	Max R	Dtr.		TE				
47	Pandey et al. (2016)	SP	BSS,BSC	Ind.	Impf, M	Un.M	Min MC Min #MB	Dtr.			DE			
48	Khatab et al. (2016)	SP	BSS,BSC	Ind.	Impf, M	Id.M	Min MC	Dtr. MBD MD					x	
49	Xu et al. (2016a)	SP	MSS,MSC	Econ.	Impf	Id.M	Max R	Dtr.			DE			
50	Hou and Qian (2016)	CP	BSS,BSC	Struct.	Pf	Id.M	Max R	Dtr.		TE				

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Table 2.3: Mapping and classification based on main characteristics

#	Article	System config.	System states	Sys. dependency	Maint. degree	Planing horizon	Optim. criteria	Stoch. param.	Notes	Solution Approaches				
										Exact alg.	Heuristic	Simulation	Solver	Deep learning
51	Guo et al. (2016)	SP	MSS,BSC	Econ.	Impf	Id.M	Max R	Dtr.			DE			
52	Khatab and Aghezzaf (2016a)	SP	BSS,BSC	Ind.	Impf	Id.M	Min MC	Dtr.					x	
53	Khatab and Aghezzaf (2016b)	SP	BSS,BSC	Ind.	Impf, M	Id.M	Min MC	Dtr. MQ					x	
54	Xu et al. (2016b)	SP	BSS,BSC	Econ.	Pf	Id.M	Max R	-			EA			
55	Cao et al. (2016a)	SP	BSS,BSC	Ind.	Pf, M	Un.M	Min MC	Dtr.	Int. mis.			x		
56	Lan et al. (2017)	S(F)	BSS,BSC	Ind.	Pf	Id.M	Max R Min MC Min GT	Dtr. MQ			NSGA-II			
57	Sharma et al. (2017)	SP	BSS,BSC	Ind.	Impf, W	Un.M	Min MC	Dtr.	size $\geq$ 20 Int. mis.		GA	x		
58	Dao and Zuo (2017a)	S	MSS,MSC	Stoch.	Impf	Id.M	Max R	Dtr. DEv				x		
59	Jinxin and Yanling (2017)	CP	BSS,BSC	Struct.	Pf, M	Id.M	Max R	Dtr.		TE				
60	Dao and Zuo (2017b)	CP	MSS,MSC	Struct.	Impf	Id.M	Max R	Dtr.			GA			
61	Khatab et al. (2017a)	SP	BSS,BSC	Ind.	Impf, M	Id.M	Min MC	Dtr. MBD MD					x	
62	Cao et al. (2017)	SP	BSS,BSC	Ind.	Impf	Id.M	Max Av.		Oc. res.		GA	x		
63	Diallo et al. (2017)	SP	BSS,BSC	Econ.	Pf	Id.M	Max R Min MC		Oc. res.				K-Nitro 9 Solver	
64	Khatab et al. (2017b)	SP	BSS,BSC	Ind.	Impf, M	Id.M	Min MC	Dtr. MD MBD RC		TE				

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Table 2.3: Mapping and classification based on main characteristics

#	Article	System config.	System states	Sys. dependency	Maint. degree	Planing horizon	Optim. criteria	Stoch. param.	Notes	Solution Approaches				
										Exact alg.	Heuristic	Simulation	Solver	Deep learning
65	Liu and Jiang (2018)	SP	BSS,BSC	Ind.	Impf, M	Un.M	Max R Min RV	Dtr.			HA			
66	Shahraki and Yadav (2018)	S	MSS,MSC	Stoch.	Impf	Id.M	Max R	Dtr. DEv				x		
67	Khatab et al. (2018a)	SP	BSS,BSC	Ind.	Impf	Id.M	Min MC	Dtr.					x	
68	Khatab et al. (2018b)	SP	BSS,BSC	Ind.	Impf	Id.M	Min MC	Dtr.					x	
69	Yang et al. (2018)	SP (F)	BSS,BSC	Ind.	Pf	Un.M	Min MC Min #MB	Dtr.	size ≥ 20		SGA			
70	Liu et al. (2018)	SP	MSS,MSC	Ind.	Impf	Un.M	Max R	Dtr. RC MBD			ACO	x		
71	Diallo et al. (2018)	CP	BSS,BSC	Ind.	Impf, M	Id.M	Max R Min MC	Dtr.	size ≥ 20	2-ph			Gurobi	
72	Khatab et al. (2018c)	SP	BSS,BSC	Econ.	Impf, M	Id.M	Max R Min MC	Dtr.	Oc. res.				x	
73	Chaabane et al. (2018)	SP	BSS,BSC	Ind.	Impf, M	Id.M	Max R	Dtr.	Oc. res.				x	
74	Chen et al. (2018a)	CP	MSS,MSC	Ind.	Impf, M	Id.M	Max R	Dtr. MQ DEv			PSO			
75	Zhao et al. (2018)	SP	MSS,MSC	Ind.	Impf	Id.M	Max R	Dtr. MQ	Oc. res.		GA			
76	Cao et al. (2018b)	SP	MSS,MSC	Ind.	Impf	Id.M	Max R	Dtr. MBD MD	Fzy prog.		HA			

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Table 2.3: Mapping and classification based on main characteristics

#	Article	System config.	System states	Sys. dependency	Maint. degree	Planing horizon	Optim. criteria	Stoch. param.	Notes	Solution Approaches: Exact alg.	Heuristic	Simulation	Solver	Deep learning
77	Duan et al. (2018)	CP	BSS,BSC	Ind.	Impf, M	Id.M	Min MC	Dtr. MQ RC	size ≥ 20	B&B	SA			
78	Zhao et al. (2019a)	P	BSS,BSC	Ind.	Pf, M	Id.M	Max R	Dtr. MD	Oc. res.			x		
79	Diallo et al. (2019b)	CP	BSS,BSC	Ind.	Impf, M	Id.M	Max R Min MC	Dtr.	size ≥ 20 Oc. res.	2-ph			Gurobi	
80	Ahadi and Sullivan (2019)	SP	MSS,BSC	Ind.	Pf	Id.M	Max R	-			HA			
81	Diallo et al. (2019a)	SP	BSS,BSC	Econ.	Impf, M	Id.M	Max R Min MC	Dtr.	Oc. res.	2-ph			Gurobi	
82	Khatab et al. (2019)	SP	BSS,BSC	Econ.	Pf	Id.M	Max R	Dtr.	Oc. res.				Lingo	
83	Chen et al. (2019)	S	MSS,MSC	Ind.	Impf, M	Id.M	Max R	Dtr. MQ	Fzy prog.		PSO			
84	Zhao et al. (2019b)	SP	MSS,MSC	Stoch.	Impf	Id.M	Max R	Dtr.			GA			
85	Wang et al. (2019)	SP	MSS,MSC	Ind.	Impf	Id.M	Min MC	-					CPLEX	
86	Zhang et al. (2019a)	SP	BSS,BSC	Ind.	Impf	Id.M	Max R Min MC	Dtr. MQ	Oc. res. Fzy prog.		ABC			
87	Chaabane et al. (2020b)	SP	BSS,BSC	Ind.	Impf, M	Id.M	Max R Min MC	Dtr. R.Av	Oc. res.				x	
88	Liu et al. (2020)	SP	MSS,BSC	Ind.	Impf, M	Un.M	Max #SM	Dtr.						DL
89	Galante et al. (2020)	SP	BSS,BSC	Ind.	Pf	Id.M	Max R	Dtr.	size ≥ 20 Oc. res.	CH				
90	Chaabane et al. (2020a)	SP	BSS,BSC	Econ.	Impf, M	Un.M	Min MC	Dtr.	Oc. res.		GA			

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Table 2.3: Mapping and classification based on main characteristics

#	Article	System config.	System states	Sys. dependency	Maint. degree	Planing horizon	Optim. criteria	Stoch. param.	Notes	Solution Approaches: Exact alg.	Heuristic	Simulation	Solver	Deep learning
91	Ikonen et al. (2020)	SP	BSS,BSC	Econ.	Pf, M	Id.M	Max R, Min MC	Dtr.	size ≥ 20	ECM			Baron	
92	Khatab et al. (2020a)	SP (SF)	BSS,BSC	Econ.	Impf, M	Id.M	Min MC	Dtr.	size ≥ 20 Oc. res.	2-ph			Gurobi	
93	Shahraki et al. (2020)	S	MSS,MSC	Stoch.	Impf	Id.M	Max R Min RV	Dtr. MQ	Fzy prog.	2-ph	GA		Gurobi	
94	Zhou et al. (2020)	SP	BSS,BSC	Ind.	Pf, M	Un.M	Max R	Dtr.			HA			
95	Jiang and Liu (2020b)	SP	BSS,BSC	Ind.	Impf, M	Un.M	Max R	Dtr. MD			SA			
96	Jiang and Liu (2020a)	CP	BSS,BSC	Ind.	Impf, M	Un.M	Max R Min RV	Dtr.			HA			
97	Ruiz et al. (2020)	SP	MSS,MSC	Econ. Stoch.	Impf, M	Id.M	Max R Min MC	Dtr.			GA&DE			
98	Zhang et al. (2020)	SP	MSS,BSC	Ind.	Impf	Id.M	Min ME	Dtr.			DGSA			
99	Khatab et al. (2020b)	SP	BSS,BSC	Ind.	Impf, M	Id.M	Min MC	Dtr. DEv	Oc. res.	2-ph				
100	Gao et al. (2021)	CP	BSS,BSC	Ind.	Impf, M	Id.M	Max R	Dtr. MD			GA			
101	Kamal et al. (2021)	SP	BSS,BSC	Ind.	Pf	Id.M	Max R Min MC	Dtr. MD	Fzy prog. size ≥ 20				Lingo	
102	Xu et al. (2021b)	SP	BSS,BSC	Ind.	Impf	Un.M	Max R	Dtr.			DE			DQN
103	Sun et al. (2021)	SP	MSS,MSC	Ind.	Impf	Id.M	Max R	Dtr. MD	Oc. res.		HA			
104	Xu et al. (2021a)	SP	BSS,BSC	Ind.	Impf	Id.M	Max R Min MC	Dtr.	Oc. res.		NSGA-III			

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Table 2.3: Mapping and classification based on main characteristics

#	Article	System config.	System states	Sys. dependency	Maint. degree	Planing horizon	Optim. criteria	Stoch. param.	Notes	Solution Approaches				
										Exact alg.	Heuristic	Simulation	Solver	Deep learning
105	Li et al. (2021)	S	MSS,MSC	Stoch.	Impf	Id.M	Max R	Dtr. MQ			GA			
106	Chen et al. (2021)	SP	MSS,MSC	Ind.	Impf	Id.M	Max Av Max PC	Dtr. RC			GA			
107	Cao and Duan (2021a)	SP	BSS,BSC	Econ.	Impf	Id.M	Max R	Dtr. MQ			GA			
108	Sun and Sun (2021)	SP	MSS,MSC	Ind.	Impf	Id.M	Max R	Dtr.			ACO			
109	Cao and Duan (2021b)	CP	BSS,BSC	Econ.	Pf	Id.M	Max R	Dtr.			SA			
110	Hesabi et al. (2022)	S	BSS,BSC	Ind.	Impf	Id.M	Min MC	Dtr.		TE				DL
111	Su et al. (2022)	SP	BSS,BSC	Ind.	Impf, M	Id.M	Max R Min MC	Dtr. MD			NSGA-II			
112	Liu et al. (2022)	SP	BSS,BSC	Ind.	Impf, M	Un.M	Min MC	Dtr. MBD MD	Oc. res. RC		GA	x		
113	O'Neil et al. (2022b)	CP	BSS,BSC	Ind.	Impf	Un.M	Max R	Dtr.	Oc. res. size ≥ 20		CG& GA			
114	Xia et al. (2022)	SP	BSS,BSC	Ind.	Impf	Un.M	Max EE	Dtr.	Int. mis		B&B			
115	Sun et al. (2022)	SP	MSS,MSC	Ind.	Impf	Id.M	Max R	Dtr. MD	Oc. res. size ≥ 20		HA			
116	Kammoun et al. (2022)	SP	BSS,BSC	Stoch.	Pf	Un.M	Min MC	Dtr.	size ≥ 20					DM
117	O'Neil et al. (2022a)	CP	BSS,MSC	Ind.	Impf	Id.M	Max R	Dtr.						DL
118	Tang et al. (2022)	S	MSS,MSC	Ind.	Impf	Id.M	Max TP	Dtr.			GA	x		
119	Ghorbani et al. (2022)	SP	BSS,BSC	Ind.	Pf	Id.M	Min MC	Dtr. DEv					Baron	

2.3.1 Categorisation of SMP modelling

2.3.1.1 System characteristics

System characteristics include features such as the reliability structure or configuration, system state, system level under consideration, and system dependency.

2.3.1.1.1 System configuration The way a system is designed and organized, known as its configuration, has a significant impact on its reliability modeling and computation as well as its maintenance policy optimisation (Wang, 2002). The SMP has been solved for various system configurations, including series, parallel, bridge, k-out-of-n and thereof combinations.

The series-parallel configuration is the most commonly considered structure in the SM literature. A large number of equipment utilized in industrial and military settings can be represented by the series-parallel configuration (Liu et al., 2009; Zhu et al., 2011; Chen et al., 2012; Sharma et al., 2017; Cao et al., 2018b; Zhao et al., 2019b; Wang et al., 2019; Liu et al., 2020). Only few studies investigate the SMP for complex reliability structures. Cassady et al. (2001b) extends the original SMP to consider subsystems of complex structures. Diallo et al. (2018) develop a two-phase approach to transform the nonlinear SMP into a multidimensional multiple-choice knapsack problem (MdMCKP). Their approach is used to optimally solve the SMP for general, large and complex reliability structures including serial k-out-of-n systems with non-identical components.

Most SM studies express their system configuration through the Reliability Block Diagram approach (RBD). However, a small number of studies in the literature use different techniques such as Dynamic Fault Tree (Maaroufi et al., 2013a), tree and leaf representation (Hou and Qian, 2016), directed graph modeling (Dao and Zuo, 2017b), Extended State Task Network (Chen et al., 2018a).

2.3.1.1.2 System level Most papers deal with single systems. Maintenance decisions are made at the system components level. However, many industries usually involve fleets of systems. For example, in the transportation industry, fleets are composed of multiple trains, buses or airplanes (Khatab et al., 2020a). In these cases, maintenance decisions must be made for all components in the fleet, adding an additional level of complexity to the optimisation process.

Several studies have presented models for optimizing the SM for fleets of assets. Cassady et al. (2003) proposed a SM model to maximize the reliability of a fleet composed of non-identical systems with non-identical missions, and different starting and ending times. Several extensions followed (Rainwater et al., 2004; Schneider and Cassady, 2004; Cassady et al., 2005b; Schneider, 2006; Schneider and Cassady, 2015). Other studies, such as (Lan et al., 2017) and (Feng et al., 2017) proposed models for the maintenance of truck fleets and military fleets respectively. (Yang et al., 2018) addressed the SMP for a fleet of equipment required to perform phased missions with short scheduled breaks. (Khatab et al., 2020a) proposed a novel variant of the fleet SMP where non-identical systems operating under several imperfect maintenance levels and multiple repair channels are available.

2.3.1.1.3 System state In reliability theory, the lifespan of components or systems is affected by ageing and wear out. Many papers in the field focus on systems that are binary-state with binary state components (BSS-BSC) where both the system and its components can only be in one of two states: failed or functioning. However, many systems in practical applications have a multi-state nature where a system has a multi-state nature with binary state components (BSS-BSC) or both the system and components are multi-state (MSC-MSS). Cassady et al. (2005b) developed a method for addressing SM decisions in MSS-BSC with non-coherent states. Khatab and Ait-Kadi (2008) generalized the SMP to multi-mission multi-states systems, and Liu et al. (2009) investigated the single mission SMP in a MSS whose components are binary-state and operate under imperfect maintenance. In these studies, capacity or productivity is often used as a measure of system performance, which is commonly used in energy transmission systems, manufacturing systems, and power generation systems.

Pandey et al. (2013a) extended the work of (Liu et al., 2009) by considering multi-state components in a MSS (MSC-MSS) rather than BSCs. Subsequent studies have further investigated the SMP for MSS. Dao and Zuo (2015) studied the SMP in a MSS under two types of stochastic dependence between components. Dao and Zuo (2017a) investigated the SMP in a MSS where components are subjected to variable loading conditions resulting in degradation depending on both their current operating state and the load applied. Shahraki et al. (2020) used a Monte Carlo simulation-based approach to calculate the reliability of a MSS for the next mission considering different levels of stochastic imperfect maintenance. By taking the expected value and variance of the system reliability as objective functions, a bi-objective SM model was proposed and solved.

2.3.1.1.4 System dependency Systems are generally made of multiple components that may have one or several interactions generally categorized into three types: economic, structural, and stochastic dependence. Economic dependence implies that costs can be reduced when several components are jointly maintained. Structural dependence is applied when several components can be structurally grouped so that the repair or replacement of one failed component implies maintenance or dismantling of the other working components in the group. Stochastic dependence occurs if the state of a component (age, failure probability, failure rate, etc.) can influence those of other components (Xu et al., 2016b).

According to Figure 2.4, the majority of SMP papers (73.9%) assume that system components are independent. However, ignoring components' interactions can result in biased predictions of system reliability and sub-optimal maintenance policies. Only 2.5% of the papers consider structural dependence (Hou and Qian, 2016; Jinxin and Yanling, 2017; Dao and Zuo, 2017b) with another 2.5% considering joint economic and stochastic dependence (Maaroufi et al., 2012, 2013a; Ruiz et al., 2020).

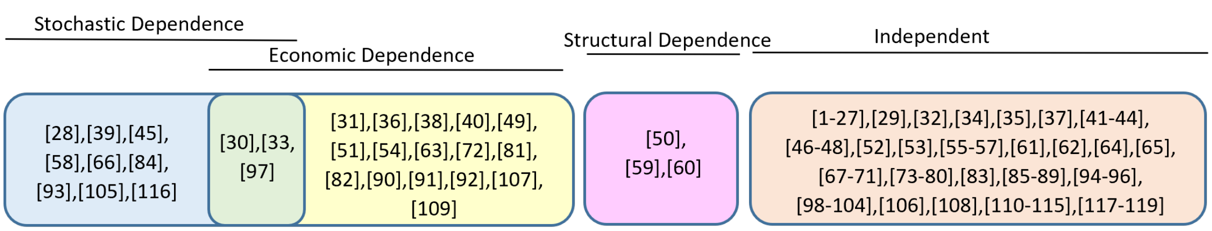


Figure 2.4: Distribution of papers vs component dependence

Several types of economic dependence have been studied in the literature, including sharing setup costs over multiple components (Maaroufi et al., 2013b; Pandey et al., 2013a), the advantage of repairing multiple identical components in each subsystem of a series-parallel system (Dao et al., 2014), and the reduction in total maintenance cost when simultaneously repairing multiple components in two subsystems (Xu et al., 2016a).

The effects of structural dependence on system reliability and maintenance strategies when resources are constrained is an under-explored topic in literature. Dao and Zuo

(2017b) examine the SM policy for multi-state systems with structural dependence and limited resources, considering multiple hierarchical levels and disassembly sequences. They find that failure to take structural dependence into account leads to over-estimations of system reliability, which become more pronounced when resources are scarce.

Stochastic dependence in SM has been studied from various perspectives including the impact of load distribution (Chen et al., 2012), variable loading conditions (Dao and Zuo, 2017a) failure isolation and propagation (Maaroufi et al., 2013a), maintainable and non-maintainable failure modes (Pandey and Zuo, 2014) on a system's performance. Studies have also examined the effect of age and degradation on failure rate (Shahraki and Yadav, 2018), used neural networks to predict state transition probabilities (Zhao et al., 2019b), considered two-way stochastic interactions and accounted for unknown factors in their models (Shahraki et al., 2020). Other studies have utilized Failure Mode Effects and Criticality Analysis (FMECA) (Li et al., 2021, 2020) and data mining techniques to assess failure effects in selective maintenance context (Kammoun et al., 2022).

2.3.1.2 Maintenance characteristics

This section deals with the attributes related to the execution of maintenance activities such as resource consumption and the effectiveness of actions carried out which can significantly affect maintenance decisions.

2.3.1.2.1 Maintenance resources Resource scarcity lies at the core of SMP as it aspires to balance the maintenance requirements and the limited available resources. Maintenance resources are typically categorized according to three factors: type, allocation, and limits.

The first factor refers to whether resources are occupied or consumed during maintenance. Consumptive resources such as time and budget are consumed and can be depleted, whereas occupied resources such as repairpersons and remanufactured spare parts can be reused. Pandey et al. (2013b) and Pandey and Zuo (2014) establish a relationship between the hazard adjustment factor, the amount of resources utilized, and the actual age of the component. They also discovered that considering imperfect maintenance allows for more efficient utilization of resources, resulting in improved reliability. Pandey et al. (2013a) discuss the SMP when some resources are scarce while others are

relatively abundant. The impact of resource variation on SM decisions and system reliability is studied. The study illustrates the importance of performing a sensitivity analysis so that resource allocation can be performed wisely to achieve the desired system reliability. As for occupied resources, most papers assume that ample repairpersons are available to perform maintenance actions during a break. However, in many real industrial maintenance applications, there is a limited number of repairpersons. In recent years, studies that consider the SMP with limited availability of repairpersons have been proposed (Diallo et al., 2017; Khatab et al., 2018c; Chaabane et al., 2018; Diallo et al., 2019b; Khatab et al., 2019; Chaabane et al., 2020b,a; Khatab et al., 2020b; Sun et al., 2021; Xu et al., 2021a; Liu et al., 2022; O'Neil et al., 2022b; Sun et al., 2022).
The second factor is related to multi-mission problems and refers to whether maintenance resources are distributed evenly or not among maintenance breaks (Jiang and Liu, 2020b; Khatab et al., 2020a), while the third factor refers to whether the model allows extending limited maintenance resources at some cost or not (Iyoob et al., 2006; Maaroufi et al., 2013a).

2.3.1.2.2 Maintenance effectiveness Maintenance keeps or restores a system to a state where its functions can be performed satisfactorily. Maintenance actions can either be corrective (CM) or preventive (PM). These PM or CM actions can also be perfect, imperfect, minimal, and worse depending on the quality and level of restoration performed. Perfect maintenance is equivalent to a component replacement whether it is working (PM) or failed (CM); restoring it to an "as good as new" (AGAN) condition. Minimal repair (MR) is a maintenance action that returns a failed component/system to a working state without affecting its failure rate. Thus, the health state after the repair is the same as it was right before failure which is called "as bad as old" (ABAO). Imperfect maintenance (IM) restores a component/system to a state somewhere between AGAN and ABAO conditions. Sometimes, the maintenance action is performed improperly, thus leading the system failure rate and actual age to increase. Hence, the system's operating condition turns to be worse than it was before maintenance.

The original SM model (Rice et al., 1998) only considered replacement of failed components. Later, Cassady et al. (2001a) expanded the model to include MR and PM. To the best of our knowledge, (Khatab et al., 2008b) was the first to include Imperfect maintenance (IM) in SMP, which is based on the age reduction concept (Malik, 1979). Several extensions followed (Liu et al., 2009; Liu and Huang, 2010; Pandey et al., 2013b; Zhu et al., 2011; Pandey and Zuo, 2013; Khatab et al., 2018b). Sharma et al. (2017) considered a scenario that is rarely considered in SM: the system's operating condition may worsen after maintenance.

2.3.1.3 Optimisation Model Characteristics

The referenced SMP models will be discussed according to the three main optimization characteristics in Figure 2.3: optimisation objective, planning horizon, and uncertain parameters.

2.3.1.3.1 Optimisation criteria Most SMP formulations are single-objective models that aim to maximize system reliability or minimise maintenance costs. Other objectives that have been considered include minimizing the number of repair times (Yang et al., 2018), environmental impacts (Khatab et al., 2018a; Zhang et al., 2020) or maximizing availability (Yu and Schneider, 2003; Hoai and Luong, 2006; Chen et al., 2021), performance capacity (Chen et al., 2021; Xia et al., 2022). In many applications, mission objectives conflict with each other, thus requiring sensitivity analysis to find trade-offs between several objectives. Multi-objective models permit to consider such trade-offs during the optimisation phase (Thibaut and Jacques, 2006; Gupta et al., 2013, 2014; Haseen et al., 2015; Lan et al., 2017; Zhang et al., 2019a; Diallo et al., 2019a; Jiang and Liu, 2020a; Shahraki et al., 2020; Yang et al., 2018; Kamal et al., 2021; Xu et al., 2021a; Su et al., 2022).

2.3.1.3.2 Planning horizon The planning horizon refers to the time horizon considered in the SMP models. The planning horizon is composed of a finite or infinite number of missions with different profiles depending on whether they are identical or not, and uninterruptible or not.

Most SMP papers assume a finite planning horizon, with the exception of the models developed by Yu and Schneider (2003) and Maillart et al. (2009). Yu and Schneider (2003) examine the impact of maintenance resource limitations on SM decisions for a serial production line operating on an infinite planning horizon, while Maillart et al. (2009) investigates SMP with multiple identical missions in both finite and infinite planning horizon cases. The long-term performance of the optimal infinite-horizon SM plan is compared to those obtained for the single mission and two-mission planning horizons. It is found that the model with an infinite number of missions only yields minimal improvement in the expected number of successful missions compared to models with one or two missions.

During the planning horizon, missions may or may not be interruptible. Interruptible missions allow for a pause to restore a failed system to a functioning state, while uninterruptible missions result in the mission being canceled or aborted if there is an interruption. Cao et al. (2016a) conclude that system availability under interruptible missions is higher than that under uninterruptible ones. Sharma et al. (2017) develop a methodology for applying assembly-level SM to a set of tanks during a war scenario in which the absence of a maintenance break is considered. Although assembly-level is more expensive than component-level maintenance, the study shows that the system state is easier to diagnose at the assembly-level. Maintenance at the assembly-level is less complicated and less time-consuming.

The majority of SMP papers concentrate on a single mission or multiple identical missions. However, in real-world scenarios, systems often face non-identical missions with varying duration and conditions, which increases the complexity of maintenance decision-making process, requiring optimisation of all components across all missions. Studies have been conducted to address this challenge by developing models that consider the specific sequence of missions with different lengths (Hou and Qian, 2015), fleet-level multi-mission systems (Yang et al., 2018), and scheduling models that find the cost-optimal number of maintenance breaks in a finite horizon composed of non-identical missions (Pandey et al., 2016). Recent studies have addressed the complexity of SMP in multi-mission systems by formulating joint maintenance and repairpersons assignment problems (Chaabane et al., 2020a), studying the SMP in MSS running multiple non-identical missions (Liu et al., 2020), developing hybrid algorithms that incorporate differential evolution and deep Q-network methods (Xu et al., 2021b), introducing solution methods for the multi-mission SMP that combine column-generation and genetic algorithms (O'Neil et al., 2022b), and developing models that consider the stochasticity of maintenance actions and duration of missions (Liu et al., 2022).

2.3.1.3.3 Non-deterministic parameters Several parameters in the SMP are inherently uncertain such as components deterioration processes, mission and break durations, quality of maintenance actions performed, system state determination and resource consumption. Neglecting the stochastic and/or uncertain nature of many parameters can lead to the overestimation of system reliability.

Degradation or deterioration
Components and systems are unavoidably subjected to age and usage degradation or /deterioration processes. To model ageing deterioration processes in a SM setting, components' lifetimes are commonly assumed to follow the exponential or Weibull distributions. Since failure rates are constant for the exponential distribution, corrective replacements of failed components are the only maintenance option available. In contrast, the Weibull distribution can be used to describe components with increasing, constant, or decreasing failure rates, and offers the possibility to include IM in SMP. In these time-dependent degradation models, usage intensity is not accounted for.

To address this limitation, various models have been proposed, such as those that evaluate component failures based on their health condition (Khatab et al., 2018b), use the non-homogeneous stochastic Poisson process to estimate average numbers of component failures (Kamal et al., 2021), consider different types of failure modes such as hard, soft, maintainable, and unmaintainable failure modes (Pandey and Zuo, 2014; Ruiz et al., 2020), and use deep learning algorithms to predict component failure probabilities. Some models use deep learning algorithms to predict component failure probabilities (Hesabi et al., 2022; O'Neil et al., 2022a), and others use statistical analysis and stochastic processes to estimate average numbers of failures and determine optimum maintenance plans (Ikonen et al., 2020). Some studies propose new variations of the SMP that account for uncertain operating conditions. These studies include developing new variants of the SMP in which the performance capacity and states transition intensities are uncertain and represented by fuzzy numbers (Cao et al., 2018b), modelling operating environments randomness as a random shock process that directly impacts the failure process of the components (Khatab et al., 2020b), and proposing stochastic programming approaches for SMP under uncertainties in future operating conditions (Ghorbani et al., 2022).

System state determination

The vast majority of SMP papers assume that system and component deterioration/ degradation levels are perfectly known. However, imperfect observations/sensors introduce additional uncertainty in the determination of components' states and effective ages. A limited number of papers in SM literature have addressed this by formulating nonlinear, discrete, chance-constrained programming optimisation models that deal with diagnostic uncertainty of built-in test equipment (Lv et al., 2011), using Bayes' theorem and probability analysis to obtain component state distributions based on uncertain diagnostic results (Haseen et al., 2015), proposing fuzzy multi-objective models to maximize fuzzy reliability of each subsystem and developing robust SM strategies to identify optimal maintenance actions for binary-state systems under imperfect observations (Jiang and Liu, 2020a).

Duration of missions and maintenance breaks

SMP papers generally assume that the duration of missions and maintenance breaks are known and constant, but this is a strong assumption that may not hold true in real-world applications and engineering practices, as it can be challenging to estimate the exact duration of missions and breaks. Stochastic mission length and maintenance duration have been scarcely investigated in the SM literature. Some studies have used probability distributions (e.g., Gamma and Triangular) to account for uncertain durations (Djelloul et al., 2015; Khatab et al., 2017b,a; Sun et al., 2021), while others have utilized discrete random variables (Gao et al., 2021) or fuzzy values (Gao et al., 2021; Cao et al., 2018b; Kamal et al., 2021). Studies have shown that the stochastic nature of failure times, mission duration, operation time, and effective age lead to uncertainty about the effective age of components at the beginning of the next mission (Jiang and Liu, 2020b). The sequence of maintenance actions also impacts the chance of completing maintenance actions if there is stochasticity (Liu et al., 2018, 2022).

Quality of maintenance action

Most papers dealing with imperfect maintenance assume that both age reduction and adjustment parameters corresponding to maintenance actions are constant. However, it is difficult to precisely evaluate the quality of maintenance actions as these are significantly affected by various factors such as the qualification and the degree of expertise of the repairperson, the maintenance methods and tools used, and natural variability. Therefore, several recent studies have started addressing this issue by considering stochastic approaches to describe the maintenance improvement by using a known probability distributions (*e.g.*, Gamma and Triangular) for the age reduction coefficient (Khatab and Aghezzaf, 2016b; Cao and Duan, 2021a; Lan et al., 2017; Shahraki et al., 2020), cognitive reliability and error analysis method (CREAM) to calculate human reliability (Zhao et al., 2018), triangle membership function to balance the relationship between maintenance action and cost (Chen et al., 2019), discrete random variables to represent the maintenance actions and their quality (Li et al., 2021), Choquet integral (Chen and Wang, 2001) based on λ-fuzzy measure to evaluate the capability of maintenance teams (Zhang et al., 2019a), and incorporating deep Q-network method to approximate the effectiveness of maintenance actions (Xu et al., 2021b).

Resource consumption

Resource consumption of maintenance actions is typically considered deterministic, but due to uncertain judgments, unpredictable conditions, or human errors, the uncertainty related to resource consumption is unavoidable, and ignoring it may lead to ineffective decisions that expose systems to risks. Few studies in the SM context have considered the uncertainty in resource consumption. Various methods have been used to account for this uncertainty. These include for example using a normally distributed maintenance duration with a chance-constrained method to restrict the feasible region so that the solution confidence level exceeds a certain probability (Ali et al., 2011a, 2013), using interval numbers for parameters such as time, cost, weight, and volume (Gupta et al., 2014), considering the repair time and cost of each component as fuzzy numbers (Haseen et al., 2015), and using homogeneous continuous-time Markov process for MSS where the time for each maintenance task can be arbitrarily distributed (Chen et al., 2021).

2.3.2 Categorisation of solution methods

The main challenge for most SMP models is that their formulations are difficult to solve optimally, especially for industrial-size problems that would allow practitioners to implement and use the SMP for their many systems and repairpersons. Rice (1999) proved that the basic SMP is NP-hard and so are all its extensions implying that computational effort increases exponentially with problem size. Different methodologies and techniques are used for solving the SMP which could be clustered into two groups: exact and heuristics approaches.

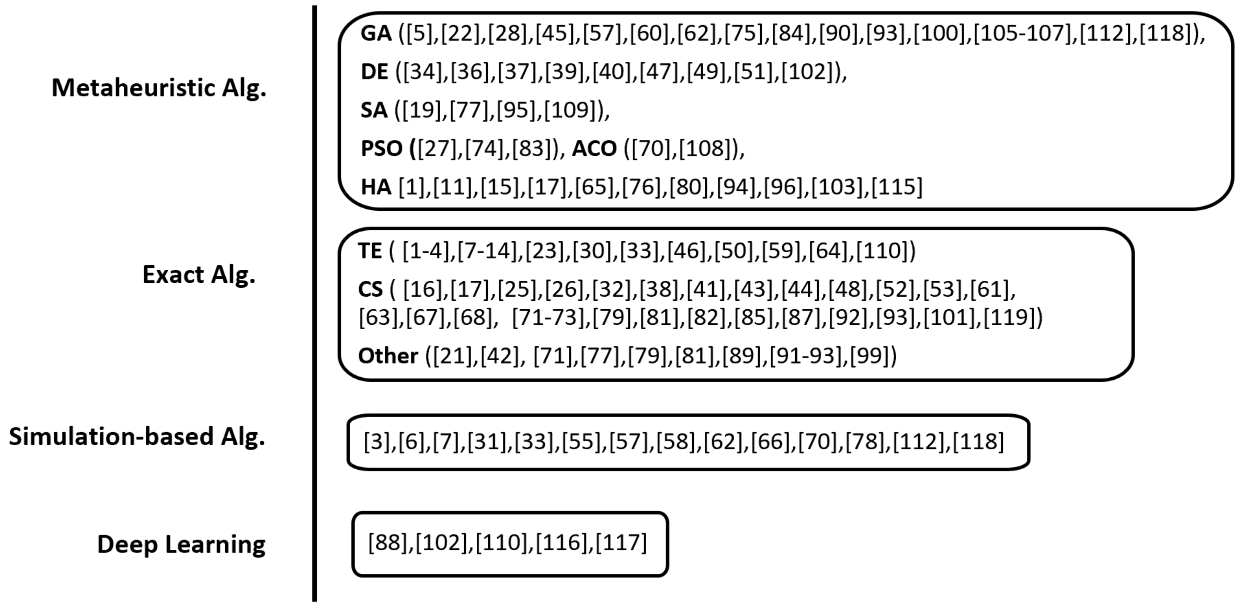


Figure 2.5: Distribution of references by solution methods

2.3.2.1 Exact algorithms

Exact solution methods constitute the majority of methods employed to solve SMOP. Such solution methods are total enumeration (Rice et al., 1998), search space reduction (Rajagopalan and Cassady, 2006), depth-first search algorithms (Cao et al., 2016b), and branch-and-bound type procedures (Xia et al., 2022), max-min approach (Schneider and Cassady, 2015), sequential construction (Galante et al., 2020), and two-phase approach (Diallo et al., 2018). Most SMP papers with exact solution methods are computationally expensive and suitable only for small to medium-sized problems (fewer than 20 components). Ikonen et al. (2020) propose two convexified SM optimisation models to improve solution efficiency, including a choice between replacement or a combination of replacement and repair. The improvements enable the convexified models to tackle large-scale SM optimisation problems with up to 1000 system components.

2.3.2.2 Heuristics, meta-heuristics and simulation

Given that the SMP is computationally expensive to solve using most of the proposed exact methods, various other approaches such as general heuristics, meta-heuristics, simulation and deep-learning-based approaches are used to quickly find near-optimal solutions.

Heuristic-based solution approaches

The original SM paper Rice et al. (1998) proposes two heuristic methods to find the components to be repaired to yield the largest reliability increase with and without considering resources. Subsequently, several studies utilized general heuristics to handle SMPs (Thibaut and Jacques, 2006; Khatab et al., 2007; Liu and Jiang, 2018; Ahadi and Sullivan, 2019; Zhou et al., 2020; Jiang and Liu, 2020a; Sun et al., 2021, 2022).

Meta-heuristic-based solution approaches

Due to their ease of use and adaptability, evolutionary algorithms such as genetic algorithms (GA) (Holland, 1975) and differential evolution (DE) (Storn, 1995) algorithms are widely used as solution approaches for the SMP. Cassady et al. (2003) first used GA to find the best maintenance strategy for a large-scale SMP. Subsequently, many studies also used GA to solve large-size instances of the SMP (Cassady et al., 2003; Liu et al., 2009; Chen et al., 2012; Dao et al., 2014; Dao and Zuo, 2015; Sharma et al., 2017; Dao and Zuo, 2017b; Zhao et al., 2018, 2019b; Chaabane et al., 2020a; Shahraki et al., 2020; Ruiz et al., 2020; Gao et al., 2021; Li et al., 2021; Chen et al., 2021; Cao and Duan, 2021a; Liu et al., 2022; Tang et al., 2022).

In addition to GA, the differential evolution algorithm, which is another evolutionary algorithm, is used intensively in SMP optimisation. Pandey et al. (2013b) are the first to use differential evolution (DE) as a solution approach for large-size instances of the SMP. Various studies followed suit and used DE (Pandey et al., 2013a; Pandey and Zuo, 2013, 2014; Dao et al., 2014; Pandey et al., 2016; Xu et al., 2016a; Guo et al., 2016; Xu et al., 2021b).

Besides evolutionary algorithms, other heuristic methods are used such as simulated annealing algorithm (SA) (Khatab et al., 2008b; Duan et al., 2018; Jiang and Liu, 2020b; Cao and Duan, 2021b), extended great deluge (EGD) (Burke et al., 2004), particle swarm optimisation (PSO) (Lv et al., 2011; Chen et al., 2018a, 2019), sequential game algorithm (SGA) (Yang et al., 2018), and ant colony optimisation algorithm (ACO) (Liu et al., 2018; Sun and Sun, 2021).

Hybrid solution approaches

Several hybrid algorithms are used to solve the SMP. Lust et al. (2009) develop an algorithm combining a construction heuristic with Tabu search in Cassady et al. (2001a). Zhu et al. (2011) put forward the same resolution algorithm to improve the quality of results and accelerate the convergence of the SMP for a machining line. Pandey et al. (2012) use a hybrid evolutionary algorithm of PSO and DE to solve a binary-state SMP. Zhang et al. (2020) propose a hybrid DE and gravitational search algorithm to solve a complex combinatorial SMP considering energy consumption. Zhang et al. (2019a) propose another two-phase method that integrates fuzzy Choquet integral (Chen and Wang, 2001) and multi-objective artificial bee-colony algorithm (Yahya and Saka, 2014) to optimise a multi-objective SMP. Xu et al. (2021b) develop a hybrid algorithm for the multi-mission SMP for a moderate-size series-parallel system by combining a discrete DE for searching the optimal maintenance action in large-scale discrete action spaces and a deep Q-network method for approximating the effectiveness of maintenance actions and facilitating the agent training. O'Neil et al. (2022b) propose a solution method for the multi-mission SMP based on a column-generation framework in which subproblems are solved using a GA. The proposed hybrid algorithm is shown to obtain near-optimal solutions and outperforms other metaheuristic solution methods. It is also shown to be capable of solving large-scale systems composed of many of both parallel and k-out-of-n:G subsystems with hundreds of components in a reasonable amount of time.

Simulation-based Approaches

Simulation-based methods offer a powerful means for evaluating system reliability due to the modelling flexibility that it offers regardless of the type and dimension of the problem (Faulin et al., 2010). Accordingly, many studies utilized simulation-based approaches to handle complex SMPs (Cassady et al., 2001a; Yu and Schneider, 2003; Schneider and Cassady, 2004; Lv et al., 2011; Maaroufi et al., 2013b,a; Cao et al., 2016a; Sharma et al., 2017; Dao and Zuo, 2017a; Cao et al., 2017; Shahraki and Yadav, 2018; Liu et al., 2018;

Zhao et al., 2019a; Shahraki et al., 2020; Yang et al., 2018). Despite the efficiency of the simulation-based methods in dealing with complex SMPs, they become relatively time-consuming as the number of SM actions increase.

In this section 2.3, the identified SMP papers were classified and reviewed according to their formulation and solutions characteristics. Features sufficiently covered and shortcomings were identified. In what follows, a selection of key representative models of the state-of-the-art SMP formulations and solution methods.

2.4 Selective maintenance models

This section begins with a general formulation of the SMP followed by the presentation of 5 models representative of recent advances in the SMP: Diallo et al. (2019b); Yang et al. (2018); Khatab et al. (2017b); Liu et al. (2018); Shahraki et al. (2020). These models have been selected to provide the maximum coverage of the characteristics explained above while offering the most recent, novel and advanced formulations and solution methods. The goal is to explain the proposed models and their respective strengths and weaknesses to assist practitioners and researchers in choosing the most suitable model for specific maintenance situations. For each of the four models, the key features and characteristics are emphasized, along with the model's definition, formulation, solution method, and limitations.

As shown in Table 2.4, the group of 5 selected SMP models cover the following features: k-out-of-n systems, single-level and fleet-level maintenance, binary or multi-state components/systems, stochastic dependence, single or multiple repairpersons, imperfect maintenance, single mission and multi-missions and non-deterministic parameters such as duration of missions, quality of maintenance, and resource consumption.

2.4.1 Generic formulation for SMP

Given a multi-component system operating a mission of duration U following a maintenance break of duration $\mathcal{T}_0$, the SMP aims to determine an optimal maintenance plan (*i.e.*, components, maintenance levels, and repair crew assignment) with one of the following two objectives:

Table 2.4: Classification of Prominent SM Models

Articles	System config.	System level	System state	Syst. dependency	# Repairpersons	Maint. degree	Optim. criteria	Planning horizon	Stoch. param.
Diallo et al. (2019b)	CP	Single	BSS, BSC	Ind.	many	Impf.,M	Max R	Id.M	Dtr.
Yang et al. (2018)	SP	Fleet	BSS, BSC	Ind.	many	Impf.	Max R	Un.M	Dtr.
Khatab et al. (2017b)	SP	Single	BSS BSC	Ind.	one	Impf.,M	Min MC	Id.M	Dtr. MD MBD
Liu et al. (2018)	SP	Single	MSS, MSC	Ind.	one	Impf.	Max R	Id.M	Dtr. RC MBD
Shahraki et al. (2020)	S	Single	MSS, MSC	Stoch.	one	Impf.	Max R, Min RV	Id.M	Dtr., MQ

- Maximize the system reliability $\mathcal{R}(U)$ during the next mission under maintenance budget $\mathcal{C}_0$. The resulting SM optimization problem (**SMP-MaxRel**) is formulated as:

$$
\begin{aligned}
\textbf{SMP-MaxRel}: \quad & \max \mathcal{R}(U) \\
\text{s.t.} \quad & \mathcal{C}(x_{ijlr}) \leq \mathcal{C}_0 \\
& \mathcal{T}(x_{ijlr}) \leq \mathcal{T}_0 \\
& x_{ijlr} \in \{0,1\}
\end{aligned}
$$

- Minimize the total maintenance cost $\mathcal{C}$ under a required minimum reliability level $\mathcal{R}_0$ during the next mission. The resulting SM optimization problem (**SMP-MinCost**) is formulated as:

$$
\begin{aligned}
\textbf{SMP-MinCost}: \quad & \min \mathcal{C} \\
\text{s.t.} \quad & \mathcal{R}(U, x_{ijlr}) \geq \mathcal{R}_0 \\
& \mathcal{T}(x_{ijlr}) \leq \mathcal{T}_0 \\
& x_{ijlr} \in \{0,1\}
\end{aligned}
$$

In the above formulations, x_{ijlr} is a binary decision variable indicating the assignment or not of repairperson r to perform maintenance level l on component j of subsystem i.

2.4.2 SMP with multiple repairpersons for complex reliability structures

The majority of SM models consider the basic series-parallel RBD and do not consider the practical industrial situations where multiple repairpersons are available to carry out maintenance actions. Indeed, in many industrial settings such as manufacturing, airline and maritime industries, inspection and maintenance tasks are assigned to several repairpersons. The resulting optimization problem is then composed of a SMP in addition to a repairpersons assignment problem (RAP). Solving these two problems sequentially (SMP first followed by RAP) will usually provide suboptimal decisions. Therefore, SMP and RAP should be considered simultaneously, leading then to the joint SM and repairpersons assignment problem (JSM-RAP) developed in (Khatab et al., 2018c; Diallo et al., 2017). This new variant of SMP is obviously more complex to solve to optimaility and efficient solution methods are needed. Dealing with such issue, a two-phase solution approach is developed in Diallo et al. (2019b) and summarized in what follows.

The main idea behind the two-phase solution approach is to formulate the Joint Selective Imperfect Maintenance and Repairperson Assignment Problem (JSM-RAP) for complex multi-component k-out-of-n systems as a binary integer programming problem (BIP). The first phase consists on generating feasible maintenance patterns each of which is a combination of components, maintenance levels and repairpersons. The second phase aims to select the appropriate mix of patterns that either maximize the system's reliability or minimize the total maintenance cost.

For illustration of maintenance patterns generation, let us consider a system where the RBD of the ith subsystem is composed of two components E_{i1} and E_{i2} in parallel (i.e., 1-out-of-2 structure). Only one repairperson is available to perform two common maintenance levels: do-nothing (l=0) and replacement (l=1). There are then 4 possible maintenance patterns defined as:

$$\begin{pmatrix}0\\0\end{pmatrix};\begin{pmatrix}0\\1\end{pmatrix};\begin{pmatrix}1\\1\end{pmatrix};\begin{pmatrix}1\\0\end{pmatrix},$$

where the first pattern means that no maintenance is performed on both components during the scheduled break. The second pattern means that only component E_{i2} is replaced, while in the third pattern 3 both component E_{i1} and E_{i2} are replaced. Dealing with the fourth pattern, only component E_{i1} is replaced. After generating the set $\mathcal{P}_i$ of patterns for each subsystem i, the BIP version of the MINLP **SMP-MaxRel** to maximize the system reliability can be formulated as follows:

$$[\mathcal{SMP}]_{BIP}: \quad \max_{x_{ip}} \ln(\mathcal{R}) = \sum_{i=1}^{I} \sum_{p=1}^{P_i} \ln(R_{ip}) x_{ip} \tag{2.1a}$$

$$\text{s.t.} \quad \sum_{p=1}^{P_i} x_{ip} = 1 \qquad \forall i \tag{2.1b}$$

$$\sum_{i=1}^{I} \sum_{p=1}^{P_i} C_{ip} x_{ip} \leq C_0 \tag{2.1c}$$

$$\sum_{i=1}^{I} \sum_{p=1}^{P_i} T_{ipr} x_{ip} \leq \mathcal{T}_0 \quad \forall r \tag{2.1d}$$

$$x_{ip} \in \{0,1\} \qquad \forall i, \forall p \tag{2.1e}$$

where $p \in \{1, \ldots, P_i\}$ is the p^{th} pattern generated for subsystem i. T_{ipr} is the total work time of repairperson r on subsystem i under pattern p, and C_{ip} is the total maintenance cost of pattern p for subsystem i.

In the above optimisation problem, Constraints (2.1b) ensure that exactly one maintenance pattern is selected per subsystem. Constraint (2.1c) guarantees that the total maintenance cost does not exceed the available maintenance budget. Constraints (2.1d) ensure that if a repairperson is hired and utilized then the corresponding total maintenance work time does not exceed the break duration. The last constraint (2.1e) defines the binary decision variable x_{ip} used in the formulation.

While the two-phase approach proposed by Diallo et al. (2019b) has been found to be effective for small to medium-sized instances of JSM-RAP, it becomes impractical to deal with industrial-sized instances where the number of components, maintenance levels, and repairpersons leads to a huge number of patterns to generate. To solve the JSM-RAP for large-scale systems, O'Neil et al. (2022b) developed a Column Generation (CG) based heuristic algorithm, where subproblems are solved using Genetic Algorithm (GA). Future extensions in this area should focus on developing better CG schemes, finding exact methods to solve the subproblems generated, embed the CG scheme in the Branch-and-Bound algorithm to guarantee optimality. Another avenue on the road map would be to extend the pattern generation scheme to SMP for multistate systems setting.

2.4.3 SMP for fleets of systems

While most SMP consider a single system (system-level SMP), only a few papers deal with the SMP for fleets composed of several systems (fleet-level SMP). The fleet-level SMP (FSMP) is naturally more complex that the system-level SMP. Indeed, the number of systems in the fleet is an additional level of combinations to be explored during the optimization process (Khatab et al., 2020a). The majority of fleet-level SM models deal with series-parallel systems subjected to a single repair channel, and operate either a single mission or an identical alternating sequence of missions and scheduled breaks. Dealing with several repair crews, Khatab et al. (2020a) develop a FSM optimization model for the joint SM and repairpersons assignment decision making. Yang et al. (2018) propose a repair frequency-based SM model for a fleet of systems working within non-identical multi-missions subjected to multiple repairing channels (integrated support stations). This approach will be discussed in what follows.

The FSM model proposed in Yang et al. (2018) considers a fleet of equipment/system required to perform M phased missions with short scheduled breaks. The preparation period of each phase starts at t_0. The start and end times of the m-th mission wave are respectively t_{1m} and t_{2m}. Maintenance for the first wave mission takes place during the entire preparation period from t_0 to t_{11}. This duration usually provides the time to repair all systems. During other mission stages, the break time is too short to fix all the equipment, meaning that some systems will not meet their initial performance for the subsequent missions. In this case, reserve systems are dispatched to be used in place of failed systems. Therefore, the dispatched equipment might vary at each stage but the fleet as a whole remains unchanged.

Maintenance operations are carried out by multiple Integrated Support Stations (ISS). Each ISS can only repair one system at a time. The study assumes that an improper maintenance schedule increases the maintenance hours or frequency leading to a need for more maintenance resources such as personnel, support equipment, etc. Consequently, the maintenance cost and equipment downtime will increase. Therefore, the proposed

FSM optimization model aims to minimize the total number of repairs ($\mathcal{N}$) of the fleet during the entire phased mission. This optimization model is formulated as follows:

$$
\begin{aligned}
[\mathcal{SMP}]_{FS}: \quad \min\ \mathcal{N} = \quad & \sum_{m=1}^{M}\sum_{s=1}^{S}\sum_{i=1}^{I} u_{msi} && && (2.2a)\\
\text{s.t.} \quad & \mathcal{F}_m \le \mathcal{F}_{0m} && \forall m && (2.2b)\\
& D_{ms} \le t_{1m} && \forall m, \forall s && (2.2c)\\
& T_q \le t_{2M} - t_0 && \forall q && (2.2d)\\
& u_{msi} \in \{0,1\} && \forall m, \forall s, \forall i && (2.2e)
\end{aligned}
$$

where $\mathcal{F}_m$ and $\mathcal{F}_{0m}$ are, respectively, the failure probability and failure probability threshold of the fleet during the m-th wave mission ($m \in \{1,2,\ldots,M\}$), D_{ms} is the end time of maintenance for the s^{th} piece of equipment ($s \in \{1,\ldots,S\}$) during the m-th wave mission, T_q ($q \in \{1,2,\ldots,Q\}$) is the total maintenance time of the q^{th} ISS, and t_0 is the start time of the preparation period for the first wave mission. The basic decision variable of the problem is u_{msi}, which is equal to 1 if the i-th subsystem of equipment s is repaired for dispatch in the m-th wave mission.

In the above FSM optimization problem, Constraints (2.2b) ensure the failure probability of the fleet does not exceed its threshold value for the m-th wave mission. Constraints (2.2c) ensure that if a piece of equipment is selected to be repaired during the m-th wave, the end time D_{ms} of maintenance of the s-th equipment for the m-th wave mission should be less than the start time of the m-th wave mission. Constraints (2.2d) force the total maintenance time for each ISS q to not exceed the end time of the last wave mission. To solve their FSM optimization problem, Yang et al. (2018) develop a sequential game algorithm with state backtracking. Results obtained from numerical experiments confirm the conclusions already obtained in (Pandey et al., 2016). These conclusions state that a high number of scheduled breaks increases maintenance cost, while a low number of maintenance breaks may increase failures during missions which in turn cause a higher maintenance cost. This suggests finding the appropriate number of scheduled breaks which minimizes the maintenance cost.

Dealing with the FSM approach proposed in Yang et al. (2018), some limitations can clearly be identified. First, the approach merely relies on a restrictive assumption according to which all equipment in the fleet share the same mission start and end times. This is indeed practically unusual as systems in the same fleet may start their missions at a different time instant. The second main drawback is related to the solution method which is capable to provide optimal solutions only for small to medium-sized applications.

From the literature review conducted, one may clearly conclude that FSMP is another stage of the road map where significant contributions can be made on both optimization problem modelling and solution methods levels. One may also observe some similarities between the FSMP and the well-known resource-constrained project scheduling problem (RCPSP). It would therefore be interesting to leverage the recent advances in RCPSP to formulate and solve large-scale FSMP. Intelligent decomposition methods such as stabilized column generation with Branch-and-bound should also be investigated for large-scale FSMP.

2.4.4 SMP under uncertain mission and maintenance break durations

As pointed out earlier, most existing SM models assume that the duration of missions and breaks are deterministic. However, this assumption may not be valid in industrial situations as it can be challenging to estimate exactly missions and breaks duration. Indeed, such duration may be impacted by unpredictable events such as environmental and operating conditions. To account for such uncertainty, it is more reasonable to consider the duration of missions and breaks as random variables with appropriate probability distributions rather than as deterministic values. The first work dealing with these uncertainties in the SMP setting appeared in Khatab et al. (2017a) and will be briefly discussed below.

In Khatab et al. (2017a), the SM model considers a system composed of i series subsystems $i \in \{1, \ldots, I\}$ each with J_i s-independent, and possibly, nonidentical parallel components $E_{ij}\,\{j = 1, \ldots, J_i\}$. During a break, several IM levels are available for each component. The break duration $\mathcal{T}_0$ and the mission duration U are uncertain and modelled as random variables each governed by a probability distribution. The resulting non-linear stochastic SM optimization model is formulated as a chance constraint programming model as:

$$[\mathcal{SMP}]_{UNC}: \quad \min \quad \sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{l=2}^{L_{ij}} C_{ijl}^{p} X_{ij} x_{ijl} + \sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{l=1}^{L_{ij}} C_{ijl}^{c}\left(1 - X_{ij}\right) x_{ijl} \tag{2.3a}$$

$$\text{s.t.} \quad \mathcal{R} \geq \mathcal{R}_0 \tag{2.3b}$$

$$Pr(TMT \leq \mathcal{T}_0) > \tau \tag{2.3c}$$

$$\sum_{j=1}^{J}\left(1 - X_{ij}\right) x_{ijl} + \sum_{j=1}^{J} X_{ij} x_{ijl} \leq 1 \tag{2.3d}$$

$$x_{ijl} \in \{0, 1\} \qquad \forall i, \forall j, \forall l. \tag{2.3e}$$

where the objective function in Eq. (2.3a) minimizes the total maintenance cost. The binary decision variable x_{ijl} is a binary decision variable: $x_{ijl} = 1$ if maintenance level l is performed on component E_{ij}, and $x_{ijl} = 0$ otherwise. Parameters $C_{ijl}^{p}(C_{ijl}^{c})$ are the unit cost of preventive (corrective) maintenance level l, and $X_{ij} = 1$ is the operational status of component E_{ij} (1 for working and 0 for failed). Equation (2.3b) refers to the required minimum reliability level. Equation (2.3c) is a chance constraint which means that the risk that the total maintenance time exceeds the break duration is less than $1 - \tau$. Constraints (2.3d) ensure that at most one maintenance level is chosen for each selected component, and constraint (2.3e) define the binary decision variable used in the formulation.

In Khatab et al. (2017a), it is shown that assuming a deterministic mission duration underestimates the true value of the system's reliability that can lead to negative economic and safety outcomes. The proposed SM approach has limitations in terms of solution methods when dealing with large-scale systems and/or limited maintenance records. Stochastic models are not efficient in solving large-scale problems as they are intractable when evaluating expected loss functions and tend to have poor performance when the sample size is small. Alternatively, Robust optimization (RO) is a more promising approach for handling uncertainty while ensuring worst-case system reliability without the need for large sample sizes but still it is crucial to have tractable formulations for RO to be effective in SMP.

2.4.5 SMP for multistate systems (MSS)

As pointed out previously, most reported works on the SMP merely rely on the assumption that the duration of missions and maintenance breaks are deterministic. However, such assumptions may not be valid in many real-life situations where unforeseen events (operation conditions, human error, etc.) may introduce uncertainty in some SMP input data (e.g, system parameters, mission and break durations, maintenance resources). Therefore, input data uncertainty should be accounted for to provide more accurate maintenance and repairpersons assignment decisions (Khatab et al., 2017b,a; Khatab and Aghezzaf, 2016b). Dealing with SMP in the MSS setting, Liu et al. (2018) develop a SM optimization model where uncertainties related to the quality of maintenance levels, and duration of breaks are considered.

In Liu et al. (2018), the MSS investigated is composed of J units and performs an alternating sequence of identical missions and scheduled breaks. Each unit j ($j \in \{1, \cdots, J\}$) evolves in a set of states $\mathcal{Y}_j = \{y_{j,1}, y_{j,2}, \ldots, y_{j,n_j}\}$ where $y_{j,1}$ refers to the worst state, y_{j,n_j} is the best state, and n_j is the number of states. At time t, the state of unit j is denoted as $G_j(t)$, where $G_j(t) \in \mathcal{Y}_j$. The state space of the system is denoted as $\mathcal{S} = \{s_1, s_2, \ldots, s_N\}$ where N is the number of the possible states. Depending on the structure and the structure-function $\phi(G_1(t), G_2(t), \ldots, G_J(t))$ of the MSS, the state of the MSS at time t can be derived from all the units' states by $G(t) = \phi(G_1(t), G_2(t), \ldots, G_J(t)) \in \mathcal{S}$. A maintenance action $a_{j,(y,z)}$ ($y < z$) when performed on unit j recovers this unit from state y to state z. The respective cost and time of a maintenance action $a_{j,(y,z)}$ are denoted as $c_{j,(y,z)}$ and $d_{j,(y,z)}$. In line with the work in (Khatab et al., 2017b,a; Khatab and Aghezzaf, 2016b), maintenance and break duration are uncertain and modelled as random variables.

To successfully operate the next mission with the highest possible probability under the limited maintenance resources, the SM for MSSs in Liu et al. (2018) consists to select the set of units and the corresponding maintenance actions to be performed during the break, and then determine the appropriate sequence planning for all the selected maintenance actions. To do so, a binary decision variable $x_{v,j,(G_j(t_0),z)}$ is used to represent the sequence of a selected maintenance action $a_{j,(y,z)}$ for unit j if its state is $G_j(t_0)$ at the beginning of the break. Roughly speaking, $x_{v,j,(G_j(t_0),z)} = 1$ if the v^{th} maintenance action is $a_{j,(G_j(t_0),z)}$ for unit j, and $x_{v,j,(G_j(t_0),z)} = 2$ otherwise. The subscript $v(v \in 1, 2, \ldots, J)$ is an integer representing the index of the selected maintenance action. If a planned maintenance action for a unit has not been completed by the end of a break, the unit will still be used and considered "as bad as old" for the next mission.

The resulting total maintenance cost C induced during a break is:

$$C = \sum_{v=1}^{J} \sum_{j=1}^{J} \sum_{z=G_j(t_0)}^{n_j} c_{j,(y,z)} \cdot x_{v,j,(G_j(t_0),z)} \tag{2.4}$$

The resulting SM optimization model aiming to maximize the reliability $\mathcal{R}$ for the next mission under maintenance budget C_0 and break is formulated as follows:

$$
\begin{aligned}
[\mathcal{SMP}]_{MSS}: \quad & \max \quad \mathcal{R} && && (2.5a)\\
\text{s.t.} \quad & \sum_{v=1}^{J} \sum_{j=1}^{J} \sum_{z=G_j(t_0)}^{n_j} c_{j,(y,z)} \cdot x_{v,j,(G_j(t_0),z)} \leq \; C_0 && && (2.5b)\\
& \sum_{v=1}^{J} \sum_{z=G_j(t_0)}^{n_j} x_{v,j,(G_j(t_0),z)} \leq 1 && \forall j && (2.5c)\\
& \sum_{j=1}^{J} \sum_{z=G_j(t_0)}^{n_j} x_{v,j,(G_j(t_0),z)} \leq 1 && \forall v && (2.5d)\\
& x_{v,j,(G_j(t_0),z)} \in \{0,1\} && \forall j, \forall v. && (2.5e)
\end{aligned}
$$

In the above optimization model, constraint (2.5b) ensures compliance with the budget C_0, constraints (2.5c) ensure that at most one maintenance action is selected for each unit, and constraints (2.5d) ensure that each selected maintenance action is used only once in the maintenance sequence plan. Finally, constraint (2.5e) refer to the binary decision variable $x_{v,j,(G_j(t_0),z)}$. The saddlepoint approximation is used to simplify the computation of the multi-dimensional integration involved in evaluating $\mathcal{R}$, and the resulting optimisation problem is solved using a tailored ant colony optimisation algorithm.

For the SMP for MSS, two main common drawbacks can be identified with most papers in the literature. First, the existing work merely relies on a single repair crew assumption. As pointed out earlier, this assumption is unrealistic and should be relaxed by considering multiple repairpersons. This will naturally lead to the issue of the joint SM and repairpersons assignment problem in the MSS setting. The second drawback identified is related to the solution methods used to solve the SMP for MSS. Indeed, a variety of interesting heuristics-based solution methods are developed. However, the resulting maintenance decisions are therefore near-optimal and may induce errors by, for example, overestimating or underestimating the maintenance cost and the reliability of the system to operate its missions. Therefore, as future research, it would be interesting to develop efficient solution methods capable of providing exact optimal maintenance and repair crews assignment decisions even for large-sized SMP in MSS setting. Another avenue on the roadmap is to develop SM optimsation models for MSS availability optimisation. This future research issue may involve a simulation-optimization scheme or resorting to Kronecker algebra (Khatab et al., 2012).

2.4.6 SMP with multiple objective functions

As mentioned in section (2.4.1), the majority of the existing SM models deal with a single objective function to either maximize system reliability (**SMP-MaxRel**) or minimize maintenance cost (**SMP-MinCost**). However, in many industrial applications, the maintenance decision-maker is usually faced with finding trade-offs between several conflicting objective functions. This led to SM models with multiple objective functions. To illustrate, the SM model proposed in Shahraki et al. (2020) is briefly discussed.

The SMP investigated in (Shahraki et al., 2020) considers a multi-state serial system where J components are s-dependent and subjected to stochastic quality of imperfect maintenance. The degradation of component j ($j = 1,\ldots,J$) is assumed gradual and governed by a continuous homogeneous time Markov process whose state space $\mathcal{Y} = \{y_0, y_1, \ldots, y_n\}$ where y_0 and y_n are the failed and the best states, respectively. The state space of the entire MSS is denoted as $\mathcal{S} = \{s_0, s_2, \ldots, s_N\}$ where $N+1$ is the number of the MSS states. At a given time t, the state of the system is determined by the random variable $G(t) \in \mathcal{S}$.

Several maintenance levels are available during the break. When maintenance action of level l is performed on a component, say j, the state z_j of component j lies between the perfect state y_n (case of renewal) and the current state y_j (case of do nothing). Accordingly, the new state z_j can be computed as $z_j = y_j + l$ with $l \in [0, n-j]$. For such maintenance level, te corresponding maintenance time and cost are denoted as $c_j(yj, zj)$ and $d_j(yj, zj)$. To select the component to maintain and the maintenance level to perform on, a binary decision variable x_{jl} is introduced such that $x_{jl} = 1$ if maintenance level l is carried out on component j, and $x_{jl} = 0$ otherwise.

The system reliability $\mathcal{R}$ during the next mission is uncertain due to the stochastic IM actions. Indeed when a maintenance action is performed on a component, the state of the entire MSS can be one of the possible states in $\mathcal{S}$ with a given probability. This uncertainty is further compounded by the stochastic dependence between components of the system. To estimate the reliability of the system Shahraki et al. (2020) used a Monte Carlo simulation.

In the SM model developed in (Shahraki et al., 2020), two objective functions are considered: the expected system reliability $\mathbb{E}[\mathcal{R}]$ that should be maximized, and the variance $\mathbb{V}[\mathcal{R}]$ of the system reliability that should be minimized. The resulting optimization problem is formulated as:

$$\max \mathbb{E}[\mathcal{R}] = \sum_{i=0}^{N} \mathbb{E}[\mathcal{R}|s_i] \Pr(G(t_0) = s_i) \tag{2.6a}$$

$$\min \mathbb{V}[\mathcal{R}] = \sum_{i=0}^{N} \mathbb{V}[\mathcal{R}|s_i] \Pr(G(t_0) = s_i) + \sum_{i=0}^{N} \mathbb{E}[\mathcal{R}|s_i]^2 \Pr(G(t_0) = s_i) - \left(\sum_{i=0}^{N} \mathbb{E}[\mathcal{R}|s_i] \Pr(G(t_0) = s_i) \right)^2 \tag{2.6b}$$

$$\text{s.t.} \quad \sum_{j=1}^{J} \sum_{l=0}^{n-j} d_j(y_j, y_j + l) \cdot x_{jl} \leq \mathcal{T}_0 \tag{2.6c}$$

$$\sum_{j=1}^{J} \sum_{l=0}^{n-j} c_j(y_j, y_j + l) \cdot x_{jl} \leq C_0 \tag{2.6d}$$

$$\sum_{l=0}^{n-j} x_{jl} = 1 \qquad \forall j \tag{2.6e}$$

$$x_{jl} \in \{0, 1\} \qquad \forall j, \forall l \tag{2.6f}$$

In the above optimization problem, Equations (2.6c) and (2.6d) refer, respectively, to the limited break duration and budget constraints. Constraint (2.6e) ensures that only one maintenance action with specified maintenance level l can be selected for each component j. Finally, constraint (2.6d) refers to the binary decision variable x_{jl}. To solve the proposed SMP, the non-dominated sorting genetic algorithm II (NSGAII) is used to find the Pareto-optimal solutions.

As with most multi-objective SM models in the SM literature, the proposed solution technique is not able to handle large-scale industrial instances and cannot ensure optimality. Also, the system configuration considered in the proposed model is basic (series structure) with similar components all modelled by homogenous CTMC. Furthermore, only one repairperson is considered. Such a formulation does not match with many real-life applications where more complex configurations should be considered.

2.5 Research gaps and future directions

In this paper, various modelling frameworks applied to the SMP were presented, and the studies were investigated in terms of their solution methodologies. In this section, research gaps and potential future research guidelines are discussed. The following are 10 areas of a road map proposed to guide the development of the SMP research topic.

1. Existing exact and heuristic methods still lack the ability to handle complex reliability structures and/or very large-scale multi-component systems and/or repairpersons and/or multiple non-identical missions. Future research should focus on developing exact or heuristic approaches that can handle these complex and large-scale cases.

2. Most fleet-level SM models consider series-parallel systems with identical binary-state components, a single repair channel, and same starting and ending times (i.e., synchronous breaks). Future research should consider complex reliability structures for each system in the fleet, and repairpersons routing for a geographically dispersed fleet. Furthermore, the fleet SMP shares many similarities with the well-known resource-constrained project scheduling problem (RCPSP). It would therefore be interesting to leverage the recent advances in RCPSP to formulate and solve large-scale FSMP. Intelligent decomposition methods such as stabilized column generation with Branch-and-bound must be developed to address large-scale FSMP.

3. Most studies dealing with multi-state systems employ Markov chains and the Universal Generating Function to model component deterioration and calculate state probabilities, while others utilize simulation. Future research should focus on developing exact solution methods for large-scale instances by incorporating a stochastic formulation and considering multiple repairpersons.

4. Research efforts should be directed towards finding a decomposition approach to remove the calculation of the multistate reliability from the optimisation phase as done by Diallo et al. (2018) in the binary state case. Accomplishing such a methodology would allow to optimally solve the multistate SMP for large-scale systems. Another avenue on the roadmap is to develop an analytical formulation of the SMP for availability optimisation. That may involve a simulation-optimization scheme or resort to Kronecker algebra.

5. Given the limited number of papers that examine the SMP in the context of structural dependence, future extensions should focus on incorporating advanced concepts of structural dependence, taking into account the presence of co-located subsystems, and improving the assignment of repairpersons to repairs. Also, it should examine the interactions between failure modes under stochastic dependence through the use of copula functions, reliability indices, and state-based interactions.

6. The majority of SM models take into account time and cost as resource constraints, while assuming an abundance of repair capacity (channels and repair crews). Future research should concentrate on addressing the joint maintenance and crew scheduling problem and creating new methods of decomposition or relaxation to solve the resulting complex combinatorial problem.

7. Few studies have addressed multi-objective optimisation problems in SMP, yet in practice maintenance, decision-makers often have multiple objectives to consider. Future research should focus on developing multi-objective formulations that align with industry needs and effective methods to solve large-scale multi-objective SMPs.

8. The majority of SMP models assume that key parameters such as mission duration, break lengths, quality of maintenance, and resource consumption are constant and known. Only a limited number of papers take into account one or two of these parameters as being stochastic and develop risk-based decision models. In reality, many of these parameters can be uncertain due to the lack of historical data or the variability occurring within the system. Therefore, it is crucial to design robust SMP models that can handle uncertainty.

9. The joint repairperson assignment extension (JSM-RAP) has proven useful. Future extensions should focus on new ideas such as workload sharing between repairpersons, and the resumability or non-resumability of maintenance actions. Learning and forgetting curves could also be integrated into the JSM-RAP formulations. A key assumption in the JSM-RAP is that all repairs can take place without repairpersons colliding. This is not always true. In naval vessels, for example, work takes place in close quarters. Thus, extending the models to consider work interruption with or without preemption to prioritize certain repairpersons would be intriguing.

10. Recent SMP studies have only just started to explore the potential of artificial intelligence (AI) and deep learning techniques, but it is becoming increasingly clear that these technologies have much to offer. Their ability to analyze large amounts of data, identify patterns, and make predictions about system behavior can help decision-makers make more informed decisions about when and how to perform maintenance. Given the potential benefits, future research should focus more on the application of AI and deep learning techniques in selective maintenance to improve decision-making and overall system performance.

2.6 Conclusion

This article reviewed the literature on selective maintenance focusing on models that have been published since the seminal works by Rice et al. (1998) and Rice (1999). These models were identified, classified, and analyzed using a proposed framework. The article also provided an overview of well-known selective maintenance models, including their underlying assumptions and methods. The classification table generated during the review highlighted several areas for future research and development, such as developing methods that can handle complex and large-scale problems, creating models that account for asynchronous break periods, incorporating more complex reliability structures and repair routing for geographically dispersed fleets, and advanced concepts of structural dependence, addressing the joint maintenance and crew scheduling problem and developing new methods of decomposition or relaxation to solve complex combinatorial problems. Additionally, there is a need for robust selective maintenance models that can handle uncertain parameters, and the application of AI and deep learning techniques to improve decision-making and system performance.

Chapter 3

Branch-and-price algorithms for large-scale mission-oriented maintenance planning problems

3.1 Introduction

Many modern systems operate according to alternating sequences of missions and break periods during which maintenance actions are performed. Such multi-component systems are encountered in traditional manufacturing, production and service industries where equipment such as production lines, aircraft, ships, and trucks operate continuously until they are interrupted to undergo maintenance. New assets such as unmanned autonomous vehicles and advanced combat/defensive systems also exhibit such patterns. The *selective maintenance* (SM) strategy introduced by Rice et al. (1998) is particularly suitable for these mission-oriented systems. To improve the ability of such systems to successfully achieve their subsequent missions, maintenance actions are carried out on components during the scheduled breaks. Limited maintenance resources such as time, budget, spare parts, and repair crews restrict the number and levels of maintenance activities that can be performed before the next mission. The decision problem that entails selecting the components to maintain and the level of maintenance actions to carry out is known as the *selective maintenance problem* (SMP). Furthermore, when the selected maintenance actions are to be performed by multiple repairpersons, potentially having different skill levels and costs, the SMP that jointly determines the assignment of tasks to repairpersons is referred to as the *joint selective maintenance and repairperson assignment problem* (JSM-RAP) (Diallo et al., 2017, 2019b). The SMP and its variants have been applied to many industrial systems such as wind turbines (O'Neil et al., 2022a; O'Neil et al., 2023a), coal conveyor systems (Liu et al., 2009), machining lines in an engine shop (Zhu et al., 2011), army tanks (Sharma et al., 2017), nuclear fuel production systems (Zhao et al., 2019b), aircraft turbine engine systems (Wang et al., 2019), and flow transmission systems (Liu et al., 2020).

Over the years, various extensions of the basic SMP with different systems structures, maintenance policies, resource limitations, modelling methods, and solution algorithms have been proposed to bring the models closer to the realities of industrial and practical settings. The reader is referred to the recent surveys by Al-Jabouri et al. (2022), Cao et al. (2018a), and Xu et al. (2015) for a detailed account of the SMP literature. These extensions can generally be clustered into four groups of characteristics related to the system, maintenance, mission and mathematical model. System characteristics include features such as system level (single system or fleet of systems) (Schneider and Cassady, 2004, 2015; Khatab et al., 2020a), system state (binary or multistate components/systems) (Meng et al., 1999; Pandey et al., 2013a; Dao and Zuo, 2017a; Yin et al., 2023), and system dependency (economic, structural, or stochastic) (Xu et al., 2016a; Dao and Zuo, 2017b; Shahraki et al., 2020). Maintenance characteristics refer to the attributes related to the execution of maintenance activities. These attributes such as repairpersons' availability (ample or limited) (Diallo et al., 2017, 2019b; Chaabane et al., 2018), and the effectiveness of actions carried out (perfect or imperfect maintenance) (Khatab et al., 2008b; Do et al., 2015; Khatab et al., 2018b), can significantly affect maintenance decisions and reliability achieved. Mission characteristics include features such as mission types (single or multi-mission) (Zhang et al., 2019b; Chaabane et al., 2020a) and planning horizon (finite or infinite) (Yu and Schneider, 2003; Maillart et al., 2009). Model characteristics include two main characteristics: optimization criteria (*e.g.*, reliability/availability maximization and/or cost/energy/emissions minimization) (Yu and Schneider, 2003; Hoai and Luong, 2006; Zhang et al., 2020), and parameter uncertainty and robustness (Jiang and Liu, 2020a,b). The main challenge for most SMP models is that their formulations are difficult to solve optimally, especially for industrial-size problems that would allow practitioners to implement and use the SMP for their many systems and repairpersons. Rice (1999) proved that the basic SMP is NP-hard, and so are all its extensions, implying that computational efforts increase exponentially with problem size. Among the solution approaches proposed for the SMP are general heuristics (Khatab et al., 2007; Lust et al., 2009; Cao et al., 2018b; Ahadi and Sullivan, 2019; Galante et al., 2020), metaheuristics (*e.g.*, genetic algorithm (Dao et al., 2014), differential evolution (Pandey et al., 2013b), and simulated annealing (Jiang and Liu, 2020b)), exact solution approaches, (*e.g.*, total enumeration (Rice et al., 1998), search space reduction (Rajagopalan and Cassady, 2006), depth-first search algorithms (Cao et al., 2016b), branch-and-bound-type procedures (Xia et al., 2022), the two-phase approach (Diallo et al., 2018)), the max-min approach (Schneider and Cassady, 2015), and machine learning (Liu et al., 2020; Kammoun et al., 2022; Hesabi et al., 2022; O'Neil et al., 2022a). However, large-scale instances of the problem, and in particular its JSM-RAP extension, are still challenging to solve due to its combinatorial and nonlinear nature. Thus, novel reformulations, approximations and solution methods that can handle real-life systems consisting of hundreds of components are still needed (Cao et al., 2018a; Diallo et al., 2019b).

This paper proposes a novel approach based on column-generation (CG) for solving large-scale instances of the JSM-RAP representative of real industry problems. The proposed approach is closely related to the two-phase approach developed by Diallo et al. (2019b) to solve moderate-size instances of the problem with imperfect repair and multiple repair channels. The two-phase approach transforms the JSM-RAP into a multidimensional multiple-choice knapsack problem (MdMCKP) by generating all feasible combinations (*i.e.*, patterns/columns) of components, maintenance levels and repairpersons, and then solving the MdMCKP to optimally select a subset of patterns that minimize the total maintenance cost or maximize the reliability for the next mission. Although this approach is shown to be efficient for small-to-medium size instances, its pattern generation scheme runs out of memory for industrial-size instances where the multiple components, maintenance levels and repairpersons immensely increase the number of combinations to explore. Furthermore, the MdMCKP is a binary integer program (BIP) and is one of the most complex members of the Knapsack Problem (KP) family, which is known to be NP-hard in general (Cacchiani et al., 2022). Thus, an MdMCKP with a large number of binary variables is extremely challenging to solve to proven optimality (Zia and Coit, 2010). Another very recent pattern-based approach for solving the JSM-RAP is proposed by O'Neil et al. (2022b). Unlike the two-phase approach of Diallo et al. (2019b), which generates all feasible patterns at the outset, they use a CG-based heuristic algorithm in which the subproblems are solved using the genetic algorithm (GA). Since a metaheuristic is used to solve the subproblems, no guarantee about the quality of the obtained solution can be provided, which is a major limitation of their approach.

Our work builds on the approaches proposed by Diallo et al. (2019b) and O'Neil et al. (2022b) while avoiding their drawbacks. To the best of our knowledge, this paper provides the first *rigorous* CG-based approach for solving the JSM-RAP. Unlike the two-phase approach (Diallo et al., 2019b), the proposed approach generates maintenance patterns and adds them to a restricted master problem iteratively and only as needed by solving a decomposable subproblem. Thus, the number of patterns considered represents a very small fraction of the total number of feasible patterns. In contrast to the GA metaheuristic used by O'Neil et al. (2022b) to solve the nonlinear subproblems, we propose two readily-solvable reformulations for the subproblem: (1) a piecewise-linear approximation (PLA) whose accuracy can be improved by increasing the number of breakpoints, and (2) an exact reformulation into a mixed-integer exponential conic optimization (ECO) problem that can be handled using off-the-shelf solvers. Also, compared to O'Neil et al. (2022b), our approach features significant improvements. Specifically, CG is embedded in a branch-and-bound (B&B) tree to construct branch-and-price (B&P) algorithms that restore solution integrality and guarantee its optimality. Moreover, a stabilization scheme is applied to accelerate the convergence of CG. Extensive numerical experiments are conducted to compare the performance of the proposed approach *vis-à-vis* state-of-the-art solution methods. The approach is then used to solve the largest JSM-RAP instances to date. In summary, the primary contribution of this paper is a branch-and-price algorithm with two readily-solvable reformulations for the subproblems, as well as a stabilization scheme to enhance the convergence of CG.

The remainder of this paper is organized as follows. Section 3.2 describes the multi-component system structure, its working assumptions, the computation of the resources available to carry out repairs, and the system reliability formulas. A nonlinear BIP formulation and a full-pattern-enumeration reformulation are developed in Section 3.3. In Section 3.4, the proposed B&P algorithms, including the two subproblem reformulations, the stabilization scheme, a heuristic to generate initial feasible solutions, and the B&B procedure, are described. Several numerical experiments and the discussion of their results are presented in Section 3.5. Conclusions are drawn and future extensions are discussed in Section 3.6.

3.2 Problem description

In this section, the multi-component system under consideration, along with its working assumptions and the notation system used in the mathematical formulation of the problem are described. The imperfect maintenance model and expressions for the total maintenance cost and duration incurred by the system components during the scheduled maintenance break and the resulting system reliability are then derived.

3.2.1 System description

The mission-oriented system under consideration is made of s subsystems in series (*i.e.*, all subsystems must function for the system to work). Each subsystem i consists of n_i statistically independent repairable binary components E_{ij} (*i.e.*, components transition between failed and functioning states) connected in parallel (*i.e.*, a subsystem functions if at least one of its components is functioning). In reliability theory, such a system is said to have a series-parallel configuration which is commonly encountered in many multi-component systems. The lifetimes of the components are not necessarily identical but follow an identical general distribution. The system performs an alternating sequence of missions and scheduled breaks of finite duration. Maintenance actions are performed during the breaks to improve the system's reliability during the following mission.

The system has completed or returned from a mission and is about to undergo maintenance activities during the scheduled break of length D_0. After the break, the system will undertake another mission of duration M. At the beginning of the current break, component E_{ij} is characterized by its current age B_{ij} and operating status X_{ij} ($X_{ij} = 1$ if functioning, $X_{ij} = 0$ if failed). Similarly, at the end of the break, component E_{ij} is characterized by its effective age A_{ij} and operating status Y_{ij} ($Y_{ij} = 1$ if functioning, $Y_{ij} = 0$ if failed).

3.2.2 Main working assumptions

The following assumptions are made:

1. The system consists of multiple, possibly non-identical and stochastically-independent repairable binary components. The components and the system are either functioning or have failed. This is a reasonable assumption and is supported by many

examples in the literature (Diallo et al., 2018; Jiang and Liu, 2020a).

2. System components do not age during the break because the age of a component is mostly operation-dependent (Diallo et al., 2019b). This is also reasonable because the break durations are typically negligible compared to the mission duration.

3. Maintenance activities are allowed only during the break, while no maintenance can be performed during the mission. For many mission-oriented systems, it is impossible to interrupt the mission to carry out any maintenance action.

4. All required limited resources (budget, repairpersons, tools) are available when needed.

5. Multiple components can be worked on simultaneously without repairpersons colliding.

3.2.3 Notation

Table 3.1 lists the indices, sets and parameters used in the mathematical formulation of the problem. Furthermore, we use the following decision variables:

$$x_{ijlr} = \begin{cases} 1 & \text{if maintenance level } l \text{ is performed on } E_{ij} \text{ by repairperson } r, \\ 0 & \text{otherwise.} \end{cases} \tag{3.1}$$

3.2.4 Maintenance levels, costs, and duration

During a scheduled break, corrective maintenance (CM) and preventive maintenance (PM) actions are performed. The former is carried out on failed components, while the latter concerns components that are still functioning.

For a failed component E_{ij}, one maintenance level among the available $(m_{ij}+1)$ CM levels $\left(l \in \left\{0, 1, \ldots,\ m_{ij}\right\}\right)$ must be chosen to be performed. The lowest level ($l = 0$) and the highest level $\left(l = m_{ij}\right)$ stand for the "Do Nothing (DN)" and the component replacement options, respectively. Level $l = 1$ refers to minimal repair (MR) which when performed brings the component to an "as bad as old" condition. Intermediate values of $1 < l < m_{ij}$ represent imperfect maintenance (IM) actions which after being performed bring the

Table 3.1: Table of notation

s	number of subsystems in series
i	index of subsystems, $i \in \mathcal{I}$ where $\mathcal{I} = \{1, \ldots, s\}$
n_i	number of components in parallel in the i^{th} subsystem
j	index of components in subsystem i, $j \in \mathcal{J}_i$ where $\mathcal{J}_i = \{1, \ldots, n_i\}$
E_{ij}	j^{th} component of subsystem i
m_{ij}	the highest maintenance level available for component E_{ij}
l	index of maintenance levels available for component E_{ij}, $l \in \mathcal{L}_{ij}$ where $\mathcal{L}_{ij} = \left\{0, \ldots, m_{ij}\right\}$
q	number of repairpersons available
r	index of repairpersons, $r \in \{1, \ldots, q\}$
p_i	number of maintenance patterns generated for subsystem i
k	index of maintenance patterns generated for subsystem i, $k \in \mathcal{K}_i$ where $\mathcal{K}_i = \{1, \ldots, p_i\}$
v_r	variable repairperson labor cost per unit of time
$t^c_{ijlr}(t^p_{ijlr})$	duration of CM (PM) when maintenance level l is performed on component E_{ij} by repairperson r
$c^c_{ijl}(c^p_{ijl})$	cost of CM (PM) when maintenance level l is performed on component E_{ij}
$B_{ij}(A_{ij})$	age of component E_{ij} at the start (end) of the break
A_{ijl}	age of component E_{ij} at the end of the break if maintenance level l is performed
$X_{ij}(Y_{ij})$	binary status parameter of component E_{ij} at the start (end) of the break (1: functioning, 0: failed)
T_{ikr}	total work time of repairperson r on subsystem i under pattern k
C_{ik}	total maintenance cost of pattern k for subsystem i
C_0	maximum maintenance budget available
D_0	break duration
M	length of next mission
$R^c_{ij}(M\|_{A_{ijl}})$	conditional reliability of component E_{ij} during the next mission given an initial age A_{ijl}
R_{ik}	reliability of subsystem i for the next mission under maintenance pattern k
F_{ik}	unreliability of subsystem i under maintenance pattern k, $F_{ik} = 1 - R_{ik}$
$\mathcal{R}$	overall system reliability during the next mission

component health condition back to somewhere between "as bad as old" and "as good as new".

The IM model is developed on the basis of the age reduction concept initially introduced by Malik (1979). Liu et al. (2009) and Liu and Huang (2010) utilize the Kijima type-II IM model in the SMP to represent imperfect repair, with age reduction depending on maintenance cost and a characteristic constant that reflects a component's relative age. The approach yields better solutions than methods that do not consider IM. Pandey et al. (2013b) redefine the age reduction model and develop a characteristic constant based on effective age and mean residual life (MRL). Zhu et al. (2011) adopt a reliability-centred maintenance approach in the SMP context using a hybrid hazard rate model for IM. Pandey and Zuo (2013) propose a new hazard-rate adjustment factor dependent on effective age and maintenance resources. Khatab et al. (2018b) extend the degradation reduction coefficient to model IM in a multi-component SMP under periodic inspection, assuming component-specific degradation processes and threshold-based failure, with maintenance actions determined by degradation reduction. The age reduction-based model (Malik, 1979) is considered in this paper, given its capacity to determine reliability in a closed-form expression, as opposed to Markovian-based approaches.

Here, IM is modelled according to the age reduction approach (Malik, 1979): when a CM of level l is performed by repairperson r on component E_{ij}, its corresponding age B_{ij} is multiplied by an age reduction coefficient θ_{ijl} $\left(0 \leq \theta_{ijl} \leq 1\right)$. Any CM action incurs a cost c^{c}_{ijl} and requires t^{c}_{ijlr} units of time.

Similarly, if component E_{ij} is still functioning, it can be subjected to a PM action of level $l \in \left\{0, 2, \ldots, m_{ij}\right\}$. Note that minimal repair ($l = 1$) is not available for working components. Intermediate values of l $\left(2 \leq l < m_{ij}\right)$ represent IM actions that rejuvenate the component by reducing its age by a factor φ_{ijl} $\left(0 \leq \varphi_{ijl} \leq 1\right)$. Any PM action incurs a cost c^{p}_{ijl} and has a duration t^{p}_{ijlr}.

According to the above IM model, the effective age A_{ijl} of a given component E_{ij} at the end of the break is computed as a function of its initial operating status X_{ij} and the maintenance level l performed, as follows:

$$A_{ijl} = B_{ij}\left(X_{ij}\varphi_{ijl} + \left(1 - X_{ij}\right)\theta_{ijl}\right). \tag{3.2}$$

The total PM and CM costs are computed as:

$$C_{PM} = \sum_{i=1}^{s}\sum_{j=1}^{n_i}\sum_{l=2}^{m_{ij}}\sum_{r=1}^{q}\left(v_r t^p_{ijlr} + c^p_{ijl}\right) X_{ij} x_{ijlr}, \tag{3.3}$$

$$C_{CM} = \sum_{i=1}^{s}\sum_{j=1}^{n_i}\sum_{l=1}^{m_{ij}}\sum_{r=1}^{q}\left(v_r t^c_{ijlr} + c^c_{ijl}\right)\left(1 - X_{ij}\right) x_{ijlr}, \tag{3.4}$$

where the term $(1-X_{ij})$ ensures that CM actions are available only for failed components. The total maintenance cost is given by $C = C_{PM} + C_{CM}$. Likewise, the total time T_r spent by each repairperson r to carry out their maintenance duties is computed as:

$$T_r = \sum_{i=1}^{s}\sum_{j=1}^{n_i}\left[\left(1 - X_{ij}\right)\sum_{l=1}^{m_{ij}} t^c_{ijlr} x_{ijlr} + \ X_{ij}\sum_{l=2}^{m_{ij}} t^p_{ijlr} x_{ijlr}\right]. \tag{3.5}$$

3.2.5 System reliability during the next mission

To compute the system reliability $\mathcal{R}$ during the next mission, we first compute the conditional reliability $R^c_{ij}(M|_{A_{ijl}})$ of component E_{ij} given that its initial age is A_{ijl}. Denote by $R_{ij}(t)$ the unconditional reliability function of component E_{ij}. Then, the reliability during the next mission if E_{ij} undergoes a maintenance action of level $l \in \{0, \ldots, m_{ij}\}$ is given by:

$$R_{ijl} = \frac{R_{ij}\left(A_{ijl} + M\right)}{R_{ij}\left(A_{ijl}\right)}. \tag{3.6}$$

Given that each component undergoes exactly one maintenance action of level l (including "Do-Nothing" when $l = 0$), and given that only one repairperson is needed to perform that maintenance action, the conditional reliability $R^c_{ij}(M|_{A_{ijl}})$ is then obtained as:

$$R^c_{ij}(M|_{A_{ijl}}) = \sum_{l=0}^{m_{ij}}\sum_{r=1}^{q} R_{ijl} x_{ijlr}. \tag{3.7}$$

Given that the i^{th} subsystem has a parallel configuration, its reliability R_i during the next mission is then given by:

$$R_i = 1 - \prod_{j=1}^{n_i}\left(1 - \sum_{l=0}^{m_{ij}}\sum_{r=1}^{q} R_{ijl} x_{ijlr}\right). \tag{3.8}$$

Finally, the reliability $\mathcal{R}$ of the whole series-parallel system is computed as follows:

$$\mathcal{R} = \prod_{i=1}^{s} R_i = \prod_{i=1}^{s}\left(1 - \prod_{j=1}^{n_i}\left(1 - \sum_{l=0}^{m_{ij}}\sum_{r=1}^{q} R_{ijl}x_{ijlr}\right)\right). \tag{3.9}$$

3.3 Mathematical formulation and pattern-based reformulation of the JSM-RAP

The JSM-RAP dealt with in this paper aims to jointly select the set of components to be maintained, the maintenance levels to be performed on the selected components, and the repairpersons to carry out the selected maintenance actions such that the system reliability for the next mission is maximized given predetermined maintenance budget and break duration. In this section, the nonlinear BIP formulation of the JSM-RAP and its reformulation with full patterns enumeration by Diallo et al. (2019b) are briefly exposed.

3.3.1 Nonlinear BIP formulation

The JSM-RAP is formulated as follows:

$$\max \quad \mathcal{R} = \prod_{i=1}^{s}\left[1 - \prod_{j=1}^{n_i}\left(1 - \sum_{l=0}^{m_{ij}}\sum_{r=1}^{q} R_{ijl}x_{ijlr}\right)\right] \tag{3.10a}$$

s.t.

$$\sum_{i=1}^{s}\sum_{j=1}^{n_i}\sum_{l=2}^{m_{ij}}\sum_{r=1}^{q}\left(v_r t^{p}_{ijlr} + c^{p}_{ijl}\right)X_{ij}x_{ijlr} + \sum_{i=1}^{s}\sum_{j=1}^{n_i}\sum_{l=1}^{m_{ij}}\sum_{r=1}^{q}\left(v_r t^{c}_{ijlr} + c^{c}_{ijl}\right)\left(1 - X_{ij}\right)x_{ijlr} \leq C_0 \tag{3.10b}$$

$$\sum_{i=1}^{s}\sum_{j=1}^{n_i}\left(\left(1 - X_{ij}\right)\sum_{l=1}^{m_{ij}} t^{c}_{ijlr}x_{ijlr} + X_{ij}\sum_{l=2}^{m_{ij}} t^{p}_{ijlr}x_{ijlr}\right) \leq D_0 \qquad \forall r \tag{3.10c}$$

$$\sum_{l=1}^{m_{ij}}\sum_{r=1}^{q} x_{ijlr} = 1 \qquad \forall i, \forall j \tag{3.10d}$$

$$x_{ijlr} \in \{0, 1\} \qquad \forall i, \forall j, \forall l, \forall r. \tag{3.10e}$$

Eqn. (3.10a) is the objective function that maximizes the system's reliability for the next mission. Constraint (3.10b) states that the total cost of maintenance cannot exceed the maintenance budget. Constraint (3.10c) ensures that the gross time of the maintenance actions assigned to each repairperson does not exceed the break duration. Constraint (3.10d) ensures that each component receives exactly one maintenance action of any given level.

The above formulation is a nonlinear BIP, which makes finding optimal solutions for real applications computationally expensive. To alleviate this computational burden, Diallo et al. (2019b) presented an equivalent linearized version of the above formulation using a complete enumeration of the maintenance patterns for each subsystem.

3.3.2 Reformulation with full patterns enumeration

A *maintenance pattern* is defined as the combination of components and their corresponding maintenance levels to be performed by repairpersons during the break. Accordingly, for a given subsystem i, a maintenance pattern can be represented as a matrix of dimension $q \times n_i$ whose elements are the maintenance levels performed on the n_i components of the subsystem. To illustrate the generation of maintenance patterns, let us consider a single subsystem ($s = 1$) composed of three ($n_1 = 3$) components in parallel such that all three are "failed" at the start of the break. Let us assume that two ($q = 2$) repairpersons are available to carry out four ($m_{ij} + 1 = 4$) common maintenance levels on the two components: Do Nothing ($l = 0$), Minimal Repair ($l = 1$), Imperfect Maintenance ($l = 2$), and Corrective Replacement ($l = 3$). The following matrix is an example of a maintenance pattern:

$$\begin{pmatrix} 0 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

The above maintenance pattern can be interpreted as follows: the first repairperson will replace the second component while the second repairperson will carry out a minimal repair on the first component and imperfect maintenance on the third component. This pattern yields the following feasible values of the decision variables: $x_{1101} = 1$, $x_{1231} = 1$, $x_{1301} = 1$, $x_{1112} = 1$, $x_{1202} = 1$, $x_{1322} = 1$, and all others are equal to 0. In general, each pattern (indexed by k) consists of a binary vector whose elements are defined as:

$$x_{ijlr}^{k} = \begin{cases} 1 & \text{if maintenance level } l \text{ is performed on } E_{ij} \text{ by repairperson } r, \\ 0 & \text{otherwise.} \end{cases} \tag{3.11}$$

Herein, we use subsystem-specific patterns. Denote by p_i the number of maintenance patterns corresponding to the i^{th} subsystem. To each pattern k ($k \in \mathcal{K}_i = \{1,\ldots,p_i\}$) correspond a total maintenance cost C_{ik} and a reliability value R_{ik}, as well as a list of total maintenance times T_{ikr} ($r = 1,\ldots,q$) spent by each repairperson r to perform their assigned maintenance actions on the components of subsystem i. These quantities are computed as follows:

$$C_{ik} = \sum_{j=1}^{n_i}\sum_{r=1}^{q}\left(\left(1-X_{ij}\right)\sum_{l=1}^{m_{ij}}\left(v_r t_{ijlr}^{c} + c_{ijl}^{c}\right)x_{ijlr}^{k} + X_{ij}\sum_{l=2}^{m_{ij}}\left(v_r t_{ijlr}^{p} + c_{ijl}^{p}\right)x_{ijlr}^{k}\right), \tag{3.12}$$

$$T_{ikr} = \sum_{j=1}^{n_i}\left(\left(1-X_{ij}\right)\sum_{l=1}^{m_{ij}} t_{ijlr}^{c} x_{ijlr}^{k} + X_{ij}\sum_{l=2}^{m_{ij}} t_{ijlr}^{p} x_{ijlr}^{k}\right), \tag{3.13}$$

$$R_{ik} = 1 - \prod_{j=1}^{n_i}\left(1 - \sum_{r=1}^{q}\sum_{l=1}^{m_{ij}} R_{ijl} x_{ijlr}^{k}\right). \tag{3.14}$$

The following binary decision variable is introduced to represent the selection of pattern k for subsystem i:

$$\lambda_{ik} = \begin{cases} 1 & \text{if pattern } k \text{ is selected for subsystem } i, \\ 0 & \text{otherwise.} \end{cases} \tag{3.15}$$

By combining Eqns. (3.3)–(3.9) and (3.12)–(3.14), the JSM-RAP (3.10) can equivalently be formulated as follows:

$$\max \mathcal{R} = \prod_{i=1}^{s}\sum_{k=1}^{p_i} R_{ik}\lambda_{ik} \tag{3.16a}$$

s.t.:

$$\sum_{i=1}^{s}\sum_{k=1}^{p_i} C_{ik}\lambda_{ik} \le C_0 \tag{3.16b}$$

$$\sum_{i=1}^{s}\sum_{k=1}^{p_i} T_{ikr}\lambda_{ik} \le D_0 \qquad \forall r \tag{3.16c}$$

$$\sum_{k=1}^{p_i} \lambda_{ik} = 1 \qquad \forall i \tag{3.16d}$$

$$\lambda_{ik} \in \{0,1\} \qquad \forall i, \forall k. \tag{3.16e}$$

Let $F_{ik} = 1 - R_{ik}$ be the *unreliability* of subsystem i when maintenance pattern k is selected. The objective function in the above model is linearized by applying the natural logarithm function to both sides of Eqn. (3.16a), leading then to:

$$\begin{aligned}\ln(\mathcal{R}) &= \sum_{i=1}^{s} \ln\left(\sum_{k=1}^{p_i}(1-F_{ik})\lambda_{ik}\right) \\ &= \sum_{i=1}^{s}\sum_{k=1}^{p_i} \ln(1-F_{ik})\lambda_{ik},\end{aligned} \tag{3.17}$$

where the last equality is valid because the variables λ_{ik} are binary and, due to Eqn. (3.16d), only one of them is equal to 1 for each subsystem.

With that, the fully-linearized formulation of the JSM-RAP becomes:

$$\max \qquad \ln(\mathcal{R}) \; = \; \sum_{i=1}^{s}\sum_{k=1}^{p_i} \ln(1-F_{ik})\lambda_{ik} \tag{3.18a}$$

s.t.:

$$(3.16\text{b}) - (3.16\text{e}). \tag{3.18b}$$

It should be noted that in the above formulation (3.18), all binary parameters x^k_{ijlr} are fully defined whenever a pattern k is selected for subsystem i through the decision variable λ_{ik}. However, except for small problem instances, generating all feasible patterns for all subsystems is prohibitively expensive.

3.4 Branch-and-price algorithms

An alternative approach that combines CG with B&B to form what is commonly known as the B&P algorithm (Bulhões et al., 2018; Dell'Amico et al., 2020; Agius et al., 2022) is proposed to solve the JSM-RAP. In what follows, the proposed approach is described in detail.

3.4.1 Column-generation

Rather than generating all feasible patterns at the outset, the CG method operates by generating them iteratively and only as needed. First, the restricted master problem is presented, followed by the initialization and column-generation process. Finally, the two reformulations for the subproblems are presented.

3.4.1.1 Formulating the restricted master problem

First, the integrality constraint in (3.18) is relaxed such that it can be solved as a linear program (LP), referred to as the *master problem*, formulated as follows:

$$[\mathcal{MP}]: \quad \max \ \ln(\mathcal{R}) = \sum_{i=1}^{s}\sum_{k=1}^{p_i} \ln(1-F_{ik})\lambda_{ik} \tag{3.19a}$$

s.t.:

$$\sum_{i=1}^{s}\sum_{k=1}^{p_i} C_{ik}\lambda_{ik} \leq C_0 \qquad (\pi) \tag{3.19b}$$

$$\sum_{i=1}^{s}\sum_{k=1}^{p_i} T_{ikr}\lambda_{ik} \leq D_0 \quad \forall r \qquad (\sigma_r) \tag{3.19c}$$

$$\sum_{k=1}^{p_i} \lambda_{ik} = 1 \quad \forall i \qquad (\theta_i) \tag{3.19d}$$

$$\lambda_{ik} \geq 0 \quad \forall i, \forall k, \tag{3.19e}$$

where the symbol between parentheses next to each constraint denotes its dual variable/multiplier.

The CG algorithm is initiated with only a small subset of patterns/columns, corresponding to the index sets $\mathcal{K}'_i \subset \mathcal{K}_i, i = 1,\ldots,r$, to construct a restricted version of $\mathcal{MP}$ referred to as the *restricted master problem* ($\mathcal{RMP}$). In every iteration, upon solving $\mathcal{RMP}$, the dual variables π, σ_r and θ_i are used to look for new columns that can potentially improve its optimal value, *i.e.*, columns with a positive reduced cost. For each subsystem, a subproblem ($\mathcal{SP}_i$) is solved to identify the column (indexed by $\bar{k}$) with the most positive reduced cost, if one exists, to be added to $\mathcal{RMP}$ in the next iteration, *i.e.*, $\mathcal{K}'_i := \mathcal{K}'_i \cup \{\bar{k}\}$.

3.4.1.2 Initializing the CG algorithm

To start the CG algorithm, an initial $\mathcal{RMP}$ must have a feasible solution to ensure that proper dual information is passed to the pricing subproblems. Accordingly, getting a "good" initial restricted master problem is crucial (Barnhart et al., 1998).

The initial $\mathcal{RMP}$ starts with one column for each subsystem i. Each initial column has $(m_{ij}+1)\times q\times n_i$ binary elements x^k_{ijlr} representing all maintenance choices for each repairperson on components in each subsystem i. To find potentially good starting columns, a two-phase heuristic is proposed as follows:

- Phase one: Extracting the optimal value of $\mathcal{RMP}$

1. Set the selected maintenance action to "Do Nothing" (*i.e.*, $l = 0$ for all initial columns).
2. Calculate the reliability R_i for each subsystem, solve the JSM-RAP in Eqns. (3.10a)–(3.10e), and find the optimal value of $\mathcal{RMP}$.

- Phase two: Generating initial columns through a greedy heuristic approach

1. Based on the reliability calculation for each subsystem in the first phase, select the most reliable component in the least reliable subsystem
2. Apply the highest maintenance level available to the selected component such that both its cost and duration satisfy the available budget or break duration.
3. Save the generated pattern and recalculate the reliability for each subsystem. Exclude any subsystem from the subsequent iterations if its reliability R_i exceeds the optimal value of $\mathcal{RMP}$ (found in phase-one).
4. Repeat steps 1 to 3 until no additional maintenance action can be applied without violating at least one of the constraints.

3.4.1.3 Generating maintenance patterns

In every iteration of the CG algorithm, a maintenance pattern is generated for the entire system by solving a pricing subproblem that maximizes the reduced cost $RC = \sum_{i=1}^{s} \ln(1 - F_i) - \sum_{i=1}^{s} C_i \pi - \sum_{i=1}^{s} \sum_{r=1}^{q} T_{ir} \sigma_r - \sum_{i=1}^{s} \theta_i$, which can be decomposed by i (subsystem) into r subproblems. In this formula, the cost C_i and the r^{th} repairperson's maintenance time T_{ir} are linear functions of x_{ijlr}. Furthermore, F_i has to be expressed as a function of x_{ijlr}. Since x_{ijlr} is binary and only one maintenance level is selected for each component (*i.e.*,

$\sum_{l=0}^{m_{ij}} \sum_{r=1}^{q} x_{ijlr} = 1$), and using Eqn. (3.14), F_i and $\ln(F_i)$, respectively, can be rewritten as:

$$\begin{aligned} F_i &= \prod_{j=1}^{n_i} \left(1 - \sum_{r=1}^{q} \sum_{l=0}^{m_{ij}} R_{ijl} x_{ijlr} \right) \\ &= \prod_{j=1}^{n_i} \sum_{r=1}^{q} \sum_{l=0}^{m_{ij}} F_{ijl}\, x_{ijlr}, \end{aligned} \tag{3.20}$$

$$\ln(F_i) = \sum_{r=1}^{q} \sum_{j=1}^{n_i} \sum_{l=0}^{m_{ij}} \ln(F_{ijl}) x_{ijlr}, \tag{3.21}$$

where $F_{ijl} = 1 - R_{ijl}$ is the unreliability of component E_{ij} when it undergoes maintenance action of level l during the scheduled break. With that, the subproblem corresponding to subsystem i ($i = 1, \ldots, s$) is formulated as follows:

$$[\mathcal{SP}_i] : \max \quad \ln(1 - F_i) - C_i \pi - \sum_{r=1}^{q} T_{ir} \sigma_r - \theta_i \tag{3.22a}$$

s.t.:

$$\ln(F_i) = \sum_{j=1}^{n_i} \sum_{l=0}^{m_{ij}} \sum_{r=1}^{q} \ln(F_{ijl}) x_{ijlr} \tag{3.22b}$$

$$C_i = \sum_{j=1}^{n_i} \sum_{r=1}^{q} \left[X_{ij} \sum_{l=2}^{m_{ij}} \left(v_r t^p_{ijlr} + c^p_{ijl} \right) + \left(1 - X_{ij} \right) \sum_{l=1}^{m_{ij}} \left(v_r t^c_{ijlr} + c^c_{ijl} \right) \right] x_{ijlr} \tag{3.22c}$$

$$T_{ir} = \sum_{j=1}^{n_i} \left[X_{ij} \sum_{l=2}^{m_{ij}} t^p_{ijlr} + \left(1 - X_{ij} \right) \sum_{l=1}^{m_{ij}} t^c_{ijlr} \right] x_{ijlr} \qquad \forall r \tag{3.22d}$$

$$\sum_{l=0}^{m_{ij}} \sum_{r=1}^{q} x_{ijlr} = 1 \qquad \forall j \tag{3.22e}$$

$$x_{ijlr} \in \{0, 1\} \qquad \forall j, \forall l, \forall r. \tag{3.22f}$$

Constraints (3.22e) and (3.22f) are carried forward from the nonlinear BIP formulation (3.10). The subproblems are nonlinear because both the objective function (3.22a) and constraint (3.22b) include logarithm functions of the decision variable F_i.

If the optimal value of a subproblem $\mathcal{SP}_i$ is strictly positive, the newly generated column is added to the corresponding subset of patterns in the $\mathcal{RMP}$ to be solved in the next iteration. This iteration between solving $\mathcal{RMP}$ and solving $\mathcal{SP}_i$ $(i = 1, \dots, s)$ continues until no columns with strictly positive reduced cost are found (*i.e.*, the Simplex optimality criterion is met for $\mathcal{RMP}$).

3.4.1.4 Solving the subproblems

As noted above, the subproblems include logarithm functions and thus cannot be readily handled by most commercial solvers. This section provides two readily-solvable reformulations for the subproblem. The first one piecewise-linearly approximates the objective function and the nonlinear constraint, whereas the second one is a reformulation into a mixed-integer ECO.

Piecewise-linear approximation (PLA)

We propose a piecewise-linear approximation for the nonlinear subproblem $\mathcal{SP}_i$ in which continuous variables are used in special ordered sets of type 2 (SOS2) (Beale and Forrest, 1976). From the objective function (3.22a) of the subproblem we define the nonlinear function $g(.)$ such that $g(F_i) = \ln(1 - F_i)$. Similarly, from constraint (3.22b), a nonlinear function $h(.)$ is defined such that $h(F_i) = \ln(F_i)$. These nonlinear concave functions are approximated by their respective piecewise-linear functions $\hat{g}$ and $\hat{h}$. Using a set of N breakpoints $\{\widehat{F}_{in}\}$ $(n = 1, \dots, N)$ for the nonlinear functions $g(F_i)$ and $h(F_i)$, and the SOS2 variables $\psi_n \in [0,1]^N$, the subproblem $\mathcal{SP}_i$ is accordingly approximated as:

$$[\mathcal{SP}_i]_{PWL} : \max \quad \sum_{n=1}^{N} \ln(1 - \widehat{F}_{in})\psi_n - C_i\pi - \sum_{r=1}^{q} T_{ir}\sigma_r - \theta_i \tag{3.23a}$$

s.t.:

$$\sum_{n=1}^{N} \ln(\widehat{F}_{in})\psi_n = \sum_{j=1}^{n_i} \sum_{l=0}^{m_{ij}} \sum_{r=1}^{q} \ln(F_{ijl}) x_{ijlr} \tag{3.23b}$$

$$\sum_{n=1}^{N} \psi_n = 1 \tag{3.23c}$$

$$\psi_n \geq 0, \text{ SOS2} \qquad \forall n \tag{3.23d}$$

$$(3.22\text{d}) - (3.22\text{f}). \tag{3.23e}$$

Mixed-integer ECO reformulation (ECO)

Alternatively, the nonlinear subproblems can be reformulated as mixed-integer ECO problems. Starting from the original subproblem formulation (3.22), note that maximizing $\ln(1-F_i)$ in Eqn. (3.22a) requires F_i to take the smallest possible value such that the equality constraint (3.22b) holds. Let us define the variable S_i such that $S_i \leq \ln(1-F_i)$. If $\ln(1-F_i)$ is replaced by S_i in (3.22a), this inequality becomes binding at optimality. Furthermore, let $Q_i = \ln(F_i)$ in (3.22b). Hence, we have $e^{S_i} \leq 1-F_i$ and $e^{Q_i} = F_i$. By adding up the two expressions we get $e^{Q_i} + e^{S_i} \leq 1$, which can be expanded as $e^{Q_i} \leq u_i$, $e^{S_i} \leq v_i$ and $u_i + v_i \leq 1$. The above results allow us to rewrite the subproblem (3.22) as follows:

$$[\mathcal{SP}_i]_{ECO}: \quad \max \quad S_i - C_i\pi - \sum_{r=1}^{q} T_{ir}\sigma_r - \theta_i \tag{3.24a}$$

s.t.

$$Q_i = \sum_{j=1}^{n_i}\sum_{l=0}^{m_{ij}}\sum_{r=1}^{q} \ln(F_{ijl})x_{ijlr} \tag{3.24b}$$

$$e^{Q_i} \leq u_i \tag{3.24c}$$

$$e^{S_i} \leq v_i \tag{3.24d}$$

$$u_i + v_i \leq 1 \tag{3.24e}$$

$$Q_i, S_i \leq 0 \tag{3.24f}$$

$$u_i, v_i \geq 0 \tag{3.24g}$$

$$(3.22c) - (3.22f). \tag{3.24h}$$

In the above formulation, inequalities (3.24c) and (3.24d) are exponential conic constraints. Such constraints can equivalently be written as $(u, 1, Q) \in K_{exp}$ and $(v, 1, S) \in K_{exp}$, respectively. The notation $(y_1, y_2, y_3) \in K_{exp}$ describes all the points satisfying the exponential cone equation $y_1 \geq y_2 e^{y_3/y_2}$, $y_1, y_2 \geq 0$ (Mosek ApS, 2021). It should be noted that, unlike the piecewise-linear approximation, (3.24) is an exact reformulation of $\mathcal{SP}_i$.

A pseudocode of the CG algorithm is provided in Algorithm 1 to summarize the steps described above.

Algorithm 1: Column-generation algorithm

Initialize: Using the two-phase heuristic (Section 3.4.1.2), generate initial columns for $\mathcal{RMP}$, $stop := 0$
while $stop = 0$ **do**
  $i := 1$, $stop := 1$
  Solve $\mathcal{RMP}$ and extract the optimal dual variables $(\pi^*, {\sigma_r}^*, \theta_i^*)$
  while $i \leq s$ **do**
    Update the objective function of $\mathcal{SP}_i$ based on the optimal dual variables
    Solve $\mathcal{SP}_i$ using either the PLA or the ECO approach (Sec. 3.4.1.4), let RC_i be its optimal value
    if $RC_i > 0$ **then**
      Add the subproblem solution column x_{ijlr}^k to $\mathcal{RMP}$, $stop := 0$
    end if
    $i := i + 1$
  end while
end while

3.4.1.5 A stabilization scheme

Despite being an efficient algorithm for solving large-scale LPs, CG often suffers from slow convergence due to the wide oscillation (*i.e.*, instability) of the optimal dual solution $(\pi^*, \sigma_r^*, \theta_i^*)$ from one iteration to the next (Brunner and Stolletz, 2014). To overcome this issue, the stabilization scheme proposed by Du Merle et al. (1999) is applied to the convexity constraint (3.19d) and its corresponding dual variable θ_i. We opt to focus only on this primal constraint/dual variable pair because preliminary experiments showed that stabilizing the other two dual variables (π and σ_r) did not lead to significant improvements, since either of them frequently assumed the value 0 (*i.e.*, the primal constraint was nonbinding). This scheme works by introducing bounds (a_i^-, a_i^+), artificial variables (h^-, h^+), and penalties (z_i^+, z_i^-) to the dual of $\mathcal{RMP}$ so that θ_i^* stays close to its best-found value. In its *primal form* (refer to Du Merle et al. (1999) for more details), the *stabilized* *restricted master problem* is stated as

$$[\mathcal{SRMP}]: \quad \max \ln(R) = \sum_{i=1}^{s}\sum_{k=1}^{p_i} \ln(1-F_{ik})\lambda_{ik} + \sum_{i=1}^{s} a_i^- h_i^- - \sum_{i=1}^{s} a_i^+ h_i^+ \tag{3.25a}$$

s.t.:

$$(3.19b), (3.19c)$$

$$\sum_{k=1}^{p_i} \lambda_{ik} - h_i^- + h_i^+ = 1 \qquad \forall i \tag{3.25b}$$

$$h_i^- \leq z_i^- \qquad \forall i \tag{3.25c}$$

$$h_i^+ \leq z_i^+ \qquad \forall i \tag{3.25d}$$

$$\lambda_{ik}, h_i^-, h_i^+ \geq 0 \qquad \forall i, \forall k. \tag{3.25e}$$

The parameters z_i^+ and z_i^- linearly penalize deviations when λ_{ik} lies outside of the interval $\left[a_i^-,\ a_i^+\right]$. Intuitively, when $z_i^+ = z_i^- = 0$ or when $\lambda_{ik} \in \left[a_i^-,\ a_i^+\right]$, the problems $\mathcal{RMP}$ and $\mathcal{SRMP}$ become equivalent. Hence, we try to set z_i^+, z_i^-, a_i^-, a_i^+ such that the penalty term vanishes as the multipliers become closer to their optimal values. In any iteration, we set $a_i^- = \bar{\theta}_i(1-\delta)$ and $a_i^+ = \bar{\theta}_i(1+\delta)$, where $\bar{\theta}$ is the best set of multipliers found so far and δ is a tolerance that is set initially to one, then doubled if no better multipliers are found. On the other hand, we initiate the penalty factors z_i^+ and z_i^- as vectors of ones, then divide them by 10 every time the upper bound does not improve. These adjustments ensure that the effect of stabilization diminishes as the algorithm converges towards the optimal multipliers. Although the description of the CG algorithm provided earlier is based on $\mathcal{RMP}$ (*i.e.*, without stabilization), it can be easily adapted to the stabilized case.

3.4.2 Branch-and-bound

Given that the integrality constraint is relaxed in $\mathcal{RMP}$, its optimal solution is likely to be fractional and the CG algorithm provides only an upper bound (*i.e.*, the *Lagrangean bound*) on the optimum value of the original problem. An easy way to obtain a feasible (binary) solution is to solve $\mathcal{RMP}$ based only on the columns generated throughout the CG iterations with the integrality constraint (3.16e) added back. However, it should be noted that the patterns that are required to construct the optimal solution for the BIP formulation of the original JSM-RAP problem (3.18) might not have necessarily been generated along the CG algorithm iteration. Hence, the feasible solution obtained using this direct method (if it exists) might not be optimal, and the corresponding objective value serves only as a lower bound on the optimum value of the original problem. If this lower bound turns out to be sufficiently close to the Lagrangean bound obtained from the CG algorithm, one can stop and declare the feasible solution as (near-)optimal. Otherwise, the CG algorithm can be embedded into a B&B tree to devise a B&P approach (Barnhart et al., 1998) for which the feasible solution serves as an initial *incumbent*.

At any given node of the B&B tree, upon applying the CG algorithm described earlier (with branching constraints, if any, added to the subproblems), we obtain the optimal solution λ^*_{ik} to $\mathcal{RMP}$ and an upper bound UB. If the problem is infeasible or the upper bound is worse (lower) than the best lower bound found so far (corresponding to the incumbent), the node needs not to be considered any further (*i.e.*, this node is pruned from the B&B tree). The node is pruned also if the optimal solution turns out to be binary, in which case both the lower bound and the incumbent are updated if necessary. These are the standard fathoming rules in B&B. The procedure terminates when all the nodes in the tree have been evaluated, and the incumbent solution is declared optimal.

Otherwise (*i.e.*, when the optimal solution is feasible and fractional and the upper bound exceeds the lower bound), we branch based on the original problem variables ($x^*_{ijlr} = \sum_{k \in \mathcal{K}'} \lambda^*_{ik} x^k_{ijlr}$) rather than the convexity variables λ^*_{ik}. In particular, we select a fractional variable $\bar{x}_{ijlr}$ and use the valid branching rule $\sum_{r=1}^{q} \bar{x}_{ijlr} \leq 0$ or $\sum_{r=1}^{q} \bar{x}_{ijlr} \geq 1$, which stipulates that the maintenance action either must or must not be performed. One of these constraints is added to the corresponding $\mathcal{SP}_i$ in each of the two child nodes. We also *warm-start* the $\mathcal{RMP}$ in the child nodes by re-using the patterns inherited from the parent node, after being *filtered* according to the branching rule, to initiate the CG algorithm. This strategy significantly improves the algorithm performance since only a few new patterns need to be generated in each node. A depth-first search (DFS) strategy is employed to select the node to explore in the B&B tree.

3.5 Numerical experiments

In this section, four sets of numerical experiments are conducted to demonstrate the validity and accuracy of the proposed approach. The first set considers two validation experiments based on literature examples from Cassady et al. (2001a); Pandey and Zuo

(2013) and Diallo et al. (2019b). The second set of experiments compares the two-phase approach developed in Diallo et al. (2019b) and the four CG-based approaches developed in this paper: the CG method that utilizes PLA to solve the nonlinear subproblems (CG–PLA), the CG method that utilizes ECO to solve the nonlinear subproblems (CG–ECO), the CG–PLA approach when embedded into a B&B procedure (BP–PLA), and the CG–ECO approach when embedded into a B&B procedure (BP–ECO). These comparisons are made on the basis of the results obtained for a moderate-size serial-parallel system with multiple repairpersons and imperfect maintenance levels. The third set of experiments demonstrates the ability of the CG–PLA and CG–ECO approaches to deal with very large JSM–RAP instances. The fourth set of experiments is similar to the third one but with additional maintenance levels for added-complexity, and aims primarily to investigate the effectiveness of the proposed stabilization scheme (Section 3.4.1.5) in accelerating the convergence of the CG algorithm.

All CG-PLA and BP-PLA experiments in this paper are run using $N = 600$ breakpoints. All experiments are run on Intel(R) Core(TM) i7 @ 1.30 GHz laptop computer with 16 GB of RAM running Windows 11. The CG-PLA and CG-ECO models are solved using the academic versions of Gurobi 9.1.1 and MOSEK 9.3, respectively. For the numerical experiments in this paper, without loss of generality, we assume that the lifetimes of components are Weibull-distributed with shape and scale parameters β_{ij} and η_{ij}, respectively. In this particular case, we have

$$R^c_{ij}(M\,|_{A_{ijl}}) = \exp\left(\left(\frac{A_{ijl}}{\eta_{ij}}\right)^{\beta_{ij}} - \left(\frac{A_{ijl}+M}{\eta_{ij}}\right)^{\beta_{ij}}\right) x_{ijlr}. \tag{3.26}$$

3.5.1 Set of experiments #1: Validation examples

To validate and demonstrate the added value of the proposed approach, two sets of experiments are conducted. The first set of experiments investigates the 2×2 series-parallel system from Cassady et al. (2001a), Pandey and Zuo (2013) and Diallo et al. (2019b) while the second set of experiments considers the 5×5 series-parallel system studied in Diallo et al. (2019b).

3.5.1.1 Experiments set #1.1: The case of a small size system with a single repairperson

The instance parameters are displayed in Table 3.2. For failed components, three CM levels are considered: $l = 0$, $l = 1$, and $l = 2$. For working components, two PM levels are considered: $l = 0$ and $l = 2$. The next mission duration is $M = 8$. Given that the original problem in Cassady et al. (2001a) did not consider multiple repairpersons, we set $q = 1$, and $v_q = 0$. Two cases will be investigated depending on whether the limited maintenance budget C_0 is accounted for or not. For comparison purposes, the resulting SMP in both cases will be solved using the two-phase approach (Diallo et al., 2019b), and the B&P algorithms (BP–PLA, BP–ECO) proposed in this paper.

Table 3.2: Data for experiment #1 (Cassady et al., 2001a; Pandey and Zuo, 2013).

E_{ij}	η_{ij}	β_{ij}	X_{ij}	B_{ij}	t^c_{ij1}	t^c_{ij2}	t^p_{ij1}	c^c_{ij1}	c^c_{ij2}	c^p_{ij1}
E_{11}	15	1.5	1	15	3	1	5	6	12	12
E_{12}	15	1.5	1	20	3	1	5	5	12	12
E_{21}	20	3	0	8	2	2	4	5	14	14
E_{22}	20	3	1	15	2	2	4	6	15	15

Let us first consider the case when the maintenance budget is not accounted for (*i.e.*, C_0 is considered infinite). In this case, for different values of the scheduled break duration D_0, the overall results are reported in Table 3.3. For each approach (two-phase, BP–PLA, BP–ECO), Table 3.3 provides the highest achievable system reliability $\mathcal{R}$ for the next mission, the total maintenance time T_1 used by the single repairperson to perform the maintenance actions, the number of nodes in the B&B tree and the CPU time for both the BP–PLA and BP–ECO approaches. As can be observed in Table 3.3, the two B&P algorithms reach the same optimal values of the two-phase approach, thus confirming their validity. Furthermore, their optimal solutions are the same as those obtained by Cassady et al. (2001a) and Pandey and Zuo (2013). One can also observe that, for such a "toy" instance, the two-phase approach CPU times are smaller than those of the two B&P algorithms since generating a very small number of feasible maintenance patterns and selecting the best among them can be done rather efficiently.

Now, let us consider the second case when both duration D_0 allotted to the scheduled

Table 3.3: Results of Experiment #1.1: The case with $C_0 = \infty$.

D_0	Two-phase			BP–PLA				BP–ECO			
	$\mathcal{R}(\%)$	T_1	$CPUt(s)$	$\mathcal{R}(\%)$	T_1	# Nodes	$CPUt(s)$	$\mathcal{R}(\%)$	T_1	# Nodes	$CPUt(s)$
16	89.25	16	0.15	89.25	16	1	0.37	89.25	16	1	0.22
12	85.89	12	0.17	85.89	12	1	0.48	85.89	12	1	0.71
9	77.53	7	0.17	77.53	7	9	3.33	77.53	7	9	4.21
5	59.71	2	0.16	59.71	2	7	2.23	59.71	2	7	4.63

break and the limited maintenance budget are accounted for. In this case, the break duration is set at $D_0 = 9$, while the maintenance budget varies. The results obtained by the two-phase approach, BP–PLA, and BP–ECO are shown in Table 3.4. Once again, the values obtained allow us to conclude that the proposed B&P algorithms are valid. When $C_0 = 25$, our results are the same as those obtained by Pandey et al. (2013b).

The results reported in Table 3.4 show, as one may expect, that decreasing the maintenance budget C_0 reduces the number of maintenance actions to be performed, and consequently increases the probability of failure risk during the next mission.

Table 3.4: Results of Experiment #1.1: The case with $D_0 = 9$.

C_0	Two-phase				BP–PLA					BP–ECO				
	$\mathcal{R}(\%)$	T_1	C	$CPUt(s)$	$\mathcal{R}(\%)$	T_1	C	# Nodes	$CPUt(s)$	$\mathcal{R}(\%)$	T_1	C	# Nodes	$CPUt(s)$
30	77.53	7	26	0.16	77.53	7	26	7	2.64	77.53	7	26	7	3.45
25	61.40	7	17	0.15	61.40	7	17	27	8.67	61.40	7	17	27	11.39
15	59.71	2	14	0.18	59.71	2	14	9	2.60	59.71	2	14	9	5.17
10	47.29	2	5	0.19	47.29	2	5	1	0.67	47.29	2	5	1	1.18

3.5.1.2 Experiments set #1.2: The case of a small size system with multiple equally-skilled repairpersons

Experiments set #1.2 considers the series-parallel system and its related data from Diallo et al. (2019b). The reliability block diagram of the system is composed of two ($s = 2$) series subsystems, each of which contains five ($n_i = 5$) *i.i.d* components E_{ij} arranged in parallel ($i = 1, 2; j = 1, \ldots, 5$). Lifetimes of components E_{ij} are Weibull distributed with respective shape and scale parameters β_{ij} and η_{ij}. These parameters are set to $\beta_{1j} = 1.5$, $\eta_{1j} = 15$, $\beta_{2j} = 3$ and $\eta_{2j} = 20$. There are $q = 2$ equally skilled repairpersons available to carry out maintenance tasks with variable cost rate $v_r = 2$. A list of four CM levels is available for failed components: $l = 0$ (DN), $l = 1$ (MR), and $l = 2$ (IM) which reduces the component age by half, and $l = 3$ (PR). For functioning components, a list of three PM levels is available: $l = 0$ (DN), $l = 2$ (IM) reduces the component age by half, and $l = 3$ (PR). Additional data related to components' status, ages and maintenance times and costs are depicted in Table 3.5. The duration of the scheduled break and that of the next mission are set to $D_0 = 5$ and $M = 8$, respectively.

Table 3.5: Parameters for experiment #1.2, source: Diallo et al. (2019b).

E_{ij}	X_{ij}	B_{ij}	t^c_{ij1}	t^c_{ij2}	t^c_{ij3}	t^p_{ij2}	t^p_{ij3}	c^c_{ij1}	c^c_{ij2}	c^c_{ij3}	c^p_{ij2}	c^p_{ij3}
E_{11}	0	15	2	5	6	2	3	5	10	14	8	10
E_{12}	1	12	2	5	6	2	3	5	10	14	8	10
E_{13}	0	10	2	5	6	2	3	5	10	14	8	10
E_{14}	1	18	2	5	6	2	3	5	10	14	8	10
E_{15}	1	20	2	5	6	2	3	5	10	14	8	10
E_{21}	0	8	4	7	8	3	5	4	8	10	5	7
E_{22}	1	15	4	7	8	3	5	4	8	10	5	7
E_{23}	0	8	4	7	8	3	5	4	8	10	5	7
E_{24}	1	15	4	7	8	3	5	4	8	10	5	7
E_{25}	0	8	4	7	8	3	5	4	8	10	5	7

The resulting JSM–RAP is solved using the two-phase, CG–PLA and CG–ECO approaches. The results obtained are reported in Tables 3.6 and 3.7 for different values of the maintenance cost C_0. These tables provide the highest achievable system reliability $\mathcal{R}$ for the subsequent mission, as well as the total cost C and the total maintenance time by each repairperson (T_1, T_2). They also show the CPU time and the gap between the optimal reliability values obtained by the two-phase approach and those obtained using CG–PLA and CG–ECO. In addition, the tables provide the size (# Nodes) of the B&B tree when the B&P algorithms are used.

The results in Table 3.6 show that 4 out of the 5 solutions obtained using CG–PLA and all the solutions obtained using BP–PLA are identical to those obtained from the two-phase approach. It should be noted that the CG–PLA approach cannot always provide optimal solutions due to the branching operation which is performed only at the final step based on a subset of potentially interesting patterns. Thus, some of the patterns required to construct the optimal BIP solution may not have been generated when optimizing the subproblems. Even using BP–PLA cannot guarantee the optimality of solutions since the subproblems solved in each iteration are only piecewise-linear approximations of the original subproblems $\mathcal{SP}_i$. Likewise, the results in Table 3.7 show that 4 out of the 5 solutions obtained using CG–ECO and all the solutions obtained using BP–ECO are identical to those obtained from the two-phase approach. It is worth noting that, unlike CG–PLA, the CG–ECO approach uses an exact reformulation of $\mathcal{SP}_i$, thus it is able to guarantee the optimality of solutions when embedded into a B&B tree (*i.e.*, BP–ECO). The numerical results in Tables 3.6 and 3.7 show that the gap values for CG–PLA and CG–ECO are usually small, not exceeding 1% and often much smaller. However, when it comes to the computation times, both CG–PLA and CG-ECO significantly outperform the exact two-phase approach.

Table 3.6: Results obtained in Experiment #1.2 (CG–PLA): The case of a 5-by-5 system with 2 identical repairpersons and $D_0 = 5$.

C_0	Two-phase				CG–PLA					BP–PLA				
	$\mathcal{R}(\%)$	C	(T_1, T_2)	$CPUt(s)$	$\mathcal{R}(\%)$	C	(T_1, T_2)	$Gap(\%)$	$CPUt(s)$	$\mathcal{R}(\%)$	C	(T_1, T_2)	# Nodes	$CPUt(s)$
50	90.09	42	(5,5)	14.08	89.11	35	(5,4)	0.98	0.82	90.09	42	(5,5)	35	16.14
40	89.11	35	(5,4)	14.12	89.11	35	(5,4)	0.00	0.99	89.11	35	(5,4)	39	16.94
30	84.47	26	(5,2)	14.12	84.47	26	(5,3)	0.00	0.83	84.47	26	(5,2)	43	20.28
20	74.65	17	(5,0)	14.15	74.65	17	(5,0)	0.00	0.86	74.65	17	(5,0)	21	8.29
10	48.94	9	(2,0)	14.14	48.94	9	(2,0)	0.00	0.74	48.94	9	(2,0)	31	11.01

Table 3.7: Results obtained in Experiment #1.2 (CG–ECO): The case of a 5-by-5 system with 2 identical repairpersons and $D_0 = 5$.

C_0	Two-phase				CG–ECO					BP–ECO				
	$\mathcal{R}(\%)$	C	(T_1, T_2)	$CPUt(s)$	$\mathcal{R}(\%)$	C	(T_1, T_2)	$Gap(\%)$	$CPUt(s)$	$\mathcal{R}(\%)$	C	(T_1, T_2)	# Nodes	$CPUt(s)$
50	90.09	42	(5,5)	14.08	89.11	35	(5,4)	0.98	2.16	90.09	52	(5,5)	29	22.01
40	89.11	35	(5,4)	14.12	89.11	35	(5,4)	0.00	2.21	89.11	35	(5,4)	37	34.53
30	84.47	26	(5,2)	14.12	84.47	26	(5,2)	0.00	2.06	84.47	26	(5,2)	41	38.71
20	74.65	17	(5,0)	14.15	74.65	17	(5,0)	0.00	1.75	74.65	17	(5,0)	37	29.27
10	48.94	9	(2,0)	14.14	48.94	9	(2,0)	0.00	1.11	48.94	9	(2,0)	31	18.15

Observations similar to those made in the previous experiments set can be made for the results reported in Tables 3.6 and 3.7. Indeed, when the budget allotted to maintenance decreases, the resulting maximal achievable system reliability for the next mission decreases. Consequently, the number of required repairpersons to carry out maintenance actions also decreases.

3.5.2 Set of experiments #2: The case of a moderate-size serial-parallel system

In this set of experiments, the moderate-size series-parallel system studied in Diallo et al. (2018) is investigated. The reliability block diagram of the system is composed of two ($s = 2$) series subsystems, where the first is composed of five ($n_1 = 5$) i.i.d. components E_{1j} ($j = 1,\ldots,5$), while the second contains eight ($n_2 = 8$) i.i.d. components E_{2j} ($j = 1,\ldots,8$) arranged in parallel, resulting in a total of 13 components. Lifetimes of components E_{ij} are Weibull-distributed with the respective shape and scale parameters β_{ij} and η_{ij} ($i = 1,2; j = 1,\ldots,n_i$). These parameters are set at $\beta_{1j} = 1.5$ and $\eta_{1j} = 15$ ($j = 1,\ldots,5$), and $\beta_{2j} = 3$ and $\eta_{2j} = 20$ ($j = 1,\ldots,8$). A list of four maintenance levels is available for all components: $l = 0$ (DN), $l = 1$ (MR: valid only for failed components), $l = 2$ (IM) that reduces the component age by half, and $l = 3$ (PR). Two cases are considered. In the first case, only one ($q = 1$) repairperson is available with a break duration of $D_0 = 20$, whereas $q = 2$ equally skilled repairpersons are available with a break duration of $D_0 = 5$ for each repairperson in the second case. In both cases, the variable cost rate of a repairperson is set to $v_r = 2$, and the duration of the next mission is $M = 8$. Additional data related to components' status, ages and maintenance times and costs are depicted in Table 3.8. For comparison purposes, the JSM–RAP is solved using the exact two-phase, CG–PLA and CG–ECO approaches.

Table 3.8: Parameters for experiment #2, source: Diallo et al. (2018).

E_{ij}	X_{ij}	B_{ij}	t^c_{ij1}	t^c_{ij2}	t^c_{ij3}	t^p_{ij2}	t^p_{ij3}	c^c_{ij1}	c^c_{ij2}	c^c_{ij3}	c^p_{ij2}	c^p_{ij3}
E_{11}	0	15	4	6	8	2	4	5	10	14	8	10
E_{12}	1	12	4	6	8	2	4	5	10	14	8	10
E_{13}	0	10	4	6	8	2	4	5	10	14	8	10
E_{14}	1	18	4	6	8	2	4	5	10	14	8	10
E_{15}	1	20	4	6	8	2	4	5	10	14	8	10
E_{21}	0	8	3	4	5	1	2	6	10	20	7	12
E_{22}	1	15	3	4	5	1	2	6	10	20	7	12
E_{23}	0	8	3	4	5	1	2	6	10	20	7	12
E_{24}	1	15	3	4	5	1	2	6	10	20	7	12
E_{25}	0	8	3	4	5	1	2	6	10	20	7	12
E_{26}	1	15	3	4	5	1	2	6	10	20	7	12
E_{27}	0	8	3	4	5	1	2	6	10	20	7	12
E_{28}	1	15	3	4	5	1	2	6	10	20	7	12

3.5.2.1 Experiments #2.1: The case of a single repairperson

Tables 3.9 and 3.10 report the maximum achievable system reliability $\mathcal{R}$ for the next mission, the total maintenance time T_1, cost C incurred, and the CPU time for different values of the maintenance budget C_0 when only a single repairperson is hired. These tables also provide the reliability gap between the exact two-phase approach solutions and those reached by CG–PLA and CG–ECO, as well as the size (# Nodes) of the B&B tree when the B&P algorithms are applied.

In terms of computation time, both CG–PLA and CG–ECO outperform the two-phase approach. On average, the exact two-phase approach takes 18.81 seconds to generate a total of 17,865 patterns and then takes around 0.2 seconds for the optimization phase. In contrast, the computation time for CG–PLA is on average less than 1 second, while the CG–ECO takes about 2.5 seconds. The total number of patterns generated for CG–PLA and CG–ECO is between 9 and 15 patterns. It is also notable that the CPU times for the three approaches remain almost constant when the maintenance budget increases (around 10 seconds for the two-phase approach, around 0.6 seconds for CG–PLA, and around 2.5 seconds for CG–ECO). Notably, the CPU times for the B&P algorithms are longer than those of the two-phase approach for this set of experiments.

Regarding the solution quality, one may observe that the gap between the exact two-phase approach and CG-based approaches (CG–PLA, CG–ECO) does not exceed 2.16%, with an average gap of around 0.4%. Meanwhile, the CG-based approaches generate only about one-thousandth of the patterns generated by the two-phase approach. Similar to previous experiment sets, the two B&P algorithms are able to solve all instances optimally. These observations comfort the conclusions from the previous sets of experiments about the validity and efficiency of the proposed approach.

Table 3.9: Results obtained in Experiments #2.1 (CG–PLA): The case of a single repairperson and $D_0 = 20$.

C_0	Two-phase				CG–PLA					BP–PLA				
	$\mathcal{R}(\%)$	T_1	C	$CPUt(s)$	$\mathcal{R}(\%)$	T_1	C	$Gap(\%)$	$CPUt(s)$	$\mathcal{R}(\%)$	T_1	C	# Nodes	$CPUt(s)$
59	94.40	14	59	19.26	92.24	10	47	2.16	0.93	94.40	14	59	101	45.74
50	92.24	10	47	19.29	92.24	10	47	0.00	0.92	92.24	10	47	119	51.79
40	88.19	8	38	19.35	86.49	6	29	1.70	0.82	88.19	8	38	305	130.66
30	86.49	6	29	19.24	86.49	6	29	0.00	0.62	86.49	6	29	65	28.31
20	76.43	2	16	19.21	76.43	2	16	0.00	0.64	76.43	2	16	73	24.38
10	70.06	1	9	19.21	70.06	1	9	0.00	0.73	70.06	1	9	87	28.98

Table 3.10: Results obtained in Experiments #2.1 (CG–ECO): The case of a single repairperson and $D_0 = 20$.

C_0	Two-phase				CG–ECO					BP–ECO				
	$\mathcal{R}(\%)$	T_1	C	$CPUt(s)$	$\mathcal{R}(\%)$	T_1	C	$Gap(\%)$	$CPUt(s)$	$\mathcal{R}(\%)$	T_1	C	# Nodes	$CPUt(s)$
59	94.40	14	59	19.26	92.24	10	47	2.16	2.11	94.40	14	59	91	70.96
50	92.24	10	47	19.29	92.24	10	47	0.00	2.43	92.24	10	47	119	116.75
40	88.19	8	38	19.35	86.49	6	29	1.70	2.09	88.19	8	38	289	216.86
30	86.49	6	29	19.24	86.49	6	29	0.00	2.06	86.49	6	29	61	49.43
20	76.43	2	16	19.21	76.43	2	16	0.00	2.08	76.43	2	16	85	70.08
10	70.06	1	9	19.21	70.06	1	9	0.00	2.05	70.06	1	9	59	54.71

3.5.2.2 Experiments #2.2: The case of multiple equally-skilled repairpersons

Experiments #2.2 use the same data as in Experiments #2.1 except that two ($q = 2$) repairpersons are now available to carry out the maintenance actions in the SM plan. The JSM–RAP is, again, solved using all five approaches: two-phase, CG–PLA, CG–ECO, BP–PLA and BP–ECO. The results obtained are shown in Tables 3.11 and 3.12 for different maintenance budgets. These results include the maximum achievable system reliability $\mathcal{R}$ for the subsequent mission, the induced total maintenance cost C, the total duration T_r consumed to perform maintenance actions for each repairperson, the CPU time and the relative gap between the optimal value of the two-phase approach and those obtained using CG–PLA and CG–ECO. The size (# Nodes) of the B&B tree is also reported when the B&P algorithms are used.

All CG-based approaches (with or without B&B) outperform the two-phase approach in terms of computation time. On average, the two-phase approach requires 823.25 seconds to generate 798,856 maintenance patterns and needs about 30 seconds more to find the optimal solution. In comparison, CG–PLA takes around 1 second and CG–ECO takes less than 4 seconds to find a solution that is at most 2.5% away from the optimal. Both approaches are capable of providing high-quality solutions very efficiently. On the other hand, all solutions obtained through the B&P algorithms are identical to the optimal solutions found using the two-phase approach. However, one may observe that the CPU times required for the B&P algorithms are significantly larger than those of the CG-based approaches without B&B. It is also notable that, in all cases, the CPU times for CG–PLA are shorter than those for CG–ECO. However, as mentioned earlier, unlike CG–PLA, CG–ECO uses an exact reformulation of $\mathcal{SP}_i$ that is capable of guaranteeing solution optimality when combined with B&B.

Finally, it is worth noting that we did not observe any stabilization effect on small/moderate-size instances. However, the stabilization appears highly effective for large-scale systems with additional maintenance levels as will be shown in the forth set of experiments presented in Section 3.5.4.2.

Table 3.11: Results obtained in Experiments #2.2 (CG–PLA): The case of 2 identical repairpersons and $D_0 = 5$.

C_0	Two-phase				CG–PLA					BP–PLA				
	$\mathcal{R}(\%)$	C	(T_1,T_2)	$CPU(s)$	$\mathcal{R}(\%)$	C	(T_1,T_2)	$Gap(\%)$	$CPUt(s)$	$\mathcal{R}(\%)$	C	(T_1,T_2)	# Nodes	$CPUt(s)$
54	89.79	54	(5,5)	857.68	87.29	38	(4,3)	2.50	1.01	89.79	54	(5,5)	121	83.15
50	89.12	50	(5,4)	853.76	87.29	38	(4,3)	1.83	1.13	88.12	50	(5,4)	243	142.42
40	87.29	38	(5,2)	848.54	86.49	29	(4,2)	0.80	1.22	88.19	38	(5,2)	313	158.07
30	86.49	29	(4,2)	848.76	86.49	29	(4,2)	0.00	0.83	86.94	29	(4,2)	41	16.74
20	76.43	16	(2,0)	855.23	76.43	16	(2,0)	0.00	0.66	76.43	16	(2,0)	55	15.84
10	70.06	9	(1,0)	853.89	70.06	9	(1,0)	0.00	1.04	70.06	9	(1,0)	53	19.15

Table 3.12: Results obtained in Experiments #2.2 (CG–ECO): The case of 2 identical repairpersons, and $D_0 = 5$.

C_0	Two-phase				CG–ECO					BP–ECO				
	$\mathcal{R}(\%)$	C	(T_1,T_2)	$CPU(s)$	$\mathcal{R}(\%)$	C	(T_1,T_2)	$Gap(\%)$	$CPUt(s)$	$\mathcal{R}(\%)$	C	(T_1,T_2)	# Nodes	$CPUt(s)$
54	89.79	54	(5,5)	857.68	87.29	38	(4,3)	2.50	2.89	89.79	54	(5,5)	95	98.29
50	89.12	50	(5,4)	853.76	87.29	38	(4,3)	1.83	3.78	88.12	50	(5,4)	223	277.71
40	87.29	38	(5,2)	848.54	86.49	29	(4,2)	0.80	3.71	88.19	38	(5,2)	253	280.31
30	86.49	29	(4,2)	848.76	86.49	29	(4,2)	0.00	3.17	86.94	29	(4,2)	41	49.22
20	76.43	16	(2,0)	855.23	76.43	16	(2,0)	0.00	1.84	76.43	16	(2,0)	73	54.39
10	70.06	9	(1,0)	853.89	70.06	9	(1,0)	0.00	1.76	70.06	9	(1,0)	91	83.47

3.5.3 Set of experiments #3: Large-scale series-parallel systems

This set of experiments aims to demonstrate the ability of the proposed approaches to deal with large-scale systems. The experiments are carried out using the series-parallel system from Ikonen et al. (2020). The basic system is composed of 100 components distributed across $s = 32$ subsystems in series. Each subsystem i ($i = 1,\ldots,s$) is composed of *i.i.d.* n_i components arranged in parallel (Figure 3.1). The components in the first two subsystems are identical. Subsystems 3 to 8, 9 to 18, 19 to 25, and 26 to 32 are identically distributed as well. This basic system has a total number NC of components where $NC = \sum_{i=1}^{s} n_i = 100$. Lifetimes of component E_{ij} are governed by a Weibull distribution whose shape and scale parameters are β_{ij} and η_{ij}, respectively. Table 3.13 summarises the overall data related to components' status and Weibull lifetimes distribution shape and scale parameters, as well as maintenance times and costs. Common lists of $m = 2$

maintenance options are available: DN and PR for functioning components and DN and MR for failed components.

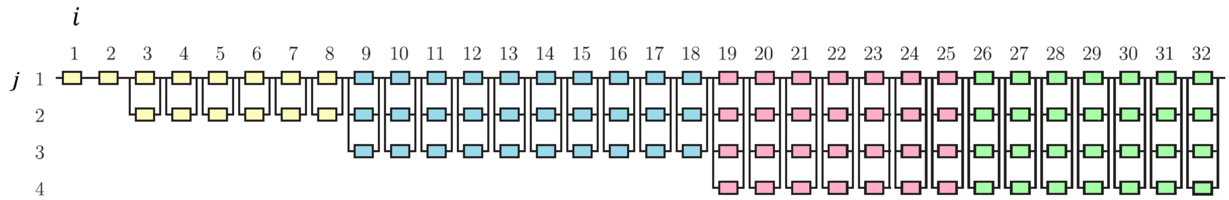


Figure 3.1: Reliability structure for the basic system with 100 components from Ikonen et al. (2020).

To date and to the best of our knowledge, the largest JSM-RAP instance comprising 700 components was solved by Ikonen et al. (2020). In this experiment, we will show that the proposed CG-based approaches (CG–PLA, CG–ECO) can deal with much larger systems. Eleven problem instances are created by duplicating the basic 100-component basic system presented above in Figure 3.1. For instance o with $1 \leq o \leq 10$, the system uses $NC_o = 100 \times o$ components obtained by duplicating the basic 100-component system o times. Instance 11 has $NC_{11} = 1500$ components obtained by duplicating the basic system 15 times. For all problem instances, the break duration is set at D_0=50. However, for any given instance o, the maintenance budget is set to $C_0 = 500 \times o$ while the number of repairpersons is set to $q = 5 \times o$. For example, for the second problem instance ($o = 2$), the number of components $NC_2 = 200$, the available maintenance budget $C_0 = 1000$, and $q = 10$ repairpersons will be available to perform the maintenance tasks.

In Algorithm 1, a heuristic method is used to find good-quality columns to initialize the solving process of the $\mathcal{RMP}$. In what follows, we will also investigate the performance of this heuristic method. We will therefore run both CG-based approaches with and without the heuristic: column-generation with piecewise linear approximation with heuristic (CG–PLAh) or without heuristic (CG-PLA), column-generation with an exponential conic reformulation with heuristic (CG–ECOh) or without heuristic (CG-ECO). All 11 instances of the problem are solved using these four approaches. The obtained results are displayed in Figures 3.2 to 3.4 and Tables 3.14 and 3.15. For each approach, the tables display the reliability values obtained ($\mathcal{R}$), the number of repairpersons used (m), the number of columns generated (Col.), the reliability gap ($\Delta\mathcal{R}$) to the reliability upper

Table 3.13: Parameters for set of experiment #3: Large-scale series-parallel system.

E_{ij} From	E_{ij} To	η_{ij}	β_{ij}	X_{ij}	B_{ij}	t^c_{ij1}	t^c_{ij2}	t^c_{ij3}	t^p_{ij2}	t^p_{ij3}	c^c_{ij1}	c^c_{ij2}	c^c_{ij3}	c^p_{ij2}	c^p_{ij3}
$E_{1,1}$	$E_{2,1}$	50	3.0	[0]	[8]	12	21	24	9	15	12	24	30	15	21
$E_{3,j}$	$E_{8,j}$	50	3.0	[0,1]	[8,15]	12	21	24	9	15	12	24	30	15	21
$E_{9,j}$	$E_{18,j}$	35	1.5	[0,1,0]	[15,12,10]	6	15	18	6	9	15	30	42	24	30
$E_{19,j}$	$E_{25,j}$	50	3.0	[0,1,0,1]	[8,15,8,15]	9	12	15	3	6	18	30	60	21	36
$E_{26,j}$	$E_{32,j}$	25	2.1	[0,1,0,1]	[6,10,6,10]	6	7.5	12	6	9	12	24	30	15	21

bound obtained from the relaxed problem (where the integrality requirement is relaxed) ($\mathcal{R}_{rel}$), and the maintenance cost (C).

Results show that the heuristic method developed and used in the first step of Algorithm 1 increases the convergence rate of the solution methods. As expected, as the number of components increases, the problem becomes more difficult to solve. Hence, more iterations are needed to reach solution convergence. Furthermore, the $\Delta\mathcal{R}$ values obtained are very small, indicating that the solutions obtained are very close to the unknown optimal solution and could possibly even be optimal in many cases.

All four approaches reach the same solutions for all instances as shown in Figure 3.2. However, CG–PLAh generated significantly fewer columns compared to the other three variants of the CG-based approach as shown in Figure 3.3, making it the fastest of the approaches as shown in Figure 3.4. Note that lines are added between data points in the plot for better visualization. They do not represent values in between the points.

The original SMP has been shown to have an exponential time complexity (Rice et al., 1998; Rice, 1999). The CPU times obtained for the current set of experiments #3 show that the proposed CG–PLAh approach has an empirical second-order polynomial time complexity as shown by the trend line in Figure 3.5. Thus, the proposed CG-based approach offers significant computation time reduction for the JSM–RAP. The reduction enabled us to solve the JSM–RAP for large-scale systems with up to 2000 components in less than one hour.

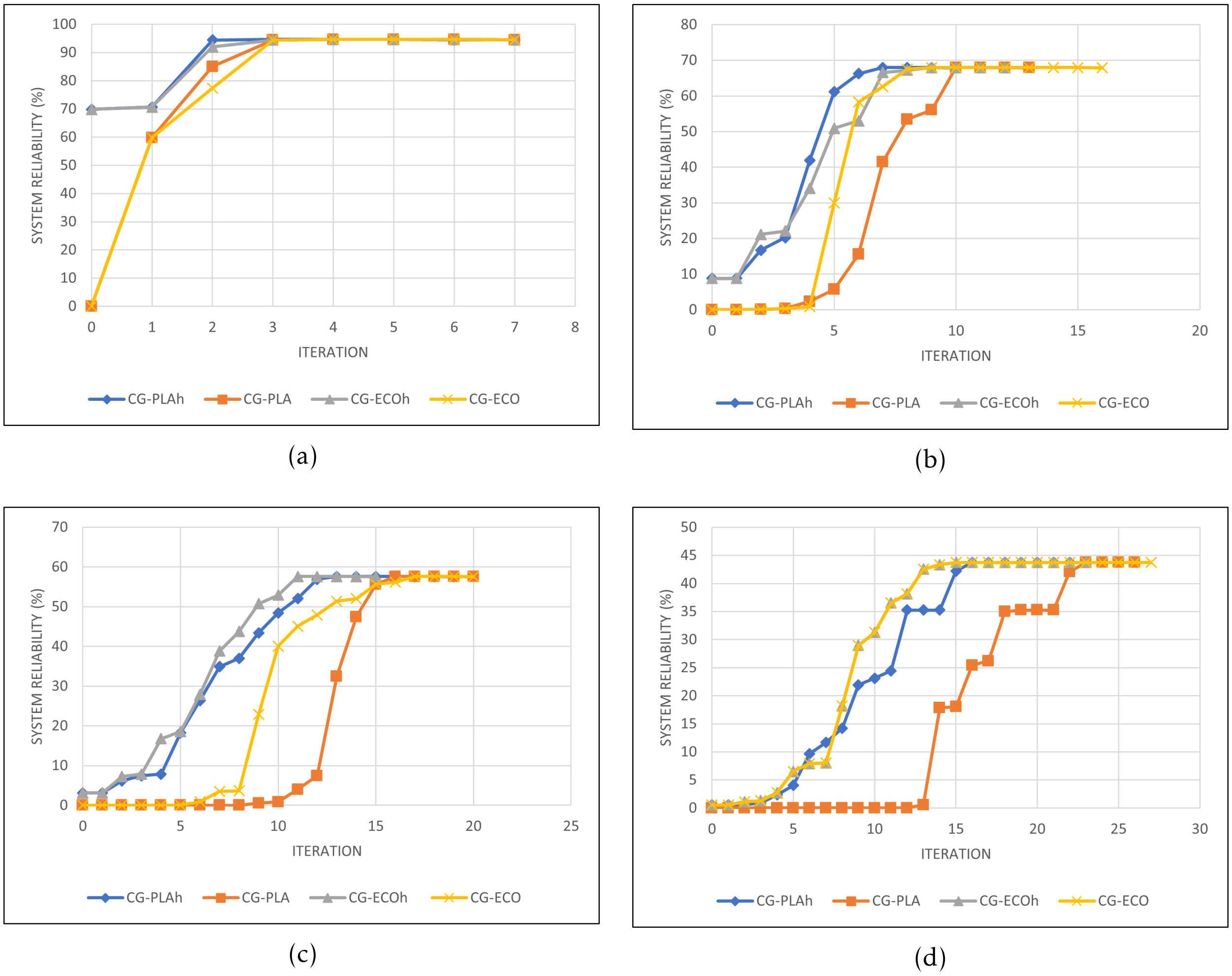


Figure 3.2: Convergence curves for different instance sizes: (a) $n = 100$, (b) $n = 700$, (c) $n = 1000$ and (d) $n = 1500$.

Table 3.14: Comparison between CG-PLAh and CG-PLA: the case of a large-scale problem with multiple repairpersons.

n	CG-PLAh					CG-PLA						
	$\mathcal{R}(\%)$	m	Col.	$\Delta R(\%)$	$CPU(s)$	$\mathcal{R}(\%)$	m	Col.	$\Delta R(\%)$	$CPU(s)$	C	$\mathcal{R}_{rel}(\%)$
100	94.51	4	108	0.12	7	94.51	4	118	0.12	10	492	94.63
200	89.50	6	276	0.06	18	89.50	6	286	0.06	16	996	89.56
300	84.75	6	414	0.00	30	84.75	7	540	0.00	33	1,500	84.75
400	80.10	8	632	0.11	52	80.10	10	856	0.11	70	1,992	80.20
500	75.85	13	815	0.05	68	75.85	11	1,200	0.05	102	2,496	75.90
600	71.83	15	1,194	0.00	142	71.83	14	1,464	0.00	128	3,000	71.83
700	67.89	16	1,617	0.09	157	67.89	14	2,205	0.09	143	3,492	67.97
800	64.29	17	1,984	0.04	250	64.29	16	3,392	0.04	344	3,996	64.33
900	60.88	20	2,781	0.00	397	60.88	18	4,014	0.00	345	4,500	60.88
1,000	57.53	21	3,300	0.08	540	57.53	20	4,600	0.08	547	4,992	57.61
1,500	43.73	31	7,215	0.00	1,771	43.73	30	10,515	0.00	1,895	7,500	43.73

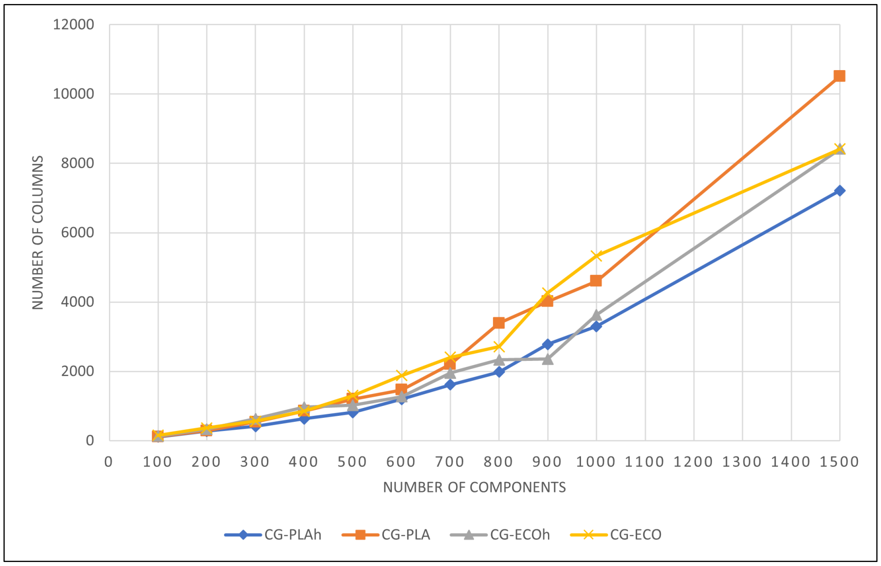


Figure 3.3: Total number of columns generated by solution method.

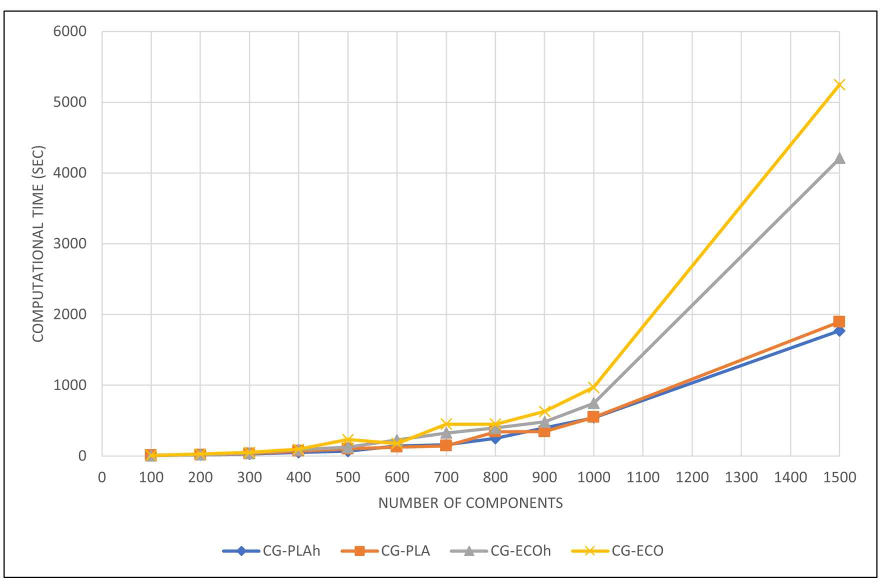


Figure 3.4: Average CPU times.

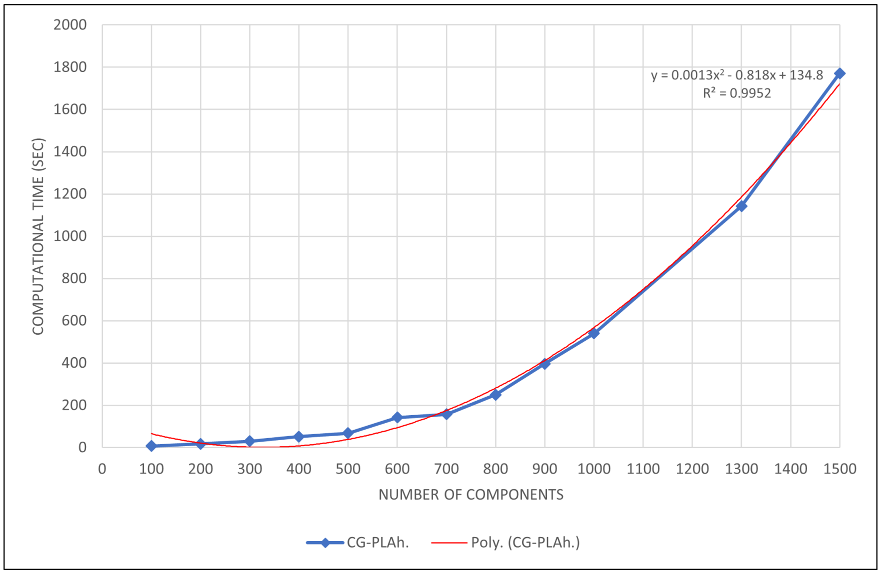


Figure 3.5: Average CPU times for CG–PLAh with trendline.

Table 3.15: Comparison between CG-ECOh and CG-ECO: the case of a large-scale problem with multiple repairpersons.

n	CG-ECOh					CG-ECO						
	$\mathcal{R}(\%)$	m	Col.	$\Delta R(\%)$	$CPU(s)$	$\mathcal{R}(\%)$	m	Col.	$\Delta R(\%)$	$CPU(s)$	C	$\mathcal{R}_{rel}(\%)$
100	94.51	4	135	0.12	8	94.51	5	158	0.12	9	492	94.63
200	89.50	6	334	0.06	19	89.50	6	366	0.06	9	996	89.56
300	84.75	10	633	0.00	38	84.75	7	564	0.00	28	1,500	84.75
400	80.10	12	964	0.10	90	80.10	10	848	0.10	97	1,992	80.20
500	75.85	10	1,020	0.05	124	75.85	12	1,305	0.05	234	2,496	75.90
600	71.83	13	1,272	0.00	227	71.83	16	1,878	0.00	178	3,000	71.83
700	67.89	19	1,953	0.09	326	67.89	18	2,408	0.09	452	3,492	67.97
800	64.29	19	2,336	0.04	397	64.29	17	2,712	0.04	452	3,996	64.33
900	60.88	20	2,358	0.00	484	60.88	25	4,266	0.00	629	4,500	60.88
1,000	57.53	21	3,630	0.08	745	57.53	23	5,330	0.08	970	4,992	57.61
1,500	43.73	31	8,415	0.00	4,209	43.73	31	8,415	0.00	5,294	7,500	43.73

3.5.4 Set of experiments #4: Large-scale series-parallel systems with additional maintenance levels

In this set of experiments, the tests conducted in the previous subsection 3.5.3 are repeated but with additional maintenance levels. The number of maintenance levels considered is expanded from two (DN, MR) to four levels (DN, MR, IM, PR) for failed components, and from two (DN, PR) to three levels (DN, IM, PR) for working components. Two experiments are conducted. The first experiment (#4.1) shows how increasing the number of maintenance levels grants more flexibility to the optimizer to find a combination of components and maintenance actions that better use the limited resources. The second experiment (#4.2) demonstrates the effectiveness of the applied stabilization scheme presented in Section 3.4.1.5 in accelerating the convergence of the CG algorithm.

3.5.4.1 Experiments #4.1: The case of a large-scale system with additional maintenance levels

The results displayed in Table 3.16 show that when additional maintenance levels are allowed, it is possible to achieve equal or slightly better results than when there are fewer maintenance levels. Another clear trend can be seen from the results displayed in Table 3.16. As the budget increases, the number of repairpersons utilized increases along with the number of components that can be replaced within the duration of the break. Thus, the maximum achievable reliability increases with the budget. When the budget allows it, additional repairpersons are added as permitted by the budget to complement the maintenance work.

3.5.4.2 Experiments #4.2: Effectiveness of the stabilization scheme on complex large-scale systems

To investigate the effectiveness of the proposed CG stabilization scheme, five problem instances are created by duplicating the basic 100-component system presented in Figure 3.1. We apply both CG-based approaches with and without stabilization (*i.e.*, CG with PLA+heuristic and stabilization (CG–PLAh-S) or without stabilization (CG-PLAh),

Table 3.16: Results obtained using CG-PLAh for a large-scale JSM-RAP with n = 100, D_0 = 50, m_0 = 5, m = 3 for working components (DN, IPM, PM) and m = 4 for failed components (DN, MR, ICM, CM).

C_0	$\mathcal{R}(\%)$	C	m	Component maintenance level (m)	Summary	$CPUt(s)$
100	35.76	99	2	**R₁**: MR($E_{1,1}$, $E_{2,1}$) **R₂**: MR($E_{9,3}$, $E_{11,3}$, $E_{15,3}$, $E_{16,3}$, $E_{18,3}$)	MR: 7 times	5.86
200	70.63	198	2	**R₁**: MR($E_{9,3}$, $E_{10,3}$, $E_{11,3}$, $E_{12,3}$, $E_{13,3}$, $E_{14,3}$, $E_{15,3}$, $E_{16,3}$, $E_{17,3}$, $E_{18,3}$) **R₂**: MR($E_{1,1}$, $E_{2,1}$, $E_{29,1}$, $E_{32,1}$)	MR: 14 times	6.63
300	79.41	300	4	**R₁**: MR($E_{1,1}$, $E_{2,1}$) **R₂**: MR($E_{3,1}$, $E_{11,1}$, $E_{17,1}$) **R₃**: MR($E_{9,3}$, $E_{10,3}$, $E_{11,3}$, $E_{12,3}$, $E_{13,3}$, $E_{14,3}$, $E_{15,3}$, $E_{16,3}$, $E_{17,3}$, $E_{18,3}$) **R₄**: MR($E_{27,3}$, $E_{28,3}$, $E_{29,3}$, $E_{30,3}$, $E_{31,3}$, $E_{32,3}$)	MR: 21 times	6.93
400	88.68	396	4	**R₁**: MR($E_{1,1}$, $E_{2,1}$, $E_{27,1}$, $E_{28,1}$, $E_{29,1}$, $E_{30,1}$) **R₂**: MR($E_{9,3}$, $E_{10,3}$, $E_{11,3}$, $E_{12,3}$, $E_{13,3}$, $E_{14,3}$, $E_{15,3}$, $E_{16,3}$, $E_{17,3}$, $E_{18,3}$) **R₃**: MR($E_{9,1}$, $E_{10,1}$, $E_{11,1}$, $E_{12,1}$, $E_{13,1}$, $E_{14,1}$, $E_{15,1}$, $E_{16,1}$, $E_{17,1}$, $E_{18,1}$) **R₄**: MR($E_{31,3}$, $E_{32,3}$)	MR: 28 times	7.04
500	94.51	492	4	**R₁**: MR($E_{1,1}$, $E_{2,1}$, $E_{26,1}$, $E_{27,1}$, $E_{28,1}$, $E_{29,1}$, $E_{30,1}$, $E_{31,1}$, $E_{32,1}$) **R₂**: MR($E_{3,1}$, $E_{4,1}$, $E_{5,1}$, $E_{6,1}$, $E_{7,1}$, $E_{8,1}$, $E_{9,1}$, $E_{10,1}$, $E_{11,1}$, $E_{12,1}$, $E_{13,1}$, $E_{14,1}$, $E_{15,1}$, $E_{16,1}$, $E_{17,1}$, $E_{18,1}$) **R₃**: MR($E_{9,3}$, $E_{10,3}$, $E_{11,3}$, $E_{12,3}$, $E_{13,3}$, $E_{14,3}$, $E_{15,3}$, $E_{16,3}$, $E_{17,3}$, $E_{18,3}$) **R₄**: MR($E_{32,3}$)	MR: 36 times	8.03
1000	95.80	996	5	**R₁**: R($E_{1,1}$, $E_{12,1}$, $E_{17,1}$,$E_{13,1}$), MR($E_{7,1}$) **R₂**: R($E_{2,1}$), MR($E_{12,3}$, $E_{13,3}$, $E_{17,3}$) **R₃**: MR($E_{3,1}$, $E_{4,1}$, $E_{5,1}$, $E_{6,1}$, $E_{8,1}$, $E_{19,3}$, $E_{20,3}$, $E_{21,3}$, $E_{22,3}$, $E_{24,3}$, $E_{25,3}$, $E_{27,3}$, $E_{31,3}$, $E_{32,3}$) **R₄**: R($E_{9,1}$, $E_{10,1}$, $E_{11,1}$, $E_{14,1}$, $E_{15,1}$, $E_{16,1}$, $E_{18,1}$) **R₅**: MR($E_{9,3}$, $E_{10,3}$, $E_{11,3}$, $E_{14,3}$, $E_{15,3}$, $E_{16,3}$, $E_{18,3}$, $E_{23,1}$, $E_{26,1}$, $E_{26,3}$, $E_{27,1}$, $E_{28,1}$, $E_{28,3}$, $E_{29,1}$, $E_{29,3}$, $E_{30,1}$, $E_{30,3}$, $E_{31,1}$, $E_{32,1}$)	MR: 37 times, R: 12 times	12.92
1700	95.92	1692	5	**R₁**: R($E_{1,1}$, $E_{2,1}$, $E_{10,1}$, $E_{10,2}$, $E_{10,3}$, $E_{11,1}$, $E_{11,2}$,$E_{11,3}$), IM($E_{24,3}$) **R₂**: R($E_{9,1}$, $E_{9,2}$, $E_{13,1}$, $E_{13,2}$, $E_{14,1}$, $E_{14,2}$, $E_{15,1}$, $E_{15,2}$, $E_{18,1}$, $E_{18,2}$, $E_{29,2}$, $E_{29,3}$) **R₃**: R($E_{12,1}$, $E_{12,2}$, $E_{12,3}$, $E_{16,1}$, $E_{16,2}$, $E_{16,3}$, $E_{17,1}$, $E_{17,2}$, $E_{17,3}$, $E_{29,4}$) **R₄**: MR($E_{3,1}$, $E_{5,1}$, $E_{7,1}$, $E_{19,1}$, $E_{20,1}$, $E_{21,1}$, $E_{22,1}$, $E_{29,1}$, $E_{30,1}$, $E_{30,3}$, $E_{31,1}$, $E_{31,3}$, $E_{32,1}$, $E_{32,3}$), R($E_{9,3}$, $E_{18,3}$) **R₅**: MR($E_{4,1}$, $E_{6,1}$, $E_{8,1}$, $E_{23,1}$, $E_{25,1}$, $E_{29,3}$, $E_{30,1}$, $E_{30,3}$, $E_{31,1}$, $E_{32,1}$), IM($E_{26,1}$, $E_{26,3}$, $E_{27,1}$, $E_{27,3}$, $E_{28,1}$, $E_{28,3}$), R($E_{13,3}$, $E_{14,3}$, $E_{15,3}$, $E_{29,2}$)	MR: 24 times, IM: 7 times, R: 36 times	12.66

and CG with ECO+heuristic and stabilization (CG–ECOh-S) or without stabilization (CG-ECOh)) on each of the five problem instances. The obtained results are displayed in Figure 3.6 and Tables 3.17 and 3.18. For each solution method, these tables display the reliability values obtained ($\mathcal{R}$), the number of iterations performed (# Iter.), the number of columns generated (Col.), and the reliability gap ($\Delta\mathcal{R}$) to the upper bound obtained from the relaxed problem ($\mathcal{R}_{rel}$).

The results obtained clearly show that applying the stabilization scheme increases the convergence rate of the solution methods by decreasing the number of iterations performed and consequently reduces the computational time and the number of generated columns. For example, in the largest instance ($n = 500$), the stabilization scheme reduces the number of iterations from 29 to 21 in the CG-PLAh case and from 37 to 25 in the CG-ECOh case. Given that the optimality gaps $\Delta\mathcal{R}$ obtained are very small, not exceeding 0.16%, we decided not to perform branching for this set of experiments.

We did not observe any significant reduction in the number of iterations in small/moderate-size instances when the stabilization scheme is applied. In fact, the number of iterations sometimes increased, a result opposite to the one expected. In contrast, stabilization is effective for large-scale systems with additional maintenance levels as demonstrated.

All four methods reach similar solutions for all instances as shown in Figure 3.6. However, CG–PLAh-S generated significantly fewer columns compared to the other three variants of the CG-based approach as shown in Tables 3.17 and 3.18, making it the fastest approach.

Table 3.17: Comparison of CG-PLAh, and CG-PLAh-S: The case of a large-scale problem with multiple repairpersons

n	CG-PLAh (Unstabilized)					CG-PLAh-S (Stabilized)					
	$\mathcal{R}$(%)	# Iter.	Col.	ΔR(%)	$CPU(s)$	$\mathcal{R}$(%)	# Iter.	Col.	ΔR(%)	$CPU(s)$	$\mathcal{R}_{rel}$(%)
100	95.92	12	384	0.00	19.7	95.92	12	384	0.00	18.9	95.92
200	92.00	16	1024	0.01	56.9	92.00	14	896	0.01	51.6	92.01
300	88.25	22	2212	0.00	270.0	88.25	18	1728	0.00	212.3	88.25
400	84.65	24	3072	0.00	355.1	84.65	21	2688	0.00	314.1	84.65
500	81.19	29	4640	0.01	447.6	81.19	21	3360	0.01	338.1	81.20

Table 3.18: Comparison of CG-ECOh, and CG-ECOh-S: The case of a large-scale problem with multiple repairpersons

n	CG-ECOh (Unstabilized)					CG-ECOh-S (Stabilized)					
	$\mathcal{R}(\%)$	# Iter.	Col.	$\Delta R(\%)$	$CPU(s)$	$\mathcal{R}(\%)$	# Iter.	Col.	$\Delta R(\%)$	$CPU(s)$	$\mathcal{R}_{rel}(\%)$
100	95.92	16	512	0.04	41.3	95.92	12	384	0.04	30.8	95.92
200	92.00	20	1280	0.05	107.2	92.00	16	896	0.05	75.0	92.01
300	88.25	30	2880	0.10	332.2	88.25	21	2016	0.10	232.5	88.25
400	88.65	32	4096	0.13	478.4	88.65	22	2816	0.13	328.9	88.65
500	81.19	37	5920	0.16	542.2	81.19	25	4000	0.16	366.4	81.20

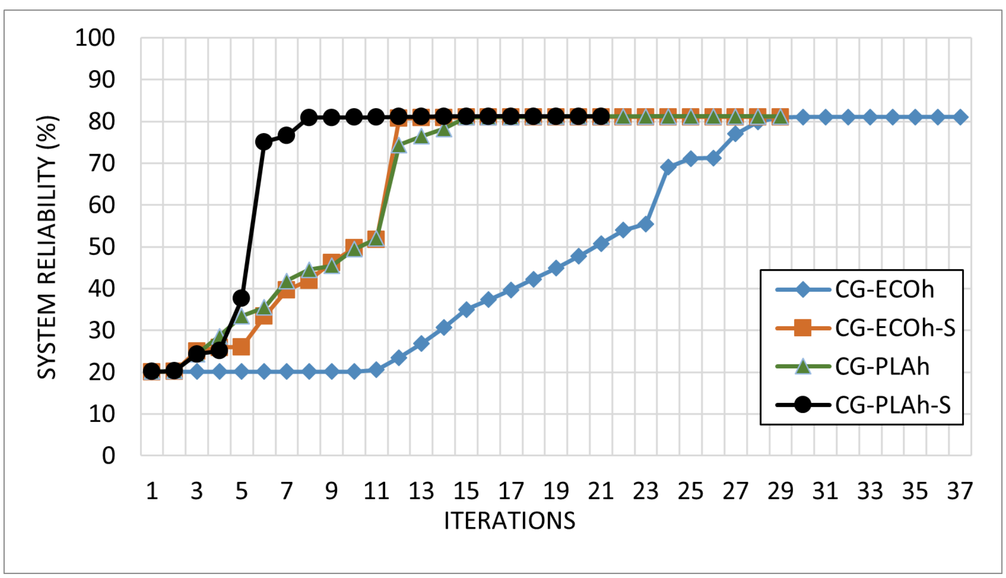


Figure 3.6: Convergence curves for instance size of 500 components with different CG-based solution approaches.

3.6 Conclusions

In this paper, large-scale instances of the joint selective maintenance and repairperson assignment problem for a series-parallel system are addressed. A column-generation-based approach that iterates between solving a restricted master problem to update the dual multipliers and solving multiple subproblems to generate maintenance patterns is developed. Two novel reformulations are proposed for the mixed-integer nonlinear subproblem: a piecewise-linear approximation and an exact reformulation into a mixed-integer exponential conic optimization problem, both can be handled directly using off-the-shelf solvers. A heuristic procedure is developed to quickly provide a feasible solution to initiate the column-generation algorithm. Furthermore, a stabilization scheme is proposed to accelerate its convergence. Finally, to restore integrality and ensure the solution optimality, column-generation is embedded into a branch-and-bound tree to devise branch-and-price algorithms.

Four sets of numerical experiments are carried out and they showed the capability of the proposed approach to deal with large-scale instances of the JSM–RAP and yield optimal solutions. The computation times of the CG-PLA and CG-ECO approaches were considerably smaller than those of the two-phase approach presented in Diallo et al. (2019b). On the other hand, the CPU times for the B&P algorithms are significantly larger than those of the CG-based approaches without B&B. However, for all experiment sets, the two B&P algorithms were able to solve all instances optimally. CG–ECO that uses an exact reformulation of $\mathcal{SP}_i$ is proven to be capable of guaranteeing solution optimality when combined with B&B. For large-scale instances, whether CG-PLA or CG-ECO is utilized, the gap between the reliability obtained from solving the JSM–RAP as a BIP problem and that obtained from the relaxed problem is usually very small, meaning that the solutions obtained are very close to the unknown optimal solution. The efficiency of the proposed algorithms is such that it was possible to solve a problem with more than double the number of components in the previous largest JSM–RAP instance solved by Ikonen et al. (2020). Finally, The effectiveness of the stabilization scheme is demonstrated in accelerating the convergence of the CG algorithm for large-scale systems with additional maintenance levels.

We are working on an extension of the current formulation to the multimission and fleet SM problem. Most existing SM models deal with reliability as the performance indicator. It would be of great value to consider system availability as well. Therefore, we are planning to study the trade-offs between system availability and the hiring of repairpersons using multi-objective optimization approaches. Future works will also focus on extending the present approach to deal with situations where the operational performance of components along with their corresponding degradation processes are both accounted for in SM modelling and optimisation. Another important research issue to investigate within JSM–RAP is the maintenance tasks scheduling problem. Finally, DFS was employed as the node selection strategy in the proposed B&P algorithm due to its lower memory consumption and because no significant difference in computation times compared to the best-first search (BFS) strategy was observed. However, this result may only be specific to the instances used in this study. A further systematic investigation is recommended to fully understand the advantages and limitations of both DFS and BFS in solving future SMPs.

Chapter 4

Distributionally-robust chance-constrained optimization of selective maintenance under uncertain repair duration

4.1 Introduction

The operation of many assets used in military and industrial applications follows alternating sequences of missions and maintenance pauses. Examples of such systems include power plants, naval vessels and aircraft. Given that such mission-oriented systems cannot undergo maintenance while in service, maintenance activities are typically executed during scheduled breaks, aiming to enhance their ability to complete subsequent missions successfully. Furthermore, given that the available maintenance resources (*e.g.*, budget, time, spare parts, personnel) are not usually sufficient to complete all required maintenance actions, the planner must determine an optimal subset of components to maintain and the level of maintenance to be executed on each component. This maintenance strategy is known as *selective maintenance* (SM), and the corresponding selection problem is the *selective maintenance problem* (SMP) (Rice et al., 1998). When multiple repairpersons possibly with different costs and skill levels are available, it is also necessary to determine the assignment of maintenance activities to each repairperson, giving rise to the *joint selective maintenance and repairperson assignment problem* (JSM–RAP) (Diallo et al., 2017, 2019b; An et al., 2021; Al-Jabouri et al., 2023; O'Neil et al., 2023b).

A lot of attention has been directed to the SMP in the last two decades. Problems with various systems structures, maintenance policies, and resource restrictions have been considered. Recent reviews on the topic include Xu et al. (2015); Cao et al. (2018a) and Al-Jabouri et al. (2022). Solution methods developed for different variants of the SMP include exact methods, (*e.g.*, enumeration (Rice et al., 1998), depth-first search algorithms (Cao et al., 2016b), branch-and-bound (Lust et al., 2009), search space reduction (Rajagopalan and Cassady, 2006), and the two-phased approach (Diallo et al., 2018)), constructive heuristics (Galante et al., 2020; Ahadi and Sullivan, 2019; Cao et al., 2018b;

Khatab et al., 2007), metaheuristics (*e.g.*, simulated annealing (Jiang and Liu, 2020b), genetic algorithms (Dao et al., 2014), and differential evolution (Pandey et al., 2013b)), the max-min approach by Schneider and Cassady (2015), and reinforcement learning by Liu et al. (2020). Despite this plethora of SMP studies, handling large-scale systems is still a challenge due to the NP-hardness of the problem resulting from its nonconvexity (Rice, 1999). Therefore, new formulations and solution algorithms that can effectively deal with the SMP and its extensions for large industrial-scale instances are required to enable real-life implementation on systems often comprised of hundreds of components.

Another important limitation of most SMP models proposed in the literature is that they assume the duration of maintenance actions to be known in advance as deterministic parameters. This assumption is quite unrealistic as the high variability of component conditions, repairperson skill levels, and other unpredictable factors such as operating conditions and human errors make it difficult to estimate the exact duration of maintenance actions accurately. A maintenance plan developed based on inaccurate duration estimates cannot usually be fully implemented, leading to low system reliability or long overtime (if the planner allows it) (Khatab et al., 2017a). Among the few studies that considered maintenance duration uncertainty is the work of Gupta et al. (2014), which addressed a case where parameter values for factors such as time, weight, volume, and cost cannot be obtained precisely but are defined within interval bounds. Two SM optimization models were developed. However, only a single repairing channel was considered with only two maintenance levels (repair or replacement). Haseen et al. (2015) addressed a SMP in which each component's repair time and cost are modelled as fuzzy numbers. However, the replacement time and cost are assumed to have fixed values. Khatab et al. (2017a) proposed a risk-neutral stochastic programming (SP) model with probabilistic constraints to handle the random maintenance durations which are assumed to follow Gamma distributions. The proposed nonlinear SP model minimizes the total maintenance cost with a constraint ensuring that the maintenance actions can be completed during the break with a pre-determined threshold probability. Liu et al. (2018) developed a SP model with the uncertainty on maintenance duration following a truncated normal distribution. A saddle-point approximation approach was used to evaluate the system's reliability.

Chance-constrained programming (CCP) was proposed as a viable alternative for handling duration uncertainty in the context of SMP (Ali et al., 2011a). Although CCP enables the planner to balance feasibility and solution quality and to control risk exposure by adjusting the confidence level, it suffers from some drawbacks that limit its usability. First, the chance constraint method requires full knowledge of the probability distributions for the duration of each maintenance action type and level. It can be challenging to determine the “true" distribution in cases when there is limited historical data, particularly for systems that have not been active for sufficient time periods or have a low frequency of failures. This situation is typical in the maintenance industry where failure events are infrequent and/or the components have not worked long enough to extract sufficient maintenance records. Moreover, such problems can be challenging to solve since chance constraints are generally nonconvex (Charnes and Cooper, 1959).

Alternatively, rather than presuming the existence of a fully-known probability distribution for the uncertain parameters, a recent modelling framework called *distributionally-robust optimization* (DRO) (Goh and Sim, 2010) involves considering an *ambiguity set* of probability distributions and solving a minimax-type problem to identify decisions that offer protection against the worst-case parameter distribution within the set (Noyan et al., 2022). In general, there are moment-based and statistical distance-based ambiguity sets in the DRO literature (Postek et al., 2016). The ambiguity sets defined by moment conditions encompass probability distributions that match certain moments, such as the empirical first and second moments. Nevertheless, such sets do not always contain the true distribution, and moment-based DRO approaches can be excessively conservative since different distributions may exhibit the same or similar lower moments. On the other hand, statistical distance-based ambiguity sets consist of probability distributions that are in the proximity of a nominal (empirical) distribution, enough to be good estimates of the true distribution. The proximity or vicinity is defined as a ball centered on the nominal distribution. Many measures of statistical distances providing an assessment of dissimilarity between two probability distributions exist and have been used to construct these balls (Pflug and Wozabal, 2007; Erdoğan and Iyengar, 2006; Zhao and Guan, 2018). A desirable characteristic of distance-based DRO methodologies is the capability to manage the level of conservatism by modifying the radius. When a prescribed level of confidence is required, selecting an appropriate radius for certain distances, like the

Wasserstein-1 metric, can ensure that the true probability distribution belongs to the ambiguity set. This advantage has significantly increased the adoption of Wasserstein distances in DRO (Noyan et al., 2022).

This paper studies the JSM–RAP with uncertain maintenance duration, aiming to address the two aforementioned issues, namely the nonlinear nonconvex nature of the problem, and the limitations of SP for handling duration uncertainty in terms of both its restrictive assumption of perfect knowledge of the probability distribution and its intractability. First, we propose a piecewise-linear approximation of the nonlinear reliability objective function such that the nominal problem can be tightly approximated as a mixed-integer linear program (MILP) without increasing the number of binary variables. Then, to handle the uncertain maintenance duration, we replace the deterministic total duration constraint with a data-driven DRCC that provides a probabilistic guarantee on the satisfaction of the total maintenance duration constraint. This guarantee is enforced for the subset of probability distributions contained in a Wasserstein ambiguity set (*i.e.*, a form of a ball in the space of probability distributions with respect to the type-1 Wasserstein metric centered at the empirical distribution) constructed based on a finite and typically small training dataset. The developed method recognizes that the statistical data samples can usually be explained by multiple distributions and alleviates the overfitting found in traditional SP based on a single distribution, which is consistently contaminated by estimation errors (Mohajerin Esfahani and Kuhn, 2018). It also enables robust maintenance plans (in the sense that they perform well for *out-of-sample* realizations) to be obtained based on a small number of training samples (in respect to the size of the uncertain problem parameters) as is usually the case in industrial/military systems with rare failure events. The DRCC is then approximated using a Conditional Value-at-Risk (CVaR) constraint such that the JSM–RAP with uncertain maintenance duration, similar to its nominal counterpart, is approximately reformulated as a MILP solved exactly and relatively easily using off-the-shelf solvers.

The article is organized as follows. Section 4.2 defines the nominal deterministic JSM–RAP for a multicomponent system, its modelling assumptions, and the system reliability computations. It also presents both the classical mixed-integer nonlinear programming (MINLP) formulation of the JSM–RAP as well as the proposed reformulation using piecewise-linear approximation (PLA). In Section 4.3, the proposed DRCC formulation that ensures maintenance plans can be completed at a given probability is developed. Several numerical experiments and their subsequent results discussion are provided in Section 4.4. Conclusions are outlined and potential ideas for extension are presented in Section 4.5.

4.2 The nominal JSM-RAP

This section provides the description and mathematical formulation of the JSM-RAP under the assumption that the maintenance duration values corresponding to each component, repairperson and maintenance level are known with certainty. This case is referred to hereinafter as the *nominal problem*. The multicomponent series-parallel system under consideration and our modelling notations and assumptions are described, then the imperfect maintenance (IM) model is defined, and the expressions for the overall duration and cost of the maintenance plan are developed. A MINLP formulation of the JSM–RAP is presented, followed by the reformulation of the nominal problem through piecewise-linear approximation.

4.2.1 System description

As depicted in Figure 4.1, the system being considered has m subsystems arranged in series (*i.e.*, each subsystem must be operational for the system to function). Each subsystem denoted by i is made up of J_i parallel repairable components E_{ij} (*i.e.*, the ith subsystem can operate as long as there is at least one functioning component among its J_i components). Subsystem components are independent with lifetimes not necessarily identically distributed. These components do not have the same age at the end of a mission when maintenance decisions must be made.

The series-parallel configuration is a widely studied structure in the SM literature, and it has been applied to model numerous industrial and military equipment. Examples of such systems include machining lines in an engine workshop (Zhu et al., 2011), coal transportation systems (Liu et al., 2009), army tanks (Sharma et al., 2017), material delivery systems (Chen et al., 2012), shell filling and shooting equipment (Cao et al., 2018b), aircraft turbine engine (Wang et al., 2019), nuclear fuel production assets (Zhao et al., 2019b), low-pressure coolant injection apparatus (Ruiz et al., 2020), and flow transmission equipment (Liu et al., 2020).

The asset/system has just concluded a mission and is switched off to go through maintenance operations during a scheduled pause of duration D_0 (see Figure 4.2). Following the maintenance pause, the system will carry out a mission that lasts V units of time.

$$X_{ij} = \begin{cases} 1 & \text{if } E_{ij} \text{ is operational at the beginning of the break.} \\ 0 & \text{otherwise.} \end{cases} \tag{4.1}$$

Likewise, after the break, the effective age of each component E_{ij} is denoted by A_{ijl} if maintenance level l is carried out, and its status is given by a binary parameter Y_{ij}, which is defined as:

$$Y_{ij} = \begin{cases} 1 & \text{if } E_{ij} \text{ is working following the end of the break} \\ 0 & \text{otherwise.} \end{cases} \tag{4.2}$$

By comparing Y_{ij} with X_{ij}, it is possible to determine whether a failed component was fixed during the pause or left unrepaired.

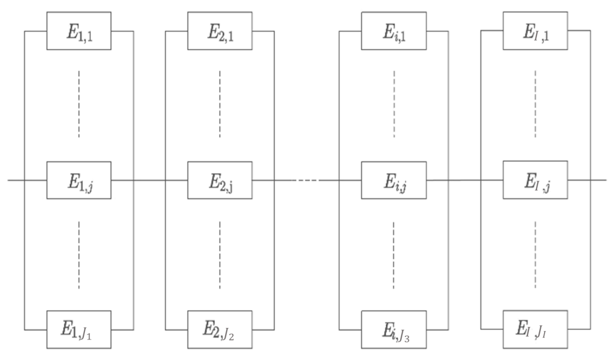


Figure 4.1: Series-parallel system structure.

4.2.2 Modelling assumptions

To formulate the problem, the following assumptions are made.

1. The asset/system comprises multiple binary components that are repairable, non-identical, and operate independently. This assumption is widely accepted and backed by multiple references (Diallo et al., 2018; Jiang and Liu, 2020a).

2. Components do not experience aging while they are inactive due to failures as their age is mainly determined by their usage. This assumption is reasonable considering

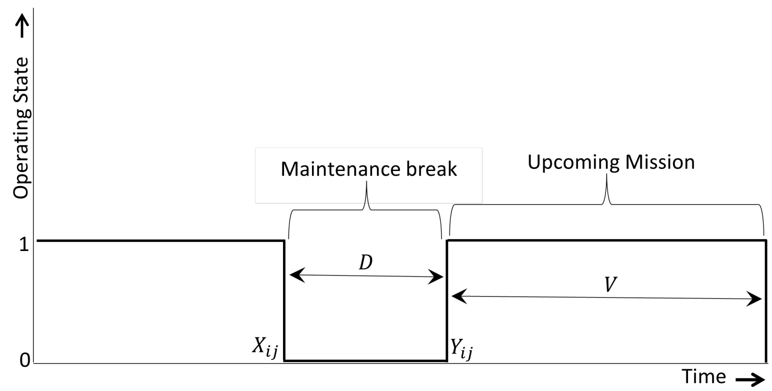


Figure 4.2: Sequence of mission and scheduled break.

that the duration of the failures is usually insignificant compared to mission length (Diallo et al., 2019b).

3. Maintenance actions can only be performed during system downtime; no maintenance is allowed during mission time. For a majority of mission-oriented assets, it is impractical or impossible to suspend the mission to conduct maintenance (Jiang and Liu, 2020b).

4. Availability of required resources such as budget, repair personnel, and tools is guaranteed at the time of requirement (Chaabane et al., 2020a).

5. Each maintenance activity is performed by a single repairperson, and any repairperson can undertake any maintenance level on any component (O'Neil et al., 2023b; Diallo et al., 2019b).

6. Concurrently working on multiple components is possible without any overlap or collision between the repairpersons involved (Diallo et al., 2019b).

Table 4.1 presents the system of notation used in this paper.

4.2.3 Modelling maintenance costs and duration

Two types of maintenance actions are performed during a break/pause: corrective maintenance (CM) and preventive maintenance (PM) (Yang et al., 2009; Al-Jabouri et al., 2023). CM is executed on failed components while PM is done on components that are still in operation.

Table 4.1: System of notations used

i	index of subsystems in series in the system considered, $i \in \mathcal{I} = \{1,\ldots,I\}$
j	index of components in subsystem i, $j \in \mathcal{J}_i = \{1,\ldots,J_i\}$
l	index of maintenance levels available for component E_{ij}, $l \in \mathcal{L}_{ij} = \left\{0,\ldots,L_{ij}\right\}$
r	index of repairpersons, $r \in \mathcal{Q} = \{1,\ldots,Q\}$
n	index of breakpoints in the PLA model, $n \in \mathcal{N} = \{1,\ldots,N\}$
k_r	index of training samples for repairperson r, $k_r \in \mathcal{K}_r = \{1,\ldots,K_r\}$
s_r	index of testing samples, $s_r \in \mathcal{S}_r = \{1,\ldots,S_r\}$
E_{ij}	j^{th} component of subsystem i
ρ	Wasserstein radius
$1-\epsilon_r$	individual chance constraint's confidence level for repairperson r
$t^{+}_{ijlr}(t^{-}_{ijlr})$	upper (lower) bound of the duration required to implement maintenance level l on component E_{ij} by repairperson r
$t^{c}_{ijlr}(t^{p}_{ijlr})$	nominal duration for performing CM (PM) on component E_{ij} at maintenance level l by repair person r
t^{k}_{ijlr}	time required to perform maintenance level l on component E_{ij} by repairperson r according to training sample k
B_{ij}	age of component E_{ij} at the start of the break
A_{ijl}	age of component E_{ij} at the end of the break if maintenance level l is performed
$X_{ij}(Y_{ij})$	binary variable indicating the status of component E_{ij} at the beginning (ending) of the break (1: working; 0: failed)
T_r	aggregate time spent by repairperson r on maintenance operations
C_0	maintenance budget cap
D_0	break duration
V	duration of the succeeding mission
$R^{c}_{ij}(V\|_{A_{ijl}})$	reliability of component E_{ij} during the subsequent mission given an initial age A_{ijl}
R	system reliability for the upcoming mission
P	an arbitrary distribution in the distributional ambiguity set
$\widehat{P}$	the empirical distribution in the distributional ambiguity set

For a failed component E_{ij}, a maintenance level among the $(L_{ij} + 1)$ CM levels ($l \in \left\{0, 1, \ldots, L_{ij}\right\}$) available must be selected. The lowest level ($l = 0$) corresponds to the Do-Nothing (DN) option while the highest level ($l = L_{ij}$) is the component replacement option. Level $l = 1$ denotes minimal repair (MR) that aims to restore the component to its original "as bad as old" condition when executed. Imperfect maintenance (IM) actions fall within the range of $1 < l < L_{ij}$ and result in the component's health condition being restored to a state between "as bad as old" and "as good as new" after completion. In this study, IM is modelled using the age reduction approach introduced in (Malik, 1979). This means that when a repairperson r performs CM of level l on component E_{ij}, the age of the component B_{ij} is multiplied by an age reduction coefficient θ_{ijl}, where $0 \leq \theta_{ijl} \leq 1$. Accordingly, the component is considered "as good as new" (*i.e.*, , replaced) if its age is reset to zero ($\theta = 0$), while it is considered "as bad as old" (minimal repair) if the age reduction coefficient is $\theta = 1$. Each CM action incurs a cost c^{c}_{ijl} and requires t^{c}_{ijlr} units of time.

Similarly, for a functioning component E_{ij}, a PM action can be performed at level $l \in \left\{0, 2, \ldots, L_{ij}\right\}$. It is worth noting that level $l = 1$ pertains to the minimal repair scenario, where a failed component is returned to working condition without affecting its failure rate. However, minimal repair is not applicable to working components, so $l = 1$ is not available for PM actions. Intermediate values of l ($2 \leq l < L_{ij}$) correspond to IM actions, which rejuvenate the component by lowering its age by a proportion φijl, where $0 \leq \varphi_{ijl} \leq 1$. For example, when $\varphi_{ijL_{ij}} = 0$, it corresponds to the perfect replacement (PR) scenario where the component's age is reset to 0. On the other hand, when $\varphi_{ijL_{ij}} = 1$, it corresponds to the "Do Nothing" scenario, where the component's age remains unchanged after the maintenance break. Every PM action incurs a cost c^{p}_{ijl} and lasts t^{p}_{ijlr} units of time.

Based on the aforementioned IM model, the computation of the effective age A_{ijl} for a given component E_{ij} at the end of the break is determined as a function of its initial operating status X_{ij} and the level of maintenance l executed. This function can be expressed as follows:

$$A_{ijl} = B_{ij}\left[X_{ij}\varphi_{ijl} + \left(1 - X_{ij}\right)\theta_{ijl}\right]. \tag{4.3}$$

When maintenance is not carried out on a component, its associated maintenance duration and cost are set to zero. However, if a component E_{ij} undergoes maintenance at level l by repairperson r, it incurs a fixed cost (c^p_{ijl} or c^c_{ijl}). Accordingly, the total costs for PM and CM are expressed by:

$$C_{PM} = \sum_{i=1}^{I}\sum_{r=1}^{Q}\sum_{j=1}^{J_i}\sum_{l=2}^{L_{ij}} c^p_{ijl}\, X_{ij}\, x_{ijlr}, \tag{4.4}$$

$$C_{CM} = \sum_{i=1}^{I}\sum_{r=1}^{Q}\sum_{j=1}^{J_i}\sum_{l=1}^{L_{ij}} c^c_{ijl}\left(1 - X_{ij}\right) x_{ijlr}, \tag{4.5}$$

where the term $(1 - X_{ij})$ guarantees that CM activities are exclusively conducted on components that have failed. The total maintenance cost C is then:

$$C = C_{PM} + C_{CM}. \tag{4.6}$$

Lastly, the total maintenance duration T_r spent by each repairperson r is determined by:

$$T_r = \sum_{i=1}^{I}\sum_{j=1}^{J_i}\left[\left(1 - X_{ij}\right)\sum_{l=1}^{L_{ij}} t^c_{ijlr} x_{ijlr} + X_{ij}\sum_{l=2}^{L_{ij}} t^p_{ijlr} x_{ijlr}\right]. \tag{4.7}$$

4.2.4 Next mission system reliability

If the unconditional reliability of component E_{ij} is denoted by $R_{ij}(t)$, then R_{ijl} the component reliability during the next mission if E_{ij} is subjected to a maintenance action of level $l \in \{0, \ldots, L_{ij}\}$ is given by:

$$R_{ijl} = \frac{R_{ij}\left(A_{ijl} + V\right)}{R_{ij}\left(A_{ijl}\right)}. \tag{4.8}$$

Considering that each component can undergo exactly one maintenance action of level l including Do-Nothing ($l = 0$) and considering that only one repair person is needed to perform that activity, $R^c_{ij}(V|_{A_{ijl}})$ the conditional reliability of component E_{ij} given that its initial age is A_{ijl} is given by:

$$R^c_{ij}(V|_{A_{ijl}}) = \sum_{l=0}^{L_{ij}}\sum_{r=1}^{Q} R_{ijl} x_{ijlr}, \tag{4.9}$$

where x_{ijlr} is a binary decision variable defined as:

$$x_{ijlr} = \begin{cases} 1 & \text{if repairperson } r \text{ performs maintenance level } l \text{ on } E_{ij} \\ 0 & \text{otherwise.} \end{cases} \tag{4.10}$$

Also, because each subsystem i is a parallel structure, its next mission reliability R_i is given by:

$$R_i = 1 - \prod_{j=1}^{J_i}\left(1 - \sum_{l=0}^{L_{ij}}\sum_{r=1}^{Q} R_{ijl}x_{ijlr}\right). \tag{4.11}$$

Lastly, the next mission reliability R of the asset (series-parallel system) is:

$$R = \prod_{i=1}^{I} R_i = \prod_{i=1}^{I}\left(1 - \prod_{j=1}^{J_i}\left(1 - \sum_{l=0}^{L_{ij}}\sum_{r=1}^{Q} R_{ijl}x_{ijlr}\right)\right). \tag{4.12}$$

4.2.5 Mixed-integer nonlinear programming formulation

The goal of the JSM–RAP addressed here is to optimize the next mission system reliability of the asset under consideration by jointly determining a list of components to maintain, the levels of maintenance to be executed, and the repairers to perform them during the break, subject to a pre-specified maintenance budget. The following MINLP formulation of the JSM–RAP was provided by Diallo et al. (2019b).

$[\mathcal{SMP}]_{MINLP}$:

$$\max_{x_{ijlr}} R = \prod_{i=1}^{I}\left(1 - \prod_{j=1}^{J_i}\left(1 - \sum_{r=1}^{Q}\sum_{l=0}^{L_{ij}} R_{ijl}x_{ijlr}\right)\right) \tag{4.13a}$$

s.t.

$$C \leq C_0 \tag{4.13b}$$

$$T_r \leq D_0 \qquad \forall r \in \mathcal{Q} \tag{4.13c}$$

$$\sum_{r=1}^{Q}\sum_{l=1}^{L_{ij}} x_{ijlr} = 1 \qquad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}_i \tag{4.13d}$$

$$x_{ijlr} \in \{0,1\} \qquad \forall (i,j,l,r) \in (\mathcal{I},\mathcal{J}_i,\mathcal{L}_{ij},\mathcal{Q}). \tag{4.13e}$$

The objective function in Equation (4.13a) maximizes the next mission system reliability. To ensure that the maintenance cost is within the available budget, constraint (4.13b) restricts the total cost of maintenance C calculated by equation (4.6) to be less than or equal to the maintenance budget. To limit the total maintenance time assigned to each repairperson T_r calculated by equation (4.7) within the break duration, constraints

(4.13c) are used. Additionally, ensuring that every component receives just one maintenance level performed by a single repairperson is enforced by constraints (4.13d).

This MINLP model is computationally expensive to solve or intractable as the problem size increases. Therefore, the first contribution of this article is to develop a piecewise-linear approximation to allow for the consideration of large-scale systems as often encountered in industry.

4.2.6 Reformulation of the nominal problem through piecewise-linear approximation

The piecewise-linear approximation proposed uses continuous variables in type 2 special ordered sets (SOS2) constraints (Beale and Forrest, 1976) to handle the non-linear formulation (4.13) so it is tightly approximated as a MILP.

First, let $F_i = 1 - R_i$ be the unreliability of subsystem i, where

$$R_i = 1 - \prod_{j=1}^{J_i} \left(1 - \sum_{r=1}^{Q} \sum_{l=0}^{L_{ij}} R_{ijl} x_{ijlr} \right). \tag{4.14}$$

Application of the natural logarithm function to to each side of Equation (4.13a) results in the linearization of the objective function

$$\ln(R) = \sum_{i=1}^{I} \ln(1 - F_i). \tag{4.15}$$

Since x_{ijlr} is binary and only a single maintenance level is chosen for every component (*i.e.*, $\sum_{r=1}^{Q} \sum_{l=0}^{L_{ij}} x_{ijlr} = 1$), and using Equation (4.14), F_i can be rewritten as

$$F_i = \prod_{j=1}^{J_i} \left(1 - \sum_{r=1}^{Q} \sum_{l=0}^{L_{ij}} R_{ijl} x_{ijlr} \right) \tag{4.16}$$

$$= \prod_{j=1}^{J_i} \sum_{r=1}^{Q} \sum_{l=0}^{L_{ij}} F_{ijl}\, x_{ijlr}, \tag{4.17}$$

which is equivalent to

$$\ln(F_i) = \sum_{r=1}^{Q} \sum_{j=1}^{J_i} \sum_{l=0}^{L_{ij}} \ln(F_{ijl}) x_{ijlr}, \tag{4.18}$$

where $F_{ijl} = 1 - R_{ijl}$ is the unreliability of component E_{ij} after undergoing maintenance level l during the planned maintenance break.

Let us define the functions $g, h : (0,1) \mapsto \mathbb{R}_-$, where $g(F_i) = \ln(1 - F_i)$ and $h(F_i) = \ln(F_i)$. To approximate these non-linear functions, a set of N breakpoints $\widehat{F}_{in}$ $(n = 1, \ldots, N)$ and SOS2 variables $\psi_i \in [0,1]^N$ are employed for their corresponding piecewise-linear functions $\hat{g}(.)$ and $\hat{h}(.)$. The piecewise-linearly approximated problem is written as follows: $[\mathcal{SMP}]_{PLA}$:

$$\max_{x_{ijlr}, \psi_{in}} \ln(R) = \sum_{i=1}^{I} \sum_{n=1}^{N} \ln(1 - \widehat{F}_{in}) \psi_{in} \tag{4.19a}$$

s.t.

$$\sum_{n=1}^{N} \ln(\widehat{F}_{in}) \psi_{in} \geq \sum_{j=1}^{J_i} \sum_{l=0}^{L_{ij}} \sum_{r=1}^{Q} \ln(F_{ijl}) x_{ijlr} \quad \forall i \in \mathcal{I} \tag{4.19b}$$

$$\sum_{n=1}^{N} \psi_{in} = 1 \qquad \forall i \in \mathcal{I} \tag{4.19c}$$

$$\psi_{in} \geq 0, \text{SOS2} \qquad \forall i \in \mathcal{I}, \forall n \in \mathcal{N} \tag{4.19d}$$

$$(4.13b) - (4.13d).$$

The objective function in Equation (4.19a) maximizes the piecewise-linearized system reliability. The constraints in Equation (4.19b) establish the linear segments that approximate the unreliability function. Equation (4.19c) ensures that the assigned weights of the breakpoints sum up to 1, while constraints (4.19d) set the SOS2 variables that define the breakpoints of the linear approximations.

4.3 Distributionally-robust chance-constrained programming model

Variability and uncertainty of the maintenance duration are inevitable due to unpredictable operating conditions, components' conditions that remain largely unknown until maintenance has started, varying repair persons' skill sets, and human errors. This raises the question of what duration values (t^c_{ijlr}, t^p_{ijlr}) to use in constraint (4.13c). If the mean duration values (or another measure of central tendency like the median) are used, the constraint would be violated, resulting in incomplete or unperformed maintenance actions, in many cases. On the other hand, if the constraint is to be satisfied for all possible realizations of the uncertain duration, a very conservative maintenance plan would be obtained with an extreme case being no maintenance performed at all. A prevalent approach for handling such parameter uncertainty is the use of probabilistic/chance constraints (Charnes and Cooper, 1959). Hence, the deterministic constraint (4.13d) would be replaced with the following chance constraint.

$$\mathbb{P}_G\left[\sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{l=1}^{L_{ij}}\tilde{t}_{ijlr}x_{ijlr} \le D_0\right] \ge 1-\epsilon_r \quad \forall r \in \mathcal{Q}, \tag{4.20}$$

which stipulates that the original constraint is satisfied with a probability of at least $1-\epsilon_r$, where $\epsilon_r \in (0,1)$ represents a *risk tolerance* parameter that prescribes the maximum acceptable violation probability of the break duration constraint for repairperson r. Note that the random variable $\tilde{t}_{ijlr}$ denotes the maintenance duration of maintenance level l performed by repairperson r on component E_{ij} and follows a known distribution G. Also, it is worth noting that, since the status (working or failed) for all components is assumed to be known at the beginning of the maintenance break, the parameter X_{ij} appears in (4.13c) and (4.7) was reduced in constraint (4.20) for simplicity and compactness.

In the context of SMP, using chance constraints for handling duration uncertainty was first proposed in (Ali et al., 2011a). Although it enables the planner to control the trade-off between feasibility and solution quality by adjusting ϵ_r, the chance constraint requires the probability distribution of the maintenance time (G) to be known. In the absence of sufficient historical data, especially for systems that have not been operational for a long time or that rarely fail, it is usually difficult to deduce the “true" data-generating distribution. Instead, one can construct a set $\mathcal{P}$ that contains the true distribution with high probability, referred to herein as the *distributional ambiguity set* (DAS), and require that the chance constraint holds for all distributions in $\mathcal{P}$ by replacing the chance constraint with a DRCC as follows:

$$\min_{P\in\mathcal{P}}\mathbb{P}_P\left[\sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{l=1}^{L_{ij}}\tilde{t}_{ijlr}x_{ijlr} \le D_0\right] \ge 1-\epsilon_r \quad \forall r \in \mathcal{Q}. \tag{4.21}$$

It is well-known that CCP problems are NP-hard, even in the simplest setting (Luedtke et al., 2010, theorem 1). Since the chance constraint (4.20) is a special case of the DRCC (4.21) (with a unitary ambiguity set), the latter is also intractable. However, tractable approximations exist in the literature for DRCCs (Chen et al., 2018b; Ji and Lejeune, 2021) and they depend on the structure of the DAS. In what follows, an illustrative example for the use of a DRCC in SMP is first presented in subsection 4.3.1. Then, the structure of the DAS used is explained in subsection 4.3.2. The Conditional Value-at-Risk (CVaR) approximation used for the proposed model is presented in subsection 4.3.3. Finally, in subsection 4.3.4, the calibration of the DAS is investigated through cross-validation (out-of-sample testing).

4.3.1 Illustrative example

To motivate the use of a DRCC to deal with the distributional ambiguity of $\tilde{t}$, let us consider a single-component system. At the beginning of the break, three maintenance levels l are available: Do Nothing ($l = 0$), Minimal repair ($l = 1$) and Replacement ($l = 2$). These options correspond to reliability levels R_0, R_1 and R_2, respectively, at the end of the next mission, where $R_2 > R_1 > R_0$. The planner wants to select a maintenance action that can be completed within a 1-hour break with 90% probability (*i.e.*, $\alpha = 0.1$ in the chance constraint). For simplicity, let us assume that the maintenance times are normally distributed and known to the planner. A replacement has a mean duration of 50 minutes and a standard deviation of 10 minutes, whereas a minimal repair has a mean duration of 30 minutes and a standard deviation of 5 minutes. However, the probability distribution parameters are unknown to the planner, who has access only to a limited sample of 3 historical realizations for each repair level. For the replacement, these realizations are 35, 45 and 55 minutes, and they are 25, 30 and 35 minutes for the minimal repair. Based on the true distribution of replacement times, option $l = 2$ is an infeasible option since it can be completed within the one-hour break time with only 84.13% probability, which is less than the required confidence level. Hence, the best feasible option is a minimal repair, leading to reliability R_1. However, the planner will base its decision on the available sample data. With sample average and standard deviation of 45 and 10 minutes, respectively, a probability of 93.32% for completing the replacement within the break time is estimated. Hence, due to sampling error, the planner would mistakenly select an infeasible option (*i.e.*, Replacement) rather than the best feasible one (*i.e.*, Minimal repair). When implementing the selected action, the planner will have two alternatives: either to violate the chance constraint (rendering the plan infeasible), or to abandon the maintenance action altogether. It should be noted that abandoning the replacement is worse than doing-nothing because valuable time and resources would have been wasted for no reliability improvement. In some cases, damage can be caused to the system in the haste of closing back the system. Often, the decision-maker would opt for costly remediations such as overtime work and/or delaying mission start.

Using a DRCC with a suitably sized DAS will prevent the selection of the infeasible replacement option. For example, if the ambiguity set admits all normal distributions which have means that are within 5 units of the sample average, the true distribution will be encompassed. In other words, the planner can "hedge" against the distributional ambiguity of the uncertain maintenance duration rather than over-relying on small data samples that often lead to out-of-sample performance disappointments.

4.3.2 Structure of the distributional ambiguity set

To construct the DAS for each repairperson, we assume that the planner has access to a finite (and usually small) number K_r of independent training samples for each repairperson (*e.g.*, historical observations or expert opinions $\hat{t}^k_{ijlr}$ with $k \in \mathcal{K}_r := \{1, \ldots, K_r\}$). These samples can be used to construct the empirical distribution $\widehat{P}^K_r$ for the (known) training samples. Furthermore, we require that all distributions in $\mathcal{P}_r$ are supported on the bounded polyhedral set $\Xi_r = \left\{\tilde{t}_r \in \mathbb{R}^{I\times J\times L} : H\tilde{t}_r \le h\right\}$, where $h \in \mathbb{R}^{O\times 1}$ and $H \in \mathbb{R}^{O\times(I\times J\times L)}$. For example, Ξ_r can be the Cartesian product of the closed interval sets $\left[t^+_{ijlr}, t^-_{ijlr}\right]$, where the lower bound t^-_{ijlr} and upper bound t^+_{ijlr} can be taken as multiples of the nominal duration values or extracted from historical observations. In this case, $h = \begin{bmatrix} t^+_r & -t^-_r \end{bmatrix}^\top$ and $H = \begin{bmatrix} \mathbb{I} & -\mathbb{I} \end{bmatrix}^\top$, where $\mathbb{I}$ is the identity matrix, $t^+(t^-) \in \mathbb{R}^{I\times J\times L}$ is a vector that represents the upper (lower) bound of the duration required to implement maintenance level l on component E_{ij} by repairperson r. For each repairperson, we use the following type-1 Wasserstein DAS:

$$\mathcal{P}^{\mathbb{W}}_r := \left\{P_r \in \mathcal{M}_r(\Xi_\mathrm{r}) : \mathbb{W}\left(P_r, \widehat{P}^K_r\right) \le \rho_r\right\} \quad \forall r \in \mathcal{Q}, \tag{4.22}$$

that contains all Ξ_r-supported distributions that are "close" to (*i.e.*, within radius ρ_r from) the empirical distribution. In this definition, $\mathcal{M}_r(\Xi_r)$ is the set of all distributions P_r supported on Ξ_r with $\mathbb{E}^P_r\left[\left\|\tilde{t}_r\right\|\right] = \int_{\Xi_\mathrm{r}} \left\|\tilde{t}_r\right\| P_r(d\tilde{t}) < \infty$, and $\mathbb{W}\left(P_r, \widehat{P}^K_r\right)$ is the Wasserstein distance between the probability distributions P_r and $\widehat{P}^K_r$. Accordingly, the Wasserstein metric (Kantorovich and Rubinshtein, 1958) is defined as:

$$\mathbb{W}(P_{r1}, P_{r2}) := \inf\left\{\int_{\Xi^2} \left\|\tilde{t}_{r1} - \tilde{t}_{r2}\right\| \Pi(d\tilde{t}_{r1}, d\tilde{t}_{r2})\right\} \quad \forall r \in \mathcal{Q}, \tag{4.23}$$

where Wasserstein distance between two probability distributions P_{r1} and P_{r2} within the space of measures $\mathcal{M}_r$ is represented by the joint distribution Π of $\tilde{t}_{r1}$ and $\tilde{t}_{r2}$. This distance represents the minimum cost of an optimal mass transportation plan, and the magnitude of the difference between $\tilde{t}_{r1}$ and $\tilde{t}_{r2}$, represented by $\|\tilde{t}_{r1} - \tilde{t}_{r2}\|$, encodes the transportation costs.

The aim is to carefully choose a radius ρ_r such that the ambiguity set $\mathcal{P}_r^{\mathbb{W}}$ contains the unknown true distribution with high confidence. Besides enabling sample data to be directly incorporated in the optimization problem, modelling the ambiguity set as a Wasserstein ball offers many practical benefits for stakeholders, *e.g.*, offering rigorous finite-sample and asymptotic consistency guarantees, and under specific regularity conditions, it affords computational tractability (Mohajerin Esfahani and Kuhn, 2018).

4.3.3 Tractable approximation

The DRCC (4.21) is known to be hard even in the special case when the DAS is unitary (in which case it reduces to a classical chance constraint). Hence, we resort to the tractable approximation proposed in (Ordoudis et al., 2021, proposition 1), which is done in two steps: firstly, by conservatively approximating the chance/Value-at-Risk (VaR) constraint using a Conditional Value-at-Risk (CVaR) constraint as shown in (Nemirovski and Shapiro, 2007), then by tractably reformulating the worst-case CVaR constraints using standard duality techniques (See the proof of Proposition 1 in Ordoudis et al. (2021) for more details). Thus, (4.21) can be approximated as follows:

$$\lambda_r \rho_r + \frac{1}{K_r} \sum_{k=1}^{K_r} s_r^k \leq 0 \qquad \forall r \in \mathcal{Q} \tag{4.24a}$$

$$\tau_r \leq s_r^k \qquad \forall r \in \mathcal{Q}, \forall k \in \mathcal{K}_r \tag{4.24b}$$

$$\|\epsilon_r(\gamma_{ijlr}^k - \sigma_{ijlr}^k) - x_{ijlr}\|_1 \leq \epsilon_r \lambda_r \qquad \forall (i,j,l,r) \in (\mathcal{I}, \mathcal{J}_i, \mathcal{L}_{ij}, \mathcal{Q}) \tag{4.24c}$$

$$\begin{aligned}&\sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{l=0}^{L_{ij}} \hat{t}_{ijlr}^k\, x_{ijlr} - D_0 + \sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{l=0}^{L_{ij}} (t_{ijlr}^{+} - \hat{t}_{ijlr}^k)\gamma_{ijlr}^k \\ &+ \sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{l=0}^{L_{ij}} (t_{ijlr}^{-} - \hat{t}_{ijlr}^k)\sigma_{ijlr}^k + (\epsilon_r - 1)\tau_r \leq \epsilon_r s_r^k \qquad \forall r \in \mathcal{Q}, \forall k \in \mathcal{K}_r\end{aligned} \tag{4.24d}$$

$$\gamma_{ijlr}^k, \sigma_{ijlr}^k \geq 0 \qquad \forall (i,j,l,r) \in (\mathcal{I}, \mathcal{J}_i, \mathcal{L}_{ij}, \mathcal{Q}). \tag{4.24e}$$

By combining the approximated DRCC (4.24) into the piecewise-linearly approximated formulation (4.19), the DRCC JSM–RAP can be written as follows:

$$\max_{x_{ijlr},\psi_{in},\gamma^k_{ijlr},\sigma^k_{ijlr},s^k_r,\lambda_r,\tau_r} \sum_{i=1}^{I}\sum_{n=1}^{N} \ln(1-\widehat{F}_{in})\psi_{in} \tag{4.25a}$$

s.t.:

$$\sum_{n=1}^{N} \ln(\widehat{F}_{in})\psi_{in} \geq \sum_{j=1}^{J_i}\sum_{l=0}^{L_{ij}}\sum_{r=1}^{Q} \ln(F_{ijl})x_{ijlr} \qquad \forall i \in \mathcal{I} \tag{4.25b}$$

$$C \leq C_0 \tag{4.25c}$$

$$\lambda_r\rho + \frac{1}{K_r}\sum_{k=1}^{K_r} s^k_r \leq 0 \qquad \forall i \in \mathcal{I} \tag{4.25d}$$

$$\tau_r \leq s^k_r \qquad \forall r \in \mathcal{Q}, \forall k \in \mathcal{K}_r \tag{4.25e}$$

$$\|\epsilon_r(\gamma^k_{ijlr} - \sigma^k_{ijlr}) - x_{ijlr}\|_1 \leq \epsilon_r\lambda_r \qquad \forall (i,j,l,r) \in (\mathcal{I},\mathcal{J}_i,\mathcal{L}_{ij},\mathcal{Q}) \tag{4.25f}$$

$$\begin{aligned} &\sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{l=0}^{L_{ij}} \Big[\hat{t}^k_{ijlr}\, x_{ijlr} + (t^+_{ijlr} - \hat{t}^k_{ijlr})\gamma^k_{ijlr}\Big] - D_0 \\ &+ \sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{l=0}^{L_{ij}} (t^-_{ijlr} - \hat{t}^k_{ijlr})\sigma^k_{ijlr} + (\epsilon_r - 1)\tau_r \leq \epsilon_r s_{kr} \qquad \forall r \in \mathcal{Q}, \forall k \in \mathcal{K}_r \end{aligned} \tag{4.25g}$$

$$\sum_{n=1}^{N} \psi_{in} = 1 \qquad \forall i \in \mathcal{I} \tag{4.25h}$$

$$\sum_{l=0}^{L_{ij}}\sum_{r=1}^{Q} x_{ijlr} = 1 \qquad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}_i \tag{4.25i}$$

$$\psi_{in} \geq 0, \mathrm{SOS2} \qquad \forall i \in \mathcal{I}, \forall n \in \mathcal{N} \tag{4.25j}$$

$$x_{ijlr} \in \{0,1\} \qquad \forall (i,j,l,r) \in (\mathcal{I},\mathcal{J}_i,\mathcal{L}_{ij},\mathcal{Q}) \tag{4.25k}$$

$$\gamma^k_{ijlr}, \sigma^k_{ijlr} \geq 0 \qquad \forall (i,j,l,r) \in (\mathcal{I},\mathcal{J}_i,\mathcal{L}_{ij},\mathcal{Q}). \tag{4.25l}$$

The resulting approximation is a MILP that has the same number of binary variables as the nominal (deterministic) problem. It can be readily handled using off-the-shelf solvers.

4.3.4 Calibrating the ambiguity set through cross-validation

The empirical distribution for each repairperson $\widehat{P}_r^K$ converges in Wasserstein metric to the unknown true distribution as K_r tends to infinity. Thus, for any given $\varepsilon_r \in (0,1)$ there is a sequence of $\rho_r^K(\varepsilon_r) \geq 0$ converging toward no violation of the chance constraint such that the Wasserstein ball of radius $\rho_r^K(\varepsilon_r)$ around $\widehat{P}_r^K$ contains the unknown true distribution with confidence $1-\varepsilon_r$ for every K_r (Xie, 2021). For implementation with a given field data size, the best Wasserstein ball radius is determined via cross-validation (Mohajerin Esfahani and Kuhn, 2018). In what follows, the cross-validation method used is explained.

For a given value of ρ_r, the out-of-sample performance (*i.e.*, reliability) of the optimal maintenance plan $\mathrm{x}^*_{train}(\rho_r)$ obtained by solving the JSM–RAP with DRCC for uncertain maintenance duration (DRSMP) in (4.25) with the training set of maintenance durations is estimated by applying the said-solution on a large number S_r of testing samples drawn at random from the same distribution of the training samples.

- If the chance constraint is satisfied for $(1-\varepsilon_r)S_r$ or more testing samples, then the maintenance plan $\mathrm{x}^*_{train}(\rho_r)$ is feasible and its reliability value is the out-of-sample reliability obtained from the DRSMP.

- If the chance constraint is not satisfied for $\varepsilon_r S_r$ or more testing samples, then the obtained maintenance plan is not feasible for the testing samples. In this case, the best subset of maintenance actions from the DRSMP solution that satisfies the chance constraint for at least $(1-\varepsilon_r)S_r$ testing samples must be identified by solving the following multidimensional multiple-choice knapsack problem (MdMCKP). The obtained objective function value is the out-of-sample reliability.

$\mathcal{U}_i$ is the powerset of patterns (*i.e.*, combinations of component and maintenance level) selected for subsystem i in the DRSMP solution under evaluation ($\mathcal{U}_i = 1,\ldots,U_i$). For each element u of the powerset, T_{riu} and R_{iu} are the corresponding maintenance work time by repairperson r and the subsystem reliability, respectively.
The following decision variables λ_{iu} and z_{rs} are used to select the best overall maintenance plan that maximizes system reliability and satisfies the chance constraint for the

required number of testing samples:

$$\lambda_{iu} = \begin{cases} 1, & \text{maintenance pattern } u \text{ is selected for subsystem } i \\ 0, & \text{otherwise,} \end{cases}$$

and

$$z_{rs} = \begin{cases} 1, & \text{if the duration of the maintenance work carried out by repairperson } r \\ & \text{using testing sample } s \text{ exceeds the break duration} \\ 0, & \text{otherwise.} \end{cases}$$

The out-of-sample reliability is determined by the subset of maintenance tasks selected through the following MdMCKP:

$$\max \sum_{i=1}^{I} \sum_{u=1}^{U_i} R_{iu} \lambda_{iu} \tag{4.26a}$$

s.t.:

$$\sum_{u=1}^{U_i} \lambda_{iu} = 1 \qquad \forall i \in \mathcal{I} \tag{4.26b}$$

$$\sum_{i=1}^{I} \sum_{u=1}^{U_i} T_{riu} \lambda_{iu} \leq D_0 + M z_{rs} \qquad \forall r \in \mathcal{Q}, \forall s \in \mathcal{S}_r \tag{4.26c}$$

$$\sum_{s=1}^{S_r} z_{rs} \leq \varepsilon_r S_r \qquad \forall r \in \mathcal{Q} \tag{4.26d}$$

$$\lambda_{iu} \in \{0, 1\} \qquad \forall i \in \mathcal{I}, \forall u \in \mathcal{U}_i \tag{4.26e}$$

$$z_{rs} \in \{0, 1\} \qquad \forall r \in \mathcal{Q}, \forall s \in \mathcal{S}_r. \tag{4.26f}$$

The maximization of the reliability for the selected subsets of maintenance patterns from the powerset of the DRSMP solution that does not satisfy the chance constraint for the required number of testing samples is achieved by (4.26a). Constraints (4.26b) requires the selection of exactly one maintenance pattern per subsystem. Constraints (4.26c) determine whether the selected maintenance pattern for repairperson r has a duration that exceeds the break duration or not. Note that M is a sufficiently large positive number. Constraints (4.26d) safeguards that the selected pattern doesn't violate the break length constraint for more than $\epsilon_r \times 100\%$ of the S_r testing samples. Constraints (4.26e) and (4.26f) define the binary decision variables. The complete cross-validation method described above is summarized as a pseudo-code in Algorithm 2.

Algorithm 2: Choosing the best range of the Wasserstein radius using cross-validation

Input data: ρ_r^{max}, ρ_r^{step}
Initialize: $\rho_r = 0$, $\mathcal{X} = \{\ \}$, $\mathcal{Y} = \{\ \}$
while $\rho_r \leq \rho_r^{max}$ **do**
  Solve DRSMP (4.25) for ρ_r optimally using K_r training samples.
  Store the optimal solution $\mathrm{x}^*_{train}(\rho_r)$ (the maintenance tasks) in $\mathcal{X}$ and the optimal value $R^*_{train}(\rho_r)$ (the in-sample reliability) in $\mathcal{Y}$.
  Compute the violation probability when the optimal solution $\varepsilon(\mathrm{x}^*_{train}(\rho_r))$ is used with S_r generated random testing samples.
  if $\varepsilon(\mathrm{x}^*_{train}(\rho_r)) \leq \epsilon_r$ **then**
    $\mathrm{x}^*(\rho_r) = \mathrm{x}^*_{train}(\rho_r)$, $R^*(\rho_r) = R^*_{train}(\rho_r)$.
  else
    Generate a powerset $\mathcal{U}_i$ of all combinations of maintenance patterns in the optimal solution $\mathrm{x}^*_{train}(\rho_r)$ for each subsystem.
    Compute the maintenance work duration T_{riu} and reliability R_{iu} corresponding to each pattern u.
    Solve the MdMCKP (4.26) that maximizes the reliability of the system while keeping the plan feasible by selecting a “subset” of the maintenance tasks.
    Store the optimal solution $\mathrm{x}^*_{test}(\rho_r)$ (the subset of the maintenance tasks) in $\mathcal{X}$ and the optimal value $R^*_{test}(\rho_r)$ (the out-of-sample reliability) in $\mathcal{Y}$.
    $\mathrm{x}^*(\rho_r) = \mathrm{x}^*_{test}(\rho_r)$, $R^*(\rho_r) = R^*_{test}(\rho_r)$.
  end if
  Store the values of $\rho_r, \mathrm{x}^*(\rho_r)$, $R^*_{test}(\rho_r)$.
  $\rho_r = \rho_r + \rho_r^{step}$.
end while
Choose the best range of the Wasserstein radius as the range that has the highest $R^*(\rho_r)$ from all iterations above ($\rho^* = \max(\mathcal{Y})$).

4.4 Numerical Experiments

The first part of this section presents the results of two sets of numerical experiments that evaluate the accuracy of the PLA-based approach (outlined in Section 4.2.6) in solving the nominal version of JSM-RAP. The second part assesses the performance of the DRCC formulation developed in Section 4.3 on an extended version of the 5-by-5 serial-parallel system from Diallo et al. (2019b).

For all numerical experiments carried out in this paper the components' lifetimes follow the Weibull distribution with scale and shape parameters of η_{ij} and β_{ij} respectively. Consequently, the conditional reliability $R^c_{ij}(V|A_{ijl})$ takes on the following form:

$$R^c_{ij}(V|_{A_{ijl}}) = \exp\left(\left(\frac{A_{ijl}}{\eta_{ij}}\right)^{\beta_{ij}} - \left(\frac{A_{ijl}+V}{\eta_{ij}}\right)^{\beta_{ij}}\right) x_{ijlr}. \tag{4.27}$$

The experiments conducted in this paper employ the Wasserstein metric induced by the L^1-norm, which implies that all resulting optimization problems are equivalent to MILP problems. The SMP-PLA experiments utilize $N = 600$ breakpoints, and all experiments are carried out on a Windows 11 laptop computer with an Intel(R) Core(TM) i7® processor operating at 1.30 GHz and equipped with 16 GB of RAM. The optimization problems are implemented in Python 3.9 and solved by Gurobi 9.1.1.

4.4.1 Computational results for the nominal problem

This subsection presents two series of numerical experiments to assess the validity of the proposed PLA-based model for solving the nominal JSM-RAP. The first set of experiments involves a comparison between the PLA-based formulation (4.19) developed in this paper, and the 2-phase approach (see Appendix 2.4.2) by Diallo et al. (2019b). This comparison relies on the findings attained for a moderately-sized serial-parallel system that has multiple repairpersons with IM levels. The second set of experiments illustrates the SMP-PLA formulation's capability to tackle large-scale JSM-RAP instances.

4.4.1.1 Experiments #1.1: equally-skilled repair persons

The series-parallel system of moderate size investigated in Diallo et al. (2018) is considered in this set of experiments. The system is made up of $s = 2$ subsystems in series, with the first subsystem comprised of n_1 independent and identically distributed (*i.i.d.*)

components E_{1j} ($j = 1,\ldots,5$), while the second subsystem has $n_2 = 8$ *i.i.d.* components E_{2j} ($j = 1,\ldots,8$) configured in parallel for a total of 13 components within the system. The shape and scale parameters of β_{ij} and η_{ij} ($i = 1,2; j = 1,\ldots,J_i$) are used to determine the Weibull distribution of the lifespan of the components. The parameter values used in this set of experiments are set to $\beta_{1j} = 1.5$ and $\eta_{1j} = 15$ ($j = 1,\ldots,5$), and $\beta_{2j} = 3$ and $\eta_{2j} = 20$ ($j = 1,\ldots,8$). A maintenance team of $Q = 2$ repairpersons with equal skill levels is available. The components in the system have four available maintenance levels: $l = 0$ (DN), $l = 1$ (MR), $l = 2$ (IM) which halves the component's age, and $l = 3$(PR). Table 4.2 provides further information about components' ages, status, maintenance costs and times. The system's break duration is set to $D_0 = 10$, and the next mission's duration is set to $V = 8$. To compare results, the JSM–RAP problem is solved by the exact 2-phase method from Diallo et al. (2019b) and the SMP–PLA approach.

Table 4.2: Parameters for Experiments #1.1, source: Diallo et al. (2018).

E_{ij}	X_{ij}	B_{ij}	t^c_{ij1}	t^c_{ij2}	t^c_{ij3}	t^p_{ij2}	t^p_{ij3}	c^c_{ij1}	c^c_{ij2}	c^c_{ij3}	c^p_{ij2}	c^p_{ij3}
E_{11}	0	15	4	6	8	2	4	5	10	14	8	10
E_{12}	1	12	4	6	8	2	4	5	10	14	8	10
E_{13}	0	10	4	6	8	2	4	5	10	14	8	10
E_{14}	1	18	4	6	8	2	4	5	10	14	8	10
E_{15}	1	20	4	6	8	2	4	5	10	14	8	10
E_{21}	0	8	3	4	5	1	2	6	10	20	7	12
E_{22}	1	15	3	4	5	1	2	6	10	20	7	12
E_{23}	0	8	3	4	5	1	2	6	10	20	7	12
E_{24}	1	15	3	4	5	1	2	6	10	20	7	12
E_{25}	0	8	3	4	5	1	2	6	10	20	7	12
E_{26}	1	15	3	4	5	1	2	6	10	20	7	12
E_{27}	0	8	3	4	5	1	2	6	10	20	7	12
E_{28}	1	15	3	4	5	1	2	6	10	20	7	12

Table 4.3 shows the maximum attainable asset reliability (R), total maintenance time (D), incurred cost (C), and CPU time (CPU_t) for different maintenance budget values C_0 when two repairpersons are available. Additionally, this table presents the relative gap between the 2-phase and SMP–PLA methods.

In terms of computational time, the SMP–PLA approach performs better than the 2-phase approach. The exact 2-phase method takes a total of 853.25 seconds on average to

Table 4.3: Outcomes of Experiments #1.1: case of two equally skilled repairpersons, and $D_0 = 10$.

C_0	2-phase approach				SMP–PLA				
	$R^*(\%)$	C^*	m^*	$CPU_t(s)$	$R(\%)$	C	m	$Gap(\%)$	$CPU_t(s)$
54	97.97	54	2	857.68	97.97	54	2	0.0	0.02
50	97.22	49	2	853.76	97.22	49	2	0.0	0.09
40	95.90	40	2	848.54	95.90	40	2	0.0	0.08
30	92.85	30	2	848.76	92.85	30	2	0.0	0.08
20	89.50	20	2	855.23	86.50	20	2	0.0	0.03
10	74.97	10	2	853.89	74.97	10	2	0.0	0.03

produce 798,856 patterns and find the optimal solution. On the other hand, SMP–PLA is much faster than the exact 2-phase approach, with an average time of under 0.1 seconds to compute. Moreover, the computation times for both techniques stay relatively stable as the maintenance budget increases (a range of approximately 10 seconds for the 2-phase approach and 0.07 second for SMP–PLA).

In terms of solution quality, it is observed that there is virtually no discrepancy between the exact 2-phase and PLA-based methods, with a 0.0% gap observed in all experiments, indicating that the solutions obtained by the PLA-based approach are optimal. Although the proposed PLA-based approach is a relaxation that provides an upper bound, and despite its tightness, it cannot guarantee optimality for the obtained solutions. Nevertheless, in all numerical experiments conducted, the optimal solution was achieved.

4.4.1.2 Experiments #1.2: large-scale series parallel systems

The aim of this series of experiments is to prove the ability of our novel PLA-based method to handle large instances of the problem. The system considered is the same one investigated by Ikonen et al. (2020). It is a large series-parallel system with 100 components spread over $I = 32$ subsystems connected in series as depicted in Figure 4.3. The total number of components in this basic system is given by NC, where $NC = \sum_{i=1}^{I} J_i = 100$. The parameter values of the system are summarized in Table 4.4. The maintenance options available for working components are PR and DN. For failed components, the options are DN and MR. Table 4.5 lists the maintenance times and costs for the components.

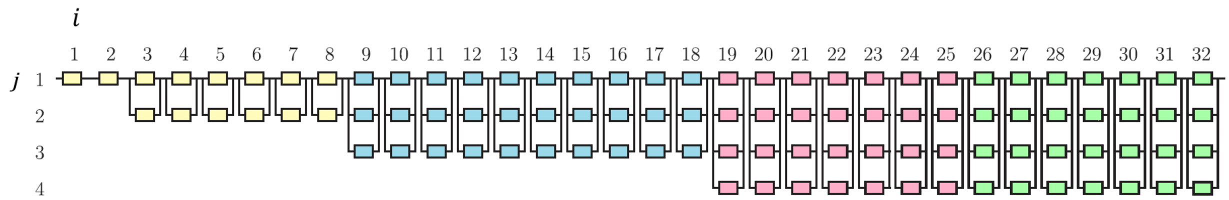


Figure 4.3: The system's reliability structure, based on the basic 100-component configuration presented in Ikonen et al. (2020).

Table 4.4: Component Lifetimes and Status Parameter Values: case of Experiments #1.2.

E_{ij}		η_{ij}	β_{ij}	X_{ij}	B_{ij}
From	To				
$E_{1,1}$	$E_{2,1}$	50	3.0	[0]	[8]
$E_{3,j}$	$E_{8,j}$	50	3.0	[0,1]	[8,15]
$E_{9,j}$	$E_{18,j}$	35	1.5	[0,1,0]	[15,12,10]
$E_{19,j}$	$E_{25,j}$	50	3.0	[0,1,0,1]	[8,15,8,15]
$E_{26,j}$	$E_{32,j}$	25	2.1	[0,1,0,1]	[6,10,6,10]

Table 4.5: Maintenance Data for Experiments Set #1.2.

E_{ij}		t^c_{ij1}	t^c_{ij2}	t^c_{ij3}	t^p_{ij2}	t^p_{ij3}	c^c_{ij1}	c^c_{ij2}	c^c_{ij3}	c^p_{ij2}	c^p_{ij3}
From	To										
$E_{1,1}$	$E_{2,1}$	4	7	8	3	5	12	24	30	15	21
$E_{3,j}$	$E_{8,j}$	4	7	8	3	5	12	24	30	15	21
$E_{9,j}$	$E_{18,j}$	2	5	6	2	3	15	30	42	24	30
$E_{19,j}$	$E_{25,j}$	3	4	5	1	2	18	30	60	21	36
$E_{26,j}$	$E_{32,j}$	2	2.5	4	2	3	12	24	30	15	21

Table 4.6: Outcomes of Experiments #1.2: large-scale instances with repairpersons and $D_0 = 50$.

n	SMP–PLA			
	$R(\%)$	C	m	$CPUt(s)$
100	94.51	492	3	1.09
200	89.50	996	6	16.01
300	84.75	1,500	6	67.14
400	80.10	1,992	8	165.48
500	75.85	2,496	13	22.72
600	71.83	3,000	15	28.02
700	67.89	3,492	16	208.33

The modular (basic) system of 100 components illustrated in Figure 4.3 was duplicated to create seven distinct problem instances. For instance o (with $1 \leq o \leq 10$), the system is expanded by repeating the 100-component system o times, resulting in a total of $NC_o = 100 \times o$ components. The system in Instance 7 comprises 7 copies of the basic system, resulting in $NC_7 = 700$ components. The duration of the break is set to $D_0 = 50$ for all problem instances. The maintenance budget and the number of repairpersons for any instance o are set to $C_0 = 500 \times o$ and $Q = 5 \times o$, respectively. For illustration, in the third instance ($o = 3$), the system has $NC_3 = 300$ components and the maintenance actions will be carried out by $Q = 15$ repairpersons with a maintenance budget of $C_0 = 1500$.

As previously demonstrated in Table 4.3, the PLA-based approach offers a substantial reduction in computation time for the SMP. With this reduction in time, the JSM–RAP can be solved for systems consisting of up to 700 components in 208.3 seconds or less as shown in Table 4.6. As anticipated, the reliability decreases as the number of series subsystems increases. Similarly, the total cost and the number of repairpersons needed increase when the number of components is increased.

4.4.2 Computational results for the DRSMP

In this subsection, two series of experiments are performed to examine the effectiveness of the DRCC formulation presented in Section 4.3. The benefits of distributional robustness in the DRSMP are demonstrated in the first batch of experiments. The second series of experiments examines the impacts of varying risk tolerances ε_r used to define the DRCC on the solution structure.

4.4.2.1 Experiments # 2.1: Benefit of considering distributional robustness

This section investigates the benefits of considering uncertainty and distributional ambiguity by comparing the developed DRSMP formulation described in Section 4.3 ($\hat{\rho}$ >0) with the nominal model (NM) and the sample-average-approximation (SAA) model (obtained by setting $\hat{\rho} = 0$ in the DRSMP). All three formulations are compared through an out-of-sample test on an extended version of the 5-by-5 serial-parallel system from (Diallo et al., 2019b). The NM uses only the nominal maintenance durations with no uncertainty, set equal to the average maintenance duration according to the available maintenance records (average of the training samples), while the SAA ($\hat{\rho}$ =0) replaces the unknown P_r with the discrete empirical distribution $\widehat{P_r^K}$, $r \in \mathcal{Q}$, that is, the uniform distribution on the known training samples (Mohajerin Esfahani and Kuhn, 2018).

The original system's reliability block diagram consists of $s = 2$ subsystems in series, each comprising five *i.i.d.* components E_{ij} in parallel ($i = 1,2; j = 1,\ldots,5$). The scale and shape parameters of η_{ij} and β_{ij} ($i = 1,2; j = 1,\ldots,J_i$) are used to determine the Weibull distribution of the lifespan of the components. The parameter values used in this set of experiments are set to $\beta_{1j} = 1.5$ and $\eta_{1j} = 15$ ($j = 1,\ldots,5$), and $\beta_{2j} = 3$ and $\eta_{2j} = 20$ ($j = 1,\ldots,5$). A maintenance team of $Q = 2$ repairpersons with equal skill levels is available. The components in the system can undergo four types of maintenance: $l = 0$ (DN), $l = 1$ (MR), $l = 2$ (IM) *i.e.*, component age is halved, and $l = 3$ (PR). Additional information regarding components' ages, statuses, maintenance costs, and times is presented in Table 4.7. The lengths of the scheduled break and the subsequent operational period are fixed at $D_0 = 10$ and $V = 8$ respectively.

The uncertain duration of maintenance action $\tilde{t}_{ijlr}$ is modelled as follows

$$\max\left\{1;\ t_{ijlr}(1-\delta)\right\} \leq \tilde{t}_{ijlr} \leq t_{ijlr}(1+\delta), \tag{4.28}$$

where δ is an adjustable variability factor and $0 \leq \delta \leq 1$.

For assets or systems that failed rarely or that have not operated long enough, acceptable reliability data is usually scant while getting supplementary data points is impossible or costly. Accordingly, this experiment relies on $K_r = 3$ training samples for each repairperson and $S_r = 2000$ testing samples for each repairperson $r \in \mathcal{Q} = \{1, 2\}$. A Wasserstein ball is constructed in the space of multivariate and non-discrete probability

Table 4.7: Parameters for Experiment #2, source: Diallo et al. (2019b).

E_{ij}	X_{ij}	B_{ij}	t^c_{ij1}	t^c_{ij2}	t^c_{ij3}	t^p_{ij2}	t^p_{ij3}	c^c_{ij1}	c^c_{ij2}	c^c_{ij3}	c^p_{ij2}	c^p_{ij3}
E_{11}	0	15	2	5	6	2	3	5	10	14	8	10
E_{12}	1	12	2	5	6	2	3	5	10	14	8	10
E_{13}	0	10	2	5	6	2	3	5	10	14	8	10
E_{14}	1	18	2	5	6	2	3	5	10	14	8	10
E_{15}	1	20	2	5	6	2	3	5	10	14	8	10
E_{21}	0	8	4	7	8	3	5	4	8	10	5	7
E_{22}	1	15	4	7	8	3	5	4	8	10	5	7
E_{23}	0	8	4	7	8	3	5	4	8	10	5	7
E_{24}	1	15	4	7	8	3	5	4	8	10	5	7
E_{25}	0	8	4	7	8	3	5	4	8	10	5	7

distributions that is centered at the uniform distribution $U(t^-_{ijlr}, t^+_{ijlr})$ on the training samples. A Beta distribution ($\alpha = 2, \beta = 2$) is used to simulate the testing samples. The upper t^+_{ijlr} and lower t^-_{ijlr} bounds of the maintenance duration for each maintenance action are determined as shown in (4.28) where the variability factor is set $\delta = 0.3$. Finally, we set $\varepsilon_r = \varepsilon = 5\%$ $\forall r \in \mathcal{Q} = \{1,\ 2\}$. Similarly, $\rho_r = \rho$ for $r = 1, 2$.

Table 4.8: Solution profiles for the nominal (NM), sample average approximation (SAA) and distributionally-robust (DRSMP) models.

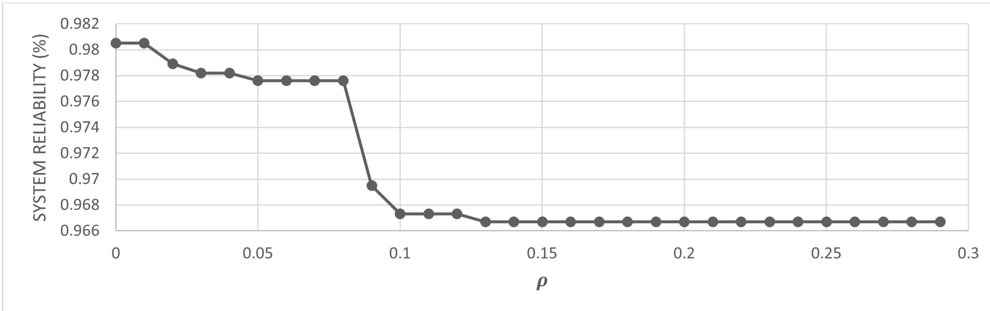
Models	In-sample (training) system reliability (%)	Out-of-sample compliance probability (%)	Out-of-sample (testing) system reliability (%)	Overtime (unit of time)
NM	98.55	24.10	89.94	0.99
SAA ($\hat{\rho} = 0$)	98.05	57.25	93.06	0.84
DRSMP ($\hat{\rho} > 0$)	97.76	99.45	97.76	0.16

We consider four indicators to evaluate the quality of obtained results: (i) the in-sample system reliability computed based on the training samples using the nominal formulation (4.19) for the NM model and the DRCC formulation (4.25) for SAA and DRSMP models, (ii) the out-of-sample system reliability computed based on the testing samples using the MdMCKP (4.26), (iii) the probability of violating the DRCC, which reports the infeasible sample size as a percentage of the $S_r = 2000$ testing samples, and (iv) the average slack or overtime values for the violated testing samples. Table 4.8 compares the results obtained. The same results are also depicted in Figures 4.4a – 4.4d.

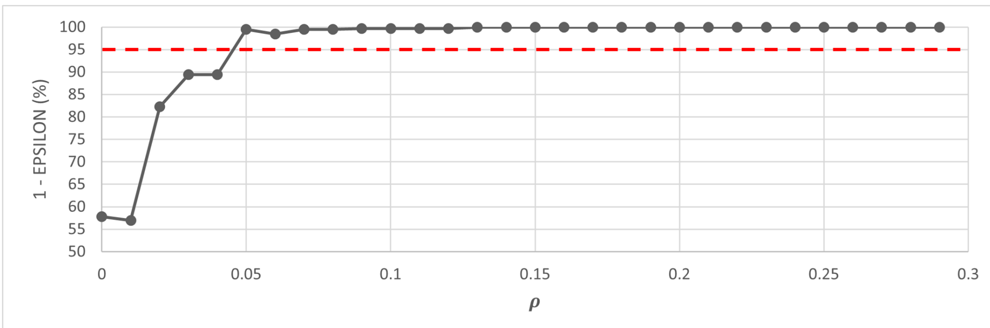
Table 4.8, which depicts the results obtained for the NM, SAA ($\hat{\rho} = 0$), and DRSMP ($\hat{\rho} > 0$) models, shows that DRSMP provides the most conservative maintenance plan ($R = 97.76\%$), followed by SAA ($R = 98.05\%$), which is expected since SAA maximizes the expected reliability based on the given training samples and focuses on the average case, while NM is a progressive approximation of SAA ($R = 98.55\%$). On the other hand, DRSMP aims to maximize the worst-case expected reliability over an ambiguity set under the DRCC. The results of the out-of-sample analysis indicate that DRSMP is superior to SAA and NM in regards to the out-of-sample compliance, out-of-sample reliability and overtime usage. The fourth column of Table 4.8 demonstrates that the out-of-sample reliability achieved from DRSMP has a larger out-of-sample reliability than SAA and NM. Indeed, the average out-of-sample reliability achieved by the DRSMP is higher than that reached by SAA and NM by 4.70% and 7.82% respectively. Also, the results for the simulation run in Figure 4.4c, illustrating the relationship between the out-of-sample system reliability and the Wasserstein radius ρ, reveal that the system reliability attains a distinct maximum within a critical Wasserstein radius range ($0.05 \leq \hat{\rho}^* \leq 0.08$). Thus, the decision-maker who overlooks ambiguity and sets $\rho = 0$ will achieve an out-of-sample reliability of 92.97%. Whereas, a more knowledgeable decision-maker who recognizes the presence of ambiguity sets $\rho = \hat{\rho}^*$ and attains a 98.05% reliability. It is also notable that the out-of-sample and in-sample reliability values are equal in the *critical range* of ρ_r, where the overtime is also the lowest.

Figure 4.4a illustrates the relationship between the DRSMP system reliability ((4.25)) and the Wasserstein radius $\rho \in \{0, 0.01, \ldots, 0.30\}$. With increasing ρ, the system reliability decreases, while the compliance $(1 - \varepsilon)$ increases as shown in Figure 4.4b. This is inline with the fact that bigger Wasserstein radii yield more conservative decisions that allow fewer maintenance actions to be implemented. For sufficiently large values of ρ, the proposed approach can guarantee that the chance constraint is satisfied (*i.e.*, the empirical violation probabilities are smaller than $\varepsilon = 5\%$).

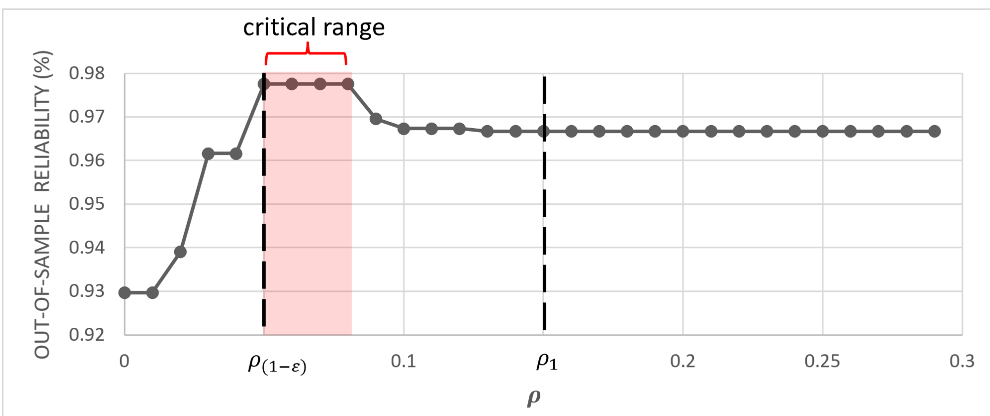
The dashed vertical line in Figure 4.4c at $\rho_{1-\varepsilon}$ shows the radius value where the violation probability falls below the predetermined maximum risk tolerance of ε (*i.e.*, radius



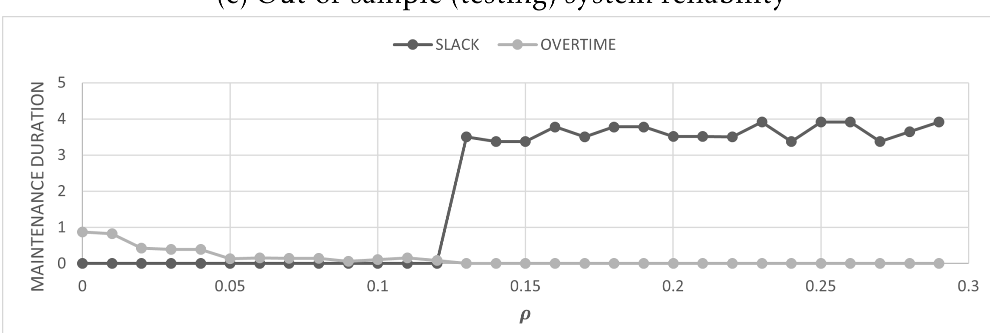
(a) In-sample (training) system reliability



(b) Out-of-sample compliance probability



(c) Out-of-sample (testing) system reliability



(d) Overtime and slack values for the violated testing samples

Figure 4.4: Choosing a Wasserstein radius using cross-validation.

threshold where the chance constraint is satisfied). At ρ_1, the chance constraint is 100% satisfied meaning that the selected maintenance plan with a radius value of $\rho \geq \rho_1$ will certainly be completed within the maintenance break.

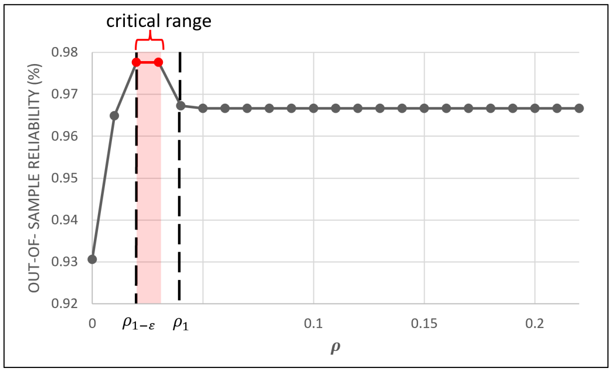
Figure 4.4d illustrates the relationship between the average overtime and slack values for the violated testing samples as the Wasserstein ball radius ρ varies. At the beginning, when $\rho = 0$, many maintenance activities are not completed which requires extending the maintenance break duration (overtime). As ρ increases, the average overtime decreases due to more conservative solutions that allow fewer maintenance actions to be implemented. When the satisfiability reaches 100%, the proposed approach can guarantee that the maintenance plan selected fits into the break duration. At this point, there is no overtime and there is even some slack. Note that for $\rho < \hat{\rho}^*$, the in-sample reliability is greater than the out-of-sample reliability (the optimizer over-promises and under-delivers), whereas for $\rho > \hat{\rho}^*$, the opposite happens (the optimizer under-promises and over-delivers), which can also be linked to the over and under usage of time.

4.4.2.2 Experiments #2.2: Analysis of the risk tolerance's impacts

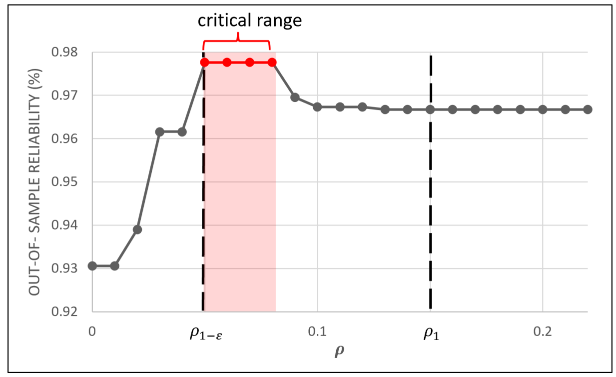
To investigate the impact of varying the risk tolerance ε on the out-of-sample system reliability, computed using the MdMCKP (4.26), the experiment presented in the previous section was repeated four times with different risk tolerance values $\varepsilon \in \{0.02, 0.05, 0.07, 0.10\}$ as depicted in Figure 4.5.

The critical range, denoted as $\Delta\hat{\rho}^*$, plays a crucial role in determining the out-of-sample system reliability. It encompasses the interval of Wasserstein radius ρ where the chance constraint is satisfied and the maximum reliability is achieved. The results presented in Figure 4.5 show that as the risk tolerance ε increases, the critical range expands. The values of $\Delta\hat{\rho}^*$ for risk tolerances of 2%, 5%, 7%, and 10% are respectively 0.02, 0.04, 0.06, and 0.08, as shown in subfigures 4.5(a) to 4.5(d). The ability of the chance constraint to accommodate a wider range of distributions contributes to the expansion of $\Delta\hat{\rho}^*$ as ε increases. Despite changes in ε, it is observed that the reliability values within the critical range remain constant.

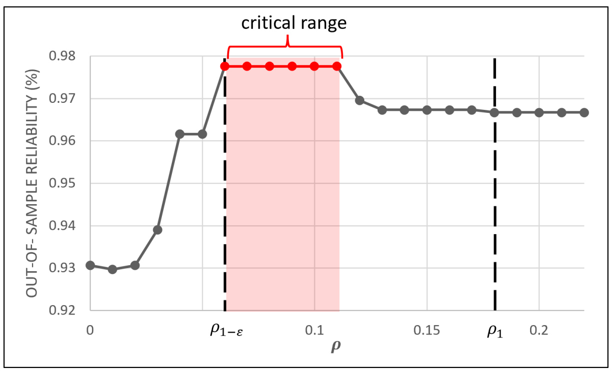
The subfigures 4.5(a) to 4.5(d) each display two dashed vertical lines at $\rho_{1-\varepsilon}$ and ρ_1 that denote the threshold of chance constraint satisfaction $(1 - \varepsilon)$ and the point where chance constraint satisfaction reaches 100%. Results of the analysis indicate that as the



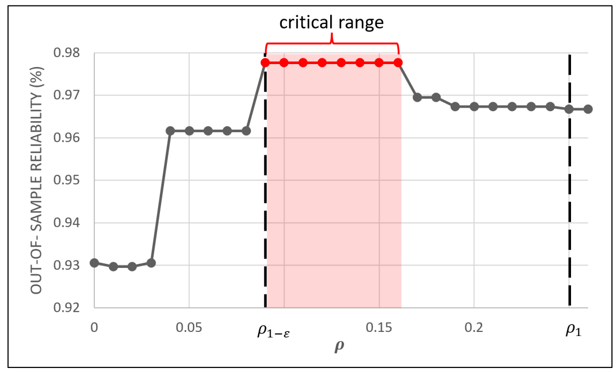
(a) ε=2%



(b) ε=5%



(c) ε=7%



(d) ε=10%

Figure 4.5: Out-of-sample (testing) system reliability for different risk tolerances ε.

risk tolerance ε increases, both $\Delta\hat{\rho}^*$ and the two dashed lines move to the right. This rightward movement implies that the algorithm is choosing higher values of the Wasserstein radius to accommodate the increased risk tolerance. A managerial insight from this experiment is that reduced risk tolerance decreases the width of the critical range where the optimal radius is located. Thus, radius calibration attempts should be careful to define an appropriate search step-size so as not to miss the critical range when the risk tolerance value ε is low.

4.5 Conclusions

This paper aims to solve the JSM–RAP with uncertain maintenance durations in a series-parallel system, with a particular focus on handling large-scale instances. A piecewise-linear approximation and a DRCC program with a Wasserstein-1 ambiguity set are proposed to deal with the problem. Three sets of numerical experiments demonstrated the ability of the proposed approaches to solve large instances of the JSM–RAP with uncertain maintenance duration. The proposed DRCC formulation ensures that effective maintenance plans with a high probability of completion and with superior out-of-sample performance of the system reliability and overtime requirement can be obtained. In addition, the computation times of the proposed PLA-based approach were significantly lower than that of 2-phase approach (Diallo et al., 2019b). The proposed method proved to be highly computationally efficient, as demonstrated by successfully solving a problem with the same number of components (*i.e.*, 700) as the previous largest JSM–RAP instance solved (Ikonen et al., 2020) with a computational time that is an order of magnitude smaller. Possible extensions of the proposed approach include deriving similar models for other variants of the SMP, *e.g.*, multimission and fleet-based SMPs. Also, future research should focus on the case in which the deterministic assumption is relaxed for other SMP parameters such as mission durations, maintenance break intervals, and quality of maintenance actions. Other ways for handling uncertainty in maintenance duration could be developed, *e.g.*, using other statistical metrics such as Phi-divergences or the Gelbrich distance to construct ambiguity sets that leverage the available information and reach more compact reformulations. Finally, besides the PLA approach developed in this article, it would be interesting to explore alternative exact or approximate reformulations of the JSM–RAP, *e.g.*, conic programming reformulations.

Chapter 5

Robust selective maintenance optimization under maintenance quality uncertainty

5.1 Introduction

In the course of their activities, many industrial and military systems, such as nuclear plants, naval vessels, aircraft, wind turbines, and power generation systems operate according to an alternating sequence of missions and scheduled breaks. During the time interval between two consecutive missions, maintenance actions can be carried out to improve the probability of successfully executing the next mission (*i.e.*, system reliability). Since there are limited resources for completing maintenance activities (*e.g.*, time, budget, spare parts, repair crews), it is often impossible to maintain all components of a system. Hence, it is necessary to select an optimal subset of components to maintain, then decide the level of maintenance actions to be performed, and assign repairpersons to the maintenance actions. This maintenance decision problem is known in the literature as the *joint selective maintenance and repairperson assignment problem* (JSM–RAP) (Khatab et al., 2018c), which is an extension of the classical *selective maintenance problem* (SMP) (Rice et al., 1998).

In general, SMPs are difficult to solve. The non-linearity of system reliability functions and the combinatorial nature of maintenance plans make finding optimal solutions for large-scale problems computationally cumbersome. Indeed, Rice (1999) proved that the basic SMP is NP-hard. The mathematical expression of the system reliability typically involves products of decision variables, which results in mixed-integer nonlinear programming (MINLP) formulations. The reader is referred to recent reviews of Xu et al. (2015); Cao et al. (2018a); Al-Jabouri et al. (2022) for a detailed account of SMP models and solution methods.

Given the important and challenging nature of the SMP, numerous solution approaches have been proposed: 1) general heuristics (Khatab et al., 2007; Lust et al., 2009; Cao et al., 2018b; Ahadi and Sullivan, 2019), 2) meta-heuristics such as genetic algorithms (Dao et al., 2014), differential evolution (Pandey et al., 2013b), and simulated annealing (Jiang and Liu, 2020b), and 3) exact solution approaches such as total enumeration (Rice et al., 1998), search space reduction (Rajagopalan and Cassady, 2006), depth-first search (Cao et al., 2016b), branch-and-bound (Lust et al., 2009), and sequential construction (Galante et al., 2020). On the other hand, canonical mathematical programming formulations that can be handled directly by efficient commercial solvers are still sparse (Cao et al., 2018a). A recent breakthrough in this direction is the two-phase approach proposed by Diallo et al. (2018), which generates maintenance "patterns" and selects an optimal subset by solving a multidimensional multiple-choice knapsack problem (MdMCKP). Nevertheless, this approach requires all feasible patterns to be generated at the outset, making it impractical for large systems. Thus, as noted by Diallo et al. (2019b), novel formulations that can be dispatched directly to efficient exact solvers are still needed.

Whereas the original SMP model by Rice et al. (1998) assumed only one type of maintenance action, namely *replacement* of failed component, later works considered multiple maintenance levels. Cassady et al. (2001a) added the option of *minimal repair*, which when performed returns a failed component to a working state without affecting its failure rate. Later, several imperfect maintenance (IM) models, in which the system can be repaired somewhere between *as good as new* and *as bad as old*, have been utilized. Khatab et al. (2008b) developed an IM model based on the age reduction concept (Malik, 1979). Pandey et al. (2013b) and Pandey and Zuo (2014) redefined the age reduction model so that the minimal repair cost does not influence the determination of the age reduction factor. In most of the IM models presented in the literature, it has been assumed that the post-maintenance reliability of a component is fully-determined by the maintenance level selected for it (*i.e.*, the maintenance quality is deterministic). This is not a realistic assumption since the maintenance quality is significantly affected by various factors beyond the control of the planner such as the qualification and the degree of expertise of the repairperson, the maintenance methods and tools used, operating conditions and other uncontrollable variability-inducing factors.

Among the few studies that considered the uncertainty in maintenance quality is the work of Khatab and Aghezzaf (2016b), which used a stochastic age reduction coefficient following a Beta distribution to describe the maintenance improvement. Lan et al. (2017)

considered the repair quality to be stochastic and following the Gamma distribution for a fleet-level series system. Zhao et al. (2018) utilized the cognitive reliability and error analysis method (CREAM) to calculate the human reliability for a multi-state series-parallel system. Chen et al. (2019) utilized a triangle membership function to balance the stochastic relationship between maintenance action and cost for a multi-state manufacturing system. The cost and time of maintenance actions were represented as fuzzy values due to the difference in qualification levels of repairpersons. Zhang et al. (2019a) proposed five criteria to evaluate the capability of maintenance teams using Choquet integral based on λ-fuzzy measure (Chen and Wang, 2001). The weight of each criterion was expressed as a fuzzy value obtained from experts or decision-makers. Shahraki et al. (2020) considered IM actions for a multi-state system as random variables. The probability of achieving the desired improved state is influenced by two factors: the history of maintenance actions performed earlier and the maintenance level. Li et al. (2021) considered the failure effects and maintenance quality uncertainty for a multi-state series system. A state transition matrix was used to describe the uncertainty of the maintenance quality.

In all of the aforementioned references, risk-neutral stochastic or fuzzy models were employed to represent maintenance quality uncertainty. Although these models lead to maintenance plans that perform well *on average*, they fail to protect the planner from adverse scenarios or to provide any performance guarantee, features that are crucial in the context of mission-based systems. For instance, a vessel that undergoes selective maintenance between missions might require a guarantee that its reliability is above a certain threshold in the presence of uncertainty about the maintenance quality. The problem is exacerbated when the probability/possibility distribution is extracted from a small set of past observations, a typical situation in the maintenance domain when the failure events are infrequent and/or the components have not worked long enough to extract sufficient maintenance records. It is often the case that the “true" post-maintenance reliability of a component is only known to belong to an interval, without any distributional knowledge. In such occasions, robust optimization (RO) provides a plausible alternative framework for hedging against uncertainty, in the sense that a guarantee on the *worst-case* system reliability can be provided. Despite the rising popularity of RO in the last two decades as an effective and tractable approach for handling uncertainty in different applications, to the best of our knowledge, it is yet to be applied to deal with maintenance quality uncertainty (or any other uncertain parameter) in the SMP. As mentioned earlier, a reason for this deficiency might be that tractable formulations of the nominal SMP do not exist yet.

In light of the research gaps highlighted above, the paper makes the following contributions.

1. We propose a novel reformulation of the JSM–RAP problem for the standard series-parallel system as a mixed-integer exponential conic program (MIECP) that can be handled directly using efficient off-the-shelf solvers. Numerical experiments conducted on benchmark JSM–RAP instances show the competitive computational performance of the proposed reformulation *vis-à-vis* state-of-the-art approaches.

2. We propose, for the first time, a RO approach to deal with the uncertain maintenance quality in the JSM–RAP. Post-maintenance reliability for the components, maintenance levels, and repairpersons are assumed to belong to non-symmetric budget uncertainty sets that enable the level of conservatism to be controlled. The robust problem is tractably reformulated as a MIECP and effectively solved. The numerical experiments conducted show that the solutions obtained for the robust problem with properly-sized uncertainty sets have better *out-of-sample* performance than their corresponding nominal problem solutions, proving the value of the proposed robust approach.

The remainder of the article is organized as follows. Section 5.2 describes the nominal version of the multicomponent system under consideration, the modelling assumptions, system reliability computations, the classical MINLP formulation of the JSM–RAP, and the MIECP reformulation. In Section 5.3, the proposed robust framework for handling maintenance quality uncertainty along with its reformulation into a MIECP are presented. Several numerical experiments and the discussion of the results are presented in Section 5.4. Conclusions are drawn and future extensions discussed in Section 5.5.

5.2 The Nominal Problem

In this section, the description and mathematical formulation of the JSM–RAP are provided under the assumption that the post-maintenance reliability (or, equivalently, the maintenance quality) values corresponding to each component, repairperson and maintenance level are known with certainty. This case is referred to hereinafter as the *nominal problem*. We first describe the multicomponent series-parallel system under consideration and our modelling notations and assumptions, then define the IM model and develop expressions for the total cost and duration of the maintenance plan. Finally, a MINLP formulation of the JSM–RAP is presented and explained along with its reformulation into a MIECP.

5.2.1 System description

As depicted in Figure 5.1, the system under consideration comprises m subsystems connected *in series* (*i.e.*, all subsystems must function for the system to work). Each subsystem i ($i = 1, \cdots, s$) consists of the J_i repairable binary (*i.e.*, operate following either a failed or a functioning state) components E_{ij}, $j \in \{0, \ldots, J_i\}$ connected *in parallel* (*i.e.*, the i^{th} subsystem functions if at least one out of its J_i components is functioning). Individual components in each subsystem have states that are statistically independent, their lifetimes are not necessarily identically distributed, and they do not have the same age at the start of the break period when maintenance decisions are to be made.

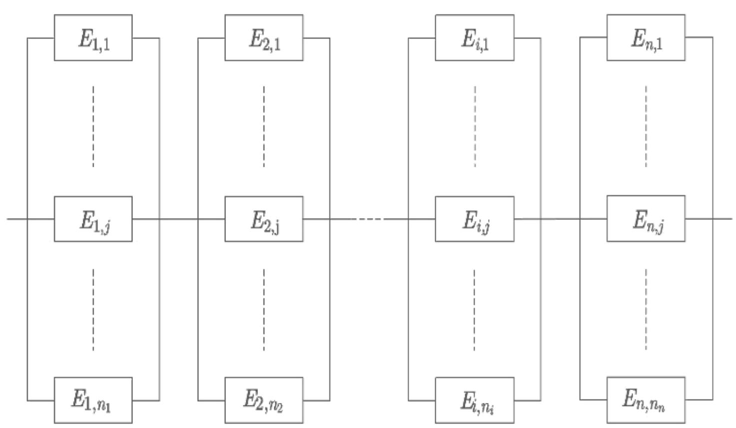


Figure 5.1: Series-parallel system structure.

The system is assumed to have just finished a mission and has been turned off during the scheduled break of length D_0 to undergo maintenance activities (Figure 5.2). The system will be used after the break to carry-out the next mission of duration M. At the end of the last mission (*i.e.*, at the beginning of the current break), each component E_{ij} is characterized by its current age B_{ij}, and its status is given by a binary state parameter

X_{ij}, defined as

$$X_{ij} = \begin{cases} 1 & \text{if } E_{ij} \text{ is working at the start of the break,} \\ 0 & \text{otherwise.} \end{cases} \tag{5.1}$$

Likewise, at the end of the break, each component E_{ij} is described by its effective age A_{ij}, and its status is given by a binary state parameter Y_{ij}, defined as

$$Y_{ij} = \begin{cases} 1 & \text{if } E_{ij} \text{ is working at the end of the break,} \\ 0 & \text{otherwise.} \end{cases} \tag{5.2}$$

Comparing Y_{ij} to X_{ij} reveals if a failed component has been repaired during the break or left as is.

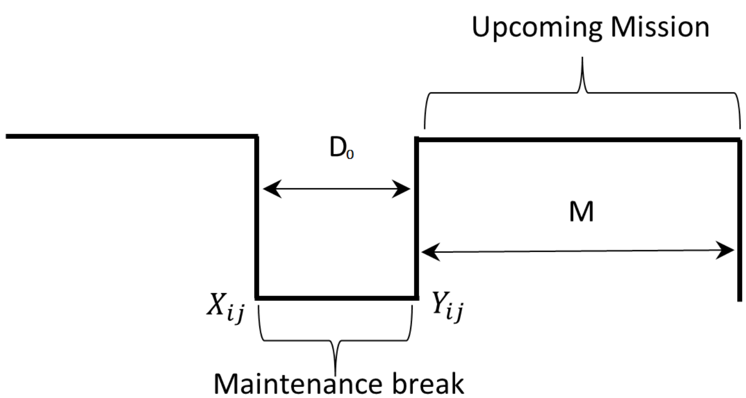


Figure 5.2: The sequence between the mission and the scheduled break.

5.2.2 Modelling notations and assumptions

Table 5.1 lists the notation used in the mathematical formulation of the nominal problem. Furthermore, the following assumptions are made to model the system.

1. The system consists of multiple, possibly non-identical and stochastically-independent repairable binary components. The components and the system are either functioning or failed. This is a reasonable assumption and is supported by many references (Diallo et al., 2018; Jiang and Liu, 2020a).

Table 5.1: Notations

i	Index of subsystems, $i \in \mathcal{I}$ where $\mathcal{I} := \{1, \ldots, I\}$
j	Index of components in subsystem i, $j \in \mathcal{J}_i$ where $\mathcal{J}_i := \{1, \ldots, J_i\}$
l	Index of maintenance levels available for component E_{ij}, $l \in \mathcal{L}_{ij}$ where $\mathcal{L}_{ij} := \left\{0, \ldots, L_{ij}\right\}$
r	Index of repairpersons, $r \in \mathcal{Q}$ where $\mathcal{Q} := \{1, \ldots, Q\}$
$t^c_{ijlr}(t^p_{ijlr})$	Nominal duration of CM (PM) when maintenance level l is performed on component E_{ij} by repairperson r
k_r	Variable labour cost per unit of time of repairperson r
$c^c_{ijl}(c^p_{ijl})$	Cost of CM (PM) when maintenance level l is performed on component E_{ij}
$B_{ij}(A_{ij})$	Age of component E_{ij} at the start (end) of the break
A_{ijl}	Age of component E_{ij} at the end of the break if maintenance level l is performed
$X_{ij}(Y_{ij})$	Status binary parameter of component E_{ij} at the start (end) of the break (1: working)
T_r	Total maintenance time spent by repairperson r to carry out their tasks
C_0	Maintenance budget available
D_0	Break duration
M	Length of the next mission
$R^c_{ij}(M\|_{A_{ijl}})$	Conditional reliability of component E_{ij} during the next mission upon being subjected to a maintenance action of level l during the scheduled break
$\mathcal{R}$	Overall system reliability during the next mission
$\bar{f}_{ijl}$	The nominal unreliability of component E_{ij} upon being subjected to a maintenance action of level l during the scheduled break.

2. The system performs consecutive *missions* separated by scheduled finite-duration breaks and the components are maintained during the breaks.

3. System components do not age during the break because the age of the components are mostly operation-driven (Diallo et al., 2019b). This is also reasonable because the break durations are typically negligible compared to the mission duration.

4. Maintenance activities are allowed only during the break, but not during the mission. For many mission-oriented systems, it is impossible to interrupt the mission to carry out any maintenance action.

5. All the required resources (budget, repairpersons, tools) are available when needed.

6. Any maintenance action can be carried out by exactly one repairperson, and any repairperson can carry out any level of maintenance on any component.

7. Multiple components can be worked on simultaneously without repairpersons colliding.

5.2.3 System reliability computations

To compute the system reliability during the next mission ($\mathcal{R}$), we first compute the conditional reliability $R^c_{ij}(M|_{A_{ijl}})$ of component E_{ij} given that its initial age is A_{ijl}. Denote by $R_{ij}(t)$ the unconditional reliability function of component E_{ij}. Then, the reliability during the next mission if E_{ij} undergoes a maintenance action of level $l \in \{0, \dots, L_{ij}\}$ is given by

$$R_{ijl} = \frac{R_{ij}\left(A_{ijl} + M\right)}{R_{ij}\left(A_{ijl}\right)}. \tag{5.3}$$

Since each component can undergo exactly one maintenance action of level l (including "Do-Nothing" when $l = 0$), and given that only one repairperson is needed to perform that maintenance action, the reliability $R^c_{ij}(M|_{A_{ijl}})$ is then obtained as

$$R^c_{ij}(M|_{A_{ijl}}) = \sum_{l=0}^{L_{ij}} \sum_{r=1}^{Q} R_{ijl}\, x_{ijlr}, \tag{5.4}$$

where x_{ijlr} is a binary decision variable defined as

$$x_{ijlr} = \begin{cases} 1 & \text{if repairperson } r \text{ performs maintenance level } l \text{ on } E_{ij}, \\ 0 & \text{otherwise.} \end{cases} \tag{5.5}$$

Given that the i^{th} subsystem has a parallel configuration, its reliability during the next mission is given by

$$R_i = 1 - \prod_{j=1}^{J_i} \left(1 - \sum_{l=0}^{L_{ij}} \sum_{r=1}^{Q} R_{ijl} x_{ijlr} \right). \tag{5.6}$$

Finally, the reliability of the whole series-parallel system is computed as

$$\mathcal{R} = \prod_{i=1}^{s} R_i = \prod_{i=1}^{I} \left[1 - \prod_{j=1}^{J_i} \left(1 - \sum_{l=0}^{L_{ij}} \sum_{r=1}^{Q} R_{ijl} x_{ijlr} \right) \right]. \tag{5.7}$$

5.2.4 Maintenance levels, costs, and duration

During a scheduled break, corrective maintenance (CM) and preventive maintenance (PM) actions are performed. The former is carried out on failed components, whereas the latter concerns components that are still functioning.

For a failed component E_{ij}, a maintenance level among the $(L_{ij} + 1)$ CM levels ($l \in \{0, 1, \ldots, L_{ij}\}$) available must be selected. The lowest level ($l = 0$) and the highest level $(l = L_{ij})$ stand, respectively, for the "Do Nothing (DN)" and the component replacement options. Level $l = 1$ refers to "minimal repair (MR)", which when performed brings the component to an "as bad as old" condition. Intermediate values of $1 < l < L_{ij}$ represent IM actions, which after being performed bring the component health condition back to somewhere between "as bad as old" and "as good as new". Here, IM is modelled according to the age reduction approach (Malik, 1979), which means that when a CM of level l is performed by repairperson r on component E_{ij}, its corresponding age (B_{ij}) is multiplied by an age reduction coefficient θ_{ijl}, $\left(0 \leq \theta_{ijl} \leq 1\right)$. Accordingly, the component becomes as good as new (replaced) if its age is reset to zero ($\theta = 0$), whereas it becomes as bad as old (minimal repair) if $\theta = 1$. Any CM action incurs a cost c^c_{ijl} and requires t^c_{ijlr} units of time.

Similarly, if component E_{ij} is still functioning, it can be subjected to a PM action of level $l \in \{0, 2, \ldots, L_{ij}\}$. Note that the level $l = 1$ refers to the minimal repair case, which when performed returns a failed component to a working state without affecting its failure rate. There is no minimal repair option with PM actions, therefore the maintenance option $l = 1$ is not available for working components. Intermediate values of l $\left(2 \le l < L_{ij}\right)$ represent IM actions which rejuvenate the component by reducing its age by a factor φ_{ijl} $\left(0 \le \varphi_{ijl} \le 1\right)$. For example, $\varphi_{ijL_{ij}} = 0$ is the perfect replacement case (*i.e.*, the age resets to 0), and $\varphi_{ij0} = 1$ is for the "Do Nothing" case (*i.e.*, the age remains the same after the maintenance break). Any PM action incurs a cost c^p_{ijl} and has a duration t^p_{ijlr}.

According to the above IM model, the effective age A_{ijl} of a given component E_{ij} at the end of the break is computed as a function of its initial operating status X_{ij} and the maintenance level l performed, as follows:

$$A_{ijl} = B_{ij}\left[X_{ij}\varphi_{ijl} + \left(1 - X_{ij}\right)\theta_{ijl}\right]. \tag{5.8}$$

Whenever a component is not selected to undergo maintenance, the corresponding maintenance cost and duration are ignored. However, if component E_{ij} is selected for maintenance and a maintenance level l is selected to be performed by repairperson r, it incurs a fixed cost (c^p_{ijl} or c^c_{ijl}) and a labour cost per unit of time k_r. Accordingly, the total PM and CM costs are computed as

$$C_{PM} = \sum_{i=1}^{I}\sum_{r=1}^{Q}\sum_{j=1}^{J_i}\sum_{l=2}^{L_{ij}}\left(k_r t^p_{ijlr} + c^p_{ijl}\right)X_{ij}x_{ijlr}, \tag{5.9}$$

$$C_{CM} = \sum_{i=1}^{I}\sum_{r=1}^{Q}\sum_{j=1}^{J_i}\sum_{l=1}^{L_{ij}}\left(k_r t^c_{ijlr} + c^c_{ijl}\right)\left(1 - X_{ij}\right)x_{ijlr}, \tag{5.10}$$

where the term $(1-X_{ij})$ ensures that CM actions are available only for failed components. The total maintenance cost C is given by

$$C = C_{PM} + C_{CM}. \tag{5.11}$$

Likewise, the total time T_r spent by each repairperson r to carry out their maintenance duties is computed as:

$$T_r = \sum_{i=1}^{I}\sum_{j=1}^{J_i}\left[\left(1 - X_{ij}\right)\sum_{l=1}^{L_{ij}} t^c_{ijlr}x_{ijlr} + X_{ij}\sum_{l=2}^{L_{ij}} t^p_{ijlr}x_{ijlr}\right]. \tag{5.12}$$

5.2.5 Mixed integer nonlinear programming formulation

The JSM–RAP aims to jointly select the set of components to be maintained, the maintenance levels to be performed on the selected components, and the repairpersons to carry out the selected maintenance actions such that system reliability for the next mission is maximized given a predetermined maintenance budget and a break duration.

Based on the description and the notations outlined earlier, the JSM–RAP was formulated by Diallo et al. (2019b) as an MINLP model:

$$\max \mathcal{R} = \prod_{i=1}^{I}\left(1 - \prod_{j=1}^{J_i}\left(1 - \sum_{r=1}^{Q}\sum_{l=0}^{L_{ij}} R_{ijl}x_{ijlr}\right)\right) \tag{5.13a}$$

s.t.

$$C \leq C_0 \tag{5.13b}$$

$$T_r \leq D_0 \qquad \forall r \in \mathcal{Q} \tag{5.13c}$$

$$\sum_{r=1}^{Q}\sum_{l=1}^{L_{ij}} x_{ijlr} = 1 \qquad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}_i \tag{5.13d}$$

$$x_{ijlr} \in \{0,1\} \qquad \forall i \in \mathcal{I}, \forall r \in \mathcal{Q}, \forall j \in \mathcal{J}_i, \forall l \in \mathcal{L}_{ij}. \tag{5.13e}$$

The objective function (5.13a) maximizes the system reliability for the next mission. Constraint (5.13b) states that the total cost of maintenance must not exceed the maintenance budget. Similarly, constraints (5.13c) ensures that the total maintenance time of the actions assigned to each repairperson does not exceed the break duration. Constraints (5.13d) ensures that each component gets exactly one maintenance level and is assigned to one repairperson. Note that the cost C and the r^{th} repairperson's maintenance time T_r are given by equations (5.11) and (5.12), respectively.

5.2.6 Exponential conic reformulation

The MINLP formulation (5.13) has an objective function that includes products of decision variables, which in general cannot be readily handled by off-the-shelf solvers. However, we show that it can be exactly reformulated as mixed-integer exponential conic program (MIECP) that can be solved by efficient solvers such as Mosek.

First, let $F_i = 1 - R_i$ be the “unreliability" of subsystem $i \in \mathcal{I}$, where

$$R_i = 1 - \prod_{j=1}^{J_i} \left(1 - \sum_{r=1}^{Q} \sum_{l=0}^{L_{ij}} R_{ijl} x_{ijlr} \right). \tag{5.14}$$

The objective function in (12) is then linearized by applying the natural logarithm function to both sides of Equation (5.13a), leading to

$$\ln(\mathcal{R}) = \sum_{i \in \mathcal{I}} \ln(1 - F_i). \tag{5.15}$$

Since x_{ijlr} is binary and only one maintenance level is selected for each component (*i.e.*, $\sum_{r=1}^{Q} \sum_{l=0}^{L_{ij}} x_{ijlr} = 1$), and using Equation (5.14), F_i can be rewritten as

$$F_i = \prod_{j=1}^{J_i} \left(1 - \sum_{r=1}^{Q} \sum_{l=0}^{L_{ij}} R_{ijl} x_{ijlr} \right) \tag{5.16}$$

$$= \prod_{j=1}^{J_i} \sum_{r=1}^{Q} \sum_{l=0}^{L_{ij}} \bar{f}_{ijl} \, x_{ijlr}, \tag{5.17}$$

or, equivalently,

$$\ln(F_i) = \sum_{r=1}^{Q} \sum_{j=1}^{J_i} \sum_{l=0}^{L_{ij}} \ln(\bar{f}_{ijl}) x_{ijlr}, \tag{5.18}$$

where $\bar{f}_{ijl} = 1 - R_{ijl}$ is the (nominal) unreliability of component E_{ij} upon being subjected to a maintenance action of level l during the scheduled break. Note that maximizing $\ln(1 - F_i)$ in Equation (5.15) requires F_i to take the smallest possible value such that the equality constraint (5.18) holds. Thus, the equality constraint (5.18) can be replaced with a “$\geq$" inequality. Accordingly, the MINLP formulation (5.13) is reformulated as follows:

$$\max \quad \ln(\mathcal{R}) = \sum_{i \in \mathcal{I}} \ln(1 - F_i) \tag{5.19a}$$

s.t.

$$\ln(F_i) \geq \sum_{r=1}^{Q} \sum_{j=1}^{J_i} \sum_{l=0}^{L_{ij}} \ln(\bar{f}_{ijl}) x_{ijlr} \quad \forall i \in \mathcal{I} \tag{5.19b}$$

$$(5.13\text{b}) - (5.13\text{e}).$$

Now, let us define the variable $S_i \leq \ln(1 - F_i)$. If $\ln(1 - F_i)$ is replaced by S_i in (5.19a), this inequality becomes binding at optimality. Furthermore, let $Z_i = \ln(F_i)$. Hence, we have $e^{S_i} \leq 1 - F_i$ and $e^{Z_i} = F_i$. By adding up these expressions we get $e^{Z_i} + e^{S_i} \leq 1$, which can be expanded as $e^{Z_i} \leq a_i$, $e^{S_i} \leq b_i$ and $a_i + b_i \leq 1$. With that, problem (5.18) can be reformulated as the following MIECP.

$$
\begin{aligned}
& \max \sum_{i \in \mathcal{I}} S_i && && (5.20a) \\
& \text{s.t.} \\
& \quad Z_i \geq \sum_{r=1}^{Q} \sum_{j=1}^{J_i} \sum_{l=1}^{L_{ij}} \ln\left(\bar{f}_{ijl}\right) x_{ijlr} && \forall i \in \mathcal{I} && (5.20b) \\
& \quad e^{Z_i} \leq a_i && \forall i \in \mathcal{I} && (5.20c) \\
& \quad e^{S_i} \leq b_i && \forall i \in \mathcal{I} && (5.20d) \\
& \quad a_i + b_i \leq 1 && \forall i \in \mathcal{I} && (5.20e) \\
& \quad Z_i, S_i \leq 0 && \forall i \in \mathcal{I} && (5.20f) \\
& \quad a_i, b_i \geq 0 && \forall i \in \mathcal{I} && (5.20g) \\
& \quad (5.13b) - (5.13e)
\end{aligned}
$$

In the above formulation, (5.20c) and (5.20d) are exponential conic constraints. Such constraints can equivalently be written as $(a_i, 1, Z_i) \in \mathcal{K}_{exp}$ $\forall i \in \mathcal{I}$ and $(b_i, 1, S_i) \in \mathcal{K}_{exp}$ $\forall i \in \mathcal{I}$, respectively. The notation $(x_1, x_2, x_3) \in \mathcal{K}_{exp}$ describes all the points in $\mathbb{R}^3$ satisfying the exponential cone equation $x_1 \geq x_2 e^{x_3/x_2}$, $x_1, x_2 \geq 0$ (Mosek ApS, 2021).

5.3 The robust problem formulation

Variability and uncertainty of the maintenance quality are inevitable due to factors such as unpredictable operating conditions, components' conditions that remain largely unknown until maintenance has started, varying repairperson skillsets, and human errors. This raises the question of what values of the post-maintenance unreliability (f_{ijlr}) to use in constraints (5.20b). Usually, the quality of maintenance actions is not known with certainty, but it rather lies within some interval or bounded uncertainty set. A common way to deal with such parameter uncertainty is to use a RO approach that requires the model constraints to hold for realizations within the uncertainty set, and maximizes the reliability function corresponding to the worst-case among these realizations.

In this section, non-symmetric budget uncertainty sets are used to represent maintenance quality uncertainty in the JSM–RAP, and we show how to tractably reformulate the robust counterpart with this set. Specifically, let $\mathcal{F}_i$, $i \in \mathcal{I}$ be subsystem-specific uncertainty sets of the form

$$\mathcal{F}_i := \left\{ \mathrm{f}_i \in \mathbb{R}_+^{Q \times J_i \times L_{ij}} \;\middle|\; \begin{array}{l} f_{ijlr} = \bar{f}_{ijl} + \hat{f}^{+}_{ijlr} w^{+}_{ijlr} - \hat{f}^{-}_{ijlr} w^{-}_{ijlr}, \forall r \in \mathcal{Q}, \forall j \in \mathcal{J}_i, \forall l \in \mathcal{L}_i \\ \sum_{r=1}^{Q} \sum_{j=1}^{J_i} \sum_{l=1}^{L_{ij}} (w^{+}_{ijlr} + w^{-}_{ijlr}) \leq \Gamma_\mathrm{i} \\ 0 \leq w^{+}_{ijlr} + w^{-}_{ijlr} \leq 1, \forall r \in \mathcal{Q}, \forall j \in \mathcal{J}_i, \forall l \in \mathcal{L}_{ij} \end{array} \right\},$$

where $w_i^{+}, w_i^{-} \in [0,1]^{Q \times J_i \times L_{ij}}, \forall i \in \mathcal{I}$ are vectors of primary uncertainty, $\Gamma_\mathrm{i} \in \left[0,\ Q \times J_i \times L_{ij}\right], \forall i \in I$, is the uncertainty budget, and $\hat{f}_i^{+}$, $\hat{f}_i^{-}$, respectively, are the maximum positive and negative deviations from the nominal unreliability $\bar{f}_i$. This is a non-symmetric variant of the budget uncertainty set introduced in Bertsimas and Sim (2004). To ensure that reliability remains between 0 and 1, the maximum positive and negative deviations must always satisfy $\hat{f}^{-}_{ijlr} \leq \bar{f}_{ijl} \leq 1 - \hat{f}^{+}_{ijlr}$. Hence, the robust counterpart of constraint (5.20b) can be written as

$$Z_i \geq \sup_{\mathrm{f}_i \in \mathcal{F}_i} \left[\sum_{r=1}^{Q} \sum_{j=1}^{J_i} \sum_{l=1}^{L_{ij}} x_{ijlr} \ln(f_{ijlr}) \right] \quad \forall i \in \mathcal{I}. \tag{5.21}$$

To tractably reformulate (5.21), we utilize the approach proposed by Ben-Tal et al. (2015) which uses Fenchel duality to decompose the problem into two parts, one that depends on the support set $\mathcal{F}_i$ and is evaluated using the support function $\delta^*(.)$, and the other depends on the functional form of the constraint and is evaluated using the partial concave conjugate (PCC) function $g_*(.,.)$. Specifically, we use the following theorem:

Theorem 1 (Ben-Tal et al. (2015), theorem 2)**.** *The vector* $\mathrm{x} \in \mathcal{X}$ *satisfies the robust constraint* $g(\mathrm{x},\mathrm{f}) \leq 0, \forall\, \mathrm{f} \in \mathcal{F}$ *if and only if* x *and* $\mathrm{v} \in \mathbb{R}^m$ *satisfy the single inequality*

$$(FRC): \quad \delta^*(\mathrm{v} \mid \mathcal{F}) - g_*(\mathrm{x},\mathrm{v}) \leq 0,$$

where δ^* *is the support function of set* $\mathcal{F}$*, defined as*

$$\delta^*(\mathrm{v} \mid \mathcal{F}) := \sup_{\mathrm{f} \in \mathcal{F}} \mathrm{f}^\intercal \mathrm{v}, \tag{5.22}$$

and, $g_*(.,.)$ *is the partial concave conjugate (PCC) with respect to the first variable and defined as*

$$g_*(\mathrm{x},\mathrm{v}) := \inf_{\mathrm{f} \in \mathcal{F}_g} \mathrm{v}^\intercal \mathrm{f} - g(\mathrm{x},\mathrm{f}), \tag{5.23}$$

and $g(.,.)$ is a mapping defined over the convex domain $\mathcal{X}_g \times \mathcal{F}_g$ with $\mathcal{X}_g \subseteq \mathbb{R}^n$ and $\mathcal{F}_g \subseteq \mathbb{R}^m$.

Ben-Tal et al. (2015) compute $\delta_i^*(.)$ and $g_*(.,.)$ for several choices of $\mathcal{F}$ and $g_*(.,.)$, respectively. One of their results states that the robust counterpart for a constraint that is the sum of separable functions $\sum_k g_k(v_k, \mathrm{f})$ can be reformulated as

$$\delta^*(v^\top|\mathcal{F}) - \sum_k (g_k)_*(v_k, f) \leq 0 \tag{5.24}$$

Accordingly, the support function $\delta_i^*(.)$ is evaluated as

$$\delta_i^*(v_i|\mathcal{F}_i) = \max_{f\in\mathcal{F}_i} f_i^\top v_i \tag{5.25a}$$

$$= \sum_{r=1}^{Q}\sum_{j=1}^{J_i}\sum_{l=1}^{L_{ij}} \bar{f}_{ijlr} v_{ijlr} +$$

$$\max_{w_{ijlr}} \sum_{r=1}^{Q}\sum_{j=1}^{J_i}\sum_{l=1}^{L_{ij}} v_{ijlr}(\widehat{f^+_{ijlr}} w^+_{ijlr} - \widehat{f^-_{ijlr}} w^-_{ijlr}) \tag{5.25b}$$

s.t.

$$\sum_{r=1}^{Q}\sum_{j=1}^{J_i}\sum_{l=1}^{L_{ij}} (w^+_{ijlr} + w^-_{ijlr}) \leq \Gamma_i \qquad (\theta_i) \tag{5.25c}$$

$$0 \leq w^+_{ijlr} + w^-_{ijlr} \leq 1 \quad \forall (j,l,r) \in (\mathcal{J}_i, \mathcal{L}_{ij}, \mathcal{Q}) \qquad (\phi_{ijlr}) \tag{5.25d}$$

Through strong duality, problem (5.25) is equivalent to

$$\min_{\theta_i, \phi_{ijlr}, u_{ijlr}, v_{ijlr}} \quad \Gamma_i\theta_i + \sum_{r=1}^{Q}\sum_{j=1}^{J_i}\sum_{l=1}^{L_{ij}} \phi_{ijlr} \tag{5.26a}$$

s.t.

$$\theta_i + \phi_{ijlr} \geq \widehat{f^+_{ijlr}} u^+_{ijlr} \qquad \forall (j,l,r) \in (\mathcal{J}_i, \mathcal{L}_{ij}, \mathcal{Q}) \tag{5.26b}$$

$$\theta_i + \phi_{ijlr} \geq \widehat{f^-_{ijlr}} u^-_{ijlr} \qquad \forall (j,l,r) \in (\mathcal{J}_i, \mathcal{L}_{ij}, \mathcal{Q}) \tag{5.26c}$$

$$\theta_i, \phi_{ijlr} \geq 0 \qquad \forall (j,l,r) \in (\mathcal{J}_i, \mathcal{L}_{ij}, \mathcal{Q}). \tag{5.26d}$$

Next, to evaluate the PCC function using (5.24), we define $g_i(\mathrm{x}, f)$ as

$$g_i(\mathrm{x}, f) = \sum_{r=1}^{Q}\sum_{j=1}^{J_i}\sum_{l=1}^{L_{ij}} x_{ijlr} \ln(f_{ijlr}) \qquad \forall i \in \mathcal{I}.$$

It is decomposable $\forall i \in \mathcal{I}$ to the $Q \times J_i \times L_{ij}$ functions

$$g_{ijlr}(x_{ijlr}, f_{ijlr}) = x_{ijlr} \ln(f_{ijlr}).$$

The PCC function for $g_{ijlr}(x_{ijlr}, f_{ijlr})$ is

$$(g_{ijlr})_*(v_{ijlr}, x_{ijlr}) = \min_{f_{ijlr}} v_{ijlr} f_{ijlr} - x_{ijlr} \ln(f_{ijlr}).$$

Since the minimization is for a convex function in f_{ijlr}, we equate its first derivative to 0 to get

$$v_{ijlr} - \frac{x_{ijlr}}{f^*_{ijlr}} = 0 \rightarrow f^*_{ijlr} = \frac{x_{ijlr}}{v_{ijlr}}.$$

By substituting f^*_{ijlr} back, we get

$$(g_{ijlr})_*(v_{ijlr}, x_{ijlr}) = x_{ijlr} - x_{ijlr} \ln\left(\frac{x_{ijlr}}{v_{ijlr}}\right).$$

Let $y_{ijlr} = -x_{ijlr} \ln\left(\frac{x_{ijlr}}{v_{ijlr}}\right)$. Note that since y_{ijlr} will have a negative sign in a $\leq$ constraint (*i.e.*, the original constraint), we want to cap it from above. So we can replace this equality with the inequality $y_{ijlr} \leq -x_{ijlr} \ln\left(\frac{x_{ijlr}}{v_{ijlr}}\right)$. Or, equivalently, $\frac{y_{ijlr}}{x_{ijlr}} \leq \ln\left(\frac{v_{ijlr}}{x_{ijlr}}\right)$. This leads to the conic exponential constraints

$$v_{ijlr} \geq x_{ijlr} e^{\frac{y_{ijlr}}{x_{ijlr}}},$$

which can be written as $(v_{ijlr}, x_{ijlr}, y_{ijlr}) \in \mathcal{K}_{\text{exp}}$. Thus, the robust constraint can be reformulated as

$$\delta^*(f_i|\mathcal{F}_i) - \sum_{r=1}^{Q} \sum_{j=1}^{J_i} \sum_{l=1}^{L_{ij}} (x_{ijlr} + y_{ijlr}) \leq Z_i \qquad \forall i \in \mathcal{I} \tag{5.27}$$

$$(v_{ijlr}, x_{ijlr}, y_{ijlr}) \in \mathcal{K}_{\text{exp}}. \tag{5.28}$$

Substituting the first term of (5.25b), and the results from (5.26) in the support function $\delta^*(f_i|\mathcal{F}_i)$ in (5.27), the robust counterparts of (5.21) can be written as (5.29h) to (5.29l).

By reintegrating the obtained robust counterparts into the ECO formulation (5.20), the robust exponential conic version of JSM–RAP that considers the uncertainty of the maintenance quality can be written as

$$\min -\sum_{i\in\mathcal{I}} S_i \tag{5.29a}$$

s.t.

$$C \le C_0 \tag{5.29b}$$

$$T_r \le D_0 \qquad \forall r \in \mathcal{Q} \tag{5.29c}$$

$$\sum_{r=1}^{Q}\sum_{l=1}^{L_{ij}} x_{ijlr} = 1 \qquad \forall i \in \mathcal{I},\ \forall j \in \mathcal{J}_i \tag{5.29d}$$

$$a_i \ge e^{Z_i} \qquad \forall i \in \mathcal{I} \tag{5.29e}$$

$$b_i \ge e^{S_i} \qquad \forall i \in \mathcal{I} \tag{5.29f}$$

$$a_i + b_i \le 1 \qquad \forall i \in \mathcal{I} \tag{5.29g}$$

$$\begin{aligned}\Gamma_i\theta_i + \sum_{r=1}^{Q}\sum_{j=1}^{J_i}\sum_{l=1}^{L_{ij}} (\bar{f}_{ijlr} v_{ijlr} \\ + \phi_{ijlr} - x_{ijlr} - y_{ijlr}) \le Z_i \qquad \forall i \in \mathcal{I}\end{aligned} \tag{5.29h}$$

$$v_{ijlr} \ge x_{ijlr} e^{\frac{y_{ijlr}}{x_{ijlr}}} \qquad \forall (i,j,l,r) \in (\mathcal{I}, \mathcal{J}_i, \mathcal{L}_{ij}, \mathcal{Q}) \tag{5.29i}$$

$$\theta_i +\ \phi_{ijlr} \ge \widehat{f}^{+}_{ijlr} u^{+}_{ijlr} \qquad \forall (i,j,l,r) \in (\mathcal{I}, \mathcal{J}_i, \mathcal{L}_{ij}, \mathcal{Q}) \tag{5.29j}$$

$$\theta_i +\ \phi_{ijlr} \ge \widehat{f}^{-}_{ijlr} u^{-}_{ijlr} \qquad \forall (i,j,l,r) \in (\mathcal{I}, \mathcal{J}_i, \mathcal{L}_{ij}, \mathcal{Q}) \tag{5.29k}$$

$$a_i, b_i, \theta_i, \phi_{ijlr}, u^{+}_{ijlr}, u^{-}_{ijlr} \ge 0 \qquad \forall (i,j,l,r) \in (\mathcal{I}, \mathcal{J}_i, \mathcal{L}_{ij}, \mathcal{Q}) \tag{5.29l}$$

$$Z_i, S_i \le 0 \qquad \forall i \in \mathcal{I} \tag{5.29m}$$

$$x_{ijlr} \in \{0,1\} \qquad \forall (i,j,l,r) \in (\mathcal{I}, \mathcal{J}_i, \mathcal{L}_{ij}, \mathcal{Q}). \tag{5.29n}$$

5.4 Numerical experiments

In this section, the validity and effectiveness of the proposed nominal and robust JSM–RAP formulations are examined. The section is divided into two parts. In the first part, two sets of numerical experiments are conducted to assess the the validity of the nominal MIECP formulation described in Section 5.2.6. In the second part, the impact of applying the robust approach described in Section 5.3 to deal with maintenance quality uncertainty on the optimal maintenance plan and system reliability is evaluated.

For all the numerical experiments, and without loss of generality, we assume that the lifetimes of components are Weibull-distributed with shape and scale parameters β_{ij} and η_{ij}, respectively. The reliability function is calculated as

$$R^c_{ij}(M|_{A_{ijl}}) = \exp\left(\left(\frac{A_{ijl}}{\eta_{ij}}\right)^{\beta_{ij}} - \left(\frac{A_{ijl}+M}{\eta_{ij}}\right)^{\beta_{ij}}\right)x_{ijlr}.$$

All experiments are run on an Intel(R) Core(TM) i7 @ 1.30 GHz laptop computer with 16 GB of RAM, running Windows 11. The problems are coded in Python 3.9 and solved using Gurobi 9.1.1 and Mosek 10.0.

5.4.1 Numerical results for the nominal problem

The first set of experiments examines the validity of the proposed MIECP formulation (5.20) by comparing its optimal solutions and values to those obtained using the exact two-phase approach from Diallo et al. (2019b) (described in 2.4.2). These comparisons are made on a moderate-size, series-parallel system with multiple repairpersons and imperfect maintenance levels. The second set of experiments aims to evaluate the computational performance of the proposed nominal formulation when used with very large JSM–RAP instances.

5.4.1.1 Experiments #1.1: Nominal MIECP model validation

For this set of experiments, we use the moderate-size, series-parallel system studied in Diallo et al. (2018). The reliability block diagram of the system is composed of $I = 2$ series subsystems. The first one is composed of the n_1 *i.i.d.* components E_{1j} ($j = 1,\ldots,5$), whereas the one second contains the $n_2 = 8$ *i.i.d.* components E_{2j} ($j = 1,\ldots,8$) arranged in parallel, resulting in a total of 13 components. The lifetimes of all component are Weibull-distributed with the respective shape and scale parameters β_{ij} and η_{ij} ($i = 1,2; j = 1,\ldots,J_i$). These parameters are set at $\beta_{1j} = 1.5$, $\eta_{1j} = 15$ ($j = 1,\ldots,5$), $\beta_{2j} = 3$ and $\eta_{2j} = 20$ ($j = 1,\ldots,8$). A list of four CM levels is available for failed components: $l = 0$ (DN), $l = 1$ (MR), and $l = 2$ (IM) which reduces the component age by half, and $l = 3$ (CR). For functioning components, a list of three PM levels is available: $l = 0$ (DN), $l = 2$ (IM) reduces the component age by half, and $l = 3$ (PR). Two repairpersons ($Q = 2$) are available, the variable cost rate of a repairperson is set to $k_r = 2$. Additional data related to components status, ages and maintenance times and costs are reported in Table 5.2. The break duration and the duration of the next mission are set to $D_0 = 10$ and $M = 8$, respectively. For comparison purposes, the JSM–RAP is solved with both the exact two-phase approach (Diallo et al., 2019b) and the MIECP formulation.

Table 5.2: Parameters for experiments #1.1, source: Diallo et al. (2018).

E_{ij}	X_{ij}	B_{ij}	t^c_{ij1}	t^c_{ij2}	t^c_{ij3}	t^p_{ij2}	t^p_{ij3}	c^c_{ij1}	c^c_{ij2}	c^c_{ij3}	c^p_{ij2}	c^p_{ij3}
E_{11}	0	15	4	6	8	2	4	5	10	14	8	10
E_{12}	1	12	4	6	8	2	4	5	10	14	8	10
E_{13}	0	10	4	6	8	2	4	5	10	14	8	10
E_{14}	1	18	4	6	8	2	4	5	10	14	8	10
E_{15}	1	20	4	6	8	2	4	5	10	14	8	10
E_{21}	0	8	3	4	5	1	2	6	10	20	7	12
E_{22}	1	15	3	4	5	1	2	6	10	20	7	12
E_{23}	0	8	3	4	5	1	2	6	10	20	7	12
E_{24}	1	15	3	4	5	1	2	6	10	20	7	12
E_{25}	0	8	3	4	5	1	2	6	10	20	7	12
E_{26}	1	15	3	4	5	1	2	6	10	20	7	12
E_{27}	0	8	3	4	5	1	2	6	10	20	7	12
E_{28}	1	15	3	4	5	1	2	6	10	20	5	12

Table 5.3 depicts the maximum achievable system reliability $\mathcal{R}$ for the next mission, the total maintenance time D, cost C incurred, and the CPU time for different values of the maintenance budget C_0 when two repairpersons are hired. This table also provides the relative gap between the exact two-phase approach solutions and the solutions obtained using the MIECP formulation.

Table 5.3: Results obtained from experiments #1.1.

C_0	Two-phase approach				MIECP				
	$\mathcal{R}^*(\%)$	C^*	m^*	$CPU(s)$	$\mathcal{R}(\%)$	C	m	$Gap(\%)$	$CPUt(s)$
54	97.97	54	2	857.68	97.97	54	2	0.0	1.89
50	97.22	49	2	853.76	97.22	49	2	0.0	2.63
40	95.90	40	2	848.54	95.90	40	2	0.0	5.61
30	92.85	30	2	848.76	92.85	30	2	0.0	3.78
20	89.50	20	2	855.23	86.50	20	2	0.0	2.83
10	74.97	10	2	853.89	74.97	10	2	0.0	0.18

From a computational time perspective, the MIECP formulation vastly outperforms the two-phase approach. The exact two-phase approach takes on average 823.25 seconds to generate 798,856 maintenance patterns and needs an additional 30 seconds on average to find the optimal solution. In contrast, the computational time with the MIECP formulation is between 0.18 and 5.61 seconds. The solutions obtained using the MIECP formulation are identical to the optimal solutions of the two-stage approach (*i.e.*, the optimality gap was always 0.0%). This confirms the validity of the proposed approach.

5.4.1.2 Experiments #1.2: Large-scale JSM–RAP instances

This set of experiments aims to evaluate the ability of the proposed MIECP formulation to deal with large-scale systems. The experiments are carried out on the series-parallel system from Ikonen et al. (2020). The basic system is composed of 100 components distributed across $I = 32$ subsystems in series. Each subsystem i ($i = 1,\ldots,I$) is composed of *i.i.d.* J_i components arranged in parallel (Figure 5.3). Components in the first two subsystems are identical. Similarly, each of the subsystems sets $\{3,\ldots,8\}$, $\{9,\ldots,18\}$, $\{19,\ldots,25\}$, and $\{26,\ldots,32\}$ consists of identical subsystems. This basic system has a total number of components $NC = \sum_{i=1}^{32} J_i = 100$. The lifetimes of all components follow Weibull distributions whose shape and scale parameters are β_{ij} and η_{ij}, respectively. The values of these parameters, along with the components' status and age data, are given in Table 5.4. A common lists of $L = 2$ maintenance options are available: DN and PR for functioning components, and DN and MR for failed components. Components maintenance times and costs are given in Table 5.5.

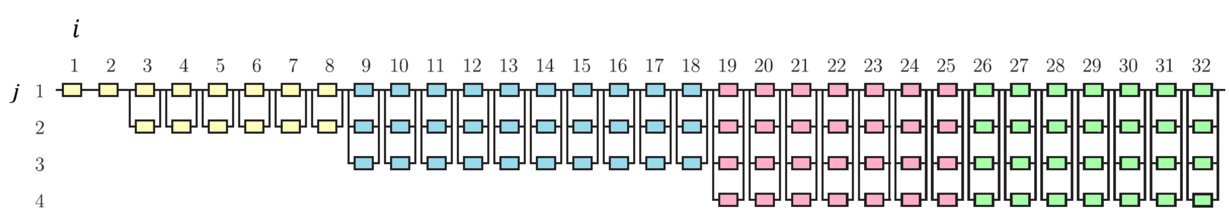


Figure 5.3: Reliability structure for the basic system with 100 components from Ikonen et al. (2020).

Five test instances are created by duplicating the basic 100-component system presented in Figure 5.3 o times, where $1 \leq o \leq 5$, such that the system has $NC_o = 100 \times o$ components. A fixed break duration of D_0=50 is used for all test instances, whereas the

Table 5.4: Parameters for experiments #1.2.

E_{ij} From	E_{ij} To	η_{ij}	β_{ij}	X_{ij}	B_{ij}
$E_{1,1}$	$E_{2,1}$	50	3.0	[0]	[8]
$E_{3,j}$	$E_{8,j}$	50	3.0	[0,1]	[8,15]
$E_{9,j}$	$E_{18,j}$	35	1.5	[0,1,0]	[15,12,10]
$E_{19,j}$	$E_{25,j}$	50	3.0	[0,1,0,1]	[8,15,8,15]
$E_{26,j}$	$E_{32,j}$	25	2.1	[0,1,0,1]	[6,10,6,10]

Table 5.5: Maintenance duration and cost values for experiments #1.2.

E_{ij} From	E_{ij} To	t^c_{ij1}	t^c_{ij2}	t^c_{ij3}	t^p_{ij2}	t^p_{ij3}	c^c_{ij1}	c^c_{ij2}	c^c_{ij3}	c^p_{ij2}	c^p_{ij3}
$E_{1,1}$	$E_{2,1}$	4	7	8	3	5	12	24	30	15	21
$E_{3,j}$	$E_{8,j}$	4	7	8	3	5	12	24	30	15	21
$E_{9,j}$	$E_{18,j}$	2	5	6	2	3	15	30	42	24	30
$E_{19,j}$	$E_{25,j}$	3	4	5	1	2	18	30	60	21	36
$E_{26,j}$	$E_{32,j}$	2	2.5	4	2	3	12	24	30	15	21

Table 5.6: Results obtained from experiments #1.2.

n	MIECP			
	$\mathcal{R}(\%)$	C	m	$CPUt(s)$
100	94.51	492	3	19.6
200	89.50	996	6	438.2
300	84.75	1500	6	122.7
400	80.01	1992	8	392.5
500	75.85	2496	13	2511.4

maintenance budget and number of repairpersons depend of o and are set to $C_0 = 500 \times o$ and $Q = 5 \times o$, respectively. For example, the second test instance ($o = 2$) has $NC_2 = 200$ components, obtained by duplicating the basic system, the available maintenance budget is set to 1000, and 10 repairpersons are available to perform the maintenance tasks. As shown in Table 5.6, the proposed MIECP formulation enabled all the test instances to be solved to proven optimality very efficiently. The largest test instance of 500 components was solved in less than 42 minutes, which clearly prove the computational effectiveness of the proposed MIECP formulation.

5.4.2 Numerical results for the robust problem

The next set of experiments uses the robust formulation developed in Section 5.3 to investigate the impact of maintenance quality uncertainty on the optimal robust maintenance plan and system reliability. The reliability block diagram of the test system is composed of $I = 3$ subsystems connected in series. Each of the first two subsystems consists of 5 *i.i.d.* components E_{ij} ($i = 1,2; j = 1,\ldots,5$) arranged in parallel, whereas the third subsystem consists of 3 *i.i.d.* components E_{3j} ($j = 1,2,3$) arranged in parallel, resulting in a total of 13 components. The lifetimes of all components are Weibull-distributed with respective shape and scale parameters set to $\beta_{ij} = 1.5$ and $\eta_{ij} = 15$. The age of all components is assumed to be 18 units of time. There are $Q = 2$ equally-skilled repairpersons available to carry out maintenance duties with a variable cost rate $k_r = 2$ ($r = 1,2$). A list of five maintenance levels is available for all components: $l = 0$ (DN), $l = 1$ (MR: valid only for failed components), $l = 2$ (IM_1), $l = 3$ (IM_2) and $l = 4$ (PR).

The duration of the scheduled break and that of the next mission are set to $D_0 = 8$ and $M = 4$, respectively. Additional data related to components' states, maintenance times and costs are displayed in Table 5.7.

To construct the uncertainty set, the post-maintenance reliability corresponding to each maintenance level is considered as an uncertain parameter in a closed interval. The upper and lower reliability bounds for different maintenance levels are illustrated in Figure 5.4. Note that the circular marker shown in each interval indicates the nominal reliability value for each maintenance level. The deviation from the nominal reliability for each maintenance level reflects the complexity degree of performing the maintenance level correctly. The deviation decreases when performing higher maintenance levels that

Table 5.7: Parameters for experiment #2

E_{ij}	X_{ij}	t^c_{ij1}	t^c_{ij2}	t^c_{ij3}	t^c_{ij4}	t^p_{ij2}	t^p_{ij3}	t^p_{ij4}	c^c_{ij1}	c^c_{ij2}	c^c_{ij3}	c^c_{ij4}	c^p_{ij2}	c^p_{ij3}	c^p_{ij4}
E_{11}	0	2	5	7	11	2	5	6	5	10	14	16	8	10	12
E_{12}	1	2	5	7	11	2	5	6	5	10	14	16	8	10	12
E_{13}	0	2	5	7	11	2	5	6	5	10	14	16	8	10	12
E_{14}	1	2	5	7	11	2	5	6	5	10	14	16	8	10	12
E_{15}	1	2	5	7	11	2	5	6	5	10	14	16	8	10	12
E_{21}	0	4	7	10	15	3	5	7	4	8	10	12	5	7	9
E_{22}	1	4	7	10	15	3	5	7	4	8	10	12	5	7	9
E_{23}	0	4	7	10	15	3	5	7	4	8	10	12	5	7	9
E_{24}	1	4	7	10	15	3	5	7	4	8	10	12	5	7	9
E_{25}	0	4	7	10	15	3	5	7	4	8	10	12	5	7	9
E_{31}	1	4	7	10	15	3	5	7	4	8	10	12	5	7	9
E_{32}	0	4	7	10	15	3	5	7	4	8	10	12	5	7	9
E_{33}	1	4	7	10	15	3	5	7	4	8	10	12	5	7	9

require more resources (time and cost) to be implemented. By definition, minimal repair (MR) returns a failed component to a working state without affecting its failure rate. Accordingly, when implementing MR, the component condition cannot be improved beyond its condition before failure, and thus MR has negative deviations only. Likewise, there is no variability in the maintenance quality if no maintenance action is implemented (*i.e.*, for DN). Since the components are assumed identical and the repairpersons are equally-skilled, the uncertainty bounds are the same for all components and repairpersons. For simplicity purposes, the uncertainty budgets are assumed to be the same for all subsystems. We test for different values of the uncertainty budget $\Gamma_i \in \{0, 0.05, \ldots, 2.00\}$.

Figure 5.5 illustrates the relationship between the worst-case system reliability, computed using the robust formulation (5.29), and the uncertainty budget Γ_i that controls the level of conservatism. As expected, the worst-case system reliability deteriorates (decreases) as the uncertainty budget is increased since more and/or larger negative deviations in the components' post-maintenance reliability are admitted. The worst-case reliability value obtained for a given uncertainty budget serves as a lower bound for the realized system reliability, thus provides a performance guarantee to the planner. For example, with $\Gamma_i = 1$, it is guaranteed that the actual system reliability equals at least 0.5597. For the purpose of providing a performance guarantee, the uncertainty set should

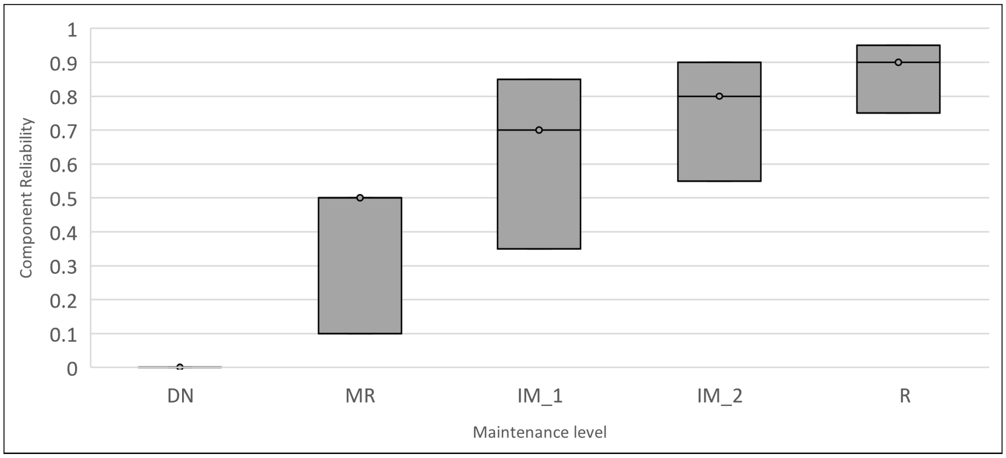


Figure 5.4: Post-Maintenance reliability bounds for each maintenance level l.

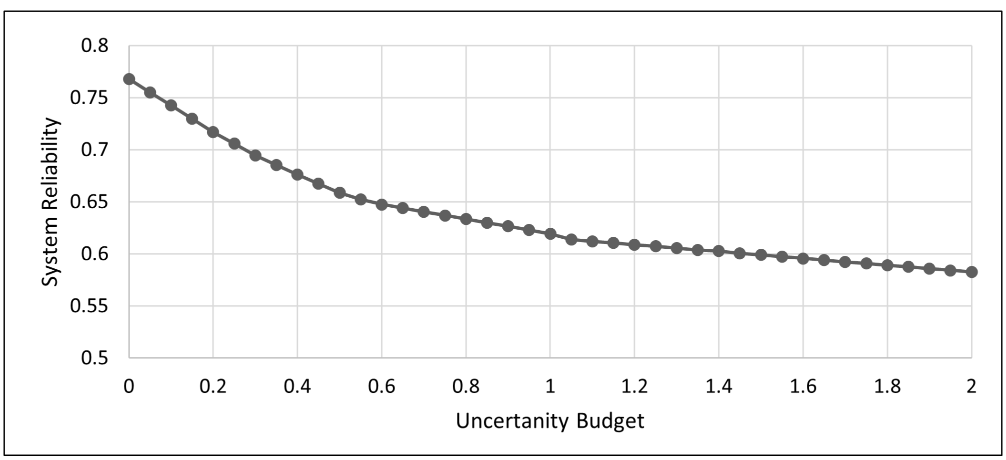


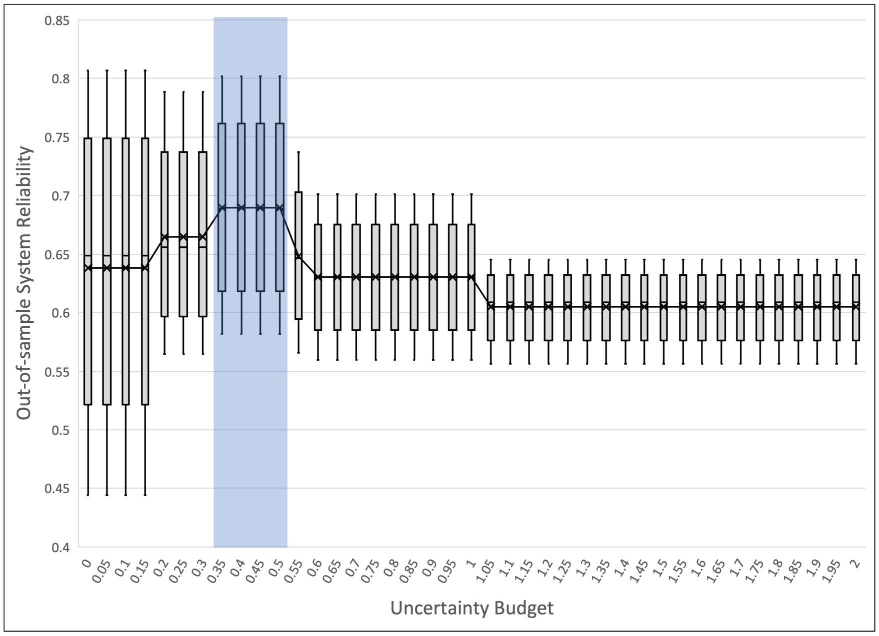
Figure 5.5: Worst-case system reliability vs. uncertainty budget.

be properly-sized based on historical data (*e.g.*, such that it includes a certain percentage of the observations) or to reflect the risk attitude of the planner.

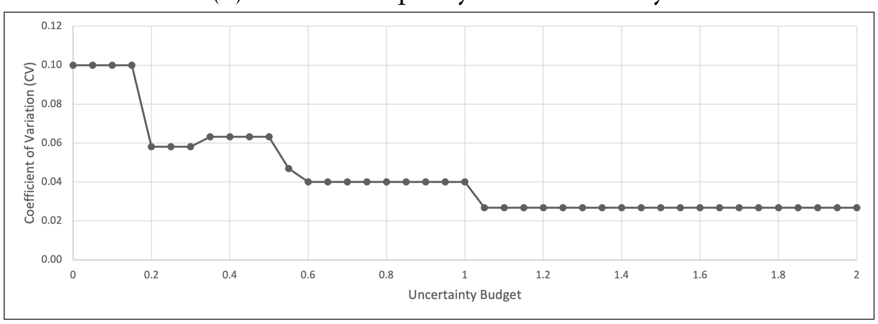
Next, we compute the out-of-sample reliability of the system based on the robust solutions (maintenance plans) obtained for each uncertainty budget values. A test sample of 1000 random post-maintenance reliability realizations for each system components is drawn and used to calculate the overall system reliability. Each realization is drawn at random and independently for each maintained component from the reliability intervals shown in Figure 5.4 while assuming that the post-maintenance reliability for each maintenance level follows a beta distribution with respective shape and scale parameters $\beta_{ij} = 2$ and $\eta_{ij} = 3$. These values are used for illustration purposes only. Any other distributions and values can be used.

Figure 5.6a illustrates the relationship between the out-of-sample system reliability and the uncertainty budget Γ_i. The bold line represents the average values whereas the box-and-whisker plots display the quartiles of the test sample's results. A closer inspection of the results reveals that the average system reliability attains a distinct maximum in the uncertainty budget range $0.35 \leq \Gamma^* \leq 0.50$. This observation indicates that a naïve operator who ignores the uncertainty related to the quality of the maintenance activities (by setting $\Gamma = 0$) achieves an average out-of-sample reliability of 63.80%, compared to a more sophisticated operator who recognizes the presence of such uncertainty (by setting $\Gamma = \Gamma^*$), thus attains a 68.99% average system reliability. If the proposed robust approach is used to improve the expected performance of the system, $0.35 \leq \Gamma^* \leq 0.50$ should be used.

Besides the improvement in average out-of-sample system reliability, there is a similar and more profound improvement in the worst-case out-of-sample reliability (*i.e.*, the lowest system reliability achieved in the test sample) when using a sufficiently-large uncertainty budget. The maximum improvement in this metric is achieved at $\Gamma = \Gamma^*$, with a worst-case reliability value of 0.5820 compared to 0.4443 in the nominal case. However, even for non-optimal uncertainty budget values, the robust maintenance plan always outperforms the nominal one in terms of the worst-case performance. These is also a significant reduction in the variability of out-of-sample system reliability when a sufficiently-large uncertainty budget is used. To better show this effect, the *coefficient of*



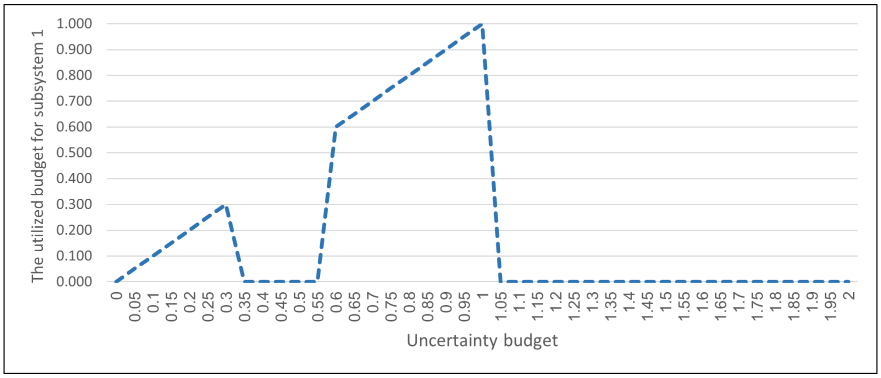
(a) Out-of-sample system reliability



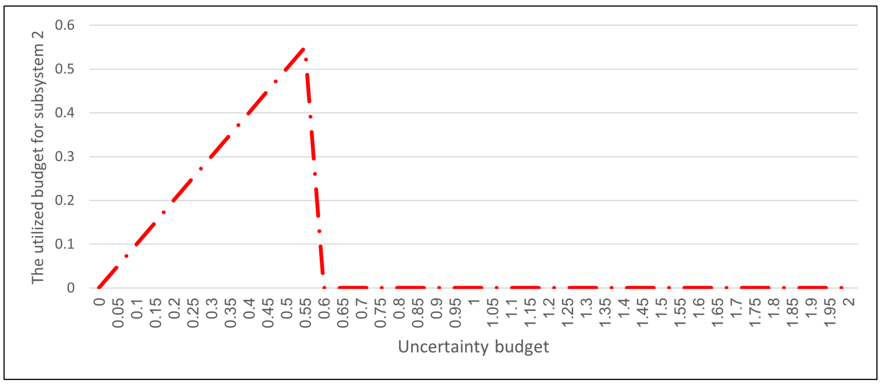
(b) Coefficient of Variation (CV)

Figure 5.6: Effect of the uncertainty budget on system performance.

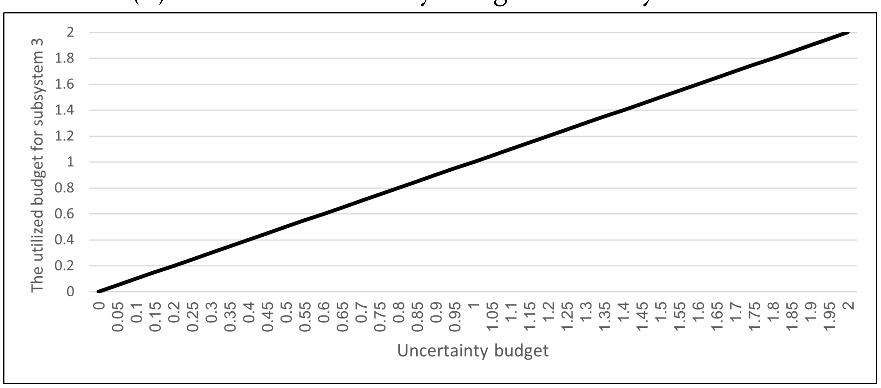
variation (CV) (*i.e.*, the ratio of the standard deviation to the average out-of-sample system reliability) is plotted vs. the uncertainty budget in Figure 5.6b. At the beginning, when the uncertainty budget is small ($0 \leq \Gamma \leq 0.15$), the variability is high with a CV value of 0.1. In other words, the nominal maintenance plan does not provide adequate protection against variability and uncertainty. As the uncertainty budget is increased, the CV decreases gradually until it reaches 0.025 for high values of Γ. This reduction is due to the hedging effect of RO, which leads to conservative solutions that perform relatively and consistently well even for bad scenarios.



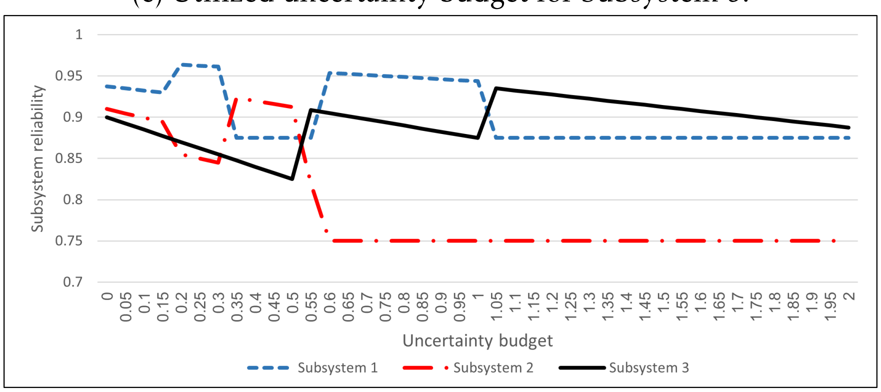
(a) Utilized uncertainty budget for subsystem 1.



(b) Utilized uncertainty budget for subsystem 2.



(c) Utilized uncertainty budget for Subsystem 3.



(d) Reliability of the subsystems

Figure 5.7: Utilized uncertainty budget per subsystem

We then investigate how the uncertainty budgets are utilized. In this context, the robust problem is perceived as a game between an adversary (*i.e.*, nature) that tries to inflict the maximum “damage" by controlling the allocation of available uncertainty budgets, and a planner that tries to protect the system from this damage by prudently selecting which maintenance actions to perform.

Figures (5.7a)-(5.7c) illustrate the utilization of uncertainty budget for each subsystem $i = (1, 2, 3)$. For the first two (5-by-5) subsystems, only one component is selected to be maintained in each, thus limiting nature’s ability to utilize more than one unit of the available budget. In contrast, the third subsystem is considered the weakest link in the system since it consists of only three parallel components. Accordingly, to maximize the system reliability, all of the components are subjected to maintenance, giving nature the opportunity to utilize the entire uncertainty budget at its disposal.

Table 5.8: Maintenance quality and quantity as uncertainty budget increases

Uncertainty Budget	Subsystem 1	Subsystem 2	Subsystem 3	# Maintenance actions performed
0.0 - 0.1	MR	IM_1, IM_1	R	4
0.2 - 0.3	MR,MR	MR	R	4
0.4	-	R	R	2
0.5	-	R	R,MR	3
0.6 - 1.0	MR	-	R,MR	3
1.1 - 2.0	-	-	R,MR,MR	3

Figure 5.7d portrays the relationship between the uncertainty budget and the resulting reliability for each subsystem. Moreover, Table 5.8 provides details about the maintenance actions performed on each subsystem at different uncertainty budgets. At the beginning ($0 \leq \Gamma \leq 0.40$), when the maintenance budget available to inflict damage is small, the maximum number of maintenance actions (within the limited break duration and budget) are performed in all subsystems since there is no significant risk of making these maintenance actions worse than planned. However, as the uncertainty budget is increased, fewer maintenance actions are performed on the “non-critical" subsystems (1 and 2), especially maintenance levels that are prone to wide variability like IM_1. Instead, the maintenance budget and repairpersons’ time are used to strengthen the weakest link in the system (*i.e.*, the third subsystem) by performing additional maintenance actions on

it, especially those with mild quality variability such as R and MR.

Finally, Figure 5.8 illustrates the relationship between the average computational time when solving the robust problem (5.29) as the uncertainty budget Γ_i was varied. As shown, the computational time for solving the 5-5-3 series parallel system with two repairpersons is fairly stable between 1.5 and 2.5 seconds.

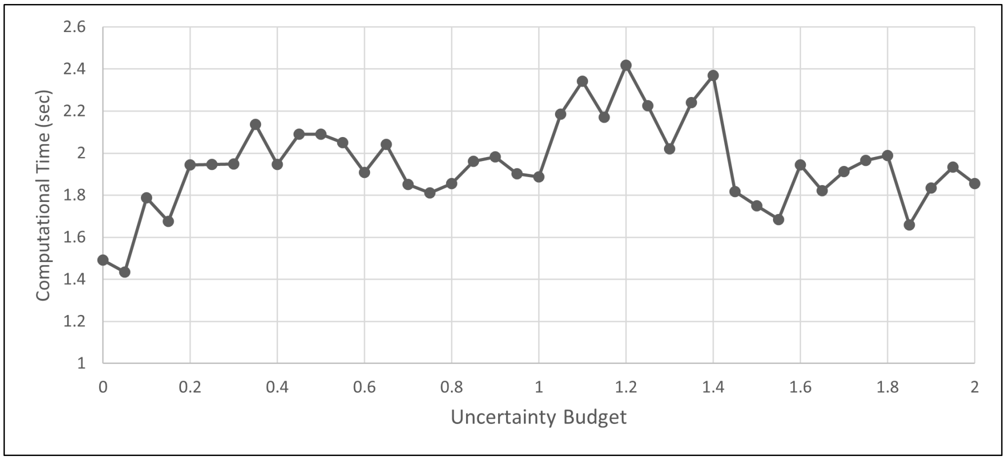


Figure 5.8: The computational performance of the robust problem.

5.5 Conclusions

In this paper, the important issue of maintenance quality uncertainty in the context of the JSM–RAP with a series-parallel system was addressed through a RO framework. We first showed how the nominal problem can be reformulated into a MIECP that can be readily handled by efficient off-the-shelf solvers. Then, we considered the robust case and used a non-symmetric budget uncertainty set to model the post-maintenance reliability of the system components such that the level of conservatism can be controlled. We were able to reformulate the robust problem into a MIECP by using the concept of Fenchel duality.

Three sets of numerical experiments demonstrated the advantages of the proposed reformulations in terms of both computational time and solution quality. In particular, the MIECP formulation enabled the nominal problem to be exactly solved much more efficiently compared to the two-phase approach of Diallo et al. (2019b). In addition, we showed how the robust formulation can be used to obtain performance guarantees, improve the average and worst-case out-of-sample performance and reduce variability with a properly-sized uncertainty budget. We also analyzed how the available uncertainty budget is utilized and how the set of maintenance actions is affected by it.

The ability to reformulate the JSM–RAP for a series-parallel systems as a MIECP opens the door for several potential extensions. On the one hand, it would be interesting to see if similar “nice" reformulations could be derived for systems with other reliability structures (*e.g.*, serial k out of n structure). Likewise, other variants of the SMP, *e.g.*, multimission and fleet SMP, and closely-related problems such as the reliability allocation problem might be amenable to such formulations. On the other hand, besides maintenance quality, other aspects of uncertainty in the SMP, including maintenance action times and costs, mission duration and break length, can be addressed similarly in a RO framework. Furthermore, starting from the nominal MIECP formulation, one can apply other ways for handling uncertainty (*e.g.*, distributionally robust optimization or robust-stochastic optimization) in case some distributional data is available to alleviate the conservatism of RO. Finally, mixed-integer linear and second-order conic approximations of the MIECP can be used to further improve the computational performance and enable extremely large SMP problems to be solved (Ye and Xie, 2021).

Chapter 6

Conclusions

This dissertation explored four themes dealing with the optimization of JSM–RAP for large-scale systems under uncertainty. The first theme provided a critical review of selective maintenance literature, identifying challenges and potential areas for future research. The second theme introduced four CG-based algorithms to effectively address the JSM–RAP for large-scale systems. The third theme presented a PLA-based approach and a DRC-W to handle uncertain maintenance duration in large-scale instances of JSM–RAP. The fourth theme addressed the optimization of JSM–RAP when the quality of maintenance actions is uncertain and impacts the post-maintenance reliability of system components. The conclusions and future research extensions for the four themes are discussed below.

While there has been a significant increase in research in the field of the SMP in the last five years, driven by advancements in robust optimization and machine learning, no recent review of the SMP literature exists. In Chapter 2 dealing with Theme 1, a state-of-the-art literature review of SM was conducted to provide a comprehensive understanding of the field and its advancements. The review examined 119 research articles related to SM optimization using a systematic classification and analysis framework to classify and analyze the models. The review also identified the limitations and potential areas for future research in the field.

As outlined at the end of Chapter 2, current methods for the SMP are not adequate for handling large systems or addressing uncertainty in maintenance parameters due to a lack of sufficient maintenance records and tractable formulations. Therefore, Theme 2 in Chapter 3 dealt with the development of four column-generation-based methods to solve large-scale instances of the JSM–RAP. The approaches employed a column-generation technique which iterated between solving a restricted master problem to update the dual multipliers and solving multiple subproblems to generate maintenance patterns. Two novel reformulations were proposed for the mixed-integer nonlinear subproblems (CG–PLA, CG–ECO), and a heuristic procedure and stabilization scheme were developed to improve the algorithm's convergence. Additionally, CG was embedded within a B&B tree to devise branch-and-price algorithms restoring integrality and ensuring solution optimality.

Numerical experiments revealed the effectiveness of the proposed approach in solving large-scale instances of the JSM–RAP and finding optimal solutions. The computation times for the CG–PLA and CG–ECO approaches were found to be significantly shorter than those of a previous two-phase approach (Diallo et al., 2019b). However, the CPU times for the B&P algorithms were longer than those of the CG-based approaches without B&B. Despite this, the two B&P algorithms were able to solve all instances optimally. The CG–ECO approach, which uses an exact reformulation of the subproblem was found capable of guaranteeing solution optimality when combined with B&B. For large-scale instances, whether CG–PLA or CG–ECO was used, the gap between the reliability obtained from solving the JSM–RAP as a BIP problem and that obtained from the relaxed problem was rather small, indicating that the solutions obtained were very close to the unknown optimal solution. The proposed algorithms were found to be efficient and able to solve a problem with more than double the number of components in the previous largest JSM–RAP instance solved. Additionally, the stabilization scheme was found to be effective in accelerating the convergence of the CG algorithm for large-scale systems with multiple maintenance levels.

Under Theme 3 in Chapter 4, we proposed an alternative approach for solving JSM–RAPs by implementing the piecewise linear approximation approach directly into the JSM–RAP BIP formulation. A distributionally robust chance-constrained program with a Wasserstein ambiguity set was then proposed along with the developed PLA-based formulation to deal with uncertain maintenance durations in the JSM–RAP.

Three sets of numerical experiments were carried out and they showed the capability of the proposed approaches to deal with large-scale JSM–RAPs with uncertain maintenance duration. The developed data-driven distributionally robust chance constraint formulation ensured that effective maintenance plans could be determined with a high probability of completion. In addition, the computation times of the proposed PLA-based approach were significantly less than the computation times of the 2-phase approach presented in Diallo et al. (2019b). The efficiency of the proposed methods was such that it was at least an order of magnitude faster than (Ikonen et al., 2020) in solving its largest JSM-RAP instance (700 components).

Theme 4 introduced the optimization of the JSM–RAP when the quality of maintenance actions is uncertain, thus leading to uncertain post-maintenance reliability of system components. A robust optimization framework was employed to represent this uncertainty through non-symmetric budget uncertainty sets, giving decision-makers the ability to manage their level of conservatism. Both the nominal and robust problems are reformulated as MIECPs that can be solved using off-the-shelf solvers. Through Fenchel duality, the proposed MIECP-based formulation enabled robust optimization to be implemented easily, leading to a tractable reformulation of the same class of the nominal problem and having the same number of discrete variables.
Three sets of numerical experiments have been performed to show the benefits of the proposed reformulations in terms of computational time and solution quality. In particular, the use of the MIECP formulation allowed us to solve the nominal problem with high accuracy and efficiency compared to the two-phase approach of Diallo et al. (2019b). Furthermore, it was demonstrated how the robust formulation can be used to achieve performance guarantees, enhance the average and worst-case out-of-sample performance, and decrease variability with a suitable uncertainty budget. The effects of utilizing the available uncertainty budget on the set of maintenance actions were also examined.

In summary, for large-scale systems, the PLA-based approaches (CG–PLA, PLA) are recommended if a short computational time with less emphasis on optimality is desired. On the other hand, if exact optimality and/or consideration of uncertainty in SMP parameters is a priority, the ECO-based approaches (CG–ECO, ECO) are recommended. When it comes to whether column generation shall be utilized or not, it depends on the problem size. The obtained numerical results indicate that for problems with up to several hundred components, employing direct methods like PLA and ECO without column generation is recommended. However, for larger-scale problems encompassing around a thousand components or more, it is preferable to utilize column generation-based techniques, including CG-PLA and CG-ECO.

The area of the SMP offers numerous opportunities for further exploration and investigation. The examination of research gaps and the proposal of a 10-point road map to guide the advancement of SMP research were emphasized in Theme 1 of the dissertation.

In this dissertation, the joint repairperson assignment extension (JSM–RAP) has been investigated, and the added value of the proposed advancements in handling large-scale systems under uncertainty has been demonstrated. Future research for JSM–RAP should aim to improve the JSM–RAP model by incorporating new ideas such as accounting for asynchronous break periods, handling complex reliability structures (*i.e.*, weighted or consecutive k – out of – n configurations), considering fleet-level selective maintenance, and incorporating stochastic and economic dependencies. The JSM-RAP currently assumes that all repairs can take place without repair persons colliding. Future research could explore the impact of space constraints, work interruption and resumption, repair prioritization, sequence-dependent maintenance duration, etc. Relaxing the deterministic assumption for other parameters and incorporating the operational performance of components into the model are also areas for future study.

Throughout this dissertation, PLA and MIECP were implemented in conjunction with a DRCC program and a RO program to address the uncertainties of maintenance duration and quality in the context of JSM-RAP. As a prospective research extension, the adoption of the proposed column generation-based techniques (CG-PLA, CG-ECO) in combination with the DRCC and RO programs should facilitate the effective resolution of larger JSM-RAP instances that incorporate uncertainties in maintenance duration and/or quality.

In addition to reliability, future research should focus on optimizing system availability and evaluating trade-offs between availability and repairperson hiring. The joint maintenance and crew scheduling problem, and methods to solve it, could also be explored. Robust-stochastic optimization and mixed-integer linear and second-order conic approximations can be used to improve computational performance and reduce conservatism in the results. By doing so, we can advance our understanding of selective maintenance optimization and ultimately provide more reliable and efficient solutions for large-scale systems under uncertainty.

Through these efforts, efficient and improved formulations that accurately reflect real-world problems for large-scale systems facing uncertainty can be provided. This will enhance the ability of practitioners to apply these solutions to their various systems, closing the divide between theoretical knowledge and actual real-world problems.

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