Asymmetric Systematic Risk and Risk Premiums Under a Regime-Switching Model

by

Ziwei Zhang

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Abstract

The purpose of this thesis is to identify asymmetry in stocks' systematic risk and market risk premiums under different financial market regimes. It is assumed that there are two unobserved regimes, bull and bear markets, in the U.S. stock market, which follow a hidden Markov process. A sample of 597 firms that are traded on multiple U.S. stock exchanges from January 1986 to October 2021 is used to test the hypotheses that systematic risk and market risk premiums are asymmetric in different market regimes. It is found that there is a strong asymmetry in the stocks' systematic risk under both the extended CAPM and the Fama and French three-factor model setting. Beta asymmetry on the two control factors (SMB and HML) in the Fama and French three-factor model are also tested. To test asymmetric market risk premiums, the cross-sectional regression is used and finds evidence that supports asymmetric market risk premium in both the extended CAPM and the Fama and French three-factor model. However, there is no asymmetric relation in either size or value coefficients in the different regimes for the Fama and French three-factor model. The main contribution to the existing literature is that a structural hidden Markov model is applied to test asymmetric parameters, especially systematic risk as well as market risk premium, in both the extended CAPM and the Fama and French three-factor model in the U.S. stock markets.

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Chapter 1. Introduction

The volatility of the stock market can increase investment risk. Beta, measuring systematic risk of a financial security, is an important indicator for investors. Investors usually examine a stock's past performance and adjust investment strategies for better allocation by analyzing beta. The market risk premium is the difference between the expected return on a market portfolio and the risk-free rate. A high market premium attracts the risk lover to invest more into the stock in order to earn higher expected returns. Hence, both systematic risk and market risk premium are essential for investors to make a better decision on investing in the stock market. Markowitz (1952) developed the theory that rational investors would like to choose multiple assets or securities, rather than invest in a single asset, because they can diversify the risk. He believed that rational investors choose the one with the highest risk-adjusted return from their feasible investment decisions. Subsequently, William Sharpe (1964), John Lintner (1965), and Fischer Black (1972) independently developed the Capital Asset Price Model (CAPM) based on Markowitz's model. The CAPM proved a linear relationship between systematic risk (beta) and expected return. In this model, beta is the only factor to consider and assumed to be constant over time.

There is no doubt that the CAPM plays an important role for managers in making an investment decision, and many scholars, such as Fama and MacBeth (1973), support the model by testing its validity. However, with more and more research related to the CAPM, some critics contend that the model is not realistic because it cannot explain anomalies, such as size and momentum effect. Therefore, later research is likely to use more control factors to explain anomalies. For example, the Fama and French three-factor model is one of the multi-factor models to modify the CAPM by considering the size of firms and book-to-market value.

The Markov regime-switching model is a popular approach used in the macroeconomics area, and it can measure data with different patterns, such as asymmetry. The traditional time-series model is used to regress a single and linear model to discover the data patterns, but it does not take multiple regimes into account. The regime-switching model has overcome this limitation. For instance, the unemployment rate is a major macroeconomic indicator used to measure the number of unemployed people expressed as a percentage of the labor force in the U.S. The unemployment rate is low during expansion, but high during contraction. In this case, it is hard to use a linear model to explain the behavior of these data. Hence, the regime-switching model is a useful approach to deal with the nonlinear time-series model by dividing the various patterns into different regimes or states. The regime-switching model is not only used in macroeconomic areas but also in financial market areas, such as testing asymmetric beta and asymmetric market risk premiums in different economic/financial strengths.

The National Bureau of Economic Research's website provides information on the business cycle in the U.S. economic market. A business cycle is a graph of the periodic growth and decline of a nation's economic activities. Growth is linked to economic expansion and contraction happens after an expansion period. Meanwhile, the terms bull and the bear market, used to describe performance of the stock market, can relate to the expansion and contraction used for characterizing economic conditions. It is generally agreed that the bull market is the time of economic growth (expansion), while the bear market is the economic decline (contraction). Beta and market risk premiums fluctuate in the bear market but are relatively stable in the bull market. Because of that, it is necessary to distinguish different patterns for each beta and market risk premium by using the regime-switching model.

Previous research focused on testing the validity of the CAPM and the Fama and French three-factor model by testing the significance of the intercept. Also, many scholars have proved beta asymmetry by using the regime-switching model. Nevertheless, no one uses the regime-switching model to test asymmetric parameters systemically in the extended CAPM and the Fama and French three-factor model. This thesis will test each parameter in both models by using the regime-switching model in time-series regression and cross-sectional regression. First, the significance of the intercept in both regression models will be tested. Secondly, the asymmetry of each coefficient will be tested. However, the thesis will focus more on the results related to asymmetric beta in time-series regression and asymmetric market risk premium in cross-sectional regression.

This thesis focuses on testing asymmetric parameters in both the extended CAPM and the Fama and French three-factor model by using the Markov switching model. In the calibration of the regime-switching model, I divide the stock market into two regimes, bull market and bear market. I use a sample of 597 firms that are traded on multiple U.S. stock exchanges during the period January 1986 to October 2021 for testing the hypotheses. Both time-series and cross-sectional regression are used for parameter estimation. The main contributions of this thesis to the current literature are two-fold: i) testing asymmetric beta and asymmetric market risk premium in both the extended CAPM and the Fama and French three-factor model via a hidden Markov model, and ii) providing an additional study to test the time-varying beta and market risk premium of the two models in the U.S. stock market to help with further research.

Section 2 reviews previous studies related to asymmetric beta and market risk premium. Section 3 introduces the models' specifications and hypotheses. Section 4 presents the source of data and testing results in the extended CAPM and the Fama and French three-factor model under different regimes. Section 5 summarizes the findings and concludes the thesis.

Chapter 2. Literature Review

Sharpe (1964), Lintner (1965), and Black (1972) independently developed the Capital Asset Pricing Model (CAPM), while studying the relationship between systematic risk and expected return on an asset. The model was based on the portfolio theory in Markowitz's (1952) paper, "Portfolio Selection". Markowitz used the mathematical method to do mean-variance analysis to help investors choose an optimal portfolio combination. Rational investors need to know they will have lower risk if they buy multiple portfolios rather than just one because portfolio combination can help them diversify risk. In other words, investors should not just focus on one stock earning and risk but need to consider the risk among the whole investment portfolios.

The CAPM assumes that beta for the given portfolio and market risk premium are both constant over time, and that all investors have the same expectation, including expected returns, standard deviations, and covariances. The model became the necessary reference to help managers make investment decisions.

Because of the importance of the CAPM, scholars used it in several ways to perform empirical tests to see the validity of the model. For example, Fama and MacBeth (1973) used the monthly percentage returns for all common stocks available for investors to trade from the New York Stock Exchange (NYSE) between January 1926 and June 1968 to test the relation between average return and risk of common stock on the NYSE. After an empirical test using the CAPM, they concluded there is a linear relationship between the risk and expected return after including unsystematic risk and the squared market beta, so the null hypothesis cannot be rejected, which proves the validity of this two-parameter model.

However, each model has shortcomings, including the CAPM, because it cannot explain all the stock market anomalies, such as size effects. With the empirical testing results of the CAPM, more and more scholars found that the model is not perfect. For example, Fama and French (1992) reached a different conclusion from Fama and Macbeth (1973). Fama and French (1992) used the non-financial stocks in NYSE, AMEX, and NASDAQ from 1963 to 1990 to evaluate the market beta, size, earnings/price ratio, leverage, and book-to-market. Their findings do not support the conclusion of the Sharpe–Linter–Black model, and their results show there is no relation between beta and average expected return, which indicates that beta is not enough to explain stock performance. Fama and French (1992) thought they reached a different conclusion from Fama and Macbeth (1973) because they choose a different sample period. Fama and MacBeth (1973) considered the period 1926–1968, while Fama and French (1992) test the model using data from period 1963–1990. They also found that beta itself had weak explanatory power between 1941 and 1990.

Moreover, the choice of a specific period is not the only shortcoming for the CAPM. The model is not realistic and cannot apply in the actual stock market because systematic risk and risk premiums are not constant over time. For example, the business cycle is an essential reason to consider that beta and expected return are related to information available at any time (Jagannathan and Wang, 1996). Stock price varies with the change of business cycle and so does the risk premium. Therefore, the unconditional CAPM cannot explain cross-sectional returns. Also, the average return can be explained by other factors, such as market volatility (Black, 1976), leverage (Bhandari, 1988), credit spread, and yield spread (Campbell, 1987).

Furthermore, the CAPM cannot explain some anomalies, such as size effect (Banz, 1981), momentum effect (Grundy and Martin, 2001), price-earnings ratio effect (Basu, 1977), and value effects (Rosenberg, Reid, and Lanstein, 1985). Banz (1981) came up with the “size effect” and indicated that firms’ shares with large market values have had smaller returns than small firms generally. Also, Grundy and Martin (2001) pointed out that the CAPM and the Fama and French three-factor model cannot explain the momentum effect. Also, Basu (1977) believed there is a relation between the investment performance of equity security and their price earnings ratio. To be specific, the price earnings ratio and return have a negative relation. Moreover, Rosenberg et al. (1985) studied the firms’ book value on the NYSE and found there is a positive relation between average return and firms’ book value on U.S. stocks.

As early researchers found, beta is time varying (Fabozzi and Frances, 1978), so it is important to take time-varying beta into account when managers make investment decisions. Because of the importance of the time-varying beta, there is much research related to it. In 1977, Bawa and Lindenberg (1977) pointed out that the CAPM assumption is not realistic because beta cannot be constant over time and there are downside and upside betas that should be considered. After that, Ang, Chen, and Xing (2006) calculated the upside (downside) beta when the rate of market return is higher (lower) than its average. They concluded that the stock with a higher downside beta usually has higher average rates of return. However, Levi and Welch (2020) doubted the results from Ang et al. (2006) because they did not consider market factor control. Levi and Welch stated that they reach a different conclusion if they use ex-ante instead of ex-post down-betas to do the test. The positive relationship between down-betas and rate of returns will disappear based on this change. Therefore, high-down beta does not mean investors can get higher average rates of return ex-post. They concluded that prevailing plain market beta is better than asymmetric down-beta for better prediction.

Time-varying beta can be tested by several methods. Bekaert and Wu (2000) used GARCH in mean parameterization to make sure conditional means, variances, and covariances are time varying. Brooks et al. (1998) used the Kalman Filter method to estimate time-varying beta in Australian industry portfolios. The Markov switching model is also an essential method to estimate time-varying beta. It is used to model the probabilities of different regimes and calculate the rates of transitions among them. Abdymomunov and Morley (2011) used the two-state Markov switching model to test beta instability in the CAPM between low- and high-volatility regimes and found time-varying beta is better than the unconditional CAPM, particularly in a market with high volatility. Also, Huang (2000) used low-risk and high-risk states to test the beta of the CAPM and pointed out that data from the high-risk regime are inconsistent with the CAPM, but data from the low-risk regime are consistent with the CAPM. Furthermore, Vendrame et al. (2018) estimated bull and bear regimes' probability by using the Markov switching approach to calculate conditional risk premiums and found that evidence for the beta from the conditional CAPM has the power to explain both the bull and bear markets.

Chapter 3. Model Specification and Testing Hypotheses

3.1 Macroeconomic model and hidden Markov modelling

Macroeconomic indicators, such as gross domestic product (GDP) and industrial production, are essential to evaluate the overall economic environment. Also, those indicators can help investors predict the trend of the future economic market. Generally, the economic market divides the business cycle into expansion and contraction periods, and the financial market divides it into the bull and bear markets. Although those regimes are unobservable, I can calculate the probabilities for each regime by using hidden Markov modelling (HMM). HMM was employed by Hamilton (1989) in the paper, "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle", and has been widely used to evaluate probability distributions over sequences of observations. In using the model, it is necessary to put all the selected macroeconomic indicators into one vector, F_t, and assume the regime follows a multivariate normal distribution for its economic regime:

$$F_t = A_{s_t} + B_{s_t} F_{t-1} + \sigma_{s_t} \varepsilon_t \tag{1}$$

In equation (1), A_{s_t}, B_{s_t}, and σ_{s_t}, are regime-dependent model parameters. ε_t is a standard multivariate normal random variable, and F_{t-1} is the observed data from last period. These quantities are used to predict future economic data, F_t.

The Bayesian information criterion is a widely used method for model selection or for determination of the optimal number of regimes. Instead of going through an estimation process, I will use just two regimes—bull market and bear market—in this thesis, in comply with the NBER analysis on U.S. economic strength. Because two regimes will be used in this thesis, s_t will take two values, 1 and 2, as equation (1) shows.

Moreover, all the macroeconomic indicators are used as observed data to calculate the conditional probabilities of the regimes. Because the switching behavior of the beta is used by the transition probability matrix (Mergner and Bulla, 2008), the changing process follows the first-order Markov chain:

p[s_t=1 |s_{t-1}= 1]=p

p[s_t=2 |s_{t-1}= 1]=1-p

p[s_t=2 |s_{t-1}= 2]=q

p[s_t=1 |s_{t-1}= 2]=1-q,

where p[s_t=j |s_{t-1}= i] is the probability that regime i changes to regime j in period t. So, we assume that the transition probability matrix of the regimes is independent on time and can be written as

$$\begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$$

where p and q are the transition probabilities staying in regime 1 and regime 2 in each period, respectively. The next step is to estimate unobservable parameters via hidden Markov model. The set of parameters is denoted as (X, J, θ). According to the set, X represents the sequence of observations, J represents the unobservable regimes over the time period, and θ represents the unknown parameters. Therefore, the maximum log-likelihood can be expressed as

$$L = max\{ln \sum_J Pr(X, J;\ \theta)\} \tag{2}$$

Based on equation (2), $Pr(X, J;\ \theta)$ represents the joint probability distribution function of X and J. To guarantee the maximal results for θ, the expectation-maximization algorithm (Dempster, Laird, and Rubin, 1977) is to be used. The expectation-maximization algorithm has two steps, called E-step and M-step. E-step determines the expected log-likelihood and M-step maximizes the expected log-likelihood from E-step. To start with E-step, I first set an initial value θ^0 for θ, and then compute the conditional distribution function, Pr(J|X; θ^0), given the observed data, X, and initial value, θ^0. Hence, I can determine the expected log-likelihood E[$ln \sum_J Pr(X, J;\ \theta)$]. The M-step is to find the θ that maximizes the expected log-likelihood. Let θ^1 be the solution, which is used as the initial value for the next iteration. It is known that the value of the objective function is monotonically increasing and the estimated θ converges to a local optimal maximizer.

By simplifying the EM algorithm above, the Markov-switching model can be expressed as:

$$\max_{\theta}\left\{\sum_{j=1}^{s_t}\sum_{t=1}^{T} Pr(s_t = j|X;\ \theta^0) lnPr(x_t|s_t = j; \theta)\right\} \tag{3}$$

In equation (3), T represents the number of time periods and x_t is the realization of X_t, and s_t stands for the unobservable regimes.

3.2 CAPM specification

Sharpe (1964), Lintner (1965), and Black (1972) built the unconditional CAPM based on Markowitz's (1952) theory. Some essential assumptions of the CAPM need to be reviewed. First, investors are risk-averse and prefer to earn returns with low uncertainty. Secondly, all investors are supposed to receive timely, relevant information, so there is no information asymmetry. Unlimited capital is available for investors' borrowing and lending at a risk-free rate of interest. Meanwhile, all investors share the same estimation of means, variance, and covariance of all assets. Also, the standard CAPM assumes there are no taxes and transaction costs. Finally, investors have the same time horizon.

Sharpe (1964), Lintner (1965), and Black (1972) independently studied the relation between systematic risk and expected return and proved the existence of a linear relation between them. The CAPM focuses on only one factor, which is the market portfolio, rather than considering multiple factors. The beta measures the sensitivity of the stock's volatility to the market portfolio, which can be described mathematically as:

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)}$$

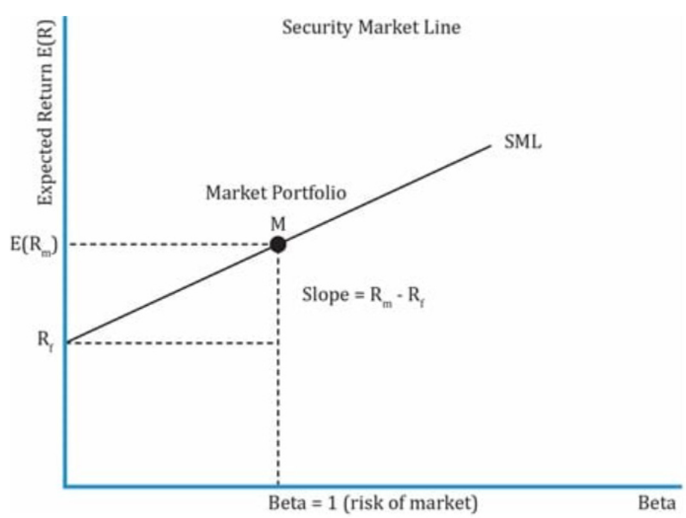
where $Cov(R_i, R_m)$ is the covariance between the return on asset i and the return on the market portfolio, and $Var(R_m)$ denoted the variance of the return on the market portfolio.

Moreover, it is worth mentioning that security market line (SML) shows the results from the CAPM. In the plot of SML in Figure 1, the x-axis stands for the beta, while the y-axis stands for the expected return, so the slope represents the market risk premium, which is $E(R_m) - R_f$. Therefore, SML (also the CAPM) can be expressed as the following equation:

$$E(R_i) = R_f + \beta_i * [E(R_m) - R_f] \tag{4}$$

- E(R_i) is the expected return on asset i;
- R_f is the risk-free rate;
- β_i measure systematic risk on asset i;
- $E(R_m) - R_f$ is the market premium, which is the difference between the rate of expected market return and the risk-free rate.

Figure 1. The plot of security market line



Time-series regression is one of the approaches used in the thesis; it was applied by Jensen (1968) to evaluate the performance of the mutual fund. Subsequently, time-series regression was used to test the CAPM by Jensen et al. (1972) who used the traditional form of the regression by adding the intercept (α) to test the model. Therefore, the new equation with intercept can be rewritten as:

$$R_{i,t} - R_f = \alpha_i + \beta_i * (R_{m,t} - R_f) + \varepsilon_{i,t} \quad (5)$$

In equation (5), $(R_{i,t} - R_f)$ and $(R_{m,t} - R_f)$ denoted the excess returns on asset i and market excess returns at time t (monthly), respectively.

Because Markov regime-switching model is used in our study, I add a new notation, s_t, to represent the different regimes. "s_t" can be regimes 1 and 2. Therefore, equation (5) can be rewritten as equation (6) under regime-switching model:

$$R_{i,t} - R_f = \alpha_{i,s_t} + \beta_{i,s_t} * (R_{m,t} - R_f) + \varepsilon_{i,t} \quad (6)$$

where $(R_{i,t} - R_f)$ is the monthly individual asset excess returns and $(R_{m,t} - R_f)$ is the monthly market excess returns. $\alpha_{i,st}$ is the intercept of the equation, and s_t represents regime 1 or regime 2, so:

$$\alpha_{s_t} = \begin{cases} \alpha_1 \text{ if } s_t = 1 \\ \alpha_2 \text{ if } s_t = 2 \end{cases} \qquad \beta_{s_t} = \begin{cases} \beta_1 \text{ if } s_t = 1 \\ \beta_2 \text{ if } s_t = 2 \end{cases}$$

α_i and β_i, $i = 1, 2$, are the conditional alphas and conditional betas in each regime. α represents the mean of abnormal returns and β stands for the mean of the systematic risk of all assets.

Based on Jensen (1972), intercept (α) was used to test whether the model is valid. Accordingly, I develop the hypotheses to test the significance of the intercept:

$$H_0: \ \alpha_1 = 0, \ H_A: \ \alpha_1 \neq 0 \quad (7)$$

$$H_0: \ \alpha_2 = 0, \ H_A: \ \alpha_2 \neq 0 \quad (8)$$

α_{s_t} is the average intercept in regime s_t. If the average of the intercept, α, is significantly different from zero in regime 1, the null hypothesis should be rejected in hypothesis equation (7). Similarly, if the average of the intercept is significantly different from zero in regime 2, it indicates that the null hypothesis will be rejected in hypothesis equation

(8). Finally, I can test the asymmetry between α_1 and α_2, which is the hypothesis equation (9).

$$H_0: \ \alpha_1 = \alpha_2, \ H_A: \ \alpha_1 \neq \alpha_2 \tag{9}$$

The next parameter is β_{m,s_t}, which is the average market beta in regime s_t. One objective of this thesis is to test the asymmetry of beta in different regimes, but one-sample for mean is used first to test the significance of β_m under two different regimes:

$$H_0: \ \beta_{m,1} = 0, \ H_A: \ \beta_{m,1} \neq \ 0 \tag{10}$$

$$H_0: \ \beta_{m,2} = 0, \ H_A: \ \beta_{m,2} \neq \ 0 \tag{11}$$

And then, two-sample for mean test related to the asymmetric systematic risk can be specified as:

$$H_0: \ \beta_{m,1} = \beta_{m,2}, \ H_A: \ \beta_{m,1} \neq \ \beta_{m,2} \tag{12}$$

In equation (12), β_{m,s_t} stands for the mean of all stocks' beta in regime s_t (s_t= 1, 2). If the null hypothesis in equation (12) is rejected in favor of the alternative, it represents the existence of asymmetric beta in the extended CAPM.

It is the time turn to the cross-sectional tests, developed by Fama and MacBeth in 1973, for asymmetric market risk premiums. The advantage of the cross-sectional tests is that they can compare the different individual assets at the same time point. A simple cross-sectional regression of the extended CAPM at any point in time t can be written as:

$$R_{i,t} - \ R_f = \gamma_{0,t,s_t} + \gamma_{1,t,s_t} * \beta_{i,m,s_t} + \varepsilon_t \tag{13}$$

- $R_{i,t} - \ R_f$ is the rate of excess return for asset i;
- $\beta_{i,m,s_t}$is the estimated from the time-series regressions for asset i;
- $\gamma_{0,t,s_t}$is the intercept in the regime s_t at time t;
- γ_{1,t,s_t} is the linear coefficient in the regime s_t at time t.

The β_i in the cross-sectional regressions are the estimated β_{i,m,s_t} from time-series regression. Let $\gamma_{0,1}$ and $\gamma_{0,2}$ be the average of intercepts over time for regimes 1 and 2. The test of the hypothesis also considers testing the significance of the average of the intercept:

$$H_0: \ \gamma_{0,1} = 0, \ H_A: \ \gamma_{0,1} \neq 0 \tag{14}$$

$$H_0: \ \gamma_{0,2} = 0, \ H_A: \ \gamma_{0,2} \neq 0 \tag{15}$$

Hypotheses (14) and (15) test whether the averages of the intercepts are significantly different from zero in different regimes. If both null hypotheses related to $\gamma_{0,1}$ and $\gamma_{0,2}$ can be rejected, it proves the average of $\gamma_{0,1}$ and the average of $\gamma_{0,2}$ are significantly different from zero. And then, the next hypothesis related to the equality between the average of $\gamma_{0,1}$ and the average of $\gamma_{0,2}$ will be developed.

$$H_0: \ \gamma_{0,1} = \gamma_{0,2} \tag{16}$$

$$H_A: \ \gamma_{0,1} \neq \gamma_{0,2}$$

The last and most important hypothesis tests the existence of asymmetric market premium, which needs to consider the relationship between the average of $\gamma_{1,1}$ and the average of $\gamma_{1,2}$. Before doing that, it is necessary to check the relationship between the average of the estimated market risk premium over all time periods, γ_1, from cross-sectional regression and market risk premium, E(R_m)-R_f, that is estimated from the time-series regression. The market risk premium that is estimated from regime dependent time-series regression and then is defined as the difference between the conditional expected market return by regime and risk-free rate; E(R_m)-R_f. For convenience, $\gamma_{m,1}$ stands for the value of market risk premium, E(R_m)-R_f, in the bull market, while $\gamma_{m,2}$ stands for the value of market risk premium, E(R_m)-R_f, in the bear market. Also, $\gamma_{1,1}$ and $\gamma_{1,2}$ are the averages of market risk premium that are estimated from cross-sectional regression for regimes 1 and 2, respectively. Hence, hypotheses (17) and (18) test whether the mean of $\gamma_{1,i}$ equals $\gamma_{m,i}$ in different regimes.

$$H_0: \gamma_{1,1} = \gamma_{m,1} \tag{17}$$

$$H_A: \gamma_{1,1} \neq \gamma_{m,1}$$

$$H_0: \gamma_{1,2} = \gamma_{m,2} \tag{18}$$

$$H_A: \gamma_{1,2} \neq \gamma_{m,2}$$

And then, I can test the relation between $\gamma_{1,1}$ and $\gamma_{1,2}$

$$H_0: \gamma_{1,1} = \gamma_{1,2} \tag{19}$$

$$H_A: \gamma_{1,1} \neq \gamma_{1,2}$$

If an asymmetric market risk premium exists, the null hypothesis should be rejected in favor of the alternative hypothesis that market risk premium is asymmetric across market regimes.

3.3 Fama and French three-factor model specification

The Fama and French three-factor model is one of the famous multi-factors models to extend the CAPM. Instead of considering all five factors (market beta, size, earnings/price ratio, leverage, and book-to-market), they focus on market, size, and value factors and successfully explain the cross-section average returns in U.S. stocks (Fama and French, 1992, 1993). SMB (small minus big) is used to measure size factor, and HML (high minus low) is used to measure value factor. According to their research, the sample of stock is divided by size and book-to-market equity. Based on the size ranking, Fama and French got two groups of stocks, big and small, and each group accounts for 50%. Also, the sample stock is divided based on the breakpoints for the bottom 30%, middle 40%, and top 30% of the ranked values of BM. Finally, six portfolios (S/L, S/M, S/H, B/L, B/M, B/H) are used by considering the two sizes and three BE/ME portfolios together. Based on the approach above, the formula for calculating SMB and HML is:

Small-minus-Big (SMB)=1/3(S/L+S/M+S/H-B/L-B/M-B/H) (20)

High-minus-Low (HML)=1/2(S/H+B/H-S/L-B/L) (21)

Because three factors—market premium, size, and value—are assumed to impact expected returns, the time-series Fama and French three-factor model can be expressed as:

$$R_{i,t} - R_f = \alpha_{i,t} + \beta_{i,m} * [R_{m,t} - R_f] + \beta_{i,s} * SMB_t + \beta_{i,h} * HML_t + \varepsilon_{i,t} \quad (22)$$

- $R_{i,t} - R_f$ is the rate of excess return on asset i at time t;
- $\alpha_{i,t}$ is the intercept on asset i at time t;
- $R_{m,t} - R_f$ is the market premium, which is the difference between the rate of expected marketed return and the risk-free rate;
- SMB_t is the rate of the difference between the return on small and big firms' stock at time t;
- $HML_t$is the rate of the difference between the return on high and low book-to-market equity ratio (BE/ME) stocks at time t;
- $\beta_{i,m}$, $\beta_{i,s}$, $\beta_{i,h}$ are the systematic risks for each factor (market premium, SMB, and HML) at time t, respectively;

In the Markov regime-switching model, s_t represents different regimes, so equation (22) can be rewritten as:

$$R_{i,t} - R_f = \alpha_{i,st} + \beta_{i,m,s_t} * [R_{m,t} - R_f] + \beta_{i,s,s_t} * SMB_t + \beta_{i,h,s_t} * HML_t + \varepsilon_{i,t} \quad (23)$$

- $\alpha_{i,st}$ is the intercept for asset i, dependent on regimes (s_t =1,2);
- $R_{i,t} - R_f$ is the excess return on asset i;
- $R_{m,t} - R_f$ is the market premium, which is the difference between the rate of expected marketed return and the risk-free rate;

- β_{i,m,s_t}, β_{i,s,s_t} , β_{i,h,s_t} are the systematic risks of the three independent variables dependent on regimes (s_t *=1,2)*;
- SMB_t is the rate of the difference between the return on small and big firms' stock at time t;
- HML_t is the rate of the difference between the return on high and low book-to-market equity ratio (BE/ME) stocks at time t.

As the hypothesis testing for the CAPM, the Fama and French three-factor model will test the intercept and beta as well. First, I test hypotheses to see whether the average of the intercept is significantly different from zero for each regime ($H_0\colon\ \alpha_1 = 0,\ H_A\colon\ \alpha_1 \neq\ 0$; $H_0\colon\ \alpha_2 = 0,\ H_A\colon\ \alpha_2 \neq\ 0$). Then, I test the asymmetry between the mean of the intercept of two regimes ($H_0\colon\ \alpha_1 = \alpha_2,\ H_A\colon\ \alpha_1 \neq \alpha_2$). Also, one-sample for mean test is used to test the significance of β_m in each regime. Finally, asymmetric beta, which is whether $\beta_{m,1}$, the average of stocks' beta in regime 1 equals $\beta_{m,2}$, the average of stocks beta in regime 2. After testing the asymmetry in the intercept and market risk premium, I would like to test the asymmetry of the other two control factors (SMB and HML).

$$H_0\colon\ \beta_{s,1} = \beta_{s,2},\ H_A\colon\ \beta_{s,1} \neq \beta_{s,2} \tag{24}$$

$$H_0\colon\ \beta_{h,1} = \beta_{h,2},\ H_A\colon\ \beta_{h,1} \neq \beta_{h,2} \tag{25}$$

Equation (24) is used to test the asymmetric size beta between two regimes, while equation (25) tests the asymmetric value beta between two regimes.

Meanwhile, the cross-sectional test developed by Fama and MacBeth (1973) is used here again to test the Fama and French three-factor model, and equation (23) is revised as:

$$R_{i,t} -\ R_f = \gamma_{0,t,s_t} +\ \gamma_{1,t,s_t} * \beta_{i,m,s_t} + \gamma_{2,t,s_t} * \beta_{i,s,s_t} + \gamma_{3,t,s_t} * \beta_{i,h,s_t} + \varepsilon_t \tag{26}$$

- $R_{i,t} -\ R_f$ is the rate of excess return for asset i;

- β_{i,m,s_t} ,β_{i,s,s_t}, β_{i,h,s_t} are estimated from the time-series regression;
- $\gamma_{0,t,s_t}$is the intercept in regime s_t at time t;
- γ_{j,t,s_t} is the linear coefficient in regime $s_t$at time t for the three factors, $j = 1,2,3$.

The hypotheses testing starts with the intercept as well in the Fama and French three-factor model. Let $\gamma_{i,j}$ be the average of coefficients over time as defined for the extended CAPM. First, I will test whether the average of $\gamma_{0,1}$ and the average of $\gamma_{0,2}$ are significantly different from zero ($H_0\colon\ \gamma_{0,1} = 0,\ H_A\colon\ \gamma_{0,1} \neq 0$; $H_0\colon\ \gamma_{0,2} = 0,\ H_A\colon\ \gamma_{0,2} \neq 0$). Secondly, I will test the hypothesis about the relationship between the mean of $\gamma_{0,1}$ and the mean of $\gamma_{0,2}$ ($H_0\colon\ \gamma_{0,1} = \gamma_{0,2},\ H_A\colon\ \gamma_{0,1} \neq \gamma_{0,2}$). Because of the other two control factors in the Fama and French three-factor model, it is still necessary to test these two factors' asymmetry after confirming the relationship between the mean of $\gamma_{1,1}$ and the mean of $\gamma_{1,2}$ ($H_0\colon\ \gamma_{1,1} = \gamma_{1,2},\ H_A\colon\ \gamma_{1,1} \neq \gamma_{1,2}$).

$$H_0\colon\ \gamma_{2,1} = \gamma_{2,2},\ H_A\colon\ \gamma_{2,2} \neq \gamma_{2,2} \tag{27}$$

$$H_0\colon\ \gamma_{3,1} = \gamma_{3,2},\ H_A\colon\ \gamma_{3,1} \neq \gamma_{3,2} \tag{28}$$

Hypothesis (27) tests the asymmetry between the coefficient of size beta in two regimes, while hypothesis (28) tests the asymmetry between the coefficient of value beta in two regimes. If both null hypotheses of (27) and (28) are rejected, then there is an asymmetric relationship between stock's sensitivities to the size and value factors in different regimes.

Chapter 4. Data Collection and Testing Results

4.1 Economic indicators

Twelve major economic indicators on the U.S. market are collected from Refinitiv Datastream for modeling the macroeconomic model. These indicators are presented in Table 1. Except for credit spread, yield spread, and U.S. T-bill second market 3-month interest rate, the rest of the indicators are calculated by using their logarithmic changes. Because the percentage of credit spread and yield spread can be negative, it is a essential to calculate their changes rather than the logarithmic ratio of the data month on month. The data of the selected indicators spans January 1986 to October 2021 (total 418 months) and are used as observed data to estimate transition matrix and posterior probabilities.

Table 1. The selected economic indicators

Code	Indicators
SP5	S&P 500 price index
LEI	Conference Board leading economic indicators index
CCI	Consumer confidence index
TIP	Total industry production
CPI	Consumer price index
YLD	Yield spread by 20 years Treasury bond yield – 2-month T-bill
CSD	U.S. credit spread
MNY	Money supply
REL	The number of housing starts
PMI	U.S. ISM purchasing managers index
UEM	Unemployment rate
ITR	U.S. T-bill second market 3-month interest rate

4.2 Sample Selection

Stocks of 630 firms on the U.S. market that traded on multiple exchanges (NYSE, NASDAQ, Non-NASDAQ OTC, NYSE MKT, and London) from January 1986 to

October 2021 were selected from Refinitiv Datastream. Excluding those with missing data, 597 firms are used for hypothesis testing. Individual stock returns were calculated as monthly logarithmic ratio and R_f is the one-month Treasury bill rate. All the data [($R_m - R_f$), SMB, and HML] of the Fama and French three factors are downloaded from Kenneth R. French's website (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

I prefer using monthly returns to daily returns because I believe that the trend of stock return is more important. It is useful for investors to study the noise of the stocks by using daily return for short-term investment, but longer investment horizons focus more on the trend instead of noise.

4.3 Estimation of the macroeconomic model

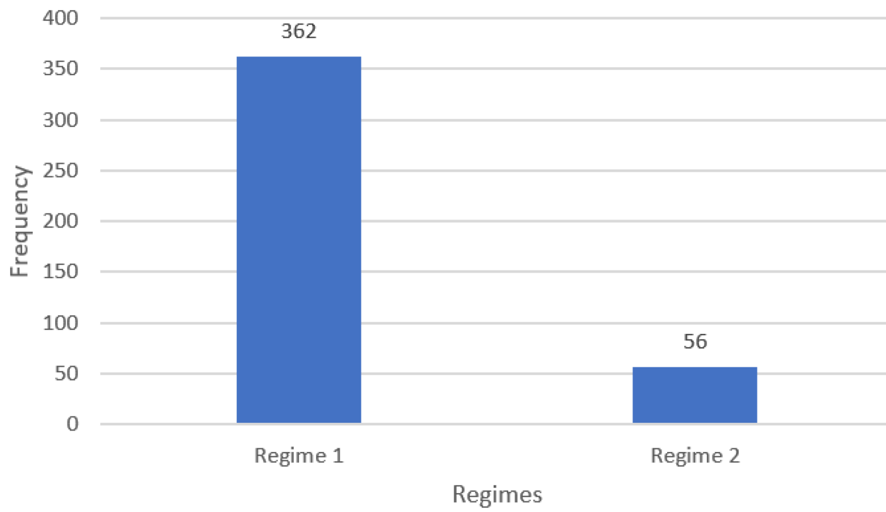
Table 2 shows the transition matrix that gives the probabilities of different states going from one to another.

Table 2. Transition matrix

	Regime 1	Regime 2
Regime 1	0.9682	0.0318
Regime 2	0.2009	0.7991

According to Table 2, the probability of staying in regime 1 is 0.9682 if it is currently in regime 1, and the probability of regime 1 transfer to regime 2 is 0.0318. The probability of regime 2 transfer to regime 1 is 0.2009, and regime 2 has 0.7991 probability of staying in regime 2 when it is in regime 2. Figure 2 shows the frequency of the inferred regime. According to Figure 2, the frequency of regime 1 is 362 months and regime 2 (bear market) is 56 months stay from January 1986 to October 2021—a total of 418 months. The duration of each regime is not consecutive, and the distribution that describes the regime by time is reported in Figure 3.

Figure 2. The frequency of inferred regime



Note: Figure 2 shows the frequency of each regime. X-axis represents the different regimes (1 and 2) and y-axis represents the frequency of each regime during the sample period January 1986 to October 2021—418 months in total.

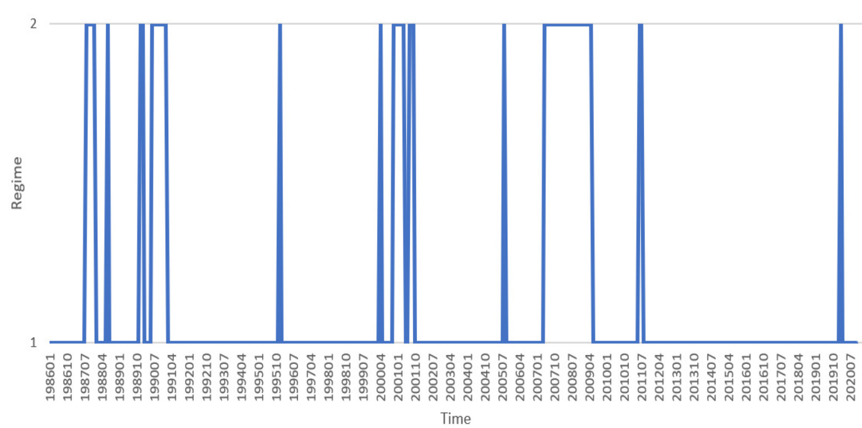
The regimes can be labeled by analyzing the economic activities over time, which are reflected in the economic indicators. Table 3 shows the change of conditional mean by month for each factor in different regimes. The second and third rows represent the economic indicators' conditional mean in the two regimes, respectively. Generally, the bull market is often connected to a strong and optimistic market attitude, while the bear market sentiments are relatively fluctuating and pessimistic. According to Table 3, most data shown in regime 1 are positive, which indicates a growing economic market. On the contrary, negative data indicate a worsening economic environment. For example, the unemployment rate is one of the important indicators of economic market condition. During the expansion period, the conditional mean of the change related to unemployment rate is relatively low (as shown in Table 3; - 0.064 in the bull market). However, the change of that rate rises dramatically in the bear market. Meanwhile, the yield spread is the difference between the 20 years Treasury bond yield and the 2-month T-bill. In general, the conditional mean of the changes related to yield spread in the bull market is lower than that in the bear market. Based on the analysis above, I can label regime 1 as bull market and regime 2 as bear market.

Table 3. The conditional mean of economic factors

	SP5	LEI	CCI	TIP	CPI	YLD	CSD	MNY	REL	PMI	UEM	ITR
Regime 1	0.127	0.037	0.061	0.030	0.024	-0.380	0.017	0.051	0.038	0.020	-0.064	0.149
Regime 2	-0.230	-0.108	-0.330	-0.051	0.033	2.034	-0.279	0.077	-0.310	-0.124	0.275	-2.161

Furthermore, because the historical data are analyzed, it is necessary to check whether the inferred regimes match with existing market categorization. I compared the estimation with the business cycle information from NBER. Figure 3 presents the inferred regimes by time; regime 1 stands for the bull market and regime 2 stands for the bear market.

Figure 3 Inferred regimes by time



Two periods are selected as examples to see whether the market classification from the two sources is consistent. First, NBER reported there are 8 months of contraction and 120 months of expansion during the period March 1991 to November 2001 (Table 4). Figure 3 shows there are 11 months in the bear market and 117 months in the bull market during this period. Secondly, NBER reported 18 months of contraction and 73 months of expansion during the period November 2001 to June 2009. According to Figure 3, it is shown that 33 months in the bear market and 58 months in the bull market during that period. The two samples show that the inferred regimes in Figure 3 are almost the same as the business cycle reported by NBER. Because of the different calculations, the market classification from the two sources is not exactly the same. I prefer to estimate the probability of the regimes rather than use the information reported in NBER. According to NBER, the recession period is defined as the dramatic decline of economic activities and lasts less than six months. Therefore, if the duration of economic activity decline is less than six months, NBER will not call it a recession period. Because of that, the regime-switching model is a suitable method to estimate the probability of different regimes to avoid bias.

Table 4. Business cycle information from NBER

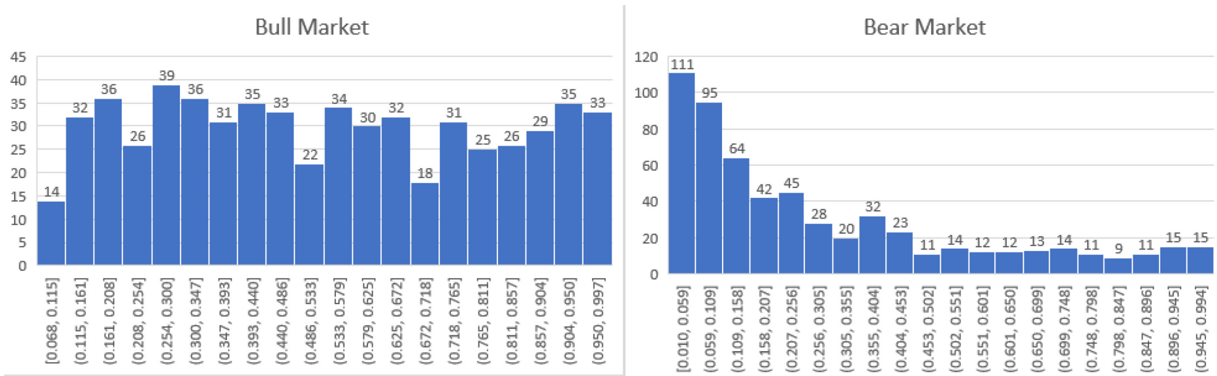
Peak month (Peak Quarter)	**Trough month (Trough Quarter)**			**Contraction**	**Expansion**
		Peak month number	Trough month number	*Duration, peak to trough*	*Duration, trough to peak*
	December 1854 (1854Q4)		660		
June 1857 (1857Q2)	December 1858 (1858Q4)	690	708	18	30
October 1860 (1860Q3)	June 1861 (1861Q3)	730	738	8	22
January 1980 (1980Q1)	July 1980 (1980Q3)	2161	2167	6	58
July 1981 (1981Q3)	November 1982 (1982Q4)	2179	2195	16	12
… …					
July 1990 (1990Q3)	March 1991 (1991Q1)	2287	2295	8	92
March 2001 (2001Q1)	November 2001 (2001Q4)	2415	2423	8	120
December 2007 (2007Q4)	June 2009 (2009Q2)	2496	2514	18	73
February 2020 (2019Q4)	April 2020 (2020Q2)	2642	2644	2	128

4.4 Parameter estimation in time-series regressions for the CAPM

4.4.1 Statistical significance of the estimated intercepts for the CAPM

Figure 4 describes the p-value distribution of α for each individual stock in both regimes. Each selected firm is associated with p-value for the estimate of α, so there are 597 p-values in total in each regime. From Figure 4, it is observed that the trend is not smooth in the bull market. No stocks have p-value below 0.05 in the bull market, which indicates no estimated intercept is statistically significant at the 5% level in the bull market. However, if I look at the trend in the bear market, it is clearly smoother than the bull market. There are 103 stocks with p-values between 0 and 0.05. Hence, those 103 stocks' estimated intercepts are statistically significant at the 5% level in the bear market.

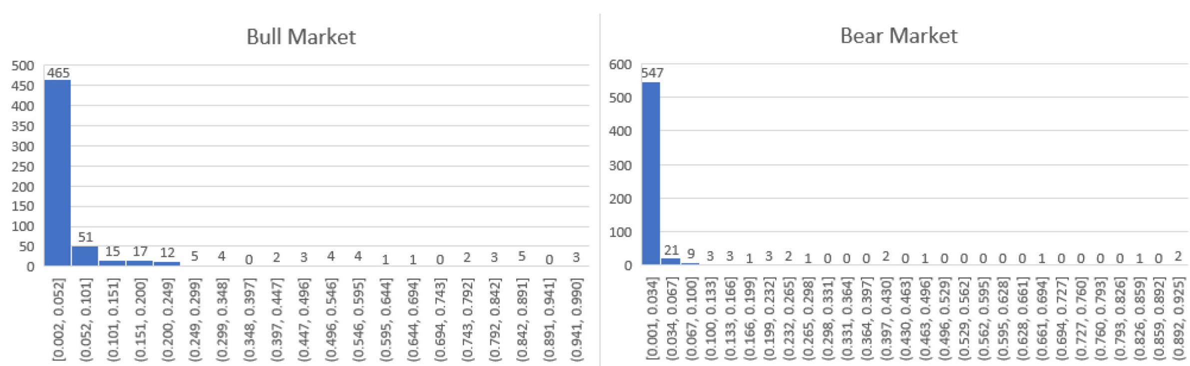
Figure 4. p-value distribution of α in different regimes (CAPM)



4.4.2 Statistical significance of the estimated β_m for the CAPM

Figure 5 shows the distribution of the p-value of β_m for each individual stock in the bear and the bull markets. The data patterns in Figure 5 indicate that there are more stocks with beta estimates having p-values smaller than 0.05 not only in the bull market but also in the bear market. Only 138 stock beta estimates have a p-value greater than 0.05, while the rest of (459) stock beta estimates have a p-value smaller than 0.05 in the bull market. 558 stocks estimated β_m are statistically significant at the 5% level in the bear market.

Figure 5. p-value distribution of β_m in different regimes (CAPM)

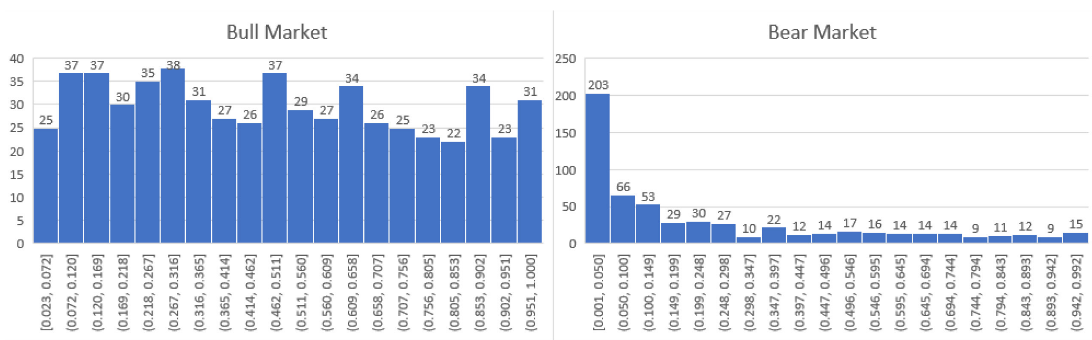


4.5 Parameter estimation in time-series regressions for the FF3 model

4.5.1 Statistical significance of the estimated intercepts for the FF3 model

The first parameter to be tested is the intercept for the Fama and French three-factor model. As seen in Figure 6, there is a flat trend shown in the bull market. In other words, there is no specific distribution, and the number of stocks' p-values at each significance level do not show a big difference. There are just 11 stocks with an estimated intercept having a p-value below 0.05, which means those stocks' estimated intercepts are statistically significant at the 5% level in the bull market. Nevertheless, there is a smooth trend shown in the bear market. There are 203 stocks with an estimated intercept having a p-value between 0.001 and 0.05. Hence, about one-third of the estimated intercepts are statistically significant at the 5% level in the bear market.

Figure 6. p-value distribution of α in different regimes (FF3)



4.5.2 Statistical significance of the estimated β_m, β_s, and β_h for the FF3 model

Figures 7–9 show the p-value distribution that relates to β_m, β_s, and β_h for each individual stock in the extended Fama and French three-factor model. According to Figure 7, which reports the distribution of β_m's p-value, more stocks with an estimated beta having a p-value between 0 and 0.05, and just a few p-values disperse between 0.05 and 1 in both regimes. This indicates that 514 stocks estimated β_m are statistically significant at the 5% level in the bull market. Similarly, 580 stocks' estimated beta have a p-value close to zero, which indicates those stocks estimated β_m are statistically significant at the 5% level in the bear market as well.

Meanwhile, the distribution of p-value about β_s and β_h indicate that majority of the stocks have a beta p-value less than 0.05 in both regimes. According to Figure 8, there are 140 stocks that have the p-value of β_s below 0.05 in the bull market, while 250 stocks have the p-value of β_s below 0.05 in the bear market. Based on Figure 9, the similar results can be observed to Figure 8. Figure 9 reports the distribution of p-value about β_h for each individual stock in different regimes. Note that there are 179 stocks with p-value smaller than 0.05 in the bull market. Looking at the distribution in the bear market, there are 362 stocks' p-value distributed between 0 and 0.05. Overall, majority of the stock have a p-value less than 0.05 the estimated all three βs' (β_m, β_s, and β_h) of stocks have p-values below 0.05 indicating that most of those estimates are statistically significant at the 5% level in both the bull and bear markets.

Figure 7. p-value distribution of $\boldsymbol{\beta_m}$ in different regimes (FF3)

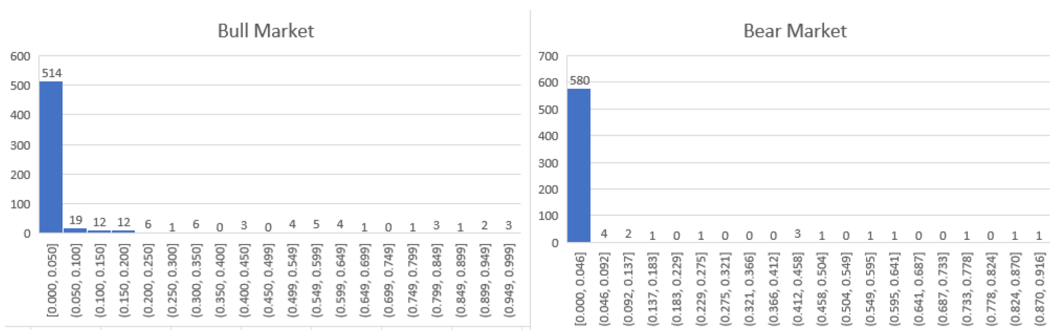


Figure 8. p-value distribution of $\boldsymbol{\beta_s}$ in different regimes (FF3)

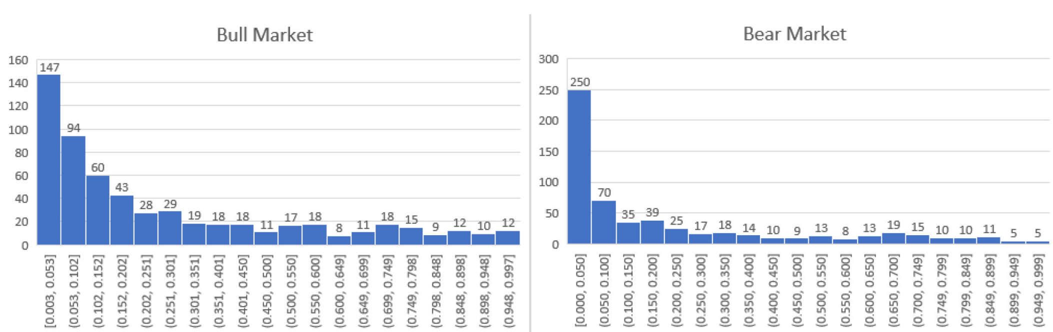
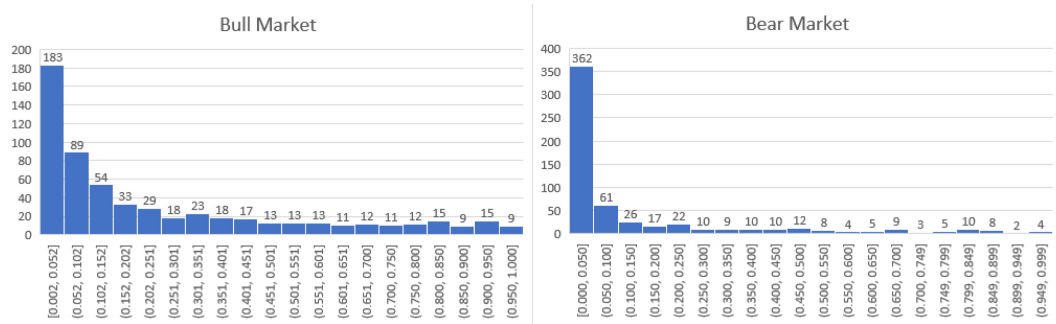


Figure 9. p-value distribution of β_h in different regimes (FF3)



4.6 Beta value distribution and statistic description

To make an investment decision, managers usually compare an individual stock's beta with the overall market volatility. A firm's stock with a higher beta has greater risk and expected returns than the overall market. If beta is greater than one, it means the stock return is in general more volatile than the market return. However, a beta smaller than one indicates the stock price is less volatile than the market. According to Table 5, the mean β_m is 0.851 in the bull market, which indicates the average value of the β_m from our 597 stocks is smaller than market volatility in the bull market. It makes sense because the financial markets in a bull market are more stable than in a bear market. Also, a mean greater than 1 (1.033) in the bear market shows the stock return is more volatile than the market. Standard deviation is a measure of how dispersed the data are in relation to the mean. The standard deviation of the estimated β_m in the bull market is 0.4, while the standard deviation of β_m in the bear market is 0.533. Figure 10 reports the value distribution of β_m in different regimes in the CAPM. Similar to the mean of β_m in the CAPM, the mean of β_m in the Fama and French three-factor model is also less than 1 in the bull market and greater than 1 in the bear market. The standard deviation of β_m (0.391) in the bull market is smaller than the standard deviation of β_m (0.555) in the bear market in the Fama and French three-factor model. Based on Figure 11, the range of β_m in the bear market is bigger than it is in the bull market.

Table 5 reports descriptive statistics about the other two control factors (β_s and β_h) for the Fama and French three-factor model as well. According to the mean of β_h, the mean is smaller and close to zero not only in the bull market (0.237), but also the bear market (0.148). Therefore, it is determined that more firms' stock is weighted toward smaller-cap stocks in both bull and bear markets. Also, the standard deviation of β_s is

0.323 in the bull market is smaller than β_s is 0.787 in the bear market. Figure 12 reports the value distribution of β_s in different regimes for the Fama and French three-factor model.

Moreover, the statistical information related to β_h are reported in Table 5. Although the mean of β_h in the bear market (0.401) is bigger than in the bull market (0.292), both numbers are positive. The standard deviation of β_h is 0.33 in the bull market and 0.663 in the bear market. Figure 13 reports the distribution of the estimated β_h in different regimes for the Fama and French three-factor model.

Table 5. Descriptive statistics for the parameters related to beta for the CAPM and the Fama and French three-factor model

	CAPM		Fama and French three-factor model					
	β_{m,s_t}		β_{m,s_t}		β_{s,s_t}		β_{h,s_t}	
	Bull Market	Bear Market	Bull Market	Bear Market	Bull Market	Bear Market	Bull Market	Bear Market
Mean	0.851	1.033	0.826	1.057	0.237	0.148	0.292	0.401
Standard Error	0.016	0.022	0.016	0.023	0.013	0.032	0.014	0.027
Median	0.875	0.993	0.843	1.026	0.213	0.119	0.286	0.368
Standard Deviation	0.400	0.533	0.391	0.555	0.323	0.787	0.330	0.663
Sample Variance	0.160	0.284	0.153	0.308	0.104	0.620	0.109	0.440
Kurtosis	0.174	19.639	0.292	27.639	3.275	18.492	5.945	7.378
Skewness	-0.043	1.637	-0.128	2.015	0.143	-1.439	-0.664	-0.969
Range	2.907	8.569	2.982	9.656	3.094	11.250	3.794	7.069
Minimum	-0.647	-2.068	-0.809	-2.413	-1.537	-7.882	-2.213	-4.419
Maximum	2.260	6.501	2.173	7.243	1.557	3.368	1.581	2.649
Count	597	597	597	597	597	597	597	597

Figure 10. The value distribution of β_m in different regimes (CAPM)

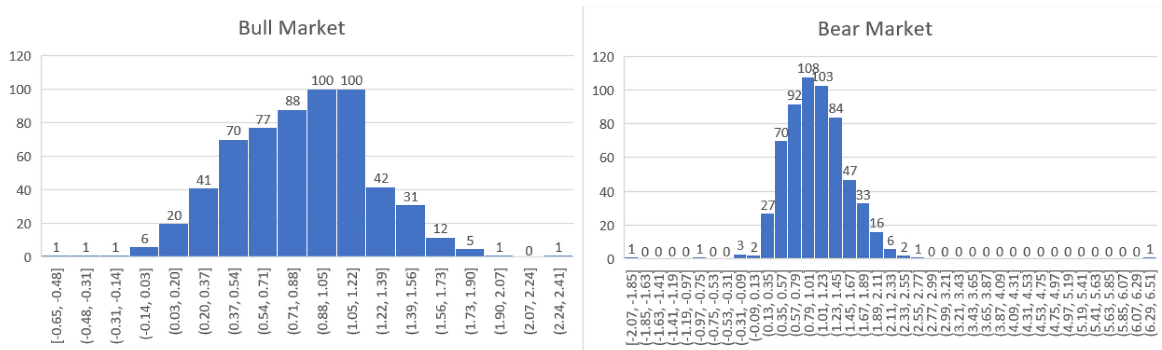


Figure 11. The value distribution of β_m in different regimes (FF3)

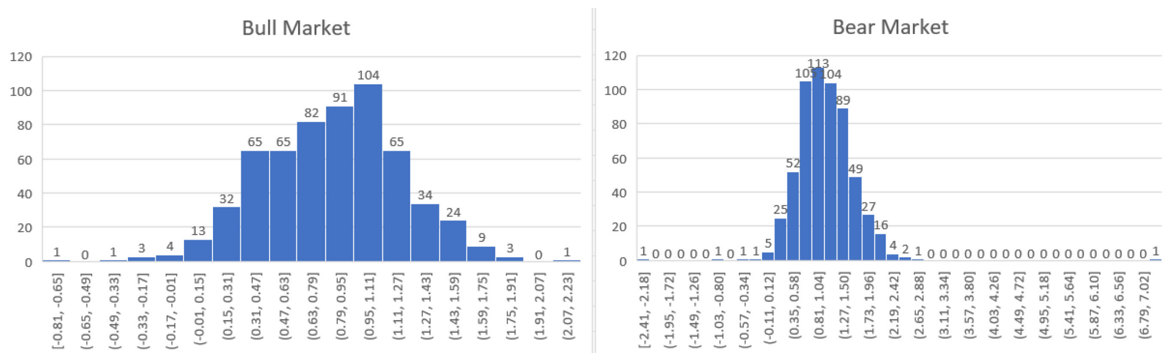


Figure 12. The value distribution of β_s in different regimes (FF3)

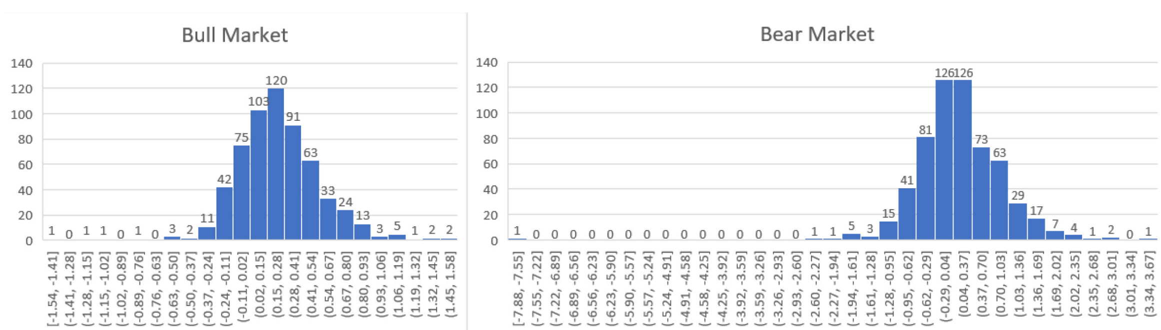
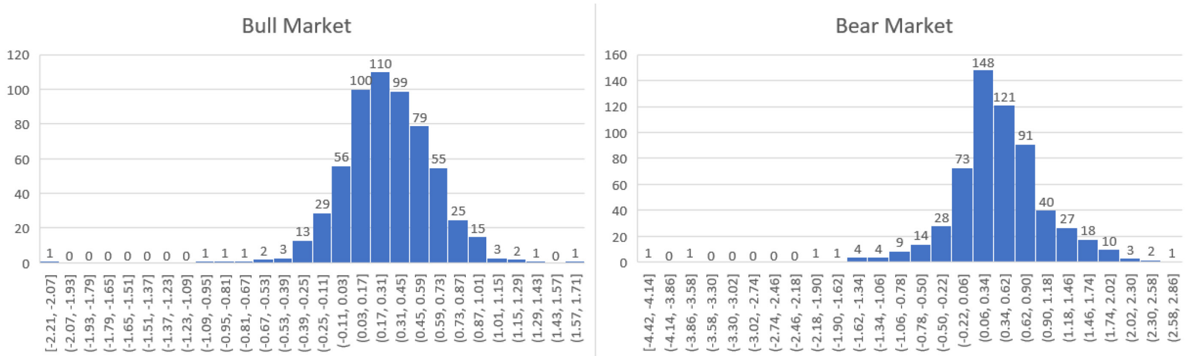


Figure 13. The value distribution of β_h in different regimes (FF3)



4.7 Hypotheses testing for significance and asymmetry

To test the hypotheses for asymmetric risk and market risk premiums, I use one-sample or paired-sample t-tests. The one-sample t-test is a parametric test of the location parameter when the population standard deviation is unknown, and paired t-test is the difference between two samples that come from a normal distribution with a mean equal to zero and unknown variance. Tables 6–9, and 11, report the outcomes from each hypothesis and include sample mean and p-value. P-value is the probability of observing a test statistic as extreme as, or more extreme than, the observed value under the null hypothesis. Small values of p cast doubt on the validity of the null hypothesis. Therefore, if the reported p-value is smaller than 0.05, the test rejects the null hypothesis at the 5% level, while there is not sufficient evidence to reject the null hypothesis at the 5% level if the reported p-value is greater than 0.05.

4.7.1 Testing results for intercept and beta

Table 6 reports the testing results for hypotheses (7) to (9) in the extended CAPM and the Fama and French three-factor model. For the CAPM, the p-value of 0.0345 is smaller than 0.05 in the bull market, so the null hypothesis of hypothesis equation (7) ($\alpha_1 = 0$) should be rejected at a 5% level, which indicates that the CAPM does not hold in the bull market. The test decision seems to conflict with the graph in Figure 4, which reported that none of the α_1 in the bull market are statistically significant. The reason is due to the sample difference. Figure 4 shows the p-value of the estimated intercept for each stock, while hypothesis (7) tests the significance of the average intercept, α_1.

Therefore, most stocks have insignificant α does not affect the test result of hypothesis (7).

However, the p-value of 0.9235 is greater than 0.05, so the null hypothesis of equation (8) (α_2 =0) should not be rejected at the 5% level in the bear market. Meanwhile, the sample mean of α_1 is positive (0.0494), while the sample mean of α_2 is negative (-0.0082).

Table 6 also reports the sample's mean and p-value for the Fama and French three-factor model. Both p-values (0.5833 and 0.0818) for the estimated intercepts are greater than 0.05, which suggests that the null hypotheses in equations (7) and (8) are not rejected at the 5% level for the Fama and French three-factor model.

The last row in Table 6 reports the results of hypothesis test for equation (9) that is related to asymmetric intercept in the extended CAPM and the Fama and French three-factor model. The p-value is 0.4986 in the extended CAPM, and the p-value is 0.0593 in the extended Fama and French three-factor model. Because both p-values are greater than 0.05, the null hypothesis of equation (9) should not be rejected in both models at the 5% level. Therefore, I can conclude that no asymmetric intercept exists in the extended CAPM or the Fama and French three-factor model.

Table 6. The results for hypotheses (7) to (9)

	CAPM		FF3	
	Mean	p	Mean	p
Hypothesis (7) ($\alpha_1 = 0$)	0.0494	0.0345	0.013	0.5833
Hypothesis (8) ($\alpha_2 = 0$)	-0.0082	0.9235	-0.0593	0.0818
hypothesis (9) (H_0: $\alpha_1 = \alpha_2$)	-	0.4986	-	0.0593

Table 7 reports the results related to the significance test [hypotheses (10) to (12)] and the asymmetric beta test [(24) and (25)] by the extended CAPM and Fama and French three-factor model. According to Table 7, p-value for both hypotheses (10) and

(11) are 0 in two models. Therefore, the average of $\beta_{m,1}$ and the average of $\beta_{m,2}$ are both statistically significant at the 5% level for the two models. Also, in the CAPM, the mean of $\beta_{m,1}$ (0.851) is smaller than 1, which indicates the average of 597 stocks' risk is lower than market volatility in the bull market. The mean of $\beta_{m,2}$ (1.033) greater than 1, so it can be concluded that the average of 597 stocks' risk is greater than market volatility in the bear market. Meanwhile, the p-value of hypothesis (12) is smaller than 0.05, so the null hypothesis of hypothesis equation (12) should be rejected in the extended CAPM. In the Fama and French three-factor model, the sample means of $\beta_{m,1}$ and $\beta_{m,2}$ provide the evidence that the sample mean has lower risk than the market volatility in the bull market and it is higher than market volatility in the bear market. The p-value of the estimated beta is 0 for the Fama and French three-factor model, so the null hypothesis of hypothesis equation (12) should be rejected for the Fama and French three-factor model. Therefore, there is strong evidence of asymmetric beta for the extended CAPM and Fama and French three-factor model.

Because of the two other control factors in the extended Fama and French three-factor model, I also test the asymmetry related to the beta of SMB and HML in both regimes and the results are reported in Table 7. Because both p-values for hypotheses (24) and (25) are smaller than 0.05, their null hypotheses of equations (24) and (25) should be rejected. Hence, there is a significant difference between the beta of SMB in bull and bear markets, and there is a significant difference between the beta of HML across market regimes as well. Also, the positive sample means, $\beta_{s,1}$ and $\beta_{s,2}$, state small-cap stocks will earn higher return in both regimes. Moreover, the positive sample means, $\beta_{h,1}$ and $\beta_{h,2}$, state that the excess return of firms are due to the high book-to-market stock.

Table 7. The results for hypotheses (10) to (12) and (24) to (25)

	CAPM			FF3		
	Mean		p	Mean		p
hypothesis (10) (H_0: $\beta_{m,1} = 0$)	0.851		0.0000	0.826		0.0000
hypothesis (11) (H_0: $\beta_{m,2} = 0$)	1.033		0.0000	1.057		0.0000
hypothesis (12) (H_0: $\beta_{m,1} = \beta_{m,2}$)	-	-	0.0000	-	-	0.0000
hypothesis (24) (H_0: $\beta_{s,1} = \beta_{s,2}$)	-	-	-	0.237 ($\beta_{s,1}$)	0.148 ($\beta_{s,2}$)	0.0025
hypothesis (25) (H_0: $\beta_{h,1} = \beta_{h,2}$)	-	-	-	0.292 ($\beta_{h,1}$)	0.401 ($\beta_{h,2}$)	0.0000

4.7.2 Results for cross-sectional regression analyses

Table 8 reports the testing results for hypotheses (14) to (16) for the extended CAPM and the Fama and French three-factor model. It is shown that both the CAPM and the Fama and French three-factor model have a p-value smaller than 0.05 for hypothesis (14). Therefore, the average of γ_0 in both models is statistically significant at the 5% level, which means the null hypothesis of equation (14) should be rejected. Nevertheless, the results for hypothesis (15) for the bear market have different conclusions compared with the results for hypothesis (14) for the bull market. P-value is greater than 0.05 for both models, implying that rejection of the null hypothesis (15) is failed at the 5% level.

Table 8 also presents the testing result for hypothesis (16) for both models. Based on the results shown in the last row of Table 8, I can conclude that the null hypothesis for both the extended CAPM and the Fama and French three-factor model should be rejected at the 5% significance level. However, there is a bias by using different sample lengths to test the intercept. The quantity of γ_0 that used to test hypotheses (14) and (15) are based on the inferred regimes. Each parameter from cross-sectional regression has two estimates because of two regimes, and the selections are based on the inferred regime. For example, if the inferred regime indicates that January 1986 is regime 1, it will select the beta from the bull market instead of the bear market. Again, 362 months are the bull market and 56 months are the bear market in terms of Figure 2. Therefore, the quantity of γ_0 is 362 in the bull market when test hypothesis (14), while the quantity of γ_0 is 56 in the bear market when test hypothesis (15). Both hypotheses (14) and (15) test statistically significant due to sample size.

Table 8. The results for hypotheses (14) to (16)

	CAPM		FF3	
	Mean	p	Mean	p
Hypothesis (14) ($H_{0:}\,\gamma_{0,1} = 0$)	2.1258	0.0081	1.8252	0.0027
Hypothesis (15) ($H_{0:}\,\gamma_{0,2} = 0$)	2.5116	0.1908	2.6498	0.1992
hypothesis (16) ($H_{0:}\,\gamma_{0,1} = \gamma_{0,2}$)	-	0.004	-	0.003

However, the bias is found if the total γ_0 (418) in both regimes is used to do the test. Table 9 reports the results for hypotheses (14) and (15) by using all (418) γ_0 in each regime to test hypotheses. Comparing Table 9 with Table 8, it is clear that there is a change in p-value for both models in hypothesis (14). The results indicate that the average of γ_0 in both models is insignificantly different from 0, which means there is not sufficient evidence to reject the null hypotheses (14) and (15). With the same samples, I re-check hypothesis (16). Both p-values are greater than 0.05 for the extended CAPM and the Fama and French three-factor model, which implies the null hypothesis of hypothesis (16) should not be rejected. Therefore, it can conclude that there is bias by using a different sample quantity to test hypotheses (16). It is clear that there is dramatic increase of sample means from Table 8 to Table 9, and the reason is because few samples are used for the test results presented in Table 8.

Table 9. The results for hypotheses (14) to (16) by using full quantity of γ_0

	CAPM		FF3	
	Mean	p	Mean	p
Hypothesis (14) ($H_{0:}\,\gamma_{0,1} = 0$)	0.1673	0.2591	0.2015	0.1568
Hypothesis (15) ($H_{0:}\,\gamma_{0,2} = 0$)	0.3506	0.1293	0.3502	0.1073
hypothesis (16) ($H_{0:}\,\gamma_{0,1} = \gamma_{0,2}$)	-	0.4218	-	0.5071

Moreover, the relation between γ_1 and the market risk premium (γ_m) in different regimes will be tested before testing the asymmetric market risk premium between the average of γ_1 and the average of γ_2. As it is mentioned in Chapter 3, market risk premium (γ_m) that is estimated from time-series regression is defined as [E(R_m)-R_f]. The numbers in Table 10 are the estimated values of the market risk premium in the bull and bear markets and they are estimated from time-series regression. Therefore, the market risk premium is 0.8237% per month in the bull market and -2.2635% per month in the bear market.

Table 10. Market premium percentage in each regime

	Bull market ($\gamma_{m,1}$)	Bear Market ($\gamma_{m,2}$)
Market risk premium	0.8237	-2.2635

Hypotheses (17) and (18) test whether the average of γ_1 in the bull market ($\gamma_{1,1}$) is close to the value of the market risk premium in the bull market ($\gamma_{m,1}$), and whether the average of γ_1 in the bear market ($\gamma_{1,2}$) is close to the market risk premium in the bear market ($\gamma_{m,2}$). Table 11 shows the results for hypotheses (17) and (18). According to the p-value of testing hypothesis (17) and (18), the null hypothesis for both models should not be rejected for both regimes. Therefore, it can conclude that the average of $\gamma_{1,1}$ is close to the $\gamma_{m,1}$ and the average of $\gamma_{1,2}$ is close to the $\gamma_{m,2}$. Also, compared with the positive sample means of market risk premium in the bull market (γ_1), the negative sample mean of the market risk premium indicates investors are willing to invest in a Treasury bill in the bear market in the extended CAPM and Fama and French three-factor model.

Meanwhile, Table 11 reports the test results of asymmetric relation of γ_1 for different regimes. The results can prove whether an asymmetric market premium exists in different regimes on the U.S. stock market. According to the row related to hypothesis (19) in Table 11, it is clear to see that both p-values for the extended CAPM and the Fama and French three-factor model are smaller than 0.05. In other words, the null hypothesis of hypothesis (19) should be rejected, which indicates the existence of asymmetric market premiums for both models in the U.S. stock market during the sample period January 1986 to October 2021.

Because there are two other control factors in the Fama and French three-factor model, the asymmetry for γ_2 and γ_3 is also tested. The results are reported in Table 17. The p-value is greater than 0.05 in hypothesis (27), so there is no asymmetry for γ_2 between the bear and bull market. Also, the result of testing hypothesis (28) related to γ_3 indicates there is no significant difference between the bull and bear market. Overall, both null hypotheses (27) and (28) cannot be rejected, so no asymmetry exists for two other control factors in the extended Fama and French three-factor model.

Table 11. The results for hypotheses (17) to (19) and hypotheses (27) to (28)

	CAPM			FF3		
	Mean		p	Mean		p
Hypothesis (17) $(H_0: \gamma_{1,1} = \gamma_{m,1})$	0.3849 $(\gamma_{1,1})$		0.9730	0.4785 $(\gamma_{1,1})$		0.9469
Hypothesis (18) $(H_0: \gamma_{1,2} = \gamma_{m,2})$	-1.3363 $(\gamma_{1,2})$		0.9159	-1.2513 $(\gamma_{1,2})$		0.9086
Hypothesis (19) $(H_0: \gamma_{1,1} = \gamma_{1,2})$	-		0.029	-		0.037
Hypothesis (27) $(H_0: \gamma_{2,1} = \gamma_{2,2})$				-0.1494 $(\gamma_{2,1})$	-0.5438 $(\gamma_{2,2})$	0.6512
Hypothesis (28) $(H_0: \gamma_{3,1} = \gamma_{3,2})$				-0.2131 $(\gamma_{3,1})$	-0.0851 $(\gamma_{3,1})$	0.8567

Chapter 5. Conclusion

This thesis focuses on studying asymmetric systematic risk and risk premiums under a regime-switching model. I assume there are two essential regimes, bull and bear markets, in the U.S. stock market and use a sample of 597 firms that were traded on multiple U.S. stock exchanges during the period January 1986 to October 2021. Twelve major macroeconomic indicators are selected, and it is assumed that regimes follow a hidden Markov chain. Based on the macroeconomic strength estimation, the probability of a regime's transit from one to another can be estimated. This thesis aims at estimating each parameter, especially systematic risk and market risk premium, via hidden Markov model, and testing whether asymmetry exists for parameters between the bull and bear market.

According to the results, the average market beta in a bull market is smaller than the average market beta in a bear market for both the CAPM and Fama and French three-factor model. This phenomenon is consistent with the fact that the risk of the most stocks is generally lower than the overall market risk in the bull market. Moreover, there is no asymmetry for α in the extended CAPM and the Fama and French three-factor model when test asymmetric parameters by using time-series regression. However, there is asymmetric beta for both models. I also check the other two control factors in the Fama and French three-factor model and find evidence that the asymmetric beta exists for SMB and HML. Furthermore, I check the asymmetric market risk premium by using cross-sectional regression. The results show the existence of asymmetric market risk premium in the extended CAPM and the Fama and French three-factor model. I also check the asymmetric relation for the coefficient of SMB and HML in different regimes by using cross-sectional regression but do not find any asymmetric relation in either SMB or HML in the different regimes for the extended Fama and French three-factor model.

The potential contributions of this thesis to the current literature are i) using the Hidden Markov Model (HMM) to test asymmetric beta and market risk premium for the CAPM and the Fama and French three-factor model, ii) providing more evidence to support the existence of asymmetric beta and market risk premiums on the U.S stock market. Early research on testing for asymmetry used GARCH models (Bekaert and Wu, 2000; Vendrame et al., 2018), but there are relatively few studies that consider using the

HMM. Moreover, the current literature's estimation of model parameters is more or less relying on linear regression, which assumes beta is constant over time (Jensen, 1972; Grauer and Janmaat, 2010). Therefore, one of the contributions of this thesis is to estimate time-varying beta by using HMM and test asymmetric beta as well as market risk premium for both the extended CAPM and Fama and French three-factor model.

Meanwhile, some research focused more on testing asymmetric beta and market risk premium in the Asian stock market. For example, Bekaert and Wu (2000) studied the asymmetric volatility and risk in the Nikkei stock market and found conditional betas do not show significant asymmetries. Therefore, another contribution of this thesis to current literature is to provide evidence for asymmetric beta and market risk premiums on the U.S stock market.

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