OPTIMAL PRODUCTION, PRICING AND AFTER-SALES SERVICE DECISIONS FOR NEW AND REMANUFACTURED PRODUCTS

by

Zhuojun Liu

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy

at

Dalhousie University
Halifax, Nova Scotia
March 2021

Table of Contents

List of Tables

List of Figures

Abstract

In 2015, the United Nations Member States adopted the 2030 Agenda for Sustainable Development with at its core 17 sustainable development goals (SDGs) to preserve the planet and its resources for future generations. Remanufacturing covers several of these SDGs and is one of the main operations of the circular economy. Remanufacturing has been adopted by (re)manufacturers to extend the life cycles of their products and reduce material, energy, and labour consumption. Remanufacturing has experienced significant growth over the past decades and is forecast to keep growing. However, some hurdles could soon hamper its development. Sales cannibalization between new and remanufactured products is one such obstacle that could prevent the remanufacturing industry from further developing. Consumers perceiving remanufactured products as having lower quality and performance than new ones could also reduce demand and force manufacturers to opt for less environmentally friendly value recovery options. To mitigate the negative effects of cannibalization, optimal pricing and retailing strategies should be designed for the competing new and remanufactured products. Appropriate after-sales services should be derived and offered to consumers to help alleviate their quality, performance, and safety concerns about the remanufactured products. This dissertation explores three themes dealing with the optimal pricing, production, retailing and after-sales service strategies for new and remanufactured products.

The first theme investigates the manufacturer's optimal pricing and production decisions for the new and remanufactured products in a dual-channel supply chain with crosschannel and intrachannel competitions. The second theme deals with the optimal pricing and production decisions for new and remanufactured products sold with base warranty. Finally, the third theme extends the models developed under the first two themes to explore the optimal strategies for competing after-sales extended warranty services offered by the (re)manufacturer and retailer. Several mathematical models are built to explore the impacts of manufacturing and remanufacturing costs, and customers willingness to pay for remanufactured products on the optimal prices, demand functions and warranty periods offered.

This research addresses key issues in the pricing, production and retailing of remanufactured products. The results obtained provide academic and managerial insights to support efficient decision-making for organizations engaging in remanufacturing.

List of Abbreviations Used

CEA	Customer's Environmental Awareness
CLSC	Closed-Loop Supply Chain
EOL	End-of-Life
EOU	End-of-Use
ERP	Extended Producer Responsibility
EW	Extended Warranty
FRW	Free Replacement Warranty
GHG	Greenhouse gas
GHGE	Greenhouse gas emissions
KKT	Karush–Kuhn–Tucker
OEM	Original Equipment Manufacturer
PRW	Pro-Rata Warranty
SC	Supply Chain
WEEE	Waste Electrical and Electronic Equipment

Acknowledgements

I wish to express my deepest gratitude to my supervisors: Prof. Claver Diallo (Department of Industrial Engineering) and Prof. Jing Chen (Rowe School of Business & Department of Industrial Engineering), who have spent their time, and provided support and guidance throughout the development of this dissertation. My sincerest gratitude is extended to the other members of my supervisory committee: Prof. Uday Venkatadri (Department of Industrial Engineering, Dalhousie) and Prof. Michael Zhang (Sobey School of Business, Saint Mary's University), who have provided valuable suggestions and feedback. I also would like to thank Prof. Guoqing Zhang, the external examiner, for taking the time to review this dissertation and providing valuable comments.

Chapter 1

Introduction

In recent years, the pressure on manufacturers to produce environmental friendly products has grown due to the increasing environmental awareness of consumers and the laws and regulations promulgated by governments. Extended Producer Responsibility (EPR) policies require manufacturers to take the financial and/or physical responsibility for the value recovery of their End-of-Use (EOU) or End-of-Life (EOL) products. Remanufacturing, a proven value recovery option and efficient alternative to sending EOU/EOL products to landfills which aims at reducing resources usage, has been adopted globally by leading original equipment manufacturers (OEMs).
Although, the remanufacturing industry has reported significant growth in revenue and number of employees over the past decade, issues such as cannibalization between new and remanufactured products, quality, performance and safety concerns from consumers regarding these remanufactured/refreshed products have become increasingly important and could risk becoming key hurdles to further growth of the remanufacturing industry. To mitigate the negative effects of cannibalization and alleviate the quality, performance and safety concerns, OEMs must judiciously decide the pricing, production, retailing and post-sales strategies for their competing new and remanufactured products. This dissertation contributes to this important decision-making by exploring the optimal pricing, production, channelling and warranty models and decisions for new and remanufactured products.

This dissertation is a thesis by articles comprised of three peer-reviewed manuscripts. The production decision focuses on investigating the manufacturer's remanufacturing decision (whether or not engaging in the remanufacturing activities); the optimal pricing decisions examines price strategies, including the wholesale price and selling price for both the new and remanufactured products; and after-sales decisions studies the optimal prices of the manufacturer and retailer on after-sale services (base and extended warranties). Chapter 1 introduces the remanufacturing process. Following a discussion on the definition of the remanufacturing in the literature, an overall introduction to the remanufacturing industry is presented. Then, the advantages and obstacles to the development of the remanufacturing industry are presented to support and justify the research motivations and goals on this dissertation. Chapter 2 investigates optimal pricing strategies of the manufacturer and the retailer for new and remanufactured products in a dual-channel supply chain under the cases with and without remanufacturing. Chapter 3 focuses on the manufacturer's optimal decisions on the price and production strategy in a closed-loop supply chain (CLSC) by considering the bundled base warranty for the products. Chapter 4 explores the optimal pricing strategy for the extended warranty (EW) offered by the manufacturer and the retailer for both products. Conclusions and extensions are presented in Chapter 5.

1.1 Introduction to remanufacturing and its processes

1.1.1 Definition of remanufacturing

Remanufacturing is a set of processes dealing with recovering value from used products. It has been defined by Ijomah (2002) as the "process of returning a used product to at least original equipment manufacturer (OEM) performance specification from the customer's perspective and giving the resultant product a warranty that is at least equal to that of a newly manufactured equivalent." This definition is used in the dissertation for two reasons. First, it distinguishes remanufacturing from repairing (or reconditioning) based on the level of quality and warranty (see Table 1.1). A similar definition is given by the Remanufacturing Industries Council (Remanufacturing Industries Council, 2020) by defining remanufacturing as "a comprehensive and rigorous industrial process by which a previously sold, worn, or non-functional product or component is returned to a 'like-new' or 'better-than-new' condition and warranted in performance level and quality."

Amezquita et al. (1995) define repairing and reconditioning as the process of transferring a damaged product to a functioning and satisfying condition. Differing from the repaired and reconditioned product, the remanufactured product usually

Table 1.1: Differences between remanufactured, repaired, and reconditioned products.

	Remanufactured product	Repaired product	Reconditioned product
Warranty level	High	Low	Medium
Quality level	High	Low	Medium

has the same or even higher quality than the new product. This is clearly stated by the British Standard (BS 8887-220:2010, 2010) which requires that the technical specification of the remanufactured product should be equivalent to the performance of the new product. This implies that the quality of a remanufactured product is higher than the quality of a repaired or a reconditioned product. Moreover, BS 8887-220:2010 (2010) states that a remanufactured product should be covered by a warranty that is equivalent to or better than the one for the new product. This is different from the repaired or reconditioned product, which has a warranty inferior to that of the new product (Ijomah, 2002). These same requirements are stated in Griffiths (2012) and Kauffman and Lee (2013).

1.1.2 Remanufacturing process

The remanufacturing process has been discussed in many studies in the literature such as (BS 8887-220:2010, 2010; Kauffman and Lee, 2013; Manufacturing, 2020). BS 8887-220:2010 (2010) classifies the remanufacturing process into 10 steps and explains the standards for each step in details, including the collection of technical documents and cores, initial inspection, disassembling, detailed inspection, remediation, replacement, reassembling, testing, and setting a warranty. Kauffman and Lee (2013) combine some of the 10 steps from BS 8887-220:2010 (2010) to result in seven steps. Atlantic Automotive Manufacturing, a leading manufacturer in the automotive aftermarket, suggests 8 key remanufacturing steps based on its practice as: core receiving and inspection, disassembling, cleaning, component preparation, assembling, quality control, final part preparation, and packing. Based on these three explanations of the remanufacturing process above, we classify the remanufacturing processes into five stages in this dissertation as shown in Figure 1.1): 1) Collection, inspection and sorting of cores; 2) Disassembly, inspection and sorting of components; 3) Reprocessing; 4) Assembling and final quality checks; and 5) Distribution, retailing and servicing.

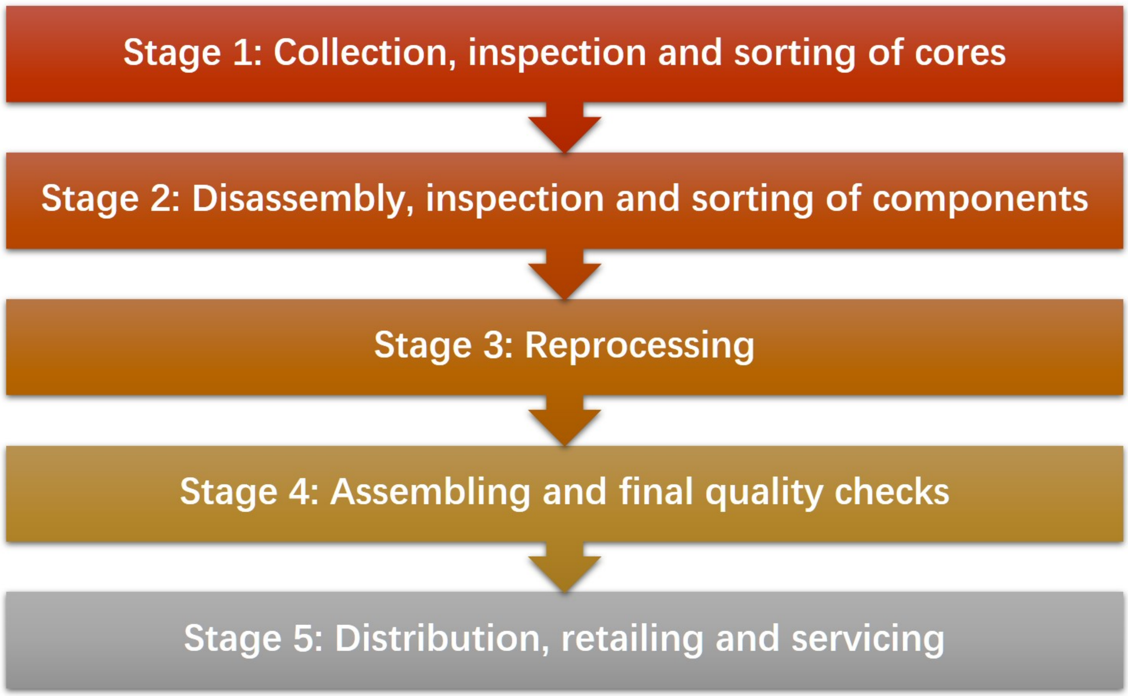


Figure 1.1: Remanufacturing stages

The first stage in the remanufacturing process is to collect the returns from consumers. The quality and quantity of the returns depend on the collection policies and the reverse channel structure adopted by the manufacturer. Based on the quality, some returns/cores with sufficiently low quality are recycled and the rest are taken to the next stage dealing with disassembly. At stage 2, the returns with sufficiently good quality are disassembled into their individual components or materials. These disassembled components/modules/subsystems are inspected, and based on their quality, they are used for repairs, refurbishment, recycling, or remanufacturing (Alqahtani and Gupta, 2017b) in Stage 3. The objective of Stage 4 is to assemble the reprocessed components/modules/subsystems to produce a remanufactured product. After assembly, the remanufactured product is tested to ensure that its quality is as good as the new product's quality (BS 8887-220:2010, 2010). After the remanufactured product is produced, the manufacturer sells it to the customer by adopting an optimal marketing strategy that includes a pricing strategy, a warranty strategy, and a choice of selling channels.

1.2 Advantages of remanufacturing

This section discusses the three types of advantages that remanufacturing brings to an organization: economic benefits, compliance with policies and regulations, and enhancement of the organization's social image.

1.2.1 Economic benefits

Due to the usage of the returns, the production costs of the remanufactured product, including costs of material, energy used, and disposal, decrease significantly as compared to the production costs of an equivalent new product.

By using the returns, the manufacturer does not need to buy large quantities of raw materials in the remanufacturing process, leading to an essential saving in the material cost. The material cost for a remanufactured product is usually 45% to 60% lower than the material cost of a new product (Saavedra et al., 2013). Giuntini and Gaudette (2001) state that remanufacturing businesses can save 14 million tons of materials every year. Xu (2010) points out that a truck engine remanufacturer in China can save around 40,000 tons of metal to produce around 50,000 engines annually. Steinhilper and Weiland (2015) also mention that a large remanufacturing factory producing auto parts saved 240 tons of copper, 440 tons of aluminum, and 2,200 tons of steel. In the United Kingdom (UK), the annual raw material saving for the remanufacturing industry is around 2.7 million tons (Gunasekara et al., 2018). Many companies involved in the remanufacturing industry enjoy the benefit of material savings. Armstrong World Industries produce tiles with more than 80% of recycled materials (Bhatia and Rajiv, 2019). Vasudevan et al. (2012) report that the production cost for HP and EPSON decreased by about 65% through remanufacturing. Volkswagen remanufactures 15,000 engines every year by using 70% of the material at half the cost from EOL products (Singhal et al., 2018).

Energy savings are also notable in remanufacturing. Compared with new products, the production of remanufactured products can use from 50% to 60% less energy (Steinhilper and Weiland, 2015; Feng et al., 2016). Boustani et al. (2010) explore the life cycle energy savings for remanufactured products and find that for washers, refrigerators, and dishwashers, the life cycle energy savings are respectively reduced by 51%, 59% and 74%, as compared to new ones. In China, 105,000-megawatt hours of energy can be saved annually by a large auto parts remanufacturer (Steinhilper and Weiland, 2015); there are around 7 million machine-tools and almost one third of them have been used for more than ten years with low energy efficiency (Cao et al., 2011). Due to economic pressures, it is not practical for the companies to replace their machine-tools with new ones (Du and Li, 2014). However, remanufacturing is considered as a cost-effective method to upgrade the quality of the machine-tools and thus to increase their energy efficiency and reduce energy consumption (Zhang et al., 2020b).

Energy savings in the remanufacturing process lead to reductions in greenhouse gases (GHG) emitted and generate substantial profit for the (re)manufacturer according to the local carbon pricing policies. According to Deng et al. (2017), the production of remanufactured products can reduce emissions by 80%. In the UK, the annual reduction in CO_2 emissions is around 0.8 million tons (Gunasekara et al., 2018). In order to reduce the greenhouse gas emissions (GHGE), many environmental policies and regulations are adopted by governments such as the cap-and-trade and carbon-cap regulations. The cap-and-trade regulation is accepted by many countries worldwide and is considered as one of the most effective policies to control the GHGE (Xu et al., 2017; Chen et al.). Under this regulation, free emission credits are allocated to each manufacturer at the beginning of a cycle and a manufacturer can trade residual credits in a carbon market. Due to the reduction in GHGE during the remanufacturing process, the allocated credits are usually more than enough for the remanufacturer who can then can trade the unused credits with other manufacturers to generate profit.

Under the Extended Producer Responsibility (EPR) regulations, manufacturers must deal with their EOL/EOU products. However, the number of returns has sharply increased rapidly due to the shortened product's life cycle and the development of online shopping. In the past decade, the life cycle of most products has dramatically shrunk. For example, in the auto industry, the life cycle of the VW Golf I was around 10 years in 1974. In 2008, the life cycle of the VW Golf VI has reduced to 5 years (Schuh et al., 2017). For mobile phones, Chiang and Trappey (2007) claim that their life cycle is shortening. Market studies show that 50% of the revenue for companies in a variety of industries are generated from the sales of new products introduced into the market within three years (Horn, 2020). Thus, this increased race to introduce new products compounded by a shortening life generate huge quantities of EOL and EOU products. This is even more dramatic for electronic products. 49 million tons of returned electronic products (e-waste) were generated in 2016 globally and this number is expected to reach 58 million tons in 2021 (Cho, 2018). In Europe, e-waste increases with an annual rate of 3% to 5%, and was predicted to reach 12.3 million tons in 2020 (Zeng et al., 2013). About 59 million tons of EOL vehicles were recorded and disposed of between 2006 and 2014 (Paterson et al., 2018). In Malaysia, e-waste is expected to increase at a rate of 21% annually, and 1.7 billion units of e-waste was estimated to be generated in 2020 (Beleya et al., 2017). Moreover, due to the development of online shopping and the promise of easy returns by retailers, the return rates for these retailers has reached high levels in the past decades, which is due to the factor that consumers are not always capable of obtain sufficiently information on the products at the time of their online purchase (Ramanathan, 2011).

According to Ramanathan (2011), in 2003, nearly 20% of online sales (a total of $96 billion) were returned to retailers. The National Retail Federation (2015) reported that the average returns rate is around 8% in the retailing industry globally. However, this rate is around 20% to 40% for online sales (Vamanan, 2020). For fashion products, the returns rate can be as high as 75% (Mostard and Teunter, 2006). High returns rate translates into high amount of returns to be processed by manufacturers. IHL Group reports that returns account for more than $ 600 billion annually worldwide, and nearly one in four return comes from North America (Hudson, 2019). Reagan (2019) also reports that in December 2019, UPS was estimated to deal with more than

1 million returns daily. Therefore, the disposal cost for the returns is non-negligible for manufacturers. According to Grau et al. (2015), the average disposal cost in Latin America and the Caribbean is about \$20.4 per ton disposed of, and in some countries such as Brazil, this unit cost is over \$30.

1.2.2 Compliance with policies and regulations

To protect the environment, many countries have adopted laws and regulations to require the manufacturers to take the responsibility for their EOL/EOU products and support value recovery activities. The Basel Convention is one of the most widespread and essential agreements to control the movement of the e-waste (European Commission, 2013). The core of this agreement is the extended producer responsibility (EPR), which is a policy to extend the manufacturer's responsibility for its own product to the post-consumer stage (De Oliveira et al., 2012). The intentions of EPR are to transfer the responsibility for dealing the EOL/EOU products from the government to the manufacturer and encourage the manufacturer to produce sustainable products that are easy to remanufacture, recycle or reuse to generate the profit (European Commission, 2013) and protect the planet. Most countries in the world have either implemented EPR (e.g., European Union, Canada, and Japan), or introduced and designed related policies (e.g., China and India) (OECD, 2014).

In Europe, the Waste Electrical and Electronic Equipment Directive (WEEE Directive) announced in August of 2005 sets collection, recycling and recovery targets for all types of electrical goods and thus requires electrical and electronic equipment manufacturers to reprocess or cleanly dispose of their EOL/EOU products (Andersen et al., 2020). Between 2011 and 2016, Sweden and Switzerland achieved the highest recycling rates for EOL/EOU products (over 60%) among all European countries (Ylä-Mella and Román, 2019). For Norway and Denmark, the rates were around 38% and 50% respectively (Andersen et al., 2020).

In the United States, Public Law 114–65, allows all federal agencies to use remanufactured products in repairing or maintaining their vehicles (Guidat et al., 2017), which is an essential support to the remanufacturing industry. The Environmental

Protection Agency (EPA) is the main environmental legislation organization in the United States, which published laws and regulations to support the remanufacturing activities both directly and indirectly with initiatives such as the Safer Choice program (Guidat et al., 2017).

As early as in 2001, the Chinese government established the National Key Laboratory, that focused on exploring new remanufacturing technologies (Williamson et al., 2012). Since 2003, many laws and regulations were put in place to foster recycling such as the Law of the People's Republic of China on Cleaner Production Promotion (2003) and the Law of the People's Republic of China on Prevention of Environmental Pollution Caused by Solid Waste (2005). In 2009, the Law of the People's Republic of China on Circular Economy Promotion pointed out that remanufacturing is considered as one method for reusing products, which gives a legal label for remanufactured products. In 2019, a new law in China allowed remanufacturers to sell used car components (Schwarck, 2019).

Similar laws and regulations to support remanufacturing can be seen worldwide, for example the eWASA Technical Guidelines on Recycling of Electrical and Electronic Equipment in South Africa and the Guidelines for Environmentally Sound Management of e-Waste in India (European Commission, 2013). These laws and regulations have fostered the creation and growth of remanufacturing activities and businesses worldwide.

1.2.3 Enhancement of the organization's social image

Adopting remanufacturing can help increase the organization's social image and boost the demand due to the increased customer's environmental awareness (CEA) (Hammami et al., 2018). In a 2019 survey, 72% of respondents indicated that they were more likely to buy a green product than five years ago (Martins, 2019). A report in 2007 from BBMG reveals that more than half of the Americans being investigated were willing to pay more for eco-friendly products and around two-thirds of the respondents indicated that they were more likely to buy products benefiting the environment (Zhang et al., 2015). A 2018 survey on social responsibility conducted in the United States shows that more than two-thirds of the youths have bought a product with some level of environmental benefit in the past year and almost 90% of young people were willing to buy a product with environmental benefits (Butler, 2018). In China, a report for the research program (customer awareness and behavior change in sustainable consumption) showed that on average half of the Chinese consumers were willing to pay a 10% premium for green products and around 70% of the consumers agreed that their purchasing behaviours are affected by the environmental issue (Li et al., 2017).

Remanufactured products are considered as green products given that they are produced from used products and can decrease energy consumption and GHGE. Engaging in remanufacturing can increase the manufacturer's social image and boost demand for its products. A 2018 survey shows that 87% of consumers showed a better attitude towards a manufacturer whose business activities yielded benefits for society and the environment, and 92% of consumers were more likely to trust a manufacturer providing a product or service with an environmental benefit (Butler, 2018).

1.3 Development of the remanufacturing industry

Due to the benefits presented in Section 1.2, remanufacturing has been adopted by many firms worldwide. Around 70 years ago, on the sole basis of financial benefits, companies in the United States and Europe began to recycle used products, which was considered as a start point for the remanufacturing industry (Xu, 2013). In the 1960s, some small auto repairing shops began to collect used auto components and reused them in the repairing service in Japan (Ikeda, 2017). As early as 1980, remanufacturing has been officially recognized as the regeneration of the wasted products (Xu, 2013). In 1981, a three year global research and development project on the integrated resource recovery was executed by the World Bank. In its report, remanufacturing was studied and classified as a part of solid-waste management (Lund, 1985). The situation of the remanufacturing industry in the developed and developing countries was also explored in this project. The British Standard Institution published the standards for the activities in the remanufacturing process in 2010 (BS

8887-220:2010, 2010). According to a survey from Global Industry Analysis, remanufactured auto parts could generate $193 billion by 2020 (Lee et al., 2017). During the 6th China Remanufacturing Summit in 2016, Joe Kripli, the president from Automotive Parts Remanufacturers Association (APRA), pointed out that the United States could still dominate the remanufacturing market in the next few years and the Asian-Pacific region would have see a substantial growth in the remanufacturing industry with a more rapid growth speed than any other regions in the world (Kripli, 2016).

The United States has the largest remanufacturing industry and more than 70 years history (Kang et al., 2018). According to a report published by the United States International Trade Commission in 2012, America had the largest remanufacturing industry in the world. As early as in 1996, remanufacturing was already adopted by 73,000 firms in America, covering 46 major industries with annual sales of $ 53 billion dollars (Lund, 1996). In 2012, the annual investment for the remanufacturing industry was around $1.2 billion, which doubled the investment in 2009 (Williamson et al., 2012). The remanufacturing auto industry in America accounted for $6.2 billion in its domestic market in 2011 (Matsumoto et al., 2017) and this number increased to $6.9 billion in 2015 (IBISWorld, 2020).

In China, remanufacturing legally started in 2008 when the government began issuing licenses to some manufacturers that allowed them to engage in the remanufacturing of auto parts, machinery, and electrical equipment (Lee et al., 2017). In 2012, the capacity of the remanufactured engines reached 110,000 units and the total remanufacturing industry in China accounted for $0.4 billion in value and this number increased to $4.89 billion in 2015 (Zhang and Chen, 2015). According to the Ministry of Industry and Information Technology, the remanufacturing industry was predicted to reach $30 billion in 2020 (Cao et al., 2020b).

The growth of the remanufacturing industry in Europe is similar to the USA, which mainly covers the aerospace, automobile parts, and machinery industries (Lee et al., 2017). The remanufacturing industry in all of Europe is estimated to have a potential value of 600 billion euros (Cattolica, 2018) and will employ 255,000 people by 2030 (Parker et al., 2015). The main remanufacturing industry in Europe was located in four countries: Germany, the UK, France, and Italy with Germany accounting for one third of the total output (Parker et al., 2015). In the UK, the annual output from remanufactured products is around $7.28 billion and the remanufacturing industry has provided more than 50,000 jobs (Cao et al., 2020a).

In Japan, remanufacturing can be traced back to 1960's. Some repair shops collected cores and components with good quality from disposed autos and used them as spare parts (Ikeda, 2017). In 2012, the market size of the auto remanufacturing industry was around $2.31 billion and with more than half from the reused auto parts (Kang et al., 2016). According to Kafuku et al. (2016), the annual value of remanufactured products in Japan was around $4.8 billion in 2016.

South Korea is also a large market for remanufactured products. Kang et al. (2018) conducted a survey in 2015 to explore the remanufacturing industry in South Korea. The result shows that the remanufacturing industry was worth around US$700 million in 2015 with a 16% increase, as compared to 2010. Auto parts accounts for the majority of the value (85%) and the rest includes toner cartridges, catalyst-coated products, and electrical and electronic equipment.

In Malaysia, the remanufacturing is a new industry for 4 major product catalogues: the vehicle components, the printer cartridge, the information and communication technology, and aerospace, generating $1.25 billion every year with a potential to be doubled when the market matures (Philippines, 2015).

In Canada, the high demand for remanufactured products due to an increasing customer's environmental awareness is the essential driving force for the development of the remanufacturing industry (Gunasekara et al., 2018). The main catalogues of remanufactured products in Canada include motor vehicle and aerospace components, medical devices and equipment (APEC and US-AID, 2013). According to a report from US International Trade Commission in 2012, Canada was one of the leading countries importing remanufactured products from the United States (Parker et al., 2015). There is therefore an urgent need for the Canadian manufacturing industry to innovate and improve its methods and decisions to compete against the American remanufacturing industry.

The numerous examples given above show how the remanufacturing industry has grown worldwide. There are significant opportunities for growth as the global economy pivots towards more sustainable economies through the 17 sustainable development goals of the United Nations. With these opportunities also come many issues and obstacles that need addressing to better support decision-making for organizations engaging in remanufacturing.

1.4 Obstacles to remanufacturing growth

Although remanufacturing has tremendous advantages and create substantial economic value, many manufacturers are still hesitant to enter this industry for several reasons including the important following three issues: (i) the cannibalization to the sales of the new product by the remanufactured; (ii) the uncertain quantity and quality of the returns; and (iii) the low consumer's perception of the remanufactured product.

1.4.1 Cannibalization of the sales of new products

One key concern disclosed by (re)manufacturers is the possibility that engaging in remanufacturing will cause the cannibalization of the sales of their new products with high profit margins by their own remanufactured products with low profit margins. Indeed, new and remanufactured products compete on the same market given that the remanufactured product has similar functions as the new one with the advantage of a lower price. Price sensitive consumers may switch from buying the new product to acquiring the remanufactured product. For example, the Sales and Marketing teams at Hewlett Packard state that the sale of one new product is lost when four remanufactured products are sold (Zhang et al., 2020a). Similar cannibalization rates are also reported by Atasu et al. (2010) and Guide and Li (2010). Moreover, due to the huge market size for the remanufactured products, this cannibalization exists regardless of the manufacturer's remanufacturing decision.

When selling the remanufactured product in an addition to the new product, a manufacturer needs to consider the competition between the different departments in the company (if the manufacturer directly sells both products) or the different members/agents in the supply chain (SC) (if the new and remanufactured products are sold through different channels). The competition is not just for the product, but also for all services offered such as the extended warranties (EW). A non-cooperative relationship between different departments in the company or different members/agents in the SC could negatively impact the manufacturer's profit (Papachristos and Adamides, 2014).

If the manufacturer only sells the new product, a third-party company may enter the market by collecting returns and producing the remanufactured product. In this case, the manufacturer faces three threats: cannibalization of the sales of new products by the remanufactured product from the third-party company, the violation of its intellectual property, and potential damages to its reputation if the remanufactured products turn out to be of poor quality (D'Adamo and Rosa, 2016). During the remanufacturing process, the third-party may learn about the manufacturer's sensitive or proprietary design and production techniques and the manufacturer cannot control the quality of the remanufactured product produced by the third-party company. Canon Inc., in April 2004, accused a third-party remanufacturer of stealing its JP3278410 patent. Similar cases have also be seen in the automotive industry (Bouchery et al., 2016).

To mitigate the negative effects of cannibalization, a price fence should be establish to prevent the customer from buying from the lower price segment (Raza and Govindaluri, 2019). This means that the manufacturer should set different prices for the new and the remanufactured products to segment the market to maximize its profit.

1.4.2 Uncertain returns

According to Seitz (2007), the main problem in remanufacturing is the uncertainty in the collection of returns. There is high variability in the quality, quantity, and costs of returned or collected products. Furthermore, the consumer's attitude towards recovered products are high variable. Unlike the forward SC, the collection of returns in the reverse SC is a process from many sources (customers) to few demand points (retailers/firms/third-party collectors) (Han et al., 2016). This "many to few" process is easily subject to high uncertainty and can be disrupted by unexpected events (e.g. disasters) as the used products are collected from geographically dispersed consumers (Han et al., 2016). Govindan et al. (2016) show that the reverse supply chain has a high uncertainty in the quality and quantity of returns, which is a difficult issue for the manufacturer to deal with. Zhao and Zhu (2018) list the factors that affect uncertainty in the reverse SC: consumer's environmental consciousness, convenience of reverse logistic, government regulations, and incentives for remanufacturing and condition/state of used products. Heydari and Ghasemi (2018) show that the uncertainty in the quality of returns can significantly affect the manufacturer's returns policy and optimal profit.

The uncertain quality and quantities of returns/cores have a significant impact on the remanufacturing production plan. For example, returns with very high quality can be resold immediately after cleaning and testing while low-quality returns must be cleaned, disassembled, and undergo several value recovering processes. As a result of these uncertainties, it is very difficult for the (re)manufacturer to forecast or estimate the remanufacturing costs and duration of production cycles. Giri et al. (2017) suggested that manufacturers should adopt the dual-channel reverse channel structure for collecting returns to ensure their quality and quantity.

1.4.3 Low consumer's perception of the remanufactured product

Although the remanufactured product is required to have a "as-good-as-new" quality, consumers still have some reservations about its quality, performance and reliability. As a result, consumers have a lower perceived value for remanufactured products (Wang et al., 2013). Guide and Li (2010) point out that the consumer would not pay the same price for the remanufactured product as for the new product. Atasu et al. (2010) state that the willingness to pay for the remanufactured product is 15% less than for the new product. Michaud and Llerena (2011) argue that the consumer has a lower perceived value on the remanufactured product when the environmental benefit of the remanufactured product is not known to them. Lee and Kwak (2020) explore the customer's valuation on the remanufactured product for six categories of items: low-end laptops, high-end laptops, smartphones, gaming consoles, printers, and water purifiers. They find that regardless of the product category, the customer perceived the value on the remanufactured product is around 83% of the value for new.

This low perceived value of remanufactured products reduces the likelihood of a consumer purchase. Therefore, to attract more customers to buy the remanufactured product and generate more profit, the (re)manufacturer needs to be proactive and increase the consumer's perception of the remanufactured product by engaging in initiatives such as offering an appropriate warranty period to signal quality and educating the consumer.

1.5 Research Objectives & Dissertation Organization

A (re)manufacturer willing to develop its remanufacturing business should begin by mitigating the negative impact of the competition between its new and remanufactured products in the SC (the cannibalization in sales of the remanufactured product). Therefore, different pricing strategies should be adopted for the new and remanufactured products to target different customer segments. The manufacturer can set a price (lower than the price of the new product) for the remanufactured product to capture the sales from the customers with low valuations to the product, and mitigate the lost sales of the new product. An extremely low price for remanufactured products, however, can decrease its profitability and lead to a significant increase in its demand, which may not be satisfied due to the limited number of returns. Therefore, the manufacturer should carefully decide the prices for the new and remanufactured product based on both the availability of returns and the demand. Abbey et al. (2015a) find that adopting a price discount to the remanufactured product (relative to the new product) can boost the demand and generate more profit for the manufacturer even in the presence of cannibalization. Choi (2017) points out that for fashion products, the retailer does not necessary need to set a lower price for the remanufactured product than the new product, and the pricing decision should be made depending on the base demand for the remanufactured product. The inventory level of the remanufactured product also plays an essential role for the manufacturer in deciding the optimal prices for the new and remanufactured products when the new product is made to order (Yan et al., 2017).

This dissertation explores three themes dealing with the (re)manufacturer's optimal pricing, production and after-sales service strategies for new and remanufactured products. Each theme is developed in a dedicated chapter. Chapter 2 investigates the manufacturer's optimal pricing and production decisions for the new and remanufactured products in a dual-channel SC. Chapter 3 deals with the optimal pricing and production decisions for new and remanufactured products sold with base warranty. Chapter 4 extends the models in the previous chapters to explore the optimal strategies for competing after-sales extended warranty (EW) services offered by the (re)manufacturer and retailer. Each chapter is self-contained and has its own introduction, literature review, model, sensitivity analysis, and conclusion sections. Chapter 5 concludes this dissertation with additional discussions. All proofs of the Lemmas and Theorems are given in Appendix.

1.5.1 Theme 1: optimal pricing and production strategies for new and remanufactured products in a dual-channel supply chain

Due to lower pricing, convenience, and various other reasons (e.g., physical distancing during pandemic, lack of nearby retailers), many customers prefer to shop through the Internet. However, other customers are still willing to shop through the traditional physical stores due to factors such as the backward logistics (ease of returns) and physical examination of product before buying (Yang et al., 2010). Thus, many manufacturers have adopted the dual-channel structure to sell their products to capture both groups of customers. The (re)manufacturer's optimal pricing strategy for the remanufactured product can become more complex when multi-channel sales are considered. Cannibalization exists between the new and remanufactured products in the same channel (intrachannel competition) and the across channels (cross-channel competition). Abbey et al. (2015a) explore a monopolistic manufacturer's optimal pricing strategy for new and remanufactured products. San Gan et al. (2015) focus the pricing problem in a SC. Chen and Chang (2013) use the dynamic programming schemes to derive the manufacturer's optimal pricing decisions. These studies consider the cannibalization between new and remanufactured products in one channel only and miss the consideration of the competition between the new and remanufactured products in the different channels. Gan et al. (2017) and He et al. (2019) consider the competition between the new and remanufactured products in the separate channels only and the channel selling costs are not included in these studies.

Moreover, the dual-channel structure can also benefit the manufacturer in the reverse SC for collecting returns. Giri et al. (2017) point out that the dual channels in the reverse SC are more effective than single channels in collecting returns. Therefore, on top of collecting returns through the retailer, many manufacturers, such as Xerox Corporation and ReCellular Inc., open a second channel to collect returns directly to ensure the quality and quantity of the returns (Batarfi et al., 2017).

Chapter 2 studies the optimal pricing strategies for both the manufacturer and the retailer, and the optimal production strategy for the manufacturer in the SC with the dual-channel structure in both the forward and reverse SCs (Theme 1). Intrachannel and crosschannel competitions between new and remanufactured products are discussed. The optimal results derived from the cases with and without remanufacturing are compared. We address the following research questions specifically.

- What pricing and production strategies should the (re)manufacturer and retailer implement in the cases with and without remanufacturing?

- Under what conditions should the (re)manufacturer carry out remanufacturing?

- How does the (re)manufacturer's remanufacturing decision affect the retailer's retail price, demand, and profit?

- How will the customer's acceptance levels for the remanufactured product and the online channel, the production and channel costs, and the proportion of high-quality returns impact the (re)manufacturer's optimal pricing and production strategy, and the retailer's pricing strategy?

The main results obtained in Chapter 2 include: (i) the manufacturer's production and optimal pricing strategies depend on both production and channel selling costs; (ii) remanufacturing is not considered by the manufacturer when the unit manufacturing cost is sufficiently low and the retailer's channel cost is sufficiently high; (iii) the introduction of remanufacturing may benefit the retailer and it hurts the retailer's profit only when the unit manufacturing cost is sufficiently high; and (iv) selling the new product online mitigates the negative effect of remanufacturing on the retailer, while a high customer's acceptance of the remanufactured product can benefit the retailer.

A manuscript resulting from Theme 1 was published in the *International Journal of Production Economics* under the following reference Liu et al. (2021): Liu, Z., Chen, J., Diallo, C., and Venkatadri, U. (2021). Pricing and production decisions in a dual-channel closed-loop supply chain with (re)manufacturing. International Journal of Production Economics, 232:107935.

1.5.2 Theme 2: optimal pricing and production strategies for new and remanufactured products with base warranties

To increase customer's the typically low perceived value of the remanufactured product, many manufacturers bundle a warranty service with the remanufactured product to boost demand. Warranty is a signal of the product's quality to customers (Cohen et al., 2011), since the manufacturer incurs a high warranty cost for poor quality product. The warranty bundled with the product also leads to a high cost. According to Yang et al. (2010), a car's warranty cost accounts for 2.3% of its revenue. This cost is likely to be higher for the remanufactured product due to its higher probability of failure during the warranty period. Therefore, the warranty cost cannot be ignored for the manufacturer when deciding the optimal price for the remanufactured products. Alqahtani and Gupta (2017a) and Alqahtani and Gupta (2017b) deal with the warranty issue in remanufacturing. However, they focus on the warranty cost evaluation only and not the price competition. Liao et al. (2015) explore the competition between new and remanufactured products with warranty. However, the relationship between the quantity sold and warranty cost is not covered in their study.

Theme 2 explores the impact of a monopolistic manufacturer's non-renewing free replacement warranty. A two-period mathematical model (selling the new product with the warranty in the first period and selling both new and/or remanufactured products with warranties in the second period) is developed to maximize the manufacturer's profit. Both the price and the warranty length affect the demand of the remanufactured product and the demand-dependent warranty cost. The limited number of returns as well as the change in the market size are considered in this chapter. Three research questions are addressed.

- How should the manufacturer set prices for the new and remanufactured product under a given warranty length?
- How does the warranty length affect the prices, demands and profit for both new and remanufactured products?
- Does an optimal warranty length exist and what factors influence it?

Chapter 3 derives the optimal pricing and production strategies for new and remanufactured products and shows the relationships between these optimal strategies and the production costs and the market change rate in the second period. The condition, under which the manufacturer should engage in remanufacturing is identified, which is related to the warranty length. The impacts of the customer's sensitivities to the warranty length for the new and remanufactured products on the optimal pricing and production strategies are discussed. The optimal warranty length for the manufacturer is shown to exist and it is affected by the unit production costs, failures rates, and sensitivity of the customer's utility to the warranties for new and remanufactured products.

A manuscript resulting from Theme 2 was published in the *International Journal of Production Economics* under the following reference Liu et al. (2020): Liu, Z., Diallo, C., Chen, J., and Zhang, M. (2020). Optimal pricing and production strategies for new and remanufactured products under a non-renewing free replacement warranty. International Journal of Production Economics, 226:107602.

1.5.3 Theme 3: optimal pricing and retailing strategies of the extended warranty for new and remanufactured products

Post-sales or after-sales services include services such as the delivery, installation of product, feedback implementations and extended warranty (EW). These services are very important and are key elements to a successful customer experience. Choudhary et al. (2011) point out that post-sales service is important for satisfying and retaining customers. Among all post-sales services, EW is a popular service for the remanufactured product, since it increases the customer's perceived value of the remanufactured product, like the base warranty discussed in the Chapter 3, but it also generates substantial profit for the company and/or retailer. Due to the economic benefit, both manufacturer and retailer are willing to provide this service (Jin and Zhou, 2020). Bian et al. (2015) focus on the competition between the EWs offered by two retailers for the same product on the market. Zhu et al. (2016) explore the impact of the remanufactured product's EW on the manufacturer's profit. These studies, however, do not consider the competition between the retailer's and the manufacturer's EWs for the new and remanufactured products.

Chapter 4 explores the EW competition between the manufacturer and the retailer in the SC. In the forward SC, the manufacturer sells the EWs for the new and remanufactured products, while the retailer offers its own EW for the new product sold in its store. In the reverse channel, a failed return is replaced by the manufacturer at a unit trade-in cost for the retailer. The optimal EW's pricing strategies for the new and remanufactured products are obtained for the manufacturer in the cases with and without the sales of the retailer's EW. The impacts of the retailer's EW on the manufacturer's pricing decisions are also examined. The numerical studies show the sensitivity of the manufacturer's and retailer's EW lengths to the optimal prices, demands, and profits. Three research questions are addressed in the chapter.

- How should the manufacturer price its EW service?
- What is the impact of the introduction of the retailer's EW on the manufacturer's decisions?
- How does the warranty length affect the manufacturer and retailer optimal decisions?

In Chapter 4, we find that (i) the manufacturer's optimal pricing strategy depends on the EW length;(ii) the introduction of the retailer's EW does not always hurt the manufacturer's profit when the unit trade-in cost is at a moderate level; (iii) there exists an optimal EW length for the manufacturer that maximizes its profit; (iv) the retailer cannot extract more profit by increasing the length of its own EW.

A manuscript resulting from Theme 3 has been submitted for publication in the *International Journal of Production Economics* with submission reference PROECO-D-20-02337.

Chapter 2

Pricing and production decisions in a dual-channel closed-loop supply chain with (re)manufacturing

2.1 Introduction

For the past two decades, online shopping has seen unprecedented growth as it offers a quick and convenient shopping experience to consumers. There are many reasons why consumers choose the online channel. For example, Jiang et al. (2013) point out that the convenience of the online transaction and efficient delivery service are the main reasons why consumers opt for online shopping. Butler and Peppard (1998) argue that online shopping provides the consumer with a variety of information, which helps the customer to find their desired product easily with at a reasonable price. Vasić et al. (2019) mention that online shopping can help the consumer to avoid the pressure from a face-to-face interaction with the retailer. The recent social distancing and reduced businesses occupancy regulations put in place by public health authorities to limit the spread of COVID-19, and the need for no-contact payment combined with curbside pick-up and free home-deliveries have fueled the most recent growth of online shopping (Grashuis et al., 2020; Tran, 2021).

In 2019, the retailing e-commerce sales amounted to more than 3.5 trillion US dollars and were forecast to reach 7 trillion by 2022 (Sabanoglu, 2019). In China, the online daily sales volume on a special promotion day (November 11th) reached $25.3 billion in 2017 (Yang et al., 2018) and this number exceeded $56 billion in 2020 (E-Commerce, 2020). The annual retailer e-commerce sales in the United States were around $470 million in 2017 and are estimated exceed $740 million in 2023 (Estay, 2020). In India, the e-commerce industry was expected to account for over 1.6% of the global GDP in 2018 with an increasing rate over 50% since 2012 (Suginraj, 2017). These numbers show that the online channel has played an essential role in the consumption market. This huge market has motivated manufacturers to develop direct online channels for selling their products in order to meet different segments of customers (Radhi and Zhang, 2018), save channel-developing cost, and obtain the latest market information Yang et al. (2018).

Although the online channels have attracted many consumers, some still prefer to shop through the tradition retail channel, as they can assess the product by touching and feeling, obtain the product immediately, and receive professional advice from the retailer directly if shopping in a physical store (Alizadeh-Basban and Taleizadeh, 2020). To reach more customers, many firms such as Dell, Apple, and Sony sell their products through dual channels: online platforms and retailers (Giri et al., 2017). For example, Apple and Philips sell only new products in their retail stores, and both remanufactured and new products through their websites (online channels) (Yang et al., 2019; Borenich et al., 2020). However, the dual-channel structure can cause competition between the two channels, which can negatively effects the profits of all members in the SC (Giri et al., 2017).

To mitigate the channel competition, it is important for the manufacturer to employ the appropriate pricing strategies for the products sold through different channels. Chiang et al. (2003) show that the retailer can benefit from the manufacturer's direct channel due to the mitigation of the double marginalization by controlling prices. Fruchter and Tapiero (2005) find that a manufacturer should set the same prices for products in both channels. Dan et al. (2012) demonstrate that the optimal pricing strategies of all members in the SC are influenced by the retailer's service quality, while Huang et al. (2012) show that the customer's preference for the online channel is a key factor in influencing the optimal prices.

In addition to the forward channel, the dual-channel setting can also benefit the manufacturer in the reverse channel. Due to the quantity and quality of used products and the inherent variation in the collecting time, it is often difficult for a manufacturer to collect sufficient returns for remanufacturing (Zhao et al., 2017). Online channels can help manufacturers obtain the appropriate quantity of returns and decrease the cost of collection and transportation, while satisfying the customers by removing the roadblocks in the traditional recycling channels due to the physical distance and time (Feng et al., 2017). Therefore, many firms, such as Hewlett Packard Corporation and Xerox (Chen and Chi, 2019), established new direct reverse channels to enable the remanufacturing process to work effectively and efficiently.

As a lower-priced substitution, the selling of a remanufactured product potentially cannibalizes the sales of an equivalent new product, and may reduce the profit of the manufacturer (Yenipazarli, 2016). Therefore, it is important for a manufacturer to carefully design their production, pricing, and channel strategies when they produce both products. Both the manufacturer and the retailer need to deal with the competition between new and remanufactured products, but also the competition between new and remanufactured products within and across channels.

An extensive literature review found that the optimal pricing and production strategies for the competing remanufactured and new products selling through the competing retail and online channels in a SC are understudied. Most papers in the literature, such as Gan et al. (2017) and He et al. (2019), only focus on the competition between new and remanufactured products in separate channels. But in practice, the SC faces the competitions between new and remanufactured products both within channels (intrachannel competition) and across competing channels (cross-channel competition). For example, Apple Inc. sells new products in both the offline and online channels, and also sells the remanufactured products in its online channel (https://www.apple.com/ca/). Similar practices are also observed for Lenovo Inc. (https://www.lenovo.com/ca) and Samsung Inc. (https://www.samsung.com/us). They sell both the new and remanufactured products through the online channel. Moreover, the channel's selling cost is ignored in most studies in the literature, such as in Gan et al. (2017), Batarfi et al. (2017), and He et al. (2019). However, the channel cost is an essential parameter to capture the difference between the online and the retail channels. Thus, the impact of these costs on the manufacturer's optimal pricing and production strategies, and the retailer's pricing decision is worth to be investigated. To fill the above research gaps, the proposed model investigates the optimal pricing and production strategies for a manufacturer selling its new product through a retailer and its direct online channel, and may sell remanufactured products through its online channel, with the consideration of both the production and the channel costs. The following questions are addressed in this chapter.

- What pricing and production strategies should the manufacturer and retailer implement in the cases with and without remanufacturing?
- Under what conditions should the manufacturer carry out remanufacturing?
- How does the manufacturer's remanufacturing decision affect the retailer's retail price, demand, and profit?
- How will the customer's acceptance levels for the remanufactured product and the online channel, the production and channel costs, and the proportion of high-quality returns impact the manufacturer's optimal pricing and production strategy, and the retailer's pricing strategy?

To answer these questions, a two-period model is developed to determine the optimal pricing and production strategies for both remanufactured and new products. It is shown that optimal pricing decisions of the manufacturer and the retailer, and the optimal production strategy of the manufacturer depend on both the channel selling cost and production cost of the new product. Numerical experiments are carried out to provide additional insights by testing the impact of the customer's acceptance level of the remanufactured product, the customer's acceptance level of the online channel, and the proportion of high-quality returns on the optimal decisions of the manufacturer and the retailer.

Our research indicates that there are key thresholds for the unit manufacturing cost of the new product and the retail channel cost, respectively. The manufacturer is not willing to engage in remanufacturing if the unit manufacturing cost for the new product is lower than a certain threshold. Moreover, the retailer's channel selling cost does not affect the manufacturer's remanufacturing decision when there are sufficient returns. When there are not enough returns, the manufacturer engages in remanufacturing if the retail channel cost is higher than a certain threshold. Furthermore, the retailer suffers a profit loss due to the introduction of the remanufactured products only when the manufacturer needs to use all returns for remanufacturing. The sensitivity analysis also shows that the customer's acceptance level for the remanufactured product has no impact on the retailer's profit when this acceptance level is lower than a threshold and the increase in the proportion of high-quality returns leads to an increase in the manufacturer's profit and a decrease in the retailer's profit when the manufacturer uses all returns for remanufacturing. The values of these key thresholds are determined and presented along with the optimal pricing and production strategies.

The contribution of this chapter to the literature is two-fold. Firstly, the optimal pricing and production strategies are explored for a SC in which remanufactured and new products are sold in a dual-channel by considering not only the channel competition (retail versus online), but also the product cannibalization/competition between the remanufactured and new products that are sold in the same channels and across channels. Hence, a two-fold product competition is considered: intrachannel and cross-channel. Secondly, our model differentiates not only the online channel and retail channel from the customer's perspective by considering a discounted perceived value on the products sold in the online channel, but also the selling costs for the two channels (different channel operating costs), which is not explored in the literature. Our results show that the channel selling cost is the essential factor affecting the optimal pricing strategies of the manufacturer and the retailer and optimal production strategy of the manufacturer.

The rest of the chapter is structured as follows. Section 2 presents the related literature review. In Sections 3 and 4, the two-period model is presented and the optimal pricing strategies of the manufacturer and retailer for the cases with and without remanufacturing are derived. The numerical experiments and the conclusion are presented in Section 5 and 6 respectively. Proofs are presented in Appendix.

2.2 Literature review

In this section, we discuss the literature on the pricing strategy for the remanufactured and new products. Comprehensive reviews of this topic can be found in Kumar and Ramachandran (2016) and Guo et al. (2017). Kumar and Ramachandran (2016) reviews the issues affecting the manufacturer's pricing decisions, which can be classified into three areas: product issues, SC issues, and formulation issues. Guo et al. (2017) focus on studies of the optimal pricing strategy for the manufacturer under different channel structures and the possible contracts to coordinate the SC.

Giri et al. (2017) focus on the revenue management aspect for remanufactured products in the closed-loop supply chain (CLSC) by considering dual channels for both forward and reverse SC. The manufacturer sells part of remanufactured products through the retailer and sells the rest through an e-channel. Similarly, a portion of the returns is collected through a third-party company and the rest of the returns is collected by the manufacturer. It is found that the manufacturer obtains the highest profit when the third party collector is the leader in the SC, while the retailer and the third-party collector obtain maximum profit when the retailer is the leader in the SC. Radhi and Zhang (2018) investigate how to price new and resale products in a dual channel. The manufacturer allows the product sold in the online store to be returned to both the physical store and the online store. Four different SC structures are discussed in the paper: centralized system, online-leading system, physical-leading system, and Nash game. Based on their study, under the Stackelberg scheme, the centralized SC sets high prices for products, as compared with the SC with competing channels. Taleizadeh et al. (2018) explore the optimal pricing, returns, quality policies, sales, and collection effort in a CLSC with dual collection channels and single or dual selling channels among the manufacturer, the retailer, and the third party collector. They find that the optimal product quality in the dual forward channels model is higher than that in the single forward channel model. When the market share of the online market is small, the wholesale price is higher under the dual forward channels configuration.

While these papers focus on the pricing issues by considering the channel competition in the CLSC, they ignore the competition between remanufactured and new products. In these papers, customers are assumed not to be able to differentiate between the remanufactured and new products, which is not practical. Although the functionalities of remanufactured and new products may be the same, customers always have a lower perceived value for the remanufactured products, as they have been used and returned before (Jiménez-Parra et al., 2014). Furthermore, due to liability and customer protection concern, sellers differentiate new products from remanufactured products to maintain their reputation. For example, in 2010, Hong Hengchang, a retailer of HP, misrepresented remanufactured computers as new ones causing incalculable damage to HP's reputation (Yan et al., 2015).

Other studies, such as Abbey et al. (2015a), Choi (2017), Liu et al. (2018), and Sun et al. (2020), investigate the optimal pricing and production strategies by considering the competition between remanufactured and new products. Abbey et al. (2015a) investigate the price competition between remanufactured and new products in several scenarios, from a monopolistic scenario to a more complex scenario with competition between a manufacturer and a third-party remanufacturer. Customers are classified into two groups based on their perceived values for the remanufactured product. The authors find that with the existence of a group of people who never buy the remanufactured product, the price for the new product should be raised to generate more profit for the manufacturer, when the remanufactured product is introduced into the market. The price competition between remanufactured and new products is also explored in Choi (2017) with the consideration of the branding investment. The remanufactured product is found to be over-priced by the retailer in the decentralized case than in the centralized case. Liu et al. (2018) explore the optimal pricing and production strategy for remanufactured and new products by considering a convex collection and inspection cost, two-quality bins for returns, and the remanufacturing losses in a two-period model. The authors find that an extremely high customer's acceptance level for the remanufactured product is not good for the manufacturer's profit. Chen et al. (2019) explore the optimal pricing strategies for the two generations of new and remanufactured products. Three production strategies in the second period (selling new products of the second generation only, selling remanufactured products of the first generation only, or sell both) are compared and the results show that selling the remanufactured product can maximize the manufacturer's profit, although it cannibalizes the sales of the new product. Sun et al. (2020) focus on the impact of warranty period on the competition of remanufactured and new products in the market with a manufacturer, a remanufacturer, and a retailer. If the repair cost of the remanufactured product increases, the remanufacturer needs to decrease the warranty length and the manufacturer can increase the wholesale price of the new product to gain more profit.

The papers mentioned above fail to consider the impacts of the channel structure on the optimal pricing decisions. With the rapid growth of information technologies and e-commerce, more customers are willing to shop online due to convenient shopping process and lower prices. To meet the needs of the customer, firms have developed their online channels in addition to offline channels to sell products (Xie et al., 2018). The dual-channel structure has become the mainstream structure in SC (Wang et al., 2020a). Therefore, it is necessary and practical to consider the channel competition in a CLSC to examining the optimal pricing and production strategy for new and the remanufactured products among the SC members.

The most related literature to our study with both competitions of channels and products can be found in Batarfi et al. (2017), Zheng et al. (2017), and Xie et al. (2017). Batarfi et al. (2017) compare a single and a dual-forward channel in a SC with a manufacturer, a retailer, and a remanufacturer by considering the optimal inventory decision and the return policy. Based on extensive numerical experiments, they conclude that the dual-forward channel is more profitable for the entire SC than the single forward-channel, and the optimal selling prices in the single channel are not affected by the introduction of a new direct channel. Zheng et al. (2017) investigate the effect of the power structure and the coordination contract for a manufacturer, a retailer and a third-party collector. Both the centralized case and decentralized cases are discussed in the paper. A modified two-part tariff contract is proved to achieve the coordination of the SC under different power structures, and all the members can be more profitable than in the decentralized case. Xie et al. (2017) focus on the coordination contracts in a dual SC with consideration of the advertisement effort. A dual revenue-sharing contract is designed, in which the manufacturer and the retailer share the cost saving from the remanufacturing and the sales revenue from the retailer. Gan et al. (2017) explore the optimal pricing strategy for new and remanufactured products in separate selling channels. Customers willingness to pay for the remanufactured product and their preferences for the online channel to acquire the remanufactured product are considered. The authors find that low customer's acceptance of the remanufactured product leads to a high retail price and a low manufacturer's profit. He et al. (2019) focus on the selling channel for the new and remanufactured products. Three cases are investigated in the paper: 1) the new product is sold by the retailer and the remanufactured product is sold by a third-party company; 2) the new product is sold by the manufacturer and the remanufactured product is sold by the third-party company; and 3) the new product is sold by the retailer and the remanufactured product is sold by the manufacturer. Both the customer's acceptance levels for the remanufactured product and the online channel are considered and their impacts on the manufacturer's and the retailer's decisions are explored.

Differing from the above papers, our study considers competition between remanufactured and new products in different channels (external competition), and their cannibalization in the same channel (internal competition) as is common in practice. For example, on top of selling its products through the retail channel, Dell also sells both its new and authorized remanufactured products through its online channel (Ovchinnikov, 2011). Gan et al. (2017) and He et al. (2019) only consider the competition between the new and remanufactured products in different channels (crosschannel cannibalization). Their setting does not consider the competition that exists in the same channel (interchannel cannibalization). Moreover, to differentiate the new and remanufactured product sold within and across channels, the proposed model considers key differentiating parameters: i) customer's acceptance levels for the remanufactured product and for the online channel for the demand aspect, and ii) production and channel selling costs for the profit aspect. Most studies in the literature, such as Batarfi et al. (2017), Zheng et al. (2017), and Xie et al. (2017), fail to consider the difference in operating costs between brick-and-mortar retailer stores and online stores that can affect the manufacturer's optimal pricing strategies and the decisions on the production of the remanufactured product. Tables 2.1 and 2.2 summarize the main differences between our model with the models mentioned in the above literature review.

Table 2.1: Summary table: planning horizon and the type of competition considered

References	Time horizon (# periods)	Intrachannel competition between new and remanufactured	Crosschannel competition between new and remanufactured	Crosschannel competition between same products
Abbey et al. (2015a)	One	√		
Giri et al. (2017)	One			√
Choi (2017)	One	√		
Batarfi et al. (2017)	Infinite	√	√	
Zheng et al. (2017)	One	√	√	
Xie et al. (2017)	One	√	√	
Gan et al. (2017)	Four	√	√	
Liu et al. (2018)	Two	√		
Radhi and Zhang (2018)	One			√
Taleizadeh et al. (2018)	One			√
Chen et al. (2019)	Two	√		
He et al. (2019)	One	√	√	
Sun et al. (2020)	One	√		
Our study	**Two**	**√**	**√**	**√**

2.3 Problem description

A CLSC with a manufacturer and a retailer is considered. The selling horizon is divided into two periods. In the first period, the manufacturer sells the new product only through both its online channel and the retail channel. In the second period, the manufacturer sells the new product through the retail channel and both the remanufactured and new products (Case B) or the new product only (Case N) through its online channel. The two cases, Case N (without remanufacturing) and Case B (with remanufacturing), are shown in Figure 2.1. Table 2.3 presents the notation used in the chapter.

In accordance with the extended producer responsibility (EPR) legislation, the

Table 2.2: Summary table: key parameters considered

References	Customer's acceptance level for remanufactured Products	Customer's acceptance level for online channel	Channel costs
Abbey et al. (2015a)			
Giri et al. (2017)		$\surd$	
Choi (2017)			
Batarfi et al. (2017)	$\surd$		
Zheng et al. (2017)		$\surd$	
Xie et al. (2017)	$\surd$		
Gan et al. (2017)	$\surd$		
Liu et al. (2018)	$\surd$		
Radhi and Zhang (2018)		$\surd$	
Taleizadeh et al. (2018)		$\surd$	
Chen et al. (2019)	$\surd$		
He et al. (2019)	$\surd$	$\surd$	
Sun et al. (2020)			
Our study	$\surd$	$\surd$	$\surd$

Table 2.3: Table of notation

Indices	
i	Index of the product types (subscript): $i = r$ (remanufactured) and $i = n$ (new)
j	Index of the planning periods (superscript): $j = I, II$
k	Index of the selling channels (subscript): $k = o$ (online) and $k = t$ (retailer)
l	Index of the SC members (subscript): $l = M$ (manufacturer) and $l = R$ (retailer)
s	Index of Cases (superscript): $s = N$ (selling new products only case) and $s = B$ (selling both remanufactured and new products case)
Parameters	
c_n	Unit production cost for the new product
c_p	Unit selling cost through the retail channel
α	Customer's acceptance level of the remanufactured product
β	Customer's acceptance level of the online channel
z	Customer's perceived value of a new product, a uniform distribution with the supporting range $[0, 1]$
c_t	Retailer's unit collection cost of the high-quality return from the customer
c_m	Manufacturer's unit collection cost of the high-quality return from the customer
c_{mt}	Manufacturer's unit collection cost of the high-quality return from the retailer
q	Fraction of products returned to the retailer
δ	Proportion/percentage of the high-quality return
h	Manufacturer's unit expected collection cost for high-quality returns
d_{ki}^{js}	Demand of product type i through channel k in period j under Case s
π_l^{js}	Profit for SC member l in period j under Case s
Decision Variables	
p_{ki}^s	Selling price for product type i through channel k under Case s
w_n^s	Wholesale price for the new product under Case s

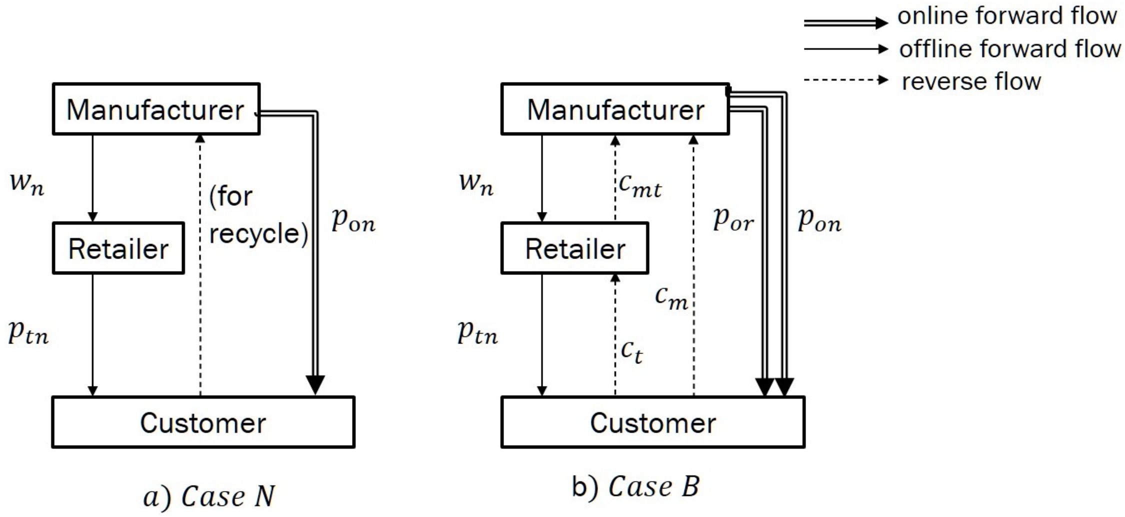


Figure 2.1: Structure of the two cases

manufacturer manages the recovery of all its products sold. Most countries in the world have either implemented EPR (e.g., European Union, Canada, and Japan), or introduced and designed related policies (e.g., China and India) (OECD, 2014). Therefore, for the reverse SC, the manufacturer is assumed to collect all end-of-life/end-of-use products directly from the customers or through the retailer. Many companies such as Xerox (Han et al., 2016) and Lexmark (Hong et al., 2015), use the direct reverse channel to recover their products. Other companies such as Hewlett Packard, collect returns through their retailers (Han et al., 2016).

Consistent with studies in the literature, such as Debo et al. (2006), each product can only be remanufactured at most once. Returned new products are classified into two bins based on their quality: high-quality and low-quality returns. The high-quality return can be remanufactured at a lower cost, compared to the low-quality return. When engaging in remanufacturing activities (Case B), in order to decrease the remanufacturing cost, the manufacturer is willing to pay a higher return/collection fee (incentive) for collecting high-quality returns directly from customers or through the retailer. In order to maximize its reverse channel profit, the retailer is also willing to pay a higher fee for collecting high-quality returns from customers and then returns them back to the manufacturer with a unit fee. In 2020, a 64GB iPhone XS can be returned for a \$50 incentive from the Apple's trade-in program in Canada, while a well-functioning one can be traded-in for up to \$400 (https://www.apple.com/ca/trade-in/). Therefore, in this chapter, the collecting/recovery costs for low-quality returns, such as the low-quality returned new product and returned remanufactured product, are considered negligible compared to the collection/recovery costs for high-quality products and are thus normalized to 0.

Therefore, for Case B, the manufacturer collects high-quality returns directly from customers with a unit cost c_m or through the retailer with a unit buyback cost c_{mt}. For the retailer, the unit collecting cost for a high-quality return is c_t. Thus, each high-quality new product returned through the retail channel leads to a net value of $c_{mt} - c_t$ for the retailer. It is assumed that a proportion δ of all returns are high-quality. A customer will return a product directly to the manufacturer with probability $1 - q$ and to the retailer with probability q. For the manufacturer, the expected collection cost for each high-quality return is $h = qc_{mt} + (1 - q)c_m$. This policy applies in both periods. However, the returns collected in the second period are not used for remanufacturing as remanufacturing takes time. The collected returns in the second period and unused returns from the first period are all recycled.

For Case N, in which the manufacturer does not engage in remanufacturing, both the retailer and the manufacturer have no incentive to collect high-quality returns with additional costs, as all returns will be recycled. It is assumed that the salvage value of a recycled product is sufficiently high to offset its collection and recycling cost. The process of the reverse SC is shown in Figure 2.2.

The unit production cost for a new product (c_n) is always higher than the cost for a remanufactured product, as remanufacturing saves both energy and raw materials in the production process (Liao et al., 2018). We normalize the unit remanufacturing cost to 0. Moreover, since the online channel generally costs less to operate due to the lower rental and labour costs (Miyatake et al., 2016), the unit selling cost in the retail channel is denoted by c_p and the unit online selling cost is normalized to 0.

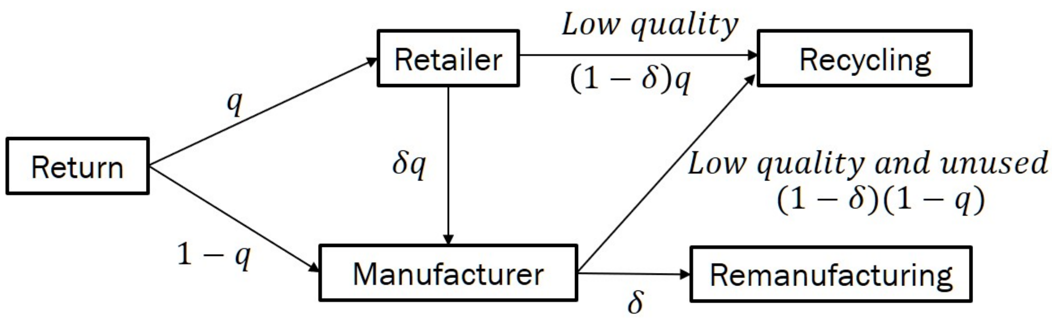


Figure 2.2: Reverse SC process

Consistent with studies in the literature (Wu and Zhou, 2017; Xu et al., 2018), it is assumed that the manufacturer is the Stackelberg leader of the SC who is the first-mover in pricing decisions. In the beginning of the first period, it sets the wholesale price (w_n) for the new product first. Then, both the retailer and the manufacturer announce the selling prices for the new product for both channels. For example, the price of the iPhone XS was announced by Apple Inc. on September 12, 2018. Then, later on the same day, retailers such as T-mobile and AT&T announced their selling prices (Liao, 2020). At the beginning of the second period, if the manufacturer sells the remanufactured product, it decides and announces the selling price for the remanufactured product. The iPhone 6s, for example, was released in September 2015 while its refurbished version was released at least a year later at the end of 2016 (Benjamin, 2020). This implies that in the first period, customers do not know the price for the remanufactured product (in the second period). Following the studies of Dou et al. (2019) and Xu and Wang (2018), customers are assumed not to postpone their purchase in the first period. The wholesale and selling prices for the new product are assumed to be the same for both periods. This assumption is common for electronics and goods. For example, the prices for the iPhone XS and iPhone XR remained unchanged on Apple's website since they launched in 2018 (Chauhan, 2019), and their refurbished versions are also currently sold on the same website several months later. The decision sequences for the manufacturer and the retailer are illustrated in Figure 2.3. It should be noted that the decision on the selling price for the remanufactured product in the second period is only made in Case B.

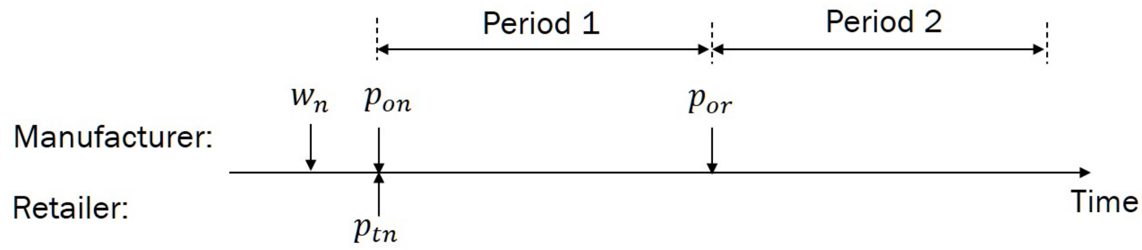


Figure 2.3: Decision sequence in the SC

The demand functions in the model are derived based on the utility theory. As in Liu et al. (2018), it is assumed that the customer's perceived value for a new product is z, which is heterogeneous and uniformly distributed on [0, 1]. Parameter α is used to represent the level of the customer's acceptance for the remanufactured product. Since the customer values the remanufactured product as the low-end version of the new product (Atasu et al., 2010), α is assumed to be between 0 and 1; $\alpha = 1$ implies that the customer equally values remanufactured and new products. Parameter β represents the level of the customer's acceptance for the online channel. A similar setting is found in Feng et al. (2017). According to Rofin and Mahanty (2018), the value of β depends on the type of products (e.g. $\beta = 0.787$ for DVD players and $\beta = 0.769$ for shoes). The customer values the online channel lower than the traditional retail channel because it takes several days to receive a purchase made online and the customer can only appreciate the product through a virtual description offered by the manufacturer's website (Chiang et al., 2003). In this chapter, β is assumed to be between 0 and 1; $\beta = 1$ represents that the customer does not differentiate between the two channels, and $\beta = 0$ means that the customer never chooses to buy products online. The potential market size is assumed to be 1 for each period. Similar assumptions can be found in Ferrer and Swaminathan (2006) and Dou et al. (2019).

For the first period, the manufacturer contracts with the retailer to sell the new product for both cases ($s = \{N, B\}$): the customer can buy the new product either from the direct online channel or the retail channel. The utilities can be presented as:

$$U_{tn}^{Is} = z - p_{tn}^{s} \tag{2.1}$$

$$U_{on}^{Is} = \beta z - p_{on}^{s}. \tag{2.2}$$

The customer will buy a new product through the retail channel when $U_{tn}^{Is} \geq 0$ and $U_{tn}^{Is} \geq U_{on}^{Is}$. $U_{tn}^{Is} \geq 0$ leads to $z \geq p_{tn}^{s}$ and $U_{tn}^{Is} \geq U_{on}^{Is}$ leads to $z \geq \frac{p_{tn}^{s} - p_{on}^{s}}{1-\beta}$. This customer will buy a new product through the online channel when $U_{on}^{Is} \geq 0$ and $U_{on}^{Is} \geq U_{tn}^{Is}$. $U_{on}^{Is} \geq 0$ and $U_{on}^{Is} \geq U_{tn}^{Is}$ lead to $z \geq \frac{p_{on}^{s}}{\beta}$ and $z \leq \frac{p_{tn}^{s} - p_{on}^{s}}{1-\beta}$, respectively. To ensure that selling the new product is profitable either from the online channel and physical retail store, it is assumed that $\frac{p_{on}^{s}}{\beta} \leq \frac{p_{tn}^{s} - p_{on}^{s}}{1-\beta} \leq 1$, which is equivalent to $\frac{p_{on}^{s}}{\beta} \leq p_{tn}^{s} \leq 1 - \beta + p_{on}^{s}$. Therefore, demands for the new and the remanufactured products in Case s ($s = \{N, B\}$) in the first period are

$$d_{tn}^{Is} = 1 - \frac{p_{tn}^{s} - p_{on}^{s}}{1-\beta}, \quad \text{and} \tag{2.3}$$

$$d_{on}^{Is} = \frac{\beta p_{tn}^{s} - p_{on}^{s}}{\beta(1-\beta).} \tag{2.4}$$

For the second period, if the manufacturer does not sell the remanufactured product ($s = N$), the demands for the new product in two channels are the same as those in the first period

$$d_{tn}^{IIN} = 1 - \frac{p_{on}^{N} - p_{or}^{N}}{1-\beta}, \quad \text{and} \tag{2.5}$$

$$d_{on}^{IIN} = \frac{\beta p_{tn}^{N} - p_{on}^{N}}{\beta(1-\beta).} \tag{2.6}$$

For $s = B$ (Case B), the customer can buy the new product through either the traditional retail channel or through the direct online channel or buy the remanufactured product through the direct online channel. The utilities for the customer are then given by:

$$U_{tn}^{IIB} = z - p_{tn}^{B}, \tag{2.7}$$

$$U_{on}^{IIB} = \beta z - p_{on}^{B}, \quad \text{and} \tag{2.8}$$

$$U_{or}^{IIB} = \alpha\beta z - p_{or}^{B}. \tag{2.9}$$

The customer will buy a new product through the retailer only when $U_{tn}^{IIB} \geq 0$, $U_{tn}^{IIB} \geq U_{on}^{IIB}$, and $U_{tn}^{IIB} \geq U_{or}^{IIB}$, which give $z \geq p_{tn}^{B}$, $z \geq \frac{p_{tn}^{B}-p_{on}^{B}}{1-\beta}$, and $z \geq \frac{p_{tn}^{B}-p_{or}^{B}}{1-\alpha\beta}$, respectively. The customer will buy a new product through the direct online channel only when $U_{on}^{IIB} \geq 0$, $U_{on}^{IIB} \geq U_{tn}^{IIB}$, and $U_{on}^{IIB} \geq U_{or}^{IIB}$, which give $z \geq \frac{p_{on}^{B}}{\beta}$, $z \leq \frac{p_{tn}^{B}-p_{on}^{B}}{1-\beta}$, and $z \geq \frac{p_{on}^{B}-p_{or}^{B}}{\beta(1-\alpha)}$, respectively. The customer will buy a remanufactured product through the direct online channel when $U_{or}^{IIB} \geq 0$, $U_{or}^{IIB} \geq U_{on}^{IIB}$, and $U_{or}^{IIB} \geq U_{tn}^{IIB}$, which give $z \geq \frac{p_{or}^{B}}{\alpha\beta}$, $z \leq \frac{p_{on}^{B}-p_{or}^{B}}{\beta(1-\alpha)}$, and$z \leq \frac{p_{tn}^{B}-p_{or}^{B}}{1-\alpha\beta}$, respectively.

To ensure that the demands for both the remanufactured and the new product in two channels are non-negative, it is required that $\frac{p_{or}^{B}}{\alpha\beta} \leq \frac{p_{on}^{B}-p_{or}^{B}}{\beta(1-\alpha)} \leq \frac{p_{tn}^{B}-p_{on}^{B}}{1-\beta} \leq 1$ ($\frac{p_{or}^{B}}{\alpha\beta} \geq \frac{p_{on}^{B}-p_{or}^{B}}{\beta(1-\alpha)}$ leads to $d_{or}^{IIB} = 0$, while $\frac{p_{on}^{B}-p_{or}^{B}}{\beta(1-\alpha)} \geq \frac{p_{tn}^{B}-p_{on}^{B}}{1-\beta}$ leads to $d_{on}^{IIB} = 0$, and $\frac{p_{tn}^{B}-p_{on}^{B}}{1-\beta} \geq 1$ leads to $d_{tn}^{IIB} = 0$). The demands for the new and the remanufactured product in Case B are

$$d_{tn}^{IIB} = 1 - \frac{p_{tn}^{B} - p_{on}^{B}}{1-\beta}, \tag{2.10}$$

$$d_{on}^{IIB} = \frac{\beta(1-\alpha)p_{tn}^{B} + (1-\beta)p_{or}^{B} - (1-\alpha\beta)p_{on}^{B}}{\beta(1-\alpha)(1-\beta)}, \quad \text{and} \tag{2.11}$$

$$d_{or}^{IIB} = \frac{\alpha p_{on}^{B} - p_{or}^{B}}{\alpha\beta(1-\alpha)}. \tag{2.12}$$

2.4 Optimal decisions

In this section, the optimal pricing strategies are derived for the manufacturer and the retailer in the Stackelberg game for both cases, $s = \{N, B\}$.

2.4.1 Case N: no remanufacturing

In this case, the customer can buy the new product only over two periods and the manufacturer will not collect returned products. This case serves as a benchmark case. Firstly, the manufacturer anticipates the wholesale price for the new product to the retailer. Secondly, without consultation with each other, the manufacturer sets the online channel selling price (p_{on}^N) and the retailer sets the physical store selling price (p_{tn}^N). The profit functions for the retailer and the manufacturer are respectively

$$\max_{p_{tn}^N \geq 0} \quad \pi_R^N = \sum_{j=I,II} \left(p_{tn}^N - w_n^N - c_p\right) d_{tn}^{jN} \quad \text{and} \tag{2.13}$$

$$\max_{p_{on}^N \geq 0} \quad \pi_M^N = \sum_{j=I,II} \left(w_n^N - c_n\right) d_{tn}^{jN} + \sum_{j=I,II} \left(p_{on}^N - c_n\right) d_{on}^{jN}. \tag{2.14}$$

By solving the objective function (2.13) and (2.14), the optimal prices are derived and summarized in Lemma 1.

Lemma 2.1. *The optimal prices for the new product in the online and retail channels are* $p_{tn}^{N*} = \frac{(c_n - w_n^N + 2)(1-\beta) + 2c_p + 3w_n^N}{4-\beta}$ *and* $p_{on}^{N*} = \frac{2c_n(1-\beta) + (c_p + 3w_n^N + 1)\beta - \beta^2}{4-\beta}$.

With prices in Lemma 1, we can derive the optimal demands for the new product in both channels as:

$$\begin{aligned} d_{tn}^{IN*} = d_{tn}^{IIN*} &= \frac{2 + c_n - w_n}{4 - \beta} - \frac{c_p(2-\beta)}{(4-\beta)(1-\beta)} \quad \text{and} \\ d_{on}^{IN*} = d_{on}^{IIN*} &= \frac{(1 + c_n - w_n)\beta - 2c_n}{\beta(4-\beta)} + \frac{c_p}{(4-\beta)(1-\beta)}. \end{aligned} \tag{2.15}$$

Lemma 1 shows that the optimal prices for the new product in both channels increase and their demands decrease, as the wholesale price increases. This implies that setting a reasonable wholesale price is an important decision for the manufacturer to maximize its profit and compete with the retailer. Due to the increase in the wholesale price (for the new product), the selling price for the new product is also increased by the retailer to ensure that the marginal profit for each new product is unchanged, which leads to a reduction in the demand for the new product in the retail channel. In the meantime, the manufacturer also increases the selling price for the new product more rapidly in the online channel to ensure a higher marginal profit for each new product, which also leads to a decrease in the demand. This result implies that a high wholesale price reduces the market share of the retailer and enhances the revenue that the manufacturer earns from the online direct channel, but it also reduces the revenue earned by the manufacturer from the wholesaling to the retailer. Increasing the wholesale price, however, cannot guarantee a profit increase in the online channel for the manufacturer. The manufacturer needs to balance the advantage and disadvantage in wholesale to obtain an optimal wholesale price.

By deriving the wholesale price and with Lemma 2.1, we have the following result.

$$\max_{w_n^N \geq 0} \quad \pi_M^N = \sum_{j=I,II} \left(w_n^N - c_n\right) d_{tn}^{jN} + \sum_{j=I,II} \left(p_{on}^N - c_n\right) d_{on}^{jN}. \tag{2.16}$$

Since $\frac{\mathrm{d}^2\pi_M^N}{\mathrm{d}w_n^{N2}} = -\frac{2\beta+16}{(4-\beta)^2} \leq 0$, π_M^N is concave in w_n^N. Thus, the optimal wholesale price is obtained by setting $\frac{\mathrm{d}\pi_M^N}{\mathrm{d}w_n^N} = 0$. The optimal prices and demands are then obtained by substituting the optimal wholesale price from Lemma 2.1.

Theorem 2.1. *Under the conditions that* $c_p \leq 1-\beta$ *and* $c_n \leq c_{n5} = \frac{(2+6c_p-\beta^2-\beta)\beta}{(1-\beta)(\beta+8)}$*, the optimal prices and demands for the manufacturer and the retailer are summarized in Table 2.4 (Strategy N):*

Table 2.4: Optimal prices and demands in Case N

Prices		Demands	
w_n^N	$\frac{\beta^2+(\beta+8)c_n-8c_p+8}{2(\beta+8)}$	d_{tn}^{jN}	$\frac{(\beta+2)(1-c_p-\beta)}{(1-\beta)(\beta+8)}$
p_{tn}^N	$\frac{(\beta+8)c_n+4c_p-\beta^2-2\beta+12}{2(\beta+8)}$	d_{on}^{jN}	$\frac{2+6c_p-\beta^2-\beta}{2(1-\beta)(8+\beta)} - \frac{c_n}{2\beta}$
p_{on}^N	$\frac{-\beta^2+(c_n-2c_p+10)\beta+8c_n}{2(\beta+8)}$		

$*j = I, II$

Theorem 2.1 states that the optimal results are feasible when the values of c_n and c_p are not extremely huge. When $c_p > 1-\beta$, the cost for selling the new product at the retailer is extremely high, and the retailer will not operate as it is not profitable. Similarly when $c_n > \frac{(2+6c_p-\beta^2-\beta)\beta}{(1-\beta)(\beta+8)}$, it is not profitable to sell the new product in the manufacturer's online channel.

The results in Table 2.4 show that an increase in the production cost c_n results in the increase of the selling price in the retail channel without affecting the optimal demand and profit of the retailer. This implies that although the increase of the production cost leads to the increase of the wholesale price, which results in the increase of the cost for the retailer, the profit for the retailer is not affected. The reason is that when the production cost increases, the retailer ensures that the profit margin for the new products is unchanged by increasing its selling price. However, the manufacturer also increases the selling price for the new product sold online due to the increase of the production cost, which causes the unchanged demand for the retailer.

Table 2.4 also shows that an increase in the retail channel cost (c_p) leads to a decrease of the manufacturer's profit, implying that the increase of the channel cost not only decreases the retailer's profit, but it also has a negative effect on the manufacturer's profit. The reason is that the increase of the retail channel cost (c_p) leads to an increase of the selling price in the retail channel, which causes the product sold in the retail channel to become less attractive to the customers. The manufacturer can then expand its market share of the new product sold online to generate more profit. However, since the selling price for the new product in the retail channel increases, the demand in this channel decreases. The manufacturer obtains a lower profit from wholesaling the product to the retailer, which leads to a decrease in its total profit. This negative effect on profit overwhelms the positive effect from expanding the market for the retailer. Therefore, the manufacturer may help the retailer to decrease the retail channel cost, which can benefit both of them.

2.4.2 Case B: with remanufacturing

For case B, following the sequence in Figure 2.3, decisions are made in three steps. Firstly, the manufacturer determines and discloses a wholesale price (w_n^B) that maximize its own profit. In the second step, based on the wholesale price offered by the manufacturer, the retailer decides and reveals its selling prices (p_{tn}^B), and meanwhile the manufacturer announces its online selling price (p_{on}^B) for the new product. In the third step, based on the number of collected returns and the prices set in the first two steps, the manufacturer decides the online selling price for its remanufactured products (p_{or}^B). Based on the game theory principles (Aumann, 2019), the problem is solved backward.

In the third step, the manufacturer needs to decide the selling price for the remanufactured product in the online channel to maximize its profit in the second period.

$$\max_{p_{or}^B \geq 0} \quad \pi_M^{IIB} = \left(w_n^B - c_n\right) d_{tn}^{IIB} + \left(p_{on}^B - c_n\right) d_{on}^{IIB} + p_{or}^B d_{or}^{IIB} - h\delta \sum_{k=t,o} d_{kn}^{IIB}, \qquad (2.17)$$

s.t.

$$\delta(d_{tn}^{IB} + d_{on}^{IB}) \geq d_{or}^{IIB}. \qquad (2.18)$$

π_M^{IIB} is the sum of four terms: the first term is the profit from wholesaling the new product to the retailer; the second term is the profit from selling the new product through the online direct channel; the third term is the profit from selling the remanufactured product, and the fourth term is the collection cost for new products sold in the second period. Constraint (2.18) ensures that the quantity of remanufactured products does not exceed the quantity of high-quality returns collected in the first period.

From the Karush-Kuhn-Tucker (KKT) conditions, the optimal prices and demands in this step can be obtained and are summarized in Lemma 2.2.

Lemma 2.2. *The optimal price for the remanufactured product sold online by the manufacturer depends on the selling price of the new product in the online channel. Specifically,*

1. *when* $p_{on}^B \leq \beta - \frac{h\delta + c_n}{2(1-\alpha)\delta}$, *then* $p_{or}^B = \frac{(2p_{on}^B - h\delta - c_n)\alpha}{2}$ *and the corresponding demand is* $d_{or}^{IIB} = \frac{h\delta + c_n}{2(1-\alpha)\beta}$;
2. *when* $p_{on}^B > \beta - \frac{h\delta + c_n}{2(1-\alpha)\delta}$, *then* $p_{or}^B = ((p_{on}^B - (1-\alpha)(\beta - p_{on}^B)\delta)\alpha$ *and the corresponding demand is* $d_{or}^{IIB} = \delta(1 - \frac{p_{on}^B}{\beta})$.

Lemma 2.2 shows the optimal selling price for the remanufactured product in the manufacturer's online channel for a given wholesale price and selling prices for the new products in both channels (online & retail). When the selling price for the new product in the manufacturer's online channel is sufficiently low, the returns in the first period are sufficient for producing the remanufactured product, resulting in a lower selling price for the remanufactured products. Otherwise, the manufacturer sets a high selling price for the remanufactured product in order to maximize the profit from selling the limited number of remanufactured products (constrained by the number of returns in the first period).

Interestingly, Lemma 2.2 shows that when the selling price for the new product is sufficiently low ($p_{on}^B \leq \beta - \frac{h\delta + c_n}{2(1-\alpha)\delta}$), the optimal demand for the remanufactured product is not affected by the selling price of the new products in both channels. This means that if the manufacturer can ensure that there is sufficient returns to produce the remanufactured product (e.g. industries with high returns rate and the manufacturer can easily collect sufficient returns for remanufacturing), it can easily design the production line and production plan for remanufactured products well in advance since the production will be constant.

In the second stage of the game, the manufacturer and the retailer need to decide the optimal selling prices for the new product in both channels simultaneously to maximize their profits over two periods.

$$\max_{p_{tn}^B \geq 0} \quad \pi_R^B = \sum_{j=I,II} \left(p_{tn}^B - w_n^B - c_p\right) d_{tn}^{jB} + (c_{mt} - c_t) q\delta \sum_{\substack{k=t,o \\ j=I,II}} d_{kn}^{jB} \tag{2.19}$$

and

$$\max_{p_{on}^B \geq 0} \quad \pi_M^B = \sum_{j=I,II} \left[\left(w_n^B - c_n\right) d_{tn}^{jB} + \left(p_{on}^B - c_n\right) d_{on}^{jB}\right] + p_{or}^B d_{or}^{IIB} - h\delta \sum_{\substack{k=t,o \\ j=I,II}} d_{kn}^{jB}. \tag{2.20}$$

After substituting the optimal p_{or}^{B*} from Lemma 2.2 into (2.19) and (2.20), the optimal prices can be derived by solving the Karush-Kuhn-Tucker (KKT) conditions.

Lemma 2.3. *The optimal selling prices for the new product sold in the online and* *the retail channels depend on the value of the wholesale price for the new product sold to the retailer:*

1. *when* $w_n^B \leq 1 - \frac{(4(1-\alpha)(1-\beta)\delta-\beta+4)C}{6\delta(1-\alpha)\beta} - \frac{c_p}{3}$,

$$p_{tn}^B = \frac{2A_1 + 2w_n^B + C - (2 + C - w_n^B)\beta}{4-\beta} \quad and$$
$$p_{on}^B = \frac{2C + (3w_n^B + A_1 - 2C)\beta - \beta^2}{4-\beta}.$$

2. *when* $w_n^B > 1 - \frac{(4(1-\alpha)(1-\beta)\delta-\beta+4)C}{6\delta(1-\alpha)\beta} - \frac{c_p}{3}$,

$$p_{tn}^B = \frac{(C\delta\alpha - 2(A_1 + w_n^B)A_2)(1-\beta) + \left(2\,C - 2\,w_n^B + 4\right)\beta - 2\,C - 4\,(A_1 + w_n^B)}{(4\,\beta - 4)\,A_2 + 2\,\beta - 8}$$
$$p_{on}^B = \frac{C\,(1-\beta)\,\delta\,\alpha + \beta^2\,(1 + 2\,A_2) + \left(2\,C - 2\,A_2 - A_1 - 3w_n^B\right)\beta - 2\,C}{(1 + 2\,A_2)\,\beta - 2\,A_2 - 4},$$

where $C = h\delta + c_n$, $A_1 = 1 + c_p$, *and* $A_2 = \delta^2\alpha(1-\alpha)$.

Using the prices in Lemmas 2.2 and 2.3, in the first stage of the game, the manufacturer decides its optimal wholesale price to maximize its optimal profit over two periods.

$$\max_{w_n^B \geq 0} \quad \pi_M^B = \sum_{j=I,II} \left(w_n^B - c_n\right) d_{tn}^{jB} + \sum_{j=I,II} \left(p_{on}^B - c_n\right) d_{on}^{jB} + p_{or}^B d_{or}^{II} - h\delta \sum_{\substack{k=t,o \\ j=I,II}} d_{kn}^{jB}. \tag{2.21}$$

By substituting the optimal p_{tn}^{B*} and p_{on}^{B*} from Lemma 2.3 into Equation(2.21), as all the constraints are linear, the optimal wholesale price can be derived by the KKT conditions. The optimal wholesale and selling prices and demands for the remanufactured and the new products are summarized in Theorem 2.2.

Theorem 2.2. *The optimal wholesale price for the new product depends on the production cost of the new product (c_n) and the retail channel cost (c_p).*

1. *Strategy B: when* $c_n \leq c_{n1}$ *and* $c_p \leq 1 - \beta$, *then* $w_n^B = \frac{\beta^2 + (C+8)\beta + 8(C+A_3)}{2\beta+16}$;

2. *Strategy BA (use all returns): when* $c_{n1} \leq c_n \leq c_{n2}$ *and* $c_p \leq 1-\beta$*, then* $w_n^B = \frac{(2-\alpha\delta)C}{4+2A_2} + \frac{2A_3(2+(1-\beta)A_2)}{8+\beta+4(1-\beta)A_2} + \frac{(1+A_2)\beta}{2+A_2}$*;*

where $A_3 = 1-c_p-\beta$ *and values of* c_{nx} $(x=1,2)$ *are summarized in Table A.1 in Appendix.*

Table 2.5: Optimal decisions for the remanufactured and new products

	Strategy B	Strategy BA
w_n^{B*}	$\frac{\beta^2+(C+8)\beta+8(C+A_3)}{2(\beta+8)}$	$\frac{(2-\alpha\delta)C}{4+2A_2} + \frac{2A_3(2+(1-\beta)A_2)}{8+\beta+4(1-\beta)A_2} + \frac{(1+A_2)\beta}{2+A_2}$
p_{tn}^{B*}	$\frac{-\beta^2+(C-6)\beta-4A_3+8C+16}{2(\beta+8)}$	$\frac{(2-\alpha\delta)C}{4+2A_2} - \frac{A_3(2+(1-\beta)A_2)}{8+\beta+4(1-\beta)A_2} + \frac{2+A_2-\beta}{2+A_2}$
p_{on}^{B*}	$\frac{\beta^2+(C+2A_3+8)\beta+8C}{2(\beta+8)}$	$\frac{(2-\alpha\delta)C}{4+2A_2} + \frac{A_3\beta}{8+\beta+4(1-\beta)A_2} + \frac{(1+A_2)\beta}{2+A_2}$
p_{or}^{B*}	$\frac{(10-2c_p-\beta)\alpha\beta}{2(\beta+8)}$	$\frac{((2-A_2+(2-3\alpha)\delta))C\alpha}{4+2A_2} + \frac{(1+\delta-\delta\alpha)A_3\alpha\beta}{8+\beta+4(1-\beta)A_2} + \frac{(1+A_2-(1-\alpha)\delta)\alpha\beta}{2+A_2}$
$d_{tn}^{IB*}/d_{tn}^{IIB*}$	$\frac{(\beta+2)A_3}{(\beta+8)(1-\beta)}$	$\frac{(2+\beta+A_2-A_2\beta)A_3}{(1-\beta)(8+\beta+4(1-\beta)A_2)}$
d_{on}^{IB*}	$\frac{8-\beta^2-6A_3-7\beta}{2(1-\beta)(\beta+8)} - \frac{C}{2\beta}$	$\frac{1}{2+A_2} - \frac{(2-\alpha\delta)\delta C}{2\beta(2+A_2)} - \frac{(3+A_2(1-\beta))A_3}{(1-\beta)(8+\beta+4(1-\beta)A_2)}$
d_{on}^{IIB*}	$\frac{8-\beta^2-6A_3-7\beta}{2(1-\beta)(\beta+8)} - \frac{C}{2(1-\alpha)\beta}$	$\frac{(3\alpha(1-\alpha)\delta-2+(2-A_2)\alpha)C}{2(1-\alpha)(A_2+2)\beta} - \frac{(3+(A_2-\alpha\delta)(1-\beta))A_3}{(8+\beta+4(1-\beta)A_2)(1-\beta)} + \frac{1-\alpha\delta}{2+A_2}$
d_{or}^{IIB*}	$\frac{C}{2\beta(1-\alpha)}$	$\frac{\delta}{2+A_2} - \frac{(2-\alpha\delta)\delta C}{2\beta(2+A_2)} - \frac{\delta A_3}{8+\beta+4(1-\beta)A_2}$

Results in Table 2.5 and Theorem 2.2 show that when c_p is extremely high ($c_p \geq 1-\beta$), there is no sales for the new product at the retailer. Similarly, when c_n is extremely high ($c_n \geq c_{n2}$), there is no sales of any product through the manufacturer's direct online channel.

Moreover, Theorem 2.2 shows that when c_n is sufficiently high ($c_{n2} \geq c_n \geq c_{n1}$), the manufacturer uses all returns as raw material to produce remanufacturing products ($\delta(d_{tn}^{IB} + d_{on}^{IB}) = d_{or}^{IIB}$). The reason is that the increase of the production cost for the new product leads to an increase of the selling prices in both channels, resulting in the decrease of the demands of new products in both channels. Furthermore, it is seen that the value of the retailer's unit collection cost (c_t) has no effect on the optimal decisions for both the manufacturer and the retailer. The reason is that the profit that the retailer gains from the reverse channel depends on the demands for products in both channels instead of the demand in its own channel. The total demand for the new product depends on the price for the new product in the online channel only. Thus, the volume of the new product sold (a fraction of which is returned to the retailer) depends on the price in the online channel but not on c_t. Therefore, there is no need for the retailer to adjust its price decisions based on c_t.

The increase in the selling cost at the retailer (c_p) leads to the switch of the manufacturer's optimal production strategy from using all returns (Strategy BA) to using a fraction of returns (Strategy B). The total demand for the new product in the first period in two channels increases as the selling cost at the retailer increases and more returns are collected for remanufacturing in the second period. The reason for this trend can be seen in the sensitivity analysis below. Lemmas 2.4 and 2.5 summarize the impacts of increasing c_n and c_p on the optimal results in Strategy B and BA, where d^{I*} represents the total demand for the new product in the first period.

Lemma 2.4. *The changes in optimal prices and demands with the increase of* c_n *and* c_p *in Strategy B are summarized in Table 2.6.*

Table 2.6: Impacts of the increase in c_n and c_p on the optimal decisions under Strategy B

	w_n^{B*}	p_{tn}^{B*}	p_{on}^{B*}	p_{or}^{B*}	$d_{tn}^{IB*}/d_{tn}^{IIB*}$	d_{on}^{IB*}	d^{IB*}	d_{on}^{IIB*}	d_{or}^{IIB*}	π_M^{B*}
c_n	↑	↑	↑	–	–	↓	↓	↓	↑	↓
c_p	↓	↑	↓	↓	↓	↑	↑	↑	–	↓

*– for no impact; ↓ for decreasing; and ↑ for increasing

There are two interesting points in Lemma 2.4. Firstly, when the selling cost in the retail channel (c_p) increases, the manufacturer decreases the wholesale price. This suggests that the manufacturer and the retailer are in a cooperative relationship in the retail channel as the manufacturer achieves more revenue from its wholesaling to the retailer. As the selling cost in the retailer's channel (c_p) increases, the retailer needs to increase the selling price (p_{tn}^{B*}), which decreases demand for the new product at the retailer and causes profit losses for both manufacturer and retailer from this channel. To maximize its profit, the manufacturer must slowdown the decrease in demand at the retailer. For that, it needs to slowdown the increase in selling price (p_{tn}^{B*}) which can be achieved by reducing the wholesale price (w_n^{B*}).

Secondly, for the remanufactured product, changes of the production cost for the new product (c_n) and the selling cost at the retailer (c_p) have different impacts on the manufacturer's optimal pricing and production strategies. The manufacturer uses different tools when c_n and c_p change. Specifically, when c_n changes, the manufacturer should keep the price for the remanufactured product unchanged and thereby letting the demand for the remanufactured product self-adjust. When c_p changes, the manufacturer should keep the demand for the remanufactured product unchanged by adjusting its price accordingly. The reason for the manufacturer's different approaches to dealing with the changes of two costs is that the change of c_n affects the price competition between remanufactured and new products, while the change of c_p affects the price competition between the online and physical channels.

When c_n increases, the price for the new product(p_{on}^{B*}) is increased by the manufacturer and the demand for the remanufactured product increases accordingly, since in comparison with the remanufactured product, the new product becomes less attractive. The increase in the price for the remanufactured product (p_{or}^{B*}) leads to an increase of the profit for the new product in the online direct channel and a larger profit loss from selling the remanufactured products. Decreasing the price for the remanufactured product (p_{or}^{B*}) results in an increase in the profit from the remanufactured product and a larger profit loss from the new product in the online direct channel. Therefore, the manufacturer should not adjust the price of the remanufactured product.

When c_p increases, the retailer increases the price for selling the new product in the physical channel. Compared to the product sold online, the new product sold at the retailer store becomes less attractive. The manufacturer thus decreases all prices in its online channel (p_{on}^{B*} and p_{or}^{B*}) to capture more market share in the online channel to maximize its profit. The increased demand is shared by the remanufactured and new products in the online channel. In addition, since Strategy B is adopted when c_n is sufficiently low ($c_n \leq c_{n1}$, Theorem 2.2), in comparison with the remanufactured product, the new product is more profitable, all increased demand are from the new product. Therefore, the demand for the remanufactured stays unchanged.

Lemma 2.5. *The impacts of the increase in c_n and c_p on optimal prices and demands under Strategy BA are summarized in Table 2.7.*

Table 2.7: Impacts of the increase in c_n and c_p on the optimal decisions under Strategy BA

	w_n^{BA*}	p_{tn}^{BA*}	p_{on}^{BA*}	p_{or}^{BA*}	$d_{tn}^{IBA*}/d_{tn}^{IIBA*}$	d_{on}^{IBA*}	d^{IBA*}	d_{on}^{IIBA*}	d_{or}^{IIB*}	π_M^{BA*}
c_n	↑	↑	↑	↑	–	↓	↓	↓	↓	↓
c_p	↓	↑	↓	↓	↓	↑	↑	↑	↑	↓

*– for no impact; ↓ for decreasing; and ↑ for increasing

As compared to Lemma 2.4, there are two different points in the sensitivity analysis in Lemma 2.5. Firstly, the manufacturer increases the price for the remanufactured product (p_{or}^{BA*}) due to the increase in c_n. The reason is that as c_n increases, both manufacturer and retailer increase the selling prices for the new product in both channels, leading to the decrease of demands in both channels. Thus, for Strategy BA, where all returns are used for remanufacturing, the decreasing demands result in the decrease of the remanufactured product that the manufacturer can offer. In order to achieve the maximal profit by selling fewer remanufactured product, the price of the remanufactured product is increased.

Secondly, when c_p increases, the total demand for the new product in the first period increases, allowing the manufacturer to produce more remanufactured products. Therefore, for Strategy BA, the manufacturer increases sales for the remanufactured product.

Under both Strategy B and Strategy BA, the increase in the selling cost at the retailer store (c_p) reduces the competitiveness of the product sold in the retail channel, which motivates the manufacturer to further decrease the prices for products sold online. The total demand for the new product in the first period increases, since more customers can afford the new product sold online due to the lower price. Therefore, the increase of c_p leads to more returns obtained by the manufacturer from the first period and its optimal strategy may switch from using all returns (Strategy BA) to partially using the returns (Strategy B).

2.4.3 Comparison of the two cases

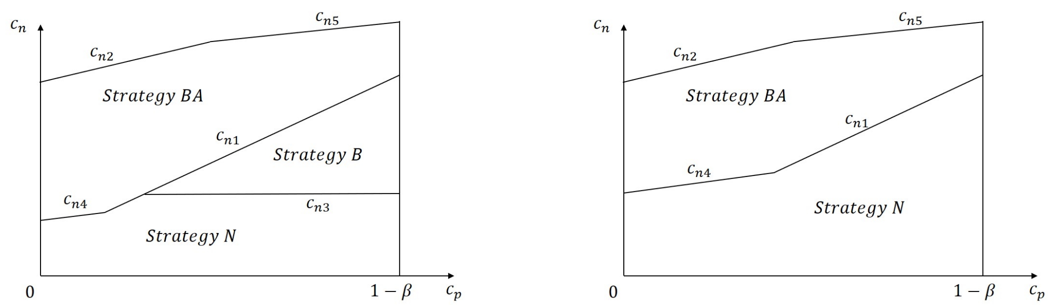
In this subsection, the optimal solutions of the two cases summarized in Theorems 2.1 and 2.2 are compared under the condition that $c_p \leq 1-\beta$ and $c_n \leq min(c_{n2}, c_{n5})$, where both optimal pricing strategies in Theorems 2.1 and 2.2 can be adopted by the manufacturer. The results of the comparison are summarized in Theorem 3 and illustrated in Figure 3 as follows.

Theorem 2.3. *Under the conditions of $c_p \leq 1-\beta$ and $c_n \leq min(c_{n2}, c_{n5})$, the optimal pricing strategy for the manufacturer depends on the production cost of the new product (c_n) and manufacturer's unit expected collection cost for the high-quality return (h). Specifically,*

1. *when the collection cost $h \leq \frac{\alpha\beta\sqrt{4+2A_2}}{2\alpha\delta+2A_2}$, the optimal pricing and production strategy is*
   - (a) *N, if $c_n \leq min(c_{n1}, c_{n3})$ and $c_n \leq max(c_{n1}, c_{n4})$;*
   - (b) *B, if $c_{n3} < c_n \leq c_{n1}$;*
   - (c) *BA, if $max(c_{n1}, c_{n4}) < c_n < min(c_{n2}, c_{n5})$;*
2. *when the collection cost $h > \frac{\alpha\beta\sqrt{4+2A_2}}{2\alpha\delta+2A_2}$, the optimal pricing and production strategy is*
   - (a) *N, if $c_n \leq max(c_{n1}, c_{n4})$;*
   - (b) *BA, if $max(c_{n1}, c_{n4}) < c_n < min(c_{n2}, c_{n5})$, where*

values of c_{nx} ($x = 1, ..., 6$) are summarized in Table A.1 in Appendix.

Theorem 2.3 shows that when the expected collection cost is sufficiently low ($h \leq \frac{\alpha\beta\sqrt{4+2A_2}}{2\alpha\delta+2A_2}$), the optimal pricing and production strategy for the manufacturer switches from Strategy N to Strategy B, and then to Strategy BA, as c_n increases. The intuition for this strategy switch is that when c_n is sufficiently low ($c_n \leq min(c_{n1}, c_{n3})$



(a) Case 1: $h \leq \frac{\alpha\beta\sqrt{4+2A_2}}{2\alpha\delta+2A_2}$ (b) Case 2: $h > \frac{\alpha\beta\sqrt{4+2A_2}}{2\alpha\delta+2A_2}$

Figure 2.4: Optimal pricing and production strategies for the manufacturer

and $c_n \leq max(c_{n1}, c_{n4})$), as compared to the new product, the remanufactured product is not profitable because of the small difference between the production cost and additional collection cost. Therefore, the manufacturer should not produce the remanufactured product. As c_n increases, the remanufactured product becomes more profitable in the market and the manufacturer starts to engage in remanufacturing (Strategy B and BA).

Moreover, when the manufacturer's expected unit collection cost for high-quality returns is sufficiently high ($h > \frac{\alpha\beta\sqrt{4+2A_2}}{2\alpha\delta+2A_2}$), Strategy B never becomes an optimal strategy for the manufacturer. This implies that the manufacturer needs to focus on increasing the total demand for the new product sold in the first period or collection rate when it decides to engage in the remanufacturing in such a case. Because the profit is constrained by the quantity of remanufactured products that the manufacturer can offer.

The retail's channel cost (c_p) also plays an important role in the manufacturer's remanufacturing decision. As c_p increases, the manufacturer has a higher probability of not selling the remanufactured product when the number of high-quality returns is not sufficient (comparing Strategy BA to Strategy N). However, when there are enough high-quality returns (the optimal solution suggests not using all returns: Strategy B), the remanufacturing decision is not affected by the value of c_p.

When the manufacturer adopts the different pricing strategies in the cases with and without remanufacturing, the optimal pricing and ordering strategy for the retailer also changes, leading to changes in its profit. Theorem 4 compares the profits of the retailer under Cases B and N (illustrated in Figure 2.5).

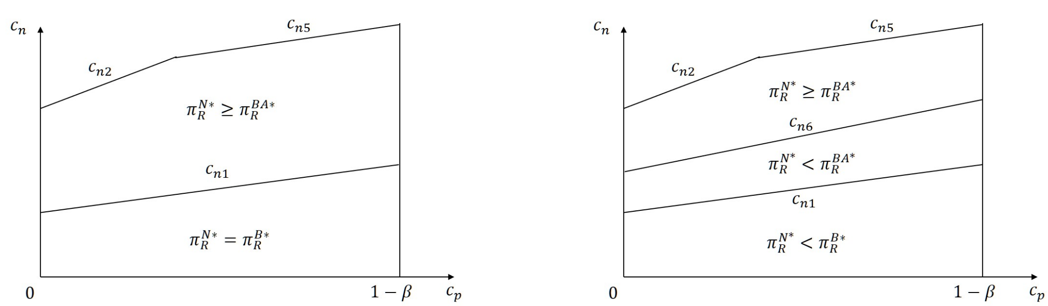
Theorem 2.4. *Under the conditions of* $c_p \leq 1-\beta$ *and* $c_n \leq min(c_{n2}, c_{n5})$*, the relationship of the retailer's optimal prices, demands, and profit under Cases N and B (Strategies B and BA) is shown as follows:*

1. *when* $c_n \leq c_{n1}$, $p_{tn}^{B*} \geq p_{tn}^{N*}$, $d_{tn}^{Bj*} = d_{tn}^{Nj*} (j = I, II)$*, and* $\pi_R^{*B} > \pi_R^{N*}$ ($\pi_R^{B*} = \pi_R^{N*}$ *when* $q = 0$*);*

2. *when* $c_{n1} < c_n \leq c_{n6}$, $p_{tn}^{BA*} \geq p_{tn}^{N*}$, $d_{tn}^{BAj*} \leq d_{tn}^{Nj*} (j = I, II)$*, and* $\pi_R^{BA*} > \pi_R^{N*}$*; this case does not exist when* $q = 0$*;*

3. *when* $c_{n6} < c_n \leq min(c_{n2}, c_{n5})$, $p_{tn}^{BA*} \geq p_{tn}^{N*}$, $d_{tn}^{BAj*} \leq d_{tn}^{Nj*} (j = I, II)$*, and* $\pi_R^{BA*} \leq \pi_R^{N*}$*;*

where values of c_{nx} $(x = 1, ..., 6)$ *are summarized in Table A.1 in Appendix.*



(a) when $q = 0$ (b) when $q > 0$

Figure 2.5: Comparison of retailer's optimal profits

When the remanufactured product is introduced into the market, the retailer increases the selling price of the new product in its channel. The demand for the new product in the retail channel stays unchanged, if c_n is sufficiently small ($c_n < c_{n1}$), and decreases if c_n is sufficiently large ($c_n > c_{n1}$). The reason is that, when c_n is sufficiently small, the introduction of the remanufactured product decreases the demand for the new product sold in the manufacturer's online direct channel, which leads to a profit loss. To cover this loss, the manufacturer increases the wholesale price for the new product. Under the pressure of the increased wholesale price for the new product, the retailer also increases its selling price for the new product to ensure that the unit profit for the new product is unchanged. Moreover, the manufacturer increases its selling price for the new product in the online channel, which not only helps counter the demand loss in the retail channel that would have occurred from the increased selling price in the retail channel, but also increases the unit profit for the new product sold online. When c_n is sufficiently large, the manufacturer increases the sales for the remanufactured product, since selling the remanufactured product becomes more profitable as compared to selling the new product. However, due to the limited number of high-quality returns, the manufacturer increases the price for the remanufactured product and thus, some customers switch to buying the new product in the online channel. The decreasing rate of the demand for the new product in the online channel due to the introduction of the remanufactured product slows down and the manufacturer does not need to set the selling price for the new product in the online channel at a sufficiently high level ($p_{tn}^{BA*} \geq p_{tn}^{B*}$), which leads to a decrease of demand in the retailer's channel.

Changes in selling price and demand for the retailer in Cases N and B lead to different profit values. The retailer obtains a higher profit when the manufacturer introduces the remanufactured product due to the extra profit gained with the recovery operations in the reverse SC. The profit from the forward SC stays unchanged for the retailer (i.e., when $q = 0$, the profits for the retailer are the same in both cases). However, when c_n increases, the manufacturer switches from Strategy B to Strategy BA by introducing more remanufactured products into the market, leading to a reduction in the retailer's profit. This profit loss increases as the value of c_n increases and when c_n is sufficiently large ($c_n \geq c_{n6}$), the retailer makes less profit in Case B than in Case N. In the meantime, the amount of high-quality returns collected in the first period has a significant impact on the manufacturer's profit. A decrease of the collection quantity leads to a decrease in the manufacturer's profit. To reduce the profit losses, the retailer can invest more in the collection service and take advantage of its location (closer to customers) to help the manufacturer collect more high-quality returns by for example, building an efficient recovery network and educating customers on proper product usage and maintenance. In return, the manufacturer shares the profit gained from the increased number of high-quality returns with the retailer. Otherwise, if the retailer does not help the manufacturer with the collection, the low number of collected high-quality returns leads to a profit loss for the manufacturer. A detailed discussion using a numerical example is provided in Section 5.3.

2.5 Numerical experiments

In this section, numerical experiments are used to examine the impacts of the customer's acceptance level for the remanufactured product (α), the customer's acceptance level for the online channel (β), and the proportion of high-quality returns (δ), on the optimal prices, demands, and profits of the manufacturer and the retailer. The following parameter values are selected: $c_n = 0.07$, $c_p = 0.04$, $q = 0.6$, $c_t = 0.007$, $c_m = 0.008$, $c_{mt} = 0.008$, $\alpha = 0.81$, $\beta = 0.85$, and $\delta = 0.4$.

2.5.1 The impact of the customer's acceptance level for the remanufactured product

In this subsection, the impact of the customer's acceptance level for the remanufactured product is examined from the results in Table 2.8.

Table 2.8: Impacts of increasing α on the optimal prices, demands, and profits

α	w_n^*	p_{tn}^*	p_{on}^*	p_{or}^*	d_{tn}^{I*}/d_{tn}^{II*}	d_{on}^{I*}	d_{on}^{II*}	d_{or}^{II*}	π_M^*	π_R^*	Strategy
0.65	0.5097	0.5851	0.4706	-	0.236258	0.2102	0.2102	-	0.3761	0.016731	N
0.67	0.5097	0.5851	0.4706	-	0.236258	0.2102	0.2102	-	0.3761	0.016731	N
0.69	0.5097	0.5851	0.4706	-	0.236258	0.2102	0.2102	-	0.3761	0.016731	N
0.71	0.5117	0.5871	0.4726	0.309	0.236258	0.2079	0.1013	0.1501	0.3764	0.016919	B
0.73	0.5117	0.5871	0.4726	0.318	0.236258	0.2079	0.0902	0.1612	0.3768	0.016916	B
0.75	0.5117	0.5871	0.4726	0.327	0.236258	0.2079	0.0773	0.1741	0.3773	0.016913	B
0.77	0.5115	0.5869	0.4723	0.337	0.236105	0.2082	0.0714	0.1777	0.3778	0.016897	BA
0.79	0.5110	0.5864	0.4719	0.348	0.236106	0.2088	0.0682	0.1779	0.3784	0.016898	BA
0.81	0.5105	0.5859	0.4713	0.359	0.236107	0.2094	0.0651	0.1782	0.3790	0.016898	BA

Table 2.8 shows that in general, as α increases, the manufacturer's optimal production strategy switches from Strategy N to Strategy B and then to Strategy BA.

The intuition is that the increase of α means that customers perception of remanufactured products is high. As compared to the new product, the remanufactured product has a lower production cost and therefore, the manufacturer starts to sell the remanufactured product and attract more customers to buy remanufactured products by adjusting the prices when α increases.

Moreover, the manufacturer's total profit increases as α increases. Unlike the results in Gan et al. (2017) and He et al. (2019), the profit of the retailer does not always decrease as α increases. It has a large increase when the manufacturer switches its optimal strategy from Strategy N to Strategy B due to the existence of the profit in the reverse SC. Then, it decreases when α increases between 0.71 and 0.75 and has a large drop when α increases to over 0.75. Then, the retailer's profit begins to increase again when $\alpha \geq 0.77$. The reason for this trend is that when α increases between 0.71 and 0.75, some customers switch from buying new products online to buying remanufactured products online and bring more profit to the manufacturer due to the higher marginal profit for the remanufactured product. In order to avoid the loss in the profit gained from the new product sold in the retail channel, the manufacturer keeps the wholesale and the online selling prices unchanged for the new product to ensure that the demand and the price for the new product in the retail channel are unchanged. However, in the reverse channel, since the total demand decreases due to the increase of α, the number of high-quality returns collected by the retailer decreases and then the associated collection profit gained by the retailer in the reverse channel also decreases. When $\alpha > 0.75$, in order to increase the number of returns, the manufacturer reduces the wholesale price for the new product which results in an increase of the retailer's profit in the forward channel. This increased profit in the forward channel offsets its profit loss in the reverse channel and leads to an overall higher total profit for the retailer. This result implies that when $\alpha \leq 0.75$, the retailer is not willing to put effort into increasing the customer's perceived value on the remanufactured product unless the manufacturer shares the portion of the increase in profit with the retailer to achieve a win-win situation. Table 2.8 shows that this win-win situation can be achieved due to the fact that the SC profit $(\pi_M^* + \pi_R^*)$ increases as α increases. When $\alpha > 0.75$, however, the retailer is willing to assist the manufacturer to increase the customer's acceptance level for the remanufactured product (e.g., promoting the sustainability virtues of remanufacturing) to enhance its own profit (the retailer's profit increases as α increases, when $\alpha > 0.75$ after an initial drop).

The wholesale price and selling prices in both channels for the new product show the same trend as α increases. When $\alpha \leq 0.75$, the change of α has no impact on all prices for the new product (when α changes from 0.69 to 0.71, all prices for the new product have a jump due to the inclusion of the collection cost as the manufacturer changes the production strategy). When $\alpha > 0.75$, the increase in α leads to a decrease in all prices of the new product. The reason is that when α is sufficiently small, although adjusting the price for the new product can increase the sales of the remanufactured product and generate more profit in the second period for the manufacturer as α increases, it can also cause a larger profit loss for the manufacturer in the first period. Therefore, in order to avoid or reduce this loss in the first period, the manufacturer should not adjust the prices for the new product. When α is sufficiently large (Strategy BA is adopted), the number of remanufactured products that can be produced is limited. Decreasing the prices for the new product cannot only attract some customers to buy the new product instead of the remanufactured product, but also increase the number of returns in the first period, which can be used for producing the remanufactured product and generate more profit for the manufacturer.

Another interesting point in Table 2.8 is that when α increases to between 0.71 and 0.75 (Strategy B is adopted), some customers switch from buying the new to buying the remanufactured product, but no new customers are attracted to buy the product in this period (as d_{tn}^{II*} and $d_{on}^{II*} + d_{or}^{II*}$ stay unchanged). This implies that in such a case, in order to maximize profit, the manufacturer will vary the selling price for the remanufactured product in the online channel based on the demands for the new product and the remanufactured product online. Customers, who originally are not willing to buy any product, still do not buy any product although their perceived values on the remanufactured product increase.

2.5.2 The effect of the customer's acceptance level for the online channel

In this subsection, the effect of the customer's acceptance level for the online channel (β) on the optimal prices, demands and profits of the manufacturer and the retailer is examined as illustrated in Figure 2.6.

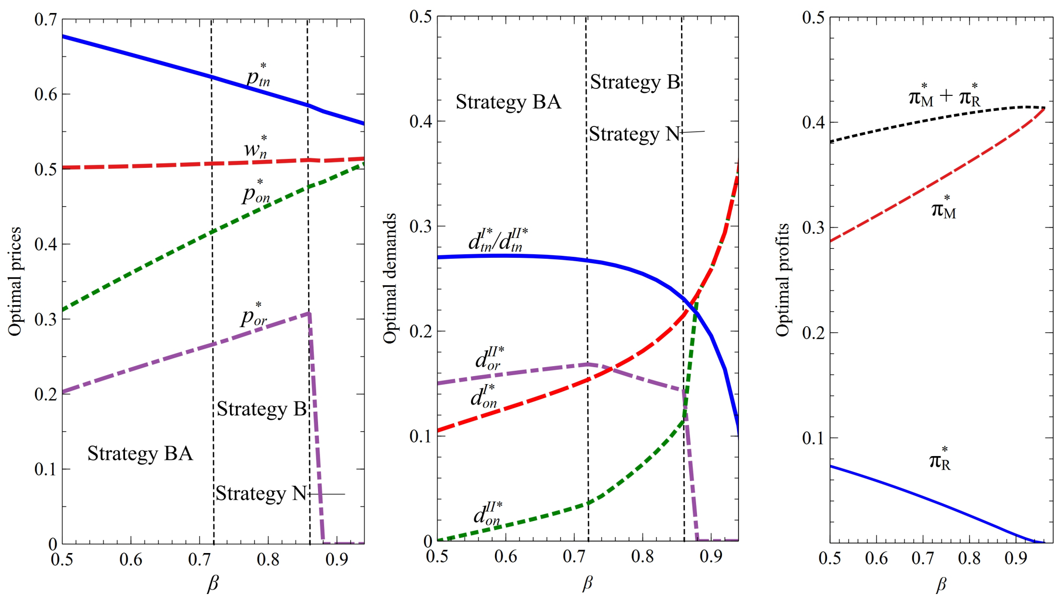


Figure 2.6: Impacts of increasing β on the optimal decisions

Figure 2.6 shows that, in general, the manufacturer's optimal production strategy switches from Strategy BA to Strategy B and then to Strategy N. This implies that the increase in β has a more positive impact on the profitability of the new product sold online than that of the remanufactured product. As β increases, fewer customers buy the remanufactured product and when β is sufficiently higher ($\beta \geq 0.88$ in this case), the demand for the remanufactured product drops to zero and the manufacturer should not engage in remanufacturing.

Moreover, the increase in β leads to an increase of the manufacturer's total profit and a profit loss for the retailer. This is because products sold in the manufacturer's online direct channel become more attractive to customers. However, the total SC profit increases at a slower pace as β increases and it decreases when $\beta \geq 0.92$, which is unlike the result in Gan et al. (2017). The reason is that on one hand, the increase in β can attract new customers, who originally were not considering a purchase, to buy the new or the remanufactured product through the online channel, as their perceived values for these products increase ($d_{tn}^{I} + d_{on}^{I}$ and $d_{tn}^{II} + d_{on}^{II} + d_{or}^{II}$ increase as β increases). On the other hand, the increase in β reduces the gap of the customer's perceived value between the retailer's store and the manufacturer's online channel, which results in intensifying the competition between both channels (especial for the new product as the gap between the selling prices for the new products sold in both channels becomes narrow). This causes a profit loss for the entire SC. This negative impact becomes larger as β increases, and results in a slight decrease in the total profit of the SC when $\beta \geq 0.92$.

2.5.3 Impacts of the proportion of high-quality returns in Strategy BA

In this subsection, the impacts of the proportion of high-quality returns (δ) on the optimal prices, demands, and profits of the manufacturer and the retailer in Strategy BA are examined as illustrated in Figure 2.7.

In general, as the proportion of high-quality returns (δ) decreases, the manufacturer's profit decreases. For the retailer, its profit from the forward SC increases while its total profit decreases as δ decreases. This happens because as δ decreases, the quantity of returns that can be used for remanufacturing (high quality) decreases, resulting in a profit loss for the manufacturer. The manufacturer can then use two different strategies to reduce its profit loss based on the value of δ: lowering the wholesale price if $\delta \geq 0.16$ or increasing the wholesale and online prices if $\delta < 0.16$.

- When $\delta \geq 0.16$, in order to reduce the profit loss, the manufacturer slows down the decrease of the quantity of high-quality returns in the first period by offering a lower wholesale price for the new product to the retailer. Moreover, it also decreases the price for the new product in the online direct channel to increase the sales such that some customers switch from buying remanufactured to buying the new product in the second period (d_{tn}^{I}, d_{on}^{I}, and d_{on}^{II} increases, while d_{or}^{II} decreases as δ decreases). Due to the lower wholesale price for the new product, the retailer's profit in the forward SC also increases. However, this increase is not sufficient to cover the losses in collection revenue due to the decreasing number of high-quality returns.

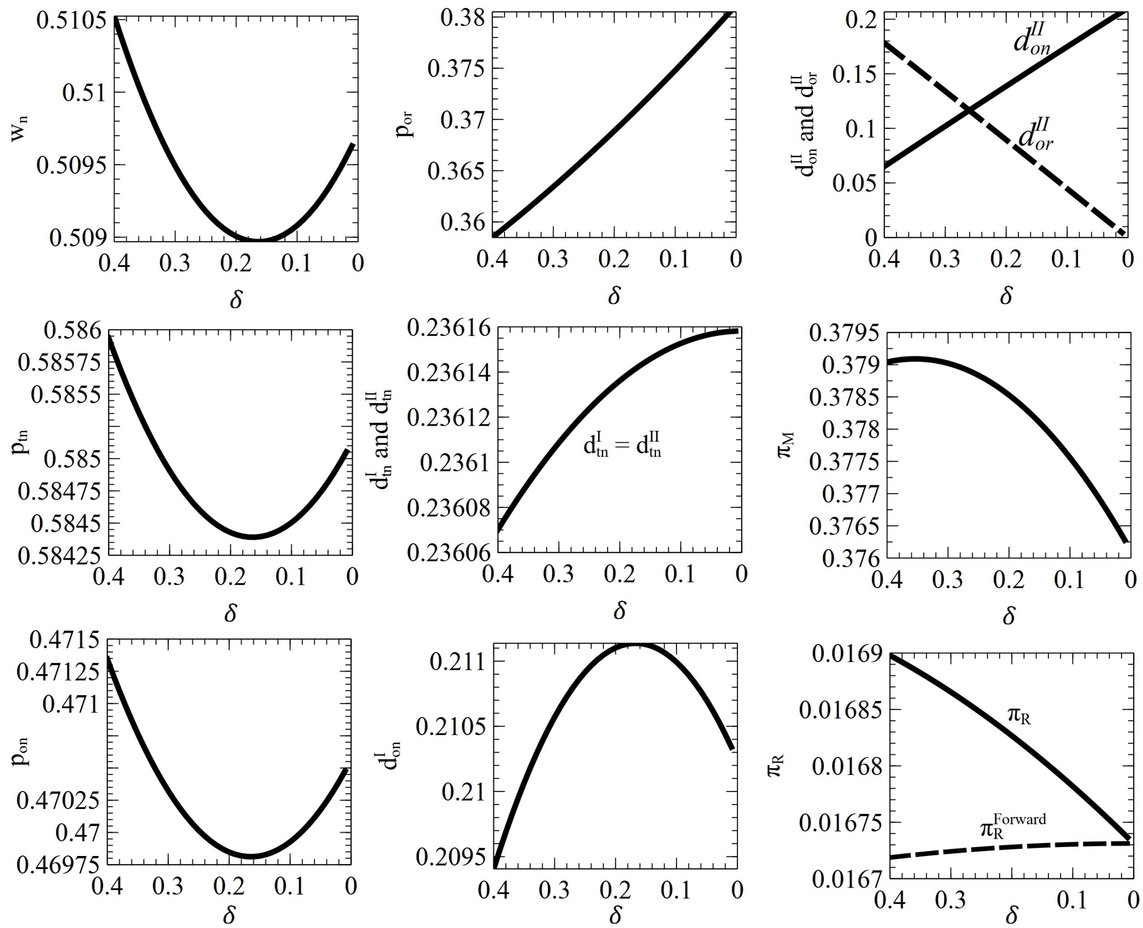


Figure 2.7: Impacts of the increase in δ on the optimal prices, demands, and profits of the manufacturer and the retailer

- When $\delta < 0.16$, the profit generated from selling the remanufactured product is low due to the low remanufacturing quantity. Therefore, the manufacturer should not sacrifice the profit from selling the new product in order to produce more returns. Then, the manufacturer's strategy is to increase the profit from selling the new product to offset the profit loss due to the decrease in δ. The wholesale price for the new product and selling price for the new product in the online direct channel are increased by the manufacturer to generate more profit and reduce the competitiveness of the new product sold by the retailer ($\delta < 0.16$). In the meanwhile, the retailer increases its selling price for the new product due to the higher acquisition cost (higher wholesale price), resulting in a slight increase in its profit, which still cannot offset its profit loss in the reverse SC.

In summary, the decrease in δ can result in profit losses for both the manufacturer and the retailer, when the manufacturer adopts Strategy BA. The total profit of the manufacturer and the retailer ($\pi_M + \pi_R$) increases as the value of δ increases. It is possible for the retailer to contract with the manufacturer at this time for a cooperation agreement, with which the retailer helps the manufacturer to increase the collection of high-quality returns by engaging in activities such as educating the customers on proper product usage and maintenance method when the product is sold or providing some simple preventive care services to customers after the product is sold. In return, the manufacturer should share the increased profit from the increased quantity of high-quality returns with the retailer.

2.5.4 Managerial implications

Managerial insights obtained from the model and numerical experiments in Chapter 2 are summarized below while addressing the research questions posed in the Introduction.

- When should the manufacturer carry out remanufacturing? Consistent with Atasu et al. (2008) and Liu et al. (2018), the production cost difference between the new and remanufactured products (equivalent to the unit production for the new products in this chapter, since the unit remanufacturing cost is normalized to 0) plays an essential role in the manufacturer's remanufacturing decision. When the cost difference is sufficiently high, it is profitable for the manufacturer to engage in remanufacturing. Moreover, it is found that the retailer's channel cost can affect the manufacturer's remanufacturing decision only when there are no sufficient high-quality returns. If the number of high-quality returns is sufficiently low, the manufacturer is more likely to engage in remanufacturing when the retailer's channel cost is low.

Due to the environmental benefits of remanufacturing such as alleviating the depletion of natural resources, reduction of energy usage and greenhouse gas emissions, government wanting to promote remanufacturing could offer subsidies or financial support to (re)manufacturers. The results of this study show that such support to the (re)manufacturers should be used to decrease the manufacturing cost and enhance the remanufacturability of the product instead of directly reducing the remanufacturing cost, if the (re)manufacturer cannot obtain enough returns. Our results also indicate that increasing the customer's acceptance level for remanufacturing is another method for promoting remanufacturing (increase the demand). Legislators and governments can invest in educating and promoting the values of buying remanufactured products. For example, on May 30, 2016, the French Parliament adopted decree 2016–703 mandating car dealerships and repair shops to offer both new and remanufactured spare parts if available to customers seeking repairs (Hogan Lovells LLP, 2016).

- How does the manufacturer's remanufacturing decision affect the retailer's selling price, demand, and profit? We find that when the manufacturer starts to produce remanufactured products, the retailer needs to increase its selling price. However, unlike the finding in Xiong et al. (2013), the manufacturer's remanufacturing activity does not always hurt the retailer's profit. When there are sufficient high-quality returns for remanufacturing, whether the manufacturer remanufactures products or not does not affect the retailer's profit in the forward SC because the manufacturer sells both the new and the remanufactured products through its direct online channel. The introduction of the remanufactured product in the market cannibalizes the demand for the new product sold online instead of the demand for the new product in the retail channel. Selling the new product online mitigates the impact of introducing the remanufactured product by the manufacturer on the retailer. Moreover, the remanufacturing activity from the manufacturer brings collection revenue to the retailer in the reverse SC.

- Which factors impact the manufacturer's and the retailer's pricing and production strategies? The customer's acceptance level for the remanufactured product (α) significantly affects the pricing and production decisions, and the profits for both the manufacturer and the retailer. When the manufacturer engages in remanufacturing, the increase of α does not affect the selling prices of the new product in both channels and the wholesale price of the new product if α is not sufficiently high. The manufacturer's profit, however, increases and the retailer's profit decreases. When the customer's acceptance level for the remanufactured product is sufficiently high, all selling and the wholesale prices for the new product decrease as α increases, and the retailer's profit begins to increase. This implies that to increase its own profit, the manufacturer should increase α through actions such as advertising the greenness of the remanufactured products, offering warranty coverage, and post-sales service.

  The proportion of high-quality returns can affect pricing and production decisions of the manufacturer and the retailer, when the manufacturer needs to use all high-quality returns for remanufacturing. The increasing proportion of high-quality returns increases the manufacturer's profit while hurting the retailer's profit in the forward SC, and generates an extra profit for the retailer in the reverse SC. Therefore, for the manufacturer, in order to maximize its profit, the remanufacturability and quality of the product should be carefully evaluated and implemented at the design stage to ensure more high-quality returns. Moreover, the retailer has the advantage in collecting returns, since it is closer to the market. The manufacturer can consider cooperating with the retailer to collect more high-quality returns that can be used for remanufacturing by increasing the unit buyback price such that the increase in the retailer's collection revenue in the reverse SC can offset its profit loss in the forward SC due to the increased proportion of high-quality products.

2.6 Conclusion

A two-period model for a CLSC with a manufacturer and a retailer to decide the optimal pricing and production strategies for remanufactured and new products is developed. Both competitions between the retailer's store and the manufacturer's online channel, and between the remanufactured and new products in the same and different channels are considered in the chapter. Demands for both product types sold in the retailer's and the manufacturer's channels are derived based on the utility theory. The production cost and the channel selling cost are both included in the discussion. The manufacturer's and the retailer's optimal pricing strategies, the manufacturer's production strategy, and the conditions associated with these strategies are identified. Numerical analysis on the impacts of the customer's acceptance level for the remanufactured product, the customer's acceptance level for the online channel, and the proportion of high-quality returns, on the optimal decisions the manufacturer and the retailer are conducted.

Comparing the optimal profits of the manufacturer and the retailer in the cases with and without remanufacturing, it is found that the manufacturer is willing to engage in remanufacturing when the production cost for the new product is sufficiently high and the retailer suffers a profit loss from the introduction of the remanufactured product when all returns are used in remanufacturing. Moreover, for the remanufactured product, it is found that the manufacturer can use different approaches to deal with the changes in the production cost of the new product and the channel selling cost of the retail channel, when sufficient returns can be collected for remanufacturing. When the production cost of the new product increases, the manufacturer increases the demand and keeps the price unchanged for the remanufactured product, while it decreases the price for the remanufactured product and keeps its demand constant as the retail channel selling cost increases.

The impact of key parameters on the manufacturer's and the retailer's optimal decisions is also examined in the chapter. When the customer's acceptance level for the remanufactured product is sufficiently low, the retailer is unwilling to assist the manufacturer to improve the customer's acceptance level for the remanufactured product, since it does not affect the retailer's optimal profit; it can decrease the retailer's optimal profit when the manufacturer switches its optimal pricing and production strategy from not using all returns to using all returns for remanufacturing. Moreover, the increase in the proportion of high-quality returns leads to an increase in the manufacturer's profit and a decrease in the retailer's profit when the manufacturer needs to use all returns for remanufacturing. It implies that the retailer is unwilling to assist the manufacturer to increase the proportion of high-quality returns unless the manufacturer provides an incentive to the retailer for its assistance in collecting the returns.

Several extensions can be made in the future. Firstly, this chapter investigates the pricing and production strategies for remanufactured and new products in a manufacturer-led SC. It is also interesting to explore the problem in a retailer-led SC, since many big retailers such as Walmart can lead the SC and squeeze the profit of their manufacturers (Giri et al., 2017). Secondly, the problem in this chapter is considered in an existing dual-channel SC and the cases with and without remanufacturing are compared. An extension would be to investigate whether it is necessary for the manufacturer to open the direct or the retail channel for selling or collecting products. Thirdly, the chapter shows that when the manufacturer needs to use all returns for remanufacturing, the introduction of the remanufactured product causes a profit loss at the retailer and the retailer is not willing to collect returns for the manufacturer, leading to a profit loss at the manufacturer. An agreement between the manufacturer and the retailer is worth investigating to coordinate the members in the SC to ensure a win-win situation. Fourthly, the chapter considers a two-period time horizon setting. It is interesting to consider the problem in the multi-period time horizon and to determine whether the optimal decisions change or not. Finally, in this chapter, the remanufactured product is sold through the online channel by the (re)manufacturer. However, some r(e)manufacturers may also sell their remanufactured products through the retail channel. Thus, our chapter can be extended to include the case where the remanufactured product is sold through both channels.

Chapter 3

Optimal pricing and production strategies for new and remanufactured products under a non-renewing free replacement warranty

3.1 Introduction

Price and customer's perceived quality are the two most essential factors that may affect the customer's purchase decision (Tang et al., 2020). High customer's perceived quality and low selling price for the remanufactured product can boost demand in the market (Abbey et al., 2015b). However, a low price may have an unintended drawback by leading the consumer to question the quality of the product (Tang et al., 2020). Therefore, decreasing the selling price should be done carefully.

The quality of the remanufactured product is a major concern for consumers (Vafadarnikjoo et al., 2018). As compared to a new product, the consumers find it difficult to assess the quality of a remanufactured product given that it uses components/modules from returns (Liao, 2018). Therefore, the consumers are concerned about the quality, performance and safety of the remanufactured product. Abbey et al. (2015c) find that the perceived quality is one of the significant factors that affects the customer's attitude and behavior towards the remanufactured product. Vafadarnikjoo et al. (2018) also point out that quality is the most significant consideration for the customers when they ponder the purchase of the remanufactured product.

As the warranty service signals the product's quality, remanufacturers usually offer lenient warranty policies to improve the perceived quality of the remanufactured products (Boulding and Kirmani, 1993). Warranty as a service contract plays an important role in business and legal transactions, especially when the quality of the product is not easily observed and valued by the customer (Esmaeili et al., 2014; Lan et al., 2014). Warranty is a contract between the manufacturer and the customer to ensure that the purchased products can be repaired, replaced, or as a compensation service to the customer when the product fails in a specified time period after purchase (Shafiee and Chukova, 2013). On the customer's side, the warranty serves as both protection and information. With the warranty, the customer can receive a redress when a product under proper use fails within a specific time period (Murthy and Djamaludin, 2002). Usually the customer can have the failed product repaired or replaced at no cost or a small cost, implying that the risk of the product's failure has been transferred from the customer to the manufacturer under the warranty (Murthy and Blischke, 2000). The other role is informative. When the customer cannot assess the quality of the product clearly, the product with a long warranty is commonly considered to be of high quality with a higher reliability than the product with a short warranty (Murthy and Djamaludin, 2002).

For the manufacturer, the warranty service can also be protective. In the warranty terms, the manufacturer usually specifies the function and the proper conditions of use of the product and the limited coverage for the product's failure due to misuse. Therefore, with the warranty, the manufacturer can protect itself from unreasonable customer claims (Shafiee and Chukova, 2013). Warranty can also serve as a promotional tool for the manufacturer. Since the consumer prefers the product with a warranty service, manufacturers typically use warranty as an advertising tool to differentiate its product from the competitor's to attract more customers.

Many types of warranties have been offered by OEMs to meet the various needs and requirements of their consumers. Warranty policies are typically classified into two groups: non-renewing and renewing warranties. Under a renewing warranty, the OEM offers a warranty identical to the original one after each legitimate claim, repair or replacement (Murthy and Jack, 2009). The warranty length is never renewed for the non-renewing warranty. Based on the cost incurred, the warranty policies can be classified into three groups: free replacement warranty (FRW), pro-rata warranty (PRW), and hybrid warranty. Under the FRW, the customer receives the repair or the replacement service during the warranty period without charge. Under PRW, the customer will pay a portion of the repair or replacement cost that is proportional to the elapsed time in the warranty period (Chien, 2010). In this chapter, we will consider the non-renewing FRW for the new and remanufactured products sold by the manufacturer and the retailer. This setting has been extensively used and explored in the literature (see (Wu et al., 2007; Elsayed, 2014; Liu et al., 2015)).

Although warranty can be beneficial to both customer and manufacturer, a generous warranty usually leads to a high cost (Shafiee and Chukova, 2013). Depending on the product type, the warranty usually costs between 2% to 10% of the product's selling price. According to an industry report, the annual warranty cost in the US auto industry reached $3.7 billion in 2016 or 3% of its revenue (Tong et al., 2017). In 2019, General Motors's annual warranty cost for the automobiles sold was nearly $3 billion, accounting for almost 3% of its total revenue (Shafiee and Chukova, 2013). For the remanufactured product, the cost may be higher because more failures might happen during the warranty period. Therefore, a trade-off should be identified between offering warranty and being profitable (Shafiee and Chukova, 2013; Chari et al., 2013).

Since warranty is an essential factor that affects the cost and the demand of a product, the following questions for the new and remanufactured product under a non-renewing FRW will be addressed in this chapter:

- How should the manufacturer set prices for the new and remanufactured products under a given warranty length?
- How does the warranty length affect the prices, demands and profit for both new and remanufactured products?
- Does an optimal warranty length exist and what factors influence it?

In this chapter, a two-period mathematical model is developed to investigate the optimal pricing and production strategies for the new and remanufactured products under a non-renewing free replacement warranty. The impacts of the sensitivity of the customer's utility to the warranty length for the new and remanufactured products on the optimal prices, demands, and profit will be analyzed. We also numerically explore how the optimal prices, demands, and profit vary in respond to the change in warranty length, and analyze the impacts of the unit production costs, the sensitivity of customer's utility to the warranty length and the failure rates of the new and remanufactured products on the optimal warranty length.

The remainder of this chapter is organized as follows. Section 2 briefly reviews the literature and summarizes the contributions of this chapter. In Sections 3 and 4, a two-period mathematical model is developed to derive the optimal pricing and production strategies for the new and remanufactured products. A sensitivity analysis is presented in Section 5. Section 6 provides the conclusion and extensions for future research. All proofs are placed in Appendix.

3.2 Literature Review

In this section, the related literature is classified in three streams: pricing strategies in the CLSC, warranty strategies for remanufactured products, joint decision of the price and warranty for the remanufactured products. Finally, we present the contributions of this chapter to the literature by differentiating it from existing studies.

3.2.1 Pricing strategy in the closed SC

Pricing is one of the most important strategic-level decisions for the remanufactured products since it controls the cannibalization between new and remanufactured products Steeneck and Sarin (2013); Bulmuş et al. (2014). Several comprehensive reviews on the pricing strategy for remanufactured products, such as Steeneck and Sarin (2013) and Kumar and Ramachandran (2016), can be referred to. Steeneck and Sarin (2013) provide a critical review on the studies of theoretical models for deciding the optimal pricing strategy: standard economic models, models with product life cycle considerations, models with variable marginal remanufacturing cost and other pricing models. Kumar and Ramachandran (2016) classify the articles based on the issues: product-related issues (such as the product categories and the quality of the returns), SC-related issues (such as the inventory strategies and the market type), and mathematical formulation-related issues (such as the time horizons and the modelling technique).

Selecting an optimal pricing strategy for the new and remanufactured products can help the manufacturer achieve maximum profit. Choi (2017) explores the optimal pricing and branding investment decisions for a fashion retailer. The retailer sells new fashion products and collect the returns, which are send to a remanufacturing factory and used as raw material for producing remanufactured products. The retailer needs to decide the branding investment and the prices for the new and remanufactured products. The study finds that the optimal price increases and the optimal brand investment decreases if the unit acquisition cost of the used fashion product and the unit remanufacturing cost increase. Phantratanamongkol et al. (2018) study the relationship between price and volume for the new and remanufactured smartphones through an empirical experiment on eBay-UK and eBay-US using daily series data from January to November 2016. It shows a significant negative contemporaneous relationship between price and volume for the new smartphones and a positive contemporaneous relationship between price and volume for the remanufactured products, which indicates that the remanufactured smartphones can be more profitable than the new ones.

Pricing strategy for remanufactured products is considered not only in a monopoly situation, but also in a SC. Zhang and Ren (2016) explore the optimal retail prices and wholesale prices for the new and remanufactured products in a SC containing an original manufacturer, a third-party remanufacturer, a retailer in a leader-follower game, and a joint decision-making game. The retail prices for both new and remanufactured products are set to be lower in a joint decision-making case than in a leader-follower game. Gan et al. (2017) study the optimal pricing strategy for new and remanufactured products with a separate sale channel. New products and remanufactured products are sold by the retailer and the manufacturer respectively. Under the separate channel, the total SC's profit can be improved and the price for the remanufactured product is higher, as compared to the case in the single-channel.

The above literature only considers the prices related factors that affect the customer's decision for buying the new or remanufactured products. The proposed chapter will also consider the effect of the warranty in the customer's purchasing decision.

3.2.2 Warranty strategy for the remanufactured products

Selecting the optimal warranty strategy is also important for the manufacturer, since it can significantly affect the total profit by controlling the demand and the cost for the new and remanufactured products.

Aksezer (2011) studies the free replacement warranty and the cost sharing warranty for second-hand vehicles. Expected warranty cost has been obtained by considering the age, usage, and the maintenance data for the second-hand vehicles. Chari et al. (2013) propose a mathematical model to decide the optimal warranty length and the age of reconditioned products, which are used to replace the failed products under a one-dimensional unlimited free-replacement warranty policy. Alqahtani and Gupta (2017b) focus on the warranty policy for a washing machine with nine components. The relevant warranty costs of the washing machine are compared in two cases: with and without a sensor used to estimate remaining lifetime. In addition to the base warranty, three EW types are offered by the manufacturer: free replacement warranty, refund warranty and a combination of these two warranties. The study shows that with the sensor, significant improvements in the total revenue and profit can be achieved and the extended free replacement warranty has the lowest average value of warranty costs. Alqahtani and Gupta (2018) expand the work in Alqahtani and Gupta (2017b) by considering the impact of the non-renewing money-back guarantee warranty policy for the remanufactured product with an embedded sensor. In the paper, the remanufactured product is sold with a warranty. If the product fails during the warranty period, it can be repaired/replaced, or collected by the remanufactured (the customer receives a certain amount of money back) when the number of repairs is over a certain level. In order to decrease the number of failures during the warranty time, preventative maintenance is provided to the products as free. It has been found that the preventative maintenance significantly reduces the total cost.

The above literature mainly focuses on obtaining the optimal warranty for the remanufactured product to minimize the total cost. In this chapter, we explore the optimal pricing for both new and remanufactured products while accounting for the warranty length in order to maximize the total profit.

3.2.3 Joint decision of price and warranty for the remanufactured product

The closest studies related to our study are the papers focusing on the joint decision of price and warranty for the new and remanufactured products.

Liao et al. (2015) investigate the impact of the warranty on the manufacturer and remanufacturer's profits, demands and prices in a competitive environment. One manufacturer produces the new product only, while the other produces the remanufactured product only. Three scenarios are discussed in the paper: 1) no warranty for both; 2) the remanufactured product has warranty, while the new product does not; 3) both have warranty. Based on their research, the (re)manufacturer can benefit by offering warranty for its own products, and its profit decreases because of the warranty policy offered by its competitor.

Chari et al. (2016b) focus on the optimal pricing strategy for the manufacturer by considering warranty length, age of reconditioned components, and the ratio of new and reconditioned components to be used. If a key component in a product fails within the warranty period, it is replaced by a spare part from a pool containing both new and reconditioned components. They find that decreasing the ratio of new and reconditioned components (increasing the proportion of the new components in the pool) leads to in an increase in the optimal price for the product and in the optimal warranty length.

San and Pujawan (2017) explore the optimal prices and warranties for the new and remanufactured products in a SC with a manufacturer and a retailer. In addition to a basic warranty offered for both new and remanufactured products, an EW is offered for the remanufactured products. A mathematical model is solved to obtain optimal prices and warranties and it is found that increasing customer demand leads to the increase of the warranty level.

Giri et al. (2018) discuss the revenue management problem for a retailer and a manufacturer producing and selling new and remanufactured products. The retailer sells the new product at the start of the selling season. Failure returns during the warranty period are replaced by new products. The returns are classified into two groups. The group with the highest quality is refurbished and returned to the customers. The other returns are remanufactured and sold in a secondary market. The study shows that increasing the warranty period leads to raising the wholesale price and the selling price. Moreover, the revenue sharing contract, in which the retailer allocates a fraction of its profit to the manufacturer to obtain lower wholesale price, creates a win-win situation for both the manufacturer and the retailer.

Different from the studies above, we develop a general model aiming to be more practical in our study. First, we consider that the customer's behavior for purchasing new and remanufactured product depends on the prices and warranties. However, unlike Liao et al. (2015), San and Pujawan (2017) and Giri et al. (2018), a concave relationship is set between the warranty length and the customer's utility for the products since the marginal customer's utility towards to products decreases as the warranty length increases. Secondly, the warranty cost is related to the demand for the products instead of being a one-time investment. Thirdly, the supply of remanufactured products is not only limited by the sales of the new products in the previous period but also controlled by the market (unlike Giri et al. (2018), in which the demand for the remanufactured product only depends on the number of returns and has no upper bound).

3.2.4 Contribution to the literature

To summarize, the study contributes to the literature in the following three ways:

- It explores the optimal pricing and production strategies for the new and remanufactured products with a non-renewing free replacement warranty by considering the limited supply of remanufactured products, the total warranty cost depending on the demand, the concave relationship between the customer's utility and the warranty length, and change in the market size. A two-period model is developed to obtain the optimal prices and the conditions for each production strategy (only new, only remanufactured or both).

- The study identifies the conditions, under which the manufacturer should engage in remanufacturing, depending on the ratio of the unit production costs and the warranty length for new and remanufactured products.
- The study investigates the sensitivity of the optimal prices, demands and the manufacturer's profit to the warranty length. Moreover, numerical experiments are used to explore how the unit manufacturing and remanufacturing costs, failures rates and the sensitivity of the customer's utility affects the optimal warranty length.

3.3 Problem description and notation

We consider a monopolistic manufacturer who produces the new product in the first period, and produce either the new, or the remanufactured, or both products in the second period, in a selling season. The manufacturer collects the failed products returned during the warranty and uses them to produce remanufactured products that are sold in the second period.

For both new and remanufactured products, the manufacturer offers a non-renewing free replacement warranty. During the warranty period, customers can return the failed new (remanufactured) product to the manufacturer and obtain a replacement of the new (remanufactured) product without any cost. The warranty lengths for both the new and remanufactured products are not renewed with the replacement.

We assume that new and remanufactured products have the same warranty length (w). This setting is common in the market. For example, Apple Inc. sells remanufactured products with the same 1-year warranty offered on their new products (Clover, 2019). Kindle from Amazon and Lenovo also have the same warranty coverage for new and remanufactured products (Livingston, 2019). In the numerical experiments section 5.3, this restriction is relaxed and different warranty length for new and remanufactured products are considered.

Following studies in Ferrer and Swaminathan (2006) and Wang et al. (2018), at the beginning of the selling season, the manufacturer needs to set the selling prices for both new and remanufactured products under the non-renewing free replacement warranty to maximize its profit over the selling season. The manufacturer, however, announces its prices sequentially at the beginning of each period. Therefore, the customers in the first period make their purchasing decisions based on their utility only and do not consider the pricing adjustment in the second period. At the start of the second period, the manufacturer announces the prices for new and remanufactured products for this period.

As in Teunter (2001); Agrawal et al. (2016), it is assumed that each new product can be only remanufactured once. That is, only the returned new product in the first period can be used as raw material for producing remanufactured products. We assume that only the returned new products sold in the first period can be used for remanufacturing, as producing remanufactured products takes some time. Moreover, after the failed new products are returned, collected, and inspected, only a proportion (θ) of returns with relatively high quality is suitable to be used in producing the remanufactured products for the second period. The returned remanufactured products and the unused returned new products are cleanly disposed of. In addition, as compared with the production and warranty cost, the disposal cost is sufficiently small, which can be normalized to 0 in this chapter.

It is assumed that the market sizes are 1 and β in the first and second periods, respectively. Specifically, $\beta > 1$ and $\beta < 1$,correspond to the market size growth and reduction in the second period, respectively; while $\beta = 1$ suggests that the market size remains unchanged in the second period.

The customer's perceived value toward a new product is assumed to be z, which is heterogeneous and uniformly distributed on $[0, 1]$ as in He et al. (2019). In the first period, the customer's utility for purchasing a new product is $U_n^I(z) = z - p_n^I + \delta_n\sqrt{w}$, where p_n^I is the price for the new product sold in the first period and δ_n represents the sensitivity of customer's utility to the warranty length of the new product. Following the study in Huang and Fang (2008), a concave relationship is assumed between the customer's utility and the warranty length. That is, as the warranty length increases, the marginal customer's utility decreases.

The customer will buy the new products only when his utility is non-negative ($U_n^I \geq 0$), which leads to $z \geq z_n^I = p_n^I - \delta_n\sqrt{w}$. Therefore, the demand of the new products in the first period (d_n^I) is:

$$d_n^I = \begin{cases} 1 - p_n^I + \delta_n\sqrt{w} & \text{if } p_n^I \leq 1 + \delta_n\sqrt{w} \\ 0 & \text{if } p_n^I > 1 + \delta_n\sqrt{w} \end{cases} \tag{3.1}$$

In the second period, the manufacturer may produce both new and remanufactured products and sell them to the customers. The customer's perceived value on the remanufactured product is discounted as it perceives it as a low-end version of the new product (Atasu et al., 2010). Parameter α represents the customer's acceptance level of the remanufactured product. Thus, the utility of the customer to buy the new or remanufactured product is given by:

$$U_n^{II}(z) = z - p_n^{II} + \delta_n\sqrt{w} \tag{3.2}$$

$$U_r^{II}(z) = \alpha z - p_r^{II} + \delta_r\sqrt{w} \tag{3.3}$$

Customers will buy the new product if and only if (iff) $U_n^{II} \geq 0$ and $U_n^{II} \geq U_r^{II}$, leading to $z = z_n^{II} \geq p_n^{II} - \delta_n\sqrt{w}$ and $z = z_{nr}^{II} \geq \frac{p_n^{II} - p_r^{II} - \delta_n\, w + \delta_r\, w}{1-\alpha}$, respectively. Similarly, the customer will buy the remanufactured product if and only if $U_r^{II} \geq 0$ and $U_r^{II} \geq U_n^{II}$. $U_r^{II} \geq 0$ leads to $z \geq \frac{p_r^{II} - \delta_r\sqrt{w}}{\alpha}$. So when $1 \geq z_{nr}^{II} \geq z_r^{II} \geq 0$ (or equivalently, $1 \geq \frac{p_n^{II} - p_r^{II} - \delta_n\, w + \delta_r\, w}{1-\alpha} \geq \frac{p_r^{II} - \delta_r\sqrt{w}}{\alpha} \geq 0$), the market can be segmented into three parts as illustrated in Figure 3.1.

Clearly, when $z_{nr}^{II} \geq z_{max} = 1 + \delta_n\sqrt{w}$, customers will never buy the new product while they will never buy the remanufactured product if $z_r^{II} \geq z_{nr}^{II}$. With the market size β, the demands for the new and remanufactured products in the second period

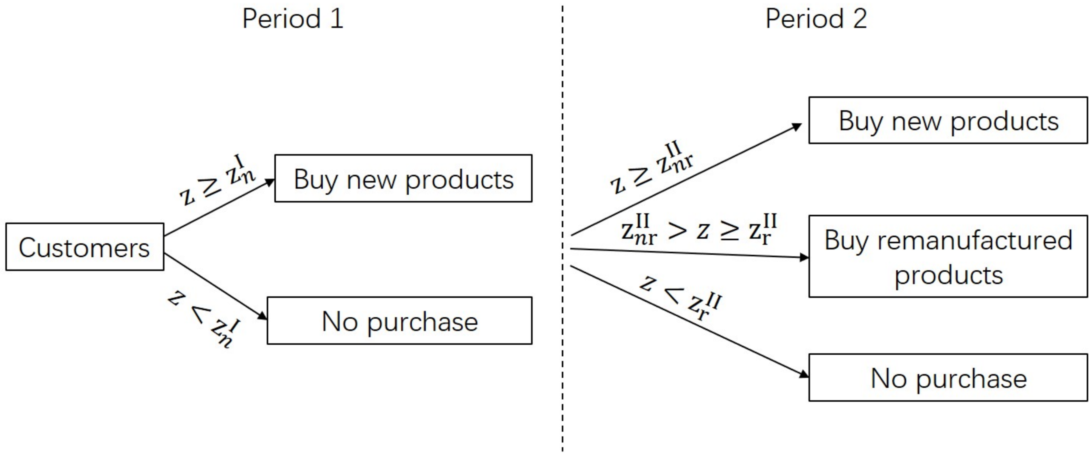


Figure 3.1: Purchasing decision segments based on customer valuation

are:

$$d_n^{II} = \begin{cases} \beta(1 - p_n^{II} + \delta_n\sqrt{w}) & \text{if } p_n^{II} < \frac{\alpha\delta_n\sqrt{w} - \delta_r\sqrt{w} + p_r^{II}}{\alpha} \\ \beta(\frac{1-\alpha-p_n^{II}+p_r^{II}+(\delta_n-\delta_r)\sqrt{w}}{1-\alpha}) & \text{if } \frac{(\alpha\delta_n-\delta_r)\sqrt{w}+p_r^{II}}{\alpha} \leq p_n^{II} \leq 1-\alpha+p_r^{II}+(\delta_n-\delta_r)\sqrt{w} \\ 0 & \text{if } p_n^{II} > 1-\alpha+p_r^{II}+\delta_n\sqrt{w}-\delta_r\sqrt{w} \end{cases} \tag{3.4}$$

$$d_r^{II} = \begin{cases} 0 & \text{if } p_n^{II} < \frac{\alpha\delta_n\sqrt{w} - \delta_r\sqrt{w} + p_r^{II}}{\alpha} \\ \beta(\frac{\alpha p_n^{II}-p_r^{II}-(\alpha\delta_n-\delta_r)\sqrt{w}}{(1-\alpha)\alpha}) & \text{if } \frac{(\alpha\delta_n-\delta_r)\sqrt{w}+p_r^{II}}{\alpha} \leq p_n^{II} \leq 1-\alpha+p_r^{II}+(\delta_n-\delta_r)\sqrt{w} \\ \beta(1-\frac{p_r^{II}-\delta_r\sqrt{w}}{\alpha}) & \text{if } p_n^{II} > 1-\alpha+p_r^{II}+\delta_n\sqrt{w}-\delta_r\sqrt{w} \end{cases} \tag{3.5}$$

The manufacturer incurs a unit cost c_r (c_n) to produce a remanufactured (new) product. When a new (remanufactured) product fails and is returned to the manufacturer, a working equivalent new (remanufactured) product is given to the customer as a compensation, which costs the manufacturer c_n (c_r). It is assumed that the failure times for new and remanufactured products follow the exponential distribution (Yedida and Sekar, 2017; Darghouth et al., 2017) with a constant rate λ_n and λ_r, respectively. This assumption is reasonable for electronic products (Wang et al., 2015). Based on the renewal theory (Blischke, 1995), the expected numbers of replacements for the new and remanufactured products that occur in the warranty period $[0, w)$ are:

$$M_n(w) = \lambda_n w \tag{3.6}$$

$$M_r(w) = \lambda_r w \tag{3.7}$$

Therefore, the average warranty costs for the new and remanufactured product are $c_n\lambda_n w$ and $c_r\lambda_r w$ respectively. It is common practice, for the manufacturers to set reserved funds aside to cover the warranty costs at the time of sale (Kim et al., 2018). Therefore, the manufacturer incurs the warranty costs once the product is sold. The notation that will be used in this chapter is summarized in Table 3.1 as follows.

Table 3.1: Table of notation

Indices	
i	Index for product type (subscript): $i = n$ for new, and $i = r$ for remanufactured.
j	Index for the planning period (superscript): $j = I, II$.
Parameters	
c_i	Unit manufacturing cost for product type i.
λ_i	Failure rate of product type i.
δ_i	Customer's sensitivity to the warranty length for product type i.
α	Customer's acceptance level for the remanufactured products.
β	Market size in the second period.
θ	Fraction of returns that can be used for remanufacturing.
w	Warranty length.
z	Customer's perceived value on the new products, a uniform distribution with supporting range $[0, 1]$.
d_i^j	Demand of product type i in period j, where $d_r^I = 0$.
Π	Total profit for both new and remanufactured products over the two periods.
Decision Variables	
p_i^j	Selling price for product type i in period j, where p_r^I does not exist.

3.4 Two-period mathematical model

In the first period, the manufacturer produces the new product only, while in the second period, it may produce both new and remanufactured products for the customer. The manufacturer decides the optimal prices for the products at the start of the selling season in order to maximize its profit over the two periods. The optimal prices can be derived by solving the following model:

$$Max \quad \Pi\left(p_n^I, p_n^{II}, p_r^{II}\right) = \sum_{\substack{i=n,r \\ j=I,II}} \left(p_i^j - c_i - c_i\lambda_i w\right) d_i^j \tag{3.8}$$

Subject to:

$$\theta\lambda_n w d_n^I \geq (\lambda_r w + 1) d_r^{II} \tag{3.9}$$

$$p_n^I, p_n^{II}, p_r^{II} \geq 0, \tag{3.10}$$

where the objective function (3.8) represents the total profit for the new and remanufactured products over the two periods. Constraint (3.9) ensures that the quantity of remanufacturable products returned in the first period is more than the number of remanufactured products sold and their warranty replacements needed in the second period (i.e., each remanufactured product sold will require on average $\lambda_r w$ replacements). Constraint (3.10) requires all prices to be non-negative.

Lemma 3.1. *The objective function (3.8) is concave in p_i^j, where $i = n, r$ and $j = I, II$.*

We define $C_n = c_n(1 + \lambda_n w)$ and $C_r = c_r(1 + \lambda_r w)$ as the total unit costs for the new and the remanufactured products, respectively. With Lemma 3.1 and since both constraints (3.9) and (3.10) are linear in terms of the prices in both periods, there exist optimal prices for new and remanufactured products in both periods, which are summarized in Table 3.2. Let $C_{r1} = \alpha + \delta_r\sqrt{w} - A_3\alpha$, $C_{r2} = \alpha + \delta_r\sqrt{w} - \frac{A_3\alpha(\beta N_r + (1-\alpha)\theta\lambda_n w)}{\beta N_r}$, $C_{r3} = \alpha + \delta_r\sqrt{w} - A_3$, $C_{r4} = \alpha + \delta_r\sqrt{w} - A_3(1 + \frac{\beta N_r^2 + \theta\lambda_n w\alpha N_r}{\theta^2\lambda_n^2 w^2\alpha})$, and $C_{r5} = \alpha + \delta_r\sqrt{w} - \frac{A_3\theta\lambda_n w\alpha}{\beta N_r}$. Developing and solving the Karush-Kuhn-Tucker conditions yields the manufacturer's optimal pricing and production strategies in Theorem 3.1 (illustrated in Figure 3.2).

Theorem 3.1. *The manufacturer's optimal production and pricing strategy in the second period is*

- *Strategy N: produce new products only iff $C_r \geq C_{r1}$ (Region I);*

- *Strategy B: produce both new and remanufactured products iff* $max(C_{r2}, C_{r3}) \leq C_r < C_{r1}$ *(partial use of returned products, Region II), and* $C_{r4} < C_r < C_{r2}$ *and* $\beta > \frac{\theta\lambda_n w\alpha}{N_r}$ *(full use of returned products, Region III);*

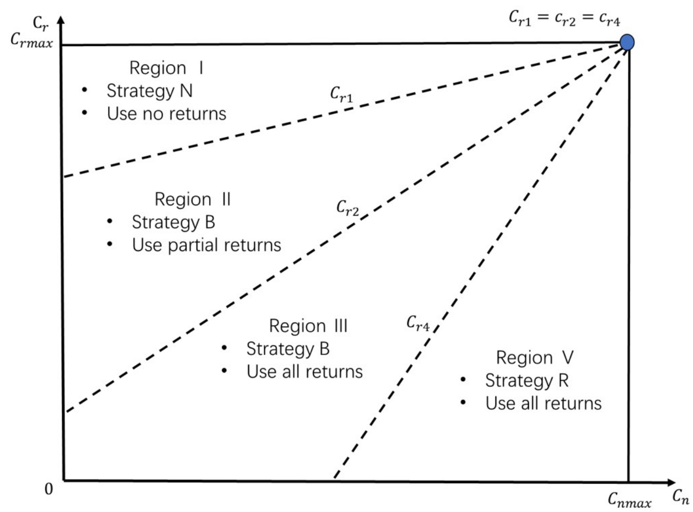
- *Strategy R: produce remanufactured products only iff* $C_{r5} < C_r < C_{r3}$ *and* $\beta \leq \frac{\theta\lambda_n w\alpha}{N_r}$ *(partial use of returned products, Region IV) and* $C_r \leq min(C_{r4}, C_{r5})$ *(full use of returned products, Region V).*

where $C_{rk}(k = 1..5)$ *are linear functions of* C_n*.*

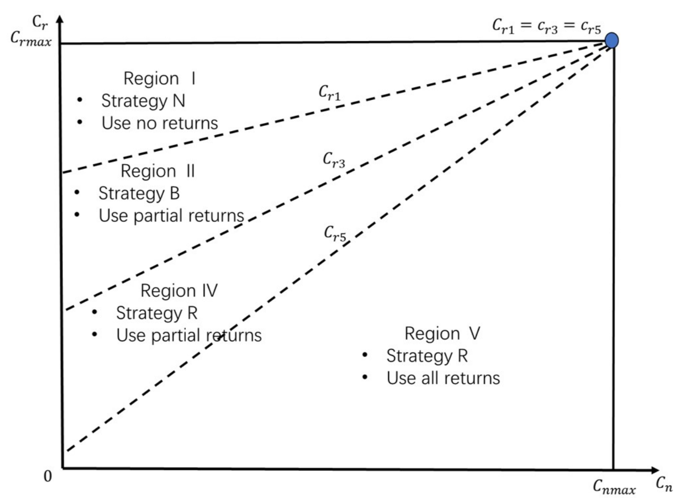
For either new or remanufactured products to be profitable, the total unit cost of each product type should be lower than its price. Setting the demand functions in (3.4) and (3.5) to zero yields the upper bounds of the total unit cost of each product, which are $C_{nmax} = 1+\delta_n\sqrt{w}$ and $C_{rmax} = \alpha+\delta_r\sqrt{w}$ as represented on Figure 3.2.

Theorem 3.1 and Figure 3.2 demonstrate that in general for the second period, as the total unit cost for the new product increases or the total unit cost for the remanufactured product decreases, the optimal production strategy for the manufacturer changes from producing new products only to producing both new and remanufactured products, and then to producing the remanufactured products only. This is due to the fact that compared with producing new products, producing remanufactured products becomes more profitable with the widening gap in the costs. The manufacturer can set lower prices for the remanufactured products and attract more customers to obtain a higher profit. Moreover, when the remanufactured products become more profitable, the number of returns from the new products sold in the first period becomes a key restricting factor for the demand (and production quantity) of remanufactured products in the second period.

The market size (β) in the second period also affects the manufacturer's selection of optimal production and pricing strategies. When the market size in the second period is sufficiently large ($\beta > \frac{\theta\lambda_n w\alpha}{N_r}$), to be more profitable, the manufacturer may use all useable returns of new products in the first period for remanufacturing, when it chooses to produce both new and remanufactured products or produce the remanufactured products only in the second period. In the other case ($\beta \leq \frac{\theta\lambda_n w\alpha}{N_r}$),



(a) Case 1: $\beta > \frac{\theta\lambda_n w\alpha}{N_r}$



(b) Case 2: $\beta \leq \frac{\theta\lambda_n w\alpha}{N_r}$

Figure 3.2: Optimal pricing and production strategies for the manufacturer

Table 3.2: Optimal prices and demands for the new and remanufactured products

	Strategy N (in Region I)	Strategy B (in Region II)	Strategy B (in Region III)	Strategy R (in Region IV)	Strategy R (in Region V)
p_n^I	$\frac{A_1}{2}$	$\frac{A_1}{2}$	$\frac{A_1}{2} + \frac{\alpha\lambda_n w\theta A_3((1-\alpha)\theta\lambda_n w+N_r\beta)-\theta\lambda_n wN_r\beta A_4}{2N_r^2\beta+2\alpha(1-\alpha){\lambda_n}^2w^2\theta^2}$	$\frac{A_1}{2}$	$\frac{A_1}{2} + \frac{\alpha{\lambda_n}^2w^2\theta^2A_3-\theta\lambda_n wN_r\beta A_4}{2N_r^2\beta+2\alpha{\lambda_n}^2w^2\theta^2}$
p_n^{II}	$\frac{A_1}{2}$	$\frac{A_1}{2}$	$\frac{A_1}{2}$	-	-
p_r^{II}	-	$\frac{A_2}{2}$	$\frac{A_2}{2} - \frac{\alpha N_r A_3((1-\alpha)\theta\lambda_n w+N_r\beta)-N_r^2\beta A_4}{2N_r^2\beta+2\alpha(1-\alpha){\lambda_n}^2w^2\theta^2}$	$\frac{A_2}{2}$	$\frac{A_2}{2} + \frac{N_r^2\beta A_4-\theta\lambda_n wN_r\alpha A_3}{2N_r^2\beta+2\alpha(1-\alpha){\lambda_n}^2w^2\theta^2}$
d_n^I	$\frac{A_3}{2}$	$\frac{A_3}{2}$	$\frac{A_3}{2} - \frac{\alpha\lambda_n w\theta A_3((1-\alpha)\theta\lambda_n w+N_r\beta)-\theta\lambda_n wN_r\beta A_4}{2N_r^2\beta+2\alpha(1-\alpha){\lambda_n}^2w^2\theta^2}$	$\frac{A_3}{2}$	$\frac{A_3}{2} - \frac{\alpha{\lambda_n}^2w^2\theta^2A_3-\theta\lambda_n wN_r\beta A_4}{2N_r^2\beta+2\alpha{\lambda_n}^2w^2\theta^2}$
d_n^{II}	$\frac{\beta A_3}{2}$	$\frac{\beta(A_3-A_4)}{2(1-\alpha)}$	$\frac{A_3\beta(\alpha\theta\lambda_n w(\theta\lambda_n w-N_r)+N_r^2\beta)-\alpha\beta{\lambda_n}^2w^2\theta^2A_4}{2N_r^2\beta+2\alpha(1-\alpha){\lambda_n}^2w^2\theta^2}$	-	-
d_r^{II}	-	$\frac{\beta(A_4-\alpha A_3)}{2\alpha(1-\alpha)}$	$\frac{\lambda_n w\theta\beta(A_3(N_r-\alpha\theta\lambda_n w)+\lambda_n w\theta A_4)}{2N_r^2\beta+2\alpha(1-\alpha){\lambda_n}^2w^2\theta^2}$	$\frac{\beta A_4}{2\alpha}$	$\frac{\lambda_n w\theta\beta(N_rA_3+\lambda_n w\theta A_4)}{2N_r^2\beta+2\alpha{\lambda_n}^2w^2\theta^2}$

* $A_1 = 1+\delta_n\sqrt{w}+C_n$; $A_2 = \alpha+\delta_r\sqrt{w}+C_r$; $A_3 = 1+\delta_n\sqrt{w}-C_n$; $A_4 = \alpha+\delta_r\sqrt{w}-C_r$; $N_r = \lambda_r w+1$

all useable returns are used only when producing only the remanufactured products in the second period (Region V). When market size in the second period increases, demand for remanufactured products also increase leading to the behavior observed above.

With the analysis above, we can derive the following result.

Lemma 3.2. *The price for the new product in the second period is independent of the remanufacturing decisions and is not lower than the price for the new product in the first period.*

Based on the results in Table 3.2, it is obvious that for Strategy N and B (in both Regions II and III), in which the manufacturer decides to produce the new products in the second period, the optimal price for the new products stays unchanged ($\frac{1+\delta_n\sqrt{w}+C_n}{2}$) regardless of the decision to remanufacture or not in the second period, and this price only depends on its own factors (the total unit production cost, the warranty length, and the customer's sensitivity to the warranty length for the new products). Moreover, the price of the new product is not lower in the second period than in the first period. The reason is that when producing the remanufactured product is sufficiently profitable (Strategy B in Region 3 and R), in order to have more raw material from returns for producing the remanufactured product to increase the total profit, the manufacturer sacrifices the profit in the first period by lowering the selling price to increase the demand of the new product.

Further analysis shows that whether or not the manufacturer engages in remanufacturing in the second period depends on the warranty length and the ratio of the unit production costs of the remanufactured and to new products ($\rho = \frac{c_r}{c_n}$). Let $w_{1,2} = \frac{\alpha\delta_n - \delta_r \pm \sqrt{(\delta_r - \alpha\delta_n)^2 - 4c_n^2(\rho-\alpha)(\rho\lambda_r - \alpha\lambda_n)}}{2(\alpha c_n\lambda_n - c_r\lambda_r)}$, $\rho_1 = \frac{\alpha\lambda_n}{\lambda_r}$, $\rho_2 = \alpha$, and $\rho_3 = \frac{\alpha(\lambda_n+\lambda_r)+\sqrt{\alpha^2(\lambda_r-\lambda_n)^2+(\alpha\delta_n-\delta_r)^2\lambda_r/c_n}}{2\lambda_r}$. We have the following results in Lemma 3.3 and Figure 3.3.

Lemma 3.3.

- *when $\rho \leq \rho_1$, the manufacturer should always produce remanufactured products.*

- *when* $\rho_1 < \rho \leq \rho_2$*, the manufacturer should produce remanufactured products only if* $0 \leq w \leq w_2$.

- *when* $\rho_2 < \rho \leq \rho_3$*, the manufacturer should produce remanufactured products only if* $w_1 \leq w \leq w_2$.

- *When* $\rho > \rho_3$*, the manufacturer should never produce remanufactured products.*

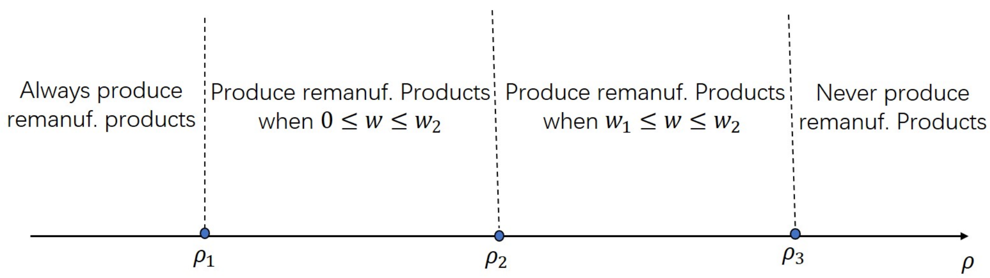


Figure 3.3: Remanufacturing decision segments based on ρ

Although a longer warranty length leads to higher warranty costs for the manufacturer, it also increases the attractiveness of the products to the customers. When the ratio of the unit production costs of the remanufactured to the new products ρ is extremely small ($\rho \leq \rho_1$), remanufacturing is always a good choice for the manufacturer to increase its profit. The intuition is that the warranty cost for the remanufactured product tends to be sufficiently lower than the warranty cost of an equivalent new product with the same warranty length, even though the remanufactured product fails more frequently during the warranty period.

When the ratio ρ is moderate ($\rho_1 < \rho \leq \rho_2$), the manufacturer should produce remanufactured products when the warranty length is sufficiently short ($w \leq w_2$). In this case, producing remanufactured products is more profitable than producing new products when warranty length is short. Although the remanufactured product has a relatively higher warranty cost in this case, compared with the new product, the positive effect of warranty length significantly increases the demand for remanufactured products. The negative marginal effect of warranty length on the demand, however, decreases as the warranty increases. Therefore, when the warranty length is sufficiently short ($w \leq w_2$), producing the remanufactured product is better for the manufacturer; when the warranty length is sufficiently long, producing the remanufactured product becomes less profitable, as compared to that of the new product due to the increase of warranty cost.

When the ratio ρ is large ($\rho_2 < \rho \leq \rho_3$), the manufacturer should not engage in remanufacturing when the warranty length is extremely small ($w < w_1$). When the warranty length is extremely small, even though the marginal effect of the warranty length on the demand is large for the remanufactured products, the gap between the positive effects of the warranty length on the profit (increasing the demands) for the new and remanufactured products cannot cover the negative effect from the gap of their production costs. Therefore, producing remanufactured products is less profitable at this time.

When the ratio ρ is extremely large ($\rho > \rho_3$), compared with the new product, the remanufactured product has a extremely higher production cost and warranty cost. The gap between the positive effect of warranty length on the profit (increasing demands) for the new and remanufactured products is insufficient to cover the negative effect from the gap of their costs regardless of the warranty length. Therefore, the manufacturer should never engage in remanufacturing.

Moreover, it is found that when the customer's acceptance level for the remanufactured products (α) increases, the threshold ρ_1, ρ_2, and ρ_3 also increases. This implies that the manufacturer is more likely to engage in remanufacturing when the customer highly value the remanufactured products.

Lemma 3.4 and Table 3.2 depict the impacts (increase, decrease, null) that increases of δ_n and δ_r have on the optimal prices and demands for the new and remanufactured products in both periods.

Lemma 3.4. *Impacts of the increase of δ_n and δ_r on the optimal prices and demands are presented in Table 3.3:*

In general, when the customer's sensitivity to warranty length for new products increases, the price for new products also increases. When the customer's sensitivity to

Table 3.3: Impacts of increases in δ_n and δ_r on the optimal prices and demands

	Strategy N (in Region I)		Strategy B (in Region II)			Strategy B (in Region III)			Strategy R (in Region IV)		Strategy R (in Region V)	
	p_n^I	p_n^{II}	p_n^I	p_n^{II}	p_r^{II}	p_n^I	p_n^{II}	p_r^{II}	p_n^I	p_r^{II}	p_n^I	p_r^{II}
δ_n	↑	↑	↑	↑	–	↑	↑	↓	↑	–	↑	↓
δ_r	–	–	–	–	↑	↓	–	↑	–	↑	↓	↑
	d_n^I	d_n^{II}	d_n^I	d_n^{II}	d_r^{II}	d_n^I	d_n^{II}	d_r^{II}	d_n^I	d_r^{II}	d_n^I	d_r^{II}
δ_n	↑	↑	↑	↑	↓	↑	↑	↑	↑	–	↑	↑
δ_r	–	–	–	↓	↑	↑	↓	↑	–	↑	↑	↑

*– for no impact; ↓ for decreasing; and ↑ for increasing

warranty length for remanufactured products increases, the price for the new product in the first period decreases if the manufacturer uses all returns for remanufacturing. Otherwise, the prices for the new product stay unchanged. Similar results apply for the remanufactured products. With the same warranty length, increasing the customer's sensitivity to warranty length for new products leads to the increase in the demand for new products. This is because the manufacturer increases the price for new products to obtain maximum profit. When the customer's sensitivity to warranty length for remanufactured products increases, the price for the new product in the second period stays unchanged since it is not affected by the manufacturer's decision to produce remanufactured products or not (Lemma 3.2). When the remanufactured products are not sufficiently profitable (Strategy N, B in Region II and R in Region IV: using partial returns), the pricing strategies in the two periods are independent and the price for the new product in the first period is not affected by the customer's sensitivity to warranty length for the remanufactured product. The reason is that the manufacturer does not sacrifice its profit in the first period to obtain more returns. When the remanufactured products are sufficiently profitable (Strategy B in Region III and R in Region V: using all returns), in order to maximize its profit over two periods, the price (demand) for the new product in the first period has the same trend as the price (demand) for the remanufactured product in the second period: increases as the customer's sensitivity to warranty length for the remanufactured product increases. The most essential factor for the pricing decision in the first period is to guarantee enough returns for producing the remanufactured product in the second period.

As the customer's sensitivity to warranty length for the new products increases, the demands for new products in both periods increase. This is due to the fact that, with the same warranty length, increasing the customer's sensitivity to warranty length for the new product can attract more customers to buy new products. In the meantime, the demand for remanufactured products decreases in Strategy B in Region II, increases in Strategy B in Region III and R in Region V, and remains unchanged in Strategy R in Region IV. The reason is that in Strategy B in Region II, increasing the customer's sensitivity to warranty length for the new product stimulates more customers to buy the new product, some of these customers originally planned to buy remanufactured products. Therefore, the demand for the remanufactured product decreases. In Strategy R in Region IV, however, all customers buy remanufactured products. Therefore, the demand for the remanufactured product is not affected. For Strategy B and R in Region III and V, in order to maximize the profit, all returns are used remanufacturing. When the customer's sensitivity to warranty length for new products increases, the demand for new products in the first period increases and more returns are collected. To use all returns, the demand of remanufactured products must increase.

The customer's sensitivity to warranty length for the remanufactured products has a positive effect on the demand for the remanufactured product and a negative effect on the demand for the new product in the second period. With the increase of the customer's sensitivity to warranty length for the remanufactured product, some customers who originally would have bought new products switch to remanufactured products. This positively affects the demand for the new product in the first period when Strategy B or R in Region III and V is adopted. The reason is that in these two strategies, the manufacturer needs to increase the demand for new products in the first period to obtain enough returns for producing the remanufactured products in the second period. Since increasing the customer's sensitivity to warranty length for remanufactured products leads to the increase in demand for the remanufactured product, the demand for the new product in the first period also increases.

3.5 Numerical studies

In this section, numerical studies are used to illustrate the major results discussed in this chapter. First, the effect of varying the warranty length (w) on the optimal prices, demands, and profit is analyzed. Then the factors that can affect the optimal warranty length are explored. Lastly, the effect of varying warranty length for the new and remanufactured products individually on the optimal solutions is investigated. The parameter values used in this section are adapted from a dataset from the Canadian aerospace remanufacturing company studied in Chari et al. (2016a). For confidentiality reasons, the data has been anonymized but proportions have been kept when possible.

3.5.1 The impact of the warranty length

In this subsection, the following parameter values are used: $c_n = 0.3$, $c_r = 0.2$, $\lambda_n = 0.3$, $\lambda_r = 0.9$, $\delta_n = 0.1$, $\delta_r = 0.15$, $\alpha = 0.9$, $\beta = 1.2$ and $\theta = 0.9$. Figure 3.4 depicts the effect of warranty length w on optimal prices, demands, and total profit.

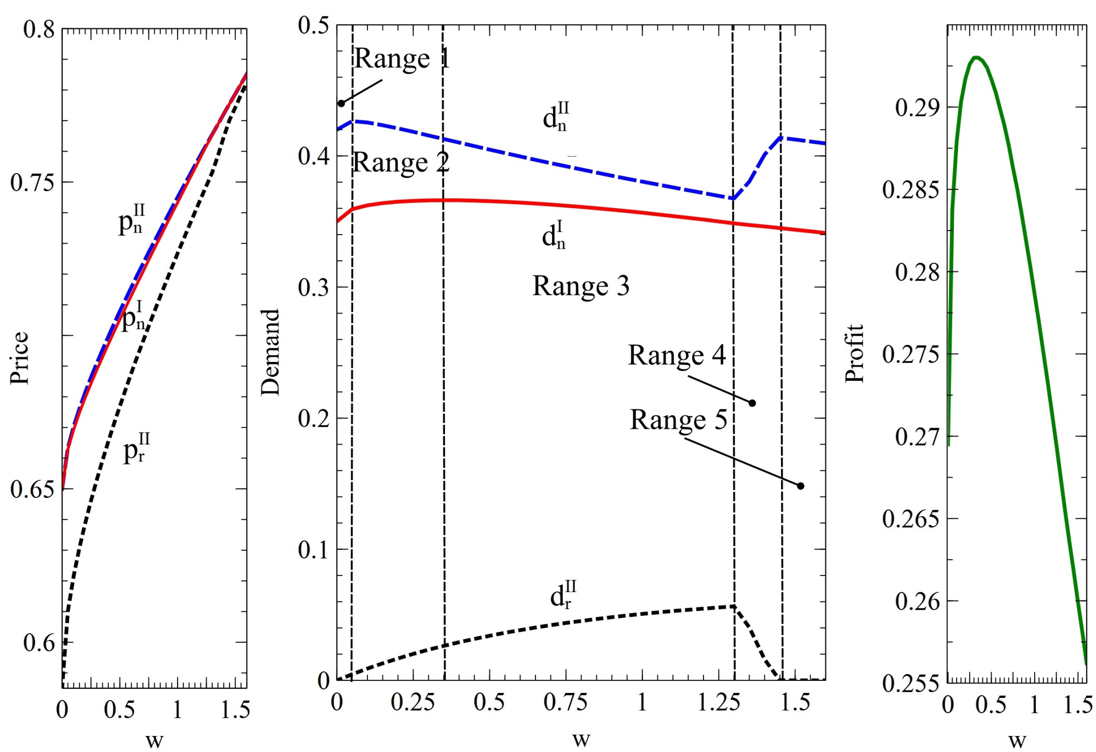


Figure 3.4: Influence of warranty length w on optimal prices, demands and profit

Figure 3.4 shows that, in general, increasing the warranty length leads to the increase of all prices for the new and remanufactured products in both periods. This implies that the increase of the warranty length increases the warranty cost for both new and remanufactured products, which forces the manufacturer to increase the prices. The result suggests that providing good post-sales service allows the manufacturer to charge a high price.

It is interesting to see that the warranty length impact demands of the new and remanufactured products differently. Specifically, when the warranty length increases, the demand for the new product in the first period and for the remanufactured product start by increasing and then decrease. The demand for the new product in the second period increases to its peak (when $w = 0.05$), then decreases to its lowest point (when $w = 1.3$). After that, it increases again and then decreases when $w \geq 1.45$. The intuition for these changes is that on one hand, the increase of warranty length can increase the warranty costs for both the new and remanufactured product, resulting in a lower profit (negative warranty cost effect). Due to the difference in failure rates of the new and the remanufactured products ($\lambda_n \leq \lambda_r$), the warranty cost of the remanufactured product increases more rapidly than that of the new product. On the other hand, the increase of the warranty length leads to the increase of demands (positive demand effect), when the prices stay unchanged. These increases in demand start to decrease when the warranty length increases. Changes in the demands for the new and remanufactured products over the two periods in Figure 3.4 can be divided into five ranges:

- Range 1 ($w < 0.05$): the positive demand effect of the increasing warranty length outweighs its negative warranty cost effect on the profit for both the new and the remanufactured products in both periods. Therefore, the manufacturer stimulates the increase in demand for both new and remanufactured products, resulting in a rapid increase in the profit.

- Range 2 ($0.05 \leq w < 0.35$): the positive demand effect of the increasing warranty length can only offset its negative warranty cost effect on the profit for the remanufactured product. For the new product, the increased warranty length leads to a lower profit. The manufacturer thus decreases the demand for the new product and enhances the demand for the remanufactured product in the second period by adjusting its pricing strategy. Therefore the total profit still increases in this warranty length range. However, as compared to the results in Range 1, the profit increases less rapidly. The demand for the new product in the first period increases to meet the need of more returns for producing the remanufactured product in the second period.

- Range 3 ($0.35 \leq w < 1.3$): the positive demand effect of the increasing warranty length cannot offset its negative warranty cost effect on the profit for both the new and remanufactured products. When the warranty length increases, the profit begins to decreases. Since the customers are more sensitive to the warranty length of the remanufactured product than that of the new product ($\delta_n \leq \delta_r$), the increase of warranty length results in less negative effect for the remanufactured product than for the new product. Therefore, the demand for the new product decreases while it increases slightly for the remanufactured product. Moreover, even though more returns are needed for producing the remanufactured products, the demand for the new product in the first period still decreases. The reason are: 1) from the analysis above, when the warranty length increases, producing the new product becomes less profitable, and thus the manufacturer decreases its production; 2) since the increase of the demand for the remanufactured product slows down, the need for returns also decreases; 3) with the increase of the warranty length, for each new product sold in the first period, more failures occur during the warranty period and the manufacturer receives more returns. Therefore, there is no need to increase the production for the new product in the first period to receive more returns.

- Range 4 ($1.3 \leq w < 1.45$): producing the remanufactured product is less profitable than producing the new product. The reason is that even though the customers are more sensitive about the warranty length on the remanufactured product than that on the new product ($\delta_n \leq \delta_r$), the marginal customer's utility on the product decreases as the warranty length increases. Moreover, since the new product's quality is higher ($\lambda_n \leq \lambda_r$), the warranty cost of the remanufactured product increases more rapidly than that of the new product with the increase of the warranty length. The manufacturer decreases the production for the remanufactured product by adjusting the prices and due to the switch of some customers who originally would buy the remanufactured product to buy the new product. The demand of the new product in the second period increases. However, producing the new product still becomes less profitable when the warranty length increases, the demand for the new product in the first period keeps decreasing.

- Range 5 ($w \geq 1.45$): there is no demand for the remanufactured product on the market (i.e., the black dotted line stays constantly at zero). When the warranty length increases, there are no switchers from remanufactured products to new. Therefore, the demand for the new product in the second period starts to decrease.

3.5.2 Sensitivity analysis for the optimal warranty length

Figure 3.4 shows that there exists an optimal warranty length ($w = 0.35$) for the manufacturer. In this subsection, changes in the optimal warranty length are analyzed by varying the unit (re)manufacturing costs (c_n and c_r), failure rates (λ_n, and λ_r) and the customer's sensitivity to warranty length (δ_n, and δ_r). The results are summarized in Table 3.4.

From Table 3.4, it is obvious that when the unit manufacturing cost for the new or the remanufactured product increases, the optimal warranty length decreases. The same conclusion holds between the failure rate for the new or the remanufactured product and the optimal warranty length. When the unit manufacturing cost or the failure rate increases, the warranty cost also increases. In order to reduce this negative impact, the manufacturer decreases the warranty cost by reducing the warranty length. Moreover, it also found that the decreasing rate of the optimal warranty length becomes smaller as the unit manufacturing cost or the failure rate for either new or remanufactured product increases. The intuition is that the decrease in the optimal warranty length leads to a negative effect on the demand, which results in a reduction in the manufacturer's profit. This negative effect becomes larger when

Table 3.4: Impacts of c_n, c_r, λ_n, λ_r,δ_n and δ_r on the optimal warranty length w

No.	c_n	c_r	λ_n	λ_r	δ_n	δ_r	$\boldsymbol{w^*}$
1	**0.3**	0.20	0.30	0.90	0.10	0.15	0.35
2	**0.5**	0.20	0.30	0.90	0.10	0.15	0.15
3	**0.7**	0.20	0.30	0.90	0.10	0.15	0.10
4	0.30	**0.10**	0.30	0.90	0.10	0.15	0.45
5	0.30	**0.20**	0.30	0.90	0.10	0.15	0.35
6	0.30	**0.30**	0.30	0.90	0.10	0.15	0.30
7	0.30	0.20	**0.30**	0.90	0.10	0.15	0.35
8	0.30	0.20	**0.40**	0.90	0.10	0.15	0.20
9	0.30	0.20	**0.50**	0.90	0.10	0.15	0.15
10	0.30	0.20	0.30	**0.40**	0.10	0.15	0.40
11	0.30	0.20	0.30	**0.70**	0.10	0.15	0.35
12	0.30	0.20	0.30	**1.20**	0.10	0.15	0.30
13	0.30	0.20	0.30	0.90	**0.02**	0.15	0.05
14	0.30	0.20	0.30	0.90	**0.05**	0.15	0.10
15	0.30	0.20	0.30	0.90	**0.08**	0.15	0.25
16	0.30	0.20	0.30	0.90	0.10	**0.15**	0.35
17	0.30	0.20	0.30	0.90	0.10	**0.30**	0.40
18	0.30	0.20	0.30	0.90	0.10	**0.45**	0.55

the warranty length becomes smaller. To decrease this negative effect, the optimal warranty length decreases slowly when the unit manufacturing cost or the failure rate increase.

Furthermore, the optimal warranty length increases with the increase of the customer's sensitivity to warranty length for either the new or the remanufactured product. This happens because when the customer's sensitivity to warranty length for either of the products increases, increasing warranty length can attract more customers and have a more positive effect on the profit. Therefore, the manufacturer increases the warranty length.

3.5.3 Sensitivity analysis on the individual warranty length

In this section, the assumption in Section 3 that the new and remanufactured products have the same warranty length is relaxed. Now, it is assumed that they have different warranty lengths (w_n and w_r) and changes in optimal prices, demands and profit for the new and remanufactured products are analyzed by varying w_n or w_r individually. Based on Equations (3.4), (3.5) and (3.8), the simultaneous change of failure rate (λ_n or λ_r) and the customer's sensitivity to warranty length (δ_n or δ_r) is the same as changing the corresponding warranty length. For example, if the failure rate is changed from λ_n to $2\lambda_n$ and δ_n to $\sqrt{2}\delta_n$ simultaneously, it is the same as changing w_n to $2w_n$.

Figure 3.5 shows the changes in the optimal prices, demands, profit when the warranty length for the new products (w_n) varies between $0.8w_n$ to $2.2w_n$ for $w_n = 1$.

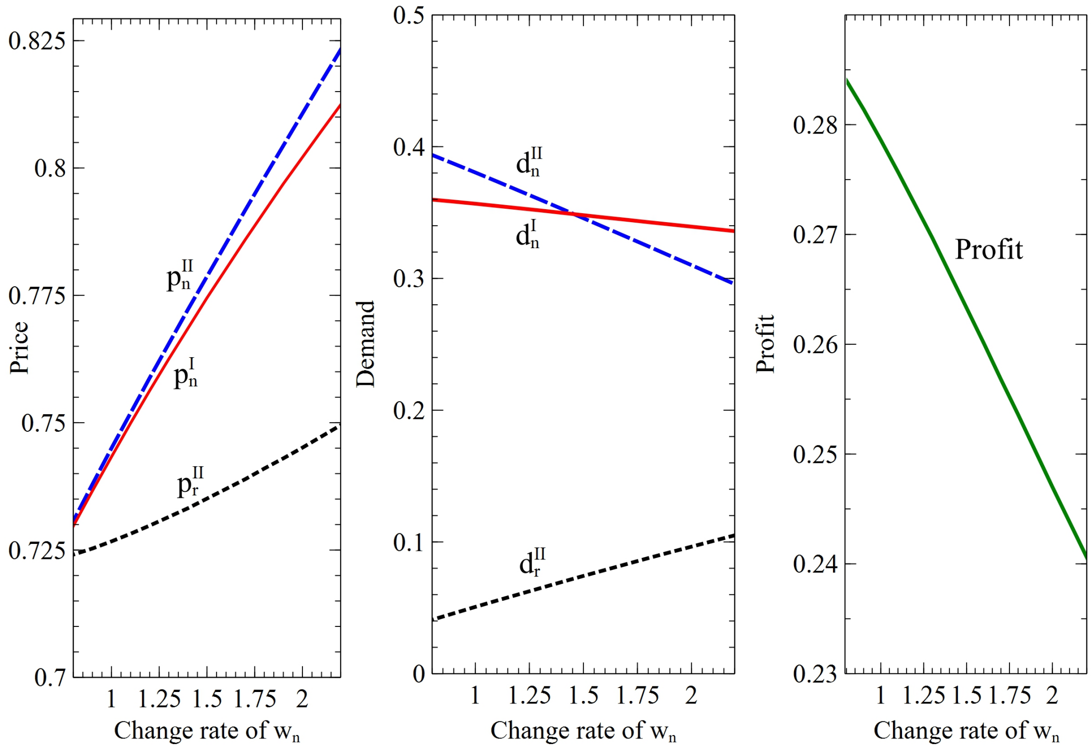


Figure 3.5: Influence of warranty length w_n on the optimal prices, demands and profit

It is observed that when the warranty length for new products (w_n) increases, the prices for the new and the remanufactured product in both periods increase. The gap between the prices for the new product and the remanufactured product in the second period increases with the increasing warranty length for the new product. The reason is that with the increase in the warranty length for the new product, producing the new product becomes less profitable as compared to producing the remanufactured product. Therefore, the manufacturer increases the gap for the prices between the new and the remanufactured product to push more customers towards the remanufactured product instead of the new product. This trend can be seen in Figure 3.5. With the increase in the warranty length for the new product, the demand for the remanufactured product increases, while the demands for the new products in both periods decrease. The intuition is that when the warranty length for the new product increases, more returns can be obtained for each new product sold in the first period. The manufacturer does not need to sacrifice the profit in the first period by increasing the production to receive more returns. Furthermore, the increase in warranty length for the new product results in a lower profit, implying that the total profit decreases as the w_n increases.

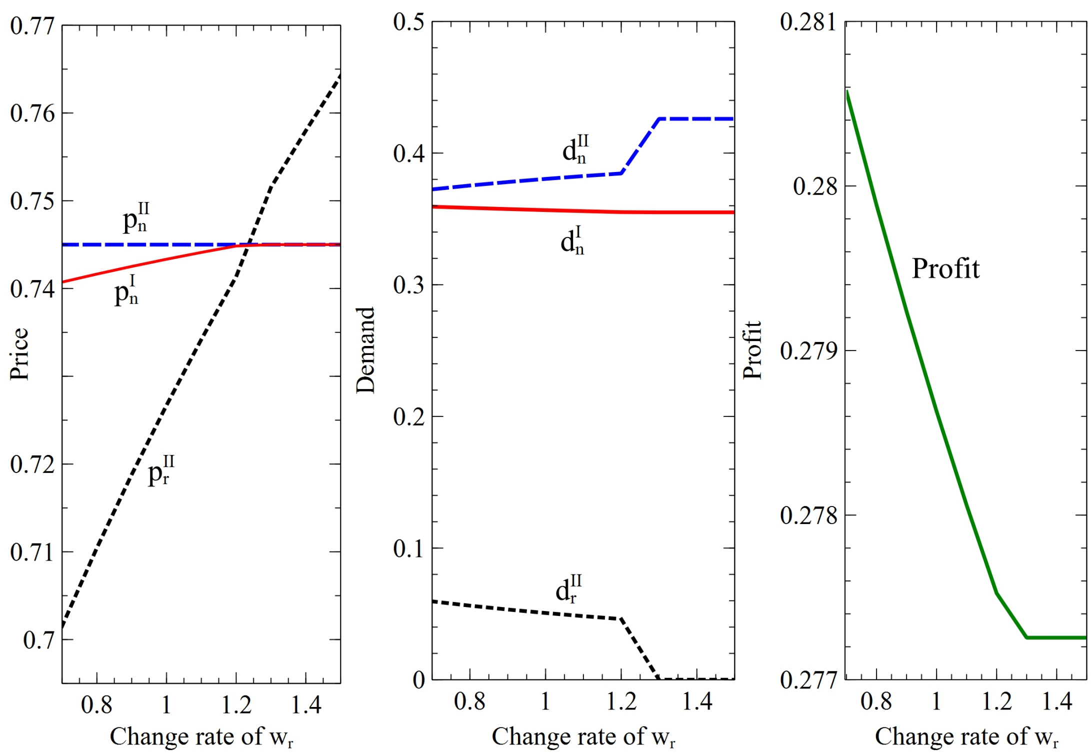


Figure 3.6: Influence of warranty length w_r on the optimal prices, demands and profit

Figure 3.6 shows the changes in the optimal prices, demands, profit when the warranty length for the remanufactured products (w_r) varies from $0.8w_r$ to $1.6w_r$ for $w_r = 1$. In general, when the warranty length for the remanufactured product increases, the price for the remanufactured product increases while the price for the new product in the second period remains unchanged. The price for the new products in the first period increases in the beginning and then stays unchanged. The intuition is that when the warranty length for the remanufactured product increases, producing remanufactured products becomes less profitable compared with producing new products. Thus, the manufacturer adjusts the prices to attract more customers to buy the new products (the demand for the new product in the second period increases, while the demand for the remanufactured product decreases until zero). Moreover, with the increased warranty length for the remanufactured product, the profit decreases in the beginning and remains unchanged after the demand for the remanufactured product drops to zero.

3.6 Conclusion

We developed a two-period model for a monopolist, who can produce remanufactured products from the failed products under warranty that are returned in the first period. Demands for both the new and remanufactured products are obtained based on the utility theory. Specifically, the customer's utility has a negative relationship with the price and a positive concave relationship with the warranty length. The non-renewing free replacement warranty is offered for the new and the remanufactured products by the manufacturer. We identified the manufacturer's optimal pricing and production strategies. We further numerically examined the impact of the warranty length on the optimal prices, demands, and profits. The effects of the unit production costs, failures rates, and customer's sensitivity to warranty length on the optimal warranty length have also been numerically investigated and discussed.

We demonstrated under which conditions the manufacturer should engage in remanufacturing. We showed that the manufacturer's decision to engage in remanufacturing or not depends on the warranty length and the ratio of the production costs ($\frac{c_r}{c_n}$). When the ratio of the production costs increases, the manufacturer reduces the warranty length if it wants to engage in remanufacturing. Moreover, when the customer's acceptance of the remanufactured product increases, the manufacturer increases the warranty length for its remanufactured product. This result implies that if the customer perceives the remanufactured products highly, it is more likely for the remanufacturer to engage in the remanufacturing operations.

We also numerically showed that there exists an optimal warranty length for the manufacturer, which is affected by the unit production costs, failures rates, and sensitivity of the customer's utility to the warranties for the new and remanufactured products. When the unit production costs and failure rates for either the new or the remanufactured products are high, the manufacturer should choose a short warranty length. When the customer highly values the warranty length in buying the product, the manufacturer should choose a long warranty length.

There are several extensions to our study. First, our model only investigated the effect of the non-renewing free replacement warranty on the pricing and production strategies for a manufacturer in the presence of the opportunity to engage in remanufacturing. Other warranty types, such as the pro-rata and refund warranties, can also be investigated. Second, our study only considers the problem in a monopoly with one manufacturer. It is interesting to explore the problem in a competitive market with competition among the manufacturers and identify the optimal pricing and production strategies for them. Third, in this chapter, the returns come only from warranted failed products in the first period. A possible extension can investigate how the optimal prices, demands, and profit change when the returns come from several other channels such as the collection of end-of-life products.

Chapter 4

Extended warranties competition in closed-loop supply chains with remanufacturing

4.1 Introduction

Two types of warranty are widely used in practice: the base warranty and the extended warranty (EW). The base warranty, which was discussed in Chapter 3, is usually sold bundled with products and is required by government regulations and industry norms. The EW, an extension to the base warranty, is a prepaid service that provides repair or maintenance service on the product for an additional time period after the base warranty expires (Rahman and Chattopadhyay, 2015). For example, on top of a one-year base warranty, Apple Inc. sells a two-year EW (AppleCare+) for all its iPad models (Tian et al., 2019). According to Ishida et al. (2019), the sales of the EW have generated nearly $16 billion worldwide with a 6.5% growth rate in 2018. The EW service is credited for a profit margin of 50-60%, nearly 18 times the margin on the product sales. For example, in 2014, the sales of the EW service by Best Buy Co., Inc., generated around half of its operating profit (Heese, 2012).

Unlike the base warranty, the EW is not free to the customer. The consumer must decide whether or not to buy the EW after purchasing the product. It is usually a difficult decision because the consumer cannot evaluate with certainty if the price paid for the EW can be offset by not paying the repair/replacement cost at a possible but uncertain product failure (Bouguerra et al., 2012). For the manufacturer, although providing EW can stimulate the demand, it may also incurs a high cost. The manufacturer needs to carefully decide the pricing strategy for the EW service to ensure that this service can be profitable (Su and Shen, 2012).

The base warranty in the CLSC has been investigated in recent years (Cao and

He, 2018; Giri et al., 2018; Liu et al., 2018; Alqahtani et al., 2019), but the EW has not been fully investigated in the remanufacturing industry. The EW is important in remanufacturing, since the warranty cost of the remanufactured product is usually higher than the warranty cost of a new product due to the lower quality and non-zero age of the remanufactured systems. The EW, however, can make the customer share a portion of the cost to reduce the firm's financial burden. Most studies only consider the single EW in the market instead of the multiple extended warranties offered by competing providers. For example, Kuik et al. (2015) explore the EW length in different warranty policies. Zhu et al. (2018) investigate the impact of the EW in the SC in the centralized and decentralized. Dan et al. (2019) study the relationship of the EW provider and the reverse channel selection. However, both manufacturer and retailer may be interested in selling their own EW. For example, many manufacturers, such as Ford, JVC, and Apple sell extended warranties directly to the customer, while retailers such Sears and Suning Commerce Group Co., Ltd. are willing to sell their own extended warranties rather than selling the manufacturer's EW (Zheng and Ai, 2017). To fill this gap, our study investigates the impact of both manufacturer and retailer extended warranties on the optimal pricing strategy for the manufacturer and the retailer and the optimal production strategy for the manufacturer. The following research question are discussed in the chapter.

- How should the manufacturer price its EW service?
- What is the impact of the introduction of the retailer's EW on the manufacturer's decisions?
- How does the warranty length affect the manufacturer and retailer optimal decisions?

The contributions of the chapter are: firstly, we explore the optimal pricing strategies of the extended warranties when they are offered by the manufacturer and the retailer. To the best of our knowledge, no previous studies have explored the competition of the extended warranties from different agents in the CLSC. Secondly, our study investigates the conditions under which the introduction of the retailer's EW may hurt the manufacturer's profit, which has not been explored in the literature. Thirdly, we examine the impact of the unit warranty replacement cost for the retailer and the retailer's EW length on decisions of the manufacturer and the retailer in the SC.

The remainder of this chapter is organized as follows. Section 2 presents a literature review on warranty in the SC. In Sections 3 and 4, the models are developed to derive the optimal pricing strategies for the manufacturer and the retailer. The sensitivity analysis to test the impact of the EW length on the optimal results are presented in Section 5. Section 6 and 7 provide the managerial implications, conclusion and extensions for the future research respectively. The proofs of all Lemmas and Theorems are in Appendix.

4.2 Literature review

In the section, the literature related to warranty policies and remanufactured products is reviewed. A comprehensive review can be found in Diallo et al. (2017), in which different warranty models in the literature from 2001 to 2016 are described. The paper shows that future extensions should focus on giving the customer the flexibility to choose the warranty policy. This paper answers this call by proposing a model where the customer is allowed to choose to buy the EW for their new products from either the manufacturer or the retailer.

Aksezer (2011) estimates the cost for the EW offered in the used car industry by considering the age, the usage, and the maintenance data. Two warranty policies are discussed: 1) free repair warranty (the cost is undertaken by the manufacturer only) and 2) cost-sharing warranty (the customer and the manufacturer share the warranty cost). Alqahtani and Gupta (2017b) focus on the calculation of the EW cost for a sensor-embedded washer, which guides the manufacturer in setting the price for the EW. Three extended warranties are considered: 1) extended free replacement warranty policy (FRW); 2) extended pro-rata warranty policy (PRW) and 3) FRW-PRW combined policy. They find that with the embedded sensor, the extended free replacement warranty policy has the lowest average warranty cost and number of warranty claims. Alqahtani et al. (2019) extend Alqahtani and Gupta (2017b) by considering the internet of things (IoT) with the warranty policies. IoT can significantly affect the manufacturer's warranty decisions, since it reduces the ambiguity of the condition and improves the estimates of the remaining life of an End-of-Use or End-of-Life product. The warranty costs are estimated under different warranty policies: the one-dimensional (renewable or non-renewable) free replacement warranty, (renewable or non-renewable) pro-rata warranty, cost sharing warranty, cost limit warranty, money back guaranty, and the combinations of above warranties. Different from the papers mentioned above, which focus on the estimation of the warranty cost for different policies, our study focuses on maximizing the profit of the SC members. Moreover the above papers do not consider the competition between providers of the extended warranties. The proposed model will include such a competition.

Warranty plays a critical role in the SC, since it affects both the cost and the customer's preference for the product. Several papers focus on the impact of warranty on the SC. Giri et al. (2018) integrate the warranty period and the concept of green innovation in the CLSC and find the optimal prices and warranty length for both the manufacturer and the retailer. Two models are considered in the paper (with or without green innovation). It is found that by considering the green innovation, both manufacturer and retailer can obtain a higher profit under the centralized and decentralized systems, and under a revenue sharing contract. Cao and He (2018) investigate the price and warranty competition problem in a SC with one retailer and two manufacturers. Four SC structures are discussed and the optimal prices and warranty periods are compared in each case: centralized, Nash game, manufacturer-leading, and retailer-leading SC. The results show that the manufacturer with high-quality product should set a higher wholesale price and a longer warranty length. Cao et al. (2020c) explore the optimal profit for the manufacturer, who sells both new and remanufactured products with different warranty lengths by considering a trade-in subsidy from the government and a carbon tax. The manufacturer needs to decide whether to offer a trade-in service for the remanufactured product or to both new and remanufactured products. They find that the optimal warranty periods are the same for both cases. Taleizadeh and Mokhtarzadeh (2020) investigate the optimal pricing problem for multiple products with two-dimensional non-renewing free replacement warranty sold through both the offline and online channels. The warranty claims and costs follow the non-homogeneous Poisson process (NHPP) and log-normal distribution respectively. The model in the problem is solved by the value at risk approach. The paper finds that the increasing warranty length and usage could affect the manufacturer's profit positively and negatively. Tang et al. (2020) focus on solving two problems: 1) whether the manufacturer should offer warranty to the new products only, or to both new and remanufactured products; 2) whether the manufacturer or the retailer should offer the warranty. They find that the warranty length impacts the prices of the new product in the first period and the remanufactured product in the second period, but it does not impact the price of the new product in the second period.

The above studies explore the base warranty only in the SC, which comes automatically with the product. Our study investigates the impact of the EW on the SC, which is a significant source of profit for the warranty provider.

Kuik et al. (2015) explore the optimal EW length for the remanufactured product with different warranty policies. Two warranty policies are discussed in the paper: 1) the manufacturer is responsible for repairing the product within a limited time period (Type-I); and 2) the manufacturer is responsible for repairing the product for limited failures (Type-II). It is found that the Type-I warranty is better than the Type-II warranty for the manufacturer. Zhu et al. (2016) study the adoption of innovation service modes for the remanufacturer: the EW and the free replacement. The data in the paper are from a leading heavy truck company in China. The empirical analysis shows that a longer EW or replacement service cannot always guarantee a higher profit for the manufacturer. Zhu et al. (2018) analyze the warranty problem for the remanufactured product in a CLSC, where the retailer sells the EW to the customer under three SC structures. They find that the increased customer's preference for the remanufactured product leads to an increased demand for the EW and the profits of each member in the SC. Dan et al. (2019) investigate the impact of the EW service and the warranty provider (manufacturer, retailer, or the third-party company) on the reverse channel selection. They find that the selection of the EW providers does not affect the manufacturer's reverse channel choice: collecting returns by the retailer is always the best choice. Afsahi and Shafiee (2020) explore the optimal length for the extended warranties and optimal price for the EW with the uncertain repairing quality for the failed returns. The failed product can experience perfect repair, imperfect repair, or minimal repair with a certain probability. The problem is solved by a meta-heuristic Monte-Carlo simulation algorithm integrated within a dynamic programming model. Jin and Zhou (2020) focus on the optimal warranty policy for the remanufactured product in a decentralized CLSC with one supplier and one manufacturer. The manufacturer can decide whether or not to sell an EW for the remanufactured product or to the customers. They find that not offering EW for the remanufactured product always hurts the supplier and the manufacturer.

Although these papers are related to the EW in the CLSC, they focus on the impact of EW on one product (either the new or the remanufactured), except for Dan et al. (2019). Secondly, the warranty provider in these paper is either the manufacturer or the retailer, which fails to reflect the common marketplace reality where both manufacturer and retailer are interested in selling the extended warranties. For example, both Apple Inc. and its retailer (Jingdong in China) offer their EW services to their iPhone customers (Zheng and Ai, 2017; Wang et al., 2020b). The competition of the warranty can significantly impact the manufacturer's and the retailer's pricing decisions. Differing from these papers, our study considers the problem with one manufacturer and one retailer in a CLSC, in which the manufacturer sells the EW for both new and remanufactured products and the retailer sells the EW for the new product only. Table 4.1 summarizes the main differences between our study and studies in the literature mentioned in this section.

Table 4.1: Main differences between our study and studies in the literature

Name	**Objective**	**Warranty type**	**Warranty provider**	**Product(s) with warranty**
Aksezer (2011)	Min cost	EW	Manufacturer	Remanufactured
Kuik et al. (2015)	Max profit	EW	Manufacturer	Remanufactured
Zhu et al. (2016)	Max profit	EW	Manufacturer	Remanufactured
Alqahtani and Gupta (2017b)	Min cost	EW	Manufacturer	Remanufactured
Giri et al. (2018)	Max profit	BW	Manufacturer	New and Remanufactured
Cao and He (2018)	Max profit	BW	Manufacturer	New
Zhu et al. (2018)	Max profit	EW	Retailer	Remanufactured
Alqahtani et al. (2019)	Min cost	BW	Manufacturer	Remanufactured
Dan et al. (2019)	Max profit	EW	Manufacturer or Retailer	New and Remanufactured
Cao et al. (2020c)	Max profit	BW	Manufacturer	New and Remanufactured
Taleizadeh and Mokhtarzadeh (2020)	Max profit	BW	Manufacturer	New
Afsahi and Shafiee (2020)	Max profit	BW and EW	Manufacturer	New
Jin and Zhou (2020)	Max profit	EW	Manufacturer	Remanufactured
Tang et al. (2020)	Max profit	BW	Manufacturer or Retailer	New and Remanufactured
Our study	Max profit	EW	Manufacturer and Retailer	New and Remanufactured

*Warranty type: EW (EW) and base warranty (BW)

4.3 Problem Description

All the notation mentioned in this chapter are summarized in Table 4.2.

Table 4.2: Notation Table

Indices	
i	Product type index (subscript): new product ($i = n$) and remanufactured product ($i = r$).
j	SC member index (subscript): manufacturer ($j = m$) and retailer ($j = t$).
k	Time period index (superscript): first period ($k = I$) and second period ($k = II$).
l	Case Index (superscript): both members offer EW ($l = B$) and only the manufacturer offers EW ($l = M$).
Parameters	
w	Length of the EW period.
c_i	Unit replacement cost by the manufacturer for product type i.
c_c	Unit trade-in cost for the retailer.
β	Customer's acceptance level of the EW offered by the retailer.
z_i	Customer's willingness to pay for product type i.
d_i^k	Demand for product type i in period k.
δ_i	Customer's sensitivity to the EW price for product type i.
λ_i	Failure rate for product type i.
D_{ij}^{kl}	Demand for EW of product type i in period k provided by member j under case l.
Π_j^l	Profit for SC member j under case l.
Decision Variables	
p_{ij}^l	Price for product type i's EW provided by member j under case l.

The EW problem in a CLSC with one retailer (t) and one (re)manufacturer (m) is considered. The EW is offered either by the manufacturer only (Case M) or by both retailer and manufacturer (Case B). The sales horizon is divided into two periods. The manufacturer produces the new product for both periods and uses returns from Period 1 to produce the remanufactured product for Period 2. Therefore, for case $l = \{M, B\}$, the manufacturer offers the EW for the new product to the customer with price p_{nm}^l in both periods and offers the EW for the remanufactured product with price p_{rm}^l in Period 2 only. The retailer offers the EW for the new product with price p_{nr}^B in both periods in Case B.

The extended warranties discussed in this chapter are assumed to be the non-renewing free-replacement warranties, which are common in practice and in the literature (Zhu et al., 2016; Alqahtani and Gupta, 2018). When a product fails during the EW period, the customer returns the failed product and receives a free replacement (identical product) from the agent, from whom the EW was bought. For both cases, if the manufacturer receives a failed new or remanufactured product from the customer within the EW period, it incurs a costs c_n (c_r) to offer a new (remanufactured) replacement to the customer. For Case B, if the retailer receives a failed new product from the customer, it can trade-in with the manufacturer for a new replacement at a cost c_c. In both cases, the failed products collected by the manufacturer from either the customer or the retailer by the end of the first period can be used to produce the remanufactured products for Period 2. It is assumed that the returned remanufactured product can not be remanufactured again due to quality issues (Debo et al., 2005; Van Loon and Van Wassenhove, 2018) and the returned new product in Period 2 arrives too late to be remanufactured (Yenipazarli, 2016; Liu et al., 2018). The return is recycled by the manufacturer and the unit recycling cost is covered by the unit salvage value. The configuration of the two cases is shown in Figure 4.1.

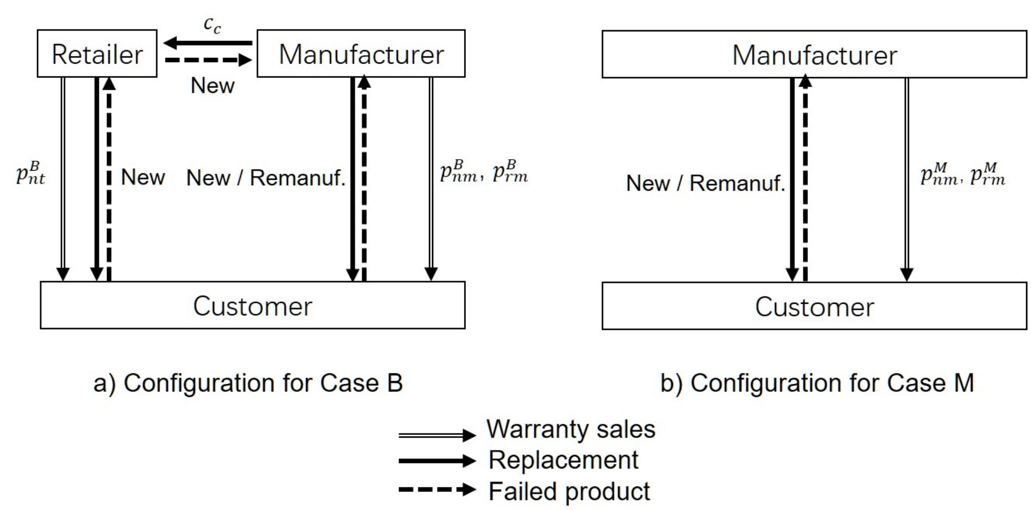


Figure 4.1: Configuration of cases B and M

The warranty length (w) is not renewed and is the same for the EW service sold by either the manufacturer or the retailer. This is also common in practice. Apple

Inc. offers a 2-year AppleCare+ service (EW) for each iPhone sold [1], while JingDong (a retailer of the iPhone in China) offers the EW option with the same length [2]. This assumption will be relaxed in the extension in subsection 4.5.2.

Following the studies by Hong et al. (2016) and Chen and Akmalul'Ulya (2019), we assume that the manufacturer is the Stackelberg leader in the SC, who has the advantage of the first move. At the beginning of the first period, the manufacturer decides and announces the price for the manufacturer's EW for the new product. Then the retailer decides and announces the price for the retailer's EW for the new product, if it offers the EW service (Case B). In the second period, the manufacturer decides and announces the price for the manufacturer's EW for the remanufactured product.

The customer has two decisions. Firstly, it decides whether or not to buy the product, and secondly, whether or not to buy the EW and from whom to buy the EW. We assume that the first decision is not affected by the second decision since it is not necessary for the customer to decide the purchase of the EW when buying the product. For example, Apple allows its customers to buy the EW (AppleCare+) up to 60 days after the purchase of the product (Zheng et al., 2018). A similar setting can also be found in Zhu et al. (2018). Our study focuses on the competition of the extended warranties offered by the manufacturer and the retailer. Let d_i^k ($i = n, r$ and $k = I, II$) denote the demand for the new and remanufactured products in the first and second periods.

Let z_n be the customer's willingness to pay for the EW sold by the manufacturer for the new product, which is uniformly distributed between 0 and 1 in Case M. For both periods, a customer buys the EW of the new product from the manufacturer with the utility $U_{nm}^M = z_n - p_{nm}^M + \delta_n\sqrt{w}$. This utility decreases as its selling price (p_{nm}^M) increases or as the EW length (w) decreases. Moreover, the marginal effect of the warranty length on the utility decreases as its value increases. δ_n represents the customer's sensitivity to the new product's EW. When the utility is non-negative,

[1] https://www.apple.com/ca/legal/sales-support/
[2] https://fuwu.jd.com/

that is $z_n \geq p_{nm}^M - \delta_n\sqrt{w}$, the customer will buy the EW service from the manufacturer. Since the demands for the new product in the first and second periods are d_n^I and d_n^{II}, the demands for purchasing the EW of the new product from the manufacturer in the first and second periods are $D_{nm}^{IM} = d_n^I(1 - p_{nm}^M + \delta_n\sqrt{w})$ and $D_{nm}^{IIM} = d_n^{II}(1 - p_{nm}^M + \delta_n\sqrt{w})$, respectively. Ensuring that the demand for the EW offered by the manufacturer is non-negative requires $p_{nm}^M \leq 1 + \delta_n\sqrt{w}$.

Let z_r be the customer's willingness to pay for the EW of the remanufactured product, which is also between 0 and 1 for the remanufactured product. In the second period, a customer buys the EW of the remanufactured product from the manufacturer only when its utility $U_{rm}^M = z_r - p_{rm}^M + \delta_r\sqrt{w} \geq 0$, which gives $z_r \geq p_{rm}^M - \delta_r\sqrt{w}$, where δ_r represents the customer's sensitivity to the remanufactured product's EW. As compared to the new product, the customer is more sensitive to the warranty offered on remanufactured products ($\delta_r \geq \delta_n$), since the customer knows less about the remanufactured product's usage history and worries more about its quality (Matsumoto et al., 2017). The demand for the remanufactured product is d_r^{II} and then the demand for the EW of the remanufactured product is $D_{rm}^{IIM} = d_r^{II}(1 - p_{rm}^M + \delta_r\sqrt{w})$. Ensuring the non-negativity of the demand for the EW of the remanufactured product offered by the manufacturer requires $p_{rm}^M \leq 1 + \delta_r\sqrt{w}$.

For Case B, β represents the acceptance level of the customer for the EW sold by the retailer, which is assumed to be between 0 and 1. $\beta = 1$ means that the customer sees no difference between the extended warranties offered by the manufacturer and the retailer, while $\beta = 0$ implies that the customer never chooses to buy the EW from the retailer. The utilities for the customer to buy the EW of the new product from the manufacturer and the retailer in both periods are $U_{nm}^B = z_n - p_{nm}^B + \delta_n\sqrt{w}$ and $U_{nt}^B = \beta z_n - p_{nt}^B + \delta_n\sqrt{w}$, respectively. The customer buys the EW of the new product from the manufacturer only when $U_{nm}^B \geq U_{nt}^B$ and $U_{nm}^B \geq 0$, which gives $z_n \geq \frac{p_{nm}^B - p_{nt}^B}{1-\beta}$. The customer buys the EW of the new product from the retailer only when $U_{nm}^B \leq U_{nt}^B$ and $U_{nt}^B \geq 0$, which gives $\frac{p_{nm}^B - p_{nt}^B}{1-\beta} \geq z \geq \frac{p_{nt}^B - \delta_n\sqrt{w}}{\beta}$. To ensure that the demand for the extended warranties of the new product offered by the manufacturer and the retailer are both positive, it is assumed that $1 \geq \frac{p_{nm}^B - p_{nt}^B}{1-\beta} \geq \frac{p_{nt}^B - \delta_n\sqrt{w}}{\beta}$. Since the demands for the new product in the first and second periods are d_n^I and d_n^{II}, the demands for purchasing the EW of the new product from the manufacturer in the first and second periods are $D_{nm}^{IB} = d_n^I(1-\frac{p_{nm}^B - p_{nt}^B}{1-\beta})$ and $D_{nm}^{IIB} = d_n^{II}(1-\frac{p_{nm}^B - p_{nt}^B}{1-\beta})$, respectively. The demands for purchasing the EW of the new product from the retailer in the first and second periods are $D_{nr}^{IB} = d_n^I(\frac{\beta p_{nm}^B - p_{nt}^B + (1-\beta)\delta_n\sqrt{w}}{(1-\beta)\beta})$ and $D_{nt}^{IIB} = d_n^{II}(\frac{\beta p_{nm}^B - p_{nt}^B + (1-\beta)\delta_n\sqrt{w}}{(1-\beta)\beta})$, respectively.

For the remanufactured product, the demand function for the EW sold by the manufacturer is similar to that in Case M, which can is written as $D_{rm}^{IIB} = d_r^{II}(1 - p_{rm}^B + \delta_r\sqrt{w})$.

The unit cost for replacing a failed new and a failed remanufactured product by the manufacturer is c_n and c_r, respectively. For the customer who buys the retailer's EW, each return costs the retailer c_c and earns the manufacturer a revenue $(c_c - c_n)$. $c_c \geq c_n$ since the manufacturer does not agree to trade in returns with the retailer, unless it can be profitable. The lifetimes of the new and remanufactured products are exponentially distributed with parameters λ_n and λ_r respectively. According to the renewal theory (Ross, 2014), $\lambda_n w$ and $\lambda_r w$ represent the expected number of failed new and remanufactured products during the EW period w.

4.4 Model

4.4.1 Case M: no retailer's extended warranty

In this case, the customer can buy the EW from the manufacturer only. The manufacturer firstly needs to decide the optimal price for the EW of the new product and then decides the optimal price of the EW for the remanufactured product. The problem is solved backward.

$$\max_{p_{rm}^M \geq 0} \sum_{i=n,r} (p_{im}^M - c_i\lambda_i w) D_{im}^{IIM} \tag{4.1}$$

s.t.

$$D_{nm}^{IM}\lambda_n w \geq d_r^{II} + D_{rm}^{IIM}\lambda_r w \tag{4.2}$$

The objective function (4.1) represents the profit obtained by the manufacturer for selling the EW for the new and remanufactured products in the second period. Constraint (4.2) implies that the number of returns collected in the first period is more than the required number of remanufactured products in the second period. With equations (4.1) and (4.2), we have the following result in Lemma 1.

Lemma 4.1. *The optimal price and demand for the EW of the remanufactured product offered by the manufacturer (p_{rm}^M and D_{rm}^{IIM}) depend on the price of the EW for the new product (p_{nm}^M). Specifically,*

- *when $p_{nm}^M \leq 1+\delta_n\sqrt{w}-\frac{(\delta_r\lambda_r w^{1.5}-\lambda_r^2 w^2 c_r+\lambda_r w+2)d_r^{II}}{2d_n^I\lambda_n w}$, then $p_{rm}^{M*}=\frac{1+c_r\lambda_r w+\delta_r\sqrt{w}}{2}$ and $D_{rm}^{IIM*}=\frac{d_r^{II}(1-c_r\lambda_r w+\delta_r\sqrt{w})}{2}$;*
- *when $p_{nm}^M \geq 1+\delta_n\sqrt{w}-\frac{(\delta_r\lambda_r w^{1.5}-\lambda_r^2 w^2 c_r+\lambda_r w+2)d_r^{II}}{2d_n^I\lambda_n w}$, then $p_{rm}^{M*}=\frac{\delta_r\lambda_r w^{1.5}+\lambda_r w+1}{\lambda_r w}-\frac{(1-p_{nm}^M+\delta_n\sqrt{w})\lambda_n d_n^I}{d_r^{II}\lambda_r}$ and $D_{rm}^{IIM*}=\frac{(1-p_{nm}^M+\delta_n\sqrt{w})\lambda_n d_n^I w-d_r^{II}}{\lambda_r w}$.*

When the price of the EW for the new product is sufficiently low ($p_{nm}^M \leq 1+\delta_n\sqrt{w}-\frac{(\delta_r\lambda_r w^{1.5}-\lambda_r^2 w^2 c_r+\lambda_r w+2)d_r^{II}}{2d_n^I\lambda_n w}$), the demand for the EW for the new product is high and the manufacturer can receive enough returns in the first period to allow for remanufacturing to take place. When the price of the EW for the new product is sufficiently high ($p_{nm}^M \geq 1+\delta_n\sqrt{w}-\frac{(\delta_r\lambda_r w^{1.5}-\lambda_r^2 w^2 c_r+\lambda_r w+2)d_r^{II}}{2d_n^I\lambda_n w}$), the manufacturer cannot receive enough returns for remanufacturing as few customers return the products, which leads to the increase in the price and the decrease in the demand for the EW of the remanufactured product.

Lemma 4.2. *When $p_{nm}^M \leq 1+\delta_n\sqrt{w}-\frac{(\delta_r\lambda_r w^{1.5}-\lambda_r^2 w^2 c_r+\lambda_r w+2)d_r^{II}}{2d_n^I\lambda_n w}$, increasing the warranty length w leads to an increased price (p_{rm}^{M*}), and the optimal demand (D_{rm}^{IIM*}) achieves its maximum at $w=(\frac{\delta_r}{2\lambda_r c_r})^2$.*

Lemma 4.2 implies that when the price for the new product's extended warranty from the manufacturer is sufficiently low, increasing in the warranty length leads to an increase in the manufacturer's price for the EW, while demand increases at first and then decreases until the profit is maximized in the second period. The reason is that the increased warranty length leads to a higher warranty cost for the remanufactured product, which pushes the manufacturer to increase the price. Moreover, the increased warranty length leads to a higher customer's perceived value to the EW of the new product and then the manufacturer can set a higher price. For the optimal demand, the increased warranty length attracts more customers to buy the EW of the new product due to the higher perceived value. However, this increase rate decreases as the warranty length increases due to the concave relationship between customer utility and warranty length. Moreover, the higher price negatively impacts the demand and the demand starts to decrease, when $w \geq (\frac{\delta_r}{2\lambda_r c_r})^2$.

After deciding the warranty price for Period 1, the manufacturer decides the optimal price for the EW of the new product (p^M_{nm}) to maximize the two-period profit.

$$\max_{p^M_{nm} \geq 0} \quad (p^M_{nm} - c_n\lambda_n w)D^{IM}_{nm} + \sum_{i=n,r}(p^M_{im} - c_i\lambda_i w)D^{IIM}_{im} \tag{4.3}$$

Let $A_1 = 1 + \delta_n\sqrt{w} - c_n\lambda_n w$, $A_2 = 1 + \delta_r\sqrt{w} - c_r\lambda_r w$, and $D_1 = (d^I_n + d^{II}_n)d^{II}_r\lambda^2_r$. Solving (4.3), yield the results that are summarized as follows.

Theorem 4.1. *Under the condition that $c_r \leq c_{r1}$, the optimal prices $(p^{M*}_{nm}, p^{M*}_{rm})$ depend on the unit replacement cost for the new product (c_n).*

- *When $c_n \leq c_{n1}$,*

$$p^{M*}_{nm} = \frac{1 + \delta_n\sqrt{w} + c_n\lambda_n w}{2}, \textit{and} \tag{4.4}$$

$$p^{M*}_{rm} = \frac{1 + c_r\lambda_r w + \delta_r\sqrt{w}}{2}, \tag{4.5}$$

- *when $c_{n1} < c_n \leq c_{n2}$,*

$$p^{M*}_{nm} = \delta_n\sqrt{w} - 0.5A_2 + 1 + \frac{(A_2 d^I_n\lambda_n w - d^{II}_r(A_1\lambda_r w + 2))d^I_n\lambda_n}{2(D_1 + \lambda^2_n d^{I2}_n)w}, \textit{and} \tag{4.6}$$

$$p^{M*}_{rm} = \frac{\delta_r\lambda_r w^{1.5} + \lambda_r w + 1}{\lambda_r w} - \frac{(A_2 D_1 w/d^{II}_r + d^I_n\lambda_n(A_1\lambda_r w + 2))d^I_n\lambda_n}{2(D_1 + \lambda^2_n d^{I2}_n)w\lambda_r}, \tag{4.7}$$

where c_{n1}, c_{n2}, and c_{r1} can be found in Table C.4 in the Appendix.

With the optimal prices in Theorem 1, demands for the new products in the first and second periods and demand for the remanufactured product in the second period are $D_{nm}^{kM*} = \frac{d_n^k(1+\delta_n\sqrt{w}-c_n\lambda_n w)}{2}$ $(k = I, II)$ and $D_{rm}^{IIM*} = \frac{d_r^{II}(1-c_r\lambda_r w+\delta_r\sqrt{w})}{2}$, when $c_n \leq c_{n1}$. When $c_{n1} < c_n \leq c_{n2}$, the optimal demands are $D_{nm}^{kM*} = \frac{(A_2D_1w+d_r^{II}d_n^I\lambda_n(A_1\lambda_r w+2))d_n^k}{2(D_1+\lambda_n^2 d_n^{I2})w}$ $(k = I, II)$ and $D_{rm}^{IIM*} = \frac{(A_2D_1w+d_r^{II}d_n^I\lambda_n(A_1\lambda_r w+2))d_n^I\lambda_n}{2(D_1+\lambda_n^2 d_n^{I2})w\lambda_r} - \frac{d_r^{II}}{\lambda_r w}$.

When the unit replacement cost for the new product is sufficiently low ($c_n \leq c_{n1}$), the manufacturer does not need to use all returns for remanufacturing. As c_n increases, the manufacturer increases the price for the new product to cover the increased warranty cost. Thus, the demand for the EW of the new product decreases and the supply for the remanufactured product and its replacement (the warranty returns in the first period) decreases. When the unit replacement cost for the new product is extremely high ($c_n \geq c_{n2}$), to provide enough returns for remanufacturing in the second period, the manufacturer should sacrifice its profit in the first period by decreasing the price of the EW for the new product to attract more demand. The increased unit replacement cost leads to a larger profit loss. When the unit replacement cost for the new product is extremely high ($c_n \geq c_{n2}$), it is not profitable for the manufacturer to sell the EW for the remanufactured product. The implication of this result is that for a firm with high production or warranty replacement costs, it is beneficial to collect warranty returns, and it is even better to build an efficient reverse network to collect end-of-use or end-of-life products.

The sensitivity of the warranty length on the threshold c_{n1} is summarized in Lemma 4.3.

Lemma 4.3. *The warranty length (w) negatively impacts the threshold c_{n1}, if the ratio of the demand for the new product in the first period to the demand for the remanufactured product is sufficiently high* ($\frac{d_n^I}{d_r^{II}} \geq \frac{w^{3/2}\delta_r\lambda_r+2\lambda_r w+8)}{\lambda_n w(\delta_n\sqrt{w}+2)}$).

c_{n1} is the threshold for the manufacturer to decide whether to use all the returns for remanufacturing or not. According to Lemma 4.3, this threshold increases as the warranty length (w) increases when the ratio ($\frac{d_n^I}{d_r^{II}}$) is sufficiently low . The reason is that the warranty length can impact both the requirement and supply for the remanufactured product and its warranty replacement. The increased warranty length leads to more replacements of the remanufactured products required in the second period (increased requirement) and more warranty returns from the failed new products in the first period (increased supply). Moreover, when the warranty length increases, the customer's utility for buying the warranty for the new and remanufactured products increases, resulting in an increase in the demand for the extended warranties for both new (increased supply) and remanufactured product (increased requirement). However, as the warranty length increases, the warranty costs for both products increase, which leads the manufacturer to hike the price for the new and remanufactured product and thus the demands both decrease (supply and requirement decrease).

When $c_n < c_{n1}$, the manufacturer will not use all returns. As c_n increases, the gap between the supply and requirement for the remanufactured product and its warranty replacement decrease and the manufacturer needs to use all returns for remanufacturing at $c_n = c_{n1}$. When the ratio $\frac{d_n^I}{d_r^{II}}$ is sufficiently small ($\frac{d_n^I}{d_r^{II}} \leq \frac{w^{3/2}\delta_r\lambda_r+2\lambda_r w+8)}{\lambda_n w(\delta_n\sqrt{w}+2)}$), increasing the warranty length results in a growing gap between the supply and requirement for the remanufactured product and its warranty replacement. Thus, the threshold c_{n1} decreases. When this ratio is sufficiently large ($\frac{d_n^I}{d_r^{II}} \geq \frac{w^{3/2}\delta_r\lambda_r+2\lambda_r w+8)}{\lambda_n w(\delta_n\sqrt{w}+2)}$), increasing the warranty length results in an imbalance between the supply and requirement for the remanufactured product and its warranty replacement (more supplies are required) when $c_n = c_{n1}$. Therefore, the threshold c_{n1} increases to achieve a new balance.

4.4.2 Case B: with retailer's extended warranty

In this case, the customer can buy the EW (EW) for the new product from either the manufacturer or the retailer. The decision sequence for the manufacturer and the retailer can be divided into three steps. In the first step, the manufacturer decides the optimal price for its EW for the new product. Then the retailer decides the optimal price for the EW of the new product in the retail's channel and finally the manufacturer determines the optimal price for the EW of the remanufactured product in the second period. The problem is solved backward.

In step 3,

$$\max_{p_{rm}^B \geq 0} \quad \sum_{i=n,r} (p_{im}^B - c_i \lambda_i w) D_{im}^{IIB} + (c_c - c_n) \lambda_n w D_{nr}^{IIB} \tag{4.8}$$

s.t.

$$(D_{nm}^{IB} + D_{nr}^{IB}) \lambda_n w \geq d_r^{II} + D_{rm}^{IIB} \lambda_r w \tag{4.9}$$

The first term, $\sum_{i=n,r}(p_{im}^B - c_i \lambda_i w) D_{im}^{IIB}$, of the objective function (4.8) represents the profit obtained by the manufacturer from selling the EW for the remanufactured and new products in the second period. The second term, $(c_c - c_n)\lambda_n w D_{nr}^{IIB}$, is the trade-in profit from the sales of the retailer's EW. Constraint (4.9) ensures that the quantity of remanufactured products and their warranty replacements in the second period does not exceed the number of returns collected in the first period.

Lemma 4.4. *The optimal price and demand (p_{rm}^B and D_{rm}^{IIB}) depends on the price of the EW for the new product in the retail channel (p_{nt}^B). Specifically,*

- *When* $p_{nt}^B \leq \beta + \delta_n \sqrt{w} - \frac{\beta d_r^{II}(\delta_r \lambda_r w^{1.5} - c_r \lambda_r^2 w^2 + w \lambda_r + 2)}{2 d_n^I \lambda_n w}$, *then* $p_{rm}^{B*} = \frac{1 + \delta_r \sqrt{w} + \lambda_r w c_r}{2}$ *and* $D_{rm}^{IIB*} = \frac{d_r^{II}(1 + \delta_r \sqrt{w} - \lambda_r w c_r)}{2}$;
- *When* $p_{nt}^M > \beta + \delta_n \sqrt{w} - \frac{\beta d_r^{II}(\delta_r \lambda_r w^{1.5} - c_r \lambda_r^2 w^2 + w \lambda_r + 2)}{2 d_n^I \lambda_n w}$, *then* $p_{rm}^{B*} = 1 + \delta_r \sqrt{w} + \frac{1}{\lambda_r w} + \frac{d_n^I \lambda_n (p_{nt}^B - \lambda_n \sqrt{w} - \beta)}{\beta d_r^{II} \lambda_r}$ *and* $D_{rm}^{IIB*} = \frac{\delta_n d_n^I \lambda_n w^{1.5} - w d_n^I (p_{nt}^B - \beta) \lambda_n - \beta d_r^{II}}{\lambda_r \beta w}$.

Lemma 4.4 shows that when the price of the EW for the new product from the retailer is sufficiently high ($p_{nt}^M > \beta + \delta_n \sqrt{w} - \frac{\beta d_r^{II}(\delta_r \lambda_r w^{1.5} - c_r \lambda_r^2 w^2 + w \lambda_r + 2)}{2 d_n^I \lambda_n w}$), the manufacturer will not receive enough returned new products for remanufacturing. Therefore, it increases the price of the EW for the remanufactured product to mitigate a negative effect on its profit.

In step 2, the retailer decides the optimal price of its EW for the new product (p_{nt}^B) to maximize its total profit over two periods.

$$\max_{p_{nt}^B \geq 0} \quad \sum_{k=I,II} (p_{nt}^B - c_c \lambda_n w) D_{nt}^{kB}. \tag{4.10}$$

The optimal price and demands for the retailer's EW in two periods obtained by solving Equation (4.10) are summarized in Lemma 4.5.

Lemma 4.5. *The optimal price p_{nt}^B and demand D_{nt}^{kB} for the retailer's EW for the new product are $\frac{c_c\lambda_n w+\beta p_{nm}^B+(1-\beta)\delta_n\sqrt{w}}{2}$ and $\frac{d_n^k(\beta p_{nm}^B-c_c\lambda_n w+(1-\beta)\delta_n\sqrt{w})}{2(1-\beta)\beta}$ respectively ($k=I,II$).*

When the price of the EW for the new product offered by the manufacturer increases, both the demands and price for the retailer's EW increase. Moreover, under condition that the price for the new product's EW is known, the optimal price for the retailer's EW increases as the warranty length increases, while the trend of the optimal demand depends on the value of the unit trade-in cost (c_c). When c_c is sufficiently low ($c_c \leq \frac{\delta_n(1-\beta)}{2\lambda_n\sqrt{w}}$), the increased warranty length leads to a decrease of the optimal demand for the retailer's EW in either the first or second period.

In step 1, the manufacturer decides the optimal price of the EW for the new product (p_{nm}^B) to maximize its profit over two periods.

$$\max_{p_{nm}^B\geq 0} \quad (p_{nm}^B - c_n\lambda_n w)D_{nm}^{IB} + \sum_{i=n,r}(p_{im}^B - c_i\lambda_i w)D_{im}^{IIB} + \sum_{k=I,II}(c_c - c_n)\lambda_n w D_{nt}^{kB}. \tag{4.11}$$

The optimal pricing strategies for Case B obtained by solving Equation (4.11) are summarized in Theorem 4.2 and illustrated in Figure 4.2.

Theorem 4.2. *Under the condition that $c_n \leq c_{n3}$ and $c_r \leq c_{r1}$, the optimal prices of the EW for the new and remanufactured products are summarized in Table 4.3.*

Table 4.3: Optimal prices and demands for the EW of the new and remanufactured products in Case B.

	$c_c \leq min(c_{c1}, c_{c2})$	$c_{c1} < c_c \leq min(c_{c3}, c_{c4})$
p_{nm}^{B*}	p_1	p_4
p_{nt}^{B*}	p_2	p_5
p_{rm}^{B*}	p_3	p_6

*p_x, c_{n3}, c_{r1}, and c_{cx} are given in Table C.4 and Table C.3 in Appendix, x=1,2,..6.

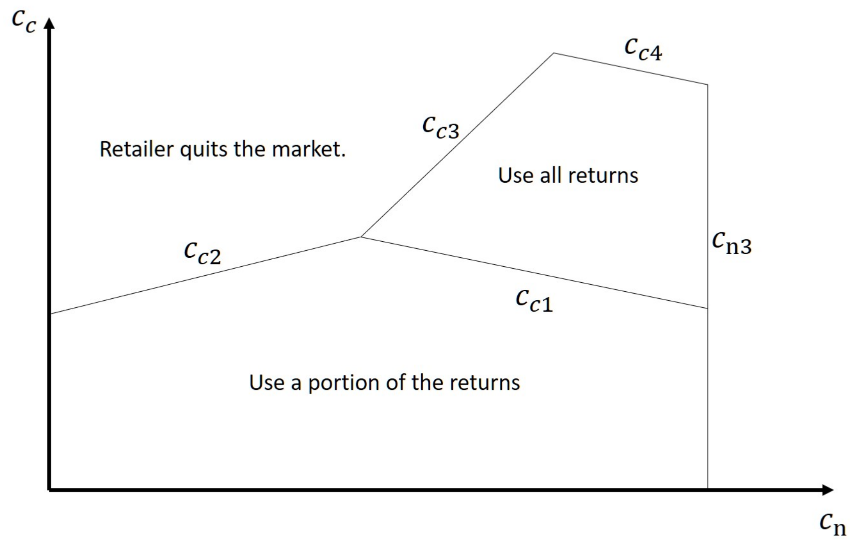


Figure 4.2: Optimal strategies for Case B

The prices in Theorem 4.2 are used to obtain the optimal demands for the new product's EW sold by the manufacturer and the retailer in both periods and for the remanufactured product's EW sold in the second period. These demands are $d_n^k d_1$, $d_n^k d_2$ and d_3 $(k = I, II)$, when $c_c \leq min(c_{c1}, c_{c2})$. When $c_{c1} < c_c \leq min(c_{c3}, c_{c4})$, the demands are $D_{nm}^{kB*} = d_n^k d_4$, $D_{nt}^{kB*} = d_n^k d_5$, and $D_{rm}^{IIB*} = d_6$ $(k = I, II)$. The value of d_x $(x = 1..6)$ can be found in Table C.3 in the Appendix.

Theorem 4.2 shows that when the unit replacement cost for the new (remanufactured) product is sufficiently high ($c_n > c_{n3}$ and $c_r > c_{r1}$), the manufacturer is not profitable selling the EW for the new (remanufactured) product. Moreover, when the unit trade-in cost is sufficiently high ($c_c > min(c_{c2}, c_{c3})$), the retailer will not sell the EW.

In general, increasing the unit trade-in cost (c_c) results in the manufacturer's optimal strategy switching from using partial returns to using all returns. The reason is that when c_c increases, the retailer needs to increase the price for the EW to cover the increased cost, resulting in a decrease in the demand for both periods. Therefore, the amount of returns in the first period decreases, which leads to a shortage of the raw materials for producing the remanufactured product and its replacement in the second period. For the retailer, increasing c_c leads to a high EW cost. Therefore, when c_c is extremely high ($c_c \geq max(c_{c2}$ and $c_{c3}))$, the retailer quits the EW market. Moreover, as c_n decreases the retailer is more likely to quit the market. This is due to the fact that, when c_n is at a low level, the manufacturer sets a low price for its EW to attract more customers. In order to compete with the manufacturer's EW, the retailer also needs to set a low price for its EW. Therefore, a little increase in c_c makes the retailer's EW unprofitable and forces the retailer to quit the market.

Lemma 4.6. *The impacts of increasing c_c on the optimal prices and demands are summarized in Table 4.4, where $k = I, II$.*

Table 4.4: Impacts of increasing c_c on the optimal prices and demands

	$c_c \leq min(c_{c1}, c_{c2})$	$c_{c1} < c_c \leq min(c_{c3}, c_{c4})$
p_{nm}^{B*}	↑	↑/↓
p_{nt}^{B*}	↑	↑
p_{rm}^{B*}	–	↑
D_{nm}^{kB*}	–	↑
D_{nt}^{kB*}	↓	↓
D_{rm}^{IIB*}	–	↓

*– for no impact; ↓ for decreasing; and ↑ for increasing

When the returns are sufficient, increasing the unit trade-in cost leads to an increase in the price for the new product's EWy sold by either the manufacturer or the retailer and a decrease in the demand for the new product's EW sold by the manufacturer, if $c_c \leq min(c_{c1}, c_{c2})$. This trend is due to the fact that the increasing trade-in cost leads to a higher replacement cost for the retailer and the profit gained by the manufacturer from the EW sold by the retailer. Therefore, the retailer increases the price of its EW and the manufacturer also increases the price of its new product's warranty to ensure that the demand for the manufacturer's EW does not change and slows down the decrease in the retailer's EW.

When the returns are not sufficient, the price for the new product's EW sold by the retailer and the price for the remanufactured product's EW sold by the manufacturer increase as the unit trade-in cost increases, if $c_{c1} < c_c \leq min(c_{c3}, c_{c4})$. In the meantime, the price for the new product's EW increases only when $d_r^{II} \geq \frac{\lambda_n^2 d_n^{I2}(1-\beta)}{2\beta\lambda_r^2(d_n^I + d_n^{II})}$.

This implies that the manufacturer takes two strategies to deal with the profit loss from the increasing trade-in cost. When the returns are not sufficient, the demand for the remanufactured product's EW is limited by the number of EW sold in the first period. The increasing trade-in cost leads to the a higher replacement cost for the retailer, which forces the retailer to increase the price that leads to the decrease in demand. Therefore, the number of returns decreases. To reduce this decrease in returns, the manufacturer increases its demand for the EW of the new product by decreasing the price (p_{nm}^B), if $d_r^{II} \leq \frac{\lambda_n^2 d_n^{I2}(1-\beta)}{2\beta\lambda_r^2(d_n^I+d_n^{II})}$, while slowing down the decrease of the retailer's demand for the new product's EW by increasing the price (p_{nm}^B), if $d_r^{II} \geq \frac{\lambda_n^2 d_n^{I2}(1-\beta)}{2\beta\lambda_r^2(d_n^I+d_n^{II})}$. The reason is that when d_r^{II} is sufficiently high, the number of returns is also high. Therefore, the increased unit trade-in cost leads to an accelerating cost for the retailer and then the price for the retailer's EW is adjusted to a higher level. The manufacturer can also significantly increase its price for the new product's EW to generate more profit in order to offset the profit loss from the trade-in service with the retailer and the sales of the remanufactured product's EW. When $d_r^{II} \leq \frac{\lambda_n^2 d_n^{I2}(1-\beta)}{2\beta\lambda_r^2(d_n^I+d_n^{II})}$, the manufacturer cannot significantly increase the price for the new product's EW to generate sufficient profit to cover the loss. Therefore, it should decrease the price to attract more customers, who originally buy the EW from the retailer, and obtain the extra profit.

4.4.3 Comparison of the two cases

In this subsection, the optimal profits from Case M and Case B are compared and results are summarized in Theorem 4.3.

Theorem 4.3. *When the failed returns are sufficient in the first period (if* $c_c \leq min(c_{c1}, c_{c2})$ *and* $c_n \leq c_{n1}$*), then*

- $\Pi_m^{M*} \geq \Pi_m^{B*}$, *if* $c_c \leq c_{c5}$ *or* $c_c \geq c_{c6}$.
- $\Pi_m^{M*} \leq \Pi_m^{B*}$, *if* $c_{c5} \leq c_c \leq c_{c6}$,

where c_{n1}*,and* c_{cx} *(where x=1,2,..6) are given in Table C.4 in Appendix.*

When the returns in the first period are sufficient for producing the remanufactured product and its warranty replacement in the second period, the introduction of the retailer's EW hurts the sales of the manufacturer's EW for the new product. However, it generates profit for the manufacturer through the trade-in service. Therefore, the impact of introducing the retailer's warranty depends on whether or not the manufacturer's profit gained by the trade-in service can offset its profit loss. According to Theorem 4.3, when the unit trade-in cost is either sufficiently small or high, the manufacturer can obtain a higher profit in Case M. It is because when the unit trade-in cost is sufficiently small, the profit gained by the manufacturer from each retailer's return is sufficiently low. When the unit trade-in cost is sufficiently high, the retailer needs to set a high selling price for the new product's EW to cover the cost, which results in a low demand. Therefore, the manufacturer is unable to obtain enough profit from the trade-in service to cover its loss.

Moreover, if the trade-in cost is an endogenous variable for the manufacturer, when $c_c = \frac{wc_n\lambda_n+\beta+\delta_n\sqrt{w}}{2\lambda_n w}$, the manufacturer can obtain its maximal profit based on Theorem 4.3. It implies that the manufacturer should encourage the retailer to sell the new product's EW by setting a reasonable trade-in cost. Then, the manufacturer can gain a higher profit, as compared to the case in which only the manufacturer sells the EW; the retailer also gains some profit from the sales of the EW, which achieves a win-win situation for both manufacturer and retailer in the SC.

Lemma 4.7. *When the failed product returns in the first period are sufficient (if $c_c \leq min(c_{c1}, c_{c2})$ and $c_n \leq c_{n1}$), then*

- $p_{nm}^{M*} \geq p_{nm}^{B*}$, *if* $w \leq \left(\frac{\delta_n+\sqrt{8\beta(c_c-0.5c_n)\lambda_n+\delta_n^2}}{2(2c_c-c_n)\lambda_n}\right)^2$;
- $p_{nm}^{M*} < p_{nm}^{B*}$, *otherwise.*

When there are sufficient returns for remanufacturing, the introduction of the retailer's EW pushes the manufacturer to decrease the price of its EW when the warranty length is sufficiently low ($w < \left(\frac{\delta_n+\sqrt{8\beta(c_c-0.5c_n)\lambda_n+\delta_n^2}}{2(2c_c-c_n)\lambda_n}\right)^2$). The reason is that the manufacturer needs to balance the profits from both the sales of the new product's EW and the trade-in service. The increased warranty length leads to an increase of the unit replacement cost for the manufacturer that decreases its profit.

However, this also results in a higher trade-in fee, that benefits the manufacturer. Therefore, when the warranty length is sufficiently high, as compared to the trade-in service, the sales of the new product's EW is less profitable for the manufacturer and then the manufacturer increases the price for the new product's EW so that more customers will buy the retailer's EW. Thus, the manufacturer can generate more profit from the trade-in service. When $w \leq \left(\frac{\delta_n+\sqrt{8\beta(c_c-0.5c_n)\lambda_n+\delta_n^2}}{2(2c_c-c_n)\lambda_n}\right)^2$, both the total trade-in fee for the EW sold by the retailer and the replacement cost for the EW sold by the manufacturer are low, as compared to the trade-in service, the sales of the new product's EW is more profitable for the manufacturer. The manufacturer decreases the price for the new product's EW to attract more customers.

Theorem 4.4. *When failed product returns in the first period are insufficient (if $c_{c1} < c_c \leq min(c_{c3}, c_{c4})$ and $c_{n1} \leq c_n \leq c_{n2}$), then*

- $\Pi_m^{M*} \geq \Pi_m^{B*}$, *if $c_c \leq c_{c7}$ or $c_c \geq c_{c8}$.*
- $\Pi_m^{M*} < \Pi_m^{B*}$, *otherwise,*

where c_{n1}, c_{n2}, and c_{cx} (where x=1,2,..8) are given in Table C.4 in Appendix.

In general, when there are insufficient returns, the introduction of the retailer's EW hurts the manufacturer's profit when the unit trade-in price is sufficiently low ($c_c \leq c_{c7}$) or sufficiently high ($c_c \geq c_{c8}$). Similar to the case with sufficient returns, when returns are insufficient, an optimal value of unit trade-in cost (c_{c9}) exists, which is between c_{c7} and c_{c8}, for the manufacturer to obtain the maximal profit. This implies that when the returns are insufficient, the manufacturer should set a moderate unit trade-in cost to entice the retailer to enter the EW market. Moreover, when c_n increases, the manufacturer has a higher probability to generate more profit in Case B than in Case M (the value interval of c_c: $c_{c8} - c_{c7}$, in which Case B generates a higher profit, decreases as c_n increases). The reason is that when c_n increases, the manufacturer has to set a higher price for the new product's EW and then the new product's EW becomes unattractive to the customers. When the retailer starts selling the new product's EW, more customers switch from buying the manufacturer' EW to buying the retailer's EW. Thus, the manufacturer suffers a huge profit loss, which can not be covered by the profit generated from the trade-in service. The implication of this result is that for industries with the high production cost (i.e., high new product's replacement cost), the unit trade-in cost will be pushed to a high level ($c_c \geq c_{c8}$). In this case, the manufacturer is better off by increasing the unit trade-in cost to over c_{c3} (see Theorem 2), such that the retailer will not have an incentive to sell the EW.

4.5 Numerical studies

In this section, the impact of the EW length (w) on the optimal prices, demands, and profits are examined. Moreover, the assumption that the manufacturer and the retailer sell the EW with the same length is relaxed. The impacts of different EW lengths on the optimal prices, demands, and profits will be examined.

4.5.1 The impact of the EW length (w) on the optimal prices, demands and the profit

In this subsection, we examine the impact of increasing the EW length on the manufacturer's and retailer's optimal decisions. The results are illustrated in Figure 4.3. The values of the parameters used in the following experiments are set as $c_n = 0.4$, $c_r = 0.2$, $d_n^I = 0.7$, $d_n^{II} = 0.5$, $d_r^{II} = 0.3$, $\lambda_n = 0.65$, $\lambda_r = 0.7$, $\delta_n = 0.7$, $\delta_r = 1$, $\beta = 0.8$, and $c_c = 0.7$. In Figure 4.3, MA refers the EW offered by the manufacturer only when all returns are used; BP refers the EW offered by both the manufacturer and the retailer when partial returns are used; and BA refers the EW offered by both the retailer and manufacturer when all returns are used.

In general, when the EW length increases, the optimal strategy for the manufacturer changes from using all the returns for remanufacturing to using a portion of the returns, and then to using all returns again. The reason is that increasing the EW length leads to an increase of the demands for the new and remanufactured product's EWs and numbers of the replacement of both products during the warranty period, when the prices stay unchanged. The number of warranty returns for each new product sold increases slower than the warranty returns of the remanufactured product within the EW period, since the remanufactured product fails more frequently due to a lower quality (i.e., higher failure rate: $\lambda_n \leq \lambda_r$). However, due to the huge difference

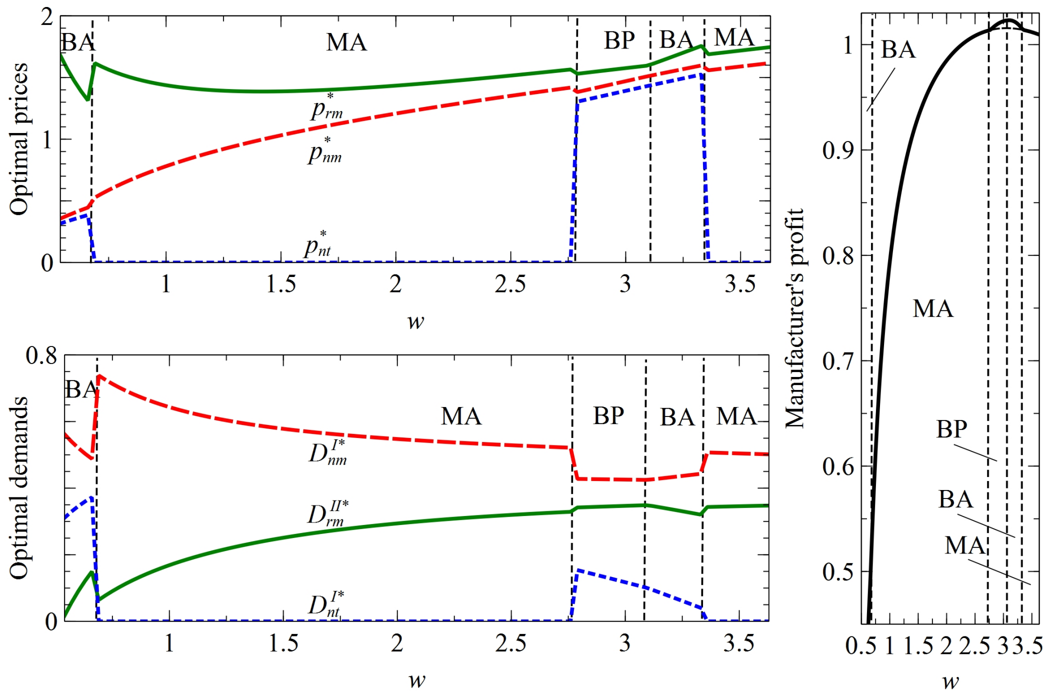


Figure 4.3: Impact of w on the optimal prices, demands, and the profits.

between the market sizes of the extended warranties for the new and for the remanufactured product (the demands for both products), the total number of returns in the first period increases faster than the number of returns required for remanufacturing (remanufactured product and its replacement) in the second period, if the EW length is sufficiently small ($w < 2.79$). Therefore, the manufacturer does not need to sacrifice the profit in the first period by setting a high selling price for the new product's EW to generate returns (D^{I*}_{nm} decreases as w increases, if $w < 2.79$.). Due to the increased warranty cost, the price for the manufacturer's EW for the new product still increases as the warranty length increases. But the increase rate slows down. When the EW length is sufficiently high ($w \geq 2.79$), the manufacturer will choose the strategy that encourages the retailer to sell the EW. Although the introduction of the retailer's EW decreases the sales of the manufacturer's EW of the new product, it attracts more customers who originally did not buy the EW. The total demand for the new product's EW in the first period increases and the manufacturer does not need to use all the returns in the second period (Strategy BP). When the EW length keeps increasing, although the demand for the new product's EW is still higher than the remanufactured product (see the red dashed line and the green solid line in the figure for the optimal demands), the increasing rate for the supply of returns from the first period is lower than the required number of returns in the second period due to the fact that more warranty replacements are needed for the remanufactured product than the new product. Therefore, when $w \geq 3.12$, the manufacturer needs to use all returns again.

Figure 4.3 also shows that when the warranty length is sufficiently short ($w \leq 0.69$) or long ($2.79 \leq w \leq 3.36$), the introduction of the retailer's EW can benefit the manufacturer. When $w \leq 0.6$, the introduction of the retailer's EW reduces the risk of shortage of returns for remanufacturing, which offsets the profit loss from the sales of the new product's EW in the first period. This generates more profit for the sales of the remanufactured product's EW, as w increases. Therefore, for the manufacturer, the profit generated from the trade-in service can cover the profit loss from the lost sales of the new product's EW due to the introduction of the retailer's EW in the market. However, when $0.69 < w < 2.79$, the manufacturer increases the price for the new product's EW to generate more profit. However, this action also decreases the competitiveness of the new product's EW from the manufacturer. If the manufacturer allows the retailer to enter the warranty selling market, the retailer can capture a significant amount of customers from the manufacturer and then the profit generated from the trade-in service cannot cover the profit loss from the sales. When $2.79 \leq w \leq 3.6$, the increasing supply of returns from the first period is lower than the required number of returns in the second period due to the fact that more warranty replacements are needed for the remanufactured products than the new products. Therefore, the manufacturer welcomes the retailer to sell the EW to relieve the shortage of returns. However, as the warranty length continues to increase, the demand for the retailer's EW decreases and when $w = 3.36$, the demand decreases to 0. Then the best strategy for the manufacturer is Case M.

Thirdly, the increased warranty length cannot always benefit the manufacturer. In Figure 4.3, there exists an optimal warranty length ($w = 3.09$) for the manufacturer to obtain the maximal profit. The increased warranty length leads to the increase of the requirement of the returns and the decrease of its supply. To satisfy all required returns, the manufacturer needs to sacrifice the profit in the first period and this sacrificed profit increases as the warranty length increases, which cannot be covered by the increased profit generated from the trade-in service when $w \geq 3.09$.

4.5.2 The impact of the retailer's EW length (w_t) on the optimal prices, demands, and the profits

In this subsection, we relax the assumption that the retailer sells the EW with the same length as that of the manufacturer. The manufacturer's warranty length is set to 1 and the impacts of varying the retailer's EW length (from 0.5 to 6) on the optimal prices, demands, and profits are summarized in Table 4.5. The values of the other parameters used in the experiment are: $c_n = 0.4$, $c_r = 0.1$, $d_n^I = 0.7$, $d_n^{II} = 0.5$, $d_r^{II} = 0.3$, $\lambda_n = 0.6$, $\lambda_r = 0.7$, $\delta_n = 0.7$, $\delta_r = 1$, $\beta = 0.8$, and $c_c = 0.6$.

Table 4.5: Optimal prices, demands, and profits as the retailer's EW length varies.

w_t	0.5	1.5	2.5	3.5	5	6
p_{nm}^*	0.474	0.579	0.676	0.791	1.043	1.315
p_{nt}^*	0.247	0.650	0.994	1.321	1.820	2.184
p_{rm}^*	1.228	1.035	1.035	1.035	1.072	1.132
D_{nm}^{I*}	0.623	0.398	0.390	0.422	0.390	0.187
D_{nm}^{II*}	0.445	0.284	0.278	0.302	0.279	0.134
D_{nt}^{I*}	0.294	0.483	0.409	0.268	0.087	0.103
D_{nt}^{II*}	0.210	0.345	0.292	0.191	0.062	0.073
D_{rm}^{II*}	0.232	0.290	0.290	0.290	0.278	0.261
Π_m^*	0.548	0.660	0.781	0.871	0.906	0.748
Π_t^*	0.034	**0.091**	0.066	0.028	0.003	0.004

The results in Table 4.5 show that the retailer can obtain a maximal profit by setting the warranty length a little bit longer than the manufacturer' EW (the manufacturer' EW is 1 and the optimal warranty length for the retailer is 1.5). Interestingly, if the retailer keeps increasing the EW length, its profit (Π_t^*) decreases. The reason is that, although the increased EW length can attract more customers to buy the retailer's EW, it also increases the warranty cost for the retailer, leading to an increase in the retailer's warranty price, which has a negative impact on the customer's utility for buying the EW. Moreover, the higher the EW length is set by the retailer, the less positive marginal effort it has on the customer's utility for buying the EW. Thus, when its positive impact cannot offset the negative impact due to the increasing price, the EW sold by the retailer is less attractive as compared to the manufacturer's EW and then the retailer's profit starts to decrease.

Moreover, the optimal production strategy for the manufacturer switches from using all returns ($w_t = 0.5$) to using partial returns ($1.5 \leq w_t \leq 3.5$) and then to using all returns again ($5 \leq w_t \leq 6$). When the retailer's EW length increases, the total demand of the new product's EW in the first period increases in the beginning and then decreases due to the increased price for the retail's EW. Therefore, when the retailer's EW is less than 0.5 or over 5, the manufacturer needs to use all returns for remanufacturing.

For the manufacturer, the increased retailer's EW length leads first to an increase and then to a decrease in profit. Due to the increased price for the retailer's EW, the manufacturer increases its price for the new product's EW and generates more profit. Moreover, it also increases the manufacturer's profit from the trade-in service, as more failures happen within the retailer's EW. When the retailer's EW length is over 5, the required number of returns is higher than its demand. To fill the gap, the manufacturer sacrifices profit in the first period to generate more returns. Moreover, the price for the remanufactured product's EW increases to reduce the required number of returns for remanufacturing. These actions contributes to the decrease in the manufacturer's profit.

4.6 Managerial implications

In this section, the managerial implications in this chapter are summarized as follows.

- **The manufacturer should carefully set the price for its EW service.** When the retailer does not sell its EW, the price for the EW sold by the manufacturer depends on the unit replacement cost of the products. The higher the replacement cost is, the higher the price for the relevant EW is. When the retailer begins to sell its own EW, the manufacturer needs to focus also on the demand for the remanufactured product and on the unit trade-in cost. When the demand for the remanufactured product is sufficiently low, the manufacturer should set the price of the EW for the new product at a low level when the unit trade-in cost is at low or extremely high levels. When the demand for the remanufactured product is sufficiently high, the price of the EW for the new product should increase continuously when the unit trade-in cost increases. For the EW of the remanufactured product, an increase of the trade-in cost leads to the increase of its price, only when the trade-in cost is at a high level.

- **The introduction of the retailer's EW will affect the manufacturer's profit depending on both the EW length and the unit trade-in cost.** When the unit trade-in cost is either sufficiently low or high, the introduction of the retailer's EW has a negative effect on the manufacturer's profit. However, when the unit trade-in cost is moderate, the manufacturer can benefit from the sales of the retailer's EW. Therefore, if the trade-in cost is an endogenous variable (can be decided by the manufacturer), in the case of either sufficient or insufficient returns, the manufacturer should set a reasonable value for the trade-in cost, which can attract the retailer to enter the warranty selling market and generate the maximum profit. For the EW length, its impact varies with the unit trade-in cost. When the warranty length is either sufficiently low or high, a higher profit is generated for the manufacturer when the retailer also sells the EW.

- **The warranty length affects the manufacturer's and the retailer's optimal decisions.** Specifically, if the EW lengths for the manufacturer and the retailer are the same, the increased warranty length results in first the increase and then the decrease of the manufacturer's profit. It implies that there is an optimal warranty length to achieve maximum profit for the manufacturer. If the retailer can set its own warranty length, it can be found that increasing its own warranty length hurts its profit. Moreover, the increased retailer's EW length can benefit the manufacturer when it is sufficiently low. However, when it is at a high level, the manufacturer's profit decreases as the retailer's EW length increases.

4.7 Conclusion

In this chapter, the optimal pricing strategies of EW (EW) for both new and remanufactured products sold by a manufacturer and a retailer are investigated by solving a two-period game theoretical model. The EW in the chapter is of the non-renewing free replacement type. The demand for the EW is derived based on the utility theory. Both market sizes (the demand for the new and remanufactured products) and the concave relationship between customer utility and warranty length are considered in the chapter. The profits of the manufacturer in both cases with and without the sales of the retailer's EW are compared and conditions under which the manufacturer should encourage the retailer to sell the EW are identified. The numerical analysis on the impacts of the EW length on optimal prices, demands, and profits for the manufacturer and the retailer are conducted.

We find that the manufacturer can benefit from the introduction of the retailer's EW when the unit trade-in cost is moderate. Moreover, the manufacturer should take different actions as the unit trade-in cost increases when returns are insufficient for remanufacturing. Numerical analysis shows that when the EW lengths for the manufacturer and the retailer are the same, the manufacturer can generate a higher profit when the retailer also sells the EW if the EW length is either sufficiently low or sufficiently high. Moreover, there exists an optimal warranty length to allow the manufacturer to maximize its profit. When the EW lengths for the manufacturer and the retailer are different, setting a long warranty length cannot benefit the retailer.

Several extensions can be investigated in future studies. Firstly, this chapter focuses on the competition between the manufacturer's and the retailer's extended warranties. The demands for the new and the remanufactured product are exogenously determined. It would be interesting to consider the competition between both products and their extended warranties and examine the impact of the EW on the products' competition. Moreover, only the non-renewing free replacement warranty is discussed in the chapter. More warranty types can be investigated in the future. Thirdly, the production of the remanufactured product and the sales of its EW are managed by the manufacturer in this chapter. Future studies can examine the case where the manufacturer outsources this business to a third party remanufacturer. The impact of the outsourcing on the optimal pricing strategies of the SC members can also be discussed.

Chapter 5

Conclusion

This dissertation explored three themes dealing with the optimal pricing, production and after-sales service strategies for new and remanufactured products. The first theme investigated the manufacturer's optimal pricing and production decisions for the new and remanufactured products in a dual-channel SC. The second theme dealt with the optimal pricing and production decisions for new and remanufactured products sold with base warranty. Finally, the third theme extended the models developed under the first two themes to explore the optimal strategies for competing after-sales extended warranty (EW) services offered by the (re)manufacturer and retailer.

Chapter 1 introduced and presented the different aspects and processes of remanufacturing. After a thorough review of the advantages of remanufacturing (economical benefits, compliance with policies and regulations, and enhancement of social and corporate image) and the obstacles to the development of the remanufacturing industry (cannibalization to the sales of the new product, uncertain returns, and low customer's perception), the motivations and research goals this dissertation were presented.

In addition to the traditional retail shops, the online shopping channel has become popular with consumers in recent years. Many manufacturers use the dual-channel structure in both the forward and reverse SC to attract more customers and increase collection rates for returns. A two-period model was developed in Chapter 2 to explore the optimal pricing strategies for new and remanufactured products sold through the online and retail channels. The optimal results showed that when there are sufficient high-quality returns for remanufacturing, a manufacturer needs to decide whether or not to engage in the remanufacturing activities based on the retail channel cost. If the retail channel cost is sufficiently low, it is not profitable for the manufacturer to produce the remanufacturing product. Moreover, the retailer may be better off with the introduction of the remanufactured product into the market. When the returns are sufficient, the introduction of the remanufactured product not only enhances the retailer's profit in the forward SC, but also generates more profit for the manufacturer (when it engages in the collecting activity for the remanufacturer) in the reverse SC. In addition, increasing the customer's perceived value on the remanufactured product can benefit both the manufacturer and the retailer.

A key concern that consumers have about remanufactured products is their quality, performance and safety, thus resulting in a lower perceived value on the remanufactured product. Therefore, many firms offer a generous warranty service to increase the customer's perceived value on the remanufactured product. Chapter 3 focused on the impact of a non-renewing warranty on a monopolistic manufacturer's optimal pricing and production strategies. We showed that if the cost saving (the ratio of the manufacturing to remanufacturing costs) for the remanufactured product is sufficiently high, remanufacturing is always a profitable activity for the manufacturer, while if the cost saving is at a moderate level, the introduction of the remanufacturing business reduces the manufacturer's profit when the warranty length for the remanufactured product is sufficiently long. Moreover, the manufacturer should carefully decide the optimal warranty length for the two products, which depends on the unit production costs, failure rates, and the sensitivity of the customer's utility to the warranty.

To further improve the post-sales service, manufacturers and retailers typically sell extended warranty (EW) services. These EW services are popular with consumers by providing additional "peace of mind" and have proven to generate significant profits for manufacturers and retailers. Chapter 4 discussed the impact of the EW for new and remanufactured products sold by the manufacturer and the retailer in the SC on their decisions. The competition between EWs for new and remanufactured products within and across channels is considered. Results show that the introduction of the EW provided by the retailer hurts the manufacturer's profit when the unit trade-in cost is either sufficiently low or high. Moreover, an optimal EW length exists that maximizes the manufacturer's profit. For the retailer, increasing its own EW length can increase its advantage in the market, but an excessive increase leads to a profit loss for both the manufacturer and the retailer.

This dissertation focus on Stage 5 (distribution, retailing and servicing) in the remanufacturing process by exploring the optimal pricing and production strategies in a dual-channel SC with base warranty, extended warranty, and product competition considerations. There are many issues in the other stages of the remanufacturing process that are worthy of investigation. For example, in the used product collecting stage, this dissertation considered adopting the dual reverse channels (collecting returns by both the manufacturer and the retailer) to ensure sufficient returned products. Moreover, it is also worthy to explore how to design an effective reverse network for collecting returns. There are many issues in this area to be discussed, such as the routing plan for the reverse logistics, the location of the remanufacturing plants, and the selection of the transportation agents and network.

At the production stage (disassembly, reprocessing, and re-assembly), this dissertation explore the optimal production strategy to assist the manufacturer in deciding whether or not it should engage in remanufacturing activities in the dual-channel supply chain with the base warranty bundled with both new and remanufactured products. The optimal disassembling sequence plan and disassembling level for the returns, that can help the manufacturer to minimize the remanufacturing cost could be further investigated. Moreover, with the development of the Internet of things (IoT), many manufacturers have started to embed sensors into their products to monitor and track product usage to facilitate remanufacturing decision-making when the products are returned (Mustajib et al., 2017). It is interesting to study how these technological developments can be leveraged to help reduce the uncertainty in the quality of returns and help improve the efficiency of remanufacturing.

At the marketing stage of remanufacturing, there are still many issues deserving to be studied. In this dissertation, optimal pricing strategies for the manufacturer and the retailer are examined in a manufacturer-dominated SC. However, the retailer can be a Stackelberg leader in the SC, such as Walmart and Costco, for increased bargaining power (Zhao et al., 2020). It is worth exploring the optimal pricing strategies for the manufacturer and the retailer in a retailer-dominated SC. Secondly, further study can focus on the idea from Chapter 2 that the manufacturer can negotiate with the retailer to achieve a cooperation agreement, with which the retailer helps the manufacturer to increase the number of returns due to its proximity with the consumer. In return, the manufacturer should share the increased profit from the increased quantity of returns with the retailer. It is worth to explore the value of the retailer's recycling activity for the manufacturer and how the profit is shared between the manufacturer and the retailer. Thirdly, in Chapters 3 and 4, the effect of the non-renewing free replacement warranty (commonly used in the literature, see Zhang et al. (2018) and Tang et al. (2020)), is studied in the closed-loop supply chain. More types of warranties can be considered in the future, such as the pro-rata warranty, the money-back guarantee warranty, and the combination of base warranties. Fourthly, the dissertation explores the optimal pricing strategy for the manufacturer in a deterministic setting. However, different from new products, an unique issue of remanufacturing is the uncertainty in the quality of returns (Yanıkoğlu and Denizel, 2021) that comes from different customers. The uncertainty results in many remanufacturing challenges, such as product design, selection of remanufacturing operations, and cost estimation (Guide Jr and Van Wassenhove, 2001). Future studies can investigate how the uncertainty in the quality of the product affects the optimal pricing and production strategy for the manufacturer.

Appendix A. Proofs for Chapter 2

Proof of Lemma 2.1

Since $\frac{\mathrm{d}^2\pi_R^N}{\mathrm{d}p_{tn}^{N2}} = -\frac{4}{1-\beta} \leq 0$ and $\frac{\mathrm{d}^2\pi_M^N}{\mathrm{d}p_{on}^{N2}} = -\frac{4}{\beta(1-\beta)} \leq 0$, π_R^N and π_M^N are concave in p_{tn}^N and p_{on}^N, respectively. Thus the optimal prices can be obtained by setting $\frac{\mathrm{d}\pi_R^N}{\mathrm{d}p_{tn}^N} = 0$ and $\frac{\mathrm{d}\pi_M^N}{\mathrm{d}p_{on}^N} = 0$, simultaneously.

Proof of Theorem 2.1

Since $\frac{\mathrm{d}^2\pi_M^N}{\mathrm{d}w_n^{N2}} = -\frac{2\beta+16}{(4-\beta)^2} \leq 0$, π_M^N is concave. We solve the equation $\frac{\mathrm{d}\pi_M^N}{\mathrm{d}w_n^N} = 0$ and obtain:

$$w_n^N = \frac{\beta^2 + (\beta+8)c_n - 8c_p + 8}{2\beta + 16}. \tag{A.1}$$

Substituting the demand obtained in Lemma 2.1 into the optimal wholesale price in Equation (A.1), we obtain the optimal demands in both channels for the new product ($j = I, II$).

$$d_{tn}^{jN} = \frac{(\beta+2)(1-c_p-\beta)}{(1-\beta)(\beta+8)} \quad and \tag{A.2}$$

$$d_{on}^{jN} = \frac{2+6c_p-\beta^2-\beta}{2(1-\beta)(8+\beta)} - \frac{c_n}{2\beta}. \tag{A.3}$$

To ensure that the new product is profitable in both channels, we check $d_{tn}^{jN} \geq 0$ and $d_{on}^{jN} \geq 0$ ($j = I, II$) , which require $c_p \leq 1-\beta$ and $c_n \leq c_{n5} = \frac{(2+6c_p-\beta^2-\beta)\beta}{(1-\beta)(\beta+8)}$.

Proof of Lemma 2.2

Since $\frac{\mathrm{d}^2\pi_M^{IIB}}{\mathrm{d}(p_{or}^B)^2} \leq 0$, π_M^{IIB} is concave in p_{or}^B and the constraint is linear. Therefore the optimal solution can be obtained by the following Karush-Kuhn-Tucker (KKT) conditions.

$$\frac{\mathrm{d}\pi_M^{IIB}}{\mathrm{d}p_{or}^B} + u_1 \frac{\mathrm{d}(\delta(d_{tn}^{IB} + d_{on}^{IB}) - d_{or}^{IIB})}{\mathrm{d}p_{or}^B} = 0, \tag{A.4}$$

$$u_1 \left(\delta(d_{tn}^{IB} + d_{on}^{IB}) - d_{or}^{IIB}\right) = 0, \tag{A.5}$$

$$\delta(d_{tn}^{IB} + d_{on}^{IB}) - d_{or}^{IIB} \geq 0, \ and \tag{A.6}$$

$$p_{or}^B, u_1 \geq 0. \tag{A.7}$$

With Equations (A.4) and (A.5), the problem can be split in two cases.

Case 1: Partially using returns for remanufacturing (i.e., $u_1 = 0$). The optimal price for the remanufactured product can be obtained by solving Equations (A.4) and (A.5).

$$p_{or}^B = \frac{(2p_{on}^B - h\delta - c_n)\alpha}{2}. \tag{A.8}$$

Substituting p_{or}^B from (A.8) into Equations (A.6) and (A.7), we obtain the conditions for the optimal solution: $p_{on}^B \leq \beta - \frac{h\delta + c_n}{2(1-\alpha)\beta}$.

Case 2: Using all returns for remanufacturing (i.e., $u_1 \neq 0$). The optimal price for the remanufactured product can be obtained by solving Equations (A.4) and (A.5).

$$p_{or}^B = ((p_{on}^B - (1-\alpha)(\beta - p_{on}^B)\delta)\alpha. \tag{A.9}$$

Substituting p_{or}^B from (A.8) into Equations (A.6) and (A.7), we obtain the conditions for the optimal solution: $p_{on}^B \geq \beta - \frac{h\delta + c_n}{2(1-\alpha)\beta}$.

With the results in Equations (A.8), (A.9), and their associated conditions, we obtain the results in Lemma 2.2.

Proof of Lemma 2.3

With the results in Lemma 2.2, there are two optimal values of p_{or}^B under different conditions. Therefore, we consider the problem under two cases.

When $p_{or}^B = \frac{(2p_{on}^B - h\delta - c_n)\alpha}{2}$, then $\frac{\mathrm{d}^2\pi_M^B}{\mathrm{d}p_{on}^{B2}} = -\frac{4}{\beta(1-\beta)} \leq 0$ and $\frac{\mathrm{d}^2\pi_R^B}{\mathrm{d}p_{tn}^{B2}} = -\frac{4}{1-\beta} \leq 0$. π_M^B

and π_R^B are concave in p_{on}^B and p_{tn}^B, respectively. We rewrite the model as:

$$\max_{p_{tn}^B \geq 0} \quad \pi_R^B = \sum_{j=I,II} \left(p_{tn}^B - w_n^B - c_p\right) d_{tn}^{jB} \quad and \tag{A.10}$$

$$\max_{p_{on}^B \geq 0} \quad \pi_M^B = \sum_{j=I,II} \left(w_n^B - c_n\right) d_{tn}^{jB} + \sum_{j=I,II} \left(p_{on}^B - c_n\right) d_{on}^{jB} + p_{or}^B d_{or}^{IIB} - h\delta \sum_{\substack{k=t,o \\ j=I,II}} d_{kn}^{jB}, \tag{A.11}$$

s.t.

$$p_{on}^B \leq \beta - \frac{h\delta + c_n}{2(1-\alpha)\delta}. \tag{A.12}$$

The optimal solutions are obtained by solving the following KKT conditions.

$$\frac{\mathrm{d}\pi_M^B}{\mathrm{d}p_{on}^B} - u_1 = 0, \tag{A.13}$$

$$\frac{\mathrm{d}\pi_R^B}{\mathrm{d}p_{tn}^B} = 0, \tag{A.14}$$

$$u_1 \left(\beta - \frac{h\delta + c_n}{2(1-\alpha)\beta} - p_{on}^B\right) = 0, \tag{A.15}$$

$$\beta - \frac{h\delta + c_n}{2(1-\alpha)\beta} - p_{on}^B \geq 0, \quad and \tag{A.16}$$

$$p_{on}^B, p_{tn}^B, u_1 \geq 0. \tag{A.17}$$

By solving Equations (A.13) to (A.17), we obtain the results: when $w_n^B \leq 1 - \frac{(4(1-\alpha)(1-\beta)\delta - \beta + 4)C}{6\delta(1-\alpha)\beta} - \frac{c_p}{3}$,

$$p_{tn}^B = \frac{2A_1 + C - (2 + C - w_n)\beta}{4-\beta} \quad and$$
$$p_{on}^B = \frac{2C + (2w_n + A_1 - 2C)\beta - \beta^2}{4-\beta}; \tag{A.18}$$

and when $w_n^B \geq 1 - \frac{(4(1-\alpha)(1-\beta)\delta-\beta+4)C}{6\delta(1-\alpha)\beta} - \frac{c_p}{3}$,

$$p_{tn}^B = \frac{A_1}{2} - \frac{C}{4(1-\alpha)\delta} \quad and$$
$$p_{on}^B = \beta - \frac{C}{2(1-\alpha)\delta}. \tag{A.19}$$

When $p_{or}^B = ((p_{on}^B - (1-\alpha)(\beta - p_{on}^B)\delta)\alpha$, then $\frac{\mathrm{d}^2\pi_M^B}{\mathrm{d}p_{on}^{B2}} = -\frac{4+2\delta^2\alpha(1-\beta)(1-\alpha)}{\beta(1-\beta)} \leq 0$ and $\frac{\mathrm{d}^2\pi_R^B}{\mathrm{d}p_{tn}^{B2}} = -\frac{4}{1-\beta} \leq 0$. π_M^B and π_R^B are concave in p_{on}^B and p_{tn}^B, respectively. In this case, the objective function is the same as Equations (A.10) and (A.11), but the constraint is changed to:

$$p_{on}^B \geq \beta - \frac{h\delta + c_n}{2(1-\alpha)\delta}. \tag{A.20}$$

The KKT conditions are also similar to those in the last case and only the conditions in (A.13) and (A.16) are changed to:

$$\frac{\mathrm{d}\pi_M^B}{\mathrm{d}p_{on}^B} + u_1 = 0 \quad and \tag{A.21}$$
$$p_{on}^B - (\beta - \frac{h\delta + c_n}{2(1-\alpha)\beta}) \geq 0. \tag{A.22}$$

By solving Equations (A.14), (A.15), (A.17), (A.21) and (A.22), we obtain the result: when $w_n^B \leq 1 - \frac{(4(1-\alpha)(1-\beta)\delta-\beta+4)C}{6\delta(1-\alpha)\beta} - \frac{c_p}{3}$,

$$p_{tn}^B = \frac{A_1}{2} - \frac{C}{4(1-\alpha)\delta} \quad and$$
$$p_{on}^B = \beta - \frac{C}{2(1-\alpha)\delta}; \tag{A.23}$$

and when $w_n^B \geq 1 - \frac{(4(1-\alpha)(1-\beta)\delta-\beta+4)C}{6\delta(1-\alpha)\beta} - \frac{c_p}{3}$,

$$p_{tn}^B = \frac{(C\delta\alpha - 2A_1A_2)(1-\beta) + (2\,C - 2\,w_n + 4)\,\beta - 2\,C - 4\,A_1}{(4\,\beta - 4)\,A_2 + 2\,\beta - 8} \quad and \tag{A.24}$$
$$p_{on}^B = \frac{C\,(1-\beta)\,\delta\,\alpha + \beta^2\,(1 + 2\,A_2) + (2\,C - 2\,A_2 - A_1 - 2\,w_n)\,\beta - 2\,C}{(1 + 2\,A_2)\,\beta - 2\,A_2 - 4},$$

where $C = h\delta + c_n$, $A_1 = 1 + w_n + c_n$, and $A_2 = \delta^2\alpha(1-\alpha)$.

With the results in Equations (A.18), (A.19), (A.23) and (A.24) and their associated conditions, we obtain the results in Lemma 2.3.

Proof of Theorem 2.2

With the results in Lemma 2.3, there are two optimal value sets of p_{tn}^B and p_{on}^B under different conditions. Therefore, we consider the problem under two cases.

When $p_{tn}^B = \frac{2A_1+C-(2+C-w_n)\beta}{4-\beta}$ and $p_{on}^B = \frac{2C+(2w_n+A_1-2C)\beta-\beta^2}{4-\beta}$, we have $\frac{\mathrm{d}^2\pi_M^B}{\mathrm{d}w_n^{B2}} \leq 0$. π_M^B is concave and becomes:

$$\max_{w_n^B\geq 0} \quad \pi_M^B = \sum_{j=I,II} \left(w_n^B - c_n\right) d_{tn}^{jB} + \sum_{j=I,II} \left(p_{on}^B - c_n\right) d_{on}^{jB} + p_{or}^B d_{or}^{II} - h\delta \sum_{\substack{k=t,o\\ j=I,II}} d_{kn}^{jB}, \tag{A.25}$$

s.t.

$$w_n^B \leq 1 - \frac{(4(1-\alpha)(1-\beta)\delta - \beta + 4)C}{6\delta(1-\alpha)\beta} - \frac{c_p}{3}. \tag{A.26}$$

The optimal solution is obtained by solving the following KKT conditions.

$$\frac{\mathrm{d}\pi_M^B}{\mathrm{d}w_n^B} - u_1 = 0, \tag{A.27}$$

$$1 - \frac{(4(1-\alpha)(1-\beta)\delta - \beta + 4)C}{6\delta(1-\alpha)\beta} - \frac{c_p}{3} - w_n^B \geq 0, \tag{A.28}$$

$$u_1\left(1 - \frac{(4(1-\alpha)(1-\beta)\delta - \beta + 4)C}{6\delta(1-\alpha)\beta} - \frac{c_p}{3} - w_n^B\right) = 0, \quad \textit{and} \tag{A.29}$$

$$w_n^B, u_1 \geq 0. \tag{A.30}$$

By solving Equations (A.27) to (A.30). We obtain the results: when $c_n \leq \frac{\delta\beta(1-\alpha)(2c_p+3\beta+6)}{(\beta+8)(1+(1-\alpha)\delta)} - h\delta$,

$$w_n^B = \frac{\beta^2 + (C+8)\beta + 8(C + A_3)}{2\beta + 16}; \tag{A.31}$$

and when $c_n \geq \frac{\delta\beta(1-\alpha)(2c_p+3\beta+6)}{(\beta+8)(1+(1-\alpha)\delta)} - h\delta$,

$$w_n^B = 1 - \frac{(4(1-\alpha)(1-\beta)\delta - \beta + 4)C}{6\delta(1-\alpha)\beta} - \frac{c_p}{3}, \tag{A.32}$$

where $C = h\delta + c_n$, $A_1 = 1 + w_n + c_n$, $A_2 = \delta^2\alpha(1-\alpha)$, and $A_3 = 1 - c_p - \beta$.

When $p_{tn}^B = \frac{(C\delta\alpha - 2A_1A_2)(1-\beta)+(2C-2w_n+4)\beta-2C-4A_1}{(4\beta-4)A_2+2\beta-8}$ and $p_{on}^B = \frac{C(1-\beta)\delta\alpha+\beta^2(1+2A_2)+(2C-2A_2-A_1-2w_n)\beta-2C}{(1+2A_2)\beta-2A_2-4}$, then $\frac{\mathrm{d}^2\pi_M^B}{\mathrm{d}w_n^{B2}} \leq 0$, π_M^B is concave. In this case, the objective function is the same as Equation (A.25), but the constraint is changed to:

$$w_n^B = 1 - \frac{(4(1-\alpha)(1-\beta)\delta - \beta + 4)C}{6\delta(1-\alpha)\beta} - \frac{c_p}{3}. \tag{A.33}$$

The KKT conditions are also similar to those in the last case and only the conditions (A.27) and (A.28) are changed to:

$$\frac{\mathrm{d}\pi_M^B}{\mathrm{d}w_n^B} + u_1 = 0 \quad \textit{and} \tag{A.34}$$

$$w_n^B - (1 - \frac{(4(1-\alpha)(1-\beta)\delta - \beta + 4)C}{6\delta(1-\alpha)\beta} - \frac{c_p}{3}) \geq 0. \tag{A.35}$$

By solving Equations (A.29), (A.30), (A.34) and (A.35), we obtain the results: when $c_n \leq \frac{\beta(1-\alpha)((A_2+2)(c_p+3-3\beta)+9\beta)\delta}{(4A_2(1-\beta)+\beta+8)(1+(1-\alpha)\delta} - h\delta$, then

$$w_n^B = 1 - \frac{(4(1-\alpha)(1-\beta)\delta - \beta + 4)C}{6\delta(1-\alpha)\beta} - \frac{c_p}{3}; \tag{A.36}$$

and when $c_n \geq \frac{\beta(1-\alpha)((A_2+2)(c_p+3-3\beta)+9\beta)\delta}{(4A_2(1-\beta)+\beta+8)(1+(1-\alpha)\delta} - h\delta$, then

$$w_n^B = \frac{(2-\alpha\delta)C}{4+2A_2} + \frac{2A_3(2+(1-\beta)A_2)}{8+\beta+4(1-\beta)A_2} + \frac{(1+A_2)\beta}{2+A_2}. \tag{A.37}$$

With the results in Equations (A.31), (A.32),(A.36), (A.37) and their associated conditions, we find that $\frac{\delta\beta(1-\alpha)(2c_p+3t\beta+6)}{(\beta+8)(1+(1-\alpha)\delta)} - h\delta \geq \frac{\beta(1-\alpha)((A_2+2)(c_p+3-3\beta)+9\beta)\delta}{(4A_2(1-\beta)+\beta+8)(1+(1-\alpha)\delta} - h\delta$ when $c_p \leq 1 - \beta$ (the necessary condition to ensure the demand for the new product in the retail channel is non-negative). This implies that $w_n^B = 1 - \frac{(4(1-\alpha)(1-\beta)\delta-\beta+4)C}{6\delta(1-\alpha)\beta} - \frac{c_p}{3}$ is not feasible.

Substituting the optimal results in (A.31) and (A.37) into the objective function in (A.25) and comparing the optimal profits in two cases, we obtain the result in Theorem 2.2.

Proof of Lemmas 2.4 and 2.5

The impacts of c_n and c_p on the optimal decision variables are examined by calculating the first order derivatives of the optimal decision variables with respect to the manufacturing cost for the new product (c_n) and the selling cost through the retailer channel (c_p).

For Strategy B :

$$\frac{\partial w_n^B}{\partial c_n} = \frac{\partial p_{tn}^B}{\partial c_n} = \frac{\partial p_{on}^B}{\partial c_n} = 0.5 \geq 0, \tag{A.38}$$

$$\frac{\partial p_{or}^B}{\partial c_n} = \frac{\partial d_{tn}^{jB}}{\partial c_n} = 0, \tag{A.39}$$

$$\frac{\partial d_{on}^{IB}}{\partial c_n} = -\frac{1}{2\beta} \leq 0, \tag{A.40}$$

$$\frac{\partial d_{on}^{IIB}}{\partial c_n} = -\frac{1}{2\beta(1-\alpha)} \leq 0, \tag{A.41}$$

$$\frac{\partial d_{or}^{IIB}}{\partial c_n} = \frac{1}{2\beta(1-\alpha)} \geq 0, \tag{A.42}$$

$$\frac{\partial \pi_M}{\partial c_n} = \frac{(2-\alpha)C}{2(1-\alpha)\beta} - 1 \leq 0, \tag{A.43}$$

$$\frac{\partial w_n^B}{\partial c_p} = -\frac{4}{\beta+8} \leq 0, \tag{A.44}$$

$$\frac{\partial p_{tn}^B}{\partial c_p} = \frac{2}{\beta+8} \geq 0, \tag{A.45}$$

$$\frac{\partial p_{on}^B}{\partial c_p} = -\frac{\beta}{\beta+8} \leq 0, \tag{A.46}$$

$$\frac{\partial p_{or}^B}{\partial c_p} = -\frac{\alpha\beta}{\beta+8} \leq 0, \tag{A.47}$$

$$\frac{\partial d_{tn}^{jB}}{\partial c_p} = -\frac{\beta+2}{(\beta+8)(1-\beta)} \leq 0, \tag{A.48}$$

$$\frac{\partial d_{on}^{jB}}{\partial c_p} = \frac{3}{(\beta+8)(1-\beta)} \geq 0, \tag{A.49}$$

$$\frac{\partial d_{or}^{IIB}}{\partial c_p} = 0, \quad and \tag{A.50}$$

$$\frac{\partial \pi_M}{\partial c_p} = -\frac{4A_3}{(\beta+8)(1-\beta)} \leq 0. \tag{A.51}$$

For Strategy BA:

$$\frac{\partial w_n^B}{\partial c_n} = \frac{\partial p_{tn}^B}{\partial c_n} = \frac{\partial p_{on}^B}{\partial c_n} = \frac{2-\alpha\delta}{4+2A_2} \geq 0, \tag{A.52}$$

$$\frac{\partial p_{or}^B}{\partial c_n} = \frac{\alpha(2-\alpha\delta)(1+\delta-\alpha\delta)}{4+2A_2} \geq 0, \tag{A.53}$$

$$\frac{\partial d_{tn}^{jB}}{\partial c_n} = 0, \tag{A.54}$$

$$\frac{\partial d_{on}^{IB}}{\partial c_n} = -\frac{2-\alpha\delta}{2\beta(2+A_2)} \leq 0, \tag{A.55}$$

$$\frac{\partial d_{on}^{IIB}}{\partial c_n} = -\frac{2-3\alpha\delta+\alpha^2\delta^2}{2\beta(2+A_2)} \leq 0, \tag{A.56}$$

$$\frac{\partial d_{or}^{IIB}}{\partial c_n} = -\frac{(2-\alpha\delta)\delta}{2\beta(2+A_2)} \leq 0, \tag{A.57}$$

$$\frac{\partial \pi_M}{\partial c_n} = -\frac{(2-\alpha\delta)(B\alpha\delta-2B+2\beta)}{2(A_2+2)\beta} \leq 0, \tag{A.58}$$

$$\frac{\partial w_n^B}{\partial c_p} = -\frac{2(2+(1-\beta)A_2)}{\beta+8+4(1-\beta)A_2} \leq 0, \tag{A.59}$$

$$\frac{\partial p_{tn}^B}{\partial c_p} = \frac{2+(1-\beta)A_2}{\beta+8+4(1-\beta)A_2} \geq 0, \tag{A.60}$$

$$\frac{\partial p_{on}^B}{\partial c_p} = -\frac{\beta}{\beta+8+4(1-\beta)A_2} \leq 0, \tag{A.61}$$

$$\frac{\partial p_{or}^B}{\partial c_p} = -\frac{\alpha\beta(1+\delta-\alpha\delta)}{\beta+8+4(1-\beta)A_2} \leq 0, \tag{A.62}$$

$$\frac{\partial d_{tn}^{jB}}{\partial c_p} = -\frac{2+\beta+(1-\beta)A_2}{(1-\beta)(\beta+8+4(1-\beta)A_2)} \leq 0, \tag{A.63}$$

$$\frac{\partial d_{on}^{IB}}{\partial c_p} = \frac{3+(1-\beta)A_2}{(1-\beta)(\beta+8+4(1-\beta)A_2)} \geq 0, \tag{A.64}$$

$$\frac{\partial d_{on}^{IIB}}{\partial c_p} = \frac{3+(1-\beta)(A_2-\alpha\delta)}{(1-\beta)(\beta+8+4(1-\beta)A_2)} \geq 0, \tag{A.65}$$

$$\frac{\partial d_{or}^{IIB}}{\partial c_p} = \frac{\delta}{\beta+8+4(1-\beta)A_2} \geq 0, \quad and \tag{A.66}$$

$$\frac{\partial \pi_M}{\partial c_p} = -\frac{2A_3(2+(1-\beta)A_2)}{(1-\beta)(\beta+8+4(1-\beta)A_2)} \leq 0. \tag{A.67}$$

Based on the derivatives in Equations (A.38) to (A.67), a positive (negative) derivative value suggests a positive (negative) relationship; a zero derivative suggests that the two parameters are independent.

Proof of Theorem 2.3

Substituting the optimal wholesale price for Theorems 2.1 and 2.2 into the corresponding objective functions (Equations (2.16) and (2.21)), respectively, we can obtain the manufacturer's optimal profits when it adopts Strategy N (π_M^N), Strategy B (π_M^B), and Strategy BA (π_M^{BA}), respectively.

Under the conditions that $c_p \leq 1-\beta$ and $c_n \leq min(c_{n2}, c_{n5})$, where both the optimal pricing strategies in Theorems 2.1 and 2.2 can be adopted by the manufacturer, $\pi_M^N - \pi_M^B \geq 0$ when $c_n \leq c_{n3}$. $\pi_M^N - \pi_M^{BA} \geq 0$ when $c_n \leq c_{n4}$.

Moreover, when $h \geq \frac{\alpha\beta\sqrt{4+2A_2}}{2\alpha\delta+2A_2}$, then $c_{n3} \geq c_{n1}$. This implies that when $h \geq \frac{\alpha\beta\sqrt{4+2A_2}}{2\alpha\delta+2A_2}$, Strategy B never becomes the optimal pricing and production strategy for the manufacturer, as Strategy B is only adopted when $c_n \leq c_{n1}$.

Proof of Theorem 2.4

Substituting the optimal selling prices in Theorems 2.1 and 2.2 into the relevant objective functions (Equation (2.13) and (2.19)), respectively, we can obtain the retailer's optimal profits when the manufacturer adopts Strategy N (π_R^N), Strategy B (π_R^B), and Strategy BA (π_R^{BA}), respectively.

Under the conditions that $c_p \leq 1-\beta$ and $c_n \leq min(c_{n2}, c_{n5})$, where both the optimal pricing strategies in Theorems 2.1 and 2.2 can be adopted by the manufacturer, we compare π_R^N to π_R^B and π_R^N to π_R^{BA} under the conditions $c_n \leq c_{n1}$ and $c_n \geq c_{n1}$, respectively. The results in Theorem 2.4 can be derived.

Threshold values for c_n

Table A.1: Threshold values for c_n

c_{n1}	$\frac{(1-\alpha)\beta\,\delta}{1+\delta\,(1-\alpha)}\left(1-\frac{A_3\sqrt{2+A_2}}{\sqrt{2(\beta+8)((1-\beta)A2+\beta/4+2)}}\right)-h\delta$
c_{n2}	$\frac{\beta\left(3-\delta\,(1-\beta)\alpha^2\right)A_3\,(2+A_2)}{2((1-\beta)A_2+\beta/4+2)(2-2\,\alpha\,\delta-A_2)(1-\beta)}-\frac{2(1-\alpha\,\delta)\beta}{2-2\,\alpha\,\delta-A_2}-h\delta$
c_{n3}	$\frac{\sqrt{2(1-\alpha)((2-\alpha)h\delta+2\,\beta\,\alpha)h\delta}-(2-\alpha)h\delta}{\alpha}$
c_{n4}	$\frac{h(2-\alpha\,\delta)^2+2\,(1+\delta\,(1-\alpha))\beta\,\alpha-\sqrt{2(2-A_2)\left(((\alpha\,\delta-2)h+\beta\,\alpha)^2+\frac{2\beta^2(1-\alpha)\alpha^2(4-3\,\alpha\,\delta+2\,\delta)A_3^2\delta}{(\beta+8)(8-4\,A_2\,\beta+4\,A_2+\beta)}\right)}}{\alpha\,(4-3\,\alpha\,\delta+2\,\delta)}$
c_{n5}	$\frac{(2+6c_p-\beta^2-\beta)\beta}{(1-\beta)(\beta+8)}$
c_{n6}	$\frac{15\left((1-\beta)A_2\left(\beta+\frac{16}{5}\right)+2/5\,\beta^2+4\,\beta+\frac{32}{5}\right)A_3{}^2\alpha\,(1-\alpha)^2\beta^2(A_2+2)\delta}{4\,(-4+\alpha^3\delta^2+(-\delta^2-2\,\delta)\alpha^2+(2\,\delta+2)\alpha)(c_{mt}-c_t)\,(\beta+8)^2((1-\beta)A_2+\beta/4+2)^2q}+\delta(1-q)(c_{mt}-c_m)$ $\frac{(A_2\,(cp-3\,\beta+3)+2\,cp+3\,\beta+6)\beta\,(1-\alpha)}{(-4+\alpha^3\delta^2+(-\delta^2-2\,\delta)\alpha^2+(2\,\delta+2)\alpha)((1-\beta)A_2+\beta/4+2)}-\delta c_{mt}$

Appendix B. Proof for Chapter 3

Proof of Lemma 3.1

The Hessian matrix (H) of Equation (3.8) is always negative semi-definite since $0 \leq \alpha \leq 1$ and $\beta \geq 0$, then the objective function (3.8) is concave (See Eiselt et al. (1987) and, Boyd and Vandenberghe (2004)).

$$H = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -\frac{2\beta}{1-\alpha} & \frac{2\beta}{1-\alpha} \\ 0 & \frac{2\beta}{1-\alpha} & -\frac{2\beta}{\alpha(1-\alpha)} \end{bmatrix} \tag{B.1}$$

Proof of Theorem 3.1

Since the objective function in (3.8) is concave and all the constraints in (3.9) and (3.10) are linear, the two period model can be solved by the Karush-Kuhn-Tucker conditions, which are listed below.

$$\frac{\partial \Pi}{\partial p_n^I} + u_1 \theta \lambda_n w \frac{\partial d_n^I}{\partial p_n^I} = 0 \tag{B.2}$$

$$\frac{\partial \Pi}{\partial p_n^{II}} - u_1(\lambda_r w + 1)\frac{\partial d_r^{II}}{\partial p_n^{II}} = 0 \tag{B.3}$$

$$\frac{\partial \Pi}{\partial p_r^{II}} - u_1(\lambda_r w + 1)\frac{\partial d_r^{II}}{\partial p_r^{II}} = 0 \tag{B.4}$$

$$u_1 \left(\theta \lambda_n w d_n^I - (\lambda_n w + 1) d_r^{II}\right) = 0 \tag{B.5}$$

$$\theta \lambda_n w d_n^I - (\lambda_n w + 1) d_r^{II} \geq 0 \tag{B.6}$$

$$p_n^I, p_n^{II}, p_r^{II}, u_1, u_2 \geq 0 \tag{B.7}$$

Based on the formulation of the demand functions in (4) and (5), the conditions above can be divided into three cases.

Case 1: Only producing new products in the second period (i.e., $d_n^{II} = \beta(1 - p_n^{II} +$

$\delta_n\sqrt{w}$) and $d_r^{II} = 0$). The optimal prices for the new product in the two periods can be obtained by solving the conditions in (B.2) to (B.7).

$$p_n^I = \frac{1 + \delta_n\sqrt{w} + C_n}{2} \tag{B.8}$$

$$p_n^{II} = \frac{1 + \delta_n\sqrt{w} + C_n}{2} \tag{B.9}$$

Based on the demand functions in (1) and (4), the optimal demands in this case are:

$$d_n^I = \frac{1 + \delta_n\sqrt{w} - C_n}{2} \tag{B.10}$$

$$d_n^{II} = \frac{\beta(1 + \delta_n\sqrt{w} - C_n)}{2} \tag{B.11}$$

Case 2: Only producing remanufactured products in the second period (i.e., $d_n^{II} = 0$ and $d_r^{II} = \beta(1 - \frac{p_r^{II} - \delta_r\sqrt{w}}{\alpha})$). The optimal prices for the new product in the two periods can be obtained by solving equations (B.2) to (B.7).

when $u_1 = 0$ ($C_r \geq 1 + \delta_n\sqrt{w} - \frac{(1+\delta_n\sqrt{w}-C_n)\theta\lambda_n w\alpha}{\beta(\lambda_r w+1)}$), then

$$p_n^{II} = \frac{1 + \delta_n\sqrt{w} + C_n}{2} \tag{B.12}$$

$$p_r^{II} = \frac{\alpha + \delta_r\sqrt{w} + C_r}{2} \tag{B.13}$$

when $u_1 > 0$ ($C_r < 1 + \delta_n\sqrt{w} - \frac{(1+\delta_n\sqrt{w}-C_n)\theta\lambda_n w\alpha}{\beta(\lambda_r w+1)}$), then

$$p_n^{II} = \frac{1 + \delta_n\sqrt{w} + C_n}{2} + \frac{\alpha{\lambda_n}^2 w^2\theta^2(1 + \delta_n\sqrt{w} - C_n) - \theta\lambda_n w(\lambda_r w + 1)\beta(\alpha + \delta_r\sqrt{w} - C_r)}{2(\lambda_r w + 1)^2\beta + 2\alpha{\lambda_n}^2 w^2\theta^2} \tag{B.14}$$

$$p_r^{II} = \frac{\alpha + \delta_r\sqrt{w} + C_r}{2} + \frac{(\lambda_r w + 1)^2\beta(\alpha + \delta_r\sqrt{w} - C_r) - \theta\lambda_n w(\lambda_r w + 1)\alpha(1 + \delta_n\sqrt{w} - C_n)}{2(\lambda_r w + 1)^2\beta + 2\alpha(1 - \alpha){\lambda_n}^2 w^2\theta^2} \tag{B.15}$$

Based on the demand functions in (1) and (4), the optimal demands in this case are:

when $u_1 = 0$ ($C_r \geq 1 + \delta_n\sqrt{w} - \frac{(1+\delta_n\sqrt{w}-C_n)\theta\lambda_n w\alpha}{\beta(\lambda_r w+1)}$), then

$$d_n^{II} = \frac{1 + \delta_n\sqrt{w} - C_n}{2} \tag{B.16}$$

$$d_r^{II} = \frac{\beta(\alpha + \delta_r\sqrt{w} - C_r)}{2\alpha} \tag{B.17}$$

when $u_1 > 0$ ($C_r < 1 + \delta_n\sqrt{w} - \frac{(1+\delta_n\sqrt{w}-C_n)\theta\lambda_n w\alpha}{\beta(\lambda_r w+1)}$), then

$$d_n^{II} = \frac{1 + \delta_n\sqrt{w} - C_n}{2} - \frac{\alpha{\lambda_n}^2 w^2\theta^2(1 + \delta_n\sqrt{w} - C_n) - \theta\lambda_n w(\lambda_r w + 1)\beta(\alpha + \delta_r\sqrt{w} - C_r)}{2(\lambda_r w + 1)^2\beta + 2\alpha{\lambda_n}^2 w^2\theta^2} \tag{B.18}$$

$$d_r^{II} = \frac{\lambda_n w\theta\beta((\lambda_r w + 1)(1 + \delta_n\sqrt{w} - C_n) + \lambda_n w\theta(\alpha + \delta_r\sqrt{w} - C_r)}{2(\lambda_r w + 1)^2\beta + 2\alpha{\lambda_n}^2 w^2\theta^2} \tag{B.19}$$

Case 3: Producing both new and remanufactured products in the second period (i.e., $d_n^{II} = \beta(1 - \frac{p_n^{II} - p_r^{II} - \delta_n\sqrt{w} + \delta_r\sqrt{w}}{1-\alpha})$ and $d_r^{II} = \beta(\frac{\left(p_n^{II} - \delta_n\sqrt{w}\right)\alpha - p_r^{II} + \delta_r\sqrt{w}}{(1-\alpha)\alpha}))$. The optimal prices for the new product in the two periods can be obtained by solving equations (B.2) to (B.7).

When $u_1 = 0$ ($C_r \geq 1 + \delta_n\sqrt{w} - \frac{(1+\delta_n\sqrt{w}-C_n)\alpha(\beta(\lambda_r w+1)+(1-\alpha)\theta\lambda_n w)}{\beta(\lambda_r w+1)}$), then

$$p_n^{I} = \frac{1 + \delta_n\sqrt{w} + C_n}{2} \tag{B.20}$$

$$p_n^{II} = \frac{1 + \delta_n\sqrt{w} + C_n}{2} \tag{B.21}$$

$$p_r^{II} = \frac{\alpha + \delta_r\sqrt{w} + C_r}{2} \tag{B.22}$$

When $u_1 > 0$ ($C_r < 1 + \delta_n\sqrt{w} - \frac{(1+\delta_n\sqrt{w}-C_n)\alpha(\beta(\lambda_r w+1)+(1-\alpha)\theta\lambda_n w)}{\beta(\lambda_r w+1)}$), then

$$\begin{aligned} p_n^{I} =& \frac{1 + \delta_n\sqrt{w} + C_n}{2} + \frac{\alpha\lambda_n w\theta(1 + \delta_n\sqrt{w} - C_n)((1-\alpha)\theta\lambda_n w}{2(\lambda_r w + 1)^2\beta + 2\alpha(1-\alpha){\lambda_n}^2 w^2\theta^2} \\ &+ \frac{(\lambda_r w + 1)\beta) - \theta\lambda_n w(\lambda_r w + 1)\beta(\alpha + \delta_r\sqrt{w} - C_r)}{2(\lambda_r w + 1)^2\beta + 2\alpha(1-\alpha){\lambda_n}^2 w^2\theta^2} \end{aligned} \tag{B.23}$$

$$p_n^{II} = \frac{1+\delta_n\sqrt{w}+C_n}{2} \tag{B.24}$$

$$p_r^{II} = \frac{\alpha+\delta_r\sqrt{w}+C_r}{2} - \frac{\alpha(\lambda_r w+1)(1+\delta_n\sqrt{w}-C_n)((1-\alpha)\theta\lambda_n w+(\lambda_r w+1)\beta)}{2(\lambda_r w+1)^2\beta+2\alpha(1-\alpha){\lambda_n}^2 w^2\theta^2} + \frac{(\lambda_r w+1)^2\beta(\alpha+\delta_r\sqrt{w}-C_r)}{2(\lambda_r w+1)^2\beta+2\alpha(1-\alpha){\lambda_n}^2 w^2\theta^2} \tag{B.25}$$

Based on the demand functions in (1) and (4), the optimal demands in this case are:

When $u_1 = 0$ ($C_r \geq 1+\delta_n\sqrt{w} - \frac{(1+\delta_n\sqrt{w}-C_n)\alpha(\beta(\lambda_r w+1)+(1-\alpha)\theta\lambda_n w)}{\beta(\lambda_r w+1)}$), then

$$d_n^{I} = \frac{1+\delta_n\sqrt{w}-C_n}{2} \tag{B.26}$$

$$d_n^{II} = \frac{\beta(1+\delta_n\sqrt{w}-C_n-(\alpha+\delta_r\sqrt{w}-C_r))}{2(1-\alpha)} \tag{B.27}$$

$$d_r^{II} = \frac{\beta(\delta_r\sqrt{w}-C_r-\alpha(\delta_n\sqrt{w}-C_n))}{2} \tag{B.28}$$

When $u_1 > 0$ ($C_r < 1+\delta_n\sqrt{w} - \frac{(1+\delta_n\sqrt{w}-C_n)\alpha(\beta(\lambda_r w+1)+(1-\alpha)\theta\lambda_n w)}{\beta(\lambda_r w+1)}$), then

$$d_n^{I} = \frac{1+\delta_n\sqrt{w}-C_n}{2} - \frac{\alpha\lambda_n w\theta(1+\delta_n\sqrt{w}-C_n)((1-\alpha)\theta\lambda_n w+(\lambda_r w+1)\beta)}{2(\lambda_r w+1)^2\beta+2\alpha(1-\alpha){\lambda_n}^2 w^2\theta^2} + \frac{\theta\lambda_n w(\lambda_r w+1)\beta(\alpha+\delta_r\sqrt{w}-C_r)}{2(\lambda_r w+1)^2\beta+2\alpha(1-\alpha){\lambda_n}^2 w^2\theta^2} \tag{B.29}$$

$$d_n^{II} = \frac{(1+\delta_n\sqrt{w}-C_n)\beta(\alpha\theta\lambda_n w(\theta\lambda_n w-(\lambda_r w+1))+(\lambda_r w+1)^2\beta)}{2(\lambda_r w+1)^2\beta+2\alpha(1-\alpha){\lambda_n}^2 w^2\theta^2} - \frac{-\alpha\beta{\lambda_n}^2 w^2\theta^2(\alpha+\delta_r\sqrt{w}-C_r)}{2(\lambda_r w+1)^2\beta+2\alpha(1-\alpha){\lambda_n}^2 w^2\theta^2} \tag{B.30}$$

$$d_r^{II} = \frac{\lambda_n w\theta\beta((\lambda_r w+1)(1+\delta_n\sqrt{w}-C_n)+\lambda_n w\theta(\alpha+\delta_r\sqrt{w}-C_r))}{2(\lambda_r w+1)^2\beta+2\alpha{\lambda_n}^2 w^2\theta^2} \tag{B.31}$$

Based on the solutions listed above (B.12 to B.31) and the profit function in (8), the optimal profit for each case can be derived. By comparing the profit in the range $C_r \geq C_{r1}$, $max(C_{r2}, C_{r3}) \leq C_r \leq C_{r1}$, $C_{r2} \leq C_r \leq C_{r4}$, $C_{r3} \leq C_r \leq C_{r5}$, and $C_r \leq min(C_{r4}, C_{r5})$, the results in Theorem 3.2 are obtained.

Proof of Lemma 3.2

With Table 2, the manufacturer produces new products in both periods only for Strategy N, B in Region II and III. In Strategy N, B in Region II, the formulation for the prices in the first and the second period are the same. For Strategy B in Region III:

$$\begin{aligned} p_n^{II} - p_n^{I} &= -\frac{\alpha\lambda_n w\theta A_3((1-\alpha)\theta\lambda_n w + N_r\beta) - \theta\lambda_n w N_r\beta A_4}{2N_r^2\beta + 2\alpha(1-\alpha){\lambda_n}^2 w^2\theta^2} \\ &= -\frac{\lambda_n w\theta(A_3\alpha((1-\alpha)\theta\lambda_n w + N_r\beta) - N_r\beta A_4)}{2N_r^2\beta + 2\alpha(1-\alpha){\lambda_n}^2 w^2\theta^2} \end{aligned} \tag{B.32}$$

For Strategy Strategy B in Region III, since $C_{r4} < C_r < C_{r2}$ and $b > \frac{\theta\lambda_n w\alpha}{N_r}$, then $A_3\alpha((1-\alpha)\theta\lambda_n w + N_r\beta) - N_r\beta A_4 \leq 0$. Thus, $p_n^{II} \geq p_n^{I}$.

Moreover, the price for the new products in the second period remains constant and equal to $\frac{C_n+1+\delta_n\sqrt{w}}{2}$ for Strategy N, B in Region II and III. Therefore, it only depends on the total unit cost, the customer's sensitivity to the warranty length and the warranty length for the new product. Thus, it is independent of any factors related to the remanufactured products.

Proof of Lemma 3.3

Based on Theorem 3.4, among all five strategies, Strategy N is the only strategy requiring the manufacturer to not produce the remanufactured product in the second period. Therefore, when $C_r \leq \delta_r\sqrt{w} - (\delta_n\sqrt{w} - C_n)\alpha$, the manufacturer should engage remanufacturing. By rearranging the condition above (using $C_n = c_n + c_n\lambda_n w$ and $C_r = c_r + c_r\lambda_r w$), we obtain:

$$(\lambda_r c_r - \lambda_n c_n\alpha)w - (\delta_r - \delta_n\alpha)\sqrt{w} + c_r - c_n\alpha \leq 0 \tag{B.33}$$

By solving the inequality in (B.33), the result in Lemma 3.3 is obtained.

Proof of Lemma 3.4

To establish how the optimal decision variables behave with respect to changes in δ_n and δ_r, the first order derivatives of the optimal prices and demands with respect

δ_n and δ_r are obtained.

For Strategy N (region I):

$$\frac{\partial p_n^I}{\partial \delta_n} = \frac{\partial p_n^{II}}{\partial \delta_n} = \frac{\partial d_n^I}{\partial \delta_n} = \frac{\partial d_n^{II}}{\partial \delta_n} = 0.5\sqrt{w} \geq 0 \tag{B.34}$$

$$\frac{\partial p_n^I}{\partial \delta_r} = \frac{\partial p_n^{II}}{\partial \delta_r} = \frac{\partial d_n^I}{\partial \delta_r} = \frac{\partial d_n^{II}}{\partial \delta_r} = 0 \tag{B.35}$$

For Strategy B (region II):

$$\frac{\partial p_n^I}{\partial \delta_n} = \frac{\partial p_n^{II}}{\partial \delta_n} = \frac{\partial d_n^I}{\partial \delta_n} = \frac{\partial p_r^{II}}{\partial \delta_r} = \sqrt{w} \geq 0 \tag{B.36}$$

$$\frac{\partial d_n^{II}}{\partial \delta_n} = \frac{\beta\sqrt{w}}{2(1-\alpha)} \geq 0 \tag{B.37}$$

$$\frac{\partial d_r^{II}}{\partial \delta_n} = \frac{\partial d_n^{II}}{\partial \delta_r} = -\frac{\beta\sqrt{w}}{2(1-\alpha)} \leq 0 \tag{B.38}$$

$$\frac{\partial d_r^{II}}{\partial \delta_r} = \frac{\beta\sqrt{w}}{2\alpha(1-\alpha)} \geq 0 \tag{B.39}$$

$$\frac{\partial p_r^{II}}{\partial \delta_n} = \frac{\partial p_n^I}{\partial \delta_r} = \frac{\partial p_n^{II}}{\partial \delta_r} = 0 \tag{B.40}$$

For Strategy B (region III):

$$\frac{\partial p_n^I}{\partial \delta_n} = \frac{\sqrt{w}}{2} + \frac{\alpha\theta\lambda_n(N_r\beta + \theta\lambda_n(1-\alpha))w^{1.5}}{2N_r^2\beta + 2\alpha(1-\alpha)\lambda_n^2 w^2\theta^2} \geq 0 \tag{B.41}$$

$$\frac{\partial p_n^{II}}{\partial \delta_n} = 0.5\sqrt{w} \geq 0 \tag{B.42}$$

$$\frac{\partial p_r^{II}}{\partial \delta_n} = -\frac{\alpha N_r\sqrt{w}(N_r\beta + \theta\lambda_n w(1-\alpha))}{2N_r^2\beta + 2\alpha(1-\alpha)\lambda_n^2 w^2\theta^2} \leq 0 \tag{B.43}$$

$$\frac{\partial p_n^I}{\partial \delta_r} = -\frac{\theta\lambda_n N_r\beta w^{1.5}}{2N_r^2\beta + 2\alpha(1-\alpha)\lambda_n^2 w^2\theta^2} \leq 0 \tag{B.44}$$

$$\frac{\partial p_n^{II}}{\partial \delta_r} = 0 \tag{B.45}$$

$$\frac{\partial p_r^{II}}{\partial \delta_r} = \frac{\sqrt{w}}{2} + \frac{\beta N_r^2\sqrt{w}}{2N_r^2\beta + 2\alpha(1-\alpha)\lambda_n^2 w^2\theta^2} \geq 0 \tag{B.46}$$

$$\frac{\partial d_n^I}{\partial \delta_n} = \frac{N_r\beta\sqrt{w}(N_r - \alpha\theta\lambda_n w)}{2N_r^2\beta + 2\alpha(1-\alpha)\lambda_n^2 w^2\theta^2} \geq 0 \tag{B.47}$$

$$\frac{\partial d_n^{II}}{\partial \delta_n} = \frac{(\alpha\lambda_n^2 w^2\theta^2 - N_r\alpha w\theta\lambda_n + N_r^2\beta)\beta\sqrt{w}}{2N_r^2\beta + 2\alpha(1-\alpha)\lambda_n^2 w^2\theta^2} \geq 0 \tag{B.48}$$

$$\frac{\partial d_r^{II}}{\partial \delta_n} = \frac{\theta\beta\lambda_n w^{1.5}(Nr - \alpha\theta\lambda_n w)}{2N_r^2\beta + 2\alpha(1-\alpha)\lambda_n^2 w^2\theta^2} \geq 0 \tag{B.49}$$

$$\frac{\partial d_n^I}{\partial \delta_r} = \frac{N_r\beta\theta\lambda_n w^{1.5}}{2N_r^2\beta + 2\alpha(1-\alpha)\lambda_n^2 w^2\theta^2} \geq 0 \tag{B.50}$$

$$\frac{\partial d_n^{II}}{\partial \delta_r} = -\frac{\alpha\beta\lambda_n^2\theta^2 w^{2.5}}{2N_r^2\beta + 2\alpha(1-\alpha)\lambda_n^2 w^2\theta^2} \leq 0 \tag{B.51}$$

$$\frac{\partial d_r^{II}}{\partial \delta_r} = \frac{\theta^2\beta\lambda_n^2 w^{2.5}}{2N_r^2\beta + 2\alpha(1-\alpha)\lambda_n^2 w^2\theta^2} \geq 0 \tag{B.52}$$

For Strategy R (region IV):

$$\frac{\partial p_n^I}{\partial \delta_n} = \frac{\partial d_n^I}{\partial \delta_n} = \frac{\partial p_r^{II}}{\partial \delta_r} = \sqrt{w} \geq 0 \tag{B.53}$$

$$\frac{\partial p_r^{II}}{\partial \delta_n} = \frac{\partial d_r^{II}}{\partial \delta_n} = \frac{\partial p_n^I}{\partial \delta_r} = \frac{\partial d_n^I}{\partial \delta_r} = 0 \tag{B.54}$$

$$\frac{\partial d_r^{II}}{\partial \delta_r} = \frac{\beta\sqrt{w}}{2\alpha} \geq 0 \tag{B.55}$$

For Strategy R (region V):

$$\frac{\partial p_n^I}{\partial \delta_n} = \frac{\sqrt{w}(N_r^2\beta + 2\alpha\lambda_n^2 w^2\theta^2)}{2N_r^2\beta + 2\alpha\lambda_n^2 w^2\theta^2} \geq 0 \tag{B.56}$$

$$\frac{\partial p_r^{II}}{\partial \delta_n} = -\frac{\theta\alpha\lambda_n N_r w^{1.5}}{2N_r^2\beta + 2\alpha\lambda_n^2 w^2\theta^2} \leq 0 \tag{B.57}$$

$$\frac{\partial p_n^I}{\partial \delta_r} = -\frac{\theta\lambda_n N_r\beta w^{1.5}}{2N_r^2\beta + 2\alpha\lambda_n^2 w^2\theta^2} \leq 0 \tag{B.58}$$

$$\frac{\partial p_r^{II}}{\partial \delta_r} = \frac{\sqrt{w}(2N_r^2\beta + \alpha\lambda_n^2 w^2\theta^2)}{2N_r^2\beta + 2\alpha\lambda_n^2 w^2\theta^2} \geq 0 \tag{B.59}$$

$$\frac{\partial d_n^I}{\partial \delta_n} = \frac{N_r^2\beta\sqrt{w}}{2N_r^2\beta + 2\alpha\lambda_n^2 w^2\theta^2} \geq 0 \tag{B.60}$$

$$\frac{\partial d_r^{II}}{\partial \delta_n} = \frac{\beta\theta N_r\lambda_n w^{1.5}}{2N_r^2\beta + 2\alpha\lambda_n^2 w^2\theta^2} \geq 0 \tag{B.61}$$

$$\frac{\partial d_n^I}{\partial \delta_r} = \frac{\theta\beta N_r\lambda_n w^{1.5}}{2N_r^2\beta + 2\alpha\lambda_n^2 w^2\theta^2} \geq 0 \tag{B.62}$$

$$\frac{\partial d_r^{II}}{\partial \delta_r} = \frac{\beta\theta^2\lambda_n^2 w^{2.5}}{2N_r^2\beta + 2\alpha\lambda_n^2 w^2\theta^2} \geq 0 \tag{B.63}$$

Based on the values in Equations (B.34) to (B.63), if a value is higher than 0, it shows a positive relationship; if a value is lower than 0, it shows a negative relationship; if a value equals zero, then the two parameters are independent.

Appendix C. Proof for Chapter 4

Proof of Lemma 4.1

Since $\frac{\mathrm{d}^2\Pi_m^{IIM}}{\mathrm{d}(p_{rm}^M)^2} = -2d_r^{II} \leq 0$, the objective function is concave. The constraint ($D_{nm}^{IM}\lambda_n w \geq d_r^{II} + D_{rm}^{IIM}\lambda_r w$) is linear and then the optimal solution of the model can be derived by solving the following Karush-Kuhn-Tucker (KKT) conditions.

$$\frac{\mathrm{d}\Pi_m^{IIM}}{\mathrm{d}(p_{rm}^M)} + u_1\frac{\mathrm{d}(D_{nm}^{IM}\lambda_n w - d_r^{II} - D_{rm}^{IIM}\lambda_r w)}{\mathrm{d}p_{rm}^M} = 0, \tag{C.1}$$

$$u_1\left(D_{nm}^{IM}\lambda_n w - d_r^{II} - D_{rm}^{IIM}\lambda_r w\right) = 0, \tag{C.2}$$

$$D_{nm}^{IM}\lambda_n w - d_r^{II} - D_{rm}^{IIM}\lambda_r w \geq 0, \ \textit{and} \tag{C.3}$$

$$p_{rm}^M, u_1 \geq 0. \tag{C.4}$$

The Equations (C.1) to (C.4) can be solved in two cases based on the value of u_1.

Case 1: $u_1 = 0$. The optimal price for the remanufactured product's EW can be obtained by solving $\frac{\mathrm{d}\Pi_m^{IIM}}{\mathrm{d}(p_{rm}^M)} = 0$, which gives $p_{rm}^{M*} = \frac{1+c_r\lambda_r w+\delta_r\sqrt{w}}{2}$. The condition of this solution is derived by checking Equation (C.3), then we obtain $p_{nm}^M \leq 1+\delta_n\sqrt{w} - \frac{(\delta_r\lambda_r w^{1.5}-\lambda_r^2 w^2 c_r+\lambda_r w+2)d_r^{II}}{2d_n^I\lambda_n w}$.

Substituting the value of p_{rm}^{M*} into the demand function $D_{rm}^{IIM} = d_r^{II}(1 - p_{rm}^M + \delta_r\sqrt{w})$, we have $D_{rm}^{IIM*} = \frac{d_r^{II}(1-c_r\lambda_r w+\delta_r\sqrt{w})}{2}$.

Case 2: $u_1 \neq 0$. The optimal price for the remanufactured product's EW can be obtained by solving Equations (C.1) and (C.2), which gives $p_{rm}^{M*} = \frac{\delta_r\lambda_r w^{1.5}+\lambda_r w+1}{\lambda_r w} - \frac{(1-p_{nm}^M+\delta_n\sqrt{w})\lambda_n d_n^I}{d_r^{II}\lambda_r}$. $u_1 \geq 0$ requires $p_{nm}^M \geq 1 + \delta_n\sqrt{w} - \frac{(\delta_r\lambda_r w^{1.5}-\lambda_r^2 w^2 c_r+\lambda_r w+2)d_r^{II}}{2d_n^I\lambda_n w}$.

Substituting the value of p_{rm}^{M*} into the demand function $D_{rm}^{IIM} = d_r^{II}(1 - p_{rm}^M + \delta_r\sqrt{w})$, which gives $D_{rm}^{IIM*} = \frac{(1-p_{nm}^M+\delta_n\sqrt{w})\lambda_n d_n^I w - d_r^{II}}{\lambda_r w}$.

With the optimal results in two cases, we obtain Lemma 4.1.

Proof of Lemma 4.2

When $p_{nm}^{M} \leq 1+\delta_n\sqrt{w}-\frac{(\delta_r\lambda_r w^{1.5}-\lambda_r^2 w^2 c_r+\lambda_r w+2)d_r^{II}}{2d_n^{I}\lambda_n w}$, the optimal price for the remanufactured product's EW is $p_{rm}^{M*}=\frac{1+c_r\lambda_r w+\delta_r\sqrt{w}}{2}$. Since $\frac{\mathrm{d}p_{rm}^{M*}}{\mathrm{d}w}=\frac{c_r\lambda_r}{2}+\frac{\delta_r}{4\sqrt{w}}\geq 0$, p_{rm}^{M*} increases as the warranty length (w) increases.

Moreover, since $\frac{\mathrm{d}^2 D_{rm}^{IIM*}}{\mathrm{d}w^2}=-\frac{d_r^{II}\delta_r}{8w^{1.5}}\leq 0$, the D_{rm}^{IIM*} can achieve its maximal value and the optimal value of w is $w=(\frac{\delta_r}{2\lambda_r c_r})^2$ by solving $\frac{\mathrm{d}D_{rm}^{IIM*}}{\mathrm{d}w}=-\frac{d_r^{II}\delta_r}{8w^{1.5}}\leq 0$.

Proof of Theorem 4.1

With Lemma 4.1, we have two optimal p_{rm}^{M*} under different conditions. Therefore, the problem can be split into two cases.

Case 1: $p_{nm}^{M}\leq 1+\delta_n\sqrt{w}-\frac{(\delta_r\lambda_r w^{1.5}-\lambda_r^2w^2c_r+\lambda_r w+2)d_r^{II}}{2d_n^{I}\lambda_n w}$. We substitute the value of p_{rm}^{M*} into the objective function and obtain $\frac{\mathrm{d}^2\Pi_m^M}{\mathrm{d}(p_{nm}^M)^2}=-2(d_n^I+d_n^{II})\leq 0$. The objective is concave and since the condition for p_{nm}^M is linear, the problem can be derived by solving the following Karush-Kuhn-Tucker (KKT) conditions.

$$\frac{\mathrm{d}\Pi_m^M}{\mathrm{d}(p_{nm}^M)}+u_1\frac{\mathrm{d}(1+\delta_n\sqrt{w}-\frac{(\delta_r\lambda_r w^{1.5}-\lambda_r^2w^2c_r+\lambda_r w+2)d_r^{II}}{2d_n^I\lambda_n w}-p_{nm}^M)}{\mathrm{d}p_{nm}^M}=0, \tag{C.5}$$

$$u_1\left(1+\delta_n\sqrt{w}-\frac{(\delta_r\lambda_r w^{1.5}-\lambda_r^2w^2c_r+\lambda_r w+2)d_r^{II}}{2d_n^I\lambda_n w}-p_{nm}^M\right)=0, \tag{C.6}$$

$$1+\delta_n\sqrt{w}-\frac{(\delta_r\lambda_r w^{1.5}-\lambda_r^2w^2c_r+\lambda_r w+2)d_r^{II}}{2d_n^I\lambda_n w}-p_{nm}^M\geq 0,\ and \tag{C.7}$$

$$p_{nm}^M, u_1\geq 0. \tag{C.8}$$

When $u_1=0$, the optimal price for the new product's EW is found by solving $\frac{\mathrm{d}\Pi_m^M}{\mathrm{d}(p_{nm}^M)}=0$, which gives $p_{nm}^{M*}=\frac{1+\delta_n\sqrt{w}+c_n\lambda_n w}{2}$. Equation (C.7) requires $c_n\leq c_{n1}$.

Substituting the value of p_{nm}^{M*} into the demand functions (D_{nm}^{IM} and D_{nm}^{IIM}), then $D_{nm}^{kM*}=\frac{d_n^k(1+\delta_n\sqrt{w}-c_n\lambda_n w)}{2}$, where $k=I,II$. Under the condition $c_n\leq c_{n1}$, the optimal demands (D_{nm}^{IM} and D_{nm}^{IIM}) are always non-negative.

When $u_1\neq 0$, the optimal price for the new product's EW is found by solving (C.5) and (C.6). Then, we have $p_{nm}^{M*}=1+\delta_n\sqrt{w}-\frac{(\delta_r\lambda_r w^{1.5}-\lambda_r^2w^2c_r+\lambda_r w+2)d_r^{II}}{2d_n^I\lambda_n w}$. $u_1\geq 0$

requires $c_n > c_{n1}$.

Case 2: $p^M_{nm} > 1 + \delta_n\sqrt{w} - \frac{(\delta_r\lambda_r w^{1.5} - \lambda_r^2 w^2 c_r + \lambda_r w + 2)d^{II}_r}{2d^I_n\lambda_n w}$. Substituting the value of p^{M*}_{rm} in Lemma 4.1 into the objective function. $\frac{\mathrm{d}^2\Pi^M_m}{\mathrm{d}(p^M_{nm})^2} = -2(d^I_n + d^{II}_n) - \frac{2d^{I2}_n\lambda^2_n}{\lambda^2_r d^{II}_r} \leq 0$ implies that the objective is concave. Since the condition for p^M_{nm} is linear, the problem can be derived by solving the following Karush-Kuhn-Tucker (KKT) conditions.

$$\frac{\mathrm{d}\Pi^M_m}{\mathrm{d}(p^M_{nm})} + u_1\frac{\mathrm{d}(p^M_{nm} - (1 + \delta_n\sqrt{w} - \frac{(\delta_r\lambda_r w^{1.5} - \lambda_r^2 w^2 c_r + \lambda_r w + 2)d^{II}_r}{2d^I_n\lambda_n w}))}{\mathrm{d}p^M_{nm}} = 0, \tag{C.9}$$

$$u_1\left(p^M_{nm} - (1 + \delta_n\sqrt{w} - \frac{(\delta_r\lambda_r w^{1.5} - \lambda_r^2 w^2 c_r + \lambda_r w + 2)d^{II}_r}{2d^I_n\lambda_n w})\right) = 0, \tag{C.10}$$

$$p^M_{nm} - (1 + \delta_n\sqrt{w} - \frac{(\delta_r\lambda_r w^{1.5} - \lambda_r^2 w^2 c_r + \lambda_r w + 2)d^{II}_r}{2d^I_n\lambda_n w}) \geq 0, \ and \tag{C.11}$$

$$p^M_{nm}, u_1 \geq 0. \tag{C.12}$$

When $u_1 = 0$, the optimal price for the new product's EW is solution of $\frac{\mathrm{d}\Pi^M_m}{\mathrm{d}(p^M_{nm})} = 0$, which gives $p^{M*}_{nm} = \delta_n\sqrt{w} - 0.5A_2 + 1 + \frac{(A_2 d^I_n\lambda_n w - d^{II}_r(A_1\lambda_r w + 2))d^I_n\lambda_n}{2(D_1 + \lambda^2_n d^{I2}_n)w}$. Equation (C.7) requires $c_n \geq c_{n1}$.

Substituting the value of p^{M*}_{nm} in this case into the demand function D^{kM}_{nm} yields the optimal demand $D^{kM*}_{nm} = \frac{(A_2 D_1 w + d^{II}_r d^I_n\lambda_n(A_1\lambda_r w + 2))d^k_n}{2(D_1 + \lambda^2_n d^{I2}_n)w}$, where $k = I, II$.

Substituting the value of p^{M*}_{nm} into Lemma 4.1, then $p^{M*}_{rm} = \frac{\delta_r\lambda_r w^{1.5} + \lambda_r w + 1}{\lambda_r w} - \frac{(A_2 D_1 w/d^{II}_r + d^I_n\lambda_n(A_1\lambda_r w + 2))d^I_n\lambda_n}{2(D_1 + \lambda^2_n d^{I2}_n)w\lambda_r}$ and $D^{IIM*}_{rm} = \frac{(A_2 D_1 w + d^{II}_r d^I_n\lambda_n(A_1\lambda_r w + 2))d^I_n\lambda_n}{2(D_1 + \lambda^2_n d^{I2}_n)w\lambda_r} - \frac{d^{II}_r}{\lambda_r w}$.

To ensure that the all demands are non-negative, it requires $c_n \leq c_{n2}$.

When $u_1 \neq 0$, the optimal price for the new product's EW is solution of (C.9) and (C.10). Then we have $p^{M*}_{nm} = 1 + \delta_n\sqrt{w} - \frac{(\delta_r\lambda_r w^{1.5} - \lambda_r^2 w^2 c_r + \lambda_r w + 2)d^{II}_r}{2d^I_n\lambda_n w}$. $u_1 \geq 0$ requires $c_n < c_{n1}$.

By combining the optimal results in two cases, we obtain Theorem 4.1.

Proof of Lemma 4.3

$\frac{\mathrm{d}c_{n1}}{\mathrm{d}w} = \frac{d^{II}_r(w^{1.5}\delta_r\lambda_r + 2\lambda_r w + 8}{2d^I_n\lambda^2_n w^2} - \frac{2 + \delta_n\sqrt{w}}{2w^2\lambda_n}$. Therefore, when $\frac{d^I_n}{d^{II}_r} \geq \frac{w^{3/2}\delta_r\lambda_r + 2\lambda_r w + 8)}{\lambda_n w(\delta_n\sqrt{w} + 2)}$, then $\frac{\mathrm{d}c_{n1}}{\mathrm{d}w} \leq 0$, suggesting that there is a negative relationship between the warranty length (w) and the threshold c_{n1}.

Proof of Lemma 4.4

Since $\frac{\mathrm{d}^2\Pi_m^{IIB}}{\mathrm{d}(p_{rm}^B)^2} = -2d_r^{II} \leq 0$, the objective function is concave. The constraint $((D_{nm}^{IB}+D_{nr}^{IB})\lambda_n w \geq d_r^{II}+D_{rm}^{IIB}\lambda_r w)$ is linear and then the optimal result of the model can be derived by solving the following Karush-Kuhn-Tucker (KKT) conditions.

$$\frac{\mathrm{d}\Pi_m^{IIB}}{\mathrm{d}(p_{rm}^B)} + u_1 \frac{\mathrm{d}((D_{nm}^{IB}+D_{nr}^{IB})\lambda_n w - d_r^{II} - D_{rm}^{IIB}\lambda_r w)}{\mathrm{d}p_{rm}^B} = 0, \tag{C.13}$$

$$u_1\left((D_{nm}^{IB}+D_{nr}^{IB})\lambda_n w - d_r^{II} - D_{rm}^{IIB}\lambda_r w\right) = 0, \tag{C.14}$$

$$(D_{nm}^{IB}+D_{nr}^{IB})\lambda_n w - d_r^{II} - D_{rm}^{IIB}\lambda_r w \geq 0, \ and \tag{C.15}$$

$$p_{rm}^B, u_1 \geq 0. \tag{C.16}$$

Equations (C.13) to (C.16) can be solved in two cases based on the value of u_1.

Case 1: $u_1 = 0$. The optimal price for the remanufactured product's EW can be obtained by solving $\frac{\mathrm{d}\Pi_m^{IIB}}{\mathrm{d}(p_{rm}^B)} = 0$, which gives $p_{rm}^{B*} = \frac{1+\delta_r\sqrt{w}+\lambda_r w c_r}{2}$. Equation (C.15) requires $p_{nt}^B \leq \beta + \delta_n\sqrt{w} - \frac{\beta d_r^{II}(\delta_r\lambda_r w^{1.5} - c_r\lambda_r^2 w^2 + w\lambda_r + 2)}{2d_n^I\lambda_n w}$.

Substituting the value of p_{rm}^{B*} into the demand function D_{rm}^{IIB}, the optimal demand is this case is $D_{rm}^{IIB*} = \frac{d_r^{II}(1+\delta_r\sqrt{w}-\lambda_r w c_r)}{2}$.

Case 2: $u_1 \neq 0$. The optimal price for the remanufactured product's EW is found by solving (C.13) and (C.14). Then we have $p_{rm}^{B*} = 1+\delta_r\sqrt{w} + \frac{1}{\lambda_r w} + \frac{d_n^I\lambda_n(p_{nt}^B - \lambda_n\sqrt{w}-\beta)}{\beta d_r^{II}\lambda_r}$. $u_1 \geq 0$ requires $p_{nt}^M > \beta + \delta_n\sqrt{w} - \frac{\beta d_r^{II}(\delta_r\lambda_r w^{1.5} - c_r\lambda_r^2 w^2 + w\lambda_r + 2)}{2d_n^I\lambda_n w}$.

Substituting the value of p_{rm}^{B*} into the demand function D_{rm}^{IIB} yields the optimal demand $D_{rm}^{IIB*} = \frac{\delta_n d_n^I\lambda_n w^{1.5} - w d_n^I(p_{nt}^B-\beta)\lambda_n - \beta d_r^{II}}{\lambda_r\beta w}$.

With the optimal results in two cases, we obtain Lemma 4.4.

Proof of Lemma 4.5

Since $\frac{\mathrm{d}^2\Pi_t^{IIB}}{\mathrm{d}(p_{nt}^B)^2} = -\frac{2(d_n^I+d_n^{II})}{\beta(1-\beta}) \leq 0$, the objective function is concave. The optimal value of p_{nt}^B can be obtained by solving $\frac{\mathrm{d}\Pi_t^{IIB}}{\mathrm{d}(p_{nt}^B)} = 0$. Therefore, $p_{nt}^{B*} = \frac{c_c\lambda_n w + \beta p_{nm}^B + (1-\beta)\delta_n\sqrt{w}}{2}$

and $D_{nt}^{kB*} = \frac{d_n^k(\beta p_{nm}^B - c_c\lambda_n w + (1-\beta)\delta_n\sqrt{w})}{2(1-\beta)\beta}$ $(k = I, II)$.

Proof of Theorem 4.2

With Lemma 4.4, we have two optimal results for the value of p_{rm}^{B*} under the different conditions. Therefore, we discuss two cases.

Case 1: $p_{rm}^{B*} = \frac{1+\delta_r\sqrt{w}+\lambda_r w c_r}{2}$. The optimal result is obtained when $p_{nt}^B \leq \beta + \delta_n\sqrt{w} - \frac{\beta d_r^{II}(\delta_r\lambda_r w^{1.5} - c_r\lambda_r^2 w^2 + w\lambda_r + 2)}{2d_n^I\lambda_n w}$. With Lemma 4.4 we have $p_{nt}^{B*} = \frac{c_c\lambda_n w + \beta p_{nm}^B + (1-\beta)\delta_n\sqrt{w}}{2}$. The condition for Case 1 can be rewritten as

$$p_{nm}^B \leq \frac{((\beta-1)\delta_n d_n^I\lambda_n - \delta_r d_r^{II}\lambda_r\beta)w^{1.5} + ((c_r\lambda_r^2 w^2 - \lambda_r w - 2)d_r^{II} + 2d_n^I\lambda_n w)\beta - c_c\lambda_n^2 w^2 d_n^I}{\beta d_n^I\lambda_n w}. \quad \text{(C.17)}$$

We substitute the value of p_{rm}^{B*} and p_{nt}^{B*} into the objective function and obtain $\frac{\mathrm{d}^2\Pi_m^B}{\mathrm{d}(p_{nm}^B)^2} = \frac{(2-\beta)(d_n^I + d_n^{II})}{\beta-1} \leq 0$. The objective is concave and since the condition for p_{nm}^B is linear, the problem can be derived by solving the following Karush-Kuhn-Tucker (KKT) conditions.

$$\frac{\mathrm{d}\Pi_m^B}{\mathrm{d}(p_{nm}^B)} - u_1 = 0, \quad \text{(C.18)}$$

$$u_1\left(2 - p_{nm}^B - \frac{c_c\lambda_n w}{\beta} + \frac{((\beta-1)\delta_n d_n^I\lambda_n - \delta_r d_r^{II}\lambda_r\beta)w^{1.5} + (c_r\lambda_r^2 w^2 - \lambda_r w - 2)d_r^{II}\beta}{\beta d_n^I\lambda_n w}\right) = 0, \quad \text{(C.19)}$$

$$2 - p_{nm}^B - \frac{c_c\lambda_n w}{\beta} + \frac{((\beta-1)\delta_n d_n^I\lambda_n - \delta_r d_r^{II}\lambda_r\beta)w^{1.5} + (c_r\lambda_r^2 w^2 - \lambda_r w - 2)d_r^{II}\beta}{\beta d_n^I\lambda_n w} \geq 0, \textit{ and} \quad \text{(C.20)}$$

$$p_{nm}^B, u_1 \geq 0. \quad \text{(C.21)}$$

When $u_1 = 0$, the optimal price for the new product's EW is solution of $\frac{\mathrm{d}\Pi_m^B}{\mathrm{d}(p_{nm}^B)} = 0$, which gives $p_{nm}^{B*} = \frac{\lambda_n w(\beta c_n - 2c_c - c_n) - (\delta_n\sqrt{w}+2)(1-\beta)}{2(\beta-2)}$. Equation (C.20) requires $c_c \leq c_{c1}$.

Substituting the value of p_{nm}^{B*} in this case into the demand function D_{nm}^{kB} yields the optimal demand $D_{nm}^{kB*} = d_n^k(\frac{2+\delta_n\sqrt{w}-wc_n\lambda_n}{4})$, where $k = I, II$. Non-negativity of D_{nm}^{IB*} and D_{nm}^{IIB*} requires $c_n \leq c_{n3}$.

When $u_1 \neq 0$, the optimal price for the new product's EW is a solution of (C.18) and (C.19). Then we have $p_{nm}^{B*} = \frac{((\beta-1)\delta_n d_n^I\lambda_n - \delta_r d_r^{II}\lambda_r\beta)w^{1.5}+((c_r\lambda_r^2w^2-\lambda_r w-2)d_r^{II}+2d_n^I\lambda_n w)\beta}{\beta d_n^I\lambda_n w} - \frac{c_c\lambda_n^2w^2d_n^I}{\beta d_n^I\lambda_n w}$. $u_1 \geq 0$ requires $c_c > c_{c1}$.

Case 2: $p_{rm}^{B*} = 1 + \delta_r\sqrt{w} + \frac{1}{\lambda_r w} + \frac{d_n^I\lambda_n(p_{nt}^B - \lambda_n\sqrt{w}-\beta)}{\beta d_r^{II}\lambda_r}$. The optimal result is obtained when $p_{nt}^B \geq \beta + \delta_n\sqrt{w} - \frac{\beta d_r^{II}(\delta_r\lambda_r w^{1.5} - c_r\lambda_r^2 w^2 + w\lambda_r + 2)}{2d_n^I\lambda_n w}$. Following Lemma 4.4, we have $p_{nt}^{B*} = \frac{c_c\lambda_n w + \beta p_{nm}^B + (1-\beta)\delta_n\sqrt{w}}{2}$. The condition for Case 2 can be rewritten as

$$p_{nm}^B \geq \frac{((\beta-1)\delta_n d_n^I\lambda_n - \delta_r d_r^{II}\lambda_r\beta)w^{1.5} + ((c_r\lambda_r^2w^2 - \lambda_r w - 2)d_r^{II} + 2d_n^I\lambda_n w)\beta - c_c\lambda_n^2w^2d_n^I}{\beta d_n^I\lambda_n w} \tag{C.22}$$

We substitute the value of p_{rm}^{B*} and p_{nt}^{B*} into the objective function and obtain $\frac{\mathrm{d}^2\Pi_m^B}{\mathrm{d}(p_{nm}^B)^2} = \frac{(\beta-2)(d_n^I+d_n^{II})}{1-\beta} - \frac{d_n^{I2}\lambda_n^2}{2d_r^{II}\lambda_r^2} \leq 0$. The objective is concave and since the condition for p_{nm}^B is linear, the problem can be derived by solving the following Karush-Kuhn-Tucker (KKT) conditions.

$$\frac{\mathrm{d}\Pi_m^B}{\mathrm{d}(p_{nm}^B)} + u_1 = 0, \tag{C.23}$$

$$u_1\left(p_{nm}^B + \frac{c_c\lambda_n w}{\beta} - 2 - \frac{((\beta-1)\delta_n d_n^I\lambda_n - \delta_r d_r^{II}\lambda_r\beta)w^{1.5} + (c_r\lambda_r^2w^2 - \lambda_r w - 2)d_r^{II}\beta}{\beta d_n^I\lambda_n w}\right) = 0, \tag{C.24}$$

$$p_{nm}^B + \frac{c_c\lambda_n w}{\beta} - 2 - \frac{((\beta-1)\delta_n d_n^I\lambda_n - \delta_r d_r^{II}\lambda_r\beta)w^{1.5} + (c_r\lambda_r^2w^2 - \lambda_r w - 2)d_r^{II}\beta}{\beta d_n^I\lambda_n w} \geq 0, \textit{ and} \tag{C.25}$$

$$p_{nm}^B, u_1 \geq 0. \tag{C.26}$$

When $u_1 = 0$, the optimal price for the new product's EW is the solution of $\frac{\mathrm{d}\Pi_m^B}{\mathrm{d}(p_{nm}^B)} = 0$, which gives $p_{nm}^{B*} = \frac{(1-\beta)(D_1\beta\delta_n + (d_n^I\delta_n(\beta+1)\lambda_n - d_r^{II}\beta\delta_r\lambda_r)d_n^I\lambda_n)w^{1.5} - A_3}{w\beta(d_n^{I2}\lambda_n^2(1-\beta)+2D_1(2-\beta))} - \frac{c_c\lambda_n w}{\beta}$. Equation (C.25) requires $c_c \geq c_{c1}$.

Substituting the value of p_{nm}^{B*} in this case into the demand function D_{nm}^{kB} yields

$D_{nm}^{kB*} = d_n^k(1+\frac{\delta_n\sqrt{w}}{2}+\frac{c_c\lambda_n w}{\beta(1-\beta)}-\frac{(2-\beta)((1-\beta)(D_1\beta\delta_n+(d_n^I\delta_n(\beta+1)\lambda_n-d_r^{II}\beta\delta_r\lambda_r)d_n^I\lambda_n)w^{1.5}-A_3)}{2\beta w(1-\beta)(d_n^{I2}\lambda_n^2(1-\beta)+2D_1(2-\beta))})$, where $k=I,II$.

Substituting the value of p_{nm}^{B*} into Lemma 4.1, the optimal price and demand for the remanufactured product's EW are $p_{rm}^{B*} = \frac{\delta_r\lambda_r w^{1.5}+\lambda_r w+1}{\lambda_r w}-\frac{(A_2D_1w/d_r^{II}+d_n^I\lambda_n(A_1\lambda_r w+2))d_n^I\lambda_n}{2(D_1+\lambda_n^2 d_n^{I2})w\lambda_r}$ and $D_{rm}^{IIB*} = \frac{(A_2D_1w+d_r^{II}d_n^I\lambda_n(A_1\lambda_r w+2))d_n^I\lambda_n}{2(D_1+\lambda_n^2 d_n^{I2})w\lambda_r} - \frac{d_r^{II}}{\lambda_r w}$. Under the condition $c_c \geq c_{c1}$, this demand is always positive.

When $u_1 \neq 0$, the optimal price for the new product's EW is solution of (C.23) and (C.24). Then we have $p_{nm}^{B*} = \frac{((\beta-1)\delta_n d_n^I\lambda_n-\delta_r d_r^{II}\lambda_r\beta)w^{1.5}}{\beta d_n^I\lambda_n w}+\frac{((c_r\lambda_r^2w^2-\lambda_r w-2)d_r^{II}+2d_n^I\lambda_n w)\beta-c_c\lambda_n^2w^2d_n^I}{\beta d_n^I\lambda_n w}$. $u_1 \geq 0$ requires $c_c < c_{c1}$.

By combining the optimal results in two cases, we obtain the optimal pricing strategies for the manufacturer's EW of the new product in Theorem 4.2. With Lemmas 4.4 and 4.5, we obtain the other optimal prices and demands in Theorem 4.2.

Proof of Lemma 4.6

The impacts of increasing c_c on the optimal prices and demands depend on the first derivatives with respect to c_c. A positive (negative) derivative suggests a positive (negative) relationship. A value of zero implies that the optimal price (demand) does not depend on the unit trade-in cost.

Table C.2: The values for the first derivatives of the optimal prices and demands with respect to c_c.

	$c_c \leq min(c_{c1}, c_{c2})$	$c_{c1} < c_c \leq min(c_{c3}, c_{c4})$
p_{nm}^{B*}	$\frac{w\lambda_n}{2-\beta}$	$\frac{w\lambda_n(2\lambda_r^2\beta d_r^{II}(d_n^I+d_n^{II})+d_n^{I2}\lambda_n^2(\beta-1))}{\beta(2\lambda_r^2(2-\beta)(d_n^I+d_n^{II})d_r^{II}+d_n^{I2}\lambda_n^2(1-\beta))}$
p_{nt}^{B*}	$\frac{w\lambda_n}{2-\beta}$	$\frac{2w\lambda_n(d_n^I+d_n^{II})\lambda_r^2 d_r^{II}}{2d_r^{II}(2-\beta)(d_n^I+d_n^{II})\lambda_r^2+d_n^{I2}\lambda_n^2(1-\beta)}$
p_{rm}^{B*}	0	$\frac{2w\lambda_n^2 d_n^I\lambda_r(d_n^I+d_n^{II})}{\beta(2d_r^{II}(2-\beta)(d_n^I+d_n^{II})\lambda_r^2+d_n^{I2}\lambda_n^2(1-\beta))}$
D_{nm}^{kB*}	0	$\frac{w\lambda_n^3 d_n^{I3}}{\beta(2d_r^{II}(2-\beta)(d_n^I+d_n^{II})\lambda_r^2+d_n^{I2}\lambda_n^2(1-\beta))}$
D_{nt}^{kB*}	$-\frac{d_n^I\lambda_n w}{\beta(2-\beta)}$	$-\frac{2\lambda_r^2 d_r^{II}d_n^I w\lambda_n(d_n^I+d_n^{II})+w\lambda_n^3 d_n^{I3}}{\beta(2d_r^{II}(2-\beta)(d_n^I+d_n^{II})\lambda_r^2+d_n^{I2}\lambda_n^2(1-\beta))}$
D_{rm}^{IIB*}	0	$-\frac{2\lambda_r d_r^{II}d_n^I w\lambda_n^2(d_n^I+d_n^{II})}{\beta(2d_r^{II}(2-\beta)(d_n^I+d_n^{II})\lambda_r^2+d_n^{I2}\lambda_n^2(1-\beta))}$

where $k = I, II$.

Proof of Theorems 4.3 and 4.4

The results in Theorems 4.3 and 4.4 are obtained by comparing the results in Theorems 4.1 and 4.2 under the conditions of sufficiently and insufficient returns respectively. Under the condition that $c_c \leq min(c_{c1}, c_{c2})$ and $c_n \leq c_{n1}$, when $c_c \leq c_{c5}$ or $c_c \geq c_{c6}$, then $\Pi^{M*} \geq \Pi^{B*}$ and when $c_{c5} \leq c_c \leq c_{c6}$, then $\Pi^{M*} \leq \Pi^{B*}$. Under the condition that $c_{c1} < c_c \leq min(c_{c3}, c_{c4})$ and $c_{n1} \leq c_n \leq c_{n2}$, when $c_c \leq c_{c7}$ or $c_c \geq c_{c8}$, then $\Pi^{M*} \geq \Pi^{B*}$, and when $c_{c7} < c_c < c_{c8}$, then $\Pi^{M*} < \Pi^{B*}$.

Moreover, for the case with sufficient returns, the optimal profit can be obtained by solving $\frac{\mathrm{d}\Pi}{\mathrm{d}c_c} = 0$. We have $\frac{\mathrm{d}\Pi^M}{\mathrm{d}c_c} = 0$ and $\frac{\mathrm{d}\Pi^B}{\mathrm{d}c_c}$ equals to 0 if $c_c = \frac{wc_n\lambda_n+\beta+\delta_n\sqrt{w}}{2\lambda_n w}$. Moreover, since $\frac{\mathrm{d}^2\Pi^B}{\mathrm{d}c_c^2} \leq 0$ and $c_{c5} \leq \frac{wc_n\lambda_n+\beta+\delta_n\sqrt{w}}{2\lambda_n w} \leq c_{c6}$, the optimal profit for the manufacturer with sufficient returns achieves at $c_c = \frac{wc_n\lambda_n+\beta+\delta_n\sqrt{w}}{2\lambda_n w}$.

For the case with insufficient returns, the optimal profit can be obtained by solving $\frac{\mathrm{d}\Pi}{\mathrm{d}c_c} = 0$. We have $\frac{\mathrm{d}\Pi^M}{\mathrm{d}c_c} = 0$ and $\frac{\mathrm{d}\Pi^B}{\mathrm{d}c_c} = 0$ if $c_c = c_{c9}$. Moreover, since $\frac{\mathrm{d}^2\Pi^B}{\mathrm{d}c_c^2} \leq 0$ and $c_{c7} \leq c_{c9} \leq c_{c8}$, the optimal profit for the manufacturer with sufficient returns reaches at $c_c = c_{c9}$.

Proof of Lemma 4.7

The result in Lemma 4.7 is obtained by comparing the optimal prices for the new product's EW from the manufacturer in Theorems 4.1 and 4.2 under the condition that $c_c \leq min(c_{c1}, c_{c2})$ and $c_n \leq c_{n1}$. we can find that $p_{nm}^{M*} - p_{nm}^{B*} \geq 0$ if $w \leq \left(\frac{\delta_n+\sqrt{8\beta(c_c-0.5c_n)\lambda_n+\delta_n^2}}{2(2c_c-c_n)\lambda_n}\right)^2$.

Threshold values used in the Chapter 4

Table C.3: The values of the optimal prices and demands in Theorem 4.2

p_1	$\frac{\lambda_n w(\beta c_n - 2c_c - c_n) - (\delta_n\sqrt{w}+2)(1-\beta)}{2(\beta-2)}$
p_2	$\frac{\delta_n(1-\beta)(4-\beta)\sqrt{w}+(wc_n\lambda_n+2)\beta(1-\beta)+4c_c\lambda_n w}{4(2-\beta)}$
p_3	$\frac{1+\delta_r\sqrt{w}+\lambda_r w c_r}{2}$
p_4	$\frac{(1-\beta)(D_1\beta\delta_n+(d_n^I\delta_n(\beta+1)\lambda_n-d_r^{II}\beta\delta_r\lambda_r)d_n^I\lambda_n)w^{1.5}-A_3}{w\beta(d_n^{I2}\lambda_n^2(1-\beta)+2D_1(2-\beta))} - \frac{c_c\lambda_n w}{\beta}$
p_5	$\frac{(1-\beta)(D_1\beta\delta_n+(d_n^I\delta_n(\beta+1)\lambda_n-d_r^{II}\beta\delta_r\lambda_r)d_n^I\lambda_n)w^{1.5}-A_3}{2w(d_n^{I2}\lambda_n^2(1-\beta)+2D_1(2-\beta))} + \frac{(1-\beta)\delta_n\sqrt{w}}{2}$
p_6	$\frac{\lambda_n d_n^I((D_1\delta_n(\beta^2-\beta-4)+d_n^I\beta\delta_r\lambda_n(\beta-1)\lambda_r d_r^{II})w^{1.5}-A_3)}{2w\beta d_r^{II}\lambda_r(d_n^{I2}\lambda_n^2(1-\beta)+2D_1(2-\beta))} + \frac{1+\lambda_r w+\delta_r\lambda_r w^{1.5}}{\lambda_r w} - \frac{d_n^I\lambda_n}{d_r^{II}\lambda_r}$
d_1	$\frac{2+\delta_n\sqrt{w}-wc_n\lambda_n}{4}$
d_2	$\frac{\delta_n\sqrt{w}(4-\beta)+c_n\lambda_n w\beta+2\beta-4c_c\lambda_n w}{4\beta(\beta-2)}$
d_3	$\frac{d_r^{II}(1+\delta_r\sqrt{w}-\lambda_r w c_r)}{2}$
d_4	$1+\frac{\delta_n\sqrt{w}}{2}+\frac{c_c\lambda_n w}{\beta(1-\beta)} - \frac{(2-\beta)((1-\beta)(D_1\beta\delta_n+(d_n^I\delta_n(\beta+1)\lambda_n-d_r^{II}\beta\delta_r\lambda_r)d_n^I\lambda_n)w^{1.5}-A_3)}{2\beta w(1-\beta)(d_n^{I2}\lambda_n^2(1-\beta)+2D_1(2-\beta))}$
d_5	$\frac{\delta_n\sqrt{w}}{2\beta} - \frac{c_c\lambda_n w}{\beta(1-\beta)} + \frac{(1-\beta)(D_1\beta\delta_n+(d_n^I\delta_n(\beta+1)\lambda_n-d_r^{II}\beta\delta_r\lambda_r)d_n^I\lambda_n)w^{1.5}-A_3}{2\beta w(1-\beta)(d_n^{I2}\lambda_n^2(1-\beta)+2D_1(2-\beta))}$
d_6	$d_r^{II}(\frac{w\lambda_n d_n^I-d_r^{II}}{\lambda_r w d_r^{II}} - \frac{\lambda_n d_n^I((D_1\delta_n(\beta^2-\beta-4)+d_n^I\beta\delta_r\lambda_n(\beta-1)\lambda_r d_r^{II})w^{1.5}-A_3)}{2w\beta d_r^{II}\lambda_r(d_n^{I2}\lambda_n^2(1-\beta)+2D_1(2-\beta))})$

$^*A_3 = (d_n^I\lambda_n((c_r\lambda_r^2 w^2 - \lambda_r w - 2)d_r^{II} + 2d_n^I\lambda_n w) + D_1 w(wc_n\lambda_n + 2))\beta(\beta-1) - 4c_c\lambda_n D_1 w^2$ and $D_1 = (d_n^I + d_n^{II})d_r^{II}\lambda_r^2$

Table C.4: The threshold values of c_n, c_r, and c_c.

	Critical value
c_{n1}	$\frac{\delta_n w+\sqrt{w}}{\lambda_n w^{1.5}} - \frac{\lambda_r d_r^{II}(\delta_r w+\sqrt{w})}{d_n^I \lambda_n^2 w^{1.5}} + \frac{d_r^{II}(\lambda_r^2 w^2 c_r-2)}{d_n^I \lambda_n^2 w^2}$
c_{n2}	$\frac{w^{1.5}\delta_n d_n^I \lambda_n + d_n^I \lambda_n w - 2d_r^{II}}{w^2 d_n^I \lambda_n^2} + \frac{1+\delta_r\sqrt{w}-\lambda_r w c_r)d_n^I}{w\lambda_r(d_n^I+d_n^{II})}$
c_{n3}	$\frac{2+\delta_n\sqrt{w}}{w\lambda_n}$
c_{r1}	$\frac{1+\delta_r\sqrt{w}}{w\lambda_r}$
c_{c1}	$\frac{d_r^{II}\beta(\beta-2)(w^{1.5}\delta_r\lambda_r-\lambda_r^2 c_r w^2+\lambda_r w+2)}{2\lambda_n^2 w^2 d_n^I} + \frac{(\beta-1)\beta c_n}{4} + \frac{6\beta-2\beta^2-\delta_n(\beta^2-\beta-4)\sqrt{w}}{4\lambda_n w}$
c_{c2}	$\frac{\beta(2+wc_n\lambda_n)+\delta_n(4-\beta)\sqrt{w}}{4w\lambda_n}$
c_{c3}	$\frac{\beta\lambda_r^2 d_r^{II}((c_n+c_r)d_n^I+c_n d_n^{II})}{2d_n^{I2}\lambda_n^2+4\lambda_r^2 d_r^{II}(d_n^I+d_n^{II})} + \frac{\delta_n\sqrt{w}+\beta}{\lambda_n w} - \frac{d_r^{II}\beta(\lambda_r w^{1.5}(\delta_n(d_n^I+d_n^{II}\lambda_r+\lambda_n d_n^I\delta_r))+2w(d_n^I+d_n^{II})\lambda_r^2+\lambda_n d_n^I(w\lambda_r+2))}{2\lambda_n w^2(\lambda_r^2 d_r^{II}(d_n^I+d_n^{II})+d_n^{I2}\lambda_n^2)}$
c_{c4}	$\frac{(\beta-1)\beta c_n}{4} + \frac{(\beta-1)d_n^I\beta c_r}{4(d_n^I+d_n^{II})} - \frac{d_n^I\beta(\beta-1)(1+\delta_r\sqrt{2})}{4w\lambda_r(d_n^I+d_n^{II})} + \frac{d_n^I\delta_n\lambda_n(4+\beta-\beta^2)w^{1.5}-2\beta(wd_n^I\lambda_n(\beta-3)-2d_r^{II}(\beta-2))}{4\lambda_n^2 w^2 d_n^I}$
c_{c5}	$\frac{2(c_n w\lambda_n+\beta+\delta_n\sqrt{w})-\sqrt{2w(c_n\lambda_n\sqrt{w}-\delta_n)^2(1-\beta)(2-\beta)}}{4\lambda_n w^2}$
c_{c6}	$\frac{2(c_n w\lambda_n+\beta+\delta_n\sqrt{w})+\sqrt{2w(c_n\lambda_n\sqrt{w}-\delta_n)^2(1-\beta)(2-\beta)}}{4\lambda_n w^2}$
c_{c7}	$\frac{\beta\lambda_r^2((c_n+c_r)d_n^I+c_n d_n^{II})d_r^{II}}{\lambda_n^2(\beta+1)d_n^{I2}+2d_r^{II}\beta\lambda_r^2(d_n^I+d_n^{II})} + \frac{\beta}{2w\lambda_n} + \frac{(\delta_n d_n^{I2}\lambda_n^2(\beta+1)-\lambda_n d_n^I d_r^{II}\beta\delta_r\lambda_r+(d_n^I+d_n^{II})\beta\delta_n d_r^{II}\lambda_r^2)w^{1.5}+\lambda_n\beta d_n^I(d_n^I\lambda_n w-(2+\lambda_r w)d_r^{II})}{\lambda_n w^2(\lambda_n^2(\beta+1)d_n^{I2}+\beta d_r^{II}\lambda_r^2(d_n^I+d_n^{II}))}$ $-\frac{\sqrt{(\beta-1)((w\lambda_n d_n^{I2}+d_r^{II}(w^2(c_n(d_n^I+d_n^{II})+d_n^I c_r)\lambda_r^2-d_n^I(w\lambda_r+2)\lambda_n-(\delta_n(d_n^I+d_n^{II})\lambda_r+\lambda_n d_n^I\delta_r)d_r^{II}\lambda_r w^{1.5})^2\beta^2(d_n^{I2}\lambda_n^2(\beta-1)+2\lambda_r^2(\beta-2)d_r^{II}(d_n^I+d_n^{II}))}}{2(\lambda_n^2(1+\beta)d_n^{I2}+2(d_n^I+d_n^{II})\beta d_r^{II}\lambda_r^2)w^2\lambda_n\sqrt{\lambda_r^2(d_n^I+d_n^{II})d_r^{II}+d_n^{I2}\lambda_n^2}}$
c_{c8}	$\frac{\beta\lambda_r^2((c_n+c_r)d_n^I+c_n d_n^{II})d_r^{II}}{\lambda_n^2(\beta+1)d_n^{I2}+2d_r^{II}\beta\lambda_r^2(d_n^I+d_n^{II})} + \frac{\beta}{2w\lambda_n} + \frac{(\delta_n d_n^{I2}\lambda_n^2(\beta+1)-\lambda_n d_n^I d_r^{II}\beta\delta_r\lambda_r+(d_n^I+d_n^{II})\beta\delta_n d_r^{II}\lambda_r^2)w^{1.5}+\lambda_n\beta d_n^I(d_n^I\lambda_n w-(2+\lambda_r w)d_r^{II})}{\lambda_n w^2(\lambda_n^2(\beta+1)d_n^{I2}+\beta d_r^{II}\lambda_r^2(d_n^I+d_n^{II}))}$ $+\frac{\sqrt{(\beta-1)((w\lambda_n d_n^{I2}+d_r^{II}(w^2(c_n(d_n^I+d_n^{II})+d_n^I c_r)\lambda_r^2-d_n^I(w\lambda_r+2)\lambda_n-(\delta_n(d_n^I+d_n^{II})\lambda_r+\lambda_n d_n^I\delta_r)d_r^{II}\lambda_r w^{1.5})^2\beta^2(d_n^{I2}\lambda_n^2(\beta-1)+2\lambda_r^2(\beta-2)d_r^{II}(d_n^I+d_n^{II}))}}{2(\lambda_n^2(1+\beta)d_n^{I2}+2(d_n^I+d_n^{II})\beta d_r^{II}\lambda_r^2)w^2\lambda_n\sqrt{\lambda_r^2(d_n^I+d_n^{II})d_r^{II}+d_n^{I2}\lambda_n^2}}$
c_{c9}	$\frac{\beta\lambda_r^2((c_n+c_r)d_n^I+c_n d_n^{II})d_r^{II}}{\lambda_n^2(\beta+1)d_n^{I2}+2d_r^{II}\beta\lambda_r^2(d_n^I+d_n^{II})} + \frac{\beta}{2w\lambda_n} + \frac{(\delta_n d_n^{I2}\lambda_n^2(\beta+1)-\lambda_n d_n^I d_r^{II}\beta\delta_r\lambda_r+(d_n^I+d_n^{II})\beta\delta_n d_r^{II}\lambda_r^2)w^{1.5}+\lambda_n\beta d_n^I(d_n^I\lambda_n w-(2+\lambda_r w)d_r^{II})}{\lambda_n w^2(\lambda_n^2(\beta+1)d_n^{I2}+\beta d_r^{II}\lambda_r^2(d_n^I+d_n^{II}))}$

References

Abbey, J. D., Blackburn, J. D., and Guide Jr, V. D. R. (2015a). Optimal pricing for new and remanufactured products. *Journal of Operations Management*, 36:130–146.

Abbey, J. D., Meloy, M. G., Blackburn, J., and Guide Jr, V. D. R. (2015b). Consumer markets for remanufactured and refurbished products. *California Management Review*, 57(4):26–42.

Abbey, J. D., Meloy, M. G., Guide Jr, V. D. R., and Atalay, S. (2015c). Remanufactured products in closed-loop supply chains for consumer goods. *Production and Operations Management*, 24(3):488–503.

Afsahi, M. and Shafiee, M. (2020). A stochastic simulation-optimization model for base-warranty and extended-warranty decision-making of under-and out-of-warranty products. *Reliability Engineering & System Safety*, 197:106772.

Agrawal, V. V., Ferguson, M., and Souza, G. C. (2016). Trade-in rebates for price discrimination and product recovery. *IEEE Transactions on Engineering Management*, 63(3):326–339.

Aksezer, C. S. (2011). Failure analysis and warranty modeling of used cars. *Engineering Failure Analysis*, 18(6):1520–1526.

Alizadeh-Basban, N. and Taleizadeh, A. A. (2020). A hybrid circular economy-game theoretical approach in a dual-channel green supply chain considering sale's effort, delivery time, and hybrid remanufacturing. *Journal of Cleaner Production*, 250:119521.

Alqahtani, A. Y. and Gupta, S. M. (2017a). Evaluating two-dimensional warranty policies for remanufactured products. *Journal of Remanufacturing*, 7(1):19–47.

Alqahtani, A. Y. and Gupta, S. M. (2017b). Warranty as a marketing strategy for remanufactured products. *Journal of Cleaner Production*, 161:1294–1307.

Alqahtani, A. Y. and Gupta, S. M. (2018). Money-back guarantee warranty policy with preventive maintenance strategy for sensor-embedded remanufactured products. *Journal of Industrial Engineering International*, 14(4):767–782.

Alqahtani, A. Y., Gupta, S. M., and Nakashima, K. (2019). Warranty and maintenance analysis of sensor embedded products using internet of things in industry 4.0. *International Journal of Production Economics*, 208:483–499.

Amezquita, T., Hammond, R., Salazar, M., and Bras, B. (1995). Characterizing the remanufacturability of engineering systems. *ASME advances in design automation conference*, 82:271–278.

Andersen, T., Jæger, B., and Mishra, A. (2020). Circularity in waste electrical and electronic equipment (weee) directive. comparison of a manufacturer's danish and norwegian operations. *Sustainability*, 12(13):5236.

APEC and US-AID (2013). Remanufacturing Resource Guide. Technical report, Nathan Associates Inc.

Atasu, A., Guide Jr, V. D. R., and Van Wassenhove, L. N. (2010). So what if remanufacturing cannibalizes my new product sales? *California Management Review*, 52(2):56–76.

Atasu, A., Sarvary, M., and Van Wassenhove, L. N. (2008). Remanufacturing as a marketing strategy. *Management Science*, 54(10):1731–1746.

Aumann, R. J. (2019). *Lectures on game theory*. CRC Press.

Batarfi, R., Jaber, M. Y., and Aljazzar, S. M. (2017). A profit maximization for a reverse logistics dual-channel supply chain with a return policy. *Computers & Industrial Engineering*, 106:58–82.

Beleya, P., Bakar, M. A. A., and Chelliah, M. K. (2017). Impact of reverse logistics in the malaysian electrical and electronics industry. *International Journal of Supply Chain Management*, 6(3):91.

Benjamin, J. (2020). Apple begins selling refurbished iphones through its online store for the first time. https://9to5mac.com/2016/11/08/apple-begins-selling-refurbished-iphones-through-its-online-store-for-the-first-time/. [Online; accessed 24-May-2020].

Bhatia, M. S. and Rajiv, K. S. (2019). Antecedents of implementation success in closed-loop supply chain: An empirical investigation. *International Journal of Production Research*, 57:7344–7360.

Bian, Y., Yan, S., Zhang, W., and Xu, H. (2015). Warranty strategy in a supply chain when two retailer's extended warranties bundled with the products. *Journal of Systems Science and Systems Engineering*, 24(3):364–389.

Blischke, W. (1995). *Product warranty handbook*. CRC Press.

Borenich, A., Dickbauer, Y., Reimann, M., and Souza, G. C. (2020). Should a manufacturer sell refurbished returns on the secondary market to incentivize retailers to reduce consumer returns? *European Journal of Operational Research*, 282(2):569–579.

Bouchery, Y., Corbett, C. J., Fransoo, J. C., and Tan, T. (2016). *Sustainable supply chains: A research-based textbook on operations and strategy*, volume 4. Springer.

Bouguerra, S., Chelbi, A., and Rezg, N. (2012). A decision model for adopting an extended warranty under different maintenance policies. *International Journal of Production Economics*, 135(2):840–849.

Boulding, W. and Kirmani, A. (1993). A consumer-side experimental examination of signaling theory: Do consumers perceive warranties as signals of quality? *Journal of Consumer Research*, 20(1):111–123.

Boustani, A., Sahni, S., Graves, S. C., and Gutowski, T. G. (2010). Appliance remanufacturing and life cycle energy and economic savings. In *Proceedings of the 2010 IEEE International Symposium on Sustainable Systems and Technology*, pages 1–6. IEEE.

Boyd, S. and Vandenberghe, L. (2004). *Convex optimization*. Cambridge university press.

BS 8887-220:2010 (2010). Design for manufacture, assembly, disassembly and end-of-life processing (MADE). The process of remanufacture. Specification. Standard, British Standard Institution.

Bulmuş, S. C., Zhu, S. X., and Teunter, R. H. (2014). Optimal core acquisition and pricing strategies for hybrid manufacturing and remanufacturing systems. *International Journal of Production Research*, 52(22):6627–6641.

Butler, A. (2018). Do customers really care about your environmental impact? `https://www.forbes.com/sites/forbesnycouncil/2018/11/21/do-customers-really-care-about-your-environmental-impact/?sh=3367ba8a240d`. [Online; accessed 29-Nov-2020].

Butler, P. and Peppard, J. (1998). Consumer purchasing on the internet: Processes and prospects. *European Management Journal*, 16(5):600–610.

Cao, H., Du, Y., and Chen, Y. (2011). Exploring a new low-carbon development paradigm for china's future manufacturing sectors. *Journal of Science and Technology Policy in China*, 2:159–170.

Cao, J., Chen, X., Zhang, X., Gao, Y., Zhang, X., and Kumar, S. (2020a). Overview of remanufacturing industry in China: Government policies, enterprise, and public awareness. *Journal of Cleaner Production*, 242:118450.

Cao, K. and He, P. (2018). Price and warranty competition in a supply chain with a common retailer. *INFOR: Information Systems and Operational Research*, 56(2):225–246.

Cao, K., He, P., and Liu, Z. (2020b). Production and pricing decisions in a dual-channel supply chain under remanufacturing subsidy policy and carbon tax policy. *Journal of the Operational Research Society*, 71(8):1199–1215.

Cao, K., Xu, B., and Wang, J. (2020c). Optimal trade-in and warranty period strategies for new and remanufactured products under carbon tax policy. *International Journal of Production Research*, 58(1):180–199.

Cattolica, D. (2018). Advancing the circular economy: Exploring landscape and developments for circular public procurement in sweden and scotland. *IIIEE Master Thesis*.

Chari, N., Diallo, C., and Venkatadri, U. (2013). Optimal unlimited free-replacement warranty strategy using reconditioned products. *International Journal of Performability Engineering*, 9(2):191–200.

Chari, N., Diallo, C., Venkatadri, U., and Aït-Kadi, D. (2016a). Production planning in the presence of remanufactured spare components: an application in the airline industry. *The International Journal of Advanced Manufacturing Technology*, 87(1-4):957–968.

Chari, N., Diallo, C., Venkatadri, U., and Khatab, A. (2016b). Modeling and analysis of a warranty policy using new and reconditioned parts. *Applied Stochastic Models in Business and Industry*, 32(4):539–553.

Chauhan, S. (2019). Apple iphone prices to remain unchanged this year: Report. https://www.techradar.com/in/news/apple-iphone-prices-to-remain-unchanged-this-year-report. [Online; accessed 11-February-2019].

Chen, C. K. and Akmalul'Ulya, M. (2019). Analyses of the reward-penalty mechanism in green closed-loop supply chains with product remanufacturing. *International Journal of Production Economics*, 210:211–223.

Chen, J. and Chang, C. (2013). Dynamic pricing for new and remanufactured products in a closed-loop supply chain. *International Journal of Production Economics*, 146(1):153–160.

Chen, J. and Chi, C. (2019). Reverse channel choice in a closed-loop supply chain with new and differentiated remanufactured goods. *Journal of Industrial and Production Engineering*, 36:81–96.

Chen, J., Venkatadri, U., and Diallo, C. (2019). Optimal (re) manufacturing strategies in the presence of spontaneous consumer returns. *Journal of Cleaner Production*, 237:117642.

Chen, W., Chen, J., and Ma, Y. Renewable energy investment and carbon emissions under cap-and-trade mechanisms. *Journal of Cleaner Production*, 278:123341.

Chiang, T. and Trappey, A. J. (2007). Development of value chain collaborative model for product lifecycle management and its lcd industry adoption. *International Journal of Production Economics*, 109(1-2):90–104.

Chiang, W. K., Chhajed, D., and Hess, J. D. (2003). Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. *Management Science*, 49(1):1–20.

Chien, Y. (2010). Optimal age for preventive replacement under a combined fully renewable free replacement with a pro-rata warranty. *International Journal of Production Economics*, 124(1):198–205.

Cho, R. (2018). What can we do about the growing e-waste problem? https://blogs.ei.columbia.edu/2018/08/27/growing-e-waste-problem/. [Online; accessed 11-Nov-2020].

Choi, T. (2017). Pricing and branding for remanufactured fashion products. *Journal of Cleaner Production*, 165:1385–1394.

Choudhary, A. I., Asif, M., Choudhry, R. M., Siddique, Z., and Mughal, A. (2011). Impact of after sale service characteristics on customer satisfaction. *Information Management and Business Review*, 3(6):360–365.

Clover, J. (2019). Apple refurbished products: Should you buy them? https://www.macrumors.com/guide/apple-refurbished/. [Online; accessed 13-May-2019].

Cohen, D., Darrough, M. N., Huang, R., and Zach, T. (2011). Warranty reserve: Contingent liability, information signal, or earnings management tool? *The Accounting Review*, 86(2):569–604.

D'Adamo, I. and Rosa, P. (2016). Remanufacturing in industry: Advices from the field. *The International Journal of Advanced Manufacturing Technology*, 86(9-12):2575–2584.

Dan, B., Xu, G., and Liu, C. (2012). Pricing policies in a dual-channel supply chain with retail services. *International Journal of Production Economics*, 139(1):312–320.

Dan, Y., Dai, Y., and Ma, Z. (2019). Reverse channel choice in a closed-loop supply chain with extended warranty service. *Industrial Engineering Journal*, 22(2):42–48.

Darghouth, M. N., Aït-Kadi, D., and Chelbi, A. (2017). Joint optimization of design, warranty and price for products sold with maintenance service contracts. *Reliability Engineering & System Safety*, 165:197–208.

De Oliveira, C. R., Bernardes, A. M., and Gerbase, A. E. (2012). Collection and recycling of electronic scrap: A worldwide overview and comparison with the brazilian situation. *Waste Management*, 32(8):1592–1610.

Debo, L. G., Toktay, L. B., and Van Wassenhove, L. N. (2005). Market segmentation and product technology selection for remanufacturable products. *Management Science*, 51(8):1193–1205.

Debo, L. G., Toktay, L. B., and Wassenhove, L. N. V. (2006). Joint life-cycle dynamics of new and remanufactured products. *Production and Operations Management*, 15(4):498–513.

Deng, Q., Liao, H., Xu, B., and Liu, X. (2017). The resource benefits evaluation model on remanufacturing processes of end-of-life construction machinery under the uncertainty in recycling price. *Sustainability*, 9(2):256.

Diallo, C., Venkatadri, U., Khatab, A., and Bhakthavatchalam, S. (2017). State of the art review of quality, reliability and maintenance issues in closed-loop supply chains with remanufacturing. *International Journal of Production Research*, 55(5):1277–1296.

Dou, G., Guo, H., Zhang, Q., and Li, X. (2019). A two-period carbon tax regulation for manufacturing and remanufacturing production planning. *Computers & Industrial Engineering*, 128:502–513.

Du, Y. and Li, C. (2014). Implementing energy-saving and environmental-benign paradigm: machine tool remanufacturing by oems in china. *Journal of Cleaner Production*, 66:272–279.

E-Commerce (2020). Crazy sale! china's singles' day bags record shopping revenues. https://www.aljazeera.com/economy/2020/11/11/crazy-sale-chinas-singles-day-bags-record-shopping-revenues. [Online; accessed 4-Dec-2020].

Eiselt, H. A., Pederzoli, G., and Sandblom, C. (1987). *Continuous optimization models*, volume 1. Walter De Gruyter Inc.

Elsayed, E. A. (2014). Life cycle costs and reliability engineering. *Wiley StatsRef: Statistics Reference Online*.

Esmaeili, M., Shamsi G, N., and Asgharizadeh, E. (2014). Three-level warranty service contract among manufacturer, agent and customer: A game-theoretical approach. *European Journal of Operational Research*, 239(1):177–186.

Estay, B. (2020). 16 online shopping statistics: How many people shop online? https://www.bigcommerce.com/blog/online-shopping-statistics/#5-essential-online-shopping-statistics. [Online; accessed 4-Dec-2020].

European Commission (2013). Equivalent conditions for waste electrical and electronic equipment (weee) recycling operations taking place outside the european union. `https://ec.europa.eu/environment/waste/weee/pdf/Final%20report_E%20C%20S.pdf`. [Online; accessed 27-Nov-2020].

Feng, L., Govindan, K., and Li, C. (2017). Strategic planning: Design and coordination for dual-recycling channel reverse supply chain considering consumer behavior. *European Journal of Operational Research*, 260(2):601–612.

Feng, Y., Tian, Y., and Zhu, Q. (2016). A combined input–output/decision making trial and evaluation laboratory method for evaluating effect of the remanufacturing sector development. *Journal of Cleaner Production*, 114:103–113.

Ferrer, G. and Swaminathan, J. M. (2006). Managing new and remanufactured products. *Management Science*, 52(1):15–26.

Fruchter, G. E. and Tapiero, C. S. (2005). Dynamic online and offline channel pricing for heterogeneous customers in virtual acceptance. *International Game Theory Review*, 7(02):137–150.

Gan, S., Pujawan, I. N., and Widodo, B. (2017). Pricing decision for new and remanufactured product in a closed-loop supply chain with separate sales-channel. *International Journal of Production Economics*, 190:120–132.

Giri, B. C., Chakraborty, A., and Maiti, T. (2017). Pricing and return product collection decisions in a closed-loop supply chain with dual-channel in both forward and reverse logistics. *Journal of Manufacturing Systems*, 42:104–123.

Giri, B. C., Mondal, C., and Maiti, T. (2018). Analysing a closed-loop supply chain with selling price, warranty period and green sensitive consumer demand under revenue sharing contract. *Journal of Cleaner Production*, 190:822–837.

Giuntini, R. and Gaudette, K. (2001). Remanufacturing, the next great opportunity for improving us productivity. *Business Horizons*.

Govindan, K., Paam, P., and Abtahi, A.-R. (2016). A fuzzy multi-objective optimization model for sustainable reverse logistics network design. *Ecological indicators*, 67:753–768.

Grashuis, J., Skevas, T., and Segovia, M. S. (2020). Grocery shopping preferences during the covid-19 pandemic. *Sustainability*, 12(13):5369.

Grau, J., Terraza, H., Velosa, R., Milena, D., Rihm, A., and Sturzenegger, G. (2015). Solid waste management in latin america and the caribbean. *Inter-American Development Bank: Washington, DC, USA*.

Griffiths, B. (2012). Manufacturing paradigms: The role of standards in the past, the present and the future paradigm of sustainable manufacturing. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, 226(10):1628–1634.

Guidat, T., Seidel, J., Kohl, H., and Seliger, G. (2017). A comparison of best practices of public and private support incentives for the remanufacturing industry. *Procedia CIRP*, 61:177–182.

Guide, Jr, V. D. R. and Li, J. (2010). The potential for cannibalization of new products sales by remanufactured products. *Decision Sciences*, 41(3):547–572.

Guide Jr, V. D. R. and Van Wassenhove, L. N. (2001). Managing product returns for remanufacturing. *Production and operations management*, 10(2):142–155.

Gunasekara, H., Gamage, J., and Punchihewa, H. (2018). Remanufacture for sustainability: A review of the barriers and the solutions to promote remanufacturing. In *2018 International Conference on Production and Operations Management Society (POMS)*, pages 1–7. IEEE.

Guo, S., Shen, B., Choi, T.-M., and Jung, S. (2017). A review on supply chain contracts in reverse logistics: Supply chain structures and channel leaderships. *Journal of Cleaner Production*, 144:387–402.

Hammami, R., Nouira, I., and Frein, Y. (2018). Effects of customers' environmental awareness and environmental regulations on the emission intensity and price of a product. *Decision Sciences*, 49(6):1116–1155.

Han, X., Wu, H., Yang, Q., and Shang, J. (2016). Reverse channel selection under remanufacturing risks: Balancing profitability and robustness. *International Journal of Production Economics*, 182:63–72.

He, P., He, Y., and Xu, H. (2019). Channel structure and pricing in a dual-channel closed-loop supply chain with government subsidy. *International Journal of Production Economics*.

Heese, H. S. (2012). Retail strategies for extended warranty sales and impact on manufacturer base warranties. *Decision Sciences*, 43(2):341–367.

Heydari, J. and Ghasemi, M. (2018). A revenue sharing contract for reverse supply chain coordination under stochastic quality of returned products and uncertain remanufacturing capacity. *Journal of cleaner production*, 197:607–615.

Hogan Lovells LLP (2016). French legal and regulatory update. `https://www.hoganlovells.com/~/media/french-content/pdf-and-publications/2016/actualites_legislatives_hl_en_may2016.pdf?la%C2%BCen`. Online; accessed 10 August 2020.

Hong, X., Xu, L., Du, P., and Wang, W. (2015). Joint advertising, pricing and collection decisions in a closed-loop supply chain. *International Journal of Production Economics*, 167:12–22.

Hong, X., Zhang, H., Zhong, Q., and Liu, L. (2016). Optimal decisions of a hybrid manufacturing-remanufacturing system within a closed-loop supply chain. *European Journal of Industrial Engineering*, 10(1):21–50.

Horn, K. (2020). The product life cycle is in decline. https://www.supplychaindigital.com/procurement/product-life-cycle-decline. [Online; accessed 11-Nov-2020].

Huang, S., Yang, C., and Zhang, X. (2012). Pricing and production decisions in dual-channel supply chains with demand disruptions. *Computers & Industrial Engineering*, 62(1):70–83.

Huang, Y.-S. and Fang, C.-C. (2008). A cost sharing warranty policy for products with deterioration. *IEEE Transactions on Engineering Management*, 55(4):617–627.

Hudson, M. (2019). The high cost of retail returns. https://www.thebalancesmb.com/the-high-cost-of-retail-returns-2890350. [Online; accessed 11-Nov-2020].

IBISWorld (2020). Auto parts remanufacturing in the us - market size 2003–2025. `https://www.ibisworld.com/industry-statistics/market-size/auto-parts-remanufacturing-united-states/`. [Online; accessed 18-Nov-2020].

Ijomah, W. (2002). A model-based definition of the generic remanufacturing business process. *University of Plymouth*.

Ikeda, A. (2017). Remanufacturing of automotive parts in japanese market. *Procedia CIRP*, 61:800–803.

Ishida, C., Pope, N., and Kaufman, P. (2019). Extended service plans and buyer perceptions and behaviors in automobile industry: An abstract. In *Academy of Marketing Science Annual Conference*, pages 31–32. Springer.

Jiang, L. A., Yang, Z., and Jun, M. (2013). Measuring consumer perceptions of online shopping convenience. *Journal of Service Management*, 24(2):191–214.

Jiménez-Parra, B., Rubio, S., and Vicente-Molina, M.-A. (2014). Key drivers in the behavior of potential consumers of remanufactured products: A study on laptops in spain. *Journal of Cleaner Production*, 85:488–496.

Jin, M. and Zhou, Y. (2020). Does the remanufactured product deserve the same warranty as the new one in a closed-loop supply chain? *Journal of Cleaner Production*, 262:121430.

Kafuku, J. M., Saman, M. Z. M., et al. (2016). Current and future issues in electronics and automobiles remanufacturing operations. In *2016 Future Technologies Conference (FTC)*, pages 415–421. IEEE.

Kang, H., Jun, Y., Kim, Y., and Jo, H. (2016). Comparative analysis on cross-national system to enhance the reliability of remanufactured products. *Procedia CIRP*, 40:280–284.

Kang, H. Y., Jun, Y. S., Jo, H. J., Baek, C. Y., and Kim, Y.-C. (2018). Korea's remanufacturing industry in comparison with its global status: A case study. *Journal of Remanufacturing*, 8(1-2):81–91.

Kauffman, J. and Lee, K. (2013). *Handbook of sustainable engineering*. Springer.

Kim, S., Kim, T., Pae, S., and Kim, S. (2018). Audit fees via an indirect payment channel and professional skepticism. *Managerial Auditing Journal*, 33(5):517–534.

Kripli, J. (2016). Remanufacturing global outlook. `https://cdn.ymaws.com/apra.org/resource/resmgr/Events/6th_China_Reman_Summit_5-27-.pdf`. [Online; accessed 25-Nov-2020].

Kuik, S. S., Kaihara, T., and Fujii, N. (2015). Stochastic decision model of the remanufactured product with warranty. In *Proceedings of the International Multi-Conference of Engineers and Computer Scientists*, volume 2.

Kumar, R. and Ramachandran, P. (2016). Revenue management in remanufacturing: Perspectives, review of current literature and research directions. *International Journal of Production Research*, 54(7):2185–2201.

Lan, Y., Zhao, R., and Tang, W. (2014). A fuzzy supply chain contract problem with pricing and warranty. *Journal of Intelligent & Fuzzy Systems*, 26(3):1527–1538.

Lee, C., Woo, W., and Roh, Y. (2017). Remanufacturing: Trends and issues. *International Journal of Precision Engineering and Manufacturing-Green Technology*, 4(1):113–125.

Lee, S. and Kwak, M. (2020). Consumer valuation of remanufactured products: A comparative study of product categories and business models. *Sustainability*, 12(18):7581.

Li, Y., Zhang, L., and Jin, M. (2017). Report on consumer awareness and behaviour change in sustainable consumption. `https://www.oneplanetnetwork.org/sites/default/files/en_report_on_consumer_awareness_and_behavior_change_in_sustainable_consumption_in_china-final.pdf`. [Online; accessed 29-Nov-2020].

Liao, B. (2018). Warranty as a competitive dimension for remanufactured products under stochastic demand. *Journal of Cleaner Production*, 198:511–519.

Liao, B., Li, B., and Cheng, J. (2015). A warranty model for remanufactured products. *Journal of Industrial and Production Engineering*, 32(8):551–558.

Liao, H., Deng, Q., Wang, Y., Guo, S., and Ren, Q. (2018). An environmental benefits and costs assessment model for remanufacturing process under quality uncertainty. *Journal of Cleaner Production*, 178:45–58.

Liao, S. (2020). Apple's iphone xs and xs max cost between $999 and $1,449, preorders start this week. https://www.theverge.com/2018/9/12/17833352/apple-iphone-xs-price-att-verizon-t-mobile-pre-order-date. [Online; accessed 25-May-2020].

Liu, B., Wu, J., and Xie, M. (2015). Cost analysis for multi-component system with failure interaction under renewing free-replacement warranty. *European Journal of Operational Research*, 243(3):874–882.

Liu, Z., Chen, J., and Diallo, C. (2018). Optimal production and pricing strategies for a remanufacturing firm. *International Journal of Production Economics*, 204:290–315.

Liu, Z., Chen, J., Diallo, C., and Venkatadri, U. (2021). Pricing and production decisions in a dual-channel closed-loop supply chain with (re) manufacturing. *International Journal of Production Economics*, 232:107935.

Liu, Z., Diallo, C., Chen, J., and Zhang, M. (2020). Optimal pricing and production strategies for new and remanufactured products under a non-renewing free replacement warranty. *International Journal of Production Economics*, 226:107602.

Livingston, A. (2019). Should I buy refurbished electronics? https://www.moneycrashers.com/buy-refurbished-electronics/. [Online; accessed 13-May-2019].

Lund, R. T. (1985). Remanufacturing: The experience of the United States and implications for developing countries.

Lund, R. T. (1996). *The remanufacturing industry: Hidden giant*. Boston University.

Manufacturing, A. A. (2020). Aae's remanufacturing process includes 8 critical process steps. `https://www.aaesteering.com/remanufacturing-process/`. [Online; accessed 26-Nov-2020].

Martins, A. (2019). Most consumers want sustainable products and packaging. `https://www.businessnewsdaily.com/15087-consumers-want-sustainable-products.html`. [Online; accessed 29-Nov-2020].

Matsumoto, M., Chinen, K., and Endo, H. (2017). Comparison of us and japanese consumers' perceptions of remanufactured auto parts. *Journal of Industrial Ecology*, 21(4):966–979.

Michaud, C. and Llerena, D. (2011). Green consumer behaviour: An experimental analysis of willingness to pay for remanufactured products. *Business Strategy and the Environment*, 20(6):408–420.

Miyatake, K., Nemoto, T., Nakaharai, S., and Hayashi, K. (2016). Reduction in consumers' purchasing cost by online shopping. *Transportation Research Procedia*, 12:656–666.

Mostard, J. and Teunter, R. (2006). The newsboy problem with resalable returns: A single period model and case study. *European Journal of Operational Research*, 169(1):81–96.

Murthy, D. and Blischke, W. R. (2000). Strategic warranty management: A life-cycle approach. *IEEE Transactions on Engineering Management*, 47(1):40–54.

Murthy, D. and Djamaludin, I. (2002). New product warranty: A literature review. *International Journal of Production Economics*, 79(3):231–260.

Murthy, D. and Jack, N. (2009). Warranty and maintenance. In *Handbook of Maintenance Management and Engineering*, pages 461–478. Springer.

Mustajib, M. I., Kurniati, N., and Ciptomulyono, U. (2017). Remanufacturing quality control strategies: A literature review and proposed conceptual framework. In *Proceedings of the Asia Pacific Industrial Engineering & Management Systems Conference*.

National Retail Federation (2015). Consumer returns in the retail industry. `https://nrf.com/resources/retail-library/consumer-returns-the-retail-industry`. [Online; accessed 11-Nov-2020].

OECD (2014). The state of play on Extended Producer Responsibility (EPR): Opportunities and Challenges.

Ovchinnikov, A. (2011). Revenue and cost management for remanufactured products. *Production and Operations Management*, 20(6):824–840.

Papachristos, G. and Adamides, E. D. (2014). Internal supply-chain competition in remanufacturing: operations strategies, performance and environmental effects. *International Journal of Logistics Systems and Management*, 19(2):187–211.

Parker, D., Riley, K., Robinson, S., Symington, H., Tewson, J., Jansson, K., Ramkumar, S., and Peck, D. (2015). Remanufacturing market study. *European Remanufacturing Network*.

Paterson, D. A., Kao, C.-C., Ijomah, W. L., and Windmill, J. F. (2018). Incorporating remanufacturing into the end-of-life vehicles directive: Current presence and the waste problem. *Journal of Remanufacturing*, 8(1-2):23–37.

Phantratanamongkol, S., Casalin, F., Pang, G., and Sanderson, J. (2018). The price-volume relationship for new and remanufactured smartphones. *International Journal of Production Economics*, 199:78–94.

Philippines, C. (2015). Remanufacturing in Malaysia - An assessment of the current and future remanufacturing industry. *Centre for Remanufacturing and Reuse for Nathan Associates Inc.*

Radhi, M. and Zhang, G. (2018). Pricing policies for a dual-channel retailer with cross-channel returns. *Computers & Industrial Engineering*, 119:63–75.

Rahman, A. and Chattopadhyay, G. (2015). Long term warranty and after sales service. In *Long Term Warranty and After Sales Service*, pages 1–15. Springer.

Ramanathan, R. (2011). An empirical analysis on the influence of risk on relationships between handling of product returns and customer loyalty in e-commerce. *International Journal of Production Economics*, 130(2):255–261.

Raza, S. A. and Govindaluri, S. M. (2019). Pricing strategies in a dual-channel green supply chain with cannibalization and risk aversion. *Operations Research Perspectives*, 6:100118.

Reagan, C. (2019). That sweater you don't like is a trillion-dollar problem for retailers. these companies want to fix it. `https://www.cnbc.com/2019/01/10/growing-online-sales-means-more-returns-and-trash-for-landfills.html`. [Online; accessed 11-Nov-2020].

Remanufacturing Industries Council (2020). What is remanufacturing? http://www.remancouncil.org/educate/remanufacturing-information/what-is-remanufacturing. [Online; accessed 10-Nov-2020].

Rofin, T. and Mahanty, B. (2018). Optimal dual-channel supply chain configuration for product categories with different customer preference of online channel. *Electronic Commerce Research*, 18(3):507–536.

Ross, S. M. (2014). *Introduction to probability models*. Academic press.

Saavedra, Y. M., Barquet, A. P., Rozenfeld, H., Forcellini, F. A., and Ometto, A. R. (2013). Remanufacturing in brazil: case studies on the automotive sector. *Journal of Cleaner Production*, 53:267–276.

Sabanoglu, T. (2019). Global retail e-commerce sales 2014-2023. https://www.statista.com/statistics/379046/worldwide-retail-e-commerce-sales/. [Online; accessed 17-Dec-2019].

San, G. S. and Pujawan, I. N. (2017). Pricing and warranty level decisions for new and remanufactured short life-cycle products. *Jurnal Teknik Industri*, 19(1):39–46.

San Gan, S., Pujawan, I. N., and Widodo, B. (2015). Pricing decision model for new and remanufactured short-life cycle products with time-dependent demand. *Operations Research Perspectives*, 2:1–12.

Schuh, G., Salmen, M., Kuhlmann, T., and Wiese, J. (2017). Highly iterative product development within the tool and die making industry. *Procedia CIRP*, 61:576–581.

Schwarck, W. (2019). 2020 – a breakthrough year for remanufacturing in china? `https://encory.com/2020-a-breakthrough-year-for-remanufacturing-in-china/`. [Online; accessed 29-Nov-2020].

Seitz, M. A. (2007). A critical assessment of motives for product recovery: the case of engine remanufacturing. *Journal of Cleaner Production*, 15(11-12):1147–1157.

Shafiee, M. and Chukova, S. (2013). Maintenance models in warranty: A literature review. *European Journal of Operational Research*, 229(3):561–572.

Singhal, D., Tripathy, S., Jena, S. K., Nayak, K. K., and Dash, A. (2018). Interpretive structural modelling (ism) of obstacles hindering the remanufacturing practices in india. *Procedia Manufacturing*, 20:452–457.

Steeneck, D. W. and Sarin, S. C. (2013). Pricing and production planning for reverse supply chain: A review. *International Journal of Production Research*, 51(23-24):6972–6989.

Steinhilper, R. and Weiland, F. (2015). Exploring new horizons for remanufacturing an up-to-date overview of industries, products and technologies. *Procedia CIRP*, 29:769–773.

Su, C. and Shen, J. (2012). Analysis of extended warranty policies with different repair options. *Engineering Failure Analysis*, 25:49–62.

Suginraj, M. (2017). Growth of online marketing in India - A study. *International Journal of Research in Management & Business Studies*, 4(3):9–14.

Sun, X., Zhou, Y., Li, Y., Govindan, K., and Han, X. (2020). Differentiation competition between new and remanufactured products considering third-party remanufacturing. *Journal of the Operational Research Society*, 71(1):161–180.

Taleizadeh, A. A. and Mokhtarzadeh, M. (2020). Pricing and two-dimensional warranty policy of multi-products with online and offline channels using a value-at-risk approach. *Computers & Industrial Engineering*, 148:106674.

Taleizadeh, A. A., Moshtagh, M. S., and Moon, I. (2018). Pricing, product quality, and collection optimization in a decentralized closed-loop supply chain with different channel structures: Game theoretical approach. *Journal of Cleaner Production*, 189:406–431.

Tang, J., Li, B., Li, K. W., Liu, Z., and Huang, J. (2020). Pricing and warranty decisions in a two-period closed-loop supply chain. *International Journal of Production Research*, 58(6):1688–1704.

Teunter, R. H. (2001). A reverse logistics valuation method for inventory control. *International Journal of Production Research*, 39(9):2023–2035.

Tian, H., Yan, Q., and Liu, C. (2019). The impact of extended warranty on base warranty: A game approach. In *2019 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, pages 830–834. IEEE.

Tong, P., Song, X., and Zixian, L. (2017). A maintenance strategy for two-dimensional extended warranty based on dynamic usage rate. *International Journal of Production Research*, 55(19):5743–5759.

Tran, L. T. T. (2021). Managing the effectiveness of e-commerce platforms in a pandemic. *Journal of Retailing and Consumer Services*, 58:102287.

Vafadarnikjoo, A., Mishra, N., Govindan, K., and Chalvatzis, K. (2018). Assessment of consumers' motivations to purchase a remanufactured product by applying fuzzy delphi method and single valued neutrosophic sets. *Journal of Cleaner Production*, 196:230–244.

Vamanan, D. (2020). How fashion ecommerce retailers can reduce online returns. `https://www.sellerapp.com/blog/amazon-business-strategy-ideas/`. [Online; accessed 11-Nov-2020].

Van Loon, P. and Van Wassenhove, L. N. (2018). Assessing the economic and environmental impact of remanufacturing: A decision support tool for oem suppliers. *International Journal of Production Research*, 56(4):1662–1674.

Vasić, N., Kilibarda, M., and Kaurin, T. (2019). The influence of online shopping determinants on customer satisfaction in the serbian market. *Journal of Theoretical and Applied Electronic Commerce Research*, 14(2):0.

Vasudevan, H., Kalamkar, V., and Terkar, R. (2012). Remanufacturing for sustainable development: Keychallenges, elements, and benefits. *International Journal of Innovation, Management and Technology*, 3(1):84.

Wang, J., Jiang, H., and Yu, M. (2020a). Pricing decisions in a dual-channel green supply chain with product customization. *Journal of Cleaner Production*, 247:119101.

Wang, S., Tomovic, M., and Liu, H. (2015). *Commercial Aircraft Hydraulic Systems: Shanghai Jiao Tong University Press Aerospace Series*. Academic Press.

Wang, X., Zhu, Y., Sun, H., and Jia, F. (2018). Production decisions of new and remanufactured products: Implications for low carbon emission economy. *Journal of Cleaner Production*, 171:1225–1243.

Wang, Y., Wiegerinck, V., Krikke, H., and Zhang, H. (2013). Understanding the purchase intention towards remanufactured product in closed-loop supply chains. *International Journal of Physical Distribution & Logistics Management*.

Wang, Y., Yu, Z., and Ji, X. (2020b). Coordination of e-commerce supply chain when e-commerce platform providing sales service and extended warranty service. *Journal of Control and Decision*, 7(3):241–261.

Williamson, I. A., Pearson, D. R., Aranoff, S. L., Pinkert, D. A., Johanson, D. S., and Broadbent, M. M. (2012). Remanufactured goods: An overview of the US and global industries, markets, and trade. *United States International Trade Commission, USITC Publication*, 4356:332–525.

Wu, C., Chou, C., and Huang, C. (2007). Optimal burn-in time and warranty length under fully renewing combination free replacement and pro-rata warranty. *Reliability Engineering & System Safety*, 92(7):914–920.

Wu, X. and Zhou, Y. (2017). The optimal reverse channel choice under supply chain competition. *European Journal of Operational Research*, 259(1):63–66.

Xie, J., Liang, L., Liu, L., and Ieromonachou, P. (2017). Coordination contracts of dual-channel with cooperation advertising in closed-loop supply chains. *International Journal of Production Economics*, 183:528–538.

Xie, J., Zhang, W., Liang, L., Xia, Y., Yin, J., and Yang, G. (2018). The revenue and cost sharing contract of pricing and servicing policies in a dual-channel closed-loop supply chain. *Journal of Cleaner Production*, 191:361–383.

Xiong, Y., Zhou, Y., Li, G., Chan, H., and Xiong, Z. (2013). Don't forget your supplier when remanufacturing. *European Journal of Operational Research*, 230(1):15–25.

Xu, B. (2010). State of the art and future development in remanufacturing engineering. *Transactions of Materials and Heat Treatment*, 31(1):10–14.

Xu, B. (2013). Progress of remanufacturing engineering and future technology expectation. *Advances in Manufacturing*, 1(1):8–12.

Xu, L. and Wang, C. (2018). Sustainable manufacturing in a closed-loop supply chain considering emission reduction and remanufacturing. *Resources, Conservation and Recycling*, 131:297–304.

Xu, L., Wang, C., and Zhao, J. (2018). Decision and coordination in the dual-channel supply chain considering cap-and-trade regulation. *Journal of Cleaner Production*, 197:551–561.

Xu, X., He, P., Xu, H., and Zhang, Q. (2017). Supply chain coordination with green technology under cap-and-trade regulation. *International Journal of Production Economics*, 183:433–442.

Yan, W., Xiong, Y., Xiong, Z., and Guo, N. (2015). Bricks vs. clicks: Which is better for marketing remanufactured products? *European Journal of Operational Research*, 242(2):434–444.

Yan, X., Chao, X., Lu, Y., and Zhou, S. X. (2017). Optimal policies for selling new and remanufactured products. *Production and Operations Management*, 26(9):1746–1759.

Yang, D., Zhang, L., Wu, Y., Guo, S., Zhang, H., and Xiao, L. (2019). A sustainability analysis on retailer's sales effort in a closed-loop supply chain. *Sustainability*, 11(1):8.

Yang, J., Zhao, H., and Wan, J. (2010). Research on the advantages and disadvantages of online shopping and corresponding strategies. In *2010 International Conference on E-Product E-Service and E-Entertainment*, pages 1–3. IEEE.

Yang, L., Wang, G., and Ke, C. (2018). Remanufacturing and promotion in dual-channel supply chains under cap-and-trade regulation. *Journal of Cleaner Production*, 204:939–957.

Yanıkoğlu, İ. and Denizel, M. (2021). The value of quality grading in remanufacturing under quality level uncertainty. *International Journal of Production Research*, 59(3):839–859.

Yedida, S. and Sekar, S. (2017). A stochastic model for remanufactured products with warranty in supply chain. *Life Cycle Reliability and Safety Engineering*, 6(3):207–214.

Yenipazarli, A. (2016). Managing new and remanufactured products to mitigate environmental damage under emissions regulation. *European Journal of Operational Research*, 249(1):117–130.

Ylä-Mella, J. and Román, E. (2019). Waste electrical and electronic equipment management in europe: learning from best practices in switzerland, norway, sweden and denmark. In *Waste Electrical and Electronic Equipment (WEEE) Handbook*, pages 483–519. Elsevier.

Zeng, X., Li, J., Stevels, A., and Liu, L. (2013). Perspective of electronic waste management in china based on a legislation comparison between China and the EU. *Journal of Cleaner Production*, 51:80–87.

Zhang, C. and Ren, M. (2016). Closed-loop supply chain coordination strategy for the remanufacture of patented products under competitive demand. *Applied Mathematical Modelling*, 40(13-14):6243–6255.

Zhang, F., Chen, H., Xiong, Y., Yan, W., and Liu, M. (2020a). Managing collecting or remarketing channels: Different choice for cannibalisation in remanufacturing outsourcing. *International Journal of Production Research*, 12:1–16.

Zhang, J. and Chen, M. (2015). Assessing the impact of china's vehicle emission standards on diesel engine remanufacturing. *Journal of Cleaner Production*, 107:177–184.

Zhang, L., Wang, J., and You, J. (2015). Consumer environmental awareness and channel coordination with two substitutable products. *European Journal of Operational Research*, 241(1):63–73.

Zhang, N., Fouladirad, M., and Barros, A. (2018). Evaluation of the warranty cost of a product with type iii stochastic dependence between components. *Applied Mathematical Modelling*, 59:39–53.

Zhang, X., Zhang, M., Zhang, H., Jiang, Z., Liu, C., and Cai, W. (2020b). A review on energy, environment and economic assessment in remanufacturing based on life cycle assessment method. *Journal of Cleaner Production*, 255:120160.

Zhao, J., Wei, J., and Li, M. (2017). Collecting channel choice and optimal decisions on pricing and collecting in a remanufacturing supply chain. *Journal of Cleaner Production*, 167:530–544.

Zhao, J., Zhou, Y.-W., Cao, Z.-H., and Min, J. (2020). The shelf space and pricing strategies for a retailer-dominated supply chain with consignment based revenue sharing contracts. *European Journal of Operational Research*, 280(3):926–939.

Zhao, S. and Zhu, Q. (2018). A risk-averse marketing strategy and its effect on coordination activities in a remanufacturing supply chain under market fluctuation. *Journal of Cleaner Production*, 171:1290–1299.

Zheng, B., Bian, Y., Sun, Y., and Ding, H. (2018). Optimal extended warranty strategy: Uniform or nonuniform pricing? *International Transactions in Operational Research*, 28:1441–1464.

Zheng, B., Yang, C., Yang, J., and Zhang, M. (2017). Dual-channel closed loop supply chains: Forward channel competition, power structures and coordination. *International Journal of Production Research*, 55(12):3510–3527.

Zheng, C. and Ai, X. (2017). Coordination of a supply chain with extended warranty when retailers compete. In *2017 International Conference on Service Systems and Service Management*, pages 1–5. IEEE.

Zhu, Q., Li, H., Zhao, S., and Lun, V. (2016). Redesign of service modes for remanufactured products and its financial benefits. *International Journal of Production Economics*, 171:231–240.

Zhu, X., Yu, L., Zhang, J., Li, C., and Zhao, Y. (2018). Warranty decision model and remanufacturing coordination mechanism in closed-loop supply chain: View from a consumer behavior perspective. *Sustainability*, 10(12):4738.