Experimental and Theoretical Investigation of Glass Ceramics: The Transparent Ferroelectric Nanocomposite LaBGeO$_5$

by

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To Zoë, for making this journey with me
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Abstract

The LaBGeO$_5$ glass-ceramic composite is a transparent ferroelectric nanocomposite (TFN) material that has come under attention for its ferroelectric properties. LaBGeO$_5$ crystals-in-glass can be formed through controlled devitrification of the glass, as well as through laser irradiation. While structural models of the glass have been proposed, they have only considered differences in the borate environment between the glass and the crystal.

Understanding the structure of the LaBGeO$_5$ glass is an important first step towards understanding the formation of the crystals-in-glass. To study the structure of the LaBGeO$_5$ glass, we made use of a combination of nuclear magnetic resonance (NMR) spectroscopy, neutron diffraction, density functional theory (DFT) calculations, and several other techniques. These techniques provided complementary data on both local order within the glass, as well as connectivity between different local structural units.

$^{11}$B NMR spectroscopy was used to establish the presence of both trigonal and tetrahedral borate units, and to identify the trigonal borate species. $^{11}$B{$^{10}$B} heteronuclear NMR spectroscopy was used to probe the connectivity between the borate units. Neutron diffraction was used to provide evidence of the presence of high-coordinate germanate units, as well as evidence of changes in the lanthanum–oxygen coordination number between the glass and the crystal. $^{139}$La NMR spectroscopy was applied to glasses for the first time in order to provide data on the La–O environment. The use of $^{17}$O NMR spectroscopy provided data regarding the connectivity of all local structural units. The $^{17}$O NMR data support a highly interconnected structure for the LaBGeO$_5$ glass, and indicated that the glass network is homogeneous.

DFT calculations were carried out to probe the stress environment in laser-written LaBGeO$_5$ crystals-in-glass. Our calculations support an anisotropic stress environment with both tensile and compressive stresses being present.

Our results support a model of the structure of the LaBGeO$_5$ glass that is radically different than the crystal structure. The new structural model should inform future studies of the LaBGeO$_5$ laser-written crystals-in-glass. Our data suggest that the nucleation mechanism in the LaBGeO$_5$ glass is heterogeneous, and we propose future work to test this hypothesis.
List of Abbreviations and Symbols Used

3QMAS triple quantum magic angle spinning

$a$ the first unit cell length

$A$ symmetric non-degenerate phonon modes

$A$ the quadrupole product prefactor; the correlation function peak area

$\bar{b}$ the coherent scattering length

$b$ the second unit cell length

BO bridging oxygen

$c$ the third unit cell length; the speed of light in vacuum; the atomic fraction; the crystallite fraction

$C$ the elastic stiffness tensor; the Raman intensity prefactor

CN coordination number

CPMG Carr-Purcell Meiboom-Gill

$C_Q$ the quadrupolar coupling constant

CSA chemical shift anisotropy

$C_z$ a MQMAS referencing convention

$d$ the lattice plane distance

$D$ the dipolar coupling constant

$d_{\text{avg}}$ the average La–O bond length

DFPT density functional perturbation theory
DFT  density functional theory

d_{\text{max}}  the maximum La–O bond length

d_{\text{min}}  the minimum La–O bond length

D(r)  the differential neutron correlation function

DSC  differential scanning calorimetry

e  the elementary charge

E  doubly-degenerate phonon modes

E  the elastic modulus; the energy

e_a  the shortest semi-major axis of a coordination ellipsoid

EA  elemental analysis

e_b  the intermediate semi-major axis of a coordination ellipsoid

EBS  electrostatic bond strength

e_c  the longest semi-major axis of a coordination ellipsoid

EC  ellipsoid character

EDS  energy dispersive spectroscopy

EFG  electric field gradient

EMPA  electron microprobe analysis

f  the quadrupolar REDOR calibration factor

FWHM  full-width at half-maximum

G  glass

G  the shear modulus
\( g^0 \) the atom number density

\( G^{(0)} \) the zeroth rotational invariant

\( G^{(1)} \) the first rotational invariant

\( G^{(2)} \) the second rotational invariant

GEM GEneral Materials Diffractometer

GGA generalized gradient approximation

GIPAW gauge-including projector augmented-wave

\( h \) the Planck constant

\( \hbar \) the reduced Planck constant

\( I \) the nuclear spin number

I the indirect nuclide

ICSD Inorganic Crystal Structure Database

\( I^\parallel_{\text{powder}} \) the parallel Raman intensity

\( I^\perp_{\text{powder}} \) the perpendicular Raman intensity

\( i(Q) \) the distinct scattering

IR infrared

\( I^{S}(Q) \) the neutron self-scattering

\( I_{\text{Total}}^{\text{powder}} \) the total Raman intensity

JTH Jollet, Torrent, and Holzwarth

\( k \) the MQMAS echo position factor; the force constant

\( K \) the bulk modulus
\( k_B \) the Boltzmann constant

L linear

LA-ICP-MS laser ablation inductively coupled plasma mass spectrometry

LBG \( \text{LaBGeO}_5 \)

LDA local-density approximation

LO longitudinal optic

\( M \) the reduced dielectric polarization vector

\( M_{2}^{\text{IS}} \) the van Vleck heteronuclear dipolar second moment

MAS magic angle spinning

MQ multiple quantum

\( M(Q) \) the modification function

MQMAS multiple quantum magic angle spinning

\( n \) the coordination number; the number of atoms

\( N \) the WURST exponent

NBO non-bridging oxygen

NCPP norm-conserving pseudopotential

NMR nuclear magnetic resonance

\( n(\omega) \) the Bose-Einstein factor

NSERC Natural Sciences and Engineering Research Council of Canada

\( \text{O} \) bridging oxygen

\( \text{O}_{666} \) oxygen triply coordinated to \( ^6\text{Ge} \)
Ob bridging oxygen

O_{ab} non-bridging oxygen

ONCVPSP Optimized Norm-Conserving Vanderbilt Pseudopotential

P the isotropic stress

∥ parallel

PAW projector augmented-wave

PBE Perdew, Burke, and Ernzerhof

PDF probability distribution function

PDF powder diffraction file

⊥ perpendicular

p_Q the Czjzek probability distribution function

P_Q the quadrupolar product

pXRD powder X-ray diffraction

q the phonon wavevector

\hat{q} the unit vector of the direction of propagation

q the incoming wavevector

Q the nuclear electric quadrupole moment; the momentum transfer; the degree of polymerization of a phosphate bonding unit

Q_{max} the maximum momentum transfer

r the internuclear distance; the correlation function peak position

R the ratio of lanthanum oxide to boron and germanium oxides
$R^2$ the coefficient of determination

$r_{\alpha \tau}$ the atomic displacement

REAPDOR rotational-echo adiabatic passage double-resonance

REDOR rotational-echo double-resonance

RF radio frequency

RMS root mean square

$r_s$ the average distance between the centroid of a coordination polyhedron and its ligands

$s$ the strain tensor; the electrostatic bond strength

$S$ the observed nuclide

$S$ the REDOR intensity of the observed nuclide with I irradiation

$S_0$ the REDOR intensity of the observed nuclide without I irradiation

SEM scanning electron microscopy

SHG second harmonic generation

ssNMR solid-state nuclear magnetic resonance

$t$ the time

$t_{ii'}(r)$ the partial correlation function

$T$ the temperature

$T^0(r)$ the average density contribution

$t_1$ the multiple quantum coherence evolution time

$T_1$ the longitudinal, or spin-lattice, relaxation time
$T_2$ the transverse, or spin-spin, relaxation time

TFN transparent ferroelectric nanocomposite

$T_g$ the glass transition temperature

TGA thermogravimetric analysis

$T_{gr}$ the reduced glass transition temperature

THF tetrahydrofuran

$T_m$ the melting temperature

TO transverse optic

$T(r)$ the total correlation function

$T_r$ the rotor spinning period

$u$ the neutron peak width

$u_{\alpha\tau}$ the phonon eigenvector

$V$ the electric field gradient tensor

$V$ volume

$v_L$ the longitudinal velocity of sound

$V_M$ the molar volume

VOCS variable offset cumulative spectrum

$v_T$ the transverse velocity of sound

$V_{xx}$ the $xx$ component of the electric field gradient tensor

$V_{yy}$ the $yy$ component of the electric field gradient tensor

$V_{zz}$ the $zz$ component of the electric field gradient tensor
\( W^* \) the work of nucleation

WCPMG WURST Carr-Purcell Meiboom-Gill

WDS wavelength dispersive spectroscopy

\( W \) the weight

WURST wideband, uniform rate, smooth truncation

\( X \) crystal

\( XC \) exchange-correlation

\( z \) the cation valence

\( Z \) the atomic number

\( \alpha \) the first Euler angle; the coefficient of thermal expansion; the ratio of \( BO_3 \) and \( \left[ BO_4 \right]^- \) in the \( LaBGeO_5 \) glass

\( \beta \) the second Euler angle; the Raman susceptibility tensor

\( \gamma \) the gyromagnetic ratio; the third Euler angle

\( \Gamma \) the Brillouin zone centre

\( \Delta \) the WURST sweep width

\( \delta_{11} \) the 11 component of the chemical shielding tensor

\( \delta_{22} \) the 22 component of the chemical shielding tensor

\( \delta_{33} \) the 33 component of the chemical shielding tensor

\( \delta_{G1-z}^{obs} \) the centre of gravity in the isotropic dimension

\( \delta_{G2}^{obs} \) the centre of gravity in the MAS dimension

\( \Delta G_z \) the free energy of crystallization
\( \delta_{ij} \) the Kronecker delta

\( \delta_{\text{iso}}^{CS} \) the isotropic chemical shift

\( \delta_{\text{MQ}}^{\text{M}} \) the shift of the isotropic axis

\( \Delta_{\text{offset}} \) the Czjzek position parameter

\( \Delta S/S_0 \) the normalized REDOR intensity

\( \epsilon \) the ellipsoid span

\( \epsilon_0 \) the vacuum permittivity

\( \epsilon_\infty \) the dielectric constant

\( \epsilon_{ij}^{\infty} \) the high-frequency dielectric

\( \epsilon_\parallel^{\infty} \) the parallel component of the dielectric constant

\( \epsilon_\perp^{\infty} \) the perpendicular component of the dielectric constant

\( \eta_Q \) the quadrupolar asymmetry parameter

\( \theta \) the scattering angle; the tip angle; the phonon wavevector angle;

\( \theta_m \) the magic angle

\( \kappa \) the chemical shift skew

\( \lambda \) the wavelength

\( \mu_0 \) the vacuum permeability

\( \mu_m \) the reduced mass

\( \nu \) the Poisson’s ratio

\( \nu_0 \) the classical vibrational frequency

\( \nu_{\text{off}} \) the offset frequency
\( \nu_Q \) the quadrupolar frequency

\( \nu_{\text{rf}} \) the RF pulse power

\( \rho \) the density

\( \sigma \) the surface energy of the nucleation interface; the stress tensor; the Czjzek breadth parameter; the chemical shielding tensor; the scattering cross-section

\( \Sigma \) the sphericity; the electrostatic bond strength sum

\( \sigma_{\text{iso}} \) the isotropic chemical shielding

\( \sigma_{\text{ref}} \) the reference isotropic chemical shielding

\( \sigma_{r_s} \) the standard deviation of the distance between the centroid of a coordination polyhedron and its ligands

\( \tau_w \) the pulse duration

\( \phi \) the WURST phase

\( \Phi \) the heterogeneous nucleation factor

\( \chi_{ij} \) the electric polarizability tensor

\( \omega \) the radio frequency amplitude; the phonon mode frequency

\( \Omega \) the chemical shift span; the solid angle; the unit cell volume

\( \omega_{\text{cf}} \) the spectrometer carrier frequency

\( \omega_i \) the transverse phonon frequency of mode \( i \)

\( \omega_L \) the laser frequency
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xxx
Chapter 1

Introduction

This dissertation focuses on the study of the LaBGeO$_5$ glass and glass-ceramic system. The LaBGeO$_5$ glass-ceramic system has provoked interest as a potential transparent ferroelectric nanocomposite (TFN) material [1]. Furthermore, single-crystal LaBGeO$_5$ lines can be induced to form in the glass via laser-writing with a femtosecond laser [2]. This transition from amorphous glass to highly-ordered single crystal represents a profound change in the long-range order of the system; however, there is no model for the atomistic changes during laser writing. This is in part due to a lack of understanding of the structure of the glass.

The current structural model suggests that the glass is divided into two different regions: “crystal-like” regions, where the structure of the glass is largely similar to that of the crystal; and regions where the structure is considerably more amorphous. The data presented in support of this conclusion are insufficient to support this model. Hence, in order to support the study of the formation of LaBGeO$_5$ single-crystal lines in the glass, we ask the following question: what is the structure of the LaBGeO$_5$ glass?

This dissertation contains nine chapters, including this introduction.

The second chapter, Chapter 2, is a description of the study of the structure of oxide glasses, a literature review for the LaBGeO$_5$ system, and why the LaBGeO$_5$ system has drawn our interest. It also contains the stated goals of this work in Section 2.5.

The third chapter, Chapter 3, describes the theory and technical underpinnings of the techniques used in this work. Particular focus is given to glass synthesis, advanced nuclear magnetic resonance (NMR) spectroscopy, neutron diffraction, and density functional theory calculations. It does not contain specific experimental details, as these are reported in the appropriate chapters.

The development of a model relating $^{139}$La NMR spectral parameters to the
structure of lanthanum oxide-based crystals is presented in Chapter 4. This model is refined and applied to the $^{139}$La NMR spectra of lanthanum oxide-based glasses in Chapter 5. However, it is not obviously possible to extract the NMR observable used in the crystalline model from the $^{139}$La NMR spectra of glass. Instead, $^{139}$La NMR spectroscopy of glasses is better suited to assessing changes in the disorder surrounding La$^{3+}$.

Chapter 6 presents a combined neutron diffraction and $^{11}$B NMR spectroscopy study of lanthanum borogermanate glasses, including LaBGeO$_5$. The neutron diffraction results support the presence of high-coordinate germanium species in the LaBGeO$_5$ glass.

The connectivity of glass bonding units is probed in Chapter 7. A combination of $^{17}$O NMR spectroscopy and $^{11}$B{$^{10}$B} heteronuclear NMR spectroscopy establish the presence (and absence) of various connectivities in LaBGeO$_5$ glass. The $^{17}$O and $^{11}$B NMR results also support a correction to the literature regarding the determination of the crystallite fraction of LaBGeO$_5$ glass-ceramic composites via high-resolution $^{11}$B NMR spectroscopy.

The Raman spectral response of the LaBGeO$_5$ crystal to isotropic and anisotropic stress is determined via density functional perturbation theory (DFPT). The results are presented in Chapter 8, and compared to micro-Raman data from the literature. A qualitative model for the residual internal stresses in the LaBGeO$_5$ glass-ceramic composite is presented in the context of the anisotropic linear thermal expansion coefficients of the LaBGeO$_5$ crystal.

Finally, the results of the overall work are discussed in Chapter 9. The “data chapters” described above (Chapters 4 to 8) were either published, or prepared to be published, as journal articles prior to the completion of this work; hence they stand largely independent from one another. Chapter 9 presents the overall conclusions that can be drawn from the combination of these chapters, as well as their limitations, and proposals for future work.

There are several appendices at the end of this work that contain supporting information for the data chapters (where applicable), details of the computational work important for reproducibility, as well as copyright permissions. The final appendix contains the bibliography.
Chapter 2

Background

Each of us is carving a stone, erecting a column, or cutting a piece of stained glass in the construction of something much bigger than ourselves.

(Adrienne Clarkson)

Glass is ubiquitous in modern life. Glass is in our homes, our cars, our phones; it is on our bodies, and sometimes in our bodies; and in the form of the fibre-optic cables which make up the backbone of the internet, it ties us all together. When glass is mentioned in common conversation, it is the former uses which generally come to mind, the soda-lime-silicate glass used in buildings. However, the term glass encompasses a wide range of material compositions, some with many different properties and uses beyond construction.

2.1 What is a Glass?

For a material to be considered a glass, it must possess two characteristics. First, a glass is amorphous, and hence structurally isotropic: it does not possess any long-range, periodic arrangement of atoms, and as such has the same properties in all directions. It is possible to break this long-range isotropy by surface treatments (e.g., tempering, ion-exchange), but in the absence of external influences or internal stresses, glasses are essentially isotropic [3]. Second, a glass exhibits a glass transition within a composition-dependent temperature range. The glass transition involves a continuous change in volume with a dramatic change in physical properties such as viscosity and thermal expansion. Both these characteristics must be present in a material for it to be considered a glass, and contribute to some of the special properties of glasses [3, 4].

Many categories of materials can exhibit these characteristics. Some examples which influence everyday life are metallic glasses — such as Metglas, typically a iron-based alloy used in electric transformers [5] — and certain plastics — such as
polycarbonate, used in applications ranging from water bottles to military aircraft [6]. The eponymous example of a glass, however, is the silicate glass. Silicate glasses are an example of the broader oxide glass family of materials. Oxide glasses are wholly or primarily composed of the oxides of metals or metalloids, such as Na₂O, SiO₂, or B₂O₃ [3]. Many oxide glasses share structural similarities across compositions, and share few with metallic or organic glasses. The particular focus of this work, LaBGeO₅, is an oxide glass. Hence this work is focused on the study of oxide glasses.

2.2 Glasses and Glass-Ceramics

2.2.1 The Structure of Oxide Glasses

Perhaps the oldest useful model of glass structure is that put forward by Zachariasen [7]. His 1932 work serves as the foundation of the random network model, though it is worth noting that a glass network cannot be truly random, as there are physical constraints on the internuclear distances of the atoms composing the network. Consider a compound consisting of a cation A and oxygen anion O with formula AₙOₘ. Zachariasen proposed that glass formation may occur in the AₙOₘ system if the following conditions are met:

1. the oxygen-cation coordination number is two or less;

2. the cation-oxygen coordination number is small, i.e., 3 or 4;

3. the cation-oxygen polyhedra share corners, not edges; and

4. at least three corners of the cation-oxygen polyhedra are shared.

Under these four conditions, Zachariasen predicted the following: that oxides with the formula A₂O or AO will not form glasses under any conditions; that oxides with the formula A₂O₃ will form a glass if the A–O coordination number is three (e.g., Fig. 2.1); and that oxides with the formulas AO₂ or A₂O₅ will form glasses if the A–O coordination number is four. These rules serve to describe some of the glass network former oxides, which are capable of forming a glass as the pure material. Typically, the glass network formers are considered to be B₂O₃, SiO₂, GeO₂, and P₂O₅, though this list is not exhaustive [3, 4]. These oxides can form a glass via the melt-quench
Figure 2.1: A schematic representation of the hypothetical two-dimensional compound $\text{A}_2\text{O}_3$. Figure 2.1a represents the periodic, crystalline arrangement, while Fig. 2.1b represents the glass network of the same compound. Note that while both have similar short-range order, the long-range behaviour of the glass and crystal are entirely different. Reprinted (adapted) with permission from W. H. Zachariasen, The Atomic Arrangement in Glass, J. Am. Chem. Soc. 54 (10) (1932) 3841–3851 [7]. Copyright 1932 American Chemical Society.
synthesis approach (Section 3.1). It is important to note that Zachariasen’s conditions are not absolute; for example, SnO$_2$ is at best considered a network modifier, and often simply precipitates from the glass, while SnO can form the majority of a glass composition [8].

Beyond the glass network formers, there are two other broad classes of oxides which can be part of the glass network. The glass network modifiers are oxides which are unable to form a glass on their own, but can be incorporated into a glass composed primarily of a network former [3]. The prototypical example of a glass network modifier is Na$_2$O, which (among other effects) can dramatically decrease the melting point of a silicate glass. Many physical properties of a glass can be modified with the addition of an appropriate glass modifier oxide. Glass network modifiers often have the chemical formula A$_2$O or AO, though A$_2$O$_3$ is not uncommon.

The final class of oxides are known as the glass network intermediates. As the name implies, the network intermediates have properties which are intermediate between those of the formers and the modifiers. In structural terms, this often manifests in behaviour which is sometimes more modifier-like, and sometimes more former-like [3]. The best-known network intermediates are perhaps Al$_2$O$_3$ and PbO, though there are others (e.g., Bi$_2$O$_3$, SnO) which qualify. The line between network modifier and network intermediate is poorly defined; the line between intermediate and former is only slightly clearer. For further discussion of the varying technical definitions of these three classes, see Ref. [3].

In oxide glasses, oxygen can generally be found in one of two broad categories [3]. The first is the bridging oxygen (BO) environment, which is indicated in this work by O. Bridging oxygen are oxygen which link two glass former (or former-like intermediate) cations. The bridged elements can be homogeneous or heterogeneous. For example, B–O–B oxygen are considered bridging, as would Ge–O–B. The second is the non-bridging oxygen (NBO) environment, frequently indicated by O$^-$. Non-bridging oxygen are bonded to a single glass former, as well as a network modifier (or modifier-like intermediate). B–O–Na would be an example of a non-bridging oxygen. There is technically a third possible environment, known as free oxygen, which consists of oxygen bonded to two network modifiers (e.g., Ca–O–Ca); however, free oxygen is not necessarily present in a glass, and if it is it tends to be present in low concentrations [9].
As a general principle, the bonding of bridging oxygen tends to be more covalent, while the bonding of non-bridging oxygen is more ionic [3].

Coordination numbers are the focus of much of the discussion in this work. As mentioned above, Zachariasen predicted that glass formers were expected to have coordination numbers of three or four [7]. In practice, this is generally true; the A-O coordination numbers of pure B$_2$O$_3$, SiO$_2$, GeO$_2$, and P$_2$O$_5$ at standard pressures are three, four, four, and four, respectively. The addition of network modifiers and intermediates introduces species with greater coordination numbers. Modifiers typically have higher coordination numbers than formers; A–O coordination numbers of six to nine are not uncommon [10–13]. Intermediates often have coordination numbers between three and six; the Al–O coordination number again is a classic example, with Al–O coordination numbers of four, five, and six all coexisting in a single glass [14, 15].

The glass formers also have the potential of showing higher coordination numbers. Six-coordinate Si–O polyhedra have been shown to exist in glasses quenched at high pressures [16]. Two of the most studied examples of network formers exhibiting higher coordination are alkali borate and alkali germanate glasses, both of which show changes in coordination in response to the addition of modifier oxide. Both alkali borate and alkali germanate glasses exhibit an “anomaly”, in which various physical properties undergo non-linear changes in response to the addition of alkali oxide [17–19].

The cause of the borate anomaly has been attributed to the changes in boron-oxygen coordination which occurs with the addition of alkali oxide [20]. Pure B$_2$O$_3$ glass is composed of threefold-coordinate BO$_3$ units, with a substantial fraction arranged into “boroxol rings” [21]. As network modifier oxide is introduced into the glass, the BO$_3$ units will convert to fourfold-coordinate [BO$_4$]$^-$ units, dramatically increasing the level of connectivity within the glass network [22]. As the modifier oxide concentration is increased further, BO$_2$O$^-$ units will form, and as it is increased further still even more depolymerized BO$_3$ species will appear. The different possible borate structural units are presented in Fig. 2.2.

The evolution of borate units is quite well understood, in no small part due to the amenable NMR properties of the $^{11}$B nuclide. The same cannot be said for the evolution of germanate structural units. Like borates, alkali germanate glasses show
Figure 2.2: Schematic representation of the basic borate structural units. A) \([\text{B} \text{O}_4]^-\). B) \(\text{B} \text{O}_3\). C) \(\text{B} \text{O}_2 \text{O}^-\). D) \(\text{B} \text{O} \text{O}_2^{2-}\). E) \(\text{B} \text{O}_3^{3-}\). In the figure, the blue circle represents tetrahedral boron, while cyan represents trigonal boron. Red circles indicate bridging oxygen (\(\text{O}^-\)), while magenta represent non-bridging oxygen (\(\text{O}^-\)). Reprinted with permission from A. C. Wright, My Borate Life: An Enigmatic Journey, Int. J. Appl. Glass Sci. 6 (1) (2015) 45-63 [20]. Copyright 2015 John Wiley and Sons.
non-linear evolution of properties with the addition of modifier oxide [23, 24]. This was termed the germanate anomaly. The study of the germanate anomaly is not as straightforward as the borate anomaly, in no small part due to the poor NMR properties of the $^{73}\text{Ge}$ nuclide [19, 25]. The precise nature of the structural rearrangements which cause the germanate anomaly have not been decisively determined, but it is generally accepted that it involves the presence of high-coordinate germanate units, \textit{i.e.}, $\text{GeO}_5$ and/or $\text{GeO}_6$ [19, 26, 27]. Since $\text{GeO}_5$ and $\text{GeO}_6$ are not obviously differentiable in glasses via neutron diffraction, high-coordinate germanium is frequently labelled as $5/6\text{Ge}$ rather than one or the other [26, 28]. As the presence of $5/6\text{Ge}$ can potentially impact the physical properties of the glass, any structural model of a germanate glass must at least consider the presence of $5/6\text{Ge}$.

### 2.2.2 Glass-Ceramics

Chemically, there are no distinctions between “glass elements” and “crystal elements”; a glass and a crystal can share the same composition, \textit{e.g.}, $\text{SiO}_2$, $\text{GeO}_2$, $\text{B}_2\text{O}_3$. The primary distinction between a crystal and a glass is the presence or absence of long-range order. At short length scales, a bond between a silicon atom and an oxygen atom in a crystal is indistinguishable from the same in a glass. It should not come at a surprise, then, that it possible for crystals and glasses to simultaneously exist in the same material. When small crystalline domains, or \textit{crystallites}, exist within a glass matrix, the resulting material is known as a glass-ceramic composite.

Crystallites most often occur in a glass when the liquid glass melt is cooled too slowly [3]. The crystalline form of a material is generally the thermodynamically stable form, whereas the glass is better described as being kinetically stable. When the temperature of the glass is above the glass transition temperature ($T_g$), the atoms composing the glass have enough energy to form nuclei. Here, nuclei are extremely small periodic collections of atoms that are the precursors of larger crystallites [3].

Nuclei can form in two different ways. The first is \textit{homogeneous} nucleation. In homogeneous nucleation, nuclei form spontaneously and stochastically. The chance of homogeneous nucleation is the same everywhere in the glass. The second is \textit{heterogeneous} nucleation. In heterogeneous nucleation, there is some surface or substrate which increases the probability of the formation of nuclei. Heterogeneous
nucleation sites include the surface of the glass, internal cracks or bubbles, or in some cases insoluble precipitates.

The formation of nuclei is governed by two considerations [3]. The first is the energetic barrier of the interface between the glass and the nucleus, which represents the change in structure between the two environments. The second is the change in the total free energy of the system. The crystal-like arrangement of the nucleus lowers the volume free energy of the system, but the introduction of a second environment results in an increase in the surface energy. The first condition imposes a minimum energy requirement on the formation of crystal nuclei; this often manifests as nucleation taking place at or near to the glass transition temperature $T_g$. The second condition affects the likelihood that a given nucleus survives to become a proper crystallite. For small nuclei, the surface energy term dominates, making nucleation energetically unfavourable. For large nuclei, the volume energy term dominates, and nuclei growth is favourable.

The work of nucleation $W^*$, for both heterogeneous and homogeneous nucleation, is given by

$$W^* = \frac{16\pi}{3} \frac{\sigma^3}{\Delta G_x^2} \Phi.$$  \hspace{1cm} (2.1)

Here, $\sigma$ is the surface energy of the interface between the nucleus and the glass, $\Delta G_x$ is the change in free energy from transforming a unit volume of the glass to a unit volume of the crystal (i.e., the free energy of crystallization), and $\Phi$ is the so-called heterogeneous nucleation factor [3]. $\Phi$ can range from 0 to 1 depending on the contact angle between the nucleus and the substrate. When $\Phi = 1$, there is no difference between the heterogeneous and homogeneous nucleation cases. When $\Phi = 0$, there is no energetic barrier to nucleation. Hence, heterogeneous nucleation is either energetically comparable or favourable when compared to homogeneous nucleation.

Both heterogeneous and homogeneous nucleation exhibit behavioural dependencies on the reduced glass transition temperature ($T_{gr}$). $T_{gr}$ is defined as $T_{gr} = T_g/T_m$, where $T_m$ is the melting temperature of the composition under consideration [29]. James and Zanotto suggest that when $T_{gr} \geq 0.6$, only heterogeneous nucleation is possible, whereas when $T_{gr} < 0.6$, both heterogeneous and homogeneous nucleation are possible [30, 31]. These predictions have been experimentally verified for silicate glasses [32].
The maximum nucleation rate is dependent on $T_{gr}$, as well as $\Phi$. Figure 2.3 shows how maximum nucleation rate can vary depending on $\Phi$. All else being equal, heterogeneous nucleation can occur at greater values of $T_{gr}$ than homogeneous nucleation, significantly expanding the range of compositions which can be induced to crystallize.

For the vast majority of systems undergoing heterogeneous nucleation, the maximum nucleation rate will occur at lower temperatures than the maximum crystalline growth rate. This is particularly advantageous for the synthesis of glass-ceramics, as this allows for the semi-independent control of the number of crystallites and their size. The examples in Fig. 2.3 show why this control is only semi-independent: for all cases, the crystallite growth rate overlaps the maximum nucleation rate. It is not generally possible to induce nucleation without at least some growth, nor to grow crystallites without inducing further nucleation. This only becomes a problem, however, if the maxima are at quite similar temperatures.

Two-phase nucleation and growth has been observed in the LaBGeO$_5$ system [33, 34]. The type of nucleation mechanism which is most likely has been hypothesized to be related to structural similarities or differences between the glass and the crystallites [35, 36]. As will be discussed below, the structural differences between the glass and crystal forms of the LaBGeO$_5$ system are a central question of this work, and hence any link between structure and nucleation is of interest. The discussion of nucleation and growth in this work is mostly qualitative due to the difficulty of probing the interface between the nuclei and the glass, which is extremely small in both length and volume. For a more rigorous discussion of the crystallization of glasses, the interested reader is directed to the literature [37–39].

2.3 How Can We Study Glass Structure?

There are many techniques used for the study of the structure of glass, but the three predominant categories are NMR spectroscopy, vibrational spectroscopy, and X-ray and/or neutron diffraction. They each provide complimentary data regarding the structure of a glass. The theory of the techniques used in this study are provided in Chapter 3; the specific technical details are presented in the chapters where they are used. This section provides context on the use of these techniques, and examples of
Figure 2.3: Possible nucleation maxima temperatures for homogeneous and heterogeneous nucleation. For both examples of heterogeneous nucleation, the maximum nucleation rate occurs at a temperature greater than that of the homogeneous nucleation rate. For example 1, $\Phi = 0.1$; for example 2, $\Phi = 0.001$. The difference between the heterogeneous and homogeneous cases is expected to be less dramatic for greater values of $\Phi$. Reprinted from Journal of Non-Crystalline Solids, 429, Jürn W.P. Schmelzer, Alexander S. Abyzov, Vladimir M. Fokin, Christoph Schick, Edgar D. Zanotto, *Crystallization of Glass-Forming Liquids: Maxima of Nucleation, Growth, and Overall Crystallization Rates*, 24–32, Copyright (2015), with permission from Elsevier [29].
the conclusions that can be drawn from their use.

2.3.1 NMR Studies of Glass

NMR spectroscopy is a powerful technique for the study of glasses. It has been used to elucidate glass structure for both short-range and intermediate-range scales [40–46]. The specific details of the application and interpretation of NMR spectroscopy substantially depend on the nuclide under investigation. Therefore, rather than attempting to provide a comprehensive overview of how NMR spectroscopy is applied to all nuclides potentially present in glass, this section describes how the elements present in LaBGeO$_5$, i.e., La, B, Ge, and O, interact with NMR spectroscopy.

Boron NMR spectroscopy is a long-standing technique for the study of borate glasses. It was first applied by Silver and Bray in 1958 to identify $[\text{BØ}_4]^{-}$ units in sodium borate glasses [40]. There are two NMR-active boron nuclides, with very distinct spectroscopic properties [47]. $^{11}$B is a quadrupolar nuclide with half-integer nuclear spin $I = 3/2$. It composes 80% of natural abundance boron. The other NMR-active boron nuclide is $^{10}$B. $^{10}$B is also quadrupolar, but has integer nuclear spin $I = 3$. It composes 20% of natural abundance boron. The most commonly used nuclide is $^{11}$B. It has the greater sensitivity — due to the greater abundance and gyromagnetic ratio — and greater resolution — due to a lesser nuclear quadrupole moment and half-integer spin.

The study of borate glass structure by use of $^{11}$B NMR spectroscopy typically involves the use of magic angle spinning (MAS) and multiple quantum magic angle spinning (MQMAS) NMR experiments. As discussed in Section 2.2.1, borate glasses often contain a variety of borate species. BO$_3$ and $[\text{BØ}_4]^{-}$ species can usually be wholly resolved by use of MAS NMR, but separating BO$_3$ species frequently requires the use of MQMAS. The presence and identity of multiple coexisting BO$_3$ species can be determined, providing insight into the nature and degree of the network polymerization [48, 49]. Additionally, $^{11}$B can detect changes in the connectivity of $[\text{BØ}_4]^{-}$ unit via changes in its chemical shift [50, 51]. $^{10}$B is not generally useful for NMR studies of glass, as the BO$_3$ environment is effectively unobservable; however, as $^{10}$B and $^{11}$B are chemically identical but spectroscopically distinct, the use of $^{11}$B{$^{10}$B} heteronuclear NMR can provide boron-boron connectivity information [52].
Although oxygen is the majority atomic constituent of oxide glasses, oxygen NMR is not routinely applied to the study of glass. There is a single NMR-active oxygen nuclide, $^{17}$O, with nuclear spin $I = 5/2$ and natural abundance 0.037%. Effective $^{17}$O NMR spectroscopy requires that the sample under investigation be enriched with $^{17}$O, typically by hydrolysis of a halide or alkoxide of the appropriate cation. However, typically samples require at least 20% $^{17}$O enrichment, which in turn requires water of greater $^{17}$O concentration; and water highly enriched with $^{17}$O is expensive, in the range of 500 CAD g$^{-1}$ to 2000 CAD g$^{-1}$. The high cost of $^{17}$O enrichment makes $^{17}$O NMR spectroscopy less feasible than NMR spectroscopy of other nuclides.

$^{17}$O NMR spectroscopy, when conducted on an enriched sample, can provide substantial information regarding the connectivity of the glass network. Bridging oxygen and non-bridging oxygen can frequently be resolved and quantified. Homonuclear (X-O-X) and heteronuclear (X-O-Y) bridges usually have different spectral properties, allowing for the investigation of network homogeneity or segregation [53]. Furthermore, different network polyhedra can sometimes be observed via $^{17}$O MAS or MQMAS NMR, which has been used to establish the presence of high-coordinate germanium in alkali germanate glasses [54–56].

Lanthanum NMR spectroscopy has not been widely applied in the literature. Natural abundance lanthanum is 100% $^{139}$La, with $I = 7/2$. The nuclear quadrupole moment of $^{139}$La is quite large [57], leading to extremely broad spectra. Only a single lanthanum-based oxide (LaAlO$_3$) has been collected under MAS conditions, and spectral collection typically requires significant signal enhancement [58–60].

Germanium is even less accessible via NMR spectroscopy than lanthanum. The only NMR-active germanium isotope is $^{73}$Ge, with low natural abundance (7.76%), a large quadrupole moment, and a low gyromagnetic ratio. While $^{73}$Ge NMR spectra of germanate glasses have been collected, the experiments were both prohibitively lengthy as well as being unable to provide insight into the structure of the glass [25]. As $^{73}$Ge NMR spectroscopy is not a viable option for the study of the Ge–O environment in glasses, we instead turn to neutron diffraction, which has been used to study germanate glasses to good effect [12, 42, 61, 62].
2.3.2 Neutron Diffraction Studies of Glass

X-ray and neutron diffraction are valuable tools for determining the structure of crystalline materials. The theory behind their use is described in Sections 3.3 and 3.8. The application of diffraction-based techniques to glasses is somewhat more complex than for crystals. In crystals, the inherent long-range order produces sharp diffraction peaks which are used to determine unit cell dimensions and ionic positions. Unit cell dimension and ionic positions are not well-founded concepts in glasses, and cannot be determined. Instead, diffraction data is used to produce correlation functions, from which bond distributions can be extracted. X-ray diffraction and neutron diffraction are complimentary techniques, as X-ray diffraction is based around the number of electrons in the system, while neutron diffraction is a function of the nuclides present. Neutron diffraction was used in this work in order to leverage the existing literature on germanate glasses [27, 28, 42, 61–67]. It is important to note that for this particular application of X-ray diffraction generally requires a synchrotron X-ray source, and is functionally distinct from powder X-ray diffraction (pXRD). While pXRD was used in this work, no synchrotron X-ray diffraction was carried out.

Neutron diffraction is well-suited for the structural study of glasses due to its high real-space resolution of mean bond lengths. For example, this property has allowed for the detection of the two different P–O bond lengths in phosphate glasses [42, 68]. In principle, if differing germanium coordination environments (i.e., GeO$_4$, GeO$_5$, GeO$_6$) existed in a germanate glass, they would be expected to possess different bond lengths. However, what is observed in alkali germanate glasses is not distinct peaks corresponding to discrete bond length distributions, but a broad tail beginning at the GeO$_4$ peak and extending to longer bond lengths [28, 64]. This is generally understood to indicate a distribution of long Ge–O bonds that are not well-localized [12]. It is useful to note that neutron diffraction can quantify coordination number via integration, and does not require strict resolution of like bonds [27, 69]. Hence the mean coordination number of germanium can be determined without requiring to distinguish between the three possible units [26].
2.3.3 Vibrational Spectroscopy Studies of Glass

Infrared (IR) spectroscopy and Raman spectroscopy have both been applied to the study of glasses. However, Raman spectroscopy tends to provide more valuable information regarding the study of glasses due to the increased resolution of Raman modes as compared to IR modes [70]. In glasses, observable Raman modes are typically the various vibrational modes of the glass network forming polyhedra. Both X–O stretching modes and X–O–X bending modes can be observed, and with very careful experimental considerations, quantified [71]. Certain structures can have extremely distinctive Raman modes, e.g., the sharp peak at \( \approx 805 \text{ cm}^{-1} \) associated with the boroxol ring breathing mode [72]. The vibrational modes of network modifiers are less frequently observed, as their bonding is more ionic and they tend to be heavier, both factors which will tend towards lower vibrational frequencies.

Raman spectroscopy is particularly pertinent to the study of LaBGeO$_5$ glass for two factors: the first is that Raman spectroscopy is frequently applied to the study of germanate glasses, and its use has the potential to provide meaningful data [73]; the second is that much of the current structural understanding of the LaBGeO$_5$ glass stems from Raman spectroscopy [74, 75].

The first structural model of the LaBGeO$_5$ glass, reported by Kratochvilova et al. in 2000, relied on Raman data to conclude that “...short range order in [the LaBGeO$_5$ glass] is the same as in the crystals...”, i.e., [BØ$_4$]$^-$, GeO$_4$, and LaO$_9$ units [74]. This was shown to be incorrect by Gupta et al. in 2004 by the use of $^{11}$B NMR spectroscopy [34]. The $^{11}$B NMR spectrum of LaBGeO$_5$ glass unambiguously shows the coexistence of both BO$_3$ and [BØ$_4$]$^-$ units in the glass. The peak associated with BO$_3$ in the Raman spectra of the LaBGeO$_5$ glass is subtle, but its presence was observable in the spectra reported by Kratochvilova et al., and was confirmed by Sigaev et al. in 2010 [75]. However, determination of coordination number by Raman spectroscopy is not always straightforward, and in particular it is difficult to definitively establish the presence or absence of high-coordinate germanium in glasses via Raman spectroscopy [19]. As such, we used Raman spectroscopy only sparingly in our study of the LaBGeO$_5$ glass.
2.3.4 DFT Studies of Materials

Density functional theory (DFT) calculations are widely used in materials science, and allow for the calculation of a wide variety of physical properties [76]. In this work, DFT is used to calculate NMR parameters of a variety of crystal structures, as well as Raman spectra of the LaBGeO₅ crystal. DFT can often be used to probe systems which would be difficult to test experimentally. However, while DFT calculations of crystalline materials are generally straightforward to run, it is difficult to extend these to amorphous materials such as glasses.

Most DFT calculations of solids take advantage of periodic boundary conditions, which effectively impose perfect long-range order. As glasses have no long-range order at all, let alone perfect long-range order, this complicates the application of periodic boundary conditions to amorphous systems. The typical method of compensating for this imposed long-range order is to use a very large simulation “box” containing hundreds or thousands of atoms. This allows for the sampling of a large number of atoms that are effectively in a disordered environment, but has two main pitfalls. The first is that for large systems, DFT computational costs scale with the cube of the size of the system; anecdotally, the structural optimization for a simple sodium silicate glass can require several core-years of calculations. The second is that the structures used for DFT calculation of glass must be rigorously validated to ensure that the model is accurately describing the physical system under investigation. This too is very computationally expensive.

To summarize, while DFT calculations on crystals are relatively straightforward, the computational costs of applying DFT calculations to amorphous materials makes the latter beyond the scope of this work.

2.4 Which Glass is Studied in this Work?

The glass (and crystal) which are the primary research target of this work have the composition LaBGeO₅. This lanthanum borogermanate composition is a synthetic analogue of the stillwellite mineral (Fig. 2.4). Named in 1955 by McAndrew and Scott, natural stillwellite has the composition (Ce, La, Ca)BSiO₅, depending somewhat on its origin [77]. The system itself is chemically flexible, and generally follows the
Figure 2.4: An example of natural stillwellite. Sample is from the mineralogy collection of the Natural History Museum of London. Photo is of the sample on display on 2016-03-26.
formula $(\text{Ln}^{3+}, \text{M}^{2+})\text{B(Si, Ge, Al, P)(O, OH, F)}_5$. There are two synthetic examples of the stillwellite structure that have garnered particular attention in the literature: LaBSiO$_5$ and LaBGeO$_5$. For reasons which will be discussed below, this work focuses primarily on LaBGeO$_5$.

2.4.1 Crystal Structure and Properties

LaBSiO$_5$ and LaBGeO$_5$ are isostructural with space group $P3_1$ [78, 79]. Part of the interest in these crystals stems from the fact that they are ferroelectric, i.e., they possess a permanent electric dipole moment which can be changed by an electric field [80, 81]. Ferroelectricity is named in analogy to ferromagnetism, though while ferromagnetism is an asymmetry in electronic spin, ferroelectricity arises from an asymmetry of charge [82]. Both ferromagnetism and ferroelectricity are dependent on both the chemical composition of a crystal and also its space group. Ferroelectric materials, in particular, can only arise in polar space groups [80]. Polar space groups are defined by their ability to support a permanent electric dipole moment, and will posses at least one Cartesian direction which differs from the others [80]. In the $P3_1$ space group, this is the $c$ axis. By the necessity of symmetry, ferroelectric materials are potentially pyroelectric and piezoelectric, and by definition all are noncentrosymmetric.

The crystal structure of LaBGeO$_5$ is displayed in Fig. 2.5; the structure of LaBSiO$_5$ is very nearly identical [78, 79, 83]. It has a single crystallographic La$^{3+}$ site, B$^{3+}$ site, and Ge$^{4+}$ site, and five distinct O$^{2-}$ sites. The ferroelectricity of the LaBGeO$_5$ and LaBSiO$_5$ are thought to originate in the [BØ$_4$]$^-$ helix which is parallel to the crystallographic $c$ axis [79, 84, 85]. The $c$ axis is the $3_1$ screw axis of the crystal. Ferroelectric materials undergo a phase transition at a critical temperature (known as the Curie temperature) to a paraelectric phase. The ferroelectric-paraelectric phase transition can be: displacive, where ionic positions are substantially changed; order-disorder, where the crystal structure loses the long-range order necessary to maintain a permanent dipole moment; or some combination of the two [82]. The phase transition in LaBGeO$_5$ is a combination of the two. The high-temperature ($T \geq 530 ^\circ C$) phase of LaBGeO$_5$ is of space group $P3_121$ [79]. In the phase transition, the La$^{3+}$ and Ge$^{4+}$ ions move to a twofold rotation axis, while the B$^{3+}$ ion (and one O$^{2-}$ ion) move towards one of two positions near, but not on, the twofold axis. The
Figure 2.5: The room-temperature crystal structure of LaBGeO$_5$ [78]. The presented view is down the crystallographic $c$ axis with a $2 \times 2 \times 2$ supercell. [BØ$_4$]$^-$ tetrahedra are coloured dark green, while GeO$_2$O$_2^{2-}$ tetrahedra are purple. LaO$_9$ polyhedra are omitted for clarity. Lanthanum ions are light green.
B$^{3+}$ and corresponding O$^{2-}$ site are half-occupancy, and this half-occupancy disrupts the periodic development of the [BØ$_4$]$^-$ helix, hence contributing to the order-disorder nature of the phase transition [79].

The spontaneous polarization of the LaBGeO$_5$ crystal has not been precisely determined. Measurements and simulations by various different techniques have returned a range of values from 2.7 µC cm$^{-2}$ to 12 µC cm$^{-2}$ [85–88].

The optical properties of LaBGeO$_5$ also are a source of scientific interest. LaBGeO$_5$ has non-linear optical properties, and in particular is capable of second harmonic generation (SHG) [89]. SHG involves the process known equivalently as frequency doubling or wavelength halving: two photons with the same frequency interact within the crystal structure, and a photon of half the original frequency is emitted [90]. The largest SHG tensor component of the LaBGeO$_5$ crystal is 0.76 pm V$^{-1}$, comparing favourably to quartz (0.3 pm V$^{-1}$) and lithium triborate (0.84 pm V$^{-1}$) [91–93]. The SHG properties of LaBGeO$_5$ have lead to interest in its use as a potential optical waveguide, or as a self-doubling laser [94].

2.4.2 Our Interest

Our interests in the LaBGeO$_5$ system are primarily structural. As displayed in Fig. 2.5, the crystal structure of LaBGeO$_5$ is known; and as discussed above, many physical properties have been measured and are understood [78, 79, 91, 95, 96]. The glass has also been subject to investigation, and some of its properties (e.g., $T_g$, shear modulus $G$, linear coefficient of thermal expansion $\alpha_L$) have been previous reported [96]. Our interest, however, stems from the formation of LaBGeO$_5$ glass-ceramic composites by laser writing of the glass.

Glass-ceramics are partially-crystallized glasses which share properties of both the crystal and the glass. In the LaBGeO$_5$ system in particular, it is the ferroelectric properties of the crystal which are desirable. The glass cannot be ferroelectric (as it is disordered on long length scales), but it is substantially more straightforward to synthesize the LaBGeO$_5$ glass than a single LaBGeO$_5$ crystal. By inducing crystallisation of the glass, a LaBGeO$_5$ glass-ceramic is formed. LaBSiO$_5$ has also been targeted as a glass-ceramic; however, recent results show that pure LaBSiO$_5$ glass cannot be formed [97]. Literature reports on LaBSiO$_5$ glass samples typically are the
result of Al₂O₃ contamination of the glass from synthesis in Al₂O₃ crucibles [97, 98].

There are two aspects of LaBGeO₅ glass-ceramics which have received interest in the literature. The first is that LaBGeO₅ can form a TFN material. The LaBGeO₅ glass is optically transparent, and if the crystallites are sufficiently small (less than approx. 40 nm) they will not significantly impede the transmission of light [1]. Many studies have been carried out with the intent of optimizing crystallite growth via heating of the bulk LaBGeO₅ glass [33, 34, 99, 100].

The second interesting aspect of the LaBGeO₅ glass is that crystallisation of the glass can be induced via laser irradiation. First reported in LaBGeO₅ by Gupta et al. in 2007, laser-induced crystallisation has been performed using both continuous-wave and femtosecond lasers of a variety of wavelengths [2, 101–104]. Recent work has focused on the use of fs lasers to produce LaBGeO₅ crystal “lines” which appear to be a single crystal [2, 103–108]. These lines can be written into complex geometries, including forking paths, without observed grain boundaries which would indicate a polycrystalline sample [107]. The nature of the change from the disorder of the glass form to the extreme order of the single-crystal form is an interesting question.

Despite many studies concerning the formation of these laser-written LaBGeO₅ glass-ceramics, several important questions remain unanswered. First among them is “what are the atomistic changes in the glass during the crystallisation process?” Answering this question requires an understanding of both the crystal structure, which is well-known, and the glass structure, which is not.

The current structural model, proposed by Sigaev et al., divides the LaBGeO₅ glass into two different types of “nanoregions”: the first with a high concentration of four-coordinated boron in “‘crystal-similar’ fragments”; and the second being considerably more disordered, with the boron being threefold coordinate [75]. Both germanium and lanthanum are expected to have the same coordination behaviour in the glass as in the crystal [75]. The current structural data available in the literature are the Raman spectra presented by Sigaev et al. and the ¹¹B NMR spectra presented by Gupta et al. [34, 74, 75]. Both are consistent with the presence of both BO₃ and [BO₄]⁻ units. However, neither speak to the connectivity of BO₃–[BO₄]⁻ units. Furthermore, ¹¹B NMR by its nature cannot provide direct information regarding the local structure of La or Ge. In principle Raman spectroscopy can provide information regarding both
La–O bonds and Ge–O bonds, but in practice the observation of La–O vibrations is impractical. GeO$_6$ can be observed in crystalline rutile-type GeO$_2$, but whether GeO$_5$ or GeO$_6$ can be detected in glasses via Raman spectroscopy was not clear at the time that the model of Sigaev et al. was reported [19, 73, 75]. In short, there are insufficient data available in the literature to produce a comprehensive model of the LaBGeO$_5$ glass structure, a problem that this work rectifies.

A second lingering question is the stress environment of the laser-written LaBGeO$_5$ lines. The thermal evolution of LaBGeO$_5$ during the laser-writing process is complex, but generally involves a portion of the glass being liquefied at high temperature, cooling sufficiently to crystallise (expected to occur above $T_g$), passing through $T_g$ (at which point the glass network ceases to easily deform), passing through the phase transition at $T \gtrapprox 530^\circ$C, and then cooling to room temperature [103]. This process induces enough stress in the sample that it cannot be carried out at room temperature; instead crystallisation is carried out with the bulk sample pre-heated to 400°C [103, 108]. The complex thermal evolution, as well as the anticipated mismatch between the thermal expansion coefficients of the glass and the crystal, would likely lead to residual internal stress on the glass-ceramic composite material. The residual stress on the laser-written crystal has the potential to affect both its ferroelectric and optical properties, the two primary potential applications of the LaBGeO$_5$ TFN material [1, 108]. As such, it is important to understand the stress environment of laser-written LaBGeO$_5$ composites.

2.5 The Central Goals of this Work

The primary goal of this work is to develop a model of the short- and intermediate-range structure of the LaBGeO$_5$ glass, and thereby allow for future investigations into the crystallization mechanism of the laser-written LaBGeO$_5$ glass-ceramic composite. To this end, both the local environments of La, B, Ge, and O, as well as the connectivity of the glass network must be probed.

The first structural model of the LaBGeO$_5$ glass assumed that all local structure in the glass was the same as the crystal, i.e., [BØ$_4$]$^-$, GeO$_4$, and LaO$_9$ [74]. However, the boron-oxygen coordination number undergoes a dramatic change between the crystal (entirely [BØ$_4$]$^-$) and the glass (both BO$_3$ and [BØ$_4$]$^-$) [34]. The current structural model of the LaBGeO$_5$ glass acknowledges this change, but concludes that only the
borate environment is changing [75]. With the knowledge that the germanium-oxygen coordination number can in principle be greater than four, there is no \textit{a priori} reason to discount the possibility of the presence of high-coordinate germanium in the LaBGeO$_5$ glass, and so the possible presence of \textit{5/6}Ge must be investigated. There is no data whatsoever on the behaviour of lanthanum in the LaBGeO$_5$ glass; however, given the established changes in the boron environment, it is entirely possible that there will be significant differences here as well. Hence, to create a model of the short- and intermediate-range structure of the LaBGeO$_5$ glass, new data must be acquired.

The data which need to be acquired can be grouped into two categories: short-range order, concerning cation-oxygen coordination; and intermediate-range order, concerning the connectivity of network unit polyhedra. In the chapters that follow, the short-range order of La, B, and Ge in the LaBGeO$_5$ glass is probed by the use of a combination of NMR spectroscopy and neutron diffraction.

The change in B coordination number has been established in the literature, but the identity (or identities) of the BO$_3$ unit (or units) has not been determined [34]. Through the use of $^{11}$B MAS and MQMAS NMR spectroscopy, the BO$_3$ environment can be characterized. This will provide data on the degree of polymerization of the borate network in the LaBGeO$_5$ glass.

As $^{73}$Ge NMR is not an effective method for characterizing germanium in glasses, we turn to neutron diffraction. Neutron diffraction provides confirmatory data regarding the boron-oxygen coordination number, but most importantly is a direct probe of the distribution of bond lengths in the germanium-oxygen polyhedra. As high-coordinate germanium is expected to have substantially greater bond lengths than four-coordinate germanium, the observation of long germanium-oxygen bond lengths would support the hypothesis that the germanium environment in the glass is significantly different than the crystal.

Neutron diffraction can, in principle, probe the lanthanum-oxygen environment in LaBGeO$_5$ glass, but given the length scales of typical La–O bonds (approx. 2.4 Å to 2.8 Å) there is the possibility of substantial peak overlap in a glass as chemically complex as LaBGeO$_5$. $^{139}$La NMR spectroscopy offers the potential to characterize the lanthanum-oxygen environment without interference from the rest of the glass network, but to date there have been no $^{139}$La NMR spectra of lanthanum-containing
glasses reported in the literature.

In order to make use of $^{139}$La NMR for the study of the LaBGeO$_5$ glass, we must first create a model of the response of $^{139}$La NMR properties to changes in structure in crystalline materials. With a working crystal model of $^{139}$La NMR behaviour, we can begin to interpret $^{139}$La NMR spectra of glasses. This will provide data regarding the lanthanum-oxygen coordination number in the LaBGeO$_5$ glass, which can then be compared to that in the LaBGeO$_5$ crystal.

The approach described above provides data regarding the short-range order of the cation components of the glass, but provides limited information regarding the intermediate-range order. The intermediate-range order of the glass network formers (i.e., boron and germanium) is more interesting than that of lanthanum, as the order of the former can be used to deduce the latter. The structural model proposed by Sigaev et al. proposes a network where three-coordinate boron is substantially segregated from four-coordinate boron [75]. To probe the intermediate-range order of the glass, we turn to $^{17}$O NMR spectroscopy and $^{11}$B{$^{10}$B} heteronuclear NMR spectroscopy. Both have been used to probe the connectivity of glass networks; in particular, the latter allows for a direct test of the model proposed by Sigaev et al. [52]. If the $^{11}$B{$^{10}$B} heteronuclear NMR provides data suggesting BO$_3$–[BO$_4$]$^-$ avoidance, then the model of Sigaev et al. becomes more plausible. However, if the data suggests that BO$_3$–[BO$_4$]$^-$ units are substantially connected, then this would contradict the Sigaev et al. model.

An additional point of interest in the LaBGeO$_5$ system is the mechanism of crystallization. It has been hypothesized that if the structures of the crystal and glass forms are similar, homogeneous nucleation is more likely [35, 36]. Conversely, if the structures of the crystal and glass forms are dissimilar, heterogeneous nucleation is more likely. Observing heterogeneous nucleation in the LaBGeO$_5$ system would support a structural model where the structure of the glass is dissimilar to the structure of the crystal.

Finally, the stress environment of a laser-written LaBGeO$_5$ sample is evaluated via the use of DFT calculations. Previous literature studies on the stress environment in laser-written LaBGeO$_5$ samples concluded that the stress was isotropically compressive, and 0.75 GPa in magnitude [108]. However, the determination of both the magnitude
and direction of the stress were based upon Raman spectral data from a LaBGeO$_5$ crystal which was isotropically compressed, which may not reflect the complex geometry of the system [109]. We conduct DFT calculations to provide insight into the response of the LaBGeO$_5$ crystal to stress, and re-evaluate the available literature data.

In summary, the goals of the work described in this document are as follows:

- obtain data on the short-range structure of boron in the LaBGeO$_5$ glass via $^{11}$B NMR spectroscopy;
- probe for the presence of high-coordinate germanium in the glass via neutron diffraction;
- develop a model relating $^{139}$La NMR parameters to short-range lanthanum structure in crystalline materials;
- apply this crystalline model to $^{139}$La NMR spectrum of the LaBGeO$_5$ glass to determine the lanthanum-oxygen environment;
- probe the connectivity of the LaBGeO$_5$ glass via $^{17}$O NMR spectroscopy;
- test the Sigaev model directly via $^{11}$B{$^{10}$B} heteronuclear NMR spectroscopy;
- examine the mechanism of crystallization of the LaBGeO$_5$ glass-ceramic composite;
- and evaluate the stress on laser-written LaBGeO$_5$ glass-ceramics via DFT calculations.
Chapter 3

Theory and Methods

The characterization of the structure and properties of oxide glasses and crystals can require a wide variety of different methods and techniques, as no single technique can provide all requisite data on a given material. As such, a wide variety of experimental and theoretical approaches are described in this document. The following chapter focuses on the theoretical underpinnings of the techniques used in later chapters. Specific experimental details are generally found within the applicable chapters.

3.1 Sample Preparation

All glasses produced for this work were produced using the melt-quench approach. The melt-quench approach generally involves three to five steps:

1. Glass reagents (often metal/metalloid oxides) are intimately mixed in stoichiometric amounts.

2. The combined reagents are calcined at a temperature lower than the melting temperature in order to drive off gaseous by-products.

3. The combined reagents are brought to a sufficiently high temperature to melt them.

4. The glass melt is “quenched” via rapid cooling through the glass transition temperature.

5. The quenched glass is annealed at a temperature near, but lower than, the glass transition temperature ($T_g$).

The first, third, and fourth steps are the fundamental steps of the melt-quench method of glass synthesis. The second and fifth steps are often useful, but are not necessary for certain applications.
The choice of reagents for glass synthesis depends on a number of factors, including stability, cost, and by-products (in particular gasses released during melting) [4]. Metal or metalloid oxides (or their precursors) are combined in stoichiometric amounts, melted at high temperature, and then “quenched” via rapid cooling of the melt. This rapid cooling of the melt passes through the $T_g$ quickly enough that the formation of crystal nuclei is inhibited, and a glass is formed [3]. The precise rate of cooling can have observable effects on the structure of the glass, but for the investigations within this work it is sufficient that the rate of cooling is high enough to prevent crystallization.

The oxide reagents or oxide precursors were combined as powders, and intimately mixed using a porcelain mortar and pestle. Glass precursors composed solely of oxide reagents (such as $\text{La}_2\text{O}_3$, $\text{B}_2\text{O}_3$, and $\text{GeO}_2$) can be placed directly into a furnace of the appropriate temperature to melt the mixture. If the glass precursor requires the use of a reagent that will be converted to the desired oxide (e.g., $\text{H}_3\text{BO}_3$, $\text{NH}_4\text{H}_2\text{PO}_4$), it can be beneficial to add a calcining step at an intermediate temperature prior to melting [4]. This calcining step allows for by-product gases (e.g., $\text{H}_2\text{O}$, $\text{NH}_3$, $\text{CO}_2$) to be removed, in order to prevent their incorporation into the glass. For glasses containing components that are volatile, or have volatile by-products, limiting the length of time spent at high temperatures can mitigate mass loss.

During glass melting, the choice of crucible becomes important. An ideal crucible is stable at the melting temperature of the glass, does not interact chemically with the glass, and is minimally wet by the glass (to aid in pouring for quenching). Crucibles are commonly produced from alumina, silica, or various metals. For glasses where the tolerance for impurities is low, it is wise to use platinum or platinum-alloy crucibles. Both Pt and Pt/Au crucibles were used in this work, as alumina contamination in lanthanum borogermanate and lanthanum borosilicate compositions has particularly severe consequences [97, 98].

Quenching can be accomplished by a variety of means, including pouring into moulds, pressing flat between metal plates or rollers, dipping the bottom of the crucible into water, or directly pouring the melt into water. The details of quenching are not explored in this work; for further details on quenching refer to the literature [3]. Glasses described herein were quenched either by pouring onto a brass plate (typically
preheated to near 400 °C), or in the case of particularly viscous glasses, by dipping the bottom of the crucible into water.

After quenching, it is usually valuable to immediately anneal the glass at a temperature near to, but below, $T_g$. Annealing the glass allows for residual thermal stresses to relax. If residual thermal stresses are allowed to persist, it is possible that the glass will fracture upon cooling to room temperature. Even if the glass is stable at room temperature, the forces introduced by e.g., cutting and polishing can interact with the residual stress and cause cracking. For the preparation of glasses for optical measurements, or for elemental analysis (Section 3.4) annealing is a recommended step. However, for analysis by techniques that do not depend on a flat surface (e.g., NMR spectroscopy, neutron diffraction) it is not strictly required.

For certain applications (e.g., neutron diffraction, $^{17}$O NMR) it may become important to isotopically enrich the sample. Enrichment with $^{11}$B is straightforward, as $^{11}$B$_2$O$_3$ ($\geq 99\%$ $^{11}$B) is commercially available. Enrichment with $^{17}$O is somewhat more complicated, as $^{17}$O is typically sold as $^{17}$O$_2$(g) or $\text{H}_2^{17}$O(l). The precise method of introducing the $^{17}$O to the glass will vary with composition, but a common method is via hydrolysis of an appropriate metal halide or alkoxide $^{[55]}$. Borane in solution is also acceptable for enrichment of borates, avoiding the by-product of hydrofluoric acid. The hydrolysis typically takes place in a double displacement reaction. For example, for the production of $^{17}$O-enriched GeO$_2$:

$$\text{Ge(OCH}_2\text{CH}_3)_4(l) + 2 \text{H}_2^{17}\text{O}(l) \rightarrow \text{Ge}^{17}\text{O}_2(s) + 4 \text{CH}_3\text{CH}_2\text{OH}(l).$$

In this work $\text{Ge}^{17}\text{O}_2$ and $\text{H}_3\text{B}^{17}\text{O}_3$ were produced with the assistance of Prof. Alex Speed of the Dalhousie University Department of Chemistry.

### 3.2 Physical Properties

#### 3.2.1 Density

Density is a particularly important physical property for the study of glass structure, as it is a quantity frequently required in the measurements of other properties (e.g., neutron diffraction, Section 3.8). The mass density of the glass samples were determined via two methods: via use of Archimedes’ principle, and via helium pycnometry.
The use of Archimedes’ principle is an elegant means of determining the density of a sample of arbitrary shape [110]. Its working equation is straightforward and requires the knowledge of only a few simple properties. Determining the density of a sample \( \rho_{\text{sample}} \) requires only measuring the weight of the sample in air \( W_{\text{air}} \), the weight of the sample in a working liquid \( W_{\text{liquid}} \), and the density of the working liquid \( \rho_{\text{liquid}} \). \( \rho_{\text{sample}} \) is then calculated by

\[
\rho_{\text{sample}} = \rho_{\text{liquid}} \left( \frac{W_{\text{air}}}{W_{\text{air}} - W_{\text{liquid}}} \right)
\]

Sample weights are typically straightforward to determine accurately, only requiring an analytical balance. The density of the working liquid may present a source of error, as it has the potential to change with temperature or with time (e.g., for hygroscopic liquids). In this work the working liquid was absolute ethanol, and its density was varied as a function of the temperature at the time of measurement. Measurements were reproducible to within \( \leq 1\% \). Accuracy was calibrated via measurement of a piece of fused quartz, which has a reliable density of 2.20 g cm\(^{-3}\).

For applications which are significantly affected by the accuracy of the density (e.g., the determination of coordination number via neutron diffraction), or for powdered samples, helium pycnometry can be used instead of Archimedes’ principle [110]. In helium pycnometry, a sample of known mass but unknown volume is inserted into a chamber of known volume. The chamber is then filled with a known volume of helium. The pressure of helium in the sample chamber can then be used to calculate the volume of the sample. Calculating the density of the sample is then straightforward. Measurements were generally reproducible to within \( \leq 0.2\\% \).

The density of the sample is closely related to the molar volume \( (V_M) \) of the material [110]. The molar volume is the ratio between the molar mass of the glass and its density. Determination of the molar mass of a sample requires accurate knowledge of both its density (as above) and its composition (see Section 3.4). The advantage of molar volume is that it removes the mass term from the density, which can make comparisons between differing compositions more straightforward.
3.2.2 Thermal Analysis

Both thermogravimetric analysis (TGA) and differential scanning calorimetry (DSC) were used occasionally in this work; however, they were not the focus of any particular portion. The description of their operation will be brief; for further detail the reader should refer to reference [110].

In TGA, the weight of a sample is recorded as it undergoes changes in temperature. The sample is placed on a precision balance inside of a furnace. As the temperature is increased, mass changes can indicate a number of different phenomenon. Adsorbed water may be released, or phase transitions can release gases (e.g., H$_2$O, CO$_2$) that were previously incorporated in the crystal structure. In this work TGA was used to determine the temperatures of several decompositions, for example

$$\text{LaPO}_4 \cdot n\text{H}_2\text{O} \rightarrow \text{LaPO}_4 + n\text{H}_2\text{O}.$$  

In DSC, the temperature of the sample (contained within a pan of known mass and thermal properties) is held at the same temperature of an empty reference pan. The difference in power provided to the pans is plotted as a function of temperature. Thermal events, such as phase changes, crystallization, decomposition, and importantly the glass transition can be observed, and their temperatures quantified, by careful analysis of the DSC curve. In this work $T_g$ was taken as the inflection point of the glass transition event.

3.2.3 Elastic Properties

The linear elastic behaviours of materials are described by the elastic stiffness tensor, $C_{ij}$ [111]. The elastic stiffness tensor is defined as

$$\sigma_{ij} = C_{ijkl}s_{kl}$$  \hspace{1cm} (3.2)

where $\sigma$ is the stress applied to the system and $s$ is strain experienced by the system. For small values of $\sigma$, the amount of strain is proportional to the applied stress, and is described by $C_{ij}$. The elastic stiffness tensor is generally sufficiently symmetric that its order can be reduced via Voigt notation [112]. For isotropic materials such as glasses,
many of the elements are zero, and most others are related by symmetry. All non-zero elements can be fully described by three terms, only two of which are independent:

\[ C_{44} = \frac{1}{2}(C_{11} - C_{12}). \]  

(3.3)

Furthermore, the elastic moduli (in this case the Young’s modulus \( E \), the bulk modulus \( K \), the shear modulus \( G \), and the Poisson’s ratio \( \nu \)) of glasses are interrelated such that there are only two independent values. With the knowledge of any two, the other two can be calculated exactly [3]. The elastic moduli can be related to \( C_{ij} \) via the following relations:

\[ C_{11} = \frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)} \]  

(3.4)

\[ C_{12} = \frac{E\nu}{(1 + \nu)(1 - 2\nu)} \]  

(3.5)

\[ C_{44} = G = \frac{E}{2(1 + \nu)}. \]  

(3.6)

The elastic moduli can be measured via a variety of means, but for glasses the ultrasonic method is particularly convenient [3]. The ultrasonic method relies upon two facts: the first is that glasses are isotropic, and hence have no directionally-dependent properties; the second is that for isotropic materials, the elastic moduli can be related to the speeds of sound in that material. There are two speeds of sound: the longitudinal velocity \( v_L \); and the transverse (or shear) velocity \( v_T \). They can both be measured in a similar fashion with only a change in transducer.

In the ultrasonic method, a transducer propagates a pulse through a glass sample of known thickness. The time required for the pulse-echo to be observed is used to calculate the velocity of the pulse. With both the longitudinal and ultrasonic velocities,
the various elastic moduli can be calculated:

\[
\nu = \frac{1 - 2 \left( \frac{v_T}{v_L} \right)^2}{2 - 2 \left( \frac{v_T}{v_L} \right)^2}
\]

(3.7)

\[
G = v_T^2 \rho
\]

(3.8)

\[
K = \rho \left( v_L^2 - \frac{4}{3} v_T^2 \right)
\]

(3.9)

\[
E = \frac{\rho v_T^2 (3v_L^2 - 4v_T^2)}{v_L^2 - v_T^2}
\]

(3.10)

The density is a term for the calculation of almost all of the moduli, and hence a precise determination of the elastic moduli requires good knowledge of the density (see Section 3.2.1). Accurate measurement of the velocities requires a homogeneous glass sample that is free of cracks or bubbles. The reliability of the method is greatest when the glass sample is prepared with parallel sides.

### 3.3 Powder X-Ray Diffraction

PXRD is a widely-used technique in the field of materials science. Typical applications include the identification of crystalline phases, the measurement of crystallite sizes in polycrystalline samples, and the quantification of samples of mixed phase [113]. As will be discussed below, the direct application of pXRD to amorphous materials (e.g., oxide glasses) is difficult due to their disordered nature. However, as oxide glasses are typically produced using polycrystalline reagents, pXRD is nevertheless a valuable tool in the synthesis and study of glassy materials.

The fundamental equation governing pXRD is known as Bragg’s law. Bragg’s law describes the diffraction of radiation of wavelength \( \lambda \) by a series of atomic planes separated by a distance \( d \). Figure 3.1 shows the diffraction from such a pair of parallel planes. Incident X-rays will be diffracted from the planes regardless of geometry. However, if the difference in path length between reflections from neighbouring planes is an integer multiple of the wavelength of the X-rays, they will interfere constructively. Hence Bragg’s law is stated as

\[
n\lambda = 2d \sin \theta
\]

(3.11)

where \( \theta \) is the scattering angle and \( n \) is an integer [114].
Figure 3.1: A schematic of Bragg’s law for a pair of parallel planes. The X-ray beams constructively interfere when the path length difference between them is an integer multiple of the wavelength, a value which is equal to $2d\sin\theta$.

As presented in Fig. 3.1, it is straightforward to visualize the effects of Eq. (3.11) for a simple pair of parallel planes. However, the atomic structure of a crystal can lead to a set of distinct and varied planes which depend on not only the relative magnitude and orientation of the lattice vectors, but also on the identity of the atoms making up the crystal. This makes X-ray diffraction a powerful tool for the determination of crystal structures. Additionally, as the X-ray diffraction pattern is characteristic of both the identity of the atoms and their relative position within the crystal structure, pXRD can be used to identify the composition and phase of unknown materials.

Typically the identification of a sample involves comparison of the experimental diffraction pattern to a database of previously reported diffraction patterns, rather than determining the structure de novo for each sample. These databases are sometimes proprietary (e.g., the International Centre for Diffraction Data’s Powder Diffraction File) and sometimes open-source (e.g., the Crystallography Open Database). In either case, phase identification is typically based primarily upon the position of the diffraction peaks, and then by their intensity. A basic knowledge of the likely composition of the sample can be quite beneficial, in order to narrow the set of possible
matches.

The effectiveness of Eq. (3.11) is conditional on a number of factors. The incident X-ray beam must be both collimated and monochromatic to ensure only reflections corresponding to a particular lattice spacing are observed. Similarly, consistent lattice spacings (i.e., long-range structural order) must exist for constructive interference to occur [113]. This latter condition severely restricts the application of pXRD to glasses, which by definition are disordered on long length scales.

With no consistent lattice spacings over long ranges, well-defined Bragg peaks are absent from X-ray diffraction patterns of glasses. Instead, broad and featureless distributions are observed when conducting pXRD of amorphous samples. It is technically possible to extract statistical information regarding the glass structure from pXRD of glasses, but this typically requires exacting experimental conditions and detailed modelling [113]. Other techniques (e.g., NMR spectroscopy, Raman spectroscopy) are frequently better suited to the restrictions imposed by the disordered structure of glass. However, pXRD has one particularly important application in the study of glasses. As Bragg diffraction peaks of crystallites are generally quite narrow (and substantially narrower than any glassy distribution), the presence of even small amounts of crystallites in a glass can be observed via pXRD. PXRD is therefore an important technique for confirming the amorphous nature of a glass sample prior to further study.

X-ray diffraction can be run on a variety of different types of instruments (varying X-ray sources, sample geometry, and detectors), and a comprehensive discussion of their relative features and merits is beyond the scope of this work [113]. A brief description of the type of instrument used in this work follows; a schematic is presented in Fig. 3.2. X-rays are generated via a Cu Kα source: low-energy electrons are emitted from a filament (typically tungsten), accelerated towards a copper anode via a strong electric field (on the order of tens of keV), and upon impacting the anode trigger the release of X-rays. The X-rays are collimated by a divergence slit, which ensures that the X-ray beam that diffracts off of the sample is reasonably parallel. The diffracted beam passes through a receiving slit (minimizing spurious reflections), and the diffracted beam is ensured to be monochromatic by passing it through a monochromator prior to detection. For X-ray diffraction, a monochromator is typically a single crystal with
Figure 3.2: A schematic of a Bragg-Brentano powder X-ray diffractometer. X-rays are collimated by a divergence slit and diffract off the sample. Diffracted X-rays are collimated by a receiving slit prior to being diffracted by a monochromator. The monochromatic, parallel beam of X-rays then reaches the detector. The scattering angle is varied by moving the X-ray source and the detector assembly (receiving slit, monochromator, detector).
well-known $d$-spacing, allowing for particular wavelengths to be selected via Eq. (3.11). The diffracted X-rays corresponding to a given scattering angle are detected via a scintillation detector. The scattering angle is controlled via simultaneous movement of the X-ray source and the detector, in a configuration known as the Bragg-Brentano geometry. With this configuration, the incident X-rays are collimated, and the detected X-rays are monochromatic, thus fulfilling the requirements of Bragg diffraction.

### 3.4 Elemental Analysis

Characterization of the composition of a given sample — typically termed elemental analysis (EA) — is an important part of understanding the structure and properties of the sample. Without careful EA, it can be difficult to derive trends based upon compositional changes, as there are a number of effects which can alter the composition of a glass as it is melting.

One common source of contamination is the crucible that the sample is prepared in. Typical crucibles for the production of glasses are made of silica, alumina, or a platinum alloy (e.g., Pt, Pt/Au, or Pt/Rh). Silica and alumina are particularly troublesome as sources of contamination, as both $\text{SiO}_2$ and $\text{Al}_2\text{O}_3$ are easily incorporated into many glass compositions. Platinum-based crucibles are generally effective at mitigating contamination, as Pt is not easily incorporated into oxide glasses.

Another frequent source of differences between nominal and actual composition is oxide volatility. Many oxides and their derivatives (e.g., $\text{P}_2\text{O}_5$, $\text{B}_2\text{O}_3$, $\text{Cs}_2\text{O}$) are known to become volatile at high temperatures [115]. Depending on the composition of the glass, it may not be possible to mitigate these losses by melting at lower temperatures. Hence EA is an important aspect of understanding the system under investigation.

There are a number of different EA techniques that are commonly used to characterize the composition of glasses. Briefly, they can be categorized into two categories: “wet” techniques, where the sample is dissolved via an appropriate solvent (frequently an acid), and the solution is characterized by common analytical methods; and “dry” techniques, where the sample is non-destructively characterized.

Most glass samples in this thesis are either LaBGeO$_5$ or a close relative. LaBGeO$_5$ is quite chemically durable, and we found it difficult to dissolve directly. It is common to use a flux to enhance the solubility of the sample: the analysed glass is melted
with a water-soluble glass, and the resulting mixed glass is analysed. However, the fluxes used in this method are typically borates; given that LaBGeO$_5$ glasses are substantially composed of boron, have high melting temperatures, and are known to have concerns regarding borate volatility, we were concerned about the reliability of this approach for our particular system. We turned instead to two dry techniques: wavelength dispersive spectroscopy (WDS) and laser ablation inductively coupled plasma mass spectrometry (LA-ICP-MS).

### 3.4.1 Electron Microprobe Analysis

Electron microprobe analysis (EMPA) is a common method of characterizing the elemental composition of glasses. As with many techniques, an electron microprobe can be coupled to different detectors. The two EMPA detectors in common use allow for energy dispersive spectroscopy (EDS) and WDS. We first discuss the operation of the electron microprobe, and then the differences in physics and application between EDS and WDS.

Both EMPA detectors operate by analysing characteristic X-rays released by the sample [116]. In order to generate the X-rays, the sample must be bombarded by free electrons. The free electrons are generally produced by heating a tungsten filament to high temperatures. The tungsten filament produces a large number of low-energy electrons that are not directionally-oriented. For EMPA to be useful, the electron beam must be both high-energy and well-collimated. Both are achieved by applying a strong ($\sim$15 keV) positive voltage to a metal plate with an aperture. The low-energy electrons are drawn to the plate (increasing their kinetic energy), and only those with appropriate directionality pass through the aperture (improving the collimation of the beam). The collimation is further improved by a series of magnetic lenses. The beam diameter at the focal point (typically the surface of the sample) is on the order of 1 to 10 µm.

Upon impacting the sample, the electron beam scatters in various ways. For EMPA, the important scattering occurs in the core electrons of the atoms in the sample. The incident high-energy electron displaces a core electron, producing a vacancy. This vacancy is filled with an electron from a higher energy level. An electron moving from a higher energy level to a lower energy level releases energy in the form of a photon.
For the core electrons, this photon is an X-ray. The precise energy of the X-ray is dependent on the electronic structure of the atom in question. As such, each element has a characteristic set of X-ray emissions in EMPA, and hence EMPA can be used to identify the elements present in a sample.

A common detection method for EMPA is EDS. In EDS, all X-rays emitted by the sample are detected nearly simultaneously, and separated based upon their energy [116]. The detector is a semiconductor block which determines the energy of an incoming X-ray by counting the number of electrons that it causes to enter the conduction band. Some of the advantages of EDS detection (as opposed to WDS detection) are that EDS collection is extremely fast, and can detect a significant number of elements simultaneously. However, the elemental specificity of an arbitrary EDS peak is not necessarily straightforward. While the set of X-ray photons emitted by an atom are characteristic of a particular element, an X-ray of a given energy (or equivalently, wavelength) could indicate one of many elements. An element will emit X-rays of a variety of energies, which are often quite disparate. This can cause substantial peak overlap between EDS peaks of different elements, hindering accurate quantification.

An additional complication of EDS is that it is frequently conducted with detectors equipped with beryllium windows. The beryllium of the window is notionally X-ray transparent, but will in fact attenuate X-rays produced by the lighter elements (\( Z \leq 11 \), i.e., sodium or lighter) [116]. If so equipped, EDS is then not suitable for the analysis of borate glasses.

For analysis of the lighter elements, WDS is preferred. Whereas EDS separates the characteristic X-rays of the sample by energy, WDS separates them by wavelength [117]. While the wavelength of the X-ray is related to its energy by the Planck-Einstein relation (i.e., \( E = hc/\lambda \)), it is usually easier to effect changes in X-ray geometry than in detector physics. By working in wavelengths, we can take advantage of Bragg’s Law (see Eq. (3.11)).

In WDS, the characteristic X-rays are generated in the same fashion as in EDS. Rather than collecting all X-rays simultaneously, however, specific X-ray wavelengths are selected. The details of X-ray collection are similar to the operation of a powder X-ray diffractometer (See Section 3.3). The important differences between WDS and EDS come from the X-ray selectivity enabled by fine control of the detector geometry.
The wavelength resolution of WDS is far superior to the energy resolution of EDS, enabling both elemental selectivity and decreasing the limit of detection. However, as different elements cannot be analysed simultaneously, WDS collection times are generally significantly greater than EDS collection times [117].

Despite the technical ability of WDS to analyse light elements, such as boron, accurate and precise quantitative determination of these elements can be quite challenging. Boron is particularly difficult to quantify reliably, as its characteristic X-rays are very low-energy, and are frequently absorbed by heavier elements present in the sample (e.g., La, Ge). It is possible to apply matrix effect corrections to attempt to compensate for the resulting low intensity of boron characteristic X-rays, but substantial uncertainties can remain despite the best efforts of the experimenter. This is an inherent complication to quantification of borate glasses present in any X-ray-based technique.

3.4.2 LA-ICP-MS

An alternative method for elemental analysis of glasses is LA-ICP-MS. In LA-ICP-MS, a laser is used to remove small amounts of material from the sample ("ablation"). The ablated material is conveyed to an argon plasma torch, heated to high temperatures by time-varying magnetic fields ("induction"). The highly-energetic argon particles in the torch ionize the constituent atoms of the sample material. The resulting ions are then directed to a mass spectrometer, which separates the ions based upon their atomic mass. A schematic of the instrument is presented in Fig. 3.3.

As a non-X-ray technique, LA-ICP-MS does not face particular challenges when quantifying light elements such as boron. However, due to potential variation in the ablation of the sample from the laser, effective use of LA-ICP-MS requires significant effort to optimize the performance of the laser for a given sample. Furthermore, accurate quantification in LA-ICP-MS is best achieved with reference materials which match the sample matrix, and with established external calibration standards [118]. LaBGeO₅ is a particularly difficult target in this context, as it is dissimilar to the geological or common industrial glasses that receive the most attention for standardization. Matrix effects are quite likely, as commercial glass are typically silicates; and even in glasses which contain lanthanum, boron, and germanium, LaBGeO₅ will
Figure 3.3: A schematic of a LA-ICP-MS instrument. A laser impacts the sample, which causes particles to be released from the surface. The particles are drawn to an inductively coupled argon plasma torch, which atomizes the particles and ionizes the atoms. The ions are drawn into a mass spectrometer, which separates them based on their mass and charge. The atoms can be identified based upon the time taken to pass through the mass spectrometer. The entire procedure is carried out under varying levels of vacuum. No part of the diagram is to scale.
contain substantially greater fractions of each. Hence WDS and LA-ICP-MS are best considered complimentary techniques, with WDS providing a reliable La/Ge ratio and LA-ICP-MS providing an overall approximation of the composition.

3.5 Nuclear Magnetic Resonance Spectroscopy

NMR spectroscopy is an incredibly useful technique in the field of chemistry. While solution-state NMR is of particular use to synthetic chemists, in part due to the spectroscopic properties of $^1$H, solid-state nuclear magnetic resonance (ssNMR) has been extensively applied to the studies of crystals and glasses. The particular utility of ssNMR when applied to amorphous materials such as glasses comes from two fundamental aspects of NMR spectroscopy. The first is that NMR spectroscopy is practically nuclide-selective. While certain nuclides can have resonances which overlap at certain field strengths (e.g., $^{137}$Ba and $^{35/37}$Cl [119]), for most investigations a given spectrum arises from the target nuclide. This selectivity is particularly valuable in chemically complex glasses. The second useful aspect of NMR spectroscopy for the study of glasses is that NMR spectroscopy primarily (though not exclusively) probes local order. As opposed to pXRD, which requires substantial long-range order, NMR spectroscopy can provide useful information regarding the local environments of crystals, glasses, intermediate states, and composites.

NMR spectroscopy, whether carried out on solution-state or solid-state samples, is a mature technique. Rather than rehashing the fundamentals of NMR spectroscopy from first principles, the novice reader is directed to one of many introductory textbooks on the subject [47, 120, 121]. The contents of this section focus on the aspects of ssNMR that have particular relevance to the work contained in later chapters, and assume a base level of experience with NMR principles.

3.5.1 Magic Angle Spinning

MAS is a fundamental technique for ssNMR. The technique involves packing a sample (typically powdered) inside a cylindrical rotor, and then spinning the rotor on a bed of air at high speeds at an angle to the magnetic field. The “magic angle” is approximately
$\theta_m = 54.74^\circ$ [122]. Its origin is the root of the second Legendre polynomial,

$$P_2(\cos \theta_m) = \frac{1}{2} (3 \cos^2 \theta_m - 1) = 0. \tag{3.12}$$

In a rotating system, many NMR interactions contain the second Legendre polynomial, and hence rotating at the solution of Eq. (3.12) will effectively negate these interactions. This results in a substantial narrowing of the peaks, enhancing their resolution. Interactions which contain the second Legendre polynomial include the first-order (but not the second-order) quadrupolar broadening, the chemical shift anisotropy, and the dipolar coupling.

It is important to note that MAS is only effective at mitigating an interaction when the rotation speed is comparable to the magnitude of the interaction (in Hz) [47]. Spinning speeds in excess of 130 kHz have been reported in the literature, but typical spinning speeds are substantially lower, typically between 4 kHz and 30 kHz. The physical maximum spinning speed of a rotor depends on its material (typically ZrO$_2$) and its diameter, with narrower rotors more capable of higher speeds. However, narrow rotors can only contain small amounts of sample. Hence rotor choice is typically governed by the amount of sample required for timely acquisition, and not simply the highest available speed.

Even the highest available MAS speeds can prove to be wholly insufficient when applied to nuclides with extremely large quadrupole moments. For example, fully narrowing the $^{139}$La NMR spectrum of LaBGeO$_5$ would require MAS spinning speeds in excess of 440 kHz at a field strength of 16.4 T. For such broad peaks, instead of attempting MAS the spectra are collected under static (i.e., non-spinning) conditions. Sensitivity enhancement techniques are frequently used as well.

### 3.5.2 Wideband, Uniform Rate, Smooth Truncation Pulses

Wideband, uniform rate, smooth truncation (WURST) pulses were first introduced by Kupče and Freeman in 1995 [123, 124]. While the full name is an accurate description for the shape and behaviour of the pulse, it is likely a “backronym”; the pulses typically resemble sausages (i.e., rounded-end cylinders). WURST pulses are intended to provide uniform adiabatic inversion of the bulk magnetization over a wide
bandwidth. The time-dependent amplitude \( (\omega_1(t)) \) of a WURST-N pulse is given by

\[
\omega_1(t) = \omega_{\text{max}} \left( 1 - \left| \cos \left( \frac{\pi t}{\tau_w} \right) \right|^N \right)
\]  

(3.13)

where \( \omega_{\text{max}} \) is the maximum radio frequency (RF) amplitude and \( \tau_w \) is the duration of the pulse. The exponent \( N \) determines the overall shape of the pulse. At low values of \( N \), the maximum RF amplitude is reached only briefly, and a WURST-2 pulse is similar to a hyperbolic secant pulse. At large values of \( N \), the WURST pulse resembles a square pulse, with sharp truncations. Intermediate values of \( N \) are required for the WURST pulse to have both smooth truncation and significant time spent at maximum RF amplitudes. WURST-80 pulses are a commonly used compromise between these two competing requirements [125].

The phase \( (\phi) \) of a WURST pulse is described by

\[
\phi(t) = \pm 2\pi \left\{ \left( \frac{\nu_{\text{off}} + \Delta}{2} \right) t - \left( \frac{\Delta}{2\tau_w} \right) t^2 \right\}.
\]  

(3.14)

\( \Delta \) is the sweep width of the WURST pulse. \( \nu_{\text{off}} \) is the offset frequency of the centre of the sweep. This phase modulation allows for the linear progression of the effective frequency of the pulse, and contributes to its wideband character. It also allows for the frequency to be swept in both positive direction (i.e., low to high frequencies) and negative direction (i.e., high to low frequencies). To ensure homogeneous excitation, it is typically advisable to run two experiments, one with each sweep direction, rather than using one or the other.

Empirically, the WURST pulses used in this work provided fairly homogeneous excitation over a range of approximately 80% of \( \Delta \), consistent with literature reports [125]. The maximum \( \Delta \) that could be achieved on the spectrometer available was 1 MHz. However, \( \Delta = 500 \text{ kHz} \) was found to generally provide more homogeneous spectra when considering the peak breadth, sensitivity, and response to changing sweep direction. WURST pulses often provide lower absolute signal-to-noise ratios than square pulses of equivalent power, but due to their greater bandwidth the resulting overall intensity is greater.
3.5.3 The WCPMG Pulse Sequence

Even with the substantial bandwidth provided by WURST pulses, many NMR line widths are so broad to prohibit their collection within a single spectral window. For example, for the spectrometers used in this work, the maximum frequency window which can be collected is 1 MHz, while the $^{139}$La NMR spectrum of LaBGeO$_5$ spans more than 1.2 MHz at 16.4 T (Chapter 4) [126]. To collect these “ultra-wide” spectra, it is typically necessary to generate a variable offset cumulative spectrum (VOCS) [127, 128]. VOCS involves collecting multiple spectra (“slices”) at various transmitter frequencies (chosen to ensure homogeneous excitation of the linewidth), summing them, and analysing the resulting spectrum. As multiple experiments must be carried out, for practical acquisition the time required for each slice must be minimized. An effective signal-enhancement method for this task is the WURST Carr-Purcell Meiboom-Gill (WCPMG) pulse sequence [123–125, 129–131]. The WCPMG pulse sequence is the combination of the well-known Carr-Purcell Meiboom-Gill (CPMG) sequence with WURST pulses.

The CPMG pulse sequence is an excitation pulse followed by a series of refocusing pulses [130, 131]. The refocusing pulses result in a “train” of echoes which persists
until coherence is lost due to transverse relaxation. When this echo train is Fourier transformed, it results in a spectrum composed of “spikelets”. The intensity manifold of these spikelets reproduce the powder lineshape of the spectrum. The density of the spikelets depends on the spacing of the echoes. A greater delay between echoes leads to more densely packed spikelets, allowing for greater resolution of spectral features. However, the greater delay between echoes reduces the total number of echoes collected, reducing the level of signal enhancement. The choice of echo spacing is therefore a balance between signal and resolution. However, even small numbers of collected echoes can lead to a substantial signal enhancement as compared to a Hahn-echo experiment. The spikelets are centred on the excitation frequency of the VOCS slice, and hence the position of the centre spikelet contains no information. This is in contrast to a MAS experiment, where the spinning speed determines the position of the spinning sidebands relative to an isotropic peak, the position of which is independent of the excitation frequency.

The above paragraph deals with the CPMG pulse sequence. The only difference between the CPMG sequence and the WCPMG sequence is that the latter uses WURST pulses, rather than “square” pulses. The WCPMG pulse sequence was first reported by O’Dell and Schurko in 2008 [129], and is displayed in Fig. 3.5. The effects of using WURST pulses in the CPMG echo train are: that the bandwidth of the experiment is increased, as per Section 3.5.2; and that the absolute number of echoes which can be collected are slightly decreased, due to the typically longer pulse lengths of WURST pulses. However, the net integrated intensity of WCPMG is generally greater than that of CPMG using square pulses. The net result in the context of VOCS acquisition is that WCPMG slices require less time to collect, and fewer are required, which together greatly decrease total experimental time for homogeneous spectral collection.
3.5.4 Multiple Quantum Magic Angle Spinning

MQMAS is an invaluable technique in ssNMR of quadrupolar nuclei. Developed in 1995 by Medek et al., MQMAS is a two-dimensional NMR experiment which has been widely applied to both crystalline and amorphous materials [132]. The primary advantage of the MQMAS experiment is the dramatic increase in resolution in spectra with multiple environments which overlap in the MAS experiment. There are a large number of variations and developments upon the basic MQMAS pulse sequence, but the fundamental experiment involves four steps [133]:

1. Multiple quantum (MQ) coherences are excited by RF pulses;
2. The MQ coherences are allowed to evolve;
3. The MQ coherences are reconverted to single quantum coherences via additional RF pulses; and
4. The signal is detected.

The evolution of the MQ coherences causes the second-order quadrupolar interactions to be averaged, resulting in very sharp half-integer quadrupole resonances. The theory behind MQ coherences is beyond the scope of this work; the reader is directed to the literature for further details.

The practical considerations of MQMAS follow [47]. Split-$t_1$ MQMAS spectra are two-dimensional NMR with a MAS axis and an “isotropic” axis. The projection of the MAS axis is generally extremely similar to a single-pulse MAS experiment, subject to lineshape distortions introduced in the MQMAS experiment. The projection of the isotropic axis is free of anisotropic quadrupolar and chemical shift contributions. Peaks in the isotropic axis are generally quite narrow (in well-ordered materials), leading the isotropic axis to also be referred to as the “high-resolution” axis. The position of a peak in the MQMAS spectrum is dependent on both its isotropic chemical shift ($\delta^\text{CS}_\text{iso}$) and its quadrupole parameters ($C_Q$, $\eta_Q$). Hence peaks which may be poorly resolved in the MAS experiment (which separates based upon $\delta^\text{CS}_\text{iso}$) can be fully resolved in MQMAS.

Resolution limits in MQMAS generally arise from one of two causes. The first is spectral. The number of data points which can be collected is restricted by the $T_2$
relaxation of the environments present in the sample. If the relaxation is fast, only a limited number of points can be collected, and hence the resolution of the isotropic axis cannot be improved. The second cause is structural. The isotropic average is free of anisotropic quadrupole and chemical shift interactions. However, if there exists a distribution of environments with similar chemical shift and quadrupole parameters (as in, for example, a glass) they will result in a relatively broad peak. Both fast relaxation and structural broadening are common in glasses, and as such MQMAS spectra of glasses are rarely as sharp and well-resolved as in crystalline materials.

Beyond providing qualitative evidence of the existence of multiple environments in a sample, MQMAS spectra can provide quantitative values for $\delta_{iso}^{CS}$ and for $P_Q$. The centres of gravity of a peak in the MAS and isotropic dimensions can be used to calculate the values of $\delta_{iso}^{CS}$ and $P_Q$ for said peak, with the precise formula depending on the referencing convention used [134]. In this work all MQMAS spectra are referenced with the $C_z$ convention [132, 134]. In the $C_z$ convention, the isotropic chemical shift and quadrupole product are calculated as follows:

$$\delta_{iso}^{CS} = \frac{10}{27} \delta_{G2}^{obs} + k_{Gz} \delta_{G1-z}^{obs} \quad (3.15)$$

$$P_Q = A \sqrt{k_{Gz} \left( \delta_{G1-z}^{obs} - k_{1z} \delta_{G2}^{obs} \right)} \quad (3.16)$$

$k_{Gz}$ and $k_{1z}$ are factors affecting the position of the MQMAS echo, and depend on the spin of the nuclide under investigation ($I$), as well as the MQ transition being excited. Their values are tabulated by Millot and Man [134]. $\delta_{G1-z}^{obs}$ and $\delta_{G2}^{obs}$ are the centres of gravity of the peak in the isotropic and MAS dimensions, respectively. $A$ is a spin- and frequency-dependent prefactor required for calculating $P_Q$, and is

$$A = I(2I - 1) \frac{\omega_{cf}}{2\pi} \sqrt{\frac{40}{3 \left[ I(I + 1) - \frac{3}{4} \right]}} \quad (3.17)$$

These equations are valid for both crystalline and amorphous materials, though in the latter they are better understood as means of a distribution, rather than as precise values. The consequences of distributions of quadrupole parameters are discussed in further detail in Section 3.5.5.
3.5.5 The Czjzek Model

Glasses are disordered materials. Rather than the well-defined bond lengths and angles present in a crystal, in glasses these properties are distributed over a range of possible values. Similarly, the electric field gradient (EFG) tensor, and its elements, will also be subject to this statistical disorder. An analytical expression for describing statistical disorder in quadrupolar environments was first formulated by Czjzek in the context of Mössbauer spectroscopy [135]. Czjzek’s model has been extended to NMR studies of quadrupolar nuclei in disordered materials via the efforts of Le Caër, Brand, Massiot, and many others [136–139]. It is known under several names, including the Czjzek model and the Gaussian Isotropic Model. A straightforward derivation of the Czjzek model is provided by de Lacaillerie et al.; rather than repeat the derivation, a summary of the important experimental considerations are provided [139].

The Czjzek model is a joint probability distribution function (PDF) of $\nu_Q$ and $\eta_Q$, where $\nu_Q$ is

$$\nu_Q = \frac{3eQ}{2I(2I - 1)}hV_{zz}. \quad (3.18)$$

The analytical expression of the joint PDF is

$$p_Q(\nu_Q, \eta_Q) = \frac{1}{\sqrt{2\pi}\sigma^5}\nu_Q^4\eta_Q\left(1 - \frac{\eta_Q}{9}\right)\exp\left[-\frac{\nu_Q^2\left(1 + \frac{\eta_Q^2}{3}\right)}{2\sigma^2}\right]. \quad (3.19)$$

Here, $\sigma$ is the standard deviation of the five independent elements of the EFG tensor. This distribution analytically describes the asymmetric lineshape of disordered quadrupolar environments in NMR spectra, and is particularly useful for quantitatively fitting partially resolved environments (e.g., $^{27}$Al NMR spectra of glasses). The validity of this PDF relies on four assumptions, quoted from de Lacaillerie et al. [139]:

1. the structural elements contribute additively to the EFG;
2. the sets of random variables defining the structural elements constitute independent random variables;
3. the number of structural elements contributing to the EFG is “sufficiently” large;
4. the ensemble of structural elements responsible for the EFG constitute a statistically isotropic solid.
The practical consequences of these assumptions are twofold. The first is that the Czjzek model is only meaningful for systems which are disordered on length scales exceeding that of the range of the quadrupole interaction. Oxide glasses easily fulfill this requirement. The second consequence is due to the third assumption; empirically, “sufficiently large” has been found to mean a coordination number of four or greater [139]. This prevents the meaningful application of the Czjzek model to $^{11}$B or $^{17}$O NMR spectra, as the coordination numbers of these nuclides are typically lower than four. In this work, the Czjzek model is only applied to $^{139}$La NMR spectra.

The Czjzek model has been widely applied to NMR studies of glasses, encompassing many quadrupolar nuclei. However, it is less straightforward to extract meaningful structural information from the Czjzek model than it is from fitting well-defined crystalline lineshapes. The Czjzek model has a single breadth parameter, $\sigma$, which is the common standard deviation of the distributions of the independent elements of the EFG tensor. The mean value of $C_Q$ (and equivalently $P_Q$ and $\nu_Q$) in the Czjzek model is zero, while the mean value of $\eta_Q$ is a constant with approximate value of 0.610. $\sigma$ can, however, be related to the standard deviations of $C_Q$ and $P_Q$ by proportionality constants. It can also be related to the most probable values of $C_Q$ and $P_Q$. Hence, while $\sigma$ cannot be meaningfully related to the well-defined $C_Q$ or $P_Q$ values of crystals, it can be used as a quantitative descriptor of the disorder surrounding a given nuclide.

The utility of the Czjzek model comes primarily from its ability to analytically fit disordered quadrupolar NMR lineshapes, and not the value of $\sigma$. Once such a lineshape has been fit, in principle both $\sigma$ and $\delta_{iso}^{CS}$ values can be extracted. The $\sigma$ values can be compared between glass samples, while the $\delta_{iso}^{CS}$ values can be compared between both glass and crystalline samples. However, care must be taken that the Czjzek position parameter, $\Delta_{\text{offset}}$, actually corresponds to $\delta_{iso}^{\text{CS}}$; a simple check of this is fitting spectra of the same sample collected at different magnetic field strengths.

### 3.5.6 Rotational-Echo Double-Resonance

First reported in 1989 by Gullion and Schaefer, the rotational-echo double-resonance (REDOR) pulse sequence is designed to reintroduce the dipolar coupling between nuclei that has been removed via MAS [140]. Dipolar coupling is the interaction between two magnetic dipoles (e.g., nuclear spins), and can occur in both homonuclear
and heteronuclear cases. Homonuclear dipolar coupling can be a significant source of spectral broadening in systems where the nuclide in question is abundant (e.g., $^1$H, $^{11}$B), and it is typically desirable to remove this interaction via MAS when possible. Mitigating the substantial dipolar coupling of $^1$H, often on the order of 50 kHz, is a major drive towards faster MAS speeds [47]. In nuclei pertinent to glasses, the dipolar coupling is usually sufficiently small to be mitigated by modest MAS spinning rates.

The REDOR experiment reintroduces the dipolar coupling between an observe nuclide (S) and an indirect nuclide (I). In the REDOR experiment, two spectra are collected under near-identical experimental conditions. Their pulse sequences are displayed in Fig. 3.6. The first experiment is a Hahn-echo of nuclide S. The second experiment is a Hahn-echo of nuclide S with $\pi$ pulses applied to nuclide I during the evolution period. The $\pi$ pulses on nuclide I invert the sign of the S-I dipolar
coupling interaction, preventing it from being averaged out during MAS, and allow for the interaction to be refocussed by the Hahn-echo pulse. The magnitude of the dipolar coupling constant can be measured by evaluating the difference in intensity of the Hahn-echo experiment without $I$ irradiation ($S_0$) and the experiment with $I$ irradiation ($S$). This difference is typically normalized to the $S_0$ intensity, and is written as $\Delta S/S_0$.

The heteronuclear dipolar coupling constant, $D$, is a function of the internuclear distance between spins. It is given by

$$D = \frac{\mu_0 \gamma_I \gamma_S h}{4\pi r^3}. \quad (3.20)$$

$\mu_0$ is the vacuum permeability, $\gamma_S$ is the gyromagnetic ratio of the observed nuclide, $\gamma_I$ is the gyromagnetic ratio of the unobserved nuclide, and $r$ is the internuclear distance between S and I [47]. Equation (3.20) allows for the internuclear distance between two spins to be determined, making it a valuable tool for structure determination. However, it has several limitations. It is well-defined only for isolated spins, and not for large ensembles. It requires a well-defined $\pi$ pulse length for complete inversion, which can prove difficult to achieve with quadrupolar nuclei [141].

For REDOR experiments on quadrupolar nuclei within crystals and glasses, rather that calculating the exact distance between isolated spins, the van Vleck heteronuclear dipolar second moment is calculated [142]. The van Vleck heteronuclear dipolar second moment $M_{IS}^2$ is defined as

$$M_{IS}^2 = \frac{4}{15} \left( \frac{\mu_0}{4\pi} \right)^2 \gamma_I^2 \gamma_S^2 \hbar^2 I(I + 1) \sum I \langle r_{IS}^6 \rangle \quad (3.21)$$

where $I$ is the nuclear spin quantum number of nuclide I [143]. The author would like to take a moment to apologize for the duplicate use of so many symbols, but is adhering to the conventions of the literature. The van Vleck second moment does not allow for precise distances to be extracted, but does allow for quantitative comparisons of the relative distances between spins S and I in systems with unknown structure. $M_{IS}^2$ can be related to the REDOR observable via

$$\Delta S/S_0 = \frac{f}{I(I + 1)\pi^2} (NT_i)^2 M_{IS}^2. \quad (3.22)$$
\( T_r \) is the rotation period of the MAS rotor, and \( N \) is the number of rotor cycles it has undergone \([142]\). It is this \( NT_r \) parameter which is under experimental control. \( f \) is a calibration factor accounting for the imperfect excitation which occurs when both the S and I nuclides are quadrupolar. It must be calibrated by comparing the experimentally determined value of \( M_2^{\text{IS}} \) to the theoretical value from a known crystal structure \([41, 52]\).

The van Vleck second moment is useful in glasses because, for small values of \( \Delta S/S_0 \), the curve of \( \Delta S/S_0 \) with respect to \( NT_r \) is independent of geometry \([144]\). A “small” value of \( \Delta S/S_0 \) is typically between \( 0 \leq \Delta S/S_0 \leq 0.2 \) \([41]\). Due to the complicated interactions present in systems where both S and I are quadrupolar, it is generally most informative to compare unknown samples to closely related model compounds, and to be cognizant about the (at best) semi-quantitative nature of the comparisons \([141]\).

### 3.5.7 Quantification in Solid-State NMR Spectroscopy

Many quantities can be derived from ssNMR spectroscopy. For glasses, two are particularly relevant: chemical shift and coordination number. Chemical shift values rarely require consideration beyond the quality of the chemical shift referencing and the quality of the fit. Mean coordination numbers in well-resolved systems (e.g., \(^{11}\text{B} \) NMR of borate glasses) are typically derived from integrated intensities. When comparing integrated intensities of different environments, care must be taken to ensure that their response to the RF pulse is homogeneous. Briefly, three factors must be considered: \( T_1 \) relaxation; sensitivity-dependence; and processing effects.

\( T_1 \) relaxation is perhaps the most straightforward factor which affects the quantification of intensities. In short, all environments in a spectrum must be fully relaxed between scans to allow for quantitative relative intensities \([47]\). This can be ensured by determining effective \( T_1 \) times for each environment (e.g., via a saturation-recovery experiment), and setting the recycle delay appropriately for the longest time. The use of short tip angles serves to generally reduce the recycle delay required \([145]\).

The sensitivity of an environment can vary substantially depending on the experiment being conducted. In this work, four different types of experiments are reported, all on quadrupolar nuclei: single-pulse; MQMAS; WCPMG; and REDOR. They are
considered in order. For single-pulse experiments, for a given environment the pulse length corresponding to maximum intensity (i.e., the 90° or \( \pi/2 \) pulse length) depends on \( C_Q \). Hence two environments with widely differing \( C_Q \) values cannot be simultaneously maximized with a single pulse length. Rather than attempting to maximize every environment simultaneously, it is best to exploit that for short pulse lengths, the sensitivity of an environment is independent of \( C_Q \) [146–148]. The maximum tip angle corresponding to this homogeneous response region is spin-dependent, and can be calculated by

\[
\theta = \frac{90}{3(I + 1/2)}
\]  

(3.23)

The \( \pi/2 \) pulse length can generally be determined from a reference sample with a highly-symmetric environment (e.g., \( \text{NaBH}_4 \)). Conveniently, reducing the tip angle also reduces the required recycle delay [145].

In MQMAS experiments, the intensity of the site is similarly dependent on \( C_Q \) [132]. However, the pulse lengths cannot be altered to allow for \( C_Q \)-independent excitation. Hence MQMAS intensities are not generally considered to have quantitative intensities.

REDOR experiments are not generally quantitative in the relative intensities between environments, and are typically conducted with pulse lengths striking a compromise with overall maximum intensity. However, this does not pose an issue to the interpretation of the experiment. The important value derived from REDOR experiment is the ratio of the change in intensity with the re-introduction of dipolar coupling. As long as the pulse lengths are consistent within a set of experiments, the ratios are not dependent on the pulse length.

WCPMG provides signal enhancement by the refocusing of large numbers of echoes. The number of echoes which can be refocused depends substantially on \( T_2 \). Hence, sites with significantly different \( T_2 \) values will experience different amounts of signal enhancement. As the \( T_2 \) of an environment within a sample cannot be known a priori, WCPMG spectra generally have non-quantitative relative intensities.

The final factor which can affect the reliability of integrated intensities is the processing required to obtain the final spectrum. This particularly concerns \(^{11}\text{B}\) single-pulse ssNMR spectra, as the MAS stator contributes a substantial background signal. Both background subtraction and phasing can affect the reported intensities of the peaks. Even with substantial care being taken during phasing and background
subtraction, errors in the integrated intensities are often on the order of 5%.

3.6 Raman Spectroscopy

Raman spectroscopy is a common technique for the study of glass structure [71]. It is (comparatively) inexpensive and technologically mature. In Raman spectroscopy, monochromatic light (typically from a moderately-powered laser) passes through some volume of a material and is scattered in all directions [149]. Much of the light is elastically scattered, and contains no pertinent structural information. Similarly, some light is absorbed and converted to heat. The balance is inelastically scattered. Inelastically scattered light has two separate frequency ranges: the Brillouin range, containing light scattered by acoustic vibrations within the sample; and the Raman range, containing light scattered by so-called “optic vibrations” [150].

Both types of inelastic scattering involve interactions with the quasiparticles known as phonons. Phonons are the quantization of collective movement (or vibration) of atoms within a crystal structure. Like photons (and indeed all particles) phonons can be modelled as a wave, a particle, or both, depending on circumstance. Phonons are grouped into two categories: acoustic, named due to their resemblance to sound waves; and optic, named such due to their interactions with visible light. It is optic phonons which interact with the laser light in Raman scattering.

In molecules, the vibrational motion of atoms can be well defined, often involving only two atoms. This allows for the use of the classical vibration equation

\[ \nu_0 = \frac{1}{2\pi} \sqrt{\frac{k}{\mu_m}} \]  

(3.24)

where \( \nu_0 \) is the vibrational frequency, \( k \) is the force constant of the bond between the two atoms, and \( \mu_m \) is the reduced mass of the two atoms [149]. In crystals and glasses, both of which are highly interconnected, vibrations often involve the motion of many atoms, and Eq. (3.24) cannot be used directly. However, its key insights still apply: atoms which are tightly bound will tend to have higher frequencies; and atoms which are heavy will tend to have lower frequencies.

In molecules, the point group symmetry determines whether a particular vibration will be Raman-active. In crystals, it is the space group which restricts the activity
of a given vibrational mode. In glasses, all modes possible for the composition are active to at least some degree, due to the absence of long-range symmetry and the disorder present even at short ranges. However, the bonds in the glass are essentially the same as the bonds present in the crystal — for example, in GeO$_2$ a Ge–O bond is essentially the same regardless of whether the GeO$_2$ is in crystalline or vitreous form. The Raman spectra of crystalline materials can therefore be used to assist in the interpretation of Raman spectra of glasses.

When high-quality data is coupled with structural modelling (e.g., force fields, molecular clusters, DFPT), the structural origin of a given Raman mode can be deduced. With knowledge of the structural origins of a mode in a control sample, changes in the frequency, intensity, or presence of modes in the Raman spectrum of an unknown can be attributed to structural changes. Structural changes which can be visible in Raman spectra include changes in atomic coordination (e.g., BO$_3$ vs. BO$_4$) and changes in next-nearest neighbour (e.g., Si–O–Si vs. Si–O–Na).

3.7 Density Functional Theory Calculations

The use of DFT calculations to complement experimental data is a common practice in the fields of both chemistry and physics. In this work the results of DFT calculations were used to validate the reliability of experimental crystal structures, assist in interpreting poorly-resolved NMR spectra, and to probe the response of Raman spectra to stress. The theoretical underpinnings of these calculations are complex and too lengthy to reproduce here, and the interested reader is directed to the literature [151, 152]. The following discussion focuses on some practical aspects of the use of DFT calculations as applied to the study of crystals. Calculations on glassy structures were not attempted, as a proper DFT handling of amorphous systems is well beyond the scope of this work.

3.7.1 A Brief Description of DFT Calculations

In a DFT calculation on an ordered crystal, an appropriate unit cell is constructed. The crystal is treated as an infinite periodic set composed of repeated unit cells. These periodic boundary conditions are amenable to being modelled by plane-wave functions. By increasing the maximum energy of the set of plane waves used, the variance of
any given term decreases. Periodic plane-waves are well equipped to model long-range phenomenon, but require extremely high energies to reproduce aperiodic interactions, *e.g.*, those close to the nucleus of an atom. To reduce the maximum plane-wave energies, the rapidly-varying atomic wavefunctions are replaced by a smoothly-varying function which matches the behaviour of the atom at long range, but is easier to model at short ranges. The details of this approximation are described in Section 3.7.2.

Rather than carrying out the calculation over all space, a set of discrete points sample the response of the plane-waves. This set of points is used for a three-dimensional integration of the response over the whole cell. These points are called k-points. The number of k-points that are required to reliably calculate a given parameter will vary with the needs of the calculation. In general, the number of k-points required to model a system are inversely proportional to the volume of the unit cell. The only detriment to increasing the number of k-points is the corresponding increase in computational time (which can be prohibitive). For the sake of expediency, it is important to test the number of k-points required for property convergence and to not overdo things.

### 3.7.2 Pseudopotentials and PAW Datasets

Modelling the behaviour of electronic states close to the nucleus is costly, due to their rapidly fluctuating nature. An effective means of reducing this cost is the use of pseudopotentials [153]. Pseudopotentials approximate the behaviour of electronic states contained within some arbitrary cut-off radius. Outside the cut-off radius, the atomic wavefunction is identical to that from an all-electron calculation. Inside the cut-off radius, the oscillating all-electron wavefunction is replaced by a smoothly-varying “pseudized” wavefunction. The cut-off radius is set such that the valence electrons (which account for the vast majority of chemically-relevant interactions) are outside the cut-off radius, while the core electrons (which are not generally chemically interesting) are inside the cut-off radius. The use of pseudopotentials has the effect of significantly reducing the computational cost of a given DFT calculation, but does not allow for the modelling of nuclear properties (*i.e.*, NMR properties).

In order to model nuclear properties while still retaining the computational efficiency of pseudopotentials, we turn to the projector augmented-wave (PAW) method. The
PAW method is superficially quite similar to the use of pseudopotentials, but is rather more complex. The details of PAW can be found in the literature [154, 155]. The practical implications of the use of PAW over pseudopotentials lie in the fact that in PAW the electronic behaviour near the core is handled explicitly. By handling the core behaviour explicitly, nuclear interactions can now be modelled.

An important factor affecting PAW is the choice of cut-off radius. The PAW formalism is only strictly correct if the cut-off radii of all PAW datasets are chosen such that there is no overlap of the resulting PAW spheres. It is not uncommon for “default” PAW datasets to have some PAW sphere overlap for an arbitrary structure. In oxide-based materials, X–O overlap is particularly common. The effects of PAW overlap range from being unnoticeable to significantly affecting the results, being particularly severe for large overlaps and nuclear properties [155]. Hence all PAW datasets used in this work were checked for overlap on the relevant structures; and custom datasets were used where overlap occurred.

3.7.3 Computation of NMR Observables

As the EFG is a strict ground-state property of the electron density, it may be computed accurately within the PAW framework. The calculation of the EFG tensor is computationally trivial, requiring only the calculation of the second spatial derivatives of the electrostatic potential. The electrostatic potential is routinely calculated in a DFT calculation, and is readily available. The EFG tensor, \( V \), is a diagonal traceless second-rank tensor, and hence can be expressed as \( V_{xx} + V_{yy} + V_{zz} = 0 \), where \( V_{xx} \), \( V_{yy} \), and \( V_{zz} \) are the diagonal tensor elements. The elements are ranked such that \(|V_{zz}| \geq |V_{yy}| \geq |V_{xx}|\). The quadrupolar symmetry parameter, \( \eta_Q \), can be derived directly from the tensor elements, and is defined as

\[
\eta_Q = \frac{V_{yy} - V_{xx}}{V_{zz}}. \tag{3.25}
\]

The other EFG NMR observable, the quadrupolar coupling constant \( C_Q \), is defined as

\[
C_Q = \frac{eQV_{zz}}{\hbar}. \tag{3.26}
\]
While $\eta_Q$ can be calculated using only the EFG tensor elements, calculation of $C_Q$ requires knowledge of the nuclear quadrupole moment $Q$. These values are determined experimentally, and can be found in the work of Pyykkö [57]. Calculating $C_Q$ can be a quick check on the validity of the input crystal structure prior to more detailed calculations: while DFT-derived $C_Q$ values are often overestimated, on the order of 10%, substantial differences from experiment (e.g., 50%) can indicate an error in the crystal structure.

Calculation of the chemical shielding tensor requires taking account of an external magnetic field, which is a subtle and difficult problem. The gauge-including projector augmented-wave (GIPAW) method, developed by Pickard and Mauri in 2001, provides one approach to this problem [156]. It allows for the calculation of the chemical shielding tensor $\sigma$ for all atoms in a given unit cell. The mathematical derivation of the GIPAW method is beyond the scope of this work, and the interested reader is directed to the literature for the details [156–158]. The practical considerations of the GIPAW method in this work follow.

The calculation of the chemical shielding tensor by GIPAW is computationally quite expensive, and scales poorly with the number of atoms in the unit cell. However, calculating the chemical shielding of the system is the only way to obtain the chemical shift. The chemical shift is (very well) approximated by the difference in shielding between the target sample and a reference composition [158]. However, as both the target system and the reference system are calculated in DFT, GIPAW results must be interpreted critically; beyond the common DFT concerns regarding input structure and exchange-correlation effects, it is possible for many subtle interactions to affect the calculated chemical shifts [157].

The chemical shielding tensor is a diagonal second-rank tensor which is not traceless. In its principal axis system, there are three unique chemical shielding tensor elements: $\sigma_{xx}$, $\sigma_{yy}$, and $\sigma_{zz}$. There are multiple notations used for describing the shielding (and equivalently, shift) tensors; the different notations and their relative advantages are discussed in the literature [159]. In this work all chemical shielding and chemical shift anisotropy (CSA) parameters are described in the Herzfeld-Berger (a.k.a. Maryland)
notation [160]. In this notation, the tensor elements are defined as

\[ \sigma_{11} \leq \sigma_{22} \leq \sigma_{33}. \]  

(3.27)

The isotropic shielding is clearly

\[ \sigma_{\text{iso}} = \frac{1}{3} (\sigma_{11} + \sigma_{22} + \sigma_{33}). \]  

(3.28)

Two additional parameters are required to describe the tensor. These are the span \((\Omega)\) and the skew \((\kappa)\), with the following definitions:

\[ \Omega = \sigma_{33} - \sigma_{11} \]  

(3.29)

\[ \kappa = 3 \left( \frac{\sigma_{\text{iso}} - \sigma_{22}}{\Omega} \right). \]  

(3.30)

\(\Omega\) is effectively a breadth parameter, while \(\kappa\) describes the axial symmetry of the tensor. \(\kappa\) will vary from +1, where it describes a prolate ellipsoid, to −1, where it describes an oblate ellipsoid.

### 3.7.4 DFT Software Used in This Work

A variety of software programs were used in this work for the purpose of carrying out DFT calculations on a variety of systems. A brief description of the software used, and the justification for their use, follows.

For pseudopotential and PAW DFT calculations, the ABINIT and QUANTUM ESPRESSO software packages were used [161–163]. Both are open-source plane-wave DFT software, and are broadly comparable in capabilities and scope. ABINIT was used for many calculations in this work due to familiarity with the software. QUANTUM ESPRESSO was used when chemical shielding calculations were required. Currently, chemical shielding calculations are typically carried out using the GIPAW method. A GIPAW module is implemented in QUANTUM ESPRESSO, but not in ABINIT.

Norm-conserving pseudopotentials (NCPPs) were generated using the OPTIMIZED NORM-CONSERVING VANDERBILT PSEUDOPOTENTIAL (ONCVPSP) code [164]. NCPPs were used for the DFPT calculations of the Raman spectra of the LaBGeO\(_5\)
crystal (Chapter 8); the code required to run the same calculations using PAW has not yet been enabled in Abinit. PAW datasets were used for all other calculations, as all other calculations were to obtain NMR properties. PAW datasets were generated using the ATOMPAW software [165]. ATOMPAW can generate PAW datasets for both Abinit and Quantum ESPRESSO. Custom PAW datasets were validated using the Elk all-electron DFT package (http://elk.sourceforge.net/).

3.8 Neutron Diffraction

Neutron diffraction is a valuable technique for both crystallography and for the study of glass structure [69]. As a diffraction-based experiment, it shares many aspects in common with X-ray diffraction, described above (Section 3.3). The greatest difference between a neutron diffraction experiment and a X-ray diffraction experiment is the nature of the diffracting beam. Whereas X-rays interact with the electron cloud surrounding each atom, neutrons interact directly with the atomic nucleus. As such, neutrons are capable of diffracting off of light elements such as H, He, or Li. However, neutron beams are much less straightforward to produce than X-ray beams. A brief description of the neutron source and detector used in this work follows. It is accompanied by a brief description of the approach for data processing.

Neutrons beams for diffraction experiments can only be generated with sufficient neutron flux via two sources: nuclear reactors and time-of-flight spallation sources. For this work, neutron diffraction was carried out at the ISIS spallation neutron source at the Rutherford Appleton Laboratory in Oxfordshire, United Kingdom. At the ISIS source, protons are accelerated by a synchrotron, which periodically collides bunches of protons with a tungsten target (Fig. 3.7). Neutrons are “spalled” from the target and channelled into several beamlines. Detectors are placed upon these beamlines, and samples placed within. The detector used for this work is the GEneral Materials Diffractometer (GEM) (Fig. 3.8), which allows for the detection of scattering over a wide range of scattering angles. Further information regarding the specifics of ISIS and GEM can be found in reference [166].

In a neutron scattering experiment, the quantity which is determined is the differential cross-section

$$\frac{d\sigma}{d\Omega} = I^S(Q) + i(Q).$$  (3.31)
Figure 3.7: The ISIS neutron source. Displayed is the containment structure of the tungsten target from which neutrons are produced. For scale, see the ladder on the right of the image. The sample access port of the GEM detector can be observed in the bottom right.
Figure 3.8: The GEM neutron detector. It is composed of eight detector banks with ZnS/$^{6}$Li scintillation detectors. The detector banks are structured to allow for the detection of scattering from a wide range of scattering angles. This image is provided courtesy of Dr. Alex C. Hannon, and is used with permission.
\[ hQ = \frac{4\pi \hbar \sin(\theta)}{\lambda} \]

is the magnitude of the momentum transferred during the scattering event, where \( \theta \) is the angle of the scattering [69, 166]. \( I^s(Q) \) is the self-scattering of the sample, and can be approximated with a knowledge of the composition of the sample and the various properties of the detector [167]. \( i(Q) \) is the distinct scattering of the sample, and is the function which contains the structurally interesting information regarding the sample. The distinct scattering can be determined by subtracting the approximation of \( I^s(Q) \) from the differential cross-section.

To obtain structural information from the distinct scattering, it is subjected to a Fourier transformation which returns the total correlation function \( T(r) \):

\[
T(r) = T^0(r) + \frac{2}{\pi} \int_0^\infty Q i(Q) M(Q) \sin(rQ) Q \, dQ. \tag{3.32}
\]

\( M(Q) \) is a modification function introduced to mitigate the effects of truncation of \( i(Q) \). Like in NMR spectroscopy, the finite nature of \( i(Q) \) can lead to ripples in \( T(r) \); this can be mitigated through a large maximum value of \( Q \) \( (Q_{\text{max}}) \), and by the use of an appropriate modification function, which will be discussed below. \( T^0(r) \) is the average density contribution to \( T(r) \), and is defined as

\[
T^0(r) = 4\pi rg^0 \left( \sum_i c_i \bar{b}_i \right)^2. \tag{3.33}
\]

\( g^0 \) is the atomic number density. \( c \) is the atomic fraction of element \( i \), and \( \bar{b} \) is its coherent neutron scattering length. The summation over \( i \) runs over the \( N \) elements present in the sample. \( T(r) \) is a weighted sum of the pairwise partial correlation functions of each element pair \( t_{ii'}(r) \):

\[
T(r) = \sum_{i,i'} c_i \bar{b}_i \bar{b}_{i'} t_{ii'}(r). \tag{3.34}
\]

In this work, two modification functions are used. The first is the simple “step” function, with

\[
M(Q) = \begin{cases} 
1, & \text{if } Q \leq Q_{\text{max}} \\
0, & \text{otherwise.}
\end{cases} \tag{3.35}
\]

The step modification function is mathematically trivial, and allows for the maximum
real-space resolution in $T(r)$. However, it will almost always lead to significant ripples on either side of real peaks in $T(r)$, due to the truncation of $Q$ during the Fourier transform [166]. These ripples can cause difficulties in fitting peaks, either from peak overlap or misidentification. To mitigate truncation effects, the Lorch modification function is commonly used (and indeed was used in this work). The Lorch function is defined as

$$M(Q) = \begin{cases} \frac{\sin(\Delta r Q)}{\Delta r Q}, & Q \leq Q_{\text{max}}, \Delta r = \pi/Q_{\text{max}}, \\ 0, & Q > Q_{\text{max}}. \end{cases}$$ (3.36)

The Lorch modification function dramatically decreases the intensity of truncation ripples, allowing for more accurate peak fitting and identification [63]. There is some loss of real-space resolution from using the Lorch function instead of the step function; however, with $Q_{\text{max}} = 50 \text{ Å}^{-1}$, the difference is only 0.04 Å.

Once $T(r)$ is calculated, much information can be derived from its constituent peaks. As per Eq. (3.34), the peaks in $T(r)$ are due to scattering from various elemental pairs. A peak maximum in $T(r)$ corresponds to the mean interaction distance between the element pair. In many cases — in oxides, typically the first X–O peak — this distance corresponds to a mean bond length. The first-order peaks typically correspond to local order, e.g., distinguishing between BO$_3$ trigonal planes and [BO$_4$]$^-$ tetrahedra. The second-order peaks can contain valuable information regarding intermediate-range order. For example, the B–B distance in borate glasses can be used to infer information regarding the presence of superstructural units [20].

The position of a peak and its area can together be used to determine the coordination number of the element pair producing the peak. The equation describing the coordination number of a peak is

$$n_{ij} = \frac{r_{ij} A_{ij}}{(2 - \delta_{ij}) c_i \bar{b}_i \bar{b}_j}. \quad (3.37)$$

Here $r_{ij}$ is the position of the peak of the interaction of the elements $i$ and $j$ [12, 69]. $A_{ij}$ is the area of the peak. $\delta_{ij}$ is the Kronecker delta, and accounts for self-interaction for a peak resulting from homonuclear scattering. As above, $c_i$ is the atomic fraction of element $i$, and $\bar{b}$ are the coherent scattering lengths. While both elements $i$ and $j$ contribute to the peak, only element $i$ is required to obtain the $i$–$j$ coordination.
number. \( n_{ij} \) and \( n_{ji} \) are related by

\[
n_{ji} = n_{ij} \frac{c_i}{c_j}
\]

(3.38)

Hence \( n_{ij} \) and \( n_{ji} \) are related but distinct values.

It is important to note how the values obtained via neutron diffraction are dependent on the composition and density of the sample under investigation. The differential cross-section measured from the sample does not depend on the knowledge of the composition (atomic and isotope fractions) nor the density. However, every subsequent processing step requires both the composition in terms of both elemental composition (in the \( c \) terms) and isotopic composition (in the \( \tilde{b} \) terms). The \( c \) and \( \tilde{b} \) terms obviously are contained within Eq. (3.37), but the \( A_{ij} \) term is also dependent on the composition and density. Errors in the composition primarily affect the vertical scale of \( T(r) \), and consequently the area of the peaks. Similarly, errors in the composition and density affect \( T^0(r) \), which in turn affects the vertical scaling of \( T(r) \). Hence an accurate set of values for the elemental and isotopic composition of the sample, as well as its density, are required to obtain accurate coordination numbers using neutron diffraction. The methods of obtaining these values are described in Section 3.4 and Section 3.2.1. Conveniently, \( r \) is not significantly affected by errors in composition or density, and hence can be analysed independently.

When considering neutron diffraction, the isotopic composition of each sample must be carefully considered. For some elements (e.g., La) there is only one naturally abundant isotope. For others (e.g., Ge) there are many isotopes, but all are amenable for diffraction (though some are more concerning than others when considering radiation safety), and scattering parameters can be taken as averages. Boron rates special consideration in the context of neutron diffraction. Two boron isotopes are naturally occurring: \(^{10}\text{B}\) and \(^{11}\text{B}\), with natural abundances of 20\% and 80\%, respectively. \(^{10}\text{B}\) is an excellent neutron absorber, with an absorption cross-section of 3835 fm\(^2\). For comparison, the absorption cross-sections of natural abundance La, Ge, and O are 8.93 fm\(^2\), 2.2 fm\(^2\), and 0.000 19 fm\(^2\), respectively [168]. The presence of \(^{10}\text{B}\) in a sample intended for analysis by neutron diffraction can significantly reduce the intensity of the detected neutron flux, and hence it is typically desirable to enrich samples intended for neutron diffraction analysis with \(^{11}\text{B}\). Thankfully, this enrichment
does not typically affect other analyses: $^{10}\text{B}$ and $^{11}\text{B}$ are chemically identical in glass; EA techniques used in this work are either isotope-agnostic, or capable of quantifying the isotopes independently; and $^{11}\text{B}$ NMR is not deleteriously affected by the absence of $^{10}\text{B}$. $^{11}\text{B}_2\text{O}_3 \geq 99\%$ $^{11}\text{B}$ is commercially available at reasonable prices (47 CAD g$^{-1}$ from Sigma-Aldrich ISOTEC at time of writing), making $^{11}\text{B}$ enrichment of borate glasses straightforward.
Chapter 4

Relating $^{139}$La Quadrupolar Coupling Constants to Polyhedral Distortion in Crystalline Structures

4.1 Context

One of the central goals of this work is to obtain information on the differences (or lack thereof) in the La–O environment between the LaBGeO$_5$ glass and crystal forms. NMR spectroscopy is frequently applied to the study of amorphous materials such as oxide glasses, as it is primarily a probe of local structure, and does not require the presence of long-range order. However, $^{139}$La NMR spectra of glasses have not been reported in the literature, in no small part because of the large nuclear quadrupole moment of lanthanum. In order to interpret $^{139}$La NMR spectra of glasses, we must first construct a model describing the relationship between spectral properties and chemical structure in lanthanum-containing crystals. This chapter assesses the state of $^{139}$La spectroscopy in the literature, and combines novel data with data from the literature to construct a model relating $^{139}$La $C_Q$ and $\delta_{iso}^{CS}$ to various structural features of lanthanum oxide-based crystals.

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on the analysis of the results and the review of the manuscript.

4.2 Abstract

A broad series of crystalline lanthanum oxide-based materials has been investigated through high-field $^{139}$La ssNMR spectroscopy and ab initio DFT calculations. The $^{139}$La ssNMR spectra of LaBGeO$_5$, LaBSiO$_5$, LaBO$_3$, LaPO$_4$·1.8H$_2$O, La$_2$(SO$_4$)$_3$·9H$_2$O, and La$_2$(CO$_3$)$_3$·8H$_2$O are reported for the first time. Both newly reported and literature values of $^{139}$La quadrupolar coupling constants $C_Q$ are related to various quantitative expressions of polyhedral distortion, including the sphericity ($\Sigma$) and ellipsoid span ($\epsilon$). The compounds were separated into two groups based upon their polyhedral distortion behaviour: compounds with the general formula LaMO$_3$, where M is a trivalent cation; and compounds with different general formulae. The $^{139}$La $C_Q$ of the LaMO$_3$ family was found to correlate best with $\epsilon$. The $^{139}$La $C_Q$ of non-LaMO$_3$ compounds correlates adequately to $\epsilon$, but is better described by $\Sigma$. The $^{139}$La isotropic chemical shift ($\delta_{iso}^{CS}$) of the non-LaMO$_3$ compounds is negatively correlated with the lanthanum coordination number (CN); there is insufficient data from the LaMO$_3$ compounds to draw conclusions relating to chemical shift. DFT calculations of NMR parameters prove to be a sensitive probe of the quality of input geometry, with predicted parameters agreeing with experiment except in cases where the crystal structure is suspect.

4.3 Introduction

Lanthanum compounds have a wide variety of applications, including nickel-metal hydride batteries [169], transparent ferroelectric nanocomposites [34], medical glassmaking [170], and both medical and environmental phosphate sequestration [171, 172]. Many of these applications involve amorphous materials, complicating the characterization of the lanthanum environment. SsNMR is an effective probe of local coordination in non-periodic solids, but has not yet been applied to $^{139}$La in amorphous lanthanum compounds. In order to increase the utility of $^{139}$La ssNMR as a structural characterization tool, we investigate the relationship between observable $^{139}$La ssNMR properties and the distortion of the lanthanum sites in lanthanum oxide-based materials.
The primary NMR-active nucleus of lanthanum, $^{139}\text{La}$, has several attractive nuclear properties including a complete natural abundance (100 %), a nuclear spin of $I = 7/2$, and a moderate gyromagnetic ratio ($\gamma = 3.801 \times 10^{-7}$ rad T$^{-1}$ s$^{-1}$), all of which contribute to a high receptivity ($1.61 \times 10^2$ as compared to $^{29}\text{Si}$). A moderately high nuclear electric quadrupole moment ($Q = 20 \text{fm}^2$)\cite{57} has been the primary barrier to the ssNMR study of $^{139}\text{La}$. Until the advent of the WCPMG pulse sequence \cite{129}, the breadth of many spectra prevented their timely acquisition, restricting investigations into relatively high symmetry compounds which yield narrow spectra \cite{58, 59, 173}. The WCPMG pulse sequence, in conjunction with VOCS collection, has recently been put to good use in the investigation of a number of both coordination and inorganic lanthanides \cite{174–176}, including LaScO$_3$ \cite{177} and LaPO$_4$ \cite{172}.

The lineshape of $^{139}\text{La}$ NMR spectra is primarily due to interactions of the nucleus with the EFG tensor. Two parameters describe this interaction: the $C_Q$, and the $\eta_Q$

$C_Q = \frac{eQV_{zz}}{h}$ \hspace{1cm} (4.1)

$\eta_Q = \frac{V_{yy} - V_{xx}}{V_{zz}}$ \hspace{1cm} (4.2)

where $Q$ is the electric quadrupole moment of the nucleus, $h$ is the Planck constant, $e$ is the elementary charge, and $|V_{zz}| \geq |V_{yy}| \geq |V_{xx}|$ are the principal components of the EFG tensor. The magnitude of the quadrupolar coupling constant has commonly been used to infer qualitative deviations from spherical symmetry while the asymmetry parameter provides information regarding axial symmetry \cite{25, 58, 60}. An additional influence on the lineshape of NMR spectra is CSA. The CSA tensor is defined similarly to the EFG tensor, with principal components $\delta_{11} \geq \delta_{22} \geq \delta_{33}$. This work uses the Herzfeld-Berger convention \cite{160} for describing the influence of CSA on NMR lineshapes.

Note that the principal directions of the CSA tensor need not coincide with those of the EFG tensor, and neither need coincide with “obvious” crystallographic directions unless satisfying the symmetry requirements of the unit cell. Nevertheless, if correlations between these tensors and crystal structures can be established for a class of materials, then extension to structurally uncharacterised samples is possible.
Indeed, many studies have attempted to link the above NMR properties to structural features: for example, Pan et al. used $^6$Li MAS NMR to probe the presence of Cr$^{3+}$ in the lithium coordination sphere [178]; $^{11}$B MAS NMR is routinely used to quantify relative proportions of three- and four-coordinate boron; Willans et al. were able to establish a link between $^{139}$La isotropic chemical shift and lanthanum CN in organic lanthanum coordination compounds using ssNMR;[176] and Michaelis and Kroeker investigated the possibility of relating $^{73}$Ge $C_Q$ to octahedral and tetrahedral distortion parameters [25].

While tetrahedral and octahedral distortion parameters are effective in describing differences between real and ideal polyhedra, it is difficult to generalize the method used by Michaelis and Kroeker to polyhedra with higher CN. Balić Žunić and Makovicky devised a method involving least-squares fitting of a sphere to the ligands of the coordination polyhedron [179] which provides a novel means of measuring distortion. Various distortion parameters are derived from the relationship between a circumscribed sphere and the ideal polyhedral shape [180]. These parameters have been successfully used to predict the coefficient of thermal expansion in $A_2M_3O_{12}$ materials [181, 182]. However, modelling the EFG tensor requires a parameter more sensitive to the specific distortion from spherical symmetry.

Balić Žunić and Makovicky [179] define the $\Sigma$ of the coordination polyhedron as

$$\Sigma = 1 - \frac{\sigma_{rs}}{r_s}$$

(4.3)

where $r_s$ is the average distance between the centroid of the coordination polyhedron and the ligands, and $\sigma_{rs}$ is the standard deviation of the same. The centroid of coordination is the point with minimal variation of position with regards to the oxygen ligands, and frequently, but not always, coincides with the position of the La$^{3+}$ cation. The sphericity describes the deviation of the ligand distances from the average distance, which models the general deviation from spherical symmetry.

Beyond considering the isotropic deviation from spherical symmetry, we also consider the anisotropy of the distortion. To this end, we fit the coordination polyhedron with a triaxial ellipsoid (Fig. 4.1). The resulting ellipsoid is defined by its three semi major axes $e_a \leq e_b \leq e_c$. The level of anisotropy of the ellipsoid is described by two parameters derived from the semi major axes: the $\epsilon$, defined by Balić Žunić and
Makovicky [183] as

$$
\epsilon = \frac{e_c - e_a}{e_a + e_b + e_c}
$$

(4.4)

and the ellipsoid character (EC), which describes how close in length \(e_b\) is to the other semi major axes. More precisely, the ellipsoid character is defined in analogy to the optical character [183]. The ellipsoid character is the cosine of the angle between the normals of the two unique circular cross-section of the ellipsoid fitting the polyhedron, as bisected by the longest major axis. Practically, the ellipsoid character ranges from -1, describing an oblate spheroid, to +1, describing a prolate spheroid, while intermediate values indicate that the ellipsoid is triaxial. \(\epsilon\) will range from 0 when the ellipsoid is spherical, and will approach 3 when \(e_c \gg e_a\).

In this work we use \(^{139}\text{La ssNMR}\) in conjunction with DFT calculations to construct an empirical model relating the distortion of the lanthanum coordination polyhedra to the observed \(^{139}\text{La } C_Q\) values. We establish that the \(^{139}\text{La } C_Q\) of compounds with the general formula \(\text{LaMO}_3\) (\(M = \text{B}^{3+}, \text{Al}^{3+}, \text{Sc}^{3+}, \text{Ti}^{3+}, \text{Cr}^{3+}, \text{Co}^{3+}\)) is dependent primarily on the ellipsoid span, while the \(^{139}\text{La } C_Q\) of other compounds is related to both the ellipsoid span and the sphericity parameter. Experimental data is reported for crystalline samples of \(\text{La}_2\text{O}_3\), \(\text{LaPO}_4 \cdot 1.8\text{H}_2\text{O}\), \(\text{LaPO}_4\), \(\text{LaBO}_3\), \(\text{LaBGeO}_5\), \(\text{LaBSiO}_5\), \(\text{La}_2(\text{SO}_4)_3 \cdot 9\text{H}_2\text{O}\), and \(\text{La}_2(\text{CO}_3)_3 \cdot 8\text{H}_2\text{O}\). The results of DFT calculations on \(\text{La}_2\text{O}_3\), \(\text{LaPO}_4 \cdot 0\text{H}_2\text{O}\), \(\text{LaPO}_4\), \(\text{LaBO}_3\), \(\text{LaBGeO}_5\), \(\text{LaBSiO}_5\), \(\text{La}_2(\text{SO}_4)_3 \cdot 9\text{H}_2\text{O}\), \(\text{La(OH)}_3\), \(\text{LaAlO}_3\), \(\text{LaCoO}_3\), \(\text{LaCrO}_3\), \(\text{LaScO}_3\), and \(\text{LaNbO}_4\) are included.

4.4 Experimental

4.4.1 Synthesis and Characterization

Samples of \(\text{La}_2\text{O}_3\) (\(\geq 99.9\%\)), \(\text{LaBO}_3\) (99.9\%), \(\text{La}_2(\text{CO}_3)_3 \cdot n\text{H}_2\text{O}\) (99.9\%), and \(\text{La}_2(\text{SO}_4)_3 \cdot 9\text{H}_2\text{O}\) (\(\geq 99.99\%\)) were purchased from Sigma-Aldrich. \(\text{LaPO}_4 \cdot n\text{H}_2\text{O}\) (99.99\%) was purchased from Alfa Aesar. The above samples were used without further purification. \(\text{LaPO}_4\) was produced by heating \(\text{LaPO}_4 \cdot n\text{H}_2\text{O}\) at 750°C for 21 h in air. Crystalline \(\text{LaBGeO}_5\) was produced via devitrification of a glass sample. \(\text{LaBGeO}_5\) glass was produced by grinding stoichiometric amounts of \(\text{La}_2\text{O}_3\), \(\text{B}_2\text{O}_3\) (99\%, Sigma-Aldrich), and \(\text{GeO}_2\) (99.998\%, Sigma-Aldrich) in a ceramic mortar and
Figure 4.1: Schematic diagram of a triaxial ellipsoid fit to the LaO$_9$ polyhedron of LaBGeO$_5$. 
pestle and heating at 1300°C for 30 min in a platinum crucible in air. The resulting glass was poured into a glass mold pre-heated to 400°C, and held at 400°C for 24 h. Crystallization was induced by heating the glass at 950°C for 12 h. The resulting ceramic was ground to powder in an agate mortar and pestle. LaBSiO₅ powder was produced by grinding stoichiometric amounts of La₂O₃, B₂O₃, and SiO₂ (Analytical, Sigma-Aldrich) in a ceramic mortar and pestle and heating from 900°C to 1300°C over 5 h in a platinum crucible in air. Identities of both commercial and synthesized samples were confirmed using pXRD. X-ray diffraction experiments were conducted using a Rigaku Ultima IV X-ray diffractometer using a copper anode X-ray tube with a diffracted beam monochromator and a scintillation detector. X-ray diffractograms are available in the supporting information. TGA was used to characterize the level of hydration of LaPO₄·1.85H₂O and La₂(SO₄)₃·9H₂O (Fig. B.8). The level of hydration of La₂(CO₃)₃·nH₂O was not determined experimentally as the ¹³⁹La NMR spectrum was not used.

4.4.2 NMR Spectroscopy

¹³⁹La NMR spectra were collected on 9.4 T (56.54 MHz ¹³⁹La frequency) and 16.4 T (98.91 MHz ¹³⁹La frequency) Bruker Avance NMR spectrometers. Samples were finely ground in an agate mortar and pestle and packed into either 4 mm (16.4 T) or 7 mm (9.4 T) outer diameter ZrO₂ rotors. All spectra were collected under static conditions. The WCPMG pulse sequence [129] was used with WURST-80 (16.4 T) or WURST-20 (9.4 T) pulses [123] of 50 µs duration, sweeping across 500 kHz at a rate of 10 MHz/ms to ensure homogeneous excitation. The number of echoes collected varied from 60 to 250 according to the T₂ relaxation of the sample. Spectral slices were collected with a transmitter offset of either approximately 100 kHz or 200 kHz, with the exact value set to an integer multiple of the spikelet separation. The number of slices depended on the breadth of the spectral peak. The number of scans per slice varied between 16 and 3192, dependent on sensitivity of the sample. Optimized recycle delays of 0.1 s to 5 s were used. Total experimental time was uniformly less than 2 h. ¹³⁹La chemical shifts were referenced to a 1.0 mol dm⁻³ aqueous solution of LaCl₃. Spectra were fit using DMFit v20110512 [184], WSOLIDS v1.19.226 [185], and QUEST v1.1.527 [186].
4.4.3 DFT Calculations

Density functional theory calculations were carried out using the Abinit code [187–190] using the PAW method [154]. The PAW datasets used varied depending on the structure being investigated. When possible, Jollet, Torrent, and Holzwarth (JTH) PAW datasets were used without alteration [191]. In systems where there was significant PAW sphere overlap, custom datasets were used to avoid this problem. Details on the datasets used for each structure are available in the supporting information (Table B.1). All calculations were performed using the Perdew, Burke, and Ernzerhof (PBE) generalized gradient approximation (GGA) exchange-correlation functional [192].

Calculations were conducted on crystal structures taken from the Inorganic Crystal Structure Database (ICSD) [193]. Optimized plane-wave cutoff energies were used, typically between 30 and 45 hartree, with PAW fine grid cutoff energies generally between 90 and 150 hartree. K-point grids were optimized for each structure, but typically had a grid spacing of $0.03 \text{ Å}^{-1}$. Specific values for each structure are available in the supporting information (Table B.2). EFG parameters were calculated on experimental geometries. Calculations were performed using the WestGrid Grex research facility, with Intel Xeon X6560 2.66 GHz cores. Between 4 and 24 cores were used depending on the fineness of the k-point grid.

4.4.4 Distortion Parameters

Distortion parameters were calculated using the IVTON software [183]. Input structures were obtained from the ICSD [193], and were also used for DFT calculations. Uncertainties in distortion parameters were either determined through propagation of error or estimated from least-squares variance.

4.5 Results

$^{139}\text{La}$ NMR spectroscopic parameters, computational results, and distortion parameters are reported in Table 4.1, Table 4.2, and Table 4.3, respectively. The presentation of the results for LaBGeO$_5$ and LaScO$_3$ are discussed as representative of the non-LaMO$_3$ and LaMO$_3$ compounds, respectively. The full results and discussion for all other
Figure 4.2: Left: Static $^{139}$La NMR spectrum of LaBGeO$_5$ at 16.4 T. Analytical simulation is show in red. The EFG parameters used are reported in Table 4.1. Right: First coordination sphere of LaO$_9$ in LaBGeO$_5$. The $V_{xx}$, $V_{yy}$, and $V_{zz}$ components are displayed as blue, green, and red, respectively. The $e_a$, $e_b$, and $e_c$ semi major axes are in yellow, magenta, and teal respectively.

compounds of interest are included in Appendix B.

4.5.1 Lanthanum Borogermanate

LaBGeO$_5$ is an example of the stillwellite rare earth mineral [77] that has been extensively studied for its ferroelectric properties [75, 195, 196]. It is a remarkably efficient glass former, given its high lanthanum content. LaBGeO$_5$ glass is an effective TFN material, due to the shared stoichiometry between crystalline and glassy phases [34]. LaBGeO$_5$ has been previously studied using Raman spectroscopy [197], computational methods [198], and $^{11}$B MAS NMR spectroscopy [34], but as of yet has not been investigated using $^{139}$La NMR. As a member of the stillwellite family, LaBGeO$_5$ is of trigonal space group $P3_1$, with three formula units per unit cell. The environment of the single lanthanum site in LaBGeO$_5$ is ninefold coordinate to oxygen, with contributions from both the GeO$_4$ and BO$_4$ tetrahedra. La–O bond lengths range from 2.41 Å to 2.74 Å, with an average of 2.60 Å. There are no obvious symmetry elements present within the lanthanum polyhedron.

The $^{139}$La ssNMR spectrum of LaBGeO$_5$ acquired at 16.4 T is presented in Fig. 4.2. The spectrum is extremely broad, spanning approximately 12 000 ppm. It is fit using a $C_Q$ of (85.5 ± 0.5) MHz and $\eta_Q$ of (0.30 ± 0.02) The spectrum is overwhelmingly
Table 4.1: Summary of the experimental $^{139}$La NMR parameters. Values with a reference were taken from the respective source.

| Compound               | $|\nu_Q|$/ MHz | $\eta_Q$/ | $\delta_{iso}$ / ppm | $\Omega$/ ppm | $\alpha$/ | $\alpha$/° | $\beta$/° | $\gamma$/° |
|------------------------|--------------|-----------|----------------------|---------------|-----------|-----------|-----------|-----------|
| La$_2$O$_3$            | 58.6 ± 0.3   | 0.00 ± 0.02 | 620 ± 30 | 500 ± 50 [60] | -1.0 ± 0.1 [60] | 150 ± 30 [60] | 90 ± 10 [60] | 180 ± 10 [60] |
| LaPO$_4$ · 1.8H$_2$O   | 33 ± 1       | 1.00 ± 0.05 | 400 ± 20 | 0.3 ± 0.1 | 15 ± 5 | 0 ± 5 | 165 ± 10 |
| LaPO$_4$               | 46.7 ± 1.0 [172] | 0.75 ± 0.01 [172] | 35 ± 4 [172] | 0 ± 0.1 | 15 ± 5 | 0 ± 5 | 165 ± 10 |
| LaBO$_3$               | 23.4 ± 0.4   | 0.68 ± 0.05 | 230 ± 10 | 350 ± 30 | 0.3 ± 0.1 | 15 ± 5 | 0 ± 5 | 165 ± 10 |
| LaBGeO$_3$             | 85.5 ± 0.5   | 0.30 ± 0.02 | 200 ± 25 | 400 ± 200 | 0.8 ± 0.3 | 75 ± 75 | 15 ± 5 | 0 ± 10 |
| LaBSiO$_3$             | 90.0 ± 0.5   | 0.35 ± 0.02 | 20 ± 20 | 200 ± 200 | 0.8 ± 0.3 | 75 ± 75 | 15 ± 5 | 0 ± 10 |
| La$_2$(SO$_4$)$_3$ · 9H$_2$O | 52.5 ± 0.5 | 0.00 ± 0.01 | -75 ± 25 | -75 ± 25 | 0.0 ± 0.1 [60] | 80 ± 7 [60] | 10 ± 10 [60] | 0 ± 10 [60] |
| La(1)                  | 36.5 ± 0.5   | 0.00 ± 0.01 | -75 ± 25 | -75 ± 25 | 0.0 ± 0.1 [60] | 80 ± 7 [60] | 10 ± 10 [60] | 0 ± 10 [60] |
| La(2)                  | 36.5 ± 0.5   | 0.00 ± 0.01 | -75 ± 25 | -75 ± 25 | 0.0 ± 0.1 [60] | 80 ± 7 [60] | 10 ± 10 [60] | 0 ± 10 [60] |
| La(OH)$_3$             | 22.0 ± 0.5 [60] | 0.05 ± 0.02 [60] | 200 ± 20 [60] | 200 ± 20 [60] | 0.0 ± 0.1 [60] | 80 ± 7 [60] | 10 ± 10 [60] | 0 ± 10 [60] |
| LaAlO$_3$              | 6.5 *[58]    | 0 ± 0.01   | 155 ± 5 [58] | 155 ± 5 [58] | 0.0 ± 0.1 [60] | 80 ± 7 [60] | 10 ± 10 [60] | 0 ± 10 [60] |
| LaCoO$_3$              | 23.8 *[59]   | 0 ± 0.01   | 230 ± 20 [59] | 230 ± 20 [59] | 0.0 ± 0.1 [60] | 80 ± 7 [60] | 10 ± 10 [60] | 0 ± 10 [60] |
| LaCrO$_3$              | 48 *[59]     | 0.15 ± 0.01 | 442 ± 5 [59] | 442 ± 5 [59] | 0.0 ± 0.1 [60] | 80 ± 7 [60] | 10 ± 10 [60] | 0 ± 10 [60] |
| LaTiO$_3$              | 53.2 *[98]   | 0.67 ± 0.04 | N.D. | N.D. | 0.0 ± 0.1 [60] | 80 ± 7 [60] | 10 ± 10 [60] | 0 ± 10 [60] |
| LaScO$_3$              | 61.6 ± 0.5 [177] | 0.10 ± 0.02 [177] | 600 ± 50 [177] | 600 ± 50 [177] | 0.0 ± 0.1 [60] | 80 ± 7 [60] | 10 ± 10 [60] | 0 ± 10 [60] |
| LaNbO$_3$              | 36 ± 2 [60]  | 0.44 ± 0.05 [60] | 255 ± 10 [60] | 255 ± 10 [60] | 0.40 ± 0.04 [60] | 90 ± 5 [60] | 50 ± 5 [60] | 270 ± 10 [60] |

* These values were provided without uncertainties; discussion of the reliability of these values is found in the Supporting Information.
Table 4.2: Summary of the calculated $^{139}$La NMR parameters.

<table>
<thead>
<tr>
<th>Compound</th>
<th>$C_Q$ (MHz)</th>
<th>$\eta_Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>La$_2$O$_3$</td>
<td>60.61</td>
<td>0</td>
</tr>
<tr>
<td>LaPO$_4$·0H$_2$O</td>
<td>−136.18</td>
<td>0.5</td>
</tr>
<tr>
<td>LaPO$_4$</td>
<td>53.67</td>
<td>0.56</td>
</tr>
<tr>
<td>LaBO$_3$</td>
<td>−28.79</td>
<td>0.57</td>
</tr>
<tr>
<td>LaBGeO$_5$</td>
<td>−89.03</td>
<td>0.32</td>
</tr>
<tr>
<td>LaBSiO$_5$</td>
<td>−109.95</td>
<td>0.05</td>
</tr>
<tr>
<td>La$_2$(SO$_4$)$_3$·9$H_2$O</td>
<td>−59.1</td>
<td>0</td>
</tr>
<tr>
<td>La(1)</td>
<td>−36.46</td>
<td>0</td>
</tr>
<tr>
<td>La(2)</td>
<td>−29.49</td>
<td>0</td>
</tr>
<tr>
<td>La(OH)$_3$</td>
<td>8.44</td>
<td>0</td>
</tr>
<tr>
<td>LaAlO$_3$</td>
<td>20.99</td>
<td>0</td>
</tr>
<tr>
<td>LaCoO$_3$</td>
<td>−47.22</td>
<td>0.32</td>
</tr>
<tr>
<td>LaCrO$_3$</td>
<td>−65.17</td>
<td>0.08</td>
</tr>
<tr>
<td>LaScO$_3$</td>
<td>39.46</td>
<td>0.5</td>
</tr>
</tbody>
</table>

quadrupolar in character, with only a minor influence from CSA. Using spectra collected at 9.4 T and 16.4 T we fit the spectra with CSA values of $\Omega = (400 \pm 200)$ ppm, $\kappa = 0.8 \pm 0.3$, $\alpha = (75 \pm 75)^\circ$, $\beta = (15 \pm 5)^\circ$, and $\gamma = (0 \pm 10)^\circ$. When fitting the spectrum acquired at the lower field strength (9.4 T), the high-field approximation is less valid due to the extreme breadth of the peak; as such, we must fit it exactly, rather than through perturbation theory [186].

DFT calculations on the experimental LaBGeO$_5$ geometry [78] yield a $^{139}$La $C_Q$ of −89 MHz and $\eta_Q$ of 0.32. This is in excellent agreement with the experimental results ($C_Q = (85.5 \pm 0.5)$ MHz and $\eta_Q = 0.30 \pm 0.02$). The $V_{zz}$ component of the EFG tensor is not directed toward any specific structural feature, but is generally aligned with GeO$_4$ tetrahedra. The La–O bond closest to the $V_{zz}$ vector links a GeO$_4$ tetrahedron with two LaO$_9$ polyhedra, but the bond and the vector are not parallel (Fig. 4.2).

The ellipsoid fit to the LaO$_9$ polyhedron is triaxial, with $e_a = (2.359 \pm 0.001)$ Å, $e_b = (2.713 \pm 0.001)$ Å, and $e_c = (2.744 \pm 0.001)$ Å. The shortest semi major axis, $e_a$, is in close proximity to the $V_{zz}$ component. The ellipsoid is significantly distorted from spherical symmetry, with $\epsilon = 0.148 \pm 0.001$. The ellipsoid is oblate in nature, with an EC of −0.87 ± 0.01.
Table 4.3: Collected distortion parameters.

<table>
<thead>
<tr>
<th>Compound</th>
<th>CN</th>
<th>Σ</th>
<th>$\epsilon_a / \text{Å}$</th>
<th>$\epsilon_b / \text{Å}$</th>
<th>$\epsilon_c / \text{Å}$</th>
<th>$\epsilon$</th>
<th>EC</th>
</tr>
</thead>
<tbody>
<tr>
<td>La$_2$O$_3$</td>
<td>7</td>
<td>0.960 ± 0.005</td>
<td>2.481 ± 0.002</td>
<td>2.481 ± 0.002</td>
<td>2.731 ± 0.002</td>
<td>0.098 ± 0.002</td>
<td>1.00 ± 0.01</td>
</tr>
<tr>
<td>LaPO$_4$ · 0 H$_2$O</td>
<td>9</td>
<td>0.965 ± 0.002</td>
<td>2.415 ± 0.001</td>
<td>2.574 ± 0.001</td>
<td>2.796 ± 0.001</td>
<td>0.147 ± 0.001</td>
<td>0.05 ± 0.01</td>
</tr>
<tr>
<td>LaPO$_4$</td>
<td>9</td>
<td>0.963 ± 0.001</td>
<td>2.331 ± 0.001</td>
<td>2.721 ± 0.001</td>
<td>2.741 ± 0.001</td>
<td>0.157 ± 0.001</td>
<td>−0.92 ± 0.01</td>
</tr>
<tr>
<td>LaBO$_3$</td>
<td>9</td>
<td>0.951 ± 0.002</td>
<td>2.359 ± 0.001</td>
<td>2.713 ± 0.001</td>
<td>2.744 ± 0.001</td>
<td>0.148 ± 0.001</td>
<td>−0.87 ± 0.01</td>
</tr>
<tr>
<td>LaBGeO$_3$</td>
<td>10</td>
<td>0.943 ± 0.004</td>
<td>2.350 ± 0.001</td>
<td>2.713 ± 0.001</td>
<td>2.808 ± 0.001</td>
<td>0.175 ± 0.001</td>
<td>−0.67 ± 0.01</td>
</tr>
<tr>
<td>LaBSiO$_3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>La$_2$(SO$_4$)$_3$ · 9 H$_2$O</td>
<td>12</td>
<td>0.960 ± 0.002</td>
<td>2.463 ± 0.001</td>
<td>2.837 ± 0.001</td>
<td>2.837 ± 0.001</td>
<td>0.138 ± 0.001</td>
<td>−1.00 ± 0.01</td>
</tr>
<tr>
<td>La(1)</td>
<td>9</td>
<td>0.9927 ± 0.0001</td>
<td>2.514 ± 0.001</td>
<td>2.514 ± 0.001</td>
<td>2.585 ± 0.001</td>
<td>0.028 ± 0.001</td>
<td>1.00 ± 0.01</td>
</tr>
<tr>
<td>La(2)</td>
<td>9</td>
<td>0.9927 ± 0.0001</td>
<td>2.551 ± 0.001</td>
<td>2.551 ± 0.001</td>
<td>2.620 ± 0.001</td>
<td>0.027 ± 0.001</td>
<td>1.00 ± 0.01</td>
</tr>
<tr>
<td>La(OH)$_3$</td>
<td>12</td>
<td>0.963 ± 0.001</td>
<td>2.67 ± 0.01</td>
<td>2.70 ± 0.01</td>
<td>2.70 ± 0.01</td>
<td>0.01 ± 0.01</td>
<td>−1.00 ± 0.01</td>
</tr>
<tr>
<td>LaAlO$_3$</td>
<td>12</td>
<td>0.931 ± 0.005</td>
<td>2.67 ± 0.04</td>
<td>2.78 ± 0.04</td>
<td>2.78 ± 0.04</td>
<td>0.04 ± 0.03</td>
<td>−1.00 ± 0.01</td>
</tr>
<tr>
<td>LaCoO$_3$</td>
<td>12</td>
<td>0.909 ± 0.009</td>
<td>2.58 ± 0.06</td>
<td>2.81 ± 0.06</td>
<td>3.03 ± 0.06</td>
<td>0.16 ± 0.05</td>
<td>−0.15 ± 0.01</td>
</tr>
<tr>
<td>LaCrO$_3$</td>
<td>8</td>
<td>0.956 ± 0.003</td>
<td>2.384 ± 0.001</td>
<td>2.462 ± 0.001</td>
<td>2.872 ± 0.001</td>
<td>0.190 ± 0.001</td>
<td>0.60 ± 0.01</td>
</tr>
<tr>
<td>LaScO$_3$</td>
<td>8</td>
<td>0.935 ± 0.006</td>
<td>2.334 ± 0.001</td>
<td>2.432 ± 0.001</td>
<td>3.032 ± 0.001</td>
<td>0.269 ± 0.001</td>
<td>0.61 ± 0.01</td>
</tr>
<tr>
<td>LaNbO$_4$</td>
<td>8</td>
<td>0.9877 ± 0.0004</td>
<td>2.419 ± 0.001</td>
<td>2.478 ± 0.001</td>
<td>2.606 ± 0.001</td>
<td>0.075 ± 0.001</td>
<td>0.32 ± 0.01</td>
</tr>
</tbody>
</table>
4.5.2 Lanthanum Scandate

LaScO$_3$ is the lanthanum perovskite most recently studied by $^{139}$La ssNMR spectroscopy, and the only one published to date that has been studied by WCPMG [177]. LaScO$_3$ is in the $Pbnm$ space group with four formula units per unit cell. The lanthanum environment is eightfold, with La–O bonds ranging from 2.40 Å to 2.88 Å, with an average bond length of 2.62 Å [177]. Like LaTiO$_3$, the LaO$_8$ polyhedron is best described as a distorted square antiprism.

Johnston et al. fit the $^{139}$La WCPMG ssNMR spectrum of LaScO$_3$ with a $C_Q$ of $(61.6 \pm 0.5)$ MHz and an $\eta_Q$ of 0.10 $\pm$ 0.02 [177], and carried out DFT calculations on the LaScO$_3$ structure reported by Liferovich and Mitchell [199]. In order to obtain the principal components of the EFG tensor, we conducted our own DFT calculations on the experimental geometry reported by Johnston et al. [177]; our calculations yielded a $C_Q$ of $-65.2$ MHz and an $\eta_Q$ of 0.08. The $V_{xx}$ component is parallel to both the crystallographic $c$ axis and the ellipsoid $e_c$ axis, hence the $V_{yy}$ and $V_{zz}$ components are in the plane defined by the $a$ and $b$ crystallographic axes. Our computed value of $\eta_Q$ is comparable with the computed value reported by Johnston et al. (0.08 vs. 0.13) as well as with their experimental value (0.08 vs. 0.10 $\pm$ 0.02), but our computed value of $C_Q$ is significantly different than their computed value ($-65.17$ MHz vs. $-51.66$ MHz) [177]. Our computed value of the $^{139}$La $C_Q$ is in good agreement with the experimental value reported by Johnston et al. ($-65.17$ MHz vs. $(61.6 \pm 0.5)$ MHz) [177]. This difference is likely due to the difference in initial starting geometries; where we used the structure reported by Johnston et al. [177], they began from the structure reported by Liferovich and Mitchell [199].

The ellipsoid used to fit the LaO$_8$ polyhedron is significantly distorted, with $e_a = (2.334 \pm 0.001)$ Å, $e_b = (2.432 \pm 0.001)$ Å, and $e_c = (3.032 \pm 0.001)$ Å (Fig. 4.3). The ellipsoid is generally prolate, with an EC of 0.61 $\pm$ 0.01, and shows the greatest distortion of all ellipsoids modelled in this study, with a $\epsilon$ value of 0.269 $\pm$ 0.001. The largest semi major axis $e_c$ is aligned with the crystallographic $c$ axis, while the other two semi major axes are slightly angled from La–O$_1$ bonds (as labelled by Johnston et al. [177]). The shortest semi major axis is very nearly parallel with a La–O bond of length 2.532 Å, while the $e_b$ axis is displaced from a La–O bond of length 2.396 Å by the same angle. Similar behaviour is observed in the isostructural LaTiO$_3$. 
Figure 4.3: First coordination sphere of LaO$_8$ in LaScO$_3$. The $V_{xx}$, $V_{yy}$, and $V_{zz}$ components are displayed as blue, green, and red, respectively. The $e_a$, $e_b$, and $e_c$ semi-major axes are in yellow, magenta, and teal respectively.
Figure 4.4: Relationship between the La CN and the $^{139}\text{La}$ isotropic chemical shifts (left) and quadrupolar coupling constants (right). Filled circles indicate compounds of the non-LaMO$_3$ family, while open circles indicate LaMO$_3$ compounds. The solid line in the left plot shows the relationship between La CN and $^{139}\text{La}\delta^{\text{CS}}_{\text{iso}}$, with $\delta^{\text{CS}}_{\text{iso}} = -135 \text{ ppm} \cdot \text{CN} + 1405 \text{ ppm}$ $(R^2 = 0.63)$ for the non-LaMO$_3$ compounds. Error bars may be obscured by the datum symbol.

4.6 Discussion

4.6.1 Coordination Number

The relationship between lanthanum CN and $^{139}\text{La}$ NMR properties has previously been investigated by Willans et al. [176]. They found no correlation between La CN and $C_Q$ or $\Omega$, but they did find a strong relationship between the La CN and $^{139}\text{La}\delta^{\text{CS}}_{\text{iso}}$. When analysing our data, we were unable to establish a generally strong relationship between the coordination number and $\delta^{\text{CS}}_{\text{iso}}$ $(R^2 = 0.11)$. The quality of the fit improves dramatically $(R^2 = 0.63)$ when considering compounds of general formula LaMO$_3$ ($M = \text{B}^{3+}, \text{Co}^{3+}, \text{Cr}^{3+}, \text{Sc}^{3+}, \text{Ti}^{3+}$) separately from the other lanthanum compounds (Fig. 4.4). The isotropic chemical shift of the non-LaMO$_3$ compounds decreases as coordination number increases, consistent with the observations of Willans et al. It is difficult to conclusively evaluate the trend of $\delta^{\text{CS}}_{\text{iso}}$ in the LaMO$_3$ compounds due to insufficient data, as the $\delta^{\text{CS}}_{\text{iso}}$ of LaTiO$_3$ has not been reported and the $\delta^{\text{CS}}_{\text{iso}}$ of LaCoO$_3$ is anomalously high; this is proposed to be caused by transferred hyperfine interactions [59]. The coordination number alone is clearly insufficient to relate the $^{139}\text{La}$ isotropic chemical shift to local structure, but it is nonetheless useful in assigning NMR peaks in structures with multiple lanthanum sites (e.g., La$_2$(SO$_4$)$_3$ · 9H$_2$O). The range of $\delta^{\text{CS}}_{\text{iso}}$ reported in this study is approximately 900 ppm, while the range of chemical shifts reported in the literature spans approximately 1200 ppm (excluding
samples affected by hyperfine interactions, e.g., LaCoO₃) [200].¹³⁹La ssNMR peaks are often broadened past this chemical shift range, limiting the potential effects of CSA in ultrawide compounds. Like Willans et al., no relationship was found between the lanthanum coordination number and ¹³⁹La CQ (Fig. 4.4).

4.6.2 Chemical Shielding Anisotropy

The quadrupolar interaction is clearly the primary influence on the lineshape of ¹³⁹La spectra. However, it is clear that CSA can have significant impact, as seen with the spectra of LaBO₃, LaScO₃, and La₂O₃. Many lanthanum compounds have only been investigated at a single field strength, preventing the collection of accurate CSA parameters. While we have investigated a few samples at two field strengths, the still too small number of data points and their considerable uncertainties render it impossible to draw any conclusions about the relationship between crystal structure and ¹³⁹La CSA at this time.

4.6.3 Quadrupolar Coupling Constant

The relationships between ¹³⁹La CQ values and selected polyhedral distortion parameters are shown in Fig. 4.5. As with the isotropic chemical shift, it is valuable to
examine the non-LaMO$_3$ and LaMO$_3$ compounds separately.

**Non-LaMO$_3$ Compounds**

The non-LaMO$_3$ compounds show a strong negative correlation with the sphericity distortion parameter, with $C_Q$ increasing as sphericity decreases. The uncertainty in the sphericity is relatively low for the non-LaMO$_3$ compounds as compared to the LaMO$_3$ compounds; this is attributed to a wider range of bond lengths present in most LaMO$_3$ compounds.

The quality of the fit of the relationship between the $\epsilon$ and $C_Q$ for the non-LaMO$_3$ compounds ($R^2 = 0.68$) is lower than both that of the LaMO$_3$ compounds ($R^2 = 0.79$) and the relationship between $\Sigma$ and $C_Q$ for the non-LaMO$_3$ compounds ($R^2 = 0.86$). The ellipsoid span is primarily dependent on the shortest and longest semi major ellipsoid axes, $e_a$ and $e_c$. There is a moderate correlation between $C_Q$ and $e_a$ for the non-LaMO$_3$ compounds (Fig. B.30, $R^2 = 0.69$), but not between $C_Q$ and $e_c$. As such, $\epsilon$ is not as good of a descriptor of distortion for the non-LaMO$_3$ compounds, but has the advantage of being generally useful for both non-LaMO$_3$ and LaMO$_3$ compounds.

In an attempt to relate $C_Q$ to specific structural features, the various distortion parameters are plotted against the longest and shortest La–O bonds ($d_{\text{max}}$ and $d_{\text{min}}$, respectively). The most relevant plots are presented in Fig. 4.6; the rest can be found in Appendix B. The distortion parameters that best correlate to the $C_Q$ of the non-LaMO$_3$ compounds are generally dependent on $d_{\text{max}}$, suggesting that the $C_Q$ of the non-LaMO$_3$ compounds might be similarly dependent. Unfortunately, this is not the case, as the relationship between $C_Q$ and $d_{\text{max}}$ of the non-LaMO$_3$ compounds is less reliable than the relationship between $C_Q$ and either $\Sigma$ or $\epsilon$ ($R^2$ values of 0.58, 0.85, and 0.68, respectively) for the non-LaMO$_3$ compounds. The quadrupolar coupling constants of the non-LaMO$_3$ compounds seem to be dependent on both $d_{\text{min}}$ and $d_{\text{max}}$, and are better fit by a combination of $d_{\text{min}}$ and $d_{\text{max}}$ than by either alone. The difference between the average La–O bond length ($d_{\text{avg}}$) and the shortest La–O bond length (Fig. 4.7) is the structural parameter which best predicts the $^{139}$La $C_Q$ for non-LaMO$_3$ compounds ($R^2 = 0.92$).
Figure 4.6: Filled circles indicate compounds of the non-LaMO$_3$ family, while open circles indicate LaMO$_3$ compounds. a) Relationship between $\Sigma$ and $d_{\text{max}}$. The solid line indicates the relationship between $\Sigma$ and $d_{\text{max}}$ for the non-LaMO$_3$ compounds, with $\Sigma = -5.71 \, \text{Å}^{-1} \cdot d_{\text{max}} + 8.23$ ($R^2 = 0.86$). The dashed line indicates the relationship between between $\Sigma$ and $d_{\text{max}}$ for the LaMO$_3$ compounds, with $\Sigma = -6.49 \, \text{Å}^{-1} \cdot d_{\text{max}} + 9.00$ ($R^2 = 0.92$). b) Relationship between $\epsilon$ and $d_{\text{min}}$. The dashed line indicates the relationship between between $\epsilon$ and $d_{\text{min}}$ for the LaMO$_3$ compounds, with $\epsilon = -0.48 \, \text{Å}^{-1} \cdot d_{\text{min}} + 2.52$ ($R^2 = 0.82$). c) Relationship between $\epsilon$ and $d_{\text{max}}$. The solid line indicates the relationship between between $\epsilon$ and $d_{\text{max}}$ for the non-LaMO$_3$ compounds, with $\epsilon = 1.92 \, \text{Å}^{-1} \cdot d_{\text{max}} + 2.5$ ($R^2 = 0.89$). Error bars may be obscured by the datum symbol.
Figure 4.7: Relationship between $C_Q$ and the difference of $d_{\text{avg}}$ and $d_{\text{min}}$. Filled circles indicate compounds of the non-LaMO$_3$ family, while open circles indicate LaMO$_3$ compounds. The solid line indicates the relationship between the bond length difference and $C_Q$ for the non-LaMO$_3$ compounds, with $C_Q = 272 \text{ MHz} \text{Å}^{-1} + 21 \text{ MHz}$ ($R^2 = 0.92$). Error bars may be obscured by the datum symbol.
LaMO$_3$ Compounds

In contrast to the strong relationship seen in the non-LaMO$_3$ compounds, the LaMO$_3$ compounds do not show a significant dependence of $C_Q$ on $\Sigma$ ($R^2 = 0.22$). The compounds that are most likely affecting the relationship between $C_Q$ and $\Sigma$ for the LaMO$_3$ compounds are LaCoO$_3$ and LaCrO$_3$. As discussed in their respective sections in the supporting information, the LaCoO$_3$ and LaCrO$_3$ coordination polyhedra are cuboctahedra with a large difference between $d_{\text{min}}$ and $d_{\text{max}}$; furthermore, the positioning of the longest and shortest La–O bonds in these structures ensures that it is difficult to fit a sphere to the oxygen positions.

The use of the ellipsoid span to relate $^{139}$La $C_Q$ and distortion is more effective than the use of $\Sigma$ for the LaMO$_3$ compounds, though it is difficult to reliably fit an ellipsoid to the lanthanum polyhedra of LaCrO$_3$ and LaCoO$_3$ for the same reasons that it is difficult to fit a sphere; this is reflected in the large uncertainties of $\epsilon$ for these compounds. The individual influences of $e_a$ and $e_c$ are examined for the LaMO$_3$ compounds, with $e_c$ having a much stronger relationship with $C_Q$ than $e_a$. This behaviour is the mirror image of the non-LaMO$_3$ compounds, and indicates the difference between the two families.

Figure 4.6 highlights additional differences between the LaMO$_3$ and non-LaMO$_3$ compounds. While the sphericity of both families is highly dependent on $d_{\text{max}}$, the ellipsoid span of the LaMO$_3$ compounds is not dependent on $d_{\text{max}}$, but instead on $d_{\text{min}}$. As such, we examine the relationship between $C_Q$ and $d_{\text{min}}$ for the LaMO$_3$ compounds (Fig. 4.8). The fit of the relationship between $C_Q$ and $d_{\text{min}}$ is comparable in quality to the relationship between $C_Q$ and $\epsilon$ ($R^2$ of 0.84 and 0.80, respectively). The $C_Q$ of the LaMO$_3$ compounds is effectively independent of $d_{\text{max}}$, with an $R^2$ of 0.06. While the $C_Q$ values of the LaMO$_3$ compounds cannot be explained solely by the shortest La–O bond lengths, they are much more dependent on this parameter than the non-LaMO$_3$ compounds. The differing influences of individual structural parameters also explains why the sphericity poorly relates to LaMO$_3$ $C_Q$: $\Sigma$ is strongly dependent on $d_{\text{max}}$, while the $C_Q$ values of LaMO$_3$ compounds are not dependent on $d_{\text{max}}$. We attribute the differences in the behaviour of the LaMO$_3$ and non-LaMO$_3$ compounds with regards to our model parameters primarily due to structural differences between the two compounds. The shortest La–O bonds in the LaMO$_3$ compounds form either
ideal or slightly distorted trigonal prisms, whereas this substructure is generally absent from the non-LaMO$_3$ coordination polyhedra. This substructure, when considered in the context of the remaining atoms in the coordination polyhedron, is difficult to fit with either a sphere or an ellipsoid, limiting the generalizability of our models.

The most generally applicable distortion parameter is the ellipsoid span, adequately relating the $C_Q$ to distortion for both the LaMO$_3$ and non-LaMO$_3$ compounds. The distortion of the non-LaMO$_3$ compounds is better described by the sphericity parameter. The $C_Q$ values of both LaMO$_3$ and non-LaMO$_3$ compounds can be related to specific structural features. The $C_Q$ of a given LaMO$_3$ compound is related to the length of the shortest La–O bond, while the $C_Q$ of a given non-LaMO$_3$ compound is related to the difference between the average La–O bond length and the shortest La–O bond length.

4.6.4 DFT Calculations

The quadrupolar coupling constants calculated with ABINIT generally show good agreement with those determined experimentally (Fig. 4.9), with deviations typically
Figure 4.9: Relationship between experimental and calculated $^{139}$La $C_Q$ values (left) and $\eta_Q$ values (right). Filled circles indicate compounds of the non-LaMO$_3$ family, while open circles indicate LaMO$_3$ compounds. The dashed line indicates a 1:1 relationship between computational results and experimental results. Error bars may be obscured by the datum symbol.

less than either 15\% or 3 MHz absolute, depending on the magnitude of $C_Q$. There are two notable exceptions to this trend: the calculated $C_Q$ values of LaBSiO$_5$ and LaPO$_4 \cdot n$H$_2$O are significantly greater than the experimental values, LaBSiO$_5$ by 22\% (20.0 MHz) and LaPO$_4 \cdot n$H$_2$O by 312\% (103.2 MHz). In both of these cases, the difference between experimental and \textit{ab initio} results are largely attributed to errors in the experimental structure. An example of the sensitivity of $C_Q$ to crystal structure is LaBO$_3$: calculations carried out on a different LaBO$_3$ structure [201] returned a $C_Q$ of $-84$ MHz and $\eta_Q$ of 0.05, with only slight structural differences from the structure used in this study. The error in the calculated $C_Q$ of LaPO$_4 \cdot n$H$_2$O has two possible explanations: firstly, the structure used in our calculations does not account for the presence of interstitial water; secondly, as discussed above, there is some debate over the structure of rare earth phosphate hydrates. The $C_Q$ predicted by the sphericity of the lanthanum site in LaPO$_4 \cdot n$H$_2$O is 107.8 MHz, consistent with the \textit{ab initio} results.

If the two results with the greatest absolute error (LaPO$_4 \cdot n$H$_2$O and LaBSiO$_5$) are excluded from the fit, the relationship between experimental and calculated $C_Q$ is very nearly 1:1, with most computational values being slightly overestimated.

The calculations of the asymmetry parameter suffer the same problems as the calculations of the quadrupolar coupling constant: the values of $\eta_Q$ for LaPO$_4 \cdot n$H$_2$O and LaBSiO$_5$ are extremely low as compared to experiment. The large deviation in $\eta_Q$ is also attributed to errors in the experimental geometry used for the DFT calculations.
The values of $\eta_Q$ for the other systems that were studied are more reliable. Most are constrained to $\eta_Q = 0$ by symmetry, which is reflected by the DFT calculations. The few that are not constrained by symmetry generally have calculated values which are reasonably close to experiment.

### 4.6.5 Differences Between LaMO$_3$ and non-LaMO$_3$ Compounds

There are no obvious differences between the LaMO$_3$ and non-LaMO$_3$ compounds aside from their general formulae. The coordination numbers of the LaMO$_3$ compounds range from 8 to 12, and examples can be found in the non-LaMO$_3$ compounds which share the same coordination numbers. A mirror plane can be found in the lanthanum polyhedra of most, but not all, LaMO$_3$ compounds, and can also be found in non-LaMO$_3$ compounds. The presence of a threefold rotation axis is not a unifying factor, as some LaMO$_3$ compounds possess this symmetry element, while others do not. Given the complex relationship between the EFG and crystal structure, it is likely that there is no single distinguishing structural feature between the LaMO$_3$ and non-LaMO$_3$ compounds, but instead a combination of features.

### 4.7 Conclusions

In this work we relate two numerical measures of the distortion of coordination polyhedra from spherical symmetry to $^{139}$La $C_Q$. The $^{139}$La WCPMG NMR spectra of LaBO$_3$, LaPO$_4$·1.8H$_2$O, LaBGeO$_5$, LaBSiO$_5$, La$_2$(SO$_4$)$_3$·9H$_2$O and La$_2$(CO$_3$)$_3$·8H$_2$O are reported for the first time, and used to supplement existing literature reports of $^{139}$La NMR parameters in solid state oxide materials. When examining a broad range of compounds, we find that the behaviour of the $^{139}$La $C_Q$ in response to deviations from spherical symmetry divides the examined materials into two families: compounds with the general formula LaMO$_3$, and the rest. The ellipsoid span, $\epsilon$, is effective in relating $C_Q$ to polyhedral distortion for both families of compounds. Non-LaMO$_3$ compounds are better described by the sphericity parameter $\Sigma$. Both families of compounds can be related to direct structural features, with the $C_Q$ of LaMO$_3$ compounds being strongly dependent on the shortest La–O bond length and non-LaMO$_3$ compounds exhibiting a more complex relationship. Isotropic chemical shifts of the non-LaMO$_3$ compounds are found to move to lower frequencies with higher coordination number, while there
are insufficient data to draw a similar conclusion for the LaMO$_3$ compounds. *Ab initio* calculations of EFG parameters provide values that generally agree with experiment, with the exceptions of LaBO$_3$, LaBSiO$_5$, and LaPO$_4$·$n$H$_2$O; the failure of *ab initio* calculations to return values consistent with experiment is attributed to errors in the reported crystal structures.

### 4.8 Acknowledgements

Financial support from the Natural Sciences and Engineering Research Council of Canada (NSERC) (Canada Grant RGPIN 261978) is gratefully acknowledged. We appreciate the assistance of Dr. M. Obrovac (TGA and pXRD) and Dr. C. Romao (IVTON). This research was enabled in part due to support provided by WestGrid ([https://www.westgrid.ca/](https://www.westgrid.ca/)).
Chapter 5

Application of $^{139}$La NMR Spectroscopy to Lanthanum Oxide-Based Glasses

5.1 Context

The previous chapter developed a model describing the relationship between various $^{139}$La spectral parameters and the local structure of lanthanum oxide-based crystals. This chapter focuses on applying that model to two lanthanum oxide-based glass series: lanthanum borogermanates and lanthanum phosphates. These series were chosen for several reasons. The lanthanum borogermanate series includes the LaBGeO$_5$ composition, which is the primary target of this dissertation. The lanthanum phosphate series forms a glass over a wide range of La$_2$O$_3$ concentrations, allowing for the possibility of a wide range of La–O coordination numbers ($n_{\text{LaO}}$). Both the lanthanum borogermanate and lanthanum phosphate series have $n_{\text{LaO}}$ values that have been determined via diffraction-based techniques; this is an important check on the predictive power of the crystal model, allowing for the model to be empirically tested. Finally, both LaBGeO$_5$ and LaP$_3$O$_9$ compositions have congruent crystal and glass forms. This potentially allows for the direct comparison of $n_{\text{LaO}}$ between the crystal and glass.

The work described in this chapter was completed by collaboration of ALP, UWZ, and JWZ. ALP synthesized most of the samples, collected the pXRD and NMR data, conducted much of the analysis, and wrote the majority of the manuscript. UWZ provided technical support in the collection of the NMR data and provided important contributions to the analysis of the $^{139}$La NMR spectra of the glass samples in particular. UWZ also provided editorial contributions. JWZ contributed to the understanding of the Czjzek distribution, contributed text describing the physical background of the distribution, and editorial input to the manuscript.
5.2 Abstract

Understanding the local environment of rare-earth ions in glasses is useful for designing effective optical glasses and functional nuclear waste glasses. Lanthanum is a particularly appealing target for NMR spectroscopy as it is both diamagnetic and chemically similar to the other lanthanide elements. However, due to its large quadrupole moment, to date $^{139}$La NMR spectroscopy has not been applied to the study of lanthanum-containing glasses. We present $^{139}$La nuclear magnetic resonance spectra of a variety of lanthanum oxide-based crystalline samples and glasses, including the LaBGeO$_5$ transparent ferroelectric composite. Our results suggest that the breadth of the $^{139}$La quadrupole distribution is indicative of the disorder in the second-neighbour shell around La. We relate the $^{139}$La isotropic chemical shift ($\delta_{\text{iso}}$) of crystalline materials to La–O coordination numbers. Our results suggest that the change in chemical shift with respect to change in CN is consistent within chemical species (with a value of $-130$ ppm CN$^{-1}$). Unfortunately, we find that the $^{139}$La $\delta_{\text{iso}}$ values appear to be masked by other broadening mechanisms in the glass samples, so we are unable to extract additionally coordination numbers by this method.

5.3 Introduction

Glasses containing rare-earth oxides are of significant scientific and industrial interest. Rare-earth oxide-based glasses have optical applications in lasers and optical amplifiers [202–205], and are a potential method for sequestering radioactive lanthanides and actinides produced in the nuclear fuel cycle [206, 207]. For many of these applications, an understanding of the local environment of the rare-earth ions in the glass is an important tool in tailoring the composition of the glass to obtain the desired set of properties.

Of the methods typically used for the characterization of the local structure of glasses (i.e., vibrational spectroscopy, NMR spectroscopy, neutron and/or X-ray diffraction), diffraction-based techniques have been the most widely used for the study of the environment of rare-earth ions. The cause for this is the high atomic weight of the lanthanides and actinides (restricting vibrational spectroscopy), and generally undesirable NMR properties of most rare-earth nuclides. Most rare-earth ions are
paramagnetic, precluding their direct study by NMR spectroscopy. Only three (i.e., $^{45}\text{Sc}$, $^{89}\text{Y}$, $^{139}\text{La}$) are reliably diamagnetic, and hence most NMR spectroscopy of rare-earth compounds have been of these nuclides.

$^{45}\text{Sc}$, $^{89}\text{Y}$, and $^{139}\text{La}$ NMR spectra of crystalline materials have been reported in the literature [47, 126, 208, 209]. However, to date only $^{45}\text{Sc}$ and $^{89}\text{Y}$ NMR spectroscopy have been applied to glasses [210, 211]. The reason for this is the large nuclear quadrupole moment of $^{139}\text{La}$ ($Q = (20.0 \pm 0.6) \text{ fm}^2$ [57]), which results in spectra that are typically quite broad, requiring step-wise collection and signal enhancement techniques to allow for timely acquisition [126]. In crystalline environments with well-defined crystallographic sites, it is nevertheless possible to separate overlapping environments via NMR lineshape fitting. This approach is less tractable in amorphous materials, where well-defined quadrupolar features are broadened and may be completely obscured by structural disorder [25].

One lanthanide oxide of particular interest is the $\text{LaBGeO}_5$ transparent ferroelectric glass-ceramic. The $\text{LaBGeO}_5$ glass crystallizes congruently, i.e., the glass and crystal share the same stoichiometry. However, there are significant differences in the local structure between the glass and the crystal. In particular, both the boron–oxygen and germanium–oxygen coordination numbers have been shown to vary [34, 212]. We have estimates of the mean La–O coordination number ($n_{\text{LaO}}$) in the $\text{LaBGeO}_5$ glass from our analysis of our neutron diffraction data, but these estimates are limited by the complex O–O coordination resulting from the distribution of Ge–O coordinations. This is a concern that is present in any amorphous system containing distributions of coordination numbers, multiple rare-earth elements, or small amounts of the element of interest (e.g., optically-relevant doping).

The extension of $^{139}\text{La}$ NMR spectroscopy to lanthanum oxide-containing glasses would allow for a useful tool in the analysis of the local environment of $\text{La}^{3+}$. As NMR spectroscopy is nuclide-specific, it allows for the characterization of local environments even in a complex system. This is particularly applicable to lanthanide or actinide glasses, where $\text{La}_2\text{O}_3$ is frequently used as a proxy for paramagnetic or radioactive oxides [206]. Furthermore, NMR spectroscopy is typically more accessible than neutron diffraction, which requires either a nuclear reactor or a spallation neutron source, available at only a few places in the world.
In this contribution we expand on our previous work on $^{139}$La NMR of crystalline materials [126], reporting the spectra of several additional crystals, and extend this work to lanthanum oxide-containing glasses. We present $^{139}$La spectra of various lanthanum borogermanate and lanthanum phosphate glasses. We are able to make structural inferences about the relative disorder around the lanthanum sites in the glasses. Unfortunately, it proves difficult to extract the isotropic chemical shift of lanthanum in the presence of the strong, disorder-broadened quadrupole interactions, so application of the crystal-based correlation found in Chapter 4 to extract coordination numbers is not possible.

5.4 Methods

5.4.1 Sample Preparation

Three glass sample series were produced: $x$LaBG ($x$La$_2$O$_3$−(100−$x$)(0.33B$_2$O$_3$−0.67GeO$_2$)), with $x = 20, 25, $ and 30; $y$LaP ($y$La$_2$O$_3$−(100−$y$)P$_2$O$_5$), with $y = 10, 15, 20, 25, $ and 30; and 25LaB ($25$La$_2$O$_3$−75B$_2$O$_3$). The synthesis and description of the $x$LaBG samples has been reported previously [212], and correspond to the samples labelled 20LaBG, 25LaBG-1, and 30LaBG-1 described therein. The $y$LaP and 25LaB glasses were produced in a similar fashion. All reagents were sourced from Sigma-Aldrich. Stoichiometric amounts of the appropriate reagents [(La$_2$O$_3$, 99.99 %), (NH$_4$H$_2$PO$_4$, ≥ 98 %), (B$_2$O$_3$, ≥ 98 %)] were thoroughly ground in a porcelain mortar and pestle, and then placed in a platinum crucible. The $y$LaP samples were first heated to approximately 400 °C for several hours, to allow for the conversion of NH$_4$H$_2$PO$_4$ to P$_2$O$_5$, before being placed into a furnace preheated to temperatures between 850 °C to 1300 °C, depending on La$_2$O$_3$ content. The 25LaB sample was placed directly into a box furnace preheated to 1300 °C and allowed to melt for 30 min. The $y$LaP and 25LaB glasses were quenched by pouring onto room-temperature brass plates.

In addition to the glasses, LaB$_3$O$_6$, LaP$_3$O$_9$, and La$_2$Sn$_2$O$_7$ crystalline powders were synthesized. The LaB$_3$O$_6$ crystalline powder was synthesized by combining stoichiometric amounts of La$_2$O$_3$ and B$_2$O$_3$ and heating in a platinum crucible for 12 h
at 850°C in air. The LaP₃O₉ crystalline powder was synthesized by combining stoichiometric amounts of La₂O₃ and solid H₃PO₄ (99.999%) under nitrogen atmosphere for 60 h at 400°C. The La₂Sn₂O₇ crystalline powder was synthesized by combining stoichiometric amounts of La₂O₃ and SnO₂ (99.9%) and heating in a platinum crucible for 4 d at 1250°C in air.

All diffraction measurements were made using a Rigaku Ultima IV X-ray diffractometer equipped with a Cu anode X-ray tube and a diffracted beam monochromator. Diffraction measurements used angular steps of 0.05°, and dwell times of 1 s.

5.4.2 NMR Spectroscopy

All ¹³⁹La NMR spectra were collected using the WCPMG pulse sequence [129], which combines WURST pulses [123, 124] with CPMG loops [130, 131]. Spectra were collected on a 16.4 T Bruker Avance spectrometer using 4 mm ZrO₂ rotors, and on a 9.4 T Bruker Avance spectrometer using custom-made 9 mm glass tubes. The spectra were collected under static conditions. 50µs WURST-80 pulses were swept across 500 kHz at a rate of 10 MHz ms⁻¹. Most spectra were too broad to allow for collection in one spectral window (1 MHz). To allow for the entire spectrum of the central transition to be collected, the VOCS method was employed [127, 128]. The transmitter was moved in VOCS steps of 100 kHz. The number of steps required was dependent on the breadth of the spectrum, with collection proceeding until either no significant signal could be observed, or until the signal consisted of nothing but a flat line (indicative of the satellite transitions). The number of scans per step varied between 16 and 32 768, depending on the La₂O₃ content of the sample, as well as the number of echoes collected per scan. The number of echoes collected per scan ranged from 5 to 100 depending on the sample, with the crystalline samples generally allowing for a greater number than the glass samples. The recycle delay was optimized for each sample, with times of 1 s proving sufficient for full relaxation in all cases. ¹³⁹La NMR spectra were referenced to a 1 mol dm⁻³ aqueous solution of LaCl₃ (0 ppm). Spectra were fit using Dmfit version 20150521 [184]. The Czjzek model, as implemented in Dmfit, was used to fit the ¹³⁹La NMR spectra of the glass samples [135, 136, 138, 139]. For the glass samples, the position parameter (Δ_offset) and the standard deviation of the electric field gradient tensor elements from the Czjzek fit (σ) are reported. For
the crystalline compounds, fits were performed with the usual chemical shift and quadrupole parameters, and the quadrupolar coupling constant \(C_Q\), quadrupolar asymmetry parameter \(\eta_Q\), and isotropic chemical shift \(\delta_{\text{iso}}\) are reported as per standard conventions [159]. We report the CSA parameters using the Herzfeld-Berger convention, where \(\Omega\) is the chemical shift span and \(\kappa\) is the chemical shift skew [160]. Some spectra are presented as the Fourier transform of the summed WCPMG echoes, which returns a continuous lineshape that is equivalent to the intensity profile of the spikelet manifold. This allows for least-squares fitting of the lineshape using DMFIT.

5.5 Results

5.5.1 X-Ray Diffraction

The results of our powder X-ray diffraction experiments are presented in Fig. 5.1. The diffraction patterns indicate that the samples are primarily composed of the structure of interest. The top diffraction pattern is consistent with that calculated from the reported crystal structure of \(\text{La}_2\text{Sn}_2\text{O}_7\) (powder diffraction file (PDF) 88-0446) [213]. We observe small impurity peaks consistent with the presence of cassiterite \(\text{SnO}_2\) (PDF 72-1147). The middle diffraction pattern is consistent with the diffraction pattern of \(\text{LaB}_3\text{O}_6\) reported in the literature (PDF 13-0246) [214]. There are minor impurity peaks identified as a trace amount of \(\text{LaBO}_3\) (PDF 12-0762). The bottom diffraction pattern is consistent with literature reports of \(\text{LaP}_3\text{O}_9\) (PDF 33-0717) [215]. There is an impurity peak at approximately 23° which we have been unable to identify conclusively. The impurity is tentatively attributed to \(\text{PtP}_2\text{O}_7\), which could have formed through the reaction of the \(\text{H}_3\text{PO}_4\) with the platinum crucible. However, we cannot be confident in the exact identification of the impurity based upon only one unique diffraction peak.

For the purposes of \(^{139}\text{La}\) NMR, these samples are sufficiently pure. As NMR is isotopically selective, the presence of impurities which do not contain \(^{139}\text{La}\) do not affect the \(^{139}\text{La}\) NMR spectrum. Hence, the presence of \(\text{SnO}_2\) does not affect the \(^{139}\text{La}\) NMR spectrum of \(\text{La}_2\text{Sn}_2\text{O}_7\). We have previously reported the \(^{139}\text{La}\) NMR spectrum of \(\text{LaBO}_3\), and its influence on the spectrum of \(\text{LaB}_3\text{O}_6\) can be accounted for [126]. While we cannot identify the impurity in the \(\text{LaP}_3\text{O}_9\) sample, it is inconsistent with
(a) La$_2$Sn$_2$O$_7$ (PDF 88-0446, squares) and SnO$_2$ (PDF 72-1147, diamonds).

(b) LaB$_3$O$_6$ (PDF 13-0246, squares) and LaBO$_3$ (PDF 12-0762, diamonds).

(c) LaP$_3$O$_9$ (PDF 33-0717, squares) and PtP$_2$O$_7$ (PDF 33-0980, diamonds). Only the major peaks of PtP$_2$O$_7$ are presented.

Figure 5.1: X-ray diffraction patterns of the crystalline compounds studied in this work. Intensities are presented on a linear scale, and are normalized such that the most intense peak has a value of 1000. The primary compounds are indicated by squares, while impurity phases are indicated with diamonds. The vertical position of a marker is indicative of its reported intensity. All reference patterns are obtained from the literature, and citations are provided in the text.
Figure 5.2: The $^{139}$La WCPMG NMR spectrum of LaB$_3$O$_6$ at 16.4 T. The LaB$_3$O$_6$ parameters used to generate the fit are reported in the text. The parameters used for the LaBO$_3$ contribution have been previously reported [126].

the presence of any known La-based crystals, and we thus neglect it.

5.5.2 $^{139}$La NMR Spectroscopy of Crystalline Compounds

We have previously reported on the relationship between various $^{139}$La NMR parameters and structural distortion in lanthanum-based oxide crystalline materials [126]. In this work we show the spectra of three additional lanthanum-based oxide crystalline materials, two of which have congruent glass forms.

We present the $^{139}$La NMR spectrum of our crystalline LaB$_3$O$_6$ sample, and our fit to the spectrum, in Fig. 5.2. The LaB$_3$O$_6$ crystal structure has a single La site, and hence we expect to fit the spectrum with a single lineshape [216]. In agreement with our X-ray results, an adequate fit of the spectrum requires the inclusion of a LaBO$_3$ peak, the $^{139}$La NMR parameters of which we have previously reported [126]. We neglect the CSA parameters in our fit, as their influence is
The $^{139}$La WCPMG NMR spectrum of LaP$_3$O$_9$ at 16.4 T. The spectral parameters used to generate the fit are reported in the text.

expected to be small compared to the quadrupolar parameters. Our fit for the LaB$_3$O$_6$ portion of the spectrum yields $\delta_{\text{iso}}^{\text{CS}} = (46 \pm 50)$ ppm, $C_Q = (54.4 \pm 0.5)$ MHz, and $\eta_Q = 0.58 \pm 0.03$. WCPMG experiments are inherently non-quantitative in relative intensities, as the signal enhancement is dependent on the spin-spin relaxation of the respective crystallographic sites. It is our experience that the La site in LaBO$_3$ is particularly enhanced by WCPMG. However, the amount of LaBO$_3$ intensity required for a reasonable fit of the spectrum is less than 5 %, consistent with the X-ray diffraction data (Fig. 5.1).

The $^{139}$La NMR spectrum of the crystalline LaP$_3$O$_9$ sample is presented in Fig. 5.3. The crystal structure of LaP$_3$O$_9$ has a single La site. We fit the spectrum with a single quadrupolar lineshape, with $\delta_{\text{iso}}^{\text{CS}} = (26 \pm 25)$ ppm, $C_Q = (43.8 \pm 0.5)$ MHz, and $\eta_Q = 0.95 \pm 0.04$. There is no indication of any La-based impurity such as La$_2$O$_3$ or LaPO$_4$.

The $^{139}$La NMR spectra of La$_2$Sn$_2$O$_7$ at 16.4 T and 9.4 T are presented in Fig. 5.4. The crystal structure of La$_2$Sn$_2$O$_7$ has a single axially symmetric La site, which restricts $\eta_Q$ to be 0 or near-0 [213]. Unlike LaB$_3$O$_6$ and LaP$_3$O$_9$, a good fit of La$_2$Sn$_2$O$_7$ can only
Figure 5.4: The $^{139}$La WCPMG NMR spectra of La$_2$Sn$_2$O$_7$ at a) 16.4 T and b) 9.4 T. The spectral parameters used to generate the fit are reported in the text. The same set of parameters is used at both field strengths.
by obtained by including the effects of CSA, requiring collecting spectra at two NMR field strengths. We fit the \( \text{La}_2\text{Sn}_2\text{O}_7 \) spectrum with the following parameters: \( C_Q = (78.3 \pm 0.3) \text{ MHz} \); \( \eta_Q = 0.00 \pm 0.02 \); \( \delta_{\text{iso}}^{\text{CS}} = (540 \pm 100) \text{ ppm} \); \( \Omega = (375 \pm 100) \text{ ppm} \); \( \kappa = 1.00 \pm 0.03 \). The EFG and CSA tensors are coaxial, consistent with the crystal structure and density functional theory calculations (not shown). The consideration of the CSA parameters is required for the accurate positioning of the step discontinuity; if the CSA parameters are neglected, the quadrupolar horns and the step discontinuity cannot be fit simultaneously.

The \(^{139}\text{La}\) isotropic chemical shift \( \delta_{\text{iso}}^{\text{CS}} \) values from this work are included in our overall dataset of known \(^{139}\text{La}\) NMR parameters and structural information, which is presented in Table 5.1. We restricted our dataset to compounds where the crystal structure is well-known, the \( \text{La–O} \) coordination numbers are well-defined, the \(^{139}\text{La}\) NMR peaks are unambiguously fit and assigned, and which do not fall into the \( \text{LaMO}_3 \) family of compounds which we have previously discussed [126]. The relationship between the \(^{139}\text{La}\) \( \delta_{\text{iso}}^{\text{CS}} \) and the \( n_{\text{LaO}} \) coordination number is presented in Fig. 5.5. The data are fit using a linear regression, with a slope of \((-130 \pm 40) \text{ ppm CN}^{-1} \) and intercept of \((1400 \pm 400) \text{ ppm} \) \( (R^2 = 0.51) \). The lanthanum phosphates \( \text{i.e.}, \text{LaPO}_4, \text{LaP}_3\text{O}_9 \) and the lanthanum sulfate nonahydrate samples have \( \delta_{\text{iso}}^{\text{CS}} \) values that are

<table>
<thead>
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<th>Compound</th>
<th>( n_{\text{LaO}} )</th>
<th>( \delta_{\text{iso}}^{\text{CS}} / ) ppm</th>
<th>( \delta_{\text{iso}}^{\text{CS}} ) Source</th>
<th>Structure Source</th>
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<tr>
<td>( \text{La(OH)}_3 )</td>
<td>9</td>
<td>260 ± 20</td>
<td>[60]</td>
<td>[217]</td>
</tr>
<tr>
<td>( \text{La}_2(\text{SO}_4)_3 \cdot 9 \text{H}_2\text{O} ) (1)</td>
<td>12</td>
<td>−175 ± 25</td>
<td>[126]</td>
<td>[218]</td>
</tr>
<tr>
<td>( \text{La}_2(\text{SO}_4)_3 \cdot 9 \text{H}_2\text{O} ) (2)</td>
<td>9</td>
<td>−75 ± 25</td>
<td>[126]</td>
<td>[218]</td>
</tr>
<tr>
<td>( \text{La}_2\text{O}_3 )</td>
<td>7</td>
<td>620 ± 10</td>
<td>[126]</td>
<td>[219]</td>
</tr>
<tr>
<td>( \text{La}_2\text{Sn}_2\text{O}_7 )</td>
<td>8</td>
<td>540 ± 100</td>
<td>This work</td>
<td>[213]</td>
</tr>
<tr>
<td>( \text{LaB}_3\text{O}_6 )</td>
<td>10</td>
<td>45 ± 50</td>
<td>This work</td>
<td>[216]</td>
</tr>
<tr>
<td>( \text{LaBGeO}_5 )</td>
<td>9</td>
<td>225 ± 25</td>
<td>[126]</td>
<td>[78]</td>
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<tr>
<td>( \text{LaBSiO}_5 )</td>
<td>10</td>
<td>225 ± 50</td>
<td>[126]</td>
<td>[220]</td>
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<tr>
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<td>295 ± 25</td>
<td>[60]</td>
<td>[221]</td>
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<tr>
<td>( \text{LaP}_3\text{O}_9 )</td>
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<td>This work</td>
<td>[222]</td>
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<td>9</td>
<td>36 ± 10</td>
<td>[172]</td>
<td>[223]</td>
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</table>
Figure 5.5: The relationship between $^{139}$La $\delta^{\text{CS}}_{\text{iso}}$ and La–O coordination number in crystalline compounds. The data used to construct this figure are reported in Table 5.1. The line indicates a linear fit to the data, with slope of $(-130 \pm 40)$ ppm CN$^{-1}$ and intercept of $(1400 \pm 400)$ ppm ($R^2 = 0.51$). The phosphate and sulphate species are highlighted.
substantially lower than the trendline. Figure 5.5 shows a moderate correlation between La–O coordination number and $^{139}$La $\delta_{iso}^{CS}$, but the deviation of the phosphates and sulfate from the trendline suggests that the relationship may be affected by composition.

5.5.3 $^{139}$La NMR Spectroscopy of Glasses

We present $^{139}$La WCPMG NMR spectra of three glass series: $x$LaBG, with composition $x$La$_2$O$_3$–(100–$x$)(0.33B$_2$O$_3$–0.67GeO$_2$); $y$LaP, with composition $y$La$_2$O$_3$–(100–$y$)P$_2$O$_5$; and 25LaB, with composition 25La$_2$O$_3$–75B$_2$O$_3$. $^{11}$B MAS NMR and neutron diffraction data from the $x$LaBG samples have been previously reported in the literature [212]. The spectra of the glass samples are presented in Fig. 5.6. The spectra are extremely broad; the narrowest spectrum has a full width at half maximum of approx. 1500 ppm. This is due in large part to the large quadrupole moment of the $^{139}$La nucleus, but also due to the structural disorder inherent in glasses. As the spectra are universally too broad to successfully narrow using MAS, they are collected under static conditions. The $x$LaBG spectra have similar shapes, with each spectrum showing an asymmetric tail to low frequencies and a noticeable increase in width at the greatest La$_2$O$_3$ content. The $y$LaP spectra show greater variation in breadth. They are markedly narrower than the $x$LaBG spectra, and also show a significant increase in width with added La$_2$O$_3$. The asymmetric shape of the $^{139}$La spectra of the glass samples are indicative of a distribution of quadrupolar parameters, that is, of electric field gradients.

In order to fit spectra with distributions of EFG parameters, we employ the Czjzek distribution [135, 136, 138, 139]. This model assumes that the values of each of the five independent elements of the EFG tensor ($C_Q$, $\eta_Q$, and the Euler angles $\alpha$, $\beta$, and $\gamma$) are normally distributed, with mean zero and standard deviation $\sigma$ (the same $\sigma$ is used for all five elements). These assumptions are typically valid for quadrupolar nuclei in disordered environments with relatively large coordination numbers, and so are reasonable to apply to the $^{139}$La case. From a Czjzek fit, the size of $\sigma$ may be correlated with the extent of disorder. In addition, the spectra show overall composition-dependent changes in chemical shift, but our tests comparing the shifts in different field strengths indicate that this shift is not due just to the isotropic chemical shift. It likely includes contributions from other interactions as well,
Figure 5.6: $^{139}$La WCPMG NMR spectra of the glass series $x$LaBG and $y$LaP, and of 25LaB. The $x$LaBG spectra are presented on the bottom; the $y$LaP spectra are in the middle; and the 25LaB spectrum is on the top. The spectra are presented as WCPMG spikelet patterns with normalized intensities. The parameters used to fit these spectra are reported in Table 5.2.
Figure 5.7: $^{139}$La WCPMG NMR spectrum of the 25LaP glass. The parameters used for the fit are reported in Table 5.2. This fit is representative of the fits of all glass samples reported here.

which we cannot quantify at this time. An example of the fit produced by applying the Czjzek distribution to our $^{139}$La NMR spectra is presented in Fig. 5.7, where we present the fit of the 25LaP sample.

The fitted $^{139}$La NMR parameters of all glass samples are presented in Table 5.2.

The greatest $\sigma$ value in the $x$LaBG samples occurs at the greatest La$_2$O$_3$ content. In the $y$LaP sample series, the $\sigma$ values are notably smaller than in the $x$LaBG samples, and increase linearly with $y$. Both observations are in line with the compositional dependence of the spectral widths noted above.

5.6 Discussion

5.6.1 The Applicability of the Crystalline Model to Glasses

Based upon the data presented in Table 5.1, we calculate a linear relationship between $^{139}$La chemical shift and La–O coordination number. This relationship is presented in Fig. 5.5. The linear fit has a slope of $(-130 \pm 40)$ ppm and an intercept of $(1400 \pm 400)$ ppm ($R^2 = 0.51$). This model does not effectively describe the behaviour
Table 5.2: $^{139}\text{La}$ $\Delta_{\text{offset}}$ and $\sigma$ values for the glass samples. Values are provided at two field strengths for the $x\text{LaBG}$ samples. Uncertainties are indicated by parentheses.

<table>
<thead>
<tr>
<th>Sample</th>
<th>La$_2$O$_3$ Content / mol%</th>
<th>$\Delta_{\text{offset}}$ / ppm</th>
<th>$\sigma$ / MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>10LaP</td>
<td>10</td>
<td>3 ± 25</td>
<td>2.9 ± 0.2</td>
</tr>
<tr>
<td>15LaP</td>
<td>15</td>
<td>109 ± 25</td>
<td>3.1 ± 0.2</td>
</tr>
<tr>
<td>20LaP</td>
<td>20</td>
<td>132 ± 25</td>
<td>3.3 ± 0.2</td>
</tr>
<tr>
<td>25LaP</td>
<td>25</td>
<td>148 ± 25</td>
<td>3.4 ± 0.2</td>
</tr>
<tr>
<td>30LaP</td>
<td>30</td>
<td>178 ± 25</td>
<td>3.6 ± 0.2</td>
</tr>
<tr>
<td>20LaBG</td>
<td>20</td>
<td>968 ± 50</td>
<td>4.3 ± 0.2</td>
</tr>
<tr>
<td>25LaBG-1</td>
<td>25</td>
<td>942 ± 50</td>
<td>4.4 ± 0.2</td>
</tr>
<tr>
<td>30LaBG-1</td>
<td>30</td>
<td>1030 ± 50</td>
<td>4.4 ± 0.2</td>
</tr>
<tr>
<td>25LaB</td>
<td>25</td>
<td>308 ± 25</td>
<td>4.0 ± 0.2</td>
</tr>
</tbody>
</table>

of the lanthanum phosphate samples included in the dataset (i.e., LaPO$_4$ and LaP$_3$O$_9$). The most likely cause for this discrepancy is a dependence of the $^{139}\text{La}$ chemical shift range on the second-nearest-neighbour of lanthanum.

Second-nearest-neighbour effects on chemical shifts are well-known in NMR spectroscopy of other elements, in both crystals and glasses. For example, Dupree et al. reported that the $^{43}\text{Ca}$ $\delta_{\text{iso}}^{\text{CS}}$ values of calcium silicates, carbonates, and oxides depend on the mean Ca–O bond length [224]. However, while all compounds were found to have the same dependence of $\delta_{\text{iso}}^{\text{CS}}$ on bond length, the shifts were offset depending on the nature of the second neighbour. Thus, the second neighbour type provided an additive constant to the shifts, while for a given neighbour type, the shifts changed with bond length in the same way.

We hypothesize that the same type of effect is operative in the $^{139}\text{La}$ shifts, namely, that all glasses show the same linear change in shift with first-neighbour oxygen coordination number, and that this linear change will be offset by different amounts, depending on the nature of the second neighbour. Hence any potential model relating $^{139}\text{La}$ $\delta_{\text{iso}}^{\text{CS}}$ to coordination number must account for composition effects. However, it is not clear how to obtain the true $^{139}\text{La}$ $\delta_{\text{iso}}^{\text{CS}}$ of the glass.

From our previous work, as well as a matter of general principles, we expect the $^{139}\text{La}$ $\delta_{\text{iso}}^{\text{CS}}$ of the glass to be related to the mean La–O coordination number of the glass [126, 225, 226]. In Table 5.2 we present the position parameters of the Czjzek model as implemented in Dmfit, which we label $\Delta_{\text{offset}}$, as well as the $\sigma$ values, of the
Figure 5.8: The $^{139}$La WCPMG NMR spectra of the 25LaBG sample at two field strengths. Top, spectrum at 9.4 T. Bottom, spectrum at 16.4 T. Parameters used to fit these spectra are reported in Table 5.2. Note that while the maxima of the spectra occur at approximately the same chemical shift value, the $\Delta_{\text{offset}}$ values are quite different.

glasses. Using the $\Delta_{\text{offset}}$ values from our 16.4 T data with our fit from Fig. 5.5 yields reasonable predicted coordination numbers for the $x$LaBG samples, consistent with those estimated from neutron diffraction (Section 6.6.2) [212]. This is coincidence. When the $\Delta_{\text{offset}}$ values at 16.4 T for the $y$LaP sample series are passed through the fit presented in Fig. 5.5, the predicted coordination numbers range between 9 and 11. From diffraction data reported by Hoppe et al., we would expect coordination numbers between $6.1 \pm 0.3$ and $8.2 \pm 0.3$ for the $y$LaP samples [10]. However, there is a more fundamental concern in applying $\Delta_{\text{offset}}$ to our crystalline model.

Spectra for the $x$LaBG samples were collected at multiple field strengths (9.4 T and 16.4 T). A comparison of the spectra of the $x = 25$ sample is presented in Fig. 5.8. Their $\Delta_{\text{offset}}$ values at the two field strengths do not agree. If $\Delta_{\text{offset}}$ was truly the isotropic chemical shift of this system (i.e., $\delta_{\text{iso}}^{CS}$), its value should be independent of field strength. As the value of $\Delta_{\text{offset}}$ is not independent of field strength, it cannot be $\delta_{\text{iso}}^{CS}$, and hence cannot be meaningfully compared to the $\delta_{\text{iso}}^{CS}$ values of the crystalline
samples, or the model described in Fig. 5.5. There are many possible interactions which could account for the discrepancy between the $\Delta_{\text{offset}}$ values at different field strengths, and there are insufficient data to allow for the determination of which interactions are responsible.

While the fit procedure outlined above yields interesting structural information about the relative disorder around the La sites, the fact that the fit parameter $\Delta_{\text{offset}}$ does not appear to be a faithful representation of $\delta_{\text{iso}}^{CS}$ means that we cannot extract coordination numbers by this method. We can speculate that $\Delta_{\text{offset}}$ contains not only the isotropic chemical shift, but also effects due to disorder broadened chemical shift anisotropy, higher-order quadrupole effects, and so forth. More detailed data will be required to test these suggestions.

### 5.6.2 Quadrupolar Interactions

While crystalline La-oxide compounds typically show La sites with significant electric field gradient interactions, the glass $^{139}$La NMR spectra are all well-fit by the Czjzek distribution. Recall that the Czjzek distribution results in a single parameter fit, assuming only that the electric field gradient tensor component values are normally distributed with mean zero and standard deviation $\sigma$ [135, 139]. This distribution thus implies that the \emph{average} quadrupole interaction for $^{139}$La in the glasses is zero. However, as has been discussed elsewhere, it can be shown that this distribution also yields standard deviations in quantities like the quadrupole product $P_Q$ and the quadrupolar frequency $\nu_Q$, as well as the most probable values of $P_Q$ and $\nu_Q$, that are non-zero [139]. There is no inconsistency here, as the EFG interaction is a signed quantity: in a disordered solid, for sites with many local interactions, it is reasonable that the average over all sites will yield both positive and negative interactions, leading to a zero mean but non-zero variance and standard deviation, just as the Czjzek model predicts. We also note that the standard deviations of $P_Q$ and $\nu_Q$, as well as their most probable values, are all proportional to the single fit parameter $\sigma$, with constants of proportionality of order 1 [139]. Therefore, to avoid the confusion of using symbols familiar from the crystalline context to describe the glasses, we will discuss the quadrupole interactions for $^{139}$La in glasses only in terms of the standard deviation of the EFG tensor elements $\sigma$. The meaning of this quantity is clear: an
increase in $\sigma$ implies an increase in the disorder in the neighbourhood of the nucleus probed.

In the LaP glass series, the increase in $\sigma$ with added La$_2$O$_3$ (Table 5.2) indicates an increase in disorder around the La sites. As the first coordination sphere is certainly oxygen, the origin of this disorder is likely due to the second neighbour distribution. As La$^{3+}$ acts as a modifier, we would expect that the increasing La$_2$O$_3$ content would be accompanied by a depolymerization of the phosphate network, and in fact the production of a variety of phosphate $Q^n$ species. Thus we interpret the increase in $\sigma$ as signalling an increase in the diversity of $Q^n$ species with added La$_2$O$_3$. This result is in agreement with the Raman spectra obtained by Brow and co-workers [227]. They concluded that as La$_2$O$_3$ content increases, the cross-linked $Q^3$ PO$_4$ tetrahedra are replaced by chain-like $Q^2$ tetrahedra. Furthermore, they observe the simultaneous existence of $Q^1$, $Q^2$, and $Q^3$ species, increasing the degree of structural disorder surrounding the La$^{3+}$ ion.

An additional potential contributor to disorder in the second coordination sphere of La$^{3+}$ is the presence of multiple chemical species. In binary lanthanum phosphates, both Brow et al. and Hoppe et al. conclude that at moderate La$_2$O$_3$ concentrations, La$^{3+}$ will begin to cluster — i.e., there will be La–O–La neighbours [10, 61, 227]. We would expect that the presence of multiple second nearest neighbour species would lead to greater variance in the EFG distribution. Likewise, the larger values of $\sigma$ observed in the LaBG glasses probably arises from the fact that both borate and germanate species appear in the second neighbour shell, which would give stronger disorder interactions than just the phosphate groups alone. This finding is in agreement with our previous work based on $^{11}$B NMR spectroscopy and neutron diffraction [212].

5.7 Conclusions

We present $^{139}$La WCPMG NMR data of a range of lanthanum oxide-based crystalline materials and glasses. The $^{139}$La NMR spectra of La$_2$Sn$_2$O$_7$, LaP$_3$O$_9$, and LaB$_3$O$_6$, and their associated fits, are reported. The $^{139}$La WCPMG NMR spectra of lanthanum oxide-based glasses are presented for the first time. The glass spectra can be well-fit using the Czjzek model, which accounts for the distribution of quadrupole parameters in disordered solids.
We find that the breadth parameter of the Czjzek model, $\sigma$, increases with increasing $\text{La}_2\text{O}_3$ content in the glasses. This is correlated with an increase in disorder surrounding the $\text{La}^{3+}$ environment. In the lanthanum phosphate glasses, the change in the disorder is attributed to changes in the $Q^n$ species of the phosphate units. In the lanthanum borogermanate glasses, the change in the disorder is attributed to both changes in the local order of the borate and germanate units, as well as potential changes in the proportion of neighbouring borate and germanate units.

The position parameter of the Czjzek fit, $\Delta_{\text{offset}}$, is found to be different from the isotropic chemical shift, $\delta_{\text{iso}}^{\text{CS}}$. As such, it is not clear how to extract $\delta_{\text{iso}}^{\text{CS}}$ values for the glass samples, and thus the correlations found from crystal models are not applicable at this point.

Our results show that $^{139}\text{La}$ WCPMG NMR spectroscopy has a complimentary role in the study of lanthanum oxide-based glasses. However, rather than obtaining coordination numbers, $^{139}\text{La}$ WCPMG NMR spectroscopy is better suited to assessing changes in disorder around the $\text{La}^{3+}$ environment.

5.8 Acknowledgements

Financial support from NSERC (Canada Grant Number RGPIN 261987) is gratefully acknowledged. Dr. Mark Obrovac is thanked for access to an X-ray diffractometer. Ms. Jocelyn Sinclair is thanked for synthesizing the $\text{La}_2\text{Sn}_2\text{O}_7$ sample.
Chapter 6

Structural Differences between the Glass and Crystal Forms of the Transparent Ferroelectric Nanocomposite LaBGeO$_5$, from Neutron Diffraction and NMR Spectroscopy

6.1 Context

The previous two chapters (Chapters 4 and 5) were concerned with determining the mean La–O coordination number in the LaBGeO$_5$ glass via $^{139}$La NMR spectroscopy. It is also possible to determine mean coordination numbers for glass modifiers in glasses via neutron diffraction, and Chapter 5 referred to estimates based on the data contained within this chapter. Furthermore, for some nuclei, NMR is not feasible while neutron diffraction is; germanium is such a case. The primary goal of the work contained within this chapter was to assess the possibility of the presence of high-coordinate germanium species (i.e., GeO$_5$ and/or GeO$_6$) within the LaBGeO$_5$ glass. Neutron diffraction has been used extensively in the literature to establish the presence of high-coordinate germanium species in alkali germanate, alkaline earth germanate, and germanophosphate glasses, making it a suitable technique for testing the LaBGeO$_5$ glass system [12, 28, 42].

An additional goal of this work was to identify the BO$_3$ species present in the LaBGeO$_5$ glass. While the presence of some kind of BO$_3$ in the LaBGeO$_5$ glass is known from the literature, the precise nature of the BO$_3$ species are present had not been determined [34]. High-resolution $^{11}$B MAS and MQMAS NMR spectroscopy was applied to a lanthanum borogermanate glass series to examine the change in the BO$_3$ environment as the amount of lanthanum oxide was varied.

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Crystal Forms of the Transparent Ferroelectric Nanocomposite LaBGeO₅, from Neutron Diffraction and NMR Spectroscopy, *J. Phys. Chem. C* 122 (36) (2018) 20963–20980 [212]. Copyright 2018 American Chemical Society. The content below has been reproduced from the preprint version of the article, with minor modifications for style and clarity. Copyright permissions for this use are contained in Appendix E. The contributions of the authors to the text are as follows. ALP synthesized all samples, collected all experimental data, performed the analyses, and wrote the majority of the manuscript. ACH provided substantial technical support in the collection of the neutron diffraction data, reviewed the neutron diffraction analysis performed by ALP, and provided additional analysis relating to the La–O coordination number (Section 6.6.2). ACH also contributed to the editorial review of the manuscript. UWZ assisted in the collection and interpretation of the NMR results, as well as contributing to the electrostatic bond strength (EBS) analysis (Section 6.6.4). JWZ assisted in the collection of the neutron diffraction data, and provided significant editorial contributions to the manuscript.

### 6.2 Abstract

A combination of pulsed neutron diffraction and high-field $^{11}$B NMR spectroscopy was used to obtain structural information on the $x$La$_2$O$_3$·(1-$x$)(0.33B$_2$O$_3$–0.67GeO$_2$) ($x = 0.00, 0.20, 0.25, 0.30$) glass system, as well as a crystallized sample of composition LaBGeO$_5$. LaBGeO$_5$ is of interest as a ferroelectric nanocomposite, as it crystallizes isochemically from the glass. The crystal structure is comprised of 4-coordinated boron and germanium, together with 9-coordinated lanthanum. Our results support a model for the glass structure that is substantially different: both NMR and neutron diffraction show that the glasses contain large fractions of 3-coordinated boron as well as 4-coordinated boron, and the neutron diffraction results show clear evidence for a significant fraction of germanium with a coordination number greater than four. Moreover, the La coordination number in the glasses is estimated as 7.4, with a typical La–O bond length of 2.52 Å, based on detailed modelling of the neutron correlation function. The composition-dependence of the boron and germanium coordination numbers provides evidence that both boron and germanium undergo non-monotonic changes in coordination similar to those observed in the “borate anomaly”
Figure 6.1: The room-temperature crystal structure of LaBGeO$_5$ [78]. The presented view is down the crystallographic $c$ axis. [B$O_4$]$^-$ tetrahedra are coloured dark green, while Ge$O_2$O$_2^{2-}$ tetrahedra are purple. LaO$_9$ polyhedra are omitted for clarity. Lanthanum ions are light green.

and “germanate anomaly”.

6.3 Introduction

TFN materials are optically interesting materials combining the excellent nonlinear optical properties of ferroelectric crystals with the ease of formability of glasses. These glass-ceramic composites have been the subject of much study over the last two decades, with a variety of compositions having been examined [1, 228–231]. One particularly interesting material is the synthetic stillwellite LaBGeO$_5$ (LBG). The LBG system is a valuable subject for the study of TFN materials, as the glass and crystal phases share the same stoichiometry. As such, the LBG system has been well-characterized, including its crystal structure [78, 79] (Fig. 6.1), second harmonic generation [232, 233], glass transition temperature [100], and Curie temperature [79, 234].

When LBG is crystallized via laser irradiation, the crystals can be made to grow in a highly directed fashion [105]. Understanding this technologically attractive
crystallization behaviour requires a knowledge of the glass structure. However, there is not a comprehensive model of the structure of the LBG glass phase. Previous structural models [74] assumed that the basic structural units present in the glass would be the same as those in the LBG crystal structure, \( i.e. \), \( \text{GeO}_4 \) and \( [\text{BO}_4]^- \) tetrahedra and \( \text{LaO}_9 \) polyhedra. Through the use of \(^{11}\text{B} \) MAS NMR, Gupta \textit{et al.} established a change in the boron–oxygen coordination environment, with roughly equal proportions of \( \text{BO}_3 \) and \( [\text{BO}_4]^− \) in the glass phase [34], where \( \text{BO}_3 \) indicates a non-specific three-coordinate boron species (Fig. 6.2). \( \text{O} \) indicates a bridging oxygen, \( i.e. \), in this case an oxygen bonded to at least two B and/or Ge atoms. The most recent direct structural investigation of the LBG glass phase concluded that while the boron-oxygen coordination environment underwent a change between the glass and crystalline phases, the germanium-oxygen coordination environment in the glass was not substantially different from that of the crystal [75]. However, there is no fundamental reason to assume that the Ge–O and La–O environments are unchanged between the LBG glass and crystal phases. Most of the structural data on the LBG glass comes from Raman and \(^{11}\text{B} \) NMR spectroscopies. The lanthanum environment is difficult to probe using Raman spectroscopy, due to very low resonant frequencies, while the interpretation of Raman data with regards to the germanium environment is an active point of discussion in the literature [19, 26]. While we have had some success with the application of \(^{139}\text{La} \) NMR spectroscopy to glassy systems, \(^{73}\text{Ge} \) NMR is not currently a useful tool in investigating the structure of germanate glasses [25]. In order to probe directly the nature of the germanium-oxygen coordination in LBG glass, we made use of neutron diffraction. Neutron diffraction has been successfully used in the literature to probe the structure of binary germanates [27, 28, 64], borates [235, 236], and lanthanoid glasses [67, 237]. As a direct probe of the length scale of bonding interactions, neutron diffraction serves as an effective means of studying the coordination environment of many glass formers. We combine the use of high-field \(^{11}\text{B} \) NMR spectroscopy and neutron diffraction to investigate the structural changes in lanthanum borogermanate glasses as a function of lanthanum content with the goal of producing a comprehensive structural model of the \( \text{LaBGeO}_5 \) glass composition.
Figure 6.2: Possible structural units under consideration in the La$_2$O$_3$–B$_2$O$_3$–GeO$_2$ glass composition. Top: the conversion of BØ$_3$ to [BØ$_4$]$^-$ to BØ$_2$O$^-$ with the addition of modifier oxide. Bottom: possible configurations of GeO$_4$, [GeO$_5$]$^-$, and [GeO$_6$]$^{2-}$. 
6.4 Experimental Procedures

6.4.1 Sample Synthesis

Glass formation has previously been reported for compositions between 20 mol% to 28 mol% La₂O₃ [238], and glasses of composition \(x\text{La}_2\text{O}_3 \cdot (1-x)(0.33\text{B}_2\text{O}_3 \cdot 0.67\text{GeO}_2)\) \((x = 0.00, 0.20, 0.25, 0.30)\) were prepared using a conventional melt-quench approach (Section 3.1). Powders of La₂O₃ (99.99 %), GeO₂ (≥ 99.99 %), and isotopically-enriched \(^{11}\text{B}_2\text{O}_3\) (99.62 atom % \(^{11}\text{B}\)) were obtained from Sigma-Aldrich and used without further purification. \(^{11}\text{B}\) enrichment is required to mitigate the effects of the high neutron absorption cross section of \(^{10}\text{B}\). Weighed powders were thoroughly ground in a porcelain mortar and pestle in air before melting. Some excess of \(\text{B}_2\text{O}_3\) was added to account for known issues with borate volatility [98, 239]. The amount of excess \(\text{B}_2\text{O}_3\) was varied based on preliminary elemental analysis results. The 25LaBG-X sample used 100.36 % of the nominal \(\text{B}_2\text{O}_3\) mass. The 0LaBG, 20LaBG, 25LaBG-1, and 30LaBG-1 samples used 116 % of the nominal \(\text{B}_2\text{O}_3\) mass. The 25LaBG-2 and 30LaBG-2 samples used 133 % of the nominal \(\text{B}_2\text{O}_3\) mass. The mixed powders were melted in covered platinum crucibles in a box furnace in air at 1300 °C for 30 min. Glasses were quenched by dipping the bottom of the crucible into room-temperature water. Significant mass loss during melting was observed; potential causes are discussed below. Mass loss was determined by comparing the masses of the reagents and crucible prior to melting with the mass of the resulting glass and crucible after quenching. The crystalline sample of \(\text{LaBGeO}_5\), 25LaBG-X, was produced by devitrification of a quenched glass. The composition of the crystal \(\text{LaBGeO}_5\) is equivalent to the \(x = 0.25\) glass. The sample was heated at 950 °C for 4 h. The crystalline phases within the sample were probed using pXRD (Fig. C.3). X-ray diffraction measurements were made using a Rigaku Ultima IV X-ray diffractometer with a Cu anode X-ray tube and a diffracted beam monochromator.

6.4.2 Sample Characterization

The elemental compositions of the samples were determined by LA-ICP-MS. The laser used a spot size of 30 µm, with a repetition rate of 7 Hz and an energy density of 3 J cm\(^{-2}\). 5 spots were analysed per sample to test for homogeneity. The observed
analytes were $^{11}\text{B}$, $^{72}\text{Ge}$, and $^{139}\text{La}$, with corrections made for the isotopic enrichment of $^{11}\text{B}$. The O content was determined by difference, assuming that all elements were in their standard oxidation states. The National Bureau of Standards standard reference material 610 (National Institute of Standards and Technology 610) glass was used as an external standard.

The density of the samples was determined by use of a Micromeritics AccuPyc II 1340 helium gas pycnometer with a 1 cm$^3$ sample chamber. The glass transition temperatures ($T_g$) were determined using a Netzsch DSC 404F1 differential scanning calorimeter. Samples were heated from room temperature to 800 °C in a platinum pan at a rate of 10 °C min$^{-1}$. The $T_g$ value was taken as the point of inflection of the $T_g$ event on the DSC curve.

### 6.4.3 $^{10}\text{B}$ and $^{11}\text{B}$ NMR

$^{11}\text{B}$ MAS and MQMAS NMR spectra were collected on a 16.4 T (224.63 MHz $^{11}\text{B}$ frequency) Bruker Avance NMR spectrometer. MAS spectra were collected at a spinning speed of (25 000 ± 5) Hz using a 2.5 mm outer diameter ZrO$_2$ rotor. Single pulses of 0.54 µs duration (15° tip angle, $\nu_{\text{rf}}$ = 77 kHz) and a recycle delay of 5 s were used to ensure quantitative excitation. Recycle delays were high enough to ensure that both three- and four-coordinate sites were fully relaxed. 256 scans were collected for each sample. To compensate for the substantial $^{11}\text{B}$ background, the spectrum of an empty rotor collected under the same conditions was subtracted from the spectrum of each sample, after careful phasing and intensity adjustment. $^{11}\text{B}$ MQMAS spectra were collected at spinning rates of (25 000 ± 5) Hz using a three-pulse sequence with full echo acquisition [240]. Spectra were collected in split-$t_1$ mode with a 2.37 ms delay between the conversion and selective pulse. Each spectrum was acquired with 166 $t_1$ increments with 96 transients per increment. All $^{11}\text{B}$ and $^{10}\text{B}$ chemical shifts were referenced to solid NaBH$_4$, with a resonance at −42.1 ppm relative to the primary shift reference (BF$_3$·Et$_2$O, 0 ppm) [159]. The MQMAS spectra were referenced using the $C_z$ convention [241]. The DMFIT 20150521 [184] software package was used to analyse the $^{11}\text{B}$ NMR data. REDOR spectra were collected at a spinning speed of (10 000 ± 5) Hz in a 3.2 mm ZrO$_2$ rotor using a HXY probehead. The conventional REDOR pulse sequence was used [140]. A sample of LBG glass with natural abundance boron was
used to allow for $^{11}\text{B}\{^{10}\text{B}\}$ interactions to be probed. The $^{10}\text{B}$ Larmor frequency at 16.4 T was 75.24 MHz. $^{11}\text{B} \pi/2$ and $\pi$ pulses were set to 3.10 $\mu$s and 6.25 $\mu$s, optimized to allow for near-maximum signal from both BO$_3$ and [BO$_4$]$^-$ sites. $^{11}\text{B}$ pulses were optimized via Hahn-echo experiments. $^{10}\text{B}$ $\pi$ pulses were optimized via maximizing the difference observed when the $^{10}\text{B}$ channel was activated. REDOR pulse powers were 80 kHz for $^{11}\text{B}$ and 40 kHz for $^{10}\text{B}$. Recycle delays for the REDOR experiments were set to 50 s, optimized to prevent relaxation effects from being mistaken for the REDOR dephasing.

### 6.4.4 Neutron Diffraction

Time-of-flight neutron diffraction data were collected at the ISIS pulsed neutron source at the Rutherford Appleton Laboratory using the GEM diffractometer [166]. Samples were loaded into thin-walled (25 $\mu$m) vanadium foil cans of 8.3 mm internal diameter, with careful consideration to minimize the exposure of the samples to atmospheric water during transport and loading. The samples were in the form of irregular fragments of maximum dimension ranging from 2 mm to 8 mm. Data were normalized by comparison to those from a vanadium-niobium rod and corrected for background signals. The distinct scattering, $i(Q)$, for each sample was combined and processed using the GudrunN [242] and ATLAS [243] program suites. As the very low momentum transfer region ($Q < 0.7 \text{Å}^{-1}$) was not directly observed, the distinct scattering for each sample was extrapolated to 0 Å$^{-1}$ by fitting a symmetric function of the form $A + BQ^2$ to the region below 0.9 Å$^{-1}$. The distinct scattering was then Fourier transformed using a modification function, $M(Q)$; either a Lorch function [63] (to minimize the effects of termination ripples) or a step function (to maximize real-space resolution) was used. All samples were transformed with a maximum momentum transfer $Q_{\text{max}} = 50 \text{Å}^{-1}$. The result of the Fourier transform of the distinct scattering is the differential neutron correlation function:

$$D(r) = \frac{2}{\pi} \int_0^\infty Q i(Q) M(Q) \sin(rQ) \, dQ.$$  \hspace{1cm} (6.1)
With the addition of an “average density” term, $T^0(r)$, we obtain the total correlation function $T(r)$:

$$T(r) = T^0(r) + D(r).$$  \hspace{1cm} (6.2)

The average density term is a function of the atomic number density of the sample, $g^0$, and the average coherent neutron scattering length of the sample, $\langle \bar{b} \rangle_{av}$:

$$T^0(r) = 4\pi r g^0 \langle \bar{b} \rangle_{av}^2.$$  \hspace{1cm} (6.3)

For all $T(r)$ reported in this paper, we followed the renormalization procedure described by Alderman et al. [27]. Renormalization constants were on the order of 5%. Peaks were fitted with resolution-broadened Gaussian functions. Average coordination numbers were calculated using

$$n_{ij} = \frac{r_{ij} A_{ij}}{(2 - \delta_{ij}) c_i \bar{b}_i \bar{b}_j},$$  \hspace{1cm} (6.4)

where $n_{ij}$ is the average coordination number of the element pair $i-j$, $r_{ij}$ and $A_{ij}$ are the peak position and peak area, respectively, of the resolution-broadened Gaussian function fitting the relevant peak, $c_i$ is the atomic fraction of element $i$, and $\delta_{ij}$ is the Kronecker delta. $T(r)$ correlation functions were fit using the Pfit program [244].

6.5 Results

6.5.1 Sample Characterization

To account for possible variations in composition, and the potentially different interactions between La$_2$O$_3$, B$_2$O$_3$, and GeO$_2$, most values are compared to changes in the molar ratio $R = [\text{La}_2\text{O}_3]/([\text{B}_2\text{O}_3] + [\text{GeO}_2])$. The ratio $R$ is chosen in analogy with alkali borate glasses, where $R = [\text{A}_2\text{O}]/[\text{B}_2\text{O}_3]$ [245]. This is intended to capture deviations in the $[\text{GeO}_2]/[\text{B}_2\text{O}_3]$ ratio, which is nominally at a constant ratio of 2:1. The densities, molar volumes, and glass transition temperatures, are presented in Table 6.1. Glass transition temperatures were determined from DSC curves presented in Appendix C (Figs. C.10 to C.15). The densities of the glasses increase with $R$. The measured density of the 0LaBG sample is consistent with data contained within the
INTERGLAD database [246], which would predict a density of \((2.98 \pm 0.07) \text{ g cm}^{-3}\).

Table 6.1: Density, molar volume, and glass transition temperatures of borogermanate samples.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Density / g cm(^{-3})</th>
<th>Molar volume / cm(^3) mol(^{-1})</th>
<th>(T_g / ^\circ\text{C})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0LaBG</td>
<td>2.95 ± 0.01</td>
<td>32 ± 1</td>
<td>406 ± 2</td>
</tr>
<tr>
<td>20LaBG</td>
<td>4.72 ± 0.01</td>
<td>29.5 ± 0.4</td>
<td>679 ± 2</td>
</tr>
<tr>
<td>25LaBG-1</td>
<td>4.96 ± 0.01</td>
<td>30.4 ± 0.4</td>
<td>683 ± 2</td>
</tr>
<tr>
<td>25LaBG-2</td>
<td>4.81 ± 0.01</td>
<td>30.7 ± 0.4</td>
<td>679 ± 2</td>
</tr>
<tr>
<td>30LaBG-1</td>
<td>5.10 ± 0.01</td>
<td>31.8 ± 0.4</td>
<td>689 ± 2</td>
</tr>
<tr>
<td>30LaBG-2</td>
<td>4.88 ± 0.01</td>
<td>32.8 ± 0.4</td>
<td>682 ± 2</td>
</tr>
<tr>
<td>25LaBG-X</td>
<td>5.16 ± 0.02</td>
<td>29.8 ± 0.5</td>
<td></td>
</tr>
</tbody>
</table>

The compositions presented in Table 6.2 were determined by LA-ICP-MS, a technique chosen to avoid the difficulty found in X-ray-based measurements in determining light atom composition.

The reported compositions deviate somewhat from the nominal compositions, typically on the order of 1 mol% to 2 mol% of the component oxides. However, it is difficult to reconcile these compositions with the observed mass loss during melting (approx. 3% of total sample mass). The combined mass of the starting reagents and crucible was compared to the mass of the resulting glass.

The La\(_2\)O\(_3\) and GeO\(_2\) starting reagents were both heated to high temperatures to assess whether the mass loss was caused by hydration or adsorbed water, but no appreciable mass loss was observed. The mass loss was thus attributed to the B\(_2\)O\(_3\) starting reagent. The amount of mass lost (approx. 3% of total sample mass, on the order of 0.5 g) suggests that the B\(_2\)O\(_3\) had become partially hydrated and converted to H\(_3\)BO\(_3\), a well-known reaction. The presence of H\(_3\)BO\(_3\) would both explain the apparent deficiency of B\(_2\)O\(_3\) in the final glass (due to less borate being provided in the batched reagents) and the significant mass loss observed upon melting (in the form of H\(_2\)O released upon melting of H\(_3\)BO\(_3\)). The compositions estimated from the mass loss are, on average, within 1 mol% to 2 mol% of the compositions from LA-ICP-MS, and are also reported in Table 6.2. The compositions as estimated from mass loss were used for analysis of the neutron diffraction data. The uncertainty surrounding the composition of the glass samples leads to uncertainty in the coordination numbers.
Table 6.2: Nominal and experimental borogermanate sample compositions. Experimental values determined from use of LA-ICP-MS and from mass loss calculations are reported.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Nominal Composition / mol%</th>
<th>Composition from LA-ICP-MS / mol%</th>
<th>Composition from Mass Loss / mol%</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>La₂O₃</td>
<td>B₂O₃</td>
<td>GeO₂</td>
<td>La₂O₃</td>
</tr>
<tr>
<td>0LaBG</td>
<td>0.0</td>
<td>33.0</td>
<td>67.0</td>
<td>0</td>
</tr>
<tr>
<td>20LaBG</td>
<td>20.0</td>
<td>26.4</td>
<td>53.6</td>
<td>21.0 ± 0.3</td>
</tr>
<tr>
<td>25LaBG-1</td>
<td>25.0</td>
<td>25.0</td>
<td>50.0</td>
<td>25.9 ± 0.2</td>
</tr>
<tr>
<td>25LaBG-2</td>
<td>25.0</td>
<td>25.0</td>
<td>50.0</td>
<td>25.2 ± 0.2</td>
</tr>
<tr>
<td>30LaBG-1</td>
<td>30.0</td>
<td>23.1</td>
<td>46.9</td>
<td>30.8 ± 0.4</td>
</tr>
<tr>
<td>30LaBG-2</td>
<td>30.0</td>
<td>23.1</td>
<td>46.9</td>
<td>30.4 ± 0.5</td>
</tr>
<tr>
<td>25LaBG-X</td>
<td>25.0</td>
<td>25.0</td>
<td>50.0</td>
<td>26.3 ± 0.5</td>
</tr>
</tbody>
</table>
derived from neutron diffraction from use of Eq. (6.4) [27].

The pXRD of the 25LaBG-X sample shows mainly the diffraction peaks of crystalline LaBGeO$_5$ with a small amount of an impurity phase tentatively identified as La$_2$Ge$_2$O$_7$. As the 25LaBG-X sample is primarily used to establish the behaviour of GeO$_4$, this impurity does not pose a significant problem: all Ge in La$_2$Ge$_2$O$_7$ is in the form of GeO$_4$. This potentially could cause estimates of the La–O coordination number to be lower than the crystal structure would imply. The presence of La$_2$Ge$_2$O$_7$ is consistent with a deficit of B$_2$O$_3$ in the sample.

### 6.5.2 $^{10}$B and $^{11}$B NMR Results

Selected $^{11}$B MAS NMR spectra are shown in Fig. 6.3, with vertical offsets for clarity. The $^{11}$B MQMAS spectra of the glass samples are shown in Fig. 6.4. The projections of the MAS and isotropic axes are present on the top and left of the spectra, respectively. The relevant spectral parameters are summarized in Table 6.3.

The BO$_3$ peak is centred between approx. 12 ppm to 16 ppm, while the [BO$_4$]$^-$ peak is centred at approx. 1 ppm. The BO$_3$ peak shows a noticeable shift to higher chemical shifts with lanthanum content. There is not a similar trend with the [BO$_4$]$^-$ peak; however, the chemical shift of the [BO$_4$]$^-$ peak in each glass sample is significantly more positive than that of crystalline LaBGeO$_5$. There is no evidence of the presence of multiple BO$_3$ peaks in any of the MQMAS spectra (Fig. 6.5). As the $P_Q$ of the [BO$_4$]$^-$ environment is very small (as is common in borate glasses), it is difficult to precisely quantify it via MQMAS. The $P_Q$ of the [BO$_4$]$^-$ environment in all glass samples is less than 600 kHz.

<table>
<thead>
<tr>
<th>Sample</th>
<th>BO$_4$</th>
<th>[BO$_4$]$^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta_\text{CS}^\ast$/ ppm</td>
<td>$P_Q^\ast$/ MHz</td>
</tr>
<tr>
<td>0LaBG</td>
<td>15.5 ± 0.5</td>
<td>2.6 ± 0.1</td>
</tr>
<tr>
<td>20LaBG</td>
<td>17.6 ± 0.5</td>
<td>2.6 ± 0.1</td>
</tr>
<tr>
<td>25LaBG-1</td>
<td>18.1 ± 0.5</td>
<td>2.6 ± 0.1</td>
</tr>
<tr>
<td>25LaBG-2</td>
<td>17.8 ± 0.5</td>
<td>2.6 ± 0.1</td>
</tr>
<tr>
<td>30LaBG-1</td>
<td>18.5 ± 0.5</td>
<td>2.7 ± 0.1</td>
</tr>
<tr>
<td>30LaBG-2</td>
<td>18.2 ± 0.5</td>
<td>2.6 ± 0.1</td>
</tr>
<tr>
<td>25LaBG-X</td>
<td>18.8 ± 0.2</td>
<td>1.4 ± 0.2</td>
</tr>
</tbody>
</table>

* Isotropic chemical shift as determined by MQMAS
† Quadrupolar product $P_Q = C_Q \sqrt{1 + \eta^2}$
‡ Location of the peak maximum in the isotropic dimension of the MQMAS experiment (using the $C_z$ convention at 16.4 T) [134].
Figure 6.3: $^{11}$B MAS NMR spectra of selected glass samples. Spectral limits are set to emphasize the central transition.
Figure 6.4: $^{11}$B MQMAS spectra of the glass samples. The projections of the MAS (horizontal) and isotropic (vertical) axes are presented at the top and the left of the figure, respectively. The spectra are of, from top-left to bottom-right: 0LaBG; 20LaBG; 25LaBG-1; 25LaBG-2; 30LaBG-1; 30LaBG-2. The isotropic dimension of all spectra are referenced per the $C_2$ convention [134].
Figure 6.5: The isotropic projections of the $^{11}$B MQMAS NMR spectra of the glass samples presented in Fig. 6.4. The projections are scaled to have approximately equal intensity of the BO$\text{}_3$ peak. The $[\text{BO}_4]^-$ peaks are truncated. Aside from the dramatic difference between the 0LaBG sample and the rest, there is little difference between the BO$\text{}_3$ peaks. The reader is directed to the online version for colour.
Figure 6.6: $^{11}\text{B}\{^{10}\text{B}\}$ REDOR NMR spectra of a natural-abundance LaBGeO$_5$ glass. The top spectrum ($S_0$) is the $^{11}$B Hahn-echo spectrum. The middle spectrum ($S$) is the $^{11}$B spectrum with $^{11}$B$\{^{10}$B$\}$ dipolar recoupling. The bottom spectrum ($\Delta S$) is the difference between the two. The difference spectrum is increased in scale by a factor of 8 for clarity. This set of spectra corresponds to a $\Delta S/S_0$ value of approx. 0.03 and Hahn-echo delay of 49 ms, and is within the $\Delta S/S_0$ range where the REDOR behavior is expected to be independent of geometry [144].

A set of $^{11}$B$\{^{10}$B$\}$ REDOR spectra are presented in Fig. 6.6. $^{10}$B and $^{11}$B are chemically identical, but spectroscopically distinct. $^{11}$B$\{^{10}$B$\}$ REDOR spectra of sodium and lithium borate glasses have been previously reported [52]. $^{10}$B, with integer spin quantum number $I = 3$, does not exhibit a sharp central transition. Therefore, due to the large quadrupole coupling of the BO$_3$ environment, it cannot be readily excited and observed using $^{10}$B NMR. Hence, $^{11}$B$\{^{10}$B$\}$ REDOR experiments selectively probe the proximity of [BØ$_4$]$^-$ environments to both BO$_3$ and [BØ$_4$]$^-$ environments. The spectrum labelled $S_0$ is the experiment without the reintroduction of the dipolar coupling, while the spectrum labelled $S$ is affected by the $^{11}$B$\{^{10}$B$\}$
Figure 6.7: The distinct scattering, \( i(Q) \), of lanthanum borogermanate glass and crystalline samples, with vertical offsets of 0.4 barns \( \text{sr}^{-1} \text{atom}^{-1} \). The error bars are smaller than the data points. The most substantial differences are observed for \( Q \) values less than 6 Å\(^{-1}\).

The dipolar coupling of the \([\text{BO}_4]^-\) environment. \( \Delta S \) is the difference between the two. In Fig. 6.6 the difference is vertically scaled by a factor of 8. Both \( \text{BO}_3 \) and \([\text{BO}_4]^-\) environments exhibit significant dephasing, indicating that both environments are close in space to \([\text{BO}_4]^-\) \([41, 247]\).

### 6.5.3 Neutron Diffraction Results

The distinct scattering for selected glass samples is shown in Fig. 6.7. While there is an obvious difference in the scattering of the \( x = 0 \) sample and the rest of the series, the \( x = [0.20, 0.25, 0.30] \) samples appear largely similar.

We present selected \( T(r) \) total correlation functions in Fig. 6.8. While the region from \( r = 0 \) Å to the first real peak should in principle have a value of 0, small
Figure 6.8: The $T(r)$ total correlation functions, in the region of interest ($r = 1$ Å to $3$ Å). The functions are vertically offset by $1$ b Å$^{-2}$ for clarity. Horizontal lines indicate the horizontal axis for the respective functions. The presented functions were Fourier transformed from the distinct scattering in Fig. 6.7 using Lorch modification functions.

Experimental errors and truncation effects lead to ripples in the measured correlation functions (see Fig. C.4).

The first real peak, generally centred at a distance of about $1.40$ Å, is due to B–O bonds. In 0LaBG and 25LaBG-X, this peak is symmetric, as the former contains only BO$_3$ units, while the latter contains only [BØ$_4$]$^-$ units. In all other compositions a distinct asymmetry can be observed in this peak; a long distance shoulder is observed regardless of choice of modification function. In order to properly fit this asymmetric peak, we use two resolution-broadened Gaussian functions with fixed positions: one peak for BO$_3$, with $r = 1.367$ Å; and one for [BØ$_4$]$^-$, with $r = 1.473$ Å. The positions for the BO$_3$ and [BØ$_4$]$^-$ peaks are based upon those obtained by fitting the $T(r)$ functions of 0LaBG and 25LaBG-X, respectively, as these samples separately contain almost entirely BO$_3$ and [BØ$_4$]$^-$. The second peak, centred at approx. 1.75 Å, is assigned to Ge–O interactions. In
Figure 6.9: The $T(r)$ correlation functions of 0LaBG (in grey) and 25LaBG-1 (open circles). The 0LaBG sample contains only GeO$_4$. The Ge–O peak in 25LaBG-1 is notably shifted in position and less symmetric than the Ge–O peak in 0LaBG. The change in total intensity is expected, as 25LaBG-1 contains significantly less Ge than does 0LaBG.

0LaBG this peak is symmetric. The addition of La$_2$O$_3$ to the glass produces substantial changes in the behaviour of this peak in the $x = [0.20, 0.25, 0.30]$ samples. This change is highlighted in Fig. 6.9, where the $T(r)$ functions of 0LaBG and 25LaBG-1 are overlaid with no vertical offset. The peak shows both a move to higher $r$ values, and a distinct asymmetric tail that extends towards high-$r$ values. This high-$r$ tail is attributed to the formation of longer Ge–O bonds, consistent with the formation of high-coordinate germanium-oxygen polyhedra, i.e., GeO$_5$ and GeO$_6$ [27, 28, 42, 64]. Our data do not have the ability to unambiguously assign this intensity to a particular species, nor to preclude a combination of the two species being present.

In order to fit this asymmetric peak which overlaps at low $r$ with the B–O peak,
Figure 6.10: An example peak fit of $T(r)$ for 25LaBG-1. The total fit (red line) is overlaid on the experimental data (open circles). The individual peaks, in blue, are in order attributed to $\text{BO}_3$, $[\text{BO}_4]^{-}$, three Ge–O peaks, and a final peak resulting from La–O, O–O, and B–B interactions. The fit residual is displayed as a dashed line.

and at high $r$ with the subsequent peak at $\sim2.4\,\text{Å}$, we follow a similar approach to Alderman et al. in their fitting of neutron diffraction data of calcium germanate glasses [12]. We first fit the B–O peak (as above) and the leading edge of the Ge–O peak, between $1.20\,\text{Å}$ to $1.80\,\text{Å}$. The residual is then fitted up to $1.95\,\text{Å}$ with a single additional peak. The leading edge of the subsequent peak (at $\sim2.4\,\text{Å}$) is fitted from $2.2\,\text{Å}$ to $2.5\,\text{Å}$. The final residual of the above 5 peaks consistently yields some remaining intensity at approximately $2.07\,\text{Å}$. This final intensity is fit with a single additional peak, and is assigned to the Ge–O interaction. An example of the fit resulting from this process is shown in Fig. 6.10.

The third peak at $\sim2.4\,\text{Å}$ arises from a combination of several different atom pairs and structural correlations, as will be discussed below. The fit to the leading edge of
this peak, performed as part of the procedure to determine the Ge–O coordination number, does not in itself provide structurally meaningful information.

Mean B–O coordination numbers, \( n_{BO} \), and mean Ge–O coordination numbers, \( n_{GeO} \), as calculated using Eq. (6.4) are reported in Table 6.4. \( n_{BO} \) and \( n_{GeO} \) of 0LaBG and 25LaBG-X were found to be approximately 3% below nominal values. Slight underestimation of coordination numbers from neutron diffraction is common, and can occur for various reasons, including \( Q \)-resolution, inadequate normalization, or errors in the multiple scattering correction [248]. The complete list of fitting parameters can be found in Table C.1.

Table 6.4: Selected mean bond lengths and coordination numbers from neutron diffraction.

<table>
<thead>
<tr>
<th>Sample</th>
<th>( r_{BO} ) / Å</th>
<th>( n_{BO} )</th>
<th>( r_{GeO} ) / Å</th>
<th>( n_{GeO} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0LaBG</td>
<td>1.367 ± 0.001</td>
<td>2.9 ± 0.1</td>
<td>1.737 ± 0.001</td>
<td>3.9 ± 0.1</td>
</tr>
<tr>
<td>20LaBG</td>
<td>1.414 ± 0.004</td>
<td>3.2 ± 0.1</td>
<td>1.787 ± 0.004</td>
<td>4.4 ± 0.1</td>
</tr>
<tr>
<td>25LaBG-1</td>
<td>1.411 ± 0.004</td>
<td>3.2 ± 0.1</td>
<td>1.785 ± 0.004</td>
<td>4.4 ± 0.1</td>
</tr>
<tr>
<td>25LaBG-2</td>
<td>1.411 ± 0.004</td>
<td>3.1 ± 0.1</td>
<td>1.788 ± 0.005</td>
<td>4.4 ± 0.1</td>
</tr>
<tr>
<td>30LaBG-1</td>
<td>1.404 ± 0.004</td>
<td>3.2 ± 0.1</td>
<td>1.788 ± 0.005</td>
<td>4.3 ± 0.1</td>
</tr>
<tr>
<td>30LaBG-2</td>
<td>1.408 ± 0.004</td>
<td>3.1 ± 0.1</td>
<td>1.784 ± 0.005</td>
<td>4.1 ± 0.1</td>
</tr>
<tr>
<td>25LaBG-X</td>
<td>1.474 ± 0.001</td>
<td>3.8 ± 0.1</td>
<td>1.744 ± 0.001</td>
<td>3.9 ± 0.1</td>
</tr>
</tbody>
</table>

6.6 Discussion

6.6.1 \( ^{10}\text{B} \) and \( ^{11}\text{B} \) NMR

Gupta et al. [34] have previously performed \( ^{11}\text{B} \) MAS NMR measurements on LaBGeO\(_5\), in both glass and crystalline states, and also on a partially crystallized sample. Their spectra show that the glass contains “about equal concentration” of BO\(_3\) and \([\text{B} \theta \text{O}_4]^-\) units, but a quantitative determination of the concentration was not reported, and thus we give the first such quantitative report. Selected \( ^{11}\text{B} \) NMR spectra are presented in Fig. 6.3, and selected parameters are presented in Table 6.3. The \( ^{11}\text{B} \) MAS NMR spectra consist of two peaks: the broad BO\(_3\) peak, with maxima at approximately 15 ppm; and the narrow \([\text{B} \theta \text{O}_4]^-\) peak, at approximately 2 ppm. The slight asymmetry apparent in the crystalline 25LaBG-X \([\text{B} \theta \text{O}_4]^-\) peak is attributed to the 3/2→1/2 satellite transition, which is not resolved from the central transition. The same
transition can be observed in the \( \text{BO}_3 \) peaks of the glass samples, where the resolution is sufficient to fit the satellite peak separately (see Fig. C.1).

The lanthanum-free 0LaBG sample contains overwhelmingly three-coordinate boron, while the crystalline 25LaBG-X sample contains only four-coordinate boron. The lanthanum-containing glass samples, even those with the same composition as the crystal, have a mixture of three- and four-coordinate boron. The proportions of \( [\text{BO}_4]^- \) decreases in response to \( R \). The change in \( n_{\text{BO}} \) is consistent with the expected evolution of the boron-oxygen environment in response to increasing modifier oxide, as displayed in Fig. 6.2. The addition of modifier oxide, in this case \( \text{La}_2\text{O}_3 \), converts \( \text{BO}_3 \) to \( [\text{BO}_4]^- \), which will in turn convert to \( \text{BO}_2\text{O}^- \) and eventually \( \text{BOO}_2^{2-} \) [22]. Identification of the type of three-coordinate species is important to assess the likelihood of whether the glass is homogeneous, or possibly contains heterogeneous regions. The isotropic chemical shift, \( \delta_{\text{iso}}^\text{CS} \), has been correlated with the amount of non-bridging oxygen in \( \text{BO}_3 \) units in crystalline materials [48]. Based upon the values from crystalline systems, the \( \delta_{\text{iso}}^\text{CS} \) value of the 0LaBG sample is consistent with either individual \( \text{BO}_3 \) units or boroxol rings, while the lanthanum-containing glasses are consistent with either \( \text{BO}_2\text{O}^- \) units or borate ring structures containing non-bridging oxygen (e.g., diborate) [20]. Our \( ^{11}\text{B} \) MAS NMR spectra have insufficient resolution to distinguish between these possibilities, largely due to the inherent structural broadening present in glassy materials. \( ^{11}\text{B} \) MQMAS NMR was used to examine the possibility of multiple coexisting \( \text{BO}_3 \) species. The isotropic dimension of the MQMAS spectrum is free from quadrupolar broadening. While this does not mitigate the structural broadening that is present in the glass, the increase in resolution has been used to evaluate \( \text{BO}_3 \) speciation in high-alkali borate glasses [49], as well as to evaluate ring and non-ring \( \text{BO}_3 \) proportions in binary borogermanate glasses [53]. For all samples studied in this work, the \( \text{BO}_3 \) peak in the isotropic dimension is easily modelled by a single Gaussian lineshape. There is no indication of multiple coexisting \( \text{BO}_3 \) environments in any sample, either via rings or varying amounts of non-bridging oxygen. In the study by Lee et al., boroxol rings cease to be apparent in MQMAS in binary borogermanate glasses as the \( \text{GeO}_2/\text{B}_2\text{O}_3 \) ratio rises above 3:2 [53]; hence it is unsurprising that our 0LaBG glass, with a \( \text{GeO}_2/\text{B}_2\text{O}_3 \) molar ratio of 2:1, does not show the presence of ring structures. Aguiar and Kroeker present \( ^{11}\text{B} \) MQMAS spectra of alkali borate glasses
with multiple coexisting BO\(_3\) units, and observe substantial asymmetric broadening in the isotropic projections of the MQMAS spectra [49]. The broadening we observe is symmetric around a mean isotropic chemical shift (Table 6.3). The significant width of the peaks in the isotropic dimension indicates the presence of a distribution of chemical shifts in the BO\(_3\) environments. The width of the isotropic peaks of the lanthanum-containing glasses are significantly narrower than that of the lanthanum-free 0LaBG sample, indicating that the lanthanum-containing glasses have less short-range disorder. The isotropic chemical shifts of each sample (Table 6.3, as derived per Millot and Man [134]) are assigned to their likely BO\(_3\) environments by comparison to literature values [48, 49]. The lanthanum-containing samples have mean \(\delta_{iso}^{CS}\) values in the range of 17.6 ppm to 18.6 ppm, which is most consistent with BO\(_2\)O\(^-\) units, and inconsistent with either BO\(_3\) or BO\(_2\)O\(^-\) units.

The change in the [BO\(_4\)]\(^-\) peak is much less drastic, and more difficult to assign. No trend is observed with the change in the \(\delta_{iso}^{CS}\) of the [BO\(_4\)]\(^-\) peak with response to \(R\) in the glass samples. However, the [BO\(_4\)]\(^-\) \(\delta_{iso}^{CS}\) for each peak is, on average, (0.7 ± 0.3) ppm more positive than the equivalent value in crystalline 25LaBG-X. While there is insufficient information available to identify the exact change in the [BO\(_4\)]\(^-\) environment responsible for this consistent change in \(\delta_{iso}^{CS}\), it is plausibly attributed to the presence of BO\(_3\) in the second coordination sphere of the [BO\(_4\)]\(^-\) unit. Attaching BO\(_3\) units to [BO\(_4\)]\(^-\) tetrahedra has been previously shown to increase the [BO\(_4\)]\(^-\) \(\delta_{iso}^{CS}\) by approximately 0.5 ppm [50]. This structural motif can be observed in the LaB\(_3\)O\(_6\) and LaB\(_5\)O\(_9\) crystals [249, 250]. However, while the effects of silicon and phosphorus substitution on the [BO\(_4\)]\(^-\) \(\delta_{iso}^{CS}\) have been established [50, 251, 252], to our knowledge there have been no similar studies on the effects of germanium substitution. Therefore, we cannot discount the possibility that the observed shift in [BO\(_4\)]\(^-\) \(\delta_{iso}^{CS}\) is due to changing connectivity between germanium and [BO\(_4\)]\(^-\).

While our \(^{11}\)B MAS experiments are insufficient to identify the nature of the connectivity of the borate species, our \(^{11}\)B\(^{10}\)B REDOR experiment provides additional evidence regarding connectivity. The REDOR pulse sequence reintroduces dipolar coupling which would otherwise be averaged away by MAS. Dipolar coupling is a through-space interaction, and is proportional to the inverse cube of the separation between nuclei. REDOR is commonly used in the study of glasses. It has been used to
characterize cation clustering in mixed-alkali silicates [253], selective site connectivity in sodium boroaluminate glasses [142], degree of polymerization in sodium tungsten phosphate glasses [254], and the number of B–O–P links in lithium borophosphate glasses [255]. As both $^{11}\text{B}$ and $^{10}\text{B}$ are quadrupolar, and $^{10}\text{B}$ is of integer nuclear spin, exact distances cannot be extracted from our data [41]. However, for small values of $\Delta S/S_0$, the dipolar dephasing is expected to reflect only short length scales, and the effect has been shown to be independent of geometry [144]. As we present spectra where $\Delta S/S_0$ is small, this implies that any dephasing observed in the $^{11}\text{B}$ spectrum is due to the presence of a $[\text{BO}_4]^{-}$ species in close proximity to either $\text{BO}_3$ or $[\text{B}O_4]^{-}$. Both the $\text{BO}_3$ and $[\text{BO}_4]^{-}$ peaks show significant dephasing, and are dephased in similar proportions. From this we conclude that both species are in close proximity to $[\text{BO}_4]^{-}$, and that neither $\text{BO}_3$–$[\text{BO}_4]^{-}$ nor $[\text{BO}_4]^{-}$–$[\text{BO}_4]^{-}$ links are preferred over the other. This supports a glass network where the borate subnetwork is not segregated, and is homogeneous.

6.6.2 Neutron Diffraction

B-O and Ge-O Coordination

Average bond lengths and mean coordination numbers determined via neutron diffraction are shown in Table 6.4. Coordination numbers were determined by the use of Eq. (6.4) applied to the resolution-broadened Gaussian functions used to fit the $T(r)$ functions. Average bond lengths were determined by integration of $rT(r)$.

The experimental values of both $n_{\text{GeO}}$ and $n_{\text{BO}}$ for the crystalline sample, 25LaBG-X, are consistent with the nominal values (4 and 4, respectively) for the reported structure of LaBGeO$_5$ [78], and similarly the experimental coordination numbers for the La-free glass, 0LaBG, are consistent with expectation (3 and 4, respectively) based on the $^{11}\text{B}$ MAS NMR results, albeit with some uncertainty reflecting the uncertainty in the sample compositions. For the La-containing glass samples, $n_{\text{BO}}$ values determined by neutron diffraction are consistently lower than those determined by $^{11}\text{B}$ MAS NMR within uncertainty (Fig. 6.11 and Fig. 6.13), and the NMR values are preferred, since they are not affected by compositional uncertainty, and these are plotted in Fig. 6.12a, together with the neutron diffraction values for $n_{\text{GeO}}$ in Fig. 6.12b. We attribute the uncertainty in $n_{\text{BO}}$ to composition, rather than fitting
errors or truncation effects in the \( T(r) \) functions or relaxation effects in the \(^{11}\)B MAS NMR, for several reasons. The maximum momentum transfer used for Fourier transformation of the distinct scattering is quite large (\( Q_{\text{max}} = 50 \, \text{Å}^{-1} \)), allowing for substantial real-space resolution and minimizing the possibility of truncation. The regions of the \( T(r) \) functions before the B–O region (\( e.g., r \geq 1 \, \text{Å}^{-1} \)) have minimal baseline effects when compared to the size of the B–O peaks (Fig. C.4). Additionally, the \(^{11}\)B MAS spectra were collected such that both BO\(_3\) and [BO\(_4\)]\(^-\) environments were allowed to completely relax between scans, preventing saturation effects that could cause errors in \( n_{BO} \).

All of the La-containing glass samples show a mean germanium coordination number which is significantly greater than 4, and this result is evidence of the presence of high-coordinate (\( i.e., 5\)-coordinate or 6-coordinate) germanium polyhedra. This conclusion is supported by the asymmetric shape of the Ge–O peaks in the \( T(r) \) correlation functions (see Figs. 6.8 to 6.10).

The \( r \)-scale of neutron diffraction data is more accurate and precise than the intensity scale [27, 69], and as such we do not expect it to have particular errors affecting the coordination number. Indeed, the average B–O bond length (\( r_{BO} \)) behaves as we would predict from the trend of the B–O coordination number according to \(^{11}\)B NMR: the mean peak position moves to shorter distances in samples with lower boron-oxygen coordination numbers (Fig. 6.13). The positions of the B–O peaks in both 0LaBG and 25LaBG-X are consistent with the bond lengths expected from bond-valence calculations for 3- and 4-coordinated boron respectively [256–258], and with similar structures from the literature [78]. The positions of the Ge–O peaks in both control samples are also consistent with the expectations for their structures. The position of the Ge–O peak in 0LaBG [(1.737 ± 0.001) Å] is consistent with GeO\(_4\) units with all bridging oxygen. The position of the Ge–O peak in 25LaBG-X [(1.744 ± 0.001) Å] is in reasonable agreement with the average Ge–O bond length in the reported LaBGeO\(_5\) crystal structure (1.753 Å) [78].

For all of the La-containing glasses, the B–O peak occurs at a longer distance compared to the La-free glass, 0LaBG, consistent with an increase in the B–O coordination number, due to the formation of some [BO\(_4\)]\(^-\) when La\(_2\)O\(_3\) is added to the glass. Furthermore, the addition of La\(_2\)O\(_3\) leads to the Ge–O peak shifting
Figure 6.11: The mean B–O coordination number as determined by $^{11}$B MAS NMR (circles) and the mean B–O coordination number as determined by neutron diffraction (triangles) as a function of the molar ratio $R = [\text{La}_2\text{O}_3]/([\text{B}_2\text{O}_3]+[\text{GeO}_2])$. The filled markers correspond to the crystalline LaBGeO$_5$ sample 25LaBG-X, while the open markers denote glass samples. In principle, all of these values should agree. However, the neutron values are usually substantially lower than those observed through NMR. This is likely due to uncertainty in the composition of the samples.
Figure 6.12: Mean coordination numbers, $n_{ij}$, as a function of the molar ratio $R = \left[ \text{La}_2\text{O}_3 \right]/\left( \left[ \text{B}_2\text{O}_3 \right] + \left[ \text{GeO}_2 \right] \right)$, with closed circles for the glass samples, and a cross for the crystalline sample. The grey lines represent a guide to the eye, with a break in the region of no glass formation [238]. a) $n_{\text{BO}}$, as determined by $^{11}\text{B}$ MAS NMR. b) $n_{\text{GeO}}$, as determined by neutron diffraction. c) $n_{\text{OX}}$ ($=n_{\text{OB}}+n_{\text{OGe}}$, where X represents any glass former cation, B or Ge), as determined from the data shown in parts a) and b). The open circles and black line are the theoretical coordination number that would be obtained for a network of corner-sharing $\text{BO}_3$ and $\text{GeO}_4$ units, with no increase in $n_{\text{BO}}$ or $n_{\text{GeO}}$. 
Figure 6.13: The average B–O bond length, $r_{BO}$, as determined by neutron diffraction as a function of the molar ratio $R = [\text{La}_2\text{O}_3]/([\text{B}_2\text{O}_3]+[\text{GeO}_2])$. The average B–O bond length decreases as $R$ increases, consistent with a reduction in $n_{BO}$. 
to longer distance and becoming asymmetric (Figs. 6.9 and 6.10), compared to the La-free glass. There are also changes in the region above 2.2 Å, due to a number of factors, as discussed below.

The germanium-oxygen peak is substantially different between the La-free and La-bearing glass samples (Fig. 6.9). The average Ge–O bond length \( r_{\text{GeO}} \) increases from \( (1.737 \pm 0.001) \) Å in 0LaBG to approximately 1.78 Å in the lanthanum-containing samples (Table 6.4). Although the presence of \( \text{La}^{3+} \) ions leads to some lengthening of the bonds to bridging oxygen (as is observed for the length of the Ge–O bond in the bridge between \( \text{GeO}_4 \) and \( [\text{B} \text{O}_4]^- \) tetrahedra in crystalline \( \text{LaBGeO}_5 \)), the major cause of lengthening of the mean Ge–O bond length is likely to be the formation of \( [\text{GeO}_5]^- \) and/or \( [\text{GeO}_6]^{2-} \) units. These units have longer mean Ge–O bond lengths than \( \text{GeO}_4 \) tetrahedra, and will likely have a significantly less symmetric distribution of bond lengths. For example, a difference is often observed between equatorial and axial bond lengths in \( [\text{GeO}_6]^{2-} \) octahedra. The longer bond lengths in these high-coordinate germanate species shifts the peak maxima to larger values of \( r \), while the distribution of bond lengths manifests as a broad tail on the high-\( r \) side of the \( \text{GeO}_4 \) peak [12, 27, 28, 42, 64]. Such a tail is observed in all lanthanum-containing glass samples (Fig. 6.8 and Fig. 6.9), and persists when the data are Fourier transformed using a step modification function, rather than a Lorch modification function (Fig. C.7).

Each lanthanum-containing glass sample has a mean value of \( n_{\text{GeO}} \) that is significantly above 4 (Table 6.4). However, whereas tetrahedral units are highly symmetric with only small differences in bond lengths, 5- and 6-coordinated units have a greater degree of asymmetry, and it is not possible to use the observed correlation function to directly differentiate between the two possible higher Ge–O coordination numbers [28, 62]. Regardless, we can still calculate reasonable estimates for the proportion of high-coordinate germanium present in our samples: from 20\% of the total germanium content (assuming only 6-coordinate germanium) to 40\% (assuming only 5-coordinate germanium).

There are many reports in the literature for binary borate [259] and germanate [55, 56, 62] glasses (especially alkali systems), studied over a wide composition range, which show that the coordination number, \( n_{\text{BO}} \) or \( n_{\text{GeO}} \), increases and then declines as increasing amounts of modifier are added to the glass; a behaviour known as the
borate anomaly or the germanate anomaly, respectively. The glass forming range for 
La-containing glasses is more limited, but nevertheless the guides to the eye shown 
in Fig. 6.12a and Fig. 6.12b strongly suggest that the two coordination numbers in 
lanthanum borogermanate glasses also follow an anomalous composition dependence, 
which we may term a borogermanate anomaly.

La–O Coordination

The germanium and boron coordination numbers in crystalline \( \text{LaBGeO}_5 \) are both 
four [78], and these coordination numbers are markedly different to those in the 
glasses, as confirmed by our experimental results (see Fig. 6.12a and Fig. 6.12b). A 
comparison of the crystalline correlation function with that for the glass with closest 
composition gives further evidence that the glass structure differs significantly from 
the crystal structure (see Fig. 6.14). In particular, the distance range \( r \sim 2.2 \text{ Å} \) to 
2.75 Å (which is typical of La–O bond lengths) shows significant differences between 
crystal and glass. This region of the correlation functions, \( T(r) \), is markedly more 
complex than the region of the first two (B–O and Ge–O) peaks, because it involves 
contributions from several different atom pairs and types of structural correlation. 
Nevertheless, it is of particular interest because it contains information about the 
La–O coordination. For example, in the determined structure of crystalline \( \text{LaBGeO}_5 \), 
four different types of distance can be identified as contributing to this region of the 
correlation function: 1) the O–O distance in \( [\text{B}O_4]^− \) tetrahedra, \( r_{O(O(B))} \approx 2.41 \text{ Å} \); 2) 
the La–O bond lengths, which are more widely distributed, from 2.40 Å to 2.74 Å, 
with a total coordination number, \( n_{\text{LaO}} \), of 9; 3) the B–B distance of 2.65 Å, arising 
from a B–O–B bridge between two \( [\text{B}O_4]^− \) tetrahedra; and 4) the O–O distance in 
\( \text{GeO}_4 \) tetrahedra, \( r_{O(O(\text{Ge}))} \approx 2.84 \text{ Å} \). For the crystal, the B–B contribution in this 
region of \( T(r) \) is small (\( \sim 5 \% \) of the area), whilst the O–O and La–O contributions 
are of similar magnitudes.

For information on the La–O coordination in the glasses to be derived, it is neces-
sary to model the other contributions to the relevant region of the correlation function. 
Generally speaking, this distance range includes La–O, O–O and X–X contributions 
(where X represents a network forming cation), but no X–O contributions. The ap-
proach used here is firstly to show that this region can be modelled well for the La-free
Figure 6.14: Comparison of the neutron correlation functions measured for crystalline LaBGeO$_5$ and for the glass sample of closest composition (samples 25LaBG-X and 25LaBG-1 respectively).
glass, secondly to show that the modelling approach yields a reasonable estimate of the La–O distribution and coordination number, \( n_{\text{LaO}} \), for the crystal, and then to use the same method to estimate the value of \( n_{\text{LaO}} \) for the La containing glasses. It is necessary to note that our model is not unique; and, as with the Ge–O peaks above, the La–O peaks are not intended to imply physically exact distributions of bond lengths, but simply a means of modelling the intensity in a convoluted function. However, as we will demonstrate, by applying constraints informed by our analysis of the B–O and Ge–O environments, we can obtain reasonable estimates of the La–O coordination in the glass.

For the glasses, the presence of BO\(_3\) units leads to an additional O–O distance from the borate units, \( r_{OO(B3)} = 2.3677\,\text{Å} (= \sqrt{3} r_{BO(B3)}) \), which does not occur in the crystal. If the B–O coordination number is known, then for a sample of composition \( x\text{GeO}_2 \cdot y\text{B}_2\text{O}_3 \cdot z\text{La}_2\text{O}_3 \) it can be shown that the coordination numbers for the two O–O distances in borate units are given by

\[
 n_{OO(B3)} = \frac{12(4 - n_{BO})}{2x + 3y + 3z} \quad (6.5)
\]

\[
 n_{OO(B4)} = \frac{24(n_{BO} - 3)}{2x + 3y + 3z} \quad (6.6)
\]

It should be emphasised that these are identities that follow directly from the B–O coordination number and the geometry of the BO\(_3\) and [BO\(_4\)]\(^-\) units, with no additional assumptions.

Similarly, the presence of higher coordinated GeO\(_m\) (\( m = 5 \) and/or 6) in the glasses leads to additional O–O distances from the germanate units, but consideration of germanate units is more complex than for borate units. Firstly, it is unknown whether the higher germanium coordination number is 5 or 6, or a mixture thereof. Secondly, whereas the higher coordinated borate unit has a simple geometry (the [BO\(_4\)]\(^-\) tetrahedron) with a very narrow distribution of O–O distances, higher coordinated germanate units are more complicated with more than one type of O–O distance (e.g., in a regular [GeO\(_6\)]\(^2-\) octahedron, there are two O–O distances, which are \( \sqrt{2} \) and 2 times the Ge–O distance). Nevertheless, if it is assumed that the higher germanium coordination is octahedral, the O–O coordination numbers for the GeO\(_4\)
unit \( r_{\text{OO(Ge4)}} \approx 2.8365 \text{ Å} = \sqrt{8/3} r_{\text{GeO(Ge4)}} \) and for the first distance in the \([\text{GeO}_6]^2^-\) unit \( r_{\text{OO(Ge6)}} \approx 2.68 \text{ Å} \) are given by
\[
    n_{\text{OO(Ge4)}} = \frac{6x(6 - n_{\text{GeO}})}{2x + 3y + 3z} \quad (6.7)
\]
\[
    n_{\text{OO(Ge6)}} = \frac{12x(n_{\text{GeO}} - 4)}{2x + 3y + 3z} \quad (6.8)
\]

In the \( \alpha \)-quartz form of GeO\(_2\) [260], the Ge–Ge distance that arises from Ge–O–Ge bridges between GeO\(_4\) tetrahedra is 3.15 Å, which is markedly longer than the distance range of interest, and hence this correlation need not be considered at this stage. However, both B–B and B–Ge distances are shorter, and need to be taken into account: a neutron diffraction study of pure B\(_2\)O\(_3\) glass shows two B–B distances of 2.364 Å inside boroxol groups (B–O–B bond angle 120°) and 2.474 Å outside boroxol groups (bond angle 130°) [261]. The B–Ge distances in the reported structure of crystalline LaBGeO\(_5\) [78] are 2.87 Å and 2.89 Å (with B–O–Ge bond angles of 121.0° and 123.9°, respectively), and in fact we have found that a successful model of the correlation function of the La-free glass requires that the B–Ge distance is shorter than this.

The measurements of the B–O and Ge–O coordination numbers for the La-free glass support a structural model comprised of only BO\(_3\) and GeO\(_4\) units. In this case, there are only two O–O contributions to the relevant region of the correlation function, with coordination numbers of \( n_{\text{OO(B3)}} = 1.700 \) and \( n_{\text{OO(Ge4)}} = 3.451 \).

The correlation function for the La-free glass was fitted in the region of interest using four peaks, as shown in Fig. 6.15, and the parameters for the fit are given in Table 6.5. The positions of the two O–O peaks were fixed at the values calculated from the fitted B–O and Ge–O bond lengths, whilst the positions of the B–B and B–Ge peaks were allowed to vary. The widths of all of the peaks except the B–B peak were allowed to vary. The B–B peak is the smallest peak, and it was found to be necessary to constrain its width; a value of 0.07 Å was used, as found in pure B\(_2\)O\(_3\) glass [261]. All of the coordination numbers were fixed; the O–O coordination numbers were fixed to values calculated using Eqs. (6.5) and (6.7), whilst the B–B and B–Ge coordination numbers were fixed to those for a chemically random network.
The fit is shown in Fig. 6.15, and it is apparent that the model assumptions that were
used give a good account of this region of the measured correlation function. The
fitted B–B distance corresponds to a B−O−B bond angle of 124.4°, typical of bond
angles in borates, and similar to the B−O−B bond angle of 129.6° in LaBGeO5. The
fitted B–Ge distance corresponds to a B−O−Ge bond angle of 121.3°; this value is
very similar to the corresponding bond angles in crystalline LaBGeO5, and hence the
main reason that the B–Ge distance in the glass is shorter than in this crystal is that
the B–O bond length is shorter (due to the smaller B–O coordination number). The
fitted peak widths are typical of the values found in oxide glasses [62, 261]. Since
the B–B and B–Ge coordination numbers were fixed at the values for a chemically
random network, the closeness of the fit is also evidence that there is not a strong
chemical ordering of the network.

Table 6.5: The parameters for a fit to the region from 2.19 Å to 2.73 Å of the correlation
function for the La-free glass. Errors for the parameters that were varied in the fit are
given.

<table>
<thead>
<tr>
<th>Atom pair i-j</th>
<th>$r_{ij}$ / Å</th>
<th>$u_{ij}$ / Å</th>
<th>$n_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>O–O$_{B3}$</td>
<td>2.3677</td>
<td>0.0701 ± 0.0009</td>
<td>1.700</td>
</tr>
<tr>
<td>B–B</td>
<td>2.419 ± 0.004</td>
<td>0.07</td>
<td>1.275</td>
</tr>
<tr>
<td>O–O$_{Ge4}$</td>
<td>2.8365</td>
<td>0.078 ± 0.011</td>
<td>3.451</td>
</tr>
<tr>
<td>B–Ge</td>
<td>2.712 ± 0.010</td>
<td>0.085 ± 0.009</td>
<td>1.725</td>
</tr>
</tbody>
</table>

The fit to the correlation function for the La-free glass shows that it is possible
to use reasonable modelling assumptions to give a good account of the O–O and
X–X correlations in the region of interest, and it yields parameters that can be
applied to the La-containing samples. The same modelling approach was then used to
simulate the non-La correlations in this region for the crystalline sample, as shown
in Fig. 6.16a, and the parameters for this simulation are given in the upper part
of Table 6.6. For the simulation, the peak widths ($u_{ij}$) determined from fitting the
correlation function of the La-free glass (Table 6.5) were used. For the LaBGeO$_5$
crystal structure, both O–O coordination numbers are 2.4 (as predicted by Eq. (6.6)
and Eq. (6.7)), whilst both X–X coordination numbers are 2.0. The O–O distance in
[B-O$_4$]$^-$ tetrahedra was calculated as $\sqrt{8/3} r_{BO(B4)}$, using a value for the B–O bond
length in [B-O$_4$]$^-$ tetrahedra, $r_{BO(B4)}$, predicted using a bond-valence calculation [258].
Figure 6.15: The fit to the region of interest of the correlation function of the La-free glass. a) The total fit (thin line) is shown together with the experimental result (thick line) and the residual (dashed line). b) The components of the fit (red for O–O peaks, blue and green for B–B and B–Ge peaks respectively) together with the experimental result (black).
In both borates and germanates, X–O–X bond angles are typically in the range from 120° to 130°, and hence the X–X distances were calculated from the bond lengths on the assumption of a bond angle of 125°. The non-La simulation was then subtracted from the experimental correlation function, to yield the difference shown in Fig. 6.16b. In the region of interest, this difference is a measure of the distribution of La–O bond lengths. As in the detailed crystallographic structural model [78], there is a smaller number of short La–O bonds (involving non-bridging oxygen), and a larger number of long La–O bonds. In fact there is an asymmetry to the larger peak, and hence the distribution was fitted with a total of three peaks to give the parameters in the lower part of Table 6.6. These fitted peaks are a means of parametrising the La–O distribution, rather than being an accurate determination of the precise bond lengths in the crystal. The total value of $n_{\text{LaO}}$ estimated in this way is 8.2; the value is a little less than, but similar to value of 9 obtained from a full crystallographic analysis [78]. The exact value obtained by this method is sensitive to the choice of parameters for the non-La simulation, especially the parameters for the O–O$_{\text{Ge4}}$ and B–Ge peaks; if these peaks are moved to longer distance (either by increasing the assumed interatomic distance, or by decreasing the assumed width), then a larger value of $n_{\text{LaO}}$ is obtained. Nevertheless, a bond-valence calculation [257] of the valence corresponding to the fitted peaks yields a total of 2.91 (see Table 6.6); this value is close to the formal La$^{3+}$ valence, indicating that the result is reasonable.

Table 6.6: Parameterisation of the region of interest of the correlation function for the crystalline sample (see Fig. 6.16). The first four sets of values were used to simulate the non-La contributions (see text for details), which were subtracted from the experimental function. The resultant difference was then fitted to give the final three sets of values for the La–O coordination, and the valence corresponding to these fit components is also given [257].

<table>
<thead>
<tr>
<th>Atom pair $i$–$j$</th>
<th>$r_{ij}$ / Å</th>
<th>$u_{ij}$ / Å</th>
<th>$n_{ij}$</th>
<th>Valence</th>
</tr>
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<tbody>
<tr>
<td>O–O$_{\text{B4}}$</td>
<td>2.412</td>
<td>0.0701</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>B–B</td>
<td>2.615</td>
<td>0.07</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>O–O$_{\text{Ge4}}$</td>
<td>2.8365</td>
<td>0.078</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>B–Ge</td>
<td>2.857</td>
<td>0.085</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>La–O</td>
<td>2.286 ± 0.002</td>
<td>0.051 ± 0.004</td>
<td>0.78 ± 0.04</td>
<td>0.58</td>
</tr>
<tr>
<td>La–O</td>
<td>2.58 ± 0.01</td>
<td>0.10 ± 0.01</td>
<td>5.85 ± 0.74</td>
<td>1.94</td>
</tr>
<tr>
<td>La–O</td>
<td>2.682 ± 0.002</td>
<td>0.05 ± 0.01</td>
<td>1.57 ± 0.73</td>
<td>0.40</td>
</tr>
<tr>
<td>Total La–O</td>
<td>2.57</td>
<td></td>
<td>8.2</td>
<td>2.91</td>
</tr>
</tbody>
</table>
Figure 6.16: The region of interest of the correlation function for the crystalline sample. a) The experimental correlation function (thin black line) together with a simulation of the non-La contributions in this region (thick black line). The individual components of the simulation are also shown; red lines are the peaks for O–O distances in $[\text{BO}_4]^-$ and GeO$_4$ tetrahedra, whilst blue and green lines are respectively the B–B and B–Ge peaks for distances arising from corner-sharing between these tetrahedra. b) The difference between experiment and the non-La simulation (thick black line), together with the components of a fit to the difference (blue lines), and the residual for the fit (thin black line).
This method of estimating the La–O correlation (simulation of non-La contributions, followed by fitting of the remainder) yields a reasonable result for the crystalline sample, and it was applied to the glasses too (see Figs. 6.17 and 6.18).

However, the non-La simulation for the glasses is more complicated. Firstly, the glasses contain more different types of structural unit (BO$_3$, [BO$_4$]$^-$, GeO$_4$, and GeO$_m$; it was assumed that $m$=6), leading to a greater number of different O–O contributions. The $n_{OO}$ coordination number for each of these was calculated using Eq. (6.5) to Eq. (6.8). As for the La-free glass, the O–O distances in BO$_3$ and GeO$_4$ units were calculated from the measured B–O and Ge–O bond lengths respectively. The O–O distances in regular [BO$_4$]$^-$ and [GeO$_6$]$^{2-}$ units were calculated from bond-valence predictions of the B–O and Ge–O bond lengths (note that the O–O distance in a [GeO$_6$]$^{2-}$ octahedron is shorter than in a GeO$_4$ tetrahedron) [258]. The widths for the four O–O peaks were assumed to be the same as determined from fitting the correlation function for the La-free glass (see Table 6.5). Secondly, the region of interest also contains X–X contributions (where X = network forming cation), and there are ten of these in total, arising from the four different structural units. The X–X coordination numbers were calculated by assuming that the NBOs are statistically distributed on the BO$_3$ and GeO$_4$ units (not the [BO$_4$]$^-$ or [GeO$_6$]$^{2-}$ units [20, 62]), and by assuming a random distribution of the different types of oxygen bridge between the units (a chemically disordered network). These are reasonable assumptions, because there is no evidence from the REDOR NMR data for an ordering of borate species, and thus there is no reason to suspect an ordering of germanate species, or between borate and germanate species. The X–X distances were calculated from the X–O bond lengths with an assumption of a X–O–X bond angle of 125°. The widths for the three B–B and four B–Ge peaks were taken from the fit for the La-free glass (Table 6.5), whilst a width $u = 0.10$ Å was assumed for the three Ge–Ge peaks. In Fig. 6.17, which shows the simulations for the samples with the least and most La$_2$O$_3$, the four simulated O–O peaks are all shown, whilst a single peak is shown for the sums of all B–B, B–Ge and Ge–Ge peaks respectively. Thus the X–X peaks in Fig. 6.17 are broader than the O–O peaks because they each include several contributions with different distances.

In the region of interest, the difference between the simulation and the experimental
Figure 6.17: The region of interest of the correlation functions for glass samples with low and high \( \text{La}_2\text{O}_3 \) content, a) 20LaBG, and b) 30LaBG-1. The thin black line is the experimental correlation function, whilst the thick black line is the total simulation of all contributions in the region of interest, other than La-O. The simulation includes component peaks due to O–O (red lines), B–B (blue line), B–Ge (green line) and Ge–Ge (cyan line) correlations.
Figure 6.18: The difference between the experimental correlation function and the simulation of the non-La contributions in the region of interest. Results for the glass samples are plotted with suitable vertical displacements, and with the La$_2$O$_3$ content increasing towards the top of the figure. The fit to the La–O peak is shown as a red line, and the individual components of the fit are shown as blue lines.
correlation function is due to La–O correlations, and this difference is larger for higher La$_2$O$_3$ content (Fig. 6.17b). For each glass, the simulation was subtracted from the experimental correlation function to yield the estimates of the La–O distribution shown in Fig. 6.18. As for the crystalline sample (Fig. 6.16), there is a main La–O peak at longer distance, with a smaller peak at shorter distance; the shorter distance peak decreases as the La$_2$O$_3$ content increases. As shown in Fig. 6.18, the estimates of the La–O distribution were fitted, yielding the results given in Table 6.7. The

Table 6.7: The parameters for fits to the La–O correlation in the glasses (errors are not given, because these values are essentially estimates). $\bar{r}$ is the average La–O bond length. $\Sigma n$ is the total La–O coordination number over all components of the fit. $\Sigma$ is the total La valence over all components of the fit.

<table>
<thead>
<tr>
<th>Glass sample</th>
<th>$r_{\text{LaO}}$ / Å</th>
<th>$u_{\text{LaO}}$ / Å</th>
<th>$n_{\text{LaO}}$</th>
<th>Valence</th>
</tr>
</thead>
<tbody>
<tr>
<td>20LaBG</td>
<td>2.22</td>
<td>0.06</td>
<td>0.40</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>2.54</td>
<td>0.13</td>
<td>7.42</td>
<td>2.77</td>
</tr>
<tr>
<td>$\bar{r}$ = 2.52</td>
<td>$\Sigma n$ = 7.82</td>
<td>$\Sigma$ = 3.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25LaBG-1</td>
<td>2.25</td>
<td>0.05</td>
<td>0.19</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>2.52</td>
<td>0.13</td>
<td>7.10</td>
<td>2.75</td>
</tr>
<tr>
<td>$\bar{r}$ = 2.52</td>
<td>$\Sigma n$ = 7.30</td>
<td>$\Sigma$ = 2.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25LaBG-2</td>
<td>2.27</td>
<td>0.05</td>
<td>0.17</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>2.53</td>
<td>0.13</td>
<td>6.75</td>
<td>2.58</td>
</tr>
<tr>
<td>$\bar{r}$ = 2.52</td>
<td>$\Sigma n$ = 6.92</td>
<td>$\Sigma$ = 2.71</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30LaBG-1</td>
<td>2.52</td>
<td>0.14</td>
<td>7.36</td>
<td>2.84</td>
</tr>
<tr>
<td>30LaBG-2</td>
<td>2.54</td>
<td>0.14</td>
<td>7.53</td>
<td>2.77</td>
</tr>
</tbody>
</table>

La–O bond lengths are typically 2.52 Å, whilst the values obtained for the total La–O coordination number vary haphazardly around a mean value of 7.4. The variation in values for $n_{\text{LaO}}$ is probably an indication of the limit of reliability of this method of estimating the correlation, but nevertheless the valence values are close to the formal valence of La$^{3+}$, indicating that the results are reasonable. The values obtained for $n_{\text{LaO}}$ are consistently smaller than the value of 8.2 obtained for the crystal using the same method (Table 6.6). The values obtained for the average La–O bond length ($\bar{r}$ in Table 6.7) are also consistently shorter than the average value for crystalline LaBGeO$_5$, 2.605 Å [78], as is expected for a smaller coordination number [258].

In crystalline LaBGeO$_5$ [78], there is one short O–O distance of 2.766 Å in the La coordination shell for a pair of oxygen that are not in the same borate or germanate
unit. This possibility has not been included in the preceding analysis, but it is worth noting that if the La–O coordination number in the glasses is smaller than in the crystal then the oxygen around a La atom are not packed as closely and hence such a short O–O distance may be less likely than in the crystal, and furthermore its statistical weight is small even for the crystal structure.

La–O coordination numbers and bond lengths in crystal structures show a wide variability; the bond lengths are usually in the range 2.3 Å to 2.9 Å, with coordination numbers varying from 7 to 12 [177, 219, 262, 263], though values as low as 6 have been observed [264]. In lanthanum borate and germanate crystals, coordination numbers from 7 to 10 are usually found [265, 266], with mean bond lengths from 2.54 Å to 2.66 Å. La–O bond lengths as short as those we have observed in the range 2.2 Å to 2.3 Å (Tables 6.6 and 6.7) have not previously been reported in crystal structures, and merit future investigation.

To summarize, while the difficulty in precisely determining the composition of our samples results in some uncertainty in our quantitative results, the general reliability of neutron diffraction allows us to make the following structural conclusions:

1. High-coordinate germanium, in the form of [GeO₅] and/or [GeO₆], is present in all lanthanum-containing glasses.

2. The fraction of germanium in the form of [GeO₅] and/or [GeO₆] is substantial, between 20% to 40% in most glass samples.

3. The lanthanum-oxygen coordination number in the LaBGeO₅ glass is lower than that of the crystal.

6.6.3 Density, \( T_g \), and Molar Volume

The variation in the glass transition temperature of the samples is very small, and not clearly related to any individual composition or structural parameter (Table 6.1). The glass transition temperature increases as the B–O coordination number decreases. This trend is contrary to what would be expected in binary alkali borate glass, where \( T_g \) reaches a maximum at approximately the same composition as \( n_{BO} \), and either decreases or remains constant as \( n_{BO} \) continues to decrease [267, 268]. The behaviour
is also different from binary lanthanum borate glasses, which experience a maximum in $T_g$ at about 25 mol% La$_2$O$_3$ [238]. The glass transition temperatures of the lanthanum borogermanate system are of similar magnitude to those of the binary lanthanum borate glasses. The binary lanthanum germanate glass composition is not well-studied, and reported composition ranges have lanthanum concentrations significantly lower than present in this work [269]. However, given the substantially higher field strength of the lanthanum cation compared to alkali or alkaline earth cations, and given the significantly narrower glass-forming region of this composition, it is unlikely that we would observe behaviour substantially similar to these simpler compositions.

The comparison of the molar volumes of the lanthanum-free glass and the lanthanum-bearing glasses is worth noting (Table 6.1). The addition of lanthanum significantly reduces the molar volume of the 20LaBG glass compared to the 0LaBG composition. The molar volume increases as the lanthanum content of the glass increases. As there is no observed trend of $n_{\text{GeO}}$ with composition, the most plausible explanation for this change is the decrease in $n_{\text{BO}}$ as [La$_2$O$_3$] increases.

6.6.4 Electrostatic Bond Strength Analysis

The $^{11}$B NMR and neutron diffraction data above suggest certain bonding arrangements: the presence of $[\text{B}O_4]^{-}$ and $\text{BO}_2\text{O}^-$; the presence of non-bridging oxygen on GeO$_4$ tetrahedra; the presence of high-coordinate germanium; and a mean lanthanum-oxygen coordination number of $\sim$7.4. We turn to EBS analysis to attempt to model the lanthanum borogermanate glass network [258, 270]. In EBS analysis, we consider the electrostatic bond strength, $s$, for each cation and coordination. $s$ is defined as $z/n$, where $z$ is the formal charge of the cation and $n$ is the number of coordinated anions (in this case oxygen). Likely oxygen environments are those for which the summation of the $s$ values of its neighbours is close to 2, the magnitude of the formal charge of oxygen. In our analysis we consider close to mean EBS sums ($\Sigma$) ranging from 1.8 to 2.2, i.e., $\pm 10\%$ of the formal charge. This can somewhat account for varying bond lengths and angles, which are not considered in this simple model [271, 272]. We also restrict oxygen to no more than 4 nearest neighbours, as numbers greater than this are geometrically unlikely in lanthanum oxide-based materials. Given the local structural units that we have observed experimentally, we can provide reasonable
estimates as to EBS values. We first consider the values of $s$ for each network forming species. The boron and germanium values are straightforward to calculate, given the limited range of available coordination numbers. Although La may occur with a range of possible coordination numbers, the range in $s$ values is actually rather small. For the likely $n_{\text{LaO}}$ range from 7 to 9, the change in $s$ for La–O bonds is from 0.43 to 0.33; hence, even large changes in the lanthanum-oxygen coordination number have smaller changes in $s$ compared to those for boron or germanium. In the notation we use, superscript prefixes denote coordination number, while subscript suffixes indicate the number of atoms of the given coordination present in the oxygen environment. $O_b$ and $O_{nb}$ indicate bridging and non-bridging oxygen, respectively. For example, $[O_b^4\text{Ge}_2]$ describes a bridging oxygen environment coordinated to two GeO$_4$ units (see Fig. 6.19a). For the purposes of this analysis, any oxygen bonded to only one B atom, or to only one Ge atom, is considered to be non-bridging.
There are many possible combinations of bonding environments to be considered. Some are perfectly charge-balanced in the absence of lanthanum, such as \([\text{O}_6^4\text{Ge}_2], \text{[O}_6^3\text{B}_2]\), or \([\text{O}_6^3\text{B}^\text{Ge}]\). Most others require coordination to at least one lanthanum ion for effective charge compensation. In Table 6.8 we present the \(\Sigma\) values for the potential bonding environments in a lanthanum borogermanate glass. While it is difficult to say which of these environments would be more or less likely on EBS analysis alone, it is notable that a very wide range of environments are plausible. \([\text{B}_4^\text{O}_4^-]\) units can plausibly be bridged to \(\text{BO}_3\) units, or to any of \(\text{GeO}_4\), \([\text{GeO}_5]^-\), or \([\text{GeO}_6]^{2-}\). The germanium bonding units are similarly versatile. This view is similar across all plausible values of \(n_{\text{LaO}}\). There are very few potential environments that can be excluded through the use of EBS analysis; nevertheless it is of note that a \([\text{O}_6^3\text{B}^4\text{La}]\) oxygen site would be substantially overbonded, with \(\Sigma = 2.25\). As \(^{11}\text{B}\) MAS NMR suggests the presence of \(\text{BO}_2\text{O}^-\) in the second coordination sphere of \([\text{BO}_4^-]\), this suggests that the minimum \(\text{La}–\text{O}\) coordination number is greater than 6, consistent with the neutron diffraction analysis. It is worthwhile to note that there are a few plausible oxygen environments that meet our criteria above that are bonded to three network-forming cations, but require no lanthanum for charge compensation. All of these environments, e.g., \([\text{O}_6^6\text{Ge}_3]\), require the presence of either \([\text{GeO}_5]^-\), \([\text{GeO}_6]^{2-}\), or both. While we cannot probe the oxygen environment directly in this experiment, the presence of such “three-coordinate oxygen” would provide additional evidence of the presence of high-coordinate germanium.

EBS analysis is of most use to us in discussing the possible environments around the non-bridging oxygen. Table 6.9 contains the EBS sums for several likely values of \(n_{\text{LaO}}\). We discount the possibility of non-bridging oxygen on four-coordinate boron, as this structure has not been generally observed [20]. The non-bridging oxygen on \(\text{BO}_3\) and \(\text{GeO}_4\) species is considered equivalent in EBS analysis, as both have \(s\) values of 1. There are plausible non-bridging oxygen configurations for all values of \(n_{\text{LaO}}\) between 7 to 9. All configurations require at least two \(\text{O}–\text{La}\) bonds (e.g., \([\text{XO}_n^7\text{La}_2]\)) with the higher coordination numbers \((n_{\text{LaO}} \geq 8)\) requiring three. In the \(\text{LaBGeO}_5\) crystal, detailed at the bottom of Table 6.9, each non-bridging oxygen is a corner of the \(\text{GeO}_2\text{O}_2^{2-}\) tetrahedron shared with two \(\text{LaO}_9\) polyhedra. However, according to the EBS analysis, these non-bridging oxygen are substantially underbonded in the
Table 6.8: Electrostatic bond strengths of various bridging oxygen environments in lanthanum borogermanates. The $n_{LaO}$ which results in a value of $\Sigma$ closest to 2 is included for applicable environments.

<table>
<thead>
<tr>
<th>O Site</th>
<th>$\Sigma$ without La / EBS</th>
<th>Best $n_{LaO}$</th>
<th>$\Sigma$ with best $n_{LaO}$ / EBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[O_b^5B_6^6Ge]$</td>
<td>2.167</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$[O_b^5Ge_6^6Ge]$</td>
<td>2.133</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$[O_b^5B_6^6Ge]$</td>
<td>2.083</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$[O_b^6Ge_3]$</td>
<td>2.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$[O_b^3B_2]$</td>
<td>2.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$[O_b^3B_4^4Ge]$</td>
<td>2.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$[O_b^3Ge_2]$</td>
<td>2.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$[O_b^3B_6^6Ge]$</td>
<td>1.667</td>
<td>9</td>
<td>2.000</td>
</tr>
<tr>
<td>$[O_b^4Ge_6^6Ge]$</td>
<td>1.667</td>
<td>9</td>
<td>2.000</td>
</tr>
<tr>
<td>$[O_b^5Ge_2]$</td>
<td>1.600</td>
<td>8</td>
<td>1.975</td>
</tr>
<tr>
<td>$[O_b^3B_4^4Ge]$</td>
<td>1.550</td>
<td>7</td>
<td>1.975</td>
</tr>
</tbody>
</table>

Crystal. In practice, this apparent underbonding is avoided mainly by a shortening of the relevant La–O bonds, so that their valence is increased [257]. The analysis in Table 6.9 implies that non-bridging oxygen in the glass are coordinated to either two or three lanthanum ions, and our estimate that the mean La–O coordination number is $\sim$7.4 implies that a NBO–La coordination number of two is the most common. Furthermore, our observation of a small number of very short La–O bonds suggests the possibility of a small number of non-bridging oxygen that are bonded to only one La$^{3+}$ ion.

Table 6.9: Electrostatic bond strengths of various non-bridging oxygen environments. Sites are arranged in order of $|2–\Sigma|$. Superscript prefixes indicate coordination number, while subscript suffixes indicate the number of a given species. X indicates either $^4$Ge or $^3$B; the electrostatic bond strength of their non-bridging oxygen environments are the same. Non-bridging oxygen on $^5$Ge or $^6$Ge are not included in the table. The site at the bottom of the table corresponds to the non-bridging oxygen environment in crystalline LaBGeO$_5$.

<table>
<thead>
<tr>
<th>O Environment</th>
<th>$\Sigma$ / EBS</th>
<th>Number of La$^{3+}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[XO_{nb}^{9}La_3]$</td>
<td>2.000</td>
<td>3</td>
</tr>
<tr>
<td>$[XO_{nb}^{8}La_3]$</td>
<td>2.125</td>
<td>3</td>
</tr>
<tr>
<td>$[XO_{nb}^{7}La_2]$</td>
<td>1.857</td>
<td>2</td>
</tr>
<tr>
<td>[$^4$GeO$_{nb}^{9}La_2$]</td>
<td>1.666</td>
<td>2</td>
</tr>
</tbody>
</table>
The EBS analysis, when combined with the $^{11}$B MAS NMR, MQMAS NMR, and neutron diffraction data, implies a reasonably homogeneous glass structure. All three-coordinate boron is in the form of $\mathrm{BO}_2\mathrm{O}^-$, implying a substantial level of connectivity within the glass network. All four-coordinate boron is in proximity to lanthanum, and the change in the $\delta_{\text{iso}}^{\text{CS}}$ of the $[\mathrm{B}\mathrm{O}_4]^-$ peak suggests that $[\mathrm{B}\mathrm{O}_4]^-$ may be connected to $\mathrm{BO}_2\mathrm{O}^-$, in which case there may be an interconnected boron network. The presence of interconnected $[\mathrm{B}\mathrm{O}_4]^-$ and $\mathrm{BO}_2\mathrm{O}^-$ also suggests a mean value of $n_{\text{LaO}}>6$, according to our EBS analysis. The amount of positive charge introduced into the system by the addition of $\mathrm{La}_2\mathrm{O}_3$ is insufficiently compensated by the observed boron species. Our neutron diffraction data indicate that germanium polyhedra compensate for this remaining charge via a combination of non-bridging oxygen and coordination numbers greater than four. Since the bond strengths for Ge and B are similar, there is nothing in the EBS analysis that precludes mixing of the two elements at the microscopic level, leading to a homogeneous structure. This is consistent with the work of Lee et al. [53], who reported no indications of network segregation in $\mathrm{B}_2\mathrm{O}_3$-$\mathrm{GeO}_2$ glasses. Despite the potential for the addition of a modifier oxide (such as $\mathrm{La}_2\mathrm{O}_3$) to promote either liquid-liquid phase separation or crystallization, our EBS analysis is consistent with a homogeneous network.

6.7 Conclusions

LaBGeO$_5$ is of importance as a ferroelectric nanocomposite, which crystallizes iso-chemically from a glass of the same composition. A knowledge of the differences and similarities between the LaBGeO$_5$ glass and crystal structures is important for determining the underlying mechanism of formation of the LaBGeO$_5$ transparent ferroelectric nanocomposite material. We have performed both $^{11}$B MAS NMR and neutron diffraction measurements on lanthanum borogermanate glasses and crystalline LaBGeO$_5$, combined with electrostatic bond strength analysis, allowing us to construct a coherent structural model of the lanthanum borogermanate glass system. These NMR spectra show the presence of both three- and four-coordinate borate units, and suggest the significant presence of the $[\mathrm{O}_4^3\mathrm{B}_4^4\mathrm{B}^n\mathrm{La}]$ oxygen environment, i.e., that the three- and four-coordinate borate units are frequently neighbours. $^{11}$B MQMAS spectra support a model where there are only single types of three- and four-coordinate
environments, and that all BO$_3$ is in the form of BO$_2$O$^-$, i.e., singly-nonbridging trigonal borate units. Our REDOR results support the presence of both BO$_3$-[BØ$_4$]$^-$ and [BØ$_4$]$^-$-[BØ$_4$]$^-$ connections, and that they exist in similar proportions. This suggests that the borate network does not preferentially group together similar bonding units. Neutron diffraction data provide evidence for the presence of a significant fraction of germanium atoms with a coordination number greater than four, as either [GeO$_5$]$^-$, [GeO$_6$]$^{2-}$, or a combination of both, as well as the presence of non-bridging oxygen on the germanium tetrahedra. A detailed modelling approach is shown to work well for the longer range region (≈2.2 Å to 2.75 Å) of the neutron correlation functions of La-free glass and crystalline LaBGeO$_5$. This approach is applied to the glass correlation functions, leading to an estimate of 7.4 for the La–O coordination number, with a typical La–O bond length of 2.52 Å. Our EBS analysis suggests the presence of multiple La$^{3+}$ ions in proximity to non-bridging oxygen sites, as well as a high degree of flexibility in network former mixing. It also constrains the [O$_6$B$_4$B$_n$La] environment to values where $n$ is 7 or greater, and hence the mean lanthanum-oxygen coordination number is greater than 6.

The structure of the LaBGeO$_5$ glass composition differs substantially from the LaBGeO$_5$ crystal; the average oxygen coordination numbers of all three cations are different from the crystal. Furthermore, the composition-dependence of the boron and germanium coordination numbers provides evidence that both boron and germanium undergo an anomaly.

6.8 Acknowledgements

Financial support from NSERC (Canada Grant Number RGPIN 261987) is gratefully acknowledged. We thank Dr. Mark Obrovac and Dr. Daniel Boyd of Dalhousie University for access to an X-ray diffractometer and helium gas pycnometer, respectively.
Chapter 7

Network Connectivity and Crystallization in the Transparent Ferroelectric Nanocomposite Material LaBGeO$_5$

7.1 Context

Through the use of $^{11}$B MQMAS and neutron diffraction (Chapter 6) we have established the local structure of the La, B, and Ge components of the LaBGeO$_5$ glass. However, determining the local order is only part of constructing a model of the structure of the glass. After establishing which short-range bonding units are present (i.e., BØ$_2$O$^-$, [BØ$_4$]$^-$, GeO$_4$, $^{5/6}$Ge, and on average LaO$_7$), the next step is to determine how these units are connected. The techniques used in previous chapters are ill-suited to determine glass network connectivity, especially in the structurally complex LaBGeO$_5$ system. Hence we turn instead to $^{17}$O NMR spectroscopy and $^{11}$B{$^{10}$B} REDOR NMR spectroscopy.

$^{17}$O MAS and MQMAS NMR is an obvious choice for probing network connectivity in oxide glasses, as all glass structural units are cation-oxygen polyhedra. The $^{17}$O NMR spectra we present in this chapter provide significant data on the presence (and absence) of various network connectivities, but, as will be discussed, the spectral resolution is limited. As a complementary technique we turned to $^{11}$B{$^{10}$B} REDOR NMR spectroscopy, which probes the connectivity between [BØ$_4$]$^-$ units and both BO$_3$ and [BØ$_4$]$^-$ units. With the overall connectivity data from our $^{17}$O NMR spectra, and the borate-specific connectivity from our $^{11}$B{$^{10}$B} REDOR NMR spectroscopy, we can construct a model for the network connectivity of the LaBGeO$_5$ glass.

This chapter also examines the crystallization of the LaBGeO$_5$ glass-ceramic. $^{17}$O NMR is equally effective in crystals and in glasses, and as such it is particularly well-suited for the study of glass-ceramic composites. Through the use of $^{17}$O NMR, along with pXRD and $^{11}$B and $^{139}$La NMR, we probe potential structural changes in either the crystalline or glass forms of LaBGeO$_5$ during the crystallization of the
LaBGeO$_5$ glass.

This chapter was originally prepared for submission in The Journal of Physical Chemistry C. The contributions of the authors to the text are as follows. ALP synthesized all of the experimental samples, conducted all of the experiments and DFT calculations, performed the analysis of the data, and wrote the manuscript. UWZ provided significant technical support with the optimization of the $^{17}$O and $^{11}$B{$^{10}$B} REDOR NMR spectroscopy, assisted in the evaluation of the $^{17}$O and $^{11}$B{$^{10}$B} REDOR data, and provided editorial contributions to the manuscript. JWZ provided significant contributions to the experimental design and substantial editorial contributions to the manuscript.

7.2 Abstract

The LaBGeO$_5$ glass-ceramic composite is a TFN material that has come under attention for its ferroelectric properties, and for its straightforward synthesis. The LaBGeO$_5$ crystals-in-glass can be formed through controlled devitrification of the glass, as well as through laser irradiation. While previous structural models of the glass have been reported, they have only considered differences in the borate environment between the glass and the crystal. We present here a comprehensive NMR study of the short-range order of crystalline LaBGeO$_5$ and LaBGeO$_5$ glass. Through a combination of $^{11}$B, $^{11}$B{$^{10}$B}, $^{17}$O, and $^{139}$La NMR spectroscopies we identify specific structural motifs in the glass. In particular, structures not present in the crystal (e.g., $\text{BO}_2\text{O}^-$, $\text{BO}_2\text{O}^-[\text{B}_4\text{O}_9]^-$, $\text{GeO}_6$) are positively identified in the glass. We present evidence in support of the presence of both $^{5/6}\text{Ge}–\text{O}–^{4}\text{Ge}$ and $^{5/6}\text{Ge}–\text{O}–\text{B}$ structures, as well as the possibility of $\text{O}_{666}$ triclusters. Quantitative $^{11}$B and $^{17}$O MAS NMR spectroscopies provide data in support of a revised equation to determine the crystallization fraction of a partially cerammed sample via moderate-field $^{11}$B NMR spectroscopy.

7.3 Introduction

Ferroelectric glass-ceramics are composite materials in which a ferroelectric crystalline phase is nucleated within a glass matrix. One example of a ferroelectric glass-ceramic is the LaBGeO$_5$ composition. As the LaBGeO$_5$ glass can be congruently
Figure 7.1: The room-temperature crystal structure of LaBGeO$_5$ [78]. The presented view is down the crystallographic $c$ axis. The unit cell is indicated by black lines; a $2\times2\times2$ supercell is shown in order to highlight the helical [BØ$_4$]$^-$ chain at the centre. [BØ$_4$]$^-$ tetrahedra are coloured dark green, while GeØ$_2$O$_2^2$ tetrahedra are purple. LaO$_9$ polyhedra are omitted for clarity. Lanthanum ions are light green.

(i.e., isochemically) crystallized, it has become a particular target for investigation into transparent ferroelectric nanocomposite materials. Many different compositions (e.g., LiNbO$_3$, BaTiO$_3$) can produce glass-ceramics that are transparent (i.e., with crystals with diameters less than the wavelength of visible light) with ferroelectric crystallites [273, 274]. However, many of these compositions produce multiple distinct crystalline phases upon devitrification, complicating their characterization and analysis. The apparently simple crystallization behaviour of LaBGeO$_5$ has made it the target of many investigations. The LaBGeO$_5$ crystal has been well-characterized, with its structure (Fig. 7.1) [78, 79], second-harmonic generation efficiency [232, 233], ferroelectric moment [88], and Curie temperature [88, 233, 275] having been reported in the literature. Similar efforts have been undertaken with regards to the glass and glass-ceramic composite, and various physical properties of both have been reported [96]. Recently, laser-written single-crystal LaBGeO$_5$ architectures in LaBGeO$_5$ glass have
been reported [107, 108]. To fully understand the mechanism and dynamics of single-crystal formation within a glassy matrix, it is necessary to have a thorough understanding of the glass structure and how it relates to the crystal structure.

There are a number of structural studies of the LaBGeO$_5$ glass reported in the literature. In 2000, Kratochvílová et al. reported Raman spectra of the glass [74]. Based upon comparison to the crystal, they concluded that the structure of the glass was similar to the crystal, albeit disordered. In 2004 Gupta et al. reported the first NMR study of the LaBGeO$_5$ glass, presenting $^{11}$B MAS NMR spectra definitively proving the presence of both BO$_3$ and [BO$_4$]$^-$ units in the glass, whereas the crystal only has [BO$_4$]$^-$ units [34]. Sigaev et al. revisited the Raman spectrum of LaBGeO$_5$ in 2010, concluding that their data confirmed the presence of BO$_3$, but also that all other structural units were unchanged from the crystal [75]. Further, they concluded that the LaBGeO$_5$ glass is composed of “crystal-like” nanoregions, where the boron is fourfold coordinate, connected by regions where the boron is threefold coordinated.

The above studies neglect to consider possible structural changes in either the germanium or lanthanum environments. Five- and six-coordinate germanium-oxygen polyhedra are known to form in germanate crystals, and have been shown to be present in alkali and alkaline-earth germanate glasses [12, 28, 55, 62]. Via neutron diffraction, we have shown that high-coordinate germanium is present in lanthanum borogermanate glasses [212]. This dramatic change in the germanium environment between the glass and the crystal suggests the possibility of radically different glass and crystal structures, on both the short-range and intermediate-range connectivity. However, only the short-range order of the LaBGeO$_5$ glass has been studied. NMR spectroscopy offers the possibility of probing the connectivity between structural units, including between high-coordinate germanate units.

We present here a study of the structural differences between the glass and crystal phases of the boron, germanium, and lanthanum local environments through a combination of $^{11}$B, $^{11}$B{$^{10}$B}, $^{17}$O, and $^{139}$La NMR spectroscopies. $^{11}$B NMR spectroscopy is a mature technique for the study of borate glasses [49, 276, 277]. $^{11}$B{$^{10}$B} REDOR experiments can probe the connectivity of the borate network in borate glasses [52]. $^{139}$La NMR spectroscopy has become more accessible with the advent of the WCPMG pulse sequence [126], but no comprehensive model of
the relationship between glass spectra and properties has yet been reported in the literature. $^{17}$O NMR spectroscopy is a very powerful technique for the study of oxide glass structure, and has previously been used to establish the presence of high-coordinate germanium in sodium germanate glasses [54–56]. $^{73}$Ge NMR studies of glass are not generally considered to be feasible, as previous attempts have produced inconclusive results [25]. With this comprehensive probe into the short-range order of the glass we intend to produce reasonable constraints on the structure of the glass in order to inform future mechanistic studies of the formation of LaBGeO$_5$ glass-ceramics.

### 7.4 Experimental Procedures

#### 7.4.1 Sample Synthesis

Crystalline Ge$^{17}$O$_2$ was synthesized by hydrolysing germanium ethoxide (Ge(OC$_2$H$_5$, ≥ 99.95 %, Sigma-Alrich) with 40 % $^{17}$O-enriched H$_2^{17}$O (Cortecnet). The ethoxide and enriched water were combined under nitrogen at a molar ratio of 1:2 in a sealed container and allowed to react for 3 d. The product was then heated to 350°C under nitrogen for approximately 3 d for improved crystallinity. The identity of the crystalline product was confirmed to be trigonal (quartz structure) GeO$_2$ via pXRD. Crystalline H$_3$B$^{17}$O$_3$ was synthesized by hydrolysing a 1 mol dm$^{-3}$ solution of borane tetrahydrofuran (THF) (BH$_3$OC$_4$H$_8$, Sigma-Aldrich) with 40 % $^{17}$O-enriched H$_2^{17}$O. The enriched H$_2^{17}$O was slowly added to 0°C borane THF solution under dry nitrogen atmosphere. Excess THF and evolved hydrogen gas were removed under vacuum. The final polycrystalline product was verified to be H$_3$BO$_3$ via both pXRD and $^{11}$B MAS NMR spectroscopy. LaBGeO$_5$ glass was synthesized by combining stoichiometric amounts of the enriched Ge$^{17}$O$_2$ and H$_3$B$^{17}$O$_3$ powders with La$_2$O$_3$ (99.99 %, Sigma-Aldrich), thoroughly grinding the mixture in a mortar and pestle, and then melting the mixture in a 95/5 Pt/Au crucible at 1250°C in a box furnace in air. The melt was quenched by pouring it onto a brass plate preheated to 400°C. Partially crystallized samples were produced by placing the powdered sample in a furnace pre-heated to 765 °C for short lengths of time (approx. 4 min). Observations (i.e., pXRD, NMR) were taken, and afterwards the sample was returned to the furnace for further crystallization. $^{17}$O NMR spectra were collected when sufficient crystallization was observed in the
powder via $^{11}$B MAS NMR. Reported samples are labelled LBG-G, for the original glass, and LBG+8, LBG+12, LBG+24, and LBG+60, where the number in the label is the total time spent crystallizing, in minutes. An additional sample was prepared by heating the LBG+60 sample for an additional 3 d; this further increased the crystal fraction, but removed the $^{17}$O, preventing the collection of $^{17}$O NMR spectra. This sample is labelled LBG+3d. LaB$_3$O$_6$ crystal powder was synthesized by combining stoichiometric amounts of La$_2$O$_3$ and B$_2$O$_3$ and heating in a platinum crucible for 12 h at 850°C. LaBGeO$_5$ samples were produced with natural abundance reagents (La$_2$O$_3$ (99.99%), GeO$_2$ (≥ 99.99%), B$_2$O$_3$ (≥ 98%), all from Sigma Aldrich) for $^{11}$B{$^{10}$B} NMR experiments, but otherwise followed the synthesis described above.

7.4.2 Sample Characterization

PXRD was carried out using a Rigaku Ultima IV X-ray diffractometer equipped with a copper anode X-ray tube, a diffracted beam monochromator, and a scintillation detector. Diffractograms were collected from finely-ground samples in air at room temperature. $2\theta$ was incremented with steps of 0.05° and a 2 s dwell time. Density of the glass sample was measured using Archimedes’ method, using absolute ethanol as the working fluid. Density measurements were controlled for variations in temperature.

7.4.3 Nuclear Magnetic Resonance Spectroscopy

$^{11}$B MAS NMR spectra were collected on a 16.4 T (224.63 MHz $^{11}$B frequency) Bruker Avance NMR spectrometer. Spectra were collected at a spinning speed of (25 000 ± 5) Hz using a 2.5 mm ZrO$_2$ rotor. Single pulses of 0.54 µs duration (15° tip angle, $\nu_{\text{rf}} = 77$ kHz) and an optimized recycle delay of 5 s were used to ensure quantitative excitation. 512 scans were collected for each sample. $^{11}$B background from the probe was subtracted from the experimental spectrum after careful phasing and intensity adjustment. Probe background spectra were collected immediately following the collection of sample spectra, using the same experimental conditions. $^{11}$B chemical shifts were referenced to solid NaBH$_4$, with a resonance at −42.1 ppm relative to the primary shift reference (BF$_3$·Et$_2$O, 0 ppm) [159].

$^{10}$B MAS NMR spectra were collected on a 16.4 T (75.24 MHz $^{10}$B frequency) Bruker Avance NMR spectrometer. Spectra were collected using a HXY 3.2 mm
probehead operating in HX mode. $^{10}$B Hahn echo spectra were collected at a spinning speed of $(10000 \pm 5)$ Hz to allow for the observation of many spinning sidebands. Pulse lengths for the $\pi/2$ and $\pi$ pulses were optimized to be $6 \mu$s and $10 \mu$s. Spectra were collected using a $20 \text{s}$ recycle delay and 256 scans.

$^{17}$O MAS and triple quantum magic angle spinning (3QMAS) NMR spectra were collected on both 9.4 T ($54.24 \text{ MHz}$ $^{17}$O frequency) and 16.4 T ($94.91 \text{ MHz}$ $^{17}$O frequency) Bruker Avance NMR spectrometers. All spectra were collected at a spinning speed of $(14000 \pm 5)$ Hz using 4 mm ZrO$_2$ rotors. MAS spectra at 16.4 T were collected using single pulses of $0.52 \mu$s duration ($10^\circ$ tip angle, $\nu_{tf} = 53 \text{ kHz}$) and optimized relaxation delays of $0.5 \text{s}$. MAS spectra at 9.4 T were collected using single pulses of $0.83 \mu$s duration ($12^\circ$ tip angle, $\nu_{tf} = 40 \text{ kHz}$). $^{17}$O MAS spectra generally required between 8192 and 16384 scans to yield spectra of sufficient quality.

3QMAS spectra were collected using a three-pulse sequence with full echo acquisition [240]. Split-$t_1$ spectra at 16.4 T were collected using optimized excitation and conversion pulses of $6.0 \mu$s and $1.5 \mu$s ($\nu_{tf} = 53 \text{ kHz}$) and a $22 \mu$s $180^\circ$ selective pulse ($\nu_{tf} = 22 \text{ kHz}$). The same parameters were used for crystalline and glassy samples. 24 slices in the F1 dimension were collected, with 1152 transients per slice. All $^{17}$O NMR spectra were referenced to tap water ($0 \text{ ppm}$). The 3QMAS spectra were referenced using the $C_z$ convention [134, 241].

Static $^{139}$La spectra were collected at 16.4 T ($98.91 \text{ MHz}$ $^{139}$La frequency) and 9.4 T ($56.52 \text{ MHz}$ $^{139}$La frequency) using the WCPMG pulse sequence [129]. The WCPMG sequence combines WURST [124] shaped pulses with the CPMG signal enhancement protocol [130, 131]. $50 \mu$s WURST-80 pulses were swept over $500 \text{ kHz}$ at a rate of $10 \text{ MHz ms}^{-1}$. Between 100 and 200 echoes were collected per transient, depending on the spin-spin relaxation time ($T_2$) of the sample. Due to the breadth of the peaks, spectra were collected piecewise by moving the transmitter in increments of $100 \text{ kHz}$, i.e., VOCS collection [127, 128]. Each subspectrum was collected by coadding 512 transients, with an optimized relaxation delay of $0.5 \text{s}$. $^{139}$La spectra were referenced to a $1 \text{ mol dm}^{-3}$ solution of LaCl$_3$ ($0 \text{ ppm}$).

$^{11}$B{$^{10}$B} REDOR spectra were collected using the conventional pulse sequence [140]. Spectra were collected using a 3.2 mm HXY probehead, with ZrO$_2$ rotors spinning at $(10000 \pm 5)$ Hz. REDOR experiments were conducted on a sample with natural
abundance oxygen. $^{11}$B $\pi/2$ and $\pi$ pulses were set to 3.10 $\mu$s and 6.25 $\mu$s and were optimized to provide a reasonable balance between optimum excitation for the BO$_3$ and [BO$_3$]$^-$ environments. $^{11}$B pulses were optimized using Hahn echo experiments. $^{10}$B $\pi$ pulses were optimized via maximizing the REDOR difference, resulting in a 12.5 $\mu$s pulse length. REDOR pulse powers were 80 kHz for $^{11}$B and 40 kHz for $^{10}$B. Recycle delays were optimized via spin-lattice relaxation ($T_1$) saturation recovery experiments, and were set to five times the longest $T_1$ observed in a given sample. These delays ranged from 50 s to 225 s. The number of scans collected varied depending on the sample, and ranged from 8 to 48.

Spectra of all nuclides studied were fitted using the int2quad module of the DMFIT 20150521 [184] software package. Fits were broadened using Lorentzian functions for crystalline environments, and Gaussian functions for amorphous environments. The glass components of $^{139}$La spectra were fitted using the Czjzek distribution [135, 136, 139], which is discussed in more detail below.

### 7.4.4 Density Functional Theory Calculations

First-principles DFT calculations were performed using the Quantum ESPRESSO code (https://www.quantum-espresso.org/), version 5.1.3 [162, 163]. In order to calculate NMR observables, the GIPAW method as implemented in version 5.1 of the QE-GIPAW package was used [156, 157]. Custom PAW datasets using the PBE exchange and correlation functional [192], based upon the JTH set [191], were constructed to avoid the issue of PAW sphere overlap [155]. PAW datasets were generated using atompaw [165]. An optimized plane-wave cutoff energy of 60 Rydberg was used with a shifted $6 \times 6 \times 6$ Monkhorst-Pack k-point grid. The latter corresponds to a k-point spacing of 0.025 Å$^{-1}$. Results from calculations on the experimental structure [78] are presented. For the EFG calculations, nuclear quadrupole moments were taken from Pyykkö [57]. To convert the chemical shielding values provided by GIPAW to experimentally-observable chemical shifts, calculated shielding values were compared to experimental shifts for known systems. Details of the conversion are reported in Appendix D.
7.4.5 Raman Spectroscopy

Unpolarized Raman spectra were collected using a Nicolet NXR 9600 Fourier-transform Raman spectrometer. A 1064 nm laser was used to excite the samples. The spectra were collected with a spectral resolution of 2 cm\(^{-1}\). The glass spectrum was collected by summing 256 scans at a power of 400 mW. The crystal spectrum was collected by summing 128 scans at a power of 70 mW. The glass spectrum is of a single piece of glass, while the crystal spectrum was collected from a powdered sample.

7.5 Results

7.5.1 X-Ray Diffraction

The X-ray diffraction results for LBG-G, LBG+12, and LBG+60 are shown in Fig. 7.2. The diffractogram for the glass sample, LBG-G, shows no sharp diffraction peaks, consistent with an amorphous material. The diffraction patterns of LBG+12 and LBG+60 are consistent with the diffraction pattern of LaBGeO\(_5\) as reported by Rulmont and Tarte [278]. Despite the substantial difference in the level of crystallization between these two samples (vide infra), the diffractograms appear similar. Both show diffraction peaks from a single crystalline phase, identified as LaBGeO\(_5\), though there is some slight variation in both peak intensity and position. Based on Rietveld refinement of the diffractograms (Appendix D.2), both the \(a\) and \(c\) lattice constants are somewhat lower in the partially crystallized samples than what is reported in the literature [78, 79, 278]. As amorphous materials are not effective in constructively scattering X-rays, it is expected that the crystallites will provide the majority of the pXRD signal regardless of the level of crystallization. The density of the LBG-G glass sample was found to be (4.89 ± 0.02) g cm\(^{-3}\), consistent with previous literature reports on this composition [96, 212]. However, one must be cautious when comparing physical properties of the LaBGeO\(_5\) glass with those in the literature, as Al\(_2\)O\(_3\) contamination of the melt from corundum crucibles is known to be an issue [98].

7.5.2 \(^{11}\)B NMR Spectroscopy

\(^{11}\)B MAS NMR spectra of LBG-G, LBG+12, and LBG+60 are shown in Fig. 7.3. The spectrum of the glass sample, LBG-G, has two peaks: one centred around
Figure 7.2: X-ray diffractograms of: glassy LBG-G, top; partially crystallized LBG+12, middle; and mostly crystallized LBG+60, bottom. Diffractograms are offset by 1000. The intensities of the LBG+12 and LBG+60 diffractograms are normalized such that the most intense peak has a value of 1000. The intensity of the LBG-G diffractogram is normalized to 100 for ease of comparison to the partially crystallized samples. The expected position and intensities of the reflections of crystalline LaBGeO$_5$ are marked with squares (PDF 41-0659) [278].
15 ppm, and a second peak centred around 1 ppm. The position, breadth, and relative intensity of these peaks are consistent with previous reported spectra from this composition [34, 212]. The peak centred around 15 ppm is consistent with the presence of BO$_3$ units, while the peak centred around 1 ppm is consistent with the presence of [BO$_4$]$^-$ units [51]. In order to identify the specific BO$_3$ species that is likely to be present, we compare the isotropic chemical shift of the observed peak to that of similar environments in crystalline samples. The isotropic chemical shift of the BO$_3$ peak is obtained by fitting the peak with a single quadrupolar environment, subject to Gaussian broadening. It should be noted that as $^{11}$B is a quadrupolar nucleus, the isotropic chemical shift of a given peak will be somewhat more positive than its centre of gravity. The parameters used to fit the glass are reported in Table 7.1.

The chemical shift of the BO$_3$ peak in the glass, $\delta_{iso}^{CS} = (18.3 \pm 0.2) \text{ ppm}$, is consistent with the chemical shift of the BO$_2$O$^-$ peak in crystalline LaB$_3$O$_6$, $\delta_{iso}^{CS} = (17.9 \pm 0.2) \text{ ppm}$ (Fig. D.5), as well with literature reports of chemical shifts of three-coordinate borate units [48]. Based upon 3QMAS spectra of lanthanum borogermanate glasses, we have previously identified this three-coordinate boron species as BO$_2$O$^-$ [212]. It is less straightforward to determine the precise environment of the
Table 7.1: $^{139}$La, $^{10}$B, $^{11}$B and $^{17}$O NMR parameters of environments in the glass. $^{139}$La and $^{11}$B values are from experiments on the LBG-G sample, and parentheses next to these values indicate experimental uncertainties. $^{17}$O values are either means taken from our experiments on the LaBGeO$_5$ crystal (Table 7.2), or means taken from the literature. For experimental data, experimental uncertainties are indicated. For literature data, standard deviations are indicated.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Environment</th>
<th>3QMAS Label</th>
<th>$\delta_{\text{iso}}^{\text{CS}}$ / ppm</th>
<th>$P_Q$ / MHz</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{139}$La</td>
<td></td>
<td></td>
<td>437 ± 30$^*$</td>
<td>4.2 ± 0.2**</td>
<td></td>
</tr>
<tr>
<td>$^{10}$B</td>
<td>BO$_4$</td>
<td></td>
<td>0.8 ± 0.8</td>
<td>1.0 ± 0.4</td>
<td></td>
</tr>
<tr>
<td>$^{11}$B</td>
<td>BO$_3$</td>
<td></td>
<td>18.3 ± 0.2</td>
<td>2.6 ± 0.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BO$_4$</td>
<td></td>
<td>2.0 ± 0.3</td>
<td>0.8 ± 0.4</td>
<td></td>
</tr>
<tr>
<td>$^{17}$O†</td>
<td></td>
<td>4$^<em>$Ge–O–4$^</em>$Ge</td>
<td>a</td>
<td>63 ± 12</td>
<td>6.7 ± 0.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4$^*$Ge–O–B</td>
<td>b</td>
<td>88 ± 5</td>
<td>5.2 ± 0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4B–O–4$^*$B</td>
<td>c</td>
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<td>6.1 ± 0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4$^*$Ge–O–$^{5/6}$Ge</td>
<td>d</td>
<td>124 ± 30</td>
<td>6.5 ± 0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>O$_{666}$ (rutile GeO$_2$)</td>
<td>e</td>
<td>156 ± 6</td>
<td>7.35 ± 0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4$^*$Ge–O–La</td>
<td>f</td>
<td>189 ± 4</td>
<td>5.2 ± 0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B–O–La</td>
<td>g</td>
<td>220 †</td>
<td>3 ± 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>O$_{666}$ (Na$_4$Ge$<em>9$O$</em>{20}$)</td>
<td>h</td>
<td>246.5 ± 0.5</td>
<td>3.75 ± 0.05</td>
</tr>
</tbody>
</table>

$^*$ This value is the position parameter of the peak, and does not correspond directly to $\delta_{\text{iso}}^{\text{CS}}$ for $^{139}$La. It is expressed in ppm and corresponds to $\Delta_{\text{offset}}$ in Chapter 5, Table 5.2.

$^{**}$ This value is the Czjzek distribution parameter $\sigma$, not $P_Q$. Both are expressed in MHz. See also Chapter 5, Table 5.2.

† $^{17}$O values for environments not observed in the LaBGeO$_5$ crystal are averages from literature reports. The uncertainties reported are an estimate of the precision of these values, and not an estimate of the range, which can vary.

‡ Average of the calculated $\delta_{\text{iso}}^{\text{CS}}$ values of the two B–O–La environments in LaBO$_3$. 
[BO$_4$]$^-$ units. The chemical shift range of the [BO$_4$]$^-$ environment only exhibits small changes in response to changes in composition, typically on the order of 2 ppm to 3 ppm [280]. Furthermore, structural disorder in the glass results in broadening of the NMR peak, which makes observing small changes in chemical shift difficult. However there is a significant change in the position of the [BO$_4$]$^-$ peak between the glass and crystal spectra. The chemical shift of the [BO$_4$]$^-$ peak in the glass is more positive than that in the crystal, with a difference of 0.7 ppm. Our REDOR results, discussed below, provide additional insight into the changes in the [BO$_4$]$^-$ environment.

The spectrum of the crystalline sample, LBG+60, shows a dominant peak in the region attributed to [BO$_4$]$^-$ units, consistent with the crystal structure of LaBGeO$_5$ [78]. There is residual BO$_3$ and [BO$_4$]$^-$ intensity due to the glass, but the change in the symmetry and breadth of the [BO$_4$]$^-$ peak allows the parameters of the crystalline [BO$_4$]$^-$ peak to be determined. The NMR parameters of this peak are reported in Table 7.2. The chemical shift of the [BO$_4$]$^-$ peak in the glass is slightly more positive than that in the crystal, with a difference of 0.7 ppm.

The spectrum of a sample with intermediate crystallization, LBG+12, is a combination of the spectra of LBG-G and LBG+60. The fitted $^{11}$B MAS NMR spectrum of LBG+12 is shown in Fig. 7.4. All partially crystallized samples can be fitted with three peaks: two from LBG-G and one from LBG+60. Furthermore, the ratio of BO$_3$/[BO$_4$]$^-$ in the glass, $\alpha$, does not appear to change due to crystallization. The crystalline fraction of the partially crystallized samples can be determined by subtracting the spectrum of the glass, LBG-G, and integrating the residual signal, or by fitting the crystalline and glass contributions of the [BO$_4$]$^-$ peak directly. Other than the relative intensity of the glass component and the crystal component, no fitting parameters are required to be changed. This suggests that there are no changes to the structure of the bulk glass during crystallization, and that $\alpha$ can be considered to be constant for a given sample. The level of crystallization of the samples is reported in Table 7.3.

7.5.3 $^{139}$La NMR Spectroscopy

$^{139}$La WCPMG NMR spectra of LBG-G, LBG+12, and LBG+60 are shown in Fig. 7.5. The spectrum of the most crystalline sample, LBG+60, is consistent with our previously
Table 7.2: Parameters used to fit experimental data, and parameters from DFT calculations, for $^{139}\text{La}$, $^{11}\text{B}$, and $^{17}\text{O}$ NMR of crystalline LaBGeO$_5$. Chemical shift anisotropy parameters are not reported, but generally are expected to be a minor contribution to the lineshape of the peak. For MAS and WCPMG spectra uncertainties derived from the fit are provided; for 3QMAS spectra, fundamental uncertainty due to the resolution of the experiment are provided.

<table>
<thead>
<tr>
<th>Site</th>
<th>MAS or WCPMG</th>
<th>3QMAS</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta_{\text{iso}}^{\text{CS}^*}$/ ppm</td>
<td>$C_Q$/ MHz</td>
<td>$\eta_Q$</td>
</tr>
<tr>
<td>La$^\dagger$</td>
<td>210 $\pm$ 25</td>
<td>85.5 $\pm$ 0.5</td>
<td>0.3 $\pm$ 0.2</td>
</tr>
<tr>
<td>B</td>
<td>1.3 $\pm$ 0.2</td>
<td>0.8 $\pm$ 0.4</td>
<td>0.8 $\pm$ 0.2</td>
</tr>
<tr>
<td>Ge$^\dagger$</td>
<td>41.3</td>
<td>0.84</td>
<td>45.8</td>
</tr>
<tr>
<td>O1</td>
<td>85 $\pm$ 2</td>
<td>5.0 $\pm$ 0.2</td>
<td>0.50 $\pm$ 0.04</td>
</tr>
<tr>
<td>O2</td>
<td>92 $\pm$ 2</td>
<td>5.0 $\pm$ 0.2</td>
<td>0.54 $\pm$ 0.05</td>
</tr>
<tr>
<td>O3</td>
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<td>5.2 $\pm$ 0.1</td>
<td>0.22 $\pm$ 0.03</td>
</tr>
<tr>
<td>O4</td>
<td>192.5 $\pm$ 0.5</td>
<td>5.1 $\pm$ 0.1</td>
<td>0.20 $\pm$ 0.03</td>
</tr>
<tr>
<td>O5</td>
<td>95 $\pm$ 2</td>
<td>5.6 $\pm$ 0.2</td>
<td>0.75 $\pm$ 0.06</td>
</tr>
</tbody>
</table>

* Experimental $^{139}\text{La}$, $^{11}\text{B}$, and $^{17}\text{O}$ chemical shifts are referenced to 1 mol dm$^{-3}$ LaCl$_3$ (0 ppm), NaBH$_4$ (−421 ppm), and tap water (0 ppm), respectively. Details on the referencing of the chemical shifts obtained from DFT calculations are provided in Appendix D.

† $^{139}\text{La}$ NMR parameters are consistent with those reported in the literature [126].

‡ $^{73}\text{Ge}$ NMR experiments were not carried out. Calculated values are included for completeness.
Figure 7.4: $^{11}$B MAS NMR spectrum of partially crystallized LBG+12. The parameters for the crystal fit are found in Table 7.2, while the parameters for the glass fit are found in Table 7.1.

Figure 7.5: $^{139}$La WCPMG NMR spectra of: glassy LBG-G, top; partially crystallized LBG+12, middle; and mostly crystallized LBG+60, bottom. As the spectra were collected using WCPMG, the signal intensity is concentrated into spikelets. The intensity envelope of the spikelets recreates the powder pattern of the static NMR spectrum.
Table 7.3: Fraction of the sample composed of LaBGeO$_5$ crystallites as determined by various means. Integrated BO$_3$ and [BO$_4$]$^-$ intensities are reported, as well as intensities determined by peak fitting. Values of $c$ as determined by fitting $^{11}$B MAS NMR and integrating $^{17}$O MAS NMR data are compared to calculated values.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Time / min</th>
<th>Integration</th>
<th>Fit</th>
<th>$^{11}$B Fit</th>
<th>$^{17}$O Integration</th>
<th>Gupta et al. [31]</th>
<th>Eq. (7.7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LBG-G</td>
<td>0</td>
<td>0.65 ± 0.03</td>
<td>0.35 ± 0.03</td>
<td>0.63 ± 0.01</td>
<td>0.37 ± 0.01</td>
<td>0</td>
<td>0.0 ± 0.3</td>
</tr>
<tr>
<td>LBG+8</td>
<td>80 ± 0.5</td>
<td>0.55 ± 0.03</td>
<td>0.45 ± 0.03</td>
<td>0.54 ± 0.01</td>
<td>0.46 ± 0.01</td>
<td>0.15 ± 0.01</td>
<td>0.13 ± 0.05</td>
</tr>
<tr>
<td>LBG+12</td>
<td>12 ± 1</td>
<td>0.46 ± 0.03</td>
<td>0.54 ± 0.03</td>
<td>0.44 ± 0.01</td>
<td>0.56 ± 0.01</td>
<td>0.29 ± 0.01</td>
<td>0.26 ± 0.05</td>
</tr>
<tr>
<td>LBG+24</td>
<td>24 ± 2</td>
<td>0.34 ± 0.03</td>
<td>0.66 ± 0.03</td>
<td>0.33 ± 0.01</td>
<td>0.67 ± 0.01</td>
<td>0.48 ± 0.01</td>
<td>0.55 ± 0.05</td>
</tr>
<tr>
<td>LBG+60</td>
<td>60 ± 3</td>
<td>0.18 ± 0.03</td>
<td>0.82 ± 0.03</td>
<td>0.17 ± 0.01</td>
<td>0.83 ± 0.01</td>
<td>0.72 ± 0.01</td>
<td>0.78 ± 0.05</td>
</tr>
</tbody>
</table>


reported spectrum of the LaBGeO$_5$ crystal [126] (Chapter 4). However, the spectrum shows poor definition of the quadrupolar features, indicating that the crystallites have limited long-range order. Furthermore, there is a substantially narrower impurity peak present in the spectrum of LBG+60, LBG+12, and all other partially crystallized samples. This peak is consistent with that of LaBO$_3$ [126]. Relative intensities are not quantitative in WCPMG spectra, as the signal enhancement due to the CPMG loops depends on the $T_2$ of each different environment. However, the signal due to LaBO$_3$ is consistently quite small, and no indication of LaBO$_3$ is seen in either the pXRD diffractograms (Fig. 7.2) or $^{11}$B MAS NMR spectra (Fig. 7.3). As such, we conclude that the amount of LaBO$_3$ present in these samples is very small (≤ 3%). This is consistent with our Rietveld refinements of or pXRD data (Appendix D.2) and will generally neglect it as a factor in our analysis. The $^{139}$La WCPMG spectrum of the glass sample, LBG-G, is fit with a Czjzek distribution [135, 136, 139]. A fit of the $^{139}$La WCPMG spectrum of LBG+12, processed such that the CPMG spikelets form a continuous lineshape, is presented in Fig. 7.6. It is important to note that the breadth parameter of the Czjzek distribution, $\sigma$, is not directly comparable to the well-defined $C_Q$, $\eta_Q$, or $P_Q$ of crystalline materials. In binary lanthanum phosphate glasses, $\sigma$ has been shown to be sensitive to changes in the $Q^n$ species making up the second coordination sphere of the La$^{3+}$ ions (Section 5.6). An increase in the diversity of the $Q^n$ species correlates to an increase in $\sigma$. If there were significant changes to the second coordination sphere of the LaBGeO$_5$ glass during crystallization, we would expect to observe changes in either $\sigma$ or the position parameter. No change in the $^{139}$La peak attributed to the glass is observed during crystallization, and so we conclude that the bulk glass does not undergo significant changes during the crystallization process.

7.5.4 $^{11}$B{$^{10}$B} REDOR NMR Spectroscopy

As has been previously reported, BO$_3$ environments cannot be effectively excited in $^{10}$B NMR spectroscopy [52]. The nuclear spin of $^{10}$B is integer ($I = 3$). Hence there is no “central transition” which is unaffected by the first-order quadrupolar broadening. Observing the spectra of integer-spin nuclei is similar to observing the satellite transitions of non-integer-spin nuclei [47]. For environments with small quadrupolar coupling constants (e.g., [BO$_4$])$^-$, this is not particularly challenging.
Figure 7.6: $^{139}$La WCPMG NMR spectrum of partially crystallized LBG+12. The experimental data is identical to that shown in Fig. 7.5, but by summing the CPMG echoes the spectrum becomes a continuous lineshape. Fitting parameters for the crystal LaBGeO$_5$ peak are reported in Table 7.2, and for the glass in Table 7.1. The spectral parameters for both LaBGeO$_5$ and LaBO$_3$ are previously reported in the literature [126].
However, for environments with large quadrupolar coupling constants (e.g., BO₃), it is quite difficult to produce appreciable excitation using a single pulse. A single pulse, centred on the [BØ₄]⁻ resonance, will effectively excite the [BØ₄]⁻ peak without exciting the BO₃ peak. Hence $^{11}$B{$^{10}$B} REDOR can be used to selectively probe [BØ₄]⁻-[BØ₄]⁻ and BO₃-[BØ₄]⁻ connectivity in crystals and glasses.

We present $^{11}$B{$^{10}$B} REDOR curves of LBG-G, crystalline LaBGeO₅ (LBG-X), and LaB₃O₆ in Fig. 7.7. The REDOR curves of the [BØ₄]⁻-[BØ₄]⁻ environment of crystalline LBG-X and the BO₃-[BØ₄]⁻ environment of LaB₃O₆ are used to interpret the behaviour of the similar environments in LaBGeO₅ glass.

Typical analysis of REDOR spectra of amorphous samples involves fitting $\Delta S/S_0$ in the region $0 \leq \Delta S/S_0 \leq 0.2$ to extract the value of the van Vleck second moment [41, 52, 142–144]. This analysis relies on the indirect nucleus (in this case $^{10}$B) being consistently excited between samples, and on the low-$\Delta S/S_0$ region of the REDOR curve following the universal REDOR lineshape [41, 52]. When we attempt to fit the curves in Fig. 7.7 to extract the van Vleck second moment, we find that the extracted value depends very strongly on the choice of points to fit, and is not reliably transferable. Our curves are effectively linear at both long and short time scales, rather than approximating the “universal” lineshape [41]. This may be due to the $^{11}$B{$^{10}$B} spin pair being $I = 3/2$ and $I = 3$, respectively. REDOR of quadrupolar spin pairs is inherently less quantitative than REDOR where one spin is $I = 1/2$, due to factors including the quadrupolar splittings of the indirect nuclei [41]. Beyond the quadrupolar nature of both nuclei, as discussed above $^{10}$B is of integer spin, and lacks a narrow central transition. This raises concerns about the consistency and homogeneity of the excitation of the [BØ₄]⁻ environments, as well as the ability to approximate the behaviour of a spin-1/2 nuclide. Hence we do not attempt a quantitative analysis of the REDOR curves, and rely on a more qualitative analysis.

The rate of change of a REDOR curve is indicative of the strength of the dipolar interaction between the observed and indirect nuclide. For an ensemble of spins such as a glass, this strength is affected both by the distance between the nuclides and by the number of nuclides in proximity. Hence a decrease in the rate of change of the REDOR curve can be attributed to a decrease in the dipolar interaction, which in turn is attributed to a decrease in the proximity of the indirect and the observe nuclides.
Figure 7.7: $^{11}\text{B}^{10}\text{B}$ REDOR NMR curves for crystalline LaB$_3$O$_6$, crystalline LaBGeO$_5$ and LaBGeO$_5$ glass. Filled data points indicate data from crystals. Open data points indicate data from the glass. Diamonds indicate [BO$_4$]$^-$–[BO$_4$]$^-$ environments. Triangles indicate BO$_3$–[BO$_4$]$^-$ environments. a) The REDOR curves at long time scales. b) The REDOR curves at short time scales.
For this interpretation to be reliable between different systems, the proportion of excited spins must be similar. This is not difficult to achieve for $^{11}$B at 16.4 T, but it is not obviously true for $^{10}$B at the same field strength. Our $^{10}$B Hahn echo NMR spectra of our samples (Figs. D.6 to D.8) are of similar breadth, and are fitted with similar values of $C_Q$ (reported in Table D.3). We conclude that the $^{10}$B $[\text{BO}_4]^{-}$ environment is consistently excited across our three samples, and hence can meaningfully interpret qualitative differences in intensity between the REDOR curves.

The REDOR curves of the $\text{BO}_3$–$[\text{BO}_4]^{-}$ and $[\text{BO}_4]^{-}$–$[\text{BO}_4]^{-}$ environments in the LBG-G sample are significant and persistent over long length scales (50 ms), but of lower magnitude than the curves for the equivalent environments in the LBG-X and LaB$_3$O$_6$ samples. From this we conclude that the $[\text{BO}_4]^{-}$ environment in LaBGeO$_5$ has both $\text{BO}_3$ and $[\text{BO}_4]^{-}$ neighbours. We also conclude that due to the reduced magnitude of the $\Delta S/S_0$ curve for the $[\text{BO}_4]^{-}$ environment in LBG-G, the $[\text{BO}_4]^{-}$–$[\text{BO}_4]^{-}$ environment is less likely to occur in LBG-G than in LBG-X. In LBG-X, the single crystallographic $[\text{BO}_4]^{-}$ site has two $[\text{BO}_4]^{-}$ neighbours; hence in the glass we expect that $[\text{BO}_4]^{-}$ has, on average, significantly fewer than two $[\text{BO}_4]^{-}$ neighbours. Similarly, in the LaB$_3$O$_6$ crystal structure, both crystallographic $\text{BO}_3$ sites have two $[\text{BO}_4]^{-}$ neighbours, and so we conclude that due to the reduced magnitude of the $\text{BO}_3$–$[\text{BO}_4]^{-}$ curve in LBG-G that, on average, $\text{BO}_3$ has significantly fewer than two $[\text{BO}_4]^{-}$ neighbours. With the relative magnitudes of the $\text{BO}_3$–$[\text{BO}_4]^{-}$ and $[\text{BO}_4]^{-}$–$[\text{BO}_4]^{-}$ $\Delta S/S_0$ curves in the LBG-G glass being similar at all time scales, it seems likely that the $\text{BO}_3$ and $[\text{BO}_4]^{-}$ environments have similar numbers of $[\text{BO}_4]^{-}$ neighbours; however, given the complicated behaviour of REDOR curves at long time scales, we cannot be certain of this interpretation.

### 7.5.5 $^{17}$O NMR Spectroscopy

$^{17}$O MAS NMR spectra of LBG-G, LBG+12, and LBG+60 collected at 16.4 T are shown in Fig. 7.8. The $^{17}$O MAS spectrum of the most crystalline sample, LBG+60, shows a small amount of residual glass intensity. The $^{17}$O MAS spectrum of the LBG-G glass sample can be subtracted from any of the spectra of the partially crystallized samples, with the glass spectrum scaled to match the intensity of the shoulder at approximately 250 ppm. When subtracting the $^{17}$O MAS spectrum of the LBG-G
Figure 7.8: $^{17}$O MAS NMR spectra of: glassy LBG-G, top; partially crystallized LBG+12, middle; and mostly crystallized LBG+60, bottom.

glass from any of the spectra of the partially crystallized sample, the same difference plot is recovered in each case. This difference plot agrees very well with the spectrum of the most crystallized sample, LBG+60. This is also true for the $^{11}$B MAS NMR spectra. That the portion of the spectrum attributed to LaBGeO$_5$ crystallites shows no change with level of crystallization in both $^{17}$O and $^{11}$B MAS NMR spectra indicates that neither the bulk glass structure nor the crystallite structure change during the crystallization process. Hence, in order to minimize error during the fitting of the $^{17}$O NMR spectrum of the LaBGeO$_5$ crystal, we subtract the spectrum of the glass sample from the spectrum of LBG+60 and fit the difference.

There are five crystallographically distinct oxygen sites in the LaBGeO$_5$ crystal structure [78]. O1 and O2 link GeO$_4$ and [BØ$_4$]$^-$ tetrahedra, and are considered BO, as they link two glass-forming elements (Ge and B). O1 and O2 are four-coordinate, singly bound to B and Ge, and loosely bound to two different La. O3 and O4 link GeO$_4$ tetrahedra with LaO$_9$ polyhedra, and are three-coordinate: singly bound to Ge, and singly bound to two different La. O3 and O4 are considered NBO, as they link a network former (Ge) with La, which often acts as a network modifier. O5 links two corner-sharing [BØ$_4$]$^-$ tetrahedra, is three-coordinate (one La and two different B
neighbours), and is considered a BO. The occupancy of all oxygen sites is one.

The $^{17}$O MAS spectrum of LBG+60 shows two major peaks with maxima at 72 ppm and 170 ppm. Integrating the two major peaks returns a 3:2 intensity ratio. Hence the peak centred at 72 ppm is attributed to the three BO sites, while the peak centred at 170 ppm is attributed to the two NBO sites. The two NBO peaks are not resolved from one another; similarly, all three of the BO peaks are strongly overlapping. Subtracting the spectrum of LBG-G from the $^{17}$O MAS spectra of the partially crystallized samples returns difference plots that are identical in all ways save their intensities. The fraction of intensity of each spectrum that is due to crystalline environments can be determined by integration of these difference plots, and these estimates are reported in Table 7.3. The crystallite fraction determined by $^{17}$O NMR independently agrees with that from $^{11}$B NMR (Table 7.3).

The 3QMAS spectrum of LBG+60 is presented in Fig. 7.9. As in the MAS spectrum, the NBO peak pair is resolved from the BO triplet. The 3QMAS spectrum has somewhat improved resolution with respect to the two NBO sites. The resolution of the three BO sites in the 3QMAS spectrum is poor. For poorly resolved 3QMAS peaks, the $P_Q$ and $\delta_{\text{iso}}^{CS}$ values of a given environment can be extracted via the procedure described by Millot and Man [134]. These values are limited in precision due to the limited resolution of the spectrum, but they do provide a reasonable estimate (and upper bound) on the $P_Q$ and $\delta_{\text{iso}}^{CS}$ of any particular site. While not wholly resolved, the extracted values of the two NBO sites support that the sites have similar $C_Q$, $\eta_Q$, and $\delta_{\text{iso}}^{CS}$ values. The extracted values of the three BO sites are not substantially different from one another. Resolution of the 3QMAS spectrum was limited by the fairly rapid decay of the 3QMAS echo.

To aid in the interpretation of the $^{17}$O NMR spectra of the LaBGeO$_5$ crystal, first principles calculations were carried out on the experimental structure [78]. The results of the calculations for the $^{17}$O NMR parameters (as well as for the $^{139}$La and $^{11}$B parameters) are presented in Table 7.2. DFT calculations returned EFG parameters in good accord with experiment. The absolute calculated isotropic chemical shifts do not agree with experiment, but the relative calculated shifts do agree with our observations. The relative shift of the NBO and BO calculated chemical shifts is correct (with $\delta_{\text{iso}}^{CS}(\text{NBO}) > \delta_{\text{iso}}^{CS}(\text{BO})$), but the calculated values of $\delta_{\text{iso}}^{CS}(\text{NBO})$ is at least
Figure 7.9: $^{17}$O 3QMAS NMR spectrum of mostly crystallized LBG+60. The diagonal is the correspondence between the MAS and isotropic dimensions, and indicates the position where peaks free from the quadrupole interaction would appear. Fitted peaks are overlaid on experimental data; the parameters of the fitted peaks are reported in Table 7.2. The top left and bottom right peaks are spinning sidebands. The projection of the isotropic dimension ($y$ axis) is processed to remove the contributions from the spinning sidebands.
20 ppm greater than any possible experimental value. Similarly, the calculated values of $\delta_{\text{iso}}^{\text{CS}}(\text{BO})$ are at least 10 ppm greater than any possible experimental value, and a 20 ppm difference would be very plausible. The calculated $\delta_{\text{iso}}^{\text{CS}}$ values for the BO sites are within 20 ppm of each other, with the two Ge–O–B sites having similar values. The relative separation of the two Ge–O–La sites is 6 ppm. Similarly, the relative separation of the two Ge–O–B sites is 5 ppm. This is lower than the resolution limit of our 3QMAS spectrum, and hence it is unsurprising that there is difficulty in resolving individual environments.

The results of our GIPAW calculations imply that we should expect similar values of $\delta_{\text{iso}}^{\text{CS}}$, $C_Q$, and $\eta_Q$ for a given pair of Ge–O–B or Ge–O–La sites. The calculated $C_Q$ and $\eta_Q$ values of the Ge–O–La sites are quite similar, as are those of the Ge–O–B sites. Both the $C_Q$ and $\eta_Q$ values of the B–O–B site are greater than of any other sites. The $\eta_Q$ values of the NBO sites are much lower than those of the BO sites. For all sites, the CSA contribution to the breadth of the peak is expected to be substantially less than the quadrupolar contribution, hence we neglect it in our fits.

With these trends, we can generate a reasonable set of fitting parameters that simultaneously fit the 16.4 T and 9.4 T MAS, Hahn echo, and 3QMAS spectra. The fit of the 16.4 T 3QMAS spectrum, and of the 16.4 T and 9.4 T MAS spectra, are shown in Figs. 7.9 and 7.10. The parameters for this fit are reported in Table 7.2. It bears mentioning that this fit is unlikely to be unique. Even with the assumption that the CSA contribution to the spectrum is negligible, there are still six independent parameters ($\delta_{\text{iso}}^{\text{CS}}$, $C_Q$, $\eta_Q$ for the two individual sites) fitting the NBO peak, and similarly nine independent parameters fitting the BO peak. However, with the significant number of constraints on the fitting process (similar integrated intensities of the peaks, simultaneously satisfying low- and high-field 1D and 2D experiments), we consider our reported parameters to be a reasonable estimate.

No similar fit can be constructed for the $^{17}$O MAS NMR spectrum of the LaBGeO$_5$ glass. There are simply too many possible environments, and insufficient resolution, to provide a quantitative estimate of any parameter. However, based upon the 3QMAS spectrum of the glass (Fig. 7.11) a qualitative model of the glass structure can be constructed. The presence of some structural elements are known to be present in the glass from other experiments (i.e., B–O–La); others are expected due to the
Figure 7.10: $^{17}$O MAS NMR spectra of the mostly crystallized LBG+60. The parameters of each peak are provided in Table 7.2. A small impurity is marked with †, and is consistent with the presence of LaBO$_3$. 

<table>
<thead>
<tr>
<th>Expt.</th>
<th>Total fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>O1 ($^4$Ge-O-$^4$B)</td>
<td></td>
</tr>
<tr>
<td>O2 ($^4$Ge-O-$^4$B)</td>
<td></td>
</tr>
<tr>
<td>O3 ($^4$Ge-O-La)</td>
<td></td>
</tr>
<tr>
<td>O4 ($^4$Ge-O-La)</td>
<td></td>
</tr>
<tr>
<td>O5 ($^4$B-O-$^4$B)</td>
<td></td>
</tr>
</tbody>
</table>

(a) Spectrum at 16.4 T.

(b) Spectrum at 9.4 T.
Figure 7.11: $^{17}$O 3QMAS NMR spectrum of the glass LBG+G. The diagonal is the correspondence between the MAS and isotropic dimensions, and indicates the position where peaks free from the quadrupole interaction would appear. The projection of the isotropic dimension (y axis) is processed to remove the contributions from spinning sidebands.
Figure 7.12: Raman spectra of the LaBGeO$_5$ crystal (solid line) and LaBGeO$_5$ glass (dotted line). Some of the Raman intensity in the glass corresponds to crystal modes, in particular the sharp mode at approx. 803 cm$^{-1}$. However, the glass also has substantial intensity which does not correspond to known LaBGeO$_5$ environments.

structure of the LaBGeO$_5$ crystal (i.e., Ge–O–La, Ge–O–B); yet others can be inferred based upon data in the literature. The resulting qualitative interpretation of the glass structure is discussed in detail below.

We do not observe any $^{17}$O intensity in the 450 ppm to 600 ppm region in any spectrum. Intensity in this region would indicate the presence of OLa$_4$ or OLa$_6$ environments as in crystalline La$_2$O$_3$ [281]. The presence of these environments could indicate partitioning of the glass into La-rich regions [282]. As we do not observe any such intensity, we conclude that La$_{3+}$ is regularly distributed in the glass.

7.5.6 Raman Spectroscopy

The Raman spectra of the LaBGeO$_5$ crystal and glass are presented in Fig. 7.12. The
spectrum of the LaBGeO$_5$ crystal is very similar to previous literature reports, and its peaks have been assigned to various vibrational modes [75, 95, 109, 278, 283]. The particular region of interest for this study is between 775 cm$^{-1}$ to 900 cm$^{-1}$, which has been assigned to both [BØ$_4$]$^-$ and GeO$_4$ tetrahedral vibrations. Notably, there is no significant Raman intensity in the crystal between 630 cm$^{-1}$ to 775 cm$^{-1}$. The Raman spectrum of the LaBGeO$_5$ glass has a broad, asymmetric, and featureless peak spanning 630 cm$^{-1}$ to 950 cm$^{-1}$. Like the spectrum of the crystal, this spectrum is consistent with literature reports [75, 109]. The large peak in the glass encompasses the intensity found in the crystal, but also has significant intensity centred at approximately 745 cm$^{-1}$; the crystal has no corresponding vibrational modes in this region.

### 7.6 Discussion

#### 7.6.1 Crystallization

The pXRD, $^{11}$B MAS NMR, $^{139}$La WCPMG NMR, and $^{17}$O MAS and 3QMAS results all support a common conclusion regarding the formation of LaBGeO$_5$ crystals from the glass: there are no substantial structural changes in either the glass or the crystal in the partially crystallized samples. In other words, the glass fraction of a partially crystallized LaBGeO$_5$ sample has the same structure as the parent glass, while the crystalline fraction is unmodified LaBGeO$_5$ crystal. The quantitative $^{11}$B and $^{17}$O MAS NMR results provide the same crystallization fraction (within error), with partially crystallized spectra being combinations of the crystal and glass spectra.

There are some noticeable differences in the pXRD and $^{139}$La WCPMG NMR results. Both become somewhat sharper as the crystallization fraction increases. This change is indicative of larger crystallites, but not of a substantial change in either the short-range order (NMR) or long-range order (pXRD) of the LaBGeO$_5$ structure. At the highest crystallization levels, a small amount of LaBO$_3$ is observed, first by $^{139}$La WCPMG NMR, then by pXRD at the highest level of crystallization (Appendix D.2). The LBG-G glass likely has a small excess of B$_2$O$_3$, allowing for non-stoichiometric crystallization in small quantities. However, LaBO$_3$ is only detected when the sample is already substantially crystallized. Finally, in the partially crystallized samples, the $a$ and $c$ lattice constants are somewhat lower than those reported in the literature.
Based upon the amount of strain determined by our Rietveld refinement (Table D.1), along with the elastic tensor of the LaBGeO$_5$ crystal (Table 8.2) [283], we conclude that the LaBGeO$_5$ crystallites are under some amount of compressive stress, on the order of about 4 GPa.

The data above provide insight into the end result of the nucleation and growth of the LaBGeO$_5$ crystallites, but cannot effectively probe the dynamics of the nucleation mechanism. The most interesting structural evolution during nucleation and growth occurs at the interface between the glass and the nucleus. By definition the nuclei are extremely small, with radii on the order of nanometres. The fraction of the total sample volume made up by the interface between nuclei and glass is extremely small, and hence effectively undetectable via either NMR or pXRD. While we cannot directly probe the changes in structure during nucleation, knowledge of the structure of the resulting glass-ceramic has practical significance.

The ability to estimate the degree of crystallization of a LaBGeO$_5$ glass-ceramic composite via NMR spectroscopy is desirable, as estimating volume fraction via techniques such as optical microscopy may prove difficult if the crystallites are sufficiently small such that the material is transparent. Gupta et al. have previously reported a model which, through the use of $^{11}$B MAS NMR spectroscopy, allows for an estimate of the crystallite fraction [34]. However, when we apply their model to our $^{11}$B MAS NMR results, the model disagrees with our direct modelling of the [BØ$_4$]$^-$ peak, as well as with the crystallization fraction estimated by our $^{17}$O MAS results. A small correction to the model of Gupta et al., when applied to our presented samples, leads to values which agree very well with our experimental results. The details of the correction follow.

We use the same notation as Gupta et al., where square brackets indicate the relative intensity of the species within. In the LaBGeO$_5$ crystal, [BO$_3$] = 0 and [BO$_4$] = 1. In the glass, both species will be present, though the precise ratio will depend on the exact composition (e.g., an excess of La$_2$O$_3$ will increase BO$_3$ at the expense of [BØ$_4$]$^-$) [212]. In a partially crystallized sample,

$$[\text{BO}_3]_{\text{expt}} + [\text{BO}_4]_{\text{expt}} = 1$$  \hspace{1cm} (7.1)
and
\[ [\text{BO}_4]\text{expt} = [\text{BO}_4]\text{cryst} + [\text{BO}_4]\text{glass}. \] (7.2)

As the crystal contains no BO\(_3\) units,
\[ [\text{BO}_3]\text{expt} = [\text{BO}_3]\text{glass}. \] (7.3)

We assume that the ratio of BO\(_3/[\text{BO}_4]\) in the glass, \(\alpha = \frac{[\text{BO}_3]\text{glass}}{[\text{BO}_4]\text{glass}}\), does not change as crystallization progresses. This assumption is well-founded based on our \(^{11}\text{B}, ^{139}\text{La}, \text{and} ^{17}\text{O} \) NMR data, none of which show any change in the structure of the LaBGeO\(_5\) bulk glass during crystallization (Section 7.5). With the assumption that \(\alpha\) is constant, the fraction of the four-coordinate boron intensity attributed to the crystallites can be calculated knowing only the fraction of the overall intensity attributed to the three-coordinate site. The fraction of the crystallites in the partially crystallized sample, \(c\), is given by
\[ c = \frac{[\text{BO}_4]\text{cryst}}{[\text{BO}_4]\text{cryst} + [\text{BO}_4]\text{glass} + [\text{BO}_3]\text{glass}}. \] (7.4)

With Eq. (7.2), this can be rewritten to remove the (potentially) inaccessible \([\text{BO}_4]\text{cryst}\) term:
\[ c = \frac{[\text{BO}_4]\text{expt} - [\text{BO}_4]\text{glass}}{[\text{BO}_4]\text{expt} - [\text{BO}_4]\text{glass} + [\text{BO}_4]\text{glass} + [\text{BO}_3]\text{glass}}. \] (7.5)

As \([\text{BO}_3]\text{glass}\) and \([\text{BO}_4]\text{glass}\) are related by \(\alpha\), Eq. (7.5) can be written as
\[ c = \frac{[\text{BO}_4]\text{expt} - [\text{BO}_3]\text{glass}}{[\text{BO}_4]\text{expt} + [\text{BO}_3]\text{glass}}. \] (7.6)

With Eqs. (7.1) and (7.3), Eq. (7.6) is reduced to
\[ c = [\text{BO}_4]\text{expt} - \frac{[\text{BO}_3]\text{expt}}{\alpha}. \] (7.7)

Equation (7.7) is used to generate the crystallite fraction reported in Table 7.3. It
Figure 7.13: A comparison of the crystal fraction, $c$, of the partially crystallized samples, as determined from both $^{11}$B and $^{17}$O MAS NMR data, and the crystal fraction predicted by model calculations. Solid points represent experimental data, while open points represent calculated values. Open circles are values calculated using the equation of Gupta et al. [34], while open squares are calculated using Eq. (7.7).

agrees very well with the fraction determined by fitting the $[\text{BO}_4]^{-}$ peak directly, and by integration of $^{17}$O MAS spectra. The only difference between our derivation and the derivation of Gupta et al. is the introduction of the $[\text{BO}_3]_{\text{glass}}$ term in Eq. (7.4). However, this term is required to accurately account for the total intensity of the glass, and prevent an overestimation of the crystal fraction. For comparison, our experimental results, the results of our equation, and the results from the equation of Gupta et al. are plotted in Fig. 7.13.

The utility of Eq. (7.7) is primarily in the (relatively) accessible data it requires. Fitting the $[\text{BO}_4]^{-}$ peak directly may not be possible except at high magnetic field strengths and spinning speeds, while the $^{17}$O enrichment required for $^{17}$O MAS NMR
is prohibitively expensive for routine studies. The ratio of BO$_3$ to [BØ$_4$]$^-$ can be quickly determined through $^{11}$B MAS NMR at moderate field strengths and spinning speeds, or potentially by Raman spectroscopy, allowing for routine characterization of the crystal fraction of LaBGeO$_5$ TFN materials.

### 7.6.2 Glass Structure

The structure of the LaBGeO$_5$ crystal is well-understood. Three structural units are present: [BØ$_4$]$^-$ tetrahedra, GeØ$_2$O$_2$$^{2-}$ tetrahedra, and LaO$_9$ polyhedra. From this, three oxygen bonding motifs are apparent: the two bridging oxygen B–O–B and Ge–O–B; and the non-bridging oxygen Ge–O–La. These three motifs are observed in the $^{17}$O MAS and 3QMAS spectra of the LaBGeO$_5$ crystal. The $^{17}$O 3QMAS spectrum of the LBG-G glass shares some similarities to that of the LBG+60 crystal, but also has substantial differences. In particular, the 3QMAS intensity in the glass has spread to more positive chemical shift values. By comparison to the crystal, some of the glass intensity can be assigned to the Ge–O–B environment ($\delta_{\text{iso}}^{\text{CS}} \approx 88$ ppm), the B–O–B environment ($\delta_{\text{iso}}^{\text{CS}} \approx 95$ ppm), and the Ge–O–La environment ($\delta_{\text{iso}}^{\text{CS}} \approx 189$ ppm). However, this only accounts for a small portion of the intensity displayed.

Beyond homonuclear (e.g., Ge–O–Ge) and heteronuclear (e.g., Ge–O–B) environments, we must also consider potentially observable differences in coordination number. For example, in binary sodium germanate crystals Du and Stebbins have observed differences in the $^{17}$O parameters between $^4$Ge–O–$^4$Ge and $^4$Ge–O–$^6$Ge oxygen sites [55], where the leading superscript indicates coordination. They also observe environments in binary sodium germanate glasses which are consistent with the $^4$Ge–O–$^6$Ge seen in similar crystals [55, 56]. Intensity in this region has been attributed to five- and/or six-coordinate germanium in various sodium germanate glasses [55]. Six-coordinate germanium in glass cannot be distinguished from a possible five-coordinate germanium environment by either spectroscopic or diffraction-based techniques, and hence both five- and six-coordinate germanium must be considered [28, 56, 73]. The notation $^{5/6}$Ge indicates either or both environments.

In addition to observing $^4$Ge–O–$^{5/6}$Ge environments in sodium germanate glasses, Du and Stebbins have observed differences between the $^3$B–O–$^3$B and $^3$B–O–$^4$B environments, as well as the $^3$B–O–Si and $^4$B–O–Si environments, in sodium borosilicate
glasses [51]. Lee et al. have reported $^{17}$O 3QMAS spectra of binary borogermanate glasses, establishing the relative positions of $^{4}$Ge−O−$^{4}$Ge, $^{4}$Ge−O−$^{3}$B, and $^{3}$B−O−$^{3}$B environments [53]. Oxygen “triclusters” (i.e., oxygen triply bonded to network forming cations such as Ge$^{4+}$, labeled O$_{666}$) are known to exist in the rutile crystal phase of GeO$_2$. With the above literature data, we can establish several trends which we will use to inform our analysis of the LBG-G $^{17}$O 3QMAS spectra. In binary germanate glasses, the BO $^{17}$O chemical shift increases as the neighbouring germanate coordination increases (i.e., $\delta_{\text{iso}}^{\text{CS}}(^{4}\text{Ge−O−}^{4}\text{Ge}) < \delta_{\text{iso}}^{\text{CS}}(^{4}\text{Ge−O−}^{5/6}\text{Ge}) < \delta_{\text{iso}}^{\text{CS}}(\text{O}_{666}))$. The opposite is true with borates, with the BO $^{17}$O chemical shift increasing as the neighbouring borate coordination decreases (i.e., $\delta_{\text{iso}}^{\text{CS}}(^{4}\text{B−O−}^{4}\text{B}) < \delta_{\text{iso}}^{\text{CS}}(^{3}\text{B−O−}^{4}\text{B}) < \delta_{\text{iso}}^{\text{CS}}(^{3}\text{B−O−}^{3}\text{B}))$. The $^{17}$O chemical shift of borates is generally greater than that of germanates for both BO and NBO.

With known oxygen environments identified by the $^{17}$O spectra of the LaBGeO$_5$ crystal, we can identify the residual intensity present in the glass. Our identification of the species present in the LaBGeO$_5$ glass is shown in Fig. 7.14. Parameters used in generating this figure are reported in Table 7.1.

The $^{4}$Ge−O−B site from the crystal lies very close to the top-right edge of the glass spectrum. If $^{4}$Ge−O−$^{4}$Ge was present in significant quantity in the glass, we would expect more intensity in this region. We therefore conclude that $^{4}$Ge−O−$^{4}$Ge links are not a substantial contributor to the glass network.

The resolution of our 3QMAS spectrum is insufficient to separate the subtle differences between the differently coordinated borate units, which are expected to vary by less than 10 ppm [51]. Given that we know that BO$_3$ units are present in the glass, we expect Ge−O−B and B−O−B peaks to broaden towards greater $\delta_{\text{iso}}^{\text{CS}}$ values. The intensity region between 100 ppm to 120 ppm can be attributed in part to $^{4}$B−O−$^{3}$B links, which we expect to be present from our REDOR results. It is also possible for $^{3}$B−O−$^{3}$B environments to be present; as our $^{11}$B{$^{10}$B} REDOR experiment only probes species with connectivity to [BØ$_4$]$^-$, it cannot detect such an environment. Hence our $^{11}$B MAS spectra, and our $^{11}$B{$^{10}$B} REDOR experiments, are consistent with the $^{17}$O 3QMAS spectrum, but are insufficient to explain the intensity present between 120 ppm to 150 ppm.

The species reported in the literature which can best explain the intensity from
Figure 7.14: Labelled $^{17}$O 3QMAS NMR spectrum of the glass LBG-G. Horizontal lines indicate mean expected positions, as well as approximate expected widths in the MAS dimension, of oxygen environments which are expected to be present in LaBGeO$_5$. Parameters for each environment are discussed in Table 7.1. Solid lines are environments present in the LaBGeO$_5$ crystal. Double lines are environments which are plausibly or possibly present based upon data from the literature. Dotted lines are environments which are unlikely to be present, again based upon data from the literature. The environments shown are: a) typical $^4$Ge–O–$^4$Ge, from literature [54, 55]; b) $^4$Ge–O–B, from Table 7.2; c) $^4$B–O–$^4$B, from Table 7.2; d) $^4$Ge–O–$^{5/6}$Ge, from literature [54, 55]; e) O$_{666}$ as in rutile GeO$_2$, from literature [54, 55]; f) $^4$Ge–O–La, from Table 7.2; g) expected B–O–La environment; h) O$_{666}$ as in Na$_4$Ge$_9$O$_{20}$, from literature [55].
120 ppm to 150 ppm is $^{5/6}$Ge–O–X, where X is another glass-forming species. The $\delta_{\text{iso}}^\text{CS}$ values of $^{5/6}$Ge–O–$^4$Ge in various sodium germanate crystals fall between 80 ppm to 150 ppm [55]. Intensity in this region has been attributed to five- and/or six-coordinate germanium in various sodium germanate glasses [55]. $^{5/6}$Ge–O–X environments, where X is either Ge or Si, have been observed in sodium germanosilicate glasses using $^{17}$O NMR spectroscopy [284]. While $^{5/6}$Ge–O–B environments have not been reported in the literature (in part due to a dearth of structural studies on borogermanates), it is plausible that they would appear in a similar chemical shift range to $^{5/6}$Ge–O–$^4$Ge. As the $^{17}$O $\delta_{\text{iso}}^\text{CS}$ of borates is generally more positive than that of germanates, it is also possible that $\delta_{\text{iso}}^\text{CS}(^{5/6}$Ge–O–$^4$Ge) < $\delta_{\text{iso}}^\text{CS}(^{5/6}$Ge–O–B). This is consistent with the intensity present between 120 ppm to 150 ppm.

There is a substantial presence of non-bridging oxygen. The peak centred at 199 ppm in the 16.4 T MAS spectrum can be partially, but not wholly, explained by a Ge–O–La NBO peak similar to that seen in the spectrum of the crystal. Assuming that the Ge–O–La NBO peak in the glass can be described by a peak near the same position as the similar peak in the crystal, an additional peak centred at approximately 215 ppm can describe the increased intensity. This is consistent with the position of the impurity peak present in the $^{17}$O MAS spectra of LBG+60 which is attributed to LaBO$_3$. Additionally, this chemical shift is consistent with the B–O–La environments predicted by DFT calculations on LaBO$_3$ and LaB$_3$O$_6$, and is consistent with the general trend of borate $^{17}$O shifts being more positive than germanate shifts. The presence of the B–O–La environment is expected due to the $^{11}$B NMR results discussed above.

There is an additional environment which must be considered: the oxygen tricluster. In the rutile phase of GeO$_2$, this environment is located at $\delta_{\text{iso}}^\text{CS} = (152.2 \pm 0.5)$ ppm, with a $P_Q$ of $(7.36 \pm 0.02)$ MHz [55]. While there is intensity in this region of the 3QMAS spectrum of LBG-G glass, the mean $P_Q$ of this region is far lower than what would be required for rutile-like O$_{666}$. It is unlikely that oxygen triclusters similar to those in rutile GeO$_2$ exist in the glass. However, an oxygen tricluster geometry has also been observed in Na$_4$Ge$_9$O$_{20}$. In Na$_4$Ge$_9$O$_{20}$, the tricluster is located at significantly more positive chemical shift ($\delta_{\text{iso}}^\text{CS} = (246.5 \pm 0.5)$ ppm) and with a much lower $P_Q$ ($(3.75 \pm 0.05)$ MHz) [55]. While there is intensity in this region in the LBG-G spectra,
it overlaps quite strongly with the expected B–O–La environment. Therefore these data cannot be used to draw a definite conclusion regarding the presence of O\textsubscript{666} triclusters in the glass. However, if O\textsubscript{666} triclusters are present, they are likely present in low amounts, as the chemical shift consistent with their presence is near to the leftmost edge of the glass signal in the quantitative MAS spectrum.

The NMR results support a LaBGeO\textsubscript{5} glass structure that is both radically different than that of the crystal and highly interconnected. The \textsuperscript{11}B and \textsuperscript{17}O NMR spectra of the glass support the presence of many species not featured in the crystal, \textit{i.e.}, B\textsubscript{O}\textsubscript{2}O\textsuperscript{−}, (\textsuperscript{5/6}Ge–O–\textsuperscript{4}Ge), and (\textsuperscript{5/6}Ge–O–B). The change in position and breadth of the \textsuperscript{139}La peak in the glass support structural changes in the La environment, though the nature of these changes is not currently certain. The [B\textsubscript{O}\textsubscript{4}]\textsuperscript{−} glass peak is of more positive chemical shift than that of the crystal, implying changes in the nature of its neighbours. Our \textsuperscript{11}B\{\textsuperscript{10}B\} REDOR experiments support this interpretation, indicating that there are fewer [B\textsubscript{O}\textsubscript{4}]\textsuperscript{−}–[B\textsubscript{O}\textsubscript{4}]\textsuperscript{−} links in the LaBGeO\textsubscript{5} glass than in the crystal, as well as the presence of BO\textsubscript{3}–[B\textsubscript{O}\textsubscript{4}]\textsuperscript{−} links. The absence of \textsuperscript{4}Ge–O–\textsuperscript{4}Ge and La–O–La environments, and the presence of both \textsuperscript{4}Ge–O–B and plausibly \textsuperscript{5/6}Ge–O–B environments, support a highly interconnected glass network. There are no indications of ordered, “crystal-like” environments present in the glass.

Direct NMR probes of internuclear distances are difficult to carry out for this composition, due to the spectroscopic properties of \textsuperscript{139}La and \textsuperscript{73}Ge. The presence of high-coordinate germanium provides a plausible explanation for the difference in glass-forming ability between the LaBGeO\textsubscript{5} and the LaBSiO\textsubscript{5} compositions. The LaBSiO\textsubscript{5} crystal is isostructural to the LaBGeO\textsubscript{5} crystal [83], but its precise composition has recently been shown to be outside the lanthanum borosilicate glass forming region when alumina contamination is avoided [97]. If six-coordinate germanium were not present, there would be no obvious structural difference between the LaBGeO\textsubscript{5} and LaBSiO\textsubscript{5} compositions to explain this difference in behaviour.

Our Raman results support the presence of high-coordinate germanium in the glass. Henderson \textit{et al.} have reported a Raman band attributed to \textsuperscript{5/6}Ge in alkali germanate glasses at a Raman shift of approx. 744 cm\textsuperscript{−1} [73]. While there is no resolved peak observed at this shift in our Raman spectrum of the LaBGeO\textsubscript{5} glass (Fig. 7.12), there is substantial intensity present. No intensity is observed at this shift in the spectrum
of the crystal. Hence the substantial intensity at approx. 744 cm$^{-1}$ in the glass can plausibly be attributed to the presence of high-coordinate germanium species.

### 7.7 Conclusions

We present $^{11}$B MAS, $^{11}$B{$^{10}$B} REDOR, $^{139}$La WCPMG, $^{17}$O MAS and $^{17}$O 3QMAS NMR spectra to provide insight regarding the structure of the glass and to probe possible structural changes during crystallization. All NMR spectra indicate substantial structural differences between the glass and the crystal. The crystal is composed of [BØ$_4$]$^-$ tetrahedra, GeO$_2$O$_2$$^2-$ tetrahedra, and LaO$_9$ polyhedra, with three oxygen environments: $^4$B–O–$^4$B, $^4$Ge–O–$^4$B, and $^4$Ge–O–La. Certain structural units not present in the crystal (e.g., BØ$_2$O$^-$, BØ$_2$O$^-$[BØ$_4$]$^-$, GeO$_6$) are positively identified in the glass. Other environments (e.g., $^{5/6}$Ge–O–B) are plausibly present as well, while others still (i.e., La–O–La and $^4$Ge–O–$^4$Ge) are absent. The diverse range of Ge–O–B environments, as well as the absence of La–O–La and $^4$Ge–O–$^4$Ge environments, suggests a highly interconnected and homogeneous glass network. Insufficient resolution precludes a quantitative analysis of the proportions of different environments. No structural changes in either the glass or crystal were observed for a range of partially-crystallized samples. A revised equation for calculating the crystallite fraction, $c$, from moderate-resolution $^{11}$B MAS NMR data is presented, supported by high-resolution $^{11}$B and $^{17}$O MAS NMR spectra.

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Chapter 8

Anisotropic Stress in Laser-Written LaBGeO$_5$ Glass-Ceramic Composites

8.1 Context

This chapter describes the results of DFPT calculations on the LaBGeO$_5$ crystal structure. Due to the complicated thermal dynamics of the laser-writing process, it is expected that laser-written LaBGeO$_5$ glass-ceramics will have some residual stress. The stresses during writing are so significant that the process needs to take place at elevated temperatures, in this case 400 °C [103]. Knorr et al. attempted to characterize this stress through analysis of micro-Raman spectra of a LaBGeO$_5$ laser-written line [108]. They informed their analysis using the data of Coussa et al., who collected Raman spectra of the LaBGeO$_5$ glass and crystal under isotropic compressive stress [109]. However, there is no reason to expect that the stress on the laser-written LaBGeO$_5$ glass-ceramic composite will be isotropic. Furthermore, Coussa et al. only reported the response of two LaBGeO$_5$ crystal Raman modes to stress. Knorr et al. report data on two modes, only one of which which was described by Coussa et al. Hence we conducted DFPT calculations on the LaBGeO$_5$ crystal to examine the response of all its Raman modes to stresses, both isotropic and anisotropic. This allowed us to construct a more physically meaningful model of the stresses present in laser-written LaBGeO$_5$ glass-ceramics.

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calculations and data analysis, and wrote the majority of the manuscript. JWZ provided substantial insights to the analysis of the data, and made significant editorial contributions to the manuscript.

8.2 Abstract

LaBGeO$_5$ glass-ceramic composite materials are of interest because the LaBGeO$_5$ crystal phase is ferroelectric, and can be grown in single-crystal form within the glass by localized heating from femtosecond laser irradiation. The crystals formed are expected to exhibit residual stress, due to the different mechanical properties of the glass and crystal phases. Recent micro-Raman data on these crystals have been interpreted as showing an isotropic stress field in the crystals. Here we reinterpret these data in light of detailed density functional perturbation calculations of the Raman spectra of the crystal phase under different stress conditions. Our results support a model where the stress in the $ab$ plane of the LaBGeO$_5$ crystal is compressive and the stress along the $c$ axis of the crystal is tensile. This model is consistent with the linear thermal expansion coefficients of the LaBGeO$_5$ crystal, which are anisotropic and of differing sign. These results indicate complexity of crystal formation in this system and possible limitations of using it in optical devices where a uniform stress state would be required.

8.3 Introduction

TFN materials are a class of glass-ceramics that combine the transparency of glass with ferroelectric properties, which can only be found in crystalline structures [1]. While there are many examples of TFN materials with widely varying compositions [274, 285], perhaps one of the most interesting is the LaBGeO$_5$ glass-ceramic. LaBGeO$_5$ is a synthetic form of the stillwellite mineral, and both the germanate and its isostructural silicate analogue are ferroelectric [88, 286]. Part of the interest in the LaBGeO$_5$ TFN system stems from its congruent crystallization, i.e., the glass and crystal have the same stoichiometry. Given that many TFN composites can contain multiple chemically distinct crystalline phases [1], the LaBGeO$_5$ system is attractive for its relative simplicity. All forms of the LaBGeO$_5$ system have come under substantial
Figure 8.1: The room-temperature crystal structure of LaBGeO$_5$ [78]. The presented view is down the crystallographic $c$ axis with a $2 \times 2 \times 2$ supercell. [BÔ$_4$]$^-$ tetrahedra are coloured dark green, while GeO$_2$O$_2^{2-}$ tetrahedra are purple. LaO$_9$ polyhedra are omitted for clarity. Lanthanum ions are light green.

investigation, including determinations of the crystal structures [78, 79] (Fig. 8.1), studies of the glass structure [34, 75], studies of the nonlinear optical properties of the system [232, 233], and studies on the crystallization of the nanocomposite material [34, 98, 287]. Perhaps one of the most interesting properties of the LaBGeO$_5$ system is the formation of LaBGeO$_5$ crystals within the bulk of the glass through the application of laser irradiation. Laser-written LaBGeO$_5$ crystals have been formed via both continuous-wave and femtosecond lasers, with the latter being used to create complex three-dimensional architectures [2, 103, 104, 288]. Recent evidence has also suggested that these laser-written crystals-in-glass maintain single-crystal character over macroscopic distances [105].

While the glass and crystal have the same chemical composition, they do not have the same structure and would not be expected to have the same mechanical properties. Therefore, the crystals induced in the glass would be expected to form under possibly significant stress fields, due to the different elastic properties of the glass host and the
Knorr et al. have provided some insight into the environment of both undoped and Er-doped LaBGeO$_5$ laser-written crystals in glass, based upon space-resolved Raman spectroscopy, and conclude that the crystal experiences isotropic compressive stress [108]. Coussa et al. report Raman spectra of both LaBGeO$_5$ glass and crystal under isotropic pressures up to 18.3 GPa [109]. However, Coussa et al. only report the response of two high-frequency peaks for the crystal, while Knorr et al. report data on both high- and low-frequency modes [108]. Furthermore, the data presented by Coussa et al. for the LaBGeO$_5$ crystal are not reported to be controlled for the orientation of the crystal [109]. The intensities and peak shifts observable in the Raman spectrum of the LaBGeO$_5$ crystal depend on both the orientation and polarization of the incident laser [95, 198]. Moreover, recent work has shown that the crystals themselves grow anisotropically [104], and in addition crystalline LaBGeO$_5$ exhibits both anisotropic and negative thermal expansion [79]. Thus the crystals, even if annealed, when cooled to ambient temperature in the glass matrix would be expected to show complex residual stress.

In order to provide a comprehensive examination of the stresses experienced by LaBGeO$_5$ laser-written crystals in glass at ambient conditions, we report DFPT-based calculations of the Raman spectra of the LaBGeO$_5$ crystal in response to changes in stress and strain, and compare them to the existing experimental data [108, 109]. Both isotropic and anisotropic stresses are considered. We also consider our results in the context of the Selsing model of stresses present in glass-ceramics [289]. The Selsing model requires knowledge of the thermal expansion coefficients of both the glass and crystal. The linear coefficient of thermal expansion of the LaBGeO$_5$ glass has been reported in the literature [96], but the thermal expansion behaviour of the LaBGeO$_5$ crystal is more complicated and the coefficients of thermal expansion of the crystal have not been previously reported. We extract the volumetric coefficient and the two linear coefficients of thermal expansion of the LaBGeO$_5$ crystal from the temperature-dependent neutron diffraction data of Belokoneva et al. [79]. The Selsing model also requires the knowledge of various elastic moduli of the glass and of the crystal. We report experimental values for the elastic moduli of the LaBGeO$_5$ glass, as well as elastic moduli for the LaBGeO$_5$ crystal derived from DFT calculations.
8.4 Theory

8.4.1 Raman Intensities from DFPT

Raman scattering consists of an incoming light wave interacting inelastically with an atomic displacement, resulting in a scattered outgoing light wave, and thus from a density functional theory standpoint can be reduced to the computation of the third derivative of the total energy with respect to two interactions of the electric field and an atomic displacement. A key result of DFPT, known as the $2n + 1$ theorem, is that such third-order terms can be computed from knowledge of only the ground and 1st-order perturbed wavefunctions [290, 291]. This result makes the ab initio computation of Raman spectra as (relatively) simple as computing the $\Gamma$-point phonons and the optical dielectric response. The reduction of the bare DFPT derivatives to experimentally relevant geometries is described in detail by Caracas and Cohen [292].

We briefly restate the relevant equations, and the context in which we interpret the results. Via DFPT, we calculate the Raman susceptibility tensor $\beta_{ij}$, with $i, j = x, y, z$. $\beta_{ij}$ is a second-rank tensor which is the derivative of the electric polarizability tensor $\chi_{ij}$ with respect to the atomic displacements $r_{\alpha\tau}$ associated with the phonon eigenvector $u_{\alpha\tau}$, scaled by the unit cell volume $\Omega$ such that:

$$\beta_{ij} = \sqrt{\Omega} \sum_{\alpha, \tau} \frac{\partial \chi_{ij}}{\partial r_{\alpha\tau}} u_{\alpha\tau},$$

(8.1)

where

$$\chi_{ij} = \epsilon_{ij}^\infty - \delta_{ij},$$

(8.2)

and $\epsilon_{ij}^\infty$ is the high-frequency dielectric tensor. The Raman intensity of a given phonon mode at a given polarization will be proportional to $\omega_L^4 \beta_{ij}^2$, where $\omega_L$ is the frequency of the incident laser. However, this requires linearly polarized incident light and the selection of polarized scattered light. In a system where the incident and scattered light are unpolarized we would expect to observe averages of the possible orientations. The equations describing the Raman intensity of a powder, in which the intensity from crystallites is averaged, should serve as a sufficient approximation for our purposes. The total Raman intensity of a powder is the sum of the orientational averages of the
parallel ($I_{\parallel}^\text{powder}$) and perpendicular ($I_{\perp}^\text{powder}$) components of the crystal:

$$I_{\text{Total}}^\text{powder} = I_{\parallel}^\text{powder} + I_{\perp}^\text{powder}. \quad (8.3)$$

The parallel and perpendicular components of the intensity can be calculated from the rotational invariants of the susceptibility tensor:

$$I_{\parallel}^\text{powder} = C \left( 10G^{(0)} + 4G^{(2)} \right),$$
$$I_{\perp}^\text{powder} = C \left( 5G^{(1)} + 3G^{(2)} \right). \quad (8.4)$$

The rotational invariants are

$$G^{(0)} = \frac{1}{3} (\beta_{xx} + \beta_{yy} + \beta_{zz})^2,$$
$$G^{(1)} = \frac{1}{2} \left[ (\beta_{xy} - \beta_{yx})^2 + (\beta_{yz} - \beta_{zy})^2 + (\beta_{zx} - \beta_{xz})^2 \right],$$
$$G^{(2)} = \frac{1}{2} \left[ (\beta_{xy} + \beta_{yx})^2 + (\beta_{yz} + \beta_{zy})^2 + (\beta_{zx} + \beta_{xz})^2 \right] + \frac{1}{3} \left[ (\beta_{xx} - \beta_{xx})^2 + (\beta_{yy} - \beta_{yy})^2 + (\beta_{zz} - \beta_{zz})^2 \right]. \quad (8.5)$$

$C$ is the prefactor

$$C = (\omega_L - \omega)^4 \frac{1 + n(\omega)}{30\omega}. \quad (8.6)$$

As mentioned above, $\omega_L$ is the frequency of the incident light (i.e., the laser frequency). $\omega$ is the frequency of the phonon mode under consideration. $n(\omega)$ is the Bose-Einstein factor $n(\omega) = \exp(h\omega/k_B T) - 1^{-1}$ at temperature $T$. Once the Raman response for each phonon mode has been calculated for a given temperature and laser frequency, the total spectrum can be plotted via convolution with an appropriate damping function, such as a Lorentzian.

### 8.4.2 Directional Dispersion of Raman Modes in Polar Crystals

The LaBGeO$_5$ crystal is a member of the polar $P3_1$ space group [78]. For polar space groups, the phonon mode frequencies depend not only on the specifics of the crystal structure, but also the relative orientation of the incident laser and the crystallographic axes. We present a brief discussion of the practical consequences of this effect in the context of Raman spectroscopy on a single crystal of LaBGeO$_5$. For a comprehensive
discussion of this phenomenon, the reader is directed to the literature [150, 293].

The LaBGeO$_5$ unit cell contains 24 atoms [78], and hence has 69 optic phonon modes, all of which are Raman-active [95]. The 69 optic modes of the LaBGeO$_5$ crystal can be decomposed into 23 A modes and 23 E modes. As a member of the trigonal polar $P3_1$ space group, the LaBGeO$_5$ crystal structure is uniaxial, and hence contains an “optically isotropic” plane and an orthogonal optic axis. The optic axis is coincident with the crystallographic $c$ axis; our choice of Cartesian coordinates places the $z$ axis in this direction. The optically isotropic $xy$ plane is hence in the crystallographic $ab$ plane. Dependent on the orientation of the phonon wavevector, $\mathbf{q}$, relative to the $c$ axis, the phonon modes of the crystal can be classified as purely longitudinal optic (LO) or transverse optic (TO). The angle between $\mathbf{q}$ and $c$ is denoted by $\theta$. For $\mathbf{q} \parallel c$ (i.e., $\theta = 0^\circ$), there will be a set of A(LO) modes and doubly-degenerate E(TO) modes. Conversely, for $\mathbf{q} \perp c$ (i.e., $\theta = 90^\circ$), the degeneracy of the E modes is broken, and three distinct symmetries can be observed: A(TO), E(TO), and E(LO) modes. One set of E(TO) modes is transverse for any $\theta$, and these modes are referred to as ordinary modes. The ordinary modes are of constant frequency for any value of $\theta$. All other modes have frequencies that depend on $\theta$; these modes are referred to as extraordinary modes. The frequencies of the extraordinary modes vary continuously and monotonically with $\theta$, and frequencies at intermediate values of $\theta$ couple two fundamental modes at $\theta = 0^\circ$ and $\theta = 90^\circ$. The symmetries of the fundamental modes do not need to be the same; A modes can couple with E modes, and TO modes can couple with LO modes. Any modes so linked are coupled by “directional dispersion” [293]. While the symmetries of the fundamental modes at the extremes of $\theta$ are well-defined, the same cannot be said for modes at intermediate values of $\theta$. These “oblique” modes can be of complex mixed A, E, LO, and TO character. The directional dispersion of the extraordinary modes can be calculated via the roots of

$$
\left| (\omega_i^2 - \omega^2)\delta_{ij} + \frac{(\hat{\mathbf{q}} \cdot \mathbf{M}_i) (\hat{\mathbf{q}} \cdot \mathbf{M}_j)}{\epsilon_0\epsilon_\infty (\hat{\mathbf{q}})} \right| = 0,
$$

where

$$
\epsilon_\infty (\hat{\mathbf{q}}) = \epsilon_\parallel \cos^2 \theta + \epsilon_\perp \sin^2 \theta.
$$

In the above equations, $\omega$ is the frequency of the extraordinary mode, $\omega_i$ is the
transverse phonon frequency of mode $i$, $\hat{q}$ is the unit vector of the direction of propagation, and $M_i$ is the reduced dielectric polarization vector of mode $i$ [95, 294].

The practical consequences of the directional dispersion of the LaBGeO$_5$ crystal are threefold. The first is that care must be taken when choosing the direction of non-analyticity of the $q \to 0$ limit in our DFPT calculations [295]. The second is that through careful analysis of the experimentally observable modes, the relative orientation of the crystal can be determined [95]. The third is that it is possible for the directional dispersion to depend on the stress experienced by the system. As such, any study of the response of the phonon frequencies of a polar crystal to stress must carefully consider the orientation of the crystal.

8.4.3 The Selsing Model

The Selsing model [289] is a simple model for estimating the isotropic thermal stress present in a glass-ceramic composite. It predicts that the stress inside the ceramic phase is given by

$$P = \frac{\Delta \alpha \Delta T}{\frac{1+\nu_G}{2E_G} + \frac{1-2\nu_X}{E_X}},$$

where subscripts G and X refer to glass and crystal values, respectively. Furthermore, $\Delta \alpha$ is the mismatch between the volumetric thermal expansion coefficients of the glass and the crystal, such that $\Delta \alpha = \alpha_X - \alpha_G$; $\Delta T$ is the difference between the glass transition temperature and room temperature; and $E$ and $\nu$ are the elastic moduli and the Poisson’s ratios of the crystal and glass, respectively. The Selsing model was derived for systems where the crystallites are spherical, isotropic, and rarefied such that their stress fields do not overlap; the latter condition typically requires a crystallized volume fraction of less than 10% [296]. In the system under investigation these conditions do not rigorously hold, but nevertheless, we use the Selsing model to provide an estimate of the order of magnitude of the stress experienced by the LaBGeO$_5$ crystals-in-glass.
8.5 Methods

8.5.1 Density Functional Perturbation Theory Calculations

All electronic structure calculations were performed using the ABINIT 8.6.3 software package [161]. NCPPs were generated using the ONCVPSP code [164]. The pseudopotentials were generated using parameters as provided from the ABINIT website (https://www.abinit.org/), except that the exchange-correlation functional was changed to the local-density approximation (LDA). The LDA with the Teter rational polynomial parametrization fit to Ceperley-Alder data was used to enable the calculation of Raman intensities. The valence spaces of the pseudopotentials were as follows: La(5s$^2$5p$^6$s$^1$5d$^2$); B(2s$^2$2p$^1$); Ge(3d$^{10}$4s$^2$4p$^2$); O(2s$^2$2p$^6$). A plane-wave energy cutoff of 40 hartree was used with a $4 \times 4 \times 4$ shifted Monkhorst-Pack grid of k-points, which resulted in a root mean square (RMS) convergence of 0.8 cm$^{-1}$ in the phonon frequencies. The choice of k-point mesh corresponds to a grid spacing of approximately 0.035 Å$^{-1}$. Structural optimizations were carried out using the Broyden-Fletcher-Goldfarb-Shanno algorithm, which was considered converged when the maximum force on any atom was less than $2 \times 10^{-5}$ hartree per bohr. The initial structural input was generated from data reported by Kaminskii et al. [78] using the Cif2cell program [297]. The crystallographic information file was obtained from ICSD [298]. The relaxed geometry was generated by allowing both cell dimensions and ionic positions to vary. Stressed geometries were generated by straining the unit cell according to the elastic constants calculated from the relaxed geometry for a given stress, and then allowing ionic positions to relax. The space group symmetry was preserved for all calculations. Raman intensities were calculated using Eqs. (8.3), (8.4) and (8.6), using values of $T = 298$ K and $\omega_L = 488$ nm, to match values provided in Knorr et al. [108]. Calculated Raman spectra were plotted with a full-width at half-maximum (FWHM) value of 15 cm$^{-1}$, consistent with widths observed by Knorr et al. [108]. Space-resolved Raman shift profiles were extracted from Fig. 6 of Knorr et al. [108] through the use of ImageJ software [299].
8.5.2 Sample Preparation and Measurements of Physical Properties

Stoichiometric amounts of La$_2$O$_3$ (Sigma-Aldrich, 99.99%), GeO$_2$ (Sigma-Aldrich, ≥99.99%), and B$_2$O$_3$ (Sigma-Aldrich, ≥98%) were intimately mixed and then melted in a 95/5 Pt/Au crucible using a 1250 °C box furnace in air for 30 min. The melt was quenched by pouring into an aluminium mould preheated to 500 °C. The resulting glass block was (5.55 ± 0.02) mm thick. Experimental elastic moduli of the glass were derived from measurement of the velocities of longitudinal and transverse ultrasonic waves in the glass. Velocities were determined via use of a Olympus 45MG ultrasonic thickness gauge. Longitudinal and transverse transducers were both operated at 5 MHz. The Poisson’s ratio of the glass is $\nu = \frac{[1 - 2(v_T/v_L)^2]}{[2 - 2(v_T/v_L)^2]}$, where $v_T$ and $v_L$ are respectively the transverse and longitudinal velocities. The shear modulus $G$ of the glass is $G = v_T^2 \rho$, where $\rho$ is the density of the glass. The other moduli can be calculated via the standard relations for a glass [3]. The density of the glass was determined by Archimedes’ method using absolute ethanol as the immersion fluid. Density measurements were controlled for temperature.

8.6 Results

The calculated phonon frequencies of the relaxed ($P = 0$ GPa) system, and their comparison to experimental values, are presented in Table 8.1. The experimental phonon frequencies are obtained from Hrubá et al. [95], and our calculated frequencies agree with their experimental values within approximately 11 cm$^{-1}$ RMS, or 0.5% less than experiment. As discussed above, the symmetry character of a given mode depends on the orientation of the incoming wavevector, $\mathbf{q}$, with the optic axis of the crystal, $c$. Frequencies for both $\theta = 0^\circ$ (A(LO), E(TO) and $\theta = 90^\circ$ [A(TO), E(TO), ELO] are provided. Calculated Raman spectra of the $\theta = 0^\circ$ orientation for selected stress fields are presented in Fig. 8.2. The relative intensities of the Raman peaks do not dramatically change when stresses of approximately 1 GPa are applied. The relative intensities of the peaks are generally consistent with literature reports on powdered samples [75, 109].

In order to make use of the Selsing model, we require reliable values for the volumetric coefficient of thermal expansion for both the LaBGeO$_5$ glass and crystal, as
Table 8.1: Phonon frequencies from experiment, and from DFPT calculations. Experimental values are obtained from Hrubá et al. [95]. Due to their low oscillator strengths, certain modes are not experimentally observable. Calculated values are from the relaxed geometry. All values are in cm$^{-1}$.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Experiment [95]</th>
<th>Relaxed</th>
<th>Experiment [95]</th>
<th>Relaxed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TO LO</td>
<td>TO LO</td>
<td>TO LO</td>
<td>TO LO</td>
</tr>
<tr>
<td>A$_1$</td>
<td>87 89</td>
<td>81.8 81.9</td>
<td>E$_1$</td>
<td>92 92.7</td>
</tr>
<tr>
<td>A$_2$</td>
<td>95 95.5</td>
<td>94.0 95.4</td>
<td>E$_2$</td>
<td>109 110</td>
</tr>
<tr>
<td>A$_3$</td>
<td>117 119</td>
<td>118.6 120.4</td>
<td>E$_3$</td>
<td>124 125</td>
</tr>
<tr>
<td>A$_4$</td>
<td>144 144.5</td>
<td>152.0 152.6</td>
<td>E$_4$</td>
<td>162 179</td>
</tr>
<tr>
<td>A$_5$</td>
<td>173 213</td>
<td>178.9 187.3</td>
<td>E$_5$</td>
<td>185 200</td>
</tr>
<tr>
<td>A$_6$</td>
<td></td>
<td>187.7 209.8</td>
<td>E$_6$</td>
<td>207 208.5</td>
</tr>
<tr>
<td>A$_7$</td>
<td>215 221.5</td>
<td>211.0 225.3</td>
<td>E$_7$</td>
<td>233 233.5</td>
</tr>
<tr>
<td>A$_8$</td>
<td>273 273</td>
<td>261.5 261.5</td>
<td>E$_8$</td>
<td>258 262</td>
</tr>
<tr>
<td>A$_9$</td>
<td></td>
<td>297.4 298.0</td>
<td>E$_9$</td>
<td>301 320</td>
</tr>
<tr>
<td>A$_{10}$</td>
<td>306 307</td>
<td>304.4 304.6</td>
<td>E$_{10}$</td>
<td>336 352</td>
</tr>
<tr>
<td>A$_{11}$</td>
<td>324 329</td>
<td>319.2 323.0</td>
<td>E$_{11}$</td>
<td>384 396</td>
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<tr>
<td>A$_{12}$</td>
<td>368 380</td>
<td>360.4 374.6</td>
<td>E$_{12}$</td>
<td>423 439</td>
</tr>
<tr>
<td>A$_{13}$</td>
<td>389 422</td>
<td>383.5 417.7</td>
<td>E$_{13}$</td>
<td>454 454</td>
</tr>
<tr>
<td>A$_{14}$</td>
<td>503 510</td>
<td>499.3 507.0</td>
<td>E$_{14}$</td>
<td>496 502</td>
</tr>
<tr>
<td>A$_{15}$</td>
<td>549 552</td>
<td>544.2 547.5</td>
<td>E$_{15}$</td>
<td>616 621</td>
</tr>
<tr>
<td>A$_{16}$</td>
<td>631 633</td>
<td>625.5 628.3</td>
<td>E$_{16}$</td>
<td>695 701</td>
</tr>
<tr>
<td>A$_{17}$</td>
<td>733 745</td>
<td>731.1 742.5</td>
<td>E$_{17}$</td>
<td>722 753</td>
</tr>
<tr>
<td>A$_{18}$</td>
<td>799 803</td>
<td>791.1 793.9</td>
<td>E$_{18}$</td>
<td>784 811</td>
</tr>
<tr>
<td>A$_{19}$</td>
<td>806 813</td>
<td>802.2 823.4</td>
<td>E$_{19}$</td>
<td>826 834</td>
</tr>
<tr>
<td>A$_{20}$</td>
<td>847 852</td>
<td>848.0 850.3</td>
<td>E$_{20}$</td>
<td>859 863</td>
</tr>
<tr>
<td>A$_{21}$</td>
<td>864 866</td>
<td>871.8 878.3</td>
<td>E$_{21}$</td>
<td>918 928</td>
</tr>
<tr>
<td>A$_{22}$</td>
<td>941 980</td>
<td>990.8 1006.8</td>
<td>E$_{22}$</td>
<td>975 1042</td>
</tr>
<tr>
<td>A$_{23}$</td>
<td>992 1050</td>
<td>1014.6 1074.9</td>
<td>E$_{23}$</td>
<td>1088 1098</td>
</tr>
</tbody>
</table>
Figure 8.2: Theoretical Raman spectra for LaBGeO$_5$ at different stress fields: a) $P = 0$ GPa; b) $\sigma_{xx} = \sigma_{yy} = -\sigma_{zz} = 1$ GPa; c) $P = 1$ GPa. The spectra are vertically offset for clarity. Intensities are calculated using Eqs. (8.3), (8.4) and (8.6), using a temperature of 298 K. The incident laser wavelength is set to 488 nm, to match the data of Knorr et al. [108]. We convolute the spectra with a Lorentzian function with a FWHM of 15 cm$^{-1}$, consistent with linewidths reported by Knorr et al.
Table 8.2: The unique elastic stiffness constants and the elastic moduli of the LaBGeO$_5$ glass and low-temperature crystal. The bulk modulus ($K$), shear modulus ($G$), elastic modulus ($E$), and Poisson’s ratio ($\nu$) of the crystal are reported as Voigt-Reuss-Hill averages. All crystal values are derived from DFT calculations on the relaxed ($P| = 0$ GPa) LaBGeO$_5$ structure.

<table>
<thead>
<tr>
<th></th>
<th>$P3_1$ Crystal</th>
<th>Glass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{11}$ / GPa</td>
<td>229.9</td>
<td>128 ± 1</td>
</tr>
<tr>
<td>$C_{12}$ / GPa</td>
<td>98.3</td>
<td>46 ± 3</td>
</tr>
<tr>
<td>$C_{13}$ / GPa</td>
<td>102.1</td>
<td></td>
</tr>
<tr>
<td>$C_{14}$ / GPa</td>
<td>-0.3</td>
<td></td>
</tr>
<tr>
<td>$C_{15}$ / GPa</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td>$C_{33}$ / GPa</td>
<td>280.7</td>
<td></td>
</tr>
<tr>
<td>$C_{44}$ / GPa</td>
<td>64.1</td>
<td>41.0 ± 0.8</td>
</tr>
<tr>
<td>$K$ / GPa</td>
<td>148.8</td>
<td>73 ± 3</td>
</tr>
<tr>
<td>$G$ / GPa</td>
<td>67.4</td>
<td>41.0 ± 0.8</td>
</tr>
<tr>
<td>$E$ / GPa</td>
<td>175.0</td>
<td>104 ± 2</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.303</td>
<td>0.26 ± 0.01</td>
</tr>
</tbody>
</table>

well as values for the glass transition temperature, the elastic modulus and Poisson’s ratio for the crystal, and the shear modulus for the glass. The glass transition temperature of the LaBGeO$_5$ glass has been reported to be about 670°C [96, 232]. Our experimental results for the elastic constants and moduli of the LaBGeO$_5$ glass, as well as the Voigt-Reuss-Hill averages [300] of the elastic moduli of the low-temperature phase of the LaBGeO$_5$ crystal, are reported in Table 8.2. The shear, bulk, and elastic moduli require knowledge of the density of the sample. The density of the glass was found to be $(4.98 \pm 0.03)$ g cm$^{-3}$. Our value for the shear modulus of the glass [(41.0 ± 0.8) GPa] is close to the values reported in the literature (approx. 39.0 GPa) [246]. To our knowledge, the elastic moduli of the LaBGeO$_5$ crystal have not been reported in the literature.

The linear thermal expansion expansion coefficient of the glass has been reported to be $\alpha_{L,G} = 7.6 \times 10^{-6}$ K$^{-1}$ over the temperature range of 20 °C to 300 °C [96]. As glass is an isotropic material, the volumetric thermal expansion coefficient is simply three times the linear coefficient. Hence the volumetric thermal expansion coefficient of the glass is $\alpha_{V,G} = 23 \times 10^{-6}$ K$^{-1}$. A preliminary value for the linear thermal expansion coefficient of the LaBGeO$_5$ crystal has been reported as $\alpha_{L,X} = 6.5 \times 10^{-6}$ K$^{-1}$ [96]. However, this value does not appear to acknowledge the anisotropy of the LaBGeO$_5$
Table 8.3: Linear ($\alpha_a$, $\alpha_c$) and volumetric ($\alpha_V$) coefficients of thermal expansion for the LaBGeO$_5$ low-temperature crystal phase ($P3_1$), high-temperature crystal phase ($P3_1\overline{2}1$), and glass. Crystal coefficients are calculated from high-resolution neutron diffraction data reported by Belokoneva et al. [79]. Glass coefficients are from Sigaev et al. [96].

<table>
<thead>
<tr>
<th>Phase</th>
<th>Temperature / °C</th>
<th>$\alpha_a$ / $10^{-6}$ K$^{-1}$</th>
<th>$\alpha_c$ / $10^{-6}$ K$^{-1}$</th>
<th>$\alpha_V$ /$10^{-6}$ K$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P3_1$</td>
<td>20–300</td>
<td>$\approx$0</td>
<td>14.1 ± 0.7</td>
<td>14.6 ± 0.9</td>
</tr>
<tr>
<td></td>
<td>350–480</td>
<td>$-3.6 \pm 0.3$</td>
<td>27 ± 1</td>
<td>16.5 ± 0.1</td>
</tr>
<tr>
<td></td>
<td>480–530</td>
<td>$-18 \pm 2$</td>
<td>38 ± 3</td>
<td>15.2 ± 0.3</td>
</tr>
<tr>
<td>$P3_1\overline{2}1$</td>
<td>550–700</td>
<td>61 ± 5</td>
<td>15 ± 1</td>
<td>20.3 ± 0.2</td>
</tr>
<tr>
<td>Glass*</td>
<td>20–300</td>
<td>7.6</td>
<td>7.6</td>
<td>23</td>
</tr>
</tbody>
</table>

* For an isotropic material such as a glass, $\alpha_a = \alpha_c = \frac{1}{3}\alpha_V$. The glass values were reported in Ref. [96] without uncertainties.

crystal, nor the negative thermal expansion reported for this system [79]. Based upon the report high-resolution temperature-dependent neutron diffraction data [79], we calculate average linear and volumetric coefficients of thermal expansion for both the high-temperature and low-temperature phases of the LaBGeO$_5$ crystal over a range of temperatures. These values are reported in Table 8.3.

8.7 Discussion

8.7.1 Selsing Model Calculations

Using the Selsing formula (Eq. (8.9)), we can estimate the magnitude and sign of the stress in the laser-written LaBGeO$_5$ crystal-in-glass. Most of the requisite data is presented in Section 8.6. With the ambient temperature assumed to be 25 °C, we calculate the residual internal thermal stress on the LaBGeO$_5$ crystal to be 0.6 GPa. As discussed in Section 8.4.3, there are a number of assumptions made by the model which do not match the system under investigation. Hence we do not expect the stresses we predict from our DFPT calculations to agree perfectly with the value from the Selsing model. We do expect that any physically plausible stress to be of a similar order of magnitude, however.
8.7.2 Crystal Orientation

The observable vibrational modes in the LaBGeO$_5$ crystal depend on the orientation and polarization of the incident light [95, 150, 293]. As such, it is important to consider the possible effects of angular variation on the behaviour of the Raman peaks of the LaBGeO$_5$ crystal in response to stress. The only empirical data on the behaviour of the crystalline LaBGeO$_5$ Raman peaks in response to stress are presented by Coussa et al. [109], who did not discuss the orientation of their sample. As such, we discuss the possible interactions of directional dispersion and stress in the sections below. However, there is substantial experimental evidence available regarding the orientation of laser-written LaBGeO$_5$ crystals-in-glass. Based upon polarized micro-Raman measurements of LaBGeO$_5$ laser-written lines, with the laser perpendicular to the direction of crystal growth, Stone et al. concluded that the $c$ axis of the crystal is parallel to the direction of crystal growth [2]. Stone et al. later confirmed this observation through the use of scanning electron microscopy (SEM) [105]. The SEM data of Stone et al. show that there is less than 1° of variation over the face of a laser-written LaBGeO$_5$ crystal. The data presented in Fig. 5 of Knorr et al. are consistent with the $c$ axis of the crystal being parallel to the direction of crystal growth [108]. Their unpolarized Raman spectra are collected parallel to the direction of crystal growth. The A(LO)$_{13}$ mode at 422 cm$^{-1}$ and E(TO)$_{19}$ mode at 826 cm$^{-1}$ are resolved from their neighbouring peaks, consistent with a $\theta = 0^\circ$ geometry. If $\theta \neq 0^\circ$, the degeneracy of the E modes is broken, and directional dispersion would lead to a complicated pattern of oblique modes. At the maximum value of $\theta = 90^\circ$, our calculations show that the E(LO)$_{11}$, E(TO)$_{12}$, and E(LO)$_{12}$ modes shift and overlap such that the peak at 422 cm$^{-1}$ is substantially diminished. A similar set of changes occurs around the E(TO)$_{19}$ mode. Therefore, regarding spectral overlap, our calculations are consistent with the conclusion of Knorr et al. that $\theta = 0^\circ$.

8.7.3 Isotropic Stress

Coussa et al. report that the Raman peak at 803 cm$^{-1}$, identified as the A(LO)$_{18}$ mode, responds to compressive isotropic stress with a slope of $(3.3 \pm 0.2)$ cm$^{-1}$ GPa$^{-1}$ [109]. Knorr et al. observe a difference in the A(LO)$_{18}$ mode of 2.5 cm$^{-1}$ between the centre and edge of their crystal sample (Fig. 8.3). Based upon the results of Coussa et
Figure 8.3: Experimental micro-Raman data for the LaBGeO$_5$ crystal-in-glass and extracted Raman shift profiles. a) Micro-Raman data of the A(LO)$_{18}$ mode, adapted from Fig. 6 of Knorr et al. [108]. b) The Raman profile of the A(LO)$_{18}$ mode. c) Micro-Raman data of the E(TO)$_6$ mode, adapted from Fig. 6 of Knorr et al. [108]. d) The Raman profile of the E(TO)$_6$ mode. The Raman profiles (b,d) are extracted from the micro-Raman data (a,c). The data points extracted from the micro-Raman data are indicated by red lines in (a,c). We set the 0 point of our x axis as the maximum of the A(LO)$_{18}$ mode, which coincides with the minimum of the E(TO)$_6$ mode and is roughly at the centre of the crystal. Positive x axis values move towards the right of the line of the inset, while negative values move toward the left. The profile was chosen to coincide with the “minor” axis of the crystal, approximately parallel with the shortest path through the crystal.
Figure 8.4: The responses of the E(TO)$_6$ (left axis, open circles) and A(LO)$_{18}$ (right axis, filled squares) modes to isotropic stress. The lines are linear fits to the data.

*al.*, Knorr *et al.* conclude that there is a stress differential of 0.75 GPa between the centre and the edge, leading to a stress gradient of 0.168 GPa $\mu$m$^{-1}$ over a length of 4.5 $\mu$m [108].

Our data for the responses of the A(LO)$_{18}$ and E(TO)$_6$ to changes in isotropic stress are presented in Fig. 8.4. Considering only the A(LO)$_{18}$ mode, our results for systems under isotropic stress are consistent with the data of Coussa *et al.* [109], with a slope for the A(LO)$_{18}$ mode of 4.72 cm$^{-1}$ GPa$^{-1}$. This would suggest a stress differential of 0.53 GPa between the edge and centre of the crystal, with a stress gradient of 0.11 GPa $\mu$m$^{-1}$. However, Coussa *et al.* do not report on the behaviour of the E(TO)$_6$ mode. Based upon changes induced by isotropic stress, our results predict that a stress differential of 0.5 GPa would result in an E(TO)$_6$ shift of 0.8 cm$^{-1}$ between the edge and centre of the crystal. Knorr *et al.* observe nearly the opposite:
Figure 8.5: Changes in the frequency of the A(LO)_{18} mode in response to changes in θ for various isotropic stresses. There is very little change in frequency for θ < 45°. For θ ≥ 45°, the A(LO)_{18} frequency increases at about the same rate regardless of stress.

A shift of −1.5 cm\(^{-1}\). Our computed slope of the E(TO)_{6} mode is both linear and positive, with a value of 1.5 cm\(^{-1}\) GPa\(^{-1}\). There is no isotropic stress, either tensile or compressive, for θ = 0° crystal orientation, that can simultaneously explain a positive shift in A(LO)_{18} and a negative shift in E(TO)_{6}, as both respond positively to compression and negatively to tension.

Therefore, explaining the laser-induced crystal growth in this system requires a reconsideration of the conclusions that θ = 0°, and/or that the stress field is isotropic. We consider first the possibility of directional dispersion. The changes in frequency of the A(LO)_{18} mode with response to changes in θ for the stresses studied are presented in Fig. 8.5, and likewise for E(TO)_{6} in Fig. 8.6. For all stresses studied, an increase in θ leads to a monotonic positive change in A(LO)_{18} and a monotonic negative change in E(TO)_{6}. It is possible to construct a combination of isotropic stress and θ ≠ 0°.
Figure 8.6: Changes in the frequency of the E(\text{TO})_6 mode in response to changes in $\theta$ for various isotropic stresses. The influence of stress on the frequency of the E(\text{TO})_6 mode decreases as $\theta$ increases.
which leads to a shift of $2.5 \text{ cm}^{-1}$ in $\text{A(LO)}_{18}$ and $-1.5 \text{ cm}^{-1}$ in $\text{E(TO)}_6$. However, we do not believe such a combination is a physically plausible explanation for the experimental data reported by Knorr et al., for the following reasons.

As discussed in Section 8.7.2, we would expect an observable change in the number of resolvable peaks as $\theta$ moved away from $0^\circ$; no such change is observed. Furthermore, any such combination of isotropic stress and $\theta$ would require $\theta = 0^\circ$ at the edge of the crystal, and $\theta \neq 0^\circ$ at the centre. Were there to be a change in $\theta$ as the crystal grows, we would expect $\theta = 0^\circ$ at the centre, and for $\theta$ to increase as the distance from the centre increases. Finally, the value of $\theta$ required to match the experimentally observed shifts is approximately $40^\circ$, while the SEM results of Stone et al. suggest that we should expect $\theta$ to be small or zero, with little to no variance across the face of the crystal-in-glass [105]. Hence, it is much more likely that a different phenomenon is responsible for the positive shift in $\text{A(LO)}_{18}$ and negative shift in $\text{E(TO)}_6$. A plausible explanation is that the crystal experiences anisotropic stress. Working on a similar system, Stone hypothesized that $\text{LaBGeO}_5$ laser-written lines experience significant tensile stress parallel to the $c$ axis of the crystal [106]. We now consider the possible effects of anisotropic stresses on the $\text{LaBGeO}_5$ crystal.

### 8.7.4 Anisotropic Stress

Anisotropic stresses are generally more complicated and more computationally expensive to calculate than isotropic stresses, as the former may break the symmetry of the crystal. However, the $\text{LaBGeO}_5$ crystal has $P3_1$ symmetry, $i.e.$, lattice constants $a = b \neq c$. If the stresses on the system are applied in a similar fashion, that is $\sigma_{xx} = \sigma_{yy} \neq \sigma_{zz}$, then the symmetry of the system is preserved. As this seems to be a plausible approximation of the system under investigation, where the constraints in the $xy$ plane are different than the constraints along the $z$ axis, we have exploited this symmetry in our calculations. The responses of the frequencies of the $\text{A(LO)}_{18}$ and $\text{E(TO)}_6$ modes to anisotropic stresses are presented in Fig. 8.7. The stresses applied are such that $\sigma_{xx} = \sigma_{yy} = -\sigma_{zz}$, $i.e.$, compressive stress in the $xy$ plane and tensile stress along the $z$ axis, or vice versa. The behaviour of the $\text{A(LO)}_{18}$ mode in response to this anisotropic stress is generally similar to its response to isotropic stress. For positive $xy$ stresses where $\sigma_{xx}, \sigma_{yy} \leq 2 \text{ GPa}$, the $\text{A(LO)}_{18}$ frequency becomes
Figure 8.7: The responses of the $E(\text{TO})_6$ (left axis, open circles) and $A(\text{LO})_{18}$ (right axis, filled squares) modes to anisotropic stress. The data are arranged according to the stress applied to the $xy$ plane. The stresses on the $z$ axis at a given $xy$ plane stress are of approximately equal magnitude and opposite sign of the $xy$ plane stress.
more positive, and vice versa for negative $xy$ stresses. However, when the stress is increased further ($\geq 2$ GPa), the $A(LO)_{18}$ shift reverses direction. The magnitude of the change in $A(LO)_{18}$ in the anisotropic case is lower than that of the isotropic case. The behaviour of the $E(\text{TO})_6$ mode in the anisotropic case is very different than in the isotropic case. The most obvious change is that in the anisotropic case, the sign of the shift of the $E(\text{TO})_6$ mode in response to changing stress is generally opposite that of the $A(LO)_{18}$ mode; in the isotropic case, both the $E(\text{TO})_6$ mode and the $A(LO)_{18}$ mode have shifts of the same sign. The changes of the $E(\text{TO})_6$ mode are very non-monotonic, with the shift reversing direction at the extremes of the strains studied. Like the $A(LO)_{18}$ mode, the magnitude of the change in $E(\text{TO})_6$ is lower in the anisotropic case than in the isotropic case.

There are two possible combinations of anisotropic stresses which could explain the experimental observations of Knorr et al. The first is compressive stress in the $xy$ plane and tensile stress along the $z$ axis, with the stress minimum at the glass-crystal interface. The second is tensile stress in the $xy$ plane and compressive stress along the $z$ axis, with the stress minimum at the centre of the crystal. In order to determine which combination is more likely, we examine the thermal expansion behaviour of the LaBGeO$_5$ glass and LaBGeO$_5$ crystal. The coefficients of thermal expansion of the LaBGeO$_5$ glass and both LaBGeO$_5$ crystal phases are reported in Table 8.3.

The temperature and crystallization dynamics of the glass-ceramic composite system are expected to be very complex. Indeed, a recent study by Stone et al. on the dynamics of laser-writing in the LaBGeO$_5$ system helps explain the preferential alignment of the crystallographic $c$ axis with the direction of laser writing, as well as the “bean” shape of the crystal cross-section [104]. The results of Stone et al. demonstrate that the direction of crystal growth can be affected by the change in thermal gradient caused by the translation of the laser beam [104], dividing the growing crystal into two regions: one that undergoes growth tangential to the centre of the laser, and one that undergoes growth radially aligned with the centre. These regions are expected to experience different thermal dynamics and growth patterns while still being crystallographically aligned. However, growth in both of these regions is expected to occur at temperatures in excess of the glass transition temperature, which in LaBGeO$_5$ glass is approximately 670°C [33]. Knorr et al. conducted their
laser writing with the glass sample within a heating chamber held at 500°C in order to mitigate thermal stresses and resultant cracking of the sample [108]. Many of the stresses from the complex thermal dynamics during laser writing may be mitigated by holding the bulk sample at this elevated temperature. Ultimately, however, it is of interest to consider the stresses in the crystals that remain at ambient conditions, where a device based on this technology would typically be used. Because the crystals grow anisotropically [104], and exhibit anisotropic thermal expansion [79], the ambient stress field could also be expected to be anisotropic. Based upon the coefficients of thermal expansion (Table 8.3) we can provide a justification as to which anisotropic stress combination is more likely.

To form the laser-written LaBGeO$_5$ crystal-in-glass, the sample is initially locally heated by the application of laser irradiation. As the system cools, it will pass through the glass transition temperature ($670\,^\circ\text{C}$) [96, 232] and a solid-solid phase transition between the high-temperature ($P3_121$) and low-temperature ($P3_1$) crystal structures ($530\,^\circ\text{C}$) [79]. The experimental data for the glass only extend to 300°C; however, based upon the behaviour of other glasses we do not expect the coefficients of thermal expansion of the glass to vary by more than 50\% [3, 301].

As the composite system cools from the glass transition temperature to the phase transition temperature, both the glass and the crystal contract in all dimensions. The volumetric coefficients of thermal expansion of the glass and the high-temperature phase of the crystal are likely similar, with the glass coefficient being somewhat higher. This would suggest that the residual stress in the crystal is uniformly compressive. However, the linear coefficients of thermal expansion suggest that tensile stress is possible: the linear coefficient of thermal expansion of the crystal $c$ axis is higher than the linear coefficient of thermal expansion of the glass.

Upon passing the phase transition temperature, the $a$ and $b$ axes of the crystal begins to expand as the temperature decreases. While there is no data on the glass coefficient of thermal expansion in this range, the thermal expansion of the glass is expected to remain positive. Hence the $ab$ plane stress experienced by the crystal is expected to remain compressive. In this temperature range the $c$ axis of the crystal continues to contract, and at a higher rate than the glass. Hence we expect the stress in the $c$ axis to remain tensile.
As the system drops below approximately 350°C, the linear coefficient of thermal expansion of the $a$ axis of the crystal approaches zero, while the $c$ axis and the glass continue to contract. The exact stress dynamics will depend on the precise (and as of yet undetermined) high-temperature behaviour of the glass, as well as the thermal gradient in the system. However, given that it is implausible for the $c$ axis of the crystal to experience anything other than tensile stress, and that even if the linear coefficient of thermal expansion of the glass is greater than expected, the negative thermal expansion present in the $ab$ plane of the crystal provides a source of compressive stress, we conclude that the most likely combination of anisotropic stress is compressive in the $ab$ plane and tensile along the $z$ axis, with a stress maximum at the centre of the crystal.

With the conclusion that the stress in the system is compressive in the $ab$ plane and tensile along the $c$ axis, we can provide an estimate on the amount of stress experienced by the system. The maximum negative shift of the E(\(\text{TO}\))$_6$ mode is between $\sigma_{xx}, \sigma_{yy} \approx 1\ \text{GPa}–2\ \text{GPa}$. Above 2 GPa the shift of the mode becomes positive. As the shift of the E(\(\text{TO}\))$_6$ mode is monotonically negative, we expect that the magnitude of the maximum stress on the system is 2 GPa or lower. This would correspond to a maximum stress gradient of 0.44 GPa $\mu$m$^{-1}$ over the 4.5 $\mu$m distance between the edge and centre of the crystal. The direction and relative magnitude of the shift of the A(\(\text{LO}\))$_{18}$ mode is consistent with this model.

The model we present above (\textit{i.e.,} compressive $ab$ stress and tensile $c$ stress) correctly returns the directions of the changes in the A(\(\text{LO}\))$_{18}$ and E(\(\text{TO}\))$_6$ modes, but underestimates the magnitude of the shifts, by 25% and 85% respectively. However, the experimental shifts are quite small: 2.5 cm$^{-1}$ and −1.5 cm$^{-1}$, respectively. It is difficult using DFT calculations to reproduce such small shifts precisely; hence we believe that the direction of the calculated shift is more reliable than the absolute values. A second point of concern is that there is no particular reason to expect that the stress gradient in the system is linear. Indeed, when the shift of the E(\(\text{TO}\))$_6$ mode is examined over the breadth of the crystal, there are indications of non-linear behaviour. Finally, it is certainly possible for the stress gradient of the $ab$ plane to be different than that of the $c$ axis. The thermal expansion dynamics of the $ab$ plane are significantly more complicated than that of the $c$ axis; the former has both positive
and negative thermal expansion in the relevant temperature regime, whereas the latter has only positive thermal expansion. The $ab$ plane is also ultimately constrained by glass, while the $c$ axis is constrained by neighbouring crystal volumes. This could lead to stresses of different magnitudes between the $ab$ plane and the $z$ axis.

While the above considerations prevent us from precisely quantifying the stress fields present in the experimental data presented by Knorr et al. [108], our results still provides compelling evidence regarding the overall nature of the stresses. Our DFT calculations support a model where the stresses experienced by the laser-written LaBGeO$_5$ crystal-in-glass are anisotropic, and best described by compressive stress in the $ab$ plane of the crystal and tensile stress along the $c$ axis of the crystal. The magnitude of these stresses is expected to be less than 2 GPa. This stress profile is consistent with the thermal expansion behaviour of the LaBGeO$_5$ glass and crystal phases, which at times undergo negative thermal expansion.

8.8 Conclusions

The observable Raman modes of the LaBGeO$_5$ crystal depend on the relative orientation of the crystal axes and the incident laser. Based upon the modes observed by Knorr et al., we conclude that the angle between the $c$ axis of the LaBGeO$_5$ crystal and the incident laser in their experiment is $\theta = 0^\circ$. Our DFPT calculations support a model where the stress in laser-written LaBGeO$_5$ crystals-in-glass is anisotropic. The observed trends in the shifts of the $A$(LO)$_{18}$ and $E$(TO)$_{6}$ modes indicate compressive stress in the $ab$ plane of the laser-written crystal and tensile stress along the $c$ axis. These stresses are between approximately 1 GPa to 2 GPa in magnitude. This anisotropic stress is supported by qualitative analysis of the relative directions and magnitudes of the coefficients of thermal expansion of the LaBGeO$_5$ glass and crystal in the temperature range of the $P3_1 21$ to $P3_1$ phase transition. The much simpler Selsing model suggests a compressive stress of a similar magnitude, 0.6 GPa, but note that the Selsing model only can yield isotropic stress. Finally, the directional dispersion of select optical phonon modes of the LaBGeO$_5$ crystal are reported. The $A$(LO)$_{18}$ and $E$(TO)$_{6}$ modes do not show complex changes in directional dispersion in response to stress. Our results show that the stress environment of laser-written LaBGeO$_5$ crystals-in-glass is more complex than previously considered, which will
need to be accounted for in applications requiring homogeneously stressed systems.

8.9 Acknowledgements

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Chapter 9

Conclusion

9.1 The Goals of this Work

The primary question motivating this work was “what is the structure of the LaBGeO$_5$ glass?” This is not a simple question, as it encompasses many different experimental approaches and techniques. Accordingly, the central goals of this work were as follows:

- obtain data on the short-range structure of boron in the LaBGeO$_5$ glass via $^{11}$B NMR spectroscopy;
- probe for the presence of high-coordinate germanium in the glass via neutron diffraction;
- develop a model relating $^{139}$La NMR parameters to short-range lanthanum structure in crystalline materials;
- apply this crystalline model to $^{139}$La NMR spectrum of the LaBGeO$_5$ glass to determine the lanthanum-oxygen environment;
- probe the connectivity of the LaBGeO$_5$ glass via $^{17}$O NMR spectroscopy;
- test the Sigaev model directly via $^{11}$B{$^{10}$B} heteronuclear NMR spectroscopy;
- examine the mechanism of crystallization of the LaBGeO$_5$ glass-ceramic composite;
- and evaluate the stress on laser-written LaBGeO$_5$ glass-ceramics via DFT calculations.

These goals were mostly achieved. This chapter will synthesize the data described in Chapters 4 to 8 and present a holistic analysis of the structure of the LaBGeO$_5$ glass.

The structure of the LaBGeO$_5$ glass can be divided into two length scales: short-range, considering only the polyhedra of the first cation-oxygen coordination sphere;
and intermediate-range, considering the connectivity between different short-range polyhedra.

We begin by discussing the crystal. There are three cations present in the LaBGeO$_5$ system: La$^{3+}$; B$^{3+}$; and Ge$^{4+}$. In the crystal, these cations are present as asymmetric LaO$_9$ polyhedra, [BO$_4$]$^{-}$ tetrahedra, and GeO$_2$O$_2$$^{2-}$ tetrahedra. This is the short-range order of the system. The intermediate-range order is well-known, but by nature somewhat lengthier to explain.

The intermediate-range order of the [BO$_4$]$^{-}$ tetrahedron is the simplest in the system. Each [BO$_4$]$^{-}$ tetrahedron shares two edges with symmetry-equivalent LaO$_9$ polyhedra. [BO$_4$]$^{-}$ tetrahedra form a helical chain around the $3_1$ crystallographic screw axis (parallel to the c axis), with each [BO$_4$]$^{-}$ unit corner-sharing with two symmetry-equivalent neighbours. Each [BO$_4$]$^{-}$ neighbour pair forms a three-membered ring with a GeO$_2$O$_2$$^{2-}$ tetrahedron.

The GeO$_2$O$_2$$^{2-}$ tetrahedron is edge-sharing with two symmetry-equivalent LaO$_9$ polyhedra, shares a corner with a third, and forms a three-membered ring with two symmetry-equivalent [BO$_4$]$^{-}$ tetrahedra. No germanium unit is connected to any other germanium unit.

The LaO$_9$ polyhedron is edge-sharing with two symmetry-equivalent [BO$_4$]$^{-}$ tetrahedra, two symmetry-equivalent LaO$_9$ polyhedra, and two symmetry-equivalent GeO$_4$ tetrahedra. It is also corner-sharing with two additional symmetry-equivalent GeO$_2$O$_2$$^{2-}$ tetrahedra. Using the notation from Section 6.6.4, there are two distinct [O$_b$$^9$La$_2$$^4$B$^4$Ge] environments with two symmetry-equivalent oxygen, two distinct [O$_{ab}$$^9$La$_2$$^4$Ge] environments with two symmetry-equivalent oxygen, and one [O$_b$$^9$La$_4$B$_2$] environment. While there are only two formally non-bridging oxygen in the system, the lanthanum polyhedron is involved with the bonding of all of the bridging oxygen in the system.

In the glass, all three cations display radical differences in both the short-range and intermediate-range order. $^{11}$B MAS NMR had previously been used by Gupta et al. to establish the presence of BO$_3$ triangles in the glass, but they did not identify the BO$_3$ species present [34]. Based upon the $^{11}$B MQMAS spectra of the $x$La$_2$O$_3$-(100-x)(0.33B$_2$O$_3$–0.67GeO$_2$) samples (where $x = 25$ corresponds to LaBGeO$_5$), we conclude that there is a single BO$_3$ species present in any appreciable
amount (Section 6.6.1). Using the $^{11}$B MQMAS spectra and our own $^{11}$B MAS spectra, we obtain the mean $\delta^{CS}_{iso}$ values of the BO$_3$ environment in the glass and correlate them to a crystalline model from the literature [48]. We conclude that the BO$_3$ units present in the glass are BO$_2$O$^-$, [BO$_4$]$^-$ units are still present in the LaBGeO$_5$ glass.

The changes in the germanium environment are less precisely known, but no less dramatic. Our neutron diffraction results (Section 6.6.2) support the partial conversion of the GeO$_2$O$_2$$^{2-}$ tetrahedra in the crystal to either [GeO$_5$]$^-$, [GeO$_6$]$^{2-}$, or both. The uncertainty in the coordination numbers determined by neutron diffraction preclude useful numerical modelling. It is not immediately obvious whether the mean number of non-bridging oxygen per GeO$_4$ unit is changed; however, the $^{17}$O MAS and MQMAS spectra of the glass support the presence of Ge–O–La environments, which are not expected to be found on [GeO$_5$]$^-$ or [GeO$_6$]$^{2-}$.

In the glass, the lanthanum-oxygen coordination number is less than it is in the crystal. Neutron diffraction data (Chapter 6) suggest that the mean $n_{LaO}$ in the glass is about 7. Unfortunately, due to differences between $\delta^{CS}_{iso}$ and $\Delta_{offset}$ in the Czjzek fit of our $^{139}$La WCPMG NMR spectra, we cannot obtain an estimate of $n_{LaO}$ via NMR spectroscopy (Chapter 5). However, our $^{139}$La WCPMG NMR spectra of LaBGeO$_5$ glass are consistent with a structure where the environment of La$^{3+}$ is disordered, and inconsistent with the presence of periodic structures. Like with the germanium results, the $n_{LaO}$ data are not sufficient to provide an estimate of the exact distribution of La-O polyhedra. However, obtaining an estimate for the mean $n_{LaO}$ in the glass informs the analysis of the intermediate-range order of the glass.

Determining the intermediate-range order of the glass is more difficult than the short-range order, as there are fewer experimental probes which are easily applicable. $^{11}$B{$^{10}$B} REDOR NMR spectroscopy is perhaps the most direct probe of the intermediate range order, at least for the borate species. Our REDOR results support a structure where the [BO$_4$]$^-$ units of the glass are approximately equally as likely to be connected to BO$_2$O$^-$ units as [BO$_4$]$^-$ units. Furthermore, the number of [BO$_4$]$^-$$^-[BO$_4$]$^-$$ connections is lower in the glass than in the crystal. A change in the [BO$_4$]$^-$$ environment is supported by the $^{11}$B MAS NMR spectra; the [BO$_4$]$^-$$ environment in the glass has a significantly different $\delta^{CS}_{iso}$ than the [BO$_4$]$^-$$ environment in the glass, consistent with a change in connectivity.
With the data above, we can begin to construct a model of the connectivity of the borate units. The $\text{BØ}_2\text{O}^{-}$ species are, on average, expected to have two $\text{La}^{3+}$ neighbours, one $[\text{BØ}_4]^{-}$ neighbour, and one other neighbour, likely a germanate. A single $\text{La}^{3+}$ neighbour is expected due to the presence of a non-bridging oxygen; the clustering of two $\text{La}^{3+}$ is expected due to the mean $n_{\text{LaO}}$ value (Section 6.6.4). One $[\text{BØ}_4]^{-}$ neighbour is expected due to the $^{11}\text{B}\{^{10}\text{B}\}$ REDOR results. The remaining neighbour is not precisely determined, as our REDOR results cannot probe possible $\text{BO}_3$–$\text{BO}_3$ connections. However, the $^{17}\text{O}$ NMR spectra support the significant presence of Ge–O–B environments, and as will be discussed below, the absence of certain Ge–O–Ge environments; additionally, in lanthanum-free borogermanate glasses, $\text{BO}_3$ and $\text{GeØ}_4$ units mix freely ([53], Section 6.6.2). Both of these points increase the likelihood of $^3\text{B}–\text{O}–\text{Ge}$ linkages.

The $[\text{BØ}_4]^{-}$ species are, on average, expected to have about one $\text{BØ}_2\text{O}^{-}$ neighbour and about one $[\text{BØ}_4]^{-}$ neighbour. As $[\text{BØ}_4]^{-}$ is not generally expected to contain non-bridging oxygen, its other two neighbours are expected to be germanate units. We might expect that $^4\text{B}–\text{O}–^4\text{Ge}$ connections are more likely than $^4\text{B}–\text{O}–^{5/6}\text{Ge}$ connections based upon the respective charges of the units. The $[\text{BØ}_4]^{-}$ species has a formal, non-local negative charge, as do both $[\text{GeØ}_5]^{-}$ and $[\text{GeØ}_6]^{2-}$. Lowenstein’s rule (i.e., tetrahedral avoidance in aluminates) suggests that charged species are less likely to be neighbours [302]. However, for a high field strength cation such as $\text{La}^{3+}$, tetrahedral avoidance has less of an impact on structure [41]. Hence we cannot identify a preference in the germanate neighbours of $[\text{BØ}_4]^{-}$ units.

Determining the connectivity of the germanate units is less straightforward than for the borate units. The most direct probe of the connectivity of germanate units is $^{17}\text{O}$ MAS and MQMAS spectroscopy. Our $^{17}\text{O}$ NMR results support the presence (or absence) of five main Ge–O–X connectivities: $^4\text{Ge}–\text{O}–\text{La}$, $^4\text{Ge}–\text{O}–^4\text{Ge}$, $^4\text{Ge}–\text{O}–\text{B}$, $^4\text{Ge}–\text{O}–^{5/6}\text{Ge}$, and $^{5/6}\text{Ge}–\text{O}–\text{B}$. We discuss them in order.

Both the $^{17}\text{O}$ MAS and MQMAS spectra clearly show the presence of Ge–O–La environments (Chapter 7). These NBO environments are not typically associated with high-coordinate germanate species [62], and so we conclude that the Ge–O–La environments are specifically $^4\text{Ge}–\text{O}–\text{La}$ environments. While our $^{17}\text{O}$ NMR are non-quantitative for various reasons, we expect that the average $\text{GeO}_4$ unit will have two
NBO present, i.e., GeO$_2$O$_2^{2−}$. This is due to simple charge compensation: between both BØ$_2$O$^−$ and [BØ$_4$]$^−$ species, the average borate unit has a single negative charge. As the number of borate and germanate units are equal, and together they must compensate for the positive charge of the La$^{3+}$ ions, the mean germanate unit must be doubly negatively charged. In four-coordinate germanate units, this would manifest as two non-bridging oxygen per bonding unit.

A connection which appears to be absent in the LaBGeO$_5$ glass is $^4$Ge–O–$^4$Ge. This environment, which we would expect to be observable via $^{17}$O MQMAS NMR, is not observed within our limit of detection. The avoidance of $^4$Ge–O–$^4$Ge is not expected from any fundamental rule, but could be due to some combination of dilution or steric effects. However, it must be accounted for in the connectivity model.

The MQMAS results support the presence of $^4$Ge–O–B environments, where the boron can either be BØ$_2$O$^−$ or [BØ$_4$]$^−$; the resolution of the MQMAS spectra is insufficient to separate the two. This supports the $^{11}$B{$^{10}$B} REDOR results described above, which suggested the presence of Ge–O–B links for both BØ$_2$O$^−$ and [BØ$_4$]$^−$. Considering the amount of intra-borate connectivity, even with conservative assumptions regarding the type and amount of high-coordinate germanium, there are more than sufficient germanate units to account for the proposed borate-germanate connectivity. However, due to a lack of quantifiable data, it is difficult to draw further conclusions regarding the $^4$Ge–O–B connectivity, and whether there exists a bias towards one of the borate units.

Connectivity between $^4$Ge and $^{5/6}$Ge is supported by the $^{17}$O MQMAS NMR spectrum. However, due to the inherently non-quantitative nature of MQMAS NMR, and the poor resolution of the $^{17}$O MAS spectrum, we cannot speculate on the probability of connectivity between these two units beyond saying that the connection does occur.

The final germanate unit connectivity we explore is $^{5/6}$Ge–O–B. This assignment is based on $^{17}$O MQMAS NMR data, and is somewhat speculative (Chapter 7). This connectivity motif has not been previously reported in the literature, but we conclude that this is due to a general lack of data on the structure of borogermanate glasses; in this case, absence of evidence is not good evidence of absence. It is possible that there could be some mutual avoidance of high-coordinate germanium and [BØ$_4$]$^−$ units, due
to both being non-locally negatively charged, but this would not apply to the $\text{BO}_2\text{O}^-$ units; also, as described above, avoidance may not apply with a high field strength cation such as $\text{La}^{3+}$.

Considering all possible connectivities for the germanate units, it becomes apparent that the germanate units are highly interconnected with the borate units, regardless of coordination. There is more connectivity between germanate units in the glass than in the crystal, as the former has $^4\text{Ge}–\text{O}–{}^{5/6}\text{Ge}$ connectivity while neither has $^4\text{Ge}–\text{O}–{}^4\text{Ge}$ connectivity. There is no obvious segregation of either borates or germanates of any coordination.

The connectivity of the lanthanum present in the LaBGEO$_5$ glass is largely described by the discussion above, but there remains two important and related points to discuss: namely, lanthanum clustering and the possibility of free oxygen ($i.e., \text{La}–\text{O}–\text{La}$ with no B or Ge neighbour). Our EBS analysis in Section 6.6.4 suggests that, within the assumptions of the EBS model, a single $\text{La}^{3+}$ ion is insufficient to compensate NBO on either $\text{BO}_2\text{O}^-$ or $\text{GeO}_2\text{O}_2^{2–}$. With a mean $n_{\text{LaO}}$ of approx. 7, at least two $\text{La}^{3+}$ are required. At first, this would appear to imply that $\text{La}^{3+}$ are clustered in the glass. However, there is no indication of the presence of free oxygen in the $^{17}\text{O}$ NMR spectra. This limits the potential size of any cluster: no lanthanum can be isolated from the borate or germanate network.

Instead of clustering, it might be more direct to describe $\text{La}^{3+}$ as being regularly distributed in the glass. Every borate and germanate structural unit described above requires one or more $\text{La}^{3+}$ ions in close proximity. There is no plausible mechanism for phase separation into a $\text{La}^{3+}$-rich phase and a $\text{La}^{3+}$-poor phase, as no plausible borate or germanate units that would support such phase separation are observed.

With the above knowledge of the short-range and intermediate-range order of the LaBGEO$_5$ glass, we can construct an overall model of its structure. The LaBGEO$_5$ glass is highly disordered. Various borate, germanate, and lanthanate polyhedra are intimately connected. Every type of borate unit is connected to every other type of unit. Only connections between the $\text{GeO}_2\text{O}_2^{2–}$ tetrahedra appear to be absent. Lanthanum ions are spread evenly throughout the glass network. There are no indications of phase separation of any sort. A plausible pictorial representation of the glass structure is presented in Fig. 9.1.
Figure 9.1: A pictorial representation of a plausible configuration of the LaBGeO$_5$ glass structure. The presented structure is manually constructed, and should only be used as a visual aid in interpreting the connectivity of the glass. Germanium ions are purple. Oxygen ions are red. Borate ions are dark green and small. Lanthanum ions are light green and large. The colours are consistent with presentations of the LaBGeO$_5$ crystal (e.g., Fig. 2.5.) Note that lanthanum-oxygen interactions are only presented for non-bridging oxygen, and do not represent the full scale of lanthanum coordination.
Our structural model contradictions the model proposed by Sigaev et al. [75]. We observe high-coordinate germanium via several means, a structural unit that was previously discounted. We do not observe “crystal-like” regions in any experiment. We show that the three-coordinate boron and four-coordinate boron are not segregated; in fact, our results indicate that the opposite is true.

Several inferences can be drawn from our model, which may be informative for future work on the LaBGeO$_5$ system and related materials. The homogeneity of the glass structure precludes the formation of clusters or structures that could provide obvious nucleation sites. The presence of high-coordinate germanate units in the LaBGeO$_5$ glass helps to explain why a LaBSiO$_5$ glass cannot be formed; silicate and germanate behaviour is similar when both are four-coordinate, but higher coordinate silicate units do not generally form at standard pressures.

The difference in structure between the LaBGeO$_5$ crystal and glass would suggest a heterogeneous nucleation mechanism [35, 52]. Thermal analysis by Sigaev et al. suggest that surface (i.e., heterogeneous) crystallization is dominant in the LaBGeO$_5$ glass [239]. This is supported by the large value for $T_{gr}$ in the LaBGeO$_5$ system; with $T_g = 938$ K and $T_m = 1467$ K, $T_{gr} = 0.64$ [96, 303]. It has been shown that, at least for silicate glasses, systems where $T_{gr} > 0.6$ will only undergo heterogeneous nucleation [32]. What, then, is the source of the nucleation sites in the LaBGeO$_5$ glass? The system has been shown to undergo bulk nucleation, both by bulk thermal heating and by local laser-heating [2, 34]. We suspect that LaBO$_3$ plays a role in the heterogeneous nucleation of LaBGeO$_5$. LaBO$_3$ is a common co-precipitant in the La$_2$O$_3$–B$_2$O$_3$–GeO$_2$ system, regardless of the precise stoichiometry. Indeed, only three crystals (i.e., LaBGeO$_5$, LaBO$_3$, and La$_2$Ge$_2$O$_7$) commonly precipitate from La$_2$O$_3$–B$_2$O$_3$–GeO$_2$ glasses. We see evidence of LaBO$_3$ in our partially-crystallized samples (Chapter 7), but no trace of La$_2$Ge$_2$O$_7$. It is possible that the greater glass-forming ability of the La$_2$O$_3$–B$_2$O$_3$ composition plays a role, as the La$_2$O$_3$–GeO$_2$ glass system only forms glasses for quite low (< 10 %) La$_2$O$_3$ contents. Stone has observed LaBO$_3$ occasionally precipitating during the laser-writing process, but not ubiquitously [106]. Lipat’ev et al. have recently suggested the possibility of the formation of Ge nanoparticles acting as heterogeneous nucleation sites for LaBGeO$_5$ [304]. The most direct way to test either hypothesis is synchrotron X-ray diffraction with micrometer spatial resolution; no
other technique has the required combination of spatial resolution and crystallographic specificity to definitively address the question of structural changes during nucleation.

9.2 Limitations of the Work

There is much work that could be done to either improve or build upon the results described herein. A major limitation of the structural model that we have constructed is that it is mostly qualitative. There are several reasons for this.

The neutron diffraction data (Chapter 6) are limited in their quantitative precision due to the uncertainty regarding the precise compositions of the samples studied. While the data unambiguously support the presence of $^{5/6}\text{Ge}$, they cannot be used to construct a meaningful quantitative model. In principle, repeating the experiment while giving exacting attention to the composition is possible, but the difficulty in accurately quantifying boron in ternary glasses remains a barrier.

The $^{17}\text{O}$ MAS NMR spectra (Chapter 7) do not have sufficient resolution between environments to allow for quantitative fitting; there are simply too many unconstrained or minimally constrained parameters for such a fit to be unique. While the peak resolution in the $^{17}\text{O}$ MQMAS NMR spectra is substantially improved, MQMAS NMR spectra are inherently non-quantitative in intensities. The same is true for the $^{11}\text{B}\{^{10}\text{B}\}$ REDOR experiments, due to the integer nature of the $^{10}\text{B}$ nuclear spin number. Rotational-echo adiabatic passage double-resonance (REAPDOR) NMR spectroscopy, a pulse sequence related to REDOR, has been suggested as an improvement over REDOR, though it is still unclear if precisely quantitative results can be achieved with integer spin nuclides.

The application of $^{139}\text{La}$ NMR to glasses (Chapter 5) is limited by two main factors. The first is that the two fit parameters in the Czjzek model ($\sigma$ and $\Delta_{\text{offset}}$) are not directly comparable to spectral parameters available in crystals. While we can still extract a quantitative indication of the level of disorder in the glass via $\sigma$, this parameter is not useful for modelling specific local structural features.

The second limitation is the strength of the $^{139}\text{La}$ NMR crystalline model (Chapter 4). Even if $\delta_{\text{iso}}^{\text{CS}}$ were to be obtained for the Czjzek model in lanthanum oxide-based glasses, any structural information it contained would be from comparison with data
from crystals. There are a limited number of suitable lanthanum oxide-based crystalline compounds for the construction of the crystal model. An ideal model compound would have easily resolved peaks corresponding to crystallographic sites with a range of coordination numbers. Given the extreme breadth of most $^{139}$La NMR peaks, this is unlikely to occur. Furthermore, in order to properly assess how $\delta_{iso}^{CS}$ responds to changes in CN in samples with differing compositions, a wide composition range in a binary system would be desirable. Unfortunately, there are very few crystal structures within a chemical series; for example, there are only two $x\text{La}_2\text{O}_3-(1-x)\text{B}_2\text{O}_3$ crystal structures which have manageable $^{139}$La NMR spectra. The lack of suitable crystal structures and series is a significant barrier to improving on the crystalline model.

The development of the $^{139}$La NMR crystalline model would also benefit from a more rigorous statistical approach. An *a priori* analysis of the statistical power of the model would be helpful to estimate the number of samples required for the model to be generally useful, regardless of composition effects. In general, pre-registering the hypothesis of the work and the methodology for the analysis is good practice, and should be considered for studies relying on statistical analysis.

### 9.3 Future Work

Despite the advances of our model, there are many avenues for further investigation. A major question which remains unanswered is the specific nucleation mechanism of the $\text{LaBGeO}_5$ glass-ceramic. While probing the nucleation mechanism of the $\text{LaBGeO}_5$ glass was an initial goal of this work, studying the formation of nuclei is quite difficult. The interface between glass and nucleus occupies a very small volume, both in absolute and relative terms. This complicates the study of the nucleation interface via NMR spectroscopy or neutron diffraction, as both techniques require significant sample volume to be effective. While we were unable to directly probe the nucleation of the $\text{LaBGeO}_5$ system, our data support a hypothesis where the nucleation is heterogeneous.

Probing the structure of a glass-crystal interface is not straightforward, due to the small size and quantity of the interface. Beyond the synchrotron X-ray diffraction mentioned above, one potentially interesting approach would be to use molecular dynamics simulations to model the interface. This is not a simple undertaking; due to the complexity of the glass structure, the number of atoms present in the simulation
would have to be quite large, on the order of thousands. Substantial validation of the simulation approach would also be required, preferably with direct comparison to experimental diffraction data (that we have conveniently provided). Even if directly modelling the interaction between crystal and glass is too computationally demanding, molecular dynamics simulations of the glass could help evaluate both our proposed structure of the glass, as well as the validity of the application of the Czjzek model to our $^{139}\text{La}$ NMR spectra of the glass.

Beyond the pure LaBGeO$_5$ glass, other examples of the stillwellite system have lingering questions. For example, does doping LaBGeO$_5$ glass (e.g., with Er$^{3+}$) cause structural changes which are significant to the crystallization process? While the LaBSiO$_5$ composition does not form a glass, other lanthanum borosilicate compositions do; is the behaviour of equivalent lanthanum borogermanates similar up until a critical point, or are there no structural similarities?

Along with the questions about the LaBGeO$_5$ system in particular, our work reveals an under-studied glass composition: the borogermanates. Compared to borosilicate glasses, there are very few structural studies of borogermanates. Our results in Chapter 6 suggest the possibility of a so-called “borogermanate anomaly”, but the miscibility gap in the lanthanum borogermanate series we studied prevented this from being thoroughly tested. Speaking generally, how does the presence of borate units affect the evolution of germanium coordination, and vice versa?

The study of the crystallization mechanism of the LaBGeO$_5$ glass-ceramic composite has an obvious route to improve upon it: the direct study of a laser-written sample. The samples studied in Chapter 7 were crushed and ground to powder in order to study them by both ssNMR and pXRD. This is not a precise recreation of the conditions of the laser-writing process. However, reproducing the laser-writing apparatus was beyond the means and scope of this work.

With the benefit of hindsight, many improvements could be made to the work contained herein. Beyond these improvements, however, a number of novel experiments could be attempted to provide new or complementary information.

To improve the experimental model of the glass structure, $^{17}\text{O}\{^{11}\text{B}\}$ REDOR NMR could potentially be useful; by selectively dephasing $^{17}\text{O}$ environments in proximity to $^{11}\text{B}$, the peaks from Ge–O–Ge and Ge–O–La could be resolved from the
Ge–O–B, B–O–B, and B–O–La environments. This could potentially identify enough constraints to make a fit of the $^{17}\text{O}$ spectrum reasonably quantitative. Additional information on the La–O environment could be obtained via the use of X-ray diffraction (which minimizes O–O interactions), or from neutron diffraction with isomorphous substitution (e.g., replacing La$^{3+}$ with chemically similar, but diffraction-distinct, Ce$^{3+}$); either would be helpful to clarify precisely which high-$r$ interactions are due to La–O polyhedra.

Exploring the differences between the lanthanum borosilicate and lanthanum borogermanate systems, as well as probing the possibility of a borogermanate anomaly, both require the same approach. Make samples (paying special attention to composition control), collect NMR spectra, collect diffraction data, and evaluate. These experiments do not need novel methods, but they do require due care and attention to ensure that quantitative results can be obtained. To probe a borogermanate anomaly in particular, a ternary borogermanate system with a wide composition range would be preferable; sodium borogermanate glasses seem to be a likely candidate.

### 9.4 Final Summary

We have produced a model for the structure of the LaBGeO$_5$ glass. This structure was constructed using $^{10}$B, $^{11}$B, $^{17}$O, and $^{139}$La NMR spectroscopies in conjunction with neutron diffraction and DFT modelling. The structure of the LaBGeO$_5$ glass is radically different from the structure of the LaBGeO$_5$ crystal, with changes in both the local structure and connectivity of all constituent elements. The stress environment in LaBGeO$_5$ laser-written crystals-in-glass is expected to be anisotropic. Our results strongly support the hypothesis of heterogeneous nucleation in this system. We hope to see this work built upon in the future.
Appendix A

PAW and Pseudopotential Generation

A.1 Software

Both PAW datasets and NCPPs were used over the course of this work. PAW datasets were generated from input based upon version 0.2 of the JTH set [191]. PAW datasets were altered to avoid PAW sphere overlap, which is disallowed by the PAW formalism [155]. Version 4.0.0.12 of the atompaw computer program was used to generate the PAW datasets [165]. NCPPs were generated from input based upon the ONCVPSP set of NCPPs distributed by the ABINIT project [164]. The NCPP were based upon the ONCVPSP-PBE-PDv0.3 set, and were converted to the LDA exchange-correlation (XC) functional using version 3.3.0 of the ONCVPSP code [165].

A.2 PAW Datasets

The details of the construction of the datasets for each atom used are described below. They are accompanied by the ATOMPW input script used for their generation. If an atom is omitted, then for all structures studied involving that element the default JTH radius was sufficient to avoid overlap. In oxide materials, PAW sphere overlap is most likely to occur between oxygen and its neighbours, particularly lighter elements such as boron.
A.2.1 Boron

Our boron dataset significantly decreases the PAW radius, from the default value of 1.7 bohr to 1.2 bohr. This is required due to the length of the typical B–O bond length, which is approximately 2.7 bohr. The boron dataset treats the $2s^22p^1$ electrons as the valence space.

```
B 5
GGA-PBE loggrid 700
2 2 0 0 0 0
2 1 1
0 0 0
1
1.20
y
0
n
y
1
n
custom rrkj
2 0 ultrasoft
1.2
1.2
1.2
1.2
```
A.2.2 Oxygen

Our oxygen dataset uses a decreased PAW radius of 1.30 bohr, down from the default value of 1.41 bohr. This helps resolve the many PAW overlaps which occur with the default radius.

```
0 8
GGA-PBE loggrid 700
2 2 0 0 0 0
2 1 4
0 0 0
c v v
v 1
1.30 y
-0.664 n y
-1.757 n
custom rrkj
2 0 ultrasoft
1.30 1.30 1.30 1.30
```
A.2.3 Silicon

Our silicon dataset uses a decreased PAW radius of 1.55 bohr, down from the default value of 1.90 bohr.

```
Si 14
GGA-PBE loggrid 1000
3 3 0 0 0 0
3 1 2
0 0 0
c
c
v
c
v
1
1.55 1.55 1.0 1.55
y
1
n
y
2
n
custom rrkj
bessel
1.55
1.55
1.55
1.55
1.55
```
A.2.4 Phosphorus

Our phosphorus dataset uses a decreased PAW radius of 1.50 bohr, down from the default value of 1.9 bohr.

```
P 15
GGA-PBE loggrid 1201
3 3 0 0 0 0
3 1 3
0 0 0
c
c
v
c
v
1
1.50
y
1.5
n
y
2
n
custom rrkj
2 0 ultrasoft
1.5
1.5
1.5
1.5
1.5
```
A.2.5 Germanium

Our germanium dataset uses a decreased PAW radius of 1.85 bohr, down from the default value of 2.30 bohr.

Ge 32

GGA-PBE scalarrelativistic loggrid 2000
4 4 3 0 0 0
4 1 2
0 0 0
c
c
c
c
v

1
1.85 1.85 1.5 1.85

y
4
n
y
2
n

custom rrkj

bessel
1.85
1.85
1.85
1.85
1.85
A.3 Optimized Norm-Conserving Vanderbilt Pseudopotentials

NCPPs were generated from the ONCVPSP-PBE-PDv0.3 set. This set uses the PBE XC functional. In order to convert a NCPP from the PBE XC functional to the LDA XC functional, we change the \texttt{iexc} flag in the input file from 4 (for PBE) to 3 (for LDA). As we did not further modify the NCPPs, we do not provide the input for their generation. The boron NCPP was obtained from the ABINIT website. The lanthanum, germanium, and oxygen NCPPs were taken as is from the \textit{ONCVPSP}-3.3.0 distribution.
Appendix B

Supporting Information for Chapter 4: Relating $^{139}$La Quadrupolar Coupling Constants to Polyhedral Distortion in Crystalline Structures

B.1 Context

This appendix was originally published as supporting information to Chapter 4, published as as Paterson, A.L., Hanson, M.A., Werner-Zwanziger, U., and Zwanziger, J.W., Relating $^{139}$La Quadrupolar Coupling Constants to Polyhedral Distortion in Crystalline Structures, J. Phys. Chem. C 119 (45) (2015) 25508–25517 [126]. Copyright 2015 American Chemical Society. The content below has been reproduced from the preprint version of the supporting information, with minor modifications for style and clarity.

B.2 Powder X-ray Diffractograms

The X-ray diffractograms are compared to PDFs obtained from the PDF-2 2002 database.
Figure B.1: PXRD diffractogram of LaPO$_4$ · 1.8H$_2$O. Circles indicate peaks from LaPO$_4$ · 0.5H$_2$O (PDF 46-1439). Squares indicate peaks from La$_2$O$_2$CO$_3$ (PDF 84-1963). Diamonds indicate peaks from La(OH)$_3$ (PDF 36-1481).

Figure B.2: PXRD diffractogram of LaPO$_4$. Circles indicate peaks from La$_2$O$_2$CO$_3$ (PDF 84-1963). All other peaks are from LaPO$_4$ (PDF 83-651).

Figure B.3: PXRD diffractogram of LaBO$_3$. All observed peaks are from LaBO$_3$ (PDF 12-0762).
Figure B.4: PXRD diffractogram of LaBGeO$_5$. All observed peaks are from LaBGeO$_5$ (PDF 41-659).

Figure B.5: PXRD diffractogram of LaBSiO$_5$. All observed peaks are from LaBSiO$_5$ (PDF 50-237).

Figure B.6: PXRD diffractogram of La$_2$(SO$_4$)$_3$·9H$_2$O (PDF 89-6401). Circles indicate peaks from an unidentified impurity.
Figure B.7: PXRD diffractogram of $\text{La}_2(\text{CO}_3)_3 \cdot 8\text{H}_2\text{O}$. Circles indicate peaks from $\text{La}_2(\text{CO}_3)_3 \cdot 8\text{H}_2\text{O}$ (PDF 73-439). Squares indicate peaks from $\text{La}(\text{CO}_3)(\text{OH})$ (PDF 26-815).
B.3 Thermogravimetric Analysis

TGA was carried out using an aluminium crucible in a NETSCH TG 209 F3 instrument under nitrogen atmosphere. 33.05 mg of LaPO$_4$·$n$H$_2$O was heated from room temperature to 500 °C based upon previous literature studies [305]. 31.71 mg of La$_2$(SO$_4$)$_3$·$n$H$_2$O was heated from room temperature to 800 °C based upon previous literature studies [306]. Based upon these plots (with 12.37 % and 22.44 % mass loss, respectively), we calculate $n$ of LaPO$_4$·$n$H$_2$O to be 1.84, and $n$ of La$_2$(SO$_4$)$_3$·$n$H$_2$O to be 9.08.
Table B.1: Atomic parameters used in this study. The electronic valence of each atom is given, along with the configuration of the PAW projectors. The PAW radius is provided in atomic units. All nuclear quadrupole moment were taken from the literature [57]. Any atom not mentioned here was taken without alteration from the JTH PAW dataset table.

<table>
<thead>
<tr>
<th>Atom</th>
<th>Valence</th>
<th>Projected</th>
<th>Radius / bohr</th>
<th>Q / b</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1s(^1)</td>
<td>3s</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>2s(^2)2p(^1)</td>
<td>2s, 2p</td>
<td>1.2</td>
<td>0.04059</td>
</tr>
<tr>
<td>O</td>
<td>2s(^2)2p(^4)</td>
<td>2s, 2p</td>
<td>1.3</td>
<td>-0.02558</td>
</tr>
<tr>
<td>Si</td>
<td>3s(^2)3p(^2)</td>
<td>2s, 2p</td>
<td>1.55</td>
<td>0</td>
</tr>
<tr>
<td>P</td>
<td>3s(^2)3p(^3)</td>
<td>2s, 2p</td>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>S</td>
<td>3s(^2)3p(^4)</td>
<td>2s, 2p</td>
<td>1.45</td>
<td>-0.0678</td>
</tr>
<tr>
<td>Ge</td>
<td>4s(^2)4p(^2)</td>
<td>2s, 2p, 1d</td>
<td>1.85</td>
<td>-0.196</td>
</tr>
</tbody>
</table>

Table B.2: Specific details regarding the calculations performed in this study. Both the plane-wave cutoff and the PAW fine grid cutoff are in hartree. All calculations save LaScO\(_3\) and LaNbO\(_4\) used shifted Monkhorst-Pack grids of the dimensions below; LaScO\(_3\) and LaNbO\(_4\) used grids not parallel to the reciprocal lattice vectors.

<table>
<thead>
<tr>
<th>System</th>
<th>ecut</th>
<th>pawecutdg</th>
<th>k-point Grid</th>
<th>Grid Spacing / Å(^-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>La(_2)O(_3)</td>
<td>30</td>
<td>140</td>
<td>10 × 10 × 7</td>
<td>0.03</td>
</tr>
<tr>
<td>LaPO(_4)·0 H(_2)O</td>
<td>45</td>
<td>150</td>
<td>6 × 6 × 6</td>
<td>0.03</td>
</tr>
<tr>
<td>LaPO(_4)</td>
<td>45</td>
<td>150</td>
<td>4 × 4 × 5</td>
<td>0.04</td>
</tr>
<tr>
<td>LaBO(_3)</td>
<td>45</td>
<td>150</td>
<td>7 × 4 × 6</td>
<td>0.03</td>
</tr>
<tr>
<td>LaBGeO(_5)</td>
<td>45</td>
<td>105</td>
<td>6 × 6 × 6</td>
<td>0.02</td>
</tr>
<tr>
<td>LaBSiO(_5)</td>
<td>45</td>
<td>150</td>
<td>5 × 5 × 5</td>
<td>0.03</td>
</tr>
<tr>
<td>La(_2)(SO(_4))(_3)·9 H(_2)O</td>
<td>25</td>
<td>75</td>
<td>3 × 3 × 4</td>
<td>0.03</td>
</tr>
<tr>
<td>La(OH)(_3)</td>
<td>40</td>
<td>150</td>
<td>7 × 7 × 12</td>
<td>0.02</td>
</tr>
<tr>
<td>LaAlO(_3)</td>
<td>35</td>
<td>110</td>
<td>8 × 8 × 8</td>
<td>0.02</td>
</tr>
<tr>
<td>LaCoO(_3)</td>
<td>40</td>
<td>150</td>
<td>9 × 9 × 9</td>
<td>0.02</td>
</tr>
<tr>
<td>LaCrO(_3)</td>
<td>45</td>
<td>150</td>
<td>6 × 6 × 4</td>
<td>0.03</td>
</tr>
<tr>
<td>LaScO(_3)</td>
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<td>40</td>
<td>0 5 3</td>
<td>0.03</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>5 0 3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5 5 0</td>
<td></td>
</tr>
<tr>
<td>LaNbO(_4)</td>
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<td>90</td>
<td>-4 0 4</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-4 -8 12</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4 4 -4</td>
<td></td>
</tr>
</tbody>
</table>
B.5 Crystal Analysis

B.5.1 Lanthanum Oxide

La$_2$O$_3$ has been previously investigated by $^{139}$La ssNMR: first by Bastow in 1994, using a frequency-swept spin-echo technique [59]; and more recently by Spencer et al. using a WURST echo [60]. Given the thorough understanding of La$_2$O$_3$ in the literature, it is a good choice as a test compound for the implementation of WCPMG and VOCS, as well as for validating computational methods. La$_2$O$_3$ crystals are hexagonal and of space group $P\overline{3}m1$ with 1 formula unit per unit cell [219]. The lanthanum environment in La$_2$O$_3$ is the simplest of all of the compounds investigated in this work, described by Spencer et al. as “pseudo-octahedral”. The LaO$_7$ coordination polyhedron is more precisely referred to as a face-capped octahedron, with two sets of three La–O bonds and the “cap” bond with lengths of 2.37 Å, 2.73 Å, and 2.46 Å respectively.

Our measured $C_Q$ for La$_2$O$_3$ is (58.6 ± 0.3) MHz with an $\eta_Q$ of 0.00 ± 0.03, shown in Fig. B.9, in excellent agreement with the most recent literature values as reported by Spencer et al. [60]. We used the same CSA values as Spencer et al. in fitting both our 9.4 T and 16.4 T (Fig. B.10) spectra to good effect.

DFT calculations of the EFG at the La$^{3+}$ site using the experimental geometry [219] yielded a $C_Q$ of 60.5 MHz and an $\eta_Q$ of 0.0, which is in good agreement with the experimental results. When examining the calculated EFG tensor, the largest component $V_{zz}$ is aligned with the La–O bond of the cap oxygen, and is parallel to the $c$ axis of the unit cell as shown in Fig. B.11. The $c$ axis in La$_2$O$_3$ possesses threefold rotational symmetry, which is consistent with both the observed and calculated $\eta_Q$ of 0.

The least-squares ellipsoid that is fit to the LaO$_7$ polyhedron in La$_2$O$_3$ is a prolate spheroid, with semi major axes $e_a = e_b = (2.481 \pm 0.002)$ Å and $e_c = (2.731 \pm 0.002)$ Å and ellipsoid character of 1.00 ± 0.01. There is a moderate departure from spherical symmetry, with $\epsilon = 0.098 \pm 0.002$. The ellipsoid is aligned with the EFG tensor, with the largest semi major axis $e_c$ parallel with $V_{zz}$, and the two other semi major axes in the plane defined by $V_{xx}$ and $V_{yy}$, perpendicular to $V_{zz}$.
Figure B.9: Static $^{139}$La NMR spectrum of La$_2$O$_3$. Analytical simulation is show in red. The EFG and CSA parameters used are given in Table 4.1.
Figure B.10: Static $^{139}$La NMR spectra of $\text{La}_2\text{O}_3$. Both spectra are set to the same frequency scale. Top: Spectrum collected at 16.4 T. Bottom: Spectrum collected at 9.4 T.
Figure B.11: First coordination sphere of $\text{LaO}_7$ in $\text{La}_2\text{O}_3$. The $V_{xx}$, $V_{yy}$, and $V_{zz}$ components are displayed as blue, green, and red, respectively. The $e_a$, $e_b$, and $e_c$ semi-major axes are in yellow, magenta, and teal respectively.
B.5.2 Lanthanum Phosphate Hydrate

LaPO$_4 \cdot n$H$_2$O, also known as rhabdophane, is a rare-earth phosphate mineral that has been proposed for use in environmental phosphate sequestration [172]. The structure of rhabdophane was first proposed by Mooney in 1950 [307]. Rhabdophane is reported to be of the hexagonal $P6_222$ space group, with zeolitic channels parallel to the $c$ axis and 3 formula units per cell. The lanthanum environment is eightfold coordinate to oxygen, with bond lengths reported: four bonds of length $2.34 \, \text{Å}$ and four of length $2.66 \, \text{Å}$. The reported bond lengths, however, do not include any possible zeolitic water contributions. There are no obvious symmetry elements in the LaO$_8$ polyhedron that would force a particular value of $\eta_Q$.

The sample of LaPO$_4 \cdot n$H$_2$O used was determined to be LaPO$_4 \cdot 1.8$H$_2$O through the use of TGA (Fig. B.8). Our $^{139}$La NMR spectrum of this sample is fit using a $C_Q$ of $(33 \pm 1)$ MHz with an $\eta_Q$ of $1.00 \pm 0.05$ (Figure S12). This conflicts with the spectrum reported by Dithmer et al. [172], where they observed a broad, largely featureless resonance. The difference may result from different methods of preparation, resulting in dramatically differing crystallinity.

The DFT calculations conducted on the experimental geometry provided by Mooney [307] yield a $C_Q$ of $-136 \, \text{MHz}$ and an $\eta_Q$ of 0.5, which is significantly at odds with the experimental results. While the difference can be explained in part by the absence of zeolitic water in the structure used for the calculation, it is unlikely for a
small amount of water to cause such a large disparity in $C_Q$. It has been suggested by Mesbah et al. that the structure of LnPO$_4 \cdot n$H$_2$O, including LaPO$_4 \cdot n$H$_2$O, is in fact monoclinic [308]. However, the structure proposed by Mesbah et al. contains six unique lanthanide sites, which are not resolved in the spectrum of LaPO$_4 \cdot 1.8$H$_2$O. Due to the extreme difference between experimental and calculated results, we place little stock in the orientation of the EFG tensor determined through our DFT calculations.

The LaO$_8$ polyhedron in LaPO$_4 \cdot n$H$_2$O as reported by Moody is poorly fit by either a sphere or an ellipsoid, and as such is not included in any structural models.

### B.5.3 Lanthanum Phosphate

Anhydrous LaPO$_4$ is better known as the mineral monazite [223]. Monazite was historically a commercial source of lanthanides, but it fell into disuse in part due to the presence of the radioactive daughter ions of thorium. The lanthanum environment in LaPO$_4$ is more complicated than the hydrated equivalent: the lanthanum site is ninefold coordinate, with significant variation in bond lengths. La–O bond lengths range from 2.47 Å to 2.78 Å, with an average bond length of 2.58 Å.

The experimental $^{139}$La ssNMR spectrum of LaPO$_4$ is presented in Fig. B.13. The spectrum was fit using two sites: one site with a $C_Q$ of $(46.7 \pm 1.0)$ MHz and an $\eta_Q$ of $0.75 \pm 0.05$, which is consistent with the values reported by Dithmer et al. ($(46.7 \pm 10.0)$ MHz and $0.75 \pm 0.03$, respectively) [172]; and one using the parameters reported above for LaPO$_4 \cdot 1.8$H$_2$O. The sample appears to have undergone hydration between heating and collection of the spectrum. There is no appreciable CSA contribution visible in the lineshape at 16.4 T.

DFT calculations of LaPO$_4$ using the experimental geometry [223] yield a $C_Q$ of 53.6 MHz and an $\eta_Q$ of 0.56. The calculations somewhat overestimates $C_Q$ and underestimates $\eta_Q$, but are consistent with the experimental results. Both the higher $C_Q$ and lower $\eta_Q$ than experiment can both be explained by an overestimation of $V_{zz}$ in the theoretical results. None of the tensor components are oriented toward any particular feature of the LaO$_9$ polyhedron.

The ellipsoid fitting the LaO$_9$ polyhedron is triaxial in nature, with $e_a = (2.415 \pm 0.001)$ Å, $e_b = (2.574 \pm 0.001)$ Å, and $e_c = (2.796 \pm 0.001)$ Å. There is significant deviation from spherical symmetry, with $\epsilon = (0.147 \pm 0.001)$ Å and a
Figure B.13: Static $^{139}$La NMR spectrum of LaPO$_4$. Total analytical simulation is shown in red. The simulations of the individual sites are in green (LaPO$_4$) and purple (LaPO$_4$·1.8H$_2$O). The EFG and CSA parameters used are reported in Table 4.1.
Figure B.14: First coordination sphere of LaO$_9$ in LaPO$_4$. The $V_{xx}$, $V_{yy}$, and $V_{zz}$ components are displayed as blue, green, and red, respectively. The $e_a$, $e_b$, and $e_c$ semi major axes are in yellow, magenta, and teal respectively.

character of $0.05 \pm 0.01$. The $V_{zz}$ component is nearest to the $e_c$ axis, but the EFG tensor is generally not aligned with the ellipsoid of best fit (Fig. B.14).

### B.5.4 Lanthanum Borate

LaBO$_3$ has previously been investigated using $^{11}$B ssNMR by Kroeker and Stebbins [48], but has not been probed by $^{139}$La ssNMR. LaBO$_3$ is a possible side product in the synthesis of LaBGeO$_5$ and LaBSiO$_5$ [286], as well as the primary phase to crystallize out of La$_2$O$_3$–B$_2$O$_3$ glasses. The room temperature phase of LaBO$_3$ is orthorhombic, with space group $Pnma$ and 4 formula units per cell [309]. The lanthanum environment is ninefold coordinate to oxygen, with four La–O bond pairs of lengths 2.41 Å, 2.49 Å, 2.77 Å, and 2.80 Å, and one La–O bond of length 2.38 Å. The average La–O bond
length is 2.59 Å. There is a mirror plane bisecting the LaO₉ polyhedron, with the independent La–O bond parallel to the plane.

Using the $^{139}$La ssNMR spectra of LaBO₃ measured at 9.4 T and 16.4 T (Figs. B.15 and B.16), the quadrupole and chemical shift parameters could be determined. The quadrupole coupling has a value of $C_Q$ of $(23.4 \pm 0.4)$ MHz with an $\eta_Q$ of $0.68 \pm 0.05$. There is a significant CSA contribution to the lineshape, with $\Omega = (350 \pm 30)$ ppm, $\kappa = 0.3 \pm 0.1$, and Euler angles $\alpha = (15 \pm 5)^\circ$, $\beta = (0 \pm 5)^\circ$, and $\gamma = (165 \pm 10)^\circ$. The magnitude of the CSA parameters is significant when compared to the quadrupolar influence on the lineshape: the measured span for the La$^{3+}$ site of LaBO₃ is comparable to La₂O₃ or LaScO₃, while the $^{139}$La $C_Q$ of La$^{3+}$ in LaBO₃ is significantly less than that in La₂O₃ or LaScO₃.

DFT calculations of LaBO₃ on the experimental geometry [309] yielded a $C_Q$ of $-28.8$ MHz with an $\eta_Q$ of 0.57. Both values agree reasonably well with experiment, though the $C_Q$ is overestimated. Calculations carried out on a different LaBO₃ structure [201] using the same parameters reported a $C_Q$ of $-84$ MHz and $\eta_Q$ of 0.05, which are even further from experiment. The $^{139}$La $C_Q$ of LaBO₃ appears to be extremely sensitive to structure, which should be considered when comparing experimental and computational values.
Figure B.16: Static $^{139}$La NMR spectra of LaBO$_3$. Both spectra are set to the same frequency scale. Top: Spectrum collected at 16.4 T. Bottom: Spectrum collected at 9.4 T.
Figure B.17: First coordination sphere of LaO$_9$ in LaBO$_3$. The $V_{xx}$, $V_{yy}$, and $V_{zz}$ components are displayed as blue, green, and red, respectively. The $e_a$, $e_b$, and $e_c$ semi major axes are in yellow, magenta, and teal respectively.

The least-squares ellipsoid which fits the LaO$_9$ polyhedron in LaBO$_3$ is significantly distorted, with $\epsilon = 0.157 \pm 0.001$ and semi major axes $e_a = (2.331 \pm 0.001)$ Å, $e_b = (2.721 \pm 0.001)$ Å, and $e_c = (2.741 \pm 0.001)$ Å. The character is nearly wholly oblate at $-0.92 \pm 0.01$. This is at odds with the experimental $\eta_Q$, as a small value of $\eta_Q$ would be expected when the system is nearly axially symmetric. The $V_{xx}$ EFG component is antiparallel to the $e_b$ semi major axis, and the $V_{zz}$ component is close to the $e_a$ axis (Fig. B.17).

B.5.5 Lanthanum Borosilicate

Another member of the stillwellite family, LaBSiO$_5$ is the silicate analogue of LaBGeO$_5$, and is isostructural to LaBGeO$_5$ [220]. Also of space group $P3_1$, the lanthanum
environment of LaBSiO$_5$ is of slightly higher coordination, forming a LaO$_{10}$ polyhedron. This difference is attributed to the difference in size between the GeO$_4$ and SiO$_4$ tetrahedra [310]. La–O bonds range in length from 2.39 Å to 2.86 Å, with an average of 2.64 Å. Like LaBGeO$_5$, there are no obvious symmetry elements present in the lanthanum polyhedron of LaBSiO$_5$.

The $^{139}$La ssNMR spectrum of LaBSiO$_5$ is presented in Fig. B.18. The lineshape is fit with a $C_Q$ of $(90.0 \pm 0.5)$ MHz and $\eta_Q$ of $0.35 \pm 0.02$. The spectrum of LaBSiO$_5$ is the broadest presented in this study, and among the broadest reported in the literature [174]. While similar in shape to the spectrum of LaBGeO$_5$, the spectrum of LaBSiO$_5$ is slightly broader and slightly less axially symmetric, likely a consequence of the higher coordination number.

The coordination ellipsoid of lanthanum in LaBSiO$_5$ is quite oblate, with $e_a =$
Figure B.19: First coordination sphere of LaO$_{10}$ in LaBSiO$_5$. The $V_{xx}$, $V_{yy}$, and $V_{zz}$ components are displayed as blue, green, and red, respectively. The $e_a$, $e_b$, and $e_c$ semi major axes are in yellow, magenta, and teal respectively.

$(2.350 \pm 0.001) \text{ Å}, e_b = (2.713 \pm 0.001) \text{ Å}, e_c = (2.808 \pm 0.001) \text{ Å}$, with a character of $-0.67$, lower than that of LaBGeO$_5$. The span of this ellipsoid is quite large at 0.175. The semi major axis $e_b$ is very nearly aligned with the $V_{yy}$ EFG component, and the $V_{zz}$ EFG component is close to the $e_a$ axis (Fig. B.19).

### B.5.6 Lanthanum Sulfate Nonahydrate

La$_2$(SO$_4$)$_3 \cdot$ 9H$_2$O is a commercially available lanthanum compound with a defined crystal structure, making it a good model compound for this investigation. La$_2$(SO$_4$)$_3 \cdot$ 9H$_2$O is of the hexagonal $P6_3/m$ space group with two formula units per unit cell [218]. There are two lanthanum sites in La$_2$(SO$_4$)$_3 \cdot$ 9H$_2$O: site La(1), with twelvefold coordination; and site La(2), with ninefold coordination. Both sites have highly symmetric polyhedra with $C_3$ rotation axes parallel to the crystallographic $c$
axis. La(1) is best described as a distorted LaO\(_{12}\) icosahedron, and La(2) as a LaO\(_9\) tricapped trigonal prism. As a result, both sites are predicted to have \(\eta_Q\) values of 0, but will have non-zero \(C_Q\) values. La(1) has two sets of bonds: three pairs of La–O bond length 2.59 Å, and three pairs of length 2.80 Å, with an average length of 2.70 Å. La(2) has two different bond lengths: the equatorial La–O bonds of length 2.51 Å, and the non-equatorial bonds of length 2.55 Å. The second coordination sphere of the two sites also has significant variation. The La(1) icosahedron edge-shares with six SO\(_4\) tetrahedra, where the La(2) site corner-shares with three SO\(_4\) tetrahedra which cap the rectangular faces of a trigonal prism. This trigonal prism is formed by water oxygen coordinating to lanthanum. La(1) has no H\(_2\)O in its near proximity, whereas the environment of La(2) is dominated by it.

The difference in the local structure of the two sites is reflected in the spectrum of La\(_2\)(SO\(_4\))\(_3\)·9H\(_2\)O (Fig. B.20). Two peaks are observed: peak 1, with isotropic chemical shift \(\delta_{\text{iso}}^{\text{CS}} = (-175 \pm 25)\) ppm, \(C_Q = (52.5 \pm 0.5)\) MHz, and \(\eta_Q = 0.00 \pm 0.02\); and peak 2, with \(\delta_{\text{iso}}^{\text{CS}} = (-75 \pm 25)\) ppm, \(C_Q = (36.5 \pm 0.5)\) MHz, and \(\eta_Q = 0.00 \pm 0.03\). Peak 1 is assigned to La(1) and peak 2 to La(2) based upon the computational results discussed below.

DFT calculations were based on the experimental geometry \([218]\) of La\(_2\)(SO\(_4\))\(_3\)·9H\(_2\)O, but there are no experimentally reported values for the position of the hydrogen atoms. We manually placed hydrogen atoms at reasonable starting locations and optimized their positions, holding the position of all other atoms (i.e., La, S, and O) constant. Our calculations yield a \(C_Q\) of −59.1 MHz and \(\eta_Q\) of 0.0 for La(1) and \(C_Q\) of −36.5 MHz and \(\eta_Q\) of 0.0 for La(2). The asymmetry parameters are confined by symmetry, but the magnitudes of the quadrupolar coupling constants agree reasonably well with the fit proposed above. The difference in the calculated \(C_Q\) of the two sites is used as a basis for assigning peak 1 to site La(1) and peak 2 to site La(2). The \(V_{zz}\) component of the EFG at La(1) is collinear to the crystallographic \(c\) axis, and as such is directed towards the neighbouring LaO\(_{12}\) icosahedron. In site 2, \(V_{zz}\) is also collinear to the crystallographic \(c\) axis, with \(V_{xx}\) and \(V_{yy}\) consequentially in the crystallographic \(ab\) plane.

Both sites can be fit independently by least-squares ellipsoids. Both are spheroids, as expected due to their axial symmetry. The ellipsoid of La(1) is oblate, with \(e_a = \ldots\)
Figure B.20: Static $^{139}$La NMR spectrum of La$_2$(SO$_4$)$_3$·9H$_2$O. Analytical simulation is show in red. The EFG parameters used are reported in Table 4.1.
(2.463 ± 0.001) Å and $e_b = e_c = (2.837 ± 0.001)$ Å. Despite the icosahedral nature of the LaO$_{12}$ polyhedron, there is significant distortion from spherical symmetry with $\epsilon = 0.138 ± 0.001$. The character of the ellipsoid is $-1.00 ± 0.01$, as expected from an oblate spheroid. Considering the LaO$_{12}$ ellipsoid, the shortest semi-major axis is aligned with the crystallographic $c$ axis, and hence collinear with $V_{zz}$. The ellipsoid of La(2) is prolate, with $e_a = e_b = (2.514 ± 0.001)$ Å and $e_c = (2.585 ± 0.001)$ Å. The La(2) ellipsoid is significantly less distorted than the La(1) ellipsoid, with $\epsilon = 0.028 ± 0.001$. As a prolate spheroid, the character of the ellipsoid is $1.00 ± 0.01$. The largest semi major axis is parallel to both $V_{zz}$ and the crystallographic $c$ axis. As a spheroid, the direction of the degenerate semi major axes is not unique, and like $V_{xx}$ and $V_{yy}$ they also lies in the $ab$ plane (Fig. B.21).

**B.5.7 Lanthanum Carbonate Octahydrate**

Despite pXRD suggesting that the sample of La$_2$(CO$_3$)$_3$·$n$H$_2$O was crystalline, and with $n = 8$, it was not possible to obtain a spectrum with sufficient resolution to separate the multiple sites present. The collected spectrum, presented in Fig. B.22, is believed to be a combination of the two sites [311] of La$_2$(CO$_3$)$_3$·8H$_2$O and the three sites [312] of lanthanum carbonate oxide (LaCO$_3$OH), the presence of which is suggested by pXRD (Fig. B.7).

Given the multi-phase nature of the sample, along with the poor spectral resolution, DFT calculations on the lanthanum carbonate octahydrate system were not attempted.

**B.5.8 Lanthanum Hydroxide**

La$_2$O$_3$ will easily convert to La(OH)$_3$ under exposure to atmospheric water [313]. It has influenced previous $^{139}$La NMR studies, and as a consequence has been well-studied using $^{139}$La ssNMR [60]. We did not collect further La(OH)$_3$ spectra in this work.

La(OH)$_3$ is in the $P6_3/m$ space group, as confirmed by both X-ray and neutron diffraction studies [217, 314]. The unit cell of La(OH)$_3$ contains two formula units. The lanthanum environment is ninefold coordinate, with only oxygen in the first coordination sphere. Like site La(2) of La$_2$(SO$_4$)$_3$·9H$_2$O, the LaO$_9$ polyhedron in La(OH)$_3$ is a tricapped trigonal prism, with a threefold rotation axis parallel to the crystallographic $c$ axis. This predicts an $\eta_Q$ of 0, which is confirmed by a previous
Figure B.21: Top: First coordination sphere of LaO$_{12}$ in La$_2$(SO$_4$)$_3$·9H$_2$O. Bottom: First coordination sphere of LaO$_9$ in La$_2$(SO$_4$)$_3$·9H$_2$O. The $V_{xx}$, $V_{yy}$, and $V_{zz}$ components are displayed as blue, green, and red, respectively. The $e_a$, $e_b$, and $e_c$ semi major axes are in yellow, magenta, and teal respectively.
We conducted DFT calculations on \( \text{La(OH)}_3 \) using the experimental geometry \([217]\), with the positions of the hydrogen atoms determined through neutron powder diffraction. Our DFT calculations yield a \( C_Q \) of \(-29 \text{ MHz}\) and an \( \eta_Q \) of 0.0, consistent with the experimental results reported by Spencer \textit{et al.}, who report an experimental \( C_Q \) of \((22.0 \pm 0.5) \text{ MHz}\) and an \( \eta_Q \) of \(0.050 \pm 0.025\). The \( V_{zz} \) component is collinear the crystallographic \( c \) axis, and is hence oriented to face the nearest neighbouring \( \text{LaO}_9 \) polyhedron.

The ellipsoid used to fit the \( \text{LaO}_9 \) polyhedron of \( \text{La(OH)}_3 \) is a prolate spheroid, with \( e_a = e_b = (2.551 \pm 0.001) \text{ Å} \) and \( e_c = (2.620 \pm 0.001) \text{ Å} \). It is very nearly spherical, with \( \epsilon = 0.027 \pm 0.010 \) and a character of \(1.00 \pm 0.01\). The longest semi major axis is parallel to the crystallographic \( c \) axis and hence collinear with \( V_{zz} \) (Fig. B.23).

### B.5.9 Lanthanum Aluminate

\( \text{LaAlO}_3 \) is a rhombohedral perovskite of space group \( R\bar{3}c \) with six formula units per unit cell. \( \text{LaAlO}_3 \) is remarkable in the NMR community for possessing one of the

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**Figure B.22:** Static \(^{139}\text{La}\) NMR spectrum of sample of nominal composition \( \text{La}_2(\text{CO}_3)_3 \cdot n\text{H}_2\text{O} \).
Figure B.23: First coordination sphere of LaO$_9$ in La(OH)$_3$. The $V_{xx}$, $V_{yy}$, and $V_{zz}$ components are displayed as blue, green, and red, respectively. The $e_a$, $e_b$, and $e_c$ semi major axes are in yellow, magenta, and teal respectively.
smallest reported $^{139}$La $C_Q$ values [58]. The coordination environment of lanthanum in LaAlO$_3$ is twelvefold coordinate, with the LaO$_{12}$ polyhedron assuming a slightly distorted cuboctahedron geometry [262]. Despite the distortion, there is a $C_3$ rotation axis parallel to the crystallographic $c$ axis, which enforces an $\eta_Q$ of 0.

The $^{139}$La ssNMR spectrum of LaAlO$_3$ was first observed by Dupree et al., where they fit the lineshape with a $C_Q$ of 6 MHz and an $\eta_Q$ of 0 [58]. Given that the peak is narrow enough to be collected through MAS NMR, these values are quite reliable. We conducted DFT calculations on the experimental geometry [262] which yielded a $C_Q$ of 8.4 MHz and an $\eta_Q$ of 0.0. This is in good agreement with the experimental results of Dupree et al. The $V_{zz}$ component of the EFG is parallel to the crystallographic $c$ axis.

The ellipsoid of LaO$_{12}$ in LaAlO$_3$ is the most spherical of all the compounds examined in this study, with $\epsilon = 0.01 \pm 0.01$. As is consistent with an $\eta_Q$ of 0, the ellipsoid is a spheroid; in particular it is an oblate spheroid, with $e_a = (2.67 \pm 0.01)$ Å and $e_b = e_c = (2.70 \pm 0.01)$ Å. The shortest semi major axis is parallel to both $V_{zz}$ and the crystallographic $c$ axis (Fig. B.24).

### B.5.10 Lanthanum Cobaltite

LaCoO$_3$ is a member of the perovskite family, and is of the rhombohedral space group $R\bar{3}c$ with two formula units per cell [263]. LaCoO$_3$ has been previously studied using $^{139}$La ssNMR by Bastow [59]. The lanthanum environment in LaCoO$_3$ is twelvefold coordinate to oxygen, forming a distorted cuboctahedron. There are three different La–O bond lengths in the LaO$_{12}$ polyhedron: three of length 2.43 Å; six of length 2.69 Å; and three of length 2.99 Å; together with an average of 2.70 Å. There is a $C_3$ rotation axis parallel to the crystallographic $c$ axis, enforcing an $\eta_Q$ of 0 on the lanthanum site.

Our DFT calculations of LaCoO$_3$ using the experimental geometry [263] yield a $^{139}$La $C_Q$ of 20.99 MHz and $\eta_Q$ of 0.00, in reasonable agreement with the experimental results of Bastow. Due to the resolution limits imposed by the frequency-stepped spin-echo technique used, Bastow estimated the $C_Q$ to be 23.8 MHz by the separation of the satellite transitions, rather than by direct simulation of the central transition. $V_{zz}$ is aligned with the $C_3$ rotation axis, and directed towards a face of a CoO$_6$ octahedron.
Figure B.24: First coordination sphere of LaO$_{12}$ in LaAlO$_3$. The $V_{xx}$, $V_{yy}$, and $V_{zz}$ components are displayed as blue, green, and red, respectively. The $e_a$, $e_b$, and $e_c$ semi major axes are in yellow, magenta, and teal respectively.
Figure B.25: First coordination sphere of LaO$_{12}$ in LaCoO$_3$. The $V_{xx}$, $V_{yy}$, and $V_{zz}$ components are displayed as blue, green, and red, respectively. The $e_a$, $e_b$, and $e_c$ semi major axes are in yellow, magenta, and teal respectively.

The ellipsoid fitting the LaO$_{12}$ cuboctahedron is oblate and somewhat distorted, with $\epsilon = 0.04 \pm 0.03$ and a character of $-1.00 \pm 0.01$. The semi major axes are $e_a = (2.67 \pm 0.04) \text{Å}$ and $e_b = e_c = (2.78 \pm 0.04) \text{Å}$. As has been observed with other compounds fit by a spheroid, the non-degenerate semi major axis is parallel to the proper rotation axis of the polyhedron (Fig. B.25). The large uncertainty in the ellipsoid span indicates that the oxygen positions are not easily fit by a triaxial ellipsoid; this is a consequence of the square (or in this distorted case, diamond) faces of the cuboctohedral geometry.

### B.5.11 Lanthanum Chromite

At room temperature, LaCrO$_3$ is in the orthorhombic $Pbnnm$ space group with four formula units per cell [315]. It experiences an antiferromagnetic phase transition at approximately 286 K [316]. The lanthanum environment is twelvefold coordinate, forming a severely distorted LaO$_{12}$ cuboctahedron. The distortion is sufficiently severe
that there are no obvious symmetry elements prescribing particular NMR behavior. The La–O bond lengths in the LaO$_{12}$ polyhedron range from 2.43 Å to 3.12 Å.

LaCrO$_3$ has been previously studied using $^{139}$La ssNMR spectroscopy by Bastow, in which he fits the LaCrO$_3$ peak with a $C_Q$ of 48 MHz and an $\eta_Q$ of 0.15 [59]. Our DFT calculations on the experimental structure [315] yield a $C_Q$ of $-47.2$ MHz with an $\eta_Q$ of 0.32. While the $C_Q$ is in good agreement with experiment, our calculations overestimate the asymmetry parameter as reported by Bastow. However, given the relatively low resolution provided by the frequency-swept spin echo technique used, it is possible that the experimental $\eta_Q$ was underestimated. The $V_{xx}$ axis is parallel to the crystallographic $c$ axis, while the $V_{yy}$ and $V_{zz}$ components of neighbouring LaO$_{12}$ polyhedra are nearly aligned (Fig. B.26).

The ellipsoid fitting the LaO$_{12}$ cuboctahedron is triaxial, reflecting the level of distortion from spherical symmetry. The span is moderately high at $\epsilon = 0.16 \pm 0.05,$
and the character of the ellipsoid is $-0.14 \pm 0.01$, reflecting a slightly oblate nature. The semi major axis $e_b$ is parallel to both the $V_{xx}$ component and the crystallographic $c$ axis, while $V_{zz}$ is closely adjacent to $e_a$ (Fig. B.26). Like LaCoO$_3$, the coordination polyhedron is poorly fit by an ellipsoid; this is also attributed to the strongly distorted cuboctahedral geometry of the local lanthanum coordination.

### B.5.12 Lanthanum Titanate

LaTiO$_3$ has been studied for its role as an antiferromagnet [194] as well as for its application in the battery material lithium lanthanum titanate [317]. LaTiO$_3$ is in the orthorhombic $Pbnm$ space group with 4 formula units per unit cell. The lanthanum environment is eightfold coordinate in a distorted square antiprismatic configuration, with bonds ranging from 2.43 Å to 2.77 Å, with an average of 2.60 Å. LaTiO$_3$ has been experimentally observed through low-temperature $^{139}$La ssNMR by Furukawa et al. They reported a quadrupole resonance frequency of 3.8 MHz and an $\eta_Q$ of 0.6 [194]. A $^{139}$La quadrupole resonance frequency of 3.8 MHz is equivalent to a $^{139}$La $C_Q$ of 53.2 MHz. The value of $\eta_Q$ remains unchanged. The measurement of the quadrupolar coupling constant was verified through nuclear quadrupole resonance [194] and a previous computational study [318].

DFT calculations of LaTiO$_3$ are complicated by the strong electron correlation at the La$^{3+}$ and Ti$^{3+}$ sites. This requires a special approach (LDA+U) [318] that is inconsistent with the rest of the DFT calculations reported in this work, and hence DFT calculations of LaTiO$_3$ were not performed.

The ellipsoid fitting the LaO$_8$ polyhedron is triaxial, with $e_a = (2.384 \pm 0.001)$ Å, $e_b = (2.462 \pm 0.001)$ Å, and $e_c = (2.872 \pm 0.001)$ Å. The ellipsoid is quite distorted from spherical symmetry, with $\epsilon = 0.190 \pm 0.001$, and a prolate character of 0.59 ± 0.01. The largest semi major axis is parallel with the crystallographic $c$ axis, and is directed towards neighbouring LaO$_8$ polyhedra (Fig. B.27).

### B.5.13 Lanthanum Niobate

LaNbO$_4$ has been thoroughly investigated using $^{139}$La ssNMR by Spencer et al. [60]; we will not duplicate their efforts here. LaNbO$_4$ has a monoclinic crystal structure of space group $C2/c$ with 4 formula units per unit cell [221]. The lanthanum environment
Figure B.27: First coordination sphere of LaO$_8$ in LaTiO$_3$. The $e_a$, $e_b$, and $e_c$ semi major axes are in yellow, magenta, and teal respectively.
in LaNbO\textsubscript{4} is eightfold coordinate, with four pairs of La–O bonds with lengths 2.48 Å, 2.49 Å, 2.50 Å, and 2.58 Å. The average La–O bond length is 2.50 Å. The lanthanum coordination polyhedron is a distorted square antiprism.

We performed DFT calculations on the room-temperature monoclinic LaNbO\textsubscript{4} crystal structure [221]. Our DFT calculations yield a $C_Q$ of 39.5 MHz and $\eta_Q$ of 0.50, in good agreement with the experimental values reported by Spencer et al. ($C_Q = (36 \pm 2) \text{ MHz}, \eta_Q = 0.44 \pm 0.05$) [60]. Our computed EFG parameters have approximately the same deviation from experiment as the computed values reported by Spencer et al. (calculated $C_Q = 33.8 \text{ MHz}, \eta_Q = 0.38$), albeit in the opposite direction.

The ellipsoid used to fit the LaO\textsubscript{8} coordination polyhedron is triaxial, with $e_a = (2.419 \pm 0.001) \text{ Å}, e_b = (2.478 \pm 0.001) \text{ Å},$ and $e_c = (2.606 \pm 0.001) \text{ Å}$. The ellipsoid span indicates a moderate level of distortion with $\epsilon = 0.075 \pm 0.001$. The ellipsoid is somewhat prolate, with a character of $0.32 \pm 0.01$. While the longest semi major axis of the ellipsoid is parallel to the crystallographic $b$ axis, the other semi major axes are not aligned with any particular structural feature (Fig. B.28).
Figure B.28: First coordination sphere of LaO$_8$ in LaNbO$_4$. The $e_a$, $e_b$, and $e_c$ semi major axes are in yellow, magenta, and teal respectively.
B.6 Additional Figures

Figure B.29: Relationship between sphericity $\Sigma$, ellipsoid span $\epsilon$, the shortest ellipsoid semi major axis $e_a$, and the longest semi major axis $e_c$ and: $d_{\text{min}}$ (a), b), d)); and $d_{\text{max}}$ (c), e)). Filled circles indicate compounds of the non-LaMO$_3$ family, while open circles indicate LaMO$_3$ compounds. The solid lines indicate the relationship between the respective distortion parameter and the bond length of the non-LaMO$_3$ family, with: a) $\Sigma = 2.78 \text{Å}^{-1} \cdot d_{\text{min}} + 0.23 \ (R^2 = 0.38)$; b) $e_c = 0.67 \cdot d_{\text{min}} + 0.81 \text{Å} \ (R^2 = 0.36)$; c) $e_a = -1.08 \cdot d_{\text{max}} + 5.35 \text{Å} \ (R^2 = 0.44)$; e) $e_c = 1.14 \cdot d_{\text{max}} - 0.39 \text{Å} \ (R^2 = 0.95)$. The dotted line in d) indicates the relationship between $e_c$ and the minimum La–O bond length of the LaMO$_3$ family, with $e_c = -0.28 \cdot d_{\text{max}} + 3.25 \text{Å} \ (R^2 = 0.62)$.
Figure B.30: Relationship between $^{139}\text{La} C_Q$ and: a) shortest ellipsoid semi major axis $e_a$; b) longest semi major axis $e_c$. Filled circles indicate compounds of the non-LaMO$_3$ family, while open circles indicate LaMO$_3$ compounds. The solid lines in indicate the relationship between $^{139}\text{La} C_Q$ and the respective distortion parameter of the non-LaMO$_3$ compounds, with: a) $C_Q = -278 \text{MHz} \cdot \AA^{-1} \cdot e_a + 734 \text{MHz}$ ($R^2 = 0.69$); b) $C_Q = 157 \text{MHz} \cdot \AA^{-1} \cdot e_c - 373 \text{MHz}$ ($R^2 = 0.42$). The dotted lines indicate the relationship between $^{139}\text{La} C_Q$ and the respective distortion parameter of the LaMO$_3$ compounds, with: a) $C_Q = -78 \text{MHz} \cdot \AA^{-1} \cdot e_a + 230 \text{MHz}$ ($R^2 = 0.35$); and b) $C_Q = 133 \text{MHz} \cdot \AA^{-1} \cdot e_c - 344 \text{MHz}$ ($R^2 = 0.81$).
Appendix C

Supporting Information for Chapter 6: Structural Differences between the Glass and Crystal Forms of the Transparent Ferroelectric Nanocomposite LaBGeO$_5$, from Neutron Diffraction and NMR Spectroscopy

C.1 Context

The contents of this appendix were published as supporting information for the manuscript making up Chapter 6, published as Paterson, A.L., Hannon, A.C., Werner-Zwanziger, U., and Zwanziger, J.W., Structural Differences between the Glass and Crystal Forms of the Transparent Ferroelectric Nanocomposite LaBGeO$_5$, from Neutron Diffraction and NMR Spectroscopy, *J. Phys. Chem. C* 122 (36) (2018) 20963–20980 [212]. Copyright 2018 American Chemical Society. Some minor changes have been made to the text for style and consistency. Copyright permissions for this use are contained in Appendix E.
Figure C.1: $^{11}$B MAS NMR spectrum of 25LaBG-2. Spectral limits are set to display the first set of spinning sidebands on either side of the central transition. The spinning sidebands are indicated by *. The slight shoulder on the positive side of the BO$_3$ peak, indicated by †, is the 3/2 $\rightarrow$ 1/2 satellite transition. Its spinning sideband is resolved from the spinning sideband of the central transition.
Figure C.2: $^{11}$B MAS NMR spectra of samples not displayed in Chapter 6. The 0LaBG and 20LaBG spectra are included for visual reference.
Figure C.3: Powder X-ray diffractogram of the 25LaBG-X sample. The sample is identified as LaBGeO$_5$ by comparison to the diffraction pattern reported by Rulmont and Tarte [278]. Impurity peaks are marked with asterisks (*). The impurity is tentatively identified as La$_2$Ge$_2$O$_7$ by comparison with the diffraction pattern calculated from the crystal structure reported by Vetter and Queyroux [265].
Figure C.4: The $T(r)$ total correlation functions of selected glass and crystal samples. The functions are vertically offset for clarity. A Lorch modification function with $Q_{\text{max}} = 50 \, \text{Å}^{-1}$ was used for the Fourier transformations. Error bars are contained within the data points.
Figure C.5: The $T(r)$ total correlation functions of selected glass and crystal samples. The functions are vertically offset for clarity. A Lorch modification function with $Q_{\text{max}} = 50 \, \text{Å}^{-1}$ was used for the Fourier transformations. The $T(r)$ functions of 25LaBG-2 and 30LaBG-2 replace those of 25LaBG-1 and 30LaBG-01, as compared to Fig. C.4 above.
Figure C.6: The density of the lanthanum-containing borogermanate glass samples, as determined through the Archimedes method, as a function of the $R = \frac{[\text{La}_2\text{O}_3]}{([\text{B}_2\text{O}_3]+[\text{GeO}_2])}$ molar ratio. The $R = 0$ sample, 0LaBG, is omitted to allow for the change in density with molar ratio to be visible. The omitted data can be found in Table 6.1.
Figure C.7: The $T(r)$ function of 25LaBG-1. Top: as transformed with a step function. Bottom: as transformed with a Lorch function. Both used $Q_{\text{max}} = 50$ Å$^{-1}$. Note the shoulder on the high-$r$ side of the left peak, attributed to the presence of [BO$_4$]$^-$ . The high-$r$ side of the centre peak shows a tail to high values of $r$ when transformed with a step function. This intensity persists when a Lorch modification function is used, and is attributed to germanium-oxygen coordinations greater than 4.
Figure C.8: The fraction of the neutron peak attributed to $[\text{BO}_4]^{-}$ as a fraction of the total area of the neutron B–O peak, as a function of the B–O coordination number as determined by $^{11}$B MQMAS. The data point marked by an X is the crystalline 25LaBG-X sample, while the open circles indicate glass samples. The dotted line is a guide to the eye that indicates correspondence between the proportion of $[\text{BO}_4]^{-}$ as determined by $^{11}$B MQMAS and the $[\text{BO}_4]^{-}$ fraction as determined by neutron diffraction. While the B–O coordination numbers as determined by neutron diffraction are inconsistent in absolute terms with the more reliable MQMAS values, the proportion of $T(r)$ area attributed to $[\text{BO}_4]^{-}$ generally follows the same trend as observed through MQMAS.
Figure C.9: The position of the fitted Ge(1) peak maximum as a function of the $R = \frac{[\text{La}_2\text{O}_3]}{([\text{B}_2\text{O}_3]+[\text{GeO}_2])}$ molar ratio.
Figure C.10: DSC scan for the 0LaBG sample.

Figure C.11: DSC scan for the 20LaBG sample.
Figure C.12: DSC scan for the 25LaBG-1 sample.

Figure C.13: DSC scan for the 25LaBG-2 sample.
Figure C.14: DSC scan for the 30LaBG-1 sample.

Figure C.15: DSC scan for the 30LaBG-1 sample.
Figure C.16: The glass transition temperature of the lanthanum-containing glass samples, as measured by DSC, as a function of the molar ratio $R = \frac{[\text{La}_2\text{O}_3]}{([\text{B}_2\text{O}_3]+[\text{GeO}_2])}$. The $R = 0$ sample, 0LaBG, is omitted for clarity.
Figure C.17: The molar volume of glass samples as a function of the molar ratio \( R = \frac{[\text{La}_2\text{O}_3]}{([\text{B}_2\text{O}_3]+[\text{GeO}_2])} \). The molar volume is at a maximum within error at both zero lanthanum content (\( R = 0 \)) and maximum lanthanum content (\( R = 0.446 \)).
Figure C.18: Isotropic chemical shifts of BO$_3$ units in the glass samples. The data points indicate the isotropic chemical shifts derived from the centres of gravity of the MQMAS spectra [134]. The error bars along the $y$ axis indicate the breath of the distribution of chemical shifts corresponding to the full-width at half-maximum of the peak in the isotropic axis of the MQMAS spectra. These values are consistent with the $\delta_{\text{iso}}$ values for BO$_2$O$^-$ reported in the literature [48, 49]. Error bars along the $x$ axis indicate the uncertainty in $R$. 
Tables

Table C.1: Collected neutron diffraction parameters. \( r \) values are peak positions. \( u \) values are the widths of the peak in \( T(r) \). \( A \) values are peak areas in \( T(r) \).

<table>
<thead>
<tr>
<th>Sample</th>
<th>0LaBG</th>
<th>20LaBG</th>
<th>25LaBG-1</th>
<th>25LaBG-2</th>
<th>30LaBG-1</th>
<th>30LaBG-2</th>
<th>25LaBG-X</th>
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<tr>
<td>B(3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>1.367</td>
<td>1.367</td>
<td>1.367</td>
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<td>( u / \AA )</td>
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<td>0.052</td>
<td>0.050</td>
<td>0.049</td>
<td>0.053</td>
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<tr>
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<tr>
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<td>1.473</td>
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<td>1.473</td>
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<td>1.927</td>
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<td>0.022</td>
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<tr>
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<td>2.060</td>
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<td>2.071</td>
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<td>0.011</td>
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<td>2.444</td>
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<td>2.448</td>
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<tr>
<td>( u / \AA )</td>
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<td>0.111</td>
<td>0.108</td>
<td>0.107</td>
<td>0.107</td>
<td>0.109</td>
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<tr>
<td>( A / b\AA^{-1} )</td>
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<td>0.444</td>
<td>0.438</td>
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Table C.2: EBS of various oxygen environments in lanthanum borogermanates, not accounting for the contribution of lanthanum. The environments are sorted according to $\Sigma$

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<th>O Environment</th>
<th>$\Sigma$ / EBS</th>
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<td>$[\text{Ob}_4\text{B}_2\text{Ge}]$</td>
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</tr>
<tr>
<td>$[\text{Ob}_5\text{Ge}_6\text{Ge}_2]$</td>
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</tr>
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<td>2.000</td>
</tr>
<tr>
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</tr>
<tr>
<td>$[\text{Ob}_3\text{B}_5\text{Ge}]$</td>
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</tr>
<tr>
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<td>$[\text{Ob}_4\text{Ge}]$</td>
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</tr>
<tr>
<td>$[\text{Ob}_5\text{Ge}]$</td>
<td>0.800</td>
</tr>
<tr>
<td>$[\text{Ob}_4\text{B}]$</td>
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<tr>
<td>$[\text{Ob}_6\text{Ge}]$</td>
<td>0.667</td>
</tr>
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</table>
Appendix D

Supporting Information for Chapter 7: Network Connectivity and Crystallization in the Transparent Ferroelectric Nanocomposite Material LaBGeO$_5$

D.1 Context

The contents of this appendix were prepared as supporting information for Chapter 7.

D.2 Powder X-Ray Diffraction

X-ray diffraction data were collected to aid in phase identification of the samples. By process of elimination based upon the sample composition, and the phases known to be present in the composition, the identities of of the crystalline components of the samples were established.

For a more rigorous identification, Rietveld refinements of the pXRD data were conducted. The phase scale, background, zero parameter, and lattice parameters were allowed to vary. Ionic positions were fixed to those reported by Kaminskii et al. [78], as those reported by Belokoneva et al. have unrealistic bond lengths and angles [79]. When positions or ionic occupancies were allowed to vary, the results were generally unphysical. Peak shape parameters were fixed to the defaults provided in the RIETICA program, as no experimental data from a standard reference material was available for the X-ray diffractometer used. The results of the refinements are presented in Table D.1; an example refinement (for sample LBG+60) is presented in Fig. D.1.

The strains reported in Table D.1 are relative to the averages of the lattice parameters reported by Kaminskii et al., Belokoneva et al., and Rulmont and Tarte [78, 79, 278]. The stresses are obtained from these strains through the use of the elastic stiffness tensor reported in Table 8.2 [283]. They should be considered a rough estimate of the stresses within the system.

Rietveld refinement was also used to quantify the fraction of LaBO$_3$ within the
Table D.1: Results of Rietveld refinements of pXRD data. Strain is reported as the ratio of the refined lattice parameters to the average of the LaBGeO$_5$ lattice parameters reported in the literature. Stresses are obtained using the elastic constants reported in Table 8.2 [283]. The sample 25LaBG-X is described in Chapter 6; all other samples are from Chapter 7.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Crystallinity / %</th>
<th>$\chi^2$</th>
<th>$a$ / Å</th>
<th>$c$ / Å</th>
<th>Strain $a$</th>
<th>Strain $c$</th>
<th>Stress / GPa $\sigma_{xx}$</th>
<th>Stress / GPa $\sigma_{zz}$</th>
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<tr>
<td>LBG+8</td>
<td>15</td>
<td>3.32</td>
<td>6.914</td>
<td>6.857</td>
<td>0.9870</td>
<td>0.9984</td>
<td>4.4</td>
<td>3.1</td>
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<tr>
<td>LBG+12</td>
<td>29</td>
<td>4.71</td>
<td>6.920</td>
<td>6.862</td>
<td>0.9879</td>
<td>0.9991</td>
<td>4.1</td>
<td>2.7</td>
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<tr>
<td>LBG+24</td>
<td>48</td>
<td>7.36</td>
<td>6.921</td>
<td>6.860</td>
<td>0.9880</td>
<td>0.9989</td>
<td>4.0</td>
<td>2.7</td>
</tr>
<tr>
<td>LBG+60</td>
<td>72</td>
<td>4.30</td>
<td>6.924</td>
<td>6.851</td>
<td>0.9885</td>
<td>0.9976</td>
<td>4.0</td>
<td>3.0</td>
</tr>
<tr>
<td>LBG+3d</td>
<td>80</td>
<td>8.30</td>
<td>6.997</td>
<td>6.868</td>
<td>0.9988</td>
<td>1.0000</td>
<td>0.4</td>
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<td>25LaBG-X</td>
<td>98</td>
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<td>6.992</td>
<td>6.863</td>
<td>0.9982</td>
<td>0.9993</td>
<td>0.7</td>
<td>0.6</td>
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</table>

Figure D.1: PXRD diffraction pattern and Rietveld refinement profile for sample LBG+60. The refinement uses lattice parameters somewhat smaller than literature values. The parameters are reported in Table D.1. The top row of peak markers are for LaBGeO$_5$; the bottom row are for the orthorhombic phase of LaBO$_3$. 
samples where it was observed. In the LBG+60 sample, room-temperature (i.e., orthorhombic) LaBO$_3$ was observed in the $^{139}$La WCPMG NMR spectrum (Fig. 7.5), but not the $^{11}$B NMR spectrum (Fig. 7.3). The $^{139}$La WCPMG NMR spectrum suggests that LaBO$_3$ makes up 4% of the sample, or 6.5% of the crystalline fraction. However, WCPMG is formally non-quantitative in intensities. The Rietveld refinement of LBG+60 suggests that (2.4 ± 0.7)% of the crystalline portion of the sample is made up of LaBO$_3$. However, when the intensity attributable to LaBGeO$_5$ is considered ((98 ± 2)%), along with the signal-to-noise ratio of the diffractogram, it is not plausible to say that LaBO$_3$ has been detected. Hence the fraction of the partially-crystallized samples made up by LaBO$_3$ is thought to be generally less than 3%.

There is what appears to be a peak at 22.65°, which has been proposed as an indication of high-temperature (i.e., monoclinic) LaBO$_3$. When the data is closely examined, the data points about 22.65° are in fact inconsistent with the behaviour of a real peak. Furthermore, the main peak of monoclinic LaBO$_3$ is located about 22.9°, and does not well fit the data. The $^{139}$La WCPMG NMR data may not be quantitative in intensity, but it is quantitative in $C_Q$, and the data is inconsistent with the monoclinic phase. Finally, the phase transition to monoclinic LaBO$_3$ occurs at (1488 ± 5)°C [214], and that temperature was never reached at any point in this study. The body of evidence suggests that LaBO$_3$ is not detected by pXRD in the LBG+60 sample.

In the LBG+3d sample, room-temperature LaBO$_3$ is detected via $^{139}$La WCPMG NMR, $^{11}$B MAS NMR, and pXRD. $^{139}$La WCPMG NMR suggests 13% of the La content is in LaBO$_3$; however, the glass fraction of LBG+3d is not detectable in the $^{139}$La WCPMG NMR spectrum. $^{11}$B MAS NMR suggests that (9 ± 1)% of the B content is in LaBO$_3$; with crystalline LaBGeO$_5$ making up (80 ± 1)% of the overall sample, LaBO$_3$ would make up about (11 ± 1)% of the crystalline content. The crystalline content attributable to LaBO$_3$ by pXRD is 15%. Of these values, the fraction derived from $^{11}$B NMR is perhaps the most reliable, as it fully accounts for the presence of the amorphous LaBGeO$_5$, the crystalline LaBGeO$_5$, and the crystalline LaBO$_3$. The estimate provided by the Rietveld refinement could be improved by a more careful model, though this would require the collection of additional data.
D.3 Density Functional Theory Calculations

The QE-GIPAW package calculates the absolute chemical shielding tensor, from which the isotropic chemical shielding ($\sigma$) is calculated. The experimental NMR observable we wish to derive from the isotropic chemical shielding ($\sigma_{iso}$) is the isotropic chemical shift ($\delta_{iso}^{CS}$). There are two generally accepted techniques used in the literature to predict chemical shift values from calculated chemical shielding values [158]. The first is to calculate the shielding of a reference compound ($\sigma_{ref}$) with known $\delta_{iso}^{CS}$, and calculate the $\delta_{iso}^{CS}$ of the compound of interest via

$$
\delta_{iso}^{CS} = \sigma_{ref} - \sigma_{iso}.
$$

The second is to fit a set of calculated $\sigma$ to their experimental $\delta_{iso}^{CS}$ values for a range of relevant systems, and interpolate the desired $\delta_{iso}^{CS}$ values via

$$
\delta_{iso}^{CS} = a(\sigma_{ref} - \sigma_{iso})
$$

where $a$ is a constant and $\sigma_{ref}$ is the $y$-intercept of the line. We use Eq. (D.2) to reference the $^{139}$La and $^{17}$O shifts, and Eq. (D.1) with LaB$_3$O$_6$ to reference $^{11}$B shifts. $^{73}$Ge chemical shifts were not referenced, as no $^{73}$Ge NMR experiments were conducted. The data used to produce the fit for the $^{17}$O and $^{139}$La references are listed in Table D.2, and plotted in Figs. D.2 and D.3.

The $\sigma$ and $\delta_{iso}^{CS}$ values for various nuclei for various crystal structures are presented in Table D.2. All calculations were conducted using the experimental crystal structures. Plane-wave cutoffs were generally set to 60 Rydberg, and shifted Monkhorst-Pack k-point grids were used. The density of the k-point grids were set to ensure k-point spacings of less than 0.03 Å$^{-1}$. 

Figure D.2: The data used to convert the calculated chemical shieldings $\sigma$ to predicted chemical shifts $\delta_{\text{iso}}^{\text{CS}}$ for $^{17}\text{O}$. The filled circles indicate data from Table D.2. The open circles indicate the O environments in LaBGeO$_5$; their experimental shifts are discussed in Section 7.5. The line is a linear fit to the filled circles.
Figure D.3: The data used to convert the calculated chemical shieldings $\sigma$ to predicted chemical shifts $\delta_{iso}^{CS}$ for $^{139}$La. The filled circles indicate data from Table D.2. The open circle indicates the La environment in LaBGeO$_5$; their experimental shifts are discussed in Section 7.5. The line is a linear fit to the filled circles.
Table D.2: Comparison between experimental $\delta_{CS}^{iso}$ and calculated $\sigma$.

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<th>$^{11}$B / ppm</th>
<th>$^{17}$O / ppm</th>
<th>Sources</th>
<th>Structure</th>
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<td>$\sigma$</td>
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D.4 NMR Spectra

Figure D.4: $^{11}$B MAS NMR spectrum of LaBO$_3$. The fit corresponds to the following set of parameters: $\delta_{\text{iso}}^{\text{CS}} = (22 \pm 1) \text{ ppm}; \ C_Q = (2.7 \pm 0.1) \text{ MHz}; \ \eta_Q \text{ fixed at 0.05. The } C_Q \text{ value agrees well with the value reported in the literature, but our } \delta_{\text{iso}}^{\text{CS}} \text{ is 1.4 ppm more positive [48].}$

Table D.3: $^{10}$B NMR parameters for the samples investigated. Parameters were obtained by fitting the spectra. Values in parentheses indicate uncertainties.

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<th>Sample</th>
<th>$\delta_{\text{iso}}^{\text{CS}} / \text{ ppm}$</th>
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<th>$\eta_Q$</th>
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<td>LBG-G</td>
<td>$0.8 \pm 0.8$</td>
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<td>LBG-X</td>
<td>$0.9 \pm 0.8$</td>
<td>$0.8 \pm 0.4$</td>
<td>$1.0 \pm 0.4$</td>
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<td>LaB$_3$O$_6$</td>
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Figure D.5: $^{11}$B MAS NMR spectrum of LaB$_3$O$_6$. The fit consists of two peaks in a 2:1 intensity ratio. The BO$_2$O$^-$ peak uses the following set of parameters: $\delta_{iso}^{CS} = (18 \pm 1)$ ppm; $C_Q = (2.7 \pm 0.1)$ MHz; $\eta_Q = 0.36 \pm 0.02$. The [BO$_4$]$^-$ peak parameters are: $\delta_{iso}^{CS} = (-1 \pm 1)$ ppm; $P_Q = (0.8 \pm 0.3)$ MHz. These values agree well with those reported in the literature [97].

Figure D.6: $^{10}$B Hahn echo NMR spectrum of LaB$_3$O$_6$. Only the [BO$_4$]$^-$ environment is excited. Fit parameters are in Table D.3.
Figure D.7: $^{10}$B Hahn echo NMR spectrum of LaBGeO$_5$ glass. Only the [BØ$_4$]$^-$ environment is excited. Fit parameters are in Table D.3.

Figure D.8: $^{10}$B Hahn echo NMR spectrum of LaBGeO$_5$ crystal. Fit parameters are in Table D.3.
Appendix E

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