SIGNAL ALIGNMENT IN COGNITIVE RADIO NETWORKS

by

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To My Parents and Family

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Abstract

Cognitive radio (CR) is a promising technique to overcome spectrum scarcity in wireless networks through dynamic spectrum access and coordinated transmissions between different users based on channel conditions. Sharing wireless spectrum at the same time and frequency introduces multiple access interference (MAI), which can be managed through the cooperation of nodes and signal alignment (SA).

In this thesis, a novel approach for eliminating interference in CR systems is introduced by adapting the SA concept used in multiple-input multiple-output (MIMO) Y channel, into an overlay CR scheme. In this system, the primary users (PUs) exchange data in a bi-directional way via the relay, and the relay uses its antennas to communicate with the secondary users (SUs). We designed advanced SA strategies to accommodate different requirements for PU and SU transmissions with different antenna configurations and different user setups. Moreover, we employed strategies to optimize bit error rate (BER) performance of systems.

List of Abbreviations and Symbols Used

3D	three-dimensional
AF	amplify-and-forward
ANC	analog network coding
AWGN	additive white Gaussian noise
BC	broadcast
BER	bit error rate
BPSK	binary phase shift keying
CDMA	code division multiple access
CR	cognitive radio
CRN	cognitive radio network
CSI	channel state information
DF	decode-and-forward
FEC	Forward error control
IA	interference alignment
IC	interference channel
i.i.d	independent identically distributed
MAC	medium access control
MAI	multiple access interference
MATLAB®	mathematical laboratory
MIMO	multiple-input multiple-output
MMSE	minimum mean square error
ML	maximum likelihood
MU-MIMO	multi-user multiple-input multiple-output
NC	network coding
PNC	physical network coding
PU	primary user
QAM	quadrature amplitude modulation
SA	signal alignment

SISO	single-input single-output
SNR	signal-to-noise ratio
SU	secondary user
SVD	singular value decomposition
TDMA	time division multiple access
TS(s)	time slot(s)
TSER	ternary symbol error rate
TWRC	two-way relay channel
WCDMA	wideband code division multiple access
WLAN	wireless local area network
WN	wireless network
WSN	wireless sensor network
XOR	exclusive or
ZF	zero-forcing

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Chapter 1

Introduction

Considering the limited spectrum resources and growing demand for wireless services, cognitive radio (CR) has attracted significant attention as a promising technique to overcome spectrum scarcity [1], [2]. Current inflexible spectrum governance policies have caused spectrum to be vastly underutilized and new intelligent spectrum utilization seems to be inevitable [3]. In cognitive radio, the secondary users (SUs) may be allowed to access the spectrum along with primary users (PUs) only when there is no or minimal interference at the PUs [4]. Sharing wireless spectrum at the same time in the same frequency band introduces multiple access interference (MAI). Hence, interference elimination techniques are one of the main areas of study in CR networks. Signal alignment (SA) has been a promising technique to handle MAI as it exploits opportunities in the time and space domains to suppress interference [5]. Moreover, cooperative relaying in wireless networks offers attractive performance enhancements especially when terminals are operating in a multi-antenna configuration [6]. Based on these premises, through advanced signal processing at the network level, this thesis exploits the opportunities for deploying SA in CR networks supported by a single relay. The relay can be a stand-alone device or one of the SUs supporting bi-directional data exchanges between PUs in order to access the PU frequency band. Specifically, the proposed transmission schemes build on multiple-input multiple-output (MIMO) techniques but they are designed to support network-level operations rather than just a single communication link.

When originally introduced, MIMO was a method for increasing the capacity of a radio link using multiple co-located transmitting and receiving antennas while exploiting fading MIMO channels through pre- and post-processing of signals at the transmitter and the receiver of a single-user channel [7]. More recently, MIMO has been applied in distributed terminal and antenna configurations referred to as virtual MIMO in multiuser channels [8]. Among many virtual MIMO applications, the

MIMO Y channel is of particular importance in this thesis as it sets the framework for distributing the information in two phases via the relay between different terminals in a broadcast radio channel. The MIMO Y channel uses SA to improve bandwidth efficiency of wireless networks [9]. In this system, three nodes exchange six messages in two phases (multiple access control -MAC- and broadcast -BC-) via the relay. The MIMO Y channel model is remarkable in terms of its high data rate under the three-user and two-way relay communication scenario. In this thesis, a novel approach for eliminating interference in CR systems has been introduced by adapting MIMO Y channel concepts into an overlay CR scheme. In this system, two users exchanging data bi-directionaly via the relay are considered as PUs, while the relay uses its antennas to send and receive independent data to and from SUs. With this technique, two groups of nodes representing PUs and SUs simultaneously communicate in the same spectrum via the common cooperating relay sharing its antennas. In the proposed setup, the data exchange for both PUs and SUs is accomplished within the same two (MAC and BC) phases.

In future wireless networks, when spectrum sharing is going to be deployed on a much wider scale within the CR paradigm, the PUs and SUs of the network will have transceivers with different antenna configurations [2]. This was the motivation to further expand our work for different antenna and different user setups and consequently find the derivation for the generalized cases of the originally developed schemes in this thesis. Having flexibility in choosing orthogonal signaling dimensions in some scenarios and having antenna diversity in others, we also introduced the capability to optimize the bit error rate (BER) performance of different links in the CR network. The proposed schemes not only perform well in terms of bandwidth efficiency but also the simulation results for different scenarios have shown a significant improvement in terms of BER over conventional MIMO Y channel approaches without signal optimization.

In non-hierarchical radio networks (without hierarchical users), to address many issues related to extending the coverage area and reducing MAI, one of the most successful and efficient solutions is the deployment of relays [10], [11]. Relays that receive and re-transmit the signals between the source and the destination terminals have been used in communication networks for a long time, but only recently with the increasing computational and processing capabilities in the terminals and relays, more advanced configurations of relay networks have been proposed. These configurations take advantage of using channel state information (CSI) in the terminals and relays to transmit highly processed signals which increase bandwidth efficiency and allow desired signal recovery in different nodes of the network. In our work related to CR, we will pursue a similar approach where data is pre- and post-processed at the PU and SU terminals and after being received by the relay, the transmitted signals in BC phase are processed versions of the received signals.

In this thesis, we focus on using SA in cooperative relay overlay networks with MIMO terminals, where multiple transmissions take place simultaneously over a common broadcast communication medium. Specifically, we use the techniques from SA to manage the utilization of signal space dimensions to accommodate both PU and SU transmissions. Interference alignment (IA) and, similarly, signal alignment (SA) are linear precoding techniques that attempt to align interfering signals in time, frequency, or space. IA has been used in communication systems with users from the same hierarchy. However, the objective of this thesis is to deploy SA in the context of cognitive radio with PUs and SUs. In our work, we build heavily on the concept of the MIMO Y channel that was proposed originally for non-hierarchical radio networks. In the conventional application of the MIMO Y channel system, which deploys SA and network coding, the bidirectional data exchange of six messages among three users (of the same type) is completed in two time slots (TSs), which significantly enhances the throughput of the network over time division multiple access (TDMA). In the first TS, called the MAC phase, three users (of the same type) send all messages simultaneously to the relay, and in the second TS, called the broadcast (BC) phase, the relay transmits messages to users. When the two nodes exchange the information through the relay using network coding principles, we refer to this communication as the two-way relay channel (TWRC) [12]. In the conventional MIMO Y channel there are three TWRCs [11]. In our system model two PUs follow the bi-directional data exchange according to TWRC principles, while SUs send and receive unicast data, to and from the relay.

To accommodate different types of users, in this thesis we allocate some signaling dimensions to exchange data between PUs, while the remaining dimensions are designated for the transmissions of the secondary users. We design precoding strategies to accommodate different requirements for PU and SU transmissions. Specifically, we first consider scenarios when PUs have to use more bandwidth than SUs, i.e., when the PUs have more data to transmit. We also analyze the situation when the PUs do not need to achieve the high bandwidth utilization and the SUs take advantage of the spatial signaling dimensions. In addition, optimization of signaling dimensions and antenna scheduling/selection are employed to improve the system performance.

In the remainder of this chapter we present the building blocks related to our research in more details: In section 1.1, we discuss CR networks; In section 1.2, single user MIMO model; in section 1.3, selection diversity; in section 1.4, network coding; in section 1.5, SA; and in section 1.6, the original MIMO Y channel. Section 1.7 describes the thesis objectives, while Section 1.8 presents the organization of the whole thesis.

1.1 Cognitive Radio Networks

CR networks have been identified as intelligent networks, taking advantage of the CSI of various links in the system, and are a promising way to improve the spectrum efficiency of wireless communications by exploiting underutilized radio resources. In these networks, the users occupying the spectrum permanently allocated for their transmissions are referred to as primary users (PUs) and the users that opportunistically use the underutilized spectrum are called secondary users (SUs). In cognitive radio network (CRN), wireless terminals use some side information to intelligently recognize the potentials of different communications to enhance and benefit the operation of various networks by reconfiguring the communication protocols. This side information usually consists of information about the activity, encoding strategies, channel conditions and sometimes transmitted data sequences of primary users whose spectrum is being utilized by SUs. To avoid interference between PUs and SUs transmitting at the same time and in the same frequency band, the mutual cooperation between terminals is necessary. Based on the application and existing constraints on spectrum usage, cognitive radio systems fall into different schemes: underlay, overlay and interweave paradigms. In underlay paradigms, the SUs are permitted to operate if the interference they cause to the PUs is below a certain threshold, or if the PUs' performance degradation is within an acceptable margin. In overlay systems, SUs and PUs transmit simultaneously at the same frequency; However, SUs can act as an intermediate for PU communications to access information about PU transmissions and also cooperate with them to improve PU communication, while obtaining spectral resources for their own communication–this is the model followed in this thesis. In interweave paradigms, SUs monitor the radio spectrum and detect the absence of a signal in PU transmissions over space, time, or frequency and opportunistically transmit during these spectral holes without interfering with PU communications [2], [4].

1.1.1 Overlay CRN

In a typical overlay system, a secondary transmitter knows the PU's message and how the message is encoded (also called codebook). If the PUs transmit based on a standard publicized codebook or broadcast their codebooks periodically, the SUs can obtain the codebook information. Specifically, the SUs can exploit the information about the PU's message and/or codebook in a variety of ways to improve the performance of both the PUs and SUs, i.e., it can be used to eliminate interference caused by PUs on the SUs' side. Moreover, the SU can cooperate with the primary system to improve its communication. In other words, an SU can dedicate part of its power to its own communication and the remainder to assist PU communications. Although the power assigned by the SU for this transmission enhances the signal-to-interference-plus-noise-ratio (SINR) for PUs, it also introduces interference to the primary system and hence decreases its SINR. Therefore, the SU's power allocation plays a role in determining the PU's SINR and its associated performance. Moreover, if the PU can decode both its own message and all or part of SU's data sequence, it can partially or completely eliminate the interference caused by the secondary user transmission. Thus, if we can suppress the interference in overlay systems, the PU's performance will remain either unchanged or be increased, while the SU will gain capacity based on the power it allocates for its own transmission. When there are multiple PUs and SUs the encoding and decoding techniques and the power allocation become more complicated [2].

In overlay systems there are complications that raise significant security and privacy concerns. These complications include overhearing the PU's transmission and sharing its data sequence with SUs, even when encrypted. Moreover, deploying encoding and decoding strategies with reasonable complexity is another practical hurdle. However, these challenges may not be applicable to some applications such as cellular overlay within the TV broadcast spectrum in which privacy and security concerns do not apply.

In licensed bands, SUs might be permitted to share spectrum with licensed PUs when they do not worsen their performance but rather improve it with cooperation. In unlicensed bands, PUs and SUs might have equal priority in the band, with SUs considered more capable. Therefore, the SUs could provide more efficient spectrum usage by exploiting their knowledge of PUs communication to reduce interference to all users. Additionally, unlike both interweave and underlay networks, in overlay networks a SU may enhance the communication of the primary system with relaying and exercising techniques to cope with interference which plays a key role to maximize the performance of both primary and secondary systems. [2]

1.2 MIMO System Model

MIMO systems in wireless communications can either be deployed to benefit from: (i) multiplexing gain which offers throughput increases in the case of high signal-to-noise (SNR) transmissions or (ii) diversity gain which provides resiliency against the fading channel [7], [13]. In a single user (SU) MIMO multiplexing configuration summarized in this section, MIMO uses multiple co-located antennas at the transmitter and the receiver for sending and receiving in multiple parallel data streams over the same radio link by exploiting multipath propagation. The MIMO system is modeled here by (i) T transmit antennas which send data streams represented by signals, X_j ($j \in \{1, \cdots, T\}$) through corresponding antennas, and (ii) R receive antennas which receive signals, Y_i ($i \in \{1, \cdots, R\}$). The channel response for the simplest flat-fading environment is modeled by a matrix with entries being scalar multiplicative gain factors for each transmit-receive antenna pair denoted by h_{ij}. The coefficient h_{ij} in the channel gain matrix $\mathbf{H}$ of size $R \times T$ represents the fading coefficient (random channel gain) from the i^{th} antenna of the transmitter towards the j^{th} antenna of the receiver which, for Rayleigh fading, is a complex Gaussian random variable [14]. Additive white Gaussian noise (AWGN) affecting the received signals is modeled by a separate noise term, N_j, added at each receive antenna. The MIMO system model is depicted in Fig. 1.1.

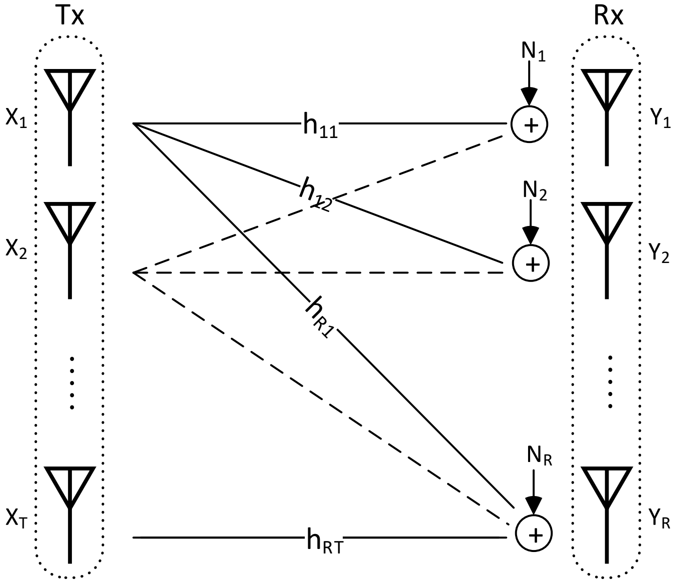

Figure 1.1: SU-MIMO system model.

Based on this SU MIMO system model, the received signals represented through linear system of equations are:

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_R \end{bmatrix} = \begin{bmatrix} h_{11} & \cdots & h_{1T} \\ \vdots & \ddots & \vdots \\ h_{R1} & \cdots & h_{RT} \end{bmatrix} \begin{bmatrix} X_1 \\ \vdots \\ X_T \end{bmatrix} + \begin{vmatrix} N_1 \\ \vdots \\ N_R \end{vmatrix} \tag{1.1}$$

or in a matrix notation they are given as:

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N} \tag{1.2}$$

where the vectors $\mathbf{Y}$ of size $R \times 1$ and $\mathbf{X}$ of size $T \times 1$ represent the signals sent and received, respectively. The vector $\mathbf{N}$ of size $R \times 1$ represents the AWGN noise.

The problem at the MIMO receiver is then how are the received signals, $\mathbf{Y}$, to be de-mixed after passing through a complex fading channel, $\mathbf{H}$? A possible solution assuming **H** is only known at the receiver is to use zero forcing (ZF) cancellation which applies to **Y** the pseudo-inverse of **H**. ZF decoding of received MIMO signals is conceptually simple and computationally not intensive but it suffers from noise enhancement which may limit the spatial stream capacity [15]. If the CSI in the form of **H** is known both to the transmitter and the receiver, a singular value decomposition (SVD) approach can be deployed to decipher the transmitted data without the noise enhancement penalty [16]. This approach requires pre-processing of the sent data at the transmitters and post-processing at the receiver in order to de-couple signals into parallel data streams. All data detection methods in MIMO channels call for a number of transmitted symbols to be less of equal to min $\big(R, T\big)$.

Finally, the representation of the received signals, **Y**, in (1.1) and (1.2) can be re-written in the form:

$$\mathbf{Y} = \sum_{j=1}^{T} \mathbf{h}_j X_j + \mathbf{N} \tag{1.3}$$

where $\mathbf{h}_j$ is the j-th column of **H** and has the size $R \times 1$. The representation in (1.3) offers geometrical interpretation of a mixing process in the MIMO channel where we are receiving signals in the R dimensional space and the transmitted scalar symbols X_j are sent along vectors (directions of) $\mathbf{h}_j$. This interpretation of sending data along the directions of $\mathbf{h}_j$ is very important in this thesis as in our work, through the pre-processing at the transmitters, we are trying to control signaling directions which is a more complex process than just working with random $\mathbf{h}_j$'s .

1.3 Selection Diversity

In fading channels when the terminals operate in a single-input single-output (SISO) antenna configuration, the channel gain may be very low (in deep fade) for a short period of time. This will lead to significant number of errors that heavily impacts the the overall average error rate of the system. The most effective way to overcome this problem is to use diversity techniques in the transmission and reception of the signals [17]. The strategy here is to provide the receiver with several replicas of the same information whose fading patterns are different and exploit the fact that it is unlikely that all signals experience bad fading conditions. Since the probability of all received signals fading simultaneously is low, this in turn leads to significant

BER improvements [18], [19]. There are several ways to provide the receiver with multiple independently fading replicas of the same information and signal combining methods. One of these methods is selection diversity in which one signal is chosen from a set of signals from diversity branches usually on the basis of received signal strength. Diversity can be achieved spatially by using multiple antennas either at the transmitter or at the receiver. Selecting the best set of antennas that result in better system performance is called antenna selection diversity an in our work we deploy this approach based on a proposed signal dimension selection criterion.

1.4 Network Coding

Network coding (NC) is a networking technique to improve capacity and performance of a system. In this technique, the transmitted data is a processed at terminals and the relay re-transmit combined version of received signals to terminals. This process is done in such a way that the receiver can recover the original message from the received signal. NC has been developed the objective of reducing total number of time slots required to exchange data. In wireless networks, there are two types of signal processing at the relay: amplify-and-forward (AF) and decode-and-forward (DF) [6]. In AF strategy, the relay re-transmit the signal after amplifying the received signal. However, the relays with DF, the relay decodes the received signal to reduce noise effects and then re-encode it before transmission. Physical-layer network coding is a wireless extension of NC that employs DF strategy at the relay. In this thesis, we take the advantage of PNC in our systems.

To illustrate a wireless system with NC, we use the model of a two-way relay channel (TWRC) [20] visualized in Fig. 1.2 [21]. In this case, there are two packets a and b that need to be exchanged between two terminals. We assume that all nodes are operating in half-duplex mode and there is no direct link between terminals as they exchange data only via the relay R. In the conventional non-NC scheme shown in Fig. 1.2(a), each of terminals A and B need one time slot (TS) to transmit a packet to the relay, and the relay needs two TSs to broadcast each packet to terminals. As a result, in this system we need four TSs to exchange two packets between two terminals. After deploying NC as in Fig. 1.2(b), we can accomplish the data exchange in three TSs. Instead of requiring two separate TSs to broadcast the packets from the relay to terminals, NC allows the relay to combine the received packets and transmit a single, XORed packet $c = a \oplus b$, in one TS. Then, both terminals A and B can recover desired packets b and a by XOR-ing packet c with their own packet they transmitted. Figure 1.2(c) shows the number of required TSs for this data exchange will be reduced to two TSs when PNC is used. Terminals A and B simultaneously send their packets a and b to the relay in one TS, and the relay broadcasts the XORed representation of packets in the second TS. The terminals recover their data of interest by XOR-ing the received data with their own data similarly as in NC.

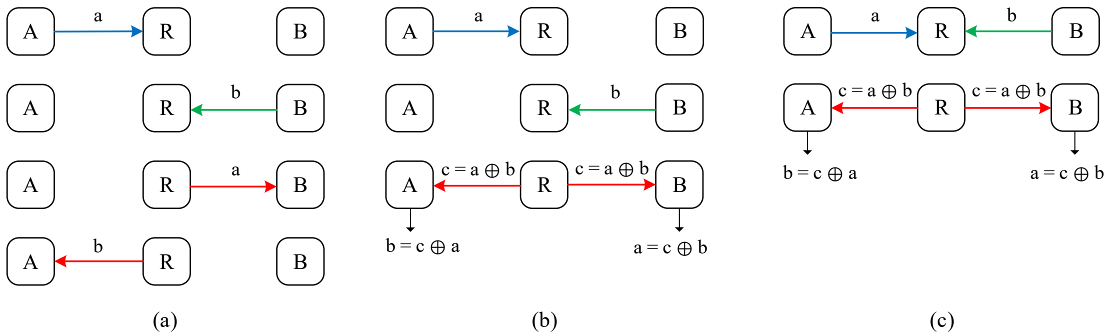

Figure 1.2: (a) Traditional non-NC vs. (b) Traditional NC vs. (c) PNC.

1.5 Signal Alignment

In wireless systems where multiple terminals transmit at the same time and frequency without orthogonal multiplexing to increase throughput, interference is inevitable. Signal alignment (SA) is a wireless communication technique that enables us to have PNC in MIMO wireless networks [5], [11]. In SA, signals are precoded at the transmitters to fully exploit the signal space so that the receivers can better utilize the spacial diversity of a MIMO network for higher throughput [22]. SA focuses on grouping signals with mutual interests by aligning them into the same dimension. For example, Fig. 1.3 shows a four-user TWRC model where the terminals, indexed by j and j' $(j, j' \in \{1, 2\})$ paired into two groups presented by different colors (red and blue). Each user is equipped with two antennas and two packets need to be exchanged via the relay which is also equipped with two antennas (each user sends one packet). Precoding vectors $\mathbf{P}_1$, $\mathbf{P}_{1'}$, $\mathbf{P}_2$ and $\mathbf{P}_{2'}$ of size 2×1 also called beamforming vectors,

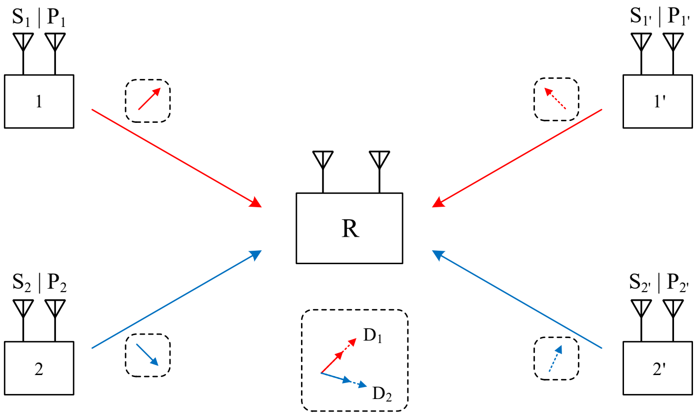

Figure 1.3: Two-paired TWRC with SA.

are designed by mutual users in such a way that when the messages pass through the channel and multiplied by the channel matrix, the signals from paired users are aligned into one dimension. Therefore, the four messages sent by the four terminals are aligned in two spatial dimensions and combined as two mutual signals so the relay can receive them with its two antennas. The two different dimensions is shown in Fig. 1.3. The relay then decodes two mutual symbols (XOR-ed version of the pair's data) and re-encodes them before broadcasting to the terminals. Each terminal is able to obtain data of interest by subtracting its own transmitted message from the received signal. Hence, SA provides exceptional bandwidth efficiency by exchanging four messages within two TSs. Furthermore, since the four messages are aligned into two dimensions, the relay requires only two antennas, while in the MIMO system without NC, the relay requires four antennas to receive four signals in the first TS.

1.6 MIMO Y Channel Model

The MIMO Y Channel is an extended model of a wireless TWRC with three users [23]. Figure 1.4 shows the minimum antenna configuration for original three user MIMO Y channel. Each user node i $i \in \{1, 2, 3\}$, is equipped with two antennas and the relay node has three antennas. Each user i sends one message to the other users, denoted as $m_{j,i}$ ($j \in \{1, 2, 3\},\ j \neq i$), where j is the destination node and i is the source node number. At the bit level, $m_{j,i} \in \{1, 0\}$, and if BPSK modulation is applied, the symbol representation of $m_{j,i}$ is $s_{j,i} \in \{1, -1\}$. It is assumed there is no direct links between user nodes and all communications between nodes are conducted through the relay station. Another assumption is that all nodes are working in half-duplex mode and have access to the perfect global channel state information (CSI). In this system, a total of six messages are being exchanged in two phase: medium access control (MAC) phase and broadcast (BC) phase.

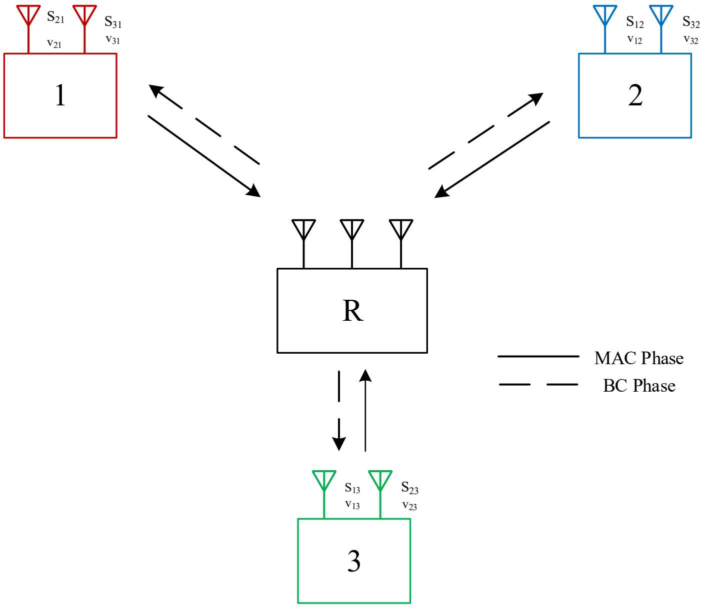

Figure 1.4: The original 3-user MIMO Y channel model.

1.6.1 MAC Phase

In the MAC phase, messages are being processed at each user node with the precoding vectors and then transmitted to the relay simultaneously. The precoding vector for each message is selected to align messages of same communication pair into one dimension in the signal space. Then, a decode-and-forward (DF) scheme is used to recover the messages at the relay. Fig. 1.5 shows the aligned dimensions in three different dimensions, denoted as $\mathbf{D}_{12}$, $\mathbf{D}_{13}$ and $\mathbf{D}_{23}$. Colors are used to differentiate between user pairs. For instance, $\mathbf{D}_{12}$ is the dimension that mutual symbols, $s_{2,1}$ and $s_{1,2}$, are aligned. The aligned dimensions are defined by the product of channel gain matrix $\mathbf{H}_{R,i}$ and the precoding vector $\mathbf{v}_{j,i}$. $\mathbf{H}_{R,i}$ matrix is of size 3×2 with each element of $h_{R,i}(m,n)$ represents the gain from transmit antenna n of the i-th terminal to the receive antenna m of the relay R. Two information symbols at user i, $s_{l,i}$ and

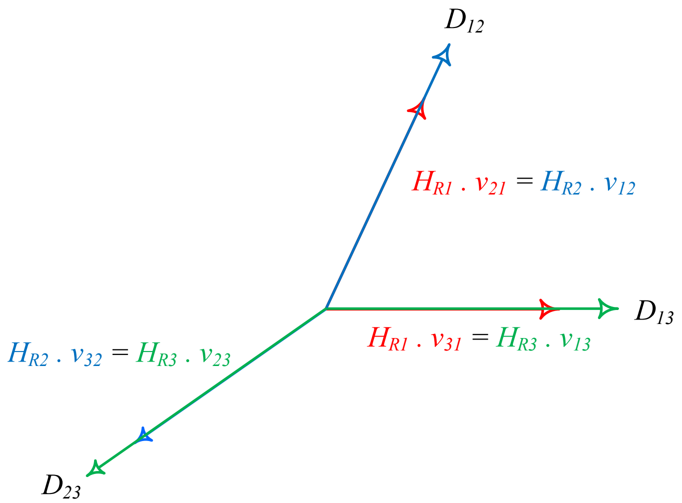

Figure 1.5: The three aligned dimensions for mutual symbols using SA in the MAC phase.

$s_{p,i}$ $(l,p = \{1,2,3\}; l,p \neq i)$, are processed by the precoding matrix $\mathbf{v}_i = [\mathbf{v}_{l,i} \mid \mathbf{v}_{p,i}]$ of size 2×2 which is a concatenation of precoding vectors $\mathbf{v}_{l,i}$ and $\mathbf{v}_{p,i}$, of size 2×1. Hence, the transmitted signal by the user i is $\mathbf{v}_i \cdot \left[\begin{smallmatrix} s_{l,i} \\ s_{p,i} \end{smallmatrix} \right]$ and the received signal from user i at the relay is $\mathbf{H}_{R,i} \cdot \mathbf{v}_i \cdot \left[\begin{smallmatrix} s_{l,i} \\ s_{p,i} \end{smallmatrix} \right]$. Since the channel matrix is determined based on the channel condition, precodig vectors are the only factors that can tailor the messages into desired signal dimensions. The common assumptions for the power constraints in MIMO Y channel are that the average power of information symbols is one and the total transmit power in the system is limited to one. When the average power of symbols is limited to one and the sum power for transmitting a pair of mutual symbols are the same, the transmit power is defined only by the square of norms of precoding vectors as follows:

$$\begin{aligned} P_T &= \sum_{i=1}^{3} \left\{ \mathbf{v}_{l,i}^H \cdot \mathbf{v}_{l,i} + \mathbf{v}_{p,i}^H \cdot \mathbf{v}_{p,i} \right\} \\ &= |\mathbf{v}_{2,1}|^2 + |\mathbf{v}_{3,1}|^2 + |\mathbf{v}_{1,2}|^2 + |\mathbf{v}_{3,2}|^2 + |\mathbf{v}_{1,3}|^2 + |\mathbf{v}_{2,3}|^2 \\ &= (|\mathbf{v}_{2,1}|^2 + |\mathbf{v}_{1,2}|^2) + (|\mathbf{v}_{3,1}|^2 + |\mathbf{v}_{1,3}|^2) + (|\mathbf{v}_{3,2}|^2 + |\mathbf{v}_{2,3}|^2) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \end{aligned} \tag{1.4}$$

The precoding vectors are designed to align mutual messages in such a way that it satisfies the following system of linear equations:

$$\begin{cases} \mathbf{D}_{12} = \mathbf{H}_{R,1} \cdot \mathbf{v}_{2,1} = \mathbf{H}_{R,2} \cdot \mathbf{v}_{1,2} \\ \mathbf{D}_{13} = \mathbf{H}_{R,1} \cdot \mathbf{v}_{3,1} = \mathbf{H}_{R,3} \cdot \mathbf{v}_{1,3} \\ \mathbf{D}_{23} = \mathbf{H}_{R,2} \cdot \mathbf{v}_{3,2} = \mathbf{H}_{R,3} \cdot \mathbf{v}_{2,3} \end{cases} \tag{1.5}$$

After precoding at every terminal and transmitting messages to the relay, the received signal at the relay is:

$$\begin{aligned} \mathbf{Y}_R &= (\mathbf{H}_{R,1} \cdot \mathbf{v}_{2,1} \cdot s_{2,1} + \mathbf{H}_{R,2} \cdot \mathbf{v}_{1,2} \cdot s_{1,2}) \\ &\quad + (\mathbf{H}_{R,1} \cdot \mathbf{v}_{3,1} \cdot s_{3,1} + \mathbf{H}_{R,3} \cdot \mathbf{v}_{1,3} \cdot s_{1,3}) \\ &\quad + (\mathbf{H}_{R,2} \cdot \mathbf{v}_{3,2} \cdot s_{3,2} + \mathbf{H}_{R,3} \cdot \mathbf{v}_{2,3} \cdot s_{2,3}) + n_r \\ &= \mathbf{D}_{12} \cdot s_{12} + \mathbf{D}_{13} \cdot s_{13} + \mathbf{D}_{23} \cdot s_{23} + \mathbf{n}_r \\ &= \mathbf{D} \cdot [s_{12}, s_{13}, s_{23}]^T + \mathbf{n}_r \end{aligned} \tag{1.6}$$

where T stands for transpose. Matrix $\mathbf{D}$ of size 3×3 is obtained by concatenating column vectors $\mathbf{D}_{12}$, $\mathbf{D}_{13}$ and $\mathbf{D}_{23}$. It is worth observing that $s_{ij} = s_{i,j} + s_{j,i}$ and if $s_{i,j} = \pm 1$ then $s_{ij} = \pm 2$ or 0.

In the next step, the relay can deploy DF techniques, e.g. zero-forcing (ZF), maximum likelihood (ML) or minimum mean square error (MMSE), to recover each aligned signal [24], [25]. In ZF approach, the data is recovered by multiplying the received signal with the inverse matrix of $\mathbf{D}$ [12],[26],[27]. When the relay receives the ternary symbols, it then encodes the data to BPSK format so that the bits that the relay broadcasts in BC phase are $(m_{2,1} \oplus m_{1,2})$, $(m_{3,1} \oplus m_{1,3})$, $(m_{2,3} \oplus m_{3,2})$, and users can easily use XOR operation to obtain the desired messages at each user node. [28] In the next subsection, we shed the light on the BC phase.

1.6.2 BC Phase

In the BC phase, a combined signal of three pairs of network-coded signals is produced by applying nulling vectors at the relay and then broadcasted to all users at the same time in one time slot. Since each user is interested in receiving only the signal of interest, the nulling vectors for each aligned aligned symbol denoted by $\mathbf{U}_{12}, \mathbf{U}_{13}$ and $\mathbf{U}_{23}$, each of size 3×1, is applied at the relay in order to remove the impact of undesired signal for each user node. Hence, each user receives the relevant data and is able to decode desired messages by conducting XOR operations.

$$\begin{aligned} \mathbf{X}_R &= \mathbf{U}_{12} \cdot s_{12} + \mathbf{U}_{13} \cdot s_{13} + \mathbf{U}_{23} \cdot s_{23} \\ &= \mathbf{U} \cdot [s_{12}, s_{13}, s_{23}]^T \end{aligned} \tag{1.7}$$

where $\mathbf{U}$ of size 3×3 is obtained by concatenating $\mathbf{U}_{12}, \mathbf{U}_{13}$ and $\mathbf{U}_{23}$. To prevent users from receiving undesired signals the nulling vectors need to satisfy the following conditions:

$$\begin{cases} \mathbf{H}_{1,R} \cdot \mathbf{U}_{23} = \mathbf{0} \\ \mathbf{H}_{2,R} \cdot \mathbf{U}_{13} = \mathbf{0} \\ \mathbf{H}_{3,R} \cdot \mathbf{U}_{12} = \mathbf{0} \end{cases} \tag{1.8}$$

In each equation of (1.8), there are three unknowns and three linear equations, so there is a solution for each nulling vector. The received signal at User 1 is then:

$$\begin{aligned} \mathbf{Y}_{1,R} &= \mathbf{H}_{1,R} \cdot \mathbf{U} \cdot [s_{12}, s_{13}, s_{23}]^T \\ &= \mathbf{H}_{1,R} \cdot \mathbf{U}_{12} \cdot s_{12} + \mathbf{H}_{1,R} \cdot \mathbf{U}_{13} \cdot s_{13} + \mathbf{H}_{1,R} \cdot \mathbf{U}_{23} \cdot s_{23} \\ &= \mathbf{H}_{1,R} \cdot [\mathbf{U}_{12} \cdot s_{12} + \mathbf{U}_{13} \cdot s_{13}] \end{aligned} \tag{1.9}$$

Users 2 and 3 also independently receive:

$$\mathbf{Y}_{2,R} = \mathbf{H}_{2,R} \cdot [\mathbf{U}_{12} \cdot s_{12} + \mathbf{U}_{23} \cdot s_{23}] \tag{1.10}$$

$$\mathbf{Y}_{3,R} = \mathbf{H}_{3,R} \cdot [\mathbf{U}_{13} \cdot s_{13} + \mathbf{U}_{23} \cdot s_{23}] \tag{1.11}$$

To recover data of interest, the received signal at User 1 is:

$$\mathbf{Y}_{1,R} = \mathbf{H}_{1,R} \cdot \mathbf{U}_1 \cdot [s_{12}, s_{13}]^T \tag{1.12}$$

Then the user i can recover aligned symbols using matrix U_1 inversion and recover the message from other users by XOR-ing its own message with the received one:

$$m_{i,j} = m_{j,i} \oplus \left(m_{i,j} \oplus m_{j,i}\right) \tag{1.13}$$

1.7 Thesis Objectives

The general objective of this work is to study methods to adapt SA principles of a relay-assisted MIMO Y channel topology to accommodate (i) bi-directional data exchanges between PUs and (ii) unicast traffic between SUs and the relay, in CR networks. Specifically, this thesis designs SA schemes for different antenna configurations and user setups where, in the multiple access (MAC) phase, users send all messages to the relay simultaneously, and, in the broadcast (BC) phase, the relay transmits messages to users on a single MIMO channel access. When spatially multiplexing two- and one-way data transmissions via a cooperating relay implementing DF processing, the objective is to work with a minimized number of antennas at all nodes in the system. The main focus is to present designs for precoding and nulling vectors in the proposed SA schemes with zero-forcing (ZF) decoding. Because of the asymmetric traffic flows in the system, there is possibility to optimize the selection of signal dimensions which is exploited to provide the diversity advantage. Since the precoding vectors generated as a solution to the SA requirements are not unique we are able to optimize the system for having the best BER performance.

The initial work in this thesis was limited to two PUs and two SUs with small numbers of antennas. However, in future wireless networks, when spectrum sharing will be deployed on a much wider scale within the CR paradigm, the PUs and SUs of spectrum will have transceivers with different antenna configurations [2]. This was the motivation to further expand our work to cover different antenna configurations and different user setups and consequently find the SA designs for the generalized case of the proposed schemes. Drawing on the flexibility in choosing orthogonal spatial dimensions in some scenarios and antenna diversity in others, we also optimized the BER performance of the systems in different network scenarios. When possible, we control the selection of spatial signaling dimensions to maximize the distance between signal points representing network coded messages and SU unicast messages. This is to improve BER performance. When possible, we design precoding vectors to utilize orthogonal signaling at the relay. This has the advantages of maximum likelihood (ML) decoding along with simple processing of the ZF decoding without suffering from noise enhancement. Joint relay antenna selection and ZF spatial multiplexing for MIMO TWRC with two nodes has been pursued in [29]. However, what is unique in this thesis is that we use specialized SA to accommodate traffic conditions from different communicating nodes, it gives us the ability to select the antennas based on virtual channel gains after pre- and post-processing at the user nodes and the relay.

In the second chapter, this thesis explores options to incorporate more users in a conventional MIMO communications and increase bandwidth efficiency by employing cognitive radio technology and then finding a way to tackle the major issue in cognitive radios: interference. We propose a novel interference mitigation technique in overlay cognitive radio by adapting SA strategies used in original MIMO Y channel scheme. We illustrate system models that can satisfy the conditions with four users (two PUs and two SUs). Then, with the flexibility in these scenarios to choose signaling dimensions, each scenario has been optimized by imposing orthogonality in signaling dimensions. This has two benefits:

1. it gives us a larger minimum distance between signaling points which can lead to significant improvement in the performance of the system in terms of BER;
2. it simplifies and improves the decoding process by deploying ZF decoding without suffering from noise enhancement which is its main disadvantage.

Furthermore, in order to have the capability of exchanging any number of messages and to identify the requirements for the antenna setting, we extend our work and analyze system models to find relationships between working scenarios and generalize them for more complicated antenna configurations.

In the third chapter, the thesis investigates the possibility of incorporating five users (two PUs and three SUs). In this case we can accommodate more SUs that utilize PU spectrum without causing interference for the PUs. In this scenario, although we don't have the freedom to modify the signaling dimensions, we have selection diversity at the relay in the MAC phase, which gives us the flexibility to choose the best antennas for the relay to receive it. Therefore we optimize this scenario to enhance the BER in the system. Afterward, the system model is generalized for any number of antenna for user nodes. In an attempt to generalize the system so it will be capable of accommodating any number of SUs in PUs' communication, after finding a relationship between working scenarios, sequences of numbers for any number of antennas and any number of SUs have been discovered. Then, a complex mathematical process and trial and error employed to transform the sequence of numbers into formulas. Hence, we are now able to accommodate as many SUs as we would like in PUs' communication by knowing the desired number of antennas at each node and find the maximum number of messages that can be exchanged in the corresponding setting.

The proposed schemes not only performed well in terms of bandwidth efficiency but also the simulation results for different scenarios have shown a significant improvement in terms of BER over conventional MIMO Y channel and non-optimized scenarios.

1.8 Thesis Organization

Results of the research described in this thesis have been partially published in the form of two conference papers [30], [31] and the following two chapters are reflecting the contributions that is partially published in these papers.

The remainder of this thesis is organized as follows:

In Chapter 2, we propose a novel scheme to eliminate interference in overlay CR networks by adapting SA techniques used in original MIMO Y channel scheme. Different scenarios with four-user nodes are described in detail in this chapter. Specifically, precoding vectors are carefully designed to meet the requirements of SA in cognitive networks with a cooperative relay. Then to further improve BER in each scenario, signaling dimensions optimized by exploiting degrees of freedom. After studying four user scenarios, the generalized extension for any number of antennas at user nodes are obtained and guidelines for further extensions are presented. Finally, we evaluate the performance of the proposed designs in terms of BER in a Rayleigh fading channel environment.

In Chapter 3, we present a unique five-user design (two PUs and three SUs) in an attempt to accommodate more SUs. Although in this category of scheme we do not have the flexibility to have orthogonal signaling dimensions in MAC phase. But we have the freedom to choose the best antennas at the reception side when the relay receives messages from four nodes. Thus, a computationally efficient method for antenna selection is deployed to optimize the system and enhance the BER. In the next step, the five-user system model is generalized so that PUs and SUs can have any number of antennas. Then, further generalization of the system are developed to be able to accommodate any number of SUs equipped with any number of antennas. After finding configurations in the generalized system model for PUs and SUs (number of messages and required antennas for each user) and verifying them, the underlying pattern for each obtained sequence of numbers are discovered and the formulas for them are developed.

Chapter 4 includes conclusion and a potential future work.

Chapter 2

Four-User Overlay CR Network in MIMO Y Channel Topology

In wireless networks, because of their broadcast attributes, signals from various transmitters sharing the wireless medium at the same time and frequency introduce multiple access interference (MAI) which is also the case in CRNs. Traditionally, MAI has been controlled using time or frequency division multiple access. Recently, to achieve higher throughput in MIMO communication systems with relays, better coordination between transmit and receive user pairs and more advanced signal processing are deployed. Different signaling schemes have appeared to successfully handle the MAI and enhance the bandwidth efficiency of relay networks in various configurations of user traffic and antenna deployments. In multi-way communications, recently, new possibilities have been created by two approaches: signal alignment (SA) and physical layer network coding. SA efficiently exploits opportunities available in time and spatial domains to suppress MAI. The physical layer network coding, through superposition of electromagnetic (EM) waves representing information signals, reduces the number of transmissions to accomplish data exchange. SA along with the physical layer network coding (PNC) allows for the increasing number of users with a significantly improved sum data rate in the system. One of the basic building blocks of relay-aided systems is the two-way relay channel (TWRC), where two source nodes exchange information through a common relay. Assuming all nodes in this system operate in half-duplex mode, for the bi-directional communications between two nodes, the network coding principles are utilized to allow information exchange of two users within two time slots. By considering multiple signal dimensions using multiple antennas, TWRC concepts have been extended to multiple source nodes, where there is no direct link between the users. In these schemes, two signals to be exchanged in the bi-directional communications between two nodes are transmitted in the same spatial dimension, and the relay jointly processes the sum of the two signals.

In this chapter, an information sharing process in the relay aided network with three users is investigated where two time slots are utilized to accomplish the data exchange between four nodes (including the relay). Specifically, using signal alignment principles along the relay node and users' spatial dimensions in the BC phase and MAC phases, respectively, (i) two users equipped with the same number of antennas are exchanging their bi-directional data via the relay according to the TWRC model in the BC and MAC phases and (ii) one user is transmitting the data to the relay in the MAC phase and receives the data from the relay in the BC phase. Depending on traffic demands and antenna configurations for different users, we design precoding vectors and signal recovery for different transmissions using similar principles to those deployed in MIMO Y and generalized MIMO Y channels. The proposed schemes have relevance to cognitive radio (CR) systems where the wireless networks intelligently utilize the information about the activity (traffic patterns), channel conditions etc. of different nodes in the system to achieve higher utilization of the shared spectrum. Specifically, this system configuration falls into the category of overlay CR where users involved in the bi-directional exchange via the relay could be considered the primary users (PUs) and the relay using redundant number of antennas for this exchange allows the secondary user (SU) transmissions by not only sharing the frequency but also the spatial dimension. It can also be viewed as a form of distributed multi-user (MU) MIMO relaying.

In the conventional MIMO Y channel scheme, to meet signal alignment conditions with precoding vectors, three homogeneous user nodes are equipped with two antennas and one relay is equipped with three antennas. However, in these systems, BER performance significantly suffers in Rayleigh fading and as they are only applicable in high SNR regions. Moreover, the antenna configuration in the original MIMO Y channel system does not offer flexibility to select spatial dimensions and controlling the minimum distance between signaling points representing network coded messages, which affects BER performance. To improve the reliability of the original Y channel model, there are few designs benefiting from transmit and receive diversity by adding more antennas [27], [32], [33]. For the traffic patterns considered in this chapter, there is a flexibility to select different beamforming vectors without increasing the number of antennas at different nodes. Specifically, the precoding vectors generated as a solution to the SA requirements are not unique and they are optimized, i.e., selected from alternative solutions to the SA requirements based on the resulting BER performance. In this chapter, we control the selection of spatial signaling dimensions to maximize the distance between signaling points representing network coded messages and SU unicast messages. This is to improve BER performance. When possible, we design orthogonal signaling at the relay having the advantage of ML decoding with simple processing and does not suffer from noise enhancement as in the Zero-Forcing (ZF) decoding.

In this chapter, we explore the integration of CR networks and SA techniques in MIMO Y channel topology. Specifically, we develop an innovative approach to mitigate interference in CRNs. Although due to antenna configuration in original MIMO Y channel we have limitations on controlling the minimum distance between signaling points, in our case we are able to design precoding vectors and optimize signaling dimensions in such a way as to simplify decoding at the relay and also further improve the BER performance. Specially, when dimensions are designed to be orthogonal it maximizes the minimum distance between signaling symbols and have the advantage of ML decoding with simple processing, which does not suffer from noise enhancement as in the ZF decoding. Furthermore, power allocation need be applied to distribute energy among the users based on their channel conditions. The introduced transmission schemes for both MAC and BC phases improve the system reliability in term of BER for PUs and also allows SU nodes to utilize the PUs' bandwidth and use it for its own communication with the relay.

This chapter is organized as follows. First, Section 2.1 introduces the general concepts for the proposed scheme, while Sections 2.1.1 and 2.1.2 demonstrate the operation of the developed schemes in detail for MAC and BC phases, respectively. Specifically, these subsections develop methods to calculate the precoding and nulling vectors. Next, Section 2.2 describes the advantages of applying orthogonality and signaling dimension optimization in the underlying system. Specific antenna configurations and corresponding traffic pattern are analyzed in Sections 2.2.1 and 2.2.2 where we reinforce the concepts from the earlier sections. Section 2.3 presents operation of the proposed scheme with higher number of antennas at various nodes. Different results are summarized and compared in Section 2.4. Finally, Section 2.5 gives a brief summary of the chapter.

2.1 System Model

Our MIMO Y channel topology with three users labeled as $k = (1, 2, 3)$ is shown in Fig. 2.1. Users 1 and 2, referred to as PUs, have M_{12} independent messages for each other denoted as $s_1(i)$ and $s_2(i)$, respectively where $i \in \{1, \cdots M_{12}\}$. Without loss of generality, the messages are represented by scalar (± 1) BPSK symbols. As there are no direct links between the users, these messages are to be exchanged via relay R in two time slots (TSs) called MAC and BC TSs. In the MAC TS, User 3, referred to as SU, sends to relay R M_3 messages represented by BPSK symbols $s_{R3}(l)$ with $l \in \{1, \cdots M_3\}$. In the BC TS, relay R sends M_3 messages to User 3 and these messages are represented by BPSK symbols $s_{3R}(l)$ again with $l \in \{1, \cdots M_3\}$. Users 1 and 2 are equipped with the same number of N_{12} antennas and User 3 is equipped with N_3 antennas. Relay R has N antennas. It is assumed that all nodes operate in half-duplex mode and all the wireless channels are Rayleigh flat fading. Also, the

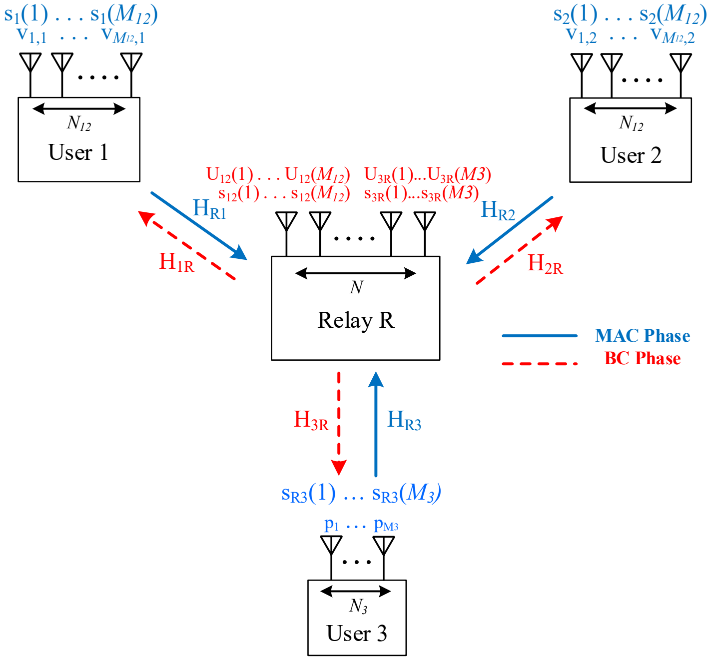

Figure 2.1: Four-user MIMO Y channel topology with TWR and unicast traffics.

common channel state information (CSI) is available to all nodes to calculate the required precoding and post-processing matrices.

In this model, the channel gain matrices $\mathbf{H}_{R,1}$ and $\mathbf{H}_{R,2}$ from Users 1 and 2 to the relay are of size $N \times N_{12}$; and $\mathbf{H}_{1,R}$ and $\mathbf{H}_{2,R}$ from the relay to Users 1 and 2 are of size $N_{12} \times N$. The channel gain matrices between User 3 and relay in the MAC and BC phases are $\mathbf{H}_{R,3}$ and $\mathbf{H}_{3,R}$ and their size is $N \times N_3$ and $N_3 \times N$, respectively. Each entry of $\mathbf{H}$ is assumed to be independent identically distributed (i.i.d.) complex Gaussian.

2.1.1 MAC Phase

In the MAC phase, for the PU k with $k = 1$ or $k = 2$, M_{12} information symbols $s_k(i)$ $(i = 1, \cdots, M_{12})$ are pre-processed by the precoding matrix $[\mathbf{v}_{1,k} \mid \cdots \mid \mathbf{v}_{M_{12},k}]$ where the precoding vectors $\mathbf{v}_{i,k}$ are of size $N_{12} \times 1$. The transmitted data vector by PU k is then:

$$\mathbf{x}_k = [\mathbf{v}_{1,k} \mid \cdots \mid \mathbf{v}_{M_{12},k}] \cdot \left[\begin{array}{c} s_k(1) \\ \vdots \\ s_k(M_{12}) \end{array} \right] = \sum_{i=1}^{M_{12}} \mathbf{v}_{i,k} \cdot s_k(i) \tag{2.1}$$

The User 3 data ($s_{R3}(l)$ with $l = (1, \cdots, M_3)$) are pre-processed in the MAC phase by the precoding matrix $[\mathbf{p}_1 \mid \cdots \mid \mathbf{p}_{M_3}]$ so that the transmitted signal at User 3 is:

$$\mathbf{x}_3 = [\mathbf{p}_1 \mid \cdots \mid \mathbf{p}_{M_3}] \cdot \left[\begin{array}{c} s_{R3}(1) \\ \vdots \\ s_{R3}(M_3) \end{array} \right] = \sum_{l=1}^{M_3} \mathbf{p}_l \cdot s_{R3}(l) \tag{2.2}$$

The standard assumptions in the MIMO Y channel model are that (i) the average power of information symbols is one and (ii) the total transmit power in the system is also limited to one, which imposes the constraints on the average transmit power for each user and their corresponding precoding vectors.

When $2 \cdot M_{12} + M_3$ messages are transmitted from the three users simultaneously to the relay, the received signal at the relay is:

$$\mathbf{Y}_R = \sum_{k=1}^{3} \mathbf{H}_{R,k} \cdot \mathbf{x}_k + \mathbf{n}_R \tag{2.3}$$

where $\mathbf{n}_R$ is the additive white Gaussian noise (AWGN) and both $\mathbf{Y}_R$ and $\mathbf{n}_R$ are of size $N \times 1$. To align the symbols $s_1(i)$ and $s_2(i)$ from Users 1 and 2 into the same dimension in the signaling space (as visible at the relay), the precoding vectors for the mutual symbols are carefully selected so that (2.3) can be represented as:

$$\begin{aligned}\mathbf{Y}_R &= \sum_{i=1}^{M_{12}} \Big(\mathbf{H}_{R,1}\mathbf{v}_{i,1} \cdot s_1(i) + \mathbf{H}_{R,2}\mathbf{v}_{i,2} \cdot s_2(i)\Big) \\ &+ \sum_{l=1}^{M_3} \mathbf{H}_{R,3}\mathbf{p}_l \cdot s_{R3}(l) + \mathbf{n}_R \\ &= \sum_{i=1}^{M_{12}} \mathbf{D}_{12}(i) \cdot \hat{s}_{12}(i) + \sum_{l=1}^{M_3} \mathbf{D}_{R3}(l) \cdot s_{R3}(l) + \mathbf{n}_R\end{aligned} \tag{2.4}$$

where $\mathbf{D}_{12}(i)$ denote the M_{12} spatial signaling dimensions of size $N \times 1$ for bi-directional transmissions between User 1 and 2 and can be seen as the effective channel gains for the aligned symbols $\hat{s}_{12}(i)$ with $\hat{s}_{12}(i) = s_1(i) + s_2(i)$. The mutual symbols $\hat{s}_{12}(i)$ are decoded at the relay as ternary symbols. The signaling dimensions $\mathbf{D}_{R3}(l)$ also of size $N \times 1$ are used for the unicast (one-way) transmissions from User 3 to relay R. According to SA requirements in (2.4), the selected precoding vectors $\mathbf{v}_{i,1}$ and $\mathbf{v}_{i,2}$ need to satisfy:

$$\mathbf{H}_{R,1} \cdot \mathbf{v}_{i,1} = \mathbf{H}_{R,2} \cdot \mathbf{v}_{i,2} \quad \text{for} \quad i \in \{1, \cdots\ M_{12}\} \tag{2.5}$$

and $\text{null}\big([\mathbf{H}_{R,1} \mid -\mathbf{H}_{R,2}]\big)$ — the null space of the concatenated matrix $[\mathbf{H}_{R,1} \mid -\mathbf{H}_{R,2}]$ — should have the dimension higher or equal to M_{12} for $\mathbf{D}_{12}(i)$ to be distinct. This is because $\left[\begin{smallmatrix}\mathbf{v}_{i,1}\\ \mathbf{v}_{i,2}\end{smallmatrix}\right]$ is the solution set of a system of linear homogeneous equations with $[\mathbf{H}_{R,1} \mid -\mathbf{H}_{R,2}]$ as the coefficient matrix.

Since the rank of any matrix $\mathbf{A}$ plus the nullity of $\mathbf{A}$ equals the number of columns of $\mathbf{A}$ [34], when we apply this theorem to $[\mathbf{H}_{R,1} \mid -\mathbf{H}_{R,2}]$, we obtain that $2 \cdot N_{12} - N \geq M_{12}$. When we have the equality in this inequality, we do not have the flexibility in choosing $\left[\begin{smallmatrix}\mathbf{v}_{i,1}\\ \mathbf{v}_{i,2}\end{smallmatrix}\right]$ and also in working with different dimensions $\mathbf{D}_{12}(i) = \mathbf{H}_{R,1}\mathbf{v}_{i,1}$ to optimize the performance of the TWRC in the MAC phase for User 1 and 2. It is actually the flexibility of working with different $\mathbf{D}_{12}(i)$ and $\mathbf{D}_{R3}(l)$ when meeting the SA requirements that motivates the work in this chapter and is exploited in Section 2.2.

Usually, the system of linear equations in (2.4) is represented in a matrix form with matrix of coefficients $\mathbf{D}$ of size $N \times (M_{12} + M_3)$ obtained by concatenating effective channel gain columns $\mathbf{D}_{12}(i)$ and $\mathbf{D}_{R3}(l)$ ($i = \{1, \cdots, M_{12}\}$ and $l = \{1, \cdots, M_3\}$).

With this representation, in the zero-forcing (ZF) approach, the recovery of the signal of interest (mutual messages $\hat{s}_{12}(i)$ and User 3 symbols $s_{R3}(l)$) at the relay is accomplished using the matrix $\mathbf{D}$ inversion (or pseudo-inversion). To decode at the relay desired signals, the number of all signaling dimensions $M_{12} + M_3 \leq N$ where in general we opt for the equality as to reduce the number of antennas at the relay. The signaling dimensions $\mathbf{D}_{R3}(l) = \mathbf{H}_{R,3}\mathbf{p}_l$ are controlled through the choice of precoding vectors $\mathbf{p}_l$ which may provide design options as presented in this chapter when attempting to maximize the distance in the constellation of received symbols at the relay.

2.1.2 BC Phase

In the BC phase, the relay forwards the aligned symbols $s_{12}(i)$ $(i = 1, \cdots, M_{12})$ as BPSK representations of XOR-ed bits between User 1 and 2; the relay also sends to User 3 its own data $s_{3R}(l)$ where $l = 1, \cdots, M_3$. A new set of precoding/nulling vectors $\mathbf{U}_{12}(i)$ and $\mathbf{U}_{3R}(l)$ of size $N \times 1$ is applied to the aligned symbols and User 3 symbols before broadcasting to users. Therefore, each user receives relevant data in the desired signal dimensions and eventually can decode intended messages using the N_{12} and N_3 antennas at User 1/2 and User 3, respectively. At User 1 and 2, bit-level XOR operations are performed on recovered symbols $s_{12}(i)$ $(i = 1, \cdots, M_{12})$ to obtain data of interest.

After precoding with the nulling matrix $\mathbf{U}=[\mathbf{U}_{12}(1)|\cdots|\mathbf{U}_{12}(M_{12})|\mathbf{U}_{3R}(1)|\cdots|\mathbf{U}_{3R}(M_3)]$ of size $N \times (M_{12} + M_3)$, the re-transmitted signal at the relay is written as:

$$\begin{aligned} \mathbf{X}_R &= \mathbf{U} \cdot [s_{12}(1), \cdots, s_{12}(M_{12}), s_{3R}(1), \cdots, s_{3R}(M_3)]^T \\ &= \sum_{i=1}^{M_{12}} s_{12}(i)\mathbf{U}_{12}(i) + \sum_{l=1}^{M_3} s_{3R}(l)\mathbf{U}_{3R}(l) \end{aligned} \tag{2.6}$$

where T stands for transpose. To realize the idea described in this section, if possible, nulling columns $\mathbf{U}_{12}(i)$ and $\mathbf{U}_{3R}(l)$ are designed to satisfy the conditions:

$$\mathbf{H}_{1,R} \cdot \mathbf{U}_{3R}(l) = 0 \quad \texttt{for} \quad l \in \{1, \cdots, M_3\} \tag{2.7}$$

$$\mathbf{H}_{2,R} \cdot \mathbf{U}_{3R}(p) = 0 \quad \texttt{for} \quad p \in \{1, \cdots, M_3\} \tag{2.8}$$

$$\mathbf{H}_{3,R} \cdot \mathbf{U}_{12}(i) = 0 \quad \texttt{for} \quad i \in \{1, \cdots, M_{12}\} \tag{2.9}$$

where $\mathbf{H}_{k,R}$ represents the channel gain matrix from the relay R to the User k which is of size $N_{12} \times N$ and $N_3 \times N$ for $k = 1, 2$ and $k = 3$, respectively. The conditions in (2.7), (2.8) and (2.9) are only desired conditions.

For User 1, the received signal $\mathbf{Y}_1$ of size $N_{12} \times 1$ is:

$$\begin{aligned}\mathbf{Y}_1 &= \mathbf{H}_{1,R}\mathbf{U} \cdot [s_{12}(1), \cdots, s_{12}(M_{12}), s_{3R}(1), \cdots, s_{3R}(M_3)]^T \\ &= \sum_{i=1}^{M_{12}} s_{12}(i)\mathbf{H}_{1,R}\mathbf{U}_{12}(i) + \sum_{l=1}^{M_3} s_{3R}(l)\mathbf{H}_{1,R}\mathbf{U}_{3R}(l) + \mathbf{n}_1\end{aligned} \tag{2.10}$$

As long as User 1 can recover from (2.10) the data of interest $s_{12}(i)$ $(i = 1, \cdots, M_{12})$, the required conditions are that $N_{12} \geq M_{12}$ and only the subsets of $N_{12} - M_{12}$ relations in (2.7) is met. Similar observations are valid for User 2; the subset of $N_{12} - M_{12}$ relations in (2.8) has to be met.

Usually, indexes l and p in (2.7) and (2.8), which define relay data to User 3 nulled at User 1 and User 2, determine the exclusive subsets to reduce the number of antennas N_{12} to meet the BC phase requirements. The implications of relaxing the conditions in (2.7) and (2.8) are such the User 1 and 2 will decode some of the data destined from relay R to User 3.

From the perspective of User 3, the received signal $\mathbf{Y}_3$ of size $N_3 \times 1$ is :

$$\mathbf{Y}_3 = \sum_{i=1}^{M_{12}} s_{12}(i)\mathbf{H}_{1,R}\mathbf{U}_{12}(i) + \sum_{l=1}^{M_3} s_{3R}(l)\mathbf{H}_{3,R}\mathbf{U}_{3R}(l) + \mathbf{n}_3 \tag{2.11}$$

where $\mathbf{n}_3$ is the AWGN noise at User 3 in the BC phase. Again to recover $s_{3R}(l)$ $(l = 1, \cdots, M_3)$ at User 3, only $N_3 - M_3$ relations have to be met in (2.9). Working with right subsets of relations in (2.7), (2.8) and (2.9) is still important in some cases to reduce the antenna number requirement for the BC phase.

With the proposed scheme in the MIMO Y channel topology for the traffic patterns considered, completing the data exchanges of $2M_{12} + 2M_3$ symbols in two TSs exhibits a huge advantage from the perspective of sum data rate over the traditional transmission schemes. In the next section, we present two examples with relatively simple antenna configurations demonstrating the potential of exploiting the options in selecting the signaling dimensions in the proposed systems. Then, in Section 2.3, we comment on some general traffic patterns and antenna configurations in the Y channel configuration considered in this thesis.

2.2 System Models for Different Antenna Configuration

In this section, we provide details for the operation of the proposed scheme with specific antenna configurations. The antenna configuration $N_{12} = 2$ and $N_3 = 1$ and $N = 2$ referred to as Scenario 1 is covered in Appendix A. This is a relatively simple case and we did not included it in the main body of the text; Though, there are some concepts presented there related to dimension selection which is easier to understand due to simplicity of the system. Scenarios 2 and 3 with antenna configurations (i) $N_{12} = N_3 = 2$ and $N = 3$ and (ii) $N_{12} = 3$, $N_3 = 2$ and $N = 4$ are discussed in subsections 2.2.1 and 2.2.2, respectively.

2.2.1 Scenario 2 with Antenna Configuration $N_{12} = 2$, $N_3 = 2$ and $N = 3$

In this subsection, we consider the following case for the general setup presented in Section 2.1 where $N_{12} = N_3 = 2$ and $N = 3$ (Fig. 2.2). Here the users and the relay are working with antenna configurations being the same as in the conventional 3-user MIMO Y channel with two-way traffic. In our case $M_{12} = 1$ and $M_3 = 2$

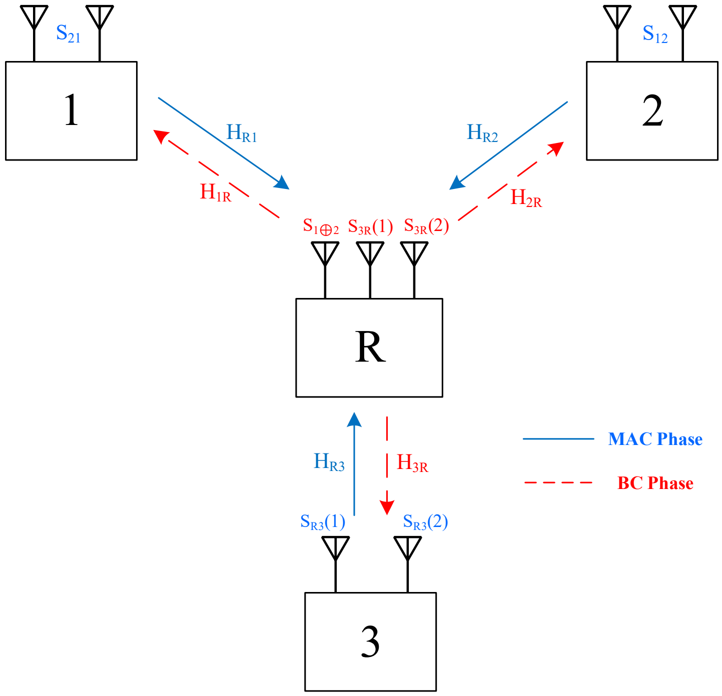

Figure 2.2: System model for Scenario 2.

which differentiate our scenario from the conventional MIMO Y channel that there is only one bi-directional exchange while in the Y channel there are three bi-directional exchanges. We use a two-way relay cooperative wireless network composed of three users and one relay which acts as a user too. There are two primary users that exchange data through the relay and two secondary users which exchange data with each other. In this system model, each user node is equipped with two antennas and the relay node is equipped with three antennas. User nodes are assumed to be synchronized and located at the same distance to the relay. Each PU sends one message to another PU and the SU sends two messages to the relay and receives two messages from the relay. Therefore, In both systems, ours and the Y channel, at the end of the BC phase the total of six messages is being sent. However, in the case of our traffic configuration one may claim the data pass through less hops. In this case, we need one precoding vector of size 2×1 for each PU and two precoding vectors of size 2×1 for SUs transmissions. Here, we use SA for primary users communications and DF for secondary users communications. Moreover, in our fading channel the channel gain matrices from User i to the relay are defined as $\mathbf{H}_{R,i}, i = 1, 2, 3$ with the size of 3×2. Each element in $\mathbf{H}_{R,i}$ is a complex normal with variance one and zero mean, $\mathcal{CN}(0, 1)$, and independent to each other. In the system analyzed in this chapter, because $[\mathbf{H}_{R,1} \mid -\mathbf{H}_{R,2}]$ is of size 3×4 the nullity of this matrix is 1 and we have only one choice for selecting the signaling direction $\mathbf{D}_{12}(1) = \mathbf{H}_{R,1}\mathbf{v}_{1,1}$ for the only bi-directional data exchange. This is because $\left[\mathbf{v}_{1,1}, \mathbf{v}_{1,2}\right]^T$ is the unique solution of a system of linear homogeneous equations as in (2.5). However, we have freedom when selecting precoding vectors $\mathbf{p}_1$ and $\mathbf{p}_2$ which define the signaling direction $\mathbf{D}_{R3}(1)$ and $\mathbf{D}_{R3}(2)$ for sending data $s_{R3}(1)$ and $s_{R3}(2)$ from User 3 to relay R. Therefore, the choices for dimensions are much more flexible and we did not have such flexibility in the original Y channel without adding the redundant antennas. We also assume that global CSI is known to all nodes and the corresponding nulling, precoding and ZF matrices are available at the corresponding nodes.

In the original MIMO Y channel described in Section 1.6, each user is equipped with two antennas, resulting in $\mathbf{H}_{R,i}$ being of size 3×2 and each precoding vector of size 2×1 (In Section 1.6 we only have one type of precoding vectors $\mathbf{v}_{i,k}$, while in this Chapter we have two types: precoding vectors for PUs, $\mathbf{v}_{i,k}$, and precoding vectors for the SU, $\mathbf{p}_l$). For each constraint in (1.5), there are four unknowns representing entries in two precoding vectors and three linear equations, which indicates a unique solution to the direction of one pair of precoding vectors. As a result, the three signaling dimensions determined in the original MIMO Y channel are fixed. After the dimensions are obtained, we also need to impose the limits on the power of the precoding vectors ($|\mathbf{v}_{j,i}|^2 = \mathbf{v}_{j,i}^H \cdot \mathbf{v}_{j,i}$) to meet the power constraint in (1.4). This power limit adds a constraint to the scaling parameters of precoding vectors, which will partially influence the effective channel gain. However, because of the Rayleigh fading channels, the effective gains $|\mathbf{D}_{12}(1)|$, $|\mathbf{D}_{R3}(1)|$ and $|\mathbf{D}_{R3}(2)|$ are different and varying, the distance between the closest aligned symbols in 3D signaling space at the relay may be large for separating some aligned symbols and small for others. The average BER performance is eventually determined by the minimum distance between the aligned symbols which cannot be controlled in the original MIMO Y channel scheme.

When the relay receives signals from three dimensions, aligned messages coming from PUs are ternary symbols of levels $\pm 2,\ 0$ for the case where terminals are suing BPSK. Despite that, messages received from SUs signaling dimensions will produce binary symbols of levels $\pm 1,\ 0$. As a result, there will be $3{\times}2{\times}2 = 12$ possible symbols received at the relay which is demonstrated in Fig. 2.3. In our proposed system model, since we have flexibility in defining SUs signaling dimensions, by providing greater separation between signaling points to have larger minimum distance, we are able to improve BER. Specifically, when we have orthogonal dimensions we will have the separation that is independent on channel conditions.

Our signal level model of the MAC and BC phase is similar to the original MIMO Y channel with SA alone, but with some modifications. In the MAC phase two paired messages of PUs are aligned into one signal dimension and pass through the corresponding channel. Besides, messages from the SU to the relay are getting aligned to separate dimensions and pass through their corresponding channels. After the relay receives signals from three dimensions from its three antennas, a decoding process like ZF or maximum likelihood will take place to obtain symbols. For the messages received from PUs, aligned symbols will produce ternary symbols of levels $\pm 2,\ 0$. Then, they get remapped from ternary symbols to binary messages to be prepared

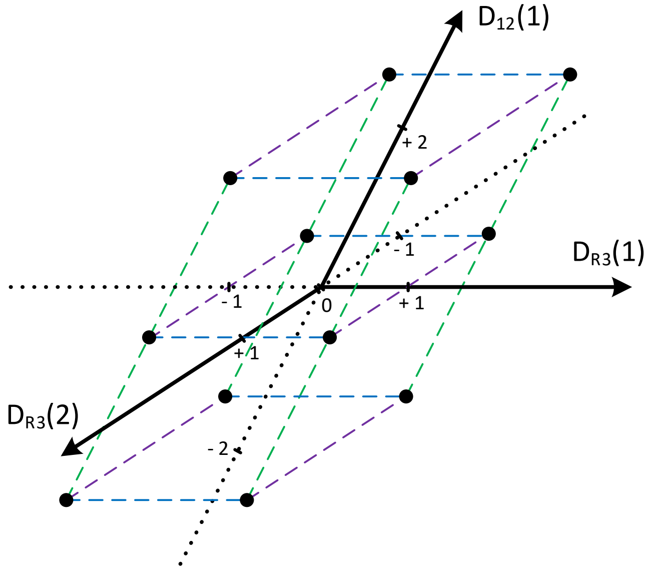

Figure 2.3: The coordinates and possible signal received at the relay through signal summation.

for the modulation at the relay before broadcasting them back to users. Table 2.1 documents how this remapping is deployed, i.e., the mapping of binary bits into BPSK symbols and remapping of ternary symbols into BPSK symbols.

Table 2.1: Mapping and re-mapping of symbols at the relay.

Source Bits		BPSK data		Received Symbols	PNC Mapping	BPSK Symbols
b_1	b_2	s_1	s_2			
1	1	+1	+1	+2	0	-1
0	1	-1	+1	0	1	+1
1	0	+1	-1	0	1	+1
0	0	-1	-1	-2	0	-1

In the BC phase, the relay modulates messages and broadcasts symbols to users. Nulling vectors are designed properly so that user nodes can recover signals. For the considered configuration, based on (2.10) and (2.11), the received signals at different nodes after applying nulling vectors at the relay are given by:

$$\begin{aligned}\mathbf{Y}_1 &= s_{12}(1)\mathbf{H}_{1R}\mathbf{U}_{12}(1)+s_{3R}(1)\mathbf{H}_{1R}\mathbf{U}_{3R}(1)+s_{3R}(2)\mathbf{H}_{1R}\mathbf{U}_{3R}(2)+\mathbf{n}_1\\ \mathbf{Y}_2 &= s_{12}(1)\mathbf{H}_{2R}\mathbf{U}_{12}(1)+s_{3R}(1)\mathbf{H}_{2R}\mathbf{U}_{3R}(1)+s_{3R}(2)\mathbf{H}_{2R}\mathbf{U}_{3R}(2)+\mathbf{n}_2\\ \mathbf{Y}_3 &= s_{12}(1)\mathbf{H}_{3R}\mathbf{U}_{12}(1)+s_{3R}(1)\mathbf{H}_{3R}\mathbf{U}_{3R}(1)+s_{3R}(2)\mathbf{H}_{3R}\mathbf{U}_{3R}(2)+\mathbf{n}_3\end{aligned} \tag{2.12}$$

Only the green portion of signals are required to be decoded at the nodes and the other signals should be or rather could be nulled. In the case of User 3, because $N_3 = 2$ and there are two messages $s_{3R}(1)$ and $s_{3R}(2)$ intended for this node, we have to apply $\mathbf{U}_{12}$ such that $\mathbf{H}_{3R}\mathbf{U}_{12} = \mathbf{0}$ and there is only one such a vector. If at User 1 and 2, we would like to null the impact of $s_{3R}(1)$ and $s_{3R}(2)$ because only $s_{12}(1)$ is desired there, we would have to satisfy $\mathbf{H}_{1R}\mathbf{U}_{3R}(1) = \mathbf{0}$ and $\mathbf{H}_{2R}\mathbf{U}_{3R}(1) = \mathbf{0}$ at the same time which is not possible. As a result, in the considered antenna setup, we design $\mathbf{U}_{3R}(1)$ and $\mathbf{U}_{3R}(2)$ so that $\mathbf{H}_{1R}\mathbf{U}_{3R}(1) = \mathbf{0}$ and $\mathbf{H}_{2R}\mathbf{U}_{3R}(2) = \mathbf{0}$ nulling the red signals in (2.12). With this User 3 decodes the data destined only to it, while User 1 and 2 recovers mutual symbol $s_{12}(1)$ of interest but they also obtain $s_{3R}(2)$ and $s_{3R}(1)$, respectively.

Regarding the choice for the signaling directions $\mathbf{D}_{R3}(1)$ and $\mathbf{D}_{R3}(2)$ once direction $\mathbf{D}_{12}(1)$ is fixed, it is highly desirable for the signaling directions to be orthogonal as this stabilizes the BER performance of the system and simplifies the decoding. In [33], in order to be able to work with orthogonal signaling dimensions, we paid the penalty of increasing the number of antennas at the users. In this section, we work with the antenna configuration that is the same as in the original MIMO channel, but the traffic patters are different which allows us to exploit the signaling space for improved performance. Specifically, we adjust $\mathbf{p}_1$ and $\mathbf{p}_2$ so that $\mathbf{D}_{12}(1) \perp \mathbf{D}_{R3}(1) \perp \mathbf{D}_{R3}(2)$. First, we find two 3D vectors defining null$\Big(\mathbf{D}_{12}(1)\Big)$ (subspace orthogonal to $\mathbf{D}_{12}(1)$ is 2D space) and then, we find $\mathbf{p}_1$ and $\mathbf{p}_2$ as the pseudo-inverse of $\mathbf{D}_{R3}(1) = \mathbf{H}_{R,3}\mathbf{p}_1$ and $\mathbf{D}_{R3}(2) = \mathbf{H}_{R,3}\mathbf{p}_2$. With signaling dimensions being orthogonal, when inverting $\mathbf{D}=[\mathbf{D}_{12}(1)|\mathbf{D}_{R3}(1)|\mathbf{D}_{R3}(2)]$ at the relay to recover data of interest, we will not suffer from noise enhancement of ZF type schemes as documented in the simulation section. After mutual messages has been received by PUs, a bit-level XOR operation is performed on s_{12} to obtain data of interest as bellow:

$$s_1(i) = s_2(i) \oplus \big(s_1(i) + s_2(i)\big) \tag{2.13}$$

2.2.2 Scenario 3 with Antenna Configuration $N_{12} = 3$, $N_3 = 2$ and $N = 4$

In this subsection, we consider the case where $N_{12} = 3$, $N_3 = 2$ and $N = 4$. In this scheme which is shown in Fig. 2.4, we have $M_{12} = 2$ and $M_3 = 2$, meaning that there are two bi-directional exchanges between PUs and User 3 sends two messages to relay R in the MAC phase and receive two messages from relay R in the BC phase. Based on (2.4), the received signal at the relay is:

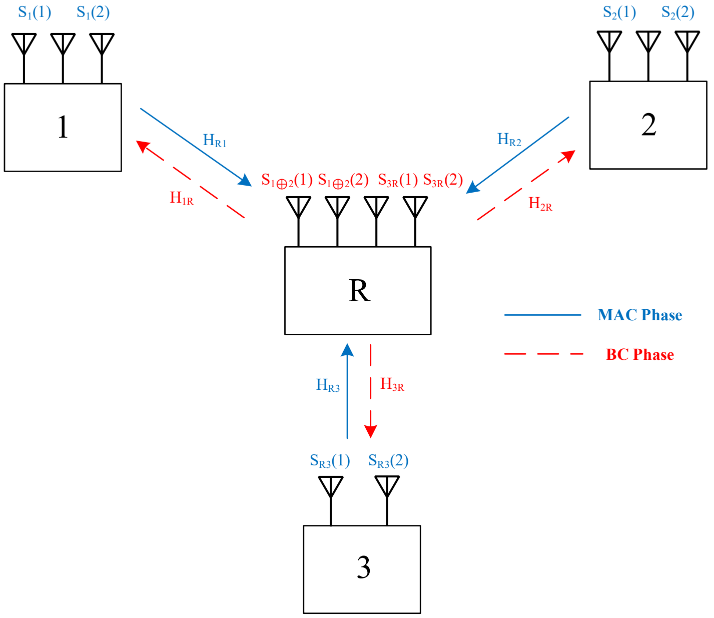

Figure 2.4: System model for Scenario 3.

$$\begin{aligned}\mathbf{Y}_R &= \mathbf{H}_{R,1}\mathbf{v}_{1,1}\cdot s_1(1)+\mathbf{H}_{R,1}\mathbf{v}_{2,1}\cdot s_1(2)+\mathbf{H}_{R,2}\mathbf{v}_{1,2}\cdot s_2(1)+\mathbf{H}_{R,2}\mathbf{v}_{2,2}\cdot s_2(2)\\ &\quad+\mathbf{H}_{R,3}\mathbf{p}_1\cdot s_{R3}(1)+\mathbf{H}_{R,3}\mathbf{p}_2\cdot s_{R3}(2)+\mathbf{n}_R\\ &= \mathbf{D}_{12}(1)\cdot \hat{s}_{12}(1)+\mathbf{D}_{12}(2)\cdot \hat{s}_{12}(2)+\mathbf{D}_{R3}(1)\cdot s_{R3}(1)+\mathbf{D}_{R3}(2)\cdot s_{R3}(2)+\mathbf{n}_R\end{aligned} \tag{2.14}$$

In this scenario because $[\mathbf{H}_{R,1} \mid -\mathbf{H}_{R,2}]$ is of size 4×6, the nullity of this matrix is 2, which determines we have only one choice for selecting the signaling dimensions $\mathbf{D}_{12}(1)$ and $\mathbf{D}_{12}(2)$ for two bi-directional data exchanges between PUs. It is worth observing that similar to previous scenario, PUs signaling dimensions are fixed. However, we have the freedom for defining $\mathbf{p}_1$ and $\mathbf{p}_2$ in such a way that $\mathbf{D}_{R3}(1)$ and $\mathbf{D}_{R3}(2)$ become orthogonal to one of PUs signaling dimensions $\mathbf{D}_{12}(1)$ or $\mathbf{D}_{12}(2)$. This will give us bigger separation between signaling points which lead to better BER. In this scenario we are not able to make all signaling dimensions orthogonal to each other.

In the BC phase, based on (2.10) and (2.11), the received signals at different users are:

$$\begin{aligned}
\mathbf{Y}_1&=s_{12}(1)\mathbf{H}_{1R}\mathbf{U}_{12}(1)+s_{12}(2)\mathbf{H}_{1R}\mathbf{U}_{12}(2)+s_{3R}(1)\mathbf{H}_{1R}\mathbf{U}_{3R}(1)+s_{3R}(2)\mathbf{H}_{1R}\mathbf{U}_{3R}(2)+\mathbf{n}_1\\
\mathbf{Y}_2&=s_{12}(1)\mathbf{H}_{2R}\mathbf{U}_{12}(1)+s_{12}(2)\mathbf{H}_{2R}\mathbf{U}_{12}(2)+s_{3R}(1)\mathbf{H}_{2R}\mathbf{U}_{3R}(1)+s_{3R}(2)\mathbf{H}_{2R}\mathbf{U}_{3R}(2)+\mathbf{n}_2\\
\mathbf{Y}_3&=s_{12}(1)\mathbf{H}_{3R}\mathbf{U}_{12}(1)+s_{12}(2)\mathbf{H}_{3R}\mathbf{U}_{12}(2)+s_{3R}(1)\mathbf{H}_{3R}\mathbf{U}_{3R}(1)+s_{3R}(2)\mathbf{H}_{3R}\mathbf{U}_{3R}(2)+\mathbf{n}_3
\end{aligned} \tag{2.15}$$

The analysis of (2.15) shows that we have to apply $\mathbf{U}_{12}(1)$ such that $\mathbf{H}_{3R}\mathbf{U}_{12}(1) = \mathbf{0}$ and there are two such vector which we have to choose one. Choosing the best nulling vector to optimize the performance can be a potential future work. The same condition applies to $\mathbf{U}_{12}(2)$ as well we have two vectors that satisfies the condition. In the considered antenna setup, we design $\mathbf{U}_{3R}(1)$ so that $\mathbf{H}_{2R}\mathbf{U}_{3R}(1) = \mathbf{0}$ and $\mathbf{U}_{3R}(2)$ so that $\mathbf{H}_{1R}\mathbf{U}_{3R}(2) = \mathbf{0}$. As a result, User 3 decodes the data transmitted only from the relay, while User 1 and User 2 recover their mutual data of interest $s_{12}(1)$ and $s_{12}(1)$, and they also obtain $s_{3R}(1)$ and $s_{3R}(2)$, respectively.

2.3 Generalized Antenna Configurations and Traffic Patterns

In this section our work is expanded to a generalized case for our 4-user overlay CRN with any number of PUs and SUs antennas. The communication scenario studied in this section was shown originally in Fig. 2.1. Table 2.2 enumerates the actual number of antennas required at the relay and the users when applying the proposed scheme for the cases that we verified the possibility of implementing the MAC and BC phases. It is notable from Table 2.2 that when the number of traffic flows keeps increasing, the number of antennas required at both the relay and users becomes high, but we aimed at the minimal configuration of antennas to support $M_{12} + M_3$ data flows at the relay, i.e., $N = M_{12} + M_3$. We performed derivations for antenna configurations and traffic patterns requirements for all additional scenarios (from 4 to 9) similarly as we have done this in Sections 2.2.1 and 2.2.2 and Appendix A. This was with the purpose to verify the feasible implementations of MAC and BC phases, which allows proper recovery of data of interest at four nodes. When working with different scenarios of network configurations reported in Table 2.2, we observed different amount of flexibility when selecting signaling dimensions. This eventually will result in different BER performance improvements of the proposed schemes depending on traffic demands in the system as presented in the next section.

Table 2.2: Antenna number and traffic requirements at the relay and user nodes.

Scenario	# of Relay Antennas N	# of User Antennas N_{12}	N_3	Traffic Patterns M_{12}	M_3
1	2	2	1	1	1
2	3	2	2	1	2
3	4	3	2	2	2
4	5	4	3	2	3
5	6	5	3	3	3
6	7	6	5	4	5
7	8	7	5	5	5
8	9	8	7	6	7
9	10	9	7	7	7

Based on the experience gained from derivations of SA conditions in different scenarios presented in Table 2.2, we claim to find general derivations for antenna configurations and traffic patterns requirements. We formalize them when the number of PUs' antennas (given now by K) is equal or greater than 4 in order to have feasible implementations of MAC and BC phases, which allows proper recovery of data of interest at four nodes. As a result, to satisfy SA conditions and existing constraints there are two cases based on whether the number of antennas in PUs are even or odd. Table 2.3 demonstrates the generalized formula for required number of antennas and corresponding number of messages that can be transmitted.

Table 2.3: Generalized antenna number and traffic requirements at relay and user nodes.

Scenario $K \geq 4$	# of Relay Antennas N	# of User Antennas N_{12}	N_3	Traffic Patterns M_{12}	M_3
K: even	K+1	K	K-1	K-2	K-1
K: odd	K+1	K	K-2	K-2	K-2

For PUs with even number of antennas, K, the relay should be equipped with $K + 1$ antennas and the SU has $K - 1$ antennas. In this case, PUs can transmit

$K-2$ messages to each other and the SU can send and receive $K-1$ messages to and from the relay. For PUs equipped with odd number of antennas, the relay need to have $K+1$ antennas with the SU having $K-2$ antennas. In this scenario, PUs can transmit $K-2$ messages to each other and the SU can transmit $K-2$ messages to and from the relay.

2.4 Performance Evaluation

In this section, we present MATLAB simulation results in terms of BER performance for our proposed schemes in Scenario 2 and 3. We illustrate the BER performances of different data traffics in the proposed orthogonal scheme and compare them with the performance of the non-optimized scheme that does not impose the orthogonality conditions or select the signaling dimensions. When reporting the BER improvements, we present the improvements in SNR between optimized (orthogonal) schemes and non-optimized schemes to achieve the target BER=10^{-3}.

Figure 2.5 shows the results for Scenario 2 discussed extensively in Section 2.2.1 when $N_{12} = N_3 = 2$ and $N = 3$. As can be observed in this figure, (i) for transmissions

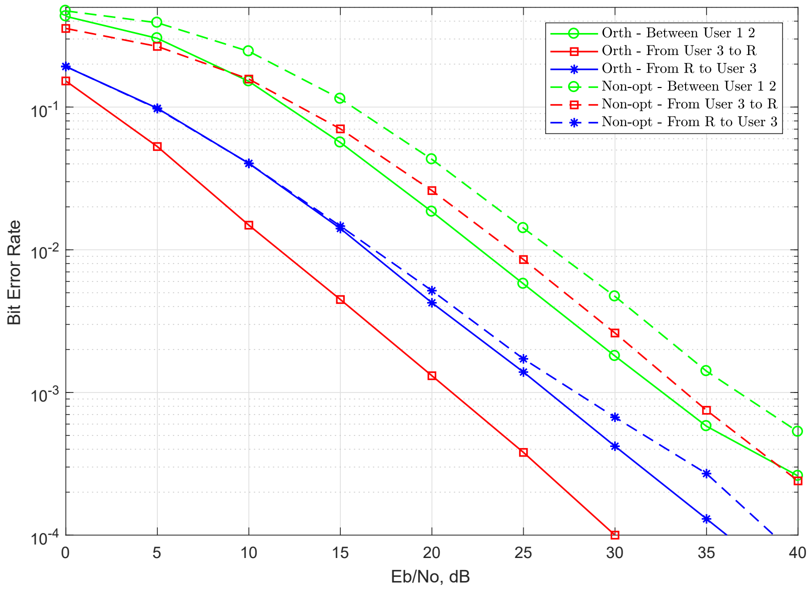

Figure 2.5: BER performance of the proposed and non-optimized schemes for different user pairs in the MIMO Y channel topology when $N = 3$, $N_{12} = N_3 = M_3 = 2$ and $M_3 = 1$ (Scenario 2).

from User 3 to the relay (red curves), there is 12 dB improvement (at the BER=10^{-3}); (ii) for transmissions between User 1 and 2 (green curves), there is 4 dB improvement; (iii) for transmissions from the relay to User 3 (blue curves), there is no improvement. As discussed in Section 2.2.1, the proposed scheme in Scenario 2 benefits mostly the MAC phase (transmissions to the relay) where by orthogonalization of signaling dimensions we eliminate the noise enhancement existing in the ZF type detection at the relay. This is the reason why there is so much improvement on the transmissions from User 3 to the relay. Data exchange between Users 1 and 2 is improved in the MAC phase but is not affected in the BC phase. Therefore the BER for this traffic is close to the average for the BER between User 3 to the relay and the relay to User 3. For the proper analysis, we we would have to consider here also the error propagation in network coding and the existence of additional ("interfering") data ($s_{3R}(2)$ or $s_{3R}(3)$) when decoding at User 1 or 2.

Figure 2.6 shows the results for Scenario 3 from Table 2.2 when $N = 4$, $N_{12} = 3$, $N_3 = 2$ and $M_3 = M_{12} = 2$. In this scenario, when deploying the optimized scheme,

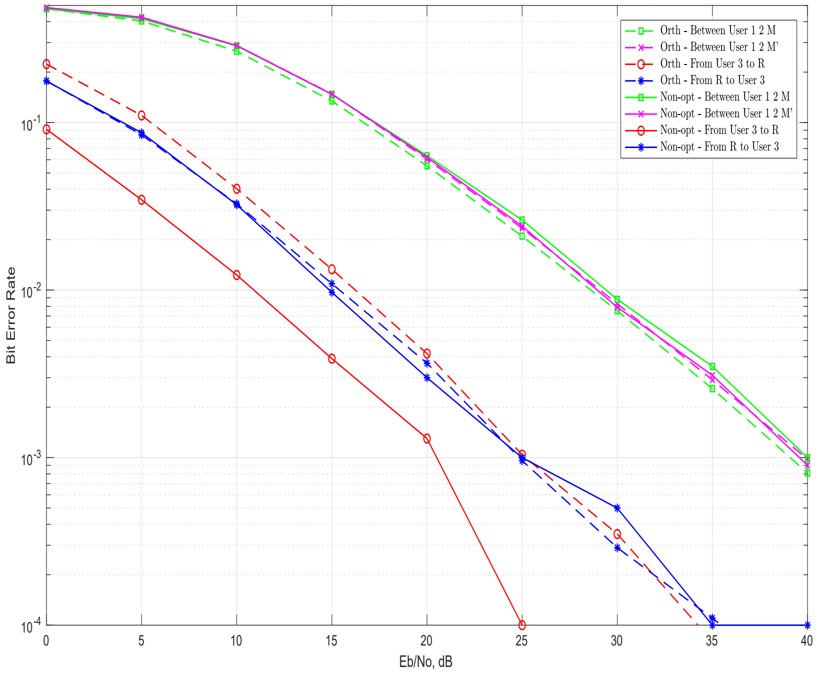

Figure 2.6: BER performance of the proposed and non-optimized schemes for different user pairs in the MIMO Y channel topology when $N = 4$, $N_{12} = 3$, $N_3 = 2$ and $M_3 = M_{12} = 2$ (Scenario 3).

there is an improvement of 5 dB on transmissions between User 3 and the relay; there is no improvements for other transmissions. The reason for the smaller benefits of the proposed scheme in Scenario 3 as compared to Scenario 2 is that here we did not have flexibility of orthogonalization of all dimensions and we only worked with signal dimension selection. As a result, the proposed scheme with optimized signaling dimensions provides BER improvements over the non-optimized scheme and the improvements depend on the antenna and traffic configuration patterns. The proposed scheme results in the uneven BER performance for user pairs which is due to different flexibility in selecting signaling dimensions for different links and user pairs. With orthogonalized dimensions, the proposed scheme provides a simple way to perform the decoding at the relay through slicing of coordinates along the orthogonal dimensions and is equivalent to maximum likelihood (ML) decoding with hyper-cube signaling similarly as presented in [33]. The scheme does not suffer from noise enhancement as in ZF decoding and complexity of ML decoding.

2.5 Summary

In this chapter, by exploiting the flexibility in selecting signaling dimensions at the user terminals and at the relay, we designed signal alignment schemes in MIMO Y channel topology. This was with the objective to improve the BER performance over non-optimized schemes. The proposed schemes differentiate from similar systems, like the conventional Y channel, by taking advantage of the traffic flow patterns considered in this chapter between three users. Specifically, when working with the minimum number of antennas to enable the desired data flow in bi-directional exchanges between two PUs and communications between one SU user and the relay (potentially the other SU), we exploited the selection of signaling dimensions to maximize the distance between signaling points and to simplify the decoding at the relay when working with orthogonal directions. All proposed schemes with optimized selection of signaling dimensions present improvement in the BER performance over the non-optimized MIMO Y channel type schemes.

Chapter 3

Multi-user Overlay CR Network with Spatial Multiplexing and Antenna Selection

In the previous chapter, we developed SA schemes to eliminate interference in the four user MIMO Y channel topology. With this technique, in new CR network settings, we were able to allow two SUs (including the relay) to transfer their data simultaneously with PUs (in the same frequency) without causing interference which increases bandwidth efficiency. In this chapter, the motivation is to incorporate more SUs in the network to communicate their data along with PUs without causing MAI. As a result, we designed a network capable of being used in a distributed MIMO channel setting for both PU and SU communications.

In future wireless networks, when spectrum sharing is going to be deployed on a much wider scale within the cognitive radio (CR) paradigm, the PUs and SUs of spectrum will have transceivers with different antenna configurations [2]. The nodes from different groups will also have different traffic requirements. MP-TWRC type systems have been studied in single and multiple antenna configurations and can accommodate a variety of communication scenarios. However, in most of these setups, all the user nodes have the same number of antennas and the same requirements for bi-directional traffic throughput [11]. There has been some limited work to accommodate asymmetric traffic in MP-TWRC models and to apply PNC in CR [10], [35], [36]. However, there is still a need to develop PNC-based CR systems supporting a diversity of data flows like uni- and bi-directional traffics which motivates the work in this chapter.

In overlay CR networks allowing concurrent primary and secondary user transmissions, there is an interest in developing signal separation over the same dimensions of time, space and relay resources [1]. This is the reason why in this work we consider two groups of nodes representing two PUs and two SUs simultaneously communicating in the same spectrum via the third SU/relay sharing its antennas. Specifically, two PUs exchange bi-directional data using a MIMO TWRC approach, while SUs transmit and receive independent data to and from the relay. In our setup, the data exchange of K messages between the PUs is accomplished within the same two phases, called MAC and BC, as the data transmission to and from the relay for the SUs. The data exchange between the PUs follows the modified MIMO TWRC approach, and to accommodate unicast traffics between the SUs and the relay, we design specialized signal precoding for both phases of data transmission.

Joint relay antenna selection and ZF spatial multiplexing for MIMO TWRC with two nodes has been pursued in [29]. However, in our work, because, initially, we have specialized SA to accommodate traffic conditions from four communicating nodes, we select the antennas based on virtual channel gains after pre- and post-processing at the user nodes and the relay.

In this chapter, in the first part, a novel approach has been introduced which can be considered as an overlay CR network with two PUs, one relay/user, and two SUs. In the second part, the scheme is extended to allow for more SUs and more complex antenna and traffic configurations. In all these SA schemes has been deployed to mitigate interference in the network and diversity selection has been used to optimize data transfer and improve BER.

This chapter is organized as follows. First, in Section 3.1, the proposed system model is presented in a simplified case of two PUs with three antennas, two SUs with single antennas and the relay with four antennas (a specific antenna configuration). Next, Section 3.2 explains the benefits of applying diversity selection and how it is implemented for BER performance optimization. Section 3.3 develops the SA for the generalized from 3.1 system configuration when the PUs are equipped with arbitrary number of antennas. Evaluation and results are presented in Section 3.4 for the system model in Section 3.3. Section 3.5 develops a signaling scheme for generalized case of the model from Section 3.3 with arbitrary configurations of antennas and traffic patters, while Section 3.6 presents the corresponding simulation results. Finally, chapter summary is provided in Section 3.7.

3.1 System Model

In this section, we have a CR network composed of four users and one relay/user node. There are two PUs (User 1 and 2) that exchange data (in a bidirectional way) via relay and two SUs (User 3 and 4) that exchange data with the relay (in a unicast way). In this system model, eight messages are being exchanged in two time slots, i.e., (i) four messages exchanged between PUs ($s_{21}(1)$ and $s_{21}(2)$ sent from User 1 to User 2; $s_{12}(1)$ and $s_{12}(2)$ sent from User 2 to User 1); (ii) two messages - s_{R3} and s_{R4} - sent from User 3 and 4 to the relay; (iii) two messages - s_{3R} and s_{4R} - sent from the relay to User 3 and 4. Moreover, PUs are equipped with three antennas, while SUs are equipped with one antenna and the relay is equipped with five antennas. This communication scenario considered is shown in Fig. 3.1. What differentiate this system model from the system model discussed in Chapter 2 is that we have different antenna configurations in MAC and BC phases in order to incorporate an additional SU and satisfy SA constraints. Specifically, for the relay, we choose four out of five antennas for our communication in the MAC phase, while we use all five antennas in the BC phase. As previously, it is assumed that (i) all nodes operate in half-duplex mode; (ii) wireless channels are quasi-static Rayleigh flat fading, and (iii) full channel state information (CSI) is available to all nodes.

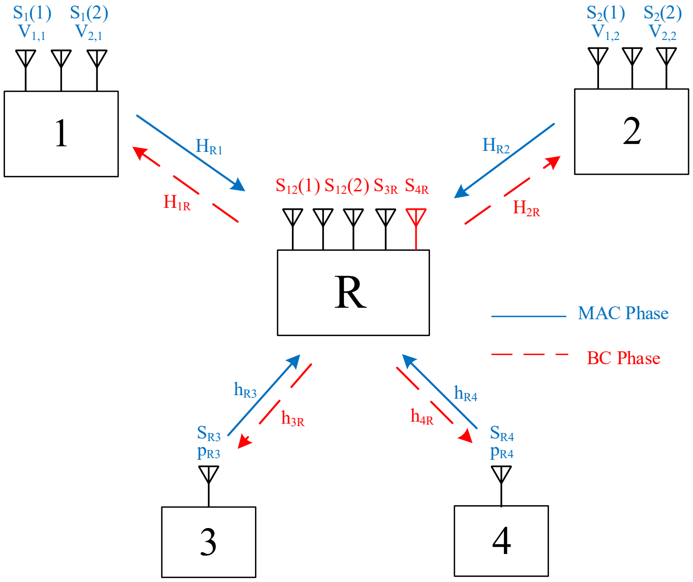

Figure 3.1: Five-user system model with TWR and unicast traffics..

In the scenario considered in this section, in the MAC phase, we need a precoding matrix of size 3×2 for each PU ($\mathbf{v}_{21}$ at PU 1 and $\mathbf{v}_{12}$ at PU 2). For SU 3 and 4, because they use single antennas, precoding matrices are just scalars, represented as $\mathbf{p}_{R3}$, $\mathbf{p}_{R4}$. Furthermore, the channel gain matrices are: (i) from PU i ($i \in 1, 2$) to the relay $\mathbb{H}_{Ri}$ of size of 5×3 and (ii) from SU j ($j \in 3, 4$) to the relay $\mathbb{H}_{Rj}$ of size of 5×1. Again, elements of each channel gain matrix are assumed to be i.i.d complex Gaussian random variables with variance one and zero mean (Rayleigh fading channel). As we need to have only four antennas for the relay in the MAC phase, after antenna selection, in the MAC phase, we work with channel gain matrices (i) $\mathbf{H}_{Ri}$ of size 4×3 and (ii) $\mathbf{h}_{Rj}$ of size 4×1 for PUs and SUs, respectively. In here, matrices $\mathbf{H}_{Ri}$ are sub-matrices (by removing one column) of original matrices $\mathbb{H}_{Ri}$ depending on the one disabled antenna at the relay, i.e., the active antenna indices decide which columns in $\mathbb{H}_{Ri}$ are used in $\mathbf{H}_{Ri}$. The criteria for the antenna selection procedure will be explained in Section 3.2.

In the MAC phase, since we are aligning four messages being exchanged between PUs into two dimensions, we will have four linear equations and six unknowns corresponding to precoding vectors $\mathbf{v}_{21}$ and $\mathbf{v}_{12}$. This will lead to two unique solutions which leaves us with two basis for the signaling directions $\mathbf{D}_1$ and $\mathbf{D}_2$ of mutual messages between PUs 1 and 2 in the MAC phase. Messages s_{R3} and s_{R4} from SUs to the relay are sent along other two separate dimensions $\mathbf{D}_3$ and $\mathbf{D}_4$. Since $\mathbf{p}_{R3} = 1$ and $\mathbf{p}_{R4} = 1$, for a given realization of $\mathbf{h}_{R3}$ and $\mathbf{h}_{R4}$, the signaling directions for $\mathbf{D}_3$ and $\mathbf{D}_4$ are known and fixed. The average power of information symbols is one for all users and this imposes constrains on $\mathbf{v}_{21}$ and $\mathbf{v}_{12}$ for PUs.

As discussed so far, in the MAC phase, messages $s_i(1)$ and $s_i(2)$ with $i = 1, 2$, that are to be exchanged between PUs, are pre-processed by the precoding matrices $\mathbf{v}_{21} = [\mathbf{v}_{1,1} \,|\, \mathbf{v}_{2,1}]$ and $\mathbf{v}_{12} = [\mathbf{v}_{1,2} \,|\, \mathbf{v}_{2,2}]$ where the precoding column vectors vectors, $\mathbf{v}_{1,i}$ and $\mathbf{v}_{2,i}$, are of size 3×1. The transmitted data vector by terminal i is then:

$$\mathbf{x}_i = [\mathbf{v}_{1,i} \,|\, \mathbf{v}_{2,i}] \cdot \left[\begin{smallmatrix} s_i(1) \\ s_i(2) \end{smallmatrix} \right] \tag{3.1}$$

After the relay receives signals in four dimensions (at four active antennas out of the five available), the received signal at the relay is:

$$\mathbf{Y}_R = \mathbf{H}_{R1} \cdot \mathbf{x}_1 + \mathbf{H}_{R2} \cdot \mathbf{x}_2 + \mathbf{h}_{R3} \cdot s_{R3} + \mathbf{h}_{R4} \cdot s_{R4} + \mathbf{n}_R \tag{3.2}$$

where $\mathbf{n}_R$ is the additive white Gaussian noise (AWGN) and both $\mathbf{Y}_R$ and $\mathbf{n}_R$ are of size 4×1. To align corresponding symbols from PUs into the same dimension, the precoding vectors are carefully selected so that (3.2) can be represented as:

$$\begin{aligned} \mathbf{Y}_R &= \Big(\mathbf{H}_{R1} \cdot \mathbf{v}_{1,1} \cdot s_1(1) + \mathbf{H}_{R2} \cdot \mathbf{v}_{1,2} \cdot s_2(1)\Big) \\ &+ \Big(\mathbf{H}_{R1} \cdot \mathbf{v}_{2,1} \cdot s_1(2) + \mathbf{H}_{R2} \cdot \mathbf{v}_{2,2} \cdot s_2(2)\Big) \\ &+ \Big(\mathbf{h}_{R3} \cdot \mathbf{p}_{R3} \cdot s_{R3}\Big) + \Big(\mathbf{h}_{R4} \cdot \mathbf{p}_{R4} \cdot s_{R4}\Big) + \mathbf{n}_R \\ &= \mathbf{D}_1 \cdot \hat{s}_{12}(1) + \mathbf{D}_2 \cdot \hat{s}_{12}(2) + \mathbf{D}_3 \cdot s_{R3} + \mathbf{D}_4 \cdot s_{R4} + \mathbf{n}_R \end{aligned} \tag{3.3}$$

where $\mathbf{D}_i$ denote the spatial signaling dimension of size 4×1 and $\hat{s}_{12}(k) = s_1(k) + s_2(k)$ is the mutual message.

Thereafter, a decoding process will take place to obtain symbols. For the messages received from PUs, aligned $\hat{s}_{12}(k)$ symbols will produce ternary symbols of levels $\pm 2,\ 0$. Then, they get remapped from ternary symbols to binary messages to be prepared for the modulation at the relay before broadcasting them back to PUs in the BC phase. According to SA requirement, the selected precoding vectors $\mathbf{v}_{k,1}$ and $\mathbf{v}_{k,2}$ need to satisfy:

$$\mathbf{H}_{R1} \cdot \mathbf{v}_{k,1} = \mathbf{H}_{R2} \cdot \mathbf{v}_{k,2} \tag{3.4}$$

for $k \in 1, 2$. Because $[\mathbf{H}_{R1} \mid -\mathbf{H}_{R2}]$ is of size 4×6, the nullity of this matrix is 2. Therefore, we have two choices that can represent signaling dimensions for PUs communications, $\mathbf{D}_1$ and $\mathbf{D}_2$. This is because $\left[\mathbf{v}_{k,1}, \mathbf{v}_{k,2}\right]^T$ can have two base solutions in the system of equations:

$$[\mathbf{H}_{R1} \mid -\mathbf{H}_{R2}] \cdot \left[\begin{smallmatrix} \mathbf{v}_{k,1} \\ \mathbf{v}_{k,2} \end{smallmatrix}\right] = \mathbf{0} \tag{3.5}$$

We do not have the option for selecting $\mathbf{D}_3$ and $\mathbf{D}_4$ which is defined by scalar $\mathbf{p}_{R3}$ and $\mathbf{p}_{R4}$ for sending data from User 3 and 4 to the relay. When selecting $\mathbf{v}_{k,1}$ and $\mathbf{v}_{k,2}$ satisfying (3.5), we have additional degree of freedom - this is because we can work with five possible realizations of $[\mathbf{H}_{R1} \mid -\mathbf{H}_{R2}]$. These realizations are allowing us to choose the best 4 out of 5 dimensions thanks to having 5 antennas at the relay. We discuss the procedure and the benefit of this flexibility in the next section.

In the BC phase, the relay modulates and forwards the aligned messages between PUs, $\hat{s}_{12}(k)$ $(k = 1, 2)$, and also sends its own messages s_{3R} and s_{4R} to User 3 and

4, respectively. A new set of precoding (called nulling) vectors is carefully designed so that user nodes can receive relevant symbols and decode data of interest without causing interference. At PUs side, a bit-level XOR operation needs to be performed on the received symbols to obtain the desired data. After precoding with the nulling matrix $\mathbf{U}$ of size 5×4, the data is broadcasted from the relay and is written as follows:

$$\begin{aligned} \mathbf{X}_R &= \mathbf{U} \cdot [\hat{s}_{12}(1), \hat{s}_{12}(2), s_{3R}, s_{4R}]^T \\ &= \mathbf{U}_1 \cdot \hat{s}_{12}(1) + \mathbf{U}_2 \cdot \hat{s}_{12}(2) + \mathbf{U}_3 \cdot s_{3R} + \mathbf{U}_4 \cdot s_{4R} \end{aligned} \tag{3.6}$$

where $\mathbf{U}_i$ ($i \in \{1,2,3,4\}$) of size 5×1 are column vectors from $\mathbf{U}$. The received signals by PUs 1 and 2 and SUs 3 and 4 are given respectively by:

$$\mathbf{Y}_1 = \mathbf{H}_{1R} \cdot \mathbf{X}_R = \hat{s}_{12}(1) \cdot \mathbf{H}_{1R} \cdot \mathbf{U}_1 + \hat{s}_{12}(2) \cdot \mathbf{H}_{1R} \cdot \mathbf{U}_2 + s_{3R} \cdot \mathbf{H}_{1R} \cdot \mathbf{U}_3 + s_{4R} \cdot \mathbf{H}_{1R} \cdot \mathbf{U}_4 + \mathbf{n}_R \tag{3.7a}$$

$$\mathbf{Y}_2 = \mathbf{H}_{2R} \cdot \mathbf{X}_R = \hat{s}_{12}(1) \cdot \mathbf{H}_{2R} \cdot \mathbf{U}_1 + \hat{s}_{12}(2) \cdot \mathbf{H}_{2R} \cdot \mathbf{U}_2 + s_{3R} \cdot \mathbf{H}_{2R} \cdot \mathbf{U}_3 + s_{4R} \cdot \mathbf{H}_{2R} \cdot \mathbf{U}_4 + \mathbf{n}_R \tag{3.7b}$$

$$\mathbf{Y}_3 = \mathbf{h}_{3R} \cdot \mathbf{X}_R = \hat{s}_{12}(1) \cdot \mathbf{h}_{3R} \cdot \mathbf{U}_1 + \hat{s}_{12}(2) \cdot \mathbf{h}_{3R} \cdot \mathbf{U}_2 + s_{3R} \cdot \mathbf{h}_{3R} \cdot \mathbf{U}_3 + s_{4R} \cdot \mathbf{h}_{3R} \cdot \mathbf{U}_4 + \mathbf{n}_R \tag{3.7c}$$

$$\mathbf{Y}_4 = \mathbf{h}_{4R} \cdot \mathbf{X}_R = \hat{s}_{12}(1) \cdot \mathbf{h}_{4R} \cdot \mathbf{U}_1 + \hat{s}_{12}(2) \cdot \mathbf{h}_{4R} \cdot \mathbf{U}_2 + s_{3R} \cdot \mathbf{h}_{4R} \cdot \mathbf{U}_3 + s_{4R} \cdot \mathbf{h}_{4R} \cdot \mathbf{U}_4 + \mathbf{n}_R \tag{3.7d}$$

PU 1 is only interested to recover the green terms in (3.7a), which contain mutual messages between User 1 and 2. But it has three antennas meaning that it is capable of receiving information from three dimensions. On the other hand, with this antenna setup it is not possible to design nulling vectors that satisfy $\mathbf{H}_{1R} \cdot \mathbf{U}_3 = 0$ and $\mathbf{H}_{1R} \cdot \mathbf{U}_4 = 0$ at the same time. The same condition applies to the second PU as well. SU 3 is interested to have the third term in (3.7c). Since SU 3 has only one antenna, it can only receive data from one dimension which means all other terms (in black) should be nulled. The same constrains apply to SU 4 as well. Hence, nulling vectors $\mathbf{U}_1$, $\mathbf{U}_2$, $\mathbf{U}_3$ and $\mathbf{U}_4$ are designed to satisfy the conditions such that:

$$\begin{cases} \mathbf{h}_{3R} \cdot \mathbf{U}_1 = 0 \quad \text{and} \quad \mathbf{h}_{4R} \cdot \mathbf{U}_1 = 0 \\ \mathbf{h}_{3R} \cdot \mathbf{U}_2 = 0 \quad \text{and} \quad \mathbf{h}_{4R} \cdot \mathbf{U}_2 = 0 \\ \mathbf{H}_{2R} \cdot \mathbf{U}_3 = 0 \quad \text{and} \quad \mathbf{h}_{4R} \cdot \mathbf{U}_3 = 0 \\ \mathbf{H}_{1R} \cdot \mathbf{U}_4 = 0 \quad \text{and} \quad \mathbf{h}_{3R} \cdot \mathbf{U}_4 = 0 \end{cases} \tag{3.8}$$

Because $\mathbf{U}_i$ ($i \in \{1, 2, 3, 4\}$) are of size 5×1 and considering the size of channel matrices, it can be verified that there exist solutions for the system of homogenous linear equations in (3.8). As a result, user nodes will receive the following signals that contain the desired message(s) for each User:

$$\begin{aligned}
\mathbf{Y}_1 &= \mathbf{H}_{1R} \cdot [\mathbf{U}_1 \cdot \hat{s}_{12}(1) + \mathbf{U}_2 \cdot \hat{s}_{12}(2) + \mathbf{U}_3 \cdot s_{3R}] && (3.9)\\
\mathbf{Y}_2 &= \mathbf{H}_{2R} \cdot [\mathbf{U}_1 \cdot \hat{s}_{12}(1) + \mathbf{U}_2 \cdot \hat{s}_{12}(2) + \mathbf{U}_4 \cdot s_{4R}] && (3.10)\\
\mathbf{Y}_3 &= \mathbf{h}_{3R} \cdot [\mathbf{U}_3 \cdot s_{3R}] && (3.11)\\
\mathbf{Y}_4 &= \mathbf{h}_{4R} \cdot [\mathbf{U}_4 \cdot s_{4R}] && (3.12)
\end{aligned}$$

To recover data of interest at User 1, the received signal can be written as:

$$\begin{aligned}
\mathbf{Y}_1 &= \mathbf{H}_{1R} \cdot \left[\mathbf{U}_1 \mid \mathbf{U}_2 \mid \mathbf{U}_3\right] \cdot [\hat{s}_{12}(1), \hat{s}_{12}(2), s_{3R}]^T \\
&= \mathbf{H}_{1R} \cdot \mathbf{U}_{1R} \cdot [\hat{s}_{12}(1), \hat{s}_{12}(2), s_{3R}]^T
\end{aligned} \qquad (3.13)$$

The recovery of the desired signals at PU 1 can be achieved using the ZF approach by inverting $\mathbf{H}_{1R} \cdot \mathbf{U}_{1R}$ and XORing mutual messages $\hat{s}_{12}(1)$ and $\hat{s}_{12}(2)$ with data sent from the PU 1 $s_1(1)$ and $s_1(2)$ that the PU has the access to. The recovery of the desired signals at the PU 2 is also similar. At SU 3 and 4, the messages from the relay can be simply decoded at each user.

3.2 Antenna Selection Strategy

The error performance of the system is mostly limited by the antenna subset with the smallest singular value gain lambda. On of the ways for selecting subset of antennas is Maximum Minimum Singular Value criterion which is based on improving post-processing SNR. In this method, for every subset of transmit antennas we compute lambda-min corresponding to the channel matrix and then we choose the subset that returns the largest lambda-min. This is because SNR-min is lower bounded by lambda-min. Therefore, by choosing a subset with largest lambda-min we can have greater SNR performance. This method does not require complex computation or calculating SNR for each subset to be able to choose the best antennas and in return, it gives near optimum performance [37], [38], [39].

3.3 Spatial Multiplexing and Antenna Selection for High Throughput PUs Traffic

The communication scenario considered in this section is a generalization (in terms of antenna and message numbers) of scenario from Section 3.1 and is shown in Fig. 3.2. in this case, we similarly have four users and there are no direct links as users communicate via and with the relay. PUs are labeled with i ($i \in 1, 2$), while SUs are labeled with l ($l \in 3, 4$). Users 1 and 2 have K independent messages for each other denoted as $s_1(k)$ and $s_2(k)$, respectively, where $k \in \{1, \cdots K\}$. These messages are to be exchanged via relay R in two time slots (TSs) called MAC and BC TSs. In the MAC TS, each User 3 and 4 sends one message (s_{R3} and s_{R4}) to relay R. Without loss of generality, all messages are represented by scalar (± 1) BPSK symbols. Users 1 and 2 are equipped with the same number of $K + 1$ antennas, and Users 3 and 4 have single antennas. In conventional MIMO TWRC with a single pair involved in bi-directional exchange, Users 1 and 2 would require K antennas. In our system model, both Users 1 and 2 require an additional antenna to deal with MAI in the BC phase and to fulfill SA conditions in the MAC phase as will be elaborated later. The

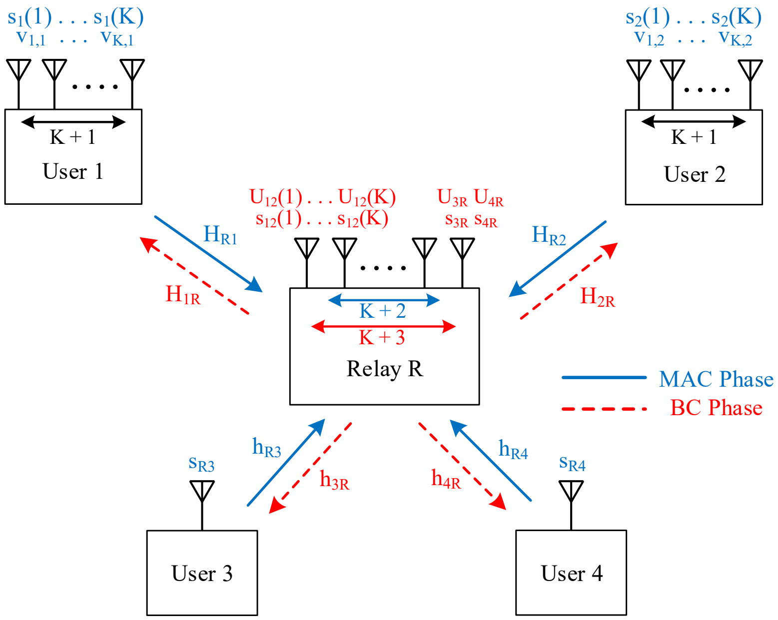

Figure 3.2: Five-user relay-assisted network with high throughput PUs and unicast SUs traffics.

relay is equipped with $K+3$ antennas to meet signal nulling conditions in the BC phase, though during the MAC phase, the relay is using $K+2$ active antennas. In the MAC phase, $K+2$ antennas are sufficient to decode $K+2$ signals of interest at the relay.

In this model, in the MAC phase, the channel gain matrices $\mathbf{H}_{R,1}$ and $\mathbf{H}_{R,2}$ from users 1 and 2 to the relay active antennas are of size $(K+2)\times(K+1)$; in the BC phase, the channel gain matrices $\mathbf{H}_{1,R}$ and $\mathbf{H}_{2,R}$ from the relay using all $K+3$ antennas to users 1 and 2 are of size $(K+1)\times(K+3)$. In the MAC phase, the channel gain column vectors between User 3 and the relay and between User 4 and the relay are $\mathbf{h}_{R,3}$ and $\mathbf{h}_{R,4}$, respectively, and their size is $(K+2)\times 1$. In the BC phase, the channel gain row vectors between User 3 and the relay and between User 4 and the relay are $\mathbf{h}_{3,R}$ and $\mathbf{h}_{4,R}$, respectively, and they are of size $1\times(K+3)$. Each entry of channel gain matrices and vectors is assumed to be independent identically distributed (i.i.d.) complex Gaussian to capture the effects of slow Rayleigh fading channels. It is assumed that all nodes operate in half-duplex mode and the full channel state information (CSI) is available to all nodes.

In the MAC phase, for the PU i with $i=1$ or $i=2$, K information symbols $s_i(k)$ ($k=(1,\cdots,K)$ are pre-processed by the precoding matrix $[\mathbf{v}_{1,i}\mid\cdots\mid\mathbf{v}_{K,i}]$ where the precoding vectors $\mathbf{v}_{k,i}$ are of size $(K+1)\times 1$. The transmitted data vector by terminal i is then:

$$\mathbf{x}_i=[\mathbf{v}_{1,i}\mid\cdots\mid\mathbf{v}_{K,i}]\cdot\begin{bmatrix} s_i(1)\\ \vdots \\ s_i(K)\end{bmatrix}=\sum_{k=1}^{K}\mathbf{v}_{k,i}\cdot s_i(k) \tag{3.14}$$

Since Users' 3 and 4 data (s_{R3} and s_{R4}) are scalars, there is no preprocessing at the SUs in the MAC phase. Also, the average power of information symbols is one and to ensure fairness between different spatial streams received at the relay, the total transmit power per antenna at all users is set to one, which imposes constraints on the norms of the precoding vectors.

When $2\cdot K$ for the PUs and 2 messages for the SUs for the total of $2K+2$ messages are transmitted from the four users simultaneously to the relay, the received signal at the relay is:

$$\mathbf{Y}_R=\mathbf{H}_{R,1}\cdot\mathbf{x}_1+\mathbf{H}_{R,2}\cdot\mathbf{x}_2+\mathbf{h}_{R,3}\cdot s_{R3}+\mathbf{h}_{R,4}\cdot s_{R4}+\mathbf{n}_R \tag{3.15}$$

where $\mathbf{n}_R$ is the additive white Gaussian noise (AWGN) ($\mathbf{n}_R\sim\mathcal{CN}(0,\sigma^2\mathbf{I})$) and both

$\mathbf{Y}_R$ and $\mathbf{n}_R$ are of size $(K+2)\times 1$. We assume here that the time synchronization at the symbol level has been established among user nodes in the MAC phase and the use of power control, so that the signals from different users arrive at the relay with the same average power. To align the mutual symbols $s_1(k)$ and $s_2(k)$ from users 1 and 2 into the same dimension in the signal space (as visible at the relay), the precoding vectors for the mutual symbols are carefully selected so that (3.15) can be represented as:

$$\begin{aligned}\mathbf{Y}_R &= \sum_{k=1}^{K}\Big(\mathbf{H}_{R,1}\mathbf{v}_{k,1}\cdot s_1(k)+\mathbf{H}_{R,2}\mathbf{v}_{k,2}\cdot s_2(k)\Big)\\ &\quad+\mathbf{h}_{R,3}\cdot s_{R3}+\mathbf{h}_{R,4}\cdot s_{R4}+\mathbf{n}_R \qquad (3.16)\\ &=\sum_{k=1}^{K}\mathbf{D}_{12}(k)\cdot\hat{s}_{12}(k)+\mathbf{D}_{R3}\cdot s_{R3}+\mathbf{D}_{R4}\cdot s_{R4}+\mathbf{n}_R\end{aligned}$$

where $\mathbf{D}_{12}(k)$ denote the K spatial signaling dimensions of size $(K+2)\times 1$ for bi-directional transmissions between User 1 and 2, and they can be interpreted as the effective channel gains for the aligned symbols $\hat{s}_{12}(k)$ with $\hat{s}_{12}(k)=s_1(k)+s_2(k)$. The mutual symbols $\hat{s}_{12}(k)$ are decoded at the relay as ternary symbols and are re-encoded into BPSK representations $s_{12}(k)$ of XOR bits between User 1 and 2 and then used in the BC phase. The signaling dimensions $\mathbf{D}_{R3}=\mathbf{h}_{R,3}$ and $\mathbf{D}_{R4}=\mathbf{h}_{R,4}$ also of size $(K+2)\times 1$ are used for the unicast transmissions from Users 3 and 4 to relay R. According to SA requirements in (3.16), the selected precoding vectors $\mathbf{v}_{k,1}$ and $\mathbf{v}_{k,2}$ need to satisfy:

$$\mathbf{H}_{R,1}\cdot\mathbf{v}_{k,1}=\mathbf{H}_{R,2}\cdot\mathbf{v}_{k,2}\quad\text{for}\quad k\in\{1,\cdots\ K\} \qquad (3.17)$$

and the null space, $\text{null}\big([\mathbf{H}_{R,1}\mid -\mathbf{H}_{R,2}]\big)$, should have the dimension higher or equal to K for $\mathbf{D}_{12}(k)$ to be distinct. This is because $\left[\begin{smallmatrix}\mathbf{v}_{k,1}\\ \mathbf{v}_{k,2}\end{smallmatrix}\right]$ is the solution set of a system of linear homogeneous equations with $[\mathbf{H}_{R,1}\mid -\mathbf{H}_{R,2}]$ as the coefficient matrix.

The rank of any matrix $\mathbf{A}$ plus the nullity of $\mathbf{A}$ equals the number of columns of $\mathbf{A}$ [34]. When we apply this theorem to $[\mathbf{H}_{R,1}\mid -\mathbf{H}_{R,2}]$, in the MAC phase, in our active antenna configuration determining the size of the matrices, we have exactly K choices for selecting $\left[\begin{smallmatrix}\mathbf{v}_{k,1}\\ \mathbf{v}_{k,2}\end{smallmatrix}\right]$. This will allow us to align the mutual symbols from the PUs at the relay, but does not offer any flexibility in working with different dimensions $\mathbf{D}_{12}(k)=\mathbf{H}_{R,1}\mathbf{v}_{k,1}$ to optimize the performance of the TWRC in the MAC phase.

The flexibility of choosing different $\mathbf{D}_{12}(k)$ and benefit from the selection diversity is only possible when we have different channel gain matrices $\mathbf{H}_{R,1}$ and $\mathbf{H}_{R,2}$. This, in turn, is controlled by selecting which $K+2$ antennas at the relay (out of the possible $K+3$ antennas) are selected as active antennas. This will be further explained in Section 3.3.1.

Usually, the system of linear equations in (3.16) is represented in a matrix form:

$$\mathbf{Y}_R = \mathbf{D} \cdot [\hat{s}_{12}(1), \cdots, \hat{s}_{12}(K), s_{3R}, s_{4R}]^T + \mathbf{n}_R \tag{3.18}$$

where T stands for transpose and the matrix of coefficients $\mathbf{D}$ of size $(K+2)\times(K+2)$ is obtained by concatenating effective channel gain columns $\mathbf{D}_{12}(k)$, $\mathbf{D}_{R3}$ and $\mathbf{D}_{R3}$. With this representation, in the zero-forcing (ZF) approach, the recovery of the signal of interest (mutual messages $\hat{s}_{12}(k)$ and Users 3 and 4 symbols s_{R3}, s_{R4}) at the relay is accomplished using the matrix $\mathbf{D}$ inversion.

In the BC phase, the relay forwards the aligned symbols $s_{12}(k)$ $(k = 1, \cdots, K)$ as BPSK representations of XOR-ed bits between User 1 and 2; the relay also sends to Users 3 and 4 its own data s_{3R} and s_{4R}. A set of precoding/nulling vectors $\mathbf{U}_{12}(k)$ is applied to the aligned symbols. Users' 3 and 4 data are preprocessed at the relay by $\mathbf{U}_{3R}$ and $\mathbf{U}_{4R}$. All the $K+2$ precoding/nulling vectors are of size $[(K+3)\times 1]$. The reason for applying the nulling vectors before sending to users is so that each user receives relevant data in the desired signal dimensions and eventually can decode intended messages without interference. This is done using all $K+1$ antennas at User 1 and 2, and single antennas at User 3 and User 4. At User 1 and 2, bit-level XOR operations are performed on recovered symbols $s_{12}(k)$ $(k = 1, \cdots, K)$ to obtain data of interest.

After precoding with the nulling matrix $\mathbf{U} = [\mathbf{U}_{12}(1)|\cdots|\mathbf{U}_{12}(K)|\mathbf{U}_{3R}|\mathbf{U}_{4R}]$ of size $(K+3)\times(K+2)$, the re-transmitted signal at the relay is written as:

$$\begin{aligned} \mathbf{X}_R &= \mathbf{U} \cdot [s_{12}(1), \cdots, s_{12}(K), s_{3R}, s_{4R}]^T \\ &= \sum_{k=1}^{K} s_{12}(k)\mathbf{U}_{12}(k) + s_{3R}\mathbf{U}_{3R} + s_{4R}\mathbf{U}_{4R} \end{aligned} \tag{3.19}$$

For PUs 1 and 2, the received signals $\mathbf{Y}_i$ $(i \in \{1, 2\})$ of size $(K+1)\times 1$ are in the form:

$$\mathbf{Y}_i = \mathbf{H}_{i,R}\mathbf{U} \cdot [s_{12}(1), \cdots, s_{12}(K), s_{3R}, s_{4R}]^T + \mathbf{n}_i \tag{3.20}$$

For SUs , the scalar received signals y_l ($l \in \{3, 4\}$) are:

$$y_l = \mathbf{h}_{l,R}\mathbf{U} \cdot [s_{12}(1), \cdots, s_{12}(K), s_{3R}, s_{4R}]^T + n_l \tag{3.21}$$

where n_l is the scalar (complex) AWGN at the single antenna of User l in the BC phase.

To allow the data recovery at PUs and SUs, the $K + 2$ nulling columns $\mathbf{U}_{12}(k)$ ($k \in \{1, \cdots, K\}$), $\mathbf{U}_{3R}$, and $\mathbf{U}_{4R}$ are designed so that:
(i) the SU l ($l \in \{3, 4\}$) using a single antenna can recover s_{lR} which requires that all the other signals, i.e., K mutual symbols $[s_{12}(1), \cdots, s_{12}(K)]$ and s_{pR} for the the SU p ($p \neq l$), when they arrive at the SU l are "zeroed" so that they do not interfere with the detection of s_{lR};
(ii) the PU i ($i \in \{1, 2\}$) using $K + 1$ antenna can recover the K mutual symbols $[s_{12}(1), \cdots, s_{12}(K)]$; for each aligned symbol $s_{12}(k)$; the PU i has the knowledge of its data represented with $s_i(k)$ and can use XOR operations to obtain the desired data (represented by $s_j(k)$, where $j \neq i$).

While the signal alignment in the MAC phase of the proposed system is in line with the approaches followed in MP-TWRC or MIMO Y channels, the signal nulling in the BC phase of the proposed system requires a specialized adaptation of similar schemes as explained in the next section.

3.3.1 Signal Nulling in the BC Phase

In the BC phase, for the underlying configuration, based on (3.20) and (3.21), the received signals at different nodes after applying nulling vectors at the relay are given by:

$$\mathbf{Y}_1 = \sum_{k=1}^{K} s_{12}(k)\mathbf{H}_{1,R}\mathbf{U}_{12}(k) + s_{3R}\mathbf{H}_{1,R}\mathbf{U}_{3R} + s_{4R}\mathbf{H}_{1,R}\mathbf{U}_{4R} + \mathbf{n}_1 \tag{3.22a}$$

$$\mathbf{Y}_2 = \sum_{k=1}^{K} s_{12}(k)\mathbf{H}_{2,R}\mathbf{U}_{12}(k) + s_{3R}\mathbf{H}_{2,R}\mathbf{U}_{3R} + s_{4R}\mathbf{H}_{2,R}\mathbf{U}_{4R} + \mathbf{n}_2 \tag{3.22b}$$

$$y_3 = \sum_{k=1}^{K} s_{12}(k)\mathbf{h}_{3,R}\mathbf{U}_{12}(k) + s_{3R}\mathbf{h}_{3,R}\mathbf{U}_{3R} + s_{4R}\mathbf{h}_{3,R}\mathbf{U}_{4R} + n_3 \tag{3.22c}$$

$$y_4 = \sum_{k=1}^{K} s_{12}(k)\mathbf{h}_{4,R}\mathbf{U}_{12}(k) + s_{3R}\mathbf{h}_{4,R}\mathbf{U}_{3R} + s_{4R}\mathbf{h}_{4,R}\mathbf{U}_{4R} + n_4 \tag{3.22d}$$

In (3.22), only the signals represented as green terms are required to be decoded at various nodes and the other signals should, or rather could, be nulled. In the case of SUs, because they receive using a single antenna and there is one message for them, either s_{3R} or s_{4R}, there is only one green term in (3.22c) and (3.22d). For User 3, this mean that we have to apply $\mathbf{U}_{12}(k)$ and $\mathbf{U}_{4R}$ such that (i) $\mathbf{h}_{3R}\mathbf{U}_{12}(k) = 0$ for $k \in \{1, \cdots, K\}$ and (ii) $\mathbf{h}_{3R}\mathbf{U}_{4R} = 0$. The signals that have to be zeroed in (3.22) are color-coded in red. In the case of PUs, because they receive using $K+1$ antennas and there are K messages for both PUs ($s_{12}(k)$ $k \in \{1, \cdots, K\}$), we have to null the impact of one (or two) symbol(s) from the SU(s). This is the reason why we have the summation term in green in both (3.22a) and (3.22b). Based on the mathematical feasibility to design the nulling vectors, we decided that for PU 1 we null the impact of s_{4R} by imposing the requirement that $\mathbf{H}_{1,R}\mathbf{U}_{4R} = \mathbf{0}_{K+1,1}$ (red term in (3.22a)). Similarly, for PU 2 we null the impact of s_{3R} by imposing the requirement that $\mathbf{H}_{2,R}\mathbf{U}_{3R} = \mathbf{0}_{K+1,1}$ (red term in (3.22b)). The black signals in (3.22a) and (3.22b), s_{3R} and s_{4R} are also going to be decoded at PU1 and PU2, respectively even though they were not intended for these users. The reason for pursuing this option is the feasibility of finding the nulling vectors meeting all the requirements.

For User 1, if we null the impact of s_{4R}, the received signal $\mathbf{Y}_1$ in (3.22a) of size $(K+1) \times 1$, can be re-written as:

$$\mathbf{Y}_1 = \mathbf{H}_{1,R}\big[\mathbf{U}_{12}(1) \cdots |\mathbf{U}_{12}(K)|\mathbf{U}_{3R}\big]\big[s_{12}(1), \cdots, s_{12}(K), s_{3R}\big]^T + \mathbf{n}_1 \tag{3.23}$$

Based on (3.23), PU 1 can recover K mutual message signals that are desired at User 1 (and s_{3R}), just by applying matrix inversion of size $(K+1) \times (K+1)$. In this ZF type processing, the decisions may be affected by the noise enhancement but this approach benefits from simplicity of linear processing which is targeted in most applications of SA.

Based on the above conditions imposed on nulling vectors, we now demonstrate that such vectors can really be found.

Regarding $\mathbf{U}_{4R}$, based on red-coloring in (3.22a), $\mathbf{U}_{4R}$ (of size $(K+3) \times 1$) has to be orthogonal to all $K+1$ rows of $\mathbf{H}_{1,R}$ where these rows are of size $1 \times (K+3)$. Also, based on (3.22c), $\mathbf{U}_{4R}$ has to be orthogonal to $\mathbf{h}_{3,R}$. Since $\mathbf{U}_{4R}$ is in $(K+3)$ dimensions (D) and has to be orthogonal to $K+2$ vectors (also in $(K+3)$-D), there exists such a unique vector obtained as a solution to: $\mathrm{null}\Big(\begin{bmatrix} \mathbf{H}_{1,R} \\ \mathbf{h}_{3,R} \end{bmatrix}\Big)$. Similarly, there is a unique solution for $\mathbf{U}_{3R} = \text{null}\left(\begin{bmatrix} \mathbf{H}_{2,R} \\ \mathbf{h}_{4,R} \end{bmatrix}\right)$. Regarding $\mathbf{U}_{12}(k)$ ($k \in \{1, \cdots, K\}$), based on (3.22c)) and (3.22d)), these K vectors have to be orthogonal, at the same time, to $\mathbf{h}_{3,R}$ and $\mathbf{h}_{4,R}$. There are $K+1$ vectors that are in the null space of $\begin{bmatrix} \mathbf{h}_{3,R} \\ \mathbf{h}_{4,R} \end{bmatrix}$ and it may seem that there is a flexibility in selecting $\mathbf{U}_{12}(k)$'s. However, when solving (3.23) through matrix inversion, we want $\mathbf{U}_{12}(k)$'s and $\mathbf{U}_{3R}$ to be independent. Since $\mathbf{U}_{3R} \perp \mathbf{h}_{4,R}$; $\mathbf{U}_{12}(k) \perp \mathbf{h}_{4,R}$ and there are $K+2$ vectors orthogonal to $\mathbf{h}_{4,R}$, when selecting $\mathbf{U}_{12}(k)$ as null$\begin{bmatrix} \mathbf{h}_{3,R} \\ \mathbf{h}_{4,R} \end{bmatrix}$, through flexibility of selecting K out of $K+1$ vectors for $\mathbf{U}_{12}(k)$'s, we ensure that non of $\mathbf{U}_{12}(k)$ is co-linear with $\mathbf{U}_{3R}$ and $\mathbf{U}_{4R}$.

3.3.2 Antenna Selection in the MAC Phase

To meet the signal nulling conditions in the BC phase as presented in 3.3.1, the relay has to be equipped with $K+3$ antennas while in the MAC phase, the relay has $K+2$ active antennas. It is therefore of interest to select a subset of available antennas at the relay for a reception during the MAC phase. The optimum relay antennas are selected by a modified Maximum-Minimum Singular Value (Max-Min SV) criterion with respect to the post-processing SNR at the relay [40]. Specifically, we identify the subset of potentially active antennas with the index k of the relay antenna that is not used during the MAC phase (there is $K+3$ of such subsets). For every subset, we extract $\mathbf{H}^k_{R,1}$ and $\mathbf{H}^k_{R,2}$ channel gain matrices from Users 1 and 2 to the relay active subset of antennas. These matrices of size $(K+2) \times (K+1)$ are derived from two channel gain matrices ($\mathbb{H}_{R,1}$ and $\mathbb{H}_{R,2}$) from User 1 and User 2 to all antennas at the relay, by selecting the same $(K+2)$ rows of $\mathbb{H}_{R,1}$ and $\mathbb{H}_{R,2}$. Matrices $\mathbb{H}_{R,1}$ and $\mathbb{H}_{R,2}$ are of size $(K+3) \times (K+1)$. Since the post-processed signal at the relay used for detection on virtual spatial streams is given by (3.18), based on $\mathbf{H}^k_{R,1}$ and $\mathbf{H}^k_{R,2}$, we calculate the corresponding matrix of effective channel gains $\mathbf{D}^k$ of size $(K+2) \times (K+2)$ as explained in Section 3.3. Now, when selecting the best active receiving antennas at the relay, we follow one of the criteria in [40] for selecting the optimal antenna subset when linear receivers are used over a slowly MIMO varying channel. Specifically, we adopt the use of the post-processing SNRs of the multiplexed streams where the antenna subset that induces the largest minimum SNR is chosen based on singular value decomposition (SVD) of the effective channel gain matrices $\mathbf{D}^k$. In this Max-Min SV method, for every $\mathbf{D}^k$ (out of possible $K+3$) we calculate the minimum singular value λ_{min}^{k} and we choose the subset of active receive antennas in the MAC phase (indexed by k) with the largest λ_{min}^{k}.

3.4 Performance Evaluation

In this section, we present the performance for our proposed topology in terms of BER. The end-to-end BER performance when $K = 2$ is demonstrated in Fig. 3.3. At BER of 10^{-3}, SUs showing a great SNR of 11 and PUs have the SNR of 40. The simulation indicates that this novel cognitive radio network scheme guarantees a non-disruptive performance for PUs when they share their spectrum and accommodate two more SUs to communicate with the relay. As we did not impose an optimization in the BC phase, the performance is better in the MAC phase for SUs.

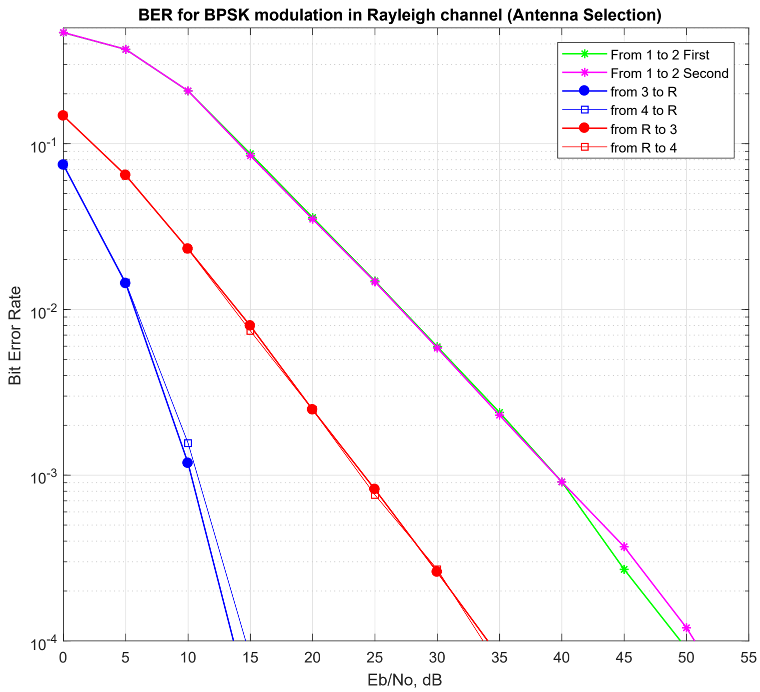

Figure 3.3: BER performance of five-user scheme with selection diversity when K=2.

We illustrate the BER performances of different data traffics in the proposed SA scheme with Max Min antenna selection and compare them with the performance of the non-optimized scheme that, in the MAC phases, always uses the first $K + 2$ antennas at the relay. The comparison between applying antenna selection into this scheme and the non-optimized scheme when K=1 has shown in Fig. 3.4. When reporting the BER improvements, we present the improvements in SNR between optimized schemes and non-optimized schemes to achieve the target BER=10^{-3}. In both figures in this section, the performance for the optimized scheme is shown using continuous lines, while the performance for the non-optimized scheme is shown using dashed lines.

It demonstrates that with employing antenna selection in this system we can achieve up to 10 dB improvement in the BER performance of PUs and 19 dB improvement for the SUs for their communications with the relay which is significant. The proposed scheme benefits mostly in the MAC phase, where by selecting $K+2$ out of $K+3$ possible receiving antennas at the relay, we exploit the spatial diversity in the MAC phase. It is worth mentioning the BER performance in these simulations are end-to-end, which means that for PUs this BER performance is for MAC and BC phases, but for SUs one is for the stream from SUs to the relay in MAC phase, and the other is for the stream from the relay to SUs in BC phase. Since we did not have an optimization in the BC phase the results show that we did not have an improvement for the data transmission from relay to SUs which is happening in BC phase.

Figure 3.5 shows the comparison between before and after antenna selection when $K=2$. In this case, the BER performance for PUs improved by 5 dB (magenta and green curves) and the performance for SUs improved by 20 dB in the MAC phase (blue curves) and no improvement in the BC phase due to not having optimization in BC phase (red curves). It is obvious that as the number of antennas increase, the BER improvement after optimization by applying antenna selection is not as much as before. This is because the minimum spatial distance between symbols decrease as the number of dimensions goes up.

In summary, the proposed scheme with optimized antenna selection at the relay in the MAC phase provides BER improvements over the non-optimized scheme and the improvements depend on the antenna and traffic configuration patterns. The proposed scheme results in the uneven BER performance for user pairs which is due to our assumption that all nodes in the system use the same transmit power.

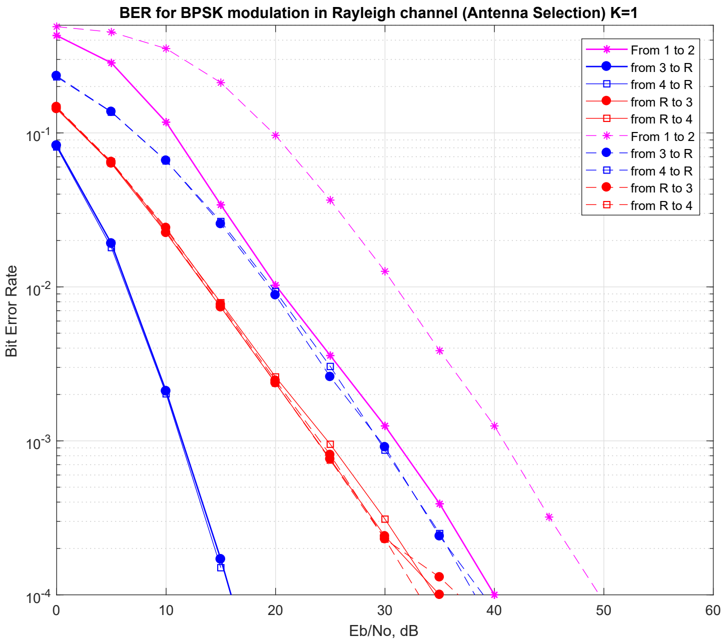

Figure 3.4: BER performance comparison for five-user scheme when K=1, with and without antenna selection.

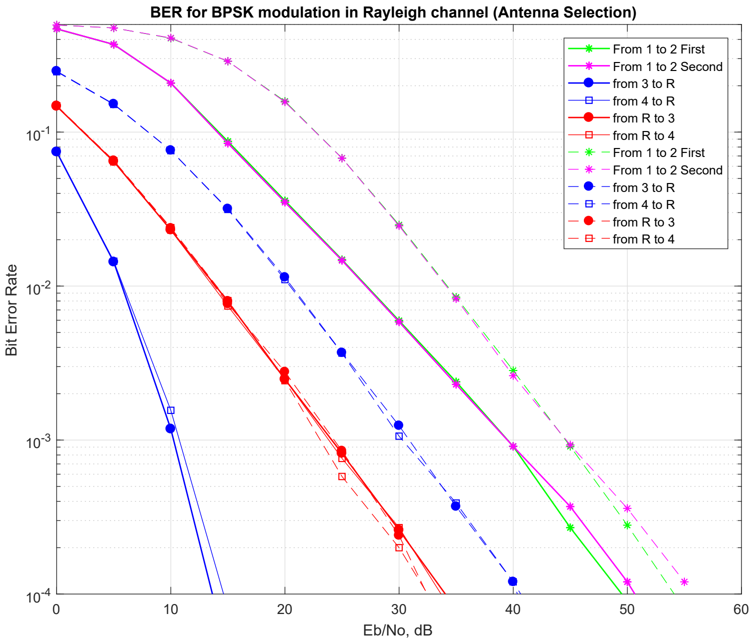

Figure 3.5: BER performance comparison for five-user scheme when K=2, with and without antenna selection.

3.5 Generalized Antenna Configuration for Desired Number of SUs Equipped with Arbitrary Number of Antennas

In this section, the contribution for the generalization of both PUs and SUs has been presented. This generalization is studying the case that PUs' communication can accommodate any number of SUs equipped with any desired number of antennas. In order to achieve that, complex computations with large number of antennas has been practiced and after sequences of numbers for each component has been found, it has been analyzed in order to find a formula for each and every sequence.

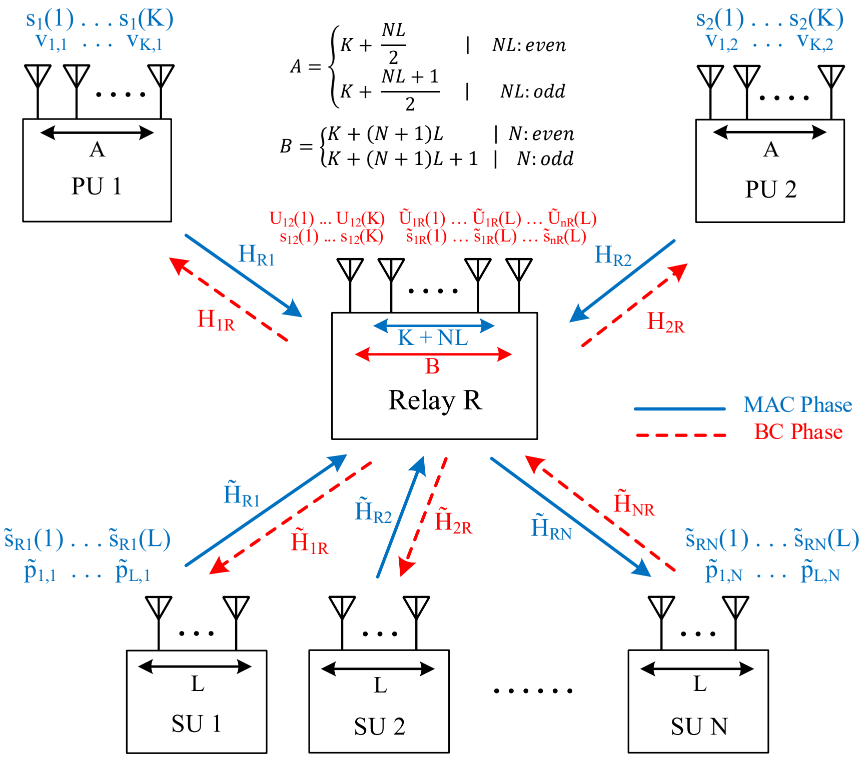

Figure 3.6: Generalized relay-assisted network with high throughput PUs and multiple high throughput SUs.

The generalized scenario is shown in Fig. 3.6, where there are two PUs, one relay and n SUs. It has to be mentioned that there are no direct link and all users communicate via the relay. PUs have K independent messages for each other denoted as $s_1(k)$ and $s_2(k)$ where $k \in \{1, \cdots K\}$. These messages are to be exchanged via relay R in two TS, MAC and BC phases. In this system there are n SUs, $n \in \{1, \cdots N\}$

equipped with L antennas. Each SU sends L messages to the relay in MAC phase, and receives L messages in BC phase. Without loss of generality, all messages are represented by scalar (±1) BPSK symbols. In this system, PUs are equipped with the same number of antenna as below:

$$A = \begin{cases} K + \dfrac{N \cdot L}{2} & N \cdot L : even \\ K + \dfrac{N \cdot L + 1}{2} & N \cdot L : odd \end{cases} \tag{3.24}$$

and SUs are equipped with L antennas. The relay R equipped with $K + NL$ antennas in the MAC phase and the number of antennas required for the BC phase to meet signal nulling conditions is as below:

$$B = \begin{cases} K + (N+1) \cdot L & N : even \\ K + (N+1) \cdot L + 1 & N : odd \end{cases} \tag{3.25}$$

In this model, in the MAC phase, the channel gain matrices $\mathbf{H}_{R,1}$ and $\mathbf{H}_{R,2}$ from PUs 1 and 2 to the relay active antennas are of size $(K + NL) \times (A)$. in the BC phase, the channel gain matrices $\mathbf{H}_{1,R}$ and $\mathbf{H}_{2,R}$ from the relay using all B antennas to PUs 1 and 2 are of size $(A) \times (B)$. In the MAC phase, the channel gain matrices between SUs and the relay are $\tilde{\mathbf{H}}_{R,N}$, and their size is $(K + NL) \times (L)$. In the BC phase, the channel gain matrices between SUs and the relay are $\tilde{\mathbf{H}}_{N,R}$, and they are of size $L \times B$. Each entry of channel gain matrices is assumed to be independent identically distributed (i.i.d) complex Gaussian to capture the effects of slow Rayleigh fading channels. It is assumed that all nodes operate in half-duplex mode and the full channel state information (CSI) is available to all nodes. In the MAC phase, for the PUs i with $i = 1$ or $i = 2$, K information symbols $s_i(k)$ $(k = (1, \cdots, K)$ are pre-processed by the precoding matrix $[\mathbf{v}_{1,i} \mid \cdots \mid \mathbf{v}_{K,i}]$ where the precoding vectors $\mathbf{v}_{k,i}$ are of size $(A) \times 1$. The transmitted data vector by terminal i is then:

$$\mathbf{x}_i = [\mathbf{v}_{1,i} \mid \cdots \mid \mathbf{v}_{K,i}] \cdot \begin{bmatrix} s_i(1) \\ \vdots \\ s_i(K) \end{bmatrix} = \sum_{k=1}^{K} \mathbf{v}_{k,i} \cdot s_i(k) \tag{3.26}$$

for the SUs n with $n = 1, \cdots, N$, L information symbols $\tilde{s}_{Rn}(l)$ $(l = (1, \cdots, L)$ are pre-processed by the precoding matrix $[\tilde{\mathbf{p}}_{1,n} \mid \cdots \mid \tilde{\mathbf{p}}_{L,n}]$ where the precoding vectors $\tilde{\mathbf{p}}_{l,n}$ are of size $L \times 1$. Therefore, the transmitted signal by each SU is:

$$\tilde{\mathbf{x}}_{Rn} = [\tilde{\mathbf{p}}_{1,n} \mid \cdots \mid \tilde{\mathbf{p}}_{L,n}] \cdot \begin{bmatrix} \tilde{s}_{Rn}(1) \\ \vdots \\ \tilde{s}_{Rn}(L) \end{bmatrix} = \sum_{l=1}^{L} \tilde{\mathbf{p}}_{l,n} \cdot \tilde{s}_{Rn}(l) \tag{3.27}$$

Also, the average power of information symbols is one and to ensure fairness between different spatial streams received at the relay, the total transmit power per antenna at all users is set to one, which imposes constraints on the norms of the precoding vectors. When $2 \cdot K$ messages for the PUs and $N \cdot L$ messages for the SUs are being exchanged in MAC Phase, the total number of $2K + N \cdot L$ messages are transmitted simultaneously to the relay, the received signal at the relay is:

$$\mathbf{Y}_R = \mathbf{H}_{R,1} \cdot \mathbf{x}_1 + \mathbf{H}_{R,2} \cdot \mathbf{x}_2 + \tilde{\mathbf{H}}_{R,1} \cdot \tilde{\mathbf{x}}_{R1} \cdots + \tilde{\mathbf{H}}_{R,N} \cdot \tilde{\mathbf{x}}_{RN} + \mathbf{n}_R \tag{3.28}$$

where $\mathbf{n}_R$ is the additive white Gaussian noise (AWGN) and both $\mathbf{Y}_R$ and $\mathbf{n}_R$ are of size $(K + N \cdot L) \times 1$. To align the mutual symbols $s_1(k)$ and $s_2(k)$ from users 1 and 2 into the same dimension in the signal space, the precoding vectors for the mutual symbols are selected in such way so that:

$$\begin{aligned} \mathbf{Y}_R &= \sum_{k=1}^{K} \Big(\mathbf{H}_{R,1} \mathbf{v}_{k,1} \cdot s_1(k) + \mathbf{H}_{R,2} \mathbf{v}_{k,2} \cdot s_2(k) \Big) \\ &+ \sum_{l=1}^{L} \tilde{\mathbf{H}}_{R,1} \cdot \tilde{\mathbf{p}}_{l,1} \cdot \tilde{s}_{R1}(l) \cdots + \sum_{l=1}^{L} \tilde{\mathbf{H}}_{R,N} \cdot \tilde{\mathbf{p}}_{l,N} \cdot \tilde{s}_{RN}(l) + \mathbf{n}_R \\ &= \sum_{k=1}^{K} \mathbf{D}_{12}(k) \cdot \hat{s}_{12}(k) + \sum_{n=1}^{N} \sum_{l=1}^{L} \tilde{\mathbf{D}}_{Rn}(l) \cdot \tilde{s}_{Rn}(l) + \mathbf{n}_R \end{aligned} \tag{3.29}$$

where $\mathbf{D}_{12}(k)$ denote the K spatial signaling dimensions of size $(K + N \cdot L) \times 1$ for bi-directional transmissions between User 1 and 2, and they can be interpreted as the effective channel gains for the aligned symbols $\hat{s}_{12}(k)$ with $\hat{s}_{12}(k) = s_1(k) + s_2(k)$. The signaling dimensions $\tilde{\mathbf{D}}_{Rn}(l)$ are also of size $(K + N \cdot L) \times 1$ are used for the unicast transmissions from SUs to relay R. According to SA requirements, the selected precoding vectors $\mathbf{v}_{k,1}$ and $\mathbf{v}_{k,2}$ need to satisfy:

$$\mathbf{H}_{R,1} \cdot \mathbf{v}_{k,1} = \mathbf{H}_{R,2} \cdot \mathbf{v}_{k,2} \quad \text{for} \quad k \in \{1, \cdots\ K\} \tag{3.30}$$

Like generalized version over PUs in the previous section, here we also only have K choices for selecting $\left[\begin{smallmatrix} \mathbf{v}_{k,1} \\ \mathbf{v}_{k,2} \end{smallmatrix} \right]$. This will allow us to align the mutual symbols from the PUs at the relay, but does not offer any flexibility in working with different dimensions to optimize the performance in the MAC phase. The flexibility is in choosing $K + N \cdot L$ antennas out of B antennas to be active antennas in MAC phase which will be discussed later. The matrix form for the received signal at the relay will be:

$$\mathbf{Y}_R = \mathbf{D} \cdot [\hat{s}_{12}(1), \cdots, \hat{s}_{12}(K), \tilde{s}_{R1}(1), \cdots\ \tilde{s}_{R1}(L), \cdots\ \tilde{s}_{RN}(L)]^T + \mathbf{n}_R \tag{3.31}$$

where T stands for transpose for the matrix of coefficients D of size $(K + N \cdot L) \times (K + N \cdot L)$ is obtained by concatenating effective channel gain columns $\mathbf{D}_{12}(k)$ and $\tilde{\mathbf{D}}_{Rn}(l)$. A matrix D inversion will be applied afterward to recover signal of interest using ZF approach.

In the BC phase, the relay forwards the aligned symbols $s_{12}(k)$ $(k = 1, \cdots, K)$ as BPSK representations of XOR-ed bits between PUs and the relay also sends its own data to SUs. A set of precoding/nulling vectors $\mathbf{U}_{12}(k)$ is applied to the aligned symbols and $\tilde{\mathbf{U}}_{nR}(l)$ to the messages from the relay to SUs, so each user receives the relevant data. All nulling vectors are of size $(B) \times 1$. At each PU, a bit-level XOR operations is performed on received symbols $s_{12}(k)$ $(k = 1, \cdots, K)$ to decode data of interest.

After precoding with the nulling matrix $\mathbf{U} = [\mathbf{U}_{12}(1)|\cdots|\mathbf{U}_{12}(K)|\tilde{\mathbf{U}}_{1R}(1)|\cdots|\tilde{\mathbf{U}}_{NR}(L)]$ of size $(B) \times (K + N \cdot L)$, the re-transmitted signal at the relay is written as:

$$\begin{aligned} \mathbf{X}_R &= \mathbf{U} \cdot [s_{12}(1), \cdots, s_{12}(K), \tilde{s}_{1R}(1), \cdots, \tilde{s}_{NR}(L)]^T \\ &= \sum_{k=1}^{K} s_{12}(k)\mathbf{U}_{12}(k) + \sum_{n=1}^{N}\sum_{l=1}^{L} \tilde{s}_{nR}(l)\tilde{\mathbf{U}}_{nR}(l) \end{aligned} \tag{3.32}$$

For PUs 1 and 2, the received signals $\mathbf{Y}_i$ $(i \in \{1, 2\})$ of size $(A) \times 1$ are in the form:

$$\mathbf{Y}_i = \mathbf{H}_{i,R}\mathbf{U} \cdot [s_{12}(1), \cdots, s_{12}(K), \tilde{s}_{1R}(1), \cdots, \tilde{s}_{NR}(L)]^T + \mathbf{n}_i \tag{3.33}$$

For SUs , received signals $\mathbf{Y}_n$ $(n \in \{1, \cdots, N\})$ are:

$$\mathbf{Y}_n = \tilde{\mathbf{H}}_{n,R}\mathbf{U} \cdot [s_{12}(1), \cdots, s_{12}(K), \tilde{s}_{1R}(1), \cdots, \tilde{s}_{NR}(L)]^T + n_n \tag{3.34}$$

where n_n is the scalar (complex) AWGN at the single antenna of User n in the BC phase.

To allow the data recovery at PUs and SUs, the $K + N \cdot L$ nulling columns $\mathbf{U}_{12}(k)$ $(k \in \{1, \cdots, K\})$, $\tilde{\mathbf{U}}_{nR}(l)$, are designed so that:

(i) the SU n using L antennas can recover $\tilde{s}_{nR}(1), \cdots, \tilde{s}_{nR}(L)$ which requires that all the other signals, i.e., K mutual symbols $[s_{12}(1), \cdots, s_{12}(K)]$ and all other messages from the Relay destined to other SUs are zeroed at the SU n, so that they do not interfere with the detection of n'th SU messages.

(ii) the PU i, using A antenna can recover the K mutual symbols $[s_{12}(1), \cdots, s_{12}(K)]$; for each aligned symbol $s_{12}(k)$; the PU i has the knowledge of its own data represented with $s_i(k)$ and can use XOR operations to obtain the desired data sent from the other PU.

3.6 Performance Evaluation

In this section, we present the BER performance simulation results for our proposed generalized scheme when i) PUs have 4 antennas exchanging 2 messages in two TSs, the relay has 8 antennas with 6 active antennas in the MAC phase, and two SUs have 2 antennas to transmit two messages to the relay in the MAC phase and receive two messages from the relay in the BC phase; ii) PUs have 4 antennas exchanging 2 messages in two TSs, the relay has 7 antennas with 5 active antennas in the MAC phase, and three SUs have 1 antennas to transmit and receive one message to and from the relay in MAC and BC phases, respectively.

In these simulations, all wireless channels are assumed to be i.i.d, and Rayleigh flat fading channels, with BPSK as the modulation mechanism, though the proposed scheme can be used for higher modulation levels. The simulation results in this section show the BER performance of system models in which the Min Max antenna selection criterion is deployed to optimize system performance. In these figures, blue lines are reserved for performance presentation of transmissions from the SUs to the relay in the MAC phase and red lines for data transmission from the relay to SUs in the BC phase. The proposed schemes offer the most benefit in the MAC phase where by selecting a specific number of antennas out of total number of antennas at the relay, we exploit the selection diversity at the receiver.

Fig. 3.7 shows the results when we have the system model described in (i). As can be observed, in order to achieve BER=10^{-3}, the PUs need to have an SNR of 43 dB. In the MAC phase, SUs need to have an SNR of 7 dB and in the BC phase they are required to have 37 dB to target the same BER. This system model is able to exchange a considerable number of messages (12 messages in two TSs). It is apparent that there is a trade-off between throughput and BER performance, as increasing the throughput deteriorates the BER performance of the system.

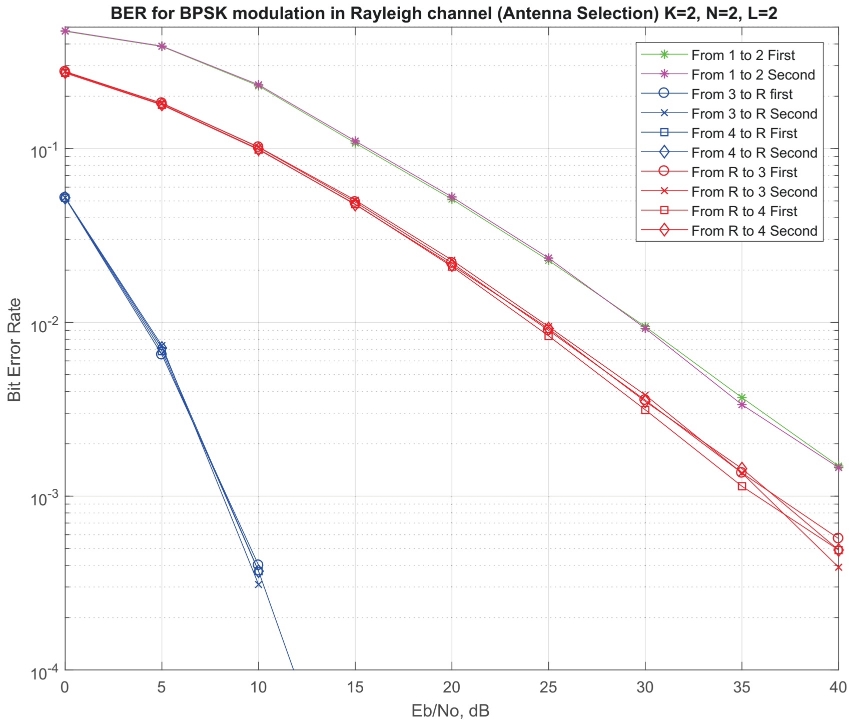

Figure 3.7: BER performance of generalized multi-user scheme with K=2, N=2, L=2

Fig. 3.8 shows the results for the system model outlined in (ii). The results show that achieving a BER of 10^{-3}, the PUs are required to have an SNR of 32 dB. Furthermore, SUs need to have the SNR of 6 dB in the MAC phase and 24 dB in the BC phase to target a BER of 10^{-3}. In this setting, 10 messages are exchanged in two TSs. To sum up, these simulation results for two system models are presented as examples for our proposed generalized scheme. These simulations verify the credibility and validity of the proposed system.

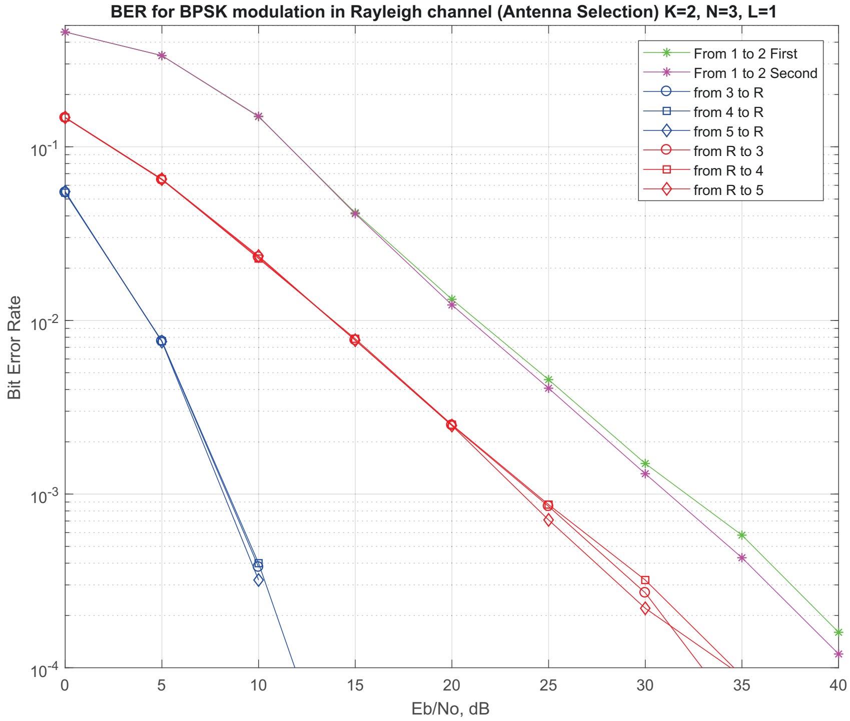

Figure 3.8: BER performance of generalized multi-user scheme with K=2, N=3, L=1

3.7 Summary

In this chapter, we designed a signal alignment scheme in a relay-assisted wireless network where two PUs with multiple antennas, and two (or more), single (or multiple) antenna, SUs send data following two- and one-way communication protocols, respectively. Specifically, the cooperating multiple antenna relay and the nodes with specialized DF processing enable data recovery from spatially multiplexed data streams in two time slots resulting in high bandwidth efficiency. The proposed scheme differentiates itself from similar systems, like the multi-pair two-way relay channel, by taking advantage of the traffic flow patterns considered in this chapter. By supporting users with heterogeneous antenna configurations, the developed system is applicable in overlay CR networks. When working with the minimum number of antennas to enable the desired data flow in bi- and uni-directional data exchanges between the nodes in the system, we exploited the redundant antenna at the relay during the MAC phase to provide the diversity advantage. The proposed scheme with optimized selection of active antennas at the relay provides an improvement in BER performance over the non-optimized scenarios.

Chapter 4

Conclusion

This chapter provides an overview of the contributions in this thesis and suggestions for future works on this topic. Section 4.1 discusses the contributions of this thesis while Section 4.2 suggests potential future works.

4.1 Thesis Contributions

The explosive growth of MIMO systems has permitted high data rates and a wide variety of applications. An example of such an application is MU-MIMO with distributed-in-space transmissions and processing of messages in terminals equipped with multiple antennas. However, limited spectrum resources urges us to find a way to efficiently tackle spectrum scarcity in wireless networks. CRNs have been identified as a promising solution for this issue.

MU-MIMO systems further leverage the benefits of conventional single link MIMO at the cost of somewhat more expensive signal processing. This thesis presents an application of a MU-MIMO concept in CR networks where MAI is eliminated by deploying SA. Specifically, building on the Y channel topology, we designed SA schemes in relay-assisted wireless networks where PUs and SUs send data following two- and one-way communication protocols, respectively. In this thesis, the cooperating multiple antenna relay and the nodes with specialized DF processing enable data recovery from spatially multiplexed data streams in two time slots (MAC and BC) resulting in high bandwidth efficiency. The proposed schemes differentiate themselves from similar systems, like the multi-pair two-way relay channel, by taking advantage of the traffic flow patterns considered in this thesis. By supporting users with heterogeneous antenna configurations, the developed systems can be applied in overlay CR networks. When working with the minimum number of antennas to enable the desired data flow in bi- and uni-directional data exchanges between the nodes in the system, we exploited the redundant antenna at the relay during the MAC phase to provide the diversity advantage. The proposed schemes, with optimized selection of active antennas or signaling dimensions at the relay, provide improvements in the BER performance over non-optimized scenarios.

Results of our research have been partially published in the form of two conference papers [30], [31] and the contributions from these papers have been summarized at the end of the respective research chapters. Below we summarize the key developments and findings in the thesis that represent common threads throughout our work.

In our first contribution, a MIMO Y channel topology was modified and adapted into the overlay CR paradigm to mitigate interference in the aforementioned system models using SA processing and enhancing BER with optimization while improving bandwidth efficiency.

In our second contribution, the presented topology was generalized to different number of antennas and messages.

In our third contribution, on the groundwork of previous contributions, a novel five-user system model was introduced which contains two PUs, two SUs and one relay/user. In this new system model not only did bandwidth efficiency increase, but BER performance also improved thanks to optimization using selection diversity for active antennas.

In our forth contribution, the five-user system model was generalized so that PUs are able to accommodate any number of SUs equipped with any number of antennas.

The research methodology followed in this thesis is that the idea is first examined and verified in smaller system configurations to test the feasibility of the proposed scheme then the scheme is expanded to generalized cases. Once the feasibility of SA algorithms in various system configurations had been verified, we exploited flexibility in choosing orthogonal spatial dimensions in some scenarios and having antenna diversity in others. This led to have the capability to optimize each scenario and this was with the objective to improve the BER performance over non-optimized schemes. Specifically, not only did the proposed schemes performed well in terms of bandwidth efficiency but the simulation results for different scenarios also showed a significant improvement in terms of BER over conventional MIMO Y channel and non-optimized scenarios.

Overall, This thesis innovates by introducing a new technique to eliminate the existing interference in CRNs due to sharing a frequency. The studies presented in Chapters 2 to 3 have demonstrated how to apply SA approaches in overlay CR networks aided by the relay and how to eliminate MAI in these networks through coordinated transmissions and signal processing. MIMO is already a well-established, powerful technique to increase capacity in point-to-point links or multi-user scenarios and this thesis demonstrates that when combined with SA it can also play an important role in supporting CR operation.

4.2 Suggested Future Work

In this section some potential future work has been suggested.

1. Different Antenna Selection Strategy

In this thesis, in the scenarios where we had the flexibility of choosing the best antennas for the receiver's reception, we deployed a sub-optimal Min-Max SVD criterion that does not involve complex computations but has near optimal solution. Therefore, it does not take many resources which means introducing less delay into the system. However, antenna selection on receiver side has been well studied and there are some effective strategies introduced in this regard that can perform better in our proposed scheme, considering the limitations and constraints [41], [42]. Thus, there is a potential for research identifying the best optimization strategy in our schemes which may lead to enhancing BER.

2. Optimizing Precoding Vectors in BC Phase

In some scenarios introduced in this thesis, there is a flexibility of choosing different nulling vectors in BC phase due to having degrees of freedom which we did not investigate. Therefore, there is a opportunity to optimize precoding vectors in BC phase that require further investigation to exploit possibility of improvement in the BC phase.

3. Power Allocation

In this study was more focused on bit level transmissions and having various power levels from different nodes was not studied. In practical systems, power control mechanisms ensure that all signals arrive within an acceptable power level and the required power level for each node is determined by the distance. Meanwhile, power control is affected by the delay in transmission of the relevant information and channel estimation accuracy. In addition, for a potential future work deploying different power allocation strategies can be a great choice as it might lead to a better and more efficient performance of the users. Moreover, receiving power at the same level for all users may not be an optimum power allocation technique in the design of our system due to different hierarchies of users and further investigations in the area of power allocation strategies in the form of cooperative communications could be considered.

4. Synchronization of Nodes

In this thesis one of our general assumptions was that user nodes have the same distance from the relay and all message transmissions are perfectly synchronized. While there are methods and network protocols to achieve synchronous operations, some technologies like OFDM which the symbol rate is very low at each subcarrier can be a solution to this problem. Due to different distance between nodes in real life scenarios, this may impose the challenge because of variability in the propagation delays of the signals.

5. Imperfect CSI

In this study, we assumed that perfect CSI is instantaneously available to all nodes so that proper preceding vectors can be calculated to implement SA. However, in practical applications CSI is usually estimated at receivers and sent to transmitters which would cause delay and inaccuracy of CSI. Although, this problem has been well studied by researchers in other schemes on how to overcome this problem, but it is important to understand the performance limits of the proposed schemes [43]–[45].

Appendix A

Scenario 1 from Chapter 2 with Four-User

In this part, we consider the case where $N_{12} = 2$, $N_3 = 1$ and $N = 2$ (Fig. A.1). In this case, we have $M_{12} = 1$ and $M_3 = 1$, which means there is only one bi-directional exchange and User 3 sends one message to relay R in the MAC phase and receives one message from relay R in the BC phase. Based on (2.4), the received signal at the relay is:

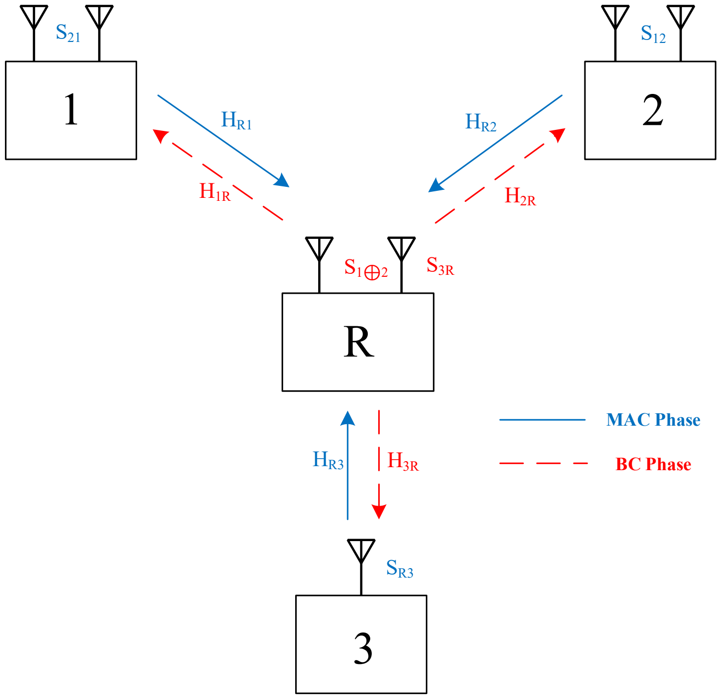

Figure A.1: System model for Scenario 1 Chapter 2

$$\begin{aligned}\mathbf{Y}_R &= \mathbf{H}_{R,1}\mathbf{v}_{1,1} \cdot s_1(1) + \mathbf{H}_{R,2}\mathbf{v}_{1,2} \cdot s_2(1) + \mathbf{H}_{R,3}\mathbf{p}_1 \cdot s_{R3}(1) + \mathbf{n}_R \\ &= \mathbf{D}_{12}(1) \cdot \hat{s}_{12}(1) + \mathbf{D}_{R3}(1) \cdot s_{R3}(1) + \mathbf{n}_R\end{aligned} \tag{A.1}$$

In this scenario, $\mathbf{p}_1$ is just a scalar so there is no possibility to manipulate $\mathbf{D}_{R3}(1) = \mathbf{H}_{R,3}$. However, because $[\mathbf{H}_{R,1} \mid -\mathbf{H}_{R,2}]$ is of size 2×4 the nullity of this matrix is 2, and we have two choices for selecting the signaling direction $\mathbf{D}_{12}(1) = \mathbf{H}_{R,1}\mathbf{v}_{1,1}$

for the one bi-directional data exchange. Let's refer to these choices as $\mathbf{D}'_{12}(1)$ and $\mathbf{D}''_{12}(1)$ as determined by two possible solutions for $\left[\mathbf{v}_{1,1}, \mathbf{v}_{1,2}\right]^T$ when solving:

$$[\mathbf{H}_{R,1} \mid -\mathbf{H}_{R,2}] \cdot \left[\begin{smallmatrix} \mathbf{v}_{1,1} \\ \mathbf{v}_{1,2} \end{smallmatrix}\right] = 0 \tag{A.2}$$

It is worth observing that in this scenario, we cannot control to have $\mathbf{D}_{12}(1) \perp \mathbf{D}_{R3}(1)$ as it is the case in the subsection 2.2.1. To decide between $\mathbf{D}'_{12}(1)$ and $\mathbf{D}''_{12}(1)$ the following observations are in order. Because along $\mathbf{D}_{12}(1)$ we are signaling using ternary symbols $(\pm 2, 0)$ and along $\mathbf{D}_{R3}(1)$ we are signaling using binary symbols (± 1), we are receiving in 2D six symbols at the relay R. Fig. A.2 shows the possible symbols received at that relay. When deciding between $\mathbf{D}'_{12}(1)$ and $\mathbf{D}''_{12}(1)$, our objective is to choose the vectors that are going to give us the biggest separation between six signaling points in order to minimize the symbol error rate (SER) at the relay in the MAC phase.

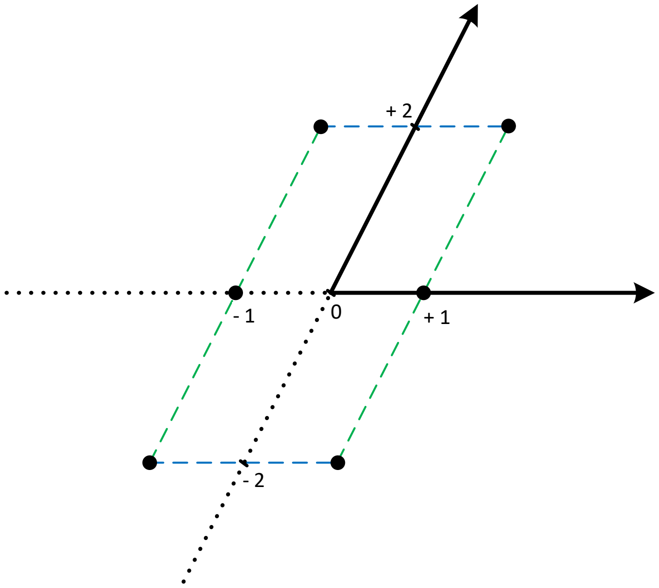

Figure A.2: Constellation for scenario 1

In this scenario in the BC phase, based on (3.20) and (3.21), the received signals at different users are:

$$\begin{aligned}\mathbf{Y}_1 &= s_{12}(1)\mathbf{H}_{1R}\mathbf{U}_{12}(1)+s_{3R}(1)\mathbf{H}_{1R}\mathbf{U}_{3R}(1)+\mathbf{n}_1\\ \mathbf{Y}_2 &= s_{12}(1)\mathbf{H}_{2R}\mathbf{U}_{12}(1)+s_{3R}(1)\mathbf{H}_{2R}\mathbf{U}_{3R}(1)+\mathbf{n}_2\\ \mathbf{Y}_3 &= s_{12}(1)\mathbf{H}_{3R}\mathbf{U}_{12}(1)+s_{3R}(1)\mathbf{H}_{3R}\mathbf{U}_{3R}(1)+\mathbf{n}_3\end{aligned} \tag{A.3}$$

The analysis of (A.3) shows that we have to apply $\mathbf{U}_{12}(1)$ such that $\mathbf{H}_{3R}\mathbf{U}_{12}(1) = \mathbf{0}$ and there is only one such vector. In the considered antenna setup, we design $\mathbf{U}_{3R}(1)$ so that $\mathbf{H}_{1R}\mathbf{U}_{3R}(1) = \mathbf{0}$ and User 1 is not affected by $s_{3R}(1)$. However, we cannot do this for User 2 which has two dimensions to detect data of interest ($s_{12}(1)$) and $s_{3R}(1)$. When simulating the performance of the proposed scheme in this configuration, we were observing improvements of 4dB at the BER=10^{-3} as compared with non-optimized scheme.

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