DOES QUANTUM MECHANICS ESTABLISH THE FINAL FAILURE OF CAUSALITY?

by

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Abstract

In the wake of relativistic field theories in physics, all of our most fundamental physical theories shared three properties regarding the way that they described the world. First, properties of objects were always determined. Second, systems always evolved deterministically. Thirdly, no causal relationship between two separate regions in space was immediate or had superluminal effect. These properties of theories are called determinacy, determinism and local causality, respectively. These properties all correspond to classical metaphysical principles about the structure of the world. It will be shown that the theory of quantum mechanics has none of these properties and so therefore introduces a tension between the classical metaphysical description of the world and the descriptions given by our most fundamental physical theories. John Stewart Bell showed that no entirely correct theory of quantum mechanics has the property of local causality. I argue that this implies that local causality is metaphysically untenable.

List of Abbreviations and Symbols Used

QM — Quantum Mechanics

EPR — Einstein, Podolsky and Rosen

$^{\mathrm{T}}$ — Property of a formal theory

$^{\mathrm{O}}$ — Property of the ontological reality

θ — Angle of measurement or of misalignment

$\mathbf{p}$ — Position observable

$\mathbf{q}$ — Velocity (or momentum for a fixed mass) observable

p_n — Precision of the measurement of a position observable for object n

q_n — Precision of the measurement of a velocity observable for object n

m— Mass

$\sim$— A relation of magnitude similarity

h— Planck's constant

i— The imaginary number

$\hbar$— Equal to $h/2\pi$

[Comp]— EPR's completeness criterion

[Real]— EPR's reality criterion

[C]— The proposition that QM is complete

[S]— The proposition that conjugate quantities can be simultaneously real

$\neg$— Negation

$\rightarrow$— Implication

$\wedge$— And

$\vee$— Or

ψ_k and ϕ_r— Wave functions describing a quantum state

S_x— System x

$P(X|Y)$— The conditional probability of X given Y

A— Alice's measurement outcome

$\hat{a}$— Alice's measurement setting

B— Bob's measurement outcome

$\hat{b}$— Bob's measurement setting

λ— A full specification of the pre-measurement state

HVT— Hidden variables theory

$(+)$— A spin up measurement for a spin-$\frac{1}{2}$ particle

$(-)$— A spin down measurement for a spin-$\frac{1}{2}$ particle

$C_x\ (a, b)$— The correlation between binary variables a and b according to x

$N(X)$— The number of trials with outcome X

$|x|$— The absolute value of a quantity x

$\vec{a}$— An axis of measurement represented as a unit vector

EPRB— The EPR experiment devised by Bohm and Aharonov

A— The outcome of a photon being absorbed in the EPRB experiment

P— The outcome of a photon passing through in the EPRB experiment

$\langle A, P, P\rangle$— A triple describing the determined state of a photon in the EPRB experiment

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Chapter 1

Introduction

The development of quantum mechanics (QM) in 1925 disrupted the standard ontology and notion of causation of classical physics. This development left physicists and philosophers scrambling to interpret the theory in ways that could restore the coherence between physics, metaphysics and epistemology. The prior coherence of the classical world view, and its tacit support from physicists, meant that the philosophical implications of QM were difficult to distinguish and isolate. Heisenberg's 1927 publication on the "indeterminacy relations" solidified the notion that some sort of uncertainty was embedded into QM theory, and perhaps into the world itself, in a way altogether unlike that found in any previous physical theory. Heisenberg himself concluded that QM theory "establishes the final failure of causality."[1]

The purpose of this thesis is to interrogate this claim by deconstructing classical causality and the connected classical intuitions before moving on to consider significant developments over the issues of indeterminacy, completeness and local causality. The first broad goal is to situate the ontological and causal commitments that characterize the classical world view, in order to analyze how they come apart in the quantum regime. These commitments are situated under the heading of what we will call "classical realism." Classical realism is not a particular view that anybody explicitly holds and supports, nor is it a view that can be narrowly attributed in full to any particular theorist. Instead, classical realism should be understood as an umbrella term for a bundle of different metaphysical and epistemological positions and attitudes that were popular up to and around the turn of the 20th century, during the period where the theory of relativity was tying up many of the loose ends of classical physics. The three components of classical realism that will looked at more closely are ontic determinacy, causal determinism and local causality. Ontic

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[1] Heisenberg, Werner. "The Physical Content of Quantum Kinematics and Mechanics" (1927), in *Quantum Theory and Measurement*, ed. John Archibald Wheeler and Wojciech Hubert Zurek (Princeton: Princeton University Press, 1983), 83.

determinacy focuses on the determination of properties of physical objects, causal determinism focuses on the regularity with which causes determine their effects and local causality focuses on how causes and effect are mediated in space and time. It is important to characterize the prevailing attitudes regarding these matters during the late classical time period because these are the attitudes that were confronted by some of the original difficulties associated with the establishment of QM theory.

As a result of the difficulties surrounding QM around the time of its introduction, some physicists started discussing the apparent conflict between the interpretations of QM and 'realism,' but without the philosophical clarity that might be hoped for. This opacity has remained to some extent up until the present day.[2] The most prominent use of the term comes from the famous 1935 paper by Einstein, Podolsky and Rosen (EPR) in which they present what has become known as the "EPR argument." In this paper the authors argue that QM theory is "incomplete," i.e., that it fails to formally capture all of the "real" physical phenomena. This argument is presented as the logical extension of a classical realist interpretation of QM theory. The argument hinges on the authors' classical conception of causality, but this remains somewhat opaque in their presentation. However, even at the time the success of QM theory led many physicists to conclude that it was not QM theory that was found wanting, but classical realism (and perhaps classical causality in particular).

Despite the large number of physicists who were willing, perhaps not surprisingly, to abandon their philosophical commitments rather than their physical theories, Einstein felt committed to the incompleteness of QM up until his death, although his particular reasons for this shifted somewhat throughout his life. As we will see, the argument given by EPR in 1935 was importantly different from the worries about the incompleteness of QM that Einstein maintained afterwards. Yet, it is clear that his worries captured the conflict between the classical world view and the new theory. Post-EPR, Einstein was able to home in on local causality as a fundamental

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[2] In "Against 'Realism'," Travis Norsen gives a number of examples. Norsen takes exception to use of the term 'realism' in physics, arguing that it has been used vacuously in the literature ever since the EPR argument and Bell's response. He considers whether 'realism' might plausibly mean naive realism, scientific realism, perceptual realism or metaphysical realism, and concludes that it has not plausibly referred to any of these ideas consistently. See:

Travis Norsen, "Against 'Realism'," *Foundations of Physics* 37, no. 3 (2006): 312-313, doi:10.1007/s10701-007-9104-1 assumption for the incompleteness argument. Einstein can thus be understood as a sophisticated representative of the classical world view.

The themes of both realism and completeness were picked up again in the early 1960s by John Stewart Bell in his responses to the EPR argument. Bell's work and the experiments that followed provide some crucial insight into how to resolve some of the metaphysical dilemmas identified by Heisenberg and EPR. As I will show in the fourth chapter, Bell argued that some of EPR's classical commitments about causality were demonstrably false. I will argue that Bell's response picks out in particular the classical commitment to local causality as demonstrably false. Consequently, any realist philosophy of science must abandon the classical commitment to local causality. I will argue that two other classical metaphysical commitments that have often been thought to be falsified by QM or by Bell's proof, *ontic determinacy* and *causal determinism*, have been incorrectly identified as such. The realist philosophers of science who find themselves committed to ontic determinacy and causal determinism may find, however, that these commitments come with baggage that they might not want to accept. In any case, local causality has been taken off of the table.

Chapter 2

The Quantum Paradigm Shift

2.1 Introduction

Thomas Kuhn introduced the idea of the paradigm shift in science. For Kuhn, a paradigm shift occurs when radical revision in theory, practice and/or world view is required in order for cutting edge science to include and consolidate anomalous phenomena. QM can be understood as new paradigm, radically revising classical physics.[1] This paradigm shift was a direct consequence of the inability to interpret QM's unique formalism from a classical world view. In §2.2, we will characterize classical realism as a set of metaphysical and epistemological positions that are inconsistent with the QM paradigm. Following this, we will consider a kind of toy (and somewhat cliché) thought experiment involving billiard balls, in order to demonstrate how the classical realist would interpret the physical theory that explains and predicts how the balls interact and move over time. This interpretation will highlight how some of the classical realist assumptions are applied in practice and what some of the vulnerabilities for interpreting the QM paradigm classically might be. In particular, we will find that interpreting *probability*, which is characteristic of predictions in QM, is one of the first roadblocks. This survey will give us the basis on which we can consider some of the early quantum phenomena with classical intuitions.

Next, we will consider the formalism developed to make sense of this phenomena. Some of the conflicts with interpreting probability will be more apparent on introducing the formalism, but more profound issues of interpretation will only become apparent when we introduce Heisenberg's indeterminacy relations in §2.4. The indeterminacy relations will build on some of the epistemic concerns that are introduced in §2.3 and it will become evident when we go on to examine the EPR paper in

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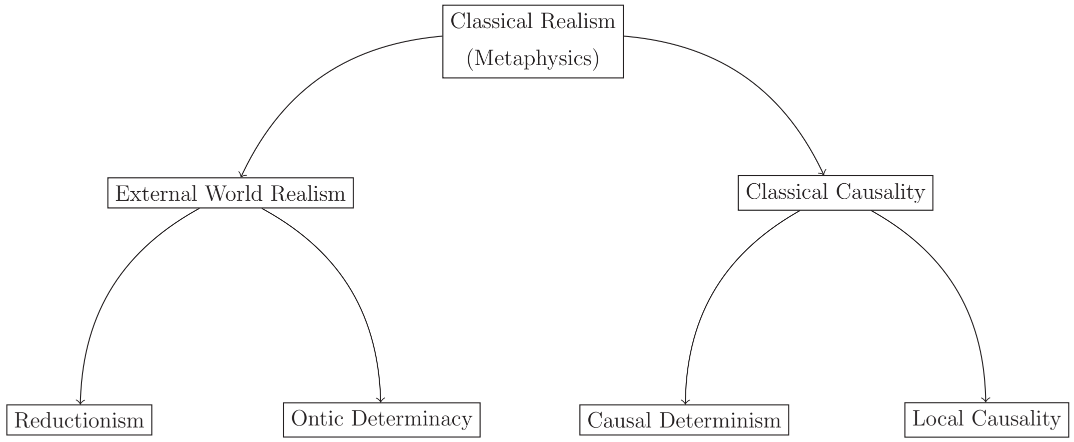
[1] Thomas S. Kuhn, *The Structure of Scientific Revolutions* (1962), (Chicago: The University of Chicago Press, 1970), 88.

chapter three that the indeterminacy relations are foundational for the incompleteness argument, which will bring the clash between the classical paradigm and the quantum paradigm to a climax.

2.2 The Classical Paradigm

Classical realism can be understood as a bundle of metaphysical and epistemological positions that share certain fundamental assumptions and are characteristic of many pre-QM philosophies of science. We can characterize the ontology of classical realism as grounded in commitments to external world realism and classical causality. These can be further subdivided into the components of reductionism, ontic determinacy, causal determinism and local causality.



The first and most apparent commitment of classical realism is external world realism. External world realism is the metaphysical position that the external world exists independent of any mind. For an external world realist, the objects and processes that we experience correspond in some significant way to objects and processes of the world, such that these objects and processes would exist even if there was no experience of them. These objects and processes are space, time, matter, energy (forces, fields, etc.) and their relations. One component of this is reductionism. Reductionism is the tenet that objects in the world are composed by other objects in a reductive way: large objects are just arrangements of smaller objects. Reductionism can also be applied to processes or interactions. Any interaction that takes place between two objects composed of smaller elements can be described in a way that refers to the process happening at a smaller scale.

The second component of external world realism is ontic determinacy. This component assumes that there are some properties of objects, notably, position and velocity, that are always determined, that is, they always have a well-defined value in nature. It is important to keep in mind that, as a metaphysical principle, this does not necessarily mean that a determined property in nature must be determinable for us in the epistemic sense. The terms ‘indeterminacy’ and ‘uncertainty’ tend to be used interchangably. I will use ‘indeterminacy$^{\mathrm{O}}$’ to refer to a metaphysical fact about the world, i.e. the negation of ontic determinacy, and ‘indeterminacy$^{\mathrm{T}}$’ as a fact about the structure of a theory, i.e. whether it ascribes determined values to all elements at all times.[2] I will use ‘uncertainty’ to refer exclusively to the epistemic state of not knowing with certainty. Likewise we might speak of indeterminacy of an element in a formal theory (indeterminacy$^{\mathrm{T}}$), but this doesn’t necessarily imply ontic indeterminacy (indeterminacy$^{\mathrm{O}}$) unless the theory is specifically interpreted in this way.

The second commitment of classical realism is classical causality. Causality is concerned with the processes in which causes bring about their effects. The first component of classical causality is causal determinism. This is the position that every event is uniquely and necessarily determined by the preceding conditions that led to it. The main idea here is that the laws of nature are not stochastic at the most fundamental level. While there may be laws that behave statistically such as those of statistical mechanics, these laws can, at least in principle, be reduced to non-statistical laws at a smaller scale. Matter interacts with regularity. Given the same kinds of matter, with the same properties, in the same relations with one another, the system will evolve in the same way. For the causal determinist, if the universe had a rewind button, it would, assuming ontic determinacy, play forward again the same way every time.

It is important to note the relationship between ontic determinacy and causal determinism. Strictly speaking, it would be possible that there could be causal determinism without ontic determinacy, but this would only be possible if the properties

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[2] A theory that ascribes determined values to all elements at all times is *determinate* and a theory that is not determinate is *indeterminate*.

that were not determined played no causal role, or if they were at least always determined whenever they were implicated as a cause. In the first case, we would think that a property that plays no causal role is not a material property at all, since the (possible) causal role of a property is fundamental to its identity as such. In the second case, it seems oddly conspiratorial to suggest that properties might blink in and out of existence, but always happen to be in existence when they are required to play a causal role. So, generally, we should expect that ontic determinacy is required for causal determinism. However, it is clear that causal determinism is not required for ontic determinacy, because we can easily construct a system with determinacy$^{\mathrm{T}}$ that evolves non-deterministically (stochastically), and there is prima facie no reason to believe (above and beyond a commitment to causal determinism itself) that the universe could not be such a system. This asymmetry means that any doubt about ontic determinacy will also imply doubt about causal determinism.

Similarly to ontic determinacy, it does not follow from causal determinism that we will be able to construct a valid deterministic theory for everything, or that we have epistemic access to the deterministic elements that must feature into such a theory. Likewise, we might speak of a formal theory being indeterministic (indeterminism$^{\mathrm{T}}$), but this doesn't necessarily imply causal indeterminism (indeterminism$^{\mathrm{O}}$) unless the theory is specifically interpreted in this way.

The second component of classical causality is local causality. This is the idea that causality takes place through a series of interactions that are extended contiguously in space. The first articulation of local causality is found in the *principle of local action*,[3] stating that "influence between events in separated regions must be mediated by some event(s) in the intervening regions."[4] The principle of local action forbids there being "action at a distance." The first instances of this principle took the form of insisting that causality is reducible to the collisions of matter. Such a characterization cannot

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[3] Isaac Newton, in light of his theory of gravity, warns against a violation of this principle. In his letters to Bentley (1692/93), he writes that "It is inconceivable that inanimate Matter should, without the Mediation of something else, which is not material, operate upon, and affect other matter without mutual Contact." See:

I. Bernard Cohen (ed.), *Isaac Newton's Papers & Letters on Natural Philosophy* (Cambridge: Harvard University Press, 1978), 302.

[4] Joe Henson, "Non-Separability Does Not Relieve the Problem of Bell's Theorem," *Found Phys* 43, no. 8 (2013): 1011, doi:10.1007/s10701-013-9730-8

make sense of the gravitational force or of the electromagnetic force. The introduction of field theory in physics allowed us to replace this crude characterization with a more accurate one: causality is extended contiguously in space by the interactions of both matter and fields. After the introduction of field theory there was no longer any need to invoke action at a distance in order to explain the gravitational force or the electromagnetic force.

The special theory of relativity put additional constraints on causality.[5] According to the special theory of relativity, no object or force can travel faster than the velocity of light.[6] This leads us to a more refined *principle of locality*, stating that "there is no superluminal [faster than light] influence."[7] It is worth noting that these two principles, the principle of local action and the principle of locality, are conceptually distinct. This is because the principle of local action does not imply that there can be no superluminal influence, all that is necessary for this principle is that the influence has contiguous causal extension over space-time. Likewise, the principle of locality does not imply that any subluminal causal influence must have extension over space-time; for the principle of locality, there could still be subluminal action at a distance. Since both the principle of local action and the principle of locality are satisfied by relativistic field theory, we can consider local causality to be the conjunction of these two principles.

Reductionism, ontic determinacy, causal determinism and local causality comprise

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[5] Despite the theory of relativity refuting Newtonian mechanics, it was not until the universal application of the concept of 'field' in general relativity to the forces of classical mechanics that the principle of local action (accepted by Newton himself) was grounded in physical theory rather than in metaphysics. While the special theory of relativity refuted the classical (meta)physical notion of absolute time or spatial reference frame, it was nevertheless in keeping with the rest of the classical world view as we have considered it. The introduction of relativity may be considered a paradigm shift in its own way, but not with regard to the concepts that we focus on.

[6] In his book *Relativity: The Special and General Theory*, Einstein writes that "[t]he success of the Faraday-Maxwell interpretation of electromagnetic action at a distance resulted in physicists becoming convinced that there are no such things as instantaneous actions at a distance (not involving an intermediary medium) of the type of Newton's law of gravitation. According to the theory of relativity, action at a distance with the velocity of light always takes the place of instantaneous action at a distance or of action at a distance with an infinite velocity of transmission. This is connected with the fact that the velocity c plays a fundamental role in this theory." See:

Albert Einstein, *Relativity: The Special and the General Theory* (1916), trans. Robert W. Lawson (Project Gutenberg, 2004): Chapter 15, <http://www.gutenberg.org/files/5001/5001-h/5001-h.htm>.

[7] Henson, "The Problem of Bell's Theorem," 1011.

the metaphysical components of classical realism. The epistemology of classical realism usually contains elements of both rationalism and empiricism, where rationalism captures the desired rigor of theoretical reasoning and empiricism captures the focus on observation as the basis to which theories are tested. Classical realism is often characterized by an optimistic epistemic attitude that the world is orderly and knowable. This is importantly related to the metaphysical components just outlined—such a metaphysics allows us to, at least in principle, know the world and learn of its regularities. The metaphysical and epistemological components of classical realism allow us to see how this bundle of philosophical principles came together to provide support for scientific reasoning while also providing an explanation for the success of science. Classical realism and classical physics were thought of as mutually reinforcing. Classical realism provided the philosophical basis on which classical physics could be realized, but it is the success of classical physics that lent credence to classical realism as a plausible philosophy. This can be illustrated by analyzing an example of how we specify a classical system and describe how it evolves.

We can use an example of a billiard ball table system in to illustrate the kind of physical analysis that is characteristic of the classical paradigm. Suppose that, for such a system, all of the locations of the balls on the table were known, as well as their radii, their mass and distribution of mass, the angle of the surface plane of the table, the coefficient of friction of the surface felt, etc. Now suppose that a measurable force was exerted on the cue ball, causing it to move in a particular direction where it would be sure to come into contact with the other balls and/or edges of the table. These facts would give us a complete description of the state of the system (where the system consists in the totality of objects under consideration) and would be enough to calculate the initial velocity of the cue ball. Principles like the conservation of momentum show how contact between balls would alter their positions and velocities, until all of the balls slow and eventually come to a stop due to air resistance and friction. We could even show, in theory, how the energy injected into the system by the initial disturbance was conserved by being dispersed into heat and sound. In this example, given a complete description of the initial conditions of the system, we would be able to calculate precisely what all of the properties of all of the balls would be at any future time.

This example illustrates how classical mechanical systems evolve deterministically over time. However, were we to set up a billiard ball table in real life, our predictions of the evolution of the system would not be perfectly precise, but would employ statistical errors. This is primarily for two reasons. First, our predictions would include statistical errors because the specification of the initial state of the experimental system would not be perfectly accurate or precise, likely due to limitations of our measuring devices.[8] Second, our predictions would also include statistical errors because our predictive model would be an idealization. Our predictive model would probably ignore physical influences that we know exist and it would probably also employ false simplifications. However these idealizations are not cause for worry because these ignored influences and false simplifications are considered negligible (for our predictive purposes and also compared to the precision of our measuring devices). We justify these idealizations because, despite the inclusion of statistical error, these models make accurate predictions.[9]

In other words, in real life, there are a number of influences that will be physically relevant in a billiard ball table experiment but that have to be ignored or merely estimated. For example, since every massive object in the universe exerts a gravitational force on every other, then these combined forces will be exerting influence on the billiard balls throughout the experiment. These forces, however, are very small and cancel one another such that the overall influence is negligible, as can be shown by the fact that our predictions are accurate despite ignoring them. Likewise, the resistance that moving balls experience is not perfectly uniform as is estimated when we assign a value to the coefficient of friction, for example. Not every ball will have exactly the same weight distribution, their surfaces will not be perfect spheres, etc. In each of these cases we find that the influences can be estimated or ignored, and we are generally happy to make such idealizations when our models accurately predict our observations.

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[8] Any limitations of our measuring devices would likewise imply that we could not measure our outcomes with perfect accuracy or precision as well.

[9] The technical distinction between accuracy and precision is important. Accuracy describes the closeness of measured values to a real value. Precision describes the closeness of measured values to other measured values. Consider a stone that weighs 100.00 kg. A scale that gives three separate measurements of 99 kg, 98 kg and 101 kg is more accurate, but less precise, than a scale that gives measurements of 97.08 kg, 96.98 kg and 97.04 kg.

In addition to these kinds of idealizations there is also the issue of disturbance as a result of measurement. The classical treatment of the measuring device as something that does not interfere with the phenomena is itself an idealization. As with the noted previous influences, this idealization is thought to be justified, since the influence on the system due to performing measurements is taken to be either non-existent, negligible or easily accounted for in our theory. From the perspective of the billiard table this is easy to understand. A number of the factors that we will take into account in our calculations can be measured before setting the table to an initial arrangement, and we do not expect that, for example, weighing the billiard balls will change their mass, or that measuring the coefficient of friction of the table with change that value, and so on. Measuring the geometrical relationships between the balls during and after the experiment is likewise considered to exert no influence on the experiment. Taking a video of the table from above could allow us to easily track the spatial relations between the balls over time without changing how they move if the camera had never been there. This idealization is one that will be bought into question in when making measurements on quantum systems.

In real life experiments, our specification of a system will usually have errors in accuracy, incomplete precision and our predictive models will usually have some idealizing assumptions. In the case of statistical errors, we use statistical theory to explain how these errors can propagate and compound as we continue to compute how a system will develop from an imprecisely specified initial state. Given such errors, it follows that our predictions will take on a statistical form. Given that errors propagate and compound, there is a practical problem that predictions of how sufficiently complex systems evolve can be extremely sensitive to errors in the specification of the initial state. The modeling of how sufficiently complex systems evolve can therefore be very difficult if there is any degree of error, even small ones, in our specification of the initial state. The same problem of addition and compounding of error occurs likewise for any idealized assumptions that we are making that in practice have only small effects in early stages of our experiments.

This classical treatment of probability is crucial to understanding the paradigm shift from the classical world view to a quantum mechanical one. It follows from causal determinism that, if we were to have a completely accurate and precise set of initial conditions, and we had predictive models with no idealizing assumptions, then we could, ignoring computational complexity, predict with certainty how any system would evolve over time and have no statistical distribution in our predicted outcomes. In other words, if we were omniscient observers of the world then we would have all of the necessary information to calculate the evolution of any system in the universe over any period of time. This view was characterize by the French physicist Pierre-Simon Laplace, who wrote in *A Philosophical Essay on Probability* (1814) :

> We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.[10]

Unlike Laplace's intellect, we are, evidently, not omniscient and not without computational limitations. While in practice perfect specification of a system may be impossible to achieve, this is nevertheless to be aspired to in our construction of experiments, models and in our general scientific practices. This is because, according to the classical world view, the real initial conditions are perfectly determined, and idealizations merely ignore small influences that are, in principle, possible to account for to the highest degree of specificity. Thus, the use of probability in classical physics is merely to account for our epistemic uncertainty and is in principle eliminable.

The billiard table illustrates how classical theory is determinate$^{\mathrm{T}}$ (properties of objects are always determined) and deterministic$^{\mathrm{T}}$ in the sense of Laplace. While formal *in*determinacy$^{\mathrm{T}}$ and *in*determinism$^{\mathrm{T}}$ do not imply *in*determinacy$^{\mathrm{O}}$ and *in*determinism,$^{\mathrm{O}}$ the story regarding determinacy$^{\mathrm{T}}$ and determinism$^{\mathrm{T}}$ is a bit different. When our best theories about the world are determinate$^{\mathrm{T}}$ and deterministic$^{\mathrm{T}}$ then this seems to justify and reinforce a metaphysical commitment to determinacy$^{\mathrm{O}}$ and determinism$^{\mathrm{O}}$

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[10] Pierre Simon LaPlace, *A Philosophical Essay on Probabilities* (1894), trans. F.W. Truscott and F. L. Emory (New York: J. Wiley, 1902), 4.

for both empiricist and rationalist epistemologies, even when these are not strictly implied.

Less transparent in the billiard table example are reductionism and local causality. However, we can use Laplace's intellect to understand both of these components of the classical world view as well. Classically, with regard to reductionism, Laplace's intellect could use the properties and interactions of all of the atoms involved in the billiard table system in order to make the same predictions as can be made on the macro scale. Indeed, strictly speaking, if Laplace's intellect were required to make predictions with complete precision and perfect accuracy then it may be required to do this in order to avoid any idealizations that are made by our theories governing macroscopic interactions. Thus, we can see how reductionism is present in analysis of classical systems.

With regard to local causality, it tells us on the one hand that all of the interactions between objects and forces in the billiard table system that Laplace's intellect would make use of would be mediated in space and time. On the other hand local causality tells us, as a consequence of the relativistic introduction of the velocity of light as an upper limit for matter and forces, that Laplace's intellect would not have to consider information about objects and forces contained in systems that are sufficiently far away from the billiard table. We will find later in the thesis that local causality becomes more central for consideration of the quantum paradigm shift. Historically, though, it was the relationship between ontic determinacy, causal determinism and their formal counterparts that were at the forefront of reflections on the inadequacy of classical realism to interpret QM theory.

2.3 The Quantum Paradigm

The most complete and historically influential axiomatic formulation of QM was introduced by John von Neumann in 1932.[11] Von Neumann's formulation was a significant step forward among the attempts to unify, in a mathematically rigorous way, the disparate theories and formalisms employed to explain various QM phenomena. While

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[11] Max Jammer, *The Philosophy of Quantum Mechanics* (John Wiley & Sons, 1974), 21.

the details of each of its five axioms are beyond the scope of this analysis, it is worthwhile to consider the primitive concepts of this formulation in order to understand where QM departs from the classical paradigm when it comes to an analysis of probability in the predicted outcomes of measurement. We will next see how experiments can be constructed in QM. A straightforward construction of an experiment involving three measurement devices will be sufficient to show how the empirically-vindicated predictions of QM clash with classical intuitions. It will become apparent that some of the idealizations that are classically valid become problematic at the quantum scale, particularly concerning the interaction between the phenomena under observation and our measuring devices. The von Neumann formalism takes the concepts of *system*, *observable*, and *state* as primitive.[12] A primitive concept is foundational and employed without interpretation in the axioms of the formulation of the theory.

A quantum 'system' is just a portion of the world under study. It at least consists in the particles that are under analysis. The issue of what to include in the system is analogous to the issue of idealization in the classical case. Recall that we recognize that every object in the universe exerts a gravitational force on every other. The mere existence of such a force is not enough in the classical case for all of the objects in the universe to be included in the billiard ball table system. As noted, we regard the influences of all of these other objects as negligible and so we idealize our system as being isolated from those influences. In QM, what must be included in a description of a quantum system in order for such a system to be considered "closed," or whether it is ever possible to describe a quantum system as closed, is problematic.[13]

An 'observable' is a property of a quantum system that can be measured and assigned a value. Quantum observables such as velocity, position and spin are analogous to classical variables, although some behave differently in fundamental ways. For example, quantum systems can have "spin" observables, which are analogous to classical angular momentum, but which differ in that they take just a directional value (of a fixed magnitude) along three orthogonal axes. Spins along an axis can take a positive or negative value (called spin up and spin down respectively), which

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[12] Jammer, *Philosophy of Quantum Mechanics*, 21.

[13] Of course, there may also be difficulties in determining when a system is closed in classical physics as well. This is why Laplace's intellect was required to know everything.

distinguishes direction. It is useful to keep in mind that although these properties may seem analogous to classical properties and are named as such, the analogy breaks down under scrutiny.

The final primitive notion is that of 'state.' A quantum state is a description of a quantum system that gives a probability distribution for each observable at any moment in time. A quantum state is described by the set of unit vectors called state vectors.[14] The Schrödinger wave equation describes how quantum states evolve over time, between measurement. The wave equation is additive across multiple systems. This additivity is captured by the notion of 'superposition.' To understand this notion, first consider a quantum system containing just two particles which have not interacted. This quantum system is reducible to two subsystems, each containing just a single particle. The principle of superposition states that the state vectors corresponding to each of these subsystems can be added together to construct a valid state vector that accurately describes the system containing both particles. Likewise, if we have a valid state vector describing a system that is composed of two subsystems and we also have a valid state vector describing one of those subsystems, then we can algebraically determine the state vector of the other subsystem.

From these three primitive notions von Neumann constructed a set of axioms capturing the theory of QM. In addition to these primitive notions, von Neumann employed other notions which are open to interpretation.[15] For our purposes the most important of these is the notion of 'measurement.' The issue here is that the state description of a quantum system in QM is of a wave, but measurements in QM are measurements of observables. This is in contrast with classical state descriptions which are simply specifications of all of the objects included in the system and their measurable properties. The wave description of a quantum state will sometimes only allow us to predict a measured value of an observable with a probability distribution. The issue is that the probability in our predictions seems to follow directly from the way in which we describe a state. Our attempts to prepare a state in order to

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[14] Each unit vector actually represents an equivalence class of vectors in a Hilbert space defined over the complex numbers. The equivalence classes are defined such that all vectors of differing magnitude but which share the same direction are equivalent. See:

John von Neumann, *Mathematical Foundations of Quantum Mechanics* (1932), trans. Robert T. Beyer (Princeton: Princeton University Press, 1955).

[15] Von Neumann, *Foundations of Quantum Mechanics*, §3.2 and §3.3.

allow us to predict an observable with certainty seems only to push the probability to predictions we might want to make about other observables. We can illustrate the case with an example.

Suppose we have a means to strip electrons from an atom's orbital shell and send them towards an apparatus that uses magnetic fields to send the electrons that have a positive spin along a chosen axis x in one direction and electrons that have a negative spin along axis x in another direction. In this way the apparatus is acting as a state preparation device that separates electrons according to their x-spin. We now have a way of separating electrons with particular spin values along axis x. If we were to place a pair of photographic plates along the paths of the separated electrons, then we could measure the proportion of the electrons that have a positive and negative spin along axis x. In actual experiments we find that the first component of our apparatus will sort the electrons randomly, but equally between the two groups: spin up and spin down along axis x.

Suppose that instead of placing photographic plates along the x-spin up path, we instead placed another spin separation device here that is set to separate incoming electrons according to spin along an axis y that it at an angle θ to axis x. Now suppose we measure the y-spin observable by placing photographic plates along the paths separated by our second device. Since we already know that the value of spin along axis x for electrons entering our second device will be up (because we have prepared them this way), then if axis y is parallel to axis x (i.e. if $\theta = 0^o$ or 180^o), we would be able to predict with certainty the spin value measured by the photographic plates. If axis y is orthogonal to axis x (i.e. if $\theta = 90^o$ or 270^o), then we would have no information available to predict the measured value. We would thus find that the electrons are again sorted randomly, but equally between y-spin up and y-spin down.

Now suppose that rather than a photographic plate we set up instead yet a third spin separation device along the path of electrons that have been prepared with x-spin up by the first device and y-spin up by the second device. If we set up this device to separate the electrons according to x-spin again and measured the result using a pair of photographic plates placed behind it, then we would expect that we could predict with certainty the outcome of the measurement, as we would have had we instead chosen to measure along this axis using photographic plates behind our second device.

But instead, we find again that our third device sorts all incoming electrons into the groups spin up and spin down randomly, but equally. It is as if choosing to separate along an orthogonal axis with our second device destroyed the preparation of the first device. Indeed, this is exactly what is predicted by the evolution of quantum states according to the Schrödinger wave equation. In QM we find that for any possible well-defined state of a quantum system, there are always some observables whose measurement cannot be predicted with certainty.

This result leads us to an important question regarding the nature of our measurement: do our devices disturb the particle in a way that changes its properties? In order to get a value for an observable, any state preparation device and any measuring device included in the experiment setup is required to interact with the system in some way. There is a worry that, for QM, these result in a kind of interaction that constitutes a disturbance of the system itself—limiting what we can know about the system. The nature of this limitation is one place where QM theory fundamentally departs from the classical paradigm.

Recall that probability in the classical paradigm was attributed exclusively to our epistemic uncertainty. This was based on the accuracy and precision of our measuring devices and on the idealizations of our predictive models. Probability in the classical paradigm was therefore in principle eliminable. In QM theory, however, the description given of the initial states themselves (state vectors) can be made very accurately and precisely. Instead, it is the very character of these states, as described by the Schrödinger wave equation, that elements of randomness with regard to predicted measurement outcomes are included. This is a fundamental departure from the classical paradigm. It is clear that probability is embedded into the formal structure of QM theory itself and that the probability found in predicted measurement outcomes is not attributable to disturbance by our devices. If there is a fundamental epistemic problem with disturbance in QM theory then it has been internalized by the structure of the theory. We can thus see why the epistemological confidence of classical realism is shaken.

In the next section we will see how Heisenberg shows that QM theory presents us with fundamental epistemic limitations on the descriptions we can give of quantum systems. These epistemic limitations imply that we can never give a description of

Interpretation	**Indeterminacy$^{\mathrm{T}}$ eliminable in principle?**	**Why?**
Soft Epistemic	Yes	b/c ontic determinacy is true
Hard Epistemic	No	b/c hard epistemic limitation
Ontological	No	b/c ontic determinacy is false

Table 2.1: Interpretations of indeterminacy$^{\mathrm{T}}$ in theories of QM

a quantum system in which the value of every observable is precisely specified, even in principle. Therefore, it is the case in QM theory that there is a fundamental indeterminacy$^{\mathrm{T}}$ of some observables. This is in major contrast with the classical realist assumption that we can always precisely specify all properties of our system in principle (because of ontic determinacy). We can interpret indeterminacy$^{\mathrm{T}}$ in QM theory in three distinct ways.

Firstly, we can interpret indeterminacy$^{\mathrm{T}}$ in our theories as a soft epistemic limitation. This is the epistemological attitude of classical realism. The idea here is that, if only we were able identify the relevant features of physical reality to a high degree of specificity, and we were able to construct our formal theories in the correct way, then we would find that the probabilities dissolve and that we are left with a deterministic$^{\mathrm{T}}$ theory. We find in QM theory that probabilities do not dissolve when we completely accurately and precisely specify the initial state of a system and this means that QM theory is indeterministic.$^{\mathrm{T}}$ For QM theory, a soft epistemic interpretation of indeterminacy$^{\mathrm{T}}$ is going to signify that it does not correctly capture the structure of the world.

Secondly, we can interpret indeterminacy$^{\mathrm{T}}$ as a hard epistemic limitation. This interpretation is agnostic about ontic determinacy$^{\mathrm{O}}$ and causal determinism.$^{\mathrm{O}}$ According to this view, formal indeterminacy$^{\mathrm{T}}$ (and indeterminism$^{\mathrm{T}}$) in QM theory does not imply that indeterminacy$^{\mathrm{O}}$ or indeterminism.$^{\mathrm{O}}$ Nevertheless, according to the hard epistemic interpretation, indeterminacy$^{\mathrm{T}}$ is in principle ineliminable from our formal predictions about measurable experimental outcomes because uncertainty is at least a fundamental feature of our epistemic access to the world if not a consequence of the very structure of the world itself.

Thirdly and finally, we can interpret indeterminacy$^{\mathrm{T}}$ as ontological. This is the view that formal indeterminacy$^{\mathrm{T}}$ (and indeterminism$^{\mathrm{T}}$) in QM theory implies that indeterminacy$^{\mathrm{O}}$ and indeterminism.$^{\mathrm{O}}$ It follows for this interpretation that uncertainty is a consequence of the structure of the world.

In a serious way, this issue about the interpretation of indeterminacy in QM theory underlies many of the worries about completeness that will be the main focus of chapter three. The issue of formal indeterminacy will be considered directly in §2.4 where we will deal with Heisenberg's development of the indeterminacy relations (also know as the uncertainty principle). We will see how indeterminacy$^{\mathrm{T}}$ is built into QM in a fundamental way and how it therefore cannot be attributed to the kind of epistemic difficulties that produce uncertainty in classical realism. In short, we will see how Heisenberg argued against having a soft epistemic interpretation of indeterminacy$^{\mathrm{T}}$ in QM theory and how this followed naturally from the way in which indeterminacy is embedded in the quantum formalism as compared to the deterministic formalism of classical physics.

The EPR incompleteness argument, discussed in chapter 3, can be understood as a kind of classical realist push back against Heisenberg's interpretation of indeterminacy in QM theory. EPR do not take the issue of uncertainty to be problematic in itself, but they argue that this uncertainty should lead us (at least initially) to the conclusion that QM is missing some feature of physical reality, the apprehension of which should, in principle, allow us to generate a new deterministic formalism that completely describes QM phenomena. In other words, EPR will argue that QM theory makes some critical idealizations with regard to formal indeterminacy.$^{\mathrm{T}}$ We will need to understand Heisenberg first before we consider the EPR incompleteness argument.

2.4 Heisenberg's Interpretation of Indeterminacy

Heisenberg's indeterminacy relations or uncertainty principle made explicit the paradigm shift away from the classical world view. Heisenberg argued that probability in QM had to at least be interpreted as a hard epistemic feature, contrary to classical realism. The indeterminacy relations set an important constraint for all later interpretations of QM, and it was this constraint that inspired the EPR argument for the incompleteness of QM theory.

The indeterminacy relations were the consequence of Bohr and Heisenberg attempting to analyze the path of an electron being observed in a Wilson cloud chamber. A Wilson cloud chamber is a measuring device that is filled with a vapor made of a mixture of water and alcohol such that any charged particle that moves through this mixture causes the vapor to condense and form a mist that will indicate the trajectory of the particle. The path of the electron through this device made a discrete sequence of droplets, rather than a continuous path of condensation, indicating to Heisenberg that the electron was not moving in a continuous trajectory but was "jumping" through the device. Furthermore, the thick formation of the clouds did not allow for precise measurement of the electron's positions during its movement through the device.[16]

These results, as well as investigations into other experiments, led Heisenberg to question some of the basic ontological assumptions of classical physics as applied at the quantum scale. In 1927, he published his article "The Physical Content of Quantum Kinematics and Mechanics" that first describes the epistemological shift required for a theory of QM. In it, he writes that:

> The mathematical scheme of quantum mechanics needs no revision... But that a revision of kinematical and mechanical concepts is necessary seems to follow from the basic equations of quantum mechanics. When a definite mass m is given, in our everyday physics it is perfectly understandable to speak of the position and the velocity of the center of gravity of this mass. In quantum mechanics, however... we have good reason to become suspicious every time uncritical use is made of the words "position" and "velocity." When one admits that discontinuities are somehow typical of processes that take place in small regions and in short times, then a contradiction between the concepts of "position" and "velocity" is quite plausible.[17]

What Heisenberg suggests here is that we must reconsider our understanding of the relation between position and velocity for quantum systems. In classical mechanics

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[16] Jammer, *Philosophy of Quantum Mechanics*, 57.

[17] Heisenberg, "Physical Content of Quantum Mechanics," 62-63.

any object will have well-defined (precisely specified in nature) properties at every point in time, and we can in theory measure all of these properties (yielding corresponding quantities) with a proper experimental setup. Heisenberg argues that QM works differently. With regard to the discontinuous trajectory of the electron through the Wilson cloud chamber, for example, he notes that each position would in fact be associated with two velocities: one by considering the distance and time taken between it and the previous position, and one by considering the distance and time taken between it and the next position. The typical way of measuring velocity by measuring the tangent of the electron's position over time will not work for a discontinuous trajectory.[18]

Heisenberg moves on to consider how to measure the position of an electron. He argues that in order for the term 'position of the electron' to have any meaning we must specify the experimental conditions under which we can measure this quantity. There are, he notes, "no shortage of such experiments, which in principle even allow one to determine the 'position of the electron' with arbitrary accuracy."[19] However, the accuracy with which we wish to measure this quantity comes with a caveat. Heisenberg informs us that the highest attainable accuracy for observing the position of an electron under a microscope is governed by the wavelength of light with which we use to measure it. Therefore, in order to increase our accuracy we should opt to use light with the smallest wavelength possible. But, Heisenberg argues, because of a phenomenon known as the "Compton effect," whenever a photon from a beam of light is reflected off of an electron in order for us to measure its position using our microscope, we know that the electron has undergone a discontinuous change in momentum as a result of this collision. The magnitude of this change is greater for a smaller wavelength of light because the energy of a photon is inversely proportional to its wavelength. We therefore find that there is a trade off for the precision at which we can define the position of an electron and its momentum (which is a function of its velocity and mass).

Likewise, consider the case of electron spins in an experimental setup similar to one considered in §2.2. The moment that we separate an electron by its spin along

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[18] Heisenberg, "Physical Content of Quantum Mechanics," 63.

[19] Heisenberg, "Physical Content of Quantum Mechanics," 64.

a particular axis x we have, in effect, prepared a new state that the electron is in. We can measure the observable x-spin with a high degree of accuracy by placing photographic plates in the separated paths. In this new state, however, an observable along any axis that is not parallel to axis x cannot be measured with high precision. In order to measure the observable y-spin ($\neq x$-spin) with a high precision we would have use another spin-separation device in order to sort the electrons by their y-spin, but recall that doing this will destroy the state we prepared earlier by separating the electrons by their x-spin, and so the observable x-spin is can no longer be measured with high precision. There is therefore a parallel trade off for the precision at which we can define the x-spin of an electron and its y-spin ($\neq x$-spin). Thus, to ask what the spins along orthogonal axes x and y are for an electron is to disregard the state that the electron may be in at any given time. It is not the case that the electron can be in a state where these quantities are simultaneously measurable with a high degree of precision. For any given state preparation, an increase in the precision of one of these observables comes with a decrease in the precision of the other. Pairs of observables that behave in such a relation are what Heisenberg calls conjugate quantities, and these kinds of relations are the indeterminacy relations.[20]

Suppose that we define quantities p_1 and q_1 as the precision with which the observables $\mathbf{p}$ and $\mathbf{q}$ are known, respectively, where $\mathbf{p}$ and $\mathbf{q}$ represent the position and velocity quantities of a quantum particle (with mass m) as matrices. Heisenberg argues that $p_1 q_1 \sim h$ is a deductive mathematical consequence of the commutation rule $\mathbf{pq} - \mathbf{qp} = -i\hbar$,[21] which holds between all conjugate quantities. The equation $p_1 q_1 \sim h$ shows that there is an upper limit for the precision with which $\mathbf{p}$ and $\mathbf{q}$ can be known. Thus it is necessarily the case in QM, in contrast to classical mechanics, that some properties (observables) of quantum particles, for example position and velocity, cannot be precisely defined simultaneously for any possible experimental setup.

Despite the indeterminacy relations that exist between conjugate properties of quantum particles, it is still the case that every conjugate property can be measured

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[20] Heisenberg, "Physical Content of Quantum Mechanics," 68.

[21] Where the '$\sim$' relation is to be read "of the same order of magnitude as." i refers to the imaginary number, which is defined as the square root of -1, allowing for the extension of the real numbers to the complex numbers. $\hbar$ refers to $h/2\pi$ where h is Planck's constant.

for a quantum particle with complete precision, with the caveat that the property's conjugate cannot be measured simultaneously with any precision at all in this case. More generally, while conjugate properties can be measured simultaneously, there is an upper bound on the total precision at which they can be measured simultaneously. For Heisenberg, this upper bound meant that these properties were not precisely definable simultaneously, for example, for a highly precise measurement of some observable quantity **a**, it did not make sense to even ask what its conjugate quantity **b** was. He took his interpretation of his discovery to reflect an important philosophical conclusion about the nature of causality and the role of physics:

> What is wrong in the sharp formulations of the law of causality, "When we know the present precisely, we can predict the future," is not the conclusion but the assumption. Even in principle we cannot know the present in all detail... As the statistical character of quantum theory is so closely linked to the inexactness of all perceptions, one might be led to the presumption that behind the perceived statistical world there still hides a "real" world in which causality holds. But such speculations seem to us, to say it explicitly, fruitless and senseless. Physics ought to describe only the correlation of observations. One can express the true state of affairs better in this way: Because all experiments are subject to the laws of quantum mechanics, and therefore to [the indeterminacy relations], it follows that quantum mechanics establishes the final failure of causality.[22]

Recall from the previous section our fourfold distinction between interpretation of formal indeterminacy$^{\mathrm{T}}$ in QM theory. This passage shows that Heisenberg thought that we must move beyond the soft epistemic interpretation of indeterminacy, which suggests that not only is there a "real" world that remains hidden (and therefore meaningful to speak of), but that this reality is in principle accessible to us. Heisenberg denies that that this reality is in principle accessible to us, because all experiments are in principle, according to QM theory, subject to the indeterminacy relations. Interestingly, Heisenberg doesn't flatly deny that there is a "real" world in which causality holds, but instead he suggests that continuing to think of QM

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[22] Heisenberg, "Physical Content of Quantum Mechanics," 83.

phenomena in this way is "fruitless and senseless," rather than strictly false. This seems to be based on his (perhaps anti-realist) claims that the goal of physics is just to describe "the correlation of observations," which seems to imply that we shouldn't expect physics to be able to explain what cannot be observed. For these reasons we should interpret Heisenberg as being committed to the hard interpretation of formal indeterminacy in QM theory, which is agnostic about the true metaphysical implications of formal indeterminacy but remains committed to the ineliminability of uncertainty.

2.5 Summary

The quantum paradigm shift was caused by the difficulties in developing a coherent classical realist interpretation of indeterminacy$^{\mathrm{T}}$ and indeterminism$^{\mathrm{T}}$ in QM theory. In particular, it seems that there is some sort of a conflict between QM and the classical assumptions of ontic determinacy and causal determinism, which are no longer reinforced by having determinacy$^{\mathrm{T}}$ and determinism$^{\mathrm{T}}$ in our best and most fundamental (in the reductionist sense) theories. In classical physics, properties are always and everywhere completely determined. Even in the classical case where we have difficulties measuring properties with precision, we assume that our lack of precision is a result of the construction of our instruments, not of the phenomena itself. Since classical realism is reductionist, having uncertainty at the smallest and most fundamental scale makes it a problem for every scale.[23] Even when we consider that there can be emergence of regularity at a higher scale, the problem of uncertainty only ever partially dissipates, rather than disappears entirely. Thus, the interpretation of indeterminacy$^{\mathrm{T}}$ as at least a hard epistemic limitation requires a major shift in the quantum paradigm.

Heisenberg's introduction and interpretation of the indeterminacy relations were the first rejection of the soft epistemic interpretation of indeterminacy in QM theory.

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[23] We can understand the Schrödinger cat thought experiment as playing on this reductionism by constructing a mechanism whereby quantum indeterminacy is scaled up to the state description of whether a cat in a box is alive or dead. See:
Erwin Schrödinger, "The Present Situation in Quantum Mechanics" (1935), in *Quantum Theory and Measurement*, eds. John Archibald Wheeler and Wojciech Hubert Zurek (Princeton: Princeton University Press, 1983), 152-167.

Heisenberg was convinced that this was a hard limitation, but while the indeterminacy relations seemed to show something about the state of affairs in QM research at the time, it was not clear that additional theory or methods could not undermine them in principle. Following Heisenberg's work, there was a fierce series of debates between Einstein, defending a kind of classical realism, and the members of the so-called Copenhagen school.[24] But Einstein was never fully convinced and his next project was to prove that QM could not be "complete," that is, could not represent all of the relevant physical phenomena. Einstein wanted to find a theory of QM that could retain determinacy$^{\mathrm{T}}$ and determinism$^{\mathrm{T}}$ which would be easy to interpret for classical realism. The next chapter will examine these arguments in detail.

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[24] It is oversimplistic to consider the members of the Copenhagen school as having a single coherent interpretation of QM, rather than a collection of interpretations that may have been equally opposed to Einstein. See:

Don Howard, "Who Invented the Copenhagen Interpretation?" *Philosophy of Science* 71, no. 5 (2004), doi:10.1086/425941

Chapter 3

The Incompleteness Argument

3.1 Introduction

While Einstein was forced to admit the fundamental character of indeterminacy$^{\mathrm{T}}$ in QM, he was not convinced that this reflected a hard epistemic limitation or the ontological reality. His challenge to Heisenberg came in a 1935 paper coauthored with Boris Podolsky and Nathan Rosen.[1] In fact, far from posing an epistemological or metaphysical problem for classical realism, Einstein, Podolsky and Rosen (EPR) argue that an examination of the workings of QM, including the indeterminacy relations, provided the resources to show that it missed an important part of reality. EPR did not argue that the empirical predictions made by QM were wrong but that, due to its structure, QM failed to include some aspects of the physical reality in its formalism. They distinguished between the "correctness" of a theory and its "completeness." A theory is *correct* when it makes accurate predictions. In this regard, EPR admitted that QM was correct. A theory is *complete*, roughly, when it includes in its formalism all of the relevant features of reality that determine the behaviour of the phenomena in question. EPR argue that QM theory is incomplete.

For EPR the goal of science is to provide theories that are both correct and complete. A theory will fail to be complete when there is some feature of physical reality that gets 'left out' of a state description in the theory (even if what *is* accounted for compares favorably with observed reality). The fact that QM was probabilistic rather than deterministic, suggested that this might be the case. But EPR went further: they argued that they had found a component of reality that was left out of the theory. They argued, contrary to Heisenberg, that conjugate quantities could be

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[1] Albert Einstein, Boris Podolsky and Nathan Rosen,"Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" (1935), in *Quantum Theory and Measurement*, eds. John Archibald Wheeler and Wojciech Hubert Zurek, (Princeton: Princeton University Press, 1983), 138-141.

simultaneously well-defined, despite the upper limit (imposed by the indeterminacy relations) on the precision with which they could be measured simultaneously.

In §3.2 we will rehearse and logically deconstruct EPR's incompleteness argument. Despite critical uptake of the EPR argument, Einstein was not entirely satisfied with it. In §3.3 we will look at Einstein's later attempts to articulate why QM theory is incomplete. Here, we can read Einstein as unconvinced that QM phenomena cannot possibly be explained classically. We will find that Einstein homes in on local causality as a necessary assumption for the incompleteness argument that remained implicit in EPR. Our exposure to Einstein's epistemology of physics will shed light on why he thought maintaining local causality was important. This discussion will give the reader the opportunity to question the importance of (and justification for) local causality. In chapter four we will find that Bell is able to give a lucid and intuitive account of local causality, but unfortunately for Einstein this will ultimately lead to a proof of Bell's theorem which shows that no correct theory that accurately describes QM phenomena can be interpreted as locally causal. Bell's proves his theorem in his response to EPR's incompleteness argument.

3.2 The EPR Incompleteness Argument

3.2.1 Overview

The EPR paper comprises two sections. In the first section, EPR rehearse what they take to be the orthodox interpretation of QM and they introduce two criteria: the completeness criterion and the reality criterion. In the second section, they introduce a thought experiment to show that, according to the predictions of QM theory itself and contrary to Heisenberg, conjugate quantities must have simultaneous reality and so QM theory is incomplete.

3.2.2 EPR Section 1

In the first section EPR argue that, while empirically "correct," QM theory fails to meet the *completeness criterion*, stating that a theory is complete only if "every element in the physical reality has a counterpart in the physical theory."[2] EPR will

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[2] Einstein, Podolsky and Rosen, "Can Quantum Mechanics Be Considered Complete?," 777.

use this criterion to establish that, if QM is complete, then conjugate properties (like positions and velocity) cannot be simultaneously real, and conversely, if conjugate properties are simultaneously real, then QM cannot be complete.

For EPR, an element is real if it meets the *reality criterion*, stating that "if, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.[3] In other words, when our physical theory includes some variable that is measurable and whose measured value is capable of being predicted with certainty, then that quantity exists as an element of reality. Note the two antecedent conditions in EPR's reality criterion. We must be able to predict with certainty, but do so with no disturbance. For ease of reference:

1. [*Comp*]: A physical theory is complete only if "every element of the physical reality has a counterpart in the physical theory."
2. [*Real*]: "If, without any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of the physical reality corresponding to this physical quantity."

The authors argue that the completeness condition is a necessary condition for a theory being complete. This means that every element of the physical reality must have a counterpart in the physical theory in order for it to be complete. They also argue that the reality criterion is a sufficient condition for the reality of a physical quantity, which means that it is enough that this condition is met in order to qualify as a real quantity (although there may be others ways to qualify as such).[4] EPR will argue that it is possible see, through the use of a thought experiment, that QM theory is incomplete because there is a real quantity that gets left of the theory (according to the reality criterion). EPR argue that QM theory itself implies that such elements exist, but that they remain inconspicuous in the formalism of the theory. Specifically, they will argue that it is the indeterminacy relations of QM theory that mask the reality of conjugate quantities, and they think that these quantities

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[3] Einstein, Podolsky and Rosen, "Can Quantum Mechanics Be Considered Complete?," 777.

[4] Einstein, Podolsky and Rosen, "Can Quantum Mechanics Be Considered Complete?," 777-778.

can be shown to be simultaneously real using QM theory despite the indeterminacy relations. Before making this argument they first summarize what they take the orthodox interpretation of the indeterminacy relations to be:

> The usual conclusion from this in quantum mechanics is that *when the momentum of a particle is known, its coordinate has no physical reality* [indeterminacy$^{\mathrm{O}}$]. More generally, it is shown in quantum mechanics that, if the operators corresponding to two physical quantities, say A and B, do not commute, that is, if $AB \neq BA$, then the precise knowledge of one of them precludes such a knowledge of the other [indeterminacy$^{\mathrm{T}}$]. Furthermore, any attempt to determine the latter experimentally will alter the state of the system in such a way as to destroy the knowledge of the first.[5]

EPR phrase their understanding of the indeterminacy relations in terms of knowledge. It is clear that they do not interpret the indeterminacy relations as reflecting an ontological fact about quantum observables. They also recognize the information-destroying aspect of QM, which they attribute to an alteration performed on the system when we attempt to prepare the system in order to measure it. We should draw our attention to the contrast between this recognition and the no disturbance condition that is an antecedent in the reality criterion. This contrast will be performing much of the work for the EPR argument.

Since the interpretation that conjugate properties are not simultaneously real is part of QM, according to EPR, they conclude that:

> Either (1) *the quantum mechanical description of reality given by the* [Schrödinger] *wave function is not complete* or (2) *when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality.*[6] For if both of them had simultaneous

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[5] Einstein, Podolsky and Rosen, "Can Quantum Mechanics Be Considered Complete?," 778.

[6] The statement "operators corresponding to physical quantities do not commute" is just a technical way of saying, equivalently, that these two quantities are conjugate.

> reality—and thus definite values—these values would enter into a complete description, according to the condition of completeness."[7]

The logical argument in favor of this conclusion is as follows: Suppose that QM is complete $[C]$ and that when the operators corresponding to two physical quantities do not commute the two quantities can have simultaneous reality $[S]$: $[C] \wedge [S]$. It follows that the two quantities which have simultaneous reality will enter into the complete description of the system in the theory. However, the indeterminacy relations in QM precludes such a description. Therefore, it is not the case [that QM is complete and that when the operators corresponding to two physical quantities do not commute the two quantities can have simultaneous reality]: $\neg([C] \wedge [S])$. The disjunction previously stated by EPR follows from applying De Morgan's Law to this negation of the conjuction: $\neg[C] \vee \neg[S]$.[8] They conclude section 1 of their article with this disjunction.

3.2.3 EPR Section 2

In section 2, EPR argue that the claim that QM is complete $[C]$, implies, contrary to the indeterminacy relations, that conjugate quantities can have simultaneous reality $[S]$: $[C] \rightarrow [S]$. It follows logically from this implication that the disjunct $\neg[C]$ is true, that is, QM is incomplete. In order to demonstrate that $[C] \rightarrow [S]$, EPR consider the following thought experiment:

> Suppose that we have two systems, I and II, which we permit to interact between t=0 to t=T, after which time we suppose that there is no longer any interaction between the two parts. We suppose further that the states of the two systems before t=0 were known. We can then calculate with the help of Schrödinger's equation the state of the combined system I+II at any subsequent time; in particular, for any time t>T... We cannot, however, calculate the state in which either one of the two systems is left

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[7] Einstein, Podolsky and Rosen, "Can Quantum Mechanics Be Considered Complete?," 778 (italics theirs).

[8] Note that we will be referring to EPR's (1) as $\neg[C]$, where $[C]$ refers to the proposition "the quantum mechanical description of reality given by the wave function *is* complete," and (2) is likewise $\neg[S]$. Thus, EPR's (1) $\vee$ (2) becomes $\neg[C] \vee \neg[S]$.

> after the interaction. This, according to quantum mechanics, can be done only with the help of further measurements.[9]

After these details EPR explain the technical composition of the Schrödinger wave equation (which we considered in §2.3). The Schrödinger wave equation describes how quantum states evolve over time. What is important for the purposes of their argument is the additive nature of the wave function describing the two-particle system after the interaction. What this means is that the state assigned to the system I+II is the addition of the states describing the systems I and II. Since we know the states of the two systems individually prior to time t=0, then we can add these states together to describe the state of the new system I+II at t=0. As the system I+II evolves during the interval t=0 to t=T according to the Schrödinger wave equation, its partial subsystems will necessarily change as well. After the interaction we will still know the state of the system I+II (the state it was at when t=T), but we will not know the states of the partial subsystems without making new measurements. Given that composition of the systems is additive, we need only, they remind the readers, to measure an observable on one of the partial subsystems in order to know the value of the same observable on the other. After being separated, the partial subsystems and their respective particles are said to be 'entangled,' because knowledge of just one of the states of these subsystems will yield knowledge of the other. Once we know the wave function for the composed system and for one of the individual states, then calculating the other individual state becomes a matter of algebra.

So, EPR use this composable nature of the wave equations to show that, after separation, a measurement might be performed on an observable p, yielding a value p_1 for (sub)system I, in order to know the value p_2 of the same observable for (sub)system II with certainty. In this case, we can say that system II is described by some state ψ_k in which the observable q_2 is known with certainty. If a measurement on a conjugate observable q had been performed instead, yielding a value q_1 for system I, then the value of the same observable for system II would likewise be known to be q_2 with certainty. In this case, we can say that system II is described by some state ϕ_r in which the observable q_2 is known with certainty. Thus, the state of system II can be described by either of the wave equations ψ_k and ϕ_r. When p and q are

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[9] Einstein, Podolsky and Rosen, "Can Quantum Mechanics Be Considered Complete?," 779.

conjugate, then it follows that two conjugate observables of system II could be known, not simultaneously but counterfactually, with certainty. EPR argue that this is in violation of the indeterminacy relations. They argue that since the two systems are no longer interacting, there is no way for the choice of measurement on system I to affect system II. It follows from this, they argue, that any value that *could have* been known with certainty must have been predetermined (ontic determinacy).

To see this, suppose we choose to measure the position p_1 of the particle in system I. We could thereby know the position p_2 of the particle in system II, even though the systems are no longer interacting. It follows that p_2 must have been predetermined prior to the measurement of p_1. Likewise, suppose we choose to measure the velocity q_1 of the particle in system I. We could thereby know the velocity q_2 of the particle in system II, even though the systems are no longer interacting. It follows that q_2 must have been predetermined prior to the measurement of q_1. It follows that both p_2 and q_2 are both predetermined before we choose to perform a measurement on system I.[10] Since the time t that we choose to measure at is arbitrarily far from the time T at which the particles are separated and no longer interacting, it follows that p_2 and q_2 must have been predetermined at time t=T, EPR argue.

Recall that [*Real*] states that "if, without any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of the physical reality corresponding to this physical quantity." Making a measurement on system I, which no longer interacts with system II, meets the no disturbance condition. An appropriate choice of measurements on system I meets the predictive certainty condition for either of two conjugate quantities in system II. Since either conjugate quantity in system II can be predicted with certainty given an appropriate measurement choice on system I, and since this measurement choice cannot influence the predicted results of a measurement performed at system II, it follows that two conjugate quantities are both simultaneous elements of reality. This, EPR concludes, demonstrates that the completeness of QM implies this simultaneous reality: $[C] \to [S]$.

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[10] We assume that this choice is free. We could, for example, let some pseudo-random process nearby determine whether we measure p_1 or q_1. The issue of freedom will be considered when we examine loopholes in the Bell tests in chapter four.

Recall, however, that $\neg[C] \vee \neg[S]$, that is, (1) *the quantum mechanical description of reality given by the wave function is not complete* or (2) *when the operators corresponding to two physical quantities do not commute* [i.e. they are conjugate quantities] *the two quantities cannot have simultaneous reality.* Thus, since it has been demonstrated in section 1 that $\neg[C] \vee \neg[S]$, and in section 2 that $[C] \rightarrow [S]$, it logically follows that QM is incomplete $\neg[C]$.

3.2.4 Summary and Implications

In summary, in the first section EPR rehearsed what they take the orthodox interpretation of the indeterminacy relations to be: that conjugate quantities cannot have simultaneous reality. They conclude that (1) QM is incomplete or (2) conjugate quantities cannot have simultaneous reality: $\neg[C] \vee \neg[S]$. The second disjunct can be understood as following straightforwardly from this interpretation of the indeterminacy relations. In the second section, EPR demonstrate that $[C] \rightarrow [S]$ by constructing and examining a hypothetical quantum experiment where, so they argue, the values of two conjugate observables must be simultaneously well defined, despite not being simultaneously measurable (contrary to the indeterminacy relations). They conclude that QM is incomplete.

EPR anticipated that this would lead some physicists to reject their reality criterion [$Real$], writing: "One could object to this conclusion on the grounds that our criterion of reality is not sufficiently restrictive. Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality *only when they can be simultaneously measured or predicted.*"[11] This is crucial because EPR are not arguing that p_2 and q_2 could ever be known simultaneously with precision, even under the experimental setup that they described. Instead, their argument relies on a counterfactual claim about which measurement is chosen at system I. Since the systems are no longer interacting, the choice of measurement at system I cannot affect what result would be found at system II, so they argue, and so any observable which could be predicted with certainty must be predetermined. In effect, EPR are arguing that the possibility of counterfactual knowledge about conjugate observables implies ontic determinacy.

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[11] Einstein, Podolsky and Rosen, "Can Quantum Mechanics Be Considered Complete?," 780.

But, EPR dismiss this further requirement on the reality criterion as a reasonable compromise since "this makes the reality of P and Q depend upon the process of measurement carried out on the first system, which does not disturb the second system in any way. No reasonable definition of reality could be expected to permit this."[12] Thus, EPR suggested that a rejection of the reality criterion results in an unacceptable notion of physical causality since it would include some sort of action at a distance (a violation of the principle of local action). They did not elaborate on this feature of their view, but they seem to acknowledge that Heisenberg's conclusion that QM theory "establishes the final failure of causality"[13] does follow from its completeness. The difference seems to be that EPR was committed to a classical realist conception of causality and therefore find this conclusion to be unreasonable, whereas Heisenberg seems more willing reject the classical conception of causality. We have also seen, though, that Heisenberg does not argue in particular that we should or can amend causality to include action at a distance, but he might have been willing to entertain the idea.

Contained in the EPR argument is the crucial stipulation that, after interaction, the quantum subsystems can be isolated such that they are no longer able to "influence" one another. The reason for believing that such a stipulation is warranted is based on the presupposition of local causality, which implies that there can be no "action at a distance." Technically speaking, the principle of local action (the first component of local causality) is enough for EPR to provide an argument in favor of this stipulation. They might argue that the quantum systems can be isolated with the use of barriers and measuring devices in order to ensure that no familiar physical medium of interaction could or does exist between the two systems. However, the relativistic introduction of the principle of locality (the second component of local causality), which states that there can be no superluminal influence, makes the justification for this stipulation more compelling. Using the principle of locality, EPR might argue, we can avoid the tricky business of using barriers and measuring devices in order to ensure that no familiar physical medium of interaction could or does exist between the two systems. Instead, using the principle of locality, all that we would

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[12] Einstein, Podolsky and Rosen, "Can Quantum Mechanics Be Considered Complete?," 780.

[13] Heisenberg, "Physical Content of Quantum Mechanics," 83.

have to do to ensure that there is no interaction between the two systems is pull them very far apart so that any possible influence between the two systems would have to be superluminal.

In effect, EPR are able to use the presupposition of local causality to argue that, since there can be no physical medium through which the two subsystems in the EPR experiment could interact, the counterfactual determinacy$^{\mathrm{T}}$ of QM theory implies ontic determinacy$^{\mathrm{O}}$ for the realist (according to the reality criterion). One can falsify this argument in two fundamental ways. First, one can argue that we must deny the reality criterion that allows EPR to derive determinacy$^{\mathrm{O}}$ from counterfactual determinacy.$^{\mathrm{T}}$ This would be done in order to deny determinacy,$^{\mathrm{O}}$ allowing one to maintain that QM theory is thereby complete. Whether this sort of argument might be advanced by an instrumentalist or a self-avowed realist is of no important consequence for us: whatever form of realism one appeals to would not be classical, because it has abandoned ontic determinacy.

The second way that one can falsify the EPR argument is to argue that we must deny local causality. Since local causality is required in order to justify EPR's stipulation that the two subsystems in the EPR experiment can no longer interact, a denial of local causality would in effect argue that, if QM theory is correct, then there must be a (non-local) causal relationship between the two subsystems. This is the line of argument that will be advanced by Bell. However, before we examine Bell's argument for why must abandon local causality, we will look more closely in the next section at Einstein's own commitment to local causality and his discovery that the principle of local action is a necessary assumption for the incompleteness argument. The next section will shed light on why Einstein feels committed to local causality from an epistemological perspective and let the reader consider the validity of this commitment. The next section will also give us some of the tools that we will need in order to understand how Bell constructs his own conception of local causality and why his conception intuitively captures the classical realist conception of it.

3.3 Einstein's Boxes and the Separation Principle

Despite the critical uptake of the EPR argument, Einstein was dissatisfied with the final paper, writing in a letter that "it has not come out as well as I really wanted;

on the contrary, the main point was, so to speak, buried by erudition."[14] Shortly after the publication of the EPR paper, Einstein clarified his own views about why he considered QM to be incomplete. These clarifications relied less on the formal method pursued by EPR and more on what Einstein took his realist commitments to amount to with regard to the structure and interpretation of physical theories. It becomes clear to Einstein that local causality was one of the fundamental assumptions on which his worries about the incompleteness of QM theory rested.

In a 1935 letter to Schrödinger, Einstein considers a thought experiment consisting of two boxes and a ball contained in one of the boxes. He then considers whether an experimenter, ignorant of the location of the ball, could suppose the following state description as complete: 'the probability that the ball is in the first box is $\frac{1}{2}$.' In the letter, Einstein wrote that:

> [O]ne cannot get at the talmudist [Bohr] if one does not make use of a supplementary principle: the 'separation principle'. That is to say: 'the second box, along with everything having to do with its contents, is independent regardless of what happens with regard to the first box (separated partial systems).' If one adheres to the separation principle... the [probabilistic] state description is an *incomplete* description of reality, or of the real states.[15]

If we consider the contents of the boxes to be defined independently from one another, and if their contents are not causally interacting, then we can consider the boxes to be *separated partial systems*. To suggest that a given system is composed of separated partial systems is to invoke the separation principle. Einstein suggests that we must invoke this principle (and therefore make these assumptions) in order to argue that a probabilistic state description is incomplete. He thinks that these assumptions are valid in the case of the boxes and balls, and suggests that they also apply (as a matter of principle) to the separated subsystems of the EPR experiment.

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[14] Einstein, *Letter to Schrödinger* (Jun 19, 1935), quoted in Don Howard, "Einstein on Locality and Separability," *Studies in the History and Philosophy of Science* 16 (1985): 175.

[15] Einstein, *Letter to Schrödinger* (Jun 19, 1935), quoted in Howard, "Einstein on Locality and Separability," 178-179.

The argument for the validity of the separation principle is easily seen in the case of the boxes. Experimenters might find that a great many predictions can be made when they assign a probability value of $\frac{1}{2}$ to each box in this case, or in some variations of such an experiment, but we know that the probability here is attributable to ignorance on the part of the experimenter about which box the ball is in. We know that any chosen box is going to have the whole ball with 50% likelihood and not half of a ball with 100% likelihood, or some fraction of a ball with some uncertain likelihood. We also know that once we see the contents of one box, we can immediately infer the contents of the other box, with respect to its containing a ball or not. Thus, Einstein concludes that the probabilistic state descriptions of the boxes are incomplete descriptions of the real states of the boxes. The real states of the boxes are that one of them contains a whole ball and the other doesn't. It is only our ignorance about which box is which that leads us to assign probabilities to them individually.

While there do seem to be some similarities to the box cases it isn't obvious that invoking the separation principle is justified in the EPR experiment. Einstein's justification for applying the separation principle in the quantum case that the separation principle is necessary for empirical testability in the physical sciences:

> For the relative independence of spatially distant things (A and B), this idea is characteristic: an external influence on A has no *immediate* effect on B; this is known as the 'principle of local action', which is applied consistently only in field theory. The complete suspension of this basic principle would make impossible the idea of the existence of (quasi-)closed systems and, thereby, the establishment of empirically testable laws in the sense familiar to us.[16]

Thus, for Einstein, the concept of a (quasi-)closed (isolated) physical system is necessary for empirical science. A closed system has no causal relationship with any element of physical reality that exists outside of the system. A quasi-closed system has only easily quantifiable causal relationships with elements of physical reality that exist outside of the system. The physical principle that guarantees the existence of

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[16] Einstein, "Quanten-Mechanik und Wirklichkeit" (1948), in *Dialectica*, quoted in Howard, "Einstein on Locality and Separability," 188.

real (quasi-)closed systems is the principle of local action, which, for Einstein, states that any causal influence between spatially distant things cannot be immediate. An action with an immediate effect is, in other words, an action with no intermediate effect. Our definition of the *principle of local action* likewise states that "influence between events in separated regions must be mediated by some event(s) in the intervening regions" and this principle forbids action at a distance.[17]

Einstein suggests that the principle of local action is *characteristic* of spatially separate things, but not *essential*. This highlights two things. First, it highlights that there is an empirical basis on which the principle of local action is founded. This empirical basis is the success of field theory. In field theory, the forces of electromagnetism and gravity are described as fields which assign a quantity (a force vector) to each point in space and time. An influence (force) between objects in spatially separated regions is therefore mediated at each point in between the two as well.

Second, it highlights that separation in space and the principle of local action are conceptually distinct.[18] This distinction is explicit in Einstein's autobiography, where, after giving a description of the incompleteness argument with separated partial subsystems S_1 and S_2, Einstein writes that "one can only avoid [the] conclusion either by assuming that the measurement on S_1 changes (telepathically) the real state of S_2, or by generally denying independent real states to things which are spatially separated from one another. Both alternatives appear to me entirely unacceptable."[19] Both of these denials correspond to distinct ways of denying the separation principle

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[17] It is possible that the principle of locality is implied by Einstein here as well, for example if we interpret 'immediate' as 'instantaneous.' But we do not need to read Einstein this way in order to make sense of what he is saying.

[18] This conceptual distinction is brought to the forefront for Howard, who argues that the separation principle contains both the locality principle and what he calls the separability principle. (Howard, "Einstein on Locality and Separability," 173.) However, Henson argues that Howard uses inadequate definitions for both locality and separability. In the former case, he argues that Howard has erroneously used a weaker definition of locality that involves no superluminal signals rather than causes. In the latter case he argues that Howard's definition of separability has been "confounded with a much weaker principle... the principle of localised events." (Henson, "The Problem of Bell's Theorem," 1011-1012.) Henson argues that it is only this weaker principle that should be attributed to Einstein on a charitable reading.

[19] Einstein, "Quanten-Mechanik und Wirklichkeit" (1948), in *Dialectica*, quoted in Howard, "Einstein on Locality and Separability," 186.

and, respectively, correspond to denying the principle of local action and the "relative independence of [the real states of] spatially distant things,"[20] which we will call *separability.*

Einstein likens the denial of the principle of local action to belief in telepathy, and argues that denying it "would make impossible the idea of the existence of (quasi-)closed systems and, thereby, the establishment of empirically testable laws in the sense familiar to us."[21] With regard to denying this principle, he states that "my physical instincts bristle at the suggestion. However, if one renounces the assumption that what is present in different parts of space has an independent, real existence, then I do not at all see what physics is supposed to describe."[22] Thus, Einstein's commitment to separability appears to be more foundational compared to the commitment to the principle of local action which is just empirically highly regarded. The foundational nature of this commitment is part of Einstein's realist epistemology of physics, as he explains:

> If one asks what is characteristic of the realm of physical ideas independently of the quantum-theory, then above all the following attracts our attention: the concepts of physics refer to a real external world, i.e. ideas are posited of things that claim a "real existence" independent of the perceiving subject (bodies, fields, etc.), and these ideas are, on the one hand, brought into as secure a relationship as possible with sense impressions. Moreover, it is characteristic of these physical things that they are conceived of as being arranged in a space-time continuum. Further, it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things "lie in different parts of space." Without such an assumption of the mutually independent existence (the "being-thus") of spatially distant things, an assumption which originates

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[20] Einstein, "Quanten-Mechanik und Wirklichkeit" (1948), in *Dialectica*, quoted in Howard, "Einstein on Locality and Separability," 188.

[21] Einstein, "Quanten-Mechanik und Wirklichkeit" (1948), in *Dialectica*, quoted in Howard, "Einstein on Locality and Separability," 188.

[22] Einstein, "Quanten-Mechanik und Wirklichkeit" (1948), in *Dialectica*, quoted in Howard, "Einstein on Locality and Separability," 191.

> in everyday thought, physical thought in the sense familiar to us would not be possible. Nor does one see how physical laws could be formulated and tested without such a clean separation.[23]

So, for Einstein, while both the principle of local action and separability are required for empirical testability, it is the latter of these two things which is grounded in the realism of "everyday thought," and which is characteristic of the concepts of physics. This explains why Einstein claims that he does not know what physics is meant to describe if we abandon this separability: he considers this to be a necessary presupposition to empirical investigation in physics. The principle of local action, on the other hand, is not characteristic of the concepts of physics but is characteristic of spatially distant things: an empirically testable, and, for Einstein, an empirically verified result. While it is true, nevertheless, that Einstein thinks that the principle of local action is necessary for empirical testability in physics (by being necessary for the real existence of (quasi-)closed systems), he does not think that it is a necessary presupposition for it.

As we have seen, if Einstein found himself forced to abandon the separation principle, then he would probably be more inclined to abandon the principle of local action compared to separability. Despite this, he thinks that a denial of the principle of local action is a repugnant conclusion because it is necessary for the real existence of (quasi-)closed systems, which is in turn necessary for the empirical testability of laws in physics. Both of these inferences are epistemologically questionable. With regard to the necessity of the principle of local action for the real existence of (quasi-)closed systems, it seems clear that without this principle, absent any other principles concerning the closure of spatial systems, there is a possibility that every spatial system is causally connected to every other. This seems to be a basis for worry about our epistemic access to the regularities of the world.[24]

With regard to the second inference that the real existence of the (quasi-)closed

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[23] Einstein, "Quanten-Mechanik und Wirklichkeit" (1948), in *Dialectica*, quoted in Howard, "Einstein on Locality and Separability," 187-188.

[24] However, it is prima-facie possible that there could be other principles of closure that restore the real existence of (quasi-)closed systems that are discontiguous over space. The principle of locality may, however, place a constraint on our ability to investigate any such principles of closure, reintroducing the epistemic worry.

system is necessary for the empirical testability of laws in physics, I will leave it to the reader to decide whether this claim is justified. When we interrogate Einstein's motivations for these inferences more, the claim seems to be a reflection of the desire for a unified field theory of physics.[25] Recall that one of the main features of classical realism is the optimistic epistemic attitude that the world is orderly and knowable. It seems that Einstein's commitment to the necessity for the (quasi-)closed system is based on a similar hopefulness, but perhaps this is not charitable enough. Hope is, of course, never a good reason to accept the truth of a claim.

Einstein's post-EPR development of the incompleteness argument unveils the principle of local action (as contained in the separation principle) as a fundamental assumption of his view. What Einstein makes explicit over time had remained more implicit in the original EPR argument, but the separation principle, and therefore the principle of local action, can be identified as an assumption in the EPR argument as well. Recall that in the EPR experiment, we are to suppose that "we have two systems, I and II, which we permit to interact between t=0 to t=T, after which time we suppose that there is no longer any interaction between the two parts." This assumes the principle of local action by assuming that we can validly stipulate that there is no interaction between the two subsystems with a sufficient experimental design (i.e. by isolating the two subsystems so that there could be no intermediated influence between the two regions). As noted in the previous section, the principle of locality lends even more justification for such a stipulation, since all that would be required given this principle is that we can separate the subsystems very far apart so that any possible influence between the two systems would have to be superluminal.[26]

We can therefore see that the principle of local action is included but is not made explicit in the original formulations of the incompleteness argument. This implicit premise will be crucial in Bell's analysis of the EPR incompleteness argument. Bell will show that local causality is inconsistent with the predictions of QM, and likewise inconsistent with the results we obtain from empirical investigation.

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[25] Albert Einstein, "Physics and Reality" (1935), *Journal of the Franklin Institute* 221, no. 3 (1936), doi:10.1016/S0016-0032(36)91047-5

[26] Hypothetically, EPR could make the incompleteness argument using just the principle of locality in this latter sense. However, as we have noted, the principle of local action and the principle of locality have been historically intertwined.

3.4 Summary

The quantum paradigm is characterized by the difficulties resulting from the attempt to interpret indeterminacy$^{\mathrm{T}}$ in QM theory classically. Heisenberg's interpretation of the indeterminacy relations called into question the validity of classical realism. The indeterminacy relations, and Heisenberg's interpretation of them, were then used by EPR to generate an argument that QM theory is incomplete. Roughly, EPR used the idea of an isolated (closed) quantum subsystem to argue that a choice of measurement would only alter which quantities we are uncertain of at a distant subsystem (indeterminacy$^{\mathrm{T}}$) but not which quantities were determined (indeterminacy^{0}). Since our measurement is isolated and we are free to measure any observable, it follows that whatever we *could* be certain about in the distant subsystem must be determined (ontic determinacy). But these determined observables do not all enter into a description of the system's state in QM theory, and so QM theory must be incomplete. In other words, there is something real and determined (ontic determinacy) that QM theory misses.

Recall again the three interpretations of indeterminacy$^{\mathrm{T}}$ in QM theory: the soft epistemic interpretation, the hard epistemic interpretation and the ontological interpretation. The first says that while we may have difficulties in practice to access the real and determined features of the world, they are in principle knowable, we just need to use the right method and instruments. This is the classical realist interpretation formal indeterminacy. The hard epistemic interpretation, on the other hand, says that these features may or may not be real and determined, but they are in principle unknowable. The ontological interpretation says that the real features of objects and processes are not always determined.

We can read Heisenberg as arguing against the classical realist's soft epistemic interpretation of indeterminacy$^{\mathrm{T}}$ as applied to quantum phenomena. Heisenberg recognized that this rejection will have ramifications for the classical conception of causality. EPR can be read as providing a counterargument in favor of classical realism. They argue that indeterminacy in QM theory is not a consequence of a failure of epistemology or metaphysics, but rather it is a consequence of QM theory failing to account for all of the real and determined properties of the world in its descriptions of an initial state.

Finally, in the last section we considered Einstein's later development of his argument for the incompleteness of QM theory, exposing some of his realist philosophical commitments as well as the assumption of the principle of local action that is necessary to the incompleteness argument. This assumption will be critically examined by Bell in his analysis of the EPR argument in the next chapter. As we just concluded, EPR were committed to the view that there are determined realities ignored by QM. Bell will respond to their incompleteness argument by trying to conceptualize what a local 'hidden variable' theory of QM would have to look like. As we will see, Bell develops a theorem showing that any local hidden variable theory would make empirically testable predictions that would turn out to be false. Understanding Bell's response will allow us to see that, Einstein's objections not notwithstanding, the local causality component of classical realism must be abandoned. Once we see that local causality cannot be maintained we will be able to judge how much of classical realism, and Einstein's realism, is left.

Before we consider Bell's proof that no local hidden variable theory can be correct, we will first consider how Bell conceptualizes local causality. I will argue that Bell's conception of local causality correctly captures the classical realist, and Einstein's, conception.

Chapter 4

Bell's Theorem and the Problem with Local Causality

4.1 Introduction

The probabilistic state description and the indeterminacy relations of QM challenge the classical conception of uncertainty, for which uncertainty is in principle eliminable with improved instrumentation and more complex calculation. The conclusion of the incompleteness argument claims that there are elements in reality that are not represented in QM theory. A theorist with classical realist sympathies is likely to take refuge in the incompleteness argument. For such a theorist, QM does not give us sufficient reason to abandon the classical conception of uncertainty, or any of the components of classical realism, until it can be more conclusively demonstrated that the soft epistemic interpretation of indeterminacy$^{\mathrm{T}}$ is not at all feasible. The incompleteness argument deserves credit as a cogent criticism of the temptation to take the QM handling of indeterminacy$^{\mathrm{T}}$ as conclusively showing that ontic determinacy is false. Even though the incompleteness argument has some merit in this regard, this chapter will prove that the argument is flawed and that Heisenberg's rejection of classical causality is ultimately correct. §4.3 will lay out the refutation of the EPR argument by John Stewart Bell in detail.

Bell was a mathematical physicist who responded to the EPR argument in 1964 – 29 years after the original EPR paper and 9 years after Einstein's death. Bell's response was both original and scientifically and philosophically fruitful. It's important to read Bell not as demonstrating that QM is complete, but rather demonstrating a flaw in the incompleteness argument. In interpreting him, we must be careful to avoid committing the "fallacy fallacy" which involves reasoning that a conclusion is false because the argument provided in favor of it is fallacious. With regard to the EPR experiment, Bell demonstrates that EPR's claim that there are determined states of the separated subsystems is empirically falsifiable. He argues that, if measurements can be performed on the separated subsystems freely (independently of each other), the predicted empirical correlations between the observed measurement results will differ for QM and any local theory. Thus, Bell demonstrates that the empirical predictions of QM would falsify the reality of these determined states. Since EPR and Bell both take for granted the correctness of QM (as one should), it is shown that the EPR argument is flawed, and that there is no such determinacy.

Taking the EPR argument as valid (in that the conclusion follows from the premises), it follows from Bell's refutation of the EPR argument that at least one of the assumptions on which it is based must be false. In §3.2, we recognized that the EPR incompleteness argument relies implicitly on local causality.[1] Bell homes in on local causality as being demonstrably false by his argument. This is because he demonstrates that it is *any local theory* of QM (i.e. any theory of QM for which the local causality is valid) that makes different predictions than QM.

Following Bell's result, there were a series of experiments known as the Bell test experiments which attempted to demonstrate the result conclusively in light of alternative explanations that are consistent with EPR. These alternative explanations are called the Bell test loopholes, and the goal of the succession of Bell test experiments was to be able to construct an experiment that might rule them out once and for all. In 2015, a number of independent Bell test experiments were published that claim to be completely loophole-free.[2,3,4] These claims will not be scrutinized in detail. Instead, §3.4 will focus on the only remaining loophole that is incapable of being tested empirically: the loophole of superdeterminism.

Superdeterminism is the view that denies that there is any kind of free choice of measurement on the part of the experimenter, an assumption that is necessary for Bell to make his argument. By "free choice of measurement," what is meant is the causal independence of choice of measurement. Recall that, in the EPR experiment, for either subsystem, we might decide before hand to use any number of pseudo-random

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[1] At least, it relies implicitly one of the components. Whichever one we think is most charitable is of no consequence, since Bell will show that both components of local causality are false.

[2] Marissa Giustina, et al., "Significant-Loophole-Free Test of Bell's Theorem with Entangled Photons," *Physical Review Letters* 115, no. 25 (2015), doi:10.1103/PhysRevLett.115.250401

[3] B. Hensen, et al., "Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres," *Nature* 526, no. 7575 (2015), doi:10.1038/nature15759

[4] Lynden K. Shalm, et al., "Strong Loophole-Free Test of Local Realism," *Physical Review Letters* 115, no. 25 (2015), doi:10.1103/PhysRevLett.115.250402

processes near the location of measurement in order to determine which observable will be measured. Thus, in order for an objection to Bell based on superdeterminism to obtain, it cannot be the case that any such pseudo-random process is causally independent from any other at the other sites of measurement. If there is no such independence, then is it possible for local causality to be consistent with the predictions of QM. §3.4 will consider the (im)plausibility of superdeterminism and whether the classical realist should be committed to this. The discussions of the chapter up to and including the section on superdeterminism will allow us to see the general implications of Bell's result. We will see that local causality must be abandoned. Since local causality is an important component of classical realism, this conclusion will frame the discussion of moving beyond classical realism for the quantum paradigm in conclusion of the chapter. Before considering Bell's response to EPR in detail, our first task will be to see how Bell conceptualizes local causality. We will see that Bell's conception captures the local causality of classical realism and of Einstein.

4.2 Local Causality and the Principle of the Common Cause

In order to frame his account of local causality, Bell relies on the concept of the light cone of a space-time event. The light cone is, roughly, the contiguous region surrounding an event wherein anything contained in it can, in principle, have interacted at a luminal or subluminal velocity.[5] The cone that extends in the past (seen in figures 4.1[6] and 4.2[7]) is the region of space-time which contains all of the things that can act as a cause of the event and the cone extending forward (not contained in figures 4.1 or 4.2) is the region that contains all of the things that the event can act as a cause of. The light cone is the upper limit on the size of Einstein's (quasi-)closed system in physics.

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[5] The light cone is defined as a cone due to the four dimensional representation of a sphere in 3d space expanding at constant rate over time (forming a hypercone in 4d space). We can visualize this in 2d space by imagining two orthogonal axis, one of which represents the coordinates of all three spatial dimensions and one of which represents the time dimension. This is what Bell does in figures 4.1 and 4.2.

[6] John Stewart, "La Nouvelle Cuisine" (1990), in *Speakable and Unspeakable in Quantum Mechanics*, ed. John Stewart Bell (Cambridge: Cambridge University Press, 2004), 240.

[7] Travis Norsen, "Bell Locality and the Nonlocal Character of Nature," *Foundations of Physics Letters* 19, no. 7 (2006), doi:10.1007/s10702-006-1055-9

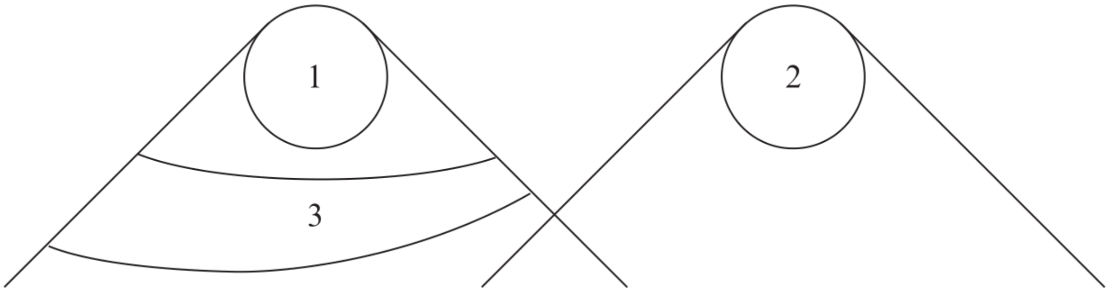

Figure 4.1: "Full specification of what happens in 3 makes events in 2 irrelevant for predictions about 1 in a locally causal theory." (Figure and caption are from Bell, "La Nouvelle Cuisine," 240.)

Bell uses the light cones of regions 1 and 2 in figure 4.1 to illustrate his concept of local causality. He argues that, for any event that happens in space-time region 1, a cause is local when it lies in the light cone of region 1, which consists in the space between the diagonal lines extending downwards from the circle enveloping region 1. Likewise, a cause for region 2 is local when it lies in the light cone of region 2. The justification is that anything outside of the light cone, even if it was traveling towards the space-time region 1 at the velocity of light, could not have reached that space-time region before the event. That is precisely what the light cone is meant to define. If there is an upper bound on the propagation of matter and fields that is equal to the velocity of light, as relatively theory seems to suggest, then it could not be possible for any such propagation to act as a cause. The propagation of matter and fields each obey both the principle of local action and the principle of locality, which we distinguished in §2.2. It follows from this that anything outside of an event's light cone should be treated as a separate system as a consequence of Einstein's separation principle, which contains the principle of local action as an assumption. Bell's framing of local causality in this way is both intuitive and empirically tractable for classical and relativistic physics. This gives us good reason to believe that Bell's conception of local causality is the same as the classical realist and Einstein's conception.

Nevertheless, it is clear that the events in regions 1 and 2 could exhibit correlations of any strength, despite not causally interacting with one another. This is because both of these regions can contain events that have been effected by a common cause that existed in their shared past (in the intersection of their light cones). Bell argues that it is this shared past which must contain any hidden determining variables for the EPR experiment (including the conjugate quantities that EPR argue are determined after separation). However, Bell argues that a complete specification of a region 3, which is a full contiguous section of the light cone in region 1, will leave any information about a cause from the shared past redundant. This is because any cause contained in the shared past will have had effects in region 3 (following the principle of local action) that will then act as a more direct cause of the events in region 1.

Crucially, Bell frames local causality as a property of *theories*. Accompanying figure 4.1, he writes that:

> A theory will be said to be locally causal if the probabilities attached to values of local beables [variables] in a space-time region 1 are unaltered by specification of values of local beables in a space-like separated region 2, when what happens in the backward light cone of 1 is already sufficiently specified, for example by a full specification of local beables in a space-time region 3.[8]

Framing local causality as a formal property of theories means that we can consider the formal structure of any theory of QM in order to see if it has this property. Consider an EPR experiment where Alice and Bob are able to make measurements on a separated pair of particles in two separated locations, respectively. If we consider a full specification of the pre-measurement state of a particle pair λ, as in figure 4.2, where λ in principle only excludes the measurement settings $\hat{a}$ and $\hat{b}$, then local causality entails that "the probability for Alice to obtain a certain outcome A for a measurement along a certain direction $\hat{a}$ is *independent* of the setting $\hat{b}$ and outcome (B) of Bob's experiment."[9] We can write this formally as:

$$P(A|\hat{a}, \hat{b}, B, \lambda) = P(A|\hat{a}, \lambda)$$

Where $P(X|Y)$ is the conditional probability of X given Y. This equation simply says that the conditional probability of Alice's measurement outcome A given her

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[8] Bell, "La Nouvelle Cuisine," 239-240.

[9] Norsen, "The Nonlocal Character of Nature," 639-640.

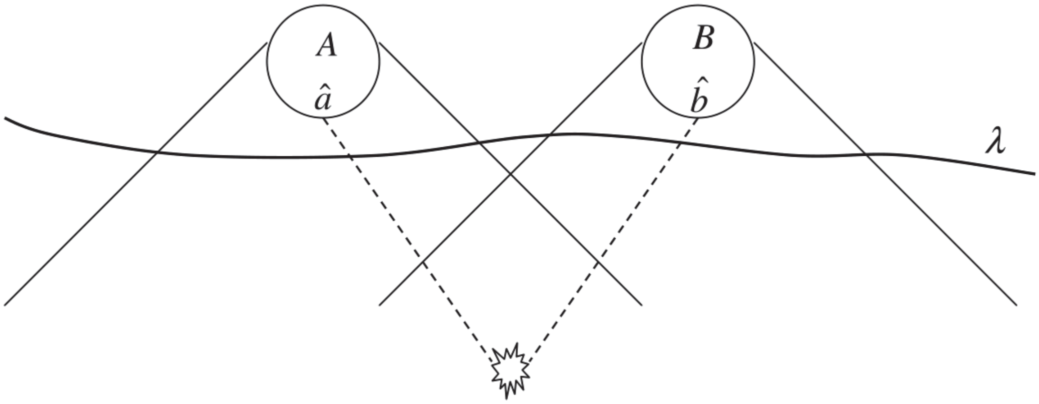

Figure 4.2: The hidden variables which determine the measurement outcomes A and B, for settings $\hat{a}$ and $\hat{b}$ respectively, must have originated in the intersection of the two light cones of the regions of space-time in which the measurements are chosen and taken. If λ is a complete specification of the state of the particle pair, then any prior information becomes redundant. (Figure is from Norsen, "The Nonlocal Character of Nature," 639.)

measurement setting $\hat{a}$, Bob's measurement setting $\hat{b}$, Bob's measurement outcome B, and a complete specification of the pre-measurement state λ, is equal to the conditional probability of Alice's measurement outcome A given her measurement setting $\hat{a}$ and a full specification of the pre-measurement state λ. In other words Bob's measurement setting $\hat{b}$ and outcome B is redundant for Alice's measurement outcome A because any correlation between A and B must be attributable to a common cause, the effects of which on A should be captured in full specification of the pre-measurement state λ.

These conditional probabilities are not empirical probabilities, they are probabilities in the predictions of some formal theory (which may or may not be empirically correct). What is contained in the term λ will depend entirely on the formal theory in question. It must, however, give us a full description of the pre-measurement particle pair and, "on that basis, a definite formal structure by which the probabilities for the relevant possible experimental outcomes can be calculated."[10] For QM theory, for example, λ would contain just the Schrödinger wave equation, because this is a full specification of the pre-measurement state according to this theory. For Bell, any theory that contains any determining "variables" (including both properties and

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[10] Norsen, "The Nonlocal Character of Nature," 641.

relations) in addition to the Schrödinger wave equation is called a hidden variables theory (HVT).

It is the kind of formal structure provided by λ that is subject to the proof of Bell's theorem given in §4.3. Bell's theorem is an inequality regarding the observed correlations between Alice and Bob's measurement pairs for entangled particles. For any theory with the property of local causality, Bell's theorem constrains the possible correlations between $(A, \hat{a})$ and $(B, \hat{b})$ in figure 4.2. Since Bell's inequality is entailed by the property of local causality, any theory which violates Bell's inequality cannot have this property. We will see that QM theory is not a locally causal theory. However, since Bell frames this problem in terms of completeness in response to EPR, it will be useful to briefly visit the problem of Einstein's boxes again in order to see how this example does not violate local causality and how QM theory does.

Suppose that Alice and Bob each have a box. Alice and Bob know that Charlie has placed a ball in just one of their boxes, but neither Alice nor Bob know whose box in particular contains the ball. Alice and Bob therefore propose the following state description: 'the probability that the ball is in Alice's box is $\frac{1}{2}$, and the probability that the ball is in Bob's box is $\frac{1}{2}$.' There is, in this state description, an indeterminacy$^{\mathrm{T}}$ in the predicted location of the ball. Now suppose that Alice and Bob separate themselves very far from one another, and each take their box with them. After they have been separated, but before looking in their boxes, Alice and Bob can each only predict that the likelihood that their box contains a ball is probability $\frac{1}{2}$.

After they have been separated, Alice looks in the first box and finds no ball. Alice now knows that Bob's box contains a ball. She knows this because she already knew that their boxes were related in a particular way: just one contained a ball. Using this information, in addition to performing her measurement by looking at the contents of her box, Alice could deduce what the contents of Bob's box had to be. There is a sense in which Alice's knowledge of the contents of her own box is "entangled" with her knowledge of the contents of Bob's box, but we do not think that anything suspicious has happened here: we do not think that Alice's looking at the contents of her box has caused Bob's box to have a ball. Instead, we know that Alice's ability to deduce the contents of Bob's box from looking at the contents of her own is due to her initial ignorance about what Charlie had done with the ball, before the boxes had ever been separated. Charlie's placing of the ball in Bob's box was the common cause that was responsible for the states of each of the two boxes.

For Bell, the explanation for why this example is not a violation of local causality is straightforward. In this case, Alice and Bob's state description was not complete, in the EPR sense of the term. There was an element of the reality, Charlie's placing of the ball, that was not specified in Alice and Bob's initial state description. Had Alice and Bob conditioned their probabilistic predictions on this event, then they would have described the state of their boxes as such: 'the probability that the ball is in Alice's box is 0, and the probability that the ball is in Bob's box is 1.' In the case where Alice and Bob condition their predictions in this way, it is clear that Bob's prediction of whether a ball is contained in his box is independent of Alice's choice to measure, and of her outcome if she so chooses. Alice's choice to measure and her outcome is therefore redundant for Bob because it does not further specify his predicted outcome of looking in the contents of his box. It is clear that local causality has not been violated.

Now suppose that Alice and Bob start the experiment again with all of the same information. First, they know that each box contains a ball with probability $\frac{1}{2}$. Second, they know that their boxes are related in a particular way: just one contains a ball. In this case, however, there is one crucial difference: these two pieces of information are a complete description of the state of the pair of boxes. Now suppose, again, that Alice and Bob separate themselves and bring their boxes with them. When Alice looks at the contents of her box, and finds no ball, she can predict with certainty that Bob's box contains a ball. In this case, Alice's measurement outcome is not redundant for Bob. If Bob knew the results of Alice's measurement, then he too would be able to predict with certainty that his box contains a ball. Without this information, however, he can do no better than assign the likelihood of his box containing a ball the probability $\frac{1}{2}$. Thus, in this case, local causality has been violated.

This example should draw our attention to the importance of the completeness postulate. The information available to Alice and Bob at each point in the experiment is identical between these two cases. In the first case, when the initial state description was known to be incomplete, conditioning on the complete description made Alice's choice to measure and her outcome redundant for Bob and thus local causality was not violated. In the second case, when the initial state description was complete, conditioning on the complete description did not make Alice's choice to measure and her outcome redundant for Bob and thus local causality was violated.

This example is not proposed with the intention of showing that QM theory is or is not complete. In the case just outlined, we had already clarified that Charlie places a ball in just one of the boxes, and so it was made explicit that the probabilistic state description was incomplete. For QM phenomena, however, there is no analogous Charlie. Bell does not argue that we should reason one way or another whether QM theory is complete. Instead, he offers the following argument: local causality can be framed as a property of theories which places a constraint on the predictions that can be made given a complete state description and a predictive formal structure. Thus, for any proposed formal theory with a proposed complete state description, we can simply ask the question: is it locally causal? If the theory's predictions are of the sort that any information pertaining to an event A which takes place outside of the light cone of a measurement B is made redundant by giving a complete specification of the pre-measurement state that lies exclusively in the past light cone of B, then the theory is locally causal. If the predictions are not of this sort, then theory is not locally causal.

Once Bell has constructed this formal parameter that can be used to determine whether a theory is locally causal or not, then he can determine whether QM theory itself is locally causal. While Bell can show that QM is not locally causal by first deriving his inequality and then showing that the predictions of QM violate this, we can see why it is not. Given just the Schrödinger wave equation as a complete state description, the situation regarding QM theory is similar to the second case of Einstein's boxes: the state description of the combined system implies that a measurement on a subsystem allows us to predict the result of a measurement performed on the other with certainty, and in a way that makes the measurement outcome for the first subsystem not at all redundant for the predicted result on the second, regardless of separation. It follows that QM theory is not locally causal.

But Bell goes even further: he shows that no theory of QM can be locally causal and correct at the same time. He does this by choosing three different measurement settings for Alice and Bob in an EPR experiment in which the two subsystems are separated as in figure 4.2. QM theory, he argues, makes different predictions than any locally causal theory, and if QM theory is entirely correct with regard to the empirical predictions it makes in this kind of experiment, then it follows that no locally causal theory of QM can make entirely correct empirical predictions.

4.3 Bell's Refutation of the Incompleteness Argument

Bell's theorem is an inequality that, for any locally causal theory, constrains the possible correlations between measurement outcomes in systems that are sufficiently separated in space. Given the robust definition of local causality as a property of theories as given in the previous section, Bell is able to show that, in an EPR experiment in which the systems have been sufficiently separated, QM theory makes certain predictions about the correlations between measurements on these two systems that violate the Bell inequality. It follows that any theory that makes the same predictions as QM theory is not locally causal.

Bell's proof is rather technical, but the basic point of his proof can be easily explained. We can take the EPR experiment already described as a basis, but it will be easier to understand the proof using a photon polarization experiment rather than an electron spin experiment. The latter kind of experiment, which is considered by Bell, will be worth considering briefly in order to justify the similarity between it and the photon polarization experiment that will be considered afterwards. Recall that in the EPR experiment we start with two spatially separated systems which are brought together to interact. During the period of interaction we can describe the state of the combined 'ensemble' system by composing the states of the individual systems prior to interaction. Next, after a period of interaction, there is a spatial separation of the systems once again, and we do not have any knowledge about the values of the observables of either subsystem without performing a subsequent measurement. The subsystems are spatially separated but remain entangled. Recall that two partial subsystems are said to be entangled when we know the state of the system that they comprise (the ensemble state) but we do not have any knowledge about the individual states of either subsystem without performing a subsequent measurement, upon which we can immediately deduce the state of both subsystems.

The particular example of an EPR experiment that Bell uses is one which was first used by Bohm and Aharanov,[11] (commonly referred to as the EPRB experiment) in which the two systems each describe a particle with spin-$\frac{1}{2}$ (such as electrons) that are brought together into what is called a singlet state. Recall from §2.3 that the spin observable is the quantum analog of angular momentum, exists along three orthogonal axis and is of a fixed magnitude but can vary in direction. A singlet state is just an ensemble of two particles with a net angular momentum (spin) of zero. The particles are then separated and move in opposite directions. Thus, we know the value of the ensemble state and we are in a position to make spin measurements on each of the entangled subsystems in a way that generates the problem discussed by EPR. When a spin-$\frac{1}{2}$ particle is measured along any axis, the result is either spin up (+) or spin down (−), indicating a direction of spin along the axis chosen for measurement. Thus, spin measurements along an arbitrary axis are binary variables.

Let us now suppose that one of the particles has its spin measured by Alice and the other has a spin measured by Bob. Since total angular momentum of the ensemble system (zero) is conserved after separation, then for any arbitrary chosen axis, a spin measurement taken by Alice allows us to predict that a measurement taken along the same axis by Bob will yield an opposite spin value. When spin measurements performed by Alice and Bob yield the same value, the measurements are said to agree. Measurements which yield opposite results are said to disagree. If an experiment is run over a number of trials then we can take any two binary variables and count the number of times they agree or disagree. Counting the number of agreements and disagreements over a series of trials allows us to measure the *correlation* between two binary variables. Two binary variables are *correlated* when they agree more or less often than they disagree. Two binary variables that agree and disagree equally are *uncorrelated*. When two binary variables always agree or always disagree, they are called *perfectly correlated*. Correlation can be quantified by the following equation:

$$C_e(a,b) = \frac{N(agree)\ -\ N(disagree)}{N(trials)} = \frac{N(+,+)\ +\ N(-,-)\ -\ N(+,-)\ -\ N(-,+)}{N(+,+)\ +\ N(-,-)\ +\ N(+,-)\ +\ N(-,+)}$$

Where $C_e\ (a,b)$ is the experimentally observed correlation ($-1 \leq C \leq 1$) between binary variables a and b and $N(X)$ is the number of trials with outcome X. Note

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[11] Bell cites: David Bohm and Yakir Aharanov, "Discussion of Experimental Proof for the Paradox of Einstein, Rosen, and Podolsky," *Physical Review* 108 (1957), doi:10.1103/PhysRev.108.1070.

that, by our convention, no correlation exists when $C = 0$, perfect correlation exists when $|C| = 1$ and some correlation exists when $0 < |C| < 1$.

We are now in a position to consider the correlations between different spin measurements taken by Alice and Bob. We can also consider the *predicted* correlations between different spin measurements by QM (C_q) and by a locally causal HVT (C_h). Recall that, for Bell, any theory that contains any determining variables (including both properties and relations) in addition to the Schrödinger wave equation is called a hidden variables theory (HVT).

Bell's proof shows that, for some appropriately chosen set of three spin measurements taken by Alice and Bob, $C_q \neq C_h$.[12] In other words, no locally causal HVT can make all of the same predictions as QM. In order to see how Bell shows that $C_q \neq C_h$ for some possible choices of three spin measurements, we need to understand what C_q is for some appropriate choice of spin measurements. Likewise, we need to understand what C_h is and why *any* local HVT predicts this.

Suppose that Alice makes a measurement along the axis $\vec{a}$ (some unit vector) and the observed result is a binary variable a. We know that if Bob makes a measurement along some $\vec{b} = \vec{a}$ and gets an observed result b, $C_q(a, b) = -1$. That is, QM predicts that a and b are perfectly correlated and always disagree. But what if Bob makes a measurement $\vec{b} \neq \vec{a}$? In order to prove what the predicted result will be for QM (in probability form), for any $\vec{b}$, we would need to get into some more technical details of QM theory. Here, it will be informative to consider a more simplified model involving the separation of photons in different directions after what is known as an atomic cascade event. This process is, for the purposes of demonstrating Bell's result that QM theory is not locally causal, structurally identical.

This kind of event can be caused by exposing calcium vapor to lasers tuned to a certain frequency. The electrons in the calcium atoms will "cascade" down to their ground state and will give off light. Each atom will give off a pair of photons traveling in opposite directions, and the polarization of these photons behaves similarly to the spin states discussed earlier.[13] Consider that we have a polarized filter acting as a

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[12] Where C_x here stands for the set of all possible correlations observed/predicted by x. For n number of chosen binary variables, noting that correlations are commutative, $|C_x| = \binom{n}{2}$.

[13] Tim Maudlin, *Quantum Non-Locality and Relativity*, (Cambridge: Blackwell, 1994), 14.

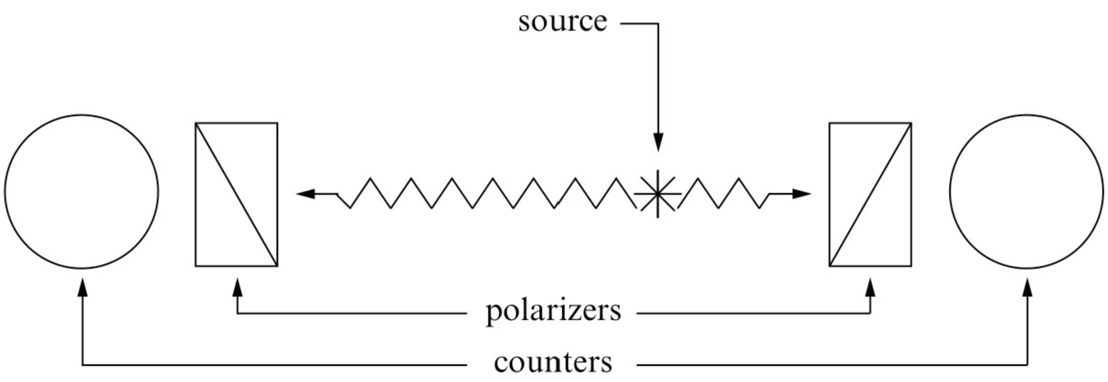

Figure 4.3: "Einstein-Podolsky-Rosen-Bohm gedankenexperiment." Figure and caption are from Bell (1988), pg. 241

measuring device on each subsystem. The photons are either absorbed or pass through the filter, so we have a measurable binary variable that can take the value of absorb A or pass P. Similarly to the electron spin experiment detailed previously, we can define agreement as both photons being absorbed or passed though (same value) and disagreement as one photon being absorbed and the other passing through (opposite value). Correlations of these measurement results are likewise determined according to previous equation.

For photons, if we have a beam of light and a polarized filter, and then if we were to set up another polarized filter after the first, the proportion of light that passes through the second filter is determined by the angle θ of misalignment between the two filters. The proportion of photons that pass through the second filter corresponds to the intensity of the beam of light after passing though the second filter, since individual photons have the same energy.[14] This fact allows us to determine that the proportion of photons that pass through the second filter which is misaligned with the first at an angle θ is equal to $\cos^2\ \theta$.[15] Likewise, for any single photon, the probability that it will pass through the second filter misaligned at angle θ is equal to the proportion of photons in a beam that will pass through, $\cos^2\ \theta$.

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[14] This is because the variable that determines the energy of a photon is its wavelength. A polarized filter does not alter the wavelength of passing light, so any change in the intensity of light passing through a polarized filter is due to the number of photons that pass through.

[15] Due to Malus' Law, which states that the intensity of polarized light passing through a second polarizer is the product of the initial intensity and $\cos^2\ \theta$.

For an individual photon, there seems to be no additional information that would allow us to predict whether it will be absorbed or pass through. The individual photon is thus like the individual electron with regard to its predicted spin measurement: the only way we can predict with certainty an individual electron's spin measurement is if we have prepared its state by sorting a beam of spin up measurements in one direction, setting up an additional spin-separation device that sorts along the same axis of the first, and then placing photographic plates behind the second device in order to make a measurement. The moment we misalign the second measuring device we can only infer about any particular particle that it will be measured spin up with a probability equal to the proportion of particles that get sorted spin up (and likewise for spin down). Recall from §2.3 that when we introduce a third measuring device into an electron spin experiment we see how a misaligned second device seems to erase the preparation obtained from the first. The same effect can be observed when we introduce a third polarized filter in the case of light beams.

The reason that the photon polarization model works is because the two photons moving in opposite directions after a cascade event act as though they have the same polarization. When Alice and Bob's filters are aligned ($\theta = 0$) we find that the photons will always agree (perfect correlation). This is similar to the fact that the spins in the previous model will always disagree (also perfect correlation) when measured along the same axis. If Alice measures a passing photon P with a polarized filter along an arbitrary axis, then, if Bob's filter is misaligned with Alice's by angle θ, and Bob's photon has the same polarization as Alice's, then Bob will measure $P\ cos^2\ \theta$ of the time. In other words, Alice and Bob's photons will agree $cos^2\ \theta$ of the time for any filter misalignment θ. It is therefore easy to see in this model that $C_q(a, b) = cos^2\ \theta$.[16]

Suppose that there are three filter settings that Alice and Bob can choose from: 0°, 30° and 60°. It follows that there are $3^2 = 9$ pairs of measurement settings between the two of them. What is the probability that Alice and Bob agree in each case? Recall that this probability is equal to the correlation observed over a sufficiently large number of trials.

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[16] In the EPRB spin-based experiment discussed prior, the probability that Alice and Bob's spin measurements would agree is $\frac{1}{2} sin^2(\theta/2)$. See:
Travis Norsen, "EPR and Bell Locality," *AIP Conference Proceedings* 844 (2006), doi:10.1063/1.2219369

Alice	Bob	Misalignment (θ)	$C_q(a, b) = cos^2\ \theta$
0°	0°	0°	1.00
0°	30°	30°	0.75
0°	60°	60°	0.25
30°	0°	30°	0.75
30°	30°	0°	1.00
30°	60°	30°	0.75
60°	0°	60°	0.25
60°	30°	30°	0.75
60°	60°	0°	1.00

Table 4.1: Possible measurement pairs for Alice and Bob

Now that we have the values for C_q, let's consider what C_h must be and why. The key lies in the property of local causality (an assumption for EPR), which states that there can be no correlation between the measurement outcomes performed at the two sites that is not made redundant by a full specification of the state at a time after their shared past. This is the formal parameter that captures EPR's claim that the two systems can no longer influence one another. It follows from local causality that any correlations between measurements performed at the two sites must have been determined before Alice or Bob choose to measure. Suppose Bob chooses to measure at $\theta = 0°$. We could predict the quantity that Bob measures in advance by measuring Alice's photon at $\theta = 0°$, because of perfect correlation. It follows that Bob's photon is determined to yield this quantity should Bob choose to measure at $\theta = 0°$. The same argument applies for each of the measurement settings that Bob can choose. Since we can predict in advance the probability of measuring Bob's photon at some binary quantity for any angle, by previously measuring Alice's photon at the same angle, it follows that the result of Bob's measurement must actually be determined.[17]

This determination allows us to put some constraints on the behavior of the photons before reaching their respective filters. First, take the photon traveling towards Alice's filter. It must be the case that, for each possible setting of Alice's filter, it is determined that the photon will either pass or absorb. Since there are only three

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[17] Note that this point about determination has the same counterfactual character as the incompleteness argument attributes to the particles in the EPR experiment. This is by design: the entire purpose of formalizing local causality is to realize this aspect of the incompleteness argument.

$\langle A, A, A\rangle$	$\langle P, P, P\rangle$
$\langle P, A, A\rangle$	$\langle A, P, P\rangle$
$\langle A, P, A\rangle$	$\langle P, A, P\rangle$
$\langle A, A, P\rangle$	$\langle P, P, A\rangle$

Table 4.2: Possible determined states

possible filter settings for Alice, this determination can be represented as a triple of binary values. The triple $\langle A, P, P\rangle$, for example, can represent the determined result that the photon will be absorbed at filter setting 0°, and pass through filter settings 30° and 60°, respectively. There are therefore 2^n possible determined states that the photon can be in with respect to Alice's measurement, where n is equal to the number of filter settings. In this case, since there are three settings, there are eight possible states that the photon can be in. This is likewise true for Bob's photon. Since we know that the photons must agree when Alice and Bob choose the same measurement setting, it follows that the two photons must be in the same determined state with respect to the three possible measurement settings. There are therefore eight possible pairs of determined states for the pair of photons.

For the purposes of measuring correlation, we can cut down the number of possible pairs of determined states in half. This is because Alice's photon (and therefore Bob's photon as well) being in state $\langle A, P, P\rangle$ is equivalent to Alice's (and Bob's) photon being in the inverse state $\langle P, A, A\rangle$. These are equivalent because the correlation depends on whether or not the values measured by Alice and Bob agree or disagree, and this value will be conserved over inverse states. To see this more clearly, suppose you are Alice, and you have randomly selected a filter setting. In the first case, suppose you will measure P, and Bob will measure either P or A. In the inverse case, you will measure A, and Bob will measure the opposite value of whatever he measured as well. Thus, if you will agree with Bob in the first case (Bob measures P), then you will agree with Bob in the inverse case (Bob measures A), and likewise if you found yourself disagreeing with Bob or if you found yourself measuring A in the first case. There are therefore four possible (equivalence classes of) pairs of determined states that Alice and Bob's pair of photons can be in corresponding to each of the rows of table 3.2. These will be referred to by the states listed in the left column: $\langle A, A, A\rangle$, $\langle P, A, A\rangle$, $\langle A, P, A\rangle$ and $\langle A, A, P\rangle$.

Measurement Settings	Misalignment (θ)	P(agree)	P(disagree)
0°, 30°	30°	0.75	0.25
30°, 60°	30°	0.75	0.25
0°, 60°	60°	0.25	0.75

Table 4.3: Probability of disagreement for measurement setting pairs

These four possible determined states exhaust the states that the pair of photons can be in with respect to their correlation. Therefore, in order for $C_h = C_q$, there must be some local mechanism which is able to sort these states into some proportion at separation such that the results of table 4.1 can be recovered. If this cannot be done then it will be shown that there is no valid local HVT of QM, that is, $C_h \neq C_q$. It will be shown that there is no possible such mechanism because there is no possible choice of proportion of these states that can match the outcomes predicted by QM, as shown in table 4.1. When Alice and Bob's filters are misaligned by 30°, then their measurements have the correlation value $C_q = cos^2\ 30 = 0.75$, meaning they agree 0.75 of the time. When Alice and Bob's filters are misaligned by 60°, then their measurements have the correlation value $C_q = cos^2\ 60 = 0.25$, meaning they agree 0.25 of the time. This information is summarized in table 4.3.

We now have enough information to show that $C_h \neq C_q$. Suppose that there is a set of variables $\{a, b, c, d\}$ such that $0 \leq a, b, c, d \leq 1$ and $a + b + c + d = 1$. Let $C_h = a\langle A, A, A\rangle + b\langle P, A, A\rangle + c\langle A, P, A\rangle + d\langle A, A, P\rangle$. When Alice and Bob's measurement settings are at 0° and 30° (irrespectively) their measurements will disagree 0.25 of the time (as can be checked by looking at table 4.3), and the photon pair must be in the determined state $\langle P, A, A\rangle$ or $\langle A, P, A\rangle$ because these are the only states that differ for these two measurement settings. It follows that $b + c = 0.25$. Likewise, when their settings are at 30° and 60° they will disagree 0.25 of the time, and the photon pair must be in the determined state $\langle A, P, A\rangle$ or $\langle A, A, P\rangle$. It follows that $c + d = 0.25$. Finally, when Alice and Bob's measurement settings are at 0° and 60° they will disagree 0.75 of the time, and the photon pair must be in the determined state $\langle P, A, A\rangle$ or $\langle A, A, P\rangle$. It follows that $b + d = 0.75$. Thus, we have the following results:

$$b + c = 0.25$$

$$c + d = 0.25$$
$$b + d = 0.75$$

Some basic algebra will allow us to solve for one these quantities. First, note that $(b+c)+(c+d) = 0.25+0.25 = 0.5$, and also that $(b+c)+(c+d) = 2c+(b+d) = 2c+0.75$. Setting these two results as equal, it follows that $2c + 0.75 = 0.5$. Solving for c, we find that $c = \frac{0.5-0.75}{2} = -0.125$. But $0 \leq c \leq 1$ because c represents the proportion of the time the photons are in state $\langle A, P, A\rangle$. The photons cannot be in this state -0.125 of time, such a quantity is meaningless. This demonstrates that there can be no set a, b, c, d that meets the previous conditions, and so there can be no proportion of pairs of possible determined states that the photons can be in to recover the results of C_q. In other words, we have shown that $C_h \neq C_q$.

While it has been shown that $C_h \neq C_q$, it has yet to be shown that $C_e = C_q$ and, consequently, that $C_e \neq C_h$. Bell takes for granted that $C_e = C_h$. We can interpret this assumption as being the same as the one taken by EPR: that QM is correct. This correctness follows from the empirical work in which the EPR argument and later Bell's argument are taking place. Nevertheless, the EPR experiment that EPR and Bell refer to can be constructed in real life, albeit without the stipulation that the partial subsystems cannot interact, since this is precisely what Bell's result brings into question. Bell's argument that the predicted correlations of QM and of any locally causal HVT are different implies that they can both be tested against reality and that they cannot both be correct. Since Bell published his response to EPR there have been experiments that have been conducted that directly support the conclusion that $C_e = C_q$.[18] These experiments are known as the Bell test experiments.

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[18] For the interested reader, Bell's actual proof, which we have avoided in light of its technical detail (in order to analyze an easier but relevantly similar case), ultimately results in a theorem that states, for any three binary variables, the following inequality holds for any locally causal theory: $C_h(a,b) - C_h(b,c) - C_h(a,c) \leq 1$. This theorem is developed on the assumption that there is perfect correlation at equal measurement settings for Alice and Bob, as was the case in our example. This assumption is not necessary in order to show the more fundamental result that $C_h \neq C_q$. Soon after Bell's original proof Bell's theorem was generalized by the four authors Clauser, Horne, Shimony and Holt (1969), who proved a more general inequality known as the CHSH inequality. Their proof abandons the assumption of perfect correlation at equal measurement settings. Another notable development by Greenberger, Horne and Zeilinger (2007) is a proof that an experiment can be set up in which Bell's result $C_e = C_q \neq C_h$ is proven for a single experimental trial. This is done by using a three particle ensemble, rather than two, in which the expected correlations C_q and C_h differ and each expected correlation is either 1 or 0. See:

In summary, it has been proven that no locally causal HVT can recover the predictions of QM. This can be seen directly from the construction of Bell's argument: he suggests that the EPR argument tacitly endorses the construction of an HVT and that it tacitly assumes local causality cannot be violated, and then he mathematically specifies the general form that *any* locally causal HVT must take and shows that *any* locally causal HVT makes different statistical predictions than QM. While we have avoided the level of technicality at which Bell himself proves this result, we can understand the argument we have just considered as showing essentially this.

First, we logically narrowed the set of possible determined states that the pair of photons can be in with regard to the predicted correlations we should expect to observe. Next, we recognized that any locally causal HVT is committed to the existence of some locally causal mechanism whereby there is a selection of some proportion of these states. Finally, when we attempted to retrieve the predicted correlations of QM, we found that algebraically there was no proportion that can successfully match these predictions. It follows that there can be no mechanism which selects for such a proportion and therefore that there can be no correct locally causal HVT of QM. This argument parallels Bell's own mathematical construction and argument in a less technical manner.

Bell interprets his result to apply not just to locally causal HVTs, but more generally to any locally causal theory. The argument for interpreting Bell's result this generally will be given in the next section. If Bell's result is this general, then this means that local causality as an ontological postulate must be abandoned if QM is correct. We need to be careful with our reasoning here. In our discussions regarding indeterminacy,$^{\mathrm{T}}$ we made sure to recognize that indeterminacy$^{\mathrm{T}}$ does not imply indeterminacy$^{\mathrm{O}}$ (and likewise for indeterminism), so why should we be committed to the claim that non-local causality$^{\mathrm{T}}$ implies non-local causality$^{\mathrm{O}}$? The reason for this is the level of generality at which Bell's proof applies. Crucially, Bell has proven that *any* theory of QM that is locally causal$^{\mathrm{T}}$ cannot be correct, but Heisenberg did

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John F. Clauser, et al., "Proposed Experiment to Test Local Hidden-Variable Theories" (1969), in *Quantum Theory and Measurement*, eds. John Archibald Wheeler and Wojciech Hubert Zurek (Princeton: Princeton University Press, 1983), 409-413.

Daniel M. Greenberger, Michael A. Horne, Anton Zeilinger, "Going Beyond Bell's Theorem," in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, ed. Menas Kafatos (Dordrecht: Kluwer, 1989), 69-72.

not prove that *any* theory of QM that is determinate$^{\mathrm{T}}$ cannot be correct.[19] All that Heisenberg argued is that even if there is a determinate structure of QM, there is a hard epistemic limitation for us in simultaneously accessing all of the determined quantities in the world.[20]

Once we have shown that Bell's proof is this general, and that we should thus conclude that local causality$^{\mathrm{O}}$ is false, then we can consider some of the alternative explanations available. The first, and most obvious, is the assumption made by both EPR and by Bell that QM theory is correct. EPR and Bell take this for granted, but it is worth noting that, up until Bell published his theorem, there had been no attempt to create an EPR experiment that would separate the two subsystems in order to test whether the predictions of QM theory would be correct in non-local conditions. Since Bell's publication there has been a series of Bell test experiments that have intended to prove just this. In 2015, three independent Bell test experiments were published that all confirmed the predictions of QM theory with statistical significance.[21,22,23] The authors of each of these experiments claim that they are entirely loophole-free, meaning that they are each able to simultaneously avoid, by experimental design, criticisms of previous Bell test experiments which might have (somewhat far-fetched but valid nonetheless) alternative locally causal explanations for the measured outcomes.

The only alternative explanation available which is able to save local causality is known as the superdeterminism loophole, and it is empirically untestable. Recall that, in Bell's definition of local causality as outlined in the previous section, we are

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[19] There are counterexamples. David Bohm's 'pilot wave' theory of QM, for example, which has been developed off of the work of Louis de Broglie, is able to exactly recover the predictions of QM theory while being both determinate$^{\mathrm{T}}$ and deterministic.$^{\mathrm{T}}$ Notably, the structure of Bohm's theory is explicitly not locally causal, which of course Bell has shown is characteristic of any entirely correct theory of QM. See:

David Bohm, "A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden' Variables, I and II" (1952), in *Quantum Theory and Measurement*, eds. John Archibald Wheeler and Wojciech Hubert Zurek (Princeton: Princeton University Press, 1983), 369-396.

[20] The non-local structure of any correct theory of QM might give us some indication for why there is such a hard epistemic limitation. There may still be a relativistic limitation on superluminal information gathering, for example, which would make it at best extremely difficult and at worst impossible to ever establish whether there are non-local variables that can be included in the structure of a correct and complete non-local theory of QM.

[21] Giustina, et al., "Significant-Loophole-Free Test with Entangled Photons."

[22] B. Hensen, et al., "Loophole-free Bell inequality violation using electron spins."

[23] Lynden K. Shalm, et al., "Strong Loophole-Free Test of Local Realism."

to suppose that a full specification of the pre-measurement state of a particle pair λ, as in figure 4.2, in principle only excludes Alice and Bob's measurement settings ($\hat{a}$ and $\hat{b}$, respectively). The superdeterminism loophole rejects that we can do this in principle. If we cannot do this, the objection goes, then it is possible for the measurement outcomes as well as the measurement settings of the EPR experiment to be events with a common cause, which also happens to recover the predictions of QM theory exactly. This objection will be considered in detail and rejected as insubstantial in §4.5.[24]

4.4 A Demonstration of Non-Local Causality

In the last section we saw a proof that parallels Bell's reasoning and that concludes that no locally causal HVT of QM can make entirely the same predictions as QM theory. It was suggested that Bell takes his result to be of great generality. For Bell, it is not just that he has shown that no locally causal HVT of QM cannot make the same predictions as QM theory, but he has shown that no locally causal theory of QM at all can. To understand why Bell takes his result so generally, we will need to consider what an HVT of QM amounts to, for Bell.

We also recognized in the last section that Bell's argument (and EPR's) is dependent on the correctness of QM theory in non-local conditions. References to recent and conclusive Bell test experiments have been provided which show that QM theory is correct in non-local conditions. The result that Bell's inequality is violated by QM phenomena in non-local conditions has been scientifically generally accepted since one of the earliest Bell test experiments produced by Aspect et al. in 1982.[25] Since QM theory is correct in non-local conditions, we should understand Bell's result to

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[24] A third alternative explanation would be a kind of anti-realism, the likes of which denies the separability defined in §3.3. The issue of formalizing separability, and whether it is a necessary assumption for local causality, is taken up by Henson. (Henson, "The Problem of Bells Theorem.") Henson concludes that Bell only relies on a very weak principle that he calls the principle of localised events. The denial of such a principle would to deny the use of a consistent application of Cartesian coordinate systems in order to describes points in space-time, a practice that Einstein rightly takes to be foundational for empirical investigation. The plausibility of such an anti-realism will not be considered in any detail.

[25] Alain Aspect, Philippe Grangier, and Roger Gérard, "Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment," *Physical Review Letters* 49, no. 2 (1982), doi:10.1103/PhysRevLett.49.91

have shown that no locally causal HVT of QM can make entirely correct predictions. The next task will be to show that Bell's proof is sufficiently general so that we can broaden this in order to understand Bell to have shown that no locally causal theory of QM can make entirely correct predictions.

When Bell discusses his result in his original 1964 publication, he claims that he has shown that "the statistical predictions of quantum mechanics are incompatible with separable predetermination."[26] We might therefore suppose that we can deny "separable predetermination," which sounds a lot like locally causal determinacy$^{\mathrm{O}}$, in order to avoid the problem that he raised. Bell also claims that "in a theory in which parameters are added to quantum mechanics to determine the results of individual experiments... there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote."[27] We should therefore admit that any correct HVT of QM is committed to a form of non-local causation. We might suppose this to just mean that we can deny that there is any correct HVT of QM if we are committed to local causality.$^{\mathrm{O}}$

It is a mistake, however, to think that we can avoid concluding that QM theory is not locally causal by denying locally causal determinacy$^{\mathrm{O}}$ (perhaps by endorsing the ontological interpretation of indeterminacy$^{\mathrm{T}}$). Recall that, for Bell's conception of local causality as a property of theories considered in §4.2, a theory λ in figure 4.2 contains a full description of the pre-measurement particle pair and a formal structure which makes predictions about how the system will evolve. Crucially, this formal structure need not be determinate$^{\mathrm{T}}$ or deterministic.$^{\mathrm{T}}$ It follows that denying locally causal determinacy$^{\mathrm{T}}$ will not relieve us of Bell's conclusion that QM theory is not locally causal.

To see this, suppose first that, in an EPR experiment like the one we considered in the previous section, the state of Alice's photon, and therefore her measurement results, are determined by a stochastic process that happens very close to the time and place of Alice's measurement. Suppose likewise for Bob's photon. If this were the case, then there is no way to make sense of the perfect correlation that is observable

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[26] John Stewart Bell, "On the Einstein Podolsky Rosen Paradox" (1964), in *Speakable and Unspeakable in Quantum Mechanics*, ed. John Stewart Bell (Cambridge: Cambridge University Press, 1988), 20.

[27] Bell, "On the EPR Paradox," 20.

when Alice and Bob choose the same measurement settings without supposing that there is some non-local causal relation between the two measurement systems. If Alice and Bob's measurement outcomes are determined by a stochastic process and their measurements systems do not interact, then we should expect to see that Alice and Bob's measurements sometimes agree and sometimes disagree at the same measurement settings (imperfect correlation). It follows that, if there is a stochastic process involved in the determination of Alice and Bob's measurement results, then it must be a process local to the particle pair system. Denying locally causal determinacy$^{\mathrm{O}}$ does not relieve us of having to admit that there is non-local causation between the two measurement systems.

Now suppose that there is some stochastic process that is local to the particle pair system that determines the states (and therefore measurement outcomes) of Alice and Bob's particles. Even when the determining process is stochastic, it should nevertheless sort all of the possible states that the particles can be in into some proportion such that the correlations predicted by QM theory can be recovered. But this is just the same reasoning that was susceptible to the proof in §4.3, which shows that no such proportion is possible and therefore there can be no such sorting process. Therefore, there is no local stochastic process that determines Alice and Bob's measurement outcomes. It is clear that Bell's proof applies equally to *any* locally causal process which determines the states of the particles. Could there be an non-locally causal HVT of QM that makes entirely correct predictions? Indeed, there are, as Bell notes in his 1964 publication when he claims that "a hidden variable interpretation of elementary quantum theory [referring to Bohm][28] has been explicitly constructed. That particular interpretation has indeed a grossly non-local structure."[29] Thus, it follows that we cannot deny that there is any correct HVT of QM.

We cannot make sense of perfect correlation at the same measurement settings without inferring either that there is a non-local causal influence between the two wings of the experiment or that the particles must be in determined states. However, if the particles are in determined states, and we take for granted that perfect correlation is recoverable, then we will find that other correlations are not recoverable as is

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[28] Bell cites Bohm, "A Suggested Interpretation of the Quantum 'Hidden' Variables."

[29] Bell, "On the EPR Paradox," 14.

demonstrated in the proof in §4.3. So what Bell shows is more general, he shows that we simply must accept that there is a non-local causal influence between the two wings of the experiment: no local theory at all can retrieve the predicted correlations of QM and so there can be no valid local theory of QM whatsoever. It follows that any correct theory of QM is necessarily a non-locally causal theory. We can see that Bell takes his result to be at this level of generality when, following his remark that Bohm's theory has a "grossly non-local structure," he writes that "[t]his is characteristic... of any such theory which reproduces exactly the quantum mechanical predictions[,]"[30] where *any theory* here includes QM theory itself, which is the theory whose formal predictive structure includes just the Schrödinger wave equation, with no additional determining variables.[31]

Commentators Travis Norsen and Tim Maudlin both argue, in line with Bell, that what Bell's Theorem shows is just that any theory of QM has to be non-locally causal.[32,33] Norsen argues that there are two common interpretations of what Bell has shown.[34] The first is the view that has been outlined here, that any theory of QM is non-locally causal. The second is the view that "use[s] Bell's theorem against the hidden variables program, by arguing that any such alternatives to orthodox theory must conflict with relativity [by being non-locally causal] and hence needn't be seriously considered."[35] Norsen considers the second interpretation to be more prominent among physicists compared to the first, despite being mistaken for the reasons we have considered. Norsen argues that since any correct theory of QM is non-locally causal, this structure cannot be held against non-locally causal HVTs like Bohm's since local causality is not recoverable in any case.

Norsen suggests that we can understand the logical structure of the EPR argument and Bell's response as a kind of one-two punch against local causality.[36] He argues that the logic of the EPR argument, which he takes to be valid, is that $LocalCausality \rightarrow$

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[30] Bell, "On the EPR Paradox," 14.

[31] Norsen, "The Nonlocal Character of Nature," 644.

[32] Norsen "The Nonlocal Character of Nature," and "EPR and Bell Locality."

[33] Maudlin, *Quantum Non-Locality and Relativity*, 6-28.

[34] Norsen, "The Nonlocal Character of Nature," 634-635.

[35] Norsen, "The Nonlocal Character of Nature," 635.

[36] Norsen, "EPR and Bell Locality."

$\neg Completeness$, since EPR take local causality for granted and then show that there must be determined states that do not figure into QM theory. Equivalently, we can understand EPR as showing that $Completeness \rightarrow \neg LocalCausality$, since they assume that QM theory is complete but then they show that there must be a kind of interaction between the wings of the EPR experiment, contrary to their assumption that there can is no such interaction (local causality). Note that this logic mirrors the relationship between completeness and local causality explored in the case of Einstein's boxes in §4.2. Norsen also argues that the logic of Bell can be understood as $\neg Completeness \rightarrow \neg LocalCausality$, since Bell is able to formalize a theory that is incomplete in the way that follows from EPR and then shows that such a theory cannot be locally causal and correct at the same time. Since QM theory is correct and is either complete or incomplete, it follows from these two arguments that, in any case, it cannot be locally causal.

Norsen suggests that there is one possible flaw in this line of reasoning. He recognizes that "one could always assert that the conclusion isn't... all that interesting, since Bell locality isn't a correct formulation of relativity's prohibition on superluminal causation." In fact, given that Norsen sees Bell's argument as sound, he claims that "if one insists on... ensuring that quantum theory doesn't turn out to be nonlocal then Bell Locality can't be the "right" definition of locality."[37] This objection is important to consider because it recognizes the motivations for theorists to value local causality in the first place: it follows from the principle of local action (field theory) and the special theory of relativity that there can be no causal influence that is faster than light (the principle of locality). Indeed, Norsen is careful to refer 'Bell locality' as opposed to 'locality' in general because he recognizes that Bell's formulation of locality as local causality is not the only formulation possible. A theorist who is committed to the relativistic prohibition of superluminal travel, as one probably should be, should think that this is a prohibition for *something*. For Bell, causality was an obvious candidate, but it is suggested that this is not the only candidate[38]

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[37] Norsen, "The Nonlocal Character of Nature," 653.

[38] Indeed, Bell is aware that his proof does not imply that a signal can be communicated at superluminal speeds, indicating that there could still be a kind locality about signals.(Bell, "La Nouvelle Cuisine," 244-245.

and that this is not strictly implied by relativity.[39]

The question, then, of what kind of locality is the "right" kind of locality, meaning the kind of locality that *is* strictly implied by relatively (if any), is of great interest.[40] However, this line of inquiry is beyond the scope of our analysis. What is important for Norsen is that "Bell's case... is, on its face, sufficiently plausible that the burden of proof should lie with those who reject this locality condition."[41] For our purposes, it is sufficient to show that Bell locality is the kind of locality that we are interested in: local causality of a general sort. This was shown in §4.2.[42] An appeal may also be made to reject the principle of locality in its causal form in order to save the principle of local action. In order for this to be done, an account of how the causal relationship between entangled particles is mediated needs to be given consideration.[43] If this can be done, then this can be understood as a rejection of Bell's framing of local causality. Despite saving the principle of local action, we still lose the classical intuitiveness of Bell's account in the process.

Finally, there is one alternative explanation available which would allow one to save local causality. This is known as the independence of measurement settings loophole, and as the superdeterminism loophole. While the other "Bell test loopholes" were focused exclusively around criticisms of the experimental setups of previous Bell test experiments, all of which have been simultaneously closed by updated experimental setups of Bell tests performed in 2015,[44,45,46] there remains this single loophole which is empirically untestable and which could possibly be invoked in order to recover local causality.[O]

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[39] Maudlin, *Quantum Non-Locality and Relativity*, 6-28.

[40] See (Maudlin, *Quantum Non-Locality and Relativity*) for an enlightening analysis of the issue.

[41] Norsen, "The Nonlocal Character of Nature," 653.

[42] A philosopher who is convinced that the special theory of relativity implies local causality may take Bell's proof as a reductio ad absurdum on this theory. The argument that we have considered, though, has only shown that local causality is an intuitive reconstruction of the classical interpretation of relativity, not that it strictly follows.

[43] Something like a non-local field may be able to do this adequately.

[44] Giustina, et al., "Significant-Loophole-Free Test with Entangled Photons."

[45] B. Hensen, et al., "Loophole-free Bell inequality violation using electron spins."

[46] Lynden K. Shalm, et al., "Strong Loophole-Free Test of Local Realism."

4.5 The Grand Conspiracy Objection

We have seen that no locally causal theory can make the same predictions as QM and that therefore no correct theory of QM is locally causal. The proof for this was given by Bell in his response to EPR. EPR and Bell both take for granted that QM is correct, but only subsequent experiments that emulate the EPR experiment are able to more conclusively show that what QM predicts in non-local conditions is correct. There is, however, one untestable loophole that can never be conclusively ruled out.

Recall that, in Bell's definition of local causality as outlined in §4.2, we are to suppose that a full specification of the pre-measurement state of a particle pair λ, as in figure 4.2, in principle only excludes Alice and Bob's measurement settings ($\hat{a}$ and $\hat{b}$, respectively). The independence of measurement settings loophole, or the superdeterminism loophole, rejects that we can do this in principle. If we cannot do this then it is possible for the measurement outcomes as well as the measurement settings of the EPR experiment to be events with a common cause, and thus it is possible that all of these events evolve in a way that reliably recovers the predictions of QM theory exactly.

The superdeterminism loophole is the explanation that the Bell test correlations match the predicted results of QM theory because Alice and Bob's measurement choices (as well as the state of their particle pair) were determined in the right way. Bell's proof relies on the fact that the measurement choices for Alice and Bob, who cannot be in communication, are not themselves independently determined. If the measurement choices are themselves determined by a common cause, then it is possible that they have been determined such that the Bell correlations are recovered as well, with no non-local causation. Bell was aware that this is an assumption that his proof relies on, writing that "it has been assumed that the settings of instruments are in some sense free variables—say at the whim of experimenters—or in any case not determined in the overlap of the backward light cones.[47] But Bell is also aware that a failure of this assumption leads to alternative local explanations for the QM correlations. He remarks that "it is supposed that an experimenter is quite free to choose among various possibilities offered by his equipment. But it might be that

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[47] John Stewart Bell, "The Theory of Local Beables" (1975), in *Speakable and Unspeakable in Quantum Mechanics*, ed. John Stewart Bell (Cambridge: Cambridge University Press, 1988) 61.

this apparent freedom is illusory. Perhaps experimental parameters and experimental results are both consequences, or partially so, of some common hidden mechanism. Then the apparent non-locality could be simulated."[48]

This might seem like a bizarre assumption on Bell's part. How did we all of a sudden start discussing the metaphysical concept of freedom, and how is this an assumption of what otherwise seems to be deductive argument about empirically verifiable theories? If Bell's proof is contingent on the existence of non-compatibilist free will, we might think, then it isn't a strong proof at all, because surely the experimenter's choices are determined. In response to this impulse, it is important to recognize that the freedom of the experimenter in this case is a much more general problem than traditional philosophical worries about the metaphysics of free will and determinism.

Recall that, throughout the thesis thus far, Alice and Bob's measurement choices were often based on pseudo-random processes. What this means is that Alice and Bob can take measurements on any process in their vicinity and use the results taken to determine which measurements they will make on the incoming quantum particles. They could also use any set of rules for assigning measurement outcomes from the pseudo-random process to measurement choices for the quantum experiment. These facts make it clear that there is more to this problem then just whether or not Alice's choices are determined.

For example, Alice could read the seconds hand on her uncalibrated watch and use this to determine her measurement settings. If she is choosing from a total of three measurement settings, 0^o, 30^o and 60^o, then she might decide to divide her watch face into three equal segments and use the seconds hand to determine her measurement setting. There are $3! = 6$ unique ways that Alice can designate her three segments to each determine one of her three measurement settings. Or perhaps Alice might choose to take her reading of the seconds hand in order to divide it by three. She could then use the three possible remainders 0, 1 and 2 similarly to how she used the segments of the watch face. Alice might just decide to use some pseudo-random number generating algorithm that she found in a computer science textbook or dice

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[48] John Stewart Bell, "Free Variables and Local Causality" (1977), in *Speakable and Unspeakable in Quantum Mechanics*, ed. John Stewart Bell (Cambridge: Cambridge University Press, 1988) 100.

that she uses from her favorite tabletop games. She might also decide to use a Geiger counter to measure background radiation or some radioactive sample and pick some digit(s) on her reading to choose her measurement settings for the incoming particles. Or she could measure weather events on a nearby planet. Or maybe she chooses to use the recent scores of her favorite sports team to determine her measurement settings, etc. Better yet, she could add the results of any number of the previous processes together to produce a new set of measurement outcomes and use that to determine her measurement settings for the incoming particles. Bob has a similar array of choices.

When we expand on the ways in which Alice and Bob can choose their measurement settings, it becomes more apparent that the number of elements of reality that will have to be coordinated with Alice and Bob's choices in just the right way in order to determine their measurement outcomes will be have to be monumental. Since Alice and Bob could agree to measure radiation from distant astronomical sources in order to determine their measurement choices, these determining elements of reality would have to be in the shared past light cone of these sources, which could theoretically extend back in time to just after the big bang.[49] These elements of reality would jointly determine the states of all of the particles that will be measured, as well as all of the events that led to Alice and Bob choosing to base their measurements on distant astronomical sources and finally all of the events leading up to those in the distant astrological sources that determine Alice and Bob's measurement settings.

So far, a causal determinist would agree that there are elements of reality that determined all of those things, but the superdeterminist goes even further: the superdeterminist argues that all of these elements of reality determine all of those things such that it merely seems as though there is non-local causation found between entangled particles, when really there is no such causation at all but just an extremely intricate common cause that explains all of these correlations. Hypothetically, this common cause could exist at the big bang event itself. The superdeterminist will invoke this explanation for every Bell test which supports the correlations of QM

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[49] An experiment has been devised and results have be confirmed for a "cosmic Bell test," for which the common cause would have to have occurred 600 years prior. See:
Johannes Handsteiner et al., "Cosmic Bell Test: Measurement Settings from Milky Way Stars," *Physical Review Letters* 118 (2017), doi:10.1103/PhysRevLett.118.060401

and, therefore, non-local causality. The result is that superdeterminism looks like a kind of conspiratorial determinism: where the initial conditions of the universe have been fine tuned in order to yield the result that our measurement settings are highly correlated with our measurement outcomes in any and every Bell test that we choose to perform, no matter how we decide to determine our measurement settings.

We cannot conclusively rule out the possibility that the universe is conspiring against our Bell tests in this way, just as we cannot rule out the possibility that the universe is conspiring against any of our experiments. However, this objection can be used against any scientific theory whatsoever. Perhaps the fine tuning of the universe seems to conspire against any (or all) of our scientific theories in order to make it seem as if the laws of nature are the way that they are, when really they are something entirely different. This kind of explanation does not really explain anything at all. In order to be seriously considered as an explanation there must be additional reasons provided for what makes it plausible and some explanation as to how it happens.[50] In absence of any additional support we are forced to accept the conclusion that local causality is false.

4.6 Summary

We have seen that Bell responds to EPR by, first, defining local causality as a property of theories in a classically lucid and intuitive way. QM theory is shown to, on inspection, not possess this property (and this result can be proven more rigorously). Bell then uses this conception of local causality in order to construct the general form of a locally causal HVT of QM. This construction, which is sufficiently general so as to include any locally causal theory of QM whatsoever, is then shown to be incapable of recovering entirely the predictions of QM theory. It follows that, since QM theory is entirely correct, no locally causal theory of QM can be entirely correct.

The construction of this argument might lead one to interpret Bell's proof as demonstrating that we can abandon one of either local causality^{O} or the correctness of hidden variables approaches to QM. If we are granted a choice between these two

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[50] There has been an attempt to develop the latter. See:
Gerard 't Hooft, *The Cellular Automaton Interpretation of Quantum Mechanics* (2015), arXiv:1405.1548v3 [quant-ph]

options, then abandoning the correctness of the hidden variables approach to QM might look more attractive. In the first place, doing this implies that there is no more work to be done in attempting to discover any hidden variables. This interpretation lends itself well to an ontological interpretation of indeterminacy.$^{\mathrm{T}}$ While this might seem more palatable than denying local causality,$^{\mathrm{O}}$ this interpretation has been shown to be mistaken.

For philosophers who find themselves with strong rationalist commitments to determinacy,$^{\mathrm{O}}$ the willingness to give this up may seem difficult to understand. For those with stronger empiricist leanings, though, this makes a lot of sense. Local causality seems to be empirically justified with the large success of relativistic field theory. In the quantum paradigm, however, belief in ontic determinacy and causal determinism are no longer reinforced by our most successful physical theories because QM is not determinate$^{\mathrm{T}}$ or deterministic,$^{\mathrm{T}}$. Furthermore, there seem to be good reasons to deny the soft epistemic interpretation of indeterminacy.$^{\mathrm{T}}$ Consequently, so the thinking of some empiricists might go, abandoning metaphysical notions for those that are empirically justified is the obvious move. For these theorists, it is the rationalist philosophers who cannot seem to abandon these metaphysical beliefs who seem misguided. Unfortunately for those who would prefer to abandon ontic determinacy and causal determinism in order to keep local causality, Bell's proof does not imply that this is a valid choice. What Bell shows is not that these two things are inconsistent, but that local causality$^{\mathrm{O}}$ alone is untenable. Rationalist philosophers who wish to maintain ontic determinacy and causal determinism for epistemological reasons can help themselves to correct theories of QM that maintain these properties at the formal level, but no philosopher can help themselves to local causality unless they want to commit themselves to the superdeterminism objection or they want to take up a form of anti-realism that rejects the separability condition described in §2.3.

Chapter 5

Conclusion

The quantum paradigm shift began with curious observations about the behavior of electrons in a Wilson cloud chamber and photons traveling through polarized filters. It soon became apparent that the use of probability in QM theory could not be attributed to the accuracy and precision of our instruments or to the idealizations that we employ in our models. The uncertainty present in our specification of initial states and the uncertainty present in our predictions would need to be reinterpreted. Heisenberg's discovery of the indeterminacy relations, which show that conjugate quantities cannot be simultaneously specified with full precision, was the first major attack on what we have called the soft epistemic interpretation of indeterminacy.$^{\mathrm{T}}$

Recall that the soft epistemic interpretation of indeterminacy$^{\mathrm{T}}$ in QM theory states that indeterminacy$^{\mathrm{T}}$ is in principle eliminable. This is the classical interpretation of indeterminacy.$^{\mathrm{T}}$ The hard epistemic interpretation of indeterminacy,$^{\mathrm{T}}$ on the other hand, states that indeterminacy$^{\mathrm{T}}$ is in principle ineliminable. For the hard epistemic interpretation, even if there is determinacy,$^{\mathrm{O}}$ there is a hard limitation for our ability to epistemically access all determined quantities simultaneously. Finally, the ontological interpretation of indeterminacy$^{\mathrm{T}}$ states that indeterminacy$^{\mathrm{T}}$ is in principle ineliminable because indeterminacy$^{\mathrm{O}}$ is a feature of reality.

For Heisenberg, we needed to recognize that indeterminacy$^{\mathrm{T}}$ is not, in principle, eliminable, contrary to the soft epistemic interpretation. Whether this was because nature itself does not have a determinate$^{\mathrm{O}}$ structure or it was because of some limitation in our ability to access this kind of underlying structure was of no consequence for Heisenberg. Since both of these explanations are consistent with what can be measured, Heisenberg remained agnostic about these philosophical quandries. He nevertheless declared that QM theory "establishe[d] the final failure of causality."

EPR present the first major classical response to Heisenberg interpretation of indeterminacy.$^{\mathrm{T}}$ EPR argue that, according to QM theory itself, conjugate quantities should be simultaneously real and completely specified. Consequently, since QM theory cannot completely specify both of these quantities simultaneously, it follows that the quantum state description and therefore the theory itself is incomplete. The incompleteness argument implies that QM theory has not established that indeterminacy$^{\mathrm{T}}$ is in principle ineliminable. We interpreted EPR's response as a defense of the soft epistemic interpretation of indeterminacy,$^{\mathrm{T}}$ because the incompleteness argument implies that QM theory makes a critical idealization over the determinacy$^{\mathrm{O}}$ of conjugate quantities. Following the EPR incompleteness argument, Einstein came to recognize that the argument is dependent on an assumption that he calls the separation principle, which in turn contains the principle of local action as a part.

The principle of local action is a classical assumption that is taken to be an empirical consequence of field theory. The principle of local action in conjunction with the principle of locality, framed in terms of superluminal causation, constitutes local causality as we have defined it.[1] We have taken local causality to be a fundamental classical realist commitment, that is, a commitment that characterizes many of the philosophies of science at the end of the classical paradigm. For Einstein, local causality was necessary for the real existence of the (quasi-)closed systems. Einstein believed that these were necessary for the empirical testability of physical laws. However, our analysis found this belief to be unsupported.

Finally, Bell responds to EPR by showing that the incompleteness argument is fundamentally flawed. He does by showing that local causality is untenable, in light of the correlations that can be demonstrated between measurements performed on quantum particles in non-local conditions. Bell argued that EPR were committed to the possible existence of a locally causal HVT of QM. He then formalized this notion and showed that it could not possibly make entirely the same predictions about statistical correlations in the EPR experiment as does QM theory. It follows that there can be no correct locally causal HVT of QM. Crucially, for Bell, the locally causal hidden variables could originate from either deterministic or stochastic

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[1] As noted in §4.3, the principle of locality framed in terms of causation may not be what actually follows from special relativity theory. This is, however, a highly plausible classical interpretation of the relativistic constraint on the velocity of light, when considered alongside field theory.

processes. What this means is that Bell has shown that there is no correct locally causal theory of QM, not that there can be no correct HVT of QM.

We can understand Bell to echo Heisenberg in response to the main question posed by this thesis: Does QM establish the final failure of causality? For Bell, the answer is yes, QM establishes the final failure of *classical* causality. This is because classical causality, as we have defined it, is composed of causal determinism and local causality. Bell has shown that local causality is false. The failure of local causality, while allowing for the construction of a correct determinate$^{\mathrm{T}}$ theory of QM, reinforces the hard epistemic limitation provided by Heisenberg, by showing that any possible determinacy$^{\mathrm{T}}$ must ascribe values to non-local variables which might be impossible to ever access epistemically. While it might be possible, in theory, to retrospectively assign a value to a non-local variable in order to reconstruct a determinate$^{\mathrm{T}}$ and deterministic$^{\mathrm{T}}$ *explanation* of how quantum systems evolve, there is no way in principle to eliminate indeterminacy$^{\mathrm{T}}$ from our *predictions*.[2]

However, Bell has not shown that causal determinism is false, nor has he shown that ontic determinacy is false. For the philosopher who feels committed to ontic determinacy and causal determinism, there can be entirely correct (explanatory) theories of QM that maintain determinacy$^{\mathrm{T}}$ and determinism,$^{\mathrm{T}}$ and so therefore lend themselves to an interpretation of QM that is consistent with ontic determinacy and causal determinism. A sophisticated form of realism for the quantum paradigm, therefore, may retain the elements of reductionism, ontic determinacy and causal determinism from classical realism, but cannot plausibly retain local causality.

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[2] This is true, at least, if signal locality follows from the special theory of relativity. This conclusion echoes that of often ignored quantum theoretician Grete Hermann who made similar claims regarding the distinction between explanation and prediction in the 1930's. See:

Grete Hermann,"The Foundations of Quantum Mechanics in the Philosophy of Nature" (1935), *The Harvard Review of Philosophy* 7, no. 1 (1999), doi:10.5840/harvardreview1999715

Bibliography

Aspect, Alain., Grangier, Philippe., Gérard, Roger. "Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment." *Physical Review Letters* 49, no. 2 (1982). doi:10.1103/PhysRevLett.49.91

Bell, John Stewart. "Free Variables and Local Causality." 1977. In *Speakable and Unspeakable in Quantum Mechanics*, edited by John Stewart Bell, 100-104. Cambridge: Cambridge University Press, 1988.

Bell, John Stewart. "La Nouvelle Cuisine." 1990. In *Speakable and Unspeakable in Quantum Mechanics*, edited by John Stewart Bell, 232-248. Cambridge: Cambridge University Press, 2004.

Bell, John Stewart. "On the Einstein Podolsky Rosen Paradox." 1964. In *Speakable and Unspeakable in Quantum Mechanics*, edited by John Stewart Bell, 14-21. Cambridge: Cambridge University Press, 1988.

Bell, John Stewart. "The Theory of Local Beables." 1975. In *Speakable and Unspeakable in Quantum Mechanics*, edited by John Stewart Bell, 52-62. Cambridge: Cambridge University Press, 1988.

Bohm, David. "A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables, I and II." 1952. In *Quantum Theory and Measurement*, edited by John Archibald Wheeler and Wojciech Hubert Zurek, 369-396. Princeton: Princeton University Press, 1983.

Bohm, David., Aharonov, Yakir. "Discussion of Experimental Proof for the Paradox of Einstein, Rosen, and Podolsky." *Physical Review* 108 (1957). doi:10.1103/PhysRev.108.1070.

Clauser, John F., Horne, Michael A., Shimony, Abner., Holt, Richard A. "Proposed Experiment to Test Local Hidden-Variable Theories" 1969. In *Quantum Theory and Measurement*, edited by John Archibald Wheeler and Wojciech Hubert Zurek, 409-413. Princeton: Princeton University Press, 1983.

Cohen, I. Bernard., ed. *Isaac Newton's Papers & Letters on Natural Philosophy.* Cambridge: Harvard University Press, 1978.

Einstein, Albert., Podolsky, Boris., Rosen, Nathan. "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" 1935. In *Quantum Theory and Measurement*, edited by John Archibald Wheeler and Wojciech Hubert Zurek, 138-141. Princeton: Princeton University Press, 1983.

Einstein, Albert. "Physics and Reality." 1935. *Journal of the Franklin Institute* 221, no. 3 (1936). doi:10.1016/S0016-0032(36)91047-5

Einstein, Albert. *Relativity: The Special and the General Theory.* 1916. Translated by Robert W. Lawson. Project Gutenberg, 2004.
<http://www.gutenberg.org/files/5001/5001-h/5001-h.htm>

Greenberger, Daniel M., Horne, Michael A., Zeilinger, Anton. "Going Beyond Bell's Theorem." In *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, edited by Menas Kafatos, 69-72. Dordrecht: Kluwer, 1989.

Giustina, Marissa,. et al. "Significant-Loophole-Free Test of Bell's Theorem with Entangled Photons." *Physical Review Letters* 115, no. 25 (2015).
doi:10.1103/PhysRevLett.115.250401

Handsteiner, Johannes,. et al. "Cosmic Bell Test: Measurement Settings from Milky Way Stars."*Physical Review Letters* 118 (2017). doi:10.1103/PhysRevLett.118.060401

Heisenberg, Werner. "The Physical Content of Quantum Kinematics and Mechanics." 1927. In *Quantum Theory and Measurement*, edited by John Archibald Wheeler and Wojciech Hubert Zurek, 62-84. Princeton: Princeton University Press, 1983.

Hensen, B. et al. "Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres." *Nature* 526, no. 7575 (2015). doi:10.1038/nature15759

Henson, Joe. "Non-Separability Does Not Relieve the Problem of Bell's Theorem." *Found Phys* 43, no. 8 (2013). doi:10.1007/s10701-013-9730-8

Hermann, Grete."The Foundations of Quantum Mechanics in the Philosophy of Nature." 1935. *The Harvard Review of Philosophy* 7, no. 1 (1999).
doi:10.5840/harvardreview1999715

Hooft, Gerard 't. *The Cellular Automaton Interpretation of Quantum Mechanics* (2015). arXiv:1405.1548v3 [quant-ph]

Howard, Don. "Einstein on Locality and Separability." *Studies in the History and Philosophy of Science* 16, no. 3 (1985): 171-201.

Howard, Don. “Who Invented the Copenhagen Interpretation?” *Philosophy of Science* 71, no. 5 (2004). doi:10.1086/425941

Jammer, Max. *The Philosophy of Quantum Mechanics*. John Wiley & Sons, 1974.

Kuhn, Thomas S. *The Structure of Scientific Revolutions*. 1962. Chicago: The University of Chicago Press, 1970.

LaPlace, Pierre Simon. *A Philosophical Essay on Probabilities* (1894). Translated by F.W. Truscott and F. L. Emory. New York: J. Wiley. 1902.

Maudlin, Tim. *Quantum Non-Locality and Relativity*. Cambridge: Blackwell, 1994.

Norsen, Travis. “Against ‘Realism’.” *Foundations of Physics* 37, no. 3 (2006). doi:10.1007/s10701-007-9104-1

Norsen, Travis. “Bell Locality and the Nonlocal Character of Nature.” *Foundations of Physics Letters* 19, no. 7 (2006). doi:10.1007/s10702-006-1055-9

Norsen, Travis. “EPR and Bell Locality.” *AIP Conference Proceedings* 844 (2006). doi:10.1063/1.2219369

Schrödinger, Erwin. “The Present Situation in Quantum Mechanics.” 1935. In *Quantum Theory and Measurement*, edited by John Archibald Wheeler and Wojciech Hubert Zurek, 152-167. Princeton: Princeton University Press, 1983.

Shalm, Lynden K. et al. “Strong Loophole-Free Test of Local Realism.” *Physical Review Letters* 115, no. 25 (2015). doi:10.1103/PhysRevLett.115.250402

Von Neumann, John. *Mathematical Foundations of Quantum Mechanics*. 1932. Translated by Robert T. Beyer. Princeton: Princeton University Press, 1955.