Numerical Analysis of Longwall Mining Layout for a Wyoming Trona Mine

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Abstract

At Solvay Mine, located in southwestern Wyoming, a subhorizontal trona seam is mined at depths of between 460-490 m using mechanized room-and-pillar and longwall mining methods. The stratigraphy at the mine generally consists of horizontally laminated (*i.e.*, bedded) sedimentary rocks comprised mostly of shales and sandstones with significantly contrasting mechanical properties. Most notably, a 43 – 82 m-thick massive, brittle sandstone unit (Tower Sandstone) is located approximately 100 m above the mining level. The Tower Sandstone unit has a tendency to promote stress arching within the overburden rock that can bridge over panel-scale mine instabilities and can lead to violent multi-panel collapse failure. One such violent collapse is the well-documented 5.1 magnitude seismic event due to a 1 x 2 km multi-panel failure on February 3, 1995. It has proven difficult to account for this arching behaviour with conventional mine design methods, such as the tributary area method. Therefore, over the past two decades or more, Solvay Mine has been utilizing numerical modelling techniques along with field instrumentation/monitoring as part of an integrated program to gain an enhanced understanding of the complex response of the overlying stratigraphy (*i.e.*, arching) to mining. In 2005 and 2006, several longwall panels in the northwest and southeast areas of the mine were instrumented and monitored during mining. Two- and three-dimensional numerical models, using *FLAC* and *FLAC3D*, were developed and calibrated on the basis of the instrumentation data, and these models were then used for mine design verification (*e.g.*, pillar and panel dimensions). This mining case study illustrates the complex excavation response due to the contrasts in stratigraphy at Solvay Mine and presents a numerical modelling study that captures the dominant aspects of these conditions. In addition, the practical use and role of numerical modelling and instrumentation within an integrated mine design methodology is demonstrated.

Keywords: longwall mining, trona, subsidence, instrumentation, numerical modelling, *FLAC*

1. Introduction

Trona is a non-marine evaporite mineral composed of sodium sesquicarbonate that has a major use in the production of glass, chemicals, paper and detergents. Trona has been mined continuously in several mines in the Green River Basin since 1947 using numerous mining methods. One of the main sources of trona production in the Green River Basin is Solvay Mine, located in western Wyoming, approximately 6.5 km east of Little America and south of Interstate 80.

At Solvay Mine, a 2.7 m-thick, sub-horizontal trona seam, known as Bed 17, is mined at depths of approximately 460 – 490 m. The general stratigraphy consists of horizontally laminated (*i.e.*, bedded) sedimentary rocks with significant contrasts in stiffness and brittleness, most notably the massive 43 – 82 m thick Tower Sandstone unit located within the bottom third of the overburden profile. The contrast in mechanical characteristics of the geological units result in complex behaviour of the mine openings and overlying strata, such as stress arching and progressive failure with mining advance. This behaviour significantly impacts the evolution of the stress distribution, yielding and subsidence within the strata. Conventional mine design methods, such as tributary area theory and empirical pillar design curves, do not account for such behaviour. The most critical issue is violent failure of the massive, brittle Tower Sandstone as a result of "stress arching" across multiple mine panels leading to a delayed failure. One example of this type of failure is the well-documented 5.1 magnitude seismic event due to a large (1 x 2 km), violent, multi-panel failure on February 3, 1995 [1-4].

For the past two decades, numerical modelling methods have been utilized at Solvay Mine to evaluate the excavation-scale and mine-scale mining response and performance. Over this period, an increasing level of experience and insight into the relatively complex rock mechanics behaviour has been gained from observations of mine response, and instrumentation and monitoring. However, given the complexity of overburden's response to mining and the consequence of inaccurate predictions in mine design, it is critical to ensure the design model is calibrated.

In 2005 and 2006, Solvay Mine completed mining four longwall panels in the northwest portion (“NW District”) of the mine and began mining a longwall panel in the southeast area (“SE District”) of the mine. An overall plan of Solvay Mine showing the location of these Districts is provided in Fig. 1. These longwall panels were monitored for surface subsidence (geodetic monitoring), pillar stresses (IRAD stressmeters), and downhole extensional strain (time domain reflectometry: TDR cable). The monitoring data was used to carry out detailed back analysis of the longwall mining to develop a “calibrated” numerical model using the two-dimensional continuum code *FLAC* [5]. The model was further verified using a three-dimensional *FLAC3D* [6] model of early stage mining of the SE District longwall panels.

This mining case study demonstrates the practical use and role of numerical modelling and instrumentation within an integrated mine design methodology. In addition, it illustrates the complex excavation response from longwall mining the relatively deep, horizontal trona seam beneath a stratigraphic profile containing units with significantly contrasting mechanical properties.

It should be noted that the original monitoring data and numerical modelling presented in this study were carried out using imperial units [7]. The imperial units have been converted to metric for this publication and many of the originally rounded imperial-based values reported may appear as “odd” values as a result of unit conversion.

2. Background

Over the last two decades numerous numerical modelling-based studies have been carried out at Solvay Mine and other mining operations in the immediate vicinity of Solvay Mine [3,8-11]. This section provides a brief overview of the general conditions at Solvay Mine, such as the geological conditions, rock mechanics conditions, and summarizes the key mining-related observations from these past studies.

2.1 Geological Setting

The Green River Formation has a maximum thickness of approximately 730 m in the Green River Basin and consists primarily of horizontally bedded fine-grained mudstones, siltstone, marlstone and oilshale [12]. The Green River Formation is comprised of the lower Tipton Shale Member, the middle Wilkins Peak Member and the upper Laney Shale Member. The bedded trona units occur within the Wilkins Peak Member. The units were deposited during the Eocene in a large lake in the Green River Basin. The evaporite units (*e.g.*, halite and trona) were formed during evaporative periods and the oilshale units were formed during high water stages when organic organisms (*i.e.*, algae) flourished. A total of twenty-two significant trona beds, including Bed 17, are present within the Wilkins Peak Member.

The Tower Sandstone is a massive, or irregularly bedded, sandstone unit that derives its name from capping the erosional remnants called “The Towers” [13] located north of Green River. The Tower Sandstone is part of the lower Laney Shale Member at the Wilkins Peak-Laney Shale contact. The Tower Sandstone is comprised of approximately 75% detrital material, of which 40% is quartz and 25% is cementitious material [14]. According to Culbertson [15], the Tower Sandstone was laid down as a marginal deposit of the fluctuating fresh water lake in the Green River Basin.

The stratigraphy at Solvay Mine, where the approximately 3.3 m thick Bed 17 trona seam is mined, is comprised of the aforementioned units. The Bed 17 trona seam is underlain by a weak oilshale layer (approximately 2 m thick) and overlain by a bedded shale unit. Approximately 20 m above Bed 17 is a 10 m-thick sandstone/mudstone layer known as the D Sandstone. The Tower Sandstone unit, a generally strong and brittle layer ranging between 40 and 82 m thick across the mine site, is encountered about 90 to 122 m above Bed 17. The Upper Wilkins

Peak member is found between the D and Tower Sandstone units and consists of bedded shale and mudstone sequences.

The Tower Sandstone is about 82 m thick in the vicinity of the NW District, while in the SE District, based on mapped stratigraphy of Shaft 6 located near the SE District, it is approximately half that thickness (41 m thick) [16]. The thickness of the Tower in the SE District depends on whether the lower portion of the unit, up to about 18 to 24 m thick and described as more fractured and with shale interbedding, is included in the interpreted thickness of the Tower Sandstone unit. Furthermore, the depth from ground surface to Bed 17 is greater in the SE District than in the NW District. A comparison of the stratigraphy at both SE and NW Districts are shown in Fig. 2.

2.2 Mine Layout

As show in Fig. 1, longwall panels in the NW District were oriented in a north-south direction. These panels were 165 m wide and 1525 m long with 39 m-wide gateroad pillars and, with the exception of the western-most panel, which was 152 m wide. In general, performance of pillars and openings in the NW District due to longwall mining was considered to be good with few signs of stress-related instability in the gateroads or the face. The longwall panels in the NW District were mined from east to west.

2.3 Past Observations of Rock Mechanics Mining Response

Although the horizontally bedded geological structure at Solvay Mine is not overly complex, the high contrasts in strength and stiffness of some of the units does result in a complex response to mining. There are several important factors that control the mining response at Solvay and other mines in the Green River area. These include: a) panel extraction ratio; b) panel span; c) barrier pillar thickness; d) strength of the floor-pillar system; and e) the thickness, strength and brittleness of the Tower Sandstone. Based on field observations, instrumentation and the results of past numerical analysis, Board and Damjanac [11] describe the mechanism of caving and subsidence for longwall mining at Solvay Mine as follows.

Near-vertical shear fracture propagation occurs above the abutments in the weak shales (immediate roof and Upper Wilkins Peak Member), resulting in uniform downward movement of the shale beds with little internal vertical straining until they contact the floor. The subsequent shale collapse opens a significant gap beneath the Tower Sandstone (up to a meter or more wide) with a span nearly the width of the panel. Bending of the Tower Sandstone into this gap occurs, with resulting formation of a zone of tension within the base of the Tower Sandstone where it has become unconfined. This tensile zone results in bulking and progressive collapse of the Tower Sandstone base. The failure of the Tower Sandstone is typically non-violent in this case, because it is essentially unconfined at its base and subject to progressive failure under low or tensile stresses. This would probably result in progressive block failures from the base of the unit that would advance, or "run," vertically upwards until the accumulation of bulked collapse material arrested further advancement of failure. Subsequent units overlying the Tower Sandstone then translate vertically downward in response to bending of the Tower Sandstone. This translation occurs with little internal strain and damage to the downward moving block. However, bedding planes within the Bridger Formation often undergo bedding plane shearing of several centimeters as the units subside. Mining of multiple panels results in the Tower Sandstone bending over all panels, creating a single subsidence trough with no evidence of the impact of chain pillars. A "break line" defining the angle of draw of the subsidence trough forms through all units at an angle of approximately 70° from the horizontal.

Within Bed 17, the contrast between the relatively strong and brittle trona, and the weak and ductile oilshale floor results in a unique load response behaviour of mine pillars and openings that has been well documented by Board et al. [3]. As the pillar punches into weak floor oilshale, the oilshale are squeezed out beneath the pillar in a ductile manner, and the pillar is then pulled apart at its base. Rib rash fractures develop at the floor line and the beds buckle in the room centers. Continued punching drives a wedge-shaped central pillar core into the floor. This core is highly confined and can result in pillar strengthening. Therefore, the pillar-floor must be treated as a "system response" to loading. This pillar load response is of greater significance for relatively slender pillars in room-and-pillar mining than for longwall mining.

One of the objectives of longwall mining at Solvay Mine is to induce a "controlled failure" of the Tower Sandstone and avoid conditions with potential for delayed, violent failure. In general, this appears to have been achieved in the NW District. Moreover, given the thickness of the overlying Tower Sandstone (82 m), the strength of the oilshale floor and roof shales, and the mining dimensions used, longwall mining in the NW District has performed adequately. The gob seems to have developed gradually without undue signs of violent collapse. Gateroads and chain pillars have shown few signs of stress-induced distress.

3. Geotechnical Conditions

There have been several rock mechanics laboratory programs carried out on the geological units in the Green River area to evaluate the intact (*i.e.*, laboratory scale) rock properties by the various mines and their consultants. The most detailed laboratory study was conducted by Tetra Tech Inc. (Salt Lake City) on behalf of the Joint Industry OGT project [17]. The data from the numerous rock mechanics studies were compiled by Weller [18] who provided recommended intact rock Unconfined Compressive Strength (*UCS*) values for the dominant geological units at Solvay Mine. The intact rock parameters are shown in Table 1.

In order to account for the presence of jointing, fractures and scale effects (*i.e.*, from laboratory size to field problem size), rock mass properties are used for analysis. Methods to determine Hoek-Brown rock mass properties have been developed [19]; however, these relationships are more appropriate for a relatively homogeneous, jointed rock mass, rather than the strongly bedded and highly variable rock layering that is found in the stratigraphy at Solvay Mine [20]. Based on the laboratory test data, estimated values of Geological Strength Index (*GSI* [21]) and past experience with numerous analysis and design projects in the Green River area, Board et al. [3] provided a range of rock mass properties for the major geological units listed in Table 2. In particular, direct observation and back-analysis of the oilshale floor-trona pillar system have been extensively used to evaluated rock properties.

Table 1 Intact rock properties from Weller [18]

Unit	Unconfined Compressive Strength (MPa)
Bridger Formation	51
Laney Shale	57
Tower Sandstone	115
Upper Wilkins Peak	16
D Sandstone	66
Roof Shale	42
Bed 17 (Trona)	45
Oilshale	33

Table 2 Estimated *in situ* rock mass properties for sandstones and shales (from [3])

Unit	*GSI*	Young's Modulus (GPa)	Cohesion (MPa)	Friction Angle (°)
Upper Wilkins Peak	40-55	5.2-6.9	0.4-0.7	25-28
Laney Shale	40-55	5.2-6.9	1.2-2.3	25-28

Roof Shale	40-55	5.2-6.9	1.8-2.3	25-28
Tower Sandstone	60-70	22.8	5.2-8.6	40-44
Bed 17 (Trona)	60-70	30	9.3	49
Oilshale	40-55	1.2	1.75	20

The rock mass parameters listed in Table 2 provide a starting point for carrying out numerical analysis for Solvay Mine. However, the 2003 – 2006 NW District longwall mining provides an opportunity to obtain a more complete understanding of the likely range of behaviour of all of the critical rock units within the stratigraphic profile. The instrumentation and monitoring program provided reliable information at both the mining level (stressmeters), at the ground surface (subsidence), and also within the stratigraphy (TDR's). A back analysis of the longwall mining that can capture all of the instrumentation data, plus match key mining observations, would be considered a calibrated model. A calibrated model is a prerequisite for model-based design [22].

The Green River Basin is a region of relatively low horizontal stresses. The *in situ* stresses have been determined based on past stress measurements (Solvay personnel, pers. comm.) at Solvay Mine and observations of mine excavation response. Based on this, a vertical stress gradient of 0.025 MPa/m with a ratio of horizontal-to-vertical stress in the range of $K = 0.3$ to 0.75 is likely. A value of $K = 0.5$ was used for the analyses.

4. Instrumentation Data from NW District Longwall Mining

Four longwall panels were mined in the NW District between 2003 and 2005. Starting from the eastern most panel (1W1N), the longwall panels were mined in a north-to-south direction, and the four panels (1W1N through 1W4N) were mined sequentially from east to west. The layout of the panels in the NW District are shown in Fig. 1 and Fig. 3. As mentioned previously, the gateroads and mining face performance was generally suitable with these mining dimensions.

4.1 Subsurface Subsidence Measurements

Subsidence data, obtained from topographic surveys, was collected from two stages of longwall mining in the NW District. The subsidence contours are shown in Fig. 3 for 2005 (2.5 panels excavated) and Spring 2006 (3 panels excavated). The longwall mining progress corresponding to these two points in time is also shown in Fig. 3. Subsidence profiles along an east-west oriented section are shown in Fig. 4.

The surface subsidence trough was elongated in a north-south direction and symmetrical with the maximum subsidence located approximately in the center of the mined-out area at each measured interval. A maximum subsidence of between 60 and 76 cm was recorded in 2005 and between 75 and 91 m was measured in 2006. In general, the subsidence profiles were relatively smooth and continuous with no surface expression of the chain pillars between panels.

Based on the author's experience from several projects in the Green River area, subsidence does not typically exceed 20 – 30 cm without significant yielding of the Tower Sandstone. Moreover, limited yielding of the Tower Sandstone would result in arching of stresses across longwall panels with a resulting smooth and shallow subsidence trough. Therefore, the steepness of the measured surface subsidence trough and magnitude of maximum subsidence are both consistent with significant yielding of the Tower Sandstone. The subsidence trough was fairly steep in both 2005 and 2006, which indicated that at least partial yielding of the Tower Sandstone unit had occurred at these mining stages.

4.2 Pillar Stress Measurements

A total of thirteen IRAD stressmeters were installed in the chain pillars along cross-cuts prior to mining the longwalls. These instruments provided a measure of the vertical stress change beginning at the time that each device was installed. The thirteen instruments were grouped by areas into Area A through D, each of which comprised between two and four stressmeters. The stressmeters and area groupings are shown in Fig. 5.

The stressmeter data for the thirteen locations is shown in Fig. 6 for the time period of March 2003 to July 2006. The stressmeters at Area A, in the east abutment, shows the least mining-induced increase in stress with all measured stress changes below 3.5 MPa. The stressmeters in Area B, within the chain pillar between Panel 1W1N and 1W2N, showed mining-induced stress changes up to about 24 MPa. Area C and D are within the north and middle area of the chain pillar between 1W2N and 1W3N, respectively. Stress changes greater than 45 MPa were measured at Area C. Data was only available until November 2005 for Area D, when panel 1W3N was partially mined, when stress reached about 17 MPa. It appears that the stress levels in Area D had not stabilized at that time and would likely continue to rise due to mining of Panel 1W3N. Both Areas C and D are located within the same chain pillar in the panel center and the shape of the data curves at Area D is similar (but slightly lower at a given time) to that of Area C. Therefore, a similar magnitude of stress change could be expected at Areas C and D.

5. Back Analysis of NW District Longwall Mining

The *FLAC* model was calibrated based on a back analysis of the mining of four longwall panels in the NW District. This section describes development of the numerical model, calibration of the model parameters and verification of the Calibrated Model by comparison with actual instrumentation data.

An important aspect of longwall mining is the behaviour of the gob, or failed rock, behind the advancing longwall face. Little is known about the extent of caved rock above the mining horizon or the properties of the fully and partially caved material [20]; however, several authors have used different methods to simulate the behaviour of this fractured and dilated material. Pappas and Mark [23] estimated gob particle size distribution using photogrammetric methods and then scaled the grain size curve down in order to perform laboratory tests on an analogous gob material. O’Connor and Dowding [24] used an early version of

UDEC software to simulate the effect of discrete fractures on caving behaviour, in particular, the interface stiffness of the gob material. Deb [25] used the Pappas and Mark approach for gob stiffness with a two-dimensional finite element model. In Deb's work gob development was simulated by replacing a given zone, upon yielding, with a zone having properties consistent with gob material (*i.e.*, stiffness and stress). Alejano [26] developed a calibrated *FLAC* model to capture longwall mining of an inclined coal seam in the English coalfields. Pierce et al. [27] have developed the Synthetic Rock Mass (SRM) method for modelling cave mining which also could be used for a sophisticated simulation of longwall gob development in three-dimensions.

According to Peng [28], the gob is usually subdivided into two zones: a lower, Fully Caved Zone and an Upper Fractured Zone. The Fully Caved Zone can be expected to extend vertically about 2 to 3 times the mining height and behaves as a granular material with a relatively high void ratio (*i.e.*, bulked material). For the Solvay Mine panels, this corresponds to a Fully Caved Zone of about 8 m above the mining seam (just below the D Sandstone) with the Upper Fractured Zone above that. Even if the Fully Caved Zone was significantly higher than this, it is relatively small compared to the full stratigraphical section at Solvay Mine. Moreover, because of the presence of the D Sandstone layer and the upper Tower Sandstone, both of which have significant potential to arch or bridge across panels, the overlying strata is dominated by arching and the subsequent yielding behaviour of the massive Tower Sandstone. Therefore, direct simulation of the gob material is likely a less critical issue at Solvay Mine than in other longwall coal mining cases discussed in the literature.

The modelling approach used in this work follows the philosophy laid out by Starfield and Cundall [29] and Hoek et al. [22], where certain simplifications were used in order to focus on an understanding of the dominant mechanism of behaviour. Rock mechanics modelling in this case, and in general, is considered a "data limited problem." It is not therefore prudent to include model details and refinement that exceed the level of understanding of the rock mechanics conditions. Therefore, overly sophisticated methods of gob simulation we not using in this stage of analysis.

5.1 Development of a Two-Dimensional Numerical Model

The two-dimensional code *FLAC* is suitable for the back analysis because the panels are quite long relative to their width and the effect of excavation advance on subsidence was considered to be minor. As mentioned above, development of the gob was not directly accounted for in the model. Moreover, use of the two-dimensional code allowed for more rapid execution of numerous models and a more comprehensive parametric study could be conducted than would be possible using a more computationally intensive three-dimensional code. Three-dimensional modelling will be described later in Section 5.2.

5.1.1 Geotechnical Model

The horizontally bedded sedimentary units at Solvay Mine are reasonably well understood from a geological perspective. However, numerous interbedded rock types are present and many of these have significantly contrasting rock mechanics characteristics (*e.g.*, the relatively strong, brittle Bed 17 trona and the weak, ductile oilshale mine floor). It would be unnecessarily difficult to capture all of the individual rock layers/units in a mine-scale numerical model. Therefore, a simplified Geotechnical Model was developed where units with similar rock mechanics behaviour (*e.g.*, strength and stiffness) were grouped together into Geotechnical Domains. The Geotechnical Domains were assigned overall representative properties of the individual subunits comprised within each and these units were then used to develop a numerical model that includes the important rock mechanics features without excessive complexity and computational burden.

The stratigraphy of the NW District described in Section 2.1 was used to develop the *FLAC* model where the Tower Sandstone unit was approximately

82 m thick. At this location Bed 17 was at a depth of approximately 470 m. The stratigraphic profile and rock mechanics characteristics of the various units are known to vary with lateral extent and there are some differences in the Geotechnical Model between the NW and SE District. The most notable differences are the thickness of the Tower Sandstone and the depth below surface of Bed 17 due to topography and the slight overall dip of Bed 17. The Geotechnical Model at both the NW and SE Districts is based on the simplified stratigraphy shown in Fig. 2.

5.1.2 Model Geometry, Boundary and Initial Conditions

In order to capture both the observations within Bed 17 and the surface subsidence for all four longwall panels in a single model, it was necessary to develop a large mine-scale model of sufficient size to include all NW District panels. In addition, a significant distance to the lateral boundaries and the bottom boundary was also required to minimize model boundary effects. The *FLAC* model for the NW District longwall panels extended approximately 3000 m horizontally and approximately 746 m vertically. Model zone sizes ranged from 1.4 to 11 m grading from small to large vertically upwards away from Bed 17. Suitably fine zone resolution in the critical Tower Sandstone unit was required to allow for accurate prediction of bending and progressive yielding. Discontinuous model interfaces, capable of yielding and separating, were built into the model at the contact of each Geotechnical Domain using *FLAC* interface logic. It was important to allow for true separation of the underlying shale from the Tower Sandstone in the model (see Section 2.3). The zones making up the four longwall panels (1W1N through 1W4N) within Bed 17, were identified within the model geometry. The individual chain pillars were not directly analyzed in the model. Instead, a solid pillar separating the longwall panels without the gateroad was used to represent the pillars. The width of the representative pillar will be termed the *gross pillar width* to quantify pillar widths. For example, an individual chain pillar width of 39 m with 4.6 m wide gateroads results in a gross pillar width of 82.5 m (*i.e.*, (39 x 2) + 4.5 = 82.5). The *FLAC* model layout is shown in Fig. 7.

The model base had fixed boundary conditions laterally and vertically (pinned boundary). The lateral boundaries were fixed laterally and free vertically (roller boundary). The vertical stresses in the model were input to correspond to a stress gradient of 0.025 MPa/m with a ratio of horizontal to vertical stresses (K) of 0.5 in both the model's in-plane and out-of-plane directions. Groundwater has not been a significant issue for the mines in the Green River area and no groundwater pressure was used in the model.

5.1.3 Constitutive Model

The constitutive models describe how the geomaterials will respond to loading, including yielding and post-peak behaviour. A detailed description of the constitutive models available in *FLAC* is described in the software manual [5]. Three main constitutive models were utilized in the *FLAC* models: Mohr-Coulomb; Strain Softening; and Ubiquitous Joint Mohr-Coulomb. The Mohr-Coulomb (*Mohr*) utilizes a Mohr-Coulomb shear failure envelope with a tensile strength cut off and perfectly plastic post-peak parameters. The Strain Softening (*SS*) model is similar to the *Mohr* model, but also includes strain softening post-peak parameters. The Ubiquitous Joint Mohr-Coulomb (*SUBI*): is similar to the *SS* model, but includes preferentially weak planes in a specified orientation that responds as if there was a continual array (*i.e.*, ubiquitous) of horizontal bedding planes. The strength of the weak planes is defined by a Mohr-Coulomb failure criterion. For the *SS* and *SUBI* models, both peak and post-peak residual strength parameters were specified. The rate at which the strength of the rock mass decreases after the peak strength is exceeded is a function of the plastic shear strain (ε^p). The strength properties are reduced to residual after the *critical* plastic shear strain (ε^p_{crit}) is achieved based on the empirically derived expression [30]:

$$\varepsilon^p_{crit} = 12.3 - (0.125 \times GSI) \quad (1)$$

Critical strain values in *FLAC* and *FLAC3D* are scale dependent and a model zone size adjustment can be made. For simplicity, a uniform value of critical strain was used in this model for each strain softening material across all zone sizes.

The trona and subsequent overlying mudstone/shale/sandstone units were modelled with the *SS* constitutive model to capture the post-peak behaviour characteristic of most of the rock units. The relatively ductile oilshale unit underlying Bed 17 was represented by a perfectly plastic *Mohr* constitutive model. In general, horizontal discontinuities (*i.e.*, bedding) were not directly represented in the numerical model; instead the *SUBI* constitutive model was used in some of the shale units. The previously described *FLAC* interfaces at the horizontal contacts between major Geotechnical Domains were assigned a perfectly plastic Mohr-Coulomb constitutive model to allow separation and/or shear between the units.

5.1.4 Excavation Sequence

Excavation was simulated in two dimensions by removing the Bed 17 zones representing a given panel. An interface was placed on the roof and floor within each excavated panel so that the roof and floor zones could “touch,” if required, and build-up resultant contact stresses. An internal pressure boundary was then applied to the rock mass roof/floor/walls equal to the initial pre-excavation pressure for both the horizontal and vertical directions, thereby maintaining the initial state of equilibrium within the model. This internal pressure was then reduced in ten equal stages, cycling the model to a state of equilibrium with each stage. This technique allowed the rock mass to respond gradually to excavation without the “shock” and resulting artificial numerical yielding of instantaneous removal of the panel material [5] allowing for a pseudo-static response. This approach staged pressure reduction also simulates, to some extent, the three-dimensional aspect of excavation advance whereby the rock mass at any given point “feels” the effect of the excavation face approach and then pass before finally relaxing to a state of fully-excavated equilibrium. This approach was used for each panel excavation until all four were excavated from east to west. Modelling was carried out using *FLAC*'s "large strain mode" where gridpoint coordinates were updated continually with model advance [5].

5.2 Model Calibration and Verification

As mentioned previously, numerous past modelling studies have been carried out to evaluate various mining/rock mechanics conditions throughout the Green River area mines. As a result, there is a reasonable level of understanding of both mechanisms and geotechnical conditions (*i.e.*, *in situ* stress and mechanical behaviour of geomaterials). However, the data available from the NW District longwall mining presented a unique opportunity for Solvay Mine to calibrate a numerical model to a high level of refinement.

5.2.1 Calibration Procedure

Numerous analyses were carried out to calibrate the *FLAC* model and the results indicated that the brittleness of the sandstone and shale units significantly impacted the model, resulting in distinctly different characteristic model behaviour. With properties that were too ductile, it was found that insufficient yielding of the Tower Sandstone occurred resulting in a subsidence trough with an insufficient magnitude of maximum subsidence compared to the actual response. Alternatively, with properties that were too brittle, it was found that yielding was pervasive through the Tower Sandstone resulting in a subsidence trough that was excessively steep with an excessive magnitude of maximum total subsidence. Moreover, for overly brittle conditions a clear trend showing the surface expression of the chain pillars was apparent. This characteristic was not observed in the actual field measurements (see Fig. 4). Therefore, the appropriate brittleness parameters were somewhere in between these two characteristic responses, and this was the key model parameter requiring calibration.

The material properties for the Geotechnical Domains (*FLAC* input parameters) were based on those described in Section 3. At least two-dozen difference *FLAC* simulations were run in order to arrive at a model that seemed to best match the instrumentation data and the observations of mining response in the NW District. Some of the key conditions that were explored in the calibration study were: the thickness of the Tower Sandstone; variation in the constitutive models; the strength parameters; and the post-peak strain softening behaviour.

There was some uncertainty regarding the thickness of the Tower Sandstone in the NW District. Models were run with a thickness of 43 m and 83 m. Models were run with both an *SS* constitutive model and with a more sophisticated *SUBI* constitutive model for the shale units to represent the pervasive bedding within those units. There was some uncertainty in the strength parameters of the rock units. The initial model parameters were based on previous modelling studies, but were refine based on the overall model response. In particular, the cohesive strength of the shales and sandstone units were varied within the range of expected values for each unit (see Table 2). The post-peak strength parameter ε^{p}_{crit} was varied, particularly the value for the Tower Sandstone, in order to "fine tune" the Calibrated Model. Equation (1) was used as a starting point for ε^{p}_{crit} (as a function of *GSI)* and further calibration was done by applying a *factor* to those initial ε^{p}_{crit} values without zone size correction.

Fig. 8 shows model-predicted subsidence profiles (shown after excavation of all four panels only) compared to actual measurements for a select number of representative *FLAC* model calibration case runs, of the more than two-dozen runs. A summary of the analysis details and results for the four Calibration Cases, corresponding to the data plotted in Fig. 8, is provided in Table 3.

Table 3. Summary of Calibration Cases

Calibration Case	Analysis details	Comments about results
1	Initial rock mechanics parameters (strength and brittleness) were used with a thin Tower Sandstone unit (43 m thick).	The subsidence profile showed yielding through the tower and the imprint of the pillars wass clearly visible.
2	Initial rock mechanics parameters with a thick Tower Sandstone (83 m) using the *SS* model for shales.	The model resulted in limited yielding of the Tower and significantly under-predicted the subsidence magnitude.
3	Thick Tower with reduced strength of the shale and increased strength of the Tower. Using the *SS* model for shale above Tower and *SUBI* model for the shale below the Tower.	The model resulted in limited yielding of the Tower and under-predicted the subsidence magnitude.
4	Thick Tower unit using the *SUBI* model for all shale units, refined strength parameters and increased brittleness (*i.e.*, reduced ε^{p}_{crit}).	The model results show a very good match of the measured subsidence, in particular the *progression* of subsidence with mining of each panel.

Calibration Case 4 was deemed to be the best match, on the basis of subsidence alone, and was considered to be the “Calibrated Model.” Moreover, the thicker Tower Sandstone unit and use of the *SUBI* model for the shale units has a geological justification. A more detailed comparison of the Calibrated Model with the field data is provided in the following sections. The constitutive models and material parameters determined for each of the Geotechnical Domains for the Calibrated Model are summarized in Table 4.

Table 4. Input parameters for *FLAC* Calibrated Model

				Peak		Residual	
Geotechnical Domain	Constitutive Model	Bulk's Modulus (MPa)	Shear Modulus (MPa)	Cohesion (kPa)	Friction Angle (°)	Cohesion (kPa)	Friction Angle (°)
Lower Shale	*MOHR*	980	535	575	28	0	25
Oilshale Floor	*MOHR*	145	65	250	20	0	25
Bed 17 Trona	*SS*	3055	1655	1340	49	0	25
Roof Shale	*SUBI*	980	535	290	28	0	25
D Sandstone	*SS*	1225	925	1150	37	0	30
Upper Wilkins Peak	*SUBI*	980	535	290	28	0	25
Tower Sandstone	*SS*	1225	925	1150	37	0	30
Laney Shale/Bridger Formation	*SUBI*	980	535	290	28	0	25

The discontinuity properties in the Calibrated Model, representing bedding planes that were accounted for using *FLAC* interfaces and the joint properties of the *SUBI* model, were: peak cohesion = 0.5 MPa and peak friction = 18°; with residual cohesion = 0 and residual friction = 18°.

5.2.2 Subsidence Comparison

Surface subsidence measurements are particularly useful for model calibration because they can be directly measured with conventional geodetic methods from ground surface with a high level of confidence. Because the unique stratigraphic profile (*i.e.*, Tower and D Sandstone units, and a trona seam with an oilshale floor) significantly affects the overall rock mass response to mining, subsidence measurements *include* the effects of the individual units (*system response*). Given a reasonable confidence level in the characteristics of the individual units, it is this system response that is most important to calibrate in the model.

The results of the fully mined calibrated *FLAC* model showing vertical displacement and model yielding is shown in Fig. 9 and the resulting model-predicted subsidence profile is shown in Fig. 10. As a result of longwall panel mining, yielding occurs in the pillar/abutment sidewalls and weak oilshale floor. As the roof relaxes downward, tensile yielding occurs at the roof-pillar intersection corner and the pillar center. Shear yielding begins to develop at the panel edge moving upwards through the roof shale. The shear yielding seems to temporarily "jump" the D Sandstone and re-occur in the Upper Wilkins Peak. Eventually, the D Sandstone and overlying units yields up to the base of the Tower Sandstone and drop into the panel, leaving a gap/separation between the Upper Wilkins Peak and the Tower Sandstone.

The Tower Sandstone bends and begins to yield in tension above the panel. The development of a gap (separation) at the base of the Tower in combination with model-predicted tensile yielding within the bottom of the Tower, indicates that dilation of the rock mass occurred. The continued bending of the Tower induces tensile yielding in the upper portion of the Tower, above the pillar sections. This combination of tensile yielding on opposite sides of the Tower "beam" reduces the effective thickness of the beam, thereby increasing its tendency for bending. Once the Tower has yielded through, a chimney-type mechanism occurs in the upper units with greatly increased surface subsidence. This can be seen above Panel 1W2N in Fig. 9. Board and Damjanac [11] also described observing a similar mechanism for longwall mining at Solvay. A fully connected vertical line of yielded elements forms at the western edge of panel 1W4N that results in concentrated vertical displacement likely as a result of the east-to-west mining sequence used in the simulation.

Subsidence profiles labeled 2005A and 2005B in Fig. 10 were measured when 2.5 panels where excavated (see Fig. 3). The model subsidence profiles corresponding to 2 (1W2N) and 3 (1W3N) excavated panels bracket the measured results for the 2005 profiles. Although the measured values are somewhat steeper on the east side than the calibrated model results, a good match to the data was achieved. The 2006 measured profile should correspond with the three mined panels stage (1W3N). The maximum magnitude of subsidence was within about 10% of the measured value and matched the trend on the east side of the model well; however, the west side model-predicted subsidence did not show as good a match.

The most notable difference between the model and the measured data was that the model appears to "hang up" in a small portion above the 1W2N – 1W3N chain pillar. This resulted in a kink in the left side of the subsidence trough, reducing the maximum magnitude of subsidence at this stage. Yielding did not seem to pass completely through the Tower Sandstone above Panel 1W3N as it did above Panel 1W2N (see Fig. 9), and this seems to be responsible for the observed effect. This could be due to the limitations of the two-dimensional model geometry, mining sequence simulation, or the constitutive model. However, this difference was relatively minor and the properties used in this model have provided an overall strong match to observations.

Development of the failure mechanism that results in this Calibrated Model response is shown in Fig. 11. After mining a single panel (Fig. 11a), inclined shearing bands develop at the panel edge resulting in minor surface subsidence (15 cm predicted). After a second panel was mined (Fig. 11b) a yielding mechanism developed due to interaction of the two mined panels that allowed for movement of the material above the inter-panel pillar into the gob with resulting increased surface subsidence (52 cm predicted). Load shedding from the inter-panel pillars to the gob material can be expected with this type of yielding mechanism.

5.2.3 Pillar Stressmeter Measurement Comparison

Basic calibration of the numerical model was primarily done using the subsidence measurements and past experience modelling studies at Solvay Mine, as described in the previous section. The stressmeter data provided an opportunity to compare the longwall panel-scale instrumentation data to the model. However, it should be stated that stressmeter data can be heavily influenced by localized influences, such as fracture formation, especially in high stress zones. The model with parameters that resulted in the best match of subsidence was compared with the stress measurements described in Section 4.2 to further evaluate the calibration.

The model indicated that unloading occurs above the excavated panels with vertical pressure of up to nearly 13.8 MPa in the gob. Maximum vertical stresses of approximately 34.5 MPa occur in the pillars and abutments. The stresses after mining all panels agrees well with a simplified tributary area estimate of σ_p = 35.1 MPa from the following equation [31]:

$$\sigma_p = \gamma z \left(1 + \frac{w_o}{w_p}\right) \tag{2}$$

Where:

σ_p - average vertical pillar stress

γ - unit weight of rock mass (0.025 MN/m^3)

z - depth below ground surface to pillar mid-height (470 m)

w_p - width of pillar (83 m)

w_o - width of opening (165 m)

In the model, vertical stress within the east side of the chain pillars is lower than the west side due to the east-to-west excavation sequence that induced yielding on the east portion of the pillars before higher stresses concentrate on the west side. This results in asymmetrical loading on the pillars with mining advance. Addressing this stress progression is one of the strengths of numerical modelling analysis. The evolution of vertical stress was recorded throughout the model runs (*i.e.*, as the model cycled through all stages of excavation) at locations within the model corresponding as closely as possible to the stressmeter locations to allow for comparison between model and actual measurements.

As a means of comparison between actual and model stresses, the average vertical stress change for the instruments, grouped by location, is shown in Fig. 12. There was no instrumentation data to match Stage 1 (one panel mine and prior to instrumentation installation). This plot shows that the model and field measurements are an overall good match for Stage 2 (after two panels were mined). At Stage 3 (after three panels were mined) the trends do not match, particularly at Area C where the instrumentation shows significantly greater stress levels. No instrumentation data was available for Stage 4 (after all panels were mined); however, as mentioned above the model achieved a reasonable match to tributary area theory at this final stage of mining.

At Area C, data for only one IRAD stressmeter location was available for comparison at Stage 3 where the greatest discrepancy occurs. The "hang up" in this region of the model discussion in Section 5.2.2 contributes to some of the discrepancy. The average measured vertical stress change at Area C was about 40 MPa, which is 7% higher than the total maximum vertical stress in the pillar. Based on the *FLAC* model-predicted stresses, and observations of good gate pillar performance, it seems unlikely that stress change at this location could approach the measured value of 41 MPa. If the pre-mining *in situ* vertical stress of 12.4 MPa is added to the measure stress change of 41 MPa, this would result in total vertical stresses of nearly 55.2 MPa at Area C during Stage 3. Based on the *UCS* = 45 MPa for Bed 17 trona, and the measured vertical stresses at Area C, the ratio of vertical pillar stress (σ_p) to *UCS* is greater than 1. Based on established pillar strength relations such as Lunder and Pakalnis [32], for a width-to-height ratio of about 14, significant stress-induced instability (*i.e.*, shearing / crushing / spalling) could be expected in the pillars. The pillar damage predicted by the Area

C recorded stresses was *not* observed within pillars at this location in the mine; therefore, it seems reasonable to conclude that the measured stress change at Area C after 2 panels were mined is likely incorrect or strongly affected by localized conditions, and that stresses from the model are within expected values. Likewise, the stressmeters at Area A (lower than anticipated magnitudes) are likely also impacted by localized fracturing and pillar yielding.

6. Three-Dimensional Back Analysis of SW District Longwall

Based on a comparison of subsidence, stress measurements and mining observations, a reasonable calibration of the *FLAC* model appears to have been achieved. The main purpose of the calibration exercise was to develop a *FLAC* model suitable to verify and refine the design of additional longwall panels in the SW District (see Fig. 1). A two-dimensional *FLAC* model of the SE District longwall panels was developed for a first stage design evaluation; however, the results of the two-dimensional analysis will not be discussed in this paper.

The two-dimensional analyses discussed thus far have been useful because various analyses could be carried out rapidly for the calibration study and the interpretation was reasonably straightforward. However, longwall mining advance is truly a three-dimensional problem; therefore, three-dimensional analyses using *FLAC3D* were conducted to compliment the two-dimensional models and provide further insight into mining performance. The main purpose of the three-dimensional analysis was to evaluate panel design dimensions for the SE District where the Tower Sandstone is thinner and where the depth from ground surface to Bed 17 is greater, compared to the NW District. In particular, to determine if reduced chain pillar width and increased longwall panel width would result in mining performance significantly different from that of the NW District longwalls.

At the time of the analysis the westernmost panel of the SE District had been mined (see Fig. 1). Because of the presence of TDR cable in a borehole located in the first mined panel (see Fig. 5) of the SE District, this provided an opportunity to further validate the Calibrated Model assumptions in a three-dimensional model.

The three-dimensional model geometry was similar to the two-dimensional model with similar zone sizes and aspect ratios. The aspect zone ratios were typically 1:1:1 (z-vertical:x-horizontal:y-horizontal) in the critical strain softening units, such as the Tower Sandstone; 1:2:2 in some of the shale units; and up to 1:4:4 in the Oilshale where perfectly plastic post-peak behaviour makes zone aspect ratio of lesser importance. The material properties and constitutive models that were calibrated in *FLAC* were also used in the *FLAC3D* model. Again, interfaces were included between major geological units to allow for separation and potential bulking behaviour. In order to increase calculation efficiency, the upper strata *above* the Tower Sandstone was not directly modelled, but instead was represented by a uniform pressure equal to the weight of the equivalent overburden. Use of symmetry boundary conditions was made along vertical planes through the middle of the panels oriented east-west (normal to panel direction) and north-south. The mining of three panels could be efficiently simulated this way. The model geometry showing stratigraphy is shown in Fig. 13.

Excavation of each panel in a manner similar to that carried out in the two-dimensional analyses was considered to be a practical simplification (*i.e.*, instantaneous excavation of each panel). This was done by removing the panel material and replacing it with stabilizing reaction forces that were reduced in ten equal stages until the excavation-induced stresses had been completely “relaxed” to a state of final equilibrium. The westernmost panel that was mined prior to the modelling exercise had a panel with of 190 m. In order to evaluate mine dimensions, *FLAC3D* models with panel widths of 190, 198 and 213 m, all with gross pillar widths of 82 m (39-m-wide chain pillars), were analyzed. In addition, a reduced pillar width model with gross pillar width of 73 m (34-m-wide chain pillars) was analyzed for a 190 m panel width.

A plot showing a section through the centre of the westernmost SE District longwall panel is shown in Fig. 14. This plot shows that the panel stayed open until 30 m of model face advance (or 60 m of actual excavated panel length due to the symmetry assumptions), before roof collapse due to delamination of the roof shale from the D Sandstone. After 107 m of advance (214 m of open symmetrical excavation), a gap can be seen in the model between the D Sandstone and the roof shale, indicating delamination near the face. At the same advancement distance the D Sandstone and Upper Wilkins Peak formation have separated from the Tower Sandstone and fallen into the panel approximately 30 – 45 m behind the face in the model. This corresponds to a 14 – 20º angle of break from the panel face to separation at the base of the Tower Sandstone. This is in agreement with the 18 – 20º degree angle of break observed in the field based on TDR measurements of early-stage mining in the SE District (see Fig. 15). After 213 m of advance (426 m of open symmetrical excavation) in the model, yielding of the Tower Sandstone into the panel can be seen approximately 122 m behind the excavation face. This trend of initial separation below D Sandstone at the mining face, followed by delamination from the base of the Tower Sandstone and then collapse of the Tower Sandstone, continues for the remainder of the panel excavation. No large “hang-up” of the Tower Sandstone followed by a significant collapse (i.e., delayed failure) seems to occur with the dimensions and stratigraphy used in the simulation.

The agreement with TDR measurements, using a three-dimensional model based on assumptions from the two-dimensional calibrated model, is particularly compelling support for the validity of the calibration assumptions.

7. Summary and Conclusions

An integrated design program utilizing instrumentation and monitoring, along with numerical modelling has been used by Solvay Mine and their rock mechanics consultant Itasca Consulting Group, Inc. for more than two decades. Over that time, qualitative and quantitative (*i.e.*, instrumentation monitoring) observations of the ground response to mining, at the scale of mine openings/pillars up to overall regional mine-scale have been collected. These observations have been used to continually refine predictive numerical models. The models suitably captured the complex behaviour of the mine due to the significant contrasts in the mechanical properties of the stratigraphic units, particularly the massive Tower Sandstone.

Longwall mining carried out in the NW and SE Districts of Solvay Mine presented an excellent opportunity to develop and validate calibrated two- and three-dimensional models. Model input parameters (*e.g.*, strength of various units) were available from numerous past modelling studies and the calibration exercise carried out in this study indicated that the most critical parameters required for calibration were the post-peak strain softening parameter ε^{p}_{crit} (*i.e.*, brittleness) of the units. After conducting more than two dozen *FLAC* models as part of this study, a Calibrated Model was arrived at that provided a strong match to instrumentation data, particularly surface subsidence, and mining observations. Moreover, a three-dimensional *FLAC3D* model using the calibration assumptions was able to capture TDR observations of behaviour within the stratigraphic units.

The calibrated two- and three-dimensional models were able to provide a reasonable match to actual ground surface subsidence, in-seam (Bed 17) pillar stresses, and TDR measurements. By obtaining a reasonable match of the measured field data throughout the stratigraphy and in both two and three dimensions, a well-calibrated model was developed. The consistent similarity in rock mechanics mechanisms and instrumentation data between field observations and the Calibrated Model allow Solvay Mine to utilize numerical-modelling as part of an ongoing mine design involving more complex conditions (*e.g.*, solution and room-and-pillar mining).

On the basis of the understanding of conditions and mechanisms gained from this study, future work can include additional complexities. For example, larger scale and more detailed three-dimensional modelling, direct modelling of gob development and greater refinement of geotechnical conditions (*e.g.*, *in situ* stresses and material properties).

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Fig. 12 Comparison of the average vertical stress for each area grouping (see Fig. 5) from stressmeter measurements compared to *FLAC* model predicted stress.

Fig. 13 Geometry of the *FLAC3D* model for evaluation of longwall mining in the SE district.

Fig. 14 *FLAC3D* model of SE Longwall with sequential excavation advancement in 15 m-long increments. Note that because of the symmetric boundary condition ("rollers") on the left side of the model, the excavation length is twice that shown in the model. The "true" excavation length is shown in the figure labels.

Fig. 15 Observations of bed separation from TDR instrumentation located in the westernmost longwall panel in the SE District (see Fig. 5).

MANUSCRIPT FIGURES

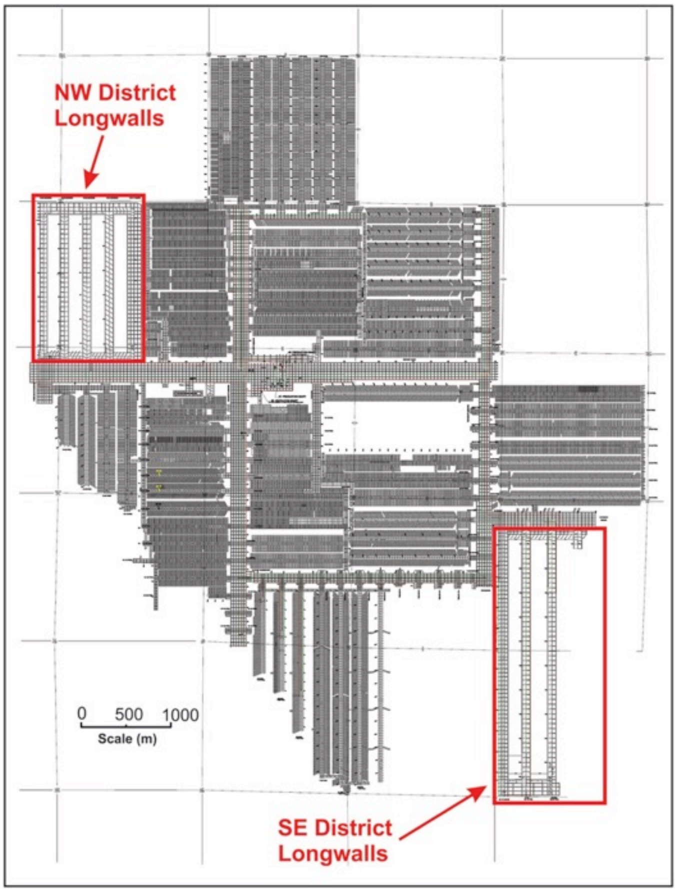


Fig. 1 Solvay Mine layout as of May 2007 showing NW District longwall panels used for model calibration and SE District longwall panels: the subject of the design study. The majority of mine is comprised of room and pillar panels.

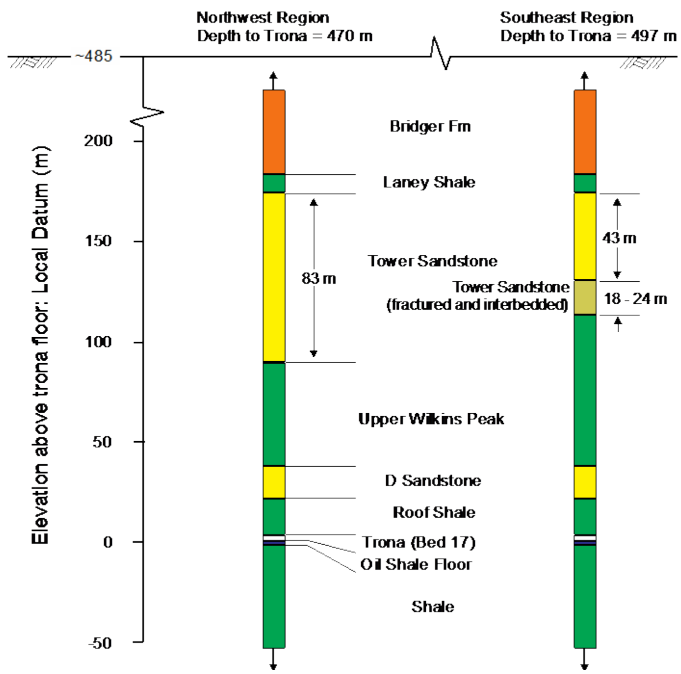


Fig. 2 Stratigraphy of the NW and SE Districts are similar; however, the Tower Sandstone is substantially thicker in the SE District and the depth from surface to Bed 17 is greater in the SE District.

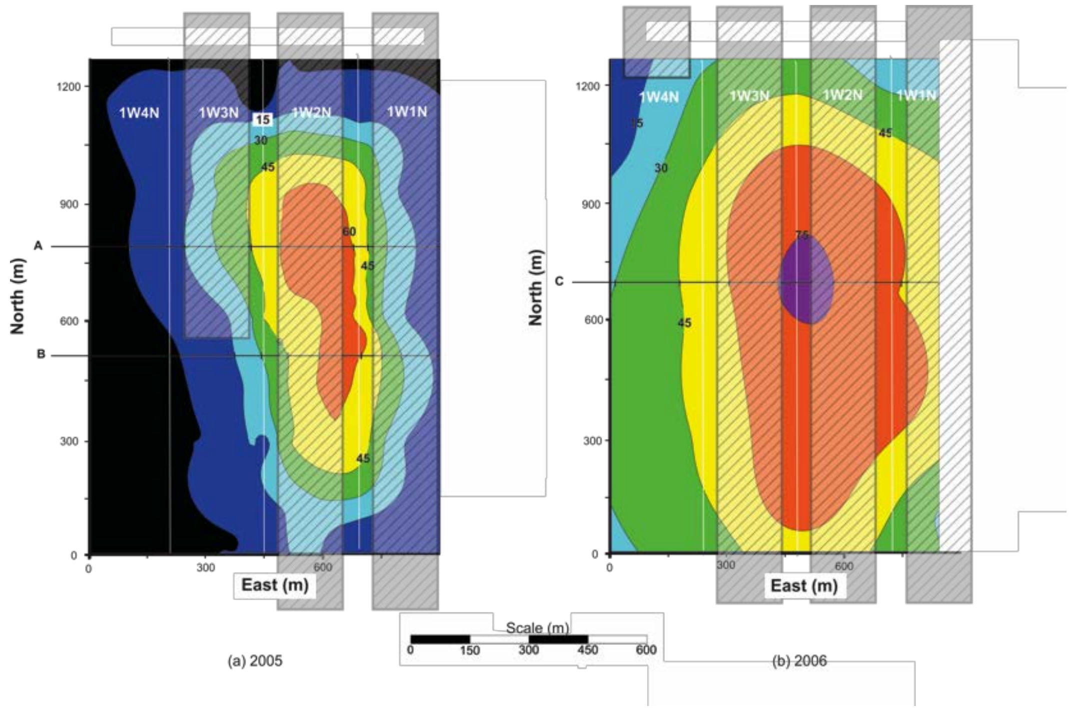


Fig. 3 Contours of surface subsidence (from geodedic surveying) for the NW District with mined out longwall panels superimposed (Modified from [16]**). Mined out portions of the longwall panels are indicated by hatching. Sections A, B and C are identified.**

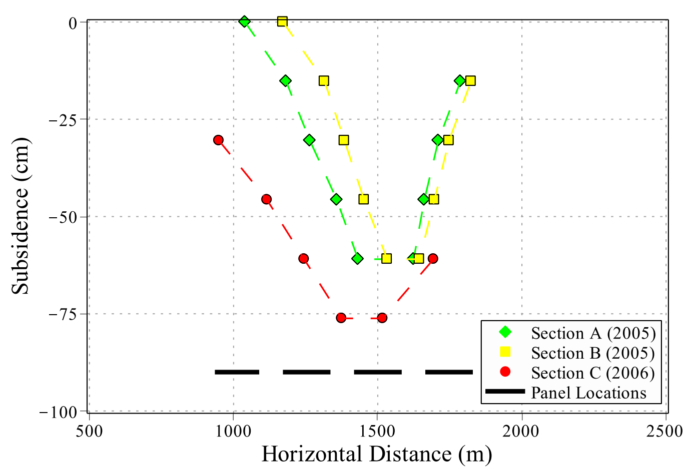


Fig. 4 Measured subsidence along section lines. The section line locations are shown in Fig. 3.

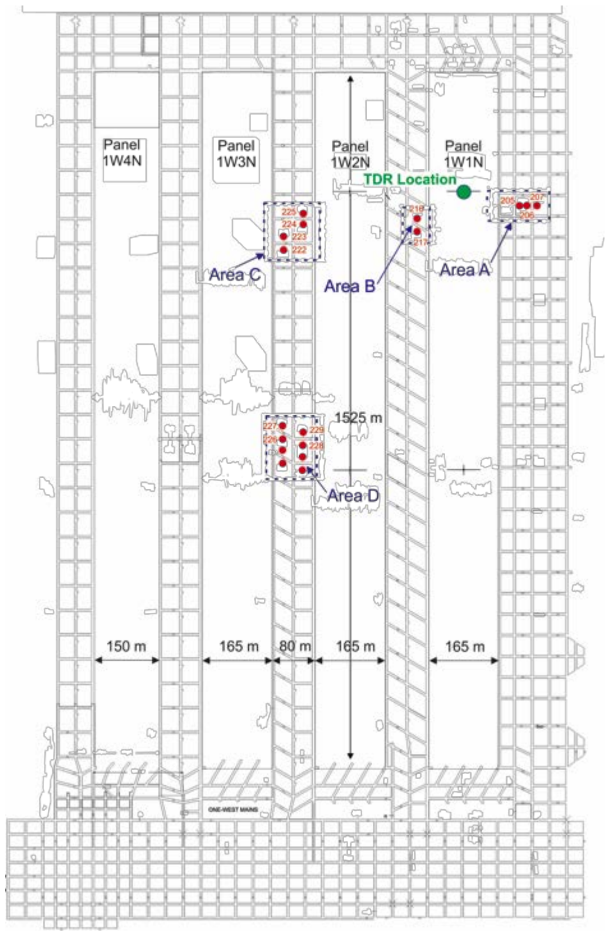


Fig. 5 Location of IRAD stressmeter cells and area grouping in the NW District longwall panels.

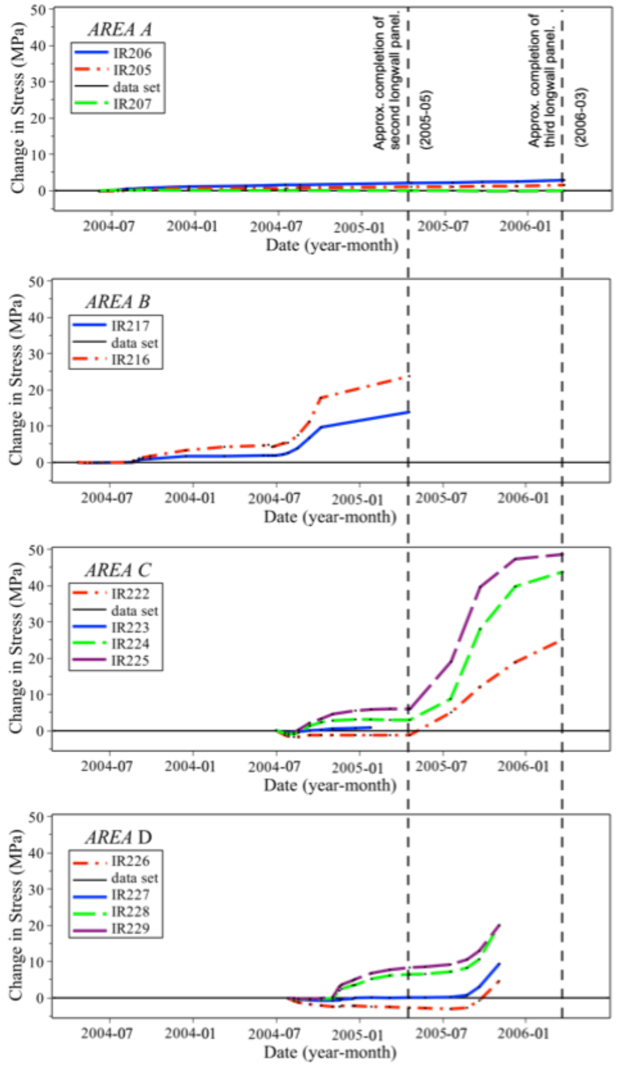


Fig. 6 Stress measurements from IRAD stressmeters (locations shown in Fig. 5) within chain pillars in NW District (original data from [16]**).**

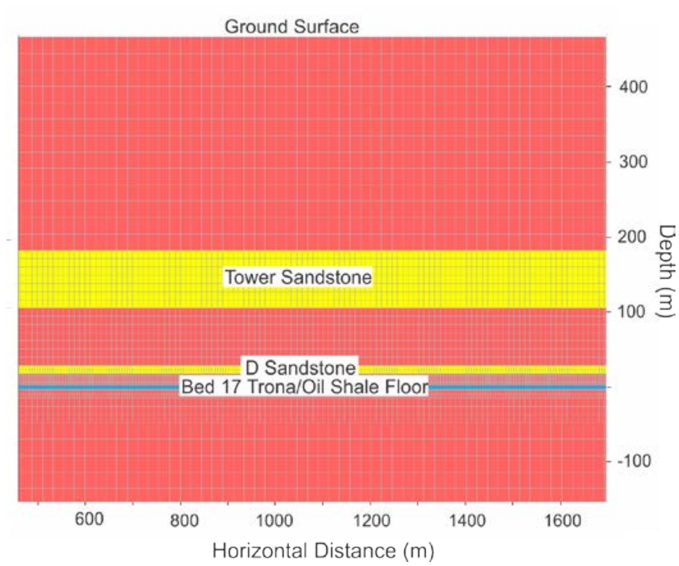


Fig. 7 *FLAC* model grid used for NW District longwall back analysis. Note that the model truncated laterally and vertically in this image.

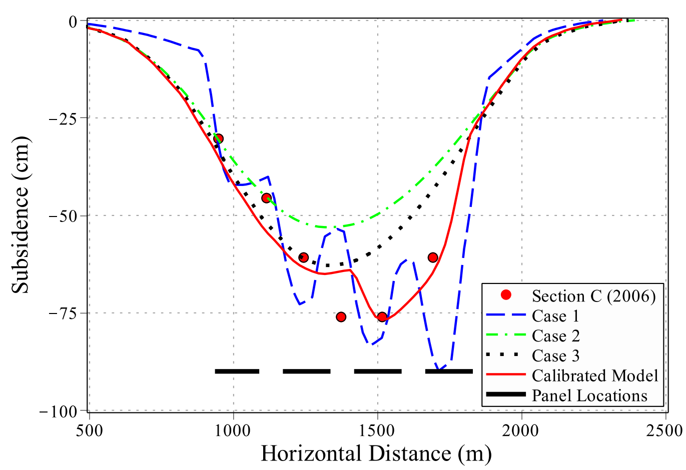


Fig. 8 Comparison of various *FLAC* model runs, including the Calibrated Model, with actual conditions for the fully mined stage along Section C (Fig. 3).

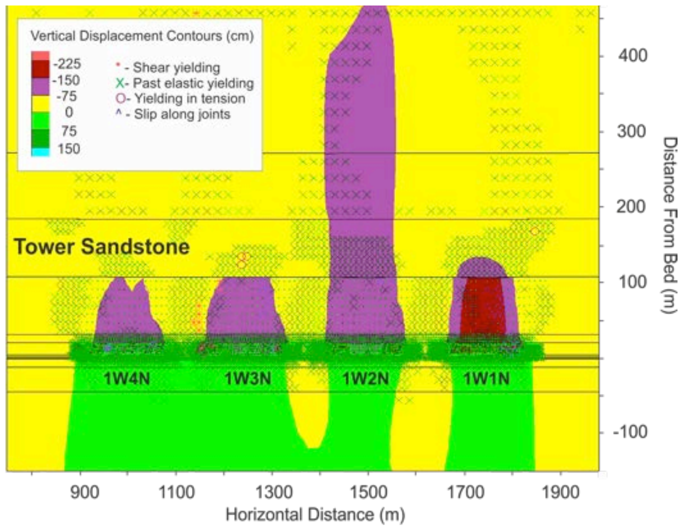


Fig. 9 ***FLAC*** **model results after excavation of all four longwall panels (panels excavated right to left). Yielding through the Tower Sandstone unit and active failure along the "leading edge" (left side of the model) resulted in a steep subsidence trough with up to 75 cm of subsidence. Note: negative values represent downward displacement.**

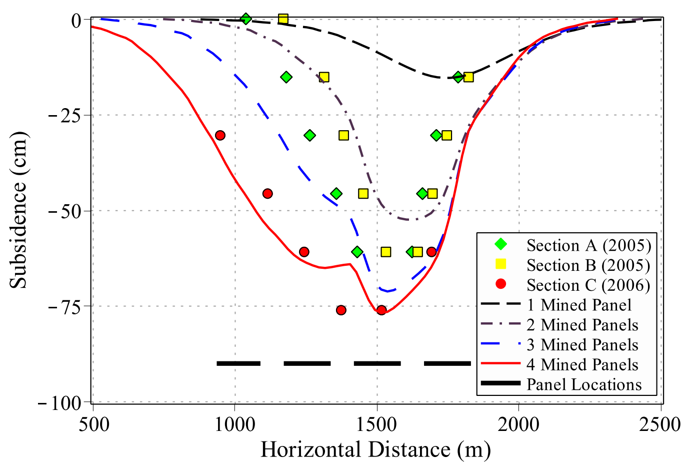
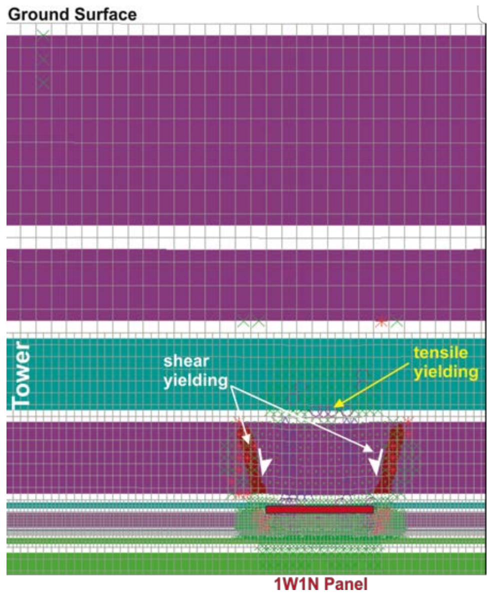
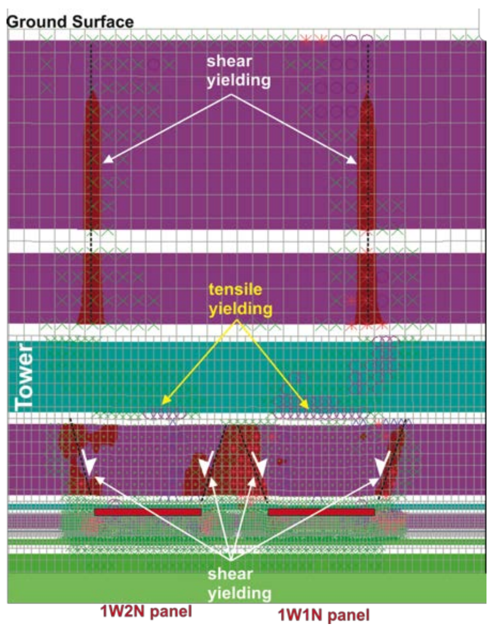


Fig. 10 Comparison between field-measured and ***FLAC*** **Calibrated Model predicted subsidence along Section C (Fig. 3).**



(a) Single panel mined with a maximum predicted subsidence of 15 cm.



(b) Two panels mine with maximum subsidence of 52 cm.

Fig. 11 *FLAC* model results showing yielding and contours of cohesion (red indicates strain softening to residual cohesion). The yielding mechanism that develops due to interaction between panels results in a wedge-like shearing above inter-panel pillars associated with load shedding into the gob. Note that circles represent tensile failure and crosses represent shear failure.

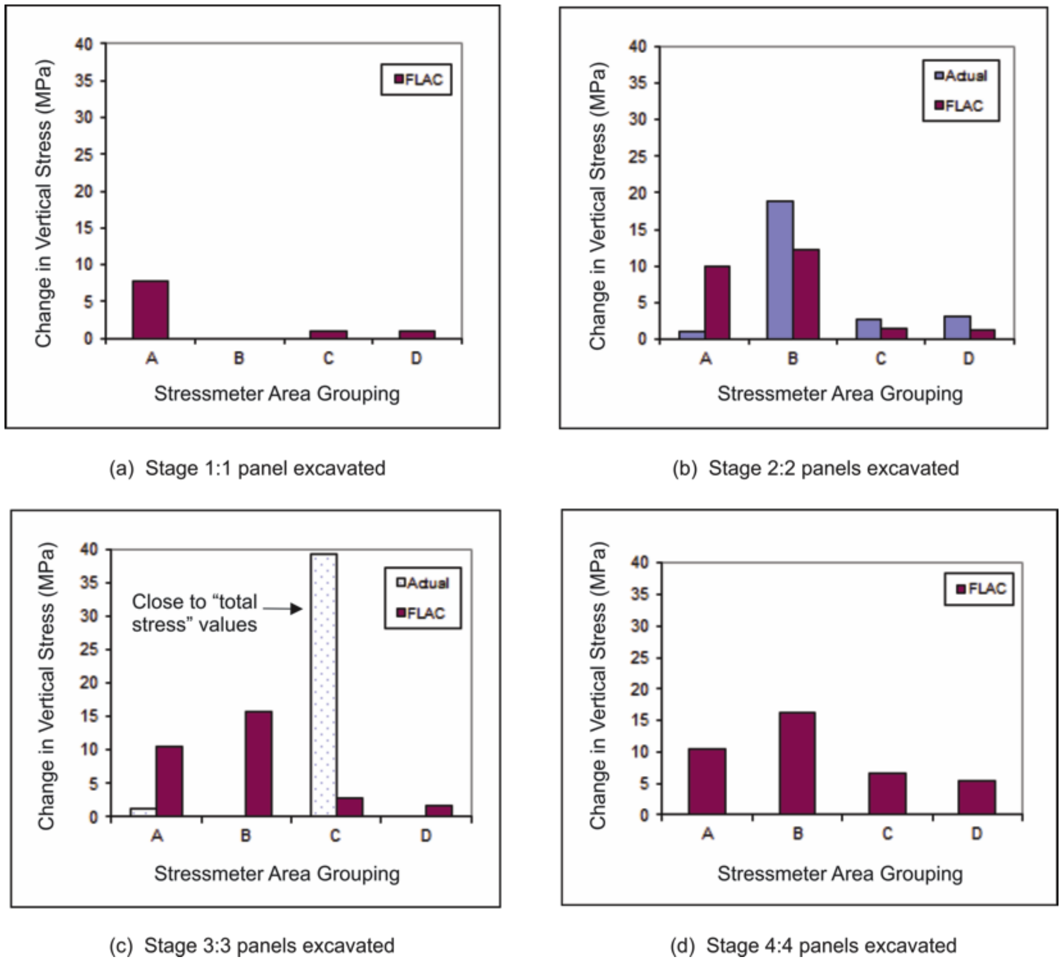


Fig. 12 Comparison of the average vertical stress for each area grouping (see Fig. 5) from stressmeter measurements compared to *FLAC* model predicted stress.

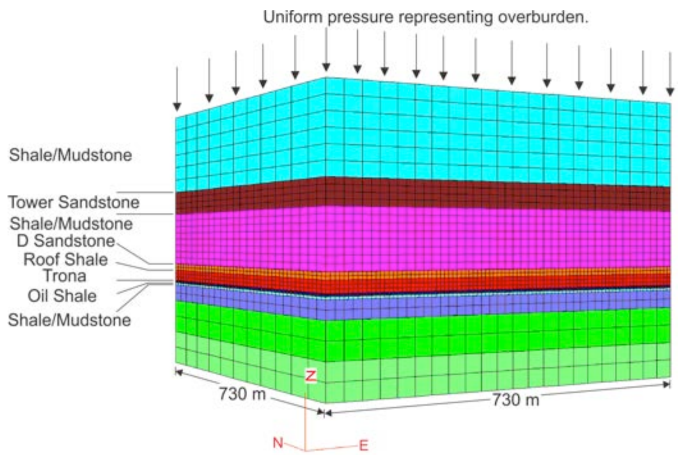


Fig. 13 Geometry of the *FLAC3D* model for evaluation of longwall mining in the SE district.

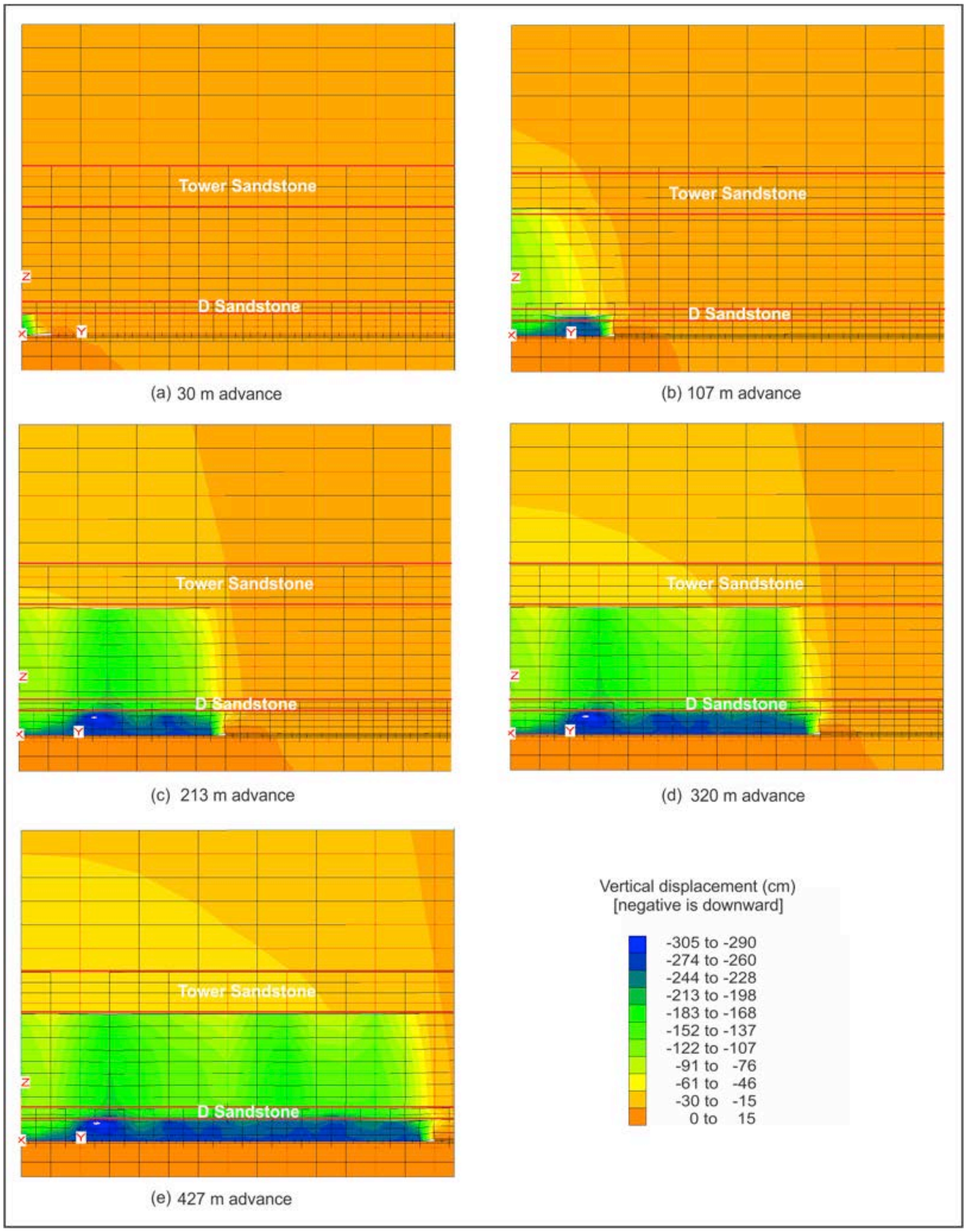


Fig. 14 *FLAC3D* model of SE Longwall with sequential excavation advancement in 15 m-long increments. Note that because of the symmetric boundary condition ("rollers") on the left side of the model, the excavation length is twice that shown in the model. The "true" excavation length is shown in the figure labels.

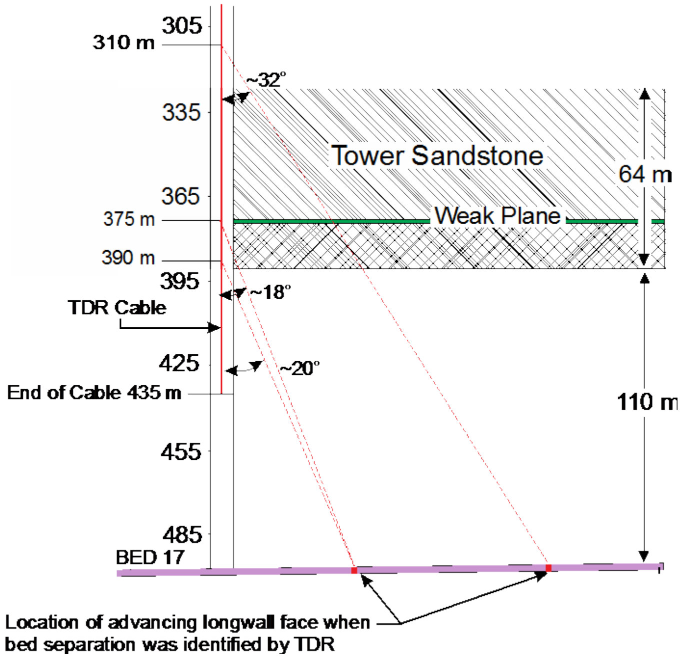


Fig. 15 Observations of bed separation from TDR instrumentation located in the westernmost longwall panel in the SE District (see Fig. 5).