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APPLICATION OF FUZZY SYSTEM TO MODEL ECONOMIC OPERATIONS OF POWER SYSTEMS

by

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List of Abbreviations

ED	Economical dispatch.
ABED	Active power balance equation.
LP	Linear programming.
FLP	Fuzzy linear programming.
FNLP	Fuzzy non-linear programming.
OPF	Optimal power flow.
TMF	Triangular membership function.
DM	Decision-making.
FLF	Fuzzy load flow.
ECED	Emission constrained economic dispatch.
DP	Dynamic programming.
ELD	Economic load dispatch.
EMS	Energy management system.
SLP	Successive linear programming.
MOSST	Multi-objective stochastic search technique.
SCED	Security constrained economic dispatch.
GA	Genetic algorithm.
LR	Lagrangian relaxation.
FLF	Fuzzy load flow.

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Abstract

The purpose of Economic Dispatch or Optimal Dispatch is to reduce fuel costs for power systems. The minimum cost is obtained if the hard constraints and soft constraints are satisfied. The hard constraints imposed on the system can not be violated; however, the soft constraints can be violated to some degree. This violation is related to power system parameters which deal with uncertainty due to fluctuations in model parameters such as load variations, noise in measurements, weather condition changes etc. For this reason there is a need more than ever for a fuzzy model to be developed to overcome this uncertainty. In this thesis the problem of fuzzy optimal economic dispatch and nonlinear optimal power flow optimization under a fuzzy load is introduced and formulated to minimize the total cost production of a network. This thesis implements three methods in formulating the economical dispatch of all thermal power systems. It starts with a simple economic dispatch problem with a fuzzy load demand neglecting transmission losses, but including generation limits. Two generation units are tested for the formulation with various α -cut representations of fuzzy numbers in illustrating the evaluation procedure and to evaluate effect of the spread on the outcome. Next a problem with a fuzzy cost function coefficient with fuzzy load demand is analyzed and formulated to minimize the total optimal production cost. To evaluate the performance and the capability of reducing cost while varying cost function coefficients, a synthetic system example of three generation units is used. Finally, a more realistic model with fuzzy load, fuzzy cost function coefficients and power losses is formulated, evaluated and tested on a three generation unit system to obtain the optimal minimum cost. The fuzzy nonlinear optimal load flow is presented when the active generation, active load, reactive generation and reactive load are considered to be fuzzy. Three formulation methods were adopted. First a system with all crisp cost function coefficient with fuzzy active, reactive power is tested on a 9-bus system for one hour. Next a fuzzy load that varies on an hourly basis for 24-hours is tested on the 9-bus system, while keeping the load and generation of the other buses unchanged. Finally, a system with a fuzzy coefficients cost function with fuzzy active and reactive power is formulated and tested to generate a minimum cost function.

Chapter 1
Introduction

In their daily operation electrical utilities face many uncertainties that affect minimizing the cost function in the economical dispatch method and the optimal power flow operation of the network. It relies entirely on the power generated by the units committed to the network, the load supplied to the consumer and the constraints set to obtain a secure and optimal network operation. This uncertainty can propagate through the time horizon; significantly affecting future transaction opportunities, fuel prices, unit availability and system demand. In practice, uncertainty arises from the imperfect knowledge of the system performance and goals of operation as well. Heuristics, intuition, experience, and linguistic descriptions are obviously important to power engineers. Virtually any practical engineering problem involves some vagueness and imprecision in the problem formulation and subsequent analysis **[38]**.

The conventional methods applied to solve the ED and the OPF problems are divided into two groups. The first group is the variational (Lagrange multiplier) approach and principle of incremental fuel cost. The second group is the direct optimization methods such as NLP approach, dynamic programming as well as simulated annealing algorithm, quadratic separable programming and reduced gradient algorithms. These solution methods have many disadvantages and limitations restricted on the constraints imposed on the model. The crisp constraint must be satisfied 100% in order to obtain an optimal solution that leads to over conservative results. One of the disadvantages of the conventional method is when permissible limits of emission and overloads are clearly specified in a power system under study, these quantities could be incorporated into the OPF as operational constraints. However, in system planning studies, these limits posed on emission or overloads would be very ambiguous, thus making such treatment difficult. Also, in actual system-operations, it is necessary to maintain the system at a proper security and emission level even when generator or transmission line tripping do occur.

To attain this goal, system operating points should not be at constraints limits but need some operational margin. Furthermore some of the operation indices are in conflicting trade off relations; successful optimization cannot be attained through any of conventional optimization approaches. On the other hand fuzzy set theory, originally presented by Zadeh is an appropriate instrument to deal with limitation restricting the constraint model, the uncertainties in the power system parameters, and vagueness and/or imprecision. The effectiveness of this approach has been demonstrated in various applications in power systems operation, planning and analysis. The major advantage of the fuzzy set theory is that it can be used to model human judgments and inexactly expressed information. Fuzzy methods do not necessarily need any data from the past. However, some data may be used as a basis for human judgment and subjective estimates. Furthermore, human judgment and decision-making are important factors during the planning period. In reality, planning engineers must make up many alternative plans to allow for these uncertainties and future fluctuations of basic parameters such as fuel cost, demand forecast. The decision-maker must select one particular plan out what is the provided alternatives based on his/her subjective judgment on many ambiguous factors. Fuzzy approach translates the uncertainty involved in the parameters into a membership function and the constraints imposed on the system can be satisfied as much as possible for the planning purpose. Transforming existing information about loads, voltage sources, power generation and phase angles into fuzzy numbers with triangular shape membership functions that measure the conformance of a variable to a concept are presented. The fuzzy arithmetic operations are rules derived from Zadeh's algebraic operations and extension principle **[46]**. As fuzzy flows are related to a feasibility idea, one would like to go a little further into an operational concept, in the sense that power generation is driven by economics and therefore uncertainty in future implies uncertainty in dispatch decisions. This thesis presents an approach to assist in dealing with this concern. In order to represent this operational feature, one must, in some way try to optimize the uncertainty in generation cost. This means combining optimization and power flow we reach a fuzzy optimal power flow framework. Fuzzy sets and fuzzy mathematical programming has been developed significantly in recent years and many scientist and engineers are solving many problem encounters in the power system planning and operation regarding uncertainty involved in the objective function and constraints by transforming the ambiguity in the parameters to a fuzzy membership function. Most of the previous work in power system was to fuzzify certain parameters in the objective function such as, emission cost, start up cost of the generator, purchase transaction…etc. In addition fuzzyifing certain constraints such as, load demand using trapezoidal membership function in the equality constraint and fuzzyifing the inequality constraint such as, system reserved demand, transmission line losses and emission constraints. The previous work approaches was to overcome the limitation restricted on the objective and the constraints. In addition violating certain parameters in system constraint will enhance the system performance and give a wide information a bout the over all system reliability and security. The fuzzy linear programming is used to transfer the fuzzy parameters into a crisp value that relies on the judgment of the decision maker. If the goal of a certain objective or constraints is not thought of much, then it must be adjusted by redefining the associated membership function. The objective of this thesis is to imply the concepts of uncertainty in the parameters of the cost function, load demand, power generation, transmission power losses, reactive power, voltage magnitude and phase angles will be expressed as fuzzy in order to obtain a total optimal minimum cost of a number ***m*** thermal units subject to satisfy the equality and inequality constraints imposed on the system. The solution steps algorithm used to obtain this objective is list as follow:

- Fuzzyifing the parameters that affect minimizing the cost function by transforming the fuzzy variable into a "TMF" representation, which is the key to (DM) when dealing with uncertainty.
- Using α -cut representation, which is used to create a family of crisp sets in order to be used in fuzzy mathematical operations.
- Introducing the violation variable into the model to relax the strict constraints.
- Using fuzzy non-linear programming approach to transform the fuzzy variable into a crisp variable in the OPF problem.

- Solving the model under different levels of system constraint violation and analyzing the generated alternatives.

1.1 Outline of the Thesis:

A) Chapter 2:

In this chapter a description and a review of unit input output curves, types of load demand and formulation of a conventional or crisp methods analysis to economic dispatch and optimal power flow of real power generation is discussed. The ED formulation is divided into three categories. ED neglecting transmission losses and generation limits, economical dispatch including generation limits neglecting losses and economic dispatch including generating limits and transmission losses respectively. Finally optimal power flow analysis is discussed and formulated to obtain a total minimum cost.

B) Chapter 3:

In this chapter fuzzy sets mathematical operations, α -cut representation, membership function mathematical formula calculation and fuzzy optimization methods formulation are presented.

C) Chapter 4:

In This chapter the load demand of a simple crisp ED optimization problem neglecting transmission losses including generation limits solution is fuzzyfied using a triangular membership function representation. The total minimum cost of the ED problem is obtained using fuzzy mathematical operation and α -cut representation. A simulated example is used to verify the proposed algorithm.

D) Chapter 5:

In this chapter the cost function coefficients and the load demand of a crisp ED optimization problem neglecting transmission loses including generation limits solution are fuzzified using a triangular membership function representation. The total minimum cost of the ED problem is obtained using fuzzy interval arithmetic representation on a triangular fuzzy number implemented by their α -cut operation. A simulated example is used to verify the proposed algorithm.

E) Chapter 6:

In this chapter the cost function coefficients, the load demand and the transmission power losses of a crisp ED optimization problem are fuzzified using a triangular membership function representation. The total minimum cost of the ED problem is obtained using fuzzy interval arithmetic representation on a triangular fuzzy number implemented by their α -cut operation. A simulated example is used to verify the proposed algorithm.

F) Chapter 7:

Fuzzy optimal power flow with fuzzy active, reactive power generation and load demand is derived using FNLP approach by Werners. The constraints imposed on the system will be violated to some degree to obtain a crisp outputs between [0, 1] to satisfy the constraints imposed on the system such that the overall production cost is minimized and the power limits and generators capacity constraints are not violated. A simulated example of 9-bus system is used to show the effectiveness of the algorithm.

G) Chapter 8:

In this chapter a fuzzy active reactive power flow and the parameter of the objective function as fuzzy is derived using FNLP approach by Lai and Hwang. The objective function coefficients will be translated into a triangular membership function then the constraint will violated to some degree to obtain a triangular optimal minimum cost membership function. A simulated example is used to show the effectiveness of the algorithm.

H) Chapter 9:

In this chapter the conclusions, thesis accomplishments and recommendations for future researches are discussed.

2.1 Literature Review

A brief review of literature on conventional artificial intelligence-based fuzzy economic dispatch and optimal power flow concepts is presented in this section.

Reference [1] is a review of recent advances in economic dispatch. This reference divides the problem into four sub-problems; (1) optimal power flow, (2) economic dispatch in relation to AGC, (3) dynamic dispatch, (4) Economic dispatch with non-conventional generation sources. The disadvantages of the techniques used in OPF are the convergence property. Some authors have suggested quasi-Newton method and explicit Newton methods while others have used sparsity oriented techniques like the Hessian-based algorithms. Real-time solutions of the OPF are one area gaining a lot of momentum in the past few years. Such a solution implies the minimization of instantaneous cost of active power generation on an operating power system subject to preventing violations of operating constraints in the event of any planned contingencies. Such an on-line implementation requires fast execution times and minimum storage allocations. Undoubtedly, these constraints elevate the nature of the OPF to a high level of complexity.

Reference [2] presents an advanced engineered-conditioning genetic approach to the economic dispatch problem. The drawback of the simple forms of GA is preventing the acceptance of the theoretic performances claimed. Thus various techniques have been studied to improve the genetic search method. The adaptive mutation prediction proposed in this paper improves the computation time online and off-line.

References [3], [4], [13] and [14] present the application of the Hopfield Neural Network to the solution of large-scale economic dispatch problems. The proposed method is quicker and more accurate than other competing methods. Reference [5] presents a fuzzy logic controlled genetic algorithm applied to power system environmental/ economic dispatch. The proposed method is tested against a conventional one phase problem structure, and is proven to be more efficient and to improve the performance it employed Genetic Algorithms technique. Reference [6] presents an algorithm for optimal power flow (OPF) which is based on P/Q decomposition of the problem and on the combined application of quadratic separable programming methods to solve the economic dispatch problem. The developed quadratic-separable programming algorithm combines the main advantages of quadratic and separable programming. On the one hand, it considers the quadratic intervals of unit curves without any approximation and gives the direction to the optimum of the quadratic function. On the other hand, it gives the technique for separable OF, which provides convergence to the optimum of the quadratic-separable problem. An economic dispatch problem with transmission line capacity constraints is solved in [7] using the Neural Network approach and the Hopfield Neural Network.
An evolutionary-programming-based algorithm is used in Reference [8] for solving the environmentally constrained economic dispatch problem. The developed algorithms is capable of dealing directly with load demand specifications in different intervals in the schedule horizon with no restrictions on the shapes of the input/output functions of the generator and the shapes of the pollution functions, which represent the emissions. The algorithm has accurately and reliable converged to the global optimum solution. In addition the speed and robustness of the algorithm are discussed.
In Reference [9] the paper proposes a practical strategy based on Quadratic programming (QP) techniques to solve the real-time economic dispatch problem. This paper has presented a practical and effective solution strategy based on QP techniques for solving the RTED problems involving multiple constraints. It has also incorporated the GP techniques in the problem formulation by defining violation variables for the constraint equations, which guarantees the best available solution as close to the desired optimal solution as possible under emergency system condition. Reference [10] proposes an

algorithm based on a dynamic queuing approach to solve the power system short term economic security dispatch. The DC power flow and limitations on power transmission are taken into consideration. The new algorithm is novel since no multiplier is introduced but it has high calculating speed and good property of convergence. The application of the interior point method to solve the economic dispatch problem with network and ramping constraints is discussed in Reference [11] by including generator, ramping limits as constraints, as well as the network line flow, both economic and security issues are treated.

A multi-objective stochastic search technique for multi-objective economic dispatch problem in power system is presented in Reference [12]. It is a highly constrained problem with both equality and inequality constraints. The genetic algorithm as well as the simulated annealing algorithm is used to solve the problem. The results indicate that the new MOSST heuristic converges rapidly to improved solutions. MOSST is a truly multi-objective technique, as it provides the values of various parameters for optimizing different objectives.

References [15] and [16] presents a genetic algorithm for solving the unit commitment problem of a hydrothermal power system. It is a two-layer approach, where in the first layer, the genetic algorithm is used to determine the on/off status of the units. The second layer uses a nonlinear programming formulation for solving the problem using Lagrangian relaxation to perform the economic dispatch while meeting the plant and system constraints. The simulation results reveal that optimal tuning of GA parameters to guarantee fast convergence and high optimal solution is difficult and depends on the studied unit commitment problem. A homogeneous linear programming based interior point algorithm for the security constrained economic dispatch problem is implemented in References [17] and [19]. However, the method is general enough to deal with any combination of outages. An optimization method that incorporates the system losses into the SCED algorithm has also been developed and it is currently being tested.

Reference [18] presents a Hopfield model based approach for the economic dispatch problem. Using this model, an energy function composing power mismatch, total fuel cost and the transmission line losses is defined. The proposed model, unlike other neural networks, requires no training. Furthermore, because the method has a Hopfield modeling framework, hardware implementation for the proposed approach is promising because of the advantage of the real time response. In addition the Computation results reveal that the proposed method is superior to the conventional lambda-iteration method in computational requirement. The integration of evolutionary programming with Tabu search and quadratic programming methods are implemented in Reference [20] to solve the non-convex economic dispatch problem. The proposed method shows that the numerical results are more effective than other previously developed evolutionary computation algorithm.

References [21] and [22] present a hybrid genetic algorithm for solving the unit commitment economic dispatch problem. The algorithm of [22] uses a fast rule-based dispatch method to evaluate possible candidates of solutions. The algorithm has been shown to significantly improve the convergence. Scheduling rules have been incorporated in a fast approximate method of evaluating solutions, accelerating the computational time of the GA to competitive levels. The knowledge-based genetic algorithm has been applied to a representative test problem and shown to obtain better solutions than Lagrangian relaxation (LR) in similar computational times.

A dynamic economic dispatch that takes the limits of the ramping of the generator units is presented in Reference [23]. It proposes two solution methods. The first is to find a feasible solution even when the load profile is non-monotonic. The second is an efficient technique for finding the optimal solution.

Reference [24] proposes fuzzy modeling of power systems to take into account the qualitative aspects and vagueness or uncertainty that are not random in nature and therefore cannot be modeled by a probabilistic approach. The advantage of this approach is that it increases quality of information obtainable from the model and its computing simplicity. The disadvantage of this approach is that the voltage possibility distributions show a slight shift regarding the value calculated with the Newton Raphson method. This paper was used to formulate the fuzzy load flow approach using the AC model.

In Reference [25] the paper presents an interactive fuzzy satisfying method for solving optimal power system rescheduling by assuming that the decision-maker has imprecise or fuzzy goal and constraints. An interactive decision-making process is formulated in which the decision-maker can learn to recognize good solutions by considering all possibilities of fuzziness. In Reference [26] fuzzy logic is used to solve the load flow problem, Improving decreased computing time of the analysis was discussed. Consequently, the repetitive solution of the proposed fuzzy load flow (FLF) method requires only 2m calculations per iteration, where m is the number of buses in the system. The solution of the load flow problem was achieved in a very short computing time by means of the implementation of the FLF method on systems of various sizes. Consequently, the FLF method can be treated as worthwhile base, which is able to homogeneously incorporate all modern control strategies of load flow designed by means of fuzzy logic control.

In Reference [27] the exponential form of a static load model relating the active and reactive power components to the bus voltage, is considered and the effects of incorporating this load model in the optimal load flow solution of several test systems are studied. The investigation in this paper reveals that when load models are incorporated in load flow studies it is seen that for some systems the difference in fuel cost, total power loss and voltage are significant, whereas for some others these differences are not significant.

The modeling of constraints in Reference [28] is an important issue in power system scheduling. Constraints can be generally classified into two categories: 1) physical limits and 2) operating limits. A schedule violating physical limit or constraint would not be acceptable. An operating limit, however, is often imposed to enhance system security but does not represent a physical bound. The problem is first converted to a crisp and separable optimization problem. Lagrange multipliers are then used to relax system-wide constraints and decompose the problem into a number of unit-wise subproblems and a membership subproblem. The method and application used in this paper contribute greatly to my thesis computation process in modeling the constraints using fuzzy sets method.

In Reference [29], a fuzzy model for power system operation is presented. Uncertainties in loads and generations are modeled as fuzzy numbers. System behavior under known (while uncertain) injections is dealt with by a DC fuzzy power flow model. This paper shows that uncertainties in load or generations (not of probabilistic type) can be incorporated into power system models so as to give a better image of system behavior. System optimal (while uncertain) operation is calculated with linear programming procedures. Imprecision in power flow analysis is modeled using fuzzy set theory in reference [30]. The fuzziness of the power generation and loads used in a power flow analysis implies fuzziness in the outputs; a method is suggested for calculating imprecision through informative statements about the system generation data and availability of generating units in the power flow analysis which depends on the imprecision of the inputs. The advantage of this technique is that the number of power flow calculation is tremendously large, where as with fuzzy power flow analysis one run will provide much wider information about the system performance. This paper was used to implement the basic foundation for fuzzy optimal power flow calculation.

Reference [31] presents a new implementation of an LP algorithm for security-constrained preventive rescheduling of real power. A number of considerations are taken into account such as: (1) Rescheduling of generation and load (if required) is used to maintain a secure condition by avoiding line overloads. (2) Transmission losses are taken into account in the constraint function. (3) The line losses are brought into the optimization in the form of an equality constraint. The algorithm has options to choose selected generators or loads to take part in the optimization. The proposed method is tested on four representative power systems, with very encouraging results.

A novel multiobjective optimization technique for dynamic generation scheduling in an interconnected system is presented in Reference [32]. In contrast to existing generator scheduling methods, the proposed approach treats economy, security, emission and reliability as competing objectives for optimal dispatch of the local system generating units. Fuzzy logic techniques are incorporated in knowledge based system to solve this difficult multicriteria problem involving multiple conflicting objectives.

Reference [33] presents fuzzy and possibility theory employed to solve uncertain power flow problems. Three fuzzy power flow models, nonlinear, linearized and multilinearization fuzzy power flow models are introduced. The solution algorithms are also described. The numerical examples of IEEE 30 bus system are provided to demonstrate the effectiveness of proposed models. This model can be used in such application situations where accurate solutions are needed.

Limiting emissions plays a great role and is discussed in great detail in Reference [34], which presents a general formulation of the optimum economic load dispatch problem in a system with thermal plants taking into account the constraints on emission of sulfur dioxide and oxides of nitrogen. The optimum mix-ratio of high and low sulfur content fuels limit the sulfur dioxide (SO_2) emission per hour. The emission of oxides of nitrogen (NO_x) is minimized by reducing the output of the generating units with the high ratio of incremental (NO_x) emission to incremental fuel cost. The method proposed for considering the pollution constraints is simple, and can easily be incorporated in an existing economic dispatch program. The algorithm is tested on a plant with four generating units, and the results are presented.

The economic dispatch problem, in presence of non-monotonically increasing incremental cost generating units, is solved by using the Newton approach shown in Reference [35]. In addition a linear transmission loss model (1), is based on load flow solutions, is established and incorporated into the economic dispatch. Test results show that fast convergence can be achieved by using this approach. The linear loss formula provides both accuracy and simplicity, and takes into account many realistic elements. A fast convergence rate is achieved by the proposed approach.

Reference [36] presents a simple and efficient economic dispatch algorithm suitable for unit commitment. It also caters to any combination of polynomial cost functions. The algorithm reduces the economic dispatch into an equivalent lossless problem from which solutions are easily obtained analytically. The equivalent lossless problem enables the generators whose outputs violate their limit constraints to be handled efficiently and correctly. The algorithm can cater to both the B matrix and Jacobian matrix loss formulation. Case studies with various test systems are presented and discussed.

The classical procedure for solving the economic dispatch problem in the presence of upper and lower limits on the generation levels may fail to lead to the constrained optimum generation schedule. In Reference [37] a simple scheme suitable for real-time applications which resolves this drawback is presented. When transmission losses are neglected, the constrained optimum can be analytically computed based on a distance measure of the unconstrained optimum schedule to the violated limits. In the presence of transmission losses, the problem is first converted into an equivalent lossless case by a simple transformation which can then be solved by the proposed algorithm.

In Reference [38], system demand reserve requirements and prices of future purchase transactions are considered as uncertain, and the integrated scheduling and transaction problem is formulated as a fuzzy mixed integer programming problem for a power system consisting of thermal units and purchase transactions. Based on the symmetric approach of fuzzy optimization and the Lagrangian relaxation technique, a fuzzy optimization-based algorithm is developed. The method and the technique used in this paper was a tremendous guide to implement the FNLP approach in OPF formulation of my thesis.

In Reference [39] a fuzzy linear programming approach is proposed to determine the amount of reactive power correction installed in the candidate load buses to maintain voltage levels of all load buses. The proposed approach in which the objective function and the constraints are modeled by fuzzy sets is applied to an example system and the results are given. In Reference [40], the sensitivity factor method is applied to the reactive power/voltage dispatch problem and combines it with the fast Newton-Raphson economic dispatch to solve the optimal power flow problem. The proposed methods have rapid and constraint convergence to the Kuhn-Tucker optimality conditions.

Reference [41], presents a method to include emission constraints in classical Economic Dispatch (ED), which contains an efficient weights estimation technique. Also, a partial closed form technique is presented to implement the Emission Constrained Economic Dispatch (ECED). Different methods of including emissions as well as their advantages and disadvantages are discussed. Sample test results are presented. The proposed two methods have the potential for on-line implementation.

Reference [42], presents a mathematical formulation for optimal power flow (OPF) taking into account the fuzzy modeling of static security constraints due to the uncertainty in bus loads. In Reference [43] the application of a fuzzy optimization technique to optimal power flow calculations is presented. The developed method has been tested on a large-scale power system. Numerical results show that this method is promising for handling uncertain constraints in practical power systems. In Reference [44], discusses considerations for the application of the fuzzy power flow for the planning and operation of practical power systems. Special attention is given to the non-linearity of the power flow problem taking in account uncertainty and the linking with a voltage stability function based on the use of eigenvalues and eigenvectors. All of these concepts have been implemented in the commercial grade interactive power flow program WINFLU which is an official tool in the Peruvian electrical sector. Tests results using a configuration of Peruvian interconnected power system (SEIN) are included and they demonstrate the validity of the fuzzy concepts as applied for a more robust planning and operation of the power system.

The techniques available in the literature lead to cumbersome computational processes that are not adequate for a large system. The methods proposed in this thesis lead to computationally efficient processes that are applicable to large system.

Chapter 2
Optimal Economical Dispatch and Characteristic of Generation Units

2.1 Introduction

The efficient and optimum economic operation and planning of electric power generation systems have occupied an important position in the electric power industry. A saving of a few percent in the operation of the system presents a significant reduction in operating cost as well as in the quantities of fuel consumed. It is no wonder that this area has warranted a great deal of attention from engineers through the years. The purpose of economic dispatch or optimal dispatch is to reduce fuel costs for the power system. Minimum fuel costs are achieved by the economic load scheduling of the different generating units or plants in the power system. By economic load scheduling we mean the requirement to find the generation of the different generators or plants so that the total fuel cost is minimum, and at the same time the total demand and the losses at any instant must be met by the total generation. However, economic load scheduling was not very important in the beginning when there were small power generating plants for each locality, such as in urban power systems, but now with the growth in the power demand and at the same time guarantees regarding the continuity of power supply to the consumer under normal conditions have lead the power systems to evolve into a complex interconnected grid system. For such system the economic dispatch problem has become increasingly important.

The objective in the economic dispatch of a power system is to minimize the cost of meeting the energy requirements of the system over some appropriate period of time in a manner consistent with reliable service. The appropriate period may be as short as a few minutes or as long as a year or more depending on the nature of the energy sources available to the system. Thus, if seasonal storage hydro is involved the appropriate period (or cycle) will generally be one year while if run-of-river and pumped storage hydro is involved the cycle may be one day or one week. These sources are frequently referred to as limited energy sources since they can not continuously maintain full output. Gas fired combustion turbines, depending on the terms of the gas supply contract, may fall into this category. Also, nuclear plants may fall into this category as a result of fuel design and circumstance which develop during the refueling cycle. Obviously, the aim in the utilization of limited energy resources is to realize the greatest possible value, during the operating cycle in terms of fuel replacement at those plants where the available fuel supply is not a limiting factor. Total operating costs generally include only the applicable fuel cost, maintenance cost, cost of transmission losses and labor cost.

2.2 Unit Input, Output Curves

The thermal generating unit input-output curve establishes the relationship between the energy input to the driven system and the net energy output from the prime mover at the electric generator. The data may be obtained from design calculations or from heat rate test.

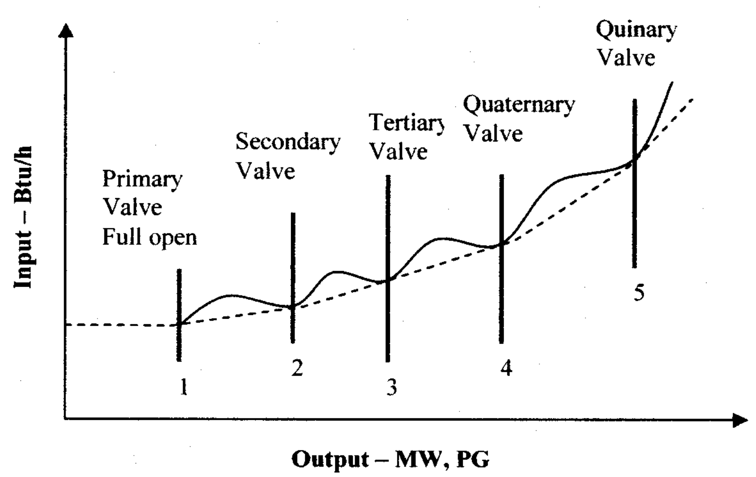


Figure (2-1) Thermal Unit–Input versus Output

Figure (2-1) shows the performance curve for a typical thermal unit. The ripples in the input-output curve are the result of the sharp increase in losses due to wire drawing effects which occur as each steam admission valve starts to open.

2.3 Incremental Fuel Cost $(\partial C/\partial P)$ [53, 55]

All units must operate at equal incremental cost for minimum plant cost in dollars per hours. The value of $\partial C/\partial P$ can be determined by the slopes of the input-output curves of the numbers of units contained in the plant. Mathematically $\partial C/\partial P$ can be calculated by taking the derivative of the cost function with respect to the power generators output. A typical plot of $\partial C/\partial P$ versus output power is shown in Fig (2-2) which is obtained from the input-output curve (slope of the curve in Fig (2-1)). The ripples in Fig (2-3) show up as the sharp spikes in Fig (2-2). The purpose of working with equal incremental fuel cost is summarized in this, suppose that the total output of the plant is supplied by two units and that the division of load between these units is such that the incremental fuel cost of one is higher than that of the other. Now suppose that some of the load is transferred from the unit with the higher incremental cost to the unit with the lower incremental cost. Reducing the load on the unit with the higher incremental cost will result in a greater reduction of cost than the increase in cost for adding that same amount of load to the unit with higher incremental cost. The transfer of load from one to the other can be continued with a reduction in total fuel cost until the two units are equal. The same reasoning can be extended to the plant with more than two units. Thus the criterion for economical division of load between units within a plant is that all units must operate at the same incremental fuel cost.

Fig (2-3) also shows the average heat rate as a function of the output level. The output level at which this curve is a minimum is the point of maximum efficiency. At this point the average heat rate is equal to the incremental input as shown in Fig (2-3). The shape of the input-output curve in the neighborhood of the valve points is difficult to determine by actual testing. However, in actual operation best economy is achieved by avoiding operation in these areas.

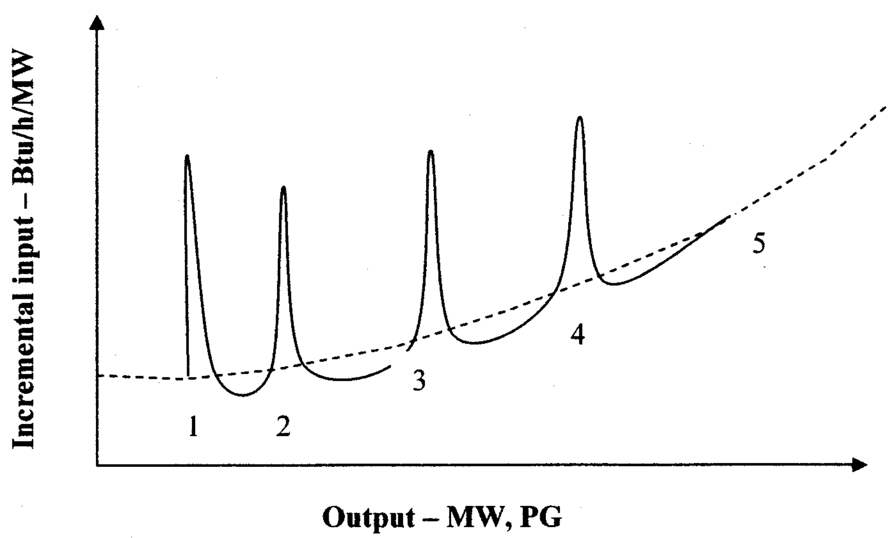


Fig (2-2) Thermal Unit–Incremental Heat Rate

Most system studies represent the performance as smooth curves, as shown by the dotted line in Figs. (2-1, 2-2) and (2-3).

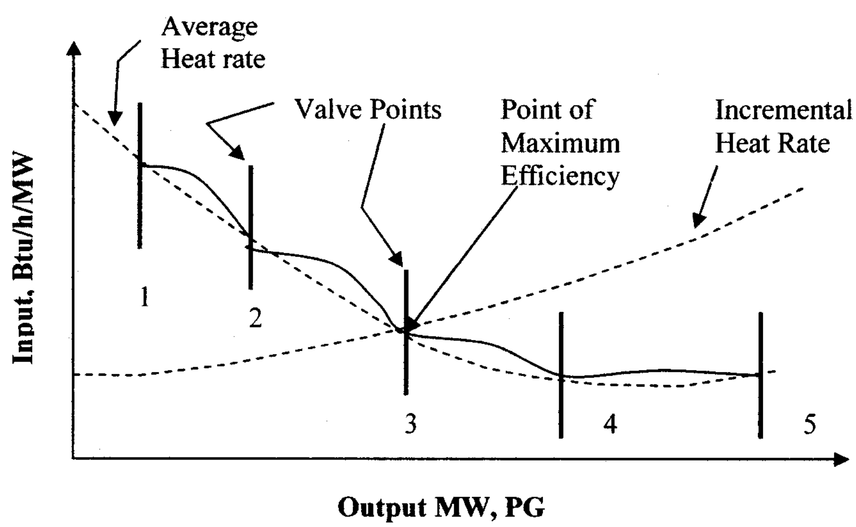


Figure (2-3) Thermal Unit Performance

2.4 General Economic Dispatch Problem Formulation

The input to the thermal plant is generally measured in Kcal/h, and the output is measured in MW. A simplified input output curve of a thermal unit known as, "heat rate curve," is given in Figure (2-4). Converting the ordinary heat-rate curve from Kcal/h to $/h results in the fuel cost curve showing in Figure (2-5). This cost curve is bounded by the generator output limits and is monotonically increasing (convex). In general, to make more electric energy requires more thermal energy and thus cost more money. Economic dispatch analysis schedules the outputs of the online generating units (those already committed) so as to satisfy the system load at least cost. Improvements in scheduling the unit outputs can lead to significant cost savings. Traditional dispatch algorithms (such as lambda iterations) are based on the concept of equal incremental cost. The total production cost of a set of generators is minimized when all the units operate at the same incremental cost.

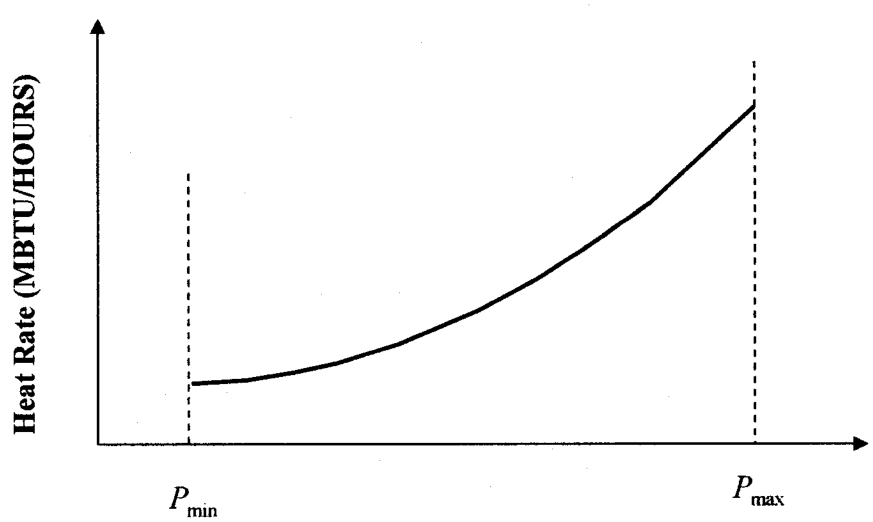


Figure (2-4) Power Output (MW) Simplified Heat Rate Curve

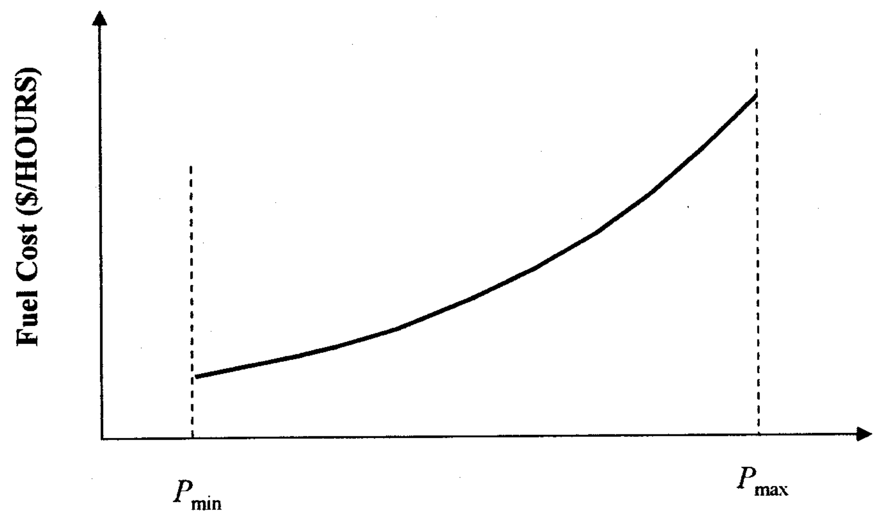


Figure (2-5) Power Output (MW) Fuel-Cost Curve

When considering transmission losses, the unit incremental costs are modified to account for incremental transmission losses **[36]**. In the classical economic dispatch a set of coordination equations is solved using the Lagrange multiplier. The algorithm uses the quadratic loss formula (B coefficients) approach to model system losses when considering transmission losses. Traditional algorithms however, require that the unit cost curves ($/h vs. MW) be convex functions. Hence, they cannot guarantee optimality for non-monotonically increasing incremental cost curves. Solution to the economic dispatch problem with non-convex unit cost functions can be achieved using dynamic programming (DP). Unlike the traditional solution, the DP solution imposes no restrictions on the generating unit characteristics. However, it suffers from the dimensionality problem: as the number of generators to be dispatched increases and higher solution accuracy is needed, the storage requirements and the execution time increases dramatically. The economic load dispatch problem (ELD) is a good example of a real world optimization problem and one of the most important problems in power system. The objective of economic load dispatch is to find the optimal combination of power generations that minimizes the total cost while satisfying the total demand. Traditionally, the cost function of each generator is approximated by a single segment quadratic function. For some present operating conditions, it is more realistic to represent the cost function for fossil fired unit by a multi-segment piecewise quadratic function. The reasons for partitioning the cost function vary. Often this is done to increase the accuracy of the functional relationship [4]. The capability of burning multiple fuels by a single generation unit results in intersecting cost curves for the same unit. These intersecting curves mean that it may be more economical to burn a certain fuel for some MW outputs and another kind of fuel for other outputs. Segmenting cost functions results from multiple sources for each generation unit. In this section the classical optimization of a continuous function is introduced. The application of constraints to the optimization problem is presented. Following this, the incremental production cost of generation is introduced. The economic dispatch of generation for minimization of the total operating cost with transmission losses neglected is obtained. Next, the transmission losses formula is derived and the economic dispatch based on the loss formula is obtained.

2.4.1 Economic Dispatch Neglecting Losses and Generating Limits [36, 41, 51]

This is the simplest economic dispatch problem as shown in Figure (2-6). The total cost function of the generators committed for the whole plant is considered.
The objective function to be minimized is the total fuel cost which can be presented as a quadratic polynomial function of real generation obtained from Figure (2-5)

$$C_{total} = \sum_{i=1}^{NG} C_i = \sum_{i=1}^{NG} \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2 \tag{2.1}$$

Where

α, β and γ = the cost function coefficient parameters.

P_{Gi} = the number of generator connected to the network.

C_{total} = the total cost of generation

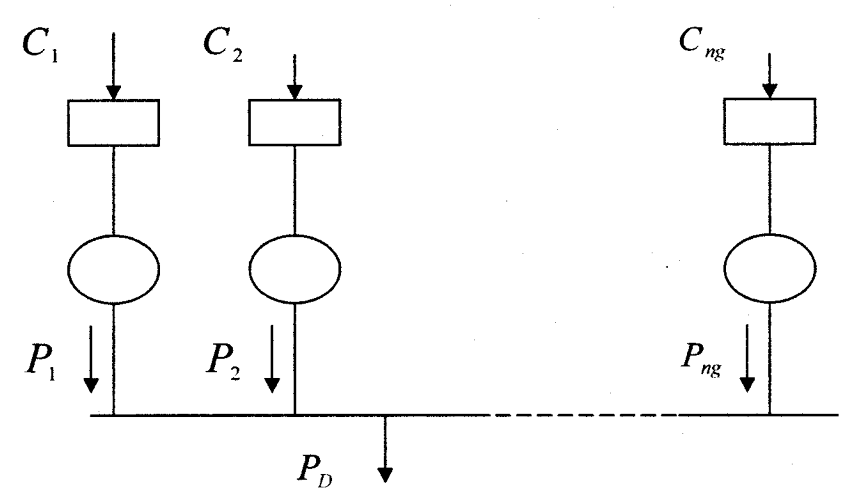


Figure (2-6) Units Connected to a Common Bus

And the equality constraint can be expressed when the requirement of the demand is met is given by:

$$\sum_{i=1}^{NG} P_{Gi} = P_{Demand} \tag{2.2}$$

Where P_{Demand} is the load demand of the customers.

The minimization of a cost function subject to equality and inequality constraints is a problem in optimization that is treated by a branch of applied mathematics called, "nonlinear programming." The most famous methods used are listed bellow:

1. Lagrangian Method
2. Karush-Kuhn-Tucker Optimality Conditions.
3. Penalty and Barrier Methods.
4. Reduced Gradient Algorithms.
5. Quadratic Programming Method.
6. Separable Programming Method.
7. Posynomial Programming Methods.

Using the Lagrangian Method as the standard approach to obtain the necessary conditions of optimality based on the Lagrangian function which is defined by:

$$L(x,\lambda)=f(x_1,\dots,x_n)+\sum_{i=1}^{m}\lambda_i h_i(x_1,\dots,x_n)$$

The variables $\lambda_1,\dots\dots\lambda_m$ are called the Lagrange multipliers

Applying this method to minimize the objective function subject to the equality constraint the equation becomes:

$$L=C_{total}+\lambda\left(P_{Demand}-\sum_{i=1}^{NG}P_{Gi}\right) \tag{2.3}$$

Then taking the derivatives of Lagrangian with respect to generator outputs P_{Gi} we get:

$$\begin{aligned}&\frac{\partial L}{\partial P_{Gi}}=0=\frac{\partial C_{total}}{\partial P_{Gi}}+\lambda(0-1)=\rightarrow\frac{\partial C_{total}}{\partial P_{Gi}}=\lambda\\&\frac{dL}{d\lambda}=0=\left(P_{Demand}-\sum_{i=1}^{NG}P_{Gi}\right)=\rightarrow\sum_{i=1}^{NG}P_{Gi}=P_{Demand}\end{aligned} \tag{2.4}$$

Since the total cost is:

$$C_{total}=\sum_{i=1}^{NG}C_i\rightarrow\frac{\partial C_{total}}{\partial P_{Gi}}=\frac{dC_i}{dP_{Gi}}=\lambda \qquad i=1,\dots\dots,NG$$

And the incremental cost becomes:

$$\lambda=\frac{dC_i}{dP_{Gi}}=\beta_i+2\gamma_i P_{Gi} \tag{2.5}$$

Then we can calculate each generated power when all plants are operating at equal incremental production cost λ as:

$$P_{Gi}=\frac{\lambda-\beta_i}{2\gamma_i} \tag{2.6}$$

Substituting this formula into equation (2.2) we get

$$\sum_{i=1}^{NG}\frac{\lambda-\beta_i}{2\gamma_i}=P_{Demand} \tag{2.7}$$

Solving for the incremental cost λ

$$\lambda=\frac{2P_{Demand}+\sum_{i=1}^{NG}\frac{\beta_i}{\gamma_i}}{\sum_{i=1}^{NG}\frac{1}{\gamma_i}} \tag{2.8}$$

The value of λ found in equation (2.8) is substituted in equation (2.6) to obtain the optimal scheduling of generation

2.4.2 Economic Dispatch Including Generating Limits Neglecting Losses [50, 51, 52].

For stable boiler operation the generators should operate within its maximum and minimum limits. The goal is to minimize the objective function subject to the equality and inequality constraints

Objective function

$$\text{MIN } C_{total} = \sum_{i=1}^{NG} C_i = \sum_{i=1}^{NG} \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}{}^2$$

Subject to

Equality constraint

$$\sum_{i=1}^{NG} P_{Gi} = P_{Demand}$$

Inequality constraint

$$P_{Gi}(\min) \le P_{Gi} \le P_{Gi}(\max) \qquad i = 1, \ldots\ldots NG \tag{2.9}$$

The upper limit on P_{Gi} is set by thermal limits on the turbine generator unit, while the lower limit is set by boiler and/or other thermodynamic considerations. A certain minimum flow of water and/or steam is required in the boiler to prevent "hot spots" from developing. The fuel burning rate must also be sufficient to keep the flame from going out ("flame out"). Other control variables, such as voltage and the phase angles across phase-shifting transformers, the turns ratios of tap-changing transformers, the admittances of variable, controllable shunt, controllable series reactors and controllable capacitors can be considered in the generalized economic dispatch problem.

The Lagrangian function for this problem is given by:

$$L = C_{total} + \lambda(P_D - \sum_{i=1}^{NG} P_{Gi}) + \sum_{i=1}^{NG} \mu_{i\max}(P_{Gi} - P_{Gi\max}) + \sum_{i=1}^{NG} \mu_{i\min}(P_{Gi} - P_{Gi\min}) \tag{2.10}$$

The multipliers λ, $\mu_{i\,\max}$ and $\mu_{i\,\min}$ are the dual variables associated with the inequality and equality constraints. Thus with the use of Lagrange multiplier, the original constrained problem has been transformed into an auxiliary unconstrained problem whose optimum solution is the same as the optimum of the original problem. The solution of this equation is the same as without generating limits as long as we do not violate their limits. If the generation exceeds the limits set by the plants, then its max or min value is set as constant by an IF statement in the iterative program, then the search continues until the difference between the sum of all generation and load demand is equal to zero. As an example consider the optimal dispatch shown in Figure (2-7).

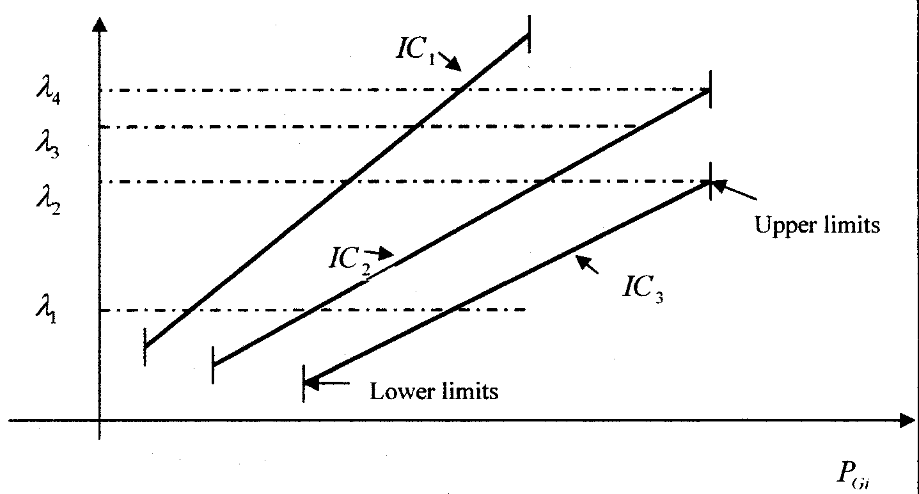


Figure (2-7) Equal Incremental Cost

Suppose that for a given P_D the system λ is λ_1. All three generators are operating in accordance with the optimal dispatch rule and the question of generator limits does not arise since each generator is operating away from any limit.

Now suppose that the power demand increases and we increase λ to provide more generation. Continuing the process in this way we reach λ_2. What if P_D increases further? P_{G3} has reached a limit and cannot be increased further. The increased load must be supplied by P_{G1} and P_{G2}. Clearly, they should operate at equal incremental cost, say λ_3. Further, increases in the load can be taken by P_{G1} and P_{G2} operating at equal IC until P_{G2} reaches its upper limit and $\lambda = \lambda_4$. Beyond this point, only P_{G1} is available to take any increase in the load.

2.4.3 Economic Dispatch Including Generating Limits and Transmission Losses [40, 49, 51, 52]

In the case of the generators that are located in one plant or are otherwise very close geographically, it is reasonable to neglect electrical line losses in calculating the optimal dispatch. On the other hand, for power stations that are spread out geographically, the transmission-line (link) losses need to be considered, and this will modify the optimal generation assignments. The total transmission loss expression is approximated by a quadratic function of the power generation given by:

$$P_L = \sum_{i=1}^{NG}\sum_{j=1}^{NG} P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^{NG} B_{0i} P_{Gi} + B_{00} \tag{2.11}$$

B_{ij} are called the losse formula coefficients and they are assumed to have constant values. Reasonable accuracy is expected when actual operating conditions are close to the base case conditions used to compute the coefficient parameters.

The goal is to minimize the objective function

$$\text{MIN } C_{total} = \sum_{i=1}^{NG} C_i = \sum_{i=1}^{NG} \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}{}^2$$

Subject to the power balance equality constraint

$$\sum_{i=1}^{NG} P_{Gi} = P_{Demand} + P_L \tag{2.12}$$

The generation equals the total load demand plus transmission losses. In addition, we have the inequality constraints:

$$P_{Gi}(\min) \le P_{Gi} \le P_{Gi}(\max) \qquad i = 1, \ldots\ldots NG \tag{2.13}$$

Substituting the objective function, equality constraints and the inequality constraints in the Lagrangian we get:

$$L = C_{total} + \lambda(P_D + P_L - \sum_{i=1}^{NG} P_{Gi}) + \sum_{i=1}^{NG} \mu_{i\,max}(P_{Gi} - P_{Gi\,max}) + \sum_{i=1}^{NG} \mu_{i\,min}(P_{Gi} - P_{Gi\,min}) \tag{2.14}$$

If the constraints are not violated, then its associated μ variable is set to zero

$$P_{Gi} < P_{Gi\,(max)} : \quad \mu_{i\,(max)} = 0 \qquad P_{Gi} > P_{Gi\,(min)} : \quad \mu_{i(min)} = 0$$

The minimum of the unconstrained function is found when taking the partial derivative of each multiplier we get:

$$\begin{aligned}
&\frac{\partial L}{\partial P_{Gi}} = 0 = \frac{\partial C_{total}}{\partial P_{Gi}} + \lambda\left(0 + \frac{\partial P_L}{\partial P_{Gi}} - 1\right) \\
&\frac{\partial L}{\partial \lambda} = 0 = P_D + P_L - \sum_{i=1}^{NG} P_{Gi} \\
&\frac{\partial L}{\partial \mu_{i\,(max)}} = P_{Gi} - P_{i\,(max)} = 0 \\
&\frac{\partial L}{\partial \mu_{i\,(min)}} = P_{Gi} - P_{Gi\,(min)} = 0
\end{aligned} \tag{2.15}$$

Where the total cost and incremental cost can be represented by:

$$\begin{aligned}
&\frac{\partial C_{total}}{\partial P_{Gi}} = \frac{\partial}{\partial P_{Gi}}\left(C_1 + C_2 + ... + C_{NG}\right) = \frac{dC_i}{dP_{Gi}} \\
&\therefore \quad \lambda = \frac{dC_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_{Gi}} = \left(\frac{1}{1 - \partial P_L / \partial P_{Gi}}\right)\frac{dC_i}{dP_{Gi}} = L_i \frac{dC_i}{dP_{Gi}}
\end{aligned} \tag{2.16}$$

Where L_i is called the transmission loss penalty factor and it is defined by:

$$L_i = \left(\frac{1}{1 - \partial P_L / \partial P_{Gi}}\right) \tag{2.17}$$

The effect of the transmission losses introduces a penalty factor that depends on the location of the plant and the minimum cost is obtained when the incremental cost of each plant multiplied by its penalty factor is the same for all plants. Taking the partial derivative of the Lagrangian with respect to generation we get:

$$P_L = \sum_{i=1}^{NG}\sum_{j=1}^{NG} P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^{NG} B_{0i} P_{Gi} + B_{00}$$

$$\frac{\partial P_L}{\partial P_{Gi}} = 2\sum_{j=1}^{NG} B_{ij} P_{Gj} + B_{0i} \qquad \frac{dC_i}{dP_{Gi}} = \beta_i + 2\gamma_i P_{Gi}$$

$$\lambda = \frac{dC_i}{dP_{Gi}} + \lambda \frac{\partial P_L}{\partial P_{Gi}} = \beta_i + 2\gamma_i P_{Gi} + 2\lambda \sum_{j=1}^{NG} B_{ij} P_{Gj} + \lambda B_{0i}$$

Then rearranging the equation we get:

$$\left(\frac{\gamma_i}{\lambda} + B_{ii}\right) P_{Gi} + \sum_{\substack{j=1 \\ j \neq i}}^{NG} B_{ij} P_{Gj} = \frac{1}{2}\left(1 - B_{0i} - \frac{\beta_i}{\lambda}\right) \tag{2.18}$$

Extending the equation to all plants result in the following linear equations (in matrix form).

$$\begin{bmatrix} \frac{\gamma_1}{\lambda} + B_{11} & B_{ii} & \cdots & B_{1n} \\ B_{21} & \frac{\gamma_2}{\lambda} + B_{22} & \cdots & B_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n1} & B_{n2} & \cdots & \frac{\gamma_3}{\lambda} + B_{nn} \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ \vdots \\ P_{Gn} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 - B_{01} - \frac{\beta_1}{\lambda} \\ 1 - B_{02} - \frac{\beta_2}{\lambda} \\ \vdots \\ 1 - B_{0n} - \frac{\beta_n}{\lambda} \end{bmatrix} \tag{2.19}$$

Or $\quad \mathbf{E\,P = D}$

Then to find the optimal dispatch for an estimated value of $\lambda^{[1]}$.

- Solve the simultaneous linear equation **E P = D**
- Then the iterative process is continued to update λ using the gradient method.
- If an approximate loss formula is used setting B_{ij} =0 and B_{00} =0 then equation

(2.11) become $P_L = \sum_{i=1}^{NG} B_{ii} P_{Gi}^2$

Then the solution of the simultaneous equations reduced to

$$P_{Gi}^{[k]} = \frac{\lambda^{[k]} - \beta_i}{2(\gamma_i + \lambda^{[k]} B_{ii})^2} \tag{2.20}$$

$$\sum_{i=1}^{NG}\left(\frac{\partial P_{Gi}}{\partial \lambda}\right)=\sum_{i=1}^{NG}\left[\frac{\gamma_i+B_{ii}B_i}{2\left(\gamma_i+\lambda^{[k]}B_{ii}\right)^2}\right] \tag{2.21}$$

2.5 Optimal Power Flow [38, 43, 50]

Static optimization of power system operation involves allocation of generation levels, voltage profiles and possibly load curtailment based on the equality and inequality constraints of the power system and pre-specified performance function (cost criterion). Normally this is referred to as the optimal load power (OPF) problem **[50]**, which is an online application used in Energy Management system (EMS) to determine real and reactive power output of the generators as well as voltages such that the overall production cost is minimized and the equipment (line, transformer, generator) capacity constraints are satisfied.

Mathematically, OPF is formulated as a constrained optimization problem

$$\begin{aligned} &\min \;\; f(x) \\ &\textit{subject to} \\ &\qquad g(x)=0 \\ &\qquad h(x)\le d \end{aligned} \tag{2.22}$$

The vector x is a vector variable that consists of a set of controllable quantities and dependent variables. The controllable quantities in power systems usually include generating unit outputs, company transactions, all phase shift transformer angles (except for the angle of the slack bus), all load bus voltage magnitudes, transformer tap positions, shunt capacitors/reactors, etc. The objective function $f(x)$ is a convex scalar function that may be the total production cost or the total active power transmission losses of the system. The equality constraints $g(x)$ represent the static load flow equations the dimension of g corresponds to that of the x vector, and the inequality constraints $h(x)$ consist of the limits on the controllable quantities and the operating limits of the power system.

When considering the network constrained active power dispatch the OPF becomes a cost minimization problem. The objective function in this case is the summation of total generation production cost. This is represented in the following equation where the set P consists of all controllable generation units.

$$f(x) = \sum_{i \in P} C_i(x) \tag{2.23}$$

The controllable quantities are the generation active power outputs. The equality constraints $g(x) = 0$ are the power flow equations, and the inequality constraints $h(x) \leq d$ in this case, include generation output limits, active power reserve margins, transmission line flows, transformer flows, and transmission corridor flow.
Traditionally, each cost function, $C_i(x)$ is modeled as a piecewise quadratic function and can be approximated as a piecewise linear function. The cost minimization OPF can be solved by the successive linear programming (SLP) algorithm which has been found to be robust and efficient. Cost minimization is the most common objective for the OPF problems, which require satisfying network constraints. In this discussion we will focus on the production cost minimization OPF problem. Extending our derivation to other objectives is straightforward.

2.5.1 Practical Considerations [43]

The conventional OPF is formulated as an optimization problem with crisp constraints. However, in practice, there are two types of inequality constraints: hard constraints and soft constraints. For example, the limits of the generating unit outputs are hard constraints because they represent physical limitations on the capacity of the generating units to produce active power. On the other hand, the limits on the transmission line flows are soft. Small violations of these limits are sometimes acceptable, especially during stressed (i.e.: emergency or peak load) situations of the system. There are usually two flow limits for each transmission line, normal and emergency limits. In general, operators desire to economically operate the system within the normal limits. When there is a real need, small violations of normal limits are allowed. However, emergency limits are never allowed to be violated and they are considered to be hard limits. These practical considerations of constraint limits are not formulated satisfactorily in a conventional OPF.
Furthermore, from an operator's point of view, minimizing cost does not entail finding a rigid optimal solution to a classical and simplistic formulation of OPF problems that

fails to model important aspects of actual operational practices. It is more appropriate to state the objectives of the OPF, which is to reduce the cost as much as possible, while satisfying the soft constraints as much as possible and while enforcing the hard constraints exactly. In a "very feasible" case, the conventional OPF can produce fairly reasonable solutions that meet the above objectives. This means that all the constraints, both soft and hard, are satisfied and cost is minimized. However, if the case is nearly feasible (or nearly infeasible), the solutions from conventional OPF may become unrealistic. Sometimes, in order to enforce a soft constraint with small violations, control variables may have to move significantly and also cost may increase considerably. Even though the solutions are mathematically correct for the formulated OPF problem, they are not consistent with practical operational practices. For infeasible cases, the conventional OPF usually cannot produce acceptable solutions even with the assistance of available relaxation procedures.

The aim in optimal load flow is to minimize the total fuel cost of generation J_F while satisfying the active and reactive power constraints of the electrical network represented by the load flow equations and the operating inequality constraints and meeting the load demand for a power system while maintaining the security of the system by keeping each device within its desired operation range at steady-state. The solution of the OPF has many great advantages over classical economic dispatch. It is capable of performing all the control functions necessary, such as monitoring system security issues including line overloads and low or high voltage problems [27]. If any security violations occur, the OPF will modify its controls to fix the problem and in return removes transmission line overloads from the system. But the greatest advantage of the OPF is the wealth of knowledge it generates about the entire system status. The standard form of the objective minimizing the cost function J_F represented as a quadratic function of real power generator is given by

$$J_F = \sum_{i=1}^{NG} (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2)$$

Where NG is the number of generator busbar α_i , β_i and γ_i are fuel cost parameters of the generating source at the ith busbars and P_{Gi} is the active power generation at the ith busbar. The factors influencing power generation at minimum cost are operating efficiencies of generators, fuel cost and transmission losses.

Generally the constraints of optimal power flow still include the entire AC power flow, generation output range, bus voltage limits, and transmission line transfer capabilities.

- The equality constraints representing the load flow equation that can be computed from the load flow analysis [27].

$$P_i = P_{G_i} - P_{d_i} = |V_i| \sum_{i=1}^{NG} |V_j| |Y_{ij}| \cos(\delta_i - \delta_j - \psi_{ij}) \tag{2.24}$$

$$Q_i = Q_{G_i} - Q_{d_i} = |V_i| \sum_{i=1}^{NG} |V_j| |Y_{ij}| \sin(\delta_i - \delta_j - \psi_{ij}) \tag{2.25}$$

Also can be written as

$$\begin{aligned} P_{Gi} - P_{di} - |V_i| \sum_{i=1}^{NG} |V_j| |Y_{ij}| \cos(\delta_i - \delta_j - \psi_{ij}) = 0 \\ Q_{Gi} - Q_{di} - |V_i| \sum_{i=1}^{NG} |V_j| |Y_{ij}| \sin(\delta_i - \delta_j - \psi_{ij}) = 0 \end{aligned} \tag{2.26}$$

The active and reactive power flows can be calculated by using the decoupled load flow approach to cut down one-third of the solution time and reduce memory requirements.

The inequality constraints apply to state, control and output variables represented as:

- Bus voltage magnitude range for all generation busses and tap transformers to satisfy legal requirements and design limitations. The bus voltage magnitude is restricted to limits

$$V_{\min} \le V \le V_{\max} \tag{2.27}$$

- Real generation range constraints of each unit must be restricted to lie within given minimum and maximum limits where boiler operation conditions determine those limits

$$P_{G_{\min}} \le P_G \le P_{G_{\max}} \tag{2.28}$$

- Reactive generation range constraints of each unit which control the voltage magnitude by varying the reactive power generated produced

$$Q_{G_{\min}} \leq Q_G \leq Q_{G_{\max}} \tag{2.29}$$

- Security constraints on line flows for specified lines

$$S_k(T_k) = \begin{cases} 0, & if\ |T_k| \leq \bar{T}_k \\ \alpha_k (|T_k| - \bar{T}_k)^2, & otherwise \end{cases} \tag{2.30}$$

- In-phase tap transformer that can be changed under load
- Phase-shifting transformers constraints on phase shifters must not be allowed to be outside a given range

Then the objective function becomes:

$$J = J_F + J_P + J_q \tag{2.31}$$

Where the subscripts p, q represent active and reactive bus constraints.

$$J_p = \sum_{i=1}^{n} \lambda_{P_i} \left[P_i - P_{G_i} + P_{d_i} \right] \tag{2.32}$$

$$J_q = \sum_{i=m2}^{n} \lambda_{q_i} \left[Q_i - Q_{G_i} + Q_{d_i} \right] \tag{2.33}$$

Where

n = total number of busbars in the system.

$m2$ = total number of voltage-controlled busbars.

P_i and Q_i = active and reactive power given as function of voltage and phase angle.

λ_{P_i} and λ_{q_i} = Lagrange multiplier corresponding to the incremental cost functions of active and reactive power delivered at ith bus.

P_{d_i} and Q_{d_i} = active and reactive power demand and the ith bus.

Q_{G_i} and P_{G_i} = reactive and active power generation at the ith busbar.

The Lagrange multiplier is used to relax "system wide constraints" into unconstrained form that matches the original objective function at feasible points.

$$L = \sum_{i=1}^{NG} (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) + \sum_{i=m1}^{n} \lambda_{P_i} \left[P_i - P_{G_i} + P_{d_i} \right] + \sum_{i=m2}^{n} \lambda_{q_i} \left[Q_i - Q_{G_i} + Q_{d_i} \right] \tag{2.34}$$

If we include the inequality constraints then the Lagrangian multiplier will become

$$L = \sum_{i=1}^{NG}(\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) + \sum_{i=m1}^{n} \lambda_{P_i}\left[P_i - P_{G_i} + P_{d_i}\right] + \sum_{i=m2}^{n} \lambda_{q_i}\left[Q_i - Q_{G_i} + Q_{d_i}\right]$$

$$+\sum_{i=1}^{NG} B_{i\max}(P_{G_i} - P_{G_{i\max}}) + \sum_{i=1}^{NG} B_{i\min}(P_{G_i} - P_{G_{i\min}}) \tag{2.35}$$

In this thesis the main goal is to use fuzzy formulations rather than crisp formulations. The classical methods of economic dispatch and power load flow optimization do not provide wide information on the system performance resulting from calculated and measured values as separate entities. Power system operation, planning, control and management are based on strict mathematical models to find a solution. The critical constraints limitations and system reliability especially in dealing with online operation which is the main requirement of OPF technology are restricted and the constraints imposed on the system have to be satisfied 100% in order to obtain a optimal solution which lead to over conservative solutions. The fuzzy formulation results of the optimal economic dispatch and optimal load flow treated in this thesis provide a wider range of information to evaluate the uncertainty in the system when the load demand, cost function coefficients and power losses are fuzzy which will lead the fuzziness to propagate throughout the entire system parameters. Fuzzifiying certain parameters in the objective function or the constraint will enhance the system performance and reliability. In addition to overcome the limitation restricted on specific power system variables such as (bus voltages, line flow, etc, may be violated to some degree in order to obtain a realistic model.

Chapter 3
Fuzzy Sets and Membership

3.1 Introduction

The term "fuzzy" was proposed by Zadeh in 1962 **[45]** and in 1965, he formally published the famous paper "Fuzzy Sets" **[46]**, developed to improve an oversimplified model, thereby, developing a more robust and flexible model in order to solve real-world complex systems involving human aspects. Furthermore, it helps the decision maker not only to consider the existing alternatives under given constraints (optimize a given system), but also to develop new alternatives (design a system).

The fuzzy set theory has been applied in many fields, such as operations research, management science, control theory, artificial intelligence/expert system, human behavior, etc.

In this chapter, we introduce principal concepts and mathematical notions of fuzzy set theory, a theory of classes of objects with non-sharp boundaries. We first view fuzzy sets as a generalization of classical crisp sets by extending the range of the membership function (or characteristic function) from [0, 1] to all real numbers in the interval [0, 1]. A number of notions of fuzzy sets such as membership function representation, support, α -cuts, convexity, and fuzzy numbers are then introduced. The formulations presented in the subsequent Chapters rely on concepts discussed in section (3-2) to (3-8) for ED problem formulation and section (3.10.1) to (3.10.3) for OPF optimization.

3.2 Membership Functions

A conventional (crisp or hard) set is a collection of distinct objects, defined in such a manner as to separate the elements of a given universe of discourse into two groups: those that belong (members) and those that do not belong (non-members). The transition of an element between membership and non-membership in a given set in the universe is abrupt and well defined. The crisp set can be defined by the so-called characteristic function.

3.3 Basic Terminology and Definition

Let X be a classical set of objects, called the universe, whose generic elements are denoted by x. The membership in a crisp subset of X is often viewed as characteristic function μ_A from X to {0, 1} such that:

$$\mu_A(x) = \begin{cases} 1 & \text{if and only if } x \in A \\ 0 & \text{otherwise} \end{cases} \tag{3.1}$$

Where {0, 1} is called a valuation set.

If the valuation set is allowed to be the real interval [0, 1], $\tilde{A}$ is called a fuzzy set proposed by Zadeh **[46]**. $\mu_A(x)$ is the degree of membership of x in A. The closer the value of $\mu_A(x)$ is to 1, the more x belongs to A. Therefore, $\tilde{A}$ is completely characterized by the set of ordered pairs:

$$\tilde{A} = \left\{ \left(x, \mu_A(x) \right) \mid x \in X \right\} \tag{3.2}$$

The characteristic function can be either a membership function or a possibility distribution. In this study, if the membership function is preferred, then the characteristic function will be denoted by $\mu_A(x)$. On the other hand, if the possibility distribution is preferred, the characteristic function will be specified as $\pi(x)$. Along with the expression of Equation (3.2), Zadeh **[48]** also proposed the following notations. When X is a finite set $\{x_1, x_2, \ldots\ldots, x_n\}$, a fuzzy set A is then expressed as:

$$\tilde{A} = \mu_A(x_1)/x_1 + \ldots\ldots + \mu_A(x_n)/x_n = \sum_i \mu_A(x_i)/x_i \tag{3.3}$$

When X is not a finite set, A then can be written as:

$$A = \int_X \mu_A(x)/x \tag{3.4}$$

Sometimes, we might only need objects of a fuzzy set (but not its characteristic function), that is to transfer a fuzzy set. To do so, we need two concepts – support and α-level cut.

3.3.1 Support of Fuzzy Set [56]

The support of a fuzzy set A is the crisp set of all $x \in U$ such that μ(x)>0. That is:

$$\text{supp}(A) = \{x \in U \mid \mu_A > 0\} \tag{3.5}$$

3.3.2 α-Level Set (α-Cut)

The α-level set (α-cut) of a fuzzy set A is a crisp subset of X and is denoted by Figure (3-1). An α-cut of a fuzzy set $\tilde{A}$ is a crisp set A, that contain all the elements of the universe **U** that have a membership grade in $\tilde{A}$ greater than or equal to α. That is:

$$\tilde{A}_\alpha = \{x \mid \mu_A(x) \geq \alpha \textit{ and } \mathrm{x} \in \mathrm{X}\} \tag{3.6}$$

If $\tilde{A}_\alpha = \{x \mid \mu_A(x) > \alpha\}$, then $\tilde{A}_\alpha$ is called a strong α-cut. Furthermore, the set of all levels, $\alpha \in [0,1]$ that presents distinct α-cut of a given fuzzy set A is called a level set of A. That is:

$$\prod\nolimits_A = \{\alpha \mid \mu_A(x) = \alpha, \text{ for some } \mathrm{x} \in \mathbf{U}\} \tag{3.7}$$

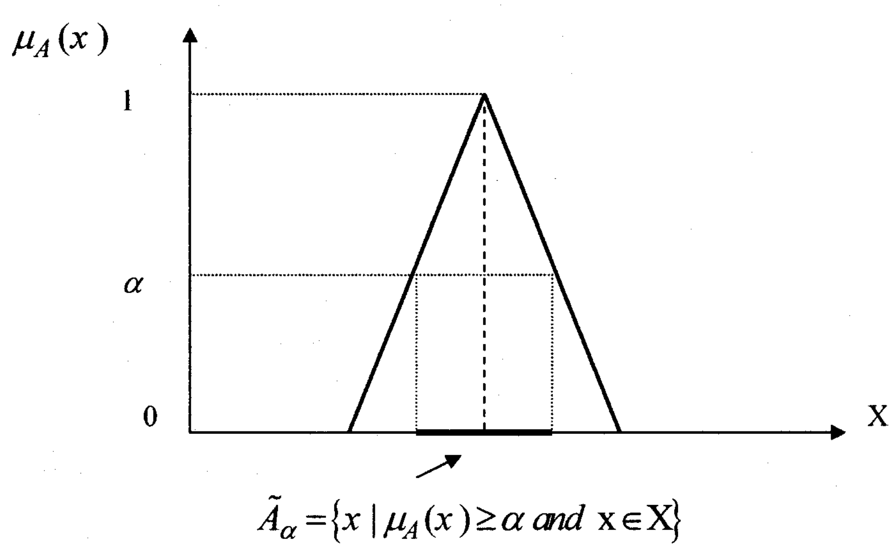


Figure (3-1) α Level–Set Representation

The fuzzy set can be viewed as comprising of a set of α-level cuts. A α-level cut of $\tilde{A}$, $\tilde{A}_\alpha$ is the crisp set obtained from $\tilde{A}$ for each $\alpha \in [0,1]$ according to equation (3.6). The α-level cut of $\tilde{A}$ can be also described as an interval of confidence at level α, that is

$$\tilde{A}_\alpha = [A_\alpha{}^L, A_\alpha{}^R]$$

Where $A_\alpha{}^L$ and $A_\alpha{}^R$ are the left and right bounds of the interval of confidences shown in Figure (3-1). The 1.0-level cut and 0.0-level cut are called the core and support of fuzzy number, respectively. The central value of a fuzzy number is defined as the mean value of its 1.0-level cut.

3.3.3 Normality

A fuzzy set A is normal if and only if $\mathrm{Sup}_x\ \mu_A(x) = 1$, that is, the supreme of $\mu_A(x)$ over X is unity. A fuzzy set is subnormal if it is not normal. A non-empty subnormal fuzzy set can be normalized by dividing each $\mu_A(x)$ by the factor $\mathrm{Sup}_x\ \mu_A(x)$. A fuzzy set is empty if and only if $\mu_A(x) = 0$ for $\forall x \in X$).

3.3.4 Convexity and Concavity

A fuzzy set A in X is convex if and only if for every pair of points x^1 and x^2 in X, the membership function of A satisfies the inequality:

$$\mu_A(\partial x^1 + (1-\partial)x^2) \geq \min(\mu_A(x^1), \mu_A(x^2)) \qquad (3.8)$$

Where $\partial \in [0,1]$ (see Figure (3.2)). Alternatively, a fuzzy set is convex if all α-level sets are convex. Dually, A is concave if its complement A^c is convex. It is easy to show that if A and B are convex, so is $A \bigcap B$. Dually, if A and B are concave, so is $A \bigcup B$.

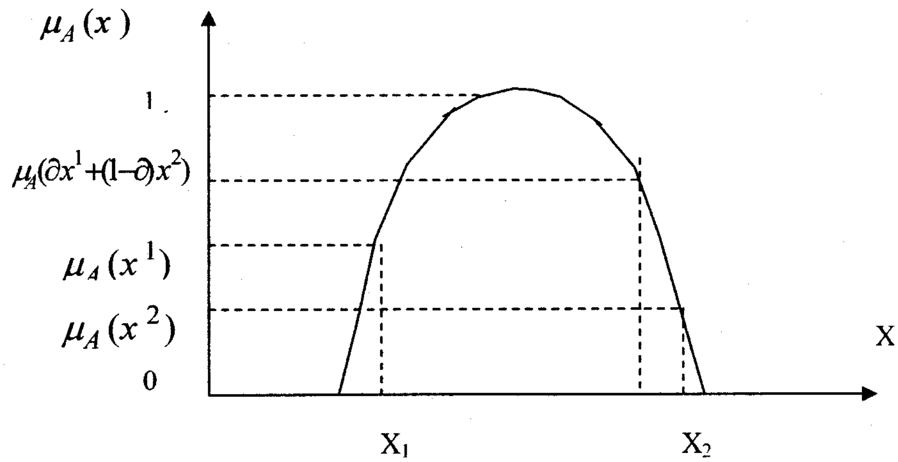


Figure (3-2) A Convex Fuzzy Curve

3.4 Basic Operation [55, 56]

This section introduces a summary of some basic set-theoretic operations which is useful in fuzzy decision-making. These operations are based on the definitions from Bellman and Zadeh [47].

3.4.1 Inclusion

Let A and B be two fuzzy subsets of X then A is included in B if, and only, if:

$$\mu_A(x) \leq \mu_B(x) \text{ for } \forall \mathrm{x} \in \mathrm{X} \tag{3.9}$$

3.4.2 Equality

A and B are called equal if and only if:

$$\mu_A(x) = \mu_B(x) \text{ for } \forall \mathrm{x} \in \mathrm{X} \tag{3.10}$$

3.4.3 Complementation

A and B are complementary if and only if:

$$\mu_A(x) = 1 - \mu_B(x) \text{ for } \forall \mathrm{x} \in \mathrm{X} \tag{3.11}$$

3.4.4 Intersection

The intersection of A and B may be denoted by $A \cap B$ which is the largest fuzzy subset contained in both fuzzy subsets A and B. When the min operator is used to express the logic "and," its corresponding membership is then characterized by:

$$\begin{aligned} \mu_{A \cap B}(x) &= \min(\mu_A(x), \mu_B(x)) \text{ for } \forall x \in X \\ &= \mu_A(x) \wedge \mu_B(x) \end{aligned} \tag{3.12}$$

Where $\wedge$ is a conjunction.

3.4.5 Union

The union ($A \cup B$) of A and B is dual to the notion of intersection. Thus, the union of A and B is defined as the smallest fuzzy set containing both A and B. The membership function of $A \cup B$ is given by:

$$\begin{aligned} \mu_{A \cup B}(x) &= \max(\mu_A(x), \mu_B(x)) \text{ for } \forall x \in X \\ &= \mu_A(x) \vee \mu_B(x) \end{aligned} \tag{3.13}$$

3.4.6 Algebraic Product

The algebraic product AB of A and B is characterized by the following membership function:

$$\mu_{AB}(x) = \mu_A(x)\mu_B(x) \text{ for } \forall x \in X \tag{3.14}$$

3.4.7 Algebraic Sum

The algebraic sum $A \oplus B$ of A and B is characterized by the following membership function:

$$\mu_{A \oplus B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x) \tag{3.15}$$

3.4.8 Difference

The difference $A - B$ of A and B is characterized by:

$$\mu_{A \cap B^c}(x) = \min(\mu_A(x), \mu_{B^c}(x)) \tag{3.16}$$

3.5 Fuzzy Arithmetic [55, 56]

The arithmetic operations in Fuzzy systems are as follows.

3.5.1 Addition of Fuzzy Numbers

The addition of X and Y can be calculated by using α -level cut and max-min convolution.

A) **α -level cut.** Using the concept of confidence intervals, the α -level sets of X and Y are $X_\alpha = [X_\alpha{}^L, X_\alpha{}^R]$ and $Y_\alpha = [Y_\alpha{}^L, Y_\alpha{}^R]$ where the result Z of the addition is:

$$Z_\alpha = X_\alpha (+) Y_\alpha = [X_\alpha{}^L + Y_\alpha{}^L, X_\alpha{}^R + Y_\alpha{}^R] \tag{3.17}$$

for every $\alpha \in [0,1]$.

B) **Max-Min Convolution.** The addition of the fuzzy numbers X and Y is represented as:

$$Z(z) = \max_{z=x+y} [\min[\mu_X(x), \mu_Y(y)]] \tag{3.18}$$

3.5.2 Subtraction of Fuzzy Numbers

A) **α -level cut.** The subtraction of the fuzzy numbers X and Y in the α -level cut representation is:

$$Z_\alpha = X_\alpha (-) Y_\alpha = [X_\alpha{}^L - Y_\alpha{}^R, X_\alpha{}^R - Y_\alpha{}^L] \tag{3.19}$$

for every $\alpha \in [0,1]$.

B) **Max-Min Convolution.** The subtraction of the fuzzy numbers X and Y is represented as:

$$\begin{aligned} \mu_Z(Z) = & \max_{z=x-y} \{[\mu_x(x), \mu_Y(y)]\} \\ & \max_{z=x+y} \{[\mu_x(x), \mu_Y(-y)]\} \\ & \max_{z=x+y} \{[\mu_x(x), \mu_{-Y}(y)]\} \end{aligned} \tag{3.20}$$

3.5.3 Multiplication of Fuzzy Numbers

A) α-level cut. The multiplication of the fuzzy numbers X and Y in the α-level cut representation is:

$$Z_\alpha = X_\alpha (.) Y_\alpha = [X_\alpha{}^L y_\alpha{}^L, X_\alpha{}^R Y_\alpha{}^R] \tag{3.21}$$

for every $\alpha \in [0,1]$.

B) Max-Min Convolution. The Multiplication of the fuzzy number X and Y is represented by Kaufmann and Gupta in the following procedure as:

1. First, find Z^1 (the peak of the fuzzy number Z) such that $\mu_z(z^1) = 1$ then we calculate the left and right legs.
2. The left legs of $\mu_z(z)$ is defined as:

$$\mu_z(z) = \max_{xy \le z} \{\min[\mu_x(x), \mu_Y(y)]\} \tag{3.22}$$

3. The right leg of $\mu_z(z)$ is defined as:

$$\mu_z(z) = \max_{xy \ge z} \{\min[\mu_x(x), \mu_Y(y)]\} \tag{3.23}$$

3.5.4 Division of Fuzzy Numbers

A) α-level cut.

$$Z_\alpha = X_\alpha (:) Y_\alpha = [x_\alpha{}^L / y_\alpha{}^R, x_\alpha{}^R / y_\alpha{}^L] \tag{3.24}$$

B) Max-Min Convolution. As defined earlier we must find the peak then the left and right leg.

1. The peak Z = X (:) Y is used.
2. The left leg is presented as:

$$\begin{aligned} \mu_z(z) &= \max_{x/y \le z} \{\min[\mu_x(x), \mu_Y(y)]\} \\ &\max_{xy \le z} \{\min[\mu_x(x), \mu_Y(1/y)]\} \\ &\max_{xy \le z} \{\min[\mu_x(x), \mu_{1/Y}(y)]\} \end{aligned} \tag{3.25}$$

3. The right leg is presented as:

$$\mu_z(z) = \max_{x/y \geq z} \{\min[\mu_x(x), \mu_Y(y)]\}$$
$$\max_{xy \geq z} \{\min[\mu_x(x), \mu_Y(1/y)]\} \tag{3.26}$$
$$\max_{xy \geq z} \{\min[\mu_x(x), \mu_{1/Y}(y)]\}$$

3.6 LR-Type Fuzzy Number [55, 56]

A fuzzy number is defined to be of the LR type if there are reference functions L and R and positive scalars as shown in Figure (3-3). Where α represents the (left spread), β represents the (right spread) and m represents the middle or some times called the (mean) or crisp value. The mathematical representation for the triangular membership function is found by letting $L(x) = R(x) = \max(0, 1-x)$

Then

$$\mu_M(x) = \begin{Bmatrix} L\left(\dfrac{m-x}{\alpha}\right) & for\ x \leq m \\ R\left(\dfrac{x-m}{\beta}\right) & for\ x \geq m \end{Bmatrix} \tag{3.27}$$

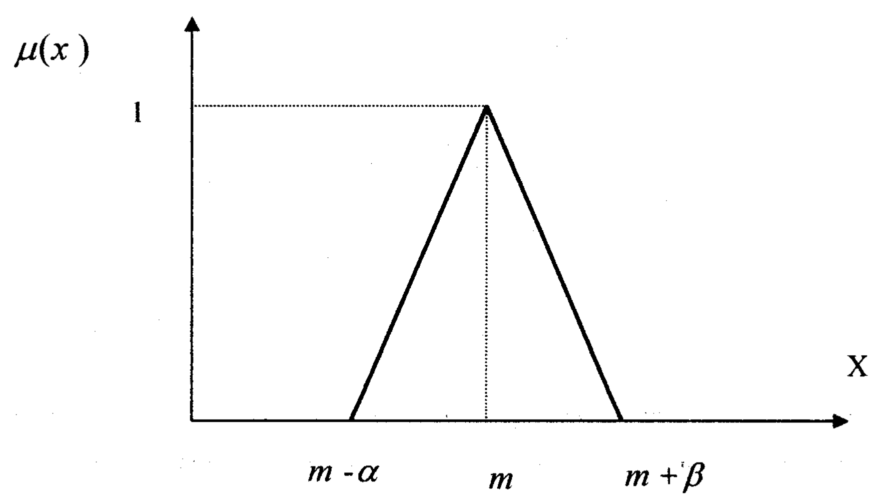


Figure (3-3) Triangular Membership Function

As the spread increases, M becomes fuzzier and fuzzier. Symbolically we write:

$$M = (m, \alpha, \beta) \tag{3.28}$$

Table (3-1) shows all the mathematical formulas used for L.R representation of fuzzy numbers.

3.7 Interval Arithmetic [55]

The interval arithmetic normally used with uncertain data obtained from different instruments. If we enclose those value obtained in a closed interval on the real line R; that is, this uncertain value is inside an interval of confidence R, $x \in [a_1, a_2]$ where $a_1 \le a_2$. Table (3-2) shows the entire fuzzy arithmetic interval used with a triangular fuzzy number. Where the fuzzy number denoted by: $X = (x^m, x^p, x^o)$ express the middle $x^m = x$, the left spread $x^p = x - \alpha$ and the right spread $x^o = x + \beta$. The condition $X > 0$ or $Y > 0$ means that the support of the fuzzy number is positive interval, i.e.: $(x^m - x^p) > 0$. Similarly $X < 0$ or $Y < 0$ means that the support of the fuzzy number is a negative interval, i.e.: $(x^m - x^p) < 0$. In Chapter (5) and (6) the tools for interval arithmetic are employed to find the fuzzy variable represented by its left, middle and right side of the triangular membership function.

3.8 Triangular Norm (t-norms) [55]

Let t: $[0,1]x[0,1] \to [0,1]$ be a function that transforms the membership functions of fuzzy sets A and B into the membership function of the intersection of A and B, that is, $t[\mu_A(x), \mu_B(x)] = \mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$

In order for the function t to be qualified as an intersection, it must satisfy at least the following four requirements:

Axiom t1: $t(0,0) = 0; t(\mu_A(x), 1) = t(1, \mu_A(x)) = \mu_A(x)$ (boundary condition).

Axiom t2: $t(\mu_A(x), \mu_B(x)) = t(\mu_B(x), \mu_A(x))$ (commutativity).

Axiom t3: *if* $\mu_A(x) \le \mu_C(x)$ and $\mu_B(x) \le \mu_D(x)$, then

$t(\mu_A(x), \mu_B(x)) \le t(\mu_C(x), \mu_D(x))$ (nondecreasing).

Axiom t4: $t[t(\mu_A(x),\mu_B(x)),\mu_C(x)] = t[\mu_A(x),t(\mu_B(x),\mu_C(x))]$ (associativity).

Table (3-1) Fuzzy Arithmetic on Triangular L-R Representation of Fuzzy Numbers

$X = (x,\alpha,\beta)$ & $Y = (y,r,\delta)$

Where for fuzzy X ..x is the middle value, α is the left spread and β is the left spread and the same apply for fuzzy Y

Image of Y:	$-Y = (-y,\delta,r)\, -Y = (-y,\delta,r)$
Inverse of Y:	$Y^{-1} = (y^{-1},\delta y^{-2},r\,y^{-2})$
Addition:	$X(+)\,Y = (x+y,\alpha+r,\beta+\delta)$
Subtraction:	$X(-)\,Y = X(+)-Y = (x-y,\alpha+\delta,\beta+r)$
Multiplication: $X>0,\ Y>0: X(\bullet)\,Y = (xy,xr+y\alpha,x\delta+y\beta)$ $X<0,\ Y>0: X(\bullet)\,Y = (xy,y\alpha-x\delta,y\beta-xr)$ $X<0,\ Y<0: X(\bullet)\,Y = (xy,-x\delta-y\beta,-xr-y\alpha)$	
Scalar Multiplication: $a>0, a\in R: a(\bullet)X = (ax, a\alpha, a\beta)$ $a<0, a\in R: a(\bullet)X = (ax, -a\beta, -a\alpha)$	
Division: $X>0,\ Y>0: X(:)\,Y = (x/y,(x\delta+y\alpha)/y^2,(xr+y\beta)/y^2)$ $X<0,\ Y>0: X(:)\,Y = (x/y,(y\alpha-xr)/y^2,(y\beta-x\delta)/y^2)$ $X<0,\ Y<0: X(:)\,Y = (x/y,(-xr-y\beta)/y^2,(-x\delta-y\alpha)/y^2)$	

Table (3-2) Fuzzy Interval Arithmetic on Triangular Fuzzy Numbers

$$X = (x^m, x^p, x^o) \;\&\; Y = (y^m, y^p, y^o)$$

Image of Y: $-Y = (-y^m, -y^o, -y^p)$

Inverse of Y: $Y^{-1} = (1/y^m, 1/y^o, 1/y^p)$

Addition: $X\,(+)\,Y = (x^m + y^m, x^p + y^p, x^o + y^o)$

Subtraction: $X\,(-)\,Y = X\,(+) -Y = (x^m - y^m, x^p - y^o, x^o - y^p)$

Multiplication:

$$X > 0,\ Y > 0 : X\,(\bullet)\,Y = (x^m y^m, x^p y^p, x^o y^o)$$
$$X < 0,\ Y > 0 : X\,(\bullet)\,Y = (x^m y^m, x^p y^o, x^o y^p)$$
$$X < 0,\ Y < 0 : X\,(\bullet)\,Y = (x^m y^m, x^o y^o, x^p y^p)$$

Scalar Multiplication:

$$a > 0, a \in R : a(\bullet)X = (ax^m, ax^p, ax^o)$$
$$a < 0, a \in R : a(\bullet)X = (ax^m, ax^o, ax^p)$$

Division:

$$X > 0,\ Y > 0 : X\,(:)\,Y = (x^m / y^m, x^p / y^o, x^o / y^p)$$
$$X < 0,\ Y > 0 : X\,(:)\,Y = (x^m / y^m, x^o / y^o, x^p / y^p)$$
$$X < 0,\ Y < 0 : X\,(:)\,Y = (x^m / y^m, x^o / y^p, x^p / y^o)$$

3.9 Fuzzy Linear Programming [56, 62]

Linear programming is concerned with the efficient allocation of limited resources to activities with the objective of meeting a desired goal such as maximizing profit or minimizing cost. The distinct characteristic of linear programming models is that the interrelations between activities are linear relationships which are the satisfactions of the proportionality and additively requirements. Symbolically, the standard linear programming problem may be stated as:

$$\begin{aligned} &\text{maximize cx} \\ &\text{subject to } Ax \le b \\ &\qquad\qquad x \ge 0 \end{aligned} \tag{3.29}$$

Where $x = (x_1,..x_n) \in R^n$ are the decision variables to be determined, $c = (c_1,..c_n)$ are called objective coefficients, and $A = [a_{ij}] \in R^{mxn}$ is called constraint matrix with its elements a_{ij} called constraints coefficients, and $b = (b_1,...b_n)^T$ are called resources. These input data (of c , b and A) are usually fuzzy/imprecise because of incomplete or non-obtainable information.

To formulate these fuzzy/imprecise numbers, we can use membership functions or possibility distributions (depending on specific problems). The function forms of membership functions and possibility distributions are depicted in Figures (3-4) and (3-5), respectively. With these fuzzy/imprecise input data, Equation (3-29) is then called fuzzy/possibilistic (linear) programming. The grade of a membership function indicates a subjective degree of satisfaction within given tolerances. On the other hand, the grade of possibility indicates the subjective or objective degree of the occurrence of an event.

It is important to realize this distinction while modeling fuzziness/imprecision in mathematical programming problems. Figures (3-4, 3-5) illustrate two cases of the preference-based membership function of fuzzy resources. When the constraints are $Ax \le \tilde{b}$, the rational preference-based membership functions can be assumed to be non-increasing. Similarly, non-decreasing functions can be assumed for $Ax \ge \tilde{b}$. For equality constraints, triangular or trapezoid functions might be appropriate. For the maximization (or minimization) problem, the preference-based membership

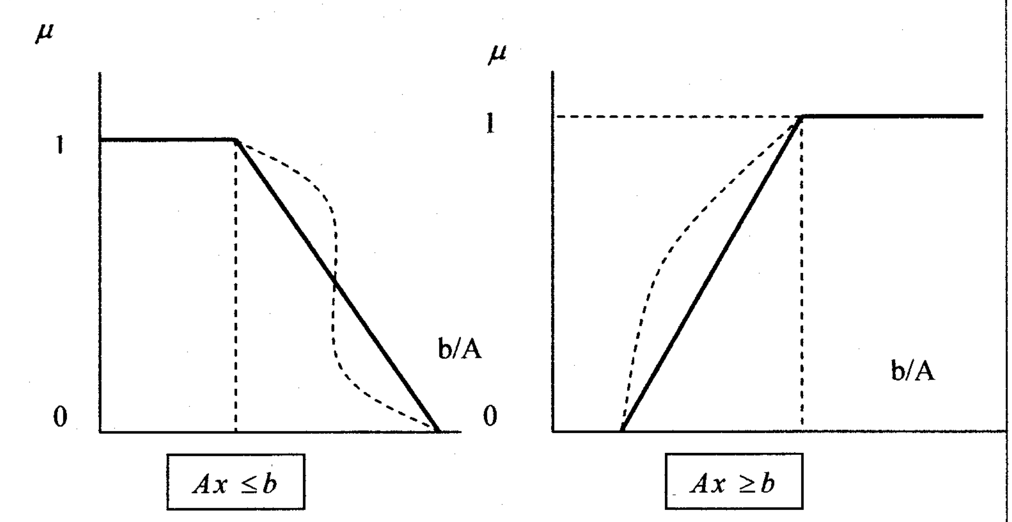


Figure (3-4) Linearly Decreasing and Increasing Membership Function

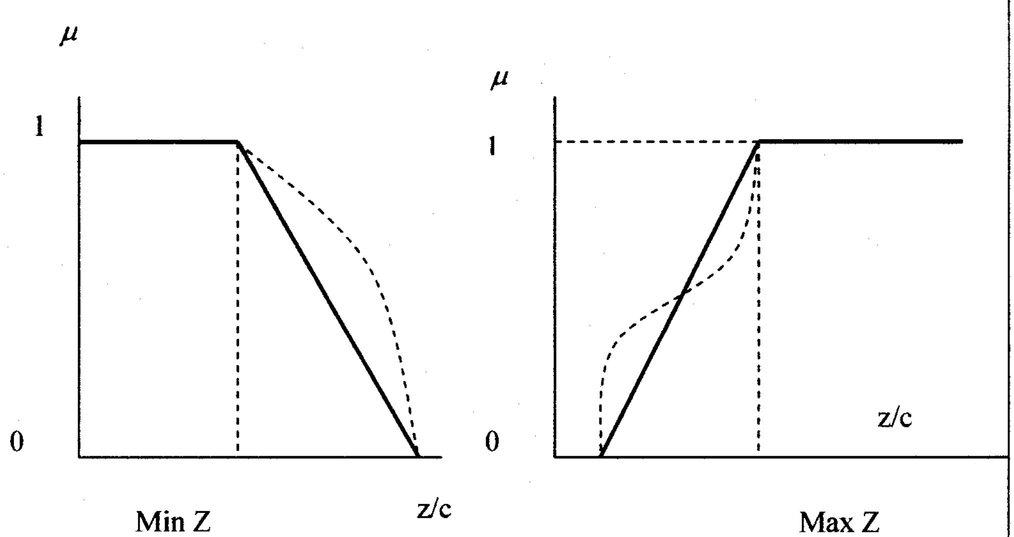


Figure (3-5) Min Z and Max Z Representation

functions of $\tilde{c}$ can be assumed to be non-decreasing as $\tilde{b}$ in $Ax \geq \tilde{b}$ or (or non-increasing as $\tilde{b}$ in $Ax \leq \tilde{b}$). As to the preference-based membership functions of $\tilde{A}$, they may be either non-increasing for $\tilde{A}x \leq b$ or non-decreasing for $\tilde{A}x \geq b$. Sometimes, triangular or trapezoidal functions might be adopted. For possibilistic (linear) programming, the possibility distributions of $\tilde{A}$, $\tilde{b}$ and/ $\tilde{c}$ are often assumed to be triangular or trapezoid membership functions.

When any of (cx) and/or Ax is a non-linear function, Equation (3-29) becomes a non-linear programming problem. If x is restricted to be an integer, then Equation (3-29) will become an integer programming problem. Both cases with fuzzy/imprecise input data as show in Figures (3-4) and (3-5) Equation (3-29) becomes fuzzy (possibilistic) non-linear and integer programming, respectively.

3.10 Fuzzy Decision

Assume that objective(s) and constraints in an imprecise situation can be represented by fuzzy sets. For an illustration, suppose that we have a fuzzy goal G and a fuzzy constraint C in a decision space X expressed as follows:

G: x should be substantially larger than 10, with

$$\begin{aligned} \mu_c(x) &= [1+(x-10)^{-2}]^{-1} && \text{if } x \geq 10 \\ &= 0 && \text{if } x<10 \end{aligned}$$

C: x should be in the vicinity of 15,

$$\mu_c(x) = [1+(x-15)^4]^{-1}$$

Then, with the assumption of the symmetry we may make decisions which satisfy both the constraint "and" the goal. That is: G and C are connected to another by the operator "and" which corresponds to the intersection of fuzzy sets. This implies that in the example the combined effect of the fuzzy goal G and the fuzzy constraint C on the choice of alternatives may be represented by the intersection $G \cap C$, with the membership function (see Figure 3-6):

$$\mu_{G \cap C}(x) = \mu_G(x) \wedge \mu_c(x) = \min\{\mu_G(x), \mu_c(x)\} \tag{3.30}$$

Then Bellman and Zadeh [47] proposed that a fuzzy decision may be defined as the fuzzy set of alternatives resulting from the intersection of the goals and the constraints. That is: the decision $D = G \cap C$ is a fuzzy set resulting from the intersection of G and C, and has its membership function.

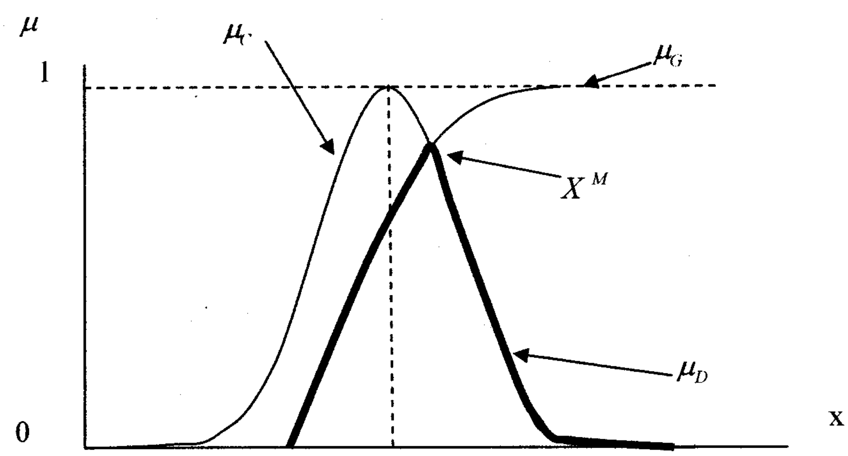


Figure (3-6) The Relationship of G, C and D

A maximizing decision then can be defined as follows:

$$\begin{aligned} \mu_D(x^M) &= \max\ \mu_D(x)\ for\ x \in \mathrm{X} \\ &= 0 \qquad\qquad \text{elsewhere} \end{aligned} \tag{3.31}$$

If $\mu_D(x)$ has a unique maximum at x^M, then the maximizing decision is a uniquely defined crisp decision which can be interpreted as the action which belongs to all fuzzy sets representing either constraints or objective(s) with the highest possible degree of membership.

3.10.1 Linear Programming with Fuzzy Resources

$$\begin{aligned} &\text{maximize } cx \\ &\text{subject to } (Ax)_i \le \tilde{b}_i, \quad i = 1,2,\ldots,m \\ &\qquad\qquad x \ge 0 \end{aligned} \tag{3.32}$$

We may also consider the following fuzzy inequality constraints:

$$\begin{aligned}&\text{maximize } cx\\&\text{subject to } (Ax)_i \tilde{\leq} b_i,\quad i=1,2,\ldots,m\\&\qquad\qquad x\geq 0\end{aligned}\tag{3.33}$$

Where $\tilde{\leq}$ is called fuzzy less than or equal to. If the membership functions of both cases are the same, then equations (3.32) and (3.33) will be the same and we will consider that both problems are equivalent in this thesis.

Let $t_i(>0)$ be the tolerance of the ***ith*** resource b_i then the fuzzy inequality $(Ax)_i \tilde{\leq} b_i$ is specified as $(Ax)_i \leq b_i + \theta t_i$ where $\theta \in [0,1]$. In other words, the fuzzy constraint $(Ax)_i \tilde{\leq} b_i$ is defined as a fuzzy set i with membership function.

$$\mu_i(x)=\begin{Bmatrix}1 & if & (Ax)_i \leq b_i\\ 1-[(Ax)_i-b_i]/t_i & if & b_i \leq (Ax)_i \leq b_i+t_i\\ 0 & if & (Ax)_i \geq b_i+t_i\end{Bmatrix}\tag{3.34}$$

Therefore, the problem becomes to find **x** such that cx and $\mu_i(x)$ for i=1,2,....,m are maximized. This is a multiple objective optimization problem.

Werner's **[57, 58]** proposed that the objective function of equation (3.32) and (3.33) should be fuzzy because of fuzzy total resources or fuzzy inequality constraints. The approach to solve this problem starts by solving the following two standard linear programming problems:

$$\begin{aligned}&\text{maximize } cx\\&\text{subject to } (Ax)_i \leq b_i,\quad i=1,2,\ldots,m\\&\qquad\qquad x\geq 0\\&\text{maximize } cx\\&\text{subject to } (Ax)_i \leq b_i+t_i,\ i=1,2,\ldots,m\\&\qquad\qquad x\geq 0\end{aligned}\tag{3.35}$$

Let x^0 and x^1 be the solutions of (3.34) and define $z^0=cx^0$ and $z^1=cx^1$. Then, the following membership function is defined to characterize the degree of optimality:

$$\mu_0(x)=\begin{Bmatrix}1 & if & cx\geq z^1\\ 1-\dfrac{z^1-cx}{z^1-z^0} & if & z^0\leq cx\leq z^1\\ 0 & if & cx<z^0\end{Bmatrix}\tag{3.36}$$

Clearly, when $cx \geq z^1$ we have $\mu_0(x) = 1$ which gives us maximum degree of optimality, when $cx \leq z^0$ we have $\mu_0(x) = 0$ which gives minimum degree of optimality, and when cx lies between z^1 and z^0 the degree of optimality changes from 1 to 0.

Since the constraints and objective function are represented by the membership functions (3.33) and (3.35), respectively, we can use the max-min method to solve this multiple objective optimization problem. Specifically, the problem becomes:

$$\max_{x \geq 0} \ (\alpha) = \min[\mu_0(x), \mu_1(x), \ldots., \mu_m(x)]$$

or equivalently

$$\begin{aligned} &\text{maximize} \quad \alpha \\ &\textit{subject to} \quad \mu_0(x) \geq \alpha \\ &\qquad\qquad\quad \mu_i(x) \geq \alpha, \quad i = 1,2,\ldots.,m \\ &\qquad\qquad\quad \alpha \in [0,1], \ x \geq 0 \end{aligned} \tag{3.37}$$

Substituting (3.33) and (3.35) into (3.36), we conclude that the fuzzy resource linear programming problem (3.32) can be solved by solving the following standard linear programming problem:

$$\begin{aligned} &\text{maximize} \quad \alpha \\ &\text{subject to} \ \ cx \geq z^1 - (1-\alpha)(z^1 - z^0) \\ &\qquad\qquad\ \ (Ax)_i \leq b_i + (1-\alpha)t_i, \quad i = 1,2,\ldots.,m \\ &\qquad\qquad\ \ \alpha \in [0,1], \ x \geq 0 \end{aligned} \tag{3.38}$$

3.10.2 Linear Programming with Fuzzy Objective Coefficients [59]

Consider the linear programming problem with fuzzy objective coefficients given as

$$\begin{aligned} &\text{maximize } \tilde{c}x \\ &\text{subject to } Ax \leq b \\ &\qquad\qquad\quad x \geq 0 \end{aligned} \tag{3.39}$$

For simplicity and without loss of much generality, we assume that the $\tilde{c}_i$'s are triangular fuzzy numbers with membership functions $\mu_{\tilde{C}_i}(x; c_i^-, c_i^0, c_i^+)$.

Symbolically, let $\tilde{c}_i = (c_i^-, c_i^0, c_i^+)$. Then (3.38) becomes:

$$\begin{aligned} &\text{maximize} \quad (c_i^- x, c_i^0 x, c_i^+ x) \\ &\text{subject to} \quad Ax \le b \\ &\qquad\qquad\quad x \ge 0 \end{aligned} \tag{3.40}$$

Where $c^- = (c_1^-, ..., c_n^-), c^0 = (c_1^0, ..., c_n^0)$ and $c^+ = (c_1^+, ..., c_n^+)$. This is a multiple objective linear programming problem. Two approaches were proposed by Lai & Hwang **[60]** to solve this problem. The first approach is simply combining the three objectives into a single objective function. For example, $c^- x, c^0 x$ and $c^+ x$ can be combined into the so-called most-likely criterion $\frac{(4c^0 + c^- + c^+)x}{6}$ (Lai and Hwang **[60]**. So (3.39) is converted into the following standard linear programming problem:

$$\begin{aligned} &\text{maximize} \quad \frac{(4c^0 + c^- + c^+)x}{6} \\ &\text{subject to} \quad Ax \le b \\ &\qquad\qquad\quad x \ge 0 \end{aligned} \tag{3.41}$$

Other weighted-sum strategies also may be used.

The second approach by Lai and Hwang suggests that the fuzzy objective is fully defined by three corner points $(c^- x, c^0 x, c^+ x)$ geometrically. Thus, maximizing the fuzzy objective can be obtained by pushing these three critical points in the direction of the right-hand side. Fortunately, the vertical coordinates of the critical points are fixed at either 1 or 0. The only considerations then are the three horizontal coordinates, as shown in Figure (3-7). Then the observation that our goal is to maximize the triangular fuzzy number $(c^- x, c^0 x, c^+ x)$. Therefore, instead of maximizing the three values $c^- x, c^0 x$ and $c^+ x$ simultaneously, we may maximize $c^0 x$ (the center), minimize $c^0 x - c^- x$ (the left leg), and maximize $c^+ x - c^0 x$ (the right leg). In this way, the triangular membership function is pushed to the right.

Thus, equation (3.39) is changed to another multiple objective linear programming problem, as follows:

$$\begin{aligned} &\text{minimize} && z1 = (c^0 - c^-)x \\ &\text{maximize} && z2 = c^0 x \\ &\text{maximize} && z3 = (c^+ - c^0)x \\ &\text{subject to} && Ax \le b \\ & && x \ge 0 \end{aligned} \tag{3.42}$$

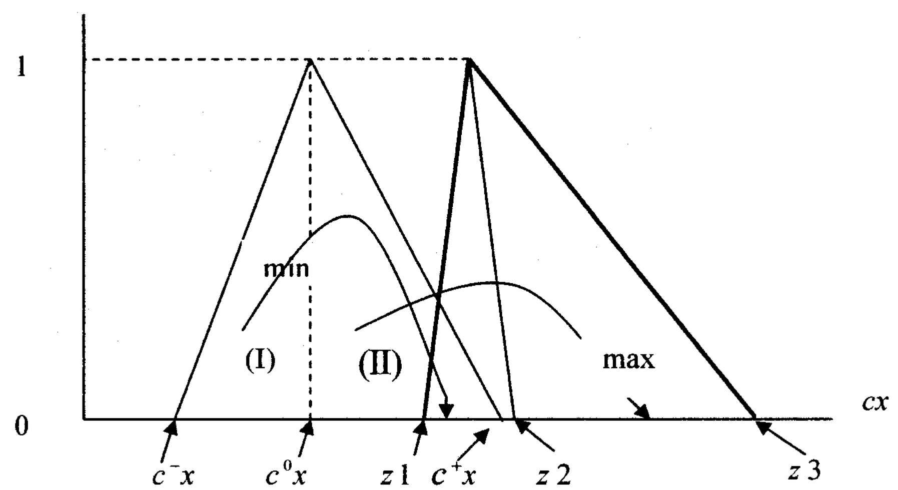


Figure (3-7) Second Approach by Lai and Hwang

A method to solve this problem is to characterize the three objective functions by membership functions and then maximize their α-cut. Specifically, we first get the solutions:

$$\begin{aligned} z_1^P &= \min_{x\in X}(c^0 - c^-)x, & z_1^N &= \max_{x\in X}(c^0 - c^-)x \\ z_2^P &= \max_{x\in X} c^0 x, & z_2^N &= \min_{x\in X} c^0 x \\ z_3^P &= \max_{x\in X}(c^+ - c^-)x, & z_3^N &= \min_{x\in X}(c^+ - c^-)x \end{aligned} \tag{3.43}$$

Where $X = \{x | Ax \le b, x \ge 0\}$. The solutions z_i^P are called Positive Ideal Solution and z_i^N are called Negative Ideal Solution. Then define the following three membership functions to characterize the three objectives:

$$\mu_{z_1}(x) = \begin{cases} 1 & if \quad (c^0 - c^-)x \le z_1^P \\ \dfrac{z_1^N - (c^0 - c^-)x}{z_1^N - z_1^P} & if \quad z_1^P \le (c^0 - c^-)x \le z_1^N \\ 0 & if \quad (c^0 - c^-)x \ge z_1^N \end{cases} \tag{3.44}$$

$$\mu_{z_2}(x) = \begin{cases} 1 & if \quad c^0 x \le z_2^P \\ \dfrac{c^0 x - z_2^N}{z_2^P - z_2^N} & if \quad z_2^N \le c^0 x \le z_2^P \\ 0 & if \quad c^0 x \ge z_2^N \end{cases} \tag{3.45}$$

$$\mu_{z_3}(x) = \begin{cases} 1 & if \quad (c^+ - c^0)x \le z_3^P \\ \dfrac{(c^+ - c^0)x - z_3^N}{z_3^P - z_3^N} & if \quad z_3^N \le (c^+ - c^0)x \le z_3^P \\ 0 & if \quad (c^+ - c^0)x \ge z_3^N \end{cases} \tag{3.46}$$

Finally, the problem is solved by solving the following standard linear programming problem:

$$\begin{aligned} &\text{maximize} \quad cx \\ &\text{subject to} \quad \mu_{z_i}(x) \ge \alpha, \quad i=1,2,3 \\ &\qquad\qquad Ax \le b, x \ge 0 \end{aligned} \tag{3.47}$$

3.10.3 Linear Programming with Fuzzy Constraint Coefficients [59]

Consider the linear programming problem with fuzzy constraint coefficients

$$\begin{aligned} &\text{maximize cx} \\ &\text{subject to } \tilde{A}x \le b \\ &\qquad\qquad x \ge 0 \end{aligned} \tag{3.48}$$

Again, for simplicity and without loss of much generality, we assume that $\tilde{A} = [\tilde{a}_{ij}]$ consists of triangular fuzzy numbers, that is, $\tilde{a}_{ij} = (a_{ij}^-, a_{ij}^0, a_{ij}^+)$ and

$\tilde{A} = (A^-, A^0, A^+)$, where $A^- = [a_{ij}^-]$, $A^0 = [a_{ij}^0]$ and $A^+ = [a_{ij}^+]$. Then the problem becomes:

$$\begin{aligned} &\text{maximize } cx \\ &\text{subject to } (A^- x, A^0 x, A^+ x) \le b_i \\ &\qquad x \ge 0 \end{aligned} \tag{3.49}$$

Using the most-likely criterion as in (3.40), we convert (3.46) into the following standard linear programming problem:

$$\begin{aligned} &\text{maximize } cx \\ &\text{subject to } \frac{(4A^0 + A^- + A^+)x}{6} x \le b \\ &\qquad x \ge 0 \end{aligned} \tag{3.50}$$

Up to this point, we have solved the three basic fuzzy linear programming problems (3.32)-(3.45). Other types of fuzzy linear programming problems are essentially combinations of the three basic problems and therefore, can be solved using similar approaches. For example, consider the problem where all the coefficients are fuzzy numbers shown below:

$$\begin{aligned} &\text{maximize } \tilde{c}x \\ &\text{subject to } \tilde{A}x \le \tilde{b} \\ &\qquad x \ge 0 \end{aligned} \tag{3.51}$$

Assume that $\tilde{c}$, $\tilde{A}$ and $\tilde{b}$ consist of triangular fuzzy numbers, that means each one will have its middle, left and right spread represented as $\tilde{c} = (c^-, c^0, c^+)$,

$\tilde{A} = (A^-, A^0, A^+)$ and $\tilde{b} = (b^-, b^0, b^+)$, then (3.48) can be converted into the following multiple objective linear programming problem:

$$\begin{aligned} &\text{maximize} \quad z1 = (c^0 - c^-)x \\ &\text{minimize} \quad z2 = c^0 x \\ &\text{minimize} \quad z3 = (c^+ - c^0)x \\ &\text{subject to} \quad A_\beta^- x \le b_\beta^-, A_\beta^0 x \le b_\beta^0, A_\beta^+ x \le b_\beta^+ \\ &\qquad x \ge 0 \end{aligned} \tag{3.52}$$

Where the given value of β is the minimal acceptable possibility given. This value can be assigned by the programmer to obtain the best solution of the problem. The obtained solution will be a triangular representation as shown in Figure (3-7). Then we can use the method explained in equation (3.43) to solve this problem. This method will be used in chapter (8) to calculate the optimal total cost of a fuzzy coefficient and fuzzy active and reactive loads.

3.11 Zimmermann's Approach – A Symmetric Model [60]

In this approach, the goal b_0 and its corresponding tolerance p_0 of the fuzzy objective are given initially, and also are for the fuzzy resources: b_i and its corresponding tolerances p_i, $\forall i$. The fuzzy objective and the fuzzy constraints are then considered without difference, and their corresponding regions can be described in the intervals $[b_i, b_i + p_i], \forall i$. Thus, Equation (3.32) can be considered as:

$$\begin{array}{ll} \textit{find} & \mathrm{x} \\ \text{such that} & cx \geq b_0 \\ & (\mathrm{Ax})_i \leq b_i, \forall i \\ & \mathrm{x} \geq 0 \end{array} \tag{3.53}$$

In fuzzy set theory, the fuzzy objective function and the fuzzy constraints are defined by their corresponding membership functions. For simplicity, let us assume that the membership function μ_0 of the fuzzy objective is a non-decreasing continuous linear function, and the membership functions $\mu_0, \forall i$, of the fuzzy constraints are non-increasing continuous linear membership functions as follows (see Figure (3-7) and (3-8).

$$\mu_0(x) = \left\{ \begin{array}{ccc} 1 & if & cx > b_0 \\ 1 - [b_0 - cx]/p_0 & if & b_0 - p_0 \leq cx \leq b_0 \\ 0 & if & cx < b_0 - p_0 \end{array} \right\} \tag{3.54}$$

$$\mu_i(x) = \left\{ \begin{array}{ccc} 1 & if & (Ax)_i < b_i \\ 1 - [(Ax)_i - b_i]/p_i & if & b_i \leq (Ax)_i \leq b_i + p_i \\ 0 & if & (Ax)_i > b_i + p_i \end{array} \right\} \tag{3.55}$$

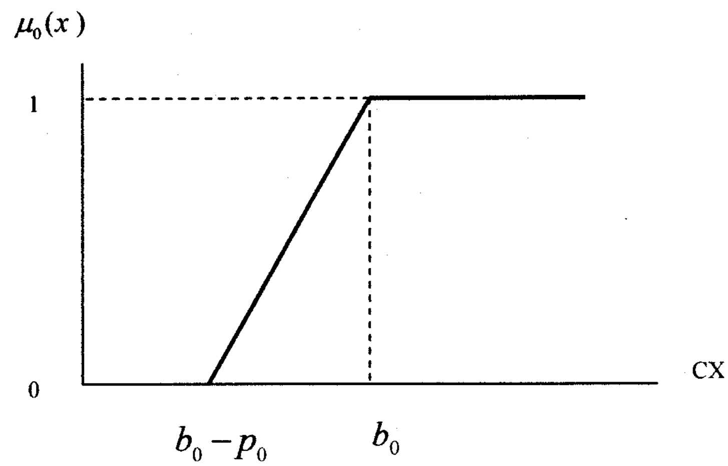


Figure (3-8) Fuzzy Linearly Increasing Objective Membership Function

Zimmermann then used Bellman and Zadeh's max-min operator to solve Equation (3.50). Thus, the optimal solution can be obtained by:

$$\max\ \mu_D(x) = \max\{\min[\mu_0(x), \mu_i(x) .... \mu_m(x)]\} \qquad (3.56)$$

Where μ_D is the membership function of the decision space D

and $\mu_D = \min(\mu_0, \mu_i, ..., \mu_m)$. If $\alpha = \mu_D$, then Equation (3.50), via the Equation (3.53) will be equivalent to:

$$\begin{aligned} &\text{maximize} \quad \alpha \\ &\textit{subject to} \quad \mu_0(x) = 1 - (b_0 - cx)/p_0 \geq \alpha \\ &\qquad \mu_i(x) = 1 - [(Ax)_i - b_i]/p_i \geq \alpha, \quad i = 1,2,....,m \\ &\qquad \mu_i(x), \forall i, \textit{and} \quad \alpha \in [0,1] \end{aligned} \qquad (3.57)$$

or

$$\begin{aligned} &\text{maximize} \quad \alpha \\ &\textit{subject to} \quad cx \geq b_0 - (1-\alpha)p_0 \\ &\qquad (Ax)_i \leq b_i + (1-\alpha)p_i, \ \forall \mathrm{i} \\ &\qquad x \geq 0 \ \textit{and} \ \alpha \in [0,1], \end{aligned} \qquad (3.58)$$

Where c, A, b_0, p_0, b_i and $p_i, \forall i$ are given initially.

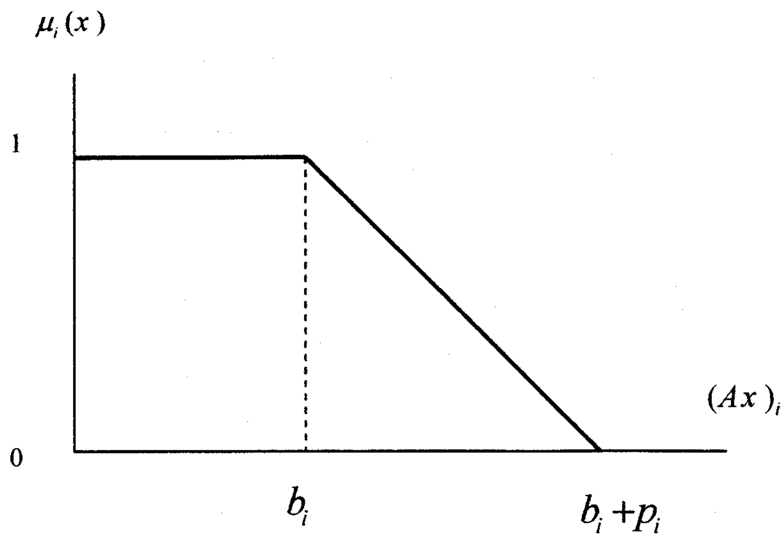


Figure (3-9) Fuzzy Linearly Decreasing Objective Membership Function

Obviously, Equation (3.55) is a crisp linear programming problem. A unique optimal solution can be obtained. It should be noted that this approach is considered as the first practical method to solve a linear programming problem with fuzzy resources and objective.

Chapter 4
Economic Dispatch of all Thermal Power Systems with Fuzzy Load

4.1 Introduction

This chapter presents a new and simple technique to solve the short-term economic dispatch problem of an all thermal electric power system, when the load demand of the system is considered to be fuzzy. The hard constraints, using this technique, are transformed to soft constraints. The membership function of the load is assumed to be triangular. A simulated example of a system consisting of two units is presented in this chapter to explain the main features of the proposed technique.

4.2 Problem Formulation

The primary objective of the ED problem of a power system that consists of ***m*** thermal units is to determine the most economic loading of the generators such that the load demand in the intervals of the generation scheduling horizon can be met and the operation constraints of the generators are satisfied. For a quadratic fuel cost function, the problem can be mathematically stated as:

Minimize

$$C_{total} = \sum_{i=1}^{NG} C_i = \sum_{i=1}^{NG} \alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}{}^2$$

Subject to satisfy the following constraints on the system

- Active power balance equation (APBE)

$$\sum_{i=1}^{NG} P_{G_i} = P_D + P_L \tag{4.1}$$

Where

P_D = is the total system demand of the network.

P_L = is the system transmission losses, which is function of the generation of each unit and system parameters related to the network model.

- The power output of any generator should not exceed its rating nor should it be lower than the minimum value necessary for stable boiler operation. Thus, the generations are restricted to lie within given minimum and maximum limits expressed as:

$$P_i(\min) \leq P_i \leq P_i(\max) \qquad i = 1, \ldots\ldots NG$$

The problem formulated above is a classical economic dispatch problem. It is well known and many techniques have been developed to solve it.

Reference [1] gives a comprehensive survey of the techniques used in solving the economic dispatch problem and the recent developments to improve the solution. The unit commitment problem and optimal power flow may be included in the problem formulation to overcome the difficulty of including the system losses in the formulation. Some techniques use the B-coefficients to express system losses.

It may be possible that fuzzy formulation for the economic dispatch problem would overcome difficulties involved in solving the problem. In the next section, we offer this formulation deriving the necessary equations based on the principle of equal incremental cost neglecting losses, where the solution of this type of problem can be found using closed-form expression. However, when losses are considered, the resulting equations as seen in the next chapter are nonlinear and must be solved iteratively. The fuzzy economic dispatch problem can be stated as

1. The power load on the system is Fuzzy $\tilde{P}_D$
2. The power generated from each unit will be fuzzy $\tilde{P}_{G_i}$

Then the optimization problem in this case is given as:

Minimize

$$\tilde{C}_{total} = \sum_{i=1}^{NG} \tilde{C}_i = \sum_{i=1}^{NG} \alpha_i + \beta_i \tilde{P}_{G_i} + \gamma_i \tilde{P}_{G_i}{}^2 \tag{4.2}$$

Subject to satisfying the following constraints

$$\sum_{i=1}^{NG} \tilde{P}_{G_i} - \tilde{P}_D \geq 0 \tag{4.3}$$

$$\tilde{P}_{G_i}(\min) \leq \tilde{P}_{G_i} \leq \tilde{P}_{G_i}(\max) \qquad i = 1, \ldots\ldots NG \tag{4.4}$$

From equation (4.3) **IF** a fuzzy system load $\tilde{P}_D = (\bar{P}_D, a_{\tilde{P}_D}, b_{\tilde{P}_D})$ is assumed to be a triangular membership function shown in Figure (4-1a), where its middle value is represented by $\bar{P}_D$ and the left and right spread are $a_{\tilde{P}_D}, b_{\tilde{P}_D}$ respectively, **THEN** the fuzzy generation $\tilde{P}_{G_i} = (\bar{P}_{G_i}, a_{\tilde{P}_{G_i}}, b_{\tilde{P}_{G_i}})$ will be of the same triangular membership function representation as shown in Figure (4-1b). The middle crisp value is represented by $\bar{P}_{G_i}$ and left, right spread of $a_{\tilde{P}_{G_i}}, b_{\tilde{P}_{G_i}}$ respectively, where $i = 1, ...NG$ represents the number of generation sources committed to the system network. The left, right side of the triangular membership function for the load demand can be calculated as $L_{\tilde{P}_D} = (\bar{P}_D - a_{\tilde{P}_D})$, $R_{\tilde{P}_D} = (\bar{P}_D + b_{\tilde{P}_D})$ respectively as shown in Figure (4-1a). The left and right side of the power generation is $L_{\tilde{P}_{G_i}} = (\bar{P}_{G_i} - a_{\tilde{P}_{G_i}})$, $R_{\tilde{P}_{G_i}} = (\bar{P}_{G_i} + b_{\tilde{P}_{G_i}})$ as shown in Figure (4-1b). In this formulation, we have translated the fuzzy load into a triangular membership function by assigning a degree of membership to each possible α-cut value of the load. Which means mapping the fuzzy variable on the [0, 1] interval. The solution of equation (4.2) will provide the generation possibility distributions corresponding to fuzzy loads for the minimum cost of operations **[42]**. In equation (4.3) the hard constraints mentioned are transferred to soft constraints by using the Lagrange multiplier. Using such equality

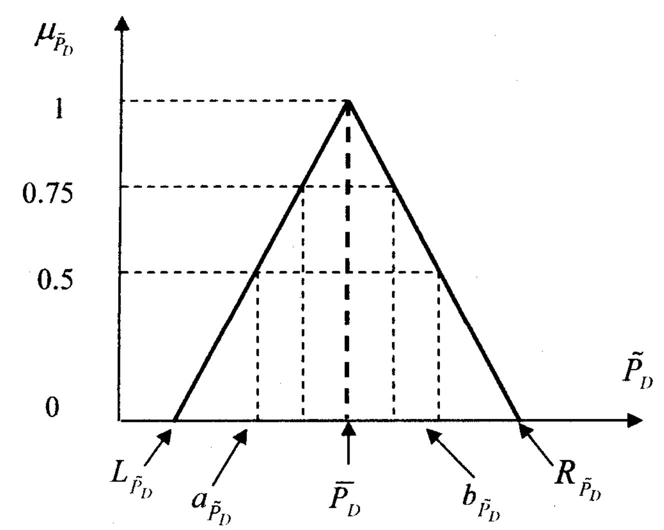


Figure (4-1a) Memberships Function for Load Demand

constraints includes implicitly the demand. The approach used in this chapter is to assume the fuzzy demand and fuzzy generation with different representations of their α -cut are expressed by (0, 0.5, 0.75 and 1) where the α -cut is used to create a family of crisp set in order to be used in fuzzy mathematical operations.

The membership formula for the load demand is expressed as:

$$\mu(\tilde{P}_D) = \begin{cases} 0 & \tilde{P}_D < L_{\tilde{P}_D} \\ \dfrac{\tilde{P}_D - L_{\tilde{P}_D}}{a_{\tilde{P}_D}} & L_{\tilde{P}_D} \le \tilde{P}_D \le \bar{P}_D \\ \dfrac{R_{\tilde{P}_D} - \tilde{P}_D}{b_{\tilde{P}_D}} & \bar{P}_D \le \tilde{P}_D \le R_{\tilde{P}_D} \\ 0 & \tilde{P}_D > R_{\tilde{P}_D} \end{cases} \tag{4.5}$$

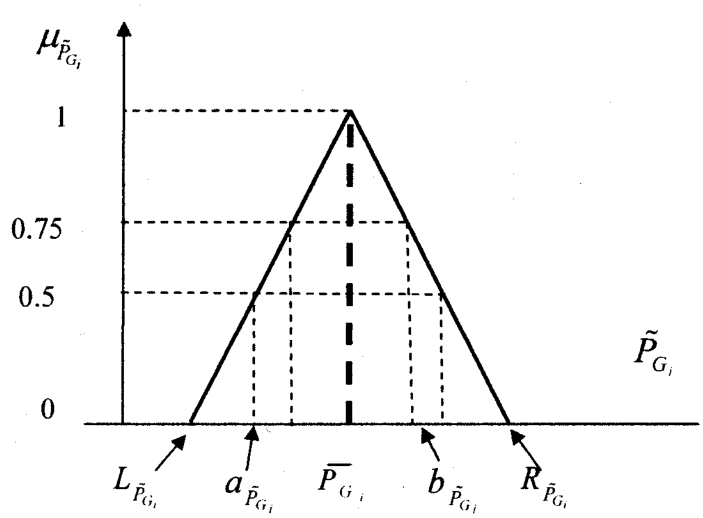


Figure (4-1b) Membership Function for Power Generation

The membership formula for the generator becomes:

$$\mu(\tilde{P}_{G_i}) = \begin{cases} 0 & \tilde{P}_{G_i} \le L_{\tilde{P}_{G_i}} \\ \dfrac{\tilde{P}_{G_i} - L_{\tilde{P}_{G_i}}}{a_{\tilde{P}_{G_i}}} & L_{\tilde{P}_{G_i}} \le \tilde{P}_{G_i} \le \bar{P}_{G_i} \\ \dfrac{R_{\tilde{P}_{G_i}} - \tilde{P}_{G_i}}{b_{\tilde{P}_{G_i}}} & \bar{P}_{G_i} \le \tilde{P}_{G_i} \le R_{\tilde{P}_{G_i}} \\ 0 & \tilde{P}_{G_i} \ge R_{\tilde{P}_{G_i}} \end{cases} \tag{4.6}$$

Below is a review of the crisp case to obtain the optimal solution using the Lagrange multiplier formula to relax "system wide constraints" into an unconstrained form that matches the original objective function at feasible points.

$$L = \sum_{i=1}^{NG} (\alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}{}^2) + \lambda (P_D - \sum_{i=1}^{NG} P_{G_i}) + \mu_i (P_{G_i}{}^m - P_{G_i}) + \psi_i (P_{G_i} - P_{G_i}{}^M) \tag{4.7}$$

Where λ, μ_i and ψ_i are fuzzy Kuhn-Tucker multipliers

Optimizing the formula by setting partial derivative to zero:

$$\frac{\partial L}{\partial P_{G_i}} = \beta_i + 2\gamma_i P_{G_i} + \lambda(-1) - \mu_i + \psi_i = 0 \tag{4.8}$$

$$\beta_i + 2\gamma_i P_{G_i} - \lambda - \mu_i + \psi_i = 0$$

$$\frac{\partial L}{\partial \lambda} = \lambda (\sum_{i=1}^{NG} P_{G_i} - P_D) = 0 \tag{4.9}$$

$$\frac{\partial L}{\partial \mu_i} = \mu_i (P_{G_i}{}^m - P_{G_i}) = 0 \tag{4.10}$$

$$\frac{\partial L}{\partial \psi_i} = \psi_i (P_{G_i} - P_{G_i}{}^M) = 0 \tag{4.11}$$

Assuming that unit ***i*** is operating within the specified limits, then μ_i and ψ_i will be equal zero.

Then from equation (4.8) we can obtain the incremental fuel cost λ as:

$$\lambda = \beta_i + 2\gamma_i P_{G_i} \tag{4.12}$$

Thus, fuzzifying the optimal solution obtained from the crisp optimization problem, then the incremental cost $\tilde{\lambda}$ can be written as:

$$\tilde{\lambda} = \beta_i + 2\gamma_i \tilde{P}_{G_i} \tag{4.13}$$

Solving for the power generation we get:

$$\tilde{P}_{G_i} = \frac{\tilde{\lambda} - \beta_i}{2\gamma_i} \qquad i = 1,..,NG \tag{4.14}$$

Then replacing equation (4.3) with (4.14) we get:

$$\sum_{i=1}^{NG} \frac{\tilde{\lambda} - \beta_i}{2\gamma_i} = \tilde{P}_D \tag{4.15}$$

Solving for $\tilde{\lambda}$ we get:

$$\tilde{\lambda} = \frac{2\tilde{P}_D + \sum_{i=1}^{NG} \frac{\beta_i}{\gamma_i}}{\sum_{i=1}^{NG} \frac{1}{\gamma_i}} \tag{4.16}$$

Substituting the middle, left and right spread representation into equation (4.16):

$$\tilde{\lambda}(\bar{\lambda}, a_{\tilde{\lambda}}, b_{\tilde{\lambda}}) = \frac{2(\bar{P}_D, a_{\tilde{P}_D}, b_{\tilde{P}_D}) + \sum_{i=1}^{NG} \frac{\bar{\beta}_i}{\bar{\gamma}_i}}{\sum_{i=1}^{NG} \frac{1}{\bar{\gamma}_i}} \tag{4.17}$$

In the above equation, we assume that the unit coefficients β_i *and* γ_i are crisp values. Using Table (3-2) from chapter (3) to implement the operation of fuzzy numbers such as addition, subtraction, division, multiplication and inversion by their α-cut operation then the crisp values in equation (4.17) is obtained by collecting all the middle crisp values of the fuel incremental cost which can be written as:

$$\bar{\lambda} = \frac{2\bar{P}_D + \sum_{i=1}^{NG} \left(\frac{\bar{\beta}_i}{\bar{\gamma}_i} \right)}{\sum_{i=1}^{NG} \frac{1}{\bar{\gamma}_i}} \tag{4.18}$$

While the left spread can be calculated from (4.16) to become:

$$a_{\tilde{\lambda}} = \frac{2(a_{\tilde{P}_D})}{\sum_{i=1}^{NG} (\frac{1}{\bar{\gamma}_i})} \tag{4.19}$$

And the right spread can be calculated as:

$$b_{\tilde{\lambda}} = \frac{2(b_{\tilde{P}_D})}{\sum_{i=1}^{NG}(\frac{1}{\bar{\gamma}_i})} \tag{4.20}$$

Equation (4.19) and (4.20) describes the fuzzy incremental fuel cost.

Substituting the middle, left and right spreads into the fuzzy generation of each unit from equation (4.14) we get:

$$\tilde{P}_{G_i} = (\bar{P}_{G_i}, a_{\tilde{P}_{G_i}}, b_{\tilde{P}_{G_i}}) = \frac{(\bar{\lambda}, a_{\tilde{\lambda}}, b_{\tilde{\lambda}}) - \bar{\beta}_i}{2\bar{\gamma}_i} \qquad i = 1,..,NG \tag{4.21}$$

The middle of the generation is calculated as:

$$\bar{P}_{G_i} = \frac{\bar{\lambda} - \bar{\beta}_i}{2\bar{\gamma}_i} = \bar{P}_{G_i} \qquad i = 1,..,NG \tag{4.22}$$

While the left spread and left side of the generation can be calculated as:

$$\begin{aligned} a_{\tilde{P}_{G_i}} &= \frac{a_{\tilde{\lambda}}}{2\bar{\gamma}_i} \\ L_{\tilde{P}_{G_i}} &= (\bar{P}_{G_i} - a_{\tilde{P}_{G_i}}) \cong \tilde{P}_{G_i(\min)} \end{aligned} \tag{4.23}$$

The right spread and right side of the generator can be calculated as:

$$\begin{aligned} b_{\tilde{P}_{G_i}} &= \frac{b_{\tilde{\lambda}}}{2\bar{\gamma}_i} \\ R_{\tilde{P}_{G_i}} &= (\bar{P}_{G_i} + b_{\tilde{P}_{G_i}}) \cong \tilde{P}_{G_i(\max)} \end{aligned} \tag{4.24}$$

The left and right sides of the generation given by equation (4.23) and (4.24) may equal the maximum and minimum limits of each thermal generator unit, or they may be included within the membership. This setting should not lead to any violation of the limit restricted on the generation as shown in equation (4.4). This means the load will be distributed evenly between the two units and satisfy the quality constraints given in equation (4.3). Using such a simplification reduces the cost calculation in the iterative method that considers the transmission line losses, even if there are some approximations. Furthermore, there is no crisp load in real time, the value of the load changes from minute to minute.

Equation (4.3) and (4.4) can be rewritten to be as:

$$\sum_{i=1}^{NG}(\bar{P}_{G_i}, a_{\tilde{P}_{G_i}}, b_{\tilde{P}_{G_i}}) - (\bar{P}_D, a_{\tilde{P}_D}, b_{\tilde{P}_D}) \geq 0 \tag{4.25}$$

$$L_{\tilde{P}_{G_i}} \leq \bar{P}_{G_i} \leq R_{\tilde{P}_{G_i}} \qquad i = 1,\ldots\ldots NG \tag{4.26}$$

The total fuzzy optimal cost function can be calculated using equation (4.2). Substituting the right and left side of the power generation for each unit to obtain the cost of each unit individually. Then the cost of all units is added to obtain the total cost. This can be verified in the following equations:

$$\tilde{C}_1 = (\bar{C}_1, L_{\tilde{C}_1}, R_{\tilde{C}_1}) = \alpha_1 + \beta_1(\bar{P}_{G_1}, L_{\tilde{P}_{G_1}}, R_{\tilde{P}_{G_1}}) + \gamma_1(\bar{P}_{G_1}, L_{\tilde{P}_{G_1}}, R_{\tilde{P}_{G_1}})(\bar{P}_{G_1}, L_{\tilde{P}_{G_1}}, R_{\tilde{P}_{G_1}}) \tag{4.27}$$

$$\tilde{C}_2 = (\bar{C}_2, L_{\tilde{C}_2}, R_{\tilde{C}_2}) = \alpha_2 + \beta_2(\bar{P}_{G_2}, L_{\tilde{P}_{G_2}}, R_{\tilde{P}_{G_2}}) + \gamma_2(\bar{P}_{G_2}, L_{\tilde{P}_{G_2}}, R_{\tilde{P}_{G_2}})(\bar{P}_{G_2}, L_{\tilde{P}_{G_2}}, R_{\tilde{P}_{G_2}}) \tag{4.28}$$

$$\tilde{C}_t = (\bar{C}_t, L_{\tilde{C}_t}, R_{\tilde{C}_t}) = (\bar{C}_1, L_{\tilde{C}_1}, R_{\tilde{C}_1}) + (\bar{C}_2, L_{\tilde{C}_2}, R_{\tilde{C}_2}) \tag{4.29}$$

Using Table (3-2) from chapter (3) the middle, left and right side of the total cost becomes:

$$\begin{aligned} \bar{C}_t &= \sum_{i=1}^{n}[\alpha_i + \beta_i(\bar{P}_{G_i}) + \gamma_i(\bar{P}_{G_i}^{\,2})] \\ L_{\tilde{C}_t} &= \sum_{i=1}^{n}[\alpha_i + \beta_i(L_{\tilde{P}_{G_i}}) + \gamma_i((L_{\tilde{P}_{G_i}})(L_{\tilde{P}_{G_i}}))] \\ R_{\tilde{C}_t} &= \sum_{i=1}^{n}[\alpha_i + \beta_i(R_{\tilde{P}_{G_i}}) + \gamma_i((R_{\tilde{P}_{G_i}})(R_{\tilde{P}_{G_i}}))] \end{aligned} \tag{4.30}$$

And if we use Table (3-1) the middle, left and right spread becomes:

$$\begin{aligned} \bar{C}_t &= \sum_{i=1}^{n}[\alpha_i + \beta_i(\bar{P}_{G_i}) + \gamma_i(\bar{P}_{G_i}^{\,2})] \\ a_{\tilde{C}_t} &= \sum_{i=1}^{n}[\beta_i(a_{\tilde{P}_{G_i}}) + \gamma_i((a_{\tilde{P}_{G_i}}\bar{P}_{G_i}) + (a_{\tilde{P}_{G_i}}\bar{P}_{G_i}))] \\ b_{\tilde{C}_t} &= \sum_{i=1}^{n}[\beta_i(b_{\tilde{P}_{G_i}}) + \gamma_i((b_{\tilde{P}_{G_i}}\bar{P}_{G_i}) + (b_{\tilde{P}_{G_i}}\bar{P}_{G_i}))] \end{aligned} \tag{4.31}$$

It is worthwhile to state here that trapezoidal membership functions as shown in Figure (4-2a), (4-2b) and (4-2c) can be used for the load demand, the fuel incremental cost and power generation of each unit instead of a triangular membership function representation. In Figure (4-2a) the peak value of the power load will occur between

$\underline{P}_D$ and $\overline{P}_D$. The minimum estimated load is a 5% deviation which will be $0.95\underline{P}_D$ and the maximum estimated load is a 10% deviation which will be $1.1\overline{P}_D$. The fuel incremental cost membership function shown in Figure (4-2b) shows the calculated incremental fuel cost λ_L corresponding to the value of $0.95\underline{P}_D$, while $\underline{\lambda}$ is calculated from $\underline{P}_D$, $\overline{\lambda}$ from $\overline{P}_D$ and λ_R from the maximum allowable 10% load deviation represented by $1.1\overline{P}_D$. Finally, the calculated trapezoidal membership function for the power generation describing the obtained fuzzy generation from a fuzzy load is shown in Figure (4-2c). The trapezoidal membership function is intended to be acknowledged in a future suggested research. The triangular membership function is a special case of the trapezoidal membership function and it will be used in our subsequent work. It is noted that the output fuzzy solution will have the same form as that of the input data this means that using triangular membership functions for the input will result in a triangular membership of the output. If a mixer of a triangular and trapezoidal are used then the outcome will be trapezoidal this follows because the trapezoidal function is the most general of forms using straight line segment. The fuzzy membership function is described by a mathematical expression involves extensive data collection that can be more complex if one uses trapezoidal or Gaussian form.

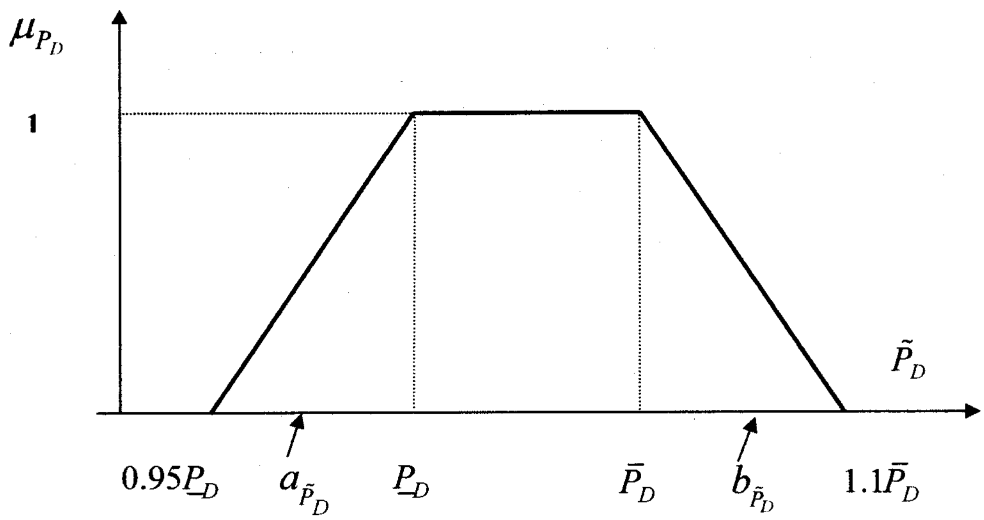


Figure (4-2a) A Trapezoidal Membership Function for Power Load

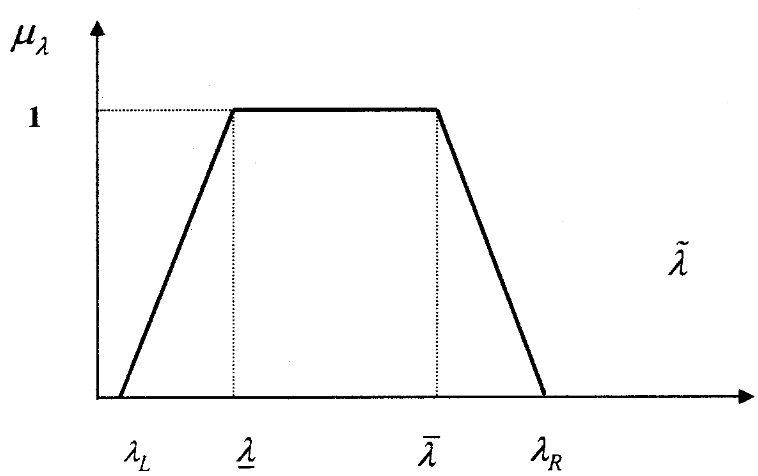


Figure (4-2b) A Trapezoidal Incremental Cost Membership Function

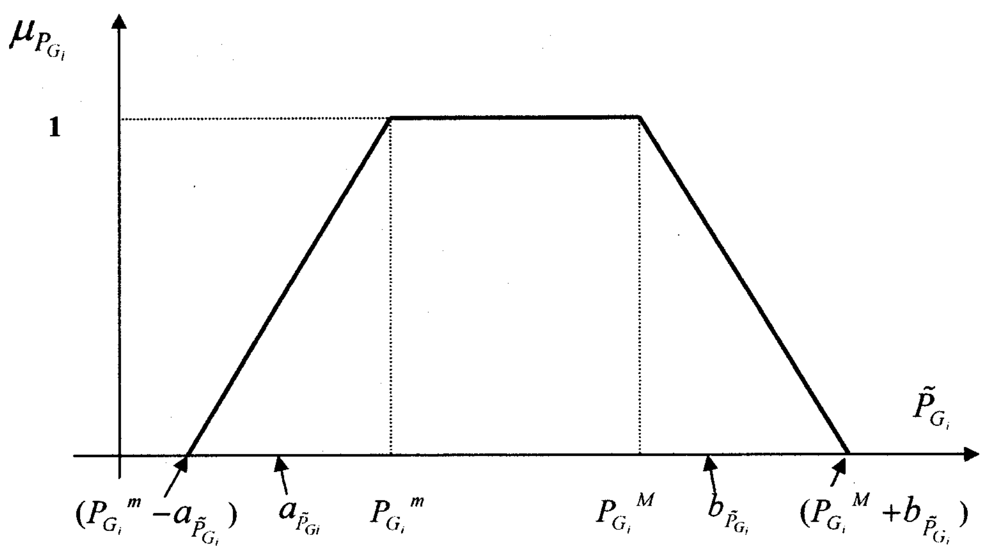


Figure (4-2c) A Trapezoidal Membership Function for Power Generation

4.3 Solution Algorithm

The load demand data was obtained from an estimated fuzzy short term load forecasting model, developed on the basis of fuzzy multiple linear regressions, to minimize the spread of the fuzzy coefficients that exist in the fuzzy winter model for weekdays and a weekend with a 20% deviation in the load demand in a 24-hour period **[63]**. The load demand will have upper, middle, and lower limits. Table (4-1) and Figure (4-1) shows the actual load demand on the system when α -cuts is equal to zero while the other left and right sides for different α -cuts values are calculated from the membership formula for the load demand in equation (4.5). In addition, the number of ***m*** thermal units feeding the load and the crisp characteristic coefficients of each unit $\alpha_i, \beta_i, and\ \gamma_i$ are known. Then a solution to the ED problem can be obtained using the following steps.

Step 1: Apply the principle of equal incremental cost to determine the optimal fuzzy dispatch and the total fuzzy cost. Then calculate the fuzzy fuel incremental cost, middle and spread, using equations (4.18), (4.19) and (4.20) for different α -cut values represented by (0, 0.5, 0.75 and 1) for hour in question.

Step 2: For each fuzzy incremental fuel cost, determine the fuzzy generation of each unit, the middle and spread, using equations (4.22), (4.23) and (4.24).

Step 3: Calculate the fuel cost of each unit that corresponds to its generation and hence the total fuel cost using equation (4.30).

4.4 Simulated Examples

The above steps are applied to a simulated example, consisting of two unit generation. The input/out fuel cost functions, for each unit, are given as:

$$F(P_{G_1}) = 200 + 7P_{G_1} + 0.008P_{G_1}^2 \quad \text{kJ/h}$$
$$F(P_{G_2}) = 180 + 6.3P_{G_2} + 0.009P_{G_2}^2 \quad \text{kJ/h}$$

The generation limit for each unit is:

$$P_{G_1\min} \le P_{G_1} \le P_{G_1\max} \ \text{MW}$$
$$P_{G_2\min} \le P_{G_2} \le P_{G_2\max} \ \text{MW}$$

Replacing the minimum and maximum limits with the left and right sides of power generation, then it can be written as:

$$L_{\tilde{P}_{G_1\min}} \le P_{G_1} \le R_{\tilde{P}_{G_1\max}} \quad \text{(MW)}$$
$$L_{\tilde{P}_{G_2\min}} \le P_{G_2} \le R_{\tilde{P}_{G_2\max}} \quad \text{(MW)}$$

Following the solution of the algorithm step by step in a simulation program for different α -cut values a number of tables are obtained and graphs are plotted to show the outcome that influences the generation and cost function when the load varies hour by hour. As an example the load demand at 10^{th} hour is a triangular membership function with middle, left and right spread. Those values can be calculated using the membership formula (4.5) for each α -cut representation then for each α -cut the incremental fuel cost is calculated for that particular hour. The power generation middle, left and right spread of each unit is obtained from equation (4.22) through (4.24) respectively then the total generation is added and tested with the load demand middle, left and right values. If they are equal then no violation has occurred. This solution algorithm is known as the analytical method. Another method to obtain the solution is the gradient method, where an iterative search solution for the fuel incremental cost is given as a guess initially and then the search continues until the total generations are equal to the load demand. In our example, the total sum of the generations of unit 1 and 2 at the 10^{th} hour is a triangular membership function with middle, left and right spread equal to the load demand at the 10^{th} hour which proves that their was no violation to the generation limit since the upper and lower values of the generation are within the 10% deviation of the load. The middle value of the total power generation and the load demand represent the conventional method or the crisp case. The fuzzy approach solution considers all the possibilities of fuzziness in the left and right spread of the load demand. Formulation of fuzzy system is to deal with the imprecise nature of the decision-maker to chose the best available solution from a wide range of solution that can be encountered do to load variation in a daily bases. The limitation restricted on the load in the conventional method will be overcome by using fuzzy sets and fuzzy mathematical operation. The middle, left and right spread value of the cost function for the 10^{th} hour is calculated for each unit from equation (4.30) then the total sum of the two units is obtained. This range of cost value is important because the variation of load happens suddenly if a large interconnected network is involved and the calculations using the standard method takes a lot of computation time and it does not provide enough information about the system performance resulting from calculating the measured values as separate entities. In fuzzy methods, variations are included in the analysis and the range of cost value is calculated hour by hour as the load changes.

Examining the graphs, the following observations are listed:

- Table (4-1) and Figures (4-3), (4-4) and (4-5) represent the fuzzy load at different α -cut values for model A with 20% deviation on weekdays. The fuzzy triangular representation of the fuzzy load is shown in Figure (4-6). It is clear that the load changes hour by hour and the left, right spread are getting closer as α -cut increases between [0,1]. The left and right spread varies in range during the day hours as shown in Figure (4-3), which is shown clearly in the triangular membership representation of the load in Figure (4-6). This variation of spread will propagate through the fuel incremental cost, the power generations and the total minimum cost.
- At each hour of the fuzzy load the existence upper, middle and lower values can be translated into a triangular membership function representation. Using the load demand mathematical formula (4.5), which is used to calculate the

left, right spreads and sides of the load for each α -cut value higher than α -cut equal to zero. Whereas the middle value of the load will remain constant because it represents the crisp case and it is the same for α -cut equal to 1 as shown in Table (4-1). The triangular membership functions as shown in Figure (4-6) follow the pattern of the fuzzy load, which validates the mathematical formula in (4.5).

- Tables (4-2) and (4-3) show the result of the two unit generation committed to the system for different α -cut values. Figure (4-7), (4-8) and (4-9) show the generation values changes according to the changes in load demand to satisfy the equality constraint shown in equation (4.3). Unit 1 is generating a little more power than unit 2 at α -cut equal to zero. At α -cut equal to 0.5 the left spread generation of unit 1 and unit 2 are generating identical power between the 10^{th} hour and the rest of the day and their crisp values are generating identical power between the 6^{th} and 10^{th} hour hours of the day. In addition their upper spreads are increasing between the 10^{th} hours and the rest of the day. At α -cut equal to 0.75 the two unit generator left and middle values are generating the same power between the 7^{th} and 11^{th} hours of the day. Whereas the two units generator right spreads are almost generating the same power between the first and the 11^{th} hours of the day. This changing in pattern of generation is very helpful in decision making. Fuzzy approach provides that information a bout the system behavior in order for the decision-maker to make their judgment according to the information provided.
- A triangular membership function representation is shown in Figure (4-10), (4-11) for each generation unit committed to the system. The left and right spread cover the limit violation restricted on the generation. If a violation occurs then it can be overcome by committing additional units to the system.
- Table (4-3) shows the total generation for different α -cut values. In addition Figure (4-12), (4-13) show the satisfaction of the equality constraint in equation (4.3) where the total generation committed is equal to the power load demand. Each output parameter such as the incremental fuel cost, power generation and minimum total cost of the ED problem has a different membership function with different spread than the input membership

function, which is the load demand. However, the power generations membership function satisfies the equality constraint and inequality constraint imposed on the system parameters. In addition the crisp or middle value of each membership function gives the same result obtained from solving the problem using the conventional method of the ED problem.

- Comparing the load demand triangular membership function in Figure (4-6) and the total power generation triangular membership function representation shown in Figure (4-14) we can say that they are identical and satisfy the equality constraints imposed on the system.
- The total fuel cost of different α -cut values calculated from equation (4-31) are shown in Table (4-5) and a plotted graph for different α -cut values are shown in Figures (4-15), (4-16) and (4-17). At the hour considered, there is a range of fuel costs for each unit as well as the total cost. Clearly the maximum and minimum value is valuable information to the operator supplying the load to know the cost of the power generated hour by hour. The total minimum cost pattern variation follows the load demand pattern variation. The fuzziness has propagated through all the parameters in the objective function and the constraints except the fixed cost function coefficients which they where selected as fixed value in this chapter.
- The fuzzy load weekend model (A) with a 20% deviation is tabulated in appendix (I). Table (P1-1) and the plotted graphs are shown in Figures (P1-1), (P1-2) and (P1-3) for different α -cut representation. Comparing the weekend model with the weekdays, we notice that the load demand is higher on the weekend than the weekdays, which makes the cost value in dollars higher on the weekend than the weekdays. If the company supplying the generator units can not provide the load demand required then other generators could be brought into the network or on a large scale interconnected network the company can buy the extra generation from other companies in the network. The same procedure is preformed in this chapter to calculate the minimum cost of the two thermal unit generators for the weekend model. The results are shown in the tables and figures in appendix (I).

Table (4-1)

Membership Function of Load Demand for (0, 0.5, 0.75, 1) α -Cut Representation for Model "A" Weekdays With 20% Deviation

Membership Function		$\mu_{P_{Load}}=0$			$\mu_{P_{Load}}=0.5$			$\mu_{P_{Load}}=0.75$			$\mu_{P_{Load}}=1$	
Daily Hours	Left Load MW	Mid Load MW	Right Load MW	Left Load MW	Mid Load MW	Right Load MW	Left Load MW	Mid Load MW	Right Load MW	Left Load MW	Mid Load MW	Right Load MW
1	257	735.9	1316	496.4	735.9	1026	616.2	735.9	880.9	735.9	735.9	735.9
2	277.6	650.6	1336	464.1	650.6	993.5	557.3	650.6	822	650.6	650.6	650.6
3	269.8	613.1	1329	441.4	613.1	970.8	527.3	613.1	792	613.1	613.1	613.1
4	274.9	599.6	1334	437.3	599.6	966.7	518.4	599.6	783.1	599.6	599.6	599.6
5	279.6	604.8	1338	442.2	604.8	971.6	523.5	604.8	788.2	604.8	604.8	604.8
6	290.4	617.1	1349	453.8	617.1	983.2	535.4	617.1	800.1	617.1	617.1	617.1
7	301.5	635.1	1360	468.3	635.1	997.7	551.7	635.1	816.4	635.1	635.1	635.1
8	295.4	731.5	1354	513.5	731.5	1043	622.5	731.5	887.2	731.5	731.5	731.5
9	299.6	915.8	1358	607.7	915.8	1137	761.7	915.8	1026	915.8	915.8	915.8
10	317.9	1002	1377	659.9	1002	1189	830.8	1002	1096	1002	1002	1002
11	320.2	1013	1379	666.6	1013	1196	839.8	1013	1105	1013	1013	1013
12	322	1015	1381	668.3	1015	1198	841.4	1015	1106	1015	1015	1015
13	338.4	1021	1397	679.7	1021	1209	850.3	1021	1115	1021	1021	1021
14	348.1	995.1	1407	671.6	995.1	1201	833.4	995.1	1098	995.1	995.1	995.1
15	377.4	979.7	1436	678.6	979.7	1208	829.1	979.7	1094	979.7	979.7	979.7
16	396.1	965.5	1455	680.8	965.5	1210	823.2	965.5	1088	965.5	965.5	965.5
17	393.7	975.1	1453	684.4	975.1	1214	829.8	975.1	1094	975.1	975.1	975.1
18	384.4	1030	1443	707.1	1030	1236	868.4	1030	1133	1030	1030	1030
19	394.2	1025	1453	709.5	1025	1239	867.2	1025	1132	1025	1025	1025
20	380	968.3	1439	674.2	968.3	1204	821.2	968.3	1086	968.3	968.3	968.3
21	393.9	955.2	1453	674.5	955.2	1204	814.9	955.2	1080	955.2	955.2	955.2
22	432.2	960	1491	696.1	960	1226	828.1	960	1093	960	960	960
23	453.1	950.7	1512	701.9	950.7	1231	826.3	950.7	1091	950.7	950.7	950.7
24	507.8	858.3	1567	683	858.3	1212	770.7	858.3	1035	858.3	858.3	858.3

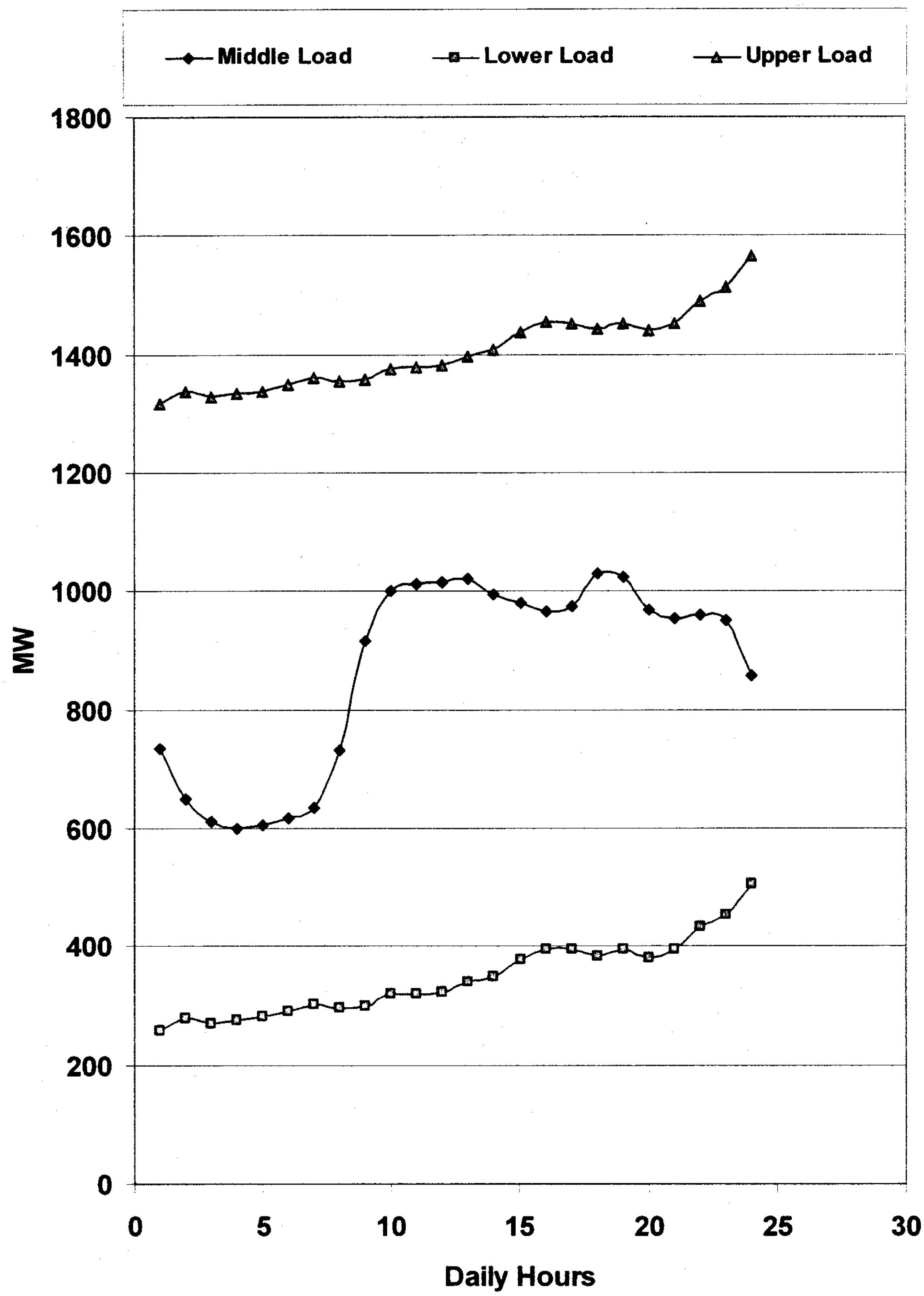


Figure (4-3) Fuzzy Load for (0- α -cut) Representation

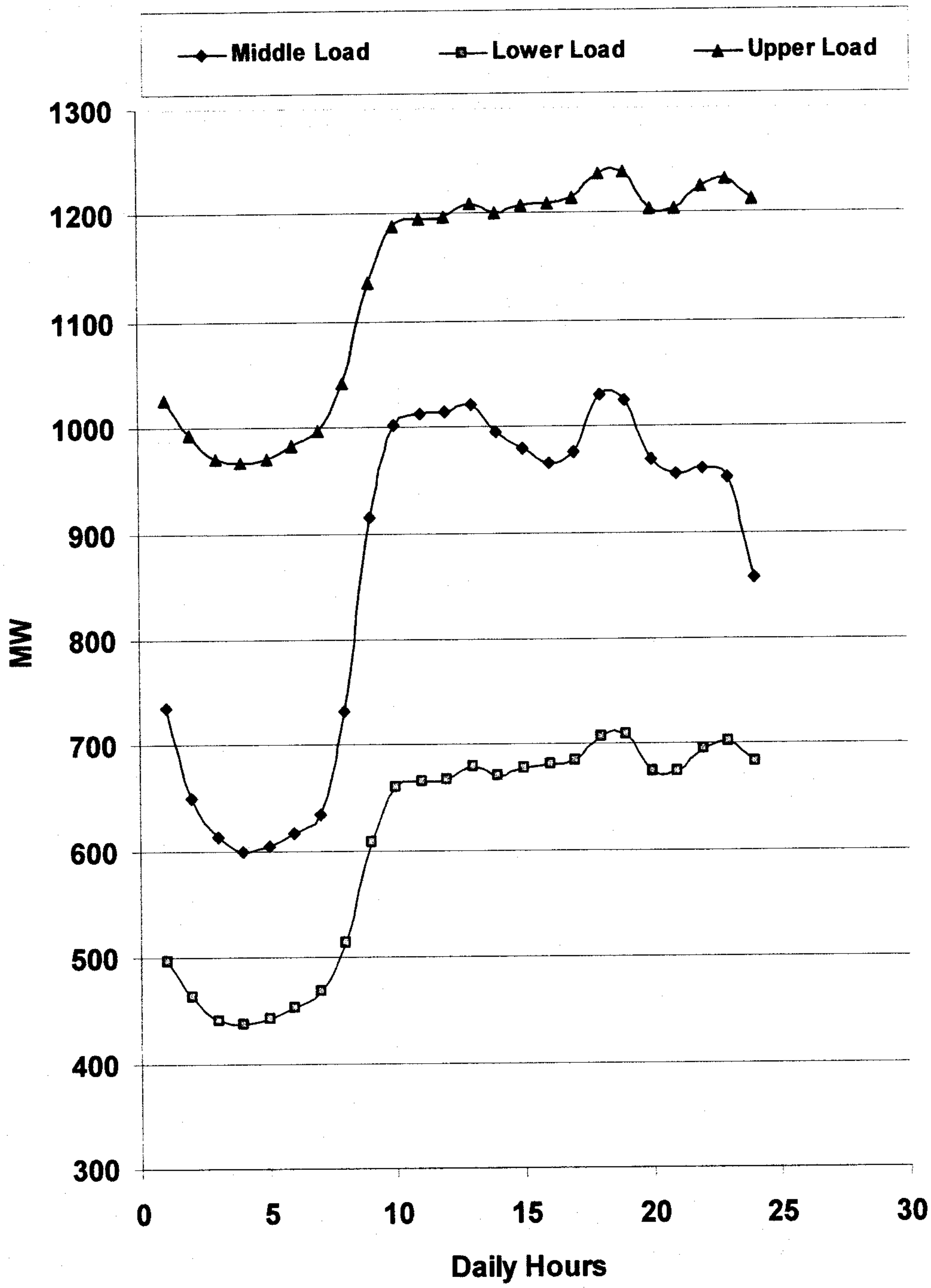


Figure (4-4) Fuzzy Load for (0.5- α -cut) Representation

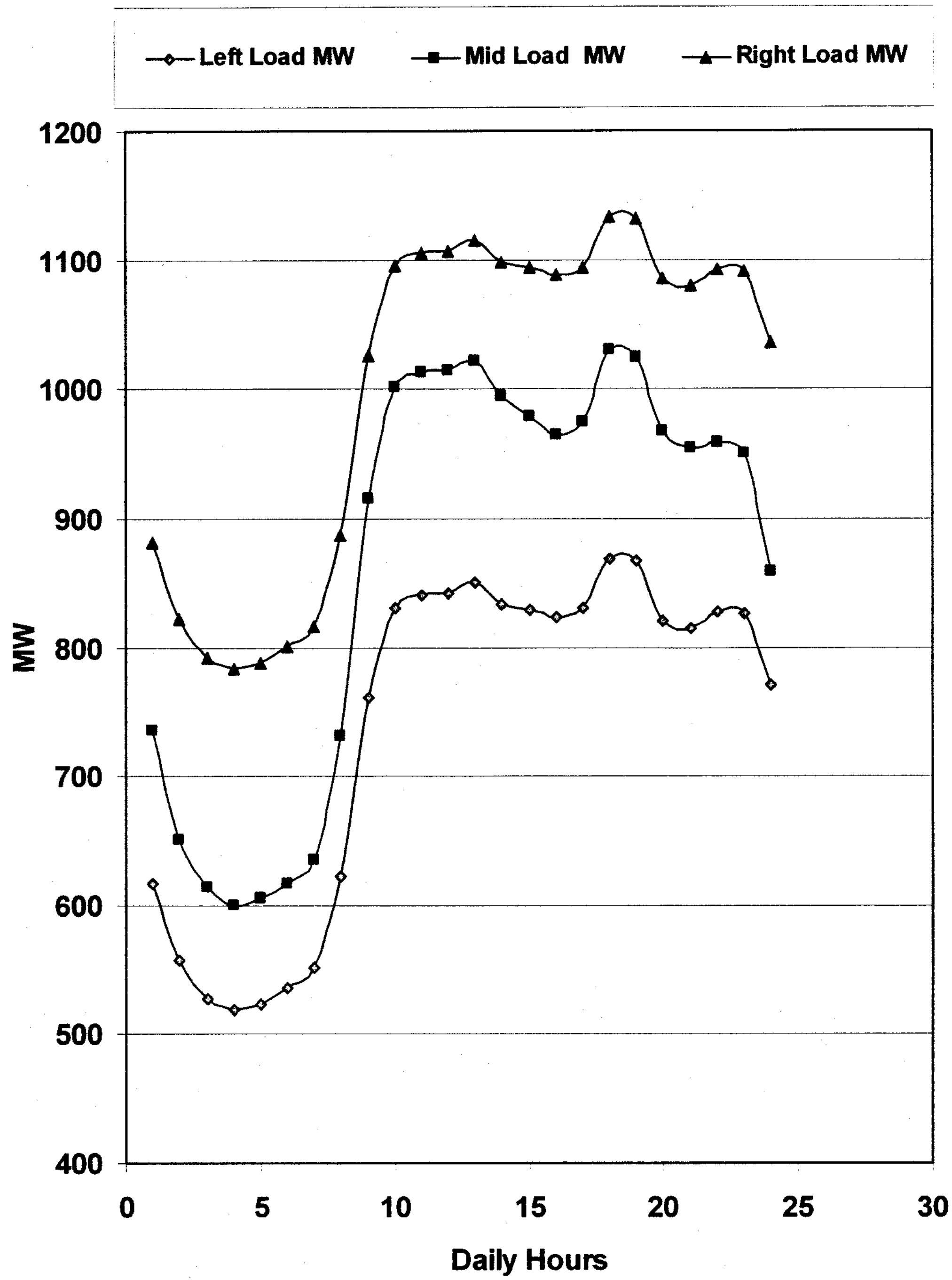


Figure (4-5) Fuzzy Load for (0.75-α-cut) Representation

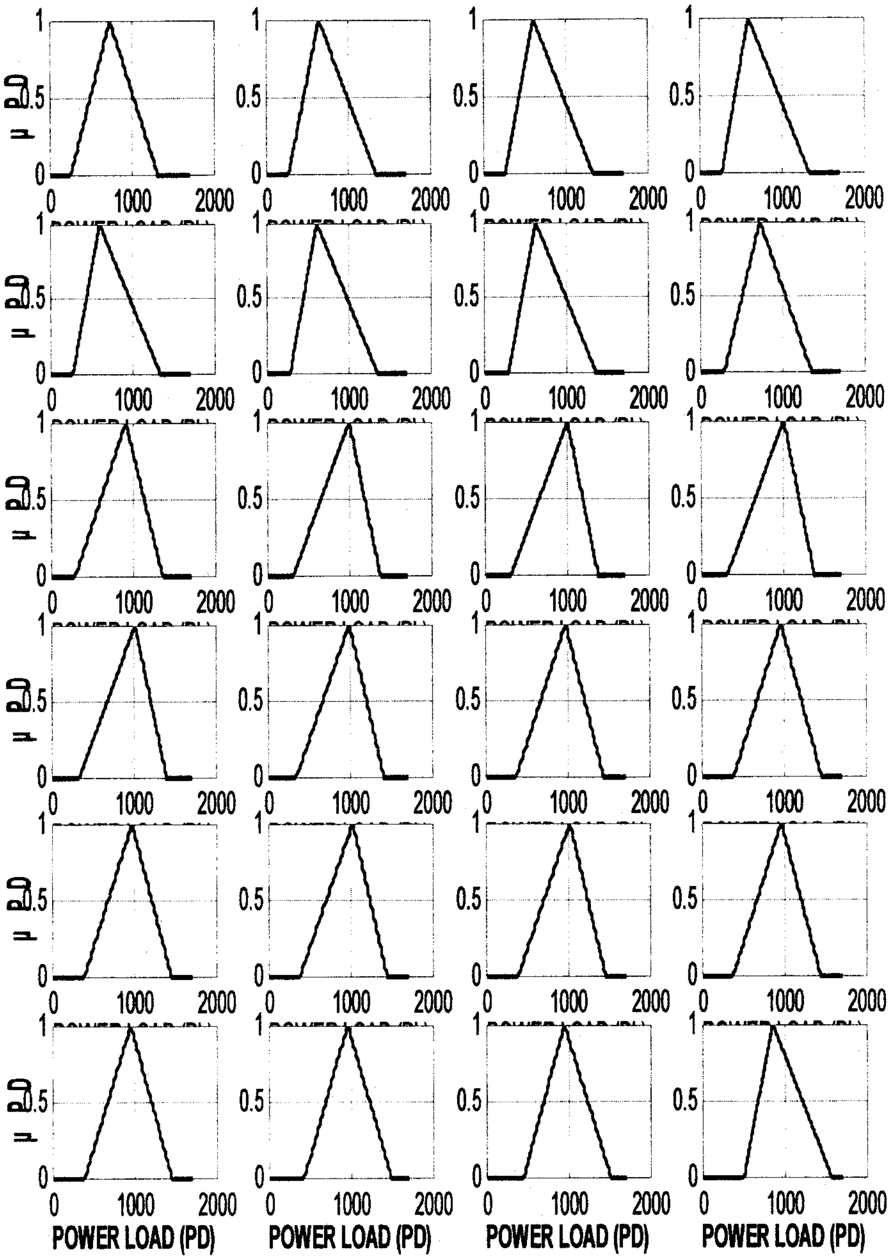


Figure (4-6) Triangular Membership Function Representation for Fuzzy Load

Table (4-2)

Membership Function of Generator #1 for (0, 0.5, 0.75, 1) α -Cut Representation Model "A" Weekdays With 20% Deviation

Membership Function	$\mu_{PG1}=0$			$\mu_{PG1}=0.5$			$\mu_{PG1}=0.75$			$\mu_{PG1}=1$		
Daily Hours	Left PG1 MW	Mid PG1 MW	Right PG1 MW	Left PG1 MW	Mid PG1 MW	Right PG1 MW	Left PG1 MW	Mid PG1 MW	Right PG1 MW	Left PG1 MW	Mid PG1 MW	Right PG1 MW
1	115.5	369	676	242.2	369	522.5	305.6	369	445.8	369	369	369
2	126.4	323.8	686.9	225.1	323.8	505.4	274.5	323.8	414.6	323.8	323.8	323.8
3	122.2	304	682.8	213.1	304	493.4	258.6	304	398.7	304	304	304
4	125	296.8	685.5	210.9	296.8	491.2	253.9	296.8	394	296.8	296.8	296.8
5	127.4	299.6	688	213.5	299.6	493.8	256.6	299.6	396.7	299.6	299.6	299.6
6	133.2	306.1	693.7	219.6	306.1	499.9	262.9	306.1	403	306.1	306.1	306.1
7	139	315.6	699.6	227.3	315.6	507.6	271.5	315.6	411.6	315.6	315.6	315.6
8	135.8	366.7	696.4	251.2	366.7	531.5	309	366.7	449.1	366.7	366.7	366.7
9	138	464.2	698.6	301.1	464.2	581.4	382.7	464.2	522.8	464.2	464.2	464.2
10	147.7	509.8	708.3	328.8	509.8	609	419.3	509.8	559.4	509.8	509.8	509.8
11	148.9	515.7	709.5	332.3	515.7	612.6	424	515.7	564.1	515.7	515.7	515.7
12	149.9	516.6	710.4	333.2	516.6	613.5	424.9	516.6	565	516.6	516.6	516.6
13	158.6	519.9	719.1	339.2	519.9	619.5	429.6	519.9	569.7	519.9	519.9	519.9
14	163.7	506.2	724.2	335	506.2	615.2	420.6	506.2	560.7	506.2	506.2	506.2
15	179.2	498.1	739.8	338.6	498.1	618.9	418.4	498.1	558.5	498.1	498.1	498.1
16	189.1	490.6	749.7	339.8	490.6	620.1	415.2	490.6	555.3	490.6	490.6	490.6
17	187.9	495.6	748.4	341.8	495.6	622	418.7	495.6	558.8	495.6	495.6	495.6
18	182.9	524.5	743.5	353.7	524.5	634	439.1	524.5	579.3	524.5	524.5	524.5
19	188.1	522	748.7	355	522	635.3	438.5	522	578.6	522	522	522
20	180.6	492	741.2	336.3	492	616.6	414.2	492	554.3	492	492	492
21	187.9	485.1	748.5	336.5	485.1	616.8	410.8	485.1	550.9	485.1	485.1	485.1
22	208.2	487.6	768.8	347.9	487.6	628.2	417.8	487.6	557.9	487.6	487.6	487.6
23	219.3	482.7	779.8	351	482.7	631.3	416.9	482.7	557	482.7	482.7	482.7
24	248.2	433.8	808.8	341	433.8	621.3	387.4	433.8	527.5	433.8	433.8	433.8

Table (4-3)

Membership Function of Generator #2 for (0, 0.5, 0.75, 1) α -Cut

Representation Model "A" Weekdays With 20% Deviation

Membership Function		$\mu_{P_{G2}}=0$			$\mu_{P_{G2}}=0.5$			$\mu_{P_{G2}}=0.75$			$\mu_{P_{G2}}=1$	
Daily Hours	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW
1	141.5	366.9	639.8	254.2	366.9	503.3	310.5	366.9	435.1	366.9	366.9	366.9
2	151.2	326.8	649.5	239	326.8	488.1	282.9	326.8	407.4	326.8	326.8	326.8
3	147.5	309.1	645.8	228.3	309.1	477.5	268.7	309.1	393.3	309.1	309.1	309.1
4	150	302.8	648.2	226.4	302.8	475.5	264.6	302.8	389.1	302.8	302.8	302.8
5	152.2	305.2	650.4	228.7	305.2	477.8	266.9	305.2	391.5	305.2	305.2	305.2
6	157.2	311	655.5	234.1	311	483.2	272.6	311	397.1	311	311	311
7	162.5	319.5	660.7	241	319.5	490.1	280.2	319.5	404.8	319.5	319.5	319.5
8	159.6	364.8	657.9	262.2	364.8	511.3	313.5	364.8	438.1	364.8	364.8	364.8
9	161.6	451.6	659.8	306.6	451.6	555.7	379.1	451.6	503.6	451.6	451.6	451.6
10	170.2	492	668.5	331.1	492	580.2	411.6	492	536.1	492	492	492
11	171.3	497.3	669.5	334.3	497.3	583.4	415.8	497.3	540.4	497.3	497.3	497.3
12	172.1	498	670.4	335.1	498	584.2	416.6	498	541.1	498	498	498
13	179.8	501	678.1	340.4	501	589.6	420.7	501	545.3	501	501	501
14	184.4	488.9	682.7	336.6	488.9	585.8	412.8	488.9	537.3	488.9	488.9	488.9
15	198.2	481.6	696.4	339.9	481.6	589	410.8	481.6	535.3	481.6	481.6	481.6
16	207	474.9	705.3	341	474.9	590.1	408	474.9	532.5	474.9	474.9	474.9
17	205.9	479.5	704.1	342.7	479.5	591.8	411.1	479.5	535.6	479.5	479.5	479.5
18	201.5	505.2	699.8	353.3	505.2	602.5	429.2	505.2	553.8	505.2	505.2	505.2
19	206.1	502.8	704.4	354.5	502.8	603.6	428.7	502.8	553.2	502.8	502.8	502.8
20	199.4	476.3	697.7	337.8	476.3	587	407.1	476.3	531.6	476.3	476.3	476.3
21	205.9	470.1	704.2	338	470.1	587.1	404.1	470.1	528.6	470.1	470.1	470.1
22	224	472.4	722.2	348.2	472.4	597.3	410.3	472.4	534.8	472.4	472.4	472.4
23	233.8	468	732.1	350.9	468	600	409.4	468	534	468	468	468
24	259.5	424.5	757.8	342	424.5	591.1	383.3	424.5	507.8	424.5	424.5	424.5

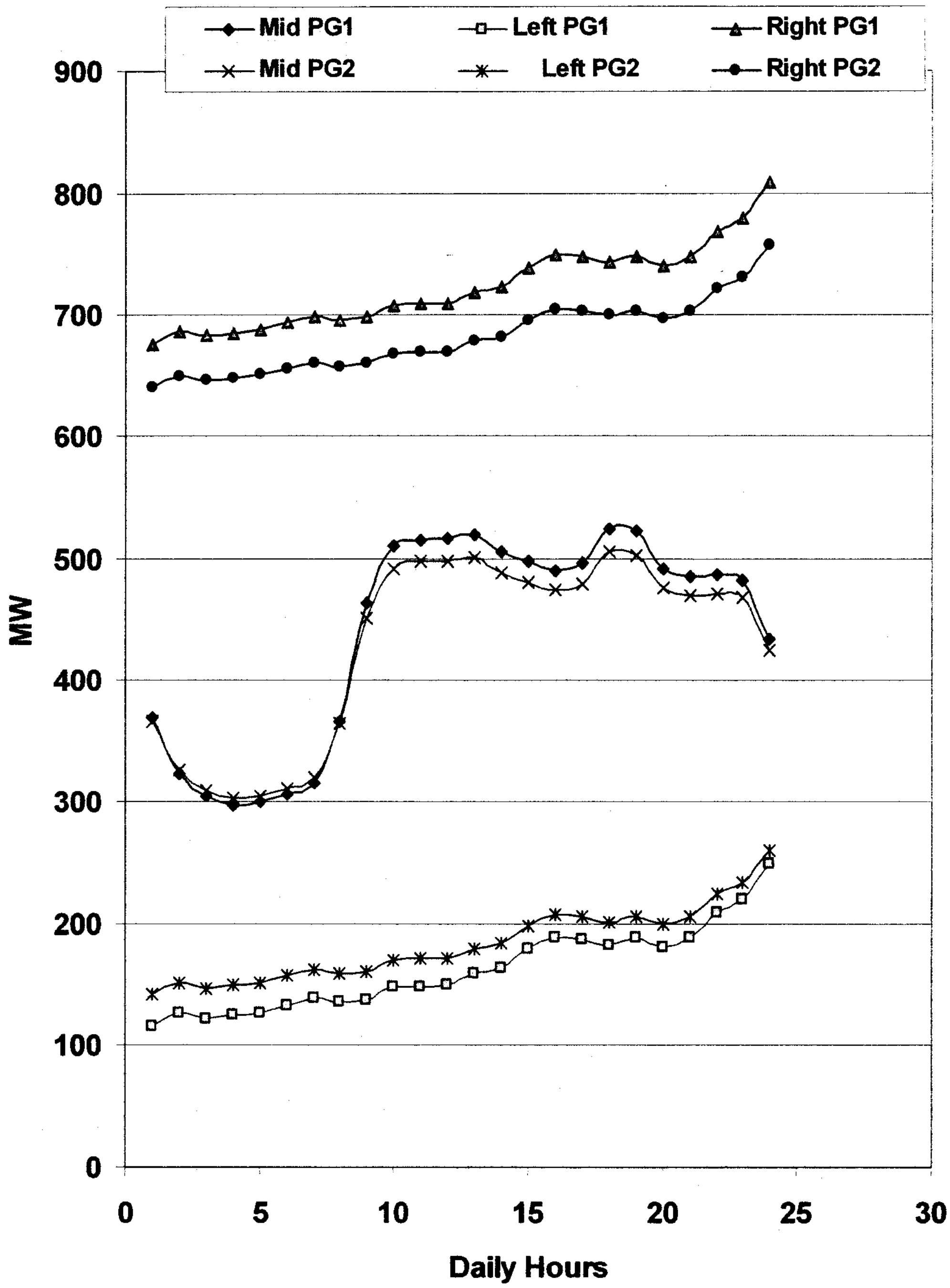


Figure (4-7) Fuzzy (0- α -cut) Representation for Power Generation of Unit 1&2

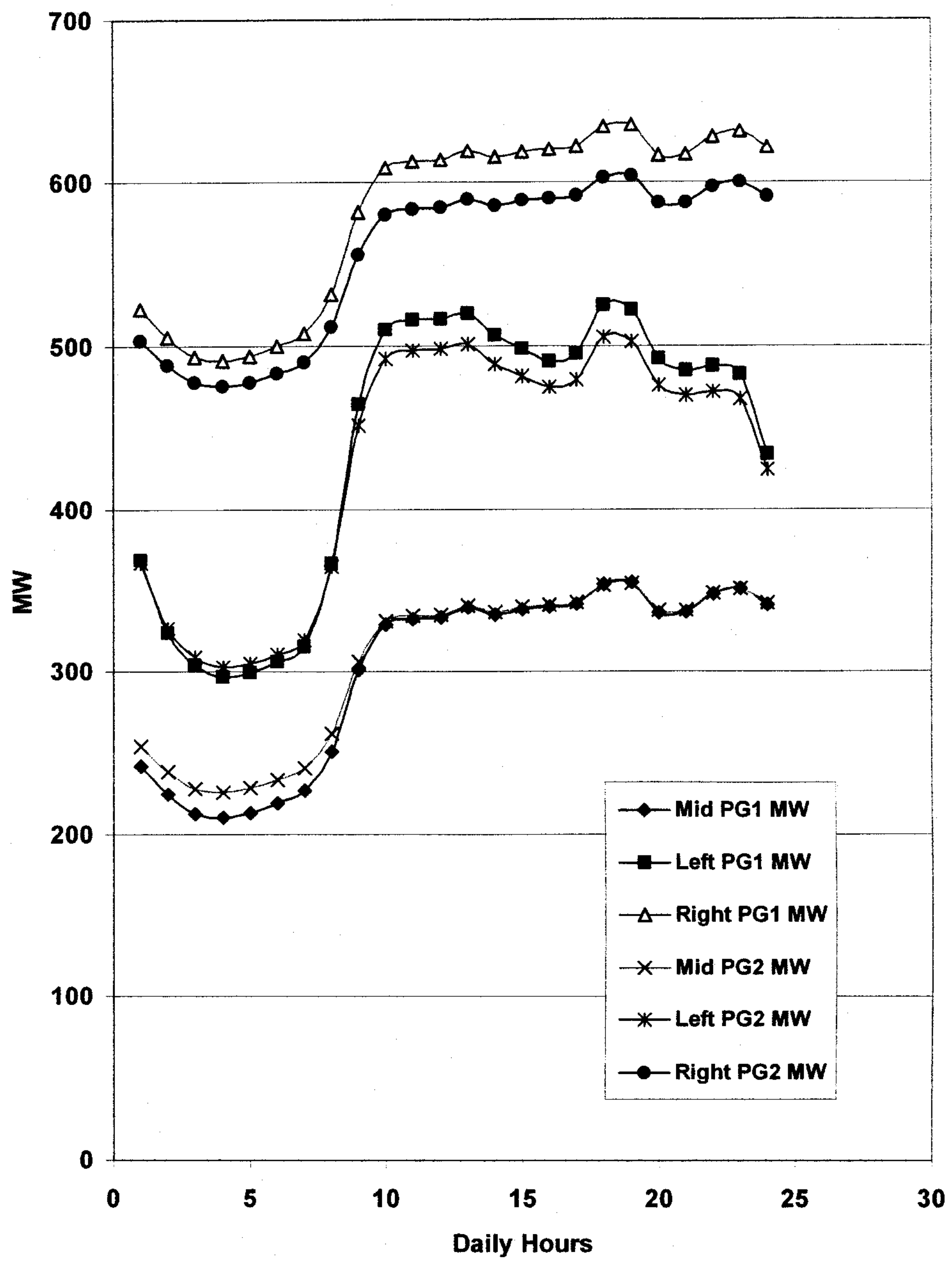


Figure (4-8) Fuzzy (0.5-α-cut) Representation for Power Generation of Unit 1&2

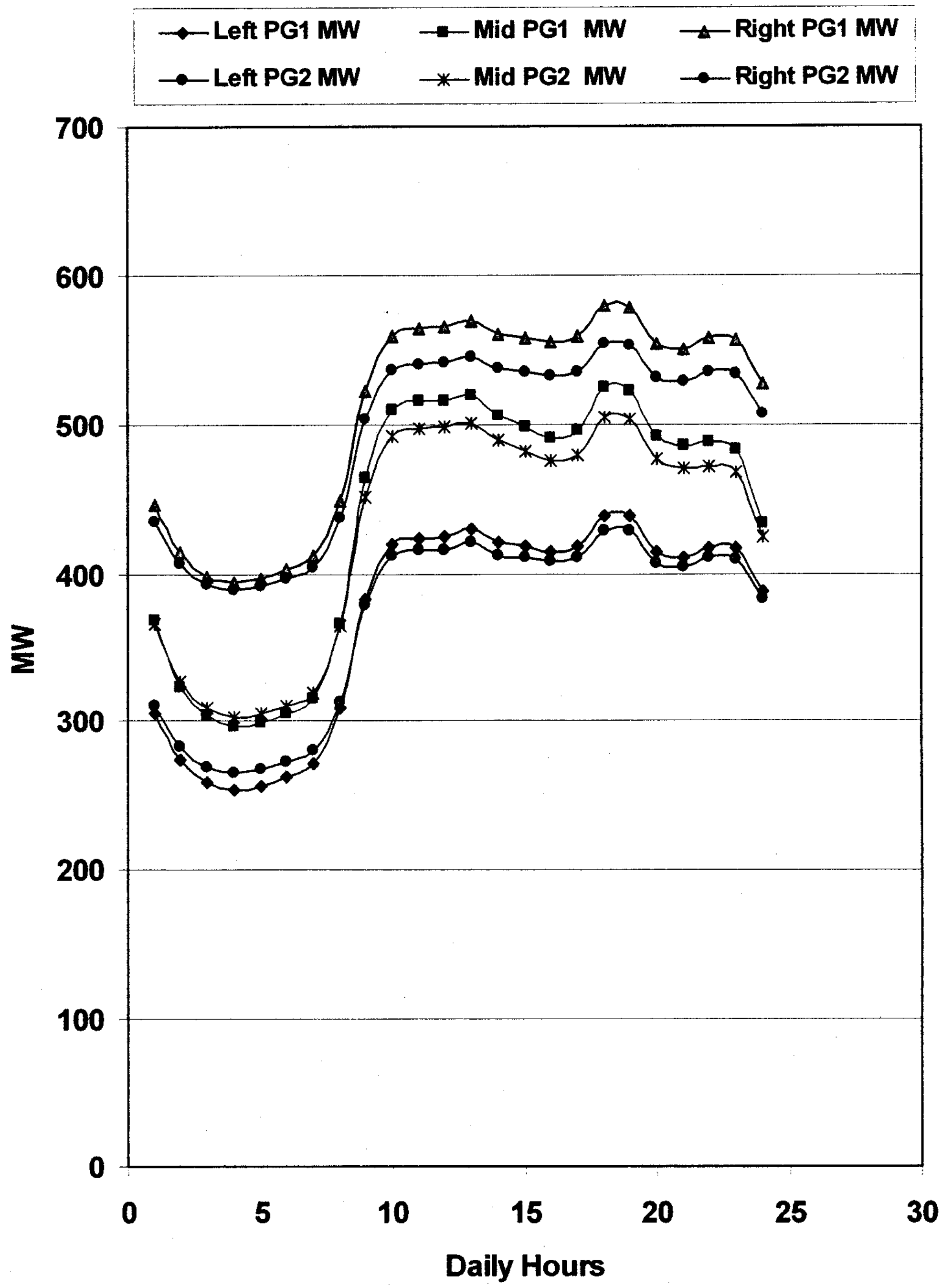


Figure (4-9) Fuzzy (0.75-α-cut) Representation for Power Generation of Unit 1&2

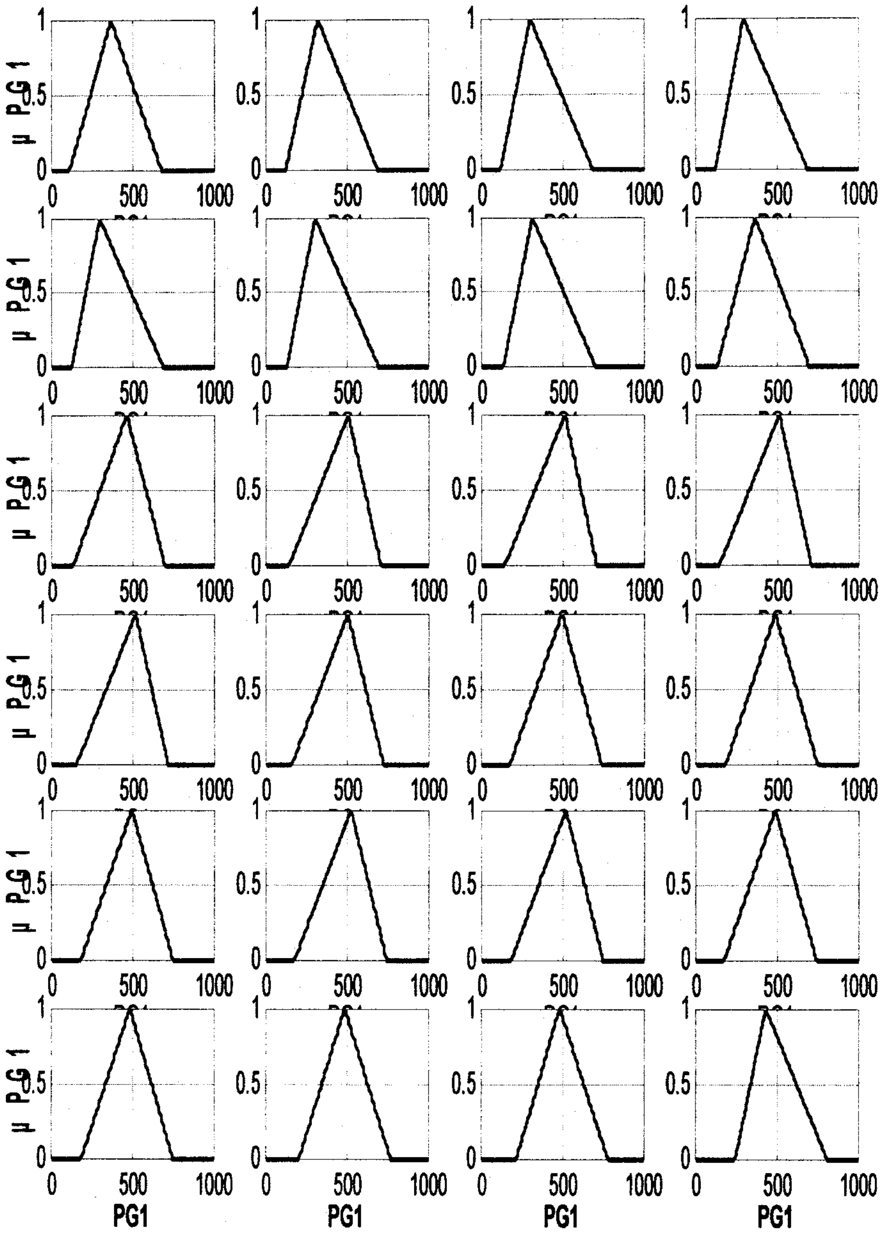


Figure (4-10) Fuzzy Triangular Membership Function for Generation #1

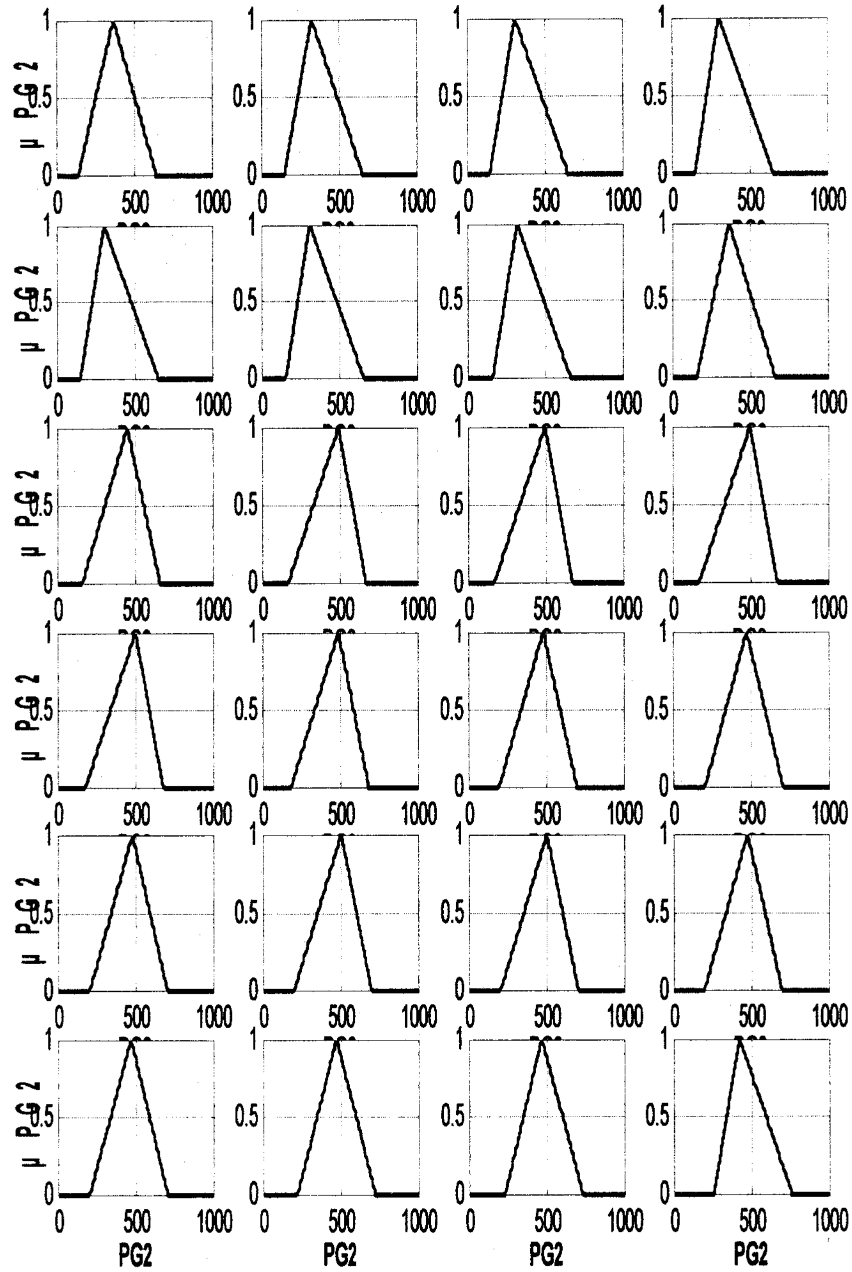


Figure (4-11) Fuzzy Triangular Membership Function for Generation #2

Table (4-4)

Membership Function of Total Generator for (0, 0.5, 0.75, 1) α -Cut Representation Model "A" Weekdays With 20% Deviation

Membership Function	$\mu_{tP_G}=0$			$\mu_{tP_G}=0.5$			$\mu_{tP_G}=0.75$			$\mu_{tP_G}=1$		
Daily Hours	Left tPG MW	Mid tPG MW	Right tPG MW	Left tPG MW	Mid tPG MW	Right tPG MW	Left tPG MW	Mid tPG MW	Right tPG MW	Left tPG MW	Mid tPG MW	Right tPG MW
1	257	735.9	1316	496.4	735.9	1026	616.2	735.9	880.9	735.9	735.9	735.9
2	277.6	650.6	1336	464.1	650.6	993.5	557.3	650.6	822	650.6	650.6	650.6
3	269.8	613.1	1329	441.4	613.1	970.8	527.3	613.1	792	613.1	613.1	613.1
4	274.9	599.6	1334	437.3	599.6	966.7	518.4	599.6	783.1	599.6	599.6	599.6
5	279.6	604.8	1338	442.2	604.8	971.6	523.5	604.8	788.2	604.8	604.8	604.8
6	290.4	617.1	1349	453.8	617.1	983.2	535.4	617.1	800.1	617.1	617.1	617.1
7	301.5	635.1	1360	468.3	635.1	997.7	551.7	635.1	816.4	635.1	635.1	635.1
8	295.4	731.5	1354	513.5	731.5	1043	622.5	731.5	887.2	731.5	731.5	731.5
9	299.6	915.8	1358	607.7	915.8	1137	761.7	915.8	1026	915.8	915.8	915.8
10	317.9	1002	1377	659.9	1002	1189	830.8	1002	1096	1002	1002	1002
11	320.2	1013	1379	666.6	1013	1196	839.8	1013	1105	1013	1013	1013
12	322	1015	1381	668.3	1015	1198	841.4	1015	1106	1015	1015	1015
13	338.4	1021	1397	679.7	1021	1209	850.3	1021	1115	1021	1021	1021
14	348.1	995.1	1407	671.6	995.1	1201	833.4	995.1	1098	995.1	995.1	995.1
15	377.4	979.7	1436	678.6	979.7	1208	829.1	979.7	1094	979.7	979.7	979.7
16	396.1	965.5	1455	680.8	965.5	1210	823.2	965.5	1088	965.5	965.5	965.5
17	393.7	975.1	1453	684.4	975.1	1214	829.8	975.1	1094	975.1	975.1	975.1
18	384.4	1030	1443	707.1	1030	1236	868.4	1030	1133	1030	1030	1030
19	394.2	1025	1453	709.5	1025	1239	867.2	1025	1132	1025	1025	1025
20	380	968.3	1439	674.2	968.3	1204	821.2	968.3	1086	968.3	968.3	968.3
21	393.9	955.2	1453	674.5	955.2	1204	814.9	955.2	1080	955.2	955.2	955.2
22	432.2	960	1491	696.1	960	1226	828.1	960	1093	960	960	960
23	453.1	950.7	1512	701.9	950.7	1231	826.3	950.7	1091	950.7	950.7	950.7
24	507.8	858.3	1567	683	858.3	1212	770.7	858.3	1035	858.3	858.3	858.3

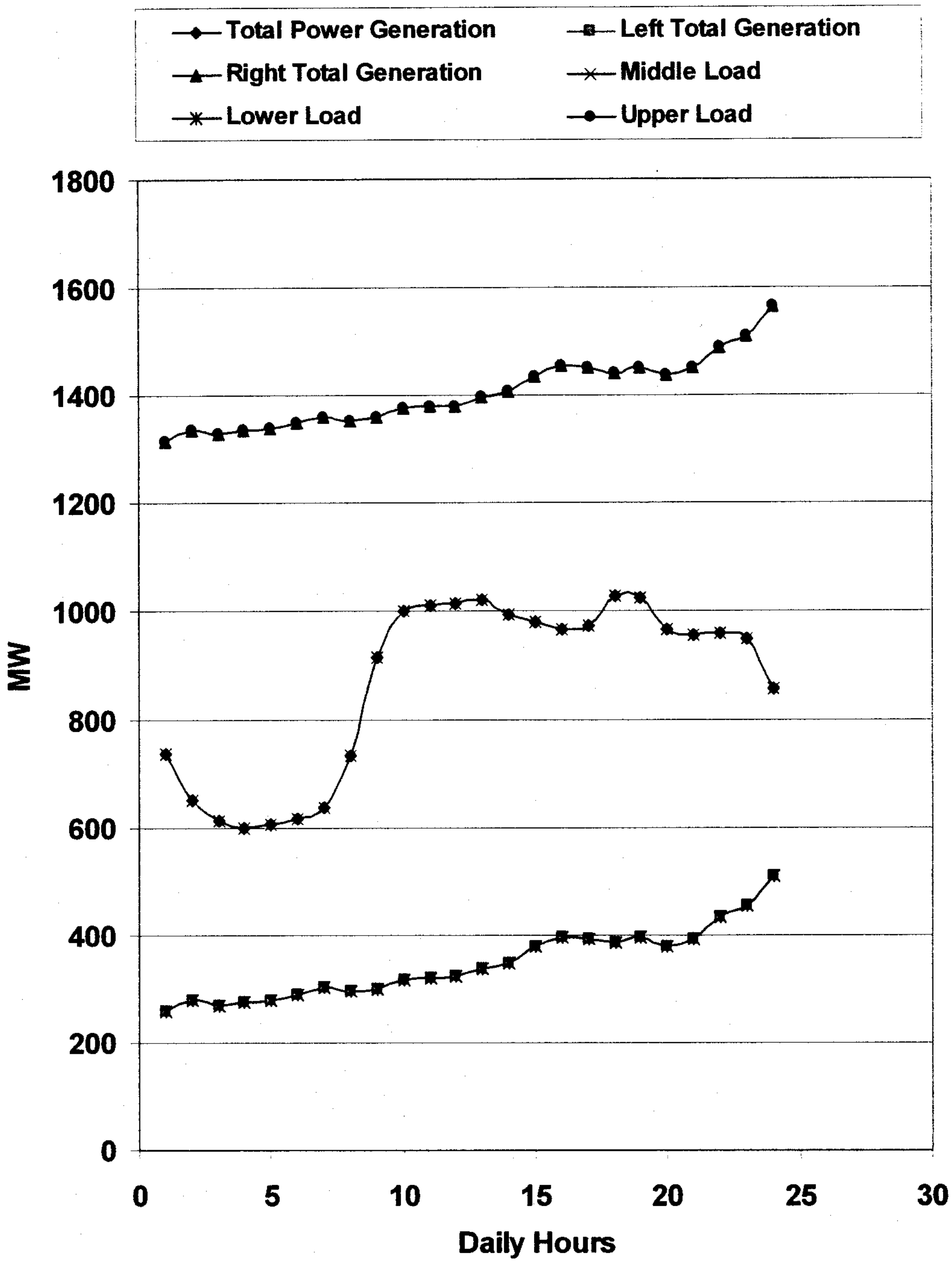


Figure (4-12) Fuzzy (0.75 - α -Cut) Representation for Total Power Generation

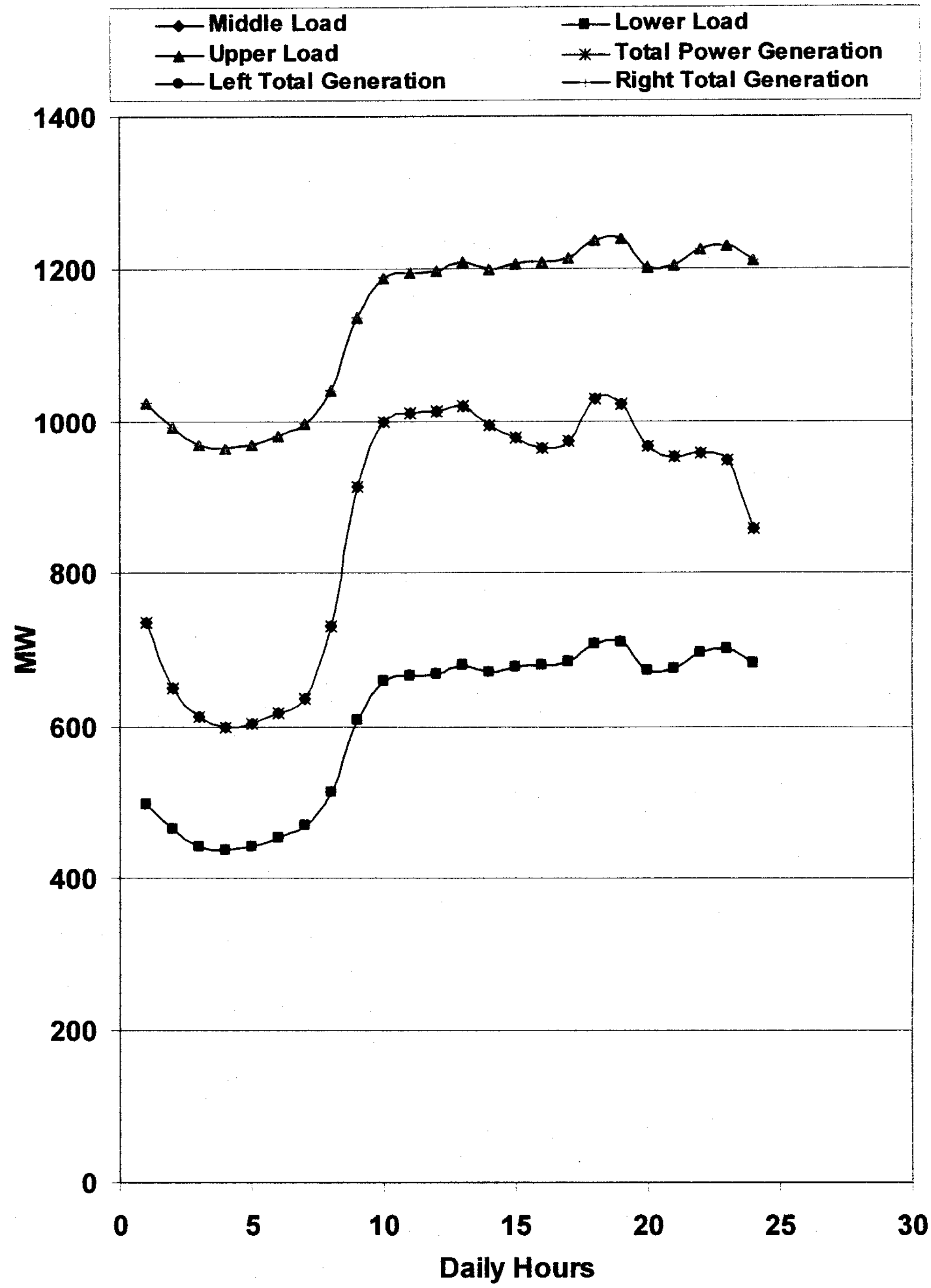


Figure (4-13) Fuzzy (0.5- α -Cut) Representation for Total Generation Versus Fuzzy Load

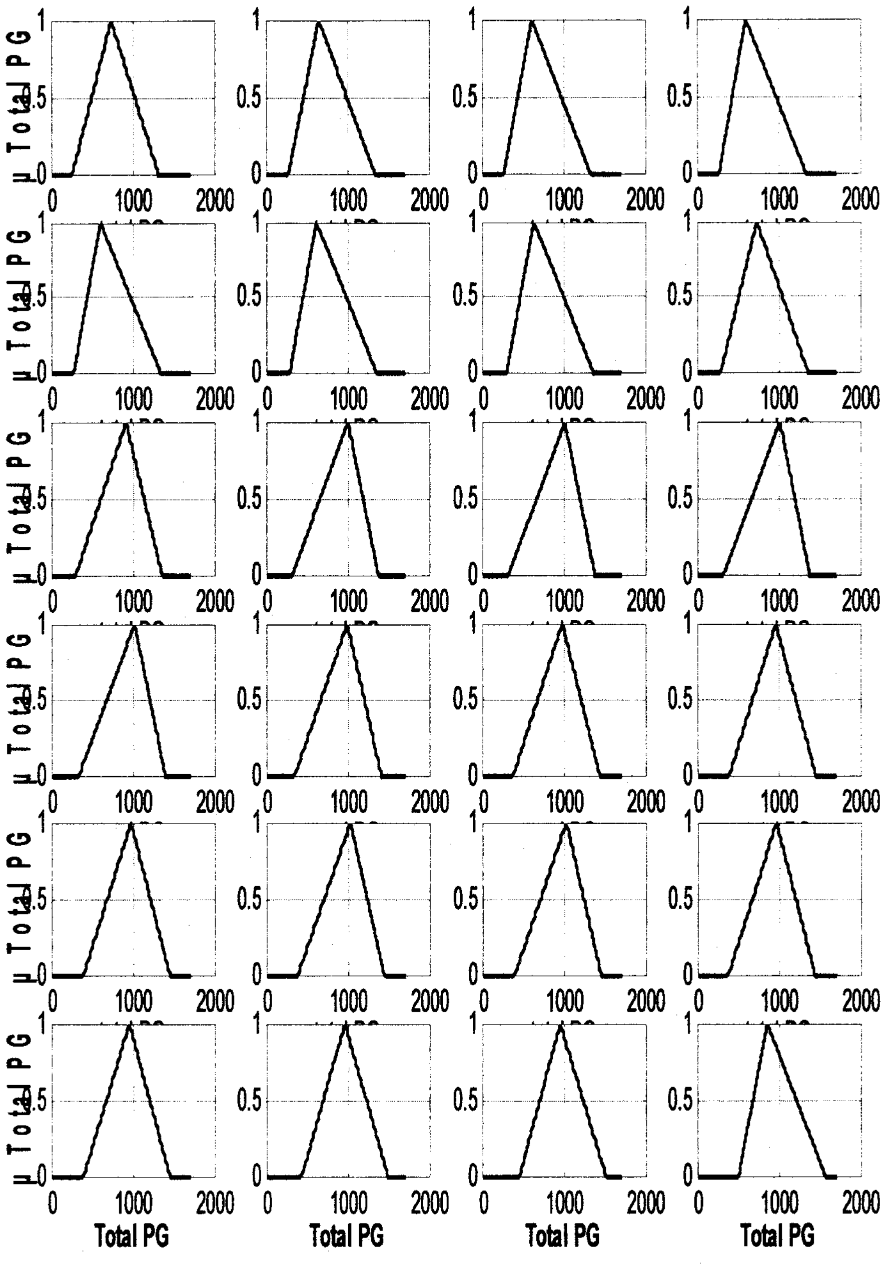


Figure (4-14) Fuzzy Triangular Membership Function for Total Generation

Table (4-5)

Membership Function of Total Cost for (0, 0.5, 0.75, 1) α -Cut Representation Model "A" Weekdays With 20% Deviation

Membership Function	$\mu_C = 0$			$\mu_C = 0.5$			$\mu_C = 0.75$			$\mu_C = 1$		
Daily Hours	Left Cost $/h	Mid Cost $/h	Right Cost $/h	Left Cost $/h	Mid Cost $/h	Right Cost $/h	Left Cost $/h	Mid Cost $/h	Right Cost $/h	Left Cost $/h	Mid Cost $/h	Right Cost $/h
1	2367	7575	16482	4728	7575	11673	6091	7575	9535	7575	7575	7575
2	2551	6505	16851	4381	6505	11180	5406	6505	8718	6505	6505	6505
3	2481	6055	16711	4143	6055	10841	5067	6055	8312	6055	6055	6055
4	2527	5895	16804	4099	5895	10779	4969	5895	8194	5895	5895	5895
5	2569	5956	16887	4151	5956	10852	5026	5956	8262	5956	5956	5956
6	2667	6102	17083	4272	6102	11025	5159	6102	8422	6102	6102	6102
7	2769	6318	17283	4425	6318	11244	5342	6318	8641	6318	6318	6318
8	2713	7519	17174	4915	7519	11936	6166	7519	9624	7519	7519	7519
9	2751	10034	17249	5990	10034	13434	7912	10034	11682	10034	10034	10034
10	2922	11306	17584	6619	11306	14296	8839	11306	12764	11306	11306	11306
11	2943	11476	17626	6701	11476	14409	8962	11476	12907	11476	11476	11476
12	2959	11501	17658	6722	11501	14437	8984	11501	12933	11501	11501	11501
13	3115	11597	17961	6863	11597	14629	9107	11597	13076	11597	11597	11597
14	3208	11205	18141	6763	11205	14493	8873	11205	12804	11205	11205	11205
15	3494	10973	18689	6849	10973	14610	8815	10973	12737	10973	10973	10973
16	3680	10761	19043	6877	10761	14649	8734	10761	12642	10761	10761	10761
17	3656	10904	18998	6922	10904	14710	8824	10904	12747	10904	10904	10904
18	3563	11732	18822	7207	11732	15096	9359	11732	13369	11732	11732	11732
19	3661	11657	19008	7238	11657	15138	9342	11657	13349	11657	11657	11657
20	3520	10803	18739	6795	10803	14536	8707	10803	12611	10803	10803	10803
21	3657	10609	19001	6799	10609	14543	8621	10609	12510	10609	10609	10609
22	4047	10680	19734	7069	10680	14909	8800	10680	12720	10680	10680	10680
23	4265	10543	20139	7141	10543	15007	8776	10543	12692	10543	10543	10543
24	4852	9218	21217	6905	9218	14686	8029	9218	11819	9218	9218	9218

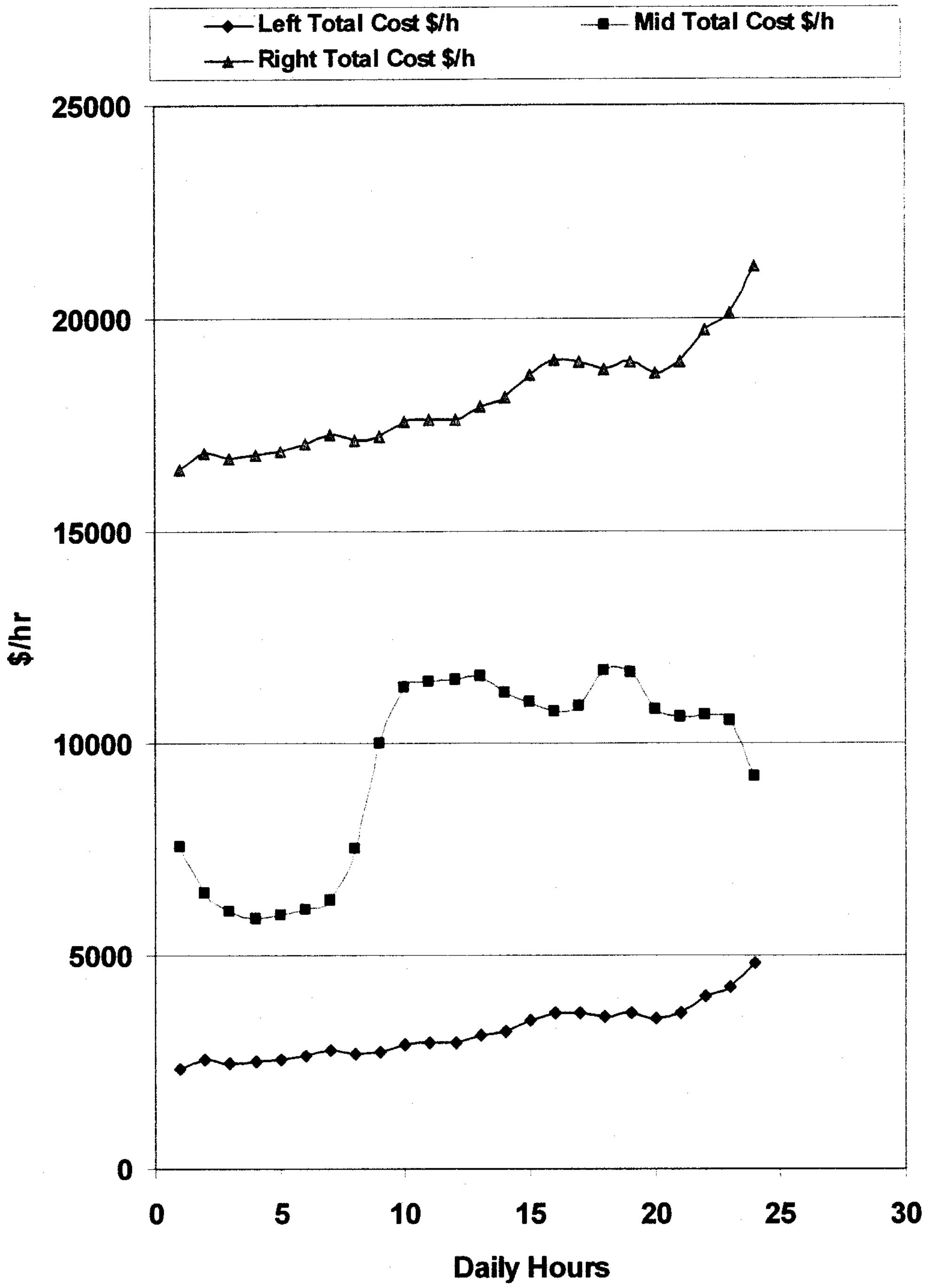


Figure (4-15) Fuzzy (0-α-Cut) Representation for Total Cost

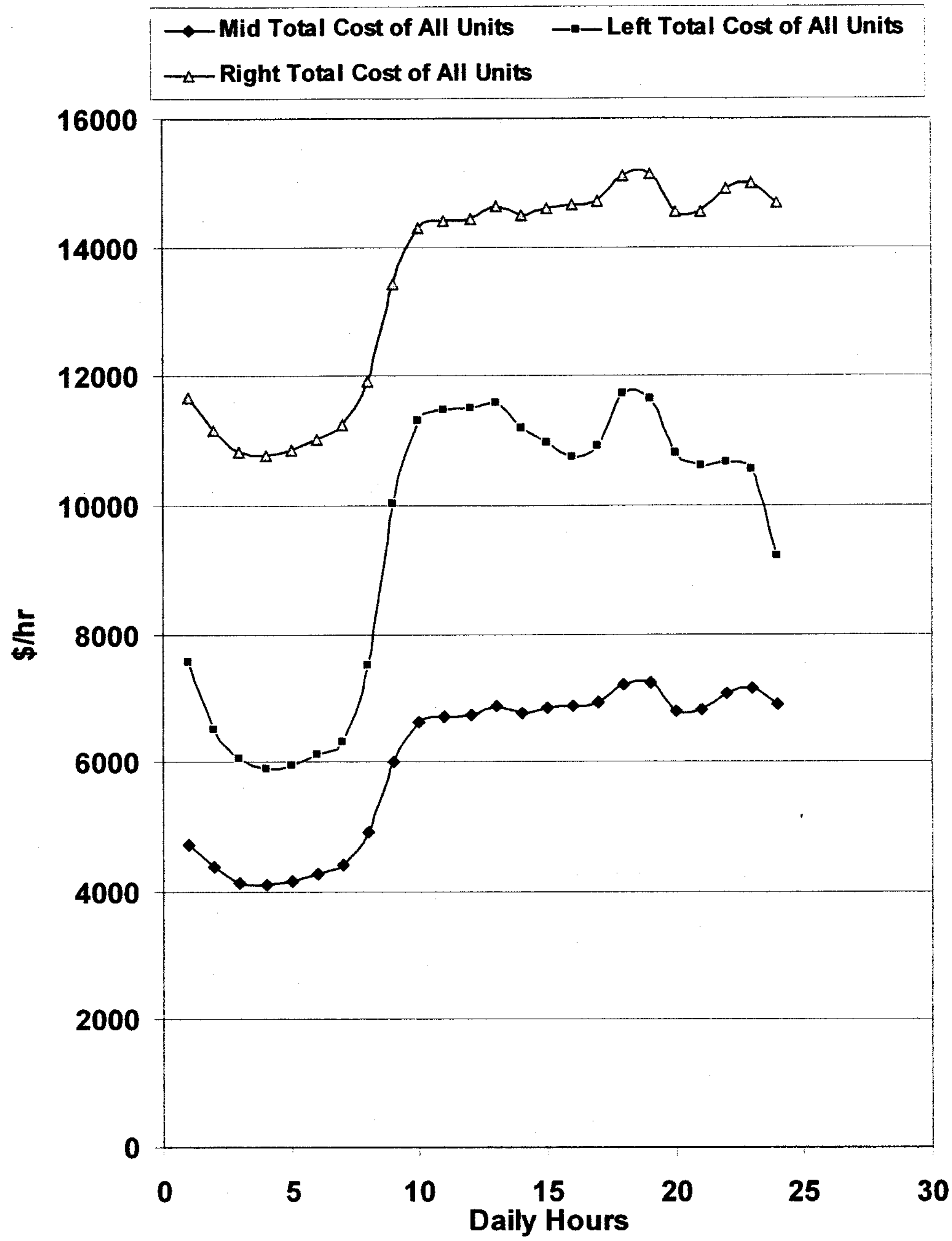


Figure (4-16) Fuzzy (0.5- α -Cut) Representation for Total Cost

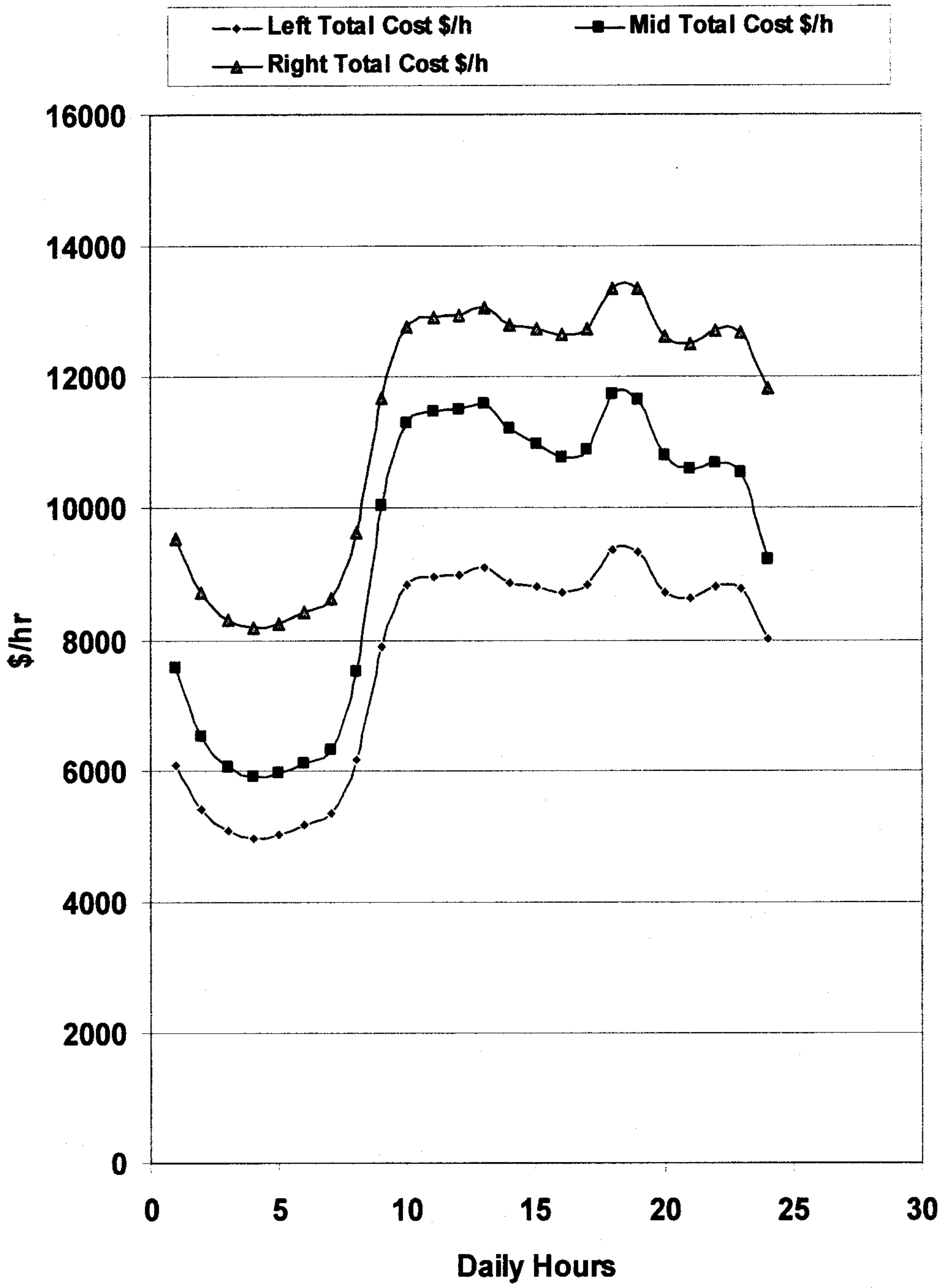


Figure (4-17) Fuzzy (0.75- α -Cut) Representation for Total Cost

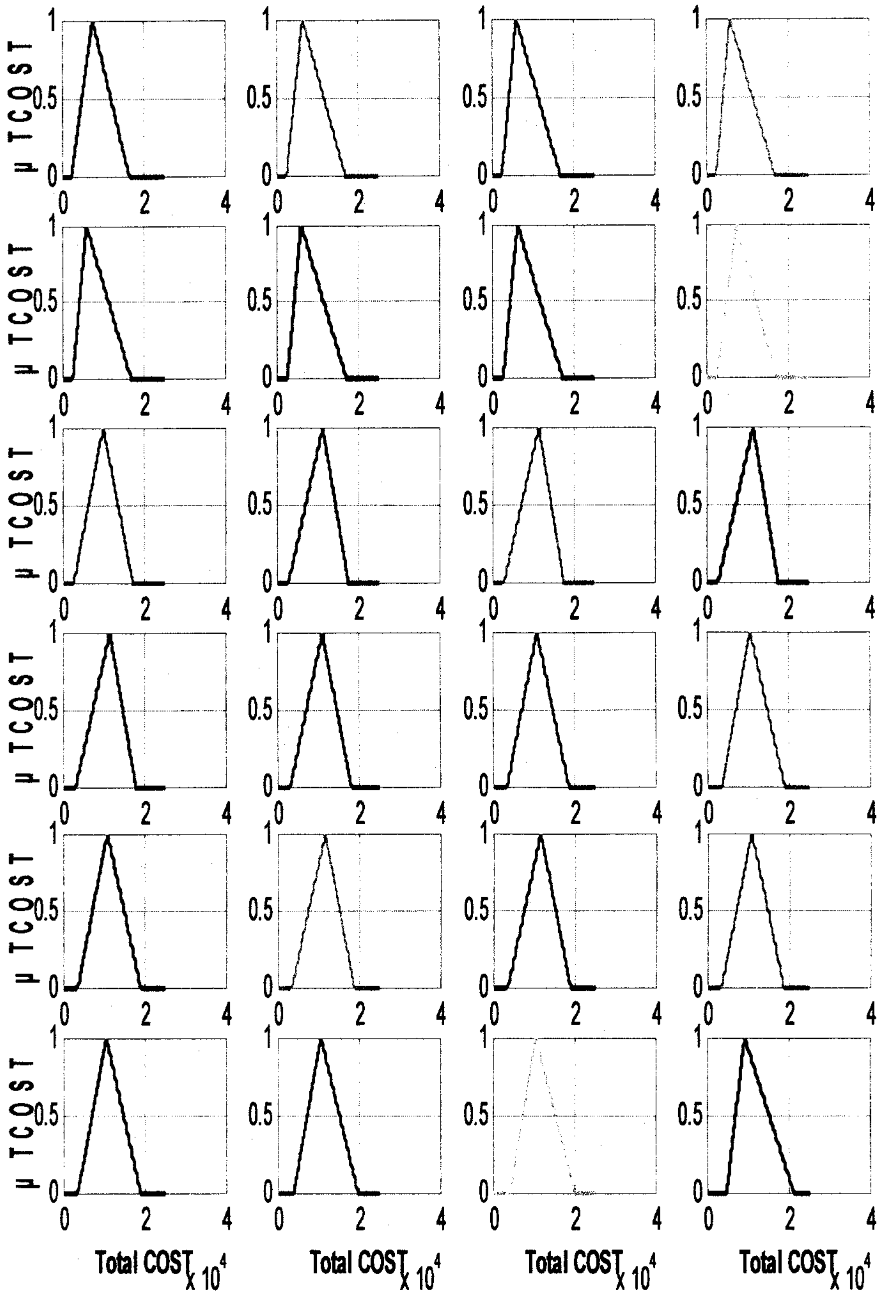


Figure (4-18) Fuzzy Triangular Membership Function for Total Cost

4.5 Conclusion

Fuzzification is simply the process of making a crisp quantity fuzzy. Recognizing that many of the quantities that we consider to be crisp and deterministic are actually not deterministic at all, they carry considerable uncertainty. If the form of uncertainty happens to arise because of imprecision, ambiguity or vagueness, then variable is probably fuzzy and can be represented by a membership function. In fact, load demand that varies hour by hour is not crisp and carries considerable uncertainty. We presented in this chapter that uncertainty loads or generations can be incorporated into power system models to give a better image of system behavior. A simple and easy technique was used to solve the economic dispatch problem using fuzzy sets. The load on the system is fuzzy and thus the fuel incremental, the costs of generation of each unit as well as the total costs are all fuzzy. The simulated example shows the effectiveness of the proposed algorithm in dealing with the system constraints.

Chapter 5

Economic Dispatch of All Thermal Power Systems with Fuzzy Load and Cost Function Parameters

5.1 Introduction

In this chapter we will consider the parameters $\tilde{\alpha}_i$, $\tilde{\beta}_i$ and $\tilde{\gamma}_i$ of the polynomial nonlinear cost function as fuzzy. The chapter starts with a simple application ignoring the power losses affecting the minimization of the cost function and the system over all performance. In the next chapter we will take into account the effect of the transmission losses in the equality constraint to achieve a more realistic economic dispatch.

5.2 Fuzzy Cost Parameters and Load Demand Neglecting Transmission Losses and Including Generator Limits

The objective is to find the minimum value of the total cost function subject to the equality and inequality constraints.

Minimize

$$\tilde{C}_{total} = \sum_{i=1}^{NG} \tilde{C}_i = \sum_{i=1}^{NG} \tilde{\alpha}_i + \tilde{\beta}_i \tilde{P}_{G_i} + \tilde{\gamma}_i \tilde{P}_{G_i}{}^2 \tag{5.1}$$

Subject to satisfying

$$\sum_{i=1}^{NG} \tilde{P}_{G_i} \geq \tilde{P}_{Demand} \tag{5.2}$$

$$\tilde{P}_{G_i}(\min) \leq \tilde{P}_{G_i} \leq \tilde{P}_{G_i}(\max) \qquad \textbf{\textit{i = 1,.......NG}} \tag{5.3}$$

The fuzzy cost function coefficient, load and generators are as follows:

1. fuzzy $\tilde{\alpha}_i = (\bar{\alpha}_i, L_{\tilde{\alpha}_i}, R_{\tilde{\alpha}_i})$
2. fuzzy $\tilde{\beta}_i = (\bar{\beta}_i, L_{\tilde{\beta}_i}, R_{\tilde{\beta}_i})$
3. fuzzy $\tilde{\gamma}_i = (\bar{\gamma}_i, L_{\tilde{\gamma}_i}, R_{\tilde{\gamma}_i})$

4. fuzzy load demand $(\bar{P}_D, L_{\tilde{P}_D}, R_{\tilde{P}_D})$
5. fuzzy power generator $(\bar{P}_{G_i}, L_{\tilde{P}_{G_i}}, R_{\tilde{P}_{G_i}})$

Substituting the middle, left and right side values into the cost function equation we get:

$$\tilde{C}_{total} = \sum_{i=1}^{NG} (\bar{C}_{ti}, L_{\tilde{C}_{ti}}, R_{\tilde{C}_{ti}}) = \sum_{i=1}^{NG} (\bar{\alpha}_i, L_{\tilde{\alpha}_i}, R_{\tilde{\alpha}_i}) + (\bar{\beta}_i, L_{\tilde{\beta}_i}, R_{\tilde{\beta}_i})(\bar{P}_{G_i}, L_{\tilde{P}_{G_i}}, R_{\tilde{P}_{G_i}}) + (\bar{\gamma}_i, L_{\tilde{\gamma}_i}, R_{\tilde{\gamma}_i})(\bar{P}_{G_i}, L_{\tilde{P}_{G_i}}, R_{\tilde{P}_{G_i}})(\bar{P}_{G_i}, L_{\tilde{P}_{G_i}}, R_{\tilde{P}_{G_i}}) \quad (5.4)$$

Subject to satisfying

$$\sum_{i=1}^{NG} (\bar{P}_{G_i}, L_{\tilde{P}_{G_i}}, R_{\tilde{P}_{G_i}}) \geq (\bar{P}_D, L_{\tilde{P}_D}, R_{\tilde{P}_D}) \quad (5.5)$$

$$L_{\tilde{P}_G} \leq \bar{P}_{G_i} \leq R_{\tilde{P}_{G_i}} \qquad i = 1, \ldots\ldots NG \quad (5.6)$$

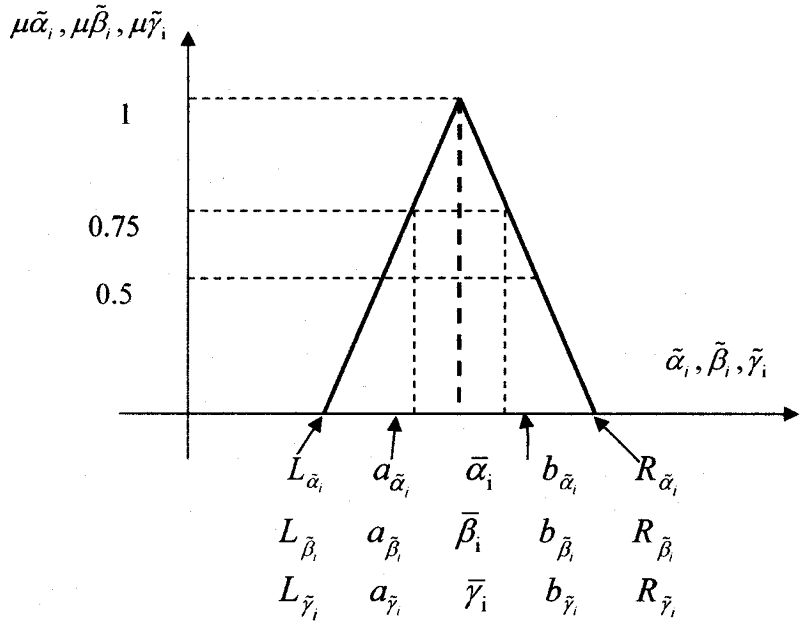


Figure (5-1) A-Triangular Membership Function for Fuzzy Cost Function Coefficient

The same method from the last chapter is applied where, the lower and upper limits of each unit generators are substituted by the left, right spreads or the left and right side of the membership function for the fuzzy generation as shown in equation (5.6). We will choose a fuzzy triangular membership function for all the fuzzy parameter values stated earlier. Since the load demand and the power generation triangular membership function were shown in Figures (4-1a) and (4-1b) in the last chapter, then we will only plot the cost function coefficient $\tilde{\alpha}_i$, $\tilde{\beta}_i$ and $\tilde{\gamma}_i$ all in one figure for simplicity but we should note that their chosen values are not the same. The percentage of deviation between the three parameters will be taken into consideration to explore the outcome of the minimum cost of the network.

The mathematical formula of $\tilde{\alpha}_i$ membership function is:

$$\mu(\tilde{\alpha}_i) = \begin{cases} 0 & \tilde{\alpha}_i < L_{\tilde{\alpha}_i} \\ \dfrac{\tilde{\alpha}_i - L_{\tilde{\alpha}_i}}{a_{\alpha_i}} & L_{\tilde{\alpha}_i} \le \tilde{\alpha}_i \le \bar{\alpha}_i \\ \dfrac{R_{\tilde{\alpha}_i} - \tilde{\alpha}_i}{b_{\alpha_i}} & \bar{\alpha}_i \le \tilde{\alpha}_i \le R_{\tilde{\alpha}_i} \\ 0 & \tilde{\alpha}_i > R_{\tilde{\alpha}_i} \end{cases} \tag{5.7}$$

The same mathematical formula representation applies to $\tilde{\beta}_i$ and $\tilde{\gamma}_i$. Using the Lagrange multiplier formula to relax "system wide constraints" into unconstrained form as in equation (2.3) – (2.7) to obtain the crisp optimization of the minimum cost function. The procedure in this chapter is to translate the fuzzy load and the fuzzy cost function coefficients $\tilde{\alpha}_i$, $\tilde{\beta}_i$ and $\tilde{\gamma}_i$ into a triangular membership function by assigning a degree of membership to each possible α -cut value of the load and the cost function coefficients. Which means mapping the fuzzy variable on the [0, 1] interval and then performing the fuzzy arithmetic operation to obtain the minimum total cost value.

Applying the fuzzy parameters into the crisp equation (2.4) we get:

$$\tilde{\lambda} = \tilde{\beta}_i + 2\tilde{\gamma}_i \tilde{P}_{G_i} \tag{5.8}$$

Setting the value of $\tilde{P}_{G_i}$ equal to $\tilde{P}_D$ as in equation (5.2) we get:

$$\sum_{i=1}^{NG} \frac{\tilde{\lambda} - \beta_i}{2\tilde{\gamma}_i} = \tilde{P}_D \tag{5.9}$$

Solving for $\tilde{\lambda}$

$$\tilde{\lambda} = \frac{2\tilde{P}_D + \sum_{i=1}^{NG} \frac{\tilde{\beta}_i}{\tilde{\gamma}_i}}{\sum_{i=1}^{NG} \frac{1}{\tilde{\gamma}_i}} \tag{5.10}$$

Evaluating the middle, left and right sides of the incremental cost into the equation

$$(\bar{\lambda}, L_{\tilde{\lambda}}, R_{\tilde{\lambda}}) = \frac{2(\bar{P}_D, L_{\tilde{P}_D}, R_{\tilde{P}_D}) + \sum_{i=1}^{NG} \frac{(\bar{\beta}_i, L_{\tilde{\beta}_i}, R_{\tilde{\beta}_i})}{(\bar{\gamma}_i, L_{\tilde{\gamma}_i}, R_{\tilde{\gamma}_i})}}{\sum_{i=1}^{NG} \frac{1}{(\bar{\gamma}_i, L_{\tilde{\gamma}_i}, R_{\tilde{\gamma}_i})}} \tag{5.11}$$

5.2.1 Fuzzy Interval Arithmetic Representation on Triangular Fuzzy Numbers

Equation (5.11) has a number of arithmetic operations such as, addition, subtraction, multiplication, division and inverse function which are all in fuzzy form. Applying fuzzy interval arithmetic operations implemented by their α-cut operation on triangular fuzzy numbers as shown in Table (3-2) while, taking into consideration that X and Y are greater than zero parameters formulation since all the input data values are greater than zero in the economical dispatch formulation.
Then the middle incremental cost becomes:

$$\bar{\lambda} = \frac{2\bar{P}_D + \sum_{i=1}^{NG} \frac{\bar{\beta}_i}{\bar{\gamma}_i}}{\sum_{i=1}^{NG} \frac{1}{\bar{\gamma}_i}} \tag{5.12}$$

The left equation for the incremental cost is:

$$L_{\tilde{\lambda}} = \frac{2L_{\tilde{P}_D} + \sum_{i=1}^{NG} \frac{L_{\tilde{\beta}_i}}{R_{\tilde{\gamma}_i}}}{\sum_{i=1}^{NG} \frac{1}{R_{\tilde{\gamma}_i}}} \tag{5.13}$$

The right side equation for the incremental cost is:

$$R_{\tilde{\lambda}} = \frac{2r_{\tilde{P}_D} + \sum_{i=1}^{NG} \frac{R_{\tilde{\beta}_i}}{L_{\tilde{\gamma}_i}}}{\sum_{i=1}^{NG} \frac{1}{L_{\tilde{\gamma}_i}}} \tag{5.14}$$

The fuzzy generation of each unit can be calculated as:

$$\tilde{P}_{G_i} = \frac{\tilde{\lambda} - \tilde{\beta}_i}{2\tilde{\gamma}_i} \qquad i = 1,..,NG \tag{5.15}$$

Evaluating the middle, left and right sides of the generation into the equation

$$\tilde{P}_{G_i} = (\bar{P}_{G_i}, L_{\tilde{P}_{G_i}}, R_{\tilde{P}_{G_i}}) = \frac{(\bar{\lambda}, L_{\tilde{\lambda}}, R_{\tilde{\lambda}}) - (\bar{\beta}_i, L_{\tilde{\beta}_i}, R_{\tilde{\beta}_i})}{2(\bar{\gamma}_i, L_{\tilde{\gamma}_i}, R_{\tilde{\gamma}_i})} \qquad i = 1,..,NG \tag{5.16}$$

The middle, left and right sides of all the generators equations are:

$$\bar{P}_{G_i} = \frac{\bar{\lambda}_i - \bar{\beta}_i}{2(\bar{\gamma}_i)} \qquad i = 1,..,NG \tag{5.17}$$

$$L_{\tilde{P}_{G_i}} = \frac{L_{\tilde{\lambda}} - R_{\tilde{\beta}_i}}{2(R_{\tilde{\gamma}_i})} \qquad i = 1,..,NG \tag{5.18}$$

$$R_{\tilde{P}_{G_i}} = \frac{R_{\tilde{\lambda}} - L_{\tilde{\beta}_i}}{2(L_{\tilde{\gamma}_i})} \qquad i = 1,..,NG \tag{5.19}$$

Substituting the power generation into cost function equation we get:

$$\begin{aligned}
\sum_{i=1}^{NG} \bar{C}_i &= \sum_{i=1}^{NG} \bar{\alpha}_i + \bar{\beta}_i \bar{P}_{G_i} + \bar{\gamma}_i \bar{P}_{G_i}^{\,2} \\
\sum_{i=1}^{NG} L_{\tilde{C}_i} &= \sum_{i=1}^{NG} L_{\tilde{\alpha}_i} + L_{\tilde{\beta}_i} L_{\tilde{P}_{G_i}} + L_{\tilde{\gamma}_i} L_{\tilde{P}_{G_i}} L_{\tilde{P}_{G_i}} \\
\sum_{i=1}^{NG} R_{\tilde{C}_i} &= \sum_{i=1}^{NG} R_{\tilde{\alpha}_i} + R_{\tilde{\beta}_i} R_{\tilde{P}_{G_i}} + R_{\tilde{\gamma}_i} R_{\tilde{P}_{G_i}} R_{\tilde{P}_{G_i}}
\end{aligned} \tag{5.20}$$

5.2.2 Fuzzy Arithmetic on Triangular L-R Representation of Fuzzy Numbers

Using Table (3-1) in chapter 3 to perform the fuzzy arithmetic calculation on triangular L-R representation of fuzzy number in equation (5.10) and (5.15) we get the following:

The middle or crisp value of the incremental cost function is:

$$\bar{\lambda} = \frac{2\bar{P}_D + \sum_{i=1}^{NG} \frac{\bar{\beta}_i}{\bar{\gamma}_i}}{\sum_{i=1}^{NG} \frac{1}{\bar{\gamma}_i}} \tag{5.21}$$

The left spread of the incremental cost becomes:

$$a_{\tilde{\lambda}_i} = \frac{(2\bar{P}_D + \sum_{i=1}^{NG} \frac{\bar{\beta}_i}{\bar{\gamma}_i})(\sum_{i=1}^{NG} a_{\tilde{\gamma}_i}\bar{\gamma}_i^{\,2}) + (\sum_{i=1}^{NG} \frac{1}{\bar{\gamma}_i})(2a_{\tilde{D}_i} + (\sum_{i=1}^{NG} (\bar{\beta}_i b_{\tilde{\gamma}_i} + a_{\tilde{\beta}_i}\bar{\gamma}_i)/\bar{\gamma}_i^{\,2}))}{(\sum_{i=1}^{NG} \frac{1}{\bar{\gamma}_i})^2} \tag{5.22}$$

The right spread of the incremental cost is:

$$b_{\tilde{\lambda}_i} = \frac{(2\bar{P}_D + \sum_{i=1}^{NG} \frac{\bar{\beta}_i}{\bar{\gamma}_i})(\sum_{i=1}^{NG} b_{\tilde{\gamma}_i}\bar{\gamma}_i^{\,2}) + (\sum_{i=1}^{NG} \frac{1}{\bar{\gamma}_i})(2b_{\tilde{D}_i} + (\sum_{i=1}^{NG} (\bar{\beta}_i a_{\tilde{\gamma}_i} + b_{\tilde{\beta}_i}\bar{\gamma}_i)/\bar{\gamma}_i^{\,2}))}{(\sum_{i=1}^{NG} \frac{1}{\bar{\gamma}_i})^2} \tag{5.23}$$

After applying the L.R. representation method, the middle power generation in equation (5.15) becomes:

$$\bar{P}_{G_i} = \frac{\bar{\lambda}_i - \bar{\beta}_i}{2(\bar{\gamma}_i)} \tag{5.24}$$

The left spread of the power generation is:

$$a_{\tilde{P}_i} = \frac{(\bar{\lambda}_i - \bar{\beta}_i)2b_{\tilde{\gamma}_i} + 2\bar{\gamma}_i(a_{\tilde{\lambda}_i} + b_{\tilde{\beta}_i})}{(2\bar{\gamma}_i)^2} \tag{5.25}$$

The right spread of the power generation is:

$$b_{\tilde{P}_i} = \frac{(\bar{\lambda}_i - \bar{\beta}_i)2a_{\tilde{\gamma}_i} + 2\bar{\gamma}_i(b_{\tilde{\lambda}_i} + a_{\tilde{\beta}_i})}{(2\bar{\gamma}_i)^2} \tag{5.26}$$

Substituting the power generation into the middle, left and right spread cost equation we get:

$$\sum_{i=1}^{NG} \bar{C}_i = \sum_{i=1}^{NG} \bar{\alpha}_i + \bar{\beta}_i \bar{P}_{G_i} + \bar{\gamma}_i \bar{P}_{G_i}^{\ 2}$$
$$\sum_{i=1}^{NG} a_{\tilde{C}_i} = \sum_{i=1}^{NG} a_{\tilde{\alpha}_i} + \bar{\beta}_i a_{\tilde{P}_{G_i}} + \bar{P}_{G_i} a_{\tilde{\beta}_i} + 2\bar{\gamma}_i \bar{P}_{G_i} a_{\tilde{P}_{G_i}} + \bar{P}_{G_i}^{\ 2} a_{\tilde{\gamma}_i} \qquad (5.27)$$
$$\sum_{i=1}^{NG} b_{\tilde{C}_i} = \sum_{i=1}^{NG} b_{\tilde{\alpha}_i} + \bar{\beta}_i b_{\tilde{P}_{G_i}} + \bar{P}_{G_i} b_{\tilde{\beta}_i} + 2\bar{\gamma}_i \bar{P}_{G_i} b_{\tilde{P}_{G_i}} + \bar{P}_{G_i}^{\ 2} b_{\tilde{\gamma}_i}$$

5.3 Simulated Example

In this section a simulated example is presented to evaluate the economical dispatch (ED) operation of power systems when the load demand is fuzzy for 24 hours and the cost function coefficients are fuzzy while ignoring the power losses affecting the minimization of the cost function and the system's overall performance. The load demand is chosen to be a triangular membership with a 10% deviation as tabulated in Table (5-1) and plotted in Figure (5-2). In addition the fuzzy cost function coefficients are a triangular membership function subjected with different percentages of deviation. The selected synthetic system example contains three thermal units and the input/out fuel cost functions, for each unit, are given as:

$$F(P_{G_1}) = 200 + 7.0P_{G_1} + 0.008P_{G_1}^{\ 2} \qquad \text{kJ/h}$$
$$F(P_{G_2}) = 180 + 6.3P_{G_2} + 0.009P_{G_2}^{\ 2} \qquad \text{kJ/h}$$
$$F(P_{G_3}) = 140 + 6.8P_{G_3} + 0.007P_{G_3}^{\ 2} \qquad \text{kJ/h}$$

The generation limits are given by the left and right sides of each unit:

$$L_{\tilde{P}_{G_1}} \le \tilde{P}_{G_1} \le R_{\tilde{P}_{G_1}} \qquad \boldsymbol{i = 1, \ldots\ldots NG}$$
$$L_{\tilde{P}_{G_2}} \le \tilde{P}_{G_2} \le R_{\tilde{P}_{G_2}} \qquad \boldsymbol{i = 1, \ldots\ldots\ldots NG}$$
$$L_{\tilde{P}_{G_3}} \le \tilde{P}_{G_3} \le R_{\tilde{P}_{G_3}} \qquad \boldsymbol{i = 1, \ldots\ldots\ldots NG}$$

This example will be implemented on the Generalized Interval Arithmetic to Fuzzy Number procedures formulated in section (5.2.1). Using the principle of equal incremental cost, we will determine the optimal fuzzy dispatch and total fuzzy cost applying the Generalized method to perform the mathematical addition, subtraction, division, inversion and multiplication to the equation as explained in the fuzzy set chapter (3). Two simulation programs were created using Matlab software. In the first program all the equations in section (5.2.1) were analyzed and debugged in a complete program set and in the second program a Matlab toolbox was used to simulate the mathematical formula. The results of the two programs were identical, which was expected. The analysis was tested on a 3% deviation for $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ then tested again on a 10% deviation. All the results were tabulated and plotted such as the fuzzy incremental cost, fuzzy generation of each unit, total fuzzy generation versus fuzzy load and total fuzzy cost.

Examining the tables and figures we can observe the following:

- Table (5-1) shows the fuzzy load with 10% deviation for model "A" representing the weekdays of a month where this fuzzy load was obtained from an estimated fuzzy short term load forecasting model, developed on the basis of fuzzy multiple linear regressions, to minimize the spread of the fuzzy coefficients that exist in the fuzzy winter model for a 24-hour period **[63]**. Figure (5-2) shows the middle, left and right sides for all the α-Cut representations of the fuzzy load. The load shows that its highest limits happen during the afternoon period while its lowest values are during the morning hours. The spread of the load is very narrow because of the 10% deviation model. The highest values represent the load at α-cut equal to zero and the middle value represent the crisp value of the load where α-cut equal to 1.
- Table (5-2) and Figure (5-2) show the fuel incremental cost calculated from equation (5-12) to (5-13) representing the middle, left and right side of the triangular membership function of the incremental fuel cost. The value of the incremental fuel cost range between 10.5 up to 14.2 which is appropriate value for a load spread of 10% deviation. The middle value of the fuel incremental cost ranges between 11.01 and 13.36. All the generators have to operate on equal incremental fuel cost in order to obtain a minimum total cost. When the fuzzy load demand increases then the generation of the units increases and the fuel incremental value increases respectively to maintain the equality of the fuel incremental cost value for all the generation in order to obtain a minimum cost. If any of the generation exceeds its limit then the plant is pegged at its

upper limit, which means that the other generators should provide the excess of increasing load.

- Table (5-3), (5-4) and (5-5) represent the power generation of each unit respectively. Figure (5-4), (5-5) and (5-6) shows the power generation of each unit for all α -cut representation. The power generation of unit 1 has a little wider spread at α -cut equal to zero than the other two generators at the peak value and at the minimum value. Unit 3 has the smallest spread than the other two generators at the peak value and at the minimum value. All the power generators follow the load demand curve shape but with different spread values of membership function. This proves that unit 1 is providing the highest power to the network which confirms that the fuzzy formulation result is compatible with the conventional formulation or crisp formulation of the ED problem except it is faster and it provides a wide range of information regarding the system performance.
- Table (5-6) represents the total generation of all units at each α -cut values. Figure (5-7) shows the total power generation of all units for all α -cut representation versus the fuzzy load demand. The results came exactly as expected, the total fuzzy generations are approximately greater than or equal to the fuzzy load. This confirms that under normal operation conditions the total generators capacity is more than the load demand. The crisp value of the total power generation and load demand are equal and satisfy the load demand in equation (5.5). The total fuzzy generations represented by the left and right sides are greater than the expected load due to the increased range of the cost function coefficients.
- Table (5-7) and Figure (5-8) represent the total minimum cost for all α -cut values. The total minimum cost has a wider spread at the peak value and at the minimum value than the power generation and the load demand. Its middle value or crisp value however, coincided with the load demand, incremental fuel cost and the power generation. Even though the load demand has only 10% deviation in the spread at the peak hour the minimum total cost has 34.4% of the spread, which means that if the load exceeds its crisp value it

would be very costly for a small variation of the cost function coefficient equal to 3% of $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$.

- If the deviation of $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ increased higher than 5% as shown in Figure (5-9) which represent the total power generation versus fuzzy load and Figure (5-10) represents the total minimum cost then the minimum, maximum total fuzzy generation are much larger than the minimum, maximum fuzzy load. In addition the total minimum cost has increased sharply with increase in coefficients. This means that it is important to keep the coefficient values in control and not to exceed their expected values. The results calculated show that the fuzzy parameters in the cost function play a great roll in the performance of the network to obtain a minimum optimal cost to the thermal generation committed. This procedure shows all the possibilities that could be encountered hour by hour for 24 hours including the minimum cost of the sudden increased load which is the main objective of the economical dispatch method.

Table (5-1)

Membership Function of Load Demand for (0, 0.5, 0.7, 1) α -Cut Representation for Model "A" Weekdays With 10% Deviation for (P_D) and 3% for (α, β, γ)

Membership Function		$\mu_{P_{Load}}=0$			$\mu_{P_{Load}}=0.5$			$\mu_{P_{Load}}=0.75$			$\mu_{P_{Load}}=1$	
Daily Hours	Left Load MW	Mid Load MW	Right Load MW	Left Load MW	Mid Load MW	Right Load MW	Left Load MW	Mid Load MW	Right Load MW	Left Load MW	Mid Load MW	Right Load MW
1	1006	1118	1229	1062	1118	1173	1090	1118	1146	1118	1118	1118
2	905.8	1006	1107	956.1	1006	1057	981.2	1006	1032	1006	1006	1006
3	849.2	943.6	1038	896.4	943.6	990.8	920	943.6	967.2	943.6	943.6	943.6
4	784.1	871.2	958.3	827.6	871.2	914.7	849.4	871.2	892.9	871.2	871.2	871.2
5	731.7	813	894.3	772.4	813	853.7	792.7	813	833.3	813	813	813
6	782.7	869.7	956.7	826.2	869.7	913.2	848	869.7	891.4	869.7	869.7	869.7
7	823.3	914.8	1006	869.1	914.8	960.5	891.9	914.8	937.7	914.8	914.8	914.8
8	880.8	978.7	1077	929.8	978.7	1028	954.2	978.7	1003	978.7	978.7	978.7
9	1042	1157	1273	1099	1157	1215	1128	1157	1186	1157	1157	1157
10	1101	1224	1346	1163	1224	1285	1193	1224	1254	1224	1224	1224
11	1095	1217	1338	1156	1217	1278	1186	1217	1247	1217	1217	1217
12	1156	1284	1413	1220	1284	1349	1252	1284	1316	1284	1284	1284
13	1133	1259	1384	1196	1259	1322	1227	1259	1290	1259	1259	1259
14	1087	1208	1329	1147	1208	1268	1178	1208	1238	1208	1208	1208
15	1040	1155	1271	1098	1155	1213	1127	1155	1184	1155	1155	1155
16	999.5	1111	1222	1055	1111	1166	1083	1111	1138	1111	1111	1111
17	984.7	1094	1204	1039	1094	1149	1067	1094	1121	1094	1094	1094
18	1002	1113	1225	1058	1113	1169	1085	1113	1141	1113	1113	1113
19	1068	1186	1305	1127	1186	1246	1157	1186	1216	1186	1186	1186
20	1025	1139	1253	1082	1139	1196	1111	1139	1167	1139	1139	1139
21	1037	1152	1268	1095	1152	1210	1123	1152	1181	1152	1152	1152
22	1104	1227	1349	1165	1227	1288	1196	1227	1257	1227	1227	1227
23	1079	1198	1318	1138	1198	1258	1168	1198	1228	1198	1198	1198
24	1001	1112	1223	1056	1112	1167	1084	1112	1140	1112	1112	1112

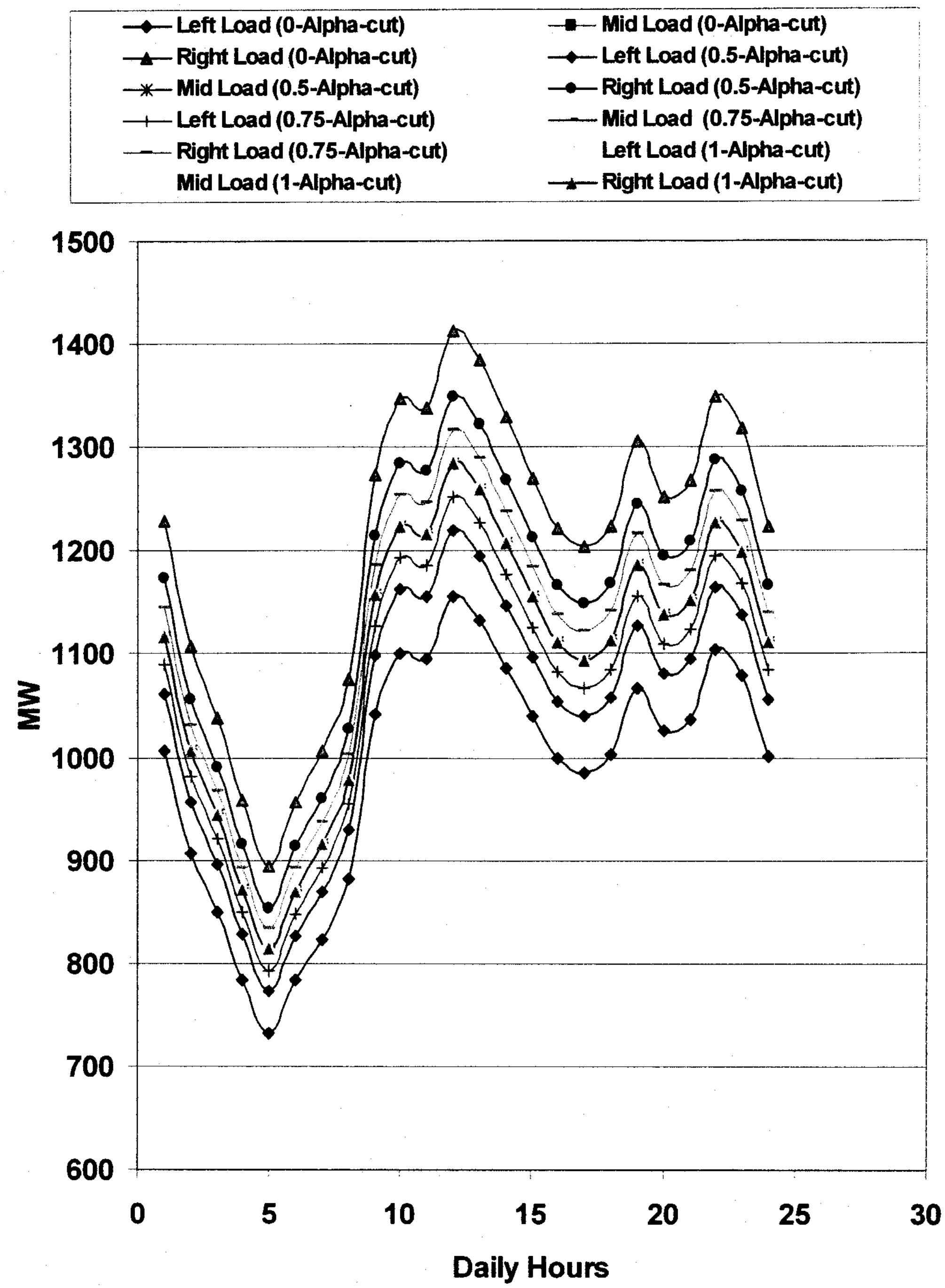


Figure (5-2) Fuzzy Load Demand for All α-Cut Representation

Table (5-2)

Membership Function of Incremental Cost for (0, 0.5, 0.7, 1) α -Cut Representation for Model "A" Weekdays With 10% Deviation for (P_D) and 3% for (α, β, γ)

Membership Function	$\mu_\lambda = 0$			$\mu_\lambda = 0.5$			$\mu_\lambda = 0.75$			$\mu_\lambda = 1$		
Daily Hours	Left λ $MW/h	Mid λ $MW/h	Right λ $MW/h	Left λ $MW/h	Mid λ $MW/h	Right λ $MW/h	Left λ $MW/h	Mid λ $MW/h	Right λ $MW/h	Left λ $MW/h	Mid λ $MW/h	Right λ $MW/h
1	11.99	12.62	13.214	12.31	12.62	12.92	12.46	12.62	12.77	12.62	12.62	12.617
2	11.44	12.03	12.588	11.75	12.03	12.305	11.9	12.03	12.166	12.03	12.03	12.031
3	11.13	11.7	12.234	11.44	11.7	11.954	11.57	11.7	11.824	11.7	11.7	11.699
4	10.78	11.32	11.827	11.08	11.32	11.55	11.2	11.32	11.432	11.32	11.32	11.317
5	10.5	11.01	11.499	10.79	11.01	11.227	10.9	11.01	11.117	11.01	11.01	11.01
6	10.77	11.31	11.818	11.08	11.31	11.54	11.19	11.31	11.424	11.31	11.31	11.309
7	10.99	11.55	12.072	11.31	11.55	11.789	11.43	11.55	11.668	11.55	11.55	11.547
8	11.31	11.88	12.432	11.63	11.88	12.143	11.76	11.88	12.014	11.88	11.88	11.884
9	12.18	12.83	13.438	12.52	12.83	13.132	12.67	12.83	12.98	12.83	12.83	12.827
10	12.51	13.18	13.812	12.86	13.18	13.501	13.02	13.18	13.339	13.18	13.18	13.178
11	12.47	13.14	13.773	12.82	13.14	13.462	12.98	13.14	13.302	13.14	13.14	13.141
12	12.8	13.5	14.153	13.16	13.5	13.836	13.33	13.5	13.667	13.5	13.5	13.497
13	12.68	13.36	14.008	13.03	13.36	13.694	13.2	13.36	13.528	13.36	13.36	13.362
14	12.43	13.09	13.722	12.78	13.09	13.412	12.93	13.09	13.253	13.09	13.09	13.094
15	12.17	12.82	13.427	12.51	12.82	13.122	12.67	12.82	12.969	12.82	12.82	12.817
16	11.95	12.58	13.175	12.29	12.58	12.874	12.43	12.58	12.727	12.58	12.58	12.581
17	11.87	12.49	13.082	12.21	12.49	12.782	12.35	12.49	12.638	12.49	12.49	12.493
18	11.96	12.6	13.19	12.3	12.6	12.889	12.45	12.6	12.742	12.6	12.6	12.595
19	12.32	12.98	13.602	12.67	12.98	13.294	12.82	12.98	13.137	12.98	12.98	12.981
20	12.09	12.73	13.335	12.43	12.73	13.031	12.58	12.73	12.881	12.73	12.73	12.73
21	12.16	12.8	13.41	12.5	12.8	13.105	12.65	12.8	12.953	12.8	12.8	12.801
22	12.52	13.19	13.827	12.87	13.19	13.516	13.03	13.19	13.354	13.19	13.19	13.192
23	12.38	13.04	13.669	12.73	13.04	13.36	12.89	13.04	13.202	13.04	13.04	13.044
24	11.96	12.59	13.182	12.29	12.59	12.88	12.44	12.59	12.734	12.59	12.59	12.587

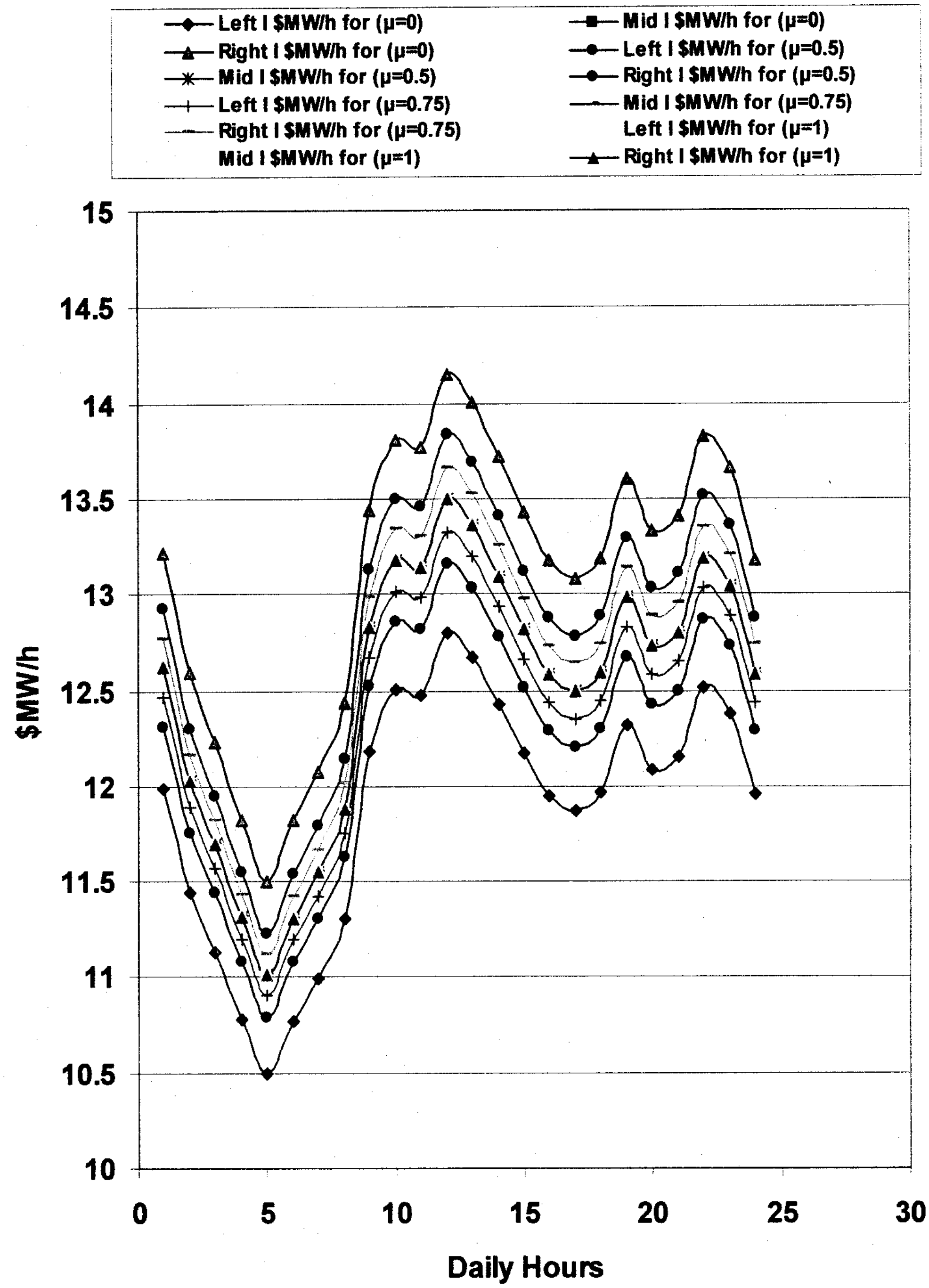


Figure (5-3) Fuzzy Incremental Cost for All α -Cut Representation

Table (5-3)

Membership Function of Generation #1 for (0, 0.5, 0.7, 1) α -Cut Representation for Model "A" Weekdays With 10% Deviation for (P_D) and 3% for (α, β, γ)

Membership Function	$\mu_{PG1}=0$			$\mu_{PG1}=0.5$			$\mu_{PG1}=0.75$			$\mu_{PG1}=1$		
Daily Hours	Left PG1 MW	Mid PG1 MW	Right PG1 MW	Left PG1 MW	Mid PG1 MW	Right PG1 MW	Left PG1 MW	Mid PG1 MW	Right PG1 MW	Left PG1 MW	Mid PG1 MW	Right PG1 MW
1	289.8	351.1	413.9	320.2	351.1	382.3	335.6	351.1	366.7	351.1	351.1	351.1
2	256.8	314.4	373.6	291.6	314.4	337.4	304.5	314.4	324.3	314.4	314.4	314.4
3	238.1	293.7	350.8	275	293.7	312.4	285.5	293.7	301.9	293.7	293.7	293.7
4	216.6	269.8	324.5	253.9	269.8	285.8	262.5	269.8	277.1	269.8	269.8	269.8
5	199.3	250.6	303.4	236.4	250.6	264.8	243.9	250.6	257.4	250.6	250.6	250.6
6	216.2	269.3	324	254.6	269.3	284.1	262.1	269.3	276.5	269.3	269.3	269.3
7	229.6	284.2	340.4	268.9	284.2	299.5	276.7	284.2	291.7	284.2	284.2	284.2
8	248.5	305.3	363.5	289	305.3	321.5	297.2	305.3	313.3	305.3	305.3	305.3
9	301.6	364.2	428.3	345.1	364.2	383.3	354.6	364.2	373.7	364.2	364.2	364.2
10	321.3	386.1	452.5	365.9	386.1	406.3	376	386.1	396.2	386.1	386.1	386.1
11	319.2	383.8	449.9	363.7	383.8	403.9	373.8	383.8	393.8	383.8	383.8	383.8
12	339.3	406.1	474.4	384.9	406.1	427.3	395.5	406.1	416.7	406.1	406.1	406.1
13	331.6	397.6	465.1	376.8	397.6	418.4	387.2	397.6	408	397.6	397.6	397.6
14	316.5	380.8	446.7	360.9	380.8	400.8	370.9	380.8	390.8	380.8	380.8	380.8
15	301	363.6	427.6	344.5	363.6	382.6	354	363.6	373.1	363.6	363.6	363.6
16	287.7	348.8	411.4	330.5	348.8	367.1	339.6	348.8	357.9	348.8	348.8	348.8
17	282.8	343.3	405.4	325.3	343.3	361.4	334.3	343.3	352.4	343.3	343.3	343.3
18	288.5	349.7	412.4	331.3	349.7	368	340.5	349.7	358.9	349.7	349.7	349.7
19	310.2	373.8	438.9	354.2	373.8	393.4	364	373.8	383.6	373.8	373.8	373.8
20	296.1	358.2	421.7	339.4	358.2	376.9	348.8	358.2	367.5	358.2	358.2	358.2
21	300.1	362.5	426.5	343.5	362.5	381.5	353	362.5	372	362.5	362.5	362.5
22	322.1	387	453.4	366.8	387	407.2	376.9	387	397.1	387	387	387
23	313.8	377.7	443.3	358	377.7	397.5	367.9	377.7	387.6	377.7	377.7	377.7
24	288	349.2	411.8	330.8	349.2	367.5	340	349.2	358.3	349.2	349.2	349.2

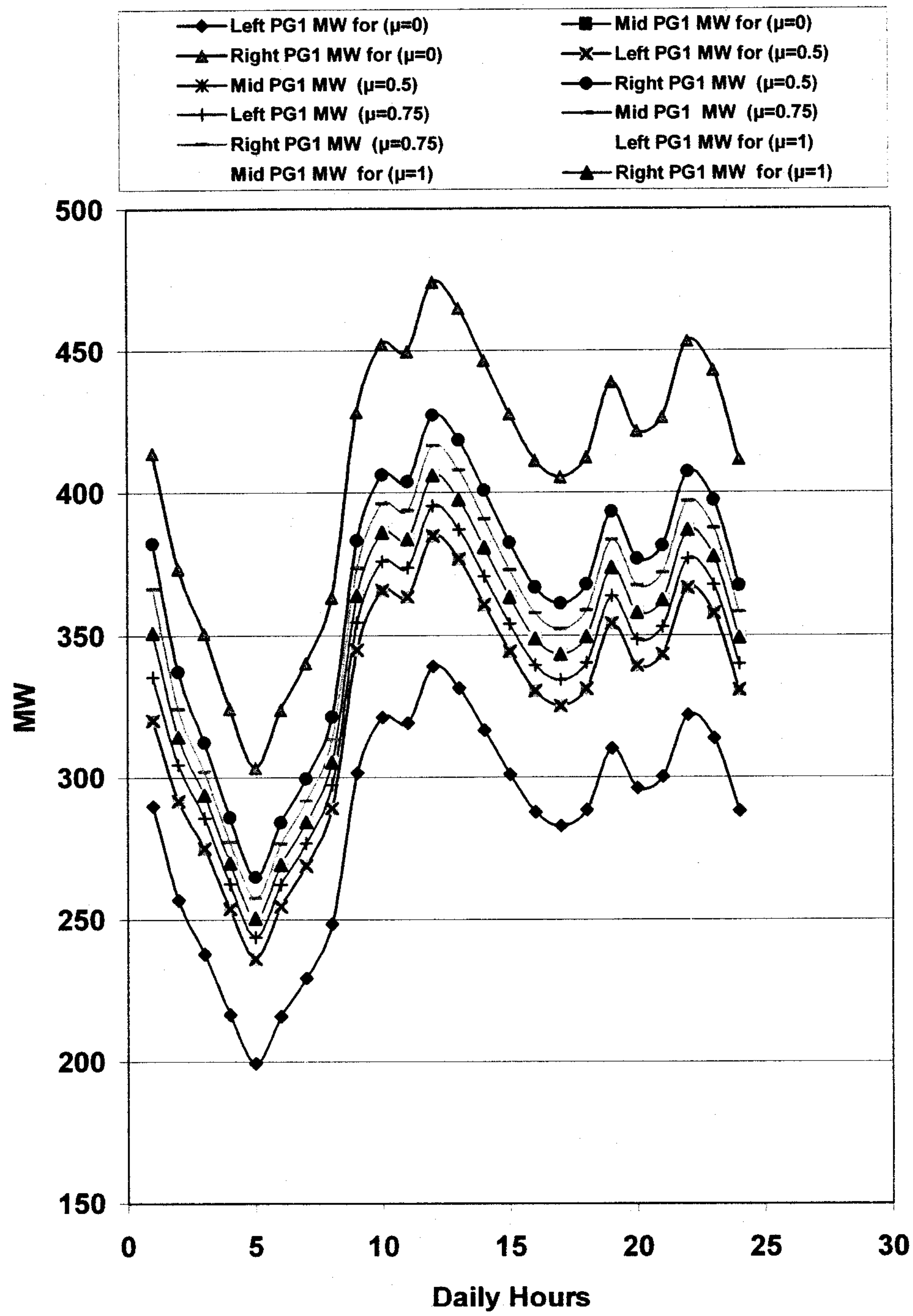


Figure (5-4) Fuzzy Power Generation of Unit #1 for All α -Cut Representation

Table (5-4)

Membership Function of Generation #2 for (0, 0.5, 0.7, 1) α -Cut Representation for Model "A" Weekdays With 10% Deviation for (P_D) and 3% for (α, β, γ)

Membership Function		$\mu_{P_{G2}}=0$			$\mu_{P_{G2}}=0.5$			$\mu_{P_{G2}}=0.75$			$\mu_{P_{G2}}=1$	
Daily Hours	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW
1	296.5	351	406.8	323.6	351	378.7	337.2	351	364.8	351	351	351
2	267.1	318.4	371	298.1	318.4	338.8	309.6	318.4	327.1	318.4	318.4	318.4
3	250.5	300	350.7	283.3	300	316.6	292.7	300	307.2	300	300	300
4	231.4	278.7	327.4	264.6	278.7	292.9	272.2	278.7	285.2	278.7	278.7	278.7
5	216.1	261.7	308.6	249	261.7	274.3	255.7	261.7	267.6	261.7	261.7	261.7
6	231	278.3	326.9	265.2	278.3	291.4	271.9	278.3	284.7	278.3	278.3	278.3
7	242.9	291.5	341.4	277.9	291.5	305.1	284.8	291.5	298.2	291.5	291.5	291.5
8	259.8	310.2	362	295.8	310.2	324.7	303.1	310.2	317.4	310.2	310.2	310.2
9	306.9	362.6	419.6	345.6	362.6	379.6	354.1	362.6	371.1	362.6	362.6	362.6
10	324.5	382.1	441.1	364.1	382.1	400.1	373.1	382.1	391.1	382.1	382.1	382.1
11	322.6	380.1	438.8	362.2	380.1	397.9	371.1	380.1	389	380.1	380.1	380.1
12	340.4	399.8	460.6	381	399.8	418.7	390.4	399.8	409.3	399.8	399.8	399.8
13	333.7	392.3	452.3	373.9	392.3	410.8	383.1	392.3	401.5	392.3	392.3	392.3
14	320.3	377.4	435.9	359.7	377.4	395.1	368.6	377.4	386.3	377.4	377.4	377.4
15	306.4	362.1	419	345.1	362.1	379	353.6	362.1	370.5	362.1	362.1	362.1
16	294.6	348.9	404.6	332.6	348.9	365.2	340.8	348.9	357.1	348.9	348.9	348.9
17	290.3	344.1	399.3	328	344.1	360.1	336.1	344.1	352.1	344.1	344.1	344.1
18	295.3	349.7	405.4	333.4	349.7	366	341.6	349.7	357.9	349.7	349.7	349.7
19	314.6	371.1	429	353.8	371.1	388.5	362.4	371.1	379.8	371.1	371.1	371.1
20	302.1	357.2	413.7	340.5	357.2	373.9	348.9	357.2	365.6	357.2	357.2	357.2
21	305.6	361.1	418	344.3	361.1	378	352.7	361.1	369.6	361.1	361.1	361.1
22	325.2	382.9	442	364.9	382.9	400.9	373.9	382.9	391.9	382.9	382.9	382.9
23	317.8	374.7	432.9	357.1	374.7	392.2	365.9	374.7	383.4	374.7	374.7	374.7
24	294.9	349.3	405	333	349.3	365.6	341.1	349.3	357.4	349.3	349.3	349.3

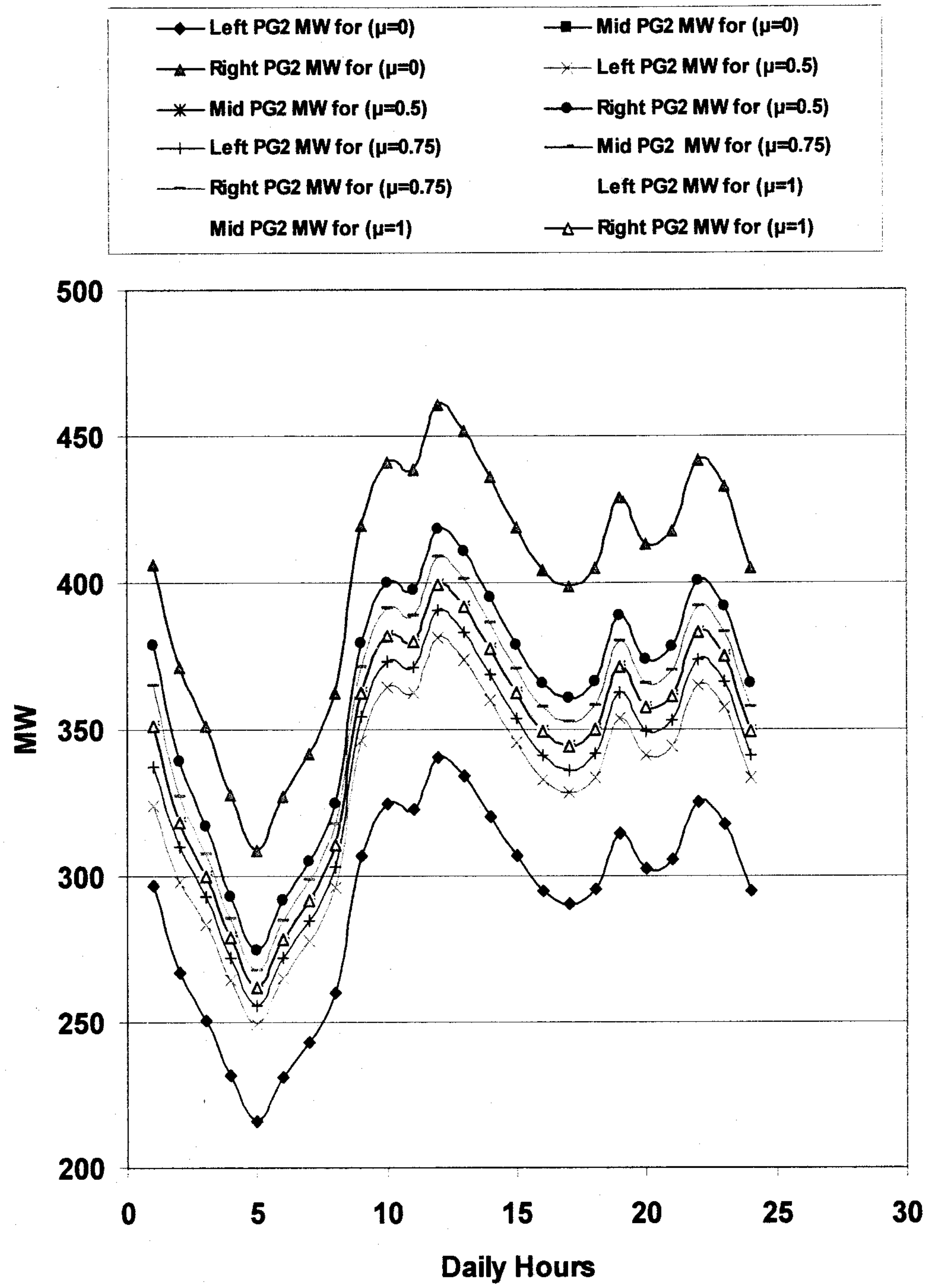


Figure (5-5) Fuzzy Power Generation of Unit #2 for All α -Cut Representation

Table (5-5)

Membership Function of Generation #3 for (0, 0.5, 0.7, 1) α -Cut Representation For Model "A" Weekdays With 10% Deviation for (P_D) and 3% for (α, β, γ)

Membership Function	$\mu_{P_{G3}} = 0$			$\mu_{P_{G3}} = 0.5$			$\mu_{P_{G3}} = 0.75$			$\mu_{P_{G3}} = 1$		
Daily Hour	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW
1	345.4	415.5	487.4	380.3	415.5	451.2	397.9	415.5	433.3	415.5	415.5	415.5
2	307.7	373.6	441.2	347.5	373.6	399.8	362.3	373.6	384.9	373.6	373.6	373.6
3	286.4	349.9	415.2	328.6	349.9	371.3	340.6	349.9	359.3	349.9	349.9	349.9
4	261.8	322.6	385.2	304.4	322.6	340.9	314.3	322.6	331	322.6	322.6	322.6
5	242.1	300.7	361	284.5	300.7	316.9	293	300.7	308.4	300.7	300.7	300.7
6	261.3	322.1	384.6	305.2	322.1	338.9	313.9	322.1	330.3	322.1	322.1	322.1
7	276.6	339.1	403.3	321.6	339.1	356.6	330.5	339.1	347.7	339.1	339.1	339.1
8	298.3	363.2	429.8	344.6	363.2	381.7	354	363.2	372.4	363.2	363.2	363.2
9	358.9	430.5	503.8	408.6	430.5	452.4	419.6	430.5	441.4	430.5	430.5	430.5
10	381.5	455.6	531.4	432.5	455.6	478.7	444	455.6	467.1	455.6	455.6	455.6
11	379.1	452.9	528.5	430	452.9	475.9	441.5	452.9	464.4	452.9	452.9	452.9
12	402	478.4	556.5	454.2	478.4	502.6	466.3	478.4	490.5	478.4	478.4	478.4
13	393.3	468.7	545.8	445	468.7	492.4	456.8	468.7	480.5	468.7	468.7	468.7
14	376	449.5	524.8	426.8	449.5	472.3	438.2	449.5	460.9	449.5	449.5	449.5
15	358.3	429.8	503	408	429.8	451.6	418.9	429.8	440.7	429.8	429.8	429.8
16	343.1	412.9	484.5	392	412.9	433.8	402.4	412.9	423.4	412.9	412.9	412.9
17	337.5	406.7	477.6	386.1	406.7	427.3	396.4	406.7	417	406.7	406.7	406.7
18	344	413.9	485.6	392.9	413.9	434.9	403.4	413.9	424.4	413.9	413.9	413.9
19	368.8	441.5	515.9	419.1	441.5	463.8	430.3	441.5	452.7	441.5	441.5	441.5
20	352.7	423.6	496.2	402.1	423.6	445.1	412.9	423.6	434.3	423.6	423.6	423.6
21	357.2	428.6	501.7	406.9	428.6	450.3	417.8	428.6	439.5	428.6	428.6	428.6
22	382.4	456.6	532.5	433.5	456.6	479.7	445	456.6	468.1	456.6	456.6	456.6
23	372.9	446	520.9	423.4	446	468.6	434.7	446	457.3	446	446	446
24	343.5	413.3	484.9	392.4	413.3	434.3	402.9	413.3	423.8	413.3	413.3	413.3

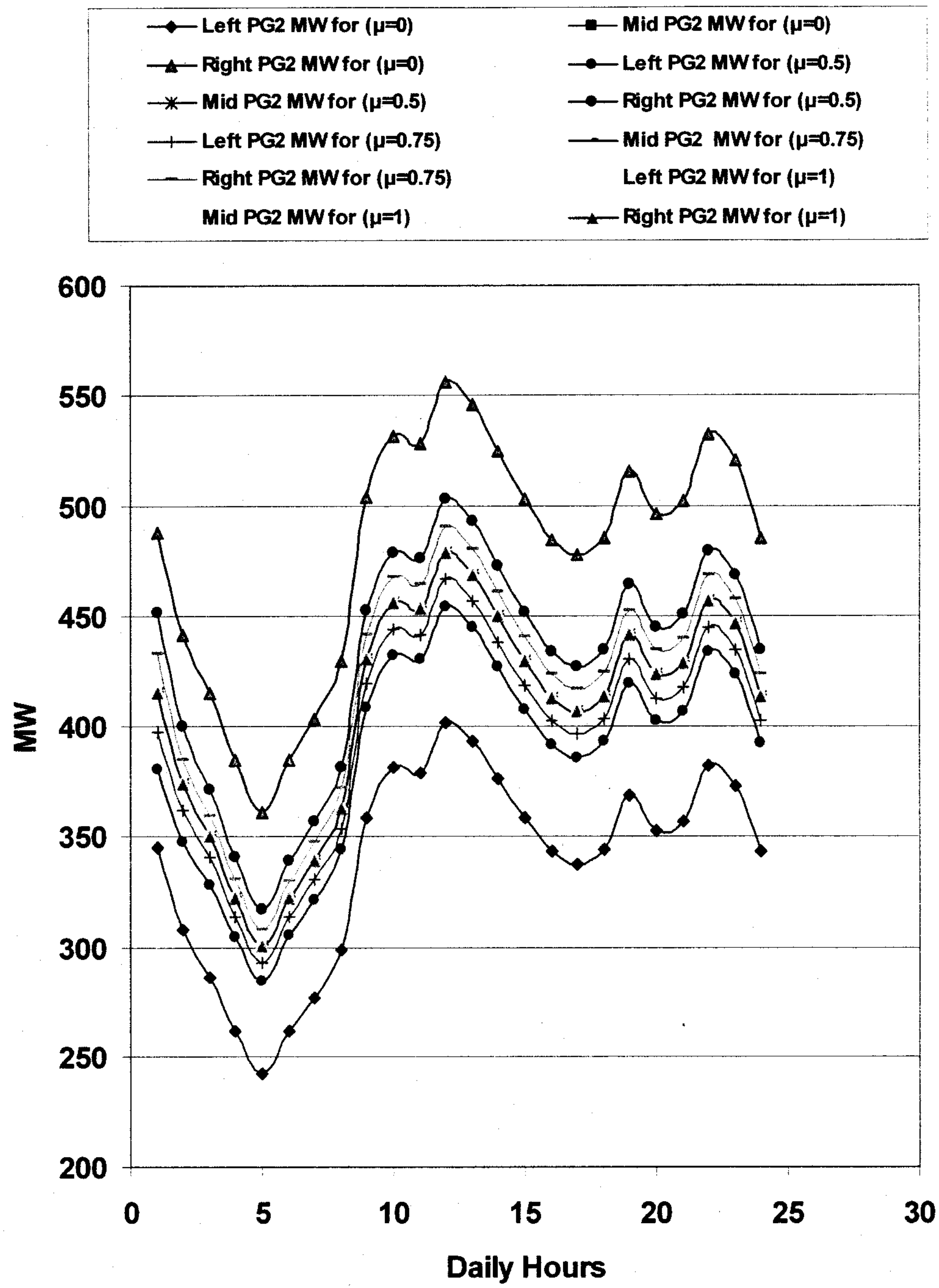


Figure (5-6) Fuzzy Power Generation of Unit # 3 for All α -Cut Representation

Table (5-6)

Membership Function of Total Generator for (0, 0.5, 0.7, 1) α -Cut Representation

For Model "A" Weekdays With 10% Deviation for (P_D) and 3% for (α, β, γ)

Membership Function	$\mu_{tP_G} = 0$			$\mu_{tP_G} = 0.5$			$\mu_{tP_G} = 0.75$			$\mu_{tP_G} = 1$		
Daily Hours	Left tPG MW	Mid tPG MW	Right tPG MW	Left tPG MW	Mid tPG MW	Right tPG MW	Left tPG MW	Mid tPG MW	Right tPG MW	Left tPG MW	Mid tPG MW	Right tPG MW
1	931.7	1118	1308	1024	1118	1212	1071	1118	1165	1118	1118	1118
2	831.6	1006	1186	937.1	1006	1076	976.5	1006	1036	1006	1006	1006
3	775.1	943.6	1117	886.9	943.6	1000	918.8	943.6	968.4	943.6	943.6	943.6
4	709.9	871.2	1037	822.8	871.2	919.5	849.1	871.2	893.2	871.2	871.2	871.2
5	657.5	813	973.1	770	813	856	792.6	813	833.4	813	813	813
6	708.6	869.7	1035	825	869.7	914.4	847.9	869.7	891.5	869.7	869.7	869.7
7	749.2	914.8	1085	868.5	914.8	961.1	891.9	914.8	937.7	914.8	914.8	914.8
8	806.7	978.7	1155	929.5	978.7	1028	954.2	978.7	1003	978.7	978.7	978.7
9	967.4	1157	1352	1099	1157	1215	1128	1157	1186	1157	1157	1157
10	1027	1224	1425	1163	1224	1285	1193	1224	1254	1224	1224	1224
11	1021	1217	1417	1156	1217	1278	1186	1217	1247	1217	1217	1217
12	1082	1284	1491	1220	1284	1349	1252	1284	1316	1284	1284	1284
13	1059	1259	1463	1196	1259	1322	1227	1259	1290	1259	1259	1259
14	1013	1208	1407	1147	1208	1268	1178	1208	1238	1208	1208	1208
15	965.7	1155	1350	1098	1155	1213	1127	1155	1184	1155	1155	1155
16	925.4	1111	1300	1055	1111	1166	1083	1111	1138	1111	1111	1111
17	910.5	1094	1282	1039	1094	1149	1067	1094	1121	1094	1094	1094
18	927.8	1113	1303	1058	1113	1169	1085	1113	1141	1113	1113	1113
19	993.6	1186	1384	1127	1186	1246	1157	1186	1216	1186	1186	1186
20	950.9	1139	1332	1082	1139	1196	1111	1139	1167	1139	1139	1139
21	962.9	1152	1346	1095	1152	1210	1123	1152	1181	1152	1152	1152
22	1030	1227	1428	1165	1227	1288	1196	1227	1257	1227	1227	1227
23	1004	1198	1397	1138	1198	1258	1168	1198	1228	1198	1198	1198
24	926.5	1112	1302	1056	1112	1167	1084	1112	1140	1112	1112	1112

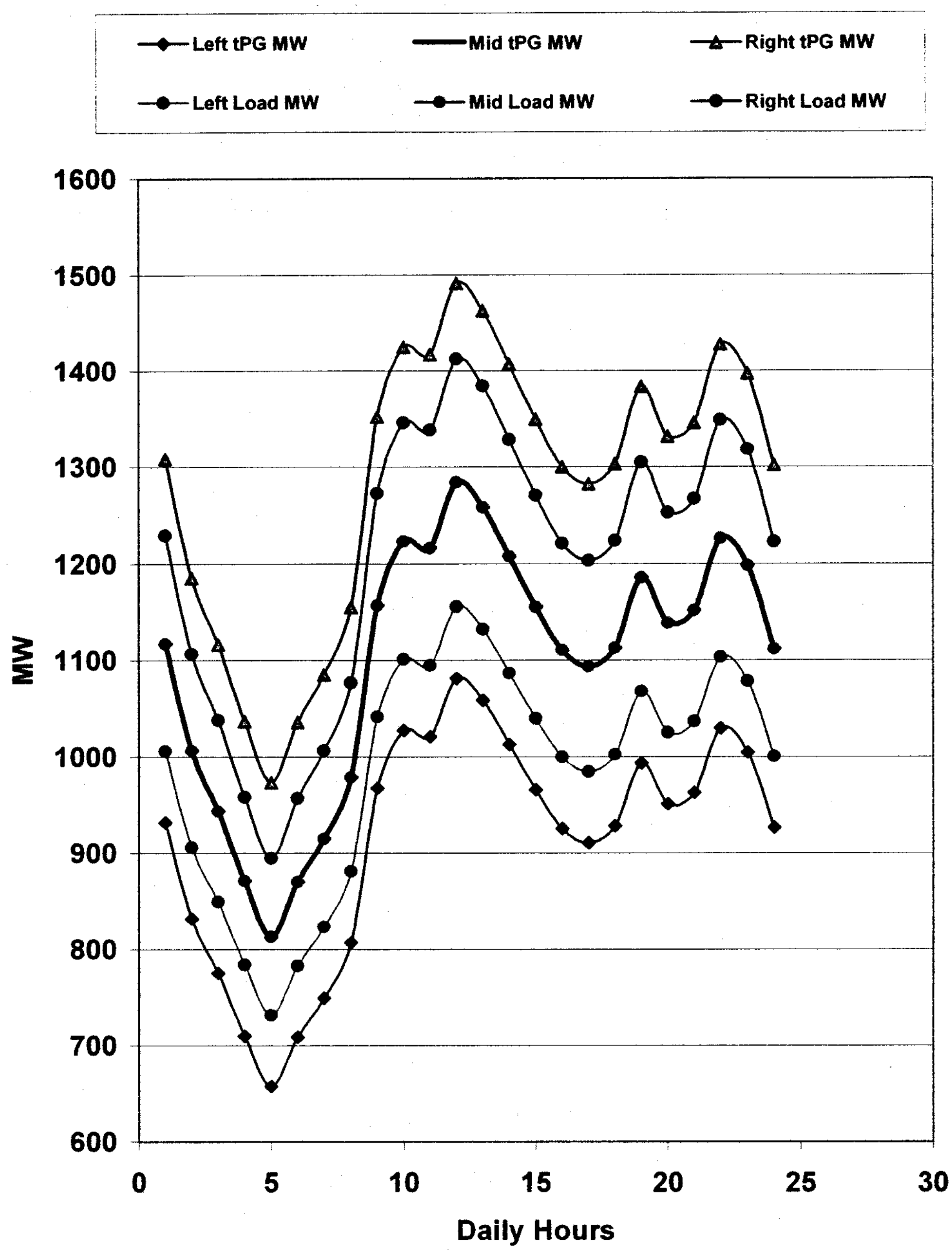


Figure (5-7) Fuzzy Total Power Generation Versus Fuzzy Load for (0- α -Cut)

Table (5-7)

Membership Function of Total Cost for (0, 0.5, 0.7, 1) α-Cut Representation for Model "A" Weekdays With 10% Deviation for (P_D) and 3% for (α, β, γ)

Membership Function	$\mu_C = 0$			$\mu_C = 0.5$			$\mu_C = 0.75$			$\mu_C = 1$		
Daily Hours	Left Cost $/h	Mid Cost $/h	Right Cost $/h	Left Cost $/h	Mid Cost $/h	Right Cost $/h	Left Cost $/h	Mid Cost $/h	Right Cost $/h	Left Cost $/h	Mid Cost $/h	Right Cost $/h
1	8755	11318	14268	9990	11318	12742	10642	11318	12018	11318	11318	11318
2	7651	9947	12593	9049	9947	10887	9570	9947	10332	9947	9947	9947
3	7050	9202	11682	8511	9202	9916	8909	9202	9499	9202	9202	9202
4	6377	8369	10665	7811	8369	8941	8119	8369	8621	8369	8369	8369
5	5853	7719	9872	7243	7719	8207	7496	7719	7945	7719	7719	7719
6	6364	8352	10644	7848	8352	8867	8107	8352	8600	8352	8352	8352
7	6780	8868	11273	8336	8868	9411	8605	8868	9133	8868	8868	8868
8	7384	9616	12188	9036	9616	10209	9327	9616	9909	9616	9616	9616
9	9161	11823	14886	11087	11823	12577	11454	11823	12196	11823	11823	11823
10	9857	12688	15945	11890	12688	13505	12287	12688	13093	12688	12688	12688
11	9783	12595	15832	11805	12595	13405	12198	12595	12998	12595	12595	12595
12	10506	13495	16933	12638	13495	14373	13064	13495	13931	13495	13495	13495
13	10228	13149	16510	12319	13149	14001	12732	13149	13572	13149	13149	13149
14	9688	12477	15687	11696	12477	13278	12084	12477	12875	12477	12477	12477
15	9142	11799	14857	11067	11799	12548	11431	11799	12171	11799	11799	11799
16	8684	11230	14161	10539	11230	11936	10882	11230	11581	11230	11230	11230
17	8517	11023	13907	10347	11023	11714	10683	11023	11367	11023	11023	11023
18	8711	11264	14202	10571	11264	11973	10915	11264	11616	11264	11264	11264
19	9464	12198	15346	11438	12198	12978	11816	12198	12586	12198	12198	12198
20	8973	11589	14600	10873	11589	12323	11229	11589	11954	11589	11589	11589
21	9110	11759	14808	11030	11759	12505	11392	11759	12130	11759	11759	11759
22	9886	12723	15988	11924	12723	13542	12321	12723	13130	12723	12723	12723
23	9589	12355	15537	11582	12355	13146	11966	12355	12748	12355	12355	12355
24	8696	11245	14179	10553	11245	11953	10897	11245	11597	11245	11245	11245

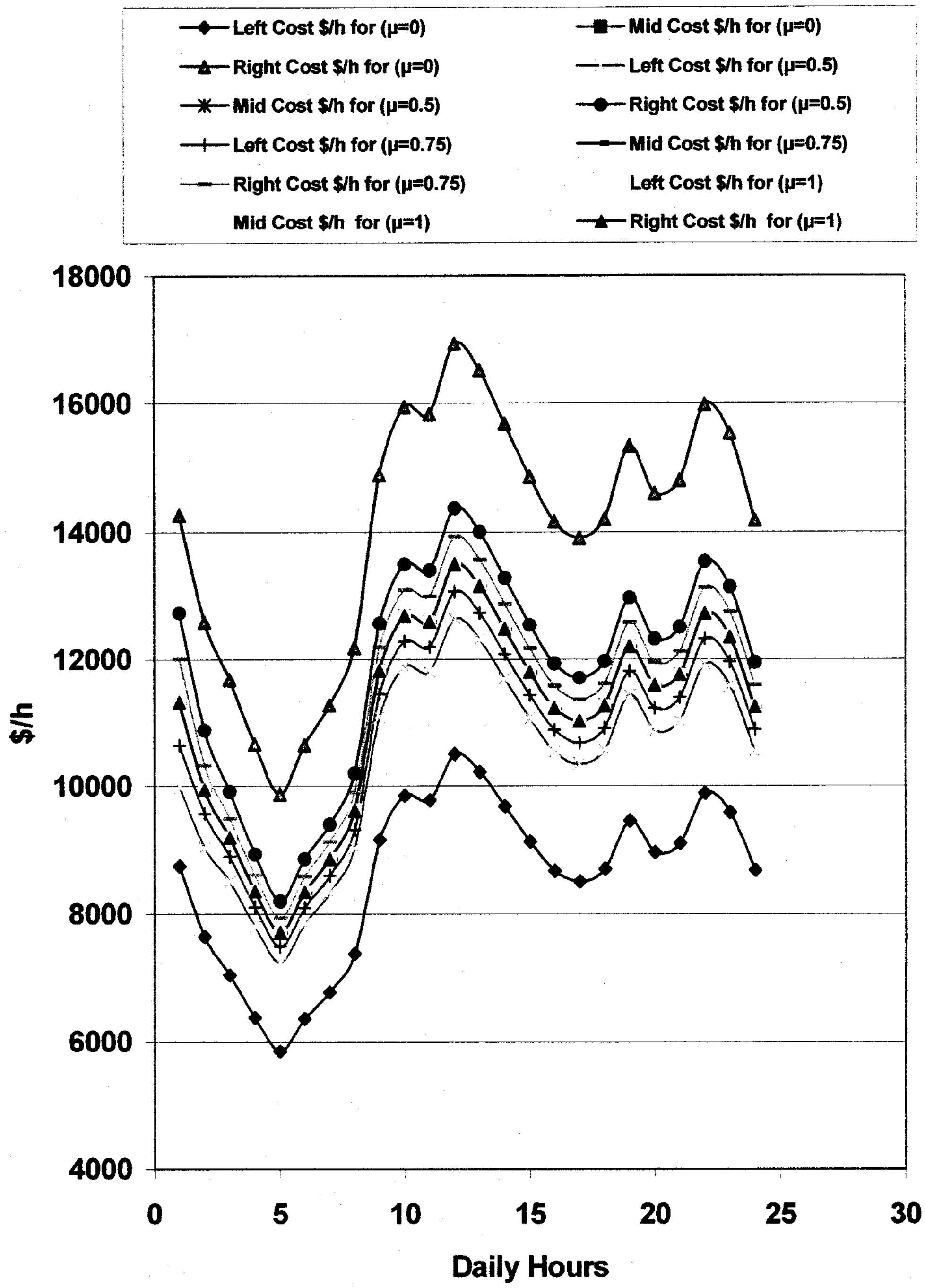


Figure (5-8) Fuzzy Minimum Total Cost for All α -Cut Representation

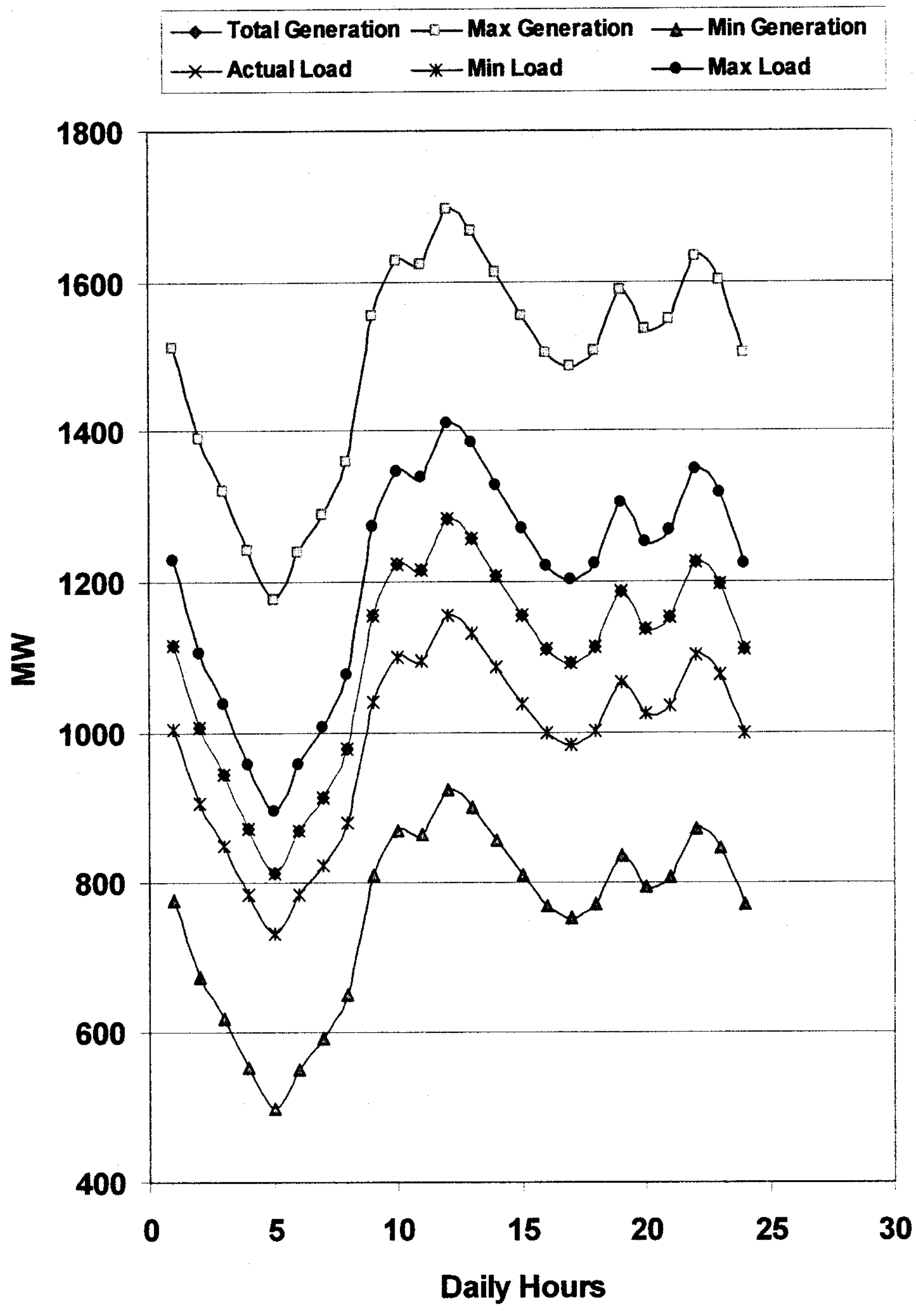


Figure (5-9) Fuzzy Total Generation Versus Fuzzy Load for 10% Deviation of (α,β,γ)

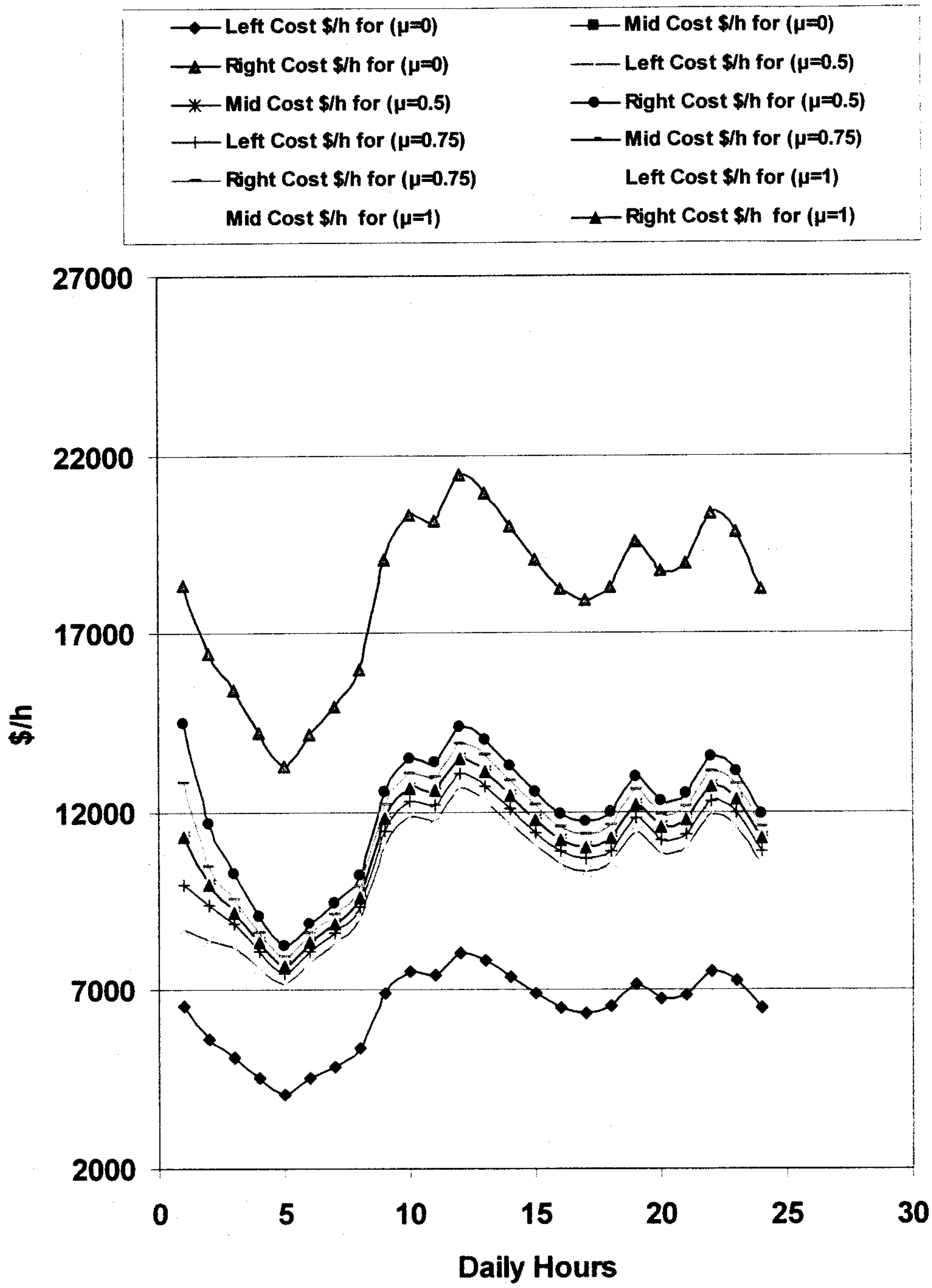


Figure (5-10) Fuzzy Minimum Total Cost for 10% Deviation of (α, β, γ) for (0-α-cut)

Chapter 6

Fuzzy Economical Dispatch Including Losses

6.1 Introduction

It is important to consider the losses in the transmission lines due to the large interconnected network where power is transmitted over long distances with low load density areas. In addition determining the distribution of load between plants needs to consider the transmission line losses where for a given distribution of loads, usually the plant with low incremental fuel cost rate has greater transmission losses than the other plants which will effect the over all economy of the system and it is wise to lower the load at that plant to achieve a minimum fuel cost.

6.2 Problem Formulation

The objective is to find the minimum value of the total cost function subject to the equality and inequality constraints.

Minimize

$$\tilde{C}_{total} = \sum_{i=1}^{NG} \tilde{C}_i = \sum_{i=1}^{NG} \tilde{\alpha}_i + \tilde{\beta}_i \tilde{P}_{G_i} + \tilde{\gamma}_i \tilde{P}_{G_i}^{\ 2} \tag{6.1}$$

Subject to satisfying

$$\sum_{i=1}^{NG} \tilde{P}_{G_i} \geq \tilde{P}_D + \tilde{P}_L \tag{6.2}$$

$$\tilde{P}_{G_i}(\min) \leq \tilde{P}_{G_i} \leq \tilde{P}_{G_i}(\max) \quad i = 1, \dots\dots NG \tag{6.3}$$

The fuzzy variable added in this case is the power losses $\tilde{P}_L = (\bar{P}_L, L_{\tilde{L}}, R_{\tilde{L}})$ denoting the middle, left and right sides of the power losses.

The total transmission losses formula is a quadratic function of the generator power output expressed as:

$$\tilde{P}_L = \sum_{i=1}^{NG}\sum_{j=1}^{NG} \tilde{P}_{G_i} B_{ij} \tilde{P}_{G_j} \tag{6.4}$$

A more general formula containing linear terms is known as Kron's loss formula is:

$$\tilde{P}_L = \sum_{i=1}^{NG}\sum_{j=1}^{NG} \tilde{P}_{G_i} B_{ij} \tilde{P}_{G_j} + \sum_{i=1}^{NG} B_{0i} \tilde{P}_{G_i} + B_{00} \tag{6.5}$$

Applying fuzzy interval arithmetic operations implemented by their α -Cut operation to obtain the power losses formula that include the middle, left and right sides of the triangular membership function, it becomes:

$$\tilde{P}_L(\bar{P}_L, L_{\tilde{L}}, R_{\tilde{L}}) = \sum_{i=1}^{NG}\sum_{j=1}^{NG} (\bar{P}_G, L_{\tilde{P}_G}, R_{\tilde{P}_{G_i}}) B_{ij} (\bar{P}_{G_j}, L_{\tilde{P}_{G_j}}, R_{\tilde{P}_{G_j}}) + \sum_{i=1}^{NG} B_{0i} (\bar{P}_{G_i}, L_{\tilde{P}_G}, R_{\tilde{P}_{Gi}}) + B_{00} \tag{6.6}$$

Using the simplest quadratic form we get:

$$\tilde{P}_L = \sum_{i=1}^{NG} B_{ii} \tilde{P}_{G_i}^{\ 2} \tag{6.7}$$

Substituting the middle, left and right sides of the generation triangular membership function into the equation we get:

$$(\bar{P}_L, L_{\tilde{L}}, R_{\tilde{L}}) = \sum_{i=1}^{NG} (B_{ii} \bar{P}_{G_i}^{\ 2}, B_{ii} L_{\tilde{P}_{G_i}}^{\ 2}, B_{ii} R_{\tilde{P}_{G_i}}^{\ 2}) \tag{6.8}$$

Then using table (3-2) to obtain the middle, left and right side of the equation:

$$\bar{P}_L = \sum_{i=1}^{NG} B_{ii} \bar{P}_{G_i}^{\ 2} \tag{6.9}$$

$$L_{\tilde{L}} = \sum_{i=1}^{NG} B_{ii} L_{\tilde{P}_{G_i}}^{\ 2} \tag{6.10}$$

$$R_{\tilde{L}} = \sum_{i=1}^{NG} B_{ii} R_{\tilde{P}_{G_i}}^{\ 2} \tag{6.11}$$

The power generation of each unit can be calculated from equation (2.20) that was derived in chapter (2) which becomes:

$$(\bar{P}_{G_i}, L_{\tilde{P}_{G_i}}, R_{\tilde{P}_{G_i}})^{[k]} = \frac{(\bar{\lambda}, L_{\tilde{\lambda}}, R_{\tilde{\lambda}})^{[k]} - (\bar{\beta}_i, L_{\tilde{\beta}_i}, R_{\tilde{\beta}_i})}{2((\bar{\gamma}_i, L_{\tilde{\gamma}_i}, R_{\tilde{\gamma}_i}) + (\bar{\lambda}, L_{\tilde{\lambda}}, R_{\tilde{\lambda}})^{[k]} B_{ii})} \tag{6.12}$$

Using Table (3-2) to perform the fuzzy set arithmetic calculation then, the middle crisp, left and right value of the equation becomes:

$$\bar{P}_{G_i}^{[k]} = \frac{\bar{\lambda}_i^{[k]} - \bar{\beta}_i}{2(\bar{\gamma}_i + \bar{\lambda}_i^{[k]} B_{ii})} \tag{6.13}$$

$$L_{\tilde{P}_{G_i}}{}^{[k]} = \frac{L_{\tilde{\lambda}}^{[k]} - R_{\tilde{\beta}_i}}{2(R_{\tilde{\gamma}_i} + r_{\tilde{\lambda}}^{[k]} B_{ii})} \tag{6.14}$$

$$R_{\tilde{P}_{G_i}}{}^{[k]} = \frac{R_{\tilde{\lambda}}^{[k]} - L_{\tilde{\beta}_i}}{2(L_{\tilde{\gamma}_i} + L_{\tilde{\lambda}}^{[k]} B_{ii})} \tag{6.15}$$

Substituting the generation values into the loss formula to calculate the power losses. Then we check the equality constraints to see if it is satisfied. If it is not satisfied then we use the iterative method shown in flow chart (6-1). Where $\sum_{i=1}^{NG} \left(\frac{\partial \tilde{P}_{G_i}}{\partial \tilde{\lambda}} \right)$ is given as:

$$\sum_{i=1}^{NG} \left(\frac{\partial \tilde{P}_{G_i}}{\partial \tilde{\lambda}} \right) = \sum_{i=1}^{NG} \left[\frac{\tilde{\gamma}_i + B_{ii} B_i}{2\left(\tilde{\gamma}_i + \tilde{\lambda}^{[k]} B_{ii} \right)^2} \right] \tag{6.16}$$

Replacing the fuzzy parameters with their middle, left and right sides into the equation we get:

$$\sum_{i=1}^{NG} \left(\frac{\partial(\bar{P}_{G_i}, L_{\tilde{P}_{G_i}}, R_{\tilde{P}_{G_i}})}{\partial(\bar{\lambda}, L_{\tilde{\lambda}}, R_{\tilde{\lambda}})} \right) = \sum_{i=1}^{NG} \left[\frac{(\bar{\gamma}_i, L_{\tilde{\gamma}_i}, R_{\tilde{\gamma}_i}) + B_{ii}(\bar{\beta}_i, L_{\tilde{\beta}_i}, R_{\tilde{\beta}_i})}{2\left((\bar{\gamma}_i, L_{\tilde{\gamma}_i}, R_{\tilde{\gamma}_i}) + (\bar{\lambda}, L_{\tilde{\lambda}}, R_{\tilde{\lambda}})^{[k]} B_{ii} \right)^2} \right] \tag{6.17}$$

The middle crisp value becomes from Table (3-2):

$$\sum_{i=1}^{NG}\left(\frac{\partial \bar{P}_{G_i}}{\partial \bar{\lambda}_i}\right)=\sum_{i=1}^{NG}\left[\frac{\bar{\gamma}_i+B_{ii}\bar{\beta}_i}{2\left(\bar{\gamma}_i+\bar{\lambda}_i^{[k]}B_{ii}\right)^2}\right] \tag{6.18}$$

The left side of the power generation becomes:

$$\sum_{i=1}^{NG}\left(\frac{\partial L_{\tilde{P}_{G_i}}}{\partial L_{\lambda i}}\right)=\sum_{i=1}^{NG}\left[\frac{L_{\tilde{\gamma}_i}+B_{ii}L_{\tilde{\beta}_i}}{2\left(R_{\tilde{\gamma}_i}+R_{\tilde{\lambda}}^{[k]}B_{ii}\right)^2}\right] \tag{6.19}$$

The right side of the power generation becomes:

$$\sum_{i=1}^{NG}\left(\frac{\partial R_{\tilde{P}_{G_i}}}{\partial R_{\tilde{\lambda}}}\right)=\sum_{i=1}^{NG}\left[\frac{R_{\tilde{\gamma}_i}+B_{ii}R_{\tilde{\beta}_i}}{2\left(L_{\tilde{\gamma}_i}+L_{\tilde{\lambda}}^{[k]}B_{ii}\right)^2}\right] \tag{6.20}$$

Since $\Delta\lambda^{(k)}$ denotes the increment of change in the incremental cost is equal to:

$$\Delta\lambda^{(k)}=\frac{\Delta\tilde{P}^{[k]}}{\sum\left(\frac{dP_{G_i}}{d\lambda}\right)^{[k]}} \tag{6.21}$$

Replacing the fuzzy parameters with their middle, left and right value into equation (6.21) we get:

$$\Delta(\bar{\lambda},L_{\tilde{\lambda}},R_{\tilde{\lambda}})^{(k)}=\frac{\Delta(\bar{P}_{G_i},L_{\tilde{P}_{G_i}},R_{\tilde{P}_{G_i}})^{[k]}}{\sum\left(\frac{d(\bar{P}_{G_i},L_{\tilde{P}_{G_i}},R_{\tilde{P}_{G_i}})_i}{d(\bar{\lambda},L_{\tilde{\lambda}},R_{\tilde{\lambda}})}\right)^{[k]}} \tag{6.22}$$

The middle or crisp value from Table (3-2) will be:

$$\Delta(\bar{\lambda}_i) = \frac{\Delta(\bar{P}_{G_i})}{\sum_{i=1}^{NG}\left[\dfrac{\bar{\gamma}_i + B_{ii}\bar{\beta}_i}{2\left(\bar{\gamma}_i + \bar{\lambda}_i^{[k]}B_{ii}\right)^2}\right]} \tag{6.23}$$

The left side becomes:

$$\Delta(L_{\tilde{\lambda}_i}) = \frac{\Delta(L_{\tilde{P}_{G_i}})}{\sum_{i=1}^{NG}\left[\dfrac{R_{\tilde{\gamma}_i} + B_{ii}R_{\tilde{\beta}_i}}{2\left(L_{\tilde{\gamma}_i} + L_{\tilde{\lambda}}^{[k]}B_{ii}\right)^2}\right]} \tag{6.24}$$

The right side becomes:

$$\Delta(R_{\tilde{\lambda}_i}) = \frac{\Delta(R_{\tilde{P}_{G_i}})}{\sum_{i=1}^{NG}\left[\dfrac{L_{\tilde{\gamma}_i} + B_{ii}L_{\tilde{\beta}_i}}{2\left(R_{\tilde{\gamma}_i} + R_{\tilde{\lambda}}^{[k]}B_{ii}\right)^2}\right]} \tag{6.25}$$

Then calculate the new value of the incremental cost

$$\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)} \tag{6.26}$$

Substituting the middle, left and right sides into equation (6.26) we get:

$$(\bar{\lambda}, L_{\tilde{\lambda}}, R_{\tilde{\lambda}})^{(k+1)} = (\bar{\lambda}, L_{\tilde{\lambda}}, R_{\tilde{\lambda}})^{(k)} + \Delta(\bar{\lambda}, L_{\tilde{\lambda}}, R_{\tilde{\lambda}})^{(k)} \tag{6.27}$$

If the value of $\Delta\lambda^{(k)}$ is very small such as 10^{-3} in a per unit system then the iteration is stopped and the power generation, the power losses and the total cost of all units are calculated. If it is not small then the iteration continues until a convergence is achieved.

6.3 Solution Algorithm

The iterative technique is used with a complete (ED) problem when the power losses are included into the system to find the optimal solution. In this method the initial guess of the incremental cost can be calculated for the middle, left and right side from (6.16), (6.17) and (6.18) assuming that the power losses are small and can be ignored then the iterative method will find the best equal incremental cost value. If this value does not satisfies the optimality condition then the iterative program repeats the process until a solution is found. The power generation equation has to be modified as explained in Chapter (2) to take into account the power losses in the network when power losse are no longer neglected and contribute to the system performance. The same simulated example of Chapter (5) is used to calculate the optimal minimum cost values of the three units committed to the network. The B_{ii} loss coefficients for this example are

$$B_{ii}(pu) = \begin{bmatrix} 0.0218 & 0 & 0 \\ 0 & 0.0228 & 0 \\ 0 & 0 & 0.0179 \end{bmatrix}.$$

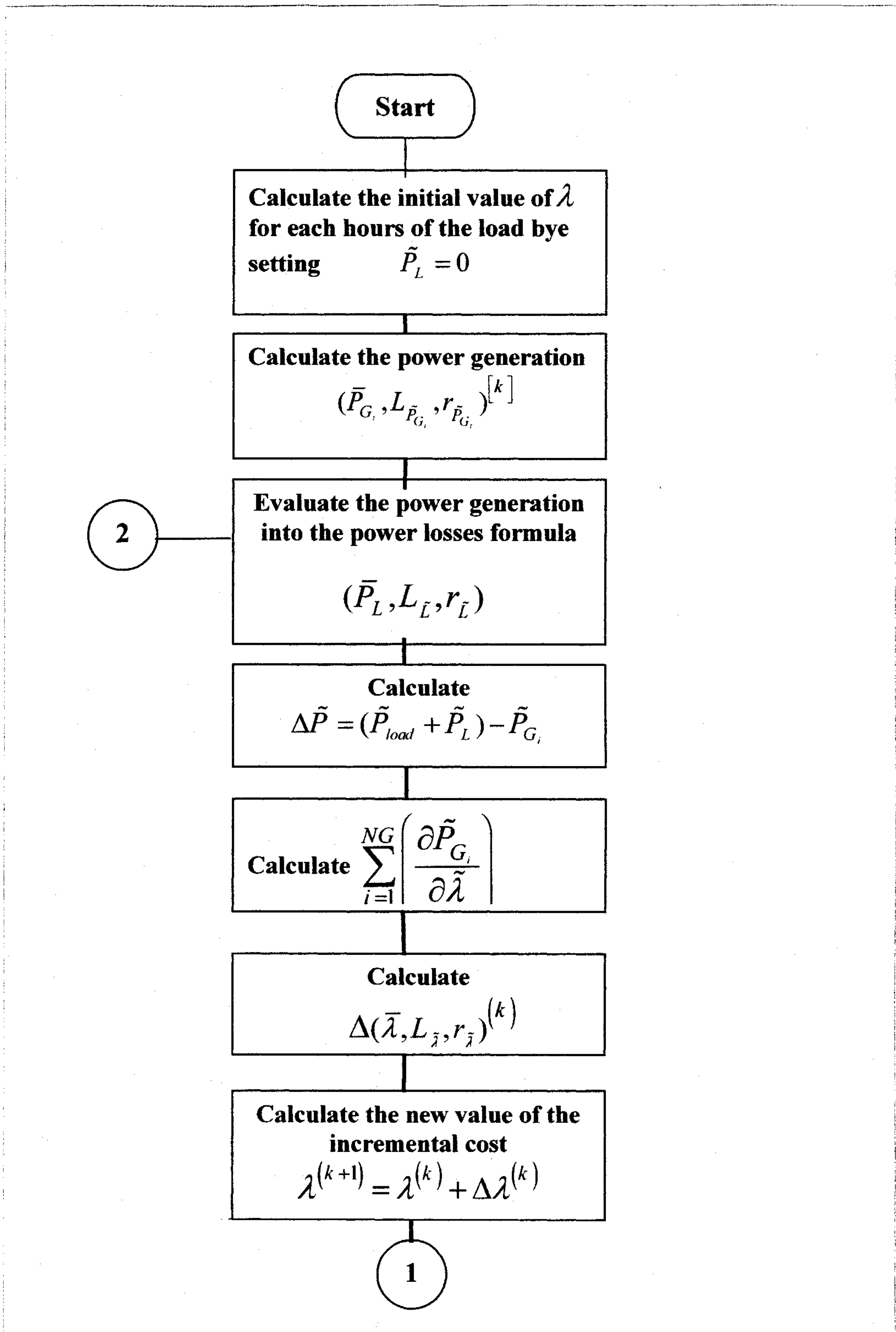
Evaluating the results obtained using the program based on the flow chart we can observe the following:

1. In the example presented, the optimal solution was found after 8 to 10 iterations for each fuzzy load for 24 hours.
2. Table (6-1) and Figure (6-1) represent the fuzzy load demand with 10% deviation which is the same load that was tested in the previous Chapter.
3. Table (6-2) and Figure (6-2) show the incremental fuel cost of the three generators. The minimum range of the fuel incremental cost is 10.5 and the maximum is 14.2 which is the same range of variation obtained in the previous Chapter. The generators will increase the generation to compensate the power losses and distribute the load demand, the power losses evenly between the generation units to maintain their equal incremental fuel cost range. In addition satisfy the constraint imposed on the system.
4. Table (6-2, 6-3, 6-4) and Figure (6-3, 6-4, 6-5) represent the power generations of unit 1, 2 and 3. The power generations spread of the three units shows almost the

same spread but with different values of middle, left and right side of the generation triangular membership function. This means that the power losses formula represented in equation (6.9 - 6.11) the square value of the power generator multiplied by the B_{ii} loss coefficients which is a small value reduced the left and right spread of the power generation and the incremental fuel cost in the iterative procedure to find the best value of the incremental fuel cost that satisfy the optimality condition.

5. Table (6-6) and Figure (6-6) show the total power generation of all the units for all α -cut representation. The total power generation satisfies the constraints set to obtain a minimum solution of the objective function in equation (6.2). Comparing Figure (5-7) from the previous Chapter and Figure (6-6) for the same deviation for α, β and γ parameters we see an increase in the total generation in Figure (6-6) to compensate the power losses of the transmission line as stated in equation (6.2).
6. Table (6-7) and Figure (6-7) show the power losses for all α -cut representation. The power losses have higher spread than the power generation for each unit, the load demand and the total power generation. This in fact is a great asset to the system operator to know all this information about the system variation on line and hour by hours.
7. Table (6-8) and Figure (6-8) show the minimum total cost for all α -cut representation. Comparing Figure (5-8) from the previous Chapter with Figure (6-8) we see an increase of cost do to considering the power losses in the formulation. This proves that when considering power losses the overall economy of the system will be affected including the upper and lower limits of the minimum cost value. The extra cost value is a result of increased power generation to balance the equality constraint set in equation (6.2) and compensate the power losses in the transmission line. In addition the spread at α -cut equal to zero has improved in Figure (6-8) when we consider losses in the formulation which means that when we consider the losses we obtain a more realistic model and limits the over approximation of the spread in the membership function

8. Table (6-9) and Figure (6-10) represents the minimum total cost for 10% deviation for the cost function α, β and γ. The cost value has increased sharply and the spread at α -cut equal to zero has increased with the increased of the cost function coefficients. Table (6-10) and Figure (6-11) show the minimum total cost for all α -cut representation for a selected 2% deviation for α and 3% deviation for β and γ. The figure shows that cost has reduced considerably with decreasing in cost function coefficients and the spread at α -cut equal to zero has reduced as well which is the nature of the quadratic equation of the cost function.



Start
Calculate the initial value of λ for each hours of the load bye setting $\tilde{P}_L = 0$
Calculate the power generation $(\bar{P}_{G_i}, L_{\tilde{P}_{G_i}}, r_{\tilde{P}_{G_i}})^{[k]}$
2
Evaluate the power generation into the power losses formula $(\bar{P}_L, L_{\tilde{L}}, r_{\tilde{L}})$
Calculate $\Delta\tilde{P} = (\tilde{P}_{load} + \tilde{P}_L) - \tilde{P}_{G_i}$
Calculate $\sum_{i=1}^{NG}\left(\frac{\partial\tilde{P}_{G_i}}{\partial\tilde{\lambda}}\right)$
Calculate $\Delta(\bar{\lambda}, L_{\tilde{\lambda}}, r_{\tilde{\lambda}})^{(k)}$
Calculate the new value of the incremental cost $\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)}$
1

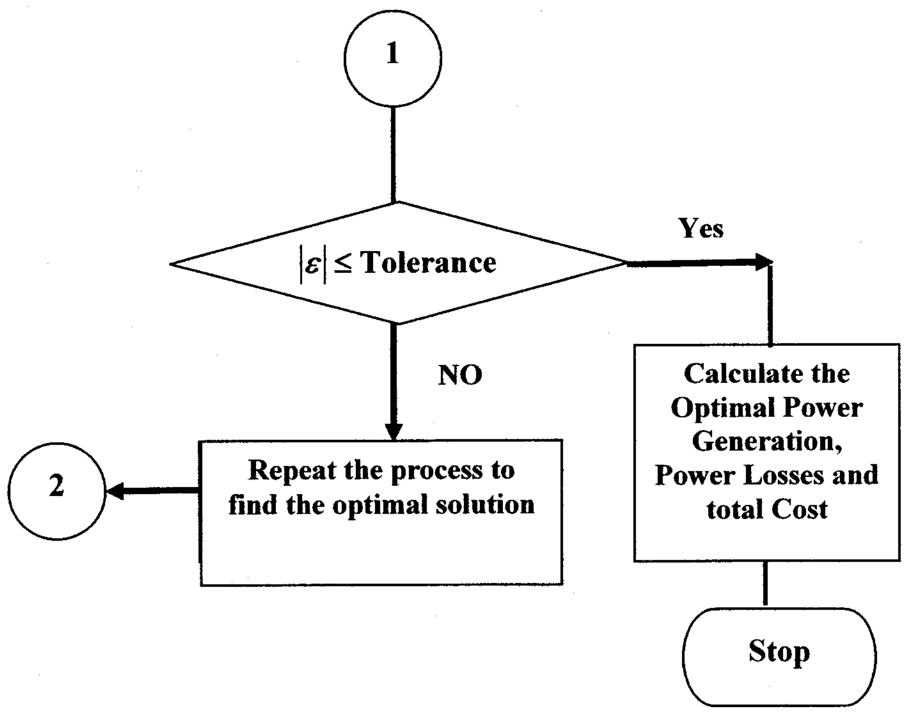


Chart (6-1) Iterative Flow-Chart of the Program

Table (6-1)

Membership Function of Load Demand for (0, 0.5, 0.75, 1) α -Cut Representation for Model "A" Weekdays With 10% Deviation for (P_D, α) and 3% for (β, γ)

Membership Function		$\mu_{P_{Load}} = 0$			$\mu_{P_{Load}} = 0.5$			$\mu_{P_{Load}} = 0.75$			$\mu_{P_{Load}} = 1$	
Daily Hours	Left Load MW	Mid Load MW	Right Load MW	Left Load MW	Mid Load MW	Right Load MW	Left Load MW	Mid Load MW	Right Load MW	Left Load MW	Mid Load MW	Right Load MW
1	1006	1118	1229	1062	1118	1173	1090	1118	1146	1118	1118	1118
2	905.8	1006	1107	956.1	1006	1057	981.2	1006	1032	1006	1006	1006
3	849.2	943.6	1038	896.4	943.6	990.8	920	943.6	967.2	943.6	943.6	943.6
4	784.1	871.2	958.3	827.6	871.2	914.7	849.4	871.2	892.9	871.2	871.2	871.2
5	731.7	813	894.3	772.4	813	853.7	792.7	813	833.3	813	813	813
6	782.7	869.7	956.7	826.2	869.7	913.2	848	869.7	891.4	869.7	869.7	869.7
7	823.3	914.8	1006	869.1	914.8	960.5	891.9	914.8	937.7	914.8	914.8	914.8
8	880.8	978.7	1077	929.8	978.7	1028	954.2	978.7	1003	978.7	978.7	978.7
9	1042	1157	1273	1099	1157	1215	1128	1157	1186	1157	1157	1157
10	1101	1224	1346	1163	1224	1285	1193	1224	1254	1224	1224	1224
11	1095	1217	1338	1156	1217	1278	1186	1217	1247	1217	1217	1217
12	1156	1284	1413	1220	1284	1349	1252	1284	1316	1284	1284	1284
13	1133	1259	1384	1196	1259	1322	1227	1259	1290	1259	1259	1259
14	1087	1208	1329	1147	1208	1268	1178	1208	1238	1208	1208	1208
15	1040	1155	1271	1098	1155	1213	1127	1155	1184	1155	1155	1155
16	999.5	1111	1222	1055	1111	1166	1083	1111	1138	1111	1111	1111
17	984.7	1094	1204	1039	1094	1149	1067	1094	1121	1094	1094	1094
18	1002	1113	1225	1058	1113	1169	1085	1113	1141	1113	1113	1113
19	1068	1186	1305	1127	1186	1246	1157	1186	1216	1186	1186	1186
20	1025	1139	1253	1082	1139	1196	1111	1139	1167	1139	1139	1139
21	1037	1152	1268	1095	1152	1210	1123	1152	1181	1152	1152	1152
22	1104	1227	1349	1165	1227	1288	1196	1227	1257	1227	1227	1227
23	1079	1198	1318	1138	1198	1258	1168	1198	1228	1198	1198	1198
24	1001	1112	1223	1056	1112	1167	1084	1112	1140	1112	1112	1112

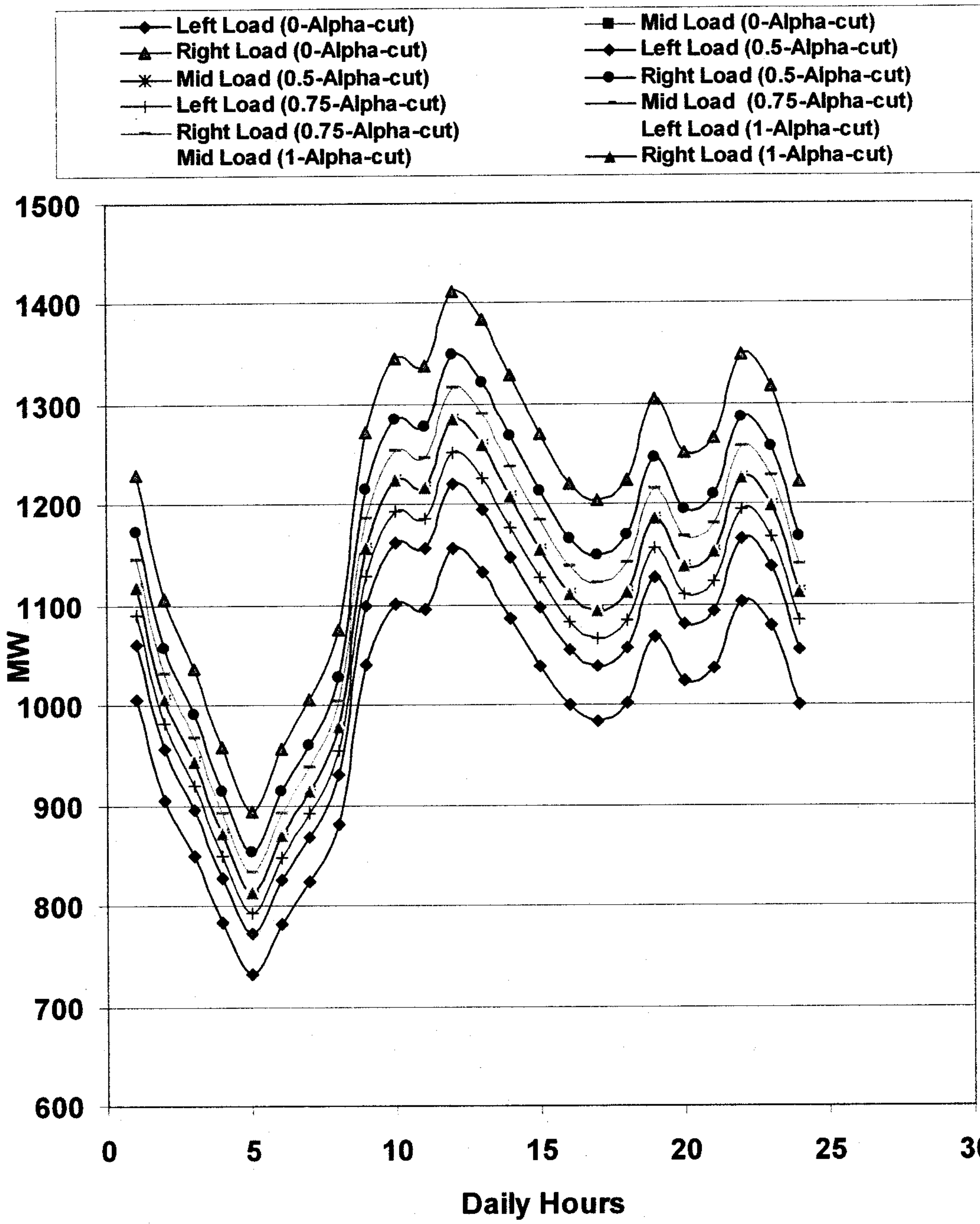


Figure (6-1) Fuzzy Load Demand for All α-Cut Representation

Table (6-2)

Membership Function of Incremental Cost for (0, 0.5, 0.75, 1) α -Cut Representation for Model "A" Weekdays With 10% Deviation for (P_D) and 3% for (α, β, γ)

Membership Function	$\mu_\lambda = 0$			$\mu_\lambda = 0.5$			$\mu_\lambda = 0.75$			$\mu_\lambda = 1$		
Daily Hours	Left λ $MW/h for (μ=0)	Mid λ $MW/h for (μ=0)	Right λ $MW/h for (μ=0)	Left λ $MW/h for (μ=0.5)	Mid λ $MW/h for (μ=0.5)	Right λ $MW/h for (μ=0.5)	Left λ $MW/h for (μ=0.75)	Mid λ $MW/h for (μ=0.75)	Right λ $MW/h for (μ=0.75)	Left λ $MW/h for (μ=1)	Mid λ $MW/h for (μ=1)	Right λ $MW/h for (μ=1)
1	11.99	12.62	13.214	12.31	12.62	12.92	12.463	12.617	12.77	12.62	12.62	12.617
2	11.44	12.03	12.588	11.75	12.03	12.305	11.895	12.031	12.166	12.03	12.03	12.031
3	11.13	11.7	12.234	11.44	11.7	11.954	11.574	11.699	11.824	11.7	11.7	11.699
4	10.78	11.32	11.827	11.08	11.32	11.55	11.202	11.317	11.432	11.32	11.32	11.317
5	10.5	11.01	11.499	10.79	11.01	11.227	10.903	11.01	11.117	11.01	11.01	11.01
6	10.77	11.31	11.818	11.08	11.31	11.54	11.194	11.309	11.424	11.31	11.31	11.309
7	10.99	11.55	12.072	11.31	11.55	11.789	11.427	11.547	11.668	11.55	11.55	11.547
8	11.31	11.88	12.432	11.63	11.88	12.143	11.755	11.884	12.014	11.88	11.88	11.884
9	12.18	12.83	13.438	12.52	12.83	13.132	12.674	12.827	12.98	12.83	12.83	12.827
10	12.51	13.18	13.812	12.86	13.18	13.501	13.016	13.178	13.339	13.18	13.18	13.178
11	12.47	13.14	13.773	12.82	13.14	13.462	12.98	13.141	13.302	13.14	13.14	13.141
12	12.8	13.5	14.153	13.16	13.5	13.836	13.328	13.497	13.667	13.5	13.5	13.497
13	12.68	13.36	14.008	13.03	13.36	13.694	13.196	13.362	13.528	13.36	13.36	13.362
14	12.43	13.09	13.722	12.78	13.09	13.412	12.934	13.094	13.253	13.09	13.09	13.094
15	12.17	12.82	13.427	12.51	12.82	13.122	12.665	12.817	12.969	12.82	12.82	12.817
16	11.95	12.58	13.175	12.29	12.58	12.874	12.434	12.581	12.727	12.58	12.58	12.581
17	11.87	12.49	13.082	12.21	12.49	12.782	12.349	12.493	12.638	12.49	12.49	12.493
18	11.96	12.6	13.19	12.3	12.6	12.889	12.448	12.595	12.742	12.6	12.6	12.595
19	12.32	12.98	13.602	12.67	12.98	13.294	12.824	12.981	13.137	12.98	12.98	12.981
20	12.09	12.73	13.335	12.43	12.73	13.031	12.58	12.73	12.881	12.73	12.73	12.73
21	12.16	12.8	13.41	12.5	12.8	13.105	12.649	12.801	12.953	12.8	12.8	12.801
22	12.52	13.19	13.827	12.87	13.19	13.516	13.03	13.192	13.354	13.19	13.19	13.192
23	12.38	13.04	13.669	12.73	13.04	13.36	12.886	13.044	13.202	13.04	13.04	13.044
24	11.96	12.59	13.182	12.29	12.59	12.88	12.44	12.587	12.734	12.59	12.59	12.587

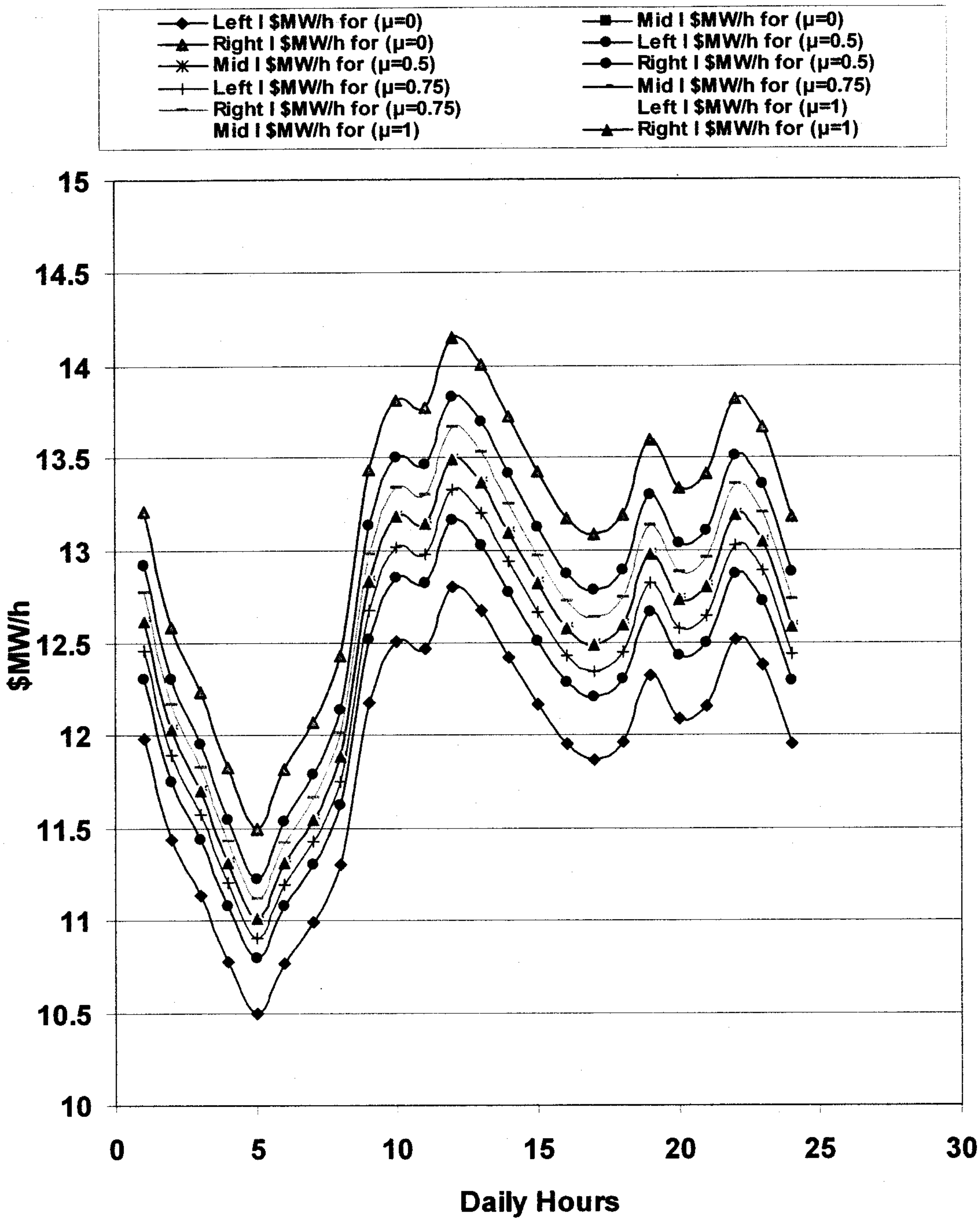


Figure (6-2) Fuzzy Incremental Fuel Cost for All α-Cut Representation

Table (6-3)

Membership Function of Generator #1 for (0, 0.5, 0.75, 1) α -Cut Representation for Model "A" Weekdays With 10% Deviation for (P_D) and 3% for (α,β,γ)

Membership Function	$\mu_{PG1}=0$			$\mu_{PG1}=0.5$			$\mu_{PG1}=0.75$			$\mu_{PG1}=1$		
Daily Hours	Left PG1 MW	Mid PG1 MW	Right PG1 MW	Left PG1 MW	Mid PG1 MW	Right PG1 MW	Left PG1 MW	Mid PG1 MW	Right PG1 MW	Left PG1 MW	Mid PG1 MW	Right PG1 MW
1	341.4	384.6	428.8	362.9	384.6	406.6	373.7	384.6	395.6	384.6	384.6	384.6
2	303.4	341.6	380.5	322.5	341.6	361	332	341.6	351.3	341.6	341.6	341.6
3	282.2	317.7	353.7	299.9	317.7	335.6	308.8	317.7	326.6	317.7	317.7	317.7
4	258.1	290.4	323.2	274.2	290.4	306.7	282.3	290.4	298.6	290.4	290.4	290.4
5	238.8	268.8	299	253.8	268.8	283.8	261.3	268.8	276.3	268.8	268.8	268.8
6	257.6	289.9	322.6	273.7	289.9	306.2	281.8	289.9	298	289.9	289.9	289.9
7	272.6	306.8	341.5	289.7	306.8	324.1	298.2	306.8	315.4	306.8	306.8	306.8
8	294.1	331	368.7	312.5	331	349.8	321.7	331	340.4	331	331	331
9	355.1	400.2	446.4	377.5	400.2	423.2	388.8	400.2	411.7	400.2	400.2	400.2
10	378.3	426.6	476.1	402.3	426.6	451.2	414.4	426.6	438.8	426.6	426.6	426.6
11	375.8	423.8	472.9	399.7	423.8	448.2	411.7	423.8	435.9	423.8	423.8	423.8
12	399.6	450.9	503.5	425.1	450.9	477	437.9	450.9	463.9	450.9	450.9	450.9
13	390.5	440.5	491.8	415.3	440.5	466	427.9	440.5	453.2	440.5	440.5	440.5
14	372.7	420.2	468.9	396.3	420.2	444.4	408.2	420.2	432.2	420.2	420.2	420.2
15	354.4	399.5	445.5	376.8	399.5	422.4	388.1	399.5	410.9	399.5	399.5	399.5
16	339	381.9	425.8	360.3	381.9	403.7	371.1	381.9	392.8	381.9	381.9	381.9
17	333.3	375.5	418.5	354.3	375.5	396.9	364.9	375.5	386.2	375.5	375.5	375.5
18	339.9	383	426.9	361.3	383	404.8	372.1	383	393.9	383	383	383
19	365.2	411.7	459.3	388.3	411.7	435.4	400	411.7	423.5	411.7	411.7	411.7
20	348.8	393	438.3	370.8	393	415.5	381.9	393	404.2	393	393	393
21	353.4	398.2	444.1	375.7	398.2	421.1	386.9	398.2	409.6	398.2	398.2	398.2
22	379.2	427.6	477.3	403.3	427.6	452.3	415.4	427.6	439.9	427.6	427.6	427.6
23	369.4	416.5	464.7	392.8	416.5	440.4	404.6	416.5	428.4	416.5	416.5	416.5
24	339.4	382.4	426.3	360.8	382.4	404.2	371.6	382.4	393.3	382.4	382.4	382.4

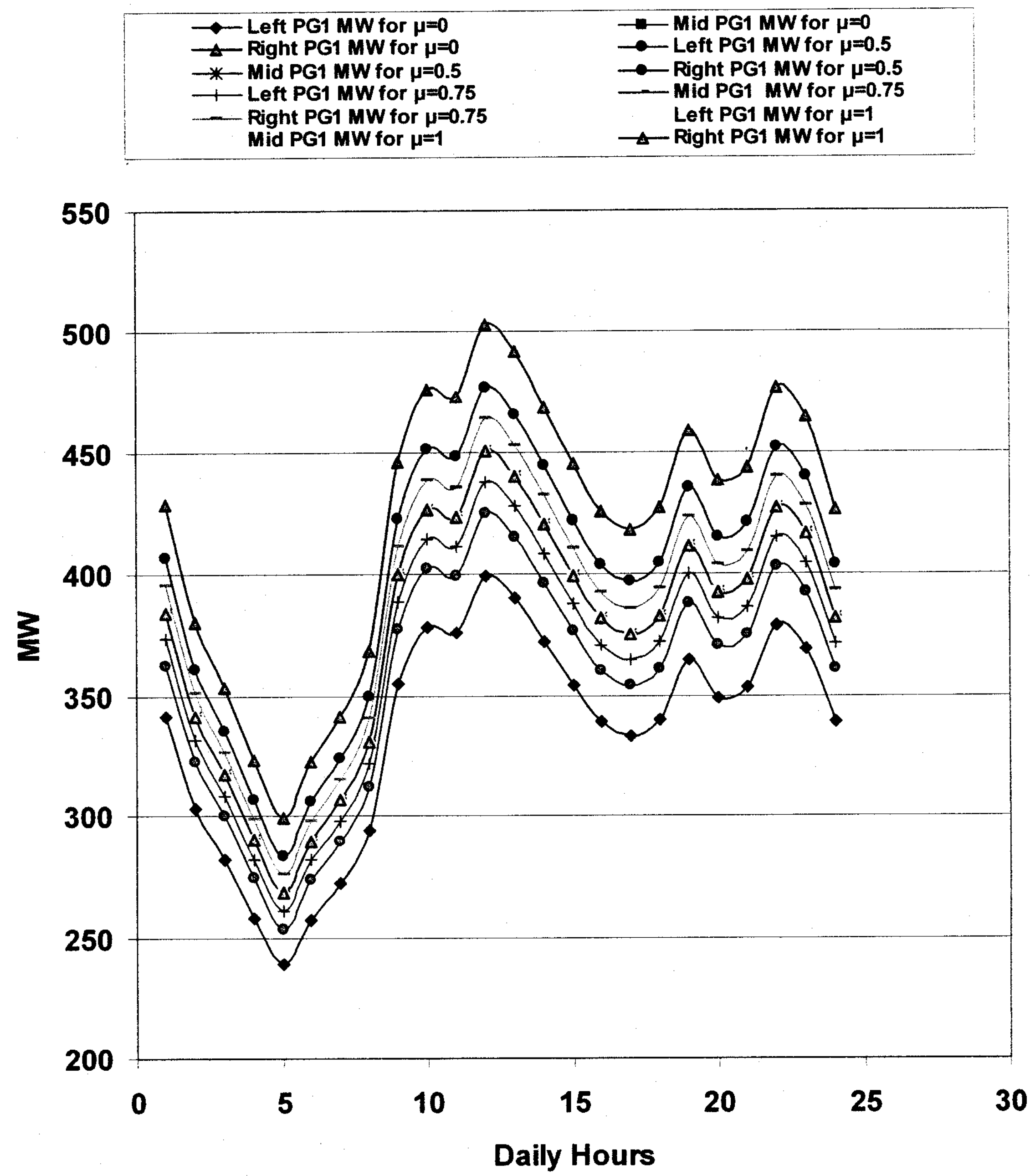


Figure (6-3) Fuzzy Power Generation of Unit #1 for All α-Cut Representation

Table (6-4)

Membership Function of Generator #2 for (0, 0.5, 0.75, 1) α -Cut Representation for Model "A" Weekdays With 10% Deviation for (P_D) and 3% for (α,β,γ)

Membership Function		$\mu_{P_{G2}}=0$			$\mu_{P_{G2}}=0.5$			$\mu_{P_{G2}}=0.75$			$\mu_{P_{G2}}=1$	
Daily Hours	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW
1	337.8	377.1	417.2	357.3	377.1	397	367.2	377.1	387	377.1	377.1	377.1
2	303.6	338.2	373.5	320.8	338.2	355.8	329.5	338.2	347	338.2	338.2	338.2
3	284.5	316.7	349.3	300.5	316.7	333	308.6	316.7	324.8	316.7	316.7	316.7
4	262.9	292.1	321.8	277.4	292.1	307	284.7	292.1	299.6	292.1	292.1	292.1
5	245.6	272.7	300.1	259	272.7	286.4	265.9	272.7	279.5	272.7	272.7	272.7
6	262.4	291.6	321.3	276.9	291.6	306.5	284.3	291.6	299.1	291.6	291.6	291.6
7	275.9	306.9	338.3	291.3	306.9	322.6	299	306.9	314.7	306.9	306.9	306.9
8	295.2	328.7	362.8	311.8	328.7	345.8	320.2	328.7	337.2	328.7	328.7	328.7
9	350.2	391.1	433.1	370.5	391.1	412.1	380.8	391.1	401.6	391.1	391.1	391.1
10	371.1	415	460	392.8	415	437.4	403.9	415	426.2	415	415	415
11	368.9	412.5	457.1	390.5	412.5	434.7	401.4	412.5	423.6	412.5	412.5	412.5
12	390.4	437	484.9	413.5	437	460.8	425.2	437	448.9	437	437	437
13	382.1	427.6	474.3	404.7	427.6	450.8	416.1	427.6	439.2	427.6	427.6	427.6
14	366	409.2	453.5	387.4	409.2	431.3	398.3	409.2	420.2	409.2	409.2	409.2
15	349.6	390.5	432.3	369.8	390.5	411.3	380.1	390.5	400.9	390.5	390.5	390.5
16	335.6	374.6	414.4	354.9	374.6	394.5	364.7	374.6	384.5	374.6	374.6	374.6
17	330.5	368.8	407.9	349.5	368.8	388.3	359.1	368.8	378.5	368.8	368.8	368.8
18	336.5	375.6	415.5	355.8	375.6	395.5	365.7	375.6	385.5	375.6	375.6	375.6
19	359.3	401.5	444.8	380.2	401.5	423.1	390.8	401.5	412.3	401.5	401.5	401.5
20	344.4	384.6	425.7	364.4	384.6	405.1	374.5	384.6	394.9	384.6	384.6	384.6
21	348.6	389.4	431.1	368.8	389.4	410.2	379.1	389.4	399.7	389.4	389.4	389.4
22	371.9	416	461.1	393.7	416	438.4	404.8	416	427.2	416	416	416
23	363.1	405.8	449.7	384.3	405.8	427.7	395	405.8	416.7	405.8	405.8	405.8
24	336	375	414.9	355.3	375	394.9	365.2	375	384.9	375	375	375

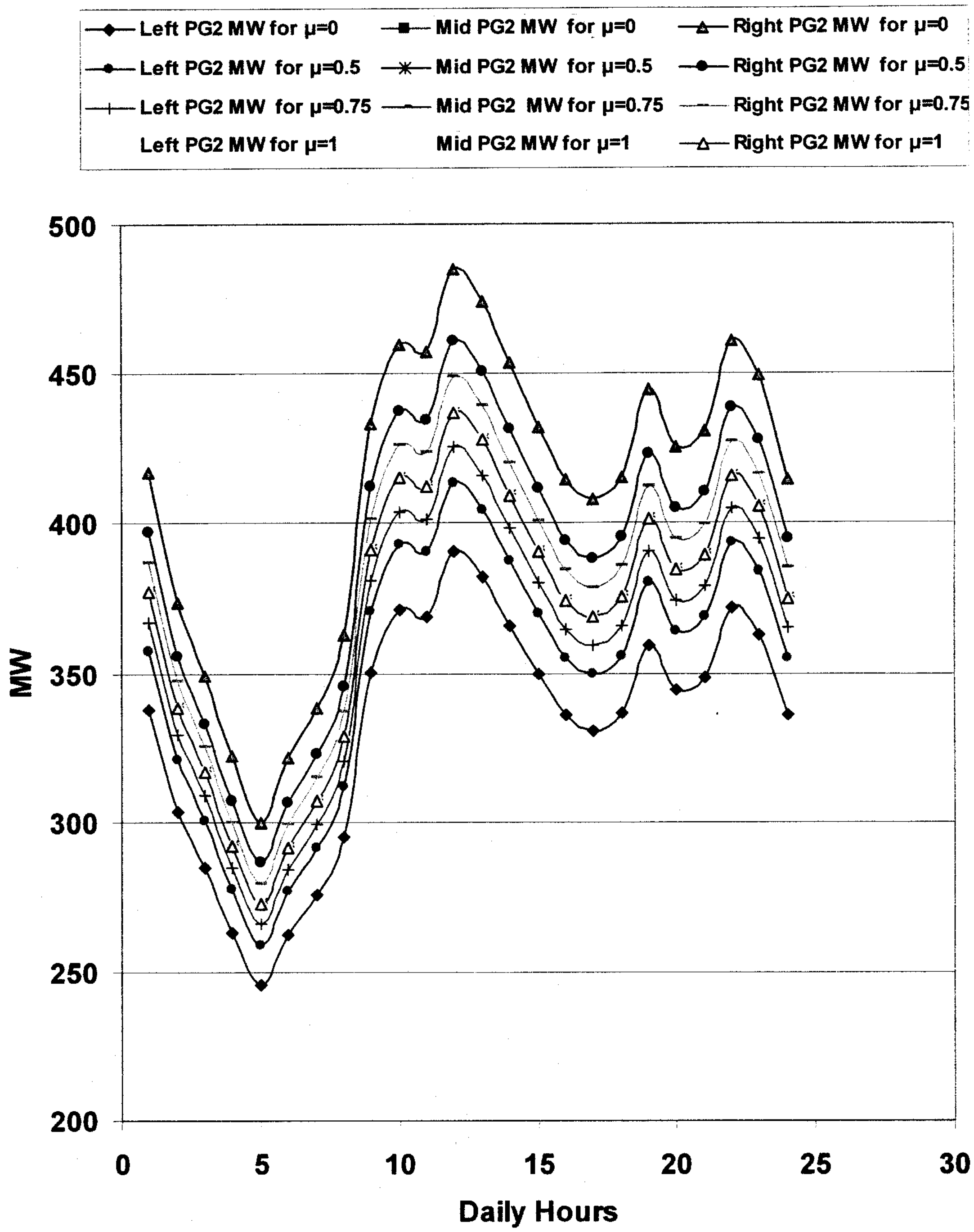


Figure (6-4) Fuzzy Power Generation of Unit #2 for All α -Cut Representation

Table (6-5)

Membership Function of Generator #3 for (0, 0.5, 0.75, 1) α -Cut Representation for Model "A" Weekdays With 10% Deviation for (P_D) and 3% for (α, β, γ)

Membership Function		$\mu_{P_{G3}}=0$			$\mu_{P_{G3}}=0.5$			$\mu_{P_{G3}}=0.75$			$\mu_{P_{G3}}=1$	
Daily Hours	Left PG3 MW	Mid PG3 MW	Right PG3 MW	Left PG3 MW	Mid PG3 MW	Right PG3 MW	Left PG3 MW	Mid PG3 MW	Right PG3 MW	Left PG3 MW	Mid PG3 MW	Right PG3 MW
1	407.7	458.1	509.5	432.8	458.1	483.7	445.4	458.1	470.8	458.1	458.1	458.1
2	363.4	407.8	453.1	385.5	407.8	430.3	396.6	407.8	419	407.8	407.8	407.8
3	338.6	379.9	421.7	359.2	379.9	400.7	369.5	379.9	390.2	379.9	379.9	379.9
4	310.5	348.1	386.2	329.3	348.1	367	338.7	348.1	357.5	348.1	348.1	348.1
5	288.1	322.9	358.1	305.5	322.9	340.4	314.2	322.9	331.6	322.9	322.9	322.9
6	309.9	347.4	385.5	328.7	347.4	366.3	338.1	347.4	356.9	347.4	347.4	347.4
7	327.4	367.2	407.5	347.3	367.2	387.2	357.2	367.2	377.2	367.2	367.2	367.2
8	352.5	395.5	439.2	373.9	395.5	417.2	384.7	395.5	406.3	395.5	395.5	395.5
9	423.8	476.3	530	450	476.3	502.9	463.1	476.3	489.6	476.3	476.3	476.3
10	450.9	507.1	564.8	478.9	507.1	535.7	493	507.1	521.4	507.1	507.1	507.1
11	448	503.9	561.1	475.9	503.9	532.2	489.8	503.9	518	503.9	503.9	503.9
12	475.9	535.6	597	505.7	535.6	566	520.6	535.6	550.8	535.6	535.6	535.6
13	465.2	523.5	583.2	494.2	523.5	553.1	508.8	523.5	538.2	523.5	523.5	523.5
14	444.3	499.7	556.4	471.9	499.7	527.8	485.8	499.7	513.7	499.7	499.7	499.7
15	423	475.4	529	449.1	475.4	502	462.2	475.4	488.7	475.4	475.4	475.4
16	404.9	454.9	505.9	429.8	454.9	480.2	442.3	454.9	467.5	454.9	454.9	454.9
17	398.3	447.4	497.5	422.8	447.4	472.2	435	447.4	459.7	447.4	447.4	447.4
18	406	456.1	507.3	431	456.1	481.5	443.5	456.1	468.8	456.1	456.1	456.1
19	435.6	489.7	545.2	462.6	489.7	517.2	476.1	489.7	503.4	489.7	489.7	489.7
20	416.3	467.9	520.5	442.1	467.9	494	454.9	467.9	480.9	467.9	467.9	467.9
21	421.7	474	527.4	447.8	474	500.5	460.9	474	487.2	474	474	474
22	452	508.4	566.2	480.1	508.4	537	494.2	508.4	522.7	508.4	508.4	508.4
23	440.5	495.3	551.5	467.8	495.3	523.1	481.5	495.3	509.2	495.3	495.3	495.3
24	405.4	455.4	506.5	430.4	455.4	480.8	442.9	455.4	468.1	455.4	455.4	455.4

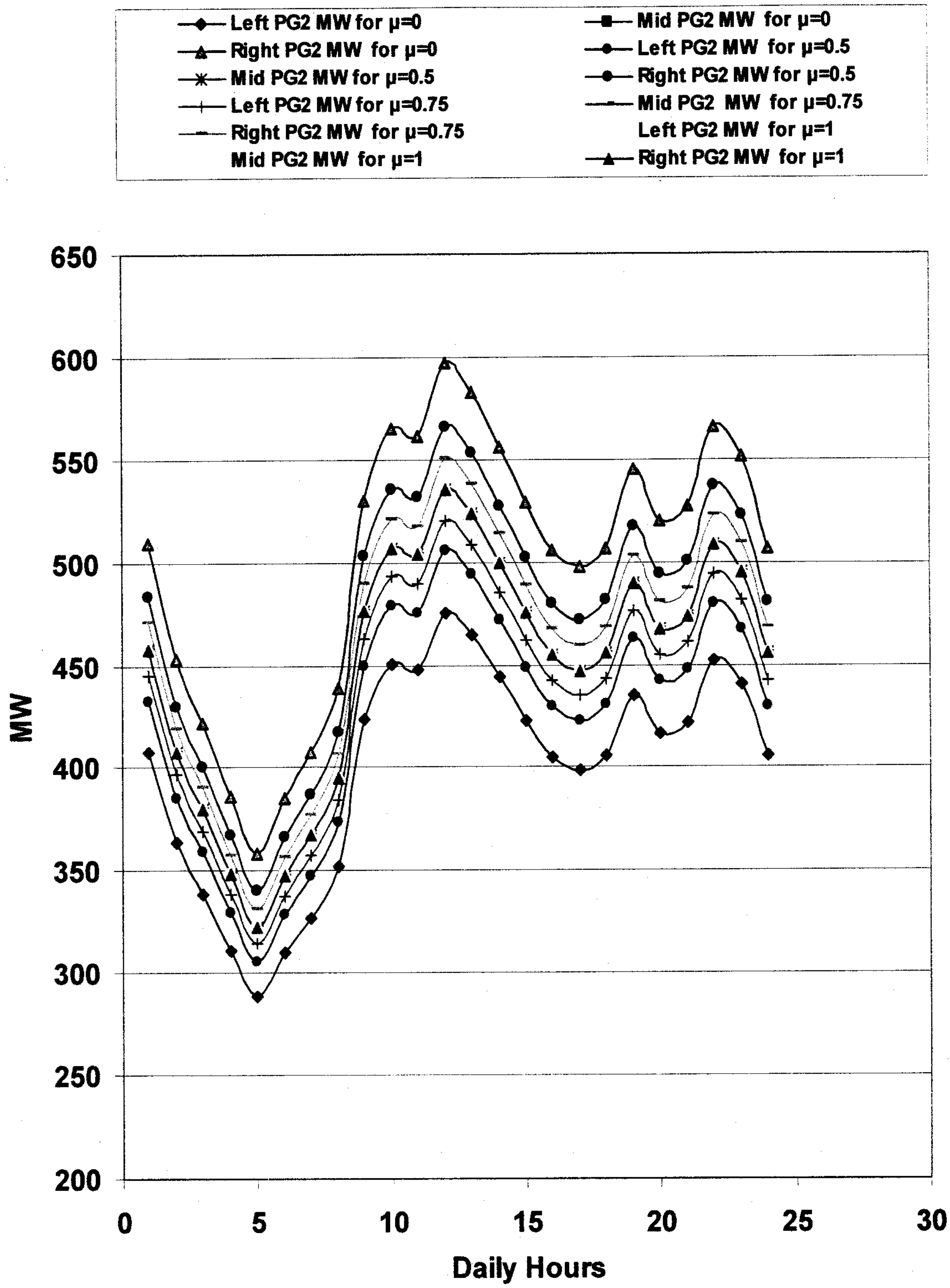


Figure (6-5) Fuzzy Power Generation of Unit #3 for All α-Cut Representation

Table (6-6)

Membership Function of Total P_G for (0, 0.5, 0.75, 1) α -Cut Representation

For Model "A" Weekdays With 10% Deviation for (P_D) and 3% for (α, β, γ)

Membership Function		$\mu_{tP_G}=0$			$\mu_{tP_G}=0.5$			$\mu_{tP_G}=0.75$			$\mu_{tP_G}=1$	
Daily Hours	Left tPG MW	Mid tPG MW	Right tPG MW	Left tPG MW	Mid tPG MW	Right tPG MW	Left tPG MW	Mid tPG MW	Right tPG MW	Left tPG MW	Mid tPG MW	Right tPG MW
1	1087	1220	1356	1153	1220	1287	1186	1220	1253	1220	1220	1220
2	970.4	1088	1207	1029	1088	1147	1058	1088	1117	1088	1088	1088
3	905.3	1014	1125	959.6	1014	1069	986.8	1014	1042	1014	1014	1014
4	831.4	930.6	1031	880.9	930.6	980.8	905.7	930.6	955.7	930.6	930.6	930.6
5	772.6	864.3	957.2	818.4	864.3	910.6	841.3	864.3	887.4	864.3	864.3	864.3
6	829.9	929	1029	879.3	929	979	904.1	929	953.9	929	929	929
7	875.8	980.8	1087	928.2	980.8	1034	954.5	980.8	1007	980.8	980.8	980.8
8	941.7	1055	1171	998.2	1055	1113	1027	1055	1084	1055	1055	1055
9	1129	1268	1409	1198	1268	1338	1233	1268	1303	1268	1268	1268
10	1200	1349	1501	1274	1349	1424	1311	1349	1386	1349	1349	1349
11	1193	1340	1491	1266	1340	1415	1303	1340	1378	1340	1340	1340
12	1266	1423	1585	1344	1423	1504	1384	1423	1464	1423	1423	1423
13	1238	1392	1549	1314	1392	1470	1353	1392	1431	1392	1392	1392
14	1183	1329	1479	1256	1329	1403	1292	1329	1366	1329	1329	1329
15	1127	1265	1407	1196	1265	1336	1230	1265	1300	1265	1265	1265
16	1080	1211	1346	1145	1211	1278	1178	1211	1245	1211	1211	1211
17	1062	1192	1324	1127	1192	1257	1159	1192	1224	1192	1192	1192
18	1082	1215	1350	1148	1215	1282	1181	1215	1248	1215	1215	1215
19	1160	1303	1449	1231	1303	1376	1267	1303	1339	1303	1303	1303
20	1110	1246	1385	1177	1246	1315	1211	1246	1280	1246	1246	1246
21	1124	1262	1403	1192	1262	1332	1227	1262	1297	1262	1262	1262
22	1203	1352	1505	1277	1352	1428	1314	1352	1390	1352	1352	1352
23	1173	1318	1466	1245	1318	1391	1281	1318	1354	1318	1318	1318
24	1081	1213	1348	1146	1213	1280	1180	1213	1246	1213	1213	1213

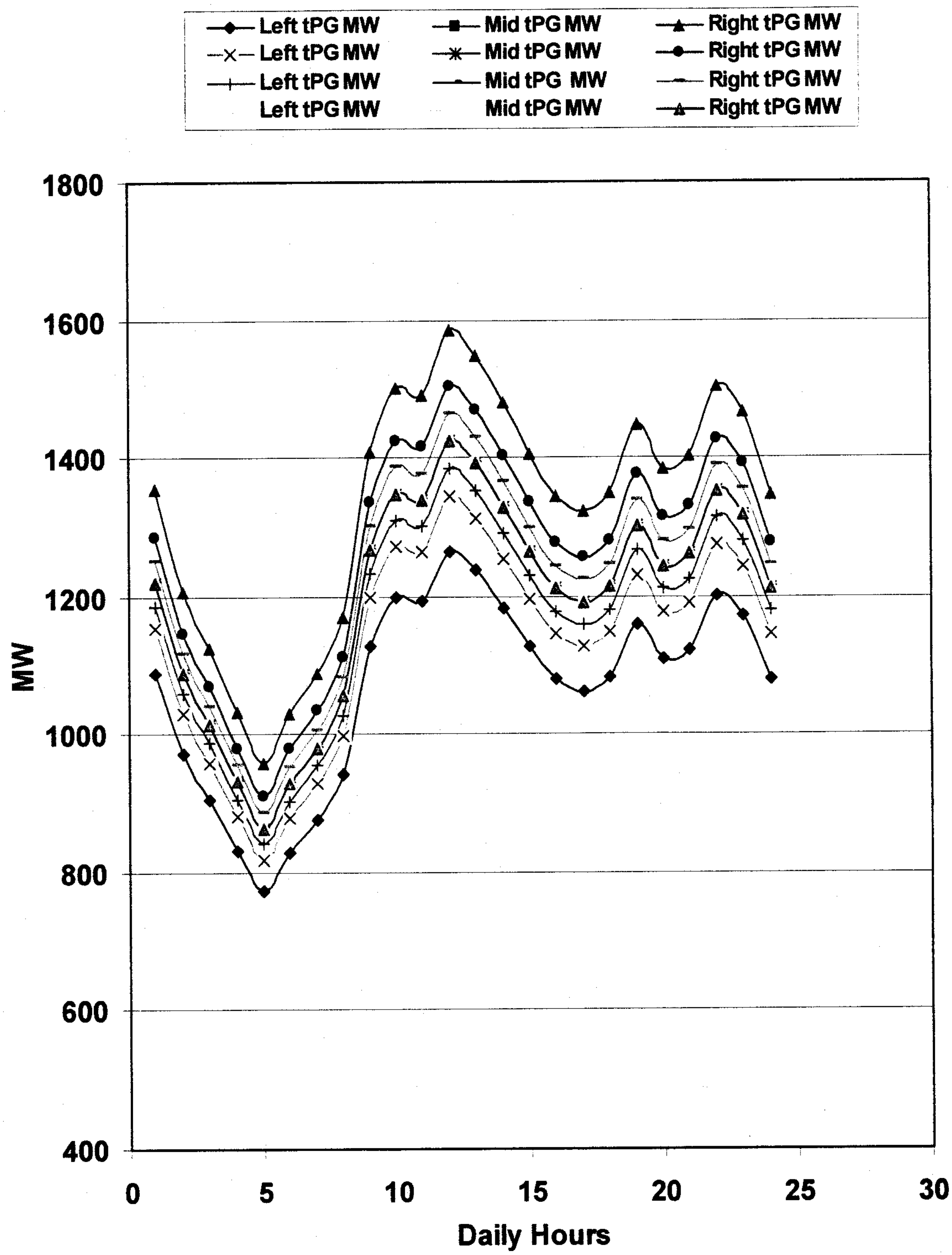


Figure (6-6) Fuzzy Total Power Generation for All α -Cut Representation

Table (6-7)

Membership Function of the Power Losses for (0, 0.5, 0.75, 1) α -Cut Representation for Model "A" Weekdays With 10% Deviation for (P_D) and 3% for (α, β, γ)

Membership Function		$\mu_{P_L} = 0$			$\mu_{P_L} = 0.5$			$\mu_{P_L} = 0.75$			$\mu_{P_L} = 1$	
Daily Hours	Left PL	Mid PL	Right PL	Left PL	Mid PL	Right PL	Left PL	Mid PL	Right PL	Left PL	Mid PL	Right PL
1	81.18	102.2	126.2	91.35	102.2	113.85	96.7	102.2	107.95	102.2	102.2	102.23
2	64.72	81.3	100.1	72.74	81.3	90.42	76.95	81.3	85.786	81.3	81.3	81.296
3	56.34	70.69	86.92	63.29	70.69	78.567	66.93	70.69	74.568	70.69	70.69	70.69
4	47.53	59.53	73.09	53.34	59.53	66.114	56.39	59.53	62.774	59.53	59.53	59.533
5	41.05	51.36	62.97	46.05	51.36	57.002	48.66	51.36	54.139	51.36	51.36	51.36
6	47.36	59.32	72.82	53.15	59.32	65.875	56.19	59.32	62.547	59.32	59.32	59.318
7	52.74	66.12	81.25	59.22	66.12	73.466	62.62	66.12	69.737	66.12	66.12	66.121
8	60.96	76.52	94.18	68.48	76.52	85.078	72.44	76.52	80.731	76.52	76.52	76.517
9	87.59	110.4	136.5	98.61	110.4	123.02	104.4	110.4	116.61	110.4	110.4	110.41
10	98.97	125	154.7	111.5	125	139.36	118.1	125	132.04	125	125	124.97
11	97.74	123.4	152.8	110.1	123.4	137.58	116.6	123.4	130.36	123.4	123.4	123.38
12	110.1	139.2	172.7	124.1	139.2	155.37	131.5	139.2	147.15	139.2	139.2	139.21
13	105.3	133	164.9	118.7	133	148.42	125.7	133	140.6	133	133	133.03
14	96.16	121.4	150.2	108.3	121.4	135.32	114.7	121.4	128.22	121.4	121.4	121.36
15	87.27	110	136	98.25	110	122.57	104	110	116.19	110	110	110.01
16	80.08	100.8	124.5	90.1	100.8	112.28	95.38	100.8	106.46	100.8	100.8	100.83
17	77.52	97.57	120.4	87.21	97.57	108.63	92.3	97.57	103.01	97.57	97.57	97.567
18	80.5	101.4	125.2	90.58	101.4	112.89	95.89	101.4	107.03	101.4	101.4	101.37
19	92.47	116.6	144.3	104.1	116.6	130.02	110.3	116.6	123.22	116.6	116.6	116.64
20	84.6	106.6	131.7	95.22	106.6	118.74	100.8	106.6	112.57	106.6	106.6	106.59
21	86.76	109.4	135.2	97.68	109.4	121.84	103.4	109.4	115.5	109.4	109.4	109.36
22	99.45	125.6	155.5	112.1	125.6	140.05	118.7	125.6	132.7	125.6	125.6	125.58
23	94.53	119.3	147.6	106.5	119.3	132.97	112.8	119.3	126.01	119.3	119.3	119.28
24	80.27	101.1	124.8	90.32	101.1	112.55	95.6	101.1	106.72	101.1	101.1	101.07

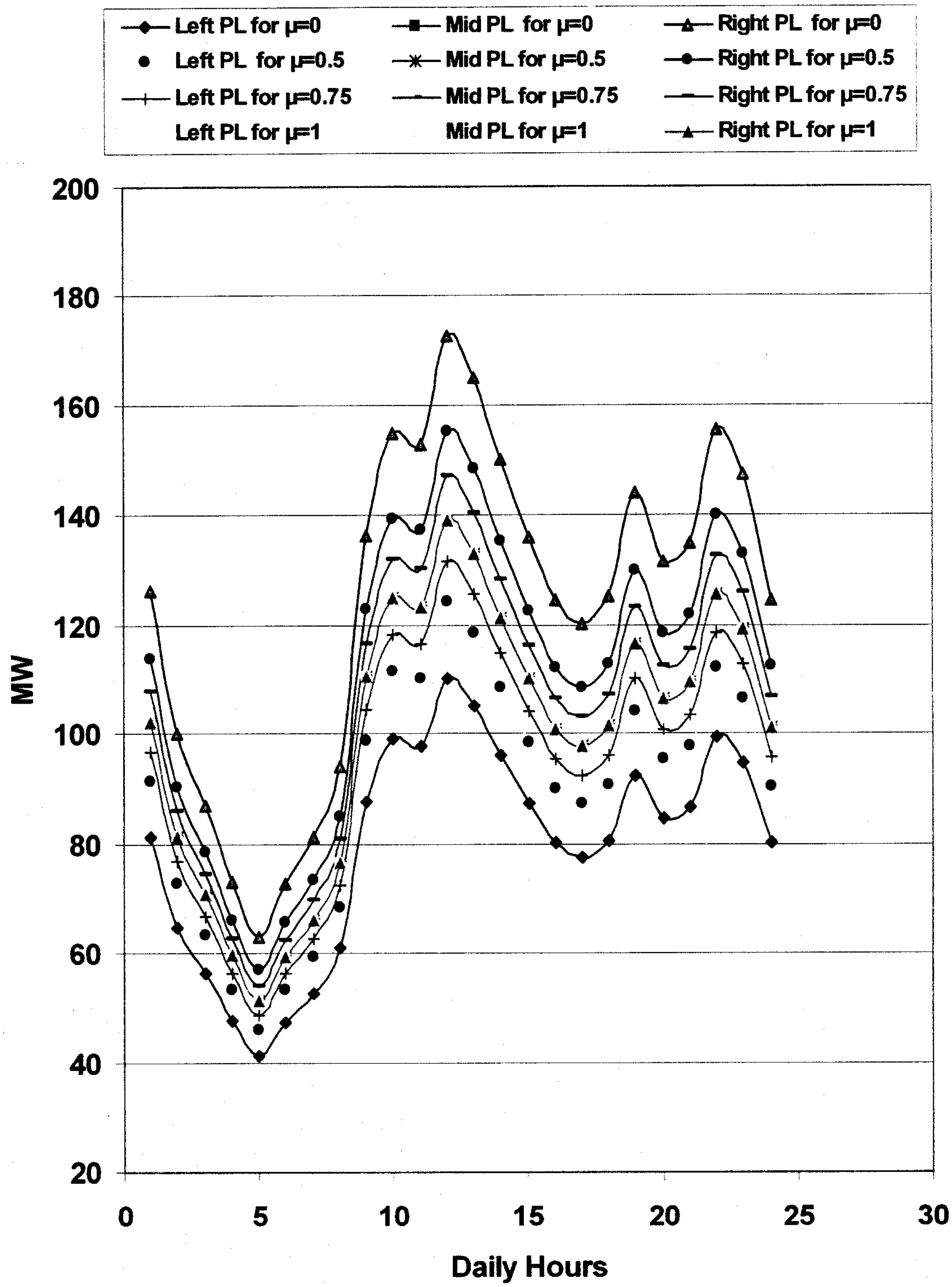


Figure (6-7) Fuzzy Power Losses for All α -Cut Representation

Table (6-8)

Membership Function of Total Cost for (0, 0.5, 0.75, 1) α -Cut Representation for Model "A" Weekdays With 10% Deviation for (P_D) and 3% for (α, β, γ)

Membership Function	$\mu_C = 0$			$\mu_C = 0.5$			$\mu_C = 0.75$			$\mu_C = 1$		
Daily Hours	Left Cost $/h	Mid Cost $/h	Right Cost $/h	Left Cost $/h	Mid Cost $/h	Right Cost $/h	Left Cost $/h	Mid Cost $/h	Right Cost $/h	Left Cost $/h	Mid Cost $/h	Right Cost $/h
1	10605	12635	14904	11592	12635	13738	12106	12635	13179	12635	12635	12635
2	9233	10943	12843	10142	10943	11781	10558	10943	11336	10943	10943	10943
3	8496	10041	11750	9355	10041	10754	9709	10041	10380	10041	10041	10041
4	7685	9051	10555	8463	9051	9659	8762	9051	9345	9051	9051	9051
5	7061	8292	9643	7771	8292	8828	8033	8292	8554	8292	8292	8292
6	7669	9032	10532	8457	9032	9624	8744	9032	9324	9032	9032	9032
7	8169	9642	11267	9021	9642	10283	9330	9642	9959	9642	9642	9642
8	8905	10541	12356	9848	10541	11258	10192	10541	10896	10541	10541	10541
9	11119	13271	15682	12349	13271	14230	12806	13271	13745	13271	13271	13271
10	12007	14375	17038	13356	14375	15436	13860	14375	14900	14375	14375	14375
11	11912	14256	16892	13248	14256	15306	13747	14256	14776	14256	14256	14256
12	12850	15425	18331	14313	15425	16586	14863	15425	15999	15425	15425	15425
13	12486	14972	17774	13901	14972	16090	14431	14972	15525	14972	14972	14972
14	11790	14105	16705	13110	14105	15140	13602	14105	14617	14105	14105	14105
15	11094	13240	15644	12321	13240	14195	12776	13240	13713	13240	13240	13240
16	10516	12525	14769	11667	12525	13415	12092	12525	12966	12525	12525	12525
17	10307	12267	14454	11431	12267	13134	11845	12267	12696	12267	12267	12267
18	10550	12567	14821	11706	12567	13461	12133	12567	13010	12567	12567	12567
19	11503	13748	16267	12785	13748	14750	13261	13748	14244	13748	13748	13748
20	10880	12976	15321	12080	12976	13907	12523	12976	13437	12976	12976	12976
21	11053	13190	15583	12275	13190	14140	12728	13190	13660	13190	13190	13190
22	12044	14421	17094	13398	14421	15486	13904	14421	14948	14421	14421	14421
23	11663	13947	16512	12967	13947	14968	13452	13947	14452	13947	13947	13947
24	10531	12543	14792	11685	12543	13435	12110	12543	12985	12543	12543	12543

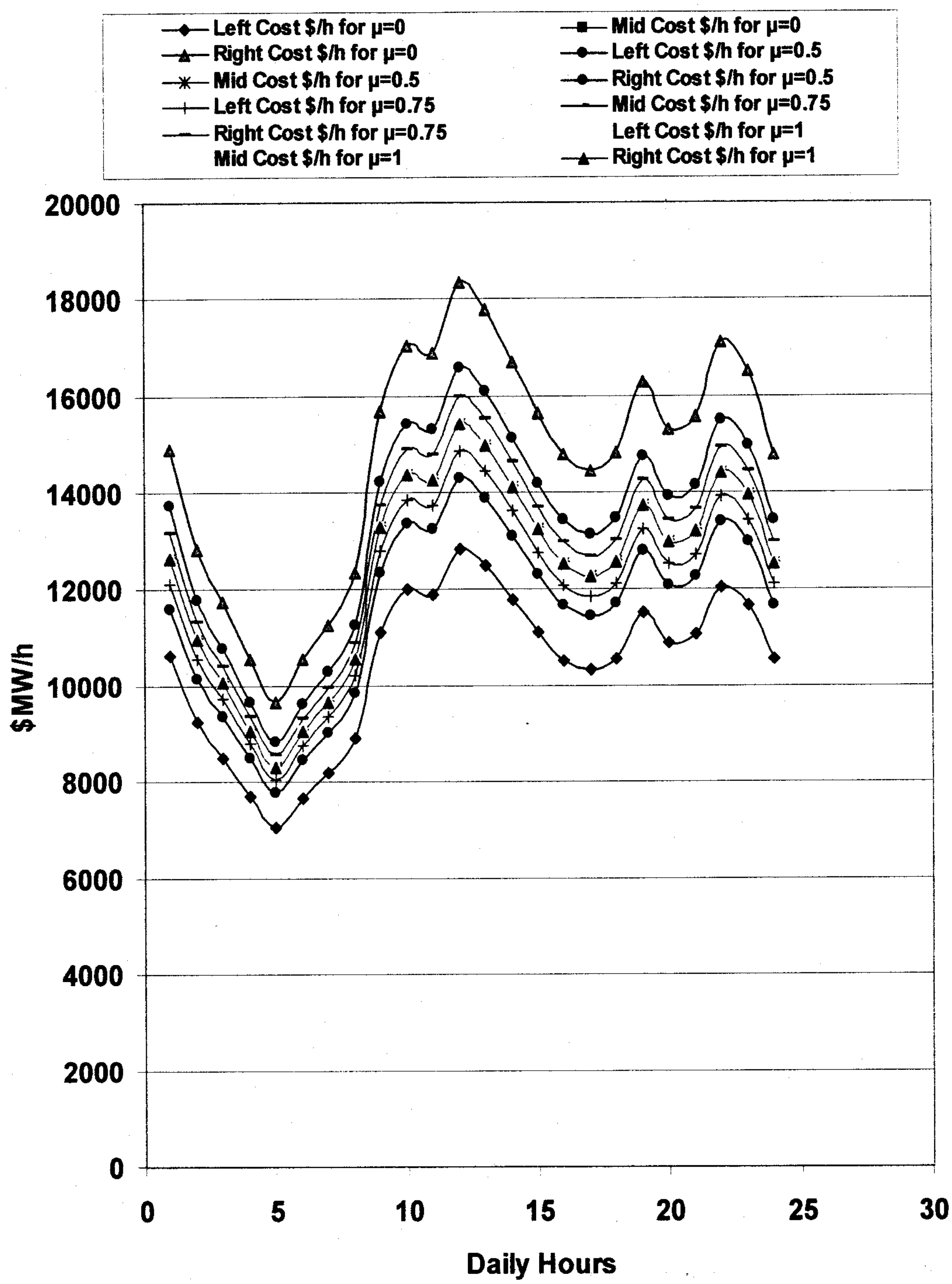


Figure (6-8) Fuzzy Min Total Cost for All α -Cut Representation

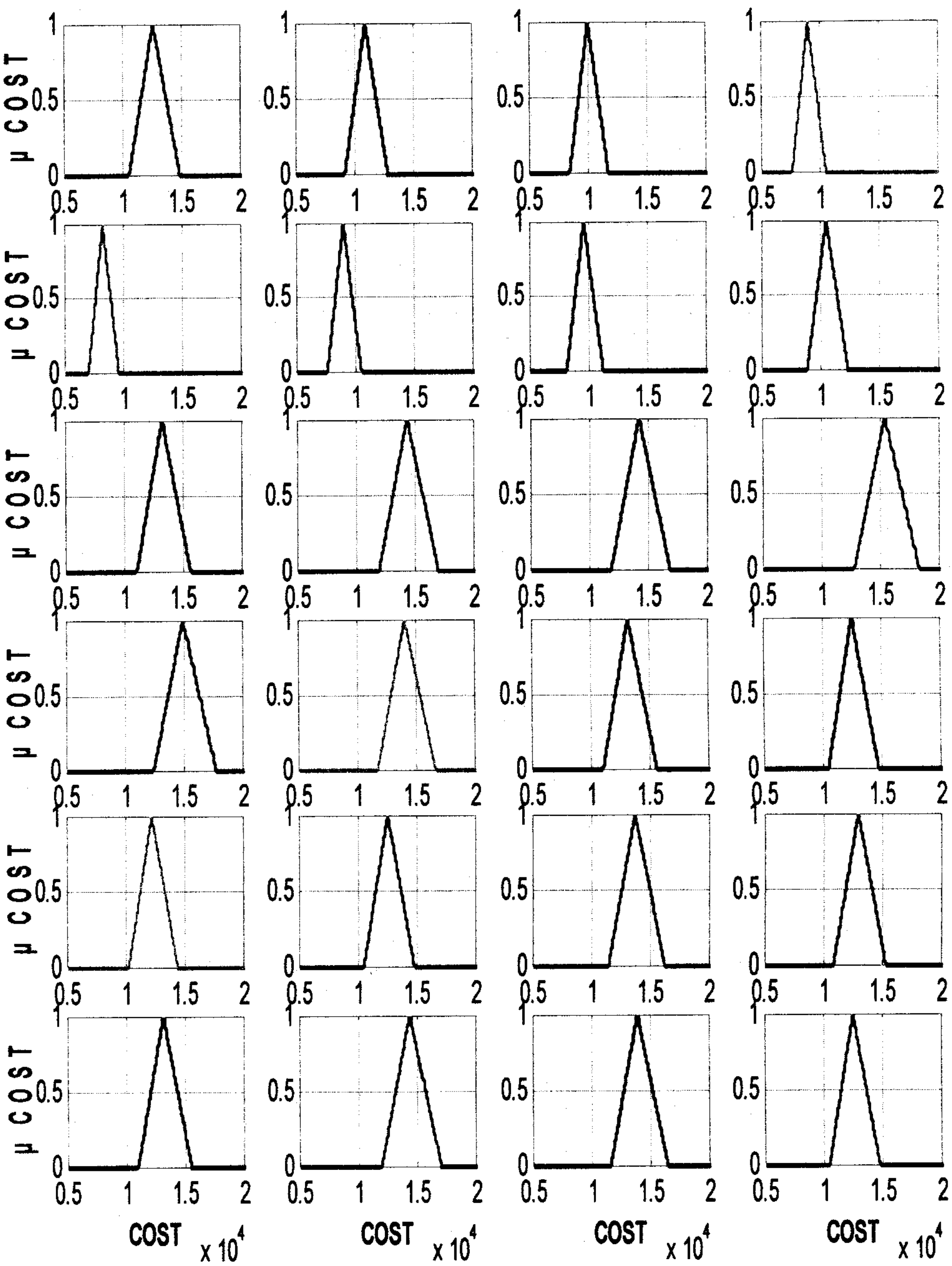


Figure (6-9) A Triangular Membership Function for Total Cost

Table (6-9)

Membership Function of Total Cost for (0, 0.5, 0.75, 1) α -Cut Representation for Model "A" Weekdays With 10% Deviation for (P_D, α) and 10% for (β, γ)

Membership Function		$\mu_C = 0$			$\mu_C = 0.5$			$\mu_C = 0.75$			$\mu_C = 1$	
Daily Hours	Left Cost \$/h	Mid Cost \$/h	Right Cost \$/h	Left Cost \$/h	Mid Cost \$/h	Right Cost \$/h	Left Cost \$/h	Mid Cost \$/h	Right Cost \$/h	Left Cost \$/h	Mid Cost \$/h	Right Cost \$/h
1	9824	12635	15925	11178	12635	14212	11892	12635	13408	12635	12635	12635
2	8555	10943	13720	9963	10943	11985	10511	10943	11385	10943	10943	10943
3	7854	10041	12529	9273	10041	10848	9698	10041	10391	10041	10041	10041
4	7105	9051	11262	8426	9051	9701	8760	9051	9348	9051	9051	9051
5	6527	8292	10293	7754	8292	8847	8032	8292	8554	8292	8292	8292
6	7090	9032	11237	8448	9032	9635	8744	9032	9324	9032	9032	9032
7	7552	9642	12017	9016	9642	10289	9330	9642	9959	9642	9642	9642
8	8252	10541	13199	9846	10541	11261	10192	10541	10896	10541	10541	10541
9	10299	13271	16757	12347	13271	14232	12806	13271	13745	13271	13271	13271
10	11120	14375	18207	13355	14375	15437	13860	14375	14900	14375	14375	14375
11	11032	14256	18051	13248	14256	15307	13747	14256	14776	14256	14256	14256
12	11919	15425	19567	14312	15425	16586	14863	15425	15999	15425	15425	15425
13	11563	14972	18994	13901	14972	16090	14431	14972	15525	14972	14972	14972
14	10919	14105	17851	13110	14105	15140	13602	14105	14617	14105	14105	14105
15	10276	13240	16716	12321	13240	14195	12776	13240	13713	13240	13240	13240
16	9742	12525	15780	11667	12525	13415	12092	12525	12966	12525	12525	12525
17	9549	12267	15443	11431	12267	13134	11845	12267	12696	12267	12267	12267
18	9774	12567	15836	11706	12567	13461	12133	12567	13010	12567	12567	12567
19	10654	13748	17383	12785	13748	14750	13261	13748	14244	13748	13748	13748
20	10079	12976	16370	12080	12976	13907	12523	12976	13437	12976	12976	12976
21	10239	13190	16651	12275	13190	14140	12728	13190	13660	13190	13190	13190
22	11154	14421	18267	13398	14421	15486	13904	14421	14948	14421	14421	14421
23	10803	13947	17645	12967	13947	14968	13452	13947	14452	13947	13947	13947
24	9756	12543	15805	11685	12543	13435	12110	12543	12985	12543	12543	12543

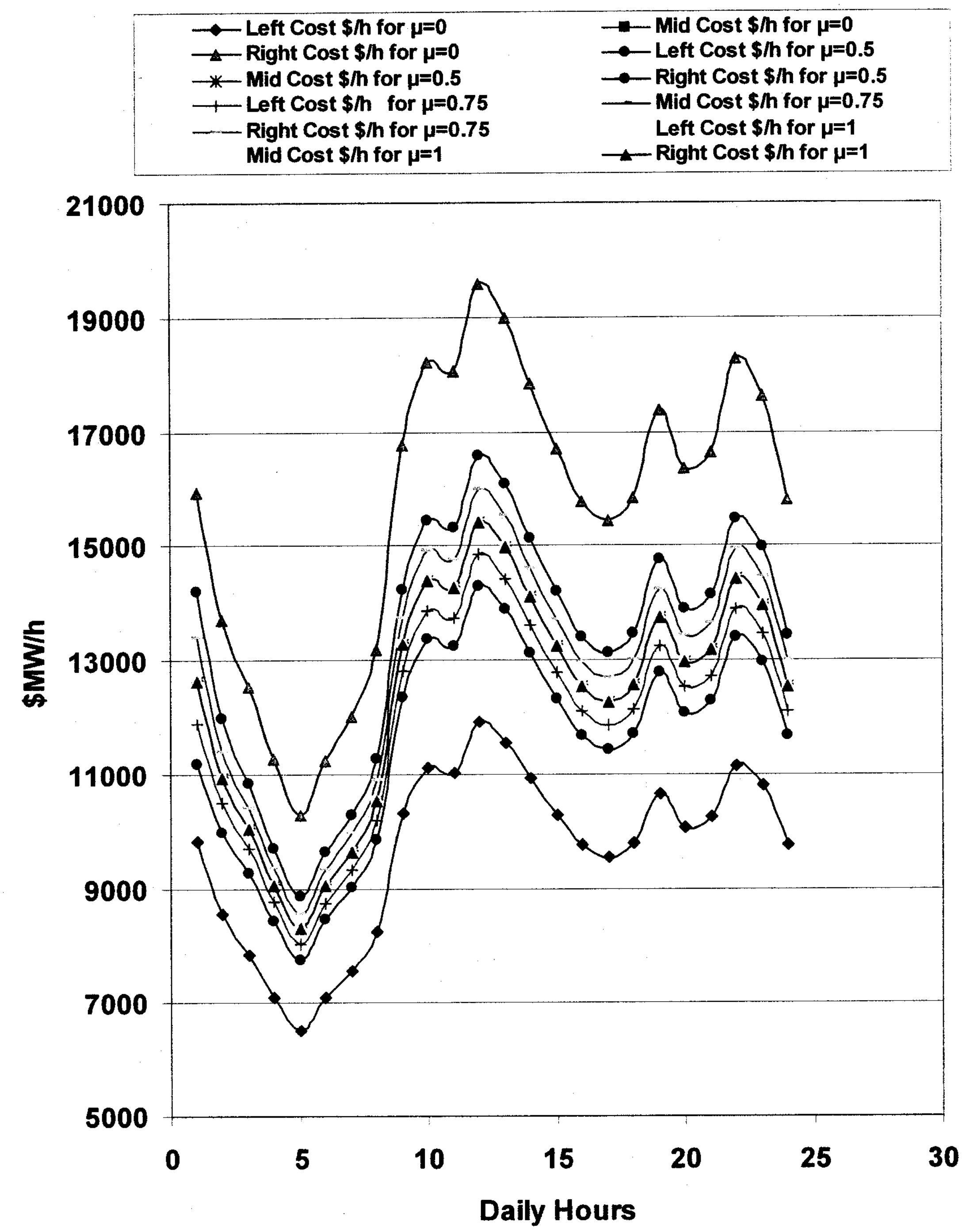


Figure (6-10) Fuzzy Minimum Total Cost for All α -Cut Representation for Table (6-9)

Table (6-10)

Membership Function of Total Cost for (0, 0.5, 0.75, 1) α -Cut Representation for Model "A" Weekdays With 2% Deviation for (α) and 3% for (β, γ)

Membership Function	$\mu_C = 0$			$\mu_C = 0.5$			$\mu_C = 0.75$			$\mu_C = 1$		
Daily Hours	Left Cost \$/h	Mid Cost \$/h	Right Cost \$/h	Left Cost \$/h	Mid Cost \$/h	Right Cost \$/h	Left Cost \$/h	Mid Cost \$/h	Right Cost \$/h	Left Cost \$/h	Mid Cost \$/h	Right Cost \$/h
1	10610	12635	14899	11594	12635	13736	12107	12635	13178	12635	12635	12635
2	9238	10943	12838	10143	10943	11779	10558	10943	11335	10943	10943	10943
3	8501	10041	11745	9356	10041	10754	9709	10041	10380	10041	10041	10041
4	7691	9051	10550	8464	9051	9659	8762	9051	9345	9051	9051	9051
5	7066	8292	9637	7772	8292	8827	8033	8292	8554	8292	8292	8292
6	7675	9032	10527	8457	9032	9624	8744	9032	9324	9032	9032	9032
7	8174	9642	11262	9021	9642	10283	9330	9642	9959	9642	9642	9642
8	8910	10541	12351	9848	10541	11258	10192	10541	10896	10541	10541	10541
9	11124	13271	15677	12349	13271	14230	12806	13271	13745	13271	13271	13271
10	12012	14375	17032	13356	14375	15436	13860	14375	14900	14375	14375	14375
11	11917	14256	16887	13248	14256	15306	13747	14256	14776	14256	14256	14256
12	12856	15425	18326	14313	15425	16586	14863	15425	15999	15425	15425	15425
13	12492	14972	17769	13901	14972	16090	14431	14972	15525	14972	14972	14972
14	11795	14105	16700	13110	14105	15140	13602	14105	14617	14105	14105	14105
15	11099	13240	15639	12321	13240	14195	12776	13240	13713	13240	13240	13240
16	10521	12525	14764	11667	12525	13415	12092	12525	12966	12525	12525	12525
17	10312	12267	14449	11431	12267	13134	11845	12267	12696	12267	12267	12267
18	10556	12567	14816	11706	12567	13461	12133	12567	13010	12567	12567	12567
19	11508	13748	16262	12785	13748	14750	13261	13748	14244	13748	13748	13748
20	10886	12976	15315	12080	12976	13907	12523	12976	13437	12976	12976	12976
21	11058	13190	15578	12275	13190	14140	12728	13190	13660	13190	13190	13190
22	12049	14421	17089	13398	14421	15486	13904	14421	14948	14421	14421	14421
23	11669	13947	16507	12967	13947	14968	13452	13947	14452	13947	13947	13947
24	10537	12543	14787	11685	12543	13435	12110	12543	12985	12543	12543	12543

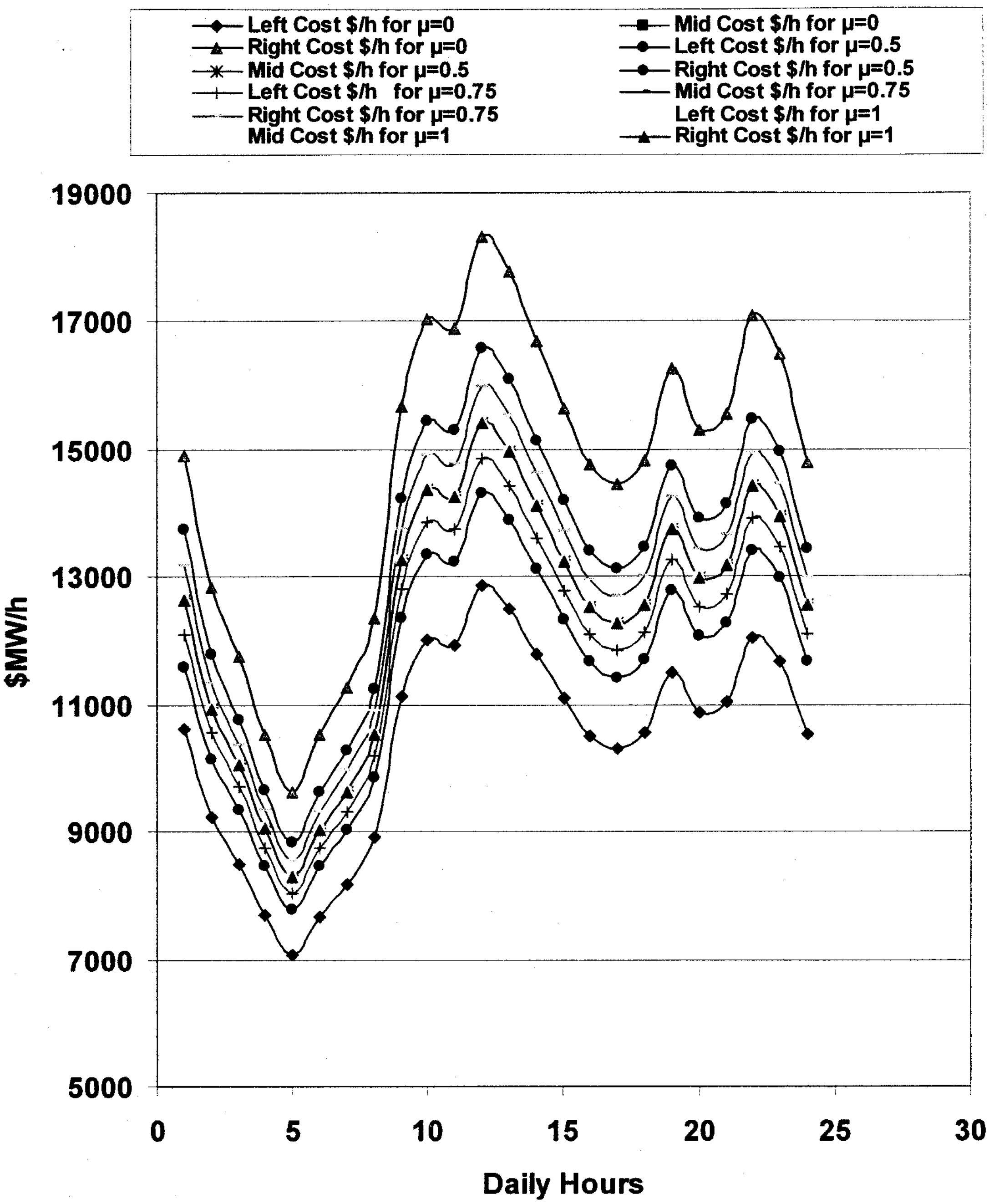


Figure (6-11) Fuzzy Minimum Total Cost for All α -Cut Representation for Table (6-10)

6.4 Conclusion

Load conditions change from time to time. The basic objective of economic dispatch operation of power systems is "the distribution of total generation of power in the network between various regional zones; various power stations in respective zones and various units in respective power stations such that the cost of power delivered is minimum." In the cost of power delivered, the cost of power generation and transmission losses should be considered. It means for every load condition, the load control center should decide the following:

a) - How much power is to be generated to meet the prevailing load condition to maintain constant frequency.

b) - How much power should each region generate?

c) - What should be the exchange of power between the regions (area)?

This aspect can be decided by the regional control center. This thesis provides all the information mentioned above. The variations of load were assumed as fuzzy, which made the output generation of each unit, the system power losses and the total network cost become fuzzy. This fuzziness provides the load control center with valuable information, which is listed below:

1. The 10% fuzzy load deviation presented gives a range of security knowledge assessment to the load control center. Knowing the minimum and maximum generation needed to compensate the load variation which occurs at each hour in question can be a great asset to the command and control engineer. If this variation can not be supplied by the unit committed to the network, then more units can be brought in to overcome the sudden variation.
2. The maximum, minimum and middle cost variation at each hour is calculated. This give the company supplying the load an optimal minimum cost generation of each unit and the total cost of all units for that particular load variation at the hour in question. This information is very helpful in decision making for the company supplying the load to the consumer. The company can decide weather to supply it if it is not costly or buy it from another company interconnected with the network.

3. In practice, the heat rate characteristic depends on thermal dynamic parameters such as ambient dry and wet bulb temperatures, operating pressures and water pumping rates. In addition the convexity of the cost curve are do to valve points, rotor metal differential temperatures, exhaust hood temperatures and rotor and shell expansion of the turbine. This applies not only to the characteristic curve of the heat rate itself, but also to upper and lower limits of generation. These variations causing discontinuity in the heat rate curve and thus in the cost curve. Considering the uncertainties involved in the variations, a fuzzy triangular membership function is chosen to model the variations. The variation of the cost function parameters that was chosen as shown in tables (6-8, 6-9 and 6-10) can effect the over all performance of the network including the total cost.
4. The power loss information is very helpful to the sub-station control room where the reactive power flow is minimized through transmission lines by compensation to minimize line losses and to maintain a stable voltage level.
5. The fuzzy load, cost function coefficients and fuzzy losses formulation using fuzzy sets operation and membership function implemented in Chapter (4), (5) and (6) enhanced the reliability of the system performance where all the information needed is available online hour by hours the load variation, the total minimum cost of the system and the power losses. The computation time is reduced where all the variation including the crisp output data which represents the conventional method solution are all included in the analysis where as in the conventional method every variation in the parameter is calculated separately. The decision maker will be in control of the operation any sudden changes outside the limit set for the membership function of the load demand has to be looked into with concern and series action. On the other hand if the load stays with in the chosen upper and lower value then the system is secure and stable.

Chapter 7
Fuzzy Optimal Power Flow with Fuzzy Active, Reactive Power Generation and Load

7.1 Introduction

Because of the vagueness, uncertainty or random nature of the data associated with load, generators, active power, reactive power, voltages and phase angles, the models assumptions could result in enormous error. It is difficult to obtain an optimal logical solution to satisfy the constraints included in the model. For this reason, a fuzzy optimal power flow model is needed more than ever to develop a realistic solution. In this chapter a fuzzy optimal power flow is discussed and analyzed to calculate the minimum total cost function for system operation. Some risk analysis has to be taken as qualitative assessment or linguistic declarations such as more or less of certain values of fuzzy variables are present in the calculations. Fuzzy nonlinear programming optimization procedures are introduced to find the optimal solution for a case study consisting of a system with several buses.

7.2 Fuzzy OPF Problem Formulation

In this chapter and the next a FNLP approach is constructed to transform the fuzzy constraints input and fuzzy objective function into a number of optimal crisp outputs between [0,1]. The values of the outputs rely on the judgment of the decision maker to choose the best optimal solution. We will start with fuzzy generation, load demand and crisp α, β and γ, then in the next chapter we will consider fuzzy coefficient parameters in the ($\tilde{\alpha}, \tilde{\beta}$ and $\tilde{\gamma}$) objective function.

7.2.1 Fuzzy Generation, Load Demand and Crisp (α β γ)

Fuzzy load demand and generation can be used instead of deterministic load and generation values. The fuzzy parameters used in this chapter are listed below:

1. Fuzzy active power generation denoted by its middle, left and right sides of a triangular number representation $\tilde{P}_{G_i} = (\bar{P}_{G_i}, L_{\tilde{P}_{Gi}}, R_{\tilde{P}_{Gi}})$
2. Fuzzy reactive power generation $\tilde{Q}_{G_i} = (\bar{Q}_{G_i}, L_{\tilde{Q}_{Gi}}, R_{\tilde{Q}_{Gi}})$
3. Fuzzy active load demand $\tilde{P}_{D_i} = (\bar{P}_{D_i}, L_{\tilde{D}_i}, R_{\tilde{D}_i})$
4. Fuzzy reactive load demand $\tilde{Q}_{D_i} = (\bar{Q}_{D_i}, L_{\tilde{D}_i}, R_{\tilde{D}_i})$

Then the FNLP optimization becomes:

$$\begin{aligned} &\min\ \tilde{J} = \sum_{i=1}^{NG} (\alpha_i + \beta_i \tilde{P}_{G_i} + \gamma_i \tilde{P}_{G_i}^2) \\ &Subject\ to \\ &\qquad P_i \cong \tilde{P}_{G_i} - \tilde{P}_{D_i} \\ &\qquad Q_i \cong \tilde{Q}_{G_i} - \tilde{Q}_{D_i} \end{aligned} \tag{7.1}$$

Or simplifying the FNLP to

$$\begin{aligned} &\min\quad \tilde{J} \\ &Subject\ to \\ &\qquad P_i(x) \cong \tilde{P}_{G_i} - \tilde{P}_{D_i} \\ &\qquad Q_i(x) \cong \tilde{Q}_{G_i} - \tilde{Q}_{D_i} \end{aligned} \tag{7.2}$$

The symbol "$\cong$" is a fuzzy equality relation in the equality constraint. It can be equivalent to two"$\tilde{\leq}$" inequality constraints representing the difference between the active, reactive power generation ($\tilde{P}_{G_i} - \tilde{P}_{D_i}$) and active reactive power load ($\tilde{Q}_{G_i} - \tilde{Q}_{D_i}$). The left, middle and right sides are represented by a triangular membership function as shown in figure (7-1) and (7-2), which represents the low level (the left side of the membership function) and high level (the right side of the membership function). $P_i(x)$ and $Q_i(x)$ are the active and reactive power flow of the network and they are a function of (x) where it contains the system voltage magnitude and phase angle shown in the power flow equation (2.23) and (2.24). Assume the difference between the active, reactive power generation and loads are $\tilde{P}k$, $\tilde{Q}k$ respectively. The result of the difference is a triangular membership function, and equation (7.2) can be simplified to:

$$P_i(x) \cong \tilde{P}k = \tilde{P}_{G_i} - \tilde{P}_{D_i} \tag{7.3}$$

$$Q_i(x) \cong \tilde{Q}k = \tilde{Q}_{G_i} - \tilde{Q}_{D_i} \tag{7.4}$$

Transforming the equality constraints into inequality constraint yields:

$$\begin{aligned} &\min \quad \tilde{J} \\ &\text{Subject to} \\ &\quad P_i(x) \lesssim \tilde{P}k_i \\ &\quad Q_i(x) \lesssim \tilde{Q}k_i \end{aligned} \tag{7.5}$$

In equation (7.5), the objective cost function $\tilde{J}$ (a quadratic function in $\tilde{P}_{G_i}$) is representing the variable cx (a linear function in x) as shown in equation (3.32) and (3.33) for fuzzy linear programming with fuzzy resources. $A(x)_i$ represents the active power flow $P_i(V_i, \delta_i)$ and reactive power flow $Q_i(V_i, \delta_i)$, where $x = (V_i, \delta_i)$ represents the magnitude voltages and phase angle. In equation (3.32) $\tilde{b}$ is equal to the difference between the active generation $\tilde{P}k_i$ and reactive load $\tilde{Q}k_i$.

The left and right linear region of the membership function shown in figure (7-1) can be mathematically described as:

$$\mu(\tilde{P}k_i) = \begin{Bmatrix} 1 & P_i = Pk_m \\ (P_i - Pk_L)/(Pk_m - Pk_L) & Pk_L < P_i < Pk_m \\ (Pk_R - P_i)/(Pk_R - Pk_m) & Pk_m < P_i < Pk_R \\ 0 & otherwise \end{Bmatrix} \tag{7.6}$$

$\mu(\tilde{P}k_i)$ is a triangular membership function and the Pk_L parameter represents the desired lowest limit that needs to be enforced and Pk_R is the highest limit that the constraint is allowed to be or the permissible width of the constraint. Physically, for soft constraints Pk_m is the normal limit and ($Pk_m + \partial Pk_i$) is the emergency limit. It is acceptable to violate the normal limit of a soft constraint "a little bit" but the emergency limit should never be violated.

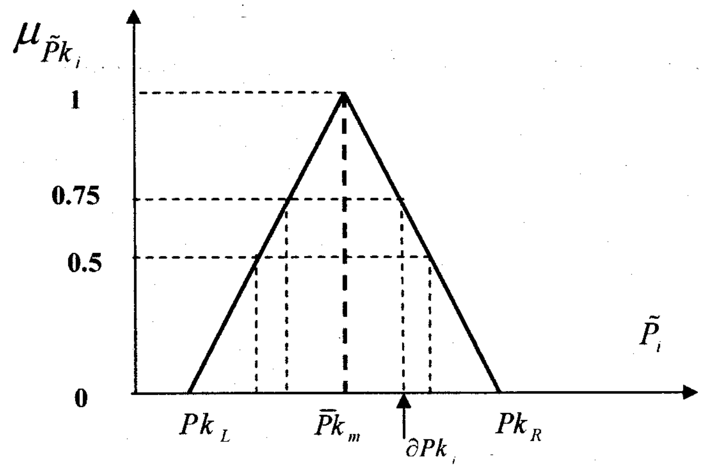


Figure (7-1) Memberships Fu[illegible] the Difference in Active Power Generator and Active Load Demand

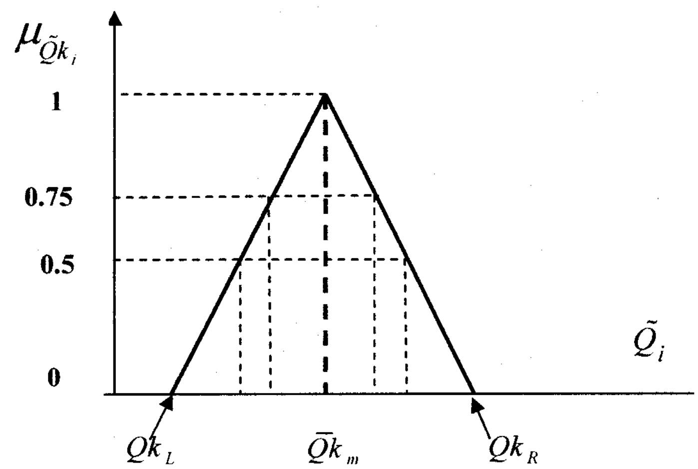


Figure (7-2) Memberships Function of the Difference in Reactive Power Generator and Active Load Demand

Similarly, the reactive power membership function is shown in figure (7-2) and its mathematical formula becomes:

$$\mu(\tilde{Qk}_i)=\begin{cases}1 & Q_i = Qk_m \\ (Q_i - Qk_L)/(Qk_m - Qk_L) & Qk_L < Q_i < Qk_m \\ (Qk_R - Q_i)/(Qk_R - Qk_m) & Qk_m < QP_i < Qk_R \\ 0 & otherwise \end{cases} \tag{7.7}$$

Representing the fuzzy objective function in equation (7.1) by its linearly decreasing membership function shown in figure (7-3). The mathematical formula for the decreasing linear region is shown in equation (7.8).

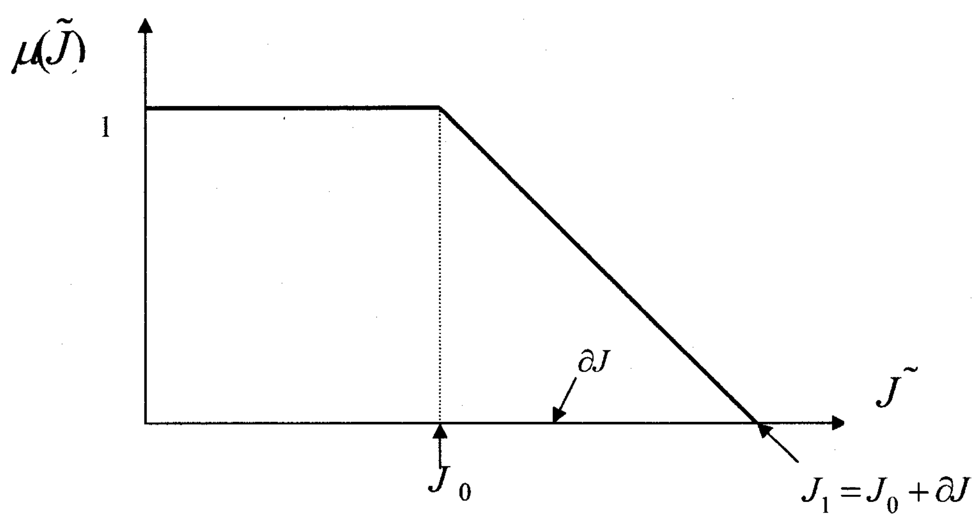


Figure (7-3) Fuzzy Objective Membership Function

$$\mu(\tilde{J})=\begin{cases}1 & J \le J_0 \\ (1-(J-J_0)/\partial J) & J_0 \le J \le J_0 + \partial J \\ 0 & otherwise \end{cases} \tag{7.8}$$

The membership function $\mu(J)$ represents a fuzzy objective function which is given a degree of satisfaction limit that can be violated to some extent without over costing the system. The difference between Zimmermann and Werner's approach is that Zimmermann assumes an initial value for the aspiration level J_0 representing the ideal cost production of the power system and the tolerance ∂J, while in Werner's approach these values are unknown and should be calculated using the method discussed in chapter (3). The optimization problem is to maximize the minimum degree of satisfaction of the fuzzy decision z among all fuzzy objective and fuzzy constraints.

$$\max\ z \quad \text{when } z = \min\left\{\mu(\tilde{J}), \mu(\tilde{P}_i), \mu(\tilde{Q}_i)\right\} \tag{7.9}$$

The fuzzy decision is defined by Bellman and Zadeh's principle as:

$$D(x) = \min\left\{\mu_0(x), \mu_1(x), \ldots\ldots, \mu_m(x)\right\}$$

Where $\mu_0(x), \mu_1(x), \ldots\ldots, \mu_m(x)$ are all the membership function of the fuzzy objective and fuzzy constraints. The x value can be any element of the n dimensional space, because any element has a degree of feasibility, which is between [0, 1]. The optimal solution of the FNLP is determined from the relationship $D(x^*) = \max_{x \in \mathbb{R}^n} D(x)$. The minimum degree of satisfaction is found by using triangular norms (t-norms) which is used extensively in fuzzy set theory to present the intersection of fuzzy sets.

Applying the approach recommended by Werner, as explained in chapter (3), we can find the minimum degree of satisfaction to the objective function with fuzzy resources represented by fuzzy active generation, active load, reactive generation and reactive load. Calculating J_0 of the objective function by using Werner's first definition then equation (3.34) becomes:

$$\begin{aligned} &\min\quad \bar{J}_0 = \sum_{i=1}^{NG} (\alpha_i + \beta_i \bar{P}_{G_i} + \gamma_i \bar{P}_{G_i}^2) \\ &\text{subject to} \\ &\qquad P_i(V_i, \delta_i) \lesssim \bar{P}k_i \\ &\qquad Q_i(V_i, \delta_i) \lesssim \bar{Q}k_i \end{aligned} \tag{7.10}$$

and J_1 can be calculated by:

$$\begin{aligned} &\min\quad \tilde{J}_1 = \sum_{i=1}^{NG} (\alpha_i + \beta_i \tilde{P}_{G_i} + \gamma_i \tilde{P}_{G_i}^2) \\ &\text{S.t} \\ &\qquad P_i(V_i, \delta_i) \lesssim \bar{P}k_i + \partial Pk_{R_i} \\ &\qquad Q_i(V_i, \delta_i) \lesssim \bar{Q}k_i + \partial Qk_{R_i} \end{aligned} \tag{7.11}$$

Then to fit the optimal fuzzy objective function solution between J_0 and J_1 approximately on the decreasing linear region then equation (3.36) becomes:

$$\begin{aligned}
&\min \quad (-z) \\
&\text{S.t} \\
&\qquad z \le \mu(\tilde{J}) \\
&\qquad z \le \mu(\tilde{P}k_i) \\
&\qquad z \le \mu(\tilde{Q}k_i) \\
&\qquad 0 \le z \le 1
\end{aligned} \tag{7.12}$$

Substituting the membership function into the optimization problem we have:

$$\begin{aligned}
&\min \quad (-z) \\
&\text{S.t} \\
&\qquad z \le (1-(J-J_0)/\partial J) \\
&\qquad z \le (P_i - Pk_L)/(Pk_m - Pk_L) \\
&\qquad z \le (Pk_R - P_i)/(Pk_R - Pk_m) \\
&\qquad z \le (Q_i - Qk_L)/(Qk_m - Qk_L) \\
&\qquad z \le (Qk_R - Q_i)/(Qk_R - Qk_m) \\
&\qquad 0 \le z \le 1
\end{aligned} \tag{7.13}$$

The calculated optimal objective function J is fuzzy corresponding to the membership grades. According to this relationship, the decision maker can then get the desired optimal solution under pre-determined imprecision allowable.

The FNLP optimization converts the fuzzy problem to a crisp one while satisfying the fuzzy objective and fuzzy constraint simultaneously. Then we can substitute the objective function, the active and reactive formula into the Lagrangian to relax the system wide constraints.

$$\begin{aligned}
L &= J + \lambda_J (J_0 - J - (1-z)\partial J) \\
&+ \sum \lambda_{P_L} (z\,(Pk_m - Pk_L) - (P_i - Pk_L)) \\
&+ \sum \lambda_{P_R} (z\,(Pk_R - Pk_m) - (Pk_R - P_i)) \\
&+ \sum \lambda_{Q_L} (z\,(Qk_m - Qk_L) - (Q_i - Qk_L)) \\
&+ \sum \lambda_{Q_R} (z\,(Qk_R - Qk_m) - (Qk_R - Q_i))
\end{aligned} \tag{7.14}$$

The NLP optimization was calculated using Matlab optimization toolbox / fuzzy toolbox. The starting search for the NLP optimization is obtained from the sensitivity analysis of the power flow calculation as discussed below.

7.3 Sensitivity Model for Power Flow Analysis [33, 42, 44]

A linearized power flow model is proposed to simplify the initial search for the optimal solution. In this model, only 1.0-level of α -cut is needed to solve equation (7.10) and (7.11) for a crisp power flow calculation. For fuzzy load flow calculation other α -cuts are computed by using the sensitivity analysis and fuzzy number operations shown in Table (3-2). Applying Taylor's series' expansions to equation (2.23) and (2.24) about the initial estimate and neglecting all higher order terms, we can obtain the following sensitivity model of system state variables.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \tag{7.15}$$

Setting J_2 and J_3 equal to zero because ΔP is less sensitive to changes in the voltage magnitude and are most sensitive to changes in phase angle $\Delta\delta$. Similarly, reactive powers are less sensitive to changes in angle and are mainly dependent on changes in voltage magnitude.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \tag{7.16}$$

Then the changes in active and reactive power become:

$$\Delta P = J_1 \Delta\delta = \left| \frac{\partial P}{\partial \delta} \right| \Delta\delta \tag{7.17}$$

$$\Delta Q = J_4 \Delta|V| = \left| \frac{\partial Q}{\partial |V|} \right| \Delta|V| \tag{7.18}$$

After a simple adjustment and assumption we arrive at the final formula of the fast decoupled power flow involving the imaginary part of the bus admittance matrix B' And B''

$$\Delta\delta = -[B']^{-1} \frac{\Delta P}{|V|} \tag{7.19}$$

$$\Delta|V| = -[B'']^{-1} \frac{\Delta Q}{|V|} \tag{7.20}$$

In fuzzy power flow calculation the inputs to the above equations should be fuzzy. Applying the fuzzy input to the sensitivity model equations we get:

$$\Delta\tilde{\delta} = -[B']^{-1} \frac{\Delta\tilde{P}}{|\tilde{V}|} \tag{7.21}$$

$$\Delta|\tilde{V}| = -[B'']^{-1} \frac{\Delta\tilde{Q}}{|\tilde{V}|} \tag{7.22}$$

Where the bus incremental active power vector is:

$$\Delta\tilde{P} = \tilde{P}_i^{Sch} - P_i^{(0)} \tag{7.23}$$

And the reactive incremental power vector is:

$$\Delta\tilde{Q}_i = \tilde{Q}_i^{Sch} - Q_i^{(0)} \tag{7.24}$$

$\tilde{P}_i^{Sch}$, $\tilde{Q}_i^{Sch}$ are the fuzzy bus active and reactive injection or scheduled vector, and

$P_i^{(0)}$, $Q_i^{(0)}$ are their initial point vector that can be calculated by using α-cut equal to 1. Updating the voltage and angle can be done using the following formulas

$$\begin{aligned} \tilde{\delta} &= \delta_1 + \Delta\tilde{\delta} \\ \tilde{V} &= V_1 + \Delta\tilde{V} \end{aligned} \tag{7.25}$$

Where the $\tilde{\delta}$, $\tilde{V}$ are the fuzzy bus voltage and angle vectors, and δ_1, V_1 are their initial point vector. The division and multiplication of the fuzzy equations can be calculated by using the triangular fuzzy operations shown in Table (3-2) in chapter (3).

7.4 Numerical Testing of a Study Case

For the purpose of estimating the performance of the algorithm, it will be tested on a 9-bus power system network of an Electric Utility Company shown in figure (7-4), all the system data are tabulated in table (7-1, 7 -2 ,7-3, 7-4) and (7-5). Taking bus 1 as the slack bus and the selected base unit is 100MVA.

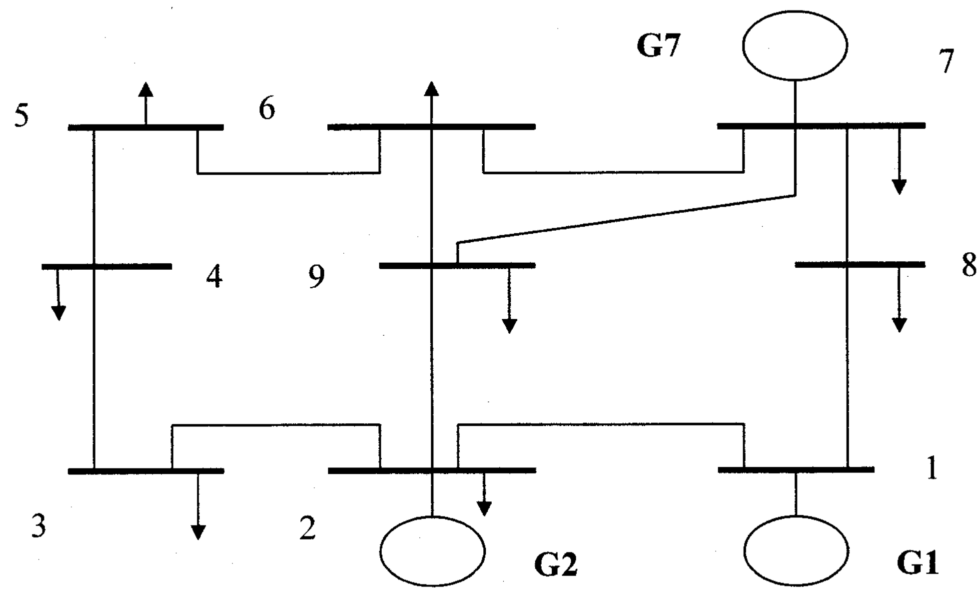


Figure (7-4) 9-Bus Power System Network

Table (7-1)

LOAD DATA		
Bus No.	Load MW	Load Mvar
1	0	0
2	20	10
3	25	15
4	10	5
5	40	20
6	60	40
7	10	5
8	80	60
9	100	80

Table (7-2)

LINE DATA				
Bus No.	Bus No.	R, PU	X, PU	1/2 B, PU
1	2	0.018	0.054	0.0045
1	8	0.014	0.036	0.0030
2	9	0.006	0.030	0.0028
2	3	0.013	0.036	0.0030
3	4	0.010	0.050	0.0000
4	5	0.018	0.056	0.0000
5	6	0.020	0.060	0.0000
6	7	0.015	0.045	0.0038
6	9	0.002	0.066	0.0000
7	8	0.032	0.076	0.0000
7	9	0.022	0.065	0.0000

Table (7-3)

SHUNT CAPACITORS	
Bus No.	Mvar
3	1
4	3

Table (7-4)

GENERATOR REAL POWER LIMITS		
Gen.	Min. MW	Max. MW
1	50	200
2	50	200
7	50	100

Table (7-5)

GENERATION DATA				
Bus No.	Voltage Mag.	Generation MW	Mvar Limits Min	Mvar Limits Max.
1	1.03			
2	1.04	80	0	250
7	1.01	120	0	100

The generators operating cost in $/h are as follows:

$$C_1 = 240 + 6.7P_{G_1} + 0.009P_{G_1}{}^2$$

$$C_2 = 220 + 6.1P_{G_2} + 0.005P_{G_2}{}^2$$

$$C_7 = 240 + 6.5P_{G_7} + 0.008P_{G_7}{}^2$$

Table (7-6) summarizes all the data output of the fuzzy non linear programming approach by Werner. The calculations of J_0 , $\tilde{J}_1$ and the optimal crisp and fuzzy cost of the decreasing linear region is tabulated. A 10%, 15% and 20% deviation is applied to the load and generator data for different α -cuts between [0, 1].

Table (7-6) Fuzzy Nonlinear Programming Parameters

α-cut	Load+Gen. Deviation %	Fuzzy P1 Left MW	Fuzzy P1 Mid MW	Fuzzy P1 Right MW	Crisp Total Cost $/h	Optimal J0 $/h	Optimal J1 $/h	Fuzzy Optimal Total Cost $/h
0	10	135.8622	150.4327	290.7173	3326.769	3276.863	3472.683	3472.683
0.1	10	137.3184	150.4327	276.593	3326.769	3276.863	3472.683	3394.544
0.2	10	138.7748	150.4327	262.4829	3326.769	3276.863	3472.683	3383.3
0.3	10	140.2314	150.4327	248.3895	3326.769	3276.863	3472.683	3379.912
0.4	10	141.6881	150.4327	234.3152	3326.769	3276.863	3472.683	3358.487
0.5	10	143.1451	150.4327	220.2629	3326.769	3276.863	3472.683	3351.335
0.6	10	144.6023	150.4327	206.2352	3326.769	3276.863	3472.683	3322.907
0.7	10	146.0596	150.4327	192.2351	3326.769	3276.863	3472.683	3305.181
0.8	10	147.5171	150.4327	178.2658	3326.769	3276.863	3472.683	3301.3
0.9	10	148.9748	150.4327	164.3305	3326.769	3276.863	3472.683	3301.7
1	10	150.4327	150.4327	150.4327	3326.769	3276.863	3472.683	3276.863
0	15	128.0601	150.4327	304.4139	3326.769	3276.863	3526.292	3526.292
0.1	15	130.3008	150.4327	288.9419	3326.769	3276.863	3526.292	3406.003
0.2	15	132.5407	150.4327	273.478	3326.769	3276.863	3526.292	3405.036
0.3	15	134.7798	150.4327	258.0251	3326.769	3276.863	3526.292	3407.006
0.4	15	137.0182	150.4327	242.5861	3326.769	3276.863	3526.292	3405.429
0.5	15	139.2557	150.4327	227.1642	3326.769	3276.863	3526.292	3401.136
0.6	15	141.4926	150.4327	211.7628	3326.769	3276.863	3526.292	3376.988
0.7	15	143.7287	150.4327	196.3851	3326.769	3276.863	3526.292	3353.122
0.8	15	145.964	150.4327	181.035	3326.769	3276.863	3526.292	3305.181
0.9	15	148.1987	150.4327	165.7162	3326.769	3276.863	3526.292	3300.467
1	15	150.4327	150.4327	150.4327	3326.769	3276.863	3526.292	3276.863
0	20	120.2932	150.4327	318.1588	3326.769	3276.863	3581.286	3581.286
0.1	20	123.3111	150.4327	301.3306	3326.769	3276.863	3581.286	3402.013
0.2	20	126.3282	150.4327	284.5052	3326.769	3276.863	3581.286	3404.289
0.3	20	129.3444	150.4327	267.6858	3326.769	3276.863	3581.286	3405.402
0.4	20	132.3598	150.4327	250.8758	3326.769	3276.863	3581.286	3406.762
0.5	20	135.3742	150.4327	234.0789	3326.769	3276.863	3581.286	3405.192
0.6	20	138.3878	150.4327	217.299	3326.769	3276.863	3581.286	3405.679
0.7	20	141.4004	150.4327	200.5401	3326.769	3276.863	3581.286	3394.544
0.8	20	144.4121	150.4327	183.8064	3326.769	3276.863	3581.286	3337.116
0.9	20	147.4229	150.4327	167.1024	3326.769	3276.863	3581.286	3326.533
1	20	150.4327	150.4327	150.4327	3326.769	3276.863	3581.286	3276.863

The minimum optimal total cost calculation is fuzzy with a linear decreasing membership function. Figure (7-5A, 7-7A and 7-10) show the membership grade versus the minimum optimal total cost. Figure (7-5B and 7-7B) show a 3D plot of Figure (7-5A and 7-7A). Figures (7-6, 7-8, 7-9) show the obtained minimum optimal total cost of the three generation versus their corresponding membership grades. Figure (7-9) shows that the decreasing values of the total optimal cost for 20% deviation does not start until α -cut is equal to 0.6. In Figure (7-8) the decreasing values for 15% deviation starts at α -cut equal to 0.5 while, the total optimal cost for 10 % deviation shown in Figure (7-6) starts at a lower α -cut level equal to 0.1 or 0.2. Increasing the deviation higher than 20% will give unsatisfactory results. The percentage of deviation for the active, reactive power load and power generation is very important in decision making where the 10% deviation gives the decision maker a range of decreasing values between [0,1]. In addition, the fuzzy OPF is higher than the crisp OPF at lower α -cut and lower than crisp OPF at higher order α -cut which gives a higher degree of satisfaction as shown in Table (6-7). We must stress here that it is up to the decision maker to choose the best degree of satisfaction needed to obtain the minimum optimal total cost of the network.

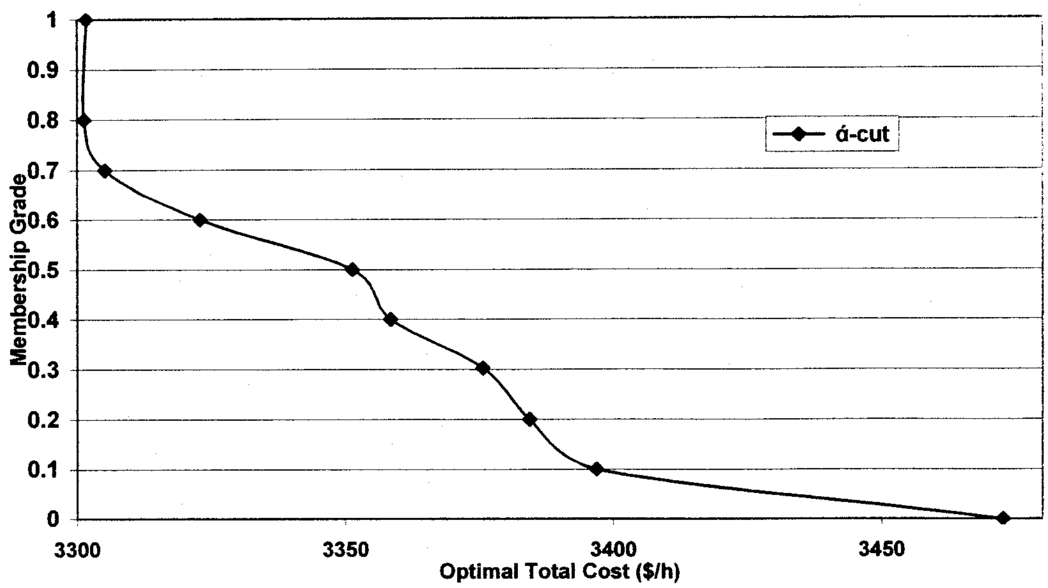


Figure (7-5A) Membership Grade Versus Optimal Total Cost for 10% Deviation

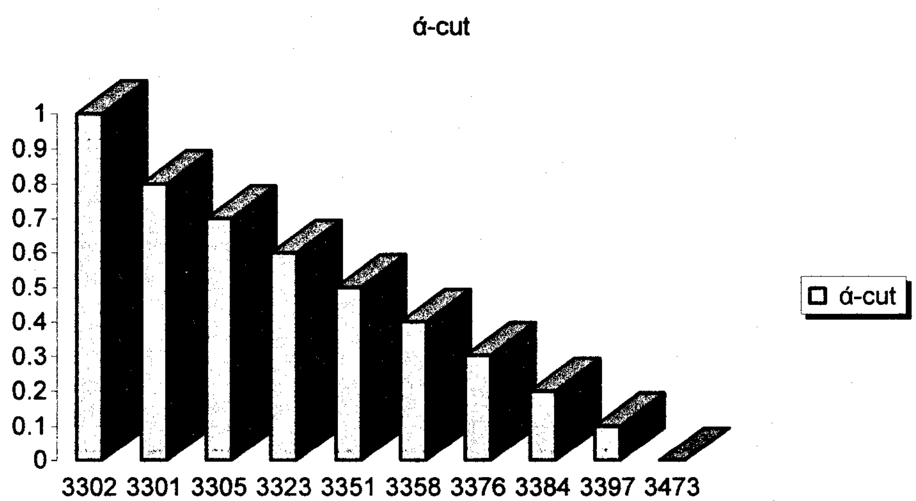


Figure (7-5B) 3D Membership Grade Versus Optimal Total Cost for 10% Deviation

Using this approach show that the limits on variables are not rigid and this makes it more suitable than standard LP for some practical cases where the small violations of the limits of the power system variables may be tolerable. The table shows that even though we had found some constraint violation we still did not go beyond the maximum limits set by J_1 the optimal solution are with in limits of the violation. This proves that fuzzy sets can represent a power system's operating conditions more realistically and so by fuzzifying certain variables, more satisfactory results can be obtained.

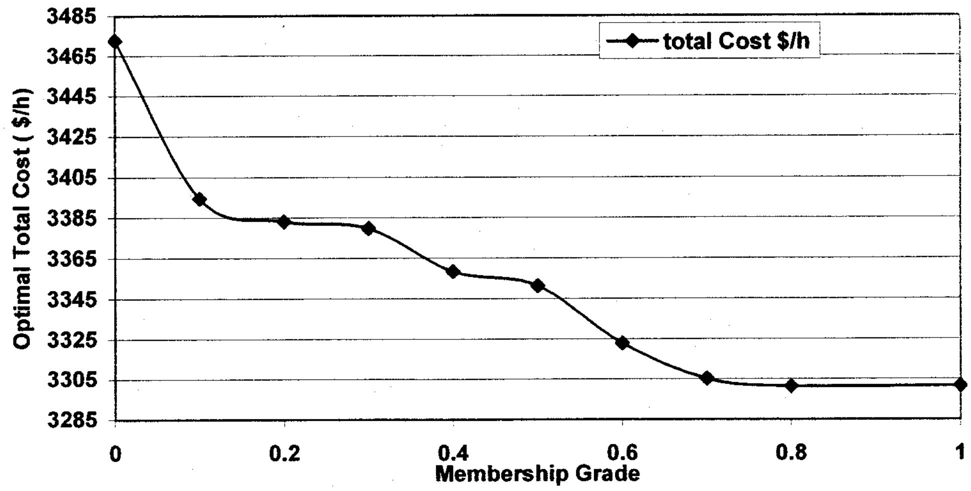


Figure (7-6) Optimal Total Cost Versus Membership Grade for 10% Deviation

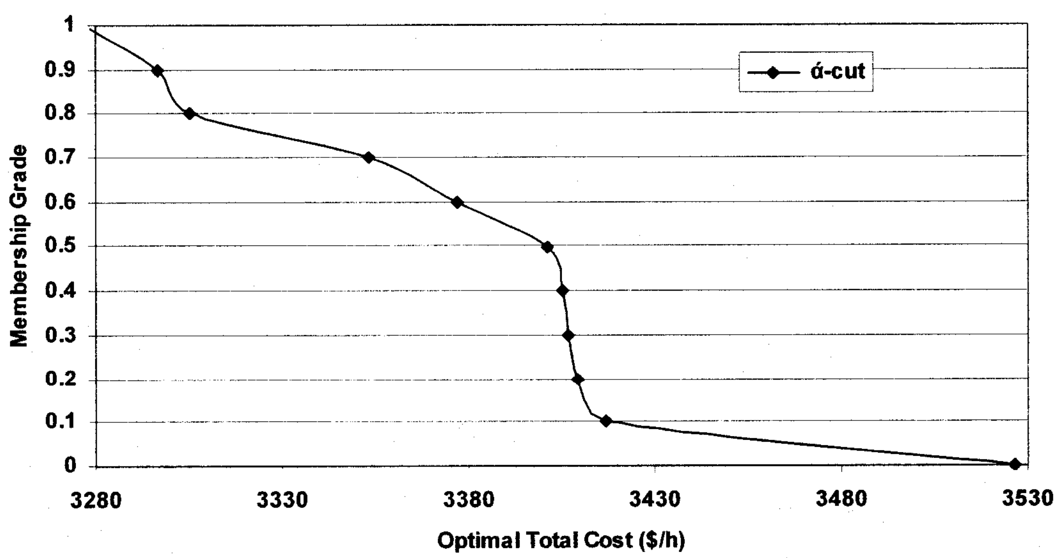


Figure (7-7A) Membership Grade Versus Optimal Total Cost for 15% Deviation

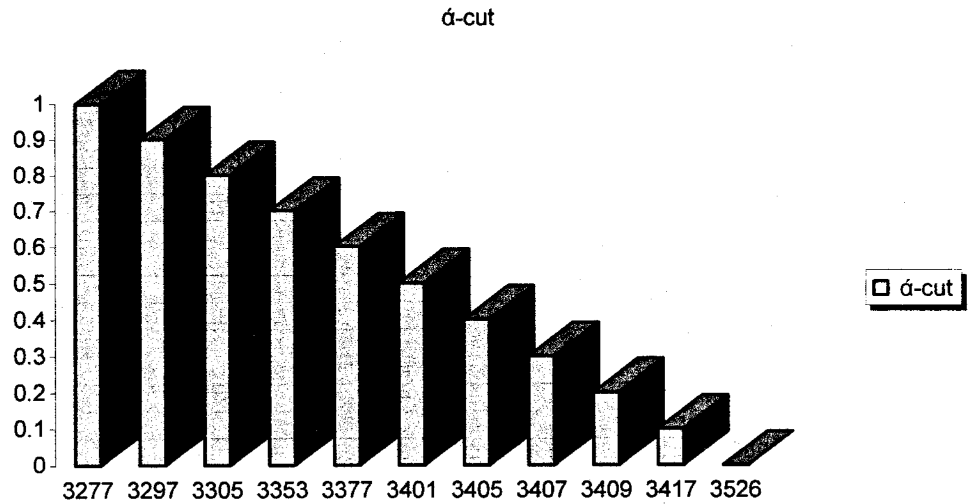


Figure (7-7B) 3D Membership Grade Versus Optimal Total Cost for 15% Deviation

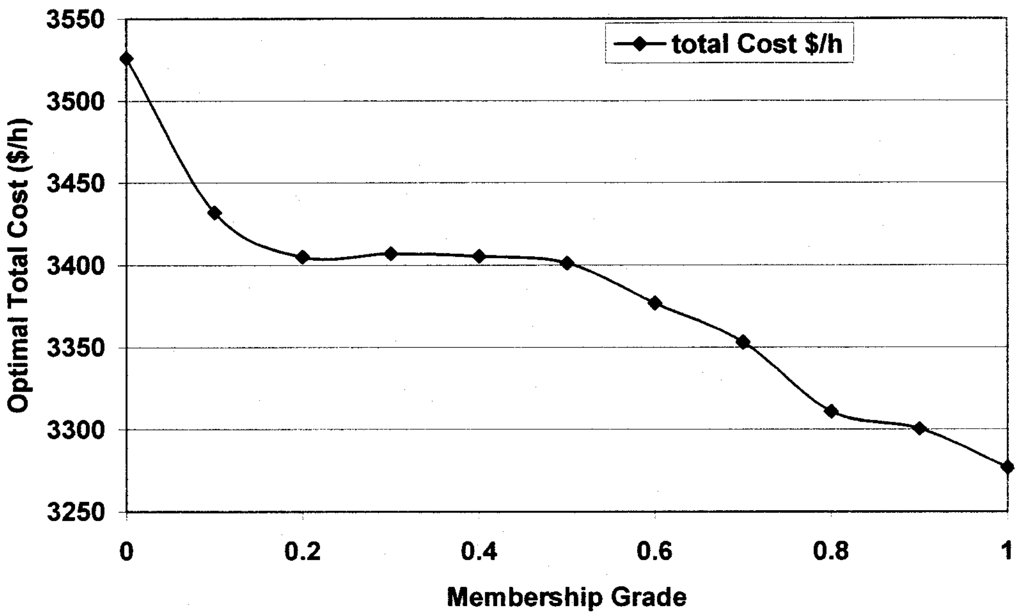


Figure (7-8) Optimal Total Cost Versus Membership Grade for 15% Deviation

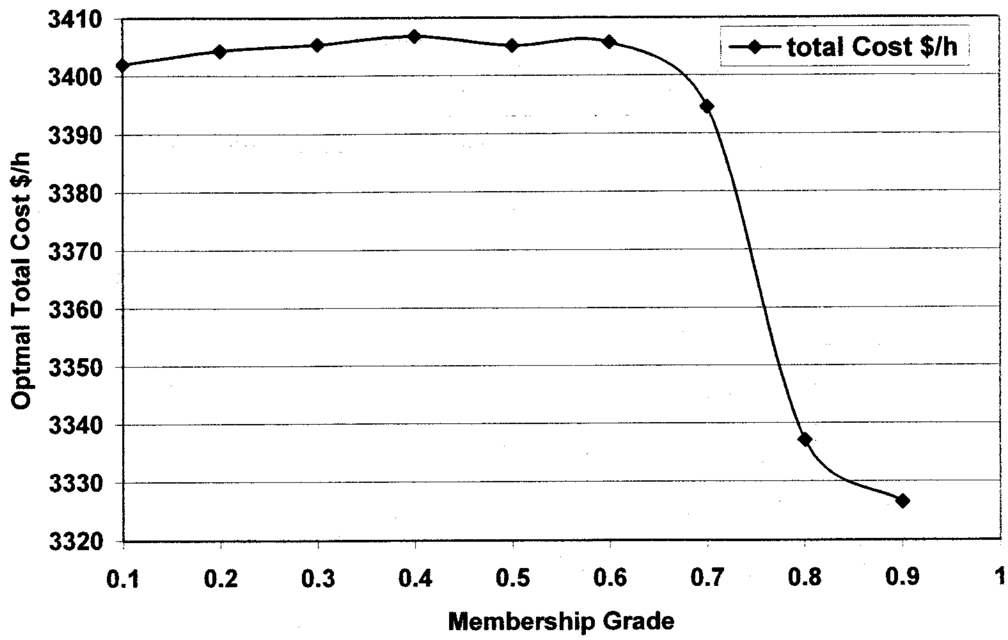


Figure (7-9) Optimal Total Cost Versus Membership Grade for 20% Deviation

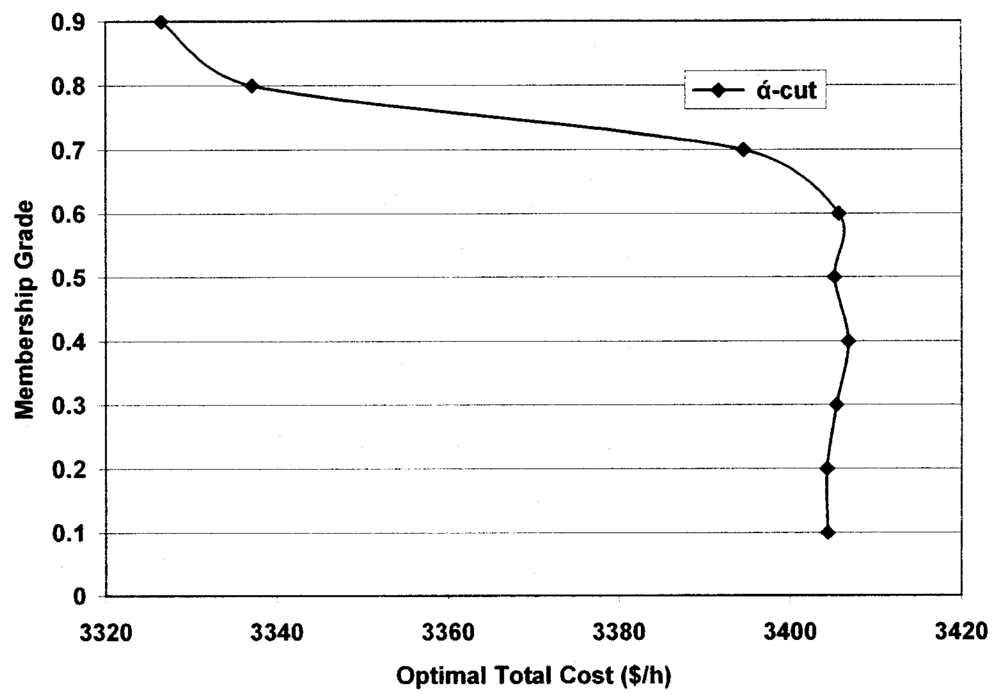


Figure (7-10) Membership Grade Versus Optimal Total Cost for 20% Deviation

Next, we will consider a fuzzy load demand that varies hour by hour for 24-hours. This means that its crisp or mean values vary at the same rate for 24-hours. The load demand of bus-9 will be selected for this operation, while keeping the load and generation of the other buses unchanged. Based on the results tabulated in Table (7-7A, 7-7B, 7-7C, 7-7D, 7-7E and 7-7F) with the same approach and calculation that was used on the last network, the minimum optimal cost is changed simultaneously with load variation of bus-9 and at each hour we get a linearly decreasing membership function as shown in Figure (7-11) for the second hour. Since 10% deviation of the load demand and generation gave the best result in the first case, it will be chosen to be the best allowable limit exceeding the load and generation expectation value. The tabulated values show the low limit J_0 and the high limit J_1 calculated by Werner's approach, the crisp minimum total cost and fuzzy minimum total cost. To analyze the reason why the fuzzy method produces a better result than the crisp OPF analysis is because data obtained by the fuzzy method can be adjusted based on the historical data or human experience. Selecting the right optimal solution from a number of available optimal values will reduce the outcome of the cost based on the input information applied to the system. When system demand varies suddenly the operator has to recalculate the minimum cost according to the sudden change. In fact, the fuzzy model calculated result of the sudden change is done once and included in the entire network hour by hour.

In this chapter an interactive decision-making process is formulated in which decision maker can learn to recognize good solution by considering all the possibilities of fuzziness. The formulation of the fuzzy model is to deal with imprecise nature of the decision maker judgment in OPF system operation. The proposed approach enables power system operator and planners to operate the system more economically for a given range of loads, while conflicting objectives such as minimum cost are modeled easily by using fuzzy sets. In addition the fuzzy approach can discriminate between different values of variables within their operating ranges which may be difficult to implement in conventional optimization techniques. The limitations on the constraint are restricted in the conventional OPF which can be overcome by using FNLP approach. The uncertainty in the OPF input data such as load values, generator characteristics and line flow limits can be translated into a fuzzy membership function then implemented into the formulation to achieve the crisp minimum objective cost of the system.

Table (7-7A) Fuzzy Nonlinear Programming Parameters for 24-Hours

ά-cut	Load+Gen. Deviation %	Fuzzy PG1 Left MW	Fuzzy PG1 Mid MW	Fuzzy PG1 Right MW	Crisp Total cost $/h	Optimal J0 $/h	Optimal J1 $/h	Optimal Cost $/h
0	10	112.0579	124.0873	136.1332	3085.164	3120.184	3198.713	3198.713
0.1	10	113.2601	124.0873	134.9279	3085.164	3120.184	3198.713	3165.561
0.2	10	114.4624	124.0873	133.7228	3085.164	3120.184	3198.713	3152.755
0.3	10	115.6649	124.0873	132.5178	3085.164	3120.184	3198.713	3146.697
0.4	10	116.8676	124.0873	131.3129	3085.164	3120.184	3198.713	3132.585
0.5	10	118.0705	124.0873	130.1083	3085.164	3120.184	3198.713	3125.875
0.6	10	119.2735	124.0873	128.9038	3085.164	3120.184	3198.713	3099.17
0.7	10	120.4767	124.0873	127.6994	3085.164	3120.184	3198.713	3082.534
0.8	10	121.6801	124.0873	126.4952	3085.164	3120.184	3198.713	3067.539
0.9	10	122.8836	124.0873	125.2912	3085.164	3120.184	3198.713	3077.559
1	10	124.0873	124.0873	124.0873	3085.164	3120.184	3198.713	3120.184
0	10	105.1821	116.4721	127.7783	3017.654	3052.128	3122.397	3122.397
0.1	10	106.3103	116.4721	126.647	3017.654	3052.128	3122.397	3097.679
0.2	10	107.4387	116.4721	125.5158	3017.654	3052.128	3122.397	3088.764
0.3	10	108.5673	116.4721	124.3848	3017.654	3052.128	3122.397	3082.82
0.4	10	109.696	116.4721	123.2539	3017.654	3052.128	3122.397	3068.971
0.5	10	110.8249	116.4721	122.1232	3017.654	3052.128	3122.397	3054.167
0.6	10	111.954	116.4721	120.9927	3017.654	3052.128	3122.397	3036.166
0.7	10	113.0833	116.4721	119.8623	3017.654	3052.128	3122.397	3019.845
0.8	10	114.2127	116.4721	118.7321	3017.654	3052.128	3122.397	3015.735
0.9	10	115.3423	116.4721	117.602	3017.654	3052.128	3122.397	3012.433
1	10	116.4721	116.4721	116.4721	3017.654	3052.128	3122.397	3052.128
0	10	101.308	112.1804	123.0689	2980.069	3014.23	3079.956	3079.956
0.1	10	102.3945	112.1804	121.9794	2980.069	3014.23	3079.956	3059.828
0.2	10	103.4812	112.1804	120.89	2980.069	3014.23	3079.956	3053.042
0.3	10	104.568	112.1804	119.8007	2980.069	3014.23	3079.956	3047.161
0.4	10	105.655	112.1804	118.7117	2980.069	3014.23	3079.956	3033.461
0.5	10	106.7421	112.1804	117.6227	2980.069	3014.23	3079.956	3026.946
0.6	10	107.8295	112.1804	116.534	2980.069	3014.23	3079.956	3000.992
0.7	10	108.917	112.1804	115.4453	2980.069	3014.23	3079.956	2984.853
0.8	10	110.0046	112.1804	114.3569	2980.069	3014.23	3079.956	2968.174
0.9	10	111.0925	112.1804	113.2686	2980.069	3014.23	3079.956	2976.145
1	10	112.1804	112.1804	112.1804	2980.069	3014.23	3079.956	3014.23
0	10	96.8481	107.2388	117.6454	2937.202	2970.998	3031.592	3031.592
0.1	10	97.8864	107.2388	116.6041	2937.202	2970.998	3031.592	3016.602
0.2	10	98.9249	107.2388	115.5629	2937.202	2970.998	3031.592	3012.208
0.3	10	99.9636	107.2388	114.5218	2937.202	2970.998	3031.592	3006.4
0.4	10	101.0024	107.2388	113.4809	2937.202	2970.998	3031.592	2992.868
0.5	10	102.0414	107.2388	112.4402	2937.202	2970.998	3031.592	2986.435
0.6	10	103.0806	107.2388	111.3996	2937.202	2970.998	3031.592	2960.794
0.7	10	104.1199	107.2388	110.3592	2937.202	2970.998	3031.592	2944.865
0.8	10	105.1594	107.2388	109.3189	2937.202	2970.998	3031.592	2927.58
0.9	10	106.199	107.2388	108.2788	2937.202	2970.998	3031.592	2934.73
1	10	107.2388	107.2388	107.2388	2937.202	2970.998	3031.592	2970.998

Table (7-7B) Fuzzy Nonlinear Programming Parameters for 24-Hours

ά-cut	Load+Gen. Deviation %	Fuzzy PG1 Left MW	Fuzzy PG1 Mid MW	Fuzzy PG1 Right MW	Crisp Total Cost $/h	Optimal J0 $/h	Optimal J1 $/h	Fuzzy Optimal Cost $/h
0	10	93.2726	103.2764	113.2959	2903.146	2936.647	2993.202	2986.321
0.1	10	94.2722	103.2764	112.2933	2903.146	2936.647	2993.202	2982.218
0.2	10	95.272	103.2764	111.2908	2903.146	2936.647	2993.202	2979.701
0.3	10	96.272	103.2764	110.2885	2903.146	2936.647	2993.202	2973.952
0.4	10	97.2721	103.2764	109.2863	2903.146	2936.647	2993.202	2960.557
0.5	10	98.2724	103.2764	108.2842	2903.146	2936.647	2993.202	2954.189
0.6	10	99.2729	103.2764	107.2824	2903.146	2936.647	2993.202	2928.799
0.7	10	100.2735	103.2764	106.2806	2903.146	2936.647	2993.202	2913.036
0.8	10	101.2743	103.2764	105.2791	2903.146	2936.647	2993.202	2895.304
0.9	10	102.2753	103.2764	104.2776	2903.146	2936.647	2993.202	2901.809
1	10	103.2764	103.2764	103.2764	2903.146	2936.647	2993.202	2871.403
0	10	96.7577	107.1386	117.5354	2936.337	2970.126	3030.617	3019.468
0.1	10	97.795	107.1386	116.495	2936.337	2970.126	3030.617	3015.729
0.2	10	98.8325	107.1386	115.4548	2936.337	2970.126	3030.617	3011.383
0.3	10	99.8702	107.1386	114.4148	2936.337	2970.126	3030.617	3005.577
0.4	10	100.9081	107.1386	113.3749	2936.337	2970.126	3030.617	2992.049
0.5	10	101.9461	107.1386	112.3351	2936.337	2970.126	3030.617	2985.617
0.6	10	102.9843	107.1386	111.2955	2936.337	2970.126	3030.617	2959.983
0.7	10	104.0226	107.1386	110.256	2936.337	2970.126	3030.617	2944.057
0.8	10	105.0611	107.1386	109.2167	2936.337	2970.126	3030.617	2926.761
0.9	10	106.0998	107.1386	108.1776	2936.337	2970.126	3030.617	2933.895
1	10	107.1386	107.1386	107.1386	2936.337	2970.126	3030.617	2909.145
0	10	99.5336	110.2145	120.9113	2962.962	2996.979	3060.65	3044.824
0.1	10	100.6009	110.2145	119.841	2962.962	2996.979	3060.65	3042.586
0.2	10	101.6684	110.2145	118.7708	2962.962	2996.979	3060.65	3036.759
0.3	10	102.7361	110.2145	117.7007	2962.962	2996.979	3060.65	3030.907
0.4	10	103.804	110.2145	116.6308	2962.962	2996.979	3060.65	3017.273
0.5	10	104.872	110.2145	115.561	2962.962	2996.979	3060.65	3010.79
0.6	10	105.9401	110.2145	114.4914	2962.962	2996.979	3060.65	2984.961
0.7	10	107.0085	110.2145	113.4219	2962.962	2996.979	3060.65	2968.906
0.8	10	108.077	110.2145	112.3526	2962.962	2996.979	3060.65	2951.979
0.9	10	109.1457	110.2145	111.2835	2962.962	2996.979	3060.65	2959.621
1	10	110.2145	110.2145	110.2145	2962.962	2996.979	3060.65	2930.519
0	10	103.4725	114.5783	125.7003	3001.028	3035.364	3103.619	3099.84
0.1	10	104.5823	114.5783	124.5874	3001.028	3035.364	3103.619	3080.94
0.2	10	105.6923	114.5783	123.4746	3001.028	3035.364	3103.619	3072.97
0.3	10	106.8024	114.5783	122.3621	3001.028	3035.364	3103.619	3067.054
0.4	10	107.9128	114.5783	121.2496	3001.028	3035.364	3103.619	3053.272
0.5	10	109.0233	114.5783	120.1373	3001.028	3035.364	3103.619	3046.719
0.6	10	110.134	114.5783	119.0252	3001.028	3035.364	3103.619	3020.615
0.7	10	111.2448	114.5783	117.9132	3001.028	3035.364	3103.619	3004.374
0.8	10	112.3558	114.5783	116.8014	3001.028	3035.364	3103.619	2988.008
0.9	10	113.467	114.5783	115.6898	3001.028	3035.364	3103.619	2996.383
1	10	114.5783	114.5783	114.5783	3001.028	3035.364	3103.619	2971.387

Table (7-7C) Fuzzy Nonlinear Programming Parameters for 24-Hours

á-cut	Load+Gen. Deviation %	Fuzzy PG1 Left MW	Fuzzy PG1 Mid MW	Fuzzy PG1 Right MW	Crisp Total Cost $/h	Optimal J0 $/h	Optimal J1 $/h	Fuzzy Optimal Cost $/h
0	10	114.5178	126.811	139.121	3109.563	3144.776	3226.321	3193.22
0.1	10	115.7463	126.811	137.8893	3109.563	3144.776	3226.321	3189.951
0.2	10	116.975	126.811	136.6577	3109.563	3144.776	3226.321	3175.829
0.3	10	118.2039	126.811	135.4263	3109.563	3144.776	3226.321	3179.219
0.4	10	119.433	126.811	134.1951	3109.563	3144.776	3226.321	3155.522
0.5	10	120.6622	126.811	132.964	3109.563	3144.776	3226.321	3148.767
0.6	10	121.8916	126.811	131.7331	3109.563	3144.776	3226.321	3121.886
0.7	10	123.1212	126.811	130.5023	3109.563	3144.776	3226.321	3105.14
0.8	10	124.351	126.811	129.2717	3109.563	3144.776	3226.321	3090.578
0.9	10	125.5809	126.811	128.0413	3109.563	3144.776	3226.321	3100.246
1	10	126.811	126.811	126.811	3109.563	3144.776	3226.321	3151.863
0	10	118.6441	131.3794	144.1315	3150.786	3186.32	3272.997	3236.322
0.1	10	119.9168	131.3794	142.8556	3150.786	3186.32	3272.997	3231.336
0.2	10	121.1897	131.3794	141.5798	3150.786	3186.32	3272.997	3214.747
0.3	10	122.4628	131.3794	140.3042	3150.786	3186.32	3272.997	3218.176
0.4	10	123.736	131.3794	139.0287	3150.786	3186.32	3272.997	3194.213
0.5	10	125.0095	131.3794	137.7534	3150.786	3186.32	3272.997	3187.382
0.6	10	126.2831	131.3794	136.4783	3150.786	3186.32	3272.997	3160.203
0.7	10	127.5569	131.3794	135.2033	3150.786	3186.32	3272.997	3143.271
0.8	10	128.8309	131.3794	133.9285	3150.786	3186.32	3272.997	3129.475
0.9	10	130.105	131.3794	132.6538	3150.786	3186.32	3272.997	3139.922
1	10	131.3794	131.3794	131.3794	3150.786	3186.32	3272.997	3193.492
0	10	118.2094	130.8981	143.6038	3146.426	3181.927	3268.058	3227.12
0.1	10	119.4774	130.8981	142.3325	3146.426	3181.927	3268.058	3226.961
0.2	10	120.7457	130.8981	141.0614	3146.426	3181.927	3268.058	3210.635
0.3	10	122.0141	130.8981	139.7904	3146.426	3181.927	3268.058	3214.059
0.4	10	123.2827	130.8981	138.5196	3146.426	3181.927	3268.058	3190.124
0.5	10	124.5515	130.8981	137.2489	3146.426	3181.927	3268.058	3183.301
0.6	10	125.8205	130.8981	135.9784	3146.426	3181.927	3268.058	3156.154
0.7	10	127.0897	130.8981	134.7081	3146.426	3181.927	3268.058	3139.242
0.8	10	128.359	130.8981	133.438	3146.426	3181.927	3268.058	3125.362
0.9	10	129.6285	130.8981	132.168	3146.426	3181.927	3268.058	3135.727
1	10	130.8981	130.8981	130.8981	3146.426	3181.927	3268.058	3189.089
0	10	122.4045	135.5419	148.6964	3188.675	3224.499	3315.931	3269.54
0.1	10	123.7175	135.5419	147.3802	3188.675	3224.499	3315.931	3267.836
0.2	10	125.0306	135.5419	146.0642	3188.675	3224.499	3315.931	3250.447
0.3	10	126.3439	135.5419	144.7484	3188.675	3224.499	3315.931	3253.91
0.4	10	127.6573	135.5419	143.4327	3188.675	3224.499	3315.931	3229.703
0.5	10	128.971	135.5419	142.1171	3188.675	3224.499	3315.931	3222.802
0.6	10	130.2848	135.5419	140.8017	3188.675	3224.499	3315.931	3195.352
0.7	10	131.5988	135.5419	139.4865	3188.675	3224.499	3315.931	3178.251
0.8	10	132.913	135.5419	138.1715	3188.675	3224.499	3315.931	3165.197
0.9	10	134.2274	135.5419	136.8566	3188.675	3224.499	3315.931	3176.366
1	10	135.5419	135.5419	135.5419	3188.675	3224.499	3315.931	3231.75

Table (7-7D) Fuzzy Nonlinear Programming Parameters for 24-Hours

á-cut	Load+Gen. Deviation %	Fuzzy PG1 Left MW	Fuzzy PG1 Mid MW	Fuzzy PG1 Right MW	Crisp Total cost $/h	Optimal J0 $/h	Optimal J1 $/h	Fuzzy Optimal Cost $/h
0	10	120.8064	133.7729	146.7565	3172.536	3208.236	3297.638	3255.12
0.1	10	122.1022	133.7729	145.4574	3172.536	3208.236	3297.638	3250.103
0.2	10	123.3982	133.7729	144.1585	3172.536	3208.236	3297.638	3235.248
0.3	10	124.6944	133.7729	142.8598	3172.536	3208.236	3297.638	3229.045
0.4	10	125.9908	133.7729	141.5612	3172.536	3208.236	3297.638	3214.593
0.5	10	127.2874	133.7729	140.2627	3172.536	3208.236	3297.638	3199.143
0.6	10	128.5842	133.7729	138.9644	3172.536	3208.236	3297.638	3180.387
0.7	10	129.8811	133.7729	137.6663	3172.536	3208.236	3297.638	3163.358
0.8	10	131.1782	133.7729	136.3683	3172.536	3208.236	3297.638	3149.984
0.9	10	132.4755	133.7729	135.0706	3172.536	3208.236	3297.638	3161.745
1	10	133.7729	133.7729	133.7729	3172.536	3208.236	3297.638	3129.038
0	10	117.6506	130.2795	142.9253	3140.828	3176.285	3261.717	3225.453
0.1	10	118.9127	130.2795	141.66	3140.828	3176.285	3261.717	3220.086
0.2	10	120.1749	130.2795	140.3949	3140.828	3176.285	3261.717	3205.353
0.3	10	121.4374	130.2795	139.1299	3140.828	3176.285	3261.717	3199.202
0.4	10	122.7	130.2795	137.8651	3140.828	3176.285	3261.717	3184.873
0.5	10	123.9629	130.2795	136.6004	3140.828	3176.285	3261.717	3169.554
0.6	10	125.2258	130.2795	135.3359	3140.828	3176.285	3261.717	3150.953
0.7	10	126.489	130.2795	134.0716	3140.828	3176.285	3261.717	3134.067
0.8	10	127.7523	130.2795	132.8074	3140.828	3176.285	3261.717	3120.081
0.9	10	129.0159	130.2795	131.5434	3140.828	3176.285	3261.717	3131.207
1	10	130.2795	130.2795	130.2795	3140.828	3176.285	3261.717	3100.062
0	10	114.4	126.6806	138.9779	3108.392	3143.596	3224.995	3189.412
0.1	10	115.6272	126.6806	137.7475	3108.392	3143.596	3224.995	3188.887
0.2	10	116.8547	126.6806	136.5172	3108.392	3143.596	3224.995	3174.722
0.3	10	118.0823	126.6806	135.2871	3108.392	3143.596	3224.995	3168.625
0.4	10	119.3101	126.6806	134.0571	3108.392	3143.596	3224.995	3154.422
0.5	10	120.5381	126.6806	132.8273	3108.392	3143.596	3224.995	3139.238
0.6	10	121.7663	126.6806	131.5976	3108.392	3143.596	3224.995	3120.796
0.7	10	122.9946	126.6806	130.3681	3108.392	3143.596	3224.995	3104.055
0.8	10	124.2231	126.6806	129.1388	3108.392	3143.596	3224.995	3089.472
0.9	10	125.4518	126.6806	127.9096	3108.392	3143.596	3224.995	3099.952
1	10	126.6806	126.6806	126.6806	3108.392	3143.596	3224.995	3067.385
0	10	111.6245	123.6073	135.6067	3080.878	3115.864	3193.865	3166.234
0.1	10	112.822	123.6073	134.406	3080.878	3115.864	3193.865	3161.256
0.2	10	114.0197	123.6073	133.2055	3080.878	3115.864	3193.865	3148.699
0.3	10	115.2175	123.6073	132.0052	3080.878	3115.864	3193.865	3142.648
0.4	10	116.4155	123.6073	130.805	3080.878	3115.864	3193.865	3128.552
0.5	10	117.6137	123.6073	129.605	3080.878	3115.864	3193.865	3121.851
0.6	10	118.8121	123.6073	128.4051	3080.878	3115.864	3193.865	3095.176
0.7	10	120.0107	123.6073	127.2054	3080.878	3115.864	3193.865	3078.56
0.8	10	121.2094	123.6073	126.0059	3080.878	3115.864	3193.865	3063.491
0.9	10	122.4083	123.6073	124.8065	3080.878	3115.864	3193.865	3073.427
1	10	123.6073	123.6073	123.6073	3080.878	3115.864	3193.865	3040.894

Table (7-7E) Fuzzy Nonlinear Programming Parameters for 24-Hours

ά-cut	Load+Gen. Deviation %	Fuzzy PG1 Left MW	Fuzzy PG1 Mid MW	Fuzzy PG1 Right MW	Crisp Total cost $/h	Optimal J0 $/h	Optimal J1 $/h	Fuzzy Optimal Cost $/h
0	10	110.6031	122.4763	134.3658	3070.795	3105.7	3182.46	3153.344
0.1	10	111.7897	122.4763	133.1762	3070.795	3105.7	3182.46	3151.124
0.2	10	112.9764	122.4763	131.9867	3070.795	3105.7	3182.46	3139.153
0.3	10	114.1632	122.4763	130.7973	3070.795	3105.7	3182.46	3133.12
0.4	10	115.3503	122.4763	129.6081	3070.795	3105.7	3182.46	3119.063
0.5	10	116.5375	122.4763	128.4191	3070.795	3105.7	3182.46	3112.38
0.6	10	117.7249	122.4763	127.2302	3070.795	3105.7	3182.46	3085.779
0.7	10	118.9125	122.4763	126.0415	3070.795	3105.7	3182.46	3069.208
0.8	10	120.1003	122.4763	124.8529	3070.795	3105.7	3182.46	3053.966
0.9	10	121.2882	122.4763	123.6645	3070.795	3105.7	3182.46	3063.703
1	10	122.4763	122.4763	122.4763	3070.795	3105.7	3182.46	3031.135
0	10	111.7917	123.7924	135.8097	3082.53	3117.53	3195.734	3165.34
0.1	10	112.991	123.7924	134.6073	3082.53	3117.53	3195.734	3162.916
0.2	10	114.1904	123.7924	133.405	3082.53	3117.53	3195.734	3150.263
0.3	10	115.3901	123.7924	132.2029	3082.53	3117.53	3195.734	3144.21
0.4	10	116.5899	123.7924	131.0009	3082.53	3117.53	3195.734	3130.107
0.5	10	117.7899	123.7924	129.7991	3082.53	3117.53	3195.734	3115.031
0.6	10	118.9901	123.7924	128.5975	3082.53	3117.53	3195.734	3096.716
0.7	10	120.1904	123.7924	127.396	3082.53	3117.53	3195.734	3080.092
0.8	10	121.3909	123.7924	126.1946	3082.53	3117.53	3195.734	3065.052
0.9	10	122.5916	123.7924	124.9935	3082.53	3117.53	3195.734	3075.02
1	10	123.7924	123.7924	123.7924	3082.53	3117.53	3195.734	3044.073
0	10	116.3225	128.8092	141.3127	3127.548	3162.902	3246.68	3209.122
0.1	10	117.5704	128.8092	140.0616	3127.548	3162.902	3246.68	3207.499
0.2	10	118.8184	128.8092	138.8107	3127.548	3162.902	3246.68	3192.818
0.3	10	120.0666	128.8092	137.5599	3127.548	3162.902	3246.68	3186.69
0.4	10	121.3151	128.8092	136.3094	3127.548	3162.902	3246.68	3172.412
0.5	10	122.5636	128.8092	135.0589	3127.548	3162.902	3246.68	3157.148
0.6	10	123.8124	128.8092	133.8087	3127.548	3162.902	3246.68	3138.612
0.7	10	125.0614	128.8092	132.5585	3127.548	3162.902	3246.68	3121.785
0.8	10	126.3105	128.8092	131.3086	3127.548	3162.902	3246.68	3107.551
0.9	10	127.5597	128.8092	130.0588	3127.548	3162.902	3246.68	3118.412
1	10	128.8092	128.8092	128.8092	3127.548	3162.902	3246.68	3086.413
0	10	113.3836	125.5552	137.7434	3098.297	3133.421	3213.571	3179.98
0.1	10	114.5999	125.5552	136.5239	3098.297	3133.421	3213.571	3178.751
0.2	10	115.8165	125.5552	135.3045	3098.297	3133.421	3213.571	3165.178
0.3	10	117.0332	125.5552	134.0853	3098.297	3133.421	3213.571	3159.098
0.4	10	118.2501	125.5552	132.8662	3098.297	3133.421	3213.571	3144.934
0.5	10	119.4672	125.5552	131.6473	3098.297	3133.421	3213.571	3129.792
0.6	10	120.6845	125.5552	130.4286	3098.297	3133.421	3213.571	3111.4
0.7	10	121.9019	125.5552	129.21	3098.297	3133.421	3213.571	3094.704
0.8	10	123.1195	125.5552	127.9915	3098.297	3133.421	3213.571	3079.941
0.9	10	124.3372	125.5552	126.7733	3098.297	3133.421	3213.571	3090.221

Table (7-7F) Fuzzy Nonlinear Programming Parameters for 24-Hours

á-cut	Load+Gen. Deviation %	Fuzzy P1 Left MW	Fuzzy P1 Mid MW	Fuzzy P1 Right MW	Crisp Total cost $/h	Optimal J0 $/h	Optimal J1 $/h	Fuzzy Optimal Cost $/h
0	10	114.2078	126.4679	138.7445	3106.482	3141.671	3222.833	3188.35
0.1	10	115.433	126.4679	137.5162	3106.482	3141.671	3222.833	3186.969
0.2	10	116.6584	126.4679	136.2879	3106.482	3141.671	3222.833	3172.916
0.3	10	117.884	126.4679	135.0599	3106.482	3141.671	3222.833	3166.823
0.4	10	119.1097	126.4679	133.832	3106.482	3141.671	3222.833	3152.627
0.5	10	120.3357	126.4679	132.6042	3106.482	3141.671	3222.833	3137.451
0.6	10	121.5618	126.4679	131.3766	3106.482	3141.671	3222.833	3119.018
0.7	10	122.788	126.4679	130.1492	3106.482	3141.671	3222.833	3102.286
0.8	10	124.0145	126.4679	128.9219	3106.482	3141.671	3222.833	3087.668
0.9	10	125.2411	126.4679	127.6948	3106.482	3141.671	3222.833	3098.11
1	10	126.4679	126.4679	126.4679	3106.482	3141.671	3222.833	3065.654
0	10	118.8118	131.565	144.3351	3152.47	3188.016	3274.903	3234.44
0.1	10	120.0863	131.565	143.0574	3152.47	3188.016	3274.903	3231.113
0.2	10	121.361	131.565	141.7798	3152.47	3188.016	3274.903	3216.335
0.3	10	122.6358	131.565	140.5024	3152.47	3188.016	3274.903	3210.165
0.4	10	123.9109	131.565	139.2251	3152.47	3188.016	3274.903	3195.791
0.5	10	125.1862	131.565	137.948	3152.47	3188.016	3274.903	3188.956
0.6	10	126.4616	131.565	136.6711	3152.47	3188.016	3274.903	3161.766
0.7	10	127.7372	131.565	135.3943	3152.47	3188.016	3274.903	3144.827
0.8	10	129.0129	131.565	134.1177	3152.47	3188.016	3274.903	3131.062
0.9	10	130.2889	131.565	132.8413	3152.47	3188.016	3274.903	3142.421
1	10	131.565	131.565	131.565	3152.47	3188.016	3274.903	3116.847
0	10	117.0671	129.6336	142.2169	3134.989	3170.401	3255.105	3216.332
0.1	10	118.323	129.6336	140.9578	3134.989	3170.401	3255.105	3214.553
0.2	10	119.579	129.6336	139.6989	3134.989	3170.401	3255.105	3199.843
0.3	10	120.8352	129.6336	138.4402	3134.989	3170.401	3255.105	3193.702
0.4	10	122.0916	129.6336	137.1816	3134.989	3170.401	3255.105	3179.395
0.5	10	123.3482	129.6336	135.9232	3134.989	3170.401	3255.105	3164.101
0.6	10	124.6049	129.6336	134.665	3134.989	3170.401	3255.105	3145.528
0.7	10	125.8618	129.6336	133.4069	3134.989	3170.401	3255.105	3128.668
0.8	10	127.1189	129.6336	132.1489	3134.989	3170.401	3255.105	3114.572
0.9	10	128.3762	129.6336	130.8912	3134.989	3170.401	3255.105	3125.581
1	10	129.6336	129.6336	129.6336	3134.989	3170.401	3255.105	3097.886
0	10	111.6988	123.6896	135.6969	3081.612	3116.604	3194.695	3166.754
0.1	10	112.8971	123.6896	134.4955	3081.612	3116.604	3194.695	3161.994
0.2	10	114.0956	123.6896	133.2942	3081.612	3116.604	3194.695	3149.394
0.3	10	115.2942	123.6896	132.0931	3081.612	3116.604	3194.695	3143.342
0.4	10	116.493	123.6896	130.8921	3081.612	3116.604	3194.695	3129.243
0.5	10	117.692	123.6896	129.6913	3081.612	3116.604	3194.695	3122.54
0.6	10	118.8912	123.6896	128.4906	3081.612	3116.604	3194.695	3095.861
0.7	10	120.0906	123.6896	127.2901	3081.612	3116.604	3194.695	3079.241
0.8	10	121.2901	123.6896	126.0898	3081.612	3116.604	3194.695	3064.185
0.9	10	122.4897	123.6896	124.8896	3081.612	3116.604	3194.695	3074.135

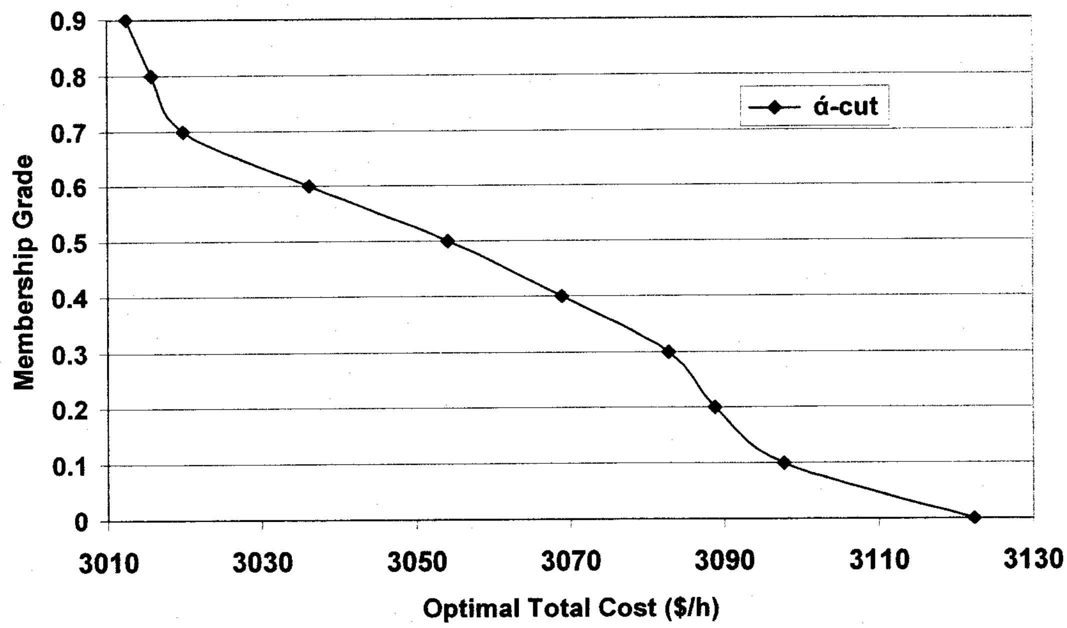


Figure (7-11) Membership Grade Versus Optimal Total Cost for the Second Hour of 24-Hours

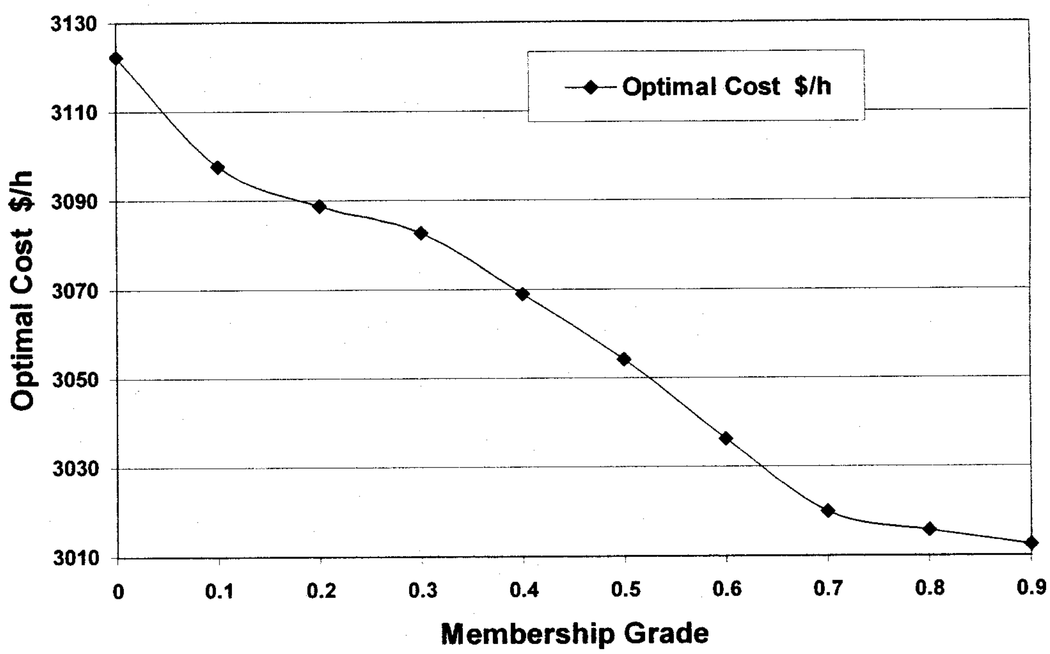


Figure (7-12) Optimal Total Cost Versus Membership Grade for the Second Hour of 24 Hours

Chapter 8

Fuzzy Active, Reactive Power Flow and the Parameter of the Objective Function as Fuzzy

8.1 Introduction

In this chapter we will transform the coefficient of the objective function α_i, β_i and γ_i into fuzzy parameters. Then the FNLP will become fuzzy objective function, $\tilde{\alpha}_i$, $\tilde{\beta}_i$ and $\tilde{\gamma}_i$ with fuzzy resources. This type of FNLP is called possibilistic programming or stochastic programming. It has been used for decision making where input data (coefficients in LP) has been given probability distributions. Many research areas such as speech recognition, robotics, medical diagnosis, analysis of rare events, decision-making under uncertainty, picture analysis, information retrieval and related areas have used this method. The pioneers in this area are Lai and Hwang [56], [59].

8.2 Problem Formulation

The methods used in chapter (3) section (3.9.3) by Lai & Hwang will be used to formulate the problem of fuzzy objective function and fuzzy resources. The procedure will start by transforming the crisp parameters of the cost function into fuzzy coefficients $\tilde{\alpha}_i$, $\tilde{\beta}_i$ and $\tilde{\gamma}_i$ with a piecewise linear triangular membership function as shown in Figure (8-1).

The mathematical formula that describes the membership function of $\tilde{\alpha}_i$ is:

$$\mu(\tilde{\alpha}_i) = \begin{cases} 1 & \alpha_i = \alpha^0 \\ 1-(\alpha^0-\alpha_i)/(\alpha^0-\alpha^-) & \alpha^- < \alpha_i < \alpha^0 \\ 1-(\alpha_i-\alpha^0)/(\alpha^+-\alpha^0) & \alpha^0 < \alpha_i < \alpha^+ \\ 0 & otherwise \end{cases} \tag{8.1}$$

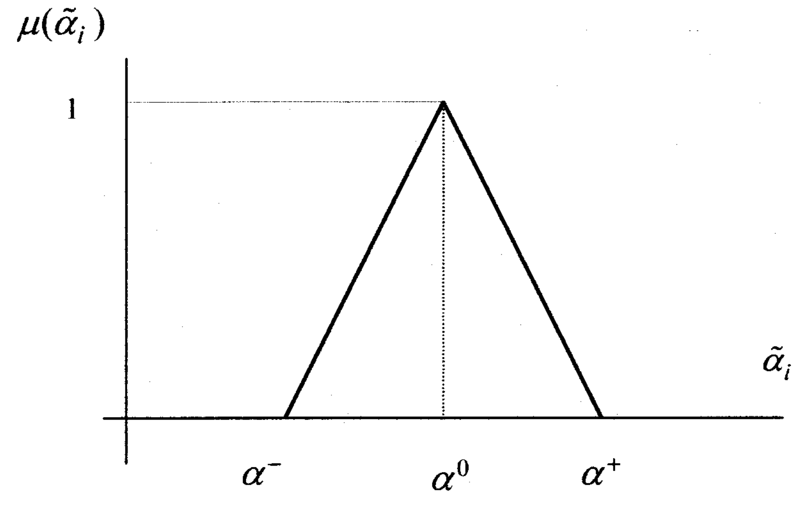


Figure (8-1) A Triangular Membership Function for $\tilde{\alpha}_i$ Coefficient

The parameter α^0 is the nominal value of α_i, having the maximum grade of membership and α^+ and α^- are the maximum and minimum allowed values of the parameter of the membership function. The same applies to $\tilde{\beta}_i$ and $\tilde{\gamma}_i$ membership functions with the same triangular shape as shown in Figure (8-2) and Figure (8-3).

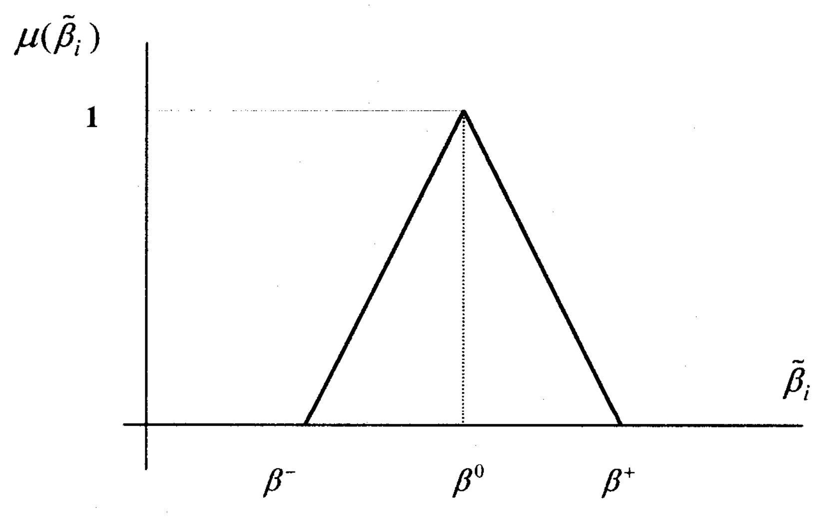


Figure (8-2) A Triangular Membership Function for $\tilde{\beta}_i$ Coefficient

The mathematical formula for $\mu(\beta_i)$ is:

$$\mu(\tilde{\beta}_i)=\begin{Bmatrix} 1 & \beta_i=\beta^0 \\ 1-(\beta^0-\beta_i)/(\beta^0-\beta^-) & \beta^-<\beta_i<\beta^0 \\ 1-(\beta_i-\beta^0)/(\beta^+-\beta^0) & \beta^0<\beta_i<\beta^+ \\ 0 & otherwise \end{Bmatrix} \tag{8.2}$$

Likewise, the mathematical formula for $\mu(\gamma_i)$ is:

$$\mu(\tilde{\gamma}_i)=\begin{Bmatrix} 1 & \gamma_i=\gamma^0 \\ 1-(\gamma^0-\gamma_i)/(\gamma^0-\gamma^-) & \gamma^-<\gamma_i<\gamma^0 \\ 1-(\gamma_i-\gamma^0)/(\gamma^+-\gamma^0) & \gamma^0<\gamma_i<\gamma^+ \\ 0 & otherwise \end{Bmatrix} \tag{8.3}$$

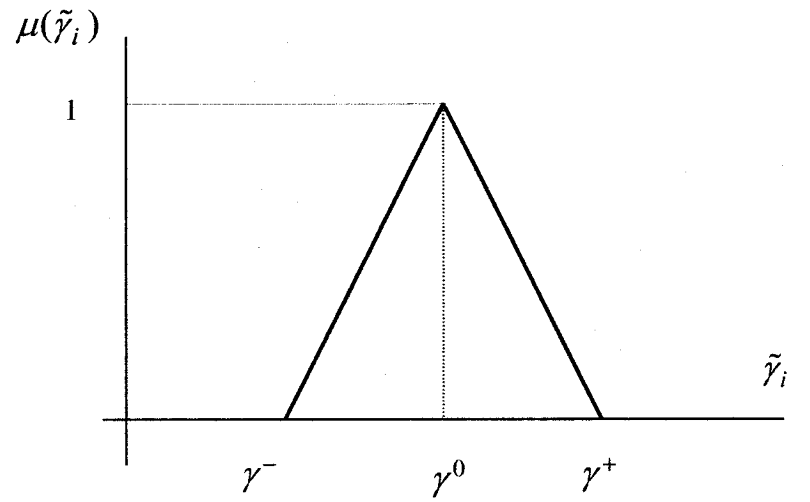


Figure (8-3) A Triangular Membership Function for $\tilde{\gamma}_i$ Coefficient

The total objective function with fuzzy coefficients is expressed as:

$$\tilde{J}_F=\sum_{i=1}^{NG}(\tilde{\alpha}_i+\tilde{\beta}_i\tilde{P}_{G_i}+\tilde{\gamma}_i\tilde{P}_{G_i}^2) \tag{8.4}$$

The optimization problem becomes non linear programming with fuzzy objective coefficients and fuzzy constraints. Applying the NLP optimization procedure, which was discussed in chapter (3), where, equation (3.48) states:

$$\begin{aligned} &\text{maximize} \quad \tilde{c}x \\ &\text{subject to} \quad \tilde{A}x \le \tilde{b} \\ &\qquad\qquad\quad x \ge 0 \end{aligned}$$

Where $\tilde{c}$, $\tilde{A}$ and $\tilde{b}$ consist of triangular fuzzy numbers each parameter will have its middle, left and right expressed as $\tilde{c} = (c^-, c^0, c^+)$, $\tilde{A} = (A^-, A^0, A^+)$ and $\tilde{b} = (b^-, b^0, b^+)$. The procedure in chapter (3) formulates the fuzzy objective function with fuzzy coefficients to maximize the possibility of obtaining a higher profit which explains maximizing the right side of the triangular membership function and minimizing the left side to avoid the risk of obtaining a lower profit. However, to minimize the cost value we maximize the left side while minimizing the right. To do this, some adjustment should be done to the formulation to obtain the optimal minimum cost. In chapter (3) Lai and Hwang's approach or strategy was to shift the triangular membership function to the right which will maximize the middle value, minimize the left spread and maximize the right spread. In this chapter we will perform the opposite approach to obtain the total minimum cost of the objective function. The triangle will be shifted to the left side which will minimize the middle value, minimize the right spread to avoid the 'risk of paying higher cost' and maximize the left spread to obtain the minimum optimal total cost value. The procedure starts by converting equation (3.52) from chapter (3) shown below into a multiple objective linear programming problem.

$$\begin{aligned} &\text{maximize} && z1 = (c^0 - c^-)x \\ &\text{minimize} && z2 = c^0 x \\ &\text{minimize} && z3 = (c^+ - c^0)x \\ &\text{subject to} && A_\beta^- x \le b_\beta^-, A_\beta^0 x \le b_\beta^0, A_\beta^+ x \le b_\beta^+ \\ & && x \ge 0 \end{aligned}$$

Our goal is to fuzzify the objective coefficients $\tilde{\alpha}_i$, $\tilde{\beta}_i$ and $\tilde{\gamma}_i$ which are a triangular membership function shown in Figures (8-1, 2, 3) with right, middle and left sides. Replacing the fuzzy objective coefficients in equation (3.52), it then becomes:

$$
\begin{aligned}
&\text{maximize} && z1=(\alpha_i^{\,0}-\alpha_i^{\,-})+(\beta_i^{\,0}-\beta_i^{\,-})P_{G_i}^{\,-}+(\gamma_i^{\,0}-\gamma_i^{\,-})(P_{G_i}^{\,-})^2 \\
&\text{minimize} && z2=(\alpha_i^{\,0})+(\beta_i^{\,0})P_{G_i}^{\,0}+(\gamma_i^{\,0})(P_{G_i}^{\,0})^2 \\
&\text{minimize} && z3=(\alpha_i^{\,+}-\alpha_i^{\,0})+(\beta_i^{\,+}-\beta_i^{\,0})P_{G_i}^{\,+}+(\gamma_i^{\,+}-\gamma_i^{\,0})(P_{G_i}^{\,+})^2 \\
&\text{subject to} && P_i(V,\delta)\,\tilde{\leq}\,\tilde{Pk}_i \\
& && Q_i(V,\delta)\,\tilde{\leq}\,\tilde{Qk}_i \\
& && x\geq 0
\end{aligned} \tag{8.5}
$$

Lai and Hwang's method to solve this problem was to characterize the three objective functions by membership functions (Z1, Z2, and Z3). For a minimum cost value some changes have to be done to go from maximization to minimization of the total cost for these parameters. The new adjustment is shown in Figure (8-4). $Z1$ is the total cost generation of the left side, Z2 is the total cost generation of the middle side and Z3 is the total cost generation of the right side. In order to obtain (Z1, Z2, Z3), we need to calculate the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) of the objective function shown in Figure (8-4). This is calculated as:

$$
\begin{aligned}
z_1^{\,P} &= \max_{x\in X}(z_1)=\max_{x\in X}[(\alpha_i^{\,0}-\alpha_i^{\,-})+(\beta_i^{0}-\beta_i^{-})P_{G_i}^{\,-}+(\gamma_i^{\,0}-\gamma_i^{\,-})(P_{G_i}^{\,-})^2] \\
z_1^{\,N} &= \min_{x\in X}(z_1)=\min_{x\in X}[(\alpha_i^{\,0}-\alpha_i^{\,-})+(\beta_i^{0}-\beta_i^{-})P_{G_i}^{\,-}+(\gamma_i^{\,0}-\gamma_i^{\,-})(P_{G_i}^{\,-})^2] \\
z_2^{\,P} &= \min_{x\in X}(z_2)=\min_{x\in X}[(\alpha_i^{\,0})+(\beta_i^{0})P_{G_i}^{\,0}+(\gamma_i^{\,0})(P_{G_i}^{\,0})^2] \\
z_2^{\,N} &= \max_{x\in X}(z_2)=\max_{x\in X}[(\alpha_i^{\,0})+(\beta_i^{0})P_{G_i}^{\,0}+(\gamma_i^{\,0})(P_{G_i}^{\,0})^2] \\
z_3^{\,P} &= \min_{x\in X}(z_3)=\min_{x\in X}[(\alpha_i^{\,+}-\alpha_i^{\,0})+(\beta_i^{+}-\beta_i^{0})P_{G_i}^{\,+}+(\gamma_i^{\,+}-\gamma_i^{\,0})(P_{G_i}^{\,+})^2] \\
z_3^{\,N} &= \max_{x\in X}(z_3)=\max_{x\in X}[(\alpha_i^{\,+}-\alpha_i^{\,0})+(\beta_i^{+}-\beta_i^{0})P_{G_i}^{\,+}+(\gamma_i^{\,+}-\gamma_i^{\,0})(P_{G_i}^{\,+})^2]
\end{aligned} \tag{8.6}
$$

And their membership function mathematical formula can be calculated as:

$$
\mu_{z_1}(x)=\begin{cases} 1 & \text{if} \quad z_1\geq z_1^{\,P} \\ \dfrac{z_1-z_1^{\,N}}{z_1^{\,P}-z_1^{\,N}} & \text{if} \quad z_1^{\,N}\leq z_1\leq z_1^{\,P} \\ 0 & \text{if} \quad z_1\leq z_1^{\,N} \end{cases} \tag{8.7}
$$

$$
\mu_{z_2}(x)=\begin{cases} 1 & \text{if} \quad z_2\leq z_2^{\,P} \\ \dfrac{z_2^{\,N}-z_2}{z_2^{\,N}-z_2^{\,P}} & \text{if} \quad z_2^{\,P}\leq z_2\leq z_2^{\,N} \\ 0 & \text{if} \quad z_2\geq z_2^{\,N} \end{cases} \tag{8.8}
$$

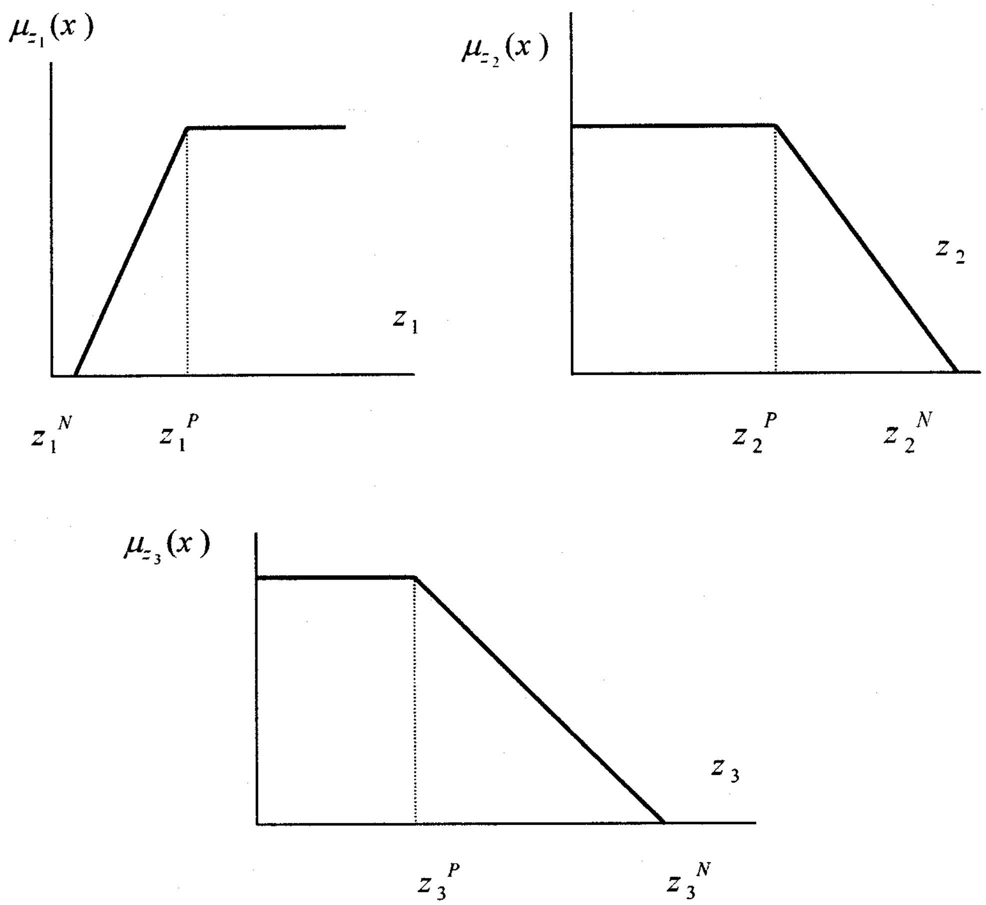


Figure (8-4) The Membership Function of the Objective z_1, z_2 and z_3

$$\mu_{z_3}(x) = \begin{cases} 1 & \text{if} \quad z_3 \le z_3^P \\ \dfrac{z_3^N - z_3}{z_3^N - z_3^P} & \text{if} \quad z_3^P \le z_3 \le z_3^N \\ 0 & \text{if} \quad z_3 \ge z_3^N \end{cases} \tag{8.9}$$

Finally the problem is solved by calculating the standard FNLP optimization as in equation (3.47) shown below:

$$\begin{aligned} &\text{maximize} \quad \alpha \\ &\text{subject to} \quad \mu_{z_i}(\mathrm{x}) \geq \alpha, \quad \mathrm{i}=1,2,3 \\ &\qquad A_\beta^{-}x \leq b_\beta^{-}, A_\beta^{0}x \leq b_\beta^{0}, A_\beta^{+}x \leq b_\beta^{+} \end{aligned}$$

In order to avoid confusion with the symbol α, β used for the fuzzy coefficient in the objective function with the equation shown above we replaced α with $\mathrm{f}(z)$, $A_\beta x \leq b_\beta$ with $P_\varphi \tilde{k}_i$ which is the difference between load and generator for active and reactive power and φ for β which is the minimal acceptable possibility value. Then the equation becomes:

$$\begin{aligned} &\text{minimize} \quad (-\mathrm{f}(z)) \\ &\text{subject to} \quad \mu_{z_i}(\mathrm{x}) \geq \mathrm{f}(z), \quad \mathrm{i}=1,2,3 \\ &\qquad P_i(V,\delta) \lesssim P_\varphi^{-}\tilde{k}_i, P_\varphi^{0}\tilde{k}_i, P_\varphi^{+}\tilde{k}_i \\ &\qquad \mathrm{Q}_i(V,\delta) \lesssim Q_\varphi^{-}\tilde{k}_i, Q_\varphi^{0}\tilde{k}_i, Q_\varphi^{+}\tilde{k}_i \\ &\qquad x \geq 0 \end{aligned} \tag{8.10}$$

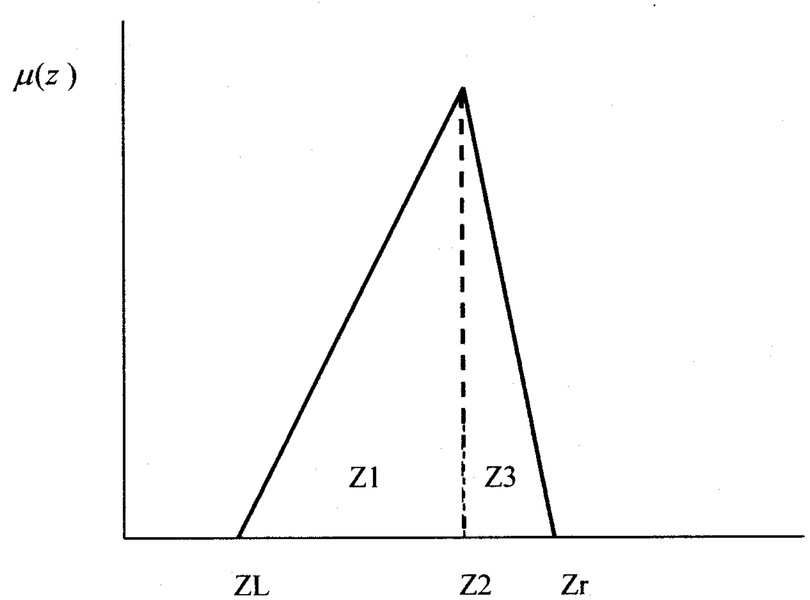


Figure (8-5) The Possibilistic Distributions of the Optimal Total Cost

The solution obtained according to equation (8.7) through (8.10) will be a triangular membership with middle, left and right representing the three parameters Z1, Z2 and Z3 shown in Figure (8-5), which was obtained from the triangular membership function shown in Figure (8-4). Notice that the left side of the triangular membership is maximized while the right side is minimized to obtain the optimal minimum cost solution of the problem.

8.3 Simulated Example

The 9-bus system that was used in chapter (7) will be used to simulate the procedure in this chapter. Two cases listed in Table (8-1) and (8-2) for various coefficients values, active reactive (load and generation) percentage deviation and minimal acceptable possibility value were tested in the formulated procedure. In the first case, the final result tabulation is shown in Tables (8-3A) and (8-3B). The graph plotted in Figure (8-6) shows the positive ideal solution and the negative ideal solution of the two triangular membership functions. The simulation provide a minimum middle optimal solution value of the triangular membership function represented by (Z2), a minimum optimal solution for the right side (ZR) which consists of adding the middle value (Z2) to the right spread of the optimal solution (Z3) and the maximum left side (ZL) which consists of subtracting the middle value (Z2) from the optimal solution of the left spread (Z1). Defining the left, middle and right sides of the triangular membership we get:

$$ZR = Z2 + Z3$$
$$Zm = Z2$$
$$ZL = Z2 - Z1$$

The final optimal solution of the total cost is shown in Figure (8-7) where the plot of the triangular membership function shows clearly the maximum left spread which leads to minimum cost values in dollars and minimizing the right spread which leads to maximum cost value. The decision maker can then chose which is the best optimal solution from the triangular membership function shown in Figure (8-7, 8-9 and 8-13). The input data shown in Table (8-1) was also tested on a higher ($\varphi = 0.75$) and a lower ($\varphi = 0.35$) minimal acceptable possibility values. Figure (8-7) and (8-9) are the results of ($\varphi = 0.55$), ($\varphi = 0.35$) respectively. The higher minimal acceptable possibility value shifted the optimal total cost triangular membership function a little to the left while the lower than 0.55 shifted the triangular to the right side but did not change the shape of the membership function where the left side is maximized and the right side is minimized as shown in Figure (8-9). $\tilde{\alpha}$ value makes the difference in the cost value. Any large deviation will make the cost value very expensive and shift the triangular to the left if the deviation on the left spread is larger than the right side, for this reason the 9-bus was tested with $\tilde{\alpha}$ coefficient that has equal deviation for the left and right sides. The result in Figure (8-10) and (8-11) clearly show that we still obtain a maximum spread to the left side and a minimum spread to the right, which shows the effectiveness of this procedure.

Table (8-1)

$\varphi = 0.55$	Fuzzy Coefficients	Left	Right
	$\tilde{\alpha}$	3%	2%
Case 1	$\tilde{\beta}$	7%	3%
	$\tilde{\gamma}$	5%	3%

Table (8-2)

$\varphi = 0.5$	Fuzzy Coefficients	Left	Right
	$\tilde{\alpha}$	2%	1%
Case 2	$\tilde{\beta}$	4%	2%
	$\tilde{\gamma}$	2%	4%

In case 2, Table (8-4A) and (8-4B) shows all the tabulated result. In Figure (8-13) the optimal total cost is shifted further to the left side due to the lower selection of the fuzzy coefficients parameter and active reactive load and generation values, where the cost function is dependent on the variation of those values from equation (8.4). This procedure shows the effectiveness of the fuzzy NLP application to formulate and evaluate the different applications in power systems regarding the randomness in the parameters involved in power system analysis.

Table (8-3A)

The Positive Ideal Solution and Negative Ideal Solution

For 10% Deviation of Active, Reactive Load & Generation

ά-Cut	ZNIS Left	ZNIS Mid	ZNIS Right	ZPIS Left	ZPIS Mid	ZPIS Right
0	17692.838	17888.11	18417.76	1976.348	3190.927	3279.655
0.1	17712.365	17888.11	18364.79	2097.806	3190.927	3270.782
0.2	17731.893	17888.11	18311.83	2219.264	3190.927	3261.909
0.3	17751.42	17888.11	18258.86	2340.722	3190.927	3253.036
0.4	17770.948	17888.11	18205.9	2462.18	3190.927	3244.163
0.5	17790.476	17888.11	18152.94	2583.638	3190.927	3235.291
0.6	17810.003	17888.11	18099.97	2705.095	3190.927	3226.418
0.7	17829.531	17888.11	18047.01	2826.553	3190.927	3217.545
0.8	17849.058	17888.11	17994.04	2948.011	3190.927	3208.672
0.9	17868.586	17888.11	17941.08	3069.469	3190.927	3199.8

Table (8-3A)

Fuzzy NLP Total Optimal Cost Function Parameters

ά-Cut	ZL	Zm=Z2	Zr
0	3171.332	3271.797	3317.374
0.1	3015.165	3109.973	3153.123
0.2	3015.165	3109.973	3153.123
0.3	3044.463	3140.333	3183.938
0.4	3044.463	3140.333	3183.938
0.5	3063.868	3160.441	3204.347
0.6	3074.234	3171.182	3215.25
0.7	3074.234	3171.182	3215.25
0.8	3074.234	3171.182	3215.25
0.9	3084.58	3181.902	3226.131

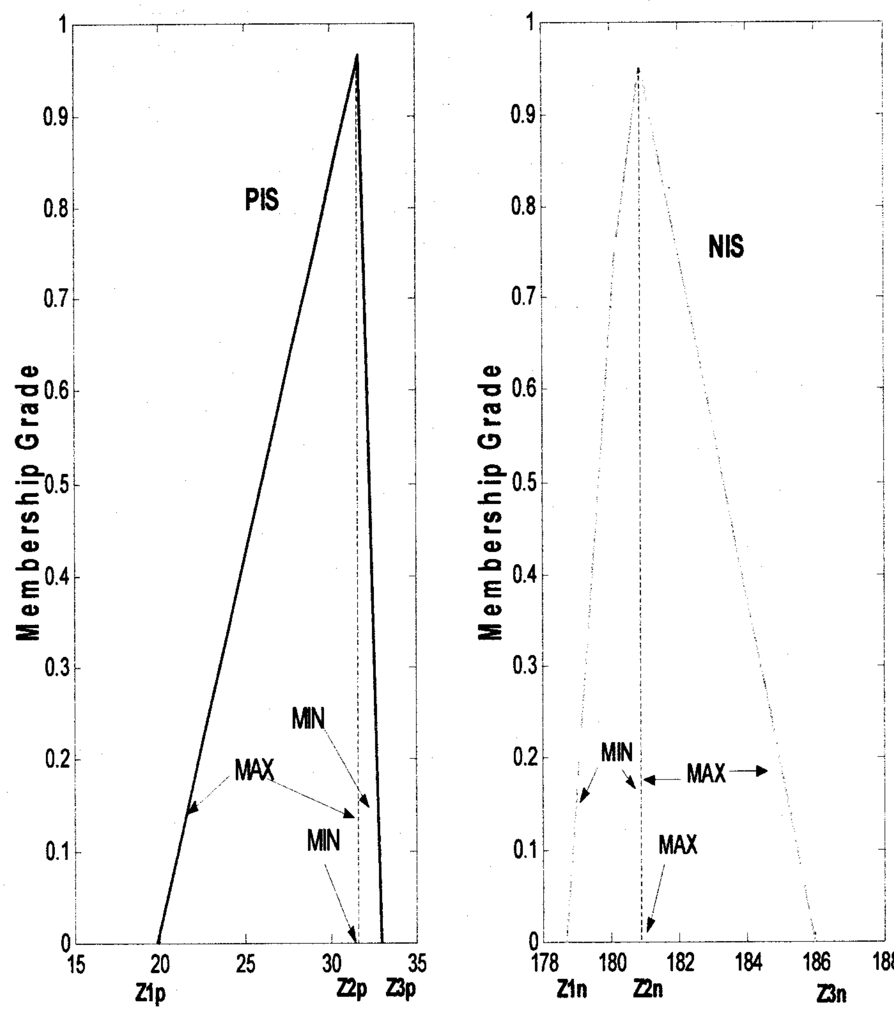


Figure (8-6) Triangular Membership Function for NIS and PIS Representation of the Total Cost (\$/h)x10^2 for $\varphi = 0.55$

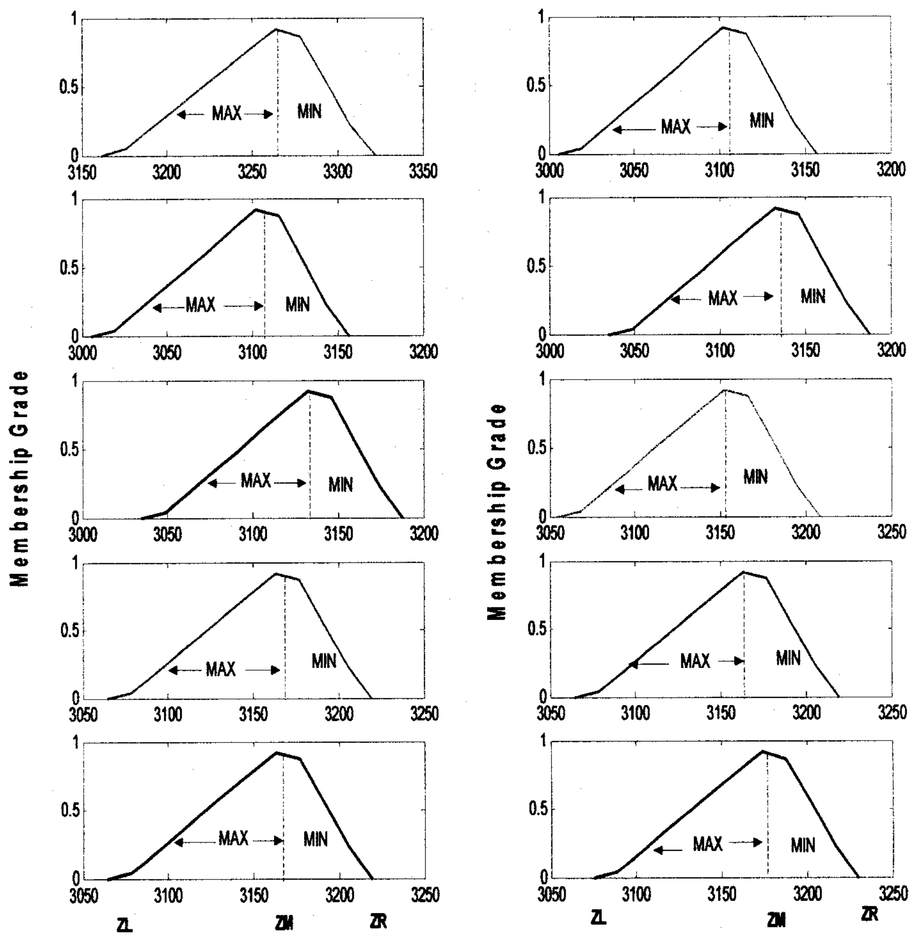


Figure (8-7) Triangular Membership Functions of the Total Optimal Cost ($/h) for $\varphi = 0.55$

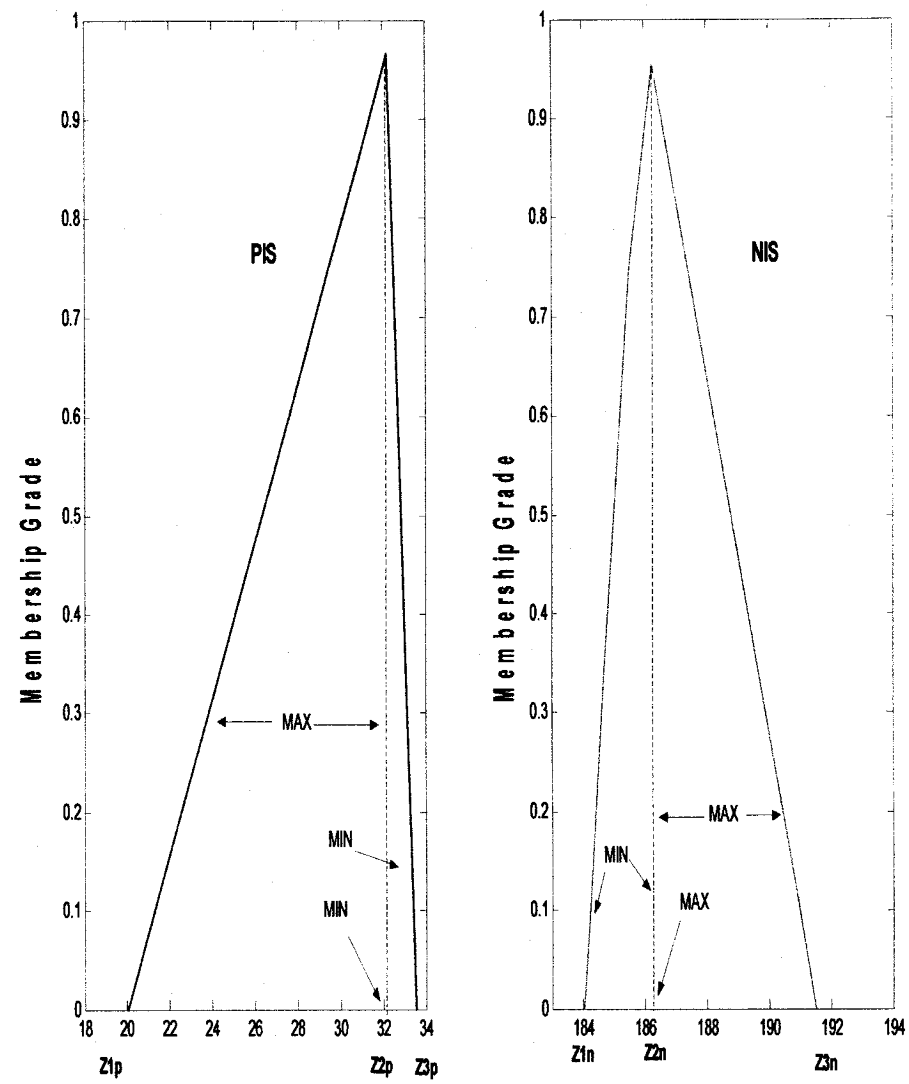


Figure (8-8) Triangular Membership Function for NIS and PIS Representation of the Total Cost (\$/h)x10^2 for $\varphi = 0.35$

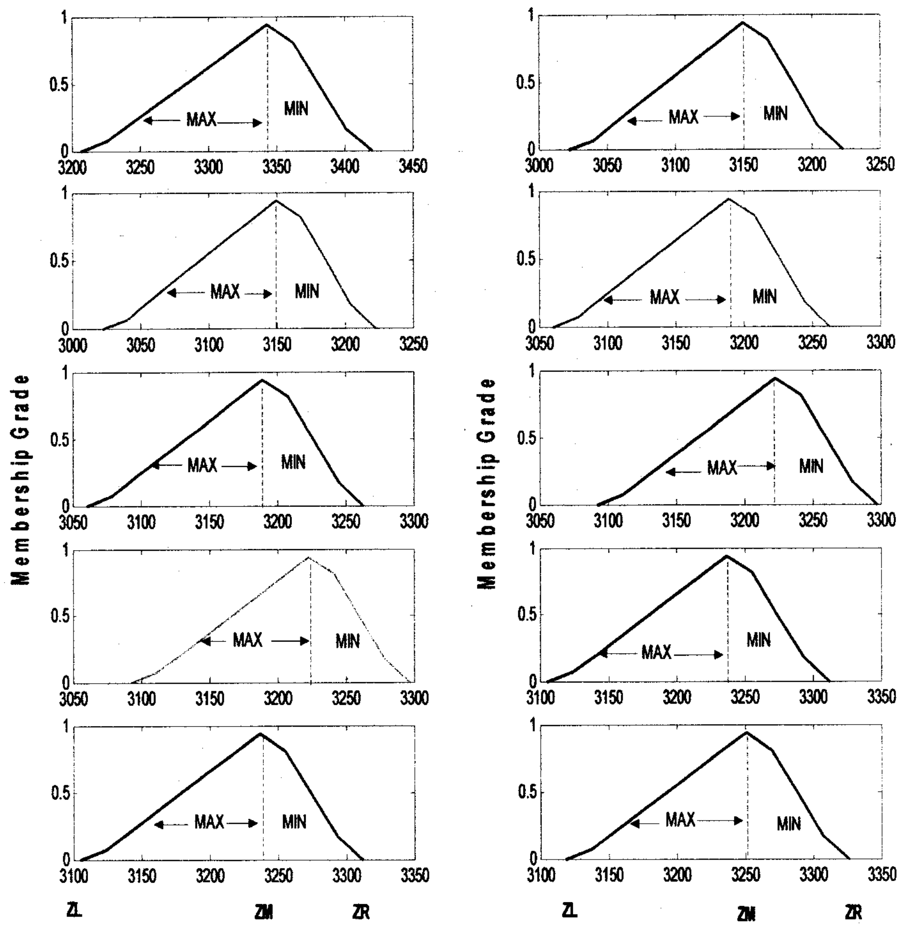


Figure (8-9) Triangular Membership Functions of the Total Optimal Cost ($/h) for $\varphi = 0.35$

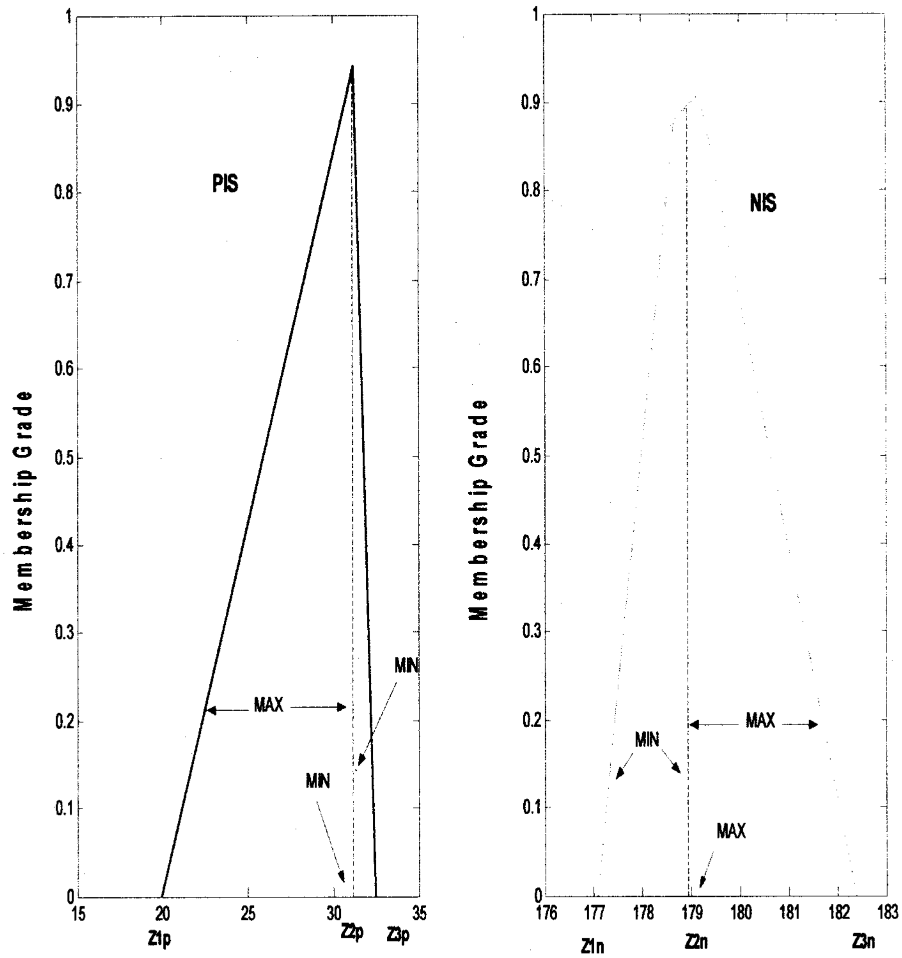


Figure (8-10) Triangular Membership Function for NIS and PIS Representation of the Total Cost (\$/h)x10^2 for Equal $\tilde{\alpha}$ Coefficient

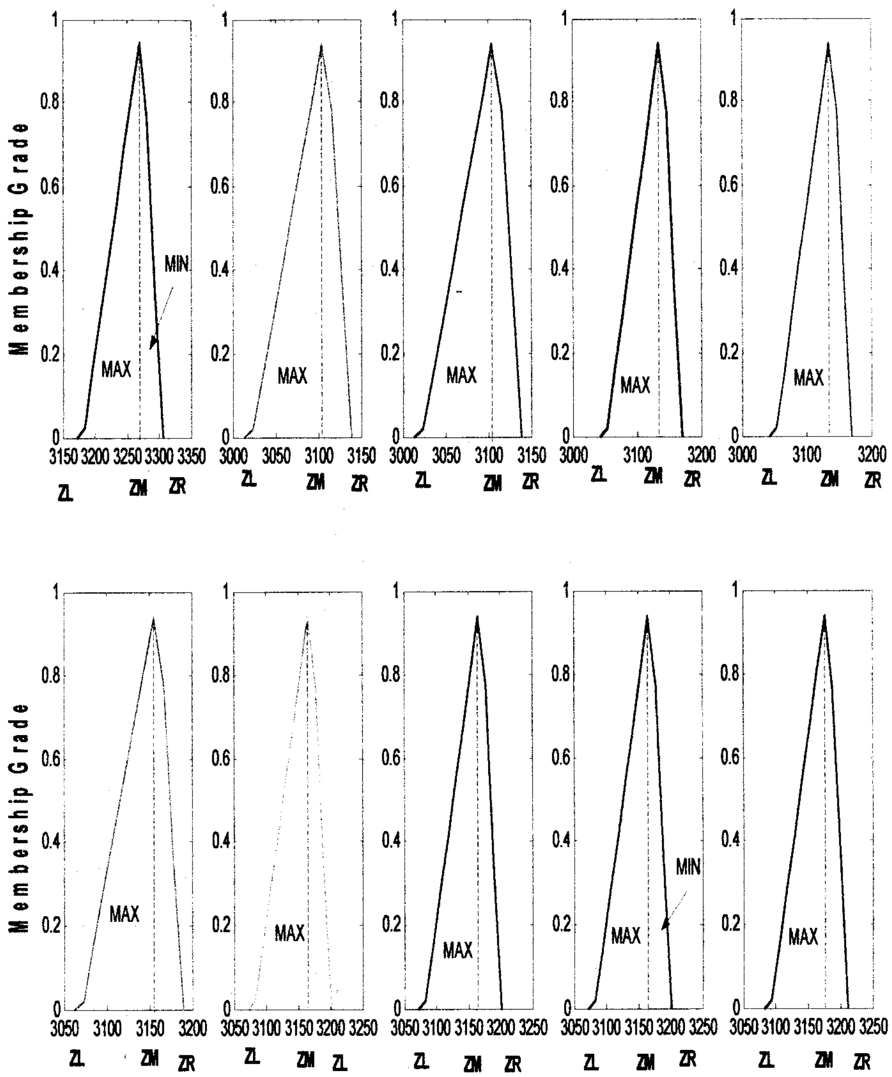


Figure (8-11) Triangular Membership Functions of the Total Optimal Cost (\$/h) for Equal $\tilde{\alpha}$ Coefficient

Table (8-4A)

The Positive Ideal Solution and Negative Ideal Solution

For (10% & 15%) Deviation of Active, Reactive Load & Generation

á-Cut	ZNIS Left	ZNIS Mid	ZNIS Right	ZPIS Left	ZPIS Mid	ZPIS Right
0	17874.26	17988.67	18351.15	2516.566	3212.406	3269.745
0.1	17885.7	17988.67	18314.9	2586.15	3212.406	3264.011
0.2	17897.14	17988.67	18278.65	2655.734	3212.406	3258.277
0.3	17908.58	17988.67	18242.4	2725.318	3212.406	3252.543
0.4	17920.02	17988.67	18206.15	2794.902	3212.406	3246.809
0.5	17931.46	17988.67	18169.91	2864.486	3212.406	3241.075
0.6	17942.9	17988.67	18133.66	2934.07	3212.406	3235.341
0.7	17954.34	17988.67	18097.41	3003.654	3212.406	3229.608
0.8	17965.78	17988.67	18061.16	3073.238	3212.406	3223.874
0.9	17977.22	17988.67	18024.91	3142.822	3212.406	3218.14

Table (8-4B)

Fuzzy NLP Total Optimal Cost Function Parameters

á-Cut	ZL	Zm=Z2	Zr
0	3358.6	3419.9	3450.66
0.1	3055.8	3111.02	3138.67
0.2	3055.8	3111.02	3138.67
0.3	3085.7	3141.44	3169.4
0.4	3085.7	3141.44	3169.4
0.5	3107.2	3163.39	3191.57
0.6	3118.1	3174.55	3202.83
0.7	3118.1	3174.55	3202.83
0.8	3118.1	3174.55	3202.83
0.9	3129	3185.65	3214.05

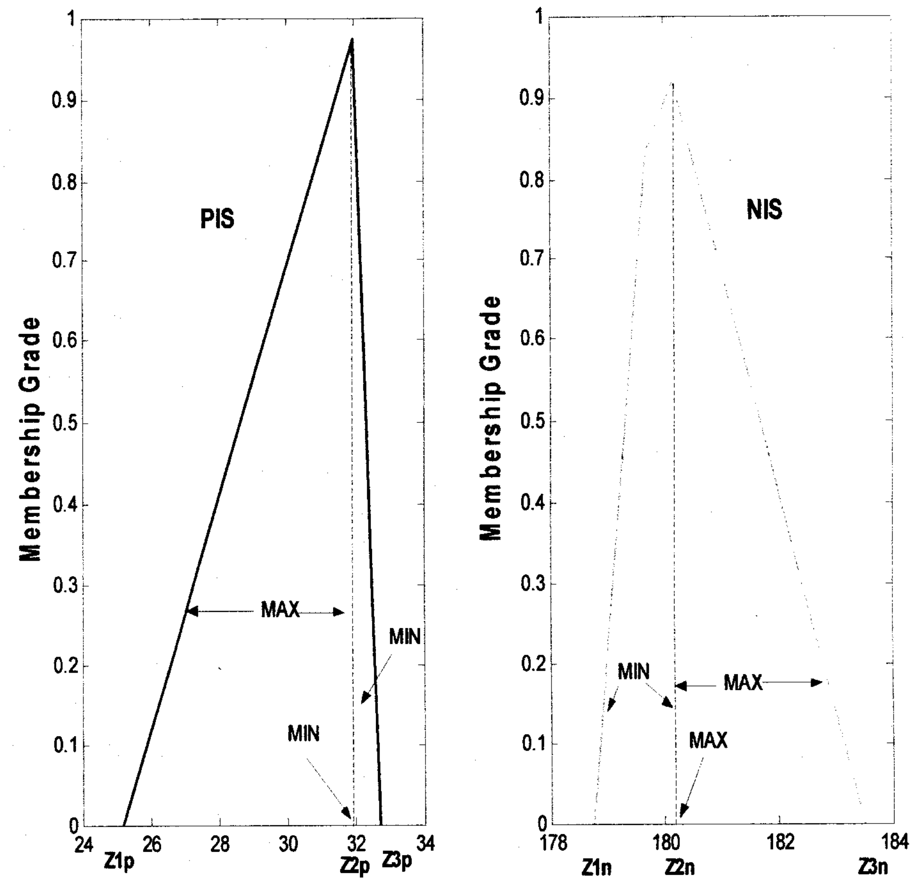


Figure (8-12) Triangular Membership Function for NIS and PIS Representation of the Total Cost (\$/h)x10^2 for $\varphi = 0.5$

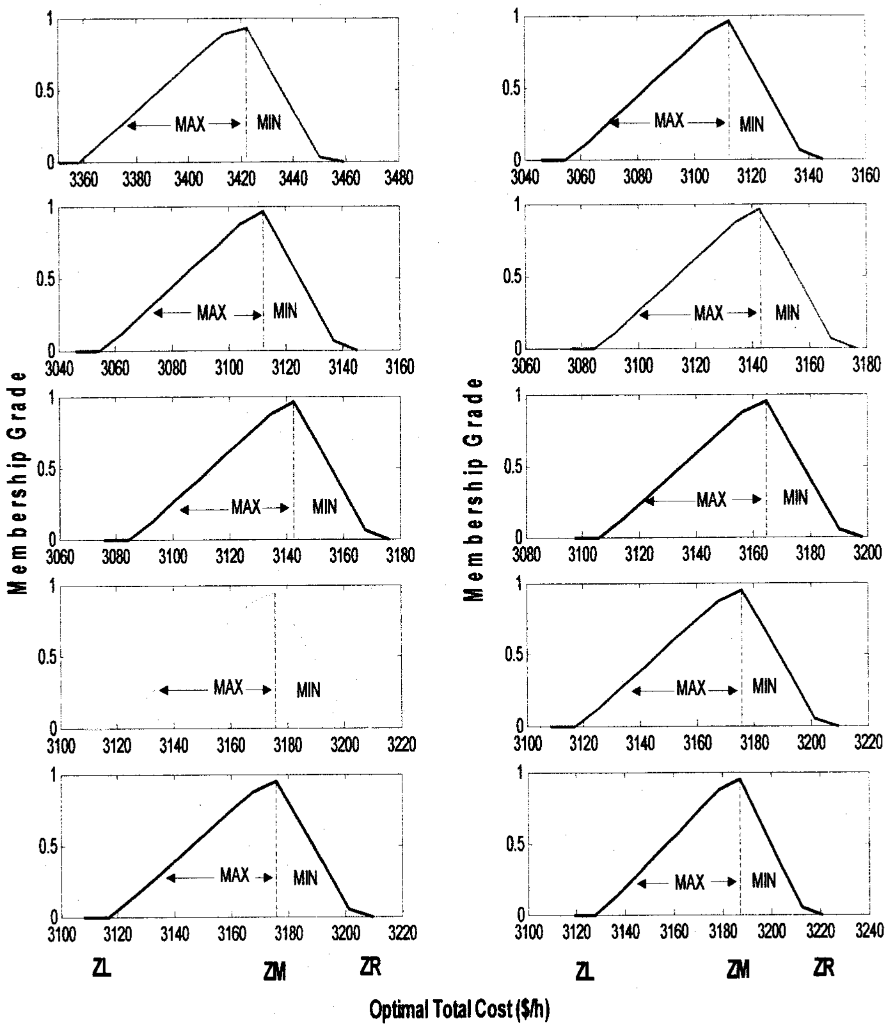


Figure (8-13) Triangular Membership Functions of the Total Optimal Cost ($/h) for $\varphi = 0.5$

Chapter 9
Conclusions and Recommendations For Future Research

9.1 Conclusions

The economic scheduling and optimal power flow are essential tools for minimizing the production cost in an electric power system. In this thesis the problem of fuzzy optimal economic dispatch and nonlinear optimal load flow optimization under a fuzzy load is introduced and formulated to minimize the total production cost of a network. This thesis implements three methods in formulating the economical dispatch of all thermal power systems. It starts with a simple economical dispatch problem with a fuzzy load demand, neglecting transmission losses, but including generation limits. Two generation units are tested for the formulation with various α -cut representations of fuzzy numbers in illustrating the evaluation procedure and to evaluate the effect of the spread on the outcome. Next, a fuzzy cost function coefficients problem with a fuzzy load demand is analyzed and formulated to minimize the total production cost. To evaluate the performance and the capability of reducing cost while varying cost function coefficients, a synthetic system example of three generation units is used. Finally, a more realistic model with fuzzy load, fuzzy cost function coefficients and power losses is formulated, evaluated and tested on a three generation unit system to obtain the optimal minimum cost. The fuzzy nonlinear optimal load flow is presented when the active generation, active load, reactive generation and reactive load are considered to be fuzzy. Three formulation methods were adopted. First a system with all crisp cost function coefficients with fuzzy active, reactive power is tested on a 9-bus system for one hour. Next a fuzzy load that varies on an hourly basis for 24-hours is tested on the 9-bus system, while keeping the load and generation of the other buses unchanged. Finally, a system with a fuzzy cost function coefficients with fuzzy active and reactive power is formulated and tested to generate a minimum cost function.

9.1.1 Conventional and Fuzzy Results Comparison

The classical methods of economic dispatch and power load flow optimization do not provide wide information on the system performance resulting from calculated and measured values as separate entities. The fuzzy formulation results of the optimal economic dispatch and optimal load flow treated in this thesis provide a wider range of information to evaluate the uncertainty in the system when the load demand is fuzzy which will lead the fuzziness to propagate throughout the entire system parameters. This information can be useful to system operators to work with a safe tolerance in meeting the consumer demand. The output of the fuzzy model is a range of upper and lower values for fuzzy load power. This range can give the system operators the ability to run the power system in a more reliable and secure way.

9.2 Thesis Contributions

1. Three fuzzy ED formulations were developed and tested. First a simple case of economic dispatch of all thermal power system with fuzzy load demand ignoring transmission power losses and including generation limits was discussed. Second, economic dispatch of all thermal power system with fuzzy load demand and fuzzy cost function coefficients ignoring transmission power losses and including generation limits was developed. Finally, economic dispatch of all thermal power system of fuzzy load demand, fuzzy cost function coefficient and fuzzy transmission power losses was considered.
2. The formulation is faster and it provides the energy control center or the system operator with a wide range of upper and lower limits, which are valuable information concerning the reliability, security and all the variation encountered in the performance of the ED power system problem.

3. A fuzzy power flow with fuzzy active, reactive generation and load formulation by using Werners approach is developed and tested with a 9-bus system. The formulation were tested with a fixed load then tested again with a varying fuzzy load for a period of 24-hours. The formulation is validated for the twenty four data sets.
4. The formulation and the analysis of the simulated example prove that fuzzifying and violating the limits of the constraint such as the active, reactive power generation and load using a triangular membership function will result in a number of optimal solutions between α -cut [0,1], which will give the decision-maker the opportunity to select the best optimal solution for any unexpected increased in load.
5. A fuzzy optimal power flow with fuzzy active, reactive power generation and load including fuzzy cost function coefficient is formulated by using FNLP approach by Lai and Hwang then simulated with a 9-bus system test.
6. The results demonstrate that we can obtain a number of optimal minimum total cost solutions for the violated objective cost function coefficients and the violated constraints using a triangular membership function representation for the constraint and the cost function coefficient including linearly decreasing and increasing membership function for the objective function. The decision-maker will decide according to their judgment and impression to select the best solution and act upon the sudden change of any unexpected variation in the cost function coefficient and constraint.

9.3 Suggestions for Future Research

1. The constraints used in this study are the fundamental equality and inequality constraints the system is subject to. However, other constraints can be included as fuzzy into the formulation. Examples of this are fuzzy start up of the generation units, transmission line loading limits, and fuzzy emission constraints. Each one can be analyzed and tested with OPF to minimize the total cost production.

2. The membership functions of the fuzzy parameters were assumed to be triangular. Other membership functions, in particular, trapezoidal and Gaussian functions may be tested and the results compared with the triangular membership functions to minimize the total cost production.
3. There is a wide range of variations in the load demand and cost function parameters. It is worth while to study the differences in variations and obtain the best suitable fixed deviation for the fuzzy parameters to offer guidance to the system operator if a defuzzification process to the fuzzy output is obtained.
4. Other optimization methods such as Genetic Algorithm, Tabu Search and Simulated Annealing can be tested with the proposed FNLP procedure.

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Appendix 1

Economic Dispatch of all Thermal Power Systems with 20% Fuzzy Load Deviation

In Chapter (4) the Fuzzy ED analysis was tested with a fuzzy winter model for the weekdays with a 20% deviation in the load demand for a 24-hour period. In this appendix the same program and analysis are used to test the weekend data of the winter fuzzy model. The fuzzy load demand for different (0, 0.5, 0.75, 1) α-cut representation are used to obtain the power generation of each unit. The obtained power generation then substituted into the cost function formula to obtain the minimum total cost of all units. All the tables and figures for all the formulas derived in Chapter (4) are shown.

Examining the graphs, the following observations are listed:

- Table (P1-1) and Figures (P1-1), (P1-2) and (P1-3) represent the fuzzy load at different α-cut values for model A with 20% deviation on weekends. It is clear that the load changes hour by hour and the left and right spread are getting closer as α-cut increases between [0,1]. The left and right spread varies in range during the day hours as shown in all the figures. This variation of spread will propagate through the fuel incremental cost, the power generations and the total minimum cost.
- Tables (P1-2) and (P1-3) show the result of the two unit generation committed to the system for different α-cut values. Comparing Figure (P1-4), (P1-5) and (P1-6) with Figure (4-7), (4-8) and (4-9) it is clear that the crisp value of unit 1 & 2 show more generated power at the early hours of the day until the 8^{th} hours of the day in the weekend model than the weekdays model. The two unit generators are generating the same power from the 8^{th} hour of the day until the 15^{th} hour for both models. The weekend model however, is generating less power than the weekday model from the 15^{th} hour until the 22^{nd} hour of the day. After the 22^{nd} hour both models are almost generating the

same crisp power value except the weekday model whose upper and lower value is much higher than the weekend model. This variation in generated power between the two models is reflected on the total minimum cost value if we compare Table (4-5) and Table (P1-5). The weekend winter model shows a higher in cost value in the early morning until the afternoon than the weekdays model. Then higher again after the 21 hour than the weekdays model. The fuzzy approach provides all the above information to the system operator to compare the weekdays and the weekend models hour by hour.

- Table (P1-4) and Figure (P1-7) and (P1-8) show the total generation for different α -cut values. The total power generations satisfy the equality constraint imposed on the system in equation (4.3) where the total generation committed is equal to the power load demand. Comparing the load demand values in Table (P1-1) and Table (P1-4) which is the total power generation of the two units for the weekend model we can see the equality constraints is satisfied.
- The total fuel cost of different α -cut values calculated from equation (4-31) are shown in Table (P1-5) and a plotted graph for different α -cut values are shown in Figures (P1-9), (P1-10) and (P1-11). The data for the weekday and weekend fuzzy load demand provides a variety of power consideration for planning and operation. The total minimum cost based on uncertainty in the load demand, incremental fuel cost and power generation for each unit is calculated for the weekend data and the weekdays data. The decision-maker will act upon the sudden change in any of the above mentioned fuzzy parameter.
- Figure (P1-12) represents the triangular membership function of the fuzzy total minimum cost. The figure show that at each our there is a crisp cost, left and right cost. The left and right spread between hours are changing from wider to a smaller spread. Clearly the variation of left and right spread is very helpful information to know the maximum and minimum spread range of minimum cost, power generation of each units and load demand.

Table (P1-1)

Membership Function of Load Demand for (0, 0.5, 0.75, 1) α -Cut Representation for Model "A" Weekend With 20% Deviation

Membership Function		$\mu_{P_{Load}}=0$			$\mu_{P_{Load}}=0.5$			$\mu_{P_{Load}}=0.75$			$\mu_{P_{Load}}=1$	
Daily Hours	Left Load MW	Mid Load MW	Right Load MW	Left Load MW	Mid Load MW	Right Load MW	Left Load MW	Mid Load MW	right Load MW	Left Load MW	Mid Load MW	Right Load MW
1	329.2	776.8	1332	553	776.8	1055	664.9	776.8	915.7	776.8	776.8	776.8
2	332.2	710	1335	521.1	710	1023	615.5	710	866.3	710	710	710
3	330.9	667.1	1334	499	667.1	1001	583.1	667.1	833.9	667.1	667.1	667.1
4	331.7	647.2	1335	489.4	647.2	991	568.3	647.2	819.1	647.2	647.2	647.2
5	324.6	639.3	1328	481.9	639.3	983.5	560.6	639.3	811.4	639.3	639.3	639.3
6	327.7	642.8	1331	485.3	642.8	986.8	564	642.8	814.8	642.8	642.8	642.8
7	340.7	657.2	1344	498.9	657.2	1001	578.1	657.2	828.9	657.2	657.2	657.2
8	349.5	689.3	1353	519.4	689.3	1021	604.3	689.3	855.1	689.3	689.3	689.3
9	356	767.5	1359	561.7	767.5	1063	664.6	767.5	915.4	767.5	767.5	767.5
10	352.7	898	1356	625.3	898	1127	761.7	898	1012	898	898	898
11	360.6	995.1	1364	677.9	995.1	1179	836.5	995.1	1087	995.1	995.1	995.1
12	365.4	1016	1369	690.8	1016	1192	853.5	1016	1104	1016	1016	1016
13	356.5	1008	1360	682.3	1008	1184	845.2	1008	1096	1008	1008	1008
14	350.8	977.9	1354	664.4	977.9	1166	821.1	977.9	1072	977.9	977.9	977.9
15	350	940.1	1353	645.1	940.1	1147	792.6	940.1	1043	940.1	940.1	940.1
16	335.1	905.1	1338	620.1	905.1	1122	762.6	905.1	1013	905.1	905.1	905.1
17	328.6	892.8	1332	610.7	892.8	1112	751.8	892.8	1003	892.8	892.8	892.8
18	319.9	915.4	1323	617.6	915.4	1119	766.5	915.4	1017	915.4	915.4	915.4
19	313.1	915.1	1316	614.1	915.1	1116	764.6	915.1	1015	915.1	915.1	915.1
20	313.8	887	1317	600.4	887	1102	743.7	887	994.5	887	887	887
21	310.9	900.2	1314	605.5	900.2	1107	752.9	900.2	1004	900.2	900.2	900.2
22	305.6	961.4	1309	633.5	961.4	1135	797.4	961.4	1048	961.4	961.4	961.4
23	296	953.1	1299	624.6	953.1	1126	788.8	953.1	1040	953.1	953.1	953.1
24	288.3	903.7	1291	596	903.7	1098	749.8	903.7	1001	903.7	903.7	903.7

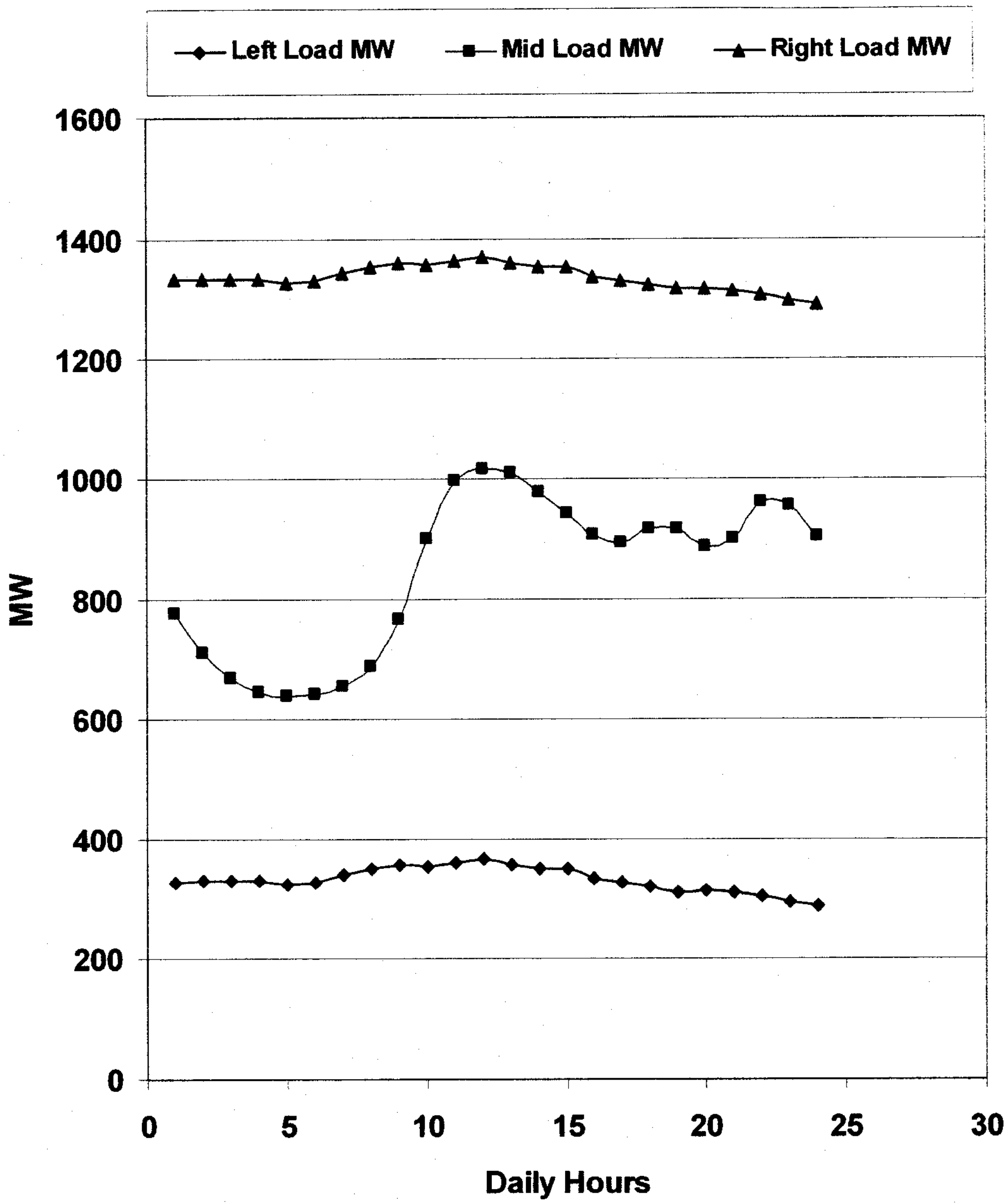

Figure (P1-1) Fuzzy Load for (0-α-Cut) Representation

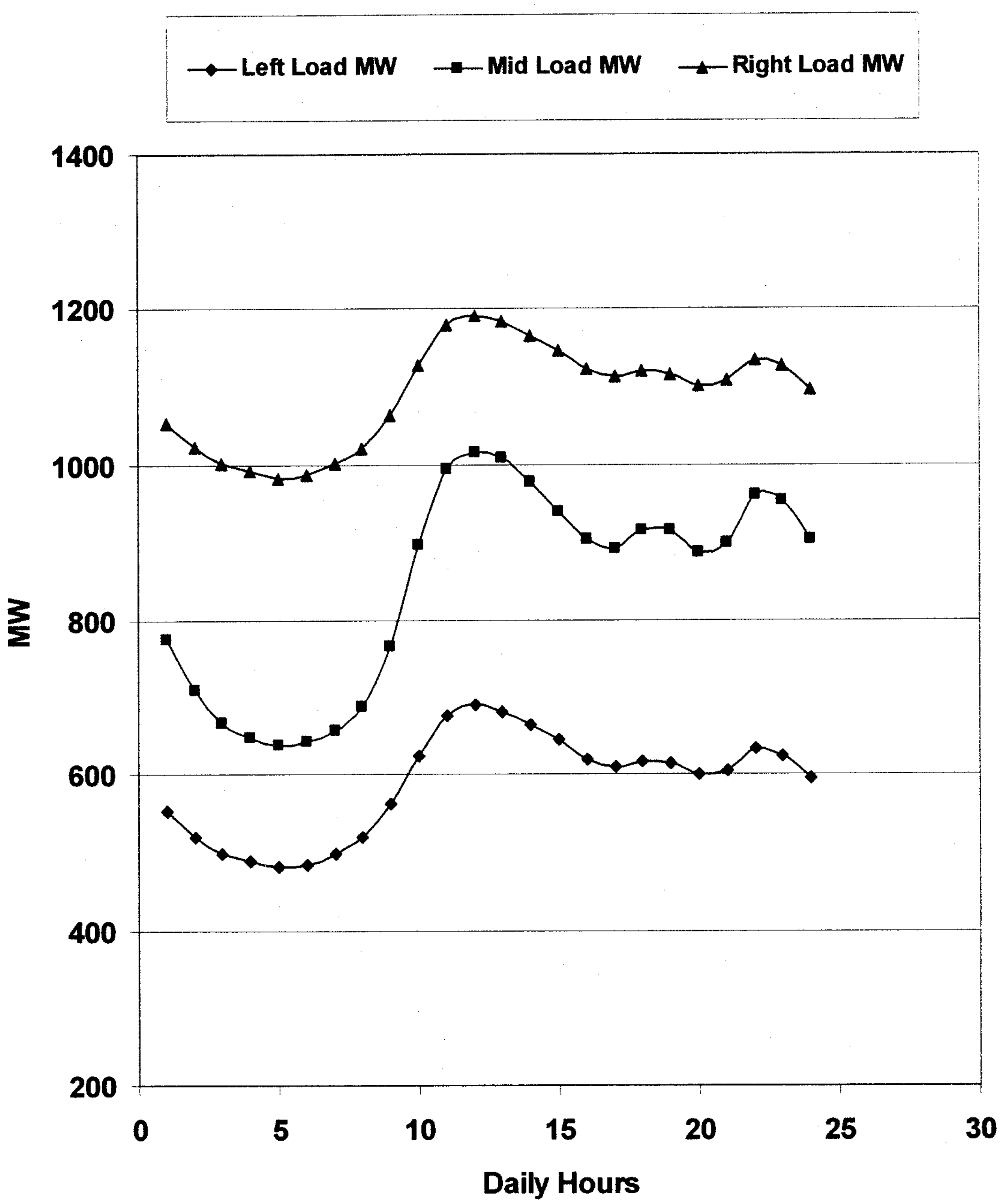

Figure (P1-2) Fuzzy Load for (0.5-α-Cut) Representation

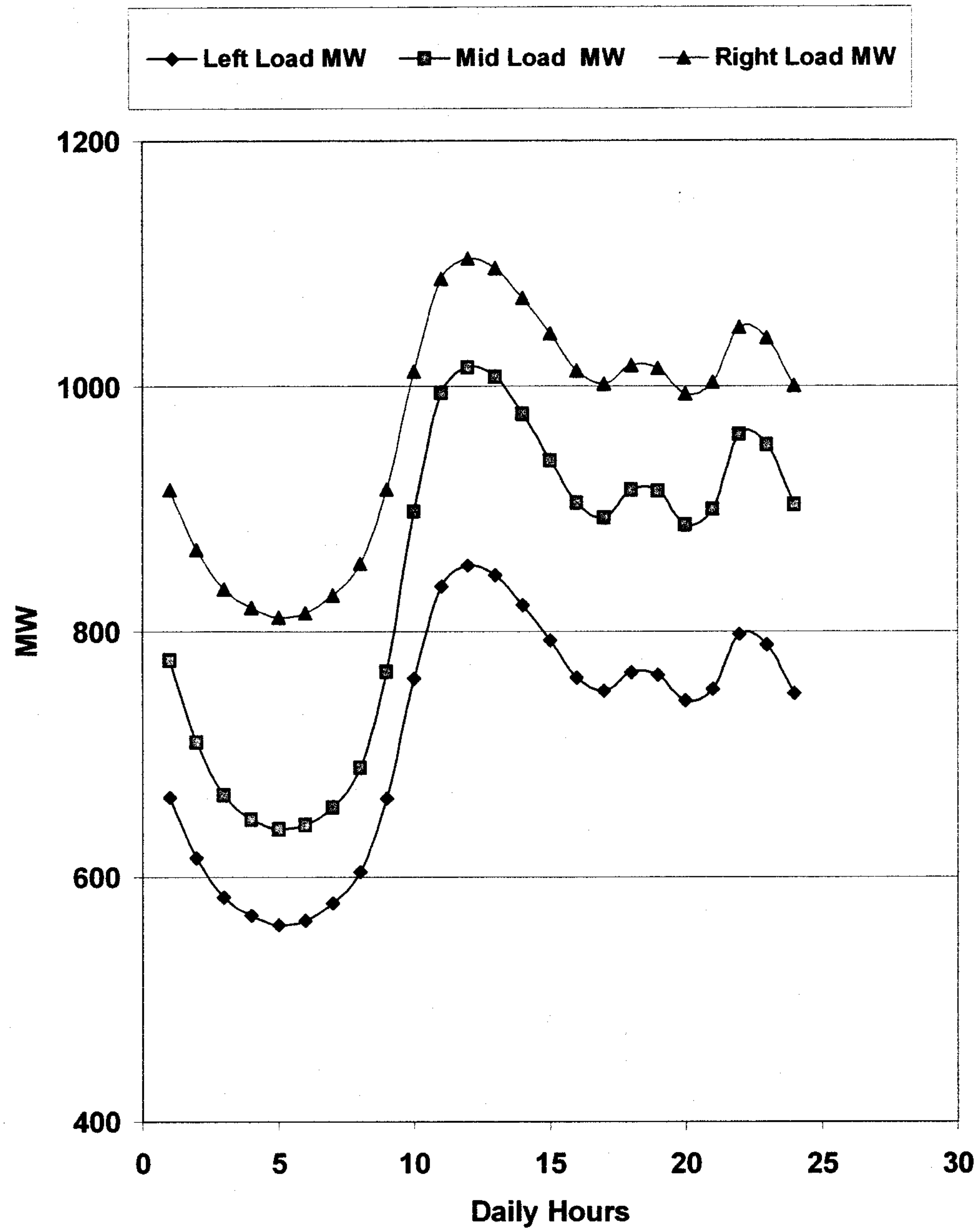

Figure (P1-3) Fuzzy Load for (0.75-α-Cut) Representation

Table (P1-2)

Membership Function of Generator #1 for (0, 0.5, 0.75, 1) α -Cut Representation for Model "A" Weekend With 20% Deviation

Membership Function		$\mu_{P_{G1}} = 0$			$\mu_{P_{G1}} = 0.5$			$\mu_{P_{G1}} = 0.75$			$\mu_{P_{G1}} = 1$	
Daily Hours	Left PG1 MW	Mid PG1 MW	Right PG1 MW	Left PG1 MW	Mid PG1 MW	Right PG1 MW	Left PG1 MW	Mid PG1 MW	Right PG1 MW	Left PG1 MW	Mid PG1 MW	Right PG1 MW
1	153.7	390.7	684.8	272.2	390.7	537.7	331.4	390.7	464.2	390.7	390.7	390.7
2	155.3	355.3	686.4	255.3	355.3	520.8	305.3	355.3	438.1	355.3	355.3	355.3
3	154.6	332.6	685.7	243.6	332.6	509.1	288.1	332.6	420.9	332.6	332.6	332.6
4	155	322	686.1	238.5	322	504.1	280.3	322	413.1	322	322	322
5	151.2	317.9	682.3	234.6	317.9	500.1	276.2	317.9	409	317.9	317.9	317.9
6	152.9	319.7	684	236.3	319.7	501.9	278	319.7	410.8	319.7	319.7	319.7
7	159.8	327.3	690.9	243.6	327.3	509.1	285.5	327.3	418.2	327.3	327.3	327.3
8	164.4	344.3	695.5	254.4	344.3	519.9	299.4	344.3	432.1	344.3	344.3	344.3
9	167.9	385.7	699	276.8	385.7	542.3	331.3	385.7	464	385.7	385.7	385.7
10	166.1	454.8	697.2	310.5	454.8	576	382.6	454.8	515.4	454.8	454.8	454.8
11	170.3	506.2	701.4	338.3	506.2	603.8	422.3	506.2	555	506.2	506.2	506.2
12	172.9	517.4	704	345.1	517.4	610.7	431.3	517.4	564	517.4	517.4	517.4
13	168.2	513.1	699.3	340.6	513.1	606.2	426.9	513.1	559.6	513.1	513.1	513.1
14	165.1	497.1	696.2	331.1	497.1	596.7	414.1	497.1	546.9	497.1	497.1	497.1
15	164.7	477.1	695.8	320.9	477.1	586.5	399	477.1	531.8	477.1	477.1	477.1
16	156.8	458.6	687.9	307.7	458.6	573.2	383.1	458.6	515.9	458.6	458.6	458.6
17	153.4	452.1	684.5	302.7	452.1	568.3	377.4	452.1	510.2	452.1	452.1	452.1
18	148.7	464	679.8	306.4	464	571.9	385.2	464	518	464	464	464
19	145.2	463.9	676.2	304.5	463.9	570.1	384.2	463.9	517	463.9	463.9	463.9
20	145.6	449	676.6	297.3	449	562.8	373.1	449	505.9	449	449	449
21	144	456	675.1	300	456	565.5	378	456	510.8	456	456	456
22	141.2	488.4	672.3	314.8	488.4	580.3	401.6	488.4	534.4	488.4	488.4	488.4
23	136.1	484	667.2	310.1	484	575.6	397	484	529.8	484	484	484
24	132	457.8	663.1	294.9	457.8	560.5	376.4	457.8	509.2	457.8	457.8	457.8

Table (P1-3)

Membership Function of Generator #2 for (0, 0.5, 0.75, 1) α -Cut Representation

For Model "A" Weekend With 20% Deviation

Membership Function	$\mu_{P_{G2}}=0$			$\mu_{P_{G2}}=0.5$			$\mu_{P_{G2}}=0.75$			$\mu_{P_{G2}}=1$		
Daily Hours	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW	Left PG2 MW	Mid PG2 MW	Right PG2 MW
1	175.5	386.1	647.6	280.8	386.1	516.9	333.5	386.1	451.5	386.1	386.1	386.1
2	176.9	354.7	649	265.8	354.7	501.8	310.3	354.7	428.3	354.7	354.7	354.7
3	176.3	334.5	648.4	255.4	334.5	491.5	295	334.5	413	334.5	334.5	334.5
4	176.7	325.2	648.8	250.9	325.2	487	288	325.2	406.1	325.2	325.2	325.2
5	173.3	321.4	645.4	247.4	321.4	483.4	284.4	321.4	402.4	321.4	321.4	321.4
6	174.8	323.1	646.9	248.9	323.1	485	286	323.1	404	323.1	323.1	323.1
7	180.9	329.9	653	255.4	329.9	491.4	292.6	329.9	410.6	329.9	329.9	329.9
8	185.1	345	657.1	265	345	501	305	345	423	345	345	345
9	188.1	381.8	660.2	284.9	381.8	521	333.3	381.8	451.4	381.8	381.8	381.8
10	186.5	443.2	658.6	314.9	443.2	550.9	379	443.2	497	443.2	443.2	443.2
11	190.3	488.9	662.4	339.6	488.9	575.6	414.2	488.9	532.2	488.9	488.9	488.9
12	192.6	498.8	664.6	345.7	498.8	581.7	422.2	498.8	540.3	498.8	498.8	498.8
13	188.4	495	660.4	341.7	495	577.7	418.3	495	536.4	495	495	495
14	185.7	480.8	657.8	333.2	480.8	569.3	407	480.8	525	480.8	480.8	480.8
15	185.3	463	657.4	324.1	463	560.2	393.6	463	511.6	463	463	463
16	178.3	446.5	650.3	312.4	446.5	548.4	379.5	446.5	497.5	446.5	446.5	446.5
17	175.2	440.7	647.3	308	440.7	544	374.4	440.7	492.4	440.7	440.7	440.7
18	171.1	451.4	643.2	311.2	451.4	547.3	381.3	451.4	499.3	451.4	451.4	451.4
19	167.9	451.2	640	309.6	451.2	545.6	380.4	451.2	498.4	451.2	451.2	451.2
20	168.3	438	640.4	303.1	438	539.2	370.6	438	488.6	438	438	438
21	166.9	444.2	639	305.5	444.2	541.6	374.9	444.2	492.9	444.2	444.2	444.2
22	164.4	473	636.5	318.7	473	554.7	395.9	473	513.9	473	473	473
23	159.9	469.1	632	314.5	469.1	550.5	391.8	469.1	509.8	469.1	469.1	469.1
24	156.3	445.9	628.3	301.1	445.9	537.1	373.5	445.9	491.5	445.9	445.9	445.9

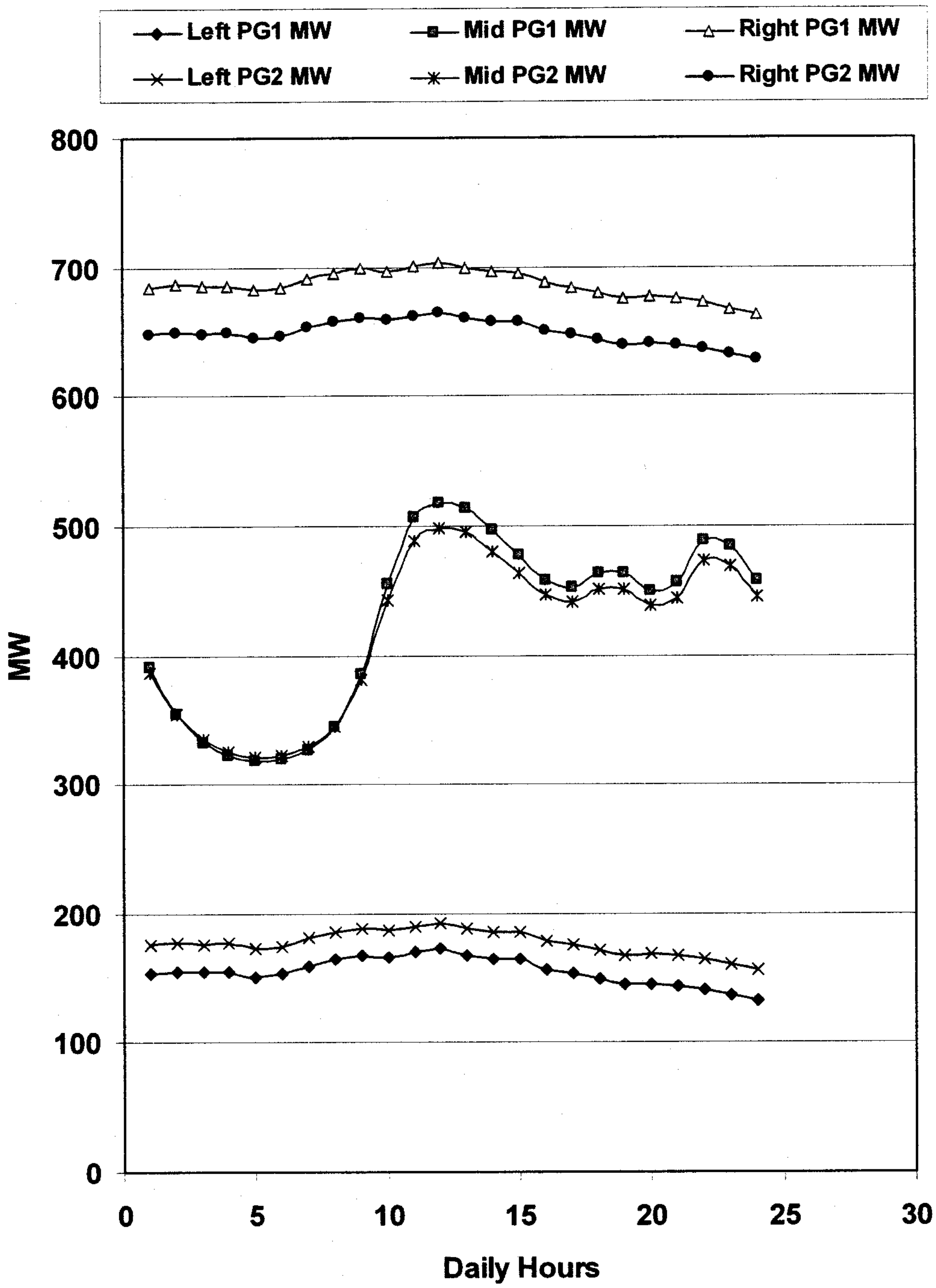

Figure (P1-4) Fuzzy (0-α-Cut) Representation for Generation of Units (1&2)

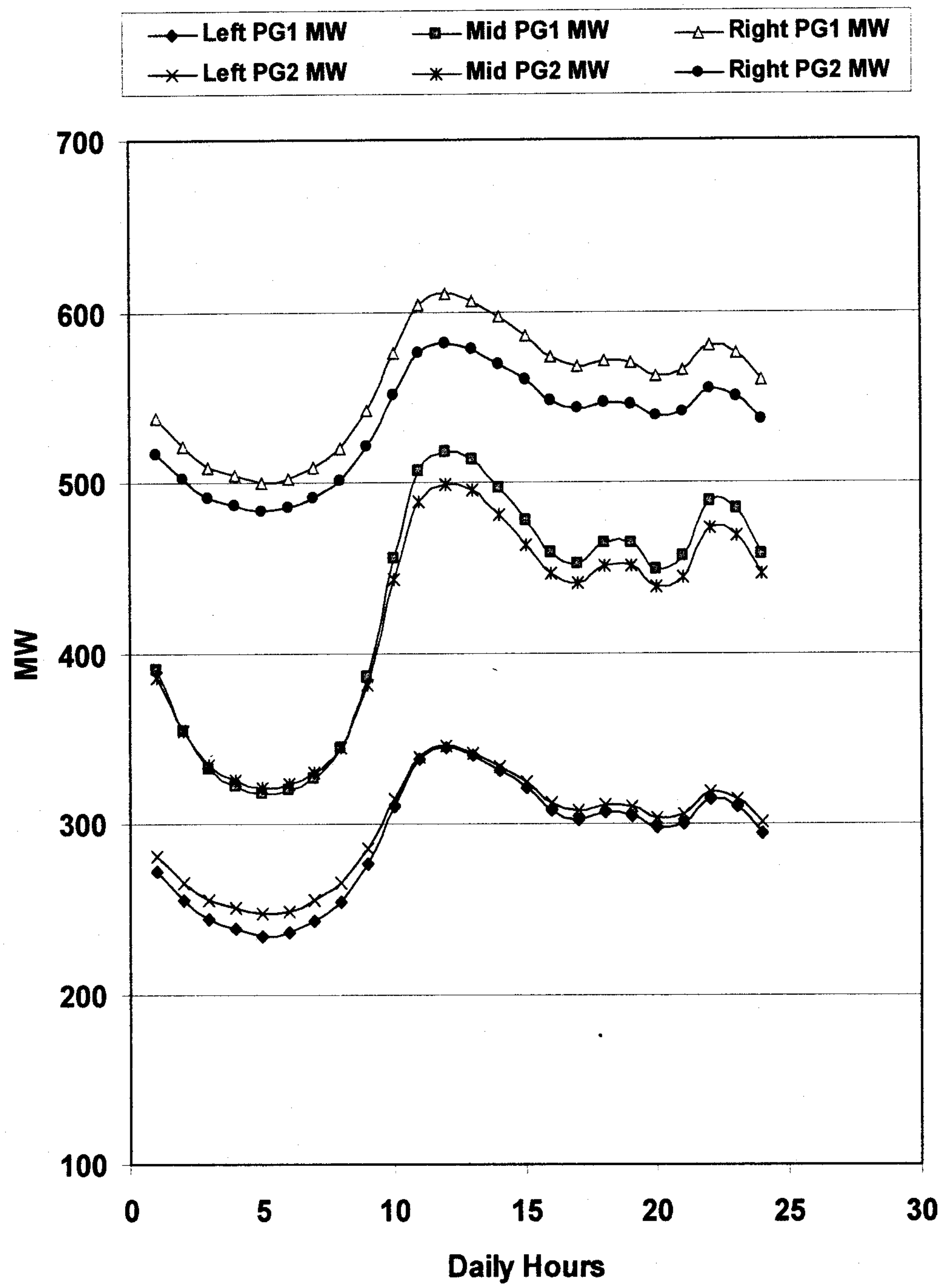

Figure (P1-5) Fuzzy (0.5-α -Cut) Representation for Generation of Units (1&2)

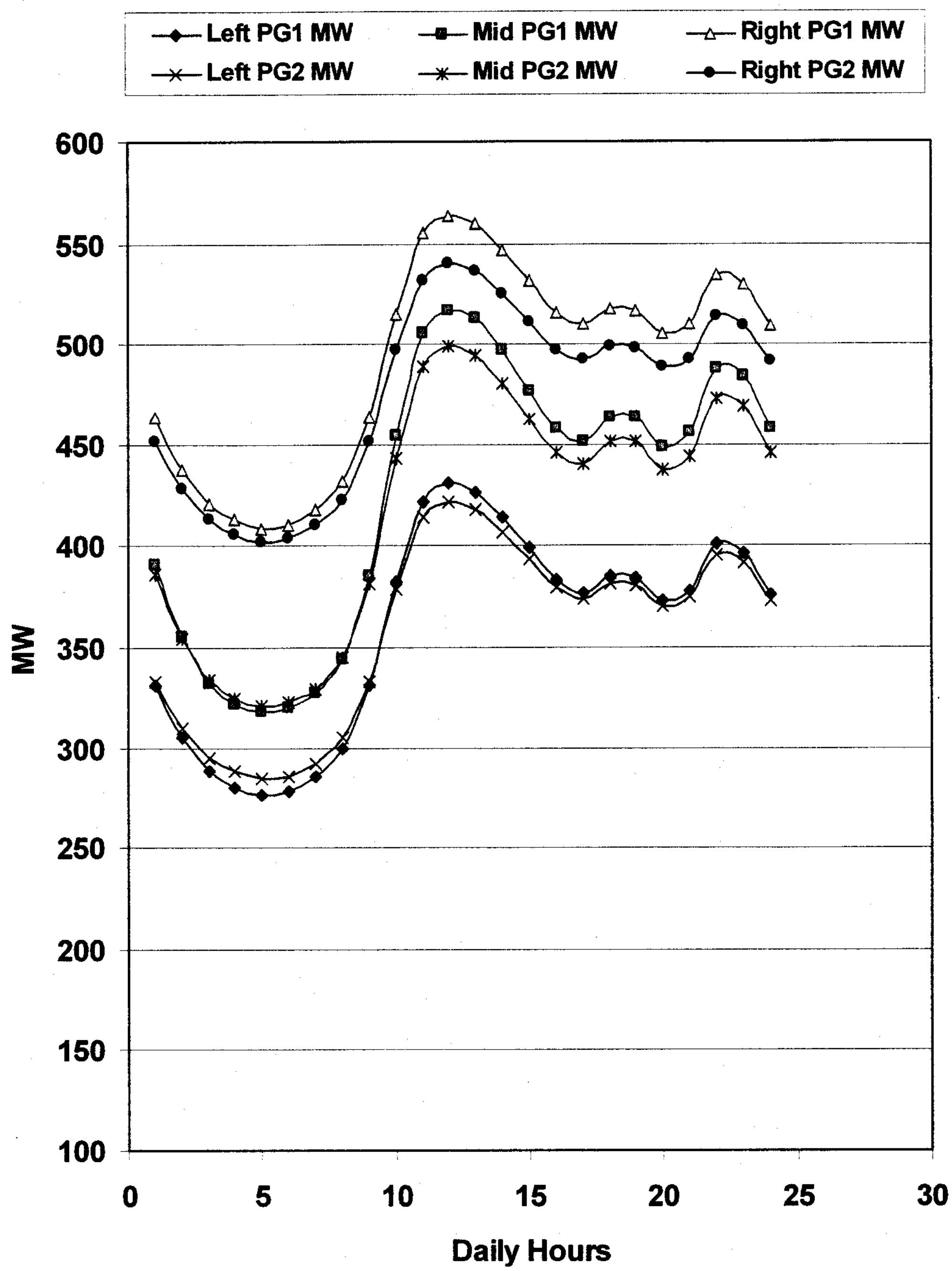

Figure (P1-6) Fuzzy (0.75-α-Cut) Representation for Generation of Units (1&2)

Table (P1-4)

Membership Function of Total Generator for (0, 0.5, 0.75, 1) α -Cut Representation For Model "A" Weekend With 20% Deviation

Membership Function		$\mu_{tP_G}=0$			$\mu_{tP_G}=0.5$			$\mu_{tP_G}=0.75$			$\mu_{tP_G}=1$	
Daily Hours	Left tPG MW	Mid tPG MW	Right tPG MW	Left tPG MW	Mid tPG MW	Right tPG MW	Left tPG MW	Mid tPG MW	Right tPG MW	Left tPG MW	Mid tPG MW	Right tPG MW
1	329.2	776.8	1332	553	776.8	1055	664.9	776.8	915.7	776.8	776.8	776.8
2	332.2	710	1335	521.1	710	1023	615.5	710	866.3	710	710	710
3	330.9	667.1	1334	499	667.1	1001	583.1	667.1	833.9	667.1	667.1	667.1
4	331.7	647.2	1335	489.4	647.2	991	568.3	647.2	819.1	647.2	647.2	647.2
5	324.6	639.3	1328	481.9	639.3	983.5	560.6	639.3	811.4	639.3	639.3	639.3
6	327.7	642.8	1331	485.3	642.8	986.8	564	642.8	814.8	642.8	642.8	642.8
7	340.7	657.2	1344	498.9	657.2	1001	578.1	657.2	828.9	657.2	657.2	657.2
8	349.5	689.3	1353	519.4	689.3	1021	604.3	689.3	855.1	689.3	689.3	689.3
9	356	767.5	1359	561.7	767.5	1063	664.6	767.5	915.4	767.5	767.5	767.5
10	352.7	898	1356	625.3	898	1127	761.7	898	1012	898	898	898
11	360.6	995.1	1364	677.9	995.1	1179	836.5	995.1	1087	995.1	995.1	995.1
12	365.4	1016	1369	690.8	1016	1192	853.5	1016	1104	1016	1016	1016
13	356.5	1008	1360	682.3	1008	1184	845.2	1008	1096	1008	1008	1008
14	350.8	977.9	1354	664.4	977.9	1166	821.1	977.9	1072	977.9	977.9	977.9
15	350	940.1	1353	645.1	940.1	1147	792.6	940.1	1043	940.1	940.1	940.1
16	335.1	905.1	1338	620.1	905.1	1122	762.6	905.1	1013	905.1	905.1	905.1
17	328.6	892.8	1332	610.7	892.8	1112	751.8	892.8	1003	892.8	892.8	892.8
18	319.9	915.4	1323	617.6	915.4	1119	766.5	915.4	1017	915.4	915.4	915.4
19	313.1	915.1	1316	614.1	915.1	1116	764.6	915.1	1015	915.1	915.1	915.1
20	313.8	887	1317	600.4	887	1102	743.7	887	994.5	887	887	887
21	310.9	900.2	1314	605.5	900.2	1107	752.9	900.2	1004	900.2	900.2	900.2
22	305.6	961.4	1309	633.5	961.4	1135	797.4	961.4	1048	961.4	961.4	961.4
23	296	953.1	1299	624.6	953.1	1126	788.8	953.1	1040	953.1	953.1	953.1
24	288.3	903.7	1291	596	903.7	1098	749.8	903.7	1001	903.7	903.7	903.7

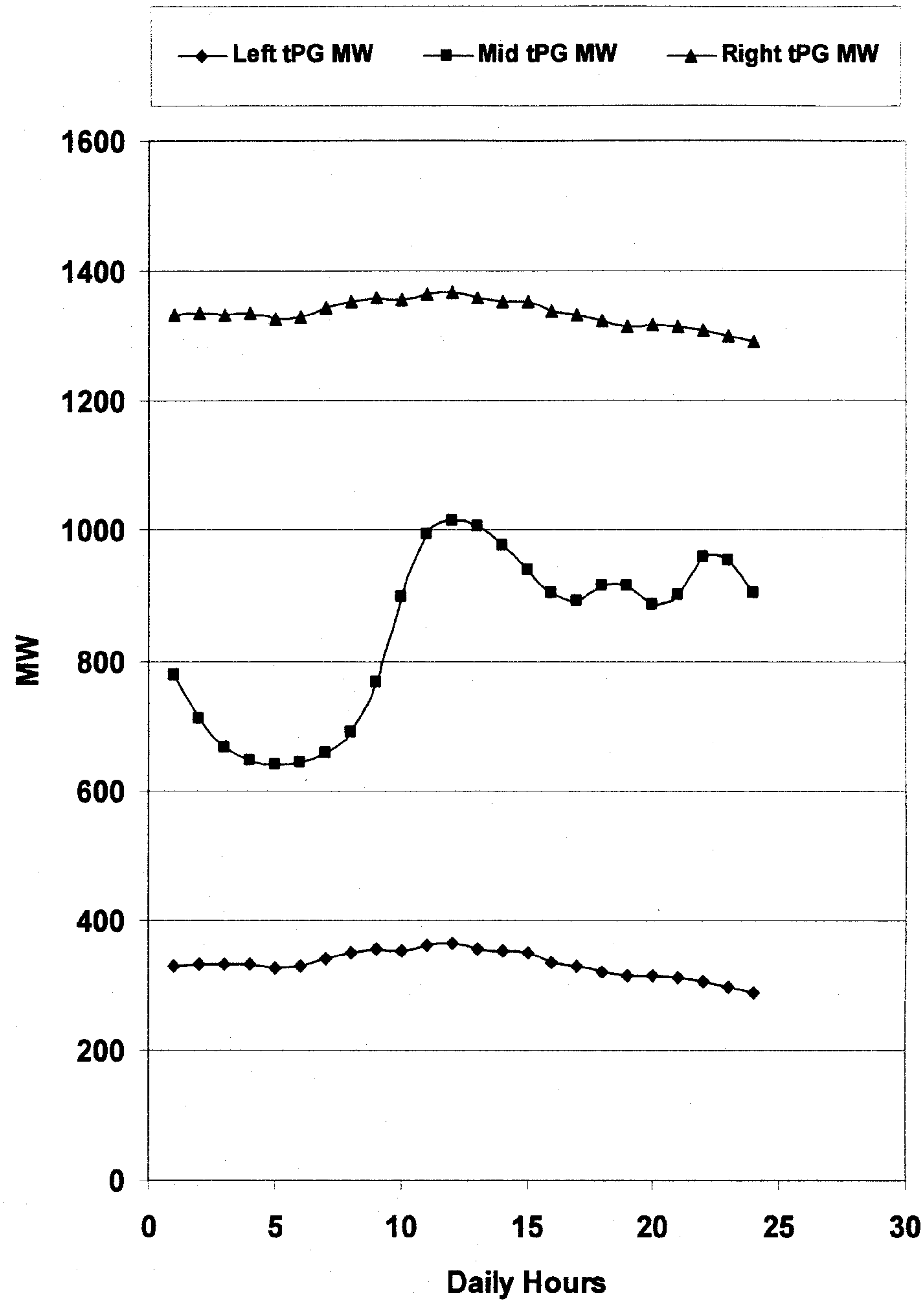

Figure (P1-7) Fuzzy (0-α-cut) Representation of Total Generation

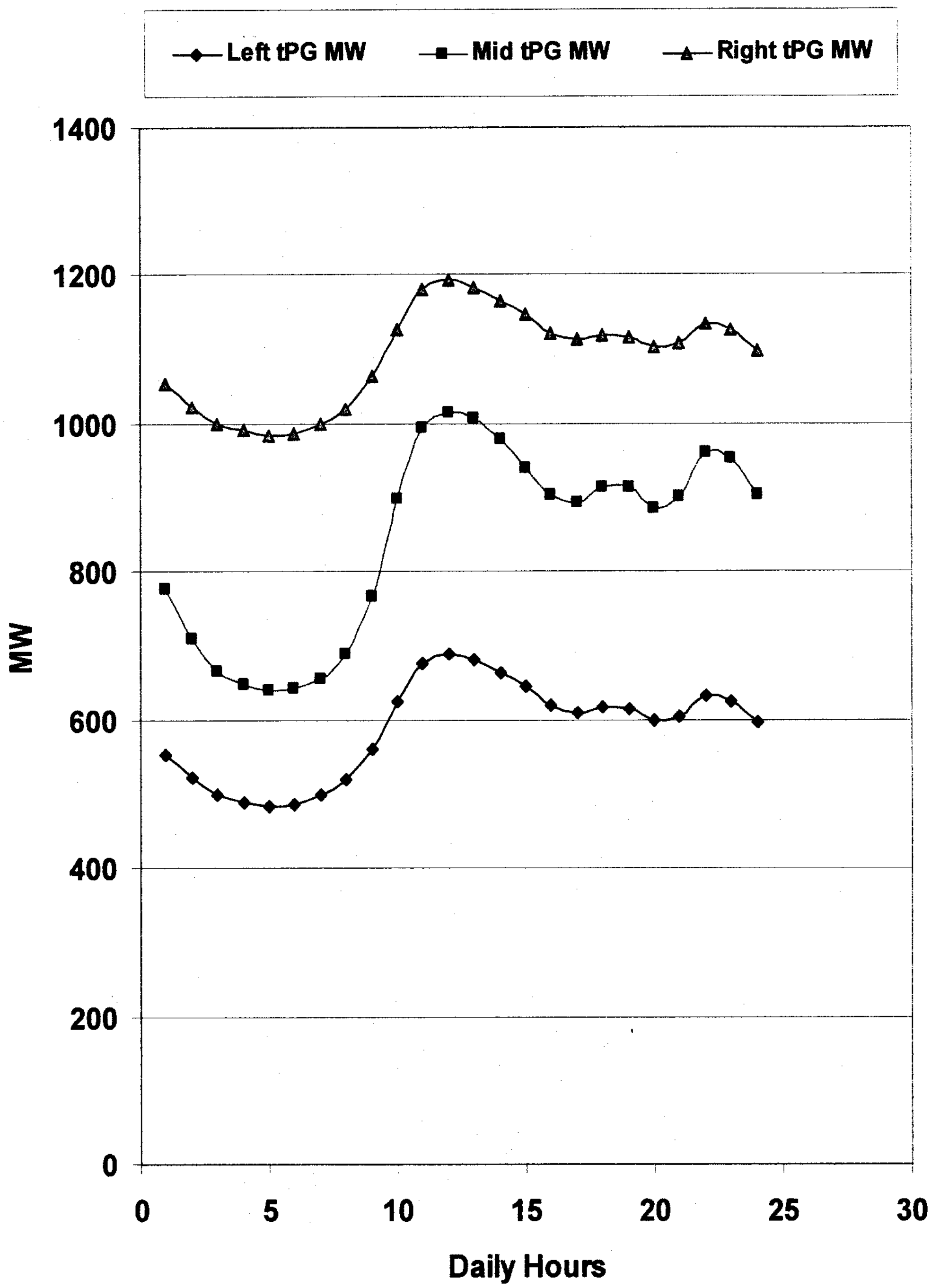

Figure (P1-8) Fuzzy (0.5-α-Cut) Representation of Total Generation

Table (P1-5)

Membership Function of Total Cost for (0, 0.5, 0.75, 1) α -Cut Representation for Model "A" Weekend With 20% Deviation

Membership Function		$\mu_C=0$			$\mu_C=0.5$			$\mu_C=0.75$			$\mu_C=1$	
Daily Hours	Left Cost $/h	Mid Cost $/h	Right Cost $/h	Left Cost $/h	Mid Cost $/h	Right Cost $/h	Left Cost $/h	Mid Cost $/h	Right Cost $/h	Left Cost $/h	Mid Cost $/h	Right Cost $/h
1	3028	8110	16779	5357	8110	12118	6680	8110	10032	8110	8110	8110
2	3056	7244	16832	4999	7244	11624	6084	7244	9330	7244	7244	7244
3	3044	6708	16810	4756	6708	11288	5702	6708	8880	6708	6708	6708
4	3051	6464	16824	4652	6464	11143	5532	6464	8678	6464	6464	6464
5	2984	6368	16696	4571	6368	11030	5444	6368	8574	6368	6368	6368
6	3014	6411	16753	4607	6411	11080	5483	6411	8620	6411	6411	6411
7	3137	6586	16986	4755	6586	11287	5644	6586	8812	6586	6586	6586
8	3221	6983	17145	4980	6983	11598	5951	6983	9174	6983	6983	6983
9	3284	7987	17263	5456	7987	12254	6677	7987	10028	7987	7987	7987
10	3252	9778	17202	6200	9778	13268	7911	9778	11468	9778	9778	9778
11	3329	11205	17347	6841	11205	14132	8916	11205	12632	11205	11205	11205
12	3376	11525	17435	7002	11525	14349	9152	11525	12904	11525	11525	11525
13	3289	11402	17273	6896	11402	14206	9036	11402	12771	11402	11402	11402
14	3234	10946	17169	6674	10946	13908	8706	10946	12390	10946	10946	10946
15	3226	10387	17154	6438	10387	13590	8320	10387	11943	10387	10387	10387
16	3083	9880	16884	6138	9880	13184	7923	9880	11482	9880	9880	9880
17	3022	9704	16768	6026	9704	13032	7781	9704	11317	9704	9704	9704
18	2940	10028	16612	6108	10028	13144	7974	10028	11542	10028	10028	10028
19	2876	10024	16491	6066	10024	13087	7949	10024	11513	10024	10024	10024
20	2883	9622	16504	5905	9622	12867	7676	9622	11196	9622	9622	9622
21	2856	9810	16451	5965	9810	12949	7795	9810	11334	9810	9810	9810
22	2806	10701	16357	6298	10701	13401	8385	10701	12019	10701	10701	10701
23	2719	10578	16188	6191	10578	13256	8270	10578	11885	10578	10578	10578
24	2648	9860	16052	5853	9860	12797	7756	9860	11288	9860	9860	9860

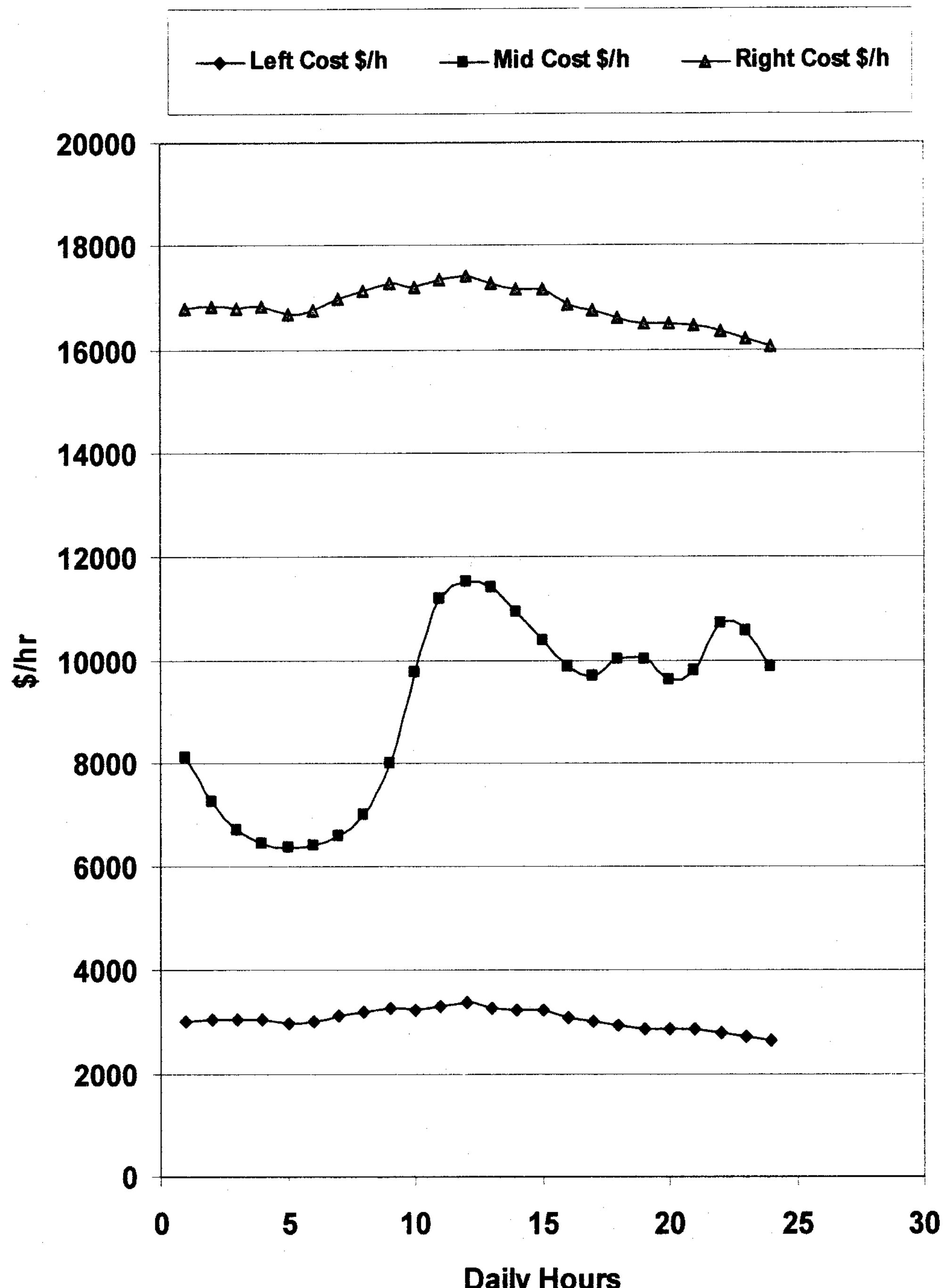

Figure (P1-9) Fuzzy (0-α-Cut) Representation of Total Cost

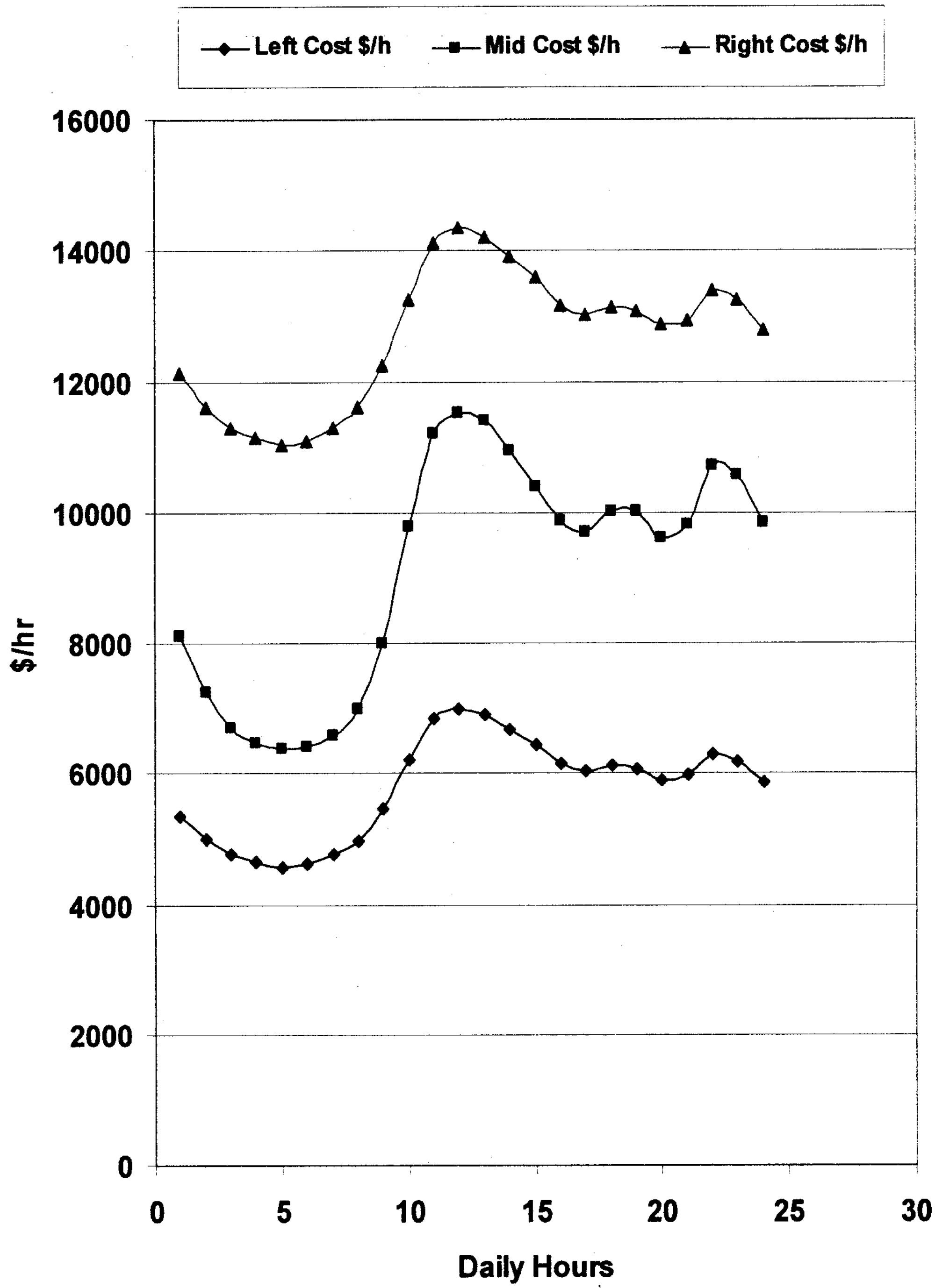

Figure (P1-10) Fuzzy (0.5-α-Cut) Representation of Total Cost

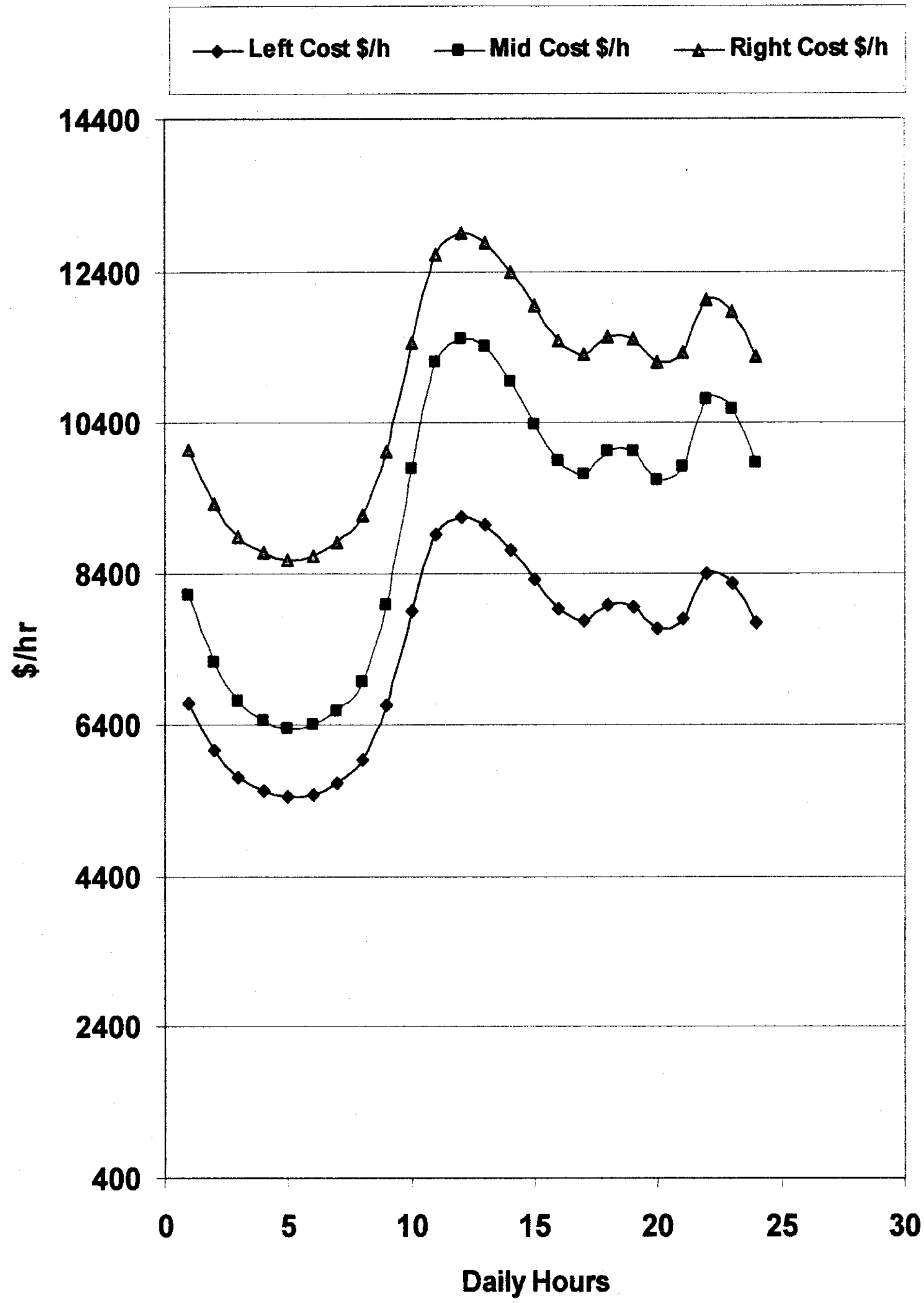

Figure (P1-11) Fuzzy (0.75-α-Cut) Representation of Total Cost

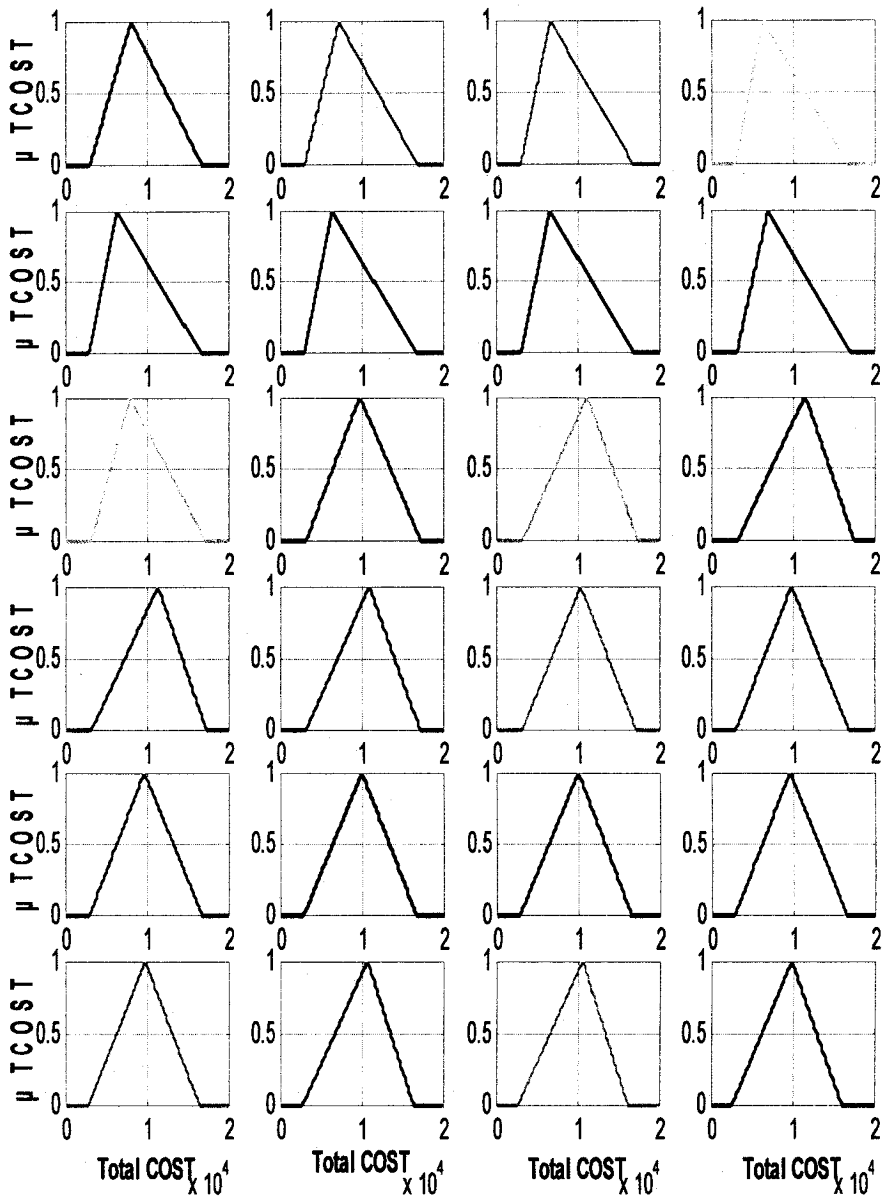

Figure (P1-12) Fuzzy Triangular Membership of Total Cost