

Phase variation of the nucleon-nucleon amplitude and the spin observables in proton-⁴He elastic scattering

R. J. Lombard and J. P. Maillet

Division de Physique Théorique, Institut de Physique Nucléaire, F-91406 Orsay CEDEX, France
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The present paper is based on a suggestion by Franco and Yin that the phase of the nucleon-nucleon scattering amplitude should vary with momentum transfer. Studying proton-⁴He elastic scattering at 1 GeV, we found this phase to be actually important for the description of spin observables. In particular, the spin rotation parameter Q is sensitive to its sign.

In some recent work by Franco and Yin,^{1,2} α -particle elastic scattering on protons, deuterons, ³He, and ⁴He has served as a test to estimate the role of the phase of the nucleon-nucleon scattering amplitude. From their systematic study at 1 GeV/nucleon incident energy, these authors conclude that indeed a phase factor $e^{i\gamma t/2}$, where t is the squared momentum transfer, was able to bring Glauber-type calculations into good agreement with experiments over a large range of t values. The nuclear densities used in these calculations were simplified, built up from single Gaussians. However, the use of more realistic densities, based on double Gaussians, improves the results only a little, as shown by Usmani *et al.*³ Thus, it cannot disprove the conclusions of Franco and Yin.

Another possibility has been advocated by Etim and Satta,⁴ namely the manifestation at large t values of a hard scattering component, which is seen in pp and $p\bar{p}$ elastic scattering at intersecting storage ring (ISR) energies. However, this model contains too many adjustable parameters to justify more than a qualitative discussion.

In this Rapid Communication, we would like to point out that spin observables constitute decisive tests for the presence of the phase factor and its value. Our calculations closely follow those of Franco and Yin.^{1,2} They are restricted to p -⁴He at 1 GeV incident energy, a case for which the polarization has been measured up to $|t| = 0.9$ (GeV/c)².

For the NN scattering matrix, use is made of the standard choice, retaining only two dominant components out of the five independent terms:

$$f(t) \cong A(t) + C(t)(\sigma_1 + \sigma_2) \cdot \hat{n}. \tag{1}$$

Details concerning the notation can be found in many papers (see, for instance, Auger *et al.*⁵). At 1 GeV the elementary proton-neutron amplitudes are poorly known, whereas phase-shift analyses are available for proton-proton scattering.⁶ Accordingly, we simply used amplitudes averaged over pp and pn data, which is sufficient in view of the points we want to raise in this work. Looking at the phase-shift analysis,⁶ we found that the use of an average slope is acceptable in the case of the spin-independent amplitude. On the contrary, this approximation is not valid for the spin-orbit component, and has a clear influence on the polarization. Consequently, the two

amplitudes A and C are parametrized by

$$A(t) = k_0 a_0 e^{(\beta_c^2 + i\gamma)t/2}, \tag{2}$$

$$C(t) = k_0 \sqrt{-t} (c_0 e^{\beta_{s0}^2 t/2} + i c_1 e^{\beta_{s1}^2 t/2}) e^{i\gamma t/2},$$

where k_0 is the incident momentum in the NN center-of-mass system. The values of the parameters are listed below:

$$a_0 = (2.75 + 12.0i) (\text{GeV}/c)^{-2}, \quad \beta_c^2 = 5.9 (\text{GeV}/c)^{-2},$$

$$c_0 = 3.0 (\text{GeV}/c)^{-3}, \quad \beta_{s0}^2 = 5.65 (\text{GeV}/c)^{-2}, \tag{3}$$

$$c_1 = 6.0 (\text{GeV}/c)^{-3}, \quad \beta_{s1}^2 = 10.3 (\text{GeV}/c)^{-2}.$$

As far as the ⁴He nuclear density is concerned, we limit ourselves to the single Gaussian model

$$\rho(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = N \delta(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4) \prod_{i=1}^4 e^{-\alpha^2 r_i^2}. \tag{4}$$

The parameter $\alpha^2 = 0.55 \text{ fm}^{-2}$ is determined from the charge radius taking into account the proton charge radius. The Glauber model is used to calculate the proton-⁴He elastic scattering amplitude, which has two components:

$$F(t) = F_1(t) + F_2(t)(\sigma_0 \cdot \hat{n}). \tag{5}$$

The three independent observables are then the differential cross section $d\sigma/dt$, the polarization P , and the spin rotation parameter Q .^{5,7} They are defined by

$$\frac{d\sigma}{dt} = \frac{\pi}{k_0^2} (|F_1|^2 + |F_2|^2),$$

$$P = 2 \frac{\text{Re}(F_2^* F_1)}{|F_1|^2 + |F_2|^2}, \quad Q = 2 \frac{\text{Im}(F_2^* F_1)}{|F_1|^2 + |F_2|^2}. \tag{6}$$

Keeping only terms linear in $C(t)$, whereas $A(t)$ contributions are included to all orders, it is possible to find compact expressions for F_1 and F_2 . It follows the prescription given by Bassel and Wilkin for a spin-

independent interaction.⁸ They read

$$\begin{aligned}
 F_1 &= k_0 e^{-t/4Aa^2} a_0 \sum_{j=1}^A \binom{A}{j} j^{-1} (2ia_0\eta_c^2)^{j-1} e^{t/4j\eta_c^2}, \\
 F_2 &= F_{20} + F_{21}, \\
 F_{2n} &= k_0 \sqrt{-t} e^{-t/4Aa^2} C_n \sum_{j=0}^{A-1} A \binom{A-1}{j} (2ia_0\eta_c^2)^j \left(\frac{\eta_{sn}^2}{\eta_{sn}^2 + j\eta_c^2} \right)^2 e^{t/4(\eta_{sn}^2 + j\eta_c^2)},
 \end{aligned}
 \tag{7}$$

with

$$\eta_c^2 = \frac{a^2}{1 + 2a^2(\beta_c^2 + i\gamma)}, \quad \eta_{sn}^2 = \frac{a^2}{1 + 2a^2(\beta_{sn}^2 + i\gamma)},$$

with $n=0$ or 1 according to the parametrization of $C(t)$.

The results of the present calculations are displayed in Figs. 1 and 2, where the three observables are plotted against $-t$. Values of γ were selected in the range given by Franco and Yin.^{1,2}

As far as the differential cross section is concerned, the calculations are checked against the two existing sets of experimental data.^{9,10} The large discrepancy, partly due to absolute normalization differences, has been recognized for a long time.¹¹ It is an experimental problem which has not been resolved so far.

The curves displayed in Fig. 1 correspond to $\gamma=0, 5,$ and 10 $(\text{GeV}/c)^{-2}$, the last value being the one advocated by Franco and Yin. In the region of the first diffraction minimum, $\gamma=5$ is qualitatively and quantitatively in better agreement with data than $\gamma=10$, which barely produces a shoulder at a place where both sets of data indi-

cate a plateau or a slight dip. Note that the set of data from Ref. 9 seems to overestimate the differential cross section even at very forward angles, where the calculations are not very sensitive to details of the model. Thus, as far as the absolute calibration is concerned, we feel more confident with the data from Ref. 10. At higher momentum transfer $\gamma=10$ seems preferred. However, contribution from neglected amplitudes should start to play a role, although it is difficult to predict *a priori* the sign and the effect of interferences.

Checking the calculated polarization against experimental data,^{9,10} we note that the virtue of γ is to smooth the typical diffractive pattern, namely the sharp hedge of P after its minimum. This effect is very difficult to obtain in varying the NN amplitude parameters. The plotted curves correspond to $\gamma=0, 5,$ and 10 $(\text{GeV}/c)^{-2}$. Again $\gamma=5$ seems to yield a reasonable compromise.

The spin-rotation parameter, on the other hand, is quite sensitive to the sign of γ . This is exemplified in Fig. 2. The three curves displayed for Q correspond to $\gamma=0, 10,$ and -15 , the three values used by Franco and Yin.

These results have to be taken with caveats. The phase factor γ has its strongest effects at places where multiple

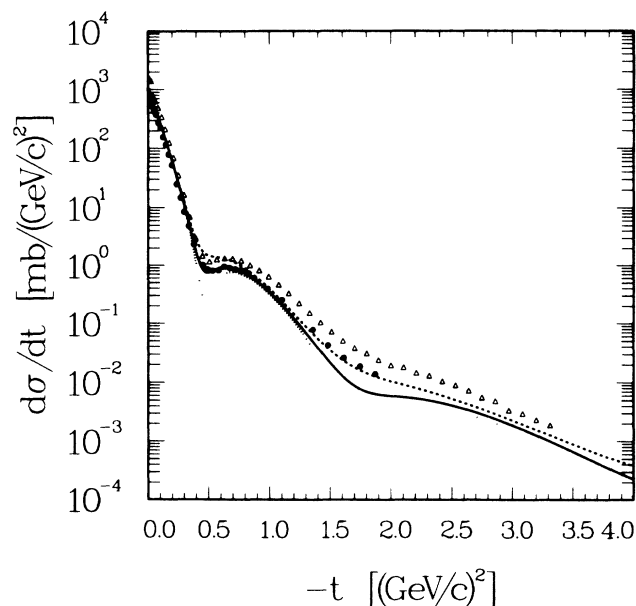


FIG. 1. Differential cross sections for elastic scattering of protons by ^4He at 1.05 GeV . (Δ , data of Ref. 9; \bullet , data of Ref. 10.) The dotted curve is the constant phase result ($\gamma=0$). The dashed and solid curves are calculations with phase variations in the NN amplitude [$\gamma=10$ and 5 $(\text{GeV}/c)^{-2}$, respectively].

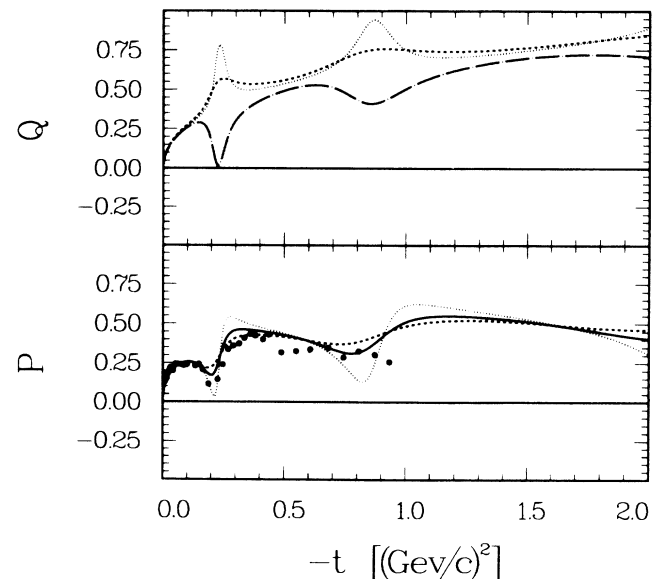


FIG. 2. Polarization P and spin-rotation parameter Q for proton- ^4He elastic scattering. Same legend as for Fig. 1. In the upper part Q the dash-dotted curve corresponds to $\gamma=-15$ $(\text{GeV}/c)^{-2}$, and shows the sensitivity of Q to the sign of γ .

scattering contributions interfere. The influence is less obvious when one term dominates. Consequently, γ is best determined at places where the calculations are sensitive to many details of the ingredients. The use of realistic densities, precise nucleon-nucleon amplitudes, coupling to inelastic channels, as well as other corrections, may eventually change our conclusions. A more severe test is now undertaken at 0.8 GeV, an energy at which both the nucleon-nucleon and proton- ${}^4\text{He}$ data are more precisely known.¹²

Finally, as pointed out by Franco and Yin, the phase factor is independent of the nucleus. Consequently, in order to get a deeper insight to the problem, it would be valuable to measure spin observables of other reactions like α - d and α - ${}^3\text{He}$. Combining several measurements, as was done for the differential cross section, may allow us to

disentangle between effects of various origin. In any case the spin observables provide definitively key information.

It should also be noted that the phase variation of the nucleon-nucleon elastic scattering with momentum transfer is a much debated question. Its effects at ISR energies have been discussed in detail by Kundrat *et al.*¹³ One wonders what kind of influence this phase variation may have on intermediate energy physics, beyond nucleon-nucleus elastic scattering. Answering this question requires a model for the origin of this phase variation, a problem we are currently trying to elucidate.

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