Diamagnetic susceptibility of a dense electron gas

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(Received 27 July 1987)

We calculate the diamagnetic susceptibility of a uniform interacting electron gas in the random-phase approximation. From this the exact high-density expansion of the diamagnetic susceptibility is obtained.

I. INTRODUCTION

The magnetic response of degenerate electronic systems has two contributions: one, known as Pauli paramagnetism, is due to spin polarization; the other, known as Landau dimagnetism, is due to the circulation of orbital currents.

The two effects have comparable magnitudes, and they must both be considered when comparing experiments with theory. For example, in a uniform noninteracting electron gas in a magnetic field $\mathbf{H}$ one finds, for the spin magnetization density,

$$\mathbf{M}_P = \chi_P^0 \mathbf{H} , \tag{1.1}$$

where $\chi_P^0 = e^2 v_F / 4\pi^2 \hbar c^2$, and v_F is the Fermi velocity. The orbital magnetization density for the same system is

$$\mathbf{M} = \chi_L^0 \mathbf{H} , \tag{1.2}$$

where $\chi_L^0 = -\frac{1}{3}\chi_P^0$.[1]

Equations (1.1) and (1.2) do not include the effect of electron-electron interactions. The spin susceptibility of a uniform interacting electron gas has been calculated by many authors in various approximations.[2] The "homogeneous" results, combined with the local spin-density functional theory,[3] have served as a cornerstone for the study of magnetism in inhomogeneous systems. In the high-density limit ($r_s \to 0$) an exact expansion is known:[4]

$$\frac{\chi_P^0}{\chi_P} \cong 1 - \frac{\alpha}{\pi} r_s + \frac{1}{2}\left[\frac{\alpha r_s}{\pi}\right]^2 \left[0.306 - \ln\frac{\alpha r_s}{\pi}\right] + O(r_s^3) \tag{1.3}$$

[$\alpha = (4/9\pi)^{1/3} \cong 0.521\,106$; r_s is the usual electron gas parameter].

On the other hand, the theory of orbital current response has been largely ignored. An early calculation[5] for the uniform electron gas, using an approximate boson Hamiltonian, gave no correction at all to the noninteracting result, Eq. (1.2). It would clearly be useful to provide a calculation of the diamagnetic susceptibility comparable to those for the spin susceptibility.

In this paper we present the first calculation of the diamagnetic susceptibility of a uniform electron gas in the random-phase approximation (RPA). The formalism employed parallels the one used by Ma and Brueckner[6] (MB) in their calculation of the exchange-correlation energy of a weakly inhomogeneous electron gas, but now it is applied to the current-current correlation functions. From this we are able to extract the *exact* high-density expansion of the diamagnetic susceptibility:

$$\frac{\chi_L}{\chi_L^0} = 1 + \frac{\alpha}{6\pi} r_s \ln r_s + 0.084\,83\alpha r_s/\pi + O(r_s^2) . \tag{1.4}$$

It is interesting to note that the leading term of the expansion is $O(r_s \ln r_s)$. This reflects the fact that the diamagnetic susceptibility diverges in the Hartree-Fock approximation, and it is necessary to include correlations to obtain a finite result.

Our calculations show that the many-body corrections in χ_L are considerably smaller than the ones in χ_P. Furthermore, the diamagnetic susceptibility is decreased by the interactions, while the spin susceptibility is enhanced. We conclude that the uniform electron gas, in agreement with one's intuition, has no tendency to break the symmetry towards a state with spontaneous orbital magnetization.

II. FORMALISM

The current induced in a homogeneous electron gas by a weak static vector potential $\mathbf{A}(\mathbf{r})$ [Fourier transform $\mathbf{A}(\mathbf{q})$] is given, in linear response theory,[7] by

$$\mathbf{j}(\mathbf{q}) = -\frac{e^2}{c}\left[\overleftrightarrow{\mathbf{P}}(\mathbf{q}) + \frac{n}{m}\right]\cdot \mathbf{A}(\mathbf{q}) \tag{2.1}$$

(n is the electron density).

The current-current correlation tensor $\overleftrightarrow{P}(\mathbf{q})$ is given by

$$P_{ij}(\mathbf{q})=-2\sum_n \frac{\langle 0\,|\,j_{pi}(\mathbf{q})\,|\,n\rangle\langle n\,|\,j_{pj}(-\mathbf{q})\,|\,0\rangle}{E_n-E_0} ,$$

$|\,n\rangle$ and E_n are the exact eigenstates and eigenvalues of the electron gas Hamiltonian;

$$\mathbf{j}_p(\mathbf{q})=\tfrac{1}{2}\sum_{\alpha=1}^{N}(\mathbf{p}_\alpha e^{i\mathbf{q}\cdot\mathbf{r}_\alpha}+e^{i\mathbf{q}\cdot\mathbf{r}_\alpha}\mathbf{p}_\alpha)$$

is the "paramagnetic" current, where $\mathbf{p}_\alpha$ and $\mathbf{r}_\alpha$ are the momentum and position operators of the αth electron. $P_{ij}(q)$ has the following properties:[7,8]

$$\text{(i)}\quad P_{ij}(\mathbf{q}=0)=-\frac{n}{m}\delta_{ij} ,$$
$$\text{(ii)}\quad P_{ij}(\mathbf{q})+\frac{n}{m}\delta_{ij}=\left[\delta_{ij}-\frac{q_iq_j}{q^2}\right]P(q) . \tag{2.2}$$

The second property means that the linear response function in Eq. (2.1) is a transverse tensor. (i) and (ii) together imply that $P(q)$ must vanish as $q\to 0$:

$$P(q)\cong P_2q^2\quad(q\to 0) . \tag{2.3}$$

The current $\mathbf{j}(\mathbf{q})$ can be written as the curl of a divergence-free magnetization $\mathbf{M}(\mathbf{q})$, i.e., $\mathbf{j}(\mathbf{q})=ic\mathbf{q}\times\mathbf{M}(\mathbf{q})$. Putting this in Eq. (2.1), using the expression $\mathbf{H}(\mathbf{q})=i\mathbf{q}\times\mathbf{A}(q)$ for the magnetic field, and taking the $q\to 0$ limit, we find

$$\mathbf{M}(q\to 0)=-\frac{e^2}{c^2}P_2\mathbf{H}(q\to 0) .$$

Thus the diamagnetic susceptibility is

$$\chi_L=-\frac{e^2}{c^2}P_2 . \tag{2.4}$$

To calculate P_2, we orient the z axis along the direction of $\mathbf{q}$, so that $P(q)=(n/m)+P_{xx}(q)$. $P_{xx}(q)$ can be expressed in terms of Green's functions (G) as follows [$\mathbf{q}\equiv q\hat{\mathbf{z}}$, $p\equiv(p_0,\mathbf{p})$ is a four-vector]:

$$P_{xx}(q)=2\int\frac{d^4p}{(2\pi)^4i}\frac{p_x}{m}R(p;\mathbf{q})\Lambda_x(p;\mathbf{q}) ,$$
$$R(p;\mathbf{q})\equiv G(p+\mathbf{q}/2)G(p-\mathbf{q}/2) . \tag{2.5}$$

The vertex correction $\Lambda_x(p,\mathbf{q})$ is defined as

$$\Lambda_x(p;q)\equiv\frac{p_x}{m}+\int\frac{d^4p'}{(2\pi)^4i}\Gamma(p,p';\mathbf{q})R(p';\mathbf{q})\frac{p_x'}{m} . \tag{2.6}$$

The spin-symmetric interaction $\Gamma(p,p';\mathbf{q})$ satisfies the Bethe-Salpeter equation

$$\Gamma(p,p';\mathbf{q})=\gamma(p,p';\mathbf{q})+\int\frac{d^4p''}{(2\pi)^4i}\gamma(p,p'';\mathbf{q})R(p'';\mathbf{q})\Gamma(p'',p';\mathbf{q}) , \tag{2.7}$$

where $\gamma(p,p';\mathbf{q})$ is the spin-symmetric irreducible electron-hole interaction.[8]

Rather than calculating Eqs. (2.5)–(2.7) and then expanding to order q^2, it is more efficient to derive an expression for P_2 directly.[9] This can be done by expanding Λ_x, R, and γ as follows:

$$\Lambda_x(p;q)\cong\Lambda_{x0}(p)+\Lambda_{x2}(p)q^2 ,$$
$$R(p;\mathbf{q})\cong R_0(p)+R_2(p)q^2 ,$$
$$\gamma(p,p';\mathbf{q})=\gamma_0(p,p')+\gamma_2(p,p')q^2 .$$

Putting these expressions in Eqs. (2.5)–(2.7) and using the Ward identity[8]

$$\Lambda_{x0}(p)R_0(p)=\frac{\partial G(p)}{\partial p_x} , \tag{2.8}$$

one finally finds (after some algebra)

$$P_2=\lim_{q\to 0}\frac{P_{xx}(q)-P_{xx}(0)}{q^2}$$
$$=2\int\frac{d^4p}{(2\pi)^4i}\Lambda_{x0}^2(p)R_2(p)+2\int\frac{d^4p}{(2\pi)^4i}\int\frac{d^4p'}{(2\pi)^4i}\frac{\partial G(p)}{\partial p_x}\gamma_2(p,p')\frac{\partial G(p')}{\partial p_x'} . \tag{2.9}$$

III. RANDOM-PHASE APPROXIMATION

To calculate P_2 in a dense electron gas, we closely follow Ma and Brueckner's[6] calculation of the coefficient q^2 in the density-density correlation function. This calculation corresponds to the random-phase approximation for the ground-state energy, and is exact in the high-density limit.

We start from the RPA expression for the self-energy [$k\equiv(\mathbf{k},\omega)$]

$$\Sigma(p)=-\lim_{\eta\to 0+}\int\frac{d^4k}{(2\pi)^4i}V(k)G_0(k+p)e^{i\eta\omega} . \tag{3.1}$$

The noninteracting Green's function is given by

$$G_0(p)=[p_0-\varepsilon_p+\mu+i\,\mathrm{sgn}(\varepsilon_p-\mu)]^{-1} ,$$
$$\varepsilon_p\equiv p^2/2m , \tag{3.2}$$
$$\mu=p_F^2/2m .$$

The RPA screened interaction is

$$V(\mathbf{k},\omega)=\frac{4\pi e^2}{k^2\epsilon(\mathbf{k},\omega)} ,$$
$$\epsilon(\mathbf{k},\omega)=1-\frac{4\pi e^2}{k^2}F_0(\mathbf{k},\omega) , \tag{3.3}$$

where $F_0(\mathbf{k},\omega)$ is the usual Lindhard function.

The knowledge of $\Sigma(p)$ determines, in a conserving approximation, the $\mathbf{q}\to\mathbf{0}$ limit of the vertex function[8]

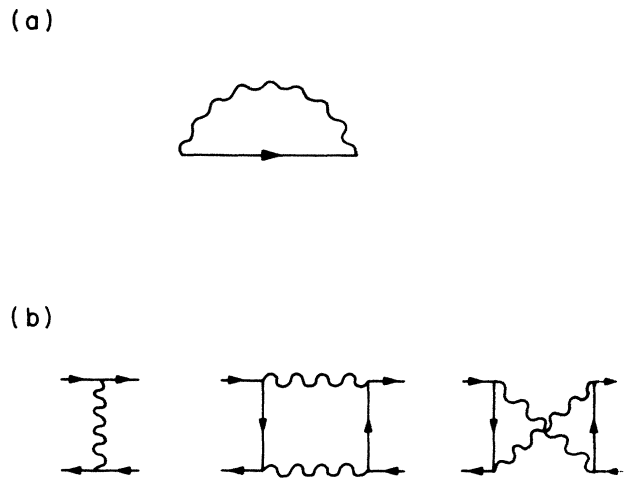

FIG. 1. (a) Self-energy diagram in RPA. The undulating line is the dynamically screened interaction in the RPA. (b) Diagrams for the irreducible electron-hole interaction generated from the RPA self-energy in a conserving approximation.

$$\Lambda_{x0}(p)=\frac{p_x}{m}+\frac{\partial\Sigma(p)}{\partial p_x}\ , \tag{3.4}$$

and the irreducible electron-hole interaction

$$\gamma=\frac{\delta\Sigma}{\delta G_0}\ . \tag{3.5}$$

Diagrams for γ are obtained by opening any G_0 line in the diagrams for Σ. This is shown in Fig. 1(b). Now, expanding the current-current correlation function up to linear terms in Σ, we get, for the interaction contribution, the five diagrams of Fig. 2. These are precisely the MB diagrams, apart from the fact that we have a current vertex p_x/m.

The coefficient of q^2 arising from the MB diagrams is

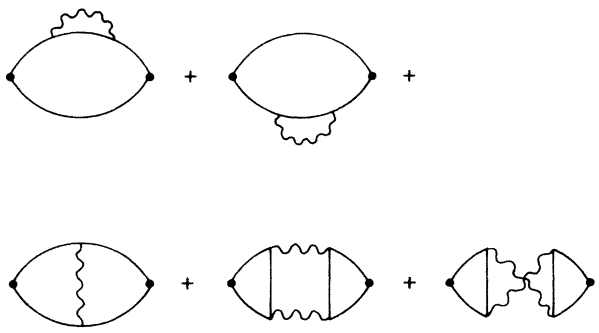

FIG. 2. RPA diagrams for the current-current correlation function, giving the exact many-body corrections in the high-density limit.

conveniently calculated with the help of Eq. (2.9), setting

$$G(p)\cong G_0(p)+G_0^2(p)\Sigma(p) \tag{3.6}$$

[the precise meaning of $G_0^2(p)$ is $-\partial G_0(p)/\partial p_0$] and collecting the terms which are linear in Σ. Three terms are found.

(i) From the first term of Eq. (2.9), putting $\Lambda_{x0}=p_x/m$, and expanding R to first order in Σ, we obtain

$$-4\int\frac{d^4p}{(2\pi)^4 i}G_0(p+q)\frac{\partial G_0(p)}{\partial p}\Sigma(p)\ .$$

This expression is then expanded to order q^2, and transformed with the help of the following identities (see Ref. 6):

$$-\frac{\partial G_0(p)}{\partial p_0}=-\frac{\partial G_0(p)}{\partial\mu}+2\pi i\delta(p_0)\delta(\mu-\varepsilon_p)\ , \tag{3.7}$$

and

$$\lim_{\delta\to 0}\frac{1}{(m-1)!}\left[-\frac{\partial}{\partial\mu}\right]^{m-1}G_0(p+\delta)\frac{1}{(n-1)!}\left[-\frac{\partial}{\partial\mu}\right]^{n-1}G_0(p)=\frac{1}{(m+n-1)!}\left[-\frac{\partial}{\partial\mu}\right]^{m+n-1}G_0(p)\ . \tag{3.8}$$

The result for the coefficient of q^2 is

$$b_1=\frac{1}{m}\int\frac{d^4p}{(2\pi)^4 i}\frac{p^2}{m^2}\Sigma(p)\left[-\frac{1}{9}\frac{\partial^3 G_0(p)}{\partial\mu^3}+\tfrac{1}{45}\varepsilon_p\frac{\partial^4 G_0(p)}{\partial\mu^4}\right]+\frac{\Sigma(k_F,\mu)}{12\pi^2 k_F}\ . \tag{3.9}$$

(ii) From the first term of Eq. (2.9), using the interaction part of Λ_{x0} and noninteracting R, the contribution to the coefficient of q^2 is

$$b_2=\int\frac{d^4p}{(2\pi)^4 i}\frac{\partial\Sigma(p)}{\partial p_x}\frac{p_x}{m^2}\left[\frac{1}{2}\frac{\partial^2 G_0(p)}{\partial\mu^2}-\tfrac{1}{15}\varepsilon_p\frac{\partial^3 G_0(p)}{\partial\mu^3}\right]\ , \tag{3.10}$$

(iii) from the last term of Eq. (2.9) using $G=G_0$ and γ given by the diagrams of Fig. 1(b). Since the first diagram is independent of q it does not contribute to γ_2. The remaining two diagrams can be written as

$$\gamma'(p,p';q)=2\int\frac{d^4k}{(2\pi)^4 i}\{V(k+\mathbf{q}/2)V(k-\mathbf{q}/2)G_0(p+k)[G_0(p'+k)+G_0(p'-k)]\}\ .$$

Inserting this in Eq. (2.9), and using the definition of the Lindhard function

$$F_0(k)=2\int \frac{d^4p}{(2\pi)^4 i} G_0(p)G_0(p+k) , \tag{3.11}$$

we find

$$\int \frac{d^4k}{(2\pi)^4 i} V(k+q/2)V(k-q/2)\left[\left[\frac{\partial F_0(k)}{\partial k_x}\right]^2 - \left[\frac{\partial F_0(k)}{\partial k_x}\right]^2\right]=0$$

for all q. Thus the last term of Eq. (2.9) does not contribute to the diamagnetic susceptibility.

Equations (3.9) and (3.10) are conveniently combined in a single expression for $P_2^{\rm int}\equiv b_1+b_2$. We next integrate by parts in Eq. (3.10), and change some derivatives with respect to μ into derivatives with respect to p, according the identity

$$\frac{\partial G_0(p)}{\partial \mu}=-\frac{\partial G_0(p)}{\partial \varepsilon_p} .$$

The final result is

$$P_2^{\rm int}=-\frac{1}{m^2}\int \frac{d^4p}{(2\pi)^4 i}\Sigma(p)\left[\frac{1}{2}\frac{\partial^2 G_0(p)}{\partial \mu^2}-\tfrac{2}{9}\varepsilon_p\frac{\partial^3 G_0(p)}{\partial \mu^3}\right] + \frac{\Sigma(k_F,\mu)}{12\pi^2 k_F} . \tag{3.12}$$

Using Eq. (3.1) for Σ, and interchanging the order of integration, this can also be written as

$$P_2^{\rm int}=\frac{1}{m^2}\lim_{\eta\to 0+}\int \frac{d^4k}{(2\pi)^4 i}V(k) \times[\tfrac{1}{2}I_1(k)-\tfrac{2}{9}I_3(k)]e^{i\eta\omega}+\frac{\Sigma(k_F,\mu)}{12\pi^2 k_F} , \tag{3.13a}$$

$$I_1(k)=\int \frac{d^4p}{(2\pi)^4 i}G_0(p+k)\frac{\partial^2 G_0(p)}{\partial \mu^2} , \tag{3.13b}$$

$$I_3(k)=\int \frac{d^4p}{(2\pi)^4 i}G_0(p+k)\frac{\partial^3 G_0(p)}{\partial \mu^3}\varepsilon_p . \tag{3.13c}$$

The integrals $I_1(k)$ and $I_3(k)$ can be calculated analytically (see Ref. 9, Appendix C). Thus, Eq. (3.13a) reduces the problem of the diamagnetic susceptibility to the evaluation of a two-dimensional integral. (The angular integration is trivial.) This is the main result of this section.

IV. HIGH-DENSITY LIMIT

In this section we calculate $P_2^{\rm int}$ in the high-density, or weak-coupling, limit. The result is exact to order $O(e^2)$.

We begin by splitting the RPA screened interaction into a statically screened Thomas-Fermi part and a dynamical part

$$V(\mathbf{k},\omega)=\frac{4\pi e^2}{k^2\epsilon(\mathbf{k},\omega)} =\frac{4\pi e^2}{k^2+\lambda^2}+4\pi e^2\left[\frac{1}{k^2\epsilon(\mathbf{k},\omega)}-\frac{1}{k^2+\lambda^2}\right] . \tag{4.1}$$

$\lambda^2=4\pi e^2\nu(0)$ is the square of the Thomas-Fermi wave vector. $\nu(0)=mk_F/\pi^2\hbar^2$ is the density of states at the Fermi energy.

The contribution of the statically screened interaction can be exactly calculated from Eq. (3.12). The reason is that the corresponding self-energy $\Sigma_{\rm TF}(\mathbf{p})$ is frequency independent. Therefore, the frequency integral in Eq. (3.12) can be done with the help of the identity

$$\lim_{\eta\to 0+}\int \frac{dp_0}{2\pi i}G_0(\mathbf{p},p_0)e^{i\eta p_0}=n_p=\Theta(\mu-\varepsilon_p) ,$$

and the momentum integral involves derivatives of $\delta(\mu-\varepsilon_p)$. The result is (reintroducing h factors)

$$(P_2^{\rm int})_{\rm TF}=\frac{1}{36\pi^2\hbar^2}[\Sigma'_{\rm TF}(k_F)+2k_F\Sigma''_{\rm TF}(k_F)] . \tag{4.2}$$

The Thomas-Fermi self-energy is given by

$$\Sigma_{\rm TF}(k)=-\frac{e^2}{\pi}\left\{k_F+\frac{k_F^2+k^2-\lambda^2}{4k}\ln\left[\frac{(k+k_F)^2+\lambda^2}{(k-k_F)^2+\lambda^2}\right] -\lambda\left[\tan^{-1}\left[\frac{k+k_F}{\lambda}\right]-\tan^{-1}\left[\frac{k-k_F}{\lambda}\right]\right]\right\} , \tag{4.3a}$$

and its derivatives with respect to k are

$$\Sigma'_{\rm TF}(k_F)=\frac{e^2}{2\pi}\left[-2+\left[\frac{2k_F^2+\lambda^2}{2k_F^2}\right]\ln\left[\frac{4k_F^2+\lambda^2}{\lambda^2}\right]\right] , \tag{4.3b}$$

and

$$\Sigma''_{\rm TF}(k_F)=\frac{e^2}{2\pi k_F}\left[2+\left[\frac{4k_F^2+2\lambda^2}{4k_F^2+\lambda^2} -\left[\frac{k_F^2+\lambda^2}{k_F^2}\right]\ln\left[\frac{4k_F^2+\lambda^2}{\lambda^2}\right]\right] . \tag{4.3c}$$

The term $\Sigma(k_F)/12\pi^2 k_F$ in Eq. (3.12) cancels out in the manipulations leading to Eq. (4.2).

Putting Eqs. (4.3b) and (4.3c) in Eq. (4.2), and expanding to order $O(e^2)$, we obtain ($\lambda^2/4k_F^2=\alpha r_s/\pi$)

$$(P_2^{\rm int})_{\rm TF}=\frac{e^2}{72\pi^3\hbar^2}\ln r_s+\frac{e^2}{18\pi^3\hbar^2}\left[1+\tfrac{1}{4}\ln\frac{\alpha}{\pi}\right] +O(e^4\ln e^2) . \tag{4.4}$$

Notice that in the Hartree-Fock limit ($\lambda\to 0$) the coefficient of e^2 becomes logarithmically infinite. This means, as pointed out in the Introduction, that the

Hartree-Fock susceptibility is not defined, and one must include correlations to obtain sensible results.

We now consider the contribution of the dynamical part of the interaction. For computational purposes, it is convenient to perform the frequency integral along the imaginary axis. However, care must be exerted because the "dynamical" interaction tends to a finite limit for large frequency:

$$\lim_{\omega\to\infty} V_{\rm dyn}(\mathbf{k},\omega)=\frac{4\pi e^2\lambda^2}{k^2(k^2+\lambda^2)}\equiv V_{\rm dyn}(\mathbf{k},\infty)\ .$$

Therefore in rotating to the imaginary axis, an additional contribution is generated, which comes from the circles "at infinity." This contribution is calculated analytically. Its value is

$$P_2^{\infty}=\frac{1}{24\pi^2k_F}\int\frac{d^3k}{(2\pi)^3}V_{\rm dyn}(\mathbf{k},\infty)=\frac{e^2\lambda}{24\pi^2\hbar^2k_F}\ . \tag{4.5}$$

Now, with the change of variables $\omega=ikv_Fy$ the integral of Eq. (3.12) takes the form

$$(P_2^{\rm int})_{\rm dyn}=\frac{e^2v_F}{m^2\pi\hbar^2}\int_0^{\infty}dk\,k^3\int_{-\infty}^{+\infty}dy\left[\frac{1}{k^2+\lambda^2R(k,y)}-\frac{1}{k^2+\lambda^2}\right]\left[\tfrac{1}{2}I_1(k,ikv_Fy)-\tfrac{2}{9}I_3(k,ikv_Fy)\right]$$
$$+\frac{\Sigma(k_F,\mu)-\Sigma_{\rm TF}(k_F)}{12\pi^2\hbar^2k_F}+P_2^{\infty}\ , \tag{4.6}$$

where $R(k,y)\equiv -F(k,ikv_Fy)/v(0)$. The last two terms in this equation combine to give a contribution which is $O(e^2\ln e^2)$ and can be neglected in the high-density limit. The integral is formally of order e^4, because the large parentheses vanish as $\lambda^2\sim e^2\to 0$. However, the coefficient of e^4 would be a divergent integral—the divergence coming from the small-k region. Thus the integral is actually $O(e^2)$.[10] To extract the coefficient of e^2 it is permissible to replace R, I_1, and I_3 by their small-k expressions[6]

$$R(k,ikv_Fy)\cong R(0,y)=1-y\tan^{-1}y^{-1}\ ,$$

$$I_1(k,ikv_Fy)\cong -\frac{m^2}{\pi^2v_Fk^2}\frac{y^2}{(1+y^2)^2}\ ,$$

$$I_3(k,ikv_Fy)\cong -\frac{m^2}{4\pi^2v_Fk^2}\frac{y^2(5+9y^2)}{(1+y^2)^3}\ ,$$

$$\tfrac{1}{2}I_1(k,ikv_Fy)-\tfrac{2}{9}I_3(k,ikv_Fy)\cong -\frac{2m^2}{9\pi^2v_Fk^2}\frac{y^2}{(1+y^2)^3}\ .$$

Using these formulas in Eq. (4.5), we find, below any arbitrarily small momentum cutoff,

$$(P^{\rm int})_{\rm dyn}=\frac{2}{9}\frac{e^2}{\pi^4\hbar^2}\int_0^{\infty}dy\frac{y^2\ln R(0,y)}{(1+y^2)^3}\ .$$

The integral is evaluated numerically: its value is $-0.332\,69$. The combination $e^2/\hbar$ can be expressed in terms of r_s and the Fermi velocity as follows: $e^2/\hbar=\alpha r_sv_F$. Finally, we combine the Thomas-Fermi, the dynamical, and the noninteracting ($P_2^0=v_F/12\pi^2\hbar$) contributions to P_2, to obtain

$$P_2=\frac{v_F}{12\pi^2\hbar}\left[1+\frac{1}{6}\frac{\alpha r_s}{\pi}\ln r_s+0.084\,83\frac{\alpha r_s}{\pi}\right]$$
$$+O(r_s^2\ln r_s)\ . \tag{4.7}$$

This leads to Eq. (1.4). It is easy to check that this result does not depend on the arbitrary decomposition of the interaction into a static and a dynamic part. Making the transformation $\lambda\to\lambda c$ in the static part of the interaction (c is an arbitrary positive constant) does not change the result.

The separation of the interaction into a static and a dynamic part [Eq. (4.1)] at *small* k is a delicate matter, which has been given considerable attention recently.[11] Here we have also carefully studied this question, by repeating the calculation *without separating* the static and the dynamic parts. Such a calculation involves contributions beyond the MB ones (in particular, all momentum transfers are involved, and not only small ones), but the final upshot is that the result does not change.

The behavior of χ_L/χ_L^0 from the high-density expansion is plotted in Fig. 3. The diamagnetic susceptibility

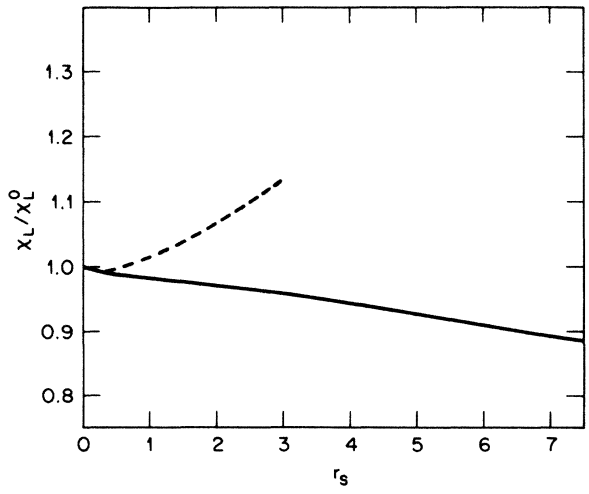

FIG. 3. Diamagnetic susceptibility ratio χ_L/χ_L^0 as a function of r_s, in the RPA. The dashed line shows, for reference, an extrapolation of the *exact* high-density formula, Eq. (4.7).

TABLE I. Comparison of many-body effects in the diamagnetic susceptibility χ_L and in the paramagnetic spin susceptibility χ_P. The noninteracting susceptibilities are $\chi_P^0 = 2.59 \times 10^{-6} r_s^{-1}$ (in cgs units) and $\chi_L^0 = -\frac{1}{3}\chi_P^0$. The values of the spin-susceptibility enhancement are taken from Table II of Ref. 2.

r_s	χ_L/χ_L^0	χ_P/χ_P^0
1	0.981	1.15
2	0.970	1.31
3	0.957	1.46
4	0.942	1.62
5	0.926	1.79
6	0.909	1.98
7	0.892	
8	0.875	
9	0.859	
10	0.842	

is *decreased* by the interactions in the very high-density limit. At larger r_s, χ_L appears to be enhanced. However, this latter is a spurious feature of the high-density expansion, as we demonstrate in the next section.

V. FULL RPA CALCULATION

We have calculated numerically the diamagnetic susceptibility in the RPA at finite r_s, using Eqs. (4.2)–(4.4) and (4.6). Our results are plotted in Fig. 3 and tabulated in Table I. As in the high-density limit, they are independent of the arbitrary decomposition of the interaction into a static and a dynamic part.

Comparing the full RPA result at finite r_s with the high-density formula, Eq. (4.7), we see that the latter is only adequate for $r_s \lesssim 0.25$. At larger r_s it begins to deviate, and for $r_s \gtrsim 0.5$ becomes even qualitatively wrong. This is different from the case of the spin susceptibility where a significant deviation occurs for $r_s \gtrsim 1.5$.

We have numerically checked the accuracy of the high-density expansion [Eq. (1.4)] by calculating the coefficient of r_s,

$$A(r_s) \equiv [\chi_L/\chi_L^0 - 1 - (\alpha/6\pi) r_s \ln r_s] r_s^{-1} .$$

TABLE II. Numerically calculated values of the coefficient of r_s in the high-density expansion for the susceptibility: $A(r_s) \equiv [\chi_L/\chi_L^0 - 1 - (\alpha/6\pi) r_s \ln r_s] r_s^{-1}$. The analytical value for $r_s \to 0$ is $A(0) = 0.01407$.

r_s	$A(r_s)$
0.1	0.0076
0.01	0.0130
0.001	0.0139
0.0001	0.0140

In the limit $r_s \to 0$, $A(r_s)$ should approach the analytical value $\simeq 0.01407$. The calculated values of $A(r_s)$ are shown in Table II. One sees that only for $r_s < 0.01$ they are in good agreement with the analytical value. Thus the range of validity of the high-density expansion appears to be very limited.

VI. SUMMARY

In this paper we have calculated the diamagnetic susceptibility of a uniform electron gas in the random-phase approximation. The calculation is exact to order $O(r_s)$. We have thus obtained the exact high-density expansion, Eq. (1.4), for the diamagnetic susceptibility. This expansion is found to be accurate for very small r_s.

Our result for the diamagnetic susceptibility has two interesting features: (1) The susceptibility is *reduced* by the many-body effects. In contrast to this, the spin susceptibility is enhanced by the many-body effects. (2) The many-body corrections in χ_L are *considerably smaller* than the corresponding corrections in the spin susceptibility. This suggests that the electron fluid has little or no tendency to spontaneously break the symmetry in favor of a magnetized state with orbital currents.

ACKNOWLEDGMENTS

This work was supported by the University of Tennessee, and by the Division of Materials Sciences, U.S. Department of Energy, under Contract No. DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

[1]Equations (1.1) and (1.2) are valid for "weak" fields $\hbar\omega_c/E_F \ll 1$, where $\omega_c = eH/mc$, and E_F is the Fermi energy. Equation (1.2) tacitly assumes $E_F \gg k_B T \gg \hbar\omega_c$, which allows one to neglect quantum oscillations.

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