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Theory of spin-fluctuation resistivity near the critical point of binary alloys and antiferromagnets*

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We present a simple theory of electrical resistivity $\rho(T)$ due to critical fluctuations in the vicinity of the Néel point of antiferromagnets and the order-disorder transition temperature of binary alloys. In the disordered phase, it is shown that the singular part of $\partial\rho/\partial T$ varies as either plus or minus the singular part of the specific heat for $T \to T_N^+$. The sign is determined by Fermi-surface geometry and the superlattice vector $\vec{Q}$ of the ordered state. The temperature range, somewhat above T_N, where short-range ($R \ll \xi$) correlations are no longer dominant is also considered. Numerical results are given for both the short-range and long-range temperature regimes. In the ordered state, it is concluded that the long-range order does not enter $\rho(T)$ directly for $T \to T_N^-$ and that $\partial\rho/\partial T$ continues to reflect more closely the specific heat. The results are compared with experiment in the representative cases of β-brass, Fe_3Al, dysprosium, and holmium. Some previously unsettled questions are answered and there is good agreement with experiment.

I. INTRODUCTION

The study of electronic transport properties at magnetic critical points has received a good deal of experimental and theoretical attention. *A priori* the following interdependent points are required for an understanding of the transport process: (i) critical properties of the spin system against which the electrons scatter, (ii) associated many-body effects such as renormalization of electron energies and velocities, and (iii) characteristic solid-state effects such as Fermi-surface anisotropy. The effect of critical fluctuations on the electrical resistivity $\rho(T)$ was first considered by deGennes and Friedel[1] using a simple model in which essentially free electrons are linearly coupled to a set of localized spins. The spin correlations were described in a conventional Ornstein-Zernicke approximation. With improved treatments of the critical fluctuations in the spin system, this s-f exchange model was widely used to discuss resistive anomalies at ferromagnetic critical points.[2,3] In particular, Fisher and Langer[3] introduced the short-range expansion for the correlation of spin fluctuations to show that $d\rho(T)/dT = \rho'(T) \propto C(T) \propto t^{-\alpha}$ for $T \to T_C^+$ [$t = (T - T_C)/T_C$ and standard notation will be used for critical exponents[4]], where $C(T)$ is the specific heat. The conclusion that $\rho'(T) \propto C(T)$ was extended to $T \to T_C^-$ by Richard and Geldart[5] who also clarified some points concerning long-range correlations of Ornstein-Zernicke (OZ) type. More recently, Richard[6] and Geldart and Richard[7] showed that the critical behavior of a variety of ferromagnets could be understood by taking into account both short-range[3] and long-range[5] correlations as well as details of the Fermi surface.

The situation has been less clear in the case of resistive anomalies at the Néel point T_N of antiferromagnets and at the order-disorder temperature T_0 of binary alloys (which have critical properties formally equivalent to those of Ising antiferromagnets[8]). The role of critical fluctuations in antiferromagnets was first studied by Suezaki and Mori[9] and later, but independently, by Geldart and Richard.[10] The former authors concluded that energy gaps in the electron dispersion relation[11,12] reflect the long-range order ($\sigma \sim |t|^\beta$) with the result that[13] $\rho'(T) \sim |t|^{\beta-1}$ for $T \to T_N^-$ and that long-range correlations dominate throughout the paramagnetic regime so that $\rho'(T) \sim t^{3\nu-\gamma-1} = t^{2\beta-1}$ if we assume the scaling relations[4] $3\nu = 2 - \alpha$ and $\alpha + 2\beta + \gamma = 2$. The latter authors also concluded that long-range correlation play a role for $T \gtrsim T_N$ (their discussion was based on a conventional OZ approximation[14] for the spin correlations) but expressed the view that *short-range* correlations dominate for $T \to T_N^+$, in contrast to the prediction of Suezaki and Mori. In the present paper, we discuss in some detail the effect of critical fluctuations on the electrical resistivity of antiferromagnets and binary alloys. The outline of the paper is as follows.

In Sec. II, we outline a formulation of the problem of calculating the resistivity in the s-f exchange model based on the variational approach to the Boltzmann equation. Appropriate choices for the spin-correlation functions for various anti-

ferromagnets are indicated. In Sec. III, we consider $\rho(T)$ for $T \to T_N^+$. The reasons for the failure of the arguments of Suezaki and Mori[9] are discussed and numerical results based on the short range expansion[3] are given. The crossover from short-range to long-range dominance with increasing temperature is considered in Sec. IV and numerical results are also given for the latter region. In Sec. V, we briefly discuss the role of energy gaps[11,12] and their effects on the temperature dependence of the resistivity for $T \to T_N^-$. Finally, in Sec. VI, we compare the present theory with experiment in the representative cases of the binary alloys β-brass and Fe_3Al and the helical rare-earth antiferromagnets dysprosium and holmium. Our basic objective in this work is to determine how well a simple conventional description of transport properties can account for the variety of observed properties and to indicate where we expect that a more sophisticated treatment will be required.

II. RESISTIVITY OF ANTIFERROMAGNETS IN *s-f* EXCHANGE MODEL

To calculate the contribution to the electrical resistivity due to spin fluctuations $\rho_s^i(T)$ when the current is in the ith crystal direction, the following standard assumptions are made: (i) Matthiessen's rule is valid. (ii) The s-f exchange interaction is weak enough to treat by the lowest-order Born approximation. (iii) The spin-fluctuation lifetime is long enough near the critical point so that the scattering is quasielastic. (iv) The resulting resistivity is adequately described by a simple variational estimate based on the Boltzmann equation. With these assumptions, the derivation of $\rho_s^i(T)$ proceeds precisely as in the case of ferromagnets[7] and can be given for a fairly general class of band structures [see Sec. II of Ref. 7]. For our immediate purposes, it is sufficient to describe the current carriers (electrons or holes) by a spherical Fermi surface. It is convenient to normalize the spin-fluctuation resistivity to its high-temperature (spin-disorder) limit $\rho_{0s}^i = \rho_s^i(T \gg T_N)$. Then, we have[3,5,7]

$$R_s^i(T) \equiv \frac{\rho_s^i(T)}{\rho_{0s}^i} = \sum_{\vec{R}} \Gamma(\vec{R}, T)\Phi^i(\vec{R}), \tag{1}$$

where

$$\Gamma(\vec{R}, T) = \langle \vec{S}_0 \cdot \vec{S}_{\vec{R}} \rangle / S(S+1) \tag{2}$$

and $\Phi^i(\vec{R})$ is the Fourier transform of

$$\Phi^i(\vec{q}) = \begin{cases} (3\pi^2/2k_F^4)(q_i^2/q), & q < 2k_F, \\ 0, & q > 2k_F. \end{cases} \tag{3}$$

Generalizations of these results to include anisotropic Fermi surfaces[7] and the effect of the finite electron mean free path[3,7] will be indicated subsequently as required. For some purposes, it is convenient to introduce the Fourier lattice transform of Eq. (2),

$$\Gamma_L(\vec{q}, T) = \sum_{\vec{R}} e^{-i\vec{q}\cdot\vec{R}} \Gamma(\vec{R}, T), \tag{4}$$

so Eq. (1) becomes

$$R_s^i(T) = \int \frac{d^3q}{8\pi^3} \Phi^i(\vec{q}) \Gamma_L(-\vec{q}, T). \tag{5}$$

We must now prescribe the type of spin-correlation function to be used in Eq. (1). Consider two interpenetrating sublattices with classical Heisenberg or Ising ferromagnetic coupling between all spins. The effect of changing the sign of all intersublattice interactions is to change the sign of the intersublattice correlation function and to leave the intrasublattice correlation functions unchanged.[6] For example, letting $J \to -J$ in a nearest-neighbor Ising model on a simple cubic lattice with basis vectors $(a\hat{x}, a\hat{y}, a\hat{z})$ leads to the replacement $\Gamma_0(\vec{R}, T) \to (\cos\vec{Q}\cdot\vec{R})\Gamma_0(\vec{R}, T)$, where $\Gamma_0(\vec{R}, T)$ is the spin-correlation function for the corresponding ferromagnet and $\vec{Q} = (\hat{x}+\hat{y}+\hat{z})\pi/a$ with the consequence that $\cos\vec{Q}\cdot\vec{R} = \pm 1$ as appropriate. In the case of order-disorder transitions (Ising-model antiferromagnet), we shall thus use

$$\Gamma(\vec{R}, T) = (\cos\vec{Q}\cdot\vec{R})\Gamma_0(\vec{R}, T), \tag{6}$$

where $\vec{Q}$ is to determined from the crystal structure and the symmetry of the *ordered* state (see also Sec. VI). For more complex magnetic systems and for nonclassical spins, we shall continue to use Eq. (6) for the following reasons. (i) Specifically quantum effects are believed to play a relatively minor role in determining the spin-correlation function near the critical point.[15] This is related to the "universality hypothesis" for systems of a given symmetry class.[16] (ii) Equation (6) satisfies the "equal-site sum rule" exactly

$$\Gamma(\vec{R} = \vec{0}, T) = \Gamma_0(\vec{R} = \vec{0}, T) = 1, \quad \text{all } T. \tag{7}$$

In spite of the innocent appearance of Eq. (7), it will be seen to play an important role. (iii) It is well known from neutron scattering[17,18] that spin correlations of wave number $\pm\vec{Q}$ (and vectors obtained by translation through reciprocal-lattice vectors) are very large and strongly temperature dependent near T_N. Clearly, $\Gamma_L(\vec{q}, T)$ must be reasonably treated for $\vec{q} \approx \pm\vec{Q}$. From Eqs. (4) and (6), we find

$$\Gamma_L(\vec{q}, T) = \tfrac{1}{2}[\Gamma_{0L}(\vec{q}+\vec{Q}, T) + \Gamma_{0L}(\vec{q}-\vec{Q}, T)], \tag{8}$$

which adequately describes the critical spin fluc-

tuations of antiferromagnets near $\pm\vec{Q}$ since the ferromagnetic correlation function peaks at zero wave number. For the helical rare-earth antiferromagnets (helimagnets) which we shall consider, the axis of the helix is along the c axis of the hcp structure and the appropriate magnetic ordering vector is seen to be

$$\vec{Q} = (2\pi/P)\hat{c}, \tag{9}$$

where the magnetic period P is approximately $9c$, $4c$, and $3.5c$ for terbium, dysprosium, and holmium, respectively, near their Néel points.[18]

Some comments are in order concerning an appropriate choice of the "corresponding" ferromagnetic correlation function in Eq. (6). We restrict present attention to $T \geq T_N$. As will be shown in Sec. III, short-range correlations are dominant for $T \to T_N^+$ just as they are for ferromagnets.[3,5-7] It is therefore appropriate to note that sums such as Eq. (1) contain implicit cutoffs so that their major contribution came from $R \lesssim R_c$, where R_c may be the finite-electron mean free path[3,5] or other electronic length scales[6,7] such as k_F^{-1}. Since the length scale of the spin correlations $\xi = \xi_0 t^{-\nu} = \kappa^{-1}$ increases indefinitely for $T \to T_N$, it is inevitable that the spin-correlation function must be correctly described in the range $R/\xi \ll 1$. This is accomplished by the scaling representation

$$\Gamma_0(\vec{R}, T) = D(\kappa R)(a/R)^{1+\eta}, \tag{10}$$

where, for small $x = \kappa R$, $D(x)$ may be expanded as[3,19]

$$D(x) = D_0 - D_1 x^{(1-\alpha)/\nu} - D_2 x^{1/\nu} + \cdots . \tag{11}$$

Quantitative statements concerning the coefficients D_j can be made using the ϵ expansion and renormalization-group methods.[20] The conclusions which we draw in the following are independent of the precise values of these coefficients and (within reasonable limits) of the critical exponents so it is sufficient to take their values from, for example, numerical work on Ising[4,21,22] or Heisenberg[23] models.

As the temperature increases in the paramagnetic regime, it is not clear that the convergence of the expansion in Eq. (11) is adequate to permit a continued description in terms of short-range correlations.[5-7] It is thus necessary, for consistency, to be convinced of the possible role played by long-range correlations [i.e., $KR \ll 1$ may not be satisfied for the relevant $R \lesssim R_c$ which enter Eq. (1)]. Several approximate forms may be suggested[5-7,22] of which the simplest which satisfies Eq. (7) and the criteria that $\Gamma_0(\vec{R}, T)$ should decrease as either R or T increases is[5-7,24]

$$\Gamma_0(\vec{R}, T) = \begin{cases} 1, & R = 0, \\ (C_0 a/R)e^{-\kappa R}, & R > 0. \end{cases} \tag{12}$$

Finally, in the ordered state below T_N, our major concern is the resistivity only in the limit $T \to T_N^-$. In this case, an appropriately modified version of Eq. (11), which exhibits the specific-heat singularity as the leading nonanalytical temperature dependence, is assumed to be valid.

The above results are applied in Secs. III–VI to study the critical behavior of the resistivity in various temperature regimes.

III. SPIN-FLUCTUATION RESISTIVITY FOR $T \to T_N^+$

As mentioned in Sec. II, sums such as Eq. (1) contain an implicit cutoff R_c due to the finite-electron mean free path (or other electronic length scales such as k_F^{-1}) limiting the range of the electronic coherence function $\Phi^i(\vec{R})$. For this reason alone, one should anticipate that in the limit of $T \to T_N^+$, the important terms in Eq. (1) are those for which $R_c/\xi \ll 1$ since $\xi \to \infty$ as $T \to T_N^+$. Thus, short-range correlations[6,10] rather than long-range correlations[9] should dominate resistive anomalies sufficiently close to the critical points of antiferromagnets just as in the case of ferromagnets.[3,5]

The conclusion of Suezaki and Mori[9] was based on an evaluation of Eq. (5), using Eq. (3) for $\Phi^i(\vec{q})$, in the form[25]

$$\frac{\partial R_s^i(T)}{\partial T} = \int \frac{d^3q}{8\pi^3} \Phi^i(\vec{q}) \frac{\partial \Gamma_L(\vec{q}, T)}{\partial T}. \tag{13}$$

Using a representation of $\partial\Gamma_L(\vec{q}, T)/\partial T$ based on scaling theory, it was concluded that its point singularity at $\vec{q} = \vec{Q}$ led to a divergence of the integral in Eq. (13) with the temperature dependence $t^{3\nu-\gamma-1}$ in the case of the electric field and net current in the crystallographic direction parallel to $\vec{Q}$. Their procedures would have been correct if the integral actually did diverge at $T = T_N$ due to the point singularity at $\vec{q} = \vec{Q}$ and if all strongly temperature-dependent components of $\partial\Gamma_L(\vec{q}, T)/\partial T$ had been adequately approximated over the required range of $\vec{q}$. To see why this procedure fails, note first that the domain of integration in Eq. (13) may be taken to be the first Brillouin zone (BZ) since $2k_F < |G_1|$ is the magnitude of the first reciprocal-lattice vector for any free-electron-like model to which Eq. (3) may be reasonably expected to apply.[26] Also, from Eq. (7), we see that

$$\int \frac{d^3q}{8\pi^3} \frac{\partial \Gamma_L(\vec{q}, T)}{\partial T} = 0. \tag{14}$$

In spite of the innocent appearance of this result, its importance should not be underestimated. In particular, it shows that the point singularity at $\vec{q} = \vec{Q}$ does *not* dominate the $\vec{q}$ integral. To appreci-

ate this in more detail, recall that $\Gamma_L(\vec{q}, T)$ is periodic in reciprocal space and, by symmetry, the singularities of $\partial\Gamma_L(\vec{q}, T)$ are symmetrically located in the BZ so the volume of integration in Eqs. (13) and (14) can be divided into a set of nonoverlapping "partial" Brillouin zones (j-PBZ) such that there is just one critical wave number $\vec{Q}_j$ in j-PBZ with Eq. (14) applying to each j-PBZ. We can write Eq. (13) as

$$\frac{\partial R_s^i(T)}{\partial T} = \sum_j \Phi^i(\vec{Q}_j) \int^{j\text{-PBZ}} \frac{d^3q}{8\pi^3} \frac{\partial \Gamma_L(\vec{q}, T)}{\partial T} + \sum_j \int^{j\text{-PBZ}} \frac{d^3q}{8\pi^3} [\Phi^i(\vec{q}) - \Phi^i(\vec{Q}_j)] \frac{\partial \Gamma_L(\vec{q}, T)}{\partial T}. \tag{15}$$

The first term is identically zero, rather than singular,[9] by virtue of the equal-site sum rule and, after angular integration, the contribution of the second term in Eq. (15) from the region $|\vec{q} - \vec{Q}_j| \lesssim \kappa$ is easily seen to be of order $t^{5\nu-\gamma-1}$ which is not singular. We conclude, from the equal-site sum rule and phase-space considerations, that any singular temperature dependence of Eq. (13) arises from $|\vec{q} - \vec{Q}| \gg \kappa$ so that a short-range representation of the spin-correlation function is required for $T \to T_N^+$.

From the above discussion, it is clear that short-range correlations dominate for $T \to T_N^+$ so that, from Eqs. (6) and (11), the appropriate correlation function may be taken to be[19]

$$\Gamma(\vec{R}, T) = (\cos\vec{Q}\cdot\vec{R})(a/R)^{1+\eta} \times [D_0 - D_1(\kappa R)^{(1-\alpha)/\nu} - D_2(\kappa R)^{1/\nu} + \cdots]. \tag{16}$$

Using this result and the Fourier transform of Eq. (3), the lattice sum in Eq. (1) has been evaluated numerically for several crystal structures and for a range of the relevant parameters. Illustrative examples of these results will now be given.

Consider a body-centered-cubic simple antiferromagnet for which, in the ordered state, ions tend to have spins up on body centers and spin down on the corners of the unit cell (or vice versa). Since there is cubic symmetry, Eq. (3) may be replaced by its average over crystallographic directions

$$\Phi(\vec{q}) = (\pi^2 q/2k_F^4)\theta(2k_F - q) \tag{17}$$

for which[3]

$$\Phi(\vec{R}) = (4k_F^4 R)^{-1} \frac{\partial^2}{\partial R^2} \frac{\cos 2k_F R - 1}{R}. \tag{18}$$

In order to discuss the effect of the finite electron mean free path, we follow Fisher and Langer[3] in replacing $\Phi(\vec{R})$ by $\Phi(\vec{R})e^{-R/l}$. More-refined treatments are possible,[6,7] but this approximation provides a qualitatively correct description of the reduction of electronic coherence at large R due to the inherent disorder of the system. For the bcc simple antiferromagnet, the appropriate $\vec{Q}$ to use in Eq. (16) is easily seen to be either $\vec{Q} = (2\pi/a)\hat{x}$ or $(2\pi/a)\hat{y}$ or $(2\pi/a)\hat{z}$ and the spin-correlation function has the required cubic symmetry. Using these results, we obtain from Eq. (1) for $T \simeq T_N^+$,

$$\frac{T_N \partial R_s(T)}{\partial T} = \pm(1-\alpha)At^{-\alpha} + B + \cdots, \tag{19}$$

where the positive (negative) sign is to be used for $\alpha > 0$ ($\alpha < 0$),[19] $\phi = (1-\alpha)/\nu - (1+\eta)$,

$$A = -|D_1|(\kappa_0 a)^{(1-\alpha)/\nu} \times \sum_{\vec{R}\neq\vec{0}} \left(\frac{R}{a}\right)^{\phi} (\cos\vec{Q}\cdot\vec{R})\Phi(\vec{R})e^{-R/l}, \tag{20}$$

and B is given by a similar form with $\alpha \to 0$ and $|D_1| \to D_2$. The lattice sum in Eq. (20) has been evaluated numerically for a range of the parameters l/a, $k_F a$, and ϕ. The results are quite insensitive to l/a and to ϕ and typical results are given in Fig. 1, for $l/a = 8$ and $\phi = 0.5$, as a function of $2k_F$. It is seen that A (which determines the slope of the resistivity at $T \simeq T_N^+$) is positive for $2k_F < Q$, whereas A is negative for $Q < 2k_F$. Thus, close to T_N^+, the singular part of the derivative of the resistivity has temperature dependence which reflects the specific-heat singularity and the resistivity slope may be *either positive or negative*

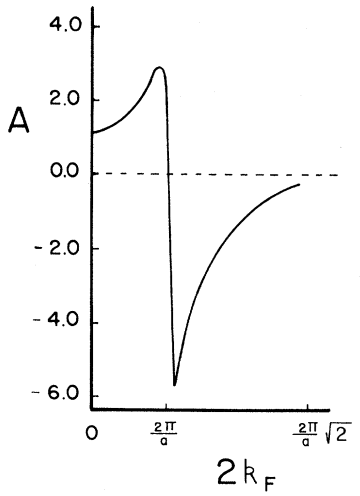

FIG. 1. Short-range expansion coefficients A plotted as a function of $2k_F$. Note that the abrupt change of sign occurs as the Fermi surface is spanned by $\vec{Q}$, of magnitude $(2\pi/a)$ in this case of a bcc simple antiferromagnet. See text of Sec. III for details. Also note that the scale of the abscissa would be linear in the effective valence if the indicated values of $2k_F$ are cubed.

depending on the ratio of the Fermi surface caliper and Q. This result will be utilized in Sec. VI in the discussion of binary alloys at their order-disorder temperature.

We now consider the case of helical (uniaxial) antiferromagnets on a hexagonal-close-packed lattice. Since there is no longer cubic system, we must distinguish between the c-axis resistivity $R_s^c(T)$ {i.e., the applied field and current flow are along the c axis of the hcp crystal and the c axis is parallel to $\vec{Q}$ [see Eq. (9)]}, and the basal plane resistivity $R_s^b(T)$, where the current is perpendicular to $\vec{Q}$. Also, the Fermi surfaces of the rare-earth helimagnets are very anisotropic.[27] As a simple model we take the dispersion relation of the current carriers (holes) to be described by

$$\epsilon_{\vec{k}} = \pm \hbar^2 [k_z^2/2m_c + (k_x^2 + k_y^2)/2m_b] + \text{const.} \qquad (21)$$

For this dispersion law, Eq. (3) is replaced by (see Ref. 7 for details)

$$\Phi^i(\vec{q}) = \begin{cases} \dfrac{3\pi^2 q_i^2}{2k_{Fc}^2 k_{Fb}^2} \dfrac{m_c/m_i}{q^*}, & q^* < 2k_{Fc}, \\ 0, & q^* > 2k_{Fc}, \end{cases} \qquad (22)$$

where $k_{Fc}(k_{Fb})$ is half the Fermi-surface caliper along the c axis (in the basal plane) and $q^* = [q_z^2 + (q_x^2 + q_y^2)m_c/m_b]^{1/2}$. The corresponding $\Phi^i(\vec{R})$ can be evaluated[6] and used in the short-range expansion to obtain[19] $(i = c, b)$

$$T_N \frac{\partial R_s^i(T)}{\partial T} = \pm (1 - \alpha) A^i t^{-\alpha} + B^i + \cdots, \qquad (23)$$

with

$$A^i = -|D_1|(K_0 a)^{(1-\alpha)/\nu} \times \sum_{\vec{R} \neq \vec{0}} \left(\frac{R}{a}\right)^{\phi} (\cos Qz) \Phi^i(\vec{R}) e^{-R/l}. \qquad (24)$$

This hcp lattice sum was also evaluated numerically to obtain both A^c and A^b for a wide range of the model parameters l, ϕ, k_{Fc}, and k_{Fb} (with $k_{Fb} > k_{Fc}$). The results were not overly sensitive to l or ϕ but were sensitive to the Fermi-surface calipers. It was found that (i) $A^b > 0$, (ii) $A^c > 0$ for $2k_{Fc} < Q$, and (iii) $A^c < 0$ for $Q < 2k_{Fc}$ except for very small values of the ratio $Q/2k_{Fc}$. The limit of small $Q/2k_{Fc}$ corresponds simply to the case of a ferromagnet.[7] These results are as anticipated. To appreciate the role played by the Fermi-surface anisotropy, we have plotted A^c and A^b in Fig. 2 as a function of the anisotropy parameter $\log_2(m_b/m_c) = 2\log_2(k_{Fb}/k_{Fc})$ for the typical case of $l/a = 6$, $\phi = 0.5$, and $Q = 2\pi/5c$. When evaluating Eq. (24) for different values of k_{Fb}/k_{Fc}, the volume of the Fermi surface was held constant so as not to be comparing systems with different densities of current carriers. The results shown in Fig. 2

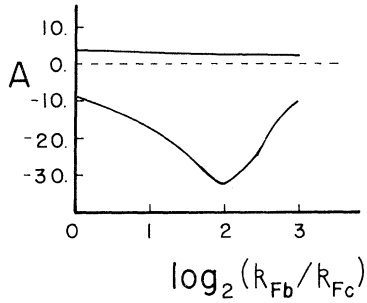

FIG. 2. Upper (lower) curve shows a plot of A^b (A^c) as a function of Fermi surface anisotropy, measured in terms of $\log_2(k_{Fb}/k_{Fc})$. The volume of the Fermi surface is held constant for different values of the anisotropy. The tendency of A^c to rise for $k_{Fb}/k_{Fc} > 4$ is due to the Fermi surface approaching the Brillouin-zone boundary of the hcp structure.

correspond to $(k_{Fb}^2 k_{Fc})^{1/3} = 1.28$. It is seen that A^b tends to be relatively small, positive and insensitive to anisotropy while A^c tends to be relatively large, negative and sensitive to anisotropy. The structure in A^c is a direct reflection of the role played by $Q/2k_{Fc}$. For example, A^c rises for large anisotropy because $Q/2k_{Fc}$ approaches unity which is roughly where A^c should become positive (see above). We shall refer to these results in Sec. VI.

Finally, it should be pointed out that the conclusion that $\partial R_s^b(T)/\partial T > 0$ for $T \to T_N^+$ can be demonstrated analytically[6] by methods similar to those employed in the corresponding discussion of ferromagnets.[7] These methods also verify that $\partial R^c(T)/\partial T$ may be either positive or negative (depending on the Fermi surface and $\vec{Q}$) but do not permit simple but reliable estimates of the *magnitude* of A^b or A^c.

IV. LONG-RANGE CORRELATIONS AND SPIN-FLUCTUATION RESISTIVITY FOR $T > T_N$

The results of Sec. III are valid for $T \simeq T_N^+$. As pointed out in Sec. II, sums such as Eq. (1) contain an implicit cutoff R_c, which is of order l or k_F^{-1}. As the temperature increases above T_N, the correlation length decreases and the values of $x < x_c = \kappa R_c$ which will be required in the short-range expansion, Eq. (11), increase correspondingly. Consequently, with increasing temperature, higher-order terms in the short-range expansion become significant and it is not at all clear that $D(x)$ continues to be adequately represented by only the first few terms in Eq. (11). In the absence of a detailed knowledge of $D(x)$ for the larger values of $x = \kappa R$ which are required, it is useful to consider alternative forms for $\Gamma_0(\vec{R}, T)$ [see Eq. (10)] which have a regime of approximate validity for intermediate or large values of κR and which might

therefore complement the short-range expansion of Sec. II. To this end, Ferer, Moore, and Wortis[22] introduced an Ornstein-Zernicke-like approximation

$$\Gamma_0(\vec{R},T)=\begin{cases}1, & R=0,\\ C_0(\kappa a)^{\eta}\dfrac{e^{-\kappa R}}{R/a}, & R\geq a,\end{cases} \tag{25}$$

which was estimated to be valid, on the basis of numerical analysis of Ising-model data, for $\kappa a > 0.1$. An alternative and simpler approximation which is also of OZ type is[5-7]

$$\Gamma_0(\vec{R},T)=\begin{cases}1, & R=0\\ C_0\dfrac{e^{-\kappa R}}{R/a}, & R\geq a.\end{cases} \tag{26}$$

Apart from the obvious but important fact that the equal-site sum rule is satisfied, the validity of Eq. (26) is otherwise limited, strictly speaking, to large values of κR.[28] For the rationale of using Eq. (25) or (26), with a view toward verifying that the essential physical features of the problem are described by either short-range or long-range (i.e., κR not necessarily small) spin correlations, see the discussion in Ref. 7.

In order to illustrate the above point, it is instructive to apply Eq. (26) to calculate the slope of the resistivity and the specific heat. From Eqs. (1), (6), and (26), we easily find

$$\frac{\partial R_s^i(T)}{\partial T}=-C_0 a\kappa'(T)\sum_{\vec{R}\neq\vec{0}}(\cos\vec{Q}\cdot\vec{R})\Phi^i(\vec{R})e^{-\kappa R}, \tag{27}$$

which can be rewritten

$$\frac{\partial R_s^i(T)}{\partial T}=C_0 a\kappa'(T)S^i(\vec{Q},\kappa), \tag{28}$$

where

$$S^i(\vec{Q},\kappa)=1-\sum_{\vec{R}} e^{-\kappa R}(\cos\vec{Q}\cdot\vec{R})\Phi^i(\vec{R}). \tag{29}$$

Consider first the case of "small" κ [e.g., $\kappa R \sim 0.1$ for the dominant terms in Eq. (29)] so Eq. (29) becomes

$$S^i(\vec{Q},0)=1-\Phi_L^i(\vec{Q}), \tag{30}$$

where $\Phi_L^i(\vec{Q})$ is the $\vec{Q}$th component of the Fourier lattice transform of $\Phi^i(\vec{R})$ and is given by

$$\Phi_L^i(\vec{Q})=\Omega_0^{-1}\sum_{\vec{G}}\Phi^i(\vec{Q}+\vec{G}), \tag{31}$$

where the sum is over reciprocal-lattice vectors, Ω_0 is the volume per ion, and $\Phi^i(\vec{Q}+\vec{G})$ is given by Eq. (3) or (22). For simplicity, we consider a Bravais lattice and an isotropic Fermi surface. Note that there are no negative contributions to the sum in Eq. (31) and that the $G=0$ term is (for $Q<2k_F$)

$$\Omega_0^{-1}\frac{3\pi^2}{2k_F^4}\frac{Q_i^2}{Q}=\frac{\Omega_c}{\Omega_0}\frac{Q}{2k_F}\times\begin{cases}1, & i=c,\\ 0, & i=b,\end{cases} \tag{32}$$

where $\Omega_c=3\pi^2/k_F^2$ is the volume per electron (hole) associated with the pocket of current carriers and can be considerably larger than Ω_0. From this simple argument, it becomes clear that the sign of $\partial R^i(T)/\partial T$ will be determined by Fermi-surface features in the regime of long-range correlations just as in the case of short-range correlations. A more quantitative evaluation of Eq. (27) is given below.

Consider now the corresponding treatment of the specific heat. We assume that the internal energy is given by a Heisenberg model with pair interactions between spins. The heat capacity per spin is, in the same OZ spirit,

$$\frac{C_s(T)}{N}=-\frac{\partial}{\partial T}\sum_{\vec{R}\neq\vec{0}}J(\vec{R})\left(S(s+1)C_0 a\frac{e^{-\kappa R}}{R/a}\right)(\cos\vec{Q}\cdot\vec{R}), \tag{33}$$

which reduces, for small κ, to

$$\frac{C_s(T)}{N}\to C_0 a\kappa'(T)S(S+1)\sum_{\vec{R}\neq\vec{0}}J(\vec{R})\cos\vec{Q}\cdot\vec{R}$$
$$=C_0 a\kappa'(T)(\tfrac{3}{2}k_B T_{N0}), \tag{34}$$

where T_{N0} is the mean-field approximation to the Néel temperature. Comparison of Eqs. (28) and (34) suggests that $\partial R_s^i(T)/\partial T$ and $C_s(T)/N$ have the same temperature dependence and that their ratio is given approximately by

$$\frac{\partial R_s^i(T)}{\partial T}\Big/\frac{C_s(T)}{N}\to\frac{2S^i(\vec{Q},0)}{3k_B T_{N0}}. \tag{35}$$

This point will be returned to in Sec. VI.

To obtain quantitative estimates of $\partial R_s^i(T)/\partial T$ in the regime where long-range correlations may be relevant, we have evaluated numerically the relevant lattice sums using the generalized OZ representation, Eq. (25), of Ferer, Moore, and Wortis[22] for various temperatures in the range where $\kappa a > 0.1$ [due to the fact that η is small, the factor of $(\kappa a)^{\eta}$ in Eq. (26) has only minor influence and essentially identical results are obtained using Eq. (25)]. The finite mean-free-path modification of Fisher and Langer[3] was also included. In Fig. 3, we plot typical results for $S^i(\vec{Q},\kappa)\equiv C_0^{-1}\partial R_s^i(T)/\partial(\kappa a)$ [see also Eq. (28)] for the bcc simple antiferromagnet, for $\kappa a=0.2$ and $l/a=8$, as a function of the caliper $2k_F$ of the isotropic Fermi surface. In Fig. 4, we have plotted typical results for $S^c(Q,K)$ and $S^b(Q,K)$ for a helical antiferromagnet on a hcp lattice, also for $\kappa a=0.2$, as a function of

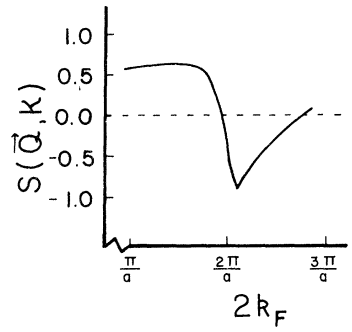

FIG. 3. Coefficient $S(\vec{Q},\kappa)$, which determines $\partial R_s(T)/\partial T$ in the regime of long-range correlations (see Sec. IV for details) for a bcc antiferromagnet, as a function of $2k_F$ for $\kappa a = 0.2$.

the anisotropy parameter $\log_2 k_{Fb}/k_{Fc}$. The model parameters were taken to be $l/a = 6$, $Q = 2\pi/5c$, and $(k_{Fb}^2 k_{Fc})^{1/3} = 1.28$ as in the corresponding discussion in Sec. III of short-range correlations. Comparison of Figs. 1 and 3 and of Figs. 2 and 4 make it clear that the overall qualitative features of $\partial R_s^i(T)/\partial T$ are very similar for both long- and short-range correlations so far as the role played by the Fermi surface is concerned. Of course, the precise temperature dependence in the two regimes will be different. This raises the following important questions:

(i) Which are the relevant features of the above model systems which are responsible for the "crossover" from short-range to long-range behavior and are these features to be found in any real materials?

(ii) In view of the expected similarity of the gross features of $\partial R_s^i(T)/\partial T$ in the short-range and long-range regimes, how is the crossover to be verified experimentally in a given system?

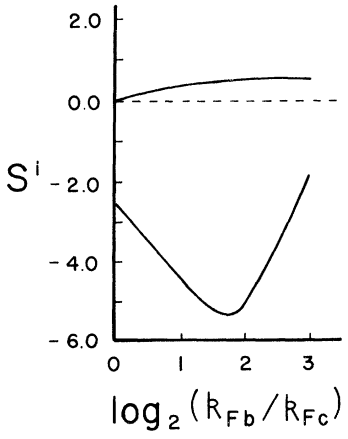

FIG. 4. Upper (lower) curve shows $S^i(\vec{Q},\kappa)$ for $i = b$ (c) for a helical antiferromagnet as a function of the ratio of the Fermi surface calipers (see text in Sec. IV for further details). The Fermi surface is spanned by $\vec{Q}$ for all model parameters in the figure.

The answer to question (i), briefly stated, is that for temperatures not too close to T_N^+ (see Sec. III) but still in the "critical" range (e.g., $t < 0.1$), the inverse correlation length κ must become comparable to other relevant inverse length scales of the problem. For materials to which the present discussion might apply, the salient inverse length scale is k_F^{-1}. Consequently, a crossover to long-range dominance can be anticipated only in systems where (a) the current carriers's Fermi surface consists of small pockets, or where (b) the Fermi-surface caliper in a given direction is small. Case (a) may be applicable to certain ferrimagnets (subject to further details of their Fermi surface) while case (b) seems applicable to the c-axis resistivity of the heavy rare-earth helimagnets. This will be discussed in Sec. VI. As regards question (ii), recall that a *rigorous* power law should be expected only in the asymptotic limit of $T \to T_N^+$. Otherwise, a strict power law is contaminated to some degree by various higher-order correction terms. The possibility of smearing or "contamination" of the transition due to sample imperfections and/or alternative competing crossovers due to, for example, long-range dipole-dipole interactions adds further complications.[29] The matter is clearly delicate and a convincing demonstration of any short range to long range crossover (which will not be sharp) will require careful analysis of high quality data. It is therefore useful to use the numerical results obtained in this work to make a quantitative comparison of the "power laws" to be expected in the two regimes. The most interesting case is that of the c-axis helimagnet with $Q < 2k_{Fc}$. For $T \simeq T_N^+$, from Eqs. (23) and (24), we find $T_N \partial R_s^c(T)/\partial T = \pm C_{SR} t^{-\alpha} + C'_{SR}$, where $C_{SR} = (1-\alpha)A^c \approx -25$ for the model parameters used in Fig. 2. Similarly, taking D_2/D_1 from Ref. 20, we may use Fig. 2 which is quite insensitive to critical exponents to obtain the rough estimate $C'_{SR} \approx -10$. In the long-range regime, use of the generalized OZ approximation[22] [which is adequately represented in its domain of validity by introducing a factor $(\kappa a)^\eta$ in Eqs. (27) and (28)] leads to $T_N \partial R_s^c(T)/\partial T \simeq C_{LR} t^{-\lambda}$, where $C_{LR} = C_0(\kappa_0 a)^{1+\eta\nu} S^c(\vec{Q},0)$ and $\lambda = 1-(1+\eta)\nu$. Estimating the various parameters from numerical data on Ising or Heisenberg models and from Fig. 4, we have $C_{LR} \approx -0.3$. This suggests that C_{SR} should be a factor of 10 or so larger in magnitude than C_{LR}. On the other hand, the short-range temperature dependence is weaker than the long-range temperature dependence since $\alpha < \lambda$ follows if we assume the scaling relations[4] $\gamma = (2-\eta)\nu$ and $3\nu = 2-\alpha$ to

write

$$\lambda = -(1+\eta)\nu + 1 = -3\nu + (2-\eta)\nu + 1$$
$$= -2 + \alpha + \gamma + 1 = \alpha + (\gamma - 1).$$

This also shows that the long range exponent can be written $-\lambda = 3\nu - \gamma - 1$ which is precisely the exponent which follows from evaluating Eq. (13) in the regime where long-range correlations dominate.[9,10] We return to these points in Sec. VI.

V. MODIFICATION OF ELECTRON ENERGIES AND RESISTIVITY FOR $T \to T_N^-$

In Secs. I–IV, we calculated the resistivity in the lowest-order Born approximation due to coupling of electrons and localized spins via the s-f exchange interaction

$$H_{sf} = \int d^3r \sum_{\vec{R}} J_{sf}(\vec{r} - \vec{R})\vec{s}(\vec{r}) \cdot \vec{S}_{\vec{R}}, \tag{36}$$

where $\vec{s}(\vec{r})$ is the electron spin density. Even though the coupling energy J_{sf} is weak on the scale of the Fermi energy, it is clear that H_{sf} plays an important role in determining effective interactions in the system and thus influences the phase transition. Consequently, in spite of the smallness of J_{sf}/ϵ_F, it is not necessarily adequate to treat H_{sf} by the simplest form of low-order perturbation theory. In particular, it is necessary, among other things, to consider possible modifications of the electronic energies (and velocities) due to the long-range order which gives rise to a new periodic average potential below the Néel point.

To be definite, consider the case of the c-axis resistivity of a helimagnet with $Q < 2k_F$. The c-axis resistivity of such systems (see Sec. VI) for $T < T_N$ shows a broad maximum at a temperature $T_m < T_N$. This characteristic feature is usually associated with energy gaps introduced in the electron energy spectrum by the long-range order[11,12] and can be described as follows. Assuming $\langle \vec{S}_{\vec{R}} \rangle \neq 0$, its effect on the electrons is to give rise to an additional Bloch potential

$$\overline{H}_{sf} = \int d^3r \sum_{\vec{R}} J_{sf}(\vec{r} - \vec{R})\vec{s}(\vec{r}) \cdot \langle \vec{S}_R \rangle. \tag{37}$$

The net one-electron Hamiltonian [Eq. (37) plus the usual kinetic energy] is then diagonalized taking account of the spiral character of $\langle \vec{S}_{\vec{R}} \rangle$ and including mixing of states $\vec{k}$ and $\vec{k} + \vec{Q}$ of appropriate spin. The new electron-dispersion law then reflects the "sublattice" magnetization $\sigma(T) \sim |t|^{\beta}$, so that $\partial R_s^c(T)/\partial T \sim -|t|^{\beta-1}$ would follow.[9] More recently, it has been suggested[30,31] that smearing effects due to gap fluctuations, normal scattering processes and/or finite temperature would lead to σ^2 (rather than σ) appearing in the "effective" density of current carriers. This would result in $\partial R_s^c(T)/\partial T \sim -|t|^{2\beta-1}$.

We disagree with this conclusion. Briefly stated, the "decoupling" of electronic and localized spins in Eq. (36) is an average field approximation which is particularly bad very close to T_N^- where fluctuations play the dominant role. For example, if this procedure were used to calculate the contribution of H_{sf} to the internal energy, the resulting specific heat would vary as $C(T) \sim |t|^{2\beta-1}$ rather than $|t|^{-\alpha'}$. This obviously incorrect result is a consequence of the mean-field approximation and its consequent symmetry breaking. Although transport processes are much more delicate, we see no reason why such methods should not lead to equally incorrect results for the resistivity. From our considerations, we feel that the spin correlations which determine the appropriate electron self-energy and thereby the modified "dispersion relation" are more likely to be of short range for $T \to T_N^-$ as a consequence of the *finite* range of the relevant electronic coherence factors. We therefore expect that the electrical resistivity will reflect the short-range temperature dependence of the internal energy and that the singular part of the temperature dependence of $\partial R_s^i(T)/\partial T$ will vary more closely as $|t|^{-\alpha'}$ for *both* $i = c$ and $i = b$. We shall return to this point in the following section where the present results are compared with experiment. Finally, it should be pointed out that the effects of the "renormalization" of the electronic spectrum must also be present above T_N just as the effects of direct spin-fluctuation scattering will certainly persist below T_N. In the neighborhood of T_N, both effects reflect the temperature dependence of short-range correlations. As the temperature decreases, the effects of inelastic scattering from magnons and phonons become relevant and the additional Bloch potential becomes more stable; the net effect is the "antiferromagnetic hump" with its maximum located somewhat below T_N.

VI. COMPARISON OF THEORY AND EXPERIMENT AND DISCUSSION

In Secs. I–V, we studied the effect of critical spin fluctuations on the resistivity of binary alloys and antiferromagnets within the context of simple models. We shall summarize the major conclusions. (i) Short-range correlations dominate for both $T \to T_N^-$ and $T \to T_N^+$. (ii) In the immediate vicinity of T_N, $\partial R_s^b(T)/\partial T > 0$ for uniaxial antiferromagnets. (iii) The sign of $\partial R_s^c(T)/\partial T$ is not universal but depends on matching of $\vec{Q}$ and features of the Fermi surface. In particular, $\partial R_s^c(T)/$

$\partial T > 0$ if Q is much less than Fermi-surface calipers (i.e., approaching the ferromagnetic limit) or if $\vec{Q}$ does not span the Fermi surface. (iv) In the regime of long-range spin correlations in the paramagnetic state, the same considerations apply. (v) Rather generally, the singular part of the temperature derivative of the resistivity varies as (either plus or minus) the singular part of the specific heat. In this section, the above conclusions are confronted with experimental results on a variety of systems in order to appraise their validity for real materials.

We first consider the case of order-disorder transitions in binary alloys (which can be formally described by a spin-$\frac{1}{2}$ simple Ising antiferromagnet[8]). One well studied case[17,32] is that of β-brass (CuZn at approximately the 50%-50% composition). In the ordered phase ($T < T_0 \simeq 739$ K), the structure is of the CsCl type ($B2$), i.e., a bcc structure with Cu ions at body centers and Zn ions at the corners of the unit cell of lattice constant a. Critical fluctuations occur[17] just as in the case of antiferromagnets and it is easy to see from the structure in the ordered state that $\vec{Q}$ is given by either of $(2\pi/a)\hat{x}$ or $(2\pi/a)\hat{y}$ or $(2\pi/a)\hat{z}$ [see also Sec. II]. Next, one must be careful to see if $\vec{Q}$ spans the Fermi surface. The appropriate Fermi surface to consider is that of the *disordered* state. Unfortunately, we know of no band-structure calculations for this phase but one can resort (with some confidence) to nearly-free-electron agruments since the following discussion shows that only rather gross symmetry considerations are involved. We take Cu and Zn to contribute one and two electrons, respectively, to the conduction band. Thus the average valence is $\frac{3}{2}$ and the average potential at each site in the disordered phase is taken to be the average of the Cu and Zn potentials in a type of virtual-crystal approximation. The reciprocal lattice is fcc and the first Brillouin zone is a rhombic dodecahedron which is $\frac{3}{4}$ filled so that the Fermi surface has substantial necks at each of the 12 faces of the zone boundary (this is a bit similar to the familiar picture of the Fermi surface of Cu). A cross section of the Fermi surface and the Brillouin zone in the ΓNG (100) plane is illustrated in Fig. 5. Other cross sections can be considered and the important conclusion is that there are *not* any segments of the Fermi surface which are spanned by $\vec{Q}$. In fact, those parts of the Fermi surface which might have been candidates are precisely those which are removed by the necks. We conclude from these results that $\partial R_s(T)/\partial T$ for β-brass should be positive and should vary as the specific heat in the critical-temperature region. This is in full agreement with the experimental work of Simons and Salamon.[32]

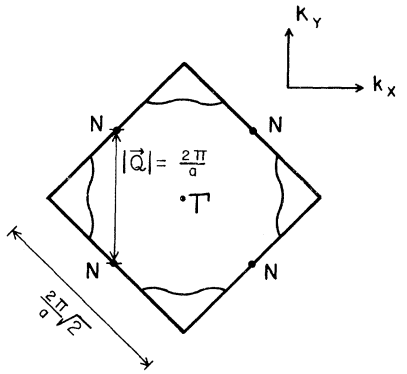

FIG. 5. Cross section of the Fermi surface of β-brass in the ΓNG plane in a virtual-crystal approximation for the disordered state. There are no sections of the Fermi surface which are spanned by $\vec{Q}$.

We now consider the binary alloy Fe_3Al which undergoes an order-disorder transition[33] at $T = T_0 \simeq 530$ K. With decreasing temperature, the transition is from the partially disordered $L2_0$ phase to the ordered $D0_3$ phase. In order to appreciate the structural considerations leading to the determination of $\vec{Q}$, consider two interpenetrating simple cubic sublattices, A and B, where the lattice points of the B sublattice are at the body-centered sites of the A sublattice. In the $D0_3$ structure, only Fe atoms are on the A sublattice sites and Fe and Al ions are evenly distributed on the B sublattice such that they are not nearest neighbors on the B sublattice. In the $L2_0$ phase, still only Fe ions are on the A sublattice but there is now no long-range order on the B sublattice. That is, the A sublattice occupancy is essentially static and temperature independent near T_0 and we need consider only critical fluctuations on the B sublattice. For the B sublattice, it is relatively easy to see that $\vec{Q}$ can be taken to be $(\hat{x}+\hat{y}+\hat{z})\pi/a$, where a is the nearest-neighbor distance on the same sublattice. We now turn to the description of the Fermi surface. In the partially disordered $L2_0$ phase, the crystal potential is due to Fe ions at corners and the average Fe and Al potentials at body centers of a simple cubic lattice having basis vectors $(0,0,0)a$ and $(\frac{1}{2},\frac{1}{2},\frac{1}{2})a$. The Brillouin zone is also simple cubic. It is important to note that, *unlike* the case of disordered β-brass, $\vec{Q}$ for Fe_3Al does *not* connect points of high symmetry on the Brillouin-zone faces, and it appears very unlikely that any symmetry imposed on the Fermi surface of Fe_3Al would be such that $\vec{Q}$ could not span the Fermi surface. In view of this fact and in the absence of detailed band-structure calculations for Fe_3Al, this point can be illustrated by the following two simple-model calculations of the Fermi surface. We take Fe and Al to contribute one and three elec-

trons, respectively, to the conduction band so that the average valence per ion is $\frac{3}{2}$. In the *partially* disordered $L2_0$ phase, Fe_3Al has an average crystal potential with the same symmetry as *ordered* β-brass. The band structure calculations of Arlinghaus[34] show that the above $\vec{Q}$ does indeed span such a Fermi surface. The same conclusion follows if we use the virtual-crystal approximation for the *totally* disordered state. In this case, the Brillouin zone and Fermi surface are similar to those in the example of disordered β-brass considered in the above discussion and we again can verify that $\vec{Q}$ spans the Fermi surface. On the basis of these results, it follows from the conclusions of previous sections that the resistivity of Fe_3Al should have negative slope in the vicinity of the order-disorder temperature. The resistivity of Fe_3Al has been measured by Thomas, Giray, and Parks[35] and is indeed found to have negative slope from about 50 K below $T_0 \simeq 530$ K to more than 200 K above T_0. The determination of the precise temperature dependence is a bit difficult and it was concluded that the data were consistent with either long-range ($2\beta - 1 \simeq -0.375$) or short-range ($-\alpha \simeq -0.1$) Ising-model critical exponents in the vicinity ($|t| \lesssim 10^{-2}$) of T_0. More precise statements concerning the temperature dependence might be possible with more-detailed numerical analysis of the data. At any rate, it is clear that the data are consistent with our results for the case where $\vec{Q}$ spans the Fermi surface. Note that the present theory provides a clear understanding of the different behavior[35] of β-brass and Fe_3Al. Very many other intermetallic compounds exhibit resistive anomalies at their order-disorder transition.[36] Those which we have studied appear to be described by the above theory.

We shall now consider the rare-earth helimagnets. Detailed measurements of the electrical resistivity of c-axis monocrystals of dysprosium have been reported.[37-39] Analysis of the data in Ref. 37 showed $R_s^c(T)$ to be described by a power law $A + Bt + Ct^{\psi}$ in the range $2.5 < T - T_N < 60$ K (where $T_N \simeq 180.33$) with the critical exponent $\psi \simeq 0.6$ which seems characteristic of long-range correlations. This power law did not extend into the region $0 < T - T_N < 2.5$ K. Qualitatively similar results were given in Ref. 38, although the breakdown of the long-range power law was not reported. These workers also suggested that the resistivity below T_N appeared to be dominated by (smeared) super zone gaps. As described in Sec. V, this implies that the critical exponents for the resistivity should be $\psi' = 2\beta$ which is certainly expected to be *less* than unity. On the other hand, the analysis of data in Ref. 39 indicated rather strongly that ψ' was substantially *larger* than unity ($\psi' \simeq 1.25 \pm 0.12$) for *both* c-axis (negative slope) and basal-plane (positive slope) monocrystals. It is evidently difficult to account for this rather weak temperature dependence with the usual theories based on energy gaps.[30,31] However, the discussion of Sec. V implies that $\psi' \simeq 1 - \alpha'$. This relation is consistent with available specific heat and resistivity data for Dy (see Ref. 39 for details).

Recently, a careful experimental study has been made of the resistivity of holmium c-axis monocrystal in the vicinity of its Néel point.[40] The slope is, as expected, negative near $T_N \simeq 130.15$. Below T_N, the critical exponent describing the singular part of the resistivity is $\psi' \simeq 1.30 \pm 0.10$. In the immediate vicinity of T_N^+ (i.e., for values of t less than 10^{-2}), the critical exponent is $\psi \simeq 1.45 \pm 0.25$ with the large uncertainty due to the small temperature range available. These values of ψ' and ψ seem to be consistent with the relatively weak temperature dependence expected when short-range correlations are dominant. However, the data for $t \gtrsim 2 \times 10^{-2}$ were described by a power law with critical exponent $\psi \simeq 0.55 \pm 0.10$ which seems more characteristic of long-range correlations. The corresponding amplitudes [in the form $T_N \partial R_s^c(T)/\partial T = Ct^{\psi-1} + C'$] were $C \approx 60, C' \approx -20$ for $T \to T_N^+$ and $C \approx -0.4$, $C' \approx 1$ for $T - T_N \gtrsim 2$ K. These results are consistent as regards both magnitude (to within a factor of 2, which is acceptable for such model calculations) and sign with those given in Secs. III and IV for helimagnets with $Q < 2k_F$ and with the suggestion that a crossover from long-range to short-range dominance occurs with decreasing temperature in the paramagnetic state. It was also shown that $T_N \partial R_s^c(T)/\partial T$ was proportional to $C_s(T)/Nk_B$ in the paramagnetic range with the constant of proportionality [see Eq. (35)] being approximately 3. The model calculations of Sec. IV led to the corresponding constant $2S^c(\vec{Q}, \kappa \approx 0)T_N/3T_{N0}$ being approximately 1. The precise value of this constant is sensitive to details of the anisotropic Fermi surface, of course, and we consider the agreement to be quite satisfactory in view of the simplicity of the model.

From the discussion of the above representative examples, it is clear that experiment and the present theory are in quite reasonable agreement in spite of the simplicity of the model calculations. This is encouraging, of course, and indicates that major physically relevant effects have been properly taken into account. We expect that the present results will provide a useful starting point in attempts to interpret resistive anomalies in a variety of magnetic systems which are more complex than those considered in this work. However, it must also be emphasized that many simplifications have been made in the above discussion and

that a number of points, such as the validity of the quasielastic approximation for large momentum transfer scattering processes and certain specifically solid-state effects associated with anistropies of the system, merit further consideration. Also, it would be particularly interesting to verify in detail the extent to which the critical properties of systems coupled by indirect exchange interactions may be described by the usual effective interactions of Ising or Heisenberg type. Answers to these questions will be required in order to proceed from the present semiquantitative but useful model calculations to a fully rigorous and general theory of resistive anomalies.

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[1]P. G. deGennes and J. Friedel, J. Phys. Chem. Solids 4, 71 (1958).

[2]For references to the literature and reviews of work in the field prior to 1970–1971, we refer the reader to M. P. Kawatra and J. I. Budnick, Int. J. Magn. 1, 61 (1970); and to R. D. Parks, AIP Conf. Proc. 5, 630 (1972).

[3]M. E. Fisher and J. S. Langer, Phys. Rev. Lett. 20, 665 (1968).

[4]M. E. Fisher, Rep. Prog. Phys. 30, 615 (1967).

[5]T. G. Richard and D. J. W. Geldart, Phys. Rev. Lett. 30, 290 (1973).

[6]T. G. Richard, Ph.D. thesis (Dalhouie University, 1973) (unpublished).

[7]D. J. W. Geldart and T. G. Richard, Phys. Rev. B 12, 5175 (1975).

[8]See, for example, K. Haung, *Statistical Mechanics* (Wiley, New York, 1963), p. 329.

[9]Y. Suezaki and H. Mori, Prog. Theor. Phys. 41, 1177 (1969).

[10]D. J. W. Geldart and T. G. Richard, in *Dynamical Aspects of Critical Phenomena,* edited by J. I. Budnick and M. P. Kawatra (Gordon and Breach, New York, 1972), p. 326.

[11]R. J. Elliott and F. A. Wedgwood, Proc. Phys. Soc. Lond. 81, 846 (1963).

[12]H. Miwa, Prog. Theor. Phys. 28, 208 (1962); and 29, 477 (1963).

[13]This result applies to the case where the current is along the crystallographic direction corresponding to the magnetic ordering vector $\vec{Q}$ (see Sec. II).

[14]In the classical OZ approximation, $\nu = \frac{1}{2}$ and $\gamma = 1$ so the critical exponent for $\rho'(T)$ is $3\nu - \gamma - 1 = -\frac{1}{2}$ rather than $\approx -\frac{1}{3}$ when more accurate critical exponents are used. Otherwise, the treatments of Refs. 9 and 10 are based on the same physical consideration for T somewhat larger than T_N in the paramagnetic state.

[15]L. P. Kadanoff *et al.*, Rev. Mod. Phys. 39, 395 (1967).

[16]See, for example, L. P. Kadanoff, in *Critical Phenomena,* edited by M. S. Green (Academic, New York, 1972), p. 100.

[17]J. Als-Nielsen and O. W. Dietrich, Phys. Rev. 153, 706, 717 (1967).

[18]See, for example, the review by W. C. Koehler, in *Magnetic Properties of Rare Earth Metals,* edited by R. J. Elliott (Plenum, London, 1972), p. 81.

[19]The signs of D_1 and D_2 in Eq. (11) are required to yield a positive specific heat with a cusp at T_N. For examples, $D_1 \gtrless 0$ for $\alpha \gtrless 0$ and $D_2 > 0$ (see also Ref. 20).

[20]M. E. Fisher and A. Aharony, Phys. Rev. Lett. 31, 1238 (1973); E. Brézin, D. J. Amit, and J. Zinn-Justin, *ibid.* 32, 151 (1974); M. E. Fisher and A. Aharony, Phys. Rev. B 10, 2818 (1974); A. Aharony, *ibid.* 10, 2834 (1974).

[21]M. E. Fisher and R. J. Burford, Phys. Rev. 156, 583 (1967).

[22]M. Ferer, M. A. Moore, and M. Wortis, Phys. Rev. Lett. 22, 1382 (1969).

[23]D. S. Ritchie and M. E. Fisher, Phys. Rev. B 5, 2668 (1972).

[24]The use of simple scaling forms in Eq. (12) and in all other instances considered in the present work precludes any dependence of $\Gamma_0(\vec{R}, T)$ upon the orientation of $\vec{R}$ with respect to the crystal axes. Correction of this and similar omissions would lead to only very minor modifications of the following results.

[25]Strictly speaking, it was a generalized susceptibility $\chi(\vec{q}, T)$ rather than $\Gamma(\vec{q}, T)$ which entered the results of Ref. 9. The following considerations are independent of the differences (thought to be minor near T_N for systems of present interest) between $\chi(\vec{q}, T)$ and $\Gamma(\vec{q}, T)$ and we conclude that short-range correlations are dominant for T_N^+ in either case.

[26]The subsequent discussion can be generalized considerably but the same conclusions are obtained.

[27]See the review of A. J. Freeman, in *Magnetic Properties of Rare Earth Metals,* edited by R. J. Elliott (Plenum, London, 1972), p. 245.

[28]See, for example, W. J. Camp and M. E. Fisher, Phys. Rev. Lett. 26, 73 (1971).

[29]There is a considerable literature on the effects of dipole-dipole interactions in various systems. See the review by A. Aharony, AIP Conf. Proc. 18, 863 (1974).

[30]S. Takada, Prog. Theor. Phys. 46, 15 (1971).

[31]T. Kasuya and A. Kondo, Solid State Commun. 14, 249 (1974).

[32]D. S. Simons and M. B. Salamon, Phys. Rev. Lett. 26, 750 (1971).

[33]L. Guttman and H. C. Schnyder, Phys. Rev. Lett. 22, 520 (1969).

[34]F. J. Arlinghaus, Phys. Rev. 186, 609 (1969).

[35]G. A. Thomas, A. B. Giray, and R. D. Parks, Phys. Rev. Lett. 31, 241 (1973).

[36]Extensive discussion of many alloys and compounds can be found in *Intermetallic Compounds,* edited by

J. H. Westbrook (Wiley, New York, 1967).
[37]K. V. Rao, Ö. Rapp, T. G. Richard, and D. J. W. Geldart, J. Phys. C 6, L231 (1973).
[38]R. A. Craven and R. D. Parks, Phys. Rev. Lett. 31, 383 (1973).
[39]K. V. Rao, Ö. Rapp, Ch. Johannesson, D. J. W. Geldart, and T. G. Richard, J. Phys. C 8, 2135 (1975).
[40]R. L. Singh and D. J. W. Geldart, Solid State Commun. 20, 501 (1976).