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Linear-response function of a semi-infinite degenerate plasma in the presence of an external magnetic field

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The static linear-response function $\chi_0(\vec{r},\vec{r}\,')$ of a semi-infinite, degenerate plasma of noninteracting electrons is calculated in the presence of an external magnetic field perpendicular to the surface. The surface is simulated by an infinite-barrier model. The work is based on the density-matrix formalism for an impurity embedded in a plasma and its image in the surface. The induced electron number density is expressed as a sum over Landau levels by a Laplace transformation of the density matrix. The Laplace-transform representation allows an analytic evaluation of the linear-response function for various magnetic field regimes. Results are presented for $\chi_0(\vec{r},\vec{r}\,')$ in the quantum strong-field limit, such that electrons occupy only the lowest Landau eigenstate, as well as the low-field limit where the response function is expressed as a series expansion in powers of the applied magnetic field strength. The intermediate-field regime for de Haas—van Alphen oscillations is also briefly discussed. These results provide a useful representation of the nonlocal static linear-response properties of a quantum plasma since $\chi_0(\vec{r},\vec{r}\,')$ is expressed in terms of known elementary and special functions.

I. INTRODUCTION

The effects due to a surface on the electronic properties of metals, semimetals, and semiconductors in the presence of an external magnetic field are of considerable interest. Both dynamic properties[1,2] (such as surface modes) and static properties[3,4] (such as charge distribution) have been studied. The electronic response functions required for these studies are usually determined, in practice, within simple models such as the infinite-barrier model (IBM) and classical infinite-barrier model (CIBM). In a self-consistent-field theory, such as the random-phase approximation (RPA), the single-particle response function is required. The evaluation of the response function for noninteracting electrons is thus a necessary first step in a self-consistent-field treatment of the electronic properties.

In this paper, we consider the semi-infinite degenerate electron gas with IBM boundary conditions, with an external magnetic field perpendicular to the surface. We give an exact analytical evaluation of the static density-density response function for noninteracting electrons. This is equivalent to a calculation of the irreducible, nonlocal polarization function $\chi_0(\vec{r},\vec{r}\,')$, in the static, i.e. zero-frequency, limit. Such results have not previously been available and we hope to obtain corresponding exact results for the frequency-dependent single-particle response function as well as for other boundary conditions. It is expected that these results will form a useful basis for further development and applications.

One application for χ_0 is the calculation of the induced electron number density $\rho_i(\vec{r})$ by an impurity embedded in the plasma. As a matter of fact, our calculation of $\chi_0(\vec{r},\vec{r}\,')$ is formulated in terms of the linear response of a noninteracting electron gas to an external perturbation $V(\vec{r}\,')$:

$$\rho_i(\vec{r})=\int dr'\chi_0(\vec{r},\vec{r}\,')V(\vec{r}\,')\ . \tag{1}$$

From the boundary conditions, the induced electron number density is due to the impurity and its image in the surface plane. This means that the density-matrix formalism used by one of the authors[5–7] in studying the static shielding of an impurity in a bulk plasma may be applied to the IBM. This provides an alternative approach to that used in Refs. 3 and 4. An advantage of the density-matrix formalism is that the sums over occupied Landau levels are expressed in terms of an inverse Laplace transform. This provides a convenient method of doing analytic calculations, as we show. A comparison of the density-matrix formalism has previously been made with the method of Horing[8,9] and Rensink[10] for a bulk degenerate plasma.[6] Although these authors[8–10] obtained approximate results for the induced electron number density in useful analytical form, further progress was limited. This is due to the fact that the sums over Landau levels involving Hermite polynomials are difficult to evaluate.

The outline of this paper is as follows. In Sec. II the linear-response formalism is presented for the IBM in the presence of an external magnetic field perpendicular to the surface. The low-magnetic-field regime is discussed in Sec. III and the quantum high-field limit is given in Sec. IV. The results for $\chi_0(\vec{r},\vec{r}\,')$ in these two limits are given in terms of known special functions. Section V deals with the intermediate-field case. Section VI consists of a summary and discussion. Our calculation involves some unfamiliar integrals involving Bessel functions of the first kind. These integrals are collected in the Appendix.

II. FORMALISM

We apply the density-matrix method employed in Refs. 5 and 6, adapted to the semi-infinite IBM. We let the point charge described by the potential $V(\vec{r})$ be embedded in a semi-infinite electron gas having single-electron eigenstates $\psi_\alpha(\vec{r})$. Then the electron density at zero temperature is (E_F denotes the Fermi energy)

$$\rho(\vec{r})=\sum_{E_\alpha \le E_F} |\psi_\alpha(\vec{r})|^2=\mathscr{L}_{E_F}^{-1}\{s^{-1}\Psi(\vec{r},\vec{r},s)\} , \quad (2)$$

where $\mathscr{L}_t^{-1}$ denotes the inverse Laplace transform (with argument t), and

$$\Psi(\vec{r},\vec{r}\,',s)=\sum_\alpha \psi_\alpha(\vec{r})\psi_\alpha^\dagger(\vec{r}\,')e^{-sE_\alpha} \quad (3)$$

is the density matrix. The latter satisfies the integral equation[11]

$$\Psi(\vec{r},\vec{r}\,',s)=\Psi_0(\vec{r},\vec{r}\,',s)-\int d\vec{r}_1 V(\vec{r}_1) \times \int_0^s ds_1 \Psi_0(\vec{r},\vec{r}_1,s-s_1) \times \Psi(\vec{r}_1,\vec{r}\,',s_1) , \quad (4)$$

where $\Psi_0(\vec{r},\vec{r}\,',s)$ is the density matrix in the absence of the potential, which for the IBM with a uniform magnetic field H applied in the z direction (normal to the surface) is

$$\Psi_0(\vec{r},\vec{r}\,',s)=(m^*/2\pi\hbar^2 s)^{3/2}(\mu_0^* Hs)\delta_{\sigma\sigma'}e^{s\mu_0 H\sigma}\operatorname{csch}(\mu_0^* Hs)$$
$$\times \exp\{-(m^*/2\hbar^2)[2i\mu_0^* H(xy'-yx')+\mu_0^* H\coth(\mu_0^* Hs)\,|\vec{\rho}-\vec{\rho}\,'|^2]\}$$
$$\times\{\exp[-m^*(z-z')^2/2\hbar^2 s]-\exp[-m^*(z+z')^2/2\hbar^2 s]\} , \quad (5)$$

where m^* is the effective mass, μ_0 (μ_0^*) is the spin (orbital) Bohr magneton, σ is a spin index, and $\vec{\rho}=(x,y,0)$.

By solving (4) to first order in V, we find the induced charge density due to the impurity is

$$\rho_i(\vec{r})=-\mathscr{L}_{E_F}^{-1}\left\{s^{-1}\int d\vec{r}_1 V(\vec{r}_1)\int_0^s ds_1 \Psi_0^{\sigma\sigma'}(\vec{r},\vec{r}_1,s-s_1)\Psi_0^{\sigma'\sigma}(\vec{r}_1,\vec{r},s_1)\right\} . \quad (6)$$

Inserting the expression (5) into (6), after a simple change of variables in the s_1 integral, we obtain our basic formula,

$$\rho_i(\vec{r})=-2(\mu_0^* H)^2(m^*/2\pi\hbar^2)^3\int d\vec{r}_1 V(\vec{r}_1)\mathscr{L}_\xi^{-1}\left\{s^{-1}\int_0^1 \frac{du}{\sqrt{uu'}}\frac{\cosh(\alpha s)}{\sinh(su)\sinh(su')}\right.$$
$$\times\exp\{-a[\coth(su)+\coth(su')]\,|\vec{\rho}-\vec{\rho}_1|^2\}$$
$$\times\{\exp[-a(z-z_1)^2/su]-\exp[-a(z+z_1)^2/su]\}$$
$$\left.\times\{\exp[-a(z-z_1)^2/su']-\exp[-a(z+z_1)^2/su']\}\right\} , \quad (7)$$

where $u'=1-u$, $\xi=E_F/\mu_0^* H$, $\alpha=m^*/m$, and $a=m^*\mu_0^* H/2\hbar^2$. In deriving this expression we have allowed for an orbital effective mass, but have set the electron g factor equal to 2. Lifting this restriction requires only a minor modification and does not appreciably affect the remainder of the calculation. We note from (1) that the response function can be extracted from (7) by the functional derivative $\delta\rho_i(\vec{r})/\delta V(\vec{r}\,')=\chi_0(\vec{r},\vec{r}\,')$.

III. INDUCED DENSITY AT LOW MAGNETIC FIELD

The low-magnetic-field regime is characterized by $\xi\to\infty$, which requires an asymptotic evaluation of the inverse Laplace transform in (7). This is easily achieved by expanding the integrand in ascending powers of s. Carrying this out and taking advantage of the $u\leftrightarrow u'$ symmetry, we obtain

$$\rho_i(\vec{r})=-2(m^*/2\pi\hbar^2)^3(\mu_0^* H)^2\int d\vec{r}_1 V(\vec{r}_1)\int_0^1 \frac{du}{(uu')^{3/2}}\mathscr{L}_\xi^{-1}\left\{s^{-3}[1-\tfrac{1}{3}as\,|\vec{\rho}-\vec{\rho}_1|^2+\tfrac{1}{2}(\alpha^2-\tfrac{1}{3}u^2-\tfrac{1}{3}u'^2)s^2+\cdots]\right.$$
$$\times\left\{\exp(-a\,|\vec{r}-\vec{r}_1|^2/suu')+\exp(-a\,|\vec{r}-\vec{r}\,'_1|^2/suu')\right.$$
$$\left.\left.-2\exp\left[-\frac{a}{s}\left[\frac{|\vec{r}-\vec{r}_1|^2}{u'}+\frac{|\vec{r}-\vec{r}\,'_1|^2}{u}\right]\right]\right\}\right\} , \quad (8)$$

where $\vec{r}\,'_1=(x_1,y_1,-z_1)$. The inverse Laplace transform can now be evaluated using standard tables,[12] and we find

$$\rho_i(\vec{r})=\rho_i^{(0)}(\vec{r})+[\rho_i^{(1)}(\vec{r})+\rho_i^{(2)}(\vec{r})](\mu_0^*H/E_F)^2+\cdots\ , \tag{9}$$

where the lowest-order term,

$$\rho_i^{(0)}(\vec{r})=-\frac{m^*k_F^2}{4\pi^3\hbar^2}\int d\vec{r}_1 V(\vec{r}_1)\int_0^1\frac{du}{(uu')^{1/2}}\Bigg\{|\vec{r}-\vec{r}_1|^{-2}J_2[k_F|\vec{r}-\vec{r}_1|/(uu')^{1/2}] + |\vec{r}-\vec{r}\,'_1|^{-2}J_2[k_F|\vec{r}-\vec{r}\,'_1|/(uu')^{1/2}] -2(|\vec{r}-\vec{r}_1|^2u+|\vec{r}-\vec{r}\,'_1|^2u')^{-1} \times J_2\left[k_F\left[\frac{|\vec{r}-\vec{r}_1|^2}{u'}+\frac{|\vec{r}-\vec{r}\,'_1|^2}{u}\right]^{1/2}\right]\Bigg\}, \tag{10}$$

contains the zero-field limit as well as a contribution due to the implicit dependence of E_F and the Fermi wave vector $k_F=(2m^*E_F/\hbar^2)^{1/2}$ on the applied field.[7] The leading terms containing explicit field dependence are

$$\rho_i^{(1)}(\vec{r})=\frac{m^*k_F^5}{96\pi^3\hbar^2}\int d\vec{r}_1 V(\vec{r}_1)(\vec{\rho}-\vec{\rho}_1)^2 \times\int_0^1\frac{du}{uu'}\Bigg\{|\vec{r}-\vec{r}_1|^{-1}J_1[k_F|\vec{r}-\vec{r}_1|/(uu')^{1/2}]+|\vec{r}-\vec{r}\,'_1|^{-1}J_1[k_F|\vec{r}-\vec{r}\,'_1|/(uu')^{1/2}] -2(|\vec{r}-\vec{r}_1|^2u+|\vec{r}-\vec{r}\,'_1|^2u')^{-1/2}J_1\left[k_F\left[\frac{|\vec{r}-\vec{r}_1|^2}{u'}+\frac{|\vec{r}-\vec{r}\,'_1|^2}{u}\right]^{1/2}\right]\Bigg\} \tag{11}$$

and

$$\rho_i^{(2)}(\vec{r})=-\frac{m^*k_F^4}{32\pi^3\hbar^2}\int d\vec{r}_1 V(\vec{r}_1)\int_0^1\frac{du}{(uu')^{3/2}}(\alpha^2-\tfrac{1}{3}u^2-\tfrac{1}{3}u'^2) \times\Bigg\{J_0[k_F|\vec{r}-\vec{r}_1|/(uu')^{1/2}]+J_0[k_F|\vec{r}-\vec{r}\,'_1|/(uu')^{1/2}] -2J_0\left[k_F\left[\frac{|\vec{r}-\vec{r}_1|^2}{u'}+\frac{|\vec{r}-\vec{r}\,'_1|^2}{u}\right]^{1/2}\right]\Bigg\}. \tag{12}$$

Quite remarkably, the u integrations can be carried out exactly, and are given in the Appendix (details will be reported elsewhere). The three terms in Eqs. (10)–(12) show the clear separation into a "classical" (cl) part, corresponding to the point charge and its image (the so-called CIBM terms) and a quantum-interference (QI) term. Accordingly, we write

$$\rho_i(\vec{r})=\delta\rho_{\rm cl}(\vec{r})+\delta\rho_{\rm QI}(\vec{r})\ . \tag{13}$$

A. Zero-field behavior

In this limit (10) can be written as the sum of

$$\delta\rho_{\rm cl}^{(0)}(\vec{r})=-\frac{m^*k_F}{4\pi^3\hbar^2}\int d\vec{r}_1V(\vec{r}_1)\left[|\vec{r}-\vec{r}_1|^{-3}\left[\frac{\sin(2k_F|\vec{r}-\vec{r}_1|)}{2\kappa_F|\vec{r}-\vec{r}_1|}-\cos(2k_F|\vec{r}-\vec{r}_1|)\right] +|\vec{r}-\vec{r}\,'_1|^{-3}\left[\frac{\sin(2k_F|\vec{r}-\vec{r}\,'_1|)}{2k_F|\vec{r}-\vec{r}\,'_1|}-\cos(2k_F|\vec{r}-\vec{r}\,'_1|)\right]\right], \tag{14a}$$

$$\delta\rho_{\rm QI}^{(0)}(\vec r)=\frac{m^*k_F}{\pi^3\hbar^2}\int d\vec r_1 V(\vec r_1)[\,|\vec r-\vec r_1|\,|\vec r-\vec r\,'_1|\,(|\vec r-\vec r_1|+|\vec r-\vec r\,'_1|)]^{-1}$$
$$\times\left[\frac{\sin[k_F(|\vec r-\vec r_1|+|\vec r-\vec r\,'_1|)]}{k_F(|\vec r-\vec r_1|+|\vec r-\vec r\,'_1|)}-\cos[k_F(|\vec r-\vec r_1|+|\vec r-\vec r\,'_1|]\right]. \tag{14b}$$

Note that there is an exact cancellation of the classical and interference terms for $\vec r=\vec 0$ as is required by the model.

B. Terms proportional to $(\mu_0^*H/E_F)^2$

The leading field-dependent terms, (11) and (12), can be written as

$$\delta\rho_{\rm cl}^{(1)}(\vec r)=\frac{m^*k_F^6}{12\pi^3\hbar^2}\int d\vec r_1 V(\vec r_1)(\vec\rho-\vec\rho_1)^2\left[\frac{\sin(2k_F|\vec r-\vec r_1|)}{(2k_F|\vec r-\vec r_1|)^2}+\frac{\sin(2k_F|\vec r-\vec r\,'_1|)}{(2k_F|\vec r-\vec r\,'_1|)^2}\right], \tag{15a}$$

$$\delta\rho_{\rm cl}^{(2)}(\vec r)=-\frac{m^*k_F^4}{4\pi^3\hbar^2}\int d\vec r_1 V(\vec r_1)\left[\alpha^2\left[\frac{\cos(2k_F|\vec r-\vec r_1|)}{2k_F|\vec r-\vec r_1|}+\frac{\cos(2k_F|\vec r-\vec r\,'_1|)}{2k_F|\vec r-\vec r\,'_1|}\right]\right.$$
$$\left.-\tfrac{1}{12}[B_0(k_F|\vec r-\vec r_1|)+B_0(k_F|\vec r-\vec r\,'_1|)]\right], \tag{15b}$$

$$\delta\rho_{\rm QI}^{(1)}(\vec r)=-\frac{m^*k_F^4}{12\pi^3\hbar^2}\int d\vec r_1 V(\vec r_1)(\vec\rho-\vec\rho_1)^2|\vec r-\vec r_1|^{-1}|\vec r-\vec r\,'_1|^{-1}\sin[k_F(|\vec r-\vec r_1|+|\vec r-\vec r\,'_1|)], \tag{15c}$$

$$\delta\rho_{\rm QI}^{(2)}(\vec r)=-\frac{m^*k_F^4}{8\pi^3\hbar^2}\int d\vec r_1 V(\vec r_1)\left[(\alpha^2-\tfrac{1}{3})\left[\frac{|\vec r-\vec r_1|+|\vec r-\vec r\,'_1|}{k_F|\vec r-\vec r_1|\,|\vec r-\vec r\,'_1|}\right]\right.$$
$$\left.\times\cos[k_F(|\vec r-\vec r_1|+|\vec r-\vec r\,'_1|)]-\tfrac{2}{3}\,\mathrm{si}\left[\frac{k_F}{2}(|\vec r-\vec r_1|+|\vec r-\vec r\,'_1|)\right]\right], \tag{15d}$$

where B_0 is defined in the Appendix. Note that there is an additional contribution proportional to H^2 arising from expanding the implicit field dependence of k_F in (10). This term is straightforwardly obtained and need not be written explicitly.

IV. QUANTUM HIGH-FIELD LIMIT

Returning to our basic formula (7), as discussed in Ref. 6, the behavior of $\rho_i(\vec r)$ in the high-field quantum limit (HFQL), where $\zeta\to 0$, can be obtained by expanding the s integrand about $s=\infty$. Each successive term in this expansion corresponds to the inclusion of one more Landau level. Here, we shall confine attention to the leading term, which corresponds to the quantum limit of the occupation of only the lowest (spin-split) Landau level. Then we have

$$\rho_i(\vec r)=-(\mu_0^*H)^2(m^*/\pi\hbar^2)^3\int d\vec r_1 V(\vec r_1)e^{-2a|\vec\rho-\vec\rho_1|^2}\mathscr{L}_{\zeta'}^{-1}\left\{s^{-1}\int_0^1\frac{du}{(uu')^{1/2}}[e^{-a|z-z_1|^2/su}-e^{-a|z+z_1|^2/su}]\right.$$
$$\left.\times[e^{-a|z-z_1|^2/su'}-e^{-a|z+z_1|^2/su'}]\right\}, \tag{16}$$

where $\zeta'=\zeta+\alpha-1$. The inverse Laplace transform is easily carried out, and we have

$$\rho_i(\vec r)=-(\mu_0^*H)^2(m^*/\pi\hbar^2)^3\int d\vec r_1 V(\vec r_1)e^{-2a|\vec\rho-\vec\rho_1|^2}$$
$$\times\int_0^1\frac{du}{(uu')^{1/2}}\left[J_0[2|z-z_1|(\zeta'a/uu')^{1/2}]+J_0[2|z+z_1|(\zeta'a/uu')^{1/2}]\right.$$
$$\left.-2J_0\left\{2\left[a\zeta'\left[\frac{|z-z_1|^2}{u'}+\frac{|z+z_1|^2}{u}\right]\right]^{1/2}\right\}\right]. \tag{17}$$

Surprisingly, the u integrations are also tractable (see the Appendix), and

$$\rho_i(\vec{r})=\delta\rho_{cl}^{HFQL}(\vec{r})+\delta\rho_{QI}^{HFQL}(\vec{r}) , \quad (18a)$$

$$\delta\rho_{cl}^{HFQL}(\vec{r})=(\mu_0^* H)^2(m^*/\pi\hbar^2)^3 \int d\vec{r}_1 V(\vec{r}_1) e^{-2a|\vec{\rho}-\vec{\rho}_1|^2}[\mathrm{si}(2k_F'|z-z_1|)+\mathrm{si}(2k_F'|z+z_1|)] , \quad (18b)$$

$$\delta\rho_{QI}^{HFQL}(\vec{r})=-2(\mu_0^* H)^2(m^*/\pi\hbar^2)^3 \int d\vec{r}_1 V(\vec{r}_1) e^{-2a|\vec{\rho}-\vec{\rho}_1|^2}\mathrm{si}[k_F'(|z-z_1|+|z+z_1|)] , \quad (18c)$$

where $k_F'=2(a\zeta')^{1/2}$.

V. INTERMEDIATE FIELDS: de HAAS—van ALPHEN TERMS

The quantum oscillatory terms in the density arise from the imaginary singularities of the s integrand in (7), which are isolated essential singularities $s_n=n\pi i/u$, $s_n'=n\pi i/u'$ $(n=\pm 1,\pm 2,\ldots)$. We express the inverse Laplace transform as a Bromwich integral, and distort the contour into a succession of small circles surrounding these points. (The singularity at $s=0$ contributes only to the steady field behavior studied in Sec. IV, and has already been taken into account.) Because of the $u\leftrightarrow u'$ symmetry we need only consider the terms arising from the singularities s_n and multiply the result by 2. Next, by evaluating the individual contour integrals by residues and pairing the $+|n|,-|n|$ terms we obtain

$$\rho_i^{osc}(\vec{r})=\frac{16}{\pi}(\mu_0^* H)^2(m^*/2\pi\hbar^2)^3$$
$$\times\int d\vec{r}_1 V(\vec{r}_1)\sum_{n=1}^{\infty}\frac{1}{n}\int_0^1\frac{du}{(uu')^{1/2}}\frac{\cos(\alpha n\pi/u)}{\sin(n\pi/u)}J_0(k_F|\vec{\rho}-\vec{\rho}_1|u^{-1/2})$$
$$\times\mathrm{Re}\{\exp[ia|\vec{\rho}-\vec{\rho}_1|^2\cot(n\pi/u)+i\zeta n\pi/u]$$
$$\times[\exp(ia|z-z_1|^2/n\pi)-\exp(ia|z+z_1|^2/n\pi)]$$
$$\times[\exp(ia|z-z_1|^2u/n\pi u')-\exp(ia|z+z_1|^2u/n\pi u')]\} . \quad (19)$$

In the interest of simplicity we consider only the case $\alpha=1$. Relying on the feature that $\zeta\gg 1$, after the substitution $x=u'/u$, we find that the dominant contribution to the x integral comes from the singularity at $x=0$. Expanding about this point leads to Fresnel integrals and, after some simplification, we find that

$$\delta\rho_{cl}^{osc}(\vec{r})=\frac{2}{\pi^4}(\mu_0^* H)^{3/2}(m^*/\hbar^2)^{5/2}$$
$$\times\int d\vec{r}_1\, V(\vec{r}_1)J_0(2k_F|\vec{\rho}-\vec{\rho}_1|)\sum_{n=1}^{\infty}n^{-3/2}[|\vec{r}-\vec{r}_1|^{-1}\cos(n\pi\zeta+a|z-z_1|^2/n\pi-\pi/4)$$
$$+|\vec{r}-\vec{r}_1'|^{-1}\cos(n\pi\zeta+a|z+z_1|^2/n\pi-\pi/4)] , \quad (20a)$$

$$\delta\rho_{QI}^{osc}(\vec{r})=-\frac{2}{\pi^4}(\mu_0^* H)^{3/2}(m^*/\hbar^2)^{5/2}$$
$$\times\int d\vec{r}_1\, V(\vec{r}_1)J_0(2k_F|\vec{\rho}-\vec{\rho}_1|)\sum_{n=1}^{\infty}n^{-3/2}[|\vec{r}-\vec{r}_1|^{-1}\cos(n\pi\zeta+a|z+z_1|^2/n\pi-\pi/4)$$
$$+|\vec{r}-\vec{r}_1'|^{-1}\cos(n\pi\zeta+a|z-z_1|^2/n\pi-\pi/4)] . \quad (20b)$$

Further discussion of the intermediate-field case will not be given here.

VI. SUMMARY

We have evaluated the static linear-response function for noninteracting electrons in the infinite-barrier model in the presence of an applied magnetic field. The calculations have been carried out in configuration space using Laplace-transform methods so that explicit sums over Landau levels are totally avoided. All quantum-interference effects are rigorously included and exact analytical results are given in closed form for both the weak-field (in the form of a series expansion in powers of the applied magnetic field strength) and in the quantum high-field limit (where only one Landau level is occupied). The intermediate de Haas—van Alphen regime of field strengths is also briefly discussed. It is expected that our new exact results will provide a convenient basis for further developments in which time-dependent external perturbations, as well as electron-electron interactions, are included.

For convenience we list the explicit forms for the linear-response function as deduced from (14) in the zero-field limit,

$$\chi_0(\vec{r}_1,\vec{r}_2)=\frac{m^*k_F}{4\pi^3\hbar^2}\left[-\frac{1}{R^3}\left[\frac{\sin(2k_FR)}{2k_FR}-\cos(2k_FR)\right]-\frac{1}{R'^3}\left[\frac{\sin(2k_FR')}{2k_FR'}-\cos(2k_FR')\right]+\frac{4}{RR'(R+R')}\left[\frac{\sin[k_F(R+R')]}{k_F(R+R')}-\cos[k_F(R+R']\right]\right], \tag{21}$$

where $\vec{R}=\vec{r}_1-\vec{r}_2$ and $\vec{R}'=\vec{r}_1-\vec{r}'_2$, while the quantum high-field limit (18) gives

$$\chi_0(\vec{r}_1,\vec{r})=(\mu_0^*H)^2/(m^*/\pi\hbar^2)^3e^{-2a|\vec{\rho}-\vec{\rho}_1|^2}\times\{\mathrm{si}(2k'_F|z-z_1|)+\mathrm{si}(2k'_F|z+z_1|)-2\,\mathrm{si}[k'_F(|z-z_1|+|z+z_1|)]\}\ . \tag{22}$$

In a self-consistent calculation of $\rho_i(\vec{r})$, (7) would be an additional term in Poisson's equation. The dynamical nature of the linear-response properties would entail factors in (6) which involve a time integral. These results may be employed in evaluating collective and dynamic aspects of bounded plasmas in the presence of an external magnetic field. This would increase our knowledge and appreciation of the nonlocal behavior of bounded plasmas under various magnetic field conditions. We hope to report such results in the future.

Note added in proof. Professor Horing has pointed out to us that a similar study is contained in the thesis of N. Yildiz (Stevens Institute of Technology, Hoboken, New Jersey). See also N. J. Horing and M. Yildiz, in *Proceedings of the Thirteenth International Conference on Semiconductors, Rome, 1976,* edited by F. G. Fumi (North-Holland, Amsterdam, 1976), p. 1129; however, these results are within the momentum representation.

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APPENDIX

A number of Bessel function integrals of an unfamiliar nature occur in this calculation. We simply record their values here and refer the interested reader to a future report for the details of the evaluations:

$$A_\nu(\lambda)\equiv\int_0^1\frac{du}{(uu')^{(3-\nu)/2}}J_\nu(\lambda(uu')^{-1/2})\ , \tag{A1a}$$

$$A_0(\lambda)=4\lambda^{-1}\cos(2\lambda)\ , \tag{A1b}$$

$$A_1(\lambda)=2\lambda^{-1}\sin(2\lambda)\ , \tag{A1c}$$

$$A_2(\lambda)=\lambda^{-1}\left[\frac{\sin(2\lambda)}{2\lambda}-\cos(2\lambda)\right], \tag{A1d}$$

$$A_3(\lambda)=\lambda^{-1}[j_0(2\lambda)-\cos(2\lambda)]\ , \tag{A1e}$$

$$B_0(\lambda)\equiv\int_0^1du\,u^{1/2}(u')^{-3/2}J_0(\lambda(uu')^{-1/2})=2\lambda^{-1}\cos(2\lambda)-2\lambda^{-1}+4J_0(2\lambda)-2\pi[J_0(2\lambda)\mathrm{H}_1(2\lambda)-J_1(2\lambda)\mathrm{H}_0(2\lambda)]+2\,\mathrm{si}(2\lambda)\ , \tag{A2}$$

$$\phi_\nu(\alpha,\beta)\equiv\int_0^1\frac{du}{(uu')^{3/2}}\left[\frac{\alpha}{u'}+\frac{\beta}{u}\right]^{-\nu/2}J_\nu\left[\frac{\alpha}{u'}+\frac{\beta}{u}\right]^{1/2}=(2\pi/\alpha\beta)^{1/2}(\alpha^{1/2}+\beta^{1/2})^{3/2-\nu}J_{\nu-1/2}(\alpha^{1/2}+\beta^{1/2})\ , \tag{A3}$$

$$\psi_0(\alpha,\beta)\equiv\int_0^1u^{1/2}(u')^{-3/2}J_0\left[\frac{\alpha}{u'}+\frac{\beta}{u}\right]^{1/2}du=2\{\alpha^{-1/2}\cos(\alpha^{1/2}+\beta^{1/2})+\mathrm{si}[\tfrac{1}{2}(\alpha^{1/2}+\beta^{1/2})]\}\ , \tag{A4}$$

$$C_0(\alpha,\beta)\equiv\int_0^1\frac{du}{(uu')^{1/2}}J_0\left[\frac{\alpha}{u'}+\frac{\beta}{u}\right]^{1/2}=-2\,\mathrm{si}(\alpha^{1/2}+\beta^{1/2})\ . \tag{A5}$$

The conventional notation for various special functions has been followed.[13]

[1]N. J. Horing and M. Yildiz, Phys. Lett. 44A, 386 (1973).
[2]N. J. Horing, M. Yildiz, F. Kortel, and T. Caglayan, J. Phys. C 6, 2053 (1973).
[3]G. Gumbs, Phys. Rev. B 27, 7136 (1983).
[4]G. Gumbs and D. J. W. Geldart, Phys. Rev. B 29, 5445 (1984).
[5]M. L. Glasser, Phys. Rev. 180, 942 (1969).
[6]M. L. Glasser, Can. J. Phys. 48, 1941 (1970).
[7]M. L. Glasser, in *Theoretical Chemistry, Advances and Perspectives,* edited by H. Eyring and D. Henderson (Academic, New York, 1976), Vol. 2, p. 67.
[8]N. J. Horing, Phys. Rev. 186, 434 (1969).
[9]N. J. Horing, Ann. Phys. (N.Y.) 54, 405 (1969).
[10]M. E. Rensink, Phys. Rev. 174, 744 (1968).
[11]A. W. Saénz and R. C. O'Rourke, Rev. Mod. Phys. 27, 381 (1955).
[12]*Tables of Integral Transforms,* edited by A. Erdélyi (McGraw-Hill, New York, 1954), Vol. 1.
[13]I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, New York, 1965).