Critical behavior of two-dimensional magnetic systems with dipole-dipole interactions

K. De'Bell and D. J. W. Geldart
Department of Physics, Dalhousie University, Halifax, Nova Scotia, Canada B3H 3J5
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The critical exponents of two-dimensional dipolar magnetic systems with isotropic in-plane ordering may be obtained from the ϵ expansion which determines the exponents of isotropic three-dimensional dipolar systems. However, a two-dimensional dipolar system with uniaxial in-plane ordering requires a new ϵ expansion with upper critical dimension $d_c = 3\frac{1}{2}$. The possible relevance to experimental studies of monolayers of gadolinium is briefly discussed.

Recent advances in techniques for the preparation of thin films and layered systems has led to renewed interest in the critical phenomena of two-dimensional systems.[1] In particular, studies of monolayer films of gadolinium on tungsten (110) surfaces have recently been reported and interpreted in terms of an Ising system with short-range (exchange) interactions.[2] However, it has recently been suggested that dipolar interactions may be relevant to the asymptotic critical behavior of bulk Gd.[3] We wish to point out that a similar situation exists in the case of films and monolayers.

We shall consider two-dimensional dipolar systems in which the order parameter lies in the two-dimensional plane. Generalizing the Hamiltonian for a two-dimensional dipolar system given by Maleev[4] to d dimensions gives

$$-\mathcal{H} = \int d^d q \sum_{\alpha,\beta} \left[(r_\alpha + q^2)\delta_{\alpha\beta} + g\frac{q^\alpha q^\beta}{q} \right] \phi_\alpha(q)\phi_\beta(-q) + \int u_{\alpha\beta}\phi_\alpha^2\phi_\beta^2 \,, \quad (1)$$

where α and β denote components parallel to the d-dimensional "plane" and the usual integration over the momentum arguments and conservation of momentum are implied in the final quartic term. The problem of a two-dimensional system with dipolar interactions in which the order parameter is perpendicular to the plane has previously been described both in the context of magnetic systems[5] and in the context of Langmuir-Blodgett films.[6]

For isotropic ordering in the plane, the derivation of the appropriate propagator is similar to that for three-dimensional dipolar systems.[7] The coefficient of $\phi^\alpha\phi^\beta$ in $\mathcal{H}$ may be decomposed into transverse and longitudinal parts

$$\left[(r+q^2)\left[\delta_{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2} \right] + (r+q^2+gq)\frac{q^\alpha q^\beta}{q^2} \right] \phi_\alpha\phi_\beta \,, \quad (2)$$

and, therefore, the Gaussian propagator is

$$G^{\alpha\beta}(\mathbf{q}) = \left[\frac{1}{r+q^2}\left[\delta_{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2} \right] \right] + \frac{1}{r+q^2+gq}\frac{q^\alpha q^\beta}{q^2} \,. \quad (3)$$

Note that the $q=0$ point is excluded in the integration of the nonanalytic term in $\mathcal{H}$ and that under a change of scale

$$q \to \mu q, \quad g \to \mu g \,.$$

The only fixed points for this system occur at $g=0$, corresponding to a system with short-ranged interactions only, and $g=\infty$. In the limit $g\to\infty$, the propagator becomes

$$\lim_{g\to\infty} G^{\alpha\beta}(q) = \frac{1}{r+q^2}\left[\delta_{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2} \right] . \quad (4)$$

This is of precisely the same form as the Gaussian propagator previously used to obtain the usual ϵ expansion for the critical exponents of bulk dipolar systems.[8] The critical exponents of this two-dimensional system's phase transition (as described by Maleev[4]) will therefore be determined by the same forms as the usual ϵ expansion, however, with $\epsilon=2$.

Turning to the case of uniaxial ordering in the plane, it is sufficient to consider a one-component field ϕ and Hamiltonian

$$-\mathcal{H} = \int d^{(d-m)}q\, d^m q_\alpha \left[r + q^2 + \frac{g q_\alpha^2}{q} \right] \phi^2(\mathbf{q},\mathbf{q}_\alpha) + \int u\phi^4 \quad (5)$$

in a convenient generalization of the physical $d=2$, $m=1$ Hamiltonian. We will express our results in terms of the minimal subtraction method, though to the order we shall work, equivalent results may be obtained by the Wilson recursion relation method.

Dimensional arguments then determine that each loop integral in the bare vertex functions is accompanied by a factor $ug^{-m/2}$ with dimensions

$$[ug^{-m/2}] = [q]^{\epsilon'} \quad (6)$$

with

$$\epsilon' = 4 - d - \frac{m}{2} \,. \quad (7)$$

The renormalized dimensionless coupling constant is then

$$u_R = ug^{-m/2}\mu^{-\epsilon'}Z_u \quad (8)$$

with μ and the renormalization constants Z, Z_u, and Z_2

chosen so that the renormalization-group condition

$$\left.\frac{\partial\Gamma^{(2)}(u_R)}{\partial q^2}\right|_{q^2=\mu^2, q^2{}_\alpha=0} = 1 + \cdots , \tag{9}$$

$$\Gamma^{(4)}(u_R)|_{\mathrm{SP}} = u_R + \cdots , \tag{10}$$

$$\Gamma^{(2,1)}(u_R)|_{\mathrm{SP}} = 1 + \cdots , \tag{11}$$

$$\Gamma^{(N,L)}(u_R) = Z^{N/2} Z_2^L \Gamma_b^{(N,L)} . \tag{12}$$

The subscript b denotes the bare function, the ellipses represent nonsingular terms, and SP denotes the symmetry point such that

$$\mathbf{q}_i \cdot \mathbf{q}_j = \frac{\mu^2}{4}(4\delta_{ij} - 1); \quad \mathbf{q}_\alpha = \mathbf{0} . \tag{13}$$

To order one loop

$$Z = 1 ,$$

$$Z_u = 1 + M_1 I_1 ,$$

$$Z_2 = 1 + M_2 I_1 ,$$

where M_1 and M_2 are determined by the multiplicity of the appropriate graph but are independent of the form of the propagator. I_1 is the one-loop integral

$$I_1 = \int\int d^{d-m}q \, d^m q_\alpha \times \frac{1}{(q^2 + q_\alpha^2/|\mathbf{q}|)[(\mathbf{q}+\mathbf{p})^2 + q_\alpha^2/|\mathbf{q}+\mathbf{p}|]} . \tag{14}$$

Evaluating the integral

$$I_1 = S_m S_{d-m} \frac{\Gamma(m/2)\Gamma(2-m/2)\Gamma[(d-m)/2]\Gamma(\epsilon'/2)}{\Gamma(2-3m/4)} \times \frac{1}{4-3m\epsilon'}[1 + O(\epsilon')] , \tag{15}$$

where

$$S_d = \frac{2(2\pi)^{d/2}}{\Gamma(d/2)} . \tag{16}$$

Having confirmed that I_1 has a simple pole at $\epsilon' = 0$, we may use the standard arguments of the minimal subtraction to obtain the correlation length exponent ν to leading order in ϵ'

$$\nu = \tfrac{1}{2}\left[1 + \frac{m_2}{m_1}\epsilon' + O(\epsilon'^2)\right] . \tag{17}$$

Thus, although ν is independent of the coefficient of ϵ' in I_1 to this order, the value of ν will depend on the value of the m through the critical dimension

$$d_c = 4 - \frac{m}{2} . \tag{18}$$

The fact that the coefficient of ϵ' in Eq. (17) is independent of m is a general feature of the one-loop calculation. The corresponding coefficients in higher-order calculations are expected to have explicit m dependence. The calculation of these higher-order coefficients (even to order two loops) presents substantial additional difficulty and has not been undertaken as yet. It is already clear, however, from this one-loop calculation that dipolar interactions may be of importance and that the usual two-dimensional Ising or Heisenberg models (with short-range interactions) may not be sufficient to account for experimentally determined critical exponents. In summary, for two-dimensional dipolar systems with isotropic ordering in the plane, the appropriate Gaussian propagator is one that has previously been used in the context of three-dimensional dipolar models. Therefore, a single ϵ expansion about $d_c = 4$ will give exponents for both $d = 2$ and $d = 3$ systems (with an appropriate resummation technique and $\epsilon = 2$ or $\epsilon = 1$, respectively). However, in the case of a two-dimensional system with uniaxial ordering in the plane, we must perform a new ϵ' expansion about the upper critical dimension $d_c = 3\frac{1}{2}$. This may be contrasted with the case of three-dimensional uniaxial dipolar systems, in which the appropriate critical dimension is 3 and, therefore, the critical behavior of the physical system may be described in terms of power laws with mean-field theory exponents and logarithmic corrections.[7]

Finally, we turn to possible experimental realizations of two-dimensional dipolar systems. As noted in the introduction, in the case of Langmuir-Blodgett films, the component of the order parameter perpendicular to the plane plays a significant role in determining the phase diagram and this has been described, within mean-field theory, in Refs. 5 and 6. In thin magnetic films, such as in Ref. 2, there may be a large number of basic interactions which influence the magnetic phase diagram. For example, spin-orbit coupling may play a significant role in determining the anisotropy of the order parameter. In addition, for thin films grown on metallic substrates, the interaction with the substrate may be of importance. Although a more complete theoretical study is called for in the case of Gd on W, in view of the possible importance of dipolar interactions in bulk Gd, such interactions should not be neglected in studies of thin films of this material. In particular, a change in the behavior of the susceptibility is observed at a reduced temperature $t \approx 10^{-2}$ in Ref. 2, and this is reminiscent of the behavior in bulk Gd at $t \approx 10^{-3}$, which has been ascribed to dipolar effects.[3]

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