A MULTIFACTOR REGIME SWITCHING MODEL FOR COUNTRY EXCHANGE-TRADED FUNDS

by

Jun Yuan

Submitted in partial fulfilment of the requirements for the degree of Master of Arts at

Dalhousie University
Halifax, Nova Scotia
August 2011

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Dated: August 29, 2011

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_________________________________
Signature of Author
To My Parents,
who support me selflessly.

To my wife, Ningjian Liang,
who always has faith in me.
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Abstract

This thesis explores the returns of country exchange-traded funds (ETFs) with regime switching risk factors. Using the Bayesian information criterion, I select the model with six risk factors and three states among other models. The estimation results show that both the returns of country ETFs and their sensitivities to risk factors are highly regime dependent. Firstly, the U.S. size and value factors are significant in explaining all selected ETFs across regimes. More specifically, small capitalization is associated with lower returns for seven ETFs in some regimes. High book-to-market ratio generates higher returns for all ETFs in most regimes. Secondly, the global stock market has a positive impact on all selected country ETFs. Thirdly, all ETFs returns are negatively correlated to market volatility in bull and bear markets. Fourthly, stronger U.S. dollar generates a higher return for US ETF and lower returns for other seven ETFs across regimes. Finally, the returns of Australia, Canada and UK ETFs, which invest heavily in materials, are positively affected by commodity prices while other ETF returns are negatively influenced by them across regimes.
List of Abbreviations and Symbols used

List of Abbreviations

AUS: Australia
BFA: BlackRock Fund Advisors
CA: Canada
CAPM: Capital Asset Pricing Model
COM: S&P Goldman Sachs Commodity Index
CS: Credit Spread
DXY: U.S. dollar index
ETF: Exchange Traded Fund
FRA: France
GER: Germany
GICS: Global Industry Classification Standard
HML: High minus low
ITA: Italy
JAP: Japan
MSCI: Morgan Stanley Capital International
S&P: Standard and Poor’s
SMB: Big minus small
US: United States
UK: United Kingdom
VIX: Chicago board options exchange volatility index
WOD: MSCI All Country World Investable Market Index
YS: Yield Spread

List of Symbols

\( \alpha_t \): Forward probability
\( \beta \): Beta
\( \delta \): Initial probability distribution
\( \Gamma \): Transition probability matrix
\( R_t \): A vector of returns on ETFs in period \( t \)
\( \rho_t \): Backward probability
\( S_t \): Market regime in period \( t \)
\( \Sigma_{st} \): A conditional variance covariance matrix
\( X_t \): Random variable
\( Z_t \): A matrix of variables in period \( t \)
Acknowledgements

My utmost gratitude goes to my advisor, Dr. Kuan Xu, for his expertise, caring, and most of all, for his guidance on this project. Dr. Kuan Xu is a great mentor who always inspires me with new ideas and encourages me to learn. The sincerest appreciation goes to my advisor Dr. Yonggan Zhao, for his encouragement and support during the development of the research model. I would not be able to complete this thesis without his enthusiastic help. I am indebted to both of my advisers.

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Jun Yuan
August 15, 2011
Chapter 1

Introduction

Investing in exchange-traded fund (ETF) has attracted a lot of attention in the market, though this kind of investment instrument has only come to existence for less than two decades. By the end of January 2011, there were 943 ETFs globally, which held $1,004 trillion assets in total\(^1\). Although international ETFs are very popular, few studies have explored the pricing mechanism for the country ETFs. This raises a variety of interesting questions: How to model the returns of country ETFs? What risk factors are important in affecting these ETFs? Are the returns on these country ETFs related to the business cycle? This thesis will answer these questions.

There is an extensive literature exploring determinants of equity returns and the appropriate pricing model. To test the determinants of equity returns, Basu (1977) and Fama and French (1992, 1993, 1996) investigated the role of fundamental factors such as market volatility, market capitalization of stocks, and book-to-market ratio. Chen et al. (1986), Campbell (1987), and Fama and French (1989) examined macroeconomic factors such as interest rate, inflation rate, yield spread and credit spread. Solnik (1974a, 1974b), Chen et al. (1986), Johnson and Soenen (2009) and Bakshi et

\(^1\)See the Investment Company Institute at http://www.ici.org.
al. (2010) examined international risk factors such as exchange rates, oil price, the commodity index and the Baltic dry index. To study asset pricing, Basu (1977) and Fama and French (1992) use the static model; Ferson and Harvey (1991, 1998), and Jagannathan and Wang (1996) use the conditional asset pricing model to allow betas vary over time; Fridman (1994), Schaller and Norden (1997), Assoe (1998), and Liu et al. (2010) use the regime switching model to incorporate the market regime changes. One major distinction among these models is the variability of betas. Do betas vary over time? Or across market states, or both? There has not been a sure answer. However, a framework with a time varying feature can certainly bring more flexibility to modelling returns. As indicated by the existing literature, both the conditional asset pricing model and the regime switching model take into account the variability of betas. Nevertheless, the latter which combines the time-varying features with the state-dependent features may shed more light on this issue. On one hand, Lewellen and Nagel (2006) find that the conditional asset pricing model performs as poorly as the traditional static model. On the other hand, Fridman (1994), Schaller and Norden (1997), Assoe (1998) and Liu et al. (2011) find that equity returns exhibit strong regime switching behaviours over time.

In this study, eight iShares MSCI country ETFs are studied with reference to a set of risk factors in a regime switching framework. This thesis differs from the literature in the context of country ETFs. First, there are only a few empirical studies investigating the performance of country ETFs. Second, for the handful existing studies, few of them adopts a multifactor CAPM which takes into account factors such as the commodity index, exchange rates, and so on. Third, few of previous studies investigate the impact of some common risk factors in the U.S. on pricing the
returns of these U.S. listed foreign assets. Fourth, although some studies consider the
regime switching (RS) model, few estimate the model with a joint distribution on the
returns of country ETFs.

The remaining of the thesis is organized as follows: Chapter 2 introduces the
sampling methodology of the underlying indices and provides a brief description of
the eight ETFs in this study. Chapter 3 reviews the existing literature. Chapter 4
explains the data. Chapter 5 discusses the mathematical background of the regime
switching model. Chapter 6 discusses estimation results. Chapter 7 concludes.
Chapter 2

Background

ETFs first appeared in the early 1990s in the U.S. and later became increasingly popular all over the world. These funds are characterized by the merits of open-end trust and tradable units. As Deville (2008) put it, ETFs “combine the creation and redemption process of open-end unit trusts with the continuous stock market tradability of close-end funds”. Since that creation and redemption are allowed continuously during a trading day, a large change in demand rarely causes a large premium or discount of the price of an ETF with respect to its net asset value (NAV). In addition, the tradability of ETFs introduces much flexibility of entering or exiting investment positions.

In this thesis, I study the iShares MSCI country ETFs. These international funds are designed to replicate the performances of corresponding market indices. They are created by sampling from the security universes of the target indices. Take the iShares MSCI Canada index fund for example, the portfolio of this ETF is constructed by sampling the securities underlying the MSCI Canada index.

This chapter introduces the methodology of constructing the ETF portfolios of
the target indices\(^1\) and provides brief descriptions concerning the selected country ETFs.

## 2.1 Index Sampling

The ETFs studied in this thesis mainly track indices created by MSCI Inc.\(^2\) Hence, it is useful to introduce the methodology of selecting the securities underlying MSCI indices. Aiming to construct an index with a broad and fair market portfolio for a target market, a four-step construction process is used by MSCI Inc. to select securities.

1. Defining the candidate universe. All listed equities and equity-like securities are eligible to be included in the universe. However, cross listed equities which represent the same company are classified as belonging to the country where the company is incorporated.

2. Calculating the free float-adjusted market capitalization for each security. This free float-adjusted market capitalization is the product of the foreign inclusion factor (FIF)\(^3\) and the security’s full market capitalization.

3. Assigning each security to one industry group at each of the four levels of the Global Industry Classification Standard (GICS). The GICS consists of 10

---

1 These target indices are constructed by MSCI Inc.
2 The only exception is Russell 3000 index fund. It follows the Russell 3000 index. The securities underlying the Russell 3000 index are selected as follows: First, ranking the U.S. common stocks from the largest to smallest market capitalization; Second, choosing the top 3000 stocks as the security universe for this index.
3 FIF is calculated based on the proportion of outstanding shares purchasable for international investors in accordance with some other rules. See the file “MSCI standard index series methodology” for details, page 11-12.
sectors, 24 industry groups, 67 industries and 147 sub-industries. This four-level classification is strictly hierarchical.

(4) Selecting securities using a “bottom up” approach. Within each industry group at each level, MSCI Inc. picks the most sizeable and liquid securities and targets an 85%\(^4\) of market representation in each industry group.

2.2 Selected Country ETFs

The country ETFs in this study are created primarily based on the MSCI indices, and their shares are traded on New York Stock Exchange (NYSE). As an agreement reached between MSCI Inc. and BlackRock Fund Advisors (BFA), MSCI Inc. is responsible for constructing the underlying indices while BFA is in charge of selecting the representative securities for the ETFs. These ETFs generally invest more than 95% of their assets in the securities and depository receipts representing the securities in the underlying indices of these funds. Owing to the high management costs of perfect replications, the funds never cover all securities in underlying indices. Thus, a representative sampling indexing strategy is adopted to manage the funds.

2.2.1 The United States

The Russell 3000 index fund is adopted as the representative of the U.S. market ETF\(^5\). It tracks the Russell 3000 index, which represents approximately 98% of the investable

\(^4\)See MSCI standard index series methodology, page 14.

\(^5\)There are five ETFs representing the broad U.S. market. They are Dow Jones U.S. index fund, MSCI USA index fund, NYSE Composite index fund, Russell 3000 index fund and S&P 1500 index fund. To achieve the best diversification effect, the Russell 3000 index fund, which has the broadest market coverage and trading history, is chosen.
U.S. stock market. The fund consists of securities from the top 3000 companies listed on U.S. stock exchanges. As of July 13th, 2011, the fund is holding 2971 stocks which are worth $3.4 billion in total. There are 42.8 million shares outstanding for this ETF. The weights of sectors in this fund are shown in Table 2.1. The two most heavily weighted sectors are Financials and Technology.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Weight</th>
<th>Sector</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financials</td>
<td>16.68%</td>
<td>Technology</td>
<td>15.90%</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>13.27%</td>
<td>Energy</td>
<td>11.56%</td>
</tr>
<tr>
<td>Health Care</td>
<td>11.49%</td>
<td>Producer Durables</td>
<td>10.97%</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>7.93%</td>
<td>Utilities</td>
<td>6.03%</td>
</tr>
<tr>
<td>Material &amp; Processing</td>
<td>4.95%</td>
<td>Product Durables</td>
<td>1.02%</td>
</tr>
</tbody>
</table>

Note: Data are retrieved from the website of iShares ETFs on July 16th, 2011. This table shows the holding distribution of iShares Russell 3000 index fund on July 13th, 2011.

2.2.2 Canada

The iShares MSCI Canada index fund tracks the MSCI Canada index. It consists of stocks traded mainly on the Toronto Stock Exchange. As of July 13th, 2011, the fund is holding 102 stocks which are worth $5.7 billion in total. There are 178.9 million shares outstanding for the fund. The weights of sectors in this fund are shown in Table 2.2. The three most heavily weighted sectors are Financials, Energy and Materials.

---

6Source: File “2010 Prospectus to shareholders- iShares Russell 3000 index fund” from the website of iShares.

7Data source: http://us.ishares.com/product_info/fund/overview/IWV.htm

8Source: File “2011 Prospectus to shareholders- iShares MSCI Canada index fund” from the website of iShares.

9Data source: http://us.ishares.com/product_info/fund/overview/EWC.htm
Table 2.2: Sector breakdown for iShares MSCI Canada index fund

<table>
<thead>
<tr>
<th>Sector</th>
<th>Weight</th>
<th>Sector</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financials</td>
<td>32.22%</td>
<td>Energy</td>
<td>27.48%</td>
</tr>
<tr>
<td>Materials</td>
<td>20.62%</td>
<td>Industrials</td>
<td>5.76%</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>3.91%</td>
<td>Telecommunication Services</td>
<td>2.93%</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>2.84%</td>
<td>Information Technology</td>
<td>1.70%</td>
</tr>
<tr>
<td>Utilities</td>
<td>1.16%</td>
<td>Health Care</td>
<td>1.11%</td>
</tr>
</tbody>
</table>

Note: Data are retrieved from the website of iShares ETFs on July 16th, 2011. This table shows the holding distribution of iShares Canada index fund on July 13th, 2011.

2.2.3 United Kingdom

The iShares MSCI United Kingdom index fund tracks the MSCI United Kingdom Index, which consists of publicly traded securities primarily on the London Stock Exchange\(^\text{10}\). As of July 13th, 2011, the fund is holding 105 stocks which are worth $1.3 billion in total. There are 72.4 million outstanding shares for this fund. The weights of sectors in this fund are shown in Table 2.3\(^\text{11}\). The two most heavily weighted sectors are Financials and Energy.

Table 2.3: Sector breakdown for iShares MSCI United Kingdom index fund

<table>
<thead>
<tr>
<th>Sector</th>
<th>Weight</th>
<th>Sector</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financials</td>
<td>19.68%</td>
<td>Energy</td>
<td>19.49%</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>14.55%</td>
<td>Materials</td>
<td>14.48%</td>
</tr>
<tr>
<td>Health Care</td>
<td>8.44%</td>
<td>Telecommunication Services</td>
<td>6.77%</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>5.60%</td>
<td>Industrials</td>
<td>5.07%</td>
</tr>
<tr>
<td>Utilities</td>
<td>4.15%</td>
<td>Information Technology</td>
<td>1.02%</td>
</tr>
</tbody>
</table>

Note: Data are retrieved from the website of iShares ETFs on July 16th, 2011. This table shows the holding distribution of iShares United Kingdom index fund on July 13th, 2011.

\(^\text{10}\)Source: File “2011 Prospectus to shareholders- iShares MSCI United Kingdom index fund” from the website of iShares.

\(^\text{11}\)Data source: http://us.ishares.com/product_info/fund/overview/EWU.htm
2.2.4 Germany

The iShares MSCI Germany index fund tracks the MSCI Germany index, which consists of stocks traded primarily on the Frankfurt Stock Exchange\textsuperscript{12}. As of July 13th, 2011, the fund is holding 53 stocks which are worth $3.7 billion in total. There are 142.5 million outstanding shares for this fund. The weights of sectors in this fund are shown in Table 2.4\textsuperscript{13}. The four most heavily weighted sectors are Consumer Discretionary, Financials, Materials and Industrials.

Table 2.4: Sector breakdown for iShares MSCI Germany index fund

<table>
<thead>
<tr>
<th>Sector</th>
<th>Weight</th>
<th>Sector</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Discretionary</td>
<td>18.03%</td>
<td>Financials</td>
<td>17.35%</td>
</tr>
<tr>
<td>Materials</td>
<td>16.58%</td>
<td>Industrials</td>
<td>15.92%</td>
</tr>
<tr>
<td>Health Care</td>
<td>10.31%</td>
<td>Utilities</td>
<td>7.27%</td>
</tr>
<tr>
<td>Information Technology</td>
<td>6.21%</td>
<td>Telecommunication Services</td>
<td>4.27%</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>3.65%</td>
<td>Other</td>
<td>0.02%</td>
</tr>
</tbody>
</table>

Note: Data are retrieved from the website of iShares ETFs on July 16th, 2011. This table shows the holding distribution of iShares Germany index fund on July 13th, 2011.

2.2.5 France

The iShares MSCI France index fund tracks the MSCI France Index, which consists of stocks traded primarily on the Paris Stock Exchange\textsuperscript{14}. As of July 13th, 2011, the fund is holding 76 stocks which are worth $483.8 million in total. There are 18.8 million outstanding shares for this fund. The weights of sectors in the underlying index

\textsuperscript{12}Source: File “2011 Prospectus to shareholders- iShares MSCI Germany index fund” from the website of iShares.

\textsuperscript{13}Data source: http://us.ishares.com/product_info/fund/overview/EWG.htm

\textsuperscript{14}Source: File “2011 Prospectus to shareholders- iShares MSCI France index fund” from the website of iShares.
are shown in Table 2.5\textsuperscript{15}. The three most heavily weighted sectors are Financials, Industrials and Consumer Discretionary.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Weight</th>
<th>Sector</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financials</td>
<td>17.83%</td>
<td>Industrials</td>
<td>16.23%</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>14.99%</td>
<td>Energy</td>
<td>11.47%</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>10.23%</td>
<td>Health Care</td>
<td>8.79%</td>
</tr>
<tr>
<td>Materials</td>
<td>7.74%</td>
<td>Utilities</td>
<td>5.81%</td>
</tr>
<tr>
<td>Telecommunication Services</td>
<td>3.49%</td>
<td>Information Technology</td>
<td>3.23%</td>
</tr>
</tbody>
</table>

Note: Data are retrieved from the website of iShares ETFs on July 16th, 2011. This table shows the holding distribution of iShares France index fund on July 13th, 2011.

2.2.6 Italy

The iShares MSCI Italy index fund tracks the MSCI Italy index, which consists of stocks traded primarily on the Milan Stock Exchange\textsuperscript{16}. As of July 13th, 2011, the fund is holding 31 stocks and has a market capitalization of $122.9 million. There are 7.65 million shares outstanding for this fund. The weights of sectors in the underlying index are shown in Table 2.6\textsuperscript{17}. The three most heavily weighted sectors are Financials, Energy and Utilities.

\textsuperscript{15}Data source: http://us.ishares.com/product_info/fund/overview/EWQ.htm
\textsuperscript{16}Source: File “2011 Prospectus to shareholders- iShares MSCI Italy index fund” from the website of iShares.
\textsuperscript{17}Data source: http://us.ishares.com/product_info/fund/overview/EWI.htm
Table 2.6: Sector breakdown for iShares MSCI Italy index fund

<table>
<thead>
<tr>
<th>Sector</th>
<th>Weight</th>
<th>Sector</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financials</td>
<td>30.69%</td>
<td>Energy</td>
<td>25.37%</td>
</tr>
<tr>
<td>Utilities</td>
<td>20.64%</td>
<td>Industrials</td>
<td>9.08%</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>6.96%</td>
<td>Telecommunication Services</td>
<td>5.44%</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>1.71%</td>
<td>Other/Undefined</td>
<td>0.11%</td>
</tr>
</tbody>
</table>

Note: Data are retrieved from the website of iShares ETFs on July 16th, 2011. This table shows the holding distribution of iShares Italy index fund on July 13th, 2011.

2.2.7 Japan

The iShares MSCI Japan index fund tracks the MSCI Japan index, which consists of stocks traded primarily on the Tokyo Stock Exchange\(^{18}\). As of July 13th, 2011, the fund is holding 311 stocks which are worth $8 billion in total. There are 751.8 million shares outstanding for this fund. The weights of sectors in the underlying index are shown in Table 2.7\(^{19}\). The three most heavily weighted sectors are Industrials, Consumer Discretionary and Financials.

Table 2.7: Sector breakdown for iShares MSCI Japan index fund

<table>
<thead>
<tr>
<th>Sector</th>
<th>Weight</th>
<th>Sector</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrials</td>
<td>20.87%</td>
<td>Consumer Discretionary</td>
<td>19.86%</td>
</tr>
<tr>
<td>Financials</td>
<td>16.77%</td>
<td>Information Technology</td>
<td>12.99%</td>
</tr>
<tr>
<td>Materials</td>
<td>7.45%</td>
<td>Health Care</td>
<td>6.01%</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>5.35%</td>
<td>Telecommunication Service</td>
<td>4.49%</td>
</tr>
<tr>
<td>Utilities</td>
<td>3.78%</td>
<td>Energy</td>
<td>1.87%</td>
</tr>
</tbody>
</table>

Note: Data are retrieved from the website of iShares ETFs on July 16th, 2011. This table shows the holding distribution of iShares Japan index fund on July 13th, 2011.

\(^{18}\)Source: File “2011 Prospectus to shareholders- iShares MSCI Japan index fund” from the website of iShares.

\(^{19}\)Data source: http://us.ishares.com/product_info/fund/overview/EWI.htm
2.2.8 Australia

The iShares MSCI Australia index fund tracks the MSCI Australia index, which consists of stocks traded primarily on the Australian Stock Exchange\textsuperscript{20}. As of July 13th, 2011, the fund is holding 73 stocks which are worth $3.1 billion in total. There are 124 million shares outstanding for this fund. The weights of sectors in the underlying index are shown in Table 2.8\textsuperscript{21}. The two most heavily weighted sectors are Financials and Materials.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Weight</th>
<th>Sector</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financials</td>
<td>41.97%</td>
<td>Materials</td>
<td>29.30%</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>9.67%</td>
<td>Energy</td>
<td>6.74%</td>
</tr>
<tr>
<td>Industrials</td>
<td>4.02%</td>
<td>Health Care</td>
<td>3.12%</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>1.69%</td>
<td>Telecommunication Service</td>
<td>1.35%</td>
</tr>
<tr>
<td>Utilities</td>
<td>0.86%</td>
<td>Information Technology</td>
<td>0.42%</td>
</tr>
</tbody>
</table>

Note: Data are retrieved from the website of iShares ETFs on July 16th, 2011. This table shows the holding distribution of iShares Australia index fund on July 13th, 2011.

2.2.9 Sector Comparison among Country ETFs

Since that the returns of securities from different sectors behave differently in response to changes of common factors, studying the compositions of ETFs would help explain the behaviours of ETF returns [see Liu et al. (2011)]. Hence, I rank the sector weights to compare the differences in the compositions of these country ETFs. The rankings are reported in Table 2.9.

\textsuperscript{20}Source: File “2011 Prospectus to shareholders- iShares MSCI Australia index fund” from the website of iShares.

\textsuperscript{21}Data source: http://us.ishares.com/product_info/fund/overview/EWA.htm
As we can see from the table, consumer discretionary, energy, financials and materials are the heaviest invested sectors across ETFs. These holding distributions may provide some implications about the behaviours of ETF returns. Firstly, Germany, Japan, U.S. and France ETFs invest heavily in the consumer discretionary sector. Since that this sector mainly provides non-essential goods and services, it tends to perform well when the market is good. Thus, the performances of these four ETFs may depend on specific market regimes. Secondly, Canada, U.K. and Italy ETFs invest in energy with the heaviest weights. This indicates that changes of energy price may have a positive impact on returns of these ETFs. Thirdly, financials is the heaviest invested sector for six ETFs. The performance of these ETFs may be subject to changes of interest rate, financial market sentiment and so on. Lastly, Australia, Canada, Germany and U.K. ETFs invest heavily in materials. It implies that an increase in prices of raw materials may lead to higher returns of these ETFs.

\footnote{These ETFs are U.S., Canada, U.K., France, Italy and Australia index funds.}
Table 2.9: Comparison of sector weights across country ETFs

<table>
<thead>
<tr>
<th>Sector</th>
<th>U.S.</th>
<th>Canada</th>
<th>U.K.</th>
<th>Germany</th>
<th>France</th>
<th>Italy</th>
<th>Japan</th>
<th>Australia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Discretionary</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>7</td>
<td>7</td>
<td>3</td>
<td>9</td>
<td>5</td>
<td>7</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Energy</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>-</td>
<td>4</td>
<td>2</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>Financials</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Health Care</td>
<td>5</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>-</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Industrials</td>
<td>-</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Information Technology</td>
<td>-</td>
<td>8</td>
<td>10</td>
<td>7</td>
<td>10</td>
<td>-</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Materials</td>
<td>9</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>7</td>
<td>-</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Producer Durables</td>
<td>6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Product Durables</td>
<td>10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Technology</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Telecommunication Service</td>
<td>-</td>
<td>6</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td>6</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Utilities</td>
<td>8</td>
<td>9</td>
<td>9</td>
<td>6</td>
<td>8</td>
<td>3</td>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

Note: Each number in the table represents the ranking of the weight of a sector held by the corresponding ETF. More specifically, a smaller number stands for a heavier weight of that sector. The symbol “-” indicates that the ETF does not hold any securities from the corresponding sector.
Chapter 3

Literature Review

The portfolio optimization proposed by Markowitz (1952) implies that risk can be mitigated by diversifying across asset classes and securities. One way to achieve this diversification is allocating assets internationally. This can be done through various instruments such as open-end trust units, closed-end country funds, or country ETFs, etc. As new investment vehicles, country ETFs have become increasingly popular. This type of fund combines the characteristics of both regular exchange listed equity and a widely diversified country specific portfolio [see Adjei (2009)]. These features bring challenges to pricing the returns of the country ETFs. In this chapter, I review the risk factors and models for pricing the mean returns of assets in the existing literature.

3.1 Risk Factors

To explore the determinants of asset returns, the existing literature has considered factors such as the book-to-market ratio [Fama and French (1992, 1993); Hou et al. (2006)], debt-equity ratio [Bhandari (1988); Hou et al. (2006)], earning-to-price ratio [Basu (1977); Fama and French (1992)], size [Banz (1981); Fama and French (1992,
1993, 1996)], momentum [Jegadeesh and Titman (1993); Carhart (1997); Chordia and Shivakumar (2002)], stock market volatility [Black (1976); French et al. (1987); Glosten et al. (1993); Ghysels et al. (2005); Ang et al. (2006)], commodity [Johnson and Soenen (2009)], oil price [Chen et al. (1986); Jones and Kaul (1996), Sadorsky (1999); Basher and Sadorsky (2006)], yield spread (YS) [Chen et al. (1986); Campbell (1987); Fama and French (1989)], credit spread (CS) [Chen et al. (1986); Keim and Stambaugh (1986)], exchange rate [Solnik (1974a, 1974b); Roll (1992); Dumas and Solnik (1995); Ferson and Harvey (1999)], and the Baltic Dry index [Bakshi et al. (2010)]. The findings regarding the usefulness of these risk factors are mixed.

The book-to-market ratio, debt-equity ratio, and earning-to-price ratio are firm specific ratios. Fama and French (2004) state that “different price ratios have much the same information about expected returns”. This implies that using one price ratio may account for all price-ratio related information. In the same paper, Fama and French also confirm the findings in Fama and French (1992) that three factors (size, book-to-market and market) can explain most of the anomalies except for the momentum effect. Part of their findings contradicts Banz (1981), who finds that “there is little difference in return between average sized and large firms”. Despite of the contradiction, the Fama-French three factors prevail in subsequent studies and are applied to new investment vehicles such as sectoral ETFs. For instance, Liu et al. (2011) and Ma et al. (2011) apply the Fama-French factors to pricing the returns on sectoral ETFs in U.S. market and find that these factors exhibit strong explanatory power.

A number of previous studies have documented a significant momentum effect. Some studies find that the momentum effect is due to stock price overreaction [e.g.
Jegadeesh and Titman (1993), while some other studies find no such phenomenon [e.g. Carhart (1997)]. Chordia and Shivakumar (2002) find that the momentum effect can be explained by macroeconomic factors, which are related to the business cycle. More specifically, they find that “returns to momentum strategies are positive only during expansionary periods. During recessions, the momentum strategy returns are negative, though statistically insignificant”. Their findings imply that the regime switching model which takes into account market regime changes may capture the momentum effect.

Previous empirical findings on volatility are quite controversial: French et al. (1987) and Ghysels et al. (2005) find a positive premium of volatility on the stock market return while Glosten et al. (1993) and Ang et al. (2006) find a negative relation between the stock market return and volatility. Koulakiotis et al. (2006) find no significant relation between the stock market returns and volatility for seven OECD countries. In Liu et al. (2011), the signs of sensitivities of returns to volatility vary across sectors and market regimes. One plausible explanation for the mixed findings is that they are resulted from different estimation methodologies. For instance, Glosten (1993) extends the GARCH model used by French et al. (1987) and finds conflicting results. Moreover, the regime switching model in Liu et al. (2011) reports mixed relations between return and volatility.

Foreign exchange risk plays an important role in modelling international assets. Roll (1992) compares stock price indices across countries and finds that exchange rates play a significant role in explaining the returns of stock market indices represented by a common currency. Dumas and Solnik (1995) find evidence supporting the existence of an exchange rate risk premium. Ferson and Harvey (1999) construct a multi-factor
conditional international CAPM to explore the determinants of returns in the global market. They find that one source of improvement in CAPM comes from the excess returns on the Euro and Yen. In light of these studies, pricing the returns of country ETFs would likely require a pricing model which takes into account foreign exchange risk.

Previous studies have also identified several material-related factors in pricing equity returns. Jones and Kaul (1996) and Sadorsky (1999) find a negative impact of oil price on real stock returns. Johnson and Soenen (2009) find that the changes of Goldman Sachs commodity prices can explain a small part of variation in stock market returns. Bakshi et al. (2010) investigate the feasibility of predicting stock market returns using the Baltic dry index (BDI). They find that the growth rate of BDI is positively and also significantly related to stock market returns. All of these three factors are related to the changes of oil price. Using one factor may account for the impacts of other factors on equity returns. Since that commodity price index not only takes into account the value of oil products, but also encompasses other raw materials such as metals, softs, etc, it may be able to explain a larger portion of equity return than the other two factors.

The literature has also examined interest related factors such as YS and CS. Chen et al. (1986) find that stock returns are negatively related to YS whereas some other studies [e.g. Campbell (1987); Fama and French (1989)] document positive risk premiums on YS. Keim and Stambaugh (1986) find a positive risk premium on CS while Fama and French (1993) find that CS factor is not significant. The conflicting

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1As an indicator of ocean transportation costs, the Baltic dry index changes simultaneously in response to the changes of oil price. Oil products account for more than 60% (Data source: S&P indices website. Link: http://www.standardandpoors.com/indices/sp-gsci/en/us) of the S&P Goldman Sachs Commodity Index (GSCI) commodity index in value.
findings above may be resulted from differences on sample periods. As found by Liu et al. (2011), the sensitivities of returns to YS and CS factors vary across market regimes. Since that different sample periods indicate different market conditions, it is reasonable to observe the loadings on YS and CS with different signs when the studied time span differs.

### 3.2 Existing Models

Based on Markowitz’s (1952) work, Sharpe (1964), Lintner (1965) and Mossin (1966) develop the Capital Asset Pricing Model (CAPM). The CAPM provides a sound and simple way to price asset returns based on their risk, however, the model suffers various criticisms relating to its underlying assumptions.

Criticism comes from Roll (1977) states that finding a portfolio covering all investable assets is extremely hard if not impossible. To tackle this limitation of CAPM, Ross (1976) proposes the Arbitrage Pricing Theory (APT), which brings more flexibility to asset pricing. Inspired by Ross’s work, studies henceforth such as Chen et al. (1986) and Fama and French (1992) use multifactor models to explain the anomalies unsolvable by the original CAPM. Their findings show that the multifactor model has stronger explanatory power than the original CAPM.

The other criticism on the original CAPM is related to the non-time-varying feature of betas. As pointed out by Bos and Newbold (1984), betas may not be constant over time. Their finding implies that a pricing model with time-varying features may shed more light on asset pricing. To incorporate time varying features into asset pricing, Ferson and Harvey (1991) propose a two-step conditional asset pricing model. Following their work, studies such as Jagannathan and Wang (1996)
and Ferson and Harvey (1998) find that the conditional model is more convincing than the traditional model with constant betas.

The feature embedded in the conditional asset pricing model is certainly appealing. However, Ghysels (1998) and Lewellen and Nagel (2006) point out that it is not superior to the traditional model with constant betas. This unfavourable evidence certainly does not disprove the superiority of the time-varying betas. Following the pioneering work of Hamilton (1989), researchers such as Fridman (1994), Schaller and Norden (1997), Assoe (1998) and Liu et al. (2011) use regime switching models to analyse stock return and find strong evidence of switching behaviours and model predictability. Moreover, Fridman (1994) also finds that the regime switching model outperforms the conditional CAPM. Therefore, utilizing the RS model may shed more light on asset pricing.
Chapter 4

Mathematical Background

This chapter discusses the specification and basic properties of the RS model, parameter estimation using the EM algorithm, the method of selecting the optimal number of regimes, standard errors’ estimation using parametric bootstrap method and future returns forecast. The mathematical specification is based on Zucchini and MacDonald (2009).

4.1 Specification

The regime switching factor model\(^1\) can be written as:

\[
R_t = Z_t \beta_{s_t} + \Sigma_{s_t} U_t,
\]

where,

\[
R_t = [R_{1t}, R_{2t}, \ldots, R_{Nt}]'
\]

is a vector of the returns on \(N\) ETFs;

\(^1\)In this thesis, it is assumed that all returns share the same market regimes. Thus the specification of the RS model is multivariate.
$Z_t$ is a matrix which has the same set of $K$ variables in each row in period $t^2$:

$$
Z_t = \begin{bmatrix}
Z_{1t} & Z_{2t} & \ldots & Z_{Kt} & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\
0 & 0 & \ldots & 0 & Z_{1t} & Z_{2t} & \ldots & Z_{Kt} & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & Z_{1t} & Z_{2t} & \ldots & Z_{Kt}
\end{bmatrix}_{N \times NK} 
$$

$s_t$ is the market regime in period $t$, when $s_t = j, j \in \{1, 2, \ldots, M\}$;

$\beta_{s_t=j} = [\beta_{11j}, \beta_{12j}, \ldots, \beta_{1Kj}, \beta_{21j}, \beta_{22j}, \ldots, \beta_{2Kj}, \ldots, \beta_{N1j}, \beta_{N2j}, \ldots, \beta_{NKj}]_{1 \times NK}$ is a column vector of coefficients conditional on the market regime in period $t$;

$U_t = [u_{1t}, u_{2t}, \ldots, u_{Nt}]'$ is a vector of error terms$^3$; $U_t \sim N(0, I)$, and $I$ is an identity matrix;

$\Sigma_{s_t}$ is a variance covariance matrix conditional on the market regimes. More specifically,

$$
\Sigma_{s_t} = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \ldots & \sigma_{1N} \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{N1} & \sigma_{N2} & \ldots & \sigma_{NN}
\end{bmatrix}_{N \times N}
$$

### 4.2 Basics of the Regime Switching Model

Let $X_t = \Sigma_{s_t}U_t = R_t - Z_t\beta_{s_t}$, which satisfies the Markov properties.

$$
\Pr(S_t|S^{(t-1)}) = \Pr(S_t|S_{t-1}) \tag{4.2.1}
$$

$$
\Pr(X_t|X^{(t-1)}, S^{(t)}) = \Pr(X_t|S_t) \tag{4.2.2}
$$

where $X^{(t-1)} = [X_1, X_2, \ldots, X_{t-1}]$, $S^{(t)} = [S_1, S_2, \ldots, S_t]$.

---

$^2$ $Z_{1t}$ is unity.

$^3$ Here I assume that there is no autocorrelation for $u_{it}$ for all $i$. However, $u_{it}$ and $u_{jt}$ can be correlated for all $i, j = 1, 2, \ldots, n$. 

The model has two essential parts: first, the probability distribution of current state only depends on the state in the previous period \((S_{t-1})\); second, the probability distribution of \(X_t\) only depends on \(S_t\).

**The likelihood function**

The general form of likelihood function is defined as follows:

\[
L_T = \Pr(X^{(T)} = x^{(T)}) = \sum_{s_1, s_2, \ldots, s_T = 1} P \left( X^{(T)} = x^{(T)}, S^{(T)} = s^{(T)} \right)
\]

\[
= \sum_{s_1, s_2, \ldots, s_T = 1} \left[ \delta_{s_1} \prod_{t=2}^{T} \gamma_{s_{t-1}, s_t} \prod_{t=1}^{T} p_{s_t}(x_t) \right]
\]

\[
= \delta \mathbf{P}(x_1) \Gamma \mathbf{P}(x_2) \ldots \Gamma \mathbf{P}(x_T) \mathbf{1}',
\]

where \(\delta\) is a row vector of the initial probability distribution of all states; \(\gamma_{s_{t-1}, s_t}\) represents the transition probability from state \(s_{t-1}\) to state \(s_t\); \(p_{s_t}(x_t) = \Pr(X_t = x_t|s_t)\); and \(\mathbf{1} = [1, 1, \ldots, 1]_{1 \times M}\).

\[
\mathbf{P}(x_t) = \begin{bmatrix}
    p_1(x_t) & 0 & \ldots & 0 \\
    0 & p_2(x_t) & \ldots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \ldots & p_M(x_t)
\end{bmatrix}
\]

for all \(t = 1, 2, \ldots, T\).

The transition probability matrix \(\Gamma\) is defined as follows:

\[
\Gamma = \begin{bmatrix}
    \gamma_{11} & \cdots & \gamma_{1M} \\
    \vdots & \ddots & \vdots \\
    \gamma_{M1} & \cdots & \gamma_{MM}
\end{bmatrix},
\]

where
\[ \sum_{i=1}^{M} \gamma_{ji} = 1 \] for all \( i, j = 1, 2, \ldots, M \).

**Forward and backward probabilities**

Two probabilities are needed for computing the conditional expectations of the EM algorithm that will be explained later in the thesis. Firstly, the forward probability is defined as follows:

\[
\alpha_t = \delta P(x_1) \Gamma P(x_2) \ldots \Gamma P(x_t) \nonumber
\]
\[
= \delta P(x_1) \prod_{\tau=2}^{t} \Gamma P(x_{\tau}). 
\tag{4.2.4}
\]

The \( j \)th component of \( \alpha_t \) is \( \alpha_t(j) \), which satisfies

\[
\alpha_{t+1}(j) = \left( \sum_{i=1}^{M} \alpha_t(i) \gamma_{ij} \right) p_j(x_{t+1}). \tag{4.2.5}
\]

Using equation (4.2.5), we can derive the following equation by induction.

\[
\alpha_t(j) = \Pr(X^{(t)} = x^{(t)}, s_t = j). \tag{4.2.6}
\]

Secondly, the backward probability is defined as follows:

\[
\rho_t' = P(x_{t+1}) \Gamma P(x_{t+2}) \ldots \Gamma P(x_T) 1' 
\]
\[
= \left( \prod_{\tau=t+1}^{T} \Gamma P(x_{\tau}) \right) 1'. 
\tag{4.2.7}
\]

The \( j \)th component of \( \rho_t \) equals to

\[
\rho_t(j) = \Pr(X_{t+1}^{T} = x_{t+1}^{T} | s_t = j), \tag{4.2.8}
\]

where \( X_{t+1}^{T} \) denotes the vector \([X_{t+1}, X_{t+2}, \ldots, X_T]\). This equation can be derived by induction.

Combining equation (4.2.6) and equation (4.2.8), we can infer

\[
\alpha_t(j) \rho_t(j) = \Pr(X^{(T)} = x^{(T)}, s_t = j). \tag{4.2.9}
\]
Following equation (4.2.9), we can infer

\[ \alpha_t \rho_t = \Pr(X^{(T)} = x^{(T)}) = L_T. \] (4.2.10)

and conditional state probability

\[ \Pr(s_t = j \mid X^{(T)} = x^{(T)}) = \alpha_t(j) \rho_t(j) / L_T \] (4.2.11)

The conditional transition probability can be written as follows:

\[ \Pr(s_{t-1} = j, s_t = i \mid X^{(T)} = x^{(T)}) = \alpha_{t-1}(j) \gamma_{ji} \rho_t(x_t) \rho_t(i) / L_T. \] (4.2.12)

### 4.3 Parameter Estimation

We construct the complete data log-likelihood (CDLL) of the observations \( x_1, x_2, \ldots, x_T \) and the missing data \( s_1, s_2, \ldots, s_T \) to estimate all parameters in the model. The CDLL is derived as follows:

\[
\log[\Pr(X^{(T)} = x^{(T)}, S^{(T)} = s^{(T)})] = \log \left( \delta_{s_1} \prod_{t=2}^{T} \gamma_{s_{t-1},s_t} \prod_{t=1}^{T} p_{st}(x_t) \right) \\
= \log \delta_{s_1} + \sum_{t=2}^{T} \log \gamma_{s_{t-1},s_t} + \sum_{t=1}^{T} \log p_{st}(x_t). \] (4.3.1)

In this setup, there are three sets of parameters:

1. The initial probability of state \( j \): \( \{\delta_{s_1}\} \)

---

\(^4\)The following two equations are needed to derive this function:

(1) \( \Pr(X^T_1, s_t, s_{t+1}) = \Pr(X^T_1, s_t) \Pr(s_{t+1} \mid s_t) \Pr(X^T_{t+1} \mid s_{t+1}) \);

(2) \( \Pr(X^T_{t+1} \mid s_{t+1}) = \Pr(X_{t+1} \mid s_{t+1}) \Pr(X^T_{t+2} \mid s_{t+1}) \).

See appendix B in Zucchini and MacDonald (2009) for details.

\(^5\)\( s_t \) is a latent random variable.
2. Transition probabilities: \( \{\gamma_{st-1, st}\} \)

3. Variance covariance matrix \( \{\Sigma_{st=j}\} \) and betas \( \{\beta_{st=j}\} \)

We estimate these parameters by maximizing the CDLL. As proposed by Hamilton (1989), the likelihood function can be maximized either through numerical maximization or the expectation-maximization (EM) algorithm. In this thesis, the EM algorithm is adopted.

First, we need to define two zero-one random variables. (1) One represents the sequence of state \( \{s_1, s_2, \ldots, s_t\} \): \( u_j(t) = 1 \) if and only if \( s_t = j \). (2) The other represents the transition from one state to the next in the next period: \( v_{ji}(t) = 1 \) if and only if \( s_{t-1} = j \) and \( s_t = i \), \( t = 1, 2, \ldots, T \) and \( i, j = 1, 2, \ldots, M \).

Then, the CDLL can then be written as

\[
\log[\Pr(X^{(T)} = x^{(T)}, S^{(T)} = s^{(T)})] = \sum_{j=1}^{M} u_j(1) \log \delta_j + \sum_{j=1}^{M} \sum_{i=1}^{M} \left( \sum_{t=2}^{T} v_{ji}(t) \right) \log \gamma_{ji} + \sum_{j=1}^{M} \sum_{t=1}^{T} u_j(t) \log p_j(x_t).
\]

**E step**

1. Assign initial values for all parameters \( \{\hat{\delta}_j\}, \{\hat{\gamma}_{ji}\}, \{\hat{\Sigma}_{st=j}\} \) and \( \{\hat{\beta}_{st=j}\} \) for all \( i, j = 1, 2, \ldots, M \).

2. Use the initial values of the parameters to compute the conditional expectations of \( v_{ji}(t) \) and \( u_j(t) \):

\[
\hat{u}_j(t) = \Pr(s_t = j|x^{(T)}) = \alpha_t(j)\rho_t(j)/L_T
\]

\[
\hat{v}_{ji}(t) = \Pr(s_{t-1} = j, s_t = i|x^{(T)}) = \alpha_{t-1}(j)\gamma_{ji}p_i(x_t)\rho_t(i)/L_T.
\]

**M step**
1. Replace \( v_{ji}(t) \) and \( u_{ij}(t) \) by \( \hat{v}_{ji}(t) \) and \( \hat{u}_{ij}(t) \) in the CDLL.

2. Maximize the CDLL w.r.t those three sets of parameters. We can split this process into three separate maximizations.

First, the term \( \sum_{j=1}^{M} \hat{u}_{j}(1) \log \delta_{j} \) depends only on \( \{\delta_{j}\} \). The solution is

\[
\delta_{j} = \hat{u}_{j}(1) / \sum_{j=1}^{M} \hat{u}_{j}(1) = \hat{u}_{j}(1).
\]

Second, the term \( \sum_{j=1}^{M} \sum_{i=1}^{M} \left( \sum_{t=2}^{T} \hat{v}_{ji}(t) \right) \log \gamma_{ji} \) depends only on \( \gamma_{ji} \). The solution is

\[
\gamma_{ji} = \frac{f_{ji}}{\sum_{i=1}^{M} f_{ji}},
\]

where \( f_{ji} = \sum_{t=2}^{T} \hat{v}_{ji}(t) \).

Third, the term \( \sum_{j=1}^{M} \sum_{t=1}^{T} \hat{u}_{j}(t) \log p_{j}(R_{t} - Z_{t}\beta_{st=j}) \) depends only on \( \{\Sigma_{st=j}\} \) and \( \{\beta_{st=j}\} \). It can be written as follows:

\[
\text{Term 3} = \sum_{j=1}^{M} \sum_{t=1}^{T} [\hat{u}_{j}(t) \log p_{j}(R_{t} - Z_{t}\beta_{st=j})]
\]

\[
= \sum_{j=1}^{M} \sum_{t=1}^{T} \hat{u}_{j}(t) \log \left( \frac{1}{(2\pi)^{n/2} |\Sigma_{st=j}|^{1/2}} e^{-\frac{1}{2} [R_{t} - Z_{t}\beta_{st=j}]^{T} \Sigma_{st=j}^{-1} [R_{t} - Z_{t}\beta_{st=j}]} \right)
\]

\[
= \sum_{j=1}^{M} \sum_{t=1}^{T} \hat{u}_{j}(t) \left( -\frac{n}{2} \log 2\pi - \frac{n}{2} \log |\Sigma_{st=j}| - \frac{1}{2} [R_{t} - Z_{t}\beta_{st}]^{T} \Sigma_{st=j}^{-1} [R_{t} - Z_{t}\beta_{st}] \right)
\]

(4.3.2)

This maximization problem can be solved numerically. Up until now, we have finished one round of the EM algorithm. We use these estimated parameters \( \{\hat{\delta}_{j}\}, \{\hat{\gamma}_{ji}\}, \{\hat{\Sigma}_{st=j}\} \) and \( \{\hat{\beta}_{st=j}\} \) as new initial values and repeat the EM steps. We keep doing this until the changes of all parameters are within a predetermined threshold.
4.4 Selection of Regimes

Given the factor model, an increase of the number of states will increase the parameters to be estimated exponentially. Although a model with more states may give a better fit for the data, we wish to find a parsimonious model. Hence, we should adopt a criterion to select an appropriate number of states.

In this thesis, the Bayesian information criterion (BIC) is used to choose the optimal number of states $M$. Let $N$ be the number of dependent variables, $K$ be the number of coefficients (alphas and betas) and $T$ be the number of observations. Then, total number of parameters to be estimated is $M^2 + (1 + K + N) * N * M - M$. Therefore, the BIC is calculated as follows:

$$BIC = [M^2 + (1 + K + N) * N * M - M]ln(T) - 2ln(L_T)$$ (4.4.1)

4.5 Bootstrap Parameters

The EM algorithm adopted in this study has a drawback. That is, it does not generate a probability distribution of the estimated parameters. Hence, we cannot conduct any statistical inference on these parameters. To tackle this limitation, a bootstrap method is adopted in this thesis.

This method is done in four steps: First, simulate a sequence of returns\textsuperscript{6} using the parameters estimated by the EM algorithm; second, use the newly generated returns to replace the original returns and estimate the parameters based on the new generated returns; third, repeat steps one and two many times to get a probability distribution for each parameter; and forth, use the empirical probability distributions

\textsuperscript{6}It refers to the returns of ETFs only.
to compute the statistics.

4.6 Forecasting

Predictive power is essential for any model. Thus, it is useful to examine the performance of the RS model on predicting future returns. In this thesis, the one period ahead prediction is calculated as follows:

$$E(R_{t+1}|s_t = j) = \sum_{i=1}^{M} Z_{t,i} \hat{\beta}_{s_t = i} \hat{\gamma}_{ji}$$  \hspace{1cm} (4.6.1)
Chapter 5

Data Description

This chapter discusses the dependent variables (returns of eight country ETFs) and independent variables (eight potential risk factors). The dataset covers the daily data of these variables from May 31, 2000 to March 31, 2011. There are 2603 observations for each variable in the dataset.

5.1 Returns of Country ETFs

In order to diversify investment globally, investors can either invest in foreign assets directly or buy the shares of international funds. The former generates high costs\footnote{These high costs are from two sources. First, the costs of collecting adequate information for specific assets are high for individual investors. Second, transaction costs of investing directly in foreign assets are also high.} that eliminate the diversification benefits while the latter can help achieve the diversification with low costs. In this regard, investing in international funds may be a better choice. Among those international funds, passively managed country ETFs invest in specific market indices and provide investors with low costs and sound international diversification opportunities. In this study, I choose to study the returns on country ETFs for international diversification while considering the market regime.
Since there are a number of country ETFs in the market, selection criterion should be set. First, all country ETFs should be managed by the same company to maintain portfolio consistency. Second, trading history must be long enough to ensure the accuracy of estimating the parameters. Third, the funds must be liquid in the sense that the ETFs shares are actively traded.

Among all country ETFs existing in the market, only iShares international index funds satisfy the above conditions. Because that the trading histories of emerging market ETFs are quite short, I choose eight developed market ETFs which account for a substantial portion of the global market capitalization in total. The eight country ETFs are for the United States (US), Canada (CA), United Kingdom (UK), Germany (GER), France (FRA), Italy (ITA), Australia (AUS), and Japan (JAP). Summary statistics are reported in Table 5.1. Among all pairs of country ETFs, CA and JAP ETFs have the lowest correlation (0.5692) while FRA and GER ETFs have the highest correlation (0.8935). These correlations indicate that investing in a portfolio of country ETFs can mitigate risk.

### 5.2 Explanatory Variables

As implied by the CAPM, asset returns are systematically related to overall market returns. Thus, the return of the total market should be priced into asset returns. In

---

2 These eight countries account for 76.52% of the MSCI all country world investable market index (ACWI IMI) in value. This index is designed to capture up to 99% of the developed and emerging investable market universe. See file “ACWI IMI factsheet” on the website of MSCI Inc.

3 The daily closing prices are retrieved from Bloomberg, which dates from May 30, 2000 to March 31, 2011. The natural logarithm difference is taken to get the daily returns. Hence, we have the daily returns from May 31, 2000 to March 31, 2011.
Table 5.1: Summary statistics of country ETFs

<table>
<thead>
<tr>
<th>Country</th>
<th>Mean</th>
<th>St.d</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>0.000009</td>
<td>0.0139</td>
<td>-0.2070</td>
<td>6.4486</td>
</tr>
<tr>
<td>Canada</td>
<td>0.000321</td>
<td>0.0167</td>
<td>-0.5810</td>
<td>5.7893</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.000023</td>
<td>0.0168</td>
<td>-0.2263</td>
<td>9.3719</td>
</tr>
<tr>
<td>Germany</td>
<td>0.000020</td>
<td>0.0187</td>
<td>-0.0279</td>
<td>7.8829</td>
</tr>
<tr>
<td>France</td>
<td>-0.000009</td>
<td>0.0180</td>
<td>-0.1275</td>
<td>5.6592</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.000108</td>
<td>0.0194</td>
<td>-0.2417</td>
<td>6.3124</td>
</tr>
<tr>
<td>Australia</td>
<td>0.000391</td>
<td>0.0194</td>
<td>-0.4023</td>
<td>7.5200</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.000137</td>
<td>0.0163</td>
<td>0.1624</td>
<td>6.1652</td>
</tr>
</tbody>
</table>

Note: These are returns of country ETFs. The daily closing prices from May 30, 2000 to March 31, 2011 are retrieved from Bloomberg. The returns are calculated by differencing the natural logarithm of daily prices. The standard deviations vary across these ETFs. The excess kurtosis is high for all ETFs returns.

Table 5.2: Correlations of returns on country ETFs

<table>
<thead>
<tr>
<th>Country</th>
<th>U.S.</th>
<th>CA</th>
<th>U.K.</th>
<th>GER</th>
<th>FRA</th>
<th>ITA</th>
<th>AUS</th>
<th>JAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CA</td>
<td>0.7113</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>0.7829</td>
<td>0.6886</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GER</td>
<td>0.7933</td>
<td>0.6692</td>
<td>0.8163</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRA</td>
<td>0.7859</td>
<td>0.6957</td>
<td>0.8386</td>
<td>0.8935</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ITA</td>
<td>0.7188</td>
<td>0.6725</td>
<td>0.7920</td>
<td>0.8274</td>
<td>0.8690</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUS</td>
<td>0.6765</td>
<td>0.6770</td>
<td>0.7026</td>
<td>0.6875</td>
<td>0.7194</td>
<td>0.6939</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>JAP</td>
<td>0.6958</td>
<td>0.5692</td>
<td>0.6606</td>
<td>0.6724</td>
<td>0.6561</td>
<td>0.5977</td>
<td>0.6134</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Note: The returns are calculated by differencing the natural logarithm of daily prices of country ETFs. The daily closing prices from May 30, 2000 to March 31, 2011 are retrieved from Bloomberg. These returns are positively correlated.
the thesis, the MSCI All Country World Investable Market Index (WOD) is adopted as the proxy of the world stock market. This index covers over 9,000 securities across large, mid and small cap size segments and across style and sector segments in 45 developed and emerging stock markets.\textsuperscript{4}

Some studies [Roll (1992); Dumas and Solnik (1995); Ferson and Harvey (1999)] find that exchange rates play an important role in pricing foreign assets. According to these findings, exchange rate should be able to explain the returns of U.S. listed country ETFs. In this thesis, I choose the U.S. dollar index\textsuperscript{5} (DXY) as the proxy for exchange rates targeting U.S. dollars. This index measures the value of the U.S. dollar against a basket of foreign currencies. An increase of the index indicates that the U.S. dollar appreciates against other currencies.

The existing literature has also documented significant risk premiums on material-related factors such as the commodity index\textsuperscript{6}, oil price and Baltic dry index [Chen et al. (1986); Johnson and Soenen (2009); Bakshi et al. (2010)]. In this thesis, I use the return on S&P Goldman Sachs Commodity Index\textsuperscript{7} (COM) instead of the changes of oil price and the Baltic dry index. This can be justified by discussions in previous chapters. Firstly, as discussed in Chapter 3, commodity index may capture more information and explain a larger portion of ETF returns than the other two factors. Secondly, the compositions of these ETFs indicate that they are potentially exposed

\textsuperscript{4}See file “ACWI IMI factsheet” on the website of MSCI Inc. The daily closing prices from May 30, 2000 to March 31, 2011 are retrieved from Bloomberg. Natural logarithm difference is computed to get its returns from May 31, 2000 to March 31, 2011.

\textsuperscript{5}The daily closing prices, which date from May 30, 2000 to March 31, 2011, are retrieved from Bloomberg. The natural logarithm difference is computed to get its change rates.

\textsuperscript{6}Johnson and Soenen (2009) use the S&P Goldman Sachs Commodity Index.

\textsuperscript{7}It is a world production weighted index based on the quantity of production of each commodity. The daily closing prices are retrieved from Bloomberg. The natural logarithm difference is computed to get its change rates.
to changes of commodity price [see Section 2.2.9 and Table 2.9].

As found by Frankel (1993), Chang et al. (1995), Russell (1998) and Gutierrez (2009), U.S. exchange traded foreign assets exhibit significant exposure to the U.S. market factor and behave like U.S. securities. I suspect that the U.S. exchange listed country ETFs may also be exposed to some U.S. common risk factors. Hence, U.S. factors such as size and value, market volatility, yield spread and credit spread are considered.

The Fama-French factors—size and value—have documented tremendous success in asset pricing [see Fama and French (1992, 1993, 1996, 1998)]. “Small minus big” (SMB) for size is the difference between the returns of small capitalization portfolio and big capitalization portfolio. It is computed as the average return on three small portfolios minus the average return on three big portfolios. “High minus low” (HML) for value is the difference between the returns of portfolio with value stocks and portfolio with growth stocks. It is computed as the average return on two value portfolios minus the average return on two growth portfolios.

\[
\text{SMB} = \frac{1}{3} (\text{Small Value} + \text{Small Neutral} + \text{Small Growth}) - \frac{1}{3} (\text{Big Value} + \text{Big Neutral} + \text{Big Growth})
\]

\[
\text{HML} = \frac{1}{2} (\text{Small Value} + \text{Big Value}) - \frac{1}{2} (\text{Small Growth} + \text{Big Growth})
\]

These two factors in this thesis are created based on returns of U.S. stocks. The SMB and HML data are retrieved from Kenneth R. French data library\(^8\).

Empirical studies record controversial results regarding to the risk premium on stock market volatility [e.g. Ghysels et al. (2005) versus Ang et al. (2006)]. Liu

\(^8\)Link: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html.
et al. (2011) reach a more comprehensive understanding of the sensitivity of equity returns to volatility. They find that the risk premium on volatility can be different in magnitudes and signs depending on specific market conditions. Following Liu et al. (2011), I use the Chicago board options exchange volatility index\(^9\) (VIX) as the proxy for market volatility.

In Chapter 3, the discussion on factors YS and CS suggests that they may be used to predict equity returns in the RS model. The existing studies on these two factors are conducted on U.S. securities. Whether YS and CS factors can be applied to pricing the returns on U.S. listed foreign assets (country ETFs) remains unanswered. Moreover, Liu et al. (2011) incorporate them into RS model to explore the returns on sector ETFs and find significant risk premiums. These motivate me to investigate the feasibility of incorporating YS and CS factors into pricing country ETF returns. The YS factor is the difference between the 20-year U.S. Treasury bond and the 3-month U.S. Treasury bill and the CS factor is the difference between return of Moody’s Baa bond and return of Moody’s Aaa bond.\(^{10}\)

The summary statistics for these eight factors are reported in Table 5.3. The correlations for country ETFs and risk factors are reported in Table 5.2 and 5.4.

---

\(^9\)The daily closing prices from May 30, 2000 to March 31, 2011 are retrieved from the website of the Chicago Board Options Exchange. The natural logarithm difference is used to get its change rates.

\(^{10}\)The data are retrieved from the Federal Reserve at St. Louis Economic research data centre. Link: http://research.stlouisfed.org/fred2.
Table 5.3: Summary statistics of risk factors

<table>
<thead>
<tr>
<th>Factor</th>
<th>Mean</th>
<th>St.d</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMB</td>
<td>0.026873</td>
<td>0.5938</td>
<td>-0.1021</td>
<td>3.3689</td>
</tr>
<tr>
<td>HML</td>
<td>0.024211</td>
<td>0.6886</td>
<td>-0.0428</td>
<td>5.9505</td>
</tr>
<tr>
<td>WOD</td>
<td>0.000027</td>
<td>0.0117</td>
<td>-0.1467</td>
<td>10.2599</td>
</tr>
<tr>
<td>VIX</td>
<td>-0.000110</td>
<td>0.0623</td>
<td>0.6503</td>
<td>4.1504</td>
</tr>
<tr>
<td>DXY</td>
<td>-0.000139</td>
<td>0.0056</td>
<td>-0.0336</td>
<td>1.3511</td>
</tr>
<tr>
<td>COM</td>
<td>0.000445</td>
<td>0.0169</td>
<td>0.0259</td>
<td>4.3720</td>
</tr>
<tr>
<td>YS</td>
<td>2.511014</td>
<td>1.6193</td>
<td>-0.4980</td>
<td>-1.3048</td>
</tr>
<tr>
<td>CS</td>
<td>1.131690</td>
<td>0.5200</td>
<td>2.6506</td>
<td>7.2500</td>
</tr>
</tbody>
</table>

Note: The SMB and HML are retrieved from the Kenneth R. French data library. WOD, DXY, COM are from Bloomberg. VIX is from the Chicago Board Option Exchange website. YS and CS are from the Federal Reserve at St. Louis Economic Research Data Centre. The data of these eight factors are from May 31, 2000 to March 31, 2011.

Table 5.4: Correlations of risk factors

<table>
<thead>
<tr>
<th>Factor</th>
<th>SMB</th>
<th>HML</th>
<th>WOD</th>
<th>VIX</th>
<th>DXY</th>
<th>COM</th>
<th>YS</th>
<th>CS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMB</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HML</td>
<td>-0.1073</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WOD</td>
<td>-0.0295</td>
<td>0.1025</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VIX</td>
<td>-0.0702</td>
<td>-0.0437</td>
<td>-0.6541</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DXY</td>
<td>0.0272</td>
<td>-0.1251</td>
<td>-0.2492</td>
<td>0.0272</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COM</td>
<td>-0.0428</td>
<td>0.1601</td>
<td>0.3283</td>
<td>-0.1532</td>
<td>-0.2731</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YS</td>
<td>0.0321</td>
<td>-0.0243</td>
<td>0.0099</td>
<td>-0.0198</td>
<td>-0.0073</td>
<td>0.0041</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>CS</td>
<td>0.0078</td>
<td>-0.0521</td>
<td>-0.0121</td>
<td>-0.0168</td>
<td>0.0003</td>
<td>-0.0326</td>
<td>0.3608</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Note: The SMB and HML are retrieved from the Kenneth R. French data library. WOD, DXY, COM are from Bloomberg. VIX is from the Chicago Board Option Exchange website. YS and CS are from the Federal Reserve at St. Louis Economic Research Data Centre. The data of these eight factors are from May 31, 2000 to March 31, 2011.
Chapter 6

Empirical Analysis

6.1 Model Selection

The preliminary analysis suggests that six factors (SMB, HML, WOD, VIX, DXY, COM) are significant in almost all eight regressions of the linear model. Hence, I use these six factors as the starting point to select the appropriate number of regimes and factors for the RS model. In this selection process, BIC is used to identify the optimal number of risk factors and market regimes\(^1\). The values of BIC are reported in Table 6.1. As we can see from the table, the model with six factors and three states has the lowest BIC value. The remaining of this chapter discusses the results of this three state six-factor model.

\(^1\)As discussed in Chapter 4, the BIC is computed based on both the number of regimes and risk factors.
Table 6.1: Values of Bayesian information criterion

<table>
<thead>
<tr>
<th>Number of Regimes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Six-factor</td>
<td>52870</td>
<td>48522</td>
<td><strong>48082</strong></td>
<td>48194</td>
<td>48608</td>
</tr>
<tr>
<td>Six-factor+YS</td>
<td>52932</td>
<td>48640</td>
<td>48256</td>
<td>48424</td>
<td>48912</td>
</tr>
<tr>
<td>Six-factor+CS</td>
<td>52931</td>
<td>48641</td>
<td>48255</td>
<td>48420</td>
<td>48888</td>
</tr>
<tr>
<td>Six-factor+YS+CS</td>
<td>52993</td>
<td>48761</td>
<td>48432</td>
<td>48651</td>
<td>49180</td>
</tr>
</tbody>
</table>

Note: Each element stands for the value of the Bayesian Information Criterion. The model in each row has different number of dependent variables. For instance, “Six-factor+YS” stands for the model with seven factors SMB, HML, WOD, VIX, DXY, COM and YS.

6.2 Transition Probability

The estimated transition probabilities across regimes are given below:

\[
\begin{bmatrix}
0.979290(1.12E-141) & 0.000671(6.97E-42) & 0.020038(1.12E-06) \\
0.004464(1.70E-53) & 0.974668(1.87E-134) & 0.020868(6.34E-40) \\
0.076179(6.78E-06) & 0.046948(5.81E-40) & 0.876872(1.38E-230)
\end{bmatrix}
\]

where p-value for each transition probability is in the parentheses. The element in \(j\)th row and \(i\)th column represents the transition probability from regime \(j\) \((j = 1, 2, 3)\) to regime \(i\) \((i = 1, 2, 3)\). As we can see from main diagonal of the matrix, all three regimes are highly persistent. More specifically, regime 1 is most persistent while regime 3 is least persistent.

6.3 Interpretation of Market Regimes

We interpret the market regimes by evaluating the performance of the first and second moments of ETF returns and the mean risk factors across regimes. These statistics are reported in Tables 6.2, 6.3 and 6.5. Regime 1 generates the highest average returns for all these ETFs while regime 3 generates the lowest returns [see Table 6.2]. For
all ETFs, the returns are least volatile in regime 1 and most volatile in regime 3 [see Table 6.5]. These results indicate that regime 1 is the calmest with highest returns (positive) while regime 3 is the most volatile one with lowest returns (negative). Furthermore, the mean WOD factor is the highest in regime 1 and lowest in regime 3. The WOD and VIX factors move in opposite directions in these two regimes [see Table 6.3]. Therefore, we can label regime 1 as the “bull” market and regime 3 as the “bear” market. The statistics indicate that regime 2 acts as an intermediate state between regime 1 and regime 3. Hence, we label it as the “transitory” market. These results are consistent with the findings based the data of the same period [see Liu et al. (2011)].

Table 6.4 reports the correlations among these country ETF returns across regimes. It is interesting to see that the correlations tend to be high in the bull and bear markets while they are relatively low in the transitory market. That is, these ETF returns are more closely correlated when the market has an upward/downward trend and less closely correlated when the market is in the transition between the bull and bear markets.
Table 6.2: Mean returns of country ETFs across regimes

<table>
<thead>
<tr>
<th>Country</th>
<th>Regime 1</th>
<th>Regime 2</th>
<th>Regime 3</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-0.002496</td>
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<td>FRA</td>
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<td>ITA</td>
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<td>0.000131</td>
<td>-0.003582</td>
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<td>JAP</td>
<td>0.000254</td>
<td>-0.000410</td>
<td>-0.001363</td>
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<tr>
<td>AUS</td>
<td>0.000923</td>
<td>0.000704</td>
<td>-0.002811</td>
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</table>

Note: The mean returns for all ETFs in regime 1 are the highest while the counterparts in regime 3 are the lowest.

Table 6.3: Mean risk factors across regimes

<table>
<thead>
<tr>
<th>Country</th>
<th>Regime 1</th>
<th>Regime 2</th>
<th>Regime 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMB</td>
<td>0.011845</td>
<td>0.081866</td>
<td>-0.032260</td>
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<tr>
<td>HML</td>
<td>0.029782</td>
<td>0.095975</td>
<td>-0.107684</td>
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<tr>
<td>WOD</td>
<td>0.000506</td>
<td>-0.000308</td>
<td>-0.001683</td>
</tr>
<tr>
<td>VIX</td>
<td>-0.000628</td>
<td>-0.000543</td>
<td>0.004321</td>
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<tr>
<td>DXY</td>
<td>-6.562E-05</td>
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</tr>
<tr>
<td>COM</td>
<td>0.000887</td>
<td>0.000702</td>
<td>-0.002597</td>
</tr>
</tbody>
</table>

Note: For SMB, HML and WOD, the means in regime 1 are the highest while the counterparts in regime 3 are the lowest. On the contrary, the VIX and COM factors behave in the opposite way. The mean DXY factor is positive in regime 3 and negative in the other two regimes.
Table 6.4: Correlations of country ETF returns across regimes

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>CA</th>
<th>GER</th>
<th>FRA</th>
<th>ITA</th>
<th>JAP</th>
<th>AUS</th>
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</tr>
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<td>Regime 2</td>
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</table>

Note: This table reports the correlations of country ETF returns across regimes.
Table 6.5: Variance-covariance estimates for residuals of the RS model

<table>
<thead>
<tr>
<th>Regime 1</th>
<th>US</th>
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<th>CA</th>
<th>GER</th>
<th>FRA</th>
<th>ITA</th>
<th>JAP</th>
<th>AUS</th>
</tr>
</thead>
<tbody>
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<td>US</td>
<td>0.0156 (1.75E-16)</td>
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<tr>
<td>UK</td>
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<td></td>
<td></td>
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</tr>
<tr>
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</tr>
<tr>
<td>GER</td>
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<tr>
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<td>0.0023 (1.33E-12)</td>
<td>0.0388 (7.45E-19)</td>
<td>0.0888 (6.14E-25)</td>
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<tr>
<td>ITA</td>
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<td>0.0014 (2.66E-12)</td>
<td>-0.0024 (1.40E-11)</td>
<td>0.0002 (1.26E-12)</td>
<td>0.1201 (4.17E-22)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Regime 2</th>
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<th>GER</th>
<th>FRA</th>
<th>ITA</th>
<th>JAP</th>
<th>AUS</th>
</tr>
</thead>
<tbody>
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<tr>
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<tr>
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<td>0.0666 (1.30E-13)</td>
<td>0.2592 (4.56E-30)</td>
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<tr>
<td>GER</td>
<td>0.0892 (1.89E-13)</td>
<td>0.1605 (3.34E-16)</td>
<td>0.0513 (1.32E-11)</td>
<td>0.3371 (4.08E-21)</td>
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<tr>
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<td>0.0623 (2.76E-13)</td>
<td>0.1926 (4.13E-20)</td>
<td>0.2364 (8.45E-26)</td>
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<tr>
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<table>
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<th>US</th>
<th>UK</th>
<th>CA</th>
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<th>FRA</th>
<th>ITA</th>
<th>JAP</th>
<th>AUS</th>
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<tr>
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</table>

Note: This table reports the variance-covariance estimates for residuals of the RS model. $p$-values are shown in the parentheses. As can be seen, all variances/covariances are significantly different from zero.
6.4 Estimates of Alpha and Beta

We now discuss the determinants of returns on country ETFs and the performance of the RS model compared to the six-factor linear model that does not consider the regimes. As we can see from Table 6.7, the estimated intercepts (alpha) for all ETFs in the six-factor linear models are statistically insignificant whereas almost all alphas are statistically significantly different from zero in the RS model\(^2\). Clearly, the RS model is able to identify the nonzero alphas by incorporating market regimes and capturing more information than the linear factor model.

Table 6.8 reports the estimated betas for both the RS model and six-factor linear model. As can be seen from the table, the RS model is different from the six-factor linear model in two aspects. Firstly, all six factors are statistically significant in the RS model whereas some factors are not significant in the six-factor linear model\(^3\). Secondly, the RS model accommodates three different market regimes and captures more information about the behaviours of ETF returns. The remaining of this section discusses the performance of each factor in the RS model.

We now discuss the two Fama-French factors, SMB and HML. Firstly, these two factors are significant for all ETFs in three regimes. It implies that the U.S. market SMB and HML factors can explain the returns of these U.S. exchange listed foreign portfolios. This finding could be interpreted as evidence confirming the previous findings that U.S. listed foreign assets are exposed to U.S. market factors and behave like U.S. securities [see Frankel (1993), Chang et al. (1995), Russell (1998), and

\(^2\)The ones for US, UK, CA and GER ETFs in the transitory market (regime 2) are not statistically significant.

\(^3\)The coefficients of SMB factor are not significant for GER, JAP and AUS. The coefficients of the HML factor are not statistically significant for FRA and JAP. The coefficients of the COM factor are not significant for UK and ITA.
Gutierrez (2009)]. Secondly, the investors do not uniformly favour stocks with small capitalizations. Other than the returns of Canada ETF, the SMB factor contributes negative premiums to all other country ETFs in some regimes. For instance, the return of UK ETF is negatively correlated to the SMB factor in all three regimes. Thirdly, the role of HML factor is more straightforward. Other than the US, Canada and Japan ETFs, the HML factor contributes positive premiums to the returns of other ETFs in all three regimes. As for the US and Canada ETFs, they are negatively correlated to the HML factor in the transitory regime. The premium on the return of Japan ETF is negative in the bull regime.

The coefficients of the WOD factor for all ETFs in all three regimes are positive. This is consistent with previous studies. Moreover, different country ETFs have different sensitivities to the WOD factor across market regimes. For instance, the coefficient of the WOD factor for FRA, JAP and AUS are greater in the bull regime than the bear regime. It suggests that these three ETFs become defensive when the market turns bear. In addition, the GER ETF tends to overreact in response to WOD factor in all three regimes.

Now, we discuss the VIX factor. The coefficients of the VIX factor for all ETFs are negative in regimes 1 and 3. In regime 2, the VIX factor contributes negative premiums to the US and Canada ETFs and positive premiums to all other ETFs. That is, the VIX risk factor is negatively correlated to the returns of all ETFs in the bull and bear regimes. Besides, it is negatively correlated to the returns of the US and Canada ETFs in the transitory regime.

The estimated betas for DXY factor are as we have expected. They are positive

\footnote{The coefficients of WOD factor are greater than one.}
for the US ETF and negative for the other seven ETFs in all three regimes. This result can be explained intuitively. An increase in DXY represents the appreciation of U.S. dollar against other currencies. The stronger the U.S. dollar, the lower the returns of foreign assets valued in the U.S. dollar. As for the positive correlation between the return of the US ETF and DXY factor, a higher value of coefficients for the DXY factor suggests that the market demands more for US dollar assets driving up the prices of these assets. Thus the return of US ETF goes up.

We now examine the COM factor. The coefficients are negative for the US, GER, FRA, ITA and JAP in all three regimes. On the contrary, the coefficients are positive for the UK (except for regime 2), CA and AUS ETFs. This phenomenon confirms part of the conjectures discussed in chapter 2. As we can see from Table 6.6, the materials and energy sectors account for 33.97%, 48.1% and 36.04% of the UK, Canada and Australia ETFs in value, respectively. Given an increase in commodity prices, the profit gained from materials and energy sectors outweighs the lose from the other sectors. As a result of the net gain, the values of the underlying assets go up and thus lead to higher returns of these ETFs. As for the other ETFs, it works in the opposite direction.

<table>
<thead>
<tr>
<th>ETF</th>
<th>CA</th>
<th>AUS</th>
<th>UK</th>
<th>ITA</th>
<th>FRA</th>
<th>GER</th>
<th>US</th>
<th>JAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>48.1%</td>
<td>36.04%</td>
<td>33.97%</td>
<td>25.37%</td>
<td>19.20%</td>
<td>16.58%</td>
<td>16.51%</td>
<td>9.32%</td>
</tr>
</tbody>
</table>

Note: This table reports the total weight of materials and energy sectors in these eight ETFs. The data are retrieved from Tables 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, and 2.8.
Table 6.7: Comparison of alpha estimates

<table>
<thead>
<tr>
<th>ETF</th>
<th>RS Model</th>
<th></th>
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<th></th>
<th>Six-Factor Model</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Regime 1</td>
<td>Regime 2</td>
<td>Regime 3</td>
<td></td>
</tr>
<tr>
<td>US</td>
<td></td>
<td>0.000346 (2.83E-40)</td>
<td>0.000493 (0.723960)</td>
<td>2.37E-05 (1.73E-45)</td>
<td>-6.07E-07 (0.9960)</td>
</tr>
<tr>
<td>UK</td>
<td></td>
<td>5.25E-05 (0.003867)</td>
<td>-8.27E-05 (0.472321)</td>
<td>-2.97E-05 (0.082513)</td>
<td>-0.000113 (0.5360)</td>
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<tr>
<td>CA</td>
<td></td>
<td>0.000370 (1.86E-08)</td>
<td>0.000382 (0.798708)</td>
<td>6.93E-05 (5.49E-20)</td>
<td>0.000093 (0.6440)</td>
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<tr>
<td>GER</td>
<td></td>
<td>-0.000139 (1.89E-27)</td>
<td>-0.001168 (0.106228)</td>
<td>0.000158 (2.08E-06)</td>
<td>-0.000081 (0.6770)</td>
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<tr>
<td>FRA</td>
<td></td>
<td>-0.000111 (1.41E-06)</td>
<td>-0.000302 (0.040297)</td>
<td>-4.82E-05 (0.001136)</td>
<td>-0.000144 (0.4180)</td>
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<tr>
<td>ITA</td>
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<td>-5.92E-05 (2.16E-22)</td>
<td>-0.000981 (0.001970)</td>
<td>-0.000127 (0.002821)</td>
<td>-0.000337 (0.0970)</td>
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<tr>
<td>JAP</td>
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<td>-0.000532 (0.042188)</td>
<td>9.69E-06 (0.000194)</td>
<td>-0.000194 (2.75E-22)</td>
<td>-0.000164 (0.4490)</td>
</tr>
<tr>
<td>AUS</td>
<td></td>
<td>0.000575 (0.016101)</td>
<td>-0.000216 (3.73E-05)</td>
<td>0.000156 (5.99E-22)</td>
<td>0.000150 (0.5330)</td>
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</tbody>
</table>

Note: This table reports the estimated alphas for the six-factor models and the Markov regime-switching (RS) model. p-values are shown in the parentheses.
<table>
<thead>
<tr>
<th>Country</th>
<th>RS Model Regime 1</th>
<th>RS Model Regime 2</th>
<th>RS Model Regime 3</th>
<th>Six-factor Model</th>
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<tbody>
<tr>
<td>U.S.</td>
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<td></td>
<td>0.0018 (9.40E-26)</td>
<td>0.0009 (1.49E-25)</td>
<td>0.0009 (1.00E-26)</td>
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<tr>
<td>Note: This table reports the estimated $\beta$ for each factor for the three- and six-factor models and the regime-switching (RS) model. $p$-values are shown in the parentheses.</td>
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6.5 Model Fitting and Prediction

To evaluate the model performance, I examine its fitness as well as prediction power.

The actual and fitted returns for each ETF are plotted with regimes implied by the RS model in Figures 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7, and 6.8. These figures show that all regime switching regressions fit very well for the eight ETFs. In particular, the fitted returns of the US, CA, FRA and AUS ETFs fit their actual returns precisely.

To examine the predictive power of the RS model, we do an out-of-sample experiment\(^5\). Figure 6.9 provides the predictions based on the RS model. Except for Japan (JAP), the RS model predicts future returns well for all other seven ETFs. This confirms the value of the RS model in fitting for and forecasting of the country ETF returns.

\(^5\)The out-of-sample period is from June 1st, 2010 to March 31st, 201. There are 203 observations in this out-of-sample experiment.
Figure 6.1: Model fitting for iShares Russell 3000 index fund returns

Note: The sample period is from May 31st, 2000 to May 27th, 2010. The vertical axis denotes the cumulative returns and the horizontal axis denotes the observation points sorted by time. The yellow three-level dash line stands for the market regimes. Market regime 3 is represented by the highest level of the dash line, while market regime 1 is represented by the lowest level of the dash line. The level in the middle stands for market regime 2.
Figure 6.2: Model fitting for iShares MSCI United Kingdom index fund returns

Note: The sample period is from May 31st, 2000 to May 27th, 2010. The vertical axis denotes the cumulative returns and the horizontal axis denotes the observation points sorted by time. The yellow three-level dash line stands for the market regimes. Market regime 3 is represented by the highest level of the dash line, while market regime 1 is represented by the lowest level of the dash line. The level in the middle stands for market regime 2.
Figure 6.3: Model fitting for iShares MSCI Canada index fund returns

Note: The sample period is from May 31st, 2000 to May 27th, 2010. The vertical axis denotes the cumulative returns and the horizontal axis denotes the observation points sorted by time. The yellow three-level dash line stands for the market regimes. Market regime 3 is represented by the highest level of the dash line, while market regime 1 is represented by the lowest level of the dash line. The level in the middle stands for market regime 2.
Figure 6.4: Model fitting for iShares MSCI Germany index fund returns

Note: The sample period is from May 31st, 2000 to May 27th, 2010. The vertical axis denotes the cumulative returns and the horizontal axis denotes the observation points sorted by time. The yellow three-level dash line stands for the market regimes. Market regime 3 is represented by the highest level of the dash line, while market regime 1 is represented by the lowest level of the dash line. The level in the middle stands for market regime 2.
Figure 6.5: Model fitting for iShares MSCI France index fund returns

Note: The sample period is from May 31st, 2000 to May 27th, 2010. The vertical axis denotes the cumulative returns and the horizontal axis denotes the observation points sorted by time. The yellow three-level dash line stands for the market regimes. Market regime 3 is represented by the highest level of the dash line, while market regime 1 is represented by the lowest level of the dash line. The level in the middle stands for market regime 2.
Figure 6.6: Model fitting for iShares MSCI Italy index fund returns

Note: The sample period is from May 31st, 2000 to May 27th, 2010. The vertical axis denotes the cumulative returns and the horizontal axis denotes the observation points sorted by time. The yellow three-level dash line stands for the market regimes. Market regime 3 is represented by the highest level of the dash line, while market regime 1 is represented by the lowest level of the dash line. The level in the middle stands for market regime 2.
Figure 6.7: Model fitting for iShares MSCI Japan index fund returns

Note: The sample period is from May 31st, 2000 to May 27th, 2010. The vertical axis denotes the cumulative returns and the horizontal axis denotes the observation points sorted by time. The yellow three-level dash line stands for the market regimes. Market regime 1 is represented by the highest level of the dash line, while market regime 2 is represented by the lowest level of the dash line. The level in the middle stands for market regime 2.
Figure 6.8: Model fitting for iShares MSCI Australia index fund returns

Note: The sample period is from May 31st, 2000 to May 27th, 2010. The vertical axis denotes the cumulative returns and the horizontal axis denotes the observation points sorted by time. The yellow three-level dash line stands for the market regimes. Market regime 3 is represented by the highest level of the dash line, while market regime 1 is represented by the lowest level of the dash line. The level in the middle stands for market regime 2.
Note: The actual and predicted returns are plotted. The sample period is from May 31st, 2010 to May 31st, 2011. The vertical axis denotes the cumulative returns and the horizontal axis denotes the observation points sorted by time.
In this study, I apply a regime switching factor model for pricing the returns of eight country ETFs. The country funds are the iShares MSCI Russell 3000 index fund (US), iShares MSCI United Kingdom index fund (UK), iShares MSCI Canada index fund (CA), iShares MSCI Germany index fund (GER), iShares MSCI France index fund (FRA), iShares MSCI Italy index fund (ITA), iShares MSCI Japan index fund (JAP) and iShares MSCI Australia index fund (AUS). Six risk factors are incorporated in the model based on the Bayesian information criterion (BIC). These factors are size (SMB), value (HML), global stock market (WOD), market volatility (VIX), U.S. Dollar (DXY) and commodity prices (COM).

The model performs well for fitting and predicting the returns of the country ETFs. It identifies three market regimes: bull market (regime 1), transitory market (regime 2) and bear market (regime 3). The bull market is characterized by positive return and low volatility while returns are negative and volatility is high in bear regime. The transitory market acts as an intermediate market regime between the bull and bear regimes. Among these three regimes, the bull regime are the most persistent while the bear regime is the least persistent.
Consistent with the previous studies, the market portfolio return has a positive premium on all country ETFs returns. This study is innovative in that it is the first research on the returns of U.S. listed country ETFs using the U.S. size and value factors (SMB and HML) and macroeconomic factors (VIX, DXY, COM). The estimation results are summarized as follows. Firstly, the U.S. size and value factors can explain the returns of the eight county ETFs selected in this thesis. That suggests that the returns of U.S. listed foreign assets may be priced as U.S. common equities. Secondly, market volatility is negatively correlated to most country ETFs returns with exceptions for UK, Germany, France, Italy, Japan and Australia ETFs in the transitory regime. Thirdly, the U.S. dollar index (DXY) is priced into the returns of these U.S. listed country ETFs as expected. More specifically, the DXY factor contributes positive premiums on the US ETF return and negative premiums on the returns of other country ETFs across regimes. Finally, the returns of UK, Canada and Australia ETFs, which heavily invest in materials and energy sectors, are positively correlated to changes of the commodity price index while the returns of other country ETFs have negative relations with these changes in all three market regimes.

The correlation statistics and behaviours of the country ETF returns identified by the RS model indicate that investing in these international funds can help mitigate portfolio risk. However, the finding that these ETF returns can be explained by U.S. market factors weakens the diversification effect. Further research is required to examine whether the weakening diversification effect persists using data with different frequencies.
Bibliography


