

DAY- AHEAD MARGINAL PRICE FORECASTING OF ELECTRIC POWER  
SPOT MARKET USING INNOVATED FORECASTING APPROACHES

by

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The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled "DAY-AHEAD MARGINAL PRICE FORECASTING OF ELECTRIC POWER SPOT MARKETING USING INNOVATED FORECASTING APPROACHES" by Mohammed H. Al-Sakhsini, in partial fulfillment of the requirements for the degree of Master of Applied Science.

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*To my beloved parents, my wife Amnah, and my son Hashim.*

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## **ABSTRACT**

Forecasting is an essential planning tool for any system, assisting decision-makers and planners to visualize and plan the future of the system according to their goals. Because of this, forecasting has been widely used in markets for cost minimization and profit maximization. In an electricity market, load and price forecasting are the two main planning tools for generation, transmission and distribution owners and consumers alike.

Over the past several decades, many techniques and approaches have been proposed and implemented for load and price forecasting. The objective of all of these methods was load and price forecasting with minimal error. However, researchers face several challenges in achieving this goal.

In the case of price forecasting, the main challenge is to forecast electricity prices accurately in a deregulated electric power market with volatile aspects. Decentralized or deregulated markets are very volatile systems. Hence, pattern following and accurate forecasting of electricity prices are difficult tasks using ordinary methods.

In this thesis, a novel approach is introduced and implemented to overcome the challenges inherent in accurate price forecasting. This novel approach involves innovations in forecasting to improve the spot power price forecasting accuracy in a competitive market. To investigate the applicability and effectiveness of this technique, Multiple Linear Regression (MLR) and Artificial Neural Networks (ANN), two well-known forecasting techniques, are developed.

## **LIST OF ABBREVIATIONS AND SYMBOLS USED**

ANN	Artificial Neural Network
AR	Auto regressive
ARIMA	auto regressive integrated moving average
ARMA	Auto regressive moving average
ATOE	average training output error
Avg_R	average residuals
BP	Back Propagation
CLR	Cubic Linear Regression
DEKF	decoupled extended Kalman filter
DINN	Direct Innovated Neural Network
DR	dynamic regression
EINN	Exact Innovated Neural Network
EMMS	Electricity Market of Mainland Spain
eMAPE	extended MAPE
EPSM	European power spot markets
FIS	fuzzy interface system
GARCH	generalized auto regressive conditional heteroskedastic approach
GRG2	Generalized reduced gradient
IF	Innovated Forecasting
IGM	Innovations' generation model
INN	Innovated Neural Network
IMLR	Innovated Multiple Linear Regression with three independent variables
IOHMM	input output hidden Markov model
IWB	initial weights and biases
LMP	Locational marginal price
LRM	long run mean
LS	Least Square
LSE	least square estimation
LSINN	Least Square Innovated Neural Network
MAE	Mean absolute error
MAPE	Mean Absolute Percentage Error
MCP	Market clearing price
MIBEL	The Portuguese exchange
MLP	multi-layer perceptron
MLR	Multiple Linear Regression
MLR_2	Multiple Linear Regression with two independent parameters
MLR_3	Multiple Linear Regression with three independent parameters
MRP	Mean Reverting Process
MSE	Mean squared error
OMEL	The Spain's power market operator -Compania O Peradora del Mercado de Electricidad
PSO	Particle Swarm Optimization
QLR	Quadratic Linear Regression
REE	The system operator of the electricity market of mainland Spain

RINN	Random Innovated Neural Network
RMSE	root mean square error
RNN	Recurrent neural networks
SLR	Simple Linear Regression
SSE	sum squared error
STEYX	residual standard deviation
STPF	Short term electricity price forecasting
TF	transfer function
TOS	Training output set
TOE	Training output error
ZMCP	Zonal market clearing price
$e_{day}$	Daily mean error
$e_{week}$	Weekly mean error
$\gamma$	Learning rate
$\alpha$	Momentum

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## **CHAPTER 1      INTRODUCTION**

Since the introduction of the electricity generation in 1882 at New York Pearl Street power station, the electric industries followed the regulated or non-competitive industrial frameworks for power generation, control and marketing. After early 90s, many countries restructured their electric power industries to be competitive or deregulated power systems. Load forecasting was the focus of electric industries during the regulation age. However, price forecasting became the main prediction issue for the deregulated electric power systems [3].

### **1.1 MOTIVATION**

In an age of deregulated power markets, the price forecasting problem has become more complex and nonlinear than ever before. The volatility and nonlinearity of this system directly affect the accuracy of price prediction, a deficiency which influences market bidding strategies and leads to an unstable market.

The motivation of this thesis is to achieve more accurate results for this nonlinear and complex problem. These results can assist power strategists to arrange more effective bids, which will in turn lead to more stable markets to benefit both producers and consumers.

### **1.2 ELECTRICITY PRICE FORECASTING**

Forecasting is a necessary scheduling and planning function in any organization. In the case of electric power corporations, electricity price forecasting is an essential planning tool applied for the purpose of optimal bidding, planning and profit maximization, especially for power producers in deregulated or competitive markets. However, price

forecasting is much more complex than load forecasting because of uncertainties in market participants' bidding strategies and operations [1].

The applications of electricity price forecasting differ depending on the forecasting period. The fundamental applications of short-term, medium-term and long-term price forecasting can be summarized as day-ahead profit maximization, bilateral contracts planning, and investment recovery confirmation, respectively [2]. Short-term electricity price forecasting (STPF) or day-ahead price forecasting for a decentralized power market is the focus of this thesis.

### **1.3 THESIS OBJECTIVES AND CONTRIBUTIONS**

The objective of this thesis is to develop algorithms to improve the accuracy of the forecasted results found by conventional regression and neural network techniques. This goal was achieved by a novel approach called Innovated Forecasting (IF). This technique takes advantage of innovations obtained from residuals in regression forecasting and from training output errors in neural network forecasting.

The contributions of this thesis can be summarized as providing a new algorithm for day-ahead marginal price forecasting in a deregulated market, implementing the proposed approach in a real power market, and significantly improving the accuracy of short-term price forecasting.

### **1.4 THESIS OUTLINE**

This thesis is organized into six chapters. Research motivation, description of electricity price forecasting, and thesis objectives and contributions are addressed in the first chapter. The second chapter presents a literature review of the electricity price forecasting approaches and evaluation. The third chapter introduces Linear Regression and offers spot power market data analyses and STPF results using regression

forecasting. Chapter 4 provides an introduction to Artificial Neural Network (ANN) modeling and improvement, and forecasts the day-ahead electricity price of the spot power market using ANN. In the fifth chapter, innovations and IF are introduced, innovated regression and neural network forecasting approaches are implemented, and the improvements of each technique are investigated. Finally, Chapter 6 presents concluding remarks about the study's findings and suggests recommendations for future work.

## **CHAPTER 2      LITERATURE REVIEW**

In this Chapter, the market clearing price and the locational marginal price of electric power is briefly defined, electricity price forecasting is introduced, and the recent electricity price forecasting techniques are classified and evaluated.

### **2.1 INTRODUCTION**

The heavy dependence of modern societies on electrical power has significantly increased the importance of reliability, efficiency and stability of this valuable power source. From the day electricity started being generated, the framework of the world's electric industry has mostly been vertically integrated, with generation, transmission and distribution ruled and regulated by government monopolistic companies [3]. However, a regulated electricity market does not necessarily result in optimal productivity and efficiency levels, particularly in a free and open market environment [4]. Indeed, a deregulated electric industry framework can decrease electricity price by encouraging competition among power providers [3]. From the producers' point of view, the main purpose of restructuring power markets is profit maximization [1]. This goal has prompted a major transformation from regulation to deregulation in the electricity markets over the past few decades.

Electric power is a unique commodity. It cannot be stored in appreciable quantities and requires constant supply and demand balance. This involves high risk to investors and makes the price of electricity volatile in nature [1]. Usually, electricity is traded in two ways: bilateral contracts trading and pool trading. The hourly bids submitted by producers' and consumers' are matched by the market operator to set the spot price in the pool trading market [3]. In the bilateral contract system, a certain amount of power is agreed to be transferred through the network between seller and buyer at a specific fixed price. A combination of pool and bilateral contracts commonly takes place in deregulated electricity markets [5].

The basic theory of electricity spot market pricing states that an hourly spot price can be defined as the sum of fuel and maintenance costs, costs to compensate for transmission losses, and capacity limitations and quality of supply (generator and network availability). Removing transmission and distribution network costs from the equation, the mathematical expression for the optimal spot price at time t is [6]:

$$p_t = \frac{\partial G_{FM}[u(t)]}{\partial u(t)} + \frac{\partial G_{QS}[u(t)]}{\partial u(t)} \quad (2.1)$$

$G_{FM}(.)$  is the total fuel and maintenance cost of generation and  $G_{QS}(.)$  is the generation quality of supply costs incurred to provide reliable energy to customers.

The lowest price that would make all the accepted buy (purchase) bids satisfied by the accepted sell (sales) bids is called the market clearing price (MCP). Considering the delivery constraints of the transmission line and the generation marginal cost, the zonal market clearing price (ZMCP) or locational marginal price (LMP) can be defined as the next MW of load supplying price at a specific location. Typically, LMP is higher than MCP, and LMP forecasting is more complex than MCP forecasting. Furthermore, LMP prediction is more significant for market participants than is MCP prediction [7].

## 2.2 ELECTRICITY PRICE FORECASTING

While load forecasting was the focus of electricity industries during the regulation age, price forecasting has become the main prediction issue for deregulated electric power systems [3]. However, because of uncertainties in market participants' bidding strategies and operations, price forecasting is much more complex than load forecasting [1].

Electricity price forecasting applications differ depending on the forecasting period. Short-term or day-ahead, medium-term and long-term are the common price forecasting periods [8]. Day-ahead profit maximization, bilateral contracts planning, and investment

recovery confirmation are, respectively, the fundamental applications of short-term, medium-term, and long-term price forecasting [2]. Among these three time horizons, Short Term Price Forecasting (STPF) is the focus of this work.

While cost minimization was the basic criterion for scheduling of energy resources in the traditional regulated market, profit maximization became the core of energy planning in the deregulated market [3]. To maximize profit, short-term electricity price forecasting, which ranges over a period of one hour to one week [5], is a very useful planning tool for producers and consumers within the new decentralized framework. From the producers' point of view, STPF enhances bidding strategies and provides optimal scheduling for energy resources [3]. Better price forecast will lead to more effective bids with lower risk and higher profit. For their part, consumers will be able to create better plans to maximize their own benefit and to protect themselves against price increases [9].

Different electricity price forecasting models use different inputs corresponding to factors that have a significant impact on electricity pricing. These influential factors include historical electricity prices, weather conditions, and transmission congestion, as presented in [1]. The most significant factors affecting electricity pricing that could be considered as input variables are listed in [10] as: available historical price and load, system operating conditions, weather conditions and temperature values, fuel prices, time indices (hour, weekday, season), etc. In [3], these factors are summarized as: historical prices and demand, bidding strategies, operating reserves, imports, temperature effects, predicted power shortfalls, and generation outages. All of the stated spot electricity price influential factors are presented in Figure 2.1.

To be accurate, forecasting should consider all possible factors that influence electricity price, such thoroughness is very complicated to achieve in a real world situation. Therefore, factors that are more important than others are usually considered in real case studies and the rest are ignored. For instance, factors such as the amount of different types of reserves, power import and predicted power shortfall do not affect the forecast at all. Moreover, unit outage information is not available for all market participants and the

effects of weather conditions or temperature could be incorporated in the demand data [11].

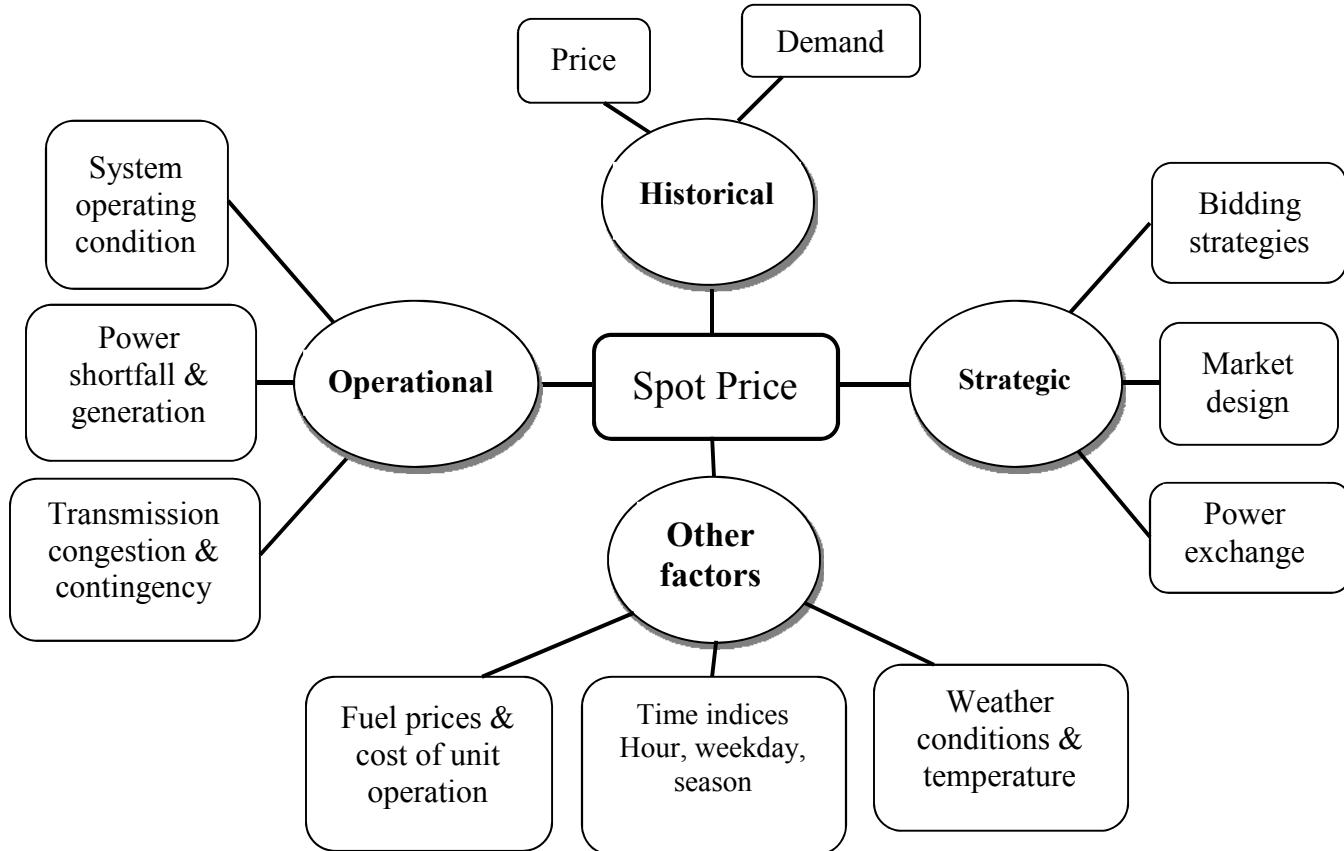


Figure 2.1 Factors affecting electricity price.

According to a study [12], adding of historical demand data to the prediction model leads only to negligible forecasting improvement in the case of ANN and ARIMA models. In general, the accuracy of predicted electricity prices is not necessarily related to more factors in the model [1].

### **2.3 TIME SERIES ELECTRICITY PRICE FORECASTING APPROACHES**

Many techniques have been proposed for price forecasting over the past two decades. These forecasting methods are classified in different ways. Statistical and non-statistical methods are the main approaches implemented for predicting electricity prices in

different time horizons as mentioned in [ 8]. The most popular category of statistical methods is the time series models. These forecasting models concentrate on the past behavior of the dependent variable [13]. The time series models include classical methods and modern methods.

### 2.3.1 Classical Models

These models are based on building a relationship between a dependent variable (electricity price) and a number of independent variables that are known or estimated [13]. Autoregressive (AR) [14] is the simplest model of this class which treats the disturbance as ideal white noise. Furthermore, the autoregressive moving average (ARMA) [15] and the autoregressive integrated moving average (ARIMA) [16] describe the correlated disturbances of assumed white noise. Considering the moments of a time series as variant where the error term does not have zero mean and constant variance as with an ARIMA process, the generalized autoregressive conditional heteroskedastic approach (GARCH) was applied for day-ahead electricity price forecasting in mainland Spain and California [17]. To improve the quality of price forecasting, a wavelet transform [18] signal processing technique was proposed to pre-process the data. In other works of this category, day-ahead electricity prices were predicted for both the Spanish and Californian electricity markets utilizing dynamic regression (DR) and transfer function (TF) models [19]. Although classical approaches are very accurate, most of them are linear and thus cannot capture nonlinear patterns; moreover, their computational cost is very high and requires a lot of information.

### 2.3.2 Modern Models

The second class of time series approaches is also known as intelligent system methods. These models are extensively used for STPF and are basically artificial neural networks (ANN). Without exploring the underlying process, neural networks map the input-output relationship [13]. The basic neural network used for electricity price prediction is the multi-layer perceptron (MLP) neural network [20], [21]. To enhance prediction, a

decoupled extended Kalman filter (DEKF) is used as a second-order learning algorithm to adjust the weights of the neural network [22], and Fourier and Hartley transforms [23] signal processing techniques are used to pre-process the data and to speed up finding the most effective data. Recurrent neural networks (RNN) [24], [25] which have feedback loops, are said to provide satisfactory results for electricity price forecasting, as it is a non-stationary time series prediction. Another learning algorithm which provides smooth transition from one observed value to another is the generalized regression neural network [26]. Historical prices together with load predicted by fuzzy logic were used as input to the neural network to forecast day-ahead spot prices of New South Wales, Australia [4]. The statistical method input-output hidden Markov model (IOHMM) is used for modeling the switching process of the spot price series of the Spanish Spot Market [27]. To gain the advantages of the neural network and fuzzy system, a neuro-fuzzy system that automatically extracts fuzzy rules from numerical data and adaptively adjusts the membership functions is introduced and applied for the Ontario, Canada, electricity market [11]. To replace the single neural network, the cascaded architecture of multiple ANNs [28] and a committee machine of neural networks [29] have been proposed. This classification is summarized in Figure 2.2.

ANNs are data driven models. Therefore, they are able to approximate nonlinear functions which are not well defined and not easily computable. Comparing ARIMA and ANN, ANNs are both easier to implement and show reasonable performance with fast consumption time.

Time series price forecasting approaches are categorized as linear, nonlinear and hybrid models [30]. For example, exponential smoothing [6] and ARIMA [16] are linear models. For nonlinear models, the most recent and well-known nonlinear model is ANN, which leads to significant improvements in the forecasting-accuracy. Hybrid models which are a combination of a linear and a nonlinear model have also been introduced in the literature.

For example, the integration of ARIMA and ANN was examined and showed accuracy improvement when applied to the Australian national electricity market [30]. Another

hybrid model relied on the fuzzy interface system (FIS), which performs input-output mapping based on fuzzy logic, and least square estimation (LSE), which provides the most accurate results [2].

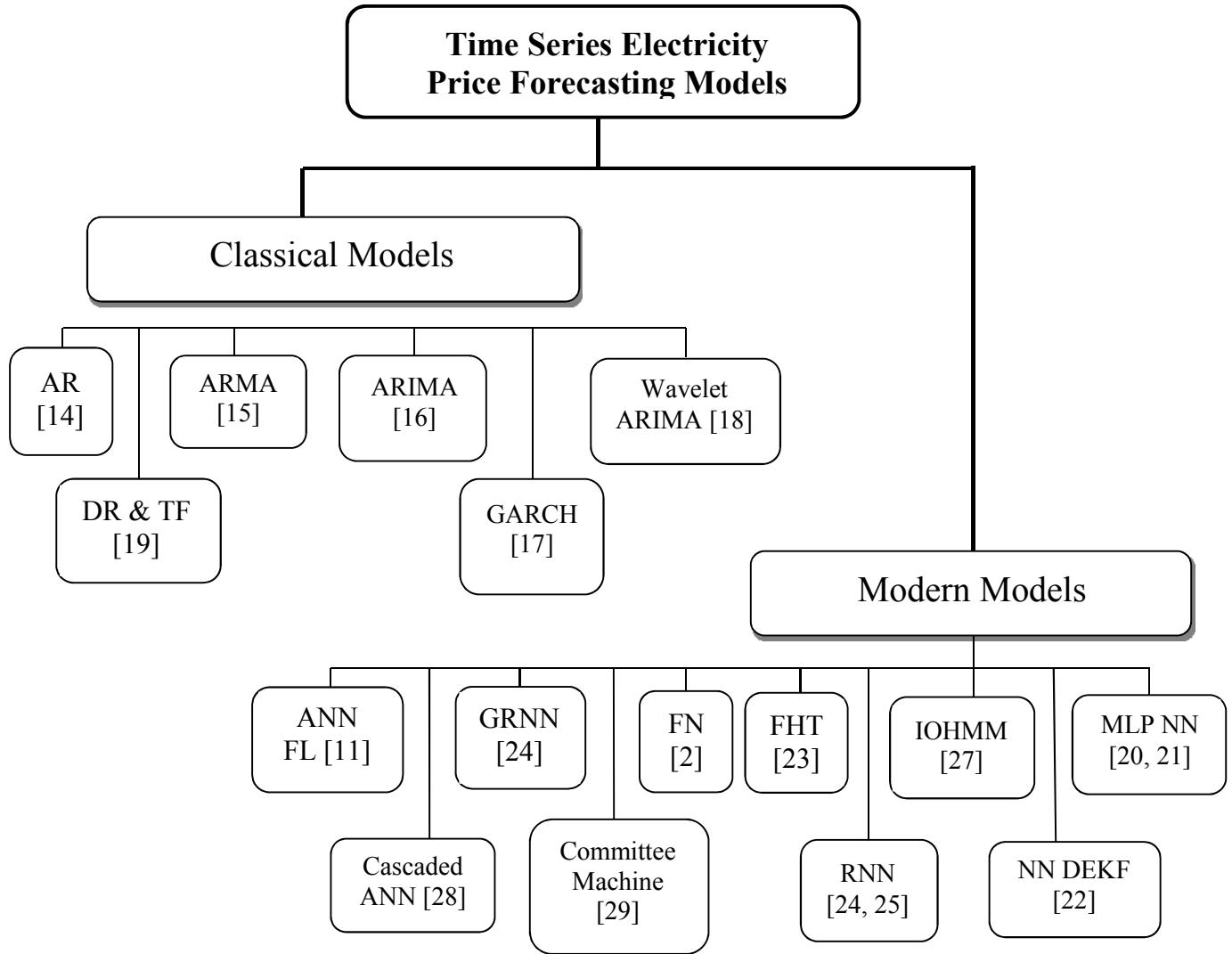


Figure 2.2 Common time series approaches for electricity price forecasting.

As stated in [30], the hybrid model is the most effective approach because it is able to capture both linear and nonlinear characteristics of the data in a real problem. These classes are presented in Figure 2.3.

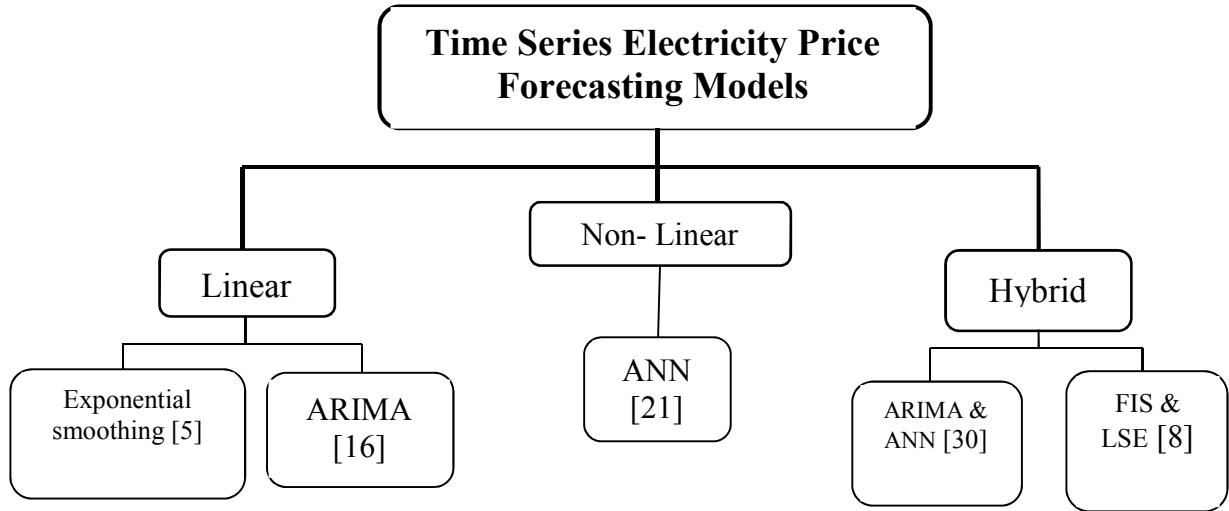


Figure 2.3 Different classifications of electricity price forecasting [30].

## 2.4 ELECTRICITY PRICE FORECASTING EVALUATION

To evaluate the performance of various forecasting models, different evaluation indices were used in the literature. The most common ones are mean absolute error (MAE) and mean absolute percentage error (MAPE) [30], [1]. The performance index utilized in this study is the mean absolute percentage error (MAPE). The extended MAPE (eMAPE) [1], root mean square error (RMSE) [30], daily mean error ( $e_{day}$ ), weekly mean error ( $e_{week}$ ) [7] and the sum squared error (S<sup>2</sup> SE) [3] are other evaluation indices used to assess the prediction accuracy. The formulas of all of these indices are as follows:

$$MAE = \frac{1}{N} \sum_{t=1}^N |(A_t - F_t)| \quad (2.2)$$

$$MAPE = \frac{1}{N} \sum_{t=1}^N \left| \frac{(A_t - F_t)}{A_t} \right| \times 100\% \quad (2.3)$$

$$eMAPE = \frac{1}{N} \sum_{t=1}^N \frac{|A_t - F_t|}{\frac{1}{N} \sum_{t=1}^N A_t} \times 100\% \quad (2.4)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^N (A_t - F_t)^2} \quad (2.5)$$

$$e_{day} = \frac{1}{24} \sum_{t=1}^{24} \frac{|A_t - F_t|}{\frac{1}{24} \sum_{t=1}^{24} A_t} \times 100\% \quad (2.6)$$

$$e_{week} = \frac{1}{168} \sum_{t=1}^{168} \frac{|A_t - F_t|}{\frac{1}{168} \sum_{t=1}^{168} A_t} \times 100\% \quad (2.7)$$

$$SSE = \sum_{t=1}^N (A_t - F_t)^2 \quad (2.8)$$

where

$A_t$  → actual value of price at t hour

$F_t$  → forecasted value of price at t hour

$N$  → number of hour for forecasting

The two most common evaluation criteria for any forecasting model are accuracy and computation time. Forecasting accuracy depends on the forecasting model and the volatility level of the forecasted market. Computation time, on the other hand, is directly related to the forecasting software and the speed of the microprocessor used for prediction. Although MATLAB is the most popular software used for forecasting, the hardware varies and thus processing speeds are different. Therefore, as a comparison of the computation time of different studies is not possible, only accuracy comparisons are taken into consideration here.

Since applying the same forecasting model for different electricity markets could result in different prediction accuracy, two different applied forecasting models for the same market are selected for the purpose of accuracy comparison. Assuming that the level of volatility is the same for the same market at all times, one applied classical model and one utilized modern model for the California electricity market, the UK power pool and Spanish electricity market are selected. The comparisons of accuracy between these markets are shown in Table 2.1.

Table 2.1 Level of Accuracy of Modern and Classical Models in Different Markets

<b>Market</b>	<b>Paper</b>	<b>Model</b>	<b>Time Horizon</b>	<b>Level of Accuracy</b>
California	[7]	ARIMA	1 DA	Average WMAPE 11%
	[31]	ANN (MLP)	Short term	WMAPE 11-13%
UK	[32]	Second order polynomial	1 DA	DMAPE 2.5-11.11%
	[33]	ANN (MLP)	1 DA	DMAPE 11.57-12.86%
Spanish	[5]	ARIMA	1 DA	WMAPE 5-27%
	[34]	ANN (MLP)	1 DA	Average WMAPE 7.5%

## **2.5 SUMMARY**

As an introduction to the topic, the definition of electricity price forecasting was discussed along with time horizons, influential variables and significance in competitive markets. In addition, various models of time series electricity price forecasting approaches were classified, an evaluation of current techniques for electricity price forecasting was presented, and a comparison between modern and classical time series prediction models were made. Since regression and neural networks are the two common basic blocks of time series electricity price forecasting, the following chapters will elucidate and investigate innovated regression and neural network forecasting.

## **CHAPTER 3      LINEAR REGRESSION**

Linear regressions are the fundamental forecasting approaches of the time series classical power price forecasting models. This chapter provides an overview of the linear regression, the data analysis of the targeted electric power market and the linear regression forecasting results.

### **3.1 INTRODUCTION**

Regression analysis is broadly used for forecasting and prediction. The earliest form of regression, which was the method of least squares, was first introduced by Legendre in 1805 and by Gauss in 1809. Both Legendre and Gauss applied this method for astronomical observations to determine the orbits of bodies around the sun. The theory was further expanded upon by Gauss in 1821.

In the nineteenth century, Francis Galton first used the term "regression" to describe a biological phenomenon. This phenomenon was the downward and slight tendency of descendants of tall ancestors towards a normal average which is also known as regression towards the mean. This work was later extended by Udny Yule and Karl Pearson to a more general statistical context and it is still an area of active research. [35]

Regression is a statistical approach to determine the relationship strength between one dependent variable usually denoted by Y and a series of other changing variables known as independent variables [36]. In other words, regression analysis could be used in any modeling or analyzing technique for several variables, where the focal point is on the relationship between a dependent variable and one or more independent variables. Regression function is a function of the independent variables to estimate the dependent variable [35].

As stated, regression is commonly used for forecasting and prediction and its use has considerable overlap with the field of machine learning. The probability distribution of the dependent variable around the regression function is a regression analysis interest. Regression analysis is also used to understand the relationship between the independent variables and the dependent variable, and to investigate the forms of these relationships [35].

### 3.2 LINEAR REGRESSION

In linear regression, the dependent variable  $y_i$  is a linear combination of the parameters and the independent variables which could be linear or nonlinear. Simple linear regression and multiple linear regression are the two basic types of linear regression. For instance, in simple regression of  $n$  data points' modeling, there is one independent variable,  $x_i$ , and two parameters,  $\beta_0$  and  $\beta_1$ , which yield a straight line:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \quad i = 1, \dots, n \quad (3.1)$$

This line is called the fitted regression line. In multiple linear regressions, there is more than one independent variable or function of independent variables. For example, the preceding regression with  $x_i^2$  term gives a parabola:

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \varepsilon_i \quad i = 1, \dots, n \quad (3.2)$$

Although the right-hand side expression is quadratic in the independent variable  $x_i$ , it is linear in the parameters  $\beta_0$ ,  $\beta_1$  and  $\beta_2$ . Therefore, this approach is still a linear regression. In the general multiple regression model, there could be  $p$  independent variables:

$$y_i = \beta_0 + \beta_1 x_{1i} + \dots + \beta_p x_{pi} + \varepsilon_i \quad (3.3)$$

$\varepsilon_i$  is the error term and the subscript  $i$  refers to a particular observation. Given a random population sample, the sample linear regression model is:

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i \quad (3.4)$$

The difference between the value of the dependent variable predicted by the model  $\hat{y}_i$  and the true value of the dependent variable  $y_i$  is known as the residual and is formulated as:

$$e_i = y_i - \hat{y}_i \quad (3.5)$$

In practice, the regression analysis methods' performance depends on the form of the data-generating process and its relation to the regression approach used. As the exact form of the data-generating process is usually not known, regression analysis makes assumptions about this process. [35] For our purposes, it is assumed that  $Y$  is normal. Hence, the values of  $Y$  follow the normal distribution density function, with the mean along the regression line and constant standard deviation. This means that, if two parallel lines with twice the value of the standard deviations above and below the regression line are drawn, a region which contains 95% of the values of  $Y$  will be obtained. Similarly, parallel lines at triple standard deviations will contain 99% of the data [37].

### 3.3 DATA ANALYSIS OF OMEL POWER MARKET

In January of 1998, the Electricity Market of Mainland Spain (EMMS) was launched. A day-ahead market, a reserve market, and a set of balancing and adjustment markets are the markets associated with this organization, and these markets are cleared based on auctions within a daily framework. Spain's power market operator (OMEL) clears most of these markets, while economic and technical issues are cleared by the system operator of the electricity market of mainland Spain (REE) [38].

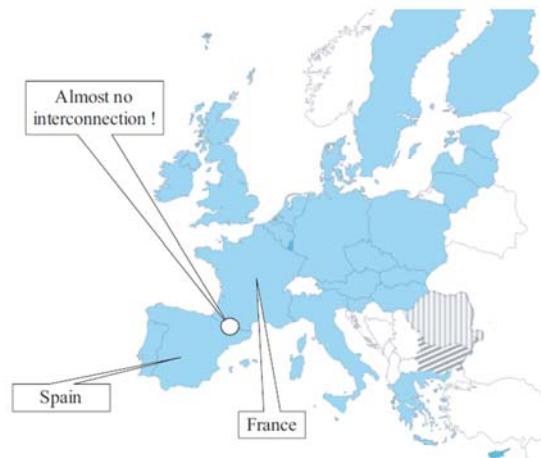


Figure 3.1 Geographical location of the electricity market of mainland Spain [38].

All seasonal marginal price data in this study were taken from OMEL. The responsibility of OMEL is the technical management of the electricity system, i.e., guaranteeing supply continuity and security. OMEL provides daily trading only (the Portuguese exchange MIBEL operates future trading). The weak physical interconnection capacities of the Iberian Peninsula with the rest of continental Europe (below 3.5% of the peak demand in Spain) is an important feature of the Spanish power system [39]. This implies that, from an electrical point of view, the Iberian Peninsula is almost an island. This fact and the geographical location of Spain are illustrated in Figure 3.1 [38].

Table 3.1 European Power Spot Market Yearly Volumes in GWh [39]

	2002	2003	2004	2005	2006	2007
EEX	31.456	49.136	59.449	85.335	87.602	117.322
IPEX	-	-	231.571	323.184	329.790	329.949
Powernext	2.623	7.478	14.128	19.670	29.600	44.212
APX NL	14.112	12	13.366	16.053	19.236	20.714
APX UK	-	-	-	-	-	10.950
NordPool	124.000	119.000	179.000	215.000	260.000	290.000
Omel	253	271	277	306	162	267
EXAA	624	1.324	1.763	1.541	1.666	2.265
Belpex	-	-	-	-	531	7.588

OMEL's trading volumes grew during 2002-2007 stage from 253 GWh to 267 GWh. These yearly volumes of OMEL power spot market compared to other European Power Spot Markets (EPSM) are shown in Table 3.1. From this table, it can be concluded that OMEL has the third largest yearly volume among all EPSM. Yearly average spot prices of some EPSM from 2002 to 2007 are illustrated in Table 3.2. The average daily spot price of OMEL and some EPSM in the 2<sup>nd</sup> quarter of 2007 are shown in Figure 3.2. It is obvious from Table 3.2 and from Figure 3.2 that the Spanish spot market is more stable compared to some EPSM.

The Spanish power market has proved to be popular with power market researchers. Among 49 price forecasting research papers done for 11 markets up to 2009, about six of these papers investigated the Spanish power market. Due to the volume of information, this competitive market was also selected for real implementation in this thesis.

Table 3.2 European Power Spot Markets Yearly Average Spot Price in €/MWH [39]

	2002	2003	2004	2005	2006	2007
EEX	22,63	29,49	28,52	45,98	50,79	37,99
Powernext	21,12	29,22	28,14	46,64	49,25	40,82
APX NL	29,91	46,47	31,58	52,39	58,10	41,92
APX UK (in £/MWh)	15,23	18,23	21,29	35,60	37,75	27,94
Nordpool	26,91	36,69	26,32	29,33	48,59	50,53
IPEX	N/A	N/A	51,60	58,59	74,75	70,99
Omel	38,21	29,74	28,46	54,78	51,53	39,34
Belpex	N/A	N/A	N/A	N/A	45,70	41,77
Average <sup>7</sup>	25,12	34,31	32,43	47,94	52,06	43,91

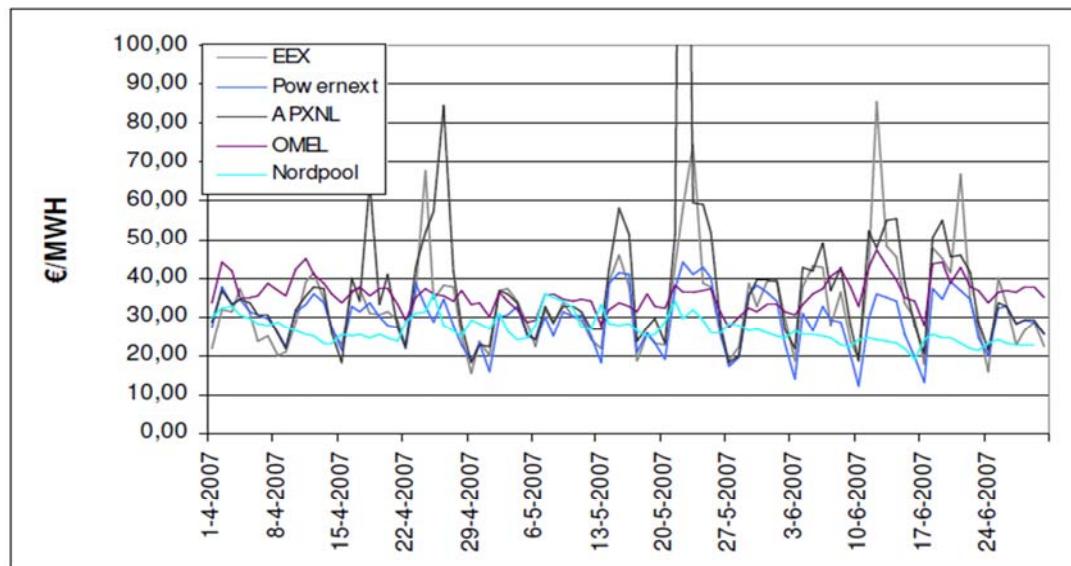


Figure 3.2 European power spot markets average daily spot price 2<sup>nd</sup> quarter 2007 [39].

As a first step for STPF research, the required hourly price data should be downloaded from the power market online database. In this case study, all hourly price data was downloaded from the official website of OMEL [40]. The downloaded data is then analyzed to calculate its statistical properties and percentage volatility. After that, the data is arranged in an Excel file according to the written forecasting programming code.

Almost two months, (60 days) worth of hourly data for each season was analyzed. The winter season ran from the 21<sup>st</sup> of December, 2008, to the 18<sup>th</sup> of February, 2009; the spring season ran from the 21<sup>st</sup> of March to the 19<sup>th</sup> of May, 2009; the summer season ran from the 21<sup>st</sup> June to the 19<sup>th</sup> of August, 2009; and the fall season ran from the 23<sup>rd</sup> of September to the 21<sup>st</sup> of November, 2009. For each season, the mean or average, variance, standard deviation, maximum value, minimum value and the difference between maximum and minimum values were calculated and listed in Table 3.3. The plot of two month series of data for each season is illustrated in Figure 3.3.

Various ways of calculating volatilities for financial markets are proposed in the literature. The standard price volatility model was the first model used to analyze the volatility of the price data series. It is started by computing average daily spot prices for a given spot market. Then, the daily percentage of price fluctuations from one day to the next, which is called the daily return of each day, is calculated.

Table 3.3 OMEL Seasonal Data Statistical Properties

	Mean	Variance	St. Deviation	Minimum	Maximum	Max - Min
Winter 4.75		1.7	1.3	0.1	10	9.9
Spring 3.71		0.29	0.53	2	5.813	3.813
Summer 3.48		0.24	0.49	1.007	4.975	3.968
Fall 3.45		0.71	0.84	0.1	8.444	8.344

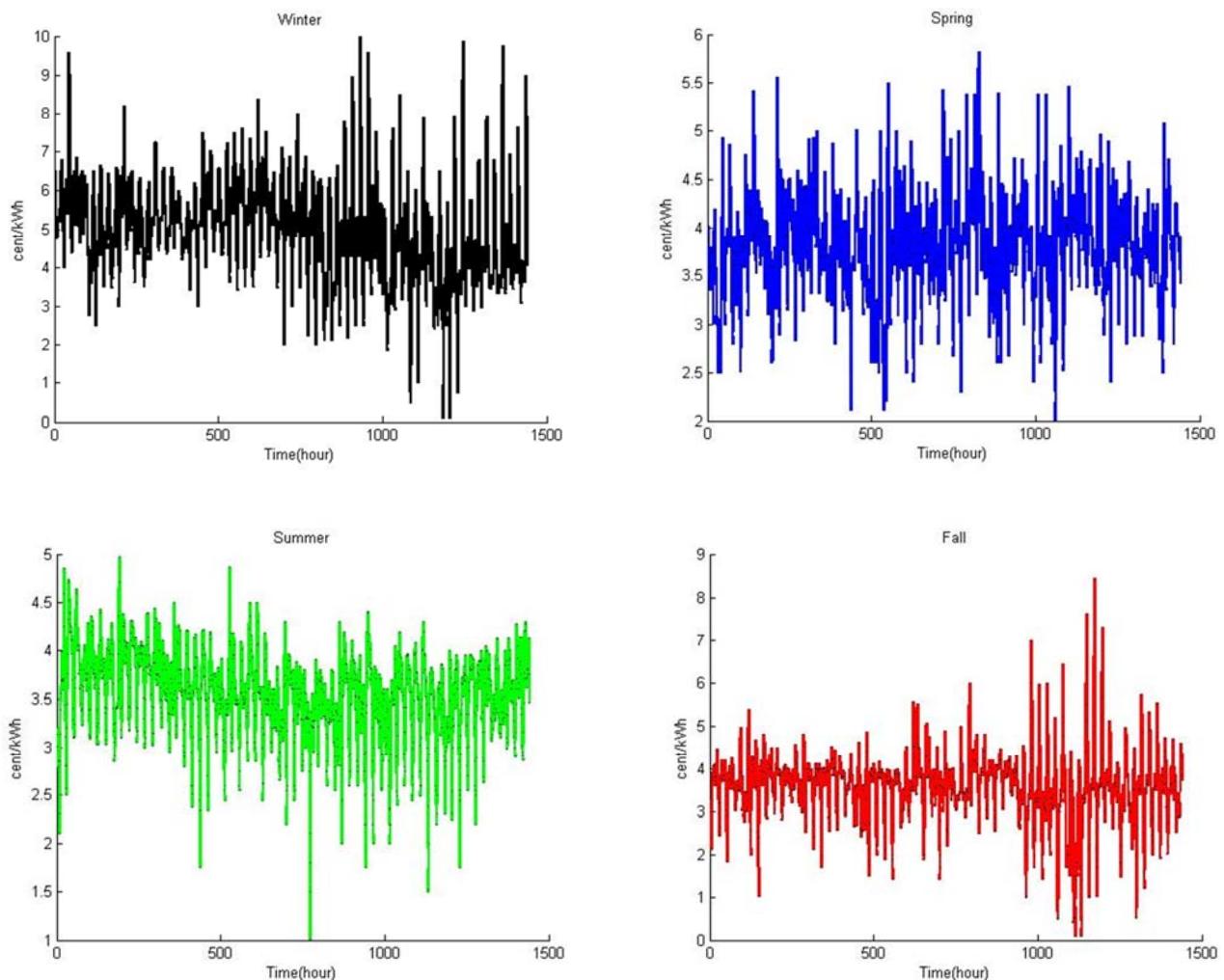


Figure 3.3 OMEL seasonal two-month hourly spot price

Next, using a two-month period as a time series, the standard deviation of the daily return is calculated. The computed standard deviation is the volatility series. In the last step, the standard deviation is annualized by multiplying standard deviation results by the square root of 260, which is the assumed annual number of trading days. This volatility calculation technique is known as the standard realized volatility calculation model and is used in most financial analysis modeling.

The methodology used for calculating volatility in this case study is known as the Mean Reverting Process (MRP). It is used most frequently in analyzing energy markets. This process assumed that prices are not totally independent of previous price levels. It

incorporates the tendency of energy prices to approach a normal equilibrium price level which is typically governed by the level of demand and cost of production. [39]

To calculate volatility of price changes and percentage volatility using MRP, particular price data series (i.e., hourly, daily or weekly) should first be selected. As mentioned earlier, a two-month hourly spot price data series for each season of the OMEL power spot market is selected. Next, the hourly price change is calculated:

$$y = s_{d+1} - s_d \quad (3.6)$$

where  $y$  is the price change,  $s_{d+1}$  is the spot price at day  $d+1$ , and  $s_d$  is the spot price at day denoted with number  $d$ . After that, linear regression is used to calculate the slope and intercept of the following linear function:

$$y = A + B * s_d \quad (3.7)$$

where  $y$  is the price change,  $A$  is the slope and  $B$  is the intercept. As a fourth step, the Long Run Mean (LRM) is calculated by:

$$LMR = - \left[ \frac{\text{Intercept}}{\text{Slope}} \right] \quad (3.8)$$

Finally, the volatility of price changes is given by the residual standard deviation calculated using the STEYX function in Excel. To obtain the percentage volatility, the residual standard deviation should be divided by the calculated LRM:

$$\%volatility = \frac{STEYX}{LRM} \quad (3.9)$$

The 2009 four season volatility calculation results of the OMEL power spot market are presented in Table 3.4. It can be concluded from Table 3.4 and from Figure 3.4 that the winter and fall seasons are more volatile than the spring and summer seasons [41].

Table 3.4 OMEL Seasonal Data Volatility Analysis

	% Slope	Intercept	LRM	STEYX	% Volatility
Winter	-21.76	1.01 4.66	0.82		17.64
Spring	-42.17	1.56 3.71	0.44		11.85
Summer	-30.84	1.08 3.50	0.35	9.93	
Fall	-28.51	0.98 3.44	0.59		17.19

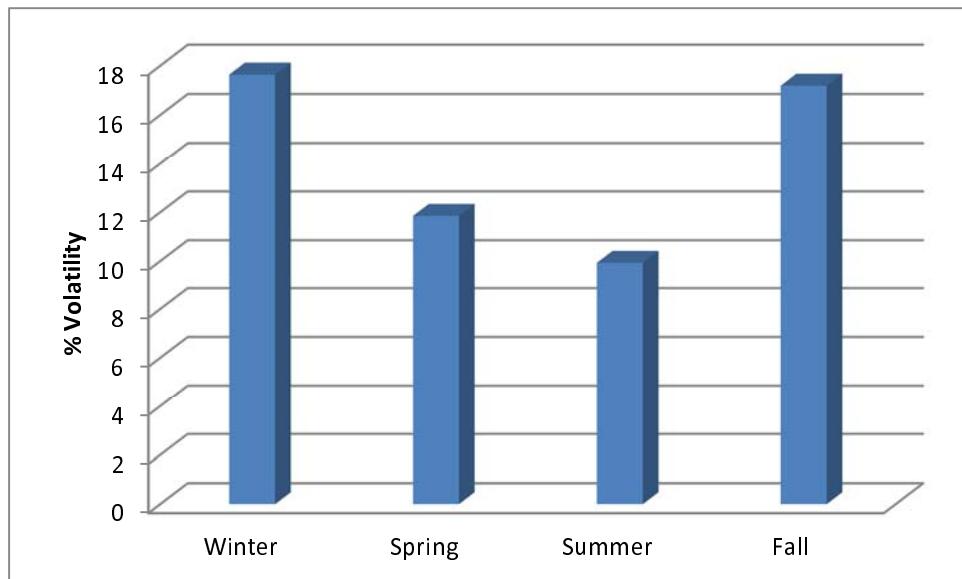


Figure 3.4 Seasonal percentage power price volatility of 2009 OMEL market.

### 3.4 OMEL LINEAR REGRESSION FORECASTING

After analyzing data volatility using MRP which is derived from Simple Linear Regression (SLR), the hourly seasonal spot power prices for weekdays and weekends are forecasted by SLR, Quadratic Linear Regression (QLR), Cubic Linear Regression (CLR), Multiple Linear Regression with two independent parameters (MLR\_2) and Multiple Linear Regression with three parameters (MLR\_3). The general equations of these linear regression approaches are:

$$\text{SLR: } y = A + B * s_d \quad (3.10)$$

$$\text{QLR: } y = A + B * s_d + C * s_d^2 \quad (3.11)$$

$$\text{CLR: } y = A + B * s_d + C * s_d^2 + D * s_d^3 \quad (3.12)$$

$$\text{MLR\_2: } y = A + B * s_d + C * s_{d-1} \quad (3.13)$$

$$\text{MLR\_3: } y = A + B * s_d + C * s_{d-1} + D * s_{d-2} \quad (3.14)$$

where  $y = s_{d+1} - s_d$  is the price change from day d+1 to day d

s is the spot price at a specific day denoted by d-2, d-1, d or d+1

A, B, C and D are constant parameters

Minitab is used in this chapter as the linear regression software tool. For each season, seven days are forecasted. These days are the 15<sup>th</sup> to the 21<sup>nd</sup> of February, 2009, for the winter season; the 16<sup>th</sup> to the 22<sup>rd</sup> of May, 2009, for the spring season; the 16<sup>th</sup> to the 22<sup>rd</sup> of August, 2009, for the summer season; and the 18<sup>th</sup> to the 24<sup>th</sup> of November, 2009, for the fall season. The weekdays' MAPE and weekends' MAPE results using different regression models and the average MAP E for weekdays and weekends regression forecasting are introduced in the following tables. The regression equations for each season are presented in the Appendix.

Table 3.5 Winter Weekdays and Weekends' MAPE Using Different Regression Models

	Weekdays' MAPE					Weekends' MAPE	
	Mon	Tues	Wed	Thu	Fri	Sat	Sun
<b>SLR</b>	16.28	5.99	11.33	7.50	7.34	11.96	11.87
<b>QLR</b>	16.09	6.18	11.52	7.38	7.41	12.11	11.95
<b>CLR</b>	15.63	4.95	9.89	7.76	5.88	12.33	9.80
<b>MLR_2</b>	15.55	4.90	11.08	6.72	5.58	11.59	11.91
<b>MLR_3</b>	12.50	3.72	6.82	6.00	5.83	12.34	11.35

Table 3.6 Spring Weekdays and Weekends' MAPE Using Different Regression Models

	Weekdays' MAPE					Weekends' MAPE	
	Mon	Tues	Wed	Thu	Fri	Sat	Sun
<b>SLR</b>	13.91	7.45	5.83	4.87	8.50	4.11	18.40
<b>QLR</b>	13.83	7.51	6.04	5.02	8.78	4.01	18.24
<b>CLR</b>	14.16	7.18	5.54	5.02	8.47	4.06	17.62
<b>MLR_2</b>	13.01	8.71	6.08	4.44	7.67	4.23	18.39
<b>MLR_3</b>	12.56	8.04	7.46	4.49	7.27	4.74	18.31

Table 3.7 Summer Weekdays and Weekends' MAPE Using Different Regression Models

	Weekdays' MAPE					Weekends' MAPE	
	Mon	Tues	Wed	Thu	Fri	Sat	Sun
<b>SLR</b>	8.07	6.06	4.43	5.06	4.90	5.26	3.28
<b>QLR</b>	8.16	6.03	4.37	4.90	4.88	5.25	3.10
<b>CLR</b>	8.20	5.57	3.71	4.92	4.75	5.53	3.07
<b>MLR_2</b>	8.38	6.39	4.68	4.97	4.71	6.01	3.42
<b>MLR_3</b>	6.48	6.27	5.67	4.79	4.80	6.33	4.38

Table 3.8 Fall Weekdays and Weekends' MAPE Using Different Regression Models

	Weekdays' MAPE					Weekends' MAPE	
	Mon	Tues	Wed	Thu	Fri	Sat	Sun
<b>SLR</b>	44.95	9.62	4.51	6.48	6.92	14.08	22.28
<b>QLR</b>	44.96	8.94	3.92	6.69	6.85	13.79	21.81
<b>CLR</b>	44.60	8.60	3.83	6.38	6.32	13.82	20.81
<b>MLR_2</b>	44.59	11.70	5.99	5.18	6.17	14.59	22.19
<b>MLR_3</b>	41.35	11.14	5.05	3.85	4.69	14.43	22.37

Table 3.9 Seasonal Weekdays and Weekends' Average MAPE Using Different Regression Models

		SLR	QLR	CLR	MLR_2	MLR_3
<b>Winter</b>	<b>Average Weekdays' MAPE</b>	9.69	9.71	8.82	8.77	6.97
	<b>Average Weekends' MAPE</b>	11.92	12.03	11.06	11.75	11.84
<b>Spring</b>	<b>Average Weekdays' MAPE</b>	8.11	8.23	8.07	7.98	7.96
	<b>Average Weekends' MAPE</b>	11.25	11.13	10.84	11.31	11.52
<b>Summer</b>	<b>Average Weekdays' MAPE</b>	5.70	5.67	5.43	5.82	5.60
	<b>Average Weekends' MAPE</b>	4.27	4.18	4.30	4.72	5.35
<b>Fall</b>	<b>Average Weekdays' MAPE</b>	14.50	14.27	13.95	14.73	13.21
	<b>Average Weekends' MAPE</b>	18.18	17.80	17.32	18.39	18.40

The shaded cells in Table 3.9 indicate the most accurate results or the lowest MAPE for weekdays or weekends of a season using a specific regression method. For weekdays, the MLR\_3 model achieves the best forecasting accuracy for three seasons. On the other hand, the CLR model leads to the most accurate forecasting results for weekends. In Chapter 5, the MLR\_3 Regression model will be used as a case study to implement innovated forecasting.

### **3.5 SUMMARY**

In this chapter, regression analysis was defined and the concept of Linear Regression illustrated. The targeted power market was then introduced and the seasonal spot power price data analyzed. Following that, the seasonal hourly day-ahead price forecasting MAPE results using different regression methods were presented.

## CHAPTER 4      ARTIFICIAL NEURAL NETWORKS

Artificial Neural Networks (ANNs) are the basic forecasting approaches of the time series modern power price forecasting models. This chapter is dedicated to introduce ANNs' modeling and recent neural network improvements. At the end, the electric power forecasting results using a neural network are presented.

### 4.1 INTRODUCTION

Among forecasting tools, Artificial Neural Networks (ANNs) have recently stepped into the spotlight because of their clear and easy model implementation coupled with acceptable performance [9]. Solving undefined relationships between input and output variables, approximating complex nonlinear functions, and implementing multiple training algorithms are the better-known advantages of these tools [1]. Furthermore, parallel data processing with no prior assumption of the model form (where the model is largely determined by the characteristics of the data) is the main source of power and effectiveness of ANN over other methodologies [30]. Hence, due to their many benefits, ANNs are increasingly being used for power systems' load and market clearing price predictions.

Artificial neural networks are like human brains, where units or neurons are highly interconnected and designed in a way to perform a particular task [3]. They are flexible computing frameworks and universal approximators that can model and approximate a broad range of nonlinear problems with a high degree of accuracy. [30] Therefore, ANN is a model that performs a nonlinear mapping from the past values ( $y_{t-1}, y_{t-2}, \dots, y_{t-p}$ ) to the future value ( $y_t$ ):

$$y_t = f(y_{t-1}, y_{t-2}, \dots, y_{t-p}, \omega) + \varepsilon_t \quad (4.1)$$

where  $\omega$  is a vector of all parameters and the function  $f$  is determined by the connection weights and network structure [30]. Equation (1) can be written in more detail as:

$$y_t = \alpha_0 + \sum_{j=1}^q \alpha_j g\left(\beta_{0j} + \sum_{i=1}^p \beta_{ij} y_{t-i}\right) + \epsilon_t \quad (4.2)$$

where the model parameters or the connection weights are  $\alpha_j$  ( $j = 0, 1, 2, \dots, q$ ) and  $\beta_{ij}$  ( $i = 0, 1, 2, \dots, p; j = 0, 1, 2, \dots, q$ ),  $p$  is the number of input nodes, and  $q$  is the number of hidden nodes. Thus, the weighted sum of neuron inputs plus a constant bias term is passed through a transfer function  $g(\cdot)$  that could be, for example, a linear, sigmoid or hyperbolic tangent. The result would be the output of the neuron, which in turn would be the input to another layer of neurons [3].

In general, training and learning are the two steps of forecasting with an artificial neural network. The success of training is highly influenced by the ANNs' adequate selection of inputs. Based on the minimization of the error measured between the output produced and the desired output, the artificial neural networks build an input-output mapping by adjusting the weights and biases at each iteration in the learning process. The error minimization process is repeated until the convergence criterion is satisfied. Known as back propagation, this training process is the network training algorithm used in this thesis. Finally, the network obtained from the learning process is tested by a new data set called the testing set. The resultant output of the network using the testing set of the data should be accurate [3].

Although a neural network has a good learning capability, its random initial conditions could lead to local extrema and slower convergence speed [42]. The inflexibility of a neural network to model the data with too few units and its over-fitting with too many units are also considered disadvantages [1].

## **4.2 ARTIFICIAL NEURAL NETWORK MODELING**

The first step in developing an ANN is data pre-processing, which is both important and time-consuming. This step is necessary for the ANN to learn the input output relationship. The data processing step includes normalizing, ranking and correlating in order to supply the ANN with the most correlated historical data in a correct style and format [43].

In developing an ANN model, combinations of different numbers of hidden layers, different numbers of units in each layer and different types of transfer function could result in different models [1]. The best known and most widely used type of artificial neural network is the multilayer perceptron. These networks could have different interconnection patterns. Feedforward networks are interconnected networks that do not form any loops, whereas recurrent networks are interconnections with one or more loops [3].

Feedforward networks are used in this study. Usually, units in the feedforward networks are arranged as an input layer, with one or more hidden layer and an output layer without feedback. Input layer units only transfer the input pattern without any processing to the rest of the network, after which information processing takes place in the hidden and output layers [3]. On the contrary, RNN uses a neuron's output as a feedback to its input. This network has been conducted to forecast LMP [24].

According to the literature, the most popular model for time series modeling and forecasting is the single hidden layer feedforward network [44]. This type of network, with sigmoid functions for hidden layers and linear functions for output layers, has been used in the literature for price forecasting [3].

The optimal selection of number of layers and neurons when training an ANN is a difficult estimation problem. Therefore, to optimize the numbers of layers and neurons, a grid search based on Genetic Algorithm is widely used. Data mining is also a valuable

ANN estimation tool [45]. However, after several combinations with different numbers of hidden layers, different numbers of units in each layer and different types of transfer functions, the best (optimal) network structure was found to be the network with one hidden layer that uses a hyperbolic tangent sigmoid transfer function and a single unit output layer with a pure linear transfer function, as stated in [3]. The selection of number of hidden nodes has no systematic rule and is data dependent as reported in [30].

The network may over-fit the data if there are too many units. Overfitting the data means fitting it to the model building sample used and poor generalization ability for data out of the sample. On the other hand, the network will not be flexible enough to model the data if there are too few units [3]. However, with adequate units in the hidden layer, this structure has been proven to be a universal mapper [46].

Probably the most significant parameter to be estimated in an ANN model is the selection of the number of input nodes or the dimension of the input vector, since the number of lagged observations (input nodes) is responsible for determining the nonlinear structure of the time series. Nevertheless, the appropriate selections of number of input nodes as well as the number of hidden nodes are usually made after several trial and error experiments [30].

In summary, ANN approaches are simple, flexible and powerful forecasting tools if enough training data is provided, the input-output samples selection is adequate, and number of hidden layers is appropriate [3].

### **4.3 ARTIFICIAL NEURAL NETWORK IMPROVEMENT**

To improve the efficiency of a neural network algorithm, it is imperative to organize the input data in a suitable manner [47]. Only one hidden layer of neurons is proven to be sufficient for a neural network to approximate any nonlinear complex function. The learning rate ( $\gamma$ ) and momentum ( $\alpha$ ) are the two parameters of the Back Propagation (BP) algorithm that could be adjusted to accelerate the learning process. The proportion of

error gradient by which the weights should be adjusted is the learning rate. The past weights' proportion of change that is used in the calculation of the new weights is determined by the momentum [3]. The values of these two parameters ( $\gamma$  and  $\alpha$ ) are selected as 0.7 and .1 in [48].

The most widely used learning algorithm is the back propagation (BP) algorithm [20], [21]. In this learning algorithm, the calculated output, achieved by passing the input through network layers, is compared to the actual output to find the error. Then, the error is propagated back to the input to adjust the biases and weights in each layer [3].

To accelerate the neural networks' learning process, some new learning algorithms other than the common BP algorithm have been utilized. The Levenberg-Marquardt algorithm, which can train a neural network 10 to 100 times faster than the common BP algorithm, and the generalized reduced gradient (GRG2), which is an efficient nonlinear training algorithm used in [49], are examples of these new learning algorithms.

ANNs recently combined with different techniques to overcome the weakness of these networks. Based on a similar day method, an ANN model is proposed to forecast day-ahead electricity prices in [50] and [43]. A relief algorithm, which is a feature selection technique, is combined with ANNs [51]. In [52], particle swarm optimization (PSO) is used for training ANN. To simplify the relationship between ANN input and output variables, the clipping technique is suggested [53]. Data pre-processing, prevention of over-fitting, partition of different transaction periods and truncation of price outliers ([25], [54], [20], [23]) are some other associated techniques with ANN to improve the forecasting accuracy or to speed up the training process.

#### **4.4 OMEL ANN FORECASTING**

For ANN forecasting in this study, the assembled data is arranged into categories as training input, training output, testing input and testing output according to the written

forecasting code in MATLAB. The training data set for each seasonal prediction is 60 days. The implemented neural network is a single hidden feed-forward network with back propagation learning algorithm and hyperbolic tangent sigmoid as the transfer function for the hidden layer. After data arrangement and network settlement, the series of data is trained. Meanwhile, the initial weights and biases (IWB) of the neural network are saved for the next iterations; they will be used for the innovated neural network (INN) in Chapter 5. All neural network specifications which are used for both ANN and INN forecasting are identical and are presented in Appendix A.9. The seasonal MAPE and the average seasonal MAPE forecasts seven days by ANN are shown in Table 4.1. The forecasted seven days of each season are the same forecasted days using regression techniques in Chapter 3. The resulting MAPE using ANN will be compared with the INN MAPE results in the next chapter.

Table 4.1 Seasonal Daily and Average MAPE Using ANN

	<b>Mon</b>	<b>Tues</b>	<b>Wed</b>	<b>Thu</b>	<b>Fri</b>	<b>Sat</b>	<b>Sun</b>	<b>Average MAPE</b>
<b>Winter</b>	21.08	6.87	9.49	7.55	6.41	13.35	11.84	10.94
<b>Spring</b>	11.53	9.76	6.67	4.29	10.00	4.39	18.08	9.24
<b>Summer</b>	8.08	6.49	4.61	5.03	6.42	5.80	4.47	5.84
<b>Fall</b>	45.88	18.13	11.38	6.01	6.56	14.40	23.14	17.93

## 4.5 SUMMARY

An introduction to Artificial Neural Networks (ANNs) was given in this chapter. Subsequently, a literature review of power price forecasting using different ANN modeling was presented and ANN forecasting improvement techniques were discussed. Such improvements include the changing neural network structure, network learning algorithm and learning parameters as well as hybrid techniques. The chapter concluded with MAPE of OMEL forecasting results using a feed-forward neural network.

## **CHAPTER 5 INNOVATED FORECASTING**

This chapter is allocated to introduce “innovations”, to model and implement the proposed innovated electric power price forecasting approaches, and to compare the results of the proposed approaches with the results of the conventional techniques.

### **5.1 INTRODUCTION**

Each approximated or fitted value brings new information which is the difference between the actual value and the estimated value. This new information is called “innovations” by Wiener and Masani [ 55]. The main function of innovations [ 56] is to lead the user to a simple and efficient predictor.

Many hybrid and sophisticated approaches were proposed and implemented in the literature to improve forecasting accuracy in nonlinear and highly volatile systems. In this thesis, Innovated Forecasting (IF) is the proposed STPF approach for the purpose of accuracy improvement.

In regression analysis, the average residuals or the average differences between the actual values and the approximated or fitted values are the regression innovations. On the other hand, ANN innovations are considered as the training output errors (TOE), which are the differences between the actual training outputs set and the resultant training outputs set after network training.

### **5.2 INNOVATED REGRESSION FORECASTING**

As stated in the introduction, the average residuals (Avg \_R) are the regression innovations utilized for prediction improvement. One-week hourly calculated innovations for each season are presented in the Appendix. These regression innovations were added

to the corresponding regression equation to get the innovated regression day-ahead power price forecasting results.

To study the effect of innovations on regression forecasting, the MLR regression technique was selected, as stated in Chapter 3. First, the MLR innovations for each season were calculated. Then, the innovated MLR day-ahead power price for castings (IMLR) were calculated by adding the innovations to the corresponding regression equation for each season. The MAPE of the regression day-ahead power price forecasting results (Old MAPE), the MAPE of the innovated regression day-ahead power price forecasting results (New MAPE), and the percentage MAPE improvements of each season are shown in Tables 5.1, 5.2, 5.3, and 5.4, respectively. As an example from each season, Monday's actual and forecasted spot power prices using MLR and IMLR are illustrated in Figures 5.1, 5.2, 5.3, and 5.4.

Table 5.1 Winter IMLR Forecasting Accuracy Improvement

Winter	Mon	Tue	Wed	Thu	Fri	Sat	Sun
<b>Old MAPE</b>	12.50	3.72	6.82	6.00	5.83	12.34	11.35
<b>New MAPE</b>	9.80	2.82	6.16	11.20	8.60	11.54	6.96
<b>%Improvement</b>	21.62	24.04	9.65	-86.48	-47.40	6.46	38.70

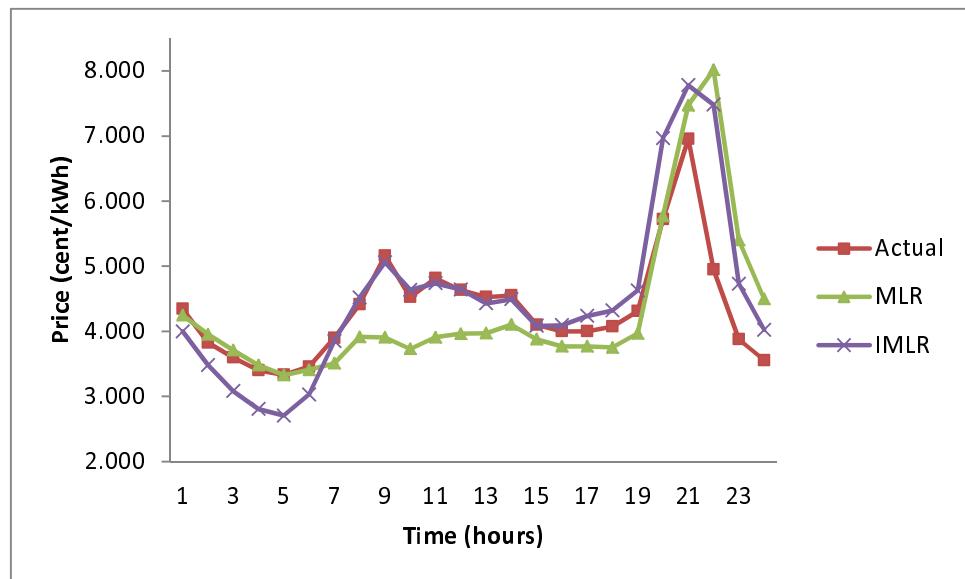


Figure 5.1 Monday day-ahead actual and forecasted power price for winter.

Table 5.2 Spring IMLR Forecasting Accuracy Improvement

Spring	Mon	Tue	Wed	Thu	Fri	Sat	Sun
<b>Old MAPE</b>	12.56	8.04	7.46	4.49	7.27	4.74	18.31
<b>New MAPE</b>	9.09	4.63	3.26	6.65	8.03	4.52	7.66
<b>%Improvement</b>	27.62	42.41	56.28	-47.93	-10.45	4.54	58.17

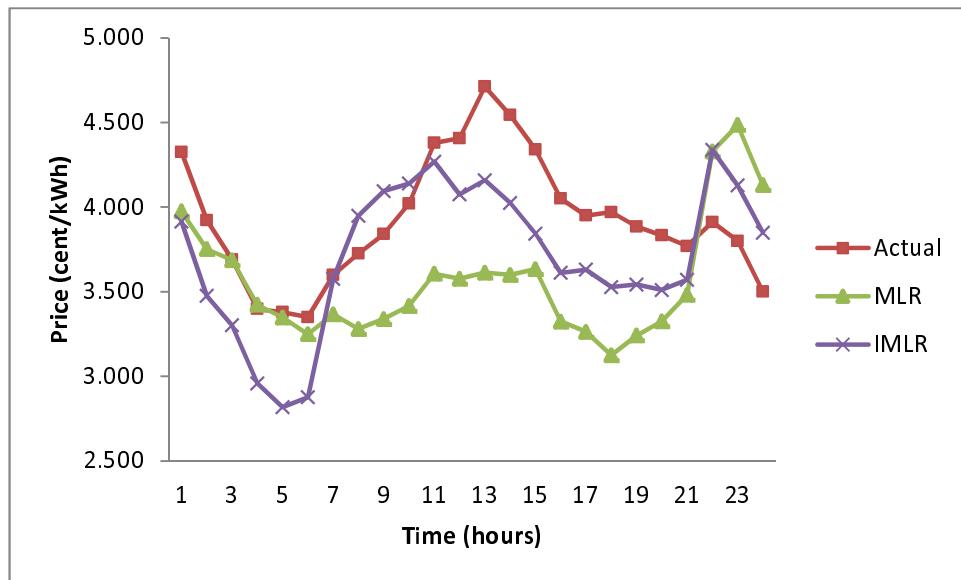


Figure 5.2 Monday day-ahead actual and forecasted power price for spring.

Table 5.3 Summer IMLR Forecasting Accuracy Improvement

Summer	Mon	Tue	Wed	Thu	Fri	Sat	Sun
<b>Old MAPE</b>	6.48	6.27	5.67	4.79	4.80	6.33	4.38
<b>New MAPE</b>	3.18	2.68	3.40	5.31	3.58	3.73	5.90
<b>%Improvement</b>	50.92	57.24	40.02	-11.03	25.37	41.15	-34.77

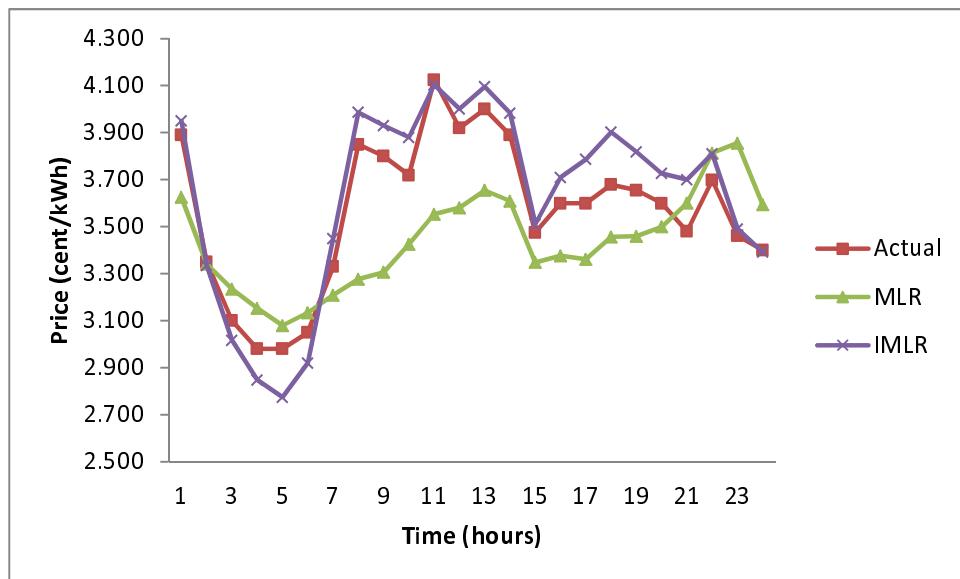


Figure 5.3 Monday day-ahead actual and forecasted power price for summer.

Table 5.4 Fall IMLR Forecasting Accuracy Improvement

Fall	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Old MAPE	41.35	11.14	5.05	3.85	4.69	14.43	22.37
New MAPE	17.64	13.03	4.39	3.53	3.51	5.16	13.66
%Improvement	57.34	-16.93	13.06	8.41	25.07	64.24	38.92

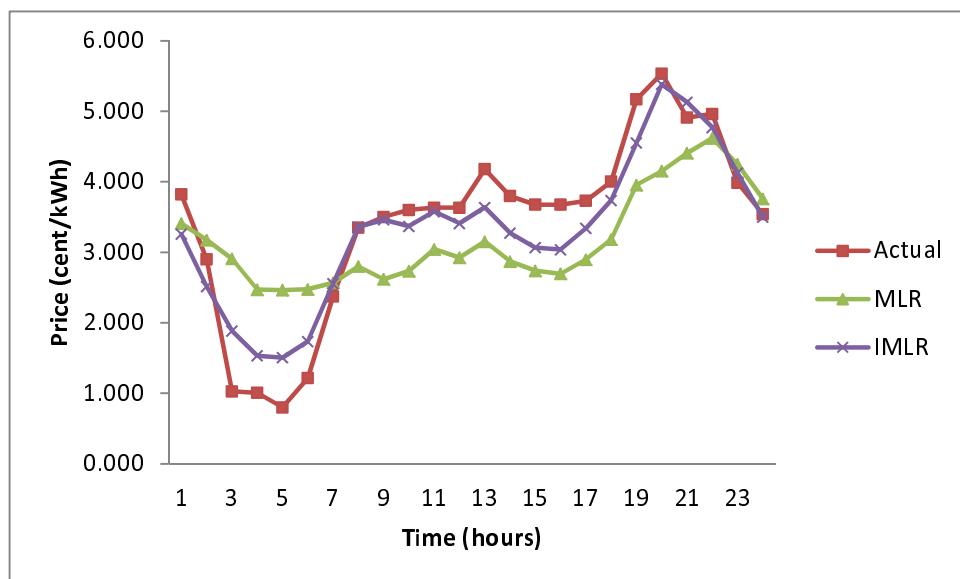


Figure 5.4 Monday day-ahead actual and forecasted power price for fall.

In Table 5.5, the seasonal and annual percentage probabilities improvement per week and the seasonal and annual average weekly percentage improvements are presented. As shown in the shaded cells of this table, the annual percentage probability improvement is 75% and the annual average percentage improvement is 33.84%. This result indicates better forecasting accuracy improvement using the proposed IMLR approach.

Table 5.5 Seasonal and Annual Improvement Probabilities and Average IMLR Forecasting Accuracy Improvements

	Winter	Spring	Summer	Fall	Annual
%Improvement Probability	71.43	71.43	71.43	85.71	75.00
%Average Improvement	20.09	37.80	42.94	34.50	33.84

### 5.3 INNOVATED NEURAL NETWORK FORECASTING

ANN is one of the most successful forecasting techniques used today in financial markets. In this section, the proposed IF approach is investigated using ANN. For Innovated Neural Networks (INNs), the training output error (TOE) is the core of the innovations. INN forecasting according to the involvement technique of innovations is classified as direct and indirect INN forecasting.

#### 5.3.1 Direct INN Forecasting

In this INN forecasting technique, innovations or average training output error (ATOE) are initially calculated after training the neural network. Then, the computed innovations are directly added to the neural network forecasted results. After that, the resultant MAPEs using ANN and INN are compared. The utilized ATOEs for each season are shown in the Appendix. Tables 5.6, 5.7, 5.8 and 5.9 present seasonal ANN MAPE (Old), INN MAPE (New) and the percentage improvements. From each season, Tuesday's actual and forecasted spot power prices using ANN and INN are illustrated in Figures 5.5, 5.6, 5.7 and 5.8.

Table 5.6 Winter Direct INN (DINN) Forecasting Accuracy Improvement

Winter	Mon	Tue	Wed	Thu	Fri	Sat	Sun
<b>Old MAPE</b>	21.08	6.87	9.49	7.55	6.41	13.35	11.84
<b>New MAPE</b>	25.18	4.96	6.42	11.29	4.57	16.33	10.05
<b>%Improvement</b>	-19.47	27.79	32.34	-49.44	28.74	-22.33	15.14

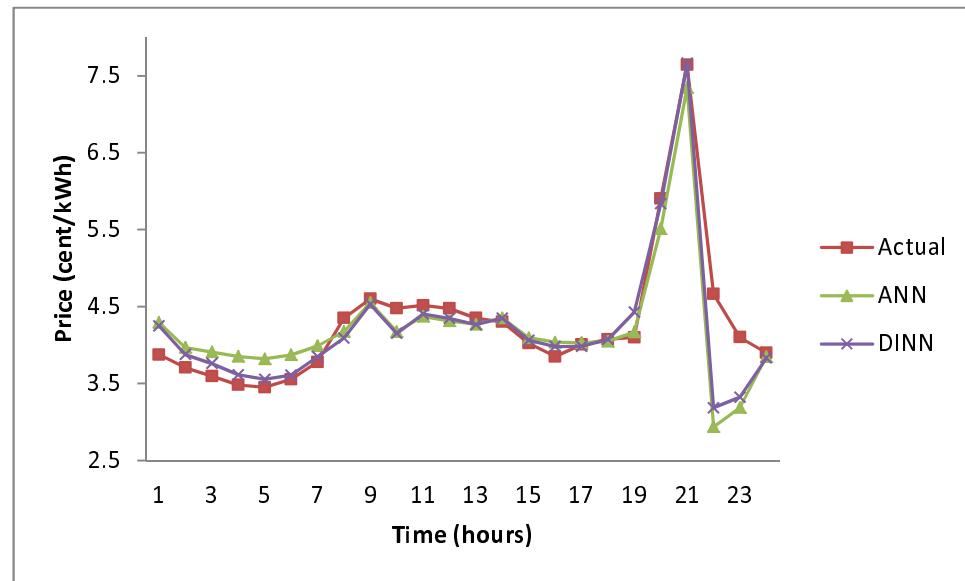


Figure 5.5 Tuesday day-ahead actual and forecasted power price for winter.

Table 5.7 Spring DINN Forecasting Accuracy Improvement

Spring	Mon	Tue	Wed	Thu	Fri	Sat	Sun
<b>Old MAPE</b>	11.53	9.76	6.67	4.29	10.00	4.39	18.08
<b>New MAPE</b>	17.65	8.94	4.22	4.74	9.67	5.18	15.14
<b>%Improvement</b>	-53.08	8.39	36.74	-10.63	3.32	-18.11	16.25

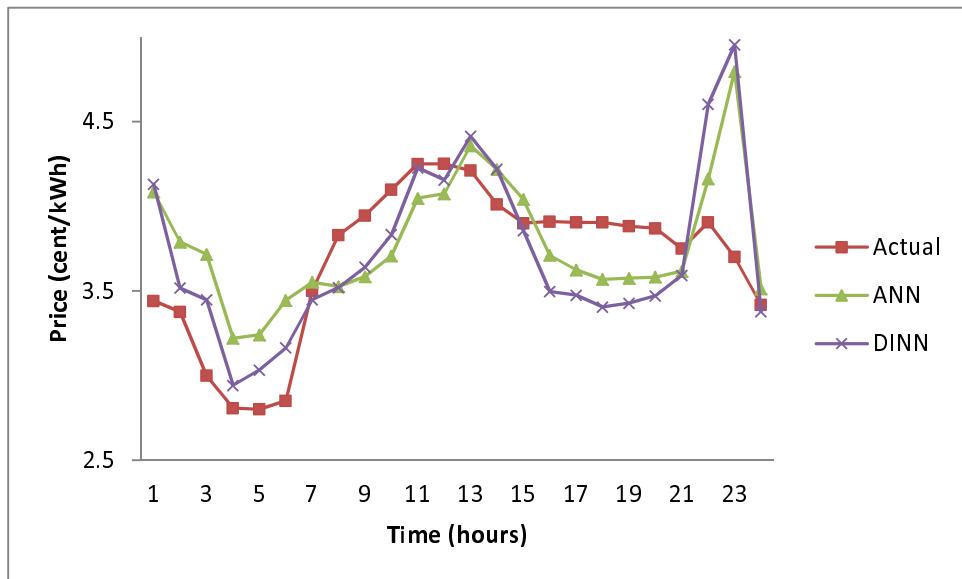


Figure 5.6 Tuesday day-ahead actual and forecasted power price for spring.

Table 5.8 Summer DINN Forecasting Accuracy Improvement

Summer	Mon	Tue	Wed	Thu	Fri	Sat	Sun
<b>Old MAPE</b>	8.08	6.49	4.61	5.03	6.42	5.80	4.47
<b>New MAPE</b>	9.67	4.87	6.59	7.68	6.30	10.55	11.85
<b>%Improvement</b>	-19.61	25.02	-43.20	-52.59	1.73	-81.82	-164.96

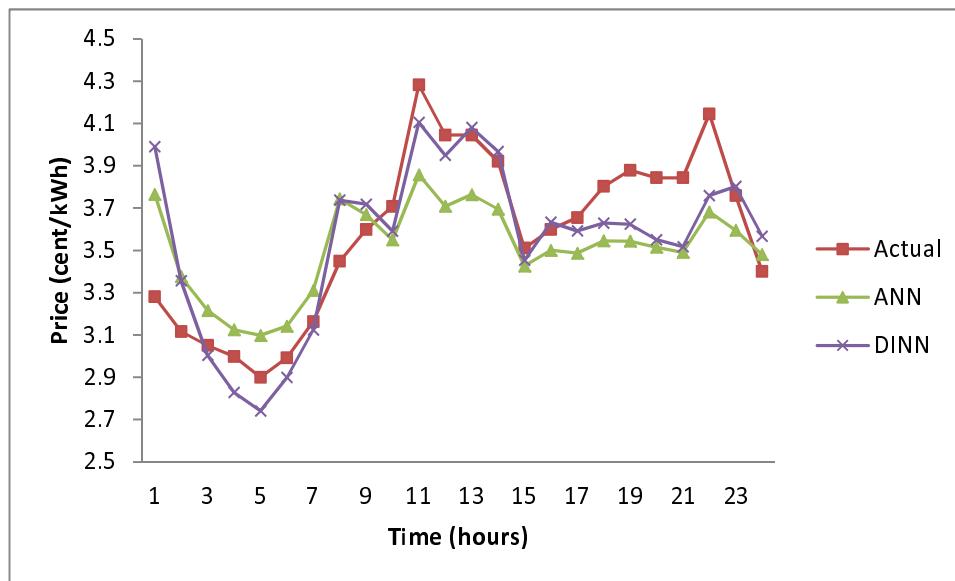


Figure 5.7 Tuesday day-ahead actual and forecasted power price for summer.

Table 5.9 Fall DINN Forecasting Accuracy Improvement

Fall	Mon	Tue	Wed	Thu	Fri	Sat	Sun
<b>Old MAPE</b>	45.88	18.13	11.38	6.01	6.56	14.40	23.14
<b>New MAPE</b>	44.32	16.69	10.86	10.70	5.52	14.95	17.38
<b>%Improvement</b>	3.40	7.94	4.63	-78.01	15.86	-3.82	24.88

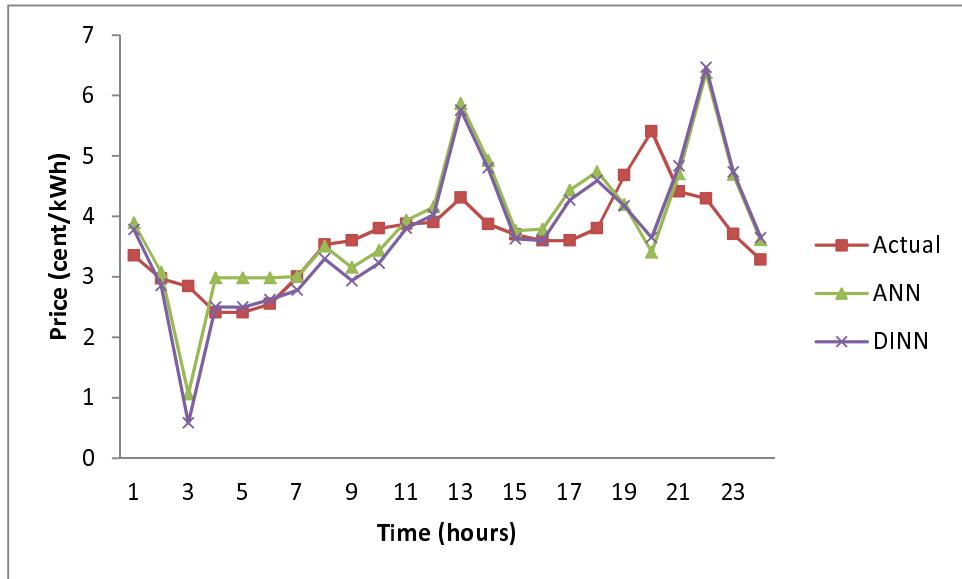


Figure 5.8 Tuesday day-ahead actual and forecasted power price for fall.

In Table 5.10, seasonal and annual percentage improvement probabilities and percentage average forecasting accuracy improvements are shown. From the shaded cells, the annual percentage improvement probability using direct INN is 53.57%. This percentage is higher for the fall, winter and spring seasons and very low for the summer season.

Table 5.10 Seasonal and Annual Improvement Probabilities and Average DINN Forecasting Accuracy Improvements

	Winter	Spring	Summer	Fall	Annual
<b>%Improvement Probability</b>	57.14	57.14	28.57	71.43	53.57
<b>%Average Improvement</b>	26.00	16.17	13.38	11.34	16.72

### 5.3.2 Indirect INN Forecasting

Another way for employing innovations to enhance neural network forecasting accuracy is indirect INN. This approach saves TOE found from the first neural network running iteration to be used in the next iteration. The proposed INN forecasting algorithm is introduced in the flow chart of Figure 5.9.

As shown in the flow chart, the same steps for ANN forecasting as stated in the previous chapter are performed first. Then, after the first neural network running iteration, the initial weights and biases of the neural network are saved and the training output error (TOE) calculated and saved for the next iteration. In the next step, the innovations' generation model (IGM) is selected. The implemented IGMs for indirect INN are exact, random or least square models. These innovations are then scaled through multiplication by the forecasting mean absolute error (MAE) and the mean squared error (MSE). Before a new training iteration, the new training output set (TOS) is produced by adding the scaled innovations to the old TOS for the next neural network training.

The neural network with the new TOS and with the same features and IWB of the first neural network is run to train the network with the new TOS. If the resulting mean absolute percentage error (MAPE) is less than the old one, the process of innovations generation and neural network running is repeated until the least MAPE or the best improvement is achieved. Convergence of this process to better forecasting accuracy results depends on the price volatility level, the neural network initial parameters and the utilized innovations' generation model.

#### 5.3.2.1 Exact Innovations

As stated earlier, the TOEs are determined and saved in a matrix. After that, the IGM is selected. If the exact IGM is selected, the innovations are directly generated from TOE. The generated innovations using this model are simply the scaled TOEs.

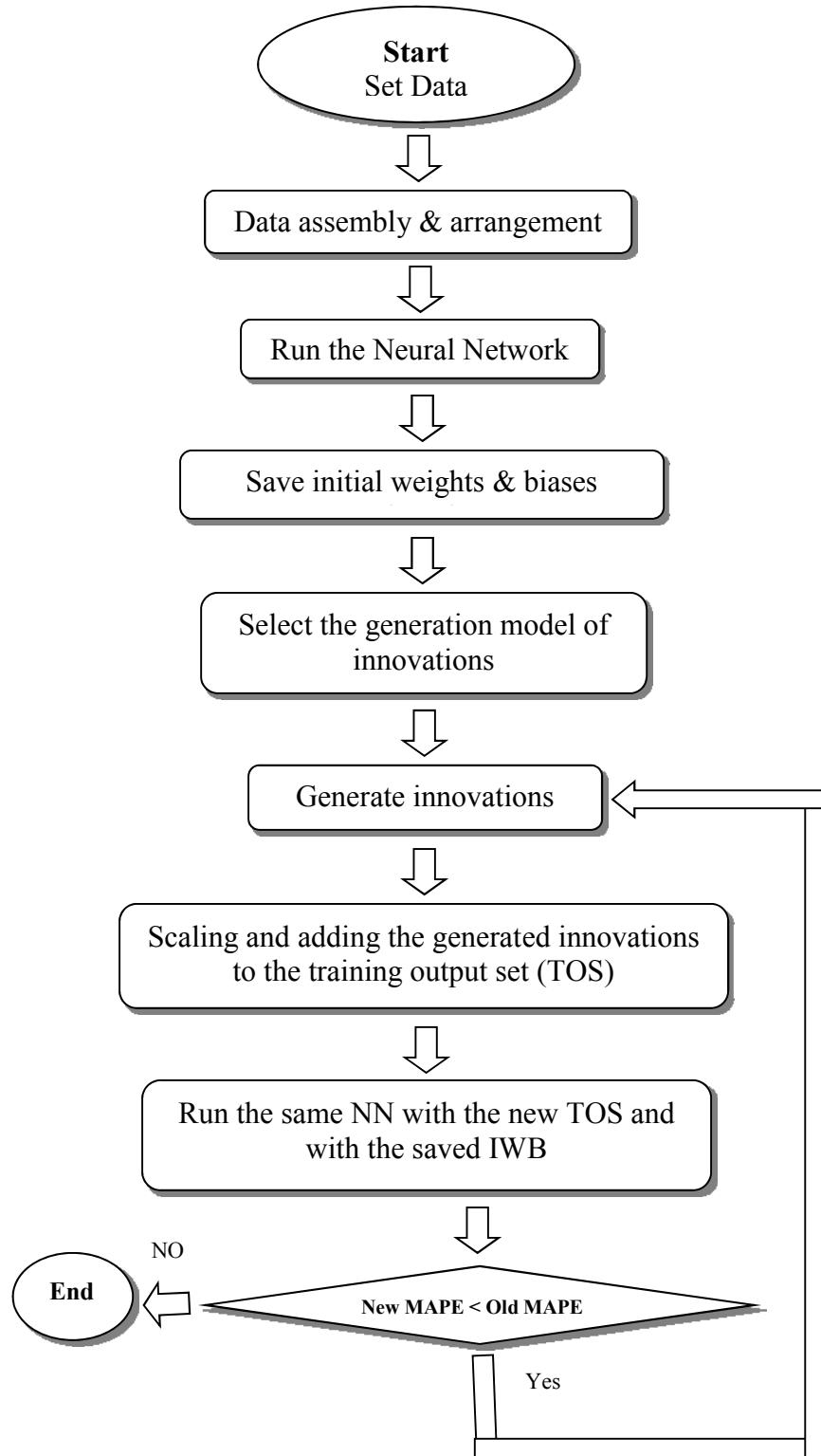


Figure 5.9 Indirect Innovated Neural Network algorithm.

These TOEs are scaled through multiplication by the MSE and the MAE of the forecasting results. In Table 5.11, seasonal and annual percentage improvement probabilities and percentage average for casting accuracy improvements using exact IGM are shown.

Table 5.11 Seasonal and Annual Improvement Probabilities and Average Indirect Exact INN Forecasting Accuracy Improvements

	<b>Winter</b>	<b>Spring</b>	<b>Summer</b>	<b>Fall</b>	<b>Annual</b>
<b>%Improvement Probability</b>	0.00	28.57	71.43	14.29	28.57
<b>%Average Improvement</b>	0.00	17.33	8.56	18.24	11.03

### 5.3.2.2 Random Innovations

To create random innovations, first a random set is generated using the “randn” function, which is treated to have the same statistical properties as the TOE set. After calculating the means and standard deviations of both the random and the TOE sets, the random set statistical treatment is applicable through the following equation:

$$r_{i(new)} = \frac{(r_{i(old)} - M(r)) * s(e)}{s(r)} + M(e) \quad (5.1)$$

where  $r_i$  is the  $i$ th number of the random set,  $M(r)$  is the mean of the random set,  $S(r)$  is the standard deviation of the random set,  $M(e)$  is the mean of TOE set, and  $S(e)$  is the standard deviation of the TOE set. After statistical treatment, the new random set and the TOE set will have identical statistical properties. The new random set is then multiplied by the MSE and MAE of the old forecasting results to generate the random innovations. Seasonal and annual percentage improvement probabilities and percentage average forecasting accuracy improvements using random IGM are presented in Table 5.12.

Table 5.12 Seasonal and Annual Improvement Probabilities and Average Indirect Random INN Forecasting Accuracy Improvements

	<b>Winter</b>	<b>Spring</b>	<b>Summer</b>	<b>Fall</b>	<b>Annual</b>
<b>%Improvement Probability</b>	0.00	42.86	57.14	28.57	32.14
<b>%Average Improvement</b>	0.00	5.99	12.90	18.59	9.37

### 5.3.2.3 Least Squares Innovations

The approach that provides an overall solution which minimizes the sum of the squares of the errors made in solving every single equation is called the Least Squares approach. Its most significant application is in data fitting. Furthermore, it is usually used to generate estimators and other statistics in regression analysis. In the least-squares sense, the best fit minimizes the sum of squared residuals which is the sum of squared differences between the value predicted by the model and the actual value. TOE is the considered residuals in this study. When the sum of squared residuals:

$$S = \sum_{i=1}^n (r_i^2) \quad (5.2)$$

is a minimum, the least squares method finds its optimum. To generate least square innovations, it is assumed that the actual and the forecasted training output data sets are dependent variables (Y) and independent variables (X), respectively. Then, the coefficients of a polynomial P(X) of degree N that best fits the data Y in a least-squares sense is worked out. After that, the created polynomial P is evaluated at X to produce the estimated Y. Lastly, the least square innovations is generated by subtracting the estimated Y from the actual Y. Seasonal and annual percentage improvement probabilities and percentage average for casting accuracy improvements using least square IGM are shown in Table 5.13.

Table 5.13 Seasonal and Annual Improvement Probabilities and Average Indirect Least Square INN Forecasting Accuracy Improvements

	Winter	Spring	Summer	Fall	Annual
%Improvement Probability	14.29	28.57	28.57	0.00	17.86
%Average Improvement	15.03	11.47	12.43	0.00	9.73

## 5.4 CONCLUDING RESULTS

Innovations were appended to M LR and A NN forecasting techniques to improve day-ahead power price forecasting. The percentage improvement probabilities and percentage average improvements of implemented direct and indirect INN approaches are illustrated in Figures 5.10 and 5.11. Direct INN showed the highest annual percentage improvement probability as well as the best annual percentage average improvement. However, the probability of improvement in the summer season is much higher using exact INN, which is an indirect INN technique. While the exact IGM is simpler than other IGMs, it showed the best annual average forecasting improvement among indirect INN techniques.

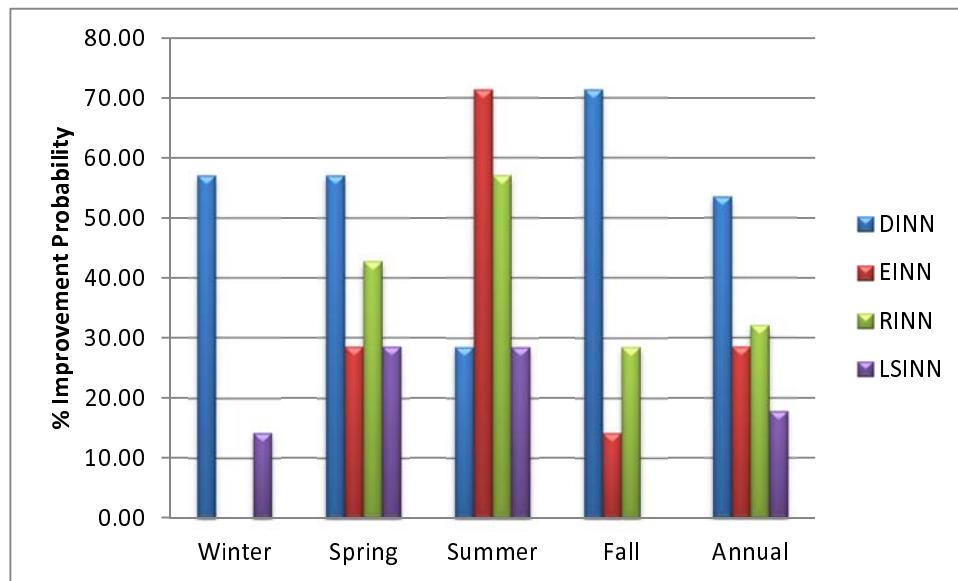


Figure 5.10 Seasonal and annual percentage improvement probabilities of direct and indirect INN.

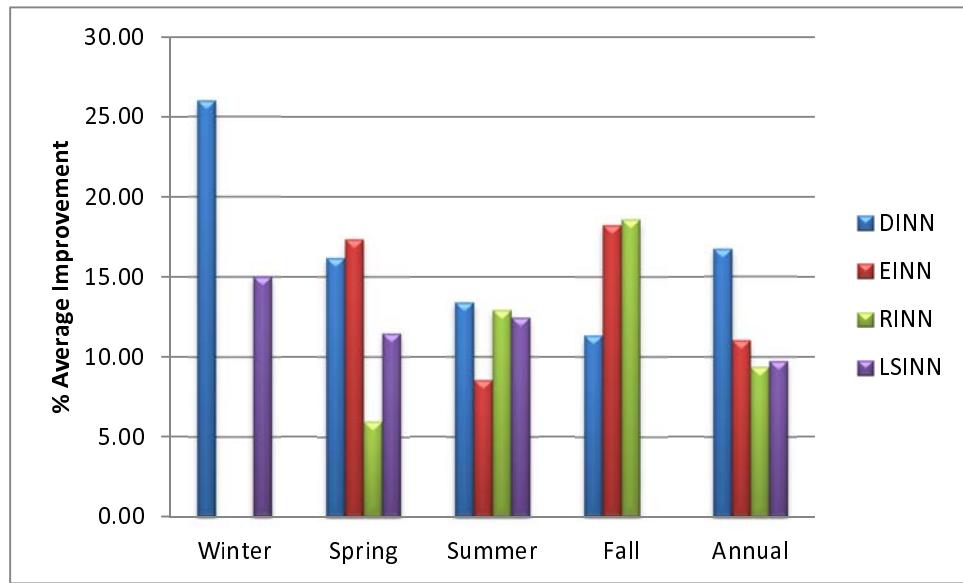


Figure 5.11 Seasonal and annual percentage average improvements of direct and indirect INN.

Further, as the direct INN can be considered the most effective INN forecasting technique, it is compared with IMLR forecasting probability improvement and average improvement in the following figures. These two different methods were used to forecast the OMEL day-ahead power price of the same seven days of each 2009 seasons, though the annual improvement probability and the annual average improvement using IMLR is much higher. The superiority of IMLR is more apparent in the summer season, where the difference in percentage improvement probability and percentage average improvement is 43% and 29% respectively.

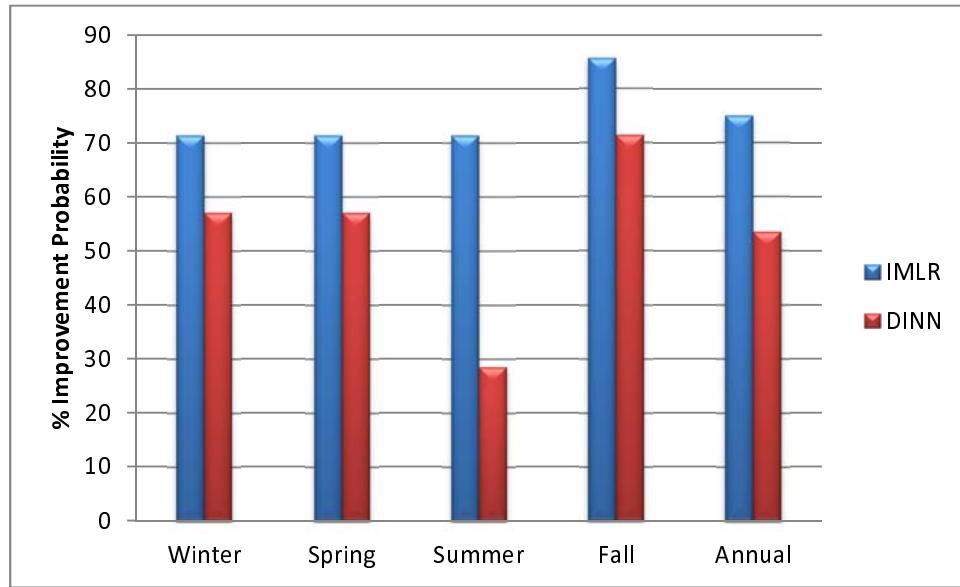


Figure 5.12 Seasonal and annual percentage improvement probabilities of DNN and IMLR.

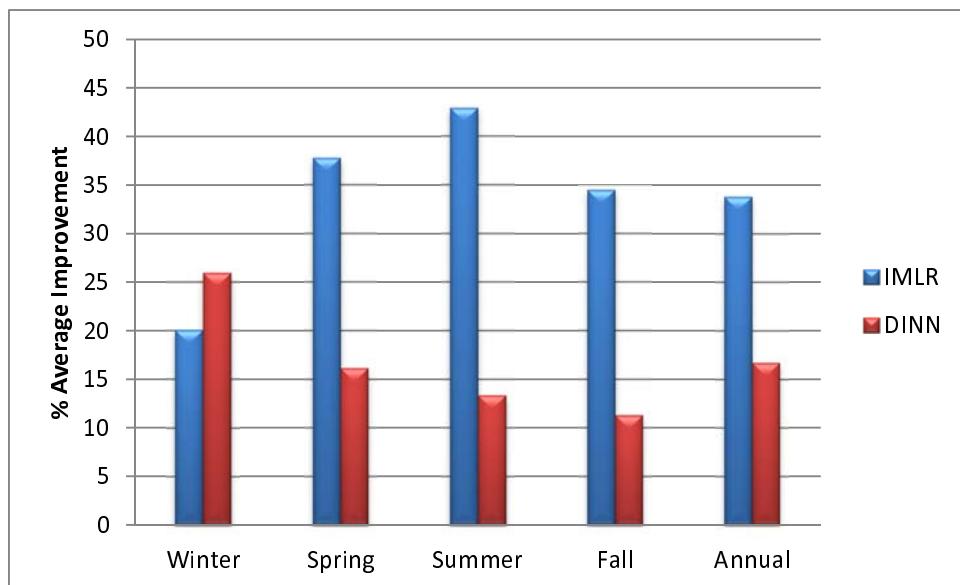


Figure 5.13 Seasonal and annual percentage average improvements of DNN and IMLR.

## **5.5 SUMMARY**

In this chapter, the idea of innovations was introduced. Then, the seasonal forecast results found in previous chapters using MLR and ANN were innovated and the new results were presented. Finally, the direct INN, indirect INN and IMLR probabilities and average forecasting improvements were investigated and compared.

## **CHAPTER 6 CONCLUSIONS**

This chapter summarizes the results of this research and emphasizes on the significance of the accuracy improvement achieved using IF approaches. Furthermore, some future works are suggested and discussed.

### **6.1 RESEARCH CONCLUSIONS**

Short-term electricity price forecasting is an important planning tool for the electric power industry, especially in deregulated and competitive power markets. Such forecasting directly influences the energy market by reducing the risk of under- or over-estimating the revenue for producers and providing better risk management [3]. Hence, improvements in the accuracy of electricity price forecasting can lead to significant financial benefits for both producers and consumers [8]. The economic incentive for the power industry raises the importance of devising new techniques to solve this significant and complex power system challenges.

This thesis started with an introduction to electricity price forecasting, research motivation, objectives and contributions, followed by a literature review of electricity price forecasting approaches. In the subsequent chapter, Linear Regression and the targeted deregulated power market were described, and seasonal spot power price data using statistical and mean reverting process techniques were analyzed. As well, the day-ahead OMEL power price was forecasted using different Linear Regression approaches. After that, an overview of ANN was introduced and used to forecast the day-ahead OMEL power price again. In the fifth chapter, innovations were integrated with MLR and ANN forecasting techniques to investigate the effectiveness and applicability of this new approach.

In this thesis, MLR and ANN were modified using their innovations to improve the seasonal day-ahead marginal power price forecasting results of the OMEL power market.

In general, our findings indicate that IMLR resulted in better price forecasting improvements. In the case of INN, direct INN came up with better annual results. In comparing direct and indirect INNs' seasonal results, we found that the probability of forecasting accuracy improvement using indirect INN is higher for the summer season, as it is less volatile. Conversely, the probabilities of forecasting accuracy improvement using direct INN are higher for other seasons which are more volatile.

The maximum achieved seasonal forecasting accuracy improvement probability is 86% for IMLR and 71% for direct INN, and the maximum accomplished seasonal forecasting accuracy average improvement is 43% for IMLR and 26% for direct INN. In one year, the maximum achieved annual forecasting accuracy improvement probability is 75% for IMLR and 53% for direct INN, and the maximum accomplished annual forecasting accuracy average improvement is 34% for IMLR and 17% for direct INN.

These results show that there are significant forecasting accuracy improvement probabilities and forecasting accuracy improvements in competitive power markets by implementing the IF approach. However, the effectiveness of this algorithm is generally affected by the volatility level of the system and is more specifically related to the network IWB and the selected IGM for INN.

## **6.2 FUTURE WORK**

For future work, the IF technique can be implemented and investigated with other simple and hybrid price forecasting techniques to improve forecasting accuracy. This new approach can also be utilized and tested as a price forecaster in other deregulated power markets.

In the case of INN, since forecasting accuracy improvement is affected by the network IWB, the optimal IWB can be found using a heuristic optimization method such as PSO. These optimal IWB would considerably enhance the probability and level of forecasting accuracy improvement.

Nonlinearity and volatility of the spot price power market data definitely affects forecasting results. To overcome this challenge, some researchers have proposed to conduct forecasting after processing the data by converting to the frequency domain. Working with innovations after data conversion is an interesting future forecasting work.

## REFERENCES

- [1] Linlin Hu, G. Taylor, Hai-Bin Wan and M. Irving. A review of short-term electricity price forecasting techniques in deregulated electricity markets. Presented at Universities Power Engineering Conference (UPEC), 2009 Proceedings of the 44th International.
- [2] G. Li, C. - Liu, C. Mattson and J. Lawarree. (2007), Day-ahead electricity price forecasting in a grid environment. *Power Systems, IEEE Transactions on* 22(1), pp. 266-274.
- [3] J. P. S. Catalao, S. J. P. S. Mariano, V. M. F. Mendes and L. A. F. M. Ferreira. An artificial neural network approach for short-term electricity prices forecasting. Presented at Intelligent Systems Applications to Power Systems, 2007. ISAP 2007. International Conference on.
- [4] V. Iyer, Chum Che Fung and T. Gedeon. A fuzzy-neural approach to electricity load and spot-price forecasting in a deregulated electricity market. Presented at TENCON 2003. Conference on Convergent Technologies for Asia-Pacific Region.
- [5] J. Contreras and J. R. Santos. Short-term demand and energy price forecasting. Presented at Electrotechnical Conference, 2006. MELECON 2006. IEEE Mediterranean.
- [6] Hui Zhou, Xinhua Wu, Wei Wang and Liping Chen. Forecast of next day clearing price in deregulated electricity market. Presented at Systems, Man and Cybernetics, 2009. SMC 2009. IEEE International Conference on.
- [7] N. M. Pindoria, S. N. Singh and S. K. Singh. (2008), An adaptive wavelet neural network-based energy price forecasting in electricity markets. *Power Systems, IEEE Transactions on* 23(3), pp. 1423-1432.
- [8] Guang Li, Chen-Ching Liu, J. Lawarree, M. Gallanti and A. Venturini. State-of-the-art of electricity price forecasting. Presented at CIGRE/IEEE PES, 2005. International Symposium.
- [9] P. Mandal, T. Senju and T. Funabashi. Neural network models to predict short-term electricity prices and loads. Presented at Industrial Technology, 2005. ICIT 2005. IEEE International Conference on.
- [10] A. Karsaz, H. R. Mashhadi and R. Eshraghian. Cooperative co-evolutionary approach to electricity load and price forecasting in deregulated electricity markets. Presented at Power India Conference, 2006 IEEE.
- [11] C. P. Rodriguez and G. J. Anders. (2004), Energy price forecasting in the ontario competitive power system market. *Power Systems, IEEE Transactions on* 19(1), pp. 366-374.

- [12] A. J. Conejo, J. Contreras, R. Espinola and M. A. Plazas. (2005, 9). Forecasting electricity prices for a day-ahead pool-based electric energy market. *Int. J. Forecast.* 21(3), pp. 435-462.
- [13] S. K. Aggarwal, L. M. Saini and A. Kumar. (2009, 1). Electricity price forecasting in deregulated markets: A review and evaluation. *International Journal of Electrical Power & Energy Systems* 31(1), pp. 13-22.
- [14] O. B. Fosso, A. Gjelsvik, A. Haugstad, B. Mo and I. W. Angensteen. (1999), Generation scheduling in a deregulated system. the norwegian case. *Power Systems, IEEE Transactions on* 14(1), pp. 75-81.
- [15] A. Kian and A. Keyhani. Stochastic pricing modeling of electricity in deregulated energy markets. Presented at System Sciences, 2001. Proceedings of the 34th Annual Hawaii International Conference on.
- [16] J. Contreras, R. Espinola, F. J. Nogales and A. J. Conejo. (2003), ARIMA models to predict next-day electricity prices. *Power Systems, IEEE Transactions on* 18(3), pp. 1014-1020.
- [17] R. C. Garcia, J. Contreras, M. van Akkeren and J. B. C. Garcia. (2005), A GARCH forecasting model to predict day-ahead electricity prices. *Power Systems, IEEE Transactions on* 20(2), pp. 867-874.
- [18] A. J. Conejo, M. A. Plazas, R. Espinola and A. B. Molina. (2005), Day-ahead electricity pricing for forecasting using the wavelet transform and ARIMA models. *Power Systems, IEEE Transactions on* 20(2), pp. 1035-1042.
- [19] F. J. Nogales, J. Contreras, A. J. Conejo and R. Espinola. (2002), Forecasting next-day electricity prices by time series models. *Power Systems, IEEE Transactions on* 17(2), pp. 342-348.
- [20] A. J. Wang and B. Ramsay. (1998, 12/7). A neural network based estimator for electricity spot-pricing with particular reference to weekend and public holidays. *Neurocomputing* 23(1-3), pp. 47-57.
- [21] B. R. Szkuta, L. A. Sanabria and T. S. Dillon. (1999), Electricity short-term forecasting using artificial neural networks. *Power Systems, IEEE Transactions on* 14(3), pp. 851-857.
- [22] Li Zhang and P. B. Luh. (2005), Neural network-based market clearing price prediction and confidence interval estimation with an improved extended kalman filter method. *Power Systems, IEEE Transactions on* 20(1), pp. 59-66.

- [23] J. D. Nicolaisen, C. W . R ichter J r. and G. B. S heble. P rice si gnal anal ysis for competitive electric generation companies. Presented at Electric Utility Deregulation and Restructuring and P ower Technolo gies, 2000. P roceedings. DRPT 2000. International Conference on.
- [24] Ying Yi Hon g and Chuan-Yo Hsiao. Locational ma rginal price forec asting in deregulated e lectric marke ts using a re current neural n etwork. Pres ented at Power Engineering Society Winter Meeting, 2001. IEEE.
- [25] Y. -. Hong and C. -. Hsiao. (2002), Locational mar ginal pric e for ecasting in deregulated electricity markets using artificial intelligence. *Generation, Transmission and Distribution, IEE Proceedings- 149(5)*, pp. 621-626.
- [26] D. F . Specht. (1991), A general regression neural network. *Neural Networks, IEEE Transactions on 2(6)*, pp. 568-576.
- [27] A. M. Gonz alez, A . M. S. Roque and J. G arcia-Gonzalez. (2005), Modeling and forecasting ele ctricity p rices with input/output hi dden markov models. *Power Systems, IEEE Transactions on 20(1)*, pp. 13-24.
- [28] L i Zhang, P. B . L uh and K. Kasi viswanathan. (2003), Ener gy clearin g price prediction a nd c onfidence inte rval e stimation with c ascaded n eural n etworks. *Power Systems, IEEE Transactions on 18(1)*, pp. 99-105.
- [29] Jau-Jia Guo and P. B. Luh. (2004), Improving market cle aring price prediction b y using a committee machine of n eural networks. *Power Systems, IEEE Transactions on 19(4)*, pp. 1867-1876.
- [30] P. Areekul, T. Sen jyu, H. To yama and A . Yona. (2010), A hybrid ARIMA and neural network model f or short-term pric e fo recasting in de regulated market. *Power Systems, IEEE Transactions on 25(1)*, pp. 524-530.
- [31] H. Y. Yamin, S. M. Shahidehpour and Z . Li. (2004, 10). Ad aptive short-term electricity p rice fo recasting using artific ial neural networks in the restructured power markets. *International Journal of Electrical Power & Energy Systems 26(8)*, pp. 571-581.
- [32] C. Kim, I . Yu and Y. H. Son g. (2002, 9). Prediction of s ystem marginal price of electricity usi ng w avelet t ransform anal ysis. *Energy Conv ersion an d Managem ent 43(14)*, pp. 1839-1851.
- [33] A. Wang and B. Ramsay. Prediction of system marginal price in the UK power pool using neural networks. Presented at Neural Networks,1997., International Conference on.
- [34] N. Amjad y. (2006), Da y-ahead pri ce forecasting o f electricity m arkets b y a ne w fuzzy neural network. *Power Systems, IEEE Transactions on 21(2)*, pp. 887-896.

- [35] Anonymous (2010, 6 December 2010). Regression analysis. *2010(25 November 2010)*, Available: [http://en.wikipedia.org/wiki/Regression\\_analysis](http://en.wikipedia.org/wiki/Regression_analysis). [Accessed: 29 Nov. 2010].
- [36] Anonymous Regression. *2010(25 November 2010)*, Available: <http://www.investopedia.com/terms/r/regression.asp>. [Accessed: 29 Nov. 2010].
- [37] W. J. Phillips February, 2010). Applied linear regression notes. Available: <http://www.math.dal.ca/courses/engm6671/notes/>. [Accessed: 11 Nov. 2010].
- [38] A. J. Conejo. The electricity market of mainland Spain: A brief critical review. Presented at Power Engineering Society General Meeting, 2007. IEEE.
- [39] Koen Rademaekers, Allister Slingenbergh, Salim Morsy. (2008, 09 December 2008). Review and analysis of EU wholesale energy markets. [Online]. Available: [http://ec.europa.eu/energy/gas\\_electricity/studies/doc/2008\\_eu\\_wholesale\\_energy\\_market\\_historical.pdf](http://ec.europa.eu/energy/gas_electricity/studies/doc/2008_eu_wholesale_energy_market_historical.pdf). [Accessed: 15 Nov. 2010].
- [40] Anonymous 2010 Available: <http://www.omel.es/en/home>. [Accessed: 14 Sep. 2010].
- [41] D. S. Carlos Blanch. (2001, June 2001). Mean reverting processes – energy price processes used for derivatives pricing & risk management. *Energy Pricing* [Online]. Available: [http://www.math.ncsu.edu/finmath/imsm2010/references/Mean\\_Reverting\\_Processes.pdf](http://www.math.ncsu.edu/finmath/imsm2010/references/Mean_Reverting_Processes.pdf). [Accessed: 11 Nov. 2010].
- [42] Qiaolin Ding , Jing Tang and Jianxin Liu. Application of new FCMAC neural network in power system marginal price forecasting. Presented at Power Engineering Conference, 2005. IPEC 2005. the 7th International.
- [43] P. Mandal, T. Senjyu, K. Uezato and T. Funabashi. Several-hours-ahead electricity price and load forecasting using neural networks. Presented at Power Engineering Society General Meeting, 2005. IEEE.
- [44] G. Zhang, B. Eddy Patuwo and M. Y. Hu. (1998, 3/1). Forecasting with artificial neural networks: The state of the art. *Int. J. Forecast.* 14(1), pp. 35-62.
- [45] C. Unsihua y-Vila, A. C. Zambroni de Souza, J. W. Marangon-Lima and P. P. Balestrassi. (2010, 2). Electricity demand and spot price forecasting using evolutionary computation combined with chaotic nonlinear dynamic model. *International Journal of Electrical Power & Energy Systems* 32(2), pp. 108-116.
- [46] S. Mukhopadhyay. (2003, 7). Neural and adaptive systems: Fundamentals through simulations: José C. Principe, Neil R. Euliano and W. Curt Lefebvre; John Wiley & Sons, Inc., USA, 2000, ISBN 0-471-35167-9. *Automatica* 39(7), pp. 1313-1315.

- [47] Jeong-Kyu Lee, Jong-Bae Park, Joong-Rin Shin and K. Y. Lee. A system marginal price forecasting based on an artificial neural network adapted with rough set theory. Presented at Power Engineering Society General Meeting, 2005. IEEE.
- [48] P. Mandal, A. K. Srivastava, M. Negnevitsky and Jung-Wook Park. An effort to optimize similar days parameters for ANN based electricity price forecasting. Presented at Industry Applications Society Annual Meeting, 2008. IAS '08. IEEE.
- [49] V. Subramanian, M. S. Hung and V. Subramanian. (1993, 09/01). A GRG2-based system for training neural networks: Design and computational experience. *ORSA Journal on Computing* 5(4), pp. 386.
- [50] P. Mandal, T. Senju, N. Urasaki, T. Funabashi and A. K. Srivastava. (2007), A novel approach to forecast electricity price for PJM using neural network and similar days method. *Power Systems, IEEE Transactions on* 22(4), pp. 2058-2065.
- [51] N. Amjadi and A. Daraeipour. Day-ahead electricity price forecasting using the relief algorithm and neural networks. Presented at Electricity Market, 2008. EEM 2008. 5th International Conference on European.
- [52] D. Srinivasan, Fen Chao Yang and Ah Choy Liew. Electricity price forecasting using evolved neural networks. Presented at Intelligent Systems Applications to Power Systems, 2007. ISAP 2007. International Conference on.
- [53] H. Mori and A. Awata. A hybrid method of clipping and artificial neural network for electricity price zone forecasting. Presented at Probabilistic Methods Applied to Power Systems, 2006. PMAPS 2006. International Conference on.
- [54] Feng Gao, Xiaohong Guan, Xi-Ren Cao and A. Papalexopoulos. Forecasting power market clearing price and quantity using a neural network method. Presented at Power Engineering Society Summer Meeting, 2000. IEEE.
- [55] T. Kailath. (1970), The innovations approach to detection and estimation theory. *Proceedings of the IEEE* 58(5), pp. 680-695.
- [56] H. Aasnaes and T. Kailath. (1973), An innovations approach to least-squares estimation--part VII: Some applications of vector autoregressive-moving average models. *Automatic Control, IEEE Transactions on* 18(6), pp. 601-607.

## APPENDIX

### Winter Regression Equations

SLR:  $y = 1.014 - 0.2176 * s_d$

QLR:  $y = 0.8209 - 0.1310 * s_d - 0.008989 * s_d^2$

CLR:  $y = 2.313 - 1.290 * s_d + 0.2538 * s_d^2 - 0.01793 * s_d^3$

MLR\_2:  $y = 0.825 - 0.354 * s_d + 0.174 * s_{d-1}$

MLR\_3:  $y = 0.537 - 0.412 * s_d - 0.0285 * s_{d-1} + 0.319 * s_{d-2}$

### Spring Regression Equations

SLR:  $y = 1.565 - 0.4217 * s_d$

QLR:  $y = 2.126 - 0.7277 * s_d + 0.04084 * s_d^2$

CLR:  $y = 9.783 - 7.050 * s_d + 1.734 * s_d^2 - 0.1474 * s_d^3$

MLR\_2:  $y = 1.33 - 0.488 * s_d + 0.131 * s_{d-1}$

MLR\_3:  $y = 1.08 - 0.516 * s_d + 0.0343 * s_{d-1} + 0.193 * s_{d-2}$

### Summer Regression Equations

SLR:  $y = 1.078 - 0.3084 * s_d$

QLR:  $y = 1.578 - 0.6176 * s_d + 0.04663 * s_d^2$

CLR:  $y = 5.457 - 4.548 * s_d + 1.321 * s_d^2 - 0.1332 * s_d^3$

MLR\_2:  $y = 0.865 - 0.435 * s_d + 0.185 * s_{d-1}$

MLR\_3:  $y = 0.661 - 0.478 * s_d + 0.0466 * s_{d-1} + 0.240 * s_{d-2}$

### Fall Regression Equations

SLR:  $y = 0.9795 - 0.2851 * s_d$

QLR:  $y = 1.263 - 0.4731 * s_d + 0.02889 * s_d^2$

CLR:  $y = 1.665 - 0.9222 * s_d + 0.1677 * s_d^2 - 0.01253 * s_d^3$

MLR\_2:  $y = 0.832 - 0.408 * s_d + 0.165 * s_{d-1}$

MLR\_3:  $y = 0.700 - 0.435 * s_d + 0.0718 * s_{d-1} + 0.159 * s_{d-2}$

**Table A.1 Winter Analyzed and Trained Data**  
 From 21<sup>st</sup> of December 2008 to 18<sup>th</sup> of February 2009

Day	Su	Mo	Tu	We	e	T	h	Fr	Sa	Su	Mo	Tu	We	e	T	h	Fr	Sa	Su
Hourly price (cent\kWh)	1	6.50	5.10	5.00	5.80	6.19	4.74	5.75	5.60	5.80	5.30	4.90	5.50	5.37	5.50	5.43			
	2	5.59	4.81	4.87	5.28	6.00	4.29	4.51	4.59	4.80	5.12	4.52	5.61	4.80	5.25	5.13			
	3	5.10	4.40	4.63	4.87	5.00	3.50	4.11	4.05	4.31	4.60	4.00	5.06	4.58	4.80	4.75			
	4	4.88	4.06	4.40	4.83	4.74	3.00	3.64	3.57	3.80	4.51	3.90	4.58	4.35	4.55	4.56			
	5	4.79	4.00	4.38	4.70	4.36	2.75	3.50	3.47	3.00	4.46	3.60	4.19	4.20	4.45	4.37			
	6	4.74	4.04	4.43	4.64	4.24	2.51	3.50	3.57	3.47	4.51	3.67	4.00	4.37	4.42	4.37			
	7	4.79	4.78	4.83	4.81	4.01	2.75	3.61	3.57	4.29	4.88	4.00	4.00	4.65	4.53	4.35			
	8	4.83	5.68	5.63	5.28	3.57	3.37	4.29	3.80	4.80	5.60	4.65	4.00	5.00	4.65	4.50			
	9	4.79	5.76	5.92	5.33	2.75	3.74	4.40	3.74	5.35	6.20	4.90	3.52	5.20	4.70	4.41			
	10	4.92	6.00	6.34	6.40	3.00	4.31	4.93	4.31	6.37	6.30	5.22	3.52	5.26	5.00	4.50			
	11	5.21	6.15	6.50	6.50	3.57	4.59	4.93	4.70	6.39	6.35	5.50	3.90	5.29	5.27	4.75			
	12	5.30	6.30	6.00	6.35	4.31	4.69	4.94	4.88	6.20	5.85	5.50	4.35	5.30	5.30	5.00			
	13	5.21	5.93	5.66	5.36	4.31	4.62	4.71	4.82	5.85	5.25	5.00	4.53	5.30	5.30	5.02			
	14	5.50	5.72	5.65	5.20	4.39	4.59	4.71	4.70	5.81	5.21	4.91	4.53	5.30	5.32	5.10			
	15	5.54	5.40	5.10	5.10	4.39	4.50	4.60	4.80	5.32	5.10	4.50	4.48	5.25	5.27	5.00			
	16	5.20	5.40	5.19	4.87	3.74	4.31	4.50	4.62	5.30	4.96	4.34	4.20	5.10	5.22	4.81			
	17	5.20	5.69	5.51	4.96	3.57	4.50	4.50	4.51	5.40	5.15	4.69	4.21	5.22	5.22	4.82			
	18	5.21	6.82	5.61	5.09	4.20	5.31	4.93	4.89	6.22	5.30	5.20	4.53	5.50	5.37	5.20			
	19	5.80	8.70	6.67	5.60	4.79	6.59	6.24	6.16	7.06	6.41	6.21	5.30	7.23	6.45	6.35			
	20	6.43	9.58	6.85	6.40	5.20	6.65	6.44	6.46	8.20	6.50	6.33	5.68	7.23	6.60	6.36			
	21	6.63	8.50	6.75	5.50	5.31	6.64	6.25	6.60	6.73	6.22	6.08	5.74	6.30	6.41	6.40			
	22	6.80	7.77	6.00	5.39	6.50	6.50	6.19	6.60	6.50	5.53	4.80	5.87	5.80	5.96	6.49			
	23	6.64	6.10	5.61	5.31	6.50	6.40	5.86	6.50	6.02	5.60	4.52	6.25	5.66	5.90	6.58			
	24	5.70	5.80	5.46	5.43	6.50	6.34	6.15	5.80	5.45	5.29	4.80	5.80	5.40	5.40	6.18			
Day	Mo	Tu	We	e	T	h	Fr	Sa	Su	Mo	Tu	We	e	T	h	Fr	Sa	Su	Mo
Hourly price (cent\kWh)	1	5.27	5.10	4.56	4.49	4.99	5.50	5.30	4.88	4.70	4.53	4.53	5.19	5.30	5.40	4.20			
	2	5.00	4.90	4.20	4.00	4.43	5.10	5.12	4.41	4.30	4.30	4.21	4.80	5.01	5.20	3.10			
	3	4.71	4.50	3.76	3.52	4.20	4.90	4.74	4.19	4.00	3.60	3.95	4.46	4.50	4.47	2.00			
	4	4.56	4.40	3.42	3.42	4.09	4.60	4.50	3.93	3.50	3.50	3.56	4.26	4.39	4.24	2.00			
	5	4.50	4.21	3.42	3.01	4.00	4.46	4.35	3.61	3.40	3.50	3.50	4.24	4.29	3.56	2.00			
	6	4.58	4.21	3.52	3.52	4.17	4.41	4.30	4.14	3.59	3.50	3.52	4.40	4.34	3.46	2.00			
	7	4.75	4.35	4.26	4.21	4.45	4.50	4.36	4.52	4.35	4.46	4.36	4.95	4.50	3.50	4.21			
	8	5.04	4.40	4.79	5.00	5.17	4.55	4.40	5.14	5.08	5.05	5.10	5.75	5.00	3.70	5.20			
	9	5.26	4.37	5.12	5.13	6.19	4.74	4.35	5.87	5.81	5.50	5.77	6.84	5.00	4.00	5.77			
	10	5.30	4.50	5.15	5.13	6.41	5.15	4.60	6.03	6.10	5.62	5.80	6.87	5.10	4.47	5.75			
	11	5.75	4.72	5.24	5.20	6.47	5.50	5.04	6.10	6.35	5.50	5.98	6.86	5.98	4.96	5.63			
	12	5.64	4.80	5.24	5.21	6.44	5.50	5.15	6.06	6.34	5.50	5.80	6.82	5.98	5.07	5.60			
	13	5.30	4.65	5.20	5.20	6.36	5.46	5.16	5.72	6.11	5.37	5.70	6.60	5.50	5.07	5.20			
	14	5.30	4.60	5.15	5.17	6.10	5.46	5.20	5.72	6.02	5.35	5.80	6.35	5.41	5.20	5.15			
	15	5.26	4.60	5.02	5.07	5.50	5.46	5.20	5.41	5.70	5.10	5.43	5.59	5.21	5.21	4.85			
	16	5.15	4.50	5.06	5.20	5.50	5.21	5.11	5.41	5.70	5.10	5.47	5.59	5.10	5.05	4.94			
	17	5.20	4.50	5.15	5.50	5.70	5.20	5.10	5.61	5.87	5.21	5.77	5.80	5.09	5.00	5.15			
	18	5.40	4.70	5.26	6.15	6.21	5.36	5.20	6.33	6.19	5.50	6.29	6.00	5.15	5.10	5.52			
	19	6.01	5.25	5.94	6.91	6.73	6.43	6.08	6.97	7.25	6.66	6.89	6.76	6.00	6.00	6.34			
	20	6.00	5.30	6.21	7.50	7.03	6.57	6.58	7.50	7.62	7.35	8.38	7.54	6.47	6.89	6.89			
	21	5.74	5.32	6.00	7.12	6.86	6.58	6.89	6.89	6.86	7.00	8.21	6.84	5.90	7.11	6.46			
	22	5.40	5.68	5.96	6.80	6.53	6.56	7.25	6.56	6.28	6.56	7.00	6.36	5.37	7.11	5.92			
	23	5.29	5.72	5.35	6.38	6.00	6.43	6.87	6.02	6.00	6.00	6.47	6.29	5.40	6.58	5.40			
	24	5.10	5.30	5.15	5.94	5.50	6.28	6.25	5.20	5.15	5.19	5.30	5.50	5.28	5.50	4.85			

Day	Tu	W	e	Th	Fr	Sa	Su	Mo	T	u	W	e	T	h	Fr	Sa	Su	Mo	Tu
Hourly price (cent/kWh)	1	4.24	4.53	4.00	3.40	4.12	4.21	3.78	3.57	3.78	4.00	4.44	4.50	4.00	4.32	3.65			
	2	4.10	4.16	3.60	3.01	3.57	3.67	3.28	3.40	3.40	3.57	4.07	4.03	3.27	3.57	3.58			
	3	3.69	3.60	3.00	2.51	3.09	3.20	3.01	3.20	3.00	3.00	3.81	3.75	2.50	3.09	3.40			
	4	3.10	3.60	2.90	2.25	2.62	2.73	2.51	2.51	3.00	2.77	3.75	3.64	2.00	3.09	3.27			
	5	3.00	3.46	2.25	2.00	2.13	2.18	2.50	2.17	2.50	2.51	3.30	3.57	1.86	2.92	3.17			
	6	3.46	3.90	3.00	2.25	2.13	2.11	2.51	2.51	2.83	2.86	3.57	3.57	2.00	3.11	3.49			
	7	4.20	4.21	3.78	3.46	2.13	2.17	3.30	3.40	3.60	3.75	4.07	3.63	2.05	3.80	3.64			
	8	4.85	5.35	4.36	4.20	2.51	3.04	4.21	4.15	4.28	4.47	5.43	3.83	2.28	4.16	4.48			
	9	5.39	5.78	5.20	5.58	2.75	3.20	5.20	4.99	5.32	5.34	5.98	3.83	2.28	4.48	5.68			
	10	5.46	6.23	5.46	6.25	3.44	3.28	5.00	5.00	5.31	4.86	6.12	4.80	2.28	4.51	5.10			
	11	5.30	5.66	5.46	5.31	3.87	3.33	5.24	5.57	5.32	5.00	6.16	5.68	3.30	4.87	5.54			
	12	5.24	5.65	5.50	4.89	3.87	3.60	4.99	5.25	5.31	4.87	5.52	5.64	3.30	5.13	5.00			
	13	5.00	5.55	5.43	4.73	3.78	3.60	4.75	4.90	4.86	4.50	5.11	4.87	3.30	4.87	4.41			
	14	5.00	5.13	5.11	4.58	3.60	3.60	4.74	4.75	4.74	4.37	4.74	4.48	3.30	4.66	4.37			
	15	4.80	4.85	4.85	4.14	3.40	3.60	4.28	4.29	4.28	4.16	4.23	4.15	3.30	4.40	4.13			
	16	4.80	4.80	4.85	4.20	3.01	3.31	4.30	4.35	4.36	4.16	4.26	3.63	3.14	4.40	4.03			
	17	5.00	4.90	4.96	4.25	3.01	3.27	4.44	4.47	4.48	4.23	4.30	3.30	3.04	4.45	4.15			
	18	5.60	5.24	5.20	4.65	3.30	3.77	4.87	4.82	4.90	4.40	4.48	3.30	3.30	4.51	4.30			
	19	6.61	6.29	5.88	5.34	4.63	5.31	6.59	6.57	6.32	5.73	5.59	4.16	4.21	5.25	4.90			
	20	8.00	6.89	6.50	6.31	5.90	6.05	7.79	8.98	10.00	8.21	7.52	5.10	5.52	8.49	6.50			
	21	7.79	6.85	6.25	6.00	6.32	6.66	7.50	7.71	8.98	9.58	7.00	5.53	7.00	7.92	6.10			
	22	6.47	6.00	5.60	5.59	6.21	6.51	6.66	6.72	6.40	7.30	6.03	5.12	7.62	6.02	5.12			
	23	5.95	5.50	5.08	5.21	6.09	5.90	5.40	5.63	5.12	5.57	4.86	4.48	6.29	4.65	4.30			
	24	5.30	4.85	4.80	4.70	5.59	4.38	4.37	4.50	4.41	4.56	4.46	4.23	5.20	4.40	3.76			
Day	We	T	h	Fr	Sa	Su	Mo	Tu	W	e	T	h	Fr	Sa	Su	Mo	T	u	We
Hourly price (cent/kWh)	1	2.51	3.26	3.87	3.91	5.20	3.31	3.23	3.85	3.61	3.83	4.56	4.45	4.35	3.88	3.70			
	2	2.00	2.41	3.42	3.37	3.87	2.42	2.42	3.50	3.41	3.45	4.15	4.14	3.83	3.71	3.61			
	3	1.00	1.00	3.25	3.00	3.25	1.00	0.75	3.21	3.22	3.28	3.83	3.80	3.60	3.60	3.55			
	4	0.50	1.15	3.03	2.51	2.85	0.10	1.00	3.04	3.06	3.17	3.60	3.46	3.40	3.48	3.17			
	5	0.50	1.00	3.00	2.42	2.42	0.10	0.75	3.00	2.88	2.96	3.47	3.31	3.33	3.45	3.08			
	6	0.50	1.65	3.00	2.43	2.43	1.00	1.50	3.22	3.07	3.17	3.47	3.33	3.46	3.55	3.36			
	7	2.42	3.13	3.42	2.51	2.41	2.55	2.93	3.80	3.47	3.41	3.53	3.38	3.90	3.78	3.61			
	8	3.62	3.97	5.36	3.00	2.43	3.91	3.72	4.66	4.70	4.54	3.60	3.46	4.42	4.35	4.20			
	9	4.33	4.02	5.68	3.00	0.10	4.50	3.83	4.50	4.51	4.51	3.45	3.45	5.16	4.60	4.20			
	10	4.75	4.24	5.25	3.13	1.84	4.15	3.87	4.51	4.31	4.22	3.55	3.31	4.53	4.48	4.15			
	11	4.70	4.33	5.41	3.50	3.17	4.51	3.94	4.82	4.71	4.54	3.83	3.46	4.82	4.52	4.15			
	12	4.30	4.33	5.20	3.40	3.46	4.23	3.85	4.64	4.48	4.54	3.83	3.55	4.64	4.48	4.04			
	13	4.23	4.27	4.48	3.25	3.51	4.05	3.80	4.48	4.26	4.48	3.76	3.59	4.53	4.35	4.00			
	14	4.03	4.17	4.30	3.38	3.72	3.74	3.48	4.48	4.07	4.48	3.83	3.82	4.55	4.30	4.00			
	15	3.51	3.94	3.83	3.13	3.51	3.34	3.31	4.00	3.54	3.88	3.61	3.75	4.10	4.02	3.61			
	16	3.51	3.97	3.80	2.51	3.01	3.34	3.30	3.97	3.50	3.90	3.58	3.55	4.00	3.85	3.64			
	17	3.88	4.15	3.65	2.51	2.51	3.48	3.55	4.05	3.61	4.00	3.55	3.50	4.00	4.01	3.80			
	18	4.22	4.25	3.90	3.37	3.22	3.81	3.81	4.08	3.83	4.00	3.55	3.47	4.07	4.07	4.00			
	19	4.83	5.33	4.33	4.21	3.61	5.18	4.28	4.54	4.15	4.48	3.95	3.60	4.31	4.10	4.10			
	20	6.04	7.92	5.33	5.57	5.38	7.94	7.94	5.76	6.72	7.77	6.12	4.99	5.72	5.90	6.91			
	21	6.04	7.92	5.25	6.50	5.50	5.77	9.86	5.77	6.79	7.94	6.79	7.82	6.96	7.64	8.99			
	22	5.58	6.91	4.67	5.80	5.71	5.37	7.88	5.30	5.50	6.00	6.12	9.76	4.95	4.66	6.91			
	23	4.41	4.60	4.20	5.42	4.80	3.94	4.38	4.10	3.88	4.32	4.29	6.14	3.88	4.10	4.35			
	24	4.20	4.00	3.91	5.33	4.00	3.83	4.54	4.15	3.90	4.48	4.31	4.52	3.55	3.90	4.05			

**Table A.2 Spring Analyzed and Trained Data**  
**From 21<sup>st</sup> of March to 19<sup>th</sup> of May 2009**

Day	Sa	Su	Mo T	u W	e	Th	Fr	Sa	Su	Mo T	u W	e T	h	Fr	Sa	
Hourly price (cent/kWh)	1	4.02	3.58	3.65	3.40	3.24	3.65	3.90	4.08	3.60	3.70	3.75	3.55	3.61	4.25	4.00
	2	3.80	3.25	3.27	3.20	3.15	3.39	3.70	4.02	3.16	3.16	3.49	3.21	3.36	3.66	3.70
	3	3.60	3.19	3.24	3.05	2.96	3.27	3.61	4.05	3.11	3.15	3.25	3.06	3.20	3.53	3.61
	4	3.59	3.00	3.00	2.98	2.50	3.09	3.52	4.00	2.60	3.00	3.16	2.84	3.14	3.41	3.39
	5	3.46	3.05	3.00	2.81	2.50	3.22	3.52	3.70	2.60	2.90	3.15	2.84	3.14	3.32	3.32
	6	3.47	3.00	3.00	3.05	3.01	3.52	3.62	3.45	2.90	3.15	3.25	3.09	3.17	3.50	3.32
	7	3.59	3.05	3.49	3.52	3.61	3.92	4.00	3.53	3.08	4.02	3.90	3.51	3.60	3.75	3.46
	8	3.47	2.60	3.90	3.90	3.92	4.13	4.23	3.36	2.62	4.60	4.50	3.72	4.40	4.09	3.53
	9	3.35	2.50	3.72	3.61	3.77	4.10	4.03	3.61	3.00	4.30	4.51	3.99	4.43	4.55	3.55
	10	3.61	3.05	3.86	3.72	3.82	4.01	4.27	3.90	3.20	4.40	4.30	4.37	4.43	4.59	3.69
	11	3.80	3.06	3.92	3.74	3.85	4.39	4.39	4.07	3.60	4.51	4.30	4.59	4.44	4.92	3.82
	12	3.75	3.00	3.90	3.65	3.85	4.20	4.18	4.05	3.75	4.42	4.22	4.48	4.38	4.78	3.80
	13	3.78	3.00	4.00	3.61	3.85	4.18	4.07	3.92	3.70	4.50	4.20	4.48	4.03	4.60	3.88
	14	3.74	2.78	3.85	3.55	3.85	4.10	4.06	3.81	3.80	4.22	4.12	4.38	3.75	4.38	3.87
	15	3.74	3.00	3.82	3.25	3.80	3.92	3.82	3.42	3.52	4.05	4.12	3.87	3.61	4.25	4.05
	16	3.50	2.50	3.82	3.20	3.61	3.86	3.70	3.16	3.16	3.90	4.00	3.74	3.57	4.16	3.70
	17	3.20	2.50	3.80	3.20	3.59	3.90	3.65	3.11	3.16	3.80	4.01	3.55	3.55	3.92	3.55
	18	3.01	2.50	3.85	3.35	3.61	3.92	3.61	3.10	3.25	3.77	3.92	3.55	3.59	4.00	3.55
	19	3.47	3.00	3.85	3.39	3.81	3.98	3.70	3.16	3.66	3.90	3.77	3.59	3.60	3.80	3.63
	20	3.77	3.58	4.16	3.72	3.90	4.23	3.86	3.70	4.00	3.61	3.71	3.78	3.57	3.87	3.80
	21	4.18	4.67	4.86	4.18	4.75	5.42	4.20	3.92	5.57	3.94	3.92	4.26	3.85	4.35	3.70
	22	4.19	4.93	4.02	3.82	4.18	4.50	3.90	3.92	5.09	4.40	4.30	4.50	4.92	5.00	4.58
	23	3.82	4.27	3.75	3.61	3.86	3.92	3.77	3.61	4.36	3.94	3.98	3.78	4.41	4.39	4.16
	24	3.61	3.72	3.41	3.39	3.80	3.73	3.70	3.61	3.61	3.61	3.73	3.61	3.99	3.83	4.16
Day	Sa	Su	Mo T	u W	e T	h	Fr	Sa	Su	Mo	Tu W	e T	h	Fr	Sa	Su
Hourly price (cent/kWh)	1	3.70	4.00	3.74	3.27	4.38	4.32	4.50	4.26	4.25	3.77	3.73	3.73	3.99	3.70	4.30
	2	3.46	3.42	3.52	3.05	4.16	4.07	4.22	3.80	3.86	3.53	3.59	3.52	3.52	3.50	4.02
	3	3.33	3.21	3.33	2.60	4.16	3.77	4.00	3.50	3.53	3.35	3.27	2.81	3.37	3.30	3.67
	4	3.25	3.10	3.18	2.28	3.84	3.46	3.00	2.93	3.14	3.10	2.77	2.50	2.95	3.51	3.37
	5	3.24	2.81	3.14	2.11	3.78	3.23	2.92	2.60	3.00	3.10	2.50	2.40	2.81	3.21	3.35
	6	3.24	2.81	3.14	2.50	3.63	3.25	2.60	2.79	3.10	3.25	2.60	2.60	3.20	3.21	3.20
	7	3.25	3.53	3.56	3.25	3.85	3.30	2.79	2.89	3.47	3.66	3.52	3.52	3.62	3.37	3.25
	8	3.24	4.00	4.00	3.60	4.00	3.46	2.81	2.50	3.70	4.06	4.02	3.90	3.93	3.52	3.08
	9	3.23	4.00	4.00	3.63	3.63	3.17	2.60	2.11	3.52	4.32	4.15	3.99	3.99	3.58	2.81
	10	3.25	4.20	4.07	3.55	3.80	3.23	3.10	2.51	3.63	4.59	4.02	4.23	3.95	4.00	3.20
	11	3.55	4.35	4.07	3.61	4.00	3.30	3.26	2.80	3.85	4.58	4.02	4.15	4.00	4.25	3.39
	12	3.60	4.35	3.94	3.70	3.81	3.47	3.25	2.92	3.83	4.40	3.86	3.86	3.94	4.26	3.50
	13	3.66	4.25	3.68	3.70	3.60	3.47	3.25	2.84	3.90	4.25	3.81	3.93	4.01	4.23	3.60
	14	3.81	4.07	3.55	3.65	3.55	3.23	3.25	3.00	3.99	3.99	3.76	3.73	4.00	4.10	3.70
	15	3.70	3.70	3.29	3.55	3.51	3.23	3.25	3.00	3.85	3.61	3.63	3.60	3.94	3.91	3.99
	16	3.44	3.63	3.25	3.55	3.39	3.17	2.80	2.87	3.85	3.60	3.60	3.57	3.80	3.63	3.51
	17	3.20	3.60	3.20	3.56	3.25	2.95	2.50	2.30	3.72	3.55	3.61	3.52	3.73	3.38	3.36
	18	3.20	3.59	3.21	3.70	3.10	2.75	2.50	2.20	3.66	3.55	3.67	3.52	3.70	3.30	3.35
	19	3.24	3.60	3.23	3.85	3.26	2.60	2.91	2.89	3.61	3.55	3.69	3.51	3.63	3.35	3.35
	20	3.50	3.65	3.29	4.05	3.52	3.00	3.22	3.04	3.73	3.55	3.76	3.52	3.73	3.35	3.35
	21	3.80	3.90	3.60	4.23	3.67	3.50	3.83	4.16	4.26	3.80	4.14	3.73	3.95	3.63	3.70
	22	4.87	4.40	4.31	5.01	4.17	4.37	5.00	5.50	5.00	4.64	4.90	4.48	4.48	4.41	5.43
	23	4.50	3.78	3.94	4.34	3.99	4.34	4.63	5.50	4.32	4.00	4.32	4.05	3.95	4.05	5.00
	24	4.31	3.65	3.85	4.15	4.01	4.23	4.50	4.62	4.10	3.85	3.86	3.83	3.73	3.60	4.41

Day	Mo	Tu	We	Th	Fr	Sa	Su	Mo	T	u	We	Th	Fr	Sa	Su	Mo
Hourly price (cent/kWh)	1	4.25	3.40	3.23	3.83	4.19	4.34	4.26	4.10	3.60	3.82	3.62	4.00	4.02	4.21	3.00
	2	3.80	3.30	2.80	3.51	3.75	4.05	4.02	3.50	3.31	3.80	3.50	4.02	3.89	3.65	2.60
	3	3.47	3.17	2.56	3.21	3.62	3.89	3.89	3.29	3.00	3.64	3.25	4.00	3.31	3.61	2.60
	4	3.36	3.00	2.40	3.10	3.35	3.60	3.33	2.95	2.74	3.50	3.00	3.64	3.00	3.28	2.00
	5	3.25	2.67	2.30	3.10	3.30	3.47	3.25	2.60	2.67	3.55	3.00	3.33	3.00	3.28	2.00
	6	3.30	2.90	2.67	3.21	3.35	3.47	3.01	3.01	3.00	3.66	3.25	3.28	3.00	3.31	2.40
	7	4.02	3.67	3.50	3.73	4.00	3.50	3.01	3.85	3.55	4.22	3.64	3.20	3.00	3.50	3.28
	8	4.50	4.00	3.82	4.10	4.50	3.56	3.01	4.16	3.66	4.12	3.89	2.40	2.60	3.27	3.59
	9	4.69	3.84	3.86	4.29	4.75	3.62	3.12	4.39	4.00	4.22	4.00	2.60	2.75	3.25	3.69
	10	4.62	4.10	3.84	4.52	5.50	3.76	3.31	4.46	4.02	4.22	4.25	3.28	3.19	3.34	4.02
	11	4.73	4.28	4.05	4.62	5.81	4.00	3.80	4.43	4.02	4.25	4.50	3.28	3.50	3.91	4.29
	12	4.30	4.05	4.04	4.50	5.50	3.91	3.64	4.02	4.00	4.22	4.43	3.30	3.63	3.90	4.30
	13	4.25	4.20	4.01	4.33	5.22	3.80	3.62	4.01	3.92	4.25	4.41	3.30	3.81	3.85	4.41
	14	4.25	4.20	4.05	4.30	4.70	3.84	3.50	3.89	3.90	4.12	4.11	3.35	4.02	3.86	4.38
	15	3.80	3.84	3.95	4.05	4.21	3.63	3.34	3.38	3.63	4.00	3.90	3.54	4.02	3.61	4.10
	16	3.60	3.73	3.94	4.03	3.89	3.33	3.18	3.34	3.60	3.90	3.89	3.30	3.86	3.30	4.00
	17	3.56	3.80	3.82	4.03	3.75	3.19	2.75	3.33	3.54	3.90	3.74	3.28	3.60	2.95	3.95
	18	3.63	3.80	4.00	4.10	3.64	3.00	2.60	3.31	3.58	3.90	3.75	3.28	3.60	2.81	3.85
	19	3.70	3.73	4.00	4.04	3.55	2.87	2.75	3.35	3.59	3.90	3.64	3.34	3.64	2.95	3.75
	20	3.91	3.75	3.90	4.04	3.60	3.27	3.00	3.40	3.63	4.00	3.60	3.60	3.81	3.25	3.71
	21	4.25	4.05	4.00	4.16	3.75	3.62	3.70	3.75	4.00	4.02	3.55	3.71	4.00	3.35	3.71
	22	4.92	5.00	5.39	5.39	4.58	4.52	5.25	4.35	4.72	4.70	3.97	5.39	5.39	4.49	4.85
	23	4.11	4.20	4.05	4.48	3.95	4.42	5.40	4.00	4.20	4.00	3.65	4.63	4.72	4.30	4.11
	24	3.60	3.65	3.85	4.10	3.73	4.00	4.66	3.66	4.02	3.80	3.59	3.91	4.02	3.35	3.81
Day	Tu	We	Th	Fr	Sa	Su	Mo	T	u	We	Th	Fr	Sa	Su	Mo	T
Hourly price (cent/kWh)	1	3.29	3.88	3.91	3.67	3.65	4.25	3.65	3.65	3.65	3.69	3.64	3.96	4.25	4.33	3.44
	2	2.98	3.75	3.67	3.51	3.50	3.92	3.38	3.50	3.50	3.50	3.50	3.70	3.86	3.92	3.38
	3	2.75	3.61	3.55	3.49	3.50	3.65	2.95	3.38	2.98	3.50	3.42	3.57	3.77	3.69	3.00
	4	2.52	3.38	3.40	3.40	3.50	3.59	2.51	3.05	2.90	3.00	3.01	3.42	3.40	3.40	2.81
	5	2.52	3.35	3.39	3.38	3.40	3.38	2.40	3.00	2.80	2.88	3.00	3.38	3.25	3.38	2.80
	6	2.65	3.36	3.43	3.40	3.39	3.25	2.50	3.05	2.98	2.88	2.88	3.40	3.09	3.35	2.85
	7	3.35	3.80	3.86	3.62	3.50	3.50	3.38	3.50	3.50	3.52	3.48	3.45	3.09	3.60	3.50
	8	3.75	4.10	4.19	3.81	3.30	2.90	3.74	3.77	3.80	3.80	3.65	3.39	2.85	3.73	3.83
	9	3.90	4.10	4.60	3.95	3.57	2.95	4.40	4.02	3.96	3.84	3.80	3.55	2.90	3.84	3.95
	10	4.11	4.29	4.62	4.10	4.00	3.21	4.50	4.18	4.14	4.15	3.90	3.80	3.00	4.02	4.10
	11	4.11	4.35	4.80	4.62	4.62	3.59	4.62	4.30	4.61	4.33	4.16	4.02	3.28	4.38	4.25
	12	4.02	4.30	4.62	4.63	4.25	3.38	4.09	4.25	4.62	4.02	4.00	4.02	3.28	4.41	4.25
	13	4.11	4.21	4.58	4.64	4.00	3.39	4.00	4.25	4.69	4.01	4.01	4.01	3.35	4.71	4.21
	14	4.00	4.04	4.10	4.71	3.95	3.50	3.77	4.20	4.33	3.80	3.90	4.25	3.35	4.54	4.01
	15	3.81	4.00	3.77	4.25	3.62	3.40	3.65	3.80	3.86	3.60	3.72	4.12	3.50	4.34	3.90
	16	3.87	3.95	3.77	4.04	3.51	3.30	3.65	3.80	3.84	3.52	3.70	3.65	2.90	4.05	3.91
	17	4.03	4.00	3.80	4.00	3.40	3.30	3.65	3.90	3.79	3.52	3.68	3.45	2.80	3.95	3.91
	18	4.11	4.00	3.88	4.00	3.50	3.30	3.70	3.90	3.80	3.60	3.72	3.42	2.50	3.97	3.91
	19	4.00	3.90	3.85	3.82	3.69	3.40	3.68	3.85	3.73	3.50	3.70	3.45	2.74	3.89	3.88
	20	3.90	3.95	3.83	3.70	3.80	3.62	3.74	3.85	3.70	3.50	3.72	3.60	2.90	3.83	3.87
	21	4.00	3.95	3.83	3.60	4.07	3.70	3.80	3.95	3.69	3.46	3.77	3.70	3.20	3.77	3.75
	22	5.47	4.72	4.35	3.89	4.97	4.89	4.50	4.35	3.85	3.79	4.40	4.12	4.67	3.91	3.91
	23	4.40	4.04	3.75	3.70	4.97	4.77	4.05	3.90	3.75	3.75	4.18	4.00	5.09	3.80	3.70
	24	3.97	3.86	3.61	3.70	4.38	4.02	3.65	3.65	3.50	3.52	3.84	3.80	4.50	3.50	3.42

Table A.3 Summer Analyzed and Trained Data  
From 21<sup>st</sup> of June to 19<sup>th</sup> of August 2009

Day	Su	Mo	T	u	W	e	T	h	Fr	Sa	Su	Mo	Tu	W	e	T	h	Fr	Sa	Su
Hourly price (cent/kWh)	1	3.21	3.44	3.41	3.64			3.61	3.39	3.87	4.26	4.10	3.72	3.95	3.51	3.42	4.30	3.96		
	2	3.14	3.11	3.29	3.45			3.39	3.21	3.65	3.95	3.70	3.43	3.41	3.30	3.29	4.00	3.47		
	3	3.01	2.81	3.10	3.39			3.14	3.03	3.41	3.66	3.41	3.30	3.20	3.16	3.15	3.61	3.35		
	4	2.85	2.58	3.10	3.39			3.11	3.11	3.36	3.40	3.18	3.24	3.17	3.04	3.04	3.50	3.35		
	5	2.80	2.50	3.10	3.23			3.08	3.03	3.17	3.24	3.10	3.17	3.06	3.00	3.02	3.42	3.16		
	6	2.85	2.81	3.11	3.23			3.11	3.03	3.17	3.17	3.10	3.18	3.17	3.10	3.15	3.35	3.16		
	7	2.51	3.21	3.23	3.40			3.39	3.25	3.08	3.06	3.43	3.41	3.41	3.50	3.41	3.32	3.13		
	8	2.11	3.45	3.61	3.58			3.72	3.55	3.03	2.85	3.80	3.61	3.87	3.83	3.98	3.21	3.00		
	9	2.30	3.76	3.72	3.62			3.82	3.60	3.40	2.91	3.90	3.80	3.85	3.95	3.87	3.49	2.95		
	10	2.85	3.97	4.29	3.80			4.19	3.79	3.66	3.10	4.25	4.10	3.82	3.84	3.87	3.80	3.34		
	11	3.14	4.33	4.60	4.06			4.36	4.05	4.03	3.25	4.39	4.32	4.05	4.39	4.44	4.10	3.55		
	12	3.39	4.53	4.62	4.03			4.26	4.10	4.01	3.40	4.30	4.19	3.97	4.06	4.24	3.96	3.71		
	13	3.56	4.73	4.64	4.06			4.32	4.44	4.12	3.42	4.25	4.32	3.98	4.14	4.30	3.98	3.87		
	14	3.71	4.72	4.50	4.00			4.17	4.40	4.12	3.40	4.10	4.26	4.03	4.40	4.34	3.87	3.88		
	15	3.99	4.40	4.00	3.76			3.86	3.90	3.81	3.41	3.85	4.04	3.84	3.83	3.70	3.58	3.66		
	16	3.70	4.20	4.05	3.69			3.76	3.79	3.80	3.40	3.95	4.04	4.10	3.95	3.76	3.66	3.50		
	17	3.68	4.29	4.05	3.69			3.76	3.79	3.96	3.40	4.05	4.14	4.04	3.85	3.75	3.55	3.41		
	18	3.59	4.20	4.06	3.71			3.80	3.85	4.00	3.43	4.10	4.24	4.06	3.86	3.74	3.55	3.33		
	19	3.70	4.25	4.06	3.76			3.72	3.79	3.96	3.51	4.08	4.14	3.97	3.75	3.64	3.50	3.41		
	20	3.68	4.12	4.00	3.70			3.65	3.71	3.71	3.69	4.03	4.12	4.04	3.80	3.75	3.49	3.49		
	21	3.70	3.97	3.76	3.58			3.60	3.73	3.90	3.96	4.05	4.03	3.97	3.74	3.66	3.53	3.54		
	22	4.26	4.10	3.81	3.76			3.73	3.75	4.07	4.37	3.87	4.05	3.90	3.80	3.66	3.55	3.71		
	23	4.86	4.15	3.86	4.30			3.94	3.92	4.30	4.98	4.08	4.14	4.05	3.84	4.10	3.80	4.50		
	24	4.60	3.69	3.60	3.87			3.76	3.82	4.10	4.72	3.75	3.82	3.80	3.55	3.73	3.60	3.97		
Day	Mo	Tu	W	e	T	h	Fr	Sa	Su	Mo	T	u	We	T	h	Fr	Sa	Su	Mo	
Hourly price (cent/kWh)	1	4.26	3.41	3.22	3.08			3.33	3.77	3.60	4.19	3.45	3.49	3.56	3.35	3.68	3.84	3.74		
	2	3.58	3.18	2.95	2.85			3.11	3.73	3.42	3.60	3.19	3.32	3.33	3.21	3.54	3.44	3.25		
	3	3.23	3.02	2.60	2.45			2.71	3.40	3.19	3.14	3.00	3.05	3.05	3.00	3.33	3.16	2.80		
	4	3.13	2.90	2.51	1.75			2.50	3.38	3.07	3.05	2.60	3.05	3.05	2.63	3.21	3.13	2.55		
	5	3.10	2.80	2.38	1.75			2.33	3.13	2.96	2.96	2.55	2.85	2.85	2.45	3.05	2.85	2.20		
	6	3.13	2.95	2.60	2.34			2.51	3.05	2.96	3.10	2.75	3.05	2.99	2.63	2.95	2.95	2.60		
	7	3.51	3.24	3.11	3.09			3.21	2.95	2.55	3.68	3.23	3.36	3.33	3.13	2.95	2.80	3.16		
	8	4.10	3.58	3.45	3.51			3.51	2.95	2.45	3.95	3.57	3.76	3.80	3.62	3.05	2.65	3.69		
	9	3.81	3.56	3.58	3.65			3.62	3.13	2.60	3.77	3.55	3.60	3.80	3.50	3.19	2.70	3.82		
	10	3.66	3.59	3.55	3.63			3.70	3.45	3.05	3.92	3.69	3.76	4.06	3.90	3.45	3.05	3.90		
	11	4.27	4.06	3.82	4.05			4.12	3.83	3.33	4.19	4.09	4.24	4.50	4.19	3.60	3.16	3.97		
	12	4.07	3.96	3.81	3.95			4.12	3.77	3.35	4.06	4.01	4.36	4.30	4.02	3.50	3.21	3.90		
	13	4.25	4.21	4.12	4.21			4.20	3.82	3.45	4.06	4.01	4.50	4.30	4.05	3.60	3.28	3.95		
	14	4.21	4.21	4.17	4.23			4.17	3.63	3.50	3.95	3.90	4.36	4.07	3.80	3.54	3.39	3.89		
	15	3.58	3.50	3.60	3.73			3.62	3.35	3.40	3.45	3.40	3.89	3.57	3.45	3.20	3.20	3.64		
	16	3.80	3.67	3.83	3.83			3.75	3.50	3.45	3.50	3.45	4.16	3.63	3.50	3.28	3.20	3.71		
	17	3.80	3.81	3.93	4.02			3.70	3.55	3.33	3.60	3.50	4.00	3.55	3.35	3.21	3.10	3.76		
	18	3.75	3.80	3.93	3.95			3.70	3.50	3.33	3.61	3.52	4.00	3.56	3.33	3.28	3.06	3.78		
	19	3.66	3.63	3.75	3.73			3.70	3.40	3.45	3.49	3.48	3.93	3.50	3.45	3.20	3.21	3.70		
	20	3.57	3.55	3.57	3.65			3.61	3.45	3.55	3.45	3.45	3.77	3.59	3.49	3.21	3.28	3.65		
	21	3.53	3.50	3.50	3.49			3.50	3.61	3.80	3.44	3.46	3.69	3.50	3.50	3.17	3.28	3.60		
	22	3.57	3.55	3.50	3.50			3.47	3.65	4.05	3.45	3.50	3.70	3.55	3.60	3.39	3.62	3.65		
	23	3.60	3.63	3.60	3.56			3.76	3.92	4.86	3.50	3.66	3.70	3.62	3.65	3.81	4.30	3.80		
	24	3.58	3.51	3.32	3.40			3.57	3.51	4.09	3.45	3.57	3.49	3.38	3.65	3.70	3.65	3.60		

Day	Tu	We	T	h	Fr	Sa	Su	Mo	T	u	We	T	h	Fr	Sa	Su	Mo	T	u
Hourly price (cent/kWh)	1	3.30	3.30	3.16	3.71	3.67	3.78	3.72	3.09	3.70	2.93	3.21	3.76	3.61	3.91	3.42			
	2	3.27	3.29	2.75	3.64	3.66	3.45	3.32	2.90	3.26	2.75	2.85	3.70	3.20	3.25	3.16			
	3	3.05	2.92	2.10	3.22	3.29	3.33	2.81	2.60	2.85	2.50	2.50	3.21	2.85	2.90	2.92			
	4	2.60	2.81	1.50	2.92	3.29	3.29	2.45	2.60	2.75	2.20	2.20	3.20	2.60	2.54	2.80			
	5	2.45	2.75	1.01	2.80	3.13	3.05	2.00	2.60	2.60	1.75	2.00	2.92	2.20	2.40	2.60			
	6	2.55	2.85	1.80	3.22	2.75	3.02	2.45	2.75	2.80	2.20	2.45	2.55	2.36	2.50	2.75			
	7	3.13	3.28	2.82	3.43	2.60	2.80	3.00	2.92	3.26	2.95	3.00	2.57	2.36	3.13	3.16			
	8	3.45	3.67	3.37	3.69	2.75	2.55	3.52	3.44	3.51	3.36	3.31	2.92	2.00	3.62	3.42			
	9	3.60	3.75	3.23	3.69	2.85	2.55	3.53	3.45	3.52	3.44	3.38	2.85	2.20	3.61	3.41			
	10	3.66	3.70	3.22	3.67	3.10	2.90	3.68	3.70	3.68	3.76	3.54	3.43	2.92	3.78	3.70			
	11	3.88	3.80	3.55	3.80	3.50	3.37	4.16	4.10	3.96	4.20	3.84	3.56	3.47	3.99	3.95			
	12	3.66	3.56	3.21	3.68	3.39	3.38	4.00	3.95	3.80	4.09	3.70	3.60	3.56	3.96	3.94			
	13	3.60	3.55	3.28	3.76	3.27	3.46	4.20	4.10	3.81	4.41	3.80	3.54	3.70	4.20	3.97			
	14	3.54	3.46	3.28	3.72	3.24	3.34	3.98	4.00	3.67	4.40	3.77	3.50	3.70	4.20	3.96			
	15	3.24	3.00	2.95	3.29	3.27	3.09	3.66	3.60	3.15	3.83	3.30	3.30	3.57	3.61	3.65			
	16	3.33	3.30	3.21	3.60	3.29	3.00	3.86	3.90	3.35	4.18	3.50	3.10	3.50	3.93	3.81			
	17	3.35	3.31	3.28	3.64	3.29	2.85	3.87	3.91	3.48	4.18	3.50	3.01	3.44	3.93	3.84			
	18	3.43	3.32	3.40	3.64	3.29	2.81	3.86	3.87	3.48	4.01	3.40	3.01	3.44	3.93	3.85			
	19	3.45	3.30	3.28	3.45	3.27	2.90	3.76	3.78	3.35	3.87	3.28	2.92	3.45	3.85	3.80			
	20	3.50	3.30	3.40	3.45	3.29	3.28	3.70	3.70	3.40	3.81	3.25	2.92	3.50	3.76	3.71			
	21	3.50	3.29	3.57	3.32	3.37	3.28	3.67	3.52	3.46	3.70	3.25	3.02	3.44	3.57	3.57			
	22	3.60	3.37	3.77	3.32	3.60	3.55	3.71	3.53	3.45	3.76	3.28	3.37	3.76	3.71	3.71			
	23	3.84	3.47	3.91	3.56	3.80	4.30	3.77	3.76	3.65	3.81	3.46	3.40	4.05	3.54	3.57			
	24	3.55	3.30	3.67	3.32	3.29	3.66	3.60	3.39	3.32	3.45	3.00	3.40	3.72	3.42	3.34			
Day	We	Th	Fr	Sa	Su	Mo	T	u	W	e	Th	Fr	Sa	Su	Mo	Tu	W	e	
Hourly price (cent/kWh)	1	3.23	3.28	3.00	3.00	3.81	3.91	3.00	3.23	3.35	3.36	3.70	3.80	3.89	3.28	3.35			
	2	2.98	3.01	2.85	2.95	3.66	3.21	2.85	2.99	2.85	3.16	3.64	3.36	3.35	3.12	3.02			
	3	2.69	2.75	2.38	2.55	3.32	2.54	2.55	2.81	2.60	2.89	3.30	3.31	3.10	3.05	2.95			
	4	2.49	2.60	1.53	2.60	3.16	2.30	2.44	2.71	2.50	2.69	3.27	3.24	2.98	3.00	2.91			
	5	2.44	2.49	1.50	2.20	2.90	2.25	1.75	2.55	2.20	2.63	3.06	3.15	2.98	2.90	2.87			
	6	2.60	2.60	2.00	2.20	2.85	2.30	2.50	2.71	2.55	2.85	3.10	3.15	3.05	2.99	2.91			
	7	3.05	3.10	2.92	2.30	2.80	2.99	2.92	3.01	2.89	3.16	3.10	3.15	3.33	3.16	3.16			
	8	3.36	3.42	3.34	2.50	2.30	3.30	3.12	3.35	3.33	3.44	3.13	3.15	3.85	3.45	3.46			
	9	3.49	3.50	3.35	2.85	2.30	3.22	3.34	3.58	3.43	3.66	3.20	3.10	3.80	3.60	3.71			
	10	3.69	3.71	3.54	3.08	2.85	3.22	3.45	3.67	3.66	3.81	3.36	3.24	3.72	3.71	3.76			
	11	3.96	4.00	3.76	3.57	2.96	3.54	3.65	3.81	3.76	4.05	3.54	3.36	4.13	4.28	4.21			
	12	3.97	4.01	3.75	3.60	3.01	3.54	3.66	3.82	3.77	3.89	3.40	3.50	3.92	4.05	4.21			
	13	4.09	4.30	3.82	3.66	3.20	3.66	3.72	3.96	3.85	3.99	3.50	3.59	4.00	4.05	4.31			
	14	3.98	4.20	3.81	3.67	3.22	3.67	3.72	3.95	3.89	4.00	3.45	3.50	3.89	3.92	4.28			
	15	3.66	3.76	3.54	3.40	3.25	3.40	3.53	3.70	3.60	3.43	3.30	3.28	3.48	3.51	3.61			
	16	3.78	3.95	3.67	3.60	3.32	3.80	3.68	3.80	3.68	3.50	3.27	3.30	3.60	3.60	3.71			
	17	3.76	3.96	3.70	3.54	3.05	3.81	3.70	3.80	3.74	3.54	3.35	3.24	3.60	3.66	3.72			
	18	3.76	3.96	3.77	3.54	3.12	3.95	3.83	3.93	3.90	3.80	3.36	3.31	3.68	3.80	3.81			
	19	3.70	3.81	3.70	3.12	3.10	3.87	3.77	3.85	3.89	3.70	3.40	3.36	3.66	3.88	3.80			
	20	3.52	3.67	3.50	3.20	3.32	3.80	3.74	3.80	3.88	3.82	3.60	3.36	3.60	3.84	3.76			
	21	3.50	3.52	3.40	3.45	3.49	3.60	3.74	3.82	3.90	3.85	3.81	3.52	3.48	3.84	3.81			
	22	3.65	3.70	3.52	3.84	3.82	3.89	3.95	3.91	4.00	3.96	4.11	3.85	3.70	4.15	4.13			
	23	3.50	3.45	3.30	3.86	4.00	3.44	3.68	3.62	3.81	3.81	4.13	4.00	3.46	3.76	3.76			
	24	3.30	3.15	3.32	3.50	3.72	3.22	3.35	3.20	3.30	3.50	3.90	3.66	3.40	3.40	3.46			

**Table A.4 Fall Analyzed and Trained Data**  
**From 23<sup>rd</sup> of September to 21<sup>st</sup> of November 2009**

Day	We	Th	Fr	Sa	Su	Mo T	u W	e T	h	Fr	Sa	Su	Mo	Tu W	e	
Hourly price (cent\kWh)	1	3.28	3.33	3.02	3.55	3.80	3.31	2.53	3.05	3.53	3.39	4.00	4.40	3.61	2.80	3.02
	2	3.08	3.20	2.63	3.49	3.54	2.90	2.40	2.96	3.46	3.18	3.89	4.00	3.08	2.76	2.65
	3	3.00	2.93	2.40	3.45	3.25	2.55	2.41	2.94	3.39	3.00	3.63	3.80	2.89	2.67	2.45
	4	2.41	2.63	1.98	3.16	3.01	2.33	2.40	2.94	3.24	2.88	3.44	3.54	2.61	2.40	1.89
	5	2.11	2.43	1.83	3.08	3.17	2.00	1.01	2.84	2.95	2.81	3.35	3.50	2.51	2.11	1.71
	6	2.37	2.51	1.98	3.01	3.17	2.56	2.37	2.88	3.05	2.88	3.42	3.43	2.66	2.40	1.99
	7	3.26	3.20	2.91	3.23	2.88	3.43	3.04	3.38	3.46	3.35	3.41	3.20	3.46	3.02	3.21
	8	3.72	3.54	3.76	3.36	3.01	4.09	3.50	3.88	3.80	3.90	3.51	3.28	4.00	3.70	3.90
	9	3.91	3.80	3.91	3.26	2.75	4.28	4.07	4.00	4.00	3.99	3.21	3.11	4.00	3.80	4.01
	10	3.80	3.76	3.70	3.44	2.98	3.93	3.69	3.96	3.98	3.70	3.49	3.13	3.80	3.84	3.98
	11	4.05	3.98	4.06	3.52	3.47	4.18	3.93	4.11	4.03	3.97	3.73	3.61	3.98	3.98	4.00
	12	4.07	4.07	4.00	3.51	3.43	4.24	4.19	4.32	4.26	4.05	3.81	3.76	4.11	4.11	4.02
	13	4.09	4.19	4.07	3.59	3.42	4.22	4.19	4.34	4.48	4.09	3.81	3.70	4.13	4.00	3.98
	14	3.92	4.09	3.91	3.72	3.43	4.09	3.93	4.09	4.23	4.01	3.98	3.70	3.98	3.84	3.89
	15	3.60	3.75	3.52	3.48	3.43	3.55	3.59	3.80	3.98	3.60	3.59	3.43	3.72	3.60	3.58
	16	3.75	3.83	3.76	3.49	3.07	3.65	3.76	3.98	3.85	3.59	3.52	3.26	3.71	3.75	3.70
	17	3.80	3.83	3.76	3.45	2.98	3.76	4.02	4.02	3.84	3.60	3.50	3.05	3.75	3.90	3.75
	18	3.83	3.83	3.81	3.50	3.07	4.00	4.11	4.03	3.98	3.89	3.59	3.00	3.93	3.98	3.92
	19	3.70	3.70	3.70	3.50	3.26	3.69	3.90	3.81	3.90	3.69	3.59	2.96	3.80	3.97	3.90
	20	3.80	3.72	3.65	3.58	3.47	3.56	3.93	3.76	3.84	3.69	3.76	3.38	3.76	3.97	3.95
	21	4.46	4.21	4.26	4.54	4.59	4.67	4.80	4.48	3.99	4.00	4.17	4.06	4.00	4.13	4.20
	22	4.41	4.00	4.00	4.97	5.37	4.32	4.50	4.24	3.85	3.90	4.57	4.79	3.98	3.98	4.03
	23	3.61	3.53	3.89	3.90	4.51	3.59	3.76	3.60	3.89	3.65	4.13	4.53	3.76	3.65	3.93
	24	3.55	3.35	3.87	3.89	3.98	3.38	3.48	3.43	3.84	3.99	4.09	3.98	3.63	3.52	3.93
Day	Th	Fr	Sa	Su	Mo	Tu W	e T	h	Fr	Sa	Su	Mo	Tu W	u	We T	h
Hourly price (cent\kWh)	1	3.31	3.22	3.92	3.85	3.84	3.31	3.20	3.21	3.00	3.81	3.88	4.24	2.95	3.42	3.16
	2	2.85	2.95	3.66	3.62	2.75	2.38	2.60	2.80	2.51	3.60	3.63	3.81	2.55	3.22	2.49
	3	2.89	3.03	3.52	3.52	2.60	2.11	2.60	2.65	2.51	3.31	3.35	3.49	2.01	3.22	2.20
	4	2.72	2.72	3.16	3.30	2.58	1.75	2.21	2.40	1.99	2.76	3.02	3.41	2.01	3.11	2.00
	5	2.55	2.56	2.93	3.23	2.40	1.50	1.99	1.89	1.40	2.63	3.02	3.40	1.89	2.89	1.40
	6	2.88	2.60	2.89	3.08	2.50	1.89	2.38	2.11	1.89	2.63	3.02	3.38	2.00	2.89	1.71
	7	3.46	3.41	2.97	2.66	2.56	2.73	3.27	3.20	2.66	2.65	2.76	3.81	3.25	3.42	2.81
	8	3.99	3.82	3.46	2.68	2.56	3.66	3.80	3.70	3.50	2.98	2.76	4.59	3.79	3.81	3.58
	9	4.13	3.92	3.38	2.63	2.53	3.95	3.92	3.84	3.71	2.80	2.76	5.50	4.01	3.98	3.93
	10	3.99	3.91	3.50	3.02	2.76	3.85	3.77	3.70	3.60	2.76	2.90	4.78	3.93	3.79	3.60
	11	4.20	4.01	3.60	3.52	3.58	3.84	3.81	3.80	3.69	3.00	3.50	4.50	4.30	3.79	3.74
	12	4.52	4.32	3.60	3.57	3.61	3.99	3.99	3.99	3.84	3.07	3.52	4.51	5.00	3.84	3.74
	13	4.52	4.22	3.60	3.58	3.50	4.00	4.16	3.99	3.95	3.15	3.56	4.48	5.07	3.87	3.53
	14	4.23	4.09	3.68	3.60	3.51	3.84	3.92	3.81	3.88	3.51	3.73	4.18	4.30	3.53	3.39
	15	3.82	3.89	3.61	3.37	3.27	3.68	3.75	3.62	3.62	3.02	3.67	3.88	3.79	3.12	3.00
	16	3.85	3.81	3.58	2.89	2.71	3.75	3.90	3.70	3.70	3.36	3.56	3.87	3.80	3.28	3.22
	17	3.84	3.76	3.50	2.68	2.56	3.80	3.80	3.67	3.64	3.25	3.45	3.79	3.78	3.40	3.22
	18	4.00	3.83	3.54	2.71	2.71	3.94	3.95	3.71	3.80	3.53	3.45	3.88	3.80	3.53	3.45
	19	3.95	3.75	3.55	3.37	3.28	3.95	3.90	3.70	3.84	3.50	3.48	3.80	3.79	3.60	3.53
	20	3.95	3.80	3.80	3.62	3.61	3.98	3.90	3.71	3.90	3.67	3.63	3.84	3.87	3.98	3.98
	21	4.43	3.95	4.20	4.49	4.40	4.11	3.99	3.95	3.99	4.35	4.51	4.01	4.76	4.51	4.87
	22	4.01	3.81	4.35	4.63	4.85	3.90	3.88	3.71	3.85	4.52	5.56	3.79	4.18	3.98	4.30
	23	3.84	3.80	3.90	4.30	4.55	3.75	3.65	3.56	3.69	3.88	5.34	3.52	3.78	3.79	3.98
	24	3.80	3.80	3.72	3.90	3.99	3.64	3.50	3.45	3.81	4.11	4.51	3.40	3.68	3.60	3.87

Day	Fr	Sa	Su	Mo T	u W	e	Th	Fr	Sa	Su	Mo T	u W	e	Th	Fr	
Hourly price (cent\kWh)	1	3.39	3.43	3.58	4.05	3.61	3.52	3.52	3.80	4.28	3.53	2.84	3.20	3.08	3.45	1.50
	2	2.60	3.45	3.43	3.54	3.23	3.10	3.05	3.52	4.18	3.34	2.01	2.58	2.58	3.17	1.00
	3	2.55	3.28	3.26	3.30	3.00	3.05	2.85	3.42	3.63	3.23	1.00	1.71	2.00	2.46	0.80
	4	2.20	3.16	3.45	3.19	2.65	3.02	2.60	3.10	3.47	3.05	1.00	1.71	2.00	2.20	0.50
	5	2.20	3.03	3.43	3.13	2.49	2.84	2.55	3.10	3.45	2.75	1.00	1.71	1.40	2.00	0.50
	6	2.49	3.02	3.42	3.16	2.65	2.85	2.76	3.05	3.44	2.65	1.01	2.00	1.50	2.21	0.51
	7	3.22	3.10	3.42	3.45	3.05	3.15	3.28	3.35	3.45	2.73	2.20	2.58	2.46	3.02	1.40
	8	3.78	3.25	3.33	3.88	3.51	3.59	3.69	3.68	3.49	2.71	3.02	3.43	3.45	3.54	3.02
	9	3.98	3.05	3.08	4.05	3.76	3.80	3.79	3.80	3.35	2.59	2.81	3.23	3.16	3.33	2.98
	10	3.80	3.31	3.05	4.20	3.83	3.83	4.08	3.92	3.50	3.02	3.00	3.20	3.23	3.30	3.05
	11	3.81	3.38	2.85	4.45	3.92	4.03	4.06	4.03	3.60	3.42	3.11	3.28	3.10	3.17	3.15
	12	3.97	3.38	3.23	4.40	3.90	4.03	4.05	4.06	3.88	3.30	3.11	3.23	3.10	3.11	3.23
	13	3.98	3.29	3.42	4.43	4.00	4.21	4.08	4.18	3.94	3.52	3.41	3.50	3.30	3.30	3.52
	14	3.58	3.39	3.50	4.18	3.90	4.06	3.98	4.09	3.98	3.45	3.30	3.30	3.08	3.09	3.36
	15	3.40	3.25	3.53	3.92	3.83	3.83	3.96	3.86	3.77	3.25	3.45	3.30	3.05	3.10	3.39
	16	3.28	3.25	3.49	3.92	3.83	3.83	3.96	3.78	3.90	3.16	3.28	3.05	2.77	3.05	3.23
	17	3.26	3.25	3.43	3.92	3.84	3.83	3.99	3.78	3.60	3.14	3.49	3.23	3.00	3.15	3.23
	18	3.48	3.38	3.26	4.15	3.92	3.92	4.08	3.86	3.90	3.30	3.59	3.71	3.25	3.39	3.61
	19	3.50	3.43	3.33	4.59	4.40	3.92	4.19	3.81	3.60	3.67	4.24	4.41	4.39	4.24	5.08
	20	3.60	3.52	3.45	4.60	4.24	4.00	4.31	3.88	3.76	4.40	7.00	5.97	5.98	5.21	6.44
	21	4.02	4.21	3.52	4.81	4.81	4.20	4.47	4.18	3.98	4.54	6.00	4.68	5.00	4.54	5.22
	22	3.80	4.98	4.15	4.10	4.41	3.98	4.19	4.19	4.31	4.26	5.22	4.47	4.61	4.46	4.78
	23	3.51	3.98	5.00	3.75	4.04	3.83	4.12	4.00	3.98	3.91	4.49	4.00	3.93	3.58	3.90
	24	3.70	4.11	6.00	3.71	3.85	3.79	3.87	4.00	3.93	3.43	3.93	3.51	3.30	3.11	3.45
Day	Sa	Su	Mo T	u W	e T	h	Fr	Sa	Su	Mo	Tu W	e T	h	Fr	Sa	
Hourly price (cent\kWh)	1	4.20	3.70	3.19	3.25	3.25	3.57	3.35	2.99	3.83	2.70	2.89	3.20	3.15	3.12	4.05
	2	3.80	3.02	1.71	2.11	2.11	3.00	3.00	2.21	3.83	2.00	2.49	2.84	2.89	2.75	3.69
	3	3.51	1.71	0.80	1.71	1.70	2.76	2.80	1.80	3.73	0.51	1.60	2.45	2.20	2.27	3.00
	4	2.89	0.50	0.10	1.00	1.01	2.76	2.60	1.71	3.26	0.51	1.20	2.31	2.00	2.00	2.86
	5	2.20	0.40	0.10	1.01	1.01	2.80	2.58	1.46	3.18	0.51	1.20	2.00	1.90	2.03	2.70
	6	1.71	0.50	0.51	1.01	1.89	3.00	2.80	1.80	3.11	1.00	2.10	2.53	2.53	2.21	2.51
	7	1.71	0.50	1.40	2.20	2.67	3.39	3.00	2.19	3.18	2.61	2.78	2.89	3.00	2.75	2.60
	8	2.00	1.50	3.35	3.49	3.51	3.91	3.65	2.61	3.19	3.69	3.44	3.48	3.49	3.25	2.76
	9	1.71	0.10	3.30	3.48	3.50	3.60	3.55	2.60	2.75	3.48	3.40	3.48	3.49	3.24	2.76
	10	2.01	0.50	3.39	3.59	3.60	3.68	3.65	2.88	3.00	3.50	3.48	3.53	3.71	3.48	3.22
	11	2.21	1.40	3.30	3.53	3.60	3.74	3.68	3.29	3.00	3.50	3.63	3.60	3.86	3.48	3.45
	12	2.20	1.50	3.18	3.53	3.53	3.68	3.68	3.62	2.95	3.51	3.65	3.57	3.80	3.63	3.20
	13	2.20	2.28	3.53	3.78	3.77	3.98	3.86	3.50	3.11	3.83	3.85	3.87	4.00	4.00	2.89
	14	2.00	1.80	3.41	3.65	3.64	3.68	3.60	3.19	2.65	3.62	3.72	3.68	3.64	3.65	2.86
	15	2.20	1.71	3.27	3.53	3.60	3.60	3.45	3.11	2.20	3.51	3.73	3.59	3.49	3.48	3.20
	16	1.75	1.50	3.20	3.49	3.57	3.47	3.20	3.02	2.11	3.39	3.60	3.48	3.40	3.40	2.89
	17	1.50	1.50	3.30	3.53	3.64	3.49	3.20	2.98	2.11	3.48	3.68	3.48	3.44	3.41	2.88
	18	2.01	1.81	3.68	3.92	4.00	3.70	3.43	3.59	2.69	3.93	3.84	3.72	3.65	3.63	2.89
	19	3.60	3.11	4.40	4.85	5.23	4.48	3.86	4.40	3.51	4.71	4.91	4.96	4.40	4.51	4.01
	20	4.40	3.78	7.61	8.44	7.30	5.12	4.14	4.96	3.88	5.72	5.32	5.55	4.73	4.71	4.60
	21	4.20	3.91	5.50	5.53	5.68	4.63	3.91	4.71	4.11	4.87	4.91	4.76	4.25	4.18	4.50
	22	4.01	4.20	5.15	5.00	5.30	4.51	3.80	4.65	4.48	4.29	4.92	4.46	4.18	3.82	4.28
	23	3.46	3.91	4.21	4.00	4.20	3.82	3.55	3.97	3.98	3.83	4.00	3.85	3.66	3.48	3.83
	24	3.25	3.49	3.53	3.40	3.53	3.51	3.30	3.86	3.48	3.45	3.50	3.30	3.10	3.25	3.73

Table A.5 Winter Regression innovations (Avg\_R) &amp; DNN Innovations (ATOE)

Mon		Tues		Wed		Thu		Fri		Sat		Sun	
Avg_R	ATOE												
-0.25	-0.07	-0.39	-0.04	-0.31	-0.12	0.04	-0.22	0.06	-0.14	0.47	-0.09	0.24	-0.07
-0.47	-0.19	-0.10	-0.09	-0.34	-0.18	0.12	-0.27	-0.11	-0.18	0.33	-0.17	0.07	-0.15
-0.63	-0.32	-0.14	-0.15	-0.42	-0.26	-0.02	-0.35	0.07	-0.25	0.30	-0.28	-0.05	-0.22
-0.67	-0.26	-0.12	-0.24	-0.37	-0.36	-0.02	-0.37	0.05	-0.29	0.17	-0.25	-0.17	-0.33
-0.62	-0.34	-0.11	-0.27	-0.34	-0.37	-0.20	-0.42	0.10	-0.32	0.15	-0.30	-0.28	-0.37
-0.38	-0.35	0.00	-0.26	-0.33	-0.38	-0.09	-0.38	-0.02	-0.27	0.04	-0.30	-0.35	-0.37
0.34	-0.35	0.04	-0.14	0.02	-0.22	-0.19	-0.27	-0.09	-0.17	-0.36	-0.16	-0.56	-0.26
0.60	-0.27	0.17	-0.09	0.34	-0.13	-0.30	-0.15	0.35	-0.05	-0.84	0.02	-0.66	-0.23
1.15	-0.18	0.33	-0.02	0.47	-0.02	-0.54	-0.04	0.73	0.12	-1.27	0.11	-0.98	-0.16
0.91	-0.26	0.20	-0.02	0.59	-0.03	-0.74	-0.03	0.83	0.04	-1.03	0.12	-0.83	-0.12
0.83	-0.22	0.14	0.03	0.33	0.02	-0.57	-0.01	0.62	0.08	-0.59	0.11	-0.65	-0.07
0.68	-0.20	0.03	0.03	0.32	0.01	-0.42	-0.01	0.49	0.06	-0.48	0.07	-0.51	-0.07
0.46	-0.19	-0.03	0.00	0.21	-0.04	-0.23	-0.06	0.42	0.00	-0.49	0.04	-0.35	-0.07
0.39	-0.19	-0.06	0.00	0.11	-0.05	-0.19	-0.08	0.33	-0.02	-0.41	0.00	-0.22	-0.09
0.21	-0.21	-0.03	-0.03	-0.07	-0.09	-0.12	-0.12	0.09	-0.04	-0.22	-0.06	-0.10	-0.11
0.32	-0.32	0.00	-0.06	0.00	-0.10	-0.19	-0.12	0.12	-0.02	-0.45	-0.05	-0.14	-0.15
0.47	-0.32	0.10	-0.05	0.10	-0.09	-0.22	-0.10	0.10	0.01	-0.60	-0.01	-0.25	-0.15
0.57	-0.18	0.07	0.02	0.15	-0.03	-0.18	-0.06	0.14	0.04	-0.63	0.07	-0.20	-0.10
0.66	0.08	0.10	0.26	0.18	0.18	-0.24	0.16	0.29	0.23	-0.43	0.23	-0.09	0.10
1.20	0.18	0.48	0.32	0.51	0.41	0.15	0.31	0.15	0.53	-0.51	0.30	-0.14	0.23
0.31	0.22	0.87	0.32	0.26	0.34	0.72	0.28	-0.19	0.34	-0.02	0.27	0.43	0.25
-0.54	0.22	0.33	0.25	-0.10	0.20	0.69	0.18	-0.21	0.39	0.24	0.20	1.23	0.18
-0.67	0.18	0.11	0.14	-0.44	0.09	0.53	0.01	-0.04	0.19	0.34	0.09	0.81	0.14
-0.48	0.02	-0.07	-0.02	-0.20	-0.05	0.34	-0.09	0.07	0.03	0.44	-0.02	0.08	0.04

Table A.6 Spring Regression innovations (Avg\_R) &amp; DNN Innovations (ATOE)

Mon		Tues		Wed		Thu		Fri		Sat		Sun	
Avg_R	ATOE												
-0.06	-0.03	-0.31	0.05	-0.17	-0.01	0.15	-0.11	0.27	0.05	0.21	0.08	0.21	0.20
-0.28	-0.12	-0.25	-0.27	-0.16	-0.20	0.11	-0.17	0.15	-0.15	0.18	-0.06	-0.01	0.00
-0.38	-0.16	-0.33	-0.27	-0.30	-0.32	0.08	-0.26	0.18	-0.12	0.12	-0.10	-0.05	-0.06
-0.46	-0.32	-0.32	-0.28	-0.40	-0.37	0.04	-0.33	0.08	-0.21	0.08	-0.13	-0.24	-0.16
-0.53	-0.37	-0.31	-0.21	-0.42	-0.45	0.08	-0.33	0.00	-0.23	0.02	-0.17	-0.27	-0.26
-0.37	-0.35	-0.23	-0.28	-0.26	-0.39	0.03	-0.32	-0.01	-0.24	-0.13	-0.13	-0.27	-0.23
0.21	-0.30	-0.01	-0.10	0.06	-0.16	0.06	-0.11	-0.06	-0.14	-0.33	-0.01	-0.33	-0.08
0.67	-0.26	0.20	-0.01	0.20	0.05	0.18	-0.04	-0.17	-0.10	-0.52	0.13	-0.63	-0.05
0.76	-0.25	0.23	0.06	0.24	0.10	0.19	-0.02	-0.12	-0.04	-0.49	0.21	-0.70	-0.20
0.72	-0.21	0.21	0.13	0.20	0.18	0.27	0.01	0.03	-0.10	-0.31	0.14	-0.69	-0.18
0.66	-0.14	0.13	0.18	0.25	0.17	0.33	0.01	0.16	-0.02	-0.17	0.16	-0.49	-0.13
0.50	-0.16	0.12	0.08	0.28	0.09	0.21	-0.02	0.18	-0.06	-0.17	0.15	-0.43	-0.03
0.55	-0.16	0.06	0.06	0.29	0.03	0.12	-0.04	0.17	-0.08	-0.17	0.24	-0.38	-0.05
0.43	-0.14	0.02	0.00	0.23	0.01	-0.01	-0.08	0.16	-0.08	-0.06	0.17	-0.29	0.07
0.21	-0.17	-0.09	-0.18	0.19	-0.16	-0.04	-0.13	0.14	-0.02	-0.06	0.09	-0.24	0.11
0.29	-0.32	-0.01	-0.21	0.20	-0.17	-0.04	-0.15	0.03	-0.14	-0.28	0.03	-0.38	-0.03
0.37	-0.37	0.08	-0.15	0.19	-0.19	-0.05	-0.14	-0.06	-0.15	-0.40	-0.02	-0.51	-0.10
0.40	-0.49	0.11	-0.16	0.24	-0.18	-0.06	-0.10	-0.11	-0.13	-0.42	-0.08	-0.58	-0.11
0.30	-0.46	0.04	-0.15	0.24	-0.18	-0.08	-0.06	-0.17	-0.08	-0.30	-0.15	-0.40	-0.16
0.19	-0.32	0.00	-0.11	0.24	-0.16	-0.07	0.00	-0.09	-0.19	-0.14	-0.13	-0.31	-0.11
0.09	-0.15	0.01	-0.02	0.22	-0.01	-0.03	0.02	-0.04	-0.20	-0.06	0.05	0.08	0.00
0.01	0.32	0.20	0.44	0.23	0.39	0.08	0.25	0.23	0.21	0.31	0.54	0.70	0.45
-0.36	0.18	0.00	0.16	-0.07	0.12	0.07	0.04	0.19	0.07	0.34	0.11	0.79	0.23
-0.28	0.20	-0.03	-0.13	-0.01	0.09	0.04	-0.03	0.06	-0.13	0.21	-0.01	0.30	0.12

Table A.7 Summer Regression innovations (Avg\_R) & DNN Innovations (ATOE)

Mon		Tues		Wed		Thu		Fri		Sat		Sun	
Avg_R	ATOE	Avg_R	ATOE										
0.33	0.32	-0.47	0.23	-0.03	0.49	-0.26	0.08	-0.01	-0.12	0.32	0.26	0.28	0.52
-0.01	0.13	-0.33	-0.02	-0.11	0.33	-0.26	-0.09	0.02	-0.34	0.38	0.07	0.07	0.48
-0.22	0.08	-0.22	-0.21	-0.18	0.13	-0.32	-0.37	-0.07	-0.42	0.28	-0.19	0.12	0.34
-0.31	0.06	-0.26	-0.30	-0.14	-0.11	-0.47	-0.44	-0.18	-0.64	0.42	0.12	0.10	0.29
-0.31	-0.06	-0.29	-0.36	-0.12	-0.01	-0.56	-0.53	-0.13	-0.63	0.30	0.09	0.05	0.19
-0.21	-0.15	-0.14	-0.24	-0.10	-0.11	-0.37	-0.36	-0.09	-0.54	-0.01	-0.12	0.03	0.34
0.24	-0.24	-0.04	-0.19	0.09	0.17	-0.15	-0.04	-0.02	-0.33	-0.39	0.13	-0.22	0.07
0.71	-0.27	0.02	-0.01	0.29	0.36	-0.03	0.16	0.05	-0.03	-0.60	0.29	-0.59	0.11
0.62	-0.23	0.08	0.05	0.31	0.36	0.00	0.20	0.02	0.04	-0.45	0.33	-0.66	0.24
0.45	-0.04	0.11	0.04	0.18	0.44	0.07	0.25	0.03	0.11	-0.26	0.40	-0.46	0.45
0.55	0.10	0.20	0.25	0.20	0.61	0.20	0.35	0.06	0.29	-0.19	0.55	-0.47	0.51
0.42	0.16	0.16	0.24	0.20	0.55	0.09	0.34	0.07	0.24	-0.19	0.52	-0.31	0.48
0.44	0.25	0.16	0.32	0.22	0.64	0.16	0.36	0.08	0.36	-0.24	0.57	-0.26	0.49
0.37	0.22	0.16	0.27	0.20	0.59	0.18	0.35	0.06	0.32	-0.26	0.56	-0.23	0.42
0.16	0.09	0.05	0.03	0.04	0.40	0.06	0.11	-0.07	0.13	-0.13	0.38	-0.12	0.31
0.33	0.05	0.08	0.13	0.15	0.41	0.06	0.21	-0.07	0.19	-0.19	0.47	-0.20	0.36
0.43	0.00	0.12	0.11	0.15	0.47	0.06	0.28	-0.10	0.24	-0.20	0.45	-0.30	0.35
0.45	-0.03	0.15	0.08	0.15	0.47	0.06	0.28	-0.09	0.21	-0.21	0.42	-0.31	0.35
0.36	0.00	0.16	0.08	0.10	0.47	0.00	0.20	-0.08	0.14	-0.25	0.41	-0.16	0.29
0.23	0.09	0.14	0.04	0.03	0.42	0.04	0.17	-0.07	0.11	-0.21	0.38	-0.03	0.31
0.10	0.12	0.09	0.03	0.03	0.39	0.03	0.17	-0.07	0.05	-0.05	0.32	0.02	0.32
0.00	0.23	0.10	0.08	-0.02	0.41	0.08	0.17	-0.10	0.07	0.12	0.32	0.19	0.39
-0.36	0.51	0.05	0.21	-0.13	0.45	0.06	0.26	0.00	0.11	0.22	0.40	0.62	0.52
-0.20	0.31	-0.05	0.09	-0.16	0.38	-0.02	0.09	0.02	-0.06	0.15	0.35	0.35	0.32

Table A.8 Fall Regression innovations (Avg\_R) &amp; DNN Innovations (ATOE)

Mon		Tues		Wed		Thu		Fri		Sat		Sun	
Avg_R	ATOE												
-0.15	-0.02	-0.47	-0.11	-0.07	-0.18	0.07	-0.33	-0.22	-0.09	0.60	-0.10	0.23	-0.14
-0.66	-0.08	-0.49	-0.22	-0.15	-0.17	0.09	-0.33	-0.29	-0.21	0.68	-0.09	0.19	-0.18
-1.02	-0.11	-0.51	-0.48	-0.03	-0.19	0.04	-0.35	-0.19	-0.23	0.50	-0.14	0.06	-0.28
-0.94	-0.17	-0.60	-0.48	-0.06	-0.23	0.04	-0.43	-0.32	-0.27	0.45	-0.18	-0.06	-0.36
-0.96	-0.22	-0.79	-0.49	-0.11	-0.28	-0.04	-0.43	-0.23	-0.30	0.37	-0.20	0.05	-0.37
-0.74	-0.25	-0.47	-0.36	-0.09	-0.22	0.01	-0.40	-0.32	-0.23	0.17	-0.17	-0.02	-0.37
-0.01	-0.30	-0.13	-0.23	0.13	-0.18	0.12	-0.37	-0.29	-0.10	-0.24	-0.12	-0.36	-0.31
0.56	-0.28	0.11	-0.21	0.25	-0.08	0.10	-0.23	-0.05	0.01	-0.54	-0.03	-0.49	-0.28
0.84	-0.30	0.26	-0.22	0.27	-0.03	0.09	-0.18	-0.04	0.09	-0.74	0.00	-0.75	-0.28
0.64	-0.20	0.21	-0.21	0.24	-0.01	0.11	-0.17	-0.03	0.04	-0.50	0.02	-0.62	-0.27
0.53	-0.09	0.19	-0.13	0.14	0.05	0.12	-0.14	-0.01	0.14	-0.40	0.03	-0.35	-0.26
0.49	-0.09	0.32	-0.13	0.09	0.04	0.15	-0.14	0.05	0.20	-0.41	0.06	-0.36	-0.18
0.49	-0.06	0.35	-0.11	0.14	0.04	0.14	-0.13	0.10	0.24	-0.54	0.07	-0.21	-0.19
0.40	-0.07	0.19	-0.12	0.10	0.00	0.07	-0.16	0.09	0.08	-0.35	0.04	-0.29	-0.19
0.33	-0.13	0.18	-0.14	0.04	-0.04	0.04	-0.24	0.04	0.02	-0.30	0.02	-0.30	-0.24
0.34	-0.19	0.25	-0.19	0.11	-0.01	0.04	-0.22	-0.05	0.03	-0.32	-0.02	-0.48	-0.21
0.44	-0.22	0.34	-0.16	0.12	0.01	0.01	-0.20	-0.09	0.03	-0.42	-0.04	-0.51	-0.28
0.55	-0.20	0.35	-0.15	0.15	0.02	0.03	-0.17	-0.02	0.04	-0.34	0.01	-0.57	-0.21
0.60	-0.08	0.46	-0.02	0.28	0.06	0.02	-0.16	0.04	0.08	-0.21	-0.02	-0.33	-0.28
1.23	0.08	0.57	0.24	0.35	0.07	-0.20	-0.15	-0.02	0.08	-0.12	0.02	-0.19	-0.20
0.73	0.24	0.33	0.13	0.22	0.07	-0.01	-0.09	-0.08	0.09	0.21	0.08	0.09	-0.13
0.15	0.22	0.14	0.09	0.05	0.05	-0.01	-0.14	-0.06	0.11	0.57	0.02	0.47	-0.11
-0.12	0.10	-0.01	0.04	-0.03	-0.01	0.02	-0.18	-0.04	0.03	0.21	0.00	0.66	-0.14
-0.26	-0.03	-0.09	0.03	-0.08	-0.04	0.03	-0.19	0.12	0.01	0.25	0.01	0.37	-0.19

Table A.9 ANN Characteristics

Number of hidden layers	1
Number of input nodes	1
Number of nodes in the hidden layers	7
Number of output nodes	1
Hidden layers transfer function	'tansig'
Output layer transfer function	'purelin'
Network training function	'trainlm'
Learning rate	0.15
Momentum constant	0.9
Performance goal	0.0001
Maximum number of epochs to train	500
Epochs between displays	100