THE RATIO OF THE ELECTRIC UNITS OF CHARGE.—BY GEORGE C. LAURENCE, B. Sc., MacGregor Fellow in Physics, Dalhousie University, Halifax, N. S.

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This work was undertaken to determine what accuracy could be attained in the measurement of the ratio of the electrical units of charge, using modern methods and apparatus available in the average laboratory. The method used was that of Rosa and Dorsay[1], that is, the capacity of a cylindrical condenser, calculated in e. s. units from its dimensions, was compared with its capacity in e. m. units measured in a Maxwell Bridge. A value of $2.9963 \times 10 \pm .05\%$ was obtained which agrees better with Rosa and Dorsay's value $2.9971 \times 10 \pm .01\%$—which is by far the best—than the values obtained by earlier experimenters (except that of Fabry and Perot; 2.9978). Accordingly it was thought worth while to describe the simplifications of the method which enabled it to be carried out in a small laboratory. For a general treatment of the method the

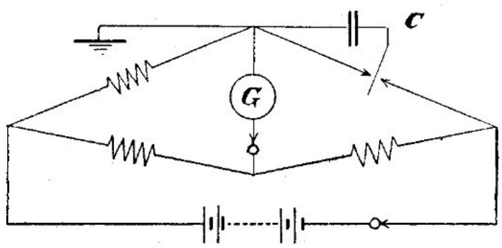

Fig. 1.

reader is referred to Rosa and Dorsay's paper,[1] the wireing diagram, however, is given in Fig. 1. The best values that have been obtained for this ratio are:

Himstead	3.0057×10 cm./sec.
Rosa	3.0000
Thomson and Searle	2.9960
Abraham	2.9913
Pellat	3.0092
Hurmezesco	3.0010
Fabry and Perot	2.9978
Rosa and Dorsay (1907)	2.9971± .01%

[1] Edward B. Rosa and N. E. Dorsay, Reprint No. 65, Bulletin of the Bureau of Standards, 1907.

Eliminating End Effects by Using a Double Condenser

The use of guard rings was obviated by making a double condenser consisting of a long and a short cylinder of equal diameter inside the one outer cylinder. (Fig. 2). By taking

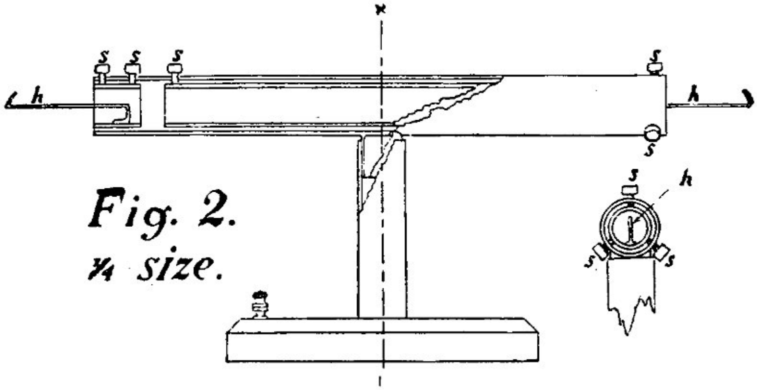

Fig. 2.
¼ size.

the difference between the capacities of the two inner cylinders end effects were satisfactorily eliminated. The cylinders were mounted so that they could be rotated about the axis x, end for end, thus putting the exposed end of the cylinder whose capacity is being measured in the same position as that previously occupied by the other cylinder, thus eliminating errors due to external electrostatic fields. While the capacity of one inner cylinder was being measured in electromagnetic units the other two cylinders were earthed. The wire hooks h, protruding from the ends of the inner cylinders served in making connections to them. The difference in capacity in electrostatic units was calculated from the expression $C=(L-S)/(2 \log R/r)$, where L and S are the lengths of the inner cylinders and r their radius, and R the radius of the outer cylinder.

An Electrical Method of Measuring the Frequency of Commutation Simultaneously with the Balancing of the Bridge

A magnetic tuning fork was used as the commutator in the Maxwell Bridge. Its frequency was measured simultaneously with the balancing of the bridge in the following manner. The fork was driven by an electric magnet and a make-and-break contact operated by a storage battery. This intermittent direct current was passed in series through the current winding of a wattmeter. The potential winding of this wattmeter was connected to the output of a motor-alternator driven at a very slightly different frequency. When alternator potential and fork current were in phase the needle of the wattmeter deflected in one direction and when out of phase in the opposite direction, so that the needle moved back and forth indicating the beat-between the two. The frequency of the beats was the differs ence between the frequency of the fork and of the alternator. The frequency of the alternator was measured with a stop-clock and a revolution counter over periods of about 15 minutes during which time the bridge was balanced and at the same time another observer counted the beats indicated by the wattmeter. The sum(or difference) of the two frequencies gave that of the fork with an error less than .05%, the stop clock being checked before and after each measurement with a good watch. A direct current ammeter and a minature lamp were also used as beat indicators, being connected so that both fork and alternator currents, superimposed on one another, flowed through them.

An Electrical Method of Adjusting the Cylinders so that they are Coaxial

A cylinder within another cylinder has a minimum capacity when it is exactly coaxial with the outside one. When the bridge was roughly balanced, decreasing the capacity of the condenser had the effect of moving the galvanometer reading to the left. So the inner cylinders were adjusted by turning the ebonite screws (s) that supported them until the galvanometer reading was moved as far to the right as it would go. By this method the cylinders could be adjusted much more easily and more accurately than by mechanical methods. This was shown by the fact that a capacity .14 m. m. f. (i. e. .12% of the total capacity, 123.50 m. m. f.) smaller was obtained with this method of adjusting, than was obtained with a mechanical method. The mechanical method consisted in adjusting the width of the space between the inner and outer cylinders at different places around the circumference by inserting a wedge therein.

Measuring the Effect of a Slight Bend in one of the Cylinders on their Capacity

The relative increase in the capacity of a cylindrical condenser caused by a small displacement x of the axis of the inner cylinder parallel to itself is $\Delta C/C = x^2/4R(R\text{-}r)$*, provided that x is small compared with $R\text{-}r$, the difference in radius of the cylinders. Hence the capacity of an element of length dy is

$$dC = (x^2/4R(R\text{-}r)+1)\ dy\ /2\ log\ (R\text{-}r) \qquad (1)$$

It was found that the inner cylinder was slightly curved, deviating at the most about .05mm. from a straight line. Considering this curve to be parabolic, taking the axis of the outer cylinder as the Y axis, and the origin at the middle of the cylinder we get in the case of the electrically adjusted cylinders $x = (y^2\text{-}h)/M$, where M is the latus rectum and h is a constant of such a size as will make the total capacity a minimum. Integrating for the whole length of the cylinder (*from* $-L/2$ *to* $+L/2$) we get for the capacity

$$C' = (L^5/80 - hL^3/6 - h^2L)\ /\ (8M^2R(R\text{-}r)\ log\ (R/r)) + C \qquad (2)$$

where C is the capacity the cylinders would have if their axes

* loc. cit., page 484

were straight. When we set $dC'/dh=O$ to find the minimum value of C' we get $h=L^2/12$ and equation (2) becomes

$$C'=L^5/(1440M^2R(R\text{-}r)\ log\ R/r)+C \tag{3}$$

In the case of the mechanically adjusted condenser the two axes intersect only at the ends of the tube i. e. where the wedge was inserted, hence $x=(y^2—L^2/4)/M$ and integrating for the whole length we get for the capacity

$$C''=L^5/(240M^2R(R\text{-}r)\ log\ R/r)+C \tag{4}$$

$$\therefore\ \ C'—C=(C''—C')/5 \tag{5}$$

The correction $(C''—C')/5$ amounted to .03 m. m. f. making C=123.50 m. m. f.. The smallness of the correction justified the assumption that the curve was parabolic.

RESULTS

The most difficult quantity to measure in finding the electrostatic capacity of the condenser was the distance between the inner and outer cylinders. This was determined to .05% by weighing the quantity of water which this space could contain. The result was checked, by two methods of direct measurement, and differed from them by less than their probable errors (.1% and .3%). The other dimensions were measured directly with calipers. The probable error in the electrostatic capacity was .07%.

Eighteen separate electromagnetic measurements of the difference in capacity of the two cylinders were made, using two different fork frequencies—90 and 126 cycles, two battery voltages—45 and 90 volts, and making similar changes in the resistances of the arms of the bridge. Two independent adjustments of the cylinders by the electrical method and two by the mechanical method were made. The results of the three measurements made with the condenser mechanically adjusted were only used in determining the size of the correction discussed in the preceeding paragraph. The mean of the fifteen results (with the cylinders electrically adjusted) had a probable error of .012%. Taking account of the errors in the resistances, the frequency, and the correction, the probable error in the electromagnetic capacity became .07%. The probable error in the ratio of the capacities is .10% and hence the probable error in the ratio of the units is .05%. The value obtained was 2.9963×10^{10} *cm./sec.*

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