Asset Pricing Model Fit for Canadian Mutual Funds

by

Tianyu Zhang

Submitted in partial fulfilment of the requirements

for the degree of Master of Science

at

Dalhousie University

Halifax, Nova Scotia

December 2022

Table of Contents

List of Tables

Abstract

Asset pricing models have been used extensively in studies to predict fund performance. However, my motivation is to test which asset pricing model is the best to evaluate mutual fund performance in Canada. In addition, I identify whether there is any difference in model performance before and after Covid-19. The competing models are the Capital Asset Pricing Model of Sharp and Lintner, Fama and French's Three-Factor Pricing Model, Carhart's Four-Factor Pricing Model, and Fama and French's Five- and Six-Factor Pricing Models. I compare frequently employed factor models using Gibbons, Ross and Shanken's methodology. I find that the Fama French Six-Factor Model is the best model for the entire period including post-Covid-19. It also performs best for all types of funds and equity funds in the pre-Covid-19 period. However, the CAPM emerges as the best performer for bond funds, and the FF5 best explains the mixed asset fund performance for pre-Covid-19.

LIST OF ABBREVIATIONS USED

CAPM	Capital Asset Pricing Model
FF3	Fama and French's Three-Factor Pricing Model
FFC	Carhart's Four-Factor Pricing Model
FF5	Fama and French's Five-Factor Pricing Model
FF6	Fama and French's Six-Factor Pricing Model

Chapter 1: Introduction

Asset pricing factor models can explain returns across assets, and a great amount of literature has verified the validity of various kinds of factor models in different countries. The results show that factor models yield different outcomes in different countries. Griffin (2002) states that country-specific three-factor models better explain average returns than international factor models. Kubota and Takehara (2018) argue that in Japan, the Fama and French five-factor model (FF5) does not fit as well as evidence indicates for the US because the coefficients of the profitability factor and investment factor are not statistically significant. Yet FF5 can justify more asset pricing anomalies in pricing Australian equities (Chiah et al., 2016). In the long-term Canadian stock market, the three-factor pricing model is augmented by a momentum factor (L'Her et al., 2004). However, the results are supportive of the Fama and French three-factor model (FF3) for the Canadian market after 2000 with both equally weighted and value-weighted portfolios (Beaulieu et al., 2016). Finally, Lalwani and Chakraborty (2019) believe that a high level of market integration is necessary when combining data from different countries, so they use data for different markets. They find that FF5 improves pricing performance for stocks in Australia, Canada, China, and the US, while FF3 or its four-factor variants are more suitable for markets in Japan, the UK, India, Malaysia, South Korea, and Taiwan (Lalwani and Chakraborty, 2019).

In this paper, I aim to focus on mutual fund performance. A mutual fund is an investment vehicle consisting of asset classes overseen by a professional manager on behalf of a pool of investors. The first open-end mutual fund in Canada was started in Montreal in 1932 during the Great Depression. There has been a tremendous increase in the market value of mutual funds in

Canada. The number of mutual funds available has increased to over 5,000 since 1932, and the amount of money Canadians invest in these funds has surpassed $1.71 trillion (IFIC, n.d.). Patterns of mutual fund performance are different from stock performance because the fund market is more rational than the stock market (Treynor & Mazuy, 1966). In addition, investors tend to allocate money based on the correct pricing risk model (Berk & van Binsbergen, 2016). An impressive amount of literature discusses the indicators of mutual fund performance evaluation. Angelidis et al. (2013) report that mutual fund manager performance is better measured by utilizing a self-reported benchmark rather than a passive portfolio with the same risk, and they prefer the return-based mutual fund performance evaluation. Also, mutual fund performance is affected by professional managers' decisions. Huij and Verbeek (2009) show that multifactor performance is affected by systematic bias, regardless of transaction costs and trading impact. They also find that factor proxies based on mutual fund returns rather than stock returns provide better standards for evaluating professional money managers. Other research establishes the importance of factor models in evaluating mutual fund performance. Christensen (2013) analyzes mutual fund performance based on the Capital Asset Pricing Model (CAPM) and multifactor models and finds that Danish mutual funds have neutral performance with non-persistent returns. Likewise, in the Portuguese equity fund market, European Union funds underperform the market significantly by utilizing CAPM and FF3 models (Leite et al., 2009).

There is extensive literature on mutual fund performance, but far too little attention has been paid to the validity of factor models on mutual fund performance. My motivation to conduct this study is to find the best fit model for the Canadian mutual fund industry in order to fill the gap. Alpha refers to a percentage measuring how funds perform compared to the benchmark index and positive alpha implies a professional fund manager who can generate added value. Huang et al. (2021) find that when returns are measured net of management and trading costs, 2.9% to 8.4% of US actively managed funds produce positive alpha. Sha and Gao (2019) highlight that FF5 outperforms other models in the Chinese mutual fund industry, and CAPM controls the estimated alpha dispersion better than other models. Likewise, Kildahl and Lunde (2018) demonstrate that while FF5 is the best model for explaining US mutual fund returns, the simplicity of CAPM performs surprisingly well. Rehman and Baloch (2016), who study a different selection of factor models, report that CAPM is preferred over FF3, which shows poor results for the value and size factors. For the Polish equity mutual fund, the Fama and French six-factor model (FF6) fits the performance of the mutual funds best with the market factor (MKT), the size factor (SMB), the profitability effect (RMW), and the investment effect (CMA) having a significantly positive impact on the performance of mutual funds (Trzebiński, 2022). What’s more, the conditional version of Carhart's (1997) four-factor asset pricing model (FFC) is recommended by Sehgal and Babbar (2017) as the best performance benchmark for assessing mutual fund performance. However, based on the US and UK mutual funds markets, modifying the standard model to account for the alpha of the benchmark index can change the traditional view of mutual fund performance (Mateus et al., 2019).

The main objective is to establish the most appropriate asset pricing model for Canadian mutual funds by evaluating the utility of applying multifactor asset pricing models to characterize their performance. The fund managers can effectively capture the factors affecting the fund market and make the best decision for investors. I assess and contrast performance of CAPM, Fama and French’s three-factor, five-factor, and six-factor models, and the four-factor model from Carhart.

Due to the massive increase in demand for cash during the pandemic (Chen et al., 2020), holding fixed-income assets now poses a greater risk from a liquidity standpoint. According to the Investment Funds Institute of Canada, one of the greatest monthly drops in Canadian mutual fund history was the stunning $14.1 billion in net redemptions reported for just March *(Canada Mutual Funds Post Record Redemptions as COVID-19 Worries Grow,* 2020*)*. Also, the total net asset value was $1.45 trillion at the end of March 2020, a dramatic decline of 11.3% from the end of 2019 (*Canada Mutual Funds Post Record Redemptions as COVID-19 Worries Grow*, 2020). This indicates that some of these losses have been recovered as a result of mutual funds having to sell investment positions to cover redemptions. Hence, I compare the pre- and post-Covid-19 periods to detect whether the pandemic affects the models' performance.

I compare widely used factor models using the approach proposed by Gibbons, Ross, and Shanken (1989). This method is popular for evaluating the model performance (Fama & French (1996), (2015), (2016)) and ranking the model performance by many recent papers (Ahmed et al. (2019), Kildahl and Lunde (2018), Sha and Gao (2019), (Trzebiński, 2022)).

The structure of this paper is as follows. Section 2 presents a review of asset pricing models. Section 3 reports the methodology used to test model performance. Section 4 describes the data and variables. Section 5 explores the empirical results, and Section 6 provides a conclusion based on the findings.

Chapter 2: Review of the asset-pricing models

The CAPM, Fama and French three-, five-, and six-factor model and Carhart's four-factor model are still predominantly used to evaluate the efficacy of mutual funds despite the introduction of roughly ten new models a few years ago (Hou et al., 2019; Mateus et al., 2019; Ahmed et al., 2019). This section aims to introduce several asset-pricing models that are used in this research.

2.1 Asset Pricing Models

The first process is to get the consumption-based pricing model. As we know, $P_0=\frac{D_1}{1+r}+\frac{D_2}{(1+r)^2}+\frac{D_3}{(1+r)^3}+\ldots\ldots,$ $P_1=\frac{D_2}{1+r}+\frac{D_3}{(1+r)^2}+\frac{D_4}{(1+r)^3}+\ldots\ldots$, so $P_0=\frac{D_1}{1+r}+\frac{1}{1+r}[\frac{D_2}{1+r}+\frac{D_3}{(1+r)^2}+\cdots]=\frac{D_1}{1+r}+\frac{P_1}{1+r}=\frac{x_1}{1+r}$, $P_0=m_1*x_1$. Because the equation is random, I take the expectation for the equation. Hence, I have

$$P_t = E_t[m_{t+1}x_{t+1}] \tag{1}$$

P_t stands for asset price, m_{t+1} stands for stochastic discount factor (SDF) which is $\beta\frac{u'(c_{t+1})}{u'(c_t)}$ and x_{t+1} stands for the value of all future payoff from the security/asset. Considering the two-period economy,

$$U = U(C_t) + \beta E_t[U(c_{t+1})] \tag{2}$$

β stands for subjective discount factor, E_t stands for conditional expectations, and C_t denotes consumption at time t. I try to maximize the utility of (2) to get the number of securities, subject to two constraints: $c_t=y_t-\theta p_t$ and $c_{t+1}=y_{t+1}+\theta x_{t+1}$, y_t is the revenue at time t and y_{t+1} is the revenue at time t+1, substitute to equation (2) solves the problem

$$U(y_t - \theta p_t) + \beta E_t[u(y_{t+1} + \theta x_{t+1})] \tag{3}$$

The first order condition is $-p_t u'(c_t) + \beta E_t[x_{t+1} u'(c_{t+1})] = 0$, so

$$p_t u'(c_t) = \beta E_t[x_{t+1} u'(c_{t+1})] \tag{4}$$

and this equation means that the marginal cost equates with the marginal utility of selling the asset and consuming payoffs.

Dividing each side of equation (1) by p_{it}, I can also get the new general pricing model as follows:

$$1 = E[m_{t+1}(1 + R_{t+1})] \tag{5}$$

where R_{t+1} is the return at time t+1.

According to the riskless asset, I can get

$$1 = E\left[m_{t+1}\left(1 + R_{ft+1}\right)\right] \tag{6}$$

Hence, the expected excess security return can be rewritten as follows:

$$E_t[r_{t+1}] = -\left(1 + E_t\left[R_{ft+1}\right]\right) Cov_t[r_{t+1}, m_{t+1}] \tag{7}$$

Where $r_{t+1}=R_{t+1}-R_{ft+1}$ is the excess return at t+1 and Cov_t is the conditional covariance operator.

2.1.1 Capital Asset Pricing Model

With the assumption states that consumption and wealth are equivalent, it can be written as:

$$m_{t+1} = a_t + b_t R_{mt+1} \tag{8}$$

Where R_{mt+1} is the excess return of market at t+1, and a_t and b_t are parameters. I can substitute (8) into equation (7) yields:

$$E[R_{t+1}] = -b_t(1 + E[R_{ft+1}])Cov[R_{t+1}, R_{mt+1}] \tag{9}$$

and

$$E[R_{mt+1}] = -b_t(1 + E[R_{ft+1}])Var(R_{mt+1}) \tag{10}$$

Dividing (8) by (9), I may get

$$r_{it} - r_{ft} = \alpha_i^{CAPM} + \beta_{i,MKT}MKT_t + \epsilon_{i,t} \tag{11}$$

where $\beta = \frac{Cov(R_c,R_p)}{Var(R_c)}$. Based on the expected utility maxim analysis of (Markowitz, 1959), the CAPM model proposed by Sharpe (1964) Lintner (1965) Black (1972) indicates a linear relationship between beta and expected returns. They argue that firm-specific risk can be diversified by a large portfolio. Beta can affect the expected returns among assets because the return of an asset is related to its sensitivity to the market. R_{it} is the return of asset I at time t, r_{ft} is the risk-free rate and MKT is the market factor in the model. Berk and van Binsbergen (2016) propose a new method of testing asset pricing models depending on quantities rather than prices or returns and infer CAPM is normally used by investors.

2.1.2 Fama and French Three-Factor Pricing Model

To date, CAPM is the standard model used to examine the market beta, which is a significant explainer of the cross-sectional expected returns. However, a few studies have highlighted the relevance of other factors and weaknesses of CAPM: Basu (1977) reports that the security price pattern does not support the efficient market hypothesis. However, Reinganum (1981) points out that anomalies are the result of the misspecification of CAPM rather than an inefficient capital market because the abnormal returns persist for two years from the portfolio formation date. Basu (1983) employs a different methodology that controls for the effect of risk on returns and shows that earnings-price ratios can significantly explain the average returns on US stocks with different firm sizes. Banz (1981) suggests the existence of the size effect for very small firms and finds that average returns on low market equity are higher than that on high market equity. What's more, the book-to-market ratio also strongly affects the cross-section of average returns in the US stock market (Rosenberg et al., 1985) and value stocks have higher expected returns. Similarly, Chan et al. (1991) find that the book-to-market ratio also has a positive relation with cross-sectional average returns on Japanese stocks. Moreover, Fama and French (1992) question the uniqueness of the factor-beta and find the combination of the book-to-market ratio absorbs the role of leverage and earnings-price ratio in average stock returns and the negative relation between market value and returns. Furthermore, Fama and French (1993) reject the term-structure variables to explain the average returns in US government and corporate bonds and add two additional factors besides CAPM: SMB and HML.

$$r_{it} - r_{ft} = \alpha_i^{FF3} + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \epsilon_{i,t} \qquad (12)$$

In equation (12), SMB (low market capitalization minus high market capitalization) is a market capitalization factor, while HML (high book-to-market ratio minus low book-to-market ratio) is a value factor.

2.1.3 Carhart's Four-Factor Pricing Model

In general, people tend to overreact to unexpected news events. Previous pricing models are based too much on earnings rather than a long-term dividend. Shiller (1981) shows that the efficient markets model does not describe movements in data and dividends do not explain observed price movements. Furthermore, Ohlson and Penman (1985) find that return volatility increase following the ex-split date is due to the overreaction. Moreover, De Bondt and Thaler (1985) use a behavioural principle to test the overreaction hypothesis, and the result shows that losing stocks earn 25% more than winning. Also, Jegadeesh and Titman (1993) identify the one-year momentum effect. Hence, Carhart (1997) adds an additional factor UMD, which captures momentum anomaly.

$$r_{it} - r_{ft} = \alpha_i^{FFC} + \beta_{i,MKT}MKT_t + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \beta_{i,UMD}UMD_t + \epsilon_{i,t} \qquad (13)$$

2.1.4 Fama and French Five- and Six-Factor Pricing Models

The previous three-factor model was directed at capturing size and value in average returns, but some researchers propose that many anomalous variables related to profitability and investment cause problems for the three-factor model. Novy-Marx (2013) reports a proxy for profitability that is related to average returns. Likewise, Aharoni et al. (2013) show that there is a statistically reliable relation between investment and average returns. As the market value of a share of stock is the discounted value of expected dividends per share, I get

$$m_t = \sum_{\tau=1}^{\infty} \frac{E(d_{t+\tau})}{(1+r)^{\tau}} \qquad (14)$$

Where m_t is the share price at time t, E ($d_{t+\tau}$) is the expected dividend per share for period t+τ, and r is the internal return on expected dividends. Dividend f can be substituted by $Y_{t+\tau}$ (total equity earnings for period t+τ) minus $I_{t+\tau}$ (investment). Miller and Modigliani (1961) report that the total market value implied by equation (14) is,

$$M_t = \sum_{\tau=1}^{\infty} \frac{E(Y_{t+\tau} - I_{t+\tau})}{(1+r)^{\tau}} \tag{15}$$

Dividing both sides by B_t, the equation is

$$(1+r)^{\tau} = \sum_{\tau=1}^{\infty} E(\frac{Y_{t+\tau}}{B_t} - \frac{I_{t+\tau}}{B_t}) \times \frac{B_t}{M_t} \tag{16}$$

The equation indicates that by fixing everything except M_t and r, expected returns are positively influenced by book-to-market equity ratio ($\frac{B_t}{M_t}$), positively influenced by profitability ($\frac{Y_{t+\tau}}{B_t}$) and negatively influenced by expected growth in book equity investment ($\frac{I_{t+\tau}}{B_t}$). Fama and French (2015) add profitability and investment factors into the model to find whether the five-factor model performs better than the three-factor model.

$$r_{it} - r_{ft} = \alpha_i^{FF5} + \beta_{i,MKT}MKT_t + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \beta_{i,RMW}RMW_t + \beta_{i,CMA}CMA_t + \epsilon_{i,t} \tag{17}$$

RMW in equation (17) is the return on stocks with robust minus weak operating profitability, and CMA is the return on stocks with conservative minus aggressive investment style. In this model, the value factor is redundant for average returns when profitability and investment factors are added. Fama and French also add the UMD factor to the six-factor model. UMD (up minus down) is a momentum factor.

$$\begin{aligned} r_{it} - r_{ft} = \alpha_i^{FF6} + \beta_{i,MKT}MKT_t + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t \\ + \beta_{i,RMW}RMW_t + \beta_{i,CMA}CMA_t + \beta_{i,UMD}UMD_t + \epsilon_{i,t} \end{aligned} \tag{18}$$

Chapter 3: Testing Methodology

The methodology for testing the factor models is explained in this section. The first analysis is based on the commonly used Gibbons et al. (1989) methodology, denoted as the GRS statistic, to test the performance of different asset pricing models. The null hypothesis proposes that the cross-section alphas (intercepts of the regressions) of the considered factors are jointly equal to zero. The GRS test ranks the models by mean-variance efficiency of asset returns, and the metric is the F-statistic of joint significance of alphas. The GRS test is computed as follows:

$$GRS = \frac{T-N-1}{N}\widehat{\alpha'}\left[\left(1+\frac{\bar{r}_m}{\hat{\sigma}_m}\right)^2\hat{\Sigma}\right]^{-1}\hat{\alpha}\ \sim F(N, T-N-1) \tag{19}$$

T is the length of the time series, N is the number of portfolios, $\widehat{\alpha'}$ is the N×1 vector of the estimated alphas, $\bar{r}_m$ is the sample mean of excess market return, $\hat{\sigma}_m$ is the estimated standard error of the excess market return and $\hat{\Sigma}$ is the estimated residual variance-covariance matrix. Moreover, the GRS test can be expanded to more factors to allow for multifactor models (Kamstra & Shi, 2020):

$$GRS = \left(\frac{T}{N}\right)\left(\frac{T-N-L}{T-N-1}\right)\left(\frac{\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}}{1+\bar{\mu}'\hat{\Omega}^{-1}\bar{\mu}}\right)\ \sim F(N, T-N-1) \tag{20}$$

L is the number of independent variables (the number of risk factors), $\bar{\mu}$ is a column vector of the factors' sample means and $\hat{\Omega}$ is the estimated covariance matrix of the factors. A set of evenly numbered portfolios with lower GRS statistics suggests that the tested portfolios deviate from the efficient portfolio less than those with higher GRS statistics. If the asset pricing model adequately accounts for the assets' return, the alpha is equal to zero. The lower value of GRS statistics represents the better performance of the model.

As a robustness test for the GRS test, I utilize several other tests. The first one to evaluate the factor models is the average absolute value of the alphas, denoted as $A|\alpha_i|$. I estimate the alpha of fund I by running the time-series regression and then computing the average of alphas (Ahmed et al., 2019). The time-series factor regression's alpha can be seen either a gauge of model mispricing or a test asset's deviation from the model. To discern between the two implications, I gauge the alphas' dispersion by $\frac{A|\alpha_i|}{A|\bar{r}_i|}$ and $\frac{A\alpha_i^2}{A\bar{r}_i^2}$, which represent, respectively, the ratio of the average absolute value of the alphas to the average absolute value of $\bar{r}_i$, and the ratio of the average squared alpha to the average squared value of $\bar{r}_i$, where $\bar{r}_i$ is the average excess return on fund i minus the average excess return on the market portfolio. These two metrics measure the dispersion of the alphas generated by a specific asset pricing model relative to the dispersion of the average excess returns on test assets (Ahmed et al., 2019). Lower values for these ratios represent superior performance of the model. The next metric for model performance assessment is $\frac{As^2(\alpha_i)}{A\alpha_i^2}$, the ratio of the average variance of sampling errors of the estimated alphas to the average squared alpha. Specifically, the metric measures the percentage of the alpha estimate dispersion that results from sampling error rather than the true alpha dispersion, and a larger ratio value denotes an improved model performance (Ahmed et al., 2019). The last metric I examine is $A(R^2)$, which is the average of the degrees of freedom-adjusted time-series regression R^2 value, and better model performance is indicated by a greater number (Ahmed et al., 2019).

Chapter 4: Data and Variables

4.1 Data

From the Thomson DataStream database I collected data on Canada-domiciled mutual fund monthly returns with total net assets of C$200 million or higher. Additionally, to prevent duplication of various classes, no-load funds that are sold without commission or sales charge are chosen, inspired by Kildahl and Lunde (2018). I collect time-series return data for bond, equity, and mixed asset mutual funds for the 68-month period beginning on January 1, 2017, and ending on August 1, 2022. I can distinguish two subperiods: the first and second time frames are January 1, 2017 to March 1, 2020, and April 1, 2020 to August 1, 2022, respectively. Distinguishing the two subperiods is necessary to investigate whether there is any difference in factor models' performance during the pre- and post-Covid-19 periods. The last restriction is that the index replication method is not full. After eliminating five missing data points, I eventually get a sample with 361 mutual funds.

Ten portfolios are created to be used as test portfolios in the GRS test. Multiple goals are served by this choice. First, it lessens the variation in return caused by idiosyncratic risk. Second, a small number of portfolios are required to ensure that the GRS test has enough degrees of freedom. Third, the GRS test has some drawbacks. For instance, when the quantity of assets increases, the test's power is reduced (Ahmed et al., 2019). I construct the portfolios on the characteristics-based sort. According to Daniel and Titman (1997), asset characteristics are better at predicting cross-sectional stock returns than factor exposures. Characteristics are easily observable and are seen to be a more accurate indicator of a portfolio's true style. In addition, the factor-based sorting method needs many successive return series, which can become noisy with inadequate observations. Hence, I build portfolios by categorizing available funds according to funds' average excess returns. I sort the funds from the lowest returns to the highest returns and categorize the funds into 10 decile portfolios.

4.2 Variables

I choose Canada's one-month Treasury bill rate as the monthly risk-free rate r_{ft}. After reinvesting all capital gains and cash distributions, the monthly mutual fund return rate is determined by dividing the net asset value (NAV) change from the net asset value at the beginning of the month. The formula is as follows:

$$r_{it} = \frac{NAV_{it} - NAV_{i,t-1}}{NAV_{i,t-1}} \times 100\% \qquad (20)$$

For each month t, r_{it} represents the monthly fund return rate. NAV_{it} is the cumulative net value of units in month t. $NAV_{i,t-1}$ is the cumulative net value in month t-1. I obtain data on SMB, HML, UMD, RMW, and CMA factors from Kenneth R. French's Internet Data Library (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). The bond fund yield is based on the monthly yield of the S&P Canada Investment Grade Corporate Bond Industry Index. The equity fund yield is based on the S&P TSX Composite index. The benchmark for mixed asset funds is composed of 30% of the S&P Canada Investment Grade Corporate Bond Industry Index and 70% of the S&P TSX Composite Index.

Chapter 5: Empirical Results

5.1 Descriptive Statistics

Table 1 shows a summary of descriptive statistics by fund type. There are 361 funds in total, including 63 bond funds, 172 equity funds, and 126 mixed asset funds. The sample period is January 1, 2017 to August 1, 2022. The average excess returns of each fund type seem consistent. Since equity combines higher returns with higher risk, it makes sense that equity funds have larger average excess returns (i.e. 0.672) than other fund types. Table 2 shows descriptive statistics of the pre-Covid-19 period for all types of funds, bond funds, equity funds, and mixed asset funds. The sample period is January 1, 2017 to March 1, 2020. Equity funds have the largest average excess returns (i.e. 0.583), and bond funds have the smallest average excess returns (i.e. 0.218). Table 3 shows descriptive statistics of the post-Covid-19 period for all types of funds, bond funds, equity funds, and mixed asset funds. The sample period is April 1, 2020 to August 1, 2022. Table 3 presents that the average excess returns of bond funds are negative because the Bank of Canada lowered its policy interest rate to boost the economy in response to the Covid-19 pandemic. The standard deviations are larger than pre-Covid-19, which suggests disturbing circumstances. Table 4 shows the correlation of factors. Although there is a significant correlation between the variables, the outcome is comparable to that of the prior study (Sha & Gao, 2019).

Table 1 Descriptive Statistics

VarName	Number of funds	Raw returns	obs	mean	sd	max	min
Total	361	0.480	24548	0.433	3.507	41.706	-30.292
Bond	63	0.099	4284	0.052	1.685	7.220	-17.070
Equity	172	0.719	11696	0.672	4.456	41.706	-30.292
Mixed	126	0.345	8568	0.298	2.558	10.627	-18.931

Note: This table shows descriptive statistics of monthly raw returns and excess returns for all types of funds, bond funds, equity funds, and mixed asset funds. The data are from the Thomson Datastream database. The sample period is January 1, 2017 to August 1, 2022.

Table 2 Descriptive Statistics for Pre-Covid-19

VarName	Number of funds	Raw returns	obs	mean	sd	max	min
Total	361	0.495	14079	0.429	2.376	16.356	-22.864
Bond	63	0.284	2457	0.218	1.026	5.531	-4.471
Equity	172	0.649	6708	0.583	3.098	16.356	-22.864
Mixed	126	0.390	4914	0.324	1.575	6.503	-9.312

Note: This table shows descriptive statistics of monthly raw returns and excess returns for all types of funds, bond funds, equity funds, and mixed asset funds. The data are from the Thomson Datastream database. The sample period is from January 1, 2017, to March 1, 2020.

Table 3 Descriptive Statistics for Post-Covid-19

VarName	Number of funds	Raw returns	obs	mean	sd	max	min
Total	361	0.460	10469	0.439	4.610	41.706	-30.292
Bond	63	-0.151	1827	-0.171	2.271	7.220	-17.070
Equity	172	0.813	4988	0.792	5.800	41.706	-30.292
Mixed	126	0.284	3654	0.263	3.466	10.627	-18.931

Note: This table shows descriptive statistics of monthly raw returns and excess returns for all types of funds, bond funds, equity funds, and mixed asset funds. The data are from the Thomson Datastream database. The sample period is April 1, 2020 to August 1, 2022.

Table 4 Correlation of Factors

	SMB	HML	RMW	CMA	UMD	Rf
SMB	1	0.184	-0.421***	0.096	-0.261**	-0.051
HML	0.252**	1	-0.312***	0.805***	-0.403***	-0.225*
RMW	-0.441***	-0.166	1	-0.202*	-0.007	-0.065
CMA	0.014	0.823***	-0.053	1	-0.306**	-0.212*
UMD	-0.360***	-0.467***	-0.056	-0.220*	1	0.077
Rf	-0.184	-0.269**	-0.106	-0.280**	0.100	1

Note: The correlation is based on data of 68 months from January 1, 2017, to August 1, 2022.*** p<0.01, ** p<0.05, * p<0.1

5.2 An Empirical Analysis of Mutual Funds for the Whole Period

Starting with Panel A of Table 5, it is apparent that all but FF6 and FF5 are rejected by the GRS test. This suggests that these three models' explanation of the average excess returns on anomaly portfolios is incomplete. FF6 has the lowest GRS statistics of 1.732 (p-value=0.098). The next best factor models are FF5 in terms of GRS statistics with a marginally larger value than FF6.

The average absolute value $A|\alpha_i|$ of 0.090 per month in FF6 is the smallest among all the factor models. Also, the value of 0.260 for the ratio $\frac{A|\alpha_i|}{A|\bar{r}_i|}$ generates the lowest dispersion, which means that the dispersion of alphas is approximately 26% as large as the dispersion of the average excess returns on portfolios. A point estimate of 0.108 for the ratio $\frac{A\alpha_i^2}{A\bar{r}_i^2}$ generates the smallest dispersion for FF6. FF5 is the second-best model in terms of the three dispersion metrics, $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, and $\frac{A\alpha_i^2}{A\bar{r}_i^2}$. The two dispersion metrics, $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, show that FF3 generates the highest dispersion (i.e. the values are 0.122 and 0.353, respectively), suggesting that FF3 cannot capture the fund returns with higher dispersion. FF6 also offers the largest value for $\frac{As^2(\alpha_i)}{A\alpha_i^2}$, which is 1.400. I notice that FF6 has the largest point estimate of the $A(R^2)$, which is 0.783, and FFC has a slightly lower value of $A(R^2)$. In summary, FF6 outperforms other models with the GRS statistics, $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, $\frac{A\alpha_i^2}{A\bar{r}_i^2}$, $\frac{As^2(\alpha_i)}{A\alpha_i^2}$, and $A(R^2)$.

Table 5 Performance of the Factor Models

Model	CAPM	FF3	FFC	FF5	FF6
Panel A: all types of funds					
GRS	2.566	2.363	2.286	1.831	1.732
p-value(GRS)	0.012	0.021	0.026	0.077	0.098
$A\|\alpha_i\|$	0.119	0.122	0.118	0.100	0.090
$\frac{A\|\alpha_i\|}{A\|\bar{r}_i\|}$	0.344	0.353	0.341	0.289	0.260
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	0.182	0.172	0.168	0.118	0.108
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	0.742	0.804	0.831	1.266	1.400
$A(R^2)$	0.774	0.778	0.780	0.776	0.783
Panel B: bond fund					
GRS	3.048	3.225	3.089	2.575	2.401
p-value(GRS)	0.004	0.002	0.004	0.013	0.020
$A\|\alpha_i\|$	0.092	0.094	0.087	0.089	0.078
$\frac{A\|\alpha_i\|}{A\|\bar{r}_i\|}$	0.724	0.740	0.685	0.701	0.614
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	0.677	0.715	0.623	0.698	0.562
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	0.616	0.605	0.694	0.681	0.854
$A(R^2)$	0.779	0.776	0.778	0.773	0.774
Panel C: equity fund					
GRS	3.091	2.986	2.897	2.418	2.302
p-value(GRS)	0.003	0.004	0.006	0.019	0.025
$A\|\alpha_i\|$	0.185	0.172	0.173	0.151	0.150
$\frac{A\|\alpha_i\|}{A\|\bar{r}_i\|}$	0.995	0.925	0.930	0.812	0.806
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	1.180	1.027	1.041	0.751	0.734
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	0.566	0.668	0.671	0.989	1.037
$A(R^2)$	0.855	0.853	0.851	0.852	0.850
Panel D: mixed asset fund					
GRS	7.519	7.186	7.150	6.339	6.146
p-value(GRS)	0.000	0.000	0.000	0.000	0.000
$A\|\alpha_i\|$	0.083	0.089	0.085	0.072	0.064
$\frac{A\|\alpha_i\|}{A\|\bar{r}_i\|}$	0.203	0.218	0.208	0.176	0.157
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	0.045	0.053	0.048	0.033	0.026
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	1.484	1.280	1.438	2.224	2.851
$A(R^2)$	0.848	0.848	0.848	0.845	0.846

Note: This table reports the performance of five competing models (CAPM, FF3, FF4, FF5, FF6, respectively) in all types of funds, bond funds, equity funds, and mixed asset funds. I employ the GRS test and derive the corresponding p-value associated with the GRS statistics. The other 5 metrics are introduced in the methodology section. The estimates are rounded to 3 decimal places. The comparison covers 361 mutual funds in Canada between January 1, 2017 and August 1, 2022.

The GRS test on the bond fund, shown in Panel B of Table 5, rejects all the asset pricing models at conventional levels of significance. Yet the rejection is weakest for FF6 with a GRS statistic of 2.401 (p-value=0.020). In terms of the magnitude of the GRS statistics, FF5 is the second-best factor model, followed by the CAPM and the FFC. FF3 appears to be the worst-performing model as it generates the largest GRS statistics.

Looking at the dispersion metrics of alpha for FF6, $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, and $\frac{A\alpha_i^2}{A\bar{r}_i^2}$ generate the smallest values of 0.078, 0.614, and 0.562, respectively. The next best model for the three dispersion measures is the FFC. FF6 still does the best with the largest point estimate of 0.854 for the model fitting indicator $\frac{As^2(\alpha_i)}{A\alpha_i^2}$, which indicates that 85% of the second moment of the alpha estimates for the model is due to sampling error. However, FF3 does the worst when judged by $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, $\frac{A\alpha_i^2}{A\bar{r}_i^2}$, and $\frac{As^2(\alpha_i)}{A\alpha_i^2}$. The largest value of $A(R^2)$ is 0.779 for CAPM. To sum up, FF6 provides the best description of the bond fund portfolios with the best performance of the GRS statistics and four metrics.

Focusing on Panel C of Table 5, the GRS test rejects all the factor models at the 5% level of significance. However, FF6 outperforms all other factor models as it generates the smallest point estimate of 2.302 (p-value=0.025) for the GRS statistics. The second-best performer is FF5 according to the magnitude of the GRS statistics, followed by FFC and FF3.

FF6 has the smallest average absolute value of 0.150% each month for alphas among all the factor models. The value of 0.806 for the ratio $\frac{A|\alpha_i|}{A|\bar{r}_i|}$ produced by FF6 is also the smallest among all the factor models, indicating that the dispersion of alphas is approximately 81% as large as the dispersion of the average excess returns on portfolios. Moreover, FF6 appears to be the top performer with 0.734 for the ratio $\frac{A\alpha_i^2}{A\bar{r}_i^2}$ and 1.037 for the ratio $\frac{As^2(\alpha_i)}{A\alpha_i^2}$. By contrast, though CAPM has the largest point estimate for $A(R^2)$ (i.e. 0.855), I notice that all the factor models outperform CAPM by generating better point estimates for the $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, $\frac{A\alpha_i^2}{A\bar{r}_i^2}$, and $\frac{As^2(\alpha_i)}{A\alpha_i^2}$. To summarize, FF6 offers the best performance for explaining the excess returns on equity funds based on the GRS statistics, $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, $\frac{A\alpha_i^2}{A\bar{r}_i^2}$, and $\frac{As^2(\alpha_i)}{A\alpha_i^2}$.

The results for the mixed asset funds in Panel D of Table 5 indicate that all the factor models are rejected easily by the GRS test. FF6 produces the smallest GRS statistics of 6.146 (p-value=0.000), followed by FF5 and FFC. The worst performer is CAPM with the largest GRS statistics.

The point estimates of $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, and $\frac{A\alpha_i^2}{A\bar{r}_i^2}$ generated by FF6 are the smallest, at 0.064, 0.157, and 0.026, respectively, followed by FF5. FF6 also generates the largest value, 2.851, for the ratio $\frac{As^2(\alpha_i)}{A\alpha_i^2}$. By contrast, for the three alpha dispersion measures, $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, and $\frac{A\alpha_i^2}{A\bar{r}_i^2}$, FF3 yields the largest point estimate (i.e., 0.089, 0.218, and, 0.053, respectively). Coincidentally, CAPM, FF3, and FFC show a superior performance with the same largest point estimate of $A(R^2)$. Taken together, FF6 is the best factor pricing model, accommodating the excess returns on balanced funds, judged by the GRS statistics, $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, $\frac{A\alpha_i^2}{A\bar{r}_i^2}$, and $\frac{As^2(\alpha_i)}{A\alpha_i^2}$.

5.3 An Empirical Analysis of Mutual Funds for Pre-Covid-19

Next, I consider the candidate factor pricing models' ability to forecast excess returns for the pre-Covid-19 period. Panel A of Table 6 reports that all the models cannot be rejected by the GRS test. FF6 is the best description of the average excess returns on total funds with the smallest GRS statistics of 1.666 (p-value=0.150), followed by FFC and FF5. The CAPM presents the worst performance based on the GRS statistics (i.e., 2.059).

Table 6 Performance of the Factor Models for Pre-Covid-19

Model	CAPM	FF3	FFC	FF5	FF6
Panel A: all types of funds					
GRS	2.059	1.860	1.684	1.830	1.666
p-value(GRS)	0.065	0.099	0.140	0.109	0.150
$A\vert\alpha_i\vert$	0.152	0.117	0.102	0.114	0.104
$\frac{A\vert\alpha_i\vert}{A\vert\bar{r}_i\vert}$	0.738	0.568	0.495	0.553	0.505
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	0.687	0.417	0.383	0.358	0.369
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	0.448	0.964	1.109	1.112	1.139
$A(R^2)$	0.647	0.637	0.646	0.641	0.646
Panel B: bond fund					
GRS	2.693	3.432	3.471	3.486	3.454
p-value(GRS)	0.019	0.006	0.006	0.006	0.007
$A\vert\alpha_i\vert$	0.085	0.094	0.099	0.091	0.097
$\frac{A\vert\alpha_i\vert}{A\vert\bar{r}_i\vert}$	0.322	0.356	2.400	0.345	0.367
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	0.146	0.146	0.160	0.141	0.156
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	0.438	0.569	0.517	0.600	0.532
$A(R^2)$	0.701	0.708	0.712	0.709	0.715
Panel C: equity fund					
GRS	3.030	2.603	2.304	2.491	2.234
p-value(GRS)	0.010	0.024	0.044	0.033	0.051
$A\vert\alpha_i\vert$	0.237	0.208	0.207	0.208	0.212
$\frac{A\vert\alpha_i\vert}{A\vert\bar{r}_i\vert}$	0.862	0.756	0.753	0.756	0.771
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	0.761	0.473	0.476	0.434	0.477
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	0.426	0.886	0.940	0.959	0.925
$A(R^2)$	0.751	0.748	0.742	0.753	0.749
Panel D: mixed asset fund					
GRS	4.019	3.031	3.133	2.848	3.209
p-value(GRS)	0.002	0.011	0.010	0.017	0.010

Table 6 Performance of the Factor Models for Pre-Covid-19

Model	CAPM	FF3	FFC	FF5	FF6
$A\|\alpha_i\|$	0.093	0.071	0.054	0.054	0.047
$\frac{A\|\alpha_i\|}{A\|\bar{r}_i\|}$	0.108	0.082	0.063	0.063	0.054
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	0.754	0.465	0.315	0.303	0.263
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	1.192	2.544	3.960	3.758	4.601
$A(R^2)$	0.742	0.732	0.733	0.743	0.739

Note: This table reports the performance of five competing models (CAPM, FF3, FF4, FF5, FF6, respectively) in all types of funds, bond funds, equity funds, and mixed asset funds. I employ the GRS test and derive the corresponding p-value associated with the GRS statistics. The other 5 metrics are introduced in the methodology section. The estimates are rounded to 3 decimal places. The comparison covers 361 mutual funds in Canada between January 1, 2017 and March 1, 2020.

The average absolute value of the alpha is 0.102% for the FFC, which is smaller than any other model. FFC also generates the smallest value of 0.495 for pricing performance metric $\frac{A|\alpha_i|}{A|\bar{r}_i|}$. The corresponding point estimates are marginally lower for FF6 (i.e. 0.104, and 0.505 for $A|\alpha_i|$, and $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, respectively). FF5 emerges as the best-performing asset pricing model with the smallest value of 0.358 for the ratio $\frac{A\alpha_i^2}{A\bar{r}_i^2}$, followed byFF6. The ratio $\frac{As^2(\alpha_i)}{A\alpha_i^2}$ for FF6 has a value of 1.139, which is significantly higher than all the other factor models. CAPM turns out to be the worst of all the measures except for the value of $A(R^2)$ (i.e. 0.647). According to the GRS statistics and $\frac{As^2(\alpha_i)}{A\alpha_i^2}$ measure, FF6 performs well as the superior model, and CAPM performs worst based on almost all the metrics for pre-Covid-19. The outcome displays the same for total funds across the entire period.

I next consider Panel B of Table 6, which shows that all models are easily rejected by the GRS test. CAPM has the smallest GRS statistic of 2.693 (p-value=0.019), which still disagrees with the null hypothesis that all intercepts from CAPM are jointly equal to zero. The next best model is FF3, followed by FF6, and FFC.

The average absolute alpha for the portfolios from CAPM is 0.085% per month, which is the smallest value across all the factor models. The $\frac{A|\alpha_i|}{A|\bar{r}_i|}$ ratio of 0.322 for CAPM is also smaller than any competing model. FF5 also does a good job by producing a slightly higher value for $A|\alpha_i|$ and $\frac{A|\alpha_i|}{A|\bar{r}_i|}$. Moreover, FF5 generates the smallest value of $\frac{A\alpha_i^2}{A\bar{r}_i^2}$, at 0.141. About 60% of the dispersion of the alpha for FF5 is attributable to the sampling error. I also find that FF6 generates the largest value of 0.715 for $A(R^2)$. All in all, CAPM offers the best performance based on the GRS statistics, $A|\alpha_i|$, and $\frac{A|\alpha_i|}{A|\bar{r}_i|}$ measures, while FF5 also does a good job in explaining the excess returns for the bond fund for pre-Covid-19. For bond funds, the best model appears differently than that for the entire time.

I find that the alphas in Panel C of Table 6 are not explained by the factor models apart from FF6, which generates the smallest value of 2.234 for the GRS statistics (p-value=0.051). FFC produces the second smallest value of GRS statistics, followed by FF5 and FF3.

FFC generates the smallest average absolute alpha, of 0.207% per month, as well as the smallest point estimate of 0.753, for the ratio $\frac{A|\alpha_i|}{A|\bar{r}_i|}$. FF5 performs well based on more than half of the metrics. For example, the ratio $\frac{A\alpha_i^2}{A\bar{r}_i^2}$ relative to the model is 0.434, which is the smallest. FF5 also generates the largest value of 0.959 for ratio $\frac{As^2(\alpha_i)}{A\alpha_i^2}$, suggesting that approximately 96% of the second moment of the alpha estimates for FF5 is attributable to sampling error, compared to 43% for CAPM. FF5 produces the largest value of $A(R^2)$ of 0.753. FF6 is ranked first, generating the smallest GRS statistics, while FF5 does a good job with the majority of the performance metrics for pre-Covid-19. For equity funds, the outcome appears to be constant with that for the entire period.

The results in Panel D of Table 6 report that the null hypothesis of zero alphas cannot be rejected by all the asset pricing models. FF5 produces smaller GRS statistics of 2.848 (p-value=0.017), compared to other factor models. FF3 emerges as the second-best model, followed by FFC and FF6.

FF6 stands out as the best with most of the best performance metrics. For the three dispersion measures, FF6 produces the smallest value of $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, and $\frac{A\alpha_i^2}{A\bar{r}_i^2}$, which are 0.047, 0.054, and 0.263, respectively. Judged by the same metrics, FF5 turns out to be the second-best model, with values of 0.054, 0.063, and 0.303, respectively. FF6 also generates the largest value of 4.601 for ratio $\frac{As^2(\alpha_i)}{A\alpha_i^2}$. For these metrics, CAPM ranks at the bottom among all the factor models. FF5 delivers the best performance for $A(R^2)$, which is 0.743. The performance of FF5 is best judged by the GRS statistics and $A(R^2)$ for pre-Covid-19. For mixed asset funds, the outcome seems different than it does over the long term.

5.4 An Empirical Analysis of Mutual Funds for Post-Covid-19

I observe from Panel A of Table 7 that the null hypothesis of the GRS test cannot be rejected for all the factor models at conventional levels of significance. FF6 emerges as the best performance model with the smallest GRS statistics of 0.896, followed by CAPM, FF5, and FFC.

FF6 delivers the best performance on virtually all the metrics. For instance, it generates the smallest average absolute alpha, smallest point estimate for ratio of $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, smallest point estimate for the dispersion metric $\frac{A\alpha_i^2}{A\bar{r}_i^2}$, and largest value of $\frac{As^2(\alpha_i)}{A\alpha_i^2}$, at 0.107, 0.149, 0.023, and 8.539, respectively. Notably, looking at the model fitting indicator, $\frac{As^2(\alpha_i)}{A\alpha_i^2}$, FF6 is far superior to other models. The sole exception is A(R^2), where CAPM stands out (i.e. 0.818). Overall, FF6 best captures the average excess returns on all types of funds for the post-Covid-19 periods, judging by the GRS statistics, $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, $\frac{A\alpha_i^2}{A\bar{r}_i^2}$, and $\frac{As^2(\alpha_i)}{A\alpha_i^2}$. For total funds, the outcome is consistent with that across the entire period.

Table 7 Performance of the Factor Models for Post-Covid 19

Model	CAPM	FF3	FFC	FF5	FF6
Panel A: all types of funds					
GRS	1.095	2.114	1.980	1.119	0.896
p-value(GRS)	0.112	0.088	0.112	0.412	0.561
$A\|\alpha_i\|$	0.217	0.225	0.217	0.156	0.107
$\frac{A\|\alpha_i\|}{A\|\bar{r}_i\|}$	0.303	0.314	0.303	0.218	0.149
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	0.111	0.110	0.103	0.050	0.023
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	1.041	1.155	1.306	3.583	8.539
A(R^2)	0.818	0.817	0.811	0.812	0.810
Panel B: bond fund					
GRS	3.626	3.238	3.227	1.870	1.697
p-value(GRS)	0.009	0.018	0.020	0.138	0.184
$A\|\alpha_i\|$	0.147	0.140	0.132	0.137	0.122
$\frac{A\|\alpha_i\|}{A\|\bar{r}_i\|}$	0.750	0.714	0.673	0.699	0.622
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	0.788	0.714	0.651	0.867	0.710
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	0.693	0.827	0.974	0.963	1.346
A(R^2)	0.806	0.779	0.792	0.788	0.780
Panel C: equity fund					
GRS	1.016	1.207	1.076	0.608	0.468
p-value(GRS)	0.468	0.356	0.435	0.784	0.883
$A\|\alpha_i\|$	0.128	0.162	0.152	0.151	0.112
$\frac{A\|\alpha_i\|}{A\|\bar{r}_i\|}$	0.298	0.377	0.353	0.351	0.260
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	0.159	0.265	0.231	0.215	0.130

Table 7 Performance of the Factor Models for Post-Covid 19

Model	CAPM	FF3	FFC	FF5	FF6
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	3.050	2.001	2.394	3.500	6.481
$A(R^2)$	0.893	0.889	0.887	0.883	0.879
Panel D: mixed asset fund					
GRS	2.927	2.595	2.489	1.427	1.188
p-value(GRS)	0.023	0.043	0.054	0.264	0.378
$A\|\alpha_i\|$	0.264	0.269	0.257	0.186	0.130
$\frac{A\|\alpha_i\|}{A\|\bar{r}_i\|}$	0.306	0.312	0.298	0.216	0.151
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	0.103	0.104	0.097	0.048	0.025
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	0.533	0.587	0.669	1.814	3.895
$A(R^2)$	0.881	0.875	0.870	0.868	0.866

Note: This table reports the performance of five competing models (CAPM, FF3, FF4, FF5, FF6, respectively) in all types of funds, bond funds, equity funds, and mixed asset funds. I employ the GRS test and derive the corresponding p-value associated with the GRS statistics. The other 5 metrics are introduced in the methodology section. The estimates are rounded to 3 decimal places. The comparison covers 361 mutual funds in Canada between April 1, 2020 and August 1, 2022.

Turning to Panel B of Table 7, I find the GRS test rejects all the factor models at the 5% significance level, except for FF6, and FF5. FF6 stands out as the best with the smallest GRS statistics of 1.697 (p-value=0.184), followed by FF5.

What's more, FF6 presents the best performance when judged by $A|\alpha_i|$ and $\frac{A|\alpha_i|}{A|\bar{r}_i|}$ (i.e., 0.122 and 0.622, respectively). Conversely, the corresponding point estimate of the metrics for CAPM is the worst across all the factor models, though the value of $A(R^2)$ is the largest overall. The point estimate of the dispersion metric $\frac{A\alpha_i^2}{A\bar{r}_i^2}$ for FFC is the smallest, at 0.651, followed by FF6. I observe that FF6 produces the largest point estimate of 1.346 for ratio $\frac{As^2(\alpha_i)}{A\alpha_i^2}$. Hence, the results suggest that FF6 has a superior ability to accommodate the excess returns on the bond fund based on the GRS statistics, $A|\alpha_i|$, and $\frac{A|\alpha_i|}{A|\bar{r}_i|}$ for post-Covid 19. The result is the same as the whole period for bond funds.

Looking at Panel C of Table 7, I see that all the factor pricing models cannot be rejected by the GRS test at conventional levels of significance. FF6 shows superior performance with the smallest GRS statistics, at 0.468 (p-value=0.883), followed by FF5, CAPM, and FFC.

Comparing the competing factor models based on the 5 metrics reveals that FF6 is the top contender with the best performance for most of the metrics, which are $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, $\frac{A\alpha_i^2}{A\bar{r}_i^2}$,and $\frac{As^2(\alpha_i)}{A\alpha_i^2}$ (i.e., 0.112, 0.260, 0.130, and 6.481, respectively). Contrarily, FF3 underperforms all other factor pricing models on virtually all metrics, except $A(R^2)$. Using the metric $A(R^2)$, FF6 is only 0.01% lower than CAPM, which is the best performer. I conclude that FF6 is the best performer in explaining the excess returns of the equity fund for post-Covid-19. For equity funds, the result remains constant over the whole time frame.

I can see that in Panel D of Table 7 the GRS test rejects only two-factor models at a 5% significance level. The null hypothesis cannot be rejected for FFC, FF5, and FF6 (p-value=0.054, 0.264, and 0.378, respectively), and FF6 presents the best result when judged by the GRS statistic of 1.188.

The factor models rank in the same order for the point estimates of three dispersion metrics, $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, and $\frac{A\alpha_i^2}{A\bar{r}_i^2}$. FF6 ranks first among the candidate factor models for all the dispersion indicators, followed by FF5, and FFC. Contrary to FF6, FF3 appears to be the worst factor model based on the three dispersion metrics. In particular, FF6 also performs best for the ratio $\frac{As^2(\alpha_i)}{A\alpha_i^2}$. The largest value of $A(R^2)$ is from CAPM. FF6 appears to be the top performer based on the

GRS statistics and the majority of metrics for post-Covid-19. For mixed asset funds, the outcome is constant during the entire time.

Chapter 6: Conclusion

This paper investigates which asset pricing model best fits the performance of Canadian mutual fund. The objectives are accomplished using asset pricing models——the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965), Fama and French's (1993) Three-Factor Pricing Model, Carhart's (1997) Four-Factor Pricing Model, and Fama and French's (2015) Five- and Six-Factor Pricing Models (2018). 361 Canadian mutual funds are analyzed from January 1, 2017, to August 1, 2022. Three categories and one combined group make up my classification of funds: bond funds, equity funds, mixed asset funds, and all types of funds. I employ six indicators to gauge how well these factor models perform: GRS statistics, $A|\alpha_i|$, $\frac{A|\alpha_i|}{A|\bar{r}_i|}$, $\frac{A\alpha_i^2}{A\bar{r}_i^2}$, $\frac{As^2(\alpha_i)}{A\alpha_i^2}$, and $A(R^2)$. Additionally, I compare the effectiveness of factor pricing models for the periods before and after Covid-19, and for the entire time.

Table 8 Best Models

Test	Total funds	Bond funds	Equity funds	Mixed funds
GRS	FF6	FF6	FF6	FF6
$A\|\alpha_i\|$	FF6	FF6	FF6	FF6
$\frac{A\|\alpha_i\|}{A\|\bar{r}_i\|}$	FF6	FF6	FF6	FF6
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	FF6	FF6	FF6	FF6
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	FF6	FF6	FF6	FF6
$A(R^2)$	FF6	CAPM	CAPM	FFC

Note: This table shows the best model (among CAPM, FF3, FFC, FF5, and FF6) of different metrics for all types of funds, bond funds, equity funds, and mixed asset funds, respectively, for the entire period. The sample period is from January 1, 2017 to August 1, 2022.

Table 9 Best Models for Pre-Covid-19

Test	Total funds	Bond funds	Equity funds	Mixed funds
$A\|\alpha_i\|$	FFC	CAPM	FFC	FF6
$\frac{A\|\alpha_i\|}{A\|\bar{r}_i\|}$	FFC	CAPM	FFC	FF6
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	FF5	FF5	FF5	FF6
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	FF6	FF5	FF5	FF6
$A(R^2)$	CAPM	FF6	FF5	FF5

Note: This table shows the best model (among CAPM, FF3, FFC, FF5, and FF6) of different metrics for all types of funds, bond funds, equity funds, and mixed asset funds, respectively, for the pre-Covid-19 periods. The sample period is January 1, 2017 to March 1, 2020.

Table 10 Best Models for Post-Covid 19

Test	Total funds	Bond funds	Equity funds	Mixed funds
GRS	FF6	FF6	FF6	FF6
$A\|\alpha_i\|$	FF6	FF6	FF6	FF6
$\frac{A\|\alpha_i\|}{A\|\bar{r}_i\|}$	FF6	FF6	FF6	FF6
$\frac{A\alpha_i^2}{A\bar{r}_i^2}$	FF6	FFC	FF6	FF6
$\frac{As^2(\alpha_i)}{A\alpha_i^2}$	FF6	FF6	FF6	FF6
$A(R^2)$	CAPM	CAPM	CAPM	CAPM

Note: This table shows the best model (among CAPM, FF3, FFC, FF5, and FF6) of different metrics for all types of funds, bond funds, equity funds, and mixed asset funds, respectively, for the pre-Covid-19 periods. The sample period is April 1, 2020, to August 1, 2022.

Table 8 shows that FF6, which produces the smallest GRS statistics and the best performance for most measures, is the best model for explaining excess returns for all types of funds, bond funds, equity funds, and mixed asset funds, during the entire period, though it does not have the largest $A(R^2)$. However, pre-Covid-19 tells a different story. Turning to Table 9, CAPM is the best-performing model for bond funds. FF5 is the best-performing model for mixed-asset funds, while FF6 performs well for four other metrics. I hypothesize that the reason for this is that the stock market is strongly related to factors in FF6. The performance of the model may be impacted when a low percentage of funds is invested in stocks. Focusing on Table 10, for post-Covid-19, FF6 still captures the excess returns best with the majority metrics for all types of funds, bond funds, equity funds, and mixed asset funds.

My research demonstrates that while FF5 has significant success in the US (Kildahl & Lunde, 2018) and China (Sha & Gao, 2019), it cannot account for the performance of Canadian mutual funds. The study reaches the same conclusion for \ Polish equity mutual funds (Trzebiński, 2022), namely that FF6 best accounts for the performance of equity mutual funds.

There are some implications of this study. First, for different fund type, we should use different factor models to evaluate the mutual fund performance. Second, we usually take alpha as a measure of fund's manager's ability. The failure of most factor models in explaining the mutual fund excess returns suggests that we should consider other explanations for the significant alpha.

To choose the optimal model for mutual funds, many other factor models should be examined in future research, such as Hou et al.'s Q-factor model, the Stambaugh and Yuan 4-factor model, and Barillas and Shanken's 6-factor model.

References

Aharoni, G., Grundy, B., & Zeng, Q. (2013). Stock returns and the Miller Modigliani valuation formula: Revisiting the Fama French analysis. *Journal of Financial Economics*, *110*(2), 347–357. https://doi.org/10.1016/j.jfineco.2013.08.003

Ahmed, S., Bu, Z., & Tsvetanov, D. (2019). Best of the Best: A Comparison of Factor Models. *Journal of Financial and Quantitative Analysis*, *54*(4), 1713–1758. https://doi.org/10.1017/S0022109018000947

Angelidis, T., Giamouridis, D., & Tessaromatis, N. (2013). Revisiting mutual fund performance evaluation. *Journal of Banking & Finance*, *37*(5), 1759–1776. https://doi.org/10.1016/j.jbankfin.2013.01.006

Banz, R. W. (1981). The relationship between return and market value of common stocks. *Journal of Financial Economics*, *9*(1), 3–18. https://doi.org/10.1016/0304-405X(81)90018-0

Basu, S. (1977). Investment Performance of Common Stocks in Relation to Their Price-Earnings Ratios: A Test of the Efficient Market Hypothesis. *The Journal of Finance*, *32(3)*, 663–682.

Basu, S. (1983). The relationship between earnings' yield, market value and return for NYSE common stocks. *Journal of Financial Economics*, *12*(1), 129–156. https://doi.org/10.1016/0304-405X(83)90031-4

Beaulieu, M.-C., Dufour, J.-M., & Khalaf, L. (2016). Identification-Robust Factor Pricing: Canadian Evidence. *L'Actualité Économique*, *91*(1–2), 235–252. https://doi.org/10.7202/1036920ar

Berk, J. B., & van Binsbergen, J. H. (2016). Assessing asset pricing models using revealed preference. *Journal of Financial Economics*, *119*(1), 1–23. https://doi.org/10.1016/j.jfineco.2015.08.010

Black, F. (1972). Capital Market Equilibrium with Restricted Borrowing. *The Journal of Business*, *45*(3), 444–455. https://doi.org/10.1086/295472

Canada mutual funds post record redemptions as COVID-19 worries grow. (2020). https://rsmcanada.com/insights/economics/canada-mutual-funds-post-record-redemptions-as-covid-19-worries-.html

Carhart, M. M. (1997). On Persistence in Mutual Fund Performance. *The Journal of Finance*, *52*(1), 57–82. https://doi.org/10.1111/j.1540-6261.1997.tb03808.x

Chan, L. K. C., Hamao, Y., & Lakonishok, J. (1991). Fundamentals and Stock Returns in Japan. *The Journal of Finance*, *46(5)*, 1739–1764.

Chen, H., Engert, W., Huynh, K., Nicholls, G., Nicholson, M., & Zhu, J. (2020, July 2). *Cash and COVID-19: The impact of the pandemic on demand for and use of cash* (No. 2020–6). Bank of Canada. https://doi.org/10.34989/sdp-2020-6

Chiah, M., Chai, D., Zhong, A., & Li, S. (2016). A Better Model? An Empirical Investigation of the Fama-French Five-factor Model in Australia: Empirical Tests on the Five-factor Model. *International Review of Finance*, *16*(4), 595–638. https://doi.org/10.1111/irfi.12099

Christensen, M. (2013). Danish mutual fund performance. *Applied Economics Letters*, *20*(8), 818–820. https://doi.org/10.1080/13504851.2012.750415

Daniel, K., & Titman, S. (1997). Evidence on the Characteristics of Cross Sectional Variation in Stock Returns. *The Journal of Finance*, *52*(1), 1–33. https://doi.org/10.1111/j.1540-6261.1997.tb03806.x

De BONDT, W. F. M., & Thaler, R. (1985). Does the Stock Market Overreact? *The Journal of Finance*, *40*(3), 793–805. https://doi.org/10.1111/j.1540-6261.1985.tb05004.x

Fama, E. F., & French, K. R. (1992). The Cross-Section of Expected Stock Returns. *The Journal of Finance*, *47*(2), 427–465. https://doi.org/10.1111/j.1540-6261.1992.tb04398.x

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, *33*(1), 3–56. https://doi.org/10.1016/0304-405X(93)90023-5

Fama, E. F., & French, K. R. (1996). Multifactor Explanations of Asset Pricing Anomalies. *The Journal of Finance*, *51*(1), 55–84. https://doi.org/10.1111/j.1540-6261.1996.tb05202.x

Fama, E. F., & French, K. R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, *116*(1), 1–22. https://doi.org/10.1016/j.jfineco.2014.10.010

Fama, E. F., & French, K. R. (2016). Dissecting Anomalies with a Five-Factor Model. *Review of Financial Studies*, *29*(1), 69–103. https://doi.org/10.1093/rfs/hhv043

Fama, E. F., & French, K. R. (2018). Choosing factors. *Journal of Financial Economics*, *128*(2), 234–252. https://doi.org/10.1016/j.jfineco.2018.02.012

Gibbons, M. R., Ross, S. A., & Shanken, J. (1989). A Test of the Efficiency of a Given Portfolio. *Econometrica*, *57*(5), 1121. https://doi.org/10.2307/1913625

Griffin, J. M. (2002). Are the Fama and French Factors Global or Country Specific? *The Review of Financial Studies*, *15*(3), 783–803.

Hou, K., Mo, H., Xue, C., & Zhang, L. (2019). Which Factors? *Review of Finance*, *23*(1), 1–35. https://doi.org/10.1093/rof/rfy032

Huang, R., Pilbeam, K., & Pouliot, W. (2021). Do actively managed US mutual funds produce positive alpha? *Journal of Economic Behavior & Organization*, *182*, 472–492. https://doi.org/10.1016/j.jebo.2019.03.006

Huij, J., & Verbeek, M. (2009). On the Use of Multifactor Models to Evaluate Mutual Fund Performance. *Financial Management*, *38*(1), 75–102. https://doi.org/10.1111/j.1755-053X.2009.01029.x

Jegadeesh, N., & Titman, S. (1993). *Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency*. 28.

Kamstra, M. J., & Shi, R. (2020). A Note on the GRS Test. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.3775089

Kildahl, T., & Lunde, M. T. (2018). *On Asset Pricing Models and Mutual Fund Performance*. 60.

Kubota, K., & Takehara, H. (2018). Does the Fama and French Five-Factor Model Work Well in Japan?: Fama and French Five-factor Model in Japan. *International Review of Finance*, *18*(1), 137–146. https://doi.org/10.1111/irfi.12126

Lalwani, V., & Chakraborty, M. (2019). Multi-factor asset pricing models in emerging and developed markets. *Managerial Finance*, *46*(3), 360–380. https://doi.org/10.1108/MF-12-2018-0607

Leite, P., Cortez, M. C., & Armada, M. R. (2009). *Measuring Fund Performance Using Multi-Factor Models: Evidence for the Portuguese Market*. 26.

L'Her, J.-F., Masmoudi, T., & Suret, J.-M. (2004). *Evidence to support the four-factor pricing model from the Canadian stock market*. 16.

Lintner, J. (1965). *Security Prices, Risk, and Maximal Gains From Diversification*. 30.

Markowitz, Wm. (1959). Variations in rotation of the earth, results obtained with the dual-rate moon camera and photographic zenith tubes. *Symposium - International Astronomical Union*, *11*, 26–33. https://doi.org/10.1017/S0074180900104164

Mateus, I. B., Mateus, C., & Todorovic, N. (2019). Review of new trends in the literature on factor models and mutual fund performance. *International Review of Financial Analysis*, *63*, 344–354. https://doi.org/10.1016/j.irfa.2018.12.012

Miller, M. H., & Modigliani, F. (1961). Dividend Policy, Growth, and the Valuation of Shares. *The Journal of Business*, *34*(4), 411–433.

Novy-Marx, R. (2013). The other side of value: The gross profitability premium. *Journal of Financial Economics*, *108*(1), 1–28. https://doi.org/10.1016/j.jfineco.2013.01.003

Ohlson, J. A., & Penman, S. H. (1985). Volatility increases subsequent to stock splits: An empirical aberration. *Journal of Financial Economics*, *14*(2), 251–266. https://doi.org/10.1016/0304-405X(85)90017-0

Our Industry | IFIC.ca. (n.d.). Retrieved October 5, 2022, from https://www.ific.ca/en/articles/who-we-are-our-industry/

Rehman, A., & Baloch, Q. B. (2016). Evaluating Pakistan™s Mutual Fund Performance: Validating through CAPM and Fama French 3-Factor Model. *Journal of Managerial Sciences*, *1*, 10.

Reinganum misspecification of CAPM.pdf. (n.d.).

Rosenberg, B., Reid, K., & Lanstein, R. (1985). Persuasive evidence of market inefficiency. *Streetwise: The Best of the Journal of Portfolio Management*, 8.

Sehgal, S., & Babbar, S. (2017). Evaluating alternative performance benchmarks for Indian mutual fund industry. *Journal of Advances in Management Research*, *14*(2), 222–250. https://doi.org/10.1108/JAMR-04-2016-0028

Sha, Y., & Gao, R. (2019). Which is the best: A comparison of asset pricing factor models in Chinese mutual fund industry. *Economic Modelling*, *83*, 8–16. https://doi.org/10.1016/j.econmod.2019.09.016

Sharpe, W. F. (1964). CAPITAL ASSET PRICES: A THEORY OF MARKET EQUILIBRIUM UNDER CONDITIONS OF RISK*. *The Journal of Finance*, *19*(3), 425–442. https://doi.org/10.1111/j.1540-6261.1964.tb02865.x

Shiller, R. (1981). Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? *The American Economic Review*, *71(3)*, 421–436. https://doi.org/10.3386/w0456

Treynor, J., & Mazuy, K. (1966). CAN MUTUAL FUNDS OUTGUESS THE MARKET? *Harvard Business Review*, *44 (4)*, 131.

Trzebiński, A. A. (2022). *Assessing the performance of mutual funds with multifactor asset pricing models*. 28.