THE THEORY OF MATHEMATICAL *SUBTRACTION* IN ARISTOTLE

by

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DEDICATION PAGE

To my Mother, Vera Romashova

Мама,

Спасибо тебе за твою бесконечную поддержку, дорогая!

TABLE OF CONTENTS

LIST OF FIGURES

ABSTRACT

The problem with most modern accounts of Aristotle's so-called 'theory of mathematical abstraction' or *aphairesis* (ἀφαίρεσις) is that it is interpreted primarily through the scope of the epistemological process of an immediate reception of mathematical forms by the soul without matter from which the former are said to be 'abstracted'. However, this interpretation is not present in Aristotle's texts. Instead, *aphairesis* presents itself as a method by which the mode of being of the objects of mathematics is explained: it elucidates the *location* and *place* of the category of quantity within a particular sensible substance, but not some kind of an abstracting activity of drawing mathematical forms or universals from matter. This latter type of the epistemological abstraction of mathematicals found in most modern commentaries was developed by later commentators of Aristotle. The analysis of the modern scholarship shows a clear trace of influence of the ancient tradition.

LIST OF ABBREVIATIONS USED

WORKS OF ARISTOTLE

APo.	*Posterior Analytics*
Cat.	*Categories*
DA	*De Anima*
Meta.	*Metaphysics*
NE	*Nicomachean Ethics*
PA	*Prior Analytics*
Phys.	*Physics*
Pol.	*Politics*
Top.	*Topics*

WORKS OF PLATO

Rep.	*Republic*
Symp.	*Symposium*
Tim.	*Timaeus*

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CHAPTER 1 INTRODUCTION

According to Heraclitus, "πάντα χωρεῖ καὶ οὐδὲν μένει, καὶ ποταμοῦ ῥοῇ ἀπεικάζων τὰ ὄντα λέγει ὡς 'δὶς ἐς τὸν αὐτὸν ποταμὸν οὐκ ἂν ἐμβαίης" (Plato, *Cratylus* 402a). This means that everything gives way and nothing remains still just like a flowing river. It is impossible to step in the same stream twice, and what is more, it is impossible to step in it even once (*Meta.* 1010a14-15). Since everything is in flux and since the matter of physical objects is constantly changing and moving, mathematics cannot be true of the sensible triangles and spheres made of bronze, wood or any other materials, as well as of those drawn on paper or in sand. It follows that when we say that this triangle is of such and such a size or that this triangle is equilateral whose internal angles are each 60°, these truths will almost never hold true of any sensible triangle. Thus, to affirm something will always result in a false statement irrelevant to its physical representative. Even if we suppose the matter of some mathematical shape to be motionless, it would still be impossible to build or draw a perfectly triangular shape due to our limited human abilities.

To save the objectivity and precision necessary for mathematics, Plato posited eternal and unchanging intermediates (τὰ μεταξύ) which later became the main focus of Aristotle's criticisms found primarily in the last two books of his *Metaphysics*, XIII and XIV. Aristotle describes Plato's intermediates in the following way: "besides sensible things and Forms he says there are the objects of mathematics, which occupy an intermediate position, differing from sensible things in being eternal and unchangeable, from Forms in that there are many alike, while the Form itself is in each case unique" (Ross, *Meta.* 987b15). Thus, whenever a Platonist postulates multiple circles, he refers neither to the Form of Circularity nor to perceptible circles, but considers perfect circles as existing in the realm of intermediates.

Aristotle rejects the concept of intermediates and Forms because for him it is impossible that substances can exist separately from their physical instantiations. Aristotle interprets Platonic separation to be ontological, in the sense that intermediates and Forms are separated from their physical representations, which in the traditional interpretation means that the former are 'placed beyond heaven.'[1] Aristotle, instead, places the objects of mathematical science *in* the sensible realm and claims that magnitudes, volumes, planes, and lines are perceptible features of sensible bodies. He agrees with Plato, however, that the mathematical features of physical objects fall short of their corresponding mathematical objects. There are no such things as perfectly circular spheres and perfectly straight lines:

> For neither are perceptible lines such lines as the geometer speaks of (for no perceptible thing is straight or curved in this way; for a hoop touches a straight edge not at a point, but as Protagoras said it did, in his refutation of the geometers), nor are the movements and complex orbits in the heavens like those of which astronomy treats, nor have geometrical points the same nature as the actual stars. (Ross, *Meta.* 997b34-998a6).

Yet he still insists that "obviously physical bodies contain surfaces and volumes, lines and points, and these are the subject-matter of mathematics" (Ross, *Physics.* 193b23–25). What are we to make of this? How can mathematics be true of the sensible world if sensible objects fail to have the precision of mathematical objects? Do the perfect objects of mathematics exist only in thinking, and would this mean that the precise objects of mathematics are simply creations of the mind? Aristotle must provide an alternative both to this mentalist interpretation of mathematics and to Plato's separately existing intermediates and Forms.

Aristotle solves these two problems by introducing the terminology of *aphairesis* (ἀφαίρεσις), or '*subtraction*' the function of which is to *reveal* or *unfold* the *spatial location* of the quantitative and continuous magnitude within a particular sensible body. Specifically,

[1] Gerald Frank Else, "The terminology of the ideas," *Harvard Studies in Classical Philology* 47 (1936): 55.

by attending to sensible bodies, we cognitively take away or subtract colour, passions, affections, motion, and change from a bronze sphere and then arrive at the continuous three-dimensional shape, such as sphericity. We may further subtract the second dimension and arrive at the idea of a two-dimensional circle. Since passions, affections, and motion are removed in thinking, mathematical truths apply within this subtracted result, a result which mind arrives at as the result of a process of thinking but which it does not create, all without needing to postulate unchanging intermediates. When we 'remove in thinking' this does not mean that the objects of mathematics are creations of mind alone. The physical representatives to which we refer, do contain surfaces, volumes, and sizes.

The problem with most modern scholarship concerning *aphairesis* is that it does not always distinguish the proper Aristotelian sense of *aphairesis* as *subtraction* from the sense of *aphairesis* as *extraction* which was developed by later Aristotelian commentators and is usually accepted by modern scholars. While *aphairesis* has two distinct meanings in the post-Aristotelian tradition, it is *always* translated as 'abstraction,' thereby making it difficult sometimes to distinguish which of the two senses the scholarship uses, either *subtraction* or *extraction.* The distinction between them is the following: *aphairesis* as *subtraction* implies the removal of many and leaving the remainder for consideration, *aphairesis* as *extraction* suggests the extraction of one while disregarding the rest. Thus, to distinguish the two senses, I will refer to *aphairesis* as *subtraction*, while I will refer to *abstraction* as *extraction.*

John Cleary (1985) was the first to point out that Aristotle's *aphairesis* should be translated as *subtraction* and that it has no relation to the epistemological process of the reception of forms by the soul without their matter, or to the abstracting activity of the mind which the modern sense of abstractionism might imply. Cleary states, that "the traditional view has been that he (Aristotle) is referring to some epistemological process of abstraction from matter, by means of which mathematical objects (along with other universals) are isolated from sensible particulars for the purposes of scientific knowledge."[2] By the "traditional view" he means de Koninck, Mansion, and others, as well as Thomas Aquinas, who Cleary claims to be the source of the new epistemological interpretation of *aphairesis* not present in Aristotle. In my thesis, I show that, in fact, it was not Aquinas who first used the term in this way. This interpretation first appears in Alexander of Aphrodisias. Cleary does not enter into a discussion of the epistemological process of abstraction in the works of the medieval and modern commentators whom he mentions. Thus, in the light of interpreting *aphairesis* as *subtraction,* I analyze key works of these commentators where the epistemological sense appears, along with other commentators not mentioned by Cleary but who introduce important developments in the understanding of *aphairesis*.

Cleary's definition of an epistemological sense as abstraction of mathematical objects (along with other universals) isolated from sensible particulars[3] is too general and not exhaustive. My analysis of the ancient and modern scholars whom Cleary mentions (and does not mention) reveals *four* specific epistemological senses of *aphairesis* found in later interpreters yet not present in Aristotle's works: abstraction of a form (essence) from matter, abstraction of a universal (e.g. man) from particulars, abstraction of a mathematical universal (e.g. circle) from particulars, or abstraction of *this* particular mathematical form (*this* circular shape) from *this* particular matter, motion, and change. Since all four uses presuppose the *taking out* of *one* from the many, I bring them all under one term, *extraction*. Furthermore, because *aphairesis* is interpreted as *extraction* or abstraction of form from matter, it sometimes gets confused with induction (*epagoge*) which in itself is, indeed, the process by which the soul receives the forms without matter. The function of induction is to collect

[2] John Cleary, "On the Terminology of "Abstraction" in Aristotle," *Phronesis* 30 (1985): 26.
[3] Ibid., 26.

particulars under a single universal (e.g. circle or man), whereas the function of *aphairesis* is to successively remove many things within a particular sensible body upon which one considers the remainder only, in this case, its mathematical properties.

In contrast to modern commentators who follow Alexander of Aphrodisias, Aquinas, and other ancient commentators in their understanding of *aphairesis*, and in contrast to those who, though they do not follow their interpretation, nonetheless speak of “The Theory of Abstraction in Aristotle,” I propose a new approach to this topic. It is in this spirit that I have entitled this thesis “The Theory of Mathematical *Subtraction* in Aristotle.” In addition, I avoid the term ‘abstraction’ because it is no longer free from the modern connotations of something being merely ideal, unrealistic, and divorced from any real situations.

CHAPTER 2 THE MEANING OF ABSTRACTION

Introduction

In the following chapter I analyze the etymology of the term ἀφαίρεσις and look into the evolution of the term, as well as investigate how it accumulated different meanings over the centuries in commentaries on Aristotle and elsewhere. Specifically, I will explore those among Aristotle's commentators who made the first shift into the meaning of Aristotle's *aphairesis* from the proper Aristotelian *subtraction* to the modern sense of *abstraction* or *extraction*. I find this shift crucial because it serves as the source of all the confusions in the modern interpretations of Aristotle's philosophy of mathematics and his so-called 'theory of abstraction' views which I shall investigate in chapter 4 of this thesis. In terms of Aristotle's standard phrase 'τὰ ἐξ ἀφαιρέσεως' or 'the things said as a result from subtraction,' the majority of scholarship always associates it with the objects of mathematics (τὰ μαθηματικά). There are however, two places in the corpus where the phrase has no relation to either geometry or arithmetics: *Posterior Analytics* I. 18 and in *Metaphysics* XIII. 2. Due to this I separate the nine uses of τὰ ἐξ ἀφαιρέσεως into two non-mathematical which have no reference to mathematics and seven mathematical uses with an obvious reference to τὰ μαθηματικά. In addition, modern scholarship generally translates both τὰ ἐξ ἀφαιρέσεως and τὰ μαθηματικά in Aristotle as 'abstract objects.' With respect to Plato, modern interpreters of his philosophy of mathematics also tend to call his intermediates and Form numbers 'abstract objects.' In fact, I suggest that Plato's mathematicals are more deserving of the term 'abstract objects,' because they are non-physical, non-spatial, non-temporal, and non-causal. On the contrary, Aristotle's objects of mathematics do not deserve to be called 'abstract,' since his objects of mathematics are integrally connected with causal matter, temporal, physical, and spatial matter.

2.1 Etymology and Evolution of the Term

The terminology of ἀφαίρεσις is derived from the Greek verb ἀφαιρεῖν, the compound of *apo* (‘from’) and *hairein* (‘to take’) which stands together as ‘to take away,’ ‘to remove,’ or ‘to subtract.’[4] The verbal noun ἀφαίρεσις can therefore be translated as ‘removal’, ‘deprivation,’ or ‘subtraction.’ The term ἀφαίρεσις appears in Aristotle’s corpus in two forms – in the form of ‘τὰ ἐξ ἀφαιρέσεως’ expression, where ‘τὰ’ designates either the objects of mathematics or the things that have no relation to geometry and arithmetics, and in the form of ἀφαιρεῖν which is applied to any direct object whether mathematical or non-mathematical. In the medieval and modern accounts of Aristotle’s philosophy of mathematics, ἀφαίρεσις is translated as ‘abstraction’ and ‘τὰ ἐξ ἀφαιρέσεως’ as ‘abstract objects’ which is often simply identified with τὰ μαθηματικά. Aristotle also uses περιαιρεῖν along with ἀφαιρεῖν where the former signifies the same operation, i.e. ‘to take away something that surrounds’ or ‘to remove.’ Since both περιαιρεῖν and ἀφαιρεῖν have nearly the same meaning, Aristotle sometimes uses them interchangeably.[5] Another verb is ἀφῑέναι – it has almost the same meaning as ἀφαιρεῖν and περιαιρεῖν which means ‘to throw away from,’ ‘to dismiss,’ or ‘to discharge.’

Even though *aphairesis* is translated similarly everywhere as ‘abstraction’ in translations of Aristotle, it has two distinct meanings in interpretations of Aristotle, *extraction*

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[4] Liddell, Henry George, Scott, Robert, Jones, Henry Stuart, and McKenzie, Roderick. *A Greek-English Lexicon*. Rev. and Augm. throughout / by Sir Henry Stuart Jones, with the Assistance of Roderick McKenzie, and with the Cooperation of Many Scholars. ed. Oxford: New York: Clarendon Press; (Oxford University Press, 1996), 285-6.
[5] Cf. *Categories* 7a31, *Metaphysics* XI.3 1061a29.

and *subtraction*. This subtle distinction often escapes the scholars of Aristotelian mathematics – they do not notice that there *are* actually two distinct meanings[6]. The sense of *extraction*, which first appeared in the ancient and medieval commentaries on Aristotle implies the reception of forms by the soul without their matter, passions, affections, and motion.[7] *Extraction* is implied when we take out *one* thing while disregarding the rest, e.g. to abstract a form (essence) from matter, to abstract a universal from particulars, to abstract a mathematical universal (e.g. a circle) from particulars, to abstract *this* particular mathematical form (this circular shape) from *this* particular matter, motion, and change. *Aphairesis* as *subtraction* means when we take away or remove many things and then study the remainder, e.g. to remove colour, passions, affections, motion, and change from a bronze triangle, and arrive at the continuous two-dimensional shape,[8] triangularity. *Extraction,* as I will show, does not exist in Aristotle. There is only one sense, *subtraction*, and people are mostly talking of another sense. Furthermore, understanding *aphairesis* in an extractionist sense may result in a possible confusion of *aphairesis* with *epagoge* which then suggests that the results of abstraction are universals.[9] However, Aristotle never employs *aphairesis* when he discusses universals or the process of induction. *Aphairesis* is rather just bringing out the space within mathematical truths upon which mathematical objects are discovered. John Thorp draws attention to this original sense of the term: "It is striking that, whereas in English we abstract

[6] Ironically, this distinction was introduced by Allan Bäck, but he has not yet himself distinguished the proper Aristotelian sense of *aphairesis* from all other senses. I will discuss this in section 4.3, chapter 4 of my thesis.
[7] Cleary (1985) calls the process of receiving forms by the soul without their matter 'epistemological' or 'psychological.' Cleary does not discuss the epistemological process of abstraction both in the works of the medieval and modern commentators whom he mentions. In addition, his definition of an epistemological sense as abstraction of mathematical objects (along with other universals) isolated from sensible particulars is somewhat general and not exhaustive.
My analysis of the ancient and modern scholars revealed four specific epistemological senses of *aphairesis* not present in Aristotle's works: abstraction of form (essence) from matter, abstraction of a universal (e.g. man) from particulars, abstraction of a mathematical universal (e.g. circle) from particulars, abstraction of *this* mathematical particular form (*this* circular shape) from *this* particular matter, motion, and change. Since all these four uses presuppose the taking out of one from the many, I bring them all under one term, *extraction.*
[8] *Metaphysics* VIII. 6 at 145a20 ff. Aristotle states that the 'round' of a 'bronze' is shape (μορφή) that the mathematician studies.
[9] Cf. Allan Bäck (2006), de Koninck (1960), and Mansion (1913).

the elements we are interested in and throw the rest away, in Greek it works the opposite way: one strips off (*aphairei*) and discards the elements that are not of interest.” [10] This latter kind of *aphairesis,* as I will show later in my analysis, is the proper Aristotelian one. *Aphairesis* as ‘extraction’ is a later concept introduced by Alexander of Aphrodisias, Philoponus, Thomas Aquinas and further accepted by Allan Bäck, Charles De Koninck, Auguste Mansion,[11] and Julia Annas. We need to keep in mind this distinction between *extraction* and *subtraction*.

One of the Aristotelian Greek commentators who first mentions *aphairesis* in his works is Alexander of Aphrodisias (AD 200). According to Ian Mueller, “the doctrine of abstractionism can be traced back to Alexander of Aphrodisias.”[12] Mueller does not define ‘abstractionism’ in Alexander, though based on Mueller’s article we may clearly establish two reasons why he gives it such a name. First, by ‘abstractionism’ Mueller means that Alexander’s reading of Aristotle’s objects of mathematics is mentalist in character.[13] This means that Alexander, according to Mueller, associated the objects of mathematics with *epinoia* (ἐπίνοια) and claimed that the mathematical body does not exist in its own right.[14] The second reason why Mueller calls Alexander “the source of abstractionist interpretation”[15] obviously lies in the fact that in his commentary on Aristotle’s *Metaphysics* III, Alexander suggests implicitly that “the way of abstraction” is Aristotle’s own view:

> Aristotle now proceeds against those who say that the mathematicals exist according to a certain proper nature, only not themselves outside of the sensibles, but rather

[10] John Thorp, "Intelligible Matter in Aristotle," *The Society for Ancient Greek Philosophy Newsletter*. 385 (2010): 2. https://orb.binghamton.edu/sagp/385

[11] Mansion’s use of mathematical *aphairesis* at times appears blurred. Sometimes he understands it as abstraction of mathematical form from matter and sometimes as abstraction of matter from mathematical form.

[12] Ian Mueller, "Aristotle's Doctrine of Abstraction in the Commentators', Aristotle Transformed, ed." *R. Sorabji, London* (1990): 467.

[13] Ibid., 469.

[14] Ibid., 467. I do not agree that Alexander’s reading of Aristotle is purely mentalist in character since he thought of mathematicals as being a part of physical nature also: ἡ γὰρ ἐν τοῖς μαθηματικοῖς νοουμένη διάστασις μετὰ τῶν τῷ λόγῳ κεχωρισμένων παθητικῶν ἡ αἰσθητὴ φύσις· ἐν ἀμφοῖν γὰρ ἡ αἰσθητὴ φύσις ἐν ὑποστάσει οὖσα φύσει. (For the extension that is thought of in the case of the mathematicals, together with the affective attributes (*pathetika*) separated by reason, is the sensible nature. For the sensible nature is naturally in existence (*hupostasis*) in both of these; transl. by Arthur Madigan).

[15] Mueller, 469.

> present in them. This view would differ from the view which says that the mathematicals are assumed and thought of by way of abstraction.[16]
>
> Εἰπὼν πρὸς τοὺς τὰ μαθηματικὰ οὐσίας λέγοντας εἶναι αὐτὰς καθ’ αὑτὰς κεχωρισμένας τῶν τε ἰδεῶν καὶ τῶν αἰσθητῶν, νῦν μέτεισιν ἐπὶ τοὺς λέγοντας μὲν τὰ μαθηματικὰ εἶναι κατά τινὰ φύσιν οἰκείαν, οὐ μὴν ἔξω αὐτὰ τῶν αἰσθητῶν εἶναι, ἀλλ’ ἐν τούτοις. διαφέροι δ’ ἄν ἡ δόξα αὕτη τῆς λεγούσης ἐξ ἀφαιρέσεως τὰ μαθηματικὰ λαμβάνεσθαί τε καὶ νοεῖσθαι.[17]

In this passage, Alexander takes the latter view to be Aristotle’s own. What I suggest marks off as the beginning of all confusions related to Aristotle’s *aphairesis* is that Alexander applies the word *aphairesis* to the process of taking out the form (not the mathematical shape, but the ‘essence’) from matter. According to Helmig, Alexander is also the first commentator who mentions two different types of abstraction in Aristotle – taking away matter and abstraction of form:

> In the discussion of Aristotle's notion of abstraction, it has become clear that he usually employs *aphairein* and its cognates in the sense of taking away matter (abstraction M) and that abstraction of form (abstraction F) does not occur in his works. However, there are at least two passages in his *De Intellectu* where *aphairein* is clearly used in the sense of abstraction of form (*De Intellectu* 110.19 and 111.16). Hence, in Alexander of Aphrodisias we notice that he prepares the ground for the modern use of the term ‘abstraction’ (*aphairesis*) as abstraction of form.[18]

Thus, we take the two passages from *De Intellectu* to make the shift in how the term is applied. In chapter 3, I will show that Aristotle never uses *aphairesis* as the taking out of form from matter, but only as the taking away of passions, affections, motion, and change within a particular sensible body so as to arrive at the continuous extension. William Dooley in his translation of Alexander’s commentary on Aristotle’s *Metaphysics* II indicates that Alexander develops the following noetic theory at length in his commentary to *De Intellectu*: “the eidos

[16] Alexander. *On Aristotle's Metaphysics 2 & 3*. Ancient Commentators on Aristotle. (Ithaca, N.Y.: Cornell University Press, 1992), 135.
[17] Michael Hayduck, ed. *Alexandri Aphrodisiensis in Aristotelis metaphysica commentaria*. Vol. 1. (de Gruyter, 1891), 200-201.
[18] Christoph Helmig, *Forms and concepts: concept formation in the Platonic tradition*, Vol. 5. (Walter de Gruyter, 2012), 155-156.

in this text later became the *species impressa* of medieval Aristotelians, the form abstracted from material things by the agent intellect that, united with the potential intellect, enables the latter to bring forth the *species expressa* or concept."[19] While Alexander does explicitly say in *De Intellectu* that forms (essences) are abstracted from matter, he never says that these are mathematical forms (shapes) that are abstracted from matter. What the mathematicians do abstract are sensible attributes.[20] In his commentary on Aristotle's *Metaphysics* III Alexander makes clear that the objects of mathematics are shapes (τὰ σχήματα), not forms (τὰ εἴδη): "Mathematicals include numbers, lengths, shapes (τὰ σχήματα); he would mean plane figures: triangle, square, circle, and the like, as well as points."[21]

Philoponus (c. 490 – c. 570) is the first Aristotelian commentator to use *aphairesis* together with the mathematical forms (τὰ εἴδη), even though Aristotle never uses the two terms together – he does not say that the results of subtraction are the forms (τὰ εἴδη). Instead, the result of subtraction is the quantitative and continuous magnitude or shape (*Metaphysics* XI. 3). Even if we assume that the latter is the same as the 'form,' Philoponus' interpretation still refers to a proper Aristotelian *aphairesis*, i.e. what is subtracted or removed is matter, not shape or form of the object because Philoponus clearly states that the forms are the things from subtraction or the results of subtraction of matter. In the passage below Philoponus states that it is more difficult to separate in thought the form of the bones from matter than the form of a sphere from its matter, bronze. This is the case because the form of bones is seen only in bones whereas the circular form of a bronze sphere is seen in other objects too, such as in an ice sphere, in a wooden sphere, in a golden ring, in the moon, etc. This is why it is impossible to define the form of bones and flesh without their physical matter, whereas the matter of a

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[19] Alexander. *On Aristotle's Metaphysics 2 & 3*. Ancient Commentators on Aristotle. (Ithaca, N.Y.: Cornell University Press, 1992), 39.
[20] Ibid., 135.
[21] Ibid., 102.

bronze sphere does not enter into a definition of a sphere.[22] Here is how Philoponus explains it in his commentary on Aristotle's *de Anima* I.1-2:

> The mathematician, too, is concerned with the forms that are inseparable from their matter, though not with all of them but only with those that can be conceptually separated. These are the so-called common objects of perception, such as magnitudes and shapes. The form of flesh and bone and similar things cannot even be separated from their matter conceptually; for when the soft and the moist and the red and anything else of which the form of flesh is made up are being thought of, their appropriate matter is being thought of simultaneously, and when their matter is being subjected to abstraction [ἀφαιρουμένης], they too are subjected to abstraction [συναφῄρηται]. The mathematician, then, states the definitions of the forms in themselves, as they are the result of abstraction [τῶν ἐξ ἀφαιρέσεως], not by taking account of the matter, but by stating them in themselves. (trans. Filip J. van der Eijk, 57,28-58,6)[23]

> Ὁ δὲ μαθηματικὸς καὶ αὐτὸς καταγίνεται περὶ τὰ εἴδη τὰ ἀχώριστα τῆς ὕλης, οὐ πάντα ἀλλ' ὅσα δυνατὰ τῇ ἐπινοίᾳ χωρίζεσθαι· ταῦτα δ' ἐστὶ τὰ καλούμενα κοινὰ αἰσθητά, οἷον μεγέθη καὶ σχήματα. σαρκὸς γὰρ εἶδος καὶ ὀστοῦ καὶ τῶν τοιούτων οὐδὲ κατ' ἐπίνοιαν χωρισθῆναι τῆς ὕλης δύναται· τὸ γὰρ μαλθακὸν καὶ ὑγρὸν καὶ ἐρυθρὸν καὶ ἐξ ὧν ἄλλων εἰδοποιεῖται ἡ σὰρξ ἅμα τῷ νοηθῆναι συνεπινοουμένην ἔχει τὴν οἰκείαν ὕλην, ἀφαιρουμένης δὲ τῆς ὕλης καὶ αὐτὰ συναφῄρηται. ἀποδίδωσιν οὖν ὁ μαθηματικὸς τῶν καθ' αὑτὰ τῶν εἰδῶν τῶν ἐξ ἀφαιρέσεως τοὺς ὁρισμοὺς οὐχ ὑπολογιζόμενος ὕλην, ἀλλ' αὐτὰ καθ' αὑτὰ ἀποδιδούς.[24]

The literal translation of τῶν εἰδῶν τῶν ἐξ ἀφαιρέσεως does not presuppose an extractionist sense of *aphairesis*, e.g. to abstract a form from matter, change, and motion. What τῶν εἰδῶν τῶν ἐξ ἀφαιρέσεως literally means is that the result of the subtraction of change, motion, and colour is the form. Now if we consider τῶν εἰδῶν τῶν ἐξ ἀφαιρέσεως in the last line translated by Ian Mueller for instance, there arises this very sense of extraction absent in Aristotle, which suggests that we abstract the forms which are "capable of abstraction:" "Therefore, the mathematician gives definitions of the *per se* (essential) forms capable of abstraction; without

[22] Even if the mathematician states the definitions of mathematical forms in themselves without any reference to matter, the mathematician yet makes a refence to intelligible matter (ὕλη νοητή), e.g. a circle is 'a plane figure' where plane is intelligible matter and 'figure' is form (*Meta.* VII.10, 1036a 1-12; VII.11, 1036b 32-1037a5; VIII.6, 1045a 33-6; XI.1). I will discuss the concept of intelligible matter in section 4.3 of my thesis.
[23] Philopon, Jean. *Philoponus: On Aristotle On the Soul 1.1-2*. (Bloomsbury Publishing, 2014), 75.
[24] Michael Hayduck, ed. *Ioannis Pholoponi in Aristotelis De Anima Libros Commentaria*, Vol. 15. (de Gruyter, 1950), 57.

taking matter into account, he gives these definitions in and of themselves."[25] The extractionist sense could have been avoided if Mueller for instance said that it is matter that is "capable of abstraction," though only on the condition if by abstraction he meant 'removal' or 'subtraction.'

Proclus (412 – 485 AD) distinguishes *aphairesis* as a 'common' tool in reaching the objects of mathematics. In his commentary to the First Book of Euclid's *Elements*, Proclus highlights two more methods for finding mathematical objects, one of which is by collection from particulars, and another is by drawing mathematicals from previously existing forms in the soul. He asks the following question: "Should we admit that they [mathematicals] are derived from sense objects, either by abstraction as is commonly said, or by collection from particulars to one common definition? Or should we rather assign to them an existence prior to sense objects, as Plato demands and as a processional order of things indicates?"[26] He further rejects Aristotle's method of abstraction due to the precision problem as there is no equality of lines from center to circumference nor is there such a thing as the rightness of angles in sensible matter.[27] He also rejects Aristotle's *aphairesis* on the basis of matter being in constant change from one state to another.[28] The collection from particulars is also rejected because the objects of perception are secondary, obscure, and less honourable.[29] Later he accepts Plato's view of the pre-existent forms in the soul which are devoid of matter and are themselves precise mathematical objects. In chapter 4 section 4.3 of this work I attempt to solve the problem of exactness and that of motion with a possible Aristotelian alternative. What is noteworthy in Proclus' account of Aristotelian *aphairesis* is that he, unlike Boethius,

[25] Ian Mueller, "Aristotle's Doctrine of Abstraction in the Commentators', Aristotle Transformed, ed." *R. Sorabji, London* (1990): 465.
[26] Proclus, *A Commentary on the First Book of Euclid's Elements*, translated by Glenn R. Morrow, (Princeton, N.J.: Princeton University Press, 1970), 10.
[27] Ibid., 11.
[28] Ibid.
[29] Ibid., 12.

Thomas Aquinas, Allan Bäck, Charles de Koninck, and Auguste Mansion, does not conflate the term with "collection from particulars to one common definition"[30] or with "assembling of the common characters in particulars"[31] and sees *aphairesis* and induction as distinct operations.[32]

In the Latin tradition, Boethius (477 AD – 524 AD) was the first to translate Aristotelian *aphairesis* in Latin as *abstractio* or *abstrahere* (*ab* – away from; *trahere* – to draw) from which the English word 'abstraction' is derived. In addition, he formulates yet another new application of *aphairesis*: as abstraction of a universal from particulars (which, in fact, is induction). The Latin word *abstractio* now becomes associated with the process of collection of particulars under a single universal. In the commentary on Porphyry's *Isagoge*, Boethius uses the term to abstract different genera and joining them into one concept, such as for instance joining a human and a horse in an image by abstraction to form a centaur, which presupposes the taking out the genera of man and horse from the things in which they exist (11.2 and 11.6).[33] Later in 11.7 he applies the term to explain how one genus is attained from the same species: "the process of conceiving genera and species involves abstracting their point of similarity from the individuals in which they exist (e.g. the similarity of humanity from individual humans different from each other)."[34] However, what seems to be described here is the process of induction, not that of abstraction. Another divergance from Aristotle is Boethius' application of *aphairesis* in his division of sciences. In his work *De Trinitate*

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[30] Ibid., 10.
[31] Ibid., 13.
[32] Cf. Guthrie who conflates *aphairesis* with induction: "Abstraction of form […] is the process whereby the natural philosopher, having observed and reflected on a number of sensible objects understands them by perceiving the *eidos* inherent in them all, constant and unchangeable" in Guthrie, W. K. C. *A History of Greek Philosophy*. 1st Pbk. ed. (Cambridge: University Press, 1990), 105. Also compare Spruit's use of *aphairesis*: "[According to Aristotle] the objects of thinking are essences existing in mind as universals abstracted from their concrete manifestations" in Leen Spruit, *Species Intelligibilis: From Perception to Knowledge*, Brill's Studies in Intellectual History v. 48-49. (Leiden; New York: Brill, 1994), 45.
[33] Boethius. *Commentary of Porphyry's Isagoge.* Trasl.by George MacDonald Ross. https://pdfslide.net/documents/boethius-commentary-on-porphyry-trg-macdonald-ross.html
[34] Ibid.

Boethius applies *aphairesis* in the division of sciences in a completely different way from Aristotle's. While Aristotle uses the term for the objects of mathematics, that is when the mathematician studies the results of mathematical subtraction from motion, change, affections, and potencies (i.e. three-dimensional quantitative and continuous), Boethius applies it only in the realm of Theology. According to Boethius, it is Theology which deals with the abstract, not mathematics. Following his interpretation, he seems to equate 'abstract' with ontologically 'separate.' Here is how he puts it:

> 1. Physics deals with that which is in motion and not abstract [*in motu inabstracta*] *anupexairetos* (for it handles the forms of bodies involving matter, which forms are not able to be actually separated from bodies; and these bodies are in motion, for when earth is carried downward and fire up, the form joined with matter has motion as well);
> 2. Mathematics deals with that which is not in motion and not abstract [*sine motu inabstracta*] (for this ponders forms of bodies without matter, and thus without motion; but these forms, since they are in matter, cannot [actually] be separated from bodies);
> 3. Theology deals with the abstract, which lacks motion and is separable [sine motu abstracta atque separabilis] (for the substance of God lacks both matter and motion). (trans. Kenyon).[35]
>
> naturalis, in motu inabstracta 'ἀνυπεξαίρετος' (considerat enim corporum formas cum materia, quae a corporibus actu separari non possunt, quae corpora in motu sunt ut cum terra deorsum ignis sursum fertur, habetque motum forma materiae coniuncta), mathematica, sine motu inabstracta (haec enim formas corporum speculatur sine materia ac per hoc sine motu, quae formae cum in materia sint, ab his separari non possunt), theologica, sine motu abstracta atque separabilis (nam dei substantia et materia et motu caret).

Boethius seem to have modeled his division on Aristotle's own division of sciences seen in *de Anima* I.I 403b10-19 and in *Metaphysics* VI.1 1026a5-25. There is no mention of *aphairesis* in *Metaphysics* VI.1, it is present only in *de Anima* I.I. In *de Anima* I.I Aristotle states that the physicist concerns himself with passive and active attributes which are inseparable both in fact and in thought. The objects of mathematics are separable by the method of subtraction only in thought. The objects of First Philosophy are separate both in thought and in fact.

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[35] Boethius. *On the Holy Trinity (de Trinitate).* Transl. by Erik C. Kenyon. https://pvspade.com/Logic/docs/BoethiusDeTrin.pdf p.3

While Boethius equates his *aphairesis* with ontological separation, Aristotle renders it as intellectual separation.

> The physicist is he who concerns himself with all the properties active and passive of bodies or materials thus or thus defined; attributes not considered as being of this character he leaves to others, in certain cases it may be to a specialist, e.g. a carpenter or a physician, in others (a) where they are inseparable in fact, but are separable from any particular kind of body by an effort of abstraction, to the mathematician, (b) where they are separate both in fact and in thought from body altogether, to the First Philosopher or metaphysician. (Smith, *Meta.* 403b10-15).
>
> ἀλλ' ὁ φυσικὸς περὶ ἅπανθ' ὅσα τοῦ τοιουδὶ σώματος καὶ τῆς τοιαύτης ὕλης ἔργα καὶ πάθη, ὅσα δὲ μὴ τοιαῦτα, ἄλλος, καὶ περὶ τινῶν μὲν τεχνίτης, ἐὰν τύχῃ, οἷον τέκτων ἢ ἰατρός, τῶν δὲ μὴ χωριστῶν μέν, ᾗ δὲ μὴ τοιούτου σώματος πάθη καὶ ἐξ ἀφαιρέσεως, ὁ μαθηματικός, ᾗ δὲ κεχωρισμένα, ὁ πρῶτος φιλόσοφος; ἀλλ' ἐπανιτέον ὅθεν ὁ λόγος.

Later in the 13th century, as proposed by Cleary, Thomas Aquinas made another shift in understanding how *ta mathematica* are attained, specifically that *aphairesis* signifies an "abstraction of form from matter."[36] This, however, is incorrect. As I have already pointed out, Alexander of Aphrodisias was the first to use *aphairesis* to explain how the form is abstracted from matter and Philoponus was the first to use the term with mathematical form (τὰ εἴδη). Thomas Aquinas was perhaps the first in the Latin tradition to use *aphairesis* to mean the extraction of mathematical form from matter. For instance, in *Summa Theologiae* (Iª q. 1 a. 1 ad 2) Aquinas distinguishes two different ways of attaining the objects of knowledge. For instance, to prove that the earth is round the astronomer abstracts matter in thought, while the physicist proves the same proposition by considering matter in his speculation. While Aristotle's application of *aphairesis* always indicates that it is matter that is subtracted, Aquinas' expression 'per mathematicum a materia abstractum' instead suggests that it is the circular form that is abstracted from matter:

> Sciences are differentiated according to the various means through which knowledge is obtained. For the astronomer and the physicist both may prove the same conclusion: that the earth, for instance, is round: the astronomer by means of

[36] John Cleary, "On the Terminology of "Abstraction" in Aristotle," *Phronesis* 30 (1985): 26.

> mathematics (i.e. abstracting from matter), but the physicist by means of matter itself. (trans. by Fathers of the English Dominican Province).[37]
>
> Ad secundum dicendum quod diversa ratio cognoscibilis diversitatem scientiarum inducit. Eandem enim conclusionem demonstrat astrologus et naturalis, puta quod terra est rotunda, sed astrologus per medium mathematicum, idest a materia abstractum; naturalis autem per medium circa materiam consideratum.

Furthermore, Aquinas follows Boethius' invention of *aphairesis* as abstraction of a universal from particulars. Aquinas in his commentary on Aristotle's *De Anima* highlights two kinds of abstraction of form: "(1) abstracting a universal from particulars and (2) abstracting mathematicals from particulars."[38] I claim that such interpretation of the term in (1) and in (2) is inadequate for two reasons: first, nowhere does Aristotle use abstraction or *aphairesis* to show how a universal may be attained from particulars,[39] and second, the statement "abstrahimus mathematica a sensibilis" suggests the sense of 'extraction' which is absent in Aristotle. It is necessary to keep in mind that when Aristotle speaks about mathematics, geometry, or universals he never uses *aphairesis* together with the concepts of matter (ὕλη) or form (εἶδος) in any of his works, and, based on Aristotle's passages where the mode of being of mathematical objects is at stake, what is abstracted, or, subtracted is change, motion, affections, and passions, but not form. According to a proper Aristotelian *aphairesis,* when the mathematicals are in question, the term means a successive process by which the mind removes certain aspects of a physical body irrelevant for mathematical investigation, thereby leaving only the quantitative and continuous extension (points, lines, planes, solids, and

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[37] Thomas Aquinas. *The Summa Theologica*. Benziger Bros. edition. Translated by Fathers of the English Dominican Province, 1947. https://dhspriory.org/thomas/summa/FP/FP001.html#FPQ1OUTP1

[38] Christoph Helmig, *Forms and concepts: concept formation in the Platonic tradition*, Vol. 5. (Walter de Gruyter, 2012), 97. Helmig seems to have taken it from Aquinas' commentary on *De Anima* I. IV: "Habemus autem duplicem modum abstractionis per intellectum: unum qui est a particularibus as universalia; alium per uqem abstrahimus mathematica a sensibilis." In Thomas Aquinas, *Doctoris Angelici divi Thomae Aquinatis sacri ordinis FF Praedicatorum opera omnia: sive antehac excusa, sive etiam anecdota...* Vol. 33. (Apud Ludovicum Vivès, 1896), 15.

[39] It is highly likely that such interpretation was influenced by *APo* I, 18, but here Aristotle states that universals are gained by induction, and not by *aphairesis*. The only two ways of learning discussed in *APo* I, 18, and 19 are induction and demonstration.

units). Christoph Helmig points out the same difference which I highlighted above: "while the object of the former [proper Aristotelian *aphairesis*] (i.e., that which is taken away) is matter or certain properties[40] (e.g. 'to abstract the matter from a bronze sphere'), the object or result of abstraction common in modern and medieval philosophy is the form itself (e.g. 'to abstract a form of a triangle from matter')."[41] Helmig is right that the modern discussions on *aphairesis* are at times blurred because of disregarding of this very crucial difference between the proper Aristotelian (the successive removal of affections, passions, etc.) and Alexander's or Aquinas' (abstraction of form from matter) types of abstraction.

I think the problem here lies in overlooking this subtle difference between two distinct processes: the process of *extracting* from things and the process of *leaving out* or *disregarding* things. It is necessary to keep in mind that the result of *aphairesis* or subtraction in mathematics according to the properly Aristotelian use of the term is not what we *extract* from things (such as extracting a form from matter), but what is *left behind* (the quantitative and continuous magnitude) after we ignore all other things unnecessary for our investigation. On the other hand, when we extract or pull out something (universals, mathematicals) from sensibles this is where the problem arises. It becomes difficult to see how Aristotle's objects of mathematics are different from Plato's. Annas rightly points out, "if abstraction is thought of as abstracting from the matter of physical objects, then the properties studied are pure

[40] Helmig does not specify what he means by 'properties' here. It can mean either *physical* properties or *mathematical* properties. By properties he may mean the active and passive *physical* properties which are always found with some body and its matter (*de Anima*, 403b10), such as heat and cold, hardness and softness, heaviness and lightness (*Meta*. 1061a30), motion and change (*Physics,* 193b35). We can either separate these properties or separate matter in thought. By removing matter, all properties seem to be removed all at once. Helmig seems to suggest that both operations, the removal of matter or the removal of properties, are interchengeable. Another interpretation of 'properties' may mean the removal matter together with some *mathematical* properties we consider unnecessary. For instance to consider such mathematical property as 'the sum of internal angles are equal to two right angles' in this particular bronze equilateral triangle, we remove its matter, bronze, and all other properties of a triangle such as for instance we can remove the property 'all the sides of an equilateral triangle are of equal length.'

[41] Christoph Helmig, *Forms and concepts: concept formation in the Platonic tradition*, Vol. 5. (Walter de Gruyter, 2012), 97.

properties of forms, and this comes dangerously close to Plato."[42] This is what happens when we interpret abstraction in Aristotle as extraction of form from physical matter. It is, however, important to note that we should not deny that Aristotle does have some kind of a theory of the soul receiving the forms without matter (*De Anima II*, 12 424a17-19). What I do deny, and in this I support Cleary and Helmig, is that nowhere in the corpus does Aristotle use *aphairesis* in the sense of abstracting particular mathematical forms (e.g. *this* circle) from particular matter, or abstracting mathematical universals (e.g. circle) from particulars, or collecting a universal (e.g. man, horse) from particulars. He rather uses the term to unfold the spatial location of the objects of mathematics within the physical body but not to explain how the intellect receives mathematical forms.

In terms of the modern accounts of Aristotle's abstraction, my particular interest will be directed to the accounts of Allan Bäck, Charles De Koninck, Auguste Mansion, and Julia Annas, whose treatment of *aphairesis* can be traced back to ancient and medieval kinds of abstractionism. Taking into consideration the thesis of this work I will thus be distinguishing two kinds of *aphairesis* which are often confused with one another: Aristotle's original sense of successive *subtraction*, and the post-Aristotelian sense of abstraction as *extraction*.

2.2 The Standard Phrase τὰ ἐξ ἀφαιρέσεως or 'Abstract Objects'

Some modern scholars such as Mure,[43] Mueller,[44] Bäck[45] and others associate Aristotle's expression 'τὰ ἐξ ἀφαιρέσεως' with the standard phrase 'abstract objects' which

[42] Julia Annas, and Aristotle. *Aristotle's Metaphysics: Books M and N*. Clarendon Aristotle Series. (Oxford [Eng.]: Clarendon Press, 1976), 33.
[43] *Posterior Analytics* 1. 18, 81a38-b9: translation by Mure in Richard McKeon, *The Basic Works of Aristotle.* Modern Library Classics. (New York: Modern Library, 2001), 136.
[44] *Posterior Analytics* 1. 18, 81a38-b9: Ian Mueller, "Aristotle on Geometrical Objects," *Archiv für Geschichte der Philosophie* 52, no. 2 (1970): 160.

they claim referrs to the objects of mathematics only and should be translated as "mathematical abstractions." What should be kept in mind is that whenever Aristotle uses the phrase 'τὰ ἐξ ἀφαιρέσεως' he always refers to the objects of geometry (with two exceptions: *Posterior Analytics* I. 18 and *Metaphysics* XIII. 2), and never to the objects of arithmetic, such as numbers and units. In addition, even though there are passages such as *Posterior Analytics* I. 18 and *Metaphysics* XIII. 2 where τὰ ἐξ ἀφαιρέσεως appears without any reference to either the objects of geometry or arithmetic, some translations nonetheless include a reference to the objects of mathematics in them. This move seems to be formed by the prejudice that τὰ ἐξ ἀφαιρέσεως in Aristotle is nothing other than the 'results of mathematical abstraction.' The other side of the scholarship on this issue, represented by Tredennick,[46] Cleary,[47] and Barnes,[48] claims that the passages where τὰ ἐξ ἀφαιρέσεως appears without any reference to mathematics have a broader reference beyond the strictly mathematical and should be translated in all these cases as "things said as a result of subtraction." In order to distinguish the two appearances of the term, whenever the 'τὰ ἐξ ἀφαιρέσεως' expression appears without any reference to mathematics, I will be calling it *non-mathematical*. Where the expression makes a clear reference either to a mathematician or to the objects of mathematics (specifically geometry), I will be referring to those instances as *mathematical*. Both expressions, as the section below shows, appear in Aristotle's corpus in nine different variations. The list which follows is full and complete.

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[45] *Posterior Analytics* 1. 18, 81a38-b9: Allan Bäck, "The Concept of Abstraction." *The Society for Ancient Greek Philosophy Newsletter.* (2006): 6. https://orb.binghamton.edu/sagp/376

[46] *Posterior Analytics* 1. 18, 81a38-b9: translation by Tredennick in Forster, E. S., and Hugh Tredennick, *Posterior Analytics Topica*. Loeb Classical Library; 391. (Cambridge, MA: Harvard University Press, 2014), 107.

[47] *Metaphysics* XIII. 2, 1077b 5-10: John Cleary, "On the Terminology of "Abstraction" in Aristotle," *Phronesis* 30 (1985): 28.

[48] *Posterior Analytics* 1. 18, 81a38-b9: translation by Jonathan Barnes in *Aristotle's Posterior Analytics.* Clarendon Aristotle Series. (Oxford: Clarendon Press, 1975), 161.

(i) τὰ ἐξ ἀφαιρέσεως λεγόμενα in the *Posterior Analytics* I. 18: no reference is made either to the mathematician or to the objects of mathematics. The context seems to indicate that the results of subtraction are particulars, by which their corresponding universal is made familiar through induction. I claim this case to be non-mathematical;

(ii) τὸ ἐξ ἀφαιρέσεως in the *Metaphysics* XIII. 2:[49] the result of subtraction is 'pale' (τὸν λευκόν) which cannot be prior in substantiality, but only in definition, to 'pale man.' Consequently, I claim this instance of *aphairesis* to be non-mathematical;

(iii) τὰ ἐξ ἀφαιρέσεως λέγεσθαι in the *De Caelo* III. 1: the reference is made to mathematics. The results of subtraction are the objects of mathematics (τὰ μαθηματικά);

(iv) ἐξ ἀφαιρέσεως ὁ μαθηματικός in the *De Anima* I. 1: the results of subtraction are attributes studied by a mathematician;

(v) τῶν ἐν ἀφαιρέσει ὄντων in the *De Anima* III. 4: the result of subtraction is the straight (τὸ εὐθὺ);

(vi) τὰ ἐν ἀφαιρέσει λεγόμενα in the *De Anima* III. 7: the result of subtraction is the hollow (κοῖλον). This excerpt where this expression appears is extremely corrupted and fragmented; possibly copied from *De Anima* III. 4 with slight changes;

(vii) τά ἐν ἀφαιρέσει λεγόμενα in the *De Anima* III. 8: the results of subtraction are the objects of thought (τὰ νοητά) of sensible spatial magnitudes;

(viii) τὰ ἐξ ἀφαιρέσεως in the *Metaphysics* XI. 3: the reference is made to a mathematician. The result of subtraction is the quantitative (τὸ ποσὸν) and continuous (συνεχές);

(ix) τὰ δι᾽ ἀφαιρέσεώς ἐστιν in the *Nicomachean Ethics* VI. 8: the things the mathematician studies exist through subtraction.

As for Plato, the modern philosophy of mathematics also tends to call his intermediates and Form numbers[50] 'abstract objects.' Balagues for instance claims "whereas

[49] The 'τὸ ἐξ ἀφαιρέσεως' expression in (ii) is the only appearance of 'the results from subtraction' terminology in the highly mathematical books XIII and XIV of the *Metaphysics*, and even when it appears in *Metaphysics* XIII. 2, the expression has no relation to the objects of mathematics. Since the main discussion in these two books is built on putting Plato's understanding of the objects of mathematics against Aristotle's and their mode of being, one would expect this expression in XIII. 2 to refer to the objects of mathematics, and, to appear at least more than once, but this is not the case. Furthermore, in chapter 3 of this book Aristotle proposes his own positive solution to the mode of being of mathematical objects; here as well as in all subsequent chapters of books XIII and XIV Aristotle makes not a single mention of τὰ ἐξ ἀφαιρέσεως or *aphairesis* in general. Due to this reason most scholars express their dissatisfaction in Aristotle's 'theory of abstraction.'

[50] There is no explicit discussion of intermediates or Form numbers (Ideas) in Plato's dialogues. The unwritten doctrines is the only testimony which throws some light on their nature of being. Aristotle in *Metaphysics* I.6 states that intermediates (τὰ μεταξύ) are the objects of mathematics existing between the sensibles and the

Mars is a physical object, the number 3 is (according to Platonism) an *abstract* object. And abstract objects, Platonists tell us, are wholly nonphysical, nonmental, nonspatial, nontemporal, and noncausal."[51] We see that Plato's objects of mathematics too did not escape being called the 'abstract objects.' Even though there is not a single use of *aphairesis* in reference to intermediates or Form numbers in Plato's works, I think that Plato's objects of mathematics are more deserving of being called 'abstract objects.' Interestingly, James Franklin, in his book *An Aristotelian Realist Philosophy of Mathematics,* always calls Plato's mathematicals 'abstract' rather than Aristotle's,[52] though he does not examine how the term appears both in Plato and Aristotle and does not explain the meaning of the term. I think that Plato's mathematicals are more justly called abstract simply because they are non-physical, non-spatial, non-temporal, and non-causal. On the contrary, Aristotle's objects of mathematics deserve far less to be called abstract since his objects of mathematics are tied up with physical, spatial, causal, and temporal quantitative and continuous matter.

Before I proceed to examine Aristotle's use of *aphairesis* I think it is necessary to first find out how the terminology appears in Plato's dialogues. I find this step crucial to my investigation because Plato is the main figure against whom Aristotle raises objections in *Metaphysics* XIII and XIV (and elsewhere in the corpus) concerning *ta mathematica* and their

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Forms: "besides sensible things and Forms he says there are the objects of mathematics, which occupy an intermediate position, differing from sensible things in being eternal and unchangeable, from Forms in that there are many alike, while the Form itself is in each case unique" (987b15). Julia Annas in her article "On the 'Intermediates'" in *Archiv fur Geschichte der Philosophie* 57, 1975. pp. 146-65 argues that Plato posited intermediates in order to solve the 'uniqueness problem': when we calculate '2+2 = 4' we cannot add the Form number Two to the same Form number Two because it is unique, but instead we add intermediate twos which are many alike. Also, why do we add intermediate twos but not sensible twos? Because sensible objects fall short of their corresponding mathematical objects (there are no perfectly straight lines and perfectly circular spheres). The same holds for geometry. Whenever a Platonist postulates multiple circles, he refers neither to the Form of Circularity nor to perceptible circles, but considers the perfect circles existing in the realm of intermediates. For an account of Plato's intermediates and Ideas please consider Findlay, John Niemeyer. *Plato: The Written and Unwritten Doctrines*. Routledge, 2012.

[51] Mark Balaguer, "Fictionalism in the Philosophy of Mathematics," *The Stanford Encyclopedia of Philosophy* (Fall 2018 Edition), Edward N. Zalta (ed.). https://plato.stanford.edu/archives/fall2018/entries/fictionalism-mathematics.

[52] James Franklin, *An Aristotelian realist philosophy of mathematics: Mathematics as the science of quantity and structure*. (Springer, 2014), 14, 15, 26, 27, 104, 105.

mode of being. Therefore, it is necessary to examine how both philosophers use *aphairesis* and whether there are any differences in how the term is applied.

CHAPTER 3 GENERAL APPLICATION OF ABSTRACTION

Introduction

Since Aristotle extensively argues against Plato's objects of mathematics, I consider it necessary to find out how Plato uses the term and whether it plays any role for him in reaching the objects of mathematics or Forms. This chapter will show that Plato uses *aphairesis* in three ways: as a simple arithmetical subtraction of things and concepts in thought, as a deprivation of physical things such as wealth, slaves etc., and as an intellectual activity of abstracting the Form of love and the Form of the good (*Rep*. 534b-c and *Symp.* 205b4) from physical appearances of which there are two instances in the dialogues. In addition, my analysis will show that Plato's *aphairesis* presents itself primarily as a simple successive removal of things from an object which suggests that it does not bear an extractionist sense (though the two mentioned passages could perhaps suggest such a reading). Contrary to Aristotle's *aphairesis*, Plato's Forms and objects of mathematics are reached by reduction (ἀνάγειν), analysis (ἀνάλυσις), difference (κατὰ διαφορὰν), opposition (κατ' ἐναντίωτιν), relation (πρός τι), by turning one's soul towards (πρὸς ἑαυτὰς ἐπιστρέφουσαι) Ideas, and by hypotheses (ὑπόθεσις). All of this is done through the four levels of the divided line: imagining (εἰκασία), belief (πίστις), thought (διάνοια), and finally understanding (νόησις).

3.1 The Instances of *Aphairein* in Plato's Dialogues

There are several passages in the Platonic corpus where *aphairesis* is used in the form of a noun (ἀφαίρεσις), verb (ἀφαιρέω), adjective (ἀφαιρετός), participle (ἀφαιρουμένων), and in other related forms. In these dialogues where the terminology is present it does not have any technical connotations: whenever it appears it has the sense of 'subtraction', and, depending on the context, it can be translated in any of its variables, such as 'subtraction', 'removal', 'deprivation', 'withdrawal', 'confiscation', 'robbery', etc. The direct objects of *aphairesis*, as seen in the corpus, include a wide variety of things: quantity (*Parmenides* 158c), number (*Cratylus* 432b), letter (*Cratylus* 393d), wickedness of the soul (*Sophist* 227d), skin from bodies (*Statesman* 288e), excess and indefiniteness (*Philebus* 26a), parts of the mixture of the Same, the Different, and of Being (*Timaeus* 35a ff.), parts of fire (*Timaeus* 63c), motions (*Timaeus* 34a), justice (*Gorgias* 519d) or injustice (*Gorgias* 520d), property (*Gorgias* 466c), passages from Homer and other poets (*Republic* III, 387b), wealth (*Republic* VIII, 565b), slaves (*Republic* VIII, 567e), satisfactions (*Republic* XI, 574a), liberty (*Laws* 697d), empire (*Laws* 695d), words (*Euthydemus* 296b), expertise at love (*Phaedrus* 257b), the art of love (*Phaedrus* 257a), the Form of Love (*Symposium* 205b), among many others.

There are other places in the dialogues where Plato uses ἀφαίρεσις together with πρόθεσις to indicate a simple arithmetical process by means of which any determinants are either removed or added. Plato provides an example of addition and subtraction in the *Theaetetus*: "Secondly, we should say that a thing to which nothing is added [ᾧ μήτε προστιθοῖτο] and from which nothing is taken away [ἀφαιροῖτο] neither increases nor diminishes but remains equal" (155a).[53] Another example is present in the *Parmenides* dialogue at 131 e: "Well, suppose one of us is going to have a part of the small. The small will be larger than that part of it, since the part is a part of it: so the small itself will be larger! And that to which the part subtracted [ἀφαιρεθέν] is added [προστεθῇ] will be smaller, not larger, than it was before." In the same dialogue at 158c Parmenides points out that the subtracted is unlimited in multitude, if it partakes of no unity: "Now, if we should be willing to subtract

[53] Also at 173a, 151c, 155c.

[ἀφελεῖν] in thought, the very least we can from these multitudes, must not that which is subtracted [τὸ ἀφαιρεθὲν], too, be a multitude and not one, if in fact it doesn't partake of the one?— Necessarily." Socrates in *Cratylus* expresses the idea that the addition of 'e', 't', and 'a' to the word 'beta' does not do any harm to the nature of the letter 'b' or any element someone wishes to name: "But it doesn't matter whether the same thing is signified by the same syllables or by different ones. And if a letter is added [πρόσκεταί] or subtracted [ἀφῄρηται], that doesn't matter either, so long as the being or essence of the thing is in control and is expressed in its name" (393d).[54] The process of subtraction is also present in the *Republic* II where the discussion of the origins of justice takes place. At 360e -361d Socrates tells us that in order to judge who is happier, the just or the unjust person, the first step in this inquiry is to "subtract [ἀφαιρῶμεν] nothing from the injustice of an unjust person and nothing from the justice of a just one" and to "take each to be complete in his own way of life." Thus, to make a judgement, both the just and the unjust should stay in their most possible complete extremes of justice and injustice with nothing being removed in the first case.[55] A simple arithmetical use of addition and subtraction of numbers is present in the *Cratylus* dialogue at 432b: "What you say may well be true of numbers, which have to be a certain number or not be at all. For example, if you add [προσθῇς] anything to the number ten or subtract [ἀφέλῃς] anything from it, it immediately becomes a different number, and the same is true of any other number you choose." The concern of what this 'different number' is is expressed by Socrates in *Phaedo* at 97b: "That I am far, by Zeus, from believing that I know the cause of any of those things. I will not even allow myself to say that where one is added [προσθῇ] to one either the one to which it is added [τὸ προσετέθη] or the one that is added [τὸ προστεθέν] becomes two, or that the one added [τὸ προστεθὲν] and the one to which it is added

[54] Also, in *Cratylus* 394b, 407b, 414c, 414e, 418b, 432a, 434d

[55] A similar expression of removing [ἀφαιρεῖ] the injustice is found in *Gorgias* dialogue at 520d 4.

[προσετέθη] become two because of the addition [τὴν πρόσθεσιν] of the one to the other."[56] Another related passage pertaining to numbers, geometry and volumes in which *aphairesis* is expressed is present in the *Timaeus* dialogue at 35b where Timaeus describes how god started to create the soul from the parts of the mixture of the Same, the Different, and Being:

> This is how he began the division: first he took one portion away [ἀφεῖλεν] from the whole, and then he took [ἀφῄρει] another, twice as large, followed by a third, one and a half times as large as the second and three times as large as the first. The fourth portion he took was twice as large as the second, the fifth three times as large as the third, the sixth eight times that of the first, and the seventh twenty-seven times that of the first. (Zeyl, *Tim.* 35b).

All these and other appearances of *aphairesis*[57] in Plato's dialogues do not carry any special or technical usage, but only a simple logical or mathematical one. Cleary suggests that this simple logical use was a standard tool in the matters of dialectic at the Academy.[58] He argues[59] that the only passage in Plato's works where *aphairesis* seems to be different from all the above, is in the *Symposium* dialogue found at 205b4: "…we divide out [ἀφελόντες] a special kind [τι εἶδος] of love, and we refer to it by the word that means the whole – 'love'; and for the other kinds of love we use other words." Cleary, however, points out that even though ἀφελόντες in this passage represents the intellectual activity of separating a form, this use is still a non-technical one,[60] namely that it does not have any technical connotations as the so-called Aristotle's theory of abstraction of form from matter.[61] At *Republic* VII, 534b-c there is another example where the separation of a form is used together with the word ἀφελὼν: "Then the same applies to the good. Unless someone can indicate in an account the form of the good from everything else [taken away], can survive all refutation, as if in a

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[56] Also, in *Phaedo* 95e.
[57] A *Word Index to Plato* by Leonard Brandwood indicates that there are 71 uses of the word πρόθεσις and 113 uses of ἀφαίρεσις in Plato's works.
[58] John Cleary, "On the Terminology of "Abstraction" in Aristotle," *Phronesis* 30 (1985): 14.
[59] Ibid., 18.
[60] Ibid.
[61] Dr. Eli Diamond pointed out to me that "this is not at all about arriving at a form – it is about how a certain linguistic convention came to be – why all the other forms of love are not called as such"

battle, striving to judge things not in accordance with opinion but in accordance with being" (trans. Grube and Reeve). Grube and Reeve, however, omit the terminology of *aphairesis* in their translation.[62] Paul Shorey, in contrast, translates ἀφελὼν as 'abstraction'[63] which supports the understanding *aphairesis* as abstraction of form from matter. When interpreting Plato's works, I therefore propose to translate the term as abstraction only when it stands together with ἰδέα, namely when it has the idea of abstracting or drawing a Form from sensible instances.[64] Thus, in Plato's dialogues, *aphairesis* is used in two senses explicitly which can be supported by a textual evidence of the term: as a simple arithmetical subtraction and, as abstraction of Form from things of which there are two technical uses, though the latter is questionable. Otherwise, there are no uses of *aphairesis* in the dialogues as an explanation of how the mind grasps the Forms. They are already present in the mind, and the sensible experience of individual sensible things serve as mere reminders for the soul. What the student of Plato needs to do is to purify his/her conception of the Forms. In the dialogues Plato also says is that the soul must be 'led' (ἀγωγῶν) and 'turned around' (μεταστρεπτικῶν) towards that which is (*Republic* VII, 525a-c). At 525b he speaks of 'rising up' (ἐξαναδύντι) out of becoming and grasping being, or 'reaching' (ἀφίκωνται) the study of the natures of the numbers themselves. In the *Republic* at 508d Plato speaks of the soul 'focusing' (ἀπερείσηται) on something illuminated by truth. Contrary to Martin's account, who sees *aphairesis* as a method with which we come to know the *One*, I may suggest that besides taking ἀνάλυσις into account, we should also look for the clues in the *Republic* VI at 510b. In

[62] "οὐκοῦν καὶ περὶ τοῦ ἀγαθοῦ ὡσαύτως: ὃς ἂν μὴ ἔχῃ διορίσασθαι τῷ λόγῳ ἀπὸ τῶν ἄλλων πάντων ἀφελὼν τὴν τοῦ ἀγαθοῦ ἰδέαν, καὶ ὥσπερ ἐν μάχῃ διὰ πάντων ἐλέγχων διεξιών, μὴ κατὰ δόξαν ἀλλὰ κατ᾽ οὐσίαν προθυμούμενος ἐλέγχειν…"

[63] "And is not this true of the good likewise – that the man who is unable to define in his discourse and distinguish and abstract from all other things the aspect or idea of the good, and who cannot as it were in battle, running the gauntlet in all tests, and striving to examine everything by essential reality and not by opinion." In Paul Shorey, *The Republic*. *Plato.* Plato in Twelve Volumes; 6-10. Cambridge, Mass.: (Harvard University Press, 1935), 207.

[64] There is no obvious and clear concept of matter in Plato's dialogues or unwritten doctrines.

this excerpt Plato elucidates the steps which allow the mind to unite with the true reality – it is the successive ascent through the four subdivisions of the levels of reality, known as the 'divided line.' The four steps include imagining (εἰκασία), belief (πίστις), thought (διάνοια), and finally understanding (νόησις). What allows the intellect to come to know the *One* (or the Good) through the last two levels of the divided line is the method of *hypotheses* (ὑπόθεσις):

> In one subsection, the soul, using as images the things that were imitated before, is forced to investigate from hypotheses, proceeding not to a first principle but to a conclusion. In the other subsection, however, it makes its way to a first principle that is not a hypothesis, proceeding from a hypothesis but without the images used in the previous subsection, using forms themselves and making its investigation through them. (Reeve, *Rep.* 510b-c).

The passage above makes it clear that the only method to raise up to the *One* is by the method of *hypothesis*, and not by *aphairesis*, though I shall not deny that there is some application of subtraction present when the mind hypothesises.

Jugrin[65] and Martin[66] see *aphairesis* in Plato (and Pythagoras) used in yet another sense, as a method by which we subtract all lower genera, differentiae, solids, planes, lines, points, and numbers upon which we then reach the *Dyad* with its *Great* and *Small* as Principles, and then, arrive at the *One* or the *Monad*, the highest principle of all things. Martin, for instance, claims that such Pythagorean and Platonic abstraction found its expression in Plotinus' and Proclus' understanding of the term:

> Abstraction is the epistemic converse of the process of physical composition…the mental process of reversion to the One. Ontologically, the Chain of Being proceeds downwards through the process of causation, but the Understanding remounts backwards from the bottom to the top. The process of remotion is called abstraction.[67]

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[65] Daniel Jugrin, "The way of ἀνάλυσις: Clement of Alexandria and the Platonic tradition," *Studia Philosophiae Christianae*52, no. 2 (2016): 77-78.
[66] John N. Martin, *Themes in Neoplatonic and Aristotelian Logic: Order, Negotiation, and Abstraction*, Ashgate. (2004): 163.
[67] Ibid.

Bäck notes rightly that Martin provides no textual support for this interpretation. The reason for this is because there actually is no textual evidence of the word *aphairesis* used in this context in Plato's dialogues. Perhaps Martin, as Bäck rightly suggests, confuses ἀφαίρεσις with ἀνάλυσις, the term which in Pythagorean, Platonic, and Aristotelian traditions generally means to reduce the compound to its principles. This is how Alexander of Aphrodisias explains it in his commentary on Aristotle's *Prior Analytics*: "the reduction of any compound to the things from which it is compounded is called *analysis*. Analysing is the converse of compounding; for compounding is a route from the principles to what depends on them, whereas analysing is a return route from the end up to the principles."[68]

In terms of the unwritten doctrines,[69] these testimonies tell us that we can come to know the Principles not by means of *aphairesis,* as Martin suggested, but by means of reduction, deduction, analysis, difference, opposition, relation, and by turning one's soul towards Ideas. Alexander of Aphrodisias, for instance, states that Plato tried 'to reduce' (ἀνάγειν) everything to Equal and Unequal.[70] Pseudo-Alexander tells us that Line is 'deduced' (συλλογίζεσθαι) from the Dyad.[71] Sextus Empiricus in the *Against the Mathematicians* points out that Pythagoreans investigate Nature from the 'analysis' (ἀνάλυσις) of the things of which Nature is a whole.[72] Later he adds that Pythagoreans 'conceive' (νοεῖται) the Principles of things by three methods: by ways of 'Difference' (κατὰ διαφορὰν) such as for instance Man, Horse, Plant, by way of 'Opposition' (κατ' ἐναντίωτιν)

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[68] Jonathan Barnes, and Aristotle. *On Aristotle's Prior Analytics 1.1-7*. Ancient Commentators on Aristotle. (Ithaca, N.Y.: Cornell University Press, 1991), 49.
[69] The sources of the unwritten doctrines include Aristotle's *Physics* and *Metaphysics*, the report of Aristoxenus, both criticisms of writing in Plato's *Phaedrus* and the *Seventh Letter*, the report of Alexander on Aristotle's *Metaphysics*, the reports of Simplicius on Aristotle's *Physics*, testimonies of Pseudo-Alexander, Sextus Empiricus, Asclepius, Themistius, and Siryanus. These unwritten doctrines constitute Plato's theories of Form numbers, the One (or the Good), and the doctrine of the Indefinite Dyad.
[70] Findlay, J. N. *Plato: The Written and Unwritten Doctrines*. International Library of Philosophy and Scientific Method. London: New York: Routledge & K. Paul; (Humanities Press, 1974), 416. The term 'reduction' also appears on pp.431, 432, and 437.
[71] Ibid., 422.
[72] Ibid., 424.

such as Good and Bad, and by 'Relation' (πρός τι).[73] Syrianus writes that the Ideas are reached by making the things to 'turn towards' (πρὸς ἑαυτὰς ἐπιστρέφουσαι) the Ideas, such as when the idea of Man perfects man with wisdom and virtue.[74]

To conclude this survey of Plato, since the terminology of *aphairesis* together with the terminology of form (ἡ ἰδέα) appears twice in Plato's works (*Rep*. and *Symp*.), is it then fair to understand Plato's *aphairesis* as an anticipation of a Thomistic use of abstraction such as abstraction of form from matter? Perhaps yes. Even if this is the case, what is abstracted is only an imperfect instantiation, a reflection of an ideal Form, therefore the soul must still turn away from physical instantiations. In terms of the objects of mathematics, it is not correct to speak of Plato's *aphairesis* as a special tool in abstracting a mathematical form from sensible instances, since there are no instances in Plato's works where *aphairesis* is used together with the forms of mathematical objects. If the opposite were the case, what would be abstracted is, again, an imperfect mathematical form. The soul rather turns away from physical mathematical objects which are always in the state of becoming and directs itself to those unchanging mathematical realities in the realm of intermediates and Form numbers. Comparing the two instances of abstraction and form in Plato, I find it problematic to speak of Aristotelian abstraction as 'abstraction of form' since there is not even a single use of both terms standing together in Aristotelian corpus. Furthermore, my analysis has shown that Plato's *aphairesis* presents itself primarily as a simple successive removal of things from an object, and that it does not bear an extractionist sense (however much *Republic* 534b-c could perhaps suggest such a reading).

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[73] Ibid., 427-428.
[74] Ibid., 452.

3.2 The Use of *Aphairein* in Aristotle's *Topics* II, III, V, VI, VIII, and *Metaphysics* VII

In this section I will investigate how Aristotle uses the terminology of *aphairesis* in the passages where it has a non-technical application, i.e. where it has no relation to the objects of mathematics but is used in various contexts according to a general application. My general thesis in this section is that ἀφαιρεῖν is not restricted to mathematical thought only but can be applied to any subject. I will show that the term does not have the sense of extraction.

Just as Plato, Aristotle uses *aphairesis* and *prosthesis* in a different variety of word forms. In terms of direct objects of *aphairesis,* there are various ways in which Aristotle uses the term in a non-technical way: to subtract irrelevant attributes in a definition and to identify the proper correlative of a relative term (*Categories* ch.7, 7a31-7b2), to remove the equivocal meaning upon which an objection relies (*Topics*, VIII.2, 158a5-10), to subtract everything superfluously added to a definition (*Topics*, VI.3, 140a20), to subtract anything added in refutations to see if absurdity follows (*Sophistical Refutations*, ch.29, 181b15), to subtract flesh from water (*Physics,* I.5, 187b27), to subtract Hermes from the stone (*Physics*, I.7, 190b7), to subtract parts of time (*Physics*, VI.7, 238a25-30), to take away that which imparts motion but is unmoved (*Physics* VIII.5, 258a27), to take away happiness (*Nicomachean Ethics* I.11, 1101b3), and others. All these uses are non-technical, i.e. are not applied to the question of the mode of being of mathematical objects.

The first use of *aphairesis* in Aristotle's *Organon* is found in *Categories* Ch.7 at 7a31-7b2. This passage discusses the error in stating a relative which is *not* a proper relative of a correlative. For instance, to claim that someone is 'a parent' in so far as s/he has 'a daughter' or 'a son' is a false relative-correlative statement. Someone is 'a parent' only in so far as s/he has ‘a child’ – this will be a proper relation. In the *Categories* Ch.7 Aristotle states that to claim that ‘master, who is biped and receptive of knowledge, is relative to slave’ will be a false relative-correlative statement. Here we need to apply the method of *aphairesis* which serves as a necessary tool in finding the proper relative of a correlative ‘slave’: we must strip off all incidental (συμβεβηκότα) attributes of a ‘master’ such as ‘biped’, ‘receptive of knowledge’, and ‘human’; only then ‘master’ will show itself to be the proper relative of ‘slave.’ Even if all incidental attributes except ‘master’ are removed, the relation will still be present because someone is a ‘slave’ only in so far as he has a ‘master’ and not because he has a ‘biped master receptive of knowledge.’ Likewise, someone is a ‘master’ only in so far as he has a ‘slave,’ but not because he has a ‘biped slave receptive of knowledge,’ or in other words, it is only *qua* ‘slave’ that someone is ‘a master.’ Therefore, one must find the subject to which the relation belongs universally or essentially.

> Further, if one thing is said to be correlative with another, and the terminology used is correct, then, though all irrelevant attributes should be removed [περιαιρουμένων], and only that one attribute left [καταλειπομένου] in virtue of which it was correctly stated to be correlative with that other, the stated correlation will still exist. If the correlative of ‘the slave’ is said to be ‘the master’, then, though all irrelevant attributes of the said ‘master’, such as ‘biped’, ‘receptive of knowledge’, ‘human’, should be removed [περιαιρουμένων], and the attribute ‘master’ alone left [καταλειπομένου], the stated correlation existing between him and the slave will remain the same, for it is of a master that a slave is said to be the slave. On the other hand, if, of two correlatives, one is not correctly termed, then, when all other attributes are removed [περιαιρουμένων] and that alone is left [καταλειπομένου] in virtue of which it was stated to be correlative, the stated correlation will be found to have disappeared. (Edghill, *Cat*. 7a31-b2).
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> ἔτι ἐὰν μὲν οἰκείως ἀποδεδομένον ᾖ πρὸς ὃ λέγεται, πάντων περιαιρουμένων τῶν ἄλλων ὅσα συμβεβηκότα ἐστίν, καταλειπομένου δὲ τούτου μόνου πρὸς ὃ ἀπεδόθη οἰκείως, ἀεὶ πρὸς αὐτὸ ῥηθήσεται· οἷον εἰ ὁ δοῦλος πρὸς δεσπότην λέγεται, περιαιρουμένων ἁπάντων ὅσα συμβεβηκότα ἐστὶ τῷ δεσπότῃ, οἷον τὸ δίποδι εἶναι, τὸ ἐπιστήμης δεκτικῷ, τὸ ἀνθρώπῳ, καταλειπομένου δὲ μόνου τοῦ δεσπότην εἶναι, ἀεὶ ὁ δοῦλος πρὸς αὐτὸ ῥηθήσεται· ὁ γὰρ δοῦλος δεσπότου δοῦλος λέγεται. ἐὰν δέ γε μὴ οἰκείως ἀποδοθῇ πρὸς ὅ ποτε λέγεται, περιαιρουμένων μὲν τῶν ἄλλων καταλειπομένου δὲ μόνου τοῦ πρὸς ὃ ἀπεδόθη, οὐ ῥηθήσεται πρὸς αὐτό·

The terminology of subtraction in this passage does not suggest a sense of extraction for two reasons: first, because we remove many – 'biped', 'receptive of knowledge', 'human' and leave only one, such as 'master' alone, and second, there is no sense of extraction because of the presence of an obvious mathematical language of successive removal (περιαιρουμένων) and remainders (καταλειπομένου).

The dialectical practice of subtracting and adding is present throughout the *Topics*. The first passage from the *Topics* VIII I have in mind is at 151b3-13, where Aristotle explains when in dialectical disputations an interrogation must and must not be made. Specifically, Aristotle explains that if someone raises an objection not in the same genus, but in an equivocal one, we must make an interrogation so that the erroneous proposition would not escape our notice. If the objector makes an interrogation in the same genus, his objection should be taken into consideration and that in which his objection consists should be removed (ἀφαιροῦντα).

> People sometimes object to a universal proposition, and bring their objection not in regard to the thing itself, but in regard to some homonym of it: thus they argue that a man can very well have a colour or a foot or a hand other than his own, for a painter may have a colour that is not his own, and a cook may have a foot that is not his own. To meet them, therefore, you should draw the distinction before putting your question in such cases: for so long as the ambiguity remains undetected, so long will the objection to the proposition be deemed valid. If, however, he checks the series of questions by an objection in regard not to some homonym, but to the actual thing asserted, the questioner should withdraw [ἀφαιροῦντα] the point objected to, and form the remainder into a universal proposition, until he secures what he requires; e.g. in the case of forgetfulness and having forgotten: for people refuse to admit that the man who has lost his knowledge of a thing has forgotten it, because if the thing alters, he has lost knowledge of it, but he has not forgotten it. Accordingly, the thing to do is to withdraw [ἀφελόντα] the part objected to, and assert the remainder [τὸ λοιπόν], e.g. that if a person has lost knowledge of a thing while it still remains, he then has forgotten it. (Pickard-Cambridge, *Top.* 151b3-13).
>
> Πρὸς δὲ τοὺς ἐνισταμένους τῷ καθόλου, μὴ ἐν αὐτῷ δὲ τὴν ἔνστασιν φέροντας ἀλλ΄ ἐν τῷ ὁμωνύμῳ, οἷον ὅτι ἔχοι ἄν τις τὸ μὴ αὐτοῦ χρῶμα ἢ πόδα ἢ χεῖρα (ἔχοι γὰρ ἂν ὁ ζωγράφος χρῶμα καὶ ὁ μάγειρος πόδα τὸν μὴ αὑτοῦ)—διελόμενον οὖν ἐπὶ τῶν τοιούτων ἐρωτητέον· λανθανούσης γὰρ τῆς ὁμωνυμίας εὖ δόξει ἐνστῆναι τῇ προτάσει. Ἐὰν δὲ μὴ ἐν τῷ ὁμωνύμῳ ἀλλ΄ ἐν αὐτῷ ἐνιστάμενος κωλύῃ τὴν

> ἐρώτησιν, ἀφαιροῦντα δεῖ ἐν ᾧ ἡ ἔνστασις προτείνειν τὸ λοιπὸν καθόλου ποιοῦντα, ἕως ἂν λάβῃ τὸ χρήσιμον. Οἷον ἐπὶ τῆς λήθης καὶ τοῦ ἐπιλελῆσθαι· οὐ γὰρ συγχωροῦσι τὸν ἀποβεβληκότα ἐπιστήμην ἐπιλελῆσθαι, διότι μεταπεσόντος τοῦ πράγματος ἀποβέβληκε μὲν τὴν ἐπιστήμην, ἐπιλέλησται δ΄ οὔ. Ῥητέον οὖν, ἀφελόντα ἐν ᾧ ἡ ἔνστασις, τὸ λοιπόν, οἷον εἰ διαμένοντος τοῦ πράγματος ἀποβέβληκε τὴν ἐπιστήμην, ὅτι ἐπιλέλησται.

Just as in *Categories* Ch.7, this passage in *Topics* VIII.I does not presuppose any sense of extracting one out of many things. The mathematical language of removal (ἀφελόντα) and leaving the remainder (τὸ λοιπόν) supports this idea.

The term *aphairesis* very often appears together with the term πρόσθεσις or addition. For example, in books II and III of his *Topics* Aristotle uses both terms *aphairesis* and *prosthesis* to find out which object is more worthy of choice by comparing two of them with the same thing and finding out which one constitutes the greater good or makes it a whole.[75] *Topics* II.11, III.3, and III.5 are the most prominent in using both *aphairesis* and *prosthesis*. All three passages have the same topic of discussion. Consider, for instance, *Topics* III.5 where the standard terminology of abstraction (ἐκ τῆς ἀφαιρέσεως) is present:

> Moreover, if in any character one thing exceeds and another falls short of the same standard; also, if the one exceeds something which exceeds a given standard, while the other does not reach that standard, then clearly the first-named thing exhibits that character in a greater degree. Moreover, you should judge by means of addition [ἐκ τῆς προσθέσεως], and see if A when added [προστιθέμενον] to the same thing as B imparts to the whole such and such a character in a more marked degree than B, or if, when added [προστιθέμενον] to a thing which exhibits that character in a less degree, it imparts that character to the whole in a greater degree. Likewise, also, you may judge by means of subtraction [ἐκ τῆς ἀφαιρέσεως]: for a thing upon whose subtraction [ἀφαιρεθέντος] the remainder [τὸ λειπόμενον] exhibits such and such a character in a less degree, itself exhibits that character in a greater degree. Also, things exhibit such and such a character in a greater degree if more free from admixture with their contraries; e.g. that is whiter which is more free from admixture with black. (Pickard-Cambridge, *Top.* 119b10-25).
>
> Ἔτι εἰ τοῦ αὐτοῦ τινος τὸ μὲν μᾶλλον τὸ δὲ ἧττον τοιοῦτο· καὶ εἰ τὸ μὲν τοιούτου μᾶλλον τοιοῦτο, τὸ δὲ μὴ τοιούτου, δῆλον ὅτι τὸ πρῶτον μᾶλλον τοιοῦτο. Ἔτι ἐκ τῆς προσθέσεως, εἰ τῷ αὐτῷ προστιθέμενον τὸ ὅλον μᾶλλον ποιεῖ τοιοῦτο, ἢ εἰ τῷ ἧττον τοιούτῳ προστιθέμενον τὸ ὅλον μᾶλλον ποιεῖ τοιοῦτο. Ὁμοίως δὲ καὶ ἐκ τῆς

[75] Cf. *Nicomachean Ethics* I.5, 1097b16-21.

ἀφαιρέσεως· οὗ γὰρ ἀφαιρεθέντος τὸ λειπόμενον ἧττον τοιοῦτο, αὐτὸ μᾶλλον τοιοῦτο. Καὶ τὰ τοῖς ἐναντίοις ἀμιγέστερα μᾶλλον τοιαῦτα, οἷον λευκότερον τὸ τῷ μέλανι ἀμιγέστερον.

The methods which Aristotle describes, are common in our daily life: it is by the methods 'from addition' and 'from subtraction' that we judge whether the thing possesses something to a greater or to a smaller degree. And based on the degree of possession, we either accept the result or refuse to proceed with it. Specifically, the idea is to explain that the way of deciding whether something is more worthy of choice is performed by means of addition of both objects of choice to the same thing, and the one which makes it a whole or a greater good is preferable. The same procedure can be done from subtraction, that is when two objects are subtracted from the same thing. That which makes the remainder a lesser good is itself the greater good and is thus more preferable. John Cleary gives a good mathematical analogy: $10 - x = 2$, $10 - y = 5$; ergo $y < x$.[76] Here as we can see one subtracted thing *x* possesses, let us say, goodness or whiteness in a greater degree than *y* because the remainder in the first set is smaller. It is clear that the standard expression 'ἐκ τῆς ἀφαιρέσεως' does not suggest any extractionist sense of one out of many, or any kind of reception of forms by the soul. The expression, together with its correlative method 'from addition' (ἐκ τῆς προσθέσεως) indicates to a simple analytical process applied in mathematics.

Another relevant use of addition can be found in *Topics* V.2. There Aristotle states that in definitions, s/he who assigns one property that denotes the essence of a body (such as 'fire' or 'liquid'), confirms a property, and s/he who adds too many properties to one thing, errs in his/her definition. For instance, to say that fire is 'the most rarefied and lightest body' is a wrong definition because either 'the most rarefied' or 'lightest' is superfluously added since both denote the same thing (131a20). Therefore, only one essential property should be

[76] John Cleary, "On the Terminology of "Abstraction" in Aristotle," *Phronesis* 30 (1985): 19.

stated in order to get a correct definition, e.g. ‘liquid’ is a ‘body adaptable to every shape’ (131a25). Later in *Topics* VI.3 Aristotle in a relevant passage claims that to arrive at a proper definition denoting the essence of the term, it is necessary to subtract or take away everything that is superfluously added to that definition, e.g. in the definition of man – ‘man is a rational animal capable of receiving knowledge,’ the ‘receptive of knowledge’ is superfluously added, and thus, should be subtracted from the definition (141a5). When this is subtracted, what remains will be a proper definition.

The language of subtraction is also present in book VII chapter 3 of *Metaphysics* where Aristotle is primarily concerned with locating and defining the substance in a sensible body. I place this passage in my chapter on non-mathematical use of *aphairesis* because the main question Aristotle there addresses concerns substance. Nonetheless, the passage also treats the process of locating the objects of mathematics (I will discuss the mathematical use later). Aristotle there states that when all affections, products, and potencies of bodies (τῶν σωμάτων πάθη καὶ ποιήματα καὶ δυνάμεις) are removed (περιαιρουμένων), what remains left is length, breadth, and depth (τὸ μῆκος καὶ πλάτος καὶ βάθος), but when these are removed, we see nothing left unless there is something bounded by these such as substance or matter (1029a 11-17). In order to locate the substance Aristotle performs a simple process of subtraction – first all affections are removed, then, once the intellect arrives at length, breadth, and depth which correspond to line, plane, and solid, these become removed as well since they belong to the category of quantities. What remains left over (ὑπολειπόμενον) would be the underlying matter – that which is not predicated of anything, but of which everything else is predicated. Later Aristotle, of course, abandons the idea that this matter is substance and proposes that substance is essence or τὸ τί ἦν εἶναι.

At this point, we can conclude that all these and other uses of relative methods of subtraction[77] and addition[78] indicate a simple logical method and appears to not be restricted to the objects of mathematics only but can also be used to find a primary subject of any given attribute, to determine the degrees of qualities, to determine a property when stating a definition, to locate the substance, among other functions. In addition, the selected passages from Aristotle have shown that the term ἀφαιρεῖν does not presuppose a sense of extraction, and even in the realm of non-mathematicals it has the meaning of consecutive removal of unnecessary objects.

[77] The term ἀφαίρεσις also appears in *Topics* I. 107a37, III. 119a3, VII. 152b8, VIII. 157b10, 161b23.
[78] The term πρόσθεσις appears in *Topics* II. 115a25-30, III. 116a7, V. 132a10-20, 134b5, 134b30, VI. 134a30, 139b17, 140a33-141a20, 143a23, 143b7, 146b30-147a2, 148a15-18, 151b25, VII. 152b10, VIII. 161a8, 161b23-27.

CHAPTER 4 TECHNICAL APPLICATION OF ABSTRACTION IN ARISTOTLE

Introduction

In the following chapter I will show that Aristotle's *aphairesis* does not presuppose any epistemological sense of extraction of one out of many. To support my point, I will look closely into those passages where Aristotle uses *aphairesis* in a more technical application together with the *qua*-terminology, both outside of a mathematical context and within it. These passages will show that *aphairesis*, when used together with the *qua*-locutor does not yet have an extractionist sense or a sense of isolation of one out of many which the application of *qua* might suggest. It still presents itself as a consecutive removal of unnecessary aspects one after another even when used together with the *qua*-filter.

Furthermore, in this chapter I shall single out two different uses of τὰ ἐξ ἀφαιρέσεως expressions, one of which is not restricted to the objects of mathematics or geometry, but instead has a completely different reference, – in the first case, to 'pale,' and in the second case, to universals. Since Aristotle agrees neither with Plato's philosophy of mathematics which posited ontologically separated intermediates and Form numbers, nor with that of Pythagoreans which stated that living bodies are themselves numbers, Aristotle has to provide his own positive alternative, which he does with his application of *aphairesis*.

My analysis will confirm that the true Aristotelian mathematical *aphairesis* is *subtraction*, that is when we take away or remove many things and then study the remainder, e.g. to remove colour, passions, affections, and motion from a bronze isosceles triangle, and consider the two-dimensional continuous shape only, such as triangularity. I agree with John Cleary that Aristotle interprets *aphairesis* as a simple mathematical or logical *subtraction* but disagree in another respect. He proposes that both τὰ ἐξ ἀφαιρέσεως expression and ἀφαίρεσις should be interpreted primarily as the logical methods of finding the primary subject of any given attribute.[79] He is right to claim that both τὰ ἐξ ἀφαιρέσεως expression and ἀφαίρεσις represent the 'logical' subtraction. While the inquiry for a primary subject can be applied only to ἀφαίρεσις,[80] I disagree that when Aristotle mentions τὰ ἐξ ἀφαιρέσεως he is looking for a primary subject of an attribute. I think the main purpose of τὰ ἐξ ἀφαιρέσεως is rather that of *uncovering* and *elucidating* the *spatial location* of the sensible magnitude, which is in all respects sensible and does not exist outside and separately (*de Anima*, 432a3) in the manner of Plato's intermediates and Form numbers. Aristotle uses the method of ἐξ ἀφαιρέσεως to oppose Plato and to show that the category of quantity immanent within the sensible body.

In addition, Aristotle's *aphairesis* should not be considered separably from his concept of *intelligible matter* (ὕλη νοητή) and the concepts of *potentiality* (δύναμις) and *actuality* (ἐνέργεια or ἐντελέχεια) if a plausible account of Aristotle's objects of mathematics is in question. I see the union of these four concepts in the following way: the objects of mathematics, while 'existing' potentially as physical continuous extension and being unfolded by successive method of subtraction, 'exist' actually in thought as a compound of intelligible matter and form.

4.1. The Use of Abstraction Outside Mathematics.

4.1.1 Two Non-mathematical Uses of τὰ ἐξ ἀφαιρέσεως.

In this section I am examining two things: the two non-mathematical instances of the 'τὰ ἐξ ἀφαιρέσεως' expression, and the connection between *aphairesis* and 'qua' terminology outside mathematics. Specifically, in the first part of this section, through an investigation of

[79] John Cleary, "On the Terminology of "Abstraction" in Aristotle," *Phronesis* 30 (1985): 32, 33, 39.
[80] Such interpretation with an application of *aphairesis* is present in *the Posterior Analytics* I.5 and elsewhere.

the two non-mathematical uses of the 'τὰ ἐξ ἀφαιρέσεως' expression, I will show that the standard use of τὰ ἐξ ἀφαιρέσεως is not restricted to the objects of mathematics. In the second part of this section I will show that *qua*-isolation has a technical application of disregarding anything that is incidental to *'F'* and therefore is also not restricted to the objects of mathematics. Let us now consider the two non-mathematical uses of τὰ ἐξ ἀφαιρέσεως which have no relation to the objects of mathematics.

(i) τὰ ἐξ ἀφαιρέσεως λεγόμενα in the *Posterior Analytics* I. 18. In the *Posterior Analytics* at 81a38-81b9 Aristotle explains that the knowledge of particulars and universals is impossible without the sense-faculty. If there is no sense-faculty, an application of induction (ἐπαγωγῇ) cannot be performed because induction develops from particulars. Furthermore, if it is impossible to perform induction, it is likewise impossible to perform demonstration (ἀπόδειξις) as it primarily depends on induction. This passage also deserves special attention since some commentators posit ἀφαίρεσις or abstraction as a separate third way of learning, on equal terms with induction and demonstration, and sometimes assimilate abstraction with induction.[81] This controversial passage is of particular interest also because some scholars tend to translate τὰ ἐξ ἀφαιρέσεως as 'mathematical abstractions' and connect the passage as a whole with the objects of mathematics. Because translators assume that the τὰ ἐξ ἀφαιρέσεως designates nothing other than the objects of mathematics, they find it unproblematic to translate the expression everywhere it occurs as mathematical abstraction, including the following passage from *Posterior Analytics* I.18:

> It is evident too that if some perception is wanting, it is necessary for some understanding to be wanting too - which it is impossible to get if we learn either by

[81] Allan Bäck, "The Concept of Abstraction," *The Society for Ancient Greek Philosophy Newsletter.* 376. (2006): 1-14. https://orb.binghamton.edu/sagp/376; Charles de Koninck, Alphonse-Marie Parent, and Émile Simard, "Abstraction from Matter (I)." *Laval Théologique et Philosophique* 13, no. 2 (1957): 133-96, and De Koninck, Charles, Alphonse-Marie Parent, and Emmanuel Trépanier. "Abstraction from Matter (II)." *Laval Théologique et Philosophique* 16, no. 1 (1960): 53-69 ; Auguste Mansion, *Introduction à la physique aristotélicienne*. Louvain: Editions de l'Institud Superieur, 1913.

> induction or by demonstration, and demonstration depends on universals and induction on particulars, and it is impossible to consider universals except through induction (since even in the case of what are called abstractions [τὰ ἐξ ἀφαιρέσεως λεγόμενα] one can make familiar through induction that some things belong to each kind, even if they are not separable, in so far as each thing is such and such), and it is impossible to get an induction without having perception - for of particulars there is perception; for it is not possible to get understanding of them; for (it can be got) neither from universals without induction nor through induction without perception. (Barnes, *APo.* 81a38-b9).
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> Φανερὸν δὲ καὶ ὅτι, εἴ τις αἴσθησις ἐκλέλοιπεν, ἀνάγκη καὶ ἐπιστήμην τινὰ ἐκλελοιπέναι, ἣν ἀδύνατον λαβεῖν, εἴπερ μανθάνομεν ἢ ἐπαγωγῇ ἢ ἀποδείξει, ἔστι δ' ἡ μὲν ἀπόδειξι ἐκ τῶν καθόλου, ἡ δ' ἐπαγωγὴ ἐκ τῶν κατὰ μέρος, ἀδύνατον δὲ τὰ καθόλου θεωρῆσαι μὴ δι' ἐπαγωγῆς (ἐπεὶ καὶ τὰ ἐξ ἀφαιρέσεως λεγόμενα ἔσται δι' ἐπαγωγῆς γνώριμα ποιεῖν, ὅτι ὑπάρχει ἑκάστῳ γένει ἔνια, καὶ εἰ μὴ χωριστά ἐστιν, ᾗ τοιονδὶ ἕκαστον), ἐπαχθῆναι δὲ μὴ ἔχοντας αἴσθησιν ἀδύνατον. τῶν γὰρ καθ' ἕκαστον ἡ αἴσθησις· οὐ γὰρ ἐνδέχεται λαβεῖν αὐτῶν τὴν ἐπιστήμην· οὔτε γὰρ ἐκ τῶν καθόλου ἄνευ ἐπαγωγῆς, οὔτε δι' ἐπαγωγῆς ἄνευ τῆς αἰσθήσεως.

G.R.G. Mure thinks that τὰ ἐξ ἀφαιρέσεως λεγόμενα is nothing other than the objects of mathematics.[82] Even a few lines later he takes ὅτι ὑπάρχει ἑκάστῳ γένει ἔνιακαὶ εἰ μὴ χωριστά ἐστιν, ᾗ τοιονδὶ ἕκαστον to mean "each subject genus possesses, in virtue of a determinate mathematical character."[83] In Greek, however, this sentence makes no reference to mathematicals. What the Greek does say is that some things inhere in each genus, even if they are not separable, insofar as each thing is such-and-such. Christoph Helmig is similarly convinced that the reference of τὰ ἐξ ἀφαιρέσεως is made to the objects of mathematics which become familiar through induction: "In *An. Post*. I.18, it is said that the only way of acquiring

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[82] It is unclear what G.R.G. Mure means by the 'objects of mathematics,' namely whether he means mathematical universals, mathematical particulars, or both. This is how he translates the passage. "It is also clear that the loss of any one of the senses entails the loss of a corresponding portion of knowledge, and that, since we learn either by induction or by demonstration, this knowledge cannot be acquired. Thus demonstration develops from universals, induction from particulars; but since it is possible to familiarize the pupil with even the so-called mathematical abstractions only through induction - i.e. only because each subject genus possesses, in virtue of a determinate mathematical character, certain properties which can be treated as separate even though they do not exist in isolation - it is consequently impossible to come to grasp universals except through induction But induction is impossible for those who have not sense perception. For it is sense-perception alone which is adequate for grasping the particulars: they cannot be objects of scientific knowledge, because neither can universals give us knowledge of them without induction, nor can we get it through induction without sense-perception." (in Ross, *APo*. I, 18).

[83] Richard McKeon, *The Basic Works of Aristotle*, Modern Library Classics. (New York: Modern Library, 2001), 136.

universal knowledge is through induction and that even mathematicals (τὰ ἐξ ἀφαιρέσεως) become familiar in this way."[84] But what kind of objects of mathematics does Aristotle mean, on this view? Ian Mueller, for instance, takes this expression to refer to mathematical truths in general: "Aristotle's point seems to be that the student is led to believe mathematical axioms by being shown that they hold in a number of particular cases."[85] Due to the controversial nature of this passage, some scholars, such as Tredennick, Barnes, and Cleary consider that the reference may be broader here. The Russian translation of the passage by Boris Fokht also supports the idea that the reference is broader by translating τὰ ἐξ ἀφαιρέσεως λεγόμενα as "так называемое отвлеченное" which means "the so-called abstract things."[86] However, to interpret the expression as "the so-called abstract things" is somewhat vague; the 'τὰ' in τὰ ἐξ ἀφαιρέσεως λεγόμενα must necessarily have a meaning behind it. If we suppose that τὰ ἐξ ἀφαιρέσεως are nothing other than the objects of mathematics, are they particular objects of mathematics (e.g. *this* circle in *this* round bronze) or universals (e.g. circle)? Can this expression also refer to non-mathematical universals, such as 'horse' and 'man'? As an experiment, let us suppose that the expression in question refers to the objects of mathematics.

If we take τὰ ἐξ ἀφαιρέσεως in *APo* I. 18 to mean the objects of mathematics, then these should be particular mathematicals (*this* circle) because the extension or the continuous magnitude of *this* particular circle cannot be reached by the standard process of induction. Particular circles (*this* circle of *this* round bronze) are reached by the step-by-step subtraction or *aphairesis* as *Posterior Analytics* I. 5, *De Caelo* III. 1, *Metaphysics* VII. 3, and *Metaphysics* XI. 3 will later indicate in my analysis. Throughout the corpus (except *Metaphysics* XIII. 2) 'τὰ' in τὰ ἐξ ἀφαιρέσεως, as I will show later in my work, is the quantitative and continuous

[84] Christoph Helmig, *Forms and concepts: concept formation in the Platonic tradition*, Vol. 5. (Walter de Gruyter, 2012), 108.
[85] Ian Mueller, "Aristotle on Geometrical Objects," *Archiv für Geschichte der Philosophie* 52, no. 2 (1970): 160.
[86] Аристотель. Аналитика — Вторая / Пер. Б. А. Фохта. (Москва, 1952), 12.

extension or magnitude attained by the successive removal of affections, passions, and motion within any particular body. The results of subtraction in τὰ ἐξ ἀφαιρέσεως *always* signify particular objects of mathematics.

However, if particular objects of mathematics are reached by subtraction, then how can we agree with Aristotle's statement that the objects of mathematics which are said as a result of subtraction become familiar through induction? In Posterior Analytics II.19 Aristotle explains that induction is a collection of particulars in the soul under one concept: "when of a number of logically indiscriminable particulars has made a stand, the earliest universal is present in the soul: for though the act of sense-perception is of the particular, its content is universal" (Mure, 100a15-100b1). Following the meaning of 'τὰ ἐξ ἀφαιρέσεως λεγόμενα ἔσται δι' ἐπαγωγῆς γνώριμα ποιεῖν,' it may suggest that *this* particular circle reached by subtraction simply falls under a universal 'circle' which is made familiar through induction. We come to know the universal 'circle' by collection from these particular circles *previously* attained by subtraction. It is the universal that becomes familiar through induction, whereas the particular is known through subtraction. In *Metaphysics* VII Aristotle acknowledges that there are two kinds of circles, one that exists simply (ὅ ἁπλῶς λεγόμενος) and other a particular circle (ὅ καθ᾽ ἕκαστον): "For 'circle' is used homonymously, meaning both the circle in general and the individual circle, because there is no name proper to the individuals"[87] (Barnes, *Meta.* 1035a33-35). Here Aristotle states that both particular and universal seem to be one and the same thing *only* homonymously as there is no proper name (μὴ εἶναι ἴδιον ὄνομα) for every particular circle. In addition, at 1036a ff. Aristotle indicates the difference between a universal circle and particular circle is that while the formula of a universal circle can be given, there is no definition of a particular circle because matter is

[87] "ὁμωνύμως γὰρ λέγεται κύκλος ὅ τε ἁπλῶς λεγόμενος καὶ ὁ καθ᾽ ἕκαστα διὰ τὸ μὴ εἶναι ἴδιον ὄνομα τοῖς καθ᾽ ἕκαστον."

unknowable in itself (ἡ δ᾽ ὕλη ἄγνωστος καθ᾽ αὑτήν). And since matter is unknowable in itself, particular circles are always defined by means of the universal formula: "the formula is of the universal; for being a circle is the same as the circle, and being a soul is the same as the soul. But when we come to the concrete thing, e.g. *this* circle, i.e. one of the individual circles, whether sensible[88] or intelligible[89] […] of these there is no definition […], they are always stated and cognized by means of the universal formula"[90] (Barnes, *Meta.* 1036a1-9).

On the other hand, if Aristotle indeed implied either mathematical universals or particular mathematicals in *APo* I. 18, one would expect him to at least make an indirect reference to mathematics either in this, in previous, or in subsequent chapters like he does throughout his works, but this is not the case. In addition, there are only fourteen references to τὰ μαθηματικά in the entire *Organon* and none of them discusses the mode of being of the objects of mathematics, therefore it is also possible that τὰ ἐξ ἀφαιρέσεως in *APo* I. 18 has no reference either to the mode of being of the objects of mathematics or to how they are attained. It does not belong to the *Analytics* to determine the ontological status of anything. Furthermore, the expression in question as it appears in *Meta*. III.2 at 1077b 1-10 does not make any direct reference to the objects of mathematics. The Greek literally says that the result of subtraction is 'pale.'

Let us now suppose the opposite, that the expression refers to non-mathematical particulars. If we take τὰ ἐξ ἀφαιρέσεως in *APo* I. 18 to mean just any particular in general (e.g. *this* horse, *this* pale) with the exception of mathematical particulars, then this passage fits well with *Meta.* XIII. 2 which has to do with definitions, where the result of subtraction is

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[88] I.e. *this* perceptible circular bronze

[89] I.e. *this* intelligible mathematical circle attained by subtraction of passions, affections, motion, and change from *this* perceptible circular bronze.

[90] "ὁ δὲ λόγος ἐστὶ τοῦ καθόλου: τὸ γὰρ κύκλῳ εἶναι καὶ κύκλος καὶ ψυχῇ εἶναι καὶ ψυχὴ ταὐτό. τοῦ δὲ συνόλου ἤδη, οἷον κύκλου τουδὶ καὶ τῶν καθ᾽ ἕκαστά τινος ἢ αἰσθητοῦ ἢ νοητοῦ […] τούτων δὲ οὐκ ἔστιν ὁρισμός […] ἀλλ᾽ ἀεὶ λέγονται καὶ γνωρίζονται τῷ καθόλου λόγῳ."

‘pale’: ‘man’ when subtracted from ‘pale man,’ becomes the result of subtraction, whereas ‘pale man’ is the result of addition in a definition. In *APo* I. 18 some things (ἔνια), such as *this* pale, the result of subtraction from ‘pale man,’ falls under its corresponding genus or universal (ὅτι ὑπάρχει ἑκάστῳ γένει) such as ‘white.’ Thus, the statement ‘τὰ ἐξ ἀφαιρέσεως λεγόμενα ἔσται δι' ἐπαγωγῆς γνώριμα ποιεῖν’ may also suggest that *this* particular pale reached by subtraction simply falls under a universal ‘white’ which is made familiar through induction.

To think of Aristotelian ‘abstraction’ being a separate third way of learning or a separate theory is also incorrect because one would expect Aristotle to fully elaborate on ‘abstraction’ as he does with induction and demonstration in the corpus. Cleary is right that the passage clearly tells us that the results of subtraction become familiar *through* induction (δι' ἐπαγωγῆς) in so far as they belong to each genus of physical objects qua (ᾗ) such-and-such.[91] And the role of induction is to collect “the one beside the many which is a single identity of them all” (*APo,* 100a8-9), whereas that of *aphairesis* is to remove objects either physically or in thought. If we consider the entire corpus, nowhere will we find Aristotle using *aphairesis* together with *epagoge* except in that controversial passage quoted above. Instead, we find that the role of *aphairesis* is to subtract items from any particular number of things or to disregard change, affections, qualities, and other things in thought in order to arrive at the remainder. Furthermore, since Aristotle has not yet developed his position on the objects of mathematics in the *Organon*, and since there is no reference to mathematics in *APo* I.18 or subsequent chapters, I am more inclined to support the non-mathematical and non-universal reference of τὰ ἐξ ἀφαιρέσεως, i.e. that it designates for instance this ‘pale’(quality)

[91] John Cleary, "On the Terminology of "Abstraction" in Aristotle," *Phronesis* 30 (1985): 17.

in Callias, this man (substance) in Socrates, this horse (substance) in Spirit, the French Trotter, etc.

(ii) τὸ ἐξ ἀφαιρέσεως in *Metaphysics* XIII. 2. In book XIII. 1 and 2 of his *Metaphysics* Aristotle concludes that the objects of mathematics can neither exist as Platonic separate individual substances outside the sensibles, nor as separate substances in sensible things, but they exist in a special sense/in a certain way (ἢ τρόπον τινὰ).[92] Then in chapter 3 Aristotle gives his own positive view about the mode of being of mathematical objects. Many scholars have found it puzzling that the term *aphairesis* or τὸ ἐξ ἀφαιρέσεως appears only once in ch.1-2 and is completely absent in ch.3 where he gives his own positive account. Aristotle's positive solution to the problem of how objects of mathematics exist is in turn filled with the 'qua' terminology (ᾗ), while the terminology of *aphairesis* is completely absent from it. Moreover, the 'τὸ ἐξ ἀφαιρέσεως' expression in chapter 2 does not make any direct reference to the objects of mathematics (only an indirect one):

> Grant, then, that they (points, lines, and planes) are prior in definition. Still not all things that are prior in definition are also prior in substantiality. For those things are prior in substantiality which when separated from other things surpass them in the power of independent existence, but things are prior in definition to those whose definitions are compounded out of their definitions; and these two properties are not co-extensive. For if attributes do not exist apart from their substances (e.g. a 'mobile' or a 'pale'), pale is prior to the pale man in definition, but not in substantiality. For it cannot exist separately, but is always along with the concrete thing; and by the concrete thing I mean the pale man. Therefore it is plain that neither is the result of abstraction [τὸ ἐξ ἀφαιρέσεως] prior nor that which is produced by adding determinants posterior; for it is by adding [ἐκ προσθέσεως] a determinant to pale that we speak of the pale man. (Ross, *Meta.* 1077b 1-10).
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> τῷ μὲν οὖν λόγῳ ἔστω πρότερα, ἀλλ᾽ οὐ πάντα ὅσα τῷ λόγῳ πρότερα καὶ τῇ οὐσίᾳ πρότερα. τῇ μὲν γὰρ οὐσίᾳ πρότερα ὅσα χωριζόμενα τῷ εἶναι ὑπερβάλλει, τῷ λόγῳ δὲ ὅσων οἱ λόγοι ἐκ τῶν λόγων: ταῦτα δὲ οὐχ ἅμα ὑπάρχει. εἰ γὰρμὴ ἔστι τὰ πάθη παρὰ τὰς οὐσίας, οἷον κινούμενόν τι ἢ λευκόν, τοῦ λευκοῦ ἀνθρώπου τὸ λευκὸν πρότερον κατὰ τὸν λόγον ἀλλ᾽ οὐ κατὰ τὴν οὐσίαν: οὐ γὰρ ἐνδέχεται εἶναι κεχωρισμένον ἀλλ᾽ ἀεὶ ἅμα τῷ συνόλῳ ἐστίν (σύνολον δὲ λέγω τὸν ἄνθρωπον τὸν

[92] I will treat this expression also in my section on the potential existence of intelligible matter.

λευκόν), ὥστε φανερὸν ὅτι οὔτε τὸ ἐξ ἀφαιρέσεως πρότερον οὔτε τὸ ἐκ προσθέσεως ὕστερον: ἐκ προσθέσεως γὰρ τῷ λευκῷ ὁ λευκὸς ἄνθρωπος λέγεται.

Here Aristotle points out that Platonists made the things which are prior in definition (τῷ λόγῳ) to be also prior in reality or in substance (τῇ οὐσίᾳ). He makes it clear that their mistake lies in thinking that 'the thing from subtraction' such as the attribute 'pale' is prior in reality to 'the thing from addition' namely, to the 'pale man.' Cleary expresses the same idea that in this context Aristotle's Greek clearly refers to whiteness and not to a mathematical object (though an indirect reference to *ta mathematica* seems to be also implied).[93] An indirect reference may imply that 'the thing from subtraction' such as point(s), line(s), and plane(s) which Aristotle mentioned earlier in 1077a35 are prior to 'the thing from addition,' such as solid, only in definition (τῷ λόγῳ) or in the order of generation (γενέσει), but not substantially (τῇ οὐσίᾳ).

Despite the fact that the terminology of *aphairesis* does not appear in *Metaphysics* XIII.3, Ross,[94] for instance, forcibly includes it into his translation.[95] Specifically, he substitutes the Greek 'ἄνευ' with 'abstraction' in the passage where the division of sciences takes place. As I have previously shown in section 1.1 of my thesis, whenever Aristotle discusses the *division of sciences*, he uses *aphairesis* only in the realm of the objects of mathematics: it is the science of mathematics that studies the results of mathematical subtraction from motion, change, affections, and potencies upon which it arrives at the three-

[93] Cleary in his article *On the Terminology of 'Abstraction' in Aristotle* claims that the use of *aphairesis* in this passage is "consistent with the more general use of the dialectical method of subtraction which we observed in the Topics" (p.28). I follow him in this regard.
[94] Richard McKeon, *The Basic Works of Aristotle*, Modern Library Classics. (New York: Modern Library, 2001), 893.
[95] Another instance of such substitution is in *Metaphysics* VII.10 at 1036b7 where Ross translates the Greek χωρίζω as 'abstraction.'

dimensional quantitative and continuous magnitude. Ross, however, in the manner of Boethius[96] also applies it in the realm of Theology or First Philosophy.

> And in proportion as we are dealing with things which are prior in definition and simpler, our knowledge has more accuracy, i.e. simplicity. Therefore a science which abstracts [ἄνευ] from spatial magnitude is more precise than one which takes it into account; and a science is most precise if it abstracts [ἄνευ] from movement, but if it takes account of movement, it is most precise if it deals with the primary movement, for this is the simplest; and of this again uniform movement is the simplest form. (Ross, *Meta.* 1078a 9-13).
>
> καὶ ὅσῳ δὴ ἂν περὶ προτέρων τῷ λόγῳ καὶ ἁπλουστέρων, τοσούτῳ μᾶλλον ἔχει τὸ ἀκριβές (τοῦτο δὲ τὸ ἁπλοῦν ἐστίν), ὥστε ἄνευ τε μεγέθους μᾶλλον ἢ μετὰ μεγέθους, καὶ μάλιστα ἄνευ κινήσεως, ἐὰν δὲ κίνησιν, μάλιστα τὴν πρώτην: ἁπλουστάτη γάρ, καὶ ταύτης ἡ ὁμαλή.

What the Greek simply and directly states is that there is more exactness in a science whose objects are *without* magnitude than with magnitude, and even more exactness when its objects have no movement. Those sciences are more exact which do take neither magnitude nor movement into account. The most precise science is First Philosophy because it does not have magnitude (studied in mathematics) and motion (studied in physics), the second, less precise science, is mathematics because it subtracts motion (studied in physics), and the least precise science is the science of physics as it takes motion into account. Therefore, when the division of sciences is in question, it would be proper to use 'abstraction' or subtraction for mathematics for the sake of consistency with *de Anima* I.1, 403b10-15 and other passages.

Aristotle uses mathematical *aphairesis* in a few ways, either to elucidate the location of the objects in mathematics as in τὰ ἐξ ἀφαιρέσεως or to find out the primary subject of any given attribute when the verb ἀφαιρεῖν is used. There are a few possible reasons I can think of why Aristotle is not using *aphairesis* in the highly mathematical books XIII-XIV, with the exception of XIII.2 (which has no relation to mathematics):

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[96] Boethius. *On the Holy Trinity (de Trinitate).* Translated by Erik C. Kenyon. https://pvspade.com/Logic/docs/BoethiusDeTrin.pdf p.3

(1) First, the location of the objects of mathematics with an application of *aphairesis* in τὰ ἐξ ἀφαιρέσεως, namely the step-by-step subtraction of change, affections, and motion has already been explained by Aristotle in *Physics* II. 2 193b20-194a10, *Metaphysics* VII. 3 1029a5-20 and in *Metaphysics* XI. 3 at 1061a30-35.

(2) Second, Aristotle in chapter 3 does not inquire into finding the primary subjects of any given attributes. In this chapter the philosopher only highlights the aspects of physical objects which a scientist can study in isolation from natural objects: he studies them '*qua* sensible', '*qua* female', '*qua* healthy', '*qua* man', '*qua* sight', '*qua* mobile', '*qua* body', '*qua* planes', etc. If he were to find out, for instance, which subject the attribute of reproductive capacity belongs to in this particular Nova Scotia Duck Tolling Retriever female dog named Sophie, he would have to refer to *aphairesis* to show that 'female' is the primary subject of the property 'reproduction.' This means he would have to subtract Nova Scotia Duck Tolling Retriever dog named Sophie from the object and consider the 'female' only. The same application of *aphairesis* Aristotle uses in the *Posterior Analytics* I. 5 where its function is to remove [ἀφαιρεθέντος] bronze and isosceles from bronze isosceles triangle in order to prove that the property of having angles equal to the sum of two right angles applies to triangle only (*APo.* I.5 74a35-b4).

(3) Finally, the fact that τὰ ἐξ ἀφαιρέσεως is absent in *Metaphysics* XIII.3 may suggest that Aristotle does not see *aphairesis* as a special separate theory which is integral to the being of mathematicals and consequently worthy of elaboration in his discussion. If he did, he would have at least explained more explicitly what he meant by the 'τὰ ἐξ ἀφαιρέσεως' expression. I claim that he understands it rather as a more general method of subtracting elements which has a particular application in mathematics.

In chapter 3 we may see only an implied successive removal of motion and dimensions which are designated by the 'qua.' For instance at 1077b30 Aristotle says: "so too in the case of mobiles there will be propositions and sciences [science of geometry and mathematics], which treat them however not qua mobile but only qua bodies (ᾗ σώματα), or again only qua planes (ᾗ ἐπίπεδα), or only qua lines (ᾗ μήκη), or qua divisibles (ᾗ διαιρετὰ), or qua indivisibles having position (ᾗ ἀδιαίρετα ἔχοντα δὲ θέσιν), or only qua indivisibles (ᾗ ἀδιαίρετα μόνον)." Aristotle states the aspects of a physical body in the manner of their substantial priority, from the more complete and the more whole (τέλειον καὶ ὅλον μᾶλλον) to the less complete and less whole. The successive removal of things is performed in progression from the least precise science, the science of physics, to the more precise science of mathematics: first, we remove motion, when motion is removed, we arrive at the idea of a solid (ᾗ σώματα), then we consider its planes, lines, points, and then finally arrive at an indivisible point having no position.

4.1.2 *Aphairesis* and the 'Qua' Method Outside Mathematics

Aristotle often uses *aphairesis* together with the 'qua' terminology, which can be translated into 'in so far as' or 'as.' It is not only mathematics that considers anything *qua* quantitative and continuous, but every science according to Aristotle can also consider things exclusively '*qua* F,' for instance, physics studies things '*qua* moving,' the science of medicine treats things '*qua* healthy,' the science of metaphysics investigates 'being *qua* being,' the science of biology may consider an animal '*qua* male' or '*qua* female' (*Metaphysics*, XIII.3).

A great example of this connection between *aphairesis* and qua-terminology first appears in the *Categories* chapter 7 where Aristotle's main statement is that all relatives have correlatives. Here *aphairesis* is used to remove any unrelated aspects when we are looking for a proper relative of a thing. Even if subtraction terminology is not present in the first three passages, its hidden function is still there: to show a proper relation we must remove ‘a bird’ from ‘a wing is necessarily relative to a bird’ and replace the former with ‘winged creature’ since the wing is not relative the bird *qua* bird:

> Sometimes, however, reciprocity of correlation does not appear to exist. This comes about when a blunder is made, and that to which the relative is related is not accurately stated. If a man states that a wing is necessarily relative to a bird, the connexion between these two will not be reciprocal, for it will not be possible to say that a bird is a bird by reason of its wings. The reason is that the original statement was inaccurate, for the wing is not said to be relative to the bird *qua* (ᾗ) bird, since many creatures besides birds have wings, but *qua* (ᾗ) winged creature. (Edghill, *Cat*. 6b36-7a3).
>
> οὐ μὴν ἀλλ' ἐνίοτε οὐ δόξει ἀντιστρέφειν, ἐὰν μὴ οἰκείως πρὸς ὃ λέγεται ἀποδοθῇ ἀλλὰ διαμάρτῃ ὁ ἀποδιδούς· οἷον τὸ πτερὸν ἐὰν ἀποδοθῇ ὄρνιθος, οὐκ ἀντιστρέφει ὄρνις πτεροῦ· οὐ γὰρ οἰκείως τὸ πρῶτον ἀποδέδοται πτερὸν ὄρνιθος· οὐ γὰρ ᾗ ὄρνις, ταύτῃ τὸ πτερὸν αὐτῆς λέγεται, ἀλλ' ᾗ πτερωτόν ἐστιν· πολλῶν γὰρ καὶ ἄλλων πτερά ἐστιν ἃ οὐκ εἰσὶν ὄρνιθες.

Likewise, in a definition of a head, ‘an animal’ is a wrong correlative since there are animals which have no head. The proper correlative in a definition of a head will be something which is ‘headed:’

> A head will be more accurately defined as the correlative of that which is 'headed', than as that of an animal, for the animal does not have a head *qua* (ᾗ) animal, since many animals have no head. (Edghill, *Cat*. 7a15-18).
>
> ὡσαύτως δὲ καὶ ἐπὶ τῶν ἄλλων, οἷον ἡ κεφαλὴ οἰκειοτέρως ἂν ἀποδοθείη κεφαλωτοῦ ἢ ζῴου ἀποδιδομένη· οὐ γὰρ ᾗ ζῷον κεφαλὴν ἔχει· πολλὰ γὰρ τῶν ζῴων κεφαλὴν οὐκ ἔχει.

An excerpt with the terminology of subtraction which I have already discussed in section 2.2 follows immediately after:

> Further, if one thing is said to be correlative with another, and the terminology used is correct, then, though all irrelevant attributes should be *removed* [περιαιρουμένων], and only that one attribute left [καταλειπομένου] in virtue of which it was correctly stated to be correlative with that other, the stated correlation will still exist. If the correlative of ‘the slave’ is said to be ‘the master’, then,

> though all irrelevant attributes of the said ‘master’, such as ‘biped’, ‘receptive of knowledge’, ‘human’, should be *removed* [περιαιρουμένων], and the attribute ‘master’ alone left [καταλειπομένου], the stated correlation existing between him and the slave will remain the same, for it is of a master that a slave is said to be the slave. On the other hand, if, of two correlatives, one is not correctly termed, then, when all other attributes are *removed* [περιαιρουμένων] and that alone is left [καταλειπομένου] in virtue of which it was stated to be correlative, the stated correlation will be found to have disappeared. (Edghill, Cat. 7a31-b2).
>
> ἔτι ἐὰν μὲν οἰκείως ἀποδεδομένον ᾖ πρὸς ὃ λέγεται, πάντων περιαιρουμένων τῶν ἄλλων ὅσα συμβεβηκότα ἐστίν, καταλειπομένου δὲ τούτου μόνου πρὸς ὃ ἀπεδόθη οἰκείως, ἀεὶ πρὸς αὐτὸ ῥηθήσεται· οἷον εἰ ὁ δοῦλος πρὸς δεσπότην λέγεται, περιαιρουμένων ἁπάντων ὅσα συμβεβηκότα ἐστὶ τῷ δεσπότῃ, οἷον τὸ δίποδι εἶναι, τὸ ἐπιστήμης δεκτικῷ, τὸ ἀνθρώπῳ, καταλειπομένου δὲ μόνου τοῦ δεσπότην εἶναι, ἀεὶ ὁ δοῦλος πρὸς αὐτὸ ῥηθήσεται· ὁ γὰρ δοῦλος δεσπότου δοῦλος λέγεται. ἐὰν δέ γε μὴ οἰκείως ἀποδοθῇ πρὸς ὅ ποτε λέγεται, περιαιρουμένων μὲν τῶν ἄλλων καταλειπομένου δὲ μόνου τοῦ πρὸς ὃ ἀπεδόθη, οὐ ῥηθήσεται πρὸς αὐτό·

Likewise, even though the excerpt that follows 7a15-18 does not make any reference to the qua-terminology, the language of ‘qua’ is still implied. It is not relative to a biped human that someone is a slave, but it is relative to the master that a slave receives this name. We must remove ‘biped’ and ‘human’ and substitute it with ‘master,’ since the two former attributes are incidental to the master as correlative to slave. The use of subtraction here is identical to what we have seen in *Topics* VI. 3 where Aristotle tells that in order to arrive at a proper definition, we must remove anything superfluously added.

The use of ‘qua’ (implied) together with *aphairesis* outside mathematics appears in *Metaphysics* VI. 2. Here Aristotle distinguishes essential being from accidental being. Accidental being is that which is neither always nor for the most part. For instance, since man is musical neither always nor for the most part, his being in a sense of being musical is accidental. Furthermore, since a man is pale neither always nor for the most part, his paleness is also accidental. However, being man *qua* man is not accidental since the man is a man always and for the most part. Being *qua* man is essential. The matter of man is the cause of the accidental. Here is how Aristotle explains it:

> Since, among things which are, some are always in the same state and are of necessity (not necessity in the sense of compulsion but that which we assert of things because they cannot be otherwise), and some are not of necessity nor always, but for the most part, this is the principle and this the cause of the existence of the accidental; for that which is neither always nor for the most part, we call accidental. […] Therefore, since not all things either are or come to be of necessity and always, but, the majority of things are for the most part, the accidental must exist; for instance a pale man is not always nor for the most part musical, but since this sometimes happens, it must be accidental (if not, everything will be of necessity). The matter, therefore, which is capable of being otherwise than as it usually is, must be the cause of the accidental. […] Let us *dismiss* [ἀφείσθω] accidental being [συμβεβηκὸς ὄντος] for we have sufficiently determined its nature. (Ross, *Meta.* VI 1026b30-1027b15).

> ἐπεὶ οὖν ἐστὶν ἐν τοῖς οὖσι τὰ μὲν ἀεὶ ὡσαύτως ἔχοντα καὶ ἐξ ἀνάγκης, οὐ τῆς κατὰ τὸ βίαιον λεγομένης ἀλλ᾽ ἣν λέγομεν τῷ μὴ ἐνδέχεσθαι ἄλλως, τὰ δ᾽ἐξ ἀνάγκης μὲν οὐκ ἔστιν οὐδ᾽ ἀεί, ὡς δ᾽ ἐπὶ τὸ πολύ, αὕτη ἀρχὴ καὶ αὕτη αἰτία ἐστὶ τοῦ εἶναι τὸ συμβεβηκός: ὃ γὰρ ἂν ᾖ μήτ᾽ ἀεὶ μήθ᾽ ὡς ἐπὶ τὸ πολύ, τοῦτό φαμεν συμβεβηκὸς εἶναι. […] ὥστ᾽ ἐπεὶ οὐ πάντα ἐστὶν ἐξ ἀνάγκης καὶ ἀεὶ ἢ ὄντα ἢ γιγνόμενα, ἀλλὰ τὰ πλεῖστα ὡς ἐπὶ τὸ πολύ, ἀνάγκη εἶναι τὸ κατὰ συμβεβηκὸς ὄν: οἷον οὔτ᾽ ἀεὶ οὔθ᾽ ὡς ἐπὶ τὸ πολὺ ὁ λευκὸς μουσικός ἐστιν, ἐπεὶ δὲ γίγνεταίποτε, κατὰ συμβεβηκὸς ἔσται (εἰ δὲ μή, πάντ᾽ ἔσται ἐξ ἀνάγκης): ὥστε ἡ ὕλη ἔσται αἰτία ἡ ἐνδεχομένη παρὰ τὸ ὡς ἐπὶ τὸ πολὺ ἄλλως τοῦ συμβεβηκότος. […] περὶ μὲν οὖν τοῦ κατὰ συμβεβηκὸς ὄντος ἀφείσθω (διώρισται γὰρ ἱκανῶς).

The use of the ‘qua’ together with *aphairesis* outside mathematics comes into sight also in *Metaphysics* VI. 4 where the science of metaphysics is said to study *being qua being* (ὂν ᾗ ὀν). There Aristotle explains that truth and falsity are not in the things themselves, but in judgements or in thought only, that is when we affirm or deny something. The falsity of a proposition, for instance, results from an improper combination or separation of two terms by the intellect, whereas when the mind comprehends the essence of something within itself, there is no truth or falsity. When the mind first comprehends the concept of man, everything that defines man as a man, comes immediately together with the concept of ‘man’ when the mind thinks it. Truth and falsity take place only when we start to combine or separate the things in thought such as essence, quality, quantity, and other things. Thus, if one further proceeds to make a statement or a definition about what man is, there may be a chance of either truth or falsity, e.g. s/he can say that (1) ‘man *is* a mortal rational animal’ or (2) ‘man is *not* mortal rational animal,' and, for instance, (3) 'man is *not* a horse' or (4) 'man *is* a horse.' In (1) and (3) the statement is true, in (2) and (4) it is wrong. Such things as 'man is *not* mortal rational animal' and 'man *is* a horse' we call false, "either because they themselves do not exist, or because the appearance which results from them is that of something that does not exist"[97] (Ross, *Meta.* 1024b25). Since falsity does not exist, another thing that must be removed besides from the accidental being is being in the sense of being true. When both accidental and being in the sense of truth are removed, only then we can consider being *qua* being:

> For falsity and truth are not in things – it is not as if the good were true, and the bad were in itself false – but in thought; while with regard to simple concepts and 'whats' falsity and truth do not exist even in thought […] But since the combination and the separation are in thought and not in the things, and that which is in this sense is a different sort of 'being' from the things that are in the full sense (for the thought attaches or *removes* [ἀφαιρεῖ] either the subject's 'what' [τὸ τί ἐστιν] or its having a certain quality or quantity or something else), that which is accidentally and that which is in the sense of being true must be *dismissed* [ἀφετέον]. […] Therefore let these be *dismissed* [ἀφείσθω], and let us consider the causes and the principles of being itself, *qua* [ᾗ] being. (Ross, *Meta.* VI 1027b25-1028a5).
>
> οὐ γάρ ἐστι τὸ ψεῦδος καὶ τὸ ἀληθὲς ἐν τοῖς πράγμασιν, οἷον τὸ μὲν ἀγαθὸν ἀληθὲς τὸ δὲ κακὸν εὐθὺς ψεῦδος, ἀλλ᾽ ἐν διανοίᾳ, περὶ δὲ τὰ ἁπλᾶ καὶ τὰ τί ἐστιν οὐδ᾽ ἐν διανοίᾳ […] ὅσα μὲν οὖν δεῖ θεωρῆσαι περὶ τὸ οὕτως ὂν καὶ μὴ ὄν, ὕστερον ἐπισκεπτέον: ἐπεὶ δὲ ἡ συμπλοκή ἐστιν καὶ ἡ διαίρεσις ἐν διανοίᾳ ἀλλ᾽ οὐκ ἐν τοῖς πράγμασι, τὸ δ᾽ οὕτως ὂν ἕτερον ὂν τῶν κυρίως (ἢ γὰρ τὸ τί ἐστιν ἢ ὅτι ποιὸν ἢ ὅτι ποσὸν ἤ τι ἄλλο συνάπτει ἢ ἀφαιρεῖ ἡ διάνοια) τὸ μὲν ὡς συμβεβηκὸς καὶ τὸ ὡς ἀληθὲς ὂν ἀφετέον. […] διὸ ταῦτα μὲν ἀφείσθω, σκεπτέον δὲτοῦ ὄντος αὐτοῦ τὰ αἴτια καὶ τὰς ἀρχὰς ᾗ ὄν.

When Aristotle gives an account of the process of combination and separation, we can see that the process of removal does not presuppose a sense of extraction of one thing from many, rather, it demonstrates a consecutive removal of many things such as quality, quantity, essence, and other things until the proposition is formed. Furthermore, to consider being *qua* being we also remove many things and leave the remainder: we take away both accidental

[97] "πράγματα μὲν οὖν ψευδῆ οὕτω λέγεται, ἢ τῷ μὴ εἶναι αὐτὰ ἢ τῷ τὴν ἀπ᾽ αὐτῶν φαντασίαν μὴ ὄντος εἶναι."

being and being in the sense of being true, to arrive at essence as the centre of first philosophy.

In the next section I will consider how *aphairesis* terminology appears within Aristotle's passages on mathematics and will examine the seven mathematical uses of 'τὰ ἐξ ἀφαιρέσεως' expression I outlined in chapter I, section 1.2.

4.2 The Use of Abstraction Within Mathematics

4.2.1 Seven Mathematical Uses of τὰ ἐξ ἀφαιρέσεως

In chapter I section 1.1 I have identified two very subtly distinct senses of mathematical *aphairesis*. One sense takes out *one* thing while disregarding the rest, e.g. when we abstract a mathematical universal (e.g. circle) from particulars, or when we abstract *this* particular mathematical circle from *this* particular matter of *this* round bronze. Such application of the term involves extraction and is common in the commentaries of Aristotle's mathematics, both ancient and modern. Another use of the term has a function of subtraction, that is when we take away or remove many things and then study the remainder, e.g. to remove colour, passions, affections, and motion from a bronze isosceles triangle, and consider the two-dimensional continuous shape only,[98] such as triangularity. In this section I examine all seven mathematical uses of 'τὰ ἐξ ἀφαιρέσεως' together with the terminology of ἀφαίρεσις as it appears throughout Aristotle's corpus. My analysis will show that Aristotle neither uses *aphairesis* as abstraction of a mathematical universal (e.g. circle) from particulars, nor as abstraction of *this* mathematical circle from *this* particular matter of *this* round bronze.

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[98] *Metaphysics* VIII. 6 at 145a20 ff. Aristotle states that the 'round' of a 'bronze' is shape (μορφή) that mathematician studies.

(iii) τὰ ἐξ ἀφαιρέσεως λέγεσθαι in the *De Caelo* III. 1. This expression in *De Caelo* III. 1 proposes an alternative to Plato's cosmological dialogue *Timaeus* where physical bodies are said to be constructed from the objects of mathematics. Aristotle objects to this saying that for the bodies to be composed from planes, there must exist indivisible magnitudes, but this is impossible because physical continuous magnitude is infinitely divisible. Aristotle starts this objection right at the beginning of *De Caelo* book I: "Now a continuum is that which is divisible into parts always capable of subdivision, and a body is that which is every way divisible" (*De Caelo.* I.1, 268a5). Since physical magnitude is indefinitely divisible, the construction of natural bodies from planes will have impossible consequences in physics which are not present in mathematics. It is only in thinking that we consider any physical object only *as if* (*qua*) it is either divisible or indivisible, and thus construct lines, planes, and solids (*Meta.* XIII.3) in thought. In mathematics, when we consider an object *qua* plane or *qua* line, we consider these *qua* divisible or *qua* indivisible so that we could have a plane and a line as distinct entities, but this does not mean that they exist as indivisible magnitudes in reality and can form a physical object. Since we can consider any object *qua* indivisible line or *qua* indivisible plane, there will present no difficulties in mathematics. The impossibility arises only in thinking that the intelligible lines and planes of any physical object can be considered as both divisible and indivisible, when in reality the continuous sensible magnitude is always divisible in every way. Here is how Aristotle explains it:

> But as for this last theory, which constructs all bodies out of planes, a glance will reveal many points in which it is in contradiction to the findings of mathematics (and unless one can replace the hypotheses of a science with something more convincing, it is best to leave them undisturbed). In addition, the composition of solids from planes clearly involves, by the same reasoning, the composition of planes from lines and lines from points (a view according to which a part of a line need not be a line); and this is something which we have already considered in the work on motion, where we concluded that there are no indivisible lines. Nevertheless, so far as they concern natural bodies, the impossibilities resulting from an assumption of

> indivisible lines are worth a little attention here. The mathematical impossibilities will be physical impossibilities too, but this proposition cannot be simply converted, since the method of mathematics is to abstract [τὰ μαθηματικά ἐξ ἀφαιρέσεως λέγεσθαι], but of natural science to add together all determining characteristics [τὰ δὲ φυσικὰ ἐκ προσθέσεως]. (tr. W.K.C. Guthrie).[99]
>
> τοῖς δὲ τοῦτον τὸν τρόπον λέγουσι καὶ πάντα τὰ σώματα συνιστᾶσιν ἐξ ἐπιπέδων ὅσα μὲν ἄλλα συμβαίνει λέγειν ὑπεναντία τοῖς μαθήμασιν, ἐπιπολῆς ἰδεῖν· καίτοι δίκαιον ἢ μὴ κινεῖν ἢ πιστοτέροις αὐτὰ λόγοις κινεῖν τῶν ὑποθέσεων. Ἔπειτα δῆλον ὅτι τοῦ αὐτοῦ λόγου ἐστὶ στερεὰ μὲν ἐξ ἐπιπέδων συγκεῖσθαι, ἐπίπεδα δ' ἐκ γραμμῶν, ταύτας δ' ἐκ στιγμῶν· οὕτω δ' ἐχόντων οὐκ ἀνάγκη τὸ τῆς γραμμῆς μέρος γραμμὴν εἶναι· περὶ δὲ τούτων ἐπέσκεπται πρότερον ἐν τοῖς περὶ κινήσεως λόγοις, ὅτι οὐκ ἔστιν ἀδιαίρετα μήκη. Ὅσα δὲ περὶ τῶν φυσικῶν σωμάτων ἀδύνατα συμβαίνει λέγειν τοῖς ποιοῦσι τὰς ἀτόμους γραμμάς, ἐπὶ μικρὸν θεωρήσωμεν καὶ νῦν· τὰ μὲν γὰρ ἐπ' ἐκείνων ἀδύνατα συμβαίνοντα καὶ τοῖς φυσικοῖς ἀκολουθήσει, τὰ δὲ τούτοις ἐπ' ἐκείνων οὐχ ἅπαντα διὰ τὸ τὰ μὲν ἐξ ἀφαιρέσεως λέγεσθαι, τὰ μαθηματικά, τὰ δὲ φυσικὰ ἐκ προσθέσεως.

Stocks, for instance, translates the last sentence "τὰ μὲν ἐξ ἀφαιρέσεως λέγεσθαι, τὰ μαθηματικά, τὰ δὲ φυσικὰ ἐκ προσθέσεως" as "mathematics deals with an *abstract* and physics with a more concrete object" (Stocks, *De Caelo.* 299a15). In my view, it would be better to give a literal translation of both expressions, as Guthrie does with τὰ δὲ φυσικὰ ἐκ προσθέσεως, that natural science considers the results of addition. Thus, we would take it to mean that the objects of mathematics are spoken about as a result of subtraction (τὰ μαθηματικά ἐξ ἀφαιρέσεως λέγεσθαι), while the objects of physics are spoken about as a result of addition (τὰ δὲ φυσικὰ ἐκ προσθέσεως). This literal translation of τὰ μαθηματικά ἐξ ἀφαιρέσεως λέγεσθαι would help to avoid any psychological abstractionist connotations which the word 'abstract' accumulated over centuries. In addition, the literal translation helps to avoid an epistemological interpretation of *aphairesis* as abstraction of a mathematical form from matter which suggests the process of an immediate reception of a form by the soul. While Aristotle discusses the reception of forms by soul without matter in *de Anima* (*De Anima* II, 12 424a17-19), this process never appears in the standard terminology of

[99] Guthrie, W. K. C. *On the Heavens*. Loeb Classical Library; No. 338. London: Cambridge, Mass.: W. Heinemann; (Harvard University Press, 1960), 263.

aphairesis. The phrase 'ἐξ ἀφαιρέσεως' simply indicates the spatial location of the quantitative and continuous magnitude within the sensible body, but not the immediate reception of a mathematical form by the soul. Once we use the literal translation, Aristotle's side-by-side use of the two relative terms, 'ἐξ ἀφαιρέσεως' and 'ἐκ προσθέσεως,' begins to make sense. The relative term 'from addition' signifies that its opposite operation, 'from subtraction', does not presuppose any extraction of one thing out of many, but rather the subtraction of many, such as affections, passions, essence, motion which then allows to arrive at the quantitative and continuous magnitude. In contrast, the science of physics does the reverse operation, it considers its objects of study as complete compound things together with affections, passions, essence, motion. As Philippe takes it : "Cette addition nous permet d'atteindre l'être physique dans sa complexité d'être physique, d'être mobile, impliquant à la fois la forme et la matière. Cette connaissance est celle que nous voyons en exercice dans la philosophie de la nature."[100] Perhaps the reason why τὰ φυσικὰ ἐκ προσθέσεως is not used very often is because physical beings are known in a more immediate way: "cette dernière expression [τὰ φυσικὰ ἐκ προσθέσεως] sera beaucoup moins souvent employée que la précédente [τὰ ἐξ ἀφαιρέσεως], le Philosophe ne s'en sert que par référence à celle de l'abstraction. Les êtres physiques peuvent être désignés d'une manière plus immédiate."[101] Thomas Aquinas is of the same opinion concerning the interpretation of Aristotle's statement:

> And this is so because mathematical things are obtained by abstraction from natural things, but natural things are by apposition to mathematical things – for they add to mathematical objects a sensible nature and motion, from which mathematics abstracts (trans. Larcher and Conway).[102]

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[100] Philippe, M. D., "Aphairesis, prosthesis, chorizein dans la philosophie d' Aristote." In: *Revue thomiste 48* (1948): 483.
[101] Ibid., 468.
[102] Thomas Aquinas, *In libros Aristotelis De caelo et mundo expositio, THE HEAVENS.* Transl. by Fabian R. Larcher and Pierre H. Conway. https://dhspriory.org/thomas/DeCoelo.htm#3-1

> quia mathematica dicuntur per abstractionem a naturalibus; naturalia autem se habent per appositionem ad mathematica (superaddunt enim mathematicis naturam sensibilem et motum, a quibus mathematica abstrahunt).

I, however, would only object to Aquinas' "*mathematica dicuntur per abstractionem a naturalibus.*" While he adequately captures the meaning of τὰ δὲ φυσικὰ ἐκ προσθέσεως as *superaddunt enim mathematicis naturam sensibilem et motum*, the meaning of τὰ ἐξ ἀφαιρέσεως τὰ μαθηματικά on the other hand is distorted, suggesting an extractionist sense. Perhaps, it would be more appropriate to take 'naturalibus' in the accusative case and interpret the statement as "to subtract sensible nature and motion from natural things."

In terms of the interpretation of τὰ δὲ φυσικὰ ἐκ προσθέσεως John Cleary proposes a different view. He takes it to mean the Pythagorean generation of physical bodies from simpler elements to more complex three-dimensional bodies: "some continuous construction which is such that its simpler elements are successively integrated into more complex structures by addition."[103] A few lines later he clarifies the statement:

> Thus the Pythagorean schema of point, line, plane, and solid, could serve as a paradigm for the sort of non-reversible relationship between physics and mathematics that Aristotle has in mind here. For example, one might say that the line is 'generated' from the point by adding one dimension, cf. *APo* 87a36-38. Similarly, if we add a new dimension in each case, the plane may be obtained from the line, and the solid from the plane, as a result of addition (ἐκ προσθέσεως). I submit that this is the proper perspective from which to view the terminology of abstraction and addition in the above passage…[104]

Then he concludes:

> Aristotle is here treating physical objects as if they were 'generated', in some logical manner, from basic mathematical structures; e.g. by the addition of three dimensions. This would also suggest that he is accepting the Platonic schema of priority as the framework for this discussion in the *de Caelo* and that he has not yet fully clarified the distinction between mathematics and physics in his own terms.[105]

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[103] John Cleary, "On the Terminology of "Abstraction" in Aristotle," *Phronesis* 30 (1985): 31.
[104] Ibid., 31-32.
[105] Ibid., 32.

Yet I argue that it is not the Pythagorean or Platonic schema of adding dimensions that is actually at stake here. The point which Aristotle makes in τὰ δὲ φυσικὰ ἐκ προσθέσεως is not the question of ‘generation’ of physical bodies from dimensions which Cleary describes, but rather that of adding affections, passions, essence, and motion to constitute a complete sensible object. Additionally, these two reverse operations distinguish the methods of both sciences and the objects that they study.

Aristotle extensively criticizes Plato for constructing physical bodies out of points, lines, and planes (the case of *Timaeus*), so he cannot adhere to the same view. Even a few lines later after the passage in question Aristotle states: “it is impossible, if two parts of a thing have no weight, that the two together should have weight […] Now if the point has no weight, clearly the lines have not either, and if they have not, neither have the planes. Therefore, no body has weight. It is, further, manifest that the point cannot have weight” (Stocks, *de Caelo*, 299a-24-30). Here Aristotle indicates another variation of the problem of why mathematical impossibilities (that there are no indivisible lines) would have consequences for physical things. A further impossibility will result if physical bodies are generated from planes, lines, and points: since points that produce lines do not have weight, whereas sensible objects do, then how can a weightless point and later a line and a plane produce physical weight in the bodies which they constitute? In addition, nowhere in the corpus does Aristotle use the expression τὰ δὲ φυσικὰ ἐκ προσθέσεως to explain the dimensional priority and generation of points, lines, planes, and solids.

Consider a passage from Aristotle’s *Physics* II.2 which fits well with the passage in *De Caelo* III. 1 under consideration, as well as into all passages where both expressions of addition and subtraction appear side by side. Here Aristotle raises a question of how the mathematician is different from the physicist. The passage in *Physics* does not make any reference to *aphairesis,* only to separation in thought (τῇ νοήσει). Nonetheless, it is worth citing because it confirms the idea that τὰ δὲ φυσικὰ ἐκ προσθέσεως does not mean the adding of less simple to more complex structures, but rather the adding of motion and matter. Mathematics, instead, while studying these same physical bodies, separates motion and matter in thought and considers the objects of mathematics only.

> We have distinguished, then, the different ways in which in which the term ‘nature’ is used. The next point to consider is how the mathematician differs from the physicist. Obviously physical bodies contain surfaces and volumes, lines and points, and these are the subject-matter of mathematics. Further, is astronomy different from physics or a department of it? It seems absurd that the physicist should be supposed to know the nature of sun or moon, but not to know any of their essential attributes, particularly as the writers on physics obviously do discuss their shape also and whether the earth and the world are spherical or not. Now the mathematician, though he too treats of these things, nevertheless does not treat of them as the limits of a physical body; nor does he consider the attributes indicated as the attributes of such bodies. That is why he separates them; for in thought they are separable from motion, and it makes no difference, nor does any falsity result, if they are separated. The holders of the theory of Forms do the same, though they are not aware of it; for they separate the objects of physics, which are less separable than those of mathematics. This becomes plain if one tries to state in each of the two cases the definitions of the things and of their attributes. 'Odd' and 'even', 'straight' and 'curved', and likewise 'number', 'line', and 'figure', do not involve motion; not so 'flesh' and 'bone' and 'man'-these are defined like 'snub nose', not like 'curved'. Similar evidence is supplied by the more physical of the branches of mathematics, such as optics, harmonics, and astronomy. These are in a way the converse of geometry. While geometry investigates physical lines but not qua physical, optics investigates mathematical lines, but qua physical, not qua mathematical. Since 'nature' has two senses, the form and the matter, we must investigate its objects as we would the essence of snubness. (R. P. Hardie and R. K. Gaye, *Phys.* 193 b25-184a10).

> Ἐπεὶ δὲ διώρισται ποσαχῶς ἡ φύσις, μετὰ τοῦτο θεωρητέον τίνι διαφέρει ὁ μαθηματικὸς τοῦ φυσικοῦ (καὶ γὰρ ἐπίπεδα καὶ στερεὰ ἔχει τὰ φυσικὰ σώματα καὶ μήκη καὶ στιγμάς, περὶ ὧν σκοπεῖ ὁ μαθηματικός)· ἔτι εἰ ἡ ἀστρολογία ἑτέρα ἢ μέρος τῆς φυσικῆς· εἰ γὰρ τοῦ φυσικοῦ τὸ τί ἐστιν ἥλιος ἢ σελήνη εἰδέναι, τῶν δὲ συμβεβηκότων καθ' αὑτὰ μηδέν, ἄτοπον, ἄλλως τε καὶ ὅτι φαίνονται λέγοντες οἱ περὶ φύσεως καὶ περὶ σχήματος σελήνης καὶ ἡλίου, καὶ δὴ καὶ πότερον σφαιροειδὴς ἡ γῆ καὶ ὁ κόσμος ἢ οὔ. περὶ τούτων μὲν οὖν πραγματεύεται καὶ ὁ μαθηματικός, ἀλλ' οὐχ ᾗ φυσικοῦ σώματος πέρας ἕκαστον· οὐδὲ τὰ συμβεβηκότα θεωρεῖ ᾗ τοιούτοις οὖσι συμβέβηκεν· διὸ καὶ χωρίζει· χωριστὰ γὰρ τῇ νοήσει κινήσεώς ἐστι, καὶ οὐδὲν διαφέρει, οὐδὲ γίγνεται ψεῦδος χωριζόντων. λανθάνουσι δὲ τοῦτο ποιοῦντες καὶ οἱ τὰς ἰδέας λέγοντες· τὰ γὰρ φυσικὰ χωρίζουσιν ἧττον ὄντα χωριστὰ τῶν μαθηματικῶν. γίγνοιτο δ' ἂν τοῦτο δῆλον, εἴ τις ἑκατέρων πειρῷτο λέγειν τοὺς ὅρους, καὶ αὐτῶν καὶ τῶν συμβεβηκότων. τὸ μὲν γὰρ περιττὸν ἔσται καὶ τὸ ἄρτιον

> καὶ τὸ εὐθὺ καὶ τὸ καμπύλον, ἔτι δὲ ἀριθμὸς καὶ γραμμὴ καὶ σχῆμα, ἄνευ κινήσεως, σὰρξ δὲ καὶ ὀστοῦν καὶ ἄνθρωπος οὐκέτι, ἀλλὰ ταῦτα ὥσπερ ῥὶς σιμὴ ἀλλ' οὐχ ὡς τὸ καμπύλον λέγεται. δηλοῖ δὲ καὶ τὰ φυσικώτερα τῶν μαθημάτων, οἷον ὀπτικὴ καὶ ἁρμονικὴ καὶ ἀστρολογία· ἀνάπαλιν γὰρ τρόπον τιν' ἔχουσιν τῇ γεωμετρίᾳ. ἡ μὲν γὰρ γεωμετρία περὶ γραμμῆς φυσικῆς σκοπεῖ, ἀλλ' οὐχ ᾗ φυσική, ἡ δ' ὀπτικὴ μαθηματικὴν μὲν γραμμήν, ἀλλ' οὐχ ᾗ μαθηματικὴ ἀλλ' ᾗ φυσική. ἐπεὶ δ' ἡ φύσις διχῶς, τό τε εἶδος καὶ ἡ ὕλη, ὡς ἂν εἰ περὶ σιμότητος σκοποῖμεν τί ἐστιν, οὕτω θεωρητέον.

The more physical branches of mathematics (τὰ φυσικώτερα τῶν μαθημάτων), such as astronomy for instance, aside from studying the sphericity of the stars and the sphericity of the fifty-five spheres (*Meta*. XII.8, 1073b20ff.) which carry on the planets and the Sun, also adds motion to its object of study so that it could give an account of the harmonious motions of all fifty-five spheres. This is what Aristotle means by saying that astronomy is the converse of geometry (ἀνάπαλιν γὰρ τρόπον τιν' ἔχουσιν τῇ γεωμετρίᾳ) because astronomy adds motion to its study while geometry removes it. The same holds for physics. While studying a snub nose physics considers it as a complex of matter, motion and change, but the science of geometry while studying the same physical lines of the snub nose, studies them *qua* lines and *qua* curved.

I can see only one possible way of how the adding of dimensions can be relevant here. The subtraction or addition of dimensions is a secondary step that happens once affections, passions, and motion have been removed from the physical body. After these are removed from the physical body, what remains is a mathematical solid. Only then in thought a mathematician can either subtract a plane from a solid or add a line to construct a plane, and further add additional planes to constitute a solid. The process of adding dimensions happens in the realm of τὰ ἐξ ἀφαιρέσεως τὰ μαθηματικά so to speak, but not in that of τὰ δὲ φυσικὰ ἐκ προσθέσεως. The expression τὰ ἐξ ἀφαιρέσεως τὰ μαθηματικά λέγεσθαι as it appears in *De Caelo* III. 1 cannot also be suggestive of extracting or abstracting of one thing out of many as most modern scholarship assumes, but it quite literally says that the objects of mathematics are said as a result of subtraction. In addition, nowhere in this passage does Aristotle say that the results of subtraction are mathematical universals, but following the passage from *Physics* II.2 it is clear that by referring to "physical bodies" which contain surfaces, volumes, points, and lines, and by referring to the mathematical spheres of the Earth, the Sun, and the moon Aristotle meant particular beings which contain particular objects of mathematics.

(iv) ἐξ ἀφαιρέσεως ὁ μαθηματικός in the *De Anima* I. 1. In this passage Aristotle distinguishes the three sciences and their methods of reaching the objects which they study, as well as the modes of being of these objects. The objects of physics exist inseparably from matter and motion both in reality and in thought, e.g. anger and fear are not separate from material substratum either in reality or in thinking. Another example is when we try to define a snub nose – there is always an aspect of matter involved, and this is why Aristotle sometimes says that it is hard to separate nose from flesh, its matter. Aristotle often says that the objects of mathematics are more separable than the objects of physics, because the former, while existing inseparably from matter and change in reality, can be separated in thought from matter, i.e. when a mathematician considers the snub nose as a concave mathematical solid. The mathematician removes various active and passive attributes which come with matter and studies the result from subtraction (ἐξ ἀφαιρέσεως) only, such as the mathematical concave solid with its planes, lines, and points. The highest science, Aristotle points out, is First Philosophy. It studies objects which exist separately from matter and motion both in reality and in thought.

> It therefore seems that all the affections of soul involve a body – passion, gentleness, fear, pity, courage, joy, loving, hating […] Hence a physicist would define an affection of soul differently from a dialectician; the latter would define e.g. anger as the appetite for returning pain for pain, or something like that, while the former would define it as a boiling of the blood or warm substance surround the heart. The latter assigns the material conditions, the former the form or formulable essence. […]

> The physicist is he who concerns himself with all the properties active and passive of bodies or materials thus or thus defined; attributes not considered as being of this character he leaves to others, in certain cases it may be to a specialist, e.g. a carpenter or a physician, in others (a) where they are inseparable in fact, but are separable from any particular kind of body by an effort of abstraction [ἐξ ἀφαιρέσεως], to the mathematician,[106] (b) where they are separate both in fact and in thought from body altogether, to the First Philosopher or metaphysician. But we must return from this digression, and repeat that the affections of soul are inseparable from the material substratum of animal life, to which we have seen that such affections, e.g. passion and fear, attach, and have not the same mode of being as a line or a plane.[107] (Smith, *de Anima.* 403a27-403b19).
>
> ἔοικε δὲ καὶ τὰ τῆς ψυχῆς πάθη πάντα εἶναι μετὰ σώματος, θυμός, πραότης, φόβος, ἔλεος, θάρσος, ἔτι χαρὰ καὶ τὸ φιλεῖν τε καὶ μισεῖν […] διαφερόντως δ' ἂν ὁρίσαιντο ὁ φυσικὸς [τε] (τὰ πάθη τῆς ψυχῆς) καὶ ὁ διαλεκτικὸς ἕκαστον αὐτῶν, οἷον ὀργὴ τί ἐστιν· ὁ μὲν γὰρ ὄρεξιν ἀντιλυπήσεως ἤ τι τοιοῦτον, ὁ δὲ ζέσιν τοῦ περὶ καρδίαν αἵματος καὶ θερμοῦ. τούτων δὲ ὁ μὲν τὴν ὕλην ἀποδίδωσιν, ὁ δὲ τὸ εἶδος καὶ τὸν λόγον. […] ἀλλ' ὁ φυσικὸς περὶ ἅπανθ' ὅσα τοῦ τοιουδὶ σώματος καὶ τῆς τοιαύτης ὕλης ἔργα καὶ πάθη, ὅσα δὲ μὴ τοιαῦτα, ἄλλος, καὶ περὶ τινῶν μὲν τεχνίτης, ἐὰν τύχῃ, οἷον τέκτων ἢ ἰατρός, τῶν δὲ μὴ χωριστῶν μέν, ᾗ δὲ μὴ τοιούτου σώματος πάθη καὶ ἐξ ἀφαιρέσεως, ὁ μαθηματικός, ᾗ δὲ κεχωρισμένα, ὁ πρῶτος φιλόσοφος; ἀλλ' ἐπανιτέον ὅθεν ὁ λόγος. ἐλέγομεν δὴ ὅτι τὰ πάθη τῆς ψυχῆς οὕτως ἀχώριστα τῆς φυσικῆς ὕλης τῶν ζῴων, ᾗ γε τοιαῦθ' ὑπάρχει <οἷα> θυμὸς καὶ φόβος, καὶ οὐχ ὥσπερ γραμμὴ καὶ ἐπίπεδον.

Even though 'τὰ δὲ φυσικὰ ἐκ προσθέσεως' is not present in the passage, its application may still be implied by its correlative 'ἐξ ἀφαιρέσεως, ὁ μαθηματικός:' the physicist studies the sensible being in its complexity together with matter, such as blood and heart (403a32) and with other active and passive properties (403b10). The mathematician, instead, subtracts them all. Furthermore, he may also subtract what the dialectician studies, i.e. all affections of the soul (anger, passion, gentleness, fear, pity, courage, joy, loving, hating). However, if he subtracts the material element first, the affections of the soul disappear together with matter.

[106] The English translation slightly distorts the original as there is no 'qua' terminology mentioned in it. Consider a more literal translation: "the mathematician studies the attributes which are on the one hand inseparable, but on the other hand he does not treat them *qua* (ᾗ) passions of such and such a body but studies them from subtraction" [my trans.].

[107] Cf. *Metaphysics* VI, 1. "For physics deals with things which exist separately but are not immovable, and some parts of mathematics deal with things which are immovable but presumably do not exist separately, but as embodied in matter; while the first science deals with things which both exist separately and are immovable." (Ross, *Meta.* 1026a13-6).

(v) τῶν ἐν ἀφαιρέσει ὄντων in the *De Anima* III. 4. In the following passage the Greek *ta mathematica* is not explicitly stated but only implied by the straight (τὸ εὐθὺ). This excerpt seems to deal with definitions. Specifically, it implies that the objects of mathematics are defined analogously to the objects of physics, i.e. in each there is a form, matter, and a compound of both: snub is a compound of curvature (form) and matter (flesh of the nose) and, likewise, a straight line is a compound of form (twoness) and matter (a continuum in one dimension).

> Again in the case of abstract objects what is straight is analogous to what is snub-nosed; for it necessarily implies a continuum as its matter: its constitutive essence is different, if we may distinguish between straightness and what is straight: let us take it to be two-ness. It must be apprehended, therefore, by a different power or by the same power in a different state. To sum up, in so far as the realities it knows are capable of being separated from their matter, so it is also with the powers of mind. (Smith, *de Anima*, 429b17-24).
>
> πάλιν δ' ἐπὶ τῶν ἐν ἀφαιρέσει ὄντων τὸ εὐθὺ ὡς τὸ σιμόν· μετὰ συνεχοῦς γάρ· τὸ δὲ τί ἦν εἶναι, εἰ ἔστιν ἕτερον τὸ εὐθεῖ εἶναι καὶ τὸ εὐθύ, ἄλλο· ἔστω γὰρ δυάς. ἑτέρῳ ἄρα ἢ ἑτέρως ἔχοντι κρίνει. ὅλως ἄρα ὡς χωριστὰ τὰ πράγματα τῆς ὕλης, οὕτω καὶ τὰ περὶ τὸν νοῦν.

Here we need to be careful not to confuse two senses of matter which appear in this passage: sensible 'matter' (e.g. bronze) which we subtract from the physical object along with other accidents in order to arrive at the extension or quantitative and continuous (shape and size), and συνεχής or the continuous as the 'intelligible matter' of the straight line, where 'twoness' is the essence of a line. The followers of "abstraction from matter" may find it puzzling that while the intellect "abstracts from matter," *ta mathematica* yet retain some kind of matter.[108] Since mathematicals depend on physical bodies, i.e. the sizes, shapes, lines, and planes are constantly changing because physical bodies are in constant flux, and consequently intelligible matter would thus also be changing. This is why intelligible matter cannot underlie

[108] I will treat this in my section on intelligible matter in section 4.3.

physical bodies, as Mueller suggests that it does.[109] I will discuss this issue in detail in section 4.3 of this chapter.

This passage in *De Anima* III. 4 is often omitted[110] in the discussions of ‘intelligible matter’ of which there are three explicit instances in the Aristotelian corpus: *Meta.* VII.10, 1036a 1-12; VII.11, 1036b 32-1037a5; VIII.6, 1045a 33-6. The reason for neglecting *De Anima* III. 4 perhaps lies in the fact that ὕλη νοητή as such is not explicitly stated there. Yet I think we have some strong evidence from other passages to assert that the continuous (συνεχής) *is* intelligible matter (ὕλη νοητή). In *Metaphysics* VIII.6 Aristotle asks:

> […] what is the cause of the unity of ‘round’ and ‘bronze’? The difficulty disappears, because the one is matter, the other form (μορφή) […] Some matter is intelligible, some perceptible, and in a formula there is always an element of matter as well as one of actuality; e.g. the circle is ‘plane figure.’ (Barnes, *Meta.* 1045a27-35).
>
> ὥστε τὸ ζητούμενόν ἐστι τί αἴτιον τοῦ ἓν εἶναι τὸ στρογγύλον καὶ τὸν χαλκόν. οὐκέτι δὴ ἀπορία φαίνεται, ὅτι τὸ μὲν ὕλη τὸ δὲ μορφή […] ἔστι δὲ τῆς ὕλης ἡ μὲν νοητὴ ἡ δ᾽ αἰσθητή, καὶ ἀεὶ τοῦ λόγου τὸ μὲν ὕλη τὸ δὲ ἐνέργειά ἐστιν, οἷον ὁ κύκλος σχῆμα ἐπίπεδον.

Thus, if ‘plane’ is an intelligible matter of a circle, in a like manner συνεχής or continuum in one dimension is an intelligible matter of the straight line.

(vi) τὰ ἐν ἀφαιρέσει λεγόμενα in the *De Anima* III. 7. This passage, though corrupted, is very similar to *De Anima* III.4. The similarity is obviously expressed in the first sentence of both excerpts: the ability of a mind to grasp *ta mathematica* by subtraction of the physical aspects of the snub nose and arriving at the hollow mathematical solid. In this passage, however, there are two new considerations which are not explicitly present in *De Anima* III.4: the objects of mathematics are separate from motion and change when the mind thinks them even though they do not exist in separation, and the mind is identical with the

[109] Ian Mueller, "Aristotle on Geometrical Objects," *Archiv für Geschichte der Philosophie* 52, no. 2 (1970): 161, 168.
[110] Cf. John Thorp’s article the "Intelligible Matter in Aristotle," *The Society for Ancient Greek Philosophy Newsletter*. 385 (2010): 1-6. https://orb.binghamton.edu/sagp/385

object of its thinking. Aristotle also asks a question whether the mind, while not existing separately from magnitude, is capable of thinking anything completely separate. This perhaps indicates that Aristotle has the unmoved prime mover in mind who exists as separate both from matter and from magnitude. It is also important to point out the place of the qua-indicator (ᾗ) in this passage. It indicates the mathematical remainder in the process of subtracting physical aspects; 'τὸ κοῖλον' is the thing existing in subtraction.

> The so-called abstract objects the mind thinks just as, if one had thought of the snub-nosed not as snub-nosed but as hollow, one would have thought of an actuality without the flesh in which it is embodied: it is thus that the mind when it is thinking the objects of Mathematics thinks as separate elements which do not exist separate. In every case the mind which is actively thinking is the objects which it thinks. Whether it is possible for it while not existing separate from spatial conditions to think anything that is separate, or not, we must consider later. (Smith, *de Anima*, 431b 13-20).
>
> τὰ δὲ ἐν ἀφαιρέσει λεγόμενα <νοεῖ> ὥσπερ, εἴ <τις> τὸ σιμὸν ᾗ μὲν σιμὸν οὔ, κεχωρισμένως δὲ ᾗ κοῖλον [εἴ τις] ἐνόει [ἐνεργείᾳ], ἄνευ τῆς σαρκὸς ἂν ἐνόει ἐν ᾗ τὸ κοῖλον-οὕτω τὰ μαθηματικά, οὐ κεχωρισμένα <ὄντα>, ὡς κεχωρισμένα νοεῖ, ὅταν νοῇ <ᾗ> ἐκεῖνα. ὅλως δὲ ὁ νοῦς ἐστιν, ὁ κατ' ἐνέργειαν, τὰ πράγματα. ἆρα δ' ἐνδέχεται τῶν κεχωρισμένων τι νοεῖν ὄντα αὐτὸν μὴ κεχωρισμένον μεγέθους, ἢ οὔ, σκεπτέον ὕστερον.

Thomas Aquinas, commenting on this passage reverses the object of subtraction. The objects of subtraction in Aristotle are motion, change, passions, and affections, whereas here Aquinas states that we abstract a mathematical form from all sensible characteristics. Aquinas seems to mean that we abstract a particular form from particular matter:

> Here he [Aristotle] proceeds in two stages: (1) he explains how we understand mathematical objects abstracted from sensible matter; and (2) he enquires whether we understand anything that is immaterial in being at 'Whether it is possible' (trans. Foster and Humphries).[111]
>
> Et circa hoc duo facit. Primo ostendit quomodo intelligit mathematica, quae a materia sensibili abstrahuntur. Secundo, inquirit utrum intelligat ea quae sunt secundum esse a materia separata, ibi, utrum autem contingat.

[111] Thomas Aquinas, *Commentary on Aristotle's De Anima*. Transl. by Kenelm Foster, O.P. and Sylvester Humphries, O.P. New Haven: Yale University Press, 1951. https://dhspriory.org/thomas/english/DeAnima.htm#37L

Later he adds that apart from abstracting the particular mathematical form from matter, we also abstract a mathematical universal along with the particular. The notion of abstraction of a mathematical or non-mathematical universal is absent in Aristotle:

> Yet in understanding them [mathematicals] we still abstract a universal from particulars, in so far as the specific nature is understood apart from the individuating principles; for these do not enter into the definition. And the mind in act is its object; for precisely in the degree that the object is or is not material, it is or is not perceived by the mind. And just because Plato overlooked this process of abstraction he was forced to conceive of mathematical objects and specific natures as existing in separation from matter; whereas Aristotle was able to explain that process by the agent intellect. (trans. Foster and Humphries).[112]
>
> abstrahit tamen circa naturalia intellectus universale a particulari simili modo, inquantum intelligit naturam speciei sine principiis individuantibus, quae non cadunt in definitione speciei. Et omnino intellectus in actu est res intellecta, quia sicut res in sui ratione habent materiam vel non habent, sic ab intellectu percipiuntur. Et quia hunc modum abstractionis Plato non consideravit, coactus fuit ponere mathematica et species separatas, loco cuius ad praedictam abstractionem faciendam Aristoteles posuit intellectum agentem.

Aquinas helpfully comments here that Plato overlooked the process of abstraction, as a result of which he was forced to posit intermediates existing separately from sensibles. Cleary, however, states the opposite. He argues that it was Aristotle who overlooked Plato's interpretation of intermediates and Forms, in a sense that for Aristotle, Plato's objects of mathematics were separated *spatially* from their physical instantiations, whereas Plato understood this separation only epistemologically (the reception of forms by the soul without matter).[113] It is difficult to say which one of these opposing views is correct. The only thing I can affirm is that Aristotle's *aphairesis* is *not* the process through which the soul receives

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[112] Ibid.
[113] For Aristotle "this separation means that Ideas are independent substances which are spatially distinct from their sensible participants. In contrast, the separation of Ideas for Plato turns out to be primarily logical and epistemological, when the relevant passages in the dialogues are scrutinised. However, the crucial point for the shape of Aristotle's own problematic is that he interprets this separation in ordinary spatial terms and thus concludes that Ideas are independent substances." John Cleary, "Aristotle's Theory of Abstraction: A Problem about the Mode of Being of Mathematical Objects." (PhD thesis. Boston University Graduate School, 1982), 143.

forms without matter. Its purpose is rather to uncover the category of quantity within any sensible body.

Earlier in his commentary to *De Anima* II. 5 and 6 Aquinas states that the universal of 'humanity' or the universal of 'man' can be also abstracted from matter.[114] Again, this is not how Aristotle uses *aphairesis* – both in *Physics* II.2, *Posterior Analytics* I.5, *De Caelo* III.1, *Metaphysics* VII.3 and XI.3, he makes it clear that this is motion, change, affections, and passions which are subtracted from the particular quantitative and continuous, not the quantitative and continuous from matter as Aquinas takes it. In addition, according to Aristotle, a universal, such as a mathematical (e.g. circle) or non-mathematical (e.g. man), is reached by induction from particulars, not by abstraction from particulars.

(vii) τά ἐν ἀφαιρέσει λεγόμενα in the *De Anima* III. 8. In this passage Aristotle states against the Platonists that neither the objects of mathematics nor the objects of physics can exist separately from sensible spatial magnitudes, and that the objects of thought (τὰ νοητά) of all these sciences are in the sensibles. This way Aristotle shows that the mode of being of the objects of mathematics which are 'spoken about in subtraction' primarily depend on sensibles. The objects of physics, such as the states and affections of sensibles (ὅσα τῶν αἰσθητῶν ἕξεις καὶ πάθη) which the physicist studies, are 'spoken about in addition.' Even if the latter expression is not in the text, its implication is still present. In addition, Aristotle says, that if any of the senses were lacking, there would be no knowledge of the sensible

[114] He states that "universal comes into being by *abstraction* from such matter [universale autem est per abstractionem ab huiusmodi materia] and all the individuating material conditions" […] Now a nature—say, human nature,—which can bethought of universally, has two modes of existence: one, material, in the matter supplied by nature; the other, immaterial, in the intellect. As in the material mode of existence it cannot be represented in a universal notion, for in that mode it is individuated by its matter; this notion only applies to it, therefore, as *abstracted* from individuating matter [abstrahitur a materia individuali]. But it cannot, as so *abstracted* [abstrahatur a materia individuali], have a real existence, as the Platonists thought; man in reality only exists (as is proved in the *Metaphysics*, Book VII) in this flesh and these bones […] Hence the mind abstracts, without any falsehood, a genus from a species [intellectus absque falsitate abstrahit genus a speciebus" (Lectio 12). In *Corpus Thomisticum. Sancti Thomae de Aquino Sentencia libri De anima, liber II*. http://www.corpusthomisticum.org/can2.html

world because the intellect is dependent on senses. The objects of mathematics, physics, and of all other sciences are like sensuous images which the mind receives without matter.

> Since according to common agreement there is nothing outside and separate in existence from sensible spatial magnitudes,[115] the objects of thought are in the sensible forms, viz. both the abstract objects and all the states and affections of sensible things. Hence (1) no one can learn or understand anything in the absence of sense, and when the mind is actively aware of anything it is necessarily aware of it along with an image; for images are like sensuous contents except in that they contain no matter. (Smith, *de Anima*, 432a3-9).
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> ἐπεὶ δὲ οὐδὲ πρᾶγμα οὐθὲν ἔστι παρὰ τὰ μεγέθη, ὡς δοκεῖ, τὰ αἰσθητὰ κεχωρισμένον, ἐν τοῖς εἴδεσι τοῖς αἰσθητοῖς τὰ νοητά ἐστι, τά τε ἐν ἀφαιρέσει λεγόμενα καὶ ὅσα τῶν αἰσθητῶν ἕξεις καὶ πάθη. καὶ διὰ τοῦτο οὔτε μὴ αἰσθανόμενος μηθὲν οὐθὲν ἂν μάθοι οὐδὲ ξυνείη, ὅταν τε θεωρῇ, ἀνάγκη ἅμα φάντασμά τι θεωρεῖν· τὰ γὰρ φαντάσματα ὥσπερ αἰσθήματά ἐστι, πλὴν ἄνευ ὕλης.

If we apply this passage from *de Anima* III.8 to the *Metaphysics* XI. 3 passage which I analyze below, we may draw the following connection. The quantitative and continuous (*Metaphysics* XI. 3) are the things that are spoken about in subtraction (*de Anima* III.8) from all the sensible qualities (*Metaphysics* XI. 3). Contrary to the epistemological interpretations of *aphairesis* as an immediate process of reception of mathematical forms or images (φάντασμά) by the soul which this passage might suggest, I do not see such application of the term both in this passage and in all other places where the term comes into sight. Again, I do agree that in the *de Anima* Aristotle does describe the process of reception of forms by the passive intellect without matter (*de Anima* III.4). I, however, refuse to accept that *aphairesis* describes any such abstracting process. In this respect I agree with Cleary.[116] He also proposes that both ἐξ ἀφαιρέσεως expression and ἀφαίρεσις should be interpreted primarily as the

[115] This does not deny the existence of immaterial beings studied by First Philosophy, such as both the sensible substance which is not predicated of anything and of which everything else is predicated, and eternal substance or Prime Mover which exists outside from sensible spatial magnitudes. By 'nothing outside and separate in existence' Aristotle means Plato's Forms and Form numbers.

[116] John Cleary, "On the Terminology of "Abstraction" in Aristotle," *Phronesis* 30 (1985): 14, 34, 36, 41.

logical methods of finding the primary subject of any given attribute.[117] While the latter application of the term is present in Aristotle, I think the main purpose of the terminology is rather that of *uncovering* and *elucidating* the *spatial location* of the sensible magnitude, which is in all respects sensible and does not exist "outside and separate" (*de Anima*, 432a3) in the manner of Plato's Forms and Form numbers. Aristotle uses both ἐξ ἀφαιρέσεως and ἐκ προσθέσεως as the tools for breaking down the sensible object into its respective categories; these correlative methods show how one sensible object can be shared by various sciences, such as mathematics and physics in particular.

(viii) τὰ ἐξ ἀφαιρέσεως in the Metaphysics XI. 3. This passage explicitly supports the following points I have been arguing for throughout my work. First, *aphairesis* is a simple successive method of subtracting items from an object in question such as weight and lightness, hardness and softness, heat and cold, motion and change,[118] upon whose removal we leave the remainder for mathematical study – the quantitative and continuous[119] in one, in two, or in three dimensions. Therefore, this does not presuppose an extractionist sense of taking out one of many. The fact that the term works as a simple method of removing things successively is supported by the mathematical language of τὰ ἐξ ἀφαιρέσεως, περιελὼν, and καταλείπει. This simple method of *aphairesis* is similar to what we have seen in the *Topics, Sophistical Refutations*, and *Physics*. Second, *aphairesis* is not an abstraction of a particular mathematical form from matter or an abstraction of a mathematical universal from particulars; the purpose of using this terminology is rather that of *uncovering* and *elucidating* the *spatial*

[117] Ibid., 32, 33, 39.
[118] Weight and lightness, hardness and its contrary, and also heat and cold and the other sensible contrarieties (βάρος καὶ κουφότητα καὶ σκληρότητα καὶ τοὐναντίον, ἔτι δὲ καὶ θερμότητα καὶ ψυχρότητα καὶ τὰς ἄλλας αἰσθητὰς ἐναντιώσεις) in *Metaphysics* XI.3 correspond to affections, products, and potencies of bodies (τῶν σωμάτων πάθη καὶ ποιήματα καὶ δυνάμεις) in *Metaphysics* VII.3.
[119] The remainder, such as the quantitative, the continuous (τὸ ποσὸν καὶ συνεχές) and dimensions (τῶν μὲν ἐφ᾽ ἓν τῶν δ᾽ ἐπὶ δύο τῶν δ᾽ ἐπὶ τρία) in *Metaphysics* XI.3 correspond to length, breadth, and depth (τὸ μῆκος καὶ πλάτος καὶ βάθος) in *Metaphysics* VII.3.

location of the particular sensible magnitude. In addition, the qua-operator denotes the results of subtraction, or the remainder in the process of subtracting the items, thus the terms are closely connected; this justifies why *aphairesis* is completely absent in *Metaphysics* XIII.3 where Aristotle gives his positive account.

> As the mathematician investigates abstractions[τὰ ἐξ ἀφαιρέσεως] (for before beginning his investigation he strips off [περιελών] all the sensible qualities, e.g. weight and lightness, hardness and its contrary, and also heat and cold and the other sensible contrarieties, and leaves [καταλείπει] only the quantitative and continuous, sometimes in one, sometimes in two, sometimes in three dimensions, and the attributes of these qua quantitative and continuous [ᾗ ποσά ἐστι καὶ συνεχῆ], and does not consider them in any other respect, and examines the relative positions of some and the attributes of these, and the commensurabilities and incommensurabilities of others, and the ratios of others; but yet we posit one and the same science of all these things – geometry) – the same is true with regard to being. For the attributes of this in so far as it is being, and the contrarieties in it qua being, it is the business of no other science than philosophy to investigate; for to physics one would assign the study of things not qua being, but rather qua sharing in movement; while dialectic and sophistic deal with the attributes of things that are, but not of things qua being, and not with being itself in so far as it is being; therefore it remains that it is the philosopher who studies the things we have named, in so far as they are being. (Ross, *Meta*. 1061a29-1061b12).
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> καθάπερ δ᾽ ὁ μαθηματικὸς περὶ τὰ ἐξ ἀφαιρέσεως τὴν θεωρίαν ποιεῖται (περιελὼν γὰρ πάντα τὰ αἰσθητὰ θεωρεῖ, οἷον βάρος καὶ κουφότητα καὶ σκληρότητα καὶ τοὐναντίον, ἔτι δὲ καὶ θερμότητα καὶ ψυχρότητα καὶ τὰς ἄλλας αἰσθητὰς ἐναντιώσεις, μόνον δὲ καταλείπει τὸ ποσὸν καὶ συνεχές, τῶν μὲν ἐφ᾽ ἓν τῶν δ᾽ ἐπὶ δύο τῶν δ᾽ ἐπὶ τρία, καὶ τὰ πάθη τὰ τούτων ᾗ ποσά ἐστι καὶ συνεχῆ, καὶ οὐ καθ᾽ ἕτερόν τι θεωρεῖ, καὶ τῶν μὲν τὰς πρὸς ἄλληλα θέσεις σκοπεῖ καὶ τὰ ταύταις ὑπάρχοντα, τῶν δὲ τὰς συμμετρίας καὶ ἀσυμμετρίας, τῶν δὲ τοὺς λόγους, ἀλλ᾽ ὅμως μίαν πάντων καὶ τὴν αὐτὴν τίθεμεν ἐπιστήμην τὴν γεωμετρικήν), τὸν αὐτὸν δὴ τρόπον ἔχει καὶ περὶ τὸ ὄν. τὰ γὰρ τούτῳ συμβεβηκότα καθ᾽ ὅσον ἐστὶν ὄν, καὶ τὰς ἐναντιώσεις αὐτοῦ ᾗ ὄν, οὐκ ἄλλης ἐπιστήμης ἢ φιλοσοφίας θεωρῆσαι. τῇ φυσικῇ μὲν γὰρ οὐχ ᾗ ὄντα, μᾶλλον δ᾽ ᾗ κινήσεως μετέχει, τὴν θεωρίαν τις ἀπονείμειεν ἄν: ἥ γε μὴν διαλεκτικὴ καὶ ἡ σοφιστικὴ τῶν συμβεβηκότων μέν εἰσι τοῖς οὖσιν, οὐχ ᾗ δ᾽ ὄντα οὐδὲ περὶ τὸ ὂν αὐτὸ καθ᾽ ὅσον ὄν ἐστιν: ὥστε λείπεται τὸν φιλοσόφον, καθ᾽ ὅσον ὄντ᾽ ἐστίν, εἶναι περὶ τὰ λεχθέντα θεωρητικόν.

In this passage Aristotle also contrasts three other sciences with the science of mathematics. While mathematics removes motion and change, the science of physics that treats of things from addition studies physical beings in their complexity together with motion and change. The science of metaphysics, in turn, removes both motion, change, and sensible special magnitudes and studies being only in so far as it is being or studies the Prime Mover. The science of dialectic removes from its consideration everything that mathematics, physics, and philosophy consider as their objects, and studies what is peculiar to it, such as logic, rhetoric, syllogisms, and reasoning in general.

We should not follow Ross in his translation of τὰ ἐξ ἀφαιρέσεως as 'abstractions.' First, this implies the idea of extracting, and secondly, the English word itself is no longer free from modern connotations of something being devoid of any content which makes it lie wide open to modern anti-abstractionist criticisms. Thomas Aquinas this time faultlessly captures the meaning of τὰ ἐξ ἀφαιρέσεως in ὁ μαθηματικὸς περὶ τὰ ἐξ ἀφαιρέσεως τὴν θεωρίαν ποιεῖται in *Metaphysics* XI. 3. He states: Aristotle "dicens quod sicut mathematica habet considerationem circa ea quae sunt ex ablatione."[120] John P. Rowan, translates Aquinas' statement in a very plausible Aristotelian way: "now the mathematician in a sense studies things which are gotten by taking something away."[121]

Philippe, analyzing the process of subtraction of sensible aspects concludes that this and all other uses of *aphairesis* are insufficient to attain mathematicals: "Remarquons bien que dans tous ces textes, le verbe est employé par Aristote pour désigner certaines opérations de l'intelligence qui ne sont pas du tout réservées au domaine des mathématiques."[122] He then adds that "Aristote, de fait, ne l'emploie pas pour exprimer l'acte par lequel on saisit les êtres mathématiques."[123] After he rejects *aphairesis* as the only method of reaching particular

[120] Thomas Aquinas, *Commentary on the Metaphysics,* Transl. by John P. Rowan. Chicago, 1961. https://dhspriory.org/thomas/Metaphysics11.htm
[121] Ibid.
[122] Philippe, M. D., "Aphairesis, prosthesis, chorizein dans la philosophie d' Aristote." In: *Revue thomiste 48* (1948): 466.
[123] Ibid.

mathematicals, he proposes that this method on its own does not isolate the separated mathematical. He claims that the second step we need to perform is isolation or extraction. [124]

> Ceci nous montre bien le caractère propre de cette abstraction mathématique: il ne s'agit pas seulement de ne pas considérer les notes individuantes et singulières de telle ou telle réalité physique, d'abandonner ses qualités physiques immédiatement sensibles, mais il faut de plus « formaliser » un aspect de ces êtres quantifiés. Autrement dit il ne faut pas les envisager formellement comme des êtres quantifiés, sujets de certaines propriétés, mais il faut isoler par la pensée cette quantité et la considérer dans son intelligibilité propre, en saisir la structure essentielle et ses diverses propriétés […] Elles [propriétés des êtres mathématiques] sont «abstraites»: elles n'existent que dans et par tel acte d'intellection qui leur donne une certaine forme en les isolant du monde physique et en les laissant dans le monde du continu."[125]

I do not agree with this interpretation. The act of successive removal of the sensible aspects brings us to the same remainder which the qua-method of isolation points to. In fact, he does not see *aphairesis* to be sufficient because its function is, indeed, not an isolation or extraction but a successive removal of physical aspects used by Aristotle to demonstrate the location of the quantitative and continuous. However, it is difficult to see why *aphairesis* alone cannot be sufficient, because whenever we subtract passions, affections, motion, and change from a physical being, the quantitative and continuous remainder also, in a sense, becomes isolated in thinking. Isn't it the case that applying the qua-method after the process of subtraction is done will be superfluous to it? When we successively subtract, we come to the knowledge of the sensible magnitude existing in an object which then becomes intelligible matter in thinking.

(ix) τὰ δι᾽ ἀφαιρέσεώς ἐστιν in the Nicomachean Ethics, VI. 8. In this passage at 1142a11 ff. Aristotle explains why young men experience no difficulty in understanding

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[124] Philippe does not explain how exactly he understands isolation in this context. It seems right to affirm that the role of isolation he attributes to 'en tant que,' though he does not say it explicitly in the body of his analysis and does not give any treatment of it. He only cites a few passages of Aristotle's uses of 'en tant que' in the footnotes. Interestingly, he seems to link this method of 'en tant que' to 'extraction:' "L'abstraction peut, en effet, jouer le rôle de cause propre et première à l'égard des êtres mathématiques, en ce sens que ces êtres ne peuvent exister que «de» , «par» et «dans» cette abstraction. Celle-ci ne les crée pas, ne les invente pas, au sens fort, mais elle leur donne leur propre mode d'être. Elle les extrait pour ainsi dire du monde physique où ils se trouvaient comme cachés et enveloppés; elle les libère." (p. 476).

[125] Philippe, M. D., "Aphairesis, prosthesis, chorizein dans la philosophie d' Aristote." In: *Revue thomiste 48* (1948): 464.

geometrical and mathematical truths. He clarifies that to understand the objects that the wise man studies, such as the principles of physics, philosophy, ethics, and politics, a young mathematician would need experience (ἐμπειρία), which he obviously lacks. Mathematics and geometry, on the other hand, need no experience – they only require the ability of logical and analytical thinking, i.e. the ability to use the method of subtraction (δι᾽ ἀφαιρέσεώς) through which a young mind can arrive at the quantitative and continuous aspect of a thing.

> What has been said is confirmed by the fact that while young men become geometricians and mathematicians and wise in matters like these, it is thought that a young man of practical wisdom cannot be found. The cause is that such wisdom is concerned not only with universals but with particulars, which become familiar from experience, but a young man has no experience, for it is length of time that gives experience; indeed one might ask this question too, why a boy may become a mathematician, but not a philosopher or a physicist. It is because the objects of mathematics exist by abstraction, while the first principles of these other subjects come from experience, and because young men have no conviction about the latter but merely use the proper language, while the essence of mathematical objects is plain enough to them? (Ross, *EN.* 1142a12-20).
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> σημεῖον δ᾽ ἐστὶ τοῦ εἰρημένου καὶ διότι γεωμετρικοὶ μὲν νέοι καὶ μαθηματικοὶ γίνονται καὶ σοφοὶ τὰ τοιαῦτα, φρόνιμος δ᾽ οὐ δοκεῖ γίνεσθαι. αἴτιον δ᾽ ὅτι καὶ τῶν καθ᾽ ἕκαστά ἐστιν ἡ φρόνησις, ἃ γίνεται γνώριμα ἐξ ἐμπειρίας, νέος δ᾽ ἔμπειρος οὐκ ἔστιν: πλῆθος γὰρ χρόνου ποιεῖ τὴν ἐμπειρίαν: ἐπεὶ καὶ τοῦτ᾽ ἄν τις σκέψαιτο, διὰ τί δὴ μαθηματικὸς μὲν παῖς γένοιτ᾽ ἄν, σοφὸς δ᾽ ἢ φυσικὸς οὔ. ἢ ὅτι τὰ μὲν δι᾽ ἀφαιρέσεώς ἐστιν, τῶν δ᾽ αἱ ἀρχαὶ ἐξ ἐμπειρίας: καὶ τὰ μὲν οὐ πιστεύουσιν οἱ νέοι ἀλλὰ λέγουσιν, τῶν δὲ τὸ τί ἐστιν οὐκ ἄδηλον;

Knowing the objects of philosophy besides mathematical objects, such as the sensible substances which exist without qualification, divine separate substance or the prime mover, being *qua* being, – all of these, require experience. Since both sensible substances and the divine unmoved mover are separable in themselves, it would be hard for a young mathematician to perceive something which is separable both from matter and from sensible spatial magnitude. While the results of mathematical separation, such as mathematical squares, spheres, triangles, and all other shapes are immediately present in the mind of a young pupil, the metaphysical separation would require a lot of effort. In a like manner, it is difficult for the young to correctly explain any physical or astronomical phenomena or theory because it requires long years of observation and physical interaction with the world to grasp the principles and causes of things and to collect the necessary data. What is more, the data or evidence collected must be true, not false. The same holds for the science of politics. Aristotle says in the *Nicomachean Ethics* I.3, that “a young man is not a proper hearer of lectures on political science [and science of ethics[126]]; for he is inexperienced in the actions that occur in life” (1095a3-5). To know what is good for the polis, one needs to be experienced in relationships and understand the principles (ἀρχαὶ) of the human good. In terms of the ‘the things that exist by subtraction’ in ‘ἢ ὅτι τὰ μὲν δι᾽ ἀφαιρέσεώς ἐστιν, τῶν δ᾽ αἱ ἀρχαὶ ἐξ ἐμπειρίας,’ Aristotle may also mean the *principles* (ἀρχαὶ) which generate the results of mathematical subtraction, but not the results of subtraction themselves. Specifically, a young mathematician is not only able to think of any physical objects as mathematical solids, but also comprehend their principles such as planes, lines, and points which are gained by the further process of subtraction from which solids are generated in thinking. This is why the essence of the objects of mathematics is plain to the youth (*EN.* 1142a20).

4.2.2 *Aphairesis* and the ‘Qua’ Method in Mathematics

I have already mentioned in my section on *aphairesis* outside mathematics that in *Metaphysics* VII.3 Aristotle highlights the term as a simple successive method of subtracting the aspects within a particular physical object so as to locate its substance. Upon the removal

[126] John Beare in his article “The Meaning of Aristotle, ‘Nicomachean Ethics’” states that “ἡ πολιτική is employed by Aristotle in three ways—(a) to embrace Ethics and Politics (the theory of individual conduct, and that of the conduct of men in masses), without distinction; (b) to designate Ethics proper; (f) to stand for Politics proper. In the sentence with which we are here most concerned, it refers primarily to Ethics, but, of course, not exclusively” in John Beare, I. "The Meaning of Aristotle, ‘Nicomachean Ethics’", 1095 a. 2." *Hermathena* 12, no. 28 (1902): 40-43.

of all determinations, what remains (ὑπολειπόμενον) is substance or matter – that which is not predicated of anything, but of which everything else is predicated. Later Aristotle abandons the idea that matter is substance and proposes that substance is essence (τὸ τί ἦν εἶναι). Apart from locating the substance, here Aristotle also uses *aphairesis* in order to arrive at the three-dimensional extension such as length (line), breadth (plane), and depth (solid). Thus, I also include this same passage in my section on *aphairesis* within mathematics. Here is how Aristotle unfolds the objects of mathematics:

> We have now outlined the nature of substance, showing that it is that which is not predicated of a stratum, but of which all else is predicated. But we must not merely state the matter thus; for this is not enough. The statement itself is obscure, and further, on this view, matter becomes substance. For if this is not substance, it baffles us to say what else is. When all else is stripped off evidently nothing but matter remains. For while the rest are affections, products, and potencies of bodies, length, breadth, and depth are quantities and not substances (for a quantity is not a substance), but the substance is rather that to which these belong primarily. But when length and breadth and depth are taken away we see nothing left unless there is something that is bounded by these; so that to those who consider the question thus matter alone must seem to be substance. (Ross, *Meta*. 1029a 8-19).
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> νῦν μὲν οὖν τύπῳ εἴρηται τί ποτ᾽ ἐστὶν ἡ οὐσία, ὅτι τὸ μὴ καθ᾽ ὑποκειμένου ἀλλὰ καθ᾽ οὗ τὰ ἄλλα: δεῖ δὲ μὴ μόνον οὕτως: οὐ γὰρ ἱκανόν: αὐτὸ γὰρ τοῦτο ἄδηλον, καὶ ἔτι ἡ ὕλη οὐσία γίγνεται. εἰ γὰρ μὴ αὕτη οὐσία, τίς ἐστιν ἄλλη διαφεύγει: περιαιρουμένων γὰρ τῶν ἄλλων οὐ φαίνεται οὐδὲν ὑπομένον: τὰ μὲν γὰρ ἄλλα τῶν σωμάτων πάθη καὶ ποιήματα καὶ δυνάμεις, τὸ δὲ μῆκος καὶ πλάτος καὶ βάθος ποσότη τές τινες ἀλλ᾽οὐκ οὐσίαι (τὸ γὰρ ποσὸν οὐκ οὐσία), ἀλλὰ μᾶλλον ᾧ ὑπάρχει ταῦτα πρώτῳ, ἐκεῖνό ἐστιν οὐσία. ἀλλὰ μὴν ἀφαιρουμένου μήκους καὶ πλάτους καὶ βάθους οὐδὲν ὁρῶμεν ὑπολειπόμενον, πλὴν εἴ τί ἐστι τὸ ὁριζόμενον ὑπὸ τούτων, ὥστε τὴν ὕλην ἀνάγκη φαίνεσθαι μόνην οὐσίαν οὕτω σκοπουμένοις.

In order to attain this extension Aristotle does not say that we must abstract mathematical extension from matter, but he rather uses a successive method of removing affections, products, and potencies of bodies within the physical object until he arrives at length, breadth, and depth. In addition, in this passage we find not just the removal of affections and potencies of bodies, but also a successive removal of dimensions themselves: “But when length, breadth, and depth are taken away we see nothing unless there is something bounded by these" (1029a14-15). Starting with the more simple dimension, such as length or line, Aristotle gradually removes the more complex ones – breadth which corresponds to plane and depth which correlates to a solid.

In *Posterior Analytics* I.5, *aphairesis* plays a key role in subtracting the subjects to which a property does not belong universally. In this excerpt Aristotle claims that in order to find what the primary subject of any given attribute is, we need to apply the process of subtraction to those subjects we consider unnecessary. For example, to find what is the primary subject to which the attribute 'the sum of internal angles equal to two right angles' belongs universally, we need to eliminate or subtract 'bronze' and 'isosceles' from 'bronze isosceles triangle,' and leave triangle as the primary subject. If one is to prove that this attribute belongs to isosceles triangle, one would be in error if one thought that one has given a universal demonstration. In this case a universal demonstration can be properly given if isosceles and triangle were the same in essence. Therefore, Aristotle points out that one should ask whether this attribute belongs to bronze isosceles triangle *qua* (ᾗ) triangle or *qua* (ᾗ) isosceles.

> We must ask "Does the property belong to its subject qua triangle [ᾗ τρίγωνον] or qua isosceles [ᾗ ἰσοσκελὲς]? When does it apply to its subject primarily? What is the subject of which it can be demonstrated universally?" Clearly the first subject to which it applies as the differentiae are removed [ἀφαιρουμένων]. E.g., the property of having angles equal to the sum of two right angles will apply to "bronze isosceles triangle"; and it will still apply when "bronze" and "isosceles" are removed [ἀφαιρεθέντος]. "But not if you remove 'figure' or 'limit'." No, but these are not the first differentiae whose removal makes the attribute inapplicable. "Then what is the first?" If it is "triangle," then it is with respect to triangularity that the attribute applies to all the rest of the subjects, and it is of "triangle" that the attribute can be universally demonstrated. (Tredennik, *APo,* 74a35-b4).
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> πότερον δ' ᾗ τρίγωνον ἢ ᾗ ἰσοσκελὲς ὑπάρχει; καὶ πότε κατὰ τοῦθ' ὑπάρχει πρῶτον; καὶ καθόλου τίνος ἡ ἀπόδειξις; δῆλον ὅτι ὅταν ἀφαιρουμένων ὑπάρχῃ πρώτῳ. οἷον τῷ ἰσοσκελεῖ χαλκῷ τριγώνῳ ὑπάρξουσι δύο ὀρθαί, ἀλλὰ καὶ τοῦ χαλκοῦν εἶναι ἀφαιρεθέντος καὶ τοῦ ἰσοσκελές. ἀλλ' οὐ τοῦ σχήματος ἢ πέρατος. ἀλλ' οὐ πρώτων. τίνος οὖν πρώτου; εἰ δὴ τριγώνου, κατὰ τοῦτο ὑπάρχει καὶ τοῖς ἄλλοις, καὶ τούτου καθόλου ἐστὶν ἡ ἀπόδειξις.

Referring to *APo*, 74a35-b4 cited above, Cleary claims that "here subtraction is a logical method of isolating the primary subject to which certain attributes belong per se and this subject is indicated by the 'qua' locution."[127] Indeed, *aphairesis* in this passage does not have a function of extracting or abstracting one out of many. It does not say that we abstract a triangle. It rather shows itself as a simple process of step-by-step removal of many such as 'bronze' and 'isosceles' from one physical object and leaving the aspect of triangularity for the study.

Throughout his corpus, Aristotle applies the qua-operator to the objects of mathematics in a similar way as he does for all other sciences: the mode of being of mathematicals in sensible things is the same as, for example, instance the mode of being of a 'female' in this particular physical body – both 'quantitative,' 'continuous,' and 'female' are not separate; and they really exist in a physical body though not as an independent entity; the difference is that *ta mathematica* are more separate in thought than the 'female.' While the result of mathematical subtraction in τὰ ἐξ ἀφαιρέσεως, such as the continuous plane or line can be designated by qua-locutor, i.e. when we consider a man *qua* solid, it does not mean that any non-mathematical qua-isolated remainder such as 'pale' or 'being' may be considered as the 'τὰ ἐξ ἀφαιρέσεως' because the seven out of nine uses of this expression point out to the objects of mathematics. While *aphairesis* showed itself as a process of the successive removal of many and leaving one for an investigation, it remains unclear whether it is right to link the qua-method with the extraction of one out of many. It seems that when we consider a man *qua* solid, we do perform an act of extraction of one thing while the rest is all at once disregarded. It is difficult to conclude whether this is the case or not.

[127] John Cleary, "Aristotle's Theory of Abstraction: A Problem about the Mode of Being of Mathematical Objects," (PhD thesis. Boston University Graduate School, 1982): 577.

4.3 Potential Existence of Intelligible Matter in Sensible Substances

Metaphysics XIII. 1 opens by setting out the objective of determining how exactly the objects of mathematics exist. Here Aristotle proposes four different views of their existence, three of which end up being rejected. The first view which states that mathematicals exist in sensible objects Aristotle rejects since it is impossible for two solids to exist in the same place (*Meta.* 1076b3); this view is about local inherence of the objects of mathematics in sensible things (Pythagoreans and Speusippus). However, in *Physics* II Aristotle says that "physical bodies contain surfaces and volumes, lines and points, and these are the subject-matter of mathematics" (193b23-25). This might suggest that Aristotle contradicts himself since he also holds that the objects of mathematics exist in sensible things. It is important to keep in mind that Pythagoreans and Speusippus, according to Aristotle, held that the objects of mathematics are substances. Aristotle does not accept that two substances can exist in the same place. The second, Platonist view which holds that mathematicals are substances and exist separately from sensible things is also rejected because substances cannot exist outside sensible bodies.[128] Apart from existing separately, Platonists taught about not just ontological priority of mathematicals over sensibles, but also of ontological priority of less complex to more complex mathematicals, e.g. points are prior to lines, and lines to planes. Such a schema of priority of one substance over others is the result of confusing physical reality with how things are usually defined. Specifically, since it is impossible to define a square without its four constituting lines, Platonists thought that lines were thus prior in reality (μᾶλλον), though they were only prior in definition (πρότερα τῷ λόγῳ), not in reality.[129] Aristotle also rejects

[128] Also discussed and rejected in *Metaphysics* III. 1 and 5, in V. 8, VIII. 2, XI. 2, and in XIV.3.
[129] Not everything that is prior in definition is prior in reality (*Metaphysics* XIII. 2, 1077a36-b11), however, sometimes priority in substance and priority in definition can coincide, when for instance the primary substance

such ontological separation based on the absurd accumulation of the objects of mathematics.[130] Beyond this primary objection to Platonic separation, Aristotle also argues that Form numbers cannot be the causes of things (*Meta.* 991bff., 1079b15, 1092b23-25, 1093b11) and that Form numbers suggest a swarm of substances (*smēnos ousiōn*),[131] e.g. if 4 is the Form of horse, man will be a part of horse if his Form is 2, and so it will follow that one substance will contain many others.[132] The objects of mathematics are not substances for Aristotle, and they are not separate ontologically. They are features of sensible objects. In *Metaphysics* XIII. 1 Aristotle also mentions the third view that mathematicals "do not exist" though he does not provide any further discussion since the complete non-existence of mathematical beings is not an option he seriously entertains. Finally, he states the fourth view, i.e. that they exist in some special sense (ἄλλον τρόπον εἰσίν):

> If the objects of mathematics exist, they must exist either in sensible objects, as some say,[133] or separate from sensible objects (and this also is said by some)[134]; or if they exist in neither of these ways, either they do not exist, or they exist only in some special sense. So that the subject of our discussion will be not whether they exist but how they exist. (Ross, *Meta.* 1076a 32-40).
>
> ἀνάγκη δ᾽, εἴπερ ἔστι τὰ μαθηματικά, ἢ ἐν τοῖς αἰσθητοῖς εἶναι αὐτὰ καθάπερ λέγουσί τινες, ἢ κεχωρισμένα τῶν αἰσθητῶν (λέγουσι δὲ καὶ οὕτω τινές): ἢ εἰ μη δετέρως, ἢ οὐκ εἰσὶν ἢ ἄλλον τρόπον εἰσίν: ὥσθ᾽ ἡ ἀμφισβήτησις ἡμῖν ἔσται οὐ περὶ τοῦ εἶναι ἀλλὰ περὶ τοῦ τρόπου.

is defined (*Metaphysics* VII.1 1028a10-35). Priority in substance vs priority in definition is also discussed in *Meta*. VII.4 at 1030b24-6, V.5 at 1018b34-7, and in XIII.2 at 1077b1-10.

[130] In *Metaphysics* XIII. II, 1076b30-34 Aristotle explains this absurd accumulation in the following way: "we find ourselves with one set of solids apart from the sensible solids; three sets of planes apart from the sensible planes—those which exist apart from the sensible planes, and those in the mathematical solids, and those which exist apart from those in the mathematical solids; four sets of lines, and five sets of points." This makes Aristotle to wonder: "with which of these, then, will the mathematical sciences deal?" (*Meta*. XIII, II 1076b 35). The same argument holds for arithmetics – there will be ideal mathematical number over sensible number. An ideal mathematical number consists of ideal units, therefore one needs to posit more units over and above to explain the previous set of units (Meta. XIII, II 1076b35-40). Please see Emily Katz' article "An Absurd Accumulation: "Metaphysics" M.2, 1076b11-36" p. 352. There she provides a clear account of the argument.

[131] Proposed by Alexander of Aphrodisias at 524.31 in his commentary to Aristotle's *Metaphysics.*

[132] I will not be discussing these two and other arguments as it goes beyond the scope of my work. For a full treatment of arguments please refer to *Metaphysics* XIII.4-10, and XIV 1-6.

[133] Pythagoreans and Speusippus

[134] Plato

Later in *Metaphysics* XIII.3 Aristotle explains that the objects of mathematics do not exist without qualification, or *simply*, i.e. in the manner of substance. Only substance alone exists without qualification. Later in the chapter he points out that mathematicals exist in the same way as 'mobile,' 'female,' 'sight' and 'voice' exist. Mathematical sciences study sensible objects too, though not *qua* sensible, but only *qua* certain possessed features: *qua* solid, *qua* planes, *qua* lines, *qua* divisibles, or *qua* indivisibles (*Meta.* 1078b 28-30):

> It has, then, been sufficiently pointed out that the objects of mathematics are not substances in a higher degree than bodies are, and that they are not prior to sensibles in being, but only in definition, and that they cannot exist somewhere apart. But since it was not possible for them to exist in sensibles either, it is plain that they either do not exist at all or exist in a special sense [ἢ τρόπον τινὰ] and therefore do not 'exist' without qualification. (Ross, *Meta.* 1077b 10-16).
>
> ὅτι μὲν οὖν οὔτε οὐσίαι μᾶλλον τῶν σωμάτων εἰσὶν οὔτε πρότερα τῷ εἶναι τῶν αἰσθητῶν ἀλλὰ τῷ λόγῳ μόνον, οὔτε κεχωρισμέν απου εἶναι δυνατόν, εἴρηται ἱκανῶς: ἐπεὶ δ᾽ οὐδ᾽ ἐν τοῖς αἰσθητοῖς ἐνεδέχετο αὐτὰ εἶναι, φανερὸν ὅτι ἢ ὅλως οὐκ ἔστιν ἢ τρόπον τινὰ ἔστι καὶ διὰ τοῦτο οὐχ ἁπλῶς ἔστιν: πολλαχῶς γὰρ τὸ εἶναι λέγομεν.

The meaning of 'ἢ τρόπον τινὰ' expression is not immediately clear. How exactly are we to understand Aristotle's statement that mathematicals exist 'in a special sense'? The two ways of existence which I think are implied by Aristotle's 'τρόπον τινὰ ἔστι' in 1077b16 is existence in the form of 'intelligible matter' or 'ὕλη νοητή'[135] (*Meta.* VII.10, 1036a 1-12; VII.11, 1036b 32-1037a5; VIII.6, 1045a 33-6; XI.1, 1059b 14-16[136]; *De Anima* III.4, 429b 17-24) and existence in the form of potentiality in physical matter and in actuality in thought

[135] By the existence in the form of intelligible matter I mean that the subtracted from matter the objects of mathematics *have* both the intelligible matter and the form, e.g. a circle is 'a plane figure' where 'circle' is the separated mathematical object, 'plane' is its matter and 'figure' is the form. Aristotle does not say that mathematical objects *are* intelligible matter, instead, they *have* intelligible matter. I will discuss it more extensively on the following pages of section 4.3 of my thesis.
[136] I do not include this passage in my general treatment of intelligible matter because the concept of ὕλη νοητή as such is absent there. Here Aristotle raises a question to what kind of science does the discussion of the matter of the objects of mathematics [περὶ τῆς τῶν μαθηματικῶν ὕλης] belongs to? He replies that it neither belongs to physics because it has a principle of motion and rest nor to science which deals with demonstrations (perhaps here he means the science of dialectic). He concludes that is it is the science of philosophy that deals with these objects. It is however unclear what kind of mathematical matter Aristotle meant it to be. Perhaps it is intelligible matter.

(*Meta*. IX.9, 1051 a21-33). How do we then put these two views together with the third, i.e. that the objects of mathematics are physical, quantitative, and continuous extension (*Meta.* XI. 3, 1061a 29-35)? In addition, how does *aphairesis* fit in this picture? It is important to address all these views in conjunction if one is to get a complete picture of the mode of being of the objects of mathematics and the method of their reception by the mind. John Thorp, for instance, while considering the concept of intelligible matter and the method of *aphairesis,* does not explain how potency and actuality can be compatible with the two former views.[137] Allan Bäck, although he gives an account of intelligible matter and *aphairesis,* does not mention the concepts of potentiality and actuality at all.[138] Michael White, in turn, treats the concepts of potentiality, actuality, and *aphairesis*, but does not examine the concept of intelligible matter.[139] Since this is the case, I see the reconciliation of these views into one picture in the following way: while 'existing' potentially as physical extension and being unfolded by the successive method of subtraction, mathematicals 'exist' actually in thought as a compound of intelligible matter and form. I will now bring together the passages to prove my point.

Existing potentially as physical extension and being unfolded by successive method of subtraction

Aristotle in *Physics* II.2 states that physical bodies contain surfaces and volumes, lines and points. These are in all respects physical surfaces, physical lines, and physical volumes. In *Metaphysics* VI.1 he also indicates that the objects of mathematics exist as embodied in matter, ὡς ἐν ὕλῃ (1026a13-6). Further, in the middle of his positive account in *Metaphysics*

[137] John Thorp, "Intelligible Matter in Aristotle," The Society for Ancient Greek Philosophy Newsletter. 385. 2010. https://orb.binghamton.edu/sagp/385
[138] Allan Bäck, "The Concept of Abstraction," The Society for Ancient Greek Philosophy Newsletter. 376. 2006. https://orb.binghamton.edu/sagp/376
[139] Michael J White, "The Metaphysical Location of Aristotle's Μαθηματικά." *Phronesis* (1993): 166-182.

XIII. 3 Aristotle says that the objects of mathematics exist in the way of matter (ὑλικῶς): "Thus, then, geometers speak correctly; they talk about existing things, and their subjects do exist; for being has two forms – it exists not only in complete reality [ἐντλεχείᾳ] but also materially [ὑλικῶς]" (Ross, *Meta.* 1078a 29031). This seems to suggest that the objects of geometry exist in the way matter does. Since Aristotle associates matter with potentiality (*Meta.* 1045a22) and since lines and surfaces are embodied in matter, it follows accordingly that quantitative and continuous extension exists potentially. As potentially existing extension cannot meet the requirements of mathematics, namely that the size of constantly changing matter alternates the size of the continuous, by stripping off sensible affections and change and merely considering the objects only *qua* quantitative and continuous (*Phys.* 193a 35) free from motion, the mathematician commits no error when his objects of mathematics become actual in thought. This means he can study any animated physical being *qua* continuous: he can study man as 'solid' or Socrates' nose as solid. Points, lines, and panes do also potentially exist in this solid. The second sense of mathematicals existing potentially in matter can be tied up with the idea that geometer's thinking can make actual what previously existed only potentially, such as when additional lines which help to prove a geometrical theorem exist only potentially in a geometrical figure and need to be actualized both in geometer's thinking and on paper by drawing, or when a sphere existing potentially in bronze becomes an actual brazen sphere.

There is a further hint in *Metaphysics* XI. 3 which explains how to understand Aristotle's statement that the objects of mathematics are embodied in matter (ὡς ἐν ὕλῃ). To reach mathematicals, one must perform the method of subtraction which would help to *locate* the objects of mathematics within a sensible body. This successive removal of unnecessary parts from a physical object explains how the quantitative and continuous is unfolded by the step-by-step removal of all sensible qualities:

> As the mathematician investigates abstractions [τὰ ἐξ ἀφαιρέσεως] (for before beginning his investigation he strips off all the sensible qualities, e.g. weight and lightness, hardness and its contrary, and also heat and cold and the other sensible contrarieties, and leaves only the quantitative [τὸ ποσὸν] and continuous [συνεχές], sometimes in one, sometimes in two, sometimes in three dimensions, and the attributes of these *qua* quantitative and continuous, and does not consider them in any other respect, and examines the relative positions of some and the attributes of these, and the commensurabilities and incommensurabilities of others, and the ratios of others; but yet we posit one and the same science of all these things – geometry). (Ross, *Meta.* 1061a 29-35).
>
> καθάπερ δ᾽ ὁ μαθηματικὸς περὶ τὰ ἐξ ἀφαιρέσεως τὴν θεωρίαν ποιεῖται (περιελὼν γὰρ πάντα τὰ αἰσθητὰ θεωρεῖ, οἷον βάρος καὶ κουφότητα καὶ σκληρότητα καὶ τοὐναντίον, ἔτι δὲ καὶ θερμότητα καὶ ψυχρότητα καὶ τὰς ἄλλας αἰσθητὰς ἐναντιώσεις, μόνον δὲ καταλείπει τὸ ποσὸν καὶ συνεχές, τῶν μὲν ἐφ᾽ ἓν τῶν δ᾽ ἐπὶ δύο τῶν δ᾽ ἐπὶ τρία, καὶ τὰ πάθη τὰ τούτων ᾗ ποσά ἐστι καὶ συνεχῆ, καὶ οὐ καθ᾽ ἕτερόν τι θεωρεῖ, καὶ τῶν μὲν τὰς πρὸς ἄλληλα θέσεις σκοπεῖ καὶ τὰ ταύταις ὑπάρχοντα, τῶν δὲ τὰς συμμετρίας καὶ ἀσυμμετρίας, τῶν δὲ τοὺς λόγους, ἀλλ᾽ ὅμως μίαν πάντων καὶ τὴν αὐτὴν τίθεμεν ἐπιστήμην τὴν γεωμετρικήν).

Commenting on Aristotle's *Metaphysics* III, 998a7 (against mathematicals as *substances* existing *in* sensibles) Alexander of Aphrodisias also highlights that the objects of mathematics are in every respect sensible, which, he mentions, may be thought of as extension or διάστασις:

> Whereas those who assume them [the mathematicals] by way of abstraction, who separate some [attributes] of the sensibles by reason (logos), leave them [the sensibles], including the [attributes] separated, all in every respect sensible, since those separated [attributes] are not capable, on their own, of making up a complete sensible nature, not even if they are thought of as having some extension (*diastasis*). For the *extension* that is thought of in the case of the mathematicals, together with the affective attributes (*pathetika*) separated by reason, is the sensible nature. (trans. Dooley. 201, 5-10).[140]
>
> οἱ δὲ ἐξ ἀφαιρέσεως λαμβάνοντες αὐτά, τῷ λόγῳ τινὰ τῶν αἰσθητῶν χωρίσαντες, καταλείπουσιν αὐτὰ σὺν τοῖς χωρισθεῖσι πάντα τὰ κατὰ τὰ ὅλα αἰσθητά, οὐκέτι ἐκείνων τῶν κεχωρισμένων αὐτῶν ἐφ᾽ αὑτῶν δυναμένων τὴν αἰσθητὴν ἀποπληροῦν φύσιν, οὐδ᾽ ἐπὶ διαστάσεως τινος νοουμένων. ἡ γὰρ ἐν τοῖς μαθηματικοῖς νοουμένη

[140] Alexander. *On Aristotle's Metaphysics 2 & 3*. Ancient Commentators on Aristotle. (Ithaca, N.Y.: Cornell University Press, 1992), 135.

> διάστασις μετὰ τῶν τῷ λόγῳ κεχωρισμένων παθητικῶν ἡ αἰσθητὴ φύσις· ἐν ἀμφοῖν γὰρ ἡ αἰσθητὴ φύσις ἐν ὑποστάσει οὖσα φύσει.

Contrary to those who think that Aristotle applies *aphairesis* to universals,[141] this and all other passages where the term appears show quite explicitly that the term is used for individuals only – the mathematician chooses a particular sensible body, strips off all its sensible qualities, and then studies its quantitative and continuous aspects. The fact that mathematicals exist potentially suggests that they are not the creations of mind alone, nor are they substances existing separately from sensibles or *in* sensibles. The extensions of all things are tangible and wholly dependent on the constant change of matter. Thus, we may affirm that the objects of mathematics exist potentially as physical extension which is unfolded by successive method of subtraction.

Mathematicals exist actually in thought

The clue to understanding how mathematicals exist actually in thought may be found in *Meta*physics IX. 9. This passage describes how the intellectual activity of a geometer can bring potential constructions to an actualization in thought to prove a theorem.

> It is an activity also that geometrical constructions are discovered; for we find them by dividing. If the figures had been already divided, the constructions would have been obvious; but as it is they are present only potentially. Why are the angles of the triangle equal to two right angles? Because the angles about one point are equal to two right angles. If, then, the line parallel to the side had been already drawn upwards, the reason would have been evident to anyone as soon as he saw the figure. Why is the angle in a semicircle in all cases a right angle? If three lines are equal the two which form the base, and the perpendicular from the centre-the conclusion is evident at a glance to one who knows the former proposition. Obviously, therefore, the potentially existing constructions are discovered by being brought to actuality; the reason is that the geometer's thinking is an actuality; so that the potency proceeds from an actuality; and therefore it is by making constructions that people come to know them (though the single actuality is later in generation than the corresponding potency). (Ross, *Meta.* 1051 a21-33).

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[141] Allan Bäck, Thomas Aquinas, de Koninck, Mansion.

εὑρίσκεται δὲ καὶ τὰ διαγράμματα ἐνεργείᾳ: διαιροῦντες γὰρ εὑρίσκουσιν. εἰ δ᾽ ἦν διῃρημένα, φανερὰ ἂν ἦν: νῦν δ᾽ἐνυπάρχει δυνάμει. διὰ τί δύο ὀρθαὶ τὸ τρίγωνον; ὅτι αἱ περὶ μίαν στιγμὴνγωνίαι ἴσαι δύο ὀρθαῖς. εἰ οὖν ἀν ῆκτο ἡ παρὰ τὴν πλευράν, ἰδόντι ἂν ἦν εὐθὺς δῆλον διὰ τί. ἐν ἡμικυκλίῳ ὀρθὴ καθόλου διὰ τί; ἐὰν ἴσαι τρεῖς, ἥ τε βάσις δύοκαὶ ἡ ἐκμέσου ἐπισταθεῖσα ὀρθή, ἰδόντι δῆλον τῷ ἐκεῖνο εἰδότι. ὥστε φανερὸνὅτι τὰ δυνάμει ὄντα εἰς ἐνέργειαν ἀγόμενα εὑρίσκεται: αἴτιον δὲ ὅτι ἡ νόησις ἐνέργεια: ὥστ᾽ ἐξ ἐνεργείας ἡ δύναμις, καὶ διὰ τοῦτο ποιοῦντες γιγνώσκουσιν (ὕστερον γὰρ γενέσει ἡ ἐνέργεια ἡ κατ᾽ἀριθμόν).

Fig.1

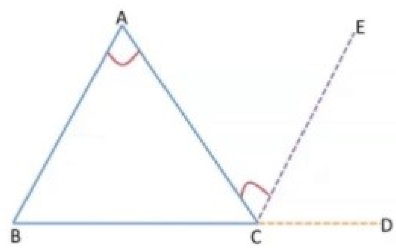
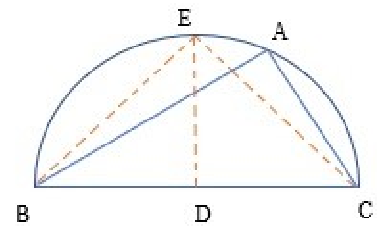


Fig.2



In the passage above potency is understood in two ways. (1) Additional lines and surfaces *CE* and *CD* in Fig.1 that prove the truth of a geometrical construction "the angles of the triangle equal to two right angles" and lines and surfaces *BED* and *DEC* in Fig.2 "the angle in a semicircle in all cases a right angle" exist potentially in the construction: "they are present only potentially." This means that the geometer needs only to recognize or *discover* (εὑρίσκεται) the right additional lines and surfaces that constitute the proof. This is possible for three reasons: a) every point or line has the potency to be extended and drawn, b) every human has an intellectual capability of discovering geometrical proofs, c) the fact that the geometer *discovers* them suggests that these additional lines in Fig.1 and Fig.2 that constitute the proof epistemically exist prior to their coming to be, but in reality they are posterior to former constructions *BAC* in Fig.1 and to *BAC* together with the semicircle in Fig.2. Potential constructions come from the geometer's actual thinking (ἐξ ἐνεργείας ἡ δύναμις) which means that geometer's thinking is prior to potential constructions. As Ross puts it: "the potentiality of the construction presupposes the activity of thought, but precedes the actuality of the construction."[142] (2) The matter, *on which* and *with which* Fig. 1 and Fig.2 were drawn, has the potency to become something else. The whole discussion of *Metaphysics* IX. 9 highlights that matter has the potency of constant change in two ways because "it can itself be acted on or because something else can be acted on by it" (Ross, *Meta.* 1046a20). This change of matter affects the size of the physical extension (quantitative and continuous) of both constructions. Fig.1 and Fig.2 become devoid of any change only in thinking. Thus, the mathematician commits no error when s/he actualizes them in thinking.

Actuality in turn is understood in two senses. (1) To be actualized is to be drawn. Additional lines and surfaces in Fig.1 and Fig.2 need to be drawn: "the reason would have been evident to anyone as soon as he saw the figure." (2) To be actualized is to be thought of. The geometer may only visually extend the base of *BC* to *CD* and visually draw *CE* parallel to *BA*. (3) To be actualized is to be understood. To draw additional lines or to simply think of them does not automatically mean that the proof can be evident to anyone. It may be evident to a geometer but not evident to a pupil. The construction becomes actualized when it is actually understood, i.e. how exactly these additional figures explain the truth of construction. Once it is actually understood, the object of thought, such as "the sum of internal angles equal to two right angles" or "the angle in a semicircle in all cases is a right angle," becomes one with the intellect (*De Anima* III, 4-8).

Bechler in his book called *Aristotle's Theory of Actuality* raises doubts and objections against Aristotle's and Lear's *qua*-filter, concluding that the mathematician nevertheless does commit an error even if he applies the filter and strips off all change and motion from mathematicals in thinking. I cite Bechler's account in full.

[142] Ross, W. D., and Aristotle. *Aristotle's Metaphysics: A Revised Text with Introduction and Commentary.* (Oxford: Clarendon Press, 1924), 273.

> This leads us at once to the troubled problem of whether Aristotle could consistently make room in his ontology for enmattered yet exact geometrical entities. It seems to me to be impossible for the simple reason that he states that what the mathematician separates from is change (*Phys.* 193a35). This clearly implies that the enmattered triangle is actually in a constant process of change, which can only mean that even if it were an exact triangle at some moment t_0, it could not be exact anymore at $t_0 \pm \Delta t$ for an infinite number of Δt*'s*, that is, almost always it will be actually inexact. But in that case, the qua-operator would not be filtering any actually-existing attribute, that is, one that exists now *in* this piece of bronze. Rather it would have to be a creative operator, which sends from the potential, as yet nonexistent *M* in *P* to its actualization as *M* in *P*. But, as we already saw, even this is not quite sufficient and actually contradicts Aristotle's view of the issue.[143]

Here he proposes another way of how understanding the stripping off of all sensible affections and change. It may be understood, he explains, as considering an enmattered constantly changing triangle as an exact triangle at some moment t_0. He then objects stating that the issue with this interpretation is that the t_0 triangle could not be exact anymore at $t_0 \pm \Delta t$ because an infinite number of Δt*'s* will be inexact almost always. I think we can answer this question in the following way: even if the triangle is not exact anymore at $t_0 \pm \Delta t$, the intellect through sense perception yet receives the triangle without any further ± Δ t after it captures the last instance of $t_0 \pm \Delta t$. It is possible that at the last moment of $t_0 \pm \Delta t$ the mind does receive an actually existing attribute which is then transmitted to the brain. And when the intellect receives the last moment of $t_0 \pm \Delta t$, the triangle undergoes no further change and becomes the t_0 triangle so to speak. The intellect may not refer back to the same physical $t_0 \pm \Delta t$ triangle, but only to the t_0 triangle the intellect retained in itself in the first place. The t_0 triangle exists actually in the mind but potentially in the sensible in the form of $t_0 \pm \Delta t$.

One might further object to this and say that even if the intellect receives the last moment of $t_0 \pm \Delta t$ and does not refer back to the same physical $t_0 \pm \Delta t$ triangle, but only to t_0 triangle the intellect retained in the first place, the triangle still undergoes alteration because there is no exact and perfect triangularity in physical objects. The mind still makes an

[143] Zev Bechler, *Aristotle's Theory of Actuality*, SUNY Series in Ancient Greek Philosophy. (Albany: State University of New York Press, 1995), 172.

adjustment of its shape and therefore mathematical abstraction is a kind of fiction because we are compelled to project exactness into sensible objects. Such is the view of Charles de Koninck (1957). Here is how he explains it:

> Both to touch and to sight the bowling-ball has the appearance of a true sphere. Actually, any visible or tangible line or sphere can offer no more than the appearance of true continuity and regularity. For it is only when we consider a line apart from any sensible example that we can be sure that it is a line; and only when we consider a sphere apart from a sensible one can we know that it is a finite solid having every point on its surface equidistant from a point within called the center. When we project this exactness into the objects of sensation, we commit an error. It is only by prescinding from *per se* sensible objects that we achieve such rigour. To proceed as if ideal and real object were the same, as when a star is taken as a point, is an example of the kind of fiction needed by mathematical physics.[144]

I think Aristotle would object to this account in the following way. Even if there is no visible or tangible perfect and exact sensible sphere, it is still possible that this perfect sensible sphere is yet *enclosed* into this inexact sensible sphere, whereas the opposite scenario does not hold true. By being enclosed, I mean that the matter of the perfect sphere occupies the actual space of the matter of an imperfect one. The fact that the human cannot build a perfectly round bronze sphere does not mean that it cannot exist at all. The perfect sphere is potentially present in the inexact one which can be brought to actuality only in thinking but not in reality. It is due to matter being in constant change and to our limited building capacities that we cannot construct a perfect object. It is the thinking activity only that can think of the enclosed sphere which at some moment in time t_0 becomes a perfect sphere in the inexact one. The discovery of such a sphere comes from the subtraction of change, affections, passions, and motion in thinking and considering the bronze sphere qua perfect sphere enclosed into the

[144] De Koninck, Charles, Alphonse-Marie Parent, and Émile Simard. "Abstraction from Matter (I)." *Laval Théologique et Philosophique* 13, no. 2 (1957): 177. Also in De Koninck, Charles, Alphonse-Marie Parent, and Emmanuel Trépanier. "Abstraction from Matter (II)." *Laval Théologique et Philosophique* 16, no. 1 (1960): 54, 57.

former one. Thus, taking the parts of analysis together I think we can rightly affirm that while mathematicals exist potentially as physical extension and are unfolded by successive method of subtraction, they exist actually in thought.

Mathematicals exist actually in thought in the form of intelligible matter

All three passages[145] where 'intelligible matter' appears have to do with definitions: mathematical objects are always defined in the way perceptible objects are defined, i.e. as compounds of matter and form where one kind of matter is perceptible and another intelligible. The former is grasped through the sense-faculty and the latter is recognized by the thinking faculty or activity.

In *Metaphysics* book VII chapter 10 Aristotle distinguishes two kinds of circles, perceptible (e.g. brazen) circles and intelligible mathematical circles. The passage also explains that the final product which the mathematician studies is the mathematical intelligible circle rather than the perceptible circle. The latter could be plausibly equated with the quantitative and continuous sensible attribute of an object. Since matter affects the continuous sensible extension because it is changeable, this perceptible circle must become intelligible in thinking. This passage also suggests that these intelligible mathematicals must have their own intelligible matter in a definition. There is however no explanation to what kind of matter this is (it only becomes clear in *Metaphysics* VII.11, and especially in VIII.6). This is how Aristotle explains the notion of intelligible matter in VII.10:

> But when we come to the concrete thing, e.g. this circle, i.e. one of the individual circles, whether perceptible or intelligible (I mean by intelligible circles the mathematical, and by perceptible circles those of bronze and of wood),-of these there is no definition, but they are known by the aid of intuitive thinking or of

[145] *Meta.* VII. 10, 1036a 1-13; *Meta.* VII.11, 1036b 32-1037a5; *Meta.* VIII. 6, 1045a 33-6.

> perception; and when they pass out of this complete realization it is not clear whether they exist or not; but they are always stated and recognized by means of the universal formula. But matter is unknowable in itself. And some matter is perceptible and some intelligible, perceptible matter being for instance bronze and wood and all matter that is changeable, and intelligible matter being that which is present in perceptible things not *qua* perceptible, i.e. the objects of mathematics. (Ross, *Meta.* 1036a 1-13).
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> τοῦ δὲ συνόλου ἤδη, οἷον κύκλου τουδὶ καὶ τῶν καθ᾽ ἕκαστά τινος ἢ αἰσθητοῦ ἢ νοητοῦ (λέγω δὲνοητοὺς μὲν οἷον τοὺς μαθηματικούς, αἰσθητοὺς δὲ οἷον τοὺς χαλκοῦς καὶ τοὺς ξυλίνους) τούτων δὲ οὐκ ἔστιν ὁρισμός, ἀλλὰ μετὰ νοήσεως ἢ αἰσθήσεως γνωρίζονται, ἀπελθόντες δὲ ἐκ τῆς ἐντελεχείας οὐ δῆλον πότερον εἰσὶν ἢ οὐκεἰσίν: ἀλλ᾽ ἀεὶ λέγονται καὶ γνωρίζονται τῷ καθόλου λόγῳ. ἡ δ᾽ ὕλη ἄγνωστος καθ᾽ αὑτήν. ὕλη δὲ ἡ μὲν αἰσθητή ἐστιν ἡ δὲ νοητή, αἰσθητὴ μὲν οἷον χαλκὸς καὶ ξύλον καὶ ὅση κινητὴ ὕλη, νοητὴ δὲ ἡ ἐν τοῖς αἰσθητοῖς ὑπάρχουσαμὴ ᾗ αἰσθητά, οἷον τὰ μαθηματικά.

The passage above supports the idea that intelligible circles exist actually in thinking, specifically where Aristotle says that when mathematicals pass out of the state of actuality (ἐκ τῆς ἐντελεχείας) it is not clear whether they exist or not. This confusion should not be taken to suggest that mathematicals exist only in thinking. It has already been shown that they also exist as sensible quantitative and continuous extension.

The concept of intelligible matter further appears in *Metaphysics* VII.11. Here Aristotle states that the lines and the continuous of such compound as ‘triangle’ are the material elements in the same way as flesh and bones are the material elements of man or bronze and stone are the material elements of the statue. In the first case the material elements are intelligible and not perceptible, in the second – perceptible and tangible. It is necessary that the objects of mathematics too have the aspect of matter in their definition, otherwise how would they be defined?

> some people [Pythagoreans] already raise the question even in the case of the circle and the triangle, thinking that it is not right to define these by reference to lines and to the continuous, but that all these are to the circle or the triangle as flesh and bones are to man, and bronze or stone to the statue. […] Regarding the objects of mathematics…for even some things which are not perceptible must have matter; indeed there is some matter in everything which is not an essence and a bare form but a 'this'. The semicircles, then, will not be parts of the universal circle, but will be parts

of the individual circles, as has been said before; for while one kind of matter is perceptible, there is another which is intelligible. (Ross, *Meta.* 1036b7-1037a5).

ἀποροῦσί τινες ἤδη καὶ ἐπὶ τοῦ κύκλου καὶ τοῦ τριγώνου ὡς οὐ προσῆκον γραμμαῖς ὁρίζεσθαι καὶ τῷ συνεχεῖ, ἀλλὰ πάντα καὶ ταῦτα ὁμοίως λέγεσθαι ὡσανεὶ σάρκες καὶ ὀστᾶ τοῦ ἀνθρώπου καὶ χαλκὸς καὶ λίθοςτοῦ ἀνδριάντος: καὶ ἀνάγουσι πάντα εἰς τοὺς ἀριθμούς, καὶ γραμμῆς τὸν λόγοντὸν τῶν δύο εἶναί φασιν. […] περὶ δὲ τὰ μαθηματικὰ… ἔσται γὰρ ὕλη ἐνίων καὶ μὴ αἰσθητῶν: καὶ παντὸς γὰρ ὕλη τις ἔστιν ὃ μὴ ἔστι τί ἦνεἶναι καὶ εἶδος αὐτὸ καθ᾽ αὑτὸ ἀλλὰ τόδε τι. κύκλου μὲν οὖν οὐκ ἔσται τοῦ καθόλου, τῶν δὲ καθ᾽ ἕκαστα ἔσται μέρη ταῦτα, ὥσπερ εἴρηται πρότερον: ἔστι γὰρ ὕλη ἡ μὲν αἰσθητὴ ἡ δὲ νοητή.

In the next passage of book VIII chapter 6 where ‘intelligible matter’ occurs for the third time, Aristotle now explains more clearly what exactly he means by intelligible matter. He highlights that there are two circles in a round bronze, perceptible brazen circles and intelligible mathematical circles. Both essences of circles have matter and form in their formula, where matter exists potentially and form actually. Thus, in a definition of the intelligible circle, e.g. ‘circle is a plane figure’, ‘plane’ is intelligible matter and ‘figure’ is actuality or the formal element. [146] Perceptible circles are defined in the same way as intelligible circles are defined: “But is the matter an element even in the formula? We certainly describe in both ways what brazen circles are; we describe both the matter by saying it is brass, and the form by saying that it is such and such a figure; and figure is the proximate genus in which it is placed. The brazen circle, then, has its matter in its formula” (Ross. *Meta.* 1033a1-4). The matter in a perceptible circle is ‘bronze’ whereas the matter of an intelligible circle is ‘plane’. However, it seems that in a definition of both perceptible and intelligible circles the formal aspect coincides, namely that perceptible circle is ‘a brazen figure’ and

[146] Alexander of Aphrodisias holds the opposite view. He thinks that the ‘figure’ stands for intelligible matter which is analogous to ‘genus’ (*In Metaphysica*, 562.14-17). Alexander seems to rely on *Meta*. VII 1038a5 where it is stated that genus is matter. This may suggest that if ‘figure’ is the genus then ‘plane’ is the differentiae or form. However, in *Metaphysics* V.28 Aristotle states that it is ‘plane’ that illustrates the genus: “There is genus in the sense in which ‘plane’ is the genus of plane figures and ‘solid’ of solids; for each of the figures is in the one case a plane of such and such a kind, and in the other a solid of such and such a kind; and this is what underlies the differentiae” (1024b1-5).

intelligible circle is ‘a plane figure.’ The difference between the two figures is that one is fully perceptible and another is fully intelligible. The mathematician cannot study perceptible figures because their sizes are being constantly alternated due to their dependence on perceptible changing matter. He studies intelligible ones as they are free from matter and change. Here is what Aristotle writes in VIII.6:

> Clearly, then, if people proceed thus in their usual manner of definition and speech, they cannot explain and solve the difficulty. But if, as we say, one element is matter and another is form [μορφή], and one is potentially and the other actually, the question will no longer be thought a difficulty. For this difficulty is the same as would arise if 'round bronze' were the definition of 'cloak'; for this word would be a sign of the definitory formula, so that the question is, what is the cause of the unity of 'round' and 'bronze'? The difficulty disappears, because the one is matter, the other form [μορφή]. What, then, causes this-that which was potentially to be actually-except, in the case of things which are generated, the agent? For there is no other cause of the potential sphere's becoming actually a sphere, but this was the essence of either. Of matter some is intelligible, some perceptible, and in a formula there is always an element of matter as well as one of actuality; e.g. the circle is 'a plane figure'. (Ross, *Meta*. 1045a20-35).
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> φανερὸν δὴ ὅτι οὕτω μὲν μετι οῦσιν ὡς εἰώθασιν ὁρίζεσθαι καὶ λέγειν, οὐκ ἐνδέχεται ἀποδοῦναι καὶ λῦσαι τὴν ἀπορίαν: εἰ δ᾽ἐστίν, ὥσπερ λέγομεν, τὸ μὲν ὕλη τὸ δὲ μορφή, καὶ τὸ μὲν δυνάμει τὸ δὲ ἐνεργείᾳ, οὐκέτι ἀπορία δόξειεν ἂν εἶναι τὸ ζητούμενον. ἔστι γὰρ αὕτη ἡἀπορία ἡ αὐτὴ κἂν εἰ ὁ ὅρος εἴη ἱματίου στρογγύλος χαλκός: εἴη γὰρ ἂνσημεῖον τοὔνομα τοῦτο τοῦ λόγου, ὥστε τὸ ζητούμενόν ἐστι τί αἴτιον τοῦ ἓνεἶναι τὸ στρογγύλον καὶ τὸν χαλκόν. οὐκέτι δὴ ἀπορία φαίνεται, ὅτι τὸ μὲν ὕλητὸ δὲ μορφή. τί οὖν τούτου αἴτιον, τοῦ τὸ δυνάμει ὂν ἐνεργείᾳ εἶναι, παρὰ τὸ ποιῆσαν, ἐν ὅσοις ἔστι γένεσις; οὐθὲν γάρ ἐστιν αἴτιον ἕτερον τοῦ τὴν δυνάμει σφαῖραν ἐνεργείᾳ εἶναι σφαῖραν, ἀλλὰ τοῦτ᾽ ἦν τὸ τί ἦν εἶναι ἑκατέρῳ. ἔστι δὲ τῆς ὕλης ἡ μὲν νοητὴ ἡ δ᾽ αἰσθητή, καὶ ἀεὶ τοῦ λόγου τὸ μὲν ὕλη τὸδὲ ἐνέργειά ἐστιν, οἷον ὁ κύκλος σχῆμα ἐπίπεδον.

The plane or intelligible matter of a separated circle in the passage above appears to be analogous to the ‘continuous’ or συνεχής of the separated straight line in *de Anima* 429b17-24 where the ‘continuous’ or συνεχής is the matter of the mathematical straight line with twoness as its form. The notion of intelligible matter, again, does not imply that the objects of mathematics exist in thinking only. They exist as physical extension. Once physical extension that depends on potentially existing matter becomes intelligible matter in thinking which is taken from the last moment of $t_0 \pm \Delta t$ in the form of two-dimensional plane, the mathematician commits no error since his object of study does not involve perceptible matter.[147]

Mueller defends a completely different view. He equates intelligible matter with extension and considers mathematical objects "to underlie physical reality."[148] Aristotle knows that sensible objects do not meet the requirements of mathematics (there are no perfectly straight lines, perfectly round spheres, etc): "with what sort of things must the mathematician be supposed to deal? Certainly not with the things in this world; for none of these is the sort of thing which the mathematical sciences demand" (1059b 8-11). If intelligible matter underlies physical reality it will follow that intelligible objects will not satisfy the conditions of mathematics which Aristotle demands, because sensible matter will be constantly affecting the size of intelligible extension. As a consequence, mathematics will not be true of intelligible mathematicals either. David Bostock also does not accept Mueller's view. He asks quite a provocative question "if it is intelligible matter that is supposed to be *perfectly* square, spherical, and so on, how can it be regarded as 'underlying' the material objects that are only *imperfect* examples of these properties?"[149] One would expect Mueller to at least say that perhaps this intelligible matter is actualized in the geometer's thinking, but he

[147] I shall not avoid citing the passage from *de Anima* III.7 for the second time which has the language of *aphairesis* and of actuality in it. It states that the hollow which potentially exist in a snub-nose, exist actually in thinking without flesh and the mind becomes one with the object of thinking: "The so-called abstract objects [τὰ δὲ ἐν ἀφαιρέσει λεγόμενα] the mind thinks just as, if one had thought of the snub-nosed not as snub-nosed but as hollow [ᾗ κοῖλον], one would have thought of an actuality [ἐνεργείᾳ] without the flesh in which it is embodied: it is thus that the mind when it is thinking the objects of Mathematics thinks as separate elements which do not exist separate. In every case the mind which is actively thinking [κατ' ἐνέργειαν] is the objects which it thinks." (Ross, *de Anima*, 431b 13-20). The definition of mathematical hollow will also include an element of matter as well as one of actuality, e.g. the hollow is 'a curved plane.'
[148] Ian Mueller, "Aristotle on geometrical objects," *Archiv für Geschichte der Philosophie* 52, no. 2 (1970): 161, 168.
[149] David Bostock, "Aristotle's Philosophy of Mathematics," *The Oxford Handbook of Aristotle*. (Oxford University Press, 2012), 477.

does not develop such a view, and actually denies it saying that for Aristotle this assumption[150] "precludes the merely mental existence of mathematical objects."[151]

As I have already shown, Aristotle does not accept the merely mental existence of mathematicals or any form of Platonism suggesting a separate existence of mathematicals. Mathematical objects exist in a 'special sense' which presupposes that they exist potentially as physical extension where the latter is unfolded by successive method of subtraction. Once the mental separation of $\pm \Delta t$ takes place, this extension becomes an intelligible matter existing in thinking as actuality without any further change involved. The function of *aphairesis* is to successively *uncover* the sensible magnitude which subsequently becomes a particular intelligible matter. Aristotle uses the term only for expository purposes to show the location of the objects of mathematics. I agree with John Thorp that whenever Aristotle discusses intelligible matter, he always refers to a particular intelligible object which remains an individual.[152] It does not have any abstractionist connotations which might suggest a sense of extracting a mathematical form from a sensible object. What perhaps might have a sense of extraction is the qua-method, which is merely the reverse operation of *aphairesis* that designates the remainder of the latter. The independence of mathematicals from motion and change is reflected both in reality, when the mind receives the last moment $\pm \Delta t$, and in definitions where no physical matter is involved when a mathematical is defined.

[150] Here he cites Zeller's statement "that the truth of knowledge keeps pace with the actuality of its object" (p.158.)
[151] Ian Mueller, "Aristotle on geometrical objects," *Archiv für Geschichte der Philosophie* 52, no. 2 (1970): 157.
[152] John Thorp, "Intelligible Matter in Aristotle," The Society for Ancient Greek Philosophy Newsletter. 385. (2010). https://orb.binghamton.edu/sagp/385

CHAPTER 5 MODERN INTERPRETERS OF ARISTOTLE'S SO-CALLED "THEORY OF ABSTRACTION"

Introduction

In this chapter I am aim to show the types of confusions that exist in modern interpretations of Aristotle's theory of abstraction which build upon, consciously or unconsciously, certain especially influential ancient commentators on Aristotle's views. The objects of my investigation will include some of the scholars mentioned by John Cleary[153] who have not yet been investigated in the light of *aphairesis* as subtraction, either by him or by anyone else. I will consider Charles de Koninck's three-volume article "Abstraction from Matter," Mansion's book *Introduction à la Physique Aristotélicienne*, and Julia Annas' account of *aphairesis* in her book *Aristotle's Metaphysics Books M and N*. I will also look into Allan Bäck's article "The Concept of Abstraction" (written several years after Cleary published his article). The works of these scholars have not yet been analyzed in the light of what I take to be the authentic interpretation of *aphairesis.* My aim is to examine whether any of the *four* previously listed types of epistemological *aphairesis* found in the works of the ancient commentators persist in modern commentary. The four different extractionist or epistemological interpretations of *aphairesis* first found in the commentaries of the ancient scholars are the following: abstraction of form (essence) from matter (Alexander of Aphrodisias), abstraction of a universal from particulars (Boethius and Aquinas), abstraction of a mathematical universal from particulars (Proclus and Aquinas), abstraction of this particular mathematical form (this circular shape) from this particular matter, motion, and

[153] Cleary, John. "On the Terminology of "Abstraction" in Aristotle." *Phronesis* 30 (1985): 13-45.

change (Philoponus and Aquinas). My analysis will show that there is a strong connection between these ancient views and the views of the modern accounts of *aphairesis* expressed by de Koninck, Mansion, Annas, and Bäck. While I shall offer in this chapter criticisms of each of these modern commentators, I will show that some of the commentators are much further from Aristotle's understanding of *aphairesis* than others, such as for instance Bäck's comparison of *aphairesis* and the qua-filter with the desire (ὄρεξις) of non-rational animals. Such a suggestion not only places *aphairesis* at the level of pure psychologism, it also drastically lowers it from the realm of rational and intellectual activity down to the biological processes of animal life.

5.1 Charles de Koninck

Charles de Koninck argues for the following two uses of Aristotle's *aphairesis* also present in Proclus, Boethius, and Aquinas, yet absent in Aristotle: *abstraction of a universal from particulars* and *abstraction of a mathematical universal from particulars*.

He starts his account of *aphairesis* with the following definition of abstraction: "both in French and in English it means, first and immediately, something far removed from what is more known to us: viz., a certain operation of the mind, or the status of something related to thought as distinguished from mere sensation."[154] De Koninck emphasizes the importance of going back to the original meaning of Aristotle's term 'abstraction' (as well as other key concepts such as 'syllogism,' 'matter,' 'form,' and 'psyche'). He goes on to say that "the neglect of primitive meanings opens the way to a philosophical jargon that all can repeat but

[154] Charles de Koninck, Alphonse-Marie Parent, and Émile Simard. "Abstraction from Matter (I)." *Laval Théologique et Philosophique* 13, no. 2 (1957): 155.

no one understands."[155] Later he adds that we should not identify the original meaning of the term with the later uses: "the emphasis which we are placing upon the original meaning of a word is not intended to suggest that this same meaning is to be identified with its subsequent uses; but rather that to neglect original meanings entirely could lead to confusion with respect to later meanings."[156]

At the very beginning of his three-volume article de Koninck cites Aquinas' prologue to Aristotle's *Physics* which is followed by an extensive commentary on why a thing is abstracted from matter and how to understand this statement:

> Since the treatise called the Physics, which it is our purpose to explain, is also the one that comes first in the study of nature, we must show, at its very beginning, what natural science is about — viz. its matter and subject. To this end, we should point out, on the one hand, that inasmuch as every science is in the intellect, and since a thing becomes intelligible in act insofar as it is more or less abstracted from matter, things, according as they are diversely related to matter, are the concern of different sciences. Again, since science is obtained by demonstration, and the middle term of demonstration is the definition, it follows, of necessity, that the sciences will be distinguished according to a difference in their mode of definition.[157]

De Koninck interprets 'abstraction from matter' in Aquinas and Aristotle in the following three ways: as abstraction of particular mathematical shape from particular matter, as abstraction of a universal from particulars, and as abstraction of a mathematical universal from particulars. De Koninck rejects the first view and accepts the latter two.

Abstraction of particular mathematical form or shape from particular matter: De Koninck also identifies an application of *aphairesis* in the process of mathematical abstraction from a particular sensible, "when we consider a sphere apart from a sensible one."[158] De Koninck rejects mathematical abstraction from sensibles due to the problem of exactness and the problem of fiction. The abstracted triangle still undergoes alteration in thinking because

[155] Ibid.
[156] Ibid., 157.
[157] Ibid., 133.
[158] Ibid., 177.

there is no exact and perfect triangularity in physical objects. The mind still makes an adjustment of its shape and therefore mathematical abstraction is a kind of fiction because we are compelled to project exactness onto sensible objects. I have already tried to resolve these two problems in section 4.3 of my thesis.

Abstraction of a universal from particulars: according to de Koninck, when a scientist states a definition, for instance, that of man, he abstracts from *this* individual sensible matter such as these bones and this flesh of Socrates, [159] - but he does not abstract from common sensible matter.[160] In the second of his 3-volume article he states that "in this type of abstraction the initial step is from the individual Socrates, Plato, etc., to man in general. After this first step, it is an easy progress from man to animal, from animal to living being."[161] This claim makes it obvious that De Koninck conflates Aristotle's *aphairesis* with induction. Such an application of the term as abstraction from individual matter in order to recognize a universal concept of man is not found in Aristotle. This interpretation of *aphairesis* falls under one of the misconceptions of the term I indicated above, namely as abstraction of a universal from particulars.

Abstraction of a mathematical universal from particulars: De Koninck's reasoning seems to suggest that mathematics can be true of universals only, that is when in abstraction from particulars we consider the common matter of mathematicals. Specifically, he writes that "the reason why complete exactness is possible in geometry is that the definitions we use are

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[159] Ibid.,166.

[160] De Koninck clarifies what he means by the 'common sensible matter' in the second volume of his article. He explains it as follows: "although the definition of animal (' a body able to sense ') differs from the definition of man (' an animal able to reason '), as being more general, they do not differ as to mode of definition, for both are with sensible matter. This may perhaps become clearer if we notice that, even when we consider Socrates as this man, or as this animal, as this living being, or as this thing, our degree of generality is widening, but we are always pointing to the same individual matter as attained in sensation. Similarly, whether man be defined as man, or animal, or living being, common sensible matter enters into each definition." Charles de Koninck, Alphonse-Marie Parent, and Emmanuel Trépanier. "Abstraction from Matter (II)," *Laval Théologique et Philosophique* 16, no. 1 (1960) : 60.

[161] Charles de Koninck, Alphonse-Marie Parent, and Emmanuel Trépanier. "Abstraction from Matter (II)." *Laval Théologique et Philosophique* 16, no. 1 (1960): 57, 59.

formally independent of, and have no reference to, the order of sense experience, and the conclusions are established as following from such definitions with necessity."[162] The exactness is possible because definitions contain no individual sensible matter.[163] I have already shown in my analysis of *aphairesis* that whenever Aristotle speaks of mathematical (and non-mathematical) abstraction, he always considers particulars and their particular intelligible matter. In terms of the latter, de Koninck does not interpret it in the way Aristotle does. He claims that the intelligible matter of this particular circle differs neither in size nor in any other way from the intelligible matter of other particular circles:

> Aristotle begins his explanation of intelligible matter by calling attention to the fact that, even in the world of mathematics, there can be individual objects, like the individual circles we describe to construct a triangle whose sides are equal. Now these circles do not differ by what they are; for one is as much a circle as the other, and they are even of the same in radius. The only difference between them is numerical.[164]

This holds true, he continues, because "the individual circles are not part of the definition of 'what circle is,' while the definition is verified in each and every one of them."[165] Now, even if it is the case that we define every intelligible circle as 'plane figure,' it is not true that whatever bronze circle we consider, the intellect receives a circle of the same radius. Whatever intelligible matter (lines, planes, solid) is separated from a particular sensible object, it receives the size and the shape of *that* particular sensible object from which it was separated. Intelligible matter is not a universal which we learn from childhood, retain in our memory and sometimes construct in our imagination. It seems likely that de Koninck conflates the particular intelligible circle with the universal concept of a circle. Throughout

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[162] Charles de Koninck, Alphonse-Marie Parent, and Émile Simard. "Abstraction from Matter (I)." *Laval Théologique et Philosophique* 13, no. 2 (1957): 148.
[163] Charles de Koninck, Alphonse-Marie Parent, and Emmanuel Trépanier. "Abstraction from Matter (II)." *Laval Théologique et Philosophique* 16, no. 1 (1960): 60, 67.
[164] Ibid., 64.
[165] Ibid.

his 3-volume article de Koninck claims that the intelligible circle, i.e. the universal concept of a circle is reached by abstraction. However, what he describes as the process of abstraction, is in fact, induction (*epagoge*). According to Aristotle a particular intelligible circle is reached by subtraction only, and it is wholly dependent on particular sensibles, whereas the universal circle is attained by the inductive process alone. In terms of abstracting universals, Ian Mueller makes quite a provocative statement: “If Aristotle thought of mathematical objects as universals separated from matter, it is difficult to see how he could distinguish legitimate mathematical separation from illegitimate separation of Platonic forms.”[166]

Highlighting the importance of going back to the original meaning of the term and stating the importance of looking “closely into the nature of mathematical abstraction as understood by Aristotle and St. Thomas,” de Koninck yet does not get behind Aquinas to the original meaning of the term as it appears in Aristotle; he cites not a single use of *aphairesis* from Aristotle’s passages. He obviously recognizes that *aphairesis* etymologically means subtraction: “The original Latin (just like the Greek ἀφαίρεσις) conveyed ‘the act of drawing or separating from,’ a meaning very near to the etymology: *ab*, *abs* (from) and *trahere* (to draw, pull, take away).”[167] De Koninck only rejects the modern sense of abstractionism as “something far removed from what is more known to us”[168] and only goes back to Thomas Aquinas’ interpretation of Aristotelian *aphairesis*, but not to Aristotle himself.

5.2 Auguste Mansion

[166] Ian Mueller, "Aristotle on geometrical objects," *Archiv für Geschichte der Philosophie* 52, no. 2 (1970): 163.
[167] Ibid.
[168] Charles De Koninck, Alphonse-Marie Parent, and Émile Simard. "Abstraction from Matter (I)" *Laval Théologique et Philosophique* 13, no. 2 (1957): 155.

Mansion assumes the following two senses of Aristotle's *aphairesis* which I have traced back to Proclus, Boethius, Philoponus, and Aquinas: *abstraction of a universal* (e.g. humanity) *from particulars* and *abstraction of a particular mathematical form from matter*. Both of these uses are accordingly absent in Aristotle's corpus.

In contrast to de Koninck, who claims that when we consider the round bronze the intellect receives a circle of the same radius, Mansion, for instance, states that even if intelligible circles are separated from sensible matter in thought, they will still be different from one another even if separated sensible circles are all of the same radius, because it is intelligible matter that makes every subtracted circle to be individuated. Indeed, it should be necessarily the case that we cannot think of a universal circle having such and such a size. If we posit a universal circle having certain size, can we legitimately call it 'universal'? In our study we always refer to particular objects and consider their size with each of them having their own individual intelligible matter. This is why, Mansion says, one mathematical circle is distinguished from another mathematical circle even if both are of the same radius. It is the intelligible matter that allows this multiplication of the same essence to be possible in various subjects:

> L'exemple classique des cercles physiques et des cercles mathématiques sert encore ici à illustrer cette conception. Le cercle, considéré comme un être de la nature, acquiert une individualité complète dès qu'il est réalisé dans tel bois réel, ou tel airain tangible: voilà sa matière sensible. Mais, même sans être projeté dans la réalité phénoménale, un cercle, conçu comme simple entité mathématique, se distingue d'un autre cercle de même rayon comme un individu d'un autre, c'est la matière intelligible qui permet cette multiplication de la même essence en des sujets divers.[169]

This account of intelligible matter is very much Aristotelian. This is however not the case with his understanding of *aphairesis*: Mansion is not consistent in his uses of it. He uses it in

[169] Auguste Mansion, *Introduction à la physique aristotélicienne*. (Louvain: Editions de l'Institut Supérieur, 1913), 81.

three different ways: as abstraction of species-form 'humanity,'[170] as abstraction of mathematical form from matter,[171] and as abstraction of matter from the mathematical form.[172] In the account below Mansion explicitly states that what we abstract from the snub nose is the geometrical form of curvature and concavity. This suggests a sense of extraction – we extract one thing (curvature) while disregarding many, i.e. the matter of nose, its passions, affections, motion and other irrelevant aspects. I will include his full account:

> D'une autre façon le camus représente tous les termes dont la défnition implique la matière en laquelle la forme de l'être est réalisée et en même temps concrétée; il s'oppose ainsi à la forme purement abstraite, ne renfermant dans sa notion aucun rapport à la matière dans laquelle on conçoit ou l'on suppose qu'elle pourrait se trouver à l'état plus concret. Le camus représentant l'être physique par opposition à l'être mathématique est simplement un cas particulier de cette classe; Aristote l'emploie fréquemment dans ce sens; et la raison en est facile à découvrir: il peut servir non seulement de terme de comparaison, mais même d'exemple proprement dit. En effet, un nez camus est un être physique réel. En outre, la forme qu'on en peut abstraire, la courbure ou la concavité est, si l'on veut, une détermination géométrique, de sorte qu'on a réuni dans le même objet les types distinctifs des deux branches du savoir qu'on veut comparer.[173]

Yet later he indicates another, this time properly Aristotelian operation of *aphairesis*, namely the 'subtraction' of physical aspects where many are subtracted and only one is left. Because the sense of 'abstraction' is not quite fitting to show the opposition to 'addition' or 'l'addition,' Mansion even uses 'retrancher' or subtraction. Mansion's sense of 'abstrait' in this passage, compared to the previous one, does play a role of subtraction even if he calls it

[170] "Transportons cette conception mutatis mutandis à la matière sensible entendue de façon universelle. Elle est incluse aussi dans un objet conçu de façon concrète, l'homme, mais non plus individuelle, Socrate, et c'est à elle qu'est dû l'état concret de l'objet considéré; en effet, n'était sa fonction de matière vis-à-vis de la forme abstraite, l'humanité, celle-ci ne serait qu'une abstraction." In Auguste Mansion, *Introduction à la physique aristotélicienne*. (Louvain: Editions de l'Institut Supérieur, 1913), 87.
[171] "En effet, un nez camus est un être physique réel. En outre, la forme qu'on en peut abstraire, la courbure ou la concavité est, si l'on veut, une détermination géométrique". In Auguste Mansion, *Introduction à la physique aristotélicienne*. (Louvain: Editions de l'Institut Supérieur, 1913), 71-72.
[172] "Ainsi c'est l'objet physique dont on abstrait certaines données pour avoir l'objet mathématique. Mansion, Introduction à la physique aristotélicienne." In Auguste Mansion, *Introduction à la physique aristotélicienne*. (Louvain: Editions de l'Institut Supérieur, 1913), 73.
[173] Ibid.

‘l'abstraction.’ For he says it is the physical object from which some things are abstracted (really subtracted) to have a mathematical object:

> Cette opposition entre les notions, qui résultent de l'abstraction ou de l'addition d'un caractère, est exprimée de façon assez défectueuse, car le fait de retrancher ou d'ajouter une note différentielle ne se réfère pas à un terme unique mais aux résultats opposés de ces opérations. Ainsi c'est l'objet physique dont on abstrait certaines données pour avoir l'objet mathématique; réciproquement c'est à ce dernier qu'on restitue les mêmes données pour en refaire un être physique.[174]

In a similar example Mansion uses it in a properly Aristotelian way, where he is employing the language of subtraction in the form of omission or exclusion: “Ce qui est caractéristique de la quantité mathématique, c'est la façon abstraite dont elle est considérée, l'exclusion de tout ce qui est sensible en dehors d'elle. ”[175] Here Mansion is clearly saying that we exclude many sensible properties and leave only one as the remainder, the mathematical quantity.

5.3 Allan Bäck

Bäck’s main move consists in reconciling and unifying the following uses of *aphairesis* he finds in Aristotle: *abstraction of a universal from particulars* (Boethius and Aquinas), *abstraction of a mathematical universal from particulars* (Proclus and Aquinas), *abstraction of this particular mathematical form from this particular matter, motion, and change* (Philoponus and Aquinas). His move to reconcile these is understandable, but also indicate how far removed he is from the original sense of the term.

In his article “The Concept of Abstraction” (2006), Bäck asks two questions which are very important for a proper understanding of Aristotle’s concept of *aphairesis*: “Does the abstraction consist in taking out something and discarding the rest? Or does it consist in

[174] Auguste Mansion, *Introduction à la physique aristotélicienne*. (Louvain: Editions de l'Institud Superieur, 1913), 73.
[175] Auguste Mansion, *Introduction à la physique aristotélicienne*. (Louvain: Editions de l'Institud Superieur, 1913), 90.

taking away something and keeping what is left? We can call the first one the selection view, and the latter the subtraction view."[176] Later in the article Bäck describes the first view as 'selective attention' and also as an 'extraction' view, and suggests that we should think of Aristotle's *aphairesis* as 'extraction.' Thus, the first view, according to Bäck, understands *aphairesis* as both selection and extraction. Both these terms mean to take something out and disregard what is left. According to Bäck, the second view, or the subtraction view, is only applied to the category of quantities, specifically, to subtraction as an arithmetic operation, i.e. when we have strictly speaking something like 4 – 2, only then we can translate *aphairesis* as 'subtraction.' He does not give much attention in his article to this kind of *aphairesis.*

How exactly abstraction as *selective attention* works, he explains in the following way:

> For it gives the intellect, and even the sense organs, an active role in locating these structures in its sense experience: it must "attend" to those features. Still, as I shall stress below, selective attention need not be a self-conscious, deliberate process. View 'attention' then as a sort of 'aiming at'. Aristotle himself seems to have this sort of conception when he attributes ὄρεξις to all animals able to perceive and imagine. [An. 413b23] We can translate 'ὄρεξις' as 'desire', but only 'desire' in a basic sense in which all animals can be said to "desire" food when they move towards a source of food. I mean 'attention' in the definition of 'abstraction' in this way too. Again, selectivity also need not imply any sort of deliberation or even of thought.[177]

What seems problematic in the account above apart from comparing *aphairesis* to the ὄρεξις of non-rational animals is that *aphairesis* presented here as not a self-conscious and deliberate process. A close textual analysis of the term has shown that it, in fact, presents a *deliberate* and *self-conscious* process used by Aristotle for didactic purposes in order to indicate the explicit spatial location of the quantitative and continuous which exists only implicitly in the sensibles.

[176] Allan Bäck, "The Concept of Abstraction" *The Society for Ancient Greek Philosophy Newsletter.* 376. (2006): 1.
[177] Allan Bäck, "The Concept of Abstraction" *The Society for Ancient Greek Philosophy Newsletter.* 376. (2006): 9

Bäck also claims that abstraction as selective attention has the advantage of "unifying two different sorts of abstraction that Alain de Libera[178] finds in Aristotle: 1) the sort in the mathematical sciences, of taking the form from the matter [in effect, what I have called 'extraction'] and 2) subtracting as opposed to adding on attributes. Selective attention performs both functions."[179] While it may be true of the second sort of *aphairesis*, there is not a single presence of the first in Aristotle's works, as I have already shown in my previous chapters.

Allan Bäck also distinguishes two kinds of 'abstract objects' in Aristotle: apart from 'mathematical *abstracta'* (e.g. *this* intelligible circle) Aristotle recognizes 'universal *abstracta*' (e.g. 'circle,' 'sight,' virtue').[180] The latter are reached by induction with some application of *aphairesis*:

> For instance, take induction as the process whereby the universals arise from the relevant singulars, and the abstraction used to generate the abstract, proper objects of mathematics as the process whereby universals inseparable *in re* in the individual substance and even *in intellectu* initially come to be treated as if they were separate. E.g., we might start off with individual physical objects and then via induction come to the general concept of body. Such a body would have color and shape (in general). Yet we may then "abstract" and treat the color and the shape as if they were separate, even though these universals necessarily go together. A non-rational animal could not make the final abstraction, Aristotle might say, although it can have experience and general notions ("primitive universals" as in Phys. 184a24-5; An. Po. 100a16) via some less ultimate processes of abstraction.[181]

While I agree that there is some kind a removal of unnecessary aspects present along with reaching a universal, what I do deny is the presence of the standard terminology of *aphairesis* in the process of reaching mathematical or non-mathematical universals.

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[178] Alain de Libera, *L'art des généralités: théories de l'abstraction*. (Editions Aubier, 1999): 30.
[179] Allan Bäck, "The Concept of Abstraction" *The Society for Ancient Greek Philosophy Newsletter.* 376. (2006): 8.
[180] Ibid., 2.
[181] Ibid., 7.

To support the concept of 'universal *abstracta*' and the role of *aphairesis* in it, Bäck refers to *de Anima* III.4, *Meta.* I.1, and *Phys*. I.1. Specifically, he says: "Aristotle explains how the universal is abstracted from the particular in his account of perception and thought [An. III.4; Metaph. I.1; Phys. I.1]."[182] Yet as I have already shown in my treatment of (iv) τῶν ἐν ἀφαιρέσει ὄντων in the *De Anima* III. 4, this passage has no discussion of a universal being attained from particular. The things existing in subtraction (τῶν ἐν ἀφαιρέσει ὄντων) are particular mathematical objects, i.e. *this* straight line. In addition, there is no reference made to τὸ καθόλου or to καθ' ἕκαστον in the passage. As for *Meta.* I.1, it is true that there are three uses of τὸ καθόλου and eight uses of ἕκαστον, but there is absolutely no mention of *aphairesis* present in the text; it is only at the end of book 1 of the *Metaphysics* Aristotle points out that universals become familiar *through induction*, but *aphairesis* is never mentioned. In terms of *Physics* I. 1, there Aristotle does not say that universals are 'abstracted' from particulars, namely that the former are attained by *aphairesis*. What Aristotle does say a few lines later in I. 2, is that the universals are attained *from induction* (ἐκ τῆς ἐπαγωγῆς). While I admit there is some kind of removal of aspects present along with the process of induction, we never see Aristotle using ἀφαίρεσις together with καθόλου, ἕκαστον, or ἐπαγωγή except in that ambiguous passage in the *Posterior Analytics* I, 18.

The second kind of *abstracta*, according to Bäck, are particular mathematicals, e.g. this intelligible circle in this particular bronze sphere. Bäck rightly indicates that when Aristotle speaks of individual mathematical circles, he refers to the intelligible matter of mathematical objects being also composed of matter and form.[183] Yet whenever Bäck mentions intelligible matter, he does not explain how this kind of matter is attained, nor how

[182] Ibid., 2.
[183] Ibid.

aphairesis is used to arrive at the idea of intelligible matter.[184] Early in his article Bäck suggests that intelligible matter cannot be attained by the subtraction view since it is only applicable to the strictly mathematical '4 - 2' usage. He only explains how the term is used when a mathematical universal is reached. But it cannot be that a mathematical universal is reached by exactly the same method as the particular mathematical in Aristotle's terms. In *Meta.* 1035a33-35 Aristotle states that both particular (e.g. *this* circle) and universal (e.g. circle) seem to be one and the same thing *only* homonymously as there is no proper name (μὴ εἶναι ἴδιον ὄνομα) for every particular circle. A mathematical universal is reached by induction from particulars, whereas we come to know the particular quantitative and continuous by the process of successive removal of passions, motion, and change. Only once these are subtracted, we arrive at the separated mathematical two-dimensional circle which is 'a plane figure', where 'plane' is intelligible matter and 'figure' is form. Whenever *aphairesis* comes into sight in the corpus treating mathematicals, it is used only to denote the quantitative and continuous magnitude which in the mind of a mathematician becomes the matter of intelligible circles.

Although Bäck does not explain how exactly intelligible matter is discovered, it is yet implied that we come to know it by 'abstraction.' But what kind of 'abstraction' is this? He concedes that the method of subtraction is present in Aristotle: to remove equals from equals which are common to all quantities (*Meta*. 1061b20) or to remove the part from a quantum (*Meta*. 1023b13-5). It is suggested that *aphairesis* in these instances highlights only a strictly mathematical usage – it is applied to quantities only: when the term is used in the category of quantum only then *aphairesis* has the meaning of 'subtraction,' but when used otherwise, it

[184] He is uncertain whether intelligible matter is a universal or particular.

should be translated as 'abstraction.' For this reason, he does not agree with Cleary's suggestion to translate every instance of the term as 'subtraction.'

If we are to understand 'abstraction' as abstraction of intelligible matter from sensible matter, then it indeed bears a sense of extraction. Based on what we have seen in the corpus, there are no examples of *aphairesis* as extraction. Instead, the use of successive *aphairesis* in the category of continuum is explicitly shown in *Posterior Analytics* I, 5, *De Caelo* III, 1, *Metaphysics* VII, 3, and in *Metaphysics* XI, 3. I suggest that it is the *qua*-method rather than *aphairesis* that may very well play the role of extraction and selective attention.

Another difficulty with Bäck's interpretation of *aphairesis* is that sometimes he sees it as an auxiliary tool with which the *qua*-method is performed, and sometimes it may seem as if he understands it as a wholly independent process not differing in any way from the *qua*-method as such. Cadavid expresses the same view that "Bäck focuses on 'ἀφαίρεσις' and understands it as 'ᾗ'."[185] To be specific, he seems to conflate three things here: *aphairesis*, the 'filtering out,' and the 'qua' itself. Bäck cites Jonathan Lear with his account of how the *qua*-filter operates:

> In order to mark off an abstract object, like 'two' or 'number', we must be able to specify the aspect that we wish to separate off. We specify an aspect like number so as to generate abstract objects. We then look at our sense perceptions, examine the phenomena, to see what content they have under this aspect. As Lear puts it, we "filter" our experience in order to get at what we have chosen to find relevant. We do not invent the phenomena, but do choose what we want to notice. Hence I suggest conceiving abstraction as selective attention.[186]

Bäck's account seems to suggest that abstraction plays the same role as the 'filtering out' of predicates of which Lear spoke. If we look closely at how Lear treats Aristotle's *qua*-method to which Bäck refers, we will indeed find there some presence of ἀφαίρεσις, though only an

[185] Eva Maria Cadavid, "Existence in a way: resolving a tension in Aristotle's philosophy of mathematics," (PhD thesis): 55.
[186] Allan Bäck, "The Concept of Abstraction." *The Society for Ancient Greek Philosophy Newsletter.* 376. (2006): 8.

implied one. There Lear uses subtraction in the sense of removal. He uses it in exactly the same way as Aristotle in *APo* I.5, 74a35-b4.

> If, for example, *b* is a bronze isosceles triangle – *Br(b) & Is(b) & Tr(b)*, then to consider *b* as a triangle – *b qua Tr* – is to apply a predicate filter: it filters out the predicates like *Br* and *Is* that happen to be true of *b*, but are irrelevant to our current concern. The filter enables Aristotle to make a different use of the distinction between incidental (*kata sumbebekos*) and essential (*kath' hauto*) predication from that which is often attributed to him. By applying a predicate filter to an object instantiating the relevant geometrical property, we will filter out all predicates which concern the material composition of the object.[187]

The implied subtraction in Lear's account plays a simple arithmetical role of removing the predicates like 'bronze' and 'isosceles.' Clearly, Lear in this passage equates subtraction with 'filtering out' of bronze and isosceles. Applying Lear's account to Bäck's statement that *aphairesis* consists in extracting *one* thing while disregarding the rest we run into two incoherences:

a) According to Lear's account in the passage above and Aristotle's passage in *APo* I. 5, 74a35-b4 we subtract two things – 'bronze' and 'isosceles' out of bronze isosceles triangle. Since the filtering out is extraction according to Bäck's account, we extract both 'bronze' and 'isosceles.' But how will this make sense if previously Bäck claimed that *aphairesis* consists in extracting *one* thing while discarding the rest?[188] It is plain that on this account we extract two things, not one.

b) If we nevertheless think of *aphairesis* as extracting *one* thing, then we extract a 'triangle' out of bronze isosceles triangle. The problem is that on this reading both *aphairesis* and the *qua*-filter turn out to be exactly the same methods not differing from one another, since the role of the *qua*-filter consists in designating one out of many. However, the Greek *aphairesis*

[187] Jonathan Lear, "Aristotle's philosophy of mathematics," *The Philosophical Review* 91, no. 2 (1982): 168-169.
[188] Allan Bäck, "The Concept of Abstraction." *The Society for Ancient Greek Philosophy Newsletter.* 376. (2006): 1.

is never an extraction: one strips off many elements that are not of interest and considers the remainder.

The role of extracting is more suitable for the *qua*-locutor – this is what I suggest enables in us the selective attention of which Bäck speaks. But the function of the proper mathematical ἀφαίρεσις is to remove the unnecessary categories within a particular sensible object upon one discovers a magnitude, to filter out predicates, and to remove the subjects which we consider incidental when we look for a primary subject of any given attribute. I especially reject the view that *aphairesis* 'need not be a self-conscious, deliberate process.' The term, as it appears in Aristotle, does not play a role of selective attention or 'aiming at' the way Bäck proposes it to be,[189] instead, it is a self-conscious and deliberate process of removing aspects used by Aristotle for didactic purposes to show where exactly the category of quantity is placed in a sensible body.

To conclude, while I may agree with Bäck that "these universals, or our knowledge of them, is somehow to be abstracted from singular things,"[190] I reject the idea of applying Aristotle's word *aphairesis* to the process of attaining 'universal *abstracta.*' Even if we see the presence of ἀφαίρεσις together with τὸ καθόλου, καθ' ἕκαστον, and ἐπαγωγή in the *Posterior Analytics* I. 18, we shall not think of the former as Aristotle's predominant tool in explaining how we reach a universal, since that passage is the one and only one instance in the whole corpus which might imply such a reading, however ambiguously. The use of *aphairesis* is directed to individuals and their particular mathematicals which are reached by successive method of subtraction. In addition, we shall not conflate *aphairesis* as it appears in Aristotle with the *qua*-method. *Aphairesis* and the *qua*-method are distinct. It seems likely that when one conceives of *this* table *qua* square, the form of the square enters into one's

[189] Ibid., 9.
[190] Ibid., 2.

mind without matter. The process of abstraction becomes immediate, almost instant. *Aphairesis*, however, is a self-conscious deliberate process removing things successively in thinking: upon the removal of passions, affections, and motion we arrive at the quantitative and continuous extension, the result of subtraction. Its function also consists in removing the subjects which we consider incidental when we look for a primary subject of any given attribute. This process is always deliberate in Aristotle and does not have any psychologistic connotations of an instant reception of forms without matter.

5.4 Julia Annas

Before treating Annas' account of Aristotle's *aphairesis*, I must mention in advance that she cites Gottlob Frege's criticism of abstractionism fairly extensively. It should be noted, however, that Frege was criticizing the whole idea of *modern* abstractionism in the philosophy of mathematics and was trying to show why it is inadequate. But this type of psychologistic abstractionism which he was criticizing is not what Aristotle understood his *aphairesis* to be. Frege's criticism of abstraction, which Annas uses as evidence to show why Aristotle's abstraction is inadequate, has nothing to do with *aphairesis.*

Julia Annas[191] mentions no use of *aphairesis* in the process of abstracting either the essence from matter (Alexander of Aphrodisias), a universal (e.g. man) from particulars (Boethius and Aquinas), or a mathematical universal from particulars (Proclus and Aquinas). She employs *aphairesis* only in the discussion of how particular *mathematical properties* and *objects* are *abstracted from matter* (Philoponus and Aquinas).

Annas begins her treatment of Aristotelian abstraction in her book *Aristotle's Metaphysics Books M and N* with the following question: "What is it that is abstracted?" She

[191] For Annas' concept of Aristotle's abstraction, see pages 28-35 of her book *Aristotle's Metaphysics M and N.*

points out that the there is an ambiguity in Aristotle's abstraction because sometimes Aristotle seem to abstract matter like bronze and, sometimes, he abstracts properties like isosceles.

> What is it that is abstracted? Sometimes it seems to be matter (e.g. the bronze of the bronze sphere). This will lead to thinking that geometry studies *properties* like roundness. Sometimes, however, what seem to be abstracted are properties (e.g. the isosceles nature of a particular triangle). This leads to thinking of the geometer as studying *objects*, and this is the view uppermost in *M* 3.[192]

I do not see any problem with applying *aphairesis* in both accounts and do not see any ambiguity here as long as we understand *aphairesis* as *subtraction*. It is true that Aristotle does say elsewhere that to study roundness we need to remove bronze. However, in *APo.* I.5 he also says that to discover in this bronze isosceles triangle what is the primary subject of the attribute 'the sum of internal angles equal to two right angles,' one must subtract isosceles, and also subtract bronze (74a35-b4). If we suppose, for instance, that there were different kinds of circles like there are different kinds of triangles (isosceles, equilateral, etc.), we would have to also subtract this differentia of the circle like we subtract the isosceles from a triangle. But there are no other kinds of circles like there are kinds of triangles. The account of *aphairesis* Annas provides in this case is the proper Aristotelian one, as subtraction. However, based on the way she links it to the *M*.3 passage, she is also forced to think of *aphairesis* as extraction.

Annas is right that book XIII.3 (M.3) is literally filled with the qua-locutor and has not a single reference to *aphairesis* in it. Since there is no *aphairesis* in the text, she connects the qua-locutor with the process of abstraction: "in M 3 what seem to be abstracted are geometrical *objects*."[193] As I have already explained, the application of *qua* suggests an immediate sense of extraction of form from matter and should not be confused with

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[192] Julia Annas, and Aristotle, *Aristotle's Metaphysics: Books M and N*, Clarendon Aristotle Series. (Oxford [Eng.]: Clarendon Press, 1976): 30.
[193] Ibid., 31.

aphairesis, which *successively* unfolds the quantitative and continuous property, finds the primary subject of attributes, and is used for didactic purposes only, i.e. to show that the category of quantity is a part of the sensible objects and is not physically separated from them in the manner of Plato's Forms and intermediates. There is no problem or ambiguity between the two senses of *aphairesis* in Annas' statement, as long as we understand *aphairesis* to have a sense of subtraction, while the extraction of properties, which is the focus of *M*.3, is performed by *qua* isolation.

Annas points to another problem when this sense of *aphairesis* is applied to number and says that it results in "really disastrous difficulties."[194] She cites Frege's account to explain what she means by this. Frege argues that once all differentiating characteristics of two different objects are removed, there will be nothing to distinguish things, and with no multiplicity, there can be no counting of number.

> If through [abstraction] the counting blocks become identical, then we now have only one counting block; counting will not proceed beyond 'one'. Whoever cannot distinguish between things he is supposed to count, cannot count them either… On the other hand, if the word 'equal' is not supposed to designate identity, then the objects that are the same will therefore differ with respect to some properties and will agree with respect to others. But to know this, we don't first have to abstract their differences.[195]

I think we can solve the problem of the two identical blocks in the following way. For these two counting blocks to become identical, they must be made not only of the same material but of the same size. However, it is impossible to build two perfectly cubical blocks of the same size. This is the case because matter is always changing, which means it constantly affects the size of the objects. Secondly, if we suppose matter was still and motionless, it would still be impossible to do so because our human ability to create perfectly straight objects is limited. Therefore, the two blocks will still be different even if they are made from the same material.

[194] Ibid.
[195] Gottlob Frege, *Philosophical Writings*, ed. Geach and Black, (Oxford, 1996), 84-85.

The two cubical forms which enter the soul without matter, will be of the two different sizes even though the senses are not able to recognize this fact.

Annas cites another remark by Frege in order to explain the problems with thinking the result of abstraction as something countable. Here Frege expresses the difficulty of finding out why the two abstracted objects still remain different. For instance, we observe two cats of a different colour sitting side by side. Then, we abstract from their colour, posture, and position, thus depriving them of any content. They, however, still remain different and it is not clear why:

> Inattention is a very strong lye; it must not be applied at too great a concentration, so that everything does not dissolve, and likewise not too dilute, so that it effects a sufficient change in the things ... Suppose there are a black and a white cat sitting side by side before us. We stop attending to their colour, and they become colourless, but are still sitting side by side. We stop attending to their posture, and they are no longer sitting (though they have not assumed another posture), but each one is still in its place. We stop attending to position; they cease to have place, but still remain different ... Finally we thus obtain from each object a something wholly deprived of content; but the something obtained from one object is different from the something obtained from another object-though it is not easy to say how.[196]

Perhaps, the reason why these abstracted cats appear to him to be different is because Frege did not mention the category of quantity in his process of abstraction (he mentioned only colour, posture, and position). The quantitative and continuous magnitude, namely the size of both cats that is what makes them to be different from each other; this is quantity that individuates them both. If we further abstract from the category of quantity (and other categories), we may come to the concept of ‘cat.’

Annas draws the following conclusion: “Number involves counting something – units of some kind; but the units have to be differentiable, so abstracting from the objects to be counted is either futile or disastrous…In any case there is a lack of correspondence in the

[196] Ibid., 85.

application of abstraction to geometry and to number."[197] Annas does not propose either her own or someone else's[198] positive solution to how *aphairesis* can be applied in Aristotle's geometry. She does resolve it for the science of arithmetic. However, what seems to be resolved is, in fact, the modern psychologistic interpretation of *aphairesis* not present in Aristotle, such as abstraction of numbers from groups. Annas accepts the premise that numbers are pure units, reached by abstraction from groups (cows, dogs). To support this idea, she cites Aristotle's passage in *Physics* IV. 14 where he states that even though the number of the two different groups, such as ten sheep and ten dogs, is the same, both decads will still be different (224a2-15).[199] However, the passage cited makes not a single reference to *aphairesis*. In fact, there are no appearances of it in the entire book IV except one single use in chapter 2 which makes no reference to numbers. Aristotle does never employ *aphairesis* in the sense of abstraction of number from groups. It is highly possible that Annas' idea has developed from its geometrical cognate, 'abstraction of form from matter.' The abstraction of numbers from groups is a relatively new notion, absent in the ancient and medieval commentators of Aristotle whom I investigated.

Annas' line of argument seems to suggest that there is no need to abstract a number from groups because number is simply the plurality of physical objects or units counted

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[197] Julia Annas, and Aristotle, *Aristotle's Metaphysics: Books M and N*, Clarendon Aristotle Series. (Oxford [Eng.]: Clarendon Press, 1976): 31.

[198] She does refer to Mueller's article (1970) were he treats *aphairesis*. Specifically, Annas cites his interpretation of his account of intelligible matter (p.167), but then she immediately rejects it for the fact that Mueller's reconstruction was built solely on the later Greek commentators. It is however unclear what is the force of this quotation from Mueller because there he makes no connection to abstraction which Annas discusses.

[199] "It is said rightly, too, that the number of the sheep and of the dogs is the same number if the two numbers are equal, but not the same decad or the same ten; just as the equilateral and the scalene are not the same triangle, yet they are the same figure, because they are both triangles. For things are called the same so-and-so if they do not differ by a differentia of that thing, but not if they do; e.g. triangle differs from triangle by a differentia of triangle, therefore they are different triangles; but they do not differ by a differentia of figure, but are in one and the same division of it. For a figure of the one kind is a circle and a figure of another kind of triangle, and a triangle of one kind is equilateral and a triangle of another kind scalene. They are the same figure, then, that, triangle, but not the same triangle. Therefore the number of two groups also-is the same number (for their number does not differ by a differentia of number), but it is not the same decad; for the things of which it is asserted differ; one group are dogs, and the other horses" (trans. Hardie and Gaye).

together, and that measure is homogeneous with the object measured.[200] We simply have to determine the unit of measurement, which should be a distinctive and indivisible entity: "we can say that the thing is 2 inches long, or weighs 5 pounds … we cannot count until we know what it is that we are to mark out as we count, whether chairs, colours, or what."[201] I agree with this. There is also no need to abstract from the matter of an object in order to count something as 'one:' that which stands as an individual is numerically one, such as for instance the things whose matter is one are numerically one (1016b 32-3) or the things whose substance is one are numerically one (1040b17).[202] Annas gives an adequate account of what Aristotle understands a number to be. Perhaps, what was wrong in the first place is to posit *aphairesis* as the source of production of pure abstract numbers deprived of any content. What Aristotle might have suggested by stating that the mathematician studies 'the results from subtraction' is that s/he can study the physical object not only *qua* an indivisible unit but can also count the number of planes and lines in the separated solid.

Such abstractionist interpretation of an immediate abstraction of mathematicals which has been discussed so far, can cause further difficulties in interpreting Aristotle's texts. For instance, commenting on 1078a 21-31, Annas concludes that the positing activity (τίθημι) of a mathematician is unreconciled with the theory of abstraction which presupposes an immediate separation. Since an abstractionist object is already present to the mind, it is unclear why the mathematician needs to 'posit' (θείη) his objects:

> Aristotle now introduces the idea that the mathematician 'posits' mathematical objects. But it is not clear what the force this has. If it means merely that he 'separates' them in thought from irrelevant properties, then there is no positing about it, in the sense of postulating; for an abstractionist the object is *already there* to be

[200] Julia Annas, and Aristotle, *Aristotle's Metaphysics: Books M and N*, Clarendon Aristotle Series. (Oxford [Eng.]: Clarendon Press, 1976): 36-37.
[201] Ibid., 36.
[202] Ibid., 38.

> studied, not stipulated. If Aristotle does mean that stipulation plays a part, then this comment is unreconciled with the theory of abstraction.[203]
>
> The best way of studying each object would be this: to separate and posit [θείη] what is not separate, as the arithmetician does, and the geometer. A man is one and indivisible as [qua] man, and the arithmetician posits [ἔθετο] him as one indivisible, then studies him neither as [qua] man nor as [qua] indivisible, but as [qua] a solid object. [204] (transl. Annas, *Meta.* XIII.3).

The positing activity of a mathematician, indeed, is unreconciled with the 'theory of abstraction,' because Annas' understanding of abstraction presupposes an immediate extraction. In locating the extension, the intellect must first separate all irrelevant aspects and then identify the quantitative and continuous. This process of locating may very well be taken as 'positing.' For him it was a logical successive and demonstrative method of the removal of sensible aspects to locate or posit the quantitative and continuous, or simply to indicate that the category of quantity is in the substance. In addition, following the line of Aristotle's thought in 1078a 21-31 to which Annas refers, by positing Aristotle may also mean 'deciding' for oneself whether s/he wants to study man *qua* geometrical or *qua* arithmetical. Depending on the science one wants to apply, s/he first needs to posit whether s/he will study a man as a unit, namely *qua* indivisible, or *qua* solid as a continuous divisible magnitude. Once the continuous divisible magnitude is separated, s/he may further continue the process of positing or deciding what part of a separated intelligible solid will s/he study, such as its first, second, or third dimension.

Frege was criticizing the whole idea of modern abstractionism in the philosophy of mathematics and was trying to show why it is inadequate. But this type of psychologistic abstractionism which he was criticizing is not what Aristotle understood his *aphairesis* to be.

[203] Ibid., 150.
[204] Ibid., 96.

As Jonathan Lear rightly points out, it is "a mistake to tar Aristotle with Frege's brush" [205] because "it differs fundamentally from the psychologistic theories that Frege scorned."[206] If we are to interpret Aristotle's *aphairesis* as Frege does applied both to numbers and to magnitudes, then the object, indeed, loses its content and becomes diluted when colour, posture and position are removed. Annas resolves this problem for arithmetic, but only its modern psychological interpretation as abstraction from groups: there is no need to abstract from groups because number is simply homogeneous with the units counted. Annas, however, cannot resolve it for the geometrical *aphairesis*. She does not provide either someone else's or her own positive solution and suggests that Aristotle has a psychological[207] doctrine of geometrical abstraction which is defined as a 'deliberate lack of attention.' To explain what she means by this psychological abstraction, she refers to Sextus Empiricus' work *Against the Mathematicians*:

> Aristotle…says…that length without breadth of which the geometers speak is not unintelligible, but that we can without any difficulty arrive at the thought of it. He rests his argument on a rather clear and indeed a rather manifest illustration of it. We grasp the length of a wall, he says, without attending also to its breadth, so that it must be possible to conceive of the length without breadth of which geometers speak.[208] (Ross, *Adv. Math.* 3.57-58).

This makes her to conclude that Aristotle's "abstraction thus comes down to deliberate lack of attention."[209] It is obvious that Annas sees the parallel between this account in *Adversus Mathematicos* and the account given by Frege where he compared abstraction to

[205] Jonathan Lear, "Aristotle's philosophy of mathematics," *The philosophical review* 91, no. 2 (1982): 184.
[206] Ibid. p. 162.
[207] Julia Annas, and Aristotle, *Aristotle's Metaphysics: Books M and N*, Clarendon Aristotle Series. (Oxford [Eng.]: Clarendon Press, 1976): 32, 33, 149.
[208] "Ἀλλ' ὅ γε Ἀριστοτέλης,καίπερ ποικίλως κατασκευασθείσης τῆς τοῦ πράγματος ἀνεπινοησίας καὶ οὐκ ἐν ὀλίγῳ κειμένων ταράχῳ τῶν γεωμετρῶν, φησὶ μὴ ἀδιανόη τον εἶναι τὸ ὑπὸ τούτων λεγόμενον μῆκος ἀπλατές, ἀλλὰ δύνασθαι χωρὶς πάσης περισκελείαςεἰς ἔννοιαν ἡμῖν ἐλθεῖν. ἵστησι δὲ τὸν λόγον ἐπί τινος ἐναργεστέρου ὑποδείγματος καὶ σαφοῦς. τὸ γοῦν τοῦ τοίχου μῆκος, φησί, λαμβάνομεν μὴ συνεπιβάλλοντες αὐτοῦ τῷ πλάτει, διόπερ ἐνέσται καὶ τὸ παρὰ τοῖς γεωμέτραις λεγόμενον μῆκος χωρὶς πλάτους τινὸς ἐπινοεῖν."
[209] Cf. Bäck's connection of *aphairesis* with 'selective attention' (p.1) and Annas' connection of the term with the 'lack of attention.'

inattention.[210] The problem with Sextus' account is that it makes no reference to *aphairesis*, thus it is illegitimate to associate it with Aristotelian *aphairesis* as such. Even if Aristotle did understand the process of grasping length without breadth the way Sextus explains it, this account has no relation to how *aphairesis* works in the Aristotelian passages we have surveyed.

Contrary to all these various modern interpretations of *aphairesis* we have seen in the accounts of de Koninck, Mansion, Bäck, and Annas, my analysis of all the instances of Aristotle's *aphairesis* has shown that it has no relation to the four epistemological senses of it as *extraction* which presuppose either the idea of receiving forms without matter or the process of induction by means of which the universal is attained. Furthermore, it is wrong to think of *aphairesis* in a psychological way through which one grasps the objects of mathematics by some kind of inattention with respect to some other aspects. It is especially wrong to compare the term to the desire of non-rational animals, as well as to think of it as a non-deliberate and non-conscious operation isolating the objects of mathematics which lay there already at hand. What possibly bears the sense of immediate abstraction is when we consider something *qua* 'F' with which *aphairesis* should not be confused. The authentic Aristotelian sense of the mathematical *aphairesis* comprises the following uses: (1) it successively uncovers the 'layers' of a sensible object for the purpose of showing that the category of quantity is a part of *this* same body and that it is not ontologically and locally separated from it in the manner of Plato's objects, (2) it helps to find the primary subject of any given attribute: we do *not* isolate the primary subject, but, instead, we successively remove all incidental subjects one after another and only then leave the primary one to be studied. In all these and other cases both mathematical and non-mathematical *aphairesis*

[210] Gottlob Frege, *Philosophical Writings*, ed. Geach and Black, (Oxford 1996), 85.

presents itself as a *simple arithmetical*, *self-conscious*, and *deliberate successive subtraction* which, contrary to inattention, may sometimes require a great amount of concentration.

CHAPTER 6 CONCLUSION

In this conclusion I will bring together the major points of my thesis which support my conclusion that Aristotelian *aphairesis* is *not* abstraction or extraction. In chapter 1 I analyzed the etymology of the term ἀφαίρεσις and systematized the nine occurrences of the 'τὰ ἐξ ἀφαιρέσεως' expression into two non-mathematical and seven mathematical uses. I then investigated how the terminology accumulated new meanings in some of the ancient commentaries on Aristotle. Specifically, Alexander of Aphrodisias claimed that forms (essences) are abstracted from matter (*De Intellectu*, 110.19 and 111.16), Philoponus used *aphairesis* for the first time together with particular mathematical forms (*In Aristotelis De Anima Libros,* I.1-2), Proclus understood *aphairesis* as the collection of particulars into one common mathematical universal (Commentary to the First Book of *Euclid's Elements*), equivalent to the abstraction of a mathematical universal from particulars. Even if Proclus was not commenting directly on Aristotelian *aphairesis*, it remains possible that his views had influenced the later Aristotelian commentators. This is the same reason why I also looked into Boethius' application of *aphairesis*. Boethius, translating *aphairesis* into Latin as *abtractio,* used it in the sense of abstracting a universal (e.g. man) from particulars (in Boethius' commentary on Porphyry's *Isagoge*, 11.2, 11.6, and 11.7). These views constitute the four epistemological senses of *aphairesis* which Cleary seems to have had in mind. Thus the three uses of *aphairesis* which Cleary attributes to Aquinas– abstraction of a particular mathematical form from matter (*Summa Theologiae,* I^{a} q. 1 a. 1 ad 2), abstraction of a universal (e.g. man) from particulars and abstraction of a mathematical universal (e.g. circle) from particulars (commentary on *De Anima* I. IV) emerge in the earlier reception of Aristotelian *aphairesis*.

In chapter 2 my analysis was directed to the general application of *aphairesis* in the works of Plato and Aristotle. Since Aristotle extensively argued against Plato's objects of mathematics, I considered it necessary to find out how Plato used the term and whether it played any role for him in reaching the objects of mathematics or Forms. My analysis has shown that Plato used *aphairesis* in three ways: as a simple arithmetical subtraction of things and concepts in thought, as a deprivation of physical things such as wealth, slaves etc., and as an intellectual activity of abstracting the Form of love and the Form of the good from their particular instantiations (*Rep*. 534b-c and *Symp.* 205b4). Since Plato used *aphairesis* together with *eidos*, I suggested that the term could possibly have had an extractionist sense for him. I then showed that ἀφαιρεῖν in Aristotle never presupposes a sense of extraction, and even in the realm of non-mathematicals it has the meaning of consecutive removal of unnecessary objects.

In chapter 3, I considered how Aristotle used *aphairesis* in a more technical way, such as in finding the quantitative and continuous in the physical object or locating and finding the substance is in this object. I examined the two non-mathematical uses of the 'τὰ ἐξ ἀφαιρέσεως' expression and concluded that neither the *Posterior Analytics* I. 18 nor the *Metaphysics* XIII. 2 passages make reference to the objects of mathematics. In the former controversial passage, the result of subtraction is a non-mathematical universal. In the latter passage the result of subtraction is 'pale' (τὸν λευκόν), though the objects of mathematics such as lines and planes are indirectly implied. The other seven occurrences of 'τὰ ἐξ ἀφαιρέσεως' do have a clear mathematical application. A close study of both mathematical and non-mathematical expressions and *aphairesis* as such, demonstrated that none of their appearances in the works of Aristotle suppose a sense of *extraction* of one out of many. Furthermore, none of the uses of the expression have the applications found in the ancient and medieval commentators: abstraction of form (essence) from matter, abstraction of universal (e.g. man) from particulars, abstraction of mathematical universal (e.g. circle) from particulars, or abstraction of *this* particular mathematical form (*this* circular shape) from *this* particular matter, motion, and change. Instead, my analysis has shown that Aristotle's non-mathematical *aphairesis* is *subtraction*, that is when we take away or remove many things and then study the remainder. Aristotle's mathematical *aphairesis* is equally also *subtraction*: we remove colour, passions, affections, and motion from a bronze isosceles triangle, and consider the two-dimensional continuous shape only, such as triangularity. Contrary to Cleary, who claims that the main function of the mathematical 'τὰ ἐξ ἀφαιρέσεως' expression and *aphairesis* terminology is to find out the primary subject of any given attribute, I argued that the main purpose of both terms is rather that of *uncovering* and *elucidating* the *spatial location* of the sensible magnitude, so as to show that the category of quantity does not exist outside and separately in the manner of Plato's intermediates and Form numbers. Finally, I argued that Aristotle's *aphairesis* should not be considered separately from his concept of *intelligible matter* (ὕλη νοητή) and the concepts of *potentiality* (δύναμις) and *actuality* (ενέργεια or ἐντελέχεια) if a plausible account of Aristotle's objects of mathematics is to be given. My analysis of these concepts has shown the following picture of how exactly we should interpret Aristotle's objects of mathematics: the objects of mathematics, while 'existing' potentially as physical continuous extension to be unfolded by the successive method of subtraction, subsequently 'exist' actually in thought as a compound of intelligible matter and form.

In chapter 4 I investigated the types of confusions that exist in modern interpretations of Aristotle's theory of abstraction, interpretations, which were built upon, consciously or unconsciously, certain especially influential ancient commentators on Aristotle's views. The scholars, whom Cleary mentioned, do indeed have an epistemological interpretation of *aphairesis* of which I identified there to be four specific uses and then subsequently united them all under the term *extraction.* My analysis has demonstrated that the commentaries of de Koninck, Mansion, Annas, and Bäck do show the traces of connection between their views and the views of Alexander of Aphrodisias, Philoponus, Proclus, Boethius, and (especially) Thomas Aquinas.[211]

Contrary to Plato's independent substances, points, lines, planes, solids, and numbers are not substances for Aristotle, but rather the properties that depend on substances. Thus, to show that the category of quantity does not exist outside the sensible body, Aristotle, by subtracting all other categories, indicates that the category of quantity is not ontologically separated in the manner of intermediates and Forms and that it is a part of a sensible body. By removing motion and change in thinking, Aristotle, perhaps, meant that there is a certain moment in time when the $t_0 \pm \Delta t$ triangle becomes a motionless t_0 triangle which can be conceived only by the intellect. The precision problem can be solved by comprehending a perfect sphere which is *enclosed* into an inexact sensible one. This means that the matter of a perfect sphere occupies the actual space of the matter of an imperfect one. Thus, the perfect sphere is potentially present in the inexact one which can be brought to actuality only in thinking but not in reality. It is the thinking activity only that can think of the *enclosed* sphere which at some moment in time t_0 becomes a perfect actual sphere in the former. There is however no such account in Aristotle's corpus of the perfect t_0 mathematicals occupying the space of the matter of imperfect ones – this is merely a reconstruction which I propose to

[211] The following outline represents the parallel between the views of the ancient and modern commentators:
($_1$) Alexander: abstraction of form (essence) from matter (N/A);
($_2$) Boethius and Aquinas: abstraction of a universal from particulars (Bäck, de Koninck, Mansion);
($_3$) Proclus and Aquinas: abstraction of a mathematical universal from particulars (Bäck, de Koninck, Mansion);
($_4$) Philoponus and Aquinas: abstraction of *this* particular mathematical form (*this* circular shape) from *this* particular matter, motion, and change (Bäck, Mansion, Annas).

explain how mathematics can be true of the sensible world and why precise mathematical objects are not merely the creations of mind alone.

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