MULTI-CRITERIA APPROACH TO MARITIME SEARCH AND RESCUE LOCATION ANALYSIS

by

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To My Loving Wife and Parents.

All this work would have been impossible without your unconditional love and

endless support.

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Abstract

Operating on the ocean can be risky, particularly in harsh weather, or under economic drivers as with the offshore industry or fishing activities. In addition to advances in safety technology and practices, a robust Search and Rescue (*SAR*) capability is a key factor for mitigating risks and improving the safety of Canadians. The Canadian Coast Guard strives to provide an acceptable maritime *SAR* service. Optimizing the efficiency of limited resources helps to ensure that the Coast Guard's maritime *SAR* services are used to best advantage. For strategic and tactical planning, this involves Location-Allocation modelling to ensure that the right assets are in the best place to respond effectively. This problem becomes more complicated when we are faced with several criteria for assessing decision outcomes, some of which are conflicting as well.

The contribution of this study is a framework of mathematical models to support efficient management of maritime *SAR* resources with regard to several criteria such as primary and backup coverage, mean access time, service equality, and cost. A scenario planning approach is adopted along with spatial density estimation to deal with uncertainty of future incidents at sea. Several models are developed in multiple phases of the study with different purposes and complexity to determine the optimal location and response allocation of *SAR* resources, aiming to achieve greater responsiveness and resource utilization.

The multi-criteria analyses, developed in different stages of this study, provide a range of good trade-off solutions. Comparing the performance of solutions obtained by the developed models with the current arrangement of the *SAR* fleet, indicates an appreciable potential improvement in terms of coverage, accessibility and efficiency of service. Such improvements can be achieved through several changes in fleet composition and/or location. Results of this study can guide decision makers with regards to *SAR* vessel acquisitions and placement in order to improve the efficiency of resources and increase the service level. More specifically, the outcome of this study provides the Canadian Coast Guard with some beneficial insights for future resource planning including fleet renewal planning, station locations for new vessels, and the arrangement of the current fleet.

List of Abbreviations Used

AHP	Analytic Hierarchy Process
CCG	Canadian Coast Guard
CCGA	Canadian Coast Guard Auxiliary
DFO	Department of Fisheries and Ocean
DND	Department of Natural Defense
FGP	Fuzzy Goal Programming
FST	Fuzzy Set Theory
GIS	Geographic Information System
GP	Goal Programming
JRCC	Joint Rescue Coordination Centre
KE	Kernel Estimation
LSCP	Location Set Covering Problem
MADM	Multi attribute Decision Making
MARIN	Maritime Activity and Investigation Network
MCDM	Multi Criteria Decision Making
MCGP	Multi-Choice Goal Programming
MCLP	Maximal Covering Location Problem
MCMCLP	Modular Capacitated Maximal Covering Location Problem
MEMTV	Medium Endurance Multi-Tasked Vessel
MEXCLP	Maximum Expected Covering Location Problem
MILP	Mixed Integer Linear Problem
MODM	Multi Objective Decision Making
MOO	Multi Objective Optimization
MSAP	Maximum Service Area Problem
MRSC	Maritime Rescue Sub-Center
NM	Nautical Mile
OPV	Offshore Patrol Vessel
RO	Robust Optimization
SAR	Search and Rescue
SAW	Simple Additive Weighting

SISAR	Search and Rescue Information Management System
SP	Stochastic Programming
USCG	United States Coast Guard
WCB	Workers Compensation Board

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Chapter 1   Introduction

1.1. Background and Problem Definition

Maritime Search and Rescue on the sea is one of the most visible humanitarian activities in Canada. The aim is to minimize loss of of life, injury, property damage and risk to the environment. Search and Rescue (*SAR*) can be categorized as an emergency response activity, and in many countries, it is considered a public service. In Canada, the Canadian Coast Guard (*CCG*) provides maritime *SAR,* operating a diverse fleet of vessels, in cooperation with the Department of National Defence (*DND*) that provides airborne services. According to *CCG* reports (Canadian Coast Guard 2014), every year on average, 97 percent of the lives at risk in maritime distress who requested emergency assistance are saved (2,200 lives per year). Another 18,000 people are helped each year in non-distress maritime incidents by the *SAR* system. Thus, the maritime *SAR* system is a vital government service.

The Canadian National Search and Rescue objective is "to prevent loss of life and injury through *SAR* alerting, responding, and aiding activities using public and private resources" (Abi-Zeid and Frost 2005). Moreover, in a *SAR* operation, the difference between life and death can sometimes be measured in minutes. Additionally, a *SAR* operation consumes considerable resources in terms of time, effort and money. Thus, emergency response actions should be well-planned and efficiently organized (Razi and Karatas 2016).

Maritime *SAR* is one of the main responsibilities of the *CCG* and it is among their strategic priorities to improve their *SAR* service level by enhancing the fleet capability and utilization. Since time is of the essence in *SAR* operations, it is paramount that their resources, including *SAR* vessels, stations, and crew, are used efficiently and effectively. To do this, one important factor is to decide where to site their resources and how to allocate incidents to the located resources. This kind of problem is known as the Location-Allocation problem. The *CCG* has many different *SAR* vessel types that were designed or purchased with specific tasks in mind, and not all are equally effective at handling specific incident types. Also, the ranges and speeds vary greatly among different types of *SAR* vessels, so response vessel characteristics need to be considered in any study on this matter.

This study falls into the field of emergency location analysis. Many researchers have studied this area and hence the literature is quite rich. In a typical *SAR* location problem, we are faced with a "server to customer" service system with mobile servers, which is similar to the ambulance location problem. While the servers represent the *SAR* vessels, the customers (demands) symbolize the maritime incidents. In this kind of location problem, the main goals are typically to respond to an incident within a minimum amount of time and/or maximize the area under coverage.

In literature, there are mathematical location models to maximize the number of incidents that can be serviced and minimize the time it would take to arrive at the incidents. However, several differences exist in the problem setting. First of all, in the case of emergency vehicle location, all response units such as ambulances are usually assumed to have the same capability and speed which is not the case in maritime *SAR*. Secondly, the method of computing distances to the incidents is quite different, as the rescue vessels are patrolling on the sea, a land-avoidance algorithm must be used to calculate the travel distance, rather than Euclidean or Manhattan distance metrics (Li 2006).

This research examines the optimal locations of *SAR* vessels in Atlantic Canada with respect to several decision criteria. The objective is to ensure the maximum likelihood of saving lives and mitigating property loss using available response resources. Access time is a common proxy measure for the likelihood of saving lives and property. The access time is defined in this study as the elapsed time from the departure from the station of a *SAR* vessel to the arrival at the scene of an incident. Coverage, which indicates the proportion of demand that can be responded to within a prespecified time limit, is another common performance metric in such problems. In addition to access time and coverage, this research also aims to address other key factors and decision criteria such as backup coverage, service equality, and cost.

Although there is extensive literature focusing on different aspects of emergency response location problems, there are few studies conducted for the particular case of maritime *SAR* with its special characteristics and conditions. Furthermore, to the best of our knowledge there is no comprehensive study that simultaneously considers different aspects of the problem including multiple objectives, uncertainty in demand, the possible changes in fleet

composition, and seasonal relocation of resources. The approach presented in this thesis integrates spatial data analysis, simulation, and multiple emergency location optimization models to develop a decision-making framework for mid- to long-term tactical and strategic decisions. Figure 1-1 summarizes the various aspects of the problem which are considered in this study, including decision criteria and exogenous factors such as uncertainty of demand.
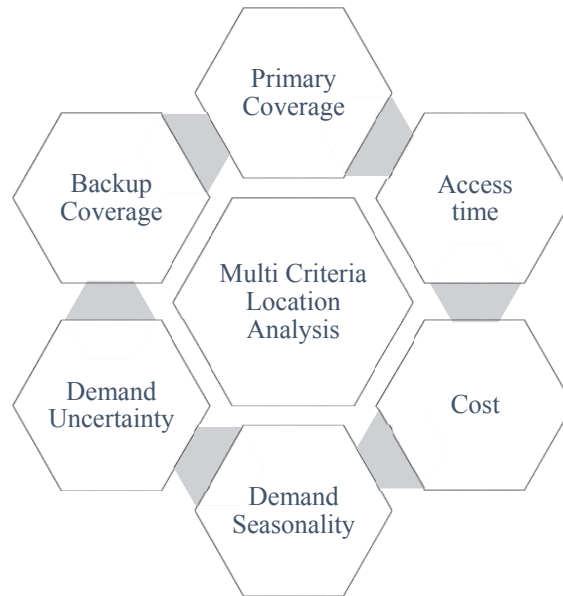
Figure 1-1- Modelling approach with different factors

1.2. Maritime Search and Rescue in Canada

Canada is a maritime nation surrounded by three oceans, whose population and economy make significant use of waterways for commercial and recreational purposes. The marine environment can be extremely dangerous as every year more than 6,000 incidents are reported in waters around Canada. To mitigate the consequences of these incidents, Canada strives to provide effective maritime Search and Rescue services.

> "Search and Rescue comprises the search for, and the provision of aid to, persons, ships or other craft which are, or are feared to be, in distress or imminent danger."
>
> (Canadian Forces 1998)

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The primary goal of the National *SAR* Program is to save lives at risk throughout Canada. This national program involves federal departments, volunteers, organizations, municipalities, provinces and territories, working together to provide this service. As part of the Department of Fisheries and Oceans (*DFO*), the *CCG* is the principal civilian maritime operational arm of the Government of Canada. It is responsible for providing maritime resources for the *SAR* mission in areas of federal responsibility. The *CCG* operates all *DFO* vessels and provides services for *SAR*, Environmental Response, Icebreaking, Marine Navigation Services, and Marine Communications and Traffic Services. Figure 1-2 demonstrates the organizational hierarchy of the *DFO* including the *CCG* structure in three regions of Canada as well as Operations and Vessel Procurement departments.

```
Minister of Fisheries and Oceans

Deputy Minister

Associate Deputy Minister        Commissioner,
                                 Canadian Coast Guard

Regional Directors General    Sector Heads    Deputy Commissioner    Deputy Commissioner
                                              Vessel Procurement      Operations

RDG, Pacific          ADM, Ecosystems
                      and Fisheries Management

RDG, Central and Arctic   ADM, Human Resources and    Assistant Commissioners    Directors General
                          Corporate Services

RDG, Quebec           Chief Financial Officer      AC, Western          DG, Operations

RDG, Gulf             ADM, Strategic Policy        AC, Central and Arctic   DG, Integrated Technical
                                                                            Services

RDG, Maritimes        ADM, Program Policy          AC, Atlantic         DG, Program Requirements

RDG, Newfoundland     ADM, Ecosystems and                               DG, Integrated Business
and Labrador          Oceans Science                                    Management Services

                      Chief Audit Executive                             DG, Major Projects

                      DG, Communications

                      Senior General Counsel

                      Executive Director Values,    ADM = Assistant Deputy Minister
                      Integrity and Conflict        RDG = Regional Director General
                      Resolution                    AC = Assistant Commissioner
                                                    DG = Director General
```

Figure 1-2- Department of Fisheries and Oceans organizational hierarchy (DFO website 2017)

The *CCG* is responsible for a number of *SAR* tasks including the detection of maritime incidents and, with the assistance of the Department of National Defence (*DND*), the coordination, control and conduct of *SAR* operations in maritime *SAR* situations within Canadian areas of federal responsibility. In addition, *CCG* is in charge of the provision of maritime resources to help with aeronautical *SAR* operations as necessary; and, when and where available, the provision of *SAR* resources to assist in humanitarian and civil incidents within provincial, territorial or municipal areas. The *CCG* also coordinates, controls and

conducts *SAR* Prevention programs to reduce the number and severity of maritime *SAR* incidents (CCG website 2017).

The *CCG*'s *SAR* Program has four important elements: management, monitoring, operation, and volunteers. The goal of management and monitoring is to ensure that the *SAR* Program operates at maximum efficiency. This is accomplished by ensuring that *SAR* coverage requirements are continuously adjusted to meet changing needs and that specialized primary *SAR* units are deployed as required. To further enhance response capabilities, *SAR* Program management cooperates with other program managers in the deployment of multi-tasked and secondary resources. These combined efforts ensure that capable emergency services will be readily available when and where they are most likely to be needed.

The following categories of vessels are used in maritime *SAR*:

- Primary *SAR* Vessels

A primary *SAR* vessel is a specially designed, equipped and crewed vessel that has *SAR* as its main responsibility. These vessels are stationed in areas that have a high risk of *SAR* incidents. These vessels maintain a maximum 30-minute state-of-readiness but are typically ready to respond the moment an alert is received.

- Multi-tasked *SAR* Vessels

Multi-tasked *SAR* vessels are tasked to deliver the *SAR* Program and at least one other operational program. They have to remain within a specific *SAR* area while they are multi-tasked to the *SAR* Program and maintain all *SAR* operational standards. Multi-tasked vessels increase fleet utilization, reduce costs to the government, and stand in for primary *SAR* vessels when necessary.

- Secondary *SAR* Vessels

Secondary *SAR* vessels are all other government vessels.

- Canadian Coast Guard Auxiliary Vessels

The Canadian Coast Guard Auxiliary (*CCGA*) is a volunteer organization made up of five regional non-profit associations and a national council which assists the Coast Guard in *SAR* response and incident prevention activities. The *CCG* assists the *CCGA* with the specialized *SAR* training necessary to become, and remain, a member.

- Vessels of Opportunity

A vessel of opportunity is any other vessel not mentioned above, close enough to provide assistance to a vessel in distress. Under the *Canada Shipping Act* and international law, every vessel at sea is required to assist in a distress situation.

Although, all vessels mentioned above can be tasked to *SAR* missions in the case of emergency need, for planning purposes only those vessels that are owned and operated by *CCG* should be included in an analysis, because other vessels are not managed by *CCG* and their positioning and allocation is not under its control. Therefore, in this study we only consider *Primary* and *Multi-Tasked SAR vessels*.

1.2.1. Rescue Co-Ordination Centres and Maritime Rescue Sub-Centres

The *CCG* jointly staffs three Rescue Co-ordination Centres (*JRCCs*) with the Canadian Forces, which are located in Victoria, British Columbia, Trenton, Ontario, and Halifax, Nova Scotia. The *CCG* also operates a Maritime Rescue Sub-Centre (*MRSC*) at Quebec City, Quebec. The function of a *MRSC* is to reduce the *JRCC*'s workload in areas of high marine activity. The *JRCCs*/*MRSCs* are responsible for the planning, co-ordination, conduct and control of *SAR* operations.

1.2.2. Canadian Coast Guard Fleet Renewal

Fleet renewal is one of four strategic and management priorities of the *CCG*. The *CCG* is always looking to modernize and renovate their fleet for improving the quality of their services. *Renewing Assets*, *Delivering Risk-Based and Client-Focused Services*, *Enhancing Capacity to Respond to Marine Incidents*, and *Advancing Workforce and Business Management Practices to Improve Program and Service Delivery* are four main strategic priorities listed in the current *CCG* strategic plan.

The Government of Canada has demonstrated a strong commitment to the *CCG* and Canada's shipbuilding industry. Since 2005 to date (2016), approximately 7 billion dollars (Canadian) have been committed for fleet investments, in addition to the procurement of small vessels and craft that *CCG* funds from its annual capital budget. These investments enable the renewal of the *CCG* fleet as current vessels reach the end of their operational lives. Furthermore, *Vessel Life Extension* and *Mid-Life Modernization* Programs are in place to determine how to best maintain the aging fleet of vessels, until new ships are delivered.

1.2.2.1. CCG Fleet Renewal Plan 2017

According to *CCG*'s Fleet Renewal Plan, there will be a comprehensive 30-year strategic investment plan which outlines asset requirements into the future. This investment plan incorporates procurement of various type of vessels to operate in different programs.

- New *SAR* Lifeboats

CCG will procure and deliver up to 15 *SAR Lifeboats* within next half decade, to replace the existing Arun-Class vessels at their home ports. The *SAR Lifeboats* are designated as primary *SAR* vessels and are specially designed, equipped and crewed for that purpose.

- Medium Endurance Multi-Task Vessels and Offshore Patrol Vessels

In October 2013, the Government of Canada announced that up to ten additional large vessels are to be built for the *CCG* fleet at an estimated cost of $3.3B. *CCG* will acquire up to five *Medium Endurance Multi-Tasked Vessels* (*MEMTV*). The *MEMTV* are large, shallow draught vessels capable of supporting many *CCG* programs primarily for the deployment, recovery and maintenance of aids to navigation. The *MEMTV* will also be capable of *SAR*, icebreaking, fisheries management and environmental response. In addition, *CCG* will acquire up to five *Offshore Patrol Vessels* (*OPV*). The *OPV* are large vessels that will be used primarily for fisheries protection, both in Canadian waters and on the high seas. The *OPV* will also be capable of *SAR*, aids to navigation support and environmental response.

These huge investment plans for renewing and modernizing the *CCG* fleet, make it crucial to study and analyze the best approach for planning their positioning and deployment in

order to ensure maximum utilization is gained. There are several models in this research that deal with Location-Allocation of *CCG's SAR* vessels. In each phase of the study, depending on the purpose of the model, either the current composition of vessels or plausible new vessels might be included in order to provide useful insights for tactical and strategic decisions on fleet mix and deployment.

1.2.3. *Maritime SAR in the Canadian Atlantic Region*

The *CCG*'s Atlantic Region encompasses approximately 2.5 million km^2 of the Northwest Atlantic Ocean and the continental shelf, and is bordered by nearly 40,000 km of coastline along Canada's East Coast provinces: New Brunswick, Newfoundland and Labrador, Nova Scotia, and Prince Edward Island. With long ice seasons and extreme weather conditions second only to the Canadian Arctic, the Atlantic Region handles the highest proportion of distress incidents and the largest percentage of maritime *SAR* cases in Canada annually, with the *SAR* zone extending halfway across the Atlantic Ocean (Canadian Coast Guard 2014). Figure 1-3 demonstrates the *CCG* regional boundaries.

The Atlantic region must also remain ready to take action against environmental pollution incidents, given that the region is home to the largest oil handling port (Canaport, approximately 9 km southeast of the city of Saint John, New Brunswick) in Canada, the periodically expanding offshore oil industry, and millions of tonnes of potentially polluting cargo and vessel fuel which transit through the region each year. To deal with the challenges it faces, the Atlantic Region is home to more Coast Guard resources than any other region in the country.

As outlined in a recent *CCG* strategic plan (Canadian Coast Guard 2014), annually, on average the Atlantic Region:

- Has 51 vessels/helicopters and 6,698 Aids to Navigation
- Employs 36.9% of *CCG*'s employees
- Breaks 22,600 km of ice-packed waterways
- Has over 111,700 commercial vessel transits
- Responds to 2,500 *SAR* incidents
- Manages 200 environmental response calls

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Figure 1-3- CCG operational regions (CCG website 2017)

Hence, Atlantic Canada is the region with the highest demand for maritime *SAR* service, which justifies the need for studying the optimal allocation of limited *SAR* resources in this area. Appendix A provides detailed information regarding the current *CCG's SAR* vessels in Atlantic region.

1.3. Data Sources

This study requires valid and real data on the demand for *SAR* services (i.e. maritime incidents) and on the available *SAR* resources. The main data source for our study is related to historical demand arising from maritime incidents, and the current *CCG's* fleet composition and arrangement in the study area.

1.3.1. Incident Data Set

The dataset being used in this study is derived from the *CCG*'s *SISAR* (Search and Rescue Program Information Management System) which collects all maritime incident information associated with *SAR* missions conducted within Canadian areas of *SAR* responsibility. *SISAR* is a quite detailed record (when filled out completely) of all the incidents where *SAR* missions were tasked. The *SISAR* database includes fields on: incident

date/time and location, incident type and cause, incident response summary (action taken), details on vessels involved, rated severity of the incident, and atmospheric conditions (wave height, wind speed, wind direction, wind-against-current, visibility, ceiling, air temperature, sea surface temperature, clouds, ice, weather comments, atmospheric conditions, tide states). In this study, the information related to location and time of incident occurrence is used and additional information such as vessel characteristics and weather condition are not considered in the models either due to low fill rate, inaccuracy or irrelevancy to the modelling approach.

The Canadian Coast Guard classifies incidents according to their type and level of severity, which are also captured in the *SISAR* database:

- M - Maritime Incidents (M1, M2, M3, M4)
- A – Aeronautical Incidents (A1, A2, A3, A4)
- H – Humanitarian Incidents (H1, H2, H3, H4)
- U – Unknown Incidents (U4).

Since this study focuses on analyzing maritime *SAR*, only maritime incidents are considered in this thesis. Humanitarian are rare events and thus are excluded from this study. We assume Maritime incidents are sub-classified according to the level of their severity as follows (Canadian Coast Guard 2000):

- M4- False alarms and hoaxes: Situations that cause the *SAR* system to react but which prove to be unjustified or fabricated, such as a mistaken report of a flare.
- M3- Incidents resolved in the uncertainty phase (Non-Distress): No distress or perceived appreciable risk to life apparent. An uncertainty phase exists when:
  1. There is doubt regarding the safety of a vessel or the persons on board;
  2. A vessel has been reported overdue at destination; or
  3. A vessel has failed to make an expected position report.
- M2- Potential Distress incidents: The potential exists for a distress incident if timely action is not taken; i.e., immediate responses are required to stabilize a situation in order to prevent distress. This incident exists when:

1. There is apprehension regarding the safety of a vessel or the persons on board;
2. Following the uncertainty phase, attempt to establish contact with the vessel has failed and inquiries addressed to the other appropriate sources have been unsuccessful; or
3. Information has been received indicating that the operational efficiency of a vessel is impaired but not to the extent to be a distress situation.

- M1-Distress incidents: Distress phase exists when:
1. A vessel or a person is threatened by grave and imminent danger and requires immediate assistance (Life-threatening situation was judged to be present or close at hand at some point during the incident);
2. Following the previous phase, further unsuccessful attempts to establish contact with the vessel and more widespread unsuccessful inquiries point to the high probability that the vessel is in distress; or
3. Information is received which indicates that the operating efficiency of the vessel has been impaired to the extent that a distress situation is very likely.

Also, 21 types of maritime incidents are identified by *CCG*, which are as listed in Table 1-1.

Table 1-1- Maritime incident types

Maritime incident types		
capsized	medical	airborne emergency
disabled	foundered	missing person(s)
disoriented	suicide	ditching
grounded	suicide attempt	stranded
false alarm	taking on water	body recovery
man overboard	forced landing	person in water
on fire	crash	other

1.3.2. SAR Stations

Based on the current situation, there are 18 onshore *SAR* stations in Atlantic Canada which are able to house *SAR* vessels. It is assumed that all stations can accommodate all vessel types and no restriction is applied in this regard.

Also, 19 potential 'offshore stations' are to be considered in our analyses. Of course, this is not a station in the traditional sense, but a central location for a vessel that spends much of its time patrolling or performing other tasks at sea. It should be noted that some *CCG* vessels have offshore patrolling responsibility with long endurance capability. These vessels usually spend most of their operational time away from inshore stations. Initially, some of these offshore stations are the centroid of each maritime subarea in Atlantic Canada (as determined by the *CCG*) and others are the actual location that some offshore vessels are currently located. So, there will be 18 onshore stations and 19 potential offshore stations considered in the analysis. One assumption in the mathematical models, represented as constraints, is that smaller *CCG* vessels, called lifeboats, cannot be located at offshore stations because their maximum range and endurance are not sufficient for offshore patrolling.

1.3.3. Land-Avoided Distances

The distances between incidents' locations and *SAR* vessels are required to perform model calculations. There are different methods for distance calculation. The most common way is calculating straight Euclidean distance. However, there is an issue for using straight or direct route calculations in this study. In some cases, it is not possible to use the straight route assumption because of land obstacles in the way. To deal with this problem, we use a previously developed land avoidance algorithm to find the shortest route between incidents and vessels while avoiding land obstacles. The error in distance calculation associated with the earth curvature is ignored since is it negligible (less than 1%) at the typical distance level in the area of interest.

1.4. Study Scope and Historical Trends

The Atlantic Canada region serves as our study area, with its maritime boundaries defined in Figure 1-4. Historical incidents with different severity levels are shown in this figure as well.
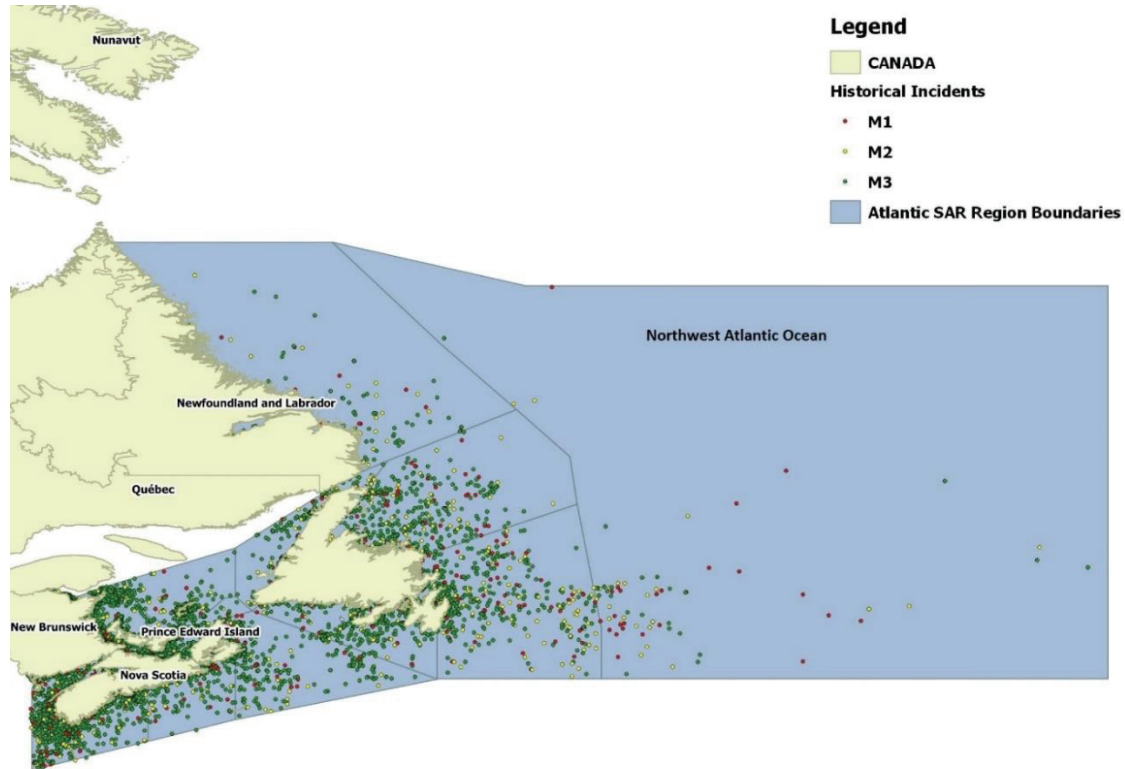
Figure 1-4- Historical incidents in the area of study (Atlantic Canada, 2005-2006,2008-2012)

The incident dataset, which has been checked and cleaned for quality control, is available from 1988 to 2013 (the data for year 2013 is incomplete, up to October), but to have a more accurate analysis, we chose the cleanest and the most reliable recent data from 2005 to 2012 for this study excluding 2007 which has data deficiencies due to a *SAR* management system change.

In Figure 1-5, the annual number of incidents (M1 to M3) in Atlantic Canada is presented over the period of study (2005 to 2012). There is no clear trend in the annual incident total given the fluctuations, although a slight decrease can be observed. As noted above, 2007 levels are lower since there was some loss in data for several months due to the system switchover. For this reason, 2007 will be excluded from the analyses.
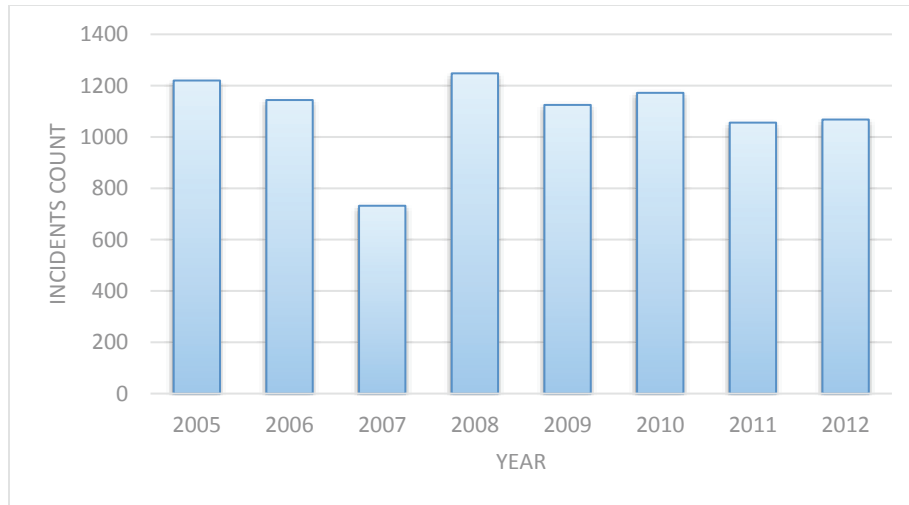
Figure 1-5- Number of incidents in the Atlantic region over the years (M1, M2, M3)

Figure 1-6 demonstrates the monthly distribution of historical incidents over the included years. It is obvious that there is a peak period starting from May until September every year, during which the majority of incidents occurred. This insight could be beneficial for the analysis, particularly when we are looking at seasonal resource planning.
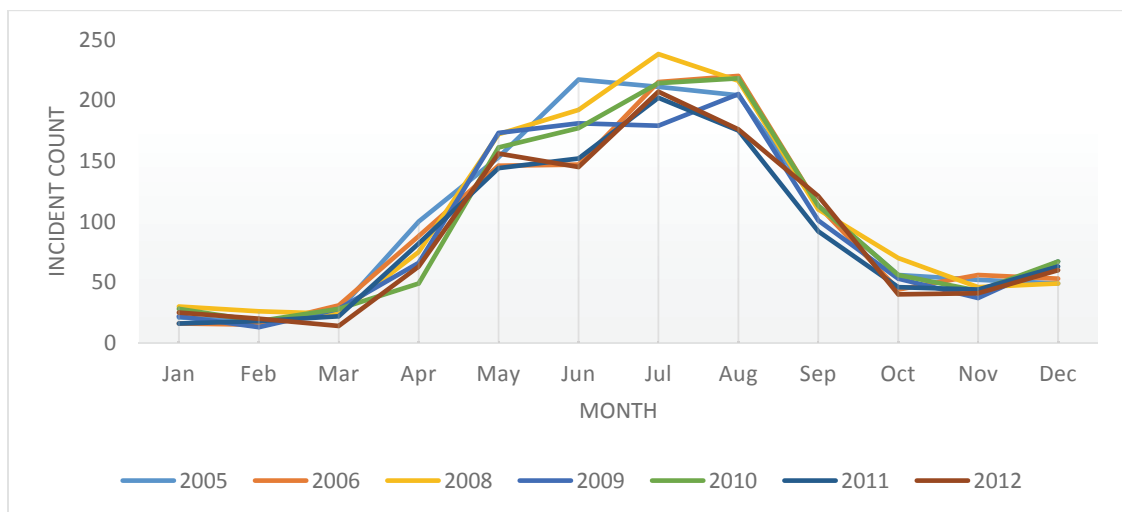
Figure 1-6- Incidents distribution by month in the Atlantic region (M1, M2, M3)

1.5. Research Objectives

Although there have been extensive studies on many aspects of emergency location analysis (as demonstrated in the next chapter with additional descriptions in subsequent chapters), some gaps still exist, particularly in the case of maritime *SAR* resource planning. In the limited research in this specific field, the proposed models typically have a single

objective while in the real case there are more than one criterion that decision makers care about. What makes this study novel in the literature is first considering several criteria as objectives in the mathematical model concurrently. Coverage, cost and mean travel time or distance are widely used in multi-criteria location analyses as model objectives. We aim to incorporate various decision criteria such as coverage, access time, service equality, and cost into our mathematical models. Moreover, other aspects of the problem including uncertainty of demand, possible changes in fleet composition, and potential seasonal rearrangement of the fleet are taken into consideration.

This research targets several objectives in particular. These objectives include:

i.   Analyze and understand different important decision criteria and service level requirements in maritime *SAR*, and figure out an appropriate methodology for measurement and incorporating them into a mathematical model;

ii.   Conduct spatial data analysis for extracting location patterns of incident occurrences and leverage that in order to find a better representation of future demand for resource positioning purposes;

iii.   Address the uncertainty involved with the future incident locations by applying an appropriate simulation methodology to account for different possible incident distribution schemes;

iv.   Develop customized multi-criteria location models to find the optimal or near-optimal solutions to the maritime *SAR* Location-Allocation Problem;

v.   Develop strategic level models for a long-term planning horizon aiming at determining the best vessel procurement and allocation strategy given the results of mathematical models on the optimal composition of *SAR* vessels and their best siting plan;

vi.   Build tactical and operational level decision models, with regard to making informed decisions on seasonal rearrangement of existing vessels as well as the demand assignment plan;

vii.   Analyze and compare the current situation of resource arrangement with the optimal solutions obtained from developed mathematical models;

viii. Extract managerial insights for use by decision makers on resource planning and policy development.

Overall, this thesis focuses on developing a framework for the multi-criteria analysis of the location and allocation of maritime *SAR* resources with the goal of improving the service quality while reducing the costs. Figure 1-7 summarizes the necessary steps that are considered for conducting this study. Ultimately, the aim of the study is to provide valuable insights assisting decision makers for defining better strategies and policies for managing Coast Guard *SAR* resources to ensure the maximum quality of response to maritime incidents given limited resources. Such outcomes are particularly beneficial in the light of the close relationship with the two *CCG* strategic priorities: (1) Enhancing capacity to respond to marine incidents and (2) fleet renewal.

| **Problem Definition** |
| Understand different relevant factors; define decision variables, objectives and constraints; choose appropriate modelling approach |

| **Spatial Demand Analysis** |
| Utilize kernel density estimation to extract patterns in historical incidents location |

| **Simulation of Demand Scenarios** |
| Simulate a set of randomly generated incident locations based on kernel density estimates |

| **Optimization** |
| Formulate and solve mathematical models to optimize the Location-Allocation of *SAR* resources |

| **Interpretation of Results** |
| Interpret and analyze the results provided by optimization and compare the optimal solutions by current situation |

Figure 1-7- Steps followed in the general methodology of the study

1.6. Study Assumptions

The following is the list of assumptions that are considered in the study:

1. Although the timing and location of incidents in the *SISAR* reports can be approximate, we consider them to be accurate.
2. The average speed used for travel time calculations of rescue vessels is assumed to be equal to their cruising speed which comes from their build specifications.
3. No time is considered to search and locate an incident in the model.
4. No environmental factors such winds, sea state, tides, etc. are taken explicitly into account.
5. No time is considered for coordination or preparation for response.

1.7. Thesis Outline

This thesis is in manuscript-based format and each phase of the research is presented as a manuscript submitted to a journal. In all the presented manuscripts, I have made the main contribution to the conception and design of the model, and the analysis and interpretation of the results. All references cited in the chapters are included in a single complete reference list at the end of thesis.

To reach the objectives defined for the research, this thesis is organized as follows:

Literature Review: This chapter traces the literature related to the general area of this study and presents some principal models in the field. The literature review that is related to more specific topics such as multi-objective optimization, stochastic optimization and uncertainty in emergency location analysis are presented in the related chapters.

Phase 1 (Multi-Criteria Analysis): This chapter examines performance of two basic popular location models for emergency response location analysis (maximal covering and *p*-median) for the case of maritime *SAR* and compares the solutions with respect to several defined decision criteria.

Phase 2 (Budgeted strategic model): The model presented in this part of thesis, combines a covering model with a *p*-median model and incorporates a fleet capital budget constraint to find optimal composition of the fleet.

Phase 3 (Multi-objective optimization): The mathematical model developed in this chapter considers three different decision criteria as objectives in order to simultaneously take into account these important and complementary criteria for improving the *SAR* service level.

Phase 4 (Multi-period planning): This part of the study extends the previous models with a seasonal vessel relocation feature to allow periodical planning for locating *SAR* vessels in response to seasonal pattern changes in demand, and improve the fleet utilization through periodical repositioning of the response vessels to address the possible variations in demand distribution.

Phase 5 (Strategic fleet planning model): This phase presents a more comprehensive model concerning strategic level decisions, by considering capital and operating costs of different vessels and aiming to optimize decisions regarding fleet renewal, procurement and decommissioning with respect to the main service level standards and requirements.

Conclusion: The final part of the thesis summarizes the results from all the phases, points out the contributions, clarifies limitations of the analyses, and makes recommendation for pursuing future works in this area.

Chapter 2   Literature Review

Location analysis is a subfield of operations research that includes a wide range of problems. The term Location Analysis refers to the modeling, formulation, and solution of a class of problems that can best be described as siting facilities in a given space in the presence of "customers". There are four components that characterize location problems, which are: (1) customers, who are presumed to be already located at points or on routes, (2) facilities that will be located, (3) the nature of the space in which customers and facilities are located, and (4) a metric that indicates distances or times between customers and facilities (ReVelle and Eiselt 2005). Location decisions relate to a system's ability to satisfy its demands in an efficient manner. Moreover, because of lasting impacts of such decisions, they will also affect the system's flexibility to meet demands as they evolve over time. For detailed and systematic introductions to the field, readers are referred to (ReVelle and Eiselt 2005).

Facility location models are used in a wide variety of applications. These include, but are not limited to, locating warehouses within a supply chain to minimize the average time to market, locating hazardous material sites to minimize exposure to the public, locating railroad stations to minimize the variability of delivery schedules, locating automatic teller machines to best serve the bank's customers, and locating a coastal *SAR* station to minimize the maximum response time to maritime accidents (Hale and Moberg 2003). Eiselt et al. (2015) present a recent classification of location analysis applications.

The space in which facilities are located can be the basis for classification of location problems. These problems are generally categorized at two levels with respect to the space of the problem in two spaces; first: planar (*n*-space) vs. network spaces; and second: continuous vs. discrete problems. All four combinations of these two levels are possible. The continuous problems deal with location problems on a continuous space (single or multiple dimensions) where any location within this space (anywhere on planar space or anywhere on a network) is considered a feasible location for a new facility. In contrast, in discrete problems, the locations must be chosen from a set of alternatives which can be in *n*-space or at network nodes.

Location problems can be categorized in several other ways based on the goal of a study. For instance, it can be divided into competitive/non-competitive problems or stochastic/deterministic problems. Another aspect by which such literature can be categorized is the desirable/undesirable facility location problem.

As mentioned in the introduction, the focus of this study is on the specific types of location models that are relevant to the application of maritime *SAR* location analysis, which falls in the field of emergency location analysis. To be more specific, the problem in this study is a non-competitive desirable facility location problem.

The Location-Allocation problem is a commonly used model which is applicable in many areas such as fire station location, ambulance location, distribution center location and warehouse location. As discussed earlier, the maritime *SAR* location problem is a "mobile server to customer" service system which is similar to ambulance location problems. In this kind of location problem, the main concern is to cover all the demands or at least try to cover as many as possible. There are two types of location problems that deal with the coverage concept: The Location Set Covering Problem (*LSCP*) and Maximal Covering Location Problem (*MCLP*). The objective of the set covering model is to find the minimum numbers of facilities required to cover all the demands. The maximum coverage location problem tries to maximize the total covered demands given a predefined number of facilities. The other type of location problem that is commonly used in the area of emergency vehicles locations involves median problems. This kind of model tries to minimize the total distance of demands from their closest server.

In this chapter, a broad background of the field of location analysis and a review of relevant literature in the area of emergency response location modelling are provided. Some basic optimization models and their extensions will be discussed as well.

2.1. Fundamental Models in Location Analysis

Facility location models are used in a broad range of applications. Mathematical location models are broadly applied not only to the private sector (e.g., industrial plants, banks, retail facilities, etc. (Sweeney and Tatham 1976)) but also in the public sector (e.g., ambulances, clinics, etc. (Brotcorne et al. 2003) and (Griffin et al. 2008)). Refer to (Owen

and Daskin 1998) for a complete overview and more examples. Other applications include locating warehouses within a supply chain to minimize the average travel time to the markets, locating hazardous material sites to minimize exposure to the public, locating railway stations to minimize the variability of delivery schedules, locating automatic teller machines to best serve the bank's customers, and locating a coastal search and rescue station to minimize the maximum response time to maritime accidents (Hale and Moberg 2003).

Aside from covering problems and median problems, center problems comprise a third popular problem type in location modeling. In the center problem, the objective is to minimize maximum distance from the facilities. Covering problems essentially are looking for maximizing the acceptability of service by defining the *coverage* concept. Such models attempt to cover the demand within a pre-specified distance via determining optimal facility locations. While median problems care about the accessibility by minimizing the total travel distance/time, the third category, the center problems, focusses on the worst-case situations and seeks to locate facilities to minimize the maximum distance from all customers to the nearest located facility.

Covering and median problems are very popular in the area of emergency location analysis which is the case in this study. Marianov and Serra (2002) pointed out that both the median and covering problems can be considered benchmarks in the development of facility location models.

2.1.1. Covering Models

Covering models attempt to maximize a predefined minimally acceptable service standard to any of the customers in the problem. Therefore, these models usually fit well with the objectives in emergency response problems. Two basic and important covering location problems in the literature according to the classification by Schilling et al. (1993) are: (1) Location Set Covering Problem where coverage is required; and (2) Maximal Covering Location Problem where coverage is optimized.

The idea of covering models started with the *LSCP* introduced by Toregas et al. (1971).This kind of problem is looking to find the minimum number of facilities required to be located

in order to cover all the customers (demand) within a given maximum covering distance/time. Later, Church and ReVelle (1974) introduced the *MCLP*. Based on their definition, the objective of the *MCLP* is to locate a fixed number of facilities to provide the service to cover as many demands as possible. A review of covering models and their applications can be found in (ReVelle et al. 2002).

2.1.1.1. Location Set Covering Problem

The *LSCP* was first introduced by Hakimi (1964) and was later formulated as an integer programming problem by Toregas et al. (1971). The objective of the *LSCP* problem is to locate the minimum number of facilities such that each demand is covered by at least one of the facilities. The general formulation of *LSCP* is as below.

$$Min \ \ Z = \sum_j y_j \tag{2.1}$$

$$s.t.$$

$$\sum_{j \in N_i} y_j \geq 1, \qquad \qquad i = 0,1,\dots,n \tag{2.2}$$

$$y_j \ \in \{0,1\}$$

where:

I: Set of demand locations

J: Set of facility locations

y_j: Binary variable for locating a facility at point j

$$N_i = \{j \mid d_{ij} \leq s\} \quad \forall i$$

d_{ij}: Distance from i to j

s: Maximum coverage distance

The constraints could be restated as below:

$$\sum_j a_{ij} y_j \geq 1, \qquad \qquad i = 0,1,\dots,n \tag{2.3}$$

where:

$a_{ij} = 1$ if : $d_{ij} \leq s$ Otherwise $= 0$

Equation (2.1) minimizes the number of located facilities, and either constraints (2.2) or (2.3) ensure that all customers are covered by at least one located facilities which is within the acceptable distance (*s*).

The *LSCP* assumes that there is no limitation on the budget, or that the number of facilities that need to be located for covering all customers is unlimited which is not realistic in most practical problems. The complete coverage requirement in *LSCP* is very restrictive. In a problem with many spatially dispersed customer locations, the requirement of complete coverage of all demands may produce solutions with a number of facilities that are unrealistic from a budgetary point of view. Given that the number of facilities is a proxy for costs, it also assumes equal costs for facilities at all potential locations, which could be far from reality. The second issue with *LSCP* is that it fails to discriminate between large demand nodes and small demand nodes. When it is impossible to cover all demand nodes within the specified service standard, it is often important to give priority to the nodes with the greater demand (ReVelle and Eiselt 2005).

2.1.1.2. Maximal Covering Location Problem

The Maximal Covering Location Problem (*MCLP*) does not attempt to cover all customers. Given a fixed number of *p* facilities, the task is to locate these facilities so as to cover the largest possible number of customers. Simply stated by Eiselt and Sandblom (2012), the *MCLP* seeks the maximum amount of coverage (in terms of population, property values, or similar parameters) given a covering distance and a specific number of facilities that can be used.

Sometimes, we cannot afford to provide the required number of facilities in order to cover all the demands. In such a situation, we are faced with a trade-off problem between increasing maximum coverage distance or accepting that some demands will not be covered. If we allow less than complete coverage to occur, we will have a new problem with the objective of maximizing coverage for a given number of resources. So, in this formulation, the covering distance and number of facilities are predetermined.

To summarize, whereas the *LSCP* has the form:

LSCP: minimize the number of facilities opened,

s.t. covering all demand,

The *MCLP* has the inverse form:

MCLP: maximize the demand covered,

s.t. a limit on the number of facilities opened as a proxy for a budget constraint.

The general *MCLP* can be formulated as below.

$$Max \quad z = \sum_i x_i \tag{2.4}$$

$$s.t.$$

$$\sum_j y_j \ = p \tag{2.5}$$

$$\sum_{j \in N_i} y_j \geq x_i, \qquad i = 0, 1, \dots, n \tag{2.6}$$

$$x_i, y_j \in \{0,1\}$$

where:

I: set of demand locations

J: set of facility locations

y_j: Binary variable of locating a facility at point *j* (Location variables)

$x_i=1$ if demand at point *i* is covered (Covering variables)

$$N_i = \{j \mid d_{ij} \leq s\} \quad \forall i$$

d_{ij}: Shortest distance from *i* to *j* (where it is assumed that all movements are along a shortest path)

s: Maximum coverage distance

p: Pre-determined number of facilities

This formulation maximizes the covered demands given *p* located facilities. The first constraint fixes the number of facilities and the second set of constraints avoids x_i taking

the value of 1 when no facilities have been located at the sites which are able to cover node i. The set N_i includes all potential facility locations which can dispatch to and reach customer i within the standard distance (s). The maximizing objective function then forces as many x_i as possible to take on the value one as long as they are within the coverage range of located facilities.

If the number of facilities needed to provide complete coverage exceeds the available resources, relaxing the requirement for covering all demand is one option. An alternative is to relax the coverage distance standard until a standard is found that allows for total coverage with the available resources. This approach is adopted by the *p*-center model (Hakimi 1964) which minimizes the maximum distance from the demand to the nearest located facility. The other common limitation of original covering problems (*LSCP* and *MCLP*) is that these models only determine whether a customer is within the coverage distance of the located facilities and thus do not explicitly allocate customers to facilities. Hence, they do not consider concerns such as the workload capacity.

2.1.1.3. Extensions to Covering Models

Numerous extensions have been introduced for the *MCLP* including consideration of partial coverage, capacity limits and stochastic factors. Schilling et al. (1980) extended the maximal covering model by considering two types of demand with different priority levels. The original covering models discussed above implicitly assume that if a demand is covered by a facility then that facility will be available to serve the demand, but in some applications, that availability assumption is problematic. Several studies attempted to provide multiple coverage to demand nodes so that if one facility is busy, others will be within the acceptable range to serve incoming demands. Daskin and Stern (1981), Hogan and ReVelle (1986) and Batta and Mannur (1990) developed an *MCLP* that contains a *secondary backup coverage* objective. However, this approach is not the best way to address the issue as it does not incorporate the stochasticity involved with the availability of facilities and the arrival of customers. Applying queuing models in location analysis is the most appropriate way to deal with congestion, although it essentially converts the problem to an operational level problem.

Probabilistic models are another stream of extensions to the *MCLP*. One of the earliest probabilistic models for ambulance location is the Maximum Expected Covering Location Problem formulation (*MEXCLP*) due to Daskin (1983). The objective in that case is to locate facilities so as to maximize the expected number of demands that a facility can cover. But the main problem in that model was using only a constant probability for the facility being busy as opposed to a probability distribution. Batta et al. (1989) extended the *MEXCLP* model by adding *correction factors* into the objective function to approximately relax the assumption of independency of facilities unavailability in *MEXCLP*. Daskin et al. (1988) studied the integration of different covering models such as multiple, excess, backup and expected covering models.

The basic location models, do not consider workload capacities. Thus, some servers may be allocated to so many tasks that they are over their maximum capacity. To solve this issue, researchers have worked on extending the fundamental models to include constraints on the capacity to balance the workload of facilities. These additional capacity constraints destroy the property that all demand of a customer is satisfied from a single facility. Also, capacity constraints make the model substantially more difficult to solve and the associated problems are usually NP-hard problems ((Current and Storbeck 1988), (Pirkul and Schilling 1991).

For uncapacitated models, in the optimal solution all demands are assigned to their closest facility. However, a constraint on the capacity of the facilities could change the assignments. The other issue that arises here is the equity of the service level in demands. This means that when capacity limits do not allow all the demands to be served in full, we must decide which demands should be covered fully, which partially, and which remain non-covered. Another issue is balancing the load across the facilities in order to avoid heavy loads on some facilities while others are idle.

Chung et al. (1983) and Current et al. (1998) were among the earliest researchers who dealt with the concept of capacitated *MCLP*, by adding a maximum capacity constraint to the model formulation. Pirkul and Schilling (1991) proposed a capacitated model that all demands are assigned to facilities, regardless of whether the demand lies within the service covering distance or not. Haghani (1996) also developed a multi-objective capacitated

MCLP wherein its objective function aims to maximize the weighted covered demand while minimizing the average distance from uncovered demands to the nearest facilities.

The capacity of different facilities can be varied based on their different characteristics. Correia and Captivo (2003) called such a problem with varied capacity constraints the modular capacitated location problem. To apply the capacitated *MCLP* model to the case of emergency facility siting problem so that the facility could have different capacity levels with varied numbers of stationed emergency vehicles, Yin and Mu (2012) proposed an extension of *MCLP* called the Modular Capacitated Maximal Covering Location Problem (*MCMCLP*). The objective of their model, similar to Haghani (1996), is to maximize the weighted covered demand and simultaneously minimize the average distance from uncovered demands to the located facilities.

2.1.2. Median Problems

Unlike covering problems, median problems deal with the allocation of customers (demands) to facilities as well as determining the facility locations. The objective of the median problem is to minimize total traveling distance/time to the customers/facilities. It is worth mentioning that for fixed demand, this also minimizes the average facility/demand distance, and with that we have an objective that maximizes accessibility. Church and ReVelle (1974) point out that one important way to measure the effectiveness of a facility location is by determining the average distance traveled by those who visit it.

These kinds of problems are applicable for establishment of the public services such as schools, hospitals, fire stations, ambulances, technical audit stations of cars, etc., although they can be of interest for private sector location problems as well since minimizing the total distance as a proxy of total transportation cost, or average distance as average access time, is appealing for decision makers in the business.

Hakimi (1964) explains the basic concepts of absolute center and absolute median. Then, he used them to discover the optimal location of a switching center in a communication network and also to find the best place for building a police station in a highway system. However, the objectives are somewhat different for these two applications. In the switching center problem, the objective is to minimize the total length of wires which makes it an

absolute median case, while in the police station problem, the objective is to minimize the maximum distance from the police station to any incident which is the form of an absolute center problem. Hakimi allowed a facility to lie anywhere along the graph's edges, but he proved that an optimal absolute median is always located at a vertex of the graph. He generalized the absolute median to find p medians on a graph in order to minimize the sum of the weighted distances. Hence, Hakimi's main contribution to location theory is that at least one absolute median is also a node-median. This reduces the search for the set of optimal solutions to the nodes. His contribution was similar to that of Dantzig for linear programming, in that it reduced the set of optimal solutions from a potentially infinite set to a finite (albeit astronomically finite) set. This result is commonly referred to as the "Hakimi theorem" or "the node property" (ReVelle and Eiselt 2005).

Although the first explicit formulation of the p-median problem is attributed to Hakimi (1964), earlier Hua (1962) proposed an algorithm for locating the 1-median on trees (and networks with cycles), and proved that locating median points on vertices is better than locating them somewhere along the edges. ReVelle and Swain (1970) first formulated the central facility location problem. The problem of central facilities location consists of locating n facilities, designating m of n communities ($m < n$) as medians in such a way that the average distance or time travelled per person is a minimum. Goldman (1971) provided simple algorithms for locating a single facility for both an acyclic network (a tree) and a network containing exactly one cycle. One important way to measure the effectiveness of a facility location is by determining the average distance travelled by those who visit it (Church and ReVelle 1974). The p-median model implicitly assumes that the cost of locating a facility at each candidate site is the same for all sites.

Marianov and Serra (2002) gave a review of the p-median problem and its extensions.

A p-median model with covering constraints:

Objective:

The objective of the model is to minimize the weighted total distance from demands to their closest facility.

Constraints:

(1) A demand at point i can only be served by facility at point j if there is one located there.

(2) Fixed number of facilities

(3) Each demand is allocated to exactly one facility.

(4) A facility can be assigned only to demands which are within its coverage distance.

The formulation of the model is as follows.

$$Min \ \ z = \sum_i \sum_j w_i d_{ij} \, x_{ij} \tag{2.7}$$

$s.t.$

$$x_{ij} \leq y_j, \qquad\qquad \forall \, i, j \tag{2.8}$$

$$\sum_j y_j = p, \tag{2.9}$$

$$\sum_j x_{ij} = 1, \qquad\qquad \forall \, i \tag{2.10}$$

$$d_{ij} x_{ij} \leq s, \qquad\qquad \forall \, i, j \tag{2.11}$$

$$x_{ij} \in \{0,1\}$$

$$y_j \in \{0,1\}$$

where:

I: set of demand locations

J: set of facility locations

x_{ij}: Binary variable for assigning a facility at point j to demand at i

y_j: Binary variable for locating a facility at point j

w_i: Demand weight at point i

d_{ij}: Distance from i to j

s: Maximum coverage distance

p: Fixed number of facilities

2.2. Other Considerations in Location Analysis

This section discusses several other important aspects and concerns in location analysis that have attracted a lot of attention among researchers in the field.

2.2.1. Dynamic Location Models

The strategic nature of facility location problems requires that any reasonable model consider some aspect of future uncertainty. Since the investment required by locating or relocating facilities is usually large, facilities are expected to remain operable for an extended time period. Thus, the problem of facility location truly involves an extended planning horizon. Decision makers must not only select robust locations which will effectively serve changing demands over time, but must also consider the timing of facility expansions and relocations over the long term (Owen and Daskin 1998). Facility location problems can be divided into two categories: static and dynamic problems. In dynamic problems, as opposed to static problems, location decisions are made in a time dependent manner (i.e. facilities can be relocated, closed, opened, and expanded during planning time horizon).

Relocation of facilities can occur in a discrete or continuous manner. In the former category, relocation is only possible at discrete pre-determined points of time (Wesolowsky 1973); while in the latter case, relocation is possible at any time during the planning horizon (Drezner and Wesolowsky, 1991). Wesolowsky (1973), Wesolowsky and Truscott (1975), and Sweeney and Tatham (1976) were pioneers in dealing with the multi-period Location-Allocation problem. Wesolowsky and Truscott (1975) extended the multi-period Location-Allocation problem, allowing facility relocation in response to predicted changes in demand. They presented an integer programming model which constrains the number of location changes in each period. A dynamic programming formulation is also presented. Sheppard (1974) presented a variety of models which extend the location of multiple facilities by determining the size of the facilities and the timing of plant construction or expansion. In order to reduce the complexity that emerges in such mathematical models,

most studies prefer discrete time periods of relocation over continuous changes. In addition, this approach is more practical in most cases when it comes to the implementation of solutions; however, choosing continuous time for relocations results in the same or better optimal solutions.

Arabani and Farahani (2012) reviewed the literature on dynamics in location analysis. Different approaches for considering dynamicity in demand, parameters and/or factors in facility location problems are examined. They categorized studies based on the method taken into account for the stochasticity of demand and its possible variations. Time dependent location problems, multi-period and simple-period location problems and location-relocation problems are among different types of problems which were reviewed.

Seyedhosseini et al. (2016) also reviewed dynamic location problems and recent advancements in the field. They covered a wide range of problems including single and multiple facility relocation problems, median and covering dynamic location problems, stochastic dynamic location problems, and fuzzy location problems.

2.2.2. *Uncertainty in Location Analysis*

In all the modelling approaches discussed so far, it is assumed that input parameters for the model are known and deterministic. This section addresses the approaches to cope with uncertainty in location problems. Uncertainty in facility location modelling may exist in different aspects of the problem such as demand locations, travel times, travel costs, demand volume, etc. There are two ways to represent the uncertainty associated with model parameters: (1) using discrete scenarios to describe the uncertain parameters where each scenario has a given probability of occurrence, and (2) using probability distributions to represent the stochasticity of parameters. A "scenario" is a complete realization of all the uncertain parameters. Each scenario fully determines the value of all the uncertain parameters.

Snyder (2006) reviewed the literature on stochastic and robust facility location models and explored a variety of approaches for optimization under uncertainty. The main approaches for optimization under an uncertain environment falls into two categories: Stochastic Programming (*SP*) and Robust Optimization (*RO*). Problems that deal with uncertain

parameters in which their variation scheme conforms to a specific probability distribution, are known as stochastic optimization problems. A common goal is to optimize the expected value of some objective function. Conversely, the cases in which parameters are uncertain, and furthermore no information about their probabilities is known, are known as robust optimization problems. In robust optimization, a set of possible future values are taken into account and typically the objective is to minimize the maximum deviation of objective values in each scenario from their best possible solution. A recent overview of different modelling approaches for dealing with uncertainty in facility location is given in (Correia and Saldanha da Gama 2015).

2.2.3. Congestion

In most of the basic location models like *MCLP*, *LSCP* and *p*-median, it is assumed that facilities have infinite capacity to respond to demands. This is not usually the case in real problems as brought up in the preceding section. Also, in real situations the nature of demands is typically variable and random. Although the facilities may be capable of coping with the average demand, there will be some peak times when they cannot provide service to all requests. Such situations are referred to as *congested systems*.

These issues can be investigated using queueing models which take into account the probabilistic nature of demand and service. In congested systems, in some cases when a facility is not able to serve all service requests, some of them can wait until the server become available. But in other cases, such as emergency systems, it is generally not reasonable to wait so if the demand is not responded within a time limit it will be assumed to be uncovered.

Marianov and Serra (1998) studied a probabilistic Maximal Covering Location-Allocation model for congested systems. They developed a Location-Allocation model for multiple server system with constraints on queues and waiting times in (Marianov and Serra 2002).

Marianov and Serra (2001) also studied the congestion concept for hierarchical facilities. Examples of hierarchical structures can be found in public health services, where hospitals correspond to the higher-level facilities, and primary health care centers are the lower level. They presented two location models for this problem. First, a queueing set covering

problem and second a queuing maximal covering problem whereby a heuristic approach was used to find the solutions for both models.

Also, it is possible to incorporate a probabilistic constraint into the model to ensure a minimum level of service availability for each demand or in a similar way include a constraint to limit the maximum waiting time of each customer.

Berman and Krass (2002) addressed facility location problems with stochastic demands and congestion for the mobile server case. Boffey et al. (2007) reviewed the studies related to congestion models in the location of immobile facility servers.

Berman et al. (2006) considered two potential sources of lost demand: (1) demand lost due to insufficient coverage; and (2) demand lost due to congestion. The objective is to minimize the number of facilities and locate them so that the amount of demand lost for either reason does not exceed certain pre-set levels. Also, Berman et al. (2007) formulated a model to maximize the expected amount of captured demand for capacitated facilities when customer demands are stochastic. It is assumed that customers travel to their closest facility for the service; In case the facility is fully utilized, they will go to the next closest facility.

An alternative common approach to cope with congestion is to consider backup coverage. Here we try to cover demands more than once in order to decrease the probability of server unavailability in case of congestion. One important concern is the possible conflict or trade-off between primary and backup coverage, whereby improving the former may hamper the latter. Hogan and ReVelle (1986) presented a maximal backup coverage model and Pirkul and Schilling (1988) proposed a capacitated maximal covering model.

2.2.4. Gradual Coverage

One problematic or maybe unrealistic assumption in *MCLP* is the 0-1 concept of coverage which means that demands within the coverage radius from the located facilities are covered completely, while demands beyond this predefined distance are not covered at all. This is the where the idea of gradual coverage comes from. The coverage is then assumed to be a gradually decaying function of distance from facilities rather than an abrupt termination. Church and Roberts (1983) were the first to suggest replacing the covered/not

covered idea by a function for modeling the level of coverage. Berman and Krass (2002) presented a generalized Maximal Covering Location Problem. They used the concept of partial coverage in their study and defined the degree of coverage as a decreasing function of the distance to the closest facility. They showed that this problem is equivalent to the uncapacitated facility location problem. Drezner et al. (2004) also formulated and solved the gradual covering problem. They presented coverage as a ring where inside its inner circle the point is fully covered, and outside the outer circle there is no coverage.

Karasakal and Karasakal (2004) applied the concept of partial coverage in *MCLP* and used Lagrangian relaxation for solving the model. They introduced coverage as a function of the distance of the demand point to the facility. It is assumed that the demand is fully covered within the minimum critical distance *S*, partially covered up to a maximum critical distance *T*, and not covered at all outside of the maximum critical distance. The coverage function is defined as below. Also, various types of coverage function are demonstrated in Figure 2-1.

$$c_{ij} = \begin{cases} 1 & if \ d_{ij} \leq S, \\ f(d_{ij}) & if \ S < d_{ij} \leq T, \left(0 < f(d_{ij}) < 1\right), \\ 0 & otherwise \end{cases} \tag{2.12}$$
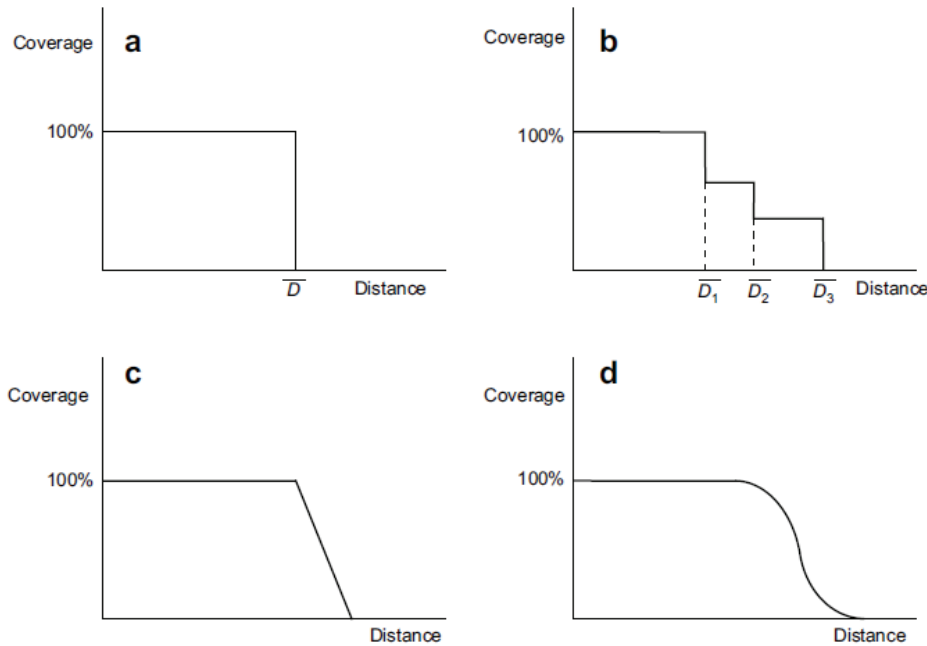
Figure 2-1- various coverage functions (Eiselt and Marianov 2009)

Eiselt and Marianov (2009) extended the *LSCP* by replacing the covered/not covered dichotomy with the gradual coverage concept. The quality of service is also incorporated in the presented model. Berman et al. (2011) studied the gradual covering location problem on a network with stochastic demand. The demand weights are assumed to be independent discrete random variables. The objective of their model is to maximize the probability that the total covered demand weight is greater than or equal to a pre-selected threshold value.

Berman et al. (2010) reviewed several recent generalizations of the basic concept of *coverage* in the maximal covering location model. This review includes gradual coverage, cooperative coverage and variable coverage radius concepts.

2.3. Multi-Criteria Location Analysis

Facility location problems usually involve multiple decision criteria. Therefore, they fall under multi-criteria decision-making (*MCDM*) problems. *MCDM* is generally classified into two categories:

- multi-objective decision-making (*MODM*), and
- multi-attribute decision-making (*MADM*)

In the former case, the process of optimizing systematically and simultaneously a collection of objective functions is called multi-objective optimization (*MOO*) or vector optimization (Marler and Arora 2004). Zimmermann (2010) defines the vector optimization problem as

Maximize $\{Z(x) \mid x \in X\}$

where $Z(x) = (z_1(x), ..., z_n(x))$ is a vector-valued function of x and X is the *solution space*.

Generally, multi-objective optimization methods can be classified into two categories: Scalarization methods and Pareto methods. In the first group of methods, the multi-objective problem is solved by translating it back to a single (or a series of single) objective, scalar problem. The formation of the aggregate objective function requires that the preferences or weights between objectives are assigned *a priori*, i.e. before the results of the optimization process are known. Alternatively, running such models multiple times while altering objective weights could approximate the non-dominated frontier, which would still be insightful for decision makers who do not know the weights in advance. The

easiest to understand and most widely used scalarization method is the weighted sum approach. The Pareto methods, on the other hand, keep the elements of the objective vector *Z* separate throughout the optimization process and typically use the concept of dominance to distinguish between inferior and non-inferior solutions.

Normally, there is no point that could simultaneously optimize all objectives at once. In other words, when we deal with a problem with multiple conflicting objectives, the concept of optimality as defined for single objective problems, ceases to exist. Hence, another concept was developed to cope with this issue by describing *efficient* or nondominated or noninferior solutions, also known as *Pareto-optimal* sets. A point *x* is an *efficient* solution if it is not possible to move feasibly from it to increase an objective without decreasing at least one of the others.

A multi-objective problem can be formulated and solved by using several mathematical methods. The most popular ones are as follows: weighted sum method, epsilon constraint method, utility function, bounded objective method, lexicographic method, goal programming (*GP*), goal attainment method, method of Geoffrion, and interactive *GP*.

The weighted sum method scalarizes the set of objectives into a single objective by multiplying each objective with a user supplied weight. The epsilon constraint method first introduced by Haimes (1971) chooses a single-objective to be optimized while every other objective is treated as a constraint. The extreme point that is computed is then used to determine the bound on the objectives, and this is repeated until there are no new solutions left. In the lexicographic method, the objective functions are arranged in order of importance and the corresponding problems are solved sequentially, each time by adding a bound on the objective value optimized in the previous round. Goal programming attempts to minimize total deviation from the predefined goals on objectives. In the value function method, a user defined utility function (a function of all objectives) which is valid over the entire feasible region is optimized. The goal attainment method aims to minimize the maximum weighted deviation from specified objective goals.

On the other hand, *MADM* is defined as a formal analysis which takes into account multiple attributes associated with certain actions or choices. The model scores and weighs these

attributes for each alternative action in a comparative mathematical analysis. Zimmermann (2001) defines a general multi-attribute decision making model as follows.

Let $X = \{x_i \mid i=1, \ldots, n\}$ be a (finite) set of decision alternatives and $G = \{g_j \mid j=1, \ldots, m\}$ a (finite) set of goals according to which desirability of an action is judged.

Determine the optimal alternative x^* with the highest degree of desirability with respect to all relevant goals g_j.

Usually *MADM* methods consist of two stages:

1. The aggregation of the judgments with respect to all goals and per decision alternative, and
2. The rank ordering of the decision alternatives according to the aggregated judgments.

2.3.1. Multi-Objective Optimization in Location Analysis

The objectives that are usually considered in multi-criteria location problems can be different, some of which are:

- Minimizing the fixed and operating cost
- Maximizing service level (required level of response in terms of time/distance)
- Minimizing average time/distance traveled
- Minimizing maximum time/distance traveled
- Minimizing the number of located facilities as a proxy of cost

Using multiple objectives in location studies started with Ross and Soland (1980) who proposed an interactive solution method to compute non-dominated solutions for multi-activity multi-facility problems to compare and choose from. Studies in this field can be categorized from various perspectives such as the application of the proposed model, the type of objective functions, or the methodology used for modelling.

2.3.1.1. Areas of Application

With respect to different applications of multi-objective optimization in location analysis, we can name practical problems like: distribution centers, fire stations, ambulance location, service centers, etc. Coverage, cost, and travelling distance/time are among the popular

location objectives and were extensively used in multi-objective location studies. Baker et al. (1989) developed a multi-criteria model for the ambulance allocation problem. They used various outcome criteria in their model, including response time, cost and workload balance. The model is then solved using an integer, non-linear goal-programming technique. In their study of heath care services, Badri et al. (1998) attempted to determine the optimal place to locate fire stations considering multiple criteria. The model incorporates multiple objectives including travel times and travel distances as well as other cost-related and quality-related objectives. These criteria include minimizing stations overlaps, and avoiding locating the facility where water availability is a problem. A tradeoff between costs and quality is provided to help decision makers. Drezner et al. (2006) have incorporated five objectives: p-median, p-center, two maximum covering and the minimum variance, in order to minimize the maximum percent deviation from the optimum of each of these objectives for casualty collection points location. Kim and Murray (2008) developed a model to maximize primary and secondary coverage for a given number of serving facilities. They proposed a bi-objective model which maximizes primary and backup coverage respectively. Balcik and Beamon (2008) minimized the sum of transportation costs and penalty costs for unsatisfied and late satisfied demands in a humanitarian relief supply problem. Burkey et al. (2012) considered efficiency, availability of the service, and equality as examining criteria to compare existing locations with optimal solutions from formulations as a maximal covering location problem and a p-median problem. The results of their study show that the existing locations provide near-optimal geographic access to health care centres. A bi-criteria problem was formulated by Kolokolov and Zaozerskaya (2013) to find the optimal locations of service centers. Trade-off methods were used for finding a subset of the Pareto-optimal solutions set.

2.3.1.2. Modelling Approaches

Moreover, different modelling methodologies were adopted by researchers to deal with multi-objective location problems. Lee et al. (1981) applied integer goal programming for facility location with multiple competing objectives. Solanki (1991) presented an approximation scheme to produce a set of Pareto solutions for a bi-objective location problem. Ogryczak (1998) looked for efficient location patterns in a multi-criteria discrete

location problem and provided a compromise method between the median and center solution concepts. Rakas et al. (2004) dealt with the location of undesirable facilities and used a weighting method to combine two objective functions. An evolutionary algorithm was utilized to deal with the multi-objective optimization in order to generate a set of trade-off solutions. Their model is divided into two parts: (1) multi-objective facility location problem; and (2) multi-objective customer allocation problem. Abounacer et al. (2014) used an epsilon-constraint method to find the Pareto frontier solutions considering three objectives in their location-transportation model for disaster response. These three objectives are total distribution time for emergency supplies, the number of agents required to operate selected distribution centers and the third one, non-covered demands.

Stochastic approaches were also utilized in multi-objective location problems that involve nondeterministic factors on demand and facility availablity. Wang et al. (2004) studied a facility location problem with stochastic demand and tried to minimize the total travel cost and waiting cost for the customer. Their model includes a restriction on the number of facilities that may be opened and an upper bound on the allowable expected waiting time at a facility. Pasandideh and Niaki (2012) also developed a bi-objective model for facility location in an M/M/1 queuing system. In this paper, a facility location problem with stochastic customer demand and immobile servers is studied. The problem was formulated using queuing theory and solved by a genetic algorithm. Rahmati et al. (2014) produced a multi-objective model for facility location-allocation problem considering stochastic demand and using a queuing framework. Their model has two objectives: first to minimize the total cost of servers, and second to minimize the total time of customers.

2.3.1.3. Fuzzy Location Optimization

Fuzzy multi-objective models have been used in several location analysis studies. In recent years, many people have brought fuzzy theory into facility location analysis. Using the fuzzy concept, we could translate the vagueness and uncertainty in the objectives' importance into numerical functions. Also, the stochastic nature of demands can be represented by fuzzy demands. Specifically, it could be helpful to apply the idea of gradual and partial coverage in the form of fuzzy values. Fuzzy set theory can also be combined with the goal programming or weighted sum methods in order to model multi-objective

problems. Bhattacharya et al. (1993) utilized a fuzzy goal programming approach for a convex multi-facility location problem with two objectives: minimizing transportation cost, and minimizing maximum distance. Shavandi and Mahlooji (2006) utilized fuzzy theory to develop a queuing Maximal Covering Location-Allocation model. Their model includes one constraint on service time or a maximum queue length to constrain the service quality. A genetic algorithm is developed to solve the model. Araz et al. (2007) also used a fuzzy goal programming approach to propose a multi-objective emergency vehicle location model which simultaneously maximizes the population covered, maximizes the population with backup coverage, and minimizes the total travel distance. Yang et al. (2007) introduced a fuzzy multi-objective programming approach to determine the location of fire stations. They also used a genetic algorithm to solve the problem and find a near-optimal solution. A maxi-min objective function was used for two objectives: (1) total fixed and operating cost and cost of incidents loss; and (2) distance from the fire station to any incident. Three types of fuzzy models are proposed by Zhou and Liu (2007) to deal with a capacitated Location-Allocation problem with fuzzy demands. They used three types of fuzzy programming models to solve a location analysis problem with fuzzy demands.

2.3.1.4. Undesirable Facilities

In some cases, researchers deal with undesirable facilities such as landfills so they might take into account different type of objectives. Ohsawa et al. (2006) developed a new bi-criteria location problem with partial covering to model siting a semi-obnoxious facility within a convex polygon. Their model considers mini-max and maxi-min objectives in Euclidean distances. Xifeng et al. (2013) considered minimum economic cost, maximum customer service reliability and minimum CO_2 emissions as objectives of their model of sustainable logistics facility problem and applied the epsilon constraint method for multi-objective optimization. A bi-objective model for locating landfills was developed by Eiselt and Marianov (2014). Their model minimizes pollution as well as cost. Another multi-objective optimization approach was proposed by Harris et al. (2014) in their study of the capacitated facility location problem with flexible store allocation for green logistics. Their model includes minimizing financial costs and CO_2 emission as objectives.

2.4. Emergency Response Location Analysis

As mentioned in previous sections, location analysis has many applications in different areas. One of the main application areas of different location models is emergency logistics. There is a very large number of studies in the literature dealing with analyzing the location of emergency services facilities such as health centers, ambulances, fire stations and Search and Rescue vessels/stations. This section concentrates on previous studies on the application of location theory in emergency services as this is the subject of this study.

Examples using location modelling in practice have included locating hospitals (Sinuany-Stern et al. 1995), emergency medical services (Pirkul and Schilling 1988), blood banks (Jacobs et al. 1996), and ambulances (Ball and Lin 1993). A review of location models specifically applied to healthcare can be found in (Daskin and Dean 2004). Also, Goldberg (2004) reviewed the literature of operations research applications in emergency services vehicles. Brotcorne et al. (2003) reviewed the evolution of models in the area of ambulance location and relocation, dividing the models into deterministic and probabilistic models. Also, dynamic models which involve ambulance relocation were discussed.

Marianov and ReVelle (1996) discussed the problems and applications in siting emergency services. They proposed a model for a queueing maximal availability location problem taking into account the randomness of servers' availability. Harewood (2002) formulated a bi-objective problem for locating ambulances on the island of Barbados. The model considers minimizing the cost of serving customers, while maximizing multiple coverage given a certain distance standard. Alsalloum and Rand (2006) extended the maximal covering problem for the case of emergency vehicle location, in two ways: First, by replacing the 0-1 coverage definition by the probability of covering a demand within a target time. Second, the minimum number of vehicles at each location that satisfies the required performance level is determined after locating the sites. Griffin et al. (2008) developed an optimization model to determine the best location and number of new Community Health Centers in a geographical network as well as what services each such center should offer at which capacity level. The objective is to maximize the weighted demand coverage of the target population subject to budget and capacity constraints.

Mathematical location modelling has been widely applied to the optimization of logistics in the humanitarian relief activities and large-scale emergency response planning. Jia et al. (2007) proposed a model in their study for determining the facility locations of medical supplies in response to large-scale emergencies. They formulated the problem as a maximal covering problem with multiple facility and quality of coverage requirements and developed three heuristics to solve the model. The results demonstrate a good capability of the model in improving the population coverage and reducing life-loss during large-scale emergencies. Balcik and Beamon (2008) integrated maximal covering facility location and inventory decision model to consider a case of facility location decisions for a humanitarian relief chain responding to quick-onset disasters. Results show the effects of pre- and post-disaster relief funding on the relief system's performance, specifically on response time and the proportion of demand satisfied.

Indriasari et al. (2010) developed a Maximal Service Area Problem (*MSAP*) utilizing the capabilities of *GIS* for emergency facilities for which accessibility is an important requirement. The objective of the *MSAP* is to maximize the total service area of a specified number of facilities. The results of the study show that the three heuristics provide better coverage than the existing coverage with the same number of fire stations within the same travel time. In another related study, Rawls and Turnquist (2010) developed an emergency response planning tool that determines the location and quantities of various types of emergency supplies to be pre-positioned, under uncertainty about time and location of natural disaster occurrence. The study presents a two-stage stochastic mixed integer program. This study was extended in (Rawls and Turnquist 2011) with additional service quality constraints. These constraints ensure that the probability of meeting all demand is at least α, and that the demand is met with supplies whose average shipment distance is no greater than a specified limit.

Yin and Mu (2012) presented a modular capacitated maximal covering model to find the optimal location and allocation of emergency vehicles. The modular capacity concept refers to the case where there are several possible capacity levels. Few studies have considered the multiple-resource multiple-depot emergency response problem considering the effects of possible secondary disasters. For instance, Zhang et al. (2012) formulated an

emergency resource location problem incorporating the constraints of multiple resources requirement as well as possible secondary disasters. They introduced the opportunity cost of the secondary disasters into the objective function to develop a model for dispatching the multiple emergency resources.

2.4.1. *Search and Rescue Location Analysis*

Focusing on works more relevant to the subject of this research, which concerns the location of *SAR* vessels, there are quite a few studies to highlight. Among them, Brown et al. (1996) developed a mixed integer model for scheduling US Coast Guard district cutters which resulted in solutions superior to manually prepared schedules. Nguyen and Kevin (2000) incorporated maximal covering and *p*-median location problems into a goal programming model to assess the level of service of the current *SAR* system (in terms of location of *SAR* aircrafts and helicopters) and compare it to the optimal solution. They also used simulation and queueing theory to examine the performance of *SAR* aircraft in terms of average time incidents spend in queue for both current and proposed solution. The proposed solution shows a significant improvement over the current situation. Also, Li (2006) applied three location models (maximal covering location problem, maximal expected covering location problem and maximal covering location problem with workload capacity) to the maritime *SAR* location problem for Atlantic Canada. A simulation model was also employed to validate, compare and improve the results. This study covers the application of general discrete location models in the case of Maritime *SAR*. Some studies have taken other analytical approaches rather than optimization. For instance, Azofra et al. (2007) proposed a tool for assignment of sea rescue resources to incidents using a gravitational modelling approach. The proposed model returns a coefficient for each possible assignment based on the appropriateness of the rescue vessel to the incident. This study only evaluates different solutions but is not attempting to propose an optimal solution. Huang and Pan (2007) developed an incident response management tool by integrating a Geographic Information System with traffic simulation and optimization of response unit assignment, but they didn't incorporate optimizing the location of resources.

Afshartous et al. (2009) took a statistical-optimization approach to obtain robust Coast Guard air station locations given uncertainty in distress call locations. They simulated distress calls and investigated the performance of the solutions obtained from the optimization problem across numerous simulation replications. Wagner and Radovilsky (2012) optimized location and allocation of U.S Coast Guard lifeboats by proposing a model that simultaneously considers balancing the workload and decreasing fleet size and operating cost while ensuring that the service requirements are met. Wex et al. (2014) developed a combined Allocation-Scheduling model to arrange the order of rescue unit response to incidents. They considered the specific requirement of incidents and different capability of rescue units in their model. This study developed a decision support model to minimize the sum of completion times of incidents weighted by their severity. Razi and Karatas (2016) also developed a multi-objective model for allocating maritime *SAR* resources in Turkey. They took advantage of the *AHP* method for ranking different incident types and aggregated the demand using a zonal distribution model. Their model has multiple objectives including minimizing response time to incidents, fleet operating cost, and the mismatch between boats' workload and operation capacity hours. A recent account of application of location modelling for maritime *SAR* is provided by Pelot et al. (2015).

As shown in this section, many studies have been performed in the area of emergency location analysis. However, there are still some gaps remaining, particularly in the case of maritime *SAR* location modelling. Most of the studies do not consider the multiple criteria in a proper way to satisfy the stakeholders' (decision makers) expectations. This shortcoming is addressed by taking a more comprehensive multi-criteria approach in this thesis. In addition, one of the most important gaps to highlight is that most studies on emergency response location analysis have relied on historical demand for planning future response which is not a realistic assumption. In this study, we aim to propose a spatial-simulation methodology to incorporate the stochasticity of future demand locations.

Chapter 3 A Maritime Search and Rescue Location Analysis Considering Multiple Criteria, with Simulated Demand

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Abstract

In this paper a multi-criteria analysis is performed on the location of maritime Search and Rescue resources. Two well-known standard location models (the maximal covering location problem and *p*-median problem) are modified and applied in accordance with our problem characteristics. The study considers several distinct response vessel types with different capabilities. Future incidents are simulated based on the underlying distribution of historical incidents. The models are formulated and solved using data from the Atlantic region of Canada. The optimal solutions of these two models, along with the current resource arrangement, are compared in terms of five decision criteria: (1) mean access time, (2) primary coverage, (3) backup coverage, (4) Gini index, and (5) maximum access time. The results indicate a significant increase in efficiency of resource utilization and availability of service based on access time and coverage criteria for the solutions provided by the optimization models compared to the current situation.

Keywords: Maritime Search and Rescue; Location analysis; Multiple criteria; Coverage; Access time; Kernel estimation

3.1. Introduction and Problem Statement

Maritime Search and Rescue is one of the most conspicuous humanitarian activities in Canada, with the aim of minimizing loss of of life, injury, property damage and risk to the environment. The marine environment can be extremely dangerous as every year more than 6,000 incidents are reported in navigable waters around Canada. Maritime Search and Rescue (SAR) is one of the main responsibilities of the Canadian Coast Guard (CCG) and, as a public service dealing with maritime incidents, it is very important that their resources, including search and rescue vessels, stations, and crew, are used efficiently. To do this, they must decide where to locate their resources and how to allocate demands to them, which is also known as the Location-Allocation problem. The Location-Allocation problem is an essential model for several important applications, including the location of ambulances, police cruisers, fire stations, distribution centres, etc. Optimizing the efficiency of resource utilization is always a major concern.

In a typical SAR location problem, we are faced with a server to customer service system, which is similar to fire station and ambulance location problems. The servers represent the SAR vessels, while the customers symbolize the incidents. In this kind of location problems, usually the main goal is to provide good coverage and service levels (e.g. minimum travel distance).

The Canadian Coast Guard has many different SAR vessel types that were designed or purchased with specific tasks in mind, and not all are equally capable of, or effective at, handling different incident types. Also, the ranges and speeds vary greatly among different types of SAR vessels, so the vessels' capabilities need to be considered.

The purpose of this study is to incorporate multiple facility types within two common location analysis formulations, viz., the maximal covering location problem and p-median problems, and then compare the solutions in terms of several decision criteria. The first model tries to maximize the coverage for a given number of resources, while in the second model the objective is to minimize the average access time to all incidents. The proposed models are applied to a real case maritime Search and Rescue location problem. In other words, the goal of the study is to examine the efficiency, accessibility and equality of SAR

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services and see if we can improve them through finding an optimal mathematical solution. Ultimately, this paper investigates some important decision criteria in maritime SAR services and will provide some helpful insights for decision makers regarding possible changes to improve the overall quality of service.

The remainder of this paper is structured as follows. First section presents relevant literature. The following section explains the decision criteria used in our study. Then, we apply the two aforementioned models using the data for the case study, followed by discussion of the results. We conclude the paper with the summary of the findings and outlook for future research in the field.

3.2. Literature Review

Facility location models are useful for a wide range of applications. Location models are applicable both to the private sector (e.g., industrial plants, banks, retail facilities, etc.), and the public sector (e.g., ambulances, clinics, etc.). In the Maritime *SAR* location problem, we are faced with a *server to customer* service system which is similar to ambulance location problems. In this kind of location problem, the main concern generally is to serve all the demands or at least try to serve as much as possible, although, like any other emergency service, response time is of great importance as we would like to reach the incidents as quickly as possible.

Covering problems and median problems are among the most popular problem types in location modelling and they are appropriately used in emergency response location analysis. The former is concerned with covering demands within a specified response distance/time standard while the latter is aiming at minimizing the system-wide average response distance/time.

3.2.1. *Maximal Covering Location Problem*

Schilling et al. (1993) classified models which use the concept of covering into two categories: (1) location set covering problems (*LSCP*), in which complete coverage is required, and (2) maximal covering location problems (*MCLP*), which maximize coverage. The idea of covering models started with the location set covering problem introduced by

Toregas et al. (1971). Later, Church and ReVelle (1974) introduced the maximal covering location problem. The task of the *MCLP* is to locate a given number of *p* facilities, so that as many customers as possible are within a pre-specified distance of any of the facilities.

Numerous extensions to the *MCLP* have been introduced including consideration of partial coverage, capacity limits and stochastic factors. Schilling et al. (1980) extended the maximal covering model by considering two types of demand with a different priority level. Daskin and Stern (1981), Hogan and ReVelle (1986) and Batta and Mannur (1990) developed an *MCLP* that contains a *secondary backup coverage* objective. A review of covering problems and their applications can be found in ReVelle et al. (2002).

3.2.2. P-Median Location Problem

One important way to measure the effectiveness of a facility location is by determining the average distance traveled (Church and ReVelle 1974). Unlike covering problems, median problems deal with the allocation of the customers to the facilities as well as deciding the facility locations. The objective of the median problem is to minimize total traveling distances or times to reach the customers (or customers to reach the facility, depending on the nature of the problem). Given that the total demand is fixed in the standard *p*-median model, this is equivalent to minimizing the average time a customer requires to reach its closest facility. These kinds of problems include the establishment of public services including schools, hospitals, firefighting, ambulance service, vehicle inspection stations, etc. where decision makers are concerned about the accessibility of resources, and Median problems are commonly used in private sector location problems as well when minimizing the total travel distances as a proxy for transportation cost is of interest.

Problems of the *p*-median type are among the most researched location models in the literature. The first explicit formulation of the *p*-median problem is attributed to Hakimi (1964). Marianov and Serra (2011) provided a review of median problems and their extensions.

3.2.3. Emergency Location Analysis

One of the main application areas of different location models is emergency logistics. There is an extensive number of studies in the literature dealing with analyzing the location of emergency service facilities such as health centers (Cho (1998) and Burkey et al. (2012)), fire stations (Badri et al. (1998) and Yang et al. (2007)), and Search and Rescue stations (Armstrong and Cook (1979) and Pelot et al. (2015)). Locating hospitals (Sinuany-Stern et al. 1995), emergency medical services (Pirkul and Schilling 1988), blood banks (Jacobs et al. 1996), and ambulances (Ball and Lin (1993), Harewood (2002), and Brotcorne et al. (2003)) are also among the example applications in this area. Goldberg (2004) provided a literature review on operations research applications for emergency services vehicles.

When we concentrate on studies in the area of *SAR* location, there is a considerable number of relevant contributions. Among those, Azofra et al. (2007) proposed a tool for assignment of sea rescue resources to incidents. They used gravitational modelling, and the model provides a coefficient for each possible assignment which represents the appropriateness of a rescue vessel for a given station location, based on the number of historical incidents, their severity, nearby infrastructure, and other factors. Their study only evaluates different solutions but does not propose an optimal solution. Huang and Pan (2007) developed an incident response management tool by integrating a geographic information system with traffic simulation and optimization of response unit assignment.

Afshartous et al. (2009) utilized a statistical-optimization approach to generate a robust solution given uncertainty in distress call locations for locating coast guard air stations. They simulated distress calls and solved the optimization problem for different simulations. However, their mathematical model is simple and lacks the consideration of different criteria. Wagner and Radovilsky (2012) undertook to optimize location and allocation of lifeboats in the U.S Coast Guard and proposed a model that simultaneously considers reduction of excess capacity and boat shortages at the stations, a decrease in the overall fleet size with an increase in boat utilization, and overall reduction of the fleet operating cost. Another significant research was performed by Pelot et al. (2015) where three location

models (maximal covering location problem, maximal expected covering location problem, and maximal covering location problem with workload capacity) were applied to the Maritime *SAR* location problem for Atlantic Canada. That study focused only on covering models so it does not consider other criteria. Moreover, relying on historical incidents for the analysis is a limitation.

Verma et al. (2013) presented a two-stage stochastic programming model which determines the optimal location and stockpile of equipment at the emergency response facilities for the case of oil-spill response. A number of scenarios were generated using the information procured from realistic sources to account for uncertainty involved with parameters' values.

Wex et al. (2014) developed a combined allocation-scheduling model to prescribe the order of rescue unit response to incidents. Their study considered the specific requirements of incidents and different capabilities of rescue units in their model. This study develops a decision support model to minimize the sum of serving times of incidents weighted by their severity. The authors propose and compare several heuristics and show that these algorithms can solve and reach a near optimal solution for a medium size problem in less than a second.

More recently, Razi and Karatas (2016) proposed a multi-objective model for allocating *SAR* resources. They used the Analytical Hierarchy Process to rank and weight different incidents and also a zonal distribution model was developed to cluster incidents and aggregate weighted demand locations. Their model features several objectives including minimizing response time to incidents, fleet operating cost and the mismatch between boats' workload and operation capacity hours. The historical incidents data was used for the analysis.

As described, there is a rich and diverse literature in the area of emergency location analysis, but some gaps still exist, particularly in the case of Maritime *SAR* resource planning. Only a few studies have attempted to consider multiple criteria. With respect to dealing with future incidents, most researchers have relied on using historical incidents for future analysis which does not take into account the stochastic nature of the demand. A

recent account of location analysis in practice, including location of emergency facilities, is provided by Eiselt et al. (2015) and particularly for Maritime *SAR* location by Pelot et al. (2015).

In this study, we aim to not only apply two common location problems to the case of maritime *SAR*, but also assess the solutions in terms of other important decision criteria and investigate their relative performance. Moreover, the underlying distribution of historical incidents is extracted and used as a basis for determining demand weights in optimization models as well as simulation of future incident locations.

3.2.4. Multi-Criteria Location Analysis

Facility location problems are usually multi-criteria decision making (*MCDM*) problems. The objectives that are typically considered in multi-criteria location problems can vary, for example:

- Minimizing fixed and operating costs
- Maximizing service level (required level of response in terms of time and/or quality)
- Minimizing average time/distance traveled
- Minimizing the number of located facilities to cover all demands
- Minimizing the maximum time/distance traveled by any customer to his closest facility

There is a significant number of studies involving multiple criteria in emergency location analysis. Baker et al. (1989) developed a multi-criteria model for the ambulance allocation problem. They used various criteria including response time, cost and workload balance in their model. A goal programming technique was used to solve their problem. Nguyen and Kevin (2000) have also applied goal programming to the case of maritime *SAR* aircraft location analysis.

Drezner et al. (2006) have incorporated five objectives of *p*-median, *p*-center, two maximum covering and the minimum variance in order to minimize the maximum percent deviation from the optimum of each of these objectives for a casualty collection point

location problem, a variant of compromise programming. In their study on location of health care services, Burkey et al. (2012) used efficiency, availability of the service and equality as their examining criteria. They compared existing locations with optimal solutions derived through a maximal covering location problem and a *p*-median problem. The results of their study show that the existing locations provide near-optimal geographic access to health care.

3.3. Assessment Criteria

This section introduces the decision criteria which are used to compare the performance of model solutions and the current situation. Basically, the decision criteria considered in this work are used to examine efficiency, service availability and service equality. As one of the main goals of this study is to incorporate multi-criteria analysis into the *SAR* resource Location-Allocation problem, five main criteria which are of considerable importance for decision makers and the public are defined and examined for the two different solutions based on the maximal covering problem and the *p*-median problem, as well as the current arrangement of resources.

These comparison criteria are as follows:

(1) **Access time:** This criterion assesses the mean time for the nearest response vessel to reach an incident.

This criterion is very common in location analysis studies. In this study, it is defined as 'mean travel time for the nearest response vessel to reach an incident. This is a widely used proxy for measuring the efficiency of service in location analysis. As mentioned before, the *p*-median problem minimizes this criterion for a given fixed number of facilities. We use time rather than distance as we are dealing with multiple type resources with different speeds, so time is a more appropriate measure of proximity than distance.

(2) **Primary Coverage:** This criterion measures the percentage of incidents to be covered within the predetermined access time by at least one *SAR* vessel.

Coverage is an obvious criterion when dealing with service location problems, especially in an emergency location problem. It is used as an index for availability of service. Coverage could have different definitions and can be measured in different ways. It can be considered as a binary function with a specified threshold or it can be a continuous function based on the proximity. It is necessary to have at least one vessel within the acceptable response time limit for areas with the potential for incident occurrence. Therefore, primary coverage is one of the main criteria of this analysis.

(3) **Backup Coverage:** This criterion expresses the percentage of incidents that are within the predetermined coverage region of at least two *SAR* vessels.

Backup coverage becomes a concern when we are faced with a congested system or a system with the possibility of resources being out of service due to maintenance etc. whereby sometimes the closest facility is unavailable to respond. So, it would be beneficial if we have another facility within range and time of coverage so as not to miss the demand. It would be considered as a secondary proxy for service availability.

Indeed, we try to cover demands more than once in order to decrease the probability of server unavailability in case of congestion. One important concern is the possible conflict or trade-off between primary and backup coverage, whereby improving the former may hamper the latter. Hogan and ReVelle (1986) presented one of the earliest maximal backup coverage models.

For the purpose of this study, in the post-modelling analysis, the access times from each grid cell (incidents are projected on mesh of grid cells, see also Section 3.4.1.3) to all vessels within their maximum range are calculated and those cells that are within the maximum access time from at least two vessels are assumed to have backup coverage.

(4) **Gini Index:** This criterion is measured as the deviation level of the access time to all incidents.

Service equality metrics have only been included in relatively few studies. The idea is to locate facilities in order to make them equally accessible to all customers. Several indexes

have been used to measure the equality, such as the range (i.e., the difference between the shortest and the longest distances between customers and assigned facilities), the variance of distances, and the Gini index.

Mulligan (1991) and Marsh and Schilling (1994) discussed the wide variety of ways that equality (or inequality) can be measured with the strengths and weaknesses of each method. For example, measures such as the range of values and maximum absolute deviation are extremely sensitive to extreme values and ignore the interior of the distribution, while measures such as the variance are not normalized, and are thus incomparable between times or jurisdictions (Burkey et al. 2012).

For evaluating this criterion we have chosen the Gini index, a very popular index in economic studies for investigating income level equity which was first suggested by Gini (1921). The Gini index always has a value between zero (indicating total equality) and one (indicating total inequality). The Gini coefficient is usually defined mathematically based on the Lorenz curve, which plots the proportion of the total income of the population (y axis) that is cumulatively earned by the bottom x% of the population (see Figure 3-1). The line at 45 degrees thus represents perfect equality of incomes. The Gini coefficient can then be thought of as the ratio of the area that lies between the line of equality and the Lorenz curve (marked A in the diagram) over the total area under the line of equality (marked A and B in the diagram); i.e., Gini = A / (A + B).
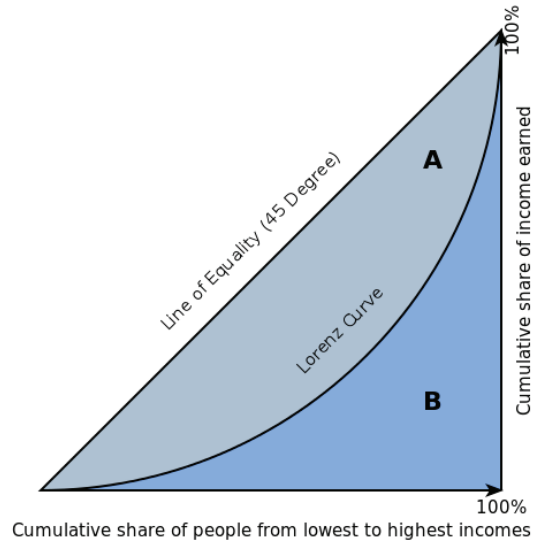
Figure 3-1- Gini index calculation using Lorenz curve

We adopt this concept for the case of emergency response access times whereby the access times to different areas (i.e. incidents) substitute for the income level of individuals in the formula. The following simplified formula (Dixon et al. 1987) is used to compute the Gini index. The index is defined as

$$G = \frac{2\sum_{i=1}^{n} i y_i}{n \sum_{i=1}^{n} y_i} - \frac{n+1}{n} \qquad (3.1)$$

where access times (y_i) are in ascending order and i will be their rank in order (i =1 to n).

(5) ***Maximum access time:*** Another relevant criterion in emergency location analysis is the maximum access time to all customers, which we would like to minimize.

This metric can be considered as an additional index for measuring equality level of service, since the range of variation of access times is smaller. The center problem is a popular type of problem in location modelling whereby the objective is to minimize the maximum distance from the located facilities to reach all customers. One issue with using this criterion is the possibility of outliers that might dramatically affect the metric value. In this study, in order to overcome the issue of rare outliers in remote areas, we take the

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average of the worst ten percent of access times to all incidents as one of the metrics for measuring equality of service provided to customers.

3.4. Case Study: Data Preparation and Modelling

To address the deficiencies discussed in the previous section in the field of maritime *SAR* location modelling, we conduct a multiple criteria decision analysis for our case study and furthermore, we propose a method for simulating future incident locations rather than just simply using the historical incident positions. This section explains the process of data preparation and mathematical modelling for the case study. All operational information used in this case study is retrieved from actual data supplied by the *CCG*. Also, necessary goals and assumptions are established in consultation with *CCG* experts.

3.4.1. *Preparation for Modelling*

This study requires real and valid data about the resources and the demand for response services. The dataset used in this study derives from the *CCG SISAR* (Search and Rescue Information Management System) database which collects information on all maritime incidents. The Atlantic Canada region serves as our research area, with the Coast Guard's administrative borders illustrated in Figure 3-2. The incident dataset, which has been checked and cleaned for quality control, is available from 1988 to 2013, but to have a more accurate analysis, we chose the most reliable recent data from 2005 to 2012 excluding 2007 which has significant accuracy issues, for this study. After performing necessary data cleaning, we obtained a refined dataset with 8,033 incident records.
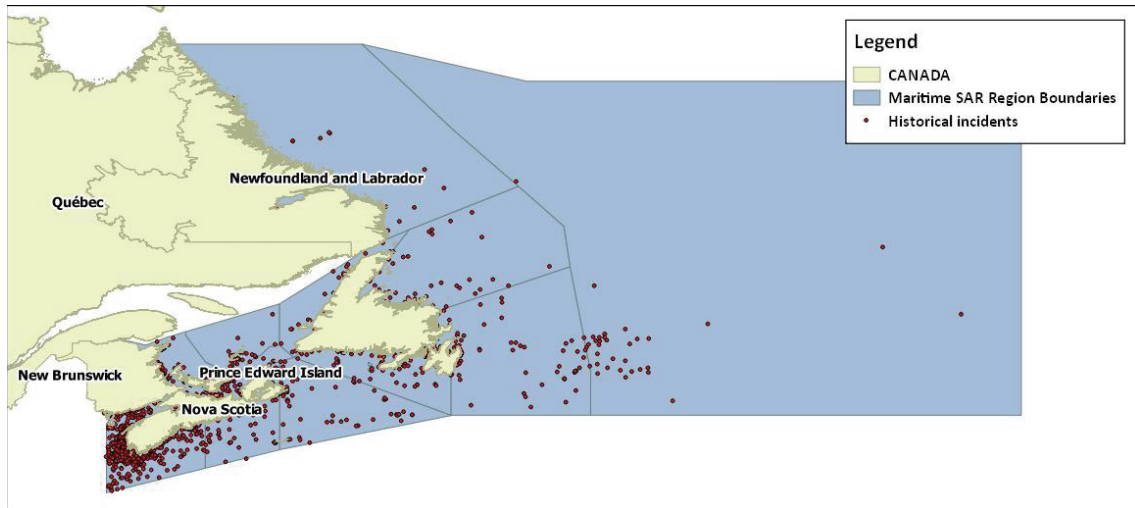
Figure 3-2- Atlantic Canada region and historical incidents

3.4.1.1. Existing SAR Stations

According to information acquired from the *CCG*, currently there are 18 inshore *SAR* stations in Atlantic Canada which are able to house *SAR* vessels. It is assumed that all stations are capable to accommodate all vessel types and no restriction is applied in this regard. Moreover, it is assumed that maximum one vessel of each category can be located in each station. Also, 19 potential offshore stations are to be considered in our analyses. Of course, this is not a station in the traditional sense, but a central location for a vessel that spends much of its time patrolling or performing other tasks at sea. Initially, these offshore stations are assumed to be at the mean point of each Maritime subarea in Atlantic Canada as determined by the Coast Guard. So, we will have 18 stations inshore and 19 potential offshore stations. One assumption in the model, represented as a constraint, is that small *CCG* vessels, called lifeboats, cannot be located at offshore stations because their maximum traveling range is not sufficient for patrol tasks and also they cannot endure for a long time offshore. This restriction is applied in the mathematical models through defining an appropriate constraint.

3.4.1.2. Vessel Types and Characteristics

As mentioned earlier, there are many different *SAR* vessels utilized by *CCG*. In this study, we use the actual information for the currently serving vessels. There are 24 vessels whose primary task is *SAR* response in Atlantic Canada. These vessels include lifeboats, multi-

tasking ships and offshore patrol vessels. Each of these vessels has its own characteristics and capabilities.

All currently available Coast Guard vessels which are capable of providing *SAR* services are categorized into four categories in order to simplify the modelling. The vessel categories with their specifications are shown in Table 3-1. The effectiveness of response thus depends on the fleet composition due to notable differences in these vessels' characteristics which affect coverage and access time.

Table 3-1- Vessel categories with characteristics

Vessel Class	Vessel type	Range (Km)	Vessel Length (m)	Cruising Speed (Km/hr)	Number available
Regular Lifeboat class	*Type 1*	185	16	26	9
Fast Lifeboat class	*Type 2*	185	15	41	7
Offshore Patrol vessel	*Type 3*	10000	60-70	31	4
Large multi-task vessel	*Type 4*	6000	80-90	22	4

3.4.1.3. Land-Avoided Distance Matrix

The distances between incidents' locations and *SAR* stations are required to perform model calculations. There are different methods for distance calculation. The most common way is calculating straight Euclidean distance. However, there is an issue for using straight or direct route calculation in this study. In some cases, it is not possible to use the straight route because of land obstacles in the way. To deal with this problem, we have to use a previously developed land avoidance algorithm by the *MARIN* (Maritime Activity and Investigation Network) research group to find the shortest route between incidents and vessels by calculating Euclidean distance between grid cells while avoiding land obstacles.

As mentioned earlier we use a mesh of square grid cells in the study area to project the incident locations in order to simplify and accelerate the computation. For simplification of calculation, it is assumed that all incidents occur at the centroid of the associated cell. To have more accurate projection of incident locations and more accurate distances, we use a variable grid cell size: smaller cells for areas close to shore with high density of

incidents (1/4 degree × 1/4 degree), larger cells farther out in the ocean (1/2 degree × 1/2 degree) and even larger cells of size (1 degree × 1 degree) in remote areas (Figure 3-3). Some grids around the shoreline are partial due to overlap with land.

The distances are collected in the matrix **D**, which includes distances between all grid cells where incidents may occur (using cell centroids) and potential stations. This matrix has been populated with land avoided distances and has 1,617 rows (grid cells) and 37 columns (stations), where d_{ij} denotes the distance of incident i from potential station j (for n incident grids and m stations). In addition, the number of incidents that occurred in each grid cell is counted. Figure 3-3 demonstrates the mesh of grid cells on the map, colour-coded based on the count of historical incidents.
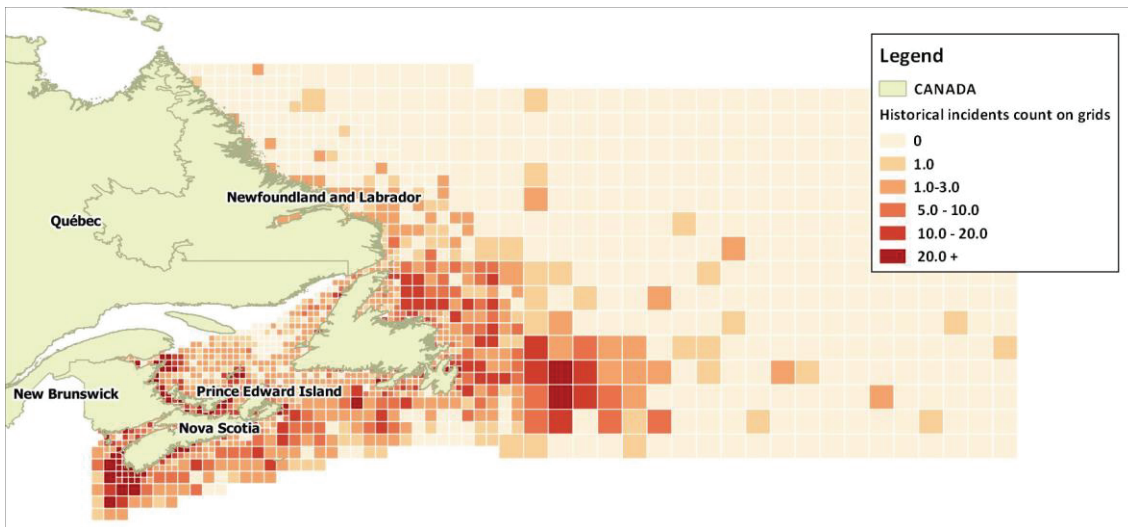
Figure 3-3- Historical incidents on grid cells

1.1.1. *Demand Density Estimation*

In order to use an appropriate estimation for potential incident locations in the future that could represent the stochasticity of demand, we attempt to use a spatial statistical method to extract the distribution of incidents that occurred in the past and use the estimated densities as a proxy of demand weights. Knowing that an incident occurred at specific point in the past does not ensure that it will happen at the same point again, nor preclude events at any other points even if there were no incidents there in the past. The historical data could be used as a strong predictor of future occurrence of incidents though. With this

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proposed approach, we take advantage of actual historical data for extracting underlying patterns and distributions.

There are several methods to find the distribution for spatial data including quadrat analysis, naive estimation, and kernel density function. Extracting patterns and distributions from historical incidents can be accomplished using a Kernel estimation (*KE*) method which is a quite popular method for analyzing spatial point patterns. For each individual point (i.e. cell centroid), the method searches neighbouring area to provide a density estimate based on the number of events (points) in the search area and their distance from that centroid. The other important advantage of the kernel density method over some alternate methods is that it works properly with gridded data and because we projected our demand (incident locations) over a mesh of grid cells, that is relevant. There is a variety of different kernels that have been used for *KE*. The quartic kernel is encountered frequently in the point pattern analysis literature (Bailey and Gatrell 1995). The *KE* with the quartic kernel can be given by:

$$\hat{\lambda}_k(x) = \sum_{d_i \leq \tau} \frac{3}{\pi \tau^2} \left(1 - \frac{d_i^2}{\tau^2}\right)^2 \tag{3.2},$$

where $\hat{\lambda}_k(x)$ represents the estimated density of kernel (*x*) and x_i are points with d_i distance from the centre of the kernel (*x*) which is less than the bandwidth τ (determining the size of the kernel). Simply stated, the kernel equation sums up a function of distances of all points that are within the search area (a circle with center *x* and radius *τ*). The bandwidth defines the level of smoothness of the kernel function whereby the larger bandwidth size results in a smoother function while the smaller one would lead to more saw-toothed profile across the grid cells. Hence, a trade-off value should be chosen depending on the density of the points to avoid over- and under-smoothing.

Using past incidents, as most other studies have done, implies the assumption that the future will behave exactly like the past, while using the procedure proposed in this study assumes that the pattern of incidents (i.e., the underlying probability distribution) remains the same but the instances will vary. This is a less restrictive assumption that will result in a more robust analysis. Furthermore, our approach does not ignore potential demand in

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areas that haven't experienced any incidents in the recorded past as kernel density estimation takes the neighbouring area of each point into account when 'distributing' the demand spatially.

To apply the kernel density estimation method, several parameters including the type of kernel function, cell size and bandwidth are required to be determined which are listed below. The QGIS software version 2.23 was used for applying the kernel density estimation.

- Kernel type: Quartic
- Cell size: (1/4 × 1/4) degree; the centre of each grid cell is used for kernel density calculation.
- Bandwidth (τ): variable size between (1/4 - 1 degrees), 1/4 degree for areas close to the shoreline with high density of incidents, 1/2 degree for areas further from shore and low incident density, and one degree for areas further offshore with a very low number of incidents in the vicinity.

The incident density estimates obtained from applying *KE* are used as demand weights (w_i) in the optimization models.

3.4.2. Study Assumptions

The following assumptions are considered in the study:

(1) The average speed used for travel time calculations of *SAR* vessels is assumed to be equal to their cruising speed which comes from their build specifications.
(2) As we focus on access time, which basically refers to travel time, the rescue operation time (the time required for *SAR* activities from arriving at the incident position until the completion of the *SAR* task, possibly including search time) is ignored in all models.
(3) No environmental impacts such winds, sea state, tides, etc., are taken into account.
(4) No time is considered for coordination or preparation of the response.
(5) The maximum access time in this study for an acceptable level of primary and backup coverage is considered to be 6 hours based on consultation with *CCG*

experts. This parameter can be varied to examine the sensitivity of model results to the maximum access time. This constraint is only applied to the maximal covering model, but not to the *p*-median model where there is no limit on access time, although it is used for calculation of primary and backup coverage in both models' solutions.

(6) We assume that the available response vessels are those in the current *CCG* fleet (fixed number of vessels in various classes).

3.4.3. *Modelling: Different Configurations*

Some modifications are made to two prototypical location models to customize them based on the real *SAR* problem. First, we solve an *MCLP* model, which includes multiple types of facilities. The objective function of the model is to maximize the number of incidents which are within a specified time (six hours in this study) from the closest facility for a given number of available facilities. In the second model, we apply a revised *p*-median model to minimize the mean access time to all incidents from their respective closest facility. We also examine two additional scenarios by adding a constraint to each of these models to restrict the location of vessels to inshore stations. This addresses the questions of whether it is possible to obtain an acceptable level of service without locating vessels offshore which apparently incurs extra operating costs.

This section provides the formulation of the models.

3.4.3.1. A Multiple Facility Type MCLP

The goal of this model is to maximize coverable demands within predetermined travel time for a given number of *SAR* vessels. The full list of variables and parameters of the model is given below.

Indices:

$i \in I:$ Grid cell index (demands)

$j \in J:$ Index for potential vessel stations

$J_S \subset J:$ Set of offshore stations

$k \in K:$ Index for vessel types

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Variables:

x_i :   Binary variable for primary coverage at cell i

y_j^k :   Integer variable for number of vessels type k located at station j

Parameters:

r^k: Coverage distance (range) of vessel type k

p^k: Available number of vessel type k

v^k: Cruising speed of vessel type k

d_{ij}:   Distance between grid cell i and station j

w_i: Number of incidents in grid cell i

t: Maximum access time for acceptable coverage

$a_{ij}^k := 1$ if: $(d_{ij} \leq r^k$ and $d_{ij}/v^k \leq t)$ else $a_{ij}^k := 0$

Parameter defining whether grid cell i is within response range and acceptable access time of vessel k positioned at j

The formulation of the proposed model is as follows.

Problem 1: Max: $Z = \frac{\sum_i w_i x_i}{\sum_i w_i}$ (3.3)

s.t.

$x_i \leq \sum_j \sum_k a_{ij}^k y_j^k, \forall i$ Primary coverage constraint (3.4)

$\sum_j y_j^k \leq p^k,$ $\forall k$ Fixed number of available vessels in each type (3.5)

$y_j^k = 0$ $\forall j \in J_s, k \in \{1,2\}$ Offshore location constraint (3.6)

The objective function (3.3) maximizes the percentage of incidents with primary coverage. Constraints (3.4) restrict the primary coverage to grid cells that are within pre-specified time from a located vessel. The maximum number of vessels in each vessel category is limited by constraints (3.5). Also, constraints (3.6) ensure that we do not locate lifeboats (vessels type 1 and 2) at the offshore stations.

3.4.3.2. Multiple Facility Type P-Median

The objective of the second model is to minimize mean access time to all incidents for a given number of *SAR* vessels. Many parameters and variables are similar to the maximal covering model. Below is the list of additional parameters and variables followed by formulation of the model.

Variables:

u_{ij}^k: Binary allocation variable for grid cell i to vessel type k located at station j

Parameters:

$b_{ij}^k := 1$ *if*: $d_{ij} \leq r^k$ *else* $b_{ij}^k := 0$

>   Parameter defining whether grid cell i is within response range of vessel k at station j

Problem 2: Min $Z = \dfrac{\sum_k \sum_i \sum_j \left(w_i u_{ij}^k \left(d_{ij}/v^k \right) \right)}{\sum_i w_i}$ (3.7)

s.t.

$\sum_k \sum_j u_{ij}^k = 1, \forall\, i$ Grid cell allocation to located vessels   (3.8)

$\sum_j y_j^k \leq p^k, \quad \forall\, k$ Fixed number of available vessels in each type (3.9)

$u_{ij}^k \leq b_{ij}^k y_j^k \quad \forall\, i, j, k$ Allocation to *SAR* vessels   (3.10)

$y_j^k = 0 \qquad \forall\, j \in J_S, k \in \{1,2\}$ Offshore locations constraint   (3.11)

The objective function (3.7) minimizes the weighted average of access time to all incidents. Constraints (3.8) require that all demand cells are assigned to exactly one located vessel. Inequalities (3.9) constrain the maximum number of available vessels in each category. Restricting the allocation of vessels to those incidents that are within the coverage range is applied by (3.10). And as in the previous model, constraints (3.11) guarantee that lifeboats are not located at the offshore stations.

These models were built in the *MPL* environment and solved by the *GUROBI 6.5* solver, producing the optimal solutions in 14.09 and 15.73 seconds respectively using a computer with *CPU* Intel *CORE* i7 and 8*GB RAM*.

3.4.3.3. The Current Arrangement of Vessels

This configuration aims to simulate the current situation of *SAR* vessel usage given the actual location of vessels, as a basis for comparison of solutions. The current location of *CCG* vessels included in this study by category is shown in Figure 3-4. Based on this information, the response allocation of resources to the forecasted incidents is simulated taking into account the vessels' characteristics (response range and speed) and the policy of allocating incidents to the closest vessel within response range. It should be noted that there is no reliable observation on actual response travel times. Even if such information were available, given that many other factors (e.g. weather and sea conditions) affect actual response times, for consistency and comparability of the results with the model solutions it is better to take our simulation approach.
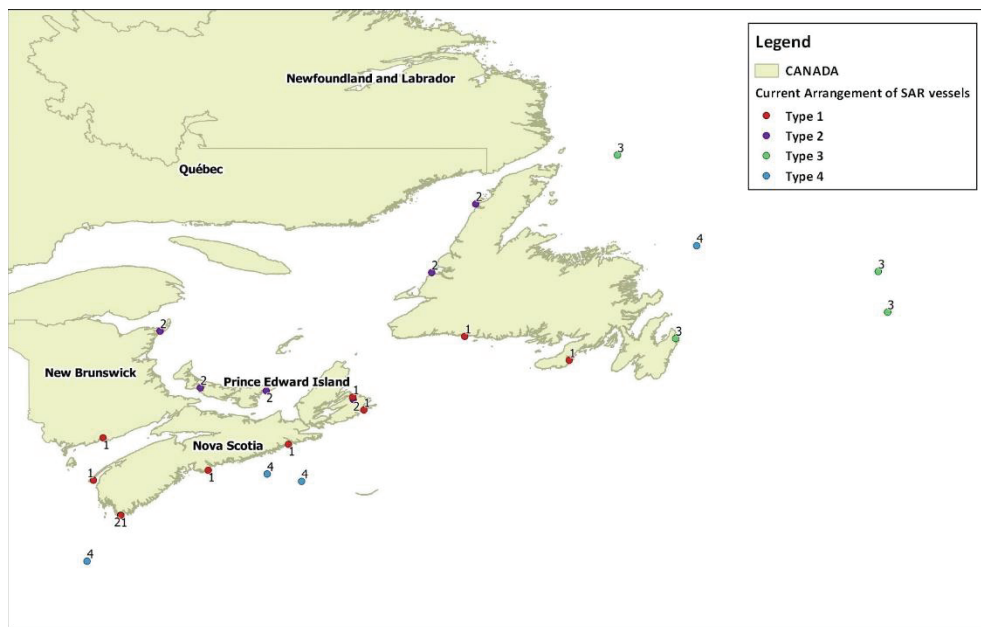
Figure 3-4- Current locations of SAR vessels in Atlantic Canada

3.4.3.4. Models with Inshore Location Restriction

By including additional constraints to restrict all vessels to locate at inshore stations, we examine the effect of excluding offshore stations from the potential vessel locations. This will basically allow us to investigate whether it is possible improve the service level compared to the current arrangement of vessels without locating any vessels in the offshore areas.

3.4.3.5. Examining Solutions' Robustness: Simulation of Incident Scenarios

The kernel density estimates are used as the basis for generating random incidents for future demand scenarios. The process of using kernel density estimations for simulating future demand is performed through the following steps:

(1) For each grid cell, the average of kernel estimates within the cell square is computed (these values are visualized in Figure 3-5).

(2) Computed incident density rates are multiplied by the grid cell area (to account for the variable size grid cells) to compute the expected number of incidents for each cell.

(3) Calculated cell incident counts are scaled (multiplied by a fixed value) so that they sum up to the average number of incidents per year. These scaled values are considered as the mean parameter of a Poisson distribution to be used for generating a random number of incidents over the mesh of grid cells in the area of interest.

(4) One hundred sets of random incident counts are generated based on the calculated Poisson rates. These randomly generated scenarios represent the stochastic future demand and are used for validation of model solutions.
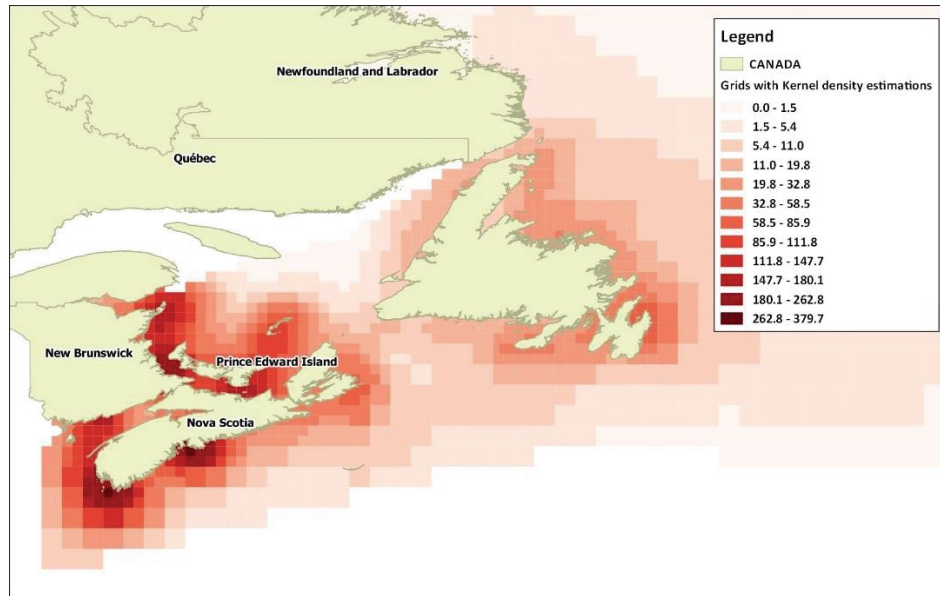
Figure 3-5- Incidents Kernel estimates on gridded map

3.4.4. Discussion of Our Results

3.4.4.1. Optimization Model Solutions

The optimal solutions found for the different scenarios were compared to each other as well as to the current arrangement of vessels in terms of discussed assessment criteria. Results of this comparison analysis are summarized in Table 3-2.

Table 3-2- Solution comparison results

Model	Primary Coverage	Backup coverage	Mean access time (hours)	Gini index	Mean access time to worst 10% (hours)
MCLP	95.42%	60.19%	2.75	0.393	8.40
p-median	94.56%	64.69%	2.50	0.404	7.77
Current arrangement	89.38%	60.18%	3.14	0.467	11.57
Inshore MCLP	89.07%	78.06%	3.23	0.474	12.51
Inshore p-median	88.79%	77.70%	2.98	0.472	11.01

As Table 3-2 shows, the *MCLP* model has slightly better performance regarding the primary coverage criterion compared to the *p*-median (95.42% vs. 94.56%), whereas the

p-median model generates a solution with significantly lower (better) access time (around 0.25 hour) and higher level of backup coverage (64.69% vs. 60.19%). Regarding the equality criteria, it is observed that the *MCLP* has a slightly better performance than the *p*-median solution in terms of the *GINI* index. This can be explained by the point that *p*-median tries to minimize the sum or mean access time so it doesn't care about the equality (fairness) that much, while *MCLP* does its best to keep all demand access times under the maximum acceptable coverage time in order to maximize the primary coverage. Although, with respect to the access time to remote incidents where we measure it by average access time to the worst 10% of incidents, *p*-median has a marginal advantage. Overall, neither of the two model solutions dominates the other one in all criteria, although *p*-median has the advantage in most cases and is competitive in others.

When it comes to comparing the models' solutions with the current situation, we see a substantial improvement in almost all indices. Primary coverage increases by at least 5%, mean access time could be lowered by up to 40 minutes, and the equality can be also improved significantly. Moreover, backup coverage can go up by 4.5% if we use the *p*-median solution. It is obvious that there is a trade-off between the different criteria, and one model with a single objective cannot perform the best in all aspects. But according to the observed results, we can say that both optimization model solutions dominate the current arrangement based on the criteria we have chosen and measured. In the case where we preclude potential offshore stations, running the two models with only inshore stations resulted in a competitive solution compared to the current arrangement where we do have some vessels patrolling offshore. The most interesting point here is that we can increase the backup coverage, to more than 78%. This might be the result of siting more vessels in the areas close to shore, which are generally the areas where more incidents occur.

The spatial variation of the vessel locations from the *MCLP* and *p*-median solutions is presented in Figure 3-6 and Figure 3-7 respectively. Although these two configurations have some differences, in general there are very similar and substantially different from the current arrangement of vessels (Figure 3-4) as the solutions obtained by both models suggests relocating about 50% of the vessels from their current station. Should this degree of change be deemed unacceptable, one could add constraints to the *MCLP* and *p*-median

formulations that limit the number of changes made to the current arrangement, and this approach could also serve to examine the tradeoff between vessel relocations and the amount of service level improvement.
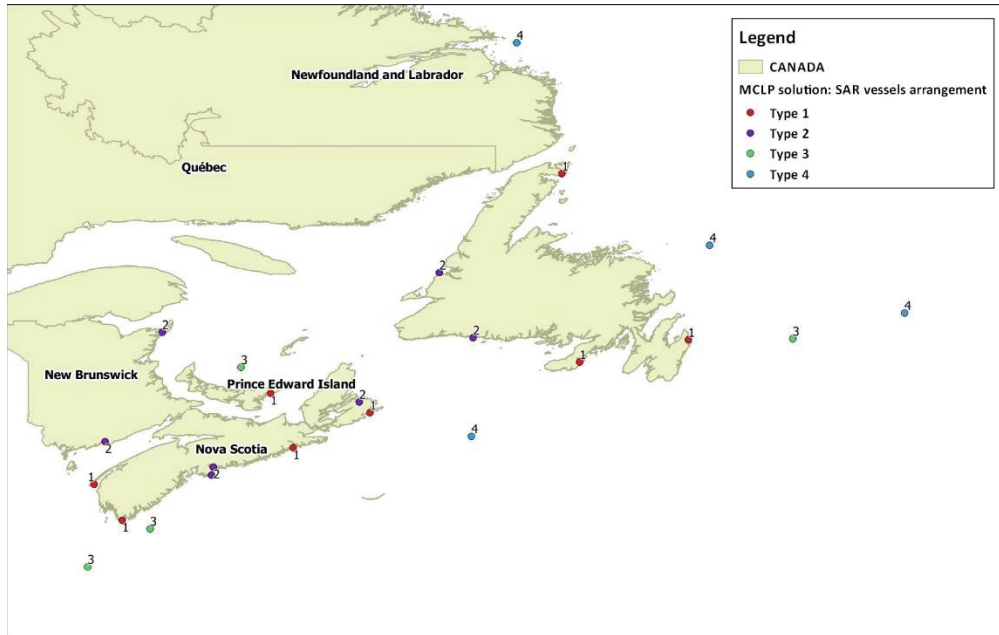
Figure 3-6- Maximal covering solution's arrangement
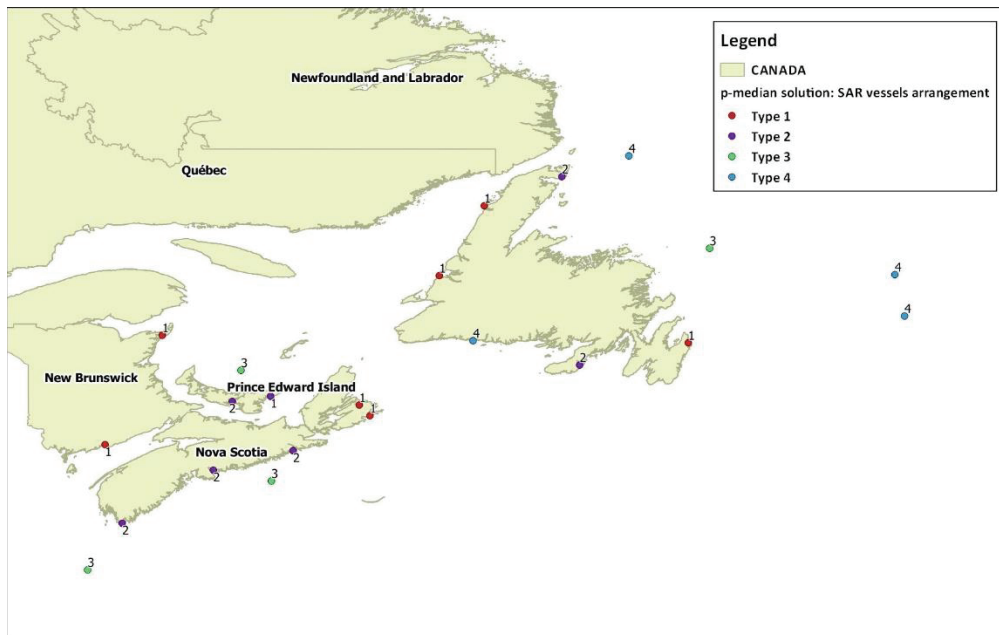
Figure 3-7- P-median solution's arrangement

3.4.4.2. Simulation validation

In order to examine the robustness of solutions over different simulated incident scenarios, three assessment metrics (criteria) are measured for each simulated scenario. That way we can see the fluctuation of metrics when incident locations change to observe how sensitive the model solutions are to the variations in demand. Figure 3-8 to Figure 3-10 demonstrate the variation of three main criteria over hundred simulated scenarios using box plots. According to these observations, the variation of metrics for the two main model solutions over different simulated scenarios is small. In addition, the observed results indicate a clear distinction of the solution performances in that the optimization model solutions dominate the current arrangement of resources with respect to the most criteria. In particular, Figure 3-8 confirms that both model solutions provide substantially better primary coverage than the current arrangement. Also, the performance of two models is very close although the *MCLP* has a slight edge as expected. The variation of metric values is very low. With respect to backup coverage, the simulation results validate our findings as *p*-median has a substantially better performance compared to other configurations (Figure 3-9), although we observe a higher level of variation compared to primary coverage. Observations on mean access times are interesting as well, where again the *p*-median solution performs the best, while the *MCLP* lies in between that and the current arrangement (Figure 3-10Figure 3-10). Overall, examining the performance of different vessel configurations over the one hundred simulated demand scenarios verifies the findings of the optimization models that used estimated demand weights as input.
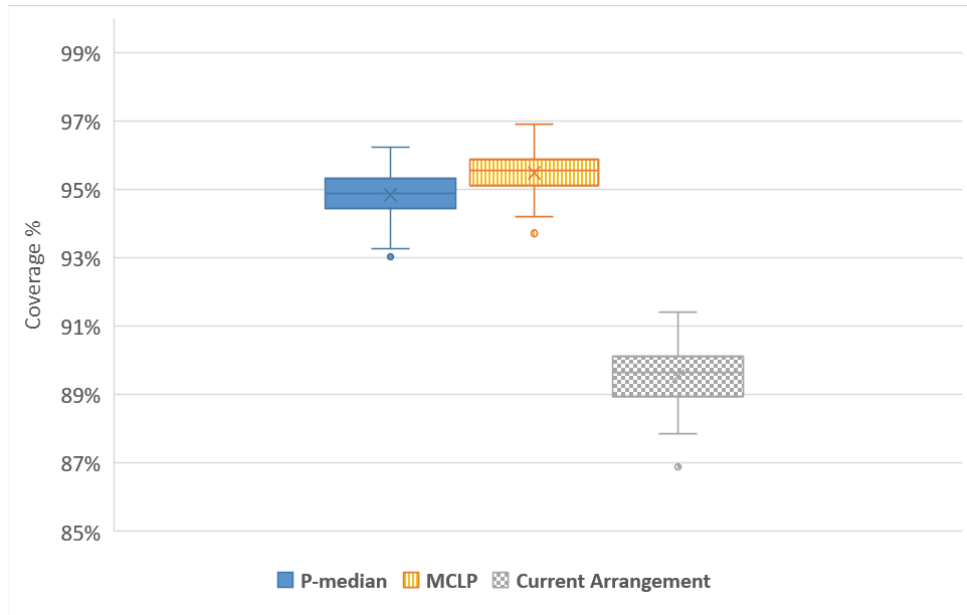
Figure 3-8- Primary coverage variations over simulated incident scenarios
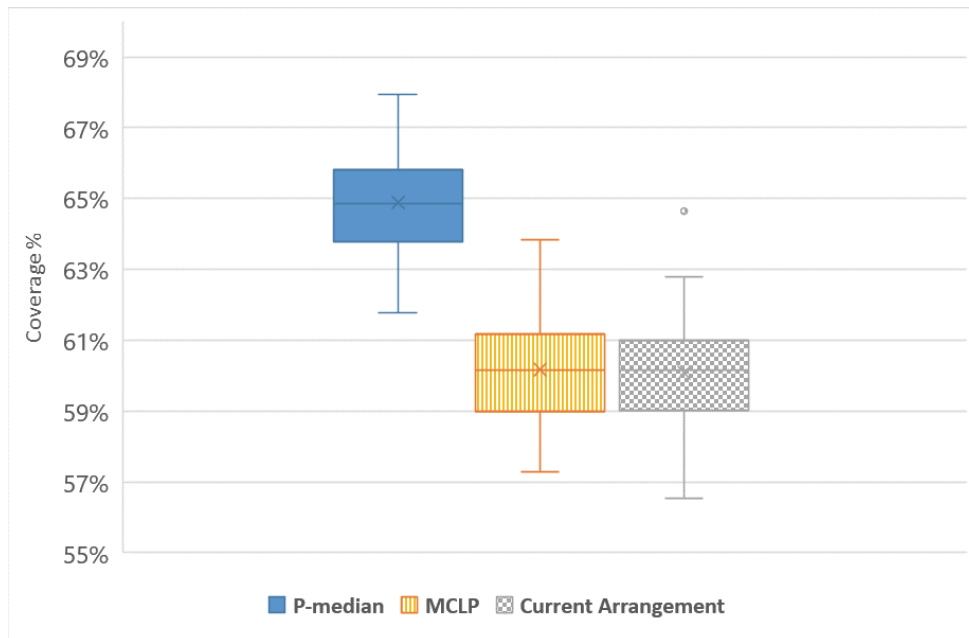
Figure 3-9- Backup coverage variations over simulated incident scenarios
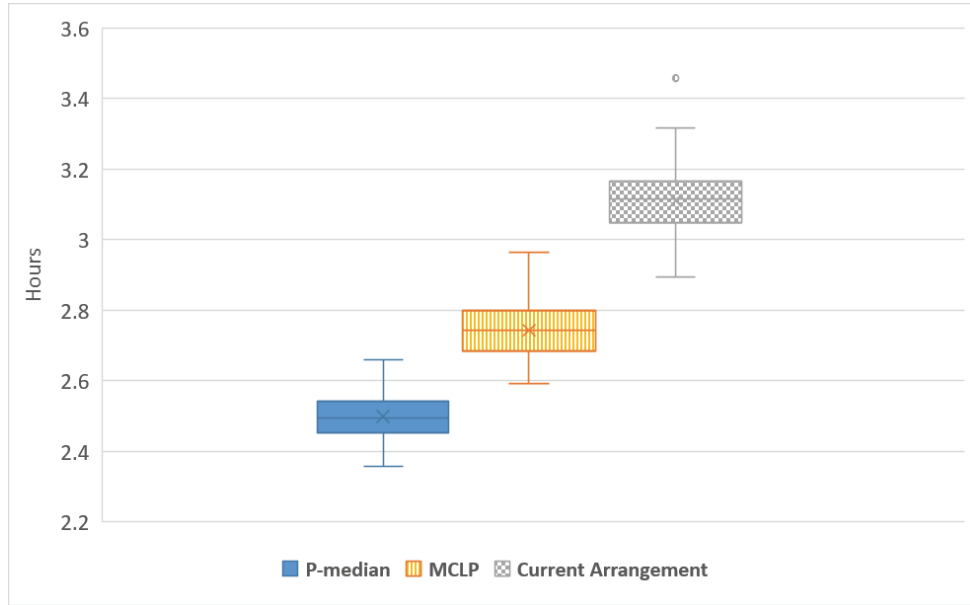
Figure 3-10- Mean access time variations over simulated incident scenarios

With regard to the distribution of access times to all incidents, it is quite important for us to reach as many incidents as possible in the minimum amount of time. Figure 3-11 presents the cumulative distribution function of access times to all incidents for the solutions of the proposed models compared to the current situation. The *p*-median solution produces access to the maximum number of incidents within two hours (more than 50% of all incidents). As we move toward six hours, the *MCLP* solution starts performing better which is not surprising as we expect that to provide better primary coverage (access within 6 hours). Moreover, it can be clearly seen that the *p*-median outperforms the current arrangement while that is not the case for the *MCLP* throughout the range.
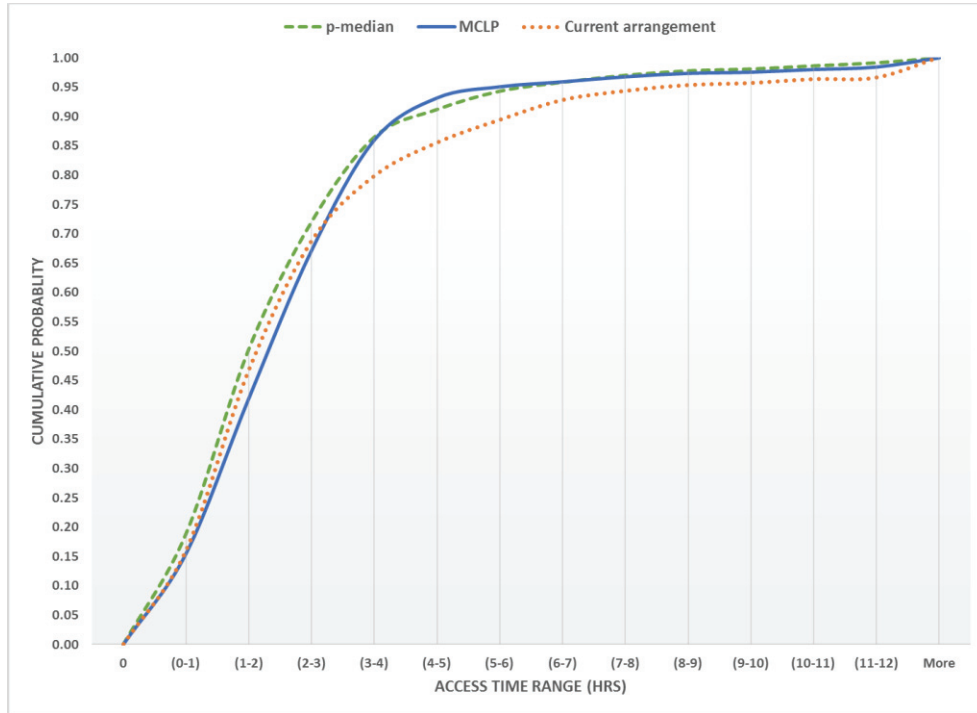
Figure 3-11- Cumulative distribution function of access times

3.4.4.3. Sensitivity to relocation

To investigate the sensitivity of the model solutions to the number of relocations required from the current vessel positions, a parametric analysis was conducted for the *p*-median model. An additional constraint is included to restrict the total number of vessel relocations from the current position, which is then altered to see how the solution changes. According to the results, the number of vessels that need to move for the *p*-median solution with no restriction is 12 vessels (50% of the fleet). Table 3-3 shows the sensitivity of the solution to this constraint. These results are useful for decision makers since it shows how much improvement is possible with fewer relocations, even though it is likely that, due to operational difficulties, it would not possible to implement all relocations suggested by the model solution. For instance, with moving only six vessels (25% of the fleet) we can drop the access time down from 3.14 to 2.67 hours thus achieving about 75% of the maximum possible reduction in mean access time (which would be obtained by 12 relocations).

Table 3-3- Sensitivity of the p-median model solution to number of relocations allowed

Number of relocations	Primary coverage	Backup coverage	Mean access time	% possible improvement in access time
12	94.56%	64.94%	2.50	100.00%
11	94.42%	61.73%	2.52	96.59%
10	94.37%	67.30%	2.55	91.26%
9	94.40%	61.27%	2.58	87.38%
8	92.89%	67.78%	2.61	83.02%
7	92.92%	63.52%	2.64	78.17%
6	92.88%	67.32%	2.67	73.82%
5	92.54%	67.31%	2.70	69.01%
4	92.88%	60.88%	2.75	60.57%
3	91.88%	64.50%	2.81	51.28%
2	91.89%	57.84%	2.88	40.99%
1	91.40%	58.91%	2.97	26.67%

3.5. Conclusion and Outlook

In this study, we applied two common location models in a maritime *SAR* Location problem. For these models, four common *SAR* vessel types which are used in practice and have different capabilities are considered. Two integer-linear optimization models with different objectives, maximizing primary coverage and minimizing mean access time, are utilized. We have chosen five performance metrics of interest for decision makers and the public to assess the solutions. We found that the *p*-median model provides a better solution in terms of three metrics (access time, backup coverage and access to furthest incidents), while the *MCLP* works slightly better for primary coverage and service equality. Overall performance of both model solutions is close. Notably, both of these optimization models provide solutions with significantly better performance versus the current arrangement of *SAR* vessels taking into account all decision criteria.

A simulation procedure based on kernel density estimation was proposed and used in this study to generate several demand scenarios to represent the stochasticity of demand in the future. The variation of models' solution performance over different demand scenarios are examined and as observed there are not large variations across simulated scenarios. Results of this study could be useful for guiding decisions with regards to *SAR* vessel placements

in order to improve the efficiency of using resources and increase the service level. Sensitivity analyses can be performed to help explore particular circumstances such as the impact of changes in service level requirements or available resources.

Decision makers can use the models to gain insights for rearrangement of current vessels, procurement of new vessels, establishing new stations or decommissioning decisions, and society would benefit from improvements in the service accessibility and effectiveness resulted from applying the proposed solutions.

There are several potential future extensions of this work. It is apparent that the two models used in this research have conflicting objectives and each of two models concentrates only on one objective and another one is ignored. One possibility is to use multi-objective optimization methods to take into account all criteria as objectives in one model, so as to obtain additional interesting trade-off solutions on the non-dominated frontier. One important factor which could make the analysis more realistic and useful is considering different characteristics of vessels and incidents which couldn't be done in this study due to unavailability of required information. Some incidents may need a special type of response or some *CCG* vessels may not be able to serve specific types of incidents. In other words, all vessels are not equally effective in responding to different types of incidents. Congestion is also an important issue whereby there is the possibility that the closest vessel is not available to respond or it is busy with another task. The models could be extended to take this into account through probabilistic and queuing models. Optimizing the fleet composition could also be included in a more comprehensive resource planning model that involves more strategic decisions with budget implications. The proposed model could be further extended to future response needs by modelling trends in incidents, incident rates, and/or exposure metrics like traffic levels.

Chapter 4   Determining the Optimal Mix and Location of Search and Rescue Vessels for the Canadian Coast Guard

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Abstract

The Canadian Coast Guard is in charge of search and rescue missions off the east coast of Canada. It accomplishes this task with a number of different vessels. In order to determine the best possible type of equipment and the optimal locations of the Coast Guard vessels, we formulate an integer optimization problem that minimizes the access time required by the Coast Guard vessels to provide service, given a budget and a required coverage. The results demonstrate a large discrepancy between the existing equipment and the vessels that would optimally be used, thus potentially providing a much improved service.

Keywords: Coast Guard; location problems; fleet configuration; *p*-median; covering models

4.1. Introduction

When driving a car on a highway, we may encounter an accident or other emergency. In such a case, given a functioning cell phone, help is usually more or less readily available within a fairly short amount of time. The situation on the oceans is generally different, mostly due to the fact that rescue facilities are more remote, much more expensive to operate, and typically considerably slower, thus rescue times are much longer, and the resulting damage to lives and property is often considerably higher. Furthermore, different incidents demand different rescue modes and equipment: a large container vessel in distress at sea will require very different response equipment than a fishing vessel near the coast that experiences a fire, or a yacht that has a man overboard.

The search and rescue problem is particularly important in the Canadian Coastal waters. As Transport Canada (2016) points out,

> "Canada has the world's longest coastline, at more than 243,000 kilometres [in excess of 150,000 mi, ed.]. Each year, 80 million tonnes of oil are shipped off Canada's east and west coasts. On any given day, there are 180 vessels (ships known as 'SOLAS vessels,' or those over 500 tonnes gross tonnage that operate internationally) operating within Canada's Exclusive Economic Zone (200 nautical miles from shore)."

These are vast areas of open water that have to be served by Coast Guard vessels, thus their positioning is crucial. Coast Guards typically have a number of different vessels with different capabilities, whose task is to respond to distress calls of different degrees of urgency, which requires a triage system. One issue that somewhat simplifies matters is that much of the traffic takes place in shipping lanes. An example for such lanes along the Canadian Coast in the North Atlantic is shown in Figure 4-1. The lane in the upper right part is the traffic that passes Cape Breton, Nova Scotia to the North and leads into the Saint Lawrence Seaway system and eventually the Great Lakes. To its south, major lanes split off along the coast of Nova Scotia to the Northeastern United States. Finally, smaller lanes

form a star-shaped pattern near the left side of the Figure. They connect the New Brunswick port of Saint John and Nova Scotia's port of Yarmouth with Boston, Baltimore, and other places in the Northeastern United States.
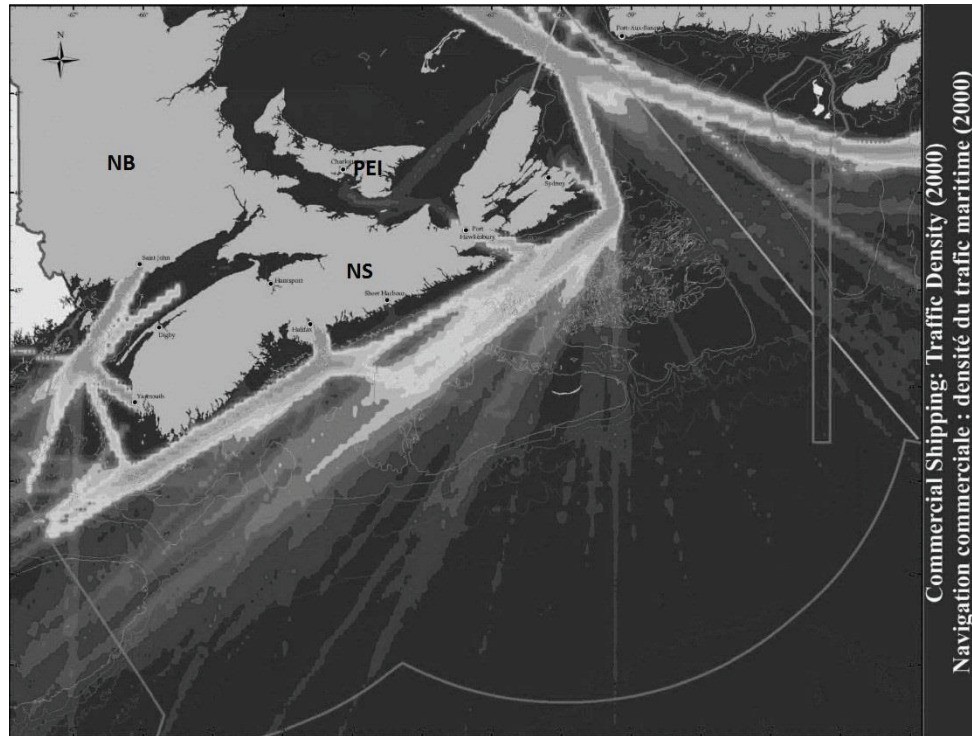
Figure 4-1- Major shipping lanes off the Eastern Canadian coast (landmass shown in gray) (Fisheries & Oceans Canada 2016)

The task of planning an efficient system for search and rescue operations requires locating appropriate equipment at appropriate places at the lowest possible cost. The question of where to locate the rescue vessels such as lifeboats, patrol vessels and multitasking ships is as important as the choice of equipment: even with the best and most appropriate equipment, if it is located very far away, it will take a long to time to respond with the associated loss of life and property. And this is the focus of this paper: what composition of search and rescue vessels do we choose and where do we locate them, in order to be able to respond as efficiently as possible?

In order to simplify matters, we will adhere as much as possible to the usual terminology used in location theory; see, e.g., Eiselt and Sandblom (2004), Eiselt and Marianov (2011) Daskin (2011), Eiselt et al. (2015) and Laporte et al. (2015).The ships or boats that are

potentially in need of assistance are referred to as "customers," while the vessels used in the search and rescue effort are referred to as "facilities." The space, in which customers and facilities are located and will locate is the Euclidean plane, which, however, has been discretized, so as to make the problem computationally tractable. Since all facilities move in water, we can safely use the ℓ_2 norm, i.e., Euclidean (straight-line) distances with avoiding land obstacles. Since the main objective of this study is to locate facilities, so as to provide customers with help as quickly as possible, we will consider two criteria. The first criterion concerns access time, while the second criterion deals with the area over which the rescue facilities can reach and provide assistance within a pre-specified time to customers. In order to operationalize and define quantitative measures for our purpose, we will use the average access time between a customer and his closest rescue facility, as well as the number of customers located within a predetermined distance D from a facility. These are best known as p-median problem (also known as multi-Weber problem or, alternatively, location-allocation problem), and a maximum covering problem. These are the starting points used in this paper.

The remainder of this paper is organized as follows. Section 4.2 discusses the basic models that are the foundation of the problem formulation applied in this paper. Section 4.3 investigates the specific situation of the Canadian Coast Guard and formulates a mathematical model that determines the optimal mix of search and rescue vessels and the locations of the vessels, so as to minimize access time while ensuring a chosen coverage level and staying within the Coast Guard's means. Section 4.4 displays the solutions of the model and discusses the results. Finally, Section 4.5 summarizes our findings and presents some potential future research strands.

4.2. The Basic Models

This section will first establish the usual p-median and maximal covering models, which form the basis of our model. This will also allow us to introduce the parameters, the variables, and the notation we will use in the model under investigation. The formulation for the model discussed in this paper is then presented.

Denote by I the set of *customers* in the model, so that $|I| = m$. Similarly, the set J with $|J| = n$ represents all *potential facility locations*. For computational reasons, we will discretize space and consider only a finite number of customer locations and possible facility locations. Furthermore, we define the *distance d_{ij}* between any two points (typically, we are interested in distances between customer points i and facility location points j) as the shortest distance between points i and j. Furthermore, we will be using the user-defined *service standard D*, which indicates a distance between customer and facility, within which a customer will be considered covered, which is why it is frequently also referred to as a *covering distance*. For instance, if a facility such as an ambulance, a fire truck, or a rescue vessel can reach a customer within, say, half an hour, and within the given context 30 minutes correspond to 20 km, then any customer, who is no farther than 20 km from the facility closest to him, will be considered covered.

Two comments regarding possible and important extensions should be made here. First of all, the model is appropriate only in case of very low demand density, i.e., a small number of incidents that require the attention of an emergency facility. Put differently, the model makes sense only if the capability of the combined forces of all vessels far surpass the number of incidents, i.e., the need. If this is not the case, *congestion* becomes an important issue, which may require serving a customer from a facility that is not closest to him. For location models with congestion, see, e.g., (Berman and Krass 2015). Secondly, coverage in practice is not a zero-one affair. If, to use the aforementioned example, the covering distance is 20 km, then a customer, who is, say 21 km from his closest aid facility, for practical purposes cannot be referred to as "not covered," even though the basic model will consider him as such. What is more realistic is what is commonly called a *gradual covering model*, (Church and Roberts (1983); Berman and Krass (2002); Drezner et al. (2004), and Eiselt and Marianov (2009)), in which the degree of coverage decreases with increasing customer–facility distance. The gradual covering model no longer uses the covered-not covered dichotomy, but assigns instead degrees of coverage: a number close to zero indicates that the distance between a customer and his closest facility is significant, whereas a number close to one shows that the facility closest to a customer is quite close.

While not probabilities *per se*, the numbers could be understood as an indicator of the likelihood that a customer could be reached and served within the covering distance D.

The basic maximal covering model is described by Church and ReVelle (1974); for a recent survey, see García and Marín (2015). Again, customers are located at sites $i \in I$, whereas facilities may be located at points $j \in J$. A total of p facilities are to be located, where the value of p is typically determined by the decision maker's budget for this purpose. The distance (assumed to be along the shortest path) between customer point i and (potential) facility site j is denoted by $d_{ij} \ \forall \ i \in I$ and $j \in J$. The set $N_i = \{j \in J: d_{ij} \leq D\}$ then denotes the set of facility sites j, from which a customer at point i is located within the covering distance D. Furthermore, with each customer point $i \in I$, we associate a weight h_i, which typically indicates the demand for service at site i, which is normally expressed by its proxy, the population at site i. As such, these weights indicate the number of incidents that requires attention from service (rescue) facilities.

We can then determine two types of variables. First, there are the binary *location variables* y_j, which assume a value of one, if we locate a rescue facility at point $j \in J$, and zero if we do not. We also need binary *covering variables* x_i, which assume a value of one, if a customer location is covered by a service facility (which is the case, if it is within the covering distance D), and zero otherwise.

The maximal covering problem can then be described as follows.

$$P_1: \text{Max } z = \sum_{i \in I} h_i x_i \tag{4.1}$$

$$\text{s.t. } x_i \leq \sum_{j \in N_i} y_j \ \forall \ i \in I \tag{4.2}$$

$$\sum_{j \in J} y_j = p \tag{4.3}$$

$$y_j = 0 \text{ or } 1 \ \forall \ j \in J \tag{4.4}$$

$$x_i = 0 \text{ or } 1 \ \forall \ i \in I \tag{4.5}$$

The objective function (4.1) in problem P_1 maximizes the coverage that the facilities provide, constraints (4.2) ensure that a customer is only considered covered if at least one facility is located within its covering distance, the single constraint (4.3) indicates that exactly p facilities are to be located, and the specifications of the variables shown in (4.4) and (4.5) indicate the binary nature of the variables in this model.

While the maximal covering problem maximizes the number of customers who will receive adequate service from the facilities, it does not say anything how good the service actually is. The p-median problem can do that. It was first put forward by ReVelle and Swain (1970), and it requires a few different definitions. The parameters h_i, d_{ij}, and p have already been defined, as have the location variables y_j. We do not need covering variables, but we need *assignment variables* x_{ij}, which are one, if the customers at point $i \in I$ are assigned to the facility at point $j \in J$ and zero otherwise. The (uncapacitated) p-median problem can then be written as

$$P_2: \text{Min } z = \sum_{i \in I} \sum_{j \in J} h_i d_{ij} x_{ij} \tag{4.6}$$

$$\text{s.t. } \sum_{j \in J} x_{ij} = 1 \ \forall \ i \in I \tag{4.7}$$

$$x_{ij} \leq y_j \ \forall \ i \in I, j \in J \tag{4.8}$$

$$\sum_{j \in J} y_j = p \tag{4.9}$$

$$x_{ij} \geq 0 \ \forall \ i \in I, \ j \in J; \ y_i = 0 \lor 1 \ \forall j \in J. \tag{4.10}$$

The objective function (4.6) of problem P_2 minimizes the sum of customer-facility distances of all customers (which is, given that each customer is actually served, equivalent to the average access time). The constraints (4.7) ensure that each customer is allocated to exactly one facility, the constraints (4.8) guarantee that service from a facility at a node n_j is provided only if there exists a facility at that node, and constraint (4.9) requires that

exactly *p* facilities are located. The specifications of the variables are expressed in constraints (4.10).

Pelot *et al.* (2015) presented several location models including maximal covering location problems, maximal expected covering location problems, and maximal covering location problems with workload capacity for the case of the maritime Search and Rescue location problem for Atlantic Canada. Also, analyzing the location of different types of coast guard vessels has recently been examined by (Akbari, Eiselt, et al. 2016; Akbari, Pelot, et al. 2016). The papers examine the improvements in coverage and access time that could be achieved by relocating the existing vessels. This paper goes one step beyond these results.

4.3. The Coast Guard Problem

While the previous section has outlined two of the most important general models in the literature, this section will show our model, which is a combination of the *p*-median and maximum covering model for the specific application of the Coast Guard. Furthermore, our model not only finds locations for vessels, but it also determines the optimal mix of different types of vessels. In order to be able to formulate the model, we will need to make a number of assumptions.

Assumption 1: Space. The space, in which incidents occur and in which Coast guard vessels are to be located is naturally continuous. In order to make the space tractable, we discretize the space. More specifically, we use a mesh of square grids. As more incidents are expected close to shore, we use a variable grid size: smaller grids close to shore (1/4 degree × 1/4 degree), larger grids farther out in the ocean (½ degree × ½ degree) and even larger grids of size (1 degree × 1 degree) in remote areas. This pattern is shown in Figure 4-2.
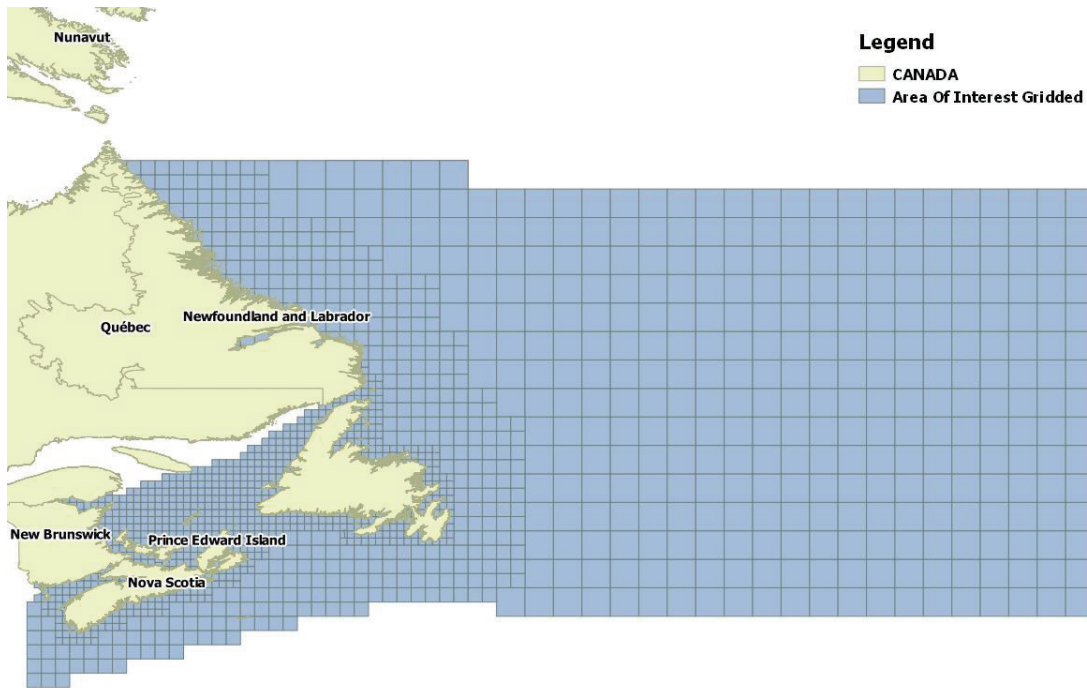
Figure 4-2- Variable grid cell sizes

Assumption 2: Customer locations. Rather than using historical data of incidents, we have chosen to use past data as a basis for a random distribution, which then, in turn, is used to generate incidents. That way, we do not assume that the future incidents occur where they happened in the past, we only assume that future incidents follow the same pattern as they did in the past. Due to the lack of data we cannot distinguish between different types of incidents. In our study, we use incident reports in the years 2005-2012 excluding 2007 which had accuracy issues, a time, for which complete sets of records is available. During those years, we have 1617 gridded customers that reported a total of 9,658 incidents of cleaned M1, M2, and M3 classes. Figure 4-3 (which was constructed on the basis of data from *SISAR*, the search and rescue information management system database provided by the Coast Guard, shows incidents off the coast of Eastern Canada. We have taken the number of reported incidents and randomly generated incidents based on patterns in the historical data.
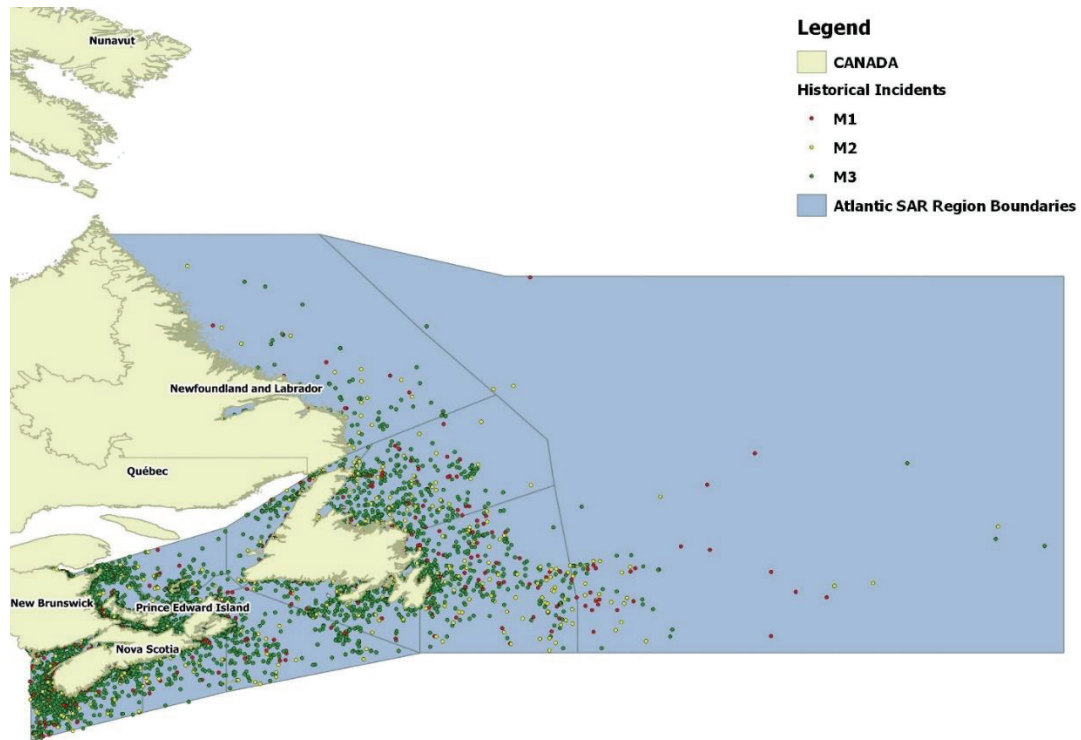
Figure 4-3- Search & rescue incidents 2005-2006, 2008-2012

Assumption 3: Classes of distress calls. The Coast Guard distinguishes between different classes of calls, *viz*., M1 to M3. In this nomenclature, "M" refers to "maritime" incidents (while "H" would symbolize a humanitarian assist, and "A" symbolizes an aircraft accident, which are excluded from this study), and the incidents range from class 1 (a distress call) to 2 (a call regarding a potential distress situation), and finally class 3 (non-distress calls). Table 4-1 shows the three incident classes and the number and relative frequency of calls in the respective categories.

Table 4-1- Incident classes recognized by the Coast Guard

Incident class	Number of incidents	Proportion
M1	657	6.8%
M2	1,086	11.24%
M3	7,915	81.96%
Grand total	9,658	100.00%

Assumption 4: Distances. Euclidean (straight-line) distances do not model paths of vessels particularly well. Even in the middle of the ocean with no obstacles present, vessels typically stick to lanes. Even more importantly, close to shore vessels may have to round islands, peninsulas, etc., so that vessels will be assumed to take the shortest route around the stretch of land that is in the way. In our paper, the distance measure of choice are Euclidean distances with land avoidance. An algorithm previously developed by the *MARIN* group is utilized for calculating land avoided distances. It uses a mesh of grids for both high seas and coastal waters. The algorithm calculates Euclidean distance while avoiding crossing the land.

Assumption 5: Existing vessels and helicopters. The Coast Guard fleet used for search and rescue can be categorized in four types of vessels at its disposal. Their ranges, sizes, speeds and cost are shown in Table 4-2. It should be noted that Lifeboats (vessels types 1 and 2) cannot be located offshore, while vessels of types 3 and 4 can. The current arrangement of existing vessels over randomly-generated incidents are visualized in Figure 4-4.

Table 4-2- Types of Coast Guard vessels

Vessel class	Vessel type	Range (km)	Vessel length (m)	Cruising speed (km/h)	Numbers available	Capital cost
Regular lifeboat class	*Type* 1	185	16	26	9	$1.2M
Fast lifeboat class	*Type* 2	185	15	41	7	$1.6M
Mid-Shore Patrol Vessel	*Type* 3	1,850	40-50	31	3	$21.6M
Large Multi-Task Vessel & Offshore Patrol Vessel	*Type* 4	6,000	60-90	22	8	$100M

Furthermore, there are 30 existing and potential stations (including 18 inshore and 12 offshore) at which Coast Guard vessels can be located. It must also be noted that the Coast Guard presently owns 22 helicopters, most of which are rather old and are in line to be replaced soon. However, after discussions with Coast Guard officials, it became clear that neither the helicopters that the Coast Guard presently owns, nor those that are to be purchased in the near future will be properly equipped for search and rescue missions. Whether or not a change of this policy can be expected in the future or is indeed desirable is unknown. Similarly, the Canadian Air Force owns some search and rescue helicopters, and while they may help out with marine incidents, they are mainly used for air incidents, but their capability limitations render them unfit to be used for general search and rescue missions of the Coast Guard.
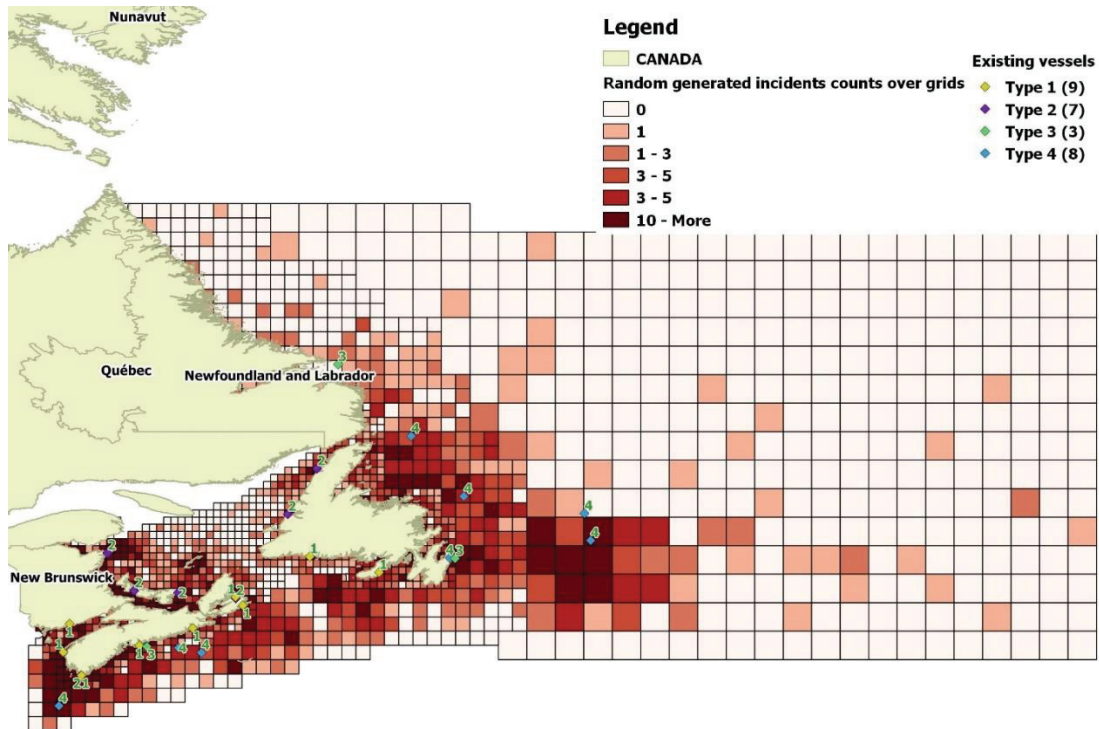
Figure 4-4- Randomly-generated incidents and existing vessel locations

Assumption 6: Budget constraint. As far as the number of types of vessels that are to be used, we are applying the following argument. First, we determine the total capital cost of all vessels, except for that of the large multi-task and offshore patrol vessel. The reason to exclude these large vessels is that they are used for missions other than search and rescue

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as well, so the Coast Guard could not simply replace them with other vessels. The cost of all Type 1 – 3 vessels (as per Table 1) is $86.8 million, which we assume are available for use. If it is assumed that more money will be available in the future, the budget could be increased by an appropriate amount. In other words, we pretend that we can sell all existing vessels for the prices shown and can purchase new vessels of the four types (actually, just Types 1 – 3, as a single Type 4 vessel costs more than is available in the budget calculated in this fashion) and locate them optimally. This way, we will simultaneously choose an optimal vessel mix for use of search and rescue missions, and locate them optimally. This will provide insight into whether or not the present fleet configuration operated by the Coast Guard is optimal, or, at least, near optimal.

<u>Assumption 7: Allocation of incidents to vessels.</u> Customers, i.e., incidents, will be allocated to vessels based on proximity. More specifically, an incident will be served by the vessel that is closest to it, where proximity is measured in terms of time, not distance. We realize that in congested models, such an allocation may result in very long wait times, as the vessel that an incident is allocated to may be busy. At present, the system is far from being congested, so that such an allocation rule is acceptable. Furthermore, we will vary the maximum acceptable access time so as to determine the sensitivity of the solution to this service parameter.

We are now ready to formulate our model. First, though, we will list all parameters and variables used in the model.

<u>The Parameters</u> This model uses the following parameters:

h_i: Number of customers at customer point i, i.e., the number of distress calls attributed to site i

$H = \sum_i h_i$: Total number of customers in the model

B: Budget (computed on the basis of existing equipment as detailed above)

c^k : Capital unit cost of vessel of type k

t_{max}: Maximum acceptable access time for coverage calculation

t_{ij}^k : Time to cover customer i from facility site j with vessel type k

D^k: Covering distance provided by vessel type k

$a_{ij}^k := 1$, if $t_{ij}^k \leq D^k$, and 0 otherwise. The parameters a_{ij}^k indicate whether or not customer i is within coverage range of vessel type k at site j

α: Required proportion of incidents covered

p^4: Fixed number of available large multitasking vessels

K_0: The set of potential offshore locations for vessel placement

The decision variables Our model has three types of decision variables. First, we will decide what types of equipment (vessels) are to be "purchased," and where to be positioned; second, we need to determine the assignment of customers to located vessels, and third, we need to determine whether or not an incident site is covered within maximum acceptable time. More specifically, the variables are

y_j^k : zero-one location variable, equals 1, if we locate vessel type k at site j, zero otherwise

x_{ij}^k : zero-one allocation variable; equals 1, if we allocate customer i to vessel type k located at site j, 0 otherwise

u_i: zero-one covering variable; equals 1, if customer i is covered, 0 otherwise

The formulation The formulation provided in the problem formulation P3 below attempts to blend p-median and covering features. The main idea is to determine the optimal mix of vessels given the present budget and to locate the vessels so as to minimize the average access time. It is also required that at least a certain proportion of incidents is covered by the search and rescue vessels within a specific time.

$$\text{P}_3: \text{Min } z = \frac{1}{H} \sum_i \sum_j \sum_k h_i t_{ij}^k x_{ij}^k \qquad (4.11)$$

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$$\text{s.t.} \sum_{k \neq 4} \sum_j c^k y_j^k \leq B \tag{4.12}$$

$$\sum_j y_j^4 = p^4 \tag{4.13}$$

$$x_{ij}^k \leq a_{ij}^k y_j^k \quad \forall i, j, k \tag{4.14}$$

$$\sum_j \sum_k x_{ij}^k = 1 \ \forall \ i \in I \tag{4.15}$$

$$\sum_i h_i u_i \geq H\alpha \tag{4.16}$$

$$u_i \leq \sum_{j,k: t_{ij}^k \leq \min\{t_{max}, D^k\}} x_{ij}^k \quad \forall i \in I \tag{4.17}$$

$$y_j^k = 0 \ \forall \ j \in \{1, 2\} \text{ and } k \in K_o \tag{4.18}$$

$$y_j^k, x_{ij}^k, u_i = 0 \text{ or } 1 \ \forall i, j, k \tag{4.19}$$

The objective (4.11) of problem P3 minimizes the weighted average of access time to all customers. Constraints (4.12) ensure that the vessels used in the model do not exceed the existing budget, where, as discussed above, Type 4 vessels are excluded. As discussed above, the budget equals the sum of the value of existing vessels (excluding large multitasking ships) plus the potential dollar amount that the Department is willing to spend on search & rescue equipment. At this point, the extra value is zero. The single constraint (4.13) ensures that the number of Type 4 vessels equals the number of such (large multitasking) vessels that presently exist. Constraints (4.14) ensure that customer i can only be served from site j by vessel k if a vessel k is actually located at site j and is within range of that vessel. Constraints (4.15) require that all customers must be allocated to exactly one located vessel. Constraints (4.16) require that at least α percent of the incidents are covered. Constraints (4.17) define coverage of a node by stating that a node is only covered if there is a vessel at site j of type k, which can reach customer i in less than minimum of D time units (max coverage of vessel k) and t_{max} (maximum desired access time). Constraints

(4.18) require that the two types of lifeboats cannot be positioned at offshore locations (i.e., those in K_o), and constraints (4.19) are the specifications of the variables.

4.4. Solutions and Discussion

The model described in the previous section was built in MPL and solved by GUROBI 6.5. The computer hardware included a CPU Intel CORE i7 and 8GB RAM. The model features 195,777 variables and 197,301 constraints and it took about one minute in average (between 15-150 seconds) to run and find the optimal solution of the problem over several model configurations. The results of a number of runs are shown in Table 4-3 (for $t_{max} = 6$ hrs) to Table 4-4 ($t_{max} = 5$ hrs), Table 4-5 ($t_{max} = 4$ hrs), and finally

Table 4-6 ($t_{max} = 3$ hrs).

Table 4-3- The solution for $t_{max} = 6$ hrs

#	Minimum coverage (α)	Actual coverage	Available budget ($M)	Excess budget ($M)	Access time (hrs)	Optimal mix of vessels		
						Type 1	Type 2	Type 3
1	89%	92.20%	$86.80	$14.60	2.2941	1	17	2
2	90%	92.20%	$86.80	$14.60	2.2941	1	17	2
3	91%	92.20%	$86.80	$14.60	2.2941	1	17	2
4	92%	92.21%	$86.80	$14.60	2.2941	1	17	2
5	92.5%	92.73%	$86.80	$3.40	2.3168	9	18	2
6	93%	93.10%	$86.80	$0.30	2.3748	0	13	3
7	93.5%	infeasible	$86.80					

Table 4-4- The solution for t_{max} = 5 hrs

#	Minimum coverage (α)	Actual coverage	Available budget ($M)	Excess budget ($M)	Access time (hrs)	Optimal mix of vessels		
						Type 1	Type 2	Type 3
1	80.0%	91.37%	$86.80	$14.60	2.2941	1	17	2
2	85.0%	91.37%	$86.80	$14.60	2.2941	1	17	2
3	90.0%	91.37%	$86.80	$14.60	2.2941	1	17	2
4	91.0%	91.37%	$86.80	$14.60	2.2941	1	17	2
5	91.5%	91.52%	$86.80	$8.20	2.3168	5	18	2
6	92.0%	infeasible	$86.80					

Table 4-5- The solution for t_{max} = 4 hrs

#	Minimum coverage (α)	Actual coverage	Available budget ($M)	Excess budget ($M)	Access time (hrs)	Optimal mix of vessels		
						Type 1	Type 2	Type 3
1	80%	87.73%	$86.80	$14.60	2.294	1	17	2
2	85%	88.12%	$86.80	$14.60	2.294	1	17	2
3	88.5%	88.67%	$86.80	$2.20	2.317	10	18	2
4	89%	infeasible	$86.80					

Table 4-6- The solution for t_{max} = 3 hrs

#	Minimum coverage (α)	Actual coverage	Available budget ($M)	Excess budget ($M)	Access time (hrs)	Optimal mix of vessels		
						Type 1	Type 2	Type 3
1	60%	66.62%	$86.80	$14.60	2.294	1	17	2
2	70%	74.29%	$86.80	$7.40	2.294	7	17	2
3	73%	74.29%	$86.80	$7.40	2.294	7	17	2
4	74%	74.29%	$86.80	$7.40	2.294	7	17	2
5	74.5%	74.62%	$86.80	$2.20	2.317	10	18	2
6	75%	75.13%	$86.80	$14.60	2.361	1	17	2
7	75.5%	infeasible	$86.80					

It is not surprising that with decreasing values of t_{max}, i.e., as conditions for coverage are tightened, the maximal possible coverage decreases as well. More specifically, given a standard of t_{max} = 6 hours, it is possible to serve 93% of the population; a number that gradually decreases to 75% for t_{max} = 3 hours (Table 3d). While coverage decreases markedly, it is noteworthy that access times are very stable. They range from 2.29 to 2.36 hours throughout the series, i.e., an increase of less than 4%. The main result, however, is the difference in the mix of vessels that the optimal result prescribes in comparison to the mix of vessels that is in use by the Coast Guard now. In all solutions with the lower coverage, the prescribed solutions call for one Type 1 vessel, 17 Type 2 vessels, and two Type 3 vessels (in addition to the mandatory Type 4 vessels). In contrast, the Coast Guard presently owns 9 Type 1 vessels, 7 Type 2 vessels, and 3 Type 3 vessels. The mean access time and coverage (given a desired covering time of 6 hours) that can be achieved by these vessels at their current positions are 3.05 hours and 86.34%. This is significantly worse than the model results which range from 2.29 hours access time (a 25% decrease) and 92% coverage (a 6.5% increase) to 2.37 hour access time and 93% coverage. Furthermore, it is worth mentioning that the existing Type 1 lifeboats are significantly older than their Type 2 counterparts, making them eligible for replacement in the not-too-distant future. In such a case, decision makers may want to consider replacing the older Type 1 boats with Type 2 vessels as suggested by the optimized solutions.

Also, a closer inspection of Tables 4-3 – 4-6 reveals that there is a significant excess budget. One may wonder why this existing budget is not spent in the model solution to achieve greater coverage or shorter access times. The reason is that the excess is not sufficient to purchase an additional Type 3 vessel (which costs $21.6 million in comparison to the available sum of $14.6 million), and while a number of Type 1 and Type 2 vessels could be purchased with the additional funds, these vessels must be based at onshore locations, where these types of vessels are already located, so that the solution cannot be improved by locating more vessels of the same type in those locations.

4.5. Summary and Outlook

This paper considers a real situation faced by the Canadian Coast guard in the Atlantic region. We first review two of the major standard generic location formulations, *viz*., the *p*-median and the maximum covering location problems. The paper then formulates the Coast Guard problem, which, in addition to combining features from both of these standard problems, also considers a possible change of the present mix of vessels for search and rescue missions. Computational experiments reveal that the present mix of vessels is very different from the optimal combination. Coverage could rise from around 86% up to 93% and mean access time could drop dramatically from 3.05 hours to 2.29 hours.

There are a number of additional features in this multimillion dollar problem that should be considered in order to make the problem "more real." Among these features is congestion. This concern was addressed by Marianov and Serra (1998, 2001), Wang et al. (2004) and Rahmati et al. (2014) in their studies by considering stochastic demand and possibility of facility unavailability. Different approaches such as queuing models and maximal probabilistic models are utilized. (Ingolfsson et al. 2008) considered randomness in ambulance availability and in the delays and the travel times for ambulance location problem. Berman and Krass (2001) covered the facility location problems with stochastic demands and congestion for the mobile server case. In other words, given that tight budgets will mean that individual vessels are used much more than is the case traditionally, it is increasingly likely that vessels which are the closest to an incident are presently busy and cannot provide the requested service. This requires concepts such as backup coverage or queuing features in the model.

Another stream of research would be the inclusion of different means of providing service. One of the obvious ideas is to include appropriately equipped helicopters as another type of "facility" in the model. The optimization would show whether or not the Coast Guard could obtain their "bang for the buck" with these different service providers.

Another extension of the model will include data concerning specific types of emergencies. In other words, individual facilities (vessel or other means of providing service) have different capabilities. For instance, some facilities may not be able to deal with fires on

boats, towing of vessels, or other emergency tasks. This would require that the Coast Guard classifies and keeps track of individual incidents in greater detail. These are data that were not available. Furthermore, with differently classified incidents, it would be possible to specify different targets. For instance, one may require to cover (within a predetermined time) 100% of all incidents that involve people, 95% of all incidents that involve fires, 50% of all incidents that require towing, etc.

Chapter 5   A Modular Capacitated Multi-Objective Model for Locating Maritime Search and Rescue Vessels

Amin Akbari, Ronald Pelot, H.A. Eiselt

Abstract

This paper presents a mathematical multi-objective model to optimize the Location-Allocation of Maritime Search and Rescue (*SAR*) vessels with regard to several criteria, including primary and backup coverage and mean access time. Atlantic Canada serves as the area of the study and the Canadian Coast Guard has provided the necessary datasets and information. A goal programming multi-objective model is developed to optimize the location and allocation of *SAR* vessels to potential future incidents in order to achieve greater level of responsiveness and coverage. Comparing the optimal solution found to the current arrangement of *SAR* vessels, shows a substantial improvement in terms of access time and coverage. The results of the study provide decision makers with valuable insights to make more informed strategic and tactical decisions for more efficient management of the *SAR* fleet.

5.1. Introduction

The Canadian Coast Guard (*CCG*) is responsible for providing maritime search and rescue (*SAR*) services in Canada and, as a public service dealing with incidents and people lives, it is very important that their resources, including *SAR* vessels and stations are used and managed efficiently. To do this, they must decide where to locate their resources and how to allocate future demands (i.e. incidents) to them, which is also known as the Location-Allocation problem. The Location-Allocation problem is an essential model for several important applications, including the location of ambulances, police cruisers, fire stations, distribution centers and so on. Optimizing the efficiency of resource utilization is always a major concern.

In a typical *SAR* location problem, we are faced with a "server to customer" service system, which is in nature similar to fire station and ambulance location problems. The servers represent the *SAR* vessels, while the customers symbolize the incidents. These situations are generally categorized as emergency location analysis problems. In this kind of location problem, the main concern is to cover all the demands or at least try to cover as many as possible and as fast as possible.

This problem becomes more complicated when we are faced with several criteria for assessing decision outcomes, some of which may be conflicting as well. There is a rich and diverse literature in the area of emergency location analysis, but some gaps still exist, particularly in the case of maritime *SAR* resource planning. Usually, research in this area considers only a single objective while in the real case, there is more than one criterion that decision makers care about. Coverage, cost and average access time or response time are widely used in multi-criteria location studies as model objectives. This study considers several criteria as objectives simultaneously in the mathematical model.

Covering problems, median problems and center problems are three popular problem types in location modeling. Covering problems are concerned with locating facilities so that as many customers as possible are within a pre-specified distance from any one of the facilities, while the median problem is aiming at minimizing the system-wide average response time. In the center problem, the objective is to minimize the maximum distance

from the facilities. Covering and median problems are extensively used in the area of emergency location analysis. Marianov and Serra (2002) pointed out that both the median and covering problems can be considered benchmarks in the development of facility location models. But, each of these general models only consider one criterion when attempting to optimize the location of facilities, while in reality it is often desired to take into consideration a couple of criteria as model objectives. In emergency location analysis, coverage is always of great importance and usually comes first. However, decision makers would still like to decrease costs through minimizing travel distances. This is also intended to increase the chance of saving people in danger through minimizing the response time to incidents. From a technical point of view, while covering problems provide acceptable service (as defined by the covering distance) to as many people as possible, median problems do not consider the number of people served, but the average service to individual customers.

The *CCG* has many different *SAR* vessels that were designed or purchased with specific tasks in mind, and not all are equally capable of, nor effective at, handling different incident types. Also, the ranges and speeds vary greatly among different types of *SAR* vessels, so the vessels' capabilities need to be considered. Resources have a predetermined capacity (maximum number of incidents handled per year) based on their type and their planned utilization. Some vessels are multi-tasking and therefore are shared among different services, including *SAR*. Planned and unplanned vessel maintenance should be considered when planning for operational requirements and capacity needs.

The purpose of this study is mainly to develop a multi-objective model for the case of maritime *SAR* resource location and attempts to incorporate different criteria that are of interest for decision makers. The goal programming optimization model aims to minimize total deviations from a predetermined target for each objective. Resource type related constraints such as capacity are considered in the model. Several demand scenarios are randomly generated based on the spatial distribution extracted from historical data. The proposed model is applied to a real world Maritime *SAR* location problem. Solutions provided by the multi-objective model are compared to the current arrangement scenario, so it helps decision makers to see exactly how they can improve what they are doing now.

Ultimately, this study seeks to propose a multi-criteria decision support model that is capable of providing helpful insights for decision makers regarding possible changes at the strategic and tactical level of managing resources to improve the overall quality of *SAR* service and effectiveness of resource utilization.

The remainder of this paper is structured as follows. Section 5.2 presents relevant literature. Section 5.3 explains the methodology used in our study for modelling a multi-objective emergency location problem. In section 5.4, the process of applying the proposed goal programming model to our case study is presented, followed by the numerical results and discussion as well as managerial aspects of using this model. The paper concludes with the summary of the findings and outlook for future research in the field.

5.2. Literature Review

5.2.1. *Multi-Objective Optimization in Location Analysis*

Facility location problems are usually multi-criteria decision-making (*MCDM*) problems. There are different approaches to model and optimize a multi-objective problem. Generally, multi-objective optimization methods can be classified into two categories: Scalarization methods and Pareto methods. In the first group of methods the multi-objective problem is solved by translating it back to a single (or a series of) objective, scalar problem. The formation of the aggregate objective function requires that the preferences or weights between objectives are assigned *a priori*, i.e. before the results of the optimization process are known. The Pareto methods, on the other hand, keep the elements of the objective vector separate throughout the optimization process and typically use the concept of dominance to distinguish between inferior and non-inferior solutions.

Using multiple objectives in location studies was started in late 1970s and early 1980s. Armstrong and Cook (1979) developed several goal programming models to allocate aircraft *SAR* resources to bases to provide the most effective level of service defined by maximum attainable probability of successfully completing *SAR* operation within predetermined amount of time for different distress levels. However, the study lacks requisite accurate data in order to apply the proposed models to this practical case. Also, it does not consider other important criteria like mean access time. Ross and Soland (1980)

worked on multi-activity multi-facility problems and proposed an interactive solution method to compute non-dominated solutions from which to compare and choose. Lee et al. (1981) studied an application of integer goal programming for facility location with multiple competing objectives. Current et al. (1985) solved their bi-objective location problem by relaxing integer terms and also used a branch-and-bound procedure.

In the 1990s, the volume of literature grew rapidly. Solanki (1991) applied an approximation scheme to generate a set of non-dominated solutions to a bi-objective location problem, while Malczewski and Ogryczak (1996) considered utility-function-based and goal programming methods and developed a new approach based on the reference point method that was applied to an interactive decision support system for multi-criteria location problems. Cho (1998) combined a Monte Carlo integer programming technique with an augmented Lagrangian algorithm to obtain an optimum global solution. Badri (1999) proposed the use of the Analytic Hierarchy Process and multi-objective goal programming methodology for the Location-Allocation problem with multiple conflicting objectives. In (Ohsawa 1999), to find the set of Pareto-optimal locations from the model, the candidate locations were examined first, based on simple geometrical methods with the help of the farthest-point Voronoi diagram, then the objective functions were minimized via solving a scalarized location model with suitable weights. Melachrinoudis and Min (2000) proposed a multi-objective mixed integer program model that generates a set of non-dominated solutions without *a priori* preference information due to its usage of a weighting generation method. Blanquero and Carrizosa (2002) suggested an algorithm which decomposed the problem and built the Voronoi cells, and constructed a finite ε-dominating set of Pareto-optimal solutions for the bi-objective problem.

In more recent studies for example, San Cristóbal (2012) utilized the goal programming method to find the optimal mix of different plant types and locations where each plant should be built in a capacity expansion planning problem of the renewable energy industry. Ho et al. (2013) integrates the Analytic Hierarchy Process (*AHP*) and multi-choice goal programming (*MCGP*) to select an appropriate house among numerous alternative locations that best suits the preferences of renters. The study obtains weights from *AHP* and implements it for each goal using *MCGP* for the location selection problem. A bi-

criteria problem was formulated by Kolokolov and Zaozerskaya (2013) to find the optimal locations of service centers. Tradeoff methods were used for finding a subset of the Pareto optimal solution set. Rahmati et al. (2013) proposed a Pareto-based meta-heuristic algorithm called multi-objective harmony search to solve a multi-objective multi-server facility location problem. Three objective functions are considered including minimizing: (*i*) sum of the aggregate travel and waiting times; (*ii*) maximum idle time of all facilities; and (*iii*) the budget required to cover the costs of establishing the selected facilities plus server staffing costs. Abounacer et al. (2014) considered three objectives in their location-transportation model for disaster response: total distribution time for emergency supplies, the number of agents required to operate selected distribution centers, and non-covered demands. They used an epsilon-constraint method to find the Pareto frontier solutions.

The idea of fuzzy programming has also been used along with the goal programming method to deal with multi-objective location problems. Bhattacharya et al. (1993) developed a fuzzy goal programming model for their convex multi-facility location problem with mini-sum (transportation cost) and mini-max (distance) objectives with rectilinear distances. Araz et al. (2007) proposed a multi-objective emergency vehicle location model which considers maximization of population covered, maximizing the population with backup coverage and minimization of total travel distance as its objectives. A fuzzy goal programming approach was used to formulate the model.

The appropriate methodology to model and find the optimal solution for a multi-criteria problem depends greatly on the structure of the problem and the decision maker's needs. Any of the aforementioned methods, including scalarization methods such as weighted methods and goal programming (given *a priori* information about objective weights) and interactive methods that are able to provide efficient solutions without *a priori* information, can be useful. A wide range of objectives/criteria has been used by researchers in location analysis including more common criteria like travel time (Badri et al. (1998), coverage (Kim and Murray (2008)) and cost (Balcik and Beamon (2008)) as well as special case-dependent criteria such as pollution in the undesirable facility location problem (Ohsawa et al. (2006) and Eiselt and Marianov (2014)).

5.2.2. Emergency Response Location Models

Emergency services are among the main applications of location studies and usually a comprehensive analysis of problems in this area requires one to consider multiple criteria. Goldberg (2004) reviewed the literature of operations research applications in emergency services vehicles and Brotcorne et al. (2003) performed a more specific review on the evolution of models in the area of ambulance location and relocation. They divided the studies into deterministic and probabilistic models.

Baker et al. (1989) developed a multi-criteria model for the ambulance allocation problem. They used various outcome criteria in their model, including response time, cost and workload balance. The model is then solved using an integer, non-linear goal-programming technique. Harewood (2002) formulated a bi-objective programming problem to locate ambulances on the island of Barbados. One objective minimizes the cost of serving customers, while the other maximizes multiple coverage given a certain distance standard. Burkey et al. (2012) used efficiency, availability of the service, and equality as their assessment criteria in the location of heath care services. They compared existing locations with optimal solutions of the maximal covering location problem and the *p*-median problem. Zhang et al. (2012) formulated an emergency resource location problem with the constraints of multiple resources as well as possible secondary disasters. They introduced the opportunity cost of the secondary disasters into the objective function to build a model for dispatching the multiple emergency resources and an effective heuristic algorithm was proposed for solving the problem.

Narrowing down the research area to location studies in maritime *SAR* problems, quite a few articles have been identified. Brown et al. (1996) developed a mixed integer model for scheduling coast guard district cutters. The proposed model provided a superior solution compared to manually prepared schedules. Nguyen and Kevin (2000) incorporated maximal covering and *p*-median location problems into a goal programming model to assess the level of service of the Canadian *SAR* system (in terms of location of *SAR* aircraft and helicopters) and compare it to the optimal solution. Afshartous et al. (2009) studied the problem of locating coast guard air stations. They utilized a statistical-optimization

approach to provide a robust solution in the presence of uncertainty in distress call locations. Distress calls are simulated and the optimization problem is solved for different simulations. The optimization model however is simple and lacks the consideration of different criteria.

Radovilsky and Koermer (2007) presented the application of integer linear programming for optimal allocation of rescue boats among the stations of the U.S. Coast Guard. The objective of their model was to minimize shortages or excess capacity at the stations. Later, Wagner and Radovilsky (2012) proposed a new model that simultaneously considers reduction of excess capacity and boat shortages at the stations, a decrease in the overall fleet size with an increase in boat utilization, and overall reduction of the fleet operating cost. Nelson et al. (2014) developed an optimization model to determine the optimal deployment assignments, operational levels and aircraft allocation among all *USCG* Air Stations.

Pelot et al. (2015) developed three location models (maximal covering location problem, maximal expected covering location problem, and maximal covering location problem with workload capacity) applied to the Maritime *SAR* location problem for Atlantic Canada. Akbari et al. (2016) presented a multi-criteria analysis on performance of solutions provided by two popular location models, *p*-median and maximal covering to the case of maritime *SAR* location. This study considers primary and backup coverage, mean access time and the Gini index as post-assessment criteria for solutions of two single objective models.

Razi et al. (2016) used optimization to determine the best allocation of helicopters for *SAR* and then validated the performance of the solution by simulating the stochastic demand. In another study Razi and Karatas (2016) designed a multi-objective model for allocation of *SAR* boats where the decision depends on several criteria such as density and type of incidents, resource capability, and business rules. The model considers minimizing response time to incidents, fleet operation cost and the mismatch between resource load and operation planned capacity.

5.2.3. Capacitated Location Models

The basic location models, particularly covering models, do not consider workload capacities. In most of the basic location models such as the maximum covering location problem (*MCLP*), the set covering location problem (*SCLP*) and *p*-median problems, it is assumed that facilities have infinite capacity to respond to demand. As a result, some servers may in reality be assigned to so many tasks that it is beyond their maximum capacity. To solve this issue, researchers have worked on extending the fundamental models to include constraints on the capacity, to balance the workload of facilities. However, an upper bound on the capacity of the facilities could change the optimal assignments generated by uncapacitated models whereby in the optimal solution all demands are assigned to their closest facility.

Chung et al. (1983) and Current and Storbeck (1988) were among the earliest researchers to deal with the concept of capacitated *MCLP*, by adding a maximum capacity constraint to the model formulation. Pirkul and Schilling (1988) proposed a capacitated model where all demands are assigned to facilities, regardless of whether the demand lies within the service covering distance or not. Haghani (1996) took into account a resource capacity constraint and proposed a capacitated multi-objective model to maximize weighted covered demand as a primary objective and to minimize the average distance from uncovered demand to the located facilities as the secondary objective. The capacity of different facilities could be varied based on their different characteristics. (Correia and Captivo 2003) called such a problem with varied capacity constraints the modular capacitated location problem. To apply the capacitated *MCLP* model to the case of an emergency facility siting problem so that the facility could have different capacity levels with varied numbers of stationed emergency vehicles, Yin and Mu (2012) proposed an extension of *MCLP* called the Modular Capacitated Maximal Covering Location Problem (*MCMCLP*). The objective of their model, similar to (Haghani 1996), is to maximize the weighted covered demand and simultaneously minimize the average distance from uncovered demands to the located facilities.

5.2.4. Uncertainty of Demand

In facility location analysis, it is typically assumed that demand locations are known at the time of making siting decisions, while in reality, there is usually, uncertainty in the location of future demand with some variations that are not necessarily predictable. As location problems usually involve strategic decisions, it is necessary to take into account variations in demand using different potential scenarios. These factors might tremendously affect the results of such long-term decisions.

Generally, there are two approaches for optimization under an uncertain environment: stochastic programming and robust optimization. In stochastic programming the value of uncertain parameters is assumed to follow probability distributions with known parameters; while, in robust optimization it is assumed that there is no information available about the probability distribution.

Snyder (2006) reviewed the literature on stochastic and robust facility location models and categorized a variety of approaches for optimization under uncertainty. In stochastic location modeling, locations are generally first-stage decisions whereas assignments of customers to facilities are second-stage. One of the main types of stochastic models is applying a scenario planning approach.

Chen et al. (2006) and Owen and Daskin (1998) applied a scenario planning approach, one of the main types of stochastic models, in which a limited number of scenarios with given probability of occurrence is considered. The other common approach is to use probability distribution functions for uncertain parameters and optimize the expected value of the objective function which increases the complexity of the problem (for example, see, Snyder (2006)). The scenario planning approach leads to more tractable model which can be solved in a reasonable amount of time.

Moreover, although the facilities may be capable of coping with the average demand, there could be some peak periods during which they cannot provide service to all requests right away. Such situations are referred to as "congested systems." These issues can be investigated using queueing models which take into account the probabilistic nature of demand and service. In congested systems, in cases when a facility is not able to serve all

service requests, some of them can wait until the server become available. But in often cases, such as emergency systems, it is generally not reasonable to wait, so if the demand is not responded to within a time limit it will be assumed to be uncovered.

A potential and common way to cope with the congestion issue is to consider backup coverage to provide multiple coverage for demands in order to decrease the probability of server unavailability in case of congestion. Backup coverage refers to the secondary coverage of a demand node and it is used as a means for handling of congestion issue in areas of high demand where the closest vehicles to respond is not available or tasked to other customer coverage. This approach works particularly well for emergency response where the customers cannot wait for a busy server to become available, thus having a backup response is an appropriate solution. One important concern is the possible conflict or tradeoff between primary and backup coverage, whereby improving the former may hamper the latter. Hogan and ReVelle (1986) presented a maximal backup coverage model.

To the best of our knowledge, there is no study in emergency location analysis and particularly in *SAR* location modelling that concurrently considers coverage, travel time and backup response, and address the stochasticity of incident locations and the capacity of response resources. Targeting these gaps in the related literature, this study aims at developing a multi-objective model for the maritime *SAR* Location-Allocation problem to analyze and compare the current resource arrangement with the optimal solution and to provide the *CCG* with advice regarding their future strategic decisions to improve the efficiency of resource utilization and response to incidents. The study contributes to the research field by:

- Considering three important criteria (primary coverage, backup coverage and mean access time) in the optimization model simultaneously;
- Categorizing facilities based on their capabilities and availabilities;
- Extracting the spatial distribution of historical incidents as a basis for generating and simulating future demand;
- Applying a variation of the goal programming method for multi-objective programming.

5.3. Methodology

The methodology used in this study is described in the following sections, explaining the way we digitize the space and simulate events based on past data. Then we develop relevant criteria for the problem, and propose the multi-objective optimization model.

5.3.1. Incidents Spatial Distribution

The nature of demand for *SAR* services is typically stochastic, but the historical recorded data are deterministic and thus using the past patterns directly for modelling the future is not an ideal approach. Knowing that an incident occurred at specific point in the past does not imply that it will happen at the same point again, nor that there cannot be events at any other points even if there were no incidents there in the past. Hence, to cope with this issue, it appears appropriate to simulate the potential incident distribution over the study region and timeframe using statistical parameters estimated from the historical demand. Therefore, finding a comprehensive approach to properly simulate the potential demand distributed over the study area is a modelling issue.

As the pattern of past incidents is a strong predictor of future, it is important to extract the underlying distribution of historical incident locations as the basis of a stochastic approach for generating future incidents. There are several methods to fit a distribution for spatial data including quadrat analysis, naive estimation, and kernel density function (Bailey and Gatrell 1995). In this study historical incidents are analyzed to extract patterns and distribution by using Kernel estimation which is a quite popular method for analyzing spatial point patterns which considers neighboring areas when calculating the density for each specific point. The other advantage of kernel density method is that it works properly with gridded data, which is the format of our demand projection.

Quadrat analysis, a common and simple way of exploring spatial patterns, includes counting points that fall within each grid square. Kernel estimation is a more sophisticated means of exploring spatial variations in terms of intensity. Such approaches are often used in the identification of clusters and hot spots. If we want to simply estimate the intensity of points over an area, we can calculate the number of events within a radius around the nodes

of a grid and divide it by the area concerned. This is called a naïve estimator. The naïve intensity estimate is given by:

$$\hat{\lambda}(o) = \frac{\#(C(o,d))}{\pi d^2},$$ (5.1)

where $\#(C(o,d))$ indicates the number of events in the circle $C(o,d)$ that has center at o and radius d. Kernel estimation (KE) can be expanded to make use of a geographical weighting scheme (a kernel function) whereby the influence of the points varies inversely to how far they are from the centre of the window (Lloyd 2010). The KE of intensity is given by:

$$\hat{\lambda}_k(o) = \sum_{i=1}^{n} \frac{1}{\tau^2} k\left(\frac{o - o_i}{\tau}\right),$$ (5.2)

where τ is the bandwidth (determining the size of the kernel) and o-o_i indicates the distance between the centre of the kernel (o) and the location o_i (i is an index for data points). There is a variety of different kernels that have been used for KE. The quartic kernel is encountered frequently in the point pattern analysis literature. The KE with the quartic kernel can be given by:

$$\hat{\lambda}_k(o) = \sum_{d_i \leq \tau} \frac{3}{\pi\tau^2} \left(1 - \frac{d_i^2}{\tau^2}\right)^2,$$ (5.3)

where d_i is the distance between the centre of the kernel (o) and the location o_i.

5.3.2. Goal Programming Multi-Objective Model

Based on the nature of our case study, our problem has some characteristics that makes weighted goal programming a good choice for modelling. First of all, the problem is multi-objective and these objectives do not necessarily have same scale. So, it seems to be a good idea to minimize the standardized deviation of objective values from their corresponding target value instead of optimizing the weighted sum of objectives. As compared to efficient methods that provide Pareto frontiers (set of non-dominated solutions) for which it is difficult for the decision maker to select one among an infinite number of efficient solutions, scalarization methods such as goal programming yield a finite number of reasonably good solutions (for variable weights associated with objectives), which is more clear to the decision maker in order to understand and choose an appropriate solution. Also,

determining the appropriate target values (goals) is desirable and also possible in our case as we have some prior information about the service level requirements and standards that decision makers are prepared to achieve. These target values are required to be sufficiently high and unattainable according to the goal programming concept. Moreover, we use one-sided deviations in the model as attempting to reach all objective targets is desirable and targets are determined such that actual outcomes cannot exceed them. For example, for primary coverage as one of main criteria in emergency response location problems, it is desired to get as close as possible to 100% which is the maximum value it could take.

On the other hand, weighted goal programming is designed for problems where all the goals are quite important, with only modest differences in importance that can be measured by assigning weights to the goals. In contrast, preemptive goal programming is used when there are major differences in the importance of the goals. In our case, none of the objectives dominates the others in terms of importance, so the preemptive method does not seem to be a good fit for modelling the problem.

In this section a multi-objective model which is a variation of goal programming is proposed with three objectives for the maritime *SAR* Location-Allocation problem: primary coverage, backup coverage and mean access time. The model considers different classes of *SAR* vessels (resources) with different speeds, capacities and potential locations. The model includes capacity constraints to reflect that resources have limited usable operating capacity. Four different *SAR* vessels are considered (two types of lifeboats, patrol vessels and large multitasking ships). Lifeboats are considered as the primary *SAR* vessels, while patrol and multitasking vessels are the secondary resources for *SAR* missions.

To avoid assuming certainty in demand, a scenario planning approach is utilized where several set of randomly generated demands (demand scenarios) are used and considered in the mathematical model. The weighted average of objective function values in all scenarios is minimized.

The three objectives in the mathematical model are defined as follows.

1) Primary coverage:

This objective is defined as the percentage of incidents to be covered within the predetermined access time (travel time) by at least one *SAR* vessel and is measured by following equation.

$$\text{Primary Coverage} = \sum_n P_n \left(\frac{\sum_i w_{in} x_i}{\sum_i w_{in}} \right), \qquad\qquad (5.4)$$

where $x_i \in \{0, 1\}$ is a primary coverage variable, which assumes a value of 1 if customer node i is covered within pre-specified access time and 0 if it is not, and w_{in} symbolizes the weight on grid i in scenario n, reflecting the number of forecasted incidents at that grid point. P_n is the probability associated with scenario n.

2) Backup coverage:

Backup coverage expresses the percentage of incidents that are within the predetermined coverage area of at least two *SAR* vessels and is calculated by relation (5.5).

$$\text{Backup Coverage} = \sum_n P_n \left(\frac{\sum_i w_{in} y_i}{\sum_i w_{in}} \right), \qquad\qquad (5.5)$$

where $y_i \in \{0, 1\}$ is a backup coverage variable, which assumes a value of 1 if customer node i is within pre-specified access time of at least two vessels and 0 if it is not.

3) Mean access time:

Mean access time is defined as "mean travel time for the nearest available response vessel to reach an incident cross all demand scenarios" and is calculated as follows.

$$\text{Mean access time} = \sum_n P_n \frac{\left(\sum_k \sum_i \sum_j \left(w_{in} u_{ijk} \left(d_{ij}/v_k \right) \right) \right)}{\sum_i w_{in}}, \qquad (5.6)$$

where u_{ijk} is a binary variable for allocation of demand at grid i to vessel type k which is located at j; d_{ij} is the distance from grid i to j and cruising speed of vessel type k is defined by v_k.

The mathematical model developed for this problem is a large scale Mixed Integer Linear Problem (*MILP*) and it is built using the constrained goal programming method. Remaining indices, parameters and variables used in the model are listed and defined below.

Indices:

$i \in I$:   Grid index (demands)

$j \in J$:   Index for potential vessel stations

$k \in K$:   Index for vessel types

$l \in L$:   Objectives' index

$n \in N$:   Index for simulated incident scenarios

$J_S \in J$:   Set of offshore stations (virtual stations offshore for patrol vessels and multitasking ships)

Variables:

x_i :   Binary variable for primary coverage at grid i

y_i :   Binary variable for backup coverage at grid i

z_{jk} :   Integer variable for number of vessels type k located at station j

u_{ijk}:   Binary allocation variable for grid i to vessel type k located at station j

ε_l :   Undesirable deviation from target value of objective l

Parameters:

λ_l :   Weight of objective l

μ_l :   Predetermined target for objective l

r_k: Coverage distance (range) of vessel type k

p_k: Available number of vessel type k

c_k: Capacity of vessel type k in terms of number of incidents that can be responded to

v_k: Cruising speed of vessel type k

d_{ij}:   Distance between grid i and station j

w_{in}:   Demand weight on grid i in scenario n

P_n: Probability of occurrence of scenario n

t: Coverage time limit for acceptable level of coverage

α_l: Minimum level for each objective value (maximum for mean access time)

a_{ijk}:   A parameter indicating whether grid i is within coverage time limit of a vessel type k at j

b_{ijk}:   A parameter indicating whether grid i is within response range of a vessel type k at j

The formulation of the proposed model is as follows:

Min:

$$E = \sum_l \lambda_l \cdot \left({}^{\varepsilon_l} / \mu_l \right) \tag{5.7}$$

s.t.

$$x_i \leq \sum_j \sum_k a_{ijk} z_{jk}, \qquad \forall\, i \in I \qquad \text{Primary coverage constraint} \tag{5.8}$$

where: $a_{ijk} := 1$ if: $d_{ij} \leq r_k$ and $d_{ij}/v_k \leq t$ else $a_{ijk} := 0$

$$y_i \leq \sum_j \sum_k (a_{ijk} z_{jk}) - x_i, \quad \forall\, i \in I \qquad \text{Backup coverage constraint} \tag{5.9}$$

$$y_i \leq x_i, \qquad\qquad \forall\, i \in I \qquad \text{Backup coverage comes after}$$
$$\text{primary coverage} \tag{5.10}$$

$$\sum_k \sum_j u_{ijk} = 1, \qquad \forall\, i \in I \qquad \text{Grid allocation to closest vessel} \tag{5.11}$$

$$\sum_j z_{jk} \leq p_k, \qquad\qquad \forall\, k \in K \qquad \text{Fixed number of available vessels}$$
$$\text{in each class} \tag{5.12}$$

$$u_{ijk} \leq b_{ijk} z_{jk} \qquad \forall\, i \in I, j \in J, k \in K \quad \text{Allocation to } SAR \text{ vessels possible}$$
$$\text{if there is a vessel within the}$$
$$\text{coverage range} \tag{5.13}$$

where: $b_{ijk} := 1$ if: $d_{ij} \leq r_k$ else $b_{ijk} := 0$

$\sum_i w_{in} u_{ijk} \leq c_k z_{jk}$ $\qquad \forall\, j \in J, k \in K, n \in N$ Capacity constraint $\qquad\qquad$ (5.14)

$z_{jk} = 0$ $\qquad\qquad \forall\, j \in J_S, k \in \{1,2\}$ Offshore location constraint $\qquad$ (5.15)

$\mu_1 - \left(\sum_n P_n \left(\frac{\sum_i w_{in} x_i}{\sum_i w_{in}} \right) \right) = \varepsilon_1$ $\qquad\qquad\qquad$ Deviation of primary coverage

from its target value $\qquad\qquad$ (5.16)

$\mu_2 - \left(\sum_n P_n \left(\frac{\sum_i w_{in} y_i}{\sum_i w_{in}} \right) \right) = \varepsilon_2$ $\qquad\qquad\qquad$ Deviation of backup coverage

from its target value $\qquad\qquad$ (5.17)

$\sum_n P_n \dfrac{\left(\sum_k \sum_i \sum_j \left(w_{in} u_{ijk} \left({}^{d_{ij}}\!/\!_{v_k} \right) \right) \right)}{\sum_i w_{in}} - \mu_3 = \varepsilon_3$ $\qquad$ Deviation of mean access time from

its target value $\qquad\qquad$ (5.18)

$\dfrac{\sum_i w_{in} x_i}{\sum_i w_{in}} \geq \alpha_1$ $\qquad\qquad \forall\, n \in N$ Minimum primary coverage in

every scenario $\qquad\qquad$ (5.19)

$\dfrac{\sum_i w_{in} y_i}{\sum_i w_{in}} \geq \alpha_2$ $\qquad\qquad \forall\, n \in N$ Minimum backup coverage in

every scenario $\qquad\qquad$ (5.20)

$\dfrac{\left(\sum_k \sum_i \sum_j \left(w_{in} u_{ijk} \left({}^{d_{ij}}\!/\!_{v_k} \right) \right) \right)}{\sum_i w_{in}} \leq \alpha_3 \ \forall\, n \in N$ $\qquad$ Maximum mean access time in

every scenario $\qquad\qquad$ (5.21)

The minimization objective function (equation 5.7) has three terms corresponding to the standardized deviation of total primary coverage, total backup coverage and mean access time from their predetermined targets value, respectively. Constraint (5.8) ensures that grid *i* can be included under primary coverage only if there is a *SAR* vessel within the maximum coverage distance and time limit. Constraint (5.9) defines the concept of backup coverage with respect to primary coverage and total number of vessels within range for each incident, and (5.10) states that backup coverage cannot happen when there is no primary coverage. Allocation of gridded demands to resources is defined in (5.11) which ensures that all grids are allocated to exactly one vessel for access time calculation. The fixed maximum number of vessels in each class is constrained by (5.12). Constraint set (5.13) limits the allocation of demands to *SAR* vessels based on the availability of having at least one vessel within its range. Vessel capacity restrictions for each scenario are applied in (5.14), and constraint

set (5.15) ensures that lifeboats (vessels type 1 and 2) cannot be located at the offshore stations (described in section 5.4.1.2). Constraints (5.16) to (5.18) are used to implement the constrained objectives concept as required in the goal programming method. Finally, constraints (5.19) to (5.21) ensure a minimum level of primary and backup coverage and maximum mean access time in every scenario in order to ensure a robust solution.

5.4. Case Study

In this section the process of implementing the developed methodology to the specific case study for maritime *SAR* in Atlantic Canada is explained.

5.4.1. *Data*

To apply the proposed model, a set of real and valid data about the resources and the demand is required. The main data source for our study is related to historical demand arising from maritime incidents. Also, some preparation and recalculation was performed to transform raw data to the format that is needed to feed into the mathematical model.

The dataset used in this study derives from the *CCG* SI*SAR*[1] database which collects information on all reported maritime incidents. The Atlantic Canada region serves as our research area, with its defined borders illustrated in Figure 5-1. The incident dataset, which has been checked and cleaned for quality control, is available from 1988 to 2013, but due to lack of quality control of the data in early years, and to have a more accurate analysis, we chose the most reliable recent data from 2005 to 2012 (excluding year 2007 which has major problems) for this study.

All incidents are categorized into several classes based on their estimated type and severity, of which the following three are relevant for this research: Class M1 comprises distress incidents, class M2 is potential distress incidents, and class M3 is non-distress incidents, where the "M" indicates maritime (versus other types, such as humanitarian assist "H" or

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[1] Search and Rescue Information Management System

an aircraft accident at sea "A"). After selecting only these three classes of incidents during the chosen study period, we obtained a refined dataset with 8,033 incident records.
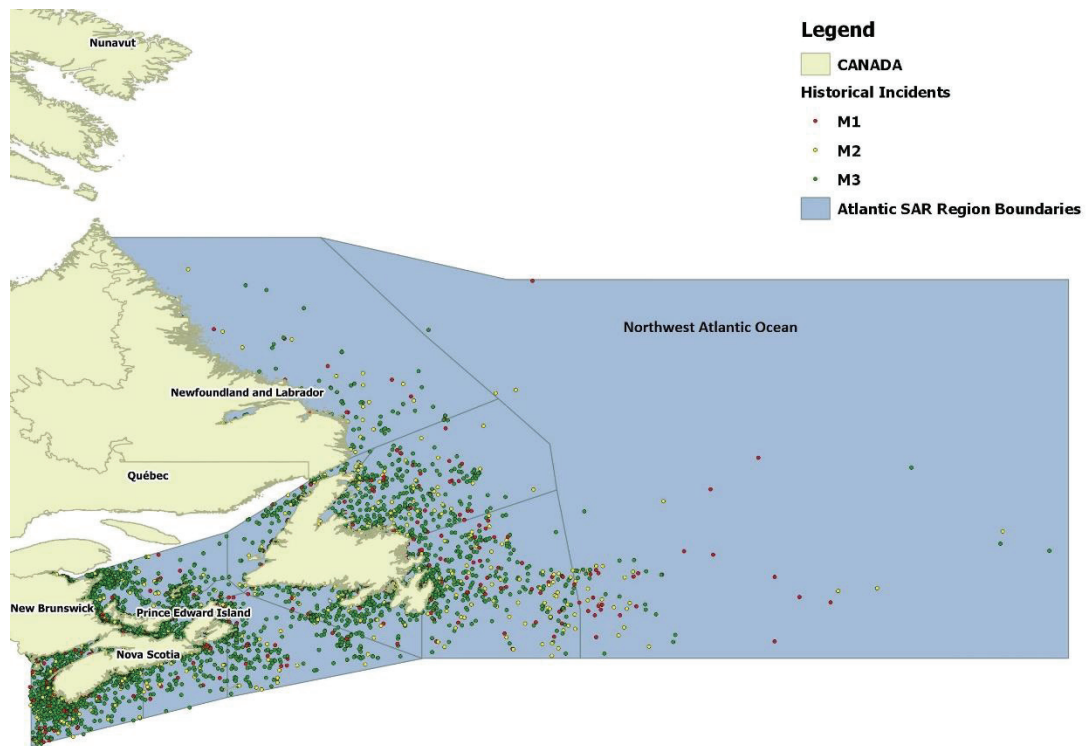
Figure 5-1- Atlantic Canada SAR region and historical incidents (2005, 2006, 2008-2012)

These historical incidents are projected on a mesh of grids generated for the study area of interest. The size of grid squares is variable such that in the areas around the shoreline where the density of incidents is higher, grids are (0.25×0.25 degrees), in the areas further out (0.5×0.5 degrees), and for areas far offshore we have bigger grids (1.0×1.0 degrees). Then, the number of incidents that occurred in each grid is counted to be used for computing incident weights later in the simulation of future incidents. Figure 5-2 shows the mesh of grids on the map, color-coded based on the count of historical incidents.
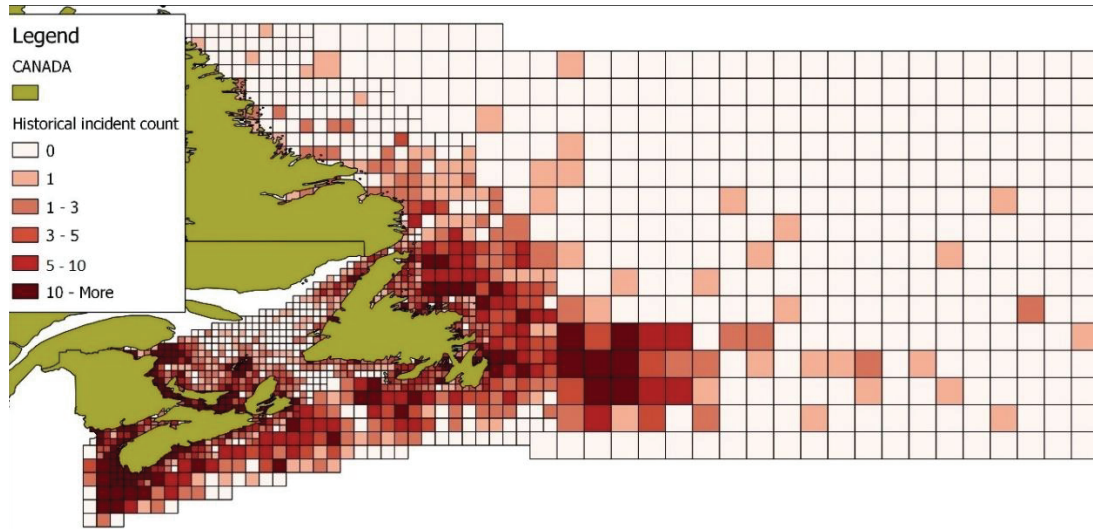
Figure 5-2- Historical incidents on grids (2005, 2006, 2008-2012)

5.4.1.1. Resources

With respect to response resources, information about operating *SAR* stations, as well as about Coast Guard vessels and their capabilities, is required.

As mentioned earlier, there are different types of *SAR* vessels utilized by the *CCG*. In this study, we use the actual information for the currently serving vessels. There are 24 vessels performing *SAR* response activities in Atlantic Canada. These vessels include lifeboats, multi-tasking ships and offshore patrol vessels. Each of these vessels has its own characteristics and capabilities, but they can be categorized into a few classes for simplifying the calculation.

All currently available Coast Guard vessels which are able to provide *SAR* services are categorized into four groups in order to simplify the modelling and reduce calculations. The vessel classes with their specifications are shown in Table 5-1.

Table 5-1- SAR Vessel classes with characteristics

Vessel Class	Vessel type	Range (Km)	Vessel Length (m)	Cruising Speed (Km/hr)	Numbers available
Regular Lifeboat class	Type 1	185	16	26	9
Fast Lifeboat class	Type 2	185	15	41	7
Offshore Patrol vessel	Type 3	10000+	60-70	31	4
Large multi-task vessel	Type 4	6000+	80-90	22	4

Different types of *SAR* vessels have different travel times as their cruising speeds are different. Moreover, their maximum coverage ranges are different as well. This requires them to be treated separately in the mathematical model as we are dealing with travel time rather than travel distance.

5.4.1.2. Potential Stations

Based on the current situation, there are 18 inshore *SAR* stations in Atlantic Canada which are able to house *SAR* vessels. It is assumed that all stations are able to accommodate all vessel types and no restriction is applied in this regard. Some vessels are capable of being positioned offshore for long periods of time. To take advantage of this capability, 19 potential offshore stations are to be considered in our analyses. Of course, this is not a station in the traditional sense, but a central location for a vessel that spends much of its time patrolling or performing other tasks at sea. Initially, these offshore stations are assumed to be at the mean point of each Maritime subarea in Atlantic Canada as determined by the Coast Guard. So, we will have 18 stations in-shore and 19 potential offshore stations. One assumption in the model, represented as a constraint, is that small *CCG* vessels, called lifeboats, cannot be located at offshore stations because their maximum traveling range and endurance are not sufficient for long patrol tasks.

5.4.1.3. Land-Avoided Distance Matrix

The distances between incidents' locations and *SAR* stations are required to perform model calculations. There are different methods for distance calculation. The most common way is calculating straight Euclidean distance. However, there is an issue for using straight or

direct route calculation in this study. In some cases, it is not possible to use the straight route because of land obstacles in the way. To deal with this problem, a previously developed land avoidance algorithm has been used to find the shortest route between incidents and vessels by calculating Euclidean distance between grids while avoiding land obstacles. Note also that great circle distances to accommodate the earth's curvature would be more accurate, but for the strategic aim of this study, and over relatively short distances, it can be ignored.

The distances are collected in the matrix D, which includes distances between all grids where incidents may occur (using grid centroids) and potential station locations. This matrix was calculated using land avoidance and has 1617 rows (grids) and 37 columns (stations), where d_{ij} denotes the distance of incident grid i from potential station j. Using a smaller grid size for areas around the shoreline corresponding to the response zone of small lifeboats (with 185 km range) helps in reducing the error in determining the coverage area.

Aggregation of demand points to a limited number of points (centroid of grids in this case) could result in some errors in measuring distances and with a consequent impact on objective functions. (Francis et al. 2009) explained different types of errors due to demand aggregation and discussed their impact on accuracy of model solutions. The scale of error depends highly on the level of aggregation and the approximation method. On the other hand, using all of the individual original demand points would increase the size of the problem and computation time and this could be an issue especially for large size problems. Moreover, predicting exact demand points is not possible due to stochasticity and continuity of demand.

5.4.1.4. Applying Kernel Density Estimation

The process of applying kernel density estimation to historical incidents is described in this section. Several parameters including the type of kernel function, cell size and bandwidth are required to be set. Parameters' values are determined as follows.

- **Kernel type:** Quartic
- **Cell size:** (0.25 × 0.25) degree; the centre of each grid is used for kernel density calculation.

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- **Bandwidth (radius):** variable size between (0.25-1.0 degrees), 0.25 degree for areas close to the shoreline with high density of incidents, 0.5 degree for areas further from shore and low incident density, and 1.0 degree for areas further offshore with very low number of incidents in the vicinity.

5.4.1.5. Simulating Future Demand

The kernel density estimates are the basis for generating random incidents for future scenarios. For each grid square, the average of kernel estimates within the grid square is calculated (Figure 5-3). These incident density rates are multiplied by the grid area (grids with variable size have variable area) to compute the incident count estimate for each grid. These calculated grid incident counts are scaled so that they sum up to the average number of incidents per year (this is calculated based on seven years' historical data). These scaled values are considered as the mean parameter of a Poisson distribution for generating a random number of incidents over the mesh of grids in the area of interest. Twenty sets of random incident counts per grid square are generated based on calculated Poisson rates. These randomly generated scenarios are used as representation of stochastic demand in the proposed model. Probability of occurrence for each simulated demand scenario is considered to be equal to 0.05 for each of the 20 scenarios as they were all generated from the same distribution.
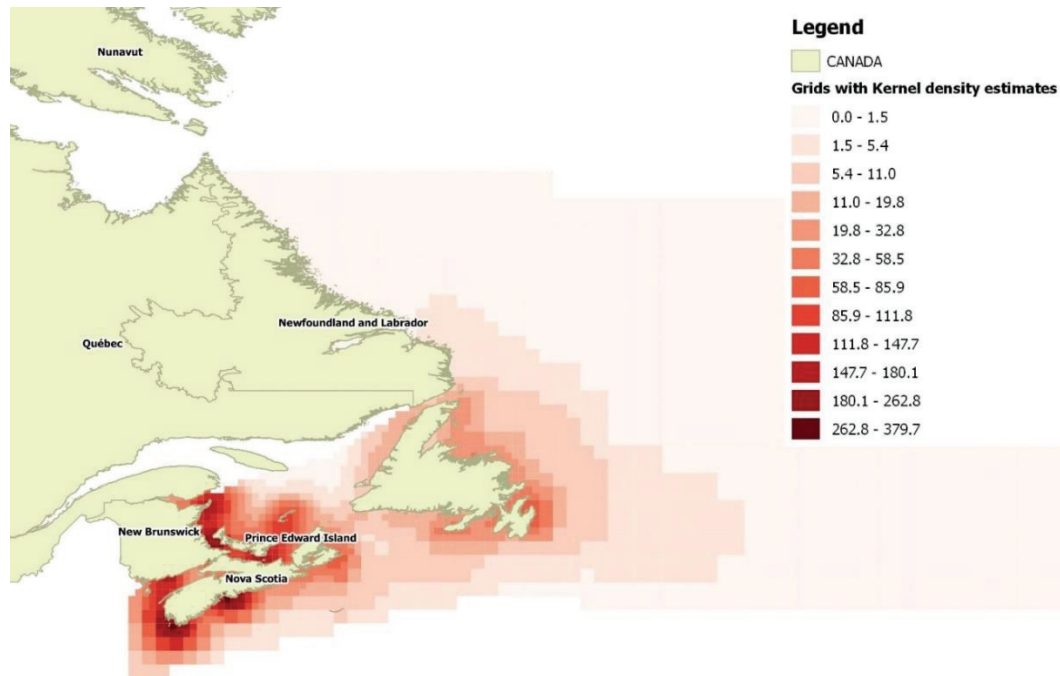
Figure 5-3- Incidents Kernel estimates on gridded map

Using past incidents for the demand, as most of other studies have done, means assuming that the future will behave exactly like the past, while using our procedure assumes that the pattern of incidents (i.e., the underlying probability distribution) remains the same. Furthermore, our approach does not ignore potential demand for the areas that haven't experienced any incidents in the recorded past. This is a less stringent approach that would result in a more reliable analysis.

5.4.2. Coverage Time Limit

The maximum access time for an acceptable level of primary and backup coverage can vary based on the predefined service level standards or expert opinion. In this study, the default value for coverage time limit is considered to be 6 hours based on consultation with *CCG* experts. This parameter will be varied in Section 5.4.4.2 to examine the sensitivity of model results to the coverage time limit. This constraint is only applicable for the primary and backup coverage concepts in the model, but not to the allocation process of the model where there is no limit on access time and all incidents are to be allocated to the closest available resource in order to calculate mean access time.

5.4.3. Resource Capacity Calculation

For each class of *SAR* vessels, a maximum capacity in terms of the number of incidents that could be responded to during the planning timeframe of analysis (one year) should be determined, and considered as a constraint in the optimization model. This is necessary to control the workload of the resources when planning. To calculate the capacity for each vessel type, several factors must be considered:

- Vessel unavailability due to maintenance: *SAR* vessels can be unavailable in some period of service due to planned and unplanned maintenance which affects their actual operational capacity.

- Vessel unavailability due to multitasking: Some vessels that perform *SAR* tasks are designed and used for multiple mandates. So, in reality they are not fully allocated to *SAR* program. This should be considered in the capacity planning of resources.

- Average response time to incidents: the average number of incidents that can be responded to by a particular vessel in a given time period based on historical observations is important in order to calculate the maximum number they can respond to over the planning horizon.

- Vessel speed: vessels have different speeds which has an impact on the duration of response to incidents and thus on the capacity of vessel.

The following equation takes into account different factors to calculate the maximum annual capacity of each vessel type:

$$c_k = (1 - mur_k) * ar_k * sar_k * air * nd, \qquad (5.22)$$

where:

c_k: Maximum number of incidents a vessel type k can respond to in the planning horizon

mur_k: Maintenance (planned and unplanned) unavailability rate of vessel type k

ar_k: Availability rate of vessel type k for *SAR* tasks

sar_k: Speed adjustment rate of vessel type k

air: Average number of incidents responded to by a vessel per day

nd: Number of days in planning timeframe (365 days)

Table 5-2- Vessel annual response capacity calculation

Vessel Class	mur_k	ar_k	sar_k	*air*	c_k
Vessel Type 1	0.15	1	0.88	1	273
Vessel Type 2	0.15	1	1	1	310
Vessel Type 3	0.18	0.55	0.81	1	98
Vessel Type 4	0.23	0.33	0.77	1	54

All these factors and their associated values are defined with consultation and information provided by *CCG* experts. The other fact that needs to be addressed concerning the capacity constraints is that according to historical incidents, there is a seasonal peak during spring and summer in the number of incidents in the area of interest. In particular, the monthly average number of incidents that occurred during the peak season is about 61% more than the annual average. To make the appropriate adjustments, we consider applying a rate to the number of incidents in the corresponding constraint in the model (i.e. by multiplying the left side of inequality (5.14) by 1.61 to increase the number of incidents with respect to the peak season). Therefore, the model ensures satisfying the demand during the peak season.

5.4.4. Numerical Results and Discussions

The proposed model was built and solved using the Gurobi 6.0.4 environment. The model has 197,457 variables and 202,999 constraints. The optimal solutions were found in a reasonable amount of time, usually in a few minutes depending on the parameters' values using a computer with Intel Core i7 CPU and 8GB RAM. In the following sections, the results generated for different scenarios are presented and discussed.

5.4.4.1. Weighted Goal Programming Model

For the first set of optimization runs, the aim is to examine the solutions for different weights on three objectives in the model. The sum of weights is always equals to one and they can vary individually in the range of (0, 1). Objective targets are fixed at 100%, 100% and two hours for primary coverage, backup coverage, and mean access time, respectively.

These are determined through consultation with experts and also given prior information about service level requirements. Table 5-3 presents details of the results for 10 different configurations. The coverage time limit for the coverage calculation is assumed to be fixed at 6 hours in all runs. These results provide good insight about the tradeoffs among objectives and how their values vary by changing the importance weights in the objective function.

Table 5-3- Weighted goal programming model solutions (varying weights)

Model Configuration	Objective Weights			Objective Targets			Coverage time limit (hrs)	Solution (objective values)			Solution time (mm:ss)
	λ_1	λ_2	λ_3	μ_1	μ_2	μ_3 (hrs)		Primary Coverage	Backup Coverage	Mean Access Time (hrs)	
1	0.4	0.3	0.3	1	1	2	6	93.83%	77.22%	2.637	2:55
2	0.4	0.2	0.4	1	1	2	6	94.11%	74.84%	2.611	2:24
3	**0.5**	**0.2**	**0.3**	**1**	**1**	**2**	**6**	**94.11%**	**74.84%**	**2.611**	**2:48**
4	0.6	0.2	0.2	1	1	2	6	93.83%	77.22%	2.637	2:59
5	0.8	0.1	0.1	1	1	2	6	93.83%	77.22%	2.637	3:54
6	0.3	0.2	0.5	1	1	2	6	94.11%	71.21%	2.578	3:24
7	0.2	0.2	0.6	1	1	2	6	94.31%	68.95%	2.563	4:03
8	0.1	0.1	0.8	1	1	2	6	94.31%	63.07%	2.534	2:09
9	0.3	0.4	0.3	1	1	2	6	93.83%	77.22%	2.637	14:15
10	0.3	0.5	0.2	1	1	2	6	93.11%	81.73%	2.751	14:49

Based on the observed results, primary coverage can vary between 93.1% and 94.3% by changing its weight, so it is not very sensitive to the weight. Mean access time has more significant variation by changing the objective weights. Its best obtained solution is 2.53 hours, while changing the weights would worsen it up to 2.75 hours. The most sensitive objective function element to the choice of weights is the backup coverage as its value varies in the range of 63.1% to 81.7% in the investigated results. So far, these results demonstrate that by using the multi-objective model it is possible to achieve solutions that are simultaneously fairly good in terms of different criteria. This will be discussed more in subsequent sections. We consider (0.5, 0.2, 0.3) as a default objectives configuration, because in the decision maker's mind, primary coverage is the most important objective, but some importance should be assigned to the other criteria, keeping in mind that access

time is significantly more important relative to backup coverage. Moreover, examining the different solutions it looks like this specific configuration of objective weights yields a reasonable tradeoff by providing a near optimal solution for all objectives.

As discussed, in order to ensure that a solution can meet the demand during peak season, we have adjusted the number of incidents in each grid (multiplying by 1.61), but this could be a pessimistic overestimate. To investigate the impact of this parameter on the solution, different model configurations were tested by altering that multiplier (from 1.61 down to 1.0 which will make it equal to the average annual demand). The results show that there will be no change in the optimal vessel locations. The only difference is with the optimal value obtained for the mean access time which drops from 2.611 to 2.600 (due to loosening the capacity constraints), a fairly negligible change. Other objectives remain unaffected.

The other thing that is important since we have nondeterministic demand, is the variation of model performance among different demand scenarios that have been generated. Clearly, we would like to have less variation or in other words a more robust solution. We attempted to control negative variations in the objective's values across different scenarios by adding constraints (5.19-21) to the model. Also, we observed the objective values for each scenario to examine whether the variation levels are under control and within confidence intervals. The results shown in Figure 5-4 indicate a relatively low level of variation in objective values among different demand scenarios for the (0.5, 0.2, 0.3) objective weights. The Backup Coverage objective is more stable, while the Primary Coverage and Mean Access Time have more fluctuation while still within reasonable range. This observation supports the robustness of solutions produced by the proposed model over different simulated demand scenarios as well as the appropriateness of the size of simulation (number of replications).
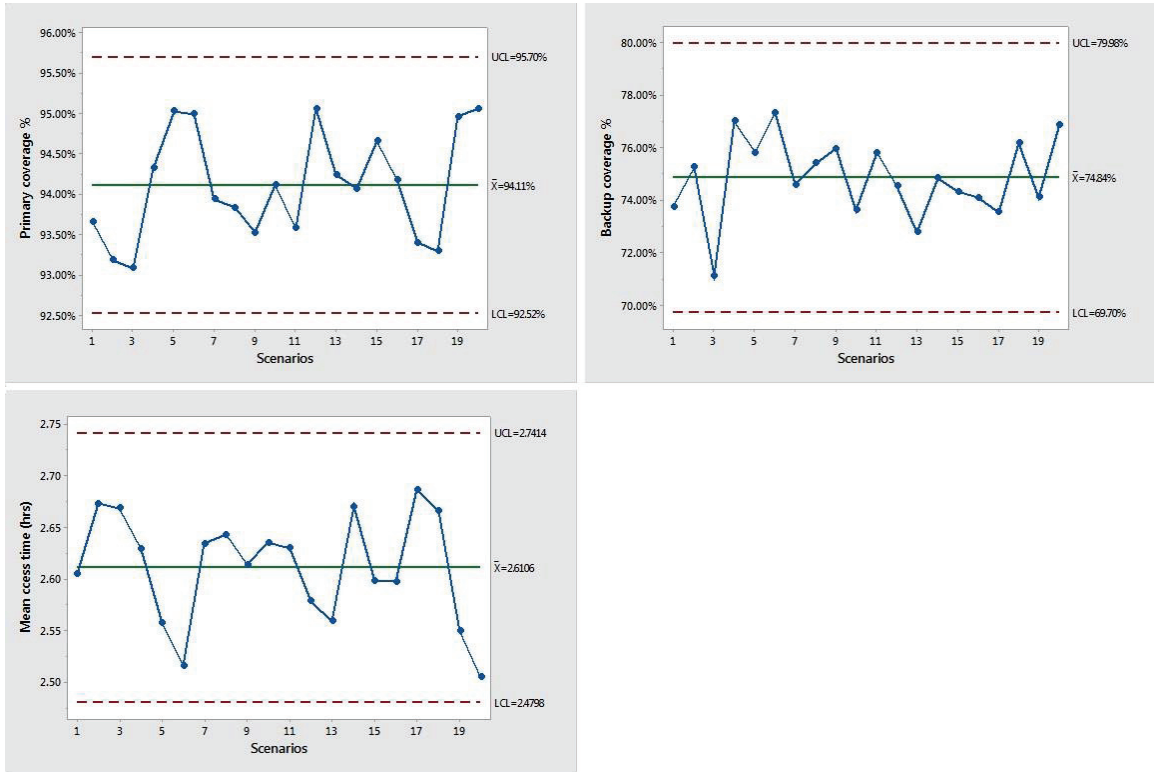
Figure 5-4- Objective values variation over demand simulated scenarios

5.4.4.2. Sensitivity Analysis on Coverage Time Limit

In the previous section the coverage time limit parameter was fixed and the changing variables were the objective weights. But coverage time limit could change based on different situations and potential service level requirement changes. In this section, objective weights are fixed at (0.5, 0.2, 0.3). Table 5-4 shows the solutions found for scenarios with coverage time limit varying between 4 and 12 hours. As shown, increasing the coverage time limit will mostly help coverage objectives improve, in fact getting very close to their target value (100%). This is expected as the coverage time limit works as a constraint on the coverage variables thus loosening that constraint will result in increase in coverage. Figure 5-5 demonstrates the performance of model solutions when increasing the coverage time limit. Moreover, we observe a slight drop in mean access time as we increase the coverage time limit. This phenomenon can be explained by considering multi-objective model behavior. The model is forced to sacrifice the mean access time for the cases with a tighter coverage time limit constraint, in order to keep reasonable objective values for primary and backup coverage that have been squeezed by the lower coverage

time limit. This is apparently done in order to minimize the total weighted deviation from targets by avoiding a sharp decline in coverage rates.

Table 5-4- Weighted goal programming model solutions (varying coverage time limit)

Model Configuration	Objective Weights			Objective Targets			coverage time limit (hrs)	Solution (objective values)			Solution time (mm:ss)
	λ_1	λ_2	λ_3	μ_1	μ_2	μ_3 (hrs)		Primary Coverage	Backup Coverage	Mean Access Time (hrs)	
1	0.5	0.2	0.3	1	1	2	4	85.64%	53.41%	2.637	3:25
2	0.5	0.2	0.3	1	1	2	5	91.88%	69.67%	2.667	6:46
3	**0.5**	**0.2**	**0.3**	**1**	**1**	**2**	**6**	**94.11%**	**74.84%**	**2.611**	**2:24**
4	0.5	0.2	0.3	1	1	2	7	95.79%	82.70%	2.614	4:45
5	0.5	0.2	0.3	1	1	2	8	97.15%	83.85%	2.585	1:53
6	0.5	0.2	0.3	1	1	2	9	97.99%	85.80%	2.570	5:10
7	0.5	0.2	0.3	1	1	2	10	98.28%	87.75%	2.553	2:14
8	0.5	0.2	0.3	1	1	2	11	98.74%	89.12%	2.536	2:37
9	0.5	0.2	0.3	1	1	2	12	99.30%	91.04%	2.536	1:27
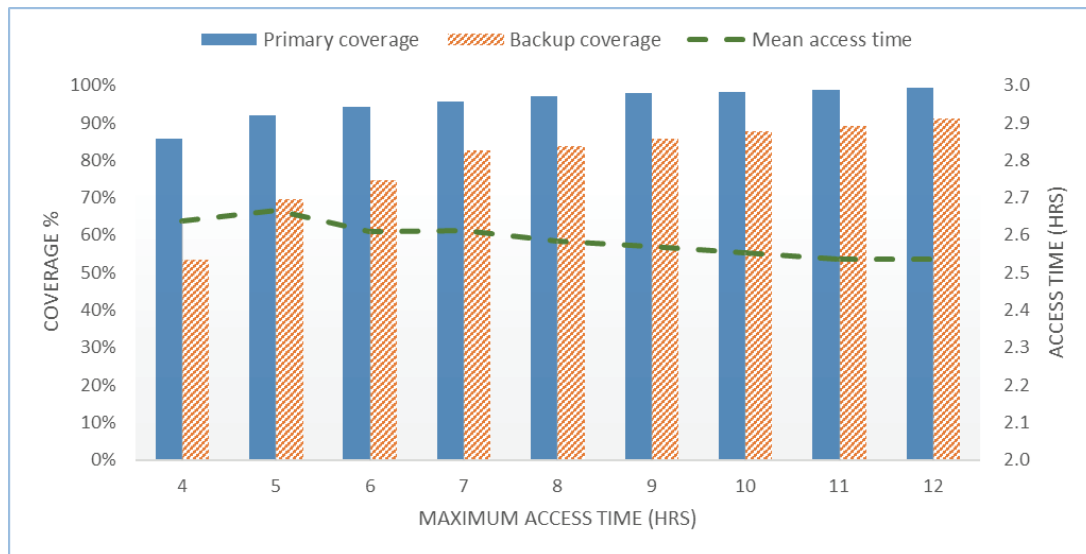
Figure 5-5- Objective values sensitivity to coverage time limit

5.4.4.3. Proposed Solution Compared to Single Objective Models' Solutions

The performance of the multi-objective goal programming model is also examined compared to two popular single objective models, Maximal covering (Church and ReVelle 1974) and *p*-median (Hakimi 1964), in terms of the three main criteria of the problem. The

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results shown in Table 5-5 demonstrate that the proposed model performs well in comparison with single objective models with respect to the purpose it is designed for, where primary coverage is only 1.1% lower than the maximal covering solution and mean access time only 0.08 hours higher than the *p*-median solution, while the backup coverage has been improved notably (14% compared to the maximal covering and 11% compared to the *p*-median). The results show that a significantly higher level of backup coverage can be provided without substantial loss of primary coverage and upsurge in access time. The main achievement of the multi-objective model, as it was anticipated, is to provide near optimal solutions for all criteria simultaneously which was overlooked in the single objective models.

Table 5-5- Proposed model solution vs. single objective models

Resource Arrangement	Solution (objective values)		
	Primary Coverage	Backup Coverage	Mean Access Time (hrs)
Maximal covering	95.23%	60.12%	2.79
p-median	94.31%	63.07%	2.53
Multi-objective model	94.11%	74.84%	2.61

5.4.4.4. Proposed Model Solution Compared to The Current Arrangement of Vessels

In order to compare the solutions provided by the multi-objective mathematical model to the current arrangement of *SAR* resources, the performance of current arrangement of *SAR* vessels across different demand scenarios is simulated. The response allocation of resources to the forecasted incidents is simulated taking into account the vessel classes' characteristics and the policy of allocating incidents to the closest vessel. In Table 5-6, the objectives' values for the simulated current arrangement scenario are compared to the goal programming model with (0.5, 0.2, 0.3) weights and given the default six hours' coverage time limit. According to the results, a remarkable improvement in terms of all criteria is provided using the multi-objective model solution versus what is obtained by the current arrangement of resources.

Table 5-6- Proposed model solution vs. the current arrangement

Resource Arrangement	Solution (objective values)		
	Primary Coverage	Backup Coverage	Mean Access Time (hrs)
Current Arrangement	89.38%	60.18%	3.14
Multi-objective model solution	94.11%	74.84%	2.61

5.4.5. Managerial Aspects

The proposed model in this study attempts to take into account different criteria that are of interest to decision makers for planning and managing limited resources efficiently and effectively. This model considers different characteristics and capability of resources as well as their operational capacities, and tries to model the problem in the closest way possible to a real case. The multi-objective model provides the desired flexibility by incorporating several variable parameters into the model for managers to play with these factors and obtain the most appropriate solution for various different business situations. The idea of goal programming helps as managers can specify their desired target values for each objective as well as their preferences across criteria. This general model can be utilized by high level decision makers in order to perform strategic resource and capacity planning. Several sensitivity analyses can be performed to examine different possible scenarios, for example decommissioning of vessels, new resource recruitment or new station development. One can see the tradeoffs among objectives' values resulting from changing different parameters in the model and thus acquire valuable insights about the anticipated impacts of strategic and tactical decisions.

5.5. Conclusion

In this study, multi-objective goal programming is applied to the Maritime *SAR* Location problem. For this model, four common *SAR* vessel types which are used in practice and have different capabilities are considered. Future demands are simulated and distributed over the study area and timeframe based on patterns extracted from historical incidents. The mixed integer-linear optimization model has three objectives: minimizing standardized deviation from: Primary coverage, Backup coverage and Mean access time. Several model runs with different parameter configuration are generated and solved and

results show the desirable performance of the model compared to common single objective models in location analysis, namely maximal covering and p-median. The proposed model provides solutions with significantly better performance versus the current response vessel arrangement taking into account three main decision criteria. Several different scenarios are examined through performing sensitivity analyses on objective weights and coverage time limit.

The results of this study could be useful for guiding decisions with regards to *SAR* vessel acquisitions and placement in order to improve the efficiency of resources and increase the service level. More specifically, the outcome of this study could provide the *CCG* with some useful insight for future resource capacity planning including fleet procurement planning and appropriate stations for placing new vessels. Also, it can be helpful for managing current operations to increase the resource utilization and effectiveness of their services. Several operational rules can be extracted from the model solution for best resource allocation policies.

There are several potential future extensions of this work. There could be different types of incidents with different severity and requirements for different types of response, as all *CCG* vessels are not equally effective at responding to different types of incidents. So, it would be more realistic if we can differentiate incident types in the model and take into account their specific response requirements. That also would require more detailed information about the capabilities of resources. In this study, we dealt with the congestion issue by incorporating backup coverage in the model as an objective, while the other common way is using queuing models. One could look at simulating the future incidents in a way that produces the exact location versus the aggregation approach which is used in this study. This could help with reducing the error in distance and coverage area calculation. The model could be further extended to future response needs by modelling trends in incidents, incident rates, and/or traffic levels. In addition, there could be a possibility to relocate some vessels periodically to balance the availability of resources responding to changing demand patterns and thus increase the utilization.

Chapter 6 γ-Robust Multiple Period Capacitated Location Model: The Case of Maritime Search and Rescue

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Abstract

This study proposes a robust multi-period p-median model with a constraint on the minimum required coverage level for optimizing the Location-Allocation of maritime search and rescue vessels over several simulated demand scenarios. Different types of capacitated vessels are considered. The possibility of seasonal relocation of vessels is taken into consideration to effectively respond to the associated changes in demand patterns. Comparing the exact optimal solution provided by the dynamic model to the current arrangement of *SAR* resources as well as to a static configuration of the model reveals a substantial potential improvement in terms of the service level, with robust performance over possible variations in demand.

Keywords: Location; Maritime Search and Rescue; Multiple period; p-median; Robust optimization

6.1. Introduction

According to many published reports, ocean activities such as fishing or marine transportation are among the most dangerous activities in the world. Since, Canada is surrounded by navigable waters, the country is subject to many risky marine operations. For example, Commercial fishing has the highest fatality rate among all industries in Canada. Workers Compensation Board (*WCB*) statistics show that 0.831 workers per 1000 died while on the job in the fishing industry, while the average across all other industries is 0.044 worker fatalities per 1000 workers (WCBNS 2012). Thus, maritime search and rescue (*SAR*) is one of the most needed rescue activities in Canada as every year more than 6,000 incidents are reported to the responsible authorities. Maritime *SAR* can be categorized as an emergency response activity. The Canadian Coast Guard (*CCG*) operates the federal government's civilian fleet within the Department of Fisheries and Oceans (*DFO*), including providing maritime *SAR* services. It is paramount that their limited resources are used efficiently. To do so, they must make informed strategic and tactical decisions on where to site their resources, how to relocate them if necessary, and how to allocate incidents to the located resources.

The Location-Allocation problem is a basic model for several important applications, including the location of ambulances, police cruisers, fire stations, distribution centers and so on. These models determine the optimal location of facilities as well as allocation of customers (demand) to the located facilities. There is a wide variety of mathematical models with different characteristics that have been formulated for such problems. Each of these models aims to provide an optimal solution with regard to single or multiple objectives (criteria) such as coverage, cost, access time, etc.

There is rich literature in the field of emergency location analysis. In a typical emergency response location problem, we are faced with a "server to customer" service system with mobile servers in contrast to the usual facility location problems wherein facilities are fixed and customers travel to those sites. So, in these problems, by facilities we refer to the emergency response vessels such as ambulances or *SAR* vessels. In such conditions, the relocation of facilities (i.e. vessels) is much easier and does not incur the regular costs

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associated with closing a facility and opening a new one in a new location. In this practical category of location problems, usually a key goal is to provide efficient response within a minimum amount of time so as to increase the chance of people surviving.

An important concern when dealing with strategic planning for any type of facility is the nature of the demand. It is crucial to consider the possible patterns and variations in demand. The spatial and temporal changes in demand in terms of volume and location should be taken into account. Overlooking these factors might dramatically affect the actual system performance. In our case, as we have different types of marine incidents with different causes and contexts, there are substantial spatial variations in demand that need to be considered for response resource planning purposes. For example, we know that the areas that most incidents occur substantially change due to variations in fishing areas, recreational zones and maritime transit corridors in different seasons.

The purpose of this study is to develop a robust multi-period model for the case of a maritime *SAR* location problem that not only incorporates different criteria that are of interest for decision makers and ensures that the solutions performs well under different demand conditions, but also allows relocation of vessels in a defined periodic manner so that we can more effectively respond to the dynamics of demand. Constraints related to resource type, such as response vessel capacity and capability, are considered in the model to make the representation more realistic. We use the scenario planning approach to deal with the demand uncertainty. Several demand scenarios are randomly generated based on the spatial distribution extracted from historical data and for different seasons of operations to account for changing seasonal patterns. The proposed model is applied to a real world maritime *SAR* location problem. Solutions provided by the proposed model are compared to the current arrangement of the fleet as well as to the solution of the model with static configuration (i.e. where the vessel arrangement is unchanged throughout the time horizon). Ultimately, this study aims to propose a decision support model that is capable of providing helpful insights for decision makers with regard to possible changes at the strategic and tactical level on managing resources to improve the overall quality of maritime *SAR* service and efficiency of resource utilization.

The remainder of this paper is organized as follows. Section 6.2 presents a critical review of relevant literature. Section 6.3 explains the proposed mathematical model. In section 6.4, the process of applying the model to our case study is presented, followed by the numerical results and discussion of the results. We conclude the paper with the summary of the findings and outlook for future research in the field.

6.2. Literature

6.2.1. Dynamic Facility Location Studies

Facility location problems can be divided into two categories: static and dynamic problems where location decisions are made just once in the static category, while in the dynamic problems location decisions are time dependent (i.e. relocation of facilities is permitted). Relocation of facilities can occur in a discrete or continuous manner. In the former category, relocation is only possible at discrete pre-determined points of time (Wesolowsky 1973); while in the latter case, relocation is possible at any time during the planning horizon (Drezner and Wesolowsky 1991).

Wesolowsky (1973), Wesolowsky and Truscott (1975), and Sweeney and Tatham (1976) dealt with the multi-period Location-Allocation problem starting with the static Location-Allocation problem, and subsequently dynamic programming was applied to introduce dynamic considerations in order to find the optimal multi-period solution. A relocation problem for public facilities was applied to a real life problem in (Min 1988). A fuzzy multi-objective model with constraints on budget and on the maximum number of relocations per period was constructed to solve the problem.

Melo et al. (2005) studied the application of a set of innovative models for the single-commodity and multi-commodity dynamic (i.e. multi-period) Location-Allocation problem. They considered gradual relocation of facilities over the planning horizon. Capacity expansion and reduction scenarios were examined to address the fluctuations in demand. More recently, Afshari et al. (2014) presented a model with an optimal approach for multi-objective, multi-period, multi-commodity, distribution-service system. Sonmez and Lim (2012) introduced a location-relocation model minimizing the initial and expected future weighted travel distance. The model considers possibility of facility relocation for

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future instances by closing some of the facilities that were located initially and opening new ones, without exceeding a given budget.

Seyedhosseini et al. (2016) point out that in most of the research on this subject there has been an emphasis on the use of deterministic parameters. Framing a problem using a probabilistic or stochastic model is the most important cause of complexity in the problem and generating the solution. In order to simplify, most authors consider the parameters as deterministic. Also, in order to reduce the complexity of the mathematical model most studies prefer discrete time periods of relocation over continuous changes. In addition, this approach is more practical in most cases when it comes to the implementation of solutions; however, choosing continuous time for relocations often results in better optimal solutions.

Arabani and Farahani (2012) have reviewed the literature on dynamics in location analysis. Different approaches for considering dynamicity in demand, parameters and/or factors in facility location problems are examined. They categorized studies based on the method taken into account for the stochasticity of demand and its possible variations. Time dependent location problems, multi-period and simple-period location problems and location-relocation problems are among different types of problems which were reviewed.

One of the most appealing practical subsets of dynamic location modelling, with a quite rich literature, is the ambulance relocation problem where the relocation or redeployment decisions are usually made in a real time manner, see e.g., Gendreau et al. (2001), Rajagopalan et al. (2008), Bélanger et al. (2014), and Moeini et al. (2014). Brotcorne et al. (2003) traced the evolution of ambulance location and relocation models including static models, probabilistic models and dynamic models. Gendreau et al. (2006) developed a Maximal Expected Coverage Relocation Problem which aims to provide optimal dynamic relocation strategy for emergency vehicle waiting sites so that the expected covered demand is maximized while the number of relocations is under control. Rajagopalan et al. (2008) also proposed a dynamic relocation/redeployment model for ambulances to determine the minimum number of ambulances required and their locations for each time cluster in which significant changes in demand pattern occur.

Seyedhosseini et al. (2016) presented the most recent comprehensive review on research studies on dynamic location problems. They classified problems in this area with respect to modelling methodology, objective function type, solution approach, and application area. They covered a wide range of problems including single and multiple facility relocation problem, median and covering dynamic location problems, stochastic dynamic location problems, and fuzzy location problems.

6.2.2. Facility Location Modelling Under Demand Uncertainty

Uncertainty in discrete facility location modelling may exist in different aspects of the problem such as demand locations, travel times, travel costs, demand volume, etc. There are two common approaches for optimization under an uncertain environment: stochastic programming (*SP*) and robust optimization (*RO*); in the first approach uncertain parameters are represented with probability distributions. In robust optimization, a set of possible future values (represented as scenarios) are taken into account and typically the objective is to minimize the largest deviation of a solution from the best possible (i.e. ideal) objective value across all scenarios. A "scenario" is a complete realization of all the uncertain parameters. Each scenario fully determines the value of all the uncertain parameters. Depending on the problem, we may have a finite or infinite number of scenarios. Minimizing regret is one of the most popular approaches taken by researchers in *RO* where the regret is generally defined as the deviation of a solution from the best possible solution. The concept of regret can be used in the same way ideal points techniques proceed, which is either by way of minimizing the average distance to the ideal point (or regret) or by a minimax objective so as to guard against the worst-case scenario.

Snyder (2006) reviewed the literature on stochastic and robust facility location models and covered a variety of approaches for optimization under uncertainty. According to this work, in risk situations there are uncertain parameters whose values are governed by probability distributions that are known by the decision maker. In uncertainty situations, parameters are uncertain, and furthermore, no information about their probabilities is known. Problems in risk situations are known as stochastic optimization problems; a common goal is to optimize the expected value of some objective function. Problems under uncertainty are

known as robust optimization problems and often attempt to optimize the worst-case performance of the system.

In stochastic programming, it is assumed that the probability distribution associated with the value of uncertain parameters is known; however, in robust optimization it is assumed that no information about probability distributions is available except limited data on the specification of intervals containing the uncertain values. According to what Owen and Daskin (1998) proposed, the uncertainty in the model parameters might arise for two reasons, as either future conditions incur planning uncertainty, or absence of knowledge about the input parameters produces the related uncertainty. As examples of applying a scenario planning approach for stochastic location programming, one can refer to Chen et al. (2006) and Owen and Daskin (1998). In scenario planning, the decision maker identifies a number of future possible scenarios and estimates the likelihood of each scenario occurring. Sheppard (1974) was among the first researchers to use scenario planning to model uncertainties in facility location. His model minimizes the expected cost over all scenarios.

In the context of stochastic facility location, the regret associated with each scenario under a given siting plan is usually defined as the difference between the objective function value of the optimal solution for that scenario and the objective function value of a chosen siting plan (Chen et al. 2006). If there is no knowledge about the probabilities of the possible outcomes, the Minimax regret principle can be useful to help people in making decisions. However, if there is knowledge about these probabilities, then the Minimax regret principle can be suboptimal. An alternative robustness measure proposed by Snyder and Daskin (2006) is "α-robustness". They present a novel robustness measure that combines the two objectives by minimizing the expected cost while bounding the relative regret in each scenario. The idea is to look for a solution that minimizes the expected cost/distance such that the relative regret in each scenario is no more than α. This study can be considered as an application of the constraint method; see e.g. Cohon et al. (1979).

Daskin et al. (1997) also proposed an α-reliable minimax regret model for a *p*-median problem. They minimized the maximum regret of the total weighted distance over a set of

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scenarios whose total probability is at least α. Their approach mixes the advantages of robust optimization by applying a regret criterion, and stochastic optimization since it minimizes the expected regret over select scenarios. Chen et al. (2006) studied a facility location problem under uncertainty associated with future events which is modelled by defining alternative future scenarios with probabilities. They presented an α-reliable mean excess model that minimizes the expected regret with respect to an endogenously selected subset of worst-case scenarios.

The two-stage nature of facility location problems (choose locations now, before we know what the future holds, and react once the uncertainty has been resolved, say, by assigning customers to facilities) has made these problems very attractive to researchers exploring approaches to decision making under uncertainty (Snyder 2006). Thus, in stochastic location modelling, locations are generally first-stage decisions whereas assignments of customers to facilities are second-stage, i.e., recourse, decisions. (If both decisions occur in the first stage, most problems can be easily reduced to deterministic problems in which uncertain parameters are replaced by their means.)

Wang et al. (2003) proposed a model which considers opening new facilities and closing some old ones that, due to a change in the distribution of customer demand, no longer provide adequate service. The model minimizes the total weighted travel distance for customers, subject to a constraint on the budget. Lim and Sonmez (2013) also consider relocating facilities where there are demand changes. Relocations are performed by closing some of the existing facilities from low demand areas and opening new ones in newly emerging areas. Different scenarios with known probabilities were used to capture demand uncertainty. Their model minimizes the expected weighted distance while making sure that relative regret for each scenario is no greater than γ. Both Wang et al. (2003) and Lim and Sonmez (2013) have not considered the capacity limitation of facilities in their models. The other limitation is that all facilities are assumed to have the same characteristics.

A review on different modelling approaches for dealing with uncertainty in facility location is provided by Correia and Saldanha da Gama (2015).

6.2.3. Search and Rescue Location Analysis

There are not many studies on the application of location modelling to *SAR* problems. Brown et al. (1996) developed a mixed integer model for scheduling US Coast Guard district cutters, whereby its solution was superior compared to manually prepared schedules. Nguyen and Kevin (2000) combined maximal covering and *p*-median location problems using a goal programming model to assess the level of service of the existing Canadian *SAR* system (in terms of location of *SAR* aircraft and helicopters) and compared it to the optimal solution of their model. Afshartous et al. (2009) studied the problem of locating Coast Guard air stations taking a statistical-optimization approach to provide a robust solution in the presence of uncertainty in distress call locations. Distress calls are simulated and the optimization problem is solved for different simulations. The optimization model however is not as comprehensive as it should be and it does not consider all different demand simulations in an integrated model.

Radovilsky and Koermer (2007) presented the application of integer linear programming for the optimal allocation of rescue boats among the stations of the U.S. Coast Guard. Their model minimizes shortages or excess capacities at the stations. In an extension to the previous study, Wagner and Radovilsky (2012) developed a new model that simultaneously considers reduction of excess capacity and boat shortages at the stations, a decrease in the overall fleet size with an increase in boat utilization, and overall reduction of the fleet operating cost. Another interesting study was conducted by Nelson et al. (2014) who developed an optimization model for determining the optimal deployment assignments, operational levels and aircraft allocation among all *USCG* Air Stations.

Pelot et al. (2015) applied three common covering problems in emergency response modelling including maximal covering location problem, maximal expected covering location problem, and maximal covering location problem with workload capacity, to the maritime *SAR* location problem for Atlantic Canada. Akbari et al. (2016) also presented a multi-criteria analysis on the performance of solutions provided by two popular location models, *p*-median and maximal covering, to the case of a maritime *SAR* location problem. This study considers primary and backup coverage, mean access time, the Gini index to

reflect the service equality level across customers, and maximum access time as post-assessment criteria for solutions of two single objective models.

Razi et al. (2016) determined the best solution for allocating helicopters to *SAR* missions using an optimization model. Simulation was used to validate the performance of the solution for the uncertain demand. Razi and Karatas (2016) conducted a study on developing a multi-objective model for allocation of *SAR* boats taking into account several factors and decision criteria such as density and type of incidents, resource capability, and business rules. Response time to incidents, fleet operation cost and the mismatch between resource load and operation planned capacity, are minimized.

To the best of our knowledge, none of studies in the area has considered the possibility of relocating vessels to deal with dynamic demand. Also, it should be noted the approach of relocating *SAR* vessels is quite different from what is usual in other emergency response activities such as ambulances as the relocation of *SAR* vessels is not as simple as the relocation of ambulances and it cannot happen very frequently. Vessels used for *SAR*, especially multitasking vessels, are often big ships such that that their relocation to a new station is not easy and involves several operational issues. The decision about where vessels are stationed, or pre-positioned, is an administrative decision which must pass through several levels of approval in the organization; this is not a quickly decided move. It may impact on the planning in other sections of the Coast Guard due to multi-tasking coordination. Depending on the nature of a relocation, it may affect from which home locations crew members are assigned, thus another logistics issue. Of course, the response range in maritime *SAR* is long and not comparable with ambulances, and thus a vessel's relocation can potentially be a long one. Furthermore, most of the proposed models assume deterministic demand, usually based on historical data. In some cases, future demand is simulated to validate the model solution performance, but not then used as an input to the optimization model.

6.3. Proposed Multi-Period Model

We present a robust multi-period capacitated p-median model which is customized for the maritime *SAR* Location-Allocation case. The p-median problem is a well-known facility location problem that was first introduced explicitly by Hakimi (1964) and formulated by ReVelle and Swain (1970).

Due to the possibility of changes in demand during different seasons, we allow relocations to occur, so vessels can be relocated from their current station to other eligible stations in a periodic manner. Also, to account for uncertainty in demand, a scenario planning approach is utilized whereby several sets of randomly generated demands (demand scenarios) at the planning horizon are used in the mathematical model. The objective function minimizes the expected weighted access time to all incidents over the demand scenarios assuring that the maximum regret across all demand scenarios stays within a prescribed upper bound (i.e. under control).

Therefore, the goal of this study is to produce a new model for the *SAR* vessel location-relocation problem with multiple types of capacitated facilities and under demand uncertainty. We develop a mathematical model to provide solutions for vessel relocation decisions that ensures good performance across all scenarios rather than finding optimal solution for each scenario. The model solution determines where to locate different types of vessels and which ones to relocate seasonally to effectively respond to demand pattern changes and balance the expected and worst case performance of the decisions.

Our model considers different classes of *SAR* vessels with different speeds, response capacities and plausible locations. It includes capacity constraints on the number of incidents that can be responded to by different types of *SAR* vessels to account for limited operational capacity of resources. Moreover, since we are dealing with a problem of locating mobile facilities (i.e. *SAR* vessels) with different speeds, we chose to use travel time instead of a regular travel distance proxy in the objective function. In addition, typically in emergency response analysis a customer is called *covered* if it is within a predetermined access time rather than prescribed range because the time is a better proxy for measuring system performance in this case.

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In our case, there is no second stage decision in contrast to common stochastic programming location models, because the allocation decision must be made before that actual demand is revealed. So, both location and allocation decisions are made prior to having complete information about the demand. In other words, the assignment of demand to resources cannot be variable for different scenarios, because even at the time of deployment of a vessel to respond to a particular incident, we still do not know which one of the demand scenarios is happening. The important fact is that we apply a scenario planning approach to come up with a solution which is robust with respect to possible demand variations, even though these specific scenarios are not guaranteed to happen. Furthermore, our model considers the capacity limits of resources and these constraints are not the same in different scenarios because in various simulated scenarios, the workload of vessels can be different due to changes in grid weights (number of incidents in each grid). Hence, we want to ensure that in all circumstances these capacity constraints are satisfied. This is another reason why scenarios cannot be simply replaced by their mean value to convert the model to a simpler deterministic version instead.

We present a new formulation for the γ-robust facility relocation problem customized for the case of emergency response facilities (particularly in maritime *SAR*) that not only concurrently incorporates the uncertainty of demand through scenario planning and relocation of facilities, but also extends the model proposed by Lim and Sonmez (2013) in at least two ways: first by considering different type of facilities (vessels) with different operational characteristics and second by including capacity constraint on facilities to account for their limited operational capacity.

The model objective is to minimize the expected mean travel time (access time) to all incident locations which are projected over a mesh of grids while ensuring that relative regret for each scenario is no more than γ. The relative regret associated with a scenario is the standardized difference between the mean access time corresponding to a given solution and the optimal mean access time for that scenario. The optimal mean access times for individual scenarios are obtained by solving the corresponding versions of our relocation problem.

The mathematical model developed for this problem is a large-scale Integer Linear Programming Problem. Indices, parameters and variables used in the model are listed and defined below.

Indices:

$i \in I$:	Demand locations
$j \in J$:	Potential vessel stations
$J_S \subset J$:	Set of offshore stations (virtual stations offshore for patrol vessels and multitasking ships)
$k \in K$:	Index for vessel types
$\omega \in \Omega$:	Index for simulated incident scenarios
$\theta \in \Theta$:	Index for relocation periods

Variables:

$x_{i\theta}$:	Binary variable for primary coverage at grid i in period θ
$z_{jk\theta}$:	The number of vessels type k located at station j in period θ
$u_{ijk\theta}$:	Allocation of customers at grid i to vessel type k located at j in period θ
v_ω:	Mean access time in scenario ω

Parameters:

r_k:	Coverage distance (range) of vessel type k
p_k:	Maximum available number of vessel type k
$c_{k\theta}$:	Capacity of vessel type k in period θ (number of incidents that can be responded to)
v_k:	Cruising speed of vessel type k
d_{ij}:	Distance between grid i and station j
π_ω:	Probability of occurrence of scenario ω
$w_{i\omega\theta}$:	Demand weight at grid i in period θ in scenario ω (number of incidents)
t:	Coverage time limit for acceptable level of coverage

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γ: Maximum relative regret (desirable robustness level, in percentage)

α: Maximum uncovered demand (as a percentage of total incidents)

v_ω^*:   Optimal objective value of the problem corresponding to individual scenario ω

The formulation of the proposed model is as follows:

Min:

$$E = \sum_\omega \pi_\omega v_\omega \tag{6.1}$$

s.t.

$$\frac{\sum_\theta \sum_k \sum_i \sum_j \left(w_{i\omega\theta} u_{ijk\theta} \left(d_{ij}/v_k \right) \right)}{\sum_\theta \sum_i w_{i\omega\theta}} = v_\omega, \; \forall\, \omega \tag{6.2}$$

$$x_{i\theta} \leq \sum_j \sum_k a_{ijk} z_{jk\theta}, \qquad \forall\, i, \theta \qquad \text{Primary coverage constraint} \tag{6.3}$$

where: $a_{ijk} := 1$ *if:* $d_{ij} \leq r_k$ *and* $d_{ij}/v_k \leq t$ *else* $a_{ijk} := 0$

$$\sum_k \sum_j u_{ijk\theta} = 1, \qquad \forall\, i, \theta \qquad \text{Unique demand allocation constraint} \tag{6.4}$$

$$\sum_j z_{jk\theta} \leq p_k, \qquad \forall\, k, \theta \qquad \text{Fixed number of available vessels in each class} \tag{6.5}$$

$$u_{ijk\theta} \leq b_{ijk} z_{jk\theta}, \qquad \forall\, i, j, k, \theta \qquad \text{Allocation to } SAR \text{ vessels possible if there is a vessel within the coverage range} \tag{6.6}$$

where: $b_{ijk} := 1$ *if:* $d_{ij} \leq r_k$ *else* $b_{ijk} := 0$

$$\sum_i w_{i\omega\theta} u_{ijk\theta} \leq c_{k\theta} z_{jk\theta}, \qquad \forall\, j, k, \omega, \theta \qquad \text{Capacity constraint} \tag{6.7}$$

$$z_{jk\theta} = 0, \qquad \forall\, j \in J_S, k \in \{1,2\}, \theta \quad \text{Offshore location constraint} \tag{6.8}$$

$$\frac{\sum_\omega \pi_\omega \sum_\theta \sum_i w_{i\omega\theta} x_{i\theta}}{\sum_\omega \pi_\omega \sum_\theta \sum_i w_{i\omega\theta}} \geq 1 - \alpha, \qquad \text{Minimum expected coverage over all scenarios} \tag{6.9}$$

$$v_\omega - v_\omega^* \leq \gamma v_\omega^*, \qquad \forall\, \omega \qquad \text{Maximum relative regret in every scenario} \tag{6.10}$$

The minimization objective function (equation 6.1) in relation with constraint (6.2) minimizes the expected mean access time for all incidents across all demand scenarios. Constraint (6.3) ensures that demand location i covered in period θ only if there is a vessel within the maximum coverage range and time in that period. Allocation of demand locations to resources is defined in (6.4) which ensures that all demand points are assigned to exactly one response vessel. The fixed maximum number of vessels in each class is constrained by (6.5). Constraint set (6.6) limits the allocation of demands to vessels based on the availability of having at least one vessel within its range. Vessel capacity restrictions for each scenario in different periods are applied in (6.7), and constraint set (6.8) ensures that lifeboats (vessels type 1 and 2) cannot be located at the offshore stations (described in section 6.4.1.2). Constraint (6.9) ensures a minimum level of expected coverage (1-α) provided. The last constraint set (6.10) is used to keep the relative regrets in each scenario below a specified threshold (γ). The robustness coefficient γ is the maximum allowable relative regret. The relative regret is computed using v_{ω}^{*}, which is an input to the model; the v_{ω}^{*} values have already been computed by solving separate deterministic capacitated p-median problems.

6.4. Case Study and Numerical Results

This section explains the process of applying the proposed model to the specific case study for maritime *SAR* in Atlantic Canada.

6.4.1. Data Preparation

6.4.1.1. Seasonal Analysis of Incidents

The location of demand for maritime incident response is typically uncertain. Although, historical locations can be a good representation of potential future demand points, those deterministic points cannot properly reflect the stochasticity of future demand. Moreover, the demand could change or fluctuate over different time periods. When dealing with strategic decision such as facility locations, it is important to pay attention to changing patterns in demand, potential peak seasons, as well as uncertainty associated with the volume and the location of demand. Therefore, choosing a comprehensive approach to properly simulate the potential future demand is a challenge.

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As the pattern of past incidents is a strong predictor of future, it appears appropriate to extract the underlying distribution of historical incident locations over different time periods as the basis of a stochastic approach for simulating future incidents. There are several methods to fit a distribution for spatial data including quadrat analysis, naive estimation, and kernel density function. In this study, historical incidents are analyzed to extract patterns and distribution by using Kernel estimation (*KE*) which is a popular method for analyzing spatial point patterns. Kernel density estimation searches neighboring areas for calculating the density of occurrence around each specific point. Hence, it has the advantage of not ignoring the potential movement in demand locations over time. Kernel estimation is a proper means of exploring spatial variations in terms of intensity. Such approaches are often used in the identification of clusters and hot spots. This method provides a density estimate for any particular point based on historical occurrences in the vicinity of that point, giving more weight to the closer occurrences. The kernel density method also works well with gridded data, which is the format of our demand projections. Kernel estimation usually uses a geographical weighting scheme (a kernel function) whereby the influence of the points varies inversely to how far they are from the centre of the window (Lloyd 2010). The *KE* of intensity is given by:

$$\hat{\lambda}_k(o) = \sum_{i=1}^{n} \frac{1}{\tau^2} k\left(\frac{o - o_i}{\tau}\right),$$
(6.11)

where τ is the bandwidth (determining the size of the kernel) and $o\text{-}o_i$ indicates the distance between the centre of the kernel (o) and the location o_i (i is an index for data points). There is a variety of different kernel functions that have been used for *KE*. The quartic kernel is encountered frequently in the point pattern analysis literature (Bailey and Gatrell 1995). The *KE* using the quartic kernel can be given by:

$$\hat{\lambda}_k(o) = \sum_{d_i \leq \tau} \frac{3}{\pi \tau^2} \left(1 - \frac{d_i^2}{\tau^2}\right)^2,$$
(6.12)

where d_i is the distance between the centre of the kernel (o) and the location o_i.

The distances between the incidents' locations and potential *SAR* stations are required to perform model calculations. There are different methods for distance calculation. The most common way is calculating straight Euclidean distances. However, in this study it is not

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always possible to use the straight (or the most direct) route because of land obstacles in the way. To deal with this problem, a previously developed land avoidance algorithm has been used to find the shortest route between incidents and vessels by calculating Euclidean distance between grids while avoiding land obstacles. Note also that great circle distances to accommodate the earth's curvature would be more accurate, but for the strategic aim of this study, and over relatively short distances, it can be ignored.

The calculated distances are collected in the matrix D, which includes distances between all grids where incidents may occur (using grid centroids) and potential station locations. This matrix has 1617 rows (gridded demand locations) and 37 columns (vessel stations), where d_{ij} denotes the distance of incident grid i from potential station j. Using a smaller grid size for areas around the shoreline corresponding to the response zone of small lifeboats (with 185 km range) helps in reducing the numerical error in determining the coverage area.

Aggregation of demand points to a limited number of points (centroid of grids in this case) could result in some errors in measuring distances and with a consequent impact on objective functions. Francis et al. (2009) explained different types of errors due to demand aggregation and discussed their impact on accuracy of model solutions. The scale of error depends highly on the level of aggregation and the approximation method. On the other hand, using all of the individual original demand points would increase the size of the problem and computation time and this could be an issue especially for large size problems. Moreover, predicting exact demand points is not possible due to stochasticity and spatial continuity of demand.

Due to some operational limitations and rules, relocation of *SAR* vessels is only possible over two operation seasons in our study area: October-April (season 1) and May-September (season 2). For many reasons related to origins of demand (incident occurrence), the second season is an expected peak season. Figure 6-1 demonstrates a huge jump in number of incidents that occurs during Spring-Summer as opposed to Fall-Winter. A similar trend is observed over different years.
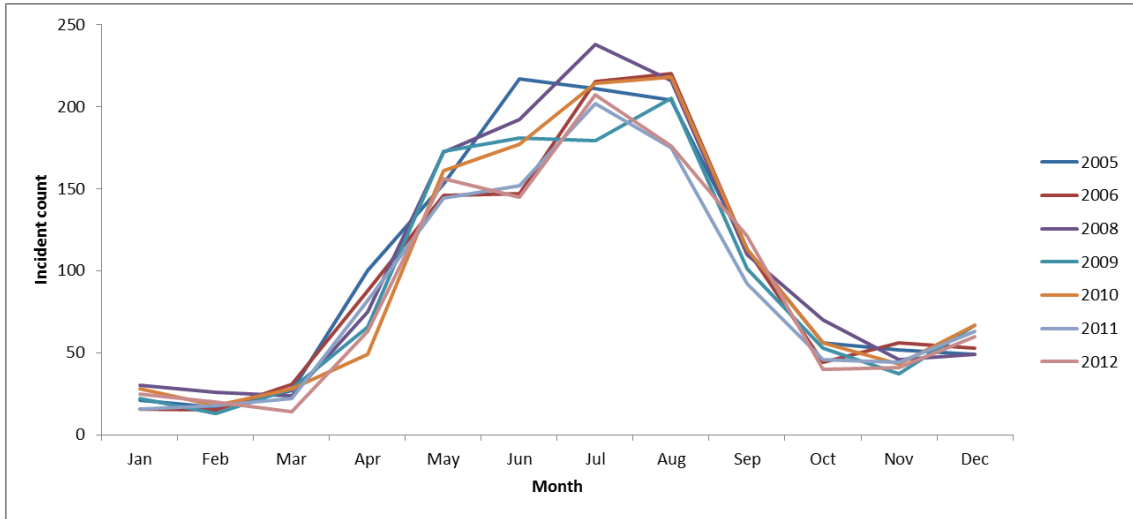
Figure 6-1- Historical incident count by month for multiple years (2007 is omitted due to data deficiencies)

For applying the kernel estimation over demand seasons, we need to determine several parameter values including the type of kernel function, cell size and bandwidth. The parameter values are listed below.

- **Kernel type:** Quartic

- **Cell size:** (0.25 × 0.25) degree; the centre of each grid is used for kernel density calculation.

- **Bandwidth (radius):** variable size between (0.25-1.0 degrees), 0.25 degree for areas close to the shoreline with high density of incidents, 0.5 degree for areas further from shore and low incident density, and 1.0 degree for areas further offshore with very low number of incidents in the vicinity.

Figure 6-2 and Figure 6-3 visualize the kernel density estimates for the two different operational seasons: Fall-Winter (season 1) and Spring-Summer (season 2). As it can be observed in these figures, the incident distribution pattern varies substantially over the two seasons, and the corresponding incidents totals are quite different.

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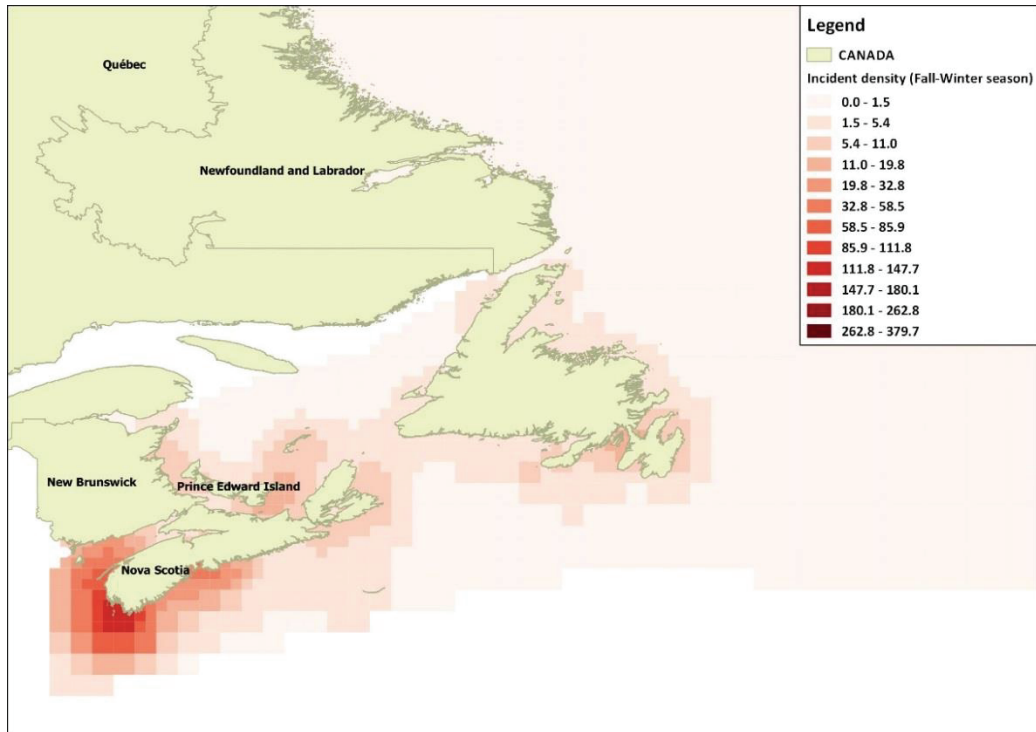
Figure 6-2- Kernel density estimation for season 1: October- April
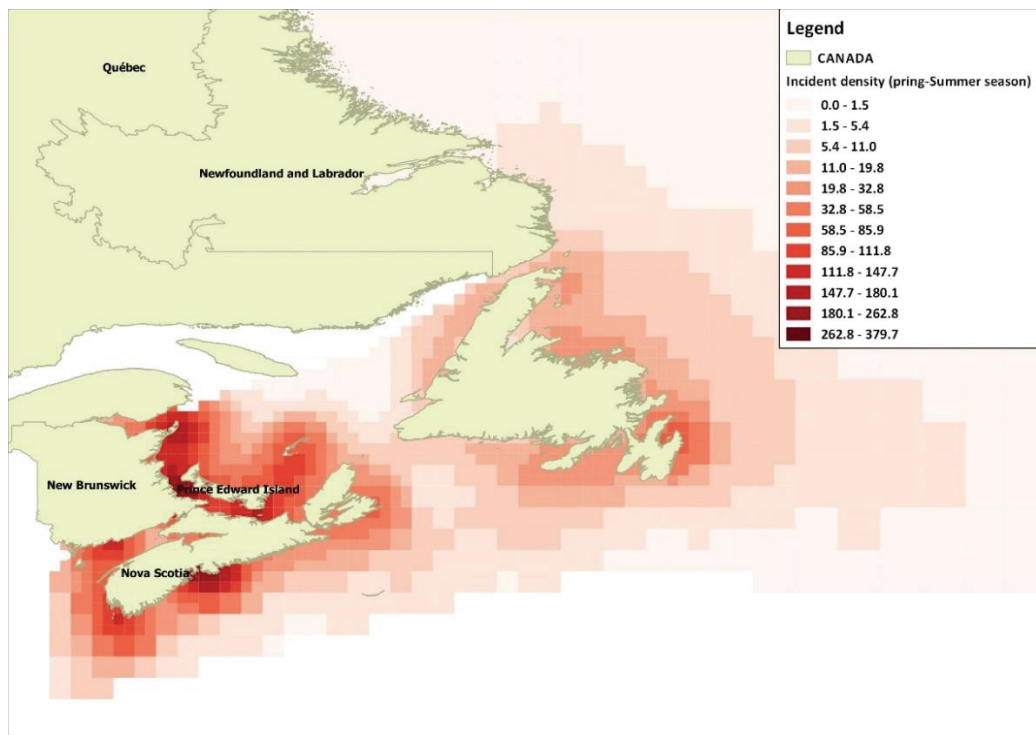
Figure 6-3-Kernel density estimation for season 2: May- September

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6.4.1.2. Vessel Classes and Characteristics

This study considers different type of *SAR* vessels which are operated by the *CCG*. There are 24 vessels performing *SAR* response activities in Atlantic Canada. These vessels include lifeboats, multi-tasking ships and offshore patrol vessels. All existing Coast Guard vessels which are able to provide *SAR* services are categorized into four groups in order to simplify the modelling and reduce calculations. The vessel classes with their specifications are shown in Table 6-1.

Table 6-1- SAR Vessel classes with characteristics

Vessel Class	Vessel type	Range (Km)	Vessel Length (m)	Cruising Speed (Km/hr)	Number available
Regular Lifeboat class	*Type 1*	185	16	26	9
Fast Lifeboat class	*Type 2*	185	15	41	7
Offshore Patrol vessel	*Type 3*	10000	40-50	31	4
Large multi-task vessel	*Type 4*	6000	60-90	22	4

Different types of *SAR* vessels have different travel times because of their different cruising speeds. Also, they have distinct maximum travel ranges as well. This requires them to be treated separately in the mathematical model as we are dealing with travel time rather than travel distance.

6.4.1.3. SAR Stations

Currently, there are 18 inshore *SAR* stations in Atlantic Canada which are able to house *SAR* vessels. It is assumed that all stations are able to accommodate all vessel types and no restriction is applied in this regard. Some vessels are capable of being positioned offshore for long periods of time. To take advantage of this capability, 19 potential and currently in-use offshore stations are to be considered in our analyses. Of course, this is not a station in the traditional sense, but a central location for a vessel that spends much of its time patrolling or performing other tasks at sea. So, we will have 18 inshore stations and 19 potential offshore stations. One assumption in the model, represented as a constraint, is that

small *CCG* vessels, called lifeboats, cannot be located at offshore stations because their maximum traveling range and endurance are not sufficient for long patrol tasks.

6.4.1.4. Capacity Calculation

For each class of *SAR* vessel, a maximum capacity in terms of the number of incidents that can be responded to during different time periods of the planning horizon must be determined for consideration as a constraint in the optimization model. This is necessary to control the workload of resources when planning, especially given the significant increase in demand over peak season. Also, it should be noted that according to the historical data, the occurrence of incidents is usually such that it does not result in congestion for the response. Therefore, our modelling approach is based on the assumption that the congestion issue is rarely expected and are we are not concerned about the wait time for the response due to servers being busy with another task.

To calculate the capacity for each vessel type, several factors must be considered:

- Vessel unavailability due to maintenance: *SAR* vessels are generally unavailable in some period of service due to planned and unplanned maintenance, which affects their actual operational capacity.

- Vessel unavailability due to multitasking: Some vessels that perform *SAR* tasks are designed and used for multiple mandates. So, in reality they are not fully allocated to the *SAR* program. This should be considered in the capacity planning of resources.

- Average response time to incidents: the average number of incidents that can be responded to by a particular vessel in a given time period based on the historical observations is important in order to calculate the maximum number they can respond over planning horizon.

- Vessel speed: vessels have different speeds, which has impact on the duration of response (i.e. transit) to incidents and thus on the capacity of vessel.

The following equation is defined to take into account different factors to calculate the maximum seasonal response capacity $c_{k\theta}$ of each vessel type:

$$c_{k\theta} = (1 - mur_{k\theta}) * ar_{k\theta} * sar_k * air * nd,$$ (6.13)

where:

> *mur_{k\theta}*: Maintenance (planned and unplanned) unavailability rate of vessel type k in season θ
>
> *ar_{k\theta}*: Availability rate of vessel type k for *SAR* tasks in season θ
>
> *sar_k*: Speed adjustment rate of vessel type k relative to the fastest vessel type
>
> *air*: Average number of incidents responded to by a vessel per day (1/day)
>
> *nd*: Number of days in a season (182-183 days)

Table 6-2- Seasonal vessel capacity (number of incidents) calculation

Vessel Class	Season 1				Season 2			
	mur_{k\theta}	*ar_{k\theta}*	*sar_k*	*capacity*	*mur_{k\theta}*	*ar_{k\theta}*	*sar_k*	*capacity*
Vessel Type 1	0.27	1	0.88	118	0.03	1	0.88	155
Vessel Type 2	0.27	1	1	134	0.03	1	1	177
Vessel Type 3	0.324	0.55	0.81	56	0.036	0.55	0.81	79
Vessel Type 4	0.414	0.33	0.77	27	0.046	0.33	0.77	44

All these factors and their associated values were defined with consultation and information provided by *CCG* experts. In order to deal with the fact that we have a seasonal peak in the number of incidents during spring and summer and to provide greater capacity level, maintenance unavailability rates are adjusted so that we have higher availability for vessels during peak season. In other words, we push the planned maintenance to happen during off-peak season to release more capacity for peak season, which also reflects the reality of their operations.

6.4.1.5. Simulation and Projection of Future Incidents

The kernel density estimates are the basis for generating random incidents for future scenarios. For each grid square, the average of kernel estimates within the grid square in each season is calculated. These incident density rates are multiplied by the grid area (to account for grids with variable area due to different sizes) to compute the incident count estimate for each grid/season. These calculated grid incident counts are scaled so that they

sum up to the average number of historical incidents in a season (this is calculated based on seven years' historical data; see Figure 6-1). These scaled values are considered as the mean parameter of a Poisson distribution for generating a random number of incidents over the mesh of grids in the study area. Ten sets of random incident counts per grid square are generated based on calculated Poisson rates. These randomly generated scenarios are used to represent the stochastic demand in the proposed model. Probability of occurrence for each simulated demand scenario is considered to be equal to 0.1 for each of the 10 scenarios as they were all generated from the same distribution.

6.4.1.6. Coverage Time Limit

The maximum access time for an acceptable level of coverage can vary based on the predefined service level standards or expert opinion about actual operations. In this study, the default value for coverage range time limit is considered to be 6 hours based on consultation with *CCG* experts. It should be noted that, this constraint is only applied to the coverage calculation in the model, but it has no impact on the allocation process of the model where there is no limit on access time and all incidents are to be allocated to the closest available resource in order to calculate mean access time.

6.4.2. Solving the Model: Different Configurations

The proposed model was built in the MPL environment and solved using the Gurobi 6.0.4 solver. The model features 446,858 variables and 449,253 constraints. The exact optimal solutions were found in a reasonable amount of time, usually between 2-10 minutes depending on the parameter values using a computer with Intel Core i7 CPU and 8GB RAM. In the following sections, the results generated for different model configurations are presented and discussed. Table 6-3 shows the model solutions for different parameter values. In one of the desirable configurations, the model solution yields 2.4845 hours mean access time and 0.9421 coverage with 10 seasonal relocations, and the maximum relative regret across demand scenarios is only 0.029. Increasing the minimum required coverage to 0.945 results to worsening the access time to 2.4885 with greater regret (0.04), and demanding an even higher coverage level makes the problem infeasible. The maximum

relative regret of solution can be improved to 0.025, but that requires reduction of coverage as well as increase in access time, which are not desirable.

Table 6-3- Model results- different configurations (without relocation limit)

Max regret γ	Min Coverage 1-α	Weighted access time (hrs)	Coverage	# of relocations	Max regret γ
0.05	0.90	2.4845	0.9201	10	0.029
0.05	**0.94**	**2.4845**	**0.9421**	**10**	**0.029**
0.05	0.945	2.4885	0.9452	11	0.040
0.05	0.95	infeasible			
0.05	**0.94**	**2.4845**	**0.9421**	**10**	**0.029**
0.025	0.90	2.4950	0.9214	10	0.025
0.02	0.90	infeasible			

Table 6-4 shows the objective function values across different demand scenarios obtained by the integrated model compared to the optimal solution for each individual scenario, followed by the relative regret in percentage. The results reveal that model solution yields relative regrets between 0.5% and 2.9% and its average is about 1.6% which is desirable.

Table 6-4- Model results: demand scenarios

Demand scenario	Optimal individual scenario objective	Optimal integrated model objective	Regret (%)
Scenario 1	2.4061	2.4332	1.1%
Scenario 2	2.5287	2.5401	0.5%
Scenario 3	2.3421	2.3590	0.7%
Scenario 4	2.3544	2.4052	2.2%
Scenario 5	2.4485	2.5006	2.1%
Scenario 6	2.5274	2.6012	2.9%
Scenario 7	2.4715	2.5106	1.6%
Scenario 8	2.5889	2.6235	1.3%
Scenario 9	2.4399	2.4793	1.6%
Scenario 10	2.3566	2.3923	1.5%

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In order to be able to compare the performance of the model solution with the current arrangement of vessels and to measure the potential improvement, we need to calculate the access time to incidents given the current siting of vessels. To do so, we fix the location of vessels where they currently are and then compute the mean access time over the different simulated demand scenarios to be comparable with the model results. The results, shown in Table 6-5, indicate a substantial improvement in access time to incidents using our model solution compared to the current arrangement of *SAR* vessels. We anticipate to see about 21% improvement in mean access time to incidents as well as 5% increase in coverage by implementing the solution suggested by our model.

Table 6-5- Model solution compared to the current arrangement

Resource Arrangement	Decision criteria	
	Mean Access Time (hrs)	Coverage
Current Arrangement	3.1424	0.8934
γ-robust *p*-median model	2.4845	0.9421

6.4.3. Sensitivity Analysis

It is generally instructive to examine the sensitivity of model solutions to possible changes in parameter values. In our case, there are several parameters that might change due to managers' decisions, policy changes or other consequential situations. For example, it is not unexpected to have new vessels joining the fleet or some old vessels being decommissioned. Hence, it is necessary to investigate the impact of these possibilities in the performance of our model. Also, these investigations can provide insightful information for decision makers.

First, we examine the sensitivity of model solutions to possible changes in available resources by altering the maximum number of vessels in each class (+1, -1) and observing the changes in the objective function value. In Table 6-6, the results show that the objective function is sensitive to changes in *class 2* and *class 4* vessels more than to the *class 1* and *class 3*. Also, *class 1* seems to be the least important class in affecting the objective

function. Decision makers can easily see which type of vessels can be a better investment for improving the service level.

Table 6-6- Sensitivity analysis on number of vessels

Vessel class	Class 1		Class 2		Class 3		Class 4	
Change	+1	-1	+1	-1	+1	-1	+1	-1
Obj. value	2.478	2.499	2.437	2.547	2.460	2.521	2.429	2.562
% dif. vs. optimal	-0.2%	0.6%	-1.9%	2.5%	-1.0%	1.5%	-2.2%	3.1%

The number of seasonal relocations is another important factor for which the impact of variations on the solution quality should be examined. Essentially, fewer relocations are better as vessel relocation involves several operational issues such as crew relocation as well as associated costs. In the original model, there is no limit on number of relocations, although it is more realistic to add such constraint not only to observe the sensitivity of model results to restricting the number of relocations, but also to consider the cost/benefit aspect. This way, it would be possible to find a tradeoff between number of relocations and the objective function value.

Table 6-7 presents the results of this analysis, altering the maximum number of relocations allowed. As can be seen, the mean access time slightly increases as we gradually restrict the number of relocations. Also, for this test we set the minimum coverage level at 94% and the maximum regret at 0.05. The results indicate a substantial improvement in access time when allowing relocation vs. the static configuration (i.e. no relocation permitted). Also, the solution for a configuration with 6 relocations seems to be a good tradeoff between the access time and number of relocations as allowing more relocations does not add much value to the objective function value and restricting it further results in a significant negative impact on the objective function value.

Table 6-7- Sensitivity analysis on relocation limit

1-α	γ	Relocation limit	Access time	Actual coverage	# of relocations
0.94	0.05	no limit	2.4845	0.9421	10
0.94	0.05	9	2.4862	0.94	9
0.94	0.05	8	2.4929	0.943	7
0.94	0.05	7	2.4989	0.943	7
0.94	**0.05**	**6**	**2.5030**	**0.9427**	**6**
0.94	0.05	5	2.5181	0.9413	5
0.94	0.05	4	2.5305	0.9418	4
0.94	0.05	3	2.5516	0.9436	3
0.94	0.05	2	2.5764	0.9418	2
0.94	0.05	1	2.5952	0.9422	1
0.94	0.05	0	2.6148	0.9437	0

6.4.4. Discussion of Our Results

Running the proposed model for our case study show that the solution performance meets the main goals for developing this model. First, our model aimed to improve the access time to potential incidents by adding an option to relocate vessels seasonally to more efficiently respond, given significant pattern changes in the demand. Comparing the multi-period model (dynamic model) solution with its static version solution confirms that substantial potential benefit can be obtained by permitting seasonal relocation. The multi-period model can also improve the mean access time from 2.615 in the static model down to 2.485 hours. Also, the solution ensures a certain level of coverage within a pre-specified amount of time (over 94%).

Moreover, the solution indicates that model has properly dealt with the robustness issue over possible variations in demand. The solution provided by our robust model can keep the maximum relative regret across all simulated demand scenarios under 0.03, which is a very reasonable level. Comparing the solution performance with the current arrangement of the *CCG SAR* fleet, shows a tremendous potential improvement both in access time and coverage. Sensitivity analyses examined the impact of changing parameters value on the

model solution. This was performed specifically for possible changes in fleet mix (altering in number of vessels in different classes). The more valuable vessel types are identified and the level of their impact is observed. These results provide the *CCG* with valuable insights on making more informed fleet renewal and procurement decisions.

6.5. Conclusion & Outlook

In this study, a multi-period robust *p*-median problem is applied to the maritime *SAR* location problem. The model considers seasonal relocation of *SAR* vessels. For this model, four common *SAR* vessel types which are used in practice and have different capabilities are considered. Future demands in two operational seasons with different demand patterns are simulated over the study area and timeframe based on incident occurrence estimates extracted from historical incidents using a kernel density estimation. Constraints on the relative regret in all demand scenarios assures a certain level of robustness over possible variations in demand. Several model runs with different parameter configuration are generated and solved. The results show the desirable performance of the model compared to the static version of the model. The proposed model provides solutions with notably better performance versus the current *SAR* vessel arrangement. Sensitivity analyses were performed on factors such as the number of vessels in different classes and the number of relocations allowed, to examine their impact on the model solution and to allow for cost/benefit tradeoffs.

The results of this study could be useful for guiding decisions with regard to *SAR* vessel arrangement, acquisitions and placement in order to improve the efficiency of resources and increase the service level, particularly in response to pattern changes in demand during different operational seasons. More specifically, the outcome of this study could provide the *CCG* with some useful insight for future resource capacity planning, including fleet recruitment planning and appropriate stations for placing new vessels. Also, it can be helpful for managing current operations to increase the resource utilization and effectiveness of their services. Several tactical and operational rules can be extracted from the model solution for best resource allocation policies.

The model proposed in this study can be further extended in several ways. We are aware of different types of incidents with different severity and requirements for different types of response, as all *SAR* vessels are not equally effective at responding to different types of incidents. Therefore, it would be more realistic if we can differentiate incident types in the model and take into account their specific response requirements. Moreover, this would provide an opportunity to add the effectiveness of response as an objective in the model. Such analysis would require more detailed information about the capabilities of resources which was not available at the time of this study. The demand simulation methodology could be further extended to future response needs by incorporating trends in incident numbers, incident rates, and/or traffic levels. It would be beneficial to add cost of relocations as a constraint in the model, and perhaps remove the limit on number of relocations. Our model assumes that the number of vessels in each class is fixed, although some sensitivity analyses were conducted to address that. But, for a more strategic level model, it is better to allow changes in fleet configuration as part of the optimization. Also, the model can be extended to incorporate more real-time decisions on vessel deployments in order to add the possibility for priority scheduling when we have incidents with different distress levels. An interesting additional feature would be to allow disruption of a response to a non-distress incident in favor of faster reaction to a distress one.

Chapter 7   A Comprehensive Strategic Model for Maritime Search and Rescue Fleet Planning

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Abstract

This study presents a comprehensive multi-objective model for decisions on the composition and Location-Allocation of maritime Search and Rescue vessels, with constraints on the minimum required primary and backup coverage levels. The model provides trade-offs between the total fleet cost and the mean access time to incidents. Various types of Search and Rescue vessels with different characteristics and operational capacity are considered. Possible variations in the distribution of future incidents are addressed by using spatial simulation. Vessels can be relocated according to predetermined seasons to more effectively respond to corresponding changes in demand patterns. The results provide a range of good trade-off solutions which can substantially improve the service level obtained compared to the current fleet composition and arrangement of Search and Rescue resources with respect to several decision criteria.

Keywords: Strategic Location analysis; Maritime Search and Rescue; Cost; Access time; Relocation; Scenario planning

7.1. Introduction

Facility location decisions are a critical element in strategic planning for a wide range of private and public firms. Emergency response planning is one of the interesting application areas for location modelling where strategic decisions greatly affect society and the environment. According to many published reports, ocean activities such as fishing or marine transportation are among the most dangerous activities in the world. Since, Canada is surrounded by navigable waters, the country is subject to many risky marine operations. Thus, maritime search and rescue is one of the most needed rescue activities in Canada. As the Canadian Forces (1998) pointed out: "Search and Rescue comprises the search for, and the provision of aid to, persons, ships or other craft which are, or are feared to be, in distress or imminent danger."

Since time is of the essence in Search and Rescue (*SAR*) operations, it is paramount that the resources, including *SAR* vessels, stations, and crew, are used efficiently and effectively. To do this, several strategic decisions have to be made in order to efficiently allocate limited resources to obtain the best service level possible. Such decisions include determining the optimal mix of vessels with different characteristics and capability, deciding the location of vessels, and optimally allocating them to the demand. The Canadian Coast Guard (*CCG*) has a variety of *SAR* vessel types that were designed or purchased with specific tasks in mind, and not all are equally effective at handling distinct incident types. Also, the ranges and speeds vary greatly among different types of *SAR* vessels. Therefore, response vessel characteristics are important factors that need to be considered in any study on this matter.

Enhancing the capacity to respond to marine incidents and renewing assets are among current strategic business priorities of the *CCG* according to their documented strategic plans (Canadian Coast Guard 2014). They are aware of the tremendous importance of enhancing the decision-making process in resource allocation matters. The development and acquisition of a new facility is typically a costly, time-sensitive project. Thus, facilities which are procured and located today are usually expected to remain in operation for an extended period of time. Several factors such as transportation and safety policy changes

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and new transit corridors might affect the demand over short- to long-term time horizon. On the other hand, changes in the environment and/or activity such as weather conditions and traffic patterns during the facility's lifetime can drastically alter the appeal of a particular site, and the effectiveness of a particular response vessel. Determining the best composition of the fleet and the vessel locations is thus an important strategic challenge, demanding that decision makers account for current requirements and uncertain future events. The resulting models can be extremely difficult to solve to optimality (most problems are classified as NP-hard).

In a maritime *SAR* operation, the difference between life and death can sometimes be measured in minutes. Additionally, a *SAR* operation consumes considerable resources in terms of time, effort and money (Razi and Karatas 2016). Maximizing the coverage as well as minimizing the access time and the cost are common objectives in emergency response location studies. There is an extensive literature on emergency location analysis. Most studies have attempted to determine the location of *SAR* vessels to maximize the coverage range given the historical incident locations. There is gap in taking a more comprehensive approach studying the strategic level decisions taking into account a broad range of parameters and factors affecting the performance of the emergency response system in the long term. Various types of response vessels with different capabilities, the possibility of vessel relocation, and uncertainty associated with the demand locations are among important factors to be considered.

This study aims at developing a comprehensive optimization model, incorporating different important decision criteria and factors for long-term fleet planning of maritime *SAR* resources. We accomplish that by employing a scenario planning approach for representing the uncertainty of future demand in order to ensure that the model solution would perform well over variations in the demand. The operational characteristics and capacity of various vessel types is taken into account. In addition, our model allows for seasonal relocation of vessels with the aim of effectively responding to significant seasonal changes in incident locations. The ultimate goal of the study is to propose a decision support model that is capable of providing helpful insights for decision makers with regard to development of long-term strategic level plans and policies on managing the limited

resources to increase the overall quality of maritime *SAR* service through improving the resource utilization.

The remainder of this paper is organized as follows. Section 7.1.1 presents a critical review of relevant literature followed by the description of the problem in section 7.2. Section 7.3 explains the proposed methodology of the study including the optimization model. In section 7.4, the process of applying the model to our case study along with the numerical results are presented, and continues with a discussion of the results. We conclude the paper with the summary of the findings and outlook for future research in the field.

7.1.1. *Related Work*

A vast literature has developed out on tackling the challenge concerned with strategic location decisions. A number of mathematical programming models were developed by researchers to represent a wide range of location problems which might have built with different objectives in mind and for various applications. Owen and Daskin (1998) reviewed various aspects and different approaches to tackle strategic location problems.

Facility location problems often involve strategic decisions that must hold for quite long time. During this time horizon, changes may occur in the underlying conditions such as demand volume and distribution. Thus, many studies attempted to address the uncertainty associated with different factors in location analysis. The most investigated source of uncertainty that affects the location modelling is the stochasticity of future demand locations.

There are two common approaches for optimization under an uncertain environment: Stochastic Programming (*SP*) and Robust Optimization (*RO*). Moreover, there are two ways for representing the uncertain parameters: first by using discrete scenarios with certain values for each parameter and each scenario has a given probability of occurrence, while in the second case uncertain parameters are represented with probability distributions. In robust optimization, a set of possible future values are taken into account and typically the objective is to minimize the worst-case scenario objective, while in stochastic programming, a common objective is to minimize expected objective value. A "scenario" is a complete realization of all the uncertain parameters. Each scenario fully

determines the value of all the uncertain parameters. Depending on the problem, there may be a finite or infinite number of scenarios (Correia and Saldanha da Gama 2015).

Sensitivity analysis attempts to quantify the effect of a change in parameter values on the optimal objective function value, while both stochastic programming and scenario planning approaches move away from reactive analyses of solution sensitivity toward models which formalize the complexity and uncertainty inherent in real-world problem instances (Owen and Daskin 1998).

Snyder (2006) presented a comprehensive review on stochastic and robust facility location models and the variety of approaches for optimization under uncertainty. Snyder points out that in risk situations there are uncertain parameters whose values are governed by probability distributions that are known by the decision maker. In uncertainty situations, parameters are uncertain, and furthermore, no information about their probabilities is known. Problems in risk situations may be modeled as stochastic optimization problems; a common goal is to optimize the expected value of some objective function. Problems under uncertainty are known as robust optimization problems and often attempt to optimize the worst-case value of the objective function.

Generally, in such problems, the objective is to determine robust facility locations which will perform well (according to the defined criteria) under a number of possible parameter realizations. Probabilistic models explicitly consider the probability distributions of the modelled random variables, while scenario planning models consider a generated set of possible future variable values. In robust optimization, it is assumed that no information about probability distributions is available except limited data on the specification of intervals containing the uncertain values.

For a comprehensive review of different modelling approaches to tackle uncertainty in facility location the readers are referred to Correia and Saldanha da Gama (2015).

Decision makers must not only select robust locations which will effectively serve changing demands over time, but must also consider the timing of facility relocations over the long term. Facility location problems can be divided into two categories with respect to timing of location decisions: static and dynamic problems. Facility locations are decided

once in the static category, while in the dynamic problems location decisions are time dependent (i.e. relocation of facilities is permitted). Relocation of facilities can occur in a discrete or continuous manner. In the former category, relocation is only possible at discrete pre-determined points of time (Wesolowsky 1973); while in the latter case, relocation is possible at any time during the planning horizon (Drezner and Wesolowsky 1991). Wesolowsky (1973), Wesolowsky and Truscott (1975), and Sweeney and Tatham (1976) are among the first researchers who dealt with the multi-period Location-Allocation problem. Arabani and Farahani (2012) reviewed different approaches for considering dynamicity in demand, parameters and/or factors in facility location problems.

Seyedhosseini et al. (2016) presented the most recent review on studies dealing with dynamic location problems. They classified problems in this area with respect to modelling methodology, objective function type, solution approach, and application area.

7.1.1.1. Search and Rescue Location Analysis

To the best of our knowledge, there is only a few studies on the application of location modelling to *SAR* problems, so that a number of aspects of the problem in the field can be the subject of further research and development. Brown et al. (1996) developed a mixed integer programming model for scheduling U.S. Coast Guard district cutters, whereby its solution was superior compared to manually prepared schedules. (Nguyen and Kevin 2000) combined two objectives used in the maximal covering location problem and the *p*-median problem using a goal programming approach to assess the level of service of the existing Canadian *SAR* system (in terms of location of *SAR* aircraft and helicopters) and compared it to the optimal solution of their model. Azofra et al. (2007) built a tool for assignment of sea rescue resources using a gravitational model based on either individual incidents or a zonal distribution representation to compute an appropriateness coefficient for each possible assignment of resources to locations. Their model does not attempt to optimize the allocation but to generate a metric for assessment of different solutions.

Afshartous et al. (2009) studied the problem of locating Coast Guard air stations and developed a statistical-optimization model to come up with a robust solution in the presence of uncertainty in distress call locations. Distress calls are simulated and the

optimization problem is solved for different simulations individually and the solutions are assessed for their similarity. Radovilsky and Koermer (2007) applied integer linear programming for the optimal allocation of rescue boats among the stations of the U.S. Coast Guard. Their model minimizes shortages or excess capacities at the stations. Wagner and Radovilsky (2012) extended the previous study by developing a new model named *BAT* that simultaneously considers reduction of excess capacity and boat shortages at the stations, a decrease in the overall fleet size with an increase in boat utilization, and overall reduction of the fleet operating cost. They presented a stochastic and robust version of their model where they used the value-at-risk concept. Nelson et al. (2014) conducted a study for maximizing the aircraft fleet operational performance for *USCG*. An optimization model was developed for determining the optimal deployment assignments, operational levels and aircraft allocation among all *USCG* Air Stations. Their model attempts to minimize the fleet operational costs subject to performance targets.

Pelot et al. (2015) examined three covering problems in emergency response modelling including maximal covering location problem, maximal expected covering location problem, and maximal covering location problem with workload capacity, to the maritime *SAR* location problem for Atlantic Canada. This study relies on historical incidents and differentiates between different distress level incidents. Akbari et al. (2016) also presented a multi-criteria analysis on the performance of solutions obtained by two popular location models, *p*-median and maximal covering, to the case of a maritime *SAR* location problem. That study considers primary and backup coverage, mean access time, the Gini index to reflect the service equality level across customers, and maximum access time as post-assessment criteria for solutions of two single objective models.

Razi et al. (2016) examined the allocation of helicopters to *SAR* missions using an optimization model with the objective of minimizing average response time to incidents. Then, a discrete event simulation was used to validate the performance of the optimization solution for the stochastic demand. Razi and Karatas (2016) proposed a multi-objective model for allocation of *SAR* boats. Their modelling approach first ranks different types of incidents according to their severity using the *Analytical Hierarchy process* and then applies a zonal distribution model for aggregating the incident locations. Finally, the

mathematical model determines the optimal allocation of boats with the objectives of minimizing response time to incidents, fleet operating cost and the mismatch between boats' workload and operation capacity hours. Their model does not account for the uncertainty involved with the incident locations though. They also defined several other metrics for assessment of the solutions.

None of the studies on Location-Allocation of *SAR* resources considers the possibility of relocating vessels in a periodic manner to tackle the dynamics of the demand. There are a lot of studies done on location-relocation models in emergency response but mostly for the case of ambulances. However, the context of vessel relocation in maritime *SAR* is quite different from what is in other emergency response activities such as ambulances. The relocation of *SAR* vessels is not as easy as the relocation of ambulances and it cannot happen very frequently (dynamically). The number of maritime incidents is usually not as high as medical emergency calls, so the chance of congestion is lower. Therefore, the *SAR* vessels do not require to be relocated very often. Also, some of the *SAR* vessels are large ships for which the relocation is more challenging. Hence, this issue should be tackled with respect to special characteristics of the problem. Furthermore, most of the models proposed for *SAR* resource allocation have relied on deterministic demand based on historical data which is a weak assumption in such a problem environment that deals with a lot of uncertain factors affecting the demand. In some cases, future demand is simulated to validate the model solution performance, but not then used as an input to the optimization model. This study aims to address these limitations and propose a comprehensive model for fleet Location-Allocation planning in maritime *SAR*.

7.2. Problem Description

This section provides information regarding the characteristics of our problem including data sources, modelling parameters and factors.

7.2.1. Historical Incidents

This study uses real and valid data on the demand. The dataset used in this study derives from the *CCG SISAR* (Search and Rescue Information Management System) database which collects information on all maritime incidents. The Atlantic Canada region serves as

our area of interest, with the Coast Guard's administrative borders illustrated in Figure 7-1. The incident dataset, which has been checked and cleaned for quality control, is available from 1988 to 2013, but to have a more accurate analysis, we chose the most reliable recent data from 2005 to 2012 (excluding 2007 which has significant deficiencies due to a system switchover) for this study. The resulting dataset contains 8,033 incident records, which is about 1,148 incidents per year.
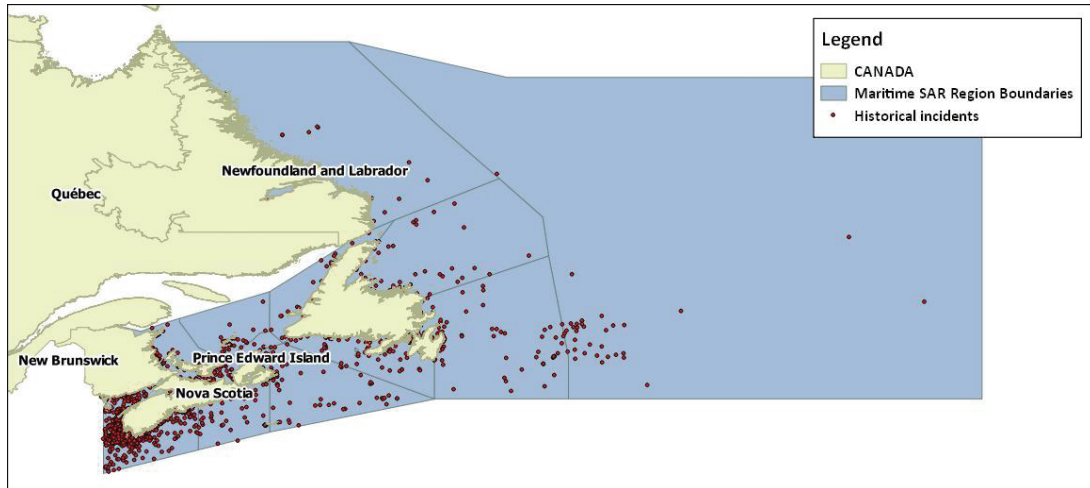
Figure 7-1- Historical marine incidents in Atlantic Canada (2005,2006,2008-2012)

7.2.2. *Vessel Classes and Characteristics*

The *CCG* operates various vessel types that were designed with specific tasks in mind for their different missions where many are primarily designed for *SAR* and others are well-suited for multi-tasking. As of December 2016, there are 24 vessels performing *SAR* response activities in Atlantic Canada, including lifeboats, multi-tasking ships and offshore patrol vessels. Moreover, the *CCG* is in the process of renewing and modernizing its fleet. They are going to replace some of the old vessels with updated versions as well as expand the fleet capacity with possible new procurements according to their current strategic plan and additional budget allocations. We would like to incorporate these new additions into the *SAR* fleet as the study aims to provide support for the strategic decisions. Existing Coast Guard *SAR* vessels included in this study can be categorized into four groups in order to simplify the modelling and reduce calculations. One new vessel class is considered in our analysis representing the new modern lifeboats to examine their impact on fleet

performance. These vessels are relatively faster than the current lifeboats in use. The vessel classes with their specifications are shown in Table 7-1.

Table 7-1- SAR Vessel classes with characteristics

Vessel Class	Vessel type	Range (Km)	Vessel Length (m)	Cruising Speed (Km/hr)	Number available
Regular Lifeboat (Arun-class)	*Type 1*	185	16	26	9
Fast Lifeboat (Cape-class)	*Type 2*	185	15	41	7
Offshore Patrol vessel	*Type 3*	10000	40-50	31	4
Large multi-task vessel	*Type 4*	6000	60-90	22	4
New lifeboat (K-class)	*Type 5*	185	15	43.5	-

As shown in Table 1, various types of *SAR* vessels have different travel times to get to a given incident because of their different cruising speeds. Also, they have different maximum travel ranges as well. These differences require them to be treated separately in the mathematical model as we are dealing with minimizing the travel time rather than the travel distance.

7.2.3. SAR Stations

Currently, there are 18 inshore *SAR* stations in Atlantic Canada which can site *SAR* vessels. It is assumed that all stations are able to accommodate all vessel types and no restrictions are applied in this regard. Some vessels are capable of being positioned offshore for long periods of time due to their long endurance and range. To leverage this potential for providing additional coverage to offshore areas and faster response, 19 offshore stations are to be considered in our analyses. Of course, this is not a station in the traditional sense, but a central location for a vessel that spends much of its time patrolling or performing other tasks at sea. Some of these virtual stations are currently considered in operation and others have been added after consulting with *CCG* experts to allow more flexibility in the analysis. Therefore, we consider 18 inshore stations and 19 potential offshore stations. An operational constraint which is considered in the model is that small *CCG* vessels, called lifeboats, cannot be located at offshore stations because their maximum traveling range and endurance are not sufficient for long patrol tasks.

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7.2.4. Operation Planning Seasons

Maritime incidents have a substantial seasonal nature due to their various causes and contexts. For example, incidents related to recreational activities mostly occur during spring and summer while the pattern is different for fishing related incidents. Also, according to historical observations, each year the total number of incidents peaks during the summer (with the highest rate in July and August). In order to be able to effectively plan the allocation of resources and also considering other operational matters, it is useful to define two operational seasons. It should be noted that this is not an unrealistic assumption as the *CCG* currently has defined operational seasons although the exact definitions might be slightly different than what we use in this study. The peak season usually starts in April with a slight jump in number of incidents and continues to increase toward summer and ends in September. Hence, we define two operation planning seasons: (1) Fall-Winter (October-March) and (2) Spring-Summer (April-September). In addition, relocation of *SAR* vessels is only assumed to be possible at the beginning of each operation season.

7.2.5. Vessel Response Capacity

In order to be able to appropriately plan the allocation of resources (vessels) to the demand (incidents), we need to have a measure to capture the actual operational capacity of each vessel type given its characteristics and availability. To do so, we considered determining a maximum capacity in terms of the number of incidents that can be responded to during different seasons for each class of *SAR* vessel. Such a capacity limit must be included in the mathematical model as a constraint. This is necessary to manage the workload of the resources when planning, especially given the significant increase in demand over the peak season. To come up with a reasonable measurement for the maximum capacity of each vessel type, the following equation is defined to take into account different factors to calculate the maximum seasonal response capacity $c_{k\theta}$ of each vessel type. It must be noted that while this capacity provides an upper bound on the service that is possible, there may be significant delays in the case where demand for service is clustered giving rise to the congestion issue.

$$c_{k\theta} = (1 - mur_{k\theta}) * ar_{k\theta} * sar_k * air * nd, \tag{7.1}$$

where:

> $mur_{k\theta}$: Maintenance (planned and unplanned) unavailability rate of vessel type k in season θ
>
> $ar_{k\theta}$: Availability rate of vessel type k for *SAR* tasks in season θ (percent of time)
>
> sar_k: Speed adjustment rate of vessel type k relative to the fastest vessel type (this rate is applied to decrease the capacity of slower vessels relative to their speed compared to the fastest vessels)
>
> air: Average number of incidents responded to by a vessel per day (1/day)
>
> nd: Number of days in a season (182 days)

Equation (7.1) computes the maximum number of incidents that a particular type of vessels can respond to during each operation season with respect to its characteristics and availability for *SAR* tasks.

Table 7-2 presents the actual parameter values and calculated vessel capacities over two operational seasons. All these factors and their associated values are determined with consultation and information provided by *CCG* experts. In order to address the concern about a seasonal peak in the number of incidents during spring and summer and to provide greater capacity level, maintenance unavailability rates are determined in such a way that we yield higher vessel availability during peak season. In other words, we limited all the planned maintenance to the off-peak season to release more capacity for peak season, which also reflects the reality of their operations. The maximum number of incidents that can be responded by different vessel types ranges from 27 to 134 in season 1 and from 44 to 177 in season 2.

Table 7-2- Seasonal vessel capacity (number of incidents) calculation

Vessel Class	Season 1				Season 2			
	$mur_{k\theta}$	$ar_{k\theta}$	sar_k	*capacity*	$mur_{k\theta}$	$ar_{k\theta}$	sar_k	*capacity*
Vessel Type 1	0.27	1	0.88	118	0.03	1	0.88	155
Vessel Type 2	0.27	1	1	134	0.03	1	1	177
Vessel Type 3	0.324	0.55	0.81	56	0.036	0.55	0.81	79
Vessel Type 4	0.414	0.33	0.77	27	0.046	0.33	0.77	44
Vessel Type 5	0.27	1	1	134	0.03	1	1	177

7.2.6. Vessels' Procurement and Operational Cost

One of the main objectives in location modelling and resource allocation analyses is minimizing the total cost including fixed (procurement) and variable (operational) costs. As we are going to plan the fleet Location-Allocation for the long run, it is desired to consider total cost associated with operating *SAR* vessels either by incorporating it as a hard budget constraint or a soft constraint as an objective in the model. Therefore, the actual annual operational and procurement cost of currently operated *SAR* vessels has been supplied by the *CCG*. For confidentiality purposes these costs are represented as scaled rates rather than actual figures which it is acceptable for use our study. The annual operational costs consist of several elements including: crew salary, maintenance, fuel, travel cost, etc. For consistency, the procurement cost of vessel, which is a fixed value, is distributed (annualized) over the operational lifetime of vessels. Table 7-3 presents the scaled cost rates for the existing vessel classes.

Table 7-3- average annual scaled cost rates for each vessel class

Vessel class	Operational lifetime (years)	Procurement cost (annualized)	Crew salary	Fuel	Refit	Maintenance	Other costs	Total cost	Total cost allocated to *SAR*
Type 1	20	0.067	0.548	0.017	0.085	0.004	0.050	0.771	0.771
Type 2	20	0.083	0.428	0.010	0.029	0.003	0.048	0.601	0.601
Type 3	35	0.511	2.343	0.693	0.906	0.020	0.466	4.939	2.716
Type 4	40	1.000	3.267	1.075	1.214	0.035	0.601	7.191	2.373

- The annualized acquisition cost of multitasking vessel is the basis for scaling the costs.

7.3. Methodology

7.3.1. Demand Pattern Analysis

When dealing with strategic decisions such as facility locations, it is important to pay attention to changing patterns in the demand, potential peak seasons, as well as uncertainty associated with the volume and the location of demand. Hence, choosing a comprehensive approach to properly simulate the potential future demand is a challenge. The location of demand for maritime incident response is typically uncertain due to its context and several non-deterministic factors. Historical locations are potentially a good representation of future demand distribution, although those deterministic points cannot properly reflect the stochasticity of future demand.

Since there is not much reliable information to properly predict the future demand distributions, so we must rely on the past distribution of incident locations for analyzing the demand. There are several methods to fit a distribution for spatial data including quadrat analysis, naive estimation, and kernel density function. In this study, historical incidents are analyzed to extract patterns and distribution by using Kernel estimation (*KE*) which is a popular method for analyzing spatial point patterns. Kernel density estimation searches neighboring areas and computes a function of the distances from incidents within search zone to the kernel centre to be used as the density of occurrence around each specific point. Thus, it has the advantage of not ignoring the potential movement in demand locations over time. The kernel density method properly works with gridded data, which is the format of our demand projections. Kernel estimation usually uses a geographical weighting scheme (a kernel function) whereby the influence of the points varies inversely to how far they are from the centre of the window (Lloyd 2010). The *KE* of intensity is given by:

$$\hat{\lambda}_k(o) = \sum_{i=1}^{n} \frac{1}{\tau^2} k\left(\frac{o-o_i}{\tau}\right), \tag{7.2}$$

where τ is the pre-specified bandwidth (determining the size of the kernel) and *o-o$_i$* indicates the distance between the centre of the kernel (*o*) and the location of an incident

o_i (i is an index for data points). There is a variety of different kernel functions that have been used for *KE*. The quartic kernel is encountered frequently in the point pattern analysis literature (Bailey and Gatrell 1995). The *KE* using the quartic kernel is given by:

$$\hat{\lambda}_k(o) = \sum_{d_i \leq \tau} \frac{3}{\pi \tau^2} \left(1 - \frac{d_i^2}{\tau^2}\right)^2,\qquad\qquad(7.3)$$

where d_i is the distance between the centre of the kernel (o) and the location o_i.

7.3.2. Distance Calculation

As one of the inputs required for the model, the matrix of distances between the incidents' locations and potential *SAR* stations must be calculated. There are various methods for distance calculation. The most common way is calculating straight Euclidean distances. However, in this study it is not always possible to use the straight (or the most direct) route because of land obstacles in the way. To cope with this problem, we use a land avoidance algorithm to find the shortest realistic route between incidents and vessels avoiding land obstacles. It worth mentioning that although, great circle distances to accommodate the earth's curvature would be more accurate, but for the strategic aim of this study, and over relatively short distances, it can be ignored.

7.3.3. Optimization Model

We develop a comprehensive model for fleet strategic planning decisions which incorporates several important factors customized for the maritime *SAR* Location-Allocation case. The model considers minimizing (1) the total annualized capital and operating cost of *SAR* vessels and (2) the mean access time to all incidents. The weighted sum method is used for scalarization of multiple objectives. Our model is a multiple period planning model to be consistent with operation seasons and moreover to provide the flexibility of relocating vessels periodically to appropriately respond to changing seasonal patterns in the demand. Multiple randomly generated seasonal incident are used in a scenario planning approach to reflect the uncertainty of the demand.

The model must consider various aspects of the problem including multiple criteria, factors and constraints. Our model considers multiple type of existing and potential *SAR* vessels with different speeds, response capacities and plausible locations. The maximum response

capacity of the vessels has been taken into account by restricting the total number of incidents can be responded to by each vessel during each season. Moreover, since we are dealing with a problem of locating mobile facilities (i.e. *SAR* vessels) with different speeds, travel time is used as opposed to the regular travel distance proxy in the objective function.

We apply the concept of coverage which is very common in emergency response location analysis where it is desired to provide acceptable level of coverage for as many customers (i.e. incidents) as possible. Typically, in emergency response analysis, a customer is called *covered* if it is within a predetermined access time rather than prescribed range because the time is a better proxy for measuring system performance in this case. In addition, congestion in service is a potential issue that should be addressed as we attempt to make strategic long-term decisions. Although it is not a great concern currently, according to expert opinion and that fact that it is not happening frequently, is a chance that it becomes more serious with a possible increase or distribution change in the demand. To cope with the issue, we consider the backup coverage concept in the proposed model in a way that we try to have more than one vessel capable of responding to each incident location (within coverage range).

The model presented in this paper extends the previous model developed in (Akbari et al. 2017) in three ways: first, by relaxing the fixed number of vessels in each class and also including the potential new vessel procurements; second, by including a cost minimization in a multi-objective function; and finally, by considering a minimum level of backup coverage to address the congestion issue. Once decided, the fleet composition is assumed to be fixed (i.e. there is no replacement until the end of the useful life of the vessels).

The mathematical model developed for this problem is a large-scale integer linear programming problem. Indices, parameters and variables used in the model are listed and defined below.

Indices:

$i \in I$:　　　　　Demand locations

$j \in J$:　　　　　Potential vessel stations

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$J_S \subset J:$ Set of offshore stations (virtual stations offshore for patrol vessels and multitasking ships)

$k \in K:$ Index for vessel types

$\omega \in \Omega:$ Index for simulated incident scenarios

$\theta \in \Theta:$ Index for relocation periods

Variables:

$x_{i\theta}$:   Binary variable for primary coverage at grid i in period θ

$y_{i\theta}$:   Binary variable for backup coverage at grid i in period θ

$z_{jk\theta}$:   The number of vessels type k located at station j in period θ

$u_{ijk\theta}$:   Allocation of customers at grid i to vessel type k located at j in period θ

$R1_{jk}, R2_{jk}$:   Binary variables for determining relocations of vessel type k at station j

RC_k: Relocations count of vessels type k

Parameters:

r_k: Coverage distance (range) of vessel type k

p_k: Minimum number of vessel type k required

ac_k: Annual cost of vessel type k

$c_{k\theta}$:   Capacity of vessel type k in period θ (number of incidents that can be responded to)

v_k: Cruising speed of vessel type k

d_{ij}:   Distance between grid i and station j

π_ω: Probability of occurrence of scenario ω

$w_{i\omega\theta}$:   Demand weight at grid i in period θ in scenario ω (number of incidents)

$t:$ Coverage time limit for acceptable level of coverage

α_1:   Maximum uncovered demand (as a percentage of total incidents)

α_2:   Maximum proportion of demand without backup coverage

ρ: Coefficient of the mean access time in the objective function

β: Maximum number of permitted relocations

The formulation of the proposed model is as follows:

Minimize:

$$\Delta = \sum_\omega \pi_\omega \left(\frac{\sum_\theta \sum_k \sum_i \sum_j \left(w_{i\omega\theta} u_{ijk\theta} \left(d_{ij}/v_k \right) \right)}{\sum_\theta \sum_i w_{i\omega\theta}} \right) + \rho \left(\sum_k ac_k \left(\sum_j z_{jk1} \right) \right) \tag{7.4}$$

s.t.

$$x_{i\theta} \leq \sum_j \sum_k a_{ijk} z_{jk\theta}, \qquad \forall\, i, \theta \qquad\qquad \text{Primary coverage constraint} \quad (7.5)$$

where: $a_{ijk} := 1$ *if:* $d_{ij} \leq r_k$ *and* $d_{ij}/v_k \leq t$ *else* $a_{ijk} := 0$

$$y_{i\theta} \leq \sum_j \sum_k \left(a_{ijk} z_{jk\theta} \right) - x_{i\theta}, \forall\, i, \theta \qquad \text{Backup coverage constraint} \quad (7.6)$$

$$y_{i\theta} \leq x_{i\theta}, \qquad\qquad\qquad \forall\, i, \theta \qquad\qquad \text{Backup coverage comes after}$$
$$\text{primary coverage} \qquad\qquad (7.7)$$

$$\sum_k \sum_j u_{ijk\theta} = 1, \qquad\qquad \forall\, i, \theta \qquad\qquad \text{Unique demand allocation}$$
$$\text{constraint} \qquad\qquad\qquad (7.8)$$

$$\sum_j z_{jk\theta} \geq p_k, \qquad\qquad \forall\, k, \theta \qquad\qquad \text{Minimum number of vessels in}$$
$$\text{each class} \qquad\qquad\qquad (7.9)$$

$$u_{ijk\theta} \leq b_{ijk} z_{jk\theta}, \qquad\qquad \forall\, i, j, k, \theta \qquad \text{Allocation to } \textit{SAR} \text{ vessels possible}$$
$$\text{if there is a vessel within the coverage}$$
$$\text{range} \qquad\qquad\qquad\qquad (7.10)$$

where: $b_{ijk} := 1$ *if:* $d_{ij} \leq r_k$ *else* $b_{ijk} := 0$

$$\sum_i w_{i\omega\theta} u_{ijk\theta} \leq c_{k\theta} z_{jk\theta}, \qquad \forall\, j, k, \omega, \theta \qquad \text{Capacity constraint} \qquad (7.11)$$

$$z_{jk\theta} = 0, \qquad\qquad\qquad \forall\, j \in J_S, k \in \{1,2\}, \theta \quad \text{Offshore location constraint} \quad (7.12)$$

$$\frac{\sum_\omega \pi_\omega \sum_\theta \sum_i w_{i\omega\theta} x_{i\theta}}{\sum_\omega \pi_\omega \sum_\theta \sum_i w_{i\omega\theta}} \geq 1 - \alpha_1, \qquad\qquad \text{Minimum expected primary coverage}$$
$$\text{over all scenarios} \qquad\qquad (7.13)$$

$$\frac{\sum_\omega \pi_\omega \sum_\theta \sum_i w_{i\omega\theta} y_{i\theta}}{\sum_\omega \pi_\omega \sum_\theta \sum_i w_{i\omega\theta}} \geq 1 - \alpha_2, \qquad\qquad \text{Minimum expected backup coverage}$$
$$\text{over all scenarios} \qquad\qquad (7.14)$$

$$\sum_j z_{jk1} = \sum_j z_{jk2}, \qquad\qquad \forall\, k \qquad\qquad \text{Fixed number of vessels in each type}$$
$$\text{in two periods} \qquad\qquad (7.15)$$

$$z_{jk2} - z_{jk1} = R1_{jk} - R2_{jk} \qquad \forall\, j, k \qquad\qquad \text{Relocation of vessel type } k \text{ from/to}$$
$$\text{location } j \qquad\qquad\qquad (7.16)$$

$$RC_k = \sum_j \frac{(R1_{jk}+R2_{jk})}{2} \qquad \forall\, k \qquad \text{Calculating number of relocations}$$

$$\text{of vessels type } k \qquad (7.17)$$

$$\sum_k RC_k \le \beta \qquad\qquad \text{Restricting number of relocations}$$

$$(7.18)$$

The minimization objective function (equation 7.4) minimizes the weighted sum of total annual cost of vessels located by the model (which the same in both seasons) and the expected mean access time for all incidents across all demand scenarios (over two seasons). Constraint (7.5) ensures that demand location i has primary coverage in period θ only if there is a vessel within the maximum coverage distance and access time in that period. Constraint (7.6) in relation with (7.7) defines the backup coverage, where backup coverage is provided for a specific location if there are more than one located vessels within the acceptable access time. Allocation of demand locations to resources is defined in (7.8) which ensures that all demand points are assigned to exactly one response vessel. The minimum number of vessels required in each class is constrained by (7.9). Constraint set (7.10) limits the allocation of demands to vessels based on the availability of having at least one vessel within its range. Vessel capacity restrictions for each scenario in different periods are applied in (7.11), and constraint set (7.12) ensures that lifeboats (vessels type 1 and 2) cannot be located at the offshore stations. Constraints (7.13) and (7.14) ensure that a minimum level of expected primary and backup coverage is provided. Equations (7.15) ensure that number of vessels in each vessel type does not vary from a period to another one. The constraint set (7.16) is used to calculate the relocation of vessels for each station and vessel type. Equation set (7.17) determine the total number of relocations for each vessel type, and inequality (7.18) restricts the total number of relocations across all vessel types.

7.4. Case Study and Numerical Results

This section explains the process of applying the proposed model to the specific case study for maritime *SAR* in Atlantic Canada.

7.4.1. *Inputs for the Optimization Model*

The calculated distances are collected in a distance matrix, which includes distances between all grids where incidents may occur (using grid centroids) and potential station locations. This matrix has 1617 rows (gridded demand locations) and 37 columns (vessel stations), where d_{ij} denotes the distance of incident grid i from potential station j. A smaller grid size is used for areas around the shoreline corresponding to the response zone of small lifeboats (with 185 km range) which reduces the numerical error in determining the coverage area and the access times.

7.4.1.1. Incidents Kernel Density Estimation

We compute the kernel density estimates for each grid and over the two predefined seasons, using the following parameter values.

- **Kernel function type:** Quartic

- **Cell size:** (0.25×0.25) degree; the centre of each grid is used for kernel density calculation.

- **Bandwidth (radius):** variable size between (0.25-1.0 degrees), 0.25 degree for areas close to the shoreline with high density of incidents, 0.5 degree for areas further from shore with low incident density, and 1.0 degree for areas further offshore with very low number of incidents in the vicinity.

Figure 7-2 and Figure 7-3 visualize the kernel density estimates for the two different operational seasons: Fall-Winter (season 1) and Spring-Summer (season 2). As it can be observed in these figures, the incident distribution pattern varies substantially over the two seasons.
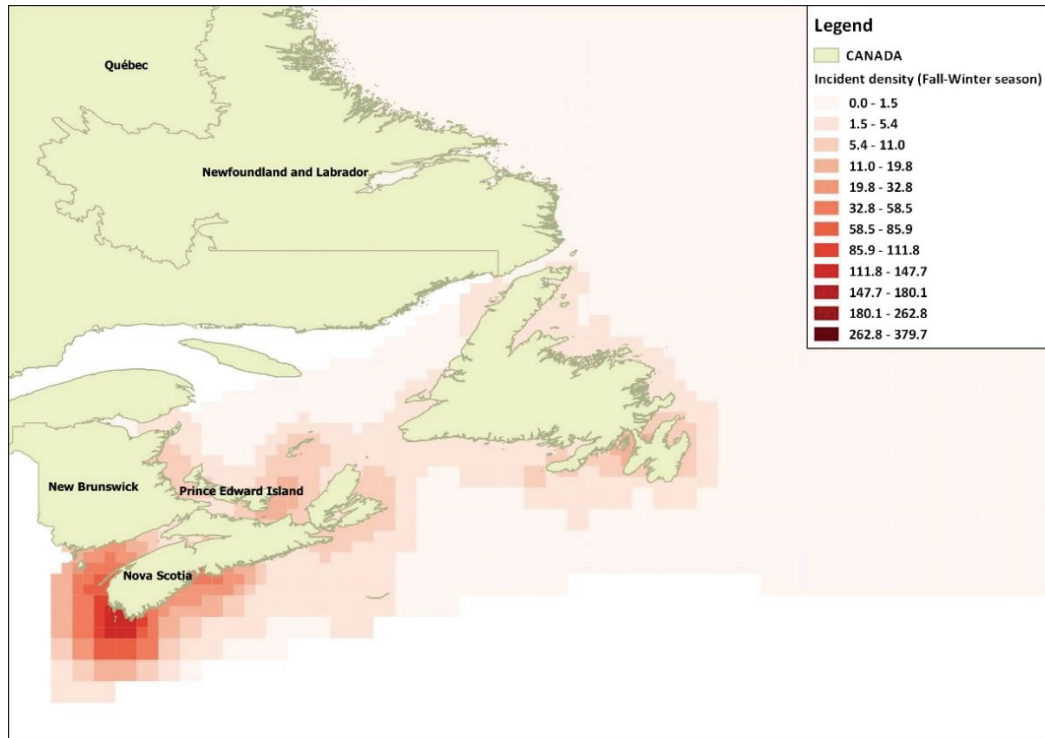
Figure 7-2- Kernel density estimation for season 1: October- April
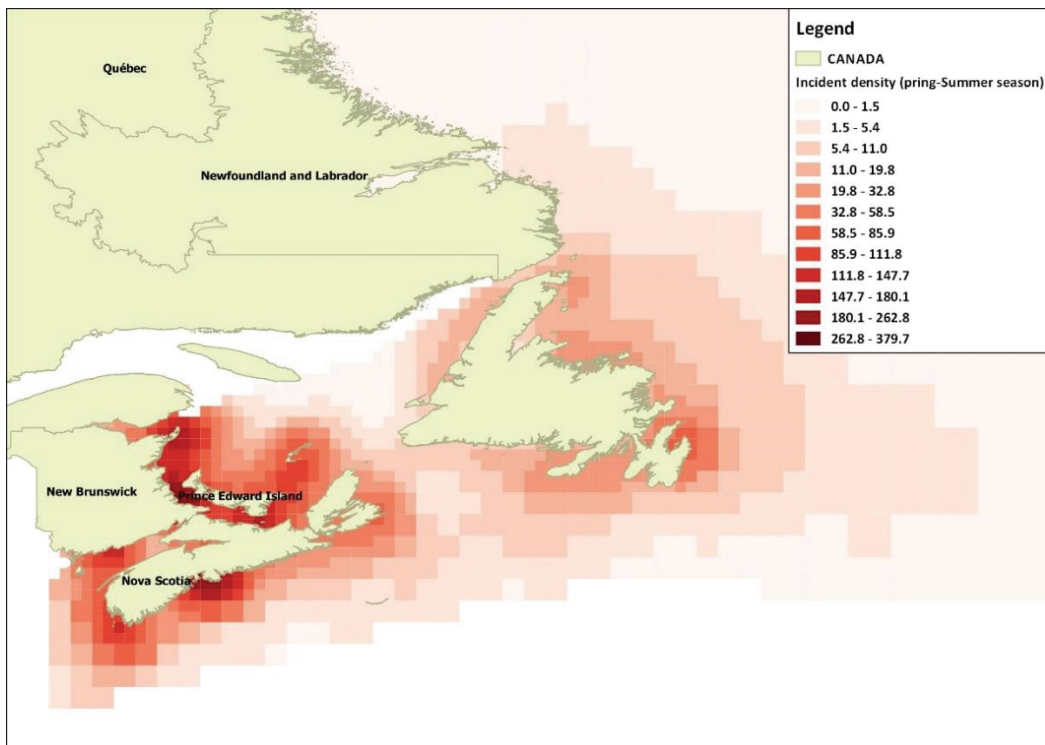
Figure 7-3-Kernel density estimation for season 2: May- September

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7.4.1.2. Future Incidents Simulation

The kernel density estimates are used as the foundation for generating random incident counts for future scenarios. For each grid square, the average of kernel estimates within the grid square in each season is calculated. These incident density rates are multiplied by the grid area (to account for grids with variable area due to different sizes) to compute the incident count estimate for each grid/season. These calculated grid incident count estimates are scaled so that they sum up to the average number of historical incidents in each season. We assume that the number of incidents in any grid follows a Poisson distribution. The scaled incident count estimates are considered as the mean parameter of a Poisson distribution for generating a random number of incidents over the mesh of grids across the study area. Ten sets of random incident counts per grid square (for each season) are randomly generated. These simulated incident scenarios are used as the representation of stochastic demand in the proposed model. Probability of occurrence for each simulated demand scenario is considered to be equal to 0.1 for each of the 10 scenarios as they were all generated from the same distribution.

7.4.1.3. Coverage Time Limit

The maximum access time for an acceptable level of primary and backup coverage can vary based on the predefined service level standards or expert opinion about actual operations. In this study, the default value for coverage range time limit is considered to be 6 hours based on consultation with *CCG* experts. It should be noted that this constraint is only applied to the coverage calculation in the model, but it has no impact on the allocation process of the model where there is no limit on access time and all incidents are to be allocated to the closest available resource in order to calculate mean access time.

7.4.2. Solving the Model: Different Configurations

The proposed model was built in the MPL environment and solved using the Gurobi 6.0.4 solver. The model features 485,728 variables and 494,794 constraints. The computation time to find the exact optimal solutions varied between 5-60 minutes in different model configurations depending on the parameter values using a computer with Intel Core i7 CPU and 8GB RAM. In the following sections, the results generated for different model

configurations are presented and discussed. There are several parameters than need to be set before attempting to solve the model. In the original model, we do not consider a minimum limitation for the number of vessels in each type and only the currently operating vessel types are considered. Also, we need to set a minimum level of primary and backup coverage. These values are determined with respect to current standards and long-term goals, and fixed at 95% and 75% respectively.

Table 7-4 presents the model solutions with respect to changing the mean access time coefficient in the objective function. There is no limitation on the number of seasonal relocations. The trade-off between total annual cost and mean access time is clear in the results. As the weight on mean access time increases in the objective function, the model yields faster access time while the cost increases. The mean access time can go down to 2.2 hours while keeping the cost under 32 units (which is less than current annual cost). The obtained solutions suggest that the optimal fleet mix most often consists of only vessels type 2 (fast lifeboats), and 3 (offshore patrol vessels) but rarely vessel type 4 (multitasking vessels). The use of vessel type 1 is not justifiable due to its capability limitations and higher costs. The other interesting fact is that enlarging the fleet size by increasing the cost often leads to a lower number of required relocations because with more vessels located, fewer relocations are needed to obtain the coverage requirements and best access times. In one of the desirable configurations, the model solution yields 2.20 hours mean access time, 95.4% primary coverage, and 75.87% backup coverage with the cost of 31.95 and only two seasonal relocations.

Table 7-4- Model results: current vessel types, different model configurations (varying objective coefficient)

	Decision criteria					**Fleet composition**			
Access time coef.	Total cost	Mean access time	Primary Coverage	Backup coverage	Number of relocations	Vessel type 1	Vessel type 2	Vessel type 3	Vessel type 4
1	26.17	2.54	95.01%	75.02%	4	0	17	5	1
5	26.17	2.54	95.01%	75.02%	4	0	17	5	1
8	26.17	2.54	95.01%	75.02%	4	0	17	5	1
10	26.51	2.51	95.01%	75.01%	7	0	16	6	0
12	28.89	2.29	95.00%	75.23%	5	0	17	6	1
15	28.89	2.29	95.00%	75.23%	5	0	17	6	1
20	29.23	2.27	95.01%	75.06%	5	0	17	7	0
30	29.23	2.27	95.01%	75.06%	5	0	17	7	0
40	31.95	2.20	95.40%	75.87%	2	0	17	8	0
50	31.95	2.20	95.40%	75.87%	2	0	17	8	0
90	34.66	2.18	95.03%	75.00%	1	0	17	9	0
100	37.38	2.15	95.04%	75.01%	4	0	17	10	0
1000	62.42	2.05	96.25%	80.09%	0	0	18	19	0

Although the results observed in the above table is a significant improvement in terms of the service level, it lacks consideration of some realistic constraints. For example, it is known that even though multitasking vessels are apparently not in the optimal composition of our model, in the real situation they cannot be eliminated because they are utilized for multiple purposes and *SAR* is only one among their tasks. So, it would make it more realistic to add a constraint into our model to ensure a minimum of number of vessels of each type. These minimum values are determined in consultation with the *CCG*. For instance, currently there are 4 patrol vessels and 4 multitasking ships operational in Atlantic Canada that are going to be operational for at least next half decade, so they need to be there regardless of what the optimal solution is. Also, the old-fashioned lifeboats (type 1) are at the end of their operational life and they are going to be replaced by new lifeboats according to the *CCG* strategic plan. Hence the minimum number of vessels for the 4 vessels types are assumed to be: (0,7,4,4) respectively. Furthermore, it is apparent that seasonal relocation of vessels entails additional cost and operational issues such as crew

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relocation which are not captured in the model. Therefore, we restrict the maximum number of relocations in the model to 5. Table 7-5 presents the solution obtained by the model including the additional constraints mentioned above. As expected, by forcing multitasking vessels into the solution, the total cost increases substantially. The minimum feasible annual cost to satisfy all model constraints, is around 30 units. With this new setting, in order to obtain 2.2 hours access time, the *CCG* would need to allocate additional budget for the fleet as the annual cost will be 36.01, a 13% increase. Whether or not this is feasible from a political point of view remains to be seen. The solution with the access time coefficient value of 20 in the objective function seems a reasonable trade-off solution as it keeps the cost at 31.17 (close to the current situation) while providing a substantial improvement in accessibility and coverage.

Table 7-5- Model results: current vessel types, different model configurations (varying objective weights) with minimum vessel numbers

	Decision criteria					Fleet composition			
Access time Coef.	Total cost	Mean access time	Primary coverage	Backup coverage	Number of relocations	Vessel type 1	Vessel type 2	Vessel type 3	Vessel type 4
1	29.97	2.40	95.02%	75.11%	5	0	16	4	4
5	29.97	2.40	95.02%	75.11%	5	0	16	4	4
8	30.57	2.32	95.01%	75.09%	5	0	17	4	4
10	30.57	2.32	95.01%	75.09%	5	0	17	4	4
12	30.57	2.32	95.01%	75.09%	5	0	17	4	4
15	30.57	2.32	95.01%	75.09%	5	0	17	4	4
20	**31.17**	**2.28**	**95.08%**	**75.01%**	**4**	**0**	**18**	**4**	**4**
30	31.17	2.28	95.08%	75.01%	4	0	18	4	4
40	31.17	2.28	95.08%	75.01%	4	0	18	4	4
50	33.29	2.24	95.14%	75.15%	3	0	17	5	4
70	36.01	2.20	95.46%	75.00%	5	0	17	6	4
85	36.01	2.20	95.46%	75.00%	5	0	17	6	4
100	41.44	2.15	95.22%	75.06%	4	0	17	8	4
1000	68.60	2.05	95.01%	75.01%	4	0	17	18	4

In order to investigate the performance of the model solution during different seasons and across various simulated scenarios, the mean access time is computed and displayed in Figure 7-4. The access time in season 2 is usually higher due to a jump in the number of incidents and the potential unavailability of the closest vessel to respond. Also, some fluctuations can be seen in mean access time across different scenarios which reflects the variations of demand distribution.
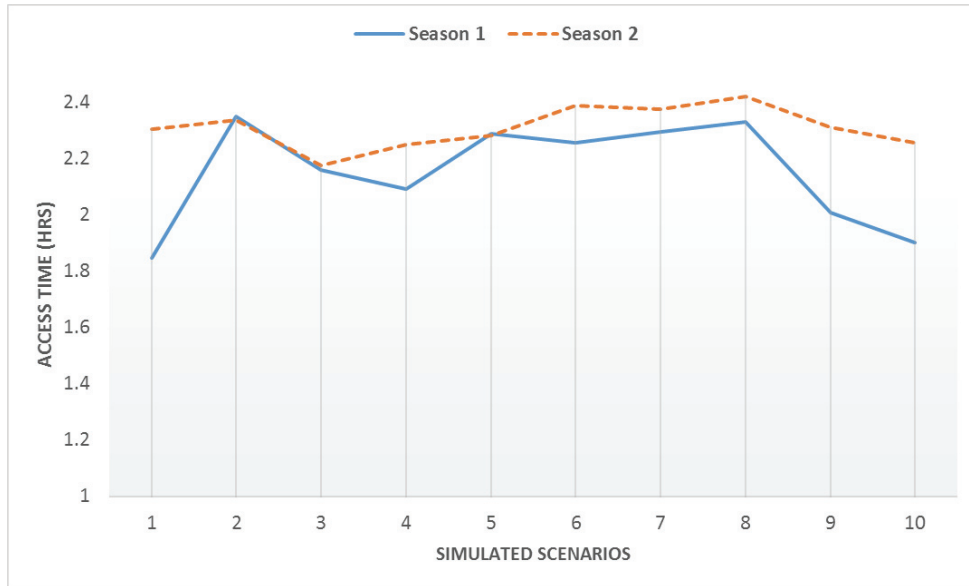
Figure 7-4- Variation of mean access times across simulated scenarios and over two seasons (model configured with current vessel types and constraints on minimum number of vessels in each vessel type and maximum relocations, ρ= 20)

For comparing the results obtained by our model with the current composition of the fleet, we configured the model by fixing the number of vessels in different classes at the existing level and ran the model with the same objective functions (from now on, the coefficient of mean access time in all presented results is fixed at 20). The results shown in Table 7-6 indicate that with the current 31.5 annual cost, the best mean access time possible (given that arrangement of vessels is optimized) is 2.57 hours and primary coverage cannot go above 94%.

Table 7-6- The optimal arrangement of existing vessels using the proposed model (ρ= 20)

	Decision criteria					**Fleet composition**			
Access time Weight	Total cost	Mean access time	Primary coverage	Backup coverage	Number of relocations	Vessel type 1	Vessel type 2	Vessel type 3	Vessel type 4
20	31.50	2.57	94.01%	75.00%	5	9	7	4	4

7.4.3. Sensitivity Analysis

Examining the sensitivity of model solutions to possible changes in parameter values is an instructive process. In particular, we would like to investigate the model performance when altering the coverage requirement constraints. Table 7-7 and Table 7-8 present the model solutions at variable levels of primary and backup coverage.

Table 7-7- Sensitivity analysis on the minimum required primary coverage

	Decision criteria					**Fleet composition**			
Minimum primary coverage	Total cost	Mean access time	Primary coverage	Backup coverage	Number of relocations	Vessel type 1	Vessel type 2	Vessel type 3	Vessel type 4
94%	30.57	2.30	94.47%	75.36%	5	0	17	4	4
95%	31.17	2.28	95.08%	75.01%	4	0	18	4	4
96%	40.24	2.21	96.00%	75.01%	4	0	15	8	4
97%	infeasible								

In order to increase the primary coverage to 96%, the fleet composition needs to change substantially (i.e. many more type 3 vessels are required). Obtaining a coverage over 97% is impossible due to the fact that there are a few remote incidents far from all potential inshore and offshore stations that are not accessible within 6 hours no matter how many vessels are located.

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Table 7-8- Sensitivity analysis on minimum required backup coverage

Minimum backup coverage	Decision criteria					Fleet composition			
	Total cost	Mean access time	Primary coverage	Backup coverage	Number of relocations	Vessel type 1	Vessel type 2	Vessel type 3	Vessel type 4
75%	31.17	2.28	95.08%	75.01%	4	0	18	4	4
77.5%	31.17	2.31	95.02%	77.54%	5	0	18	4	4
80%	31.78	2.31	95.02%	80.09%	4	0	19	4	4
82.5%	32.38	2.31	95.02%	82.59%	5	0	20	4	4
85%	33.58	2.28	95.10%	85.65%	4	0	22	4	4

Increasing the backup coverage above the default rate (75%) is possible and it can go above 85% by employing a few more vessels type 2 which incurs about 8% additional cost.

7.4.4. Incorporating New Lifeboats in the Model

As discussed earlier, the *CCG* is in the process of renewing part of its fleet. More specifically for vessels conducting *SAR* tasks, some vessels are at the end of their operational life and also do not have acceptable up-to-date capability to perform effective *SAR* operations. Among *SAR* vessels in the Atlantic Canada region, 9 old Arun-class lifeboats are currently due for replacement. A new class of lifeboats are under consideration to substitute for these older-model boats, with their characteristics presented in section 7.2.2. In this section, we examine the impact of including these new vessels and the feasibility of their acquisition is examined. As these new lifeboats are different and there are no historical observations on their operational costs, the model is configured with a variable range of costs for the new lifeboats (proportional to the cost of the current fast lifeboats, which are competitive). The results shown in Table 7-9 suggest that these new lifeboats can improve the service level at any cost not more than 1.5 times the cost of lifeboats type 2. If the new vessels have the same cost level, they have complete advantage in the model as they are faster. The solution with 0.66 annual cost (second row) is appealing. This solution suggests replacing the old lifeboats with the new class and also adding two more lifeboats of type 2 leads to significant improvement in access time (2.20 vs. 2.28) compared to the results shown in Table 7-5, with only a slight increase in cost.

The trade-off between the number of lifeboats type 2 and new lifeboats (type 5) based on variable cost estimates for new lifeboats is demonstrated in Figure 7-5.

Table 7-9- Model results, incorporating new lifeboats at variable annual cost levels

	Decision criteria					**Fleet composition**				
Vessel type 5 cost	Total cost	Mean access time	Primary coverage	Backup coverage	Number of relocations	Vessel type 1	Vessel type 2	Vessel type 3	Vessel type 4	Vessel type 5
0.60	31.17	2.20	95.04%	75.00%	5	0	7	4	4	11
0.66	31.71	2.20	95.15%	75.13%	5	0	9	4	4	9
0.72	32.01	2.22	95.02%	75.00%	5	0	11	4	4	7
0.78	31.53	2.25	95.15%	75.13%	5	0	16	4	4	2
0.84	31.41	2.27	95.15%	75.13%	3	0	17	4	4	1
0.90	31.48	2.27	95.15%	75.13%	3	0	17	4	4	1
0.96	31.17	2.28	95.08%	75.01%	4	0	18	4	4	0

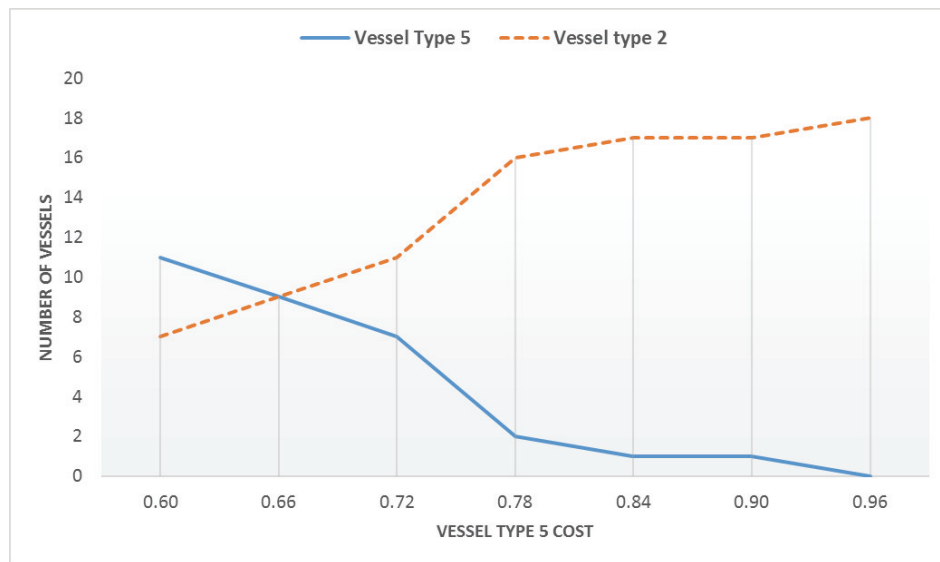*Figure 7-5- Trade-off between the annual cost of new lifeboats and their optimal numbers in the model's solution*

Figure 7-6 presents the similar results to Figure 7-4, for the new configuration of the model including the new lifeboats. Comparing these two figures shows a similar trend across scenarios and the two seasons but with a significant shift down in access time by inclusion of new lifeboats.

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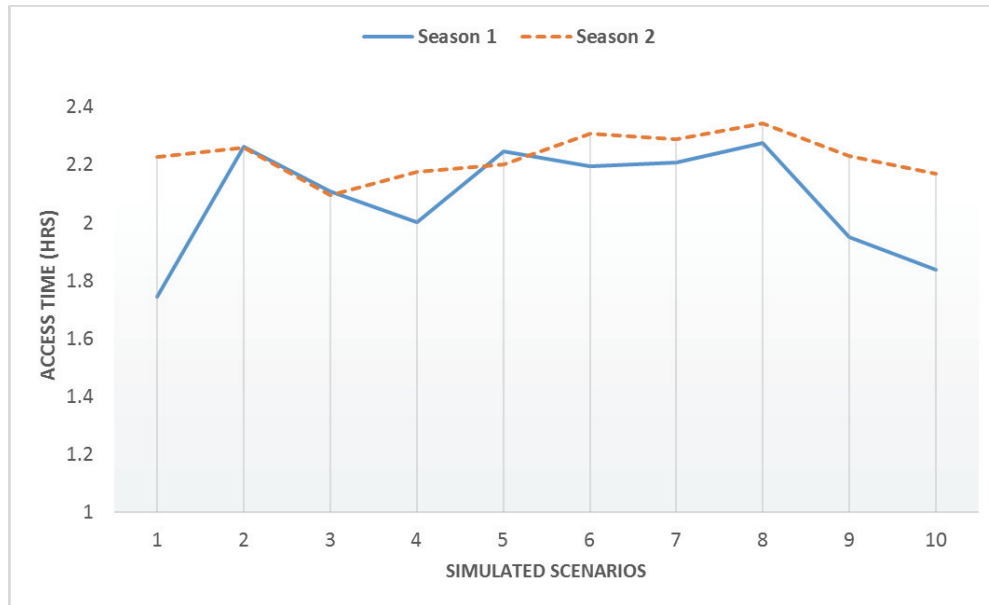
Figure 7-6- Variation of mean access times across simulated scenarios and over two seasons (model configured incorporating new lifeboats, ρ= 20)

7.4.5. Summary of Our Results

In order to be able to compare the performance of the model solution with the current situation and to measure the potential improvement, the access time to simulated incidents is calculated given the current arrangement of *SAR* vessels. The summary of the results of the proposed model compared to the current arrangement of vessels in terms of several decision criteria are shown in Table 7-10.

Table 7-10- Model solutions compared to the current arrangement

Resource Arrangement	Decision criteria			
	Total fleet annual cost	Mean access time (hrs)	Primary coverage	Backup coverage
Current situation	31.50	3.14	89.38%	60.18%
The proposed model with current vessel types	31.17	2.28	95.08%	75.01%
The proposed model including new vessels	31.71	2.20	95.15%	75.13%

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Comparing the solutions performance with the current arrangement of the *CCG SAR* fleet, shows a tremendous potential improvement both in access time and coverage. In particular, holding the annual costs in the same level, it is possible to drop the mean access time to potential incident locations by almost one hour (30%) and increase the primary and backup coverage by 6% and 15% respectively.

7.5. Conclusion and Outlook

This study attempted to develop a comprehensive fleet planning model for strategic decisions on procurement, location, and allocation of maritime *SAR* vessels with a case study on Atlantic Canada. The proposed model incorporates several decision criteria such as fixed and operational cost, access time to incidents, and primary and backup coverage, to address different aspects of the problem. A multi-objective function minimizes the weighted sum of the total annual fleet cost and mean access times. A multi-period location/relocation approach is taken to allow seasonal relocation of vessels within the area of interest to provide more flexibility to respond to periodic variations in demand distribution. Future demands in two operational seasons with different demand patterns are simulated over the study area and timeframe based on incident occurrence estimates extracted from historical incidents using a kernel density estimation. Four *SAR* vessel types which are used in practice with various characteristics are considered. Also, the consequence of substituting some old lifeboats with a new class of lifeboats is examined. Several model runs with different parameter configurations are generated and solved. The results point out that at current cost level (similar budget) the composition and location arrangement of vessels can be optimized with a significant improvement in terms of access time and coverage. Sensitivity analyses were conducted to observe solution changes with respect to variable service level requirements.

The results of this study could be useful for guiding strategic decisions with regard to *SAR* vessel arrangement, acquisitions and placement which has a long-term impact on the efficiency of using limited resources and attainable service level. The outcome of this study could provide the *CCG* with some useful insight for future resource planning, including fleet renewal plans and determining appropriate stations for placing new vessels. Also, it

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can be helpful for managing current operations to increase the resource utilization and effectiveness of their services. Finally, several tactical and operational rules can be extracted from the model solution for best resource allocation policies.

The model proposed in this study can be further extended in several ways. There are different types of incidents with different severity levels, thus requiring variable levels and types of response, and all *SAR* vessels are not equally effective at responding to different types of incidents. Therefore, it would be more realistic if we can differentiate incident types in the model and take into account their specific response requirements. Moreover, this would make it possible to quantify and consider the effectiveness of response as an objective in the model. Such analysis would require more detailed information about the capabilities of resources, which was not available at the time of this study. Also, the demand simulation methodology can be further extended to estimate future response needs by incorporating trends and forecasts on exposure factors such as incident rates, and/or traffic levels. Given the availability of actual data, the cost of relocations can also be added into the model either as a constraint or as a part of the objective function.

Chapter 8   Conclusion

8.1. Summary and Findings

Location-Allocation problems are fundamental models for several important applications, including emergency response logistics planning. Optimizing the efficiency and effectiveness of resource utilization is always a major concern. Maritime *SAR*, categorized within public emergency response activities, is a potential application of location analysis to deal with managing limited resources and their locations. This problem becomes more complicated when we are faced with several objectives which are sometimes conflicting as well. This research developed a framework of mathematical models to optimize the Location-Allocation of maritime *SAR* resources with regard to several criteria including primary and backup coverage, mean access time, service equality, and cost. These criteria were chosen and defined in order to represent and quantify broader qualitative concepts such as acceptability, effectiveness, and efficiency of maritime *SAR* services.

The main objectives achieved by this study can be summarized as follows:

- Determine and incorporate various important decision criteria in maritime *SAR* services in a framework of mathematical models as a decision support tool;
- Analyze spatial and seasonal trends and patterns in incident occurrences and leverage the extracted knowledge for planning a more effective response;
- Consider the uncertainty involved with future incident locations and adopt spatial density estimation methods and a simulation approach to account for different possible incident distribution schemes;
- Apply optimization models to determine the optimal or efficient solutions to the maritime *SAR* Location-Allocation Problem;
- Develop various operational to strategic level decision support models to assist decision makers with short- to long-term decisions regarding effectively and efficiently managing *SAR* resources;
- Assess the current situation of resource arrangement and compare it with the solutions suggested by the developed mathematical models in order to reveal the potential area of improvements;

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- Provide and support decision makers with insightful information extracted from the outputs of the comprehensive analysis to enhance the resource planning and policy development processes.

To achieve these objectives, several phases of statistical and mathematical modelling were conducted as summarized below.

In the first phase of the study, two common location models, the Maximal Covering Location Problem and *p*-median problem, were modified and applied to the maritime *SAR* Location problem. Five metrics were defined and measured to assess the solution with respect to acceptability, accessibility, and equality of the service. The results indicate that the *p*-median model provides a better solution in terms of three metrics: access time, backup coverage, and access to furthest incidents, while the *MCLP* works slightly better for primary coverage and Gini index. The solutions provided by both optimization models dominate the current arrangement of *SAR* vessels. A simulation procedure based on the underlying distribution of historical incidents was used to validate the performance of the obtained solution across possible variations in the demand.

The second phase of the research aimed at combining the advantages of the *MCLP* and *p*-median formulations into a new model minimizing the mean access time as the objective, constrained to providing a certain level of coverage. The presented model also considers a possible change of the present mix of vessels for *SAR* missions. A budget limitation was applied to restrict the capital cost associated with any fleet composition to be the same as capital cost of the current fleet. The results are particularly instructive for making strategic level decisions for procurement of new vessels.

While the models developed in previous phases were all single objective, a multi-objective model was presented in the third paper which adopted a goal programming approach to incorporate three decision criteria in the optimization process. Moreover, a set of demand scenarios were considered in the optimization model to account for the stochasticity of incident locations. The proposed model also extends the previous models by inclusion of the annual vessels response capacity. The primary coverage, backup coverage and mean access time to incidents were incorporated into a model that minimizes the weighted

standard deviations of objective values from their pre-specified targets. A series of good trade-off solutions were obtained by this model which decision makers can choose from according to their different perspectives. These solutions perform well with respect to all decision criteria, which was a major limitation to the previously applied single objective models.

The fourth phase addressed possible seasonal variations in the demand in terms of the incident location and frequency, which historical records show can vary substantially over different seasons. The model proposed in this section considers seasonal relocation of vessels to effectively respond to such changes in demand patterns. Moreover, a robust optimization approach was adopted to ensure that the solution performs at a satisfactory level across several simulated demand scenarios. The solution obtained by the presented multi-period (i.e. dynamic) model yields notably faster access time versus the static solution and the current *SAR* vessel arrangement, with only a few seasonal vessel relocations.

Finally, in the last phase of the research, the aim was to develop a comprehensive strategic model that incorporates the main advantages of all models proposed in previous steps. In particular, this model integrates following four main elements of previous models: (1) several decision criteria including primary coverage, backup coverage, access time, and cost are incorporated into the model either explicitly as part of objective function or implicitly with inclusion of appropriate constraints; (2) various *SAR* vessels types with different characteristics and capacity are taken into account; (3) the uncertainty of future incident occurrence is addressed using a scenario planning approach; and (4) the model allows periodic fleet relocations to respond to seasonal demand variations. In addition, since this model is meant to be a decision support tool for strategic long-run fleet planning, the possibility of changing the fleet composition via decommission of old vessels and/or acquisition of new vessels of the current type or new variants, is considered. The results indicate that when holding the capital and operational cost of *SAR* fleet at the current level (similar budget), a substantial improvement in service level is achievable by modifying the composition and location arrangement of vessels.

Decision makers can choose among the proposed models in this study with respect to the inputs and information they require for making challenging operational, tactical, and strategic decisions. The outcomes of the modelling would be instructive for them in regard to decisions on the rearrangement of the current fleet, procurement of new vessels, establishing new lifeboat stations, or decommissioning old vessels. Such informative results can be used as inputs for future resource capacity and utilization planning. The *CCG*, in particular, would receive some valuable insight for their future resource planning, including fleet renewal plans and siting of new vessels. In addition to support the design of long-term resource planning strategies and policies, the knowledge extracted from the models' solutions would greatly assist managers to optimize operational procedures and rules for more efficient fleet operations management. Also, society would gain benefits from the potential improvement in the service accessibility and effectiveness resulting from the implementation of solutions suggested by developed models.

8.2. Limitations

Due to the fact that this study relies on historical data for modelling the future state of maritime *SAR* system, the quality of data obtained and the methods to process the data impose some limitations on the results.

Although the *SISAR* dataset collects a reasonable amount of information related to maritime incidents with an acceptable level of accuracy in most cases, some shortcomings were identified. Some of the fields in the database are not filled in completely as they were not made mandatory for completion by operators. For instance, there is a field defined for recording the primary cause of an incident which has a very low fill rate. This limitation, in particular, impedes differentiation between incidents resulting from different causes (environmental conditions, technology failure, human error, etc.).

Also, on the response side, the recorded data have some major limitations. For example, there are no comprehensive records on tracking the *SAR* vessels' locations from the time they depart for a mission until they return to the base station, although most of these vessels are equipped with location tracking systems. It would be very helpful to have such

information in hand which is specifically required for implementing more dynamic (real-time) models such as queuing models for real-time deployment of vessels.

Moreover, there is not much information available about the characteristics and requirements of different types of incidents. Utilizing a triage system to conduct a needs analysis according to different types of incidents, and thus relate it with the operational capability of *SAR* vessels, would enable researchers to consider more realistic response requirements and more properly deal with the response effectiveness concept. Using the rapidly proliferating information systems similar to OnStar (developed by General Motors) for better communication, possibly coupled with video, can simplify the triage.

Traffic levels and patterns can be important factors related to the incident occurrences. This study did not incorporate the traffic as an exposure measure for predicting the location and frequency of maritime incidents due to limitations in accessing reliable and recent data. Hot spots (risky area) could be identified by relating the number of incidents to the traffic level (i.e. define incident rate as a proportion of a number of incidents relative to the traffic). This information would be beneficial for providing a more efficient response to high-risk areas. Obtaining such data and, more importantly, a proper forecast of future trends and changes in traffic, would enhance the accuracy and reliability of the future demand used for long-term planning.

The impact of environmental factors on the occurrence of incidents nor on the response provided by *SAR* resources is examined in this research. Harsh weather or sea conditions might increase the chance and/or severity of an incident as well as adversely affect response operations. In order to be able to consider these factors, one needs to integrate relevant datasets and conduct sufficient statistical analyses to first investigate the relevancy of these factors and then use the outcomes as parameters of an optimization model. Rezaee, Pelot, and Ghasemi (2016) studied the effect of extreme weather conditions on the fishing incidents occurrence. The effect of extreme weather conditions on incidents severity is examined in (Rezaee, Pelot, and Finnis 2016).

8.3. Recommendations for Future Work

There are several streams for future work on extending this research. One important element which could make the analysis more realistic and interesting for the decision makers is to consider characteristics of different incidents with different response requirements and try to translate those requirements into operational rules. On the other hand, because there are different types of vessels used for *SAR* tasks which are not all equally capable of performing various tasks, it would be appealing to measure the capability and effectiveness of each individual vessel type in responding to a specific incident type. To do so, additional data collection and analysis are required on the capability of various vessels in responding to different incidents with diverse needs. Quantifying such qualitative factors makes it feasible for possible inclusion in a mathematical model as one of the decision criteria. This was not possible in this study due to unavailability of the required information, as discussed in the previous section.

As discussed throughout the thesis, congestion is an important issue whereby there is a possibility that the closest vessel is not able to respond due to being busy with another task or unavailable for other reasons (e.g. maintenance). In this study, I dealt with this problem by using backup coverage as one of the criteria in the analysis whereby it is desired to have more than one vessel within coverage range/time so if the closest vessel is not available, another one can fulfill the task. Other approaches such as probabilistic and queuing models can be taken to more effectively address this concern. Also, such a modelling approach makes it possible to prioritize the incidents based on their severity level. So, for example, if a vessel is in the middle of a response to a non-distress incident and a distress incident is reported in the vicinity, then the response to the original incident can be disrupted in order to provide faster response to a more severe incident. Or in the same vein, a response vessel's availability for a new incident might be affected if it is returning from dealing with another incident. Conducting an extended study to examine the performance of the system in such situations, can provide insightful inputs for optimizing the operational procedures that might involve congestion, priority processing and even service preemption policies.

The approach taken in this study for modelling and simulating future demand can be further extended to account for trends in incidents, incident rates, and/or exposure metrics like traffic levels. In addition, other factors such as environmental changes (e.g. climate change) which can alter maritime activities and influence incident rates can have a potentially significant effect on the demand patterns in the future. Hence, integrating other data sources related to these factors into a more comprehensive methodology for forecasting future trends would enhance the accuracy and reliability of the representation of the demand distribution in the analysis and thus provide more robust solutions. Furthermore, the distinction can be made between incidents with different severity levels in order to set faster response requirement for more severe incidents as well as accounting for possible differences in their distribution patterns which can affect the optimal arrangement of *SAR* vessels.

Another stream of research would be the inclusion of other means of providing *SAR* service other than vessels operated by the *CCG*. The *DND SAR* helicopters and auxiliary *SAR* vessels are among the possible complementary *SAR* resources to take into consideration as other types of "facilities" in the model. Of course, such inclusion means conducting sufficient analysis on the capability of these additional resources compared to the usual primary resources in response to various incidents. Also, the study of resource sharing among different agencies, which might even involve vessels from other countries (primarily the US in our southern waters, or other Arctic countries in the North is an interesting subject to study.

Moreover, the presented models can be integrated with operational plans which involve crew scheduling. Also, decisions regarding determining the optimal locations for new lifeboat station development can be one additional element of strategic planning models.

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Appendix A - Canadian Coast Guard SAR Fleet in Atlantic Region

The *CCG* fleet in the Atlantic region supports the operational requirements associated with the four provinces of New Brunswick, Nova Scotia, Prince Edward Island, and Newfoundland and Labrador as well as adjacent waters, including the southern Gulf of Saint Lawrence. The vessels that are utilized for the Search and Rescue program, either primarily or as multi-tasked vessels, can be categorized into four classes as described here.

A.1. Arun-Class Search and Rescue Lifeboats

An Arun-class lifeboat is a small, shore-based, self-righting vessel capable of Search and Rescue operations up to 100 NM from shore; its top speed is approximately 25 knots. Ten 15.77 metre (51.7 ft) boats were built for the Canadian Coast Guard. They were built with aluminum hulls. They are considered "high endurance" lifeboats staffed by a crew of four. The first vessels of this class were ordered in 1990.

The existing Arun-class lifeboats in Atlantic region are listed below. These vessels came into service between 1985-96, but are currently near the end of their useful life and are going to be replaced by new lifeboats.

- **CCGS** *W.G. George*
- **CCGS** *W. Jackman*
- **CCGS** *Westport*
- **CCGS** *Courtney Bay*
- **CCGS** *Bickerton*
- **CCGS** *Sambro*
- **CCGS** *Spindrift*
- **CCGS** *Spray*
- **CCGS** *Goeland*
- **CCGS** *Clarks Harbour*

The summary of vessel specifications is as follows:

Length: 15.8 (Metres)

Breadth: 5.2 (Metres)

Draft: 1.3 (Metres)

Freeboard: Not Applicable

Gross Tonnage: 43.0 (Tons)

Net Tonnage: 32.0 (Tons)

Cruising Range: 200 (Nautical Miles)

Endurance: 2 (Days)

Cruising Speed: 14.0 (Knots)

Maximum Speed: 20.0 (Knots)

Fuel Capacity: 3.20 (Cubic Metres)

Fuel Consumption: 2.50 m^3/d

Figure A-1- A typical Arun-class life boat

A.2. Cape-Class Search and Rescue Lifeboats

Cape-class motor lifeboats have total lengths of 14.61 m (47 feet 11 inches) and beam lengths of 4.3 m (14 feet). They are constructed from marine-grade aluminium; ships have draughts of 1.37 m (4 feet 6 inches). They contain two Caterpillar 3196 diesel engines providing a combined 900 shaft horsepower. They have two 710 mm × 910 mm (28 by 36 inches) four-blade propellers, and each ship's complement is four crew members and five passengers.

These lifeboats have a maximum speed of 25 knots (46 km/h; 29 mph) and a cruising speed of 22 knots (25 mph). Cape-class lifeboats have a fuel capacity of 400 US gallons (1,500 l; 330 imp gal) and a range of 200 nautical miles (370 km; 230 mi) when cruising. They are capable of operating at wind speeds of 50 knots (93 km/h; 58 mph) and wave heights of 9.1 m (30 feet). They can two ships with displacements of up to 150 tonnes (170 short tons).

The Atlantic region currently operates several cape-class lifeboats which are listed here. The lifeboats were launched between 2002-2005.

- **CCGS** *Cape Fox*
- **CCGS** *Cap Nord*
- **CCGS** *Cape Spry*
- **CCGS** *Cape Norman*
- **CCGS** *Cap Breton*
- **CCGS** *Cape Edensaw*

The specification of these vessels is provided below.

Length: 14.6 (Metres)
Breadth: 4.3 (Metres)
Draft: 1.4 (Metres)
Freeboard: 0.8 (Metres)
Gross Tonnage: 33.8 (Tons)

Net Tonnage: 25.3 (Tons)

Cruising Range: 200 (Nautical Miles)

Endurance: 1 (Days)

Cruising Speed: 22.0 (Knots)

Maximum Speed: 25.0 (Knots)

Fuel Capacity: 1.60 (Cubic Metres)

Figure A-2- A Typical Cape-class lifeboat

A.3. Offshore Patrol Vessels

The *CCG*'s offshore patrol vessel is approximately 70 metres long. It can operate beyond 120 nautical miles (220 km; 140 mi) including outside the Exclusive Economic Zone, has a top speed greater than 20–25 knots (37–46 km/h; 23–29 mph) and can stay at sea for up to six weeks. It can operate year-round in Canadian waters, except the Arctic archipelago, and has a minimal ice capability to transit light ice-infested waters. It carries two rigid-hulled inflatable boats, up to 11 metres (36 ft) long, and can accommodate a helicopter

with minimal hangar capabilities. This vessel is designed to support law enforcement, and it has a program operations room. It is primarily used for fisheries enforcement and Search and Rescue.

There are currently four offshore patrol vessels operating in Atlantic region:

- **CCGS** Cape Roger
- **CCGS** Cygnus
- **CCGS** Leonard J. Cowley
- **CCGS** Sir Wilfred Grenfell

The summary of specifications for a typical offshore patrol vessel is as follows:

- **Length:** 70 (Metres)
- **Breadth:** 14.2 (Metres)
- **Draft:** 4.5 (Metres)
- **Freeboard:** 2.9 (Metres)
- **Gross Tonnage:** 2188.0 (Tons)
- **Net Tonnage:** 655.0 (Tons)
- **Cruising Range:** 12600 (Nautical Miles)
- **Endurance:** 35 (Days)
- **Cruising Speed:** 17.0 (Knots)
- **Maximum Speed:** 18.0 (Knots)
- **Fresh Water:** 32.00 (Cubic Metres)
- **Fuel Capacity:** 420.00 (Cubic Metres)
- **Fuel Consumption:** 12.00 m^3/d

Figure A-03- Leonard J Cowley, an offshore patrol vessel operating in the Atlantic region

A.4. High Endurance Multi-Tasked Vessels

This category encompasses large highly adaptable multi-tasked vessels, approximately 80 metres long, with an icebreaking capability to work in the southern and western Arctic, for escort operations in the Great Lakes, St. Lawrence River and Gulf of St. Lawrence and Atlantic Coast in winter. It also has a crane, a large cargo hold and deck capacity, a helicopter hangar that will accommodate a *CCG* helicopter, and it can launch and recover rigid-hull inflatable boats and two utility craft. It is capable to deliver many of the Government of Canada programs. It is also formally referred to as a Type 1100 vessel.

Currently, there are four high endurance multi-tasked vessels in Atlantic region:

- **CCGS** Ann Harvey
- **CCGS** Edward Cornwallis
- **CCGS** George R. Pearkes
- **CCGS** Sir William Alexander

The detailed specifications of these vessels are as follows.

- **Length:** 83.0 (Metres)
- **Breadth:** 16.2 (Metres)
- **Draft:** 6.2 (Metres)

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- **Freeboard:** 1.8 (Metres)
- **Gross Tonnage:** 3853.6 (Tons)
- **Net Tonnage:** 1528.0 (Tons)
- **Cruising Range:** 8200 (Nautical Miles)
- **Endurance:** 120 (Days)
- **Cruising Speed:** 12.0 (Knots)
- **Maximum Speed:** 16.5 (Knots)
- **Fresh Water:** 100.00 (Cubic Metres)
- **Fuel Capacity:** 783.70 (Cubic Metres)
- **Fuel Consumption:** 14.00 m^3/d

Figure A-04- Ann Harvey, a high endurance multi-tasked vessel operating in the Atlantic region

The information presented in this appendix is obtained from the *CCG* website (http://www.ccg-gcc.gc.ca/Fleet/Home).

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