FUZZY SYSTEM APPLICATIONS FOR SHORT-TERM ELECTRIC LOAD FORECASTING

By

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TABLE OF CONTENTS

Page

LIST OF TABLES

LIST OF FIGURES

Figure | Page

List of Abbreviations

E_{LAV}	Percentage error between actual and least absolute, estimated or predicted value
E_{LS}	Percentage error between actual and least error square, estimated or predicted value
H (t)	Humidity factor in percent
LAV	Least absolute value
LES	Least error squares
(P.)	Figure or table in appendix
t	Time in hours
t_F	Number of days where data are taken for specified hour
T (t)	Temperature deviation in °C, at time t
T_a (t)	Average dry bulb temperature in °C, at time t
T_d (t)	Dry bulb temperature in °C, at time t
V (t)	Speed of wind in km/hr at time t
W (t)	Wind cooling factor at time t
Y (t)	Load in MW at time t
Z_{LAV}	Least absolute value load forecast
Z_{LS}	Least error square load forecast
[ ]	Reference

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Abstract

Load forecasting is an important function in economic power generation, allocation between plants (Unit Commitment Scheduling), maintenance scheduling, and for system security applications such as peak shaving by power interchange with interconnected utilities. In this thesis the problem of fuzzy short term load forecasting is formulated and solved. The thesis starts with a discussion of conventional algorithms used in short-term load forecasting. These algorithms are based on least error squares and least absolute value. The theory behind each algorithm is explained. Three different models are developed and tested in the first part of the thesis. The first model (A) is a regression model that takes into account the weather parameters in summer and winter seasons. The second model (B) is a harmonics based model, which does not account for weather parameters, but considers the parameters as a function of time. Model (B) can be used where variations in weather parameters are not available. Finally, model (C) is created as a hybrid combination of models A and B. The parameters of the three models are estimated using the two static estimation algorithms and are used later to predict the load for twenty-four hours ahead. The results obtained are discussed and conclusions are drawn for these models. In the second part of the thesis new fuzzy models are developed for crisp load power with fuzzy load parameters and for fuzzy load power with fuzzy load parameters. Three fuzzy models (A),(B) and (C) are developed. The fuzzy load model (A) is a fuzzy linear regression model for summer and winter seasons. Model (B) is a harmonic fuzzy model, which does not account for weather parameters. Finally fuzzy load model (C) is a hybrid combination of fuzzy load models (A) and (B). Estimating the fuzzy parameters for the three models turns out to be one of linear optimization. The fuzzy parameters are obtained for the three models. These parameters are used to predict the load as a fuzzy function for twenty-four hours ahead. Prediction results are obtained and presented using data from Nova Scotia Power and Environment Canada.

Chapter 1
Introduction

1.1 Background

Economic development, throughout the world, depends directly on the state of the availability of electric energy, especially since most industries depend almost entirely on its use. The availability of a source of continuous, cheap and reliable energy is of foremost economic importance.

Electrical load forecasting is an important tool used to ensure that the energy supplied by utilities meets the load plus the energy lost in the system. To this end, a staff of trained personnel is needed to carry out this specialized function.

Load forecasting is always defined as basically the science or art of predicting the future load on a given system, for a specified period of time ahead. These predictions may be just for a fraction of an hour ahead for operation purposes, or as much as twenty years into the future for planning purposes

The load forecasting can be categorized into three subject areas namely [1].

(a) Long–range forecasting, which is used to predict loads as distant as fifty years ahead, so that expansion planning can be facilitated.

(b) Medium range forecasting which is used to predict weekly, monthly and yearly peak loads up to ten years ahead, so that efficient operational planning can be carried out and,

(c) Short range forecasting, which is used to predict loads up to a week ahead, so that daily running and dispatching costs can be minimized.

In the above three categories, an accurate load model is developed to mathematically represent the relationship between the load and influential variables such as time, weather, economic factors etc...

The precise relationship between the load and these variables is usually determined by their role in the load model. Once the mathematical model is constructed, the model parameters are determined by estimation techniques

Exrapolating the mathematical relationship to the required lead-time ahead and giving the corresponding values of influential variables to be available or predictable,then forecasts can be made. Since factors such as weather and economic indices are increasingly difficult to be accurately predicted for longer lead times ahead, the greater the lead-time, the less accurate the prediction is likely to be.

The final accuracy of any forecast thus depends on the load model employed, the accuracy of predicted variables and the parameters assigned by the relevant estimation technique. Since different methods of estimation will result in different values of estimated parameters, it follows that the resulting forecasts will differ in prediction accuracy.

Over the past fifty years, the parameter estimation algorithms used in load forecasting have been limited to those based on the least error squares minimization criterion, even though estimation theory indicates that algorithms based on the least absolute value criteria are a viable alternative **[44]**.

In this thesis, the fuzzy systems theory is implemented to estimate the load model parameters, which are assumed to be fuzzy parameters having certain middle and spread. Different membership functions, for load parameters, are used namely, triangular membership and trapezoidal membership functions. The problem of load forecasting in this thesis is restricted to short-term load forecasting, and is formulated as a fuzzy linear estimation problem. The objective is to minimize the spread of the available data points, taking into consideration the type of the membership of the fuzzy parameters, subject to satisfying constraints on each measurement point, to insure that the original membership is included in the estimated membership.

1.2 Outline of the Thesis

In this thesis, fuzzy system theory is implemented to estimate the fuzzy load model parameters, considered to be fuzzy parameters with certain middle and certain spread.

Chapter Two gives a literature review, the state of the art, of the various algorithms used during the past decades for short term load forecasting. A brief discussion for each algorithm is presented in this chapter. Advantages and disadvantages of each algorithm are discussed. Reviewing the most recent publications in the area of short term load forecasting indicates that most of the available algorithms treat the parameters of the proposed load model as crisp parameters, which is not the case in reality.

Chapter Three proposes different load models used in load modeling for 24 hours. Three models are proposed in this chapter, namely model A, B and model C. Model A is a multiple linear regression model of the temperature deviation, base load and either wind-chill factor for winter load or temperature humidity factor for the summer load. The parameters of load A are assumed to be crisp parameters in this chapter. The term crisp parameters means clearly defined parameter values without ambiguity.

Load model B is a harmonic decomposition model that expresses the load at any instant, t as a harmonic series. In this model, however, the weekly cycle is accounted for, by use of a daily load model, whose parameters are estimated seven times weekly. Again, the parameters of this model are assumed to be crisp.

Load model C is a hybrid load model that expresses the load as the sum of a time-varying base load and a weather dependent component. This model is developed with the aim of eliminating the disadvantages of the other two models by combining their modeling approaches.

After finding the parameter values, they are used to determine the electric load from which these parameter values are extracted and this value is called the estimated load. Then, the parameter values are used to predict the electric load for a randomly chosen day in the future and it is called the predicted load for that chosen day.

Chapter Four presents the theory involved in different approaches that use parameter estimation algorithms. In the first part of the chapter, the crisp parameter estimation algorithms are presented, these include the least error squares (LES) algorithm, the least absolute value (LAV) algorithm. The second part of the chapter presents an introduction to fuzzy set theory and systems followed by a discussion of fuzzy linear regression algorithms. Different cases for the fuzzy parameters are discussed in this part. The first case is for the fuzzy linear regression of the linear models having fuzzy parameters with non-fuzzy outputs, while the second case is for the linear regression of fuzzy parameters with fuzzy output, and finally the third case is for fuzzy parameters formulated with fuzzy output of LR-type.

Chapter Five presents the fuzzy modeling of electric loads for short term load forecasting. The models proposed in chapter three for crisp parameters are used in this chapter. Fuzzy model A, employs a multiple fuzzy linear regression model. The membership function for the model parameters is developed, where triangular membership functions are assumed for each parameter of the load model. Two constraints are imposed on each load measurement to insure that the original membership is included in the estimated membership.

Fuzzy model B, which is a harmonic model, has been proposed as well in this chapter. This model involves fuzzy parameters having certain median and a certain spread. Finally, a hybrid fuzzy model C, which is the combination of the multiple linear regression model A and the harmonic model B, is presented in this Chapter.

The models proposed, in chapter five, are used to estimate the load on an actual power system in operation, Nova Scotia Power Inc. The data for five years of the electric load together with the weather data are used in the forecasting process.

Chapter Six gives the results obtained for the different proposed static models. The accuracy of forecasting is discussed in this chapter. A comparison study is performed between the least error squares algorithm and least absolute value.

Chapter Seven discusses the fuzzy short-term load forecasting results. In this chapter the fuzzy parameters of the three maodels A, B, and C are estimated based on fuzzy system theory and are used to predict ahead the load power.

Chapter Eight gives a discussion of the results obtained and recommendations for the future research. A list of references and appendices are given at the end of the thesis.

Chapter 2
Short-Term Load Forecasting (STLF): The State of The Art

2.1. Introduction

Short-term load forecasting is an integral part of power system operation which is essential for securing an inexpensive supply of reliable electric energy. It is used to predict load demands up to a week ahead so that the day to day operation of a power system can be efficiently planned and so that the operating costs are minimized.

Short-term load forecasting can be performed in one of two modes, namely on and off-line forecasting. This categorization, as the names suggest, stem from the areas of application of the load predictors.

Off-line load forecasting is primarily implemented in the scheduling of the large generating units whose "start up" times may vary from a few hours ahead to a few days ahead. The scheduling process is termed unit commitment and ensures that there is sufficient operating generation capacity to meet the variable load demand with specified reliability **[1]**. When load forecasting is poor, incorrect scheduling may occur, resulting in higher daily operational cost caused by use of higher cost quick start units in the event of under-scheduling, or alternatively result in the uneconomic operation of large generating units in the event of over-scheduling **[44]**.

On-line operation of a power system, the economic load dispatching to various generating units makes the generating mix dependant on calculations to minimize the cost function which is based on the characteristics of the generating units. These calculations are based on values of load demand predicted a few hours in advance, and as such the optimum generating mix is dependant upon the accuracy of the on-line forecasts.

It has been recognized for long that accurate short-term load predictors as well as a load model are basic necessities for the optimum economic operation of power systems.

A prerequisite to the development of an accurate load-forecasting model is the understanding of the characteristics of the load to be modeled. This knowledge of load behavior is gained from experience with the load and through statistical analysis of past load data. Utilities with similar climatic and economic environments usually experience similar load behavior and load models developed for one utility can usually be modified to suit another.

2.2 Literature Review

A review of the literature on short term load forecasting indicates that many factors should be included in the load prediction model. Reference [1] reviews the short-term load demand modeling and forecasting for off-line and on-line implementation. Included also in [1] is a review of most techniques used at that time, the merits and drawbacks of each approach are presented. Reference [2] presents an algorithm based on curve fitting of past load growth for forecasting distribution system loads. The proposed algorithm in this reference uses clustering of historical load at the small area level as the forecast algorithm. References [3, 4] compare fourteen methods of forecasting future distribution system loads in terms of forecasting accuracy, data needs and resources. The tests of different forecast methods were carried out in as uniform a manner as possible. This reference claims that the selection of a forecast method is based on a great deal more criteria than those discussed in the reference. Data availability is usually an important factor, choice of a distribution load forecasting method may also be constrained by many other factors, including available computer resources and the level of expertise of the users.

Reference [5] reviews some of the existing studies on one-to-twenty four hour load forecasting algorithms, and presents an expert system based algorithm as an alternative. This algorithm is developed on the logical and syntactical relationships between weather and load, and prevailing daily load shapes. It has been found in this reference that the proposed algorithm is robust and accurate and has yielded results that are equally good, if not better, when compared to the regression based forecasting techniques.

Reference [6] presents an adaptive nonlinear predictor with orthogonal escalator structure for short-term load forecasting. The proposed method in this reference uses a nonlinear time-varying functional relationship between load and temperature. Parameters in both linear and nonlinear parts of the predictor are updated systematically using a scalar orthogonalization procedure. Matrix operations are avoided, in this reference, which results in a more robust and better numerical property algorithm. This reference claims that there is no need for seasonal off-line model calibration or modification since the proposed adaptive algorithm has good model tracking ability.

Analysis and evaluation of five short-time load-forecasting techniques are performed in Reference [7]. The five techniques are; (1) Multiple linear regression, (2) Stochastic time series, (3) General exponential smoothing, (4) State space and Kalman filter and (5) Knowledge based approach.

The use of a statistical decision function is implemented in Reference [8]. A hierarchical classification algorithm is applied to hourly temperature readings to divide the historical database into seasonal subsets. These subsets are identified statistically to fit a response function for each season. For a given day, an appropriate model is selected by performing discriminate analysis. It has been found in this reference that the proposed algorithm is less sensitive to extreme values than other algorithms. Also, the parameters should be updated periodically using the most recent seasonal subsets.

Reference [9] presents a robust model for forecasting power system hourly load. The method exploits the convenience of the auto-correlation function, and the partial auto-correlation function of the resulting differences previous load data identifying a sub-optimal model. The algorithm used in identifying the parameters of the proposed load is the iteratively reweighted least squares. Three-way decision variables in identifying an optimal model and the subsequent parameter estimates are used in this reference. These variables are; (1) the weighting function; (2) the tuning constant and; (3) the sum of the squared residuals.

Reference [10] presents formulation and analysis of short term load forecasting rule based algorithm. Load parameters are classified into weather and non-weather related values. The rules are the product of identifiable statistical relationship and expert knowledge. The forecasting algrithm puts smaller weight on the temperature effect and depends on the natural diversity of the load with a reduced or enlarged base.

A knowledge-based expert system is proposed in Reference [11] for short-term load forecasting of a power system. The expert system is developed using a 5-year database. Eleven load shapes, each with different means of load calculations, are established in this reference. The effect of weather variables, such as temperature and humidity on load forecasting is examined. The effect of thermal build-up is also studied. The proposed expert system is used to forecast the hourly loads of a power system over a whole year using the past five-year data base. This reference claims that the developed expert system can serve as a valuable assistant to system operators in performing their daily load forecasting duties.

Reference [12] describes a linear regression-based model for the calculation of the short-term system load forecasts. The model, in this reference, has the merits of; (1) innovative model building, including accurate holiday modeling by using binary variables; (2) temperature modeling by using heating and cooling degree functions; (3) robust parameter estimation and parameter estimation by using weighted least squares linear regression techniques; (4) the use of reverse" errors-in-variables" techniques to mitigate the effects of potential errors in the explanatory variables on load forecasts; and (5) distinction between time-independent daily peak load forecasts and the maximum of the hourly load forecasts in order to prevent peak forecasts from being negatively biased. Taken together, the above merits result in accurate, robust and adaptive response to changing conditions algorithm.

Reference [13] develops a composite load model for 1-24 hours ahead prediction of hourly electric loads. The load model, in this reference, is composed of three components: the nominal load, the type load and the residual load. The Kalman filter algorithm is used to estimate the parameters of the nominal load together with the exponentially weighted recursive least square method. The type load component is extracted for weekend load prediction and updated by an exponential smoothing method. The auto regressive model predicts the residual load and the parameters of the models are estimated using the recursive least square method.

In the last two decades ANN found wide applications in power system analysis and control. One of the successful applications is short-term load forecasting. References [14] and [15] present an approach using artificial neural network (ANN) for short-term load forecasting. In Reference [14] a neural network based on self-organizing feature maps to identify those days with similar hourly load patterns. The load patterns of several days in the past are averaged to obtain the load pattern of the day under study. The averaging days are of the same type of the day under study. In Reference [15] a feedforward multilayer neural network is designed to predict daily peak and valley loads. Once the peak load and valley load and the hourly load patterns are available, the desired hourly loads can be readily computed. The authors of these two references point out that the self-organizing feature map is capable of identifying a new type of load pattern before the operators can recognize the new day type.

An adaptive load-forecasting algorithm for one-hour-ahead time period has been developed in Reference [16]. The major enhancement is the ability to forecast total hourly system load as far ahead as five days. An important benefit of the adaptive algorithm is the ability to predict load shapes in addition to daily peak loads. System operators are able to utilize the predicted load shapes of several-hour-old one- day-ahead or five-day-ahead forecasts, even when the individual hourly errors are rather large.

Reference [17] presents an ANN method to forecast the short-term load for a large power system. The load is assumed to have two distinct patterns: weekday and weekend patterns. A nonlinear load model is proposed together with several structures of ANN. This reference claims that the neural network, when grouped into different load patterns gives good load forecast. It is found that the back propagation algorithm is robust in estimating the weights in nonlinear equation.

A multilayer neural network with an adaptive learning algorithm is proposed in Reference [18] for short-term load forecasting. Effects of learning rate, momentum and other factors on the efficiency and accuracy of the back propagation-momentum learning method are studied in this reference. The proposed adaptive learning algorithm converges much faster than the learning rate and the initial value of the momentum will not affect the conventional back propagation momentum learning method and the convergence property of the adaptive learning algorithm.

Reference [19] presents an improved neural network approach to produce short-term electric load forecasts. In this approach a minimum distance measurement is used to identify the appropriate historical patterns of load and temperature readings to estimate the network weights. By using this strategy the problem of holidays and drastic changes in weather patterns, are overcome. This algorithm also includes a combination of linear and nonlinear terms which map past load and temperature inputs into the load forecast output. This reference demonstrates that even a simple three-layer network produces results which are quite favorable compared to those typically seen in the literature, with smaller absolute errors.

Reference [20] reviews short term load forecasting techniques to find a standard for comparison. Size of error can be used as a measure for comparison standard.

Reference [21] presents a non-fully connected ANN model for short-term forecasting. The model used in this reference consists of one main ANN and three supporting ANNs. The main ANN is used to provide the models` basic forecast reference. Three supporting ANNs are added to increase the learning capacity of the proposed model. These supporting ANNs enable the model to better extract the relationships among different input categories, and achieve improved accuracy. In addition, three feedforward connections are established in the main ANN. These feedforward connections provide the most recent load and temperature references and greatly improve learning efficiency. It is found that the model, compared with a fully connected ANN, requires less training time and has better performance.

Reference [22] presents an expert system using fuzzy set theory for short-term load forecasting . The uncertainties in weather variables and statistical model are taken into account by using fuzzy set theory. Also incorporated into the system are the operator's heuristic rules. Two approaches based on the minimum-maximum algorithm and the equal-area criterion algorithm are proposed to determine the most desirable change in peak load from separate sources of fuzzy information.

The ANN model, in Reference [23] is claimed to be a useful tool for short-term load forecasting. Radically different from statistical methods, these models have shown promising results in load forecasting. Reference [23] concludes that, on the basis of the results obtained, there is no firm criterion to select a suitable network structure for a set of hourly load and temperature data. Models are not unique, and systems with different load characteristics require different structures. However, once a model is identified for a given system, the model need not be modified frequently. Neural network models are sensitive to bad data, so that intelligent data filtering techniques need to be designed in order to maintain acceptable accuracy in the ANN models based load forecasts.

Reference [24] presents a generalized short-term load-forecasting algorithm. This algorithm combines features from a knowledge base and statistical techniques. The technique is based on a generalized model for the weather-load relationship, which makes it site-independent. However, adding the site-dependent characteristics easily customizes it. Such characteristics are formulated in the form of selection and adjustment rules. Once added, these rules are expected to improve the performance of the algorithm for a specific site. The technique in this reference has been proven to be fairly robust, inherently updateable, and allows operator intervention if necessary. It does not require more than three years of past data.

Based on the attractive features of both distributed artificial intelligence and existing load forecasting techniques a distributed problem solving system for short-term load forecasting is presented in Reference [25]. Such a distributed paradigm is a multi-agent system, each processing agent of which can compute autonomously and cooperate with other agents to reason an accurate and satisfactory solution for load forecasting. The designed load forecasting system solves problems using three basic modules: a backboard module, knowledge sources, and a control mechanism. In this reference, the existing techniques are embedded in the domain knowledge source.

Reference [26] presents an algorithm using an unsupervised/supervised learning concept and the historical relationship between the load and temperature for a given season, day type and hour of the day to forecast hourly electric load with a lead time of 24 hours. An additional approach using functional link net, temperature variables, average load and the last one - hour load of previous day is introduced and compared with the ANN model with one hidden layer load forecast. Examination of load shapes indicated that the five working days, Saturdays, Sundays and holidays should be separately treated.

References [27] and [28] present the applications of ANN to short-term load forecasting. Reference [27] investigates the effectiveness of ANN in short-term load forecasting. It has been shown that the application of a combined solution using artificial neural networks and expert systems yields a good short-term load forecast which neither system alone can provide.

Reference [28] applies another type of neural network, called the radial basis function (RBF) network to the SLF. The results obtained using both radial basis function network and back propagation network indicate that the RBFN model performs better than the BPN model. It is claimed that the RBFN model can also compute reliability measures, which is an added advantage of the RBFN model. These measures provide confidence intervals for the forecasts and an extrapolation index to determine when the model is extrapolating beyond its original training data.

Reference [29] presents an adaptive neural network based short-term load forecasting system. The system accounts for seasonal and daily characteristics, as well as abnormal conditions such as cold fronts, heat waves, holidays and other conditions. The algorithm in this reference is capable of forecasting load with a lead-time of one hour to seven days. The adaptive mechanism is used to train the neural networks when on-line.

Reference [30] presents an adaptive auto-regressive moving average (ARMA) model for SLF of a power system. In this reference, the Box-Jenkins transfer function is considered as one of the better accurate methods, but it has limited accuracy without adapting the forecasting errors available to update the forecast. The adaptive approach first derives the error learning coefficients by virtue of minimum mean square error (MMSE) theory and then updates the forecasts based on the one-step-ahead forecast errors and the coefficients. The proposed algorithm in this reference can deal with any unusual system condition. It is shown that the proposed adaptive ARMA is more accurate than the conventional Box-Jenkins approach.

Reference [31] presents a survey for applying fuzzy systems in power systems. It discusses five forecasting methods. These methods are already presented in reference [24].

Reference [32] presents a highly adaptable and robust short-term load-forecasting algorithm. Adaptive general exponential smoothing augmented with power spectrum analysis is used to account for the changing base load component. The algorithm includes an adaptive auto-regressive modeling technique enhanced with partial auto-correlation analysis to model the random component of the load. The load consists of a base load, weather-sensitive load and random load components. The Akaike information criterion (AIC) is employed to generate model parsimony. The weighted recursive least square estimation algorithm with variable forgetting factors is applied to estimate model parameters. A nonlinear weather-sensitive model is used to present the influence of weather changes on energy consumption. This reference claims that the approach has the capacity to better track load changing patterns and the human intervention of this technique is a minimum, which enhances the suitability of the approach for online applications

Reference [33] presents a hybrid model for short-term load forecast that integrates artificial neural networks with fuzzy expert systems. The load is obtained in two steps. In the first step, the ANN's are trained with load patterns corresponding to the desired forecasted hour, and the trained ANN's obtains the provisional forecasted load. In the second step, the fuzzy expert systems modify the provisional forecasted load considering the possibility of load variation due to changes in temperature and the nature of the day if it is a holiday.

References [34, 35] present a fuzzy system for SLF. The fuzzy system has the net structure and training procedures of a neural network and is called neural fuzzy network (FNN). An FNN initially creates a rule base from historical load data. The parameters of the rule base are then turned through a training process, so that the output of the FNN matches the available historical load data adequately. Once trained, the FNN can be used to forecast future load.

Reference [36] proposes an optimal fuzzy inference method for short-term load forecasting. This reference constructs an optimal structure of the simplified fuzzy inference that minimizes model errors and the number of membership functions to grasp the nonlinear behavior of power system short-term loads. Simulated annealing and the steepest descent method identify the model parameters in this reference.

Reference [37] proposes an evolutionary programming (EP) approach to identify the parameters of an auto-regressive moving average with exogenous variable (ARMAX) model for one day to one-week ahead hourly load demand forecasts. The surface of the forecasting error function possesses multiple local minimum points. Solutions of the traditional gradient search based identification technique therefore may stall at the local optimal points, which results in an inadequate model. By simulating natural evolutionary process, the EP algorithm offers the capability of converging towards the global extreme of a complex error surface. The results obtained using this approach indicate that this algorithm provides a method to simultaneously estimate the appropriate order and parameter values of the ARMAX model for diverse types of load data.

Reference [38] presents the application of ANN to determine the short-term load forecasting while paying attention to accurate modeling of holidays. A single neural network with 24 output is used for the short term forecasting for all day types.

Reference [39] compares three techniques; fuzzy logic (FL), neural networks (NN) and auto-regressive (AR) for very short term load forecasting. The authors find a simple satisfying dynamic forecaster to predict the very short term load trends on line. FL and NN are good candidates for short term load forecasting. A neural network technique for electric load forecasting based on weather compensation is presented in References [40] and [41]. The method is a nonlinear generalization of Box and Jenkins approach for non-stationary time-series prediction. A nonlinear auto-regressive integrated (NARI) model is identified to be the most appropriate model to include the weather compensation in short-term electric load forecasting. A weather compensation neural network based on an NARI model is implemented for one-day ahead electric load forecasting. This weather compensation neural network can accurately predict the change of electric load consumption for the coming day. Based on the results obtained, the authors claim that this methodology is capable of providing more accurate load forecast.

Previous experience with basic ANN architectures have shown that, even though these architectures provide results comparable with these obtained by human operators for most normal days, they show some deficiencies in the accuracy when applied to "anomalous" load conditions occurring during holidays and long weekends [42]. Reference [42] proposes a specific procedure based upon a combined unsupervised/supervised approach. In the unsupervised stage a preventive classification of historical load data by mean of a Kohonen self organizing map is provided, while in the supervised stage, the proper forecasting activity is obtained by using a multi-layer perception with a back-propagation learning algorithm.

Reference [43] proposes a method for short-term load forecasting which would help demand side management. The proposed method is based on Kalman filtering algorithm with the incorporation of a "fading memory". The load is forecasted in two stages. In the first stage the mean is first predicted, while in the second stage, a correction is incorporated in real time using error feedback from the previous hours. The authors claim that the proposed algorithm is suitable for developing countries where the total load is not large, especially at substation levels, and the data available are grossly inadequate. In this reference, the fading-memory Kalman filter assigns variable weight to past data. This causes reduction of the dependence on data for back into the past, and also improves the accuracy of prediction to a certain extent. Also, it was suggested that the space for storage and the time taken for computation are both significantly low and make this method highly suitable for use in small computers.

Reference [44] compares two linear static parameter estimation techniques as they apply to the twenty-four hour off-line forecasting problem. Three 24-hours load models are used. The least error squares and the least absolute value based linear programming algorithm are the two parameter estimation approaches used to estimate the parameters of the three models. The three load models are (1) a multiple linear regression model, (2) a harmonic decomposition model and (3) a hybrid multiple linear regression/harmonic decomposition model. It is concluded that if the data source is free of errors, both techniques produce the same degree of accuracy for the three models. However, if the data source is contaminated with gross errors, then the use of the least absolute value criterion, will result in greater prediction accuracy.

A method of forecasting the hourly load demand on power system and uses threshold auto-regressive models with a stratification rule is presented in [45]. By using the threshold model algorithm, fewer parameters are required to capture the random component in load dynamics. Based on the results obtained, the authors conclude that : (a) the optimum stratification rule attempts to remove any judgmental input and to render the threshold process entirely mechanistic, (b) the simplicity of the proposed threshold auto-regressive model varies under different perspectives, such as the piecewise linear algorithms and the threshold procedures of the stratification to effectively handle non-stationary. Therefore, the simplicity consists of finding architectures, which are auto-regressive to model the non-linearity of the series, and economical in terms of parameters, (c) the high level of achievement is due primarily to a more accurate AR modeling in a threshold model, and the threshold AR model's ability to respond rapidly to sudden changes.

Reference [46] develops a forecasting model for one-day ahead. This model identifies a "normal" or weather-insensitive load component and a weather-sensitive load component-linear regression analysis of past load and weather is used to identify the normal load model. The weather-sensitive component of the load is estimated using the parameters of the regression analysis. In this reference, an automated load forecasting system is presented that includes adaptability to changing operational conditions, computational economy and robustness. Also, presented in this reference is the monthly error statistics of forecast load for only one day ahead for recorded weather conditions.

Reference [47] presents a functional-link network based short-term electric load forecasting system for real time implementation. The load and weather parameters are modeled as nonlinear ARMA process and parameters of this model are obtained using the functional approximation capabilities of an auto-enhanced functional link network. The adaptive mechanism with a nonlinear learning rule is used to train the link network on-line. The results obtained in this reference indicate that the functional link net-based load forecasting system produces robust and more accurate load forecasts in comparison to simple adaptive neural network or statistical based approach.

Reference [48] describes a load forecasting system called ANNSTLF (Artificial Neural Network Short-term Load Forecasting). This system is suggested to be used now by many utilities across North America. The effects of temperature and relative humidity on the load are considered. ANNSTLF contains also forecasts that can generate the hourly temperature and relative humidity forecasts needed by the system. ANNSTLF is based on a multiple ANN strategy that captures various trends in the data. The building block of the forecasters is a multilayer neural network trained with the error back-propagation learning rule. To adjust the ANN weights during on-line forecasting, an adaptive scheme is employed. The forecasting models are site independent and only the number of hidden layer nodes of ANN's need to be adjusted for a new database.

Reference [49] presents a "Quasi Optimal" neural network to solve the short-term load forecasting problem. Rules for building a "quasi optimal" neural network to solve the STLF are derived. It is demonstrated that the "quasi optimal" neural network is superior to an automated Box-Jenkins seasonal ARIMA model in solving the STLF problem. Most significantly, the authors demonstrate how orthogonal fractional factorial designs can be used to understand how technical issues that arise in creating a neural network affect singularly, and in pairs, the performance of the network is solving the STLF problem.

An algorithm using cascaded learning algorithm together with the historical load and weather data is presented in [50] to forecast half-hourly load for the next 24-hours. This cascaded neural network algorithm (CANN's) includes peak, minimum, and daily energy prediction as additional input data for the final forecast stage. These additional input data are predicted using the first ANN's model.

The use of a weighted least square procedure when training a neural network to solve the short-term load forecasting problem is presented in [51]. It is shown that a neural network that implements the weighted least squares procedure outperforms a neural network that implements the least squares procedure during the on-peak period for the two performance criteria specified; mean absolute error and cost, and during the entire period for the cost criterion. This reference has shown the potential benefit of using a cost-based weighted least squares training approach.

Reference [52] postulates that the load can be modeled as the output of some dynamic system influenced by a number of weather, time and other environmental variables. Recurrent neural networks, being members of a class of connectionist models exhibiting inherent dynamic behavior, can thus be used to construct empirical models for this dynamic system. This reference claims that due to the nonlinear dynamic nature of these models, the behavior of the load prediction system can be captured in a compact and robust representation.

Reference [53] presents a self organizing model of fuzzy auto regressive moving average with exogenous (FARMAX) variables for one day ahead hourly load forecasting of power systems. A comparison between the existing and ARAMAX model values shows reduction in error for forecasting results.

An efficient modeling technique based on fuzzy curve notation is presented in References [54] to generate fuzzy models for short-term load forecasting. The steps in this approach are: (a) prediction of the load curve extremals (peak and valley loads) using separate fuzzy models, (b) formulation of the representative day load curve to the forecasted peak values to obtain the predicted day load curves, and (c) transformation of the representative day load curve to fit the forecasted peak and valley loads in order to obtain the final next days' load curve forecast.

References [55] presents an approach to short-time load forecasting by the application of non-parametric regression. The method is derived from a load model in the form of a probability density function of load and load affecting factors. A load forecast is a conditional expectation of load given the time, weather conditions and other explanatory variables. This forecast can be calculated directly from historical data as a load average of past observed loads with the size of the local neighborhood and the specific weights on the load defined by a multivariate product kernel. The procedure requires a few parameters that can be easily calculated from historical data by applying the cross-validation technique.

Reference [56] describes a method for input variable selection for artificial neural network (ANN) based short-term load forecasting (STLF). The method is based on the phase-space embedding of a load time-series. The accuracy of the method is enhanced by the addition of temperature and cycle variables. This reference compare it favorably to the ones reported in the literature, indicating that a more parsimonious set of input variables can be used in STLF without sacrificing the accuracy of the forecast. This allows more compact ANNs, smaller training sets and easier training.

Reference [57] studies a short-term electric load forecasting technique using a multi-layered feedforward ANN and a fuzzy set-based classification algorithm. The hourly data is subdivided into various classes of weather conditions using the fuzzy set representation of weather variables and then the ANNs are trained and used to perform the load forecasting up to 120 hours ahead accurately.

Reference [58] presents an architecture which is substantially changed from the earlier neural network techniques. It includes only two ANN forecasters, one predicts the base load and the other forecasts the change in load. The final forecast is computed by adaptive combinations of these two forecasts. The effects of humidity and wind speed are considered through a linear transformation of temperature. This algorithm significantly improves the accuracy of the holiday forecasts.

Reference [59] presents a method that is suitable for power system operational planning studies. Bayesian estimation is used to predict multiple step ahead peak forecasts using peak and average temperature forecasts as explanatory variables. Furthermore, the authors claim that better results can be obtained, with more attention paid to model identification.

Reference [60] describes the application of ANN in forecasting short term load using a multilayer perceptron. ANN combines both time series and regression approaches to predict load demand. A functional relatioship between weather variable and electrical load is not needed because ANN can generate the functional relationship in learning and training the data.

A fuzzy modeling method is developed in Reference [61] for short-term load forecasting. In this method, identification of the premise part and consequent part is separately accomplished via the orthogonal least square (OLS) technique. The OLS is first employed to partition the input space and determine the number of fuzzy rules and the premise parameters. In the sequel, a second orthogonal estimator determines the input terms that should be included in the consequent part of each fuzzy rule and calculate its parameters. Different models are developed for each day type in every season.

Reference [62] presents a self-supervised adaptive neural network to perform STLF for a large power system covering a wide service area with several heavy load centers. The self-supervised network is used to extract corrolation features from temperature and load data. The authors` design provides a good adaptability using a rapid, on-line training mode that is crucial in applications, where the source statistics are non-stationary or where the forecaster is used with different power systems.

The behaviour of electric power systems and networks varies considerably due to their characteristics. There does not appear to be one forecasting method that fits all power systems. In fact, the electric load on each system may be forecasted using different techniques to suit different situations.

Chapter 3

Short Term Load Forecasting

3.1 Introduction

In short-term load forecasting, the future load on a power system is predicted by extrapolating a pre-determined relationship between the load and its influential variables, namely time and/or weather. Determining this relationship is a two-stage process that requires (a) identifying the relationship between the load and the related variables, and (b) quantifying this relationship through the use of a suitable parameter estimation technique.

A prerequisite to the development of an accurate load-forecasting model is an in depth understanding of the characteristics of the load to be modeled. This knowledge of the load behavior is gained from experience with the load and through statistical analysis of past load data. Utilities with similar climatic and economic environments usually experience similar load behavior and load models developed for one utility can usually be modified to suit another [44].

The review of the literature on short-term load modeling of chapter 2, indicates that the load supplied by a power system is dynamic in nature and directly reflects the activities and conditions in the surrounding environment. This load can be separated into a standard or base load, a weather dependent load and a residual load. In the following sections the characteristics of each of these components are reviewed in turn [7, 44].

3.2 Base Load [44]

This load results from the business and economic conditions of the service area, and is the largest component of total system load. It accounts for approximately 90% of total load and can be spectrally decomposed into four distinct components, namely:

(a) A long-term component that reflects the economic growth of the area and is usually directly proportional to the growth of the national economy.

(b) A seasonal component that results from changes in electricity demand from one season to another. In North America this load pattern is characterized by midwinter and midsummer peaks inter-spaced by troughs occurring during the central spring and autumn seasons.

(c) A weekly load cycle that results from the consumption pattern of one day of the week being characteristically different from the others. Weekly business cycles and repetitive local activities are the main reasons for this aspect of load behavior that is characterized by relatively constant mid-week demands and smaller weekend loads.

(d) A daily load cycle that results from the basic daily similarity of consumer activities. Low early morning demand peaking at mid-afternoon high usually characterizes this load cycle.

3.3 Weather Dependant Load [44]

The weather contributes significantly to the dynamics of the load, and much effort was spent to find a viable relationship between the weather and the load, so that an accurate load model could be developed. The survey of the literature in the second chapter, indicates that each utility has its own load model that depends on the weather of that load the utility is serving, and a load model for a utility does not necessarily suit (fit) the load of another utility.

The effects of weather on load are usually modeled by expressing the load as a linear regression of explanatory meteorological factors such as temperature, wind speed, humidity etc. While it is recognized that an extremely wide variety of explanatory weather variables are required to totally represent the effects of weather, studies have shown that a few basic meteorological factors usually account for most of the weather dependent load.

The specific weather variables that are normally used to model weather dependent load are dry bulb temperature, wind-speed, humidity and daylight illumination. The latter is usually the least significant of these weather variables and since its metering is difficult and costly, it is usually omitted from most models. The general effects of these weather variables on load are summarized next **[7, 44]**.

3.3.1 Temperature

In most load environments, dry bulb temperature is the most significant weather variable and usually accounts for the largest percentage of weather dependent load. Deviations of temperature from the norm can result in major changes in the load pattern. These changes however, do not occur immediately, but are rather delayed due to thermal storage in buildings.

The effects of temperature on load pattern are not uniform and are different from one utility to another and from one season to the next. A decrease in temperature below room temperature during the winter season means an increase in the heating load, but an increase in the temperature above room temperature during summer means increasing of air conditioning load (increasing the cooling load).

Temperature effects are usually modeled by considering the load to be a function of the effective temperature or temperature deviation, rather than the actual temperature. This stems from the realization that the general effects of base temperature are already included in the seasonal load cycle and only deviations from the norm will result in load changes.

In other words, each utility company designs the base load according to the normal temperature of the environment of that load, and any temperature deviation will lead to changes in the load.

3.3.2 Wind Speed

A factor that can contribute significantly to the weather dependent load is wind. Wind effects are especially prevalent during winter and are a direct consequence of the cooling power of the wind. The cooling effect of the wind depends on the wind speed and the dry bulb temperature. The heat loss from a building is proportional to the product of the square root of the wind speed and the temperature deviation from the comfort level of approximately $18^{O}C$. This effect is relatively small in post winter seasons and for simplicity, are usually only included in winter models **[44]**.

Some researchers prefer to use the wind-chill factor as a means of representation of the wind in their models, since a wind-chill factor is often strongly correlated with winter load [7]. Others contend that the wind-chill factor is only a measure of the discomfort level of the wind and temperature and as such, is not a true index for gauging the resulting load response [7, 44]. High wind-chills however, have the psychological effect of causing people to turn up their thermostats.

3.3.3 Humidity

A weather variable that greatly influences air conditioning and other related cooling loads in summer, is the level of humidity in the atmosphere. The effects of high humidity are generally only noticeable when the temperature is quite high, usually above room temperature. The humidity effects can be considered in the load model by representing it as a function of relative humidity, the temperature humidity index or the dew point temperature. The most common variable used in the literature is the humidity index.

The temperature humidity index is a measure of the discomfort level or equivalent heat stress in summer and depends on both the temperature and relative humidity, and normally shows greatest correlation with summer load and only influences the load above a predetermined cutoff temperature.

3.3.4. Illumination

Daytime illumination has a small effect on the load model, compared to the other two previously discussed factors. Surveying the literature shows that in most cases this factor is often omitted from most load models.

Low daytime illumination can cause an increase in daytime lighting load and advance the effects of nightfall, thereby altering the evening load pattern. This factor is influenced by such weather conditions as cloud cover, dust, fog, haze etc… is the measure for the level of luminous radiation received at ground level.

3.4. Residual Load [44]

This load component occurs in load modeling and usually accounts for a small percentage of total load and results from irregularities in the behavior of the consuming public. Abnormal consumer demands, though quite frequent in occurrence, are very difficult to model and predict and are not accounted for, in most load models.

The common factors of unpredictable load behavior range from public response to major television events, strikes, storms, disasters, time changes etc.

3.5 Short-Term Load Forecasting Models [7, 9, 12, 16, 50]

Reviewing short-term load forecasting methods indicates that the most important modeling techniques used, can be classified in one of the following categories :

(1) Multiple linear regression,

(2) General exponential smoothing,

(3) Stochastic time series,

(4) Expert systems approach, and

(5) State space model.

These models are classified on the basis of the name of the underlying mathematical technique used to estimate the parameters of the model. The preceding classifications are not unique and the one used with one utility is not necessarily suitable for another. However, one can combine these models or can use one model to initiate another model to predict certain parameters from past history. With unknown information about the load, these techniques can be combined to improve the accuracy of the forecast. Also, each model possesses distinct advantages and disadvantages compared to each other. In the following subsections, the first three methods are reviewed, while the last two methods are beyond the scope of this research.

3.5.1 Multiple Linear Regression [44]

This is the earliest technique of load forecasting methods. Here, load is expressed as a function of explanatory weather and non-weather variables that influence the load. The influential variables are identified on the basis of correlation analysis with load, and their significance is determined through statistical tests such as the False and True tests.

Mathematically the load model using this approach can be written as:

$$y(t) = a_0 + \sum_{i=1}^{n} a_i x_i(t) + r(t) \tag{3.1}$$

where y(t) is the load value at time t, $x_1(t),\ldots,x_n(t)$ are explanatory variables, r(t) is the residual load at time t and a_i are the regression parameters relating the load y(t) to the explanatory variables. Previous analysis that uses this model treats a_i as a crisp number

If the number of observations equals exactly the number of parameters to be estimated, then r(t) is forced to zero. Eq. (3.1) becomes

$$y(t) = a_0^* + \sum_{i=1}^{n} a_i^* x_i(t) \tag{3.2}$$

where the asterisk indicates the optimal estimated values of the parameters.

The multiple linear regression technique has found greatest application as an off-line forecasting method and is generally unsuitable for on-line forecasting, as it requires many external variables that are difficult to introduce into an on-line algorithm [44].

These models are relatively simple to apply but require extensive initial analysis to identify the regressors and their place in the model. Also because the relationship between the load and weather variable is time specific, this model requires continuous re-estimation of its parameters to perform accurately.

3.5.2 General Exponential Smoothing [7, 44]

In this technique the load is modeled using a time dependent fitting function that satisfies the relationship:

$$f(t) = L\, f(t-1) \tag{3.3}$$

where f(t) is the fitting function at time t, and L is a constant matrix called the transition matrix [44]. Mathematically the model is expressed as:

$$y(t) = \beta(t)\, f(t) + r(t) \tag{3.4}$$

where y (t) =load at time t, β (t) = coefficient vector at time t and r (t) = residual load or noise at time t.

The parameter vector is estimated from a data window of previous observations using least errors square minimization technique(LES). The estimated parameter vectors are obtained by minimizing the cost function

$$J = \sum_{j=0}^{N-1} w^j [y(N-j) - f(-j)\, \beta]^2 \tag{3.5}$$

where w is called the weighting factor, and (1-w) is called the smoothing constant. The parameter vector that minimizes the cost function J can be written as :

$$\beta\hat{}(N) = F^{-1}(N)\, h(N) \tag{3.6}$$

where

$$F(N) = \sum_{j=0}^{N-1} w^j f(-j) f^T(-j) \tag{3.7}$$

and

$$h(N) = \sum_{j=0}^{N-1} w^j f(-j) y(N-j) \tag{3.8}$$

The forecast at a lead time *l*, is then given by :

$$y\hat{}(N+l) = f(l)\, \beta\hat{}(N) \tag{3.9}$$

and the parameters of the forecasts can be updated using

$$\hat{\beta}(N+1) = L^T \hat{\beta}(N) + F^{-1} f(0)[y(N+1) - \hat{y}(N)] \tag{3.10}$$

and

$$\hat{y}(N+1+l) = f^T(l)\,\hat{\beta}(N+1) \tag{3.11}$$

This method can be used for both on and off-line forecasting though its recursive nature and generally poor long-range accuracy makes it much more suitable for on-line forecasting. The low accuracy encountered for longer lead times stems from the fact that this technique cannot use related weather information and so this technique cannot account for weather related load changes. Simplicity, re-cursiveness and economical usage , however, make this technique a very attractive forecasting tool in practice.

3.5.3 Stochastic Time Series [7, 44]

In this method the load is modeled as the output of a linear filter driven by white noise. Depending on the characteristics of the linear filter, different load models can be formulated.

The autoregressive and moving average processes are the two simplest forms of stochastic time series and though neither of these processes is usually individually capable of accurately modeling the load, they form the basis for development of more complex processes.

In the autoregressive (AR) process the current value of load is expressed linearly in terms of previous values and a random noise. The order of this process depends upon the oldest previous value for which the load is regressed. The moving average process on the other hand expresses the load linearly in terms of current and previous values of a white noise series and again the order of the series depends upon the oldest previous value.

The auto-regressive and the moving average processes are usually combined to give the popular ARMA or auto-regressive moving average process, which has found widespread use in the power industry. In the ARMA process, the load at any instant is expressed as a linear combination of its past values and a white noise series. The order of this process is specified by the order of the AR and MA series included in its composition **[1]**.

Time series used for AR, MA or ARMA models are referred to as stationary processes when their means and covariances are stationary with respect to time. So if the process being modeled is non-stationary, then it is firstly transformed to a stationary series before being modeled by AR, MA or ARMA process [1].

Making a non-stationary process into a stationary one is accomplished by the method of differencing and the order of a differencing process refers to the number of times the process has been differenced before achieving stationarity. Differenced processes modeled as AR, MA or ARMA are now called integrated processes and are relabeled ARI, IMA and ARIMA.

The auto-regressive integrated moving average or ARIMA process, like the ARMA process is a very popular load modeling technique that produces very accurate load forecasts. For longer lead times, however, a seasonal or periodic component must be included into these processes. This results in what is known as a seasonal process and the abbreviations SARMA and SARIMA are now used [44].

The lack of weather input into time series models usually limits their forecasting ability. By expressing these processes in transfer functions form makes it possible to add some weather information. This is usually limited to the single-most influential variable, that is temperature, which generally accounts for most of weather induced load [1].

The popularity of the stochastic time series approach in on-line forecasting stems mainly from the level of accuracy available and their ease of on-line implementation. The identification process of the time series models is a major disadvantage since the process requires extensive analysis of raw load data through the use of range-mean, correlation and auto-correlation analysis.

3.5.4 Qualities of Forecasting Models

The review of short-term load forecasting methods indicates that depending on the forecasting technique employed, many different load models can be developed to predict the same load. For these models to be considered good or efficient, however their formulation must feature certain basic qualities and their performance must be within tolerable limits.

The literature indicates that some of preferred qualities in a load-forecasting algorithm include adaptiveness, recursiveness, economy, robustness and accuracy [44].

Adaptivness

The parameters of a short-term load-forecasting model are usually estimated from a fixed window of data and are only accurate for a specified period of time ahead. As the forecast period elapses and new measurements become available, the algorithm should be able to automatically update its data window and recompute its estimates.

Recursiveness

As new data such as weather and load measurements become available the algorithm should be able to correct its forecasts and prediction for the next step.

Computational Economy.

The pursuit of accuracy can lead to very complicated models that require the use of excessive computing facilities. A forecasting algorithm however, should attempt to be computationally efficient with regards to execution time and care utilization.

Robustness.

An algorithm should be robust to miss-specification and erroneous data. i.e. reasonable forecasts should be produced even if the model is predicting for conditions for which it was not designed, or even if its database is contaminated with bad or anomalous data.

Accuracy

The performance of a short-term load-forecasting algorithm depends largely upon the forecasting lead-time as well as upon such factors as load behavior and model type.

For a model with a 24-hours prediction period errors in the range of 2-3 % are considered normal, whereas for models with lead-time of one hour the same error is considered large. Models with longer lead-time than 24-hours show reduced accuracy and for a lead-time of one week, accuracies within 10% are to be expected.

3.6 Load Forecasting Models [44]

In short-term load forecasting, the future load on a power system is produced by extrapolating a pre-determined relationship between the load and its influential variables, namely time and/or weather information. Determination of this relationship is a two stage process that requires

(a) identifying the relationship between the load and related variables, and (b) quantifying this relationship through the use of a suitable parameters estimation technique.

In order to study the effects of parameter estimation techniques on short-term load forecasting accuracy , it is necessary to identify and develop suitable load models that will allow for the application of these estimation techniques.

In the next sections, load models are developed for crisp parameters. These models will be used in both summer and winter forecasting modes and as such, where applicable, winter and summer load formulations are included. In chapter five, fuzzy models are developed for winter and summer loads and the techniques used to estimate these fuzzy parameters are discussed in chapter four. In this part of the chapter, crisp models are presented and discussed. These three models are developed in [44] for off-line load models. The parameters are assumed to be crisp. Modifications will be carried out, if necessary, on these models for the fuzzy type models, as will be seen in subsequent chapters.

The models will be referred to as A, B and C, respectively. Model A is developed on the basis of multiple linear regression, whereas model B is developed on harmonic basis, furthermore, model C is a hybrid one that embodies both properties of models A and B. These models are developed to forecast for twenty-four hours ahead .

3.6.1 Model A

This is a multiple linear regression model that expresses the load at any discrete time instant t as a function of a base load and a weather dependent component. The base load is assumed to be constant for each discrete time interval. The variable part of the load is weather dependant .

This model will be used for both winter and summer load forecast simulations, and since the relationship between load and weather differs significantly over these two seasons, a different load formulation will be required in each case. This will result in two load models, namely a winter model and a summer model.

These models are based on the assumption that a common daily base load cycle is experienced by week days and that a constant but different base load cycle is experienced by weekend days, namely Saturday and Sunday. As such, two models are required to predict loads over a complete week, i.e. one for predicting weekday loads and one for predicting weekend loads. Correlation analysis of load and temperature deviations from the norm indicates that the load to be modeled depends on both immediate and previous values of temperature deviations. This correlation however, is strongest for immediate values of temperature deviations and dies out in approximately 72 hours.

The wind-chill and wind cooling factors also display similar relationship in winter, as does the temperature and humidity in summer. The wind-cooling factor however, was selected in favor of wind-chill factor, as it generally results in smaller prediction errors during forecast trial [44].

Based on early analyses, initial winter and summer models were formulated and tested in off line simulation mode. The following two load models formulations were selected [44].

3.6.1.1 Winter Model

Mathematically, the load at any discrete instant t, where t varies from one to twenty-four, can be expressed as:

$$\begin{aligned} Y(t) = {} & a_0(t) + a_1(t)\,T(t) + a_2(t)\,T^2(t) + a_3(t)\,T^3(t) \\ & + a_4(t)T(t-1) + a_5(t)T(t-2) + a_6(t)T(t-3) \\ & + a_7(t)\,W(t) + a_8(t)\,W(t-1) + a_9(t)W(t-2) \end{aligned} \tag{3.12}$$

Where

$Y(t)$ = load at time t, $t=1,2\ldots24$

$T(t)$ = temperature deviation at time t

$W(t)$ = wind cooling factor at time t

$a_0(t)$ = base load at time t, and

$a_1(t), a_2(t), \ldots, a_9(t)$ are the regression parameters to be estimated at time t.

The temperature deviation at the instant t, is calculated as the difference between the dry bulb temperature at time t, and the average dry bulb temperature of the previous twenty weekdays (four weeks) temperature measurements, corresponding to the same discrete instant, i.e.

$$T(t) = T_d(t) - T_a(t) \tag{3.13}$$

Where

T_d (t) is the dry bulb temperature at time t, in ^{O}C

T_a (t) is the average dry bulb temperature at time t,

$$T_a (t) = [T_d(t-24)+ T_d (t-48) +\ldots+ T_d (t-480)] /20 \quad (3.14)$$

It should be noted, that equations (3.13) and (3.14) refer to a database consisting only of weekday temperature recordings.

The wind-cooling factor is calculated from

$$W (t) = [18- T_d (t)] [V (t)]^{1/2} \quad (3.15)$$

Where V (t)= wind speed in km/h at time t

3.6.1.2 The Summer Model

The winter equivalent of the load model given by equation (3-12) can be modified to become

$$\begin{aligned} Y (t) = {} & a_o(t)+ a_1 (t) T (t) +a_2 (t) T^2(t) + a_3(t)T^3(t) \\ & + a_4(t)T(t-1) + a_5 (t)T(t-2) + a_6(t)T(t-3) \\ & + a_7(t) H(t) + a_8(t) H(t-1) + a_9(t)H(t-2) \end{aligned} \quad (3.16)$$

where

Y (t) = load at time t

T (t) = temperature deviation at time t.

a_0 (t) = base load at time t.

$a_1(t),a_2(t),\ldots,a_9(t)$ are the regression parameters to be estimated at time t.

The temperature deviation is calculated as for the winter model.

The humidity factor H(t), that replaces the wind cooling factor in the winter model, is given by

$$H(t) = 0.55\, T_d(t) + 0.2\, T_p(t) + 5.05 \quad (3.17)$$

where

$T_p(t)$ = dew point temperature at time t, in ^{O}C.

The humidity factor H (t) is set to zero if the dry bulb temperature is less than twenty-five degrees Celsius, since at temperatures less than room temperature, the humidity effects are negligible.

Equations (3.12) and (3.16) give the multiple linear regression models for the load in winter and summer day. As such, it is required to estimate twenty-four parameters (24 sets of the parameters) to predict the next day hourly load profile.

Equations (3.12) and (3.16) can be written in compact form as:

$$Y(t)= f^{T}(t)x(t) \tag{3.18}$$

Where f (t) is a fitting function given by

$$f(t) = \begin{bmatrix} 1 \\ T(t) \\ T^2(t) \\ T^3(t) \\ T(t-1) \\ T(t-2) \\ T(t-3) \\ W(t) \\ W(t-1) \\ W(t-2) \end{bmatrix} \tag{3.19}$$

in winter, and

$$f(t) = \begin{bmatrix} 1 \\ T(t) \\ T^2(t) \\ T^3(t) \\ T(t-1) \\ T(t-2) \\ T(t-3) \\ H(t) \\ H(t-1) \\ H(t-2) \end{bmatrix} \tag{3.20}$$

in summer. Moreover x(t) is the parameters vector to be estimated and is given by

$$X(t) = \begin{bmatrix} a_0(t) \\ a_1(t) \\ . \\ . \\ . \\ a_7(t) \\ a_8(t) \\ a_9(t) \end{bmatrix} \qquad (3.21)$$

In this chapter, the parameter vector of equation (3.21) is assumed to be crisp (vector with constant values at time t). In chapter five, this vector will be assumed to be fuzzy (A vector with certain middle and certain spread).

The corresponding parameters X(t) at any given discrete interval are estimated using the previous four weeks of weekday data corresponding to the discrete instant. The overdetermined system of equations corresponding to the estimates at the instant t, will read

$$\begin{bmatrix} y(t-24) \\ y(t-48) \\ y(t-960) \end{bmatrix} = \begin{bmatrix} f(t-24) \\ f(t-48) \\ f(t-960) \end{bmatrix} X(t) \qquad (3.22)$$

Equation (3.22), which involves crisp parameters estimation, can be solved using an appropriate estimation technique. After estimating the parameter vector X(t), it can be substituted into equation (3.12) or (3.16) to obtain the load prediction for time t.

3.6.2 Model B

The load type of this model is expressed as a function of a constant base load and a Fourier harmonic series. It was discovered from studying early load data that there is a presence of a weekly load cycle that is characterized by distinct daily periodicities.

In this model however, the weekly cycle is accounted for, by the use of a daily load model, whose parameters are estimated seven times weekly. Since this load does not take weather into consideration , a single load model will suffice for both winter and summer simulations.

Therefore the load at any time t is

$$y(t) = a_o + \sum_{i=1}^{N} [a_i \sin(i\omega t) + b_i \cos(i\omega t)] \quad (3.23)$$

$$y(t) = a_o + \sum_{i=1}^{N} c_i \sin(i\omega t + \phi_i) \quad (3.24)$$

where $c_i = \sqrt{a_i^2 + b_i^2}$

$\tan \phi_i = b_i / a_i$

Equation (3.23) is the most suitable form to model the load, since it is a linear equation in the parameters to be estimated. In equation (3.23) :

$y(t)$= the load at time t

N = number of harmonics to be chosen

$\omega = 2\pi / 24$

a_o = constant base load for each day of the week, and

a_i, b_i, $i=1, \ldots, N$ are the parameters corresponding to the harmonics in the load composition.

To predict the hourly load profile for any day of the week, an overdetermined system of equations is set up using data from the previous four weeks corresponding to the day in question.

Equation (3.23) can be written as

$$y(t) = f^T(t)\mathbf{X} \quad (3.25)$$

where

$$f(t) = \begin{bmatrix} 1 \\ \sin\omega t \\ \cos\omega t \\ . \\ . \\ . \\ \sin N\omega t \\ \cos N\omega t \end{bmatrix} \tag{3.26}$$

and

$$X = \begin{bmatrix} a_0 \\ a_1 \\ b_1 \\ . \\ a_N \\ b_N \end{bmatrix} \tag{3.27}$$

The overdetermined system of equations can now be written as:

$$\begin{bmatrix} y(t-168) \\ . \\ . \\ y(t-192) \\ y(t-336) \\ . \\ . \\ y(t-672) \end{bmatrix} = \begin{bmatrix} f^T(t-168) \\ . \\ . \\ f^T(t-192) \\ f^T(t-336) \\ . \\ . \\ f^T(t-672) \end{bmatrix} [X] \tag{3.28}$$

Having obtained the parameter vector x, then equation (3.23) can be used to forecast for the next twenty-four hours.

3.6.3 Model C

This model consists of the sum of a time-varying base load and a weather dependent load. This model is developed to eliminate the disadvantages of the previous two models A and B.

Model A has the advantage of being weather responsive, but suffers the disadvantages of requiring (a) twenty-four separate parameter estimates in order to predict the next day load, and (b) the use of weekday and weekend both with winter and summer formulations.

Model B requires using of a single model formulation and hence it estimats a single parameters vector in order to predict the next day load, however, it suffers the disadvantage of being weather insensitive.

Models A and B are combined to form model C to obtain a computationally efficient and weather sensitive model. This new model will eliminate the use of separate weekday and weekend models, as is the case with model A. Also by limiting the weather input to temperature only, a single load model could be used for both winter and summer load forecast simulations. The main disadvantage of model C is its assumption of constant relationship between load and weather for all times of the day. However, if there is a set of parameters for every hour, the model becomes computationally inefficient.

Mathematically load model C can be expressed at any discrete time instant as

$$y(t) = a_o + \sum_{i=1}^{N} [a_i \sin(i\omega t) + b_i \cos(i\omega t)] + c_o T(t) + c_1 T(t-1) + c_2 T(t-2) + c_3 T(t-3) \quad (3.29)$$

where T(t) is the temperature deviation at time t, and is given by

$$T(t)=T_d(t)-T_c(t) \quad (3.30)$$

where $T_c(t)$ is the average dry bulb temperature for the discrete instant t, calculated from the previous twenty-eight daily temperature measurement corresponding to the discrete instant, i.e.

$$T_c(t) = [T_d(t-24) + \ldots\ldots + T_d(t-672)]/28 \quad (3.31)$$

Equation (3.29) can be written in vector form as

$$y(t) = f^T(t)X \tag{3.32}$$

where

$$f^T(t) = [1 \ \sin\omega t \ \cos\omega t \ldots \sin N\omega t \ \cos N\omega t \ T(t) \ T(t-1) \ldots\ldots T(t-3)] \tag{3.33}$$

and

$$X^T = [a_o \ a_1 \ b_1 \ \ldots\ldots\ldots a_N \ b_N \ c_o \ c_1 \ \ldots\ldots c_3] \tag{3.34}$$

and the parameters vector X can be estimated as for model B, i.e. from the system of equations given by:

$$\begin{bmatrix} y(t-168) \\ \cdot \\ \cdot \\ y(t-192) \\ y(t-336) \\ \cdot \\ \cdot \\ y(t-672) \end{bmatrix} = \begin{bmatrix} f^T(t-168) \\ \cdot \\ \cdot \\ f^T(t-192) \\ f^T(t-336) \\ \cdot \\ \cdot \\ f^T(t-672) \end{bmatrix} [X] \tag{3.35}$$

The next day forecast can then be done, by substituting for X and the predicted values of temperature deviation into equation (3.29).

3.7 Conclusions

In this chapter models used for short-term load forecasting are presented. Three models are proposed, namely model A, B and C. Model A is a multiple linear regression model. From model A two models are derived, the first can be used for winter load forecasting while the second can be used for summer load forecasting. Model B is a Fourier series model. It is not weather sensitive. Finally model C is a combination of the multi-regression model A and the Fourier series model B. In this model C the effects of temperature deviations are taken into account.

In the three models, the parameters to be estimated are assumed to be constants during the time interval considered and have crisp values. In chapter five, the fuzzy load models used for short term load forecasting are presented. The parameters in these models are assumed to be fuzzy numbers having certain middle and spread values.

Chapter 4
Static State Estimation

4.1 Introduction

The purpose of this chapter is to study the static state estimation problem. The first part of this chapter discusses the static estimation problem, when the observations available are crisp measurements. Two techniques are discussed for the estimation process. The first technique is based on the least error squares (LES) criterion, while the second technique is based on the least absolute value (LAV) criterion. In the second part of this chapter, the fuzzy estimation problem is discussed. Two problems are discussed in this section. The first problem, the output data are non-fuzzy data, while the parameters of the explanatory function are fuzzy parameters. In the second problem, the output data are fuzzy and the parameters of the explanatory function are fuzzy.

State estimation is the process of assigning a value to unknown system state variables and filtering out erroneous measurements before they enter into the calculation process. A familiar criterion in state estimation is the least error squared (LES) which is the minimization of the sum of squares of the difference between the estimated and true (measured) value of the function. Another technique of state estimation is based on minimizing the absolute value of the difference between the measured and calculated quantities, and it is called the least absolute value (LAV). These techniques require excessive computer memory space and long computer time The main advantage of the LAV algorithm is its ability to reject the bad data points in the estimation process, i.e. it is insensitive to the outliers. A non-iterative method was developed in **[64 - 65]** to solve the least absolute value problem. This method uses the least error squares solution as a starting point. The steps of this method are explained within this chapter.

The main objectives of this chapter are to introduce the static estimation problem and the different techniques used to solve it.

4.2. Static Estimation Problem, Crisp Linear Estimation [63, 64]

The static estimation problem can be simply stated as: given the system measurement linear equation

$$\underline{z} = H\underline{\theta} + \underline{v} \qquad (4.1)$$

Where $\underline{z}$ is a m×1 vector of system measurements (known).

$\underline{\theta}$ is a n×1 vector of parameters to be estimated (unknown).

H is a m×n matrix describes the mathematical relationship between the measurements and the system parameter vector (known)

and $\underline{v}$ is a m×1 vector of measurement errors (unknown) to be minimized.

It is required to estimate the parameter vector $\underline{\theta}$, which minimizes the error vector $\underline{v}$ in some sense.

The best parameter estimate $\hat{\theta}$ must be chosen to minimize a given cost function. A general form of the cost function is

$$J_P(\underline{\theta}) = \left\{ \sum_{i=1}^{m} \left| z_i - H_i \theta \right|^P \right\}^{\frac{1}{P}} \qquad (4.2)$$

or

$$J_P(\underline{\theta}) = \left\{ \sum_{i=1}^{m} \left| v_i(\theta) \right|^P \right\}^{\frac{1}{P}} \qquad (4.3)$$

Where

$J_P(\underline{\theta})$	is the cost function to minimized.
p	is some number ≥ 1, which defines the nature of the cost function.
z_i	is the i th measurement.
H_i	is the row of H corresponding to the i th measurement.
v_i	is the residual of the i th measurement; that is, $v_i = z_i - H_i\underline{\theta}$.

For p = 2, the cost function is the sum of the squares of the residuals while for p=1 the cost function is the sum of the absolute values of the residuals .

If the number of measurements (m) equals the number of unknown parameter (n), then an estimation of $\underline{\theta}$ can be obtained as in (4.4)

$$\hat{\underline{\theta}} = [H]^{-1} \underline{z} \tag{4.4}$$

For this type of estimation, the estimated parameters vector exactly fits the measurements set, i.e.

$$\underline{z} - H\hat{\underline{\theta}} = \underline{v} = 0 \tag{4.5}$$

4.3 Linear Least Error Squares (LES) Estimation [75]

If the number of measurements (m) exceeds the number of system parameters (n), i.e. m<n, then the measurement errors can be filtered out in the estimation process and good estimates can be obtained. In the LES, the objective is to minimize the sum of the squares of the residuals.

As mentioned, for p = 2, Equation (4.2) can be rewritten in vector form as

$$J_2(\underline{\theta}) = \frac{1}{2}\{(\underline{z} - H\underline{\theta})^T(\underline{z} - H\underline{\theta})\}^{\frac{1}{2}} \tag{4.6}$$

or

$$J_2(\underline{\theta}) = \frac{1}{4}(\underline{z} - H\underline{\theta})^T(\underline{z} - H\underline{\theta}) \tag{4.7}$$

It should be noted that minimizing the sum of the squares is equivalent to minimizing the square root of the sum of the squares.

Setting the first derivative of Equation (4.7), $dJ_2(\theta)/d\theta$, to equal zero, yields

$$-H^T\underline{z} - H^T\underline{z} + 2H^T H\hat{\underline{\theta}} = 0 \tag{4.8}$$

which gives

$$\hat{\underline{\theta}} = [H^T H]^{-1} H^T \underline{z} \tag{4.9}$$

or

$$\hat{\underline{\theta}} = [\mathrm{H}]^{+} \underline{\mathrm{z}} \tag{4.10}$$

Where $H^+ = \left[H^T H\right]^{-1} H^T$ is the left pseudo-inverse of H, and $\hat{\underline{\theta}}$ is the optimal or best LES estimate of $\underline{\theta}$. It should be noted that the second-order partial derivative is

$$\frac{\partial^2 J'(\underline{\theta})}{\partial \underline{\theta}^2} = H^T H \tag{4.11}$$

This matrix is positive definite as long as H is of full-column rank, the rank of H equals n. Therefore the value of $\hat{\underline{\theta}}$ given by Equation (4.10) is unique and minimizes $J_2(\underline{\theta})$.

An LES estimator finds the mean value of a set of measurements **[68].** The mean value is generally accepted to be the best estimate when the set of measurements has a Gaussian error distribution. However, for other error distributions the LES will not produce the best estimate **[66].** The LES estimate is also adversely affected by the presence of bad data; most LES estimators use some form of bad data suppression.

4.4. Weighted Linear Least Error Squares Estimation (WLES) [75]

In the LES explained in section 4.3 above, if all measurements are treated equally, then the less accurate measurements will affect the calculation process as much as the more accurate measurements.

As a result, the final set of data obtained from the least error squares estimation process will still contain large error due to the influence of bad measurements. By introducing a weighting matrix to distinguish the more accurate measurements from the less accurate ones, the calculation process can then force the results to coincide with more accurate measurements. A sensible way of choosing the weights is to make them inversely proportional to the variance of the measurements. This approach means that larger weighting is placed on measurements with smaller variance (more accurate) and smaller weighting on measurements with larger variance (less accurate).

The cost function to be minimized, in this case, is given as:

$$J_2(\underline{\theta}) = \sum_{i=1}^{m} \frac{(z_i - H_i \underline{\theta})^2}{\sigma_i^2} \tag{4.12}$$

or

$$J_2(\theta) = \sum_{i=1}^{m} w_i (z_i - H_i \underline{\theta})^2 \tag{4.13}$$

where σ_i is the standard deviation of the i th measuring device.

σ_i^2 is the variance of the i th measurement.

w_i is the weight assigned to the i th measurement.

In vector form, equation (4.13) can be written as

$$J_2(\underline{\theta}) = (\underline{z} - H\underline{\theta})^T W (\underline{z} - H\underline{\theta}) \tag{4.14}$$

Similarly, it can be shown that the weighted least error squares estimation is given by

$$\hat{\underline{\theta}} = \left[H^T W H\right]^{-1} H^T W \underline{z} \tag{4.15}$$

4.5 Constrained Least Error Squares Estimation [68]

The constrained linear least error squares problem is to find the state vector $\hat{\underline{\theta}}$ that minimizes cost function

$$J_2(\theta) = \frac{1}{2}(\underline{z} - H\underline{\theta})^T (\underline{z} - H\underline{\theta}) \tag{4.16}$$

subject to satisfying the linear constraints given by

$$C\underline{\theta} = \underline{d} \tag{4.17}$$

where C is an $\ell \times n$ matrix which represents the relation between $\underline{\theta}$ and $\underline{d}$.

$\underline{d}$ is an $\ell \times 1$ vector, which represents the constraints measurements.

An augmented cost function can be formed by adjoining equation (4.17) the equality constraints to equation (4.16) via LaGrange's multiplier λ to obtain:

$$J_2(\underline{\theta}) = \frac{1}{2}(\underline{z} - H\underline{\theta})^T (\underline{z} - H\underline{\theta}) + \underline{\lambda}^T (C\underline{\theta} - \underline{d}) \tag{4.18}$$

The cost function of equation (4.18) is a minimum when

$$\partial J_2(\underline{\theta})/\partial\underline{\theta} = 0 = \frac{1}{2}[-2H^T\underline{z} + 2H^T H\underline{\theta}] + C^T\lambda$$

which gives:

$$\hat{\underline{\theta}} = \left[H^T H\right]^{-1}\left[H^T\underline{z} - C^T\underline{\lambda}\right] \tag{4.19}$$

The LaGrange's' multiplier $\underline{\lambda}$ is to be determined such that the equality constraints of equation (4.17) are satisfied. Pre-multiplying equation (4.19) by C, then

$$\underline{\lambda} = \left[C[H^T H]^{-1} C^T\right]^{-1}\left[C[H^T H]^{-1} H^T\underline{z} - \underline{d}\right] \tag{4.20}$$

Thus the state $\underline{\theta}$ can be obtained by substituting equation (4.20) into (14.17) to obtain

$$\hat{\underline{\theta}} = [H^T H]^{-1}\left[H^T\underline{z} - C^T[C[H^T H]^{-1} C^T]^{-1}[C[H^T H]^{-1} H^T\underline{z} - \underline{d}]\right] \tag{4.21}$$

It can be noticed that if C = 0, there are no constraints, then $\underline{\theta}$ turns out to be the optimal estimate for the unconstrained least error squares estimates given by equation (4.9).

4.6 Recursive Least Error Squares Estimation [75]

The previous estimators are "batch processing" algorithms, in that all measurements are processed together to provide the estimate of a constant vector. If a new measurement is obtained, then the first way is to append the new data to $\underline{z}$ and repeat the entire process. The second way is to use the prior estimate as the starting point for a sequential estimation algorithm that assigns proper relative weighting to the old and new data.

Given $\underline{z}_1$ measurements vector corresponding to m_1 measurements, H_1 measurements matrix and W_1 weighing matrix then the resulting estimate $\hat{\underline{\theta}}_1$ are:

$$\underline{z}_1 = H_1\underline{\theta}_1 + v_1 \tag{4.22}$$

$$\hat{\underline{\theta}}_1 = \left[H_1^T W_1 H\right]^{-1} H_1^T W_1\underline{z}_1 \tag{4.23}$$

The new measurement $\underline{z}_2$ with dimension m_2, is

$$\underline{z}_2 = H_2\hat{\underline{\theta}}_2 + v_2 \tag{4.24}$$

W_2 is $m_2 \times m_2$ containing the expected squared errors in the new measurement. The cost function for all (m_1+m_2) measurements

$$\underline{z} = \begin{bmatrix} \underline{z}_1 \\ \underline{z}_2 \end{bmatrix} \tag{4.25}$$

can be partitioned as

$$\underline{J}(\underline{z}_1, \underline{z}_2) = \frac{1}{2}\left(\underline{z}_1 - H_1\hat{\underline{\theta}}_2\right)^T \left(\underline{z}_2 - H_2\hat{\underline{\theta}}_2\right) \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix} \begin{bmatrix} \underline{z}_1 - H_1\hat{\theta}_2 \\ \underline{z}_2 - H_2\hat{\theta}_2 \end{bmatrix} \tag{4.26}$$

where $\hat{\underline{\theta}}_2$ is the state estimate obtained by using all data.

Taking the derivative of $J(\underline{z}_1, \underline{z}_2)$ and setting it equal to zero provides the least-squares estimates $\hat{\underline{\theta}}_2$:

$$\hat{\underline{\theta}}_2 = \hat{\underline{\theta}}_1 + \underline{K}_2(\underline{z}_2 - H_2\hat{\underline{\theta}}_1) \tag{4.27}$$

where K_2 is the recursive weighted-least-squares estimator gain matrix.

$$\underline{K}_2 = \underline{P}_1 H_2^T (H_2 \underline{P}_1 H_2^T + W_2^{-1})^{-1} \tag{4.28}$$

and

$$\underline{P}_1^{-1} = H_1^T W_1 H_1 \tag{4.29}$$

Equation (4.27) looks like a digital filter, and measurements taken over a period of time could update the estimate as they occur. Redefining k as a time index and letting the observation vector at time k have r components, the recursive mean-value estimator is

$$\hat{\underline{\theta}}_k = \hat{\underline{\theta}}_{k-1} + K_k(\underline{z}_k - H_k\hat{\underline{\theta}}_{k-1}) \tag{4.30}$$

with

$$K_k = \underline{P}_{k-1} H_k^T (H_k P_{k-1} H_k^T + W_k^{-1})^{-1} \tag{4.31}$$

and

$$P_k = (P_{k-1}^{-1} + H_k^T W_k H_k)^{-1} \tag{4.32}$$

Note that K_k is a $(n \times r)$ gain matrix, which P_k is a $(n \times n)$ matrix that represents the estimation error at the K_{th} sampling instant.

4.7 Nonlinear Least Error Squares Estimation [68]

In the previous sections, the linear least error squares estimation problem is discussed and there is a direct linear relationship between the measuring value and the estimation parameters so that the solution is obtained directly without any iteration. If the relationship between the measurements and the estimate parameters is nonlinear, the cost function needs to be linearized by using first order Taylor series expansion . In this section, the solution for the nonlinear parameter estimation problem is to be found using the linear least error squares algorithm explained earlier in the previous sections.

The nonlinear least error squares problem is to estimate the parameter vector $\underline{\theta}$ which minimizes

$$J_2(\underline{\theta}) = \sum_{i=1}^{m} \frac{(z_i - f_i(\underline{\theta}))^2}{\sigma_i^2} \tag{4.33}$$

The gradient of $J_2(\underline{\theta})$ is given by:

$$\nabla J_2(\underline{\theta}) = grad(\ J_2\ ) = \begin{bmatrix} \frac{\partial J_2(\underline{\theta})}{\partial \theta_1} \\ \frac{\partial J_2(\underline{\theta})}{\partial \theta_2} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial J_2(\underline{\theta})}{\partial \theta_m} \end{bmatrix} \tag{4.34}$$

This can be written as:

$$\nabla J_2(\theta) = 2 * \begin{bmatrix} \frac{\partial f_1}{\partial \theta_1} & \frac{\partial f_1}{\partial \theta_2} & . & . & . & . \\ \frac{\partial f_2}{\partial \theta_1} & \frac{\partial f_2}{\partial \theta_2} & . & . & . & . \\ \frac{\partial f_m}{\partial \theta_1} & \frac{\partial f_m}{\partial \theta_2} & . & . & . & . \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma_1^2} & . & . \\ . & \frac{1}{\sigma_2^2} & . \\ . & . & \frac{1}{\sigma_m^2} \end{bmatrix} \begin{bmatrix} z_1 - f_1(\theta) \\ z_2 - f_2(\theta) \\ z_m - f_m(\theta) \end{bmatrix}$$

(4.35)

Equation (4.35) can be written in compact form as

$$\nabla J_2(\underline{\theta}) = 2H^T W \Delta \underline{z} \tag{4.36}$$

where the m×n matrix H is defined as

$$H = \begin{bmatrix} \frac{\partial f_1}{\partial \theta_1} & \frac{\partial f_1}{\partial \theta_2} & . & . & . & \frac{\partial f_1}{\partial \theta_n} \\ \frac{\partial f_2}{\partial \theta_1} & \frac{\partial f_2}{\partial \theta_2} & . & . & . & \frac{\partial f_2}{\partial \theta_n} \\ . & . & . & . & . & . \\ \frac{\partial f_m}{\partial \theta_1} & \frac{\partial f_m}{\partial \theta_2} & . & . & . & \frac{\partial f_m}{\partial \theta_n} \end{bmatrix} \tag{4.37}$$

$$= Jacobian \quad of \quad f(\underline{\theta})$$

$$W = \text{m} \times \text{m weighting matrix} = \begin{bmatrix} \frac{1}{\sigma_1^2} & . & . \\ . & \frac{1}{\sigma_2^2} & . \\ . & . & \frac{1}{\sigma_m^2} \end{bmatrix}$$

and $\Delta \underline{z} = m \times 1$ difference matrix between the measured values and the estimated values.

To make $\nabla J(\underline{\theta})$ equal zero, the Newton-Raphson method is implemented.

Thus :

$$\Delta \underline{\theta} = \left[\frac{\partial \nabla J(\underline{\theta})}{\partial \underline{\theta}} \right]^{-1} \left[-\nabla J_2(\underline{\theta}) \right] \tag{4.38}$$

The Jacobian matrix of $\nabla J_2(\underline{\theta})$ is calculated by treating [H] as a constant matrix. Thus,

$$\frac{\partial \nabla J_2(\underline{\theta})}{\partial \theta} = -2H^T W[-H]$$

Then

$$\Delta\hat{\underline{\theta}} = \left[H^T W H\right]^{-1}\left[H^T W \Delta \underline{z}\right] \tag{4.39}$$

The procedures of the algorithm for solving the nonlinear state estimation problem can be stated as follows:

Step 1. Assume initial guesses for $\underline{\theta}$.

Step 2. Compute the measurement vector $\Delta \underline{z}$ using these initial guesses.

Step 3. Calculate the matrix H as well at these guesses.

Step 4. Solve for $\Delta\underline{\theta}$ using equation (4.39).

Step 5. If $\Delta\underline{\theta}$ satisfies a certain specified terminating criterion, terminate the iteration, otherwise go to step 6.

Step 6. Update the parameter vector $\underline{\theta}$ as
$\underline{\theta}_n = \underline{\theta}_o + \Delta\underline{\theta}$ and go to step 2.

4.8 Properties of Least Error Squares Estimation [63 - 68, 75]

The least error squares are the best estimates (maximum likelihood) when the measurement errors obey Gaussian or normal distribution and the weighting matrix equals to the inverse of the covariance matrix. Also, for the estimates where the measurements errors does not obey a Gaussian distribution and the number of measurements greatly exceeds the number of unknown parameters, the method of least error squares yields very good estimates.

There are many estimation cases where the errors distribution is not a Gaussian distribution and the number of measurements does not greatly exceed the number of unknown parameters. In these cases, the least error squares estimation results are adversely affected by bad data. These cases have been recognized and addressed by several researchers, who have proposed different ways of refining the least error squares method in order to make estimation less affected by presence of bad data.

4.9 Least Absolute Value State Estimation (LAV) [63 - 65]

In contrast to the LES the least absolute value estimation is based on minimizing the sum of the absolute value of the residuals. There is a basic difference between the two techniques. Using least absolute value the best approximation is determined by interpolating a minimum subset of the available measurements. While using the least error squares, the best approximation is derived from the mean of the available measurements when the error statistics are Gaussian.

The purpose of this section is explain least absolute value approximation theory. Then the techniques to obtain LAV state estimation are discussed . After that, an algorithm based on LAV is introduced to obtain the best state estimation.

The cost function in the case of LAV is given by, for p = 1 in equation (4.2)

$$\bar{J}_1(\underline{\theta}) = \sum_{i=1}^{m} \left| z_i - H_i \underline{\theta} \right| \tag{4.40}$$

As mentioned earlier, the minimum of $J_1(\hat{\underline{\theta}})$ corresponds to the best LAV estimate $\hat{\underline{\theta}}$, of the system parameters.

Important characteristics of the LAV solution are given by the following theorems:

Theorem 1.

If the column rank of the m×n matrix H is k, k n (for maximum rank k = n), then there exists a vector $\hat{\underline{\theta}}$ corresponding to a best approximation that interpolates at least k points of the measurement set.

This theorem states that, if there are m measurements $z_i,\ i = 1,2,\ldots,m$ *and* n unknowns, then the optimal hyper plane z based on LAV will pass through at least n points of the measurements set. This is in contrast to the least square approximation, which does not necessarily pass through any of the measurement points of the set $\underline{z}$.

Theorem 2.

If N_1 is the number of measurement points above the optimal hyper plane under LAV plane and N_2 is the number of points below the hyper plane, provided that n+1 points do not lie on a hyper plane in n-dimension, then

$$|N_1 - N_2| \leq n$$

These two theorems state the interpolation property of the LAV solution. Since the LAV solution interpolates data points, it will reject bad data points, provided that none of the bad data points are among the points interpolated. Thus, the problem reduces to selecting n (n = the number of the parameter variables to be estimated) data points to minimize the LAV cost function and to find $\underline{\hat{\theta}}$. The popular method of finding $\underline{\hat{\theta}}$ has been through linear programming. The formulation of the linear programming problem can be carried out as explained in the next subsection.

4.9.1 Least Absolute Value (LAV) Based on Linear Programming [66 -72, 75]

In this section, a technique is presented to solve the LAV estimation problem. The formulation of this technique is :

Minimize the cost function of

$$J_1(\underline{\theta}) = \sum_{i=1}^{m} v_i \tag{4.41}$$

Subject to

$$v_i \geq 0 \tag{4.42}$$

and

$$v_i + \left| z_i - \sum_{j=1}^{n} H_{ij}\theta_i \right| \geq 0, \qquad i = 1,....,m \tag{4.43}$$

Equation (4.43), can be written as

$$v_i \geq z_i - \sum_{j=1}^{n} H_{ij}\theta_i , \qquad i = 1,.....,m \tag{4.44}$$

and

$$v_i \geq \sum_{j=1}^{n} H_{ij}\theta_i - z_i\,, \qquad i = 1,\ldots\ldots, m \tag{4.45}$$

Thus, the linear programming problem is to minimize (4.41) subject to satisfying the constraints given by equation (4.44) and (4.45). It can be noted from equations (4.44) and (4.45), that if any of the constraints is negative, the other will be positive, and v_i must be positive in order to obey the linear programming requirements.

The main steps of this algorithm are :

- Select n points from the set of measurements.
- Evaluate the cost function; and
- Select new points, which decrease the cost function. When the cost function becomes a minimum, the LAV solution has been reached. It has been shown that the size of the matrix to be stored and manipulated is [2 (m + n) × n]

The main disadvantages of linear programming technique are:

- It is an iterative technique, which requires considerable computing time.
- It needs a large size of memory to store and manipulate a matrix of size 2 (m + n) ×n;
- The frequent inaccessibility of the linear programming algorithm within a statistical package.
- The solution obtained may not be unique.

Some of the more recent algorithms have been attempted to overcome these difficulties and researches continue in this area.

4.9.2 An LAV Algorithm [63 - 65]

Given the measurement equation described in equation (4.1), the following are the main steps in this LAV algorithm for unconstrained problem.

Step 1. Calculate the LES solution, as defined earlier, using the equation

$$\underline{\hat{\theta}} = \left[H^T H\right]^{-1} H^T \underline{z}$$

Step 2. Calculate the LES residuals generated from this solution as

$$v_i = z_i - H_i \underline{\hat{\theta}}$$

Step 3. Calculate the standard deviation of the calculated residuals as

$$\sigma^2 = \frac{1}{m-n+1}\sum_{i=1}^{m}(v_i - v_{av})^2 = \text{variance}$$

$$\sigma = \sqrt{\text{variance}} = \left[\frac{1}{m-n+1}\sum_{i=1}^{m}(v_i - v_{av})^2\right]^{\frac{1}{2}}$$

Step 4. Reject the outliners having residuals greater than the standard deviation σ, provided the system is observable.

Step 5. Recalculate the new LES estimates, using the rest of the measurements and calculate the new corresponding residuals for these measurements.

Step 6. Select the n measurements that correspond to the smallest least error squares residual and form the corresponding $\hat{\underline{z}}$ and $\hat{H}$.

Step 7. Solve for the least absolute value estimate $\underline{\theta}^*$ using

$$\underline{\theta}^* = \left[\hat{H}\right]^{-1}\hat{\underline{z}}$$

4.10. Constrained LAV Estimation [68]

The constrained state estimation problem can be handled by the LAV technique. If there are m measurements and ℓ constraints, $n \succ \ell$, the technique will interpolate at least n points of the given measurements. The constraints represent good measurements so that the residuals of the least error square solution for these constraints will be zero. The least absolute value technique must interpolate the ℓ constraints before interpolation of $n - \ell$ of the other measurements. The total number of the interpolated points will equal $(n - \ell + \ell = n)$. Thus the method will select directly the ℓ constraints and the $n - \ell$ measurements corresponding to the smallest LS residuals.

The number of constraints should be less than the number of unknowns otherwise the least absolute value will interpolate the n points from the constraints only.

The solution technique may use the method proposed in the pervious section for LS parameters estimation with constraints and then proceed in the same manner as the LAV technique to obtain the least absolute value optimal solution.

4.11 Fuzzy linear Estimation [77, 79, 80, 83]

In this section, a formulation of the fuzzy linear estimation problem is presented. The problem is formulated as a linear programming problem. The objective is to minimize the spread of the data points, taking into consideration the type of the membership function of the fuzzy parameters to satisfy the constraints on each measurement point and to insure that the original membership is included in the estimated membership. Different models are developed for a fuzzy triangular membership and the fuzzy numbers of LR-type. The fuzzy parameters linear estimation model or fuzzy regression model can be described by the following equation

$$\mathbf{Y} = \mathbf{f}(\mathbf{x}, \mathbf{A}) = \mathbf{A}_1\mathbf{x}_1 + \mathbf{A}_2\,\mathbf{x}_2 + \ldots + \ldots\,\mathbf{A}_n\,\mathbf{x}_n \tag{4.46}$$

At any observation j; j = 1,2..., m, equation (4.46) can be written as

$$\mathbf{Y}_j = \mathbf{f}(\mathbf{x}, \mathbf{A}) = \mathbf{A}_1\mathbf{x}_{1j} + \mathbf{A}_2\,\mathbf{x}_{2j} + \ldots + \ldots + \mathbf{A}_n\,\mathbf{x}_{nj} \tag{4.47}$$

In fuzzy regression, the differences between the observed and the estimated values are assumed to be due to the inherent ambiguity in the system. Therefore, the above fuzzy regression model is built in terms of possibilities. It evaluates all observed values as possibilities the system must contain. The model in equation (4.46) is named as a possibilstic regression model. In this model Y_j is the observation at measurement j. This output observation may be a non-fuzzy or a fuzzy observation, A_i, i=1,2,..., n are the fuzzy coefficients of the model in the form of (p_i, c_i), where p_i is the middle and c_i is the spread, or it may take the form of LR-type as (p_i, c_i^L, c^R_i) and x_{ij} is the input to the model where i =1,..., n and j = 1,2,.., m . In this section, three cases for the output Y_j are studied:

4.11.1 Non-fuzzy output ($Y_j = m_j$)

In this model the output Y_j is a non-fuzzy observation, but the model coefficients A_i $i=1,2,\ldots, n$ are fuzzy parameters either in the form of $A_i = (p_i, c_i)$, or $A_i = (p_i, c_i^L, c^R_i)$, $i=1,\ldots, n$ for the LR-type and the input x_{ij} is a non-fuzzy input. The membership functions for each type of A_i are given in Figures (4.1) and (4.2)

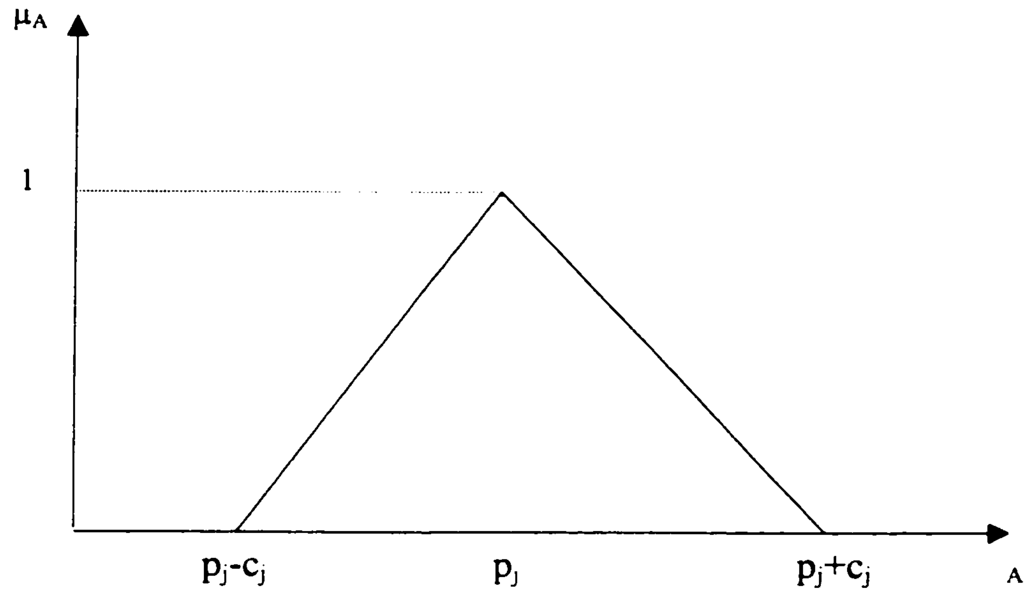


Figure 4.1 Membership Function of the Fuzzy Parameters A_j

The equation that describes this membership can be written mathematically, for the triangular fuzzy number, as

$$\mu_A(x)=\begin{cases} 1-|p_j-a_i|/c_j & p_j-c_j \le a_i \le p_j+c_j \\ 0 & \text{otherwise} \end{cases} \tag{4.48}$$

While the membership function of A_j of the LR-Type is assumed to be trapezoidal function as shown in Figure 4.2

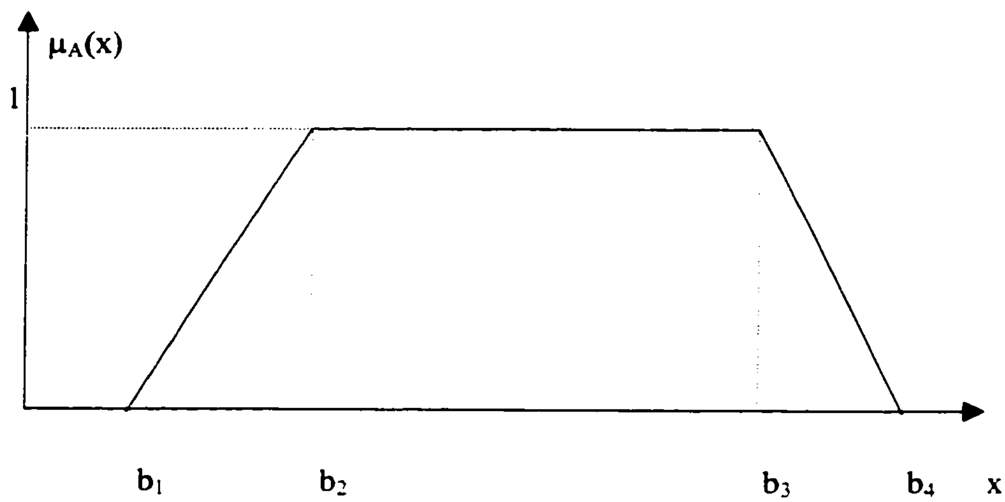


Figure 4.2 Trapezoidal Membership Function of A_j

Note that if $b_2 = b_3$, we obtain the triangular membership. In general the membership function for the LR-type can be described as

$$\mu_{A_j} = \begin{cases} L(p_j - x / c_j^L) & \text{for } x \le p_j \\ R(x - p_j / c_j^R) & \text{for } x \ge p_j \end{cases} \tag{4.49}$$

p_j is called the middle of A_j or the mean. c_j^l is the left spread and c_j^R is the right spread.

Equation (4.46) can now be written as

$$Y_j = (p_1, c_1)x_{1j} + (p_2, c_2)x_{2j} + \ldots + (p_n, c_n)x_{nj} \quad, j=1,\ldots, m \tag{4.50}$$

for the first type of the fuzzy coefficients, and

$$Y_j = (p_1, c_1^L, c_1^R)x_{1j} + (p_2, c_2^L, c_2^R)x_{2j} + \ldots + (p_n, c_n^L, c_n^R)x_{nj} \quad, j=1,\ldots, m \tag{4.51}$$

for the second type of the fuzzy coefficients.

In the non-fuzzy output data regression described by equations (4.50) and (4.51), the parameters are to be found $A_i = (p_i, c_i)$ or $A_i = (p_i, c_i^L, c_i^R)$ that minimize the spread of the fuzzy output for all data sets. In mathematical form, this can be described as :

Minimize :

$$J_1 = \sum_{j=1}^{m} \sum_{i=1}^{n} | c_i x_{ij} | \tag{4.52}$$

such that the fuzzy regression model contains all observed data in the estimated fuzzy numbers resulted from the model. This can be expressed mathematically as:

$$y_j \geq \sum_{i=1}^{n} p_i x_{ij} - (1 - \lambda) \sum_{i=1}^{n} c_i x_{ij} \tag{4.53}$$

$$y_j \leq \sum_{i=1}^{n} p_i x_{ij} + (1 - \lambda) \sum_{i=1}^{n} c_i x_{ij} \tag{4.54}$$

Note that the first term in the right hand side of equations (4.53) and (4.54) represents the estimated middle of the fuzzy coefficients, while the second term represents the estimated spread of these coefficients and λ is the level of fuzziness and is specified by the user.

For the fuzzy coefficients of the LR-type, the cost function to be minimized is

$$J_1 = \sum_{j=1}^{m} \sum_{i=1}^{n} | (2m_j - 2p_j x_{ij} + c_i^L x_{ij} - c_i^R x_{ij}) | \tag{4.55}$$

Subject to satisfying the following two constraints on each data point

$$y_j \geq \sum_{i=1}^{n} p_i x_{ij} - (1 - \lambda) \sum_{i=1}^{n} c_i^L x_{ij} \qquad , j=1,\dots, m \tag{4.56}$$

$$y_j \leq \sum_{i=1}^{n} p_i x_{ij} + (1 - \lambda) \sum_{i=1}^{n} c_i^R x_{ij} \qquad , j=1,\dots, m \tag{4.57}$$

The problem formulated in equations (4.52) to (4.54) and that formulated in equations (4.55) to (4.56) are linear optimization problems, which can be solved by linear programming using the simplex method. However, if the sum of the absolute value deviations in equations (4.52) and (4.57) is to be minimized, subject to satisfying the inequality constraints given by equations (4.53), (4.54) and equations (4.56) and (4.57), then the problem is least absolute value linear optimization and can be solved by using the software package available in the IMSL/STAT library.

4.11.2 Fuzzy Output

If the output is a fuzzy number, it may be represented by a fuzzy number in the form of $Y_j = (m_j , \alpha_j)$ in case of triangular membership function or $Y_j = (m_j , \alpha_j^L, \alpha_j^R)$, j=1,..., m, in case of trapezoidal membership function. For triangular membership function, equation (4.47) can be written as

$$Y_j = (m_j , \alpha_j) = (p_1, c_1) x_{1j} + (p_2, c_2) x_{2j} +\ldots+ (p_n, c_n) x_{nj} \quad ,j=1,2,\ldots, m \tag{4.58}$$

which can be written as

$$(m_j,\alpha_j) = (p_1 x_{1j} + p_2 x_{2j} +\ldots+ p_n x_{nj}, c_1 x_{1j} + c_2 x_{2j} +\ldots + c_n x_{nj}) \quad ,j=1,2,\ldots, m \tag{4.59}$$

$$(m_j , \alpha_j) = (\sum_{i=1}^{n} p_i x_{ij}, \sum_{i=1}^{n} c_i x_{ij}) \tag{4.60}$$

Equation (4.60) is valid when

$$m_j = \sum_{i=1}^{n} p_i x_{ij} \quad ,j=1, 2,\ldots,m \tag{4.61}$$

$$\alpha_j = \sum_{i=1}^{n} c_i x_{ij} \quad ,j= 1, 2,\ldots,m \tag{4.62}$$

Given the fuzzy output $Y_j = (m_j , \alpha_j)$, it is required to find the fuzzy parameters (p_i, c_i), i=1, 2,.., n that minimize the cost function

$$J_1(p_i, c_i) = \sum_{j=1}^{m} \left| \left\{ m_j - \sum_{i=1}^{n} p_i x_{ij} + \alpha_j - \sum_{i=1}^{n} c_i x_{ij} \right\} \right| \quad (4.63)$$

subject to satisfying the following constraints on each measurement point

$$m_j - (1-\lambda)\,\alpha_j \geq \sum_{i=1}^{n} p_i x_{ij} - \sum_{i=1}^{n} c_j x_{ij} \qquad ,j=1,\ldots,m \quad (4.64)$$

$$m_j + (1-\lambda)\,\alpha_j \leq \sum_{i=1}^{n} p_i x_{ij} + \sum_{i=1}^{n} c_j x_{ij} \qquad ,j=1,\ldots,m \quad (4.65)$$

If the fuzzy output is of the LR-type, then equation (4.58) can be written as

$$(m_j, \alpha_j^L, \beta_j^R) = \left(\sum_{i=1}^{n} p_i x_{ij}, \sum_{i=1}^{n} c_i^L x_{ij}, \sum_{i=1}^{n} c_i^R x_{ij} \right) \quad (4.66)$$

Equation (4.66) can be separated into the following equations

$$m_j = \sum_{i=1}^{n} p_i x_{ij} \qquad ,j=1,\ldots,m \quad (4.67)$$

$$\alpha_j^L = \sum_{i=1}^{n} c_i^L x_{ij} \qquad ,j=1,\ldots,m \quad (4.68)$$

$$\beta_j^R = \sum_{i=1}^{n} c_i^R x_{ij} \qquad ,j=1,\ldots,m \quad (4.69)$$

The objective function to be minimized is given **[81]** as:

$$J_1 = 0.25 \sum_{j=1}^{m} \left| \left\{ 4m_j - 4\sum_{i=1}^{n} p_i x_{ij} - \alpha_j^L + \sum_{i=1}^{n} c_i^L x_{ij} - \beta_j^R - \sum_{i=1}^{n} c_i^R x_{ij} \right\} \right| \quad (4.70)$$

Subject to satisfying the following constraints

$$m_j - (1-\lambda)\, c_j^L \geq \sum_{i=1}^{n} p_i x_{ij} - \sum_{i=1}^{n} c_i^L x_{ij} \qquad ,j=1,\ldots,m \quad (4.71)$$

and

$$m_j + (1-\lambda)\, c_j^R \geq \sum_{i=1}^{n} p_i x_{ij} + \sum_{i=1}^{n} c_i^R x_{ij} \qquad ,j=1,\ldots,m \quad (4.72)$$

Again the problem formulated in equations (4.63) to (4.65) and that formulated in equation (4.70) to (4.72) for LR-type, all are linear optimization problems subjected to a set of linear constraints. These problems can be solved using the standard linear programming using the simplex method. However, if the objective functions are minimization of the sum of the absolute value of the deviation, then the least absolute value optimization technique based on linear programming is used to solve the problems formulated above.

4.12 Conclusions

This chapter discusses static estimation problem formulation. In the first part crisp static estimation is discussed. Two techniques are used. The first is based on the least error squares (LES) algorithm, while the second technique is based on the least absolute value (LAV) algorithm.

In the second part, fuzzy static estimation is discussed. The objective of the estimation problem is to minimize the spread of the measurement data (observations) constrained to satisfying two constraints on each measurement and to consider the measurement membership in the proposed model.

Chapter 5

Fuzzy Short-Term Load Modeling

5.1. Introduction

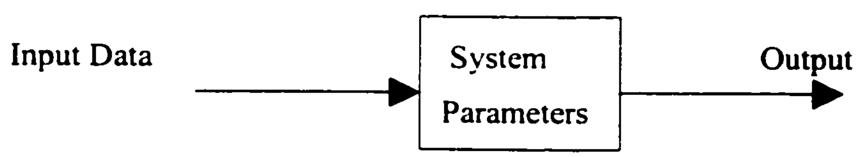
Most of the work on off-line short- term load models available today assumes that the parameters of the model are constant crisp values. This assumption is to some extent true, as long as there are no big changes in weather parameters from day to day. The load power is characterized by both uncertainty and ambiguity.

In this chapter, the load models used in chapter 3 are reformulated to account for fuzziness of the load characteristics. In the first section the input is assumed to be crisp, while the load model parameters are expressed as fuzzy numbers having certain middle and spreads. Three models are used in this section, namely fuzzy load models A, B and C. The fuzzy load model A is a multiple linear regression model. This model takes into account the weather parameters. The fuzzy load model B is a harmonic model, and does not account for the weather parameters. The fuzzy load model C is a hybrid model, that combines models A and B and takes into account the weather parameters.

In the second section the input data are assumed to be fuzzy numbers having certain middles and spreads. The parameters of the load model are fuzzy. The fuzzy numbers used for the fuzzy variables in this chapter are assumed to have a symmetrical triangular membership function.

5.1.1 Background

The following system is considered:



- If the input data are crisp (non-fuzzy) and the system parameters A_i (i, 1,.., n) are crisp (non-fuzzy), then the output is also crisp (non-fuzzy) with an error deviation between the actual and the estimated or predicted values. (The static cases in chapter 4)

- If the input data are crisp (non-fuzzy) and the system parameters are fuzzy and follow a membership function (e.g. Triangular Membership Function) then the output is fuzzy and follow the same membership as in the system parameters [77] [80].
- If the input data are fuzzy and the system parameters are fuzzy, then the output is fuzzy. The output will have some resemblence of shape of the membership function used.
- The membership functions used in this thesis are triangular membership functions with fuzzy numbers having a certain middle and equal left and right spreads [77].
- The objective of the fuzzy parameters estimation is to minimize the spreads of the fuzzy parameters. If spreads of zero are attained, then the output is crisp with an error deviation from the actual value. If the spreads are minimized, then the output will follow the shape of triangular membership function [77] and the output value will be in a range between upper and lower values.

5.2 Crisp Data

$$(Y_j(t) = m_j(t), j = 1,.., m; t=1, 2,...,..., 24)$$

The input data of the load model are assumed to be crisp values, while the load parameters are fuzzy.

5.2.1 Multiple Fuzzy Linear Regression model

The load, in this model, can be expressed mathematically as:

$$Y_j(t) = A_o + \sum_{i=1}^{n} A_i\, x_{ij}(t), \qquad j=1,..., m \tag{5.1}$$

where $Y_j(t)$ is the value of the load power at time t.

A_o is the fuzzy base load having a triangular membership with a middle p_o and spread c_o, as shown in Figure 5.1a.

A_i are the fuzzy coefficients having a triangular membership with a middle p_i and spread c_i as shown in Figure 5.1b.

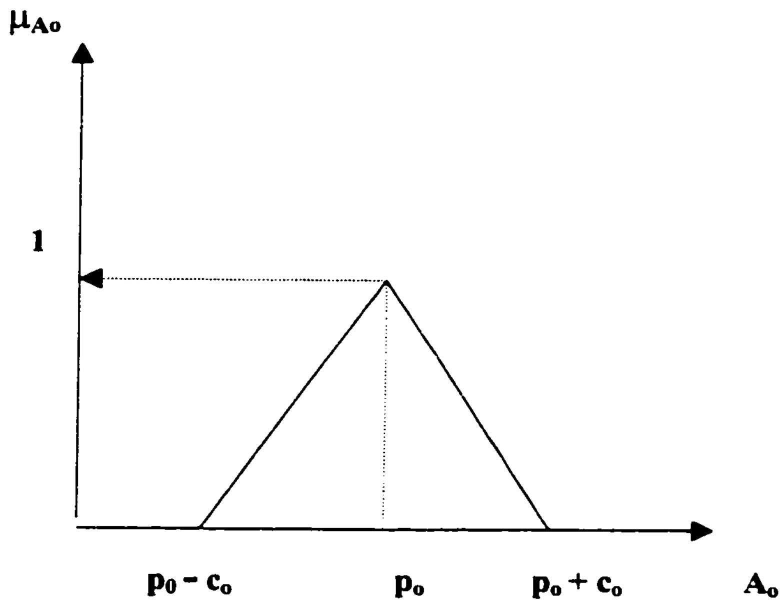


Figure (5.1a) Membership Function of A_o

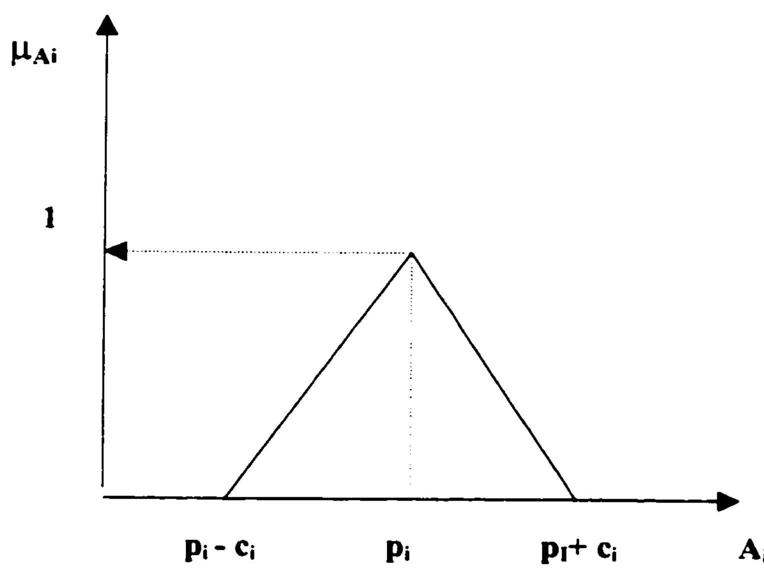


Figure (5.1b) Membership Function of A_I

Equation (5.1) can be rewritten as:

$$Y_j(t) = (p_{yj}(t), c_{yj}(t)) = m_j(t) = (p_o, c_o) + \sum_{i=1}^{n} (p_i, c_i)\, x_{ij}(t) \tag{5.2}$$

As shown in chapter four, for the output data described by equation (5.2), the coefficients $A_o(p_o, c_o)$ and $A_i(p_i, c_i)$ are to be found such that the spread of the fuzzy output is minimized for all data sets. In mathematical form, this can be described as:
Minimize:

$$J = \left| \sum_t \left\{ c_0 + \sum_{j=1}^{m} \sum_{i=1}^{n} c_i\, x_{ij}(t) \right\} \right| \tag{5.3}$$

where $t \in [0, t_F]$, t_F is the number of days for which data are taken at the hour in question. The fuzzy regression model in equation (5.3) contains all observed data in the estimated fuzzy numbers resulting from the model. This can be expressed mathematically as:

$$y_j(t) \geq [p_o + \sum_{i=1}^{n} p_i x_{ij}(t)] - (1-\lambda)[c_o + \sum_{i=1}^{n} c_i x_{ij}(t)] \quad ; j=1,.., m \tag{5.4}$$

and

$$y_j(t) \leq [p_o + \sum_{i=1}^{n} p_i x_{ij}(t)] + (1-\lambda)[c_o + \sum_{i=1}^{n} c_i x_{ij}(t)] \quad ; j=1,\dots, m \tag{5.5}$$

Note that the first term of the right hand side of equations (5.4) and (5.5) represents the estimated middle of the fuzzy coefficients, while the second term represents the estimated spread of these coefficients. λ is the level of fuzziness and is specified by the user. As λ increases, the fuzziness of the output increases. In the above equations m is the number of observations and n is the number of fuzzy parameters used in the model.

In the following subsections two multiple fuzzy linear regression models are developed. The first model can be used to predict the load during the winter season, while the second model can be used to predict the load during the summer season. The only difference between the two models is that the winter model considers the wind-cooling factor as an explanatory variable, while the summer model considers the humidity factor as an explanatory variable.

5.2.1.1 Fuzzy Model A (Winter Model)

The fuzzy winter model, equation (3.12), can be written in fuzzy form as:

$$\underset{\sim}{Y}_j(t) = \underset{\sim}{A}_o + \underset{\sim}{A}_1\, T_j(t) + \underset{\sim}{A}_2\, T^2_j(t) + \underset{\sim}{A}_3 T^3_j(t) + \underset{\sim}{A}_4 T_j(t-1) + \underset{\sim}{A}_5 T_j(t-2)$$
$$+ \underset{\sim}{A}_6 T_j(t-3) + \underset{\sim}{A}_7\, W_j(t) + \underset{\sim}{A}_8\, W_j(t-1) + \underset{\sim}{A}_9\, W_j(t-2) \qquad ;j=1,\ldots,m \qquad (5.6)$$

Where $Y_j(t)$ is the load power j; j=1,..., m at time t; t=1,2,..., 24 and is assumed to be given as non-fuzzy data. $T_j(t)$ is the jth temperature deviation from nominal at time t and is given by equation (3.13). $W_j(t)$ is the wind cooling factor at time t and is given by equation (3.15), and $\underset{\sim}{A}_o, \underset{\sim}{A}_1,\ldots, \underset{\sim}{A}_9$ are load model fuzzy coefficients having middles $p_o, p_1,\ldots, p_9$ and spreads $c_o, c_1,\ldots, c_9$.

Equation (5.6) can be written as:

$$Y_j(t) = (p_o, c_o) + (p_1, c_1)T_j(t) + (p_2, c_2)\, T^2_j(t) + (p_3, c_3)\, T^3_j(t)$$
$$+ (p_4, c_4)T_j(t-1) + (p_5, c_5)T_j(t-2) + (p_6, c_6)T_j(t-3) + (p_7, c_7)W_j(t)$$
$$+ (p_8, c_8)W_j(t-1) + (p_9, c_9)W_j(t-2) \qquad ;j=1,\ldots, m \qquad (5.7)$$

In fuzzy linear regression, the spread of the fuzzy coefficients are to be minimized. This results in an objective function which can be expressed mathematically as:

$$J = \Bigg| \sum_t \Big\{ c_o + \sum_{j=1}^{m} [\, c_1 T_j(t) + c_2 T^2_j(t) + c_3 T^3_j(t) + c_4 T_j(t-1) + c_5 T_j(t-2) + c_6 T_j(t-3)$$
$$+ c_7 W_j(t) + c_8 W_j(t-1) + c_9 W_j(t-2)\,] \Bigg| \qquad (5.8)$$

where $t \in [0, t_F]$, t_F is the number of days for which data are taken at the hour in question. Subject to satisfying the two inequality constraints on each load power given as:

$$y_j(t) \geq p_o + p_1\, T_j(t) + p_2\, T^2_j(t) + p_3\, T^3_j(t) + p_4\, T_j(t-1) + p_5\, T_j(t-2)$$
$$+ p_6\, T_j(t-3) + p_7\, W_j(t) + p_8\, W_j(t-1) + p_9\, W_j(t-2) - (1-\lambda)(c_o + c_1\, T_j(t)$$
$$+ c_2\, T^2_j(t) + c_3\, T^3_j(t) + c_4\, T_j(t-1) + c_5\, T_j(t-2) + c_6\, T_j(t-3)$$
$$+ c_7\, W_j(t) + c_8\, W_j(t-1) + c_9 W_j(t-2)) \qquad ,j=1, 2,\ldots, m \qquad (5.9)$$

$$y_j(t) \le p_o + p_1 T_j(t) + p_2 T^2_j(t) + p_3 T^3_j(t) + p_4 T_j(t-1) + p_5 T_j(t-2)$$
$$+ p_6 T_j(t-3) + p_7 W_j(t) + p_8 W_j(t-1) + p_9 W_j(t-2) + (1-\lambda)(c_o + c_1 T_j(t)$$
$$+ c_2 T^2_j(t) + c_3 T^3_j(t) + c_4 T_j(t-1) + c_5 T_j(t-2) + c_6 T_j(t-3)$$
$$+ c_7 W_j(t) + c_8 W_j(t-1) + c_9 W_j(t-2)) \quad , j = 1, 2, \ldots, m \qquad (5.10)$$

The optimization problem formulated in equations (5.8 - 5.10) is linear and can be solved using linear programming based on simplex method available in the IMSL/STAT library.

Having identified the middle and spread of each coefficient, then the fuzzy load model for the winter season can be obtained using equation (5.6) or equation (5.7).

5.2.1.2 Fuzzy Model A (Summer Model)

The summer fuzzy model for the short-term load forecasting can be written as:

$$\underset{\sim}{Y}(t) = \underset{\sim}{A}_o + \underset{\sim}{A}_1 T(t) + \underset{\sim}{A}_2 T^2(t) + \underset{\sim}{A}_3 T^3(t) + \underset{\sim}{A}_4 T(t-1) + \underset{\sim}{A}_5 T(t-2)$$
$$+ \underset{\sim}{A}_6 T(t-3) + \underset{\sim}{A}_7 H(t) + \underset{\sim}{A}_8 H(t-1) + \underset{\sim}{A}_9 H(t-2) \qquad (5.11)$$

where

$\underset{\sim}{Y}(t)$ is the summer load power at time t.

T(t) is the temperature deviation at time t given by equation (3.13)

$\underset{\sim}{A}_o, \underset{\sim}{A}_1, \ldots, \underset{\sim}{A}_9$ are the fuzzy load coefficients having certain middle $p_o, p_1, \ldots, p_9$ and certain spread $c_o, c_1, \ldots, c_9$ at time t.

H(t) is the temperature humidity factor given by equation (3.17)

The summer load model stated in equation (5.11) takes into account the temperature deviation and the temperature humidity factor for each hour and at three and two hours before.

Equation (5.11) can be rewritten as :

$$\underset{\sim}{Y}_j(t) = (p_o, c_o) + (p_1, c_1) T_j(t) + (p_2, c_2) T^2_j(t) + (p_3, c_3) T^3_j(t)$$
$$+ (p_4, c_4) T_j(t-1) + (p_5, c_5) T_j(t-2) + (p_6, c_6) T_j(t-3)$$
$$+ (p_7, c_7) H_j(t) + (p_8, c_8) H_j(t-1) + (p_9, c_9) H_j(t-2) \qquad (5.12)$$

In fuzzy linear regression, the parameters $A_i = (p_i, c_i)$, i=1,..,...,9 are to be found that minimize the spread of the fuzzy output for all data set. This can be expressed mathematically as:

Minimize

$$J = \Big| \sum_{t} \{ \ c_o + \sum_{j=1}^{m} [c_1 T_j(t) + c_2 T^2_j(t) + c_3 T^3_j(t) + c_4 T_j(t-1) + c_5 T_j(t-2) +$$

$$+ c_6 T_j(t-3) + c_7 H_j(t) + c_8 H_j(t-1) + c_9 H_j(t-2)] \Big| \qquad (5.13)$$

where $t \in [0, t_F]$, t_F is the number of days for which data are taken at hour in question. Subject to satisfying the following inequality constraints at j ; j=1,..., m

$$\begin{aligned} y_j(t) \geq\ & p_o + p_1 T_j(t) + p_2 T^2_j(t) + p_3 T^3_j(t) + p_4 T_j(t-1) + p_5 T_j(t-2) \\ & + p_6 T_j(t-3) + p_7 H_j(t) + p_8 H_j(t-1) + p_9 H_j(t-2) - (1-\lambda)[c_o + c_1 T_j(t) \\ & + c_2 T^2_j(t) + c_3 T^3_j(t) + c_4 T_j(t-1) + c_5 T_j(t-2) + c_6 T_j(t-3) \\ & + c_7 H_j(t) + c_8 H_j(t-1) + c_9 H_j(t-2)] \qquad , j = 1, 2, \ldots, m \end{aligned} \qquad (5.14)$$

$$\begin{aligned} y_j(t) \leq\ & p_o + p_1 T_j(t) + p_2 T^2_j(t) + p_3 T^3_j(t) + p_4 T_j(t-1) + p_5 T_j(t-2) \\ & + p_6 T_j(t-3) + p_7 H_j(t) + p_8 H_j(t-1) + p_9 H_j(t-2) + (1-\lambda)[c_o + c_1 T_j(t) \\ & + c_2 T^2_j(t) + c_3 T^3_j(t) + c_4 T_j(t-1) + c_5 T_j(t-2) + c_6 T_j(t-3) \\ & + c_7 H_j(t) + c_8 H_j(t-1) + c_9 H_j(t-2)] \qquad , j = 1, 2, \ldots, m \end{aligned} \qquad (5.15)$$

The problem formulated in equations (5.13) to (5.15) is linear and can be solved by the linear programming optimization package available in the IMSL/STAT library.

Having obtained the fuzzy parameters $A_i = (p_i, c_i)$, i=1,..., 9, then the load can be predicted for the next twenty four hours using equation (5.11).

5.2.2 Fuzzy Load Model B

This is a harmonic decomposition model and does not account for weather conditions. It does not account for temperature deviation, wind cooling factor nor humidity factor. Thus this model can be used for both winter and summer simulations.

The fuzzy load at any time t therefore, can be written as:

$$\underline{Y}(t) = \underline{A}_o + \sum_{i=1}^{n} (\underline{A}_i \sin i\omega t + \underline{B}_i \cos i\omega t) \tag{5.16}$$

where

$\underline{Y}(t)$ is the load power at time t and it is assumed to have crisp values.

$\underline{A}_o$, $\underline{A}_i$ and $\underline{B}_i$ are fuzzy parameters having certain middles and spreads, and are given as: $\underline{A}_o = (p_o, c_o)$, $\underline{A}_i = (p_i, c_i)$, and $\underline{B}_i = (\alpha_i, b_i)$

The model described in equation (5.16) can be written as:

$$\underline{Y}(t) = (p_o, c_o) + \sum_{i=1}^{n} [(p_i, c_i) \sin i\omega t + (\alpha_i, b_i) \cos i\omega t] \tag{5.17}$$

Note that the middles and the spreads are constants and are estimated seven times weekly.

The objective is to find the fuzzy parameters that minimize the spread of the load power . Mathematically, this can be written as :

Minimize:

$$J = \left| \sum_t \{ c_o + \sum_{j=1}^{m} \sum_{i=1}^{n} [c_i x_{ij}(t) + b_i y_{ij}(t)] \right| \tag{5.18}$$

where

$x_{ij}(t) = (\sin i\omega t)_j$, $j = 1, \ldots, m$; $i = 1, \ldots, n$

$y_{ij}(t) = (\cos i\omega t)_j$, $j = 1, \ldots, m$; $i = 1, \ldots, n$

m, n are the number of observations and harmonics chosen in the model, respectively.

$t \in [0, t_F]$, t_F is the number of days for which data are taken at the hour in question.

Subject to satisfying the inequality constraints given by:

$$y_j(t) \geq [p_o + \sum_{i=1}^{n} (p_i \sin i\omega t + \alpha_i \cos i\omega t)]_j - (1-\lambda)[c_o + \sum_{i=1}^{n} (c_i \sin i\omega t + b_i \cos i\omega t)_j \quad (5.19)$$

$$y_j(t) \leq [p_o + \sum_{i=1}^{n} (p_i \sin i\omega t + \alpha_i \cos i\omega t)]_j + (1-\lambda)[c_o + \sum_{i=1}^{n} (c_i \sin i\omega t + b_i \cos i\omega t)_j \quad (5.20)$$

The optimization problem formulated in equations (5.18) to (5.20) is a linear optimization problem and can be solved using the simplex method of linear programming .

Having obtained the fuzzy load parameters, the load for the next twenty-four hours can be predicted using equation (5.16)

5.2.3 Fuzzy load Model C.

This is a fuzzy hybrid model that takes into account weather dependent components. The base load in the model is a time-varying function and takes the form of Fourier's coefficients. This model can be considered as a combination of fuzzy load model A and fuzzy load model B. Here the weather input is limited only to temperature deviation , and the model is used for both winter and summer load forecast simulations.

The fuzzy load model in this case, can be written mathematically as:

$$\underline{Y}_j(t) = \{\underline{A}_o + \sum_{i=1}^{n} [\underline{A}_i \sin i\omega t + \underline{B}_i \cos i\omega t]\}_j + \{\underline{C}_0 T_j(t) + \underline{C}_1 T_j(t-1) + \underline{C}_2 T_j(t-2) + \underline{C}_3 T_j(t-3)\} \quad (5.21)$$

where

$\underline{A}_o$, $\underline{A}_i$ and $\underline{B}_i$ are the weather independent fuzzy parameters having certain middles and certain spreads.

$\underline{C}_o$, $\underline{C}_1$, $\underline{C}_2$ and $\underline{C}_3$ are the temperature dependent fuzzy parameters.

The terms in the first brace in equation (5.21) can be considered as the base load which depends only on time, while the terms in the second brace are the temperature dependent load terms.

Equation (5.21) can be written as:

$$\underline{Y}(t) = (p_o, c_o) + \sum_{i=1}^{n} [\,(p_i, \alpha_i)x_i(t) + (b_i, \beta_i)\, y_i(t)]_j + [(\gamma_o, s_o)T_j(t) + (\gamma_1, s_1)T_j(t-1) + (\gamma_2, s_2)T_j(t-2) + (\gamma_3, s_3)T_j(t-3)] \qquad (5.22)$$

where in equation (5.22), the first letter in the parameters brackets indicates the middle of that parameter and the second letter indicates the spread of this parameter.

In fuzzy regression, the fuzzy model parameters are to be found to minimize the spread of the output. In mathematical form, this can be expressed as:

$$J = \left| \sum_t \Big\{ c_o + \sum_{j=1}^{m} \sum_{i=1}^{n} [\alpha_i\, x_{ij}(t) + \beta_i y_{ij}(t)] + \sum_{j=1}^{m} [s_o T_j(t) + s_1 T_j(t-1) + s_2 T_j(t-2) + s_3 T_j(t-3)] \right| \qquad (5.23)$$

where $t \in [0, t_F]$, t_F is the number of days for which data are taken at the hour in question.

Subject to satisfying the following two constraints on the output so that the fuzzy regression model could contain all the observed data j, j=1,..., m in the estimated fuzzy numbers resulting from the model. This can be expressed mathematically as:

$$y_j(t) \geq [p_o + \sum_{i=1}^{n} (p_i x_{ij}(t) + b_i y_{ij}(t)) + \gamma_o T_j(t) + \gamma_1 T_j(t-1) + \gamma_2 T_j(t-2) + \gamma_3 T_j(t-3)] - (1-\lambda)[\, c_o + \sum_{i=1}^{n} (\alpha_i x_{ij}(t) + \beta_i y_{ij}(t)) + s_o T_j(t) + s_1 T_j(t-1) + s_2 T_j(t-2) + s_3 T_j(t-3)]$$

$$, j=1,\ldots, m \qquad (5.24)$$

$$y_j(t) \le [p_o + \sum_{i=1}^{n} (p_i x_{ij}(t) + b_i y_{ij}(t)) + \gamma_o T_j(t) + \gamma_1 T_j(t-1) + \gamma_2 T_j(t-2) + \gamma_3 T_j(t-3)]$$

$$+ (1-\lambda)\, [c_o + \sum_{i=1}^{n} (\alpha_i x_{ij}(t) + \beta_i y_{ij}(t)) + s_o T_j(t) + s_1 T_j(t-1) + s_2 T_j(t-2) + s_3 T_j(t-3)]$$

$$, j=1,\ldots, m \qquad (5.25)$$

The problem formulated in equation (5.23) to (5.25) is a linear optimization problem and can be solved using linear programming based on the simplex method explained in chapter 4. Having identified the fuzzy model parameters, the load for the next twenty-four hours can be predicted using equation (5.22)

5.3 Fuzzy Data: Fuzzy Power Load

In section 5.2 the load power data is assumed to be non-fuzzy, while the parameters of the load power are fuzzy. Different linear optimization problems were derived with different load models. In this section, the load data are assumed to be fuzzy power values having certain middle and certain spread $\underline{Y}_j(t) = [m_j(t), \alpha_j(t)]$, where $m_j(t)$ is the middle of the load power at the time t in question during the observation j, and $\alpha_j(t)$ is the spread of the load power at time t and observation j. Using this formulation of fuzzy number means that a triangular membership function is assumed, as shown in Figure (5.1a) and (5.1b).

5.3.1 Multiple Fuzzy Linear Regression, Model A

The fuzzy model for the load power can be expressed mathematically as :

$$\underline{Y}_j(t) = [m_j(t), \alpha_j(t)] = \underline{A}_o + \sum_{i=1}^{n} \underline{A}_i\, x_{ij}(t) \qquad , j=1,.., m \qquad (5.26)$$

which can be rewritten as:

$$[m_j(t), \alpha_j(t)] = (p_o, c_o) + \sum_{i=1}^{n} (p_i, c_i)\, x_{ij}(t) \qquad , j=1,..., m \qquad (5.27a)$$

or, it can be separated as:

$$[m_j(t), \alpha_j(t)] = [\{p_o + \sum_{i=1}^{n} p_i x_{ij}(t)\}, \{c_o + \sum_{i=1}^{n} c_i x_{ij}(t)\}] \qquad , j = 1,\ldots, m \qquad (5.27b)$$

Equation (5.27b) is only valid when*:

$$m_j(t) = p_o + \sum_{i=1}^{n} p_i x_{ij}(t) \qquad ,j=1,..,..,m \tag{5.28}$$

$$\alpha_j(t) = c_o + \sum_{i=1}^{n} c_i x_{ij}(t) \qquad ,j=1,..,..,m \tag{5.29}$$

The problem turns out to be: Given the fuzzy load power at time t

$\underset{\sim}{Y}_j(t) = [m_j(t),\alpha_j(t)]$, it is required to find the fuzzy parameters $\underset{\sim}{A}_o$ and $\underset{\sim}{A}_i$ that minimize the cost function given by:

$$J=\left|\sum_t\left\{ \sum_{j=1}^{m} \{m_j(t) - p_o - \sum_{i=1}^{n} p_i x_{ij}(t) + \alpha_j(t) - c_o - \sum_{i=1}^{n} c_i x_{ij}(t)\} \right.\right| \tag{5.30}$$

where $t \in [0, t_F]$, t_F is the number of days for which data are taken at the hour in question.

Subject to satisfying the following constraints on each measurement point:

---

* Given two fuzzy numbers $M_1 = (m_1, \alpha_1, \beta_1)_{LR}$ and $M_2 = (m_2, \alpha_2, \beta_2)_{LR}$ in terms of LR functions **[78]** that follow triangular membership function.

where

m_1 and m_2 are the centers of the membership function

α_1 and α_2 are left side spreads

β_1 and β_2 are right side spreads

Then

$$M_1(m_1, \alpha_1, \beta_1)_{LR} + M_2(m_2, \alpha_2, \beta_2)_{LR} = (m_s, \alpha_s, \beta_s)_{LR}$$

where

$m_s = m_1 + m_2$

$\alpha_s = \alpha_1 + \alpha_2$

$\beta_s = \beta_1 + \beta_2$

The center of the sum is equal to the sum of the centers and each of the spreads of the sum are the sum of the respective spreads.

$$m_j(t) - (1-\lambda)\,\alpha_j(t) \geq (p_o + \sum_{i=1}^{n} x_{ij}(t)) - (c_o + \sum_{i=1}^{n} c_i\, x_{ij}(t)) \qquad ,j=1,\ldots,m \qquad (5.31)$$

$$m_j(t) + (1-\lambda)\,\alpha_j(t) \leq (p_o + \sum_{i=1}^{n} x_{ij}(t)) + (c_o + \sum_{i=1}^{n} c_i\, x_{ij}(t)) \qquad ,j=1,\ldots,m \qquad (5.32)$$

The problem formulated in equations (5.30) to (5.32) is a linear optimization problem. This problem can be solved using linear programming. In the next subsections two multiple linear regression models are discussed, one for the winter and one for the summer .

5.3.1.1 Fuzzy Winter Model

Two factors affect this model. The first is the temperature deviation. The more temperature deviation the more load power is needed. While the second factor is the wind-cooling factor, as the wind cooling factor increases, the load power increases. The load power data in this model is assumed to be a fuzzy power unlike the load model in equation (5.6), where the load power is assumed to be crisp (non-fuzzy). Equation (5.7) can be written as

$$\begin{aligned} \underline{Y}_j(t) &= (m_j(t), \alpha_j(t)) \\ &= (p_o, c_o) + (p_1, c_1)T_j(t) + (p_2, c_2)\,T^2_j(t) + (p_3, c_3)\,T^3_j(t) \\ &\qquad + (p_4, c_4)T_j(t-1) + (p_5, c_5)T_j(t-2) + (p_6, c_6)T_j(t-3) \\ &\qquad + (p_7, c_7)W_j(t) + (p_8, c_8)W_j(t-1) + (p_9, c_9)W_j(t-2) \end{aligned} \qquad (5.33)$$

Equation (5.33) can be rewritten as:

$$\begin{aligned} m_j(t) &= p_o + p_1T_j(t) + p_2\,T^2_j(t) + p_3T^3_j(t) + p_4T_j(t-1) + p_5T_j(t-2) + p_6T_j(t-3) \\ &\qquad + p_7W_j(t) + p_8W_j(t-1) + p_9W_j(t-2) \qquad ,j=1,\ldots,m \end{aligned} \qquad (5.34)$$

$$\begin{aligned} \alpha_j(t) &= c_o + c_1T_j(t) + c_2\,T^2_j(t) + c_3T^3_j(t) + c_4T_j(t-1) + c_5T_j(t-2) + c_6T_j(t-3) \\ &\qquad + c_7W_j(t) + c_8W_j(t-1) + c_9W_j(t-2) \qquad ,j=1,\ldots,m \end{aligned} \qquad (5.35)$$

Given the fuzzy load power $(m_j(t), \alpha_j(t))$ at any time t, it is required to determine the middle and the spread of each parameter that minimize the following cost function :

$$J=\left|\sum_{t}\left\{\sum_{j=1}^{m}\left[m_j(t)-\{p_o+p_1T_j(t)+p_2T^2_j(t)+p_3T^3_j(t)+p_4T_j(t-1)+p_5T_j(t-2)\right.\right.\right.$$

$$+p_6T_j(t-3)+p_7W_j(t)+p_8W_j(t-1)+p_9W_j(t-2)\}$$

$$+\alpha_j(t)-\{c_o+c_1T_j(t)+c_2T^2_j(t)+c_3T^3_j(t)+c_4T_j(t-1)+c_5T_j(t-2)+c_6T_j(t-3)$$

$$\left.\left.\left.+c_7W_j(t)+c_8W_j(t-1)+c_9W_j(t-2)\}\right]\right|\qquad(5.36)$$

where $t\in[0,\ t_F\ ]$, t_F is the number of days for which data are taken at the hour in question.

Subject to satisfying the following two constraints at each measurement point.

$$m_j(t)-(1-\lambda)\,\alpha_j(t)\geq[\,(\text{RHS of equation 5.34})-(\text{RHS of equation 5.35})],\quad j=1,\ldots,m\qquad(5.37)$$

$$m_j(t)+(1-\lambda)\,\alpha_j(t)\leq[\,(\text{RHS of equation 5.34})+(\text{RHS of equation 5.35})],\quad j=1,\ldots,m\qquad(5.38)$$

RHS in the above two equations stands for the right hand side of.

The problem formulated in equations (5.36) to (5.38) is one of linear optimization. This problem can be solved using standard linear programming .

Having identified the fuzzy parameters of the fuzzy winter model, the load in a winter day can be predicted. The middle of the load can be predicted at any hour t using equation (5.34), while the spread can be predicted using equation (5.35).

5.3.1.2 Fuzzy Summer Model

The load in this model is a function of the temperature deviation and the humidity factor. The load power as well as the load model parameters are assumed to be fuzzy numbers. Mathematically, this can be expressed as:

$$\underline{Y}_j(t)=(m_j(t),\alpha_j(t))$$

$$=\underline{A}_o+\underline{A}_1T_j(t)+\underline{A}_2T^2_j(t)+\underline{A}_3T^3_j(t)+\underline{A}_4T_j(t-1)$$

$$+\underline{A}_5T_j(t-2)+\underline{A}_6T_j(t-3)+\underline{A}_7H_j(t)+\underline{A}_8H_j(t-1)+\underline{A}_9H_j(t-2)$$

$$,\ j=1,\ldots,m\qquad(5.39)$$

where

$\underset{\sim}{Y}_j(t)$ is the fuzzy load power i; i= 1,...,m, at time t. This power has a middle $m_j(t)$ and a spread $\alpha_j(t)$

$\underset{\sim}{A}_o, \underset{\sim}{A}_1, \ldots, \underset{\sim}{A}_9$ are the fuzzy load parameters at time t with certain middle $p_o,\ldots,p_9$ and certain spread $c_o, c_1, \ldots, c_9$.

$T_j(t)$ is the temperature deviation at time t , j=1,..., m.

$H_j(t)$ is the humidity factor given by equation (3.17)

Equation (5.39) can be written as:

$$\begin{aligned}\underset{\sim}{Y}(t) &= (m_j(t), \alpha_j(t)) \\ &= (p_o, c_o) + (p_1, c_1)\,T(t) + (p_2, c_2)\,T^2(t) + (p_3, c_3)\,T^3(t) \\ &\quad + (p_4, c_4)\,T(t-1) + (p_5, c_5)\,T(t-2) + (p_6, c_6)\,T(t-3) \\ &\quad + (p_7, c_7)H(t) + (p_8, c_8)H(t-1) + (p_9, c_9)H(t-2)\end{aligned} \qquad (5.40)$$

provided that the memberships for the fuzzy numbers are triangular memberships. Equation (5.40) can be rewritten as two equations:

$$\begin{aligned}m_j(t) &= p_o + p_1T_j(t) + p_2T_j^2(t) + p_3T_j^3(t) + p_4T_j(t-1) + p_5T_j(t-2) \\ &\quad + p_6T_j(t-3) + p_7H_j(t) + p_8H_j(t-1) + p_9H_j(t-2) \qquad , j=1,\ldots,m\end{aligned} \qquad (5.41)$$

$$\begin{aligned}\alpha_j(t) &= c_o + c_1T_j(t) + c_2T_j^2(t) + c_3T_j^3(t) + c_4T_j(t-1) + c_5T_j(t-2) \\ &\quad + c_6T_j(t-3) + c_7H_j(t) + c_8H_j(t-1) + c_9H_j(t-2) \qquad , j=1,\ldots,m\end{aligned} \qquad (5.42)$$

In the fuzzy optimization linear problem, the model fuzzy parameters are to be found to minimize the spread of the fuzzy load power.

Mathematically, this can be expressed as:

Minimize:

$$J = \left|\sum_t \left\{\sum_{j=1}^{m}\left[\left(m_j(t) - \text{RHS of equation } 5.41\right) + \left(\alpha_j(t) - \text{RHS of equation } 5.42\right)\right]\right\}\right| \qquad (5.43)$$

where $t \in [0, t_F]$, t_F is the number of days for which data are taken at the hour in question.

Subject to satisfying the following constraints:

$$m_j(t) - (1-\lambda)\,\alpha_j(t) \geq [\text{ (RHS of equation 5.41) } - \text{ (RHS of equation 5.42)}], \quad , j=1,\ldots,m \qquad (5.44)$$

$$m_j(t) + (1-\lambda)\,\alpha_j(t) \leq [\text{ (RHS of equation 5.41) } + \text{ (RHS of equation 5.42)}], \quad , j=1,\ldots,m \qquad (5.45)$$

The optimization problem formulated in equations (5.43) to (5.45) is one of linear optimization and can be solved using linear programming .

Having obtained the fuzzy load parameters, then equation (5.39) can be used to predict the fuzzy load power at any hour t in question.

5.3.2 Fuzzy Load Model B

This model does not account for weather conditions in the load and it can be expressed as:

$$\underline{Y}_j(t) = (m_j(t), \alpha_j(t)) = \underline{A}_o + \sum_{i=1}^{n} [\,(\underline{A}_i \sin i\omega t + \underline{B}_i \cos i\omega t)\,]_j \qquad ,j=1,\ldots,m \qquad (5.46)$$

The only difference between equation (5.17) and (5.46) is the load power $\underline{Y}_j(t)$ at time t. In (5.17) the load power is assumed to be a crisp value, while in (5.46) it ia assumed to be a fuzzy value having a middle $m_j(t)$ and a spread $\alpha_j(t)$. Equation (5.46) can be written as :

$$(m_j(t), \alpha_j(t)) = (p_o, c_o) + \sum_{i=1}^{n} [(p_i, c_i) \sin i\omega t + (b_i, \beta_i) \cos i\omega t]_j \qquad ,j=1,\ldots,m \qquad (5.47)$$

which can be split into

$$m_j(t) = p_o + \sum_{i=1}^{n} [(p_i \sin i\omega t + b_i \cos i\omega t]_j \qquad ,j=1,\ldots,m \qquad (5.48)$$

$$\alpha_j(t) = c_o + \sum_{i=1}^{n} [\,c_i \sin i\omega t + \beta_i \cos i\omega t]_j \qquad ,j=1,\ldots,m \qquad (5.49)$$

The load fuzzy parameters are to be found that minimize the spread of the fuzzy load power. This can be expressed mathematically as:

$$J = \left| \sum_t \{ \sum_{j=1}^{m} [(m_j(t) - \text{RHS of equation 5.48}) + (\alpha_j(t)\text{-RHS of equatio5.49})] \right| \qquad (5.50)$$

where $t \in [0, t_F]$, t_F is the number of days for which data are taken at the hour in question.

Subject to satisfying the following two constraints as:

$$m_j(t) - (1-\lambda)\,\alpha_j(t) \geq [\text{RHS of equation 5.48 - RHS of equation 5.49}]; \quad ,j=1,\ldots,m \qquad (5.51)$$

$$m_j(t) + (1-\lambda)\,\alpha_j(t) \leq [\text{RHS of equation 5.48 + RHS of equation 5.49}]; \quad ,j=1,\ldots,m \qquad (5.52)$$

The problem formulated in equations (5.50) to (5.52) is one of linear optimization that can be solved using linear programming .

Having identified the middle and the spread of fuzzy parameters, then the harmonic load model described in equation (5.47) can be used to predict the load at any hour t. Note that the load power obtained in this case is independent of the weather conditions, and depends only on the hour in question. The next model, model C, combines the fuzzy load model A and the fuzzy load model B. This model takes weather conditions into account.

5.3.3 Fuzzy Load Model C

The fuzzy load model A derived earlier has the advantage of being weather responsive, the fuzzy coefficients of this model depend on the weather conditions. These conditions include temperature deviation and cooling factor.

Model B is weather insensitive. The fuzzy coefficients of this model depend only on the time in question.

In this section, the two models A and B are combined into one fuzzy model. The resulting model is weather sensitive. This fuzzy model is suitable for all weekdays and can be used for both winter and summer load forecast simulations. Its main disadvantage is the assumption that the relation between load and weather is constant throughout the day.

The fuzzy model for the load in this case can be expressed mathematically as:

$$\underline{Y}_j(t)=(m_j(t), \alpha_j(t)) =\{\underline{A}_o + \sum_{i=1}^{n} (\underline{A}_i \sin i\omega t + \underline{B}_i \cos i\omega t)\}_j + \{\underline{C}_o T_j(t) + \underline{C}_1 T_j(t-1)$$

$$+ \underline{C}_2 T_j(t-2) + \underline{C}_3 T_j(t-3)\}_j \qquad ,j=1,\ldots,m \qquad (5.53)$$

where $m_j(t)$, $\alpha_j(t)$ is the middle and spread of load power j, j=1,...,m at time t.

$\underset{\sim}{A}_o$, $\underset{\sim}{A}_i$, and $\underset{\sim}{B}_i$ are the weather independent fuzzy parameters with certain middles and spreads.

$\underset{\sim}{C}_o$, $\underset{\sim}{C}_1$, $\underset{\sim}{C}_2$, and $\underset{\sim}{C}_3$ are the temperature dependent fuzzy parameters with certain middles and spreads.

The LHS of equation (5.53) is the fuzzy load power. The terms in the first bracket in the RHS of equation (5.53) can be considered as the fuzzy base load, and it depends only on time, while the second bracket is the temperature dependent fuzzy load terms.

Equation (5.53) can be written as

$$(m_j(t), \alpha_j(t)) = \{ (p_o , c_o) + \sum_{i=1}^{n} [(p_i , \theta_i)x_i(t) + (b_i , \beta_i)y_i(t)]\}_j + \{(\gamma_o , c'_o)T_j(t) + (\gamma_1, c_1)T_j(t-1) + (\gamma_2 , c_2)T_j(t-2) + (\gamma_3 , c_3)T_j(t-3)\}_j \quad , j=1,\ldots,m \qquad (5.54)$$

For simplicity let :

$$x_i(t) = \sin i\omega t \qquad , i=1,\ldots,n \qquad (5.55a)$$

$$y_i(t) = \cos i\omega t \qquad , i=1,\ldots,n \qquad (5.55b)$$

In equation (5.54), the first letter in all small brackets of the equations indicates the middle of the parameter, while the second letter indicates the spread of that parameter. A triangular membership is used for each parameter .

In the fuzzy model developed in equation (5.54), the fuzzy model parameters are to be found to minimize the spread of the output. Mathematically, the fuzzy linear optimization problem can be expressed as:

Minimize:

$$J = \Big| \sum_{t} \Big\{ \sum_{j=1}^{m} [m_j(t) - \{p_o + \sum_{i=1}^{n} [p_i\, x_i(t) + b_i\, y_i(t)]_j + \gamma_o T_j(t) + \gamma_1 T_j(t-1) + \gamma_2 T_j(t-2) + \gamma_3 T_j(t-3)\} + \{\alpha_j(t) - [c_o + \sum_{i=1}^{n} \{\theta_i\, x_i(t) + \beta_i y_i(t)\}_j + c'_o\, T_j(t) + c_1 T_j(t-1) + c_2 T_j(t-2) + c_3 T_j(t-3)\}] \Big| \qquad (5.56)$$

where $t \in [0, t_F]$, t_F is the number of days for which data are taken at the hour in question.

Subject to satisfying the following two constraints for each measurement point given as:

$$m_j(t) - (1-\lambda)\,\alpha_j(t) \geq \{p_o + \sum_{i=1}^{n} \{[p_i\, x_i(t) + b_i\, y_i(t)]\}_j + \gamma_o T_j(t) + \gamma_1 T_j(t-1) + \gamma_2 T_j(t-2)$$
$$+ \gamma_3 T_j(t-3)\} - \{c_o + \sum_{i=1}^{n} \{\theta_i\, x_i(t) + \beta_i y_i(t)\}_j + c'_o\, T_j(t) + c_1 T_j(t-1)$$
$$+ c_2 T_j(t-2) + c_3 T_j(t-3)\} \qquad , j = 1,\ldots,m \qquad (5.57)$$

$$m_j(t) + (1-\lambda)\,\alpha_j(t) \leq \{ p_o + \sum_{i=1}^{n} [p_i\, x_i(t) + b_i\, y_i(t)] + \gamma_o T(t) + \gamma_1 T_j(t-1) + \gamma_2 T_j(t-2)$$
$$+ \gamma_3 T_j(t-3)\} + \{c_o + \sum_{i=1}^{n} \{\theta_i\, x_i(t) + \beta_i y_i(t)\}_j + c'_o\, T_j(t) + c_1 T_j(t-1)$$
$$+ c_2 T_j(t-2) + c_3 T_j(t-3)\} \qquad , j=1,\ldots,m \qquad (5.58)$$

The problem formulated in equations (5.56) to (5.58) is one of linear optimization and can be solved by linear programming .

Having obtained the middle and spread of each fuzzy parameters, then the load power at any hour in question can be calculated using equation (5.54)

5.4 Conclusions

In this chapter a new formulation for fuzzy short-term load forecasting models is presented, where in the first part of the chapter, the load power is considered given as a crisp (non-fuzzy) data while the load model parameters are fuzzy having certain middles and spreads. The problem turns out to be one of linear optimization .

In the second part of the chapter, the load power is considered to be a fuzzy power data having certain middles and spreads. Three different fuzzy models A, B, and C are developed and new fuzzy equations are obtained. The resulting optimization problem is linear and can be solved using linear programming .

Chapter 6

Load Forecasting Computational Results

Static State Estimation

6. 1 Introduction

In the previous chapters different models are developed for short-term load forecasting during the summer and winter seasons. In chapter three the models are derived on the basis that the load powers are crisp in nature, while in chapter five the models are developed on the basis of fuzzy load powers. For sake of comparison, the data available from Nova Scotia Power Inc. are used to forecast the load power in the crisp case as well as in the fuzzy case. In the first part of this chapter, the results obtained for the crisp load power data for the different load models developed in chapter three are shown. In the second part, the results obtained for the fuzzy load powers for the different fuzzy load models developed in chapter five are shown. A comparison is done at the end of the chapter for the two cases.

6. 2 Description of the Data

Nova Scotia Power Inc. supplied the data used in this study for the years 1994 and 1995 hourly load power, while the Atlantic Climate Center of Environment Canada supplied the hourly weather conditions for the same two years that were extracted from Environment Canada's Archives. These data include hourly dry bulb temperatures, the wind speed and the percentage humidity recorded at Shearwater Airport at Halifax. A standard record format has been adopted for climatological data. Each record consists of station identification. date (year, month and day) and element number followed by the data repeated 24 times. The element number identifies each data type and implies the units and decimal position. The element numbers are described in Table (6.1) as they appeared in the data from Enviroment Canada. If there are missing data for a certain hour, denoted by -999, the average value of the hour before and hour after are used to replace this missing point.

Table 6.1 The Elemens and Their Units

Element	Units	Description
78	0.1 deg C	Dry Bulb Temperature
76	km/h	Wind Speed
80	percent	Humidity

6. 3 Off-Line Simulation (Static Load Forecasting Estimation)

In this section, the three off-line load models for crisp load power data that were developed in chapter three have been used to predict the next day hourly load profile for selected periods for winter and summer of 1994.

For each load model, the least errors square (LES) and least absolute value (LAV) algorithms are used to estimate the load model parameters. The results for each load model parameters are given in table format, while the final forecasts for LES and LAV together with the actual load are given in the form of curves. Furthermore the estimated parameters for each model are used to predict the load 24 hours ahead for the same time period.

The following abbreviations are used in this section

z = Actual recorded load.

z_{LS} = Load forecast made from least errors square.

z_{LAV} = Load forecasted using the least absolute value algorithm.

The percentage errors corresponding to the forecasted loads are given by:

$$\varepsilon_L = [(z - z_{LS}) / z] \times 100$$

and

$$\varepsilon_{LAV} = [(z - z_{LAV}) / z] \times 100$$

6. 4 Model A

Model A has been described in chapter three for crisp load powers. It is a multiple linear regression model whose parameters are constants during the hour considered. A parameter estimate from the available data is obtained for every hour.

An excessive volume of computations is associated with a single twenty-four hours load prediction. The days are chosen for the prediction process in a random way . The model is applied to different days in the same period of time (same time) for the same season. The parameters estimated for each of these days are not reported because the obtained predictions for more days are essentially the same.

Two approaches are applied. The first approach estimates a given parameter for every hour in question in the day. The days are chosen randomly. The second approach assumes the model parameters to be constants during the whole day studied. The estimated parameters in the two approaches are used to predict the load for one day ahead of a working day and a weekend day in summer and winter.

6. 4. 1 Model Parameters Estimation for Every Hour in a Summer Weekday (24 sets) :

Tables (6.2) and (6.3) give the estimated parameters for a summer weekday using the LES and LAV algorithms. While Table 6.4 gives the estimated load and percentage errors in the estimates using the least errors squares (LES) and least absolute value (LAV) algorithms. Figure (6.1) gives a comparison between actual and estimated loads, while Figure (6.2) gives the errors in the estimated powers compared to actual load. From these tables and figures the following remarks are noted :

- LES estimates the actual load value with a maximum error of 9.1% (underestimated) at hour 24, and a minimum error of 0.1% (overestimated) at hour 1. Most error values are below 4% (19 hours).
- LAV estimates the actual load value with a maximum error of 10.6% (underestimated) at hour 23 and a minimum error of 0% at hours 3, 10, 18, 22.
- Since many error values are less then 4% for both algorithms, the estimated power values during the day (even with wide variations in the weather data) are still acceptable. If the redundancy in the estimated parameters is increased, the errors in the LAV estimates will be decreased.

Table (6.2) Estimated parameters for summer weekday using LES

Hour	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
1.0	1485.72	-0.06	0.67	-0.03	-85.81	125.68	-26.89	-5021	145.44	-167.20
2.0	1087.29	-77.95	0.09	-0.16	0.05	91.86	0.37	133.90	9.98	-159.39
3.0	1430.12	69.93	0.16	-0.15	-51.12	0.89	4.8	-122.4	86.21	5.42
4.0	1329.39	-101.96	-0.54	-0.19	171.69	-47.67	0.27	172.15	-296.75	97.17
5.0	1524.05	3.91	0.21	-0.09	-141.83	154.46	6.87	34.12	215.41	-238.16
6.0	183.12	-209.76	2.83	-0.05	46.45	142.08	16.73	407.91	-33.91	-357.56
7.0	2126.05	18.25	1.47	0.14	16.32	-15.47	7.32	15.62	-54.59	-16.73
8.0	2255.80	357.16	3.53	0.36	-216.20	-33.64	-80.21	-695.73	490.80	158.28
9.0	1676.21	-10.22	21.37	1.49	-100.93	93.19	87.58	167.32	-215.71	9.87
10.0	675.81	92.49	9.87	0.76	-17.63	-147.9	69.54	-195.15	60.49	146.96
11.0	-13.12	352..99	-6.15	-0.89	-579.85	233.87	-5.45	-1114.21	1787.35	-631.76
12.0	906.51	37.41	-2.21	-0.3	77.14	-65.40	-49.86	-2.45	-486.20	501.33
13.0	2402.79	-160.45	0.2	0.05	56.89	63.64	66.33	321.47	-107.42	-266.44
14.0	1070.16	202.14	-1.96	-0.15	-359.94	155.71	-9.96	-393.21	729.93	-340.41
15.0	3054.72	-807.87	-6.31	-0.45	239.56	671.98	-58.39	910.25	316.52	-1302.35
16.0	3572.46	-233.72	-14.14	-0.88	-100.83	338.87	5.27	470.43	-151.57	-403.66
17.0	3634.89	-398.62	-11.87	-0.64	417.67	-141.99	115.88	718.68	-871.51	65.02
18.0	2637.68	35.35	0.41	-0.04	-286.61	235.38	34.42	2.97	621.69	-679.62
19.0	138.92	230.39	-2.47	-0.21	-13.45	-97.77	-125.81	-467.09	184.76	260.46
20.0	2270.39	-371.77	-9.7	-0.63	769.15	-352.97	-47.10	646.96	-1304.62	618.39
21.0	2196.61	52.98	-0.35	0.21	-140.15	140.63	-57.59	-70.79	286.13	-254.96
22.0	401.09	-395.79	0.83	-0.30	368.14	-38.64	73.38	906.26	-821.55	-46.64
23.0	2079.17	66.14	2.11	0.08	-44.17	-1.21	4.87	-25.46	29.71	-44.55
24.0	1664.7	85.43	0.4	-0.05	-54.52	7.19	-19.68	-143.52	123.10	-9.41

Table (6.3) Estimated parameters for summer weekday using LAV

Hour	A_o	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
1.0	3121.23	8.04	-3.64	0.23	-199.75	212.04	7.7	-65.25	467.30	-482.31
2.0	1468.80	-23.16	-1.02	-0.27	0.73	42.86	0.92	35.08	2.53	-66.94
3.0	2255.94	-339.70	-17.13	-1.57	332.40	5.29	-4.95	701.62	700.61	-51.41
4.0	1303.64	-59.84	0.03	-0.17	146.48	-61.04	0.20	92.20	-253.86	134.58
5.0	2325.94	-55.66	4.65	0.11	132.66	-66.12	35.54	289.02	-347.47	-2.51
6.0	-50.26	-189.05	2.17	-0.03	59.67	98.03	22.82	369.27	-57.10	-288.02
7.0	40313.83	3180.82	11.05	18.78	-2127.89	-2978.19	2410.72	-3911.42	2112.78	350.55
8.0	2065.19	400.32	2.95	0.47	-204.63	-99.20	-87.98	-773.30	516.02	219.04
9.0	5167.72	-158.56	39.85	2.56	-347.79	456.89	280.96	802.52	-471.99	-517.27
10.0	-2688.83	-660.06	19.79	1.08	1571.86	-1197.89	252.98	1122.56	-2374.36	1367.30
11.0	1078.60	140.32	-5.51	-0.72	-229.54	132.16	-28.66	-674.78	1060.20	-383.87
12.0	-379.85	64.34	-1.51	-0.39	291.02	-300.76	-74.02	-64.05	-1006.76	1139.02
13.0	2717.30	-216.48	-0.44	0.00	133.62	28.22	87.19	414.68	-192.20	-286.04
14.0	1535.7	-36.68	-18.33	-0.99	-284.97	271.08	-30.666	-124.18	491.70	-389.96
15.0	2736.55	-773.48	-5.98	-0.44	351.08	517.49	-57.37	895.09	46.65	-1005.28
16.0	211.73	114.98	-3.27	-0.12	-174.79	13.87	1.5	-358.85	416.69	-35.45
17.0	-835.93	-46.97	-7.14	-0.17	419.89	-524.28	41.55	-67.83	-946.26	1068.56
18.0	-23585.22	3485.4	-15.7	-0.41	675.49	-5438.4	699.05	-6189.0	-920.71	7850.56
19.0	3671.00	-201.02	-9.82	-0.49	-58.98	228.14	9.67	400.88	107.68	-586.85
20.0	2175.94	-159.33	-10.15	-0.93	584.64	-382.53	-18.69	191.58	-891.17	656.41
21.0	3323.57	391.81	-5.9	-0.01	-342.10	-124.78	61.03	-701.63	693.20	-81.93
22.0	794.65	-179.54	2.27	0.13	133.18	-20.76	55.26	525.48	-349.20	-161.09
23.0	1152.33	-206.01	3.58	0.35	261.59	-96.35	35.88	344.41	-370.59	18.70
24.0	1180.20	-442.01	3.12	-0.33	328.66	112.57	25.73	851.90	-300.86	-557.56

Table (6.4) Estimated load and percentage error for summer weekday, 24 parameters sets, Model A

Daily hour	Actual load(MW)	LES Estimate	LAV Estimate	% LES error	% LAV error
1.0	674.0	674.9	674.7	-0.1	-0.1
2.0	609.9	618.8	610.4	-1.5	-0.1
3.0	559.6	569.0	559.7	-1.7	0.0
4.0	537.7	550.7	538.0	-2.4	-0.1
5.0	536.8	516.9	501.3	3.7	6.6
6.0	535.6	548.6	536.5	-2.4	-0.2
7.0	545.6	555.7	546.4	-1.9	-0.1
8.0	574.4	600.7	598.3	-4.6	-4.2
9.0	668.9	689.7	695.8	-3.1	-4.0
10.0	787.3	778.5	786.9	1.1	0.0
11.0	875.1	844.6	849.2	3.5	3.0
12.0	909.4	894.5	893.9	1.6	1.7
13.0	925.0	928.2	924.9	-0.3	0.0
14.0	903.0	894.5	903.7	0.9	-0.1
15.0	876.0	855.4	846.8	2.3	3.3
16.0	848.7	869.2	870.8	-2.4	-2.6
17.0	848.3	820.7	829.0	3.3	2.3
18.0	884.8	896.7	884.6	-1.3	0.0
19.0	880.9	896.0	906.3	-1.7	-2.9
20.0	837.9	894.9	887.3	-6.8	-5.9
21.0	805.1	873.7	870.2	-8.5	-8.1
22.0	824.4	833.4	824.4	-1.1	0.0
23.0	876.2	797.2	783.4	9.0	10.6
24.0	815.6	741.6	739.4	9.1	9.3

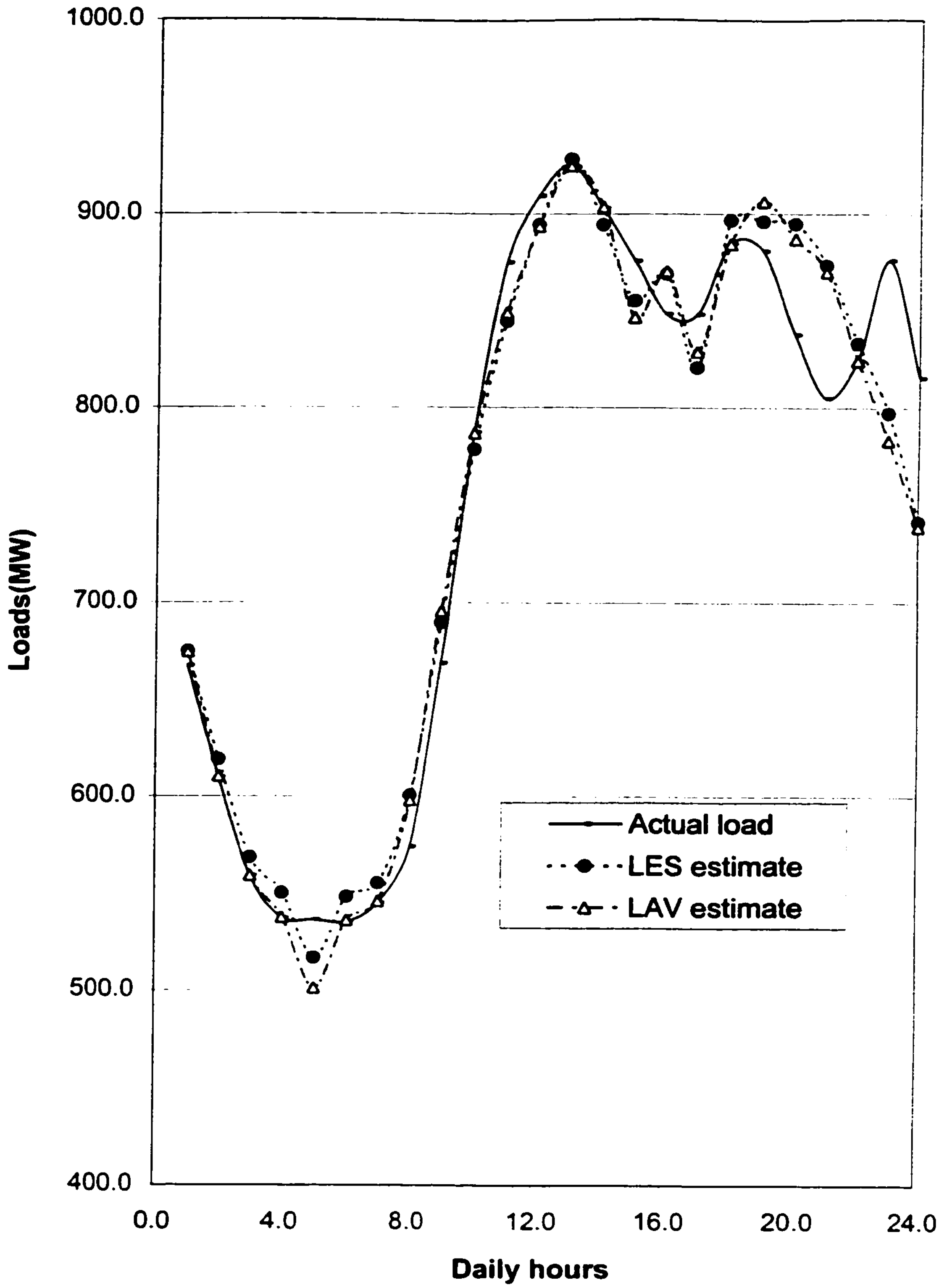


Figure (6.1) Estimated load for a summer weekday using 24 parameters sets, Model A

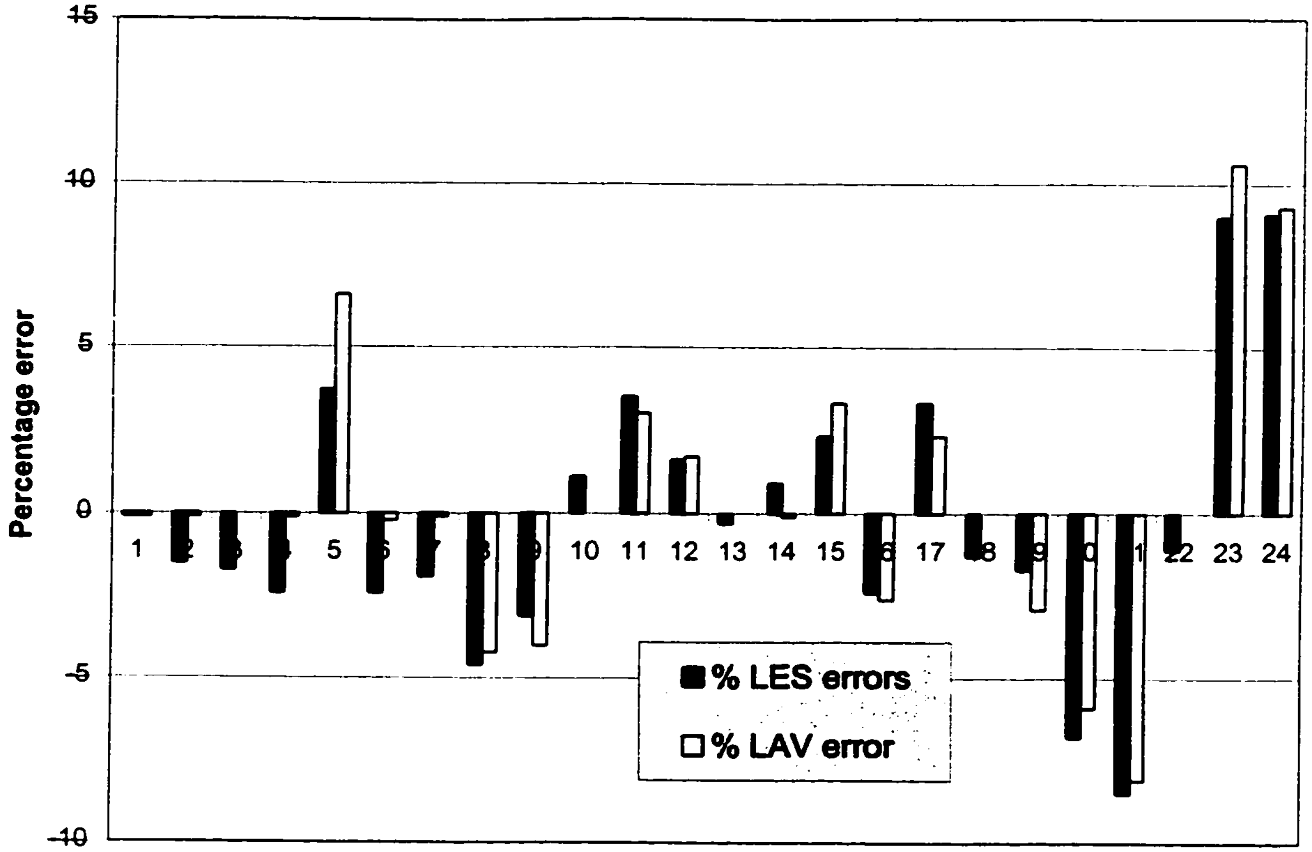


Figure (6.2) Estimated load error for a summer weekday using 24 parameters sets, Model A

The estimated parameters during the 24 hours are used to predict the load 24-hour ahead. Table 6.5 gives the predicted load power 24- hours ahead using the estimated load parameters given in Tables 6.2 and 6.3. Figure 6.3 gives the predicted load for 24 hours ahead. Figure 6.4 shows the error in the predicted load. Examining Table 6.5, Figure 6.3 and Figure 6.4 reveals the following :

- LES predicts the load 24 hours ahead with a maximum error of 10.7% (overpredited) at hour 9 and a minimum error of 0% at hour 4. Most error values are below 4% (15 hours).

- LAV predicts the load 24 hours ahead with a maximum error of 39.5% (overpredicted) at hour 7 and a minimum error of 0.1% (overpredicted) at hour 21. There are several hours where the errors are over 4%. LAV needs more data to decrease errors` values.
- Since the levels of error in LES prediction are less than LAV prediction for the load, LES predicted value represents the load better than LAV prediction. The errors` levels in LAV prediction can be reduced if the redundancy in the estimated parameters is increased.

Table (6.5) Predicted load and percentage error for summer weekday, 24 parameters sets, Model A

Daily hour	Actual load(MW)	LES Prediction	LAV Prediction	% LES error	% LAV error
1.0	681	686.8	583.9	-0.9	14.3
2.0	622.7	610	613.9	2	1.4
3.0	586.5	573.8	630.7	2.2	-7.5
4.0	569.7	569.5	564	0	1
5.0	572.3	566.2	556.8	1.1	2.7
6.0	569.7	573.6	570.2	-0.7	-0.1
7.0	594.2	573.2	828.8	3.5	-39.5
8.0	661.5	677.6	665.6	-2.4	-0.6
9.0	783.8	867.8	814.1	-10.7	-3.9
10.0	900.1	921.5	855.3	-2.4	5
11.0	979.6	945.1	938.4	3.5	4.2
12.0	1020	953.6	961.5	6.5	5.7
13.0	1047	966.5	944	7.7	9.8
14.0	1032	924.3	854.4	10.4	17.2
15.0	1016	1018	1020	-0.1	-0.3
16.0	1003	951.2	921.7	5.1	8.1
17.0	1011	923.2	869.7	8.7	14
18.0	1044	979	1007	6.2	3.5
19.0	1022	931.6	967.4	8.9	5.4
20.0	956.6	937.2	887	2	7.2
21.0	922.9	907.4	923.9	1.7	-0.1
22.0	957.2	1010	992.1	-5.5	-3.6
23.0	958.8	967.4	916.4	-0.9	4.4
24.0	874.5	872.7	860.1	0.2	1.6

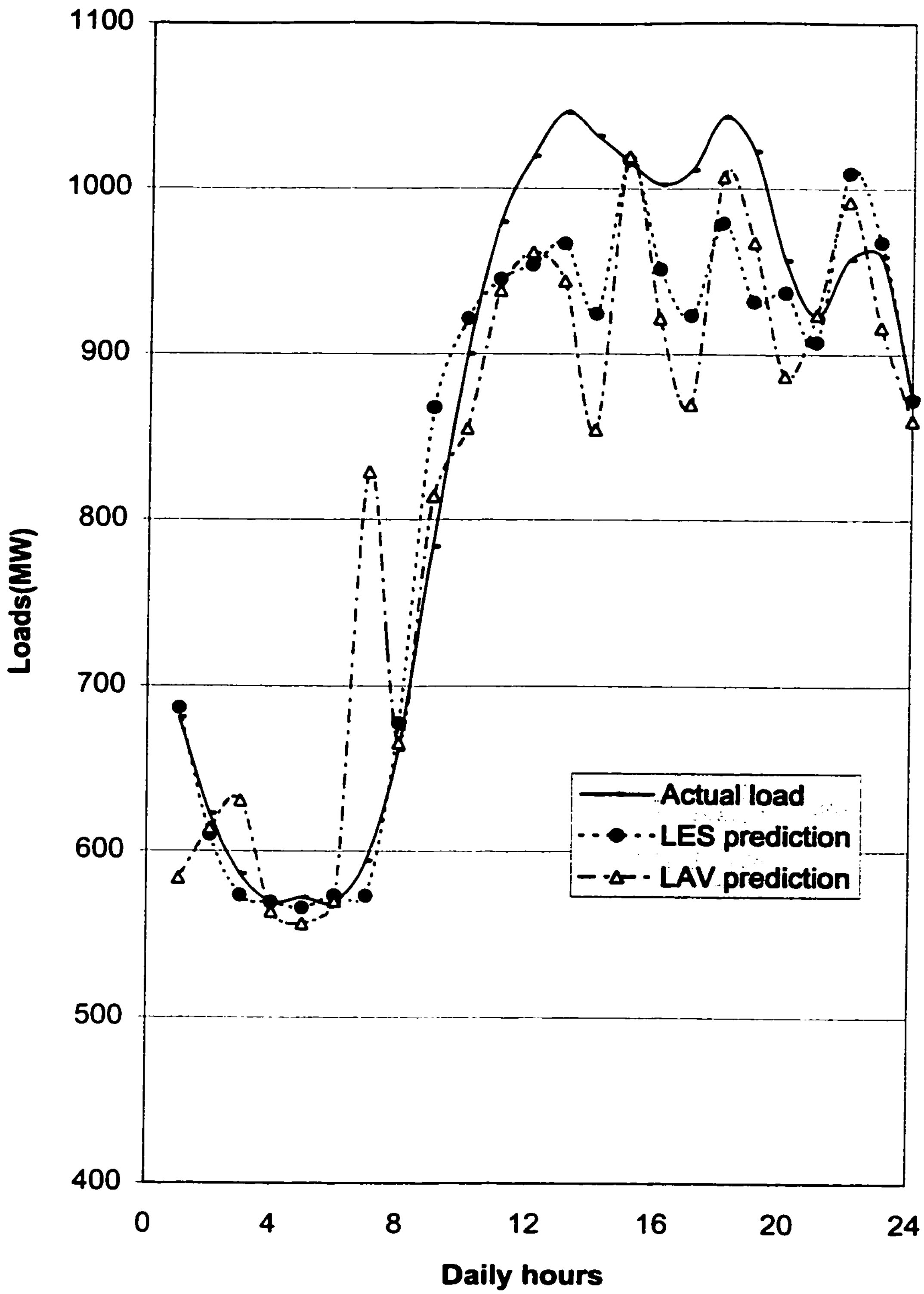


Figure (6.3) Predicted load for a summer weekday using 24 parameters sets, Model A

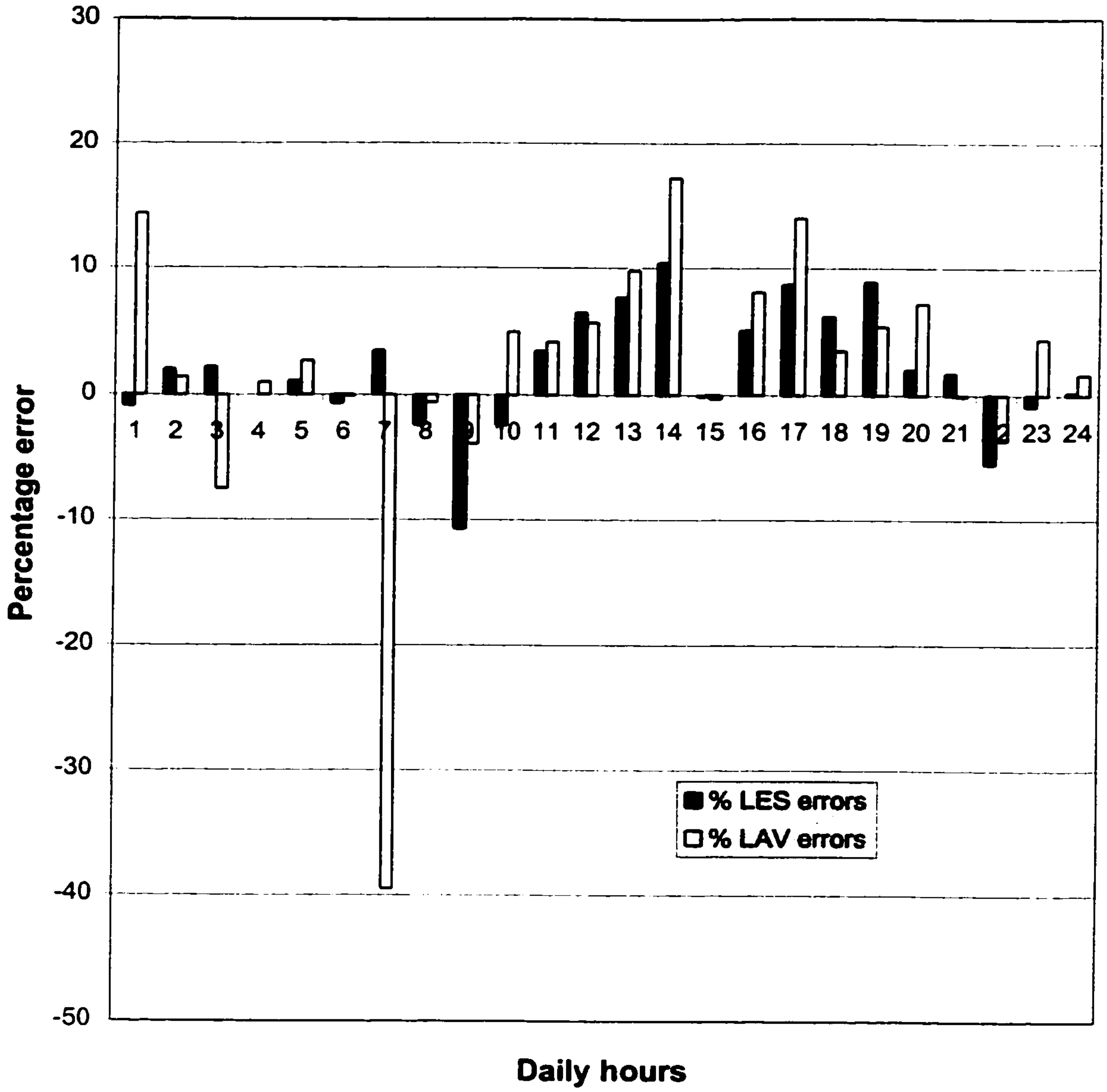


Figure (6.4) Predicted load error for a summer weekday using 24 parameters sets, Model A

6. 4. 2 Estimation of Constant Model Parameters for a Summer Weekday (1 set) :

In the second approach the load parameters are assumed to be constants during the day in question where there is only one group of parameters instead of 24 groups. Table (6.7) gives the estimated load and percentage error for a summer weekday. Figures (6.5) and (6.6) show the estimated load and the error in the estimated load . Examining these tables and figures reveals the following:

- LES estimates the load with a maximum error of 10.2% (overestimated) at hour 22 and a minimum error of 0.1% (overestimated) at hour 16. Most error values are below 4% (17 hours).
- LAV estimates the load with a maximum error of 17% (overestimated) at hour 7 and a minimum error of 0% at hours 8 and 14. Error values under 4% are at 14 hours.
- Both LES and LAV estimations for the load are showing range of errors as the 24 parameter sets. The estimations deviate from the actual load with acceptable range of errors.

The performance of two approaches for a summer weekday as explained, are also examined for a summer weekend.

Table (6.7) Estimated load and percentage error for summer weekday, one parameters set, Model A

Daily hour	Actual load(MW)	LES Estimate	LAV Estimate	% LES error	% LAV error
1	674	691.9	679.6	-2.7	-0.8
2	609	624	624	-2.3	-5.9
3	559.7	572	582	-2.3	-3.98
4	537.7	554.4	540	-3.1	-0.4
5	536.8	567.5	561.7	-5.7	-4.6
6	535.6	536.4	533.9	-0.2	0.3
7	545.6	586.2	638.3	-7.4	-17
8	574.4	566.5	574.6	1.4	0
9	668.9	685.6	694.1	-2.5	-3.8
10	787.3	830.7	680.1	-5.5	13.6
11	875.1	787.5	828	10	5.4
12	909.4	878.7	859.2	3.4	5.5
13	925	915.2	902.2	1.1	2.5
14	903	933.3	903	-3.4	0
15	876	900.9	903.5	-2.8	-3.1
16	848.7	849.5	858.3	-0.1	-1.1
17	848.3	801.6	868.2	5.5	-2.3
18	884.8	851.9	871.2	3.7	-1.5
19	880.9	877.5	874.3	0.4	0.8
20	837.9	869.9	826.3	-3.8	1.4
21	805.1	841.9	827.2	-4.6	-2.7
22	824.4	908.9	723.4	-10.2	12.3
23	876.2	887.3	871.6	-1.3	0.5
24	815.6	820.1	756.6	-0.6	7.2

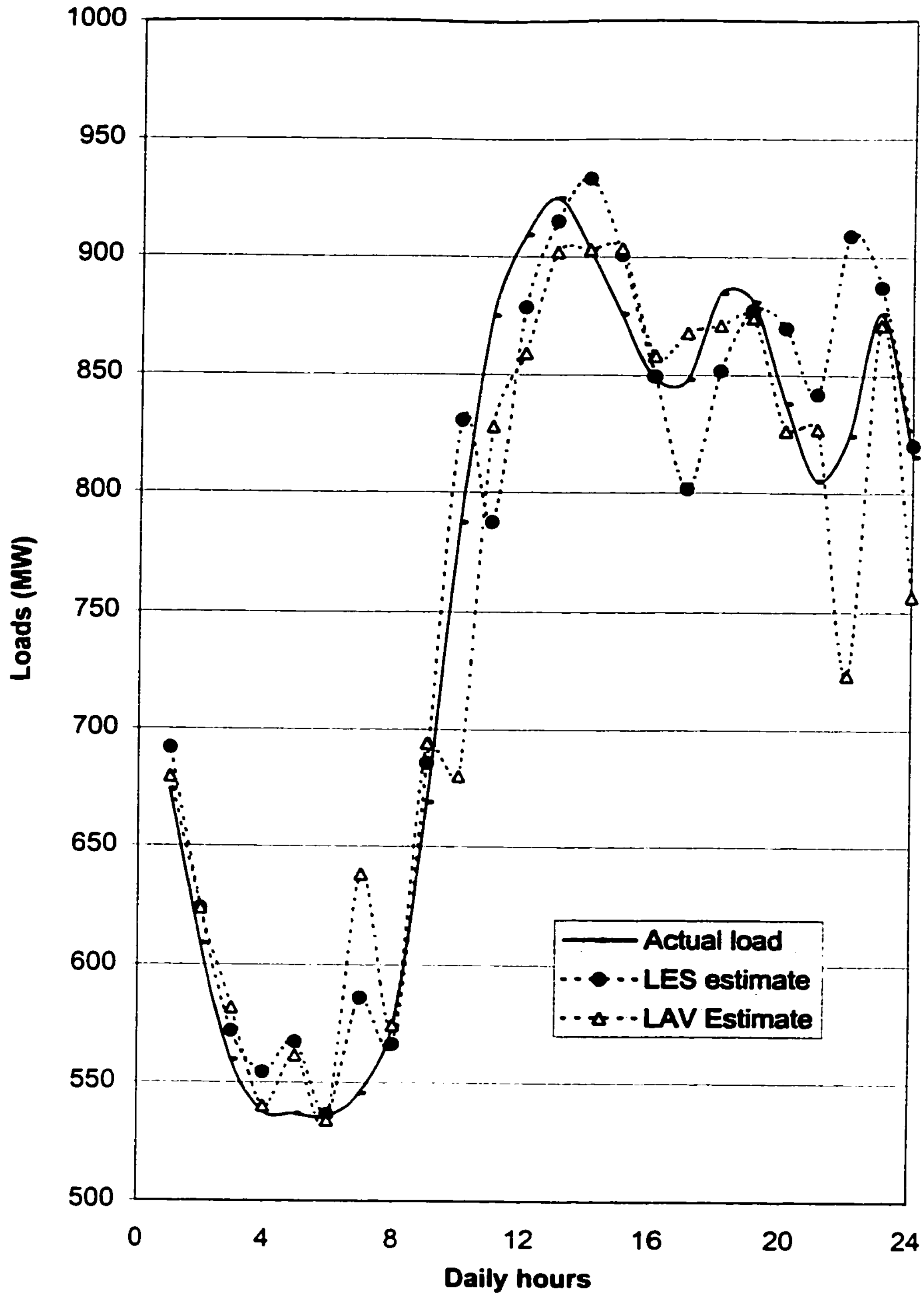


Figure (6.5) Estimated load for a summer weekday using one parameters set, Model A

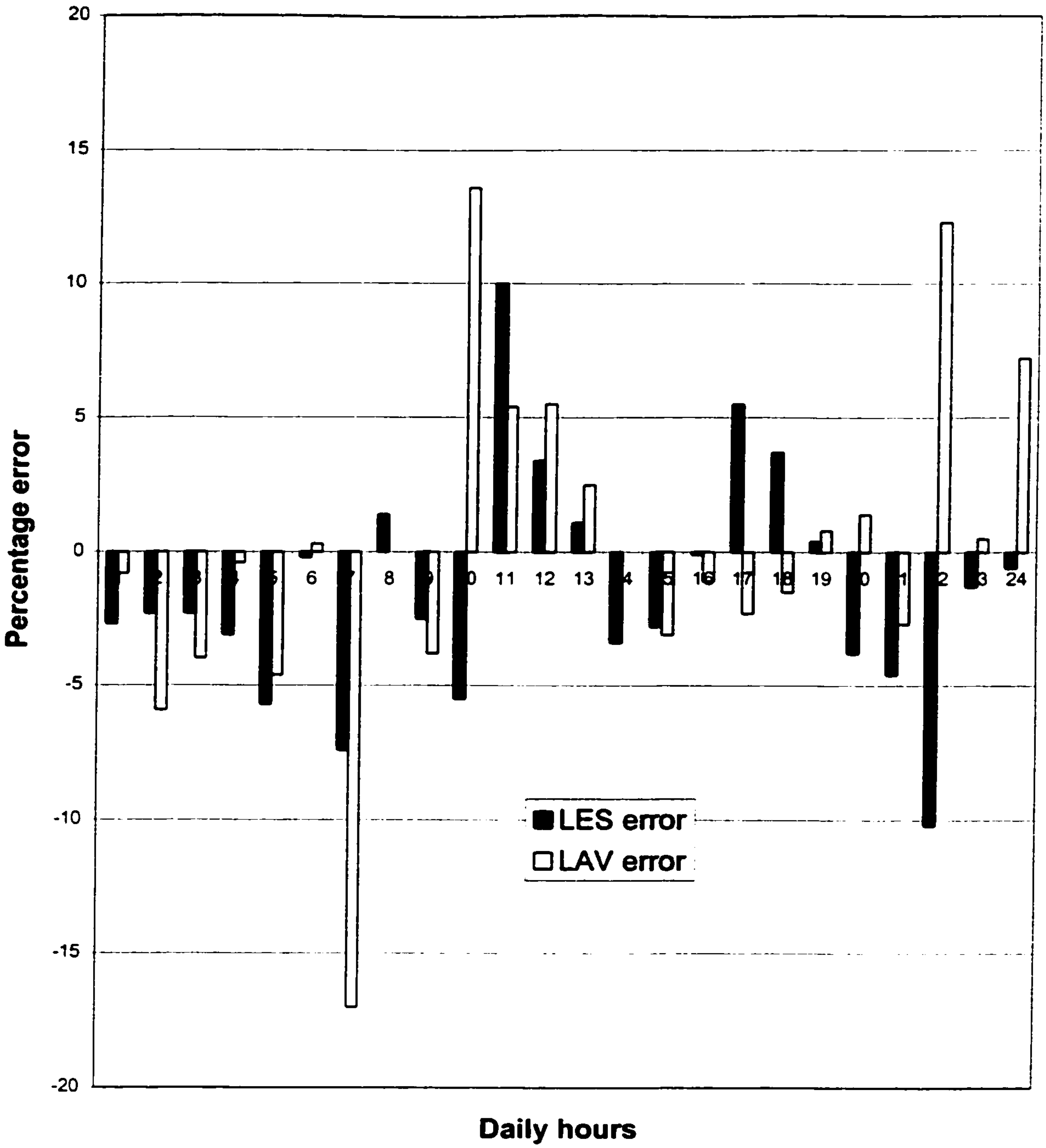


Figure (6.6) Estimated load error for a summer weekday using one parameters set, Model A

6. 4. 3 Model Parameters Estimation for Every Hour in a Summer Weekend Day (24 parameters sets) :

Tables (6.8) and (6.9) give the estimated parameters using LES and LAV techniques for a summer weekend day. Table (6.10) together with Figures (6.7) and (6.8) give the estimated load and and the percentage errors in this estimate using the sets of parameters from Tables (6.8) and (6.9). Examining these tables and figures reveals the following:

- LES estimates the load for a weekend day with a maximum error of 4.4% (overestimated) at hour 22 and a minimum error of 0.1% (overestimated) at hour 2. So, the estimated load values are good due to small error values.
- LAV estimated load value has a maximum error of 10.2% (underestimated) at hour 24 and a minimum error of 0% at houre 2 and 12. Since most of the rest of errors' values are under 4% (either overestimated or underestimated), the estimated load value is good.

The parameters sets are used to predict a load one week ahead. Table (6.11) and Figures (6.9) and (6.10) give the predicted load for a weekend ahead and the percentage error in this prediction. Examining these tables and figures reveals the following::

- The maximum error in LES predicted load is 7.34% (overpredicted) at hour 20, while the minimum error is 0.1% (underpredicted) at hour 18. Most of the rest of the errors are less than 4% (overpredicted or underpredicted) in value.
- LAV predicts the load with a maximum error of 19.34% (overpredicted) at hour 24 and a minimum error of 0.31% (overpredicted) at hour 15. Most of the rest of the errors are less than 4% (overpredicted or underpredicted) in value.
- Due to the small values of errors, both LES and LAV give an acceptable load predictions.

Table (6.8) Load parameters estimate for a summer weekend day using LES

Hour	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
1	1435.81	72.27	0.66	0.00	-15.49	-38.55	-2.57	-120.13	33.65	61.44
2	1363.24	134.85	4.32	0.28	-135.02	62.94	-42.81	-99.64	125.57	-50.92
3	1555.77	-22.88	2.15	0.11	87.01	-35.92	-5.51	54.71	-126.25	38.45
4	1489.99	55.25	3.15	0.28	-30.05	29.55	-36.80	-127.88	79.99	15.58
5	2163.66	-147.40	1.13	0.06	264.62	-71.51	-9.86	250.78	-538.07	231.55
6	1920.35	-142.49	1.26	0.11	-48.00	197.89	19.50	310.40	19.44	-376.30
7	569.98	-171.00	-0.83	-0.11	3.40	152.48	18.05	357.35	-8.27	-349.69
8	1773.11	78.76	0.86	0.07	-107.69	70.32	-20.87	-149.73	215.78	-106.17
9	2041.33	91.35	4.78	0.40	-66.78	-8.24	9.20	-173.76	177.87	-49.36
10	2251.05	27.74	2.38	0.18	-31.19	33.50	-2.82	-33.24	37.36	-55.18
11	2064.25	-0.33	1.93	0.15	16.52	4.58	-0.72	-23.67	1.75	-18.64
12	2034.49	-1.58	-0.23	0.04	-4.29	14.03	0.29	-6.96	-32.39	1.21
13	1825.83	0.74	3.63	0.31	1.15	-43.52	54.15	-5.23	27.41	-53.81
14	2045.94	53.70	-0.51	-0.08	-55.92	-2.17	27.68	-51.46	60.65	-45.87
15	2070.46	43.23	-0.36	-0.05	-24.57	0.08	1.43	-61.04	37.44	-15.53
16	1959.00	-14.17	0.01	-0.03	64.39	-35.62	5.98	34.31	-94.15	24.01
17	2141.73	12.86	-0.01	-0.04	-40.99	72.82	-21.00	8.05	50.52	-100.60
18	2350.85	-61.65	0.25	-0.04	97.26	-8.61	3.18	195.08	-194.68	-47.91
19	3034.16	12.43	0.13	-0.04	-103.41	158.93	-29.70	17.99	216.46	-304.28
20	3588.45	121.73	0.10	-0.04	-52.50	19.87	-41.18	-268.64	263.70	-89.13
21	3251.25	108.27	1.10	0.06	-12.62	-38.94	-13.51	-144.86	-30.30	91.70
22	832.83	-273.51	3.67	0.29	275.32	-22.36	10.56	720.83	-603.88	-111.68
23	1843.92	18.56	-0.58	-0.12	-12.09	16.12	0.13	-11.66	-64.77	41.59
24	1781.55	-22.47	0.24	-0.05	8.62	15.08	20.15	23.22	-6.55	-50.93

Table (6.9) Load parameters estimate for a summer weekend day using LAV

Hour	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
1	1925.05	351.61	-4.97	-0.36	-501.12	202.56	-13.70	-661.78	522.29	99.18
2	2858.66	126.66	16.21	1.38	-214.21	43.40	102.83	168.38	-3.25	-243.29
3	1370.18	234.06	8.07	0.80	-130.52	-96.20	9.76	-416.12	429.09	-41.91
4	1440.79	89.07	3.48	0.33	26.36	-51.51	-47.43	-166.05	-62.77	197.41
5	1856.32	-78.48	-0.20	-0.03	266.78	-136.75	-23.14	58.76	-372.25	268.50
6	1979.72	-590.94	1.21	-0.06	65.92	484.29	85.22	1244.87	-299.76	-990.48
7	488.05	-164.70	-1.35	-0.21	-58.97	197.10	33.37	362.71	97.66	-457.65
8	1921.76	-81.29	1.85	0.06	78.69	76.50	-49.68	217.72	-204.45	-64.07
9	2006.14	97.51	4.84	0.45	-130.47	41.26	11.56	-214.22	319.58	-149.95
10	1859.74	34.84	4.08	0.32	-24.69	3.40	4.57	50.38	16.38	-2.60
11	2062.51	-2.05	1.04	0.10	29.80	-9.71	-2.02	-28.19	-28.92	17.36
12	2031.34	-1.28	-0.22	0.04	-12.48	22.79	-1.19	-13.88	-17.58	-6.67
13	1713.77	-16.20	4.29	0.37	0.81	-42.64	68.38	28.04	40.97	-97.89
14	2336.05	353.96	-1.07	-0.12	-340.39	-3.77	25.98	-709.18	735.74	-69.28
15	2154.70	65.69	-0.70	-0.03	109.28	-147.41	0.50	-77.23	-304.63	342.11
16	1987.46	-35.39	0.26	0.01	211.14	-157.83	-1.20	57.93	-453.88	358.91
17	1437.82	-20.57	0.49	0.01	114.03	-57.97	-22.64	-6.55	-143.12	129.69
18	2077.61	23.43	0.30	-0.19	36.99	-36.48	5.74	40.00	-66.28	-13.54
19	2318.04	-558.93	-20.51	-0.86	163.89	348.72	114.86	717.74	216.48	-957.97
20	3669.66	131.49	-0.14	-0.04	-44.87	3.95	-42.49	-300.70	272.71	-69.35
21	3006.87	124.95	1.56	0.08	-150.75	33.42	33.90	-188.26	258.75	-142.83
22	735.76	-312.10	4.40	0.36	298.03	-12.86	13.16	838.73	-700.76	-128.88
23	2122.03	-97.16	-0.56	-0.08	255.28	-177.09	42.80	115.07	-353.68	193.96
24	-2521.94	-1930.21	-10.17	-0.07	1420.66	1174.25	-857.40	5104.95	-4591.60	-370.97

Table (6.10) Estimated load and percentage error for summer weekend day, 24 parameters sets, Model A

Daily hour	Actual load(MW)	LES Estimate	LAV Estimate	% LES error	% LAV error
1.0	758.1	759.3	796.2	-0.2	-5
2.0	683.3	684.3	683.2	-0.1	0
3.0	640.8	629.3	660.6	1.8	-3.1
4.0	614.2	612.9	622.4	0.2	-1.3
5.0	597.3	607.6	598.2	-1.7	-0.2
6.0	586.8	597.3	618.7	-1.8	-5.4
7.0	590.3	568.3	568.5	3.7	3.7
8.0	601.3	613.9	592.5	-2.1	1.5
9.0	667.3	659.8	655.3	1.1	1.8
10.0	764.1	759.1	765.8	0.7	-0.2
11.0	848.8	843	825.8	0.7	2.7
12.0	885.7	884	885.8	0.2	0
13.0	907.7	901.3	903.7	0.7	0.4
14.0	897.2	904.6	986.6	-0.8	-10
15.0	869.5	857.4	903.1	1.4	-3.9
16.0	842.4	834.6	810.3	0.9	3.8
17.0	835.5	829.8	834.2	0.7	0.2
18.0	853.8	850.5	851.8	0.4	0.2
19.0	857.7	851.8	787.6	0.7	8.2
20.0	823.9	811.3	809.4	1.5	1.8
21.0	801.8	812.7	821.6	-1.4	-2.5
22.0	823.4	859.6	863.4	-4.4	-4.9
23.0	835.3	828.1	816.6	0.9	2.2
24.0	783.1	767.4	703.2	0.2	10.2

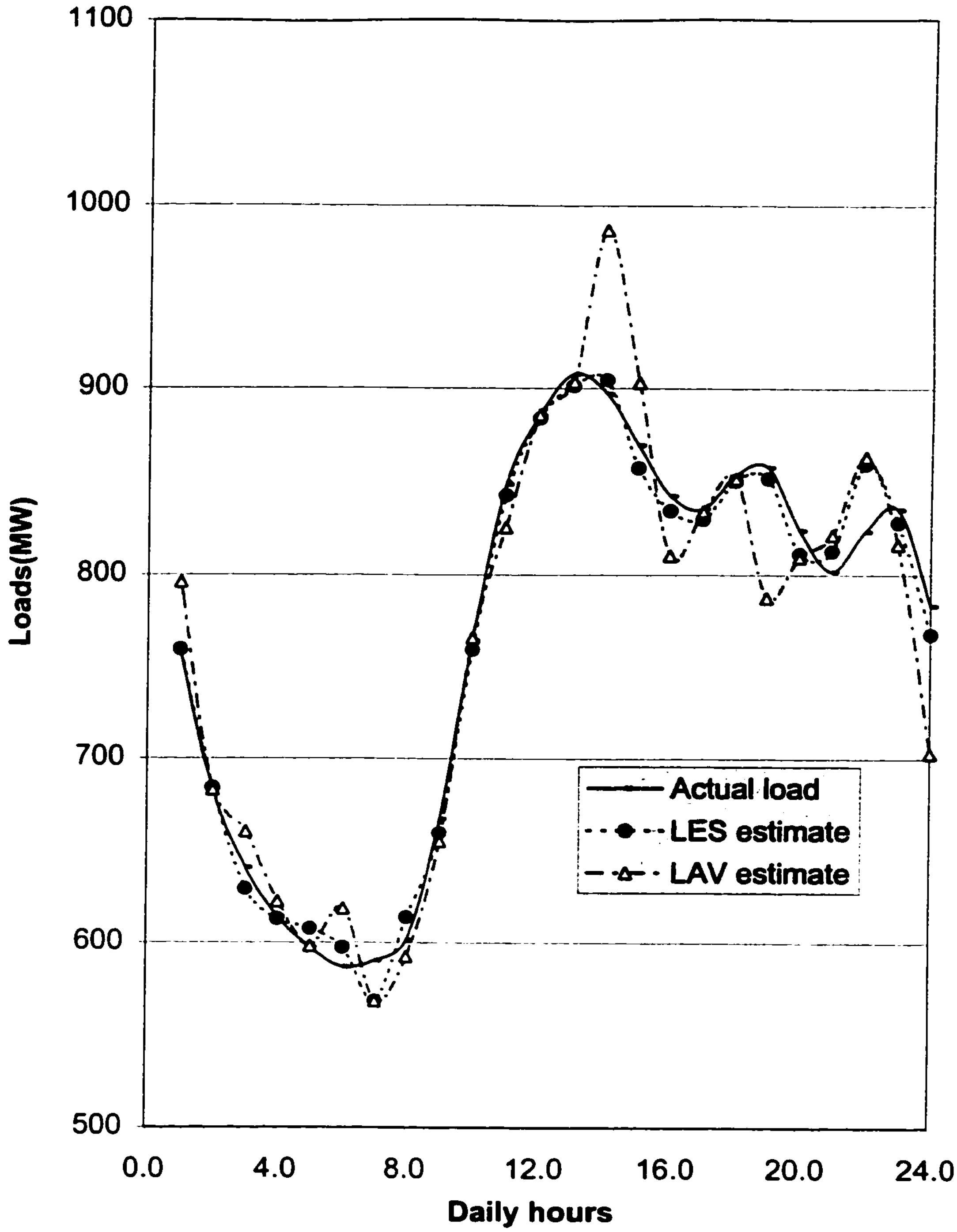


Figure (6.7) Estimated load for a summer weekend day using 24 parameters sets, Model A

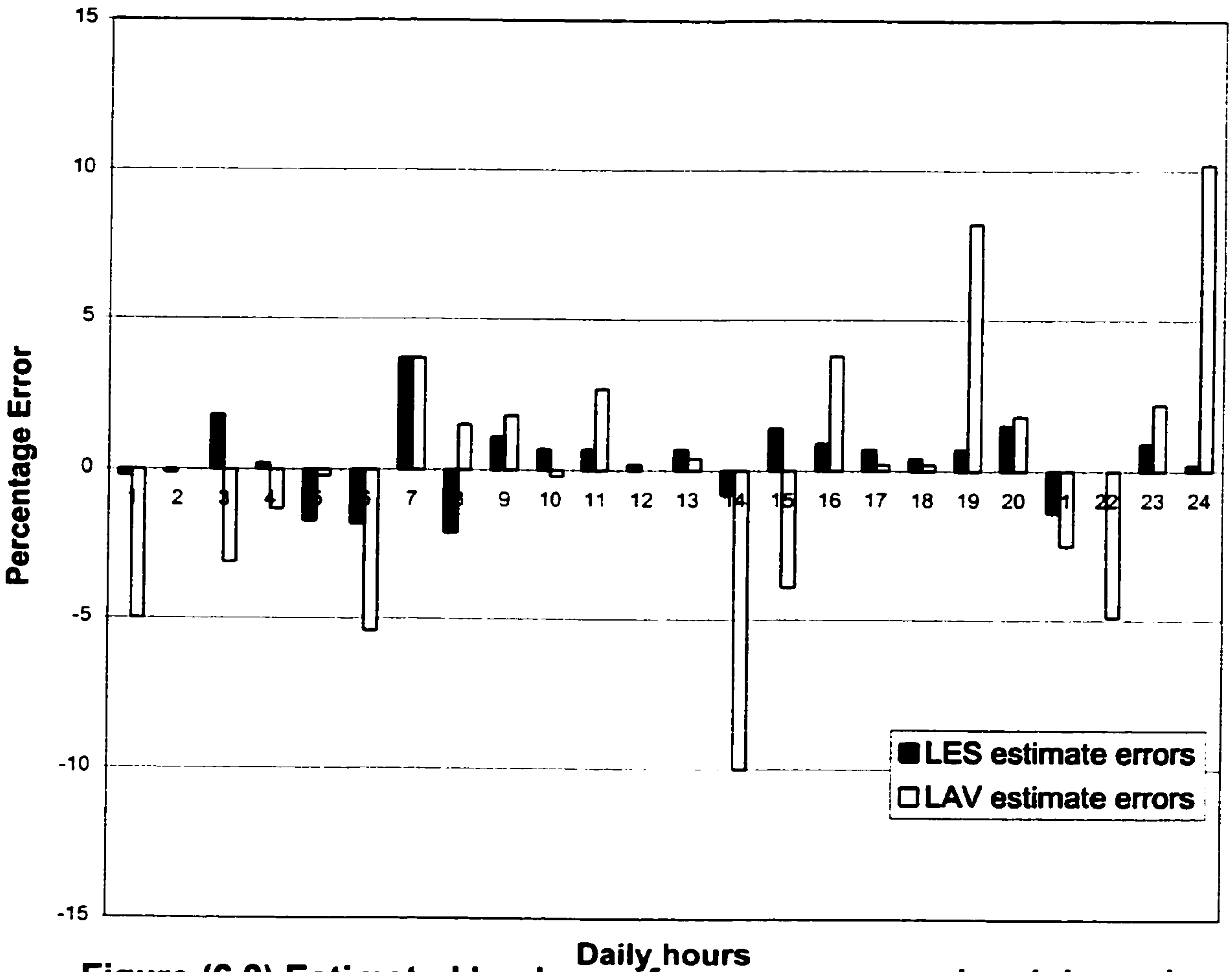


Figure (6.8) Estimated load error for a summer weekend day using 24 parameters sets, Model A

Table (6.11) Predicted load and percentage error for summer weekend day, 24 parameters sets, Model A

Daily hour	Actual load(MW)	LES Prediction	LAV Prediction	% LES error	% LAV error
1.0	716.8	736.1	761.7	-2.7	-6.27
2.0	637.7	655.6	643	-2.81	-0.84
3.0	598.5	603	618.7	-0.75	-3.38
4.0	573.7	588.9	581.3	-2.66	-1.32
5.0	558	565.6	555	-1.36	0.54
6.0	550.7	571.1	551.4	-3.7	-0.12
7.0	560.7	587.5	569	-4.78	-1.49
8.0	585.7	587.5	576.4	-0.37	1.58
9.0	659.2	645.5	649.5	2.08	1.48
10.0	762.4	750	748.2	1.63	1.86
11.0	843.9	833.5	829.4	1.23	1.72
12.0	875	861.6	861.8	1.53	1.51
13.0	881.2	867.2	867.2	1.59	1.58
14.0	863.2	864.8	847	-0.19	1.87
15.0	831.4	847.1	834.1	-1.89	-0.32
16.0	805.3	808.6	813.2	-0.42	-0.98
17.0	795.6	799.3	804.2	-4.46	-1.08
18.0	814	813.2	816.7	0.1	-0.33
19.0	808	826.2	903.6	-2.26	-11.83
20.0	766.4	822.6	833.5	-7.34	-8.76
21.0	748.2	791.5	759.2	-5.78	-1.47
22.0	823.3	800.2	793.2	2.81	3.66
23.0	801.8	807.6	840.1	-0.72	-4.78
24.0	744.5	753.9	888.5	-1.62	-19.34

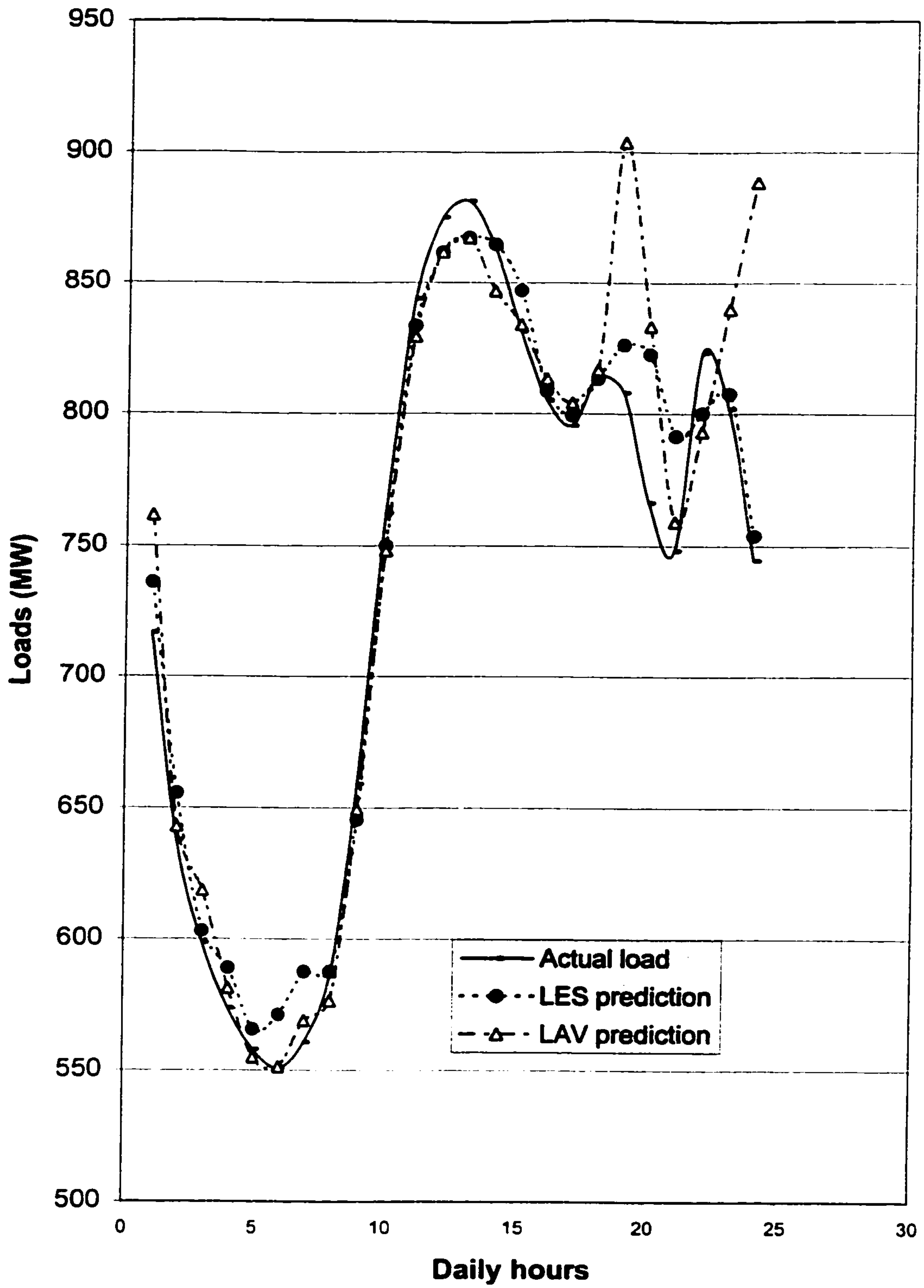


Figure (6.9) Predicted load for a summer weekend day using 24 parameters sets, Model A

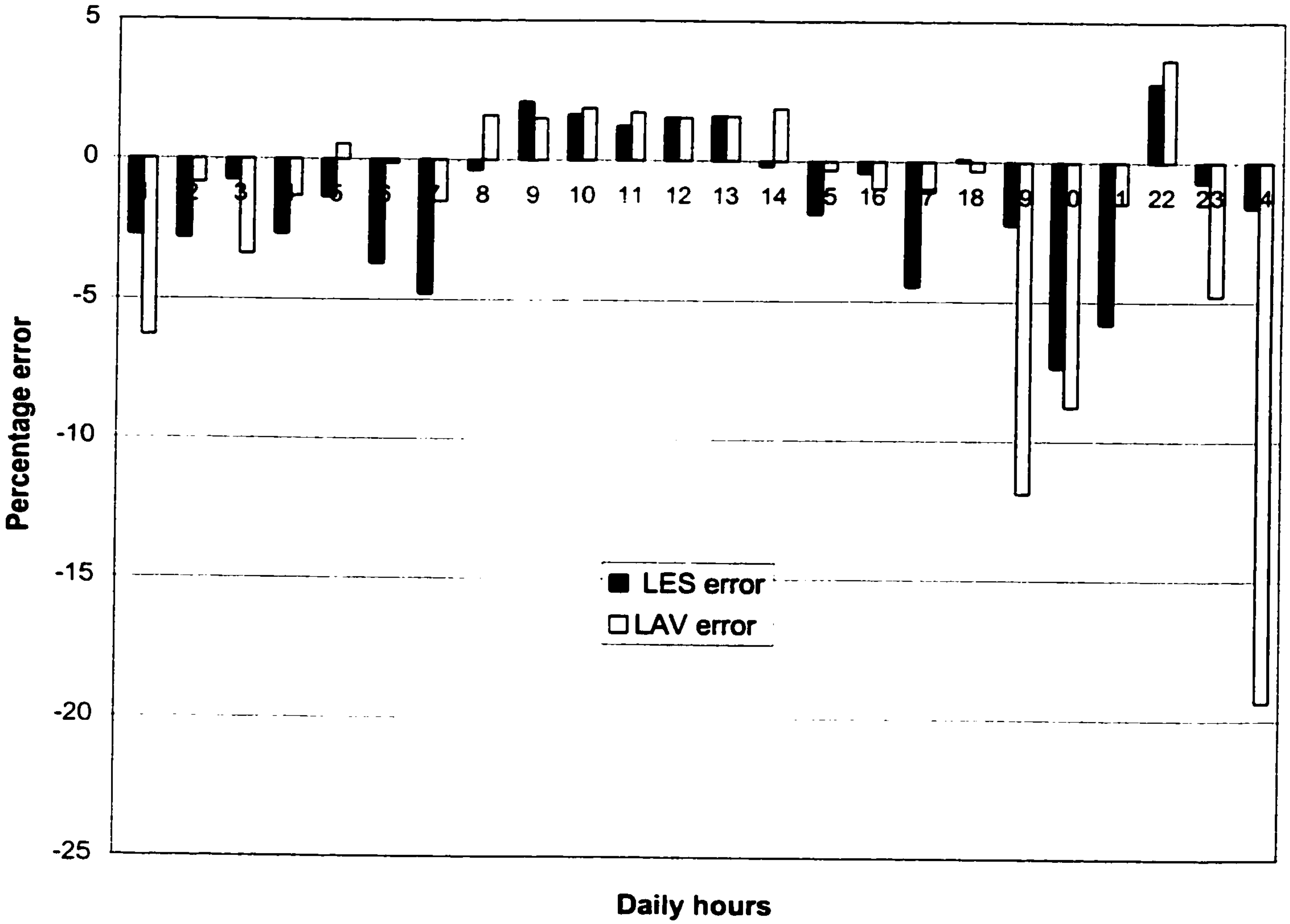


Figure (6.10) Predicted load error for a summer weekend day using 24 parameters sets, Model A

6. 4. 4 Estimation of Constant Model Parameters for a Summer Weekend Day (1 set) :

Table (6.12) gives the estimated parameters for a summer weekend day, while Table (6.13) and Figures (6.11 - 6.12) give the estimated load and percentage errors in this estimate using one set of parameters given in Table (6.12) for LES and LAV techniques. Examining these tables and figures reveals the following:

- Maximum error in LES load estimation is 15% (underestimated) at hour 8, while the minimum error is 0.1% (overestimated) at hour 12. Most of the rest of the errors (12 hours) are less than 4% (overestimated or underestimated) in value.
- Maximum error in LAV load estimation is 15% (underestimated) at hour 8, while the minimum error is 0.1% (overestimated) at hour 12. Most of the rest of the errors (12 hours) are less than 4% (overestimated or underestimated) in value.
- Better estimated values can be obtained by using more data to reduce the errors.

The one set of parameters are used to predict the load one week ahead. Table (6.14) and Figures (6.13-6.14) give the obtained results. Examining these tables and figures reveals the following:

- Maximum error in LES load prediction is 9.4% (underpredicted) at hour 1, while the minimum error is 0.1% (overpredicted) at hour 12. Most of the rest of the errors (15 hours) are less than 4% (overpredicted or underpredicted).
- Maximum error in LAV load prediction is 9.5% (overpredicted) at hour 9, while the minimum error is 0.1% (overpredicted) at hour 3.
- For both tehniques LES and LAV, the predicted load can be represented better by using more data to reduce the errors.

Table (6.12) Estimated parameters for a weekend day using LES and LAV algorithm

Parameter	LES estimate	LAV estimate
A_o	-3892.0	-3991.2
A_1	568.47	592.35
A_2	-123.38	-131.14
A_3	6.10	6.53
A_4	-8.30	-8.12
A_5	-4.97	-0.96
A_6	-0.89	-0.71
A_7	159.08	140.93
A_8	-84.80	-36.55
A_9	52.49	24.88

Table (6.13) Estimated load and percentage error for summer weekend day, one parameters set, Model A

Daily hour	Actual load(MW)	LES Estimate	LAV Estimate	% LES error	% LAV error
1.0	758.1	687.1	687.1	9.4	9.4
2.0	683.3	672.4	677.8	1.6	0.8
3.0	640.8	678.1	673.8	-5.8	-5.2
4.0	614.2	664.1	655.6	-8.1	-6.7
5.0	597.3	634.3	623.9	-6.2	-4.5
6.0	586.8	553.4	531	5.7	-9.5
7.0	590.3	590.9	585.9	-0.1	0.7
8.0	601.3	691.6	691.6	-15	15
9.0	667.3	714.5	730.7	-7.1	9.5
10.0	764.1	753.2	759	1.4	0.7
11.0	848.8	842.3	854.3	0.8	-0.7
12.0	885.7	886.4	886.5	-0.1	-0.1
13.0	907.7	882.8	901	2.7	0.7
14.0	897.2	868.5	893.3	3.2	0.4
15.0	869.5	863.5	868.6	0.7	0.1
16.0	842.4	916.6	914.5	-8.8	-8.6
17.0	835.5	845.1	841	-1.1	-0.7
18.0	853.8	900.2	900	-5.4	5.4
19.0	857.7	821.5	806.6	4.2	6
20.0	823.9	787.5	793.6	4.4	3.7
21.0	801.8	809.1	807.3	-0.9	-0.7
22.0	823.4	794.6	791.3	3.5	3.9
23.0	835.3	759.7	760.3	2.1	9
24.0	783.1	733.9	722.3	6.3	7.8

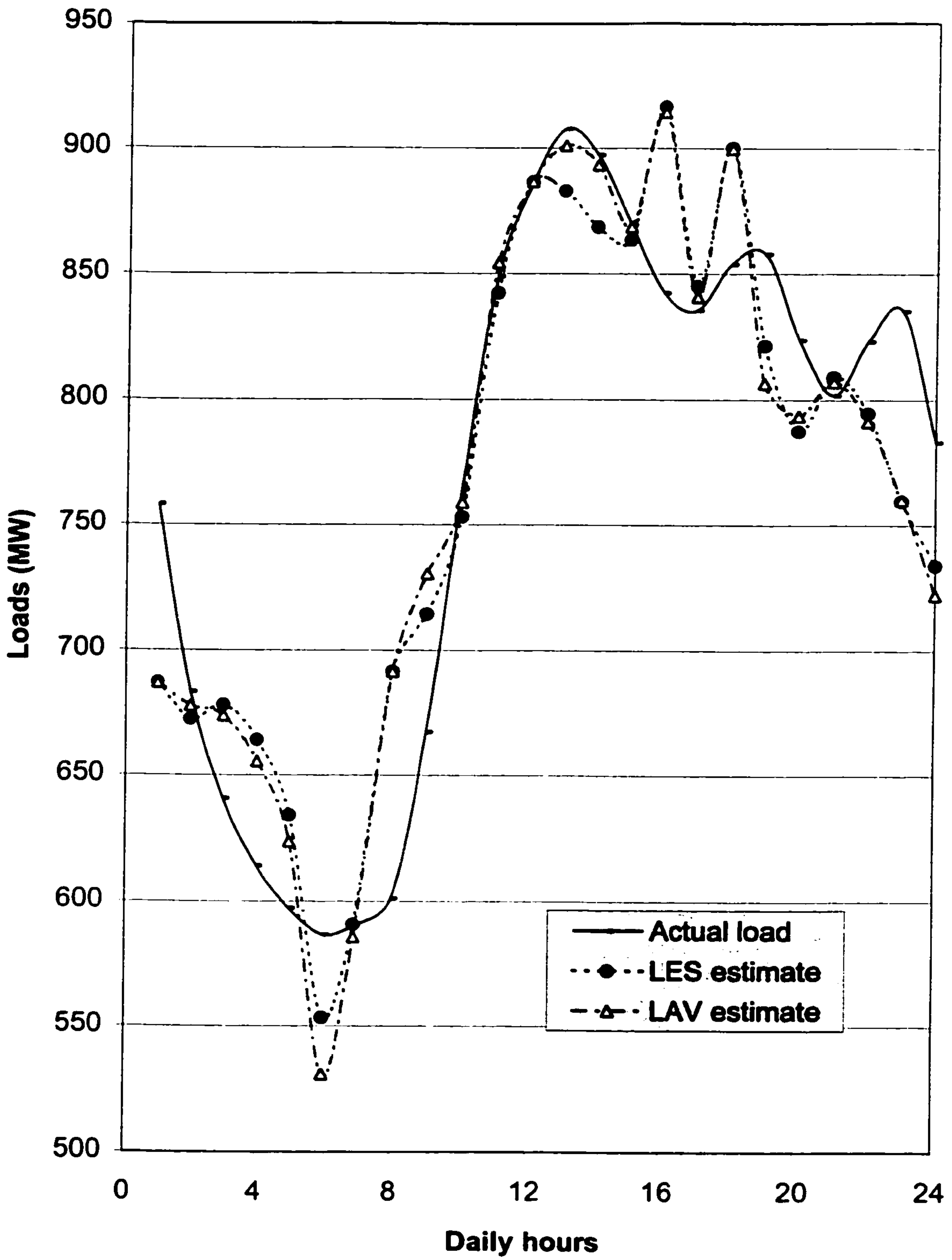


Figure (6.11) Estimated load for a summer weekend day using one parameters set, Model A

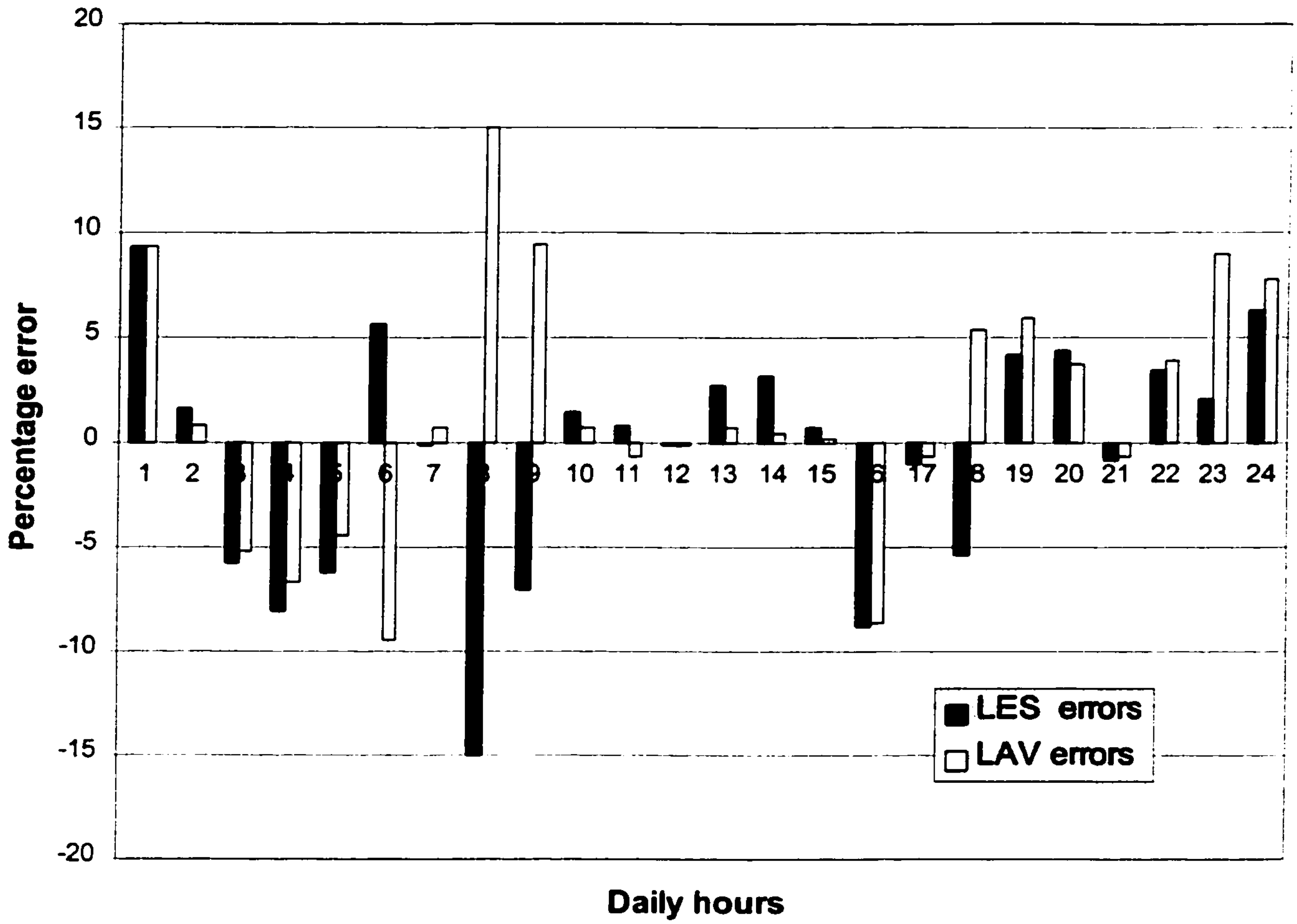


Figure (6.12) Estimated load error for a summer weekend day using one parameters set, Model A

Table (6.14) Predicted load and percentage error for summer weekend day, one parameters set, Model A

Daily hour	Actual load(MW)	LES Prediction	LAV Prediction	% LES error	% LAV error
1.0	758.1	687.1	687.1	9.4	9.4
2.0	683.3	672.4	677.8	1.6	0.8
3.0	640.8	678.1	673.8	-5.8	-5.2
4.0	640.8	678.1	673.8	-5.8	-5.2
5.0	640.8	678.1	673.8	-5.8	-5.2
6.0	640.8	678.1	673.8	-5.8	-5.2
7.0	640.8	678.1	673.8	-5.8	-5.2
8.0	640.8	678.1	673.8	-5.8	-5.2
9.0	667.3	714.5	730.7	-7.1	-9.5
10.0	764.1	753.2	759	1.4	0.7
11.0	848.8	842.3	854.3	0.8	-0.7
12.0	885.7	886.4	886.5	-0.1	-0.1
13.0	907.7	882.8	901	2.7	0.7
14.0	897.2	868.5	893.3	3.2	0.4
15.0	869.5	863.5	868.6	0.7	0.1
16.0	869.5	863.5	868.6	0.7	0.1
17.0	869.5	863.5	868.6	0.7	0.1
18.0	869.5	863.5	868.6	0.7	0.1
19.0	869.5	863.5	868.6	0.7	0.1
20.0	869.5	863.5	868.6	0.7	0.1
21.0	801.8	809.1	807.3	-0.9	-0.7
22.0	823.4	794.6	791.3	3.5	3.9
23.0	835.3	759.7	760.3	9.1	9
24.0	783.1	733.9	722.3	6.3	7.8

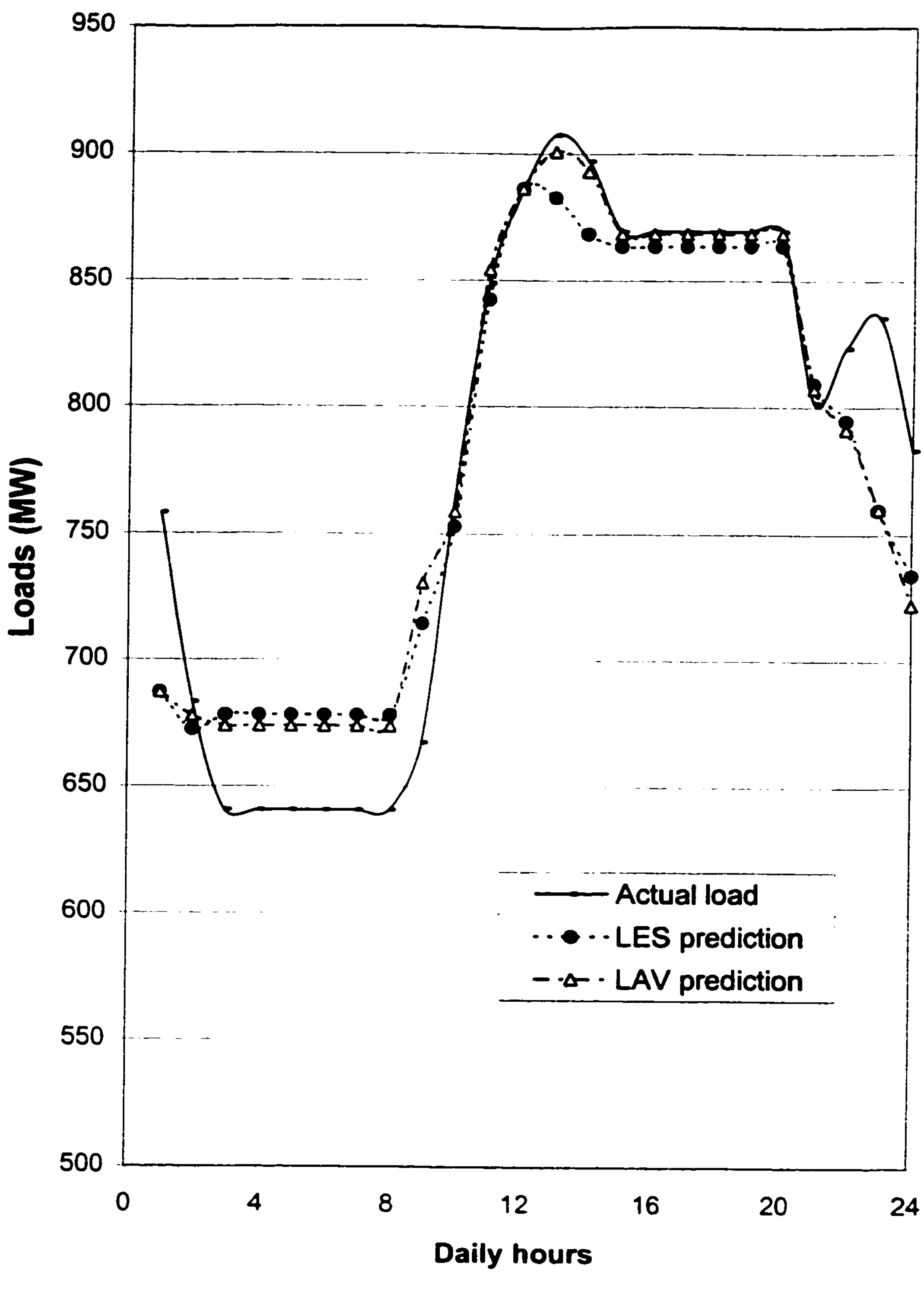


Figure (6.13) Predicted load for a summer weekend day using one parameters set, Model A

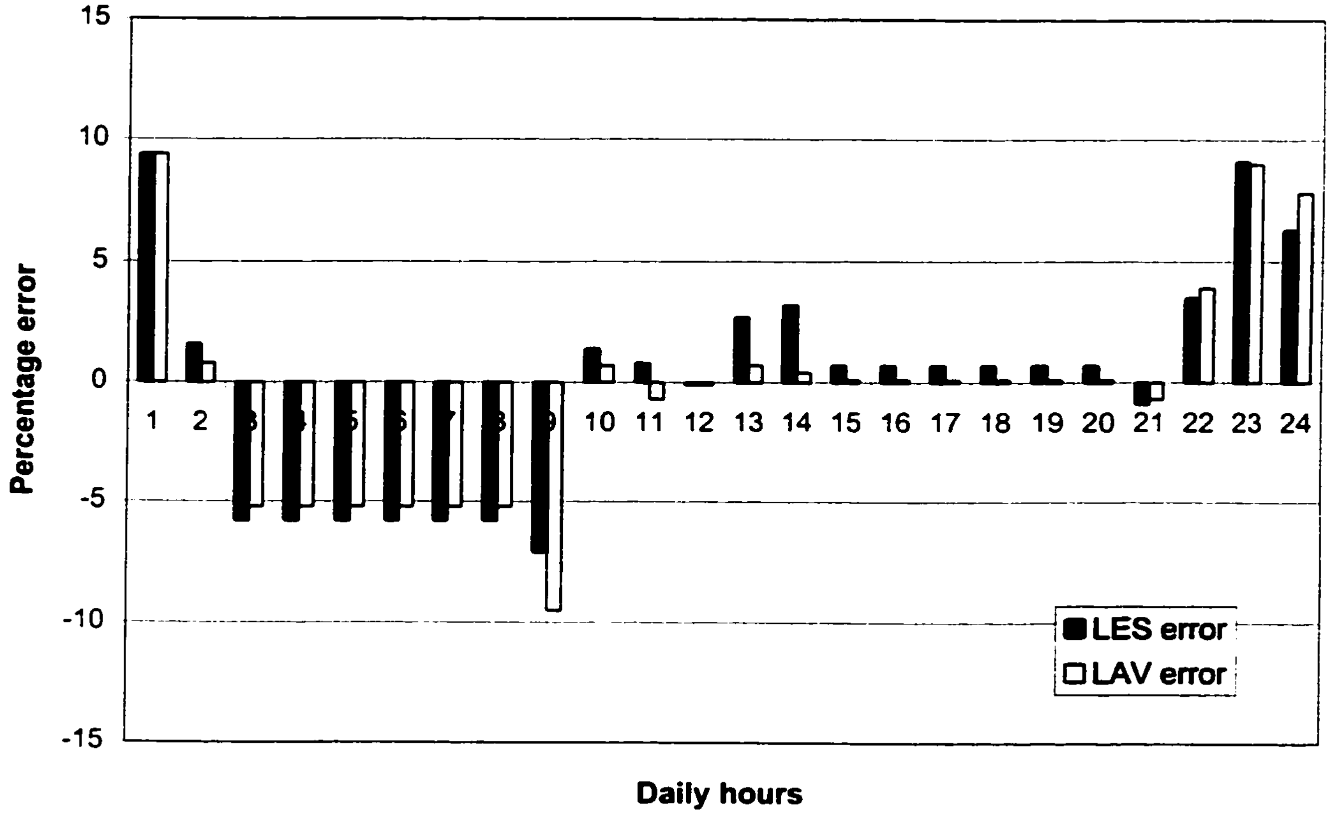


Figure (6.14) Predicted load error for a summer weekend day using one parameters set, Model A

6. 4. 5. General Remarks For Summer Model A

The two approaches LES and LAV give acceptable predicted load values. There are errors involved between actual and estimated load, actual and predicted load. To reduce errors more quality data has to be used. The scope here is to show how LES and LAV algorithms are applied as predicting tools. Estimated parameters values obtained using 24 sets or 1 set of parameters are producing estimated and predicted values deviating with errors from actual value. The results obtained using 24 sets or 1 set contain error values. Thus, by using one set of parameters will be more economical with less effort and computing time. These algorithms are to be compared to the fuzzy algorithm in the section on results and technique.

6. 4. 6 Winter Predictions

Appendix 3 and 4 give the results obtained for winter weekday and winter weekend using model A. The same arguments for summer results can be said for winter results. The estimated and predicted load values deviate from the actual load values. Tables (6.6a) and (6.6b) give a brief summary for estimated and predicted errors. The errors are to be reduced by using more quality data, so that the predicted values can resemble and predict the data as accurate as possible. The usage of either 24 sets or 1 set of parameters give high error values in the predicted load. Thus, using one set of parameters will be more economical and time saving

Table (6.6a) Estimated and predicted errors for a winter weekday

Algorithm	LES %		LAV %	
Type of day	Weekday		Weekday	
Parameters set	24	1	24	1
Estimated load maximum error	5.04	5.2	-8.64	-6.1
Estimated load minimum error	-0.1	0	-0.02	0
Predicted load maximum error	-6.48	-11.63	10.41	-17.99
Predicted load minimum error	0.07	0.41	-0.41	0.03

Table (6.6b) Estimated and predicted errors for a winter weekend day

Algorithm	LES %		LAV %	
Type of day	Weekend		Weekend	
Parameters set	24	1	24	1
Estimated load maximum error	-5.62	-15.22	-8.86	-11.6
Estimated load minimum error	-0.15	-0.09	0.01	0.04
Predicted load maximum error	25.81	-27.12	26.26	23.13
Predicted load minimum error	-0.41	-0.09	-1.5	0.2

6. 5 Model B

It is a weather insensitive model, that depends only on the hour (time) considered. The model proposed in chapter 3 is used to predict the load power for 24-hours ahead for one working day and one weekend day in summer and winter . The model is applied to the data from Nova Scotia power and Environment Canada Weather. Harmonics included are from the lowest number to thirteen. Nine harmonics are used in the static estimation process since it was found to produce the lowest error.

6. 5. 1 Summer Weekday

Table (6.15) gives the estimated load parameters for a summer weekday, while Table (6.16) and Figures (6.15-6.16) give the estimated load for a summer weekday and the percentage error in this estimate using the LES and LAV algorithms. Examining these tables and figures reveals the following :

- LES estimates the load with a maximum error of 26.2% (overestimated) at hour 2 and a minimum error of 2.5% (underestimated) at hour 18.
- LAV estimates the load with a maximum error of 87,4% (overestimated) at hour 3 and a minimum error of 0% at hour 24.
- Estimated load error values of LAV from hour 4 to hour 22 are very small compared to the estimated load error values of LES (for example at hour 4, LAV error is 0.05% while LES error is 20.5%, and at hour 20, LAV error is 0.06% while LES error is 14.8%). LAV algorithm gives a better load estimate than LES estimate for the hours 4 to 22. The large error values for some hours in LAV estimation are caused by bad data. More data can help screen these bad points and reduce error values.

Table (6.15) Load parameters for a summer weekday, Model B

Parameter	LES estimate	LAV estimate
A_o	777.09	681.07
A_1	1.72	-168.82
B_1	12.74	186.35
A_2	-13.75	147.90
B_2	14.83	-35.02
A_3	-46.07	-84.19
B3	27.08	-93.95
A_4	-2.18	-169.25
B_4	17.64	328.01
A_5	83.61	266.86
B_5	91.62	22.80
A_6	-7.84	-335.86
B_6	10.54	-62.62
A_7	14.35	243.44
B_7	15.06	199.83
A_8	11.67	32.27
B_8	75.36	-147.38
A_9	5.67	88.30
B_9	19.20	437.50

The parameters estimated for model B are used to predict the load 24-hours ahead for the same weekday, during the same season. Table 6.17 and Figures (6.17-6.18) show the predicted load and the error in this prediction. Examining the table and figures reveals the following:

- LES predicts the load with errors larger than 14% in 11 instances. The range of errors is from 27.54% (overpredicted) at hour 2 as the highest to 0.11% (underpredicted) at hour 12 as the lowest.

Table (6.16) Estimated load and percentage error for summer weekday, Model B

Hour	Actual load(MW)	LES Estimate	LAV Estimate	% LES Error	% LAV Error
1	768.1	913.95	1424.26	-19	-85.4
2	678.3	855.77	795.24	-26.2	-17.2
3	636.1	774.79	1192.31	-21.8	-87.4
4	609.9	735	609.59	-20.5	0.05
5	598	702.72	599.75	-17.5	-0.29
6	595.6	712.31	594.91	-19.6	0.12
7	608.4	652.75	610.12	-7.3	-0.28
8	661	710.12	661.39	-7.4	-0.06
9	788.6	886.08	787.26	-12.4	0.17
10	908.4	1015.51	907.89	-11.8	0.06
11	982.9	1045.47	984.1	-6.4	-0.12
12	1015	1013.59	1014.47	0.14	0.05
13	1028.9	1022.18	1028.36	0.65	0.05
14	1011.1	918.6	1011.58	9.2	-0.05
15	988.6	873.21	989.87	11.7	-0.13
16	984.4	905.99	981.62	8	0.28
17	997.2	956.31	997.88	4.1	-0.07
18	1002.4	977.58	1003.96	2.5	-0.16
19	977.2	839.29	978.01	14.1	-0.08
20	929.4	791.66	929.93	14.8	-0.06
21	894.8	772.42	894.63	13.7	0.02
22	907.7	750.66	865.09	17.3	4.69
23	946.7	795.58	393.67	16	58.42
24	882.9	781.31	882.92	11.5	0

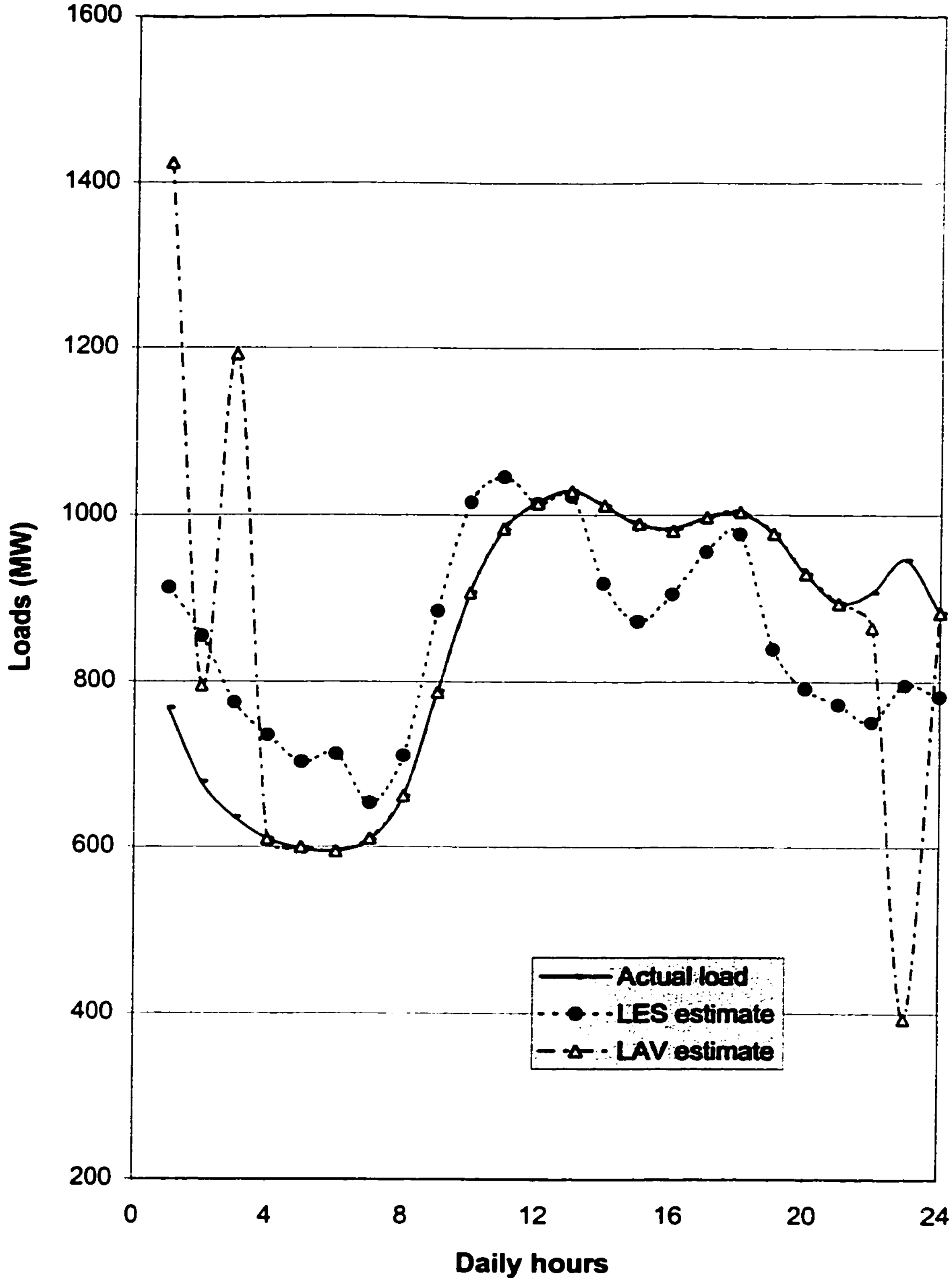


Figure (6.15) Estimated load for a summer weekday , Model B

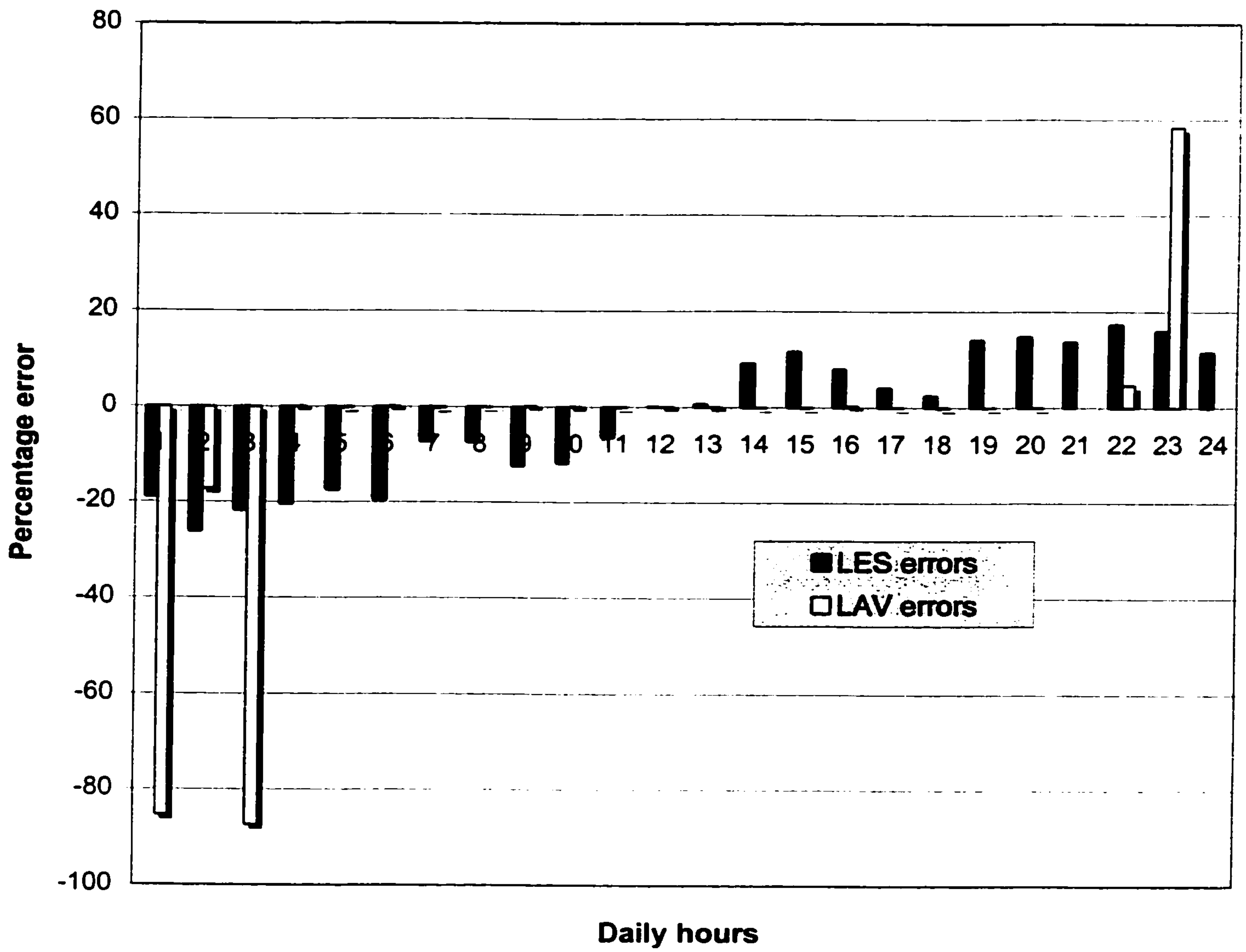


Figure (6.16) Estimated load error for a summer weekday, Model B

- LAV predicts the load with errors less than 0.6% in 19 instances. The errors range is from the highest 20.98% (overpredicted) to the lowest 0.08% (overpredicted).
- LAV error values are less than LES error values in every hour except hour 23 where LES error is 16.46% (underpredicted) and LAV error is 18.16% (overpredicted). This indicates that LAV gives in this case better representation than LES.

6. 5. 2 Summer Weekend Day

The same model is used to forecast a weekend day load, Table (6.18) and Figures (6.19 - 6.20) give the estimated load and the percentage error in this estimate. Examining the table and figures reveals the following:

- LES estimates the load with a maximum error of 17.78% (overestimated) at hour 2 and aminimum error of 0% at hour12.
- LAV estimates the load with a maximum error of 13.88% (overestimated) at hour 2 and a minimum error of 0.05% (overestimated) at hour 17.
- LAV estimated load errors areless with a wide magin than the LES estimated load errors in all the cases except for hour 1.
- Since LAV estimated errors are less than 0.22% in 19 instances, LAV gives better estimates than LES.

Table (6.19) and Figures (6.21 - 6.22) give the predicted load for a weekend day one week ahead. Examining the table and figures reveals the following:

- LES predicted load error has a maximum of 20.07% (overpredicted) at hour 2 and a minimum of 0.47% (undrpredicted) at hour 12.
- LAV predicts the load with a maximum error of 2.3% (underpredicted) at hour 2 and a minimum error of 0% at hours 0, 3.
- Since all LAV predicted load errors are less than LES predicted load errors, LAV gives load predictions better than LES load prediction.

Table (6.17) Predicted load and percentage error for summer weekday, Model B

Hour	Actual load(MW)	LES Prediction	LAV Prediction	% LES Error	% LAV Error
1	674	815.4	815.41	-20.98	-20.98
2	609.9	777.8	685.1	-27.54	-12.32
3	559.7	688	554.72	-22.92	0.89
4	537.7	657.6	538.2	-22.3	-0.09
5	536.8	637.6	535.4	-18.77	0.27
6	535.6	647.95	533.32	-20.98	0.42
7	545.6	587.13	543.41	-7.61	0.4
8	574.4	619.43	579.1	-7.84	-0.81
9	668.9	762.6	667.5	-14.01	0.22
10	787.3	889.3	789.5	-12.95	-0.28
11	875.1	933.54	872.4	-6.68	0.31
12	909.4	908.44	910.1	0.11	-0.08
13	925	919.11	926.71	0.64	-0.18
14	903	813.7	902.6	9.89	0.05
15	876	766.6	876.7	12.49	-0.08
16	848.7	776.1	851.6	8.55	-0.35
17	848.3	808.8	852.8	4.65	-0.53
18	884.8	860.4	884.2	2.76	0.07
19	880.9	746.7	860.2	15.23	2.31
20	837.9	707.95	836.2	15.51	0.21
21	805.1	692.1	807.3	14.04	-0.27
22	824.4	673.8	921.3	18.27	-11.75
23	876.2	731.99	1035.31	16.46	-18.16
24	815.6	719.32	815.1	11.8	0.07

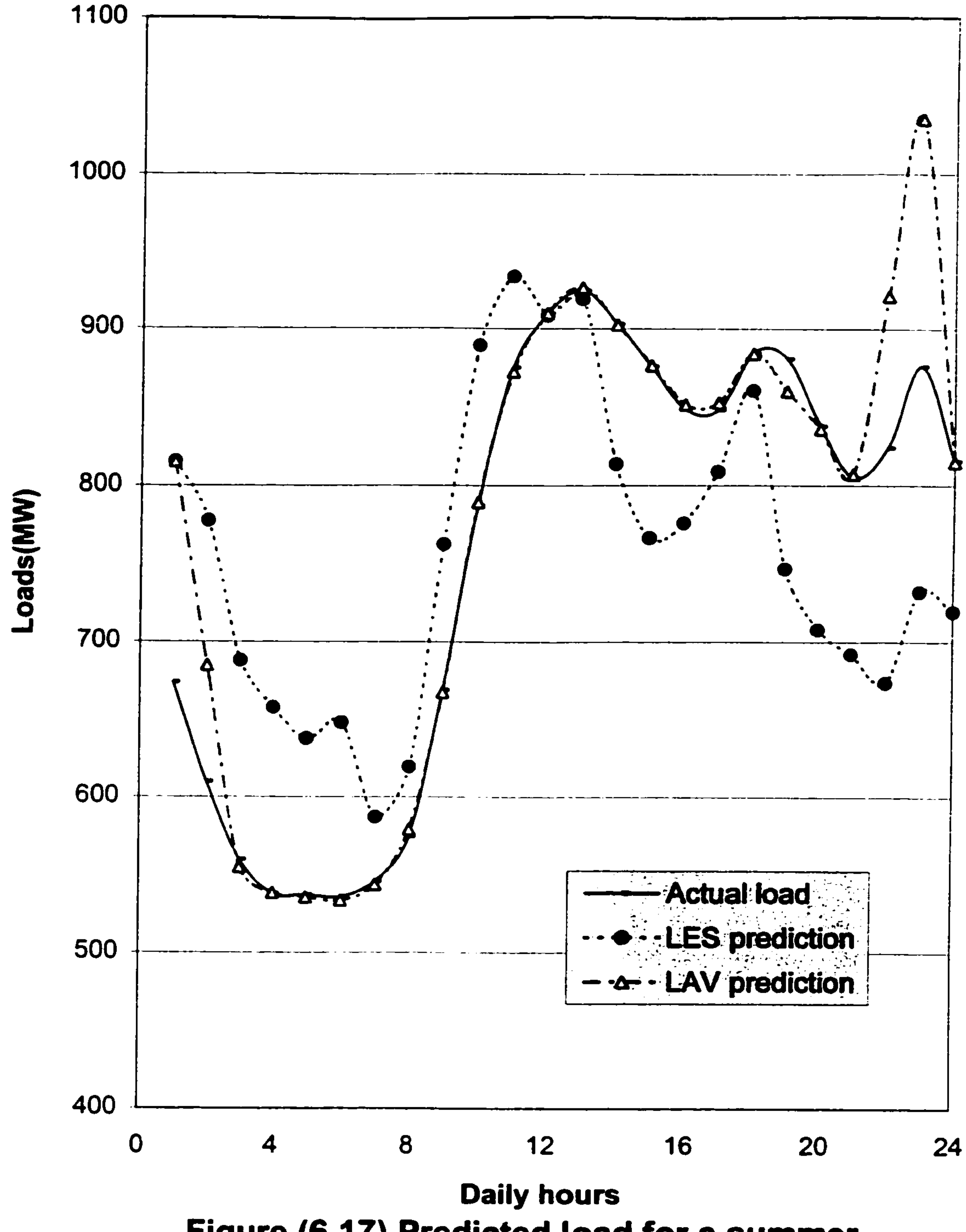


Figure (6.17) Predicted load for a summer weekday, Model B

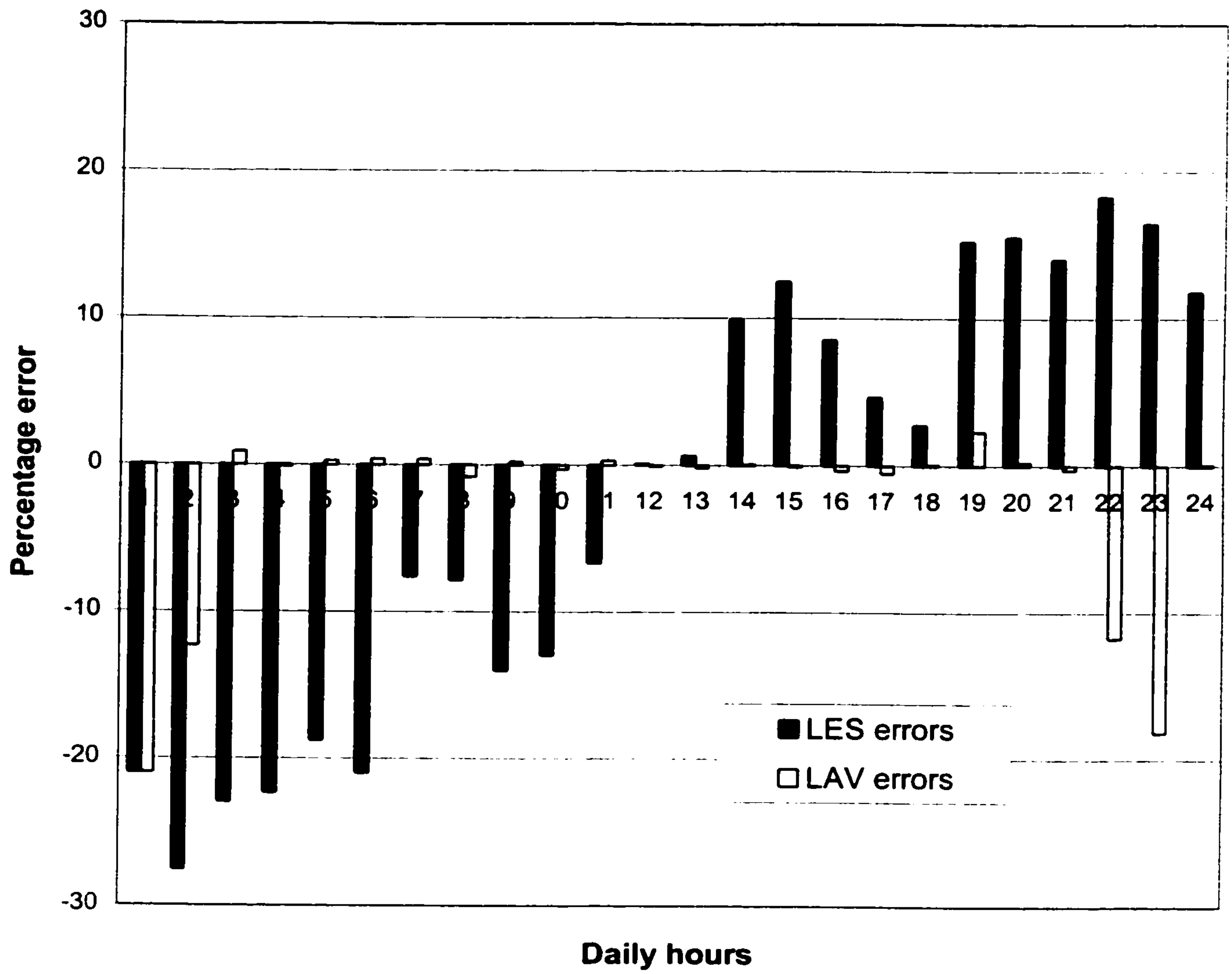


Figure (6.18) Predicted load error for a summer weekday, Model B

Table (6.18) Estimated load and percentage error for summer weekend day, Model B

Hour	Actual load(MW)	LES Estimate	LAV Estimate	% LES Error	% LAV Error
1	758.1	856.5	856.5	-12.98	-13
2	683.3	804.8	778.17	-17.78	-13.88
3	640.8	736.7	696	-14.97	8.5
4	614.2	705.6	613.83	-14.88	0.06
5	597.3	668.2	598.55	-11.92	-0.21
6	586.8	658.88	586.38	-12.28	0.07
7	590.3	619.78	591.35	-4.99	-0.18
8	601.3	635.87	601.51	-5.75	-0.04
9	667.3	737.25	666.33	-10.48	0.15
10	764.1	837.83	763.88	-9.65	0.03
11	848.8	889.14	849.58	-4.75	-0.09
12	884.7	885.69	885.42	0	0.03
13	907.7	902.45	907.21	0.58	0.05
14	897.2	832.8	897.6	7.18	-0.04
15	869.5	790.19	870.32	9.12	-0.09
16	842.4	789.04	840.56	6.33	0.22
17	835.5	804.79	835.9	3.68	-0.05
18	853.8	837.54	855	1.9	-0.14
19	857.7	768.37	858.17	10.42	-0.05
20	823.9	730.4	824.31	11.35	-0.05
21	801.8	716.3	801.68	10.66	0.01
22	823.4	709.57	760.58	13.82	7.63
23	835.3	732	771.6	12.37	7.6
24	783.1	720.21	783.2	8.03	-0.01

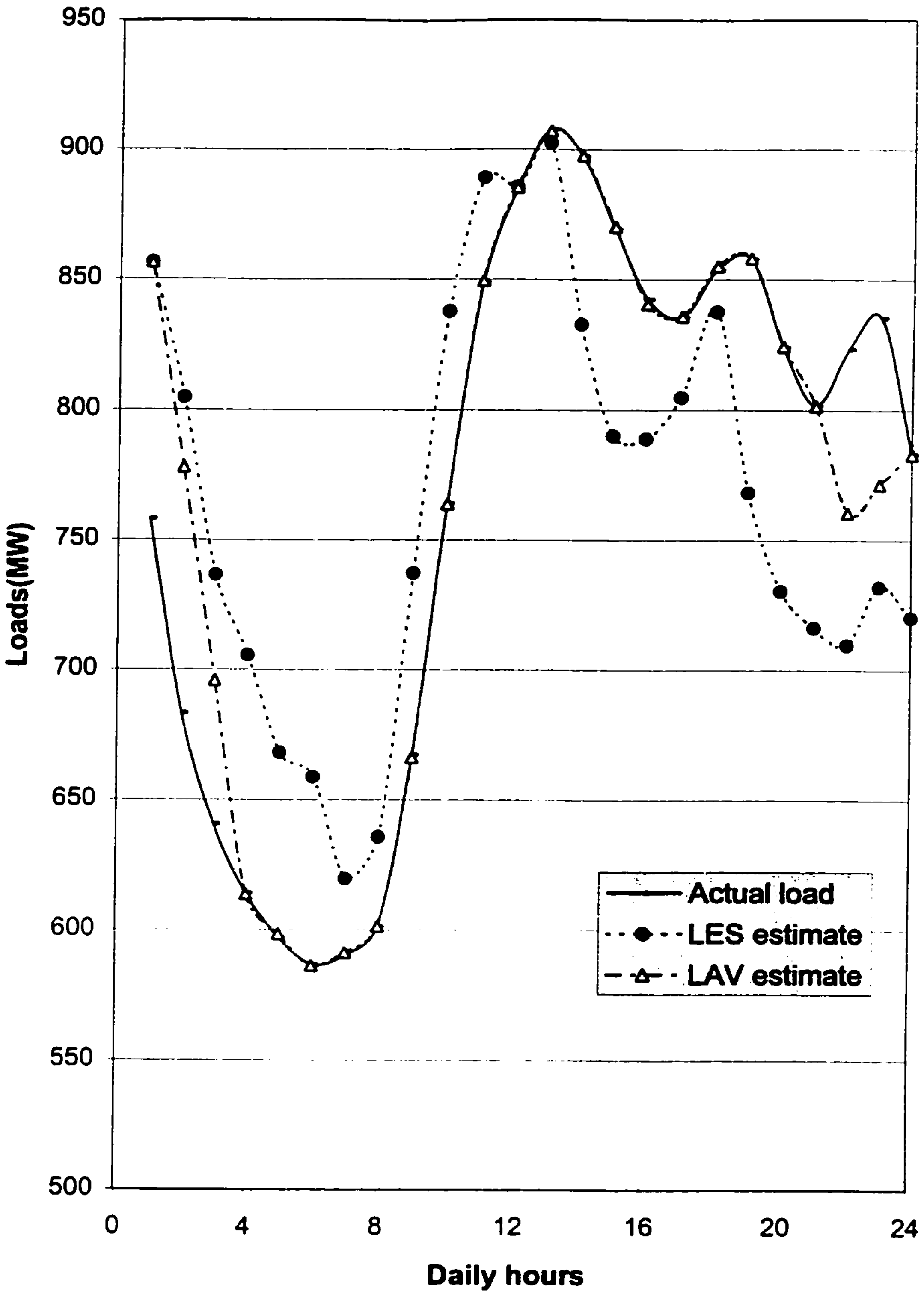


Figure (6.19) Estimated load for a summer weekend day, Model B

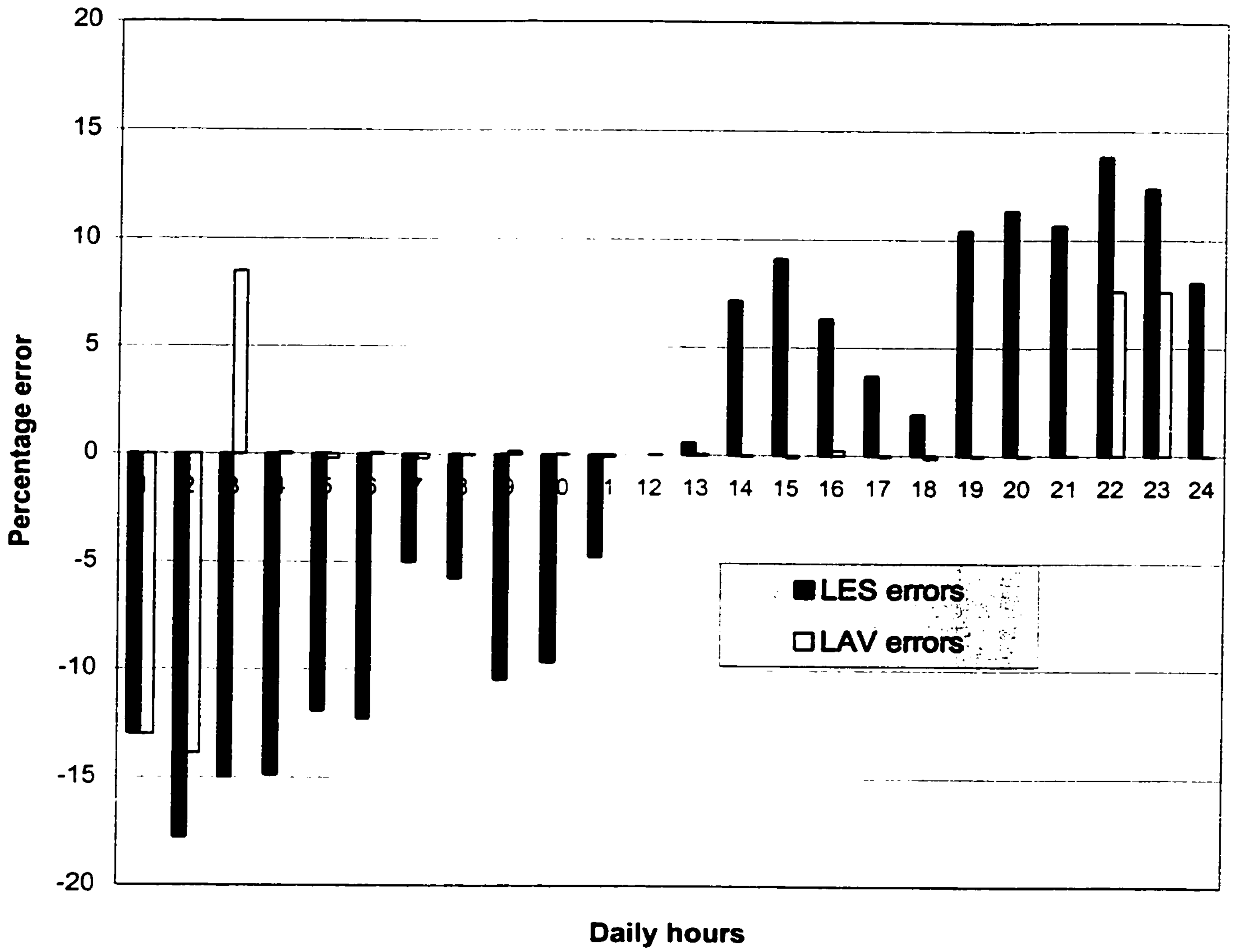


Figure (6.20) Estimated load error for a summer weekend day, Model B

Table (6.19) Predicted load and percentage error for summer weekend day, Model B

Hour	Actual load(MW)	LES Prediction	LAV Prediction	% LES Error	% LAV Error
1	950.4	1089.5	950.4	14.64	0
2	874.5	1050.04	894.7	-20.07	2.3
3	838.7	979.43	839	-16.78	0
4	814	929.21	813.16	-14.15	0.1
5	806.3	858.62	809.71	-6.49	-0.42
6	800.4	872.44	800.31	-9	0.01
7	811.1	862.16	811.44	-6.3	-0.04
8	833.2	895.18	833.49	-7.44	-0.03
9	918.6	1017.23	916.64	-10.74	0.21
10	1038.1	1121.45	1041.85	-8.03	-0.36
11	1116.9	1157.88	1120.92	-3.67	-0.36
12	1148.4	1143	1149.4	0.47	-0.09
13	1158	1140.82	1155.9	1.48	0.18
14	1135.1	1041.03	1133.6	8.29	0.13
15	1098.2	991.95	1095.1	9.68	0.28
16	1074.5	1004.23	1072.3	6.54	0.21
17	1072.9	1038.91	1075.8	3.17	-0.27
18	1110.2	1110.88	1116.8	-0.06	-0.6
19	1135.1	1021.23	1132.1	10.03	0.27
20	1141.2	996.94	1148.4	12.64	-0.6
21	1159.6	1023.25	1164.8	11.76	-0.45
22	1112.8	960.62	1127.3	13.68	-1.31
23	1060.7	961.22	1063.46	9.38	-0.26
24	981.7	925	980.7	5.78	0.1

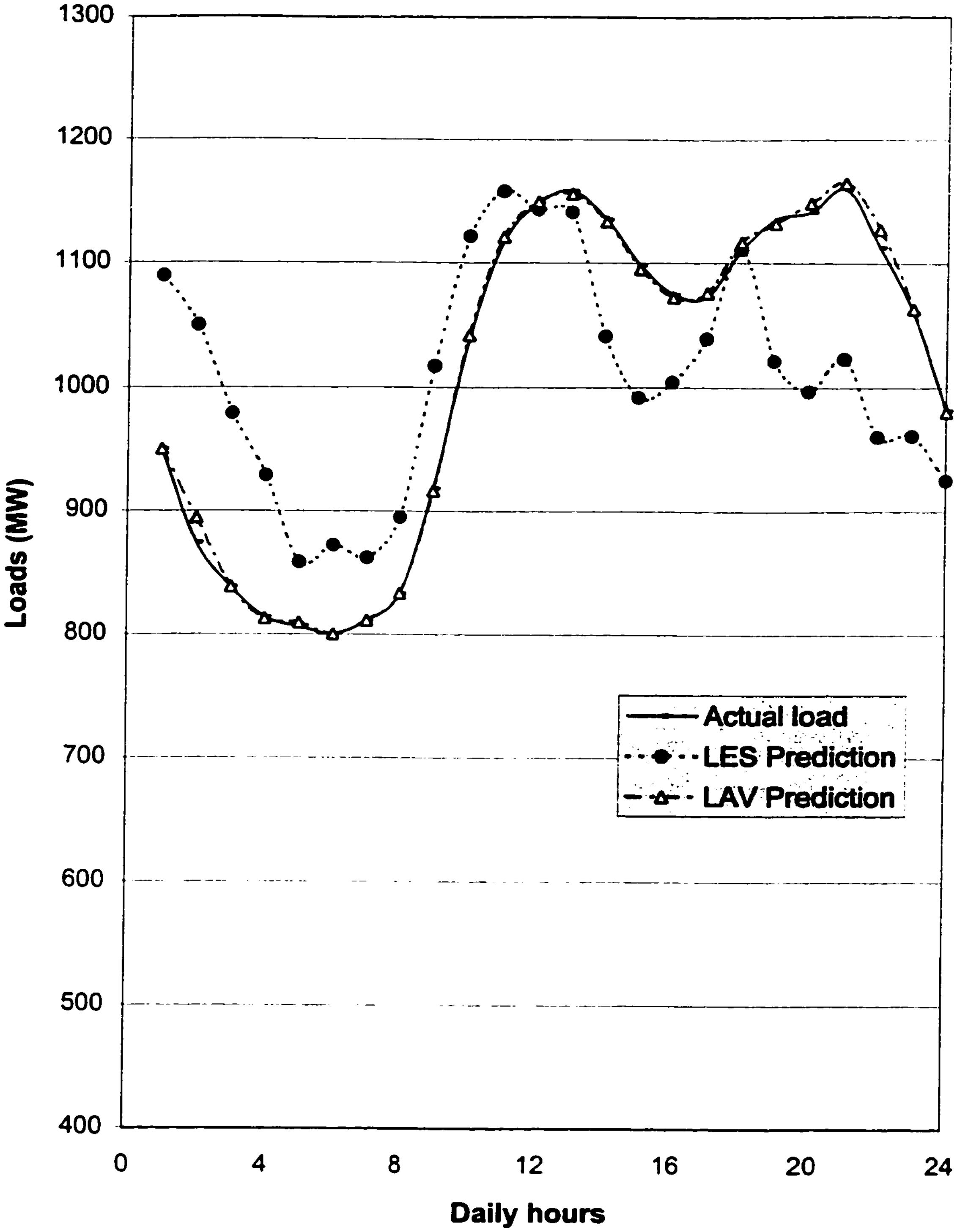


Figure (6.21) Predicted load for a summer weekend day, Model B

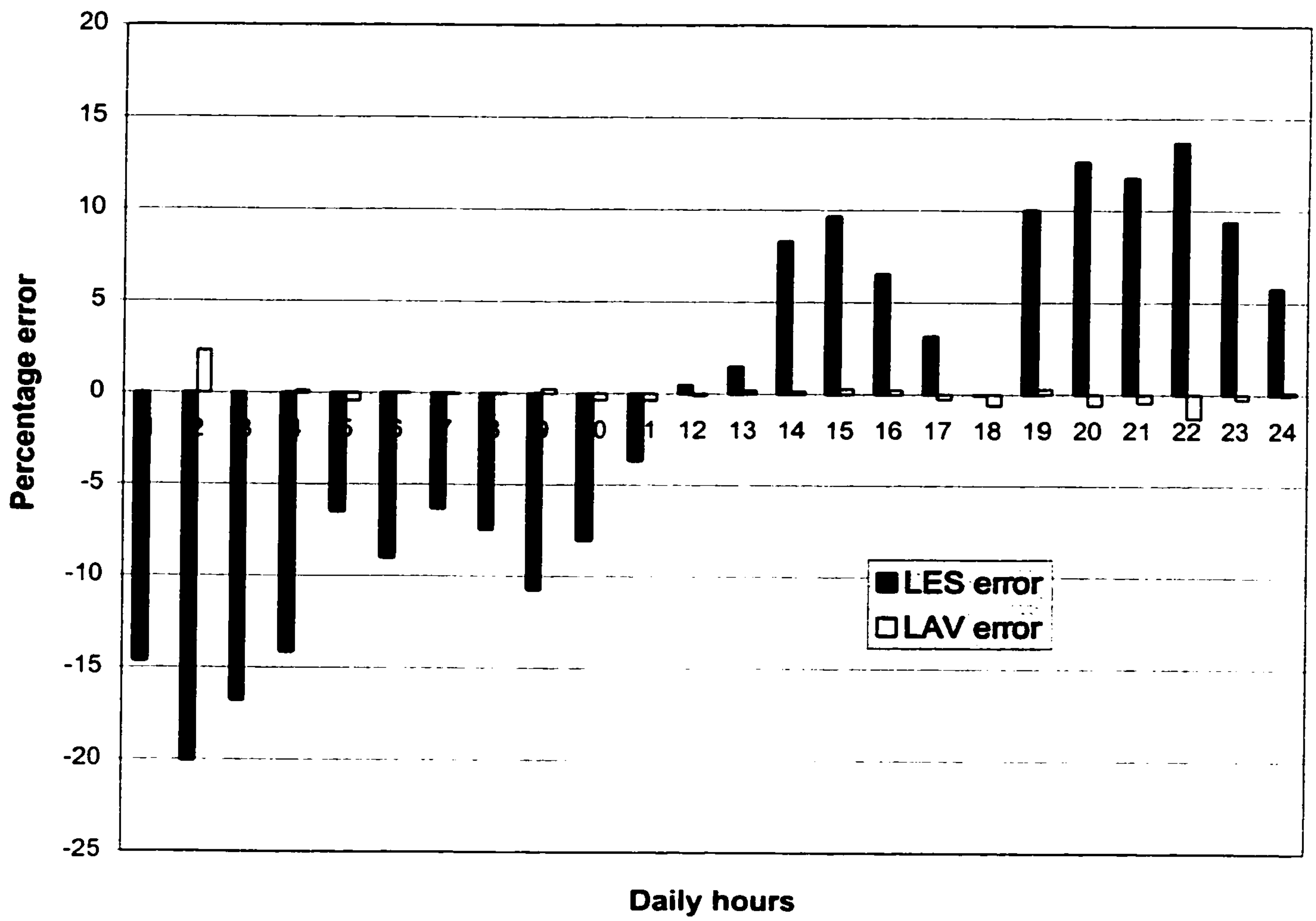


Figure (6.22) Predicted load error for a summer weekend day, Model B

6. 5. 3 General Remarks For Summer Model B

The two approaches give acceptable load predictions. Model B is not weather responsive. More quality data have to be used to reduce the error values. The LES and LAV tools present predicted load values for both weekday and weekend days. These algorithms are to be compared to the fuzzy algorithm in results and technique later on in this thesis.

6. 5. 4 Winter Predictions

Appendix 3 and 4 give the results for a weekday and a weekend day during the winter season. Model B is a non-sensitive weather model. This feature is reflected upon the results. They show deviations in estimated and predicted load values from the actual load. The error levels range from high to low values. The same argument applies for either weekday or weekend day.

6. 6 Model C

Model C is a combination of a harmonic, weather insensitive model, and a multiple linear regression model which accounts for weather parameters. In other words it's a hybrid of models A and B. The parameters of model C are estimated for a weekday and a weekend day during the summer season and winter season. Table (6.21) and Figures (6.23 - 6.24) give the estimated load and the percentage errors in the estimate during this summer weekday. Furthermore, Table (6.22) and Figures (6.25 - 6.26) give predicted load for a summer weekday 24 hours ahead. Examining these tables and figures reveals the following:

- From Table (6.20), parameter A_o has the largest value, since it represents the basic load while the rest of the parameters represent the variations in the load from other factors. A_o is 1020.16 at LES estimation and 1023.68 at LAV estimation.
- The results given in Table (6.21), estimated load, indicate that the parameters estimates for model C are accurate, since the errors in the estimated load power values are very small, for both LES and LAV

techniques. LES estimated load error goes from the highest of 0.36% (underestimated) at hour 4, to the lowest of 0% at hour 18, while LAV estimated load error goes from the highest of 1.86% (overestimated) at hour 22, to the lowest of 0.01% (overestimated) at hour 16.

- From Figure (6.24), it is noted that the maximum error obtained at hour 22 by the LAV algorithm is 1.86% (overestimated) and LES algorithm is 0.28% (overestimated) which is small and acceptable.
- Both Table (6.22), predicted load for 24-hours ahead and Figure (6.27), giving a comparison between the predicted and actual load, show that the load is being overpredicted. LES predicted load error has the highest value of 22.45% (overpredicted) at hour 4 and the lowest value of 0.51% (underpredicted) at hour 3. LAV predicted load error has the highest value of 24.17% (overpredicted) at hour 4 and the lowest value of 0.7% (underpredicted) at hour 24.

6. 6. 1 General Remarks For Summer Model C

Model C considers all days of the week and does not distinguish between weekday and weekend days. The two approaches using model C give good load predictions. Over prediction takes place more than under prediction. So the loads are over predicted. More data have to be used to reduce error values. LES and LAV algorithms are applied as predicting tools, and later in this thesis these algorithms are to be compared to the fuzzy algorithm in results and technique.

Table (6.20) Load parameters for a summer or winter day, Model C

Parameter	LES estimate	LAV estimate
A_o	1020.16	1023.68
A_1	3.78	2.97
B_1	-2.85	-1.53
A_2	-18.56	-12.83
B_2	24.52	27.64
A_3	-24.86	-24.56
B3	-10.3	-18.43
A_4	3.38	6.5
B_4	2.25	3.73
A_5	17.12	31.64
B_5	17.10	16.77
A_6	-5.34	-14.59
B_6	11.36	6.51
A_7	-0.44	3.43
B_7	5.5	12.72
A_8	27.35	27.26
B_8	78.71	76.95
A_9	-3.57	-12.48
B_9	-7.25	-16.88
C_0	-27.56	-35.72
C_1	-4.28	9.34
C_2	4.04	5.8
C_3	-7.52	-14.47

Table (6.21) Estimated load and percentage error for summer day, Model C

Hour	Actual load(MW)	LES Estimate	LAV Estimate	% LES Error	% LAV Error
1	749.8	747.5	749.92	0.31	-0.02
2	666.2	667.3	665.42	-0.17	0.12
3	621.9	622.4	621.4	-0.08	0.08
4	598.5	596.4	598.85	0.36	-0.06
5	587.4	588.8	587.33	-0.23	0.01
6	586.4	586.8	586.62	-0.06	-0.04
7	603.2	604.4	603.65	-0.21	-0.07
8	656.8	656.4	656.31	0.06	0.08
9	787.2	785.8	787.56	0.18	-0.05
10	897.4	898.6	897.56	-0.13	-0.02
11	969.8	970.5	969.58	-0.07	0.02
12	1008	1006.2	1006.61	0.18	0.14
13	1019.7	1020.5	1020.4	-0.08	-0.07
14	1002.5	1003.7	1002.4	-0.12	0.05
15	990.1	988.95	989.99	0.12	0.01
16	971.7	970.98	971.78	0.07	-0.01
17	965.5	967.1	966.02	-0.17	-0.05
18	987.4	987.4	987.99	0	-0.06
19	968	969.94	968.24	-0.2	-0.03
20	923.7	922.42	924.16	0.14	-0.05
21	888.8	888.47	889.09	0.04	-0.03
22	900.1	902.66	916.85	-0.28	-1.86
23	940.5	938.93	940.7	0.17	-0.02
24	875.7	874.93	876.01	0.09	-0.04

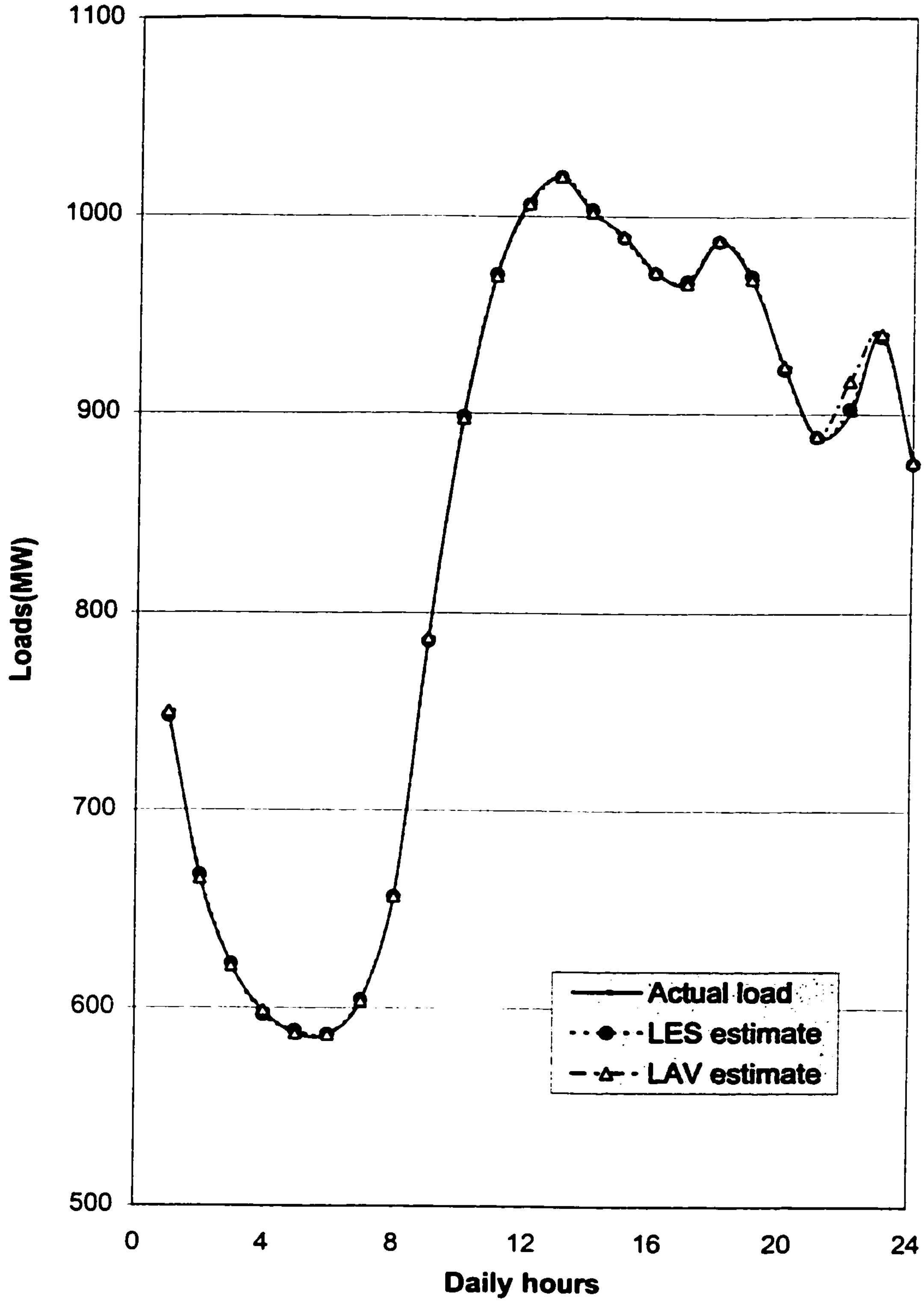


Figure (6.23) Estimated load for a summer day, Model C

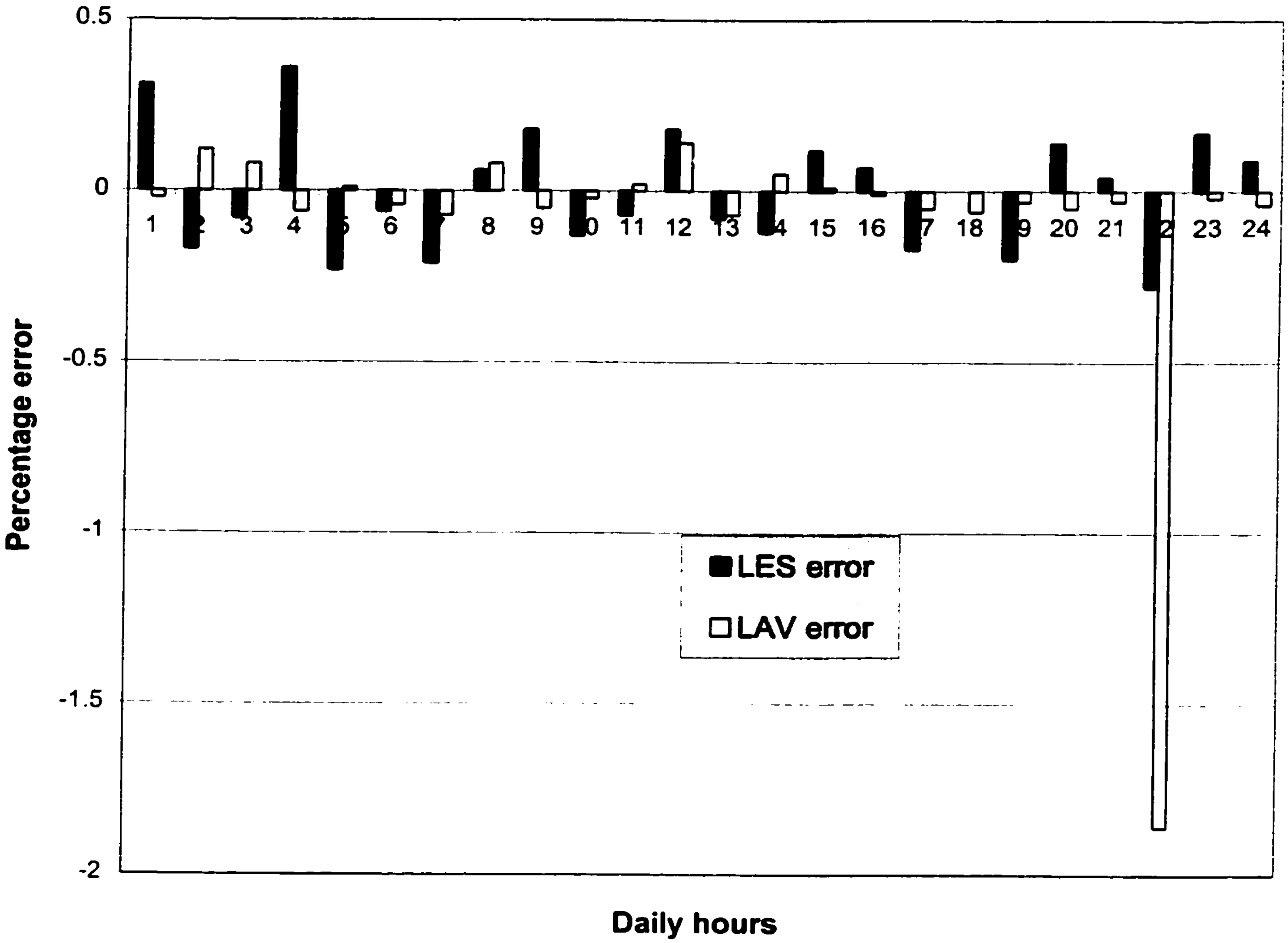


Figure (6.24) Estimated load error for a summer day, Model C

Table (6.22) Predicted load and percentage error for summer day, Model C

Hour	Actual load(MW)	LES Prediction	LAV Prediction	% LES Error	% LAV Error
1	744.7	723.46	725.41	2.85	2.59
2	670.8	774.04	777.23	-15.39	-15.87
3	634.3	631.08	631.69	0.51	0.41
4	613.4	751.12	761.63	-22.45	-24.17
5	607.3	632.5	635.28	-4.15	-4.61
6	608.9	666.03	671.54	-9.38	-10.29
7	628.1	521.71	519.44	16.94	17.3
8	697.2	830.98	839.66	-19.19	-20.43
9	817.9	858.27	865.72	-4.94	-5.85
10	934.9	994.62	1000.39	-6.39	-7.01
11	997.4	1107.61	1115.68	-11.05	-11.86
12	1030.4	1155.39	1165.91	-12.13	-13.15
13	1069	1213.89	1226.49	-13.55	-14.73
14	1054.7	1121.25	1129.7	-6.31	-7.11
15	1043.3	1250.67	1268.26	-19.88	-21.56
16	1028.5	1112.96	1126.19	-8.21	-9.5
17	1033.2	1119.93	1131.55	-8.39	-9.52
18	1058.1	906.4	908.96	14.34	14.09
19	1036.2	1158.78	1169.03	-11.83	-12.82
20	970.1	1027.05	1037.87	-5.87	-6.99
21	930.2	1013.43	1023.91	-8.95	-10.07
22	962.9	941.63	961.87	2.21	0.11
23	996.4	1014.58	1023.29	-1.82	-2.7
24	925.4	912.85	918.96	1.36	0.7

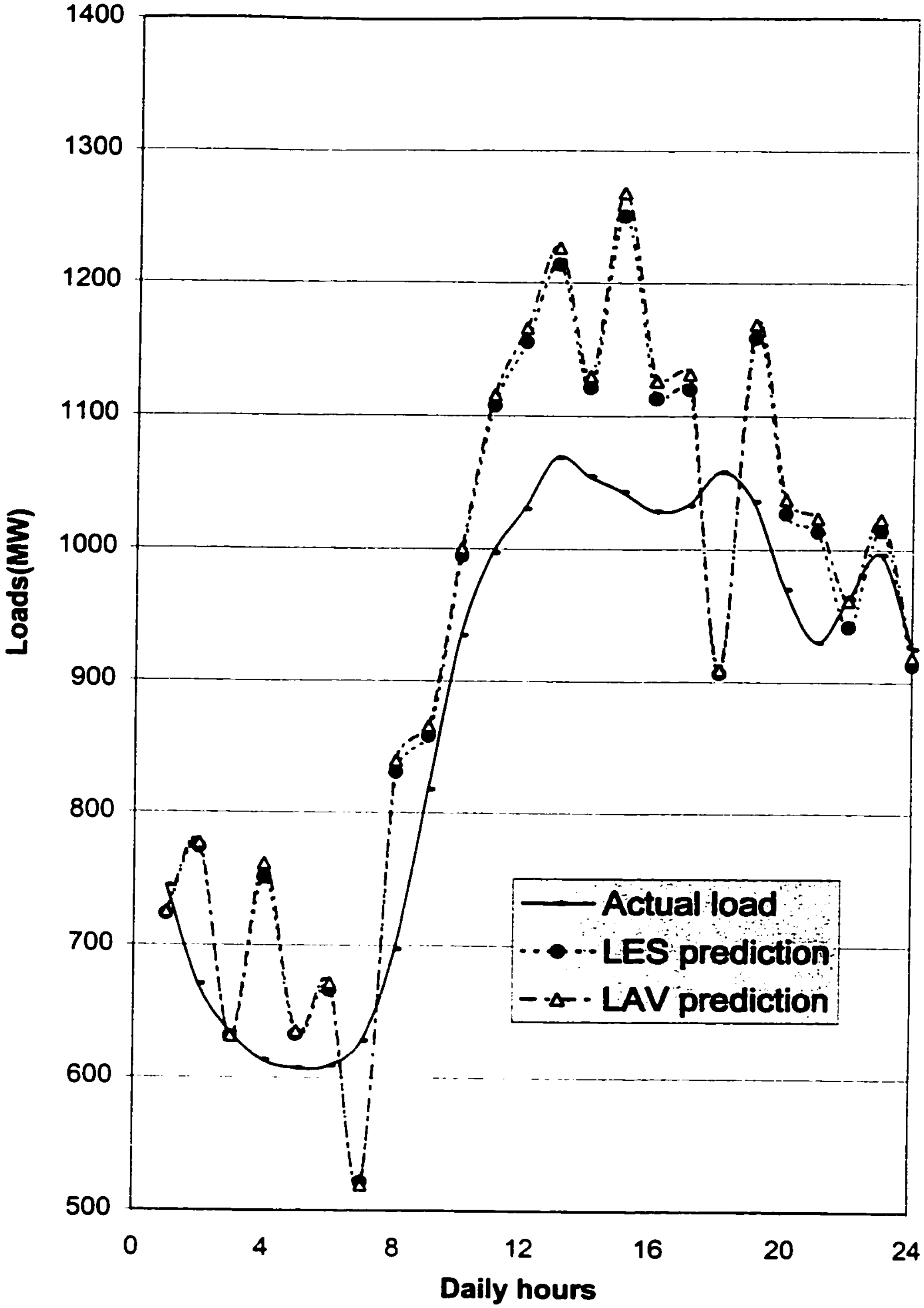


Figure (6.25) Predicted load for a summer day, Model C

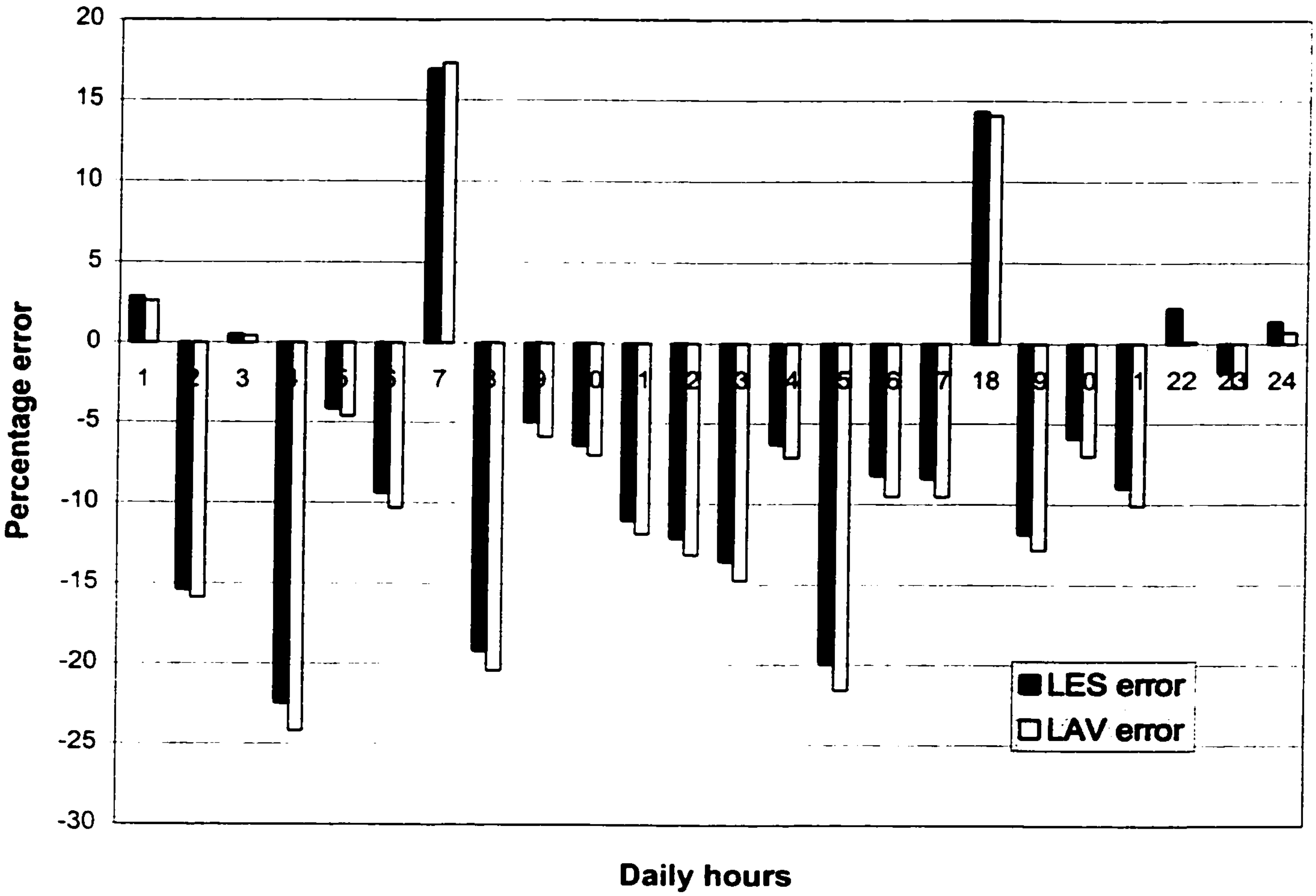


Figure (6.26) Predicted load error for a summer day, Model C

6. 6. 2 Winter Predictions

Appendix 3 and 4 give the prediction results for a winter day. Examining these results, the same remarks can be reached as those for the summer day. Table (P3.13) shows the estimated load results to be very good. LES estimated load has a maximum error of 0.43% (overestimated) at hour 3, and a minimum of 0% at hour 13. While LAV estimated load has a maximum error of 2.29% (overestimated) at hour 3, and a minimum error of 0% at hours 11,16,19 and 21. The level of errors are small and acceptable. Table (P3.14) exhibits the predicted load results. LES predicted load has a maximum error of 13.01% (underpredicted) at hour 1, and a minimum of 1.12% (overpredicted) at hour 15. LAV predicted load has a maximum of 12.97% (underpredicted) at hour 2, and a minimum of 0.3% (ovrepredicted) at hour 15. In general, since model C accounts for weather and time, it exhibits better results.

6. 7 Concluding Remarks

In this chapter, the LES and LAV parameter estimation algorithms are used for static estimation for the parameters of different load models. Three models are used namely A, B and C. These models are used to predict load power for the next 24-hours in a weekday and a weekend ahead for a weekend day. It has been found that model A gives acceptable load predictions.

Model A possesses the advantage of being weather sensitive, but suffers the following: (1) It needs 24 separate parameters sets in order to predict the load 24 hours ahead as accurate as possible, and this needs more computing time, it is also found that one set of parameters gives acceptable results, (2) The use of separate models for weekday and weekend day both with summer and winter formulations. Model B does not account for weather effects, but is a function of the hour (Time) considered and produces acceptable results and takes less computing time. It can only be used for a case where the weather variations are small during the day.

Model C is the most suitable model, since it takes into account both time and weather during summer and winter seasons. It eliminates the use of separate models for both weekday and weekend day.

Chapter 7
Load Forecasting Computational Results
Fuzzy Linear Regression

7.1 Introduction

In chapter 6 the short-term load forecasting problem is discussed, and the LES and LAV parameter estimation algorithms are used to estimate the load model parameters. The error in the estimates is calculated for both techniques. The three models, proposed earlier in chapter 3, are used in that chapter, to present the load in different days for different seasons. In this chapter, the fuzzy load models developed in Chapter 5 are tested. The fuzzy parameters of these models are estimated using the past history data for summer weekday and weekend days as well as for winter weekday and weekend days. Then these models are used to predict the fuzzy load power for 24 hours ahead, in both summer and winter seasons. The results are given in the form of Tables and Figures for the estimated and predicted loads.

7.2 Fuzzy Load Model A

The developed fuzzy model A for summer in Chapter 5 is tested in this section. First, the load power data are assumed to be crisp values, and the load parameters are fuzzy. Then, the load power data and the load parameters are both assumed to be fuzzy. It is found that nine fuzzy parameters are enough to model this type of load.

7.2.1 Load Parameters For A Summer Weekday

Table (7.1) gives the estimated fuzzy parameters for three cases. In the first case, the load power has crisp values. The other two cases, the load power is fuzzy data and it is assumed that load power has deviated by 5% and 20% from the original case to simulate the fuzziness in these values. Examining this table reveals the following:

Table (7. 1) Fuzzy parameter for a summer weekday, Model A

Parameters	Crisp load		5 % load Deviation		20 % load Deviation	
	Middle	Spread	Middle	Spread	Middle	Spread
A_0	0.0	335.69	0.0	288.4	0.0	391.91
A_1	0.0	0.0	8.5	0.0	0.0	0.0
A_2	0.0	0.0	0.0	0.0	0.0	0.0
A_3	0.0	0.0	0.0	0.0	0.00807	0.0
A_4	0.0	0.0	3.75	0.0	3.229	0.0
A_5	0.0	0.0	0.579	0.0	0.612	0.0
A_6	0.0	0.0	0.0	0.0	0.0	0.0
A_7	22.94	0.0	1.919	0.0	22.92	0.0
A_8	0.0	0.0	0.0	0.0	0.0	0.0
A_9	1.03	0.0	21.82	0.0	0.0	0.0

- The only fuzzy parameter is A_o, which conforms with the assumption that the load power has a crisp value, and the spreads of the parameters are to be minimized.
- It can be noted that three parameters are adequate to represent the load for the crisp case, six parameters for the 5% load deviation case and five parameters for 20% load deviation case, since the output of the linear optimization problem produces only these parameters.
- The middle of some parameters (A_1) are not zeros at 5% load deviation and are zeros for 20 % load deviation and vice versa (A_3 has a middle value at 20% load deviation but the middle is zero at 5% load deviation).

- Among the values of the parameters, the A_o parameter is the largest one. This parameter represents the base load. The extra power component that comes from other parameters represents the variation in the load power due to the variation in weather conditions.

7.2.2 Load Estimation For A Summer Weekday

Using the estimated fuzzy parameters mentioned in Table (7.1), Figures (7.1 – 7.3) give the actual and the estimated load during the same period of time for the three load power conditions. Examining these figures reveals the following:

- The estimated fuzzy load contains the given load values within the allowable range specified by spreads in the parameters.
- The estimation results are good since the given load has never gone outside the range given by the spreads of the fuzzy parameters.
- The problem involving crisp values for load power at any hour, mentioned in chapter 6, is now solved, by transforming the load at the hour into a soft load, and a range of lower load to upper load is allowed.
- As the load deviation percentage increases, the spread between the upper load and the lower load increases.

7.2.3 Load Prediction For A Summer Weekday

The estimated fuzzy parameters are used to predict the load 24-hours ahead for a summer working day. Figures (7.4-7.6) give the results obtained for the three fuzzy ranges for this day. Examining these Figures reveals the following:

- The estimated parameters produce good predictions for the load at every hour in question.
- The given load is within the range produced by the estimated parameter spreads.

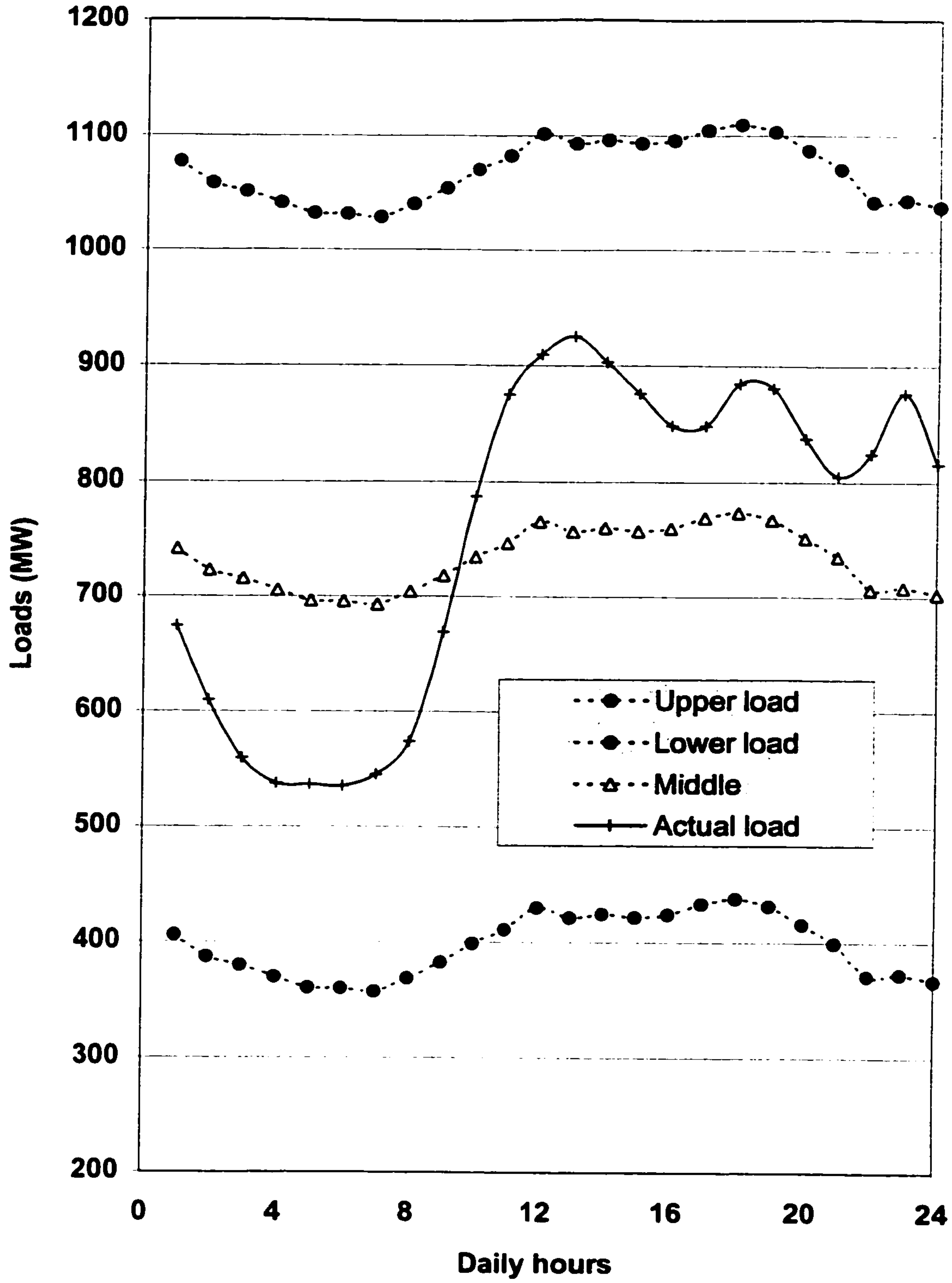


Figure (7.1) Estimated load for a summer weekday, crisp load, Model A

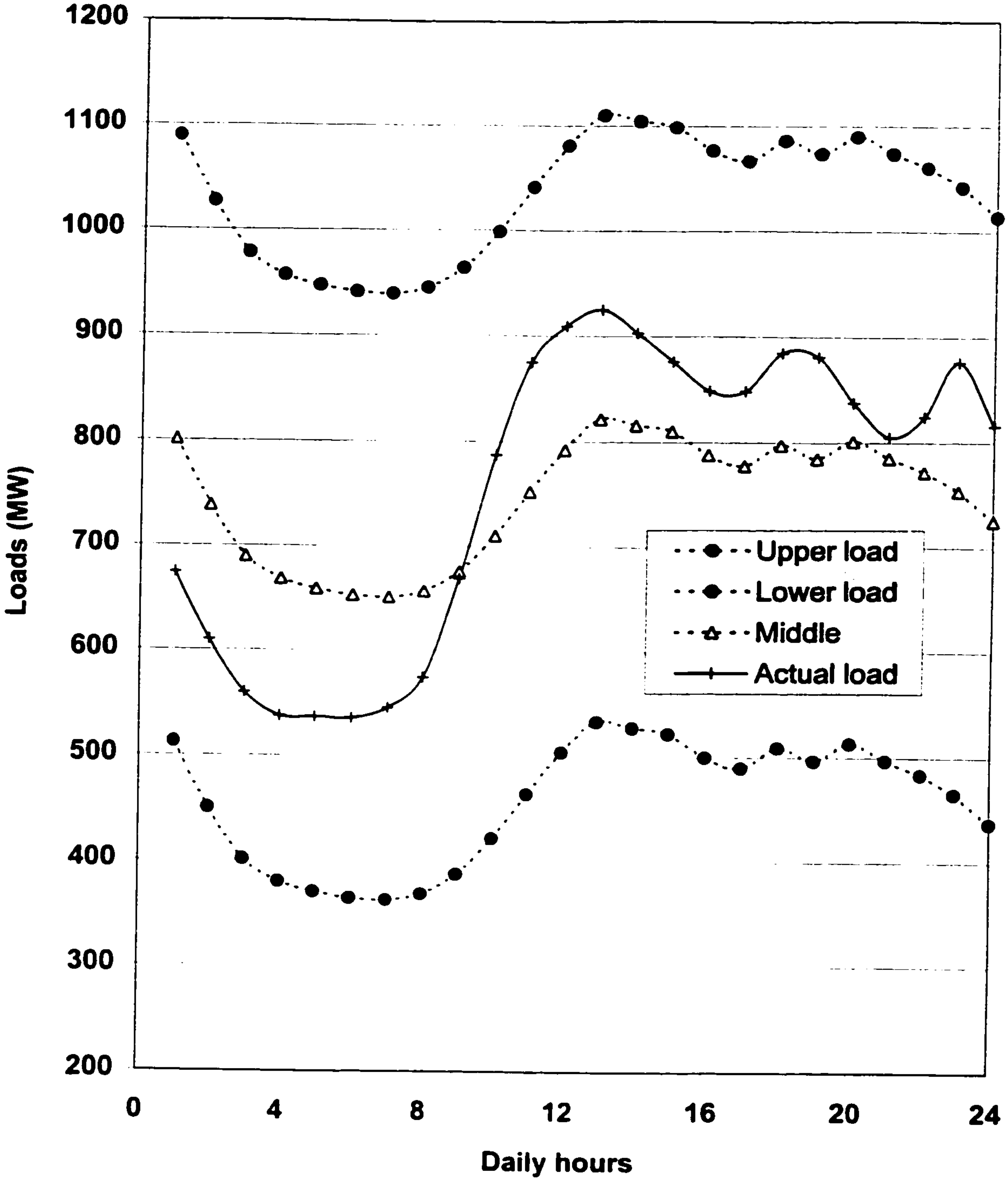


Figure (7.2) Estimated load for a summer weekday, (5 % load deviation), Model A

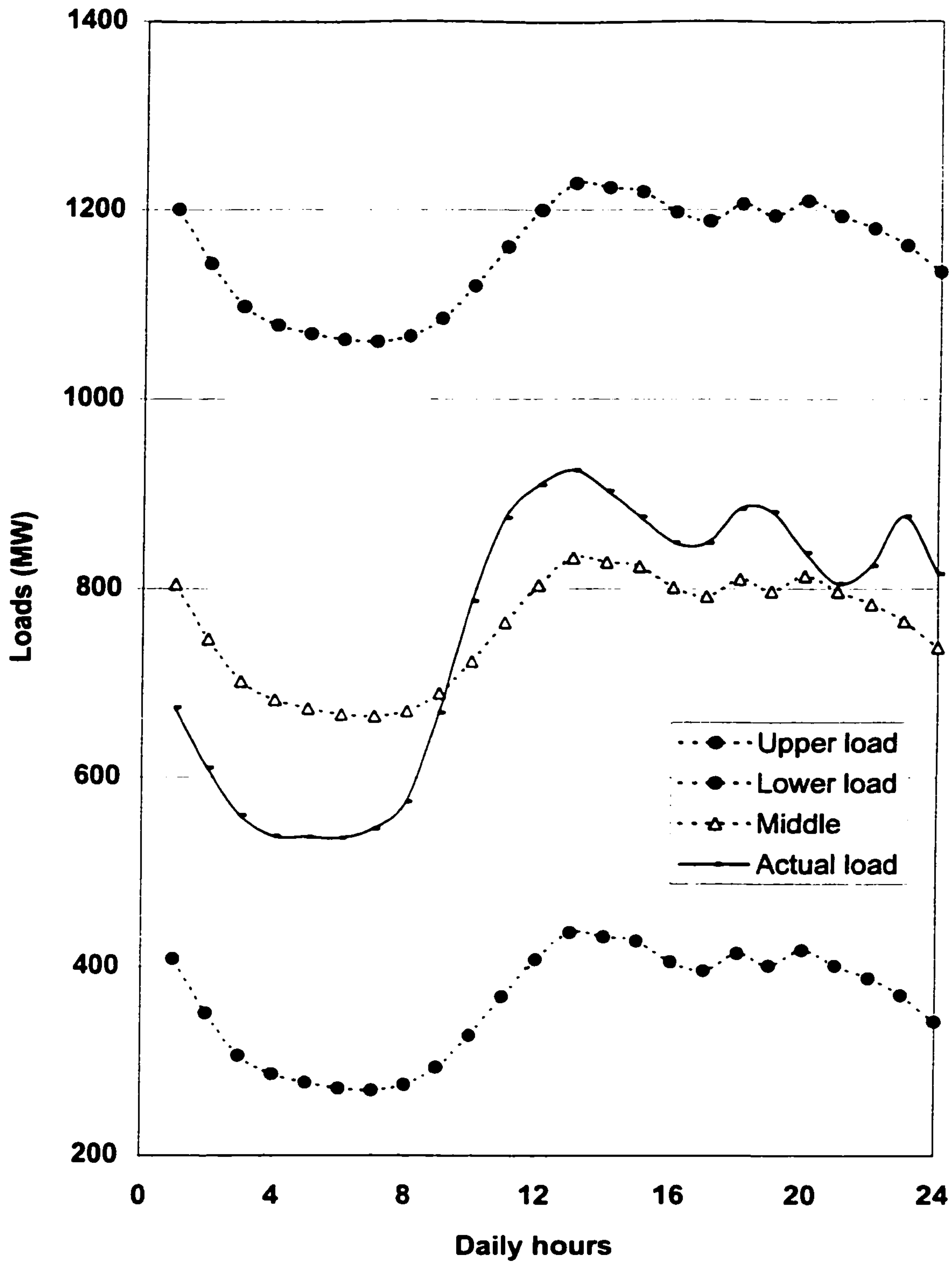


Figure (7.3) Estimated load for a summer weekday, (20 % load deviation), Model A

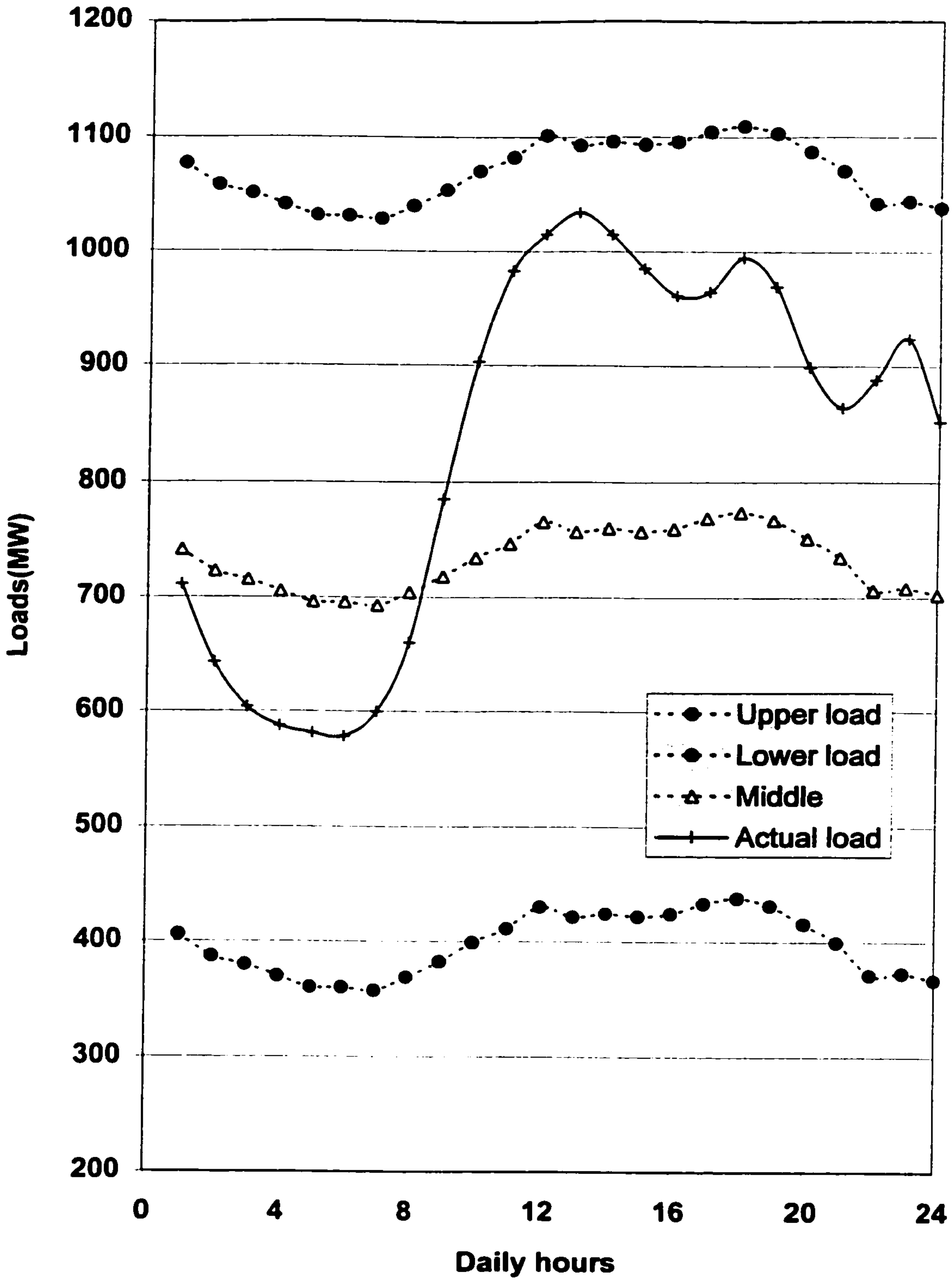


Figure (7.4) Predicted load for a summer weekday, crisp load, Model A

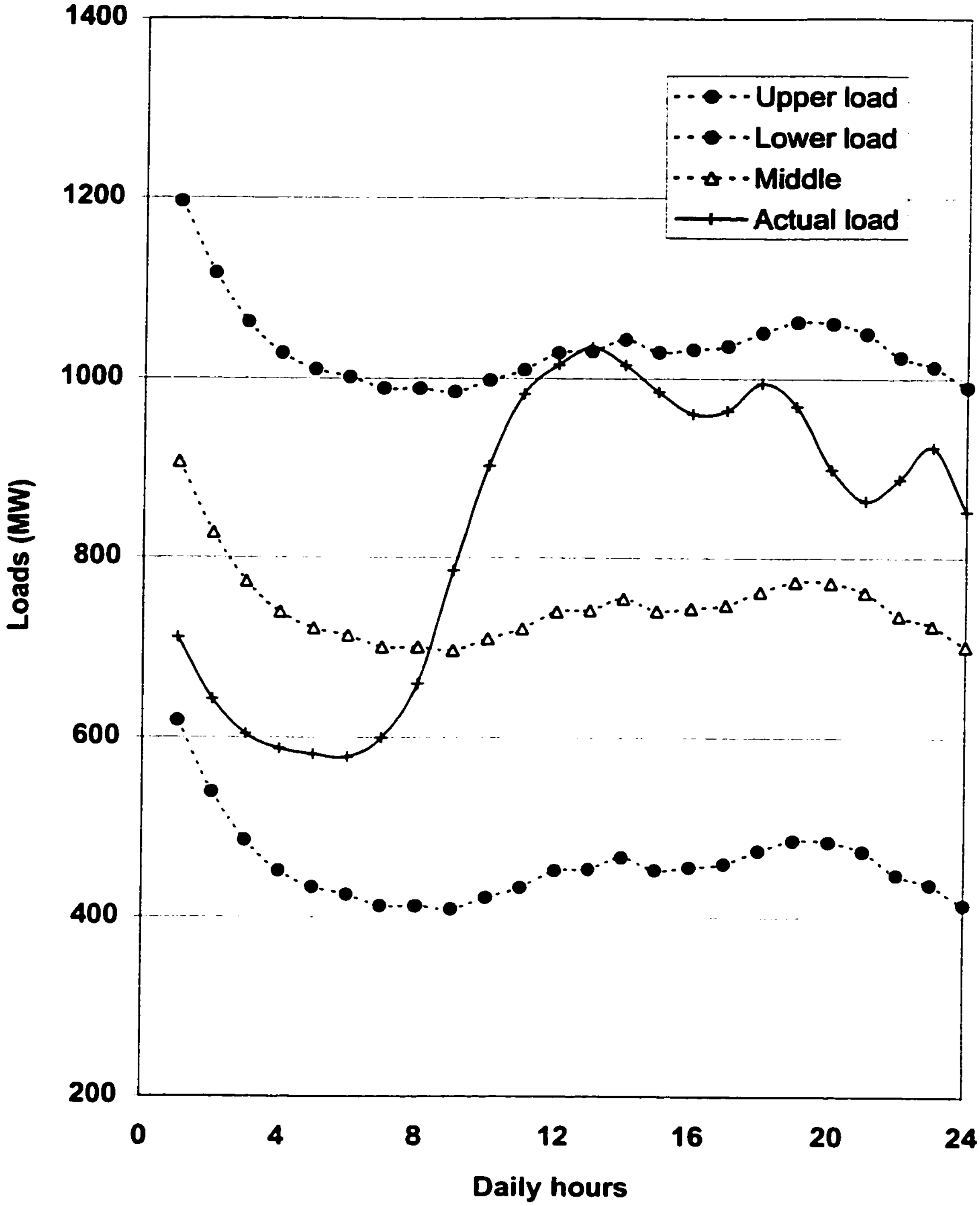


Figure (7.5) Predicted load for a summer weekday, (5% load deviation), Model A

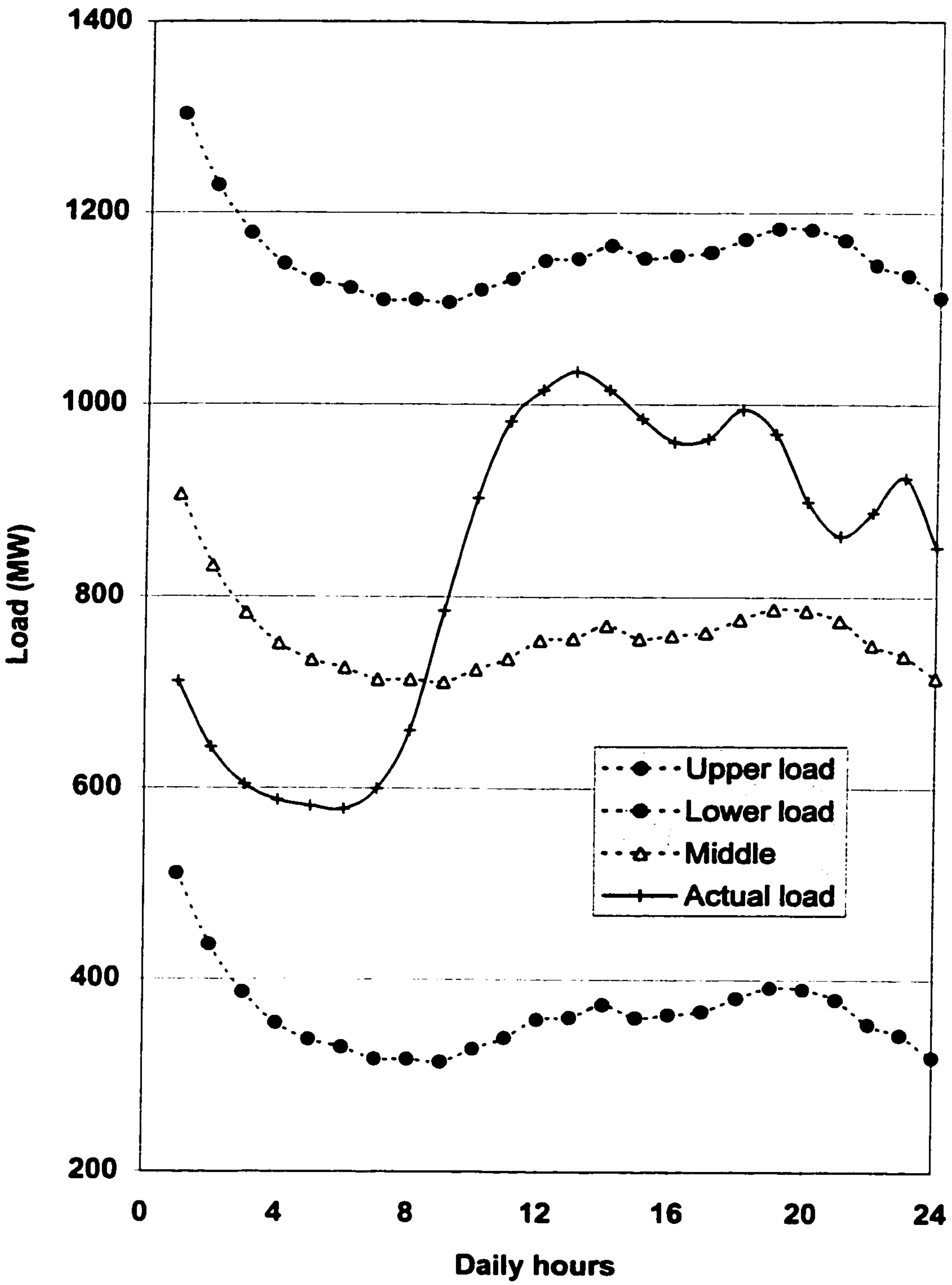


Figure (7.6) Predicted load for a summer weekday, (20% load deviation), Model A

- The actual load deviates a very small amount from these ranges. These deviations can be neglected for such type of forecasting.
- At a given hour, the upper and lower values can be considered as constraints on the load at this hour.

Table (7. 2) Fuzzy parameters for a summer weekend day, Model A

Parameters	Crisp load		5 % load Deviation		20 % load Deviation	
	Middle	Spread	Middle	Spread	Middle	Spread
A_0	0.0	247.328	0.0	283.0	0.0	391.9
A_1	0.0	0.0	0.0	0.0	0.0	0.0
A_2	0.0	0.0	0.0	0.0	0.0	0.0
A_3	0.0069	0.0	0.0072	0.0	0.0081	0.0
A_4	2.779	0.0	2.891	0.0	3.23	0.0
A_5	0.307	0.0	0.383	0.0	0.612	0.0
A_6	0.0	0.0	0.0	0.0	0.0	0.0
A_7	22.576	0.0	22.662	0.0	22.92	0.0
A_8	0.0	0.0	0.0	0.0	0.0	0.0
A_9	0.0	0.0	0.0	0.0	0.0	0.0

7.2.4 Load Estimation For A Summer Weekend Day

The proposed fuzzy model is used as well to predict the load on a summer weekend day. The fuzzy parameters are estimated first. Table (7.2) gives the estimated fuzzy parameters, while Figures (7.7-7.9) depict the results for the load deviation ranges. Examining the table and figures reveals the following:

- Among the parameters, $\underset{\sim}{A}_o$ is the only parameter showing fuzziness for the crisp and the other two cases, since it has spread values. The objective is to minimize the spread of each fuzzy parameter.
- Five fuzzy parameters are adequate to model this type of load for this specific day and season.
- The actual load is in the range given by the estimated spread and does not cross the border of the estimated load.
- The actual load lies between the upper and lower fuzzy ranges of the loads.

7.2.5 Load Prediction For A Summer Weekend Day

The estimated fuzzy parameters are used to predict the load ahead in a weekend day. The results obtained are given in Figures (7.10 – 7.12). Examining these figures reveals the following:

- A good load prediction is obtained for a specified weekend day.
- A range is allowed for the load power to vary at every specified hour, and this range increases as the load deviation increases.
- The actual load never crosses the limits determined by the spreads of the load parameters.These limits are an upper load and a lower load.
- At a given hour, the upper and lower load powers can be considered as constraints on the actual load at this hour.
- The actual powers, 24 hours ahead, in all curves do not violate the upper and lower constraints power load.

In conclusion, the proposed fuzzy load model A, is adequate to present the load for the summer weekday and weekend days.

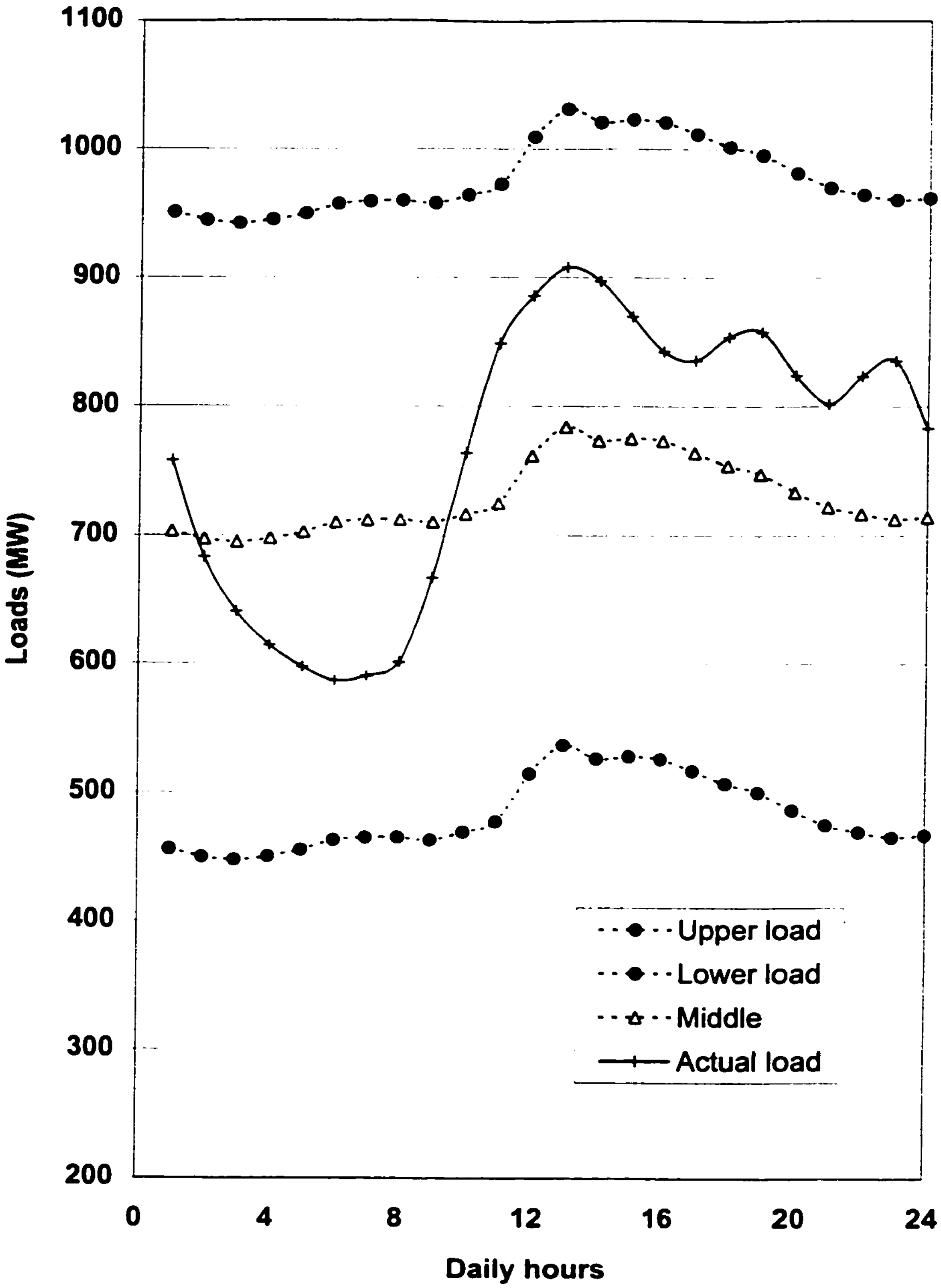


Figure (7.7) Estimated load for a summer weekend day, crisp load, Model A

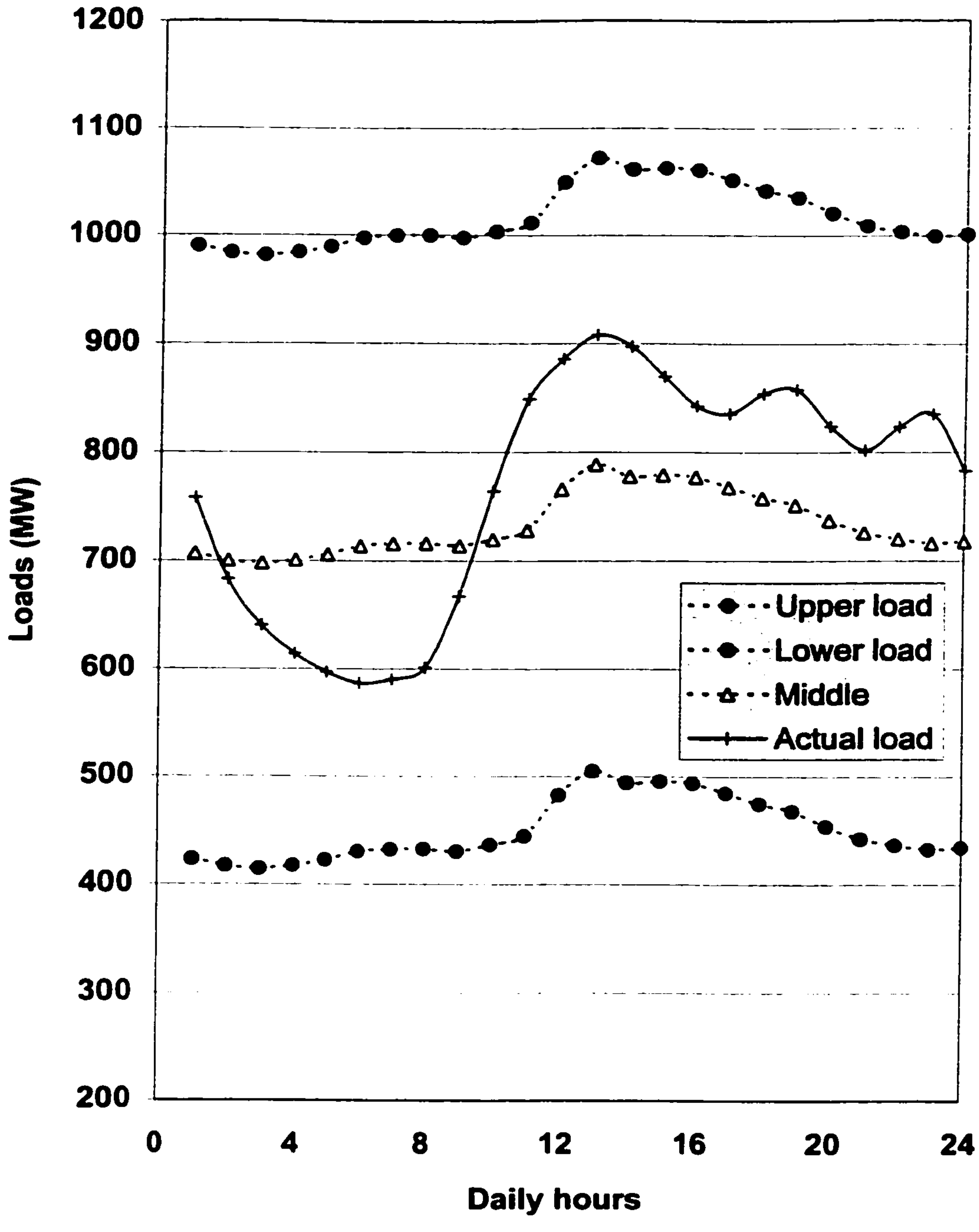


Figure (7.8) Estimated load for a summer weekend day, (5% load deviation), Model A

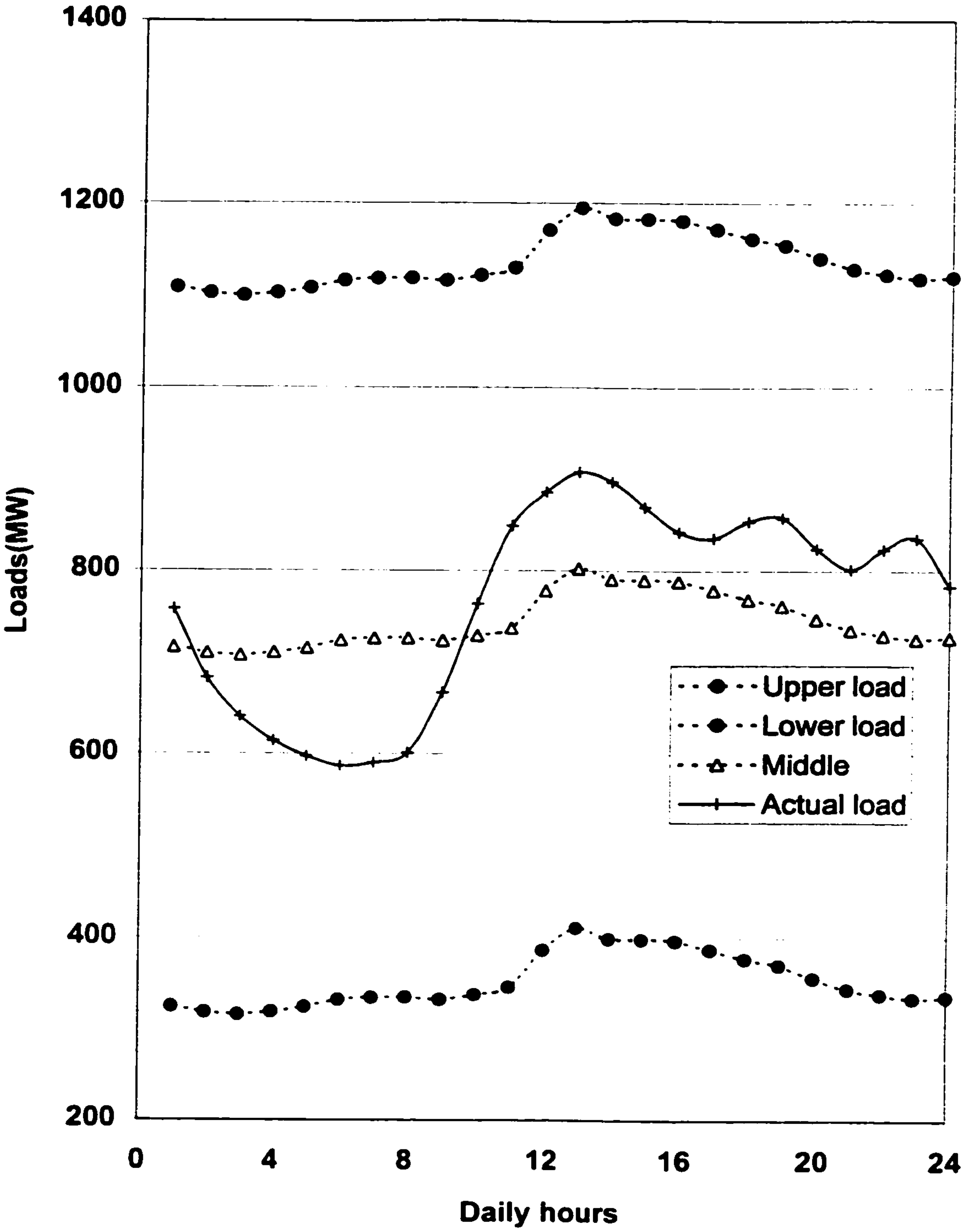


Figure (7.9) Estimated load for a summer weekend day, (20% load deviation), Model A

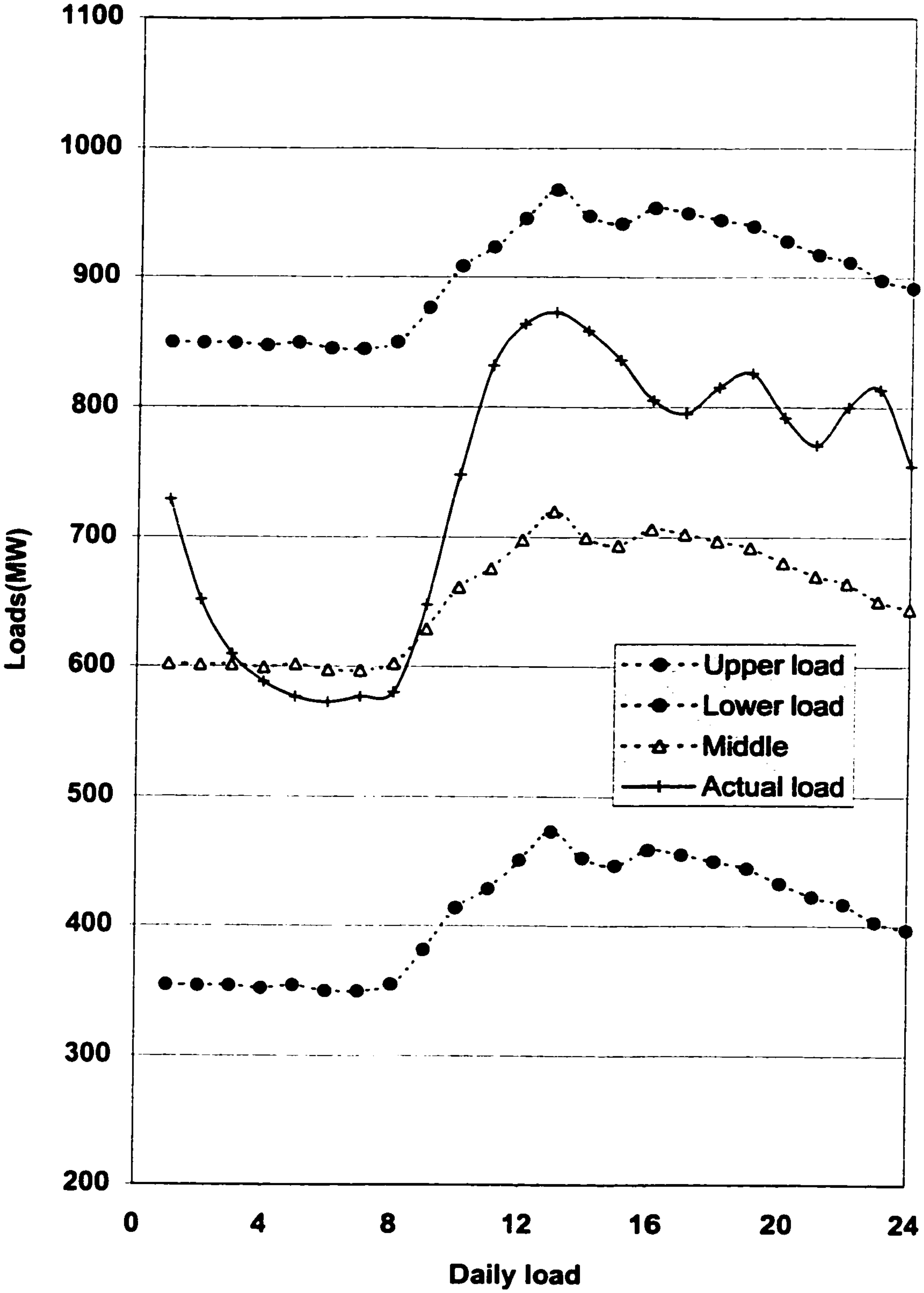


Figure (7.10) Predicted load for a summer weekend day, crisp load, Model A

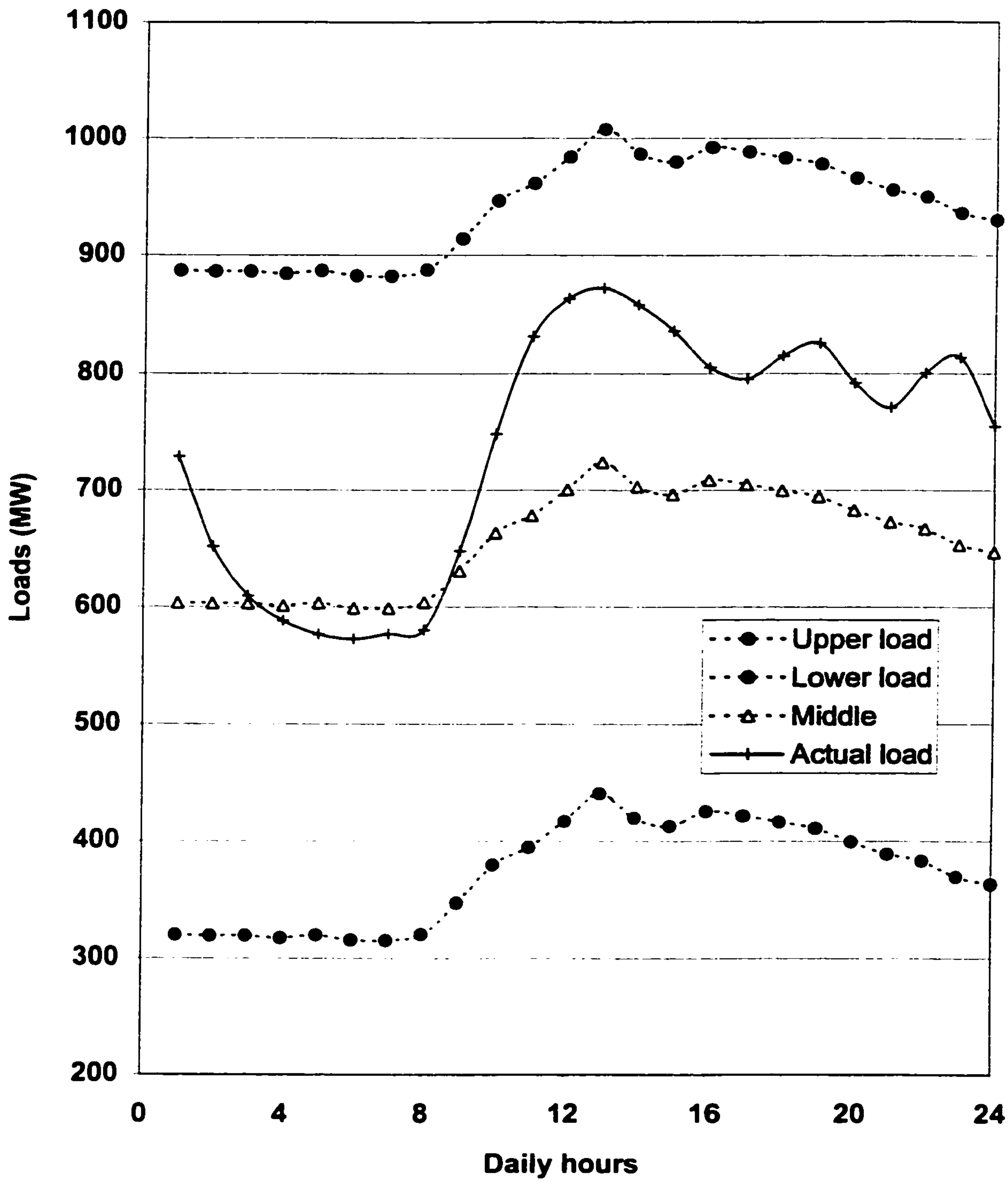


Figure (7.11) Predicted load for a summer weekend day, (5% load deviation), Model A

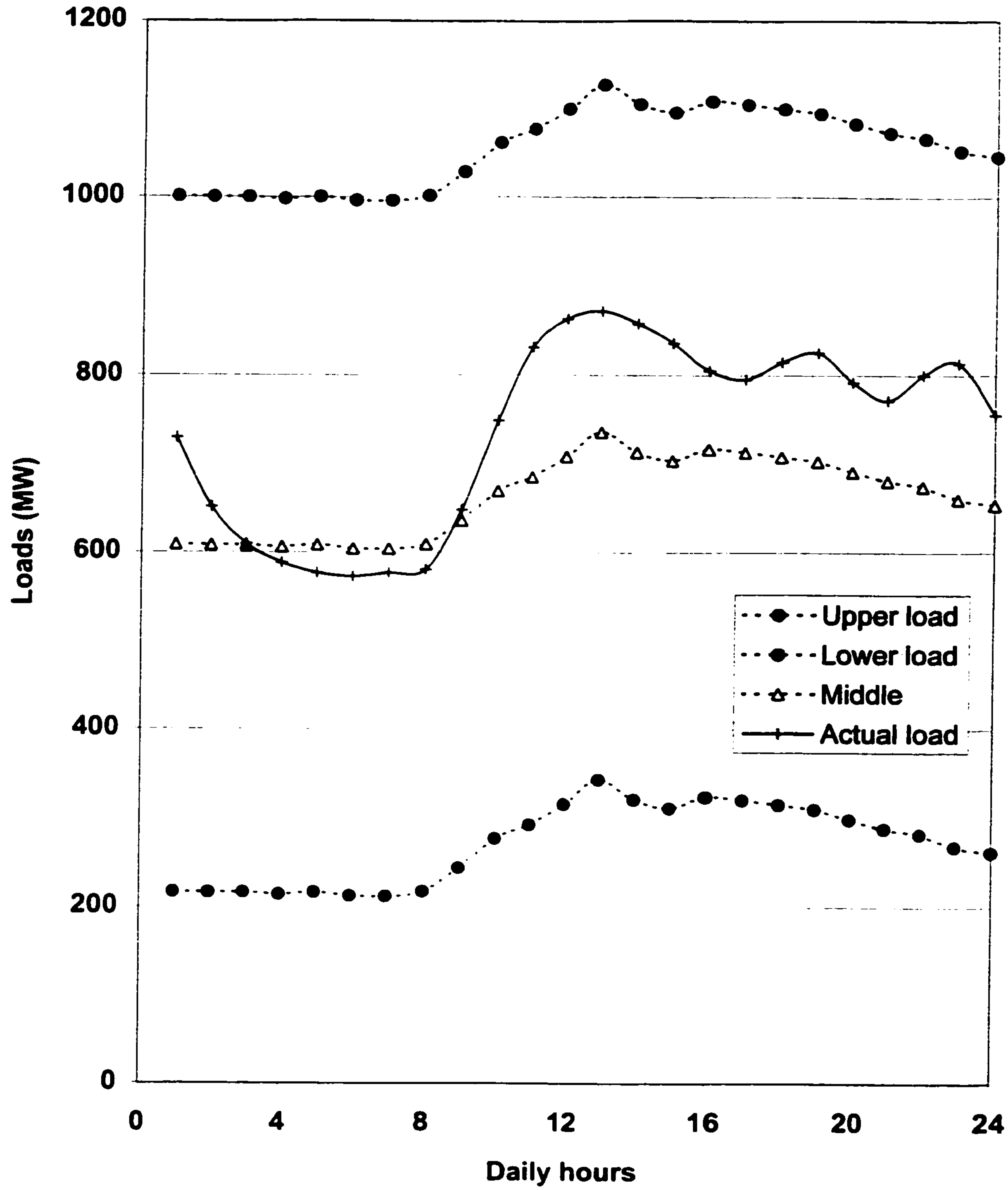


Figure (7.12) Predicted load for a summer weekend day, (20% load deviation), Model A

7.2.6 Load Estimation And Prediction For Winter Weekday And Winter Weekend Day

The results for a winter weekday and a winter weekend are given in appendices 1 and 2. The same concluding remarks can be reached for the data listed.

7.3 Fuzzy Load Model B

This model is a harmonic model and it is not sesitive to the weather parameters, (temperature, wind speed, humidity..., etc). Nine parameters are chosen for the sine term and nine for the cosine term beside to the base load parameter. The load deviation for this load model takes a value of 0% (crisp load power), 5%, 10% and 20 % to simulate the fuzziness of the load power.

7.3.1 Load Parameters For Model B

Table (7.3) gives the variation of the fuzzy parameters at percentage load deviations. Examining this table reveals the following:.

- Among the load parameters, only parameters A_o, the base load parameter, and A_5 are fuzzy.
- Parameter A_5 has a zero value at the middle and a different spread value in all cases considered. For (e.g. 20% load deviation case):

  The upper parameter value = middle + spread

  $$= \quad 0 \quad + 20.196 = 20.196$$

  The lower parameter value = middle – spread

  $$= \quad 0 \quad - 20.196 = 20.196$$

  The membership for A_5 is a line on the x-axis centered at the origin with a zero middle value and a spread of (20.196).

  The spread increases with the increase of the degree of fuzziness:

Table (7.3) Fuzzy parameters for a summer day load, Model B

Parameter	Crisp load		5% load deviation		10% load deviation		20% load deviation	
	Middle	Spread	Middle	Spread	Middle	Spread	Middle	Spread
A_o	874.32	258.7	875.99	299.306	879.498	340.848	886.038	424.722
A_1	1.594	0.0	1.402	0.0	0.0	0.0	0.0	0.0
A_2	28.95	0.0	28.544	0.0	26.8084	0.0	23.770	0.0
A_3	0.0	0.0	0.355	0.0	1.7189	0.0	3.4214	0.0
A_4	45.81	0.0	45.502	0.0	44.6506	0.0	42.304	0.0
A_5	0.0	14.43	0.0	18.5504	0.00.0	19.411	0.00.0	20.196
A_6	23.40	0.0	23.336	0.0	24.1051	0.0	24.9042	0.0
A_7	13.5	0.0	12.826	0.0	10.7958	0.0	7.3645	0.0
A_8	14.3	0.0	14.083	0.0	12.2520	0.0	11.0160	0.0
A_9	108.9	0.0	104.869	0.0	103.782	0.0	100.678	0.0
B_1	66.22	0.0	65.528	0.0	65.1667	0.0	63.921	0.0
B_2	24.80	0.0	23.676	0.0	22.1530	0.0	18.937	0.0
B_3	9.43	0.0	8.582	0.0	9.42273	0.0	9.601	0.0
B_4	29.16	0.0	28.407	0.0	27.5010	0.0	25.045	0.0
B_5	3.93	0.0	2.192	0.0	1.81461	0.0	2.0200	0.0
B_6	1.02	0.0	0.0	0.0	0.0	0.0	0.0	0.0
B_7	54.67	0.0	51.593	0.0	50.220	0.0	47.1926	0.0
B_8	11.62	0.0	11.832	0.0	11.5480	0.0	12.288	0.0
B_9	2.70	0.0	1.7536	0.0	3.2094	0.0	5.8511	0.0

For 0% load deviation, the spread is 14.43,

5% load deviation, the spread is 18.5504,

10% load deviation, the spread is 19.411, and

20% load deviation, the spread is 20.196

This indicates the fuzziness effect in load's nature, where increasing the degree of fuzziness, the spread increases, then the range between upper and lower limits increases.

- For A_o (e.g. 20% load deviation):

  The upper parameter value = 886.038 + 424.722 = 1310.76

  The lower parameter value = 886.038 – 424.722 = 461.316

  Predicted base load will fall in the range between (1310.76 and 461.316). Both spreads from A_o and A_5 contribute to the total spread between upper and lower load values.

  The total spread = 424.722 + 20.196 [sin (5ωt)]

  The effect of the large middle and spread values of A_o shows the fuzziness in the large range where predicted load should lie in it.
- Parameter A_o has a large middle and spread, because A_o represents the base load while the other parameters (either fuzzy or not), are contributing to the excess power variations due to other load factors.
- Both the middle and spread of the base load parameter increase due to the increase in load deviation.
- All load parameters follow the same pattern of variation at each load deviation.

7.3.2 Load Estimation And Prediction

The estimated and predicted loads for a summer day, either weekday or weekend day, are given in Figures (7.13) to (7.20) for the ranges of load deviation. Examining these figures reveals the following:

- The load model B estimates and predictes the load power at any week day in any season given that the actual load does not violate the upper and lower load values.
- As the load deviation increases, the range between the upper and the lower loads increases due to increases in the spread of the fuzzy parameters.

In conclusion, model B is as good as Model A. Despite the fact that model B does not account for the weather variation, the predicted load does not violate the upper or lower load limits.

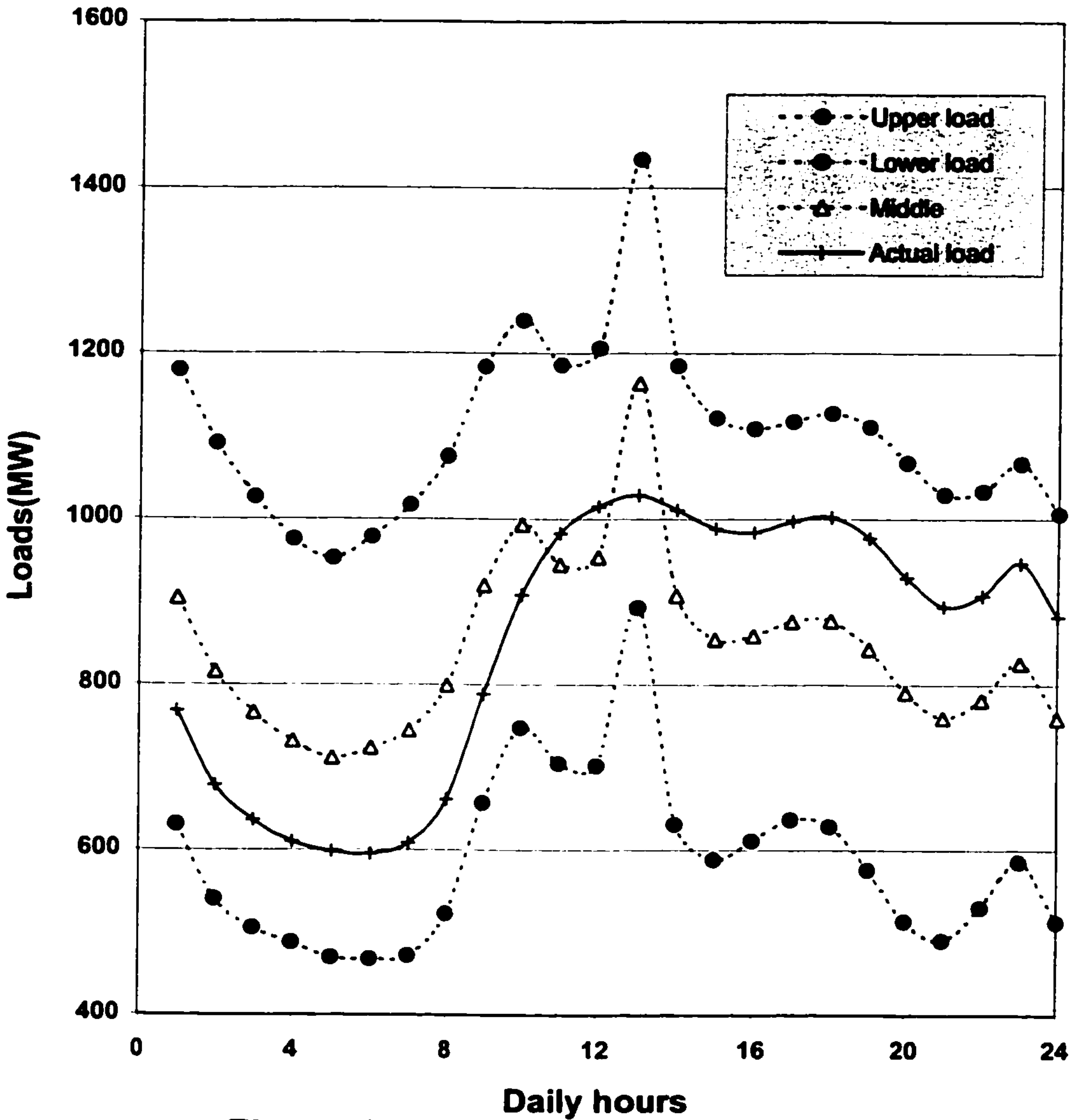


Figure (7.13) Estimated load for a summer weekday, crisp load, Model B

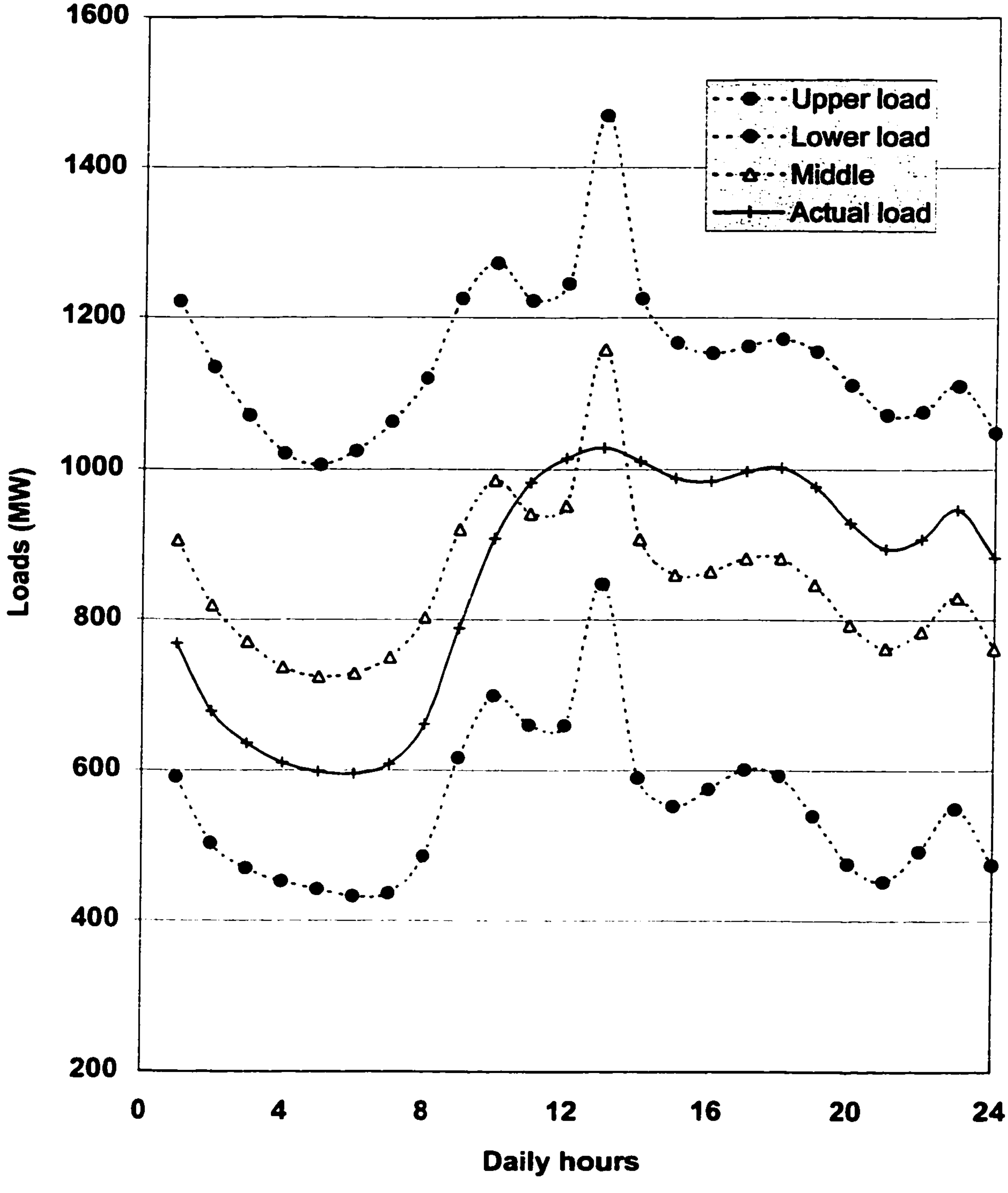


Figure (7.14) Estimated load for a summer weekday, (5% load deviation), Model B

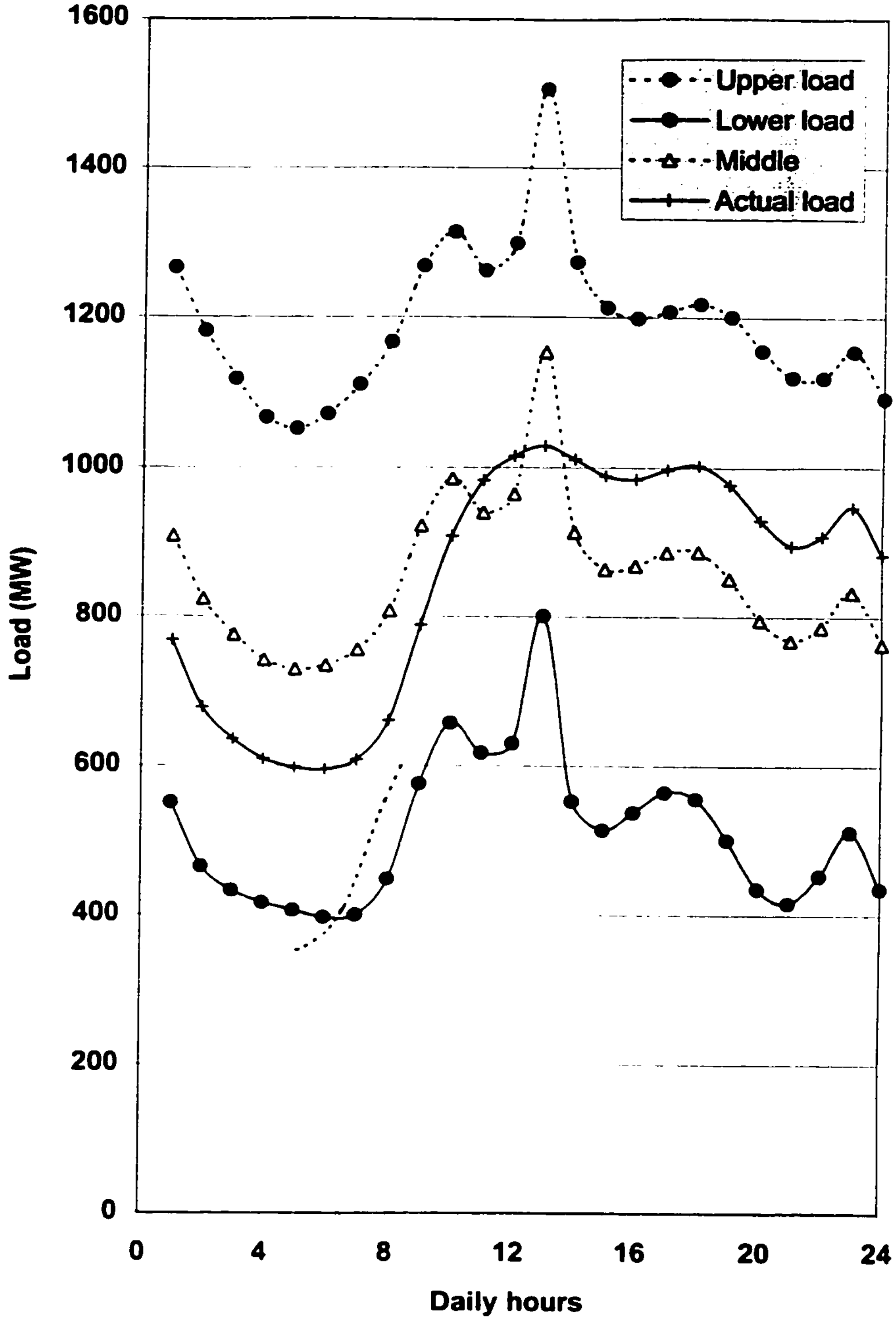


Figure (7.15) Estimated load for a summer weekday, (10% load deviation), Model B

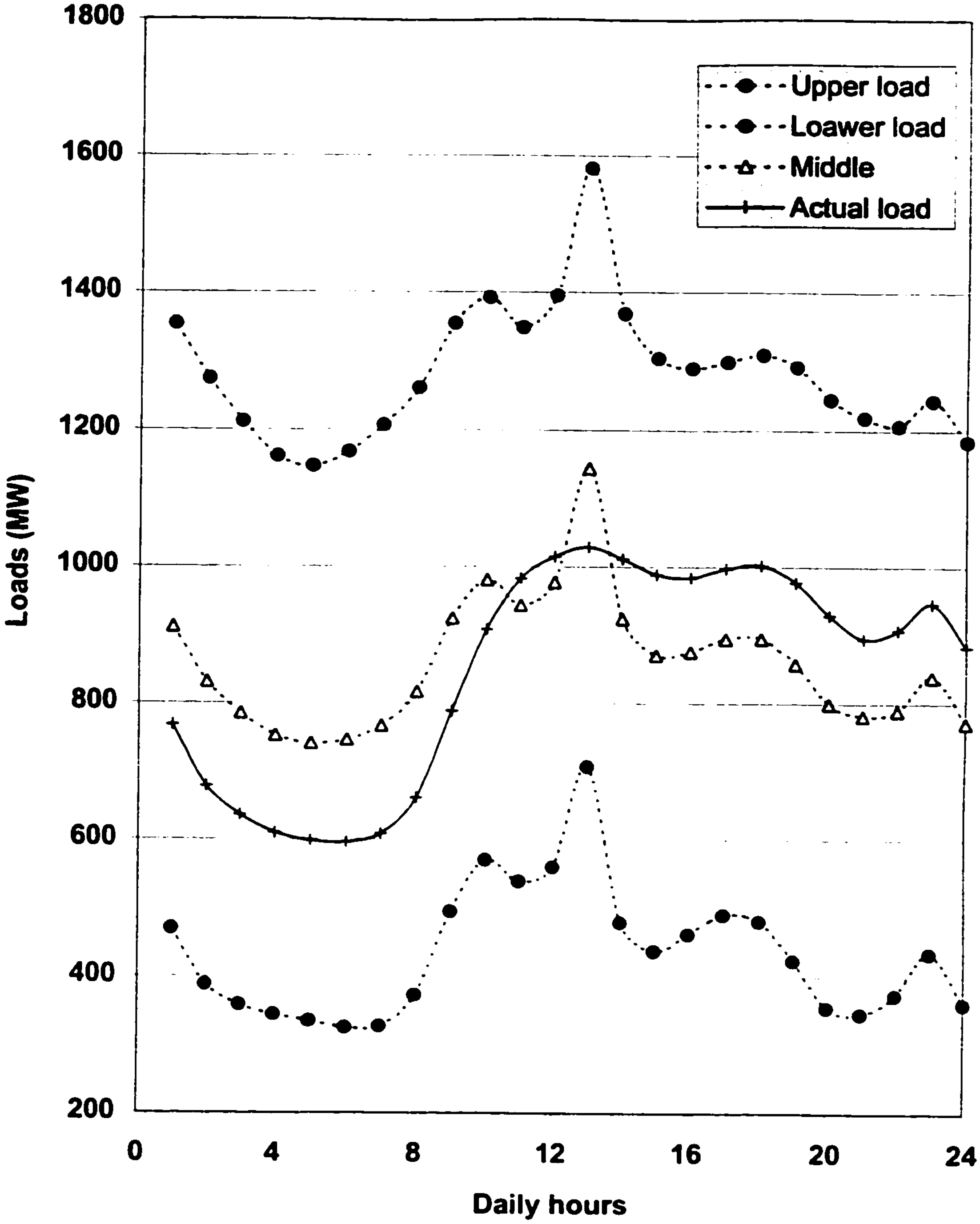


Figure (7.16) Estimated load for a summer weekday, (20 % load deviation), Model B

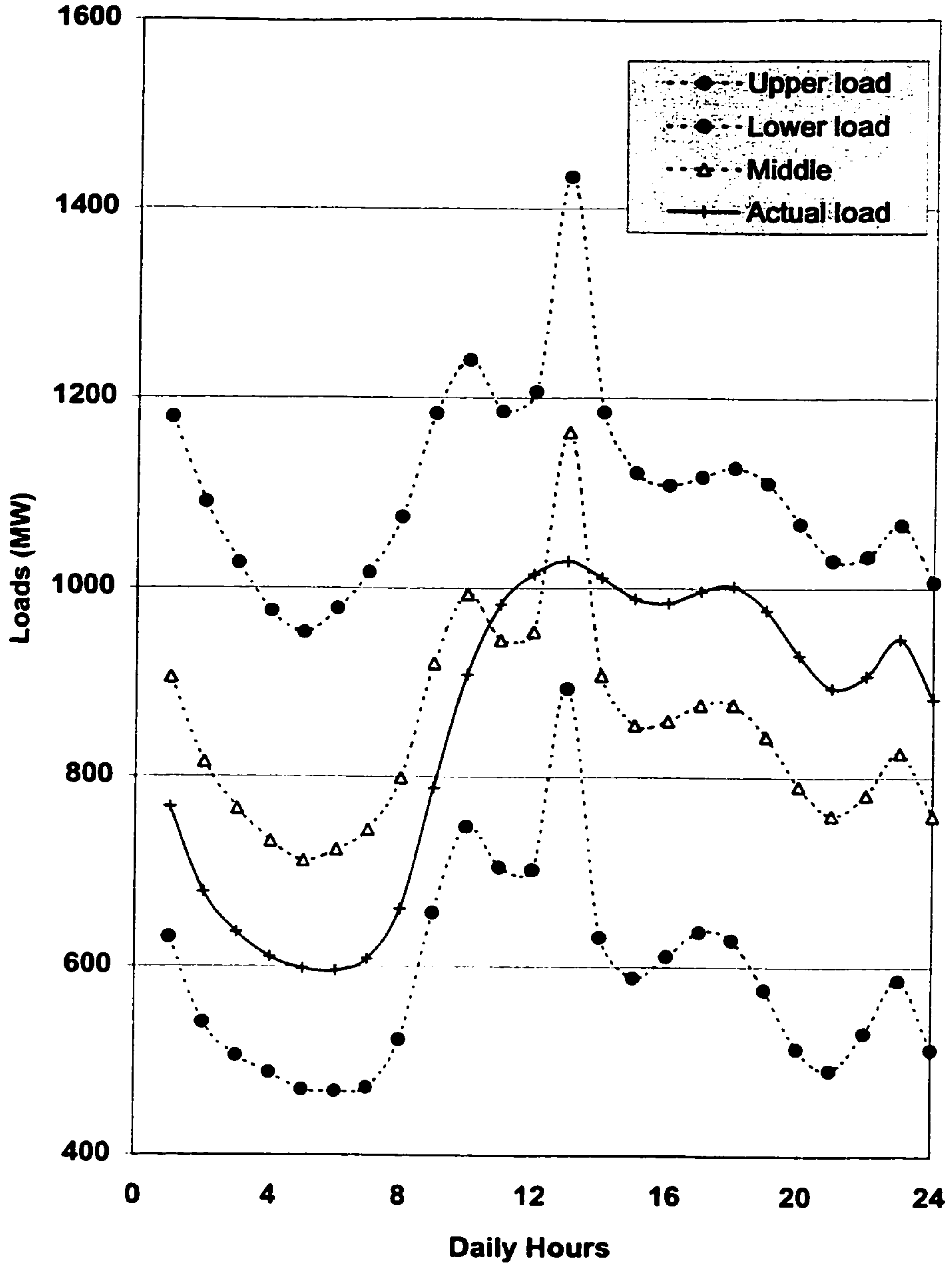


Figure (7. 17) Predicted load for a summer weekday, crisp load, Model B

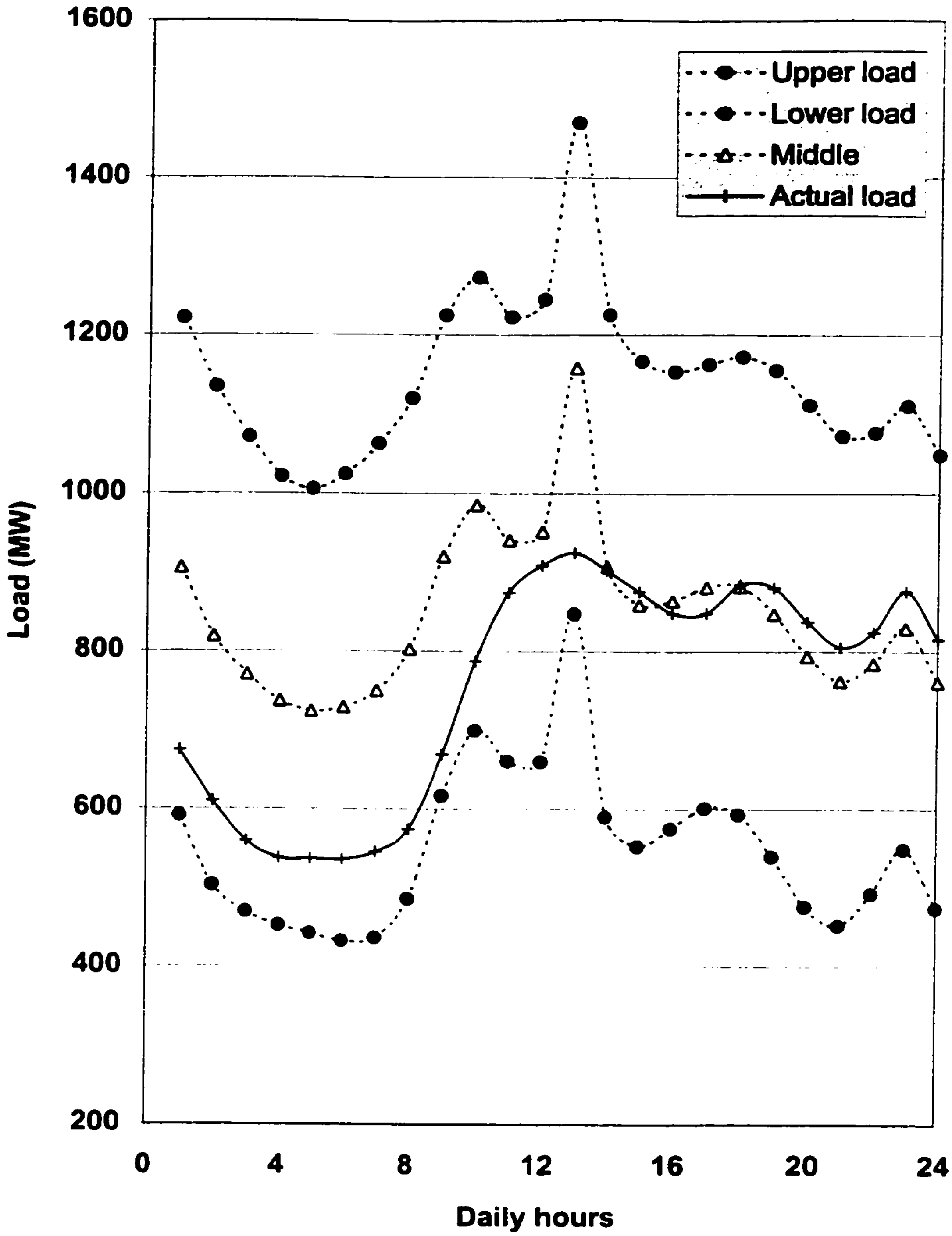


Figure (7.18) Predicted load for a summer weekday, (5% load deviation), Model B

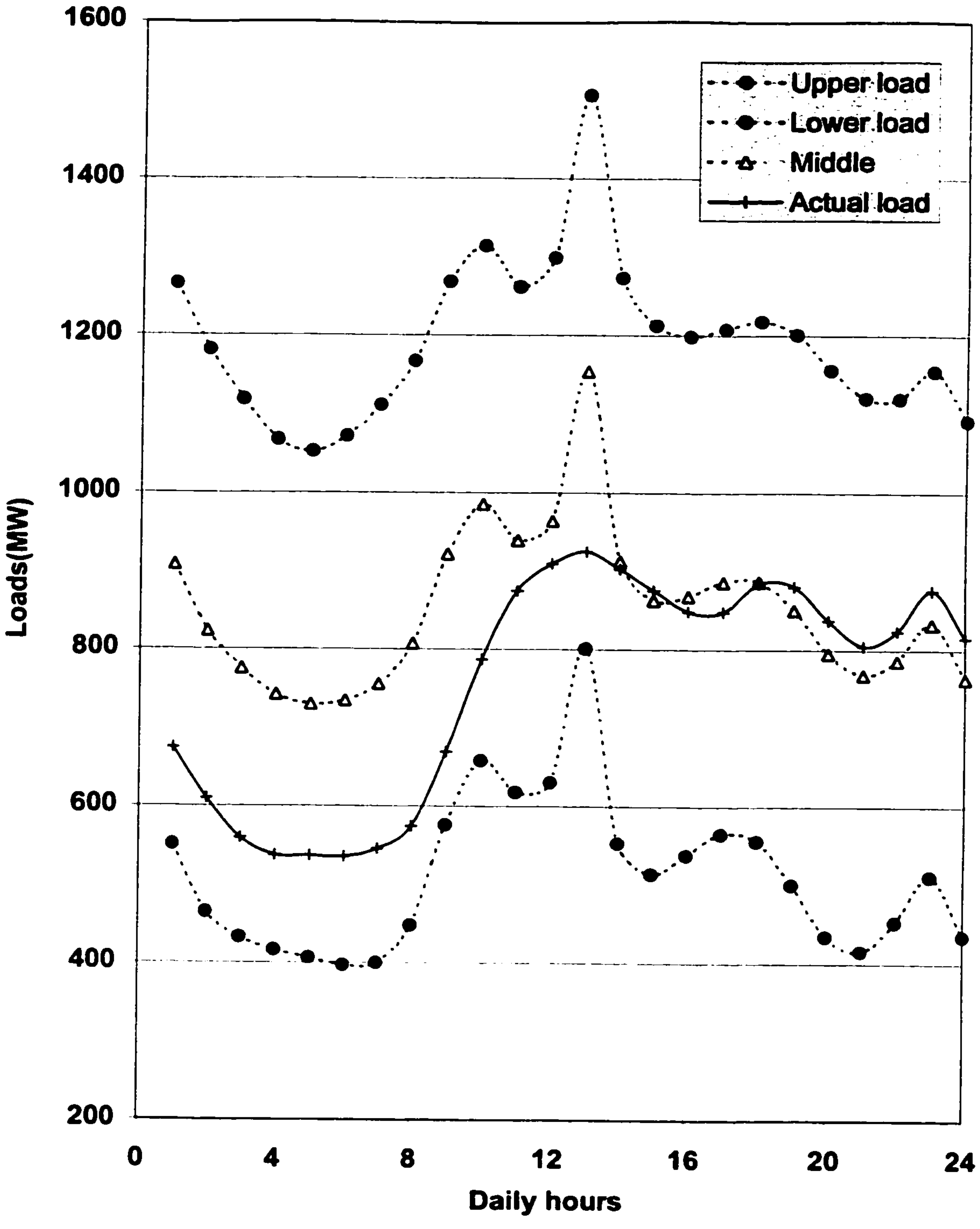


Figure (7.19) Predicted load for a summer weekday, (10 % load deviation), Model B

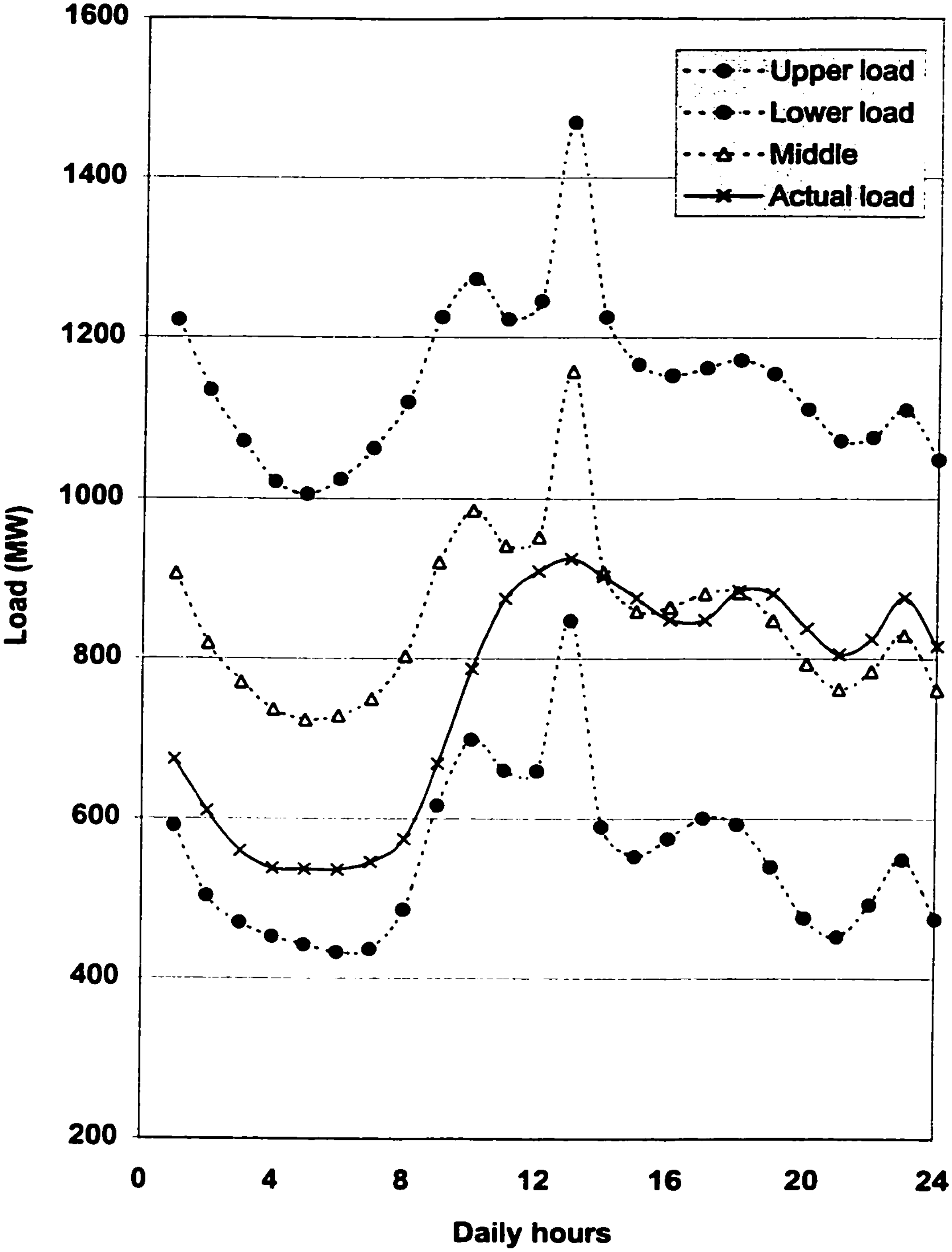


Figure (7.20) Predicted load for a summer weekday, (20% load deviation), Model B

7.4 Fuzzy Load Model C

The developed fuzzy model C in Chapter 5 is tested for summer weekday or weekend days. This model is a hyprid combination of models A and B. A summer weekday load data are used to estimate the fuzzy parameters of the model. These parameters are then used to predict the load power one day ahead. The load deviation that creates fuzziness is changed from 0%, 5 % to 20 %, with a degree of fuzziness of 50 %.

7.4.1 Load Parameters For Model C

Table (7.4) gives the estimated 24 fuzzy parameters, 23 parameters and the base load parameter, at different load deviations. Examining this table reveals the following:

- Most of the load prameters are crisp, since the spreads are zeros. There are three fuzzy parameters, and they are the same parameters in the three cases of the load deviation (A_o, A_4, B_8).
- As the load deviation increases, the spreads of these parameters increase to include the parameter memberships in the solution.
- Large middle and spread values for A_o in the three cases, since A_o is representing the base load.

7.4.2 Load Estimations and Predictions For A Summer Day

The estimated parameters are used to predict the load power 24 hours ahead, for either a weekday or weekend day. Figures (7.21-7.28) give the estimated and predicted loads at the given load deviations. Examining these figures reveals the following:

- At load deviation equals 5% the actual load is greater than the upper limit for two hours only by about 1.7 % and 4.1 %, which is still an acceptable amount. However, if the load deviation is increased to 10 % the actual load does not violate the upper load.
- The estimated load using the fuzzy parameters for all load deviation does not violate neither the upper nor the lower load.

Table (7.4) Fuzzy parameters for a summer day load, Model C

Parameters	Crisp load		5% load deviation		10% load deviation		20% load deviation	
	Middle	Spread	Middle	Spread	Middle	Spread	Middle	Spread
A_0	515.54	46.58	520.62	84.5043	520.385	124.141	529.00	205.698
A_1	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A_2	0.0	0.00	0.408	0.00	2.5126	0.00	3.0646	0.00
A_3	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A_4	12.14	14.60	17.631	19.00	15.132	12.925	15.346	12.618
A_5	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A_6	37.0	0.00	36.372	0.00	36.816	0.00	37.101	0.00
A_7	4.59	0.00	8.1702	0.00	8.097	0.00	5.5347	0.00
A_8	0.0	0.00	0.00	0.00	0.0182	0.00	3.8353	0.00
A_9	33.91	0.00	34.568	0.00	33.06	0.00	31.653	0.00
B_1	31.99	0.00	24.101	0.00	24.0741	0.00	19.783	0.00
B_2	0.47	0.00	0.614	0.00	0.447	0.00	0.451	0.00
B_3	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B_4	13.85	0.00	10.476	0.00	9.667	0.00	7.02	0.00
B_5	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B_6	31.41	0.00	27.631	0.00	25.8138	0.00	21.295	0.00
B_7	52.04	0.00	51.294	0.00	48.9959	0.00	45.185	0.00
B_8	16.86	12.87	16.531	17.918	17.1895	21.881	18.279	21.5500
B_9	20.42	0.00	16.403	0.00	20.160	0.00	23.135	0.00
C_0	16.12	0.00	14.576	0.00	16.620	0.00	15.600	0.00
C_1	0.0	0.00	0.00	0.00	0.000	0.00	0.00	0.00
C_2	1.13	0.00	1.578	0.00	0.2810	0.00	1.955	0.00
C_3	13.39	0.00	12.710	0.00	13.565	0.00	12.066	0.00

- The actual load violates the upper limit load in:
  - Crisp load case, Figure (7.25).
  - 5% load deviation case, Figure (7.26).
  - 10% load deviation case, Figure (7.27).
  - This violation decreases as the load devation increases. For example, in Figure (7.28), the actual load does not violate the upper load, since the load deviation is increased to 20 %, which increases the fuzziniess of the load.
- Since the load varies between the upper and lower values, the estimated parameters can sufficiently be used to predict the load for any day in the week in any season. The load parameters must be updated from weekday, weekend day and from season to the other.

7.4.3 Load Estimation and Prediction For A Winter Day

The results obtained for the winter weekday are reported in the appendices 1 and 2.The same conclusions can be made.

7.5 Conclusion

In this chapter, the fuzzy short term load forecasting problem is solved. The three models developed in chapter 5 are implemented to predict the load. The three models are used to estimate the load power at any day in any season, based on fuzzy optimization rules. The predicted load lies between upper and lower limits. It has been shown that the actual load never violates these limits.

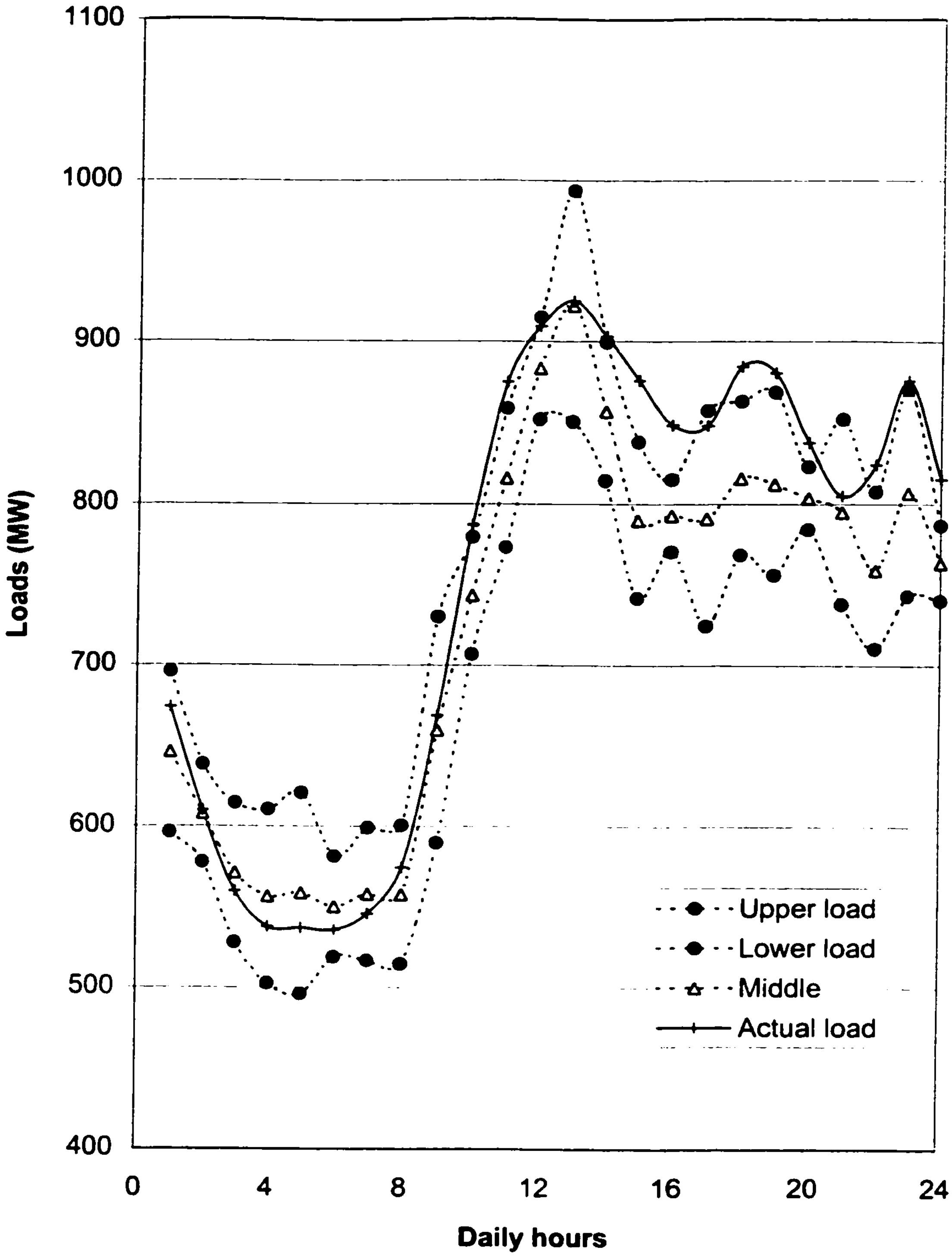


Figure (7.21) Estimated load for a summer day, crisp load, Model C

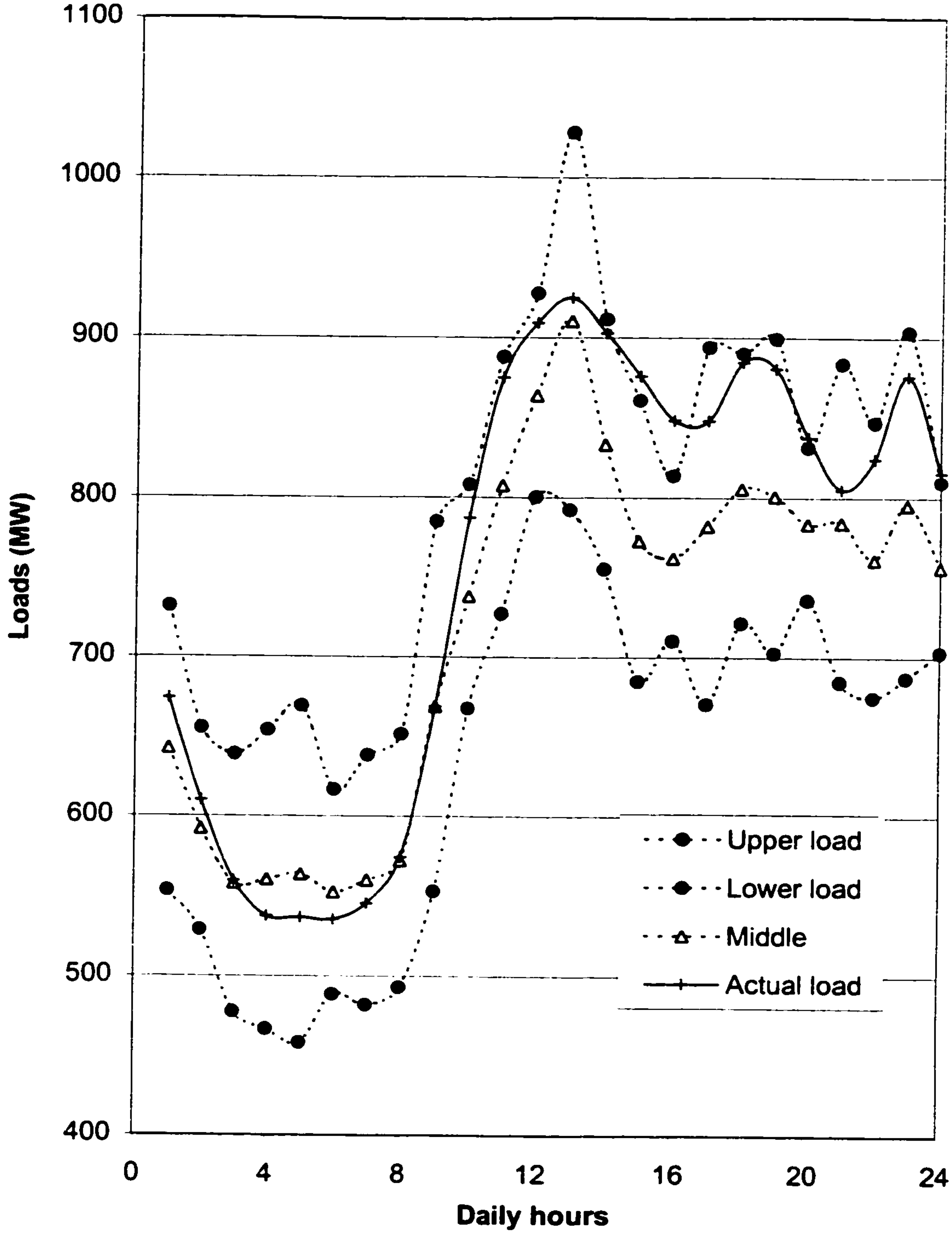


Figure (7.22) Estimated load for a summer day, (5 % load deviation), Model C

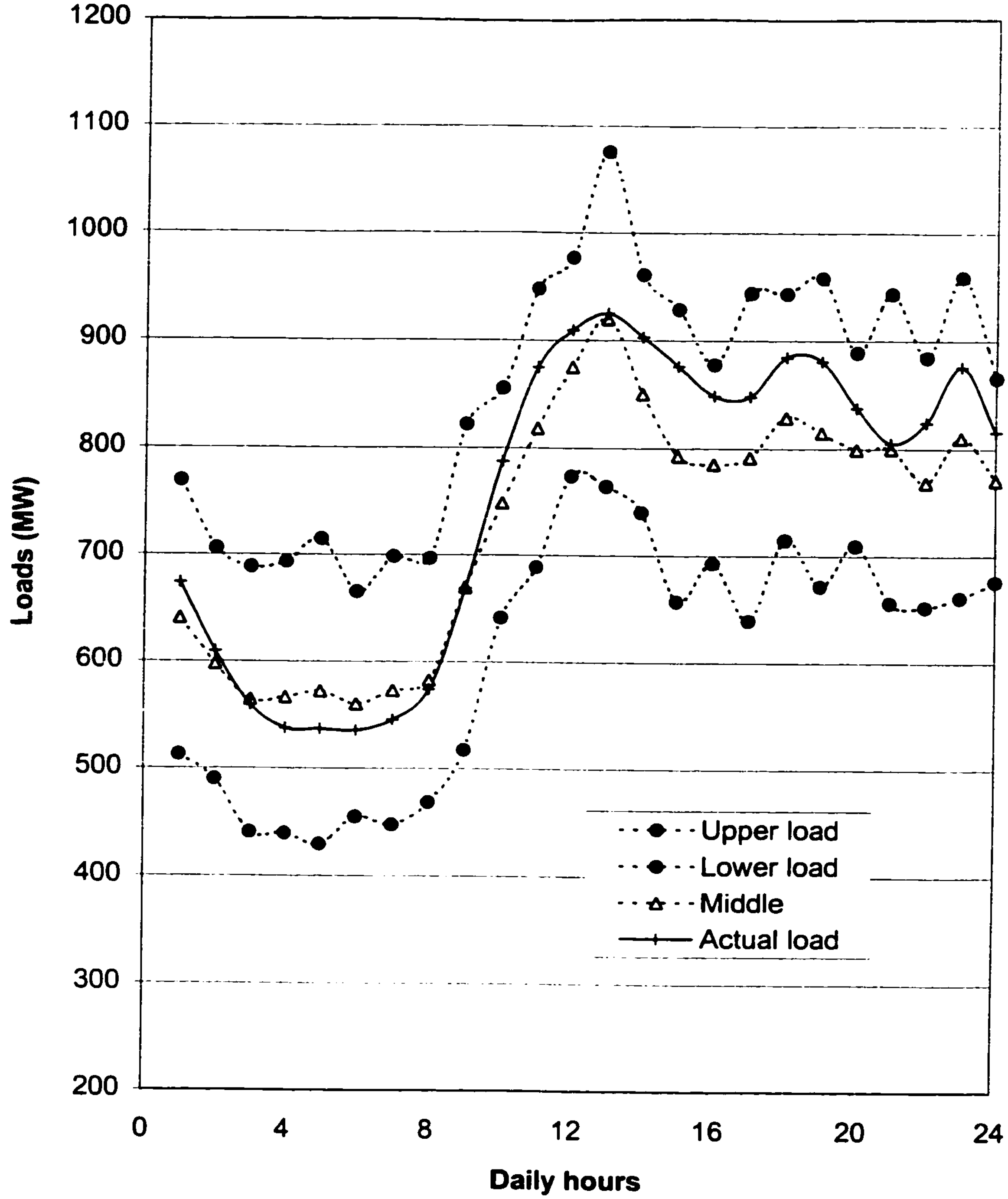


Figure (7.23) Estimated load for a summer day, (10% load deviation), Model C

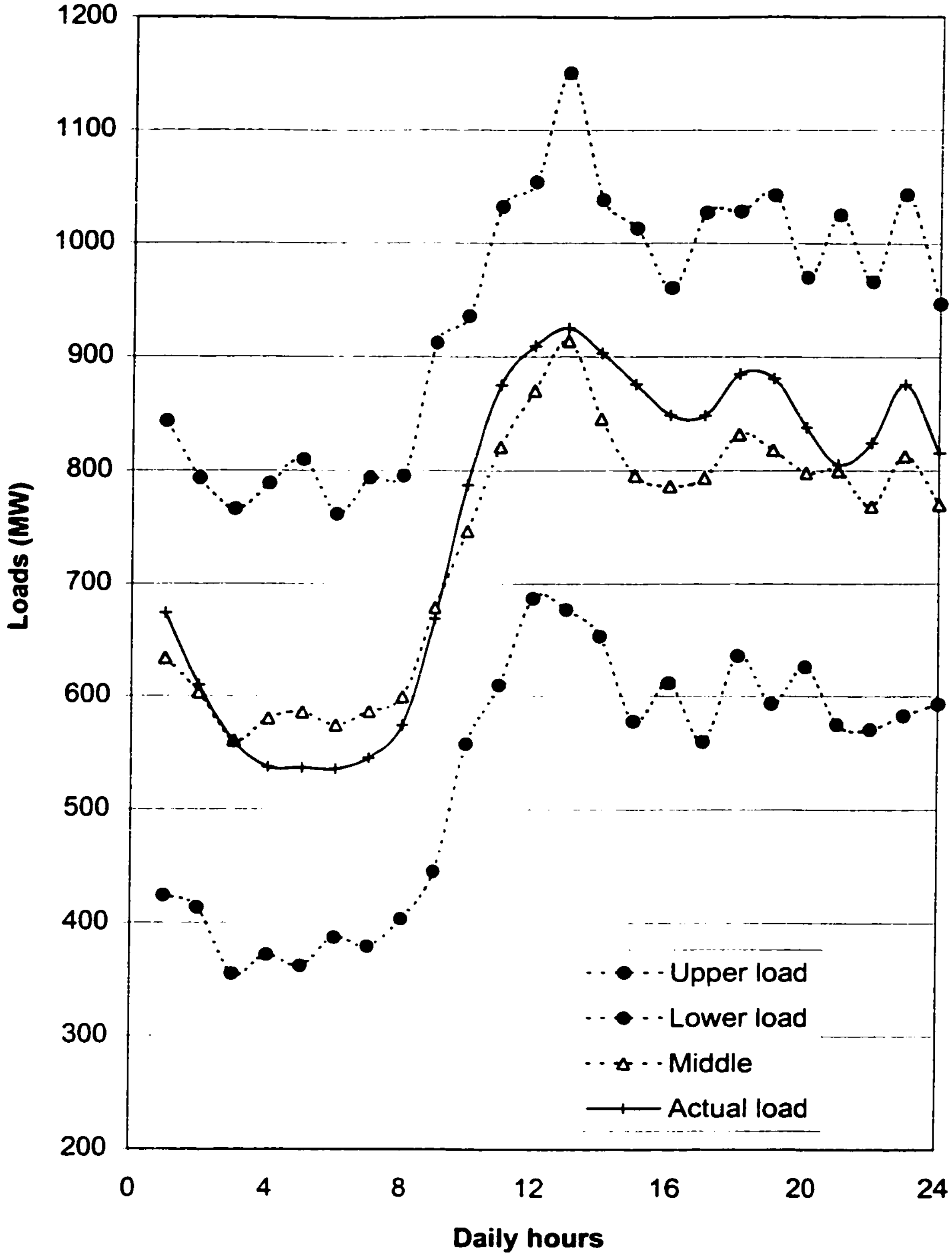


Figure (7.24) Estimated load for a summer day, (20 % load deviation), Model C

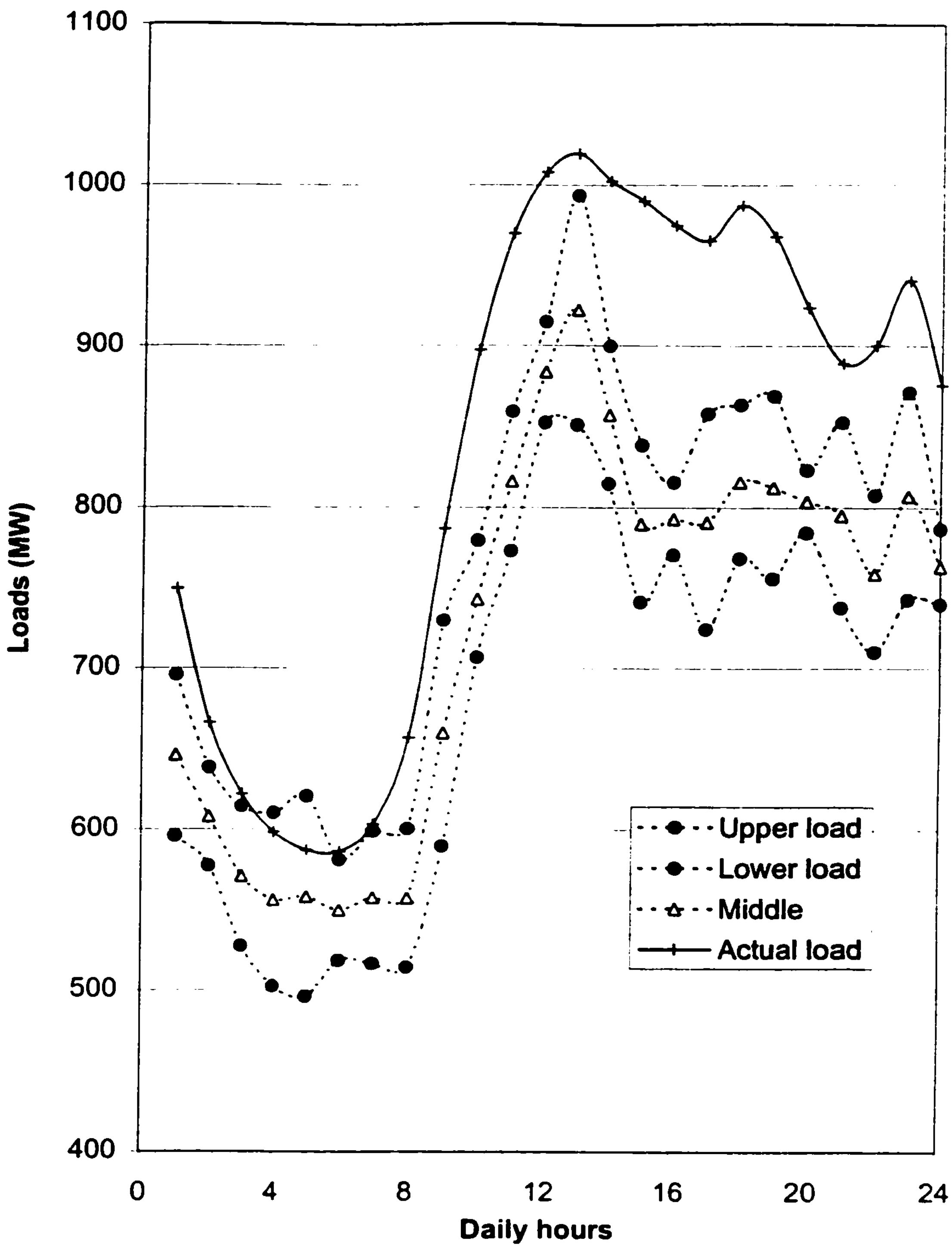


Figure (7.25) Predicted load for a summer day, crisp load, Model C

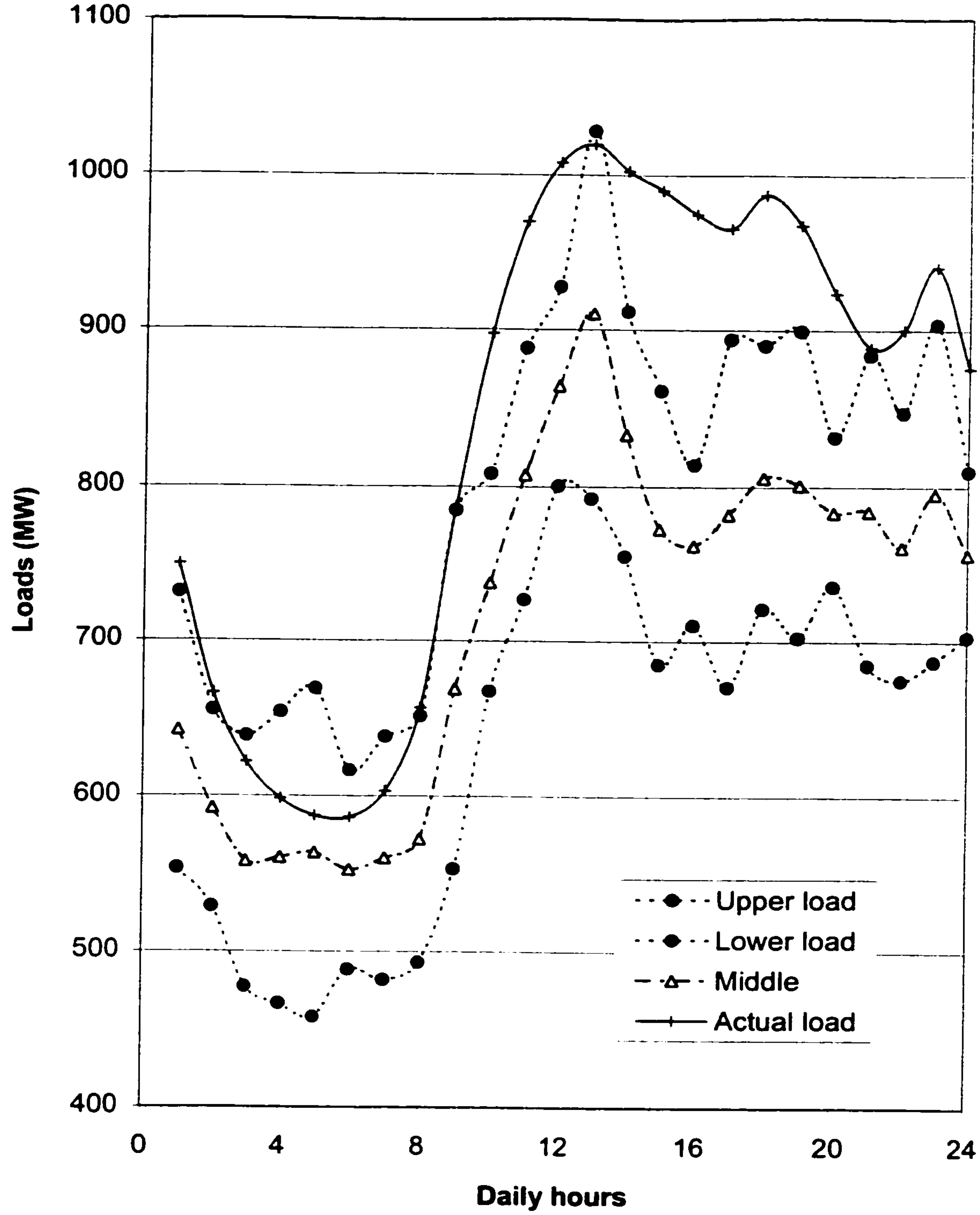


Figure (7.26) Predicted load for a summer day, (5 % load deviation), Model C

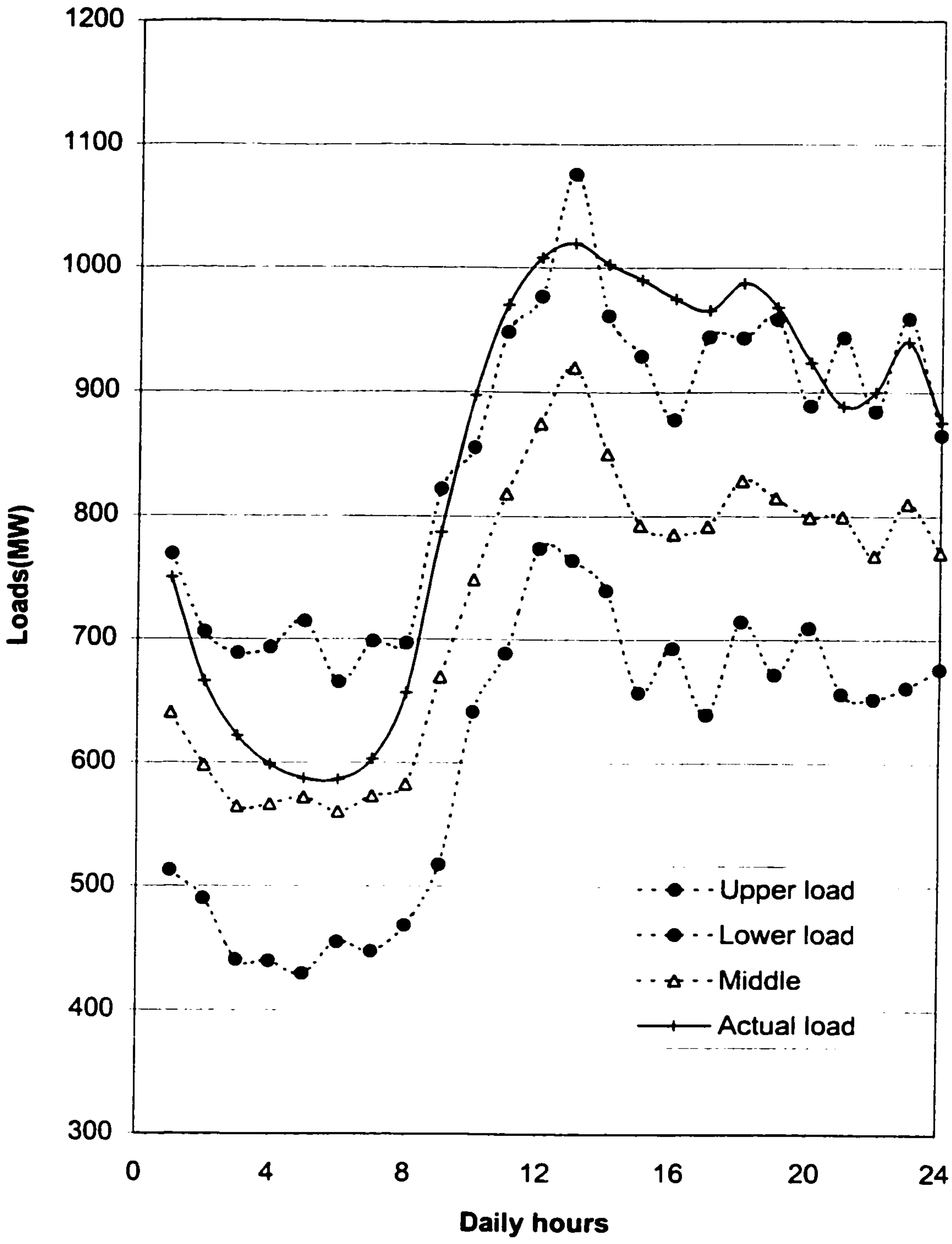


Figure (7.27) Predicted load for a summer day, (10 % load deviation), Model C

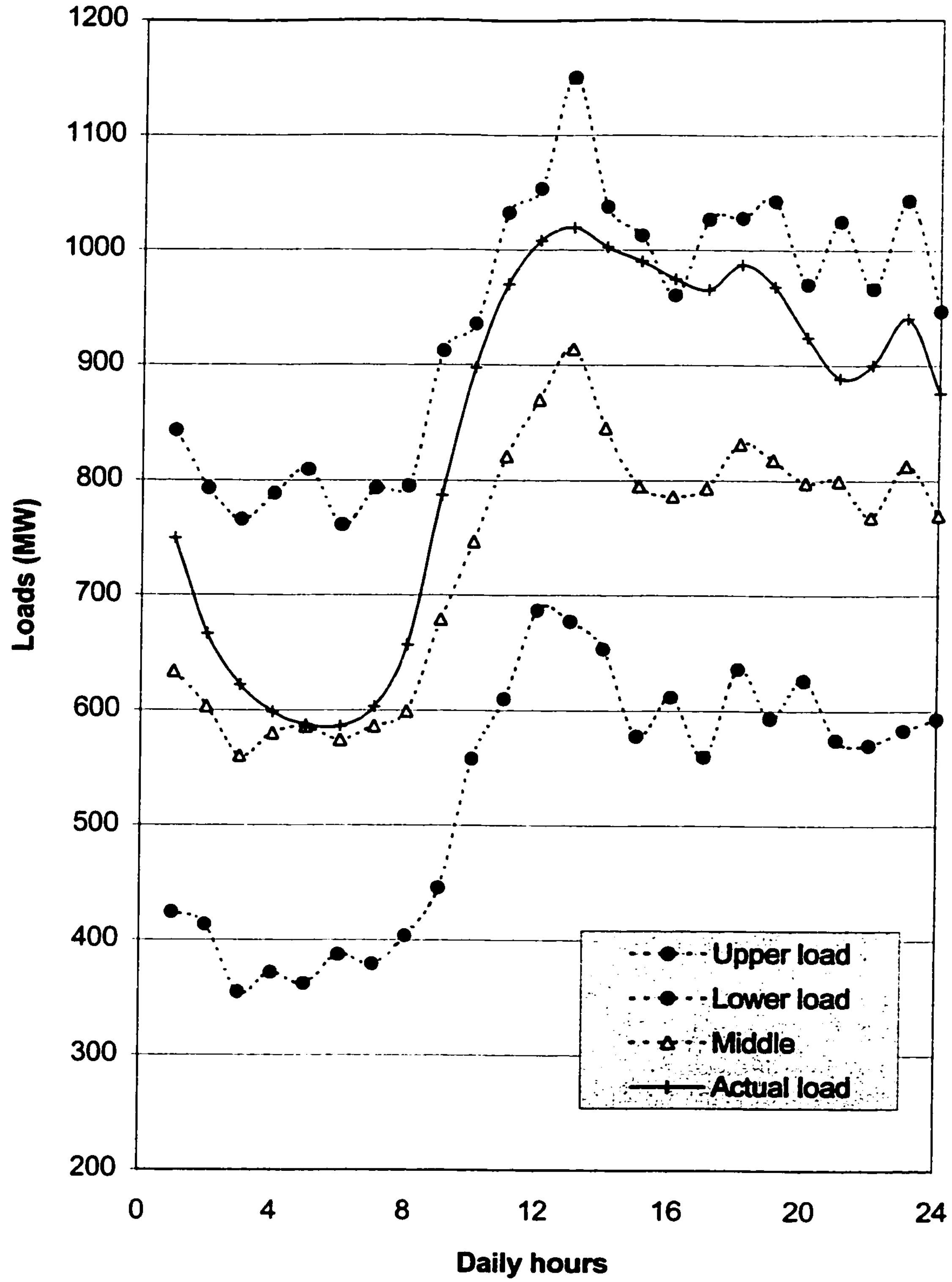


Figure (7.28) Predicted load for a summer day, (20 % load deviation), Model C

Chapter 8
Conclusions and Recommendations For Future Research

8.1 Conclusions

Load forecasting is an important tool in power applications. In this thesis, fuzzy short term load forecasting is formulated and solved. There are many good models used for electric load forecasting. The models proposed in the thesis represent an addition to the existing models in short term electric load forecasting and demonstrate the applicability of fuzzy techniques.

The thesis starts with a discussion of conventional algorithms used in short-term load forecasting. These algorithms are based on least error squares and least absolute value. The theory behind each algorithm is explained.

Three different models are developed and tested in the first part of the thesis. The first model (A) is a regression model that takes into account the weather parameters in summer and winter seasons. The second model (B) is a harmonics based model, which does not account for weather parameters, but considers the parameters as a function of time. Model (B) can be used where variations in weather parameters are not appreciable. Finally, model (C) is created as a hybrid combination of models A and B the parameters of the three models are estimated using the two static estimation algorithm and are used later to predict the load for twenty-four hours ahead. The results obtained are discussed and conclusions are drawn for these models.

In the second part of the thesis new fuzzy models are developed for crisp load power with fuzzy load parameters and for fuzzy load power with fuzzy load parameters. Three fuzzy models (A),(B) and (C) are developed. The fuzzy load model (A) is a fuzzy linear regression model for summer and winter seasons. Model (B) is a harmonic fuzzy model, which does not account for weather parameters. Finally fuzzy load model (C) is a hybrid combination of fuzzy load models (A) and (B). Estimating the fuzzy parameters for the three models turns out to be one of linear optimization. The fuzzy parameters are obtained for the three models. These parameters are used to predict the load as a fuzzy function for twenty-four hours ahead. Prediction results are obtained and presented using data from Nova Scotia power and Environmental Canada.

8.1.1 Static and Fuzzy Results Comparison

It is acknowledged that LES (Least Error Square) and LAV (Least Absolute Value) estimates and predictions deviate from the actual values with errors. Results from static models contain range of errors caused by the type of data, type of models used and how many factors are represented. It is been found that the range of errors is sometimes large. With more data and more representing variables, the results will clearly be improved. LES and LAV are represented here as predicting tools. It is preferred if the error is overpredicted since the system operator will work with a safe tolerance in meeting the consumer demand.

The fuzzy estimates offer a range of values that can be useful to system operators. These estimates are more reliable than static estimates.

8.1.2 Main Contributions

The thesis contributes the following:

1. Three new short-term load-forecasting static models are developed and tested.
2. The hybrid model C is a new innovation by the author of this thesis.
3. A comparison between two static estimation algorithms is performed for three short-term load-forecasting models for crisp power and crisp load parameters. The two estimation algorithms are based on LES (Least Error Squares) and LAV (Least Absolute Value). It has been shown that using static state estimation to predict the load 24 hours ahead may produce large errors in the obtained estimates.

4. Three short-term load forecasting fuzzy models are developed and tested. The testing is made for fuzzy load parameters. The input data are taken to be crisp while the output load powers are tested for three states of fuzziness.
5. It has been shown that the three proposed fuzzy models are suitable and adequate to short-term load forecasting. The output of the fuzzy models is a range of upper and lower values for the predicted load power. This range can give the system operators the ability to run the power system in a more reliable and secure way.

8.2 Suggestions for Future Research

1. During the course of this study, especially the fuzzy load modeling calculation, the degree of fuzziness λ is assumed to be equal 50 %. However, this degree of fuzziness depends on the experience of the working operator in the field of fuzzy systems and fuzzy load forecasting. It is worth while to study the effects of this degree of fuzziness on the load range for the hour in question to obtain the optimal range for the system dispatch.
2. The forecasting of the load power in this thesis is performed off-line, it is worthwhile to develop a dynamic fuzzy load forecaster to predict the load on – line, or at least one hour ahead. This requires the availability of weather data and load history in advance for the hour in question.
3. The weather information used in this thesis was obtained from Environmental Canada. It was recorded for a small part of Nova Scotia Province (Shearwater Airport Halifax), while the demand load was recorded for the whole province. It is worthwhile for future research to study fuzzy load forecasting using the weather information data for the whole province. Better results will be obtained for load forecasting.
4. The membership functions of the fuzzy parameters were assumed to be triangular. An investigation into the validity of this assumption is needed. In particular trapezoidal and Gaussian functions may be compared.

5. It is appropriate to examine the practicality of solving the forecasts and modeling problems for fully fuzzy set of variables.
6. A worthwhile effort would be to examine the combination of neural network in pre-forecasts followed by fuzzy forecasting or other appropriate combination.

References

[1] Mohamed A. Abu-El-Magd and N. K. Sinha, "Short-Term Load Demand Modeling and Forecasting: A Review", IEEE Transactions on Systems, Man and Cybernetics, Vol. 12, No. 3, pp. 370- 382, May/June 1982.

[2] M. L. Willis, A. E. Schauer, J.E.D. Northcote and T.D. Vismor, "Forecasting Distribution System Loads Using Curve Shape Clustering", IEEE Transactions on Power Apparatus and Systems, Vol. 102, No. 4, pp. 893-901, April 1983.

[3] M. Lee Willis, R.W. Powell and D. L. Wall, " Load Transfer Coupling Regression Curve Fitting for Distribution Load Forecasting ", IEEE Transactions on Power Apparatus and Systems, Vol. 103, No. 5, pp. 1070-1076, May 1984.

[4] M. Lee Willis and J.E.D Northcote-Green, "Comparison Tests of Fourteen Distribution Load Forecasting Methods", IEEE Transactions On Power Apparatus and Systems, Vol. 103, No. 6, PP. 1190-1197, June 1984.

[5] S. Rahman and R. Bhatnager, "An Expert System Based Algorithm for Short Term Load Forecast", IEEE Transactions on Power systems, Vol. 3, No. 2, pp.392-399, May 1988.

[6] Q. C Lu, W. M. Grady, M. M. Crawford and G.M. Anderson, "An Adaptive Nonlinear Predictor With Orthogonal Escalator Structure for Short-Term Load Forecasting", IEEE Transactions on Power Systems, Vol. 4, No. 1, pp 158-164, February 1989.

[7] Ibrahim Moghram and Saifur Rahman, "Analysis and Evaluation of FIVE Short-Term Load Forecasting Techniques", IEEE Transactions on Power Systems, Vol. 4, No. 4, pp. 1484-1491, October 1989.

[8] N. F. Hubele and Chuen-Sheng Cheng, "Identification of Seasonal Short-Term Load Forecasting Models Using Statistical Decision Functions", IEEE Transactions on Power Systems, Vol. 5, No. 1, pp. 40-45, February 1990.

[9] M.E. El-Hawary and G..A.N Mbamalu, "Short-Term Power System Load Forecasting Using the Iteratively Reweighted Least Squares Algorithm", Electric Power Systems Research, Vol 19, pp. 11-22, 1990.

[10] S. Rahman, "Formulation and Analysis of a Rule-Based Short-Term Load Forecasting Algorithm", IEEE Proceedings, Vol. 78, No. 5, pp. 805-816, May 1990.

[11] Kun-long Ho, Yuan-Yih Hsu, Chih- Chien Lising and Tssu-Shin Lai, "Short Term Load Forecasting of Taiwan Power System Using a Knowledge Based Expert System", IEEE Transactions on Power Systems, Vol. 5 No. 4, pp. 1214-1221, November 1990.

[12] Alex D. Papalexopoulos and Timothy C. Hesterberg, "A Regression-Based Approach To Short-Term Load Forecasting", IEEE Transactions on Power Systems, Vol. 5 No. 4, pp. 1535-1550, November 1990.

[13] J.H. Park, Y.M. Park and K.Y.Lea, "Composite Modeling for Adaptive Short-Term Load Forecasting", Power IEEE Transactions on Systems, Vol. 6, No. 2, pp. 450-457, May 1991.

[14] Yuan-Yih Msu and Chien-Chuen Yang ,"Design of Artificial Neural Networks for Short-Term Load Forecasting. Part I: Self-Organizing Feature Maps for Day Type Identification Peak Load and Valley Load Forecasting", IEE Proceedings, Vol. 138, No. 5, pp. 407-413, September 1991.

[15] Yuan-Yih Msu and Chien-Chuen Yang "Design of Artificial Neural Networks for Short-Term Load Forecasting. Part II: Multilayer Feedforward Networks for Peak Load and valley Load Forecasting", IEE Proceedings, Vol. 138, No. 5, pp. 414-418, September 1991.

[16] W. M. Grady, L. A. Groce, T. M. Huebner, Q. C. Lu and M. M. Crawford, " Enhancement, Implementation and Performance of an Adaptive Short-Term Load Forecasting Algorithm", IEEE Transaction on Power Systems, Vol. 6, No. 4, pp. 1404-14710, November 1991.

[17] K. Y. Lee, Y. T. Cha and J.H. Park, "Short-Term Load Forecasting Using an Artificial Neural Network", IEEE Transactions on Power System, Vol. 7, No. 1, pp124-132, February 1992.

[18] Kun-Long Ho, Yuan-Yih Msu and chien-Chuen Yang, "Short-Term Load Forecasting Using a Multilayer Neural Network With an Adaptive Learning Algorithm", IEEE Transactions on Power Systems, Vol. 7, No. 1, pp. 141-149, February 1992.

[19] T. M. Peng, N. F. Huebele and G.G. Karady, "Advancement in The Application of Neural Networks for Short-Term Load Forecasting", IEEE Transactions on Power Systems, Vol. 7, No. 1, pp. 250-257, February 1992.

[20] S. Rahman and Irislav Drezga, "Identification of a Standard for Comparing Short-Term Load Forecasting Techniques", Electric Power System Research, Vol. 25, pp. 149-158, 1992.

[21] Shin Tzo Chen, David C. Yu and A. R. Moghaddamjo, "Weather Sensitive Short-Term Load Forecasting Using Non-fully Connected Artificial Neural Network", IEEE Transactions on Power Systems, Vol. 7, No. 3, pp. 1098-1105, August 1992.

[22] Y. -Y. Hsu and K. L. Ho, "Fuzzy Expert Systems: An Application to Short-Term Load Forecasting", IEE Proceedings- C, Vol. 139, No. 6, pp. 471-477. November 1992.

[23] C. N. Lu, H.T. Wu and S. Vemuri, "Neural Network Based Short Term Load Forecasting", IEEE Transactions on Power systems, Vol. 8, No. 1, pp. 336-342, February 1993.

[24] S. Rahman and O. Hazim, "A Generalized Knowledge Based Short-Term Load Forecasting Technique", IEEE Transactions on Power Systems, Vol. 8, No. 2, pp. 508-514, May 1993.

[25] Jiann- Liang Chen, Ronlon Tsai and Sheau-Shing Liang, "A Distributed Problem Solving System for Short-Term Load Forecasting", Electric Power Systems Research, Vol. 26, pp. 219-224, 1993.

[26] M. Djukanovic, B. Babic, D. J. Sobajic and Y. -H. Pao, "Unsupervised/ Supervised Learning Concept for 24-Hour Load Forecasting", IEEE Proceedings-C, Vol. 140, No. 4, pp.311-318, July 1993.

[27] Azzam-ul-Asar and James R. McDonald, "A Specification of Neural Network Applications in the Load Forecasting Problem", IEEE Transactions on Control Systems Technology, Vol. 2, No. 2, pp. 135-141, June 1994.

[28] D. K. Ranaweara, N. F. Hubele and A.D. Pagalexopoules, "Application of Radical Basis Function Neural Network Model for Short-Term Load Forecasting", IEE Proceedings-Generation, Transmission and Distribution, Vol. 142, No. 1, pp.45-50, January 1995.

[29] O. Mohamed, D. Park, R. Merchant, T. Dinh, C. Tong, A. Azeem, J. Farah and C. Drake, "Practical Experiences with an Adaptive Neural Network Short-Term Load Forecasting System", IEEE Transactions on Power Systems, Vol. 10, No. 1, pp. 254-265, February 1995.

[30] Jiann-Fuh Chen, Wei-Ming Wang and Chao-Ming Huang, "Analysis of an Adaptive Time-Series Autoregressive Moving-Average (ARMA) Model for Short-Term Load Forecasting", Electric Power Systems Research, Vol.34, pp. 187-196, 1995.

[31] Dipti Srinivasan, A. C. Liew and C. S. Chang, "Applications of Fuzzy Systems in Power systems", Electric Power Systems Research Journal, Vol. 35, pp. 39-43, 1995.

[32] A. A. El-Keib, X. Ma and H. Ma, "Advancement of Statistical Based Modeling Techniques for Short Term Load Forecasting", Electric Power Systems Research Journal,Vol. 35, pp. 51-58, 1995.

[33] Kwang-Ho Kim, Jong-Keun Park,Kab-Ju Hwang and Sung-Hak Kim, "Implementation of Hybird Short-Term Load Forecasting System Using Artificial Neural Networks and Fuzzy Expert Systems", IEEE Transactions on Power Systems, Vol. 10, No. 3, pp. 1534-1539, August 1995.

[34] A. G. Bakirtzis, J. B. Theocharis, S. J. Kiartzis and K. J. Satsios, "Short Term Load Forecasting Using Fuzzy Neural Networks", IEEE Transactions on Power Systems, Vol. 10, No. 3, pp. 1518-1524, August 1995.

[35] J. A. Momoh and K. Tomsovic, "Overview and Literature Survey of Fuzzy Set Theory in Power Systems", IEEE Transactions on Power Systems, Vol. 10, No. 3, pp. 1676-1690, August 1995.

[36] Hiroyuki Mori and Hidenori Kobayashi, "Optimal Fuzzy Inference for Short Term Load Forecasting", IEEE Transactions on Power Systems, Vol. 11, No. 1, pp. 390-396, February 1996.

[37] Hong-Tzer Yang, Chao-Ming Haung and Ching-Lien Haung, "Identifications of ARMAX Model for Short Term Load Forecasting: An Evolutionary Programming Approach", IEEE Transactions on Power systems, Vol. 11, No. 1, pp. 403-408, February 1996.

[38] A. G. Bakirtzis, V. Petrldls, S. J. Klartzls,M. C. Alexladls and A. H. Malssis, "A Neural Network Short Term Load Forecasting Model for The Greek Power System", IEEE Transactions on Power Systems, Vol. 11,No. 2, pp. 858-863, May 1996.

[39] K.Liu, S. Subbaratan, R. R. Shoults, M. T. Manry, C. Kwan, F. L. Lewis and J. Naccarino, "Comparison of Very Short-Term Load Forecasting Techniques", IEEE Transactions on Power Systems, Vol. 11, No. 2, pp. 877-882, May 1996.

[40] T. W. S. Chow and C.T. Leung, " Nonlinear Autoregressive Integrated Neural Network Model for Short Term Load Forecasting", IEE Proceedings-Gener. Transm. Distrib. , Vol. 143, No. 5, pp. 500-506, September 1996.

[41] T. W. S. Chow and C.T. Leung, "Neural Network Based Short-Term Load Forecasting Using Weather Compensation", IEEE Transactions on Power Systems, Vol. 11, No. 4, pp. 1736-1742, November1996.

[42] R. Lamedica, A. Prudenzi, M. Sforna, M. Caciotta and V. Orsolini Cencellli, "A Neural Network Based Technique for Short-Term Forecasting of Anomalous Load Periods", IEEE Transactions on Power Systems, Vol. 11, No. 4, pp. 1749-1756, November 1996.

[43] S. Sargunaraj, D.P. Sen Gupta and S. Devi, "Short-Term Load Forecasting for Demand Side Management", IEE Proceedings. Gener. Transm. Distrib. Vol. 144, No. 1, pp. 68-74, January 1997.

[44] S.A. Soliman, S. Persaud, K. EL-Nagar and M. E.EL-Hawary, "Application of Least Absolute Value Parameter Estimation Based on Linear Programming to Short-Term Load Forecasting", Electrical Power and Energy Systems, Vol. 19, No. 3, pp. 209-216, 1997.

[45] S. R. Huang, "Short-Term Load Forecasting Using Threshold Autoregressive Models", Online no. 19971144, IEE Proceedings, 1997.

[46] O. Hyde and P. F. Hodnett, "An Adaptive Automated Procedure for Short-Term Electricity Load Forecasting", IEEE Transactions on Power Systems, Vol.12, No. 1, pp. 84-94, February 1997.

[47] P. K. Dash, H. P. Satpathy, A.C. Liew and S. Rahman, "A Real-Time Short-Term Load Forecasting System Using Functional Link Network", IEEE Transactions on Power Systems, Vol. 12, pp. 675-680, May 1997.

[48] Alireza Khotanzad, Reza Afkhami-Rohani, Tsun-Liang Lu, Alireza Abaye, Malcom Davis and Dominic J. Maratukulam, " ANNSTLF-A Neural- Network Basic Electric Load Forecasting System", IEEE Transactions on Neural Networks, Vol. 8, No. 4, pp. 835-846, July 1997.

[49] M. Hisham Choueiki, Clark A.Mount-Campbell and Stanley C. Ahalt, "Building a Quasi Optimal Neural Network to Solve Short-Term Load Forecasting Problem", IEEE Transactions on Power Systems, Vol. 12, No. 4, pp. 1432-1439, November 1997.

[50] A. S. Al-Fuhaid, M. A. EL-Sayed and M. S. Mahmoud, "Cascaded Artificial Neural Networks for Short-Term Load Forecasting", IEEE Transactions on Power Systems, Vol. 12, No. 4, pp. 1524-1529, November 1997.

[51] M. Hisham Choueiki, Clark A. Mount-Campbell and Stanley C. Ahalt, "Implementing a Weighted Least Squares Procedure in Training a Neural Network to Solve the Short-Term Load Forecasting Problem", IEEE Transactions on Power Systems, Vol. 12, No. 4, pp. 1689-1694, November 1997.

[52] J. Vermaak and E. C. Botha, "Recurrent Neural Networks for Short-Term Load Forecasting", IEEE Transactions on Power Systems, Vol.13, No.1, pp.126-132, February 1998.

[53] Hong-Tzer Yang and Chao-Ming Haung, "A New Short-Term Load Forecasting Approach Using Self-Organizing Fuzzy ARMAX Models", IEEE Transactions on Power Systems, Vol.13, No.1, pp. 217-225, February 1998.

[54] S. E. Papadakis, J. B. Theocharis, S. J. Kiartzis and A. G. Bakertzis, "A Novel Approach to Short-Term Load Forecasting Using Fuzzy Neural Networks", IEEE Transactions on Power Systems, Vol.13, No.2, pp. 480-492, May 1998.

[55] W. Charytoniuk, M. S. Chen and P. Van Olinda, "Nonparametric Regression Based Short-Term Load Forecasting", IEEE Transactions on Power Systems, Vol.13, No.3, pp. 725-730, August 1998.

[56] I. Drezga and S.Rahman, "Input Variable Selection for ANN-Based Short-Term Load Forecasting", IEEE Transactions on Power system, Vol.13, No.4, pp.1238-1244, November 1998.

[57] M. Daneshdoost, M. Lotfalian, G. Bumroonggit and J. P. Ngoy, "Neural Network with Fuzzy Set-Based Classification for Short-Term Load Forecasting", IEEE Transactions on Power Systems, Vol.13, No.4, pp. 1386-1391, November 1998.

[58] Alireza Khotanzed, Resa Afkhami-Rohani and Dominic Maratukulam, "ANNSTLF-Artificial Neural Network Short-Term Load Forecasting-Generation Three", IEEE Transactions on Power Systems, Vol.13, No.4, pp. 1413-1422, November 1998.

[59] Andrew P. Douglas, Arthur M. Breipohl, Fred N.Lee and Rambabu Adapa, "The Impacts of Temperature Forecast Uncertainty on Bayesian Load Forecasting", IEEE Transactions on Power Systems, Vol.13, No.4, pp.1507-1513, November1998.

[60] Raj Aggarwal and Yonghua Song, "Artificial Neural Networks In Power Systems-Part 3: Examples of Applications in Power Systems", Tutorial: ANNs in Power Systems, Power Engineering Journal, pp. 279-287, December 1998.

[61] P.A. Mastorocostas, J.B. Theocharis and A.G. Bakirtzis, "Fuzzy Modeling for Short-Term Load Forecasting Using the Orthogonal Least Squares Method", IEEE Transactions on Power Systems, Vol.14, No.1, pp. 29-36, February 1999.

[62] Hyeonjoonj Yoo and Russell L. Pimmel, " Short-Term Load Forecasting Using a Self-Supervised Adaptive Neural Network", IEEE Transactions on Power Systems, Vol.14, No.2, pp. 779-784, May 1999.

[63] G. S. Christensen, A. H. Rouhi and S. A .Soliman, "A New Technique for the Unconstrained and Constrained Linear LAV Parameter Estimation", Canadian Journal of Electrical & Computer Engineering. Vol. 14, No. 1, PP. 24-30, 1989.

[64] S. A. Soliman and G. S. Christensen, "A New Technique for Curve Fitting Based on Weight Least Absolute Value Estimation", Journal of Optimization Theory and Application, JOTA, Vol. 62, No. 2, PP. 281-299, 1989.

[65] S. A. Soliman and G. S. Christensen and A. Rouhi, "A New Technique for Curve Fitting Based on Minimum Absolute Deviation", Computational Statistics and Data Analysis, Vol. 6, No. 4, PP. 341-351, 1988.

[66] E. J. Schlossmacher, " An Iterative Technique for Absolute Deviations Curve Fitting", Journal of American Statistical Association., Vol. 68, No. 344, PP. 857-869, 1973.

[67] P. D. Robers and A. Ben-Israel, "An Iterative Programming Algorithm for Discrete Linear L_1 Approximation Problems", Journal of Approximation Theory, Vol. 2, PP. 323-336, 1969.

[68] S. A. Soliman and G. S. Christensen and M. Y. Mohamed, "Power System State Estimation Based on LAV", Control and Dynamic Systems", "Analysis and Control System Techniques for Electric Power Systems", Edits by C.T. Leondes", Vol. 44, No. 4, Academic Press, 1991.

[69] I. Barrodale and F.DK. Roberts, "An Improved Algorithm for Direct L_1 Linear Approximation", SIAM Journal of Numerical Analysis, Vol. 10, No. 5, PP. 839-849, 1973.

[70] V. A. Sposito and M. L. Hand, "Using an Approximate L_1 Estimator" Communications in Statistics, Simulation and Computation, Vol. B6 (3), PP. 236-268, 1977.

[71] I. Barrodale and F.DK Roberts, " An Efficient Algorithm for Discrete L_1 Linear Approximation with Linear Constraints", SIAM Journal of Numerical Analysis, Vol. 15, No. 3, PP. 603-611, 1978.

[72] G. F. Mc Cormick and V. A. Sposito, "Using the L_2-estimator in L_1-estimation", SIAM Journal of Numerical Analysis, Vol. 13, No. 3, PP. 337-343, 1976.

[73] W.D. Fisher, "A Note on Curve Fitting with Minimum Deviations by Linear Programming", Journal of American Statistical Association, Vol. 56, PP. 359-362, 1961.

[74] I. Barrodale and A. Young, "Algorithms for Best L_1 and L_∞ Linear Approximations on a Discrete Set", Numerische Mathematik, Vol. 8, PP. 295-306, 1966.

[75] R. F. Stengel, Stochastic Optimal Control: Theory & Application, John Wiley & Sons, Inc. New York, 1986.

[76] A. J. Wood and B. F. Wollenberg, Power Generation, Operation and Control, John Wiley and Sons, New York, 1984.

[77] T. J. Ross, Fuzzy Logic with Engineering Applications, McGraw-Hill, Inc. New York, 1995.

[78] Mohamed E. El-Hawary, Electric Power Applications of Fuzzy Systems, IEEE Press, 1998.

[79] J. Nazarka and W. Zalewski, " An Application of the Fuzzy Regression Analysis to the Electrical Load Estimation ", Electrotechnical Conference MELECON'96, pp. 1563-1566, Vol.3, IEEE Catalog #96CH35884, 13-16 May 1996.

[80] H. Tanaka, S. Uejima, and K, Asai, "Linear Regression Analysis with Fuzzy Model ", IEEE Transmission On System Man, and Cyber., Vol. SMC-12, vol. 6, pp.903-907, 1982.

[81] P. T. Chang and E. S. Lee, "Fuzzy Least Absolute Deviations Regression Based on the Ranking of Fuzzy Numbers", IEEE World Congress on Fuzzy Systems, IEEE Proceeding, pp.1365-1369, Vol.2, 1994.

[82] J. Watada and Y. Yabuchi, "Fuzzy Robust Regression Analysis", IEEE World Congress on Fuzzy Systems, IEEE Proceeding, pp. 1370-1376, Vol. 2, 1994.

[83] R Alex, and P.Z. Wang, "A New Resolution of Fuzzy Regression Analysis", IEEE International Conference on Systems, Man, and Cybernetics, Vol.2, pp.2019-2021, 1998.

[84] H. Ishibuchi, and M. Nii, " Fuzzy Regression Analysis by Neural Networks with Non-Symmetric Fuzzy Number Weights", IEEE International Conference on Neural Networks, Vol. 2, pp. 1191-1196, 1996.

[85] H. Ishibuchi, and M. Nii, "Fuzzy Regression Analysis with Non-Symmetric Fuzzy Number Coefficients and Its Neural Network Implementation" Proceedings of the 5th IEEE International Conference on Fuzzy Systems, pp. 318-324, Vol.1, 1996.

[86] S. Ghoshray, "Fuzzy Linear Regression Analysis by Symmetric Triangular Fuzzy Number Coefficients", Proceedings of IEEE International Conference On Intelligent Engineering Systems, pp. 307-313, 1997.

[87] Miin-Shen Yang and Cheng-Msiu Ko, "On Cluster-Wise Fuzzy Regression Analysis", IEEE Transactions on Systems, Man and Cybernetics-Part B:Cybernetics, Vol. 27, No. 1, pp. 1-13, February 1997.

[88] Hideo Tanaka and Hackwan Lee, "Interval Regression Analysis by Quadratic Programming Approach", IEEE Transactions on Fuzzy Systems, Vol. 6, No. 4, pp. 473-481, November 1998.

[89] C. J. Huang, C. E. Lin and C. L. Haung, "Fuzzy Approach for Generator Maintenance Scheduling", Electric Power System Research, Vol. 24, pp. 31-38, 1992.

[90] I.M. Altas and A. M. Sharaf, "A Fuzzy Logic Power Tracking Controller for a Photovoltic Energy Conversion Scheme", Electric Power systems Research, Vol.25, pp. 227-238, 1992.

[91] C.S. Chen and J.N. Sheen, "Applying Fuzzy Mathematics to the Evaluation of Avoided Cost for a Load Management Program", Electric Power Systems Research, Vol. 26, pp. 117-125, 1993.

[92] Nikola Rajakovic and Slobodem Ruzic, "Sensitivity Analysis of an Optimal Short Term Hydro-Thermal Schedule", IEEE Transactions on Power Systems, Vol. 8, No. 3, pp. 1235-1241, August 1993.

[93] Rasool Kenarangui and Alireza Seifi, " Fuzzy Power Flow Analysis", Electric Power Systems Research, Vol. 29, pp. 105-109, 1994.

[94] J. Y. Fan and J.D. McDonald, "A Real-Time Implementation of Short-Term Load Forecasting for Distribution Power Systems", IEEE Transactions On Power Systems, Vol. 9, No. 2, pp. 988-944, May 1994.

Appendix 1

Winter Tables: Fuzzy Case

Model A:

Table (P1.1) Estimated load for a winter weekday, (20 % load deviation), Model A

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	735.9	1315.764	786.363	256.962
2	650.6	1336.382	806.9808	277.5798
3	613.1	1328.587	799.1863	269.7853
4	599.6	1333.742	804.3407	274.9397
5	604.8	1338.393	808.9922	279.5912
6	617.1	1349.205	819.804	290.403
7	635.1	1360.268	830.8672	301.4662
8	731.5	1354.234	824.8327	295.4317
9	915.8	1358.382	828.9814	299.5804
10	1001.8	1376.737	847.3363	317.9353
11	1013	1379	849.5992	320.1982
12	1014.6	1380.76	851.3592	321.9582
13	1020.9	1397.229	867.8283	338.4273
14	995.1	1406.91	877.5085	348.1075
15	979.7	1436.202	906.8009	377.3999
16	965.5	1454.934	925.5329	396.1319
17	975.1	1452.545	923.1443	393.7433
18	1029.7	1443.242	913.8412	384.4402
19	1024.8	1453.048	923.6472	394.2462
20	968.3	1438.842	909.441	380.04
21	955.2	1452.671	923.2699	393.8689
22	960	1491.015	961.614	432.213
23	950.7	1511.884	982.4832	453.0822
24	858.3	1566.572	1037.171	507.7695

Table (P1.2) Predicted load for a winter weekday, (20% load deviation), Model A

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	748.8	1433.813	904.4123	375.0113
2	655.9	1434.945	905.5437	376.1427
3	621.5	1438.716	909.3153	379.9143
4	606.2	1434.693	905.2924	375.8914
5	604.1	1433.31	903.9094	374.5084
6	606.6	1430.419	901.0179	371.6169
7	625	1437.208	907.8066	378.4056
8	723.9	1432.305	902.9037	373.5027
9	913.8	1441.105	911.7039	382.3029
10	1004.4	1464.489	935.0875	405.6865
11	1026.9	1460.843	931.4417	402.0406
12	1025.6	1476.306	946.905	417.504
13	1021.9	1495.289	965.8884	436.4874
14	992.4	1485.232	955.8309	426.4299
15	972.1	1491.266	961.8654	432.4644
16	946.2	1501.701	972.3	442.899
17	949.4	1508.49	979.0888	449.6878
18	986.3	1510.753	981.3517	451.9507
19	966.8	1536.273	1006.872	477.4715
20	913.5	1557.394	1027.993	498.5922
21	877.1	1554.628	1025.227	495.8264
22	889.4	1547.085	1017.684	488.2833
23	935.5	1543.942	1014.541	485.1404
24	875.6	1523.953	994.5522	465.1512

Table (P1.3) Estimated load for a winter weekday, (5% load deviation), Model A

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	735.9	1171.989	767.3909	362.7927
2	650.6	1194.385	789.7866	385.1884
3	613.1	1185.848	781.2501	376.6518
4	599.6	1191.447	786.849	382.2508
5	604.8	1196.578	791.9793	387.3811
6	617.1	1208.244	803.6458	399.0475
7	635.1	1220.145	815.5464	410.9482
8	731.5	1213.567	808.9683	404.3701
9	915.8	1218.228	813.6301	409.0318
10	1001.8	1238.197	833.5988	429.0006
11	1013	1240.624	836.0258	431.4276
12	1014.6	1242.264	837.6659	433.0677
13	1020.9	1259.998	855.3998	450.8016
14	995.1	1270.451	865.8528	461.2545
15	979.7	1302.363	897.7643	493.166
16	965.5	1322.842	918.2436	513.6454
17	975.1	1320.457	915.8586	511.2603
18	1029.7	1310.515	905.9163	501.3181
19	1024.8	1320.925	916.3272	511.729
20	968.3	1305.3	900.7019	496.1037
21	955.2	1319.778	915.1799	510.5816
22	960	1361.398	956.7994	552.2012
23	950.7	1384.028	979.4294	574.8312
24	858.3	1443.742	1039.143	634.545

Table (P1.4) Predicted load for a winter weekday, (5% load deviation), Model A

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	748.8	1299.978	895.3793	490.781
2	655.9	1301.191	896.5928	491.9945
3	621.5	1305.342	900.7439	496.1457
4	606.2	1300.957	896.3585	491.7602
5	604.1	1299.509	894.9107	490.3125
6	606.6	1296.337	891.7388	487.1405
7	625	1303.618	899.0198	494.4215
8	723.9	1298.253	893.6552	489.0569
9	913.8	1307.727	903.1289	498.5307
10	1004.4	1333.018	928.4202	523.822
11	1026.9	1329.144	924.5454	519.9471
12	1025.6	1345.94	941.3422	536.7439
13	1021.9	1366.654	962.0558	557.4575
14	992.4	1355.69	951.0922	546.494
15	972.1	1362.269	957.6703	553.0721
16	946.2	1373.424	968.8262	564.228
17	949.4	1380.705	976.1072	571.509
18	986.3	1383.132	978.5342	573.936
19	966.8	1410.893	1006.295	601.6963
20	913.5	1433.757	1029.159	624.5605
21	877.1	1430.862	1026.263	621.665
22	889.4	1422.878	1018.279	613.6812
23	935.5	1419.79	1015.192	610.5933
24	875.6	1398.139	993.5407	588.9425

Table (P1.5) Estimated load for a winter weekend day, (20 % load deviation), Model A

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	776.8	1332.376	830.7912	329.2061
2	710	1335.332	833.7467	332.1616
3	667.1	1334.108	832.5232	330.9381
4	647.2	1334.858	833.2725	331.6873
5	639.3	1327.748	826.1624	324.5773
6	642.8	1330.898	829.3125	327.7274
7	657.2	1343.858	842.2724	340.6873
8	689.3	1352.656	851.0706	349.4855
9	767.5	1359.129	857.5435	355.9583
10	898	1355.821	854.2363	352.6512
11	995.1	1363.775	862.1894	360.6043
12	1016.2	1368.613	867.0279	365.4428
13	1008.1	1359.703	858.1177	356.5326
14	977.9	1354.002	852.4168	350.8317
15	940.1	1353.171	851.5858	350.0007
16	905.1	1338.23	836.6452	335.0601
17	892.8	1331.781	830.1959	328.6108
18	915.4	1323.021	821.4362	319.851
19	915.1	1316.243	814.6578	313.0727
20	887	1316.993	815.408	313.8228
21	900.2	1314.049	812.4639	310.8788
22	961.4	1308.722	807.1371	305.552
23	953.1	1299.203	797.6177	296.0326
24	903.7	1291.464	789.8784	288.2933

Table (P1.6) Predicted load for a winter weekend day, (20% load deviation), Model A

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	786	1346.21	844.6251	343.0399
2	711.3	1342.504	840.9189	339.3338
3	670.9	1331.717	830.1323	328.5472
4	653	1332.811	831.2263	329.6411
5	645.1	1332.12	830.5352	328.95
6	646	1324.543	822.9579	321.3728
7	659	1317.92	816.3345	314.7494
8	687.6	1315.135	813.5496	311.9645
9	767.7	1306.185	804.6002	303.0151
10	889.4	1308.836	807.251	305.6659
11	968.7	1304.631	803.0461	301.461
12	989.2	1311.826	810.2408	308.6557
13	983.5	1313.216	811.6306	310.0454
14	952.3	1321.447	819.8619	318.2768
15	911.5	1326.293	824.7076	323.1225
16	873.4	1336.028	834.4432	332.8581
17	859.6	1341.86	840.2751	338.69
18	888	1339.287	837.7015	336.1164
19	911.5	1329.935	828.35	326.7649
20	899.8	1319.202	817.6172	316.0321
21	889.4	1316.834	815.2491	313.664
22	922.6	1303.98	802.3948	300.8097
23	900.4	1329.948	828.3627	326.7776
24	835.8	1320.157	818.5723	316.9872

Table (P1.7) Estimated load for a winter weekend day, (5 % load deviation), Model A

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	776.8	1197.809	818.4391	439.0692
2	710	1201.219	821.8492	442.4793
3	667.1	1200.542	821.1721	441.8021
4	647.2	1200.075	820.7051	441.3351
5	639.3	1192.373	813.0028	433.6329
6	642.8	1195.785	816.4152	437.0453
7	657.2	1209.825	830.4552	451.0853
8	689.3	1217.553	838.1832	458.8132
9	767.5	1223.011	843.6414	464.2714
10	898	1219.795	840.4249	461.0549
11	995.1	1228.388	849.0179	469.648
12	1016.2	1231.29	851.9202	472.5503
13	1008.1	1222.987	843.6172	464.2473
14	977.9	1219.795	840.4249	461.055
15	940.1	1218.644	839.2739	459.9039
16	905.1	1203.098	823.7279	444.358
17	892.8	1197.971	818.6008	439.2308
18	915.4	1187.932	808.5621	429.1921
19	915.1	1182.77	803.4	424.0301
20	887	1184.363	804.9928	425.6229
21	900.2	1180.994	801.6241	422.2542
22	961.4	1175.262	795.8923	416.5223
23	953.1	1166.613	787.2429	407.8729
24	903.7	1157.237	777.8669	398.4969

Table (P1.8) Predicted load for a winter weekend day, (5% load deviation), Model A

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	786	1210.677	831.3076	451.9377
2	711.3	1205.9	826.5298	447.1599
3	670.9	1196.521	817.1506	437.7807
4	653	1198.223	818.8534	439.4835
5	645.1	1197.44	818.0698	438.6999
6	646	1189.704	810.3345	430.9646
7	659	1184.206	804.8357	425.4658
8	687.6	1180.378	801.0081	421.6381
9	767.7	1171.985	792.6147	413.2448
10	889.4	1173.958	794.5875	415.2176
11	968.7	1169.659	790.2896	410.9196
12	989.2	1177.446	798.0762	418.7062
13	983.5	1179.021	799.6508	420.2808
14	952.3	1185.808	806.4385	427.0686
15	911.5	1189.627	810.2576	430.8876
16	873.4	1200.271	820.9011	441.5312
17	859.6	1207.217	827.8467	448.4767
18	888	1204.591	825.2209	445.851
19	911.5	1194.832	815.4623	436.0924
20	899.8	1184.677	805.3068	425.9369
21	889.4	1180.792	801.4216	422.0517
22	922.6	1171.357	791.9873	412.6174
23	900.4	1198.294	818.9243	439.5544
24	835.8	1184.288	804.9178	425.5479

Model B

Table (P1.9) Predicted load for a winter weekday, (20% load deviation), Model B

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	776.8	1354.124	912.2347	470.3455
2	710	1274.516	831.8185	389.1215
3	667.1	1212.3	785.7601	359.2201
4	647.2	1161.758	753.1927	344.6273
5	639.3	1147.11	741.1701	335.2302
6	642.8	1167.671	746.5836	325.4966
7	657.2	1207.364	767.4948	327.6252
8	689.3	1261.066	816.9554	372.8446
9	767.5	1354.7	924.7267	494.7534
10	898	1392.019	981.03	570.041
11	995.1	1348.908	943.9782	539.0481
12	1016.2	1395.226	977.572	559.9182
13	1008.1	1581.876	1144.632	707.3882
14	977.9	1368.704	923.9869	479.2703
15	940.1	1304.163	870.7567	437.3499
16	905.1	1289.748	875.9318	462.1153
17	892.8	1299.183	894.657	490.1308
18	915.4	1309.851	895.4289	481.0066
19	915.1	1292.027	857.8129	423.5983
20	887	1244.799	799.8808	354.9622
21	900.2	1218.812	782.3753	345.9391
22	961.4	1206.893	790.0472	373.2014
23	953.1	1243.734	839.006	434.2779
24	903.7	1183.662	772.0673	360.4725

Model C

Table (P1.10) Estimated load for a winter day, (0% load deviation), Model C

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	1117.6	1117.352	1049.314	981.2758
2	1006.4	1002.796	959.6642	916.5328
3	943.6	947.7949	893.3424	838.8899
4	871.17	930.072	847.0901	764.1083
5	813	886.8589	806.3678	725.8766
6	869.7	935.3588	884.3027	833.2465
7	914.8	944.6049	899.8886	855.1723
8	978.7	1053.5	981.386	909.2726
9	1157.3	1169.818	1082.987	996.1563
10	1223.8	1199.176	1134.987	1070.799
11	1216.8	1190.083	1147.857	1105.631
12	1284.3	1223.936	1165.634	1107.332
13	1258.6	1203.112	1118.093	1033.073
14	1207.8	1053.874	976.5532	899.232
15	1155.4	975.6566	927.5439	879.4313
16	1110.6	913.4152	866.661	819.9069
17	1094.1	954.7243	878.988	803.2518
18	1113.3	1023.045	937.1199	851.1946
19	1186.4	1046.106	985.9926	925.8795
20	1139	1003.872	961.873	919.8738
21	1152.3	1055.361	993.2103	931.0594
22	1226.5	1098.469	1012.091	925.7128
23	1198.4	1031.638	957.7131	883.7883
24	1111.8	1026.037	980.4147	934.7927

Table (P1.11) Predicted load for a winter day, (0% load deviation), Model C

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	883.7	1117.352	1049.314	981.2758
2	806.5	1002.796	959.6642	916.5328
3	779	947.7949	893.3424	838.8899
4	772.4	930.072	847.0901	764.1083
5	780.2	886.8589	806.3678	725.8766
6	795.6	935.3588	884.3027	833.2465
7	843.3	944.6049	899.8886	855.1723
8	966.8	1053.5	981.386	909.2726
9	1145.8	1169.818	1082.987	996.1563
10	1225.8	1199.176	1134.987	1070.799
11	1220.9	1190.083	1147.857	1105.631
12	1188.1	1223.936	1165.634	1107.332
13	1174.1	1203.112	1118.093	1033.073
14	1130.2	1053.874	976.5532	899.232
15	1108.7	975.6566	927.5439	879.4313
16	1082.2	913.4152	866.661	819.9069
17	1105	954.7243	878.988	803.2518
18	1148.7	1023.045	937.1199	851.1946
19	1146.9	1046.106	985.9926	925.8795
20	1120.2	1003.872	961.873	919.8738
21	1128.4	1055.361	993.2103	931.0594
22	1164.8	1098.469	1012.091	925.7128
23	1126.5	1031.638	957.7131	883.7883
24	1026.5	1026.037	980.4147	934.7927

Table (P1.12) Estimated load for a winter day, (5% load deviation), Model C

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	1117.6	1173.625	1049.911	926.197
2	1006.4	1055.183	961.504	867.825
3	943.6	1007.615	898.3017	788.9886
4	871.17	998.3513	855.9619	713.5724
5	813	952.1627	815.037	677.9114
6	869.7	993.3805	889.2535	785.1266
7	914.8	1000.306	901.8365	803.3668
8	978.7	1111.747	983.4883	855.2292
9	1157.3	1228.142	1083.423	938.7039
10	1223.8	1254.239	1133.437	1012.635
11	1216.8	1239.701	1144.984	1050.267
12	1284.3	1274.589	1163.387	1052.185
13	1258.6	1262.633	1118.719	974.8044
14	1207.8	1116.402	979.9797	843.5577
15	1155.4	1030.421	930.3224	830.2241
16	1110.6	969.5775	871.4403	773.3031
17	1094.1	1019.414	885.0286	750.6428
18	1113.3	1087.438	941.9798	796.5219
19	1186.4	1101.045	987.7631	874.4816
20	1139	1056.232	962.2513	868.2704
21	1152.3	1114.093	995.4145	876.7358
22	1226.5	1160.676	1016.154	871.6326
23	1198.4	1092.559	962.5981	832.6372
24	1111.8	1082.281	982.8972	883.5136

Table (P1.13) Predicted load for a winter day, (5% load deviation), Model C

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	883.7	1173.625	1049.911	926.197
2	806.5	1055.183	961.504	867.825
3	779	1007.615	898.3017	788.9886
4	772.4	998.3513	855.9619	713.5724
5	780.2	952.1627	815.037	677.9114
6	795.6	993.3805	889.2535	785.1266
7	843.3	1000.306	901.8365	803.3668
8	966.8	1111.747	983.4883	855.2292
9	1145.8	1228.142	1083.423	938.7039
10	1225.8	1254.239	1133.437	1012.635
11	1220.9	1239.701	1144.984	1050.267
12	1188.1	1274.589	1163.387	1052.185
13	1174.1	1262.633	1118.719	974.8044
14	1130.2	1116.402	979.9797	843.5577
15	1108.7	1030.421	930.3224	830.2241
16	1082.2	969.5775	871.4403	773.3031
17	1105	1019.414	885.0286	750.6428
18	1148.7	1087.438	941.9798	796.5219
19	1146.9	1101.045	987.7631	874.4816
20	1120.2	1056.232	962.2513	868.2704
21	1128.4	1114.093	995.4145	876.7358
22	1164.8	1160.676	1016.154	871.6326
23	1126.5	1092.559	962.5981	832.6372
24	1026.5	1082.281	982.8972	883.5136

Table (P1.14) Estimated load for a winter day, (10% load deviation), Model C

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	1117.6	1229.683	1051.127	872.5706
2	1006.4	1111.278	967.0101	822.7422
3	943.6	1071.517	904.9275	738.3383
4	871.17	1066.356	865.0006	663.6457
5	813	1012.47	822.2262	631.9821
6	869.7	1048.724	890.5163	732.3086
7	914.8	1059.512	903.2762	747.0405
8	978.7	1167.856	984.9442	802.0327
9	1157.3	1284.741	1085.945	887.1494
10	1223.8	1315.252	1136.116	956.9797
11	1216.8	1294.429	1144.169	993.9083
12	1284.3	1326.631	1164.099	1001.566
13	1258.6	1320.511	1119.519	918.528
14	1207.8	1180.547	984.0671	787.587
15	1155.4	1087.571	935.0442	782.517
16	1110.6	1025.568	876.0086	726.4489
17	1094.1	1083.584	889.7103	695.8363
18	1113.3	1148.609	944.8365	741.0643
19	1186.4	1152.285	987.6144	822.9443
20	1139	1113.382	965.0074	816.6323
21	1152.3	1177.339	999.8724	822.4058
22	1226.5	1222.244	1022.885	823.5271
23	1198.4	1153.569	969.8036	786.0383
24	1111.8	1141.033	984.1565	827.2798

Table (P1.15) Predicted load for a winter day, (10% load deviation), Model C

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	883.7	1229.683	1051.127	872.5706
2	806.5	1111.278	967.0101	822.7422
3	779	1071.517	904.9275	738.3383
4	772.4	1066.356	865.0006	663.6457
5	780.2	1012.47	822.2262	631.9821
6	795.6	1048.724	890.5163	732.3086
7	843.3	1059.512	903.2762	747.0405
8	966.8	1167.856	984.9442	802.0327
9	1145.8	1284.741	1085.945	887.1494
10	1225.8	1315.252	1136.116	956.9797
11	1220.9	1294.429	1144.169	993.9083
12	1188.1	1326.631	1164.099	1001.566
13	1174.1	1320.511	1119.519	918.528
14	1130.2	1180.547	984.0671	787.587
15	1108.7	1087.571	935.0442	782.517
16	1082.2	1025.568	876.0086	726.4489
17	1105	1083.584	889.7103	695.8363
18	1148.7	1148.609	944.8365	741.0643
19	1146.9	1152.285	987.6144	822.9443
20	1120.2	1113.382	965.0074	816.6323
21	1128.4	1177.339	999.8724	822.4058
22	1164.8	1222.244	1022.885	823.5271
23	1126.5	1153.569	969.8036	786.0383
24	1026.5	1141.033	984.1565	827.2798

Table (P1.16) Estimated load for a winter day, (20% load deviation), Model C

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	1117.6	1341.807	1050.709	759.6112
2	1006.4	1226.967	974.957	722.9469
3	943.6	1196.721	915.1665	633.6118
4	871.17	1198.373	879.9313	561.4897
5	813	1136.893	835.7927	534.6922
6	869.7	1164.667	894.9226	625.1779
7	914.8	1173.425	901.817	630.2086
8	978.7	1278.405	983.1068	687.8086
9	1157.3	1398.555	1087.813	777.0703
10	1223.8	1434.492	1139.156	843.8197
11	1216.8	1402.996	1139.749	876.5016
12	1284.3	1425.355	1154.119	882.8824
13	1258.6	1430.091	1113.992	797.8934
14	1207.8	1302.322	987.6617	673.0016
15	1155.4	1204.581	942.3026	680.0242
16	1110.6	1147.248	889.0203	630.7924
17	1094.1	1214.996	903.5651	592.1339
18	1113.3	1269.902	949.7	629.4979
19	1186.4	1259.544	986.0875	712.6309
20	1139	1223.917	963.7767	703.6367
21	1152.3	1294.554	1000.43	706.3053
22	1226.5	1339.801	1027.696	715.5914
23	1198.4	1269.075	973.8017	678.5282
24	1111.8	1254.081	982.1103	710.1401

Table (P1.17) Predicted load for a winter day, (20% load deviation), Model C

Daily hours	Actual load (MW)	Upper load (MW)	Middle load (MW)	Lower load (MW)
1	883.7	1344.002	1052.904	761.8068
2	806.5	1228.994	976.9838	724.9737
3	779	1204.884	923.3297	641.775
4	772.4	1202.089	883.647	565.2054
5	780.2	1137.512	836.412	535.3115
6	795.6	1164.442	894.6974	624.9526
7	843.3	1176.578	904.9697	633.3613
8	966.8	1276.66	981.3616	686.0634
9	1145.8	1388.196	1077.454	766.7114
10	1225.8	1423.288	1127.952	832.6163
11	1220.9	1400.012	1136.765	873.5177
12	1188.1	1429.521	1158.285	887.0485
13	1174.1	1434.482	1118.383	802.2847
14	1130.2	1308.965	994.305	679.6449
15	1108.7	1209.479	947.2005	684.9222
16	1082.2	1147.586	889.358	631.1302
17	1105	1213.589	902.1577	590.7264
18	1148.7	1271.591	951.389	631.1869
19	1146.9	1257.911	984.4548	710.9983
20	1120.2	1223.917	963.7767	703.6367
21	1128.4	1294.61	1000.486	706.3616
22	1164.8	1336.986	1024.881	712.7765
23	1126.5	1269.019	973.7454	678.4718
24	1026.5	1253.011	981.0406	709.0704

APPENDIX 2

WINTER FUZZY FIGURES

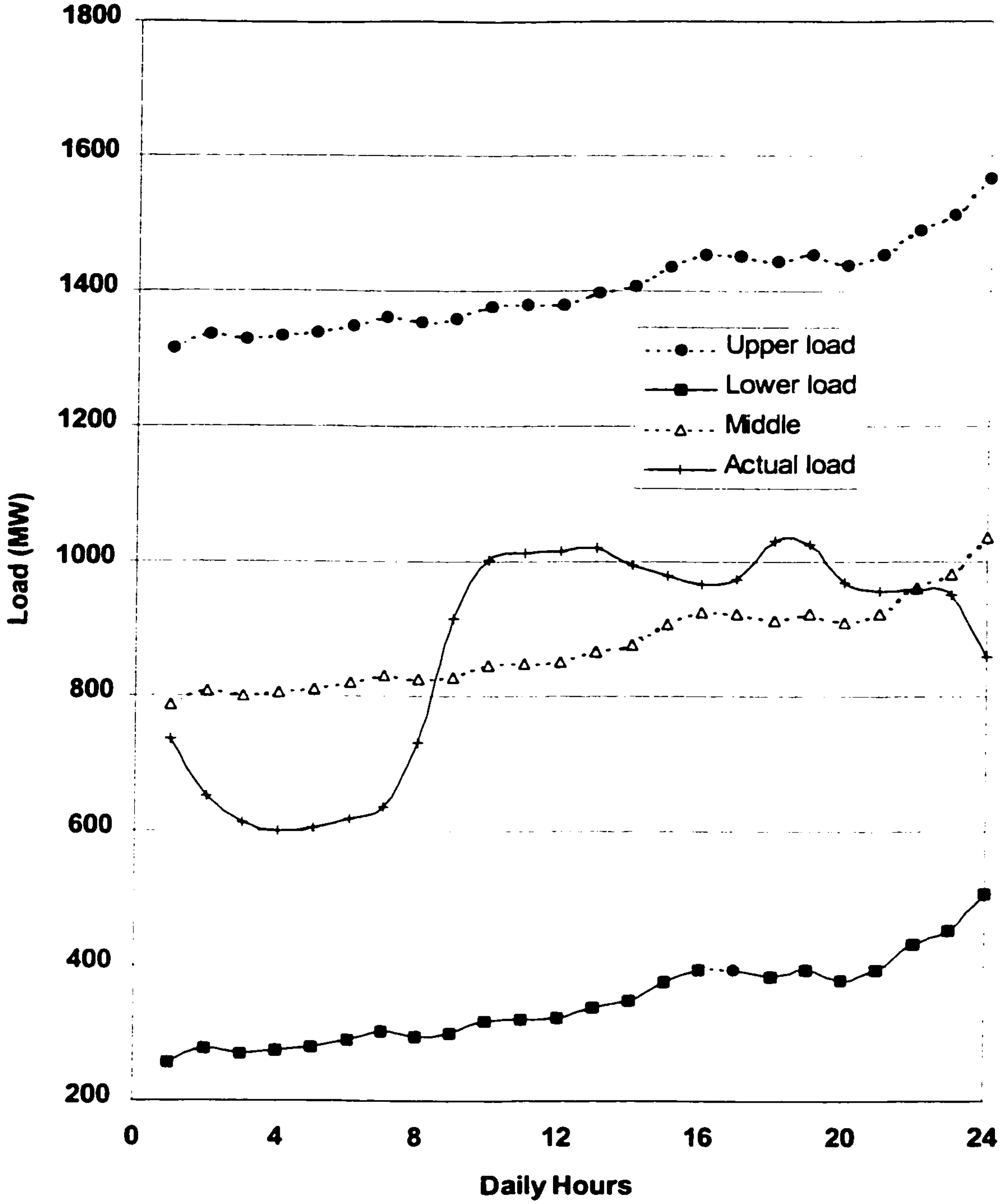


Figure (P2.1) Estimated load for a winter weekday (20% load deviation), Model A

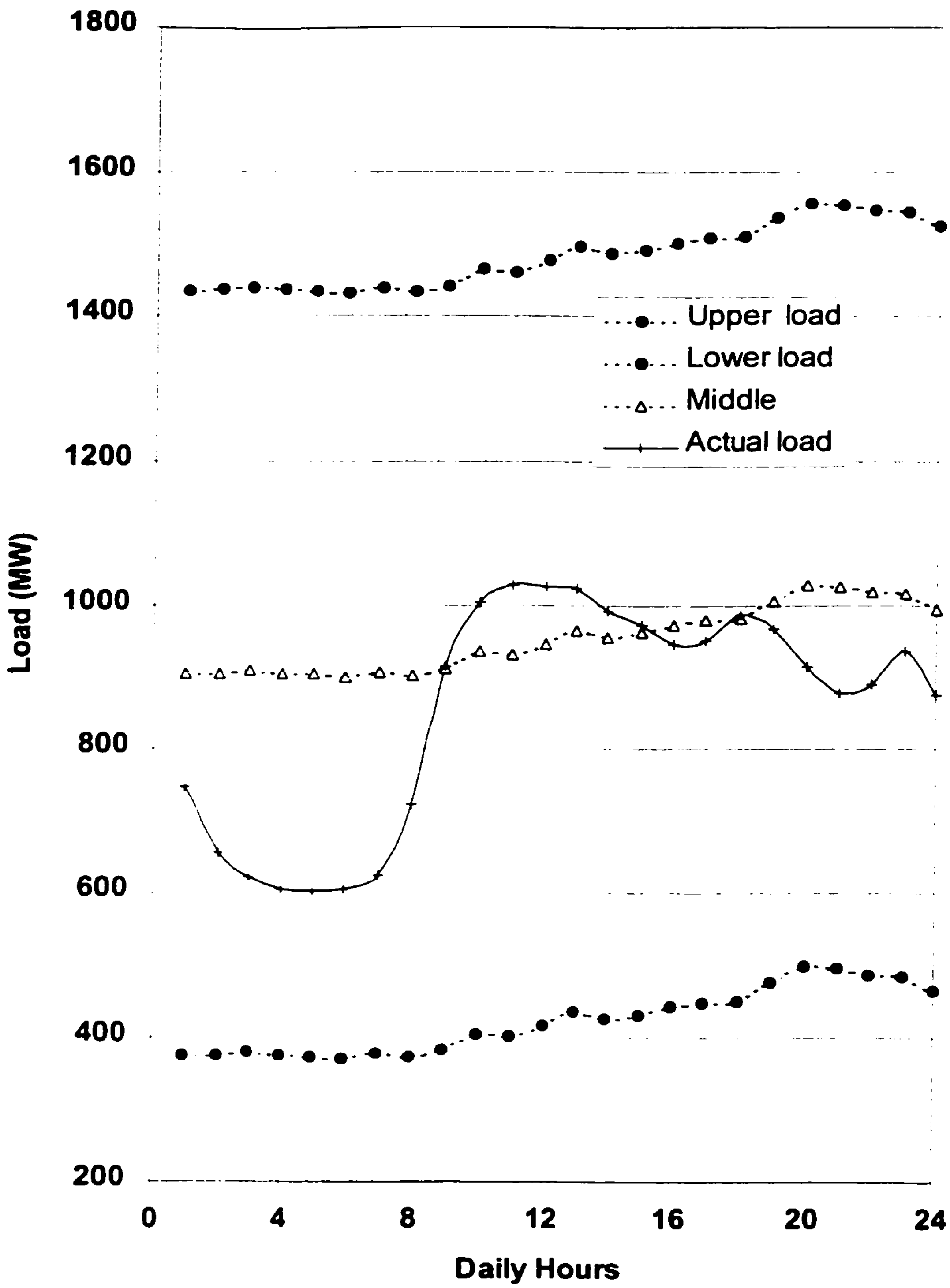
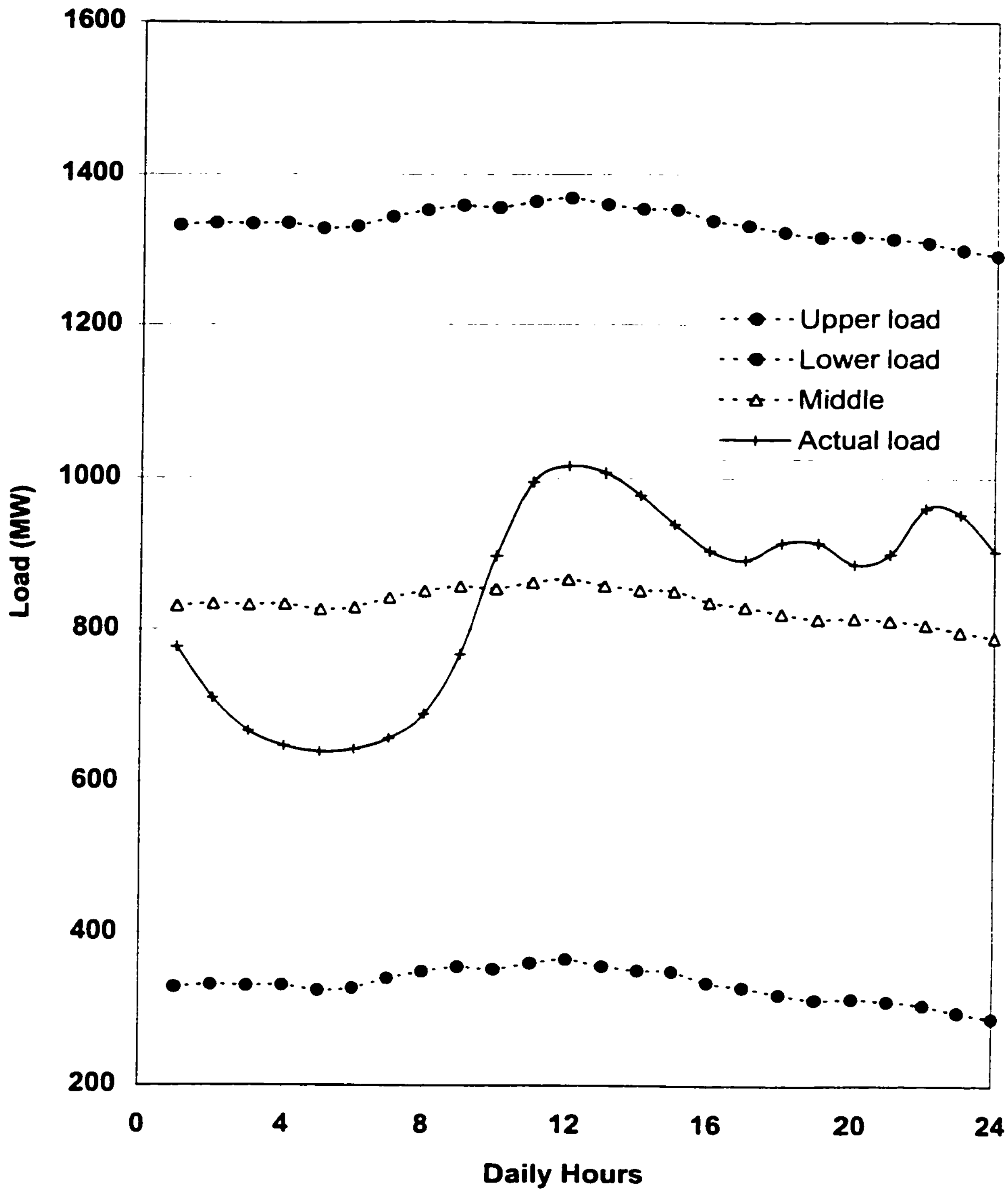


Figure (P2.2) Predicted load for a winter weekday (20%load deviation), Model A



Figure(P2.3) Estimated load for a winter weekend day (20% load deviation), Model A

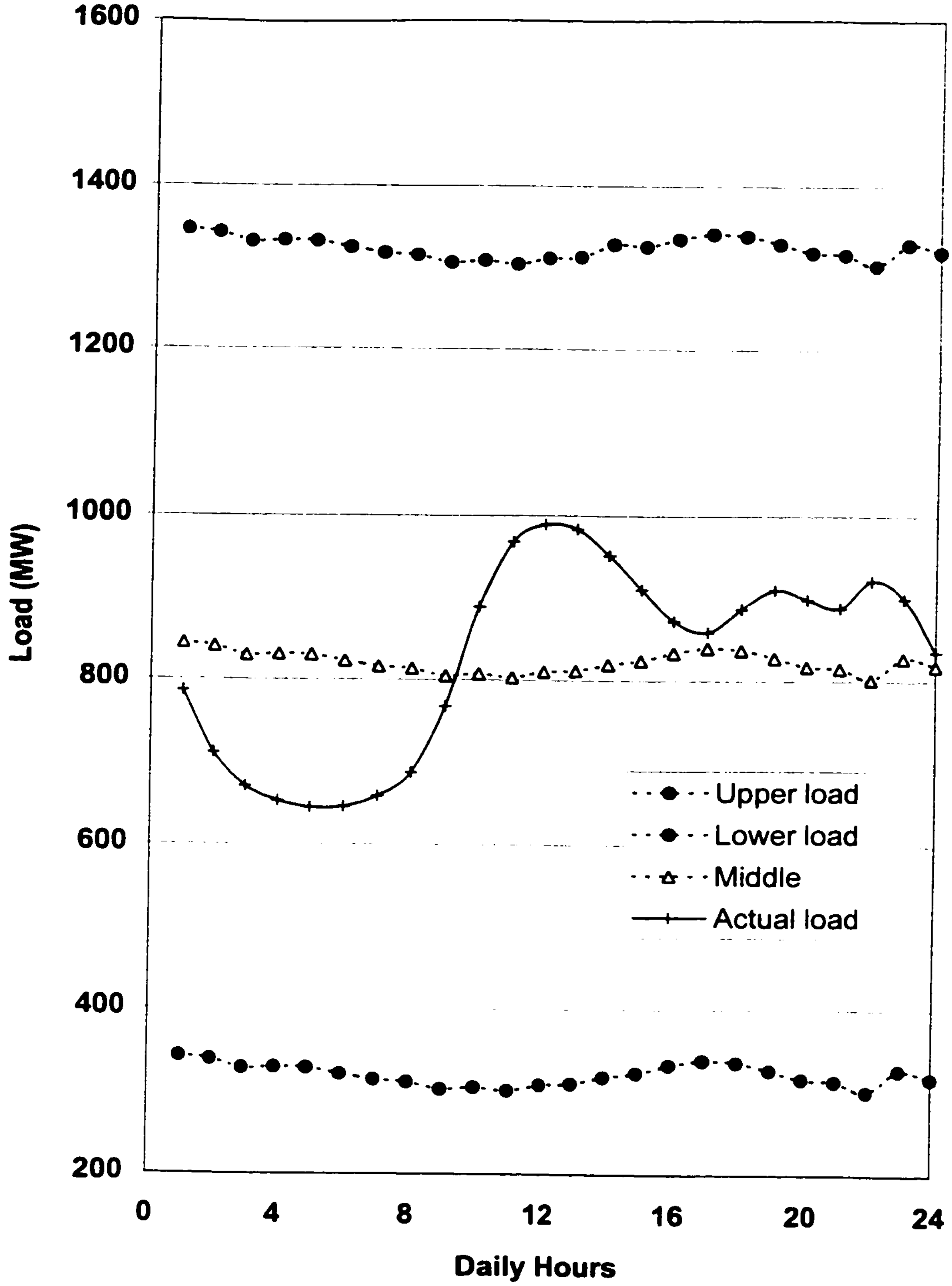


Figure (P2.4) Predicted load for a winter weekend day (20% load deviation), Model A

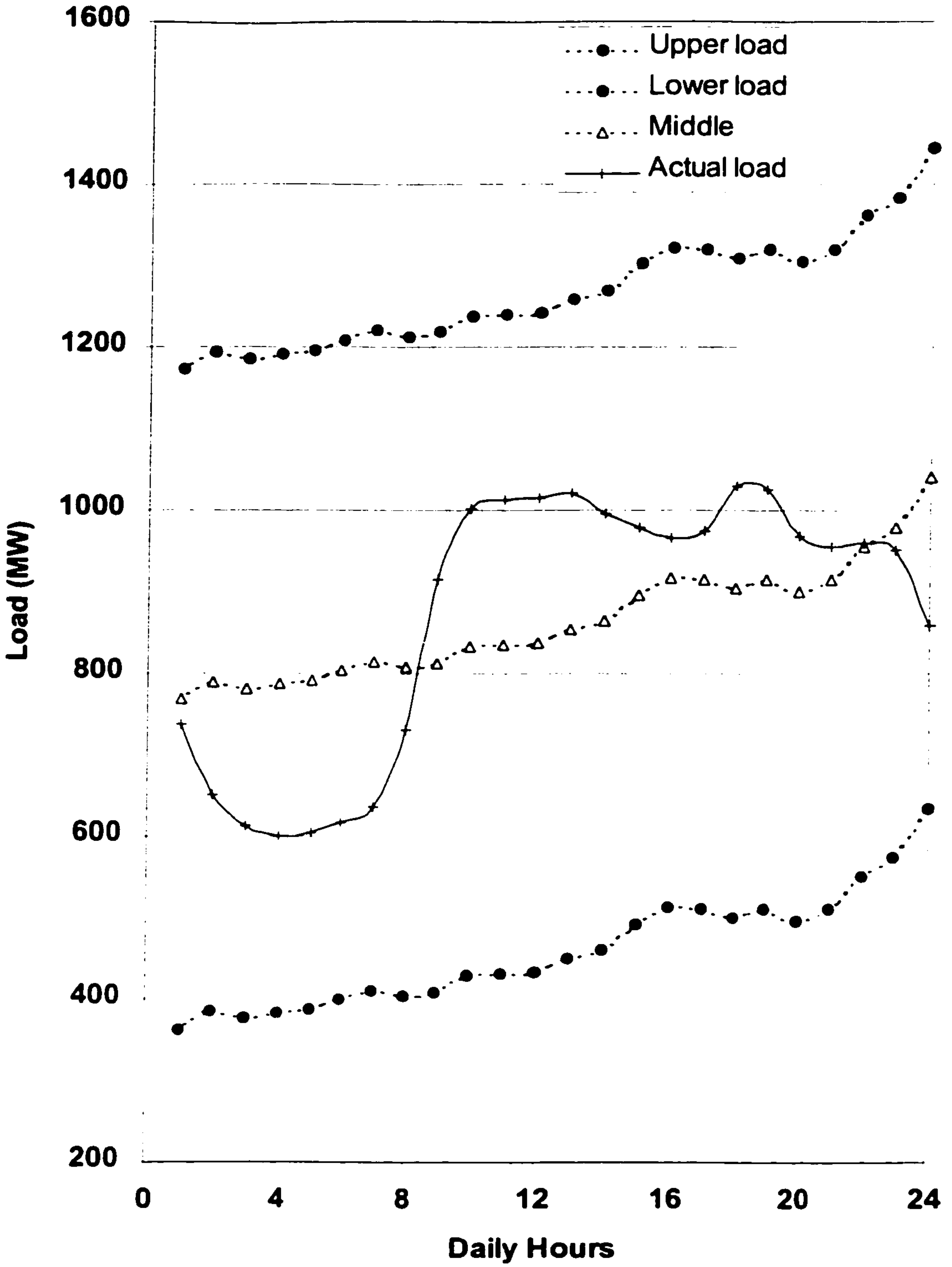


Figure (P2.5)Estimated load for a winter weekday (5% load deviation), Model A

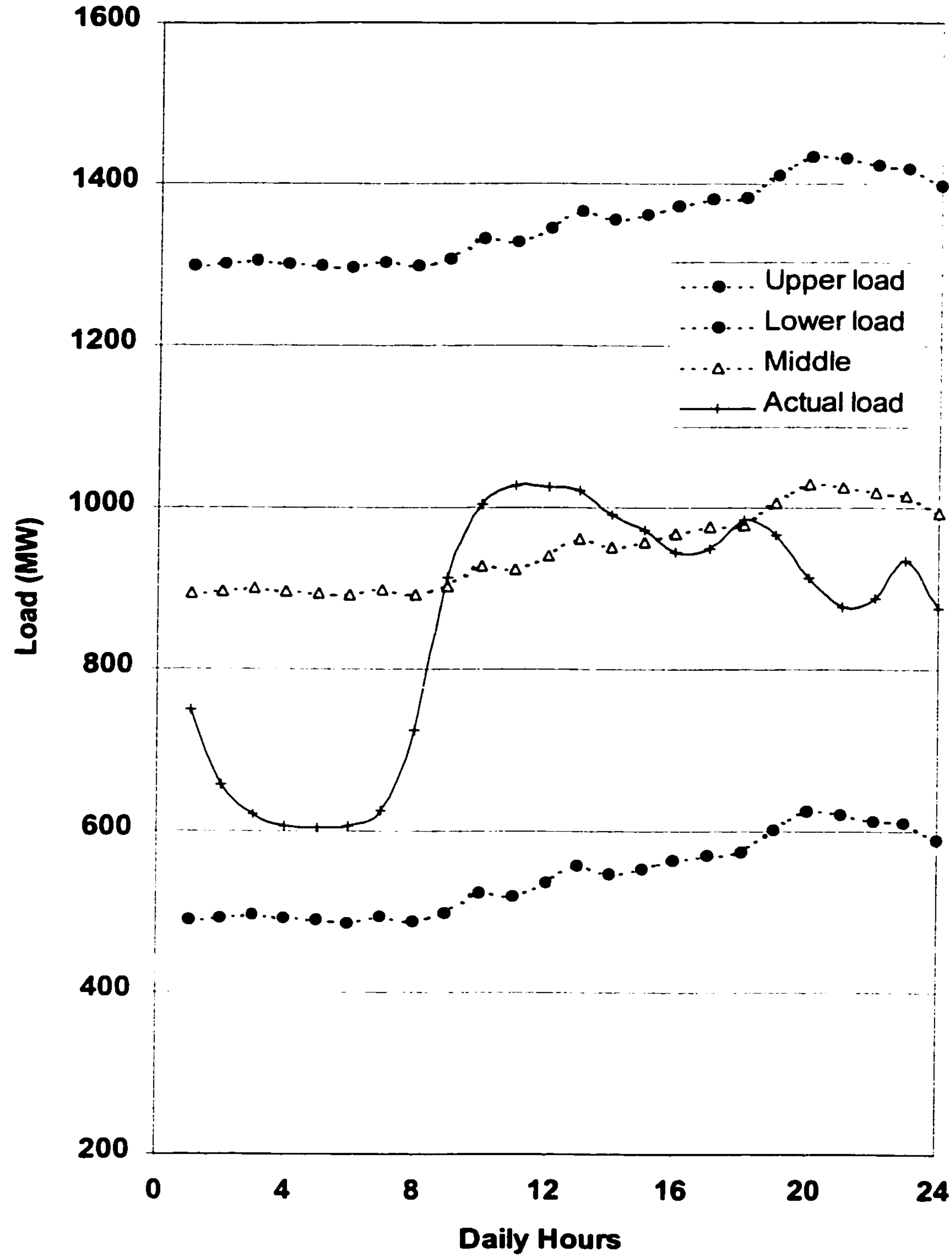


Figure (P2.6) Predicted load for a winter weekday (5% load deviation), Model A

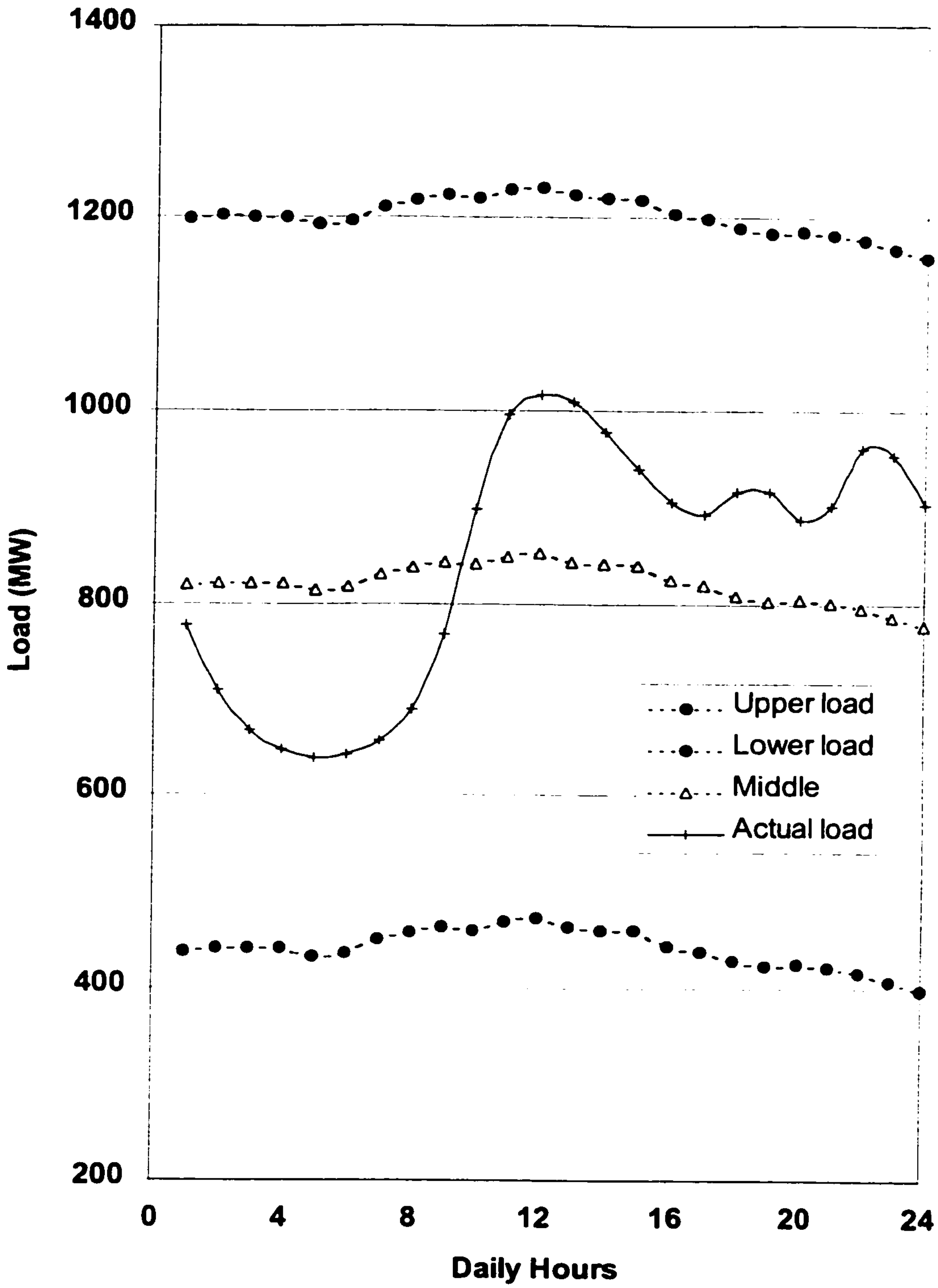


Figure (P2.7) Estimated load for a winter weekend day (5% load deviation), Model A

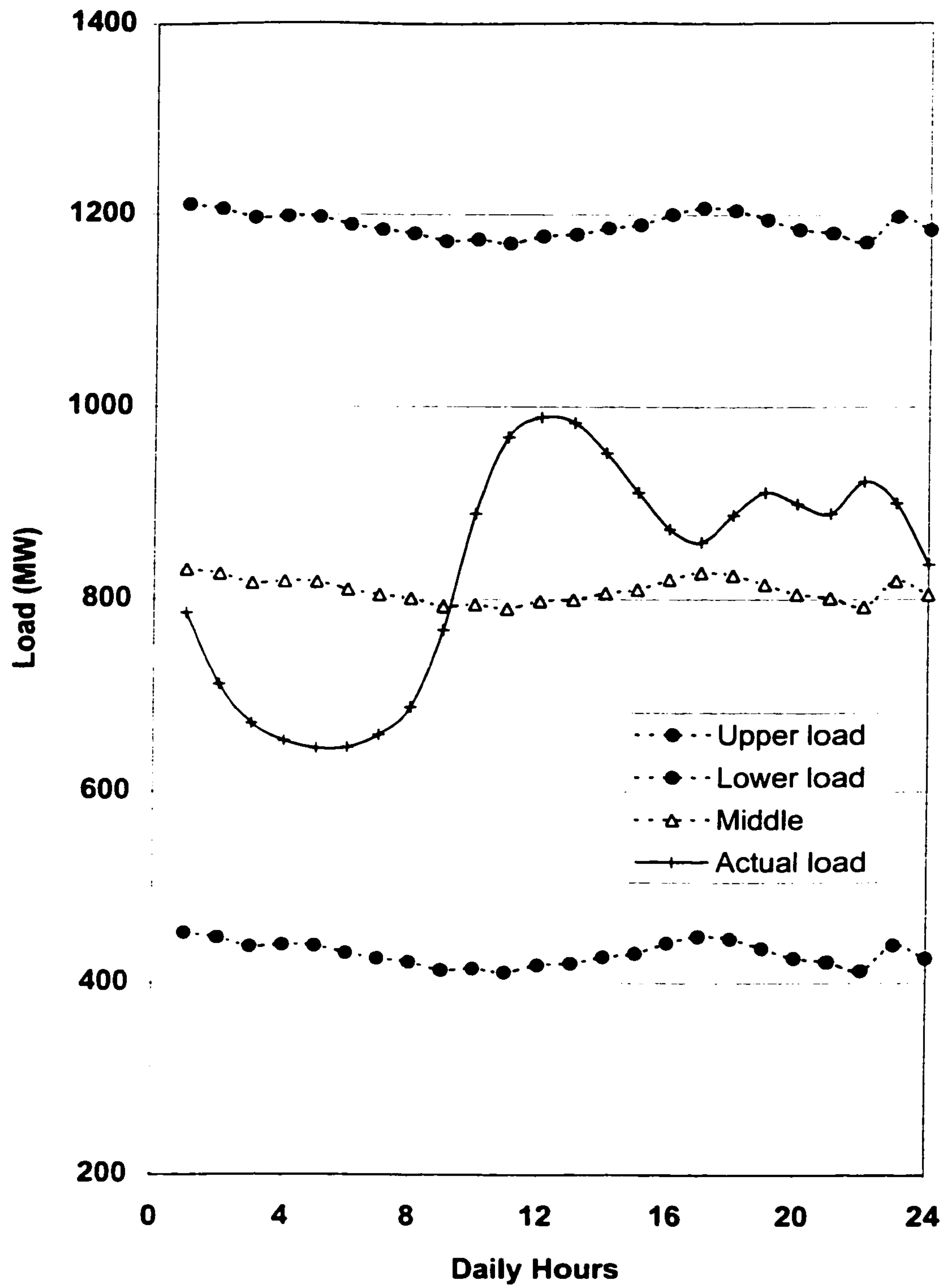


Figure (P2.8) Predicted load for a winter weekend day (5% load deviation), Model A

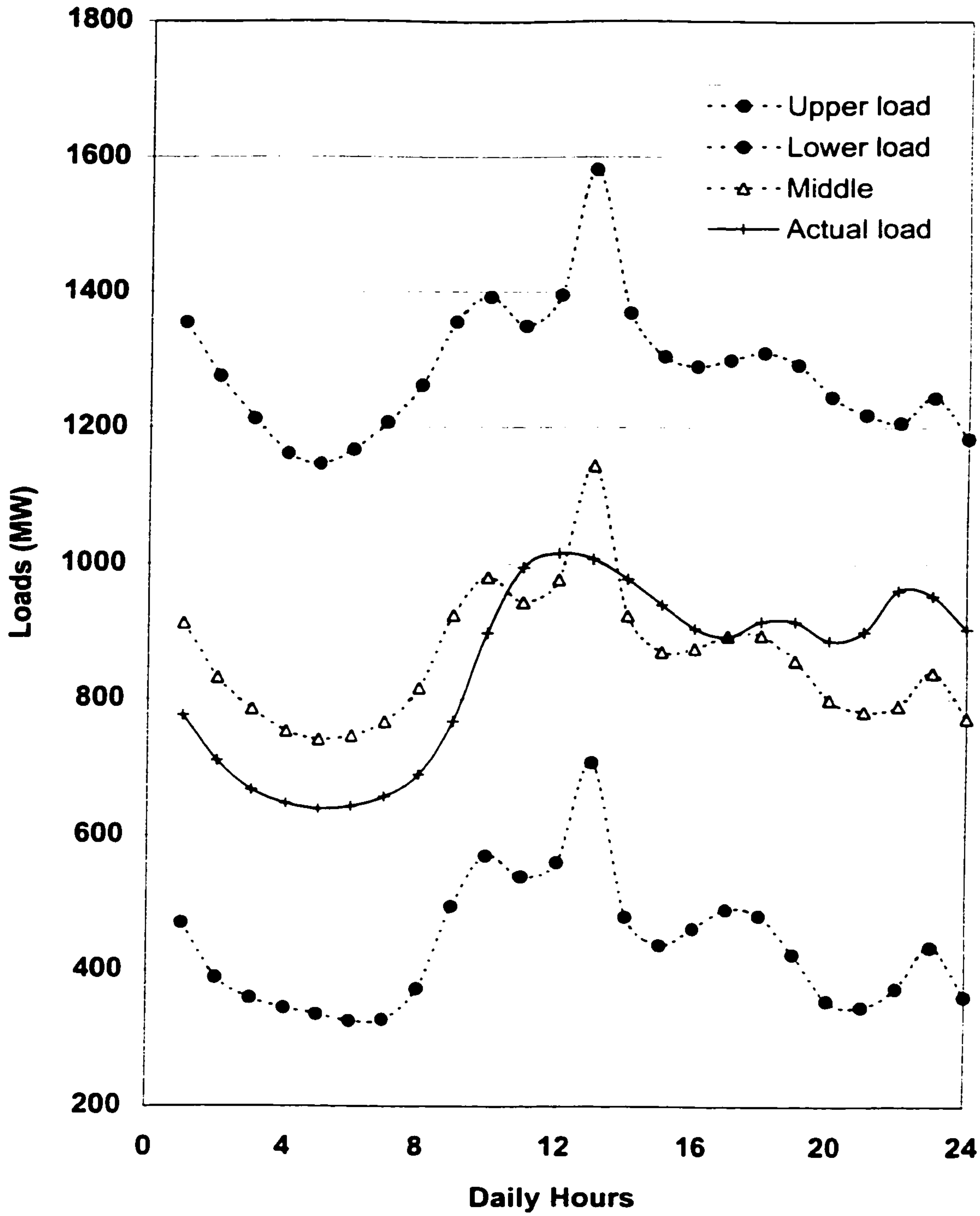


Figure (P2.9) Predicted load for a winter weekday (20 % load deviation) using the parameters of the summerday, Model B

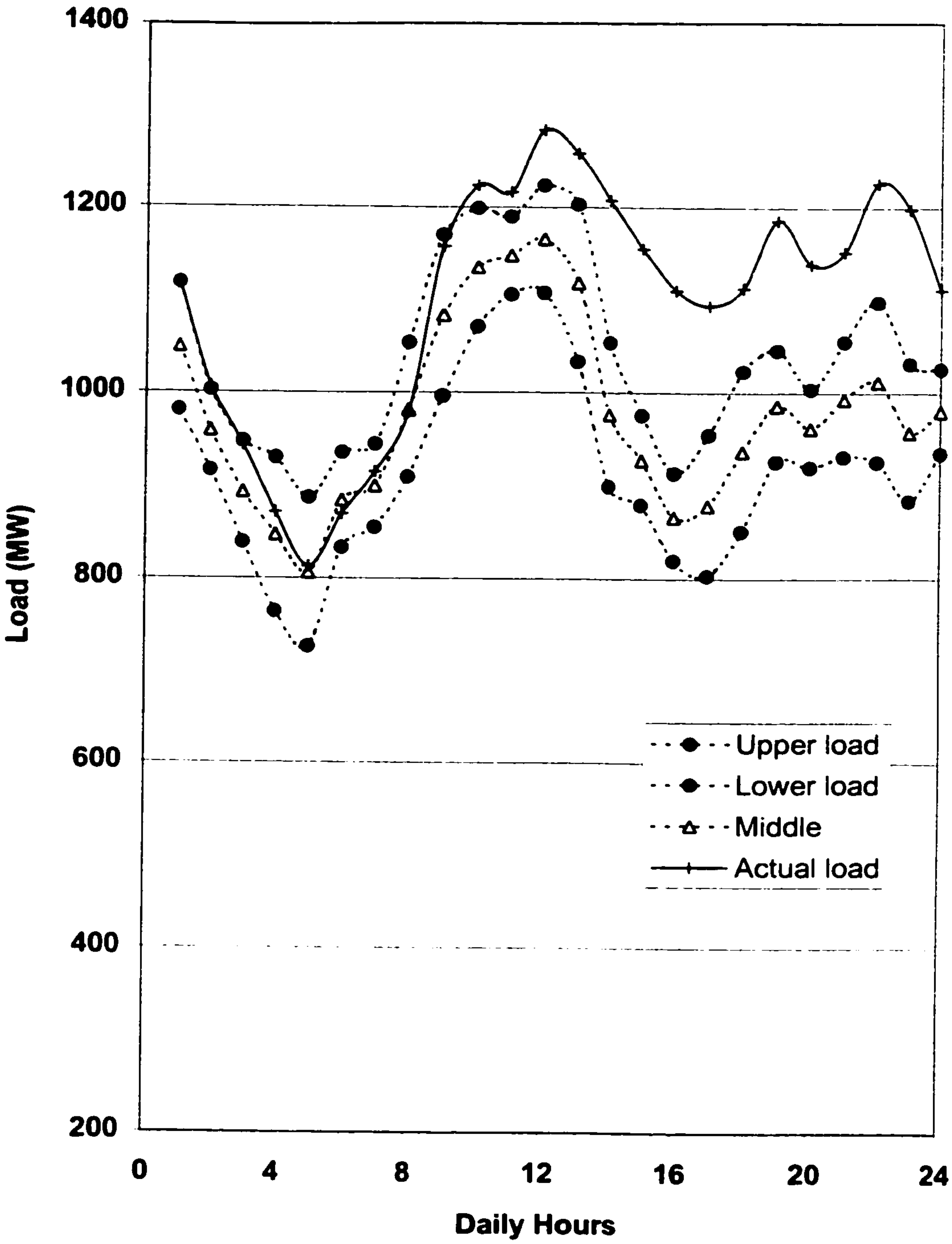


Figure (P2.10) Estimated load for a winter day (0% load deviation), Model C

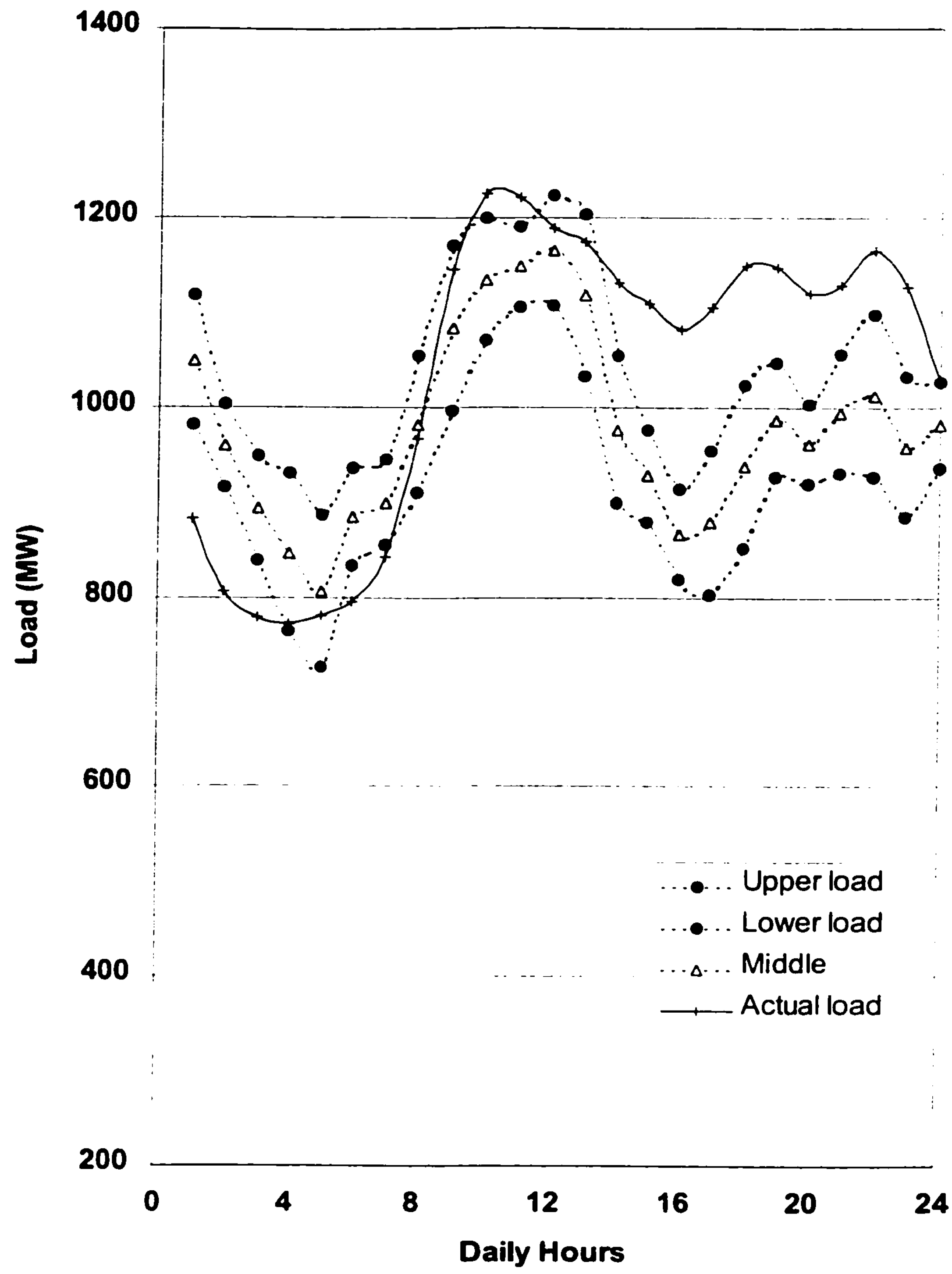


Figure (P2.11) Predicted load for a winter day (0% load deviation), Model C

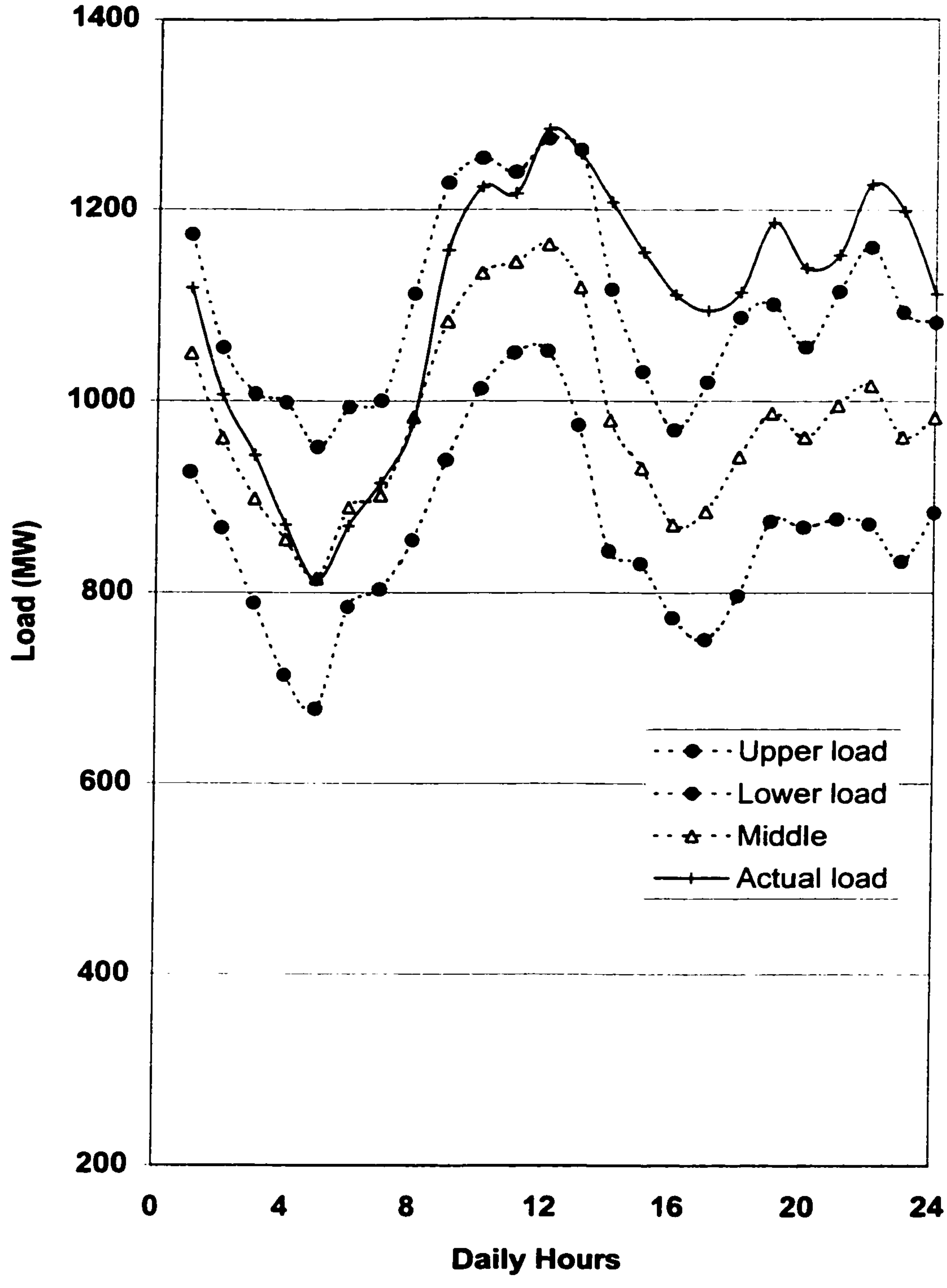


Figure (P2.12) Estimated load for a winter day (5% load variation), Model C

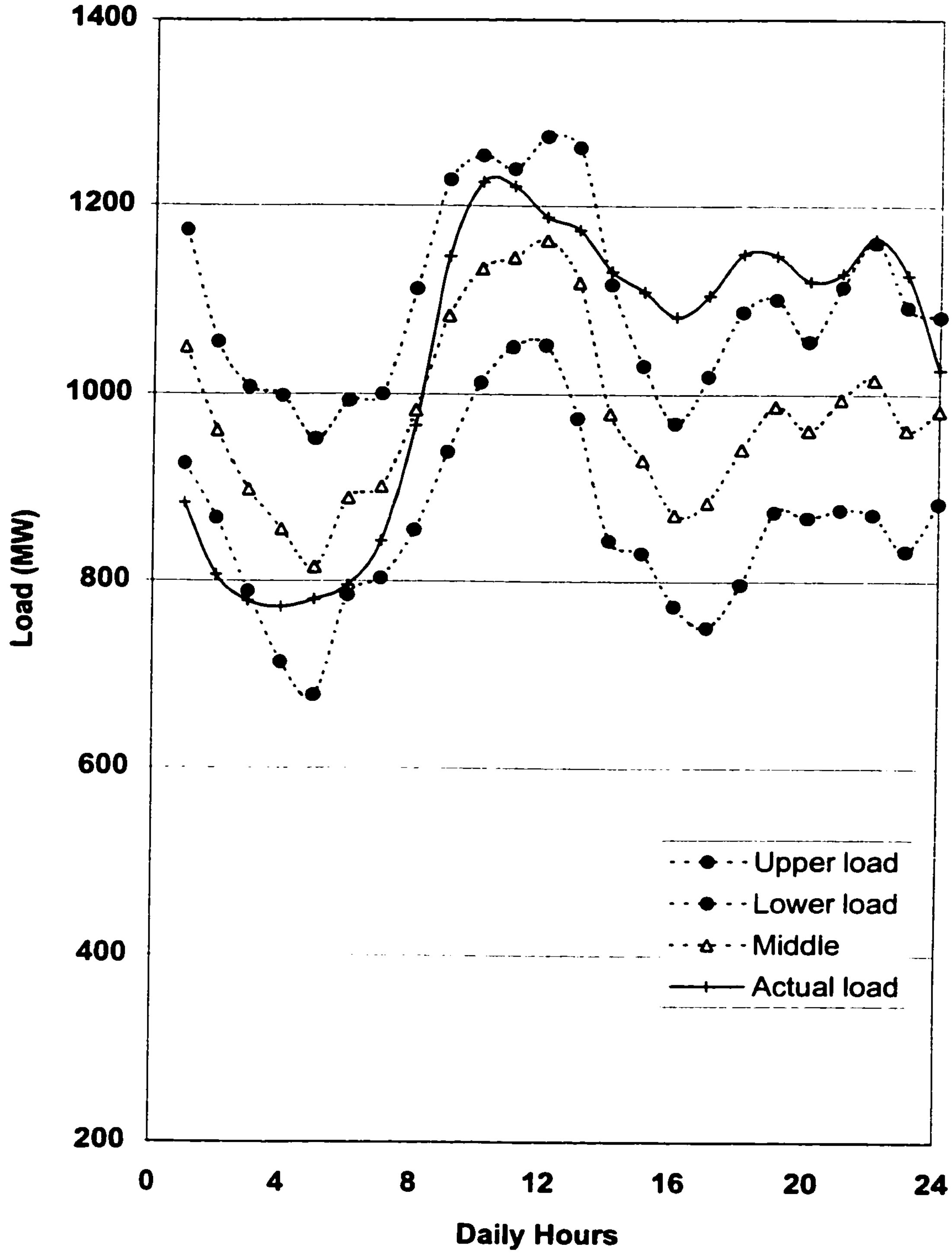


Figure (P2.13) Predicted load for a winter day (5% load deviation), Model C

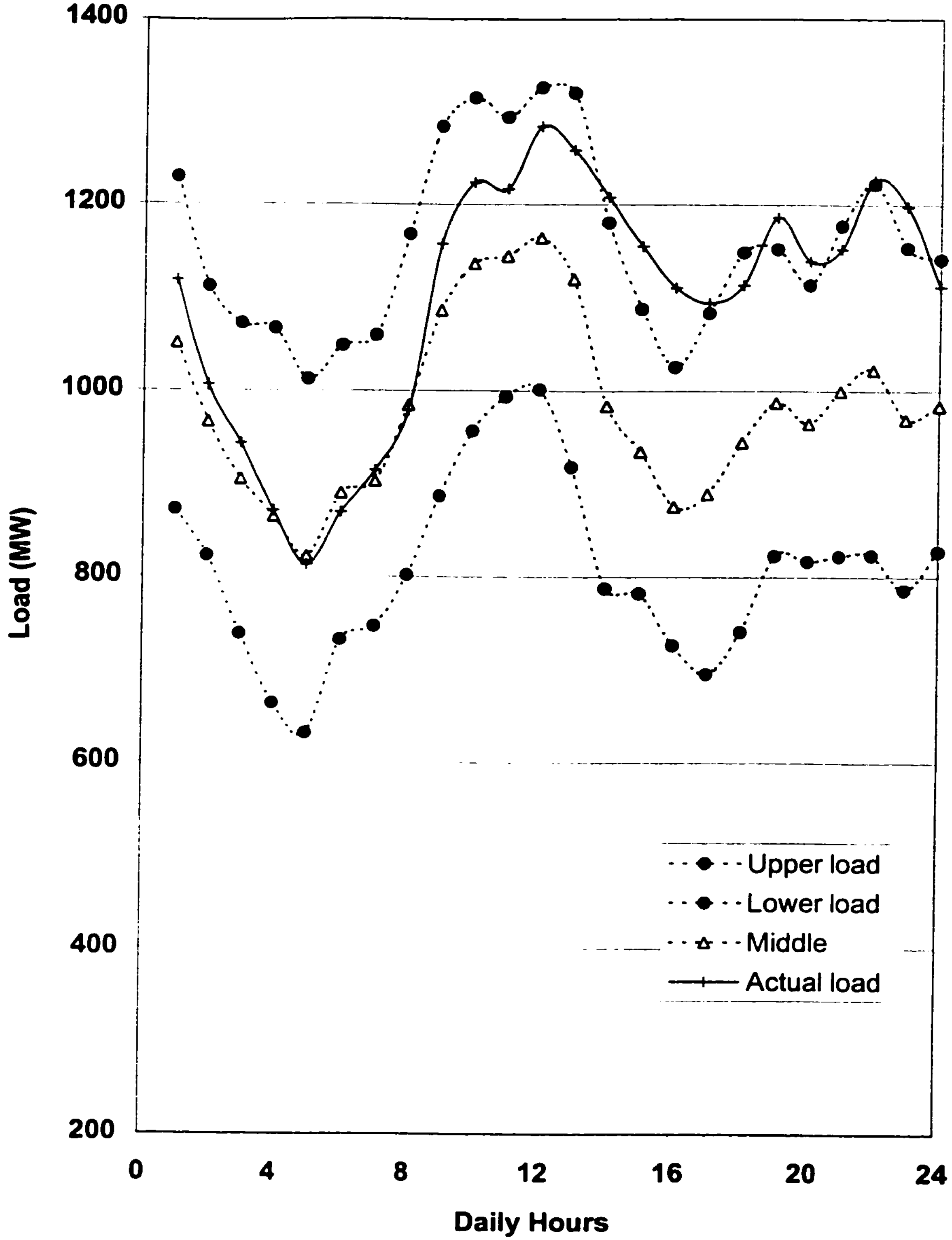


Figure (P2.14) Estimated load for a winter day (10% load deviation), Model C

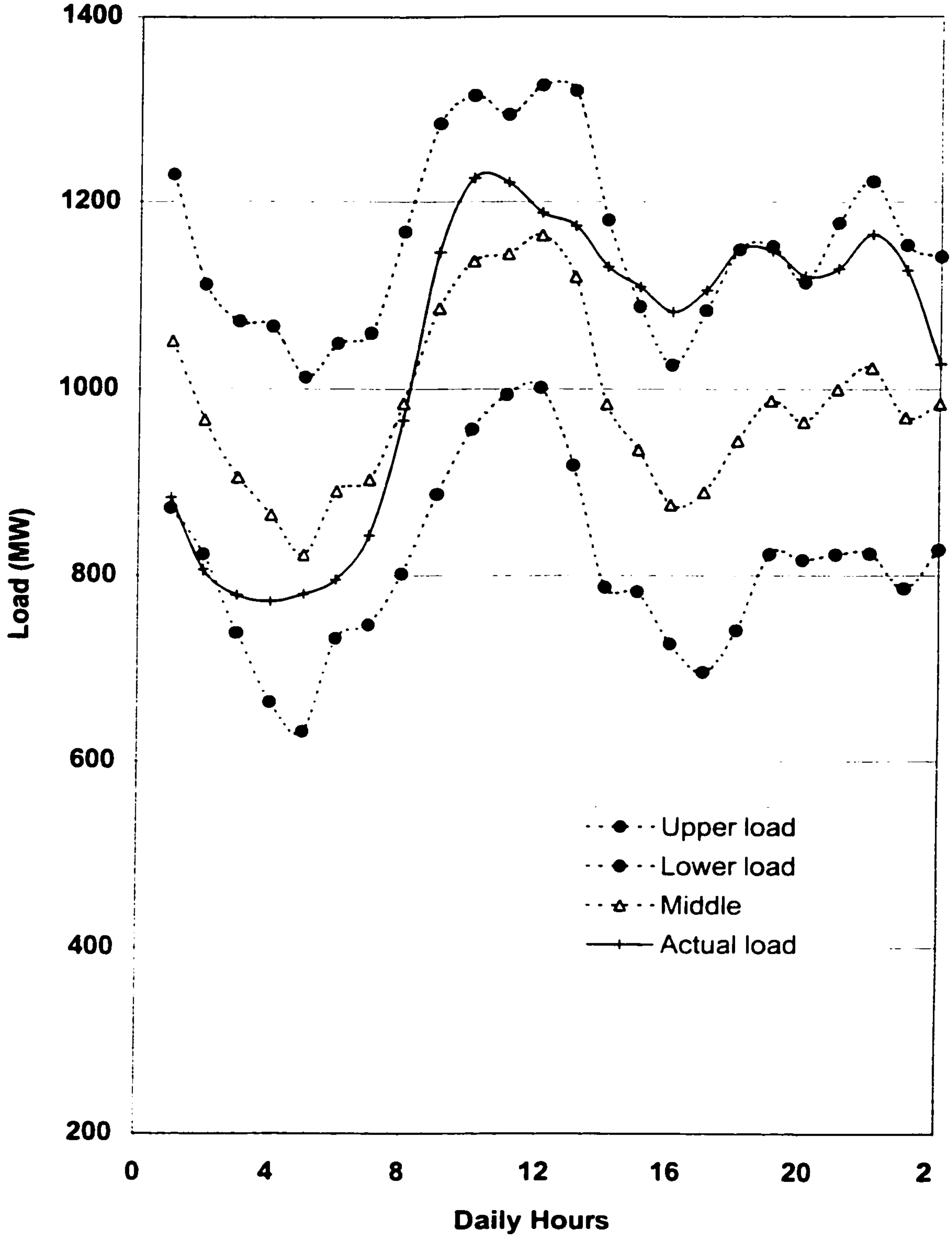


Figure (P2.15) Predicted load for a winter day (10% load deviation), Model C

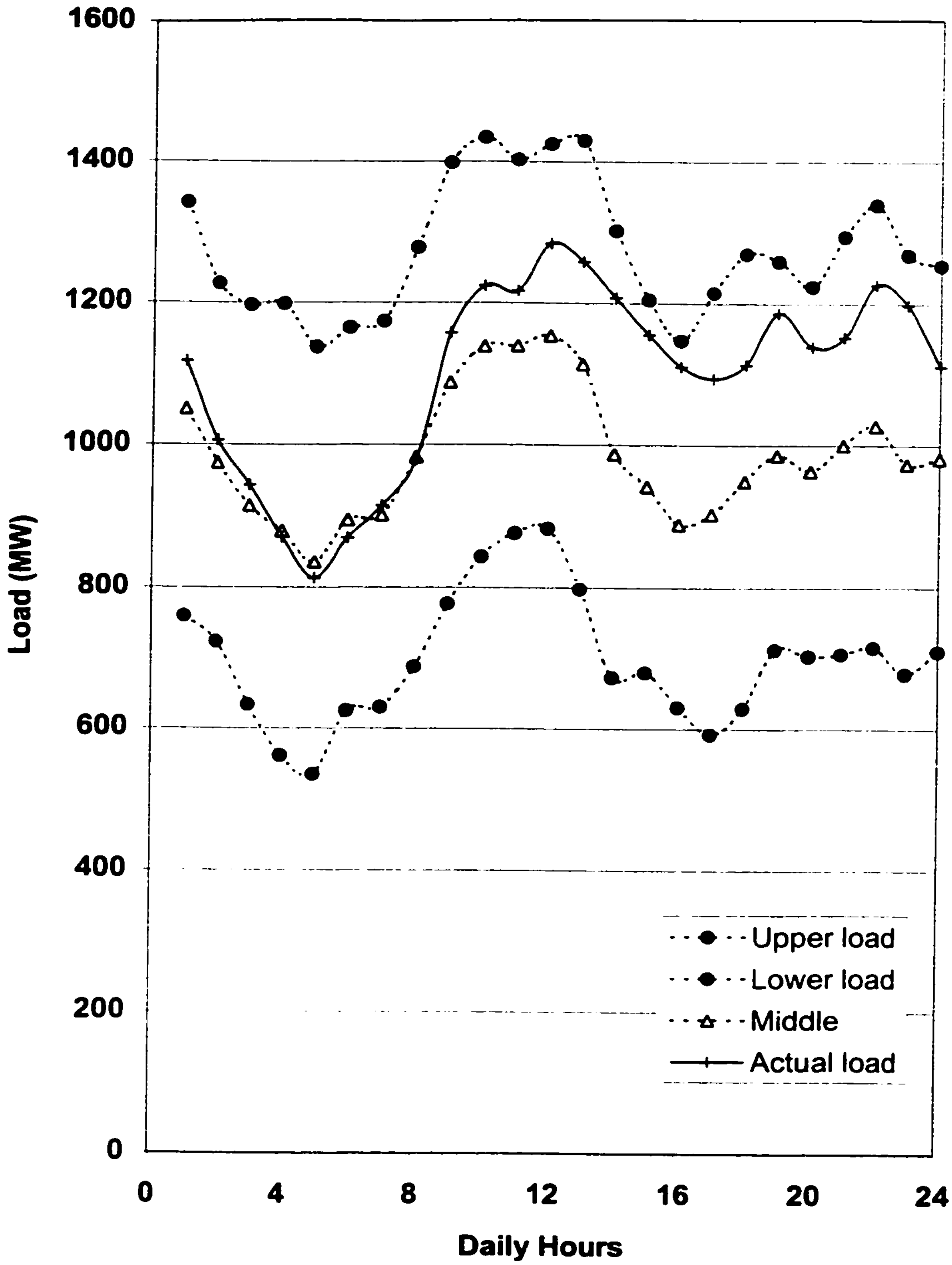


Figure (P2.16) Estimated load for a winter day (20% load deviation), Model C

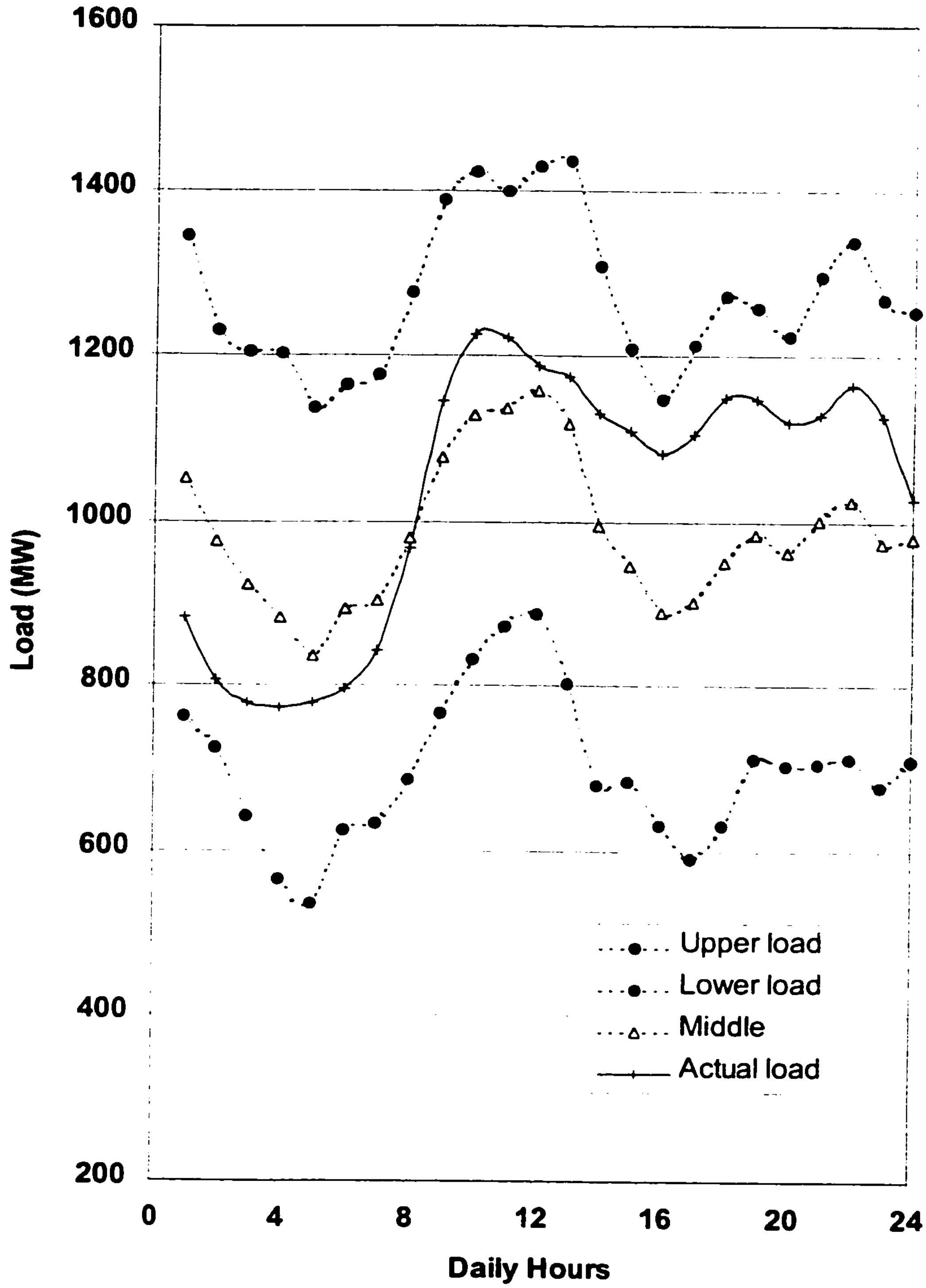


Figure (P2.17) Predicted load for a winter day (20% load deviation), Model C

Appendix 3

Winter Tables: Static Estimation (Crisp Case)

Model A:

Table (P3.1) Estimated load and percentage error for a winter weekday using 24 parameters sets, Model A

Daily hours	Actual load(MW)	LES Estimation	LAV Estimation	% LES error	% LAV error
1	943.4	896.3	943.5	5	-0.02
2	850.5	834	863.1	1.94	-1.48
3	811.2	797.3	807.2	1.72	0.5
4	793.6	769.5	792.4	3.03	0.16
5	794.6	754.5	863.3	5.04	-8.64
6	810.3	816.2	816.6	-0.72	-0.78
7	855.2	874.1	845.9	-2.21	1.09
8	991	996.6	994.6	-0.57	-0.36
9	1198.5	1198.9	1193.4	-0.03	0.43
10	1302.3	1304.5	1305.8	-0.17	-0.27
11	1331.8	1331.7	1332.5	0.01	-0.05
12	1344.7	1345.2	1343.4	-0.04	0.1
13	1366.1	1365.5	1364.9	0.04	0.09
14	1346.2	1345.8	1345.1	0.03	0.08
15	1332.7	1333.7	1334.1	-0.07	-0.11
16	1320.6	1321.7	1321.3	-0.08	-0.05
17	1341.4	1342.8	1342.7	-0.1	-0.1
18	1405.2	1407.6	1408.6	-0.17	-0.24
19	1403.3	1405.3	1423.5	-0.14	-1.44
20	1380.9	1384.3	1385.2	-0.25	-0.31
21	1406.5	1407	1410	-0.03	-0.25
22	1440.9	1443.9	1443.7	-0.21	-0.19
23	1390.9	1387.5	1368.7	0.25	0.3
24	1281.1	1282.5	1283.4	-0.11	-0.18

Table (P3.2) Estimated load and percentage error for a winter weekday using one parameters set, Model A

Daily hours	Actual load(MW)	LES Estimation	LAV Estimation	% LES error	% LAV error
1	1331.8	1330.2	1331.8	0.1	0
2	1344.7	1381.1	1360.9	-2.7	-1.2
3	1366.1	1340.3	1366	1.9	0
4	1346.2	1348.3	1346.2	-0.2	0
5	1332.7	1366.2	1360.1	-2.5	-2.1
6	1320.6	1288.9	1309.4	2.4	0.9
7	1341.4	1342.3	1341.4	-0.1	0
8	991	1034.3	1041.6	-4.4	-5.1
9	1198.5	1136	1134	5.2	5.4
10	1302.3	1261.7	1274.5	3.1	2.1
11	1331.8	1330.2	1331.8	0.1	0
12	1344.7	1381.1	1360.9	-2.7	-1.2
13	1366.1	1340.3	1366	1.9	0
14	1346.2	1348.3	1346.2	-0.2	0
15	1332.7	1366.2	1360.1	-2.5	-2.1
16	1320.6	1288.9	1309.4	2.4	0.9
17	1341.4	1342.3	1341.4	-0.1	0
18	1405.2	1416	1405.3	-0.8	0
19	1403.3	1432.7	1425.3	-2.1	-1.6
20	1380.9	1391.8	1380.9	-0.8	0
21	1406.5	1373.8	1397.1	2.3	0.7
22	1440.9	1413.8	1413.3	1.9	1.9
23	1390.9	1390.7	1390.8	0	0
24	1281.1	1317.4	1359.6	-2.8	-6.1

Table (P3.3) Predicted load and percentage error for a winter weekday using 24 parameters sets, Model A

Daily hours	Actual load(MW)	LES Prediction	LAV Prediction	% LES error	% LAV error
1	779.2	795.6	835.7	-2.11	-7.25
2	698.6	724.8	737.7	-3.75	-5.6
3	665.8	674.9	703.4	-1.36	-5.65
4	658.5	669.7	700.5	-1.71	-6.38
5	660.1	697.6	707.4	-5.68	-7.17
6	674.7	715.4	706.3	-6.03	-4.68
7	714.3	745	767.3	-4.3	-7.43
8	827.4	808.7	741.3	2.26	10.41
9	1003.5	1033.4	1031.4	-2.98	-2.78
10	1065.1	1089.7	1098.9	-2.31	-3.17
11	1062.1	1063.7	1073.2	-0.15	-1.05
12	1044.2	1043.4	1060.4	0.07	-1.55
13	1030.9	1033.3	1043.9	-0.23	-1.26
14	996.2	1006	1005	-0.99	-0.89
15	971.3	977.6	975.3	-0.65	-0.41
16	946.9	954.8	951.3	-0.84	-0.46
17	946.9	959.3	958.8	-1.31	-1.26
18	975.9	1001.6	996.9	-2.63	-2.16
19	965.8	993	1016.8	-2.82	-5.28
20	917.2	976.6	977.4	-6.48	-6.57
21	902.1	941.5	879.2	-4.37	2.53
22	971.3	996.6	976.3	-2.61	-0.51
23	981.8	987.5	995.1	-0.58	-1.35
24	898.1	907.1	908.5	-1	-1.16

Table (P3.4) Predicted load and percentage error for a winter weekday using one parameters set, Model A

Daily hours	Actual load(MW)	LES Prediction	LAV Prediction	% LES error	% LAV error
1	883.7	913.6	886.2	-3.38	-0.29
2	806.5	900.3	887.7	-11.63	-9.15
3	779	933.3	949.9	-19.8	-17.99
4	772.4	824.8	850.5	-6.79	-9.18
5	772.4	824.8	850.5	-6.79	-9.18
6	772.4	824.8	850.5	-6.79	-9.18
7	772.4	824.8	850.5	-6.79	-9.18
8	966.8	949.5	968.5	1.79	-0.17
9	1145.8	1175	1144.8	-2.55	0.09
10	1225.8	1213.3	1223.7	1.02	0.17
11	1220.9	1209.2	1220.3	0.96	0.05
12	1188.1	1144.9	1169.4	3.64	1.6
13	1174.1	1085.3	1091.5	7.57	7.57
14	1130.2	1037.4	1053.3	8.21	7.3
15	1108.7	1047.5	1069.9	5.52	3.63
16	1082.2	1069.2	1080	1.2	0.2
17	1082.2	1069.2	1080	1.2	0.2
18	1082.2	1069.2	1080	1.2	0.2
19	1082.2	1069.2	1080	1.2	0.2
20	1120.2	1065.5	1056	4.88	6.08
21	1128.4	1138.3	1128.1	-0.88	0.03
22	1164.8	1160	1162.2	0.41	0.23
23	1126.5	1205.4	1220.7	-7	-7.72
24	1026.5	1225.7	1225.7	-19.4	-16.25

Table (P3.5) Estimated load and percentage error for a winter weekend day using 24 parameters sets, Model A

Daily hours	Actual load(MW)	LES Estimation	LAV Estimation	% LES error	% LAV error
1	776.8	791.8	793.5	-1.93	-2.15
2	710	721.5	709.1	-1.62	0.12
3	667.1	697.8	717.8	-4.6	-7.6
4	647.2	672.5	609.1	-3.9	5.89
5	639.3	675.2	681.7	-5.62	-6.63
6	642.8	652.2	642.7	-1.46	0.01
7	657.2	662.9	679.6	-0.87	-3.4
8	689.3	694.7	716.9	-0.78	-4.01
9	767.5	774.1	767.8	-0.86	-0.03
10	898	925.2	898.4	-3.03	-0.05
11	995.1	1054.6	1056.6	-5.98	-6.18
12	1016.2	1042.2	1015.8	-2.56	0.04
13	1008.1	1025.9	1008.3	-1.77	-0.02
14	977.9	1014.8	984	-3.77	-0.62
15	940.1	964	960.5	-2.54	-2.17
16	905.1	919.1	969.2	-1.55	-7.09
17	892.8	885.1	894.8	0.86	-0.22
18	915.4	946.3	996.5	-3.37	-8.86
19	915.1	940.7	957.7	-2.8	-4.6
20	887	902.1	918.8	-1.7	-3.58
21	900.2	891.2	897.4	1	0.31
22	961.4	930.4	892.3	3.22	7.19
23	953.1	954.5	958.4	-0.15	-0.56
24	903.7	889.2	899.3	1.6	0.49

Table (P3.6) Estimated load and percentage error for a winter weekend day using one parameters set, Model A

Daily hours	Actual load(MW)	LES Estimation	LAV Estimation	% LES error	% LAV error
1	776.8	797.7	780.9	-2.69	-0.53
2	710	764.6	785.3	-7.68	-10.6
3	667.1	755.1	745.1	-14	-11.6
4	647.2	745.7	705	-15.22	-8.94
5	639.3	648.6	634.2	-1.45	0.8
6	642.8	630.6	640.5	1.89	0.35
7	657.2	662.1	660.7	-0.75	-0.53
8	689.3	731.2	717.6	-6.07	-4.1
9	767.5	811.5	799.9	-5.73	-4.22
10	898	865.4	894	3.63	0.45
11	995.1	879.6	928.2	11.61	6.7
12	1016.2	1053.7	962.5	-3.69	5.28
13	1008.1	1030.5	1006.1	-2.22	0.2
14	977.9	935.6	973.7	4.33	0.43
15	940.1	871.3	900.8	7.32	4.18
16	905.1	935.6	955.6	-3.37	-5.58
17	892.8	814.3	892.4	8.8	0.04
18	915.4	834.5	829.1	7.7	9.4
19	915.1	854.8	859.4	6.59	6.1
20	887	900.7	890.7	-1.54	-0.41
21	900.2	931.5	905.1	-3.48	-0.54
22	961.4	933.3	965.9	2.93	-0.47
23	953.1	974.8	956.6	-2.28	-0.37
24	903.7	904.5	898.7	-0.09	0.55

Table (P3.7) Predicted load and percentage error for a winter weekend day using 24 parameters sets, Model A

Daily hours	Actual load(MW)	LES Prediction	LAV Prediction	% LES error	% LAV error
1	814.4	790.8	788.4	2.9	3.2
2	738.5	716.3	708.6	3.01	4.05
3	712	674.9	678.2	5.21	4.74
4	699.9	638	659.2	8.85	5.81
5	705.5	634.7	647.6	10.03	8.21
6	717.1	619.8	614.7	13.57	14.28
7	756.4	658.6	664.8	12.93	12.11
8	866.6	677	679.4	21.88	21.6
9	1035.1	767.9	763.3	25.81	26.26
10	1095.8	850.7	824.7	22.37	24.74
11	1085.7	980.7	965.3	9.67	11.09
12	1057.4	979.3	1004.3	7.38	5.02
13	1042.7	1033.3	1023	0.9	1.89
14	1003.3	1024.6	1038.9	-2.12	-3.55
15	976.7	983	992.6	-0.64	-1.63
16	951.6	977.1	974.3	-2.68	-2.38
17	959.5	978.5	973.9	-1.98	-1.5
18	1000.5	989.2	1051.6	1.13	-5.1
19	1006.2	1025.3	1062.6	-1.9	-5.6
20	973.4	1027.6	1035.5	-5.57	-6.38
21	972.2	1013.3	1018	-4.23	-4.71
22	1021	1060.8	1048.2	-3.9	-2.67
23	997.3	1001.4	1013.8	-0.41	-1.65
24	909.5	938.3	939.7	-3.16	-3.32

Table (P3.8) Predicted load and percentage error for a winter weekend day using one parameters set, Model A

Daily hours	Actual load(MW)	LES Prediction	LAV Prediction	% LES error	% LAV error
1	776.8	797.7	780.9	-2.69	-0.53
2	710	764.6	785.3	-7.68	-10.6
3	667.1	848	813.2	-27.12	-21.9
4	647.2	848	813.2	-27.12	-21.9
5	639.3	848	813.2	-27.12	-21.9
6	642.8	630.6	640.5	1.89	0.35
7	657.2	662.1	660.7	-0.75	-0.53
8	689.3	731.2	717.6	-6.07	-4.1
9	767.5	811.5	799.9	-5.73	-4.22
10	898	865.4	894	3.63	0.45
11	995.1	879.6	843.4	11.61	15.25
12	1016.2	1053.7	962.5	-3.69	5.28
13	1008.1	1030.5	1006.1	-2.22	0.2
14	977.9	935.6	973.7	4.33	0.43
15	940.1	871.3	900.8	7.32	4.18
16	905.1	871.3	900.8	7.32	4.18
17	892.8	871.3	900.8	7.32	4.18
18	915.4	761.6	703.7	16.8	23.13
19	915.1	854.8	798.7	6.59	12.72
20	887	900.7	890.7	-1.54	-0.41
21	900.2	931.5	905.1	-3.48	-0.54
22	961.4	933.3	965.9	2.93	-0.47
23	953.1	974.8	956.6	-2.28	-0.37
24	903.7	904.5	898.7	-0.09	0.55

Model B :

Table (P3.9) Estimated load and percentage error for a winter week day, Model B

Daily hours	Actual load(MW)	LES Estimation	LAV Estimation	% LES error	% LAV error
1	943.4	1199.08	780	-27.1	17.3
2	850.5	1157.67	715	-36.1	15.9
3	811.2	1072.45	650	-32.2	19.9
4	793.6	1023.73	793.24	-29	0
5	794.6	969.21	797.75	-22	-0.4
6	810.3	1014.77	809.27	-25.2	0.1
7	855.2	929.75	857.75	-8.7	-0.3
8	991	1085.1	991.43	-9.5	0
9	1198.5	1375.46	1196.94	-14.8	0.1
10	1302.3	1486.53	1301.21	-14.1	0.1
11	1331.8	1446.21	1333.82	-8.6	-0.2
12	1344.7	1344.73	1344.4	0	0
13	1366.1	1350.99	1365.11	1.1	0.1
14	1346.2	1182.07	1347.21	12.2	-0.1
15	1332.7	1132.32	1334.5	15	-0.1
16	1320.6	1174.25	1315.78	11.1	0.4
17	1341.4	1264.18	1342.01	5.8	0
18	1405.2	1366.85	1407.81	2.7	-0.2
19	1403.3	1162.88	1404.19	17.1	-0.1
20	1380.9	1144.36	1382.12	17.1	-0.1
21	1406.5	1174.45	1406.14	16.5	0
22	1440.9	1154.32	1339.92	19.9	7
23	1390.9	1134.51	887	18.4	36.2
24	1281.1	1099.54	1281.07	14.2	0

Table (P3.10) Predicted load and percentage error for a winter week day, Model B

Daily hours	Actual load(MW)	LES Prediction	LAV Prediction	% LES error	% LAV error
1	1096.8	1217.08	1304.44	-10.97	-18.93
2	1014.4	1172.24	1144.71	-15.56	-12.8
3	988.9	1086.51	984.98	-9.87	0.4
4	983.3	1038.37	983.15	-5.6	0.02
5	991.3	1012.94	992.67	-2.18	-0.14
6	1003.1	1063.71	1002.5	-6.04	0.06
7	1050.2	1107.28	1051.01	-5.43	-0.08
8	1187.1	1236.25	1187.86	-4.14	-0.06
9	1384	1450.19	1382.71	-4.78	0.09
10	1433	1491.2	1434.98	-4.06	-0.14
11	1404.4	1428.25	1406.94	-1.7	-0.18
12	1354.1	1340.07	1354.2	1.04	-0.01
13	1319.2	1303.76	1319	1.17	0.02
14	1261.3	1186.46	1259.87	5.93	0.11
15	1221.7	1133.64	1219.76	7.21	0.16
16	1184.5	1139.18	1183.88	3.83	0.05
17	1187.9	1180.8	1190.33	0.6	-0.2
18	1240.1	1249.37	1243.48	-0.75	-0.27
19	1307.1	1204.12	1200.94	7.88	8.1
20	1361.9	1221.47	1200.94	10.31	11.8
21	1349.6	1249.4	1199.1	7.42	11.2
22	1313.9	1225.59	1313.89	6.72	0
23	1262.5	1202.34	1264.48	4.76	-0.16
24	1166.7	1128.54	1166.23	3.27	0.04

Table (P3.11) Estimated load and percentage error for a winter weekend day, Model B

Daily hours	Actual load(MW)	LES Estimation	LAV Estimation	% LES error	% LAV error
1	916.8	899.66	780	1.9	14.9
2	846.7	846.56	836.88	0	15.5
3	810.4	769.84	650	5	19.8
4	801.4	748.77	652.21	6.6	18.6
5	796.9	699.11	647.98	12.3	18.7
6	808.4	706.74	645.76	12.6	20.1
7	826	696.44	659.44	15.7	20.2
8	869.7	729.43	687.89	16.1	20.9
9	939.8	847.3	765.89	9.8	18.5
10	1031.8	959.46	892.76	7	13.5
11	1092.4	999.63	972.25	8.5	11
12	1093.8	986.42	990.06	9.8	9.5
13	1072.2	973.71	981.79	9.2	8.4
14	1030.2	876.4	950.87	14.9	7.7
15	981.6	827.58	908.69	15.7	7.4
16	937.4	823.16	871.66	12.2	7
17	912	830	862.19	9	5.5
18	928.4	881.36	893.76	5.1	3.7
19	938.2	816.56	908.56	13	3.2
20	915.2	790.67	901.3	13.6	1.5
21	930.8	795.04	894.04	14.6	3.9
22	1004.6	798.32	954.98	20.5	4.9
23	994.3	810.64	887	18.5	10.8
24	942.9	789.79	835	16.2	11.4

Table (P3.12) Predicted load and percentage error for a winter weekend day, Model B

Daily hours	Actual load(MW)	LES Prediction	LAV Prediction	% LES error	% LAV error
1	950.4	1089.5	938.83	-14.64	1.21
2	874.5	1050.04	896.94	-20.07	-2.56
3	838.7	979.43	805.05	-16.78	4.01
4	814	929.21	813.16	-14.15	0.1
5	806.3	858.62	809.71	-6.49	-0.42
6	800.4	872.44	800.31	-9	0.01
7	811.1	862.16	811.44	-6.3	-0.04
8	833.2	895.18	833.49	-7.44	-0.03
9	918.6	1017.23	916.64	-10.74	0.21
10	1038.1	1121.45	1041.85	-8.03	-0.36
11	1116.9	1157.88	1120.92	-3.67	-0.36
12	1148.4	1143	1149.4	0.47	-0.09
13	1158	1140.82	1155.89	1.48	0.18
14	1135.1	1041.03	1133.63	8.29	0.13
15	1098.2	991.95	1095.1	9.68	0.28
16	1074.5	1004.23	1072.26	6.54	0.21
17	1072.9	1038.91	1075.77	3.17	-0.27
18	1110.2	1110.88	1116.84	-0.06	-0.6
19	1135.1	1021.23	1132.05	10.03	0.27
20	1141.2	996.94	1148.43	12.64	-0.63
21	1159.6	1023.25	1164.81	11.76	-0.45
22	1112.8	960.62	1127.33	13.68	-1.31
23	1060.7	961.22	1063.46	9.38	-0.26
24	981.7	925	980.73	5.78	0.1

Model C :

Table (P3.13) Estimated load and percentage error for a winter day, Model C

Daily hours	Actual load(MW)	LES Estimation	LAV Estimation	% LES error	% LAV error
1	943.4	942.15	943.82	0.13	-0.04
2	850.5	850.56	849.79	-0.01	0.08
3	811.2	814.68	829.75	-0.43	-2.29
4	793.6	791.68	793.89	0.24	-0.04
5	794.6	792.57	794.7	0.26	-0.01
6	810.3	810.78	810.07	-0.06	0.03
7	855.2	856.41	855.73	-0.14	-0.06
8	991	992.96	991.31	-0.2	-0.03
9	1198.5	1197.02	1198.36	0.12	0.01
10	1302.3	1301.02	1301.63	0.1	0.05
11	1331.8	1333.28	1331.78	-0.11	0
12	1344.7	1343.98	1344.41	0.05	0.02
13	1366.1	1366.08	1366.82	0	-0.05
14	1346.2	1347.41	1346.24	-0.09	0
15	1332.7	1332.93	1333	-0.02	-0.02
16	1320.6	1318.58	1320.56	0.15	0
17	1341.4	1342.45	1341.48	-0.08	-0.01
18	1405.2	1406.36	1405.07	-0.08	0.01
19	1403.3	1404.3	1403.34	-0.07	0
20	1380.9	1380.14	1381.19	0.06	-0.02
21	1406.5	1402.9	1406.43	0.26	0
22	1440.9	1443.07	1440.71	-0.15	0.01
23	1390.9	1392.63	1390.79	-0.12	0.01
24	1281.1	1280.05	1281.77	0.08	-0.05

Table (P3.14) Predicted load and percentage error for a winter day, Model C

Daily hours	Actual load(MW)	LES Prediction	LAV Prediction	% LES error	% LAV error
1	1117.6	972.23	974.73	13.01	12.78
2	1006.4	875.64	875.91	12.99	12.97
3	943.6	904.36	922.82	4.16	2.2
4	871.1	905.33	912.45	-3.93	-4.75
5	813	861.62	867.75	-5.98	-6.73
6	869.7	848.86	851	2.4	2.15
7	914.8	847.76	847.99	7.33	7.3
8	978.7	922.48	919.01	5.74	6.1
9	1157.3	1053.34	1049.43	8.98	9.32
10	1223	1103.54	1095.86	9.77	10.4
11	1216.8	1124.58	1113.24	7.58	8.51
12	1284.3	1166.26	1157.16	9.19	9.9
13	1258.8	1162.27	1152.49	7.67	8.45
14	1207.8	1158.97	1147.51	4.04	4.99
15	1155.4	1168.28	1158.85	-1.12	-0.3
16	1110.6	1114.91	1106.27	-0.39	0.39
17	1094.1	1138.9	1127.06	-4.09	-3.01
18	1113.3	1239.31	1228.27	-11.32	-10.33
19	1186.4	1222.02	1211.05	-3	-2.08
20	1139	1205.36	1196.66	-5.83	-5.06
21	1152.3	1217.28	1210.77	-5.64	-5.07
22	1226.5	1253.05	1240.43	-2.16	-1.14
23	1198.4	1220.65	1209.09	-1.86	-0.89
24	1111.8	1066.59	1057.3	4.07	4.9

Appendix 4
Winter Static Figures

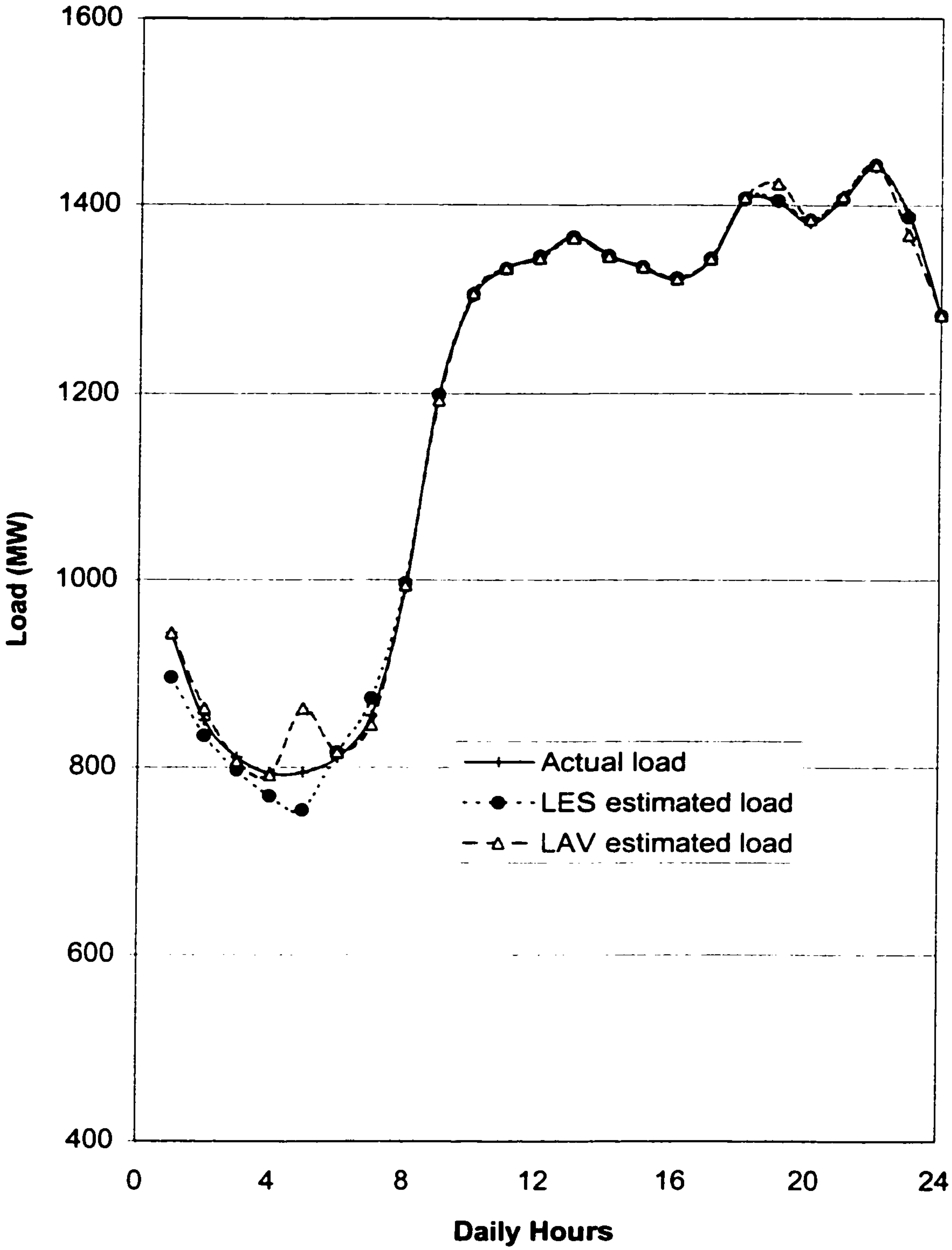


Figure (P4.1) Estimated load for winter weekday using 24 parameters sets, Model A

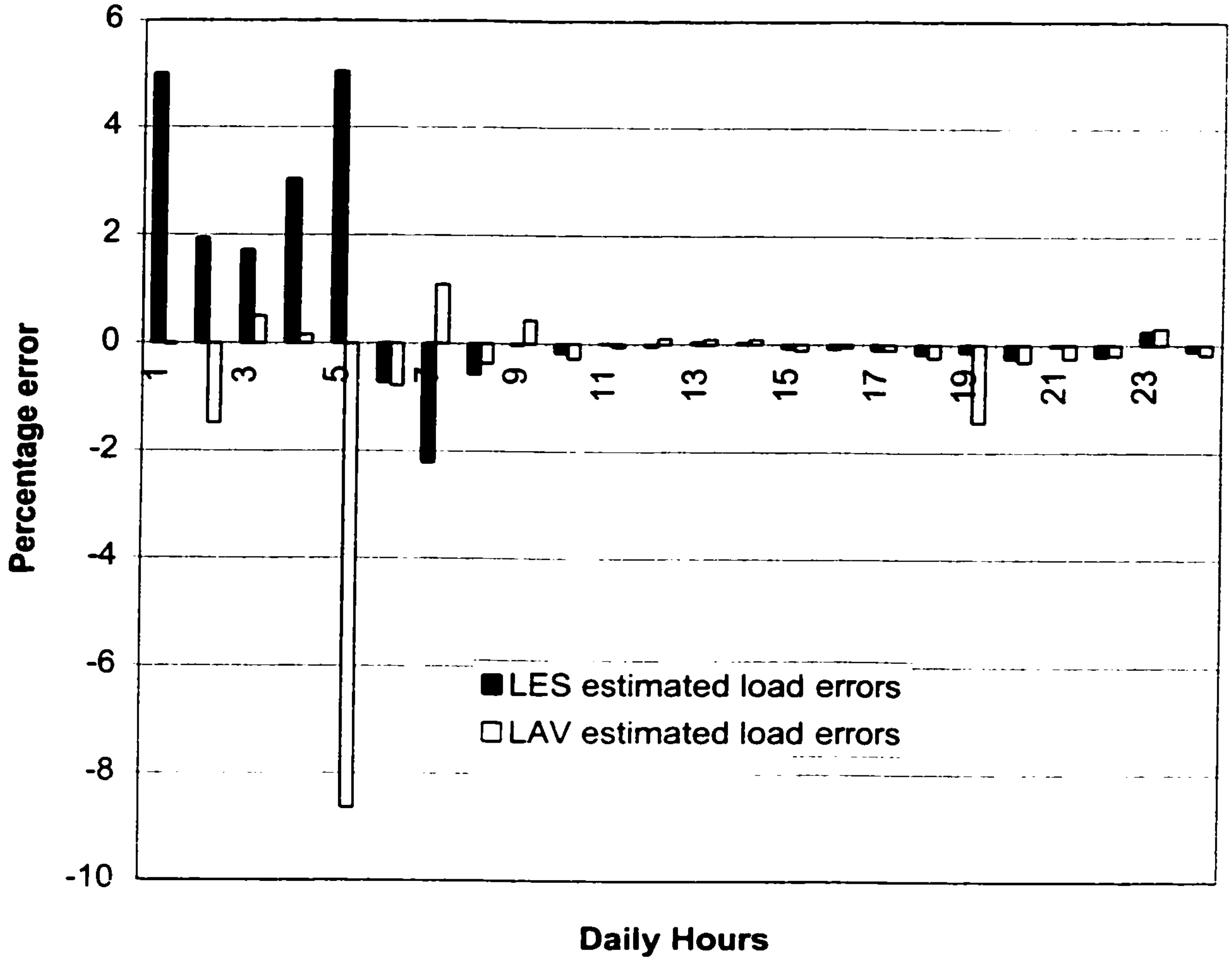


Figure (P4.2) Estimated load error for a winter weekday using 24 parameters sets, Model A

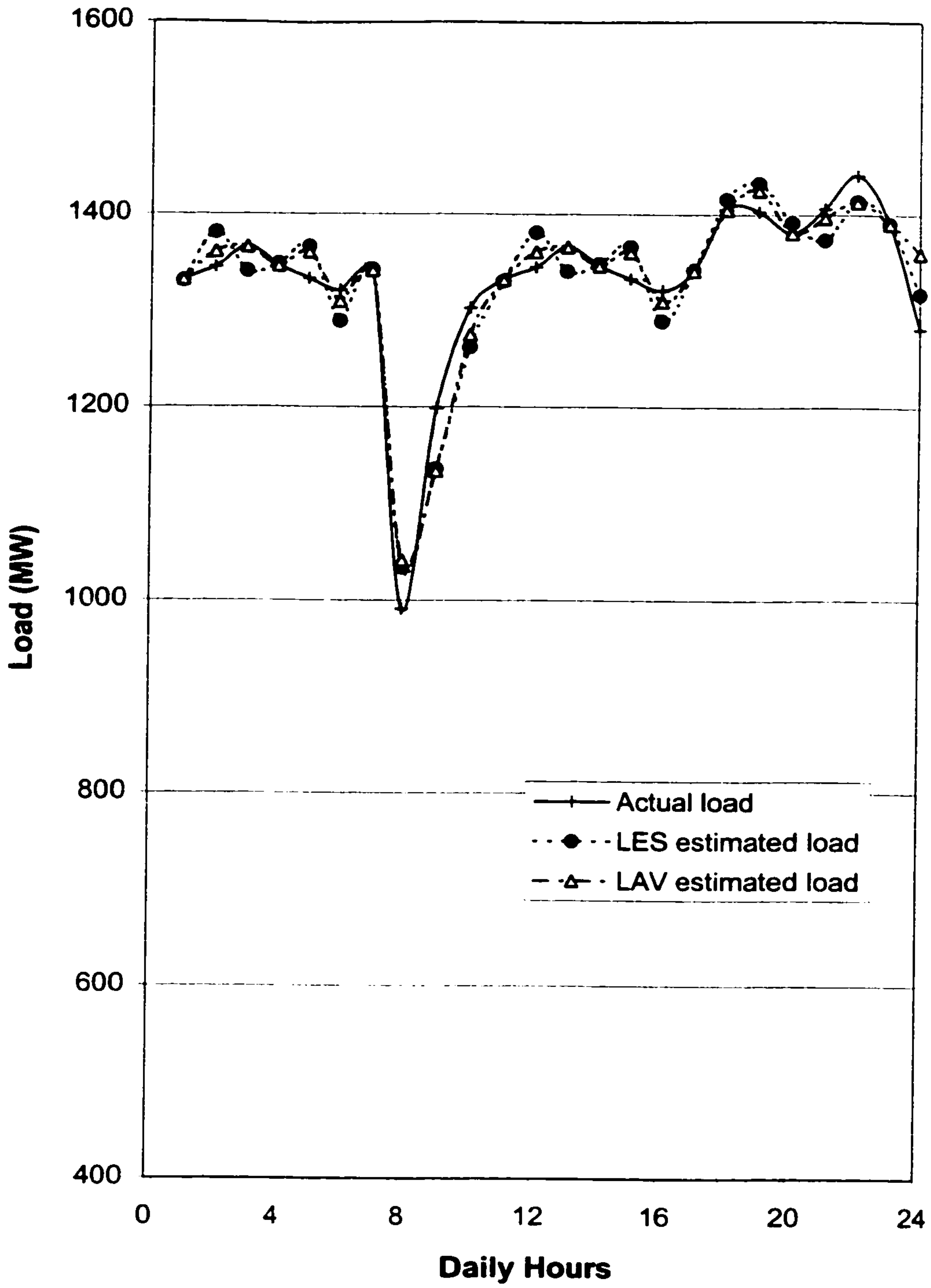


Figure (P4.3) Estimated load for winter weekday using one parameters set, Model A

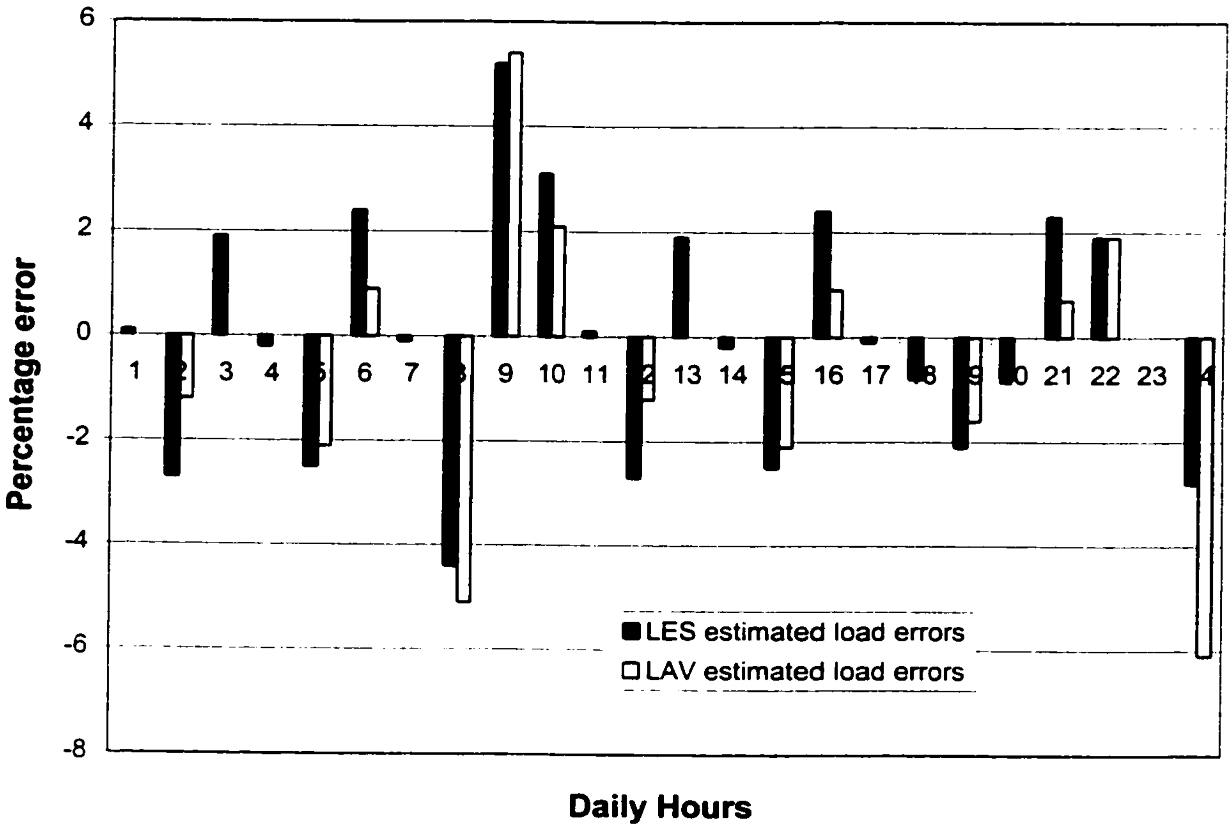


Figure (P4.4) Estimated load error for a winter weekday using one parameters set, Model A

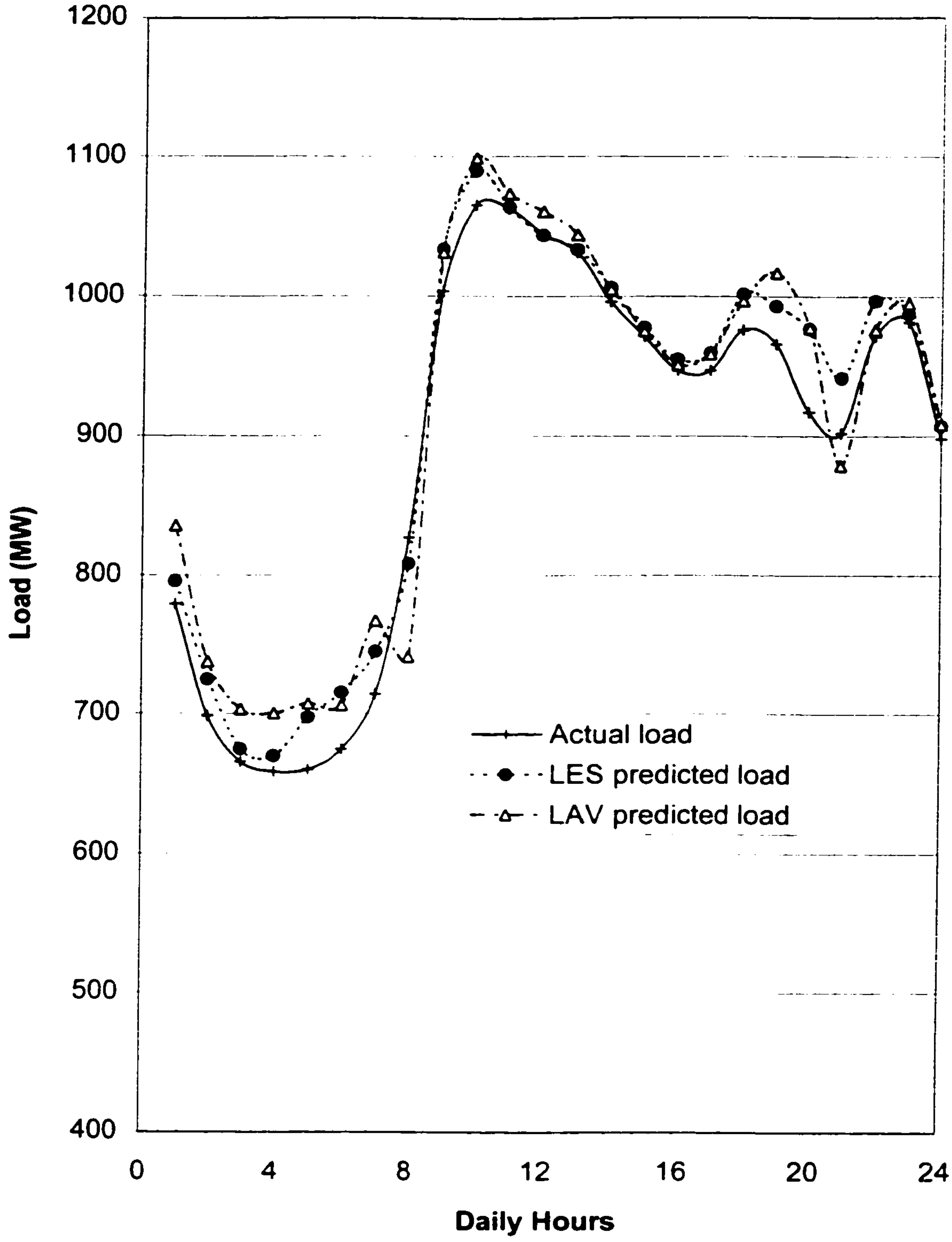


Figure (P4.5) Predicted load for a winter weekday using 24 parameters sets, Model A

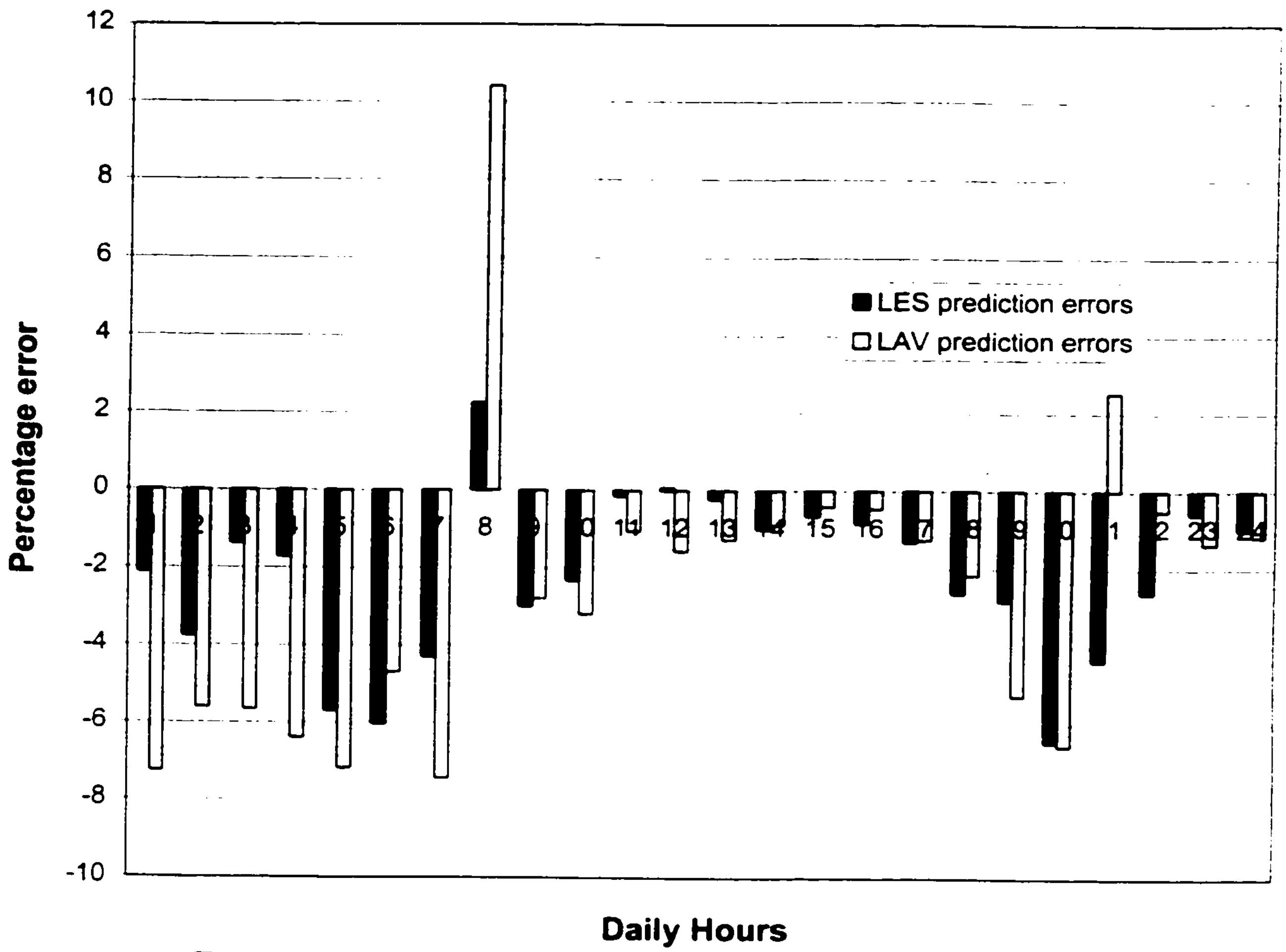


Figure (P4.6) Predicted load error for a winter weekday using 24 parameters sets, Model A

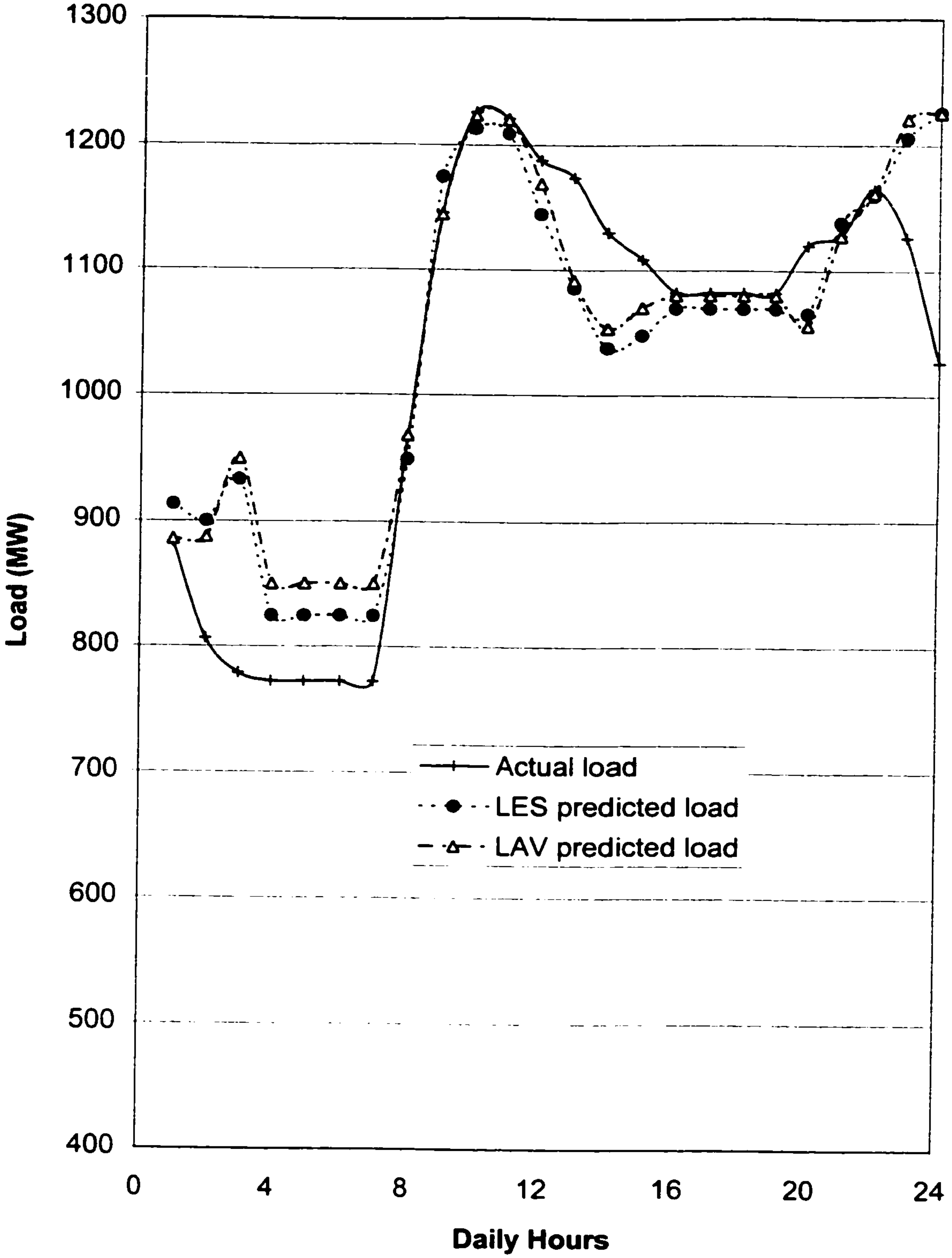


Figure (P4.7) Predicted load for winter weekday using one parameters set, Model A

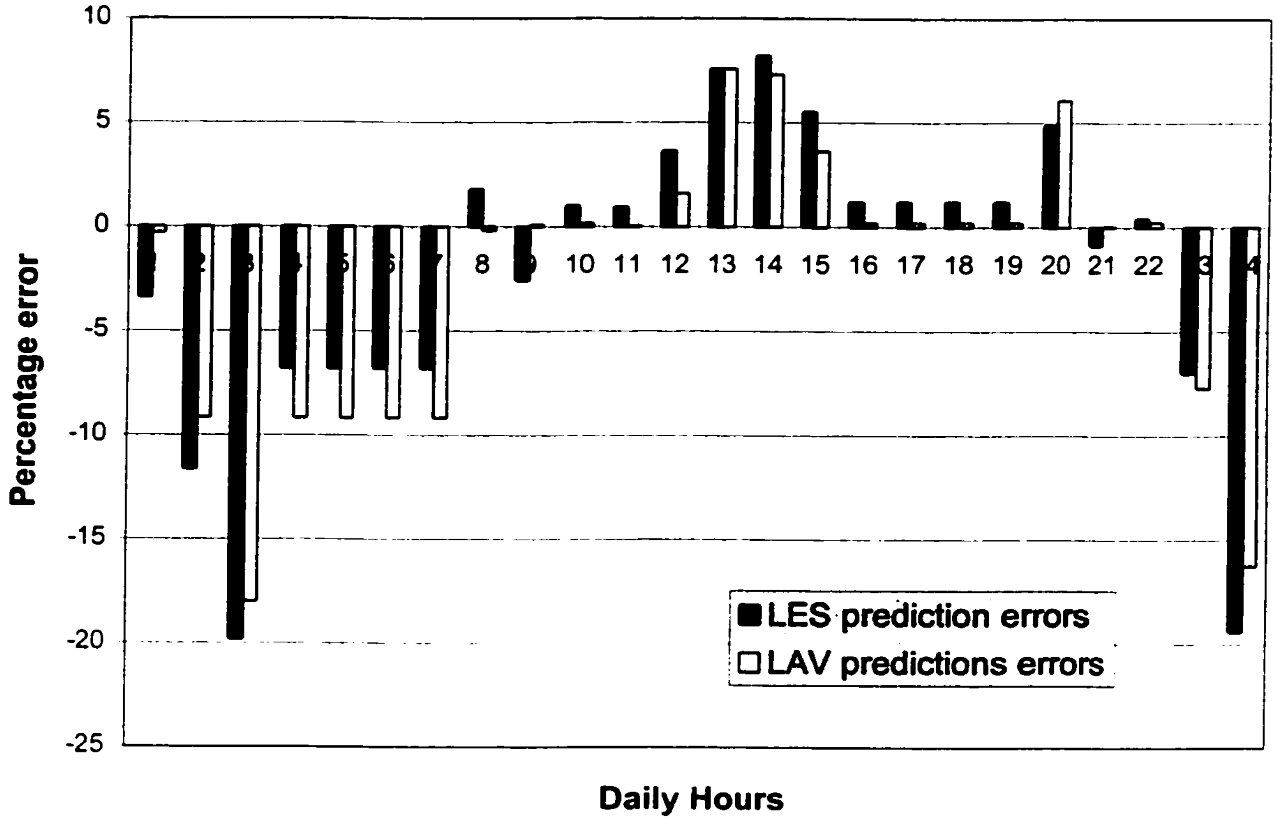


Figure (P4.8) Predicted load error for a winter weekday using one parameters set, Model A

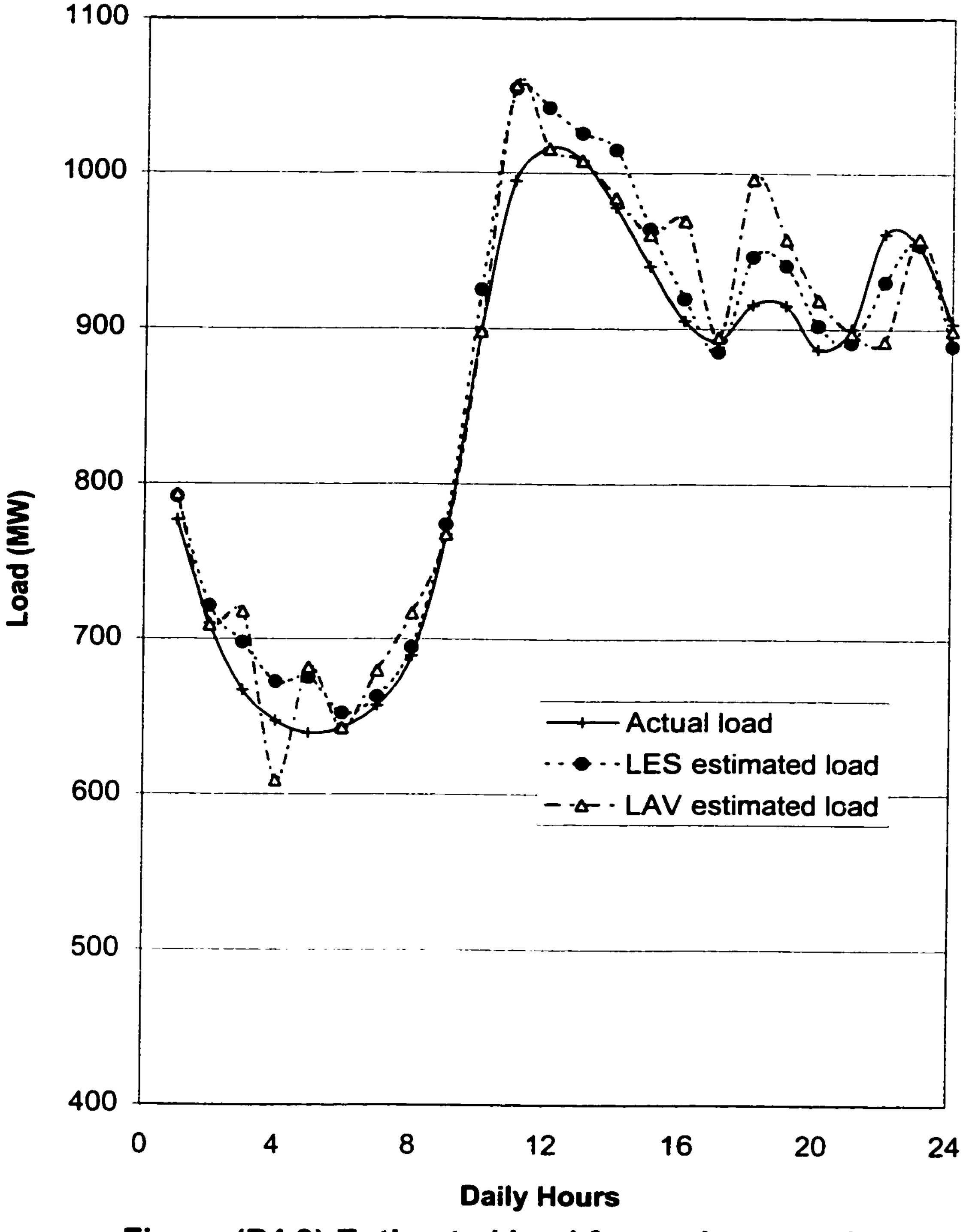


Figure (P4.9) Estimated load for a winter weekend day using 24 parameters sets, Model A

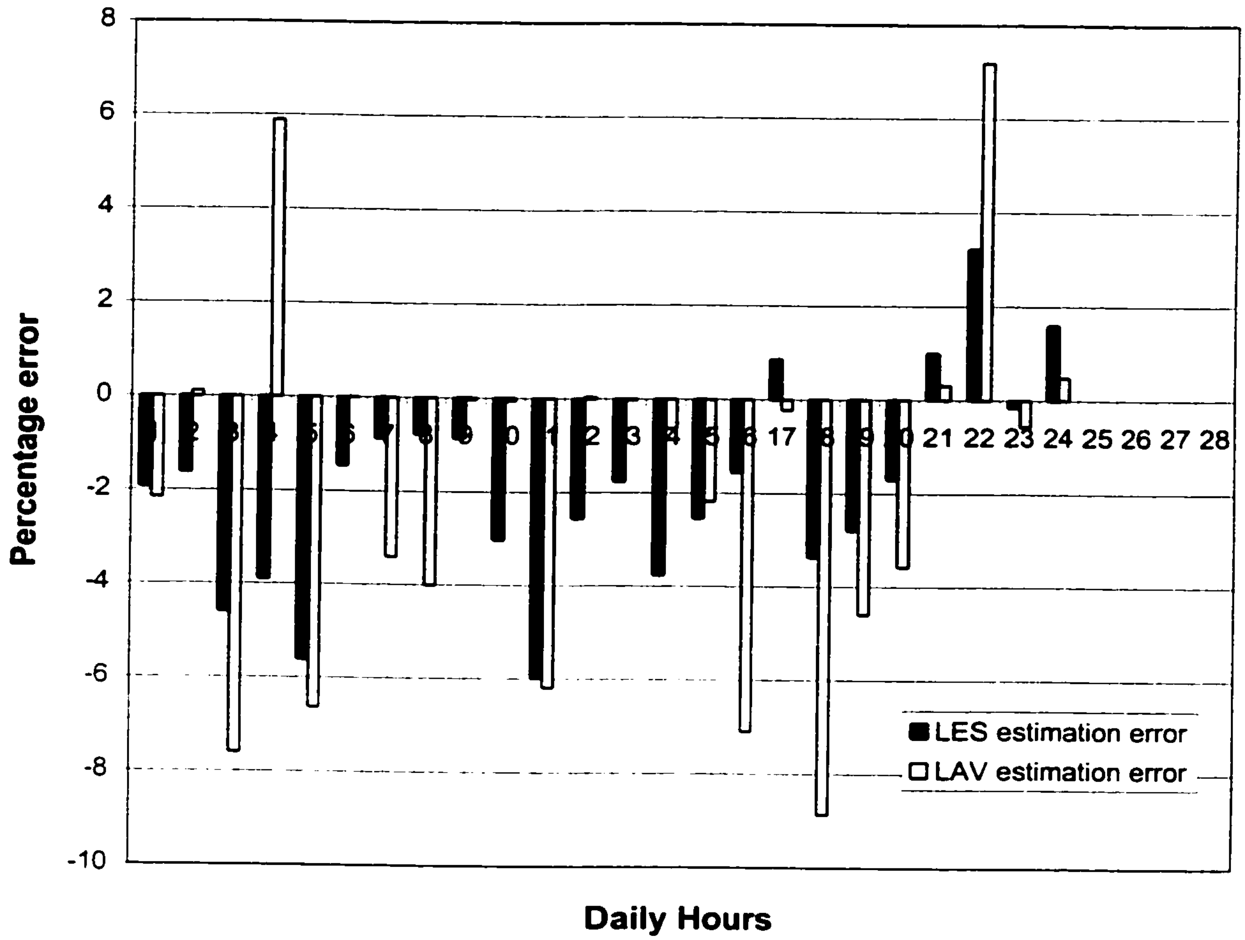


Figure (P4.10) Estimated load error for a winter weekend day using 24 parameters sets, Model A

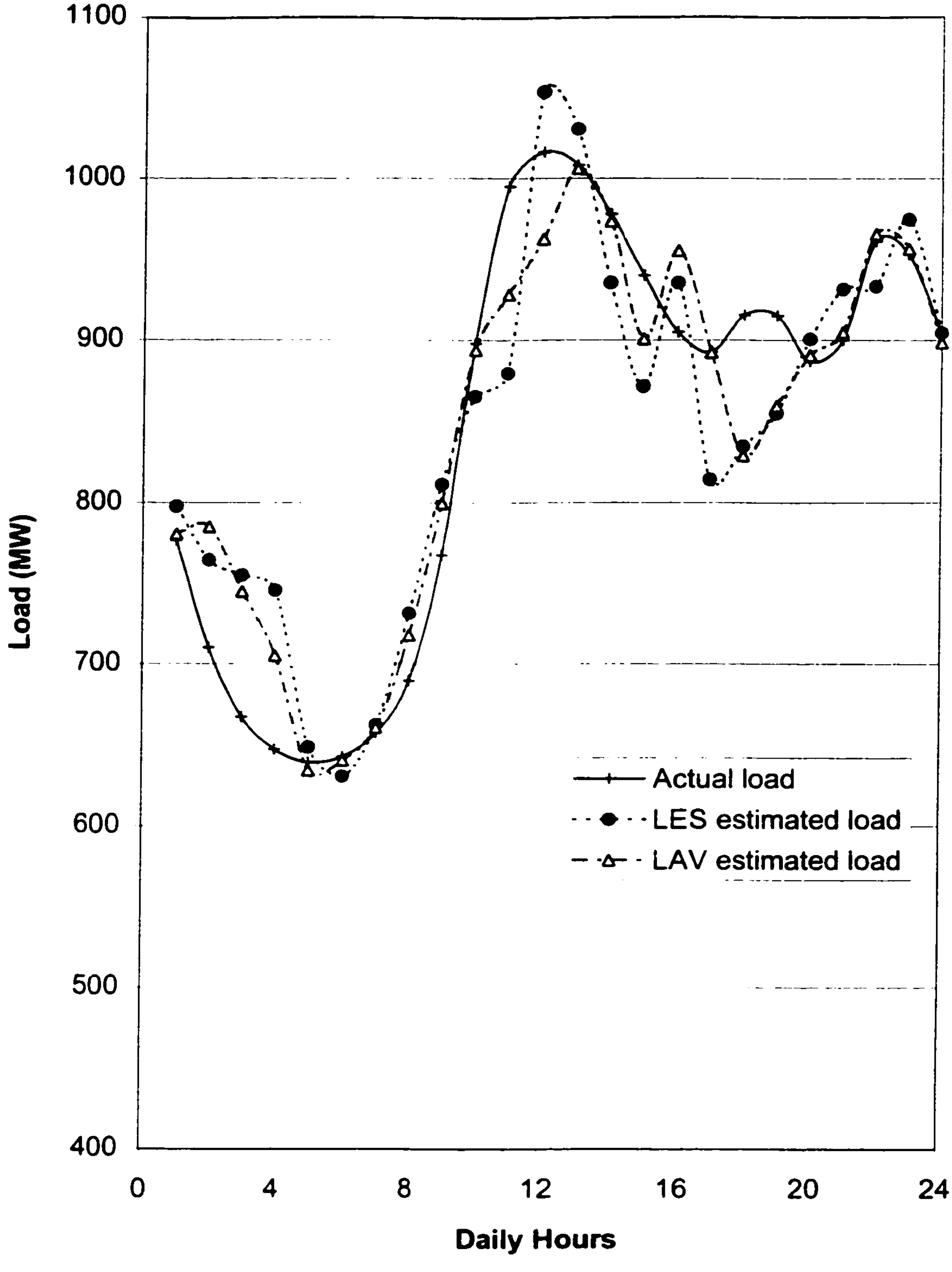


Figure (P4.11) Estimated load for a winter weekend day using one parameters set, Model A

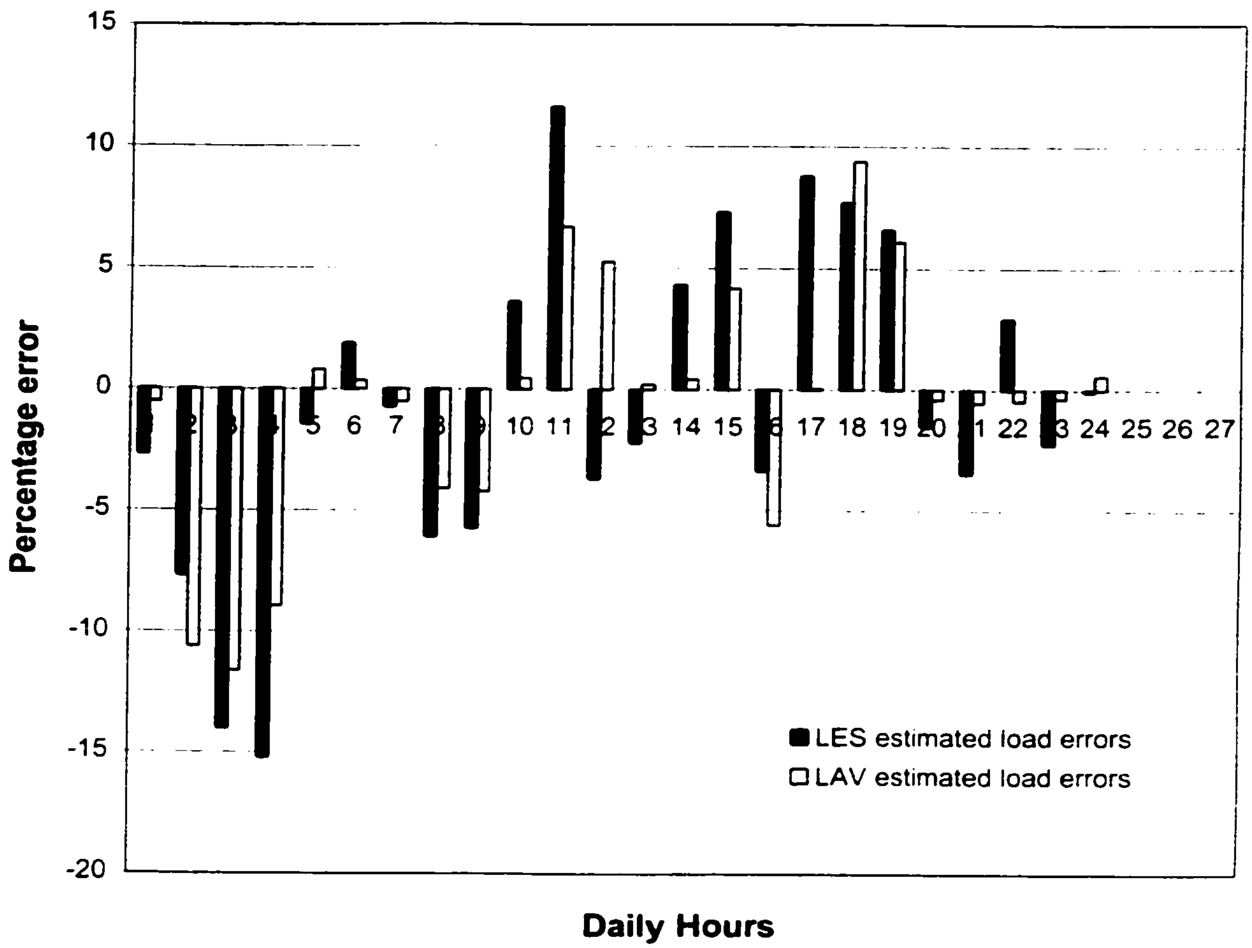


Figure (P4.12) Estimated load error for a winter weekend day using one parameters set, Model A

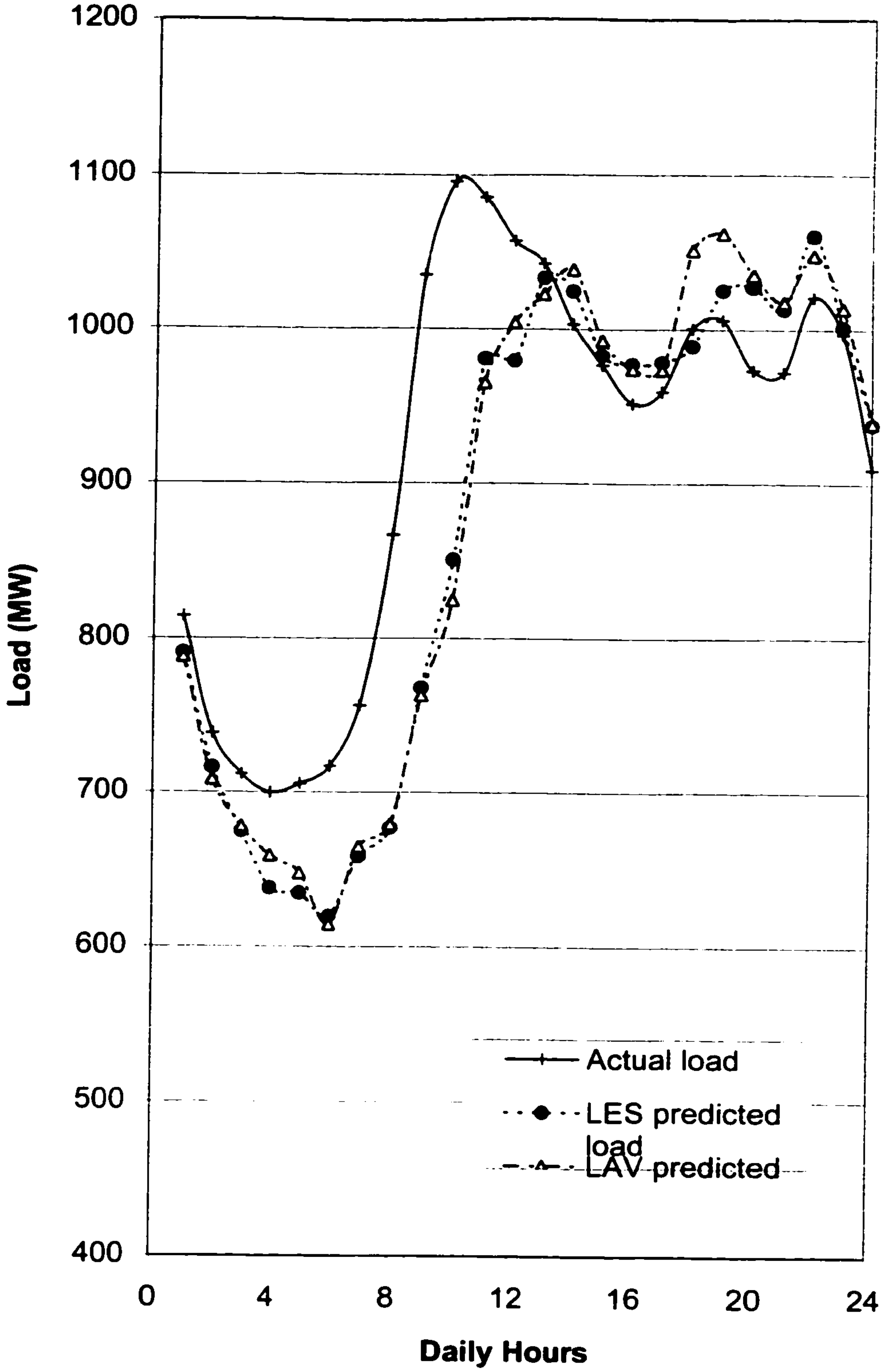


Figure (P4.13) Predicted load for a winter weekend day using 24 parameterts sets, Model A

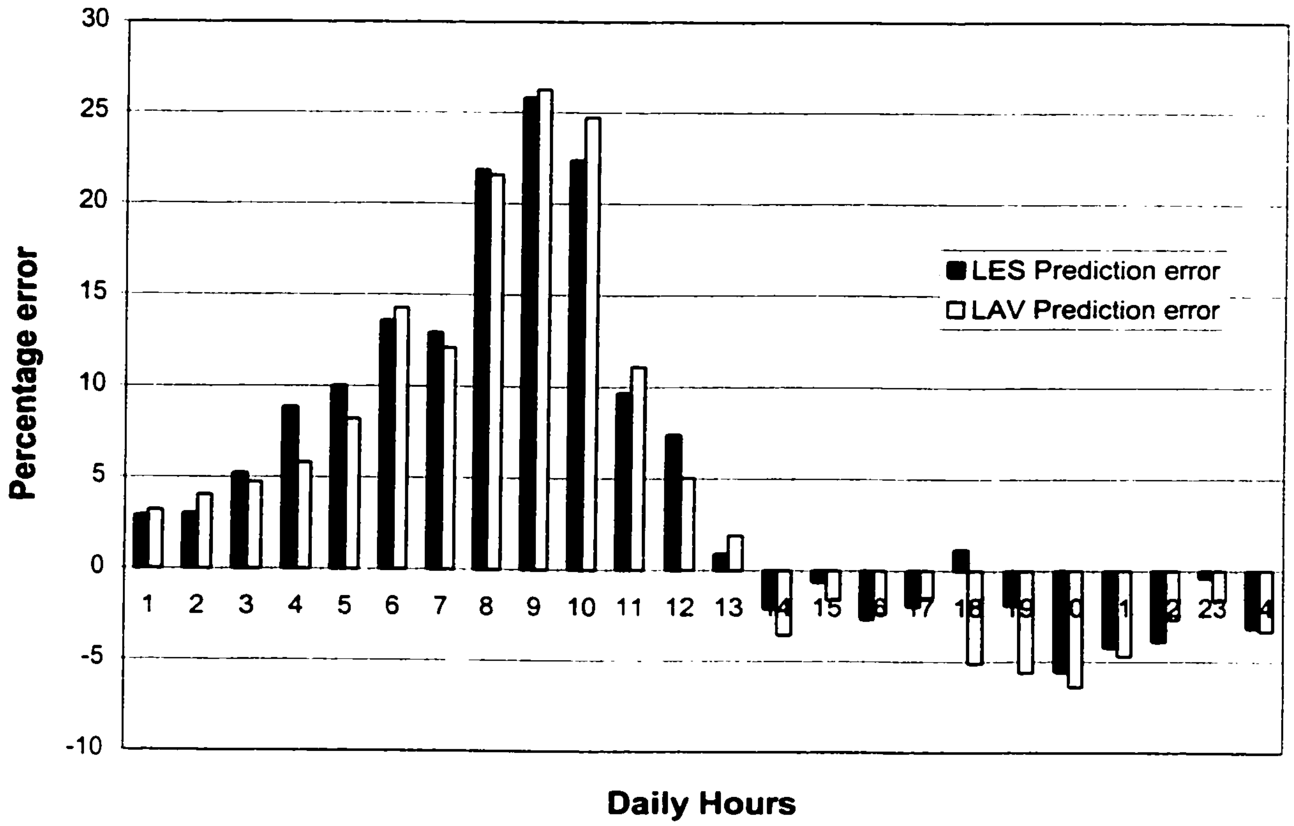


Figure (P4.14) Predicted load error for a winter weekend day using 24 parameters sets, Model A

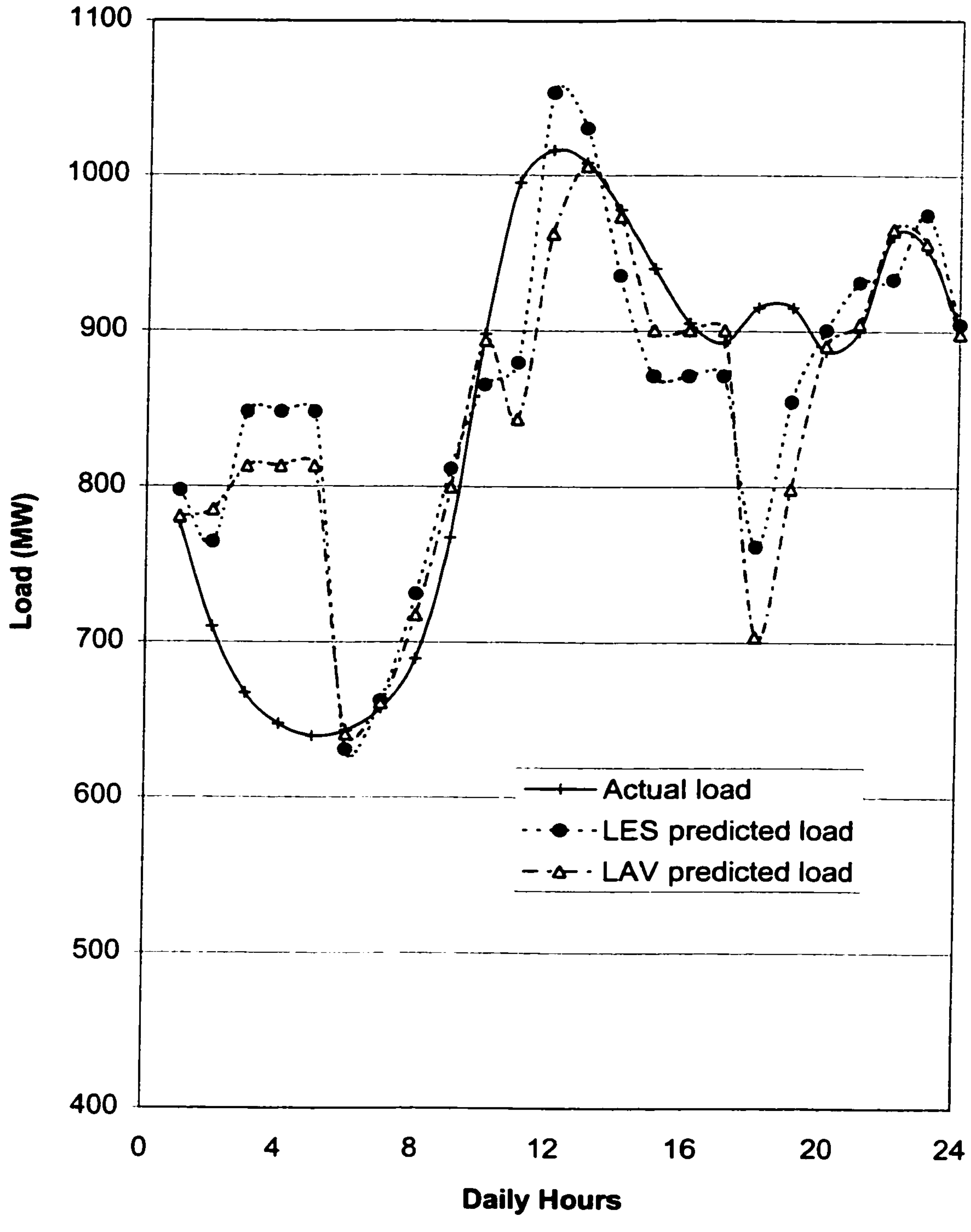


Figure (P4.15) Predicted load for a winter weekend day using one parameters set, Model A

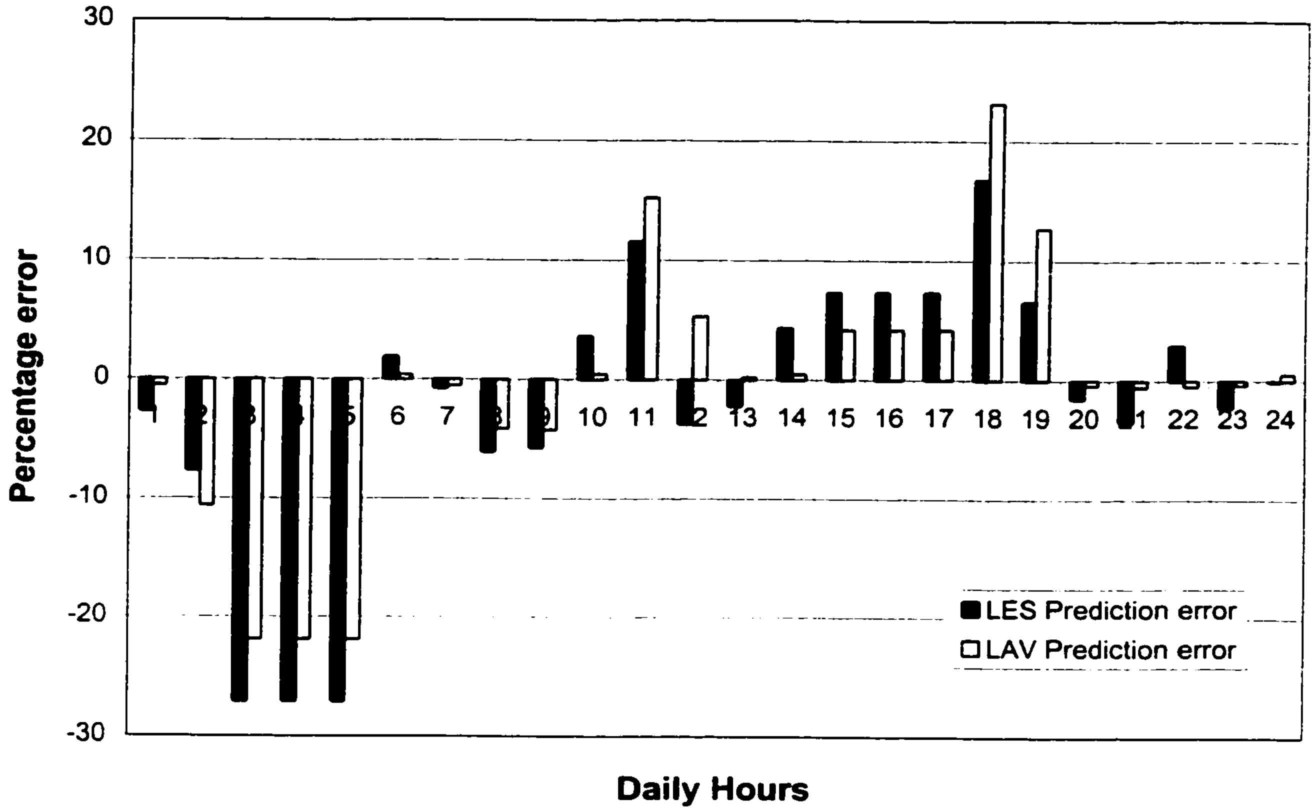


Figure (P4.16) Predicted load error for a winter weekend day using one parameters set, Model A

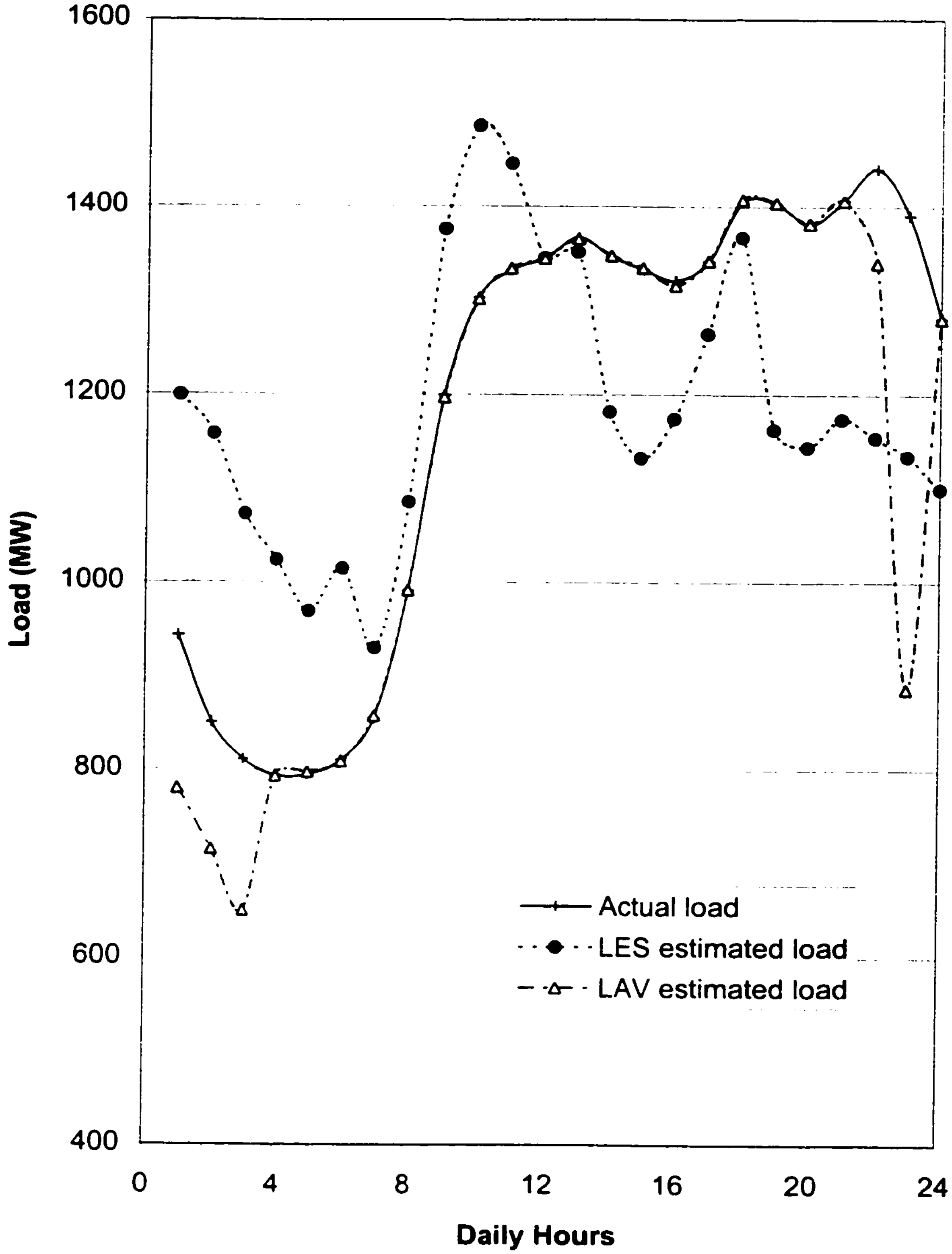


Figure (P4.17) Estimated load for a winter weekday, Model B

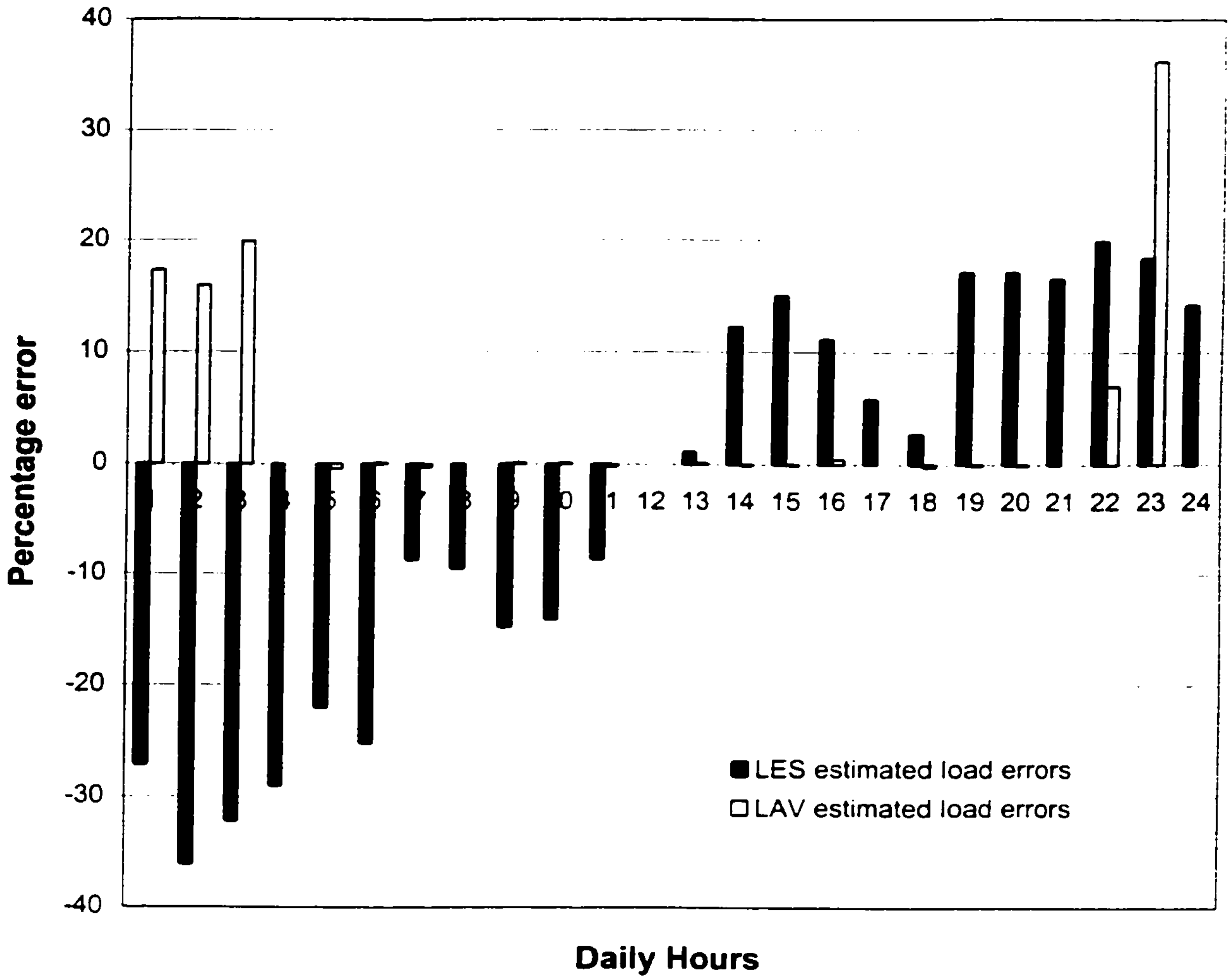


Figure (P4.18) Estimated load error for a winter weekday, Model B

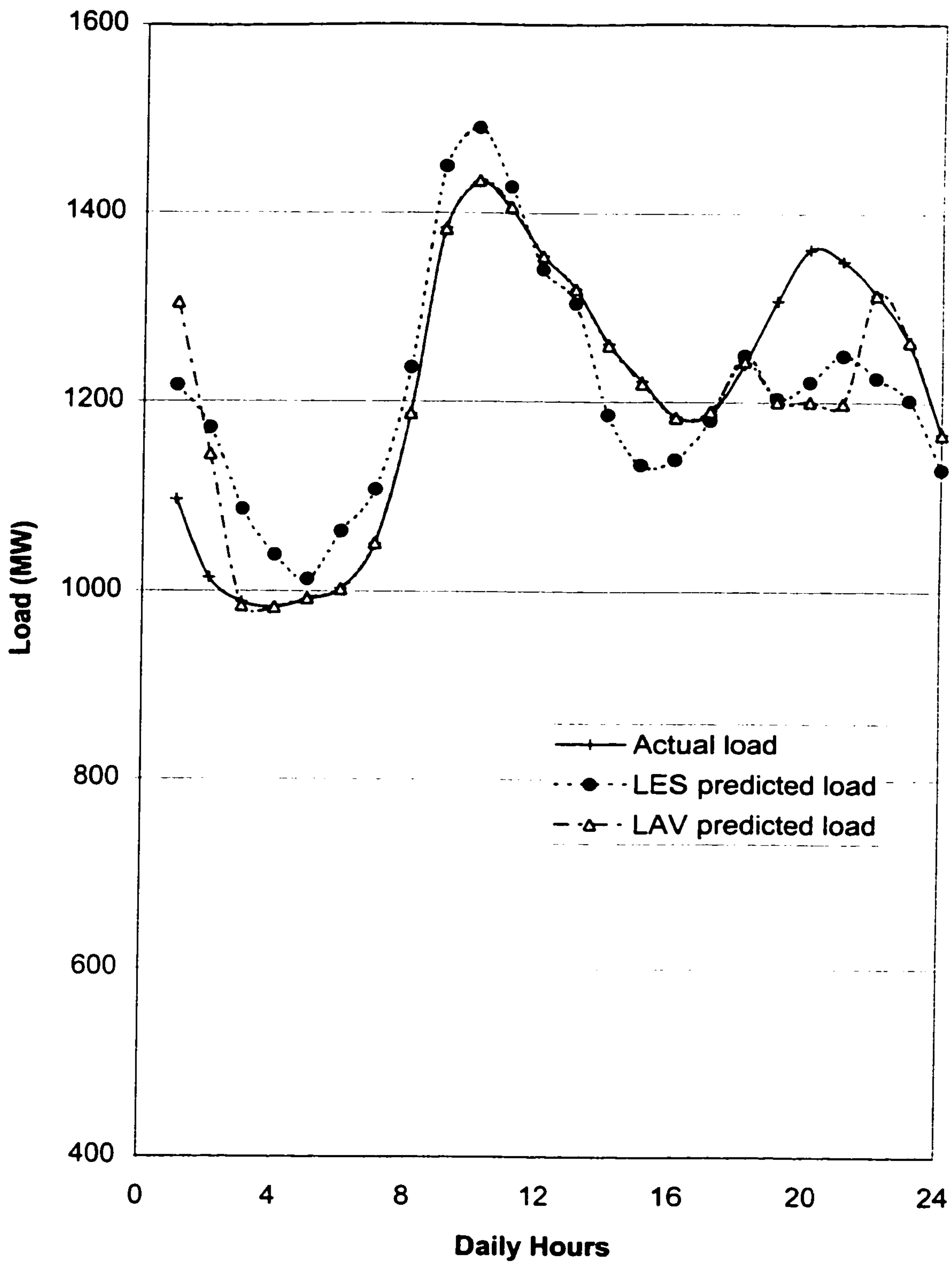


Figure (P4.19) Predicted load for a winter weekday, Model B

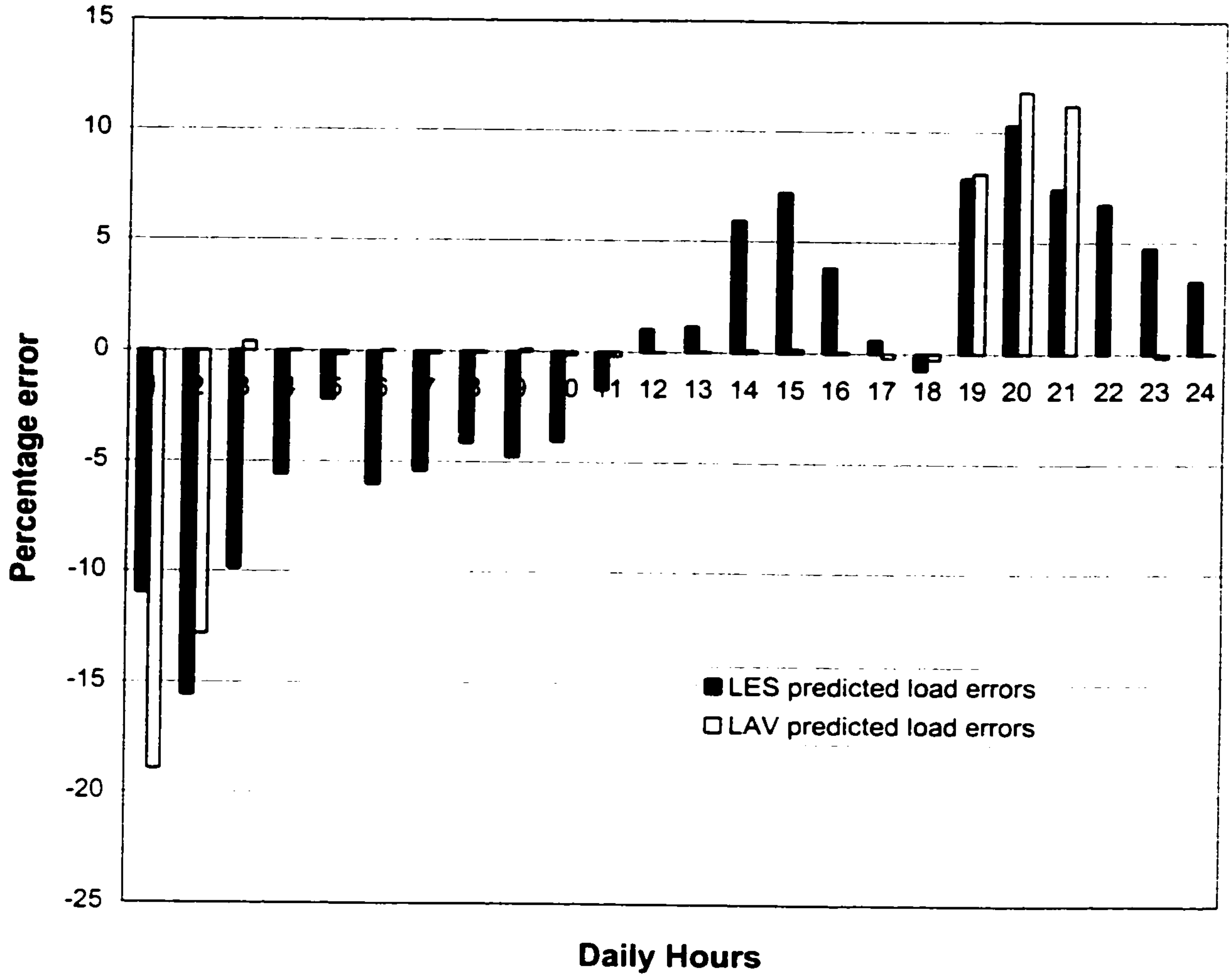


Figure (P4.20) Predicted load error for a winter weekday, Model B

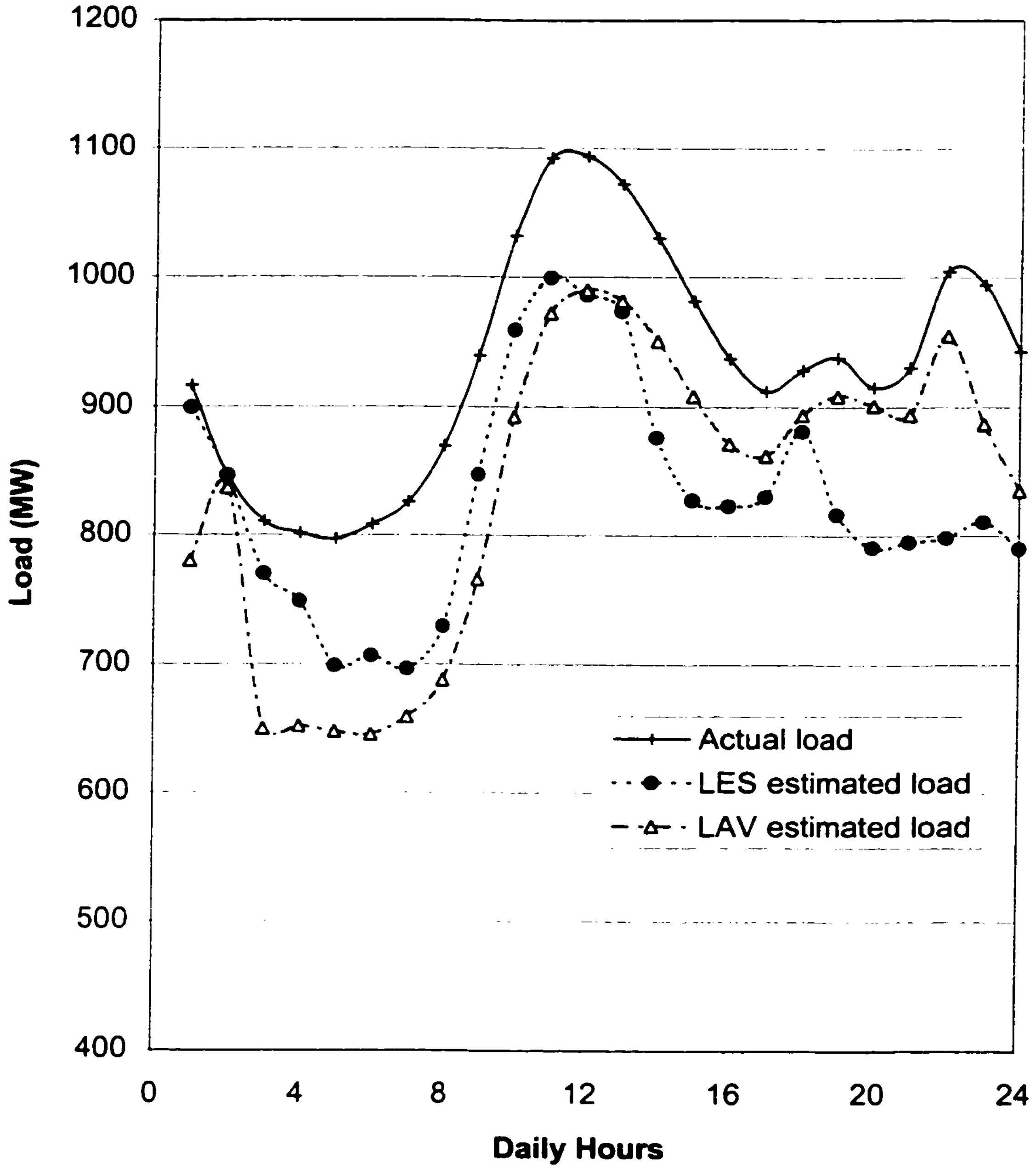


Figure (P4.21) Estimated load for a winter weekend day, Model B

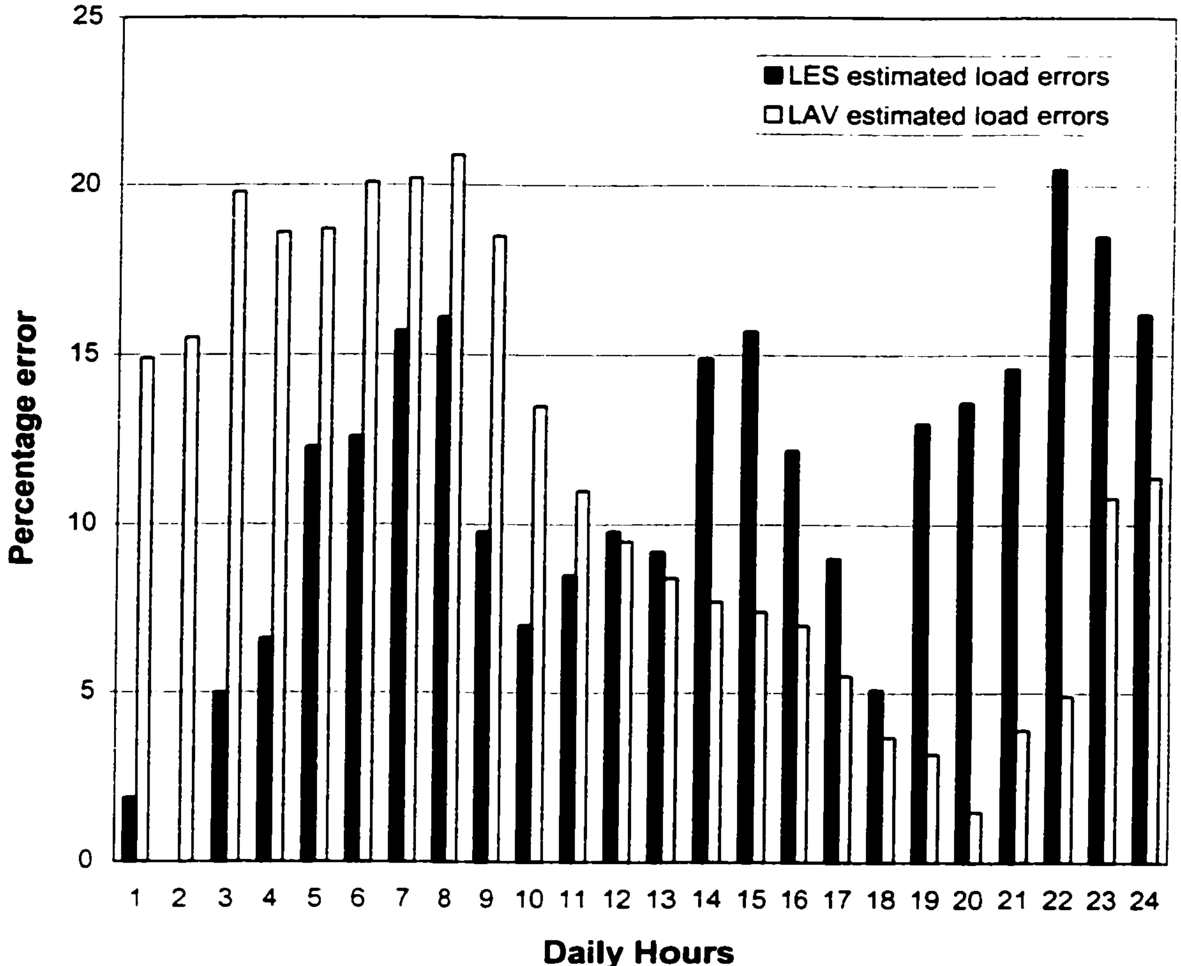


Figure (P4.22) Estimated load error for a winter weekend day, Model B

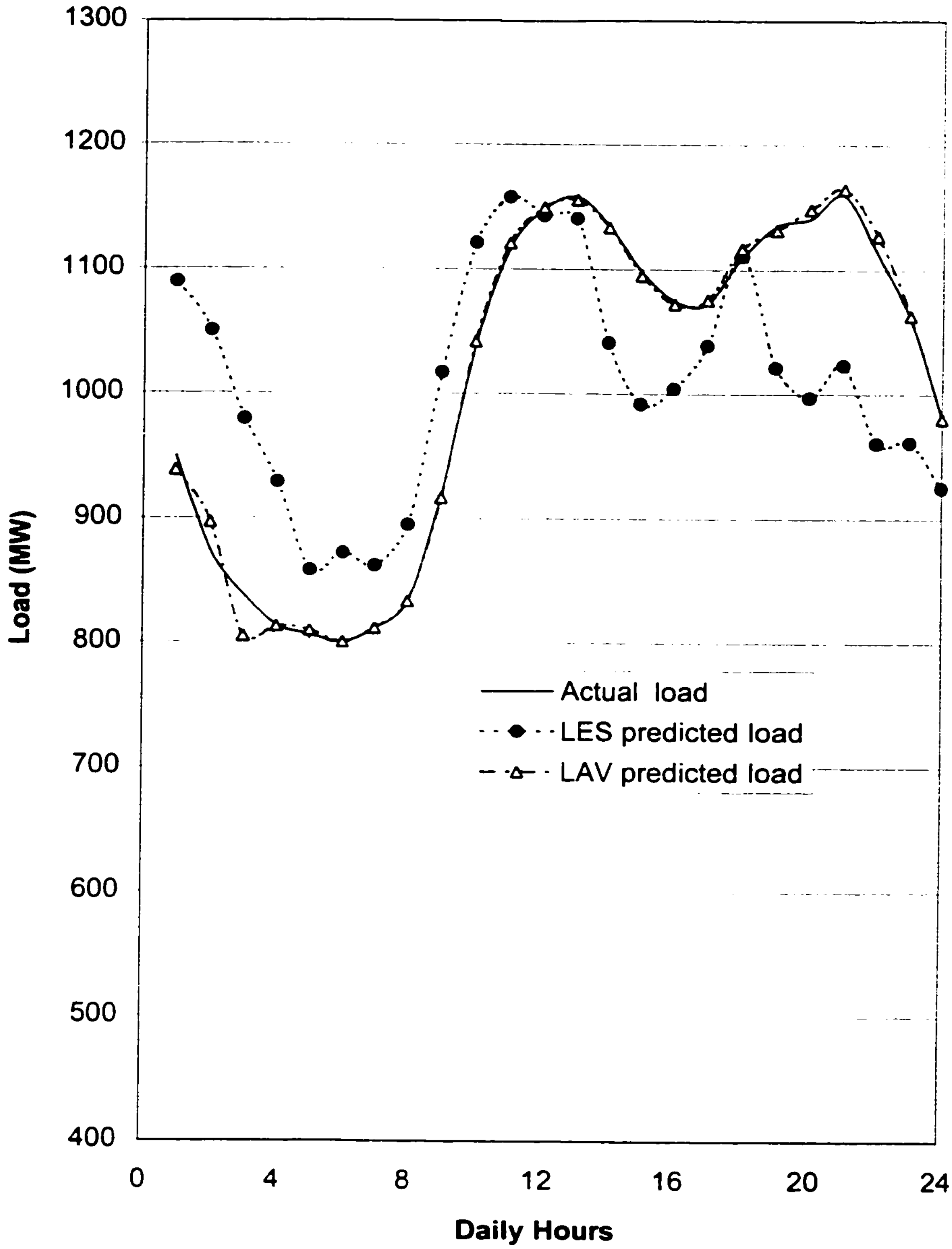


Figure (P4.23) Predicted load for a winter weekend day, Model B

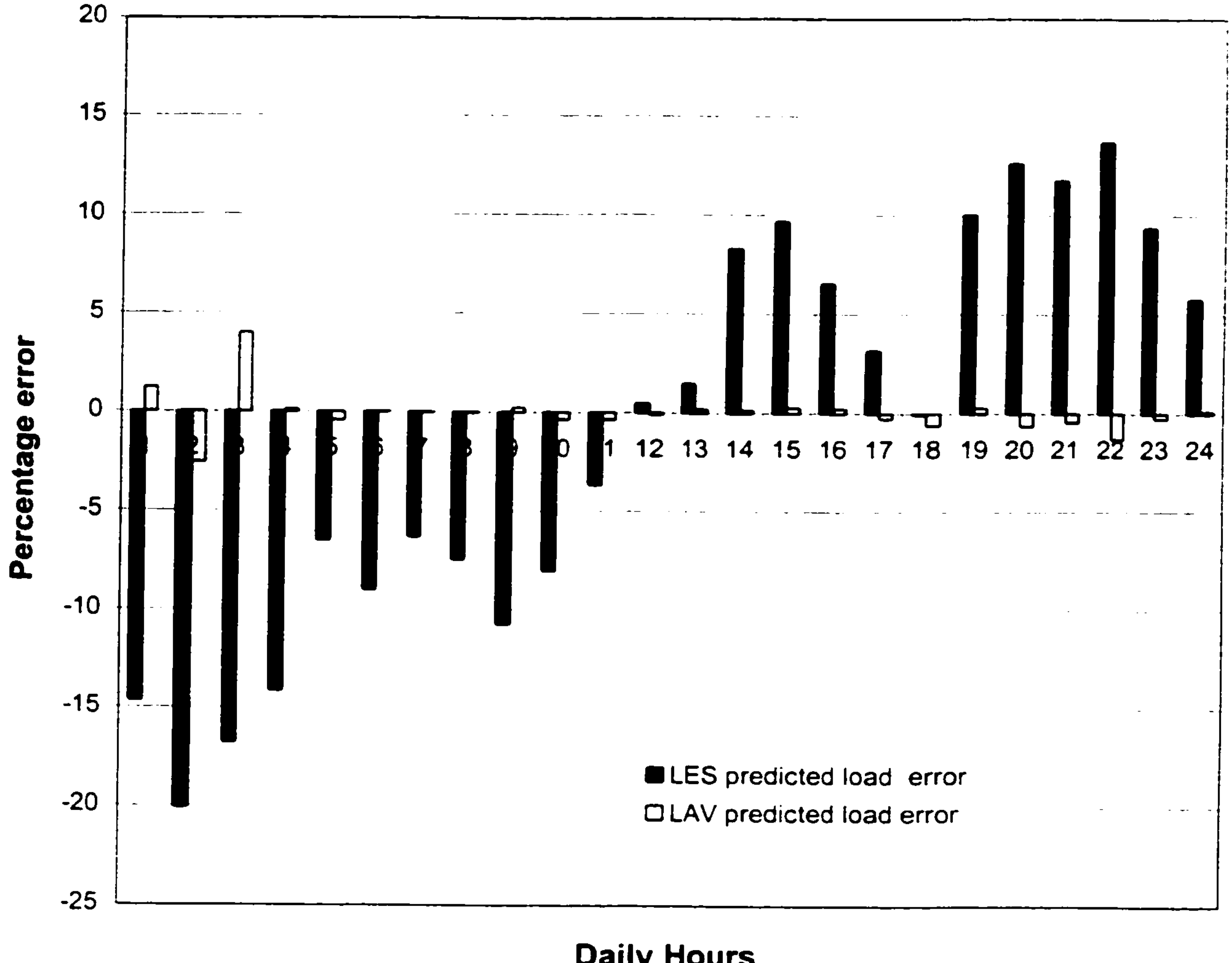


Figure (P4.24) Predicted load error for a winter weekend day, Model B

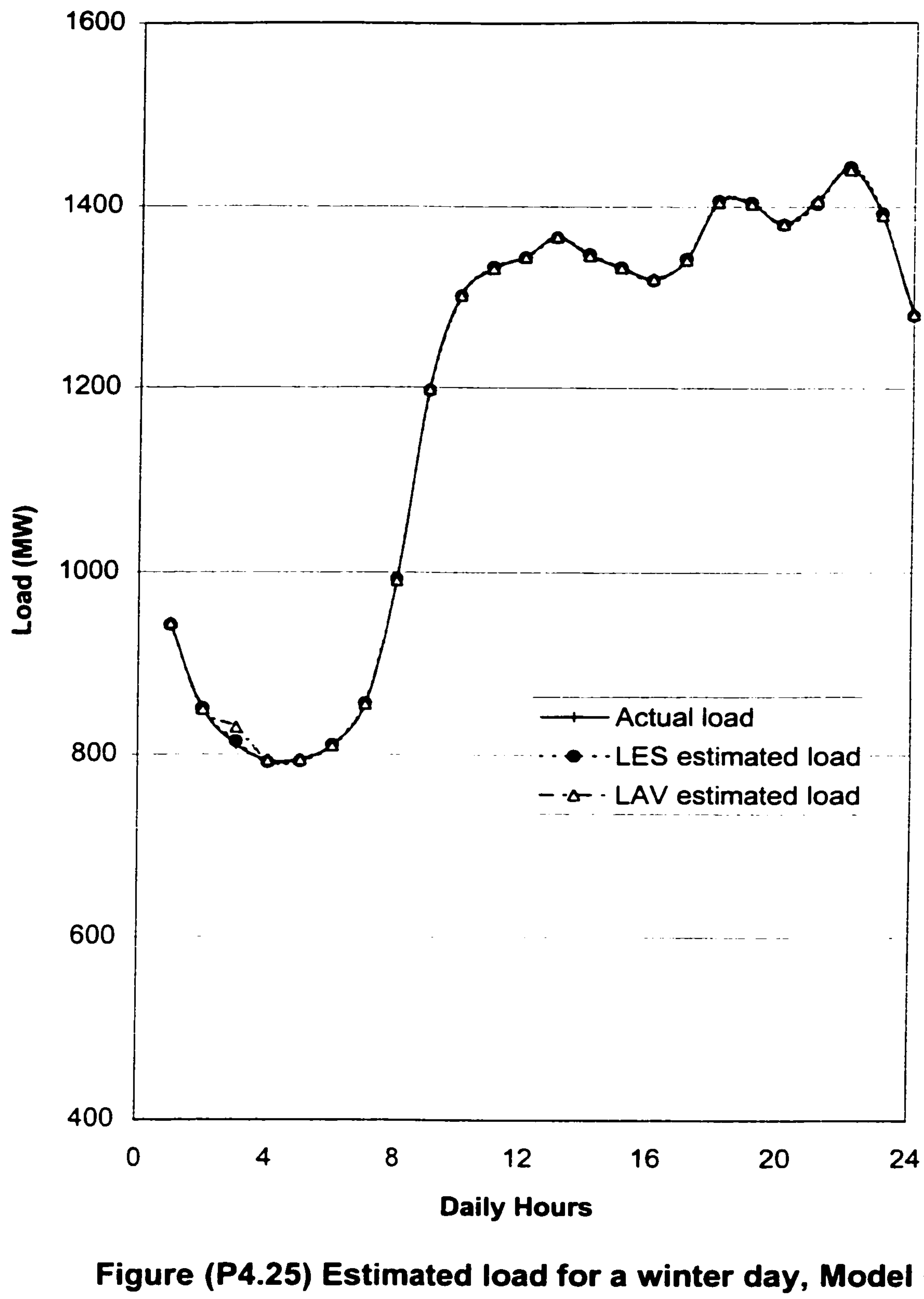


Figure (P4.25) Estimated load for a winter day, Model C

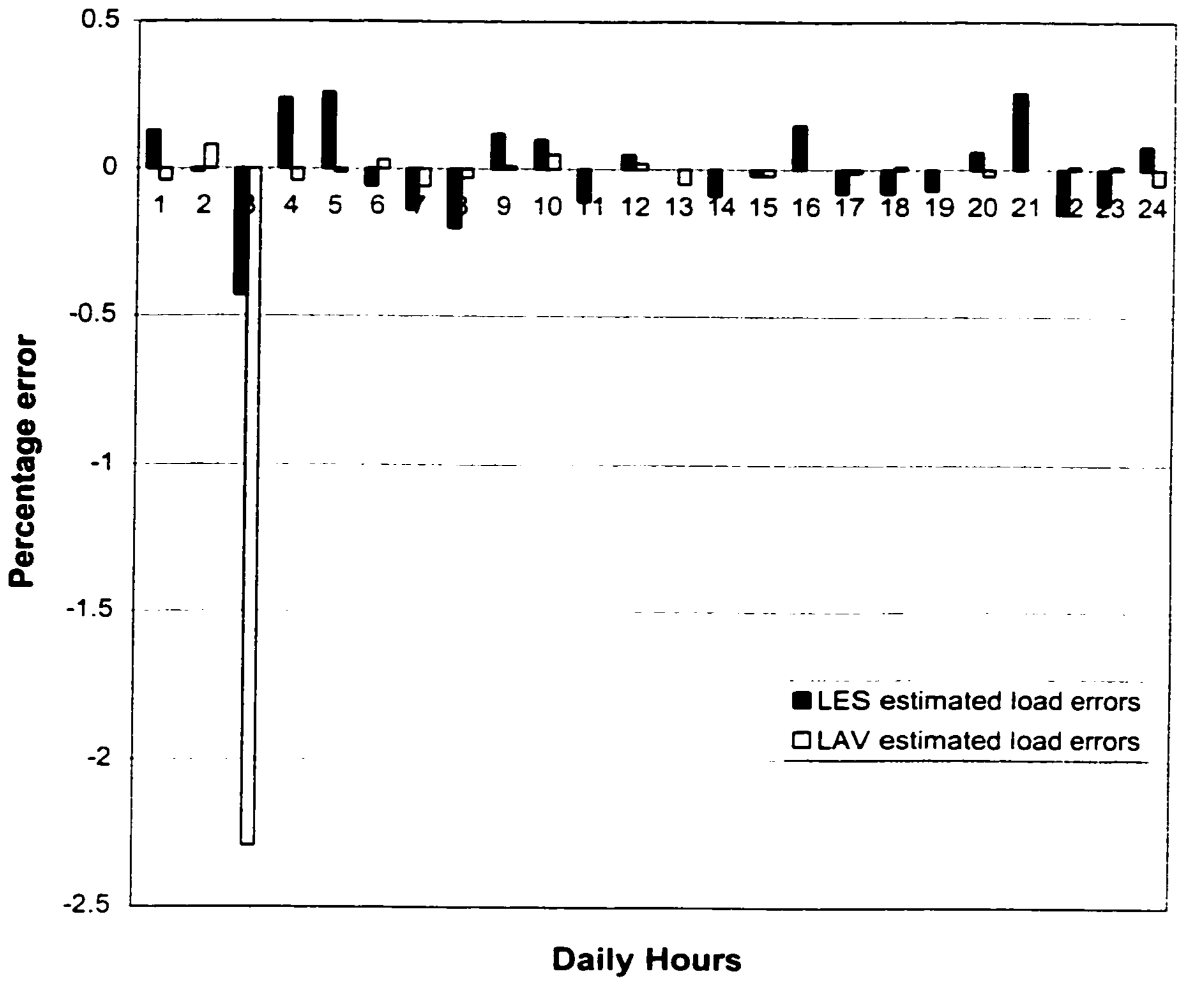


Figure (P4.26) Estimated load error for a winter day, Model C

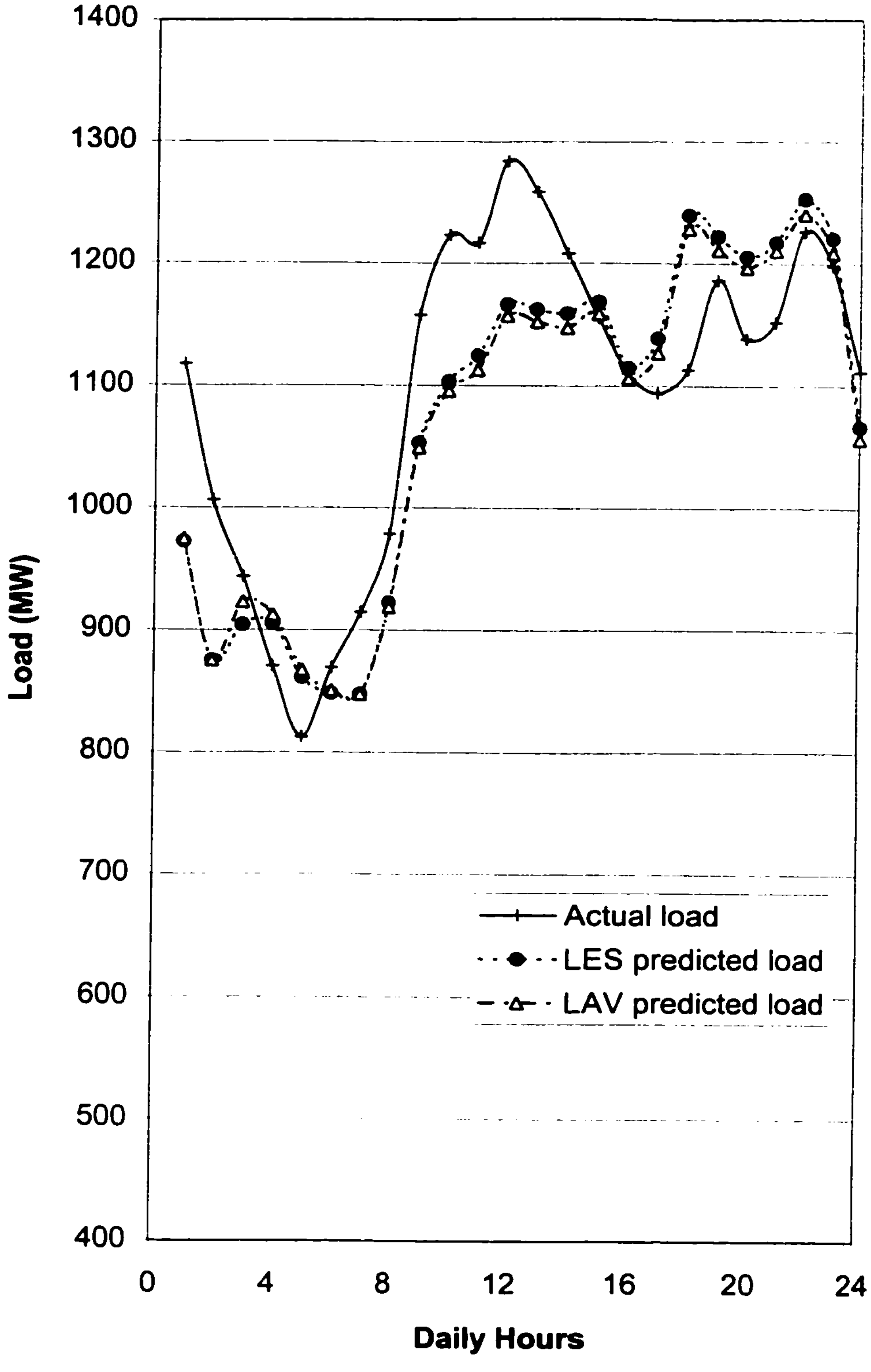


Figure (P4.27) Predicted load for a winter day, Model C

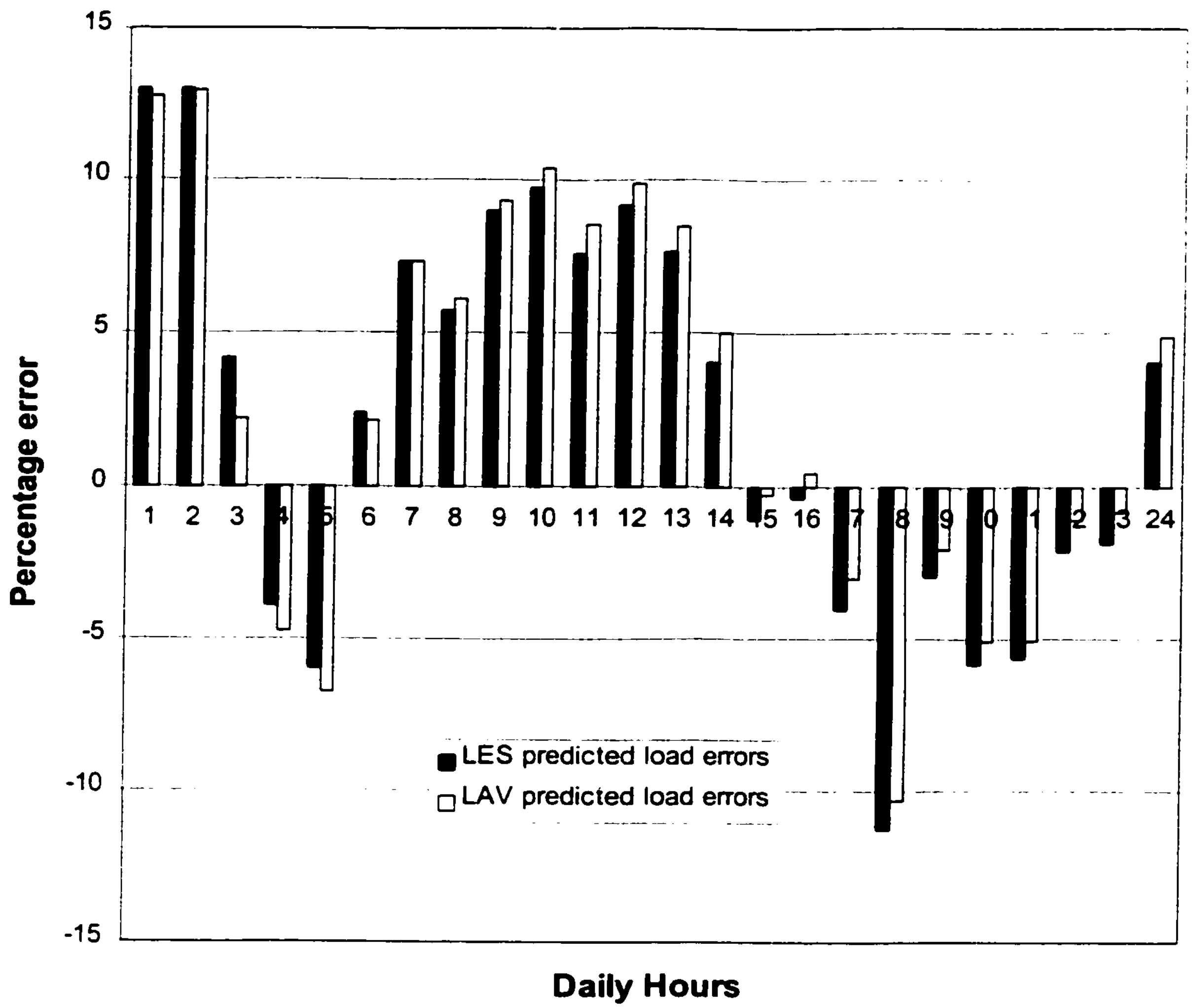


Figure (P4.28) Predicted load error for a winter day, Model