OPTIMAL DISTRIBUTED GENERATION SIZING AND PLACEMENT VIA SINGLE- AND MULTI-OBJECTIVE OPTIMIZATION APPROACHES

by

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This thesis is dedicated

To the soul of my Father,
May Allah forgive him and grant him his highest paradise.

To my beloved Mother,
May Allah bless her and elongate her live in his obedience.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

Numerous advantages attained by integrating Distributed Generation (DG) in distribution systems. These advantages include decreasing power losses and improving voltage profiles. Such benefits can be achieved and enhanced if DGs are optimally sized and located in the systems. In this thesis, the optimal DG placement and sizing problem is investigated using two approaches. First, the optimization problem is treated as single-objective optimization problem, where the system's active power losses are considered as the objective to be minimized. Secondly, the problem is tackled as a multi-objective one, focusing on DG installation costs. These problems are formulated as constrained nonlinear optimization problems using the Sequential Quadratic Programming method. A weighted sum method and a fuzzy decision-making method are presented to generate the Pareto optimal front and also to obtain the best compromise solution. Single and multiple DG installation cases are studied and compared to a case without DG, and a 15-bus radial distribution system and 33-bus meshed distribution system are used to demonstrate the effectiveness of the proposed methods.

LIST OF ABBREVIATIONS AND SYMBOLS USED

List of Appreciations

ABC	Artificial Bee Colony
AFC	Alkaline Fuel Cell
BFGS	Method for updating the Hessian Matrix
CHP	Combined-Heat and Power
DG	Distributed Generation
FC	Fuel Cell
GA	Genetic Algorithm
GDP	Gross Domestic Product
HRA	Hereford Ranch Algorithm
KKT	Karush Khun Tuker conditions
MCFC	Molten Carbonate Fuel Cell
MT	Micro-turbine
NLP	Nonlinear Optimization Problem
PAFC	Phosphoric Acid Fuel Cell
PEMFC	Proton Exchange Membrane Fuel Cell
PSO	Particle Swarm Optimization
PV	Photovoltaic
QP	Quadratic Programming

SA	Simulated Annealing
SOFC	Solid Oxide
SQP	Sequential Quadratic Programming
T&D	Transmission and Distribution
THD	Total Harmonic Distortion

List of Symbols

f(x)	The objective function
h(x)	The vectors of equality constraints
g(x)	The vectors of inequality constraints
x	The vector of *n* decision or unknown variables
P_{Loss}	The total real power loss
NS	The total number of branches,
G_k	The conductance of the *k-th* branch
V_i	Voltage magnitude at bus *i*
δ_i	Voltage angle at bus *i*
P_{DGi}	Active power delivered by DG at bus *i*
Q_{DGi}	Reactive power delivered by DG at bus *i*
P_{Di}	Active power demand at bus *i*
Q_{Di}	Reactive power demand at bus *i*
$\lvert Y_{ij}\rvert$	The magnitude of the *ij-th* element of the admittance matrix
φ_{ij}	The angle of *ij-th* element of the admittance matrix
NB	Total number of buses
S_{ij}^{max}	Apparent power maximum allowable for branch *i j*
S_{ij}	Apparent power flow transmitted from bus *i* to bus *j*
P_{DGi}^{max}	The maximum DG outputs of unit *i*

P_{DGi}^{min}	The minimum DG outputs of unit *i*
V_i^{max}	The upper bounds of the voltage at bus *i*
V_i^{min}	The lower bounds of the voltage at bus *i*
δ_i^{max}	The upper bounds of the voltage angle at bus *i*
δ_i^{min}	The lower bounds of the voltage angle at bus *i*
$\nabla f(x)$	The gradient of the *f(x)*
$\nabla^2 f(x)$	The Hessian of the *f(x)*
H	The Hessian of the *f(x)*
$\mathcal{L}$	Langrange function
λ	The equality Lagrange multiplier
μ	The inequality Lagrange multiplier
A	Active set
B	The Hessian of the Lagrange function
v_λ	The Newton's steps toward a KKT solution point
v_μ	The Newton's steps toward a KKT solution point
VD	The voltage deviation index
V_{ref}	The voltage reference
$F(x)$	The vector of objective functions
S	The feasible area in the decision space
Z	The feasible area in the objective space

C_{Inv}	Investment cost
$C_{M\&O}$	Maintenance and operation cost
DG_i^{max}	The maximum capacity of DG at bus *i*
d	Discount rate
T	Planning period (year)
nDG	Number of DG placements in the network
β_t	Present worth
f_i^{max}	The maximum values of the objective function *i*
f_i^{min}	The minimum values of the objective function *i*
μ_i^k	Membership function
Nf	The number of objective functions
Nk	The number of non-dominated solutions

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CHAPTER 1 INTRODUCTION

1.1 MOTIVATION

One of the largest consumer markets in the world is the electric power industry. For instance, in the United States, 3% of America's Gross Domestic Product (GDP) is spent on electric energy purchases, which are increasing faster than the rate of economic growth. The cost of electricity is estimated at around 50% for fuel, 20% for generation, 5% for transmission and 25% for distribution [1]. Distribution systems must deliver electricity to each customer's service entrance at an appropriate voltage rating. The X/R ratio for distribution levels is low compared to transmission levels, causing high power losses and a drop in voltage magnitude along radial distribution lines. Studies [2] have indicated that approximately 13% of the total power generated is consumed as real power losses at the distribution level. Such non-negligible losses have a direct impact on the financial issues and overall efficiency of distribution utilities. Traditionally, distribution power losses are minimized through proper dispatch of reactive power control devices, which can be done by deploying automatic voltage regulators (tap changing transformers) and shunt capacitors installed at low voltage buses [3].

The installation of Distributed Generation (DG) units is becoming more prominent in distribution systems due to their overall positive impacts on power networks. Some major advantages of integrated DGs include reducing power losses, improving voltage profiles, reducing emission impacts and improving power quality. Because of these benefits, utility companies have started to change their electric infrastructure to adapt to the introduction of DGs in their distribution systems.

Nonetheless, in order to maximize benefits, solution techniques for DG deployment should be obtained using optimization methods, since installing DG units at non-optimal places and in inappropriate sizes may cause an increase in system power losses and costs. Moreover, installing DG units is not straightforward, and thus the placement and sizing of DG units should be carefully addressed. Investigating this optimization problem is the major motivation of the present thesis research.

1.2 Thesis Objectives

The main goal of this thesis is to solve the optimal DG placement and sizing problem in distribution networks. This problem is treated both as a single-objective and a multi-objective optimization problem. Both problems are formulated as constrained nonlinear optimization problems and are solved using the Sequential Quadratic Programming (SQP) deterministic method.

The single-objective optimization problem aims to find the optimal place and size of DG by using the total real power losses as a particular objective to be minimized. In a similar fashion, the multi-objective optimization method is proposed to consider the cost aspects of DG installation, where the total real power losses and the total DG installation cost are considered as objectives that should be minimized simultaneously. The multi-objective optimization problem aims to find the Pareto front, which consists of a set of trade-off solutions. Each solution gives a particular place and size for the DG unit to be installed. As a result, the decision-maker can select the proper solution according to subjective preferences. In addition, a fuzzy decision-making procedure for order preference is used to guide the decision-maker to the best compromise solution among all acceptable solutions. The impact of integrating single and multiple DGs is also investigated in this work. Two topologies of distribution test networks (radial and meshed) are selected to validate the proposed methods and the results are presented.

1.3 THESIS OUTLINE

This thesis is divided into five chapters. The motivation for this thesis and the objectives that are aimed to be achieved are addressed in Chapter 1. Chapter 2 presents an introduction of the power system networks structure, followed by a brief overview of the DG, including its definition, technology and benefits. Various optimization techniques that were used earlier for DG allocation are also presented in Chapter 2. The third and fourth chapters deal with the proposed optimization problems. In Chapter 3, the optimization problem is formulated to handle single-objective problems, and the problem objective and its constraints, as well as the SQP algorithm used as a solver, are presented. The proposed algorithm is then applied in two distribution test systems with single and multiple DG installation cases and the simulation results presented and discussed. In Chapter 4, a multi-objective optimization problem is proposed. The Pareto optimality principle, the weighted sum method and the fuzzy decision-making method are introduced. Simulation results for the multi-objective optimization approach are presented and discussed using the same test systems and cases considered in Chapter 3. The final part of the thesis, Chapter 5, provides conclusions and offers suggestions for future work.

CHAPTER 2   OVERVIEW

2.1 INTRODUCTION

Electrical power systems generate electricity and deliver it to customers in a three-stage process of generation, transmission and distribution. Each stage has its own characteristics, as illustrated in Figure 2.1 [1].

***Generation*:** Electricity is produced at central power generation plants (also referred to as generating stations) by converting mechanical energy into electrical energy. A prime mover, such as a steam turbine that burns coal, oil or nature gas, is used to bring the mechanical power to a synchronous generator in order to produce electricity. Other plants use nuclear fuel or non-thermal sources such as hydraulic and wind turbines. These plants are usually located at the energy sources, away from heavily populated areas. The electricity is produced at a low voltage level between 11 kV and 30 kV due to insulation requirements and practical design limitations [4]. The generation plants are connected to transmissions networks via generation substations, where electric power is stepped up to a higher voltage (normally between 115 kV to 1100 kV) to be transported over long distance.

***Transmission*:** The transmission system is divided into two parts: transmission and sub-transmission systems. The transmission system transfers electric power from a generation plant to a sub-transmission system through generation substations at voltage levels of 230 kV or higher. The sub-transmission system then transfers the electrical power at voltage levels between 69 kV to 138 kV to the distribution substation and on to the distribution networks.

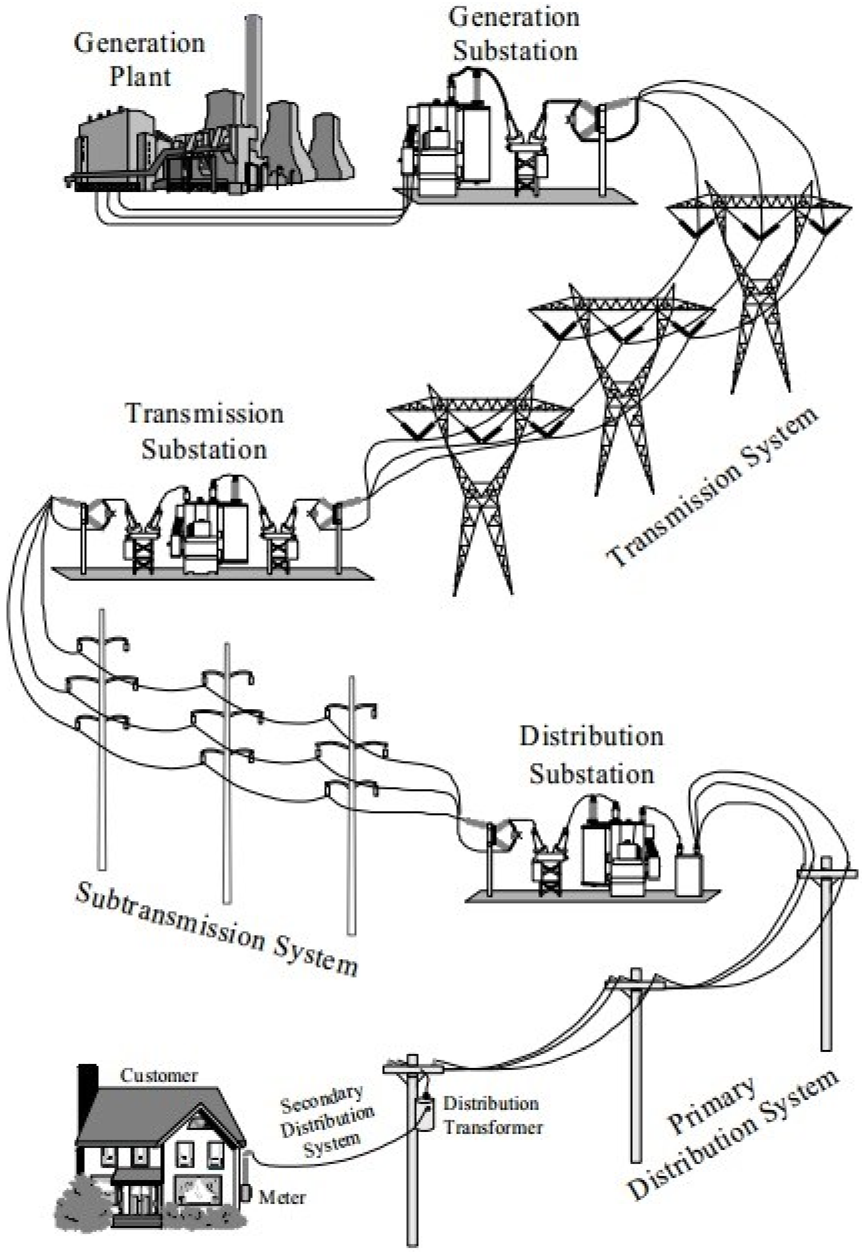

Figure 2.1: Electric power systems [1].

Distribution: Distribution is the final stage of delivering electricity to customers. First, the distribution substations step down the voltage from 4 kV to 34.5 kV. Then, the primary distribution system transfers the electric power from the distribution substation to the distribution transformers. Some industrial customers are served directly from the primary distribution system. The distribution transformers step down the voltage again to utilization levels, namely: 120/240 V for single phase, 120/208V for three phases or 277/480V for three phases. Lastly, the secondary distribution systems distribute the power to the customers' service-entrance equipment. The distribution networks are either overhead or underground conductors. Two types of feeders are utilized in the distribution systems: overhead and underground feeder. Seventy percent of new customers in Canada and the United States are supplied via underground distribution systems [4].

2.2 DISTRIBUTED GENERATION

The term Distributed Generation, or DG, refers to the use of small-scale electric power generators dispersed within the distribution network level, whether located on the utility system near customers or at an isolated site not connected to the power grid [1]. The efficiency of DG technologies is high, e.g. 40 to 55% for fuel cells, compared to 28 to 35% for traditional large central power generators [5].

2.2.1 DISTRIBUTED GENERATION DEFINITION

Distributed generation is a new approach in the power industry. In fact, it is so new, neither a standard definition nor a standard name for it have been agreed upon. Nevertheless, various definitions and names have been used in the literature. Some researchers define DG by rating DG units, whereas others define DG in terms of the technology used. DG also appears under different names, depending on the country. For instance, in some parts of North America, the term Dispersed Generation is used, while in South America, Embedded Generation has been coined. Meanwhile, in Europe and some Asian countries, DG stands for Decentralized Generation [6].

After studying and analysing several papers, T. Ackermann et al. [7] proposed a general definition for DG, suggesting that the most apt definition would be “an electric power source connected directly to the distribution network or on the customer site of the meter”. However, this definition does not mention any capacity criterion or the technologies used to build and run these sources. Therefore, two additional categories are suggested in classifying as well as defining DG [7]. The first category classifies DG based on its capacity, and the second category classifies DG based on its technology.

Table 2.1: DG category based on capacity.

Categories	Ratings
Micro-distributed generation	~1 W < 5 kW
Small-distributed generation	5 kW < 5 MW
Medium-distributed generation	5 MW < 50 MW
Large-distributed generation	50 MW < 300MW

Table 2.2: DG category based on technology.

Renewable DG
Modular DG
Combined heat and power (CHP) DG

2.2.2 DISTRIBUTED GENERATION TECHNOLOGY

Various DG technologies are involved in power systems. Some of these technologies have been in use for a long time while others are newly emerging. Nonetheless, the features that all DG technologies have in common are to increase efficiency and decrease costs related to installation, running and maintenance. DG technologies are loosely categorized into two types: renewable technologies (e.g., photovoltaic and wind turbine) and non-renewable technologies (e.g., mini and micro-turbines, combustion turbines and fuel cells). DG technologies have a significant impact on the selection of the appropriate size and place of a DG unit to be connected to a grid or customer loads. The following sections provide details on the most popular DG technologies currently in the market:

2.2.2.1 Fuel Cells

Fuel Cells (FC) are classified as non-traditional generators. They are electrochemical devices that convert chemical energy from a fuel directly into electrical energy by combining oxygen, as an oxidant, and hydrogen, as a fuel, without combustion

[6]. The hydrogen is usually procured from a fossil fuel "natural gas" while air is used as a source for oxygen. The result of this electrochemical process is high-current/low-voltage DC power. To connect the fuel cell to the grid, a DC/AC converter and filter system current are used to convert the output to AC power. Water (H_2O) and heat are by-products of the process. This heat, which often exceeds 1,000 ^{0}F, converts the water to steam, which can then be used to perform other work [5]. Regardless of the auxiliary systems, FCs have no moving parts and no combustion, making them silent devices [5]. FCs are divided into five types depending on the chemical reaction: Alkaline (AFC), Molten Carbonate (MCFC), Phosphoric Acid (PAFC), Proton Exchange Membrane (PEMFC) and Solid Oxide (SOFC) [8]. The characteristics of those types are summarized in Table 2.3 [5, 9].

Table 2.3: Characteristics of fuel cell types.

	PEMFC	**AFC**	**PAFC**	**MCFC**	**SOFC**
Electrolyte	Polymer membrane	KOH & H_2O Phosp. Acid	H_3PO_4 Lithium carb.	$LiKaCo_3$ Zirconia	Stabilized
Internal Temp.	85^{0}C	120^{0}C	190^{0}C	650^{0}C	1000^{0}C
Efficiency	30%+	32%+	~40%	~42%	~45%
Applications	Car, space	Car, other	DG	Large DG	Very large DG
Installed Cost $/kW	$1,400	$2,700	$2,100	$2,600	$3,000
Advantages	Solid electrolyte reduces corrosion, low temperature, quick start-up	Cathode reaction faster in Alkaline electrolyte, high performance	Up to 850/. efficiency m co-generation of electricity	Higher efficiency, fuel flexibility, inexpensive catalysts	Higher efficiency, fuel flexibility, inexpensive catalysts, solid electrolyte advantages like PEXI

2.2.2.2 Micro-turbines

Micro-turbines (MT) are small electricity generators that burn fuel such as natural gas, propane and fuel oil to create high-speed rotation that is transferred to an electrical generator via a main shaft. MT consists of three basic components: a compressor, a turbine generator, and a recuperator [10]. In present energy markets, MT generators are the most improved and most attractive devices in distributed power generation equipment [11]. Their capacity ranges from 20 kW to 500 kW and their efficiency is more than 80% when the CHP application is used in the system. Also, the NO_x emissions of MT are very low compared to large-scale turbines [8].

2.2.2.3 Photovoltaic

Photovoltaic (PV) technology converts solar energy directly into electricity using semi-conductor solar cells. These cells are manufactured in small sizes of usually around one square centimetre. When the solar cells are exposed to direct sunlight, each cell generates less than one watt of DC power, with the lowest voltage around 0.5 V.

Normally, a panel or module can be formed by electrically connecting twelve solar cell units in series to provide 12 V. In the same way, a group of modules can be connected together in parallel to increase the output to the needed power [5]. PV systems are divided into three sizes based on the power they produce (the small size is less than 10 kW; the medium size is 10 kW to 100 kW; and the large size is more than 100 kW). The large size is appropriate for the distribution network level [8]. Despite the high initial price of PV systems (US $6,000-10,000/kW [8]), the most significant features are that no fuel is needs to operate them, and they are very clean and quiet [5].

2.2.2.4 Wind Turbines

Wind turbines are among the most popular renewable electrical sources in the world. A large number of wind turbine systems have already been installed and connected to the grid, generating globally around 238,000 megawatts of electricity in 2011, and many new systems are being planned [12]. Manufacturers offer wind turbines in a capacity range from less than 5 to over 1,000 kW [13]. Wind turbines are usually integrated to the transmission voltage level and combined to make a wind farm. However, wind turbines are sometimes considered distributed generation, because the size and location of some small wind farms make them suitable for connection at the distribution voltage level [14]

Wind turbines consist of a rotor, turbine blades, generator, drive or coupling device, shaft and nacelle. The energy of the raw wind turns the blades and the common shaft, producing electrical power. Like PV systems, wind turbines require no fuel, no emissions, and produce DC power that needs AC/DC inverters to be connected to the grid. Moreover, small wind turbines can be combined with PV and battery systems to cover loads of 25 to 100 kW [6]. The main drawbacks of wind turbines are their high initial costs and unpredictability of energy production. As well, they are not suited to CHP applications [5].

2.2.3 DISTRIBUTED GENERATION BENEFITS

Distributed generation promises several potential positive impacts, both economical and technical. The major benefits of the integrations of DG into electric power networks are as follows [6, 15-17]:

- DG units are usually installed near the load site on the radial distribution networks. Thus, part of the transmission power is replaced by the injected DG power, causing a reduction in transmission and distribution line losses, which minimizes costs related to loss.
- Injecting active and reactive power by DG units improves system voltage profiles and the load factor, which minimizes the number of required voltage regulators, capacitors and their ratings and maintenance costs. However, the amount of improvement depends on the size and location of the DG unit.
- Increases in power demands as a result of load growth can be covered by DG units without needing to increase existing traditional generation capacity; it also reduces or delay the need for building new T&D lines, upgrades the present power systems and reduces T&D network capacity during the planning phase.
- DGs are flexible devices that can be installed at load centres rather than at substations, where difficulties due geographical constraints or scarcity of land availability may occur. In addition, DG locations are not restricted by the government's choice for potential locations, as is the case when selecting new substation locations.
- DG technology is available in a wide capacity range (i.e., from ten kW up to 15 MW), so it can be installed on medium and/or low voltage distribution networks, giving it flexibility for sizing and siting.

- DG plants require a short period of time to install and pose less of an investment risk due to their modular characteristics, which enables them to be easily assembled anywhere, such as with FC-MT and MT-batteries. Each modular can be operated immediately after its installation, independent of other modules, and is not affected by other modular's operation failure. In addition, the total capacity can be increased or decreased by adding or removing more modules, respectively.
- DG technologies produce electric power with few emissions (and sometimes zero emissions). This feature makes them more environmentally friendly compared to traditional power plants.
- DGs can help in system service continuity and reliability, as there are many generation spots, not just one large centralized generation site. This is particularly useful in the case of end-user customers with low reliability since, when combined with DGs, there will be new customer classifications (e.g., those with high need for reliability with high service costs and those with low need for reliability with lower service costs).

On the other hand, integrating DG units may lead to negative impacts on a distribution system, especially for large scale installations if they are not optimally handled. For instance, DG may result in high voltage causing currents that exceed the line's thermal limit, harmonic problems, noticeable voltage flicker and instability of the voltage profile of some electricity customers. In addition, the bi-directional power flows can lead to voltage profile fluctuation and change the short circuit levels sufficiently to cause fuse-breaker miscoordination [15, 18]

2.3 OPTIMIZATION TECHNIQUES FOR DG ALLOCATION

Solution techniques for DG deployment can be obtained via optimization methods in order to maximize DG benefits. Several optimization techniques have been presented by researchers in determining the optimal location and size of DG. Such optimization methods can be classified into deterministic methods such as analytical and SQP methods and heuristic methods such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Artificial Bee Colony (ABC), etc., or into single- and multi-objective, based on the number of objectives. The major objective of DG placement techniques used in the literature is to minimize power system losses. However, other objectives, like improving the voltage profile and reliability and maximizing DG capacity and cost minimization have also been considered.

Hong Cui et al. [19] sought the optimal allocation of DG in a smart grid via a multi-objective optimization model. The objectives for the proposed method were to minimize operational costs of DG and network active power loss and to maximize environmental benefits. First, the optimal DG placement was determined by performing a network power loss sensitivity analysis, where a bus with high sensitivity was selected to install a DG unit. To solve the sizing problem, fuzzy theory was proposed. The multi-objective planning was converted into single-objective planning by employing the fuzzy optimization theory.

Acharya et al. [20] proposed an analytical method to determine the optimal capacity of DG. The optimal sizes corresponding to each network bus were calculated using a direct equation derived from the sensitivity factor equation. In addition, an effective methodology based on an exact loss formula was applied to determine the optimal site of DG that minimizes total power losses. The method carried out the load flow to times, for the base case, without DG, and with DG, and considered installing only a single DG that injects active power.

Kashem et al. [21] developed a deterministic methodology based on the SQP algorithm to identify the optimal size and placement of DG in distribution systems. The authors proposed a combined objective function that aims to reduce power loss at minimal DG cost. This function is a highly nonlinear constrained multi-objective problem which consists of two parts. The first part involved the total savings contributed by the loss reduction, while the second part was the total cost of all DG units. The proposed method was tested using different percentages of loss reduction, from 5% to 25%, and different maximum numbers of DG as well. In addition, actual historical data from Tasmanian customers' demand was used in the test.

AlHajri et al. [22] determined the optimal location and size of DG using a new methodology based on the Fast SQP. The DG optimal sizing problem was formulated as one of constrained nonlinear programming subject to nonlinear equality and inequality constraints in addition to boundary restrictions imposed on the system. Minimizing the real power losses of a radial distribution network was the objective for solving the DG size optimization problem, while all possible combinations for allocating DG were investigated to find the best DG placement. Integrated single- and multiple-DG units were discussed where the DG units were modeled as a negative load delivering an active and reactive power to the distribution system, regardless of the system voltage.

Alinejad-Beromi et al. [23] presented a method for optimal allocation of DG for voltage profile improvement and loss reduction. GA was used as the optimization technique. Load flow was applied for decision-making which combined appropriately with GA. The GA method was also introduced by Celli and Pilo [17] to minimize the cost of power loss in a distribution network during a predetermined period of study. The proposed method considers all technical constraints such as feeder capacity limit, feeder voltage profile and three-phase short circuit current in the network nodes. The planning study period is 20 years , the DG units under consideration range between 100 to 500 kVA with a 100 step, and the maximum level of DG penetration is 20% of total power demands.

Gandomkar et al. [24] used the Hereford Ranch Algorithm (HRA) to determine DG place and size that minimized distribution power losses, with the condition that the number of DGs and total capacity of DGs are known. The parent selection algorithm for generating offspring affects the ability of GA in three aspects: finding a correct solution for a variety of problems; preserving diversity to prevent premature convergence; and improving convergence time. To overcome the defects of existing GA, the authors applied HRA to search for optimal DG site and size in distribution feeders. HRA uses sexual differentiation and selective breeding in choosing parents for genetic strings. In terms of both solution quality and number of iterations, the proposed HRA performs better than individual GAs.

The same authors proposed another method [25] that employs a new hybridized algorithm for the evaluation of DG site and size in medium voltage systems. This algorithm was used to minimize distribution power losses. The GA was correlated by

Simulated Annealing (SA) metaheuristic methods and employed for DG allocation. Unlike in [24], the proposed algorithm deals with a fixed number of DGs and specific total capacity of DGs. The assumption made for the maximum total capacity and maximum number of DGs was 0.1 power demand and one-third of the number of nodes for each network, respectively. The researchers concluded that the proposed combined GA-SA method is a better form than the individual GA approach in terms of both solution quality and number of iterations.

Kim et al. [26] employed a hybridized method that determines DG optimal places to be installed and their capacities in distribution networks simultaneously. The authors combined the GA with fuzzy set theory. The proposed algorithm considers an objective to reduce power loss costs of distribution systems and the constraints with the number or size of DG and the bus voltage deviation. The original objective function and constraints are transformed into the equivalent multi-objective functions and modeled with fuzzy sets to evaluate their imprecise nature. Without any transformation for this nonlinear problem to a linear model or other methods, the GA and goal programming were applied to obtain the global solution.

Sedighi et al. [27] and Alinejad-Beromi et al. [28] utilized PSO to evaluate the optimal place and size for single and multiple DGs in order to achieve different goals, including improving voltage profile, reducing active power loss and related DG costs and minimizing THD (Total Harmonic Distortion) in distribution networks. An overall objective function with composing constraints and goals was formulated by using properly weighted factors. The authors in the two proposed papers applied load flow and harmonic load flow algorithm to evaluate fitness values sensitivity in the PSO algorithm process.

A novel algorithm that combined GA and PSO was presented by Moradi and Abedinie [29]. The combined method was formulated as a multi-objective constrained optimization problem that aimed to obtain the optimal size and site for DG in distribution systems where the proposed objectives were to minimize the network power losses, maximize the voltage stability, and improve voltage regulation in a given radial distribution system. Penalty coefficients were used to convert these three objective functions into a scalar objective. Further, the GA was used to determine the optimal site of DG, while PSO was used to determine its optimal size. The proposed combined GA and PSO achieved a better solution quality with fewer iterations compared to cases where either method was applied alone. However, one drawback for the proposed combined GA and PSO is that it is extremely time-consuming.

Abu-Mouti et al. [30] proposed a new metaheuristic optimization approach, based on an ABC algorithm, to determine the optimal location, size and power factor of DGs in a distribution system. The new approach was mathematically formulated as a constrained nonlinear optimization problem where the objective is to minimize the total real power of the network, subject to nonlinear equality and inequality constraints. The DG capacities were bounded between 10% and 80% of the total load and approximated to discrete values with a 100-step interval between sizes. In addition, the power factor of the DG units was set to operate at practical values, including unity, 0.95, 0.90, and 0.85 towards the optimal result.

Abu-Mouti et al. [31] proposed a heuristic technique to find the optimal DG siting and sizing. Here, the total system power loss for radial distribution networks was the optimization objective. The problem was divided into two sub-problems and each part was treated independently. The authors employed a sufficient sensitivity analysis based on power flow in the first portion to determine the best bus to allocate the DG unit while the optimal DG capacity was chosen by using a heuristic curve-fitted technique. However, the proposed method did not address multiple DG installation.

2.4 SUMMARY

This chapter opened with an overview of the electrical power system, along with a brief discussion about electrical power system stages (i.e., generation, transmission and distribution). This was followed by a review of DG definitions, technologies and benefits. Next, a variety of optimization techniques for DG allocation were presented. Most of the proposed methods to solve the DG optimization problem considered single objectives, the main one of which was to minimize a system's power loss in DG placement techniques.

CHAPTER 3 OPTIMAL DISTRIBUTED GENERATION SIZING AND PLACEMENT VIA SINGLE-OBJECTIVE OPTIMIZATION APPROACH

3.1 Introduction

Installing DG in an electrical distribution system has numerous positive impacts, but these impacts can be further enhanced if the DG units are installed at a proper place and in a proper size. Non-optimal placement and sizing of DG units can cause significant negative repercussions on distribution systems. In this chapter, the optimal DG placement (or siting) and sizing problem is investigated using a single objective function that is subjected to equality and inequality constraint equations. The optimal DG size problem is handled via the SQP deterministic method and by performing this method at all candidate buses. The bus with a minimum DG size will be selected as the optimal location to install the DG. The proposed technique succeeds in solving single and multiple DG installations for both radial and meshed distribution systems.

3.2 Problem Formulation

An optimization problem can be mathematically defined as the minimization or maximization of a function (called the objective function) while satisfying a number of equality and/or inequality constraints on its variables [32]. The general optimization problem can be formulated as:

$$\underset{x \in R^n}{Min/Max}: \quad f(x) \tag{3.1}$$

$$subject\ to: \quad h_i(x) = 0, \quad i = 1,2, \dots, n \tag{3.2}$$

$$g_i(x) \leq 0, \quad i = 1,2, \dots, m \tag{3.3}$$

$$x^{min} \leq x \leq x^{max} \tag{3.4}$$

where

f(x) : the objective function, a function of *x* that we want to maximize or minimize.

h(x), *g(x)* : the vectors of equality and inequality constraints that the unknowns must satisfy.

x : the vector of *n* decision or unknown variables and $x=[x_1, x_2, \dots, x_n]$.

This kind of optimization is called a single-optimization problem, since *f(x)* is only one objective function. On the other hand, a multi-optimization problem has more than one objective function, as illustrated in the following chapter.

3.2.1 Problem Objective

The objective function to be minimized to solve the optimization problem is the total active power loss of a distribution system.

$$Minimize: \ P_{Loss}(x) \tag{3.5}$$

where P_{Loss} is the total real power loss, which can be expressed in the following equation:

$$P_{loss} = \sum_{k=1}^{NS} G_k \left(|V_i|^2 + |V_j|^2 - 2|V_i||V_j| cos(\delta_i - \delta_j) \right) \tag{3.6}$$

where

NS : the total number of branches,

G_k : the conductance of the *k-th* branch which connects the sending bus *i* and the receiving bus *j*,

V_i, V_j : voltage magnitude at bus *i* and *j*,

δ_i, δ_j : voltage angle at bus *i* and bus *j*.

3.2.2 CONSTRAINTS

The objective function is minimized subject to various operational constraints to satisfy the electrical requirements for the distribution network and constraints on DG operation. These constraints are discussed as follows:

Power Balance Constraints*:* Power balance is given by nonlinear power flow equations, which state that the sum of complex power flows at each bus in the distribution system injected into a bus minus the power flows extracted from the bus should equal zero.

$$P_{DGi} - P_{Di} - \sum_{j=1}^{NB} |V_i||V_j||Y_{ij}| \, cos(\delta_i - \delta_j - \varphi_{ij}) = 0 \tag{3.7}$$

$$Q_{DGi} - Q_{Di} - \sum_{j=1}^{NB} |V_i||V_j||Y_{ij}| sin(\delta_i - \delta_j - \varphi_{ij}) = 0 \quad (3.8)$$

where

P_{DGi}, Q_{DGi} : active and reactive power delivered by DG at bus i,

P_{Di}, Q_{Di} : active and reactive power demand at bus i,

$|Y_{ij}|$: the magnitude of the *ij-th* element of the admittance matrix,

φ_{ij} : the angle of the *ij-th* element of the admittance matrix,

NB : the total number of buses.

Power Flow Constraints: The power flow constraint is used to ensure that they do not approach their thermal limits. The following constraint checks for the absolute power flow both at the sending and receiving ends of a particular line to be within the upper limit of the line.

$$S_{ij} \leq S_{ij}^{max} \quad (3.9)$$

$$S_{ji} \leq S_{ji}^{max} \quad (3.10)$$

where

S_{ij}^{max} : apparent power maximum allowable for branch *i j*,

S_{ij} : apparent power flow transmitted from bus *i* to bus *j*.

***Generation Capacity Constraints**:* Limiting the DG size so as not to exceed the power supplied by the substation and the output power of each DG unit is constrained by lower and upper limits.

$$\sum_{i=1}^{nDG} (P_{DGi} + jQ_{DGi}) \leq P_{ss} + jQ_{ss} \tag{3.11}$$

$$P_{DGi}^{min} \leq P_{DGi} \leq P_{DGi}^{max} \tag{3.12}$$

where P_{DGi}^{min} and P_{DGi}^{max} are the minimum and maximum operating outputs of unit *i*, respectively.

Bus voltage limit: Bus voltage magnitudes and phase angles of the radial distribution system are to be bounded between maximum and minimum values, imposed by a system operator. The boundary constraint can by expressed as follows:

$$|V_i^{min}| \leq |V_i| \leq |V_i^{max}| \tag{3.13}$$

$$\delta_i^{min} \leq \delta_i \leq \delta_i^{max} \tag{3.14}$$

where: $|V_i^{min}|$, $|V_i^{max}|$, δ_i^{min} and $\delta_i^{max}$are the lower and upper bounds of the bus voltage $|V_i|$ and the bus voltage angle δ_i, respectively.

3.2.3 Mathematical Models of DG Units

A DG unit can be modelled as either a PV or PQ bus in the distribution system. If DGs have control over the voltage by regulating the excitation voltage (synchronous generator DGs) or if the control circuit of the converter is used to control P and V independently, then the DG unit may be modelled as a PV type. Other DGs, like induction generator-based units or converters used to control P and Q independently, are modelled as PQ types. The most commonly used DG model is the PQ model [33]. In this work, the PQ-DG units are represented as a negative PQ load model delivering active and reactive power to a distribution system. The DG reactive power can be calculated by the following equation:

$$Q_{DGi} = P_{DGi} \times tan\left(cos^{-1}(PF_{DGi})\right) \tag{3.15}$$

3.2.4 Sequential Quadratic Programming

Since the objective function and its constraints are naturally nonlinear equations, the optimization problem is classified as a Nonlinear Optimization Problem (NLP) [32]. The DG optimization problem is performed using a conventional Sequential Quadratic Programming (SQP) method also known as Iterative Quadratic Programming and Recursive Quadratic Programming, meaning that one Quadratic Programming (QP) subproblem is solved at each major iteration. According to the accuracy, efficiency and percentage of successful solutions of the SQP method over a large number of test problems, it is considered as the best nonlinear programming method for constrained optimization [34].

The main idea of SQP is to model the optimization functions at the current point, x^k, by making a quadratic model of the objective function and linear models of the constraints using Taylor's expansion. These are then solved at each iteration to find a new search direction, d, with a better solution, x^{k+1}. This method closely resembles Newton's method for unconstrained minimization [35]. By applying Taylor's expansion to the general optimization problem (3.1-3), we get:

$$f(x) \approx f(x^k) + \nabla f(x^k)^T(x - x^k) + \frac{1}{2}(x - x^k)^T \nabla^2 f(x^k)(x - x^k) \tag{3.16}$$

$$h(x) \approx h(x^k) + \nabla h(x^k)^T(x - x^k) \tag{3.17}$$

$$g(x) \approx g(x^k) + \nabla g(x^k)^T(x - x^k) \tag{3.18}$$

where ∇ refers to the gradient of the *f(x)*, and ∇^2 is the Hessian of the *f(x)*.

Setting:

$$d = (x - x^k) \tag{3.19}$$

$$H^k = \nabla^2 f(x^k) \tag{3.20}$$

Thus, the QP subproblem will have the form:

$$minimize\text{:} \qquad f(x^k) + \nabla f(x^k)^T d + \tfrac{1}{2} d^T H^k d \tag{3.21}$$

$$subject\ to\text{:} \qquad h(x^k) + \nabla h(x^k)^T d = 0 \tag{3.22}$$

$$g(x^k) + \nabla g(x^k)^T d \leq 0 \tag{3.23}$$

The SQP applies the Lagrange multipliers method to the QP subproblem, starting by transforming the constrained optimization problem to a Lagrangian function and then satisfying conditions (called Karush-Khun-Tuker (KKT) conditions) and solving the unknown variables from the derived equations through Quasi-Newton method in each iteration. The Lagrangian function for this problem can be written as follows:

$$\mathcal{L}(x,\lambda,\mu) = f(x) + \lambda^T h(x) + \mu^T g(x) \tag{3.24}$$

where

λ : the equality Lagrange multiplier,

μ : the inequality Lagrange multiplier.

The KKT conditions state that, at the optimal point solution, the gradients of the Lagrange function are equal to zero, as follows:

$$\nabla f(x) + \lambda^T \nabla h(x) + \mu^T \nabla g(x) = 0 \tag{3.25}$$

$$h(x) = 0 \tag{3.26}$$

$$g(x) \leq 0 \tag{3.27}$$

$$\mu^T g(x) = 0 \ , \mu \geq 0 \tag{3.28}$$

The active set method [36] applies to the inequality constraints to partition it into two groups. The first group is to be treated as active and the second group as inactive. Let A be a set of i, such that $g_i(x) = 0$. The necessary conditions for the inequality constraints then become:

$$\nabla f(x) + \lambda^T \nabla h(x) + \mu^T \nabla g_i(x) = 0 \tag{3.29}$$

$$g_i(x) = 0, \quad i \in A \tag{3.30}$$

$$g_i(x) < 0, \qquad i \notin A \tag{3.31}$$

$$\mu_i \geq 0, \qquad i \in A \tag{3.32}$$

$$\mu_i = 0, \qquad i \notin A \tag{3.33}$$

The Lagrange multipliers for the inactive inequality constraints are set to zero. Therefore, they will be considered as equality constraints in the Lagrange function.

The QP subproblem is formulated as:

$$minimize: \qquad f(x^k) + \nabla f(x^k)d + \frac{1}{2}d^T\nabla^2\mathcal{L}(x,\lambda,\mu)d \tag{3.34}$$

$$subject\ to: \qquad h(x^k) + \nabla h(x^k)^T d = 0 \tag{3.35}$$

$$g_A(x^k) + \nabla g_A(x^k)^T d \leq 0 \tag{3.36}$$

where $\nabla^2\mathcal{L}(x,\lambda,\mu)$ is the Hessian of the Lagrange function.

The local convergence of the SQP method follows from the application of Newton's method to the nonlinear system given by the Kuhn-Tucker-Karush (KKT) conditions:

$$\begin{pmatrix} \nabla\mathcal{L}(x_k,\lambda_k,\mu_k) \\ h(x_k) \\ g_A(x_k) \end{pmatrix} = 0 \tag{3.37}$$

The QP subproblem solution is obtained by solving the Quasi-Newton, as follows:

$$\nabla^2\mathcal{L}(x_k,\lambda_k,\mu_k)\begin{pmatrix} d \\ v_\lambda \\ v_\mu \end{pmatrix} = -\nabla\mathcal{L}(x_k,\lambda_k,\mu_k) \tag{3.38}$$

$$\begin{pmatrix} \nabla^2 \mathcal{L}(x,\lambda,\mu) & \nabla h(x_k) & \nabla g(x_k) \\ \nabla h(x_k)^T & 0 & 0 \\ \nabla g(x_k)^T & 0 & 0 \end{pmatrix} \begin{pmatrix} d \\ v_\lambda \\ v_\mu \end{pmatrix} = -\begin{pmatrix} \nabla f(x) + \lambda^T \nabla h(x) + \mu^T \nabla g_A(x) \\ h(x_k) \\ g(x_k) \end{pmatrix} \tag{3.39}$$

The Newton step from the iterate k is thus given by:

$$\begin{pmatrix} x_{k+1} \\ \lambda_{k+1} \\ \mu_{k+1} \end{pmatrix} = \begin{pmatrix} x_k \\ \lambda_k \\ \mu_k \end{pmatrix} + \begin{pmatrix} d \\ v_\lambda \\ v_\mu \end{pmatrix} \tag{3.40}$$

where $v_\lambda$and v_μ are the Newton's steps toward a KKT solution point.

These formulae may be rearranged by moving the $\{\lambda^T \nabla h(x) + \mu^T \nabla g_A(x)\}$ term to the left-hand side of (3.39), giving:

$$\begin{pmatrix} \nabla^2 \mathcal{L}(x,\lambda,\mu) & \nabla h(x_k) & \nabla g(x_k) \\ \nabla h(x_k)^T & 0 & 0 \\ \nabla g(x_k)^T & 0 & 0 \end{pmatrix} \begin{pmatrix} d \\ \lambda_{k+1} \\ \mu_{k+1} \end{pmatrix} = -\begin{pmatrix} \nabla f(x) \\ h(x_k) \\ g(x_k) \end{pmatrix} \tag{3.41}$$

The Newton-KKT system solves the equations starting by estimated solution points to get the search direction and new values for the Lagrange multipliers in order to be utilized in the next iteration. The process is repeated iteratively until an optimal solution, x*, is reached or certain convergence criteria are satisfied.

The Hessian of the Lagrangian function in the QP subproblem is to be calculated in every iteration. The Quasi-Newton method approximates the Hessian matrix (B) instead to calculate it. The most widely used formula, and the one considered to be most effective, is the BFGS update formula, named for its inventors, Broyden, Fletcher, Goldfarb, and Shanno [32]. Using this scheme, we set:

$$r_k = \theta_k y_k + (1 - \theta_k) B_k s_k \tag{3.42}$$

where

$$s_k = x_{k+1} - x_k \tag{3.43}$$

$$y_k = \nabla\mathcal{L}(x_{k+1}, \lambda_{k+1}, \mu_{k+1}) - \nabla\mathcal{L}(x_k, \lambda_k, \mu_k) \tag{3.44}$$

$$\theta_k = \begin{cases} 1 & if\ s_k^T y_k \geq 0.2 s_k^T B_k s_k \\ \dfrac{0.8 s_k^T B_k s_k}{s_k^T B_k s_k - s_k^T y_k} & if\ s_k^T y_k < 0.2 s_k^T B_k s_k \end{cases} \tag{3.45}$$

Then we can update B_{k+1} using,

$$B_{k+1} = B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} + \frac{r_k r_k^T}{s_k^T r_k} \tag{3.46}$$

3.3 Software Tools Used

The proposed optimal DG size and placement in the distribution systems was coded in MATLAB® Version 7.8.0.347 (R2009a). MATLAB is a high-level language and interactive environment for numerical computation, visualization, and programming. Programmers and users of MATLAB can analyze data, develop algorithms, and create models and applications, using the language, tools, and built-in math functions to explore multiple approaches and solve technical computing problems faster than with spreadsheets or traditional programming languages, such as C/C++ or Java™ [37].

Files written in MATLAB are called m-files. For this work, we used the MATLAB's optimization function (*fmincon*) to find the optimal solution with the SQP method. This function seeks a constrained minimum of a single function of multivariables, starting at an initial estimate [38]. For a problem specified by:

$$Min\, f(x),\ \ subject\ to: \tag{3.47}$$

$$C(x) \leq 0 \tag{3.48}$$

$$Ceq(x) = 0 \tag{3.49}$$

$$A.x \leq b \tag{3.50}$$

$$Aeq.x = beq \tag{3.51}$$

$$lb \leq x \leq ub \tag{3.52}$$

where *f(x)* is the objective function to be minimized. $C(x) \leq 0$, and $Ceq(x) = 0$ are nonlinear inequality and equality constraints. $A \cdot x \leq b$ and $Aeq \cdot x = beq$ are linear inequality and equality constraints. *lb* and *ub* are lower bound and upper bound.

The syntax of the *fmincon* function is as follows:

$$[x, fval] = fmincon(fun, x0, A, b, Aeq, lb, ub, nonlcon, options) \tag{3.53}$$

The attempt starts at *x0* and tries to find a minimizer *x* of the function described in *fun*, where *fun* and *nonlcon* are the objective function and nonlinear constraints.

3.4 SIMULATION RESULTS

Two different distribution systems were used to test the proposed optimization method in finding the optimal DG size and place. The first system is a 15-bus radial distribution system and the second system is a 33-bus meshed distribution system. Various scenarios, including single and multiple DG installations are analyzed using these systems. The following analysis is performed with the test systems and presented accordingly:

- Determining the optimal size and placing of DG.
- The effect of DG allocation on a voltage profile.
- The effect of the number of DG unit installations.

A voltage deviation index was calculated in all tests and cases to show improvements in the voltage profiles. The voltage deviation is mathematically formulated as follows:

$$VD\% = \left(\sum_{i=1}^{NB} (V_{ref} - V_i) \div NB\right) \times 100 \quad (3.54)$$

where V_{ref} is the voltage reference ($V_{ref} = 1$ *p.u.*),

The assumption made in the test is that all available DGs are of 4 MW capacities with a 0.85 power factor, and that the bus voltages are to be maintained within ±10% of the nominal voltage throughout the optimization process.

3.4.1 RADIAL DISTRIBUTION SYSTEM (15-BUS)

The first test was applied on an existing rural distribution feeder. This system consists of 15 buses and 14 branches at 11 kV voltage level. The capacity of the system is 1226.4 kW real power and 1251.2 kvar reactive power. The full network parameters are given in Appendix A [39]. Figure 3.1 shows the single line diagram of the radial distribution system under study, with its lateral branches. The optimization problem is investigated for single and multiple DG installation, as follows:

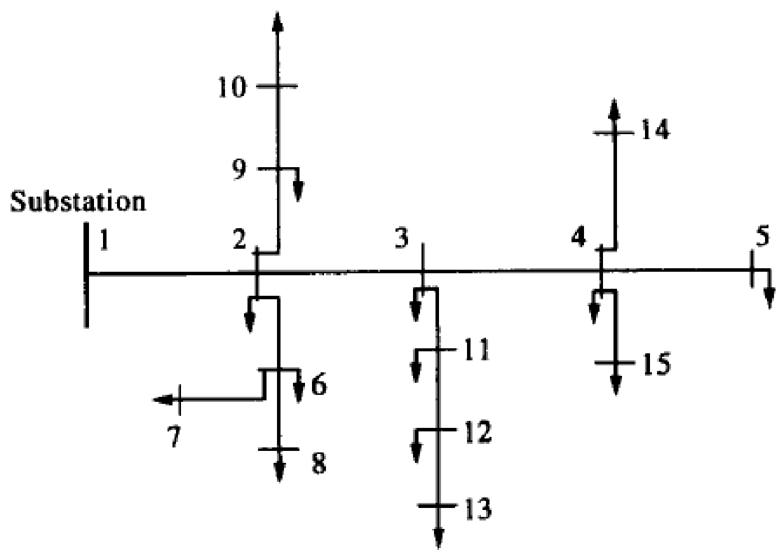

Figure 3.1: A single-line diagram of a 15-bus radial distribution system [39].

3.4.1.1 Case 1: Installing One DG

The proposed method was applied to a 15-bus radial distribution system by installing one DG at each candidate bus. All buses are considered as candidate buses in this test and in all subsequent tests. Table 3.1 shows the DG optimal size and corresponding real power losses and voltage deviation at all of the system buses.

Figure 3.2 shows the corresponding total real power losses for installing the optimal DG size at each bus of the system. From the figure, we can determine that the best bus for optimal DG allocation is at bus 3. Installing the DG at bus 3 with a size of 1192.965 kW caused a reduction in real power losses from 61.7945 kW to 17.25 kW, which is about a 72.085% reduction. Figure 3.3 shows the improvement in the voltage profile after installing the DG unit at bus 3. Here we can see that voltage deviation improved from 4.185% to 1.047%.

Table 3.1: Results of installing one DG.

DG bus	DG size (kW)	P loss (kW)	Losses reduction%	VD %
2	1226.4	25.908	58.075	2.019
3	1192.965	17.25	72.085	1.047
4	1012.799	18.948	69.336	1.201
5	726.561	30.264	51.025	1.952
6	795.812	31.625	48.823	2.281
7	662.002	35.2	43.037	2.54
8	628.8	37.133	39.909	2.616
9	700.201	42.145	31.798	2.726
10	487.805	47.572	23.016	3.103
11	830.574	25.071	59.428	1.627
12	585.706	33.399	45.952	2.146
13	467.566	38.487	37.718	2.48
14	655.675	32.458	47.475	2.129
15	798.721	25.961	57.989	1.748

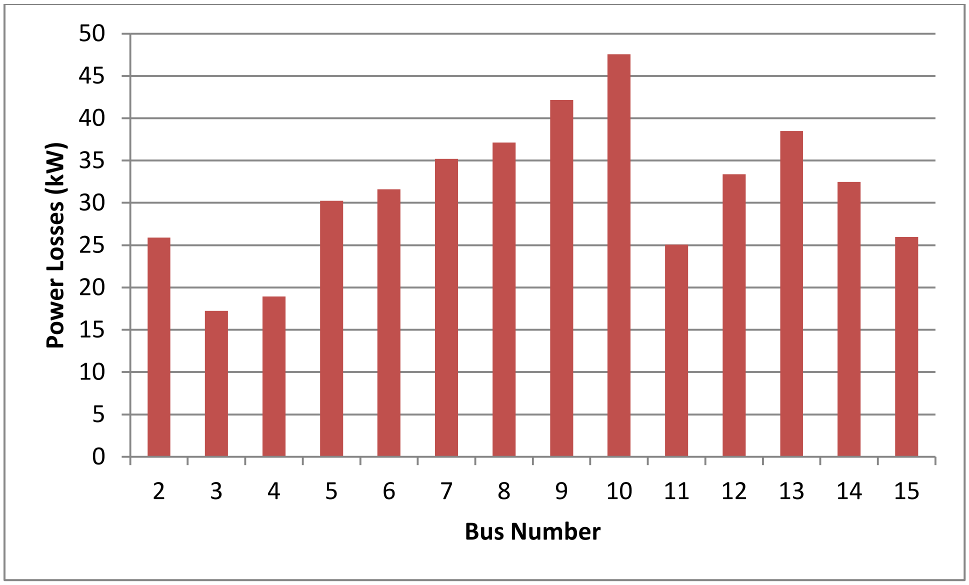

Figure 3.2: Total real power losses per DG placement.

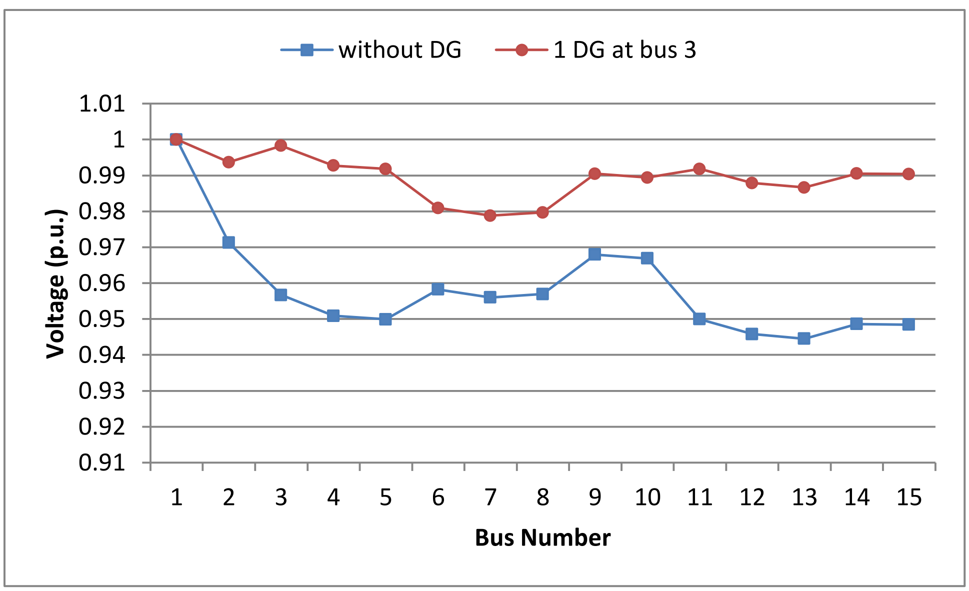

Figure 3.3: Voltage profiles of 15-bus radial distribution system.

3.4.1.2 Case 2: Installing Two DGs

An additional decision variable was added to handle the second DG, and all combinations for allocating two DGs in a distribution system were investigated. The best ten solutions for optimal DG allocation are listed in Table 3.2. It is clear that installing DGs at bus 4 with a 760.062 kW capacity and at bus 6 with a 466.338 kW capacity gives the optimal solution. However, other solutions can be taken as alternatives should the DGs be unable to be installed at the optimal location. At the optimal solution, the total real power losses are reduced from 61.7945 kW (at no DGs in the system) to 9.1 kW, which is about an 85.273% reduction. This reduction is higher than installing only a single DG; in other words, installing two DGs would give better results in reducing the total real power losses. The voltage profile was also improved, as shown in Figure 3.4, where the voltage deviation was reduced from 4.185% to 0.822%.

Table 3.2: Best ten optimal solutions for installing two DGs.

DG1 bus	DG1 size (kW)	DG2 bus	DG2 size (kW)	P loss (kW)	Losses reduction %	VD%
4	760.062	6	466.338	9.1	85.273	0.822
4	817.659	7	408.741	9.677	84.34	0.759
3	827.426	6	398.974	10.235	83.438	1.039
3	879.976	7	346.424	10.424	83.132	0.997
4	841.418	8	384.982	10.678	82.72	0.746
3	905.778	8	320.622	11.234	81.821	0.991
6	554.466	11	647.798	12.179	80.291	0.866
6	564.081	15	619.926	12.554	79.684	0.945
7	505.401	11	632.358	13.431	78.265	0.983
7	497.542	15	622.524	13.797	77.672	1.05

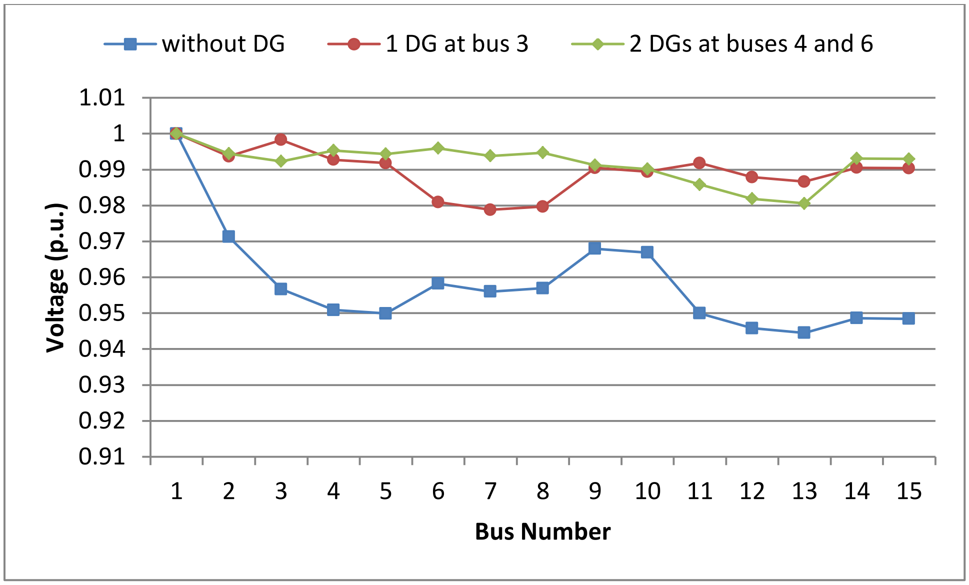

Figure 3.4: Voltage profiles of 15-bus radial distribution system.

3.4.1.3 Case 3: Installing Three DGs

In this test, in addition to the decision variables for the voltage magnitude and its angles, there are three decision variables for each DG. The optimization problem was solved and the best ten solutions are provided in Table 3.3. The results show that the total real power losses were reduced even more compared to the previous scenarios. The optimal locations for the DGs were at bus 4, 6 and 12, with sizes of 575.153 kW, 426.505 kW and 224.743 kW, respectively. The total real power loss for this solution is 6.103 kW, (which is about a 90.124% reduction) and the voltage deviation is 0.677%. Figure 3.5 shows the improvement in the radial system voltage profiles in this case as well as in previous cases.

Table 3.3: Best ten optimal solutions for installing three DGs.

DG1 bus	DG1 size (kW)	DG2 bus	DG2 size (kW)	DG3 bus	DG3 size (kW)	P Loss (kW)	Losses reduction %	VD%
4	575.153	6	426.505	12	224.743	6.103	90.124	0.677
4	500.021	6	409.547	11	316.832	6.151	90.046	0.743
4	534.348	7	360.156	11	331.897	6.407	89.631	0.685
4	616.757	7	375.197	12	234.446	6.448	89.566	0.614
4	615.646	6	436.622	13	174.132	6.548	89.404	0.682
6	449.584	11	396.471	15	380.346	6.759	89.063	0.721
4	661.954	7	383.111	13	181.335	6.942	88.766	0.617
7	398.217	11	422.29	15	405.894	7.238	88.287	0.655
4	549.972	8	336.798	11	339.63	7.262	88.249	0.672
4	635.901	8	351.676	12	238.823	7.346	88.112	0.599

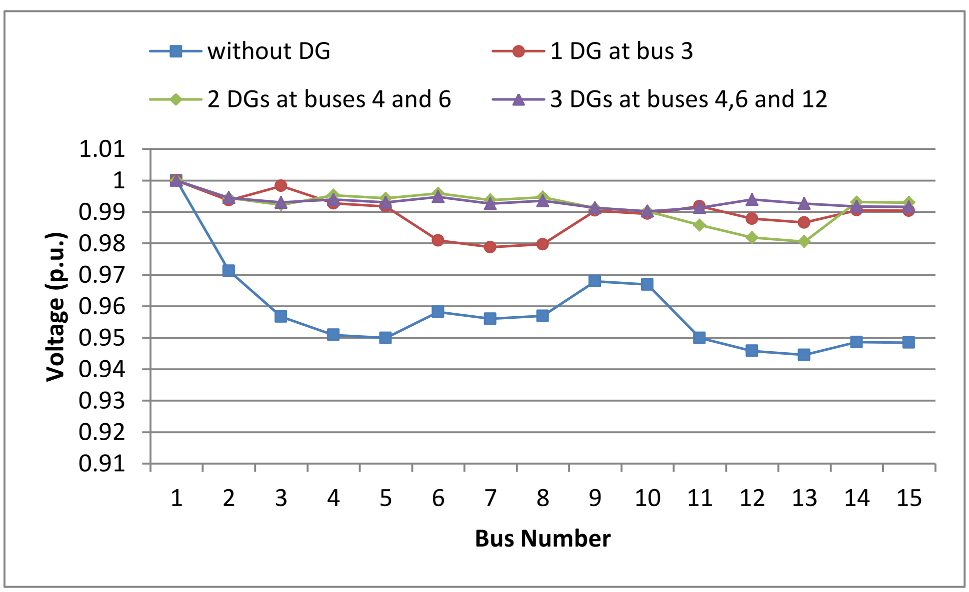

Figure 3.5: Voltage profiles of 15-bus radial distribution system.

3.4.2 MESHED DISTRIBUTION SYSTEM (33-BUS)

In the second test, a meshed distribution system was used to investigate the proposed optimization problem in finding the optimal DG size and place. The 33-bus meshed distribution system is a 12.66 kV voltage level and has 33 bus and 37 branches. The total active and reactive loads are 3715 kW and 2300 kvar, respectively. The corresponding single line of the meshed distribution system is shown in Figure 3.6 and the system's parameters are provided in Appendix B [40] . The optimization problem was solved for single and multiple DG installations.

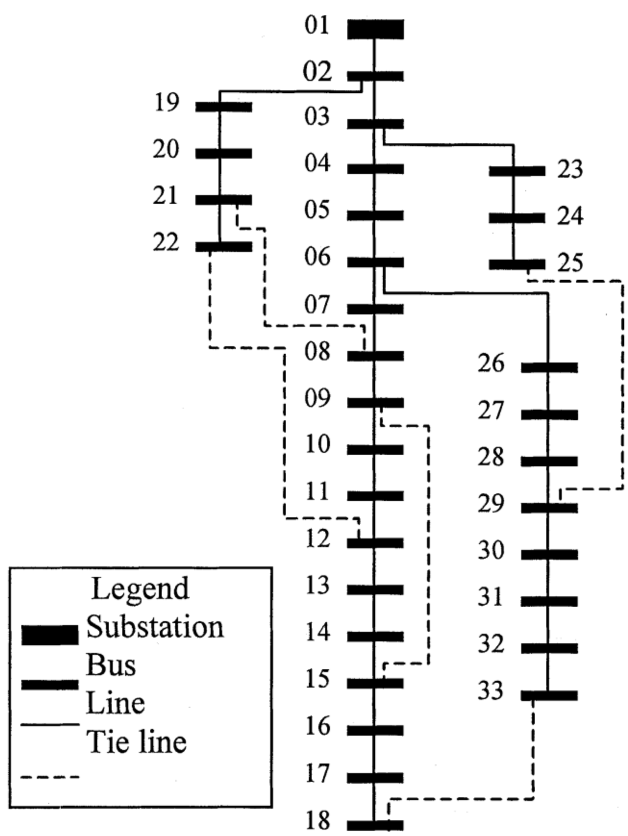

Figure 3.6: A single-line diagram of a 33-bus meshed distribution system [41].

3.4.2.1 Case 1: Installing One DG

At all of the 33 buses, the optimal DG sizing problem was solved for installing a single DG. The results are listed in Table 3.4. Figure 3.7 shows the corresponding total real power losses for installing an optimal DG size at each bus of the system. The figure shows that the minimal total real power loss is at bus 29. By locating the single DG at bus 29 with power output of 2357.809 kW, the total real power loss is reduced from 123.35 kW at no DG installed to 30.889 kW, which is an approximate reduction of 74.96% in losses. As shown in Figure 3.8, voltage profiles are also improved, with voltage deviation being reduced from 3.08% to 0.966%.

Table 3.4: Results of installing one DG.

DG bus	DG size (kW)	P loss (kW)	Losses reduction%	VD %
2	3711.208	110.94	10.068	2.8
3	3006.457	79.382	35.65	2.075
4	2446.301	78.283	36.541	2.014
5	2247.294	74.005	40.009	1.884
6	2320.597	56.086	54.535	1.35
7	2207.223	58.638	52.466	1.361
8	1959.784	62.382	49.431	1.316
9	1773.961	62.564	49.283	1.249
10	1584.946	68.674	44.331	1.384
11	1586.072	68.693	44.315	1.38
12	1612.039	68.204	44.711	1.361
13	1492.105	67.157	45.56	1.359
14	1555.862	63.139	48.817	1.253

15	1670.43	57.997	52.986	1.117
16	1599.876	57.7	53.226	1.155
17	1616.504	52.098	57.767	1.104
18	1690.069	46.871	62.005	1.037
19	2236.962	112.415	8.872	2.788
20	1688.086	90.315	26.788	2.101
21	1775.509	81.825	33.669	1.831
22	1538.33	82.292	33.291	1.811
23	2406.231	75.128	39.098	2.017
24	2227.792	55.917	54.671	1.654
25	2283.963	38.425	68.851	1.23
26	2217.459	56.624	54.099	1.377
27	2119.876	56.62	54.102	1.401
28	2093.641	47.743	61.298	1.269
29	2357.809	30.889	74.96	0.966
30	2160.214	31.076	74.809	0.979
31	1884.635	37.653	69.477	0.98
32	1828.466	39.395	68.065	0.974
33	1763.651	42.671	65.409	0.992

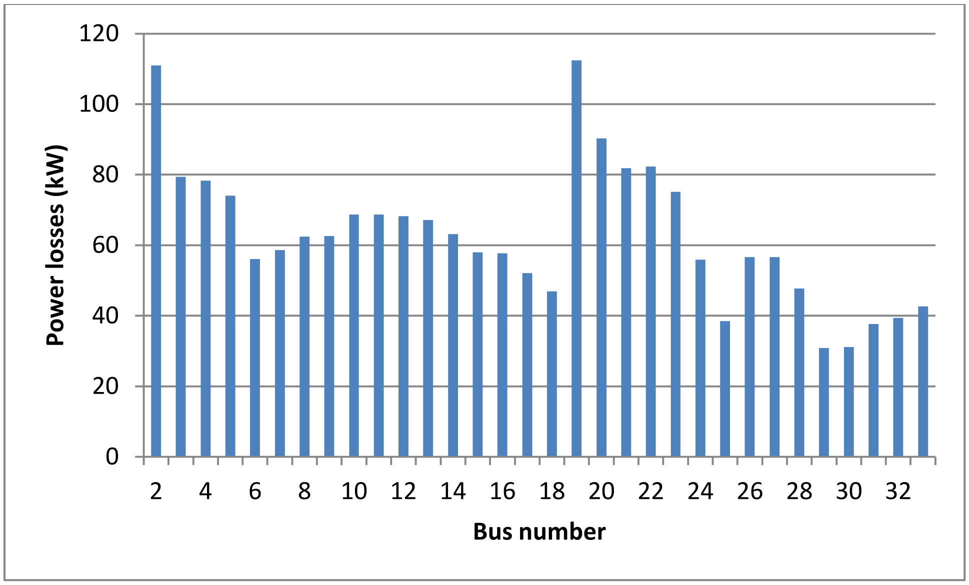

Figure 3.7: Total real power losses per DG placement.

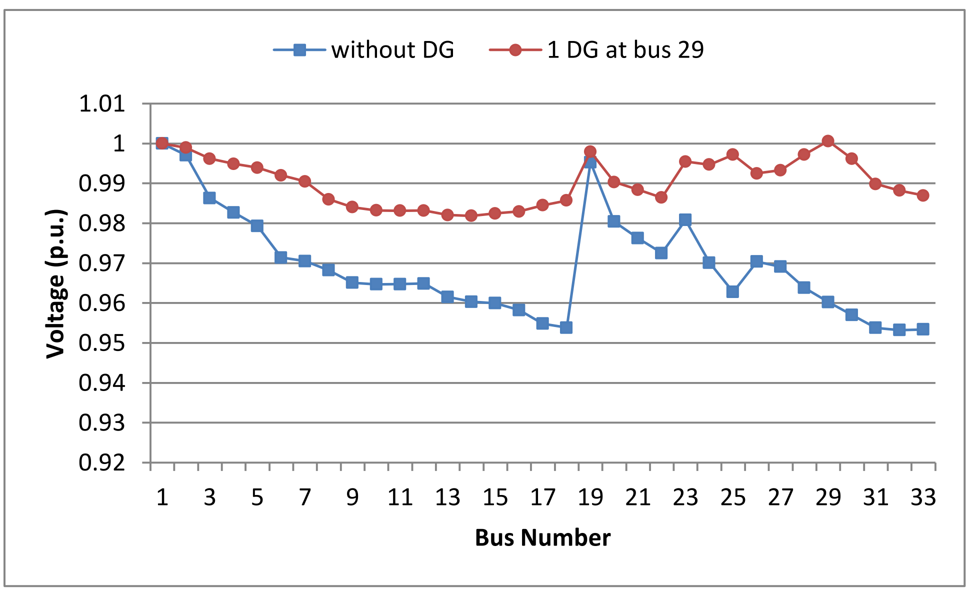

Figure 3.8: Voltage profiles of 33-bus meshed distribution system.

3.4.2.2 Case 2: Installing Two DGs

In this test, all combinations for installing two DGs were examined. Table 3.5 shows the best ten solutions for optimal DG size and placement. Installing two DG units at 15 with an output of 919.063 kW and at 29 with an output of 1831.496 kW caused a reduction in the total real power system to a minimum value. This value, as shown in the mentioned table, is 15.673 kW, which signifies an 87.295% reduction in the system's losses compared to original losses. Moreover, a significant improvement in the voltage profile occurred, as shown in Figure 3.9, where the voltage deviation was 0.355%.

Table 3.5: Best ten optimal solutions for installing two DGs.

DG1 bus	DG1 size (kW)	DG2 bus	DG2 size (kW)	P loss (kW)	Losses reduction %	VD%
15	919.063	29	1831.496	15.673	87.295	0.355
9	979.217	29	1877.888	16.145	86.912	0.384
14	857.256	29	1883.2	16.295	86.791	0.382
8	1076.381	30	1721.038	16.532	86.598	0.435
9	973.616	30	1720.61	16.595	86.548	0.399
12	895.387	29	1936.432	16.785	86.393	0.387
8	1057.124	29	1873.093	17.005	86.215	0.447
11	878.399	29	1940.05	17.007	86.214	0.397
13	818.129	29	1923.609	17.077	86.156	0.409
10	873.192	29	1944.719	17.099	86.139	0.4

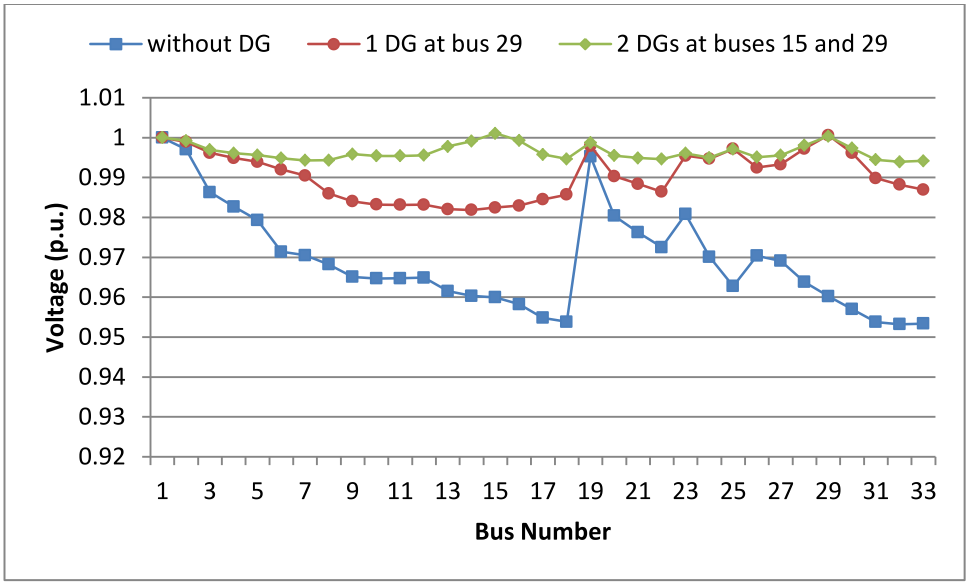

Figure 3.9: Voltage profiles for 33-bus meshed distribution system.

3.4.2.3 Case 3: Installing Three DGs

The optimal DG sizing and placement for integrating three DG units was investigated for all DG combinations in the system. The optimal solution that minimizes the objective function was found at buses 8, 25 and 32, with a real power output of 913.298 kW, 1213.427 kW and 873.196 kW, respectively. Here, total real power losses were reduced to 9.517 kW. This indicates a reduction of about 92.285% from the pre-installation case. By installing the DGs at the optimal allocation in the system, the voltage deviation is reduced from 3.08% to 0.266%. Figure 3.10 shows the voltage profile improvement for these and previous cases.

Table 3.6: Best ten optimal solutions for installing three DGs.

DG1 bus	DG1 size (kW)	DG2 bus	DG2 size (kW)	DG3 bus	DG3 size (kW)	P Loss (kW)	Losses reduction %	VD%
8	913.298	25	1213.427	32	873.196	9.517	92.285	0.266
8	936.123	25	1165.527	31	901.811	9.786	92.067	0.285
9	809.616	25	1278.621	32	829.962	9.956	91.93	0.25
8	936.928	25	1260.045	33	814.669	10.02	91.878	0.248
9	842.59	25	1219.51	31	860.615	10.045	91.857	0.26
8	891.256	29	1296.079	32	746.085	10.401	91.569	0.254
9	911.923	24	882.823	30	1289.332	10.47	91.513	0.317
12	747.983	25	1301.664	32	845.15	10.471	91.512	0.251
11	734.686	25	1299.729	32	851.105	10.552	91.446	0.256
8	875.131	29	1366.401	33	697.877	10.558	91.441	0.238

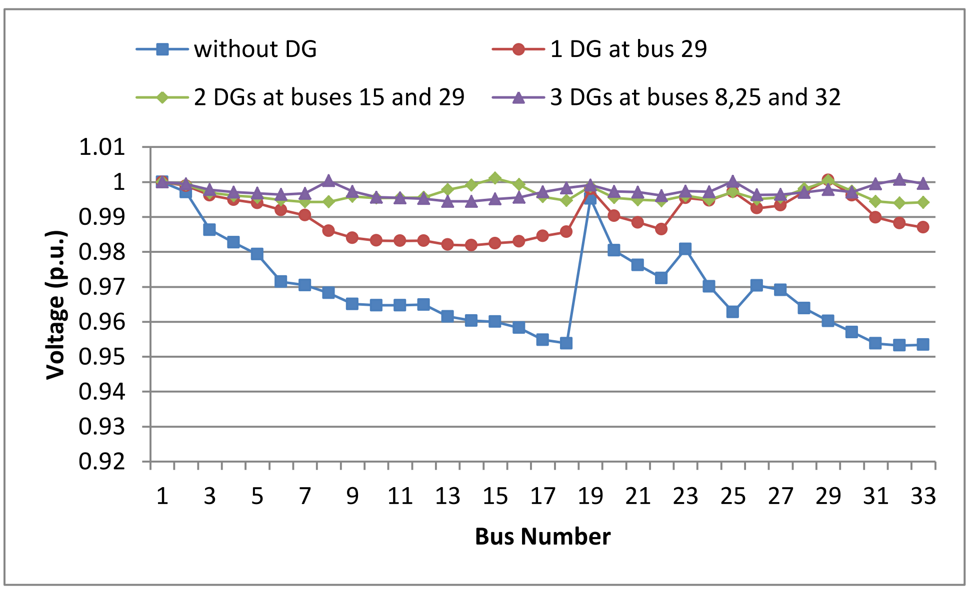

Figure 3.10: Voltage profiles for 33-bus meshed distribution system.

3.5 ROUNDING OFF THE DG SIZES

The sizes of the DG units obtained in the results are not practical; therefore, they should be set to operate at discrete (practical) values which may have an effect on the optimality. The DG sizes are approximated to the nearest integer values with a 100 kW step interval between sizes, and the total real power losses are calculated. Table 3.7, Table 3.8 and the rest of the tables in Appendix C show that the change in the total real power losses is small and has no effect on the optimality results.

Table 3.7: Rounding off results: 15-Bus system, Case (1).

DG bus	DG size (kW)	P loss (kW)	**Rounding off results**	
			DG1 size (kW)	P loss (kW)
2	1226.4	25.908	1200	26.148
3	1192.965	17.25	1200	17.251
4	1012.799	18.948	1000	18.955
5	726.561	30.264	700	30.303
6	795.812	31.625	800	31.625
7	662.002	35.2	700	35.281
8	628.8	37.133	600	37.181
9	700.201	42.145	700	42.145
10	487.805	47.572	500	47.58
11	830.574	25.071	800	25.117
12	585.706	33.399	600	33.414
13	467.566	38.487	500	38.587
14	655.675	32.458	700	32.578
15	798.721	25.961	800	25.961

Table 3.8: Rounding off results: 33-Bus system, Case (1).

DG bus	DG size (kW)	P loss (kW)	**Rounding off results**	
			DG size (kW)	P loss (kW)
2	3711.208	110.94	3700	110.944
3	3006.457	79.382	3000	79.383
4	2446.301	78.283	2400	78.299
5	2247.294	74.005	2300	74.031
6	2320.597	56.086	2300	56.091
7	2207.223	58.638	2200	58.639
8	1959.784	62.382	2000	62.406
9	1773.961	62.564	1800	62.577
10	1584.946	68.674	1600	68.678
11	1586.072	68.693	1600	68.697
12	1612.039	68.204	1600	68.207
13	1492.105	67.157	1500	67.159
14	1555.862	63.139	1600	63.184
15	1670.43	57.997	1700	58.015
16	1599.876	57.7	1600	57.7
17	1616.504	52.098	1600	52.106
18	1690.069	46.871	1700	46.873
19	2236.962	112.415	2200	112.418
20	1688.086	90.315	1700	90.317
21	1775.509	81.825	1800	81.833
22	1538.33	82.292	1500	82.317
23	2406.231	75.128	2400	75.128
24	2227.792	55.917	2200	55.927
25	2283.963	38.425	2300	38.429
26	2217.459	56.624	2200	56.628
27	2119.876	56.62	2100	56.626

28	2093.641	47.743	2100	47.743
29	2357.809	30.889	2400	30.915
30	2160.214	31.076	2200	31.104
31	1884.635	37.653	1900	37.658
32	1828.466	39.395	1800	39.413
33	1763.651	42.671	1800	42.699

3.6 SUMMARY

In this chapter, a deterministic method to find optimal DG sizing and placement in a distribution network was proposed, where the total real power losses of the network were employed as the objective to be minimized. The proposed method was formulated as a constrained nonlinear programming problem and applied to two different distribution systems topologies (15-bus radial distribution systems and 33-bus meshed distribution systems) to show its applicability. Additionally, single and multiple DG installation cases were performed for each test system and compared to the case without DG. The results demonstrated that DG size and placement have a significant influence in minimizing power losses as well as improving voltage profiles. It was also demonstrated that integrating multiple DGs reduces the system power losses more than integrating only one DG.

CHAPTER 4 OPTIMAL DISTRIBUTED GENERATION SIZING AND PLACEMENT VIA MULTI-OBJECTIVE OPTIMIZATION APPROACH

4.1 INTRODUCTION

As concluded in Chapter 3, total real power losses are reduced and voltage profiles improved when more than a single DG are installed in distribution networks. Thus, distribution planners should increase DG numbers to enhance these positive impacts. However, costs related to DG units, such as purchase, installation, operation and maintenance costs, should also be considered. In this chapter, the optimal DG sizing and placement are solved via a multi-objective optimization approach. The total real power losses and the overall DG installation cost are two important factors that this approach considers as objective functions. A set of optimal solutions commonly called Pareto front is obtained using the SQP deterministic method as a solver, with the weighted sum method used to combine the objectives. In addition, a fuzzy decision making method is applied to guide the decision-maker to the compromise trade-off solutions among two different objective functions.

4.2 PROBLEM FORMULATION

The single-optimization problem explained in section 3.2 deals with no more than one objective. Therefore, a multi-objective optimization concerns the minimization of a vector of objectives, while satisfying a number of equality and inequality constraints or bounds. The multi-objective optimization problem can be mathematically formulated as follows:

$$\underset{x \in R^n}{minimize}: \qquad F(x) = [f_1(x), f_2(x), \ldots, f_k(x)] \tag{4.1}$$

$$subjec\ to: \qquad h_i(x) = 0, \qquad i = 1,2, \ldots, n \tag{4.2}$$

$$g_i(x) \leq 0, \qquad i = 1,2, \ldots, m \tag{4.3}$$

$$x^{min} \leq x \leq x^{max} \tag{4.4}$$

where *F(x)* is a vector of *k* objective functions.

We called a solution for any choice of values of $x=[x_1, x_2, \ldots, x_n]$, and a solution that satisfies all of the constraints is called a feasible solution. Therefore, a set of all feasible solutions makes a feasible area in the decision space, and can be defined by the set of *S*, as follows:

$$S = \{x \in R^n | h_i(x) = 0, g_i(x) \leq 0\} \tag{4.5}$$

We refer a 'point' as the corresponding objective vector to the set of *S*. The feasible domain contained in the objective space is defined as the set of *Z* [42, 43].

$$Z = \{z \in R^k | z_1 = f_1(x), z_2 = f_2(x), \ldots, z_k = f_k(x), x \in S\} \tag{4.6}$$

4.2.1 PROBLEM OBJECTIVES

The objective functions adopted in the optimization problem are the total real power losses function and the total cost of DG installation cost. Both objective functions are subject to equality and inequality constraints. The first objective and the constraints are previously delineated in section 3.2. The second objective is the total DG installation cost.

$$F_2 = \min DG_{cost} \tag{4.7}$$

The total cost of DG installation depends on the number, type and the capacity of units. It can be expressed by aggregating investment costs, represented by fixed costs such as DG unit cost, investigation fee, site preparation for DG installation, construction costs, monitoring equipment costs, etc. and the operation and maintenance costs which represent the running costs. This cost is not related to DG placement and is the same for all DG placements. Hence, the total cost of DG installation can be mathematically formulated as [44]:

$$Cost = \sum_{i=1}^{nDG} \left(C_{Inv_i} \times DG_i^{max}\right) + 8760 \times \sum_{t=1}^{T} \sum_{i=1}^{nDG} \left(\beta_t \times C_{M\&O_i} \times P_{DG_i}\right) \tag{4.8}$$

$$presentworth: \beta_t = \frac{1}{(1+d)^t} \tag{4.9}$$

where:

C_{Inv} : Investment cost $\left(\frac{M\$}{MVA}\right)$,

$C_{M\&O}$: Maintenance and operation cost $\left(\frac{\$}{MWh}\right)$,

P_{DG_i} : Generated power by DG source installed in bus i (MW),

$\mathrm{DG}_{\mathrm{i}}^{\mathrm{max}}$: Selected capacity of DG for installation in bus i (MVA),

d : Discount rate,

T : Planning period (year),

nDG : Number of DG placements in the network.

4.2.2 Pareto Optimality Principle

The solution of multi-objective problems is generally different from that of single-objective optimization problems. In multi-objective optimization problems, the optimum solution may not exist with respect to all objectives. In other words, a solution may be optimal for one or some objectives while not being optimal for the rest, resulting in a set of incomparable solutions. The solutions in this set are known as non-dominated solutions or Pareto optimal solutions [43]. Vilfredo Pareto generalized the concept of the Pareto optimal, originally introduced by Francies Ysidro [45]. The principle states that, for multi-objective optimization problems, a solution x_1 is said to dominate the other solution x_2 if the following two conditions are true:

1- The solution x_1 is not worse than the solution x_2 in all objectives.

$$\forall\ i \in \{1, 2, \dots, k\} \rightarrow f_i(x_1) \leq f_i(x_2) \tag{4.10}$$

2- The solution x_1 is strictly better than x_2 in at least one objective.

$$\exists\, j \in \{1, 2, \dots, k\} \rightarrow f_j(x_1) < f_j(x_2) \tag{4.11}$$

If any of the above conditions is violated, the solution x_1 does not dominate the solution x_2. If x_1 dominates the solution x_2, x_1 is called a non-dominated solution. The set of all non-dominated solution points is called the Pareto optimal set and the front obtained by mapping these non-dominated particles into the objective space is called the Pareto optimal front or simply Pareto front [46]. Figure 4.1 depicts a Pareto set for a two-objective minimization problem.

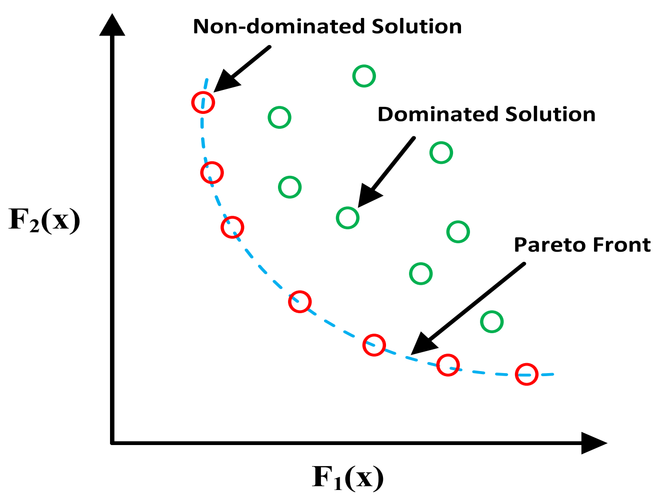

Figure 4.1: Pareto Optimal Front [46].

4.2.3 Weighted Sum Method

Multi-optimization problems are involved in many real-world applications [47]. The idea of most classical multi-optimization methods is to transform the optimization problem from a multi-objective one into a single-objective one, and then to solve the problem by using traditional scalar-valued optimization methods. Classical methods include ε -constraint method and weighted-sum methods. In the ε –constraint method, one of the objective functions is considered a master objective while the other objectives are added to the constraints with acceptable bound values, ε. The weighted sum method was proposed by Zadeh in 1963 [45] and it is the simplest and most popular of all classical methods [47]. It transforms the set of objectives into a single objective function by multiplying each objective with a proper positively-weighted factor depending on the priority of each objective and the decision-maker's preferences. These weights range from 0 to 1, the total summation should be equal to one, and the summing up of all weighted objective functions should be such that further objective functions can be added. The mathematical problem formulated for the weighted sum method is as follows [45]:

$$min\ F = \sum_{i=1}^{k} w_i f(x)_i \tag{4.12}$$

$$Where: \quad w_i \geq 0 \quad and \quad \sum_{i=1}^{k} w_i = 1 \tag{4.13}$$

The solution of the aggregation function in (4.12) gives only one point in the objective space. Therefore, each set of selection weights generates one Pareto point. Thus, by changing the values of the weights, a trade-off between objectives can be determined over the range of values of weights. In a case when the decision-maker has insufficient knowledge or no prior information, the problem should be solved iteratively with incrementally weighting values in order to generate the distributed solutions on the entire Pareto optimal set [48].

4.2.4 FUZZY DECISION MAKING

After obtaining the Pareto optimal set solution, the ultimate goal of the decision-maker is to choose one best compromise solution among the Pareto optimal front. A fuzzy satisfying method, which represents the goals of each objective function, is applied to find the best compromise solution. For each solution in the Pareto optimal front *k*, a simple linear membership function μ_i^k is considered for each of the objective functions. The membership function is defined as follows [49]:

$$\mu_i^k = \begin{cases} 1 & f_i^k \le f_i^{min} \\ \dfrac{f_i^{max} - f_i^k}{f_i^{max} - f_i^{min}} & f_i^{min} < f_i < f_i^{max} \\ 0 & f_i^{max} \le f_i^k \end{cases} \tag{4.14}$$

where f_i^{max} and f_i^{min} are maximum and minimum values of the objective function *i*.

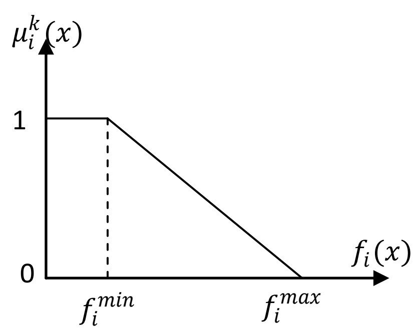

Figure 4.2: Linear type membership function [50].

The membership function μ_i^k ranges from 0 to 1, where $\mu_i^k = 0$ indicates incompatibility of the solution with the set, while $\mu_i^k = 1$ means full compatibility. Figure 4.2 illustrates the graph of this membership function. For each member of the non-dominated set, k, the normalized membership value is calculated using the following equation:

$$\mu^k = \frac{\sum_{i=1}^{nf} \mu_i^k}{\sum_{k=1}^{nk} \sum_{i=1}^{nf} \mu_i^k} \tag{4.15}$$

where

nf : the number of objective functions.

nk : the number of non-dominated solutions.

The non-dominated solution, having the maximum value of the membership, μ^k, can be chosen as the best compromise solution. Moreover, arranging all of the normalized membership values in descending order provides the decision-maker with a priority guidance list of non-dominated solutions. Figure 4.3 simplifies the idea of the compromise solution among Pareto front solutions.

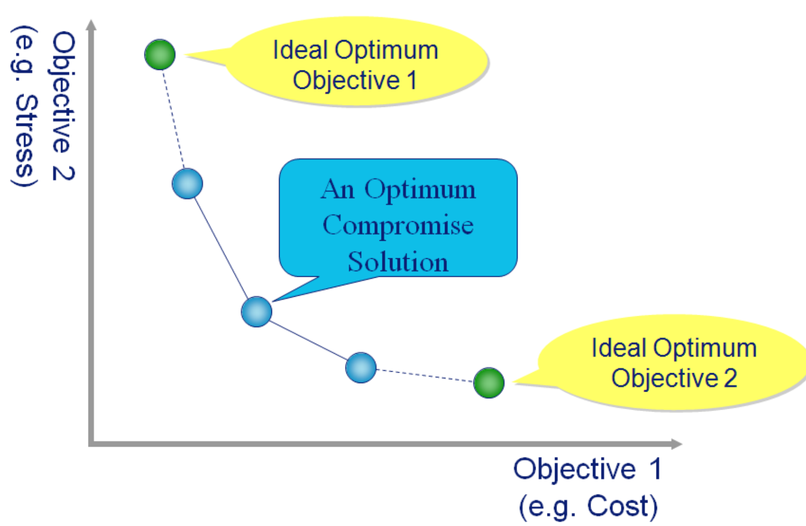

Figure 4.3: The optimum compromise solution [51].

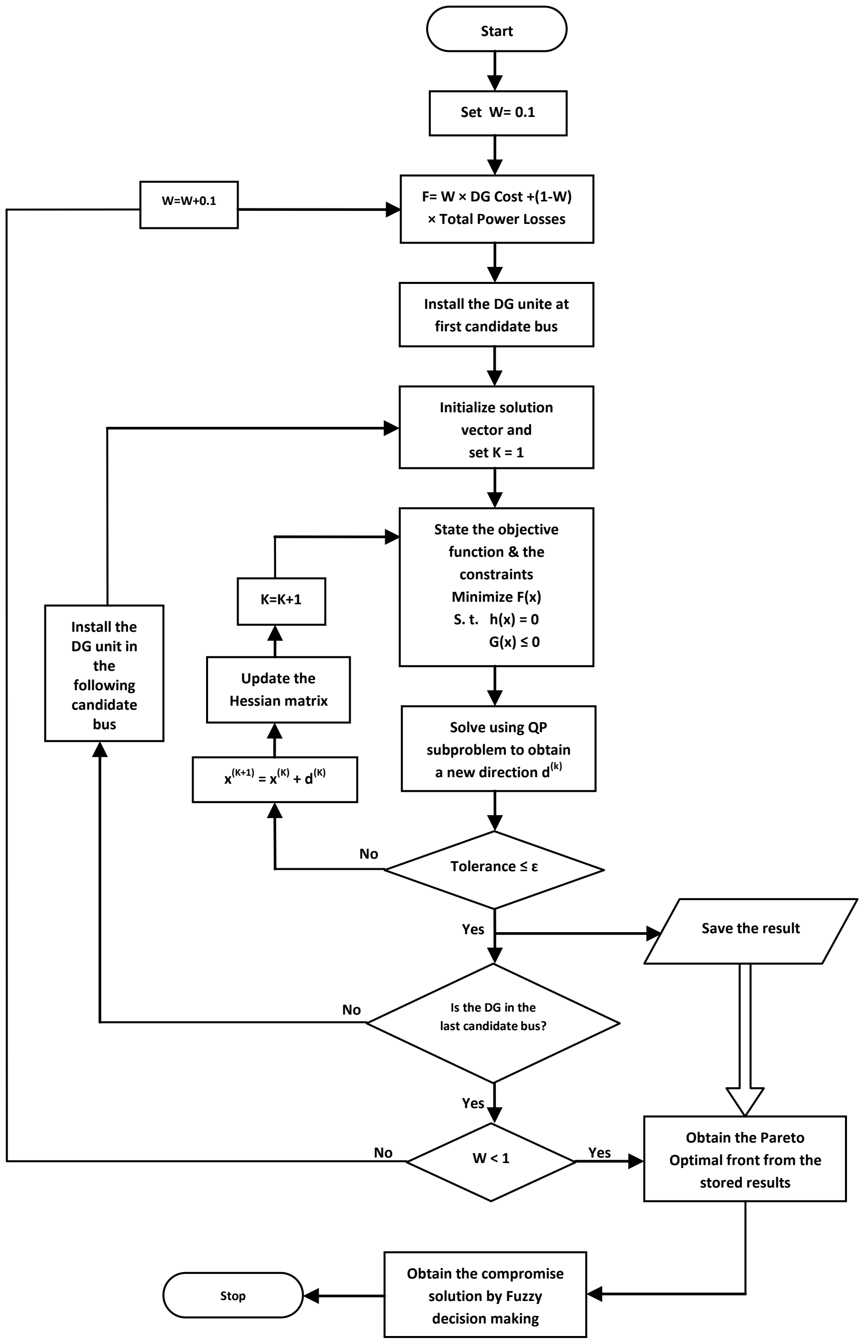

Figure 4.4: Flowchart of the multi-objective optimization problem.

4.3 Simulation Results

Based on the preceding sections that illustrated in the flowchart shown in Figure 4.4, two main objectives are investigated. The same distribution systems used in the previous chapter are used again here to investigate the proposed multi-objective method to ascertain optimal DG sizing and placement. The assumption made in the simulation is that all available DGs are of 2 MW capacities with a 0.85 power factor. For DG cost data [52], the fixed or investment cost is 0.5 M$/MW and the variable costs consist of maintenance and operation is 50 $/MWh. The planning period is 20 years, with a 12.5% discount rate. Single and multiple DG installation cases were considered in the tests.

4.3.1 Radial Distribution System (15-Bus)

The first system is a 15-bus radial distribution system. It was tested for the single-objective optimization technique proposed in the previous chapter. The data of the system are given in Appendix A. A multi-objective optimization problem is applied to obtain the Pareto optimal set of non-dominated solutions and the compromise solution among them. Single and multiple DG installation cases are considered in the test, which ended with a comparative study of the two cases.

4.3.1.1 Case 1: Installing Single DG

In this case, the multi-objective optimization problem is solved by integrating a single DG to a 15-bus radial distribution system to find the optimal location and size. Figure 4.5 shows the non-dominated solutions set. The non-dominated solutions or the Pareto optimum front consists of 57 trade-off solutions. As shown in Figure 4.5, A and B represent the two ends of the Pareto optimal front. At point A, the DG size is 9.02 kW at bus 6. This solution gives the minimum cost 1028.6 k$ and the maximum power loss at 61.086 kW among all solutions. On the other hand, in the solution at point B, the DG size is 1193.6 kW at bus 3 and the lowest reduction in real power loss drop from 61.7945 kW at no DG installation to 17.25 kW, which is about 72.085%. However, the cost of DG installation is relatively high compared to the other optimum solution, due to the high amount of power generated from the DG unit, as shown in Table 4.1. Between the two extreme solution points, A and B, we have a set of trade-off optimum solutions, where the decision-maker can choose any one. By applying the fuzzy decision-making method, the best compromise solution was investigated, as shown in Figure 4.4. In the best compromise solution, the DG size is 537.38 kW at bus 4 and the total real power loss is reduced to 27.909 kW. This signifies a 54.84% reduction in distribution network losses.

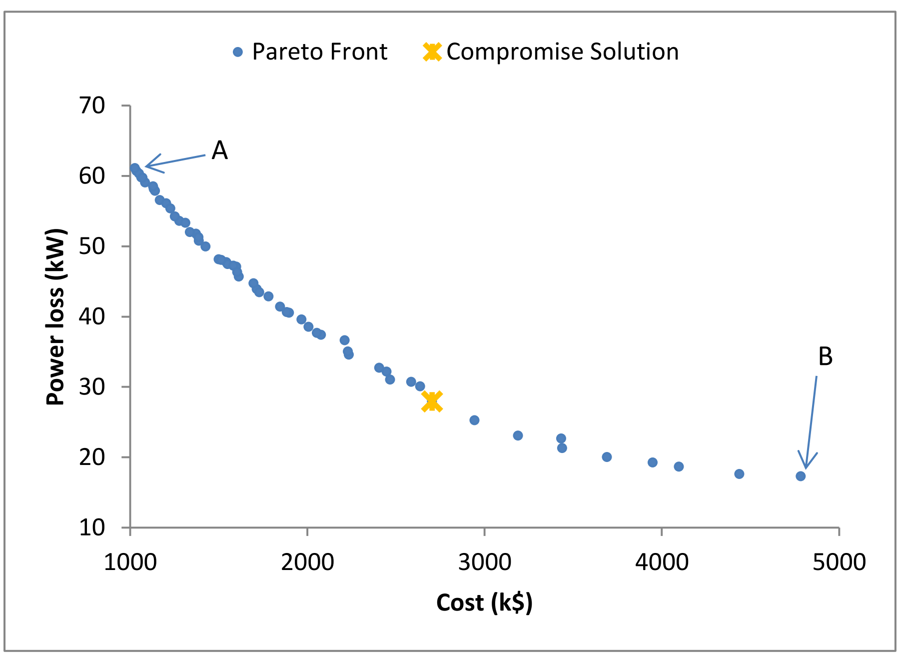

Figure 4.5: Pareto front of Case 1 (One DG unit).

Table 4.1: Results of Case 1.

	DG1 bus	DG1 size (kW)	Cost (k$)	P Loss (kW)
A	6	9.02	1028.6	61.086
B	3	1193.6	4785.8	17.25
Compromise Solution	4	537.3828	2704.417	27.909

4.3.1.2 Case 2: Installing Multiple DGs

In this case, the multi-objective problem is solved by installing two DG units at one time and three DG units at another time. The Pareto front for installing two DG units and three DG units are shown in Figure 4.6 and Figure 4.7, respectively. From these figures, we can see that multiple DG installations decreased real power losses more than that of a single DG installation. However, multiple DG installations may result in unnecessary additional costs. Table 4.2 provides details of the best compromise solution points.

Table 4.2: Compromise solution of Case 2.

	DG1 bus	DG1 size (kW)	DG2 bus	DG2 size (kW)	DG3 bus	DG3 size (kW)	Cost (k$)	P Loss (kW)
2 DGs installation	12	196.85	15	273.5	-	-	3491.8	28.56
3 DGs installation	7	164.38	12	177.22	15	232.84	4821.9	22.27

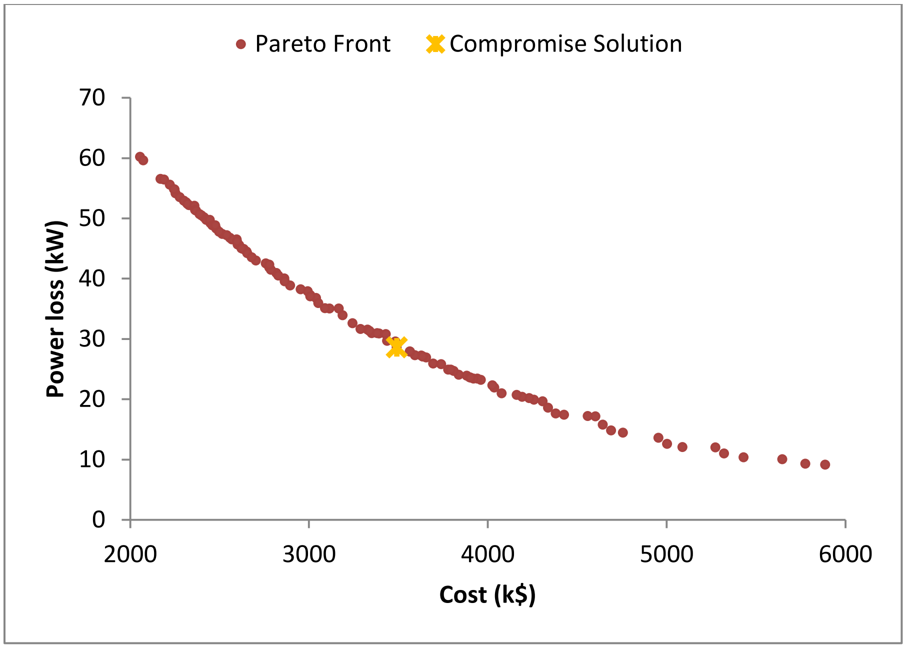

Figure 4.6: Pareto front of Case 2 (Two DG units).

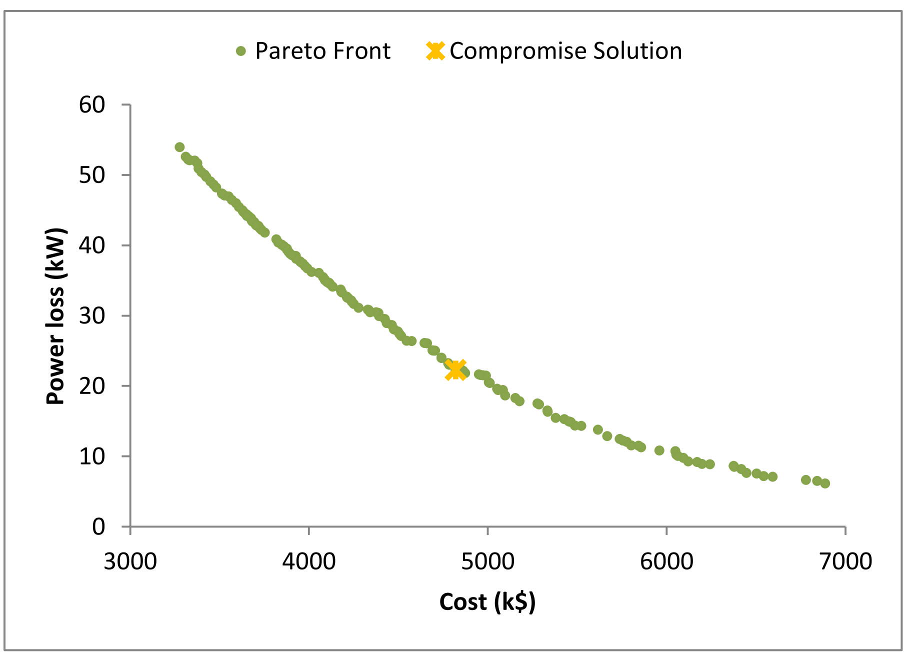

Figure 4.7: Pareto front of Case 2 (Three DG units).

4.3.1.3 Comparison of the Cases

A comparison of the optimal results of the two previous cases is shown in Figure 4.8, where the Pareto optimal fronts of single DG and multiple DG installation cases are combined in the same objective space. The figure shows that there are some solution points from different curves dominated by other points. Therefore, we obtained a new Pareto front for the combined fronts. The generated Pareto optimal front consists of solution points from different cases. The best compromise solutions can be obtained by applying the fuzzy decision-making method to the generated Pareto optimal front. This point was investigated and results are listed in Table 4.3.

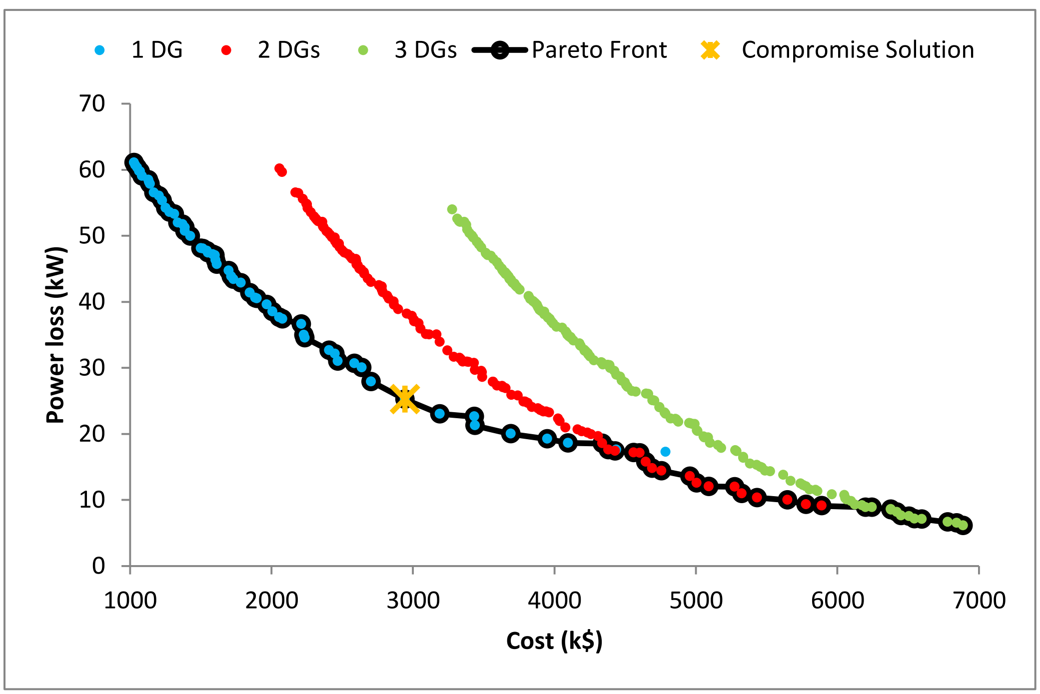

Figure 4.8: Comparison of the cases.

Table 4.3: Compromise solution of all cases.

DG1 bus	DG1 size (kW)	DG2 bus	DG2 size (kW)	DG3 bus	DG3 size (kW)	Cost (k$)	P Loss (kW)
4	613.1739	-	-	-	-	2944.8	25.24

4.3.2 MESHED DISTRIBUTION SYSTEM (33-BUS)

The second test system is 33-bus meshed distribution system which was tested in the previous chapter for the single-objective optimization problem. Data for the system are given in Appendix A. The multi-objective optimization problem is implemented to generate the non-dominated solutions set or the Pareto front as well as the compromise solution. Single and multiple DG installation cases and comparison studies are considered, as follows.

4.3.2.1 Case 1: Installing Single DG

In this case, the multi-objective optimization problem is investigated by locating a single DG unit in the 33-bus meshed distribution system. The two proposed objectives are converted into a single objective by using the weighting factors. The non-dominated solution set is obtained using the proposed method, and the best compromise solution is also investigated using the fuzzy decision-making method. The Pareto front and the compromise solution point are shown in Figure 4.9. In the compromise solution, the DG size is 928.02 kW at bus 31. The total real power loss is reduced to 58.68 kW and the total cost is 3943.4 k$.

Table 4.4: Compromise solution of Case 1.

	DG1 bus	DG1 size (kW)	Cost (k$)	P Loss (kW)
Compromise Solution	31	928.02	3943.41	58.68

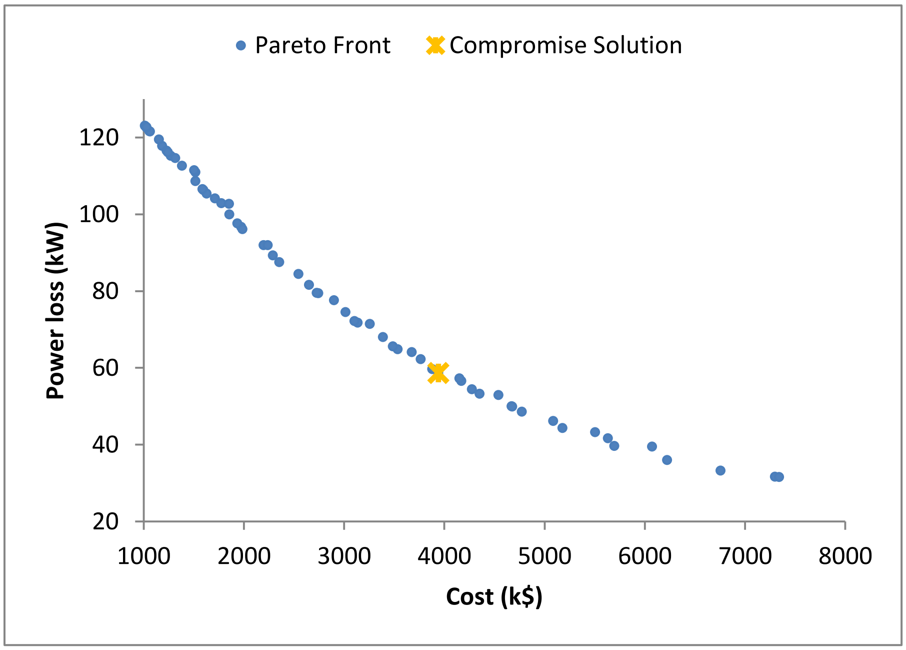

Figure 4.9: Pareto front of Case 1 (One DG unit).

4.3.2.2 Case 2: Installing Multiple DGs

The multi-objective problem in this case is implemented by installing two DG units first and then by installing three DG units. Figure 4.10 and Figure 4.11 show the trade-off curves for installing two DG units and three DG units, respectively. Integrating three DGs reduced the total real power up to 92% in some instances. However, additional costs have to be paid to achieve this reduction. The compromise solutions for both cases are listed in Table 4.5.

Table 4.5: Compromise solution of Case 2.

	DG1 bus	DG1 size (kW)	DG2 bus	DG2 size (kW)	DG3 bus	DG3 size (kW)	Cost (k$)	P Loss (kW)
2 DGs installation	30	693.56	33	541.41	-	-	5917	43.396
3 DGs installation	14	252.7	30	597.44	32	478.54	7214.2	38.6

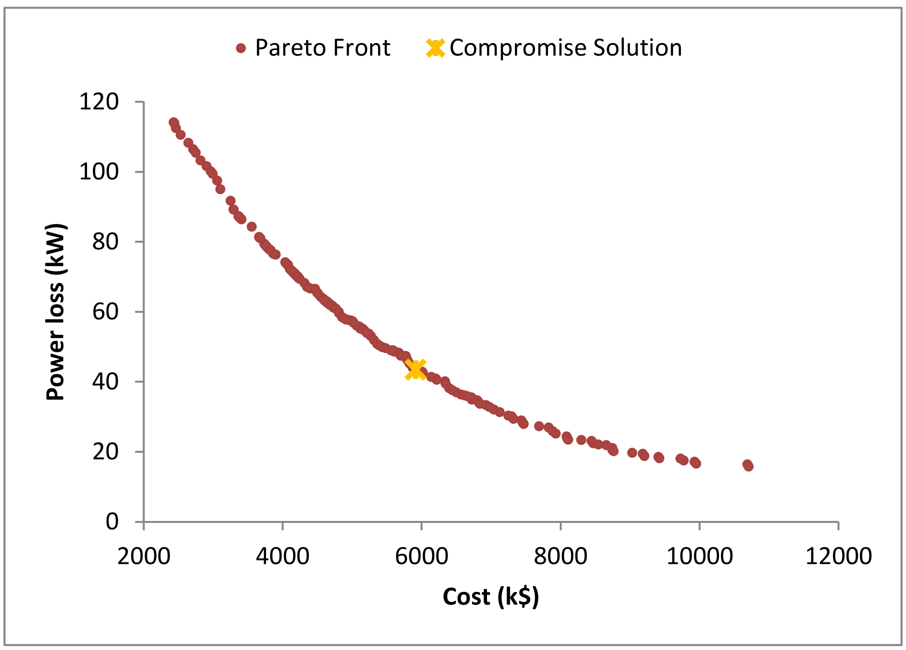

Figure 4.10: Pareto front of Case 2 (Two DG units).

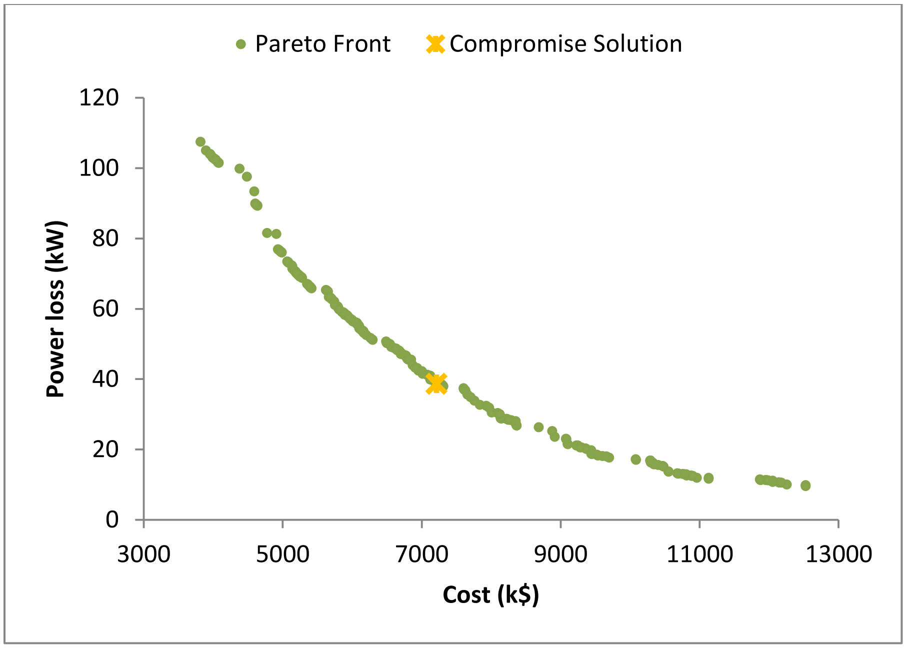

Figure 4.11: Pareto front of Case 2 (Three DG units).

4.3.2.3 Comparison of the Cases

A comparison study was done by combining the Pareto optimal front for each case in the same objective space as shown in Figure 4.12. A number of overlapping solution points appear in the figure and a new Pareto front for the combined Pareto fronts is generated. The resulting Pareto optimal front consists of various solution points from different cases. By applying the fuzzy decision-making method to the obtained Pareto optimal front, the best compromise solution was found to be as shown in Table 4.6.

Table 4.6: Compromise solution of all cases.

DG1 bus	DG1 size (kW)	DG2 bus	DG2 size (kW)	DG3 bus	DG3 size (kW)	Cost (k$)	P Loss (kW)
30	1318.89	-	-	-	-	5183.1	44.33

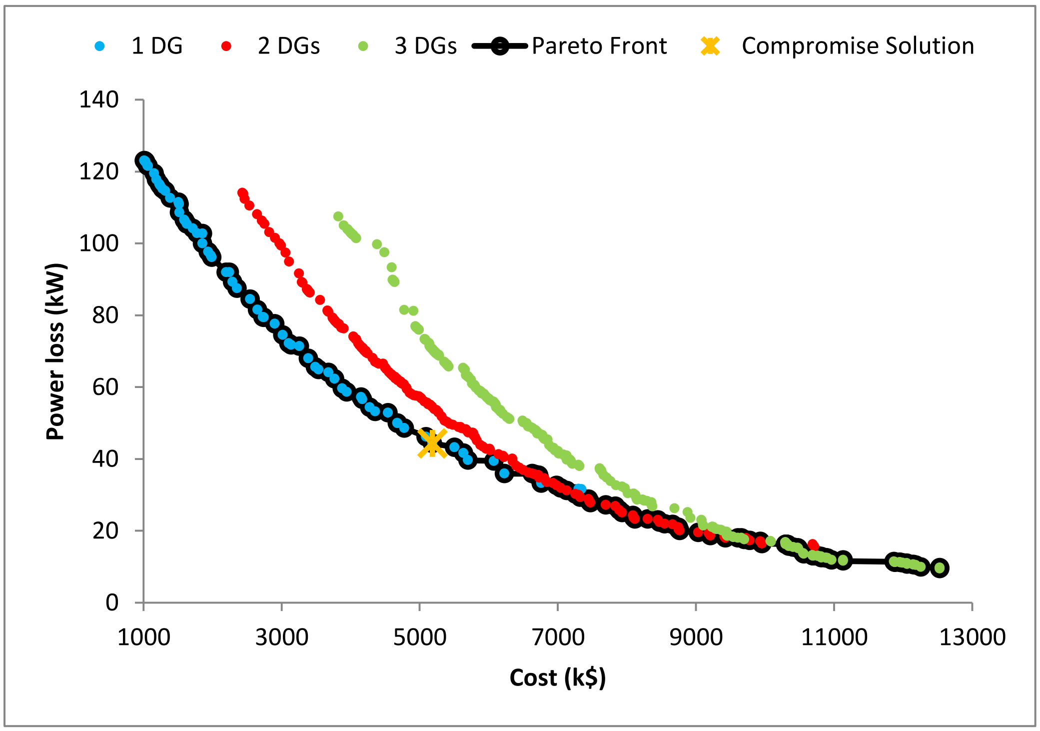

Figure 4.12: Comparison of the cases.

4.4 Summary

In this chapter, a multi-objective optimization method was applied to find optimal DG sizing and placing. The optimization problem had two objectives: the total DG installation cost and the total real power loss for a system. The weighting sum method was successfully used to generate a set of acceptable trade-off solutions. Moreover, the fuzzy decision-making method was employed to provide the decision-maker with the best compromise solutions from the trade-off curve. The method was applied to a 15-bus radial distributed system and a 33-bus meshed distribution system, where the results demonstrated that the proposed approach is efficient and applicable for solving multi-objective optimization problems for DG allocation.

CHAPTER 5 CONCLUSION AND FUTURE WORK

5.1 CONCLUSION

The installation of DG units in power distribution networks is becoming more prominent. Consequently, utility companies have started to change their electric infrastructure to adapt to DGs due to the benefits of DG installation on their distribution systems. These benefits include reducing power losses, improving voltage profiles, reducing emission impacts and improving power quality. Additional benefits are avoiding upgrading the present power systems and preventing a reduction of T&D network capacity during the planning phase. Nevertheless, achieving these benefits depends highly on the capacity of the DG units and their installation placement in the distribution systems.

In this thesis, the optimal placement and sizing of DGs within distribution networks was investigated. An optimization problem was formulated as a constrained nonlinear optimization problem and solved using the SQP deterministic method. This problem was tackled as a single-objective optimization problem in Chapter 3 and as a multi-objective optimization problem in Chapter 4.

The single-objective optimization problem attempted to determine a DG's optimal place and size by using total real power losses as an objective to be minimized. Single and multiple DG installation cases were studied using two different topology distribution systems, a 15-bus radial distribution system and a 33-bus meshed distribution system. The results were compared to a case without DG. It was shown that choosing proper DG size and place has a significant impact on minimizing power losses and improving voltage profiles. The results also showed that integrating multiple DGs reduces real power losses in a system more than by integrating only a single DG.

Additionally, a multi-objective optimization method was proposed in order to allow cost considerations of DG installation. Two objectives to be minimized were considered in this approach: total real power loss and overall DG installation cost. The weighted sum method was adapted to convert the multi-objective problem to a single one and was solved to generate the Pareto optimal front of the optimal set of solutions. A fuzzy decision-making procedure for order preference was used in order to guide the decision-maker to the best compromise among the acceptable solutions. The impact of integrating single and multiple DGs in the multi-objective optimization problem was investigated using the same distribution test systems utilized in the single-objective optimization problem. The results demonstrated and emphasized that multiple DG installations decreased total real power losses more than single DG installations. However, multiple DG installations may result in unnecessary additional costs.

The following points are the major contributions of this thesis:

- Including additional advantages in reducing power losses and improving voltage profile.
- Allowing more flexibility to the decision maker.

5.2 Future Work

Different areas of this thesis can be further explored and extended. Some are presented below:

- The study examined distribution test systems that have constant active power demands with constant power factors. Consequently, the study could be extended to investigate systems with varying load levels and load factors.
- Balanced distribution test systems were considered in this work. The optimization problem could be investigated using unbalanced distribution systems.
- The PQ-DG type model was considered in the proposed methods. Future work could include the PV-DG type model.
- The optimal DG size and placement problem could be investigated using DG with different practical values of power factor, such as 0.9, 0.95 and unity, or using DG with unspecified power factors.
- The multi-objective optimization problems proposed in this thesis could be solved using other aggregation methods such as Goal Programming and ε-constraint methods.
- Various objectives could be included in the multi-objective approach, such as maximizing grid stability and minimizing gas emissions.

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APPENDIX A

Table A.1: Line data of 15-bus radial distribution system.

Branch number	Sending bus	Receiving bus	R (Ω)	X (Ω)
1	1	2	1.35309	1.32349
2	2	3	1.17024	1.14464
3	3	4	0.84111	0.82271
4	4	5	1.52348	1.0276
5	2	9	2.01317	1.3579
6	9	10	1.68671	1.1377
7	2	6	2.55727	1.7249
8	6	7	1.0882	0.734
9	6	8	1.25143	0.8441
10	3	11	1.79553	1.2111
11	11	12	2.44845	1.6515
12	12	13	2.01317	1.3579
13	4	14	2.23081	1.5047
14	4	15	1.19702	0.8074

Table A.2: Load data of 15-bus radial distribution system.

Nodes	S (kVA)
1	0
2	63
3	100
4	200
5	63
6	200
7	200
8	100
9	100
10	63
11	200
12	100
13	63
14	100
15	200

Power factor of the load is taken as $\cos\theta = 0.70$.

Real power load = PL = kVA* $\cos\theta$.

Reactive power load = QL = kVA* $\sin\theta$.

APPENDIX B

Table B.1: Data for 33-bus meshed distribution system.

Branch number	Sending bus	Receiving bus	R (Ω)	X (Ω)	P (kW)	Q (kvar)
1	1	2	0.0922	0.047	100	60
2	2	3	0.493	0.2511	90	40
3	3	4	0.366	0.1864	120	80
4	4	5	0.3811	0.1941	60	30
5	5	6	0.819	0.707	60	20
6	6	7	0.1872	0.6188	200	100
7	7	8	1.7114	1.2351	200	100
8	8	9	1.03	0.74	60	20
9	9	10	1.044	0.74	60	20
10	10	11	0.1966	0.065	45	30
11	11	12	0.3744	0.1238	60	35
12	12	13	1.468	1.155	60	35
13	13	14	0.5416	0.7129	120	80
14	14	15	0.591	0.526	60	10
15	15	16	0.7463	0.545	60	20
16	16	17	1.289	1.721	60	20
17	17	18	0.732	0.574	90	40
18	2	19	0.164	0.1565	90	40
19	19	20	1.5042	1.3554	90	40
20	20	21	0.4095	0.4784	90	40
21	21	22	0.7089	0.9373	90	40
22	3	23	0.4512	0.3083	90	50
23	23	24	0.898	0.7091	420	200

24	24	25	0.896	0.7011	420	200
25	6	26	0.203	0.1034	60	25
26	26	27	0.2842	0.1447	60	25
27	27	28	1.059	0.9337	60	20
28	28	29	0.8042	0.7006	120	70
29	29	30	0.5075	0.2585	200	600
30	30	31	0.9744	0.963	150	70
31	31	32	0.3105	0.3619	210	100
32	32	33	0.341	0.5302	60	40
33*	21	8	2	2	-	-
34*	15	9	2	2	-	-
35*	22	12	2	2	-	-
36*	33	18	0.5	0.5	-	-
37*	29	25	0.5	0.5	-	-

* Tie Lines, V base= 12.66 kV, S base=10 MVA.

APPENDIX C

Table C.1: Rounding off results: 15-Bus system, Case (2)

DG1 bus	DG1 size (kW)	DG2 bus	DG2 size (kW)	P loss (kW)	**Rounding off results**		
					DG1 bus	DG2 bus	P loss (kW)
4	760.064	6	466.336	9.1	800	500	8.937
4	817.658	7	408.742	9.677	800	400	9.743
3	827.428	6	398.972	10.235	800	400	10.478
3	879.974	7	346.426	10.424	900	300	10.716
4	841.418	8	384.982	10.678	800	400	10.756
3	905.779	8	320.621	11.234	900	300	11.431
6	554.466	11	647.798	12.179	600	600	12.306
6	564.074	15	619.923	12.554	600	600	12.607
7	505.402	11	632.359	13.431	500	600	13.57
7	497.535	15	622.516	13.797	500	600	13.872

Table C.2: Rounding off results: 15-Bus system, Case (3)

DG 1 bus	DG1 size (kW)	DG2 bus	DG2 size (kW)	DG3 bus	DG3 size (kW)	P loss (kW)	**Rounding off results**			
							DG1 size (kW)	DG2 size (kW)	DG3 size (kW)	P loss (kW)
4	575.149	6	426.501	12	224.75	6.103	600	400	200	6.347
4	500.02	6	409.546	11	316.835	6.151	500	400	300	6.373
4	534.217	7	360.219	11	331.965	6.407	500	400	300	6.722
4	616.757	7	375.199	12	234.444	6.448	600	400	200	6.697
4	615.648	6	436.621	13	174.131	6.548	600	400	200	6.797
6	449.581	11	396.472	15	380.348	6.759	400	400	400	6.997
4	661.952	7	383.11	13	181.338	6.942	700	400	200	6.846
7	398.212	11	422.287	15	405.901	7.238	400	400	400	7.346
4	549.962	8	336.802	11	339.636	7.262	500	300	300	8.327
4	635.903	8	351.675	12	238.822	7.346	600	400	200	7.709

Table C.3: Rounding off results: 33-Bus system, Case (2)

DG1 bus	DG1 size (kW)	DG2 bus	DG2 size (kW)	P loss (kW)	**Rounding off results**		
					DG1 bus	DG2 bus	P loss (kW)
15	919.0775	29	1831.523	15.673	900	1800	15.703
9	979.2185	29	1877.905	16.145	1000	1900	16.167
14	857.2567	29	1883.203	16.295	900	1900	16.353
8	1076.37	30	1716.282	16.532	1100	1700	16.539
9	973.8647	30	1721.95	16.595	1000	1700	16.605
12	895.3799	29	1936.455	16.785	900	1900	16.803
8	1057.134	29	1873.092	17.005	1100	1900	17.06
11	878.3735	29	1940.054	17.007	900	1900	17.028
13	818.1293	29	1923.62	17.077	800	1900	17.101
10	873.1759	29	1944.737	17.099	900	1900	17.121

Table C.4: Rounding off results: 33-Bus system, Case (3)

DG 1 bus	DG1 size (kW)	DG 2 bus	DG2 size (kW)	DG 3 bus	DG3 size (kW)	P loss (kW)	**Rounding off results**			
							DG1 size (kW)	DG2 size (kW)	DG3 size (kW)	P loss (kW)
8	913.288	25	1213.37	32	873.194	9.517	900	1200	900	9.529
8	936.067	25	1165.512	31	901.822	9.786	900	1200	900	9.81
9	809.6	25	1278.409	32	830.068	9.956	800	1300	800	9.976
8	936.923	25	1260.042	33	814.661	10.02	900	1300	800	10.011
9	842.772	25	1219.656	31	860.9	10.046	800	1200	900	10.084
8	891.251	29	1296.065	32	746.08	10.401	900	1300	700	10.442
9	911.975	24	882.821	30	1289.313	10.47	900	900	1300	10.484
12	747.99	25	1301.653	32	845.154	10.471	700	1300	800	10.612
11	734.697	25	1299.751	32	851.122	10.552	700	1300	900	10.598
8	875.129	29	1366.375	33	697.873	10.558	900	1400	700	10.597