Analytical structure of the wave-number-dependent susceptibility of many-fermion systems at low temperature and long wavelength. II

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It is shown that the nonanalytic structure of the wave-number-dependent susceptibility noted by Geldart and Rasolt is not removed by including repeated particle-particle scattering.

In a study of the wave-number- and temperature-dependent paramagnetic susceptibility, $\chi(\vec{k},T)$, Geldart and Rasolt[1] (to be referred to as GR) evaluated the expansion coefficient $\alpha(T)$ which appears in the long-wavelength expansion

$$\chi(\vec{k},T)=\chi(\vec{0},T)-\alpha(T)k^2+O(k^4) \tag{1}$$

for three model many-fermion systems and showed that $\alpha(T)\sim\ln T$ at low temperature. We emphasize that this fact does not interfere in any way with $\chi(\vec{k}\to\vec{0},T)=\chi(T)=$ uniform susceptibility. Neither does it invalidate $\chi(\vec{k},T\to 0)\equiv\chi_g(\vec{k})=$ ground state nonuniform susceptibility *provided* that no k^2 expansion has been attempted. However, it does not imply that $\chi_g(\vec{k})$ is not analytic in its k dependence. The nonanalyticity in T of the uniform susceptibility was previously known[2-6] while the new fact that the *nonuniform* susceptibility $\chi_g(\vec{k})$ is nonanalytic in k and cannot be expanded in powers of k^2 required a study of $\chi(\vec{k},T)$ when both $\vec{k}$ and T are small and comprises the major contribution of GR. In order to avoid confusion, one must understand that a given graph in the perturbation expansion for $\chi(\vec{k},T)$ for real systems gives a well-defined function of $\vec{k}$ and T even when either or both of these are small. However, attempts to construct *derivatives* of that function at small $\vec{k}$ and T encounter problems due to the confluent singularities of repeated propagators, or energy denominators, which are thereby generated. As pointed out in GR, this is the origin of $\alpha(T)\sim\ln T$. In fact, the higher-order expansion coefficients in Eq. (1) exhibit even more singular behavior at low T since more energy denominators occur. Alternatively, attempts to expand $\chi_g(\vec{k})$ in powers of k^2 generate expansion coefficients represented by a sequence of increasingly divergent integrals.[7] The point is that the rate of convergence of the k^2 expansion is strongly T dependent, and the form which is consistent with the above facts and describes $\chi(\vec{k},T)$ when both $\vec{k}$ and T are small is

$$\delta\chi(\vec{k},T)\equiv\chi(\vec{k},T)-\chi(\vec{0},T)\approx-\alpha''k^2\ln\left(\frac{k^2}{k_0^2}+\frac{T}{T_0}\right)+\cdots, \tag{2}$$

where α'' is a model-dependent constant.[1]

Recently this question has been reconsidered[8] with the tentative conclusions that (i) the limits of $\vec{k}\to\vec{0}$ and $T\to 0$ cannot be interchanged in the nonuniform susceptibility $\chi(\vec{k},T)$, (ii) the $T\to 0$ limit of statistical mechanics as formulated by Kohn, Luttinger, and Ward[9,10] requires extensive modification, and (iii) the inclusion of repeated particle-particle scattering via a t matrix restores analyticity to $\chi(\vec{k},T)$. These conclusions are incorrect. It is straightforward to show that the $\vec{k}\to\vec{0}$ and $T\to 0$ limits can indeed be interchanged order by order in perturbation theory so that (i) is wrong. The necessity for (ii) then disappears, of course, and it is easy to see that the work of Kohn, Luttinger, and Ward[9,10] does not require extensive revision.

Although $\alpha(T)$ was not explicitly calculated in Ref. 8, the gist of their procedures is that

$$\delta\chi(\vec{k},T)\approx\frac{-\alpha''k^2\ln(k^2/k_0^2+T/T_0)}{1-\bar{I}\ln(k^2/k_0^2+T/T_0)}, \tag{3}$$

where $\bar{I}\propto IN_0(0)=$ constant. This result is *not* analytic in k^2 for $T=0$ and cannot be expanded in powers of k^2. Note also that it is *a priori impossible* that all logarithms should disappear upon inclusion of a particle-particle t matrix since the rigorous result of GR to order I^2 must be contained in Eq. (3) at *finite* T where no problems can arise. Consequently, (iii) is also wrong. However, there is one aspect of Eq. (3) which does differ substantially from Eq. (2). The k^2 "expansion coefficient" of Eq. (3) at $T=0$ is

$$-\alpha''\ln(k^2/k_0^2)/[1-\bar{I}\ln(k^2/k_0^2)]$$

and is finite for $k \to 0$ (although not analytic, of course).

In summary, the structure of $\chi(\vec{k}, T)$ for many-fermion systems contains logarithmic contributions[11] at long wavelength and low temperatures and is therefore not analytic in these variables.

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[7]When a power expansion of $\chi_g(\vec{k})$ is attempted while maintaining some form of limiting procedure, it can be shown that the k^2 and k^4 expansion coefficients are given by divergent integrals of the form $\ln\delta$ and $1/\delta$, respectively, where $\delta \to 0^+$. This structure is consistent with the $T \to 0$ limit of the k^2 expansion of Eq. (2). Furthermore, the divergence of the k^4 expansion coefficient is such that it cannot be removed by inclusion of repeated particle-particle scattering which is consistent with the observations following Eq. (3).

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[11]For the practical implications of these logarithmic contributions in applications to magnetized systems using gradient expansions, see Sec. III B of Ref. 1.