WHAT ARE THEY COUNTING ON? AN INVESTIGATION OF THE ROLE OF WORKING MEMORY IN MATH DIFFICULTIES IN ELEMENTARY SCHOOL-AGE AND UNIVERSITY STUDENTS

by

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The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled "WHAT ARE THEY COUNTING ON? AN INVESTIGATION OF THE ROLE OF WORKING MEMORY IN MATH DIFFICULTIES IN ELEMENTARY SCHOOL-AGE AND UNIVERSITY STUDENTS" by Melissa McGonnell in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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To Mum, Dad, Mike, Mark, and Matt, who I’ve always been able to count on,

Francine for showing me what courage really is,

and

Danielle, Hannah, and Simon for bringing joy to everything, including math

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ABSTRACT

Math difficulties (MD) are nearly as common as difficulties with reading. Despite this, MDs have received much less attention from researchers and we have yet to define a core cognitive process for MD. Knowledge about a core cognitive process would assist with early identification and remediation of MDs. Working memory has been identified as one cognitive process that is strongly associated with math difficulties. Most research examining the association between working memory and math calculation skills has been predicated on Baddeley and Hitch's (1974) multicomponent model of working memory. Results of studies are inconclusive with respect to which component of Baddeley and Hitch's model is most associated with math calculation skills. The wide variety of tasks that have been used to measure the components of Baddeley and Hitch's model may be one reason for the lack of consistent findings. In the Introduction, common tasks used to measure the components of Baddeley and Hitch's model are described and discussed. The Automated Working Memory Assessment Battery (AWMA) is suggested as a measure that adequately assesses all components of Baddeley and Hitch's model. The AWMA was used in two studies examining the role of the components of working memory in math calculation skill in elementary-school (Study 1) and university (Study 2) students. Participants in Study 1were 94 (42 female) elementary-school children (*M* age = 9 years 1 month; *Range* 6 years 0 months – 11 years 8 months). Participants in Study 2 were 42 university students (*M* age 20 years 9 months; *Range* 18 years 6 months to 22 years 11 months). In both studies, the visuospatial sketchpad (short-term visuospatial memory) emerged as the component of working memory that explained the most variance in math calculation scores. In elementary-school children, phonological processing was also important. Evidence points to a developmental path emphasizing both verbal and visuospatial skills in math calculation skills of younger children and a more specific role for visuospatial memory in adults (university students). Explicit instruction using visuospatial strategies in the teaching of math calculation skills will be important at all ages.

LIST OF ABBREVIATIONS AND SYMBOLS USED

ASRS	Adult ADHD Self-Report Scale
APA	American Psychiatric Association
ADHD	Attention-Deficit Hyperactivity Disorder
AWMA	Automated Working Memory Assessment Battery
CANTAB	Cambridge Neuropsychological Test Automated Battery
CTOPP	Comprehensive Test of Phonological Processing
CPRS-R	Conners' Parent Rating Scale - Revised
DSM-IV-TR	Diagnostic and Statistical Manual for Mental Health Disorders – 4th Edition, Text Revision
DSM-5	Diagnostic and Statistical Manual for Mental Health Disorders – 5th Edition
GPA	Grade Point Average
Δ	Increment of change
IDEA	Individuals with Disabilities Act
IQ	Intelligence quotient
MD+RD	Math and reading disability
MLD	Math learning disability; math disability
MD	Math difficulty
M	Mean
R^2	Multiple correlation squared
N	Number of participants
n	Number of participants in a subsample
r	Pearson Product Moment Correlation
p	Probability (for significance testing)
PASAT	Paced Auditory Serial Addition Test
RD	Reading disability
t	Sample value of the *t*-test statistic
SD	Standard deviation
SE	Standard error
β	Standardized regression coefficient
S-CPT	Swanson-Cognitive Processing Test
B	Unstandardized regression coefficient
WAIS-III	Wechsler Adult Intelligence Scale – Third Edition
WISC-IV	Wechsler Intelligence Scale for Children – Fourth Edition
WISC-III	Wechsler Intelligence Scale for Children – Third Edition
WMS-III	Wechsler Memory Scale – Third Edition
WJ-III	Woodcock Johnson Tests of Achievement – Third Edition
WMTB-C	Working Memory Test Battery for Children

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CHAPTER 1: INTRODUCTION

Math is an integral part of daily life. Sometimes we are very aware that we are using our math skills (e.g., when we calculate sale prices or measure ingredients when we cook). More often, however, the fundamental importance of math is less obvious than is the role of reading in our lives. The act of reading often requires an overt behavioural change: the opening of a book or an electronic document or the decision to attend to text on a sign or on a menu. The “act of math” can be a much more subtle experience as we are often unaware that we are using our numeracy skills (e.g., when we check the time in the morning and decide that we have time to stay in bed just a little longer). Perhaps the fact that we are often less conscious of the importance of math in our everyday lives provides a partial explanation for the imbalance evident in the amount of research attention that has been paid to learning about math as compared with reading. In the ten years from the mid-1960s to the mid-1970s, studies examining challenges with reading outnumbered similar studies about math 100:1 (Gersten, Clarke, & Mazzocco, 2007). More recently, interest in math has been growing, but between 1996 and 2005, the number of studies focused on difficulty with reading still outnumbered math studies 14:1 (Gersten et al., 2007). This lack of attention to challenges with math achievement is surprising given that difficulty with math is nearly as common as difficulty with reading. Population incidence studies have reported cumulative incidence rates of 5.3% to 11.8% for reading (Katusic, Colligan, Barbaresi, Schaid, & Jacobsen, 2001) and 5.9% to 13.8% for math (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005) by the age of 19.

Research into difficulty with reading has resulted in the development of a general and long-standing consensus (e.g., Vellutino & Scanlon, 1998) about the core behavioural deficit (word reading) and the deficient cognitive process (phonological processing) that are associated with reading difficulty. The same degree of consensus has not yet been achieved in the area of math difficulty. The focus of this dissertation is the search for a cognitive process for math that would parallel phonological processing for reading. While a number of possibilities such as phonological processing (e.g., Fuchs, Compton, Fuchs, Paulsen, Bryant, & Hamlett, 2005) and attention (e.g., Robinson, Menchetti, & Torgesen, 2002) have been suggested, working memory is one cognitive process that has been strongly implicated in math learning (e.g., Geary, 2004). Most researchers (e.g., Andersson, 2010; Berg, 2008; Booth & Siegler, 2008; Chong & Siegel, 2008; Fuchs, et al., 2010; Geary, Hamson, & Hoard, 2000; Gropper & Tannock, 2009; Jordan, Glutting, & Ramineni, 2010; Murphy, Mazzocco, Hanich, & Early, 2007; Rasmussen & Bisanz, 2005; Seethaler & Fuchs, 2006; Swanson, Jerman, & Zheng, 2008; Tolar, Lederberg, & Fletcher, 2009; Vukovic & Siegel, 2010) have used Baddeley and Hitch's (1974) multicomponent model of working memory as a framework for these investigations. This model proposes that working memory is best conceptualized as containing three components: the phonological loop (short-term verbal storage), the visuospatial sketchpad (visuospatial short-term storage), and the central executive (verbal and visuospatial storage and processing).

A broad variety of tasks which purport to measure the components of Baddeley and Hitch's working memory model have been used in previous research and, as a result, choosing a task or tasks for use in research can be a daunting prospect. Additionally, studies investigating the role of working memory in math do not always include measures which tap into all components of Baddeley and Hitch's model. The goal of this dissertation was to add to the literature seeking to define a core cognitive deficit associated with math difficulties. Working memory was the primary cognitive process examined in the two empirical studies that comprise this dissertation. Both studies used a measure (the Automated Working Memory Assessment; AWMA) that is predicated on Baddeley and Hitch's model.

Two empirical studies are included in this dissertation. The first examined the role of working memory in math skills of elementary school-aged children and the second examined the same question in a population of university students. This methodology will help to ascertain whether similar or different working memory components contribute to math calculation skills across a broad range of development. These studies are contained in Chapters 2 and 3. This general introduction provides a preface to these studies by including background information necessary to provide a full context for these studies. This information includes a clarification of the terms used to label groups of those with math challenges, some detail about Baddeley and Hitch's model of working memory, an examination of the tasks commonly used by researchers to examine the role of working memory in math challenges, and an explanation of the benefits of using the AWMA.

Terms and Operational Definitions

In a survey of studies investigating challenges with learning math, differences in the vocabulary used to describe these challenges are readily apparent. The *Diagnostic and Statistical Manual of Mental Disorders, Fourth Edition, Text Revision* (DSM-IV-TR; APA, 2000) uses the term *Mathematics Disorder*, but in practice the terms *dyscalculia*, *math disability* (or *math learning disability*), and *math difficulty* are more commonly used. In general, the terms dyscalculia and math disability/math learning disability (MLD) are all used somewhat interchangeably to imply that challenges with math have a biological basis (i.e., are inherent to the individual and not the result of environmental factors) while the term math difficulty (MD) simply implies poor achievement that can result from either biological or environmental factors (Mazzocco, 2007). Mazzocco acknowledged that in terms of level of math achievement, these two groups overlap to some degree. (See Barbaresi et al. [2005] for a discussion of the overlap of groups identified with math learning difficulties as a result of the use of different diagnostic formulas.) However, Mazzocco also stated that difficulty with math achievement is not sufficient evidence to allow one to conclude that an individual has a MLD because some challenges with math result from purely environmental circumstances such as poor instruction. She further noted that individuals with MLD might not actually demonstrate clear difficulty with math since some might achieve at or above grade level as a result of extremely high levels of effort.

It has been argued (e.g., Geary, Hoard, Nugent, & Byrd-Craven, 2008; Mazzocco, 2007; Murphy, et al., 2007) that including individuals with MD in studies purporting to investigate the basis for MLD could blur important differences (e.g., in cognitive abilities) that would be noticeable if a more purely defined group were used and, as a result, it is important for researchers to distinguish between these groups. Practically speaking, however, determining whether someone has a MLD and distinguishing MLD and MD groups is difficult. [For a detailed discussion of the complexities of this issue, see Fletcher, Lyon, Fuchs, and Barnes (2007).] Adding to the complexity is the fact that the diagnostic criteria in the DSM-IV-TR (APA, 2000) state that those with Mathematics

Disorder must have math ability below what is expected given age, education, and measured intelligence and that this must interfere with academic achievement. Central to this definition is the assumption that IQ is predictive of academic achievement. It has long been argued, however, that IQ is not a good predictor of achievement or of response to intervention and that it should not, therefore, be considered when defining reading disability in particular (e.g., Gresham & Vellutino, 2010; Siegel, 1989; Stanovich, 1991; Stuebing, Fletcher, LeDoux, Lyon, Shaywitz, & Shaywitz, 2002; Vellutino, Scanlon, & Lyon, 2000). Mazzocco and Myers (2003) also found that most children with low math achievement did not have an IQ-achievement discrepancy and that children determined to have a math disability using an IQ-achievement discrepancy definition did not necessarily continue to meet criteria for this diagnosis over time. Recent changes to Individuals with Disabilities Education Improvement Act (IDEA, 2004) in the United States have mandated that states cannot require discrepancy between ability (i.e., intelligence) and achievement when determining whether individuals have learning disabilities. Proposed revisions to the forthcoming DSM-5 have noted that the wording of the diagnostic criteria needs to be consistent with the changes to IDEA (APA, 2010).

In practice, most researchers use IQ scores as a simple means of excluding individuals with extreme ability scores rather than as a result of any theoretically driven belief about the influence of overall IQ on math skills as this is currently not known (Geary et al., 2008). Moreover, researchers rarely include a discrepancy between ability and math achievement as an inclusion criterion for participants in their studies. As well, the terms MLD and MD are used somewhat indiscriminately. Typical practice involves researchers stating the term they will use and then operationally defining their use of the chosen term for the purposes of their study. In this context, the term *math disability* (MLD) has been used to refer to a wide range of difficulty including achievement below the 45th percentile (approximately equivalent to a standard score of 99; Geary, Bow-Thomas, & Yao, 1992) and achievement below the 2nd percentile (approximately equivalent to a standard score of 70; Desoete & Royers, 2005). Others have defined MLD as achievement below the 10th percentile and MD as achievement between the 11th and 25th percentile (e.g., Chong & Siegel, 2008). Fletcher (2005) included those with achievement below the 26th percentile in an MLD group while Jordan, Hanich, & Kaplan (2003b) designated the 35th percentile as the cut-off point for designation with MD. In summary, cut-off points for inclusion in MLD groups are typically, but not always, lower than those for inclusion in MD groups and cut-off points chosen by researchers vary widely.

Not all research methodology involves comparing groups with MLD or MD to one another or to those who are typically achieving. Some researchers investigate math achievement more broadly, by including individuals with a wide range of math achievement and employing statistical analyses that capitalize upon the dimensional nature of the association between math achievement and other academic and cognitive skills. This is the approach taken in the two studies which comprise this dissertation.

Baddeley and Hitch’s Multicomponent Model of Working Memory

Much of the research examining the associations between math skills and working memory is predicated on Baddeley and Hitch’s model (Baddeley & Hitch, 1974). Their choice of the term *working* memory was meant to represent the strong functional role that Baddeley and Hitch believed memory played in humans’ ability to process information and learn (Baddeley, 1996; Baddeley & Hitch, 1974; Baddeley & Hitch, 1994). In a review of the results of their own and others' memory research, Baddeley and Hitch (1974) concluded that there was good evidence for a multicomponent working memory system. In general, they described working memory as a workspace with a limited capacity that was divided between the demands of short-term information storage and information processing. They concluded that there was a fixed amount of space that was always dedicated to short-term storage and would never be used for processing but that if this storage space became overloaded, some of the more flexible processing capacity could be dedicated to temporary storage function (Baddeley & Hitch, 1974).

Baddeley and Hitch further concluded that there was evidence for two short-term storage systems. One system was able to store limited amounts of verbal material. Baddeley and Hitch (1974) referred to this system as the *phonemic response buffer* or the *phonemic loop*. It was later termed the *articulatory loop* (Baddeley, 1981), but its name was finally changed to the *phonological loop* to clarify the fact that this system is still present and functioning when individuals do not or cannot audibly articulate information (Baddeley, 2002). Baddeley and Hitch (1974) also postulated that a similar and separate system existed for visual information storage. This was later termed the *visuospatial* (sometimes visuo-spatial, visual-spatial, and visual spatial; Baddeley, 1996) *scratchpad* (Baddeley, 1981), but its name was later changed to the *visuospatial sketchpad* to better reflect the system's visuospatial characteristics (Baddeley, 2002). The phonological loop and the visuospatial sketchpad are often referred to as the *slave systems* (Baddeley, 1996) presumably to reflect the fact that their activity is directed by the central executive.

Baddeley and Hitch (1974) termed the flexible processing capacity the *central executive*. Their initial conceptualization of this system was somewhat vague and later described by Baddeley (1996) as a *ragbag* (i.e., merely a recognition that there was a great deal of complex activity yet to be explained by the model). In 1974, Baddeley and Hitch described the central executive as responsible for providing additional short-term storage capacity when the verbal and/or visual buffers were taxed, for organizing the rehearsal routines responsible for maintaining information in the buffers, and for recoding material (e.g., through chunking) to reduce the demands on the storage capacity of the buffers. They also mentioned that the central executive could have a role in directing attention. Since 1974, the conceptualization of each system has developed somewhat. An explanation of the current general understanding of each system follows.

The Phonological Loop. The phonological loop has been described as the most straightforward working memory component (Baddeley & Hitch, 1994) and as the component that has received the most attention from researchers (Baddeley, 1996; Baddeley & Hitch, 1994). The phonological loop is responsible for our short-term memory for verbal or speech-like information. It is hypothesized to contain a phonological store, which is only able to maintain information for a very brief time (approximately two seconds), and an articulatory rehearsal system which allows this memory trace to be renewed using subvocal rehearsal (Baddeley, 2003; Baddeley & Hitch, 1994). The limited capacity (or span) of the phonological loop is related to the fact that this rehearsal takes place in real time. As a result, as the number (or complexity) of items and the time it takes to rehearse them increases, the possibility that earlier items will fade from the phonological store before they can be rehearsed (i.e., before approximately two seconds have passed) increases (Baddeley, 2003).

The Visuospatial Sketchpad. The visuospatial sketchpad is conceptualized as being responsible for short-term retention of visual and spatial information. It was included in the model to account for evidence that Baddeley and Hitch had accumulated which indicated that the short-term storage of visual information was not impeded by phonemic processing. Hence, Baddeley and Hitch (1974) concluded that visual information must have its own storage system. The visuospatial sketchpad has received much less attention from researchers than has the phonological loop and consequently, is less well understood (Baddeley & Hitch, 1994; Baddeley, 1996). It has been described as a limited capacity system which can hold three to four objects in the short term. This accounts for the phenomenon of *change blindness*, which refers to the fact that individuals do not always notice when objects in a scene change or disappear if the number of visuospatial items in the scene exceeds this capacity (Baddeley, 2002). No sub-systems analogous to the phonological store and the articulatory rehearsal system were described by Baddeley and Hitch in 1974, but others have reported evidence supporting the conclusion that the visuospatial sketchpad should be considered to consist of a *visual cache* (analogous to the phonological store), which is responsible for short-term storage, and an *inner scribe* (analogous to the articulatory rehearsal system), which is responsible for dynamic rehearsal of visual information (e.g., Logie & Pearson, 1997).

An ongoing debate with respect to the visuospatial sketchpad concerns whether information is processed and recalled visuospatially or visually *and* spatially. Some researchers (e.g., Darling, Della Sala, & Logie, 2007; Della Sala, Gray, Baddeley, Allamano, & Wilson, 1999) have concluded that it is possible to design tasks that specifically tap into either visual or spatial information and have interpreted this as evidence supporting the recommendation that the visuospatial sketchpad should be reconceptualised as two separate systems: a visual system that processes static patterns and/or object appearance and a spatial system that processes dynamic spatial information and/or object locations (Baddeley, 2003; Darling et al., 2007; Della Sala et al., 1999). While investigations of this sort are easily noted in cognitive and neuropsychological literature, separate examination of the role of visual and/or spatial short-term memory in math (or other academic achievement) is rarer.

The Central Executive. Baddeley (2002) has acknowledged that the central executive was not well described in the 1974 model. At that time, it was conceived as a system that could supply additional storage capacity if the slave systems were overloaded and that also had an unspecified processing function. More recently, Baddeley and Hitch (1994) have described the central executive as the most complicated but the least understood working memory component. It is currently considered to provide attentional control functions in that its role is to focus attention, divide attention, and control task switching (Baddeley, 2002). At one time, the central executive was also considered to provide for cross-communication between the phonological loop and visuospatial sketchpad and with long-term memory. More recently, Baddeley (2000) proposed an addition to the working memory model, the *episodic buffer*. He described the episodic buffer as a limited capacity system which is responsible for integrating information across systems and with long-term memory. The addition of the episodic buffer removes the cross-system (and cross verbal/visuospatial modality) communication function from the central executive, which is now conceptualized more as an attentional control system.

Clarification of the Term *Working Memory*

In 1974, Baddeley and Hitch deliberately chose the term *working* memory to represent the functional aspect of this type of memory and to replace the use of the terms *short-term memory* or *short-term store* which represented unitary rather than multicomponent conceptualizations. Baddeley and Hitch's use of the term working memory was meant to encompass all components (phonological loop, visuospatial sketchpad, central executive and perhaps episodic buffer) rather than referring to any one individual component of their model. Importantly, however, the term working memory is not universally used in this manner. In animal literature, for example, the term working memory generally connotes memory for a task over time (e.g., Brady, Saul, & Wiest, 2010), a conceptualization rather close to long-term memory. A more common use of working memory in human research is as a term used to imply the inclusion of both storage and processing functions. In this sense, the term working memory, rather than central executive, is often used to describe tasks that require both processing and short-term storage of information. As well, the term short-term memory is often, but not always, used to describe tasks which require short-term storage but not processing of information. Some researchers, however, use the term working memory as Baddeley and Hitch did originally, to describe both types of tasks (i.e., tasks that require only short-term storage and tasks that require short-term storage and processing). This terminological inconsistency is readily apparent in an examination of studies of the role of working memory in math skills and is a source of potential misunderstanding when attempts are made to compare results across studies.

Working Memory Tasks in Studies of Math Skills

Conflicting results abound in the literature examining the role of working memory in math skills of children and adults. A review of this literature revealed a number of inconsistencies with respect to terminology (as discussed above) used to denote the component of Baddeley and Hitch's working memory model that is being investigated but also great variety in the tasks used to operationalize the various components of working memory. Adding to the complexity of this situation, researchers sometimes use the same label for tasks which do not have exactly the same demands. Sometimes researchers note that tasks have been altered to accommodate participant characteristics (e.g., age) or to allow group administration, but sometimes there is no stated reason for alterations to tasks. The same task can also be assigned different labels across research studies and tasks with very similar names, which could be assumed to have the same task demands, sometimes have subtle or quite significant differences. A more significant difficulty has to do with occasionally inconsistent choice of tasks used to represent each of the working memory constructs. This inconsistency is particularly noticeable in tasks that are sometimes described as measuring the visuospatial sketchpad (short-term visuospatial memory) and at other times described as tapping into the visuospatial central executive (visuospatial working memory).

With the goal of helping to clarify the tasks used in previous research that has examined math skills and working memory, five tables are provided. Table1.1 provides a list of labels used in each study to describe the tasks which were employed to tap into the components of Baddeley and Hitch's working memory model. The central executive is divided into tasks assessing verbal and visuospatial components. Four other tables (Tables 1.2, 1.3, 1.4, and 1.5) provide a list of tasks commonly used to assess each of the components of Baddeley and Hitch's model. This list is not exhaustive but is meant to provide a general guide to the types of tasks that have been and continue to be used. Commercially available tasks are listed first in the tables. The remaining tasks are presented in an order that approximates frequency of use. Examples of studies (since the year 2000) which have used the tasks are also included in Tables 1.2, 1.3, 1.4, and 1.5, so that the reader can read the original description of the tasks. A shortened description of each task is provided in the tables. (Descriptions of commercially available measures are not provided.) A summary of concerns about these tasks is presented next.

Concerns About Memory Tasks

Phonological Loop (Verbal Short-Term Memory). A variety of labels were used by researchers to describe tasks which measure the phonological loop or verbal short-term memory (see Table 1.1) in the included studies. A number of studies simply labelled this construct short-term memory (i.e., without specifying *verbal* short-term memory). Commonly, but not always (e.g., Booth & Siegler, 2008), measures of visuospatial memory were not included in these studies so it could be argued that specifying that verbal memory is being measured is not absolutely necessary. However, this omission could also point to an underlying belief that the visuospatial system is relatively unimportant. *Phonological short-term memory* (or simply *phonological memory*) and *phonological processing* were also used as labels for these tasks. Locuniak and Jordan (2008) simply used the term *memory span* while Dennis and Barnes (2002) used the term *immediate memory* to denote their tasks. Kyttälä, Aunio, and Hautamäki (2010) used the term *verbal working memory: short-term storage* to refer to their phonological loop tasks. This term seems to reflect Baddeley and Hitch's original conception of the phonological loop as being the storage component of the larger working memory system.

As noted by Baddeley (2003), tasks which are designed to tap into the phonological loop commonly require immediate serial recall of verbally presented information (e.g., digits, words, nonwords) and these tasks were readily apparent in the research studies examining the role of working memory in math (see Table 1.2). Some researchers chose to use standardized tasks (commonly from the Wechsler Intelligence Scale for Children; WISC), but researcher-designed versions of serial recall tasks were also prevalent. The most commonly used tasks were serial recall of digits and serial recall of words or nonwords. There was some variety of labels ascribed to these tasks. In the case of digit and word recall tasks, these differences seem unlikely to cause confusion. There was more variability in the naming of tasks which assessed serial recall of nonwords (i.e., nonword repetition, pseudoword span, phonetic memory) which has the potential to cause confusion in interpretation. Several more novel tasks were also located. These included serial recall of food and animal words and sentence repetition.

Serial recall tasks involving the *visual* presentation of digits on a computer screen were also used. Dennis and Barnes (2002) required participants to type their response on the computer's numeric keypad while Noël, Désert, Aubrun, and Seron (2001) required a verbal response. These tasks could be classified as tapping the phonological loop if participants named the digits they saw (highly likely in the Noël et al. study given the response requirements) or as tapping the visuospatial sketchpad if participants only stored the visual form of the digit presented. Both groups of researchers seemed to recognize this possibility as the tasks were described as measuring *immediate memory* (Dennis & Barnes, 2002) and *memory span* (Noël et al., 2001) without verbal or visuospatial specifiers.

Across these tasks, information about the rate of item presentation was not consistently provided and neither was it always specified whether items were pre-recorded or read by a researcher. Rate of presentation is important because the span of the phonological loop is limited by time (Baddeley, 2003). Slower presentation of items could result in lower spans because the memory trace of initially presented items could fade before the final items are presented. Consistency in presentation rate is therefore important and it could be argued that this is best achieved using a recording. Using a pre-recorded presentation in serial nonword recall tasks is of particular importance given that the potential for inconsistent pronunciation would be higher.

Visuospatial Sketchpad (Visuospatial Short-Term Memory). A wide variety of labels were also used for the tasks measuring this component of Baddeley and Hitch's model (see Table 1.1). Many authors did use the terms *visuospatial sketchpad* or *visuospatial short-term memory*. As noted above, Dennis and Barnes (2002) used the term *immediate memory* to describe their task, while Noël et al. (2001) used the term *memory span*. Berg (2008) and Rasmussen and Bisanz (2005) used the term *visual-spatial* working *memory* to label forward Corsi span tasks which are generally considered a measure of the visuospatial sketchpad (i.e., short-term visuospatial memory) as they do not have a processing component. As was the case with their label for phonological loop tasks, Kyttälä et al. (2010) used a term (*visuo-spatial working memory: short-term storage*) that reflected Baddeley and Hitch's original conception of the visuospatial sketchpad as being the short-term storage component of the larger working memory system. Finally, Gropper and Tannock (2009) and LeFevre, Skwarchuk, Fast, Smith-Chant, Bisanz, Kamawar, and Penner-Wilger (2010) used the term *spatial*, rather than *visuospatial*, to describe their tasks. This label is technically correct as the tasks used in these studies were modifications of Corsi span which have been demonstrated to specifically tap spatial rather than visual memory (e.g., Della Sala et al., 1999). However, the use of a variety of labels for the same tasks has the potential to make it difficult to compare results across studies.

A number of tasks (see Table 1.3) were used to measure the visuospatial sketchpad (visuospatial short-term memory). The most common of these was the (forward) Corsi Blocks task or a modification of this task (often referred to as Corsi span tasks). The original Corsi Block task (Milner, 1971) involved having a participant replicate sequences which an examiner tapped on a pattern of 1 ¼ inch black blocks fastened to a black board. Current modifications include electronic presentation of a variety of images (e.g., video of a version of the original task, matrices with sequences of colours or dots appearing) and/or including story information (e.g., about frogs and lily pads) to provide context to the task for children. Pattern span tasks were the second most common type of task employed to measure the visuospatial sketchpad. These tasks are similar to the Corsi span tasks but remove the spatial component by presenting all locations to be recalled simultaneously (i.e., as a pattern) in a matrix of gradually increasing size.

While both Corsi span and pattern span tasks purport to measure the visuospatial sketchpad, recent evidence (noted above) points to the possibility that this memory component can be (and perhaps should be) fractionated into spatial (Corsi span tasks) and visual (pattern span tasks) components (e.g., Della Sala et al., 1999). In theory, individuals could have very different spatial and visual memory abilities. The fact that some studies only assess the spatial aspect of the visuospatial sketchpad while others only assess the visual aspect makes it problematic to draw conclusions about the role of the visuospatial sketchpad across studies.

A final difficulty in studies purporting to assess the visuospatial sketchpad occurred in two studies (Swanson, 2006; Swanson et al., 2008). The tasks labelled by the authors of these studies as measuring this memory component were clearly described as containing an element of processing which would seem to indicate that they would be better labelled as measuring the visuospatial central executive. In addition, other studies by the same lead author (H. L. Swanson) have utilized very similar tasks and labelled them as measures of the visuospatial central executive. This is a significant inconsistency that makes cross-study interpretation challenging.

Verbal Central Executive (Verbal Working Memory). A number of authors specified that they were measuring the verbal central executive or verbal working memory in the labels that they ascribed to their tasks assessing this memory component (see Table 1.1). Others used similar terms such as *auditory-verbal working memory* (Gropper & Tannock, 2009) and *phonological working memory* (Hecht, Torgesen, Wagner, & Rashotte, 2001). Keeler and Swanson (2001) used the label *verbal processing*. A number of other researchers simply used the labels *working memory* or *central executive* (or *executive processing*; Swanson et al., 2008) without specifying that it was the verbal component of working memory that was being assessed. Geary, Hoard, Byrd-Craven, and DeSoto (2004) used the label *visual counting span/working memory* for their task, but this task (counting span) is generally considered to be a verbal working memory task as the goal is to recall the names of the numbers. This is also how the task is conceptualized on the Working Memory Test Battery for Children (WMTB-C; Pickering & Gathercole, 2001) which the same authors have used in other studies.

With respect to specific tasks (see Table 1.4), many authors have chosen to use backwards serial tasks to assess verbal working memory. Commonly, these involve participants restating sequences of digits or words in backwards order and researchers have used both commercially available standardized measures from the Wechsler Intelligence Scales for Children (WISC) or the Wechsler Adult Intelligence Scales (WAIS) as well as experimenter designed versions of these tasks. The Letter-Number Sequencing subtest from the WAIS or Wechsler Memory Scale (WMS), which requires resequencing of digits and letters, is also sometimes used. In the case of these tasks, the processing demand is the result of the necessity of reordering the items on the list. One novel list-based task that employs a different processing demand was noted. Andersson (2007) used what he termed the Animal Dual-Task which required participants to listen to lists of words, determine whether each word they heard was an animal or not, and then correctly state the list of all words presented.

Another commonly used task is based on the reading span test first devised by Daneman and Carpenter (1980). In this original task, participants were shown 8 x 5 inch index cards on which sentences (13-16 words in length) were typed in a single line.

Participants were asked to read the sentence (e.g., When at last his eyes opened, there was no gleam of triumph, no shade of anger.) aloud at their own pace. At the end of each set of sentences (range 2-6 sentences), participants were asked to recall the last word of each sentence in the order presented. Miller and Bichsel (2004) and Tolar et al. (2009) used reading span tasks that were very similar to Daneman and Carpenter's task. Tolar et al.'s task also included a specific processing requirement as participants had to determine whether or not the sentence they read made sense. It is important to note, however, that while these tasks definitely tap verbal working memory, they are confounded by reading ability. Individuals with relatively poor reading skills would be disadvantaged in the completion of these tasks. This makes the suitability of these tasks for research investigating academic achievement questionable.

Most other versions of this span task require individuals to listen to sentences read to them. While this strategy does relieve the cognitive load required by reading the sentences, it does add another potential dimension of inconsistency related to the rate at which sentences are read to the participant. As previously noted, verbal storage is limited by the passage of time. Therefore, if sentences are not read at a consistent pace, the processing demands could vary considerably from one participant to another within a study. This difficulty could be managed by using a recording to present the sentences. While some studies may have employed this strategy, how sentences are presented is often not specified in task descriptions.

Another modification of Daneman and Carpenter's task, commonly used by Swanson and colleagues, requires that participants attend to digits embedded in sentences that are read to them. This task has been labelled as Auditory Digit Sequencing from the

Swanson-Cognitive Processing Test (Berg, 2008; Swanson, 2004; Swanson & Sachse-Lee, 2001) and as Digit/Sentence Span (Keeler & Swanson, 2001; Swanson, 2006; Swanson & Beebe-Frankenberger, 2004; Swanson et al., 2008). In several studies (e.g., Berg, 2008; Keeler & Swanson, 2001; Swanson, 2006; Swanson & Beebe-Frankenberger, 2004), the task also required participants to hear descriptions of possible strategies (i.e., rehearsal, chunking, associating, and elaborating) which could be used to retain the digits in memory and, at some point, indicate which strategy was most like the one they used to remember the digits. Swanson et al. (2008) used the same label for a task that was identical except that it did not require strategy choice, significantly reducing time and processing demands inherent in the task. Again, this subtle difference in tasks that have been given the same label makes interpreting results across studies problematic.

The third type of commonly used task is based on Case, Kurland, and Daneman's Counting Span Test (Case, Kurland, & Goldberg, 1982). The original version of this task presented participants with white cards (minimum span 1 card) containing green dots which the participant was asked to count. After the last card was seen, the participant was asked to state the number of dots counted on each card. Some studies continued to employ the original cards but more recently, computerized presentations have become available. Another common adaptation to this task is to increase the processing demands by presenting a number of colours of dots or a variety of shapes in different colours and requiring participants to only count one specifically defined type of shape (e.g., the yellow dots in an array of blue and yellow dots). In Case et al.'s (1982) original version of this task, counting speed of the participants was also assessed. This seems important as the speed at which participants can count the items in the array would contribute to processing difficulty and therefore to task performance. Despite this fact, counting speed is not typically assessed when this task is used. In one study (Geary et al., 2004), this task is described as a *visual* counting span task by the authors, but the task is generally considered to be a verbal working memory task because both the processing and recall demands are verbal despite the visual display of the stimuli.

A number of other verbal central executive tasks utilized in research employ categorization of words as the processing dimension of the task. In these tasks, participants hear lists of words which must be reorganized and restated in categories. The categories and words within the categories can be in any order. For example, if participants heard the words *cat, apple, banana, frog,* they could reply *cat, frog, apple, banana* or *banana, apple, cat, frog*. Swanson and colleagues have used the label Semantic Association for these tasks during which participants are also asked a process question (e.g., ‘Which word, *saw* or *level*, was on the list?’). Berg (2008) used the Semantic Categorization task (from the Swanson-Cognitive Processing Test; Swanson, 1995) which requires participants to indicate which strategy (e.g., categorization, thinking of something to associate with each word) was most like the one they used to recall the words. Noël (2009) used the label Category Span for her task which contained lists of food and animal words and required participants to first state the food words followed by the animal words on the list.

Three other tasks stand out as being unique. A Story Retelling task was used by Swanson and Sachse-Lee (2001) and by Wilson and Swanson (2001). This could perhaps be considered a verbal working memory task by virtue of the amount of detail contained in the story (i.e., general memory load) rather than the requirement of processing the information in some way. In fact, in this task, processing of the information seems to be discouraged by the requirement that information be recalled in the original order. The Paced Auditory Serial Addition Test (PASAT) was employed by Gropper and Tannock (2009). This task required that the participant listen to digits presented verbally (by computer) at a specific rate, add the digit just heard to the previously heard digit, and state the answer. When the next digit is heard, the participant must ignore the answer just provided and again add the digit just stated by the computer to the last digit stated by the computer. The processing element of this task requires rapid addition of pairs of digits. As a result, this task is confounded with math skills. Murphy et al. (2007) used the Contingency Naming Test in their study. This task required participants to view two shapes and provide a response based on whether the shapes did or did not match, and to reverse the response rule when a backwards arrow appeared over the shape. The authors used the broad label *working memory* for this task. The decision to include it under tasks measuring the *verbal* central executive in Table 1.4 was based on the degree to which verbal skills are required to process the directions and provide responses in the task; however, the stimuli for the task are visual in nature, so it could also be argued that this task should be included in measures of visuospatial working memory.

Visuospatial Central Executive (Visuospatial Working Memory). Again, some studies labelled tasks measuring this memory component clearly as either *visuospatial central executive* or *visuospatial working memory* tasks (see Table 1.1). Keeler and Swanson (2001) used the term *visual-spatial processing* to label their task while Kyttälä et al. (2010) used the term *non-verbal central executive* to label their task. Both terms could be said to clearly describe the memory component being measured. Others (e.g., Andersson, 2007; McGlaughlin, Knoop, & Holliday, 2005; Swanson & Beebe-Frankenberger, 2004) used the labels *working memory* or *central executive* without specifying that the task in question was targeting the visuospatial domain. Andersson (2010) used the term *visual working memory* to describe the Visual-Matrix task. This was an adaptation of the pattern span task that required participants to view a matrix wherein some squares contained dots, answer a process question (e.g., 'Were there any dots in the first column?') after the matrix was taken away, and finally draw the location of the dots they saw. In this case, the term could be said to be accurate as pattern span is considered to tap visual (rather than spatial) memory and the addition of the process question is considered to tap working memory; however, Andersson (2007) used the term *central executive* (without specifying its visual nature) to describe a task with essentially the same demands. Miller and Bichsel (2004) used *visual working memory* as a label for their task despite the fact that the task clearly contains a spatial component (i.e., participants must determine the location of holes in an unfolded paper). Gropper and Tannock (2009) used the term *spatial working memory* which could be considered an accurate term as their tasks required the determination of location. Finally, and confusingly, Swanson (2006) and Swanson et al. (2008) labelled their tasks as measuring both the *visual-spatial sketchpad* and *visual-spatial working memory*. Again, the variety of labels ascribed to similar tasks makes it difficult to compare findings across studies.

The visuospatial central executive (visuospatial working memory) was represented by relatively few tasks (see Table 1.5) as compared to the other components of Baddeley and Hitch's model. The most common task used in the studies reviewed was the Visual Matrix or Visual Matrix Span task (described above). These tasks do require the storage of visual information (i.e., patterns of dots) and, in this sense, are similar to pattern span tasks that have been shown to measure visual memory (e.g., Della Sala et al., 1999). The authors of the studies which use these tasks contend that they are measures of the central executive (working memory) because they have included a processing requirement. In this case, the participant is asked a yes/no question about the location of dots within the matrix just viewed. While it can be argued that answering this question does require that the contents of the matrix be processed, it is also true that listening and responding to the question require that at least some information is processed in verbal rather than in visual or spatial form. As a result, this task does not seem to represent an ideal measure of visuospatial working memory.

Swanson and colleagues have frequently used tasks called Mapping or Mapping and Directions to assess visuospatial working memory. Here, participants are shown a map that depicts the route of a bicycle through a town for a short time. The route is represented by lines and directional arrows while dots represent stoplights. When the map is taken away, a process question is asked (e.g., 'Were there any stoplights on the first street?') and participants are asked to draw the route (lines and arrows) and stoplights (dots) on a blank map. These tasks seem to require the recall but not the manipulation of visuospatial information. Again, Swanson and colleagues contend that the task assesses visuospatial working memory because they have included a process question. However, as noted above, this process question could also be conceptualized as an interference task (as it overlays additional verbal information) rather than a processing task and since it is verbal in nature, could also be said to tap the resources of verbal rather than strictly visuospatial memory. As such, these tasks do not seem to be ideal measures of visuospatial working memory.

Gropper and Tannock (2009) utilized the Spatial Working Memory Task from the Cambridge Neuropsychological Testing Automated Battery (CANTAB). This is a computerized task which requires participants to search for a token under boxes without looking under boxes where tokens have been found before. This task requires active processing of the location of objects and therefore seems to be appropriate for assessing visuospatial working memory.

A Comprehensive Memory Measure for Research

The need for a measure of working memory that can assess all aspects of Baddeley and Hitch's model is clear. Perhaps in response to this need, Pickering and Gathercole (2001) developed the Working Memory Test Battery for Children (WMTB-C). This measure gathered together many of the commonly used working memory measures in one commercially available instrument, standardized the administration, and provided norms. More recently, the Automated Working Memory Assessment (AWMA; Alloway, Gathercole, & Pickering, 2004), a computer based assessment very similar to the WMTB-C was developed and published. The original version of the AWMA was normed for use with children aged 4-11 years and contained twelve subtests, three which measured the phonological loop (short-term verbal memory), three which measured the visuospatial sketchpad (short-term visuospatial memory), and six which measured the central executive (three for verbal working memory and three for visuospatial working memory). An updated version of the AWMA (Alloway, 2007) is now available. It retains the same twelve tasks as the original version of the AWMA and is normed for use with individuals up to the age of 22. The value of this measure as a research tool is enhanced by the fact that it incorporates multiple measures of each component of Baddeley and Hitch's model of working memory and that its computerized format improves the ability to administer the subtests in a standardized way. Raw and standardized scores are available for each individual subtest and combined standardized scores are also available for each working memory component.

The AWMA has been used extensively in research to describe and differentiate patterns of memory strengths and challenges in children with a variety of health and educational challenges. Performance on the AWMA has also been shown to be related to academic achievement (e.g., Alloway, 2009; Alloway, Gathercole, Adams, Willis, Eaglen, & Lamont, 2005). Recently, the AWMA was used in a study specifically examining math skills in 7- and 8-year-old children in Italy (Alloway & Passolunghi, 2011). Findings indicate that visuospatial short-term memory (the visuospatial sketchpad) was the only memory component that predicted significant additional variance in math skills after accounting for vocabulary knowledge. To our knowledge, no study has yet used the AWMA to examine the association between math skills and working memory in a broad elementary-age population or in university-aged students. The general objective of this dissertation was to address this gap in the literature. Specifically, the goal was to determine which component of Baddeley and Hitch's model of working memory, as measured by the AWMA, best predicts math skills in elementary- and university-age students.

This goal is addressed in the studies contained in Chapters 2 (elementary-age students) and 3 (university students). Chapter 4 includes an integrated discussion of the results from these two studies along with an exploration of the implications for the assessment (clinical implications) and remediation (educational implications) of MD and conclusions about the research utility of the AWMA.

Table 1.1

Descriptive Labels Used for Working Memory Components in Math and Working Memory Studies

	Working Memory Component			
	Short-term Memory		Working Memory/Central Executive	
Study	Verbal (Phonological Loop)	Visuospatial (Visuospatial Sketchpad)	Verbal	Visuospatial
Andersson, 2007	+	*	Central executive	Central executive
Andersson, 2010	Short-term memory	*	+	Visual working memory
Berg, 2008	Short-term memory	Visual-spatial working memory	+	+
Booth & Siegler, 2008	Short-term memory	*	*	*
Chong & Siegel, 2008	*	*	Working memory	*
De Smedt et al., 2010	Phonological short-term memory	*	*	*
Dennis & Barnes, 2002	Immediate memory[a]	Immediate memory[a]	Working memory[a]	Working memory[a]
Fuchs et al., 2005	*	*	Working memory	*
Fuchs et al., 2010	+	+	Central executive	*
Geary et al., 2009	+	+	Central executive	*

	Memory Component			
	Short-term Memory		Working Memory/Central Executive	
Study	Verbal (Phonological Loop)	Visuospatial (Visuospatial Sketchpad)	Verbal	Visuospatial
Geary et al., 2004	*	*	(Visual) counting span/working memory	*
Geary et al., 2007	+	+	Central executive	*
Gropper & Tannock, 2009	*	Spatial working memory	Auditory-verbal working memory	Spatial working memory
Hecht, 2006	*	*	Working memory	*
Hecht et al., 2001	Phonological memory	*	Phonological working memory	*
Hoard et al., 2008	+	+	Central executive	*
Holmes et al., 2008	*	+	*	*
Jordan et al., 2010	Short-term memory	*	Working memory	*
Keeler & Swanson, 2001	*	*	Verbal processing	Visual-spatial processing
Krajewski & Schneider, 2009	Phonological loop	Visual-spatial sketchpad Visual-spatial working memory	Central executive	*

	Memory Component			
	Short-term Memory		Working Memory/Central Executive	
Study	Verbal (Phonological Loop)	Visuospatial (Visuospatial Sketchpad)	Verbal	Visuospatial
Kyttälä et al., 2010	Verbal working memory: short-term storage	Visuo-spatial working memory: short-term storage	+	Visuo-spatial working memory *and* Non-verbal central executive
LeFevre et al., 2010	*	Spatial attention	*	*
Locuniak & Jordan, 2008	Memory span Short-term recall	*	Memory span Active memory	*
McGlaughlin et al., 2005	*	Working memory[b]	Working memory[b]	Working memory[b]
Meyer et al., 2010	+	+	Central executive	*
Miller & Bichsel, 2004	*	*	+	Visual working memory
Murphy et al., 2007	*	*	Working memory[c]	*
Noël, 2009	+	+	Central executive	*
Noël et al., 2001	Memory span	Memory span	*	*
Osmon et al., 2006	Verbal span	*	*	*

	Memory Component			
	Short-term Memory		Working Memory/Central Executive	
Study	Verbal (Phonological Loop)	Visuospatial (Visuospatial Sketchpad)	Verbal	Visuospatial
Rasmussen & Bisanz, 2005	+	Visual-spatial working memory	Central executive	*
Seethaler & Fuchs, 2006	*	*	Working memory	*
Swanson, 2004	Speed and phonological processing	*	+	+[d]
Swanson, 2006	+	*	+	Visual-spatial sketchpad AND Working memory
Swanson & Beebe-Frankenberger, 2004	Short-term memory[e]	*	Working memory	Working memory[d]
Swanson et al., 2008	Short-term memory[e]	*	Executive processing	Visual-spatial sketchpad AND Working memory
Swanson & Sachse-Lee, 2001	Phonological processing	*	+	+[d]
Tolar et al., 2009	*	*	Working memory	*
Vukovic & Siegel, 2010	Short-term memory	*	Working memory	*
Wilson & Swanson, 2001	*	*	+	+[d]

Note. + Tasks were described using term heading the column; * No tasks described by this label were included in the study.
[a] Tasks could tap either verbal or visuospatial domains.

[b] Wechsler Memory Scale-Third Edition (WMS-III) Working Memory Index score was used. This index contains a measure of verbal working memory (Letter-Number Sequencing), the visuospatial sketchpad (forward portion of the Spatial Span subtest), and visuospatial working memory (backward portion of the Spatial Span subtest).

[c] This task was described as a working memory task by the authors. The decision to include it under tasks measuring verbal working memory was based on the degree to which verbal skills are required to process the directions and provide responses in the task; however, the stimuli for the task are visual in nature, so it could be argued that this task should be included in measures of visuospatial working memory.

[d] Tasks only require the recall of visuospatial information and not manipulation of this information. They do contain a processing requirement that has verbal demands. Tasks are classified as measures of the visuospatial sketchpad AND visual-spatial working memory by Swanson (2006) and Swanson et al., (2008).

[e] Includes Backwards Digit Span task which is typically classified as a verbal working memory task.

Table 1.2

Phonological Loop (Verbal Short-term Memory) Tasks Commonly Used in Studies of Math and Working Memory

Task Name	Brief Task Description	Sample Study(s) Using Task
WISC-III Forward Digit Span	Only digits forward trials used	Swanson, 2004 Swanson, 2006 Swanson & Beebe-Frankenberger, 2004 Swanson, Jerman, & Zheng, 2008 Swanson & Sachse-Lee, 2001 Vukovic & Siegel, 2010
WISC-III Backward Digit Span	Only digits backward trials were used	Swanson & Beebe-Frankenberger, 2004 Swanson, Jerman, & Zheng, 2008
WISC-IV Digit Span Forward	Only digits forward trials used	Jordan, Glutting, & Ramineni, 2010 Locuniak & Jordan, 2008
WISC-IV Forward Digit Span	Only digits forward trials used	Booth & Siegler, 2008
Digit Span Forward	Participants heard a list of single-digit numbers (maximum span 9) read by a researcher and were asked to repeat the digits in the correct order.	Berg, 2008
Digit Span Forward (German)	Participants heard monosyllabic single-digit number words at a rate of one per second and were asked to repeat them.	Krajewski & Schneider, 2009

Task Name	Brief Task Description	Sample Study(s) Using Task
Oral Digit Span	Lists containing 2-9 digits were presented orally (one per second) to participants who repeated the digits they heard in the correct order.	Noël, Désert, Aubrun, & Seron, 2001
Digit Span	Task was described as an adaptation of the WISC-R Digit Span subtest. Participants were asked to repeat (in the same order) lists (span range 1-9) of digits spoken by a researcher. Presentation rate was controlled by having the researcher read the digits from a timed computer display.	Andersson, 2007
Digit Span	Participants were asked to repeat lists of digits (minimum span 3) presented orally (one per second) by a researcher.	Andersson, 2010
Digit Span	Participants were read lists of digits (span range 2-7) and asked to state them in the order presented.	Rasmussen & Bisanz, 2005
Digit Span	Participants heard a recording of a series of 2-9 digits (1 per second) and were asked to repeat them in the same order.	Hecht, Torgesen, Wagner, & Rashotte, 2001
Word Span Forward	A researcher read lists of common one-syllable words (maximum span of 9) to the participants who were asked to repeat them in the same order.	Berg, 2008
Word Span	A researcher read lists of 2-8 nouns (1-3 syllables in length) to participants who were asked to repeat the words in the correct order.	Swanson, 2006

Task Name	Brief Task Description	Sample Study(s) Using Task
Word Span	Participants heard lists of 2-8 nouns (1-2 syllables in length) and were asked repeat the lists in the correct order.	Swanson & Beebe-Frankenberger, 2004 Swanson, Jerman, & Zheng, 2008
Word Span	Participants heard lists (2-5 words) of one-syllable words (1 per second) and were asked to repeat them in the correct order.	Noël, 2009
Nonword Repetition	Participants heard a nonword consisting of syllables containing one consonant followed by one vowel (span of 2-5 syllables) and were asked to repeat it.	Noël, 2009
Nonword Repetition Test	Participants were asked to immediately restate the nonword presented by the recorded female voice. The test contained 16 nonwords of 1-4 syllables (4 nonwords at each length) that followed an alternating consonant vowel-structure. The words contained a total of 90 phonemes.	De Smedt, Taylor, Archibald, & Ansari, 2010
Nonword Repetition Test (Finnish)	Participants were read nonwords (span 2-7 syllables) and were asked to immediately restate them.	Kyttälä, Aunio, & Hautamäki, 2010
Phonetic Memory (Pseudoword Span)	Participants heard lists (span range of 2-7) of one-syllable nonwords read by a researcher and were asked to repeat the words in the correct order.	Swanson, 2006
Phonetic Memory (Pseudoword Span)	Participants heard lists (span range 2-6) of one-syllable nonwords and were asked to repeat the lists in the correct order.	Swanson & Beebe-Frankenberger, 2004 Swanson, Jerman, & Zheng, 2008

Task Name	Brief Task Description	Sample Study(s) Using Task
Food and Animal Word Span	Participants heard lists (span range 2-5 words) of one-syllable food and animal words (1 per second) and were asked to repeat them in the correct order.	Noël, 2009
Memory for Sentences	Participants heard 19 recorded sentences (4-21 words in length) and were asked to repeat them verbatim.	Hecht, Torgesen, Wagner, & Rashotte, 2001
Woodcock-Johnson-Revised Memory for Sentences Memory for Words		Osmon, Smerz, Braun, & Plambeck, 2006

Note. WISC-III = Wechsler Intelligence Scale for Children – Third Edition; WISC-IV = Wechsler Intelligence Scale for Children – Fourth Edition.

Table 1.3

Visuospatial Sketchpad (Visuospatial Short-term Memory) Tasks Commonly Used in Studies of Math and Working Memory

Task Name	Brief Task Description	Sample Study(s) Using Task
WMS-III Spatial Span	Forward portion of task	McGlaughlin, Knoop, & Holliday, 2005
Corsi Block*	This task used a board containing 9 randomly positioned blocks. A sequence of blocks (maximum span 9) was tapped at a rate of one block per second and the participant was asked to replicate the sequence by tapping the same blocks in the correct order.	Berg, 2008
Corsi Block	This task uses a board with 9 cubes glued on. A sequence of blocks was tapped at a rate of one block per second and the participants were asked to replicate the sequence by tapping the same blocks in the correct order.	Noël, 2009
Block Recall	This task uses a board containing 9 randomly positioned blocks. A sequence of blocks (smallest span one block) was tapped at a rate of one block per second and the participants were asked to replicate the sequence by tapping the same blocks in the correct order.	Holmes, Adams, & Hamilton, 2008
Corsi Blocks	Participants were shown 9 black cubes randomly distributed on a black board. Sequences of blocks (beginning with a span of 2 blocks) were tapped out (1 block per second) and the participant was asked to tap the same sequence.	Kyttälä, Aunio, & Hautamäki, 2010

Task Name	Brief Task Description	Sample Study(s) Using Task
Modified Corsi Block Task	Participants were shown a small figure passing through fields on a board and were asked to remember and trace its path. There were 16 different trials/paths.	Krajewski & Schneider, 2009
Corsi Span*	Participants were shown a paper with nine shaded spots and told that the spots represented stones in a pond while their finger would represent a frog jumping from stone to stone. The task was to make their own finger jump the same path as the finger of the researcher (span range 2-6 spots).	Rasmussen & Bisanz, 2005
Spatial Attention	Participants were shown an array of 9 lily pads on a computer screen. A frog appeared on a sequence of 2-6 lily pads (for one second on each pad) and the participant was asked to tap the pads on which the frog appeared in the correct order. The researcher clicked on each location pointed to by the participant so the computer could record the responses.	Lefevre, Fast, Skwarchuk, Smith-Chant, Bisanz, Kamawar, & Penner-Wilger, 2010
Spatial Span Task (from the Cambridge Neuropsychological Testing Automated Battery; CANTAB)	Participants were shown an array of white squares on a computer screen which briefly changed colour in sequence (span 2-9 squares). Participants were asked to touch the boxes on the screen in the order that they changed colour.	Gropper & Tannock, 2009
Visual-Spatial Span	Participants saw a matrix containing 4-24 squares on a computer screen. One by one, half of the squares turned black for 1 second in a sequence. Participants were asked to draw the order of appearance of the dark squares on a blank paper matrix.	Noël, Désert, Aubrun, & Seron, 2001

Task Name	Brief Task Description	Sample Study(s) Using Task
Visual Patterns Test	Participants saw matrices containing black and white squares for 2 seconds and then, after a .5 second delay, were asked to recall the location of the black squares on a blank matrix. The smallest matrix was 2 x 2 and had 2 black squares.	Holmes, Adams, & Hamilton, 2008
Matrix Task	Participants were shown 20, 3 x 3 matrices one at a time for 5 seconds each. On each matrix, some cells were blacked out. After seeing each matrix, participants were shown a blank matrix and asked to indicate which cells had been black on the original matrix they had seen.	Krajewski & Schneider, 2009
Matrix Task	Participants were shown paper cards containing matrices on which half the squares were marked with a black dot. The smallest matrix had 4 squares. When shown a blank matrix, they were asked to point to where the dots had been on the original matrix.	Kyttälä, Aunio, & Hautamäki, 2010

Note. WMS-III = Wechsler Memory Scale – Third Edition; * This task was labelled as measuring visual-spatial working memory by the author(s) but is most often described as a measure of the visuospatial sketchpad (short-term visuospatial memory).

Table 1.4

Verbal Central Executive (Verbal Working Memory) Tasks Commonly Used in Studies of Math and Working Memory

Task Name	Brief Task Description	Sample Study(s) Using Task
WISC-III Backward Digit Span	Only digits backward trials used	Swanson, 2006
WISC-IV Digit Span Backward	Only digits backward trials used	Jordan, Glutting, & Ramineni, 2010 Locuniak & Jordan, 2008
WAIS-III Digits Backwards	Only digits backward trials used	Tolar, Lederberg, & Fletcher, 2009
WAIS-III Digit Span	Both forward and backward trials used	Gropper & Tannock, 2009
WAIS-III Letter-Number Sequencing		Tolar, Lederberg, & Fletcher, 2009 Gropper & Tannock, 2009
WMS-III Letter-Number Sequencing		McGlaughlin, Knoop, & Holliday, 2005
WJ-III Numbers Reversed		Seethaler & Fuchs, 2006
Digit Span Backward (German)	Participants repeated mono-syllabic single-digit numbers (presented 1 per second) in reverse order.	Krajewski & Schneider, 2009
Backward Digit Span	Participants were read lists of digits (span range 2-7) and asked to state them in backwards order.	Rasmussen & Bisanz, 2005

Task Name	Brief Task Description	Sample Study(s) Using Task
Backwards Word Recall (Finnish)	Participants were read lists (smallest span = 2 words) of common 2-syllable words and asked to restate the words in backwards order.	Kyttälä, Aunio, & Hautamäki, 2010
Reverse-Word Span	Participants were read lists of one-syllable words and asked to restate them in backwards order.	Noël, 2009
Animal Dual-Task	Participants listened to sequences of lists of 4 one- and two-syllable words (1 word per second with a 2 second pause at the end of the list) and were asked to tap the table if a word they heard was the name of an animal. Presentation rate was controlled using a timed computer display of words for the researcher to read. At the end of the each sequence of 2-4 word lists, participants were asked to state the last word of each list in the correct order.	Andersson, 2007
Sentence Span	Participants listened to groups of sentences (maximum span = 8) read to them and were asked to remember the last word of each sentence. There was a 5 second pause at the end of each sentence. After each group of sentences, participants answered a question about a sentence and then were asked to state the remembered words in the correct order.	Swanson, 2004 Swanson & Sachse-Lee, 2001

Task Name	Brief Task Description	Sample Study(s) Using Task
Listening Sentence Span	Participants listened to groups of sentences (maximum span = 8) read to them and were asked to remember the last word of each sentence. There was a 2 second pause at the end of each sentence. After each group of sentences, participants answered a question about a sentence and then were asked to state the remembered words in the correct order.	Swanson, 2006
Listening Sentence Span	Participants listened to groups of sentences read to them and were asked to remember the last word of each sentence. There was a 5 second pause after each sentence. After each group of sentences, participants answered a question about a sentence and then were asked to state the remembered words in the correct order.	Swanson & Beebe-Frankenberger, 2004
Listening Sentence Span	Participants listened to groups of 2-6 sentences read to them and were asked to remember the last word of each sentence. After each group of sentences, participants answered a question about a sentence and then were asked to state the remembered words in the correct order.	Swanson, Jerman, & Zheng, 2008
Listening Span	Participants were read sequences of 3-word sentences at a rate of approximately one word every 0.8 seconds and asked to determine whether the sentence did or did not make sense. Then, the participants were asked to recall the first word of each sentence in the sequence in the correct order.	Andersson, 2010

Task Name	Brief Task Description	Sample Study(s) Using Task
Phonological Working Memory	Participants answered *yes* or *no* to a series of short questions (4 sets of each span of 2-4 sentences) and then stated the last word of each sentence heard in the correct order.	Hecht, Torgesen, Wagner, & Rashotte, 2001
Reading Span	Participants were shown (on a computer) a sentence followed by a '?' and a capital letter (e.g., The tugboat had never been so in love. ? H). Participants read the sentence out loud, indicated whether the sentence makes sense (yes/no), and read the letter out loud. At the end of a trial (consisting of 2-5 sentences), participants were shown a screen containing only a "?" and were asked to write down the sequence of letters they had read in the correct order on a piece of paper.	Tolar, Lederberg, & Fletcher, 2009
Verbal Working Memory	Participants were asked to read groups of sentences (shown on a computer) aloud and then recall the final words (low-imagery nouns). Sentences ranged from 9-16 words in length and sentence groups ranged from 2-5 sentences in length.	Miller & Bichsel, 2004
Auditory Digit Sequencing (from Swanson-Cognitive Processing Test; S-CPT)	Participants were read a sentence that contained a street address (e.g., 4876 Green Street) and were asked a process question (e.g., What was the name of the street?). Then they were asked to choose a strategy (rehearsal, chunking, associating, or elaborating) from a display card to help remember the information and were asked to state the digits in the address. Digits in the sentence were read at 2 second intervals.	Berg, 2008

Task Name	Brief Task Description	Sample Study(s) Using Task
Auditory Digit Sequencing (from Swanson-Cognitive Processing Test; S-CPT)	Participants were shown a card with pictures depicting four possible strategies (rehearsal, chunking, associating, and elaborating) they could use in this task. These strategies were explained. Participants heard a sentence that contained a street address (e.g., 4876 Green Street) and were asked a process question (e.g., What was the name of the street?). Then they were asked to state which strategy on the card was closest to the strategy they would use to remember these digits. Finally, they were asked to state the number in the address. Digits in the sentence were read at 2 second intervals.	Swanson, 2004 Swanson & Sachse-Lee, 2001
Digit/Sentence Span (from S-CPT)	Participants were shown a figure with pictures demonstrating 4 possible strategies they could use in the task. These strategies (rehearsal, chunking, association, and elaboration) were explained. Participants were read a sentence that contained a street address (e.g., 4876 Green Street), were asked a process question (e.g., What was the name of the street?), and had 10 seconds to indicate which strategy was most like the one they used to remember the information. Then they were asked to state the digits in the address in the correct order. Digits (span 3-14) in the sentence were read at 2 second intervals.	Keeler & Swanson, 2001 Swanson, 2006 Swanson & Beebe-Frankenberger, 2004
Digit/Sentence Span	Participants were read a sentence that contained numerical information (e.g., 8651 Elm Street), were asked a process question (e.g., What was the name of the street?), and then were asked to state the numbers in the sentence (span range 2-14 numbers). Digits in the sentence were read at 2 second intervals.	Swanson, Jerman, & Zheng, 2008

Task Name	Brief Task Description	Sample Study(s) Using Task
Counting Span	Participants were shown an irregular array of yellow and blue dots on a card and asked to count the yellow dots out loud. Sets of cards (2-5) were presented and at the end of each set, participants were asked to state the number of yellow dots they had counted on each card in the correct order.	Chong & Siegel, 2008 Rasmussen & Bisanz, 2005 Vukovic & Siegel, 2010
Counting Span	Participants were shown (on a computer) an array of circles and squares and asked to state the number of circles. On each trial, participants repeated this process with 2-7 screens. At the end of each trial, participants were asked to state the number of circles they had counted on each screen in the correct order.	Hecht, 2006
Counting Span	Participants viewed (on a computer) an array of dark blue circles, dark blue squares, and light green circles. The participants counted the dark blue circles out loud and repeated the total which they were asked to remember. A series (2-6) of arrays appeared in each trial. At the end of each trial, a screen with three question marks appeared and the participants were asked to write down the digits they remembered in the correct order.	Tolar, Lederberg, & Fletcher, 2009
Visual Counting Span/Working Memory	Participants were shown a series of white 5 x 8 inch index cards containing ½ inch red and blue dots in a random pattern. The total number of dots ranged from 3-16. Cards were shown one at a time (beginning with a sequence of two cards) to the participants who were asked to count the red dots on each. At the end of each sequence, the participants were asked to recall the number of dots counted on each card in the correct order.	Geary, Hoard, Byrd-Craven, & DeSoto, 2004

Task Name	Brief Task Description	Sample Study(s) Using Task
Semantic Association	A list of words was presented verbally at a rate of one word every 2 seconds. A process question (e.g., “Which word, saw or level, was said in the list of words?”) was asked. The participants were asked to recall the words, in any order, in categories. There were 8 lists which ranged in length and complexity from 2 words in 2 categories to 4 words in 5 categories.	Wilson & Swanson, 2001
Semantic Association	A list of words was presented verbally at a rate of one word every two seconds. A discrimination question (e.g., “Which word, saw or level, was said in the list of words?”) was asked. The participants were asked to recall the words together in categories. There were 8 lists which ranged in length and complexity from 2 words in 2 categories to 4 words in 5 categories. All items in one list had to be recalled and the process question answered correctly for words to count.	Swanson & Beebe-Frankenberger, 2004 Swanson, Jerman, & Zheng, 2008
Semantic Categorization (from Swanson-Cognitive Processing Test; Swanson, 1995)	Participants were read a set of words (including a category name and the words in the category) at a rate of one word every two seconds. Sets ranged from 1 category with 2 words to 8 categories with 3 words each. Then, participants were asked to choose a strategy (top-down subordinate organization, interitem discrimination, interitem associations, or subjective organization) to help them remember the categories and words. A process question (e.g., Which word, *rose* or *violet*, was presented?) was then asked and participants were asked to recall the categories and the words.	Berg, 2008

Task Name	Brief Task Description	Sample Study(s) Using Task
Category Span	Lists of one-syllable food and animal words were read to the participant who was asked to repeat them in categories: food first and then animals.	Noël, 2009
Story Retelling	Participants were asked to remember the sequence of events in a paragraph. The paragraph contained 11 sentences and each sentence contained 2 story "units" and 8-11 words. The paragraph was read, a process question was asked (e.g., "Was the person who jumped out of the plane a man or a woman?") and the participants were asked to recall the story. To be counted as correct, sentences had to be recalled in the correct order and contain both story units but not necessarily all the words.	Swanson & Sachse-Lee, 2001 Wilson & Swanson, 2001
Contingency Naming Test *	Participants were shown stimuli that had an inner and outer shape that matched (e.g., a small circle inside another larger circle) or did not match (e.g., a small square inside a larger circle). If the two shapes matched, the participant named the colour of the shape. If the two shapes did not match, the participant named the outer shape. If a backward arrow appeared over the shape, the naming rules had to be reversed.	Murphy, Mazzocco, Hanich, & Early, 2007
Paced Auditory Serial Addition Test (PASAT)	A computer auditorally presented sets of numbers one after the other at a specific rate (2.4 seconds between digits for one set and 1.6 seconds between digits for a second set). Participants were required to add each new number to the previously heard number and state the answer.	Gropper & Tannock, 2009

Note. WISC-III = Wechsler Intelligence Scale for Children – Third Edition; WISC-IV = Wechsler Intelligence Scale for Children – Fourth Edition; WAIS-III = Wechsler Adult Intelligence Scale – Third Edition; WMS-III = Wechsler Memory Scale – Third Edition; WJ-III = Woodcock Johnson – III; * This task was described as a working memory task by the authors. The decision to include it under tasks measuring verbal working memory was based on the degree to which verbal skills are required to process the directions and provide responses in the task; however, the stimuli for the task are visual in nature, so it could be argued that this task should be included in measures of visuospatial working memory.

Table 1.5

Visuospatial Central Executive (Visuospatial Working Memory) Tasks Commonly Used in Studies of Math and Working Memory

Task Name	Brief Task Description	Sample Study(s) Using Task
WMS-III Spatial Span	Backward portion of task	McGlaughlin, Knoop, & Holliday, 2005
Visual Matrix	A matrix containing dots was shown for 5 seconds and taken away. A process question (e.g., “Were there any dots in the first column?”) was asked. Then, participants were asked to draw the location of the dots on a blank matrix. The degree of difficulty ranged from 2 dots in 4 squares to 12 dots in 45 squares.	Berg, 2008 Swanson, 2004 Swanson & Sachse-Lee, 2001 Wilson & Swanson 2001
Visual Matrix	Groups of participants viewed an overhead projection of a matrix containing a number of dots for 5 seconds. Participants were asked a process question (e.g., Are there any dots in the first column?) and circled *yes* or *no* in their booklets. Then participants were asked to draw the location of the dots they had seen in the projection in their booklets. Matrix size ranged from 4 squares with 2 dots to 45 squares with 12 dots.	Swanson & Beebe-Frankenberger, 2004
Visual Matrix*	Groups of participants viewed an overhead projection of a matrix containing a number of dots for 5 seconds. Participants were asked a process question (e.g., Are there any dots in the first column?) and circled *yes* or *no* in their booklets. Then participants were asked to draw the location of the dots they had seen in the projection in their booklets. Matrix size ranged from 4 squares with 2 dots to 45 squares with 12 dots.	Swanson, 2006 Swanson, Jerman, & Zheng, 2008

Task Name	Brief Task Description	Sample Study(s) Using Task
Visual-Matrix Span	This task was described as a computerized version of Swanson's (1992) visual-matrix span task. The participants were shown a matrix with 2 cm white squares, some of which contained black dots (1 cm diameter), for 5 seconds, were asked a process question (e.g., Were there any dots in the first column'), and then were asked to draw the dots in the correct squares on a blank matrix. The matrices ranged in size from 6 squares with 2 dots to 56 squares and 9 dots.	Andersson, 2007
Visual-Matrix Span	A piece of paper showing a matrix containing dots was shown for 5 seconds and taken away. A process question (e.g., Were there any dots in the first column?) was asked. Then, participants were asked to draw the location of the dots on a blank matrix. The first matrix had 9 squares and 2 dots. Complexity was gradually increased by increasing the size of the matrix or the number of dots.	Andersson, 2010
Visual Working Memory (Paper folding)	Participants were shown (on a computer) a rectangle representing a piece of paper. The paper was folded (on the screen) 1-3 times and a circle representing a holepunch appeared. The participants were asked to consider where holes would appear if the paper were to be unfolded. A second, unfolded piece of paper containing holes then appeared and participants were asked to determine whether it correctly represented the location of the holes punched in the previous display.	Miller & Bichsel, 2004

Task Name	Brief Task Description	Sample Study(s) Using Task
Spatial Working Memory Task (from the Cambridge Neuropsychological Testing Automated Battery; CANTAB)	In this computer based task, participants were asked to search for a token that was hidden under a box without looking under boxes where tokens had previously been found. Arrays contained 4, 6, or 8 boxes.	Gropper & Tannock, 2009
Mapping and Directions	Participants were given an explanation of 4 strategies (elemental, global, sectional, and backward) they could use to help remember information in this task and shown a card containing a picture representing the strategy. The participants were required to recall a sequence of directions on a map that did not contain any verbal labels. A map containing streets (spaces between squares) and stoplights (dots) as well as the route of a bicycle (represented with lines connecting the stoplight dots) was presented for 10 seconds. A process question (e.g., Were there any dots on the first street?) was asked and participants then had 10 seconds to choose a strategy to help remember the bicycle route. Then, participants were asked to draw the route (lines and dots) on a map containing only the street information. A total of 9 maps with 4-19 stoplights were shown.	Keeler & Swanson, 2001
Mapping and Directions	Participants were shown a map for 10 seconds. It depicted a route (lines with arrows) that a bicycle took. Stoplights were shown with dots. A process question (e.g., Were there any stoplights on the first street?) was asked. Then participants were asked to draw the route of the bicycle (lines, arrows, and dots) on a blank map. A total of 9 maps with 4-19 stoplights were shown.	Swanson, 2004 Swanson & Beebe-Frankenberger, 2004 Swanson & Sachse-Lee, 2001

Task Name	Brief Task Description	Sample Study(s) Using Task
Mapping and Directions*	Participants were shown a map depicting a route (lines with arrows) that a bicycle took. Stoplights were shown with dots. A process question (e.g., Were there any stoplights on the first street?) was asked. Then participants were asked to draw the route of the bicycle (lines, arrows, and dots) on a blank map. Difficulty ranged from a map with 2 dots and 3 lines to a map with 20 dots and 23 lines.	Swanson, Jerman, & Zheng, 2008
Mapping and Directions	The participants were required to recall a sequence of directions on a map that did not contain any verbal labels. A map containing streets (spaces between squares) and stoplights (dots) as well as the route of a bicycle (represented with lines connecting the stoplight dots) was presented for 10 seconds. A process question (e.g., Were there any dots on the first street?) was asked and participants were shown illustrations of possible strategies they could use to remember the bicycle route. Then, participants were asked to draw the route (lines and dots) on a map containing only the street information. A total of nine maps were shown and the number of stoplights on the maps ranged from 4-9.	Wilson & Swanson, 2001
Mapping*	Participants were provided information about possible strategies to use to remember information in this task. Then, participants were shown (for 5 seconds) a street map with dots indicating stoplights and lines and arrows to indicate the path and direction a bicycle travelled. A process question (e.g., Were there any stoplights on the first street?) was asked and participants were then asked to draw the path of the bicycle (using dots, lines, and arrows) on a blank street map.	Swanson, 2006

Note. WMS-III = Wechsler Memory Scale – Third Edition; * These tasks were included under the heading "Visual-spatial sketchpad" in the tasks and materials section; however, the author referred to them as working memory tasks within the text of the article. Very similar tasks have been categorized as measuring visuospatial working memory in other research by the same lead author. Given that the tasks require processing, they were included as visuospatial working memory tasks here.

CHAPTER 2:

COUNTING ON MEMORY: BADDLEY AND HITCH'S MODEL OF WORKING MEMORY AND MATH CALCULATION SKILLS IN CHILDREN

Melissa McGonnell, Penny Corkum, Joan Backman, Shannon A. Johnson, and Fiona Davidson

The manuscript based on this study is presented below in Chapter 2. The reader is advised that Melissa McGonnell developed the research hypotheses, research methodology, and approach to data analysis for this study. She was responsible for all participant recruitment, data collection, and data entry (with assistance from research assistants). She completed all of the background research for this manuscript and was responsible for all aspects of the writing processes. She received editorial feedback from her dissertation committee members. This paper will be submitted for peer review.

Abstract

Math difficulties (MD) occur in about 3-7% of children. To date, findings about the role of working memory in MD have been conflicting. The Automated Working Memory Assessment Battery (AWMA) was used to investigate which component of Baddeley and Hitch's model of working memory was most related to math calculation skills in elementary-school children. Participants were 94 (52 male) children (*M* age = 9 years 1 month; *Range* = 6 years 0 months to 11 years 8 months). As hypothesized, math calculation scores were correlated with all four working memory components (phonological loop, visuospatial sketchpad, verbal and visuospatial central executive), but in a regression analysis, the visuospatial sketchpad (along with phonological processing) explained the most variance in math calculation scores. Short-term visuospatial memory should be assessed in children having difficulty with math and visuospatial strategies should be employed in the teaching of math calculation skills.

The importance of math literacy in today's technologically demanding society cannot be overstated. Unfortunately, many children have difficulty acquiring necessary numeracy skills. We have known for some time that significant difficulties with math (or learning disabilities in mathematics; MLD) are quite common, with prevalence estimates ranging from about 3-7 % of children (Badian, 1999; Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Kosc, 1974; Shalev, Manor, & Gross-Tsur, 2005). Despite this, MLDs have received much less attention from researchers than have reading disabilities (Geary, Hamson, & Hoard, 2000; Robinson, Menchetti, & Torgesen, 2002; Swanson & Jerman, 2006). In fact, research into MLDs has recently been described as still being in the early stages of development (Swanson, 2007).

One consequence of the fact that MLD has been under-investigated as compared to reading disability (RD) is that researchers have developed a widely accepted theory of RD, but no parallel theory to explain MLD has yet been articulated. Theories of learning disabilities require a specific description of an observable core behavioural deficit and an understanding of which associated cognitive processes may be deficient (Torgesen, 1999). Research into reading disabilities has addressed these considerations and resulted in wide acceptance of word-level reading as the core observable behavioural deficit and phonological processing as the cognitive process that is most commonly deficient (Lyon, Shaywitz, & Shaywitz, 2003; Shankweiler et al., 1995; Stanovich & Siegel, 1994). The same level of agreement has not been reached with regard to either the core behavioural deficit or the associated cognitive process for MLDs. As noted by Robinson et al. (2002), the identification of a core behavioural deficit and related deficient cognitive process for RDs has allowed for early identification of children with reading problems and the development of effective remediation techniques to assist them. Identifying behavioural and cognitive deficits for math has the potential to provide similar benefits for children experiencing trouble learning about that subject. The goal of the current study was to investigate working memory as a possible cognitive deficit for MLD. To that end, we first turn to the work that has been done toward identifying a core behavioural deficit for math.

A Core Behavioural Deficit for Math

Geary (2007) divided mathematical competencies into those that are primary and those that are secondary in nature. Primary competencies (e.g., counting and estimation) are biologically based and begin to emerge in children before they even begin school; however, he noted that most mathematical skills (i.e., observable behaviours) are considered secondary in nature as they do not develop without specific formal instruction, which usually begins at school entry. In fact, the development of math skills seems to be generally more reliant on teaching than is the development of reading skills. Once basic reading skills (e.g., knowledge of sound-symbol correspondences and basic sight words) are established, children are able to continue to use them independently as the demands of reading tasks increase (e.g., when presented with unfamiliar or more complex words). In addition, because reading involves language, something to which children are exposed on a daily basis, the process of learning to read is reinforced as children speak and hear others speak. The knowledge they gain from reading can be integrated with and reinforced by experiences with oral language. The learning of math skills does not exactly parallel this process. It can be argued that math information is present in the world around us just as language is, but the fact remains that new math concepts (e.g., fractions, decimals, percentages) and new multi-step processes (e.g., for long division) usually need to be specifically introduced through formal instruction, and the practice of math skills is not as integrated into daily life as is our practice of language. As well, although we build upon fundamental math skills when learning new concepts or procedures (e.g., using knowledge of single-digit multiplication facts when learning to calculate the area of a circle), some specific teaching (e.g., about the formula and possibly its derivation) is still often necessary to acquire this new information. As a result, it is harder for children to independently build on their knowledge of mathematics than on their knowledge of language.

Despite these challenges, which add to the complexity of defining a core behavioural deficit for MLD, some level of agreement has been reached. In one of the most recognized and cited reviews of the neuropsychological and cognitive literature related to MLDs, Geary (1993) noted that researchers in both fields, working separately, had consistently found the same core behavioural deficit in individuals with both acquired (as the result of brain injury) and developmental MLD. This deficit was in basic computational skills involving the use of computational procedures and the retrieval of math facts (i.e., one-digit addition, subtraction, multiplication, and occasionally division problems where the answers are typically known and can be retrieved directly from memory without the need for specific computation; Zamarian, López-Rolón, & Delazer, 2007). Developmentally and in most math curricula, basic fact mastery often precedes the learning of more complicated computational procedures (Fuchs et al., 2006). In studies with younger children (e.g., Fuchs, Compton, Fuchs, Paulsen, Bryant, & Hamlett, 2005; Krajewski & Schneider, 2009), measures that specifically test math fact knowledge are often used; however, measures that tap the broad range of calculation skills required in the classroom (i.e., math facts *and* computational procedures) are also commonly used with children (e.g., Andersson, 2010; Berg, 2008; Fuchs et al., 2010; Jordan, Glutting, & Ramineni, 2010; Swanson, Jerman, & Zheng, 2008). This second approach was used in the current study.

Defining Deficit

It has been argued (e.g., Mazzocco, 2007) that researchers should distinguish between children with math disabilities and children with low math achievement (sometimes referred to as math difficulties; MD) because of the possibility that the challenges with math come from very different sources. Mazzocco (2007) noted that a variety of environmental factors (e.g., poverty, poor attendance, lack of appropriate instruction) can contribute to low math achievement, while math disabilities are assumed to result from inherent cognitive deficits which interfere with the development of math skills. Consequently, simply using low achievement to categorize participants as having a math disability could blur research findings. Despite this argument, the approach of categorizing participants as typically achieving or as having a math disability solely on the basis of a single measure of math achievement is commonly used. Unfortunately, there is no universally agreed upon method to define deficit in these studies. Typically, a cut-off point is used, but the actual point chosen ranges widely from as high as the 46th percentile (approximately equivalent to a standard score of 99; e.g., Geary, Bow-Thomas, & Yao, 1992) to as low as the 2nd percentile (approximately equivalent to a standard score of 70; e.g., Desoete & Roeyers, 2005; Passolunghi, Marzocchi, & Fiorillo, 2005). Others (e.g., Ellis, 1985; Fletcher, Lyon, Fuchs, & Barnes, 2007; Lipka, Lesaux, &

Siegel, 2006, Stanovich, 1988) advocate against a categorical approach, believing that math (and reading) disabilities are best conceptualized as existing on a “continuum of severity” (Fletcher et al., 2007; p. 28) and noting that there is a significant body of research that argues against the use of arbitrary cut-off points to distinguish groups with and without disabilities. This latter approach of considering math ability across a wide spectrum of ability was used in the current study. The term MD will henceforth be used to refer to those with a broad range of difficulties with math.

Proposed Cognitive Deficits

Attention. Numerous studies (e.g., Fuchs et al., 2005; Shalev et al., 2005) have documented the association between inattention and MDs and the importance of continuing to investigate the role of attention in MDs has been noted (e.g., Fletcher, 2005; Raghubar, Barnes, & Hecht, 2010). A specific association between MD and Attention-Deficit Hyperactivity Disorder (ADHD) has also been established, with estimates of the proportion of children with ADHD who also have MD ranging from 11-33% (Capano, Minden, Chen, Schachar, & Ickowicz, 2008; Mayes & Calhoun, 2006; Mayes, Calhoun, & Crowell, 2000; Monuteaux, Faraone, Herzig, Navsaria, & Biederman, 2005; Semrud-Clikeman, Biederman, Sprich-Buckminster, Lehman, Faraone, & Norman, 1992). Consequently, a large proportion of any population of children with MDs will also have ADHD and we therefore believed it was important to include children with ADHD in the current study. Despite the common co-morbidity of ADHD and MD, studies examining factors relating to math achievement often fail to make any mention of whether participants were screened for ADHD/attention difficulties or whether these participants were deliberately excluded or included (e.g., Hecht, Torgesen, Wagner, & Rashotte, 2001; Swanson & Beebe-Frankenberger, 2004), even when assessing attention as a variable predicting math performance (e.g., Fuchs et al., 2005).

Phonological Processing. Phonological processing is commonly accepted as a primary core cognitive deficit in RDs and RDs often co-occur with MDs. For example, Barbaresi et al. (2005) found RDs in 45-65% of their sample of children with MDs. However, as noted by Jordan (2007), early studies of MD did not commonly consider reading skills of the participants. More recently, research has compared children with MD only to children with math and reading difficulties (MD+RD). In general, researchers have found that both groups have challenges with math calculation skills but that children with MD+RD have more difficulty with word problems than children with MD only (e.g., Fuchs & Fuchs, 2002; Geary et. al, 2000; Jordan & Hanich, 2003; Jordan, Hanich, & Kaplan, 2003b). Some (Geary, 1993, Robinson et al., 2002; Rourke & Conway, 1997) have proposed that verbal skills (i.e., semantic memory or phonological processing) could also explain MDs in some children (i.e., children with MD+RD) but that visuospatial deficits might be the source of difficulty for children with MD only. Others (e.g., Cirino, Fletcher, Ewing-Cobbs, Barnes, & Fuchs, 2007; Geary et al., 2000; Jordan, Hanich, & Kaplan, 2003a) have found evidence that basic math skills develop similarly in children with MD and children with MD+RD. This could be considered evidence against phonological processing as a primary core cognitive deficit in children with MD, a conclusion also reached by Jordan (2007). Swanson and Jerman (2006) noted, however, that MD research to date has a number of methodological problems (including significant variability in cut-off scores as discussed above) which, at this point, make it difficult to definitely conclude whether MDs and RDs have similar or different deficiencies in core cognitive processes. Therefore, we cannot rule out a role for phonological processing in math achievement.

Working Memory. A large number of studies (e.g., Fuchs, et al., 2005; Geary et al., 2000; Swanson, 2004) have implicated working memory as the potential core cognitive deficit underlying learning challenges in the area of mathematics. Most studies conceptualize working memory using Baddeley and Hitch's multicomponent model (Baddeley, 2002; Baddeley & Hitch, 1974; Baddeley & Hitch, 2000), which proposes that working memory is best understood as having three components. The first component, the *phonological loop,* is theorized to contain two parts: the phonological store, which has the ability to hold verbal information for a very brief time, and the articulatory rehearsal system, which allows us to briefly increase the capacity of the phonological store through verbal rehearsal. The second component, the *visuospatial sketchpad,* is the system that allows us to hold or manipulate visual and/or spatial information received from our senses or accessed from long-term memory. The third component, the *central executive*, can access stored verbal or visuospatial information through either of the other systems, but it also has an attentional control system which is responsible for maintaining or shifting focus.

In some conceptualizations of Baddeley and Hitch's model, the first two components are described as having purely *storage* functions (or as *short-term* memory) while the central executive is described as having both storage and *manipulation* functions (or as *working* memory) for both modalities (verbal and visual). To be clear, in the context of this study, we will use the terms *phonological loop*, *visuospatial sketchpad*, and *verbal* and *visuospatial central executive* when referring to the components of Baddeley and Hitch's working memory model.

In Baddeley and Hitch's original 1974 model, the central executive was described in somewhat vague terms (i.e., as having a role in processing without specifying anything about a mechanism through which this would occur) with most focus on providing evidence to support the existence of the phonological loop and, to some extent, the visuospatial sketchpad. More recently, Baddeley (2002) has elaborated on the central executive's role in allocating (focussing and dividing) attentional resources. Some (e.g., Engle, 2002, Engle, 2010) argue that working memory capacity is, in essence, only the ability to control attention. Others (e.g., Cowan, 2008) conceptualize working memory more as did Baddeley and Hitch, as including both short-term memory and a variety of processing abilities (including attention) that make it possible to use short-term memory to accomplish tasks. What is common to all these conceptualizations is the inclusion of an attention function as a part of the working memory system. As a result, it is important to be aware that any measure of working memory will, at some level, also be a measure of attentional capacity.

To date, no agreement has been reached as to which components of Baddeley and Hitch's model are important in MD. Two recent meta-analyses have attempted to summarize the literature with respect to patterns of working memory difficulties in children with learning disabilities and ADHD. Swanson and Jerman (2006) examined the results of 28 studies published between 1970 and June 2003 which compared cognitive functioning of children with MD to that of children with RD, both MD and RD, and/or average achieving (AVE) children. They concluded that one of the most robust differences was that children with MD had less well developed verbal working memory (central executive) than AVE children but also noted that children with MD had relatively weaker visual-spatial (authors' term) working memory (central executive) abilities than children with RD only. Notably, however, Swanson and Jerman did not use Baddeley and Hitch's model to develop their classification of tasks used in the studies they included in their meta-analysis. The authors included three categories of verbal tasks (short-term memory for words, short-term memory for numbers, and working memory-verbal) and two visuospatial categories (problem solving – visual-spatial and working memory – visual-spatial). Unfortunately, it is not possible to determine whether measures specific to the visuospatial sketchpad were included in one or the other (or both) of Swanson and Jerman's visual-spatial categories or whether, in fact, the studies in this meta-analysis did not incorporate such measures. In either case, the classification system used by Swanson and Jerman clearly illustrates the lack of consistency in how the components of Baddeley and Hitch's model are operationally defined, another possible explanation for the variability in research findings to date.

Martinussen, Hayden, Hogg-Johnson, and Tannock (2005) conducted a similar meta-analysis of the literature examining working memory impairments in children with ADHD (who, as noted previously, often have MD) and found evidence for weaknesses in the visuospatial sketchpad and the visual-spatial (authors' term) component of the central executive but much smaller deficits in the phonological loop and the verbal component of the central executive. Interestingly, this association between the visuospatial sketchpad and math skills is consistent with visuospatial weaknesses identified in children with nonverbal learning disabilities who typically have impaired math skills (e.g., Harnadek & Rourke, 1994; Rourke, 1993).

As noted above, some variability in research findings with respect to working memory may be the result of the fact that although the majority of studies have been based (sometimes loosely based) on Baddeley and Hitch's model, researchers have not always included measures which tap all working memory components. Additionally, researchers frequently use only one task to measure functioning of a component despite the fact that the relationship between learning difficulties and working memory is more likely to be captured if multiple measures are used (Geary et al. 2000; Martinussen & Tannock, 2006).

In 2001, Pickering and Gathercole published the Working Memory Test Battery for Children (WMTB-C), an instrument that assesses all elements of Baddeley and Hitch's model. Even more recently, the Automated Working Memory Assessment (AWMA; Alloway, Gathercole, & Pickering, 2004), a computer based assessment very similar to the WMTB-C was developed. The original version of the AWMA (an updated version was published in 2007) contains twelve subtests, three which measure the phonological loop (short-term verbal memory), three which measure the visuospatial sketchpad (short-term visuospatial memory), and six which measure the central executive (three for verbal working memory and three for visuospatial working memory), and was normed for use with children aged 4-11 years. This measure's utility as a research tool is enhanced by the fact that it incorporates multiple measures of each component of Baddeley and Hitch's model of working memory and that its computerized format improves the ability to administer the tests in a standardized way.

To our knowledge, only one other study has used the complete AWMA battery to specifically examine relationships between working memory and math achievement in children. Alloway and Passolunghi (2011) used the Italian adaptation of the AWMA to investigate the contribution of the four working memory components and vocabulary to math skills in seven- and eight-year-old typically developing Italian children. After accounting for vocabulary knowledge, verbal short-term memory (the phonological loop) was the only memory component to account for additional variance in math computation scores in seven-year-old children. Visuospatial short-term memory (the visuospatial sketchpad) was the only component to account for additional variance in the math computation scores of the eight-year-old children.

Current Study

Deficits in the various components of Baddeley and Hitch's model of working memory have been found to be related to MD in children; however, there is considerable disagreement about which components of this model contribute to math achievement. As described above, there are several key reasons for the contradictory results to date. Firstly, studies have conceptualized MD both as a continuum of ability and by using a categorical approach with a variety of cut-off points. Secondly, studies examining the role of memory in math skills have not always incorporated measures of all components of Baddeley and Hitch's working memory model or have only used one measure of each working memory component. Finally, the potential confounding effects of attention (or comorbid ADHD) and phonological processing skills (or comorbid RD) have not typically been considered when examining working memory in MDs.

The goal of the current study was to examine how the components of Baddeley and Hitch's model of working memory are associated with math calculation skills in elementary school-aged children who are in the relatively early stages of formal math instruction. This study used a measure to assess the broad range of calculation skills required in the classroom (i.e., math facts and computational procedures) and employed the AWMA, a standardized measure based on Baddeley and Hitch's model which includes multiple measures of each component of children's working memory skills. The objective was to determine which components of Baddeley and Hitch's model contributed to math calculation skills in elementary school children after accounting for variability from attention problems and phonological processing. Our sample included children with variability in math skills, phonological processing ability, and attention, which was necessary to meet this objective. We expected that math calculation would be correlated with all four components of memory assessed by the AWMA, but based on the findings of Alloway and Passolunghi (2011), Martinussen et al. (2005), and Swanson and Jerman (2006), we hypothesized that after accounting for phonological processing and attention, math calculation skill would be best predicted by the two visuospatial memory components (the visuospatial sketchpad and the visuospatial central executive).

Method

Participants

Participants were recruited in rural and urban areas of Nova Scotia, Canada using mailings to parents of children in one school board encompassing a primarily rural area, parents of children attending several private schools and tutoring programs, and parents of children who had been rigorously diagnosed with ADHD at a hospital clinic or through a private psychological practice. We recruited participants from diverse locations and from both clinical and non-clinical settings with the goal of ensuring that our sample included children with a broad range of math skills, phonological processing ability, and difficulty with attention. Participants were all between the ages of 6 and 11 and had an IQ of at least 80 as estimated from two subtests (Block Design and Vocabulary) of the Wechsler Intelligence Scale for Children – 4th Edition (WISC-IV; Wechsler, 2003). Children did not have any known general medical or neurological conditions (e.g., seizure disorder, epilepsy, cerebral palsy) and did not take psychostimulant medications (for ADHD symptoms) on the day of their participation in the study.

Data were collected from a total of 104 children. Due to experimenter error, data from ten children were incomplete and were not included in the analyses for this study. Data from the remaining 94 participants, 27 of whom were diagnosed with ADHD, were included in analyses. The participants (52 male, 42 female) ranged from 6 years, 0 months to 11 years, 8 months of age (*M* = 9 years, 1 month; *SD* = 1 year, 7 months) and were attending grades primary through six (primary: n = 5; grade 1: n = 12; grade 2: n = 16; grade 3: n = 17; grade 4: n = 22; grade 5: n = 18; grade 6: n = 4).

Measures

Descriptive Measures.

Estimated IQ. Two subtests (Vocabulary and Block Design) of the *Wechsler Intelligence Scale for Children – 4th Edition* (WISC-IV; Wechsler, 2003), a widely used measure of cognitive ability for children, were used to estimate general cognitive ability. This procedure has been shown to be a highly reliable and valid means of estimating IQ (Sattler, 2008).

Demographic Information. A background questionnaire was used to obtain descriptive information about the participants.

Academic Measures.

Math Calculation and Word Reading. The Calculation and Letter-Word Identification subtests of the *Woodcock-Johnson Tests of Achievement – 3rd Edition* (WJ-III; Woodcock, McGrew, & Mather, 2001) were used to assess math calculation and reading skills. The WJ-III is a widely used, comprehensive, standardized measure of academic achievement normed for use with anyone over the age of 24 months. Grade-based standard scores were used for analyses.

Cognitive Measures.

Attention. The *Conners' Parent Rating Scale – Long Form (*CPRS-R*;* Conners, 1997) is a commonly used questionnaire that has been widely reported to be effective in identifying externalizing problems such as those associated with ADHD and to have good reliability (e.g., internal consistencies of .73 to .96; test-retest reliabilities of .47 to .85) and adequate validity (Sattler, 2002). The CPRS-R was completed by parents of children who participated in this study. The T-scores from the DSM-IV Inattentive subscale were used as a measure of attention difficulties.

Phonological Processing. The *Comprehensive Test of Phonological Awareness* (CTOPP) is a widely used measure of phonological processing which is normed for use with individuals up to the age of 24 years (Wagner, Torgesen, & Rashotte, 1999). It has good reliability (most internal consistency coefficients exceed .80; test-retest coefficients range from .68 to .97) and validity (correlations with the Woodcock Reading Mastery Test-Revised range from .58 to .73). The Phonological Awareness composite score (a standard score obtained by combining scores from the Elision and Blending Words subtests) was used as a measure of phonological processing.

Working Memory. The experimental version of the *Automated Working Memory Assessment Battery* (AWMA; Alloway et al., 2004) was used to assess working memory. The AWMA is a computer-based assessment of working memory that has been validated against Baddeley and Hitch's model. This version of the AWMA has been determined to be a reliable means of assessing memory in children from 4 to 11 years of age (Alloway, Gathercole, & Pickering, 2006). Test-retest reliability estimates range from .64 to .84. The AWMA contains 12 subtests organized to assess all components of Baddeley and Hitch's model of working memory. Standard scores for each subtest and for the phonological loop, the visuospatial sketchpad, the verbal central executive, and the visuospatial central executive are available. (Note: A version of the AWMA with norms extending to 22 years 11 months is now commercially available.)

The phonological loop (verbal short-term memory) is assessed with three subtests. On the *Digit Recall* subtest, the child hears a list of digits and has to recall them in the correct order. On the *Word Recall* subtest, the child hears a list of words and has to recall them in the correct order. On the *Nonword Recall* subtest, the child hears a sequence of nonsense words (nonwords) and has to recall each sequence in the correct order.

The visuospatial sketchpad (visuospatial short-term memory) is assessed with three subtests. On the *Dot Matrix* subtest, the child is shown the position of a red dot in a series of four by four matrices and has to indicate recall of the positions by tapping the squares in a blank matrix on the computer screen. On the *Mazes Memory* subtest, the child views a maze with a red path drawn through it and then uses his/her finger to trace the same path on a blank maze presented three seconds later on the computer screen. On the *Block Recall* subtest, the child views a video of a series of blocks being tapped and reproduces the sequence in the correct order by tapping an image of the blocks on the computer screen.

The central executive is assessed using six subtests. Three of these subtests assess the verbal central executive. On the *Listening Recall* subtest, the child hears a series of individual sentences and judges if each sentence is true or false. At the end of the trial, the child recalls the final word of each sentence in the correct order. On the *Counting Recall* subtest, the child counts the number of circles on a series of arrays of circles and triangles and then has to recall the counted numbers in sequence. On the *Backwards Digit Recall* subtest, the child hears a list of digits and has to recall them in backwards order. Three other subtests are used to assess the visuospatial central executive. On the *Odd-One-Out* subtest, the child sees three shapes, each in a box presented in a row, and must identify the shape that is the odd-one-out. Then the child recalls the location of each odd-one-out shape, in the correct order, by tapping the correct box on the screen. On the *Mister X* subtest, the child views a picture of two Mister X figures. The child identifies whether the Mister X with the blue hat (who is sometimes rotated) is holding the ball in the same hand as the Mister X with the yellow hat. At the end of each trial, the child has to recall the location of each ball held by the blue Mr. X, in the correct order, by pointing to a diagram with six compass points. On the *Spatial Span* subtest, the child views a picture of two shapes where the shape on the right has a red dot above it. The child identifies whether the shape on the right is oriented in the same or opposite direction as the shape on the left. At the end of each trial, the child has to recall the location of each red dot on the shape, in the correct order, by pointing to a picture with three compass points.

One adaptation was made to the AWMA for this study. During pilot testing with typically developing children, it became obvious that children were distracted by the British accent of the female voice on some of the verbal subtests (particularly Nonword Recall). Therefore, to prevent any possible confounding effects, five verbal subtests (Digit Recall, Word Recall, Nonword Recall, Listening Recall, and Backwards Digit Recall) were recorded with a female voice with a local accent. These recordings were accessed on the same computer and played during the administration of the AWMA maintaining the same order of subtest administration. Responses were recorded manually and later transferred to the AWMA for scoring.

Procedure

All data were collected individually, by one of two female researchers, in one session lasting between 1 ½ and 2 hours. The child first completed the AWMA and then the Calculation and Letter-Word Identification subtests of the WJ-III followed by the two CTOPP subtests. The two subtests of the WISC-IV were administered last. Parents completed the CPRS-R and the background questionnaire while their child participated in the study. To thank children for their time and effort, they were provided with a certificate of participation and allowed to choose a small prize. To thank parents/guardians for their time, an individual research feedback report was provided to them a few weeks after their research appointment.

Results

Descriptive Statistics

Estimated IQ for the sample (N = 94) ranged from 80-135 (M = 102.2; SD = 12.8). Socio-economic status was estimated using parents' report of the household total annual income. Parents indicated the range into which their family's total income fell using a 7-point Likert scale divided in $10,000 increments from 1 (*up to $20,000 per year*) to 7 (*more than $70,000 per year*). Ninety-one of the ninety-four families (96.8%) provided information about their annual income. The median income level was 6 ($60,000 to $70,000 per year) which is similar to the median income in Nova Scotia in 2007 ($59,200; Statistics Canada, 2010). Nine families (9.9%) reported income of less than $30,000 or less than half the median income level. This is a commonly used marker of low-income and is similar to the 2006 low-income prevalence in Nova Scotia (12.1%; Nova Scotia Department of Community Services, 2008).

Before conducting further analyses, children with a diagnosis of ADHD (ADHD group) were compared to children without a diagnosis of ADHD (non-ADHD group) to ensure that combining these groups was appropriate. The groups did not differ in terms of average age, t (92) = 1.48, p = .14, average IQ, t (92) = .81, p = .42, distribution of grade, χ^2 (6, N = 94) = 9.36, p = .15, or family income, χ^2 (6, N = 91) = 2.28, p = .89. The groups did differ in terms of proportion of males to females, χ^2 = (1, N = 94) = 10.49, p = .001, as 81.5% of the ADHD group were male while 44.8% of the non-ADHD group were male. The groups also differed in terms of the proportion of individuals who scored below the 25th percentile (SS = 90) on the math measure, χ^2 (1, N = 94) = 13.08, p < .001, as 81.5% of the ADHD group but only 40.3%. of the non-ADHD group scored below this point. (See Table 2.1 for complete statistics.) It is common for boys to outnumber girls in samples of children with ADHD and our ratio of 4.4:1 is very similar to those in other studies with much larger samples (e.g., Capano et al., 2008). The proportion of children with ADHD who have academic difficulties is also often much higher than is the case in populations of children who do not have ADHD. For example, Barkley (1998) reported statistics that would indicate that children with ADHD have learning difficulties with math 1.5 to 10 times more often than children without ADHD. Hence, the differences in our two groups are believed to reflect actual population differences and as a result, analyzing grouped data was determined to be appropriate.

Academic and Cognitive Variables

Scores on the Calculation subtest of the WJ-III ranged from 57-135 (M = 90.1; SD = 16.0). Mean scores on the cognitive variables (four working memory components, phonological processing, attention problems, estimated IQ) and on the Letter-Word Identification subtest of the WJ-III were all in the broadly average range (see Table 2.2). Scores on the Calculation and Letter-Word Identification subtests were highly correlated ($r = .62, p < .001$).

Regression Analysis

Hierarchical regression was used to determine which memory component(s) best explained children's math calculation scores. All relevant assumption checks were conducted.[1] (The hierarchical regression was conducted with identified outliers removed and the pattern of the results did not change significantly.) All variables used in the regression analyses were significantly correlated with scores on the Calculation subtest of the WJ-III (see Table 2.3). We were primarily interested in the ability of memory to predict math calculation scores after accounting for variability due to phonological processing and attention problems which have both been shown to be related to math skills; however, age was significantly negatively correlated (r = -.36, p < .01) with standard scores on the Calculation subtest of the WJ-III, indicating that, in our sample, older children had more difficulty with math calculation than younger children. As a result, age (in months) was added as an additional predictor variable. IQ, which was significantly correlated with math calculation scores (r = .425, p < .001), was not included as a predictor variable as recommended in a recent critical review of this practice (Dennis, Francis, Cirino, Schachar, Barnes, & Fletcher, 2009).

Scores (grade-based[2] standard scores) on the Calculation subtest of the WJ-III were entered as the dependent variable, age (in months) was entered as a predictor in step 1, phonological processing (CTOPP Phonological Awareness Composite standard score) and attention problems (DSM-IV Inattention subscale T-score) were entered in step 2. The standard scores for the four working memory components were then entered in step 3. All three steps (see Table 2.4) significantly predicted scores on the Calculation subtest of the WJ-III. In step 1, age accounted for 12.6% of the variability in scores on the Calculation subtest. In step 2, phonological processing and attention problems explained an additional 26.8% of the variance. In step 3, the only working memory component that was a significant predictor of calculation scores was the visuospatial sketchpad which explained an additional 10.2% of unique variance in calculation scores. With the addition of the visuospatial sketchpad in this step (3), the contribution of attention problems was no longer significant (p = .23). The same regression analysis was conducted including only the data from the non-ADHD participants. The pattern of results was identical.

Discussion

The objective of this study was to determine which component or components of Baddeley and Hitch's (1974) model of working memory were related to math skills in elementary school-aged children. We expected that two visuospatial working memory components (the visuospatial sketchpad and visuospatial central executive) would be most associated with MDs. The visuospatial sketchpad (short-term visuospatial memory) emerged as the working memory component which had the strongest association with calculation skill in elementary-aged children, but our results also pointed to a role for phonological processing and attention.

The Role of Working Memory

Consistent with our predictions, all four working memory components of Baddeley and Hitch's model (tested using the AWMA) were correlated with scores on the Calculation subtest of the WJ-III, with *r*s ranging from .42 to .53. Based on the conclusions of two recent meta-analyses (Martinussen et al., 2005; Swanson & Jerman, 2006) and a recent study using the AWMA (Alloway and Passolunghi, 2011), we predicted that after accounting for variance due to age, attention problems, and phonological processing, both visuospatial working memory components would predict math calculation skills; however in our regression analysis, the only working memory component to account for additional variance in math calculation scores was the visuospatial sketchpad. A closer examination of the methodology of the two meta-analyses reveals three possible explanations for this seeming inconsistency.

Firstly, Swanson and Jerman (2006) did not include a category dedicated to visuospatial short-term memory (the visuospatial sketchpad) in their meta-analysis and no information is provided about the reason for this omission. Failure to include measures of the visuospatial sketchpad has been noted as a problem in MD research generally (Raghubar et al., 2010), so it is possible that too few (or none) of the studies included in the meta-analysis specifically assessed the visuospatial sketchpad. Swanson and Jerman did identify what they termed visual-spatial *working* memory as a cognitive skill which distinguished children with MD from children with RD and from children who were typically achieving. Importantly, the example they provided to illustrate how visual-spatial working memory tasks were defined in their meta-analysis was the Visual Matrix task which has been used as a measure of the visuospatial sketchpad (e.g., Krajewski & Schneider, 2009; Kyttälä, Aunio, & Hautamäki, 2010; Swanson et al., 2008) and is similar to the Dot Matrix subtest, one of the visuospatial sketchpad measures on the AWMA. Given the lack of a visuospatial sketchpad category and the similarity in task demands, it seems logical to conclude that our results are at least partially consistent with Swanson and Jerman's (2006) conclusions as both confirmed the importance of some form of visuospatial memory.

A second point is related to the importance of attention in the central executive. Our results are only partially consistent with the results of the Martinussen et al. (2005) meta-analyies of memory difficulties in children with ADHD. Martinussen et al. did organize the tasks in the studies they examined based on a system that closely parallels Baddeley and Hitch's model and the structure of the AWMA. Tasks that involved visuospatial storage only were considered to measure the visuospatial sketchpad, while tasks that required storage and manipulation were considered to measure the visual component of the central executive. Martinussen et al. concluded that children with

ADHD have greater deficits in visual (both the visuospatial sketchpad and visuospatial central executive) than in verbal working memory. In our analyses however, the ability of the visuospatial central executive to explain variability in math calculation scores was reduced by the fact that attention problems were entered first in our regression analysis. This pattern of results is consistent with Martinussen and Tannock (2006) who found that attention problems were better predictors of the verbal and visuospatial central executive than of verbal and visuospatial short-term memory.

A third possibility is related to the validity of Baddeley and Hitch's model. Previous research (e.g., Engle, Tuholski, Laughlin, & Conway, 1999) has found that although they are related, it is possible to differentiate the verbal components of Baddley and Hitch's model (i.e., the phonological loop and the verbal central executive); however, Miyake, Friedman, Rettinger, Shah, and Hegarty (2001) have demonstrated that the visuospatial components of Baddeley and Hitch's model (i.e., the visuospatial sketchpad and the visuospatial central executive) cannot be reliably separated as they are both equally related to executive functioning abilities and recommended that memory models be adjusted to account for this fact. Hence, our data may indicate that math calculation skills are related to visuospatial memory in broad terms (as predicted) rather than specifically to short-term visuospatial memory.

Our findings are consistent with those of Alloway and Passolunghi (2011) who found the visuospatial sketchpad to be the working memory component that best predicted math computation scores of eight-year-old children and the majority (71.2%) of the participants in our study were eight-years-of-age or older at the time of participation.

A similar pattern of results was noted by Meyer, Salimpoor, Wu, Geary, and Menon (2010).

The Role of Attention

Scores for parent report of attention problems were significantly negatively correlated with math calculation scores in the full sample of children, but the results of the regression analyses did not support the conclusion that attention problems are the core cognitive deficit in children with MDs. Although attention problems explained a significant proportion of the variance in calculation scores when initially entered into our regression models, this proportion was reduced and was no longer significant in step 3 after variance due to scores on both phonological processing and the visuospatial sketchpad were accounted for. This could be considered to point to these cognitive processes being more important than attention problems in explaining math calculation skills in children.

It is important to note, however, that this study did not include a cognitive measure of attention problems but relied on parent report. Hence, our scores for attention problems represent parents' subjective perceptions and results may have been different had an objective measure been utilized. It is also important to consider that some theories of memory (e.g., Cowan, 2008) specifically describe attention as important to the functioning of short-term memory (i.e., the visuospatial sketchpad). In Cowan's model, short-term memory is described as the subset of the activated portion of long-term memory that is the current *focus of attention*. Hence, attention is central to short-term memory and, by extension, a measure of short-term memory (or the visuospatial sketchpad) would also be a measure of attention. This conceptualization of memory and attention as enmeshed constructs would explain the fact that the introduction of the visuospatial sketchpad to the model reduced the proportion of variance explained by the attention problems measure but would also point to phonological processing, short-term visuospatial memory, *and* attention as important to math calculation skills.

Practically speaking, children who are able to sustain attention to instructions or to the task at hand learn more effectively than children who have poor sustained attention. Some facets of children's attention can be improved using attention state training (e.g., meditation or mindfulness) or through direct practice. (For a review, see Tang and Posner, 2009). It is also possible to make it easier for children to sustain attention by tailoring instructions and tasks to reduce the load on their cognitive processing resources (i.e., working memory load) and improve their ability to use the attention resources they possess.

The Role of Phonological Processing

Phonological processing contributed significantly to the variance in calculation scores when it was entered simultaneously with attention problems (after accounting for age) and it continued to account for a significant proportion of variance in calculation scores even after all four working memory components were entered in the third step. It seems logical that phonological processing has a role to play in the development of basic math calculation skills in elementary school-age children. At a very fundamental level, an introduction to most learning, including how to approach paper and pencil math tasks, is provided with language, as is most corrective feedback. As well, a broad association between verbal skills and academic learning is well established and it would be surprising if a relationship between phonological processing, a basic skill necessary for much language processing, and math skills were not found. The importance of phonological processing is also consistent with Alloway and Passolunghi's (2011) finding of the relationship of the phonological loop to the early development of math computation skills.

The Importance of Verbal and Visuospatial Skills

The idea that both verbal and visuospatial abilities are important to math calculation skill is consistent with many theories about mathematics including Dehaene's (1992) triple-code model of numerical abilities and Geary's (1993) theory of three subtypes of math disabilities. Dehaene's model is based on the notion that numerical information can be coded using verbal abilities (used when we count or when we use verbal repetition to aid the development of memory for basic math facts) or visual abilities (used when we compare quantities or approximate values), as well as with a notational system (i.e., written numerals) that is useful and often necessary for complex calculation. Geary posited that MD could result from difficulties with fact retrieval (which would be highly associated with RD and possibly result from difficulties with phonological information), difficulties with visuospatial skills resulting in poor conceptualization and interpretation of numerical information, or difficulties with using procedures (e.g., borrowing or carrying using the notational system) effectively.

The importance of both verbal and visual memory is also consistent with literature examining the development of basic numerical understanding in children. Children as young as four or five months can distinguish between small quantities (two versus three) and understand simple addition ($1 + 1 = 2$) and subtraction ($2 – 1 = 1$) problems presented in visual format (Starkey & Cooper, 1980; Wynn, 1992), indicating that the ability to visually conceptualize quantity is present early and pre-verbally in children. Between two- and three-years-of-age, children develop the ability to solve simple, nonverbal addition and subtraction problems (Huttenlocher, Jordan, & Levine, 1994). Huttenlocher et al. concluded that preschool children develop a mental model for numbers well before they are introduced to conventional knowledge (e.g., number words, counting, Arabic digits) and that as verbal information about numbers is acquired, it is mapped onto these mental models somewhat later. As children begin to speak, they begin to use number words as part of their vocabulary, to recite number words, to recall the correct order of number words, to link these verbal representations with specific visual quantities, and to accurately distinguish quantities (Krajewski, 2008, as cited in Krajewski & Schneider, 2009), all tasks which require the use of verbal and/or visuospatial memory. Critically, these early numerical abilities (sometimes termed *number sense*), which can involve linking verbal and visual math concepts, have been found to be predictive of math achievement and MDs (e.g., Gersten, Jordan, & Flojo, 2005; Krajewski & Schneider, 2009).

Interestingly, children are sometimes better able to solve problems presented in nonverbal form than as problems with digits and operation signs. Sherman and Bisanz (2009) compared the ability of grade two children to solve equivalence problems presented using math symbols (e.g., 5 + 3 = 2 + ___) and to their ability to solve similar problems using a nonsymbolic, visual presentation of the quantities involved. The children in this study were much more accurate when solving the nonsymbolic problems. Sherman and Bisanz further demonstrated that experience with problems in nonsymbolic form improved children's accuracy with the same type of problem presented using math symbols. These results seem to support Huttenlocher et al.'s (1994) conclusions about a nonverbal mental model which forms an essential foundation for the development of math calculation skills in children.

The body of literature that has examined strategy use in children engaged in math calculation also illustrates the importance of both verbal and visual skills. It is generally acknowledged that children have access to and can use a variety of strategies to solve math calculation problems. Initially, children often use various counting procedures such as counting on their fingers, counting nonverbally, and *counting on* from one of the numbers, but eventually they develop the ability to retrieve answers more directly from their long-term memory (Siegler & Shrager, 1984). Geary, Hoard, Byrd-Craven, & De Soto (2004) documented that elementary school-aged children with MD rely more heavily on counting verbally or with fingers than do typically achieving children who rely more on direct retrieval. Geary et al. also reported that the reliance of children with MD on these less mature strategies is linked to working memory capacity. While Geary et al. (2004) used a task that specifically assessed only the verbal component of the central executive, Robinson et al. (2002) proposed that difficulties with retrieval could result from inefficient phonological processing of math information *or* from challenges with conceptual processing of math facts (i.e., difficulties with number sense).

Limitations and Future Directions

The participants in the current study were children from all grades in elementary school (primary through grade six) who had a wide range of math, reading (phonological processing), working memory, and attention skills. Overall, the participants in this study performed in the broadly average range (with wide variability) on measures of reading, working memory, phonological processing, and attention problems. Thus, we were successful in recruiting children with both strengths and weaknesses in math, as well as in the other cognitive areas of interest. Overall, however, math skills were somewhat lower than performance on other measures. The overall mean standard score on the WJ-III Calculation subtest was approximately 90. While this score does fall in the broadly average range, it is toward the bottom of that range and therefore might be considered to be somewhat lower that would be expected, particularly given the scores on other measures. One explanation for this is that we may have over-recruited children with MD, possibly as a result of the inclusion of children with ADHD. We do not think this is the case, however, because the math scores of our sample seem to be representative of the math skills of children in Nova Scotia, Canada. Only 63% of grade 6 children in Nova Scotia met expectations on a recent province-wide math assessment (Nova Scotia Education Evaluation Services, 2010) and 13-year-old Nova Scotia students scored about half of one standard deviation below the Canadian average on math measures on the most recent Pan-Canadian Assessment (Council of Ministers of Education, Canada, 2008). This would correspond to an average standard score of approximately 92 which is very similar to our average. Hence, we consider our sample of children to be representative of the math skills of children in Nova Scotia. That said, it is possible that our results might not generalize to other populations of children.

It is also important to note that some studies (Alloway & Passolunghi, 2011; Meyer et al., 2010) have found that the pattern of importance of verbal and visuospatial abilities to math skills changes across grades. While our study included children in all elementary grades, there were insufficient numbers in each grade to allow for any analyses of developmental trends regarding the relationships between specific working memory components and math skills. Future research with the AWMA should include sufficient numbers of children across grade levels to enable a more detailed analysis of developmental patterns.

In our regression analysis, phonological processing was entered as a control variable with attention problems in step 2. Our rationale for this was that phonological processing has been suggested as a possible core deficit for math difficulties as well as reading difficulties and as a possible reason why children often have challenges with both of these academic areas. As well, previous research (e.g., De Smedt, Taylor, Archibald, & Ansari, 2010) has shown that phonological processing is related to math calculation. Entering phonological processing first allowed us to determine whether memory explained additional unique variance in math calculation scores. It could be argued, however, that entering phonological processing in this way provided a control for general verbal ability in the regression analysis and that no such control was employed to control for general nonverbal (visuospatial) ability. The ability of the visuospatial sketchpad to account for variance in math calculation scores could also be interpreted as signalling a role for visuospatial *ability* generally rather than for visuospatial *memory* specifically in math calculation. Including controls for both general verbal and nonverbal abilities in future research could provide clarity on this point.

In this study, we deliberately chose to focus on the relationship between general math calculation skill (as measured by the WJ-III Calculation subtest) and the components of Baddeley and Hitch’s working memory model. It would be interesting to determine whether the visuospatial sketchpad is also important in other types of math skills (word problems, geometry, interpretation of graphs) and for subtypes of math calculation skills (e.g., basic addition facts, basic multiplication facts, speeded fact retrieval, addition requiring carrying, subtraction requiring borrowing). This would be an important direction for future research.

General Conclusions

The results of this study point to an important role for visuospatial sketchpad (short-term visuospatial memory) in math calculation skills of elementary school-age children but also to a role for phonological processing in math calculation skills of children in this age range. These findings are consistent with the conclusions of a meta-analysis of the literature examining memory processes in children with MDs and nonverbal learning disabilities, with research literature that has examined the development of numerical understanding in children and of their use of strategies in solving math problems, and with theories about how numerical abilities develop in children and theories about causes of MD in children. As a result, it is important that those who assess children's math skills also include measures of visuospatial and verbal/phonological skills and that their recommendations focus on the use of teaching strategies that encourage children to strengthen areas of relative weakness and to maximize their ability to make use of compensatory strategies. It is quite easy to find suggestions about how to incorporate visuospatial strategies in math teaching (e.g., Rapp, 2009), and programs which provide a template for helping children integrate visual and verbal math information are commercially available (e.g., On Cloud Nine®; Bell & Tuley, 1997). Unfortunately, research on the *effectiveness* of these strategies is sparse and there is a general lack of information about effective intervention for those with MD (Gersten et al., 2005).

A recent meta-analysis of the literature which investigated interventions for children with learning disabilities in math, located only 42 studies that met criteria for inclusion (Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2009). Gersten et al. found evidence to support the use of explicit instruction along with the use of visual representations and student verbalizing of their reasoning processes. Interestingly, some research (e.g., Holmes, Gathercole, & Dunning, 2009) has also shown that an intervention designed to improve various aspects of memory can also result in improvements in math skills even though they were not specifically targeted by the intervention (See Klingberg, 2010, for a synthesis of this literature). Importantly, Gersten et al. (2005) specifically noted that it is unlikely that one type of approach will be effective for all children and Gersten et al.'s (2009) meta-analysis also found that providing ongoing (i.e., formative rather than summative) feedback to teachers and students about student progress and challenges is an important component of effective intervention.

The recognition of phonological processing as the core cognitive deficit in RDs has resulted in the development of numerous programs that have been shown to effectively teach specific phonemic skills (e.g., sound/symbol correspondences) to children and to improve reading skills. Number sense has been proposed by some (e.g., Jordan, 2007) as a potential core cognitive deficit for MD and the teaching of number sense specifically has been shown to improve math skills (e.g., Fuchs, Fuchs, & Karns, 2001). However, Robinson et al. (2002) suggested that number sense is a skill that can be learned (analogous to the specific phonemic skills that can be taught) rather than an intrinsic cognitive process itself and they suggest that "...one place to look for intrinsic processing weaknesses that might interfere with the acquisition of basis number sense would be in that family of cognitive-processing skills that are based primarily in the right hemisphere of the brain" (p. 87). Consistent with this, the results of the current study indicate that visuospatial short-term memory (the visuospatial sketchpad) may be the core cognitive deficit for math calculation skills. It is important to note, however, that this study was not longitudinal in design and it is therefore not possible to conclude that there is a causal link between difficulties with math and the visuospatial sketchpad. Future research should employ a longitudinal design to examine the suitability of specific interventions targeted to improving children's visuospatial short-term memory and/or their use of these memory skills to develop number sense and to learn about math.

Footnotes

[1] The sample size (N = 94) was adequate based on the recommendation of 10-15 participants per predictor. With six predictors (independent variables), a sample size of 60-90 participants would be needed. All variables were continuous, quantitative variables with non-zero variance. An examination of outliers in the dependent and independent variables revealed that all cases fell within three standard deviations of the mean; however, some cases fell more than two standard deviations from the mean. The differences between means and trimmed means were all less than one point. Hence, any outliers had minimal effects. With respect to normality, only one Kolmogorov-Smirnov test was significant indicating that the scores for Attention Problems were not normally distributed. It could be argued that difficulties with attention are not normally distributed in the population (i.e., that most children do not have difficulty with attention and a few do) and that the skewed distribution present in the data accurately reflects the underlying nature of this characteristic and represents what would be seen in the population. As a result, the data were not transformed. With respect to multicolinearity, an examination of correlations between the all variables revealed no variables with correlations greater than .9 (largest r = .761). All independent variables of interest were significantly correlated with the dependent variable (range from r = - .356 to r = .538). All Tolerance values were greater than .10, with (lowest Tolerance = .760) and all VIF (variance inflation factors) were less than 10 (largest VIF = 1.316). Casewise diagnostics indicated no standardized residuals greater than 2.5. Hence, all cases are within 2.5 standard deviations of prediction by the model. The maximum Mahalanobis Distance = 9.739. With df = 6, the

critical value = 22.458. The maximum value for Cook's Distance = .079. (Cases with values > 1 are problematic).

[2] Grade-based rather than age-based norms were used. Math curricula are grade-based, but children enter school based on reaching a certain age by a certain date. Hence, children in any grade range across one year or more in age. As a result, it was determined that using age-based scores could unfairly penalize children who were among the oldest in a grade and also unfairly advantage children who were among the youngest.

Table 2.1

Descriptive and Group Difference Statistics for ADHD and non-ADHD Groups

	Group			
	ADHD	Non-ADHD		
	N = 27	N = 67	*t*	*p*
Age	9 years: 5 months (1 year: 3 months)	8 years: 11 month (1 year :7 months)	1.48	.14
IQ (standard scores)				
M (SD)	100.5 (12.4)	102.9 (13.0)	.81	.42
			χ^2	*p*
Gender				
Male	22	30	10.49	.001
Female	5	37		
Math				
Scored above 90	5	40	13.08	< .001
Scored below 90	22	27		
Grade				
P	0	5	9.36	.15
1	3	9		
2	3	13		
3	7	10		
4	4	18		
5	9	9		
6	1	3		
Family Income				
< 20 000	1	4	2.28	.89
20 001 – 30 000	0	4		
30 001 – 40 000	2	5		
40 001 – 50 000	3	6		
50 001 – 60 000	3	8		
60 000 – 70 000	4	9		
70 000 +	14	28		

Table 2.2

Descriptive Statistics (N = 94) for Academic and Cognitive Variables

	Mean (*SD*)	Range
Age (months)	9 yrs 1 mo (1 yr 6 mos)	6 yrs 0 mos – 11 yrs 8 mos
Estimated IQ	102.2 (12.8)	80-135
Calculation	90.1 (16.0)	57-135
Reading	98.0 (16.7)	61-148
Memory		
Phonological Loop	100.3 (16.6)	64-141
Visuospatial Sketchpad	104.3 (18.7)	57-138
Verbal Central Executive	92.6 (16.5)	60-143
Visuospatial Central Executive	96.7 (17.7)	59-147
Phonological Processing	95.2 (15.0)	57-139
Attention Problems (T-scores)	58.3 (13.4)	41-90

Note. All scores are standard scores except where indicated; Calculation = Calculation subtest of WJ-III; Reading = Letter-Word Identification subtest of the WJ-III; Phonological Loop = AWMA Verbal Short Term Memory Composite; Visuospatial Sketchpad = AWMA Visuospatial Short-Term Memory Composite; Verbal Central Executive = AWMA Verbal Working Memory Composite; Visuospatial Central Executive = AWMA Visuospatial Working Memory Composite; Phonological Processing = CTOPP Phonological Awareness Composite; Attention Problems = CPRS-R DSM-IV Inattention Subscale.

Table 2.3

Correlations Between Math, Age, and Cognitive Variables

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1) Calculation	-						
(2) Age	-.36**	-					
(3) Phonological Processing	.54**	-.26*	-				
(4) Attention Problems	-.44**	.25*	-.33**	-			
(5) Phonological Loop	.53**	-.12	.62**	-.45**	-		
(6) Verbal Central Executive	.49**	-.02	.59**	-.39**	.66**	-	
(7) Visuospatial Sketchpad	.42**	.18	.30**	-.31**	.44**	.63**	-
(8) Visuospatial Central Executive	.44**	.08	.42**	-.34**	.49**	.76**	.70**

Note. ** $p < .01$; * $p < .05$.

Table 2.4

Stepwise Regression for Math Calculation

	B	*SE*	β	*t*
Step 1				
Age	-.31	.08	-.36	-3.65***
Step 2				
Age	-.16	.08	-.19	-2.15*
Phonological Processing	.43	.10	.41	4.58***
Attention Problems	-.30	.11	-.25	-2.86**
Step 3				
Age	-.26	.07	-.31	-3.55***
Phonological Processing	.23	.11	.22	2.01*
Attention Problems	-.13	.11	-.11	-1.21
Phonological Loop	.16	.11	.17	1.47
Verbal Central Executive	-.03	.14	-.03	-.19
Visuospatial Sketchpad	.21	.10	.25	2.16*
Visuospatial Central Executive	.09	.12	.10	.79

Note. R^2 = .126 for Step 1; ΔR^2 = .268 for Step 2; ΔR^2 = .102 for Step 3; * = $p < .05$; ** = $p < .01$; *** $p < .001$.

CHAPTER 3:

ADDING IT UP: THE ROLE OF WORKING MEMORY IN MATH CALCULATION SKILLS IN UNIVERSITY STUDENTS

Melissa McGonnell, Penny Corkum, Shannon Johnson, & Joan Backman

The manuscript based on this study is presented below in Chapter 2. The reader is advised that Melissa McGonnell developed the research hypotheses, research methodology, and approach to data analysis for this study. She was responsible for all participant recruitment, data collection, and data entry (with assistance from research assistants). She completed all of the background research for this manuscript and was responsible for all aspects of the writing processes. She received editorial feedback from her dissertation committee members. This paper will be submitted for peer review.

Abstract

Math skills are important in today’s society, but math difficulties (MD) are common in university students and can affect their choice of degree program and career. Our understanding of the processes that contribute to MD in university students is limited, but working memory has been implicated. The Automated Working Memory Assessment Battery (AWMA) was used to investigate which component of Baddeley and Hitch’s (1974) model of working memory was most related to math calculation skills in university students. Participants were 42 (26 female) university students (*M* age = 20 years, 9 months; *Range* = 16 years, 6 months to 22 years, 11 months). Overall, younger students scored higher on the math calculation measure. Math calculation scores were correlated with three of the four working memory components (phonological loop, visuospatial sketchpad, verbal central executive), but in a regression analysis, the visuospatial sketchpad explained the most variance in math calculation scores. Short-term visuospatial memory should be assessed in students having difficulty with math and math instruction should incorporate explicit visuospatial strategies.

Estimates of the prevalence of learning disabilities in adults vary widely. In 2006 in Canada, approximately 2.5% of those 15 years-of-age and older self-identified as having a learning disability (Statistics Canada, 2007). Prevalence estimates at post-secondary institutions in the United States range up to as high as 10% (Vogel, Leonard, Scales, Hayeslip, Hermansen, & Donnells, 1998) and more and more students with learning disabilities are attending university[1] (Kirby, Silvestri, Allingham, Parrila, & La Fave, 2008). In a recent survey of university students' self-reported learning difficulties, difficulties with math concepts ranked second, after difficulties with writing but before problems with reading, which ranked third (Rachal, Daigle, & Rachal, 2007). In Canada, the results of the 2003 International Adult Literacy and Skills Survey indicated that more than half (55%) of Canadians 16 years-of-age and older had numeracy skills that were below the level of competence necessary for coping with the demands of today's society (Statistics Canada, 2005).

Difficulty with math can have many consequences for university students. Most obviously, math difficulties could influence their choice of courses in university and/or result in students having trouble completing the requirements for their degree (McGlaughlin, Knoop, & Holliday, 2005), but poor math literacy has other far-reaching implications for adults in today's society. Good math skills are necessary for coping with the level of today's technology and have been linked to better chances of employment and to higher paying jobs (Shapka, Domene, & Keating, 2006). Those who experience academic success also achieve higher levels of occupational satisfaction and are healthier (Hazell, 2007; McDonough, Sacker, & Wiggins, 2005). In contrast, those who experience educational challenges have a higher incidence of substance abuse and of mental and physical health difficulties (Mun, Windle, & Schainker, 2008; Redmond & Hosp, 2008).

Despite the frequency of reported difficulty with math and its importance for success in school and beyond, there has been considerably less research focus on math difficulties (MD) as compared to difficulties with reading (Geary, Hamson, & Hoard, 2000; Robinson, Menchetti, & Torgesen, 2002). The efforts of researchers in the field of reading disabilities have resulted in a wide acceptance of word-level reading as the core measureable behavioural deficit and of phonological processing as the primary associated cognitive deficit. As a result of research investigating reading disabilities, early identification has become more common and we have also been able to develop effective interventions for reading difficulties. Unfortunately, the same level of agreement has not been reached with regard to the core behavioural deficit and/or associated deficient cognitive process for math and thus, our ability to identify MDs and implement effective interventions also lags behind.

A Core Behavioural Deficit for Math

Geary (2007) noted that there are a number of biologically based *primary* math abilities (e.g., the ability to quickly discriminate a quantity of two from a quantity of three) but that the vast majority of math abilities (what he termed *secondary* abilities) are not acquired until formal math instruction is begun at school age. Similarly, our ability to distinguish the sounds of language and to speak are biologically driven primary abilities while our ability to read is a secondary ability we acquire only with formal instruction. Logically, then, we should seek to identify a core behavioural deficit for math amongst the secondary math abilities as researchers have done with reading. Difficulty with developing rapid and automatic knowledge of single-digit math facts (e.g., 9 + 2 = 11; 7 x 8 = 56) has been identified as one possibility (Jordan, Hanich, & Kaplan, 2003a; Robinson et al., 2002). The rationale for choosing this skill includes the fact that those with MD commonly have problems with fact mastery (Swanson & Jerman, 2006) and these difficulties are often persistent across time (Jordan et al., 2003a). Additionally, fact mastery has been described as fundamentally necessary to the development of higher level math skills (Robinson et al., 2002). Geary (1993) noted that basic computational skills, involving both fact mastery as well as computational procedures, had been identified as a core behavioural deficit by researchers in individuals with both acquired (as a consequence of brain injury) and developmental MD. Given that the current study examined math skills in university students, we chose to use an instrument that would measure broad math calculation skills (including questions that require the use of computational procedures) rather than only fact mastery. Despite the fact that there is some agreement about an appropriate core behavioural deficit for MD, many researchers have continued to focus on other areas of math such as applied math skills/word problems (e.g., Miller & Bichsel, 2004), mental calculation (e.g., Noël, Désert, Aubrun, & Seron, 2001), or algebra (e.g., Tolar, Lederberg, & Fletcher, 2009). Although it is also important to gather information about what factors contribute to these higher-order math skills, the fact that math skills are defined in a wide variety of ways in the research literature makes it difficult to draw general conclusions across studies.

A Core Cognitive Deficit for Math

The concentrated effort dedicated to the investigation of reading difficulties has led to the conclusion that phonological processing is the most common core cognitive deficit in those who have difficulty developing effective word-reading skills. The debate as to which cognitive process or processes are connected to the development of math skills is ongoing. It has been theorized that phonological processing might also be related to challenges with learning math (Hecht, Torgesen, Wagner, & Rashotte, 2001). Challenges with attention and/or a diagnosis of Attention Deficit/Hyperactivity Disorder (ADHD) have also been found to be associated with general academic underachievement (e.g., Barry, Lyman, & Klinger, 2002; Todd, Sitdhiraksa, Reich, Ji, Joyner, Heath, & Neuman, 2002) and with MDs specifically (Fuchs, Compton, Fuchs, Paulson, Bryant, & Hamlett, 2005). A meta-analysis of the literature examining the association between ADHD and academic achievement found that achievement in reading, spelling, and math continues to be negatively impacted in adults with ADHD (Frazier, Youngstrom, Glutting, & Watkins, 2007). More specifically, inattentive (but not hyperactive) symptoms of ADHD have been found to be associated with achievement (i.e., GPA) in college students (Frazier et al., 2007). Many researchers (e.g., Fletcher, 2005; Raghubar, Barnes, & Hecht, 2010) have stressed the importance of continuing to investigate the role attention plays in MDs. Unfortunately, most studies investigating cognitive processes involved in MDs with adults fail to include measures of phonological processing or attention.

The Role of Working Memory

Although there is little agreement about a core cognitive deficit associated with MD, many researchers have focussed on the role of working memory and most of this research has been predicated on Baddeley and Hitch's multicomponent model (Baddeley, 2002; Baddeley & Hitch, 1974; Baddley & Hitch, 1994). This model proposes that working memory is not a single construct but that it is best conceptualized as having three components. The first is the central executive (verbal and visuospatial working memory) which controls the maintenance and switching of attention and the ability to spread attention across multiple tasks. This system can also access information from other systems and is associated with two other components (sometimes referred to as slave systems). These are the phonological loop (verbal short-term memory) and the visuospatial sketchpad (visuospatial short-term memory). The phonological loop is theorized to contain two components: a phonological store, which can hold information for approximately two seconds, and the articulatory rehearsal system, which allows us to rehearse information verbally to preserve it in memory for a little longer. The visuospatial sketchpad is the system that allows us to retain, manipulate, and connect visual and/or spatial information that is obtained from our senses or accessed from long-term memory.

In adult populations, the majority of research examining the association between the components of Baddeley and Hitch's working memory model and math skills has used dual-task methodology. In this paradigm, a participant is asked to complete a math task while simultaneously completing a second task designed to engage concurrent processing or to increase memory load. If performance on the math task decreases as the demands of the second task increase, the interpretation is that both tasks require use of the same cognitive resource. Conflicting results have emerged from studies using this paradigm. Some researchers (e.g., De Rammelaere, Stuyven, & Vandierendonck, 2001; Lemaire, Abdi, & Fayol, 1996) have concluded that the central executive is most critical to math skill. Others (e.g., Imbo, & Vandierendonck, 2007; Seitz & Shumann-Hengsteler, 2000) have implicated both the central executive and the phonological loop.

A number of factors have contributed to these seemingly contradictory findings. These include the fact that researchers operationalize the core behavioural deficit for math in a variety of ways (as previously mentioned), but another difficulty arises from the fact that studies do not always include measures of all components of Baddeley and Hitch's model. For example, no task loading on the visuospatial sketchpad was included by Lemaire et al. (1996), Imbo and Vandierendonck (2007), or De Rammelaere et al. (2001). This omission is unfortunate given that a number of studies (e.g., Cirino, Morris, & Morris, 2002; Cirino, Morris, & Morris, 2007; Greiffenstein & Baker, 2002; Osmon, Smerz, Braun, & Plambeck, 2006) have emphasized the importance of visuospatial abilities to math skills. It is also important to note that, while dual-task studies have a number of strengths in terms of the degree of experimental control they are able to maintain over the type of math skill under scrutiny and the degree of load of the secondary memory task, this methodology sacrifices the ability to generalize results to typical applications of math skills in everyday life and to the types of math tasks commonly required in the classroom.

An additional problem with interpreting the results of dual-task studies is that performance on the experimental math tasks has typically been reported, but many studies have not included information about scores on standardized math measures (e.g., De Rammaelaere, et al., 2001; Imbo & Lefevre, 2010). Hence, it is difficult to determine whether results are generalizable to individuals across a wide range of math abilities.

Correlational and Group Differences Studies

Studies that investigate MD in adults using other methodologies are relatively rare. Only seven studies (Hecht, 2006; McGlaughlin et al., 2005; Miller & Bichsel, 2004;

Noël et al., 2001; Prevatt, Welles, Li, & Proctor, 2010; Tolar et al., 2009; Wilson & Swanson, 2001) were located. Some studies compared math and memory skills in typically achieving adults to those of adults with MD and others used correlational methods to examine the relationship between the various components of working memory and math skills. Unfortunately, the results of these studies are also somewhat contradictory. There are a variety of possible explanations for these conflicting results including the use of different populations, varying operational definitions of math skills, and which working memory components were included. There was also some variability in the terms used to refer to the components of Baddeley and Hitch's working memory model. For the purpose of describing these studies in a consistent manner, tasks which require simple storage of information have been considered to measure short-term memory (the phonological loop or the visuospatial sketchpad), while tasks that require both storage *and* processing or manipulation of information have been considered to measure the central executive.

The participants in most studies were university/college students (Hecht, 2006; McGlaughlin et al., 2005; Noël et al., 2001; Prevatt et al., 2010; Tolar, et al., 2009) or a mix of college students, college graduates, and other adults with some or no college education (Miller & Bichsel, 2004); however, Wilson and Swanson (2001) included both children and adults (ages ranging from 11 to 52 years). No study measured phonological processing skills, although Wilson and Swanson (2001) did include descriptive information about the reading skills of their participants. One study (McGlaughlin et al., 2005) measured symptoms of ADHD using the Conners' Adult ADHD Rating Scale and found that their participants with and without math difficulties did not differ on this measure.

The seven studies operationalized math skills using a variety of measures including all four math subtests (Prevatt et al., 2010) from the Woodcock-Johnson Tests of Achievement (WJ; Woodcock, McGrew, & Mather, 2001), only the Calculation (Hecht, 2006) or Calculation and Applied Problems (Miller & Bichsel, 2004) subtests from the WJ, the Math Fluency subtest of the WJ (McGlaughlin, et al., 2005), the math subtest of the Wide Range Achievement Test-Revised (Wilson & Swanson, 2001), mental calculation of single-digit addition and multiplication problems (Hecht, 2006) or of three-digit addition problems (Noël et al., 2001), and measures of algebra skill (Tolar et al., 2009).

With respect to working memory, two of these seven studies included measures of verbal working memory only. Hecht (2006) investigated the phonological loop alone and Prevatt et al. (2010) measured the verbal central executive alone. Both were found to be related to a variety of math skills. Four of the studies (McGlaughlin et al., 2005; Miller & Bichsel, 2004; Noël et al., 2001; Wilson & Swanson, 2001) included measures of both visuospatial and verbal memory. McGlaughlin et al. combined one measure of the verbal central executive and one measure of the visuospatial sketchpad to obtain a general working memory measure, which was found to be related to math skills. This makes it impossible to differentiate the importance of these two components of working memory. Two other studies (Wilson & Swanson, 2001; Noel et al., 2001) concluded that their results suggested that verbal memory was more related to math skills than was visuospatial memory. Wilson and Swanson (2001) found that both the visual-spatial (authors' term) and the verbal central executive were related to math skills but noted that the relationship between math skills and the verbal central executive was stronger. Of interest is the fact that verbal central executive measures employed by Wilson and Swanson in this study were very demanding in terms of language processing but were not correlated with a measure of word reading skills which ranged from the average to the well above average level in their sample. This is a somewhat puzzling finding and leads to some question about the appropriateness of these verbal central executive measures. Noël et al. (2001) found that both the phonological loop and visual-spatial span (the visuospatial sketchpad) were negatively correlated with the number of errors their participants made. However, manipulating the visual similarity of digits in visually presented math problems had no effect on accuracy or reaction time while increasing the phonological similarity of digits increased reaction time and decreased accuracy, leading Noël et al. to conclude that the phonological loop was more implicated in participants' ability to solve mental calculation problems.

Several other studies have concluded that visuospatial memory is more highly associated with math skills than verbal memory. The primary focus of Miller and Bichsel's (2004) study was the relationship between anxiety and math skills; however, they determined that while both the visual (authors' term) and the verbal central executive were related to math skills, the relationship between math skills and the visual central executive was slightly stronger. Tolar et al. (2009) examined a number of models of algebra achievement which only included measures of the verbal (not visuospatial) central executive but also included two measures of what they termed 3D spatial visualization. These measures required participants to recall previously viewed items in order to perform the tasks correctly. As such, these tasks could be considered to be measures of the visuospatial central executive as they required participants to hold and manipulate visuospatial information to complete the tasks. Tolar et al. concluded that the verbal central executive was more strongly related to computational fluency (i.e., timed math calculation skills) than to algebra skills and posited that the verbal central executive also influenced the early development of 3D spatial visualization abilities; however, in their sample of university students, they found that 3D spatial visualization mediated the effect of the verbal central executive on algebra achievement and concluded in general that 3D spatial abilities have the most direct effect on the ability to solve complex math problems.

Taken together, the findings from the four studies that utilized measures of both verbal and visuospatial memory point to the relationship between visuospatial memory and the math calculation skills of university students (adults) being stronger than the relationship between verbal memory and these math skills. Two of the four studies (Miller & Bichsel, 2004; Tolar et al., 2009) reached this conclusion directly. The other two studies (Noël et al, 2001; Wilson & Swanson, 2001) concluded that verbal memory had a more important role to play, but characteristics of the participants (average to above average reading skills; Wilson & Swanson, 2001), the memory tasks (Wilson, & Swanson, 2001), and the math task used (complex mental math; Noël et al., 2001) point to other possible explanations for the association found between verbal memory and math skills.

While there are a number of reasons (as described above) for the somewhat contradictory findings of this very limited body of research, one of the primary difficulties is related to the fact that no study investigated Baddeley and Hitch's full working memory model, a problem also noted in the dual-task literature, and three studies did not include any measures of visuospatial working memory. Additionally, several of these studies used single tasks, although the use of multiple tasks has been recommended (e.g., Geary et al., 2000; Martinussen & Tannock, 2006) and, across studies, tasks with very different demands were used, at least in theory, to measure the same working memory component. For example, Miller and Bichsel (2004) used a paper-folding task (requiring participants to choose the correct unfolded version of a previously folded and hole-punched visual image) to measure the visuospatial central executive, while Wilson and Swanson (2001) used both a Visual Matrix (requiring participants to reproduce the location of dots in a matrix) and a Mapping and Directions task (requiring participants to reproduce a route drawn on a map) to measure the same working memory component.

A Measure for Baddeley and Hitch's Full Working Memory Model

In an effort to develop one instrument that would reliably assess all aspects of Baddeley and Hitch's working memory model, Pickering and Gathercole (2001) created the Working Memory Test Battery for Children (WMTB-C). A computer-based pilot-version of the WMTB-C, the Automated Working Memory Test Battery (AWMA; Alloway, Gathercole, & Pickering, 2004) has been used in research for a number of years. This pilot-version was normed for use with children younger than twelve. More recently, an updated version of the AWMA (Alloway, 2007) has been published. This version retains the same twelve tasks as the original version of the AWMA but is normed for use with individuals up to the age of 22 and includes subtests which measure the verbal and visuospatial working memory components of Baddeley and Hitch's model.

The Current Study

The primary goal of the current study was to examine how the components of Baddeley and Hitch's model of working memory are related to math calculation skills in university students. Difficulties with math calculation are common and under-investigated in this population and math calculation has been identified as a good target behaviour for theoretical development. We utilized the AWMA, a standardized measure based on Baddeley and Hitch's model which includes multiple measures of each working memory component, and included measures of attention problems and phonological processing as these cognitive abilities have yet to be included together in studies investigating the role of working memory in calculation skills in university students. Based on the findings of previous research, we predicted that the two visuospatial working memory components of Baddeley and Hitch's model (the visuospatial sketchpad and the visuospatial central executive) would be more related to math calculation skills than would the two verbal working memory components. We also predicted that this relationship would remain after accounting for variance from other cognitive abilities (phonological processing and attention problems).

Method

Participants

Participants (N = 42) for this study were recruited from the university population in five post-secondary institutions (four universities and one degree granting college) in Halifax, Nova Scotia, Canada using posters placed around the campuses and through an online experimental participation system at one university. All participants received an individual feedback report about their performance on the math and working memory measures in the study. Participants recruited through the online experimental participation system also received course credit. All participants were university (or degree granting college) students under 23, spoke English as their first language, and did not have any known general medical or neurological conditions.

Measures

Academic Measures.

Math Calculation and Word Reading. The Calculation and Letter-Word Identification subtests from the *Woodcock-Johnson Tests of Achievement – 3rd Edition* (WJ-III; Woodcock, et al., 2001) were used to assess math and reading skills. The WJ-III is a widely used, comprehensive, standardized measure of academic achievement normed for use with anyone over the age of 2 years. The one-year test-rest reliabilities range from .75 to .89. Validity estimates are also adequate as the WJ-III achievement clusters for reading and mathematics show correlations ranging from .56 to .69 with the reading and mathematics composite scores on the Wechsler Individual Achievement Test – Revised.

On the Calculation subtest, individuals were asked to solve mathematics problems on paper. These began simply (1 + 3) and gradually became more difficult, asking individuals to multiply, divide, and to solve problems with fractions and algebra. Testing concluded when individuals made six consecutive errors. The Letter-Word Identification subtest required individuals to read a list of real words beginning with words which were simple for individuals to read and continued until the individual made six consecutive errors.

Cognitive Measures.

Working Memory. The *Automated Working Memory Assessment Battery* (AWMA; Alloway, 2007) was used to assess working memory. The AWMA is a computer-based assessment of working memory that contains 12 subtests which assess all four components of working memory described in Baddeley and Hitch's model. Test-retest reliability estimates range from .69 to .90. The AWMA can be used with individuals ranging in age from 4 to 22 years.

The phonological loop (verbal short-term memory) is assessed with three subtests. On the *Digit Recall* subtest, the individual verbally recalls sequences of digits of gradually increasing length. The *Word Recall* and *Nonword Recall* subtests follow the same format with individuals hearing and being asked to verbally recall sequences of words and nonsense words respectively.

The visuospatial sketchpad (visuospatial short-term memory) is assessed with three subtests. On the *Dot Matrix* subtest, the individual is shown sequences of a red dot appearing in a four by four matrix and demonstrates recall of the sequence of positions by tapping the squares on the computer screen. On the *Mazes Memory* subtest, the individual sees a maze with a red path drawn through it and after a three second delay, uses his/her finger to trace the path just seen on the computer screen. On the *Block Recall* subtest, the individual first sees a video showing a series of blocks being touched and is asked to reproduce the sequence in the correct order on the blocks shown on the computer screen.

The central executive is assessed using six subtests. Three of these subtests assess the verbal central executive. On the *Listening Recall* subtest, the individual is read a series of sentences one at a time and indicates whether each is true or false. After all sentences of one trial are read, the individual recalls the final word of each sentence in the correct order. On the *Counting Recall* subtest, the individual counts the number of circles in a series of arrays of circles and triangles and then has to recall the tally of numbers in the series in the correct order. On the *Backwards Digit Recall* subtest, individuals verbally recall each sequence of digits presented in backwards order. Three other subtests are used to assess the visuospatial central executive. On the *Odd-One-Out* subtest, three shapes, each in a box, are presented in a row, and the individual identifies the odd-one-out shape. At the end of each trial, the individual indicates recall of the order of the location of each odd-one-out shape by tapping the sequence in boxes on the screen. On the *Mister X* subtest, the individual sees two Mister X figures and indentifies whether the Mister X with the blue hat is holding the ball in the same hand as the Mister X with the yellow hat. The Mister X with the blue hat may also be rotated. At the end of each trial, the individual points to the location of each ball in the correct order. On the *Spatial Span* subtest, the individual sees two shapes side by side. The shape on the right, which is sometimes rotated, has a red dot above it and the individual must indicate whether this shape is oriented in the same or opposite direction as the shape on the left. At the end of each trial, the individual points to the location of each red dot, in the correct order, on a picture with three compass points.

One adaptation was made to the AWMA for this study. During pilot testing, it was noted that individuals were distracted by the British accent of the female voice on some of the verbal subtests (particularly Nonword Recall). To avoid possible confounding effects, the instructions and items for five verbal subtests (Digit Recall, Word Recall, Nonword Recall, Listening Recall, and Backwards Digit Recall) were recorded using a female voice with a local accent. Subtests were administered in the same order and participants' responses were recorded manually.

Phonological Processing. The *Comprehensive Test of Phonological Processing* (CTOPP; Wagner, Torgesen, & Rashotte, 1999) is a widely used measure of phonological awareness and rapid naming which is normed for use with individuals up to the age of 24 years. It has good reliability (most internal consistency coefficients exceed .80; test-retest coefficients range from .68 to .97) and validity (correlations with the Woodcock Reading Mastery Test-Revised range from .58 to .73). On the Elision subtest, individuals were asked to repeat words after adding or deleting sounds (e.g., Say *cup*. Now say *cup* without saying /k/.) On the Blending Words subtest, individuals were presented with groups of sounds and asked to state what word to the sounds they heard make (e.g., /s/ - /un/ would require the individual to say *sun*). Scores from these two measures were combined to obtain a phonological awareness composite score.

Attention. The *Adult ADHD Self-Report Scale* (ASRS v1.1; Kessler et al., 2005) was designed as a quick screening instrument for use clinically and in research. It contains 18 questions which reflect the 18 symptoms of ADHD in the Diagnostic and Statistical Manual for Mental Disorders – 4th Edition: Text Revision (DSM-IVTR; APA, 2000). Individuals are asked to rate how frequently (on a 5-point Likert scale ranging from *Never* to *Very Often*) they have experienced each symptom presented in the past six months. This scale cannot be used to diagnose ADHD but was used to obtain descriptive information about self-reported symptoms of difficulty with attention. A score for attention was created by totalling the number of critical symptoms (i.e., the first six symptoms on the scale) individuals rated in the clinical (as defined by this measure) range.

Descriptive Measures.

Estimated IQ. The Wechsler Adult Intelligence Scale – Third Edition (WAIS-III; Wechsler, 1997a) is a widely used measure of cognitive ability for individuals over 16. Two subtests (Vocabulary and Block Design) of the WAIS-III were used to estimate general cognitive ability. This procedure has been shown to be a highly reliable (r = .916) and valid (r = .874) means of estimating IQ (Sattler, 2008).

Demographic Information. A demographic questionnaire was used to obtain descriptive information about the participants including age and family income.

Procedure

When students arrived for their research appointment, the study was explained and consent was obtained. All data was collected individually in one session lasting between 2 ½ and 3 hours. Students first completed the AWMA and then the math and reading subtests of the WJ-III followed by the two subtests of the CTOPP. The two subtests of the WAIS- III were then administered. Finally, students were asked to complete the ASRS, and the demographic information form.

Results

The 42 participants in this study (26 female and 16 male) represented a broad cross-section of the university population. Students ranged in age from 18 years, 6 months to 22 years 11 months (*M* = 20 years, 9 months; *SD* = 1 year, 5 months). Participants were recruited from across attendance years (first year: n = 8; second year: n = 11; third year: n = 10; fourth year: n = 11; fifth year: n= 2) and were registered in a variety of degree programs (Bachelor of Arts: n = 19; Bachelor of Science: n = 16; Bachelor of Commerce: n = 4; Bachelor of Engineering: n = 1; Bachelor of Fine Arts: n = 2). Over half of the students (62%) indicated that at least one math credit was required to complete their degree. Estimated IQ ranged from 90 to 143 (*M* = 120.6; *SD* = 13.1) and all participants' word reading skills were within one standard deviation of average (Range = 87-115). All but two participants scored within one standard deviation of average on the phonological processing measure (*M* = 102. 2; *SD* = 9.6; Range = 79-115). The mean number of symptoms endorsed in the critical range was 2.6 (*SD* = 1.9; Range 1-6) on the ASRS with 17 participants (40.5%) endorsing 4 or more symptoms in the critical range. Socio-economic status was estimated using participants' report of the household total annual income. Participants indicated the range of their family's total income using a 7-point Likert scale divided in $10,000 increments from 1 (*up to $20,000 per year*) to 7 (*more than $70,000 per year*). The median income level was 7 (more than $70,000 per year). Participants in this study demonstrated a wide range of math calculation and memory skills. Group means were all in the average range. (See Table 3.1 for complete statistics.)

Pearson's correlations were used to examine relationships between math calculation skills, age, and the cognitive variables (four working memory components, attention problems, and phonological processing) of interest (see Table 3.2). Math calculation scores were significantly correlated with three of the four AWMA working memory components (phonological loop, verbal central executive, and visuospatial sketchpad). The correlation between math calculation scores and the visuospatial central executive approached significance (*p* = .053) Math calculation scores were not significantly correlated with either phonological processing or with attention problems. There was a significant negative correlation between math calculation scores and age indicating that in our sample, older students scored more poorly than younger students. All four working memory components were significantly correlated with one another and all except the visuospatial sketchpad were significantly correlated with phonological processing. Attention problems were not significantly correlated with any of the working memory components or with phonological processing.

Next, a hierarchical stepwise regression was used to determine which working memory component was most related to math calculation skills. All relevant assumption checks were conducted.[2] Age was entered in the first step and the three working memory variables (phonological loop, visuospatial sketchpad, verbal central executive) that were correlated with math calculation scores were entered in step two. Phonological processing and attention variables were not included in this analysis as they were not significantly correlated with scores for math calculation or any of the working memory components. Both steps were significant (see Table 3.3). In the first step, age explained a significant amount (11%) of unique variance in math calculation scores (p = .04). In step two, the only working memory component to emerge as a significant predictor was the visuospatial sketchpad which explained an additional 10% of the unique variance in math calculation scores. The contribution of age was no longer significant (p = .06) in this step.

Discussion

The purpose of this study was to investigate the role that working memory plays in math calculation skills in university students. The results indicated that while three of the four components (phonological loop, verbal central executive, visuospatial sketchpad) of Baddeley and Hitch’s (1974) working memory model were significantly correlated with math calculation skills, consistent with our prediction the visuospatial sketchpad was the working memory construct most strongly associated with math calculation scores in our sample of university students. We had also predicted that the visuospatial central executive would predict significant unique variance in math calculation scores, but this was not the case in our sample. No other cognitive process investigated (i.e., neither phonological processing nor attention) was found to be related to math calculation scores in our sample.

The Role of Working Memory

The results of our correlational analyses indicate that math calculation skills are generally related to both verbal and visuospatial memory and that both short-term memory and the central executive are important. This finding is consistent with theories about the development of number skills (e.g., Clark & Campbell, 1991) and about how we process numbers (e.g., Dehaene, 1992). In general, these theories propose that numerical information can be coded and recalled either verbally or visually and that over time, these two systems can learn to interact causing verbal presentations to activate visual representations and vice versa. As a result, weaknesses in either system can cause difficulties with math. Our results are also consistent with findings of brain imaging research which has found a complex network of brain regions including both left- (i.e., verbal) and right- (i.e., visuospatial) lateralized systems to be involved in math calculation in adults (e.g., Zago, Petit, Turbelin, Andersson, Vigneau, & Tzourio-Mazoyer, 2008).

At first glance, our results also seem to point to a particular importance for the visuospatial sketchpad (i.e., short term visuospatial memory). This finding could be related to the importance of visuospatial skills in conceptualizing math knowledge generally and higher-order (i.e., fractions, algebra, linear equations) math concepts in particular. In order for the university students who participated in this study to score in the average range on the math calculation task, answering a number of higher-order math questions (e.g., problems with fractions, basic algebra, exponents, and integers) correctly was necessary. Most participants were not math majors and anecdotally commented that it had been some time since they had worked on problems of this sort. Therefore, their ability to correctly solve these problems supports the contention that these students had a strong conceptual understanding of the math involved rather than the notion that they were simply employing rote memory to recall a simple algorithm. Students with strong visuospatial skills may have better ability to develop these conceptualizations and therefore to retain math knowledge over longer periods of time, even when it is not practiced regularly.

The specific importance of the visuospatial sketchpad to math skills is consistent with Rourke and Strang's (1978) finding of a specific visual-spatial weakness in children who had impaired math skills coupled with good reading and spelling skills, later described as having nonverbal learning disabilities (e.g., Harnadek & Rourke, 1994; Rourke, 1993). Geary (1993) proposed a visuospatial deficit as one of the three theoretical cognitive deficits that could result in math disabilities. More recently, Geary (2010) noted that research findings have been mixed with respect to substantiating the existence of this specific deficit. Geary posited that this could be due to the fact that math disabilities resulting from a specific visuospatial deficit were rarer (and therefore difficult to detect in research samples) or to the fact that the math tasks employed in studies did not require strong visuospatial skill; however, another possible explanation for the mixed evidence is the simple fact that visuospatial tasks have been included in research with much less regularity than have verbal tasks. To illustrate, of the seven, non dual-task adult studies we located, only one (Noël et al., 2001) included an isolated measure of the visuospatial sketchpad. The strength of the relationship between the visuospatial sketchpad (i.e., visuospatial short-term memory) and math skills in the current study points to the importance of ensuring that visuospatial as well as verbal tasks are included in future research.

Our findings did not support the predicted role for the visuospatial central executive in math calculation skills in our sample of university students. Our prediction for a role for the visuospatial central executive was based on the findings of Miller and Bischel (2004), Tolar et al. (2009), and Wilson and Swanson (2001); however, none of these studies included a measure of the visuospatial sketchpad. Hence, it is possible that what these studies found was a relationship between math skills and the *storage* function (akin to the visuospatial sketchpad) of the visuospatial central executive rather than with both the storage and manipulation functions.

It has been suggested, however, that the visuospatial sketchpad (i.e., short-term visuospatial memory) and the visuospatial central executive are not, in fact, dissociable constructs as they are both strongly related to attentional control (e.g., Miyake, Friedman, Rettinger, Shah, & Hegarty, 2001). Miyake et al. suggested that it might be appropriate to reconceptualize theoretical models of working memory to better account for visuospatial memory as a singular construct. If this conceptualization is correct, our results would be better considered to indicate a role for visuospatial memory in general rather than for the visuospatial sketchpad in particular.

Anecdotally, it is important to note that many participants in this study clearly recruited verbal memory resources to assist with their performance of the visuospatial tasks of the AWMA. (e.g., sub-vocalizing *words* to help recall *locations*). The utilization of such integrated or cross-domain strategies is not surprising, but it does make the interpretation of our results more complex. It is possible, for example, that the individuals in our study who did well on the tasks used to assess the visuospatial sketchpad were able to do so because they possessed strong ability to integrate visuospatial and verbal information and that it is this ability to integrate information across domains that enabled them to develop very good math skills. This interpretation is consistent with theories about numerical processing (e.g., Clark & Campbell, 1991; Dehaene, 1992) and with the results of brain imaging studies (e.g., Zago, et al., 2008). A definitive conclusion is beyond the scope of the current investigation but should be the focus of future research.

Attention and Phonological Processing

In our sample, math calculation scores were not significantly correlated with either attention or phonological processing. This is somewhat surprising as attention (Fuchs et al., 2005; Shalev, Manor, & Gross-Tsur, 2005) and phonological processing (Geary, 1993, Robinson et al., 2002) have been found to be related to math skills in children.

With respect to attention, there seem to be three possible explanations for this finding. Firstly, it is possible that the measure used (ASRS) was not sensitive enough to detect attention difficulties in our sample; however, the ASRS has been used to substantiate current symptoms and subtype university students with ADHD (e.g., Gropper and Tannock, 2009), so this does not seem to be the most probable explanation. A second possibility is that the score used in our analyses (i.e., the six critical item screening score) did not specifically detect attention problems because it included only four items that ask about inattentive symptoms and also two items that ask about hyperactive-impulsive symptoms of ADHD. To investigate this possibility, a total score for the nine inattentive items on the ASRS was calculated. This score was not significantly correlated with math calculations scores (p = .56), making this second explanation also unlikely. Finally, this result may simply be related to the fact that attention is not related to math calculation skills (as measured by the Calculation subtest of the WJ-III) in university students.

With respect to phonological processing, most participants (90.5%) scored in the average or above average range on this measure (CTOPP) and all but one participant had a reading score in the average or above average range. This contrasts with the much wider range (standard scores ranging from 74-134) of math calculation scores in our sample. It is not surprising therefore, that phonological processing was not related to math calculation skills. This indicates that phonological processing is unlikely to be the core cognitive deficit for math difficulties; however, it does not preclude the possibility that in those with reading difficulty, challenges with phonological processing could also contribute to math difficulties. Given that the vast majority of individuals in our sample had intact reading skills, this possibility could not be investigated in our study, but it should be the focus of future research.

Limitations and Future Directions

As the participants in the study were all university students who had generally good reading and attention skills, it is important to note that the results may not generalize to other adult populations particularly those who are older, less educated, or who have difficulty with reading or attention. Before definitive conclusions about the role of the visuospatial sketchpad in math calculation skills can be made, it will be important to examine the role of working memory in math calculation in these populations as well as in samples which include high school students and post-university populations (i.e., adults who have been away from school and environments which require the practise of paper and pencil math skills).

Based on the results of this study, the relationship noted between math calculation skills and the visuospatial sketchpad can only be said to apply to broad paper and pencil based math calculation skills. It is possible that other facets of working memory are more important for other types of math calculation. Future research should focus on examining the role of all components of working memory and other cognitive processes in mental math calculation and to the fluency of math calculations. As noted above, future research should also include tasks which require the integration of both verbal and visuospatial skills to enable an analysis of whether it is the ability to fluidly cross informational domains that actually contributes most significantly to the development of math skills.

Implications for Teaching and Assessment

While it is important to note that the results of this study indicate a relationship between current math calculation skills and visuospatial abilities, the fact that this study was not longitudinal in design means that it is not possible to determine the exact role that the visuospatial sketchpad plays in the acquisition of math calculation skills. That said, the findings of this study do point to the importance of encouraging students to use and strengthen visuospatial skills as they learn about math. This should not, however, be interpreted as an endorsement of the practice of simply using visuospatial math activities (e.g., diagrams, number lines, and base-ten or other manipulative materials) in the classroom. It cannot be assumed that simple exposure to visuospatially based math information will develop other math skills or that students with relatively weak visuospatial abilities will be able to connect these activities with other types of mathematics information or different math tasks such as paper and pencil calculation. For example, Wilson, Dehaene, Dubois, and Fayol (2009) demonstrated that an intervention designed to help children develop better number sense (i.e., understanding of the meaning of numbers and how they relate to one another) by training rapid connection of visually presented arrays of objects and digits did enhance that skill, but it did not affect other similar skills (e.g., the ability to determine which of two visually presented arrays contained more objects). The participants in this study were all children experiencing difficulty with math. It is possible that those with good visuospatial skills would show broad benefits from exposure to visuospatial math activities. For example, Sherman and Bisanz (2009) demonstrated that grade two children who gain experience with complex calculation problems using manipulatives (i.e., in a visual format) also improve their ability to solve the same type of problem presented using math symbols. It is possible that those with less well-developed visuospatial skills would have more difficulty with this type of activity and also experience difficulty connecting them to other sorts of math activities such as paper and pencil computation. Consequently, it will be important to make deliberate and explicit connections between visuospatial information and oral or written math and to recognize that how these connections are best made will vary to some degree from individual to individual.

Robinson et al. (2002) suggested that interventions with those having difficulty with math should be predicated on the source of difficulty. Determining the source of the challenge with math is, therefore, an important initial stage in designing an effective intervention. Given the results of this study, it will be important to include an evaluation of visuospatial skills and of working memory in assessments for math difficulties because information about strengths or weaknesses in these skills will be important for explaining specific challenges with math and for planning effective remediation.

Footnotes

[1] In Canada, the term *college* is not used synonymously with the term *university* as is often the case in the United States. Therefore in this article, the term *university* refers to both Canadian universities and American colleges and universities.

[2] The sample size (N = 24) was adequate based on the recommendation of 10-15 participants per predictor. With four predictors (independent variables), a sample size of 40-60 participants would be needed. All variables were continuous, quantitative variables with non-zero variance. An examination of outliers in the dependent and independent variables revealed that all cases fell within three standard deviations of the mean; however, one case in one variable fell more than two standard deviations from the mean. The differences between means and trimmed means were all less than one point. Hence, any outliers had minimal effects. With respect to normality, only two Kolmogorov-Smirnov tests were significant indicating that the scores for Attention Problems and Age in Months were not normally distributed. It could be argued that difficulties with attention are not normally distributed in the population (i.e., that most individuals do not have difficulty with attention and a few do) and that the skewed distribution present in the data accurately reflects the underlying nature of this characteristic and represents what would be seen in the population. As a result, the data for this variable were not transformed. The age range of the participants in this study was somewhat restricted (18 years and 6 months to 22 years and 11 months) as they were all undergraduate students. Consequently, the non-normality of the distribution of age is considered to accurately reflect the nature of the population and this variable was not transformed. With respect to

multicolinearity, an examination of correlations between the all variables revealed no variables with correlations greater than .9 (largest r = .649). All independent variables of interest were significantly correlated with the dependent variable (range from r = - .323 to r = .361). All Tolerance values were greater than .10, with (lowest Tolerance = .772) and all VIF (variance inflation factors) were less than 10 (largest VIF = 1.296). Casewise diagnostics indicated no standardized residuals greater than 2. Hence, all cases are within 2 standard deviations of prediction by the model. The maximum Mahalanobis Distance = 4.687. With df = 4, the critical value = 18.467. The maximum value for Cook's Distance = .154. (Cases with values > 1 are problematic).

Table 3.1

Descriptive Statistics (N = 42) for Academic and Cognitive Variables

	Mean (*SD*)	Range
Math Calculation	102.1 (17.3)	74-134
Reading	97.6 (6.3)	87-115
Memory		
Phonological Loop	99.9 (15.2)	69-133
Visuospatial Sketchpad	96.5 (11.5)	73-118
Verbal Central Executive	99.5 (18.2)	72-131
Visuospatial Central Executive	99.2 (12.3)	78-134
Phonological Processing	102.2 (9.6)	79-115
Attention Problems (number critical symptoms)	2.6 (1.9)	0-6

Note. Math Calculation = Calculation subtest of WJ-III; Reading = Letter-Word Identification subtest of the WJ-III; Phonological Loop = AWMA Verbal Short Term Memory Composite; Visuospatial Sketchpad = AWMA Visuospatial Short-Term Memory Composite; Verbal Central Executive = AWMA Verbal Working Memory Composite; Visuospatial Central Executive = AWMA Visuospatial Working Memory Composite; Phonological Processing = CTOPP Phonological Awareness Composite; Attention Problems = ASRS number of critical symptoms rated in clinical range; All means except Attention Problems represent standard scores.

Table 3.2

Correlations Between Math, Age, Working Memory, Attention, and Phonological Processing Measures

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1) Math Calculation	-						
(2) Age	-.32*	-					
(3) Phonological Processing	.14	-.004	-				
(4) Attention Problems	.07	-.13	-.26	-			
(5) Phonological Loop	.34*	-.09	.45**	-.24	-		
(6) Verbal Central Executive	.32*	-.38*	.50**	-.15	.65**	-	
(7) Visuospatial Sketchpad	.36*	-.14	.07	.02	.36*	.34*	-
(8) Visuospatial Central Executive	.30	-.17	.34*	-.03	.47**	.64**	.45**

Note. * $p < .05$; ** $p < .01$.

Table 3.3

Hierarchical Regression for Math Calculation Skills

	B	*SE*	β	*t*
Step 1				
Age	-.34	.16	-.32	-2.16*
Step 2				
Age	-.29	.15	-.28	-1.93
Visuospatial sketchpad	.49	.22	.32	2.23*

Note. R^2 = .11 for Model 1; Δ R^2 = .10 for Model 2; * $p < .05$.

CHAPTER 4: DISCUSSION

This chapter focuses on summarizing the findings from the two studies (Chapters 2 and 3) to determine which component of Baddeley and Hitch's model of working memory, as measured by the AWMA, best predicts math calculation skills. This question was examined in elementary school-aged children (Chapter 2) and in university students (Chapter 3). Hence, it is important to examine the results of these two studies in relation to each other to enable some comment on the developmental nature of the association between working memory and math calculation skills. This discussion also includes comment on the implications for intervention with individuals who have difficulty acquiring math skills and concludes with a discussion of the utility of the AWMA as a research tool.

Which Component of Baddeley and Hitch's Model of Working Memory Is Most Associated with Math Skills?

The Automated Working Memory Assessment Battery (AWMA; Alloway, 2007) was used in the studies in this dissertation to measure the components of working memory as defined by Baddeley and Hitch's model (Baddeley & Hitch, 1974). The broad goal of both studies was to determine which working memory component was most associated with math calculation skills. The focus was on math calculation skills because this facet of math has been identified, from both practical and theoretical perspectives, as a likely core behavioural deficit for math difficulties (akin to word reading for reading difficulties). Consistent with predictions based on previous literature, the results of the two studies (contained in Chapters 2 and 3) indicated that the visuospatial sketchpad is the memory component which is most associated with math calculation skills in elementary school-aged children and in university students.

In elementary-school aged children, all four working memory components (phonological loop, visuospatial sketchpad, verbal central executive, and visuospatial central executive) were correlated with math calculation scores, but after accounting for variance due to age and phonological processing, the visuospatial sketchpad emerged as the only memory component to account for unique variance in math calculation scores. In university students, three of the four memory components (the phonological loop, the visuospatial sketchpad, and the verbal central executive) were significantly correlated with math calculation scores and the correlation with the visuospatial central executive approached significance. In the regression analysis, the visuospatial sketchpad again emerged as the only memory component to account for unique variance in math calculation scores. These findings seem to point to a general role for memory in learning math skills and to a particular importance for the visuospatial sketchpad, or visuospatial memory broadly, in acquiring these skills. This finding is consistent with theories about the acquisition of math skills.

Theoretical Importance of the Visuospatial Sketchpad

The importance of the visuospatial sketchpad is supported by theories about numerical abilities (e.g., Dehaene, 1992) and math disabilities (e.g., Geary, 1993). Dehaene believed that we manage numerical information using three codes. The auditory verbal code is used when we say or write number words, such as *seven* and the visual Arabic code is used when we write digits, such as *7*, but it is the analogue magnitude code, which Dehaene described as an internal number line (a visuospatial construct), that is used when we reason about math and understand quantity. Hence, in Dehaene’s view, it is the visuospatial representation of quantity (on a number line) that is central to our ability to understand math. Geary (1993) also identified visuospatial skills as being central to fundamental understanding of math. He noted that impaired visuospatial abilities had the potential to interrupt math abilities at both a functional (e.g., by hampering the ability to line up digits for multi-digit paper and pencil computations) and an abstract (e.g., by hindering the ability to conceptualize quantity) level. Geary described three possible subtypes of math disability, one of which was visuospatial. He indicated that this type of math disability would be behaviourally demonstrated as difficulty with spatial representation of numbers (e.g., difficulty lining up digits for multi-digit calculations) and place value error. Given the findings of the two studies in this dissertation, the effects of visuospatial skills could be more pervasive than predicted by Geary in 1993 and interestingly, Geary has more recently (Geary, 2010) described the relationship between working memory and math as more complex than originally anticipated.

Developmental Importance of the Visuospatial Sketchpad

Nonverbal, or visuospatial, skill has been shown to be fundamental to the development of early math skills. Wynn (1992) demonstrated that even infants understand quantity. Huttenlocher, Jordan, and Levine (1994) showed that between the ages of 2 and 3 years, children develop the ability to solve simple addition and subtraction problems when the presentation and the response are in nonverbal format. Huttenlocher et al. proposed that children first develop a nonverbal mental model for numbers which is then connected with math language as verbal skills develop and with the symbols of math (e.g., digits and operation signs) as math instruction begins, often at school entry.

These early nonverbal math abilities are sometimes termed *number sense*. Number sense is the early developing ability to conceptualize quantity and is what Geary (2007) described as a primary mathematical competency (i.e., an ability that is biologically based and emerges without specific, formal instruction). As conceptualized by Dehaene (1992), number sense is strongly related to visuospatial ability and number sense has recently been identified as an important foundation in the development of math skills (e.g., Jordan, Glutting, & Ramineni, 2010).

An early number sense skill is the ability to subitize, a term first proposed by Kaufman, Lord, Reese, and Volkmann (1949) to describe the ability to quickly (and without counting) identify the numerical quantity of a small (i.e., less than seven) group of items. It is this ability that lets us determine that all of the groups of items represented in Figure 4.1 represent the numerical quantity of five, regardless of the shape of the objects, the organization of objects, the proximity of the objects, or the colour of the objects. To be able to recognize all these arrays as representing the quantity of five, one has to employ both the visual (i.e., pattern recognition) and the spatial (i.e., location recognition) abilities which are part of the visuospatial sketchpad (Della Sala, Gray, Baddeley, Allamano, & Wilson, 1999; Logie & van der Meulen, 2008). Hence, the very concept of number seems to be visuospatial in nature.

At a more pragmatic level, visuospatial skills are necessary in the perception of written digits (see Figure 4.2). When digits are placed immediately adjacent to one another as in the top line of illustration 4.2 a), we automatically perceive them as the numeral twenty-three; however, if the digits are gradually moved apart, our perception changes to the point where on the bottom line, we perceive two separate numerals, two and three. Illustrations 4.2 b) and 4.2 c) demonstrate the necessity of visuospatial skills for lining up multi-digit problems appropriately so that we can correctly execute complex paper and pencil calculations. Visuospatial skills also seem to be employed in the recognition of patterns (i.e., groups of three digits) when we read longer multi-digit numbers (illustration 4.2 d).

In terms of correctly completing paper and pencil math calculation problems, it might be expected that weaker visuospatial abilities would lead to more visuospatially based errors (e.g., overcrowding numbers, misaligning numbers) in those with math difficulties. Raghubar, Cirino, Barnes, Ewing-Cobbs, Fletcher, and Fuchs (2009) tested this possibility in grade 3 and 4 children with and without math and/or reading difficulties. Frequency of visuospatially based errors was not related to math achievement but was related to reading achievement. It may be that visuospatially based errors are a relatively rare type of problem experienced by those with math difficulties and/or that visuospatially based errors are common only in a subgroup of individuals with both math and reading difficulties. This would be an interesting area for future research. Raghubar et al. noted that no cognitive measure of working memory was included in their study and that it would be interesting to examine the relationship between visuospatially based errors and working memory.

While the studies in this dissertation focussed solely on math calculation skills, visuospatial abilities have been shown to be related to the ability to apply math calculation knowledge to problem solving in children with a broad variety of math achievement levels (van Garderen, 2006). This could be interpreted as indicating that visuospatial abilities continue to be important as children develop the ability to apply early knowledge to new situations.

Incorporating Visuospatial Information in Instruction

Based on the results of the two studies which comprise this dissertation and on an examination of rudimentary skills and abilities necessary to work with numbers and digits, it is only possible to conclude that there is a relationship between individuals' current math calculation skills and their short-term visuospatial memory, not that poor visuospatial memory *caused* difficulties with math. That said, it seems logical to conclude that poor visuospatial skills will make it more difficult to learn about mathematics. Hence, it will be important to attend to visuospatial abilities and to ensure that visuospatial information and the building of associated skills are included in mathematics instruction. At a basic level, this can be achieved by incorporating the use of visuospatial materials (i.e., manipulatives, diagrams, number lines) in the process of learning and teaching about math. Ideally, these experiences encourage children to make connections between digits and quantity as well as between the step-by-step procedures that are used in many math calculations (e.g., long division) and the reasons why these steps are necessary.

Interventions which employ a wide variety of visuospatially based strategies have been shown to be effective at teaching math concepts in general. The use of manipulatives in a tutoring program has been shown to improve children's performance on measures of math computation and applications of math including word problems (Fuchs et al., 2005). A visual demonstration of the inverse relationship between addition and subtraction, which involved manipulative materials, has also been shown to be effective at improving children's ability to apply this knowledge to more complex problems (e.g., 7 – 2 + 2; Nunes, Bryant, Hallett, Bell, & Evans, 2009). Liang and Sedig (2010) demonstrated that the use of a computerized spatial visualization tool improved the scores of students in grades three through twelve on a geometry test and also had positive effects on qualitative and quantitative measures of their engagement in the process of learning. In general, it seems that encouraging the use of visuospatial abilities and strategies is of benefit to math learning.

Visuospatial Strategies and Math Difficulties

The use of visual strategies (including concrete manipulatives, number lines, and diagrams) was one of the instructional components identified as effective in a recent meta-analysis of the literature examining math instruction for students with learning disabilities (Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2009). It is important to note, however, that while the use of visuospatial strategies is important, simply providing exposure to or the opportunity to use manipulatives, diagrams, and/or number lines will not necessarily assist those who are having difficulty learning about math. It is also necessary to attend to the way in which these strategies are employed. Generally, those having difficulty with learning require explicit instruction in the classroom environment. The importance of explicit instruction was identified in the Gersten et al. (2009) meta-analysis and by Fuchs, Fuchs, Powell, Seethaler, Cirino, and Fletcher (2008) as one of six principles necessary for instruction to be effective.

Explicit instruction occurs when teachers directly provide specific, step-by-step instruction about the topic at hand and then monitor and encourage students' use of this information. Fuchs et al. (2008) particularly noted that while many students benefit from discovery-based or constructivist learning, students who struggle with math require more explicit instruction. In the case of the use of visuospatial strategies then, it will be important for teachers to provide explicit connections between the manipulatives (or number lines or diagrams) and the math concepts in question. Including an instructional component that makes these connections explicit has been shown to be an important part of effective intervention (e.g., Dowker, 2001).

Connected with the concept of the importance of explicit instruction is the idea that those with relative weakness or impairment in the visuospatial system may find it extremely difficult to benefit from instruction provided using this modality. Hence, they could fail to benefit even though explicit explanations of the connections between the concepts and procedures illustrated by visuospatial strategies are provided. Assuming that the provision of visuospatially-based instruction will assist with the learning of math could be just as damaging to the development of math skills as ignoring this form of instruction altogether. It will be important to consider individual differences and recognize that some individuals will require a highly scaffolded approach (i.e., the gradual addition of small amounts of new information coupled with intensive practice) and that others may need help developing compensatory mechanisms (e.g., verbally-based acronyms such as BEDMAS to assist with remembering order of operations) to cope with relatively weak visuospatial abilities.

A Developmental Perspective

The fact that the visuospatial sketchpad emerged as the working memory component that best predicted math calculation skills in both elementary school-aged and university students speaks to its potential to be of central importance in the development of these skills. It is important to note, however, that the visuospatial sketchpad was not the only factor that was found to be related to math calculation skills in the studies in Chapters 2 and 3. In both elementary school-age children (Chapter 2) and university students (Chapter 3), age was significantly negatively correlated with math calculation scores. This indicates that older participants in both studies scored less well than younger participants. In children, this finding could be considered an artefact of the measure itself in that there may be a floor effect as, for example, children in grade primary only need to be able to add one to another digit to score well. This would not seem to be a reasonable explanation for the same finding in the population of university students who had all completed high school and had been exposed to all the concepts on the Calculation subtest of the WJ-III. It should be acknowledged that in order to score in the average range, university students would only have to correctly complete problems involving fractions, decimals, percent, and integers (as well as more basic problems), problems that continue to be practiced in the activities of daily life. However, in order to score *very well* on the math calculation measure, students would, at a minimum, have to correctly solve complex algebra problems and possibly problems with logarithms, trigonometry, and matrices. It is possible that the older university students had not taken math regularly in several years while the younger university students were more recent high school graduates and had, therefore, had more recent practice with the more complex problems resulting in higher scores. Whatever the explanation, in both populations, math calculation was more difficult for older students. This finding seems to be consistent with the results of population incidence studies (e.g., Barbaresi, Katusic, Colligan, Weaver, &

Jacobsen, 2005) which have found that the incidence of math difficulties increases with age.

There were other differences between the results of the two studies as well. In university students, attention problems were not significantly correlated with calculation scores while these two measures were significantly negatively correlated in children. This finding could be the result of the fact that there were a significant number of children with ADHD included in the child study and only one individual with this (self-reported) diagnosis amongst the university student participants. It could also be related to the fact that different measures were utilized in the two studies, parent-report in the child study and self-report with the university students. It is also possible, however, that attention is less related to math calculation skills as we get older, or that math calculations become less cognitively demanding with more practice, so attention matters less. Research has shown that in those with typically developing math abilities, strategies used to solve simple problems change with age from those that are more active, more verbal, and more attentionally demanding (e.g., counting on fingers or counting verbally) to simple retrieval from long-term memory, an act that requires much less sustained attention (Geary, Brown, & Samaranayake, 1991).The fact that attention is not related to calculation skills in university students could help to explain the fact that their math calculation scores were not significantly correlated with scores for the visuospatial central executive since the central executive functions are currently conceptualized as having primarily attentional functions including focussing attention, dividing attention, and controlling task switching (Baddeley, 2002).

A final difference between the results of the elementary school-age and university student studies has to do with the role of phonological processing. Phonological processing was not significantly correlated with math calculation skills in university students and was therefore not included in the regression analysis. Phonological processing was significantly correlated with math calculation scores in the child study and continued to account for a unique proportion of the variance in math calculation scores even after the visuospatial sketchpad was added to the regression in step 3. It seems then that phonological processing, and by extension the phonological loop, may have a role to play in the early development of math calculation skills. This conclusion is consistent with the findings of Alloway and Passolunghi (2011) and Meyer, Salimpoor, Wu, Geary, & Menon (2010) who found evidence for a switch from stronger reliance on the phonological loop in second grade to the visuospatial sketchpad in grade three.

This finding is not consistent in the literature, however. In preschoolers, the phonological loop (Noël, 2009), visuospatial sketchpad (Rasmussen & Bisanz, 2005; Simmons, Singleton, & Horne, 2008), and the central executive (Noël, 2009; Vukovic and Siegel, 2010) have been implicated. In early grades, the phonological loop (Fuchs et al., 2005; Passolunghi, Mammarella, & Altoè, 2008; Rasmussen & Bisanz, 2005) and the visuospatial sketchpad (Gathercole & Pickering, 2000) have been found to be important. In later grades, the central executive and visuospatial sketchpad (Holmes, Adams, & Hamilton, 2008) have been found to be important. As noted in Chapter 1, there are a number of possible explanations for the lack of consensus in the literature to date. One strong possible explanation relates to inconsistencies in the tasks used to measure the various components of Baddeley and Hitch's model of working memory. The studies in

Chapters 2 and 3 used the AWMA, a measure which represents a possible antidote to the vagaries present in the literature at the moment. What follows is a discussion of the utility of the AWMA as a research tool.

The AWMA as a Tool to Measure Working Memory

The two studies in this dissertation utilized the Automated Working Memory Assessment (AWMA; Alloway, 2007) to measure Baddeley and Hitch's components of working memory (Baddeley & Hitch, 1974). The AWMA contains twelve subtests, three which measure each of the four working memory components (phonological loop, visuospatial sketchpad, verbal central executive, and visuospatial central executive). Researchers employ a wide variety of measures to assess working memory and, as noted in Chapter 1, this can make it challenging to compare results across studies. The AWMA seems to present a possible alternative to the somewhat disorganized approach that has been used to identifying working memory measures. As such, a comparison of the subtests of the AWMA to measures commonly used in research over the past decade is provided below.

Measures of the Phonological Loop.

The phonological loop is typically measured with serial recall tasks that require individuals to repeat a sequence of verbally presented items. Most commonly, these items consist of digits and words. The AWMA's three subtests (Digit Recall, Word Recall, and Nonword Recall) are just such tasks.

Measures of the Visuospatial Sketchpad.

The visuospatial sketchpad is commonly assessed with tasks that have a primarily visual (e.g., Corsi span) or primarily spatial (e.g., pattern span) orientation. The AWMA contains two Corsi span tasks, Block Recall and Dot Matrix, which primarily assess visual components of the visuospatial sketchpad. The third subtest is Mazes Memory. On this task, individuals are shown a red line representing the path through the maze for a short time. Then, they are asked to trace the same path on the computer screen with their finger. This task seems to have a primarily visual component (i.e., the task is to recall the pattern of the path), but it could also be argued that it has a spatial component as there is a dynamic, location-based aspect to the response required. The AWMA does not contain a pure pattern span task.

Measures of the Verbal Central Executive.

There are three common task types that are used to assess the verbal central executive and the AWMA contains one of each type. The first task type is a backwards span task where individuals hear a sequence of items (commonly words and digits) and are asked to repeat them in backwards order. The AWMA task is Backwards Digit Span. The second type of task is based on the reading span test first devised by Daneman and Carpenter (1980). The AWMA Listening Recall subtest is a task of this type. The third commonly used verbal central executive measure is typically based on the Counting Span Test (Case, Kurland, & Goldberg, 1982). The AWMA Counting Span subtest is a computerized version of this task which has visual stimuli but requires verbal counting (the processing component) and verbal storage.

Measures of the Visuospatial Central Executive.

The visuospatial central executive tasks identified in the literature primarily consisted of tasks that could be used to assess the visuospatial sketchpad with the addition of a verbal processing component. Two exceptions were tasks that employed a backwards span approach to a Corsi blocks task and a task that required individuals to maintain spatial information while conducting a spatial search so as to not return to search in the same location twice. The AWMA tasks are somewhat different from these measures. All of the AWMA tasks (Spatial Span, Mr. X., and Odd-One-Out) require that the individual distinguish one visual stimulus from others in some way while at the same time storing spatial (location) information to be recalled later. The Odd-One-Out task requires that individuals make visual distinctions between different shapes as the processing task while the other two subtests require individuals to make judgements about spatial orientation.

Conclusions about the AWMA's Utility as a Research Tool.

The AWMA provides access to measures that are commonly used by researchers to assess the phonological loop, the visuospatial sketchpad, and the verbal central executive. The tasks used by the AWMA to measure the visuospatial central executive are more novel. In addition, the computerized administration format of the AWMA helps to ensure standardized administration. This is particularly important in measures that assess working memory because increased time (e.g., to present items) can result in extra memory load and reduced performance. If this load is not standardized across participants (e.g., if different experimenters read lists of digits at different rates or the same experimenter reads lists differently on different days), the validity of the scores and the research conclusions could be questioned.

In addition, the AWMA provides normative scoring for all twelve subtests and for each of the working memory components assessed. Knowledge gained about individuals who participate in research is not always provided to the individual participants and, in reality, would not always be useful to them. For example, there would be no benefit to providing information about raw scores to individuals as there is no context for interpreting whether they did well or poorly on the task. The norm-referenced, standard scores available from the AWMA represent the opportunity to provide valuable information to individuals, parents, and/or educators about areas of relative strength and weakness amongst the components of Baddeley and Hitch's model of working memory. Working memory deficits are difficult to detect with simple observation and there are few tools available to specifically assess working memory in the classroom (Alloway, Gathercole, Kirkwood, & Elliot, 2008). Instead, individuals must wait, often for very long periods of time, to receive formal assessment. Researchers using the AWMA, however, could obtain data that is useful for both the study in question and the individual participants.

The commercially available version of the AWMA comes with a booklet titled *Understanding Working Memory: A Classroom Guide*, which provides a beginning level of information about working memory and its implications for instruction. Additionally, in the United Kingdom, teachers can purchase the AWMA directly from the publisher and so have direct access to the information it can provide. The AWMA cannot be purchased directly by teachers (without additional qualifications) in North America.

The AWMA does not include a pure short-term visual memory task (i.e., a pattern span task) amongst the subtests assessing the visuospatial sketchpad. This could be viewed as a weakness of this measure given that the fractionation of the visuospatial sketchpad is currently being considered (Baddeley, 2003; Darling, Della Sala, & Logie, 2007; Della Sala et al., 1999). The measures included in the AWMA to assess the visuospatial central executive are quite different from the tasks located in a review of the literature of the past decade. The AWMA's tasks have both visual and spatial storage demands but more purely spatial processing components. This could also be viewed as a weakness of this instrument; however, given that most of the tasks located in the literature demand *verbal* processing, the AWMA could also be considered to represent an advance in the measurement of the visuospatial central executive.

General Conclusions and Future Directions

The finding of a strong relationship between the visuospatial sketchpad and math calculation skills across two very different age groups (elementary school-aged children and university students) speaks to the general importance of short-term visuospatial storage in the learning of math skills. As a result, appropriate incorporation of visuospatial strategies (e.g., diagrams, number lines, and manipulatives) in the teaching of math is strongly recommended. It is important, however, that those using such strategies utilize them while bearing in mind the recommendations of Fuchs et al. (2008) and Gersten et al. (2009) about the equal importance of explicit instruction.

While the results of the studies in this dissertation speak to the general importance of the visuospatial sketchpad to math skills, the results do not allow for a determination of whether or not individuals with relatively strong and relatively weak short-term visuospatial memory would benefit differently from the use of such strategies. Future research should address this question by assessing math skills and visuospatial short-term memory, providing explicit math instruction with developmentally- and curriculum-appropriate visuospatial components (e.g., number lines, manipulatives, diagrams), and assessing progress with math skills along with the relationship of this progress to short-term visuospatial memory.

The fact that the AWMA was administered many times, with many individuals across a broad range of ages and abilities during the course of collecting the data reported in Chapters 2 and 3 afforded the opportunity to amass a considerable amount of anecdotal information about how participants approached the tasks of the AWMA. In general, it was noted that participants often used verbal strategies to assist with visuospatial tasks (e.g., using the words *left*, *centre*, and *right* subvocally to assist with recall of location on the Odd-One-Out subtest) and sometimes used visualization to assist with verbal tasks (e.g., visualizing digits or locations on a telephone keypad to assist with the Backwards Digit Span task). An interesting direction for future research would be to investigate strategies individuals employ to complete tasks and whether those who are better able to integrate their verbal and visuospatial memory skills also do better on memory tasks and/or have better math skills. This would require interviewing individuals about their approaches to the tasks and evaluating the relationship between self-reports of the integration of verbal and visuospatial strategies with performance on working memory measures.

It is notable that there seems to be more agreement about what tasks best tap verbal than visuospatial components of working memory. This is particularly true with respect to tasks that tap the visuospatial central executive. Research directed at developing and testing the value of such tasks is recommended.

In conclusion, the information presented in this dissertation provides insight into the importance of the visuospatial sketchpad (visuospatial short-term memory) to math calculation skills and about the usefulness of the AWMA as a research tool. It is hoped that this information can be used by educators and researchers to help further our knowledge about how individuals learn about math and how to help those who find math difficult. While math is of central importance in today's society, the fact remains that functional numeracy is not as valued as is functional literacy. It would be somewhat challenging, although not impossible, to find individuals who would happily admit that reading had always been difficult for them or that they could never manage reading in school, but it is relatively easy to find people who readily confess to disliking math, to never being able to *do* math in school, and to having found math instruction boring or incomprehensible. It is also possible to find abundant evidence of the difficulties individuals face with everyday math skills. This is easily noticed on any shopping trip during which one might encounter individuals who cannot calculate sales prices or determine which product is the best buy and sales people who are unable to calculate correct change if they accidentally tender an incorrect payment on their cash register. Good teaching about math skills is necessary to remedy this situation. Good teaching requires understanding of and enthusiasm about math on the part of those teaching about it. Research tells us that good teaching should also include thoughtful and individualized instruction and the findings of this dissertation indicate that this instruction should definitely incorporate visuospatial approaches.

Figure 4.1

Possible Visual Representations of the Concept of Five

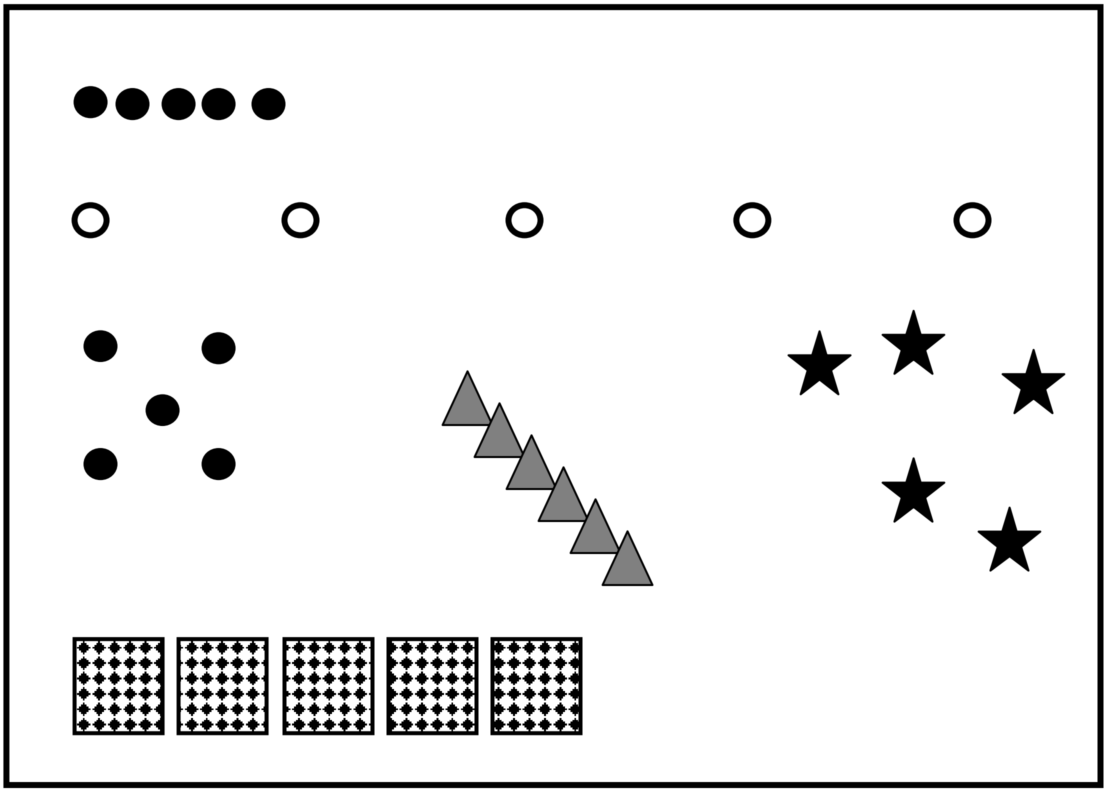

Figure 4.2

Illustration of the Necessity of Spatial Skills for the Perception of Written Numerals

a) 23
$2\;3$
$2\quad 3$
$2\qquad\qquad\qquad 3$

b)
$$\begin{array}{r} 456 \\ \underline{+\,235} \end{array}$$

c)
$$\begin{array}{r@{}cccc} & & 2 & 3 & 8 \\ \cline{2-5} 12\,) & 2 & 8 & 5 & 6 \\ & \underline{2} & \underline{4} & & \\ & & 4 & 5 & \\ & & \underline{3} & \underline{6} & \\ & & & 9 & 6 \\ & & & \underline{9} & \underline{6} \\ & & & & 0 \end{array}$$

d) 197 432

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