MARITIME SEARCH AND RESCUE VESSELS LOCATION-ALLOCATION PROBLEM WITH A COST-EFFECTIVENESS MEASURE

by

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To my lovely family.
For their pure love, endless support, inspiration, and encouragement throughout my studies.

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ABSTRACT

The East Coast of Canada experiences roughly a thousand maritime incidents per year, many requiring search and rescue (SAR) response from the Canadian Coast Guard (CCG). We developed a location-allocation model to examine different options for improving the SAR coverage by repositioning some vessels across two operational seasons in Atlantic Region. The criteria include the capital and operating costs, insufficiency probability, primary and backup coverage, and the effectiveness of a response resource in addressing a particular kind of incident. The data comprises maritime incidents that occurred over the years 2014-2016 drawn from the CCG SISAR database, information on the CCG vessel characteristics, costs, operational limitations, and expert opinion on their respective capabilities. The results present the trade-off between cost and coverage, and the effectiveness of the response as a function of the vessel capabilities and allocations. A comparison with the current fleet configuration is made and sensitivity analysis examines alternatives arrangements.

LIST OF ABBREVIATIONS USED

ABC	Artificial Bee Colony
ACP	Advanced Care Paramedic
CAF	Canadian Armed Forces
CASARA	Civil Air Search and Rescue Association
CCG	Canadian Coast Guard
CCGA	Canadian Coast Guard Auxiliary
CGFO	Coast Guard Fleet Order
CPI	Consumer Price Index
DEvS	Discrete Event System
Dew	Dewatering
DF	Direction Finder
DFO	Department of Fisheries and Oceans
DMB	Data Marker Buoy
DND	Department of National Defence
DP	Dynamic Programming
DSS	Decision Support System
EMS	Emergency Medical Services
End	Endurance
FA	First Aid
FLI	Finnish Lifeboat Institute
FLIR	Forward Looking Infrared
FPE	Fire Protective Equipment
FRC	Fast Rescue Craft
FTA	Fault Tree Analysis
GA	Genetic Algorithm
GDP	Gross Domestic Product
GIS	Geographical Information System
GMDSS	Global Maritime Distress and Safety System
HM	Harmony Memory
HMCR	Harmony Memory Consideration Rate
HMS	Harmony Memory Size
HS	Harmony Search
HSGA	Hybrid Algorithm of Harmony Search and Genetic Algorithm
INLP	Integer Non-Linear Programming
INSARAG	International Search and Rescue Advisory Group
IRB	Inshore Rescue Boat
JRCC	Joint Rescue Co-ordination Center
KD	Kernel Density
KE	Kernel Estimation
LMSAR	Lead Minister for Search and Rescue
MARIN	Maritime Activity and Risk Investigation Network
MARS	Maritime Risk and Safety Research Group

MASRES	Marine Search and Rescue Simulator
MILP	Mixed Integer Linear Program
MRSC	Maritime Rescue Sub-Center
NB	New Brunswick
NHMS	New Harmony Memory Size
NI	Number of Iterations
NL	Newfoundland
NM	Nautical Mile
NS	Nova Scotia
NSP	National Search and Rescue Plan
OFA	Occupational First Aid
OSC	On-Scene Coordination
PAR	Pitch Adjustment Rate
PCP	Primary Care Paramedic
PEI	Prince Edward Island
PI	Performance Indicator
PSA	Particle Swarm Algorithm
PSO	Particle Swarm Optimization
R/R	Redundancy/Robustness
RAMSARD	Risk-Based Analysis of Maritime Search and Rescue Delivery
RCMP	Royal Canadian Mounted Police
Rec	Recovery
RI	Risk Indicator
S	Search
SA	Simulated Annealing
SAGA	Hybrid Algorithm of Simulated Annealing and Genetic Algorithm
SAR	Search and Rescue
SCBA	Self-Contained Breathing Apparatus
SISAR	Search and Rescue Program Information Management System
SK	Sea Keeping
SLDMB	Self-Locating Data Marker Buoy
SME	Subject-Matter Expert
Sp	Speed
Tow	Towing
UAV	Unmanned Aerial Vehicle
VR	Virtual Reality

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1. CHAPTER 1 INTRODUCTION

1.1 Background and Problem Definition

Maritime search and rescue (SAR) is one of the most important humanitarian activities in Canada. According to Statistics Canada, this country has the world's longest coastline (about 243,000 Km) and numerous marine activities are underway at all times in Canada. Marine-related activities including commercial fishing, offshore oil and gas exploration/extraction, marine transportation, and ocean tourism (recreational fishing, cruise ship, coastal tourism) attract thousands of people every year to work in this sector and are considered one of the main parts of the Canadian economy. According to government reports, the marine industry has contributed close to 20 billion dollars in GDP just in 2018. In this regard, a lot of marine incidents occur all across the country each year and this makes SAR an essential service. Effective SAR plays a crucial role in saving lives and can decrease the number of victims drastically. According to Canadian Coast Guard annual reports (Government of Canada, 2021), around 2200 lives per year are saved in distress or emergency maritime incidents, and a further 18,000 people are helped in non-distress maritime incidents every year.

In most countries, SAR is provided as a public (government) service. In Canada, the Canadian Coast Guard (CCG) is responsible for providing the rescue-vessel aspect of maritime SAR. The Department of National Defence (DND) is in charge of the associated airborne services.

Various SAR goals and objectives have been stated in the literature, but the most important objectives of SAR are the reduction of loss of life and injury, supplemented by lessening property damage and risk to the environment (Akbari, Eiselt, et al., 2018a). According to Canada's national search and rescue plan (NSP), the goal is a Canada where the critical importance of search and rescue is reflected in a multi-jurisdictional approach to promoting individual, collective, and organizational behaviors that minimize the risk of injury or loss of life while maintaining timely and effective response services. Many Canadian agencies, such as DND and the RCMP (Royal Canadian Mounted Police), are involved in supporting this objective (Abi-Zeid & Frost, 2005). To achieve this goal, two primary objectives need to be addressed:

- Prevention: through many means including educating individuals and organizations
- Response: providing effective and capable SAR response across Canada

The response phase is crucial because quick and effective response can save lives and even minutes matter in responding to SAR operations. Furthermore, SAR operations use a considerable amount of resources (time, money, and human resources), so it is necessary to plan them efficiently (Razi & Karatas, 2016a).

Due to different environmental conditions and maritime activity mixes in various parts of the country, and also for ease of SAR management across the country, CCG has divided Canada into four operational regions: Western, Central, Atlantic, and Arctic. Figure 1 illustrates the CCG regional boundaries (Government of Canada, 2013).

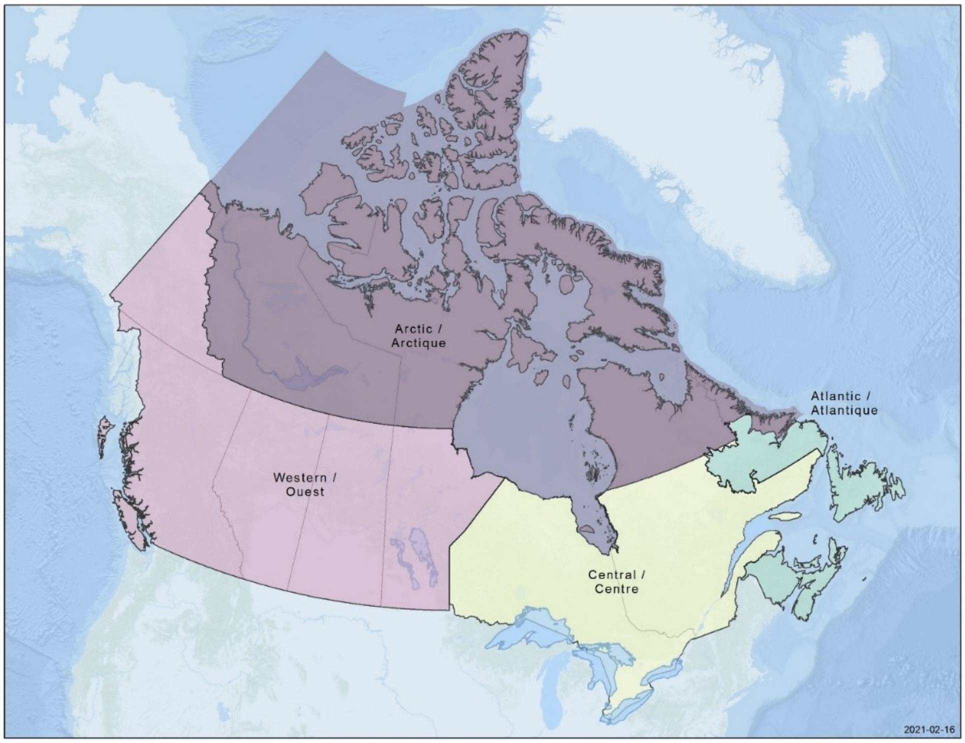

Figure 1. CCG regional boundaries

As SAR is one of the main responsibilities of the CCG in Canada, it is very important for them to continuously aim to improve their service level. This can occur through enhancing the SAR fleet or implementing better plans to utilize the current vessels. Changing the current fleet to introduce new response vessels, which is considered to be a strategic decision, can impose huge costs for the government, while implementation of a resource location-allocation plan can help to improve the system efficiency at lower cost. A location-allocation plan helps us to decide where to establish SAR stations, which resources (i.e., different SAR vessels) to assign to each lifeboat station, and also which vessel(s) would be ideally dispatched to each demand grid (the forecasted incident occurrences, or demand, are spatially represented by a mesh of grid squares as described in chapter 4). Different SAR vessels exist in the Canadian fleet with different capabilities and characteristics (range, speed, length), and vessels are designed to respond most effectively to specific category(ies) of incidents. Thus, it is important that such capability differences be considered when conducting research on maritime SAR effectiveness.

Many studies have been conducted in the field of emergency location analysis. Generally, some remote ‘servers’ (i.e., vessels in the SAR context) and ‘customers’ (maritime incidents in the SAR context) are considered in these problems and the goal is usually minimizing the service time (which is significant to save the lives) or maximizing the coverage. Some models also try to maximize the number of covered demand points (incidents), where ‘covered’ means that a response resource can reach an incident within a specific time or distance limit.

One of the points that is usually omitted by the researchers is the difference between vessels. They often assume that all of the response vehicles have similar characteristics (for example in the ambulance problem) which is rarely a realistic assumption in the maritime SAR problem. There is also a limited number of papers that have differentiated between various groups of incidents (such as capsized, disabled, grounded, etc.). In this thesis, different incidents have been categorized based on requiring similar response characteristics, and the allocation of vessels to incidents is done with respect to the response capability of each vessel that serves each group of incidents. Another common issue in emergency response research is the way that distances are calculated, either with Euclidean or Manhattan distance metrics. However, these methods are not applicable in our problem, because SAR vessels are travelling on the sea, which is different than following routes on land. In this regard, a land-avoidance matrix algorithm has been used in this thesis for the distance calculation between points on the ocean. This algorithm, developed by MARS (Maritime Risk and Safety Research Group) at Dalhousie University, calculates the distance between incidents and stations by avoiding land obstacles.

This thesis investigates the optimal location of SAR stations and vessels along with the optimal allocation of vessels to each incident (by demand grid) in the Atlantic region of Canada. We have considered four different criteria in the optimization model:

- Minimizing Insufficiency probability
- Minimizing Cost
- Maximizing Effectiveness rating
- Minimizing uncovered demand

Insufficiency probability is defined as when the actual number of incidents exceeds the expected value of incidents based on historical averages. This value has been raised to the total number of potential response vessels' power (number of vessels which are available and in range of a specific incident) in the optimization model. It is obvious that the more potential vessels available in the area the less the insufficiency probability will be. This idea has been derived from a reliability concept in parallel systems.

In this model, cost consists of the annual costs of response vessels (operational and annualized procurement) and relocation costs if vessels are repositioned between seasons. The effectiveness rating is defined to reflect the degree to which a response vessel's capabilities match an incident's characteristics. Uncovered demand measures the number of incidents that are beyond reach of the response vessels based on transit time or range limits.

Due to the considerable difference between the number of incidents at various time of the year, two different operational seasons have been considered in this study:

- Season 1: Fall and Winter (October to March inclusive)
- Season 2: Spring and Summer (April to September inclusive)

In addition to seasonality of demand, vessel relocations between stations have also been investigated in this research. The seasonal changing pattern of incidents might require CCG managers to change the location of some vessels and assign them to another station periodically. In this thesis, relocations are only allowed at the end of each season.

Maritime SAR planning is all about considering some unknown factors (Breivik & Allen, 2008) and among them the location and kinds of incident are of utmost importance. A stochastic approach was selected in that study to deal with demand uncertainty whereby historical data have been utilized to obtain the mean and distribution function of incidents in each grid square.

This thesis proposes an optimization model, and its results can be used as a decision-making tool for CCG members. The findings of this thesis will help them to make better tactical (or even strategic) decisions. Figure 2 shows the main elements of the proposed model.

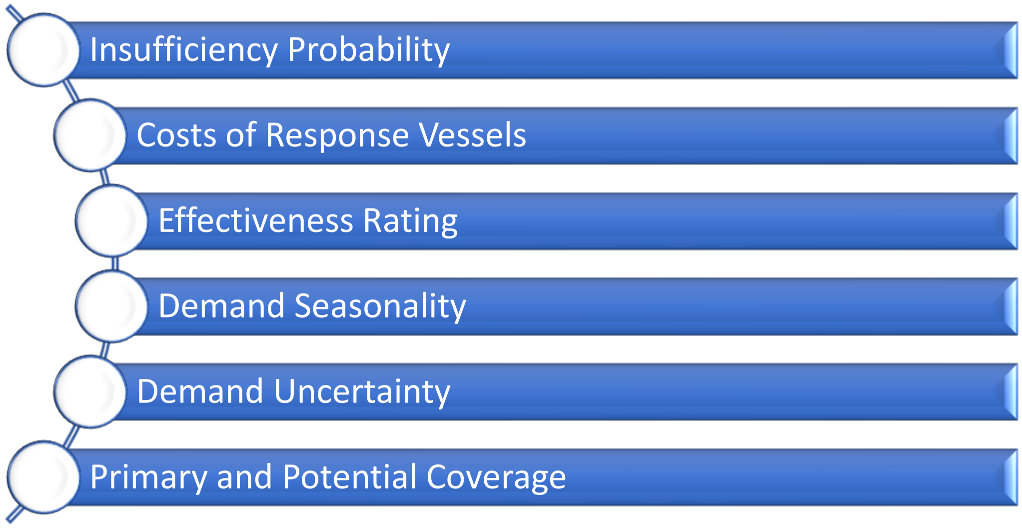

Figure 2. Main elements of the proposed optimization model

To provide more context and details to help position the SAR model developed in this research, further elaboration is provided next on several aspects of Maritime SAR in Canada.

1.2 Maritime search and rescue in Canada

SAR is shared responsibility in Canada, comprising a wide range of government and volunteer organizations. CCG and DND are the two principal agencies for delivering maritime SAR services. In the following sections, a brief explanation of these two organizations will be provided (Government of Canada, 2019).

1.2.1 Overview of the Canadian Coast Guard

As part of the Department of Fisheries and Oceans (DFO), the Canadian Coast Guard (CCG) is the principal civilian maritime operational arm of the Government of Canada. The Canadian Coast Guard operates all DFO vessels and provides services for SAR, Environmental Response, Icebreaking, Marine Navigation Services, and Marine Communications and Traffic Services. The Coast Guard also provides maritime support and services to departmental programs in Science and Fisheries Conservation and Protection, as well as to other agencies at all levels of government (Government of Canada, 2019).

The Canadian Coast Guard is responsible for some SAR tasks. These include the recording of maritime incidents and, with the assistance of the Department of National Defence (DND), the coordination, control and conduct of SAR operations in maritime SAR situations within Canadian areas of federal responsibility; the provision of maritime resources to complement the aeronautical SAR operations as necessary; and, when and where available, the provision of SAR resources to assist in humanitarian and civil incidents within provincial, territorial or municipal areas. The CCG also coordinates, controls and conducts boating Prevention programs to reduce the number and severity of maritime SAR incidents.

Provider of the primary maritime SAR response element, the Canadian Coast Guard augments it using multi-tasked and secondary SAR vessels. Furthermore, the CCG oversees the activities of the Canadian Coast Guard Auxiliary (CCGA), a volunteer organization (Government of Canada, 2019).

The CCG, as the main provider of maritime SAR service in Canada, has "defined" several objectives for saving and protecting lives in the maritime environment (Government of Canada, 2019):

- Save 100% of lives at risk.
- Reduce the number and severity of SAR incidents.
- Minimize loss of life, injury, property damage and risk to the environment.
- Support and involve the Canadian Coast Guard Auxiliary.
- Maintain the highest professional standards.
- Provide national leadership and effective SAR Program management.
- Provide international SAR leadership.
- Maximize SAR system efficiency through innovation.
- Promote volunteerism.
- Increase awareness of the SAR Program.
- Assist in the development of the National SAR Program.
- Foster co-operative SAR agreements.
- Provide humanitarian aid and civil assistance where possible.

1.2.2 Department of National Defence

In 1976, the Prime Minister of Canada appointed the Minister of National Defence as the lead minister for SAR (LMSAR). The LMSAR is responsible for the coordination of the National SAR Program (NSP) and the development of national SAR policies in conjunction with other Ministers. The LMSAR is the designated national spokesperson and is charged with ensuring that the national SAR system operates effectively.

DND delivers primary air SAR services for both air and maritime incidents; provides a high level of secondary SAR support from its aircraft; and co-ordinates the activities of the Civil Air Search and Rescue Association (CASARA), a volunteer organization. Under the SAR program, DND and the Canadian Coast Guard coordinate the response to air and maritime SAR incidents through the Joint Rescue Co-ordination Centres (JRCCs) (Government of Canada, 2019).

1.2.3 Rescue Co-ordination Centres and Maritime Rescue Sub-Centres

The Canadian Coast Guard jointly staffs three Rescue Coordination Centres (JRCCs) with the Canadian Armed Forces (i.e., DND). The JRCCs are in Victoria (British Columbia), Trenton (Ontario), and Halifax (Nova Scotia). The Canadian Coast Guard also operates two Maritime Rescue Sub-Centre (MRSC) in Quebec City, Quebec and St. John's, Newfoundland. The function of a MRSC is to reduce the JRCC's workload in areas of high marine activity. These centres are staffed by SAR Coordinators who operate 24 hours a day, seven days a week, year-round. The maritime area for which the Canadian JRCCs/MRSCs are collectively responsible for is more than 5.3 million square kilometers.

The JRCCs/MRSCs are responsible for the planning, coordination, conduct and control of SAR operations. JRCCs/MRSCs have highly trained staff, detailed operational plans and an effective communications system. Once a JRCC/MRSC is notified that a person(s) is in danger, the SAR Coordinator begins to organize the rescue. All available information about the person(s) in danger is gathered and recorded and the positions of potential assisting resources in the area of the incident are determined. SAR Coordinators are trained to evaluate various situations and send the most effective resources to deal with a particular incident. In complex and major incidents, many resources are often sent or tasked to assist (Government of Canada, 2019). Figure 3 shows the relationship between the mentioned centers:

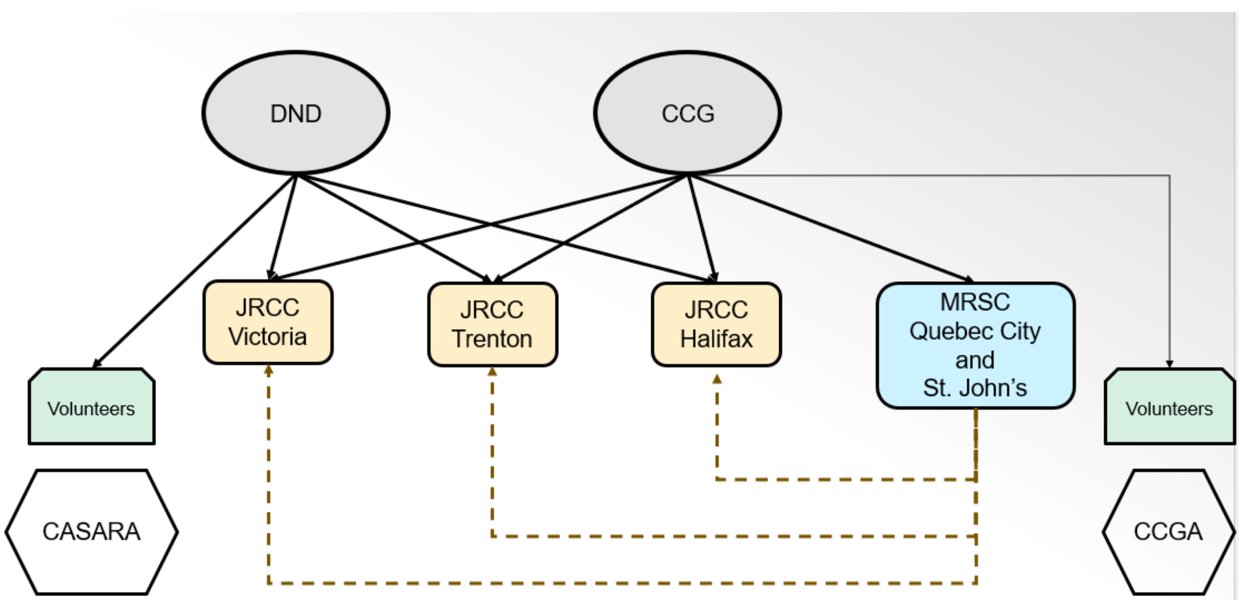

Figure 3. Maritime SAR-related centers in Canada

1.2.4 Vessels' category in maritime SAR

Following categories of response vessels are used in maritime SAR (Government of Canada, 2019):

1.2.4.1 Primary SAR Vessels

A primary SAR vessel is a specially designed, equipped, and crewed vessel that has SAR as its main responsibility. These vessels are located in relatively high-risk areas where SAR incidents are more probable to happen. Their appearance is similar to other CCG vessels (red and white) and the word "RESCUE/SAUVETAGE" are visible on them.

1.2.4.2 Multi-tasked SAR Vessels

Multi-tasked SAR vessels are other Canadian Coast Guard vessels that are available to deliver the SAR Program while also being assigned to at least one other operational program. They have to remain within a specific SAR area while they are multi-tasked to the SAR Program and maintain all SAR operational standards. Multi-tasked vessels increase efficiency, reduce costs to the government and stand in for primary SAR vessels when necessary.

1.2.4.3 Secondary SAR Vessels

Secondary SAR vessels comprise all other government vessels which can be called upon to respond to any maritime emergency if needed.

1.2.4.4 Additional SAR resources

In addition to the vessels mentioned, there are two main volunteer organizations that provide help in SAR operations. The first one is The Canadian Coast Guard Auxiliary (CCGA) which consists of five non-profit associations and a national council. The second one is the Civil Air Search and Rescue Association (CASARA) which operates under the supervision of DND and provides assistance to the Canadian Coast Guard. Moreover, in accordance with the *Canada Shipping Act* and international law, all the vessels close to a SAR incident are required to assist any vessel in distress (Legislative Services, 2019). Such nearby vessels are called "vessels of opportunity".

1.3 Maritime SAR in the Atlantic region

The CCG Atlantic region of Canada includes 2.5 million squared kilometers of Northwest Atlantic Ocean, and this region has about 40,000 km shoreline across the four East coast provinces (New Brunswick, Nova Scotia, Prince Edward Island, and Newfoundland and Labrador). Figure 4 represents the location of the Atlantic region (Government of Canada, 2020).

Figure 4. Atlantic region map

Extensive fishing activities in the region, a substantial variety of recreational boating during the summer and shoulder seasons, aquaculture operations, significant shipping to ports in the region and traffic lanes to other regions such at the St. Lawrence River and the US Eastern Seaboard all contribute to the need for effective SAR service. That is why the Atlantic region is home to a large number of CCG resources. All the mentioned operations justify the need for investigating the optimal location of CCG stations and the allocation of SAR resources in the Atlantic region of Canada, which can provide insights for the most effective development and deployment of the SAR fleet.

1.3.1 Data sources

As a practical and applicable study, this research requires valid and reliable data. Otherwise, its results and findings cannot be used as a decision-making tool and may suggest unrealistic options involving exorbitant costs for the Canadian Coast Guard vessel location problem. In this section, the main sources of data which were used in this thesis will be explained.

1.3.2 Historical Incidents

One of the inputs used in the optimization model is the mean number of incident occurrences in the area of interest. Obtaining the incidents' distribution function requires historical data from several preceding years in the Atlantic region. Hence, a database referred to as SISAR (Search and Rescue Program Information Management System) was used. SISAR, developed by the CCG decades ago, integrates all SAR incidents into a national database. After each SAR incident, one record is added to the SISAR database, and it thus provides a full account of all SAR incidents in Canada's coastal areas and navigable inland waters (Stoddard & Pelot, 2020). The main objective of this database is easy access to necessary information for supporting SAR planning, management, and operations (Marven et al., 2007). SISAR collects considerable details on each incident, including fields such as incident ID, incident date, latitude and longitude of incident occurrence, incident type, description of incident, response summary, severity of incident, and atmospheric conditions (wave height, wind speed, wind direction, wind-against-current, visibility, ceiling, air temperature, sea surface temperature, clouds, ice, weather comments, tide states).

In this database, incidents are classified into these four broad categories:

- M - Maritime Incidents (M1, M2, M3, M4)
- A – Aeronautical Incidents (A1, A2, A3, A4)
- H – Humanitarian Incidents (H1, H2, H3, H4)
- U – Unknown Incidents (U4)

Each category is also sub-classified based on the severity level, ranging from 4 (least) to 1 (most severe). Given that the focus of this research is on maritime incidents, the four sub-categories of the "M" group are "explained" as follows (Rezaee & Carrol, 2014):

- <u>M4- False alarms and hoaxes</u>: Situations that cause the SAR system to react which proves to be unjustified or fabricated, such as a mistaken report of a flare sighting.

- <u>M3- Incidents resolved in the uncertainty phase (Non-Distress)</u>: No distress or perceived appreciable risk to life apparent. An Uncertainty phase exists when:

    a. There is doubt regarding the safety of a vessel or the person on board;
    b. A vessel has been reported overdue at destination; or
    c. A vessel has failed to make an expected position report.

- <u>M2- Potential Distress incidents</u>: The potential exists for a distress incident if timely action is not taken, i.e., immediate response is required to stabilize a situation in order to prevent distress. This incident type exists when:

    a. There is apprehension regarding the safety of a vessel or the person on board;
    b. Following the uncertainty phase, attempt to establish contact with the vessel has failed and inquiries addressed to other relevant sources have been unsuccessful; or
    c. Information has been received indicating that the operational efficiency of a vessel is impaired but not to the extent to be a distress situation.

- <u>M1-Distress incidents</u>: Distress phase exists when:

    a. A vessel or a person is threatened by grave and imminent danger and requires immediate assistance (life-threatening situation was judged to be present or close at hand at some point during the incident);
    b. Following the previous phase, further unsuccessful attempts to establish contact with the vessel and more widespread unsuccessful inquiries point to the high probability that the vessel is in distress; or
    c. Information is received which indicates that the operating efficiency of the vessel has been impaired to the extent that a distress situation is very likely.

An important facet of this research is to allocate the most suitable SAR resource to distinct incident types, as the required response capability varies. Twenty-one different incident types are defined in the SISAR database. Table 1 lists the SISAR incident types.

Table 1. SISAR incident types

SISAR incident types		
Capsized	Disabled	Disoriented
Grounded	False Alaram	Man Overboard
On Fire	Medical	Foundered
Suicide	Suicide Attempt	Taking on Water
Forced Landing	Crash	Airborne Emergency
Missing Person(s)	Ditching	Stranded
Body Recovery	Person in Water	Overdue

1.3.3 Study Scope and Historical trends

Our area of interest in this thesis is the Atlantic region of Canada. Figure 5 shows this area along with the incidents that occurred from 2014 to 2016.

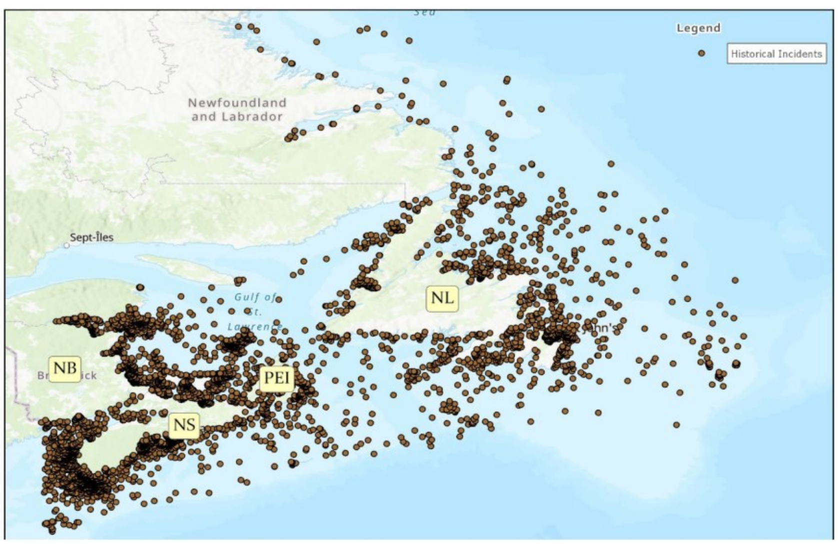

Figure 5. Atlantic region SAR incidents from 2014 to 2016

These incidents were extracted from the SISAR database. The newest SISAR data that were available were from 2014 to 2016. Since the emphasis of this thesis is maritime SAR, false alarms and airborne related incidents were deleted from our dataset (i.e., forced landing, crash, airborne emergency, ditching). Some other incident types such as suicide, suicide attempt, missing person, and body recovery are also ignored due to their relatively rare occurrence (just 30 incidents in 3 years). The remaining incident types were categorized into four groups based on similar characteristics and similar response vessels which will be explained in Chapter 3 of this thesis.

Another modification applied to the dataset is deleting the incidents which were responded to by inshore rescue boats (IRBs). IRBs are small, usually inflatable boats which are used to aid near-shore boaters in distress, along with providing public education on boating safety topics (like hypothermia, rules of navigation, personal watercraft, personal floatation devices, pleasure craft courtesy checks, boating restrictions and regulations, required safety equipment aboard a vessel, and proposed changes to required safety equipment) (see Figure 6 (Fisheries and Oceans Central & Arctic Region, 2019)). IRBs are seasonal, have a limited range of operations, and are generally not based at a lifeboat station, thus it was decided not to include them in the analysis of SAR vessel optimal location.

Considering the fact that icebreakers are rarely used in SAR operations, the incidents which were responded to by these vessels were also omitted from our incidents database. There are five icebreakers in the Atlantic region (Louis S. St-Laurent, Henry Larsen, Terry Fox, Captain Molly Kool, Jean Goodwill) which have served for just 8 incidents in the Atlantic Region in four years according to SISAR.

Figure 6. CCG IRB crew on Georgian Bay

After refining and cleaning the SISAR database, 2641 incidents were identified in the Atlantic region of Canada from 2014 to 2016 inclusive. Figure 7 shows the number of incidents in each year in the Atlantic region.

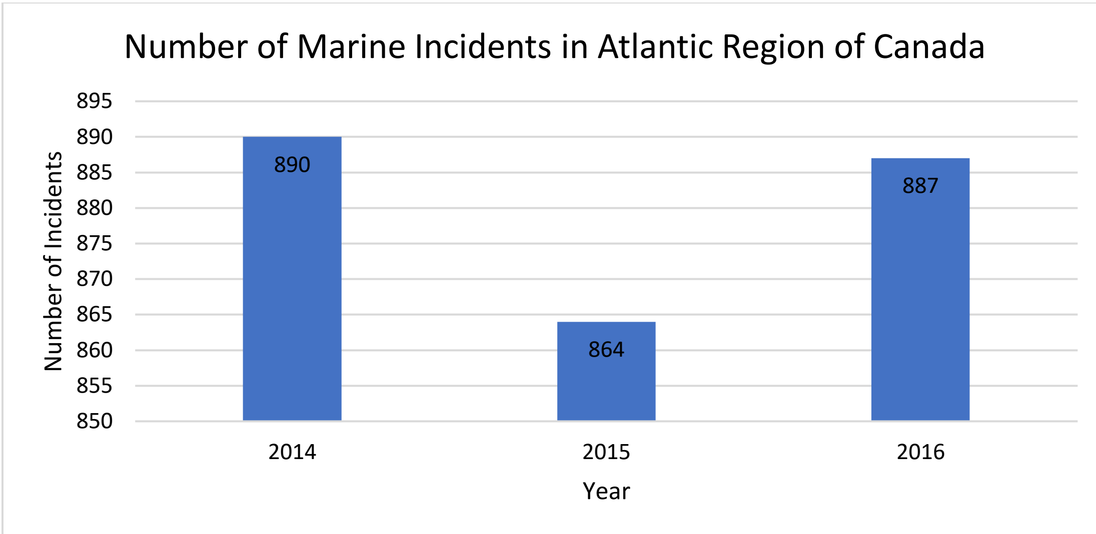

Figure 7. Number of marine incidents in Atlantic region of Canada

Figure 8 illustrates the distribution of incidents across different months. As shown, the number of incidents soars from April to September (due to more marine activities in warmer seasons) and decreases in the fall and winter. This fluctuation can have an impact on the allocation of SAR vessels in different stations. In this regard, two operational seasons are considered for our study to address this difference and decide about the relocation of the vessels at the end of each period.

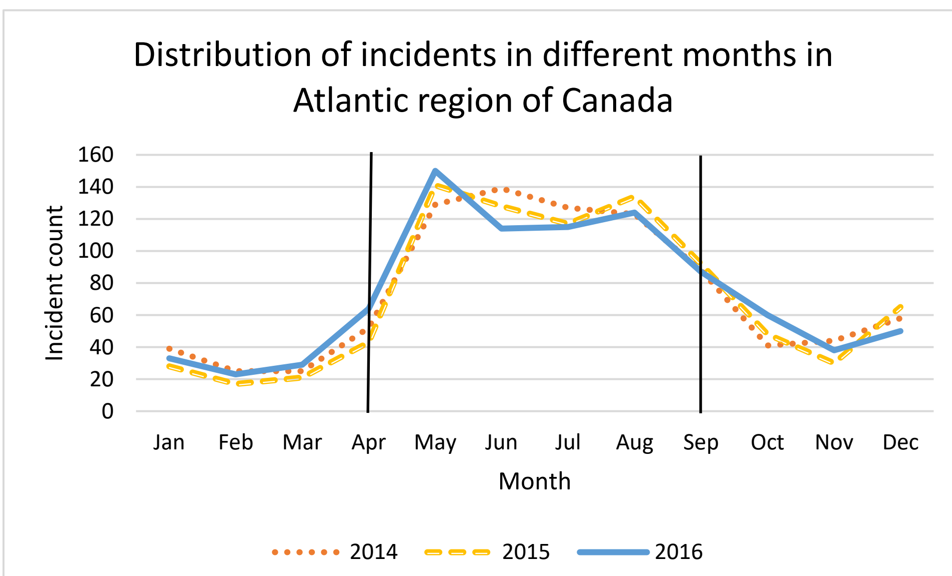

Figure 8. Monthly incident distribution in Atlantic region of Canada

1.3.4 SAR stations

This research considers where response vessels are generally 'pre-positioned' when SAR calls come in. Thus, we consider the existing SAR stations, referred to here as 'onshore' stations, as well as a set of potential 'offshore stations' which denotes central locations where a SAR vessel would generally be positioned to conduct patrolling or other tasks at sea. Considering the current SAR fleet deployment, there are 19 onshore SAR stations in the Atlantic region of Canada. Two of these stations have been established in the past two years (N.L. Old Perlican in 2019 and N.L. Twillingate in 2020). These stations can accommodate different kinds of SAR vessels. Furthermore, 19 potential offshore stations have been considered in this thesis. The location of these stations has been determined through consultation of CCG experts, whereby the centroid of some marine subareas in the Atlantic region and also some offshore patrolling locations were determined. It is important for an offshore vessel to have sufficient endurance and maximum range to be qualified to be located at an offshore station. This issue is dealt with as a constraint in the optimization model, which consequently does not allow lifeboats to be assigned to offshore stations.

1.3.5 Land-avoided distances

For running our model, we need to calculate the distance between incidents and stations (i.e., response vessels). The most common method is to calculate based on straight Euclidean distance which is not applicable for this study since there are some land obstacles at sea which impede direct routes. Therefore, a land-avoidance algorithm was used in this thesis. This method was developed by the *MARIN* (Maritime Activity and Risk Investigation Network) research group to find the shortest route between incidents and vessels.

1.4 Research Objectives

Many papers have been published about SAR from the perspective of different fields. This includes studies from mechanical engineering, computer science, industrial engineering and medical studies to name but a few. Despite the rich literature, there are still a lot of research gaps to examine. The focus of our research is on the location-allocation problem in marine search and rescue with the goal of finding the optimal location of SAR stations and the optimal allocation of SAR vessels to each station. In other words, the possibility of better SAR resource allocation through four defined criteria has been investigated in this thesis:

- Insufficiency probability
- Vessels' annual cost
- Effectiveness rating
- Primary coverage

We have added some new concepts to previous works. The contributions include:

- Model development
  - Defining response vessel effectiveness ratings for matching with incidents
  - Defining Insufficiency Probability
  - Using a reliability concept for potential coverage in the optimization model
- Molel-solving
  - Formulating the problem for using dynamic programming
  - Comparing heuristics methods for solving the problem

- Model application
    - Producing the optimal solution for the given conditions and criteria
    - Comparing and contrasting multiple solutions using sensitivity analysis

Figure 9 shows the research steps which have been followed in this thesis.

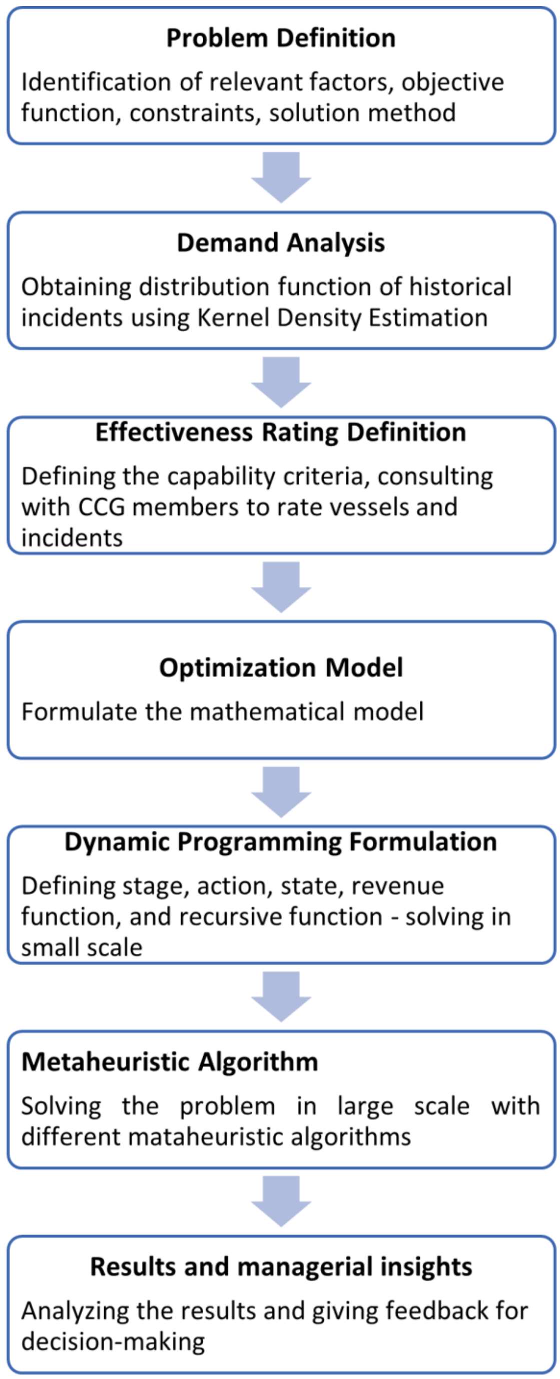

Figure 9. Research steps

1.5 Thesis Assumptions

The main assumptions of this thesis are as follows:

1. The location and time of incidents which have been extracted from the SISAR database are considered to be accurate.
2. The average speed of the response vessels is assumed to be equal to their stated cruising speed.
3. Environmental factors such as wind, temperature, and sea states have not been considered in this thesis.
4. No preparation, coordination, nor search time are taken into account.
5. Even though each incident is unique, there are sufficient similarities to be able to group them into categories by type for the purpose of effective response analysis.
6. Even though response vessels of a given class are not necessarily identical, they are similar enough to group them into 5 categories of response vessels for the purpose of effective response analysis.

1.6 Thesis Outline

This thesis is organized as follows. In chapter 2, previous studies in this field are reviewed in order to determine the research gaps and necessity of the current research. Chapter 3 covers the explanation of methodology and research steps, which includes the optimization model and solution methods. In chapter 4 the data collection and calculation processes are discussed in detail. In chapter 5 the model outputs are presented along with thorough sensitivity analysis, followed by proposing managerial insights, conclusion, and future studies in chapter 6.

2. CHAPTER 2 LITERATURE REVIEW

In the introduction chapter, we discussed the research necessity and overview of the problem. In this chapter it is essential to scrutinize the subject literature to appreciate the results to date and clarify the research gap. The research done in this thesis includes a multi-dimensional problem which requires us to present the literature review in different sections. A systematic and thorough literature review will help us to understand the problem better and investigate different approaches. Different sections of the literature review will be discussed according to the following topics.

- The main objective of this study is determining the location of SAR vessels; therefore, it is of great importance to have a review of previous research in location-allocation models. Studies in this field are so diverse that it is hard to cover all the aspects, but we have tried to present a comprehensive view of location problems.

- Another subject which will be covered in this chapter is emergency response problems. The essence of the proposed problem in this thesis is search and rescue, and effective response is vital to urgent problems (especially in non-deterministic situations). Looking at the research trends, we can conclude that despite the importance of saving lives, emergency response-related problems have received less attention in the literature in comparison to economy and production problems.

- The solution method presented in this thesis is unique and includes dynamic programming. Dynamic programming is one of the most powerful solution methods which is able to solve a broad range of problems by breaking down large ones into their sub-problems and applying numerical optimization. One of the factors that makes Dynamic programming less common is its difficult implementation relative to other methods (such as heuristics and meta-heuristics). Another reason is the curse of dimensionality in large scale problems. To overcome this issue, meta-heuristics methods have been used to solve the case study in this thesis. Solution methods (especially dynamic programming) and meta-heuristics methods will be discussed in this section of the literature review.

- The main focus of this thesis is problems related to maritime search and rescue, and we have investigated the search and rescue problem in Atlantic region shores of Canada as our case study. In this regard, it is necessary to have a review of maritime search and rescue problems.

- For calculation of demand and the associated spatial analysis, Kernel Density (KD) estimation was utilized. Taking this into consideration, spatial analysis techniques should be studied in the literature review.

- Finally, after analyzing the previous research from the above sections, the research gap and the contributions of this thesis will be presented.

2.1 Location Models

The location problem can be defined as art and science of finding proper locations as part of a more general system. The objective of location models is selecting the location in a way that results in the best outcome (in accordance with the problem's criteria). A comprehensive review of location models will be presented next.

Equipment location, locating trapped miners, and even location of railways have been among the oldest favorite problems of researchers, and their trace can be seen in books published in the 19^{th} and early 20^{th} century ((Winsor, 1884), (Durkin, n.d.), (Wellington, 1887)). Location problems have been so broad that (Krzyzanowski, 1927) prepared a review of the studies on this field in 1927. When discussing "location" in problems, it usually turns the mind towards factory location or equipment locations, but one of the studies done by (Ullman, 1941) has presented locations for establishing new cities considering urban constraints. This research can show us the extent of location problems and it implies that the scope of the problem ranges from the location of one part in an electronic circuit to the location of a big city in a region.

Location problems became more popular in the second half of the 20^{th} century with the advent of dominant location models such as maximal covering, set covering, and P-median. Some of the seminal papers on these topics are described next.

(Church & ReVelle, 1974) presented research on maximal covering location problems. They believed that if we address factories and private equipment, it would be reasonable to set the objective to reducing the costs; but in problems related to public services, it is necessary to pay more attention to social justice and maximum coverage for all parts of society. The proposed model in their study is so significant that it is still being used in current research about public services (like fire department, emergency units, etc.). It is worth mentioning that the model presented in this thesis is based on the maximal covering model.

Problems related to uncertainty in locating emergency services have been studied for a long time. In a research by (Aly & White, 1978) emergency services' location problem has been addressed in a way that locations of incidents (fires, accidents, or customers) are random variables. The presented model was a covering problem and some examples have been mentioned for showing the importance of using probabilistic assumptions in these kinds of models.

Location problems became more extensive in 21st century and it is hard to cover all the aspects of these problem in this chapter. Therefore, we will narrow it down and focus on covering location problems. In 2002, (Berman & Krass, 2002) discussed the maximal covering problem in a more generalized way in which they considered partial coverage of customers as well. They defined a function for coverage, and they considered the distance of a customer from a facility as criteria for coverage. At the end, they also showed the solvability of their model under different condition.

Partial coverage attracted a lot of attention from the researchers of this field. In 2004, a maximal covering problem with partial coverage was presented by (Karasakal & Karasakal, 2004). According to this study, maximal covering location problems are problems for locating some of the facilities to cover maximum number of demand points. It is worth mentioning that this definition got more complete in the following years with introduction of problems which requires all the demand grids (not some of them) to be covered (like fire stations location problems or SAR location problems the has been presented in this thesis). Authors of this paper believed that considering a crisp boundary for coverage will lead to unreliable results and suggested using partial coverage. The importance of reviewing this paper is that the essence of partial coverage is similar to the potential coverage concept which has been used in this thesis. This problem has been solved with Lagrangian relaxation method and they have made a comprehensive comparison at the end of this paper.

Consideration of the uncertainty in location models has been done with different approaches. For example (Araz et al., 2007) presented a covering problem for determining the optimal location of vehicles for emergency services. One of the distinguishing features of this study was addressing the uncertainty with the fuzzy concept. They developed a multi-objective model and solved that with fuzzy goal programming, also using lexicographic linear programming. Another important specification of this paper is the consideration of backup coverage. Three objectives mentioned in their optimization model were maximizing covered population by a vehicle (ambulance or fire department vehicle), maximizing population with backup coverage, and minimizing the travel distance. The findings of this research showed satisfactory solutions in all the model's objectives.

In 2008, a comprehensive review paper was published by (Daskin, 2008). In this paper, he presented a classification for location-related studies, and then categorized all the papers accordingly. Given the complete discussion and conclusions in this paper, it gives a proper introduction and overview to all researchers interested in this field.

In recent years, emergency facility location problems have been a hot topic for researchers. Some of the newly published papers will be reviewed below.

(B. Zhang et al., 2017) published a paper related to the covering location problem of emergency service facilities with consideration of uncertainty. They assumed uncertainty in the demand and formulated the problem based on both set covering and maximal covering approaches. A case study was also investigated for showing the interaction with uncertainty in the real world. In another paper published by (B. Zhang et al., 2018), the location-routing problem for emergency service facilities was developed. They assumed that the information in the model is uncertain, which includes travel distance, customer demand, and cost of opening a depot. The proposed multi-objective model of this paper was converted to a single objective using the main objective method and solved with a hybrid algorithm combining simulation and a genetic algorithm (GA). Numerical examples presented in this paper have shown the effectiveness and robustness of the suggested algorithm.

One of the newest contributions in this field is a study by (Mohri & Haghshenas, 2021) about the ambulance location problem for covering the road crashes which are random and rare in practice. They defined a network based on the real-world case study and assigned the crashes to the edge of the network (not the nodes), because crashes usually occur on the roads between the cities. They solved the problem considering partial coverage and they presented some managerial insights at the end. Another study by (Hajipour et al., 2021) investigated the covering location problem of fire stations under uncertainty. The main objective of the model was covering demand grids in emergency situations like natural disasters or war. They considered demand and coverage radius to be uncertain and addressed them with fuzzy logic. Since the problem fits in the NP-hard category, meta-heuristics algorithms were applied to solve the problem. Findings showed that PSO (particle swarm optimization) algorithm outperformed ABC (artificial bee colony) for obtaining the optimum solution.

In summarizing the articles mentioned, we can conclude that despite the increasing attention of the researchers to emergency service problems, there is still space to work in this field and implement different models on real case studies. One of the gaps in the current literature is that approximate solution methods are used by most of the researchers which could be otherwise addressed by applying exact solutions like dynamic programming to the problem.

2.2 Emergency Response

Emergency response is among the problems that have been receiving a lot of attention from scholars in recent years, because they are directly related to people's mortality and morbidity. This topic covers a broad range of problems like police department response, fire department service, Red Cross units, and SAR operations. We will review some of these problem areas.

The phrase "emergency response" was originally used mostly in terms of having a proper reaction to accident occurrences in different places (like mines). After 1970 and with the increasing utilization of computers, researchers started working more on planning emergency situations with consideration of uncertainty which is an inherent characteristic of urgent situations. Another important factor which is crucial in emergency situation is fast response; for example in the study done by (Drabek, 1985), the authors conducted a comprehensive investigation into the management of disasters and efficient response in emergency situations. This work which discussed different kinds of disasters such as tornados, hurricanes, chemical spills, and terrorist threats showed that response in the initial hours after incident occurrence is of great importance and can prevent life and financial losses.

At the end of 20th and early 21st century, studies in this field had astounding progress and included a broader range of incidents like earthquakes. (Tierney & Goltz, 1997) investigated the situation of Kobe city in Japan after the devastating earthquake and fire in 1923. They studied all the statistics related to this disaster and suggested miscellaneous ways to be more prepared for natural catastrophes and decrease life and financial loss at the same time. In another comprehensive study done by (Fiedrich et al., 2000), proper and effective resource allocation for the response to earthquake disasters was discussed. The main objective of this research is finding the best allocation of available resources in the initial stages of search and rescue after earthquake disasters which leads to minimizing the fatalities. An optimization model (considering detailed information of operational areas) was developed and solved with approximate solution methods.

The start of the 21st century was a milestone in publishing important papers on emergency response which provided a basis for other studies in upcoming years. One of the main reasons for this trend was the September 11 attacks which acted as a trigger for more research in this area. In one of the studies after September 11 attacks, (Perry & Lindell, 2003) investigated the concept of community preparedness and they also scrutinized its relationship with training, exercises, and terrorism emergency response plans. It was mentioned in the paper that governments all around the world have invested a lot of money in writing terrorism emergency response plans which resulted in neglecting the planning process itself. The outcome of this paper was 10 different planning process guidelines which can be applied to all environmental threats. In another study (Manoj & Baker, 2007) pointed out that the communication problem is the primary challenge in response to natural and human-made catastrophes. They believed that better communication systems in such accidents can result in better response in emergency situations. The authors of this study considered the terrorist attacks of September 11 and also Hurricane Katrina as two main examples and claimed that lack of radio interoperability was one of the main problems in the two mentioned incidents. They categorized communication challenges into three different categories (technological, sociological, and organizational) and stated that attention to these three areas is the key factor for developing an effective disaster communication system.

In recent years and especially after 2010, the emergency response field became a trending issue for Industrial Engineering experts, and they have developed many mathematical and decision-making models for analyzing problems in this field. In the following, some of these papers will be reviewed.

(Li et al., 2011) presented all the covering models for emergency response and had a comprehensive review of these problems. Their focus is emergency medical services (EMS) in which they are trying to find the best solution for locating EMS facilities in order to effectively deal with demand in different areas (similar to the objective being pursued in this thesis in a different context). After reviewing different types of covering models (set covering, maximal covering, double standards, etc.), the authors also summarized the methods for solving the models, namely heuristic algorithms, simulation, and exact methods.

One of the interesting papers published in 2013 is the work of (Sommer et al., 2013). The authors in this paper presented a model for describing, analyzing, and planning the learning process in emergency situations. They believed that decision making is of great importance in urgent conditions, and they emphasized the learning process which leads to making reasonable decisions. In accordance with this research, by learning from an emergency situation and implementing the obtained skills in similar conditions, one can ensure that the best way was selected by the responder to face the incident. Another study which has its focus on the decision-making process is (Y. Liu et al., 2014). They claimed that decision-making in emergency situations is usually risky with a considerable amount of uncertainty because the lack of data and changing scenarios are common in these situations. In this research, Fault Tree Analysis (FTA) was used to show the logical relation between factors that result in an emergency situation. With consideration of available response actions, the probability of each scenario was calculated. At the end, a case study of N1H1 disease has been presented to show the feasibility and validity of the method.

Most of the recent studies on emergency response problems have been dedicated to the current global situation under the influence of the Covid-19 virus pandemic. These studies contain a broad range of subjects such as development of web-based platforms for delivering healthcare services, preventive measures to delay the spread of the virus, and healthcare waste management during pandemic (Krausz et al., 2020), (Erkhembayar et al., 2020), (L. Yang et al., 2021).

As explained, emergency response problems have been a research interest of scholars over the years, and they have not lost their importance up to now. The reason is that these problems are directly related to enhancing people’s lives and decreasing the fatalities of incidents. In this thesis, we discuss the case study of the emergency response problem for SAR units in the Atlantic region of Canada with a new approach to uncertainty in these problems. We have also considered backup coverage along with primary coverage to minimize Insufficiency probability of SAR missions in this region. Another contribution of this research is formulating and solving the problem with dynamic programming which is a recognized exact method for solving optimization models. In the following section, papers related to this method will be reviewed.

2.3 Dynamic Programming

One of the most efficient solution methods for optimization problems is Dynamic Programming. The reason for this efficiency is the numerical optimization approach (such as meta-heuristic methods) that is used in this method. This feature becomes even more important when the intended model is non-linear with constraints and discrete variables because dynamic programming is among the few exact solution methods that can solve these problems. Dynamic programming can be used for solving models as long as we are able to break down the problem into sub-problems and also calculate the objective function value for different variables (regardless of model essence).

The presented model in this thesis has all the mentioned features above, thus it is required to use a method like dynamic programming to solve it. In the following we will review some of the papers related to this method and its application.

The dynamic programming method was presented by (Bellman, 1951) in the middle of the 20^{th} century for the first time. In this study the author discussed some linear problems which were solved by a new method called linear dynamic programming. This paper just considered some basic problems, but other researchers used Bellman's work to further develop this method.

In the following years, some scholars tried to use this method to solve their problems, but Bellman was the only person who strived to introduce this method as an approved theory to science society. Between 1950 to 1955, several papers were published by this researcher about dynamic programming ((Bellman, 1953), (Bellman, 1952), (Bellman, 1954)).

In the last decade of the 20th century, dynamic programming became a common method for solving industrial engineering problems, and quite a few books were written by researchers (other than Bellman) about this method. One of them is (DEWHURST, 1992) which explains outlines of this method along with the mathematical basics behind dynamic programming. It is worth mentioning that some applications of this solution method are also mentioned in this book. Another book by (Sniedovich, 1991) notes that dynamic programming can be used as the main solution method for problems in different subjects like operations research, management, and economics. This book tries to answer basic questions about dynamic programming and its applications.

After acceptance of dynamic programming as a valid solution method, many researchers tried to enhance this method to be more efficient and less computationally intensive. (Tsitsiklis & Van Roy, 1996) developed a framework to help solve large-scale problems with dynamic programming. The authors tried to use feature-based compact representation to decrease the computational volume. Findings showed that despite the success of this framework, a disadvantage is that integrating compact representation with dynamic programming can be difficult. In another study by (Rust, 1996), the authors explored numerical methods for solving dynamic programming problems. It is stated in the paper that dynamic programming is widely accepted and used in economics, because it has the ability to deal with sequential and uncertain decision-making problems. (de Farias & Van Roy, 2003) tried to use a linear programming to present a method for enhancing approximate dynamic programming. "Approximate dynamic programming" is a new term which uses methods aimed at reducing mathematical calculations of dynamic programming to the detriment of precision. This study used linear programming to do so, and the authors guaranteed the performance by applying error bounds to prevent deviation from the original dynamic programming method. A paper by (Iyengar, 2005) introduced a robust formulation for discrete time dynamic programming to decrease the sensitivity of the optimal policy to underlying probabilities. The authors assigned a set of conditional measures to each state-action pair and concluded that if these sets have the "rectangularity" property, the main results of dynamic programming will extend to robust counterparts. In an interesting paper by (Doerr et al., 2011), they discussed the possibility of combining evolutionary algorithms with dynamic programming. The authors developed an approach for constructing evolutionary algorithms in a way that its representation enables them to use dynamic programming. The main trigger of this research was decreasing solution time, and they claimed that approximation based on evolutionary algorithms can reduce this time for a wide range of dynamic programming problems.

(Maxwell et al., 2010) is among the few studies which has used dynamic programming for emergency response problems. In this research, the authors formulate the ambulance redeployment problem with dynamic programming. The aim of this problem is reassigning idle ambulances to demand grids in order to maximize the number of calls while considering a delay threshold. To deal with curse of dimensionality, they approximated the value function and parameterized it with a small number of parameters. The parameters' tuning process was done with simulated cost of the system. Implementation of this method on two metropolitan areas has shown considerable improvement in performance.

A recent study by (Karimi & Sadjadi, 2020a) uses dynamic programming to solve a non-linear inventory problem for deteriorating items. The importance of this paper is that they simply formulated a complex and non-linear problem (with capacity constraints) as a knapsack problem and solved it with dynamic programming. According to this paper, solving problems with complex objective functions can be easy with dynamic programming because the basis of this method is numerical calculations. It should be mentioned that the steps and assumptions of dynamic programming formulation in this thesis have been done based on this paper.

After many years since the introduction of dynamic programming, researchers are still using this method in different applications like pruning algorithms, scheduling, renewable energy, military medical evacuation, power distribution, etc. ((Alzubi et al., 2020), (J. Wang et al., 2020), (Ding et al., 2020), (Jenkins et al., 2021), (Hu et al., 2021))

To summarize, dynamic programming is a powerful method for solving complex problems that can be considered as a set of sequential decisions. Two advantages of this method are providing the global optimum and its strength in solving non-linear models. One of the contributions of this thesis is formulating the search and rescue problem with dynamic programming which has not been done by other researchers in this field. Moreover, having a power term in our objective function along with different constraints and discrete variables, makes it hard to find an alternative for dynamic programming as a solution method. Therefore, dynamic programming has been used to solve the proposed model in this thesis.

2.4 Search and Rescue (SAR)

Search and rescue is a subject that encompasses a broad range of research areas. The main areas of SAR are marine SAR, aerial SAR, and land SAR. Land SAR includes road accidents or natural disasters like flood and earthquake operations, but incidents that occur in the sea or near shore fall into the marine SAR category. It is noticeable that aerial SAR can be used in both mentioned categories. The presented optimization model and also the case study of this thesis is related to marine SAR. Thus, we will review some papers to cover SAR concepts in general, then we will focus on marine SAR and related papers. The classification of papers in this section is based on two main phases of SAR which are "search" and "rescue". First, search papers will be reviewed and after that rescue-related research will be discussed.

(Kierstead & DelBalzo, 2003) used genetic algorithm (GA) for designing a search path. As opposed to prior research, they considered space and time to be continuous and GA helped them to perform detection. The comparison between their model and navy's existing method demonstrated a considerable improvement (%46) in search performance.

(Ryan & Hedrick, 2005) investigated an algorithm for planning the path of UAVs in SAR missions. It is mentioned in the paper that the current flight path contains a lot of sudden turns that is not possible to do with SAR UAVs. The performance of the proposed algorithm was tested using simulation.

(Falcon & Abielmona, 2012) worked on risk management for SAR. They focused on the literature gap about combining risk-driven analysis to SAR frameworks. The authors proposed a multi-objective optimization for searching among available actions and resources and validated this method through simulation in Atlantic regions of Canada.

In 2018, (Shih et al., 2018) proposed a method for faster searching in SAR missions. They emphasized the importance of time for increasing the survival rate and saving more lives. They used a greedy search algorithm for path planning, and in the same scenarios, their method outperformed the previous one in terms of speed.

In another research study by (Atif et al., 2021) the impact of localization with unmanned aerial vehicles (UAV) on the SAR missions was studied. In this research, the location of the trapped victims is determined through UAV systems and is sent to SAR crew. Using the results of this paper can lead to faster SAR missions.

(Albrigtsen et al., 2015) conducted interesting research on backup SAR resources in the Arctic. Due to the harsh conditions in the Arctic, some current SAR units might not be enough to serve the incident in that area. Considering this situation, they proposed a system called the “Buddy System” in which two or more SAR resources collaborate to support each other. At the end of the paper, they investigate two case studies, and in both the “Buddy System” improved survival probability.

(Karatas et al., 2017) combined optimization and simulation approaches to develop a method for the dispatching plan of SAR helicopters. They proposed an integer linear programming model to get initial deployment of SAR helicopters. Then, they used the model’s output as the simulation’s input to seek better alternative plans with considering some other constraints. Findings showed that using this hybrid method (in comparison to each of them alone) will result in better solutions.

(Okita et al., 2021) conducted a survey exploring the importance of classification of international SAR teams by the International Search and Rescue Advisory Group (INSARAG). The authors pointed out it is crucial to have this classification because it will improve the response to different situations by the proper SAR team.

In a paper conducted by (Alhaqbani et al., 2021), a fish-inspired algorithm was used for task allocation of UAV vessels in SAR missions. One of the main challenges for this research was time constraints (both running time and rescue time), which was resolved with the proposed algorithm. Rescue time in emergency missions is really crucial for victim survival. This research’s findings show that the proposed method can rescue all the survivors, while previous methods could only rescue 75% of them at best.

2.5 Marine SAR

Marine SAR has been examined by researchers from early 40’s. At that time, this subject was not discussed directly, and researchers considered it to be a peripheral study. For example, in (Michell, 1948), which is about marine navigation between New York and New London, it is stated that radio can have considerable impact on marine transportation. It is also mentioned that radio navigation makes search operations much faster and more reliable. This paper did not provide further explanation about SAR operations, but this study became a basis for commencing more extensive studies.

(Hypher, 1980) discussed SAR in Canada. He mentioned that despite the low population of Canada, there are enormous amounts of marine activities in Canadian shorelines for leisure, shipping, and fishing, and that these marine activities in turn may cause distress or ship disasters which require SAR response operations. This paper summarized the characteristics of Canadian SAR operations and discussed the response doctrine which is in place in Canada.

At the end of 20th century, most of the researchers focused on applications of different SAR methods in real operations and used empirical trial and error to find the best SAR strategy. For example, in 1990 (Redfern, 1990) studied the Merchant Ship Search and Rescue Manual and tried to improve it based on his experience. This manual was published in 1986 for guiding the shipmasters dealing with search and rescue tasks. After some years, the writer felt that this document could be improved in some areas (handling man overboard, conducting searches, and coordinating surface search selection) and published his findings in this paper.

Early years of 21st century was a turning point in marine SAR studies in accordance with the number of published studies in this field. In 2005, the performance of a new sensor system (EO/IR) in marine SAR was investigated. It is stated that one of the most challenging tasks in marine SAR is finding small targets in the sea (like finding a drowning person in high seas). The findings of this paper showed that utilization of this sensor can increase the mission success probability and improve search efficiency. This paper is important to us because it has proposed the concept of success probability in a mission. In this thesis, this concept has been used in another way as well, and one of our objectives in optimization model is maximizing success probability (or minimizing insufficiency probability) in SAR missions.

In another interesting study by (Yong et al., 2010) the application of Virtual Reality (VR) in marine SAR simulators was studied. They believed that with this technology SAR operators can experience a distress situation in advance and be more prepared for the operations in real life. The marine search and rescue simulator (MASRES) which is based on Full Mission Ship Handling Simulator and GMDSS (Global Maritime Distress and Safety System) was evaluated in this paper by three realism indexes (behavioral, environmental, and physical realisms).

In his thesis, (Ashpari, 2012) investigated the impact of different factors on SAR effectiveness. He considered physical characteristics of a SAR offshore vessel (like Maximum speed and number of helicopters and UAVs onboard) and also some environmental factors (like visibility, wind direction, etc.). Ultimately, they quantified all the factors to obtain a function based on probability of detection to see which of them has the most impact on SAR performance.

In recent years, the trend in studies has moved toward mathematical modeling and utilization of decision-making techniques and simulation. In 2013, (Breivik et al., 2013) published a review paper to explain the history of SAR at sea and also summarize the results of studies in this field from 1944 to 2012.

(Shi et al., 2014) developed an accessibility model to evaluate the SAR performance in the context of the scattered Nansha Islands in China. 19 islands were considered as rescue goals and eight SAR bases in four countries were selected for sending resources. Since they had more patrol vessels, China could serve the islands in minimum time. They concluded that it is vital to found some SAR bases on selected islands to decrease the access time.

(Sonninen & Goerlandt, 2015) applied a visual data mining analysis to SAR missions related to recreational boats. They built a database considering wind and wave conditions to compare mission types and investigate miscellaneous engaged SAR organizations. Two other parameters were also considered, namely travel distance to incident and occurrence time.

(Lee & Morrison, 2015) developed a mixed integer linear program (MILP) for generating effective SAR operation plans. They focused on UAVs as SAR vessels and considered access to fuel stations as an objective for persistent SAR operations.

(Goerlandt et al., 2015) developed a risk-informed SAR performance evaluation. They defined seven risk indicators (RIs) and four performance indicators (PIs) with consultation of Finnish Lifeboat Institute's (FLI) experts and investigated all of them in predetermined sub-areas in Finnish part of the Gulf of Finland. Findings of this research showed that response performance is great in most of the locations.

In another paper published by (L. Wei & wenyuan, 2016), the selection method of SAR ships was studied. In this research, the authors implemented three kinds of multi-objective selection methods to choose the best vessels for conducting SAR missions which is crucial for effective rescue. One of the methods used is fuzzy similar priority ratio evaluation which shows the importance of considering uncertainty in these problems. It should be noted that in the papers published in these years, uncertainty has not lost its significance and scholars have tried to include uncertainty in scientific and theoretical methods.

In research conducted by (Razi & Karatas, 2016a), they proposed a mixed integer model for boat allocation with minimizing response time, cost, and workload imbalance. They classify different types of incidents based on severity and implemented their model on Aegean Sea in collaboration with the Turkish Coast Guard. Findings of this research showed more effective utilization of the SAR resources in comparison to previous allocations.

(Malyszko & Wielgosz, 2016) proposed a conceptual framework and decision support system (DSS) for SAR operations. They investigated different factors which impact SAR decisions and showed them in a flowchart. The working procedures and system principles were explained.

(K. Wang & Liu, 2016) conducted research for evaluating the SAR capability in Zhoushan Sea area in China. They identified four categories of risk factors, namely: human factors, environmental factors, rescue factors, and SAR communications factors. Using fuzzy comprehensive evaluation, they obtained an evaluation index for each factor and claimed that the findings can be used to improve SAR capability in the Zhoushan Sea.

(Lv et al., 2018) proposed an improved Particle Swarm Algorithm (PSA) to generate the search plan for maritime static targets. First, they divided the area into sub-regions and implemented their method on those. Findings showed that success probability increased by 16.4% in comparison to the original PSA.

(X. Zhou et al., 2019a) studied the possibility of founding SAR bases on islands to improve the coverage of the system. They chose some candidate islands for SAR bases and calculated the response time with maximal covering location problem. They implemented their proposed framework in the South China Sea and concluded that if island bases enter the system, the primary coverage will approximately increase by 18%.

(Guo et al., 2019) studied long-range maritime SAR which is more challenging compared to short-range. In contrast to previous research, they investigated the multiple resources in different modes (airborne and marine vessels) and developed an integer non-linear programming (INLP) model to allocate the fleet in a sustainable manner. Due to being NP-hard, Genetic Algorithm II (NSGA-II) was used to solve the model. After implementation in the Chinese Bohai Sea, findings of this paper provided valuable information with coast guard members.

(X. Zhou et al., 2020) developed a multi-step comprehensive framework for evaluation of SAR capability in the South China Sea. In the first and second steps, they calculated response time and demand based on GIS models and historical data. At the final step, they evaluate SAR capability in accordance with three criteria (primary coverage, weighted coverage, mean access time). Two scenarios were considered for this assessment. One of them is with the collaboration of different countries bordering South China Sea and another one is without that. Findings showed that cooperation of the different countries will increase the SAR coverage drastically.

(H. Liu et al., 2021) presented an SAR model based on a helicopter response plan with uncertainty. Uncertain factors in this study are captured in a multi-agent method which uses Discrete Event System (DEvS) for considering the actions on each mission. In this project the Monte Carlo method was used for calculation of incidents' distribution probability and robustness of response plan to get the best results.

(Karatas, 2021) developed a dynamic multi-objective location allocation model for SAR planning. The presented model is a linear mixed-integer programming which aims to optimize response time, workload balance, and cost. In this paper demand is considered uncertain and rescue vessels contain aerial and land vessels. The authors used goal programming with simulated demand for solving their model.

We can roughly divide the marine SAR studies (in terms of uncertainty) into three periods. In the first one (1950 to 1980), researchers discussed uncertainty, but they did not put forward any solution for addressing this issue. In the second period (1980 to 2000), researchers focused on computer simulation to overcome the uncertainty problem. In the third period (2000 up to now), the emphasis of the research has been on mathematical models, stochastic and fuzzy models, and decision-making theories.

Searching the literature on marine SAR articles, we can find quite a few studies showing the importance of this subject in Canada. For example (Ford & Clark, 2019) studied the Canada's role in Arctic SAR considering the climate change in this area. The authors argued that given substantial climate change in the Arctic and increasing international transportation through Arctic routes, all the countries in the area should be prepared to provide SAR services. The authors believe that Canada has not been successful, and more training and resources should be provided for this country to enhance SAR in the Arctic.

(Akbari, Eiselt, et al., 2018b) proposed a multi-criteria model for maritime SAR problem in Atlantic Canada. They modified their model based on P-median and maximal covering problems and considered mean access time, primary coverage, and backup coverage as their criteria in the objective function. The findings showed significant improvements in resource efficiency in comparison to the present situation.

In research done by (Akbari, Pelot, et al., 2018a), they developed a multi-objective location allocation model for SAR vessels. They considered coverage and access time for the objectives and used the goal programming method to solve the problem. Implementation of this model in the Atlantic region of Canada has shown better access time and coverage comparing to previous SAR deployment. It should be mentioned that the model presented in this paper was the basis for developing the model in this thesis.

In another study by (Kikkert & Lackenbauer, 2021), the authors investigated Canadian Coast Guard Auxiliary (CCGA) which provides marine SAR and boating safety in Canada's Maritime provinces. They mentioned that even though in 2015 there were only nine auxiliary units, 20 active units were established in 2020 (most located in Inuit Nunangat) and they are planning for future expansion. The objective of this study is discussing the expansion of this plan and its impact on marine safety in Canada's Arctic.

Another recent study which investigates the marine SAR station selection problem is (Hadi et al., 2021). In this study, the authors used the set-covering method for selecting the optimum location of SAR stations. They also proposed a conceptual framework for designing the infrastructure with respect to historical data on routes and station locations. The proposed model was implemented on the western part of Indonesia, and the findings have shown that the new plan has increased the coverage rate in that area (number of covered incidents / total number of incidents). The importance of this research is addressing back-up coverage along with primary coverage due to lack of sufficient SAR units in that area.

Taking all the reviewed papers into account, we can analyze marine SAR problems from the following points of view:

- Uncertainty: in this thesis we have addressed this issue by calculation of demand probabilities in each grid with respect to historical data.
- Coverage: in this thesis potential coverage has been considered with a new approach (based on a reliability concept) to reduce the insufficiency probability. It should be mentioned that potential (backup) coverage comes after primary coverage as a support for the SAR system.
- Capability: one of the most challenging issues in marine SAR is dispatching the most capable vessel to the scene with respect to incidents' characteristics. In this thesis an effectiveness rating table has been defined to match the incidents and SAR vessels based on valid criteria.
- Case study: in this thesis a real case study of marine SAR in Atlantic region of Canada has been investigated.

2.6 Spatial Analysis (Kernel Density)

In this section we will briefly review the history of spatial analysis and its applications along with the Kernel Density method which has been used in this thesis. Ground mapping has a long history in geography, as studies show that Egyptians used ground mapping from 1400 BC which this reflects the fundamental importance of this subject.

Basic studies about spatial analysis started from the early 1950s with increasing attention to maps and their applications. (Birdsell, 1950) conducted a survey about human race trends and changes with respect to spatial analysis. This paper shows the diversity of spatial analysis application to different fields. It was mentioned that with extraction of data from geographical maps and utilizing spatial analysis method, a lot of useful insights can be obtained in different domains.

In 1999, (Goodchild & Longley, 1999) published a paper and developed a model for addressing the future of geographical information systems (GIS) and spatial analysis. It is stated that the advent of GIS technology was a huge breakthrough in the field of spatial analysis, because it provides massive databases with low cost and high speed. After the development of GIS technology in different domains, many researchers started writing practical books for explaining the basics of spatial analysis and its application in various fields. One of them is (Stillwell & Clarke, 2003) in which the authors provided a lot of case studies and best practices from different businesses and also public sectors. The focus of this book is showing the implementation of spatial analysis and investigating GIS components along with spatial methodologies in practice. Another interesting book published in 2014 by (Dale & Fortin, 2014) which argues that spatial analysis is one of the most important parts of ecology. This book provided a guideline for ecologists to choose the best spatial analysis methods. It is stated that due to the wide range of methods in this field, selecting the right method would be challenging even for experts.

Spatial analysis problems are still a hot topic for the researchers after thousands of years and the volume of published papers in the last two years can be proof of this claim. It should be mentioned that this field (like other fields) has been influenced by the Covid-19 pandemic and quite a few papers have implemented spatial analysis on the outbreak regions of this virus. (Franch-Pardo et al., 2020) reviewed 63 papers about geographical dimensions of Covid-19 pandemic. This research classified geospatial analysis into five categories (spatiotemporal analysis, health and social geography, environmental variables, data mining, and web-based mapping). It has been mentioned that obtaining information about spatiotemporal dynamic of this virus is vital for decreasing the spread and can be helpful for planning and decision making. The authors emphasized that it is necessary to deal with Covid-19 as an interdisciplinary subject (medicine, mathematics, and social science) and face it with a global approach, because international collaboration is needed to prevent the spread of this disease.

In most of the published papers, historical data for the location of incidents have been defined as points on a map. So, the calculation of density of demand requires a method to assign a weight to areas near the incidents based on the distance and number of occurrences. Kernel density estimation, which is one of the most common methods of density calculation, has been used in this thesis. Utilizing this method by researchers has an extensive background, but from the late 70s many studies were done to improve this method and enhance the convergence rate ((Devroye & Wagner, 1979), (Terrell & Scott, 1980)). Implementation of this non-parametric method for density function estimation has been popular in recent years and in different subjects like routing and Covid-19 risk identification ((S. Zhang et al., 2021), (Shi et al., 2021)).

All the mentioned studies highlight the importance of Kernel Density estimation as one of the most common and efficient methods to fit a distribution for spatial data. In this thesis, we have implemented this method with ArcGIS Pro software to conduct the heat map analysis of the marine incidents and fit a distribution to our historical data. The details of this method and the utilized parameters will be explained in the methodology chapter.

In the next section we will summarize the highlighted papers in Table 2 to show the research gap and introduce our contributions to fill this gap. In other words, Table 2 shows the position of our research within the literature on this subject.

Table 2. Literature review table

Paper	Uncertainty	Solution Method	Model	Special Constraints	Incident type definition
(Razi & Karatas, 2016)	Using historical data without considering uncertainty	Exact, decision-making theory techniques	Multi-objective mathematical modeling	Boat capacity	Yes, binary vessel-incident matching
(Akbari, Eiselt, et al., 2018a)	Using distribution function of demand grids	Approximate, Simulation with random scenarios	P-median, MCLP	No	No
(Akbari, Pelot, et al., 2018b)	Using distribution function of demand grids	Approximate, Simulation with random scenarios	Mathematical modeling with Goal programming	Off-shore stations and capacity constraints	No
(Hadi et al., 2021)	Using real data without considering uncertainty	Approximate, mathematical analysis	Mathematical modeling with simple analysis	No	No
(X. Zhou et al., 2019b)	Estimation of demand with historical data	Approximate, simple heuristic method	Mathematical modeling, MCLP	No	No
(Karatas et al., 2017)	Generating random data	Approximate, mathematical optimization and simulation	Mathematical modeling	Helicopter capacity	Yes
(Karatas, 2021)	Generating random data	Approximate, simulation	Multi-objective Mathematical modeling, MILP	Boats' workload, Station capacity	Yes
(Lee & Morrison, 2015)	Defining probability of success for missions	Exact, using IBM ILOG CPLEX	Mathematical modeling, MILP	Number of UAVs eligible to fly in each period	No
(X. Zhou et al., 2020)	Estimation of demand using historical data	Approximate, heuristic	Mathematical modeling	No	No
Present Research	Probabilistic demand grids/ obtaining distribution function using historical data and KD estimation	Exact, Dynamic programming for small scale Approximate, meta-heuristics for large scale	Binary location-allocation model	Vessels' capacity, Seasonal and offshore stations	Yes, Incident-vessel matching with defining effectiveness ratings

The contributions of the current research are as follows:

- Developing a binary location-allocation optimization model for SAR operations
- Developing an exact solution method with a dynamic programming formulation

- Defining new groups of incidents based on similar characteristics
- Introducing new effectiveness ratings for assessing the capability of each response vessel and incident type
- Consideration of potential coverage with utilizing a reliability concept
- Solving the model with meta-heuristics algorithms in large scale
- Implementation of the model with real data in Atlantic region of Canada

3. CHAPTER 3 METHODOLOGY

3.1 Demand Pattern Analysis

One of the biggest challenges of optimization problems is dealing with demand uncertainty. In this thesis, the location and volume of the maritime SAR incidents (i.e., the demand) are uncertain due to the context and many non-deterministic factors involved in the problem such as weather, variable activity locations, ocean currents, etc.

Due to the fact that the available information for predicting the future demand pattern is not reliable, we used the historical data for analyzing the current demand and predicting future demand. To address stochasticity better, we should find a suitable distribution for future demand, because deterministic incident points cannot be an appropriate reflection of future.

The method that has been used to fit a distribution for our data is Kernel Estimation (KE). KE is a popular and effective method for analyzing spatial point patterns. The mechanism of KE is that it searches the neighboring areas around the kernel center and computes a function of distance from all incidents within a predetermined bandwidth. The calculated value is considered to be the density of occurrence around that point. Therefore, this method does not ignore potential changes in demand location. Another advantage of Kernel estimation is its ease of implementation with gridded data (like the data we have in this thesis). KE uses a function to determine how the influence and importance of each point varies (inversely) with the distance from the center. The general formulation of KE is as follows:

$$\widehat{\lambda_k}(O) = \sum_{i=1}^{n} \frac{1}{\tau^2} K(\frac{O - O_i}{\tau})$$

In this formulation τ is the predetermined bandwidth and $O - O_i$ shows the distance between kernel center (O) and the incident location (O_i).
Different kernel functions have been used for KE. One of the most common functions for analyzing point patterns is quartic (Bailey & Gatrell, 1995) which is given by:

$$\widehat{\lambda_k}(O) = \sum_{d_i \le \tau} \frac{3}{\pi\tau^2} (1 - \frac{d_i^2}{\tau^2})^2$$

where d_i is the distance between the center of the kernel (O) and the incident location O_i.

3.2 Response capability

Introducing effectiveness ratings to the objective function of our optimization model is one of the contributions of this thesis. An effectiveness rating determines the capability of a vessel for responding to a specific kind of incident. In other words, the purpose of this rating is matching the response vessels and incidents in such a way that the appropriate vessel (in terms of required capabilities) is dispatched to the incident. In this regard, it is necessary to categorize both the incidents and vessels for simplifying the model.

3.3 Incidents' classification

As mentioned in Chapter 1, there are 20 different incident types in the SISAR database, but some of them have been ignored in the categorization process due to being airborne related incidents (the focus of our thesis is on maritime incidents), false alarms, or scarce occurrences. After consultation with subject-matter experts (SMEs), we came up with a four-group classification of incidents. The criteria for this classification were having similar characteristics and requiring similar response vessels. Table 3 shows the final classification of maritime incidents.

Table 3. Defined groups of maritime incidents

Group 1: Disabled, Disoriented, Overdue
Group 2: Capsized, Foundered, Taking on water, On fire
Group 3: Medical, Man Overboard, Person in Water, Stranded (Person)
Group 4: Grounded

3.4 Vessels' classification

In accordance with CCG website, there are 26 active SAR vessels in the Atlantic region of Canada, each with distinct capabilities and specifications. Consulting with SMEs guided us through the classification process, and we thus obtained five vessel classes. The specifications of each class are shown in Table 4:

Table 4. Vessel classes' specifications

Vessel Class	Vessel Type	Range (Km)	Vessel Length (m)	Cruising Speed (Km/hr)	Number Available
Medium Endurance lifeboats (Cape Class)	Class 1	185	14.6	37	5
High Endurance lifeboats (Bay Class)	Class 2	231	15.8 - 19	37	11
Mid-shore Class	Class 3	1852	42.8	26	2
Large Multi-Task Class	Class 4	6019	69.7 - 83	28	5
Off-shore Class	Class 5	9260	62.4 - 72	22	3

3.5 Matching Vessels and incidents

The first element needed for matching incidents and vessels is a reliable capability measure (criteria) to assess them based on that. To this end, the CCG Risk-based Analysis of Maritime SAR Delivery (RAMSARD) document was used in this thesis. RAMSARD is a new risk-based methodology for the Search and Rescue Needs Analysis to help identify the operational risks in each SAR area, including any need for additional capacity or capabilities from partners. In this document, which was first implemented in 2018, some SAR capabilities were introduced for both marine and air SAR. There are 11 SAR capabilities as shown in Table 5:

Table 5. RAMSARD SAR capabilities

A	Speed
B	Endurance / Range
C	Sea keeping
D	Search
E	Survivor Recovery/Care/Transportation
F	First Aid / Medical
G	On Scene Coordination
H	Towing
I	Fire Protective Equipment
J	Dewatering
K	Redundancy

This document contains a ratings table for each of the mentioned capabilities, which rates SAR vessels from 1 to 7. A vessel that gets rate 7 is the most capable and the one that is rated 1 would be the least capable on that capability.
These tables are reproduced provided in Table 6 to Table 16:

Table 6. RAMSARD rating table for speed capability (source: RAMSARD, 2018)

Capability	**A – Speed (Sp)**
Rating	**Criteria**
7	Vessel able to make 40 knots or greater.
6	Vessel able to make 35 knots or greater.
5	Vessel able to make 30 knots or greater.
4	Vessel able to make 25 knots in fair conditions, or major vessel able to launch independent Fast Rescue Craft (FRC) that can make 25 knots.
3	Vessel able to make 20 knots.
2	Vessel able to make 15 knots.
1	Vessel able to make 10 knots or less.

Table 7. RAMSARD rating table for Endurance/Range capability (source: RAMSARD, 2018)

Capability	B – Endurance / Range (End)
Rating	**Criteria**
7	Vessel range of at least 800 NM and greater than 40 hours of continuous operation.
6	Vessel range of at least 600 NM and 30 hours of continuous operation.
5	Vessel range of at least 400 NM and 20 hours of continuous operation.
4	Vessel range of at least 200 NM and 10 hours of continuous operation.
3	Vessel range of less than 200 NM and 10 hours of continuous operation.
2	Vessel range of less than 100 NM and 5 hours of continuous operation.
1	Vessel range of less than 50 NM and 3 hours of continuous operation.

Table 8. RAMSARD rating table for Sea Keeping capability (source: RAMSARD, 2018)

Capability	C – Sea Keeping (SK)
Rating	**Criteria**
7	Vessel able to operate effectively in storm conditions (winds of 50-55 knots) and sea state 10 (9-12.5 meters in open sea).
6	Vessel able to operate effectively in a strong gale (winds of 45 knots) and sea state 9 (7-10 meters in open sea).
5	Vessel able to operate effectively in a gale (winds of 35-40 knots) and sea state 8 (5.5-7.5 meters in open sea)
4	Vessel able to operate effectively in a near gale (winds of 30 knots) and sea state 7 (4-5.5 meters in open sea).
3	Vessel able to operate effectively in a strong breeze (winds of 25 knots) and sea state 6 (3-4 meters in open sea).
2	Vessel able to operate effectively in a fresh breeze (winds of 20 knots) and sea state 5 (2-2.5 meters in open sea).
1	Vessel able to operate effectively in a moderate breeze (winds of 15 knots) and sea state 4 (1-1.5 meters in open sea).

Table 9. RAMSARD rating table for Search capability (source: RAMSARD, 2018)

Capability	**D – Search (S)**
Rating	**Criteria**
7	Vessel has all equipment noted below; at least 20 feet height of eye; and sufficient crew to conduct visual and electronic searches simultaneously.
6	Vessel has all equipment noted below and Forward Looking Infrared (FLIR).
5	Vessel has baseline equipment plus Self Locating DMB (SLDMB).
4	Vessel has the following equipment and attributes: electronic navigation equipment sufficient to conduct extended search in restricted visibility; enclosed bridge with at least 8 feet height of eye; Data Marker Buoy (DMB); Direction Finder (DF); binoculars; search light with minimum candle power; and night vision equipment.
3	Vessel has electronic navigation equipment, but does not carry one of the following: enclosed bridge with at least 8 feet height of eye; DMB; DF; binoculars; search light with minimum candle power; or night vision equipment.
2	Vessel has electronic navigation equipment, but does not carry two of the following: enclosed bridge with at least 8 feet height of eye; DMB; DF; binoculars; search light with minimum candle power; or night vision equipment.
1	Vessel does not have electronic navigation equipment or does not carry: enclosed bridge with at least 8 feet height of eye; DMB; DF; binoculars; search light with minimum candle power; nor night vision equipment.

Table 10. RAMSARD rating table for Survivor Recovery, Care and Transportation capability (source: RAMSARD, 2018)

Capability	**E – Survivor Recovery, Care and Transportation (Rec)**
Rating	**Criteria**
7	Vessel can carry more than 50 survivors in a sheltered location.
6	Vessel can carry more than 25 survivors in a sheltered location.
5	Vessel can carry more than 12 survivors in a sheltered location.
4	Vessel can carry less than 12 survivors in a sheltered location.
3	Vessel can carry less than 12 survivors in an exposed location.
2	Vessel can carry less than five survivors in a sheltered location.
1	Vessel can carry less than five survivors in exposed location.

Table 11. RAMSARD rating table for First Aid / Medical Training, Space and Equipment capability (source: RAMSARD, 2018)

Capability	**F – First Aid / Medical Training, Space and Equipment (FA)**
Rating	**Criteria**
7	Doctor of Emergency Medicine or equivalent.
6	Advanced Care Paramedic (ACP) or equivalent (e.g., Physician's Assistant).
5	Primary Care Paramedic (PCP) or equivalent (e.g., CAF SAR Technician).
4	CCG Rescue Specialist or equivalent (e.g., Emergency Medical Responder [3-week training course]) with SAR first aid equipment as per CGFO 207 or equivalent, and sheltered space for at least one stretcher patient.
3	Advanced first aid training (e.g., Marine Advanced First Aid, Medical First Responder, Advanced Wilderness First Aid, OFA 3 [1- or 2-week course]) or no shelter for at least one stretcher patient.
2	Standard first aid training (e.g., Marine Basic First Aid, Standard First Aid [2-day course]).
1	No first aid training (vessel may have First Aid trained person on board, but there is no requirement that this be carried).

Table 12. RAMSARD rating table for On-Scene Coordination capability (source: RAMSARD, 2018)

Capability	**G – On-Scene Coordination (OSC)**
Rating	**Criteria**
7	Vessel has capability to co-ordinate air search in addition to the following attributes: sufficient communications equipment (minimum 2 VHF - FM radio sets); an enclosed bridge with space sufficient to lay out marine chart; and personnel trained as On-Scene Coordinator.
6	In addition to the attributes below Vessel has sufficient crew to conduct simultaneous visual and electronic searches in addition to the following attributes: sufficient communications equipment (minimum 2 VHF - FM radio sets); an enclosed bridge with space sufficient to lay out marine chart; and personnel trained as On-Scene Coordinator.
5	Vessel has sufficient crew to have a full navigational watch and an On-Scene Coordinator in addition to the following attributes: sufficient communications equipment (minimum 2 VHF - FM radio sets); an enclosed bridge with space sufficient to lay out marine chart; and personnel trained as On-Scene Coordinator.
4	Vessel has the following attributes: sufficient communications equipment (minimum 2 VHF - FM radio sets); an enclosed bridge with space sufficient to lay out marine chart; and personnel trained as On-Scene Coordinator.
3	Vessel is missing one of the attributes from rating level 4 criteria
2	Vessel is missing two of the attributes: from rating level 4 criteria

Capability	**G – On-Scene Coordination (OSC)**
Rating	**Criteria**
1	Vessel is missing three of the attributes: from rating level 4 criteria

Table 13. RAMSARD rating table for Towing capability (source: RAMSARD, 2018)

Capability	**H – Towing (Tow)**
Rating	**Criteria**
7	Vessel is fitted for towing large displacement hull vessels and has a bollard pull of greater than 50 tonnes
6	Vessel is fitted for towing large displacement hull vessels and has a bollard pull of 20 to 50 tonnes
5	Vessel is fitted for towing displacement hull vessels greater than 36 feet and has a bollard pull of less than 20 tonnes
4	Vessel is fitted for towing a displacement hull vessel of at least 36 feet in 30 knot winds
3	Vessel is fitted for towing a displacement hull vessel of at least 30 feet in 20 knot winds
2	Vessel is fitted for towing a planning hull vessel of at least 24 feet in 20 knot winds
1	Vessel is not fitted for towing (no tow post or tow line)

Table 14. RAMSARD rating table for Fire Protective Equipment capability (source: RAMSARD, 2018)

Capability	**I – Fire Protective Equipment (FPE)**
Rating	**Criteria**
6	Vessel has: • capability to refill self-contained breathing apparatus (SCBA) bottles on board; • spare SCBAs and bottles that can be transferred to casualty; • external fire monitor(s) to provide protective spray to allow safe approach; • capacity to rig fire hoses to provide protective spray to allow safe approach; • additional extinguisher(s) that can be transferred to casualty
5	Vessel has: • spare SCBAs and bottles that can be transferred to casualty; • external fire monitor(s) to provide protective spray to allow safe approach; • capacity to rig fire hoses to provide protective spray to allow safe approach; • additional extinguisher(s) that can be transferred to casualty
4	Vessel has: • external fire monitor(s) to provide protective spray to allow safe approach; • capacity to rig fire hoses to provide protective spray to allow safe approach; • additional extinguisher(s) that can be transferred to casualty
3	Vessel has: • capacity to rig fire hoses to provide protective spray to allow safe approach; • additional extinguisher(s) that can be transferred to casualty
2	Vessel has additional extinguisher(s) that can be transferred to casualty
1	Vessel carries no additional fire protective equipment

Table 15. RAMSARD rating table for Dewatering capability (source: RAMSARD, 2018)

Capability	**J – Dewatering (DeW)**
Rating	**Criteria**
4	Vessel has a high-capacity submersible pump that can be deployed to another vessel
3	Vessel has two dewatering pumps, including one that can be deployed to another vessel
2	Vessel has a dewatering pump (minimum 3.5 hp) that can be deployed to another vessel
1	Vessel has no portable dewatering capability

Table 16. RAMSARD rating table for Redundancy / Robustness capability (source: RAMSARD, 2018)

Capability	**K – Redundancy / Robustness (R/R)**
Rating	**Criteria**
7	Vessel: • has double hull; • stability condition for deck icing; • three independent means of position fixing; or two compasses or VHF DF; and • Is twin screw and has backup steering system
6	Vessel: • stability condition for deck icing; • has three independent means of position fixing; or two compasses; or VHF DF; and • is twin screw and has backup steering system.
5	Vessel: • has double hull; • has three independent means of position fixing; or two compasses; or VHF DF; and • is twin screw and has backup steering system.
4	Vessel: • has three independent means of position fixing; or two compasses; or VHF DF; and • is twin screw and has backup steering system.
3	Vessel is twin screw and has backup steering system.
2	Vessel is twin screw.
1	Vessel has no redundancy of systems.

It should be mentioned that the highest rating for “Dewatering” and “Fire protective equipment” is 4 and 6 respectively, because vessels with the most advanced ratings are not available in Canada.

3.6 Vessels' capability Ratings Table

In this step we create a table to rate our vessel classes based on RAMSARD capabilities. After holding some meetings with SMEs, and with due attention to RAMSARD tables and specifications of each vessel class, ratings have been assigned. Table 17 shows the details of the ratings for the five vessel classes listed in Table 18.

Table 17. Vessels' capability ratings table

	A (Sp)	B (End)	C (SK)	D (S)	E (Rec)	F (FA)	G (OSC)	H (Tow)	I (FPE)	J (DeW)	K (R/R)
C1	3	3	5	6	4	4	4	5	3	3	4
C2	3	3	6	6	5	4	5	5	3	3	4
C3	2	7	6.5	6.5	6	4	6	5.5	5	4	5
C4	2	7	7	5	7	4	6	6	5	4	6
C5	2	7	7	5	7	4	6	6	4	4	6

Table 18. Defined vessel classes

C1	C2	C3	C4	C5
Medium Endurance Lifeboat	High Endurance Lifeboat	Mid-shore Vessels	Large Multi-task Vessels	Off-shore Vessels

3.7 Incidents' capability ratings table

In this section, we want to determine how much relevant each response vessel capability is for group of incidents. In other words, the incidents' capability ratings table shows the relative importance of each capability in response to each group of incidents. For quantifying this subject, we defined a scale from 0 to 10. Zero indicates no relevance and 10 indicates very high relevance for a given capability for that particular group of incidents. This scale is shown in detail in Figure 10.

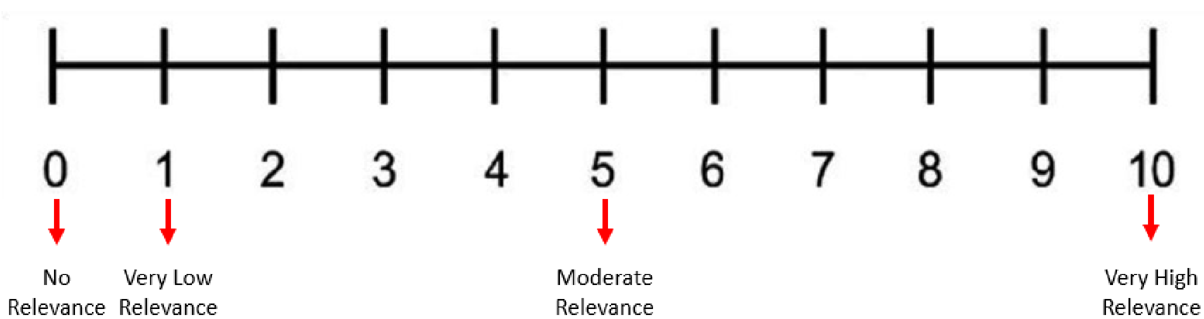

Figure 10. incident-capability relevance scale

As in the previous section, we asked our group of SMEs to guide us through assigning rates to our four defined groups of incidents (G1 to G4). Due to the importance of "Speed" and "Sea Keeping" capabilities for responding to all SAR operations, we reached a consensus to consider the highest rating (10) for them. Table 19 is the finalized incidents' capability ratings table for the four incident groups listed in Table 20.

Table 19. Incidents' capability ratings table

	A (Sp)	B (End)	C (SK)	D (S)	E (Rec)	F (FA)	G (OSC)	H (Tow)	I (FPE)	J (DeW)	K (R/R)
G 1	10	6	10	9	2	3	9	10	2	3	8
G 2	10	7	10	6	9	9	6	7	10	10	8
G 3	10	7	10	8	8	9	8	1	2	2	7
G 4	10	8.5	10	1	7	4	3	9	2	8	10

Table 20. Defined incident groups

G1	G2	G3	G4
Disabled Disoriented Overdue	Capsized Foundered Taking on water On fire	Medical Man Overboard Person in Water Stranded (Person)	Grounded

3.8 Final Rating Table

To obtain the final rating table which is used as an input to the optimization model, we will multiply the elements of each row (element by element) and calculate their sum. The obtained value will be one element of the final table and shows the relative capability of each vessel class in response to each group of incidents. The calculation procedure and final rating table are provided in Table 21 and Table 22 respectively.

Table 21. Final rating table calculation process

	Vessel class 1 (k indices)	.	Vessel class 5
G1 q indices	$Ra_{qk} = \sum_{z=Capability\ 1}^{Capability\ 11} Vessel's\ Rating_z \times Incident's\ Rating_z$ For $q = 1$ and $k = 1$		
…			
G4			

Table 22. Final rating table

	C1	C2	C3	C4	C5
G1	305	326	380.5	387	385
G2	360	385	467.5	484	474
G3	294	320	376.5	385	383
G4	282.5	302.5	368.5	393.5	391.5

3.9 Optimization Model

The model developed in this thesis is the extension of the work presented by (Akbari, Pelot, et al., 2018b). The proposed model is a non-linear programming problem with binary variables. The three parts that have been included in the objective function are minimizing insufficiency probability, minimizing cost, and maximizing effectiveness ratings.

Insufficiency probability happens when the number of actual incidents in a grid is more than the expected value of incident occurrence. The number of potential vessels (vessels that can be dispatched to an incident considering time and distance constraints) have been used to reduce the insufficiency probability, and a concept of reliability in parallel systems has been utilized to formulate the problem. In short, the insufficiency probability increases the more the actual number of incidents exceeds the expected number, but then can be decreased with the addition of more response resources.

Operational and procurement costs of SAR vessels can be very important for the CCG to offer a reliable service while simultaneously managing its costs. Consequently, these costs have been considered in the objective function. Another factor which has been included in the objective function is maximizing the effectiveness ratings. These ratings prioritize that the dispatched vessel is very capable for serving a given incident. The calculation process of ratings was presented in the previous section.

Indices, parameters, and variables used in the model are listed and defined below.

3.9.1 Indices

$i \in I$	Demand Locations
$j, j' \in J$	Potential Vessel Stations
$l \in L$	Vessel number
$k \in K$	Index "for" vessel types
$\theta \in \phi$	Index "for" relocation periods
$q \in Q$	Index "for" incident groups

3.9.2 Variables

$x_{ijkql\theta}$	Binary variable for main coverage at grid i from station j in period θ by vessel type k number l for incident group q
$y_{ijkql\theta}$	Binary variable for potential coverage at grid i from station j in period θ by vessel type k number l for incident group q

$z_{jkl\theta}$	Binary variable for vessel type k number l located at station j in period θ
$u_{kl\theta}$	1 if vessel type k number l has been relocated at the beginning of period θ, otherwise 0
av_{ijkl}	1 if vessel type k number l from station j is in range to respond grid i, otherwise 0

3.9.3 Parameters

rng_{kl}	Coverage distance (range) of vessel type k, number l
Ra_{qkl}	Rating of vessel type k, number l in dealing with incident group q
$ac_{kl\theta}$	Cost of vessel type k, number l in period θ
rc_{kl}	Cost of relocation of vessel type k, number l
$c_{kl\theta}$	Response_Capacity of vessel type k, number l in period θ (Number of incidents that can be responded to in a period by a vessel)
sp_{kl}	Cruising speed of vessel type k, number l
d_{ij}	Distance between grid i and station j
$E(i,q,\theta)$	Expected value of incident group q in period θ for grid i
$\varphi(i,q,\theta)$	Number of actual incidents group q in period θ for grid i
t	Coverage time limit for acceptable level of coverage
ρ	Coefficient of the costs in the objective function
μ	Coefficient of the ratings in the objective function
β	Coefficient of the total coverage in the objective function
von_{kl}	1 if vessel type k number l is onshore, otherwise 0
sof_j	1 if station j is offshore, otherwise 0
$as_{j\theta}$	1 if station j is available in period θ, otherwise 0
M	Large number

The formulation of the proposed model is described in detail below.

3.9.4 Model formulation

$Minimize$:

$$\Delta = \sum_{i,\theta,q}\left(\left(p(\varphi(i,q,\theta) > E(i,q,\theta))^{\sum_{j,k,l} y_{ijkql\theta}}\right)\right) + \rho\left(\sum_{j,k,l,\theta} ac_{kl\theta} \,.\, z_{jkl\theta} + \sum_{k,l,\theta} rc_{kl}.\, u_{kl\theta}\right) - \mu \sum_{i,j,k,\theta,l,q}\left(x_{ijk\theta lq}\ \ Ra_{qkl}\right) + \beta \frac{\sum_{i,j,k,\theta,l,q}\left(-\left\lceil\frac{-E(i,q,\theta)}{M}\right\rceil - x_{ijk\theta lq}\right)}{\sum_{i,\theta,q} - \left\lceil\frac{-E(i,q,\theta)}{M}\right\rceil} \tag{1}$$

$s.t.$

$$\sum_{j,k,l} x_{ijk\theta lq} \leq M \cdot E(i,q,\theta) \qquad \forall\, i,\theta,q \tag{2}$$

$$\sum_{i,q} x_{ijk\theta lq} \leq M.z_{jkl\theta} \qquad \forall\, j,k,l,\theta \tag{3}$$

$$y_{ijkql\theta} \leq z_{jkl\theta} - x_{ijkql\theta} \quad \forall\, i,j,k,l,q,\theta \tag{4}$$

$$\sum_{q,\theta} x_{ijk\theta lq} \leq M.av_{ijkl} \qquad \forall\, i,j,k,l \tag{5}$$

$$\sum_{q,\theta} y_{ijk\theta lq} \leq M.av_{ijkl} \qquad \forall\, i,j,k,l \tag{6}$$

$$av_{ijkl} \leq sgn\left(rng_{kl} - d_{ij}\right) + 1 \quad \forall\, i,j,k,l \tag{7}$$

$$av_{ijkl} \leq sgn\left(t - \frac{sp_{kl}}{d\ _{ij}}\right) + 1 \quad \forall\, i,j,k,l \tag{8}$$

$$\sum_{j} z_{jkl\theta} \leq 1 \qquad \forall\, k,l,\theta \tag{9}$$

$$\sum_{i,j,k} x_{ijk\theta lq}.E(i,q,\theta) \leq c_{kl\theta} \qquad \forall\ k,l,\theta \tag{10}$$

$$von_{kl}\,.z_{jkl\theta} \leq 1 - sof_j \qquad \forall\ j,k,l,\theta \tag{11}$$

$$u_{kl0} = 0 \quad \forall\ k,l \tag{12}$$

$$u_{kl\theta} = z_{jkl\theta}\,.\sum_{j\prime \neq j} z_{j'kl(\theta-1)} \qquad \forall\ j,k,l,\theta \geq 1 \tag{13}$$

$$\sum_{k,l} z_{jkl\theta} \leq as_{j\theta} \qquad \forall\, j,\theta \tag{14}$$

$$x_{ijkql\theta}\,, y_{ijkql\theta}\,, z_{jkl\theta}\,, u_{kl\theta}\,, av_{ijkl} \in \{0,1\} \tag{15}$$

Objective function (1) minimizes: insufficiency probability (first part); normalized costs (second part); and lack of primary coverage (fourth part); along with maximizing normalized capability ratings (third part).

Constraint 2 guarantees that there is no allocation when we do not have demand for a grid. Constraint 3 shows that a vessel must be available at a station in order to dispatch to a grid.

Potential coverage can be done when the vessel is available at the station and also no primary coverage is done from the same station in the same period. It is assumed that insufficiency probability is defined for potential coverage, because in accordance with the expression $p(\varphi(i,q,\theta) > E(i,q,\theta))$ we already have at least $E(i,q,\theta)$ demand coverage. Thus, primary coverage should not be calculated in this section, and constraint 4 is a guarantee for that.

According to constraints 5 and 6, coverage is achieved when the demand grid is in range of station coverage.

Constraints 7 and 8 state that av_{ijkl} can be 1 if and only if the sign functions' output are 0 or 1 (see Figure 11). In other words, constraint 7 ensures that the demand grid is in coverage distance (range) of the vessel and constraint 8 guarantees that the vessel can respond to the demand grid within the coverage time limit.

Based on constraint 9 each vessel should be located in one station throughout the period and relocations are allowed only at the end of each period.

Constraint 10 is the Response-Capacity constraints of vessels. The definition of the vessel response-capacity is the number of incidents that can be responded in a period.

Constraint 11 states that an onshore vessel cannot be located at offshore stations. This constraint is applied due to the fact that onshore vessels do not have sufficient capability (range) to serve from an offshore station.

Constraint 12 is a logical constraint for determining a time origin for vessel relocations. According to constraint 13, if vessel number l in period θ is located at station j and it was located at another station other than j in previous period, vessel relocation has occurred for this vessel.

Constraint 14 states that if a station is not available in a period, the station will not be activated in that period. Constraint 15 shows the variables' type. In our optimization problem all of them are binary.

Sign function (Sgn(x)):

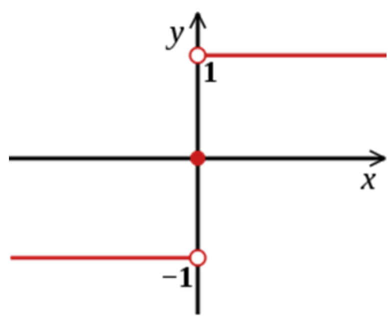

Figure 11. Sign function

One of the most important concepts that has been used in this thesis is addressing SAR missions' insufficiency probability. The development of this concept is inspired by reliability problems in production layout design. We assumed that the vessels which can "potentially" serve a demand grid are like parallel system machines in a production line which can serve the products at the same time. It is worth mentioning that in this case "potential vessels" might not serve the grids, but they can decrease the insufficiency probability in urgent situations.

Mission insufficiency probability has been shown with the following mathematical expression in this thesis:

$$p(\varphi(i,q,\theta) > E(i,q,\theta))^{\sum_{j,k} y_{ijkq\theta}}$$

It defines the insufficiency as when the actual number of incidents is more than the value our model has planned for (the expected value of incident occurrence). It should be noted that the model guarantees serving all the grids up to $E(i,q,\theta)$. Therefore, insufficiency happens when the actual number of incidents exceeds this value.

This expression was developed using reliability concept (Misra, 1972). If the probability of failure for each component is Q_i, the failure probability for the whole system would be $\prod Q_i$. In this regard, if the mission insufficiency probability for a SAR unit is $p(\varphi(i,q,\theta) > E(i,q,\theta))$ and assuming that grid i is covered by $\sum_{j,k} y_{ijkq\theta}$ SAR units, the probability of insufficiency in this grid equals to:

$$\prod_{1}^{\sum_{j,k} y_{ijkq\theta}} p\big(\varphi(i,q,\theta) > E(i,q,\theta)\big) = p(\varphi(i,q,\theta) > E(i,q,\theta))^{\sum_{j,k} y_{ijkq\theta}}$$

3.10 Dynamic Programming

In this thesis, the backward Dynamic Programming method has been used to solve the proposed model at a small scale. Due to the model non-linearity and being binary, common exact solution software (like GAMS and LINGO) is not able to solve the problem. Thus, Dynamic Programming has been utilized as our exact solution method (Bellman, 1966).

Dynamic Programming can solve a vast domain of problems without considering the nature of the variables (continuous, discrete, …) nor presence or absence of constraints. One of the main reasons for choosing this method is that it finds the global optimum solution and it does not need to prove convexity or concavity of the model. Also unlike other methods (like KKT, differentiation, …) adding constraints is simple in this method (Karimi & Sadjadi, 2020b). In the following section, the method developed for solving the model is presented.

3.10.1 Backward Dynamic Programming

In this method we start with the last stage, and we move backward stage by stage to reach the first one. The proposed dynamic programming formulation for our problem is explained below.

For implementing the Dynamic Programming method, it is necessary to break down a problem into separated sub-problems in such a way that solving each part is independent from the others, except in resource allocation. Each sub-problem is called a "stage". It is also required that each stage just be related to its previous and its next stage. In this thesis each stage (which is shown as $n(\theta, i, q)$) includes the period number, demand grid, and incident group. For example, $n = 5(2,4,1)$ means in stage 5, demand grid 4 and incident group 1 has been considered in period 2. So, it is obvious that the maximum possible number of stages in the solution would be:

$$Number\ of\ incident\ groups\ \times Number\ of\ demand\ grids\ \times Number\ of\ periods$$

One of the most notable characteristics of the Dynamic Programming is that changing the order of stages will not affect final solution and can be considered arbitrarily. It is important to mention that due to the relocation cost which is applied to the system at the end of each period, it is necessary to go over all stages in one period and then go to next period. In other words, the order of stages in one period can be considered arbitrarily but the order of periods itself should be followed sequentially. For example, $n = 5(2,4,1)$ and $n = 6(2,4,2)$ is not different from $n = 6(2,4,2)$ and $n = 5(2,4,1)$ and the final solution would be the same. Note that the stage is defined when there is a demand for that grid or in other words, $E(i,q,\theta) > 0$.

In each stage one "action" should be done which is equivalent to determining the variables. In this thesis, in each stage n, action "a" (which is shown by $a_n(k,l,j)$) includes determining the dispatch of unit l from station j. For example, if in $n = 5(2,4,1)$ we have $a_5(2,3,5) = 1$ as an action, it means that SAR vessel type 2, number 3 from station 5 will be dispatched to demand grid 4 with incident group 1 in period 2. In each stage, optimum action is shown with a_n^* which is equivalent to the main variable x_{452132} turning to 1. It is worth mentioning that all the constraints related to av_{ijkl} in the model will be considered in this section. For all the allocations here, it is crucial to consider offshore constraints (constraint 11).

It is possible that there is no assignment in a stage, so in each stage $a_n(0,0,0)$ is defined which is called action 0 in stage n. it should be noted that in each stage (like n) we have:
$a_n(0,0,0) + a_n(k,l,j) = 1$
Which means in each stage we have either an action or action 0 but we cannot have both in one stage.

In this method, the "state" of each stage should be certain. In the thesis, "state" is defined as the remainder and available resources and is shown by $s_n(k,l,\theta)$. In fact, "state" at the beginning of each stage represents the remainder response-capacity of each SAR unit. Allocation of each action in each stage is done according to this value. So, there is no possibility of generating an infeasible solution. For example, if the remainder response-capacity of vessel type 1, number 2 at the beginning of stage 3 in period 1 is 100, we have $s_3(1,2,1) = 100$.
Each state should be updated at the end of each stage. This act is done by deducting the expected value of demand for the action taken in each stage. According to backward dynamic programming, at the final stage the following equation is true for each period:
$s_n(k,l,\theta) = c_{kl\theta}$

In each stage, it is necessary that a set of feasible states be considered at the same time. In this regard, we define a new notation as follows:
$s'_{nm}\{s_n(k,l,\theta)\}$ is the m^{th} combination of feasible states in stage n.
It should be noted that all the possible combinations of feasible states in each stage should be considered in the calculations.

The objective function value in accordance with state $s'_{nm}\{s_n(k,l,\theta)\}$ in stage $n(\theta,i,q)$ with action $a_n(k,l,j)$ is called "revenue". The decisions are made based on this measure. In this thesis, revenue is the objective function for each demand grid which can be written as in the equation below. We separated the objective function into two parts:
$r_{ni'}\big(\mathrm{n}(\theta,i,q), s'_{nm}\{s_n(k,l,\theta)\}, a_n(k,l,j)\big) = p\big(\varphi(i',q,\theta) > E(i',q,\theta)\big) \cdot a'''_n(k,l,i',j) +$
$(1 - a'''_n(k,l,i',j))$

$r2\big(\mathrm{n}(\theta,i,q), s'_{nm}\{s_n(k,l,\theta)\}, a_n(k,l,j)\big) = \rho\big(ac_{kl\theta} \cdot a_n(k,l,j) + rc_{kl} \cdot a''_n(k,l,j)\big) - \mu \cdot$
$a_n(k,l,j) \cdot Ra_{qkl} + \beta \cdot a_n(0,0,0)$

In which:
$a''_n(k,l,j) = 1$, if vessel type *k* number *l* had been in a station other than *j*, otherwise 0
$a'''_n(k,l,i',j) = 1$, if grid i' (all the grids except the grid we work on in the current stage) had not been under coverage of vessel type *k* number *l* in previous stages and this vessel is able to cover the grid i', otherwise 0.

It should be noted that calculation of these values is simple, and it does not increase the computational complexity of the model. Constraints related to av_{ijkl} will be addressed in this section.

In each stage, there are I objective function elements which can be written as follows:
$r\big(\mathrm{n}(\theta,i,q), s'_{nm}\{s_n(k,l,\theta)\}, a_n(k,l,j)\big) = \sum_{i'} r_{ni'}\big(\mathrm{n}(\theta,i,q), s'_{nm}\{s_n(k,l,\theta)\}, a_n(k,l,j)\big) +$
$r2\big(\mathrm{n}(\theta,i,q), s'_{nm}\{s_n(k,l,\theta)\}, a_n(k,l,j)\big)$

It should be noted that if an action is infeasible (in terms of distance, time, or other constraints), its revenue will be given a very large number (infinity), and with respect to the minimization problem infeasible solutions will be omitted.

In Dynamic Programming, it is assumed that each stage is only related to its previous and subsequent stage. This relation is through the "Transition Function". In this thesis the transition function (which is shown with v) is defined as follows:

$v_n(k,l,\theta) = s_n(k,l,\theta) - a_n(k,l,j) \cdot E(i,q,\theta)$

According to the mentioned explanations, it can be inferred that $v_n(k,l,\theta) = s_{n-1}(k,l,\theta)$, and we put $v_n(k,l,\theta) \geq 0$ to ensure the feasibility of the response-capacity constraint. So, there is no $v_n(k,l,\theta)$ in the first stage of each period.

The most important part of this method's calculation is the "Recursive Function". In other words, the difference between the revenue and recursive function is that the revenue function calculates the objective function in a specific stage, but the recursive function considers the current stage along with all the previous ones. Note that the recursive function has its best value for state $s'_{nm}\{s_n(k,l,\theta)\}$ with the optimal action $a_n^*(k,l,j)$ among all the available actions. Total amount of the objective function is calculated with this function as follows:

$f_{ni'}(n, s'_{nm}\{s_n(k,l,\theta)\}) = r_{ni'}\big(\mathrm{n}(\theta,i,q), s'_{nm}\{s_n(k,l,\theta)\}, a_n^*(k,l,j)\big) \cdot f_{(n-1)i'}(n-1, \{v_n(k,l,\theta)\})$

$f2(\mathrm{n}(\theta,i,q), s'_{nm}\{s_n(k,l,\theta)\}) = r2\big(\mathrm{n}(\theta,i,q), s'_{nm}\{s_n(k,l,\theta)\}, a_n^*(k,l,j)\big) + f2\big(\mathrm{n} - 1(\theta,i,q), v_n(k,l,\theta)\big)$

Therefore

$f(n, s'_{nm}\{s_n(k,l,\theta)\}) = Minimize\{\sum_{n,i}(f_{ni}(n, s'_{nm}\{s_n(k,l,\theta)\}) + f2(\mathrm{n}(\theta,i,q), s'_{nm}\{s_n(k,l,\theta)\})\}$.

For the first stage we have:

$f_{(n-1)i'}(n-1, \{v_n(k,l,\theta)\}) = 1$ and $f2\big(\mathrm{n} - 1(\theta,i,q), v_n(k,l,\theta)\big) = 0$

Generally, given a stage, if the coverage is happening for the first time in a grid its $f_{(n-1)i'}$ is assumed to be 1; and if it is not the first time then $f_{(n-1)i'}$ will be calculated according to the previous stage. With respect to the mentioned rule, $f_{(n-1)i'}$ equals to 1 for the first stage of each period.

And the final value of objective function would be the best value of $f(n = I, s'_{nm}\{s_n(k,l,\theta)\})$.

3.11 Metaheuristic Algorithms

In accordance with the large scale of our case study (due to high numbers of demand grids and, as a result, stages) and the curse of dimensionality of dynamic programing, DP is unable to find an optimum solution in a reasonable amount of time. Therefore, metaheuristic algorithms have been used to solve the problem. In other words, dynamic programming is used as a validation of other methods at a small scale. We first consider a problem with a limited number of grids, stations, and vessels. Then we solve that problem in accordance with our DP formulation and also with various metaheuristic algorithms. Considering that the dynamic programming provides us with the global optimum, the results of the metaheuristic algorithms (in comparison to dynamic programming) will show their capability to solve the problem at a large scale.

3.11.1 Optimization algorithms

The optimization methods and algorithms have been divided into two groups: Exact algorithms; Approximate algorithms.

Exact algorithms are able to find exact optimum solutions, but they are not efficient enough for large NP-hard problems, and their running times increase exponentially with the scale of the problem.

Approximate algorithms in computer science and operational research are methods for finding approximate solutions for optimization problems. These algorithms often are used for NP-hard problems which are common in optimization. They can often find good solutions (close to optimal) in acceptable running times for NP-hard problems. Approximate algorithms are classified into three groups: Heuristics, Meta-heuristics, and Hyper-heuristics.

Heuristics have been developed to solve NP-hard problems in a more efficient way and timely manner. They are usually problem-specific and cannot be applied to other problems. Two major drawbacks of heuristics are getting stuck in local optima and speed of convergence. Meta-heuristics have been presented to solve these issues. They are problem-independent and adjustable to wide variety of problems. The main difference between meta-heuristics and hyper-heuristics is the search space. Meta-heuristics search within a problem space to find a direct answer, but hyper-heuristics search across different heuristics to find the best method or sequence of methods to solve the problem.

In this thesis, following metaheuristic algorithms have been used to solve the problem in large scale:

- Genetic Algorithm (GA)
- Simulated Annealing (SA)
- Hybrid algorithm of SA and GA (SAGA)
- Harmony search (HS)
- Hybrid algorithm of HS and GA (HSGA)
- HSGA with improved strategy (HSGA I)
- HSGA with improved strategy (HSGA II)

In chapter 5, performance of the mentioned algorithms will be assessed. The mechanism of each algorithm along with their process flowchart is explained hereunder.

3.11.2 Genetic Meta-Heuristic Algorithm (GA)

GA is a search algorithm which is inspired by Charles Darwin's theory of natural selection and mechanisms of population genetics. Its basic idea has been derived from the biological process of survival and adaptation in which the fittest individuals will be selected to reproduce and make offspring for the next generation (Abuiziah & Shakarneh, 2013). In nature, a combination of better chromosomes will lead to better generation, and some mutation occurs which might also cause a better next generation. GA is used to solve problems using these concepts using the following steps:

- Forming the problem solutions as chromosomes
- Determining the fitness function
- Producing the initial population (i.e., potential solutions)
- Introducing the selection operator
- Introducing genetic operators (Crossover and Mutation)

In GA, a set of solutions is produced in the first step which is called "initial population". Each solution is represented as a chromosome. Then, better chromosomes will be selected by somehow blending the initial ones, and genetic operators will produce new offspring. At the end, the current population will be combined with the new offspring and this process is repeated until the termination condition has been reached. Some of the common termination conditions are as follows:

- ✓ The solution has met the minimum Objective Value criterion
- ✓ Predetermined number of generations reached
- ✓ Allocated budget (computation time / money) reached
- ✓ Manual inspection
- ✓ Different combination of above conditions

Figure 12 shows the flowchart of this algorithm, and it clarifies how it works (Ab Wahab et al., 2015):

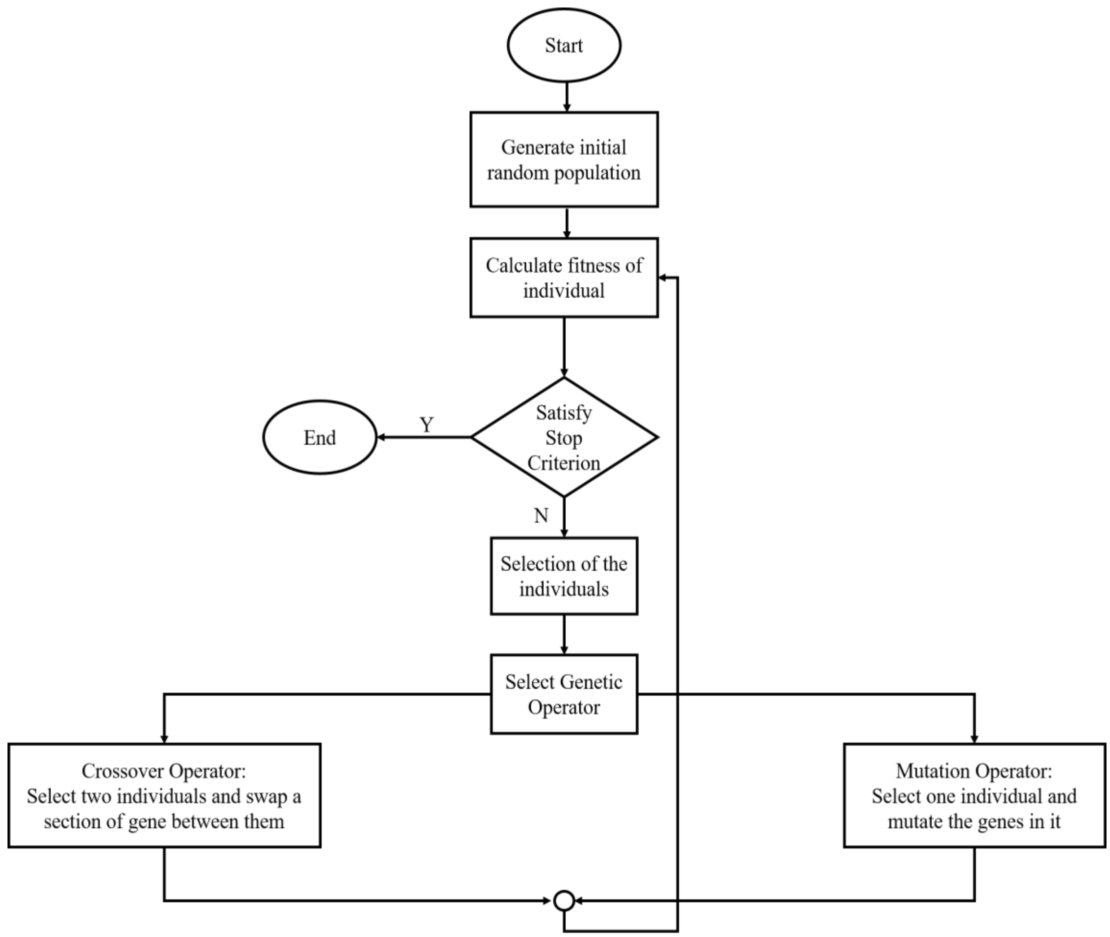

Figure 12. GA flowchart

GA, like all the other methods, has some strength and weaknesses. In this section, some of them will be mentioned.

GA advantages:

- ✓ The concept is understandable
- ✓ It is a population-based method
- ✓ It uses objective function information, not derivatives
- ✓ The transition rules are probabilistic
- ✓ It is robust circumventing local optimum points (i.e., mutation operator can help to escape from a local optimum)
- ✓ It works well on mixed discrete/continuous problems

GA disadvantages:

- ✓ Designing the algorithm and implementation can be hard
- ✓ It is time-consuming
- ✓ The stopping criterion is not clear in many problems
- ✓ It is not effective for single right or wrong measures (like decision problems)

3.11.3 Simulated Annealing Algorithm (SA)

The simulated annealing algorithm is an optimization method which mimics the slow cooling of metals. The name of this algorithm comes from the annealing technique in metallurgy (Aleksendrić & Carlone, 2015). Annealing is the process of heating and controlled cooling of different material in order to increase the crystals' size and reduce the defects. This method can be used for hard computational optimization problems where the exact algorithms cannot be implemented. It is usually applied to problems which have discrete search space, and it has good performance to find an approximate global optimum which could be beneficial for many practical problems. The following procedure should be followed to implement the SA algorithm.
Starting with an initial solution $s = S_0$ and initial temperature $t = t_0$, we will determine a temperature reduction function. Usually there are three types of temperature reduction rules as shown in Figure 13:

1. **Linear Reduction Rule:** $t = t - \alpha$
2. **Geometric Reduction Rule:** $t = t * \alpha$
3. **Slow-Decrease Rule:** $t = \frac{t}{1+\beta t}$

Figure 13. SA temperature reduction rules

β is an arbitrary constant in the last rule.
Then, we pick one of the neighborhood solutions (solution that are close to the current one) and calculate the difference in objective functions between the new and old (i.e., current) solution Δf. If the objective value for the new solution is better, then we accept it and substitute it for the current solution. Otherwise, we will accept the new solution with the probability of exp $(-\frac{\Delta f}{t})$. The acceptance probability function in this method is defined as $P(e, e_{new}, T)$ in which $e = E(s)$ and $e_{new} = E(S_{new})$ and T is temperature of the system that decreases by the time. E represents the energy of the system. It is obvious that states with smaller energy are better than the states with greater energy. It should be noticed that the probability function is positive all the time (even when e_{new} is bigger than e) because this feature prevents the algorithm from getting stuck at a local optimum. This process repeats for the predetermined number of iterations to reach thermodynamic balance. After that t will be decreased gradually with respect to reduction rules until the termination condition is met. Figure 14 shows the flowchart of this algorithm for a minimization formulation (A.-H. Zhou et al., 2018):

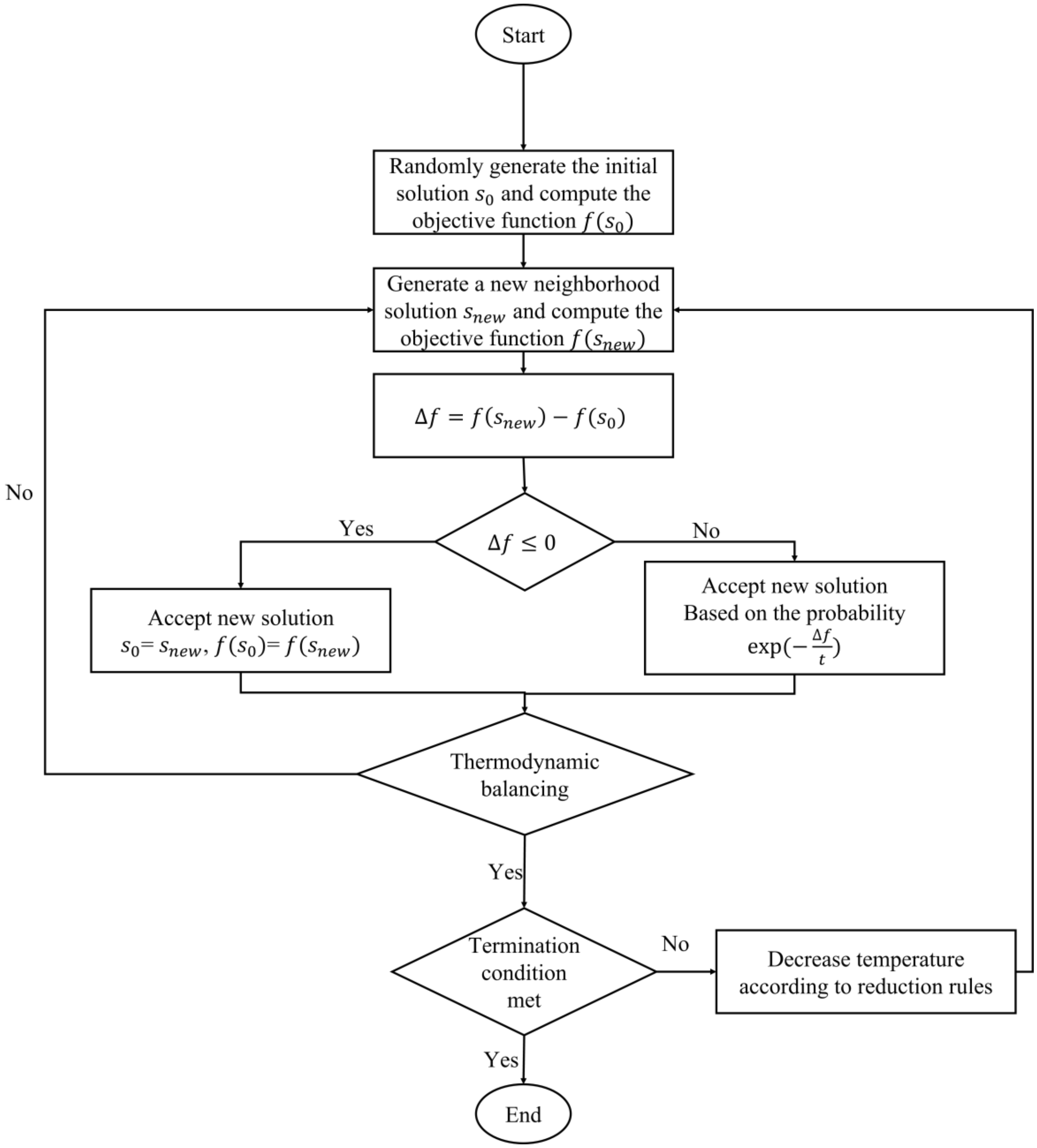

Figure 14. SA flowchart

Some of the strengths and weaknesses of SA algorithm are as follows:
Strengths:

- The ability to deal with highly nonlinear models and many constraints as a robust and general technique
- The flexibility and ability to approach the global optimum

Weaknesses:

- Computation time can be high when generating quality solutions

- Tuning the parameters can be rather delicate
- It is always possible to get stuck in local optimum which can be avoided by appropriate annealing schedule (i.e., rate of decreasing temperature) and starting point.

3.11.4 SAGA Hybrid Algorithm

In previous sections it was mentioned that there are significant differences between GA and SA. GA starts from a set of solutions, does information exchange, and makes new population using its operators. On the other hand, SA just works on a single solution at a time. Another difference is the selection strategy. GA uses the same strategy throughout the algorithm, but SA updates the temperature parameter which is used for solution evaluation. Both of them have some advantages and disadvantages. SA cannot get an overall view of the search space due to using one candidate solution and this algorithm is slow because of the sequential nature. SA usually finds good quality solutions in a neighborhood, but it is likely to stuck in local optima and it takes longer for this algorithm to escape in comparison to GA. GA tends to discover the search space much faster but it has difficulty in finding the exact minimum (Elhaddad, 2012). A combination of GA and SA algorithms can be beneficial for us to take the advantages of the two algorithms in order to enhance the quality of solutions in an acceptable amount of time. At the beginning of SAGA all the initialization parameters (such as population size, number of variables, mutation and crossover rates, selection method, annealing schedule, and temperature function) are defined. Then the SA part of the algorithm is activated, and it generates an initial population for the GA algorithm. In other words, SA produces a start point for GA in this algorithm which has a great impact on the quality of the final solution. Figure 15 shows the flowchart of this algorithm.

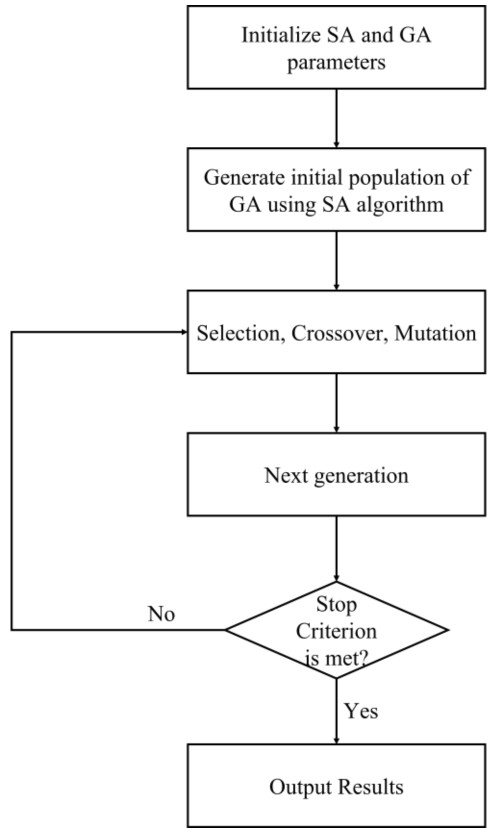

Figure 15. SAGA flowchart

3.11.5 Harmony Search Algorithm (HS)

HS was first presented by (X.-Y. Yang et al., 2010). This algorithm imitates the process of music improvisation in which musicians are adjusting their pitches in their memory to reach the perfect state of harmony. The analogy between musical instruments and optimization problems is summarized as follows:

- ith musical instrument → ith decision variable
- Harmony H_j created by all musical instruments → jth solution vector
- Evaluation → Objective function
- Experience → Harmony memory
- Practice → Iteration

The process of HS starts with initializing the parameters which are harmony memory size (HMS), harmony memory consideration rate (HMCR), pitch adjustment rate (PAR), iteration number (NI), and new harmony memory size. Then we put random initial solution vectors into harmony memory (HM). Based on probability of HMCR we start the search and select process in HM for every solution vector, otherwise values within the possible range will be searched randomly by a probability of 1-HMCR. When searching harmony memory, the components of a new solution vector will be changed by a probability of PAR. After generating a new solution vector, if it is better than the worst old one in HM, we will include the new vector and remove the old one. Figure 16 shows the flowchart of this algorithm (Du, 2012):

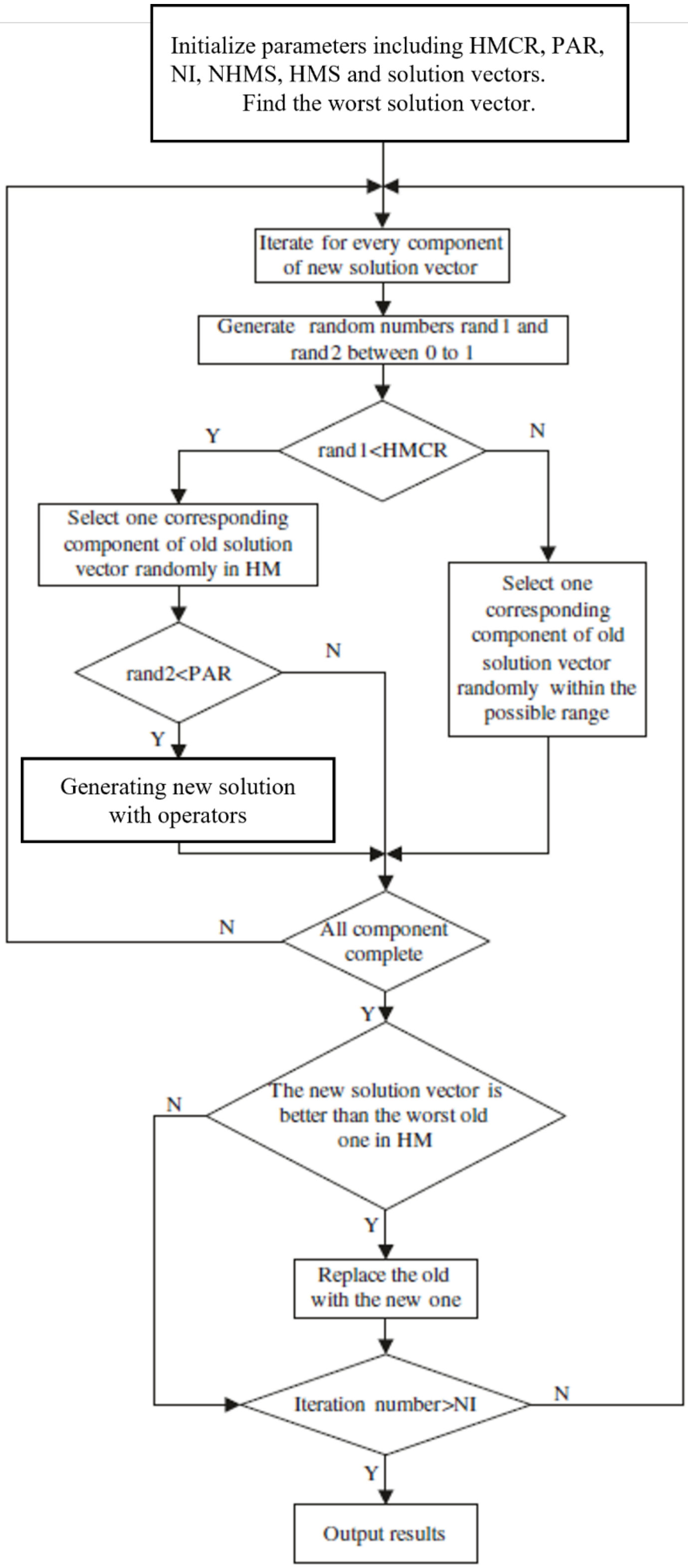

Figure 16. HS flowchart

This algorithm also has some strengths and weaknesses like the previous ones. Some of them are mentioned below:

Advantages (Abdel-Raouf & Abdel-Baset Metwally, 2013):

- Fewer mathematical requirements
- Generation of the new vector after considering all of the existing vectors will result in more flexibility and produce better solutions (unlike the GA which just considers two parent vectors)
- Good at identification of high-performance solution space regions in a reasonable time

Disadvantages

- Low in precision (got stuck at local solutions)

3.11.6 HSGA

Harmony search and Genetic algorithm are alike in the representation of an optimization problem. For example, chromosomes in GA and harmony in HS represent a feasible solution. Furthermore, sets of feasible solutions are expressed as populations in GA and harmony memories in HS. Considering these analogies, an HSGA hybrid algorithm has been introduced in which harmony and chromosomes are integrated as well as harmony memory and population. The main process of this algorithm is the initialization of GA and HS parameters (size of population, crossover and mutation probability, HMS, HMCR, PAR, NI, and NHMS). In the next step, the fitness value of all the individuals in the population is calculated, and the next population is generated using selection, crossover, and mutation operators. This new population is considered as harmony memory and chromosomes will also be considered as harmony. With the HS algorithm, a new solution vector will be generated and the fitness value for each individual will be calculated again. This process will be repeated until the termination condition is met. Figure 17 shows the flowchart of this hybrid algorithm (Du, 2012).

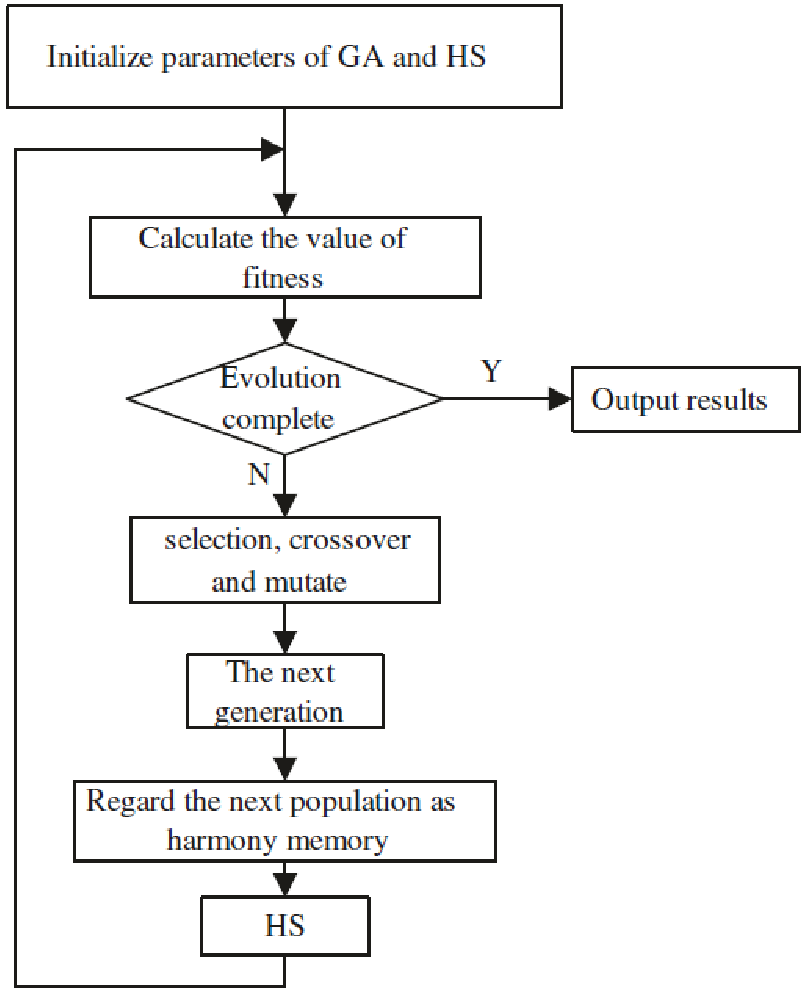

Figure 17. HSGA flowchart

3.11.7 Improved strategies of HSGA

After comparing this hybrid algorithm with GA, we can conclude that the convergence speed and searching ability have been improved. On the other hand, running time and complexity of the hybrid algorithm have been increased because in every iteration both GA and HS are involved in improving the solution. With this in mind, two strategies have been put forward by (W. S. Wei et al., 2013).

3.11.7.1 Strategy One (HSGA 1)

The idea behind this strategy is that HS is not involved in all the iterations. It is just operated when the iteration number is the integral multiple of N (fixed value). This strategy reduces the number of HS operations in order to improve the operation time. Figure 18 shows the flowchart of this HSGA1.

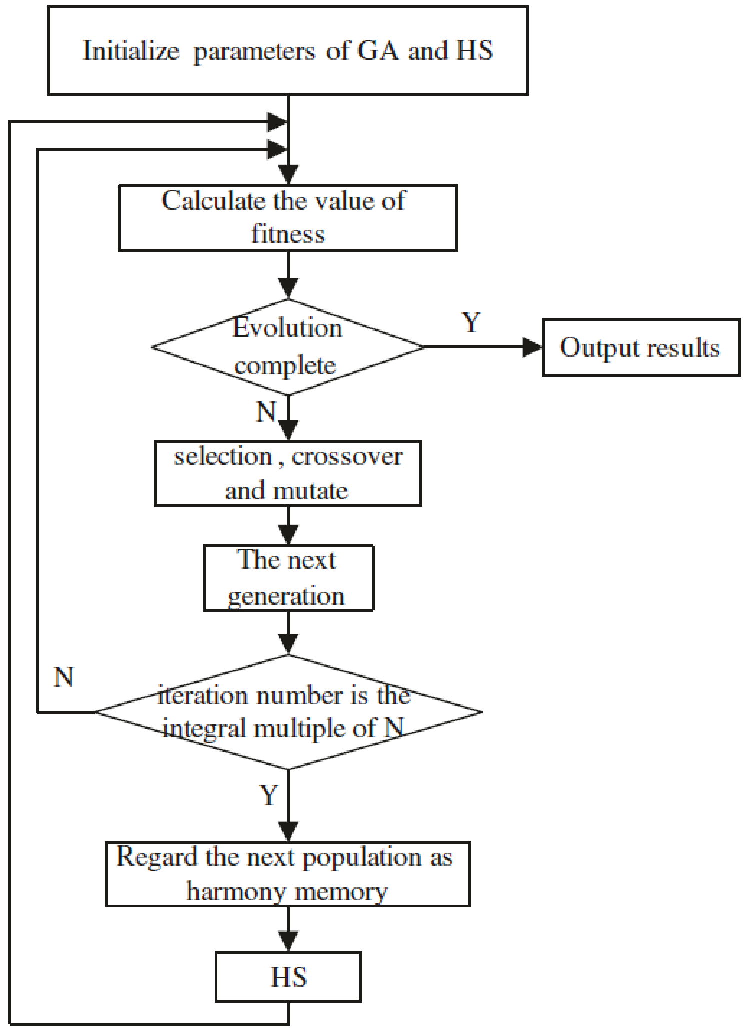

Figure 18. HSGA I flowchart

3.11.7.2 Strategy two (HSGA 2)

In this strategy only M (fixed value) chromosomes will randomly be selected in each iteration to enter HS which will reduce the complexity of the process. Figure 19 represents the flowchart of this algorithm.

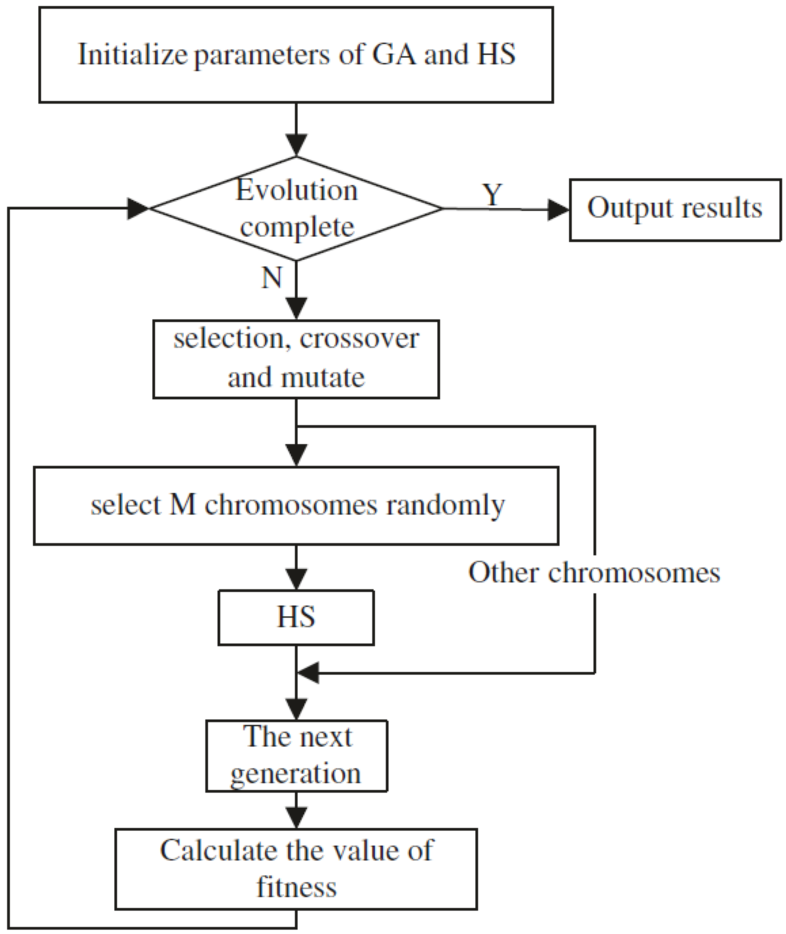

Figure 19. HSGA II flowchart

Using any metaheuristic algorithm requires determining some attributes for the intended problem as follows:

- Representation (coding of problem variables)
- Generation of initial members
- Members selection for entering into operators
- Operators
- Members selection for entering next stage or iteration
- Termination condition

In this section, these attributes will be explained for all the presented metaheuristic methods.

3.11.8 Representation:

We have used a two-part representation for each period (i.e., season) and for each of the methods. The first part is a $i \times q$ matrix wherein each element is a random value between 0 and 1. For converting this representation to problem variables, we multiply the random values (for the grids with non-zero demand) by the number of available vessels for covering each grid and round up the obtained value. This value will be the vessel number which will be dispatched to the grid considering capacity constraints. This representation makes the use of operators easier and never generates infeasible solutions.

The second part of the representation is an array with l rows where each element shows the station number of each vessel. In this part, seasonal stations and offshore constraints have been considered in allocation process.

One example of this representation for a period is shown in Figure 20.

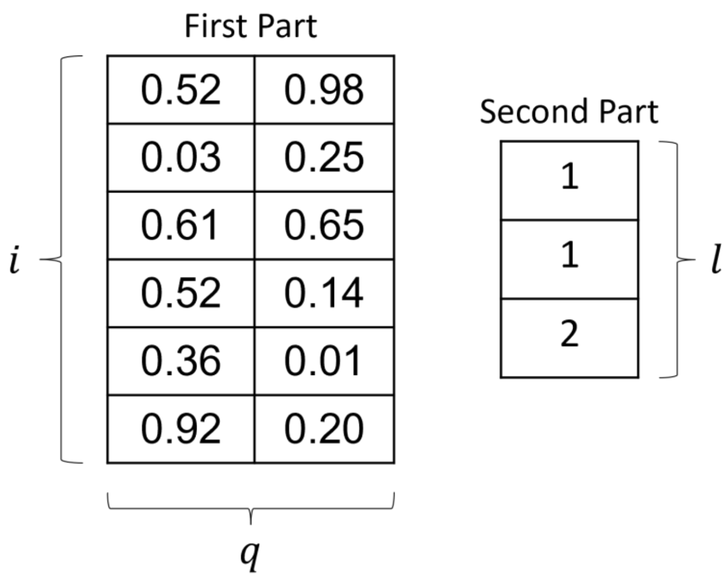

Figure 20. Two-part representation for metaheuristic methods

3.11.9 Generation of initial members

In all the methods, the initial member or members are generated randomly. This causes more diverse search in the solution space.

3.11.10 Members selection for entering into operators

In the SA algorithm, the optimization process is being done with just one member so members selection is meaningless in this algorithm. In the other algorithms selection would be random, because by doing so we will give a chance to low quality members to be selected and they might contain a good gene to improve the results.

3.11.11 Operators

In this section all the operators of independent methods will be explained. Operators of the hybrid algorithms consist of the same operators as the independent methods.

<u>SA method</u>

3.11.11.1 Insertion operator:

In this operator two rows are randomly selected from the first part, then all of the rows from the second selected row (i.e. green) to the bottom are moved up to just before the first selected row (i.e. blue). The same procedure is then applied (independently) to the second part. One example of this operator has been shown in Figure 21.

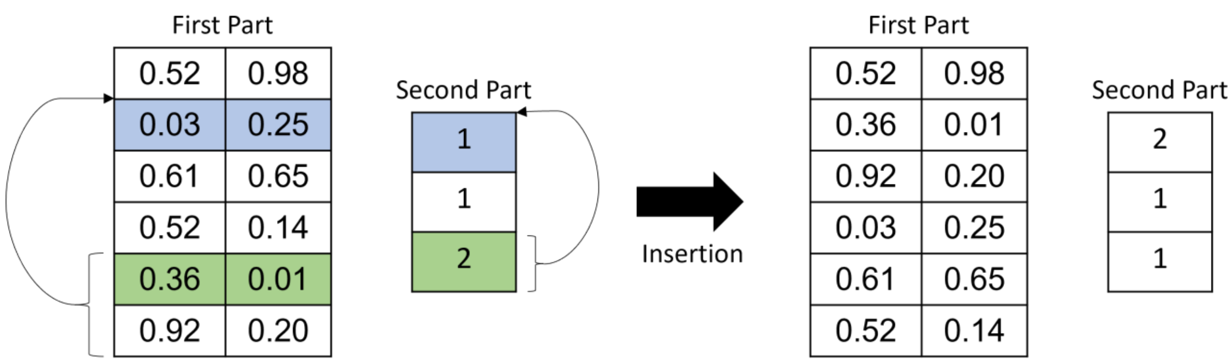

Figure 21. Insertion operator

3.11.11.2 Reversion Operator:

In this operator two rows are randomly selected from the first and second part, and then the position of those rows are interchanged in each part respectively. One example of this operator has been shown in Figure 22.

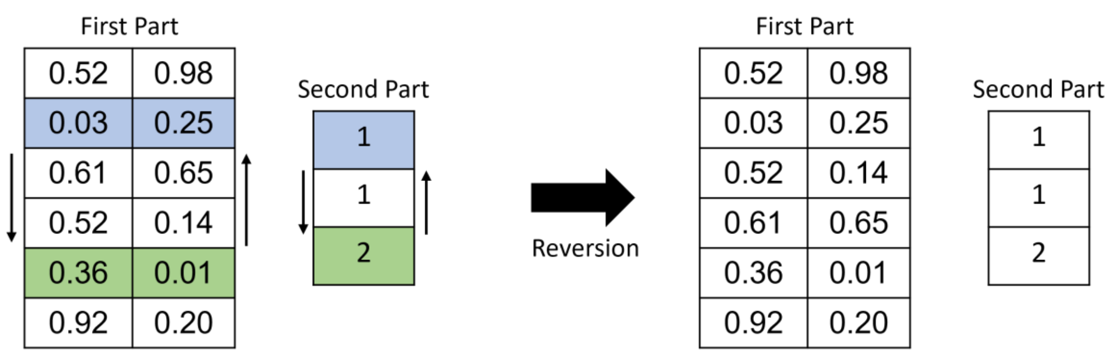

Figure 22. Reversion operator

3.11.11.3 Swap Operator:

In this operator two rows are randomly selected from the first part, and their elements are swapped. Similarly, two rows are randomly selected from the second part, and their elements are swapped One example of this operator is shown in Figure 23.

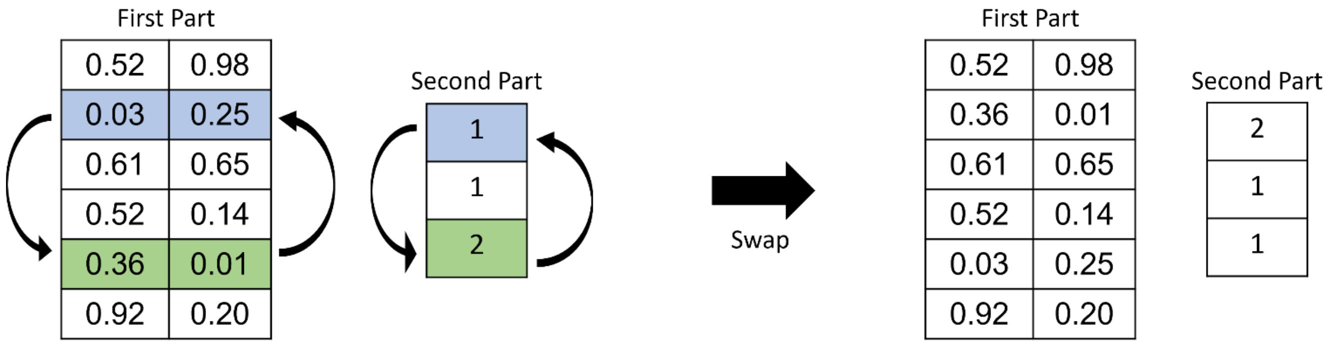

Figure 23. Swap operator

GA method

3.11.11.4 Two-point Crossover

GA operators contain mutation and crossover. In this method we have used a mutation similar to the insertion operator in the SA method, and two-point crossover. In this kind of crossover, two parents are selected along with two rows from the first and second part of each parent. The elements between the chosen rows are swapped to make two children. Figure 24 illustrates this process.

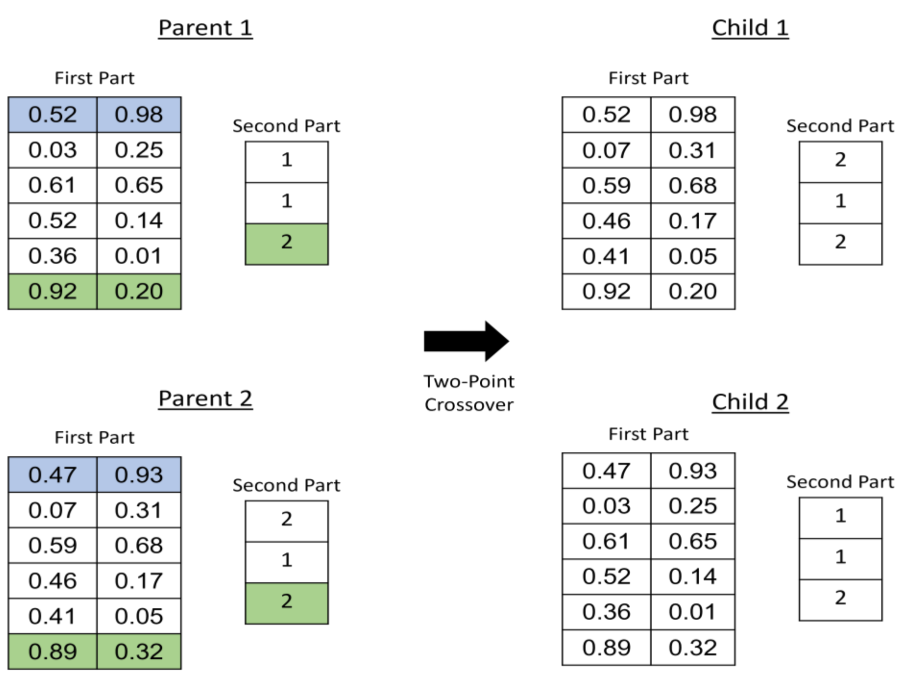

Figure 24. Two-point crossover

HS method

The HS method has only one operator which acts like swap operator in SA. In this operator one member is selected from harmony memory and is swapped with the intended member of the new solution. It worth mentioning that the activation of this operator is probabilistic, and it is possible that the harmony memory is not improved in an iteration.

All of these operators act separately for each iteration. Furthermore, infeasible solutions might be generated after applying each of these operators. Thus, infeasible solutions will be modified and changed to feasible ones using a function.

It should be mentioned that members are selected randomly for the next iteration in all the mentioned methods, because low-quality members might contain good genes. Moreover, the termination condition is reaching a predetermined number of iterations in all of the methods.

4. CHAPTER 4 MODEL INPUTS

4.1 Area of Interest

As mentioned in previous sections, our case study in this thesis is optimizing location and allocation of SAR vessels and stations in Atlantic region of Canada. The Atlantic region of Canada encompasses four provinces, namely Nova Scotia, New Brunswick, Newfoundland, and Prince Edward Island (see Figure 25Figure 25. Atlantic region of Canada).

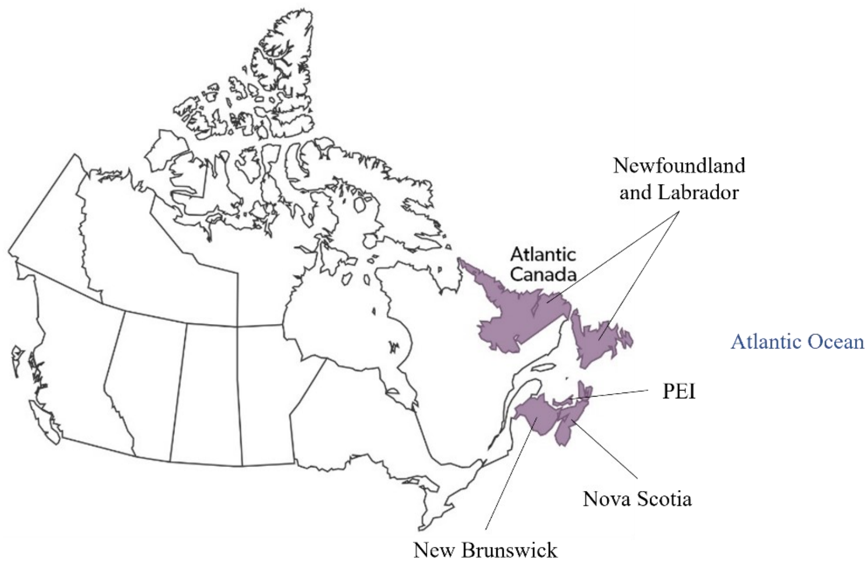

Figure 25. Atlantic region of Canada

Our historical data contain all the maritime incidents from 2014 to 2016 (most recent available at the time of this research) which are extracted from the SISAR database. In total, 2641 incidents were identified in this period in Atlantic region, and these data were used for the demand

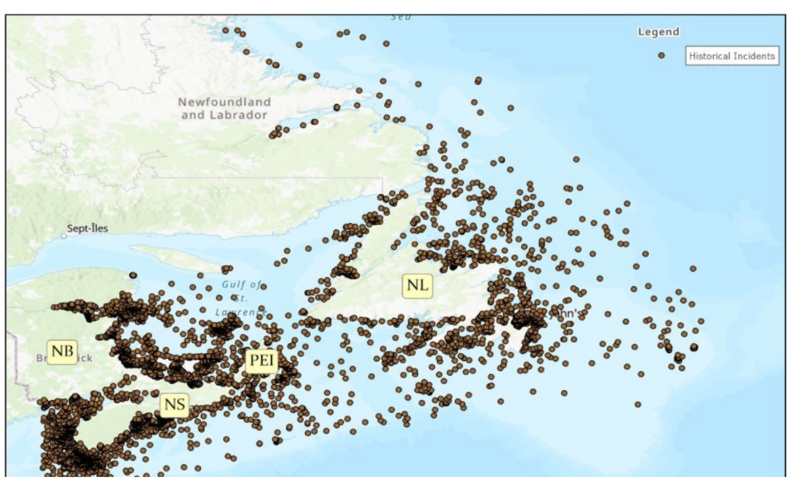

Figure 26. Historical SAR incidents in Atlantic region

calculation in this thesis. The data cleaning and refining process was explained in the introduction chapter. Figure 26 shows all the historical incidents in our area of interest.
Demand Grids

The first thing for analyzing spatial data is dividing the area of interest into different grids. In this thesis, we have considered three different grid sizes in our area based on their distance from shore. The smaller grid size is used for areas around the shoreline which is related to smaller lifeboats' response range (range of 185 Km). Based on this, 1617 demand grids have been defined in the Atlantic region of Canada using Arc GIS Pro software as shown in Figure 27.

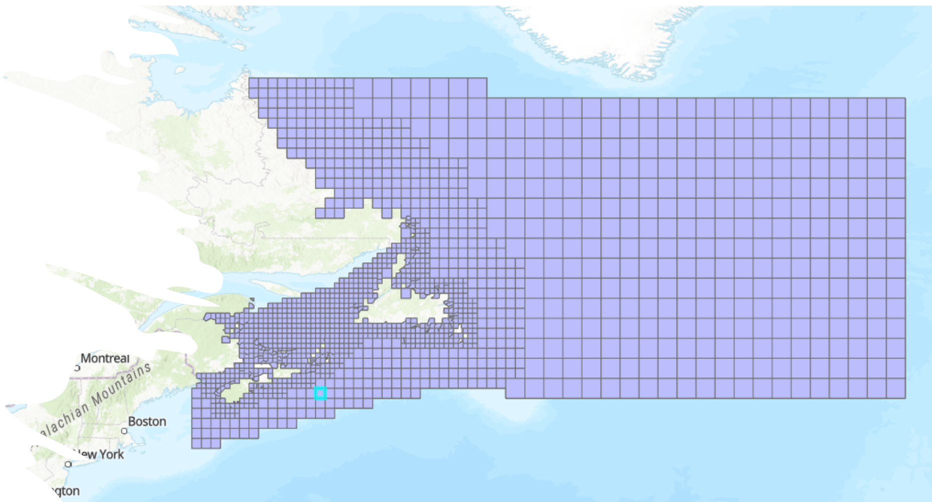

Figure 27. Defined demand grids in area of interest

4.2 Operation Planning Seasons

In accordance with the maritime incidents' historical data, it can be observed that their occurrence is seasonal. As an example, most of the recreational-related incidents occur during warm seasons (spring and summer), and the number of incidents is higher approximately from April to September. To effectively address these changes, we have considered two operational seasons:

- Season 1: Fall and Winter (October to March)
- Season 2: Spring and Summer (April to September)

This is not an unrealistic assumption, because the CCG has also considered operational seasons for managing the operations better which might be a little different than our defined seasons. It should be mentioned that we assumed the relocation of SAR vessels can be done only at the beginning of each operational season.

4.3 SAR Stations

At the time of writing this thesis, there are 19 onshore SAR stations in the Atlantic Region of Canada that are able to accommodate different types of vessels. Two of these stations are newly founded and they are home ports respectively for two Bay Class lifeboats, namely Conception Bay at the Twillingate station, and Sacred Bay at the Old Perlican station. We have not applied any restrictions to stations, and it is assumed that onshore stations are able to house all kinds of SAR vessels. The only constraint we have applied to onshore stations is operation seasonality. After consultation with CCG representatives, we learned that some specific stations are not active in winter season, and they operate only in warmer seasons. Hence, we have added a constraint (constraint 14) to address this unavailability. The list of seasonal stations is provided below:

- St. Anthony Station NL
- Twillingate Station NL
- Old Perlican Station NL
- Lark Harbour Station NL
- Port Au Choix Station NL
- Shippagan Station NB
- Souris Station PEI
- Summerside Station PEI

Some of the SAR vessels have the capability to patrol far from the shore for a long time. With this in mind, 19 potential offshore SAR stations have been considered in our analysis. The location of these stations has been determined in consultation with CCG members (i.e., the centroid of some marine subareas in Atlantic region and also some offshore patrolling locations). These stations are considered to be central locations for offshore vessels to spend much of their time patrolling or performing other tasks at sea. Therefore, it is important for an offshore vessel to have sufficient endurance and maximum range to be qualified for being located at an offshore station. We dealt with this issue as a constraint (constraint 11) in the optimization model which does thus not allow lifeboats (Cape class and Bay class) to be positioned at offshore stations.

4.4 Land-avoided distances

For running our model, we need to calculate the distance between incidents and stations (vessels). The most common method for travel is calculating path length based on straight Euclidean distance which is not applicable in our study. The reason is that there are some land obstacles in coastal areas which hinder us from using this method. Therefore, a land avoidance algorithm was used in this thesis. This method was developed by the *MARIN* (Maritime Activity and Investigation Network) research group at Dalhousie University to find the shortest route between incidents and vessels. Therefore, our distance matrix has 1617 rows (demand locations) and 38 columns (onshore and offshore stations). Each element of this matrix shows the distance between the corresponding grid and station.

4.5 Kernel Density (KD) Estimation of the incidents

In this thesis, we have computed the KD estimates for each of the demand grids over the two predetermined seasons (Season one: October-March / Season two: April-September) and for four groups of incidents. Implementation of KD requires the determination of some parameters. The following parameters were used for our calculation in Arc GIS Pro software:

- ✓ Kernel function type: Quartic
- ✓ Cell size: (0.25×0.25) degree; the centre of each grid is used for kernel density calculation.

- ✓ Bandwidth (radius): variable size between (0.25-1.0 degrees), 0.25 degree for areas close to the shoreline with high density of incidents, 0.5 degree for areas further from shore with low incident density, and 1.0 degree for areas further offshore with very low number of incidents in the vicinity.

Figure 28 to Figure 35 visualize the KD estimates over the Fall-Winter (season 1) and Spring-Summer (season 2) respectively for each of the incident groups.

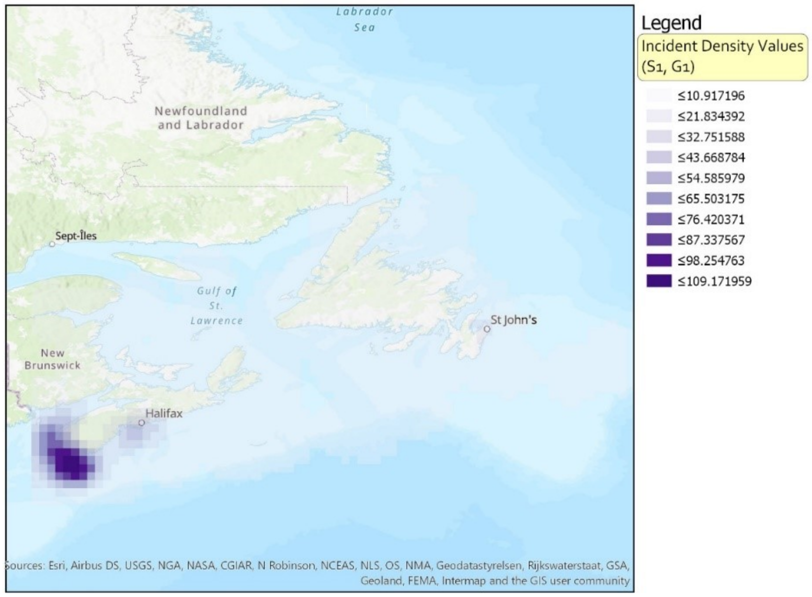

Figure 28. KD estimates heatmap for season 1 and incident group 1

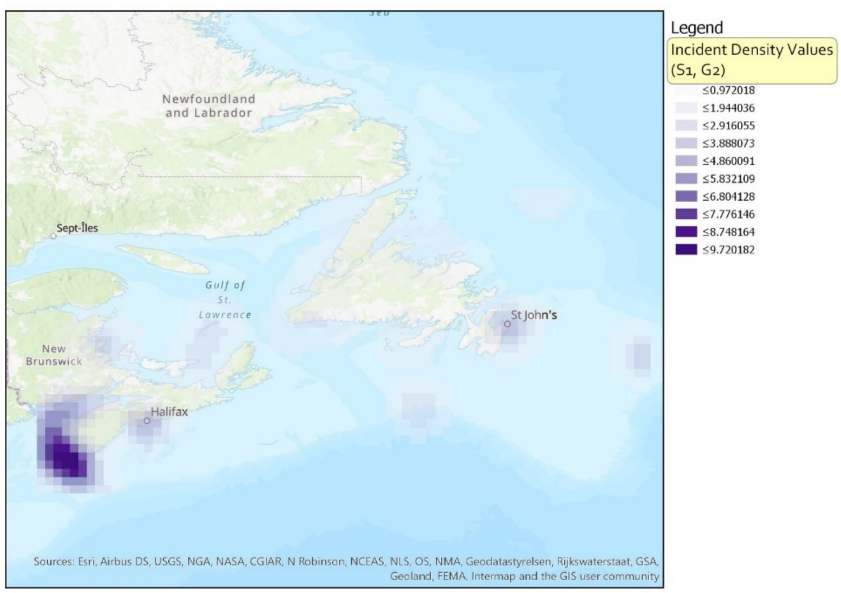

Figure 29. KD estimates heatmap for season 1 and incident group 2

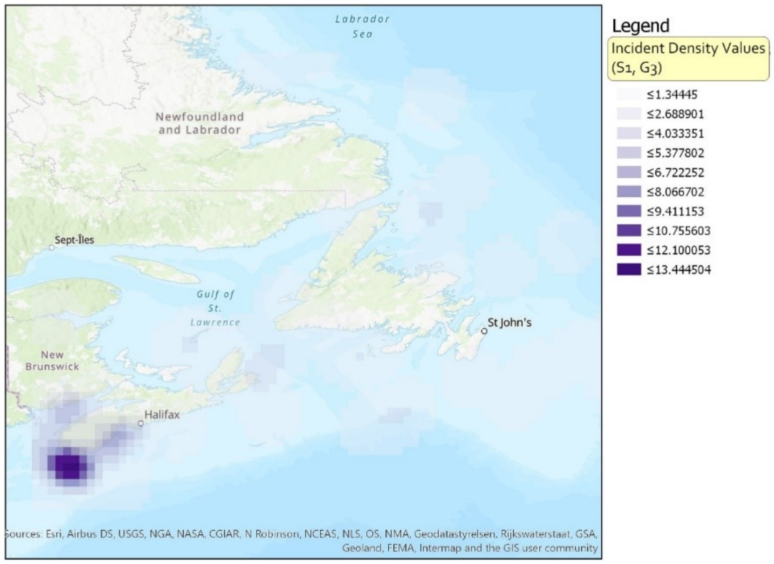

Figure 30. KD estimates heatmap for season 1 and incident group 3

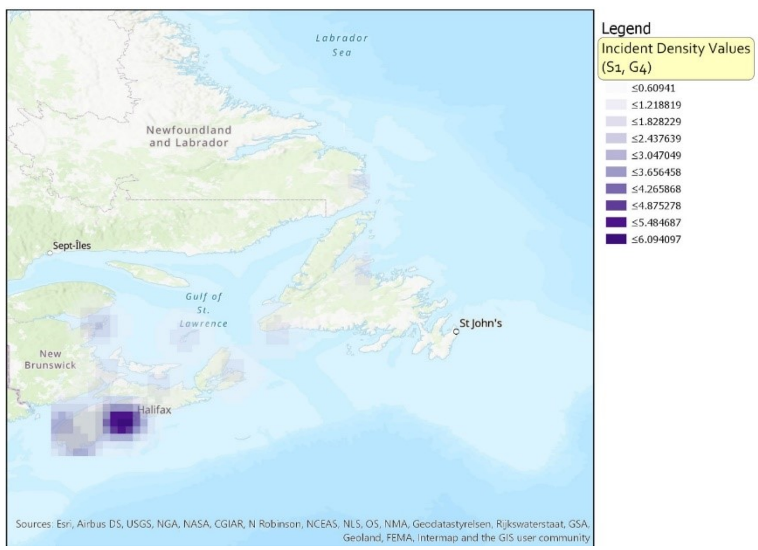

Figure 31. KD estimates heatmap for season 1 and incident group 4

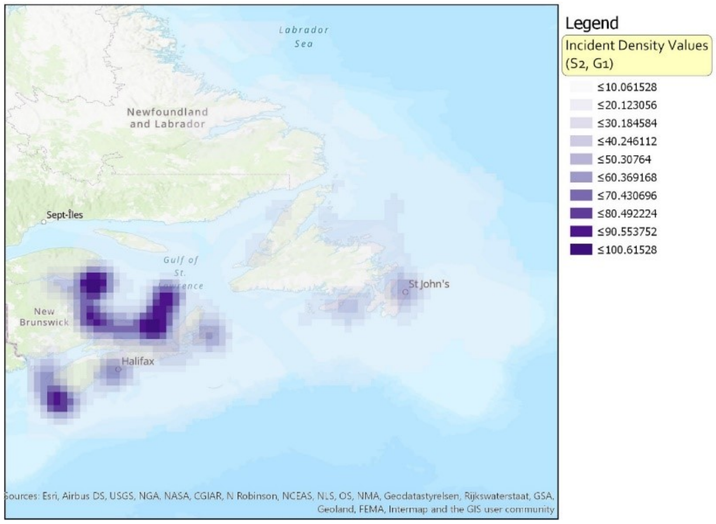

Figure 32. KD estimates heatmap for season 2 and incident group 1

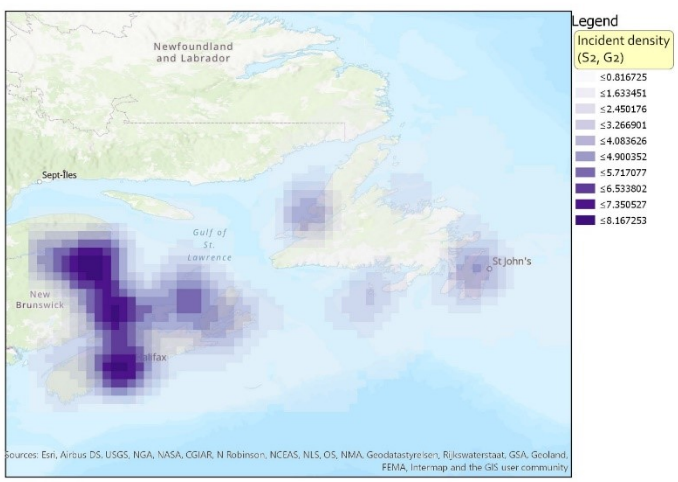

Figure 33. KD estimates heatmap for season 2 and incident group 2

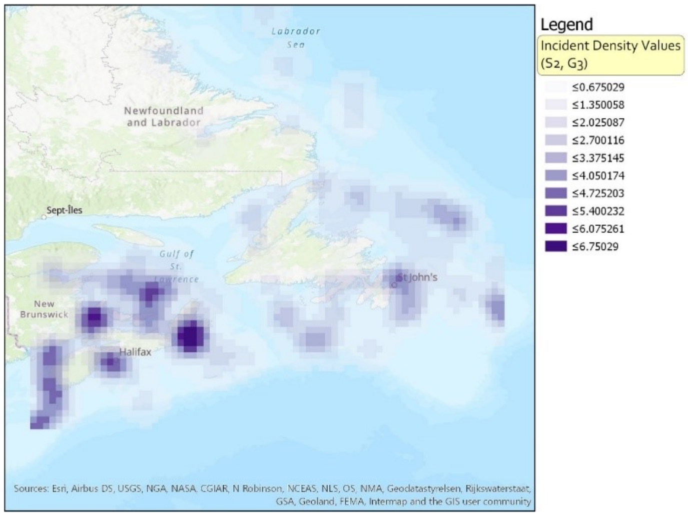

Figure 34. KD estimates heatmap for season 2 and incident group 3

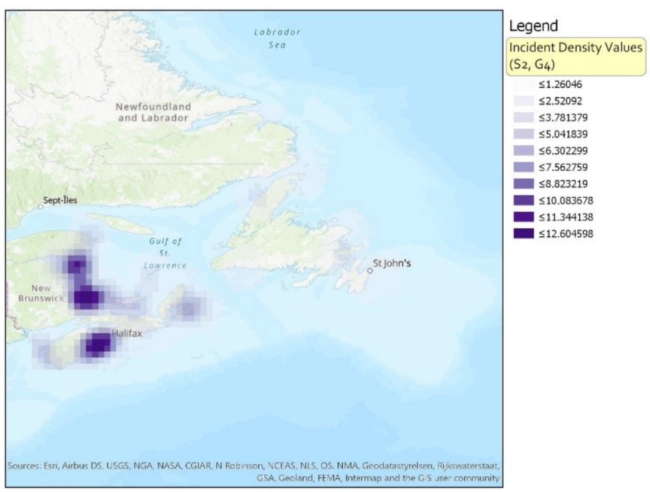

Figure 35. KD estimates heatmap for season 2 and incident group 4

4.6 Vessel classes and characteristics

In this thesis, due to the difference in characteristics, we have differentiated between SAR vessels. “Speed” and “range” of each vessel are two of the input parameters of our proposed optimization model. These parameters are used for assessing the capability of vessels for being located in offshore stations and also measuring their ability to cover a given demand grid. All the active SAR vessels in the Atlantic region of Canada (26 SAR vessels) have been identified and classified into five different classes after consultation with CCG subject matter experts. This classification was done considering similar characteristics and response capability of vessels, and data that have been extracted from the CCG website. It is worth mentioning that science vessels, specialty vessels, and icebreakers are excluded from our data because they are relatively rarely used in SAR operations in the Atlantic area. Moreover, one mid-shore vessel (Corporal McLaren M.M.V.) was out of service due to maintenance at the time of writing this thesis and two spare lifeboats (Spray from the Bay Class, and Cape Edensaw from the Cape Class) have not been considered in this research because they are only used for backfilling. Table 23 shows the vessel classes along with their characteristics.

Table 23. Vessel classes and characteristics

Vessel Class	Vessel Type	Range (Km)	Vessel Length (m)	Cruising Speed (Km/hr)	Number Available
Medium Endurance lifeboats (Cape Class)	Class 1	185	14.6	37	5
High Endurance lifeboats (Bay Class)	Class 2	231	15.8 - 19	37	11
Mid-shore Class	Class 3	1852	42.8	26	2
Large Multi-Task Class	Class 4	6019	69.7 - 83	28	5
Off-shore Class	Class 5	9260	62.4 - 72	22	3

4.7 Response- Capacity

It is obvious that the capacity of the response vessels is limited, and we should include capacity constraints to control the workload of the vessels. The definition of the capacity is a bit different in this thesis than some other studies. The maximum response-capacity is defined as the number of incidents that can be responded to in a period (operational season). For calculation of response-capacity, we have used the method developed by (Akbari, Pelot, et al., 2018b). They have mentioned some factors which have impact on the response-capacity calculation as follows:

- Vessel unavailability due to maintenance: *SAR* vessels are generally unavailable in some period of service due to planned and unplanned maintenance, which affects their actual operational capacity.

- Vessel unavailability due to multi-tasking: Some vessels that perform *SAR* tasks are designed and used for multiple mandates. So, in reality they are not fully allocated to the *SAR* program. This should be considered in the capacity planning of resources.
- Average response time to incidents: the average number of incidents that can be responded to by a particular vessel in a given time period based on the historical observations is important in order to calculate the maximum number they can respond over planning horizon.
- Vessel speed: vessels have different speeds, which has impact on the duration of response (i.e., transit) to incidents and thus on the capacity of vessel.

The following equation is proposed by (Akbari, Pelot, et al., 2018b) to calculate maximum response-capacity for vessel type k, number l, in period θ:

$C_{kl\theta} = (1 - mur_{kl\theta}) \times ar_{kl\theta} \times sar_{kl} \times air \times nd$

where:

$mur_{kl\theta}$: Maintenance unavailability rate of vessel type k, number l, in period θ

$ar_{kl\theta}$: Availability rate of vessel type k, number l for *SAR* tasks in period θ

sar_{kl}: Speed adjustment rate of vessel type k, number l relative to the fastest vessel type

air: Average number of incidents responded to by a vessel per day (1/day)

nd: Number of days in a season (182-183 days)

After consultation with CCG experts, the response-capacity of each vessel class in each season was calculated. Table 24 shows the response capacity table:

Table 24. Response capacity table

Vessel Type	**Season 1**				**Season 2**			
	$mur_{kl\theta}$	$ar_{kl\theta}$	sar_{kl}	$Capacity$	$mur_{kl\theta}$	$ar_{kl\theta}$	sar_{kl}	$Capacity$
Class 1 (Cape)	0.1	1	1	163.8	0.01	1	1	180.18
Class 2 (Bay)	0.1	1	1	163.8	0.01	1	1	180.18
Class 3 (Mid-Shore)	0.324	0.2	0.702	17.273	0.036	0.2	0.702	24.632
Class 4 (Multi-Task)	0.414	0.5	0.756	40.314	0.046	0.5	0.756	65.631
Class 5 (Off-shore)	0.324	0.2	0.594	14.616	0.036	0.2	0.594	20.843

As mentioned before, the basis of the response-capacity calculation is inspired by (Akbari, Pelot, et al., 2018b) and SMEs guided us through the calculation of different parameters. The calculation process of each parameter is explained in the following:

$ar_{kl\theta}$: As we can see in the table, $ar_{kl\theta}$ has been considered 1 for classes 1 and 2 because lifeboats are fully allocated to SAR missions. Other values for this parameter have been assigned based on data analysis and expert comments.

sar_{kl}: For the sar_{kl}, the fastest vessels (lifeboats) were given a 1 and other vessels' speed ratings were calculated accordingly.

$mur_{kl\theta}$: according to the CCG, they tend to do most of the maintenance tasks in the colder season (season 1), so the maintenance unavailability rate has been considered more in season 1. Due to the similarities between mid-shore and offshore class, we have decided to consider equal values for this parameter in these two classes. It should be mentioned that the presence of two backup (spare) lifeboats (Spray and Cape Edensaw) has decreased the maintenance unavailability rate to 10% and 1% for season 1 and season 2 respectively.

air: based on the SISAR data and CCG expert comments, on average each vessel can respond to one incident per day. So, this parameter has been considered 1 for all vessel classes.

4.8 Vessels' Procurement and Operational Cost

As mentioned before, one of the objectives of our optimization model is minimizing the total cost of the SAR vessels. Total cost of each vessel consists of fixed (procurement) and variable (operational) costs. Operational costs include the following items:

- Crew salary
- Fuel
- Refit
- Maintenance
- Other costs (Provisions, Travel, Training)

Fixed cost is the acquisition cost which was paid at the start of the launch year of each vessel. In accordance with CCG cost data, it is assumed that the annual operational cost values are based on real 2019 dollars. That is, 2019 (the launch year of the newest vessel in our database) has been considered as the base year. It should be mentioned that due to confidentiality purposes, the cost data are represented as scaled (i.e., relative) rates, which is acceptable for our study. In this scale, the highest cost (acquisition cost of "George R. Pearkes") has been taken as a baseline (1) and other costs were estimated based on that. One of the noticeable points in the cost database provided by CCG is that inflation has not been applied to the acquisition costs. This means, for example. that the acquisition cost of the "Cape Roger" vessel, which was launched in 1977, was calculated based on 1977 actual dollars. Thus, it is required to convert this cost to 2019 real dollars first and then calculate the annual cost considering the lifetime of the vessel. We used Consumer Price Index (CPI) for 'all-items' to calculate the inflation rate (Government of Canada, 2007) and the average discount rate has been considered to be approximately 4% for each year. Public discount rate can vary between countries and applications, but they are typically quite low. We have elected to use 4% which is in line with published figures (Benefits Canada, 2019). The CPI is a measure that examines the weighted average of prices of a basket of consumer goods and services (such as transportation, food, medical care, etc.) and it is calculated by taking price changes for each item over time and averaging them using a weighting function. In accordance with (Government of Canada, 2007), the year 2002 has been considered as the base year and thus the CPI for that year is 100. Values of the CPIs (for all-items) in different years has been shown in the Table 25:

Table 25. CPI index table

Year	1975	1976	1977	1978	1979	1980	1981	1982	1983
CPI	29	31.1	33.6	36.6	40	44	49.5	54.9	58.1
Year	1984	1985	1986	1987	1988	1989	1990	1991	1992
CPI	60.6	63	65.6	68.5	71.2	74.8	78.4	82.8	84
Year	1993	1994	1995	1996	1997	1998	1999	2000	2001
CPI	85.6	85.7	87.6	88.9	90.4	91.3	92.9	95.4	97.8
Year	2002	2003	2004	2005	2006	2007	2008	2009	2010
CPI	100	102.8	104.7	107	109.1	11.5	114.1	114.4	116.5

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019
CPI	119.9	121.7	122.8	125.2	126.6	128.4	130.4	133.4	136

The calculation process for "Cape Roger, 1977" costs is explained below.

$$Inflation\ Rate\ between\ 1977\ and\ 2019 = \frac{CPI_{2019} - CPI_{1976}}{CPI_{1976}} = 3.372$$

$$Acquisition\ Cost\ of\ Cape\ Roger\ in\ real\ 2019\ dollars\ considering\ calculated\ inflation\ rate$$
$$= Acquisition\ cost_{1977} \times (1 + Inflation\ rate) = 0.429 \times (1 + 3.372) = 1.878$$

$$Annual\ equivalent\ of\ acquisition\ cost\ over\ vessel's\ lifetime =$$
$$PMT(discount\ rate, number\ of\ years, present\ value) = PMT(4\%, 43, 1.878) = 0.092$$

$$Total\ annual\ cost\ of\ \text{Cape Roger}$$
$$= Annualized\ acquisition\ cost + Annual\ operational\ cost = 0.092 + 0.105$$
$$= 0.197$$

The cost calculations have two important assumptions:

- It is assumed that acquisition cost was paid at the end of year 0 (e.g., 1976 for the Cape Roger)
- The acquisition cost has been distributed over: Max [vessel's nominal lifetime, no. years in service]

After calculating the annual cost of each vessel, we obtained the average for each vessel class and finally got the total cost allocated to SAR (after applying availability rate which was discussed earlier). Table 26 summarizes the costs for different vessel classes.

Table 26. Final cost table for different vessel classes

Vessel Type	Operational Lifetime (Years)	Procurement Cost (annualized)	Operational Cost	Total Cost	Total cost allocated to SAR
Class 1 (Cape)	20	0.004	0.018	0.022	0.0223
Class 2 (Bay)	20	0.005	0.018	0.024	0.0241
Class 3 (Mid-Shore)	25	0.029	0.048	0.077	0.0155
Class 4 (Large Multi-Task)	35	0.107	0.161	0.268	0.134
Class 5 (Off-shore)	35	0.0819	0.120	0.202	0.040

4.9 Relocation Cost

As discussed earlier, it is possible for vessels to be relocated from one station to another at the end of each period. After consultation with CCG members, they mentioned that the cost of these temporary relocations (which is called “repositioning” in practice) is negligible, because it just involves salary and fuel cost. In this regard, we have considered the relocation cost as different percentages of total cost to do sensitivity analysis and investigate the impact of relocation cost on the solutions.

4.10 Demand distribution function

In accordance with the methodology chapter, Kernel Density estimates were used in this thesis for assessing the incidents’ density in each grid and for different seasons and groups of incidents. First, the heatmap of incidents (for different seasons and different groups of incidents) was depicted using Arc GIS pro software. Since the outputs were raster files, the data were converted to points using the “raster to point” function and the weight for each of our defined grids was calculated with the “spatial analysis” toolbox in this software. Then, these incident density rates are multiplied by the grid area (for addressing the different grid sizes) and scaled so that they sum up to the average number of historical incidents in each season. It is assumed that the number of incidents in each grid follows a Poisson distribution, and the calculated density rates have been considered as the mean parameters of the Poisson distribution.

4.11 Coverage time limit

One of the parameters of the optimization model is t which is the maximum time limit for acceptable coverage. It is true that this parameter can vary based on defined service level and expert opinion, but in this thesis this parameter has been considered to be 6 hours in accordance with consultation with CCG experts. It should be mentioned that this time limit has been applied to the problem for both the allocation process and coverage calculations.

4.12 Effectiveness ratings

As discussed in the methodology chapter, RAMSARD criteria have been utilized to assess the capability of vessels in response to different groups of incidents. Based on the described calculation process in the previous chapter, Table 27 has been used as the final capability rating table in the optimization model.

Table 27. Capability rating table

	C1	C2	C3	C4	C5
G1	305	326	380.5	387	385
G2	360	385	467.5	484	474
G3	294	320	376.5	385	383
G4	282.5	302.5	368.5	393.5	391.5

5. CHAPTER 5 RESULTS AND DISCUSSIONS

For assessing the efficiency of an optimization model and solution methods, it is essential to do verification and validation to make sure that the outputs are reliable. In the previous chapter the optimization model and solution methods were presented, and in this chapter the numerical results will be discussed. To do so, first we will validate the suggested meta-heuristic methods with a small-scale problem, and we will show that the output results of these methods are reliable. Then, parameter adjustment will be done to determine the best set of parameters for obtaining the best results. Afterwards, sensitivity analysis of the main parameters will be carried out on a medium-sized problem and finally the case study (large scale real-world SAR problem in Atlantic region of Canada) will be presented.

It is necessary to mention that all the calculations (except for the large-scale problem) were implemented on a system with the following configuration:

CPU: Intel Core i7-2670QM Processor (2.2GHz up to 3.10GHz), Ram: 8 GB DDR3-SDRAM

5.1 Methods validation

In order to validate the meta-heuristic methods, we designed a small-scale problem to solve with an exact solution method and compare the results with the other methods. In the small-scale problem, we have considered two grids, two stations, two vessels, and two incident groups. Dynamic programming was used as our exact method for solving this problem with the formulation described in the chapter 3. Indices and parameters for the small-scale problem are shown below. This configuration has been considered for validation of the proposed meta-heuristic methods.

Indices

$i \in I$	2
$j, j' \in J$	2
$l \in L$	2
$k \in K$	2 (vessel class 1 with *l*=1 and vessel class 2 with *l*=2)
$\theta \in \phi$	2
$q \in Q$	2

Parameters

Table 28. Small-scale problem parameters

Parameter	Values			
$Ra_{\ qkl}$	$Ra_{\ 111} = 0.4$	$Ra_{\ 211} = 0.6$	$Ra_{\ 122} = 0.3$	$Ra_{\ 222} = 0.7$
$ac_{kl\theta}$	$ac_{\ 111} = 120$	$ac_{\ 112} = 140$	$ac_{\ 221} = 140$	$ac_{\ 222} = 100$
$c_{kl\theta}$	$c_{\ 111} = 110$	$c_{\ 112} = 100$	$c_{\ 221} = 120$	$c_{\ 222} = 110$
d_{ij}	$d_{\ 11} = 35$	$d_{\ 21} = 55$	$d_{\ 12} = 45$	$d_{\ 22} = 40$
$E(i,q,\theta)$	$E(1,1,1) = 90$	$E(2,1,1) = 70$	$E(1,2,2) = 60$	
$\varphi(i,q,\theta)$	$p\big(\varphi(1,1,1) > E(1,1,1)\big) = 0.2$	$p\big(\varphi(2,1,1) > E(2,1,1)\big) = 0.1$	$p(\varphi(1,2,2) > E(1,2,2)) = 0.3$	
rng_{kl}	$rng_{11} = 50$	$rng_{22} = 100$		
rc_{kl}	$rc_{11} = 20$	$rc_{22} = 30$		
sp_{kl}	$sp_{11} = 10$	$sp_{22} = 12$		
t	$t = 5$			
ρ	$\rho = 0.01$			
μ	$\mu = 1$			
von_{kl}	$von_{kl} = 0$			
sof_j	$sof = 0$			
$as_{j\theta}$	$as_{j\theta} = 1$			
β	β = 5			

5.1.1 Dynamic Programming steps and results

With respect to these parameters, each vessel can be located at each station (there are no offshore constraints in this example). Also, considering time and distance constraints, vessel 1 cannot be dispatched from station 1 to grid 2. So, we have three stages in this problem as follows:

- ✓ $n_1(1,1,1)$
- ✓ $n_2(1,2,1)$
- ✓ $n_3(2,1,2)$

Note that the changing the order of stages 1 and 2 does not have any impact on the final solution; also stage 3 can be before two other stages, but with respect to the rules (i.e., all the stages related to one period should be addressed continuously), we cannot change the order of stage 3 and 2.

Stage 1:

$n_1(1,1,1)$, $i = 1$, $q = 1$, $\theta = 1$

available actions:

$a_1(1,1,1)$, $a_1(1,1,2)$, $a_1(2,2,1)$, $a_1(2,2,2)$

according to the constraints we have:

$a''_1(1,1,1) = 0$, $a''_1(1,1,2) = 0$,
$a''_1(2,2,1) = 0$, $a''_1(2,2,2) = 0$,
$a'''_1(1,1,2,1) = 0$, $a'''_1(1,1,2,2) = 0$,
$a'''_1(2,2,2,1) = 0$, $a'''_1(2,2,2,2) = 0$.

In this stage the following states and combinations are available:

$s_1(1,1,1)$, $s_1(2,2,1)$,
s'_{11}: $\{s_1(1,1,1) \geq 90$, $s_1(2,2,1) < 90\}$,
s'_{12}: $\{s_1(1,1,1) < 90$, $s_1(2,2,1) \geq 90\}$.

Considering $a_1(1,1,1)$ and s'_{11} we have:

$r_{12}\big(n_1(1,1,1), s'_{11}, a_1(1,1,1)\big) = p\big(\varphi(2,1,1) > E(2,1,1)\big) \cdot a'''_1(1,1,2,1) + \big(1 - a'''_1(1,1,2,1)\big) = 0.1 * 1 + (1 - 1) = 0.1$

$r2\big(n_1(1,1,1), s'_{11}, a_1(1,1,1)\big) = 0.01 * \big(ac_{111} * a_1(1,1,1) + rc_{11} * a''_1(1,1,1)\big) - 1 * a_1(1,1,1).\ Ra_{111} + 5.a_1(0,0,0) = 0.01 * (120 * 1 + 20 * 0) - 1 * 1 * 0.4 + 5 * 0 = 1.2 - 0.4 = 0.8$

$$r\big(n_1(1,1,1), s'_{11}, a_1(1,1,1)\big) = \sum_{i'} r_{1i'}\big(n_1(1,1,1), s'_{11}, a_1(1,1,1)\big) + r2\big(n_1(1,1,1), s'_{11}, a_1(1,1,1)\big) = 0.9$$

Considering $a_1(1,1,1)$ and s'_{12} , it leads to an infeasible solution.

Considering $a_1(1,1,2)$ and s'_{11} we have:

$r_{12}\big(n_1(1,1,1), s'_{11}, a_1(1,1,2)\big) = p\big(\varphi(2,1,1) > E(2,1,1)\big).a'''_1(1,1,2,2) + \big(1 - a'''_1(1,1,2,2)\big) = 0.1 * 1 + (1 - 1) = 0.1$

$r2\big(n_1(1,1,1), s'_{11}, a_1(1,1,2)\big) = 0.01 * \big(ac_{111} * a_1(1,1,2) + rc_{11} * a''_1(1,1,2)\big) - 1 * a_1(1,1,2).\ Ra_{111} + 5.a_1(0,0,0) = 0.01 * (120 * 1 + 20 * 0) - 1 * 1 * 0.4 + 5 * 0 = 1.2 - 0.4 = 0.8$

$r\big(n_1(1,1,1), s'_{11}, a_1(1,1,2)\big) = \sum_{i'} r_{1i'}\big(n_1(1,1,1), s'_{11}, a_1(1,1,2)\big) + r2\big(n_1(1,1,1), s'_{11}, a_1(1,1,2)\big) = 0.9$

Considering $a_1(1,1,2)$ and s'_{12}, it leads to an infeasible solution.

This is stage 1 and there is no $v_n(k,l,\theta)$ in this stage so we can calculate recursive functions as follows:

$f_{1i'}(n_1(1,1,1), s'_{11}) = r_{1i'}\big(n_1(1,1,1), s'_{11}, a *_n (k,l,j)\big) \cdot f_{(n-1)i'}(n-1, \{v_n(k,l,\theta)\}) = 0.1 * 1 = 0.1$

$f2(n_1(1,1,1), s'_{11}) = r2\big(n_1(1,1,1), s'_{11}, a *_n (k,l,j)\big) + f2\big(\mathrm{n}-1(\theta,i,q), v_n(k,l,\theta)\big) = 0.8 + 0 = 0.8$
Therefore
$f(n_1(1,1,1), s'_{11}) = Minimize\{\sum_{ni}(f_{ni}(n_1(1,1,1), s'_{11}) + f2(n_1(1,1,1), s'_{11})\} = 0.9$

Considering $a_1(2,2,1)$ and s'_{11}, it leads to an infeasible solution.

Considering $a_1(2,2,1)$ and s'_{12} we have:
$r_{12}\big(n_1(1,1,1), s'_{12}, a_1(2,2,1)\big) = p\big(\varphi(2,1,1) > E(2,1,1)\big) \cdot a'''_1(2,2,2,1) + \big(1 - a'''_1(2,2,2,1)\big) = 0.1 * 1 + (1-1) = 0.1$
$r2\big(n_1(1,1,1), s'_{12}, a_1(2,2,1)\big) = 0.01 * \big(ac_{221} * a_1(2,2,1) + rc_{22} * a''_1(2,2,1)\big) - 1 * a_1(2,2,1) \cdot Ra_{122} + 5 \cdot a_1(0,0,0) = 0.01 * (140 * 1 + 30 * 0) - 1 * 1 * 0.3 + 5 * 0 = 1.4 - 0.3 = 1.1$
$r\big(n_1(1,1,1), s'_{12}, a_1(2,2,1)\big) = \sum_{i'} r_{1i'}\big(n_1(1,1,1), s'_{12}, a_1(2,2,1)\big) + r2\big(n_1(1,1,1), s'_{12}, a_1(2,2,1)\big) = 1.2$

Considering $a_1(2,2,2)$ and s'_{11}, it leads to an infeasible solution.

Considering $a_1(2,2,2)$ and s'_{12} we have:
$r_{12}\big(n_1(1,1,1), s'_{12}, a_1(2,2,2)\big) = p\big(\varphi(2,1,1) > E(2,1,1)\big) \cdot a'''_1(2,2,2,2) + \big(1 - a'''_1(2,2,2,2)\big) = 0.1 * 1 + (1-1) = 0.1$
$r2\big(n_1(1,1,1), s'_{12}, a_1(2,2,2)\big) = 0.01 * \big(ac_{221} * a_1(2,2,2) + rc_{22} * a''_1(2,2,2)\big) - 1 * a_1(2,2,2) \cdot Ra_{122} + 5 \cdot a_1(0,0,0) = 0.01 * (140 * 1 + 30 * 0) - 1 * 1 * 0.3 + 5 * 0 = 1.4 - 0.3 = 1.1$
$r\big(n_1(1,1,1), s'_{12}, a_1(2,2,2)\big) = \sum_{i'} r_{1i'}\big(n_1(1,1,1), s'_{12}, a_1(2,2,2)\big) + r2\big(n_1(1,1,1), s'_{12}, a_1(2,2,2)\big) = 1.2$

This is stage 1 and there is no $v_n(k,l,\theta)$ in this stage so we can calculate recursive functions as follows:
$f_{12}(n_1(1,1,1), s'_{12}) = r_{12}\big(n_1(1,1,1), s'_{12}, a_1^*(k,l,j)\big) \cdot f_{(n-1)2}(n-1, \{v_1(k,l,\theta)\}) = 0.1 * 1 = 0.1$
$f2(n_1(1,1,1), s'_{12}) = r2\big(n_1(1,1,1), s'_{12}, a_n^*(k,l,j)\big) + f2\big(\mathrm{n}-1(\theta,i,q), v_n(k,l,\theta)\big) = 0.8 + 0.1 = 0.9$
Therefore
$f(n_1(1,1,1), s'_{12}) = Minimize\{\sum_{ni}(f_{ni}(n_1(1,1,1), s'_{12}) + f2(n_1(1,1,1), s'_{12})\} = 0.9$
We have $a_n^*(k,l,j)$= {$a_1(1,1,1)$,$a_1(1,1,2)$}

Stage 2:
$n_2(1,2,1)$ $, i = 2, q = 1$ $, \theta = 1$
available actions:
$a_2(1,1,2)$, $a_2(2,2,1)$, $a_2(2,2,2)$
$a_2(1,1,1)$ is unavailable since $d_{21} > rng_{11}$ and $\frac{d_{21}}{rng_{11}} > t$

according to the constraints we have:

$a''_2(2,2,1) = 0,\qquad a''_2(2,2,2) = 0\,,$
$a'''_2(1,1,1,2) = 0,\qquad a''_2(1,1,2) = 0,$
$a'''_2(2,2,1,1) = 0,\qquad a'''_2(2,2,1,2) = 0\ \ .$
In this stage we are at the last grid of the first period, so the following states and combinations are available:
$s_2(1,1,1),\quad s_2(2,2,1),$
$s'_{21}\text{:}\{s_2(1,1,1) = 110\,,\qquad s_2(2,2,1) = 100\,\},$

Considering $a_2(1,1,2)$ and s'_{21} we have:
$r_{21}\big(n_2(1,2,1), s'_{21}, a_2(1,1,2)\big) = p\big(\varphi(1,1,1) > E(1,1,1)\big)\cdot\ a'''_2(1,1,1,2) + \big(1 - a'''_2(1,1,1,2)\big) = 0.2 * 0 + (1-0) = 1$
$r2\big(n_2(1,2,1), s'_{21}, a_2(1,1,2)\big) =\ 0.01 * \big(ac_{111} * a_2(1,1,2) + rc_{11} * a''_2(1,1,2)\big) - 1 * a_2(1,1,2)\cdot\ \ Ra_{111} + 5.\,a_2(0,0,0) = 0.01 * (120 * 1 + 20 * 0) - 1 * 1 * 0.4 + 5 * 0 = 1.2 - 0.4 = 0.8$
$r\big(n_2(1,2,1), s'_{21}, a_2(1,1,2)\big) = \sum_{i'} r_{1i'}\big(n_2(1,2,1), s'_{21}, a_2(1,1,2)\big) + r2\big(n_2(1,2,1), s'_{21}, a_2(1,1,2)\big) = 1.8$
We can calculate $v_n(k,l,\theta)$ as below:
$v_2(1,1,1) = s_2(1,1,1) - a_2(1,1,2)\cdot E(2,1,1) = 110 - 70 * 1 = 40$
$v_2(2,2,1) = 100$

Considering $a_2(2,2,1)$ and s'_{21} we have:
$r_{21}\big(n_2(1,2,1), s'_{21}, a_2(2,2,1)\big) = p\big(\varphi(1,1,1) > E(1,1,1)\big)\cdot\ a'''_2(2,2,1,1) + \big(1 - a'''_2(2,2,1,1)\big) = 0.2 * 1 + (1-1) = 0.2$
$r2\big(n_2(1,2,1), s'_{21}, a_2(2,2,1)\big) =\ 0.01 * \big(ac_{221} * a_2(2,2,1) + rc_{22} * a''_2(2,2,1)\big) - 1 * a_2(2,2,1)\cdot\ \ Ra_{122} + 5\cdot a_2(0,0,0) = 0.01 * (140 * 1 + 30 * 0) - 1 * 1 * 0.3 + 5 * 0 = 1.4 - 0.3 = 1.1$
$r\big(n_2(1,2,1), s'_{21}, a_2(2,2,1)\big) = \sum_{i'} r_{1i'}\big(n_2(1,2,1), s'_{21}, a_2(2,2,1)\big) + r2\big(n_2(1,2,1), s'_{21}, a_2(2,2,1)\big) = 1.3$
We can calculate $v_n(k,l,\theta)$ as below:
$v_2(1,1,1) = 110$
$v_2(2,2,1) = s_2(1,1,1) - a_2(2,2,1).\,E(2,1,1) = 100 - 70 * 1 = 30$

Considering $a_2(2,2,2)$ and s'_{21} we have:
$r_{21}\big(n_2(1,2,1), s'_{21}, a_2(2,2,2)\big) = p\big(\varphi(1,1,1) > E(1,1,1)\big)\cdot\ a'''_2(2,2,1,2) + \big(1 - a'''_2(2,2,1,2)\big) = 0.2 * 0 + (1-0) = 1$
$r2\big(n_2(1,2,1), s'_{21}, a_2(2,2,2)\big) =\ 0.01 * \big(ac_{221} * a_2(2,2,2) + rc_{22} * a''_2(2,2,2)\big) - 1 * a_2(2,2,2)\cdot\ \ Ra_{122} + 5\cdot a_2(0,0,0) = 0.01 * (140 * 1 + 30 * 0) - 1 * 1 * 0.3 + 5 * 0 = 1.4 - 0.3 = 1.1$
$r\big(n_2(1,2,1), s'_{21}, a_2(2,2,2)\big) = \sum_{i'} r_{1i'}\big(n_2(1,2,1), s'_{21}, a_2(2,2,2)\big) + r2\big(n_2(1,2,1), s'_{21}, a_2(2,2,2)\big) = 2.1$
We can calculate $v_n(k,l,\theta)$ as below:

$v_2(1,1,1) = 110$
$v_2(2,2,1) = s_2(1,1,1) - a_2(2,2,2) \cdot E(2,1,1) = 100 - 70 * 1 = 30$
The recursive function can be calculated as below:
$f_{21}(n_2(1,2,1), s'_{21}) = r_{21}\big(n_2(1,2,1), s'_{21}, a *_2 (k,l,j)\big) \cdot f_{(2-1)1}(2-1, \{v_2(k,l,\theta)\}) = 0.2 * 1 = 0.2$
$f_{12}(n_2(1,2,1), s'_{21}) = 0.1$
$f2(n_2(1,2,1), s'_{21}) = r2\big(n_2(1,2,1), s'_{21}, a *_n (k,l,j)\big) + f2\big(\mathrm{n}-1(\theta,i,q), v_n(k,l,\theta)\big) = 1.1 + 0.8 = 1.9$
Therefore
$f(n_2(1,2,1), s'_{21}) = Minimize\big\{\sum_{n,i}(f_{ni}(n_2(1,2,1), s'_{21}) + f2(n_2(1,2,1), s'_{21})\big\} = 0.2 + 0.1 + 1.9 = 2.2$
Note that in this stage $a_n^*(k,l,j) = a_2(2,2,1)$.

Stage 3:
$n_3(2,1,2)$, $i = 1, q = 2, \theta = 2$
available actions:
$a_3(1,1,1)$, $a_3(1,1,2)$, $a_3(2,2,1)$, $a_3(2,2,2)$
according to the constraints we have:
$a''_3(1,1,1) = 0$, $a''_3(1,1,2) = 0$,
$a''_3(2,2,1) = 0$, $a''_3(2,2,2) = 0$,
$a'''_3(1,1,2,1) = 0$, $a'''_3(1,1,2,2) = 0$,
$a'''_3(2,2,2,1) = 0$, $a'''_3(2,2,2,2) = 0$
In this stage we are at the last grid of the second period, so the following states and combinations are available:
$s_3(1,1,2)$, $s_3(2,2,2)$,
$s'_{31}: \{s_3(1,1,2) = 120$, $s_3(2,2,2) = 110\}$,

Considering $a_3(1,1,1)$ and s'_{31} we have:
$r_{32}\big(n_3(2,1,2), s'_{31}, a_3(1,1,1)\big) = 1$
$r2\big(n_3(2,1,2), s'_{31}, a_3(1,1,1)\big) = 0.01 * \big(ac_{112} * a_3(1,1,1) + rc_{11} * a''_3(1,1,1)\big) - 1 * a_3(1,1,1) \cdot Ra_{211} + 5 \cdot a_3(0,0,0) = 0.01 * (140 * 1 + 20 * 0) - 1 * 1 * 0.6 + 5 * 0 = 1.4 - 0.6 = 0.8$
$r\big(n_3(2,1,2), s'_{31}, a_3(1,1,1)\big) = \sum_{i'} r_{1i'}\big(n_3(2,1,2), s'_{31}, a_3(1,1,1)\big) + r2\big(n_3(2,1,2), s'_{31}, a_3(1,1,1)\big) = 1.8$

Considering $a_3(1,1,2)$ and s'_{31} we have:
$r_{32}\big(n_3(2,1,2), s'_{31}, a_3(1,1,2)\big) = 1$
$r2\big(n_3(2,1,2), s'_{31}, a_3(1,1,2)\big) = 0.01 * \big(ac_{112} * a_3(1,1,2) + rc_{11} * a''_3(1,1,2)\big) - 1 * a_3(1,1,2) \cdot Ra_{211} + 5 \cdot a_3(0,0,0) = 0.01 * (140 * 1 + 20 * 1) - 1 * 1 * 0.6 + 5 * 0 = 1.6 - 0.6 = 1$
$r\big(n_3(2,1,2), s'_{31}, a_3(1,1,2)\big) = \sum_{i'} r_{1i'}\big(n_3(2,1,2), s'_{31}, a_3(1,1,2)\big) + r2\big(n_3(2,1,2), s'_{31}, a_3(1,1,2)\big) = 2$

Considering $a_3(2,2,1)$ and s'_{31} we have:
$r_{32}\big(n_3(2,1,2), s'_{31}, a_3(2,2,1)\big) = 1$
$r2\big(n_3(2,1,2), s'_{31}, a_3(2,2,1)\big) = 0.01 * \big(ac_{222} * a_3(2,2,1) + rc_{22} * a''_3(2,2,1)\big) - 1 * a_3(2,2,1) \cdot Ra_{222} + 5 \cdot a_3(0,0,0) = 0.01 * (100 * 1 + 30 * 0) - 1 * 1 * 0.7 + 5 * 0 = 1 - 0.7 = 0.3$
$r\big(n_3(2,1,2), s'_{31}, a_3(2,2,1)\big) = \sum_{i'} r_{1i'}\big(n_3(2,1,2), s'_{31}, a_3(2,2,1)\big) + r2\big(n_3(2,1,2), s'_{31}, a_3(2,2,1)\big) = 1.3$

Considering $a_3(2,2,2)$ and s'_{31} we have:
$r_{32}\big(n_3(2,1,2), s'_{31}, a_3(2,2,2)\big) = 1$
$r2\big(n_3(2,1,2), s'_{31}, a_3(2,2,2)\big) = 0.01 * \big(ac_{222} * a_3(2,2,2) + rc_{22} * a''_3(2,2,2)\big) - 1 * a_3(2,2,2) \cdot Ra_{222} + 5 \cdot a_3(0,0,0) = 0.01 * (100 * 1 + 30 * 1) - 1 * 1 * 0.7 + 5 * 0 = 1.3 - 0.7 = 0.6$
$r\big(n_3(2,1,2), s'_{31}, a_3(2,2,2)\big) = \sum_{i'} r_{1i'}\big(n_3(2,1,2), s'_{31}, a_3(2,2,2)\big) + r2\big(n_3(2,1,2), s'_{31}, a_3(2,2,2)\big) = 1.6$

This is the first stage of second period and there is no $v_n(k,l,\theta)$ in this stage so we can calculate recursive functions as follows:
The recursive function can be calculated as below:
$f_{12}(n_3(2,1,2), s'_{31}) = 0.1$
$f_{21}(n_3(2,1,2), s'_{31}) = 0.2$
$f_{32}(n_3(2,1,2), s'_{31}) = 1$
$f2(n_3(2,1,2), s'_{31}) = r2\big(n_3(2,1,2), s'_{31}, a*_3\,(k,l,j)\big) + f2\big(\mathrm{n}-1(\theta,i,q), v_n(k,l,\theta)\big) = 0.3 + 1.9 = 2.2$
Therefore
$f(n_3(2,1,2), s'_{31}) = Minimize\big\{\sum_{n,i}(f_{ni}(n_3(2,1,2), s'_{31}) + f2(n_3(2,1,2), s'_{31})\big\} = 0.2 + 0.1 + 1 + 2.2 = \mathbf{3.5}$
Note that in this stage $a_n^*(k,l,j) = \{a_3(2,2,1), a_3(2,2,2)\}$

Final value of objective function is 3.5 and optimal solutions are shown in Table 29 (there are two optimal solutions with equal objective function).

Table 29. Dynamic programming final solutions

Solution \ Stage	1	2	3
1	$a_1(1,1,1)$	$a_2(2,2,1)$	$a_3(2,2,1)$
2	$a_1(1,1,2)$	$a_2(2,2,1)$	$a_3(2,2,1)$

Note that given the high penalty of action 0 in this example, it was not given any value to this action in all the stages.

5.1.2 Metaheuristic methods results

The results of solving the described example with different metaheuristic methods in MATLAB are summarized in Table 30:

Table 30. Metaheuristic methods' results for small-scale problem

Method	Objective Function	Elapsed time (Seconds)
SA	3.5	2.044566
GA	3.5	10.946567
HS	3.5	14.938023
SAGA	3.5	55.246449
HSGA	3.5	14.654833
HSGA1	3.5	10.035562
HSGA2	3.5	1415.610515

As shown in the table, the output of all the methods is the same and it is equal to the output of the dynamic programming method. Therefore, these methods are valid and can be used for the real-world case study. The only difference here is the running time, in which the SA method outperformed the others.

Before continuing, note that a medium-sized example was designed for investigating sensitivity analysis and parameter adjustment. The Small-scale problem is not able to show the difference between different configurations because it gives the best value all the time. On the other hand, the Large-scale problem is computationally intensive, and it is not appropriate for parameter adjustment which needs large numbers of runs. Thus, an example with the parameters listed in Table 31 has been designed for sensitivity analysis and parameter adjustment.

Table 31. Medium-sized problem parameters

Parameter	Value
Number of Grids	10
Number of Stations	5
Number of Vessels	5
Number of Vessel Classes	2
Number of Periods	2
Incident Groups	2
rng_{kl}	random between 5 and 10
$Ra_{\ qkl}$	random between 10 and 30
$ac_{kl\theta}$	random between 200 and 300
rc_{kl}	$0.05 * ac_{kl\theta}$
$c_{kl\theta}$	random between 2 and 3
sp_{kl}	random between 20 and 30
d_{ij}	random between 5 and 15
$E(i, q, \theta)$	random between 0 and 10
t	0.5
ρ	0.01
μ	0.01

Parameter	Value
von_{kl}	1
sof_j	0
$as_{j\theta}$	1
β	100

5.2 Parameter adjustment

Each of the meta-heuristic methods has some parameters which are independent from the main parameters of the problem and have a considerable impact on the final results. Determining these parameters in a way that results in the best output is called parameter adjustment. The most common statistical method (Taguchi method) was used in this thesis for parameter adjustment.

5.2.1 Taguchi Method

The Taguchi method, used in a broad range of engineering fields (like Mechanical and Material Engineering), tries to decrease the impact of noise factors in experiments. This method implements the algorithm with different combinations of parameters and suggests the best parameter set for achieving the best results. Minitab software has been used for implementing Taguchi method.

5.2.1.1 Parameter adjustment for the SA method

The SA method has four independent parameters as follows:

A. Max number of iterations: this parameter shows the number of loops in the algorithm, and it is one of the main parameters in most of the meta-heuristic methods. In fact, this parameter acts as the termination condition.
B. Max number of sub-iterations: this parameter shows the number of iterations in each main loop of the algorithm.
C. Initial temperature: SA method is based on the slow cooling behavior of metals and this parameter represents the starting temperature of the process.
D. Cooldown factor: this parameter represents the coefficient of temperature reduction in each step.

In this part, we consider three levels for each parameter. According to the array selector table, an L9 orthogonal array is used which means 9 experiments with different levels of parameters will be performed and the output will be utilized in the Taguchi method. Different levels of the parameters are summarized in Table 32:

Table 32. Levels of SA parameters

SA Parameter	Level 1	Level 2	Level 3
A (No. of iterations)	50	75	100
B (No. of sub iterations)	100	150	200
C (Initial temperature)	20	30	40
D (Cooldown factor)	0.9	0.95	0.99

In this part, it is necessary to obtain the objective function value for each experiment. Since the Taguchi method is used in maximization problems, the reverse of the objective function value will be calculated and used as Taguchi input. This value for all the experiments has been calculated in Table 33.

Table 33. SA objective function for different experiments

Parameter / Experiment	A	B	C	D	Obj Function
1	1	1	1	1	0.0607151
2	1	2	2	2	0.0485435
3	1	3	3	3	0.0508393
4	2	1	2	3	0.0723721
5	2	2	3	1	0.0752313
6	2	3	1	2	0.0543319
7	3	1	3	2	0.0547678
8	3	2	1	3	0.0555062
9	3	3	2	1	0.0758719

After implementing the Taguchi method in Minitab software, the means and signal-to-noise (SN) ratio plots were obtained (see Figure 36 and Figure 37).

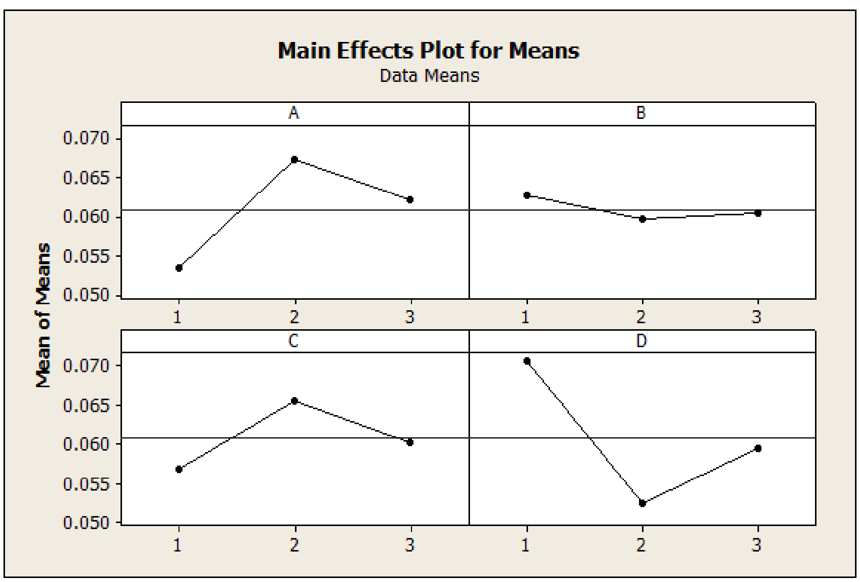

Figure 36. SA Taguchi means plot

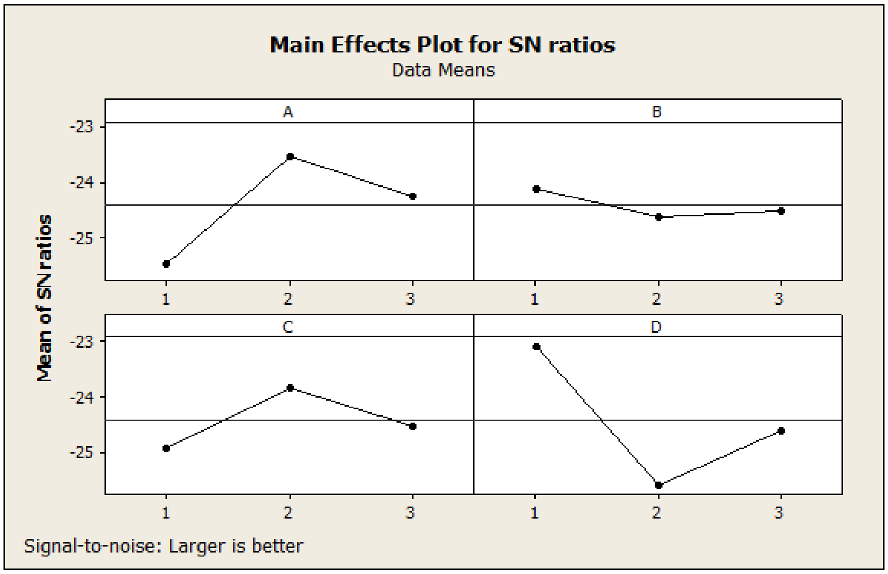

Figure 37. SA Taguchi SN ratio plot

Our objective function is maximization (using the reverse of the objective function value) and a level of parameters which put these plots at the highest value would be the best level for the problem. So, the levels in Table 34 will be selected for SA method in this thesis:

Table 34. Selected levels for SA parameters

SA Parameter	Level	Value
A) Max number of iterations	2	75
B) Max number of sub-iterations	1	100
C) Initial temperature	2	30
D) Cooldown factor	1	0.9

These results show that the number of iterations is not an important factor in this problem as level 2 and 1 have been selected for parameters A and B respectively. We can also conclude that high levels of sub-iterations make the method get stuck in local optima and it does not improve the final result.

5.2.1.2 Parameter adjustment for the GA method

GA method has four parameters as follows:

A. Max number of iterations: this parameter shows the number of loops in the algorithm, and it is one of the main parameters in most of the meta-heuristic methods. In fact, this parameter acts as the termination condition.
B. Number of populations: GA method is a population-based method and the number of populations in each iteration can have a considerable impact on the final result.
C. Crossover rate: this parameter shows the percentage crossover and generating new off-spring in each iteration.
D. Mutation rate: this parameter demonstrates the percentage (probability) of mutation in each iteration.

Like the SA method, three levels of parameters have been considered for the GA method. The defined levels are shown in Table 35:

Table 35. Levels of GA parameters

GA Parameter	Level 1	Level 2	Level 3
A (No. of iterations)	50	75	100
B (No. of populations)	50	75	100
C (Crossover rate)	0.8	0.85	0.9
D (Mutation rate)	0.2	0.25	0.3

Given four parameters with three levels, the L9 orthogonal array was selected to design the experiments. Table 36 shows the objective function value for different configurations of parameter levels.

Table 36. GA objective function for different experiments

Parameter / Experiment	A	B	C	D	Obj Function
1	1	1	1	1	0.0565533
2	1	2	2	2	0.0562313
3	1	3	3	3	0.0577229
4	2	1	2	3	0.0764355
5	2	2	3	1	0.0759432
6	2	3	1	2	0.0759669
7	3	1	3	2	0.0534205
8	3	2	1	3	0.0764872
9	3	3	2	1	0.0760558

After implementing the Taguchi method in Minitab, and inserting the previous table as input, the means and signal to noise plots were obtained (See Figure 38 and Figure 39).

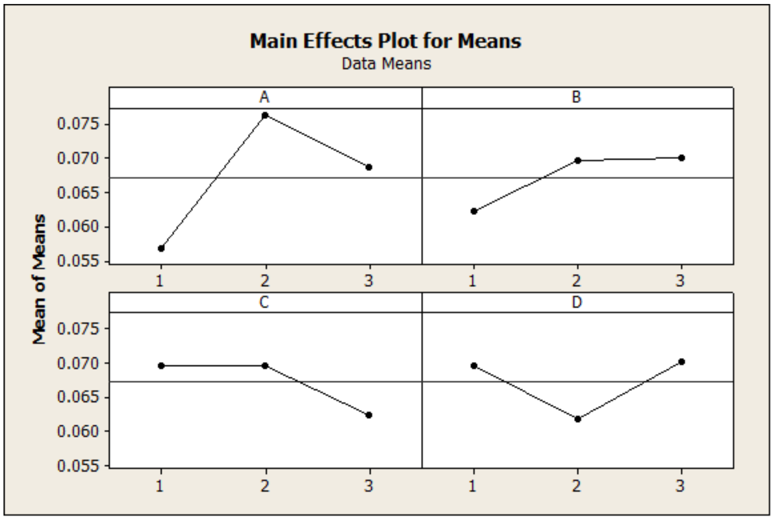

Figure 38. GA Taguchi means plot

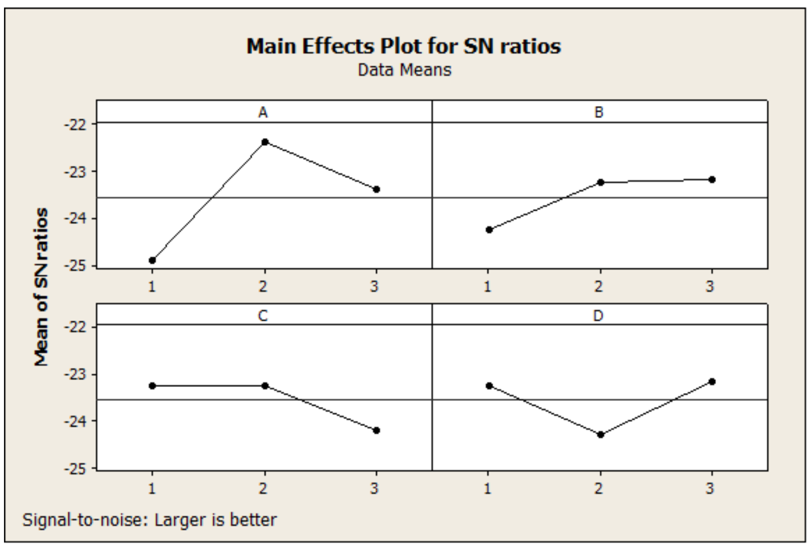

Figure 39. GA Taguchi SN ratio plot

Our objective function is maximization and a level of parameters which put these plots at the highest value would be the best level for the problem. So, the levels in Table 37 will be selected for the GA method in this thesis.

Table 37. Selected levels for GA parameters

GA Parameter	Level	Value
A) Max number of iterations	2	75
B) Number of populations	3	100
C) Crossover rate	2	0.85
D) Mutation rate	3	0.3

These results show that the number of iterations and crossover rate are in a middle level. Moreover, the highest level of population and mutation rate demonstrate the effort of the GA method to escape from local optima and generate diverse solutions.

5.2.1.3 Parameter adjustment for the HS method

HS method has five parameters as follows:

A. Harmony memory size (HMS): this is one of the most important parameters of HS and shows how many elements will enter harmony memory.
B. New harmony memory size (NHMS): this parameter shows the number of new solutions which are allowed to enter harmony memory in different iterations.
C. Max number of iterations (NI): this parameter shows the number of loops in the algorithm, and it is one of the main parameters in most of the meta-heuristic methods. In fact, this parameter acts as the termination condition.
D. Harmony memory consideration rate (HMCR): this parameter shows the probability of using harmony memory elements for generating new solutions. This parameter acts almost like the crossover rate parameter in the GA algorithm, and it helps the convergence of the method.
E. Pitch adjusting rate (PAR): this parameter shows the probability of using new elements for generating solutions. This parameter acts like mutation rate in GA algorithm and helps the method's divergence.

In this part three levels have been defined for each of the mentioned parameters. Table 38 shows the various levels for parameters.

Table 38. Levels of HS parameters

HS Parameter	Level 1	Level 2	Level 3
A (Harmony memory size (HMS))	10	15	20
B (New harmony memory size (NHMS))	50	75	100
C (Max number of iterations (NI))	50	75	100
D (Harmony memory consideration rate (HMCR))	0.7	0.75	0.8
E (Pitch adjusting rate (PAR))	0.05	0.07	0.09

Considering five parameters with three levels, an L27 orthogonal array was selected for designing the experiments which means 27 different experiments with various combinations of parameters. Table 39 demonstrates the objective function value for each set of suggested parameters.

Table 39. HS objective function for different experiments

Parameter / Experiment	A	B	C	D	E	Obj Function
1	1	1	1	1	1	0.050930
2	1	1	1	1	2	0.038507

Experiment	A	B	C	D	E	Obj Func
3	1	1	1	1	3	0.047899
4	1	2	2	2	1	0.042747
5	1	2	2	2	2	0.047112
6	1	2	2	2	3	0.048739
7	1	3	3	3	1	0.052561
8	1	3	3	3	2	0.044437
9	1	3	3	3	3	0.048489
10	2	1	2	3	1	0.048601
11	2	1	2	3	2	0.043207
12	2	1	2	3	3	0.046433
13	2	2	3	1	1	0.048830
14	2	2	3	1	2	0.045999
15	2	2	3	1	3	0.046198
16	2	3	1	2	1	0.046136
17	2	3	1	2	2	0.045061
18	2	3	1	2	3	0.046530
19	3	1	3	2	1	0.052092
20	3	1	3	2	2	0.048921
21	3	1	3	2	3	0.043053
22	3	2	1	3	1	0.045980
23	3	2	1	3	2	0.052357
24	3	2	1	3	3	0.048933
25	3	3	2	1	1	0.052555
26	3	3	2	1	2	0.047859
27	3	3	2	1	3	0.049261

Implementing the Taguchi method in Minitab with the mentioned inputs, the results were obtained as shown in Figure 40 and Figure 41.

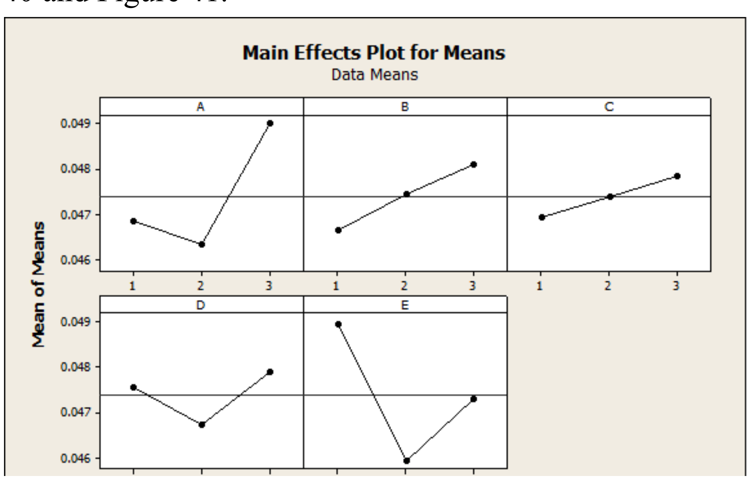

Figure 40. HS Taguchi means plot

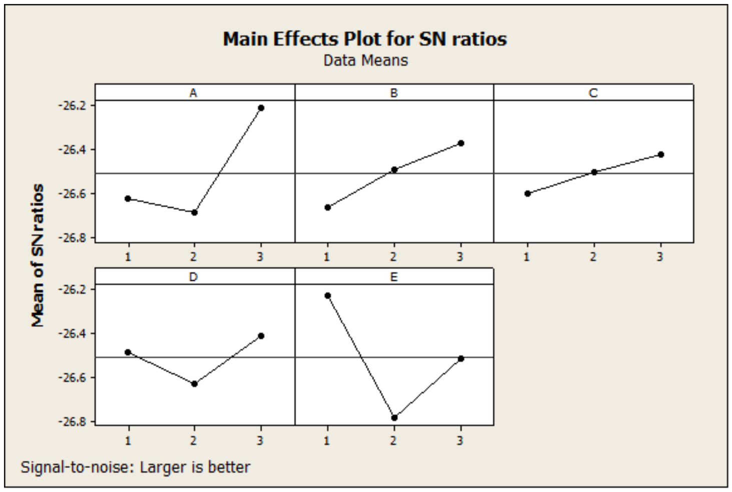

Figure 41. HS Taguchi SN ratio plot

Our objective function is maximization and a level of parameters which put these plots at the highest value would be the best level for the problem. So, levels in Table 40 will be selected for HS method in this thesis.

Table 40. Selected levels for HS parameters

HS Parameter	Level	Value
A) Harmony memory size (HMS)	3	20
B) New harmony memory size (NHMS)	3	100
C) Max number of iterations (NI)	3	100
D) Harmony memory consideration rate (HMCR)	3	0.8
E) Pitch adjusting rate (PAR)	1	0.05

These results show that this method is highly dependent on repetition and practice for finding the best results and this fact is in line with its musical nature.

It should be mentioned that the adjusted parameters for each method will be used for the hybrid methods as well.

5.3 Sensitivity Analysis

In this section, we will investigate the impact of different parts of the optimization model on the final results. This enables us to control the parameters to reach acceptable results. Notice that the analysis in this section was done with the medium-sized example because the small-scale problem is not able to generate various solutions and the large-scale problem is computationally intensive and cannot solve the problem in a reasonable amount of time.

The values of different parts of the objective function have been considered as a reference. Reference values have been obtained from solving the model with the GA algorithm and tuned parameters from the previous section. GA algorithm has been used because it is quite fast and precise in finding the optimum solution. Table 41 shows the reference values.

Table 41. Reference values for sensitivity analysis

1st part (Sum of Insufficiency Probability)	2nd part (Cost)	3rd part (Effectiveness Rating)	4th part (Percentage of primary coverage)
2.94	1726.1	744.9	95

5.3.1 First part sensitivity analysis (Insufficiency probability)

Since the first part of the objective function does not have any coefficient, coefficient 1 has been considered for this part and six different states were studied for the coefficient: Zero (i.e., not considering this part), -50% of the reference coefficient, -25% of the reference coefficient, +25% of the reference coefficient, +50% of the reference coefficient, and infinity (i.e., just considering this part).Table 42 shows the results that were obtained after running codes in different states of the first part.

Table 42. First part coefficient sensitivity analysis table

First part coefficient	1st part (Sum of Insufficiency Probability)	2nd part (Cost)	3rd part (Effectiveness Rating)	4th part (Percentage of primary coverage)
inf	1.14	1959.8	762.08	100
+50%	2.71	1727.5	730.04	95
+25%	2.9	1751.9	754.30	95
Reference	2.94	1726.1	744.90	95
-25%	2.73	1751.9	735.20	95
-50%	2.84	1758.2	744.80	95
0	1.19	1973.9	765.90	100

In this part, the impact on different parts of the objective function will be explained in detail.

Impact on the first part

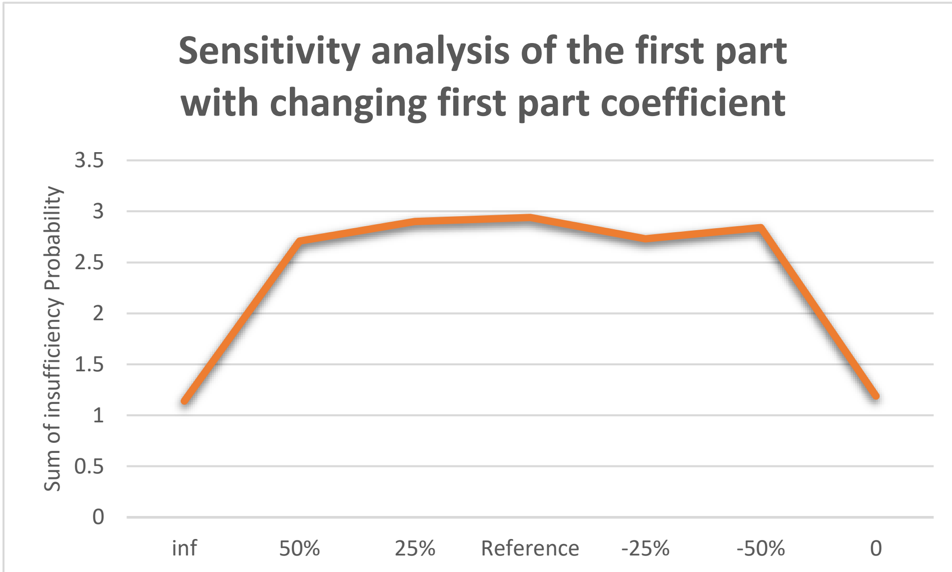

Figure 42. First part coefficient sensitivity analysis with first part variation

Results of this section are appreciable. They show that the change in the first part of the objective function (which is insufficiency probability) has a parabolic relationship with the first part coefficient. When we just consider this part in objective function, we will get the best result as expected; but as we increase the importance of other parts (specially the effectiveness rating which goes up with increasing coverage), the insufficiency probability decreases again. Generally, as we will show later, increasing response coverage leads to decreasing insufficiency probability.

Impact on the second part

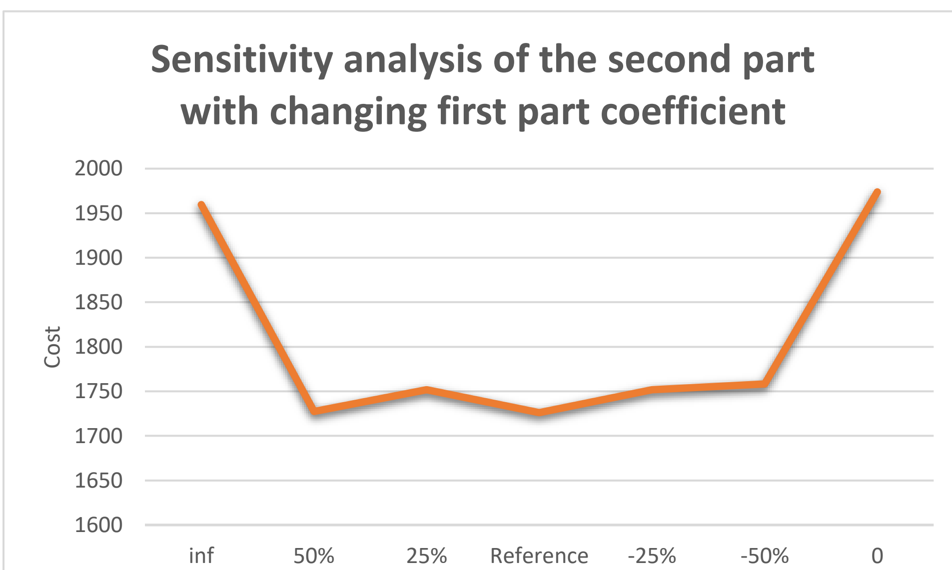

Figure 43. Second part coefficient sensitivity analysis with first part variation

This section acts in the opposite way to the previous graph because a decline of the insufficiency probability (and consequently increase of coverage percentage) results in an increase of the number of vessels dispatched and their costs. Soaring costs requires the investigation of experts and policymakers. Considering the significance of SAR and its direct relation to human lives, they would have to decide whether it is reasonable to pay extra for improving the coverage and insufficiency probability or not.

<ins>Impact on the third part</ins>

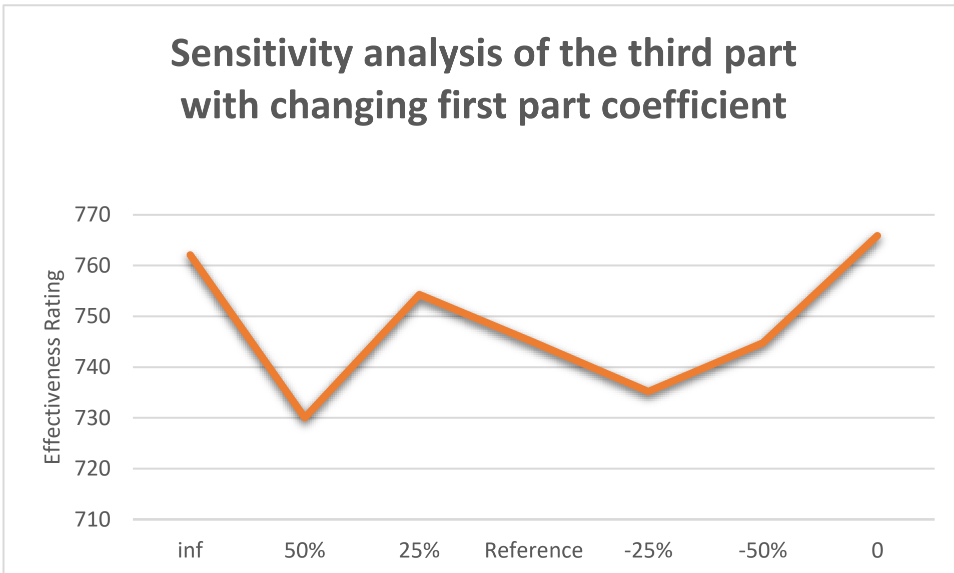

Figure 44. Third part coefficient sensitivity analysis with first part variation

The results show that the change in the first part coefficient does not have a meaningful correlation with effectiveness ratings. One of the reasons for this behavior might be the probabilistic nature of the metaheuristic method which generates different solutions.

Impact on the fourth part

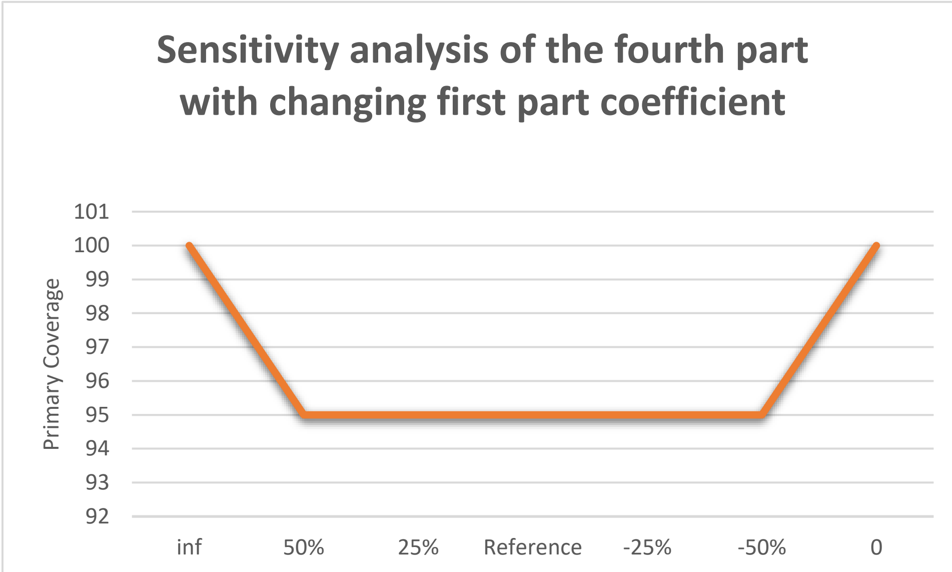

Figure 45. Fourth part coefficient sensitivity analysis with first part variation

Figure 45 shows that the coverage has a negative correlation with insufficiency probability which means that an increase of the coverage provides more backup coverage for the demand grids, and as a result decreases the insufficiency probability in the model.

5.3.2 Second part sensitivity analysis (Annual costs)

The coefficient of the second part of the objective function is ρ which will be studied in six different states as follows: Zero (i.e. not considering this part), -50% of the reference coefficient, -25% of the reference coefficient, +25% of the reference coefficient, +50% of the reference coefficient, and infinity (i.e. just considering this part). Table 43 shows the results that were obtained after running codes in different states of the second part.

Table 43. Second part coefficient sensitivity analysis table

Second part coefficient	1st part (Sum of Insufficiency Probability)	2nd part (Cost)	3rd part (Effectiveness Rating)	4th part (Percentage of primary coverage)
inf	20.1	806.95	338.34	42.5
+50%	2.73	1713.4	728.84	95
+25%	2.73	1726.3	726.68	95
Reference	2.94	1726.1	744.9	95
-25%	0.98	1971.5	754.27	100

Second part coefficient	1st part (Sum of Insufficiency Probability)	2nd part (Cost)	3rd part (Effectiveness Rating)	4th part (Percentage of primary coverage)
-50%	1.75	1975.1	758.96	100
0	1.13	2163	773.07	100

The impact on different parts of the objective function will be explained below.

<u>Impact on the first part</u>

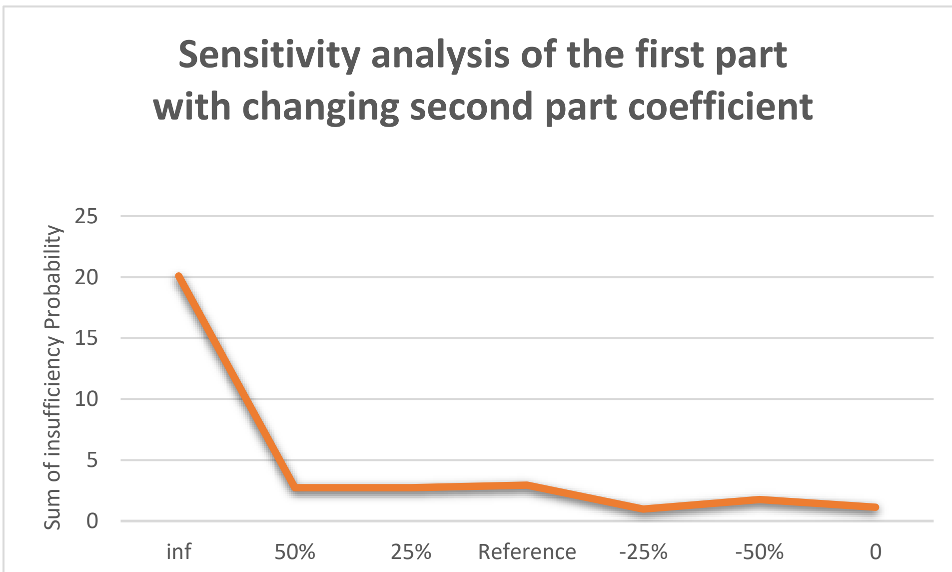

Figure 46. First part coefficient sensitivity analysis with second part variation

Figure 46 shows that decreasing the cost coefficient will reduce the insufficiency probability. The main reason for this correlation is focusing on primary and backup coverage improvement which requires more vessels to dispatch, which will increase the costs in the model.

Impact on the second part

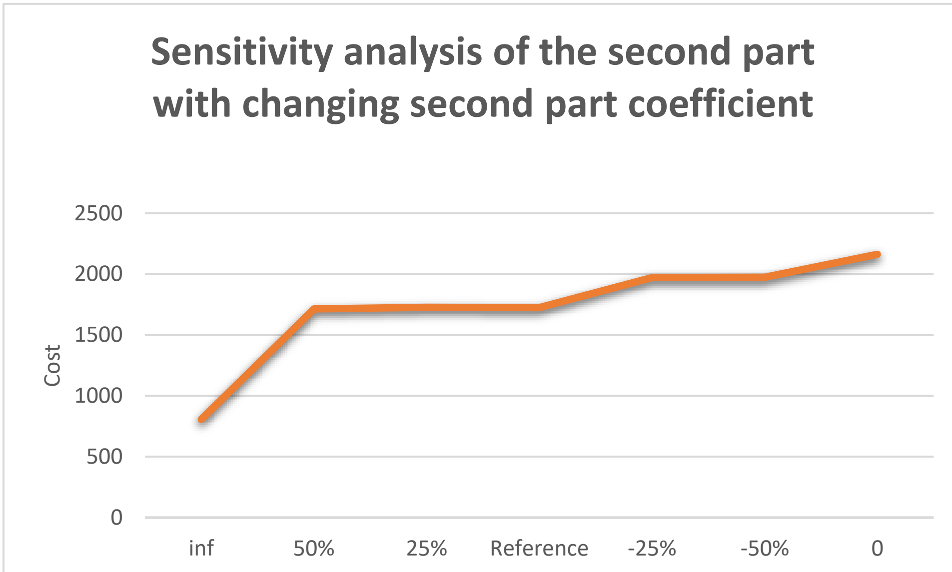

Figure 47. Second part coefficient sensitivity analysis with second part variation

As we expected, with decreasing the importance of cost coefficient, vessel costs increase and consequently the coverage has been raised. These results show that CCG policymakers can reduce the insufficiency probability by about 95% by accepting the increase of costs by 2.7 times. Considering the limited budget of CCG for SAR, this trade-off should be discussed with authorities and SMEs to investigate its feasibility.

Impact on the third part

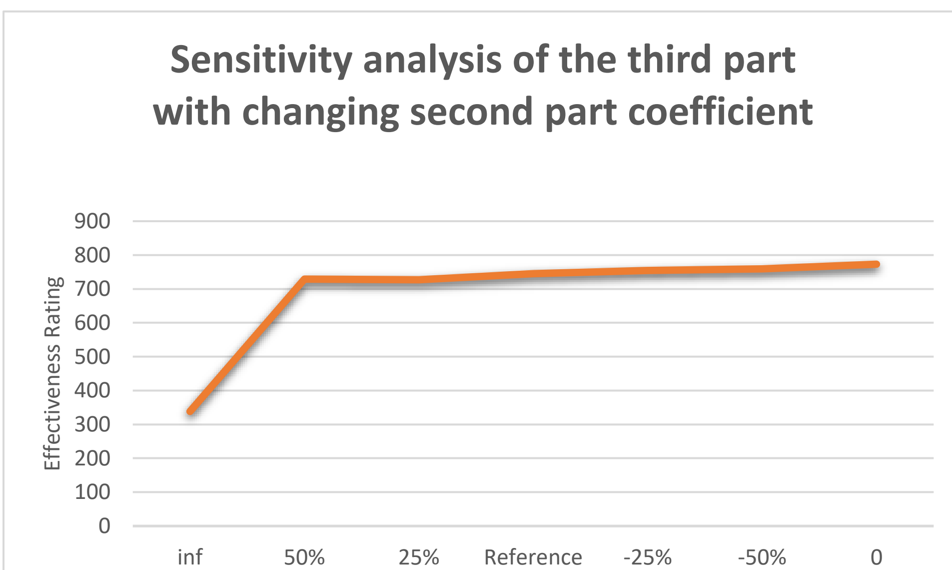

Figure 48. Third part coefficient sensitivity analysis with second part variation

The results of this section demonstrate that decreasing the importance of the cost coefficient can improve the effectiveness ratings considerably at first, but then the slope of the graph decreases significantly which reduces the subsequent effect of cost on effectiveness ratings.

Impact on the fourth part

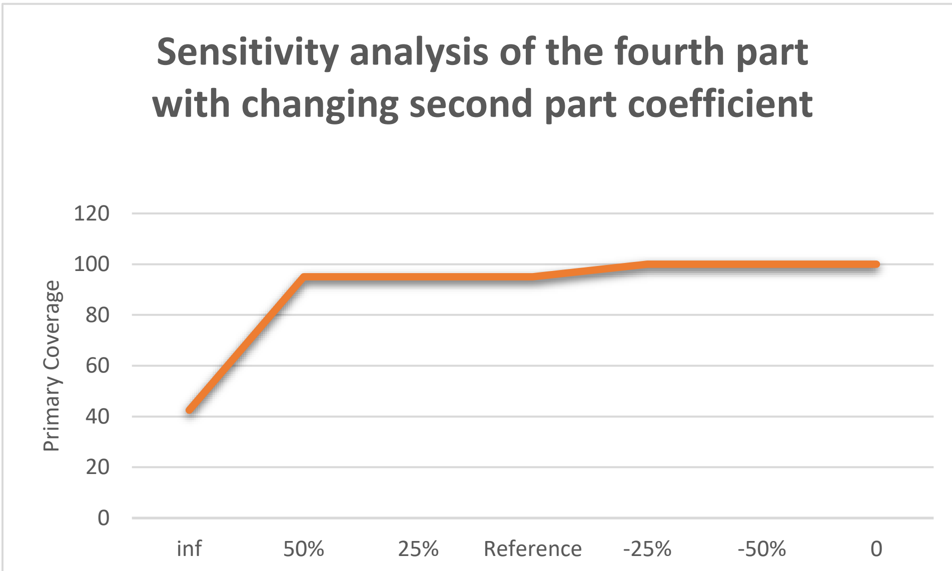

Figure 49. Fourth part coefficient sensitivity analysis with second part variation

This section's results are aligned with the previous section. They show that decreasing the cost importance results initially in an impressive improvement in coverage, and then this trend stops at a specific point. It should be mentioned that when we set the cost coefficient to infinity, the coverage percentage goes to zero. Thus, we set a minimum coverage of 40% to run the problem.

5.3.3 Third part sensitivity analysis (Effectiveness Rating)

The coefficient of the third part of the objective function is μ which will be studied in six different states as follows: Zero (i.e. not considering this part), -50% of the reference coefficient, -25% of the reference coefficient, +25% of the reference coefficient, +50% of the reference coefficient, and infinity (i.e. just considering this part). Table 44 shows the results that were obtained after running codes in different states of the third part:

Table 44. Third part coefficient sensitivity analysis table

Third part coefficient	1st part (Sum of Insufficiency Probability)	2nd part (Cost)	3rd part (Effectiveness Rating)	4th part (Percentage of primary coverage)
Inf	2.11	2264	793.96	100
+50%	1.02	1949.5	756.03	100

Third part coefficient	1st part (Sum of Insufficiency Probability)	2nd part (Cost)	3rd part (Effectiveness Rating)	4th part (Percentage of primary coverage)
+25%	1.17	1923.8	748.89	100
Reference	2.94	1726.1	744.90	95
-25%	2.76	1751.9	738.39	95
-50%	2.65	1751.9	733.88	95
0	2.57	1948.3	707.73	95

The impact on different parts of the objective function will be explained below.

Impact on the first part

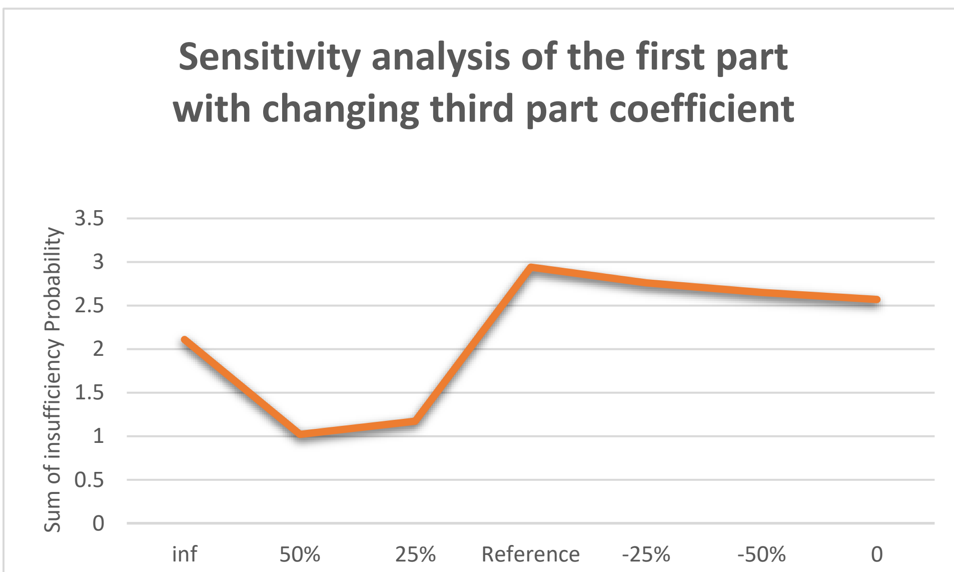

Figure 50. First part coefficient sensitivity analysis with third part variation

The results obtained from this part show the lack of meaningful correlation between effectiveness rating coefficient and insufficiency probability. As we can see, the insufficiency probability fluctuates with different states and there is not any specific trend in this graph.

Impact on the second part

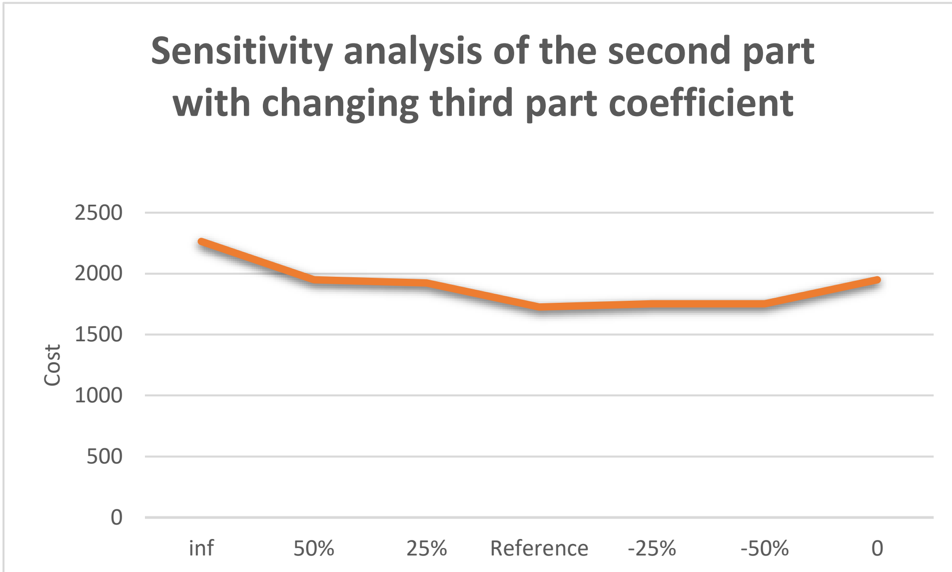

Figure 51. Second part coefficient sensitivity analysis with third part variation

As we mentioned earlier, increasing the importance of effectiveness ratings leads to higher vessel costs which is consistent with this section's output and shows the reliability of the results.
Impact on the third part

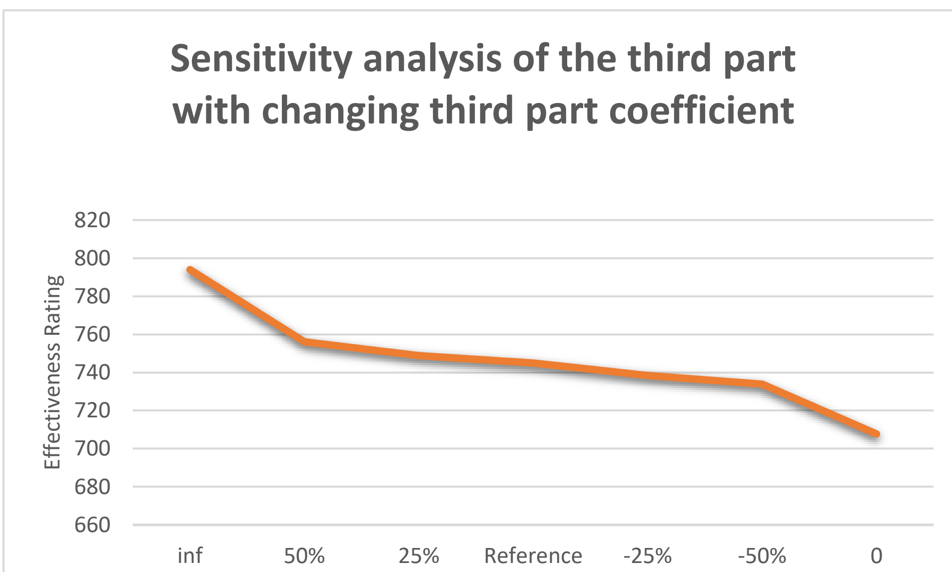

Figure 52. Third part coefficient sensitivity analysis with third part variation

Results of this section are almost predictable. The effectiveness rating which is one the most important innovations of this thesis, demonstrates that the quality of coverage is not the same in different configurations. This means that the capability of the dispatched vessels is different, and with increasing the importance of these ratings we can improve the quality of SAR services.

Impact on the fourth part

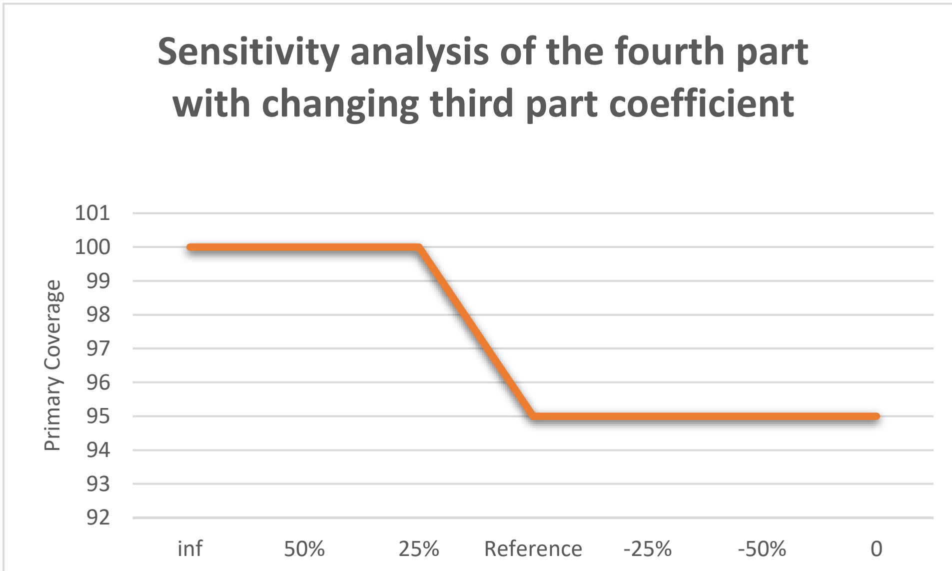

Figure 53. Fourth part coefficient sensitivity analysis with third part variation

Figure 53 shows that increasing the importance of the effectiveness rating will enhance the coverage, because the model tries to dispatch the most effective vessel to demand grids and it increases the number of covered grids.

5.3.4 Fourth part sensitivity analysis (Percentage of Primary Coverage)

The fourth part of the objective function has been considered with a penalty coefficient β which will be studied in six different states as follows: Zero (i.e., not considering this part), -50% of the reference coefficient, -25% of the reference coefficient, +25% of the reference coefficient, +50% of the reference coefficient, and infinity (i.e. just considering this part). Table 45 was obtained after running codes in different states of the fourth part.

Table 45. Fourth part coefficient sensitivity analysis table

Fourth part penalty	1st part (Sum of Insufficiency Probability)	2nd part (Cost)	3rd part (Effectiveness Rating)	4th part (Percentage of primary coverage)
Inf	2.01	2264.0	768.96	100
+50%	1.2	1959.9	766.68	100
+25%	1.3	1985.5	743.43	97.5

Fourth part penalty	1st part (Sum of Insufficiency Probability)	2nd part (Cost)	3rd part (Effectiveness Rating)	4th part (Percentage of primary coverage)
Reference	2.94	1726.1	744.90	95
-25%	3.04	1761.1	744.75	95
-50%	2.79	1714.7	746.98	95
0	4.91	1502.3	703.11	85

The impact on different parts of the objective function will be explained below.

Impact on the first part

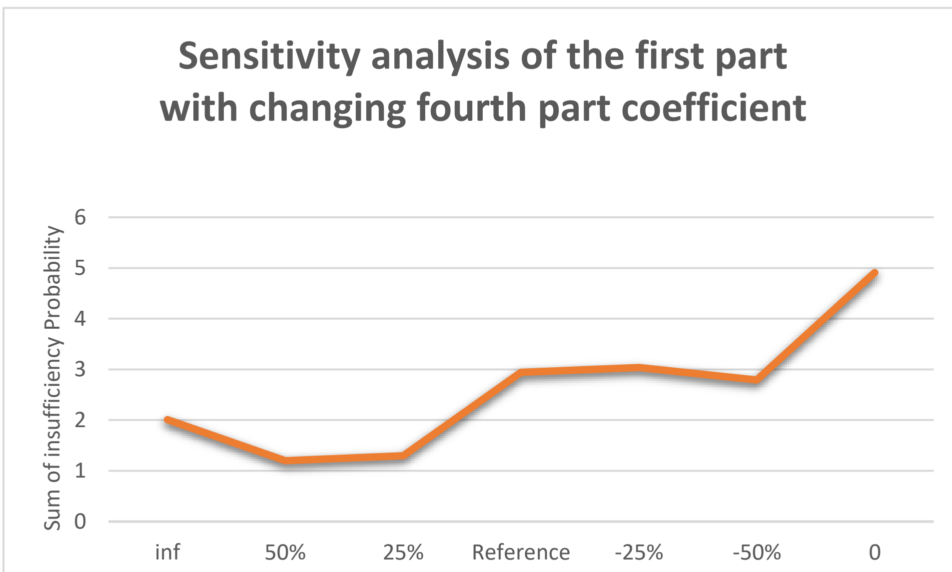

Figure 54. First part coefficient sensitivity analysis with fourth part variation

It can be generally concluded from the above graph that emphasizing the primary coverage will decrease the insufficiency probability, because covering more demand grids means more vessels to dispatch and consequently providing more backup coverage.

Impact on the second part

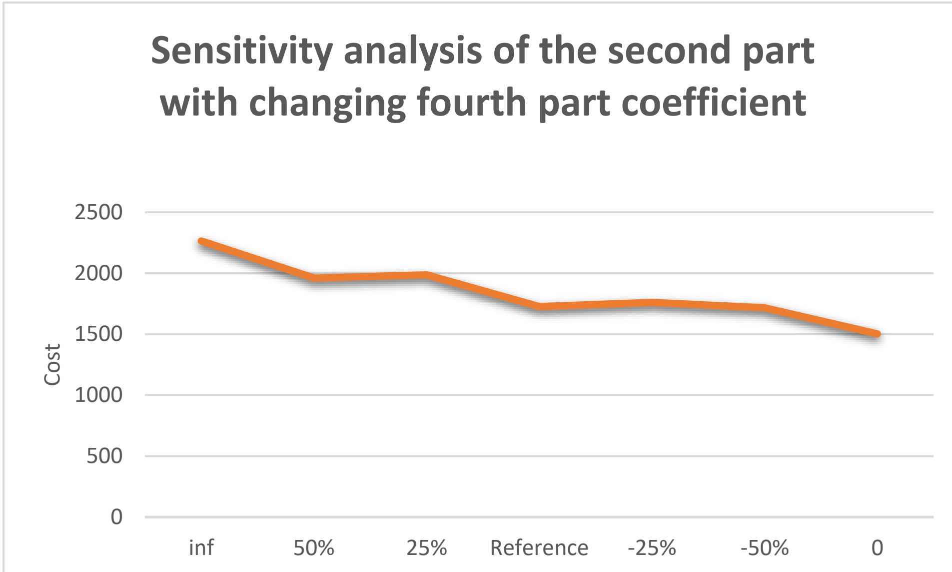

Figure 55. Second part coefficient sensitivity analysis with fourth part variation

As expected, the costs and coverage coefficient have a positive correlation. It means that sending more vessels for improving the coverage will result in increasing the vessel costs. This is one the most important trade-offs in this problem and the decision-makers should determine their desirable level of coverage considering their defined budget.

Impact on the third part

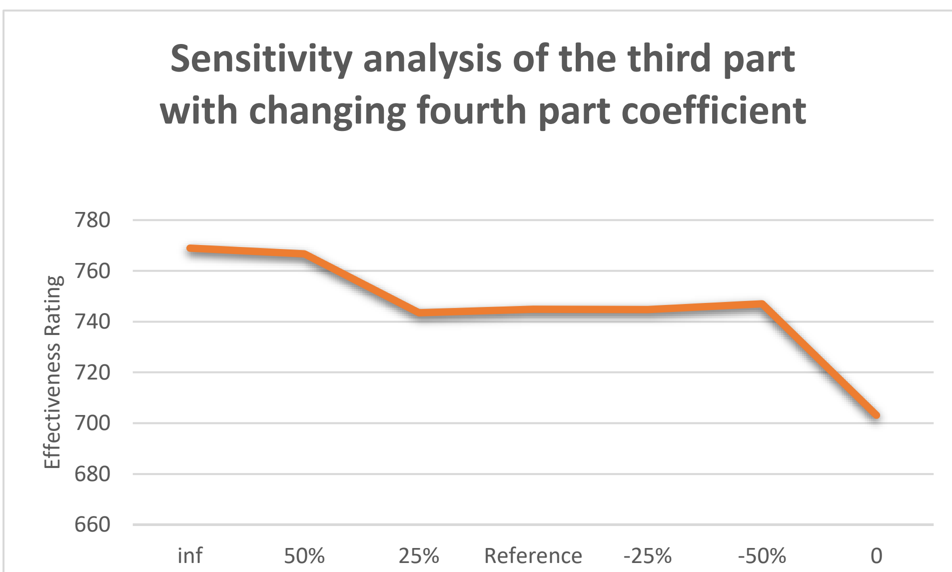

Figure 56. Third part coefficient sensitivity analysis with fourth part variation

In accordance with Figure 56, increasing the coverage will help to enhance the effectiveness ratings, because the model tries to dispatch the most effective vessels to increase the objective function value.

<u>Impact on the fourth part</u>

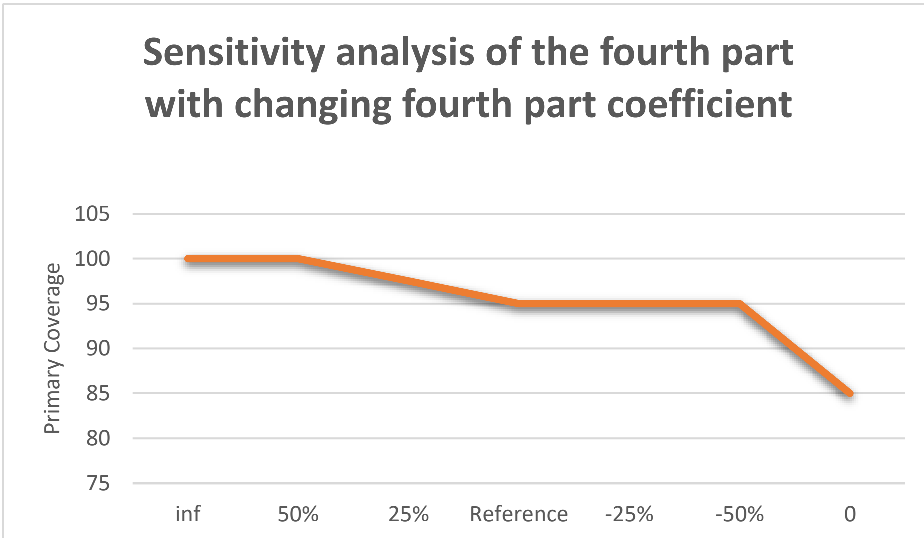

Figure 57. Fourth part coefficient sensitivity analysis with fourth part variation

The output of this section is easy to explain, because increasing the importance of coverage obviously increases the coverage percentage. As we can see, we will get the full coverage at the left side of the graph.

5.3.5 Overall conclusion from sensitivity analysis of the coefficients

With due attention to the stated results, we can conclude that acceptance of higher levels of costs in this problem will enhance the coverage, which consequently increases effectiveness ratings and lowers the insufficiency probability. Moreover, we can see that emphasis on the effectiveness ratings and insufficiency probability can have considerable impact on the outputs.

5.3.6 Relocation cost sensitivity analysis

In this section, we investigate the impact of the relocation cost (rc_{kl}) on the objective function. We have considered zero relocation cost to track the difference in objective function (in comparison to the reference configuration). In the reference configuration, relocation cost has been considered 5% of the annual vessel costs ($rc_{kl} = 0.05 \times ac_{kl\theta}$). This decision has been made after consultation with CCG experts. They mentioned that relocation cost consists of fuel and salary and does not impose considerable cost to the system. Table 46 shows the results of this change.

Table 46. Relocation cost sensitivity analysis table

rc_{kl}	1st part (Sum of Insufficiency Probability)	2nd part (Cost)	3rd part (Effectiveness Rating)	4th part (Percentage of primary coverage)
$0.05 \times ac_{kl\theta}$	2.94	1726.1	744.9	95
$rc_{kl} = 0$	4.58	1715.9	718.2	90

These results show that the effect of this parameter on the objective function is not considerable and only a small percentage of total vessel costs are related to relocation cost. It should be noticed that the cost decrease might have happened due to primary coverage reduction; but generally, considering the limited number of relocations and its small share of total cost, we can conclude that this parameter has no considerable effect on the objective function.

5.3.7 Coverage time limit (t) sensitivity analysis

In this section, the effect of the coverage time limit (i.e., t parameter) on the problem is analyzed. We have used the same reference configuration as in previous sections. The results obtained after running codes in different states of t parameter are shown in Table 47.

Table 47. Coverage time limit sensitivity analysis

Parameter t	1st part (Sum of Insufficiency Probability)	2nd part (Cost)	3rd part (Effectiveness Rating)	4th part (Percentage of primary coverage)
inf	1.21	1985.5	768.32	100
Reference	2.94	1726.1	744.90	95
-10%	3.07	1761.1	729.40	92.5
-20%	4.62	1741.4	696.51	90
-30%	7.42	1690.4	659.07	85
-40%	14.26	1398.8	524.68	65
-50%	29.21	1124.5	278.28	35
-60%	37.65	469.8	79.40	10
-70%	40	0	0	0

The impacts on different parts of the objective function are explained below.

Impact on the first part

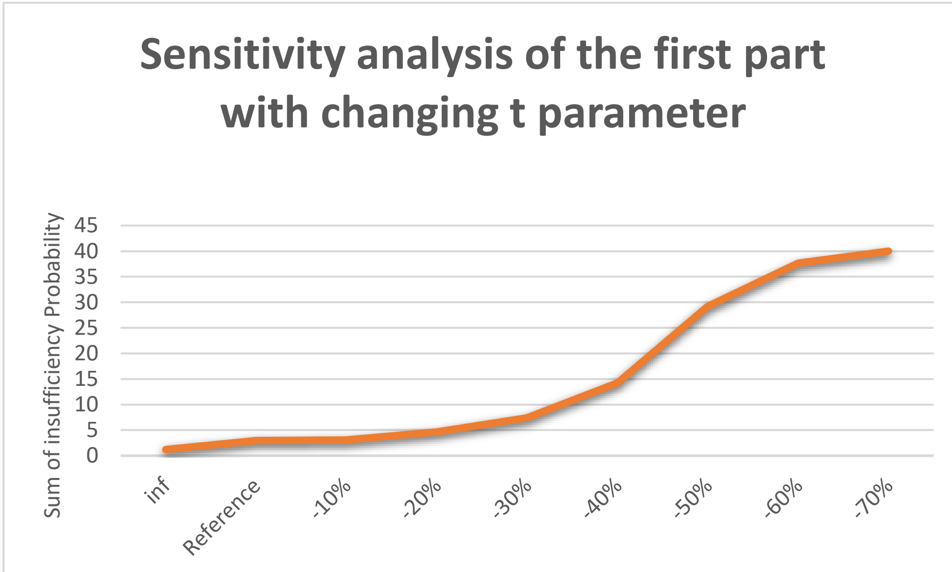

Figure 58. First part sensitivity analysis with t parameter variation

Results of Figure 58 show that the insufficiency probability decreases with increasing the access time limit, because increasing the access time limit makes more vessels eligible to be dispatched to demand grids. Therefore, primary and backup coverage will be enhanced, and the insufficiency probability will be decreased.

Impact on the second part

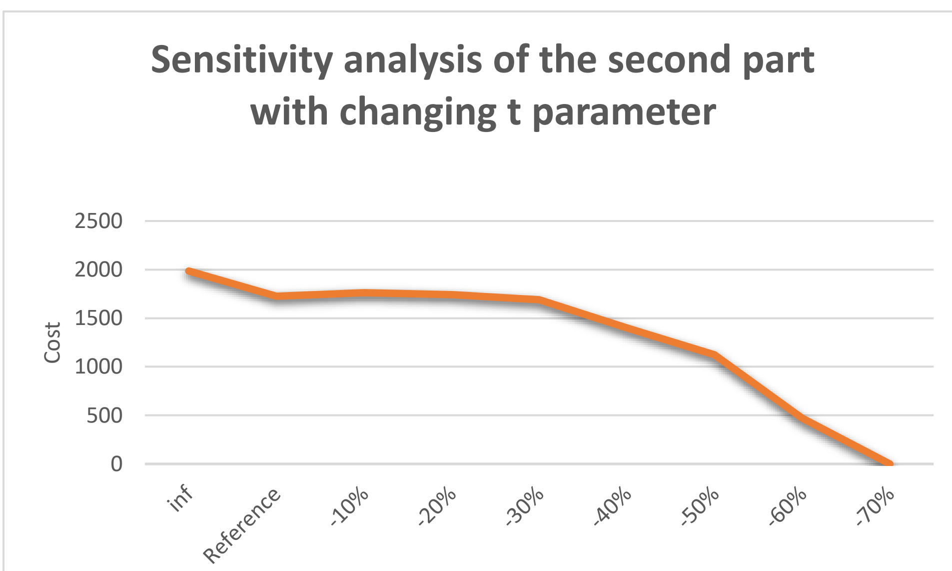

Figure 59. Second part sensitivity analysis with t parameter variation

Figure 59 which is consistent with the previous one, demonstrates that the elimination of the access time limit constraint will in many cases increase the number of eligible vessels able to reach any given incident and consequently impose higher levels of cost to the system. Thus, there is a positive correlation between this parameter and cost in this problem.

Impact on the third part

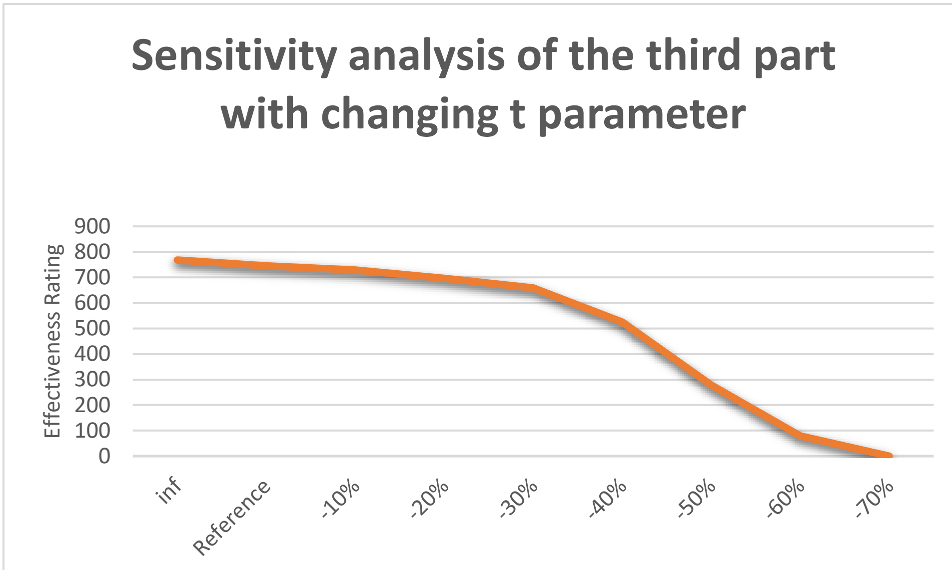

Figure 60. Third part sensitivity analysis with t parameter variation

Like the previous section, effectiveness ratings have a positive correlation with the coverage time limit parameter, because with increasing coverage and dispatching more vessels to different grids, the model tries to increase the quality of service and send the most effective vessels to incidents, which makes the ratings go higher.

Impact on the fourth part

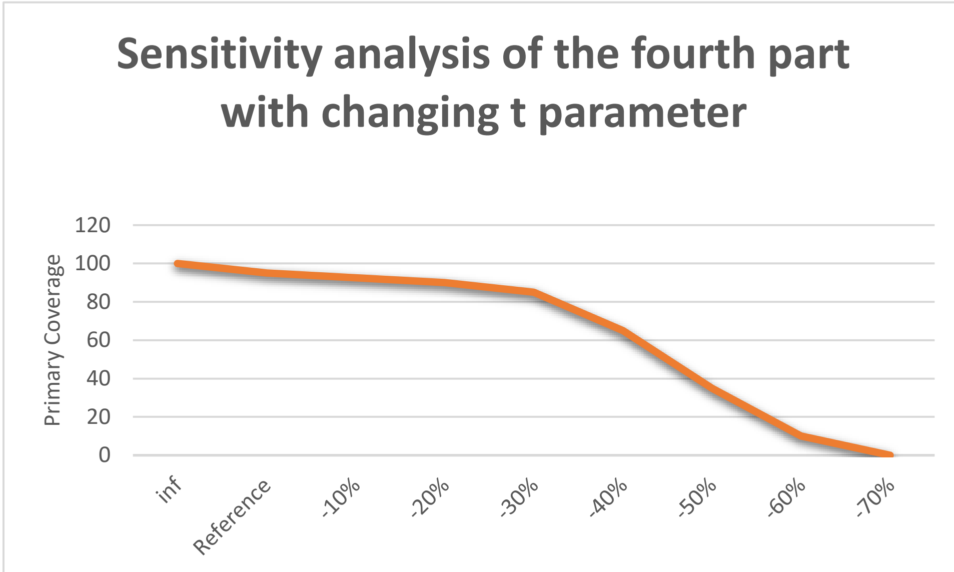

Figure 61. Fourth part sensitivity analysis with t parameter variation

This sections' results corroborate previous ones and show that the model and method are reliable and work properly. As we can see in the graph in Figure 61, the coverage percentage goes up with increasing the access time limit, because more vessels will be located in range of different incidents and more vessels will be dispatched to demand grids which improves the coverage to 100%. We will discuss the managerial insights derived from all the presented sensitivity analyses in chapter 6.

5.4 Case study results

After adjusting the parameters and numerical analysis, the results of the model applied to the real world can be shown. The ultimate goal of this study is solving real world problems and improving the quality of life. It is obvious that implementing a model in a real case study can improve the value of the research and show the reliability of the model. As mentioned before, our case study is the location-allocation of SAR vessels in the Atlantic region of Canada. This large-scale problem has the following key parameters.

- Number of periods: 2
- Number of demand grids: 1617
- Number of stations: 38
- Number of SAR vessels: 26
- Number of SAR vessel classes: 5
- Number of incident groups: 4

Note that the objective function coefficients have been set to equalize the importance of the different criteria in this case study, and if the decision-makers are willing to consider a different preference, they can be changed accordingly. Results obtained from the different methods are summarized in Table 48.

Table 48. Case study results table

	Sum of Insufficiency probability	Total costs	Ratings	Primary Coverage	Obj function
SA	881.57	1.056	1712989	84.39	1729.521
GA	606.00	1.199	1818483	89.31	1219.436
HS	716.03	1.170	1756747	85.57	1544.875
HSGA	891.23	1.163	1687703	83.96	1830.234
HSGA1	577.30	1.200	1864846	89.91	1141.410
HSGA2	496.15	1.140	1827807	90.5	1007.654
SAGA 1st	503.02	1.218	1839289	91.36	1005.129
SAGA 2nd	501.58	1.164	1864254	91.51	956.885
SAGA 3rd	471.24	1.143	1840709	91.97	897.850
Current	798	1.201	1718480	86.49	1605.262

As shown in Table 48, the best coverage is obtained from the SAGA method. Due to the importance of coverage in SAR problems and considering the better results and also running time of SAGA (half of HSGA II's running time), this method was used for more runs to explore better performance, with two of the best outputs of this method summarized in Table 48. As we can see in the table, the outputs of HSGA 1, HSGA2 and SAGA (2nd and 3rd) are better than the current configuration of the SAR fleet (without consideration of offshore stations and with respect to current homeports of the vessels) in terms of all parts of the objective function.

The running time of the different methods (for the best obtained result) is summarized in Table 49. It should be mentioned that case study was implemented on a system with the following configuration:

CPU: AMD Ryzen 9 3950X 16-Core Processor 3.49 GHz, Ram: 64.0 GB

Table 49. Running time of different methods

Method	SA	GA	HS	SAGA	HSGA	HSGA1	HSGA2
Run time (seconds)	1893.56	2201.84	5128.47	191972.70	5078.20	5521.58	383663.51

Calculation of time complexity for different methods (which is known as O function) showed that Dynamic programming complexity is directly proportional to the cube of demand grids (i^3), square of incident groups (q^2), number of periods (θ), and number of vessels (l); while all the other methods are proportional to the square of demand grids (i^2).

Next, we will compare the methods in terms of different parts of the objective function.

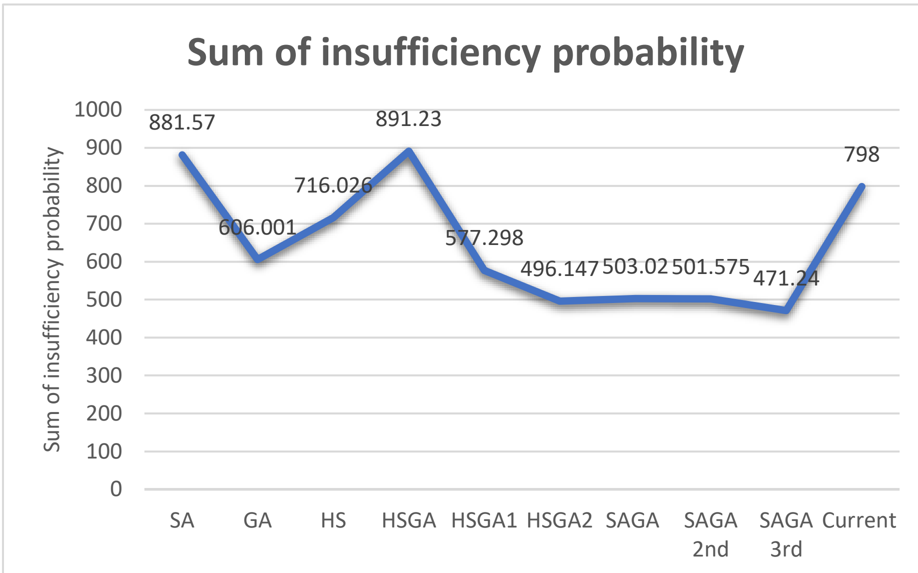

Figure 62. Insufficiency probability comparison for different methods

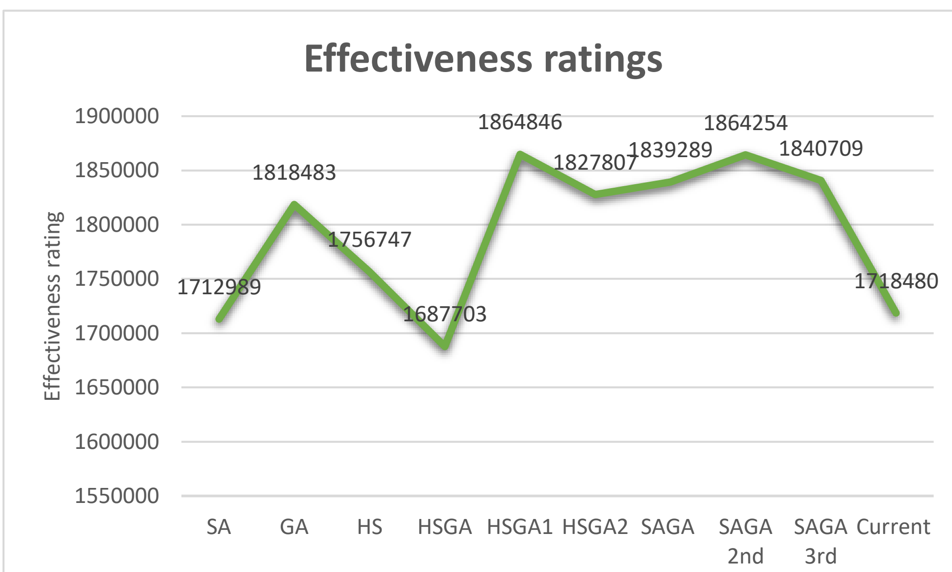

Figure 63. Effectiveness rating comparison for different methods

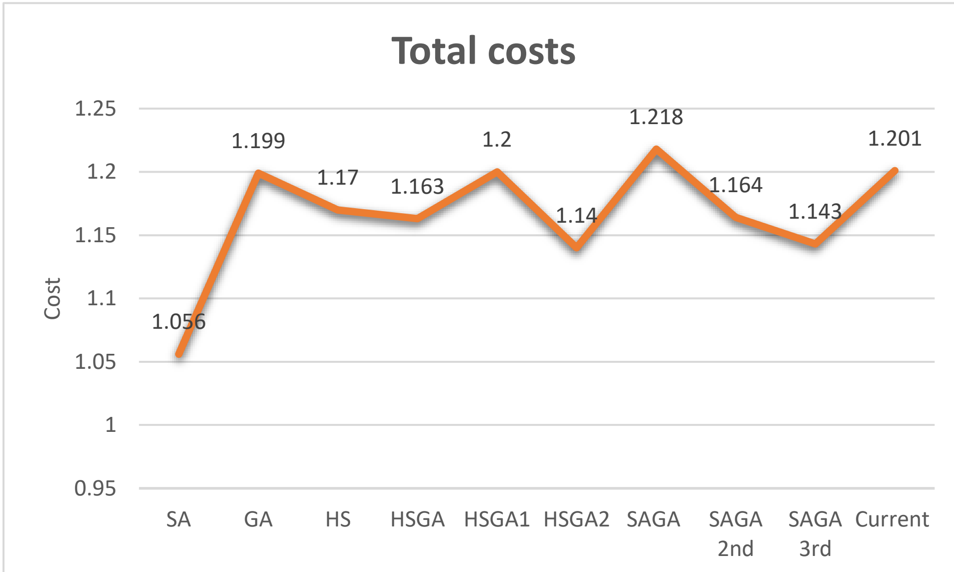

Figure 64. Total cost comparison between different methods

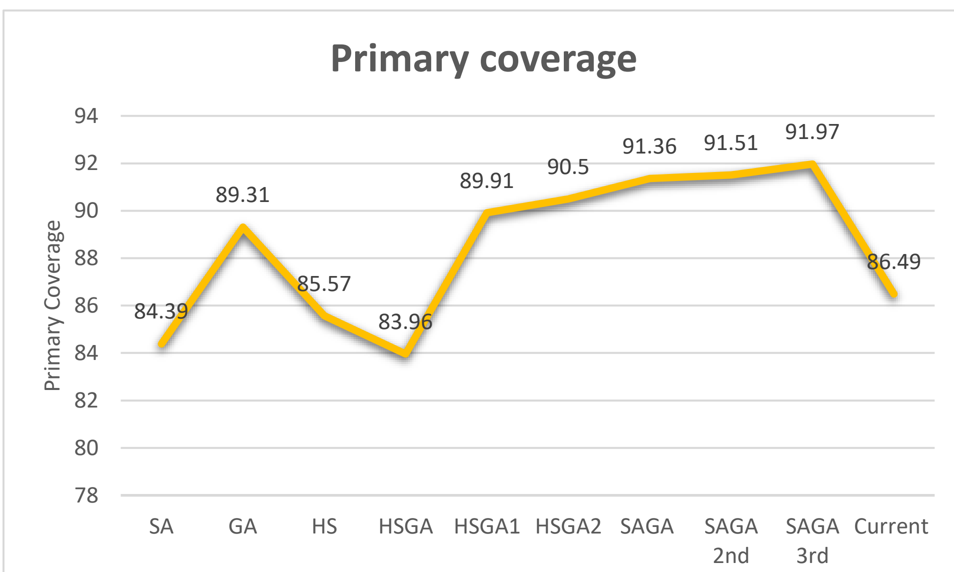

Figure 65. Primary coverage comparison between different methods

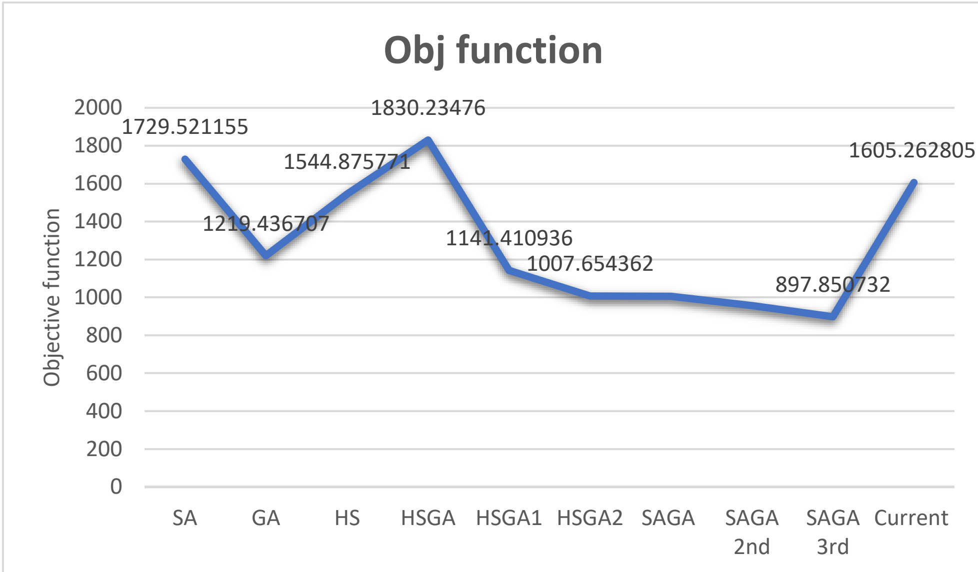

Figure 66. Objective function comparison between different methods

The results of this section show that hybrid methods outperform basic methods in most cases and that GA has the best performance in comparison to SA and HS (see Table 48). We can conclude that using hybrid methods can be suitable approaches for making strategic and long-term decisions (due to higher running time) and using the GA algorithm would be the most reasonable option when we need to find a local optimum quickly.

The best result and configuration in this case study was obtained from the third SAGA with 92% coverage which is comparable to previous studies, but with the benefit of the current study incorporating effectiveness of response to four different groups of incidents. In this solution, we have 20 relocations which is considerable. Some of the reasons behind this high number of relocations are the presence of scattered demand grids and time/distance constraints which require the model to do a lot of relocations to provide appropriate coverage. Moreover, low relocation cost (5% of the vessels' operational cost) definitely has an impact on the number of relocations.

Figure 67 shows the location of the SAR stations in the Atlantic region (along with assigned numbers) and Table 50 represents the optimum location of SAR vessels in period 1 and 2. The numbers in the Table 50 demonstrates the vessel class at each station.

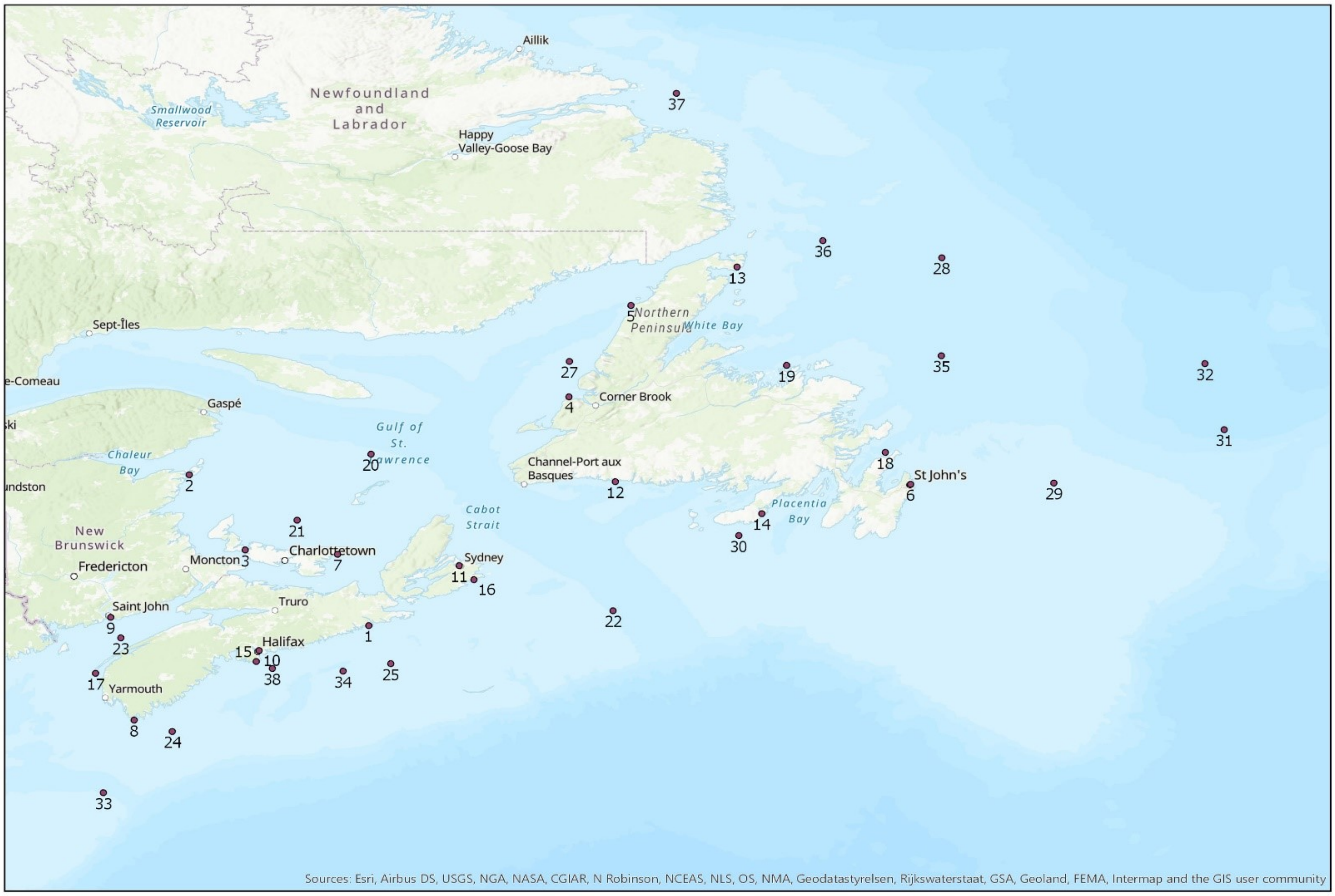

Figure 67. Location of SAR stations in Atlantic Region of Canada

Table 50. Allocation of SAR vessels to SAR stations (colored numbers shows the vessel class)

Station Number	Current Status	Period 1	Period 2
1	2	1	2
2	1		2
3	1	2	
4	1		2
5	1		2
6	4,4,5,5,5	2,2	4
7	1		2,2
8	2	2,4	4
9	2		2
10	3,3,4,4,4	1,2,2	1
11		2	5
12	2		1,5
13	2		2
14	2	1,2	1
15	2	1	2,5
16	2	2	1,2
17	2	2	
18	2	1,4	2
19	2	2	1
20			3
21			
22		4	
23			
24			
25			
26			4
27			
28			
29			4
30			
31		4	
32		5	
33		5	
34			
35		5	
36			
37		3	4
38		3,4	3

To better picture the differences between Vessel allocation in the two periods, the current and optimum SAR vessel locations in the two periods are shown respectively in Figure 68, Figure 70, and Figure 69. The most apparent changes are the broader distribution of vessels in the busy summer season. In reality, the CCG does often position vessels offshore during summer periods, but this practice is not formalized and thus "standardized" positions are not available for reporting. So, we have compared the proposed configuration in operational season 1 and 2 with the current configuration without consideration of offshore stations in the current status.

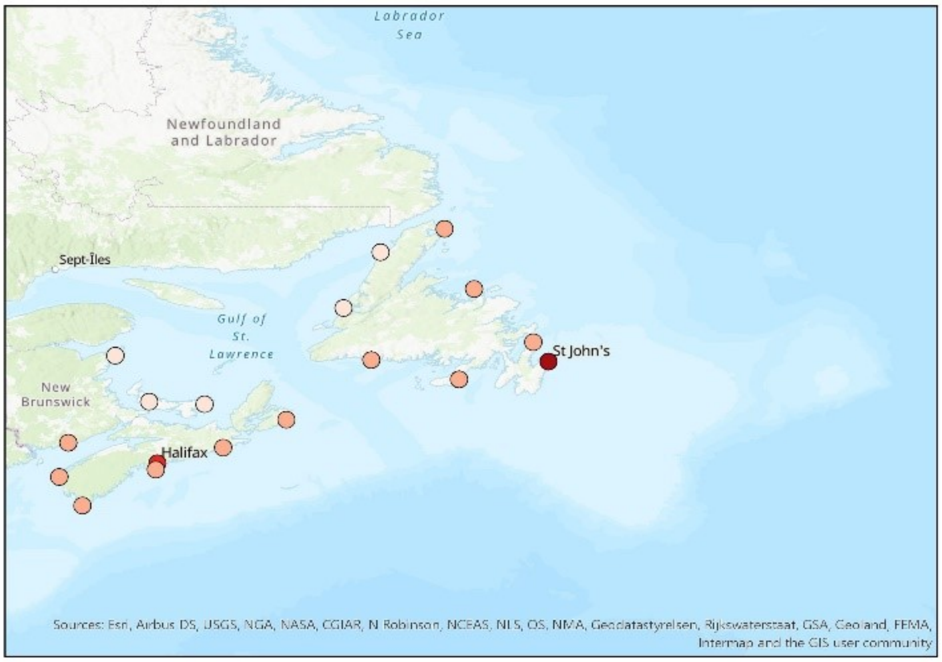

Figure 68. Current SAR vessels configuration in Atlantic region

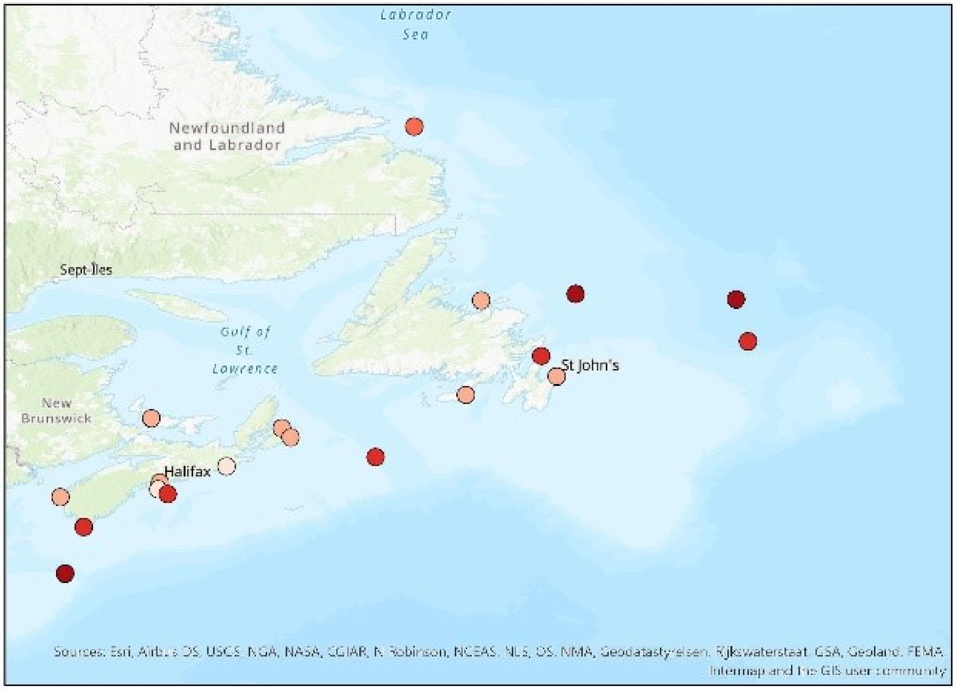

Figure 70. SAR vessels configuration from Oct to Mar (operational season 1)

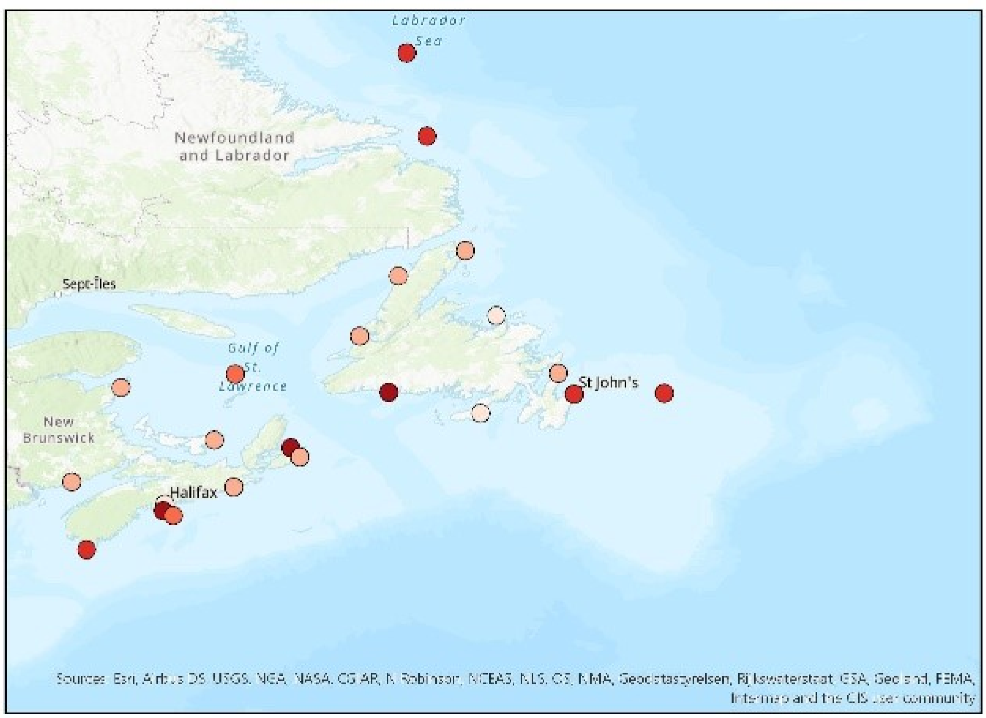

Figure 69. SAR vessels configuration from Apr to Sep (operational season 2)

As we can see on the map, the location of vessels in the optimal solution is more scattered in comparison to the current configuration. For quantifying the dispersion of vessels' locations, we have calculated the standard deviation of their coordinates (Latitude and Longitude) in Table 51.

Table 51. Standard deviation of vessels location coordinates

Configuration / StD	Current status	Operational season 1	Operational season 2
StD of Latitude	2.077	2.648	3.081
StD of Longitude	4.984	5.840	4.254

According to Table 51, in operational season 1, the vessel locations are more horizontally scattered (standard deviation of longitude is bigger) and in operational season 2, the points are more dispersed along the vertical axis (standard deviation of latitude is bigger) which shows easier access to the northern part of the Atlantic region in warmer seasons of the year.

5.5 Sensitivity analysis of coefficients in the case study problem

In this section, we run the case study problem (SAGA method) with unequal importance of coefficients. In each run, only one part of the objective function has been considered in order to investigate the impact on the other parts. Table 52 summarizes the results.

Table 52. Sensitivity analysis of case study problem

Value / Considered part	Sum of Insufficiency Probability	Total cost	Effectiveness ratings	Primary coverage	Objective function
Sum of insufficiency probability (1st part)	255.23	1.325	1820805	89.81	908.3
Total cost (2nd part)	5907	0	0	0	11504.4
Effectiveness ratings (3rd part)	567.23	1.253	1981010	89.03	1168.6
Primary coverage (4th part)	424.11	1.310	1812315	92.31	932.0

As we can see in Table 52, when focusing on the first part of the objective function, the insufficiency probability decreases because the model tries to improve the backup coverage. With consideration of the second part (cost), no vessel will be dispatched to demand grids and all the grids will be uncovered. So, the objective function considers the penalty of uncovered grids along with the sum of the insufficiency probability of non-zero demand grids. Emphasis on the third part (effectiveness ratings) will result in better effectiveness ratings and sending more effective vessels to demand grids but it will slightly decrease the primary and backup coverage. And lastly, by increasing the importance of primary coverage coefficient the highest coverage will be obtained to the detriment of higher costs and lower effectiveness ratings.

In this chapter, we discussed the numerical results and mathematical analysis along with parameter adjustments and sensitivity analysis. We also showed that the developed model is applicable in the real world and can be solved with the suggested methods. In the next chapter we will investigate the managerial insights and propose some ideas for future research.

6. CHAPTER 6 CONCLUSION AND FUTURE RESEARCH

In previous chapters, first we discussed the research overview and a brief introduction to search and rescue. In the second chapter, the literature review and research gaps were presented, and the necessity for this research were described. The third chapter proposed the precise methodology and optimization model as well as exact and approximate (meta-heuristic) solution methods. In the fourth chapter, all the required data for implementation of the model in the real world (case study in the Atlantic region of Canada) were presented and explained. Finally in chapter five of this thesis, numerical results, sensitivity analysis, and the case study solution were discussed.

In the current chapter, we present and discuss the managerial insights from this thesis. These explanations and discussions are necessary for the researchers who are interested in this field of study and also for managers or decision-makers who are in charge of determining SAR policies and strategic plans.

The results obtained from this thesis have been divided into two parts. The first part is related to mathematical results extracted from numerical outputs, parameter adjustment, and sensitivity analysis. The second part concern managerial insights resulting from in-depth analysis of the first part. Finally, at the end of this chapter some suggestions are provided for future research in order to improve the optimization model and solution methods.

6.1 Mathematical results

- The results showed that dynamic programming can be used for solving complex problems in this domain and, regarding its ability to give the global optimum solution, it can be reliable for non-linear models.
- Solving the small-scale problem demonstrated that all the introduced meta-heuristic methods are reliable, but some of them have advantages over other methods. For example, even though the outputs of SAGA and HSGA II are acceptable, their execution time might be 700 times longer than the SA method.
- Parameter adjustment of the SA method demonstrated that a high number of iterations decreases algorithm divergence and causes the method to get stuck in local optima. In addition, according to the results, decreasing the initial temperature at a fast rate would result in better outputs.
- In accordance with the GA method parameter adjustment, a high mutation rate and population would give us the best outputs, which emphases the importance of divergence for escaping from local optima.
- Unlike SA and GA, results of the harmony search (HS) algorithm demonstrated that a high number of iterations can improve the results, which is consistent with the musical nature of this algorithm.

- Sensitivity analysis of the objective functions' coefficients showed that vessels' costs have positive correlation with coverage and effectiveness ratings, and negative correlation with insufficiency probability.
- Regarding the importance of human lives in SAR, one of the best results occurs when we place emphasis on the effectiveness ratings which is one of the contributions of the current thesis. This emphasis causes dispatching the most effective vessels to the incidents and enhancing the quality of SAR services.

6.2 Managerial insights

- Executive managers who are in charge of determining SAR policies should be consulted for determining the coefficients of the objective function. If costs are of great importance for them, they should accept the decrease of coverage and increase of insufficiency probability (which may result in greater loss of life in the Atlantic region). Even though such a course is not in line with this research's objectives, lack of budget and economic situations might require the managers to consider the costs as a high priority despite potential negative impact on coverage.
- As mentioned in the sensitivity analysis section, relocation costs do not have a considerable effect on the solution, because in accordance with SMEs' comments they are negligible in comparison to other costs of the vessels. Moreover, the final results of the case study showed a lot of relocations between the seasons which are due to low relocation costs, scattered demand grids, and time/distance constraints.
- One of the important factors in SAR problems is the coverage time limit (access time limit). Decreasing the access time limit with high coverage and low insufficiency probability requires the managers to establish new stations or to add more (or faster) vessels to CCG SAR fleet. This would increase the cost drastically.
- According to the results, the insufficiency probability and effectiveness ratings had considerable impact on the results. Placing emphasis on these two factors enhanced the different parts of the objective function, and the only negative impact was on the costs which is justifiable with consideration of the vital role of SAR in saving lives. Lowering insufficiency probability improves the backup coverage, and effectiveness ratings enhances the service quality through sending the best suited response vessels to the incidents.
- With increasing the number of SAR vessels in the Atlantic region of Canada, we can reach close to 100% coverage with very low insufficiency probability, because having more vessels as primary coverage will increase the number of potential vessels which can also serve the uncovered grids as backup coverage. These changes would impose significant costs to the system which might exceed a predetermined budget.

6.3 Future research suggestions

In this section some of the ideas for improving and developing the current research are presented. They can be useful for scholars who are interested in pursuing this subject and creating new contributions to this field of study.

- Using simulation and investigating different probable scenarios
- Utilizing more meta-heuristic methods to assess their ability in finding better solutions
- Implementing different uncertainty approaches like fuzzy numbers, robust optimization, distributed robust optimization, etc.
- Considering more uncertain parameters in the problem like costs, vessel speed, etc.
- Developing a multi-objective model to assess different criteria and comparing the results with this thesis
- Analyzing the establishment of new stations and adding new SAR vessels to the current fleet and investigating the impact on different parts of the objective function
- Developing a framework for estimation of required rescue time based on incident group
- Consideration of more groups of incidents and vessel classes to match them in a more precise way.
- Utilizing linearization methods to make the objective function and constraints linear and solve the optimization model with Gurobi solver to make a comparison between the metaheuristics and new solutions.

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