# OPTIMIZATION MODELING AND ANALYSIS OF TRUCK ALLOCATION SYSTEM IN SURFACE MINING OPERATIONS

by

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*To*

*My parents, Meilan Liu and Wu Wen*

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# ABSTRACT

In the surface mining industry, trucks and shovels are the essential components in the whole operation system. Optimal allocation of trucks/shovels resources is critically necessary in the surface mining operations not only for satisfying the mining production profit goal, but also for minimizing the mining operation cost. Also, many factors in the operation process make the truck allocation decision making plagued with uncertainties, vagueness and complication. In the past, Interval-parameter Linear Programming (ILP) has been widely used to deal with uncertainties and to assist optimal decision making in a variety of system planning and management problems. However, the existing ILP solution algorithms, i.e., best-worst case algorithm and 2-step algorithm, are found to be ineffective through a validity checking process. Moreover, the results from ILP cannot reflect the linkage between decision risks and the system return.

In this study, a Risk Explicit Interval-Parameter Linear Programming (REILP) model and a fuzzy-REILP model are developed to generate the least cost strategies while minimizing the decision risks. The developed methods are then applied to the optimal planning of the truck allocation system in an open-pit mine case, and this is the first attempt of using ILP-based optimization techniques to the surface mining industry. This method is specifically designed to deal with extensive uncertainties existed in the truck allocation system and to provide decision supports to the surface mining operators and planners. In the developed methods, the ILP is used to reflect uncertainties existed in both objective function and constraints. Based on the basic ILP, a risk function is defined to assist in finding solutions with minimum system cost while minimizing the decision risk, under certain aspiration levels. The aspiration level could be conservative, medium or aggressive, and can thus be presented as a fuzzy set to reflect the preference of decision makers. Three sets of solutions are obtained accordingly. Besides, the model was also solved under the aspiration level from 0 to 1, with a step of 0.1, for providing a comprehensive decision support.

This approach can effectively reflect dynamic, interactive, uncertain characteristics, as well as the interactions between overall cost and risk level of the mining truck allocation system. The results can effectively reflect the tradeoff between decision risks and the system return, and thus provide valuable information to support the decision-making process related to the planning of the truck allocation, and timing and routing of the mine-hauling activities.

# LIST OF ABBREVIATIONS AND SYMBOLS USED

| | |
|---|---|
| BWC | Best-Worst Case Algorithm |
| CDF | Cumulative Distribution Function |
| FLP | Fuzzy Linear Programming |
| FMP | Fuzzy Mathematical Programming |
| FREILP | Fuzzy Risk Explicit Interval-Parameter Linear Programming |
| ILP | Interval-Parameter Linear Programming |
| IPMP | Interval-Parameter Mathematical Programming |
| LP | Linear Programming |
| MILP | Mix-Integer Linear Programming |
| PDF | Probability Distribution Function |
| REILP | Risk Explicit Interval-parameter Linear Programming |
| SLP | Stochastic Linear Programming |
| SP | Stochastic Programming |
| TSM | Two-Step Method |

| | |
|---|---|
| $A^{\pm}$ | A set of interval parameters of ILP constraints |
| $B^{\pm}$ | A set of right-hand side of parameters of ILP |
| $C^{\pm}$ | A set of interval parameters of ILP objective function |
| $a_i$ | Interval parameters of ILP constraints |
| $b_i$ | Right-hand side of parameters of ILP |
| $c_j$ | Interval parameters of ILP objective function |
| $f^{\pm}$ | Value of objective function |
| $f^{\pm}_{jop}$ | Optimum objective function interval |
| $f^{+}_{jop}$ | Upper bound of optimum objective function |
| $f^{-}_{jopt}$ | Lower bound of optimum objective function |
| $k_1$ | Number of positive $c_j$ parameters |
| $\lambda_0$ | Aspiration level |
| $\lambda_{ij}$ | Risk level variables |
| $\xi$ | Risk function |
| $x_j$ | Decision variables of ILP |
| $x^{\pm}_{jop}$ | Optimum decision variable intervals of $x_j$ |
| $x^{+}_{jopt}$ | Upper bound of optimum decision variable $x_j$ |
| $x^{-}_{jopt}$ | Lower bound of optimum decision variable $x_j$ |
| $X^{\pm}$ | A set of decision variables of ILP |
| $X^{\pm}_{opt}$ | Optimum decision variable values |
| $X^{-}_{opt}$ | Lower bound of optimum decision variable values |
| $X^{+}_{opt}$ | Upper bound of optimum decision variable values |
| $^{-}$ | "-" superscript represents the lower bound of an interval-parameter or variable |
| $^{+}$ | "+" superscript represents the upper bound of an interval-parameter or variable |

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# CHAPTER 1 INTRODUCTION

## 1.1 STATEMENT OF THE PROBLEM

### 1.1.1 Truck Allocation in the Mining Operation System

In the surface mining industry, trucks and shovels are the essential components in the whole operation system. Subsequently, the associated costs from truck and shovel technologies play a dominant role in the total mining operating cost.

Generally, the open-pit mining operation system consists of four major components: exploration process, ore mining, waste removal, and operation office. The exploration process involves ore mine explorations, ore seams, and trial pits. As the operation moves towards ore mining, ores mined from the pits will be hauled to the ore crushing area from where the ores will be conveyed by conveyer belts to the ore surge pile awaiting being washed. The waste generated from the mining process will be hauled by trucks to the crushing area and then be transported via belt systems to the waste-dumping site.

This study will focus on the cost reduction and cost control associated with the ore mining operations through optimal analysis of the ore and waste truck allocations in the mining industry.

### 1.1.2 Optimization Modeling of Truck Allocation Problems

The application of optimization techniques to the open-pit mining operation system was first proposed by Richard in 1980 (Richard, 1980), and a variety of optimization models have been developed since then (Usama, 1996; Beaumont, 1998; Samanta, 2002; Walster, 2003; Pan, 2012; Mitra and Saydam, 2012). Since many factors in the mining operation process are plagued with uncertainties, the deterministic optimization programming techniques have been recognized insufficient to model a complex system, and several uncertainty-based optimization techniques have been developed to reflect system uncertainties, vagueness, and complication. They include fuzzy mathematical programming approaches (FMP) (Huang et al., 1995; Hammah and Curran, 1998; Emmanuel, 2001; Kannan et al., 2003), stochastic mathematical programming approaches (SMP) (Inuiguchi and Ramık, 2000; Hansen and Walster, 2003; Goel and Grossmann, 2004; Zhou et al., 2009), interval-parameter mathematical programming (IPMP) (Huang et al., 2001; Maqsood and Huang, 2003; Zhou et al., 2009), and some hybrid or integrated programming methods (Samanta et al., 2002; Chung, 2002; Goel and Grossmann, 2004; Krause and Musingwini, 2007; Barr, 2012; Burt and Caccetta, 2014). Mixed solutions and results have been generated when different approaches were used to reflect the system uncertainties in different applications. Comparing to fuzzy or stochastic programming approaches, in terms of data quality and requirements, IPMP does not require the information of membership functions or the distribution of parameters, which could be very challenging to collect in the practical cases. Moreover, fuzzy and stochastic approaches often lead to more complicated sub-models, which makes the real-world applications unrealistic or impractical. Interval-parameter Linear Programming (ILP) is one kind of IPMP, and it can effectively deal with uncertainties without leading to more complicated sub-models. The ILP approach has been widely used in many different areas in the past, such as civil/environmental engineering and economics (Huang et al., 1993; Inuiguchi and Sakawa, 1994; Goel and Grossmann, 2004; Chinneck and Ramadan, 2000; Huang and Cao, 2011; Sahoo, 2012; Chung et al., 2013; Torkamani and Askari-Nasab, 2015). However, the ILP approach has never been applied in the optimization of the truck allocation system in the surface mining industry. As an extension of previous efforts, in this study, the ILP technique is attempted to be applied to the mining industry for the first time to enhance the decision making process in the truck allocation system.

For solving the ILP models, three solution algorithms have been developed to facilitate the use of ILP, including Monte-Carlo simulation, Best-Worst Case analysis (BWC) and 2-step interactive algorithm. Monte Carlo simulation algorithm (or Monte Carlo experiment) is a computerized mathematical technique that relies on repeated random sampling to obtain numerical results (Rubinstein and Marcus, 1985) while accounting for risks in quantitative analysis framework. It randomly generates values for each parameter within their interval ranges in order to form a classic LP model. Monte-Carlo simulation needs to be run by sufficient times (i.e., millions of times) in order to make the results reliable, which makes it impractical and unrealistic for many real-world problems with a large number of decision variables and constraints. BWC and two-step algorithms have been developed to overcome the gaps (Huang et al., 1993; Chinneck and Ramadan, 2000). Both algorithms reformulate the original model into two sub-models using extreme constraints to represent the most conservative and the most aggressive situations, respectively. The main difference between the two algorithms is that the 2-step algorithm differentiates the selection of extreme parameter values with different signs after reformulating the objective functions of two sub-models, while the BWC treats all the parameters equally without discrimination. Both algorithms provide an interval solution space, and each point in the interval solution space can be used to form a decision alternative for implementation. However, theoretically, the development of both algorithms is associated with flaws, and infeasible and/or non-optimal solutions can be generated from their interval spaces. This would lead to significant risks associated with the decision alternatives in practical decision making process. In this study, validity checking is performed to prove the infeasibility and non-optimality issues of the ILP algorithms, indicating that the ILP modeling results could bring the decision risks to its practical implementation and improving its applicability is desired.

Moreover, the results obtained from the ILP lack of a linkage and tradeoff analysis between decision risks and system return. The decision makers would much prefer to know the system returns along with the associated decision risks rather than the system returns only. Hence, it is desirable to formulate a model which can effectively reflect the linkage between the system return and decision risks. Rui et al. (2010) preliminarily explored this modeling issue by developing a risk explicit interval-parameter linear programming (REILP) model. However, the REILP method still has some deficiencies, such as infeasibility problem, the risk function formulation problem and the difficulty of selecting the pre-set aspiration levels. This study will address these deficiencies and extend/evolve the REILP method to a new application in the mining industry.

### 1.1.3 Open-Pit Mining Operation System of Anshun Yalong Project

Guizhou Province is located in the southwestern part of China and has over a thousand coal mines in operations in 2015 (China Mining Association, 2015). The city of Anshun produces the largest amount of coal every year, contributing to over 60% of the total coal production in the province (China Mining Association, 2015). As the study case in this thesis, Yalong project is an open-pit coal mining project in the city of Anshun, which has started operation since 2010.

Considering the distributions of operating costs for the activities in an open-pit mining production cycle, loading and hauling costs could take up to 70% of the total (Javad, 2009). Yalong Group Corporation has strived to establish an efficient truck allocation system since 2013 through various available programs using heuristic rules or the methods based on the dispatchers' experiences. The heuristic rules may work well for small mining operations, but may not work for large and complicated mining operations in most cases (China Mining Association, 2014). During the implementation of these programs and continuous increase of the coal production rate, one major concern from the decision makers and stakeholders is the cost reduction and control. In Guizhou, the total annual capital cost for the truck allocation system is around 60.9 million Canadian dollars under the current coal production level, i.e., the yearly coal production is around 173.65 million tonnes in Guizhou (National Energy Administration, 2015). Furthermore, if a higher production rate is desired, or if the equipment or trucks need to be replaced or repaired, additional cost would be occurred.

There is always a tradeoff between the system cost and associated coal production rate. The total cost will be unaffordable if the cost for truck allocation system is not well controlled and managed. In the past, very few studies have reported a comprehensive truck allocation system study and addressed the associated system uncertainties (Jordan, 2015), and none of them have focused on using the ILP method to reflect the uncertainties. For Yalong project, no mathematical programming methods have been applied to investigate its truck-shovel dispatching and waste hauling problems. This study will be the first attempt through conducting a comprehensive truck allocation study to provide better and more reliable decision support information for local coal mining managers.

## 1.2 RESEARCH OBJECTIVE

As an extension of previous efforts on truck allocation system of the open-pit mine, this study attempts to address the optimal truck allocation problem under various ore production and waste hauling constraints. A number of optimization modeling approaches including ILP, Risk Explicit Interval-parameter Linear Programming (REILP) and Fuzzy Risk Explicit Interval-parameter Linear Programming (FREILP) will be developed and applied to the Yalong project as a case study for generating optimal truck allocation solutions and more importantly, analyzing the risks associated with the generated decision alternatives. This study entails the following objectives:

(1) Validity checking of two ILP solution algorithms, i.e., BWC algorithm and the Two-Step interactive algorithm

A numerical example will be formulated and solved by the Monte-Carlo simulation, BWC and Two-Step algorithms, respectively, for investigating the validities of BWC and Two-Step algorithms, and the focus will be on the feasibility and optimality of the interval solutions of the ILP model.

(2) Development of REILP and Fuzzy REILP models

The formulation of REILP model will be based on the ILP model solutions and could better reflect complex connections between system return and decision risks. The proposed FREILP model is designed to minimize the decision risks of the truck allocation system while the total system cost is maintained at a minimum level with the aspiration level being preferably selected by the decision makers. In addition, problems of model infeasibility and risk function formulation will also be discussed.

(3) Application of the developed FREILP model to the truck allocation planning for surface mine industry

The modeling results could provide mining operators scientific bases for generating practical truck allocation schemes, and thus both mine/waste hauling and operating cost goals could be achieved. In this application, three options based on different aspiration levels of decision makers will be provided, including aggressive schemes, medium schemes, and conservative schemes.

## 1.3 STRUCTURE OF THE THESIS

The structure of this thesis is organized as follows:

Chapter 1 introduces the surface mine truck allocation problem and the application of mathematical programming to the mining truck allocation system. Problems associated with the current modeling studies are discussed, leading to the need of a new approach that could provide decision support for cost-effective truck allocation planning.

Chapter 2 presents a comprehensive literature review with respect to the previous efforts in using optimization techniques to solve the truck allocation problem. The focuses have been placed on discussing uncertainty-involving optimization techniques and their applications to the truck allocation system management. A summary has been provided in order to identify the knowledge gaps based on these previous studies.

Chapter 3 introduces a numerical example to conduct the validity checking of the existing ILP algorithms with focuses on checking the feasibility and optimality of the ILP model solutions.

Chapter 4 presents the development of the REILP model along with the selection of the aspiration levels and the construction of the constraint-wise risk function. A numerical example is provided in this chapter to illustrate the REILP model solution process.

Chapter 5 discusses the details of the FREILP model formulation and development. The solution process for solving the FREILP model is also provided.

Chapter 6 presents the application of the developed ILP, REILP and FREILP models to the case study of the Yalong Project. The model input data and the model development are provided in details in this chapter.

Chapter 7 presents the modeling results obtained from Chapter 6. The implications of the FREILP modeling results are also discussed.

Chapter 8 is a summary of this study along with the conclusion. Recommendation for further work is also provided in this chapter.

# CHAPTER 2 LITERATURE REVIEW

## 2.1 CONVENTIONAL HAUL TRUCK ALLOCATION SYSTEM AND PLANNING METHODS

Since the 1980's, much effort has been devoted to the development of the mathematical programming models to support the decision-making process of truck allocation system and to evaluate the relevant operational and investment policies. Turpin (1980) firstly proposed an economic optimization technique for open-pit mining operation system. After that, a number of open-pit mining operation related deterministic optimization models have been developed. Rommelfanger et al. (1989) applied the Linear Programming (LP) technique to investigate the related costs of truck and shovel fleets for surface mining operations. Sturgul and Li (1997) also used the LP approach to optimize the truck and shovel operation system. His research is based on performance estimation models developed by a project in Utah. Usama (1996) developed the Mix-Integer Linear Programming (MILP) approach and applied it to the planning of the truck allocation system. Beaumont (1998) applied the MILP in the open-pit mining operation system in America. Huang et al. (2001) applied the stochastic programming approach to determine the optimal truck hauling amount and the truckload for the open-pit mining project in Shandong, China; Chung (2002) applied the integrated linear programming approach to the surface mining operation system. The cost and truck-shovel resources were considered while formulating and selecting the optimal system management alternatives. Krause and Musingwini (2007) developed an integrated truck-shovel operation system model by using the LP approach to assist in identifying the optimal truck resources management strategy which could meet the balance of the cost control and the operation system management objectives.

Since the open-pit mining operation system has become more advanced and integrated, it has been realized that the above deterministic optimization approaches are insufficient to formulate proper models for the complex operation system problems. In surface mining operation systems, many truck hauling related processes need to be considered by the decision maker and stakeholder, including ore hauling, waste hauling, truckload selection and truck resources (Caccetta and Hill, 2003). There are many factors in these processes that interact with each other with multi-period, multi-layer and multi-objective features (Cetin, 2004). Meanwhile, the temporal and spatial variations of different system components may further multiply the uncertainties in the whole operation system (Pan, 2012). Hence, these factors are not only closely connected with uncertainties but also hard to be evaluated in the precise term.

Since the deterministic data and crisp model constraints are required in the deterministic optimization approach, it is desired to develop the optimization approach with the reflection of the uncertainties. In the past, a variety of uncertainty-handling techniques, have been developed and applied to different cases, such as fuzzy and stochastic mathematical programming approaches. However, their applications to solving the surface mining operation problems are limited and reviewed in next section.

## 2.2 OPTIMIZATION APPROACHES THAT DEAL WITH UNCERTAINTIES

### 2.2.1 Fuzzy Mathematical Programming Approach

Fuzzy mathematical programming approach is based on the fuzzy set theory and formalized by Zadeh in 1965. The fuzzy set is different from the classical set, which the membership can only take values of 0 or 1. The fuzzy set can be presented by the membership function which takes values in the range of [0, 1].

Fuzzy mathematical programming approach has been developed and applied to many different optimization applications, including surface mining operation system. Bascetin and Kesimal (1999) applied the fuzzy mathematical programming to the optimization of inventory stockpiles and mine production. Caccetta and Hill (2003) extended an efficient optimization model for the long-term production planning at LKAB's Kiruna mine by using the fuzzy mathematical programming approach. Chung et al. (2005) applied the fuzzy programming approach to the optimal equipment selection of the surface mining hauling system in Istanbul, Turkey. In this study, the fuzzy objectives for the decision makers were quantified under multiple types of truck-shovel operation management alternatives. More recently, Liu et al. (2010) developed a fuzzy linear programming model for a review of operation research in mine planning. It can deal with the uncertainties which are denoted as fuzzy sets in the left-hand side and right hand side of the constraints and the objective function.

Fuzzy mathematical programming technique can be sorted into two major categories due to its characteristics. They are fuzzy possibility programming approach and fuzzy flexibility programming approach (Inuiguchi and Sakawa, 1994). For the fuzzy flexibility programming approach, the flexibility in the constraints and the fuzziness in the objectives are denoted by the fuzzy sets and presented as the fuzzy constraints and the fuzzy objective, which can be expressed as the membership grades. However, the fuzzy flexibility programming approach cannot express the uncertainties as ambiguous coefficients in both of the constraints and objective functions (Inuiguchi and Ramık, 2000; Schultz, 2003). In the fuzzy possibility programming approach, the fuzzy parameters are addressed in the programming model and denoted as fuzzy sets along with their possibility distributions. However, there is certain limitation for the application of the fuzzy possibility programming approach. Huang et al. (1993) mentioned that when many uncertain parameters are expressed as fuzzy sets in a model, the interactions among these uncertainties may lead to serious complexities, especially for the large scale practical cases (Zhou et al., 2009). Meanwhile, the membership function of the fuzzy set, as the critical role of the fuzzy mathematical programming approach, numerically expresses the degree of each element in its belonging fuzzy set. In addition, the membership function of the parameter is hard to define and the inaccurate membership function will lead to undesirable results.

### 2.2.2 Stochastic Mathematical Programming Approach

Stochastic mathematical programming approach is developed based on the probability theory. The random elements are addressed in order to account for the probability uncertainty in the coefficients in the stochastic programming model. Chung et al. (2005) developed a stochastic programming model for the mine truck allocation which allows for analyzing stochastic information in the truck allocation management. The inherent uncertainty in a model can be denoted as stochastic elements in the constraints matrix, the right hand side stipulations or the objective function (Chung et al. 2005). However, the programming model will be extremely hard to solve if all of the parameters in this model are denoted as random variables. In addition, it also may lead to some infeasibility problems.

Chance-constrained programming approach is one of the stochastic mathematical programming techniques which contains the random distributions of the right hand side parameters ($b_i$). All of the constraints are not required to be satisfied in the chance-constrained approach. Instead of this, a certain level of violation of the constraint with the random distribution under certain circumstances can be allowed in this approach (Steuer et al., 1981), as presented in the following equation:

$$\mathrm{P}[g_i(\bar{x}) \geq b_i] \geq p \quad (2.1)$$

The probability which the $i^{th}$ constraint satisfied is $p$, where $0 \leq p \leq 1$. And the $b_i$ can be determined by its distribution and possibility $p$.

Generally, chance-constrained programming approach always combines with other uncertainty handling approaches, due to many uncertain factors other than the right hand side stipulations existed in the practical cases. Guo et al. (2010) combined the chance-constrained approach and mixed-integer linear programming approach into a general optimization modeling framework for the long term production planning of an open-pit mine. Zhou et al. (2009) developed a fuzzy chance-constrained programming model for the shovel-truck-crusher system in the open-pit mine. Guo et al. (2010) developed an inexact fuzzy stochastic mixed-integer programming model to the surface mining operation management design.

The stochastic mathematical approach could provide a complete view of the effects of the uncertainties along with the relationships between the uncertain inputs and resulting solutions for the decision makers and stakeholders (Chinneck and Ramadan, 2000). However, the availability of sufficient data to obtain the Probability Distribution Functions (PDF) for the random parameters is always problematic in the real-world cases. Moreover, even if the random distribution functions are available, the large scale stochastic programming models are still hard to solve with all uncertainties being denoted as PDFs (Krause and Musingwini, 2007).

### 2.2.3 Interval-parameter Mathematical Programming Approach

Both fuzzy and stochastic mathematical programming approaches can effectively reflect the uncertainties in the model. However, they require a significant amount of data to obtain the membership functions and PDFs, which in many real-world cases it is impractical or impossible. In addition, even data are sufficient but its distribution is hard to specify; as a result, the modeler or decision maker would rather select the fluctuation interval of the uncertain parameters than specifying its distribution (Huang et al., 2001). In the surface mining industry, the hourly ore production rate often fluctuates within a certain interval, but it is difficult to obtain sufficient data to present it as a reliable distribution function (Caccetta and Hill, 2003). Hence, the interval-parameter mathematical programming approach becomes a popular and alternative method for dealing with the uncertainties in the constraints and objectives of the model (Huang et al., 2001). Comparing to the fuzzy and stochastic mathematical programming models, the interval-parameter mathematical programming approach does not require the membership function or the distribution of the uncertain parameters. This programming approach is based on the interval analysis. And it only requires extreme bounds (upper and lower bounds) of the uncertain parameters. The interval-parameter analysis was firstly proposed by Moore in 1979, and was then developed into interval-parameter mathematical programming approach (Moore, 1979). This programming approach indicates the intrinsic vagueness of the informational characteristics during the parametric estimation (Hansen and Walster, 2003).

In the past decades, the interval-parameter programming approaches have been widely introduced in many fields due to its simplicity (Richard, 1980; Huang et al., 1992; Emmanuel, 2001; Hansen and Walster, 2003; Zhou et al., 2009; Liu et al., 2010; Huang and Cao, 2011). Among them, the Interval-parameter Linear Programming (ILP) approach becomes a critical member of the interval-parameter mathematical programming approach. ILP approach does not require any distributions when dealing with uncertain parameters expressed as intervals, and it does not lead to more complicated sub-models either. The ILP approach can deal with the uncertain parameters in the objective function (Iishibuchi and Tanaka, 1990), right- and left- hand side of the constraints, and any combinations of above all (Huang et al., 1992 & 1995). The application of the ILP to the surface mining area include hypothetical case examples in equipment selecting problem in the open-pit mining operation system (Hammah and Curran, 1998; Chinneck and Ramadan, 2000; Chung et al., 2005), truck-shovel allocation problem (Beaumont, 1998; Cetin, 2004; Javad, 2009), and the long term planning for the open-pit mining operation system (Liu et al., 2010). It has also been applied to practical cases, such as the long term production scheduling optimization for the surface mining operation (Huang et al., 2001), the mine truck allocation scheduling in South America (Chung et al., 2005), and the optimization of the shovel-truck system for surface mining (Ercelebi and Bascetin, 2009).

There are three algorithms which have been used to solve the ILP programming models, including Monte Carlo simulation algorithm, Two-Step algorithm and BWC algorithm. Monte Carlo algorithm requires the repeated random samplings in order to compute the results (Rubinstein and Marcus, 1985), which means high computational requirements. Hence, it is not realistic to solve real-world large scale models which involve numerous uncertain parameters and variables. Huang et al. (1992) developed the Two-step algorithm and Tong (1994) developed the BWC algorithm. Comparing to the Monte Carlo algorithm, Two-step and BWC methods are easier to use when solving the ILP models since they generate two deterministic sub-models corresponding to the upper and lower bounds of the objective function. In 2009, Rosenberg conducted a research on a few ILP models and concluded that the Two-step algorithm could not provide a good performance in some cases (Zhou et al., 2009). Therefore, the validity checking is desired to be conducted for these two algorithms in order to examine the feasibility and optimality of the obtained results.

### 2.2.4 Hybrid and Mixed Mathematical Programming Approach

Previously, many hybrid optimization approaches have been developed to account for the model uncertainties, and the ILP approach has been widely used to combine with other optimization techniques due to the flexibility and the simplicity. Chung et al. (2005) proposed an interval based possibilistic programming model for the optimal surface mining operation planning while minimizing the system cost. Javad (2009) proposed a fuzzy flexible programming model and applied to the queuing network model for shovel-truck-crusher systems in open-pit mining. Guo et al. (2010) proposed a fuzzy interval-parameter mixed integer programming model to analyze the hybrid system for surface coal mine production. This research presented how uncertainties could be quantified by specific membership functions and the intervals in the multi-objective programming model. Liu et al. (2010) developed a mixed optimization approach to improve the ILP method and the mixed-integer programming approach for a better performance prediction of gob gas ventholes for sealed and active longwall mines. In these applications, almost all of the ILP-based programming models are solved eventually by the ILP solution algorithm. Consequently, once there are any fundamental flaws in the ILP solution process, the results of these ILP based studies may become problematic, and its validity checking is necessary. In addition, the corresponding risks associated with the decisions generated from the ILP models need to be examined for better supporting the decision-making process.

## 2.3 SUMMARY OF PREVIOUS STUDIES ON OPEN-PIT MINING OPERATION SYSTEM

In summary, although the uncertainty-handling approaches have been applied to the fields of surface mining operation, the validity of the ILP modeling results need to be thoroughly checked, and furthermore, the risks of implementing the decisions generated from the ILP modeling results have never been studied in the past. For the first time in the surface mining industry, the Risk Explicit Interval-parameter Linear Programming (REILP) approach will be developed and applied to the truck allocation process to improve the operation efficiency and effectiveness in the open-pit mining operation system.

Moreover, when calculating the ore/waste truck cycle time, the hauling speed cannot be exactly same every time in the real world situation. This variable can be affected by the drivers or weather conditions or different road conditions. Therefore, an interval range can be figured out for the truck cycle time in order to generate better optimal solutions. Meanwhile, some other variables also need to be considered as interval parameters, such as the waste hauling amount and the cost coefficients of different truck types.

In this study, the ILP and the REILP approaches will be applied in the truck allocation problem for the Anshun Yalong coal mine project. In addition, considering different decision makers may have different preferences on decision making process, this study will develop a Fuzzy Risk Explicit Interval Linear Programming (FREILP) model to improve the applicability for this case study.

# CHAPTER 3 ILP VALIDITY CHECKING

## 3.1 EXISTING ILP SOLUTION ALGORITHMS

In the past decades, the uncertainty-based linear programming (LP) models have been widely used in assisting optimal decision making in various fields (Huang et al., 1992; Huang et al., 1995; Sturgul and Li, 1997; Krause and Musingwini, 2007). The intrinsic uncertainties in LP can be expressed as probability, possibility, and interval formats, and thus several types of LP were developed and applied in previous studies, such as Interval LP (ILP) and some hybrid models (Huang et al., 1995; Sturgul and Li, 1997; Javad, 2009). In the ILP models, interval numbers ($\pm$) were introduced to model parameters for describing the potential uncertainties (max $C^{\pm}X^{\pm}$, s.t. $A^{\pm}X^{\pm} \leq B^{\pm}$ and $X \geq 0$ ) (Moore, 1979) and the information regarding the probabilistic distribution of the parameters is unknown to the modelers. It is expected that the solution of ILP provides the range of values for each decision variable and objective function.

**Definition 3.1.1**: An ILP model is defined as follows (Huang et al., 1992):

a) For maximizing problems:

Max

$f^{\pm} = C^{\pm}X^{\pm}$

Subject to

$A^{\pm}X^{\pm} \leq B^{\pm}$

$X^{\pm} \geq 0$

For minimizing problems:

Min

$f^{\pm} = C^{\pm}X^{\pm}$

Subject to

$$A^{\pm}X^{\pm} \geq B^{\pm}$$

$$X^{\pm} \geq 0$$

Where,

$$A^{\pm} = \{a_{ij}^{\pm}\}, i = 1, \dots, m\, ; j = 1, 2, \dots, n$$

$$B^{\pm} = [b_1^{\pm},\ b_2^{\pm}, \dots,\ b_m^{\pm}]$$

$$C^{\pm} = [c_1^{\pm},\ c_2^{\pm}, \dots,\ c_n^{\pm}]$$

$$X^{\pm} = [X_1^{\pm},\ X_2^{\pm}, \dots,\ X_n^{\pm}]$$

"-" indicates the lower bound of the interval parameter or variable,
"+" indicates the upper bound of the interval parameter or variable.

Since interval parameters exist in the objective function and constraints, the ILP models need to be transformed into its deterministic equivalent forms for being solved by mathematical programming software, such as LINGO. The optimal solutions of the interval-parameter linear programming model are:

$$f_{opt}^{\pm} = [f_{opt}^{-}, f_{opt}^{+}]$$

$$X_{opt}^{\pm} = [x_{1opt}^{\pm}, x_{2opt}^{\pm}, \dots, x_{nopt}^{\pm}]$$

$$x_{jopt}^{\pm} = [x_{jopt}^{-}, x_{jopt}^{+}], j = 1, 2, .., n$$

In the past, three solution algorithms have been developed for transforming the original ILP models into the deterministic models, i.e., Monte Carlo simulation, two-step method and best-worst case (BWC) algorithm. A detailed explanation for each algorithm is provided below, and a numerical example is then created for checking the validity of the ILP algorithms.

### 3.1.1 Monte Carlo Simulation Algorithm

Monte Carlo simulation algorithm (or Monte Carlo experiments) is a computerized mathematical technique that relies on repeated random sampling to obtain numerical results (Rubinstein and Marcus, 1985) while letting people to account for risks in quantitative analysis and decision making process. This method has been widely used in various fields, such as civil or environmental engineering, industrial engineering, research and development, project management, finance, and etc.

Monte Carlo simulation can offer the decision-maker with a range of possible outcomes and the probabilities for any choice of action. And it also shows the extreme possibilities — the outcomes of going broken or the most conservative decision — along with all possible consequences for middle-of-the-road decisions (Steuer, 1981).

By using the Monte Carlo method to solve the ILP model, it sets values for parameters at random within their ranges to form the classic linear programming model (Hartman, 1992). The operations of solving the ILP model by the Monte Carlo simulation algorithm are described as follows (Cetin, 2004):

[Step 1]: First, select random feature to every single interval parameter ($a_{ij}$, $b_i$ and $c_j$) with probability distribution functions (PDF) in the interval linear programming (ILP) model. Then, convert each probability distribution function into its cumulative distribution function (CDF).

Figure 3.1 is an example of a parameter with a normal distribution, and Figure 3.2 shows the corresponding CDF converted from the normal distribution as shown in Figure 3.1.

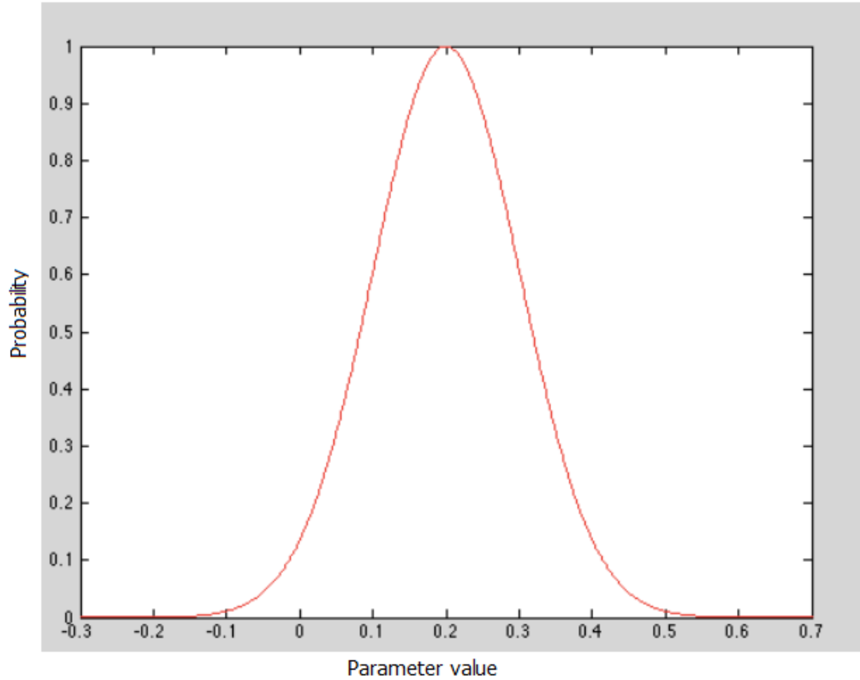


*Figure 3.1 The PDF curve of a random parameter*

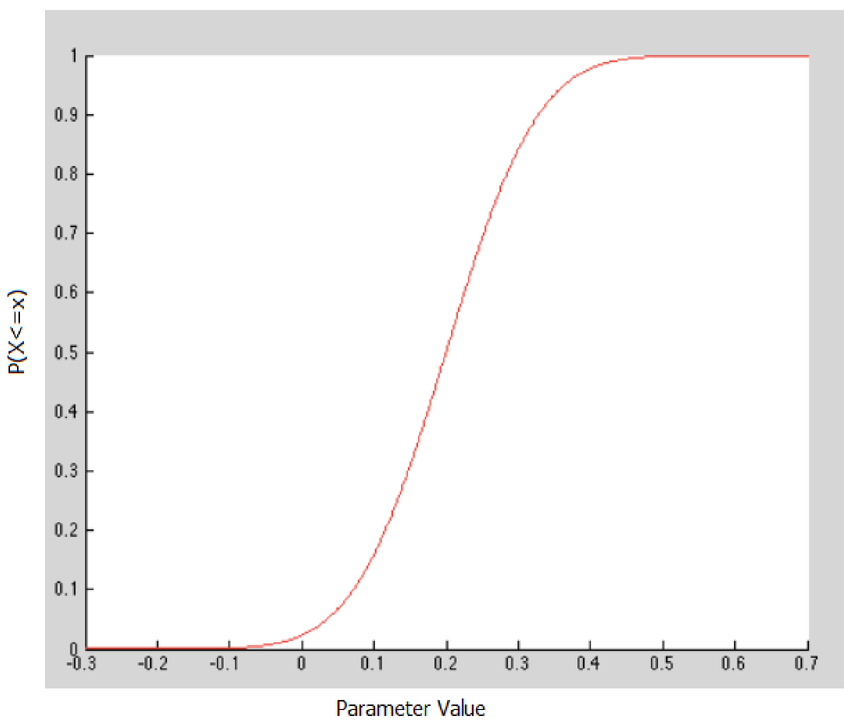


*Figure 3.2 The CDF curve of a random parameter converted from its PDF*

In terms of PDF forms, the normal (or Gaussian) distribution has been frequently used in the ILP model in the past, while some other distributions such as discrete probability distributions or uniform distribution were also used to show the various distribution characteristic of different parameters.

[Step 2]: Generate a random number between 0 and 1 by using the random number generator in the programming software and then denote it as *r*.

[Step 3]: Make the generated random number *r* be connected with the CDF curve for every parameter to get a set of deterministic values for all parameters in $a_{ij}$, $b_i$ and $c_j$.

Figure 3.3 shows how to locate a parameter value using the generated *r*.

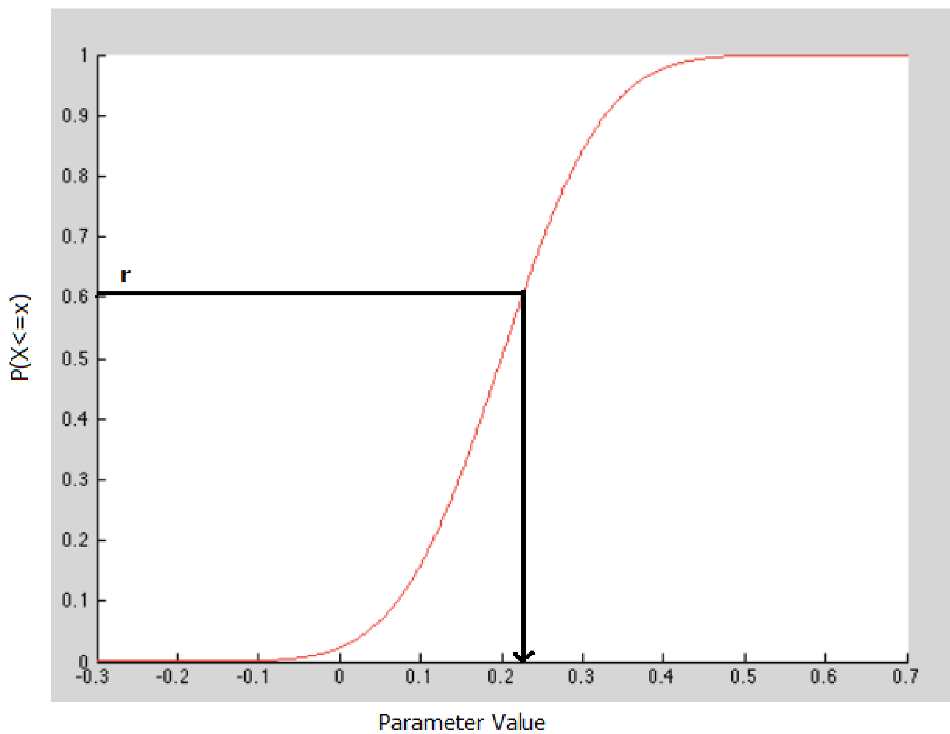


*Figure 3.3 Setting an exact value for the random parameter using r.*

[Step 4]: Replace the interval parameters $a_{ij}$, $b_i$ and $c_j$ by using the set of deterministic values which we get from Step 3, and create a classic deterministic linear programming model.

[Step 5]: Solve the classic deterministic linear programming model by software and generate a set of corresponding deterministic solutions.

[Step 6]: Run the process above sufficient times by repeating Steps 2, 3, 4 and 5 in order to obtain the distribution of the solution for each specific decision variable.

Since the Monte Carlo simulation algorithm could offer us the solid results via simulating the practical situations in the real world, this method can be a really successful algorithm for solving the ILP model (Krause and Musingwini, 2007), although it requires large numbers of computational runs of random or pseudo random numbers. Usually, thousands or even millions of times of simulations have to be conducted in order to obtain the meaningful distribution of the solutions. Therefore, it would become unrealistic for solving the intricate practical problems along with a large number of uncertain parameters and variables in the real world. In addition, for most of practical problems, it is nearly impossible to acquire plenty of data to formulate the distributions for the interval parameters (Emmanuel, 2001), which makes the algorithm less feasible and applicable.

### 3.1.2 Two-Step Algorithm

Two step algorithm was first introduced by Huang et al. in 1992. It is an interactive method. For a maximization problem, the first sub-model is formulated to solve for the upper bound of the objective function, and then the sub-model corresponding to the lower bound of the objection function is formulated and solved (Huang et al., 1992).

The first sub-model is formulated as:

For the *n* interval coefficients $c_j$ (*j=1,2,3,...,n*) in the original objective function $f^{\pm} = C^{\pm}X^{\pm}$ in an ILP model, assume $k_1$ of these coefficients be non-negative, and $k_2$ be negative, let the n coefficients be reordered such that $c_j^{\pm} \geq 0$ $(j = 1, 2, \dots, k_1)$, and $c_j^{\pm} < 0$ ($j = k_1 + 1$, $k_1 + 2$, ..., n ). The sub-model corresponding to the upper bound of the objective function (when the objective is to be maximized) is formulated as:

Max

$$f^{+} = \sum_{j=1}^{k_1} c_j^{+}\, x_j^{+} + \sum_{j=k_1+1}^{n} c_j^{+}\, x_j^{-} \tag{3.10}$$

Subject to

$$\sum_{j=1}^{k_1} |a_{ij}|^{-} Sign(a_{ij}^{-}) x_j^{+} + \sum_{j=k_1+1}^{n} |a_{ij}|^{+} Sign(a_{ij}^{+}) x_j^{-} \leq b_i^{+}, \forall i \tag{3.11}$$

$$x_{ij}^{\pm} \geq 0, \forall j \tag{3.12}$$

The above sub-model 3.10 to 3.12 is a classic linear programming model which can be solved by simplex algorithm or any other existing algorithms. The sub-model corresponding to the lower bound of the objective function can then be formulated:

Max

$$f^{-} = \sum_{j=1}^{k_1} c_j^{-}\, x_j^{-} + \sum_{j=k_1+1}^{n} c_j^{-}\, x_j^{+} \tag{3.13}$$

Subject to

$$\sum_{j=1}^{k_1} |a_{ij}|^{+} Sign(a_{ij}^{+}) x_j^{-} + \sum_{j=k_1+1}^{n} |a_{ij}|^{-} Sign(a_{ij}^{-}) x_j^{+} \leq b_i^{-}, \forall i \tag{3.14}$$

$$x_{ij}^{\pm} \geq 0, \forall j \tag{3.15}$$

$$x_j^{-} \leq x_{jopt}^{+}, j = 1,2,3,\dots,k_1 \tag{3.16}$$

$$x_j^{+} \geq x_{jopt}^{-}, j = k_1 + 1, k_1 + 2, \dots, n \tag{3.17}$$

The sub-model (3.13 to 3.17) is also a classic linear programming model which can be solved by simplex algorithm or any other existing algorithms. Where, $x_{jopt}^{+}$ $(j = 1,2,3,\ldots,k_1)$ and $x_{jopt}^{-}$ $(j = k_1 + 1, k_1 + 2, \ldots, n)$ are the optimal solutions generated from the sub-models 3.10 to 3.12; $x_{jopt}^{+}$ $(j = 1,2,3,\ldots,k_1)$ and $x_{jopt}^{-}$ $(j = k_1 + 1, k_1 + 2, \ldots, n)$ are the optimal solutions generated from the sub-models 3.13 to 3.17.

Therefore, after solving the two sub-models respectively, we can get the optimal solutions for the original ILP model, which are:
$x_{jopt}^{\pm} = [x_{jopt}^{-}, x_{jopt}^{+}]$ and $f_{opt}^{\pm} = [f_{opt}^{-}, f_{opt}^{+}]$.

For a minimization problem, the sub-model for solving the lower bound of the objective function should be formulated and solved first.

Min

$$f^{-} = \sum_{j=1}^{k_1} c_j^{-} x_j^{-} + \sum_{j=k_1+1}^{n} c_j^{-} x_j^{+} \quad (3.18)$$

Subject to

$$\sum_{j=1}^{k_1} |a_{ij}^{+}| Sign(a_{ij}^{+}) x_j^{-} + \sum_{j=k_1+1}^{n} |a_{ij}^{-}| Sign(a_{ij}^{-}) x_j^{+} \le b_i^{+}, \forall i \quad (3.19)$$

$$x_{ij}^{\pm} \ge 0, \forall j \quad (3.20)$$

The sub-model for solving the upper bound of the objective function can then be formulated as:

Min

$$f^{+} = \sum_{j=1}^{k_1} c_j^{+} x_j^{+} + \sum_{j=k_1+1}^{n} c_j^{+} x_j^{-} \quad (3.21)$$

Subject to

$$\sum_{j=1}^{k_1} |a_{ij}^{-}| Sign(a_{ij}^{-}) x_j^{+} + \sum_{j=k_1+1}^{n} |a_{ij}^{+}| Sign(a_{ij}^{+}) x_j^{-} \le b_i^{-}, \forall i \quad (3.22)$$

$$x_j^{+} \ge x_{jopt}^{-}, j = 1,2,3,\ldots,k_1 \quad (3.23)$$

$$x_j^- \leq x_{jopt}^+, j = k_1 + 1, k_1 + 2, \ldots, n \quad (3.24)$$

$$x_{ij}^\pm \geq 0, \forall j \quad (3.25)$$

Similar to the former maximization model, the sub-model (3.18 to 3.25) is also a classic linear programming model which can be solved by simplex algorithm. Where, $x_{jopt}^+$ $(j = 1,2,3,\ldots,k_1)$ and $x_{jopt}^-$ $(j = k_1 + 1, k_1 + 2, \ldots, n)$ are the optimal solutions generated from the sub-models 3.18 to 3.20; $x_{jopt}^+$ $(j = 1,2,3,\ldots,k_1)$ and $x_{jopt}^-$ $(j = k_1 + 1, k_1 + 2, \ldots, n)$ are the optimal solutions generated from the sub-models 3.21 to 3.25.

Therefore, after solving these two sub-models respectively, the optimal solutions for the minimized ILP model can be obtained, which are:
$x_{jopt}^\pm = [x_{jopt}^-, x_{jopt}^+]$ and $f_{opt}^\pm = [f_{opt}^-, f_{opt}^+]$.

Below is a numerical example to illustrate the 2-step solution algorithm:

Max

$[50, 60]x_1^\pm$ - $[70, 90]x_2^\pm$

Subject to

$[4, 6]x_1^\pm + x_2^\pm \leq [150, 200]$

$16x_1^\pm + [5, 7]x_2^\pm \leq [280, 360]$

$x_1^\pm + [3, 4]x_2^\pm \leq [90, 110]$

$x_1^\pm - 10x_2^\pm \leq -0.5$

$x_1^\pm, x_2^\pm \geq 0$

According to the two-step algorithm, the first sub-model is formulated to maximize the upper bound of the objective function:

Max

$f^+ = 60x_1^+ - 70x_2^-$

Subject to

$4x_1^+ + x_2^- \leq 200$

$16x_1^+ + 7x_2^- \leq 360$

$x_1^+ + 4x_2^- \leq 110$

$x_1^+ - 10x_2^- \leq -0.5$

$x_1^\pm, x_2^\pm \geq 0$

The optimal solution of the first sub-model is:

$x_1^+ = 21.54$

$x_2^- = 2.20$

$f^+ = 1159.44$

Then, the second sub-model can be formulated for the lower bound of the objective function:

Max

$f^- = 50x_1^- - 90x_2^+$

Subject to

$6x_1^- + x_2^+ \leq 150$

$16x_1^- + 5x_2^+ \leq 280$

$x_1^- + 3x_2^+ \leq 90$

$2x_1^- - 10x_2^+ \leq -0.5$

$x_1^+ \geq x_1^-$

$x_2^- \leq x_2^+$

$x_1^\pm, x_2^\pm \geq 0$

The optimal solution of the second sub-model is:

$x_1^- = 16.46$

$x_2^+ = 3.34$

$f^- = 522.09$

Combining the solutions from both sub-models, the optimal solution for the original numerical example is as follows:

$x_1^\pm$ = [16.46, 21.54]

$x_2^\pm$ = [2.20, 3.34]

$f^\pm$ = [522.09, 1159.44]

### 3.1.3 Best-Worst Case Algorithm (BWC)

The best-worst case (BWC) algorithm also includes a two-step procedure to solve two corresponding sub-models for the ILP model (Tong, 1994; Chinneck and Ramadan, 2000). However, the obvious difference between the two-step algorithm and BWC is that the two-step algorithm needs to distinguish the selection of extreme parameter values (such as their lower and upper bounds) for decision variables with different signs (which is negative or positive) in the objective function, while the BWC method treats all parameters without any distinctions. It is worth to mention that the original BWC algorithm was designed for the minimization problem. However, since the ILP model could be easily transformed into its corresponding canonical format, this method has been widely used and developed in various areas in the past.

For the BWC algorithm, we usually need to formulate and solve the best-case sub-model and then the worst-case sub-model for the objective function, respectively.

For a maximization problem, the first step in the BWC is to formulate its best-case sub-model for its corresponding upper bound of the objective function, which is expressed as follows (from 3.26 to 3.28):

Max

$$f^{+} = c_j^{+} x_j \tag{3.26}$$

Subject to

$$a_{ij}^{-} x_j \leq b_i^{+}, \forall i \tag{3.27}$$

$$x_j \geq 0, \forall j \tag{3.28}$$

Then, we could formulate its corresponding worst-case sub-model as follows (from 3.29 to 3.31):

Max

$$f^{-} = c_j^{-} x_j \tag{3.29}$$

Subject to

$$a_{ij}^{+} x_j \leq b_i^{-}, \forall i \tag{3.30}$$

$$x_j \geq 0, \forall j \tag{3.31}$$

For a minimization problem, the best-case sub-model should be firstly formulated as (from 3.32 to 3.34):

Min

$$f^- = c_j^- x_j \tag{3.32}$$

Subject to

$$a_{ij}^+ x_j \geq b_i^-, \forall i \tag{3.33}$$

$$x_j \geq 0, \forall j \tag{3.34}$$

Then, its corresponding worst-case sub-model is formulated as follows (from 3.35 to 3.37):

Min

$$f^+ = c_j^+ x_j \tag{3.35}$$

Subject to

$$a_{ij}^- x_j \geq b_i^+, \forall i \tag{3.36}$$

$$x_j \geq 0, \forall j \tag{3.37}$$

In general, the best-case and worst-case sub-models in the BWC algorithm represent two different extreme situations for both the objective function and all the constraints of the original ILP model, respectively, i.e., for the maximization problem, the best case sub-model (from 3.26 to 3.28) represents the most ideal situation (the largest) for the objective function of the original ILP model while its constraints delimit the largest decision space for the optimal solution; the worst case sub-model (from 3.29 to 3.31) represents the least value for the objective function of the original ILP model, and its constraints define the smallest, which is also the narrowest, decision space.

On the contrary, for the minimization problem, the best-case sub-model (3.32 to 3.34) represents the lowest value for the objective function of the original ILP model, and its constraints delimit the narrowest decision space for the optimal solution. The worst-case sub-model (3.35 to 3.37) gives the most ideal condition (the largest value) for the objective function, and its constraints delimit the largest decision space for the optimal solution.

Below is a numerical example to illustrate the BWC algorithm:

Max

$$f = [50,60]X_1 - [70,90]X_2 \quad (3.38)$$

Subject to

$$[4,6]X_1 + X_2 \leq 150 \quad (3.39)$$

$$6X_1 + [5,7]X_2 \leq 280 \quad (3.40)$$

$$X_1 + [3,4]X_2 \leq 90 \quad (3.41)$$

$$[1,2]X_1 - 10X_2 \leq -1 \quad (3.42)$$

(1) Best case sub-model:

Max

$$f^+ = 60X_1 - 70X_2 \quad (3.43)$$

Subject to

$$4X_1 + X_2 \leq 150 \quad (3.44)$$

$$6X_1 + 5X_2 \leq 280 \quad (3.45)$$

$$X_1 + 3X_2 \leq 90 \quad (3.46)$$

$$1X_1 - 10X_2 \leq -1 \quad (3.47)$$

(2) Worst case sub-model:

Max

$$f^- = 50X_1 - 90X_2 \quad (3.48)$$

Subject to

$$6X_1 + X_2 \leq 150 \quad (3.49)$$

$$6X_1 + 7X_2 \leq 280 \quad (3.50)$$

$$X_1 + 4X_2 \leq 90 \quad (3.51)$$

$$2X_1 - 10X_2 \leq -1 \quad (3.52)$$

After solving the two sub-models in Lingo, we could obtain the optimal solutions as follows:

$$f^{\pm} = [764.6774, 1930.732]$$

$$X_1^{\pm} = [24.1774, 36.5609]$$

$$X_2^{\pm} = [3.7561, 4.9355]$$

## 3.2 VALIDITY CHECKING FOR TWO-STEP AND BWC ALGORITHMS

Through examining how the two sub-models are formulated in both two-step and BWC algorithms, it is apparent that both algorithms ignore some of the system uncertainties when reformulating the sub-model constraints and this treatment could be a potential flaw of both algorithms and could very possibly lead to feasibility and optimality concerns towards the generated interval optimal solutions. This concern has triggered off a desire to check the validity of both algorithms for specifying the nature of the problem. In this study, a numerical example is designed to illustrate the validity checking process for both algorithms, and the focus of the validity checking is on the investigation of any infeasible solutions existed in the generated interval optimal solution and any optimal solutions missing from it.

### 3.2.1 A Numerical Example for Validity Checking

In this study, a minimization problem with 2 decision variables and 2 constraints was designed to illustrate the validity checking for both BWC and two-step algorithms.

Min

$$\mathrm{f} = [3, 4]X_1 + X_2 \tag{3.53}$$

Subject to

$$2X_1 - [2.4, 2.8]X_2 \geq [6, 8] \tag{3.54}$$

$$2X_1 - [3, 4]X_2 \geq [10, 12] \tag{3.55}$$

$$X_1, X_2 \geq 0 \tag{3.56}$$

Before solving this ILP model by two-step and BWC algorithms, the first step is to generate a large number of event models by using the Monte-Carlo Simulation method as described in Section 3.1.1. Each event model is a classic deterministic LP model which can be easily solved. By solving these event models, a large number of solution sets for decision variables can be produced and the solution ranges of the objective function and decision variables can then be obtained. The larger the numbers of the event models are solved, the better the solution resolution and accuracy could be obtained. The solution obtained from the Monte Carlo Simulation is then used as the near-real solution of the original ILP model to be compared with the solutions from 2-step and BWC algorithms for performing the validity checking. In this study, one million event models were generated and solved by the Monte-Carlo simulation method, and the obtained interval solutions are:

$$\mathrm{f} = [11.97, 20.4]$$

$$X_1 = [3.78, 4.94]$$

$$X_2 = [0.35, 1.08]$$

**(1) Two-step Algorithm Solution**

According to the two-step algorithm, two sub-models corresponding to $f^-$ and $f^+$ could be formulated as follows:

Sub-model 1:

Min

$$f^- = 3X_1^- + X_2^- \quad (3.57)$$

Subject to

$$2X_1^- - 2.8X_2^- \geq 6 \quad (3.58)$$

$$2X_1^- + 4X_2^- \geq 10 \quad (3.59)$$

$$X_1^-, X_2^- \geq 0 \quad (3.60)$$

Sub-model 2:

Min

$$f^+ = 4X_1^+ + X_2^+ \quad (3.61)$$

Subject to

$$2X_1^+ - 2.4X_2^+ \geq 8 \quad (3.62)$$

$$2X_1^+ + 3X_2^+ \geq 8 \quad (3.63)$$

$$X_1^+ \geq X_{1opt}^- \quad (3.64)$$

$$X_2^+ \geq X_{2opt}^- \quad (3.65)$$

Where,

$X_{1opt}^-$ and $X_{2opt}^-$ are the optimal solutions of $X_1$ and $X_2$ from sub-model 1. Both sub-models are classic linear programming models and can be solved by Lingo.

The optimal solutions obtained by the two step algorithm are:

$$f = [12.06, 20.3]$$

$X_1 = [3.82, 4.89]$

$X_2 = [0.59, 0.74]$

**(2) BWC Algorithm Solution**

Based on the BWC algorithm, two sub-models corresponding to the best-case and worst-case situations can be formulated as follows:

Sub-model 1:

Min

$$f^- = 3X_1^- + X_2^- \tag{3.66}$$

Subject to

$$2X_1^- - 2.4X_2^- \geq 6 \tag{3.67}$$

$$2X_1^- + 4X_2^- \geq 10 \tag{3.68}$$

$$X_1^-, X_2^- \geq 0 \tag{3.69}$$

Sub-model 2:

Min

$$f^+ = 4X_1^+ + X_2^+ \tag{3.70}$$

Subject to

$$2X_1^+ - 2.8X_2^+ \geq 8 \tag{3.71}$$

$$2X_1^+ + 3X_2^+ \geq 12 \tag{3.72}$$

$$X_1^+, X_2^+ \geq 0 \tag{3.73}$$

These two sub-models are also deterministic linear programming models which can be solved by Lingo. The optimal interval solutions obtained by the BWC algorithm are:

$f = [11.88, 20.55]$

$X_1 = [3.75, 4.97]$

$X_2 = [0.63, 0.69]$

### 3.2.2 Result Interpretation and Validity Checking

**(1) Optimality Checking**

Theoretically, although a large number of event model runs were implemented, the optimal solution space provided by the Monte-Carlo simulation should be narrower than the real solution space of the original example model, at most get very close to it.

Before checking the validity of the optimal interval solutions obtained by two-step and BWC algorithms, two facts should be noted: (1) every optimal solution generated by solving Monte-Carlo simulation even model represents a subset of true optimal solution sets of the original model, and solution infeasibility is not an issue; (2) the optimal solution spaces provided by both 2-step and BWC algorithm should completely include the optimal solution space from the Mont-Carlo simulation method (Pei, 2011). Mathematically, it yields:

$X_{1opt}^{-} \leq 3.78 \leq 4.94 \leq X_{1opt}^{+}$

$X_{2opt}^{-} \leq 0.35 \leq 1.08 \leq X_{2opt}^{+}$

$f_{opt}^{-} \leq 11.97 \leq 20.41 \leq f_{opt}^{+}$

Based on these two facts, if the optimal solution space provided by two-step or BWC algorithm does not cover the interval ranges of Monte-Carlo simulation results, i.e., the above relationship cannot be satisfied, this could lead to two significant consequences: (1) some optimal solution pairs are missing from the two-step algorithm or BWC algorithm; (2) the optimal solutions produced by both algorithms might include some pair points which are infeasible.

Table 3.1 shows the optimal solutions obtained from these three algorithms for comparison.

Table 3.1 Optimal solutions from three algorithms

| Algorithms | $X_1$ | | $X_2$ | | f | |
|---|---|---|---|---|---|---|
| | $X_1^-$ | $X_1^+$ | $X_2^-$ | $X_2^+$ | $f-$ | $f+$ |
| Monte Carlo simulation | 3.78 | 4.94 | 0.35 | 1.08 | 11.97 | 20.41 |
| Two-step algorithm | 3.82 | 4.89 | 0.59 | 0.74 | 12.06 | 20.3 |
| BWC algorithm | 3.75 | 4.97 | 0.625 | 0.69 | 11.88 | 20.55 |

The optimal solutions generated from three algorithms are summarized in the table above (Table 3.1) for comparison. In this table, the optimal intervals of the results from the two-step algorithm are all narrower than the ones from Monte Carlo algorithm. It can be seen that some pairs of optimal solutions may be missing from the two-step algorithm. As a result, two-step algorithm fails the validity checking in terms of solution optimality (optimal solutions are missing and incomplete).

For the optimal solution from BWC algorithm in this table, it shows that the decision variable $X_1$ has a full range coverage to $X_1$ from Monte Carlo simulation. Whereas the range of decision variable $X_2$ is narrower than $X_2$ from two-step algorithm, which means some of the optimal solutions are still missing from BWC algorithm. This indicates that BWC algorithm fails in the optimality checking as well. According to the fundamental theory of BWC algorithm, the two reformulated sub-models should be able to represent the two extreme conditions for the original LP model, and the optimal solution should be able to cover the full optimal solutions. However, the optimality checking results shows a disproof of the theory and more insightful research is needed and out of the scope of this study. Moreover, Table 3.1 shows that the BWC algorithm can provide a more complete optimal range of the objective function than the other two algorithms.

**(2) Feasibility Checking**

Figure 3.5 shows the optimal solution space and the feasible decision space generated by the two-step method. The line ADHK represents the boundary of the decision space given by the constraint (3.58), while the line BEIL represents the boundary of the decision space given by the constraint (3.62) from two-step sub-models. The two red dotted lines next to them stand for the two constraints of the BWC sub-models which are reformulated by the same original constraints. On the other hand, the line CDEF and the line GHIJ represent the boundaries of the feasible decision region generated by constraint (3.59) and constraint (3.63), respectively. Since the constraints (3.68) and (3.72) from BWC sub-models are same as constraint (3.59) and (3.63) from two-step sub-models, they are shown by the same lines in this figure. Figure 3.4 also shows the feasible decision space delimited by the two-step algorithm, which is under the line ADHK and BEIL, and above the line CDEF and GHIJ. Thus, the feasible decision space can be divided into several different sub-regions, including: (1) the space of BEF is the absolute feasible space which satisfies all the constraints in the model; (2) the space above the line ADHK and the space under the line GHIJ are the infeasible space since they violate at least one of the constraints; (3) the space which is bounded within the lines of ADHIJ and BEF are the softly feasible space, which means the solutions within this area cannot be guaranteed to satisfy all of the constraints; (4) the middle region which is bounded by the points DHIE represents the feasible optimal solution space generated from the two-step method.

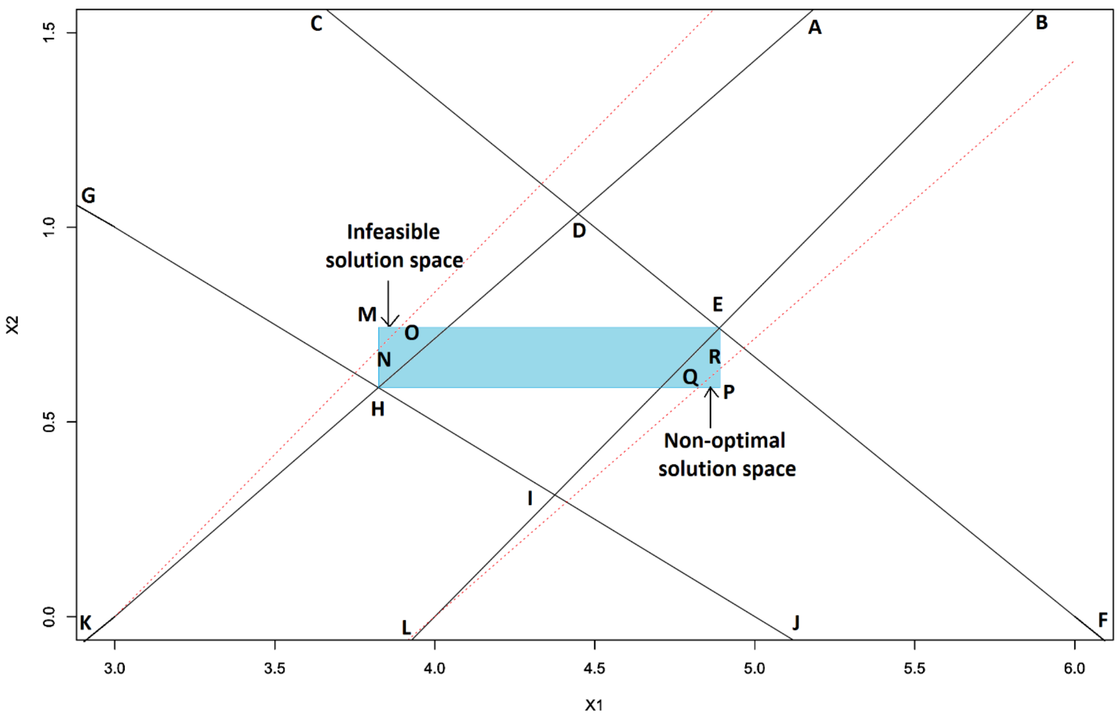


*Figure 3.4 Classification of feasible and optimal solution space for the two-step algorithm*

The blue rectangular in the middle of Figure 3.4 represents the optimal solution space from the two-step method. It can be used to explain the result of infeasibility checking. From Figure 3.4, it can be seen that the optimal solutions generated from the two-step algorithm are mostly located in the softly feasible space. The blue triangle MON is located in the infeasible space, which means the two-step algorithm does generate infeasible solutions for the original model. System failure can be caused if the decision alternatives are produced from this infeasible solution space and implemented in practice. Meanwhile, another blue triangle PQR in this figure demonstrates the non-optimal solution region, which means the solutions interpreted from this region are valid solutions but not optimal.

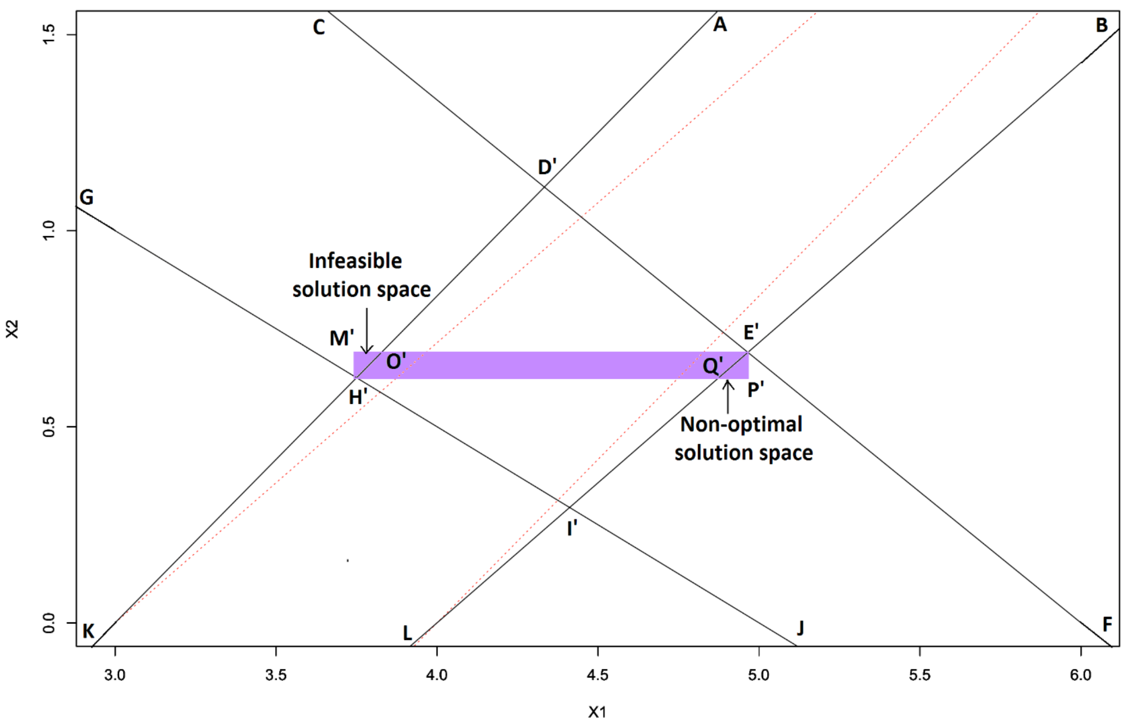


*Figure 3.5 Classification of feasible and optimal solution regions for the BWC algorithm*

Figure 3.5 above shows the optimal solution region and the feasible solution region generated by the BWC approach. The lines AD'H'K and BE'I'L stand for the constraint (3.67) and constraint (3.71), respectively. The line CD'E'F represents the constraint (3.68) while the line GH'I'J stands for the constraint (3.72). Two red dotted lines in this figure stand for the two constraints of the sub-models from two-step method which are reformulated by the same original constraints. Figure 3.5 shows that: (1) the space of BE'F is the absolute feasible space which can satisfy all the constraints in the model; (2) the space above the line AD'H'K and the space under the line GH'I'J are the infeasible space since they violate at least one of the constraints; (3) the region which is bounded within AD'H'I'J and BE'F are the softly feasible space, which means the solutions from this area cannot be guaranteed to satisfy every constraint; (4) the middle region which is bounded by the points D'H'I'E' represents the feasible optimal solution space generated from the BWC method.

The purple rectangular in the middle of Figure 3.5 demonstrates the optimal solution space from the BWC method. It can be used to show the result of infeasibility checking. Similar to Figure 3.4, in Figure 3.5, it can be seen that the optimal solutions generated from the BWC method are mostly located in the softly feasible space. The purple triangle M’O’H’ represents the infeasible space, which means the BWC method does generate infeasible solutions for the original model just as two-step algorithm. Similar system failure can be caused if the decision alternatives are produced from this infeasible solution space and implemented in practice. In the meantime, another purple triangle E’ P’Q’ in this figure represents the non-optimal solution area, which means the solutions interpreted from this region are valid but not optimal.

Figures 3.4 and 3.5 shows that both the two-step method and BWC method fail the feasibility checking for their ‘optimal’ solutions, and that infeasible solutions are generated and included in their optimal solution ranges and also some optimal solutions are missing. Therefore, decisions formulated from the generated solutions of both methods are always associated with risks in the decision-making and implementing process, which might be able to cause the failure of the entire plan and system. In a brief summary, the validity checking results could help the planners and decision-makers better understand the strength and weakness of the modeling results, and this has triggered off a desire to keep an eye on the problem and explore the measures to improve the process, for example, introducing the decision risk to the optimization process is an option.

# CHAPTER 4 RISK EXPLICIT INTERVAL LINEAR PROGRAMMING MODEL (REILP)

## 4.1 INTRODUCTION

According to the validity checking results presented in Chapter 3, the ILP solutions generated from both two-step algorithm and BWC algorithm have various deficiencies. It is proved that the optimal solutions of ILP model are usually located in the softly feasible decision region, however, the optimal solutions are not always valid and some solutions may have risks of violating some of the constraints. In the meantime, the ILP model is not able to reflect the relationship between decision risks and system return if the infeasible or invalid solutions are used to generate decision alternatives. Therefore, if the optimal solutions from the ILP model need to be used into practical cases, the potential risks associated with the decisions should be minimized rather than being remained unknown to the decision makers.

In order to overcome these shortcomings and limitations of the solutions from the ILP model while maintaining its strengths, a Risk Explicit Interval Linear Programming (REILP) approach was developed to explore the tradeoff between system return and decision risk for improving the feasibility and applicability of the ILP approach (Rui et al., 2010). The fundamentals of the REILP method is presented below.

An event model of a general ILP model can be formulated as follows:

Max

$$f = \sum_{j=1}^{n} [\, c_j^- + \lambda_0 (c_j^+ - c_j^-)] x_j \tag{4.1}$$

Subject to

$$\sum_{j=1}^{n} [\, a_{ij}^+ - \lambda_{ij} (a_{ij}^+ - a_{ij}^-)] x_j - [b_i^- + \eta_i (b_i^+ - b_i^-)] \le 0, \forall i \tag{4.2}$$

$$x_j \ge 0, \forall j \tag{4.3}$$

$$0 \leq \lambda_0 \leq 1 \quad (4.4)$$

$$0 \leq \lambda_{ij} \leq 1, \forall i, j \quad (4.5)$$

$$0 \leq \eta_i \leq 1, \forall i \quad (4.6)$$

The above model represented by equations (4.1) to (4.6) is a classic LP model, which corresponds to a specific set of crisp value of each coefficient given $\lambda_0$, $\lambda_{ij}$ and $\eta_i$. By re-arranging terms in equations (4.1) to (4.6), the model becomes:

Max

$$f = \sum_{j=1}^{n} [c_j^- x_j + \lambda_0 (c_j^+ - c_j^-) x_j] \quad (4.7)$$

Subject to

$$\sum_{j=1}^{n} a_{ij}^+ - b_i^- \leq \sum_{j=1}^{n} \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-), \forall i \quad (4.8)$$

$$x_j \geq 0, \forall j \quad (4.9)$$

$$0 \leq \lambda_0 \leq 1 \quad (4.10)$$

$$0 \leq \lambda_{ij} \leq 1, \forall i, j \quad (4.11)$$

$$0 \leq \eta_i \leq 1, \forall i \quad (4.12)$$

Let $d = \lambda_0 (c_j^+ - c_j^-) x_j$, and $e = \sum_{j=1}^{n} \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-)$, where *i=1, 2, ..., m*, the model can be reformulated as:

Max

$$f = \sum_{j=1}^{n} [c_j^- x_j + d] \quad (4.13)$$

Subject to

$$\sum_{j=1}^{n} a_{ij}^+ - b_i^- \leq e, \forall i \quad (4.14)$$

$$x_j \geq 0, \forall j \quad (4.15)$$

In the model above (4.13 to 4.15), when *d* and *e* equal to 0, it becomes the worst case sub-model of the BWC algorithm. It represents the most pessimistic situation of the original objective function, indicating that the optimal solutions generated under this condition would have zero risk of violating any of the constraints, due to its formulation satisfying the tightest constraints (i.e., risk = 0 when *d* and *e* equal to zero).

On the other hand, when $e$ is greater than zero, the constraints are relaxed by a level of *e* to search for its corresponding optimal solutions to achieve higher system return. However, in the meantime, the optimal solutions come along with a certain level of risk of violating the constraints as well. Accordingly, higher $e$ means that the optimal solutions associated with higher risk. When both of $\lambda_{ij} = 1\,(\forall i, j)$ and $\eta_i = 1\,(\forall i)$ , *e* reaches its maximum value, and the model represents the most optimistic situation and the decision risks might be highest. Hence, *e* $(\forall i)$ can be used to evaluate the risk level of a decision to represent its possibility of violating the corresponding constraints.

**Definition 4.1:** Function $e = \sum_{j=1}^{n} \lambda_{ij}\,(a_{ij}^{+} - a_{ij}^{-})x_j + \eta_i(b_i^{+} - b_i^{-})$ is defined as the Risk Function for the constraint *i* in the ILP method.

The original ILP model is to maximize the system return. Since the system return and decision risk represent two conflicting factors in practical decision making process, a sound and satisfactory decision can be obtained only through minimizing the risk function while maximizing the system return. This leads to a multi-objective optimization problem:

Max

$$f = \sum_{j=1}^{n} c_j^{-} x_j + d \tag{4.16}$$

Min

$$\xi = \oplus_i[\sum_{j=1}^{n} \lambda_{ij}\,(a_{ij}^{+} - a_{ij}^{-})x_j + \eta_i(b_i^{+} - b_i^{-})] \tag{4.17}$$

Subject to

$$\sum_{j=1}^{n} a_{ij}^{+} x_j - b_i^{-} \leq \xi_i, \forall i \tag{4.18}$$

$$x_j \geq 0, \forall j \tag{4.19}$$

In the model above, $\oplus$ indicates a general arithmetic operator (i.e., simple addition or weighted addition or simple arithmetic mean or weighted arithmetic mean). The subscript for $\oplus_i$, $i$, suggests that the operator be applied across constraints to obtain a unified risk function for the entire optimization problem. $\lambda_{ij}$ and $\eta_i$ are model variables, while $0 \leq \lambda_{ij}, \eta_i \leq 1$. For each individual constraint, the constraint-wise risk function $\sum_{j=1}^{n} \lambda_{ij} (a_{ij}^{+} - a_{ij}^{-}) x_j + \eta_i (b_i^{+} - b_i^{-})$ can differ from that of another constraint by order of magnitude due to different categories of $b_i$ as well as the incorporation of interactions among $\lambda_{ij}$, $a_{ij}^{+}$, $a_{ij}^{-}$, $x_j$, $\eta_i$, $b_i^{+}$ and $b_i^{-}$ in the function. Hence, it requires to convert the constraint-wise risk function into comparable magnitude (Yong et al. 2010). And a simple method through scaling each constraint-wise risk function by $1/b_i^{-}$ can be a feasible choice, which actually indicates a fractional risk factor from the most pessimistic case (Hua et al., 2010). For practical cases, more refined approaches can be developed in order to better reflect different decision environments.

The above multi-objective optimization model (4.16 to 4.19) can then be reformulated into a general optimization model through minimizing the single objective risk function:

Min

$$\xi = \oplus_i \left[\sum_{j=1}^{n} \lambda_{ij} (a_{ij}^{+} - a_{ij}^{-}) x_j + \eta_i (b_i^{+} - b_i^{-})\right] \tag{4.20}$$

Subject to

$$\sum_{j=1}^{n} (c_j^{-} x_j + d) \geq f_{opt}^{-} + \lambda_0 (f_{opt}^{+} - f_{opt}^{-}), \forall i \tag{4.21}$$

$$\sum_{j=1}^{n} a_{ij}^{+} x_j - b_i^{-} \leq \xi_i, \forall i \tag{4.22}$$

$$\lambda_0 = \lambda_{pre} \tag{4.23}$$

$$0 \leq \lambda_{ij} \leq 1 \tag{4.24}$$

$$x_j \geq 0, \forall j \tag{4.25}$$

$$0 \leq \eta_i \leq 1, \forall i \tag{4.26}$$

**Definition 4.2:** The developed risk-minimization optimization model (4.20 to 4.26) is derived from the original ILP model and defined as the Risk Explicit Interval-parameter Linear Programming (REILP) model.

In the REILP model, $\lambda_0$ is defined as the aspiration level preset by the decision maker, indicating the degree of aggressiveness, or in other words, the aspiration level of decision makers given the uncertainties in the optimization model. It takes the value between 0 to 1. Normally, when $\lambda_0 = 0$, the model represents the least aggressive case and the most conservative and safest solutions will be obtained without facing any risks of violating the constraints. However, when $\lambda_0 = 1$, it represents the most aggressive situation, and the most optimistic but risky solutions will be obtained, and the solutions have the highest risks of violating some or all of the constraints. In practical cases, the decision makers usually prefer to choose the value of $\lambda_0$ between 0 and 1, to find the optimal solutions with least risk level but with a desired degree of aggressiveness.

The risk-minimization REILP model (4.20 to 4.26) is a non-linear programming model. Its non-linearity is produced by introducing the risk level variables (i.e., $\lambda_0$ or $\lambda_{ij}$) to reflect the intricate non-linear interactions of uncertainties between different variables and terms in a constraint (Pei, 2011). If a large $\lambda_{ij}$ is associated with a small $x_j$, the large $\lambda_{ij}$ would have small contribution to the risk in the decision. On the other hand, if the $\lambda_{ij}$ is associated with a large $x_j$ it would result in significant contribution to the overall risk of the decisions.

## 4.2 THE ASPIRATION LEVEL

As described above, the REILP model could provide the decision makers more reliable and practical implementation schemes through minimizing the decision risks while maximizing the system return. The improvement over the two-step algorithm or BWC algorithm is that the risks associated with the possible optimal solutions and decisions derived from them would be explicitly incorporated into the decision making process (Pei, 2011). In the REILP, the preset of aspiration level $\lambda_0$ needs to be further discussed.

In the REILP model, the objective function of the original ILP model was converted into a constraint (Rui et al. 2010):

$$\sum_{j=1}^{n}( c_j^- x_j + d) \geq f_{opt}^- + \lambda_0(f_{opt}^+ - f_{opt}^-) \quad (4.27)$$

Where,

$$d = \lambda_0(c_j^+ - c_j^-)x_j \quad (4.28)$$

Then we can get:

$$\sum_{j=1}^{n}[ (c_j^- + \lambda_0(c_j^+ - c_j^-))x_j] \geq f_{opt}^- + \lambda_0(f_{opt}^+ - f_{opt}^-) \quad (4.29)$$

In equation (4.29), the aspiration level $\lambda_0$ appears in both sides of this inequality constraint and represents the preference of the decision maker and needs to be preset. In this constraint, it is assumed that the system return coefficient $c_j$ has the same changing rate (i.e., $\lambda_0$ ) from its lower bound as $f_{opt}$ (i.e., $(c_j^+ - c_j^-)$ and $(f_{opt}^+ - f_{opt}^-)$). However, this is not always true in a real-world decision making problem, and $c_j$ and $f_{opt}$ might take different rates changing with their own intervals. A better formulation for the inequality (4.29) should be:

$$\sum_{j=1}^{n}[ (c_j^- + \lambda_j(c_j^+ - c_j^-))x_j] \geq f_{opt}^- + \lambda_0(f_{opt}^+ - f_{opt}^-) \quad (4.30)$$

Where, $\lambda_j$ is the new specific changing rate for $c_j$, while $j = 1, 2, 3, ..., n$. Meanwhile, $\lambda_j$ and $\lambda_o$ take different values in most cases.

In general, the aspiration level ($\lambda_o$) is capable of reflecting the relationship between the system return and the corresponding decision risks. In real-world problems, the decision makers usually choose different values of the aspiration level based on their own needs or the available resources.

## 4.3 THE RISK FUNCTION

In the REILP model, the risk function was formulated as:

$$e = \sum_{j=1}^{n} \lambda_{ij} (a_{ij}^{+} - a_{ij}^{-}) x_j + \eta_i (b_i^{+} - b_i^{-}) \tag{4.31}$$

This risk objective function is able to reflect the risks of violating the constraints of $a_{ij}$ and $b_i$. However, it is not able to reflect the risk of violating the constraint of $c_j$, and it cannot directly reflect the relation between the risk function and the aspiration levels as well (Pei, 2011).

Furthermore, when the original objective function of the ILP model was converted into a constraint in the REILP model, the risk of violating this constraint should also be considered. Usually, if the decision maker chooses a higher aspiration level, it means that the decision maker prefers an aggressive decision and a higher system return, and the risk level of the entire system will be elevated. In general, the risk function without considering the risk level of parameters $c_j$ and aspiration level $\lambda_0$ is not sufficient.

## 4.4 SOLUTION PROCESS OF THE REILP MODEL

A complete solution process for solving the REILP model is presented below (Rui et al. 2008):

Step 1: Use the BWC algorithm to reformulate the original ILP model into two sub-models and solve two sub-models to get the upper and lower bounds of the objective function.

Step 2: Generate a risk minimization model based on the REILP modeling approach by using the solutions of objective function obtained in Step 1.

Step 3: Following Step 2, solve the model for a series of prescribed, discrete aspiration levels to obtain the corresponding optimal solutions for the optimal risk levels and decision variables, and the optimal solutions allow the prescribed discrete aspiration levels to be achieved with respectively minimized risk levels.

Step 4: Normalize the obtained risk levels such that the Normalized Risk Level (NRL) equals to 0 for the most pessimistic condition and 1 for the most optimistic condition.

Step 5: Plotting the optimal NRL against the corresponding aspiration level would facilitate the understanding of the relationship between decision risk and system return. The decision makers are then in a desirable position to make decent and sound decisions based on their interpretation of risks and aspiration preferences.

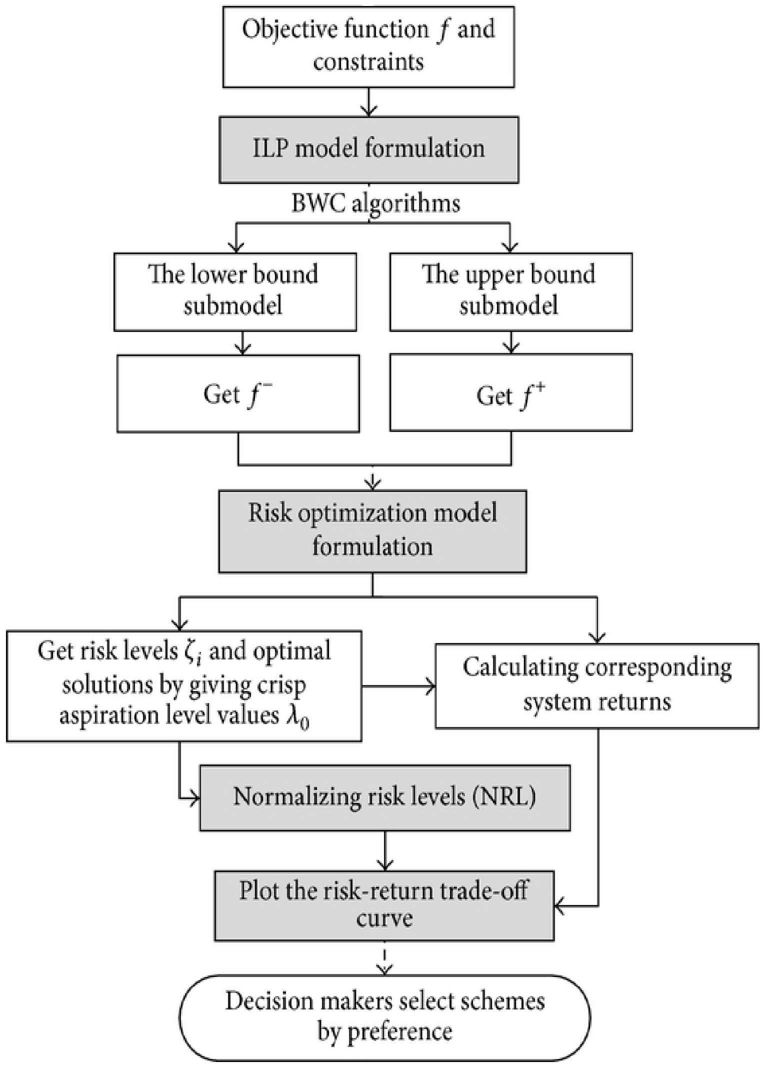


*Figure 4.1 Algorithmic process of REILP model*

*(Source: Zhang et al., 2014)*

The REILP approach improves the traditional ILP method through obtaining crisp solutions at each desired aspiration level (or risk tolerance level) for directly formulating implementation schemes. Hence, the solutions can be adopted by the decision makers directly for making decisions with explicit risk-reward trade-off information.

## 4.5 THE NUMERICAL EXAMPLE AND DISCUSSION

A numerical example is given below to illustrate the developed REILP approach through demonstrating the optimal decision making on land-use management for nutrient loading control and maximum profits gain.

In this hypothetical case, a factory manufactures two different products, denoted as $X_1$ and $X_2$ , by using Machine A and Machine B. The producing time of each unit of $X_1$is 38 min to 42 min for Machine A; 28 min to 33 min for Machine B. Meanwhile, it takes 22 min to 26 min by using Machine A and 30 min to 35 min by using Machine B to produce each unit of $X_2$. Usually, Machine A can be used 135 hours to 140 hours per week. Machine B can work 140 hours to 161 hours per week. For the coming week, there is a stock of 90 units of $X_1$ and 108 units of $X_2$ now. The weekly demand is at least 75 units of $X_1$ and 95 units of $X_2$. In addition, the net benefits of each unit of $X_1$ and $X_2$ are [$72, $78] and [$80, $88], respectively. The manufacturer desires a planning scheme on the production of both products in order to maximize the weekly stock and net profit at the same time.

This manufactural planning and management problem could be formulated as the following ILP model:

Max

$$f = [72, 78] * (X_1 + 90 - 75) + [80, 88] * (X_2 + 108 - 95) \quad (4.32)$$

After re-arranging the terms in equation (4.32), the objective function becomes:

Max

$$f = [1080, 1170] * X_1 + [1040, 1144] * X_2 \quad (4.33)$$

Subject to

$$[30, 32] * X_1 + [22, 26] * X_2 \leq [135, 140] * 60 \quad (4.34)$$

$$[28, 33] * X_1 + [30, 35] * X_2 \leq [140, 161] * 60 \quad (4.35)$$

$$X_1 \geq 0 \quad (4.36)$$

$$X_2 \geq 0 \quad (4.37)$$

By implementing the BWC algorithm, we can get its two sub-models as follows:

(1) Best-case sub-model:

Max

$$f^+ = 1170 * X_1 + 1144 * X_2 \quad (4.38)$$

Subject to

$$30 * X_1 + 22 * X_2 \leq 140 * 60 \quad (4.39)$$

$$28 * X_1 + 30 * X_2 \leq 161 * 60 \quad (4.40)$$

$$X_1 \geq 0 \quad (4.41)$$

$$X_2 \geq 0 \quad (4.42)$$

(2) Worst-case sub-model:

Max

$$f^- = 1080 * X_1 + 1040 * X_2 \quad (4.43)$$

Subject to

$$32 * X_1 + 26 * X_2 \leq 135 * 60 \quad (4.44)$$

$$33 * X_1 + 35 * X_2 \leq 140 * 60 \quad (4.45)$$

$$X_1 \geq 0 \quad (4.46)$$

$$X_2 \geq 0 \quad (4.47)$$

Solving these two sub-models in Lingo, we could get the optimal solutions as follows:

$f^{\pm}$ = [274120, 382278]

$X_1^{\pm}$ = [139, 249]

$X_2^{\pm}$ = [5, 192]

According to the REILP approach, this ILP model can be converted into a risk explicit ILP model:

Max

$$1080 * X_1 + 1040 * X_2 + r_0(1170 - 1080) * X_1 + r_0(1144 - 1040) * X_2 \qquad (4.48)$$

Min

$$\xi = \begin{array}{l} r_1(32-30)X_1/8100 + r_2(26-22)X_2/8100 + r_3(8400-8100)/8100 \\ +r_4(33-28)X_1/8400 + r_5(35-30)X_2/8400 + r_6(9660-8400)/8400 \end{array} \qquad (4.49)$$

Subject to

$$[1080 + r_0(1170 - 1080)]X_1 + [1040 + r_0(1144 - 1040)]X_2 \geq 274120 + r_0(382278 - 274120) \qquad (4.50)$$

$$32X_1 + 26X_2 - 8100 \leq r_1(32 - 30)X_1 + r_2(26 - 22)X_2 + r_3(8400 - 8100) \qquad (4.51)$$

$$33X_1 + 35X_2 - 8400 \leq r_4(33 - 28)X_1 + r_5(35 - 30)X_2 + r_6(9660 - 8400) \qquad (4.52)$$

$$X_1, X_2 \geq 0 \qquad (4.53)$$

$$0 \leq r_0, r_1, r_2, r_3, r_4, r_5, r_6 \leq 1 \qquad (4.54)$$

Where, $r_0$ is the aspiration level preset by the decision maker.

The solutions of the above model are presented in Table 4.1.

Table 4.1 Optimal solutions under different aspiration levels

| Aspiration Level ($r_0$) | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Profit ($) | 274120 | 285257 | 295811 | 306660 | 317807 | 328290 | 339037 | 349842 | 360662 | 371472 | 382278 |
| $X_1$ (unit) | 249 | 206 | 167 | 128 | 89 | 54 | 19 | 14 | 4 | 70 | 139 |
| $X_2$ (unit) | 5 | 58 | 106 | 154 | 202 | 245 | 288 | 300 | 317 | 256 | 192 |
| Risk Function | 0.00 | 0.05 | 0.10 | 0.14 | 0.19 | 0.23 | 0.27 | 0.32 | 0.37 | 0.44 | 0.51 |

According to Section 4.2, $c_j$ and $f_{opt}$ may have different changing rates rather than the same $\lambda_0$. Hence, in order to improve this current risk function, we introduce the specific $\lambda_j$ into the REILP model, and a better formulation for the constraint (4.50) should be:

$$\sum_{j=1}^{n}[(c_j^- + \lambda_j(c_j^+ - c_j^-))x_j] \geq f_{opt}^- + \lambda_0(f_{opt}^+ - f_{opt}^-) \tag{4.55}$$

By introducing the specific $\lambda_j$ into the current example model, we will use $r_7$ for $X_1$, $r_8$ for $X_2$. The new risk function will be re-formulated as follows:

Min

$$\begin{aligned}\xi = \; & r_1(32-30)X_1/8100 + r_2(26-22)X_2/8100 + r_3(8400-8100)/8100 \\ & + r_4(33-28)X_1/8400 + r_5(35-30)X_2/8400 + r_6(9660-8400)/8400\end{aligned} \tag{4.56}$$

Subject to

$$[1080 + r_7(1170-1080)]X_1 + [1040 + r_8(1144-1040)]X_2 \geq 274120 + r_0(382278 - 274120) \tag{4.57}$$

$$32X_1 + 26X_2 - 8100 \leq r_1(32-30)X_1 + r_2(26-22)X_2 + r_3(8400-8100) \tag{4.58}$$

$$33X_1 + 35X_2 - 8400 \leq r_4(33-28)X_1 + r_5(35-30)X_2 + r_6(9660-8400) \tag{4.59}$$

$$X_1, X_2 \geq 0 \tag{4.60}$$

$$0 \leq r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8 \leq 1 \tag{4.61}$$

Where, $r_0$ is the preset aspiration level defined by the decision maker.

The above model was solved by LINGO, and the solutions were presented in Table 4.2.

Table 4.2 New optimal solutions under different aspiration levels

| Aspiration Level ($r_0$) | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $r_1$ | 0.90 | 0 | 0 | 0.08 | 0 | 0 | 0 | 0.04 | 0.13 | 0.49 | 0.98 |
| $r_2$ | 0 | 0 | 0.41 | 0.14 | 0 | 0.08 | 0 | 0.04 | 0.13 | 0.23 | 1 |
| $r_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.70 | 1 |
| $r_4$ | 0.50 | 0 | 0 | 0.24 | 0.39 | 0.49 | 0.49 | 0.50 | 0.50 | 0.10 | 1 |
| $r_5$ | 0 | 0 | 0.12 | 0.46 | 0.42 | 0 | 0.48 | 0.73 | 0.95 | 0.10 | 1 |
| $r_6$ | 0 | 0 | 0 | 0 | 0.25 | 0.77 | 0.85 | 0.90 | 0.90 | 0.10 | 1 |
| $r_7$ | 0.50 | 0.84 | 1 | 1 | 0.99 | 0.10 | 0.96 | 0.99 | 0.98 | 1 | 1 |
| $r_8$ | 0.98 | 0.83 | 0.33 | 0.99 | 0.98 | 1 | 0.10 | 1 | 1 | 1 | 1 |
| $X_1$ (unit) | 0 | 193 | 253 | 220 | 162 | 128 | 71 | 36 | 12 | 31 | 139 |
| $X_2$ (unit) | 240 | 55 | 0 | 43 | 112 | 156 | 224 | 269 | 303 | 293 | 192 |
| Profit ($) | 274120 | 284936 | 296010 | 306567 | 317383 | 328199 | 339015 | 349831 | 360646 | 371462 | 382278 |
| Risk Function | 0 | 0 | 0 | 0.05 | 0.10 | 0.16 | 0.21 | 0.27 | 0.33 | 0.41 | 0.51 |

The results show that the values of aspiration level $\lambda_0$ and $r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8$ are all in the reasonable range from 0 to 1. Besides, the values of profit and the risk function have rational relationship. If the profit increases, the corresponding risk level will increase simultaneously.

## 4.6 IMPROVEMENT OF THE NUMERICAL EXAMPLE

In Section 4.3, the risk objective function of the REILP model was formulated as:

$$\xi = \sum_{j=1}^{n} \lambda_{ij} (a_{ij}^{+} - a_{ij}^{-})x_j + \eta_i(b_i^{+} - b_i^{-}) \tag{4.62}$$

This risk objective function can reflect the risks of violating the constraints of $a_{ij}$ and $b_j$. However, it cannot reflect the risk of violating the constraint of $c_j$, and it cannot reflect the relationship between the risk function and the aspiration level $\lambda_0$ as well (Pei, 2011).

When the objective function of the original ILP model was converted into a constraint of the REILP model, the risk of violating this constraint should also be considered. In order to take this into account, an improved risk objective function could be formulated as follows:

The original risk function is formulated as:

$$\xi_i = \sum_{j=1}^{n} \lambda_{ij} (a_{ij}^{+} - a_{ij}^{-})x_j + \eta_i(b_i^{+} - b_i^{-}) \tag{4.63}$$

$$\xi = \oplus_i[\sum_{j=1}^{n} \lambda_{ij} (a_{ij}^{+} - a_{ij}^{-})x_j + \eta_i(b_i^{+} - b_i^{-})] \tag{4.64}$$

This risk function was converted into a constraint as:

$$\sum_{j=1}^{n}[c_j^{-} + \lambda_j(c_j^{+} - c_j^{-})x_j] \geq f_{opt}^{-} + \lambda_0(f_{opt}^{+} - f_{opt}^{-}) \tag{4.65}$$

An improved risk function with considering the risk of violating the above constraint can then be formulated as:

$$\xi = \oplus_i\left[\sum_{j=1}^{n} \lambda_{ij} \left(a_{ij}^{+} - a_{ij}^{-}\right)x_j + \eta_i(b_i^{+} - b_i^{-})\right] + \oplus_k\left[\sum_{j=1}^{n} \lambda_j \left(c_j^{+} - c_j^{-}\right)x_j + \lambda_0(f_{opt}^{+} - f_{opt}^{-})\right] \quad (4.66)$$

If we scale the risk function by：

$\oplus_i = 2/(b_i^{+} + b_i^{-})$

$\oplus_k = 2/(f_{opt}^{+} + f_{opt}^{-})$

Then we can get:

$$\xi = \sum_{i=1}^{m} \frac{2}{b_i^{+} + b_i^{-}} \left[\sum_{j=1}^{n} \lambda_{ij} \left(a_{ij}^{+} - a_{ij}^{-}\right)x_j + \eta_i(b_i^{+} - b_i^{-})\right] + \frac{2}{f_{opt}^{+} + f_{opt}^{-}}\left[\sum_{j=1}^{n} \lambda_j \left(c_j^{+} - c_j^{-}\right)x_j + \lambda_0\left(f_{opt}^{+} - f_{opt}^{-}\right)\right] \quad (4.67)$$

By using the new risk objective function (4.67) to solve the model by LINGO, the new optimal solutions can be obtained (as presented in Table 4.3).

Table 4.3 Improved optimal solutions under different aspiration levels

| Aspiration Level ($r_0$) | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $r_1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.52 | 0.58 | 0.98 |
| $r_2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.09 | 0.15 | 1 |
| $r_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| $r_4$ | 0 | 0 | 0 | 0.32 | 0.40 | 0.40 | 0.49 | 0.50 | 0.50 | 1 | 1 |
| $r_5$ | 0 | 0 | 0 | 0.34 | 0.43 | 0.36 | 0.48 | 0.76 | 1 | 1 | 1 |
| $r_6$ | 0 | 0 | 0 | 0 | 0.24 | 0.64 | 0.84 | 0.90 | 0.86 | 1 | 1 |
| $r_7$ | 0 | 0.48 | 0.95 | 0.98 | 0.99 | 0.95 | 0.98 | 0.60 | 0.48 | 1 | 1 |
| $r_8$ | 0 | 0.09 | 0.50 | 0.94 | 0.98 | 0.98 | 0.99 | 1 | 0.97 | 1 | 1 |
| $X_1$ (unit) | 249 | 249 | 249 | 206 | 163 | 115 | 72 | 24 | 0 | 31 | 139 |
| $X_2$ (unit) | 5 | 5 | 5 | 58 | 111 | 170 | 223 | 282 | 316 | 293 | 192 |
| Profit ($) | 274120 | 284936 | 295752 | 306567 | 317383 | 328199 | 339015 | 349831 | 360646 | 371462 | 382278 |
| Risk Function | 0 | 0.06 | 0.13 | 0.22 | 0.30 | 0.39 | 0.48 | 0.57 | 0.66 | 0.77 | 0.90 |

# CHAPTER 5 FUZZY REILP APPROACH (FREILP)

## 5.1 INTRODUCTION

In the REILP model, $\lambda_0$ is the aspiration level preset by the decision makers, representing the degree of aggressiveness and the preference of the decision makers. Therefore, the aspiration level $\lambda_0$ is uncertain. Normally, when $\lambda_0 = 0$, the model represents the least aggressive case and the most conservative also safest solutions will be obtained without facing any risks of violating the constraints; when $\lambda_0 = 1$, the model represents the most aggressive situation, and the most optimistic solutions will be obtained, which comes along with the highest risk of violating some or all of the constraints. Usually, in the practical cases, the decision makers would prefer to choose the value of $\lambda_0$ between 0 and 1, comparing to these two extreme $\lambda_0$ values (0 or 1), to make the decision risk and the system return in a preset degree of balance.

However, in real-world cases, it is hard for the decision makers or the stakeholders to preset the aspiration level at a specific value, or they may have different respective preferences. Therefore, in order to preset the aspiration levels more practical for real world applications, Fuzzy Risk Explicit Interval Linear Programming (FREILP) was developed in response to this need.

## 5.2 INTRODUCTION TO FUZZY SET THEORY

The fuzzy set theory was first proposed as an extension of the classical notion of set (Zadeh and Klaua, 1965). Comparing to the classical set, which has only binary forms to reflect the object either belonging to or not belonging to the set, the fuzzy set becomes more realistic and humanized for practical problems by allowing the gradual assessment of the membership of the object in the set. In Zadeh's research, the fuzzy set actually includes the classical set because the situations of the classical set can be seen as the special situations in the membership function of the fuzzy set as it only takes the value of 0 or 1. Since then, the fuzzy set has been introduced in various areas from mathematics to engineering.

**Definition 5.1:** In general, if we let $X$, a collection of objects, denoted by $x$, the fuzzy set $\widetilde{A}$ in $X$ is a set of ordered pairs as:

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) \mid x \in X\} \quad (5.1)$$

In function 5.1, $\mu_{\widetilde{A}}(x)$ is defined as the membership function of $x$ in $\widetilde{A}$, while it maps $X$ to the membership space $M$. Normally, the range of this membership function is a subset of the non-negative real numbers whose supreme is finite. Meanwhile, the elements will not be listed if they have zero degree of the membership function (Pei, 2011).

Zadeh (1975) and Zimmermann (1985) developed the $\alpha$-level set theorem, which becomes a critical part of the fuzzy set theory and its application. This method generates the crisp sets which associate with their corresponding different $\alpha$ level. It also represents the distinct grades of membership function.

**Definition 5.2:** The $\alpha$-level set (all called α-cut) is defined as follows (Zadeh, 1973):

$$A_\alpha = \{(x \in X), \mid \mu_{\widetilde{A}}(x) \geq \alpha\} \quad (5.2)$$

The $\alpha$-cut set is obtained by drawing a horizontal line of the $\alpha$-level value on the membership function curve. The curve above the line includes all the elements in the $\alpha$-level set (also called $\alpha$-cut). Figure 5.1 presents the membership function curve for an example fuzzy set (i.e., real numbers close to 10). When $\alpha$-cut = 0.5, the horizontal line cuts the membership function curve into two parts at two intersection points of x = 9 and x = 11, respectively. The membership grades for all the real numbers in between 9 and 11 are greater than 0.5.

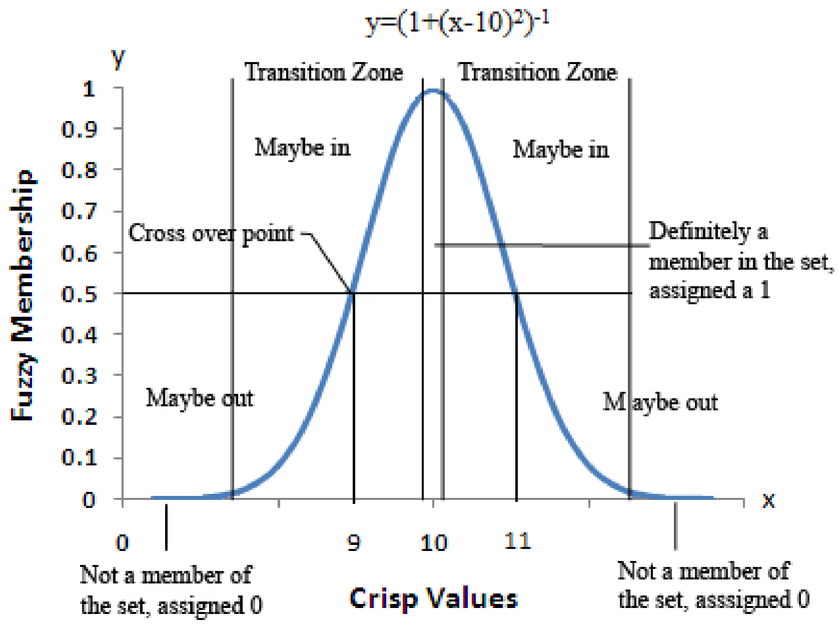


*Figure 5.1 An α-cut example for the fuzzy set of real numbers close to 10*

## 5.3 MEMBERSHIP FUNCTIONS OF THE FUZZY ASPIRATION LEVEL IN THE REILP MODELING

In order to account for the fuzzy nature associated with the aspiration level, in this study, three levels of the aspiration value ($\lambda_0$) were considered, including aggressive aspiration level, medium aspiration level and conservative aspiration level. The development of their membership functions are based on the "young people" membership function given in Equation 5.3. Equations 5.4, 5.5 and 5.6 are the membership functions for the conservative, medium, and aggressive aspiration levels, respectively. Their plots are presented in Figures 5.2, 5.3, and 5.4, respectively.

$$\tilde{A}(x) = \begin{cases} 1, & 0 \leq x \leq 25 \\ \left[1 + \left(\frac{x-50}{5}\right)^{-2}\right]^{-1}, & 25 \leq x \leq 100 \end{cases} \quad (5.3)$$

(a) The membership function of the conservative aspiration level ($\lambda_0$):

$$\tilde{A}(\lambda_0) = \begin{cases} 1, & 0 \leq \lambda_0 \leq 0.25 \\ [1 + (4\lambda_0 - 1)^2]^{-1}, & 0.25 \leq \lambda_0 \leq 1 \end{cases} \tag{5.4}$$

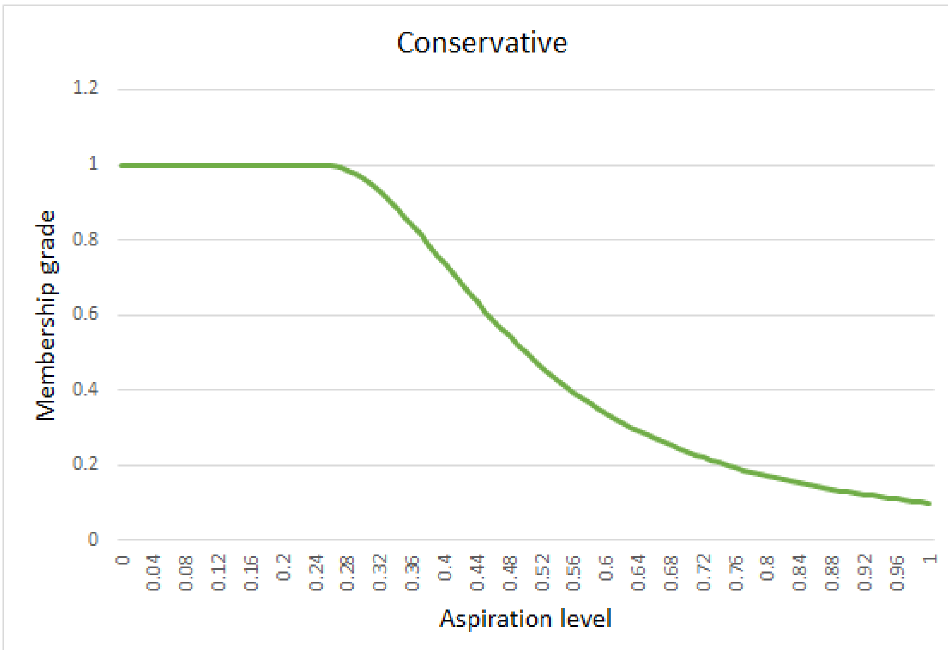


*Figure 5.2 The plot of the membership function of the conservative aspiration level*

(b) The membership function and plot of the medium aspiration level ($\lambda_0$):

$$\tilde{B}(\lambda_0) = \begin{cases} [1 + (15 - 40\lambda_0)^2]^{-1}, & 0 \leq \lambda_0 \leq 0.375 \\ 1, & 0.375 \leq \lambda_0 \leq 0.625 \\ [1 + (40\lambda_0 - 25)^2]^{-1}, & 0.625 \leq \lambda_0 \leq 1 \end{cases} \tag{5.5}$$

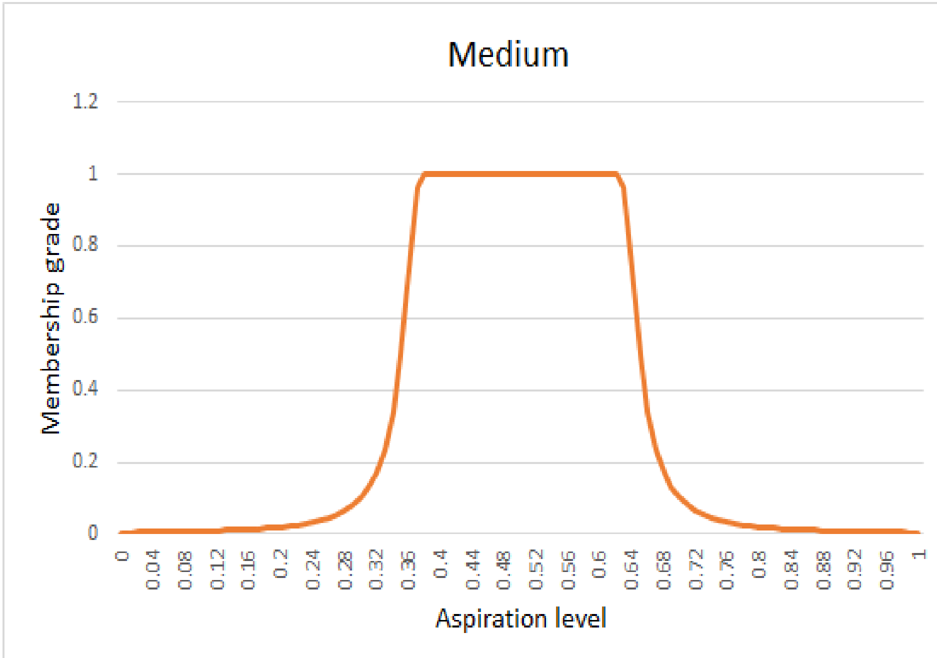


*Figure 5.3 The plot of the membership function of the medium aspiration level*

(c) The membership function and plot of the aggressive aspiration level ($\lambda_0$):

$$\widetilde{C}\,(\lambda_0) = \begin{cases} [1 + (15 - 20\lambda_0)^2]^{-1}, & 0 \leq \lambda_0 \leq 0.75 \\ 1, & 0.75 \leq \lambda_0 \leq 1 \end{cases} \tag{5.6}$$

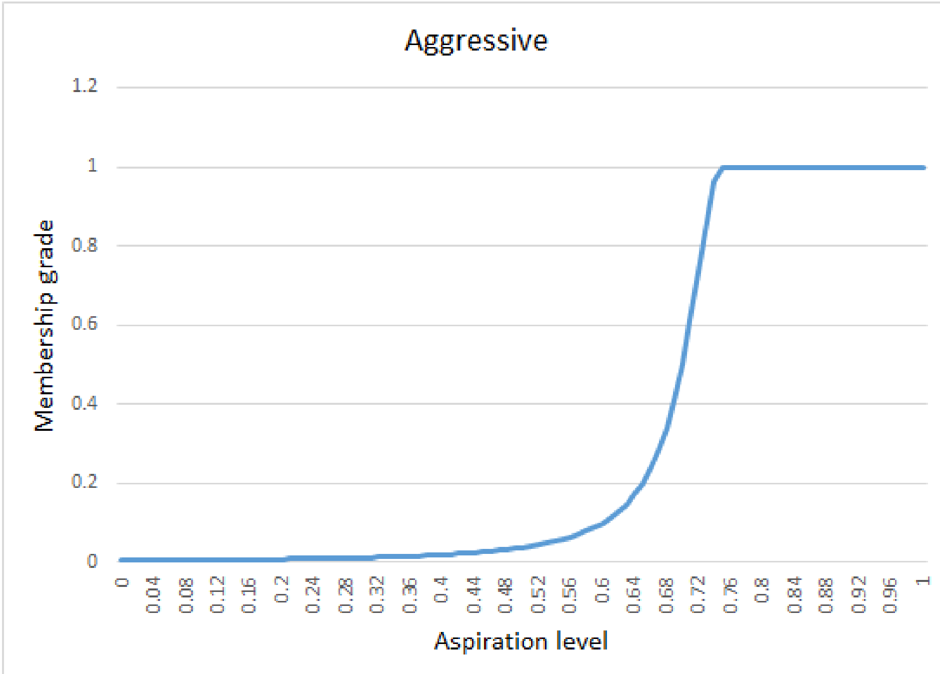


*Figure 5.4 The plot of the membership function of the aggressive aspiration level*

## 5.4 FREILP MODEL FORMULATION

With the defined fuzzy aspiration levels above, the FREILP model can be formulated as follows:

(1) For a maximization problem:

Max

$$f^{\pm} = \sum_{j=1}^{n} c_j^{\pm} x_j^{\pm} \tag{5.7}$$

Subject to

$$\sum_{j=1}^{n} a_{ij}^{\pm} x_j^{\pm} \leq b_i^{\pm}, \forall i \tag{5.8}$$

$$x_j^{\pm} \geq 0, \forall j \tag{5.9}$$

In the FREILP model, the objective is to minimize the risk of violating any of the constraints. So the model can be re-formulated as:

Min

$$\xi = \sum_{i=1}^{m} \frac{2}{b_i^+ + b_i^-} [\sum_{j=1}^{n} \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-)] + \frac{2}{f_{opt}^+ + f_{opt}^-} [\sum_{j=1}^{n} \lambda_j (c_j^+ - c_j^-) x_j + \lambda_0 (f_{opt}^+ - f_{opt}^-)] \quad (5.10)$$

Subject to

$$\sum_{j=1}^{n} [c_j^- + \lambda_j (c_j^+ - c_j^-) x_j] \geq f_{opt}^- + \lambda_0 (f_{opt}^+ - f_{opt}^-) \quad (5.11)$$

$$\sum_{j=1}^{n} a_{ij}^+ x_j - b_i^- \leq \sum_{j=1}^{n} \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-), \forall i \quad (5.12)$$

$$0 \leq \lambda_{ij}, \lambda_j, \eta_i \leq 1 \quad (5.13)$$

$$x_j \geq 0, \forall j \quad (5.14)$$

(2) For a minimization problem:

Min

$$f^{\pm} = \sum_{j=1}^{n} c_j^{\pm} x_j^{\pm} \quad (5.15)$$

Subject to

$$\sum_{j=1}^{n} a_{ij}^{\pm} x_j^{\pm} \geq b_i^{\pm}, \forall i \quad (5.16)$$

$$x_j^{\pm} \geq 0, \forall j \quad (5.17)$$

The FREILP model for the minimization problem can be re-formulated as follows:

Min

$$\xi = \sum_{i=1}^{m} \frac{2}{b_i^+ + b_i^-} [\sum_{j=1}^{n} \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-)] + \frac{2}{f_{opt}^+ + f_{opt}^-} [\sum_{j=1}^{n} \lambda_j (c_j^+ - c_j^-) x_j + \lambda_0 (f_{opt}^+ - f_{opt}^-)] \quad (5.18)$$

Subject to

$$\sum_{j=1}^{n}[\, c_j^+ - \lambda_j(c_j^+ - c_j^-)x_j] \geq f_{opt}^+ - \lambda_0(f_{opt}^+ - f_{opt}^-) \tag{5.19}$$

$$b_i^+ - \sum_{j=1}^{n} a_{ij}^- \, x_j \geq \sum_{j=1}^{n} \lambda_{ij} \, (a_{ij}^+ - a_{ij}^-)x_j + \eta_i(b_i^+ - b_i^-), \forall i \tag{5.20}$$

$$0 \leq \lambda_{ij}, \lambda_j, \eta_i \leq 1 \tag{5.21}$$

$$x_j \geq 0, \forall j \tag{5.22}$$

In both of the FREILP models above, the aspiration levels $\lambda_0$ are treated as the fuzzy sets.

# CHAPTER 6 PLANNING AND MANAGEMENT OF TRUCK ALLOCATION SYSTEM FOR AN OPEN-PIT MINE CASE STUDY

In the surface mining industry, truck and shovel are the critical components in the entire operation system. Subsequently, the associated costs from truck and shovel technology plays a dominant role in the total mining operating cost. In terms of the distributions of operating costs among the relevant activities in open-pit mining production cycle, loading and hauling costs could take up to 70% of the total system cost (Ercelebi and Bascetin, 2009). In the past, the mining companies usually implement the truck dispatching system through heuristic rules or the dispatchers' experiences to allocate the truck resources based on their needs or production requirements. The heuristic rule may work well for small mining operations, however, it is usually not good enough for large and complicated mining operations in most real-world cases. Therefore, more effective methods through combining the mathematical programming models and the heuristic rules into a general modeling framework under uncertain decision-making environment is desired for mining operators to develop a sound and optimal truck allocation scheme for achieving minimized operating costs and thus potentially maximizing the mining profits.

In this study, both REILP and FREILP modeling approaches are applied to the planning and management of the truck allocation system for the Anshun Yalong surface mining project in Guizhou, China. This application will be used not only for testing the applicability of both modeling approaches on the practical case, but also providing the decision makers and the stakeholders with the applicable implementation schemes.

## 6.1. STUDY CASE BACKGROUND

The study case is the Yalong Coal Surface Mining project, located in Anshun, Guizhou Province, China. Surface mining, as its name implies, is a big group of mining categories in which soil and rock overlying the mineral deposit (the overburden) are removed. It is also known as open-pit mining, strip mining and mountaintop removal mining. Surface mining is opposite to underground mining in which drillings are needed to reach the ore layer and the ores are conveyed out via the built tunnels and belts.

Figure 6.1 shows a schematic diagram of a surface coal mining operation.

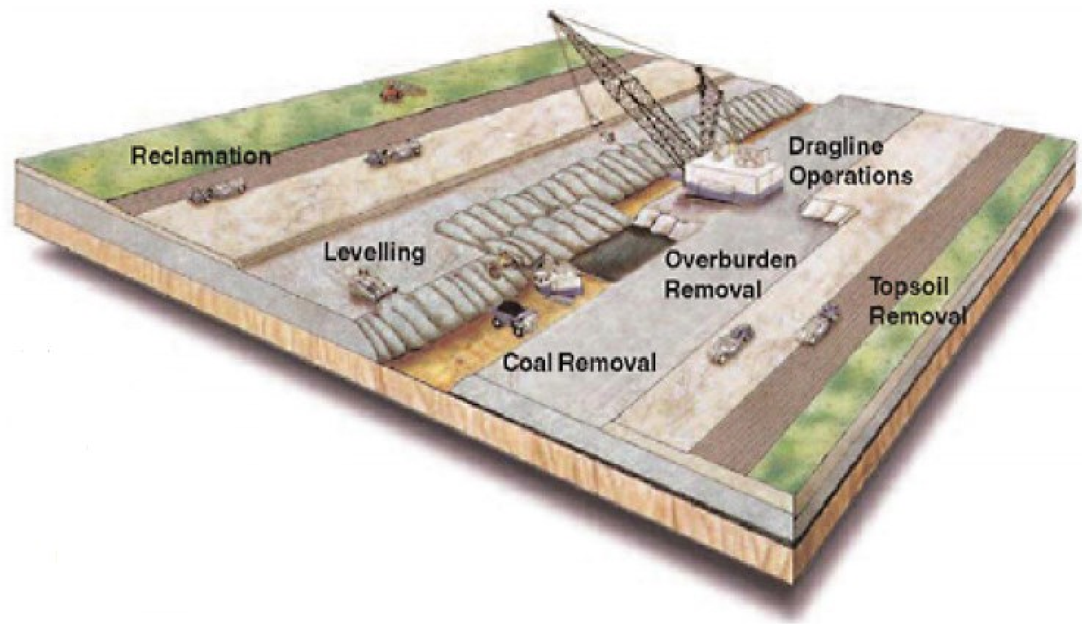


*Figure 6.1 Schematic diagram of an area surface coal mine operations*

*(Source: Royal Utilities, 2012)*

A surface coal mining operation system consists of four phases, i.e., exploration, waste mining, coal mining, and mining camp. The exploration phase is to locate coal seams through trial pits. After the coal seams are located, the overburden materials will be excavated and removed as the wastes from the ground surface by shovel, and then be transported to waste dumping areas by truck, and eventually be conveyed by waste belts to the waste dumping ground. Coal seams and layers now become exposed and are shoveled and transported to the coal crushing areas by trucks. The crushed coal will then be conveyed via coal belts to the "Run of Mine" (ROM) coal surge pile. The coal will be washed and sent to the coal camp afterwards. The entire process is controlled and managed by the operation administration office.

Figure 6.2 shows a schematic diagram of the truck allocation system for the case study. The main components considered in a truck allocation system include coal shovel sites, coal dumping/crushing areas and waste dumping places. For this study case, there are three surface coal shovel sites, two coal dumping/crushing areas, and one waste dumping place. In this project, the coals from two dumping/crushing areas will be conveyed to the next surge pile by using the same belt for cost-saving purpose. As indicated, the distance from three shovels sites to two dumping areas is all same at 4.1 miles. The distance from the shovel sites to the waste crushing place is 5.5 miles.

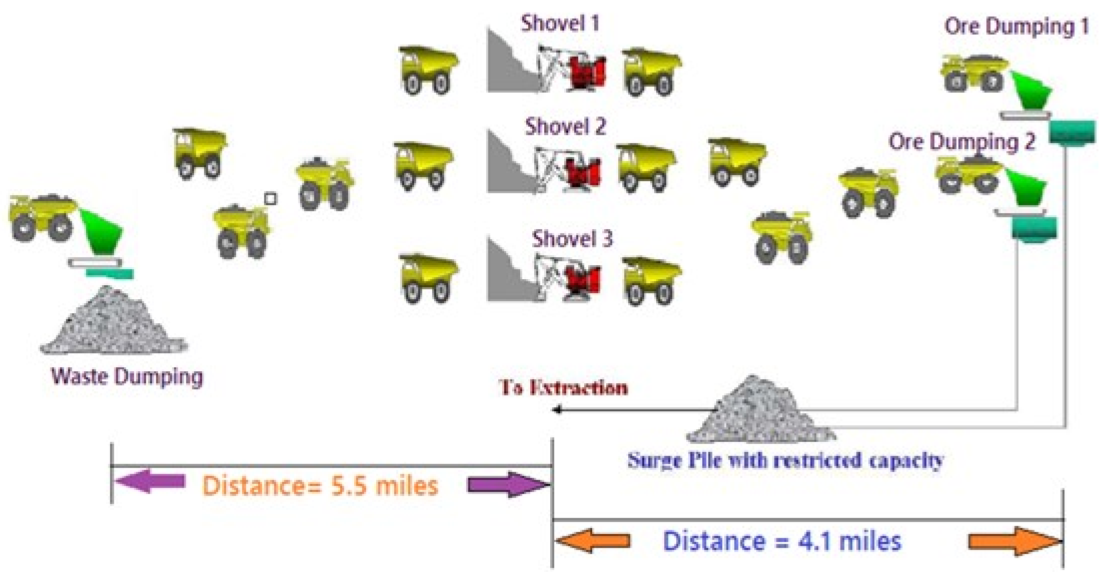


*Figure 6.2 Truck-shovel operation system of Yalong project*

The coal mining and hauling process starts from coal shoveling at coal pit, then load and transport the coal to the ore dumping/crushing areas by trucks, and then convey the crushed coal by the conveyor belt to the coal surge pile. As shown in Figure 6.3, the surge pile is a transitional storage place for the coal materials and acts as a buffer between the later extraction plant (if needed) and the mine production line. Usually, the surge pile is huge enough to feed the extraction plant in order to make the mine production line and the extraction plant to be relatively independent with each other.

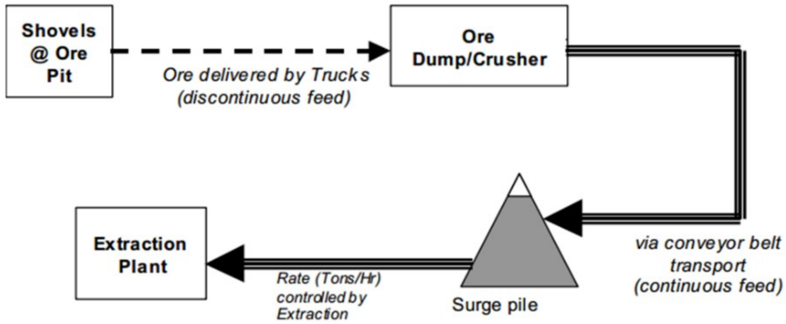


*Figure 6.3 Ore mining flow chart*

*(Source: George, 2002)*

In this study, the truck allocation problem will be formulated as a mixed-integer linear programming model. The model uncertainties will be reflected by the ILP approach. Then, a REILP model will be developed to provide more effective decision support for the decision maker and the stakeholder through considering the balance of decision risk and system benefit. Furthermore, the FREILP model will be formulated to address the preference and aspiration levels of the decision makers for providing more realistic and practical decision support.

## 6.2. MODEL INPUT DATA

Figure 6.4 to Figure 6.7 present a few 3D simulated coal mining picture, including parallel coal shoveling sites, coal dumping process, parallel coal dumping areas, and a coal surge pile.

*Figure 6.4 Parallel coal shoveling sites*

*(Source:  Mycosm - 3D simulation)*

*Figure 6.5 Coal dumping from trucks*

*(Source:  Mycosm - 3D simulation)*


*Figure 6.6 Parallel coal dumping places*

*(Source: JoyGlobal, 2014)*

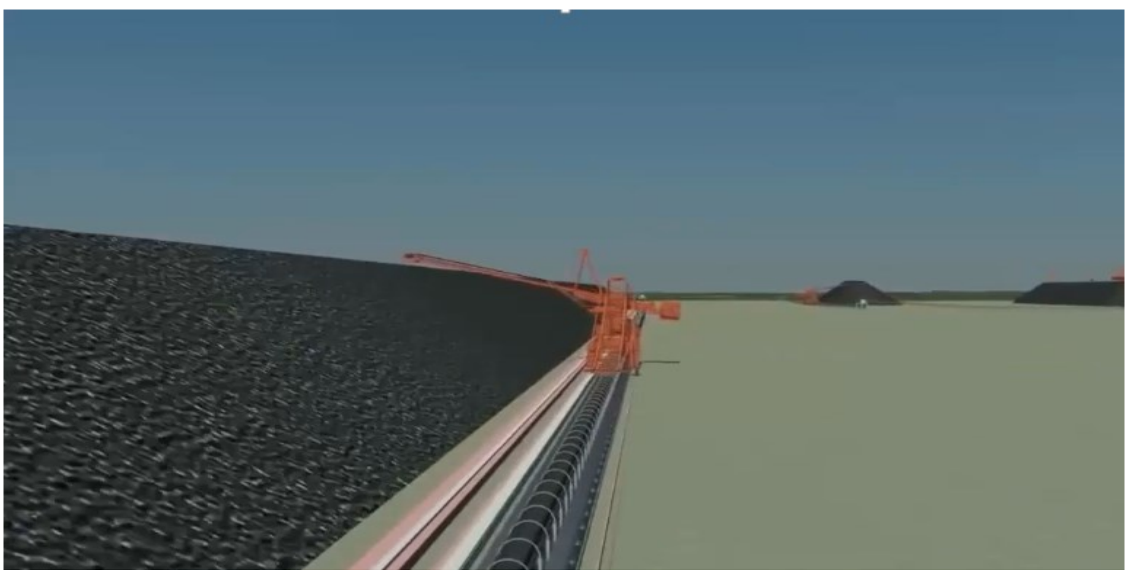

*Figure 6.7 A coal surge pile*

*(Source:  Mycosm - 3D simulation)*

In this study, three types of trucks will be used for coal and waste hauling, with a capacity of 240 Tonnes, 320 Tonnes and 360 Tonnes, respectively. They are denoted by subscript '$j$' ($j = 1$ for 240 Tonnes trucks, $j = 2$ for 320 Tonnes trucks, and $j = 3$ for 360 Tonnes trucks).

Truckload is denoted as '$L$', where $L(s,d,j)$ indicates the coal truckload and $L(s,j)$ indicates the waste truckload while 's' indicating the shovel locations 1, 2 & 3. In the term $L(s,d,j)$, '$d$' indicates the coal dumping place 1 & 2.

Three shovel locations used in this case all have a shoveling capacity of 6,000 Tonnes per hour ($Tph$), which is denoted by $C_{shovel}$. Three shovels are all denoted as '$s$' (where, $s = 1$, $s = 2$ and $s = 3$, representing three shovels accordingly). Two coal dumping places are denoted by '$d$' (where, $d = 1$ and $d = 2$). The coal truck cycle time and the waste truck cycle are denoted by $t(s,d,j)$ and $t(s,j)$, respectively, and they are all expressed at minutes.

In addition, the cost coefficient ($K_j$) is used to indicate the cost of each type of truck. In this study, the truck with larger capacity would cost more. $V_{ore}$ represents the hourly ore hauling rate from the shovel sites to the ore dumping places, while $V_{extraction}$ representing the hourly ore extraction rate from the surge pile at the extraction plant.

The following Table 6.1 shows the important thesis conventions used in this study.

Table 6.1 Important thesis conventions

| Symbols and Abbreviations | Description |
|---|---|
| *240T, 320T, 360T* | Short form reflecting the capacity-based category of trucks, i.e., 240T trucks, 320T trucks or 360T trucks |
| *Tph* | Tonnes per hour |
| *j* | Truck category index based on truck capacity, *j* = 1 for 240T trucks, *j* = 2 for 320T trucks, *j* = 3 for 360T trucks |
| *t(s,d,j)* | Cycle time of the ore trucks of type *j* (minutes) |
| *t(s,j)* | Cycle time of the waste trucks if type *j* (minutes) |
| *L(s,d,j)* | Ore truckload of truck of type *j* (Tonnes) |

| | |
|---|---|
| $L(s,j)$ | Waste truckload of truck of type *j* (Tonnes) |
| $W_m$ | Minimum amount of waste required to be moved over the time period in order to keep the operation system balanced |
| $R_j$ | Truck resource limitation of type *j* |
| $X(s,d,j)$ | Decision variables: number of ore trucks per hour for truck of type *j* (sending from Shovel '*s*' to Ore Dumping '*d*') |
| $Y(s,j)$ | Decision variables: number of waste trucks per hour for truck of type *j* (sending from Shovel '*s*' to Waste Dumping) |
| $K_j$ | Cost coefficient of different truck types |
| $V_{ore}$ | Hourly rate of ore hauled from Shovel to Dump (Tonnes) |
| $V_{extraction}$ | Hourly rate of ore extracted from Surge Pile (Tonnes) |
| $C_{shovel}$ | Hourly shovel capacity (three shovel capacities are the same in this case) (Tonnes) |

In a truck allocation system, the truck cycle time plays a critical role for the formulation of ore and waste removal constraints. The truck cycle time is a sum of the loading time from shovel to truck, travelling time of trucks from shovel sites to dumping/crushing areas after being loaded, waiting time of trucks at dumping/crushing areas, dumping time, travelling-back-to-shovel-site time, plus waiting time at the shovel. Figure 6.8 shows all elements included in the truck cycle time, and this cycle time model is applicable to both ore trucks and waste trucks.

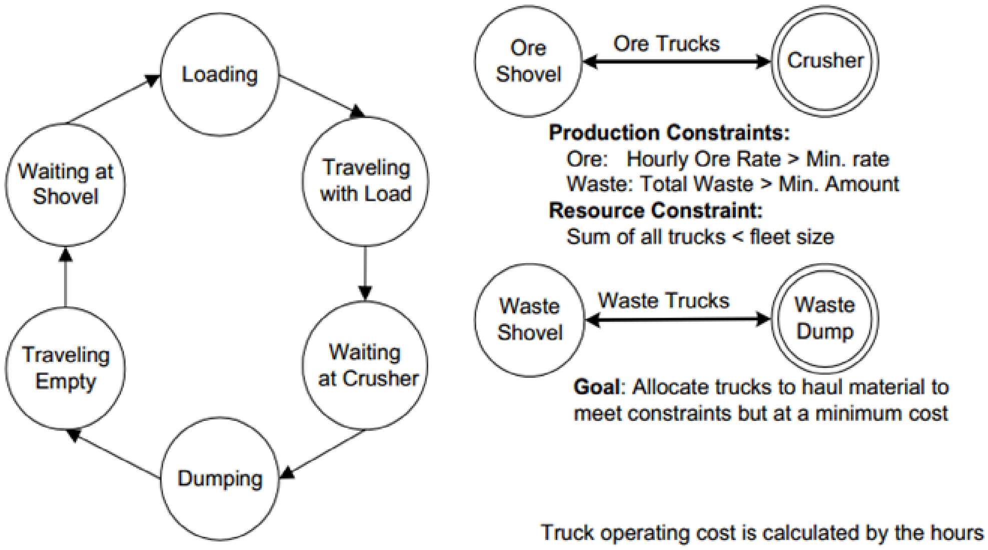


*Figure 6.8 Elements in a truck cycle time model*

*(Source: George, 2002)*

In this study, the truck cycle time equals to a total of loading time, haul time, dump time and return time in a unit minutes. Table 6.2 represents the calculated cycle times for different ore- and waste-hauling time, and they will be used as the fundamental parameters in the model formulation.

Table 6.2 Parameters for calculating the truck cycle time

| | Load Time (min) | Dump Time (min) | Haul Speed (mph) | Distance (miles) | Haul Time (min) | Return Time (min) | Return Speed (mph) | Cycle Time (min) |
|---|---|---|---|---|---|---|---|---|
| Ore | | | | | | | | |
| 240T | 3.5 | 2.5 | 31 | 4.1 | 8 | 6 | 41 | 20 |
| 320T | 5 | 3 | 27 | 4.1 | 9 | 7 | 35 | 24 |
| 360T | 6 | 4 | 23 | 4.1 | 11 | 7 | 31 | 28 |
| Waste | | | | | | | | |
| 240T | 4 | 2 | 31 | 5.5 | 10 | 8 | 41 | 24 |
| 320T | 4 | 7 | 27 | 5.5 | 12 | 9 | 35 | 32 |
| 360T | 7.5 | 4.5 | 23 | 5.5 | 14 | 10 | 31 | 36 |

### 6.2.1 Input Data for LP and ILP Model

The mean value of each parameter is used in the basic (classical) LP model. The following Table 6.3 shows the input data which are used in the classical LP model in this study.

Table 6.3 Input data for LP Model

| | |
|---|---|
| Ore truck cycle time (min) | $t(s,d,j)$ = 20, 24, 28 min |
| Waste truck cycle time (min) | $t(s,j)$ = 24, 28, 32 min |
| Deterministic truckload (in order of 240T, 320T, 360T trucks) | $L(s,d,j)$ = {220, 290, 327} Tonnes<br>$L(s,j)$ = {220, 290, 327} Tonnes |
| Hourly ore constraint (Tph) | 10,000 for each shovel |
| Minimum waste constraint (Tph) | $W_{min}$ = 2,500 from each shovel |
| Truck fleet size (240T, 320T, 360T) | $R_j$ = 35, 15, 15 |
| Hourly rate of ore extracted from surge pile | $V_{extraction}$ = 12,000 Tph |
| Minimum volume requirement for surge pile in order to keep system balanced | $V_{min}$ = 14,000 Tonnes |
| Cost coefficients (in order of 240T, 320T, 360T trucks) | $K_j$ = 1, 1.33, 1.5 |

For the ILP model of this study, the interval parameters include ore and waste truck cycle time (min), cost coefficients ($K$) for three types of trucks, the minimum volume requirement for the ore surge pile in order to constantly feed the extraction plant at required balance and the minimum waste removal amount in each hour. Table 6.4 presents the interval parameters used in the ILP model.

Table 6.4 Interval parameters used in ILP model

| | Truck Cycle Time (min) | Cost Coefficient *(K)* | $V_{\min}$ *(Tonnes)* | $W_{min}$ *(Tonnes)* |
|---|---|---|---|---|
| Ore | | | [ 6500 , 7500 ] | [ 2000 , 3000 ] |
| 240 T | [ 16 , 24 ] | [ 0.8 , 1.2 ] | | |
| 320 T | [ 20 , 28 ] | [ 1.1 , 1.5 ] | | |
| 360 T | [ 24 , 32 ] | [ 1.3 , 1.7 ] | | |
| Waste | | | | |
| 240 T | [ 20 , 28 ] | [ 0.8 , 1.2 ] | | |
| 320 T | [ 24 , 32 ] | [ 1.1 , 1.5 ] | | |
| 360 T | [ 28 , 36 ] | [ 1.3 , 1.7 ] | | |

## 6.3. MODEL DEVELOPMENT

### 6.3.1. Objective Function of Basic LP Model

The objective of this study is to minimize the total system cost of truck allocation system. The total system cost is determined by the ore hauling cost and the waste hauling cost under the limited truck resources. Hence, the objective function is given as follows.

Minimize Total Cost = (I) + (II)

Where,

$$(I) = \sum_{s=1}^{3} \sum_{d=1}^{2} \sum_{j=1}^{3} K\,(j) X(s,d,j) \tag{6.1}$$

$$(II) = \sum_{s=1}^{3} \sum_{j=1}^{3} K\,(j) Y(s,j) \tag{6.2}$$

(6.1) represents the total ore hauling cost in the entire surface mining operation system. (6.2) is the total waste hauling cost in the surface mining operation system.

$X(s,d,j)$ denotes the allocated ore hauling trucks running from shovel places '*s*' to ore dumping places '*d*'. *s* = 1, 2, 3 represent three shovel places, respectively, while *d* = 1, 2 indicating two ore dumping places in this case. '*j*' represents three types of trucks: $j = 1$ for 240T trucks, $j = 2$ for 320T trucks, $j = 3$ for 360T trucks. $Y(s,j)$ is the allocated waste hauling trucks running from shovel places '*s*' to waste dumping place (there is only one waste dumping place in this case). $K_j$ denotes the cost coefficients (in order of 240T, 320T, 360T trucks) in this model.

### 6.3.2. Model constraints

(1) $V_{ore} - V_{extraction} \geq V_{min}$ (6.3)

This constraint is the minimum volume requirement for the coal surge pile in order to keep the operation system balanced, which means the hauled ore to the surge pile deducting the extracted ore from the surge pile must satisfy the minimum volume requirement.

(2) $V_{ore} = \sum_{s=1}^{3}\sum_{d=1}^{2}\sum_{j=1}^{3}\frac{60}{t(s,d,j)}L(s,d,j)X(s,d,j)$ (6.4)

This constraint gives the total hauled ore amount, which equals to the total hauled by three types of trucks '*j*', from three shovel places '*s*' to the two ore dumping places '*d*'. As mentioned previously, *t(s,d,j)* is the ore truck cycle time of type of *j*; *L(s,d,j)* is the ore truckload of truck type of *j*.

(3) $\sum_{d=1}^{2}\sum_{j=1}^{3}\frac{60}{t(s,d,j)}L(s,d,j)X(s,d,j) \leq C_{shovel}$ (6.5)

This constraint indicates that the hourly hauled ore amount at each shovel place must be less than or equal to the hourly shovel capacity (three shovel capacities are the same in this case).

(4) $\sum_{s=1}^{3}\sum_{j=1}^{3}\frac{60}{t(s,j)}L(s,j)X(s,j) \geq W_{min}$ (6.6)

This constraint gives the hourly hauled waste requirement amount at each shovel place, which means the hourly hauled waste must satisfy the minimum requirement in order to avoid over accumulation of the waste at each shovel place and to keep the entire operation system being balanced. As mention previously, *L(s,j)* is the waste truckload of truck type $j$; $W_{min}$ is the minimum amount of waste required to be hauled in each hour.

(5) $X(s,d,j) + Y(s,j) \leq R_j$ (6.7)

$X(s,d,j) \geq 0$ (6.8)

$Y(s,j) \geq 0$ (6.9)

Constraint (6.7) represents the availability of the truck resources in this case. Integer $X(s,d,j)$ is the number of trucks allocated to ore hauling, and integer $Y(s,j)$ is the number of trucks allocated to waste hauling. $R_j$ is the total available truck resource for each truck type $j$.

This mixed-integer LP model is a deterministic model and the mean values of each parameter are introduced in the model in order to get the optimal solution under the basic circumstance, which means any uncertainties are not considered in this LP model.

### 6.3.3. ILP Model Development

Based on the classical LP model formulated (Model 6.1 to 6.9), the corresponding ILP model for this case can be developed with some uncertain parameters being reflected by interval numbers. In this study, there are four parameters being considered as intervals, and they are the ore truck cycle time, $t^{\pm}(s,d,j)$; the waste truck cycle time (min), $t^{\pm}(s,j)$; the cost coefficients $K^{\pm}$(j) for three types of trucks; the minimum volume requirement, $W^{\pm}_{min}$, for the ore surge pile; and the minimum waste removal requirement. Where, "+" indicates the upper bound of this parameter, while "-" indicating the lower bound of this parameter.

The formulated ILP model for the study case is presented below:

Min

$f^{\pm} = \sum_{s=1}^{3} \sum_{d=1}^{2} \sum_{j=1}^{3} K^{\pm}(j) X(s,d,j) + \sum_{s=1}^{3} \sum_{j=1}^{3} K^{\pm}(j) Y(s,j)$ (6.10)

Subject to

$$V_{ore}^{\pm} - V_{extraction} \geq V_{min}^{\pm} \tag{6.11}$$

$$V_{ore}^{\pm} = \sum_{s=1}^{3} \sum_{d=1}^{2} \sum_{j=1}^{3} \frac{60}{t^{\pm}(s,d,j)} L(s,d,j) X(s,d,j) \tag{6.12}$$

$$\sum_{d=1}^{2} \sum_{j=1}^{3} \frac{60}{t^{\pm}(s,d,j)} L(s,d,j) X(s,d,j) \leq C_{shovel} \tag{6.13}$$

$$\sum_{s=1}^{3} \sum_{j=1}^{3} \frac{60}{t^{\pm}(s,j)} L(s,j) X(s,j) \geq W_{min}^{\pm} \tag{6.14}$$

$$X(s,d,j) + Y(s,j) \leq R_j \tag{6.15}$$

$$X(s,d,j) \geq 0 \tag{6.16}$$

$$Y(s,j) \geq 0 \tag{6.17}$$

By using the BWC algorithm to solve this ILP model, this ILP model can be reformulated into two sub-models under best-case situation and worst-case situation. These two sub-models can be run by Lingo in order to get the optimal solutions under the interval-parameter condition.

(a) Best case sub-model:

Min

$$f^{-} = \sum_{s=1}^{3} \sum_{d=1}^{2} \sum_{j=1}^{3} K^{-}(j) X(s,d,j) + \sum_{s=1}^{3} \sum_{j=1}^{3} K^{-}(j) Y(s,j) \tag{6.18}$$

Subject to

$$V_{ore}^{+} - V_{extraction} \geq V_{min}^{-} \tag{6.19}$$

$$V_{ore}^{+} = \sum_{s=1}^{3} \sum_{d=1}^{2} \sum_{j=1}^{3} \frac{60}{t^{-}(s,d,j)} L(s,d,j) X(s,d,j) \tag{6.20}$$

$$\sum_{d=1}^{2} \sum_{j=1}^{3} \frac{60}{t^{-}(s,d,j)} L(s,d,j) X(s,d,j) \leq C_{shovel} \tag{6.21}$$

$$\sum_{s=1}^{3} \sum_{j=1}^{3} \frac{60}{t^{-}(s,j)} L(s,j) X(s,j) \geq W_{min}^{-} \tag{6.22}$$

$$X(s,d,j) + Y(s,j) \leq R_j \tag{6.23}$$

$$X(s,d,j) \geq 0 \tag{6.24}$$

$$Y(s,j) \geq 0 \tag{6.25}$$

(b) Worst case sub-model:

Min

$$f^{+} = \sum_{s=1}^{3}\sum_{d=1}^{2}\sum_{j=1}^{3} K^{+}(j)X(s,d,j) + \sum_{s=1}^{3}\sum_{j=1}^{3} K^{+}(j)Y(s,j) \tag{6.26}$$

Subject to

$$V_{ore}^{-} - V_{extraction} \geq V_{min}^{+} \tag{6.27}$$

$$V_{ore}^{-} = \sum_{s=1}^{3}\sum_{d=1}^{2}\sum_{j=1}^{3} \frac{60}{t^{+}(s,d,j)} L(s,d,j)X(s,d,j) \tag{6.28}$$

$$\sum_{d=1}^{2}\sum_{j=1}^{3} \frac{60}{t^{+}(s,d,j)} L(s,d,j)X(s,d,j) \leq C_{shovel} \tag{6.29}$$

$$\sum_{s=1}^{3}\sum_{j=1}^{3} \frac{60}{t^{+}(s,j)} L(s,j)X(s,j) \geq W_{min}^{+} \tag{6.30}$$

$$X(s,d,j) + Y(s,j) \leq R_j \tag{6.31}$$

$$X(s,d,j) \geq 0 \tag{6.32}$$

$$Y(s,j) \geq 0 \tag{6.33}$$

### 6.3.4. REILP Model Development

Based on the REILP methodology developed in Chapter 4, a multi-objective REILP formulation of the study case can be formulated as follows:

Min

$$\sum_{s=1}^{3}\sum_{d=1}^{2}\sum_{j=1}^{3} K(j)X(s,d,j) + \sum_{s=1}^{3}\sum_{j=1}^{3} K(j)Y(s,j) \tag{6.34}$$

Min

$$\xi = \oplus^{i}\left[\sum_{j=1}^{n} \lambda_{ij}\,(a_{ij}^{+} - a_{ij}^{-})x_j + \eta_i(b_i^{+} - b_i^{-})\right] \tag{6.35}$$

Subject to

$$\sum_{j=1}^{n}(c_j^- \, x_j + d) \geq f_{opt}^- + \lambda_0(f_{opt}^+ - f_{opt}^-), \forall i \tag{6.36}$$

$$\sum_{j=1}^{n} a_{ij}^+ \, x_j - b_i^- \leq \xi_i, \forall i \tag{6.37}$$

$$\lambda_0 = \lambda_{pre} \tag{6.38}$$

$$0 \leq \lambda_{ij} \leq 1 \tag{6.39}$$

$$x_j \geq 0, \forall j \tag{6.40}$$

Where,

$$d = \lambda_0(c_j^+ - c_j^-)x_j \tag{6.41}$$

$\lambda_0$ is defined as the range of [0, 0.1, 0.2, 0.3, ..., 1] to obtain the risks under different aspiration levels. In this multi-objective programming model (6.34 to 6.41), the first objective function (6.34) is to minimize the total operating cost while minimizing the risk as in its second objective function (6.35). $X(s, d, j)$ and $Y(s, j)$ denote the allocated number of trucks for ore hauling and waste hauling. $K_j$ represents the cost coefficients (in order of 240T, 320T, 360T trucks) in this model.

### 6.3.5. FREILP Model Development

As described in Chapter 5, in the practical situation, the decision makers or stakeholders may not be able to define the aspiration level at a specific value, or they may have different respective preferences. Therefore, in order to make the aspiration level more applicable for the real world cases, the REILP model is reformulated into a FREILP model (as shown below) through incorporating three different aspiration categories.

Min

$$\begin{aligned} \xi = \quad & \sum_{i=1}^{m} \frac{2}{b_i^+ + b_i^-} \left[\sum_{j=1}^{n} \lambda_{ij} \, (a_{ij}^+ - a_{ij}^-)x_j + \eta_i(b_i^+ - b_i^-)\right] \\ & + \frac{2}{f_{opt}^+ + f_{opt}^-} \left[\sum_{j=1}^{n} \lambda_j \, (c_j^+ - c_j^-)x_j + \lambda_0(f_{opt}^+ - f_{opt}^-)\right] \end{aligned} \tag{6.42}$$

Subject to

$$\sum_{j=1}^{n}[c_j^+ - \lambda_j(c_j^+ - c_j^-)x_j] \geq f_{opt}^+ - \lambda_0(f_{opt}^+ - f_{opt}^-) \tag{6.43}$$

$$b_i^+ - \sum_{j=1}^{n} a_{ij}^- x_j \geq \sum_{j=1}^{n} \lambda_{ij} (a_{ij}^+ - a_{ij}^-)x_j + \eta_i(b_i^+ - b_i^-), \forall i \tag{6.44}$$

$$0 \leq \lambda_{ij}, \lambda_j, \eta_i \leq 1 \tag{6.45}$$

$$x_j \geq 0, \forall j \tag{6.46}$$

In this FREILP model, the aspiration level $\lambda_0$ is treated as the fuzzy set. In order to represent the aspiration levels under conservative, medium and aggressive situations, the $\alpha$-cut levels will be selected before running the FREILP in Lingo. In this study, three different $\alpha$-cuts are selected as $\alpha$-cut = 0.5, $\alpha$-cut = 0.6 and $\alpha$-cut = 0.7.

The detailed process for solving this REILP model is presented below:

(1) Define the aspiration level as conservative, medium or aggressive according to the preference of the decision maker, and choose an $\alpha$-cut level to obtain two crisp values from the membership function curve.

(2) Two $\alpha$-cut crisp values are then used as the aspiration level inputs for solving the formulated FREILP model.

(3) Run the FREILP model in Lingo and generate the optimal solutions.

Determination of the interval aspiration levels under $\alpha$-cut levels of 0.5, 0.6 and 0.7 for three different scenarios are presented in Figures 6.9 to Figure 6.11, respectively.

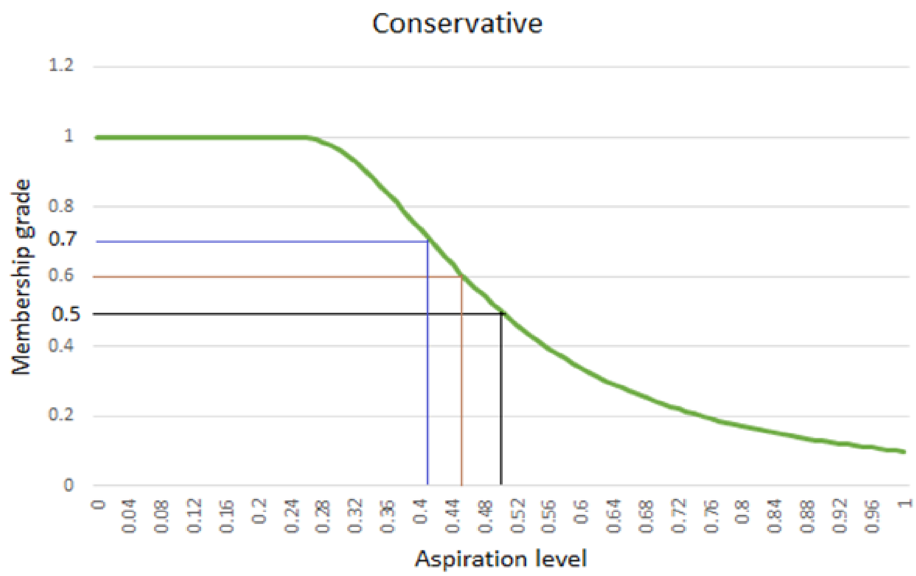


*Figure 6.9 Scenario 1 – conservative situation*

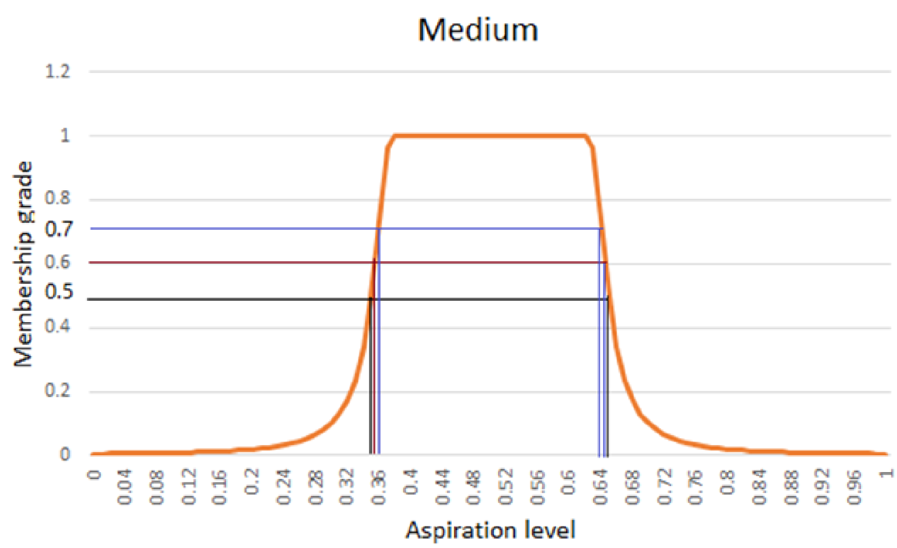


*Figure 6.10 Scenario 2 – medium situation*

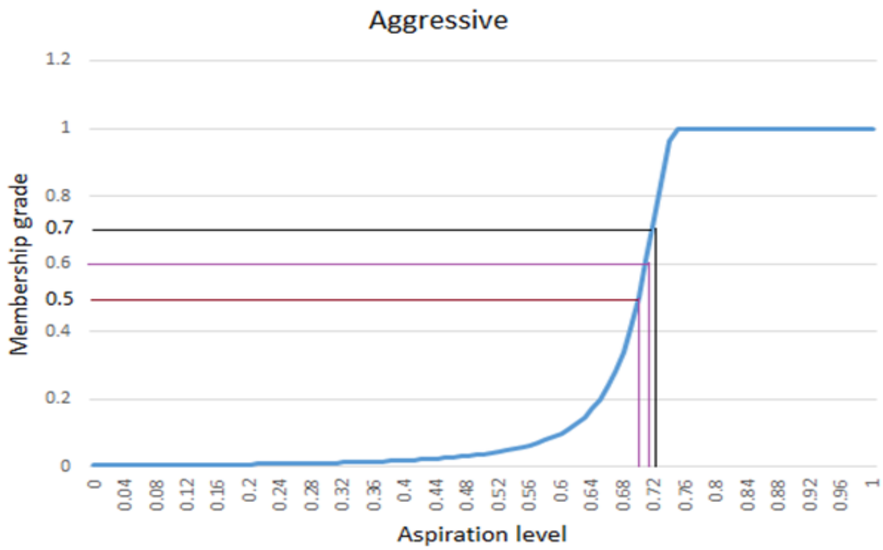


*Figure 6.11 Scenario 3 – aggressive situation*

# CHAPTER 7 RESULTS AND DISCUSSIONS

According to the modelling approach described in Chapter 6, the truck allocation case problem was firstly formulated and solved as a deterministic LP model. The case problem was then reformulated as an ILP model with some of the model parameters being represented as interval number. The ILP model results were then used to formulate the case problem as a REILP model, and 11 preset aspiration levels ranging from 0 to 1 with a step increment of 0.1 were chosen to solve the REILP model. The REILP model was then transformed into the FREILP model through incorporating the fuzzy set theory as described in Chapter 6. The FREILP model was solved under three α-cut levels representing the conservative, medium and aggressive decision preferences, respectively. The solutions from these models could provide valuable insights for the decision-makers and stakeholders to support their decision-making process.

## 7.1 LP AND ILP RESULTS

The deterministic LP model formulated for the case problem is given in the model from (6.1) to (6.9) in Chapter 6. Solving the model by LINGO software, the optimal solutions can be obtained, as shown in Table 7.1. The results consist of the allocation pattern of different trucks to different dumping sites as well as the total truck allocating cost in a unit of 59.45

As indicated in Table 7.1, there are a total of 35 240T trucks, 15 320T trucks, and 15 360T trucks available for ore and waste hauling for this project. Among the 35 240T trucks, 10 of them are used for ore hauling from Shovel 1 to Ore Dumping Site 1, and another 10 of them are used for ore hauling from Shovel 3 to Ore Dumping Site 2, and 5 of them are chosen for each Shovel place for waste hauling; while for 15 320T trucks, none of them have been used for hauling waste, and the number of them chosen for ore hauling are 4 for Shovel 1 to Ore Dumping Site 2, 2 for Shovel 2 to Ore Dumping Site 2, and 1 for Shovel 3 to Ore Dumping Site 1; among all the 360T trucks, only 3 of them are used for hauling ore from Shove 2 to Ore Dumping Site 2.

Table 7.1 Deterministic LP model solutions

| Truck Allocation # | To Dump 1 | To Dump 2 | Waste Dumping |
|---|---|---|---|
| From Shovel 1 | | | |
| 240T Truck | 10 | 0 | 5 |
| 320T Truck | 0 | 4 | 0 |
| 360T Truck | 0 | 0 | 0 |
| From Shovel 2 | | | |
| 240T Truck | 0 | 0 | 5 |
| 320T Truck | 8 | 2 | 0 |
| 360T Truck | 0 | 3 | 0 |
| From Shovel 3 | | | |
| 240T Truck | 0 | 10 | 5 |
| 320T Truck | 1 | 0 | 0 |
| 360T Truck | 0 | 0 | 0 |
| Total System Cost (unit) | 59.45 | | |

The ILP model formulated for the case problem is given in the model from (6.10) to (6.17) in Chapter 6. By using the BWC algorithm, the ILP optimal solutions can be obtained, as shown in Table 7.2. It is indicated that the unit of total truck hauling cost changed from $f_{opt}$ = 59.45 (unit) in the LP model to $f_{opt}$ = [30.8, 90] (unit) in the ILP model, and a different truck allocation pattern was obtained.

As shown in Table 7.2, for ore hauling from Shovels 1, 2 and 3 to two dumping sites, any truck number combinations of [28, 29] for 240T, [3, 14] for 320T and [0, 4] for 360T trucks can be used for generating a truck resource allocation scheme for achieving the least-cost strategy. In the meantime, [6, 6] of 240T, [3, 1] of 320T and [0, 11] 360T trucks have been allocated for waste hauling from three shovel places. Comparing to the optimal solutions of LP model (as shown in Table 7.1), the significant change of the ILP result is the allocation and usage of 360T trucks, which has increased from only 3 to as many as 11 out of 15, and increased cost-hauling efficiency could be achieve.

Table 7.2 ILP solutions solving by BWC algorithm

| Truck Allocation # | Best-case results | | | Worst-case results | | |
|---|---|---|---|---|---|---|
| From Shovel 1 | To Dump 1 | To Dump 2 | Waste Dumping | To Dump 1 | To Dump 2 | Waste Dumping |
| 240T Truck | 4 | 7 | 2 | 1 | 12 | 2 |
| 320T Truck | 0 | 1 | 1 | 2 | 1 | 0 |
| 360T Truck | 0 | 0 | 0 | 0 | 0 | 4 |
| From Shovel 2 | | | | | | |
| 240T Truck | 10 | 0 | 2 | 1 | 4 | 2 |
| 320T Truck | 0 | 2 | 1 | 8 | 1 | 1 |
| 360T Truck | 0 | 0 | 0 | 0 | 0 | 3 |
| From Shovel 3 | | | | | | |
| 240T Truck | 2 | 5 | 2 | 8 | 3 | 2 |
| 320T Truck | 0 | 0 | 1 | 1 | 1 | 0 |
| 360T Truck | 0 | 0 | 0 | 2 | 2 | 4 |
| Total System Cost | 33.8 | | | 90 | | |

## 7.2 REILP RESULTS

In the REILP model from (6.34) to (6.41), the aspiration level $\lambda_0$ represents the degree of aggressiveness and is a value preset by the decision maker. In this study, $\lambda_0$ is defined in the range of [0, 1] with a step increment of 0.1, and thus the decision risks under a series of aspiration levels can be obtained.

The REILP model was solved by the LINGO software. Figure 7.1 is an example of the LINGO solver info page when $\lambda_0$ = 0.

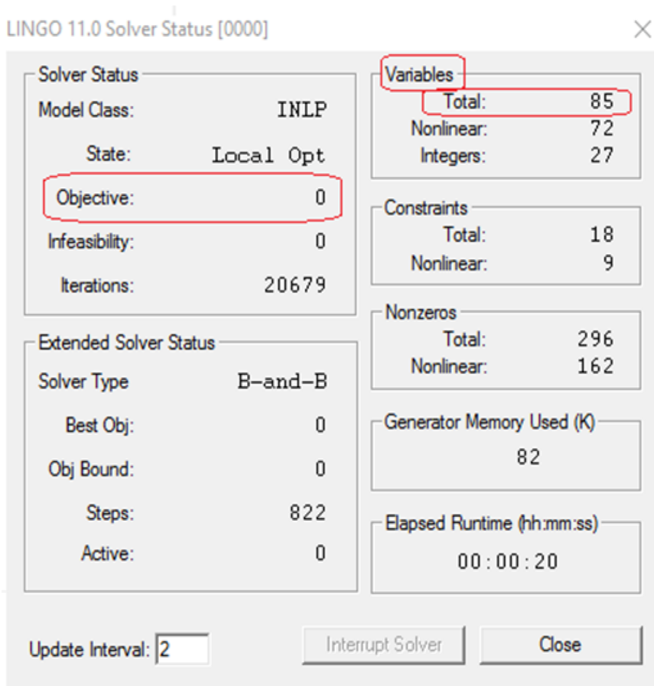


*Figure 7.1 A LINGO solver info page when $\lambda_0 = 0$*

Figure 7.1 shows that there are a total of 85 variables (including 27 integer decision variables) and 18 constraints for the REILP model. It also shows that, when $\lambda_0 = 0$, the value of this objective function is 0 as well.

Table 7.3 shows the optimal solution of the REILP model under the aspiration levels ranging from 0 to 1 with the step increment of 0.1.

As described in Chapter 6, *X(s,d,j)* represents the number of different types of trucks allocated from 3 shovel places to 2 ore dumping sites; *Y(s,j)* represents the number of 3 trucks allocated from 3 shovel places to the waste dumping site. For example, *X111* = 12 under the aspiration level of 0.1, indicating that a total of 12 *240T* trucks are allocated for ore hauling from Shovel 1 to the Ore Dumping Site 1. Similarly, *Y13* = 3 under the aspiration level of 0.1, indicating that a total of 3 *360T* trucks will be used for waste hauling from Shovel 1 to the Waste Dumping Site.

Table 7.3 presents a detailed truck allocation scheme as well as the values of risk function and total hauling cost under different preset aspiration levels. When the aspiration level = 0, the total hauling cost reaches its highest value of 90 with a risk function value of 0. On the other end, when the aspiration level = 1, the total hauling cost would decrease to 33.8, however, with a highest risk function value of 0.326. The modeling results echo that, if the decision maker wants to spend less money to achieve a higher system return, the risks of violating the system constraints will be elevated.

Table 7.3 REILP solutions under different aspiration levels

| Aspiration Level | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| X111 | 16 | 12 | 12 | 7 | 14 | 18 | 3 | 16 | 0 | 12 | 0 |
| X211 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 12 | 0 | 16 |
| X311 | 3 | 0 | 0 | 0 | 0 | 0 | 7 | 0 | 0 | 0 | 0 |
| X112 | 0 | 3 | 0 | 7 | 1 | 0 | 0 | 0 | 3 | 1 | 0 |
| X212 | 0 | 1 | 0 | 4 | 4 | 0 | 1 | 0 | 0 | 2 | 0 |
| X312 | 0 | 3 | 8 | 0 | 1 | 0 | 6 | 0 | 0 | 0 | 0 |
| X113 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| X213 | 4 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| X313 | 1 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| X121 | 0 | 0 | 0 | 0 | 0 | 0 | 14 | 2 | 5 | 3 | 16 |
| X221 | 8 | 14 | 14 | 1 | 13 | 3 | 1 | 10 | 0 | 13 | 0 |
| X321 | 3 | 0 | 0 | 18 | 0 | 0 | 1 | 0 | 7 | 0 | 0 |
| X122 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| X222 | 5 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 3 | 0 | 0 |
| X322 | 6 | 8 | 3 | 0 | 6 | 15 | 0 | 4 | 0 | 0 | 0 |
| X123 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| X223 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| X323 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Y11 | 0 | 3 | 3 | 2 | 2 | 4 | 3 | 1 | 5 | 0 | 1 |
| Y12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 1 | 3 | 2 |
| Y13 | 6 | 3 | 3 | 4 | 4 | 2 | 3 | 0 | 0 | 0 | 0 |
| Y21 | 2 | 3 | 3 | 4 | 3 | 4 | 2 | 3 | 6 | 2 | 2 |
| Y22 | 0 | 0 | 1 | 0 | 1 | 0 | 4 | 3 | 0 | 2 | 1 |
| Y23 | 4 | 3 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 |
| Y31 | 3 | 3 | 3 | 3 | 3 | 4 | 3 | 3 | 0 | 2 | 0 |

| Aspiration Level | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Y32 | 3 | 0 | 0 | 1 | 2 | 0 | 3 | 3 | 5 | 3 | 3 |
| Y33 | 0 | 3 | 3 | 2 | 1 | 2 | 0 | 0 | 0 | 0 | 0 |
| Risk Function | 0 | 0.008 | 0.019 | 0.032 | 0.042 | 0.059 | 0.069 | 0.082 | 0.121 | 0.242 | 0.326 |
| System Cost | 90 | 84.08 | 78.52 | 72.92 | 67.28 | 61.5 | 55.92 | 50.5 | 44.96 | 39.42 | 33.80 |

Based on the solutions provided in Table 7.3, with the aspiration level increasing, the value of risk function is increased from 0 to 0.326 while the total system cost is decreased from its highest 90 to its lowest 33.80. The relationship between the aspiration level, risk function and total system cost is plotted in Figure 7.2. A negative correlation between system cost and decision risk can be observed.

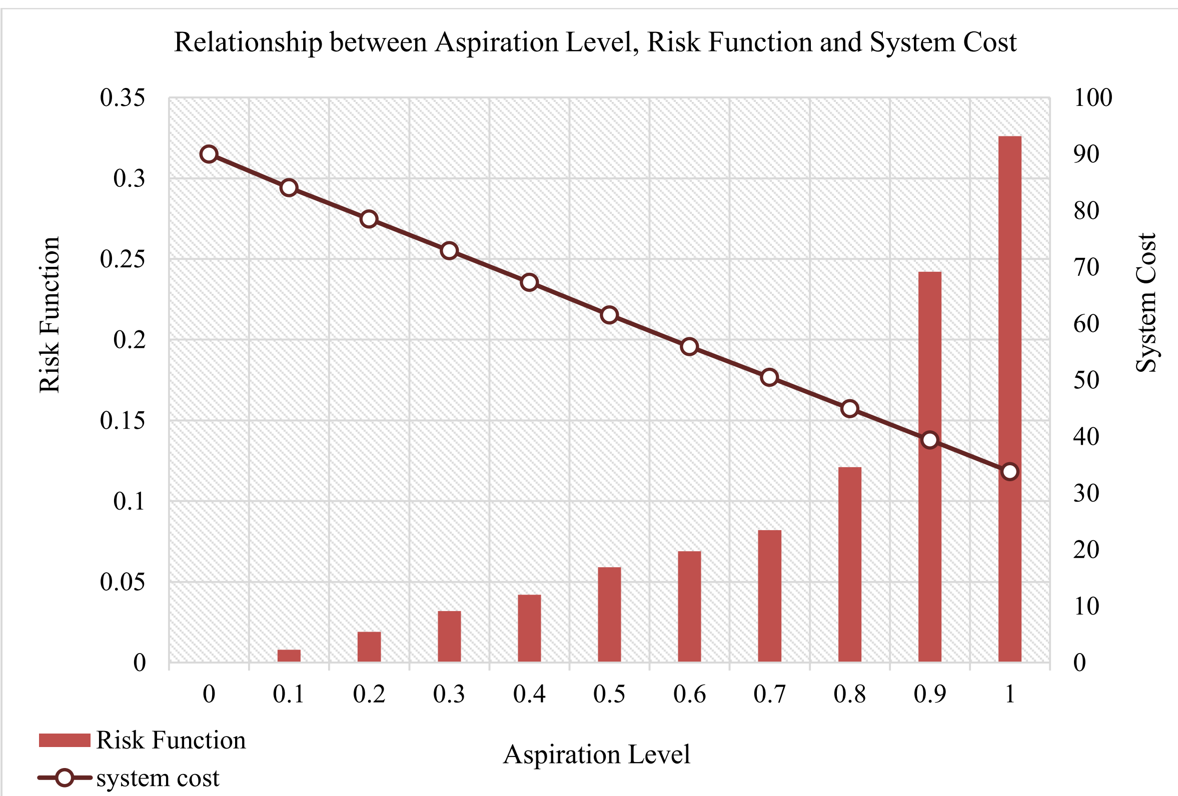


*Figure 7.2 Relationship between aspiration level, risk function and system cost*

Figure 7.2 also indicates that a negative correlation exists between the system cost and the aspiration level. In the REILP formulation, the cost constraint is transformed from the original objective function (minimizing the total system cost), and is proportionally subject to the aspiration level in the REILP model, as given by:

$$f_{opt} = f_{opt}^{-} + \lambda_0 (f_{opt}^{+} - f_{opt}^{-}) \tag{7.1}$$

Also, their relationship is a nonlinear one, as indicated by:

Min

$$\begin{aligned} \xi = \quad & \sum_{i=1}^{m} \frac{2}{b_i^{+} + b_i^{-}} \left[ \sum_{j=1}^{n} \lambda_{ij} \, (a_{ij}^{+} - a_{ij}^{-}) x_j + \eta_i (b_i^{+} - b_i^{-}) \right] \\ & + \frac{2}{f_{opt}^{+} + f_{opt}^{-}} \left[ \sum_{j=1}^{n} \lambda_j \, (c_j^{+} - c_j^{-}) x_j + \lambda_0 (f_{opt}^{+} - f_{opt}^{-}) \right] \end{aligned} \tag{7.2}$$

By using this new objective function (7.2) to replace the one in the REILP model (give the Model number here), while maintaining all the same constraints, the new REILP model can be formulated. Solving the model with the same set of aspiration levels, the new optimal solutions can be obtained.

Figure 7.3 shows the LINGO solver info page for the new REILP model when the aspiration level is 1. In this new REILP model, there are a total of 112 decision variables while the number of constraints keeps unchanged.

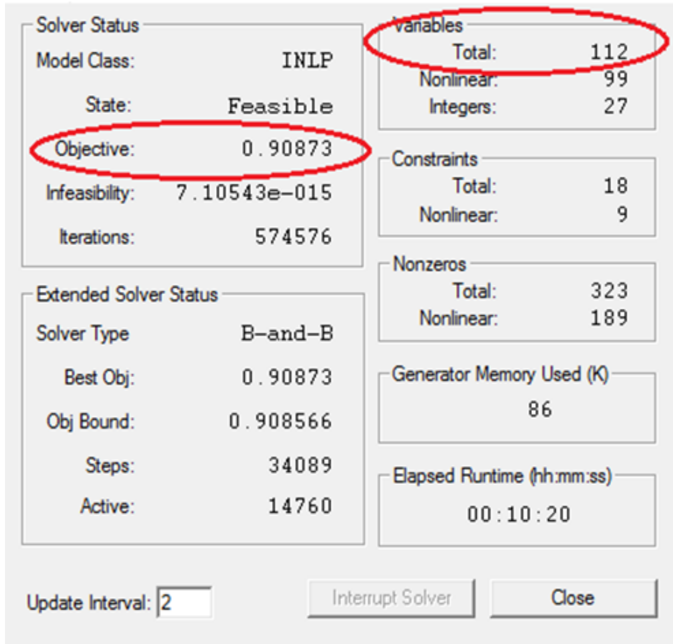


*Figure 7.3 A new LINGO solver status at $\lambda_0 = 1$*

Table 7.4 presents the optimal solutions of the new REILP model under the aspiration levels ranging from 0 to 1 with the step increment of 0.1

Table 7.4 New REILP optimal solutions under the same aspiration levels

| Aspiration Level | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| X111 | 5 | 0 | 8 | 0 | 1 | 3 | 1 | 7 | 0 | 13 | 0 |
| X211 | 5 | 0 | 12 | 4 | 2 | 1 | 4 | 3 | 9 | 1 | 15 |
| X311 | 0 | 13 | 0 | 3 | 8 | 3 | 8 | 2 | 3 | 1 | 0 |
| X112 | 1 | 0 | 2 | 8 | 1 | 0 | 4 | 0 | 0 | 0 | 0 |
| X212 | 6 | 7 | 1 | 1 | 0 | 1 | 1 | 0 | 3 | 1 | 0 |
| X312 | 1 | 1 | 0 | 4 | 6 | 7 | 0 | 4 | 0 | 0 | 0 |
| X113 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| X213 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| X313 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| X121 | 3 | 15 | 6 | 9 | 0 | 14 | 0 | 11 | 11 | 4 | 9 |
| X221 | 0 | 0 | 0 | 9 | 15 | 6 | 13 | 0 | 5 | 10 | 0 |
| X321 | 17 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 3 |
| X122 | 0 | 2 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 2 |

| Aspiration Level | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| X222 | 4 | 5 | 0 | 1 | 0 | 0 | 0 | 4 | 0 | 0 | 0 |
| X322 | 0 | 0 | 6 | 0 | 3 | 0 | 2 | 0 | 0 | 2 | 1 |
| X123 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| X223 | 0 | 0 | 4 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| X323 | 0 | 1 | 4 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| Y11 | 3 | 2 | 3 | 3 | 4 | 0 | 3 | 3 | 0 | 3 | 3 |
| Y12 | 2 | 0 | 0 | 0 | 2 | 4 | 3 | 3 | 5 | 2 | 1 |
| Y13 | 1 | 4 | 3 | 3 | 1 | 2 | 0 | 0 | 0 | 0 | 0 |
| Y21 | 0 | 2 | 3 | 3 | 3 | 3 | 2 | 6 | 3 | 2 | 0 |
| Y22 | 1 | 0 | 3 | 1 | 3 | 2 | 4 | 1 | 3 | 1 | 3 |
| Y23 | 5 | 4 | 0 | 2 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| Y31 | 2 | 3 | 3 | 3 | 2 | 3 | 3 | 3 | 2 | 0 | 2 |
| Y32 | 0 | 0 | 2 | 0 | 0 | 0 | 1 | 3 | 2 | 2 | 1 |
| Y33 | 4 | 3 | 1 | 3 | 4 | 3 | 2 | 0 | 1 | 1 | 0 |
| Risk Function | 0 | 0.091 | 0.182 | 0.272 | 0.363 | 0.454 | 0.545 | 0.636 | 0.727 | 0.818 | 0.909 |
| System Cost | 90 | 84.08 | 78.52 | 72.72 | 67.28 | 61.50 | 55.92 | 50.50 | 44.96 | 40.36 | 33.80 |

As indicated in Table 7.4, the new REILP model has a risk function value range of [0, 0.909] while the former REILP model has a range of [0, 0.326]. It is apparent that, under the same aspiration level, the risk value of the new REILP model is higher than the former model. However, the total system cost is maintained in a similar range of [90, 33.8]. In addition, the allocation pattern of the truck resources has changed as well.

The relationship between the aspiration level, the value of risk function and the total system cost for the new REILP model is plotted in Figure 7.4. In Figure 7.4, the total system cost and the value of risk function still have the obvious negative correlation. The correlations among the aspiration level, decision risk and system cost are found to be same between two REILP models.

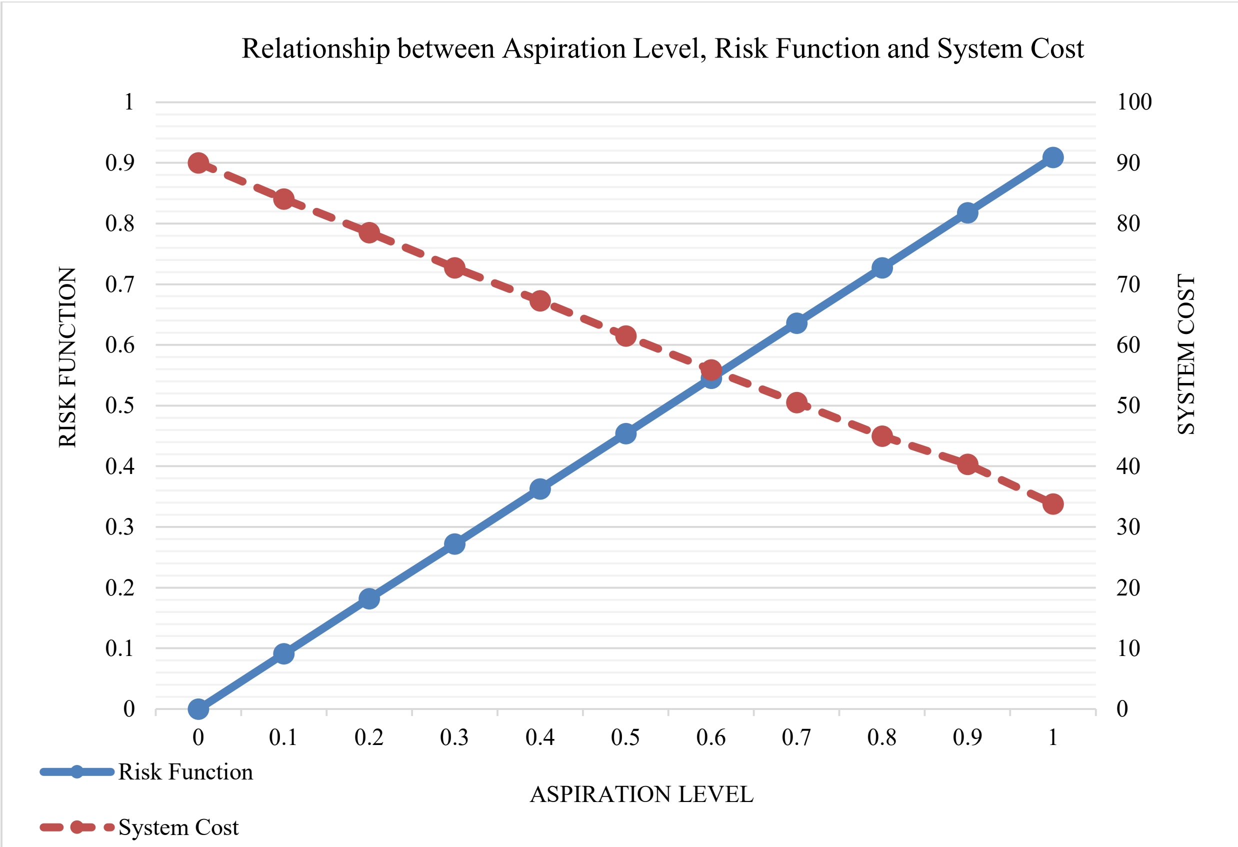


*Figure 7.4 Correlation among the aspiration level, risk function and system cost for the new REILP model*

Table 7.5 Distribution of allocation cost from the new REILP model

| | Truck Hauling Cost (unit) | | |
|---|---|---|---|
| From Shovel 1 | To Dump 1 | To Dump 2 | Waste Dumping |
| 240T Truck | [0, 13] | [0, 15] | [0, 4] |
| 320T Truck | [0, 10.67] | [0, 2.67] | [0, 6.67] |
| 360T Truck | [0, 7.50] | [0, 1.50] | [0, 6] |
| Total Cost (unit) | [0, 31.17] | [0, 19.17] | [0, 16.67] |
| From Shovel 2 | | | |
| 240T Truck | [0, 15] | [0, 15] | [0, 6] |
| 320T Truck | [0, 9.33] | [0, 6.67] | [0, 5.33] |
| 360T Truck | [0, 1.50] | [0, 6] | [0, 7.50] |
| Total Cost (unit) | [0, 25.83] | [0, 27.67] | [0, 18.83] |
| From Shovel 3 | | | |

| | Truck Hauling Cost (unit) | | |
|---|---|---|---|
| | To Dump 1 | To Dump 2 | Waste Dumping |
| From Shovel 3 | | | |
| 240T Truck | [0, 13] | [0, 17] | [0, 3] |
| 320T Truck | [0, 9.33] | [0, 8] | [0, 4] |
| 360T Truck | [0, 1.50] | [0, 6] | [0, 6] |
| Total Cost (unit) | [0, 23.83] | [0, 31] | [0, 13] |

Table 7.5 presents the distribution of truck allocation costs for 240T, 320T and 360T trucks from each shovel site to each ore dumping site and waste dumping site. The cost values (unit) are formed based on the highest and the lowest values of each allocation pattern obtained from all of the event models under 11 degrees of aspiration levels from Table 7.4. Among all the costs, Shovel 2 contributes the most to the total waste hauling cost at the range of [0, 18.83]. Meanwhile, the value of ore hauling cost (unit) from Shovel Site 1 to Ore Dumping Site 1 contributes more than hauling from other 2 shovel sites to the same ore dumping site. In addition, the value of ore hauling cost at Shovel Site 3 takes the dominant place among the total ore hauling costs from 3 shovel sites to Ore Dumping Site 2.

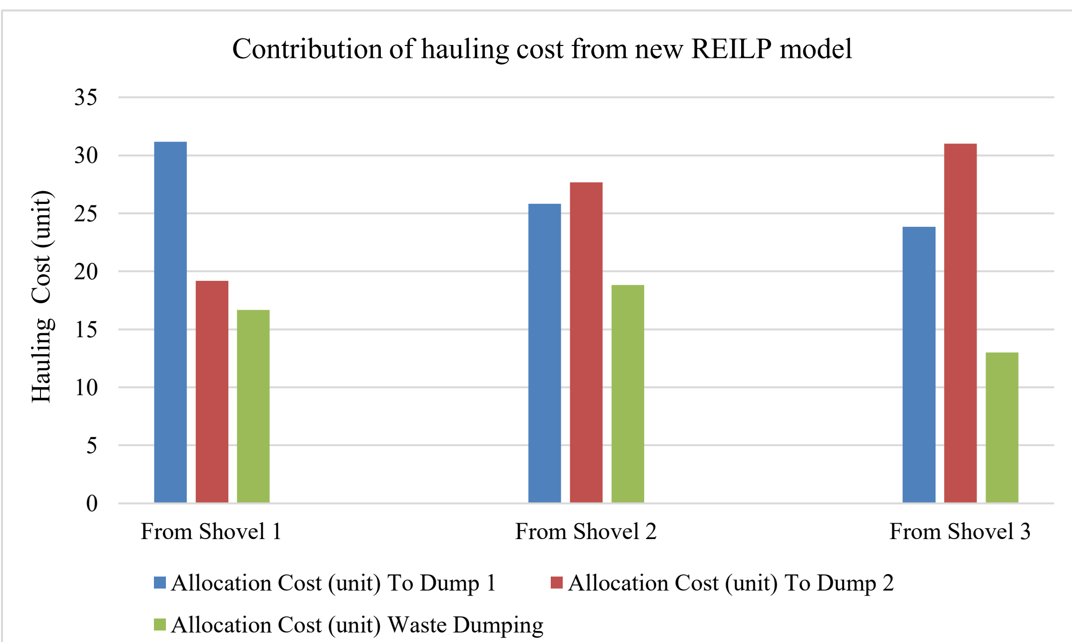


*Figure 7.5 Contribution of hauling cost based on new REILP model results*

Figure 7.5 illustrates the contribution of ore/waste hauling cost (unit) from three shovel sites. It is obvious that the total waste hauling cost (unit) is always lower than the ore hauling costs (unit) among three shovel sites.

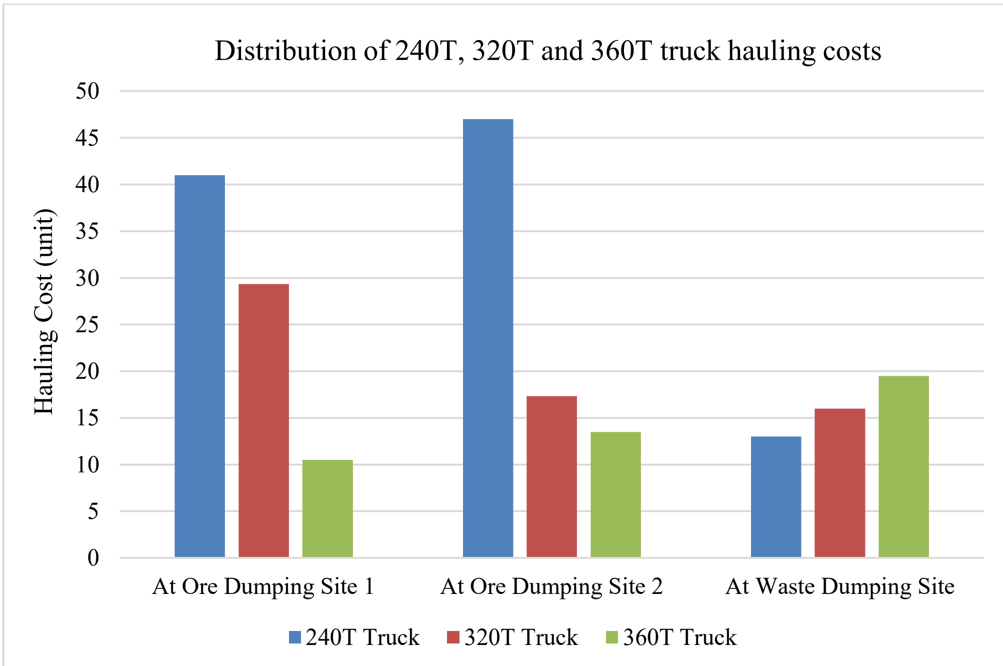


*Figure 7.6 Distribution of 240T, 320T and 360T truck hauling cost based on new REILP model results*

Distributions of 240T, 320T and 360T truck hauling cost based on new REILP optimal solutions from Table 7.4 are illustrated in Figure 7.6. In this figure, the blue bar represents the unit value of 240T hauling cost and the orange bar stands for the unit value of 320T truck hauling cost while the grey bar represents the unit value of 360T truck hauling cost. It can be observed that 240T truck contributes the most at two ore dumping sites and waste dumping site. In the meantime, the total unit value of 360T truck hauling cost takes the lowest place among all of the hauling costs.

## 7.3 FREILP RESULTS UNDER DIFFERENT $\alpha$-CUTS

In this study, in order to represent the aspiration levels under conservative, medium and aggressive scenarios, three different $\alpha$-cuts are selected, i.e., $\alpha$-cut = 0.5, $\alpha$-cut = 0.6 and $\alpha$-cut = 0.7, respectively. Figure 7.7 provides an example on how to determine the interval aspiration levels for medium scenario under three selected α-cuts. For example, when $\alpha$-cut = 0.5, the interval aspiration level is [0.35, 0.65]. Determination of the interval aspiration levels under $\alpha$-cut levels of 0.5, 0.6 and 0.7 for medium scenario is illustrated in Figure 7.7.

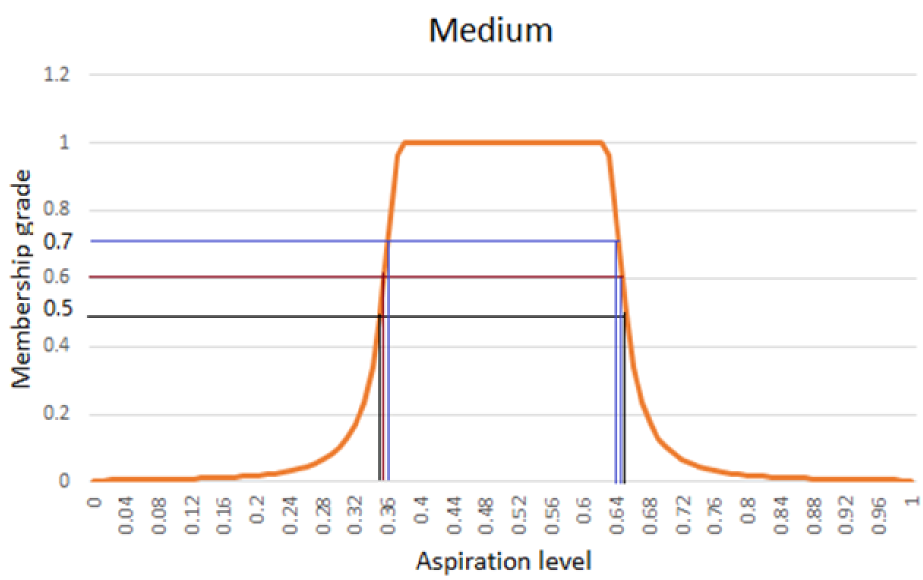


*Figure 7.7 Determination of the interval aspiration levels for medium scenario under α-cut levels of 0.5, 0.6 and 0.7*

### 7.3.1 Scenario 1: $\alpha$-cut = 0.5

Table 7.6 presents the optimal solutions of the FREILP model for α-cut = 0.5 in terms of risk function and total system cost (unit), ore hauling cost (unit) and waste hauling cost (unit) under conservative, medium and aggressive conditions. As shown in Table 7.6, when α-cut = 0.5, the corresponding conservative, medium and aggressive interval aspiration level values are [0.00, 0.30], [0.35, 0.65] and [0.70, 1.00], respectively.

Under the conservative condition, the risk function is in the range of [0.000, 0.272], and its corresponding total system cost (unit) is in the range of [90.00, 72.72]. As the condition changes from 'conservative' to 'medium', the risk increase from [0.000, 0.272] to [0.318, 0.590], while the total system cost (unit) decreases from [90.00, 70.72] to [70.18, 53.44]. Furthermore, when the condition changes from 'medium' to 'aggressive', the value of risk continue to increase to a range of [0.636, 0.909], and the total system cost (unit) continues to decrease to [50.50, 33.80].

Table 7.6 FREILP solution when $\alpha$-cut = 0.5

| **0.5-cut** | **Conservative** | **Medium** | **Aggressive** |
|---|---|---|---|
| Aspiration Level | [ 0.00, 0.30 ] | [ 0.35, 0.65 ] | [ 0.70. 1.00 ] |
| Risk Function | [0.000,0.272] | [0.318, 0.590] | [0.645, 0.909] |
| Total System Cost (unit) | [90.00, 72.72] | [70.18, 53.44] | [50.50, 33.80] |
| Ore Hauling Cost (unit) | | | |
| 240T Truck | [30.00, 26.00] | [27.00, 28.00] | [28.00, 29.00] |
| 320T Truck | [18.57, 21.10] | [19.74, 4.91] | [8.09, 0.00 ] |
| 360T Truck | [ 7.50, 3.00 ] | [ 0.00, 0.00 ] | [ 0.00, 0.00 ] |
| Waste Hauling Cost (unit) | | | |
| 240T Truck | [ 5.00, 9.00 ] | [ 8.00, 7.00 ] | [ 7.00, 3.00 ] |
| 320T Truck | [13.93, 1.62] | [ 4.94, 13.53] | [7.41, 1.80 ] |
| 360T Truck | [15.00, 12.00] | [10.50, 0.00] | [ 0.00, 0.00 ] |

### 7.3.2 Scenario 2: $\alpha$-cut = 0.6

Table 7.7 illustrates the optimal solutions when $\alpha$-cut = 0.6 in terms of risk function, ore hauling cost (unit), waste hauling cost (unit) and total allocation system cost (unit) under conservative, medium and aggressive situations, respectively. In this table, when $\alpha$-cut = 0.6, the corresponding conservative, medium and aggressive interval aspiration levels are [0.00, 0.29], [0.36, 0.65] and [0.71, 1.00], respectively.

Comparing to the results in Table 7.6 (when $\alpha$-cut = 0.5), the current value of risk function changes to the range of [0.000, 0.263] while its total system cost changes to the range of [90.00, 73.46]. Meanwhile, for the medium condition, the value of risk function increases from [0.000, 0.263] under conservative condition to [0.327, 0.590], while its total system cost (unit) decreases from [90.00, 73.46] to [69.75, 53.44]. In addition, when the condition changes from 'medium' to 'aggressive', the value of risk function increases to the range of [0.645, 0.909], which achieves the highest value range of risk function among conservative, medium and aggressive conditions. In the meantime, the total system cost (unit) decreases to its lowest range of [50.08, 33.80] comparing to the conservative and medium conditions.

Table 7.7 FREILP solution when $\alpha$-cut = 0.6

| **0.6-cut** | **Conservative** | **Medium** | **Aggressive** |
|---|---|---|---|
| Aspiration Level | [ 0.00, 0.29 ] | [ 0.36, 0.65 ] | [ 0.71, 1.00 ] |
| Risk Function | [0.000, 0.263] | [0.327, 0.590] | [0.645, 0.909] |
| Total System Cost (unit) | [90.00, 73.46] | [69.75, 53.44] | [50.08, 33.80] |
| Ore Hauling Cost (unit) | | | |
| 240T Truck | [30.00, 26.00] | [27.00, 28.00] | [28.00, 29.00] |
| 320T Truck | [18.57, 21.78] | [19.40, 4.92] | [ 7.87, 0.00 ] |
| 360T Truck | [ 7.50, 3.00 ] | [ 0.00, 0.00 ] | [ 0.00, 0.00 ] |
| Waste Hauling Cost (unit) | | | |
| 240T Truck | [ 5.00, 9.00 ] | [ 8.00, 7.00 ] | [ 7.00, 3.00 ] |
| 320T Truck | [ 13.93, 1.68 ] | [ 4.85, 13.52] | [ 7.21, 1.80 ] |
| 360T Truck | [15.00, 12.00 ] | [ 10.50, 0.00 ] | [ 0.00, 0.00 ] |

### 7.3.3 Scenario 3: $\alpha$-cut = 0.7

When $\alpha$-cut is selected as 0.7, the corresponding optimal solutions will be changed. Table 7.8 presents the optimal solutions at the stage of $\alpha$-cut = 0.7 in terms of the value of risk function, ore hauling cost (unit), waste hauling cost (unit) and total system cost (unit) under conservative, medium and aggressive conditions. As shown in Table 7.8, when $\alpha$-cut = 0.7, the aspiration are [0.00, 0.28], [0.35, 0.64] and [0.72, 1.00] under conservative, medium and aggressive conditions.

Under current conservative condition, the value of the risk function is in the range of [0.000, 0.254], while its corresponding total system cost (unit) is in the range of [90.00, 73.96]. However, when the condition changes from ‘conservative’ to ‘medium’, the inverter value of risk function changes from [0.00, 0.254] to [0.315, 0.581], while the unit value of total system cost decreases from [90.00, 73.96] to [70.18, 53.64]. Moreover, when the condition changes from ‘medium’ to ‘aggressive’, the value of risk function continues to increase to the range of [0.654, 0.909], while the unit value of total system cost continues to decrease to a lower range of [49.39, 33.80].

Table 7.8 FREILP solution when $\alpha$-cut = 0.7

| **0.7-cut** | **Conservative** | **Medium** | **Aggressive** |
|---|---|---|---|
| Aspiration Level | [ 0.00, 0.28 ] | [ 0.35, 0.64 ] | [ 0.72. 1.00 ] |
| Risk Function | [ 0.00,0.254] | [0.318, 0.581] | [0.654, 0.909] |
| Total System Cost (unit) | [90.00, 73.96] | [70.18, 53.64] | [49.39, 33.80] |
| Ore Hauling Cost (unit) | | | |
| 240T Truck | [30.00, 26.00] | [27.00, 28.00] | [28.00, 29.00] |
| 320T Truck | [18.57, 26.89] | [22.54, 4.97] | [ 7.51, 0.00 ] |
| 360T Truck | [ 5.00, 2.00 ] | [ 0.00, 0.00 ] | [ 0.00, 0.00 ] |
| Waste Hauling Cost (unit) | | | |
| 240T Truck | [ 5.00, 9.00 ] | [ 8.00, 7.00 ] | [ 7.00, 3.00 ] |
| 320T Truck | [ 13.93, 2.07 ] | [ 5.64, 13.67] | [ 6.88, 1.80 ] |
| 360T Truck | [10.00, 8.00 ] | [ 7.00, 0.00 ] | [ 0.00, 0.00 ] |

## 7.4 DISCUSSION

Based on Table 7.7, the optimal solutions at $\alpha$-cut = 0.6 will be used for further result discussion and analysis in this section.

Table 7.9 provides the detailed truck allocation scheme for conservative, medium and aggressive conditions under 0.6-cut. As the condition changes from ‘conservative’ to ‘aggressive’, the unit value of total system cost decreases gradually from the range of [90.00, 73.46] to [50.08, 33.80], while the value of risk function increases from the interval value of [0.000, 0.263] to [0.645, 0.909]. Meanwhile, it is indicated that the unit value of total system cost is in the range of [69.75, 53.44] while the corresponding value of risk function is in the range of [0.327, 0590].

Table 7.9 Truck allocation solutions under 0.6-cut

| 0.6-cut | Conservative | | Medium | | Aggressive | |
|---|---|---|---|---|---|---|
| Aspiration Level | 0 | 0.29 | 0.36 | 0.65 | 0.71 | 1 |
| X111 | 5 | 1 | 9 | 13 | 3 | 0 |
| X211 | 5 | 1 | 0 | 1 | 0 | 15 |
| X311 | 0 | 1 | 0 | 1 | 13 | 0 |
| X112 | 1 | 6 | 2 | 1 | 0 | 0 |
| X212 | 6 | 0 | 2 | 0 | 0 | 0 |
| X312 | 1 | 3 | 3 | 0 | 0 | 0 |
| X113 | 5 | 0 | 1 | 0 | 0 | 0 |
| X213 | 0 | 0 | 0 | 0 | 0 | 0 |
| X313 | 0 | 6 | 1 | 0 | 0 | 1 |
| X121 | 3 | 10 | 4 | 0 | 10 | 9 |
| X221 | 0 | 14 | 9 | 0 | 0 | 0 |
| X321 | 17 | 0 | 5 | 9 | 0 | 3 |
| X122 | 0 | 0 | 1 | 0 | 4 | 2 |
| X222 | 4 | 0 | 3 | 7 | 3 | 0 |
| X322 | 0 | 0 | 0 | 0 | 0 | 1 |
| X123 | 0 | 0 | 0 | 2 | 0 | 0 |
| X223 | 0 | 0 | 0 | 0 | 0 | 0 |
| X323 | 0 | 0 | 1 | 0 | 0 | 0 |
| Y11 | 3 | 1 | 3 | 3 | 3 | 3 |
| Y12 | 2 | 0 | 2 | 1 | 3 | 1 |
| Y13 | 1 | 5 | 1 | 2 | 0 | 0 |
| Y21 | 0 | 2 | 2 | 3 | 3 | 0 |
| Y22 | 1 | 4 | 1 | 3 | 3 | 3 |
| Y23 | 5 | 0 | 3 | 0 | 0 | 0 |
| Y31 | 2 | 3 | 3 | 3 | 3 | 2 |
| Y32 | 0 | 2 | 1 | 3 | 2 | 1 |
| Y33 | 4 | 1 | 2 | 0 | 1 | 0 |
| Risk Function | 0.000 | 0.263 | 0.327 | 0.590 | 0.645 | 0.909 |

| 0.6-cut | Conservative | | Medium | | Aggressive | |
|---|---|---|---|---|---|---|
| Aspiration Level | 0 | 0.29 | 0.36 | 0.65 | 0.71 | 1 |
| Total System Cost (unit value) | 90.00 | 73.46 | 69.75 | 53.44 | 50.08 | 33.80 |

Table 7.10 presents different truck hauling costs in terms of their lower and upper levels under conservative, medium and aggressive conditions. As indicated in this table, for ore hauling, the unit value of total system cost of 240T truck takes the dominant place among the other total hauling costs, while the unit value of total system cost of 320T truck takes the second place. However, for waste hauling, the unit value of total system cost of 320T truck contributes the most than the other two types of trucks, while the unit value of total system cost of 240T truck contributes the least.

Table 7.10 Truck hauling costs under 0.6-cut

| 0.6-cut | Conservative | | Medium | | Aggressive | |
|---|---|---|---|---|---|---|
| | Lower Level | Upper Level | Lower Level | Upper Level | Lower Level | Upper Level |
| Ore Hauling Cost (unit value) | | | | | | |
| 240T Truck | 30.00 | 26.00 | 27.00 | 28.00 | 28.00 | 29.00 |
| 320T Truck | 18.57 | 21.78 | 19.40 | 4.92 | 7.87 | 0.00 |
| 360T Truck | 7.50 | 3.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Waste Hauling Cost (unit value) | | | | | | |
| 240T Truck | 5.00 | 9.00 | 8.00 | 7.00 | 7.00 | 3.00 |
| 320T Truck | 13.93 | 1.68 | 4.85 | 13.52 | 7.21 | 1.80 |
| 360T Truck | 15.00 | 12.00 | 10.50 | 0.00 | 0.00 | 0.00 |
| Total System Cost (unit value) | 90.00 | 73.46 | 69.75 | 53.44 | 50.08 | 33.80 |
| Risk Function | 0.00 | 0.26 | 0.33 | 0.59 | 0.65 | 0.91 |

### 7.4.1 Conservative Risk Decision Support

If the decision maker chooses the conservative decision, the ore/waste truck allocation schemes based on the lower and upper level values from Table 7.9 are shown in Table 7.11 and Figure 7.8. Generally, all of the trucks are allocated more to ore hauling than waste hauling. In Figure 7.8, it is illustrated that 240T trucks are relatively used more than 320T and 360T trucks for both of ore hauling and waste hauling purposes. In addition, only 5 360T trucks are used for ore hauling under this conservative decision making condition.

In Table 7.11, the unit value of total system cost achieves its highest value at 90 under the lower level of aspiration value (aspiration level = 0), while the corresponding value of risk function reaches its lowest at 0. Therefore, it is the most conservative solution with no risk for the decision maker for this project. Meanwhile, when aspiration = 0.29, the unit value of total system cost is 73.46 and its corresponding value of risk function is 0.263.

Table 7.11 Ore/waste truck allocation scheme under conservative condition

| Conservative Condition | Aspiration Level = 0 | Aspiration Level = 0.29 |
|---|---|---|
| X111 | 5 | 1 |
| X211 | 5 | 1 |
| X311 | 0 | 1 |
| X112 | 1 | 6 |
| X212 | 6 | 0 |
| X312 | 1 | 3 |
| X113 | 5 | 0 |
| X213 | 0 | 0 |
| X313 | 0 | 6 |
| X121 | 3 | 10 |
| X221 | 0 | 14 |
| X321 | 17 | 0 |
| X122 | 0 | 0 |
| X222 | 4 | 0 |
| X322 | 0 | 0 |
| X123 | 0 | 0 |
| X223 | 0 | 0 |

| Conservative Condition | Aspiration Level = 0 | Aspiration Level = 0.29 |
|---|---|---|
| X323 | 0 | 0 |
| Y11 | 3 | 1 |
| Y12 | 2 | 0 |
| Y13 | 1 | 5 |
| Y21 | 0 | 2 |
| Y22 | 1 | 4 |
| Y23 | 5 | 0 |
| Y31 | 2 | 3 |
| Y32 | 0 | 2 |
| Y33 | 4 | 1 |
| Risk Function | 0.000 | 0.263 |
| System Cost | 90.00 | 73.46 |

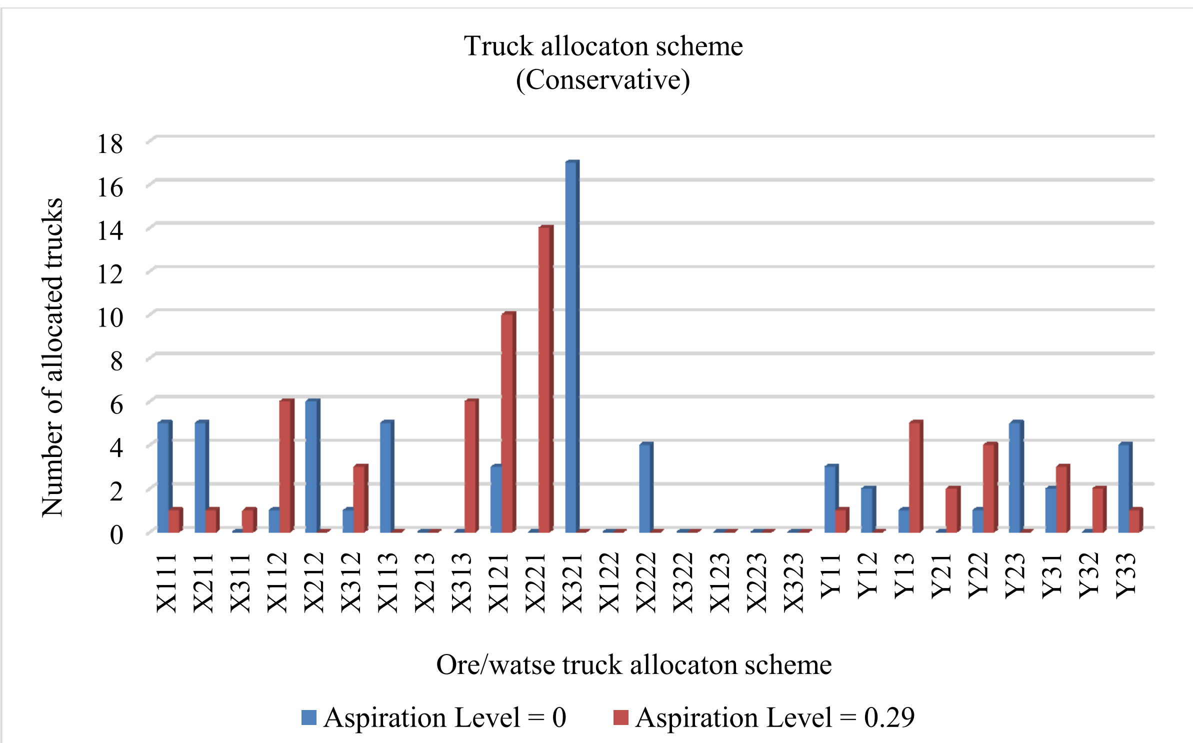


*Figure 7.8 Ore/waste truck allocation scheme under conservative condition*

Table 7.12 provides the detailed ore/waste truck hauling costs in terms of 240T, 320T and 360T truck categories. It is shown that the unit value of total system cost is in the range of [90.00, 73.46], while its corresponding interval value of risk function is in the range of [0.000, 0.263]. However, for ore hauling, the unit value of total hauling cost of 240T truck is in the range of [30.00, 26.00], while the unit value of total hauling cost of 320T is in the range of [18.57, 21.78] and the total hauling cost of 360T truck is in the interval value of [7.50, 3.00]. On the other hand, for waste hauling, the unit value of total hauling cost of 240T truck is in the range of [5.00, 9.00] and the one of 320T truck is in the range of [13.93, 1.68]. Meanwhile, the unit value of total hauling cost of 360T truck is in the range of [15.00, 12.00].

Table 7.12 Ore/waste truck hauling costs under conservative condition

| 0.6-cut | Conservative Condition | |
|---|---|---|
| | Aspiration Level = 0 | Aspiration Level = 0.29 |
| Ore Hauling Cost (unit value) | | |
| 240T Truck | 30.00 | 26.00 |
| 320T Truck | 18.57 | 21.78 |
| 360T Truck | 7.50 | 3.00 |
| Waste Hauling Cost (unit value) | | |
| 240T Truck | 5.00 | 9.00 |
| 320T Truck | 13.93 | 1.68 |
| 360T Truck | 15.00 | 12.00 |
| Total System Cost (unit value) | 90.00 | 73.46 |
| Risk Function | 0.000 | 0.263 |

The ore/waste hauling costs are presented in Figure 7.9 in terms of 240T, 320T and 360T truck categories under conservative decision making condition. In this figure, the blue bar represents the conservative condition when aspiration level = 0, while the orange bar represents the conservative condition when aspiration level = 0.29. As illustrated in Figure 7.9, the total unit value of ore hauling cost is higher than waste hauling cost. Moreover, it is shown obviously that the unit value of total system cost under aspiration level = 0 is higher than the total system cost under aspiration level = 0.29. Therefore, it will cost more if the decision maker chooses the zero risk condition (which aspiration level = 0).

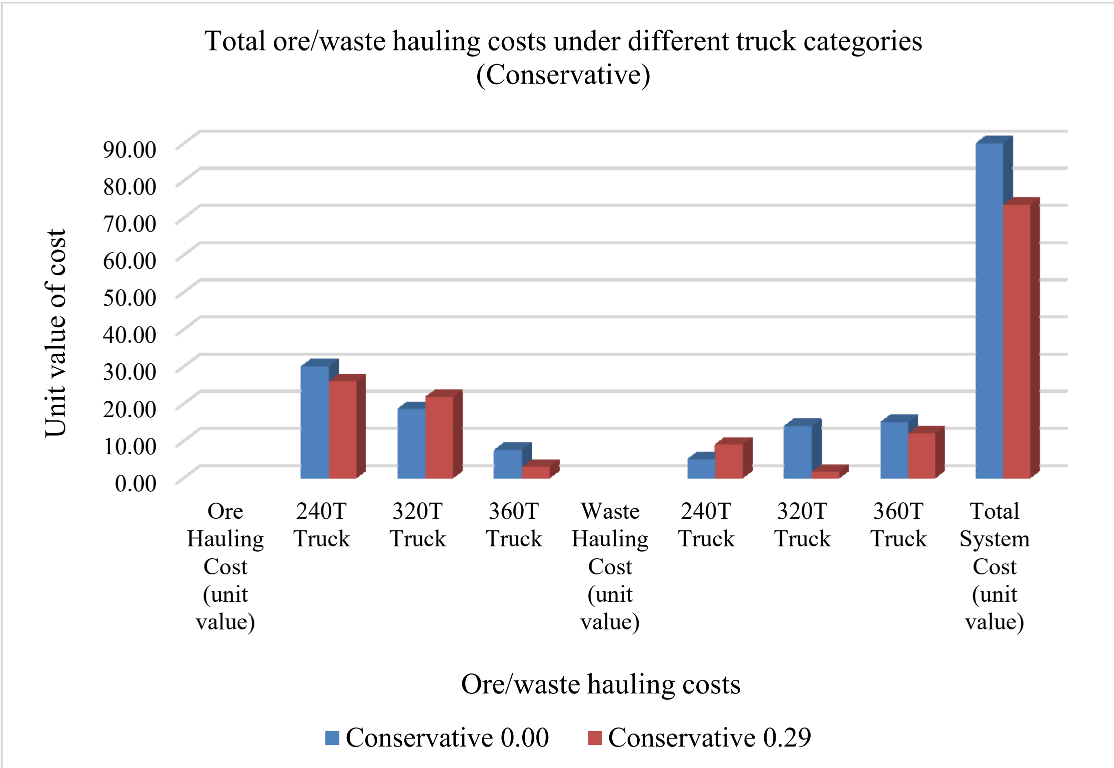


*Figure 7.9 Total ore/waste hauling costs under conservative condition*

### 7.4.2 Medium Risk Decision Support

However, if the decision maker chooses the medium-risk decision, the ore/waste truck allocation schemes based on the lower and upper level values from Table 7.9 are shown in Table 7.13 and Figure 7.10. As shown in Table 7.13, the interval unit value of total system cost is in the range of [69.75, 53.44], while its value of risk function is in the range of [0.327, 0.590]. In addition, as illustrated in Figure 7.10, 240T trucks are allocated more than 320T and 360T trucks for both of ore hauling and waste hauling. In the meantime, 360T trucks are used the least for ore hauling and waste hauling comparing to other two types of trucks. There are only 3 360T trucks are used for ore hauling purpose under medium decision making condition.

Table 7.13 Ore/waste truck allocation scheme under medium condition

| | Aspiration Level = 0.36 | Aspiration Level = 0.65 |
|---|---|---|
| X111 | 9 | 13 |
| X211 | 0 | 1 |
| X311 | 0 | 1 |
| X112 | 2 | 1 |
| X212 | 2 | 0 |
| X312 | 3 | 0 |
| X113 | 1 | 0 |
| X213 | 0 | 0 |
| X313 | 1 | 0 |
| X121 | 4 | 0 |
| X221 | 9 | 0 |
| X321 | 5 | 9 |
| X122 | 1 | 0 |
| X222 | 3 | 7 |
| X322 | 0 | 0 |
| X123 | 0 | 2 |
| X223 | 0 | 0 |
| X323 | 1 | 0 |
| Y11 | 3 | 3 |
| Y12 | 2 | 1 |
| Y13 | 1 | 2 |
| Y21 | 2 | 3 |
| Y22 | 1 | 3 |
| Y23 | 3 | 0 |
| Y31 | 3 | 3 |
| Y32 | 1 | 3 |
| Y33 | 2 | 0 |
| Risk Function | 0.327 | 0.590 |
| System Cost | 69.75 | 53.44 |

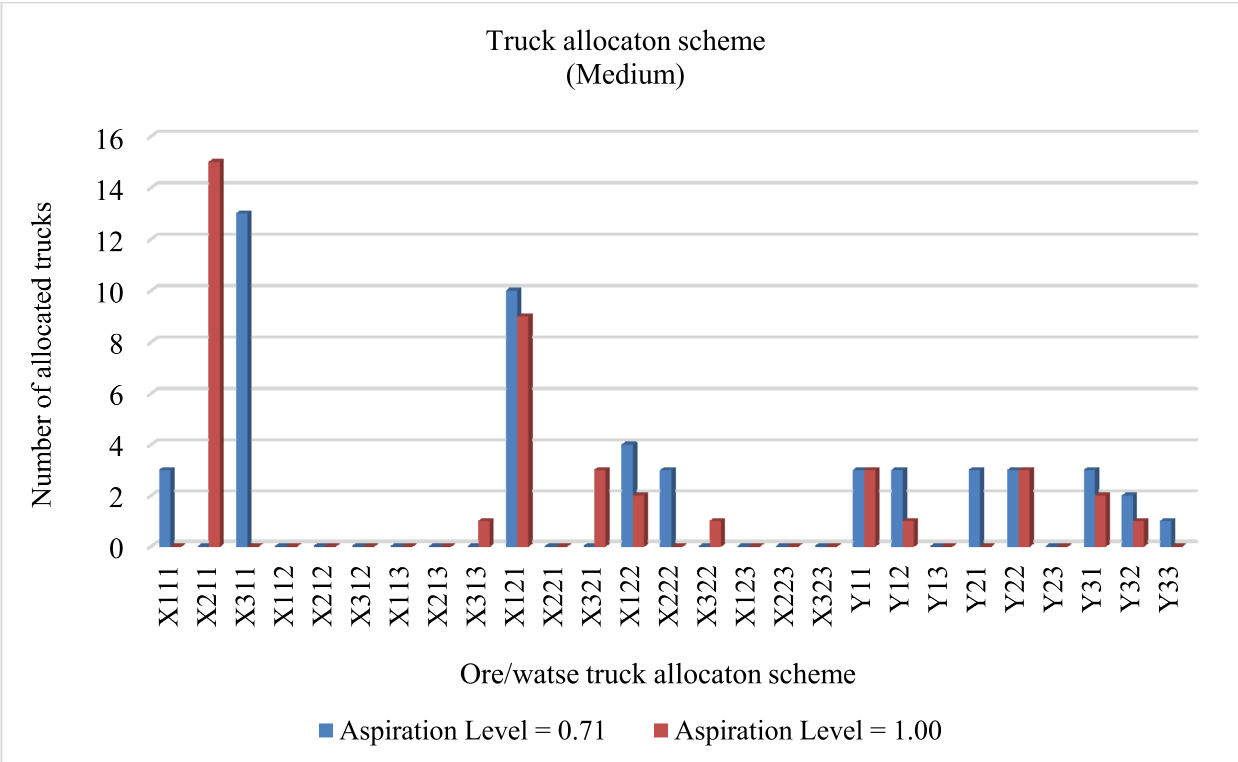


*Figure 7.10 Ore/waste truck allocation scheme under medium condition*

Table 7.14 illustrates the ore/waste truck hauling costs in terms of 240T, 320T and 360T truck categories. It is shown that the unit value of total system cost is in the range of [69.75, 53.44], while its corresponding interval value of risk function is in the range of [0.327, 0.590]. Furthermore, for ore hauling, the unit value of total hauling cost of 240T truck is in the range of [27.00, 28.00], while the interval unit value of total hauling cost of 320T is in the range of [19.40, 4.92]. However, there is no 360T truck allocated for ore hauling under medium decision making condition.

On the other hand, for waste hauling, the unit value of total hauling cost of 240T truck is in the range of [8.00, 7.00] and the one of 320T truck is in the range of [4.85, 13.52]. Meanwhile, the unit value of total hauling cost of 360T truck is in the range of [10.50, 0.00].

Table 7.14 Ore/waste truck hauling costs under medium condition

| 0.6-cut | Medium Condition | |
|---|---|---|
| | Aspiration Level = 0.36 | Aspiration Level = 0.65 |
| Ore Hauling Cost (unit value) | | |
| 240T Truck | 27.00 | 28.00 |
| 320T Truck | 19.40 | 4.92 |
| 360T Truck | 0.00 | 0.00 |
| Waste Hauling Cost (unit value) | | |
| 240T Truck | 8.00 | 7.00 |
| 320T Truck | 4.85 | 13.52 |
| 360T Truck | 10.50 | 0.00 |
| Total System Cost (unit value) | 69.75 | 53.44 |
| Risk Function | 0.327 | 0.590 |

Figure 7.11 provides the ore/waste hauling costs in terms of 240T, 320T and 360T truck categories under medium decision making condition. In this figure, the blue bar represents the medium condition when aspiration level = 0.36, while the orange bar represents the medium condition when aspiration level = 0.65. As shown in Figure 7.11, the total unit value of ore hauling cost is still higher than waste hauling cost. Moreover, the unit value of total ore hauling cost of 240T truck contributes the most comparing to 320T and 360T trucks. Meanwhile, when aspiration level = 0.36, the unit value of total system cost is higher than the one at the aspiration level of 0.65 under medium decision making condition.

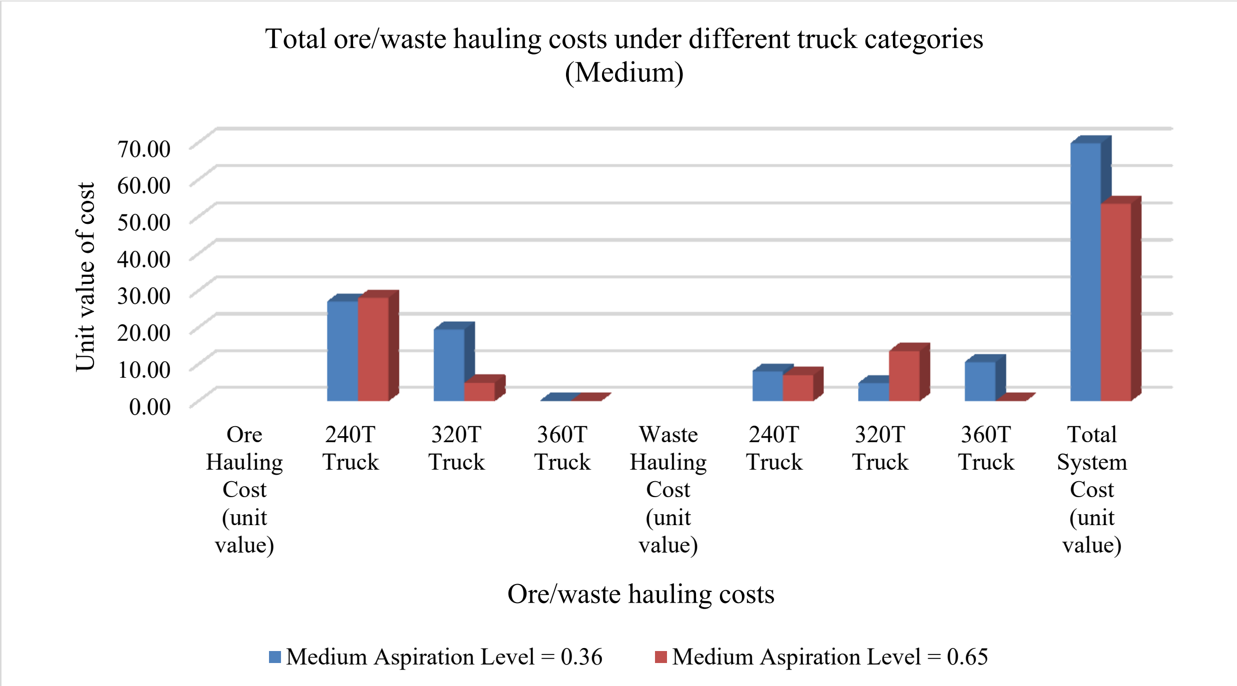


*Figure 7.11 Total ore/waste hauling costs under medium condition*

### 7.4.3 Aggressive Risk Decision Support

If the decision maker chooses the aggressive decision, the ore/waste truck allocation schemes based on the lower and upper level values from Table 7.9 are shown in Table 7.15 and Figure 7.12. In Table 7.15, the unit value of total system cost is in the range of [50.08, 33.80], while the corresponding value of risk function is in the range of [0.645, 0.909].

As shown in Figure 7.2, 240T trucks are used more than 320T and 360T trucks for ore hauling purposes. In addition, there is no 360T used for ore hauling under aggressive decision making condition. However, the similar numbers of trucks of 240T, 320T and 360T are allocated for waste hauling.

Table 7.15 Ore/waste truck allocation scheme under aggressive condition

| | Aspiration Level = 0.71 | Aspiration Level = 1 |
|---|---|---|
| X111 | 3 | 0 |
| X211 | 0 | 15 |
| X311 | 13 | 0 |
| X112 | 0 | 0 |
| X212 | 0 | 0 |
| X312 | 0 | 0 |
| X113 | 0 | 0 |
| X213 | 0 | 0 |
| X313 | 0 | 1 |
| X121 | 10 | 9 |
| X221 | 0 | 0 |
| X321 | 0 | 3 |
| X122 | 4 | 2 |
| X222 | 3 | 0 |
| X322 | 0 | 1 |
| X123 | 0 | 0 |
| X223 | 0 | 0 |
| X323 | 0 | 0 |
| Y11 | 3 | 3 |
| Y12 | 3 | 1 |
| Y13 | 0 | 0 |
| Y21 | 3 | 0 |
| Y22 | 3 | 3 |
| Y23 | 0 | 0 |
| Y31 | 3 | 2 |
| Y32 | 2 | 1 |
| Y33 | 1 | 0 |
| Risk Function | 0.645 | 0.909 |
| System Cost | 50.08 | 33.80 |

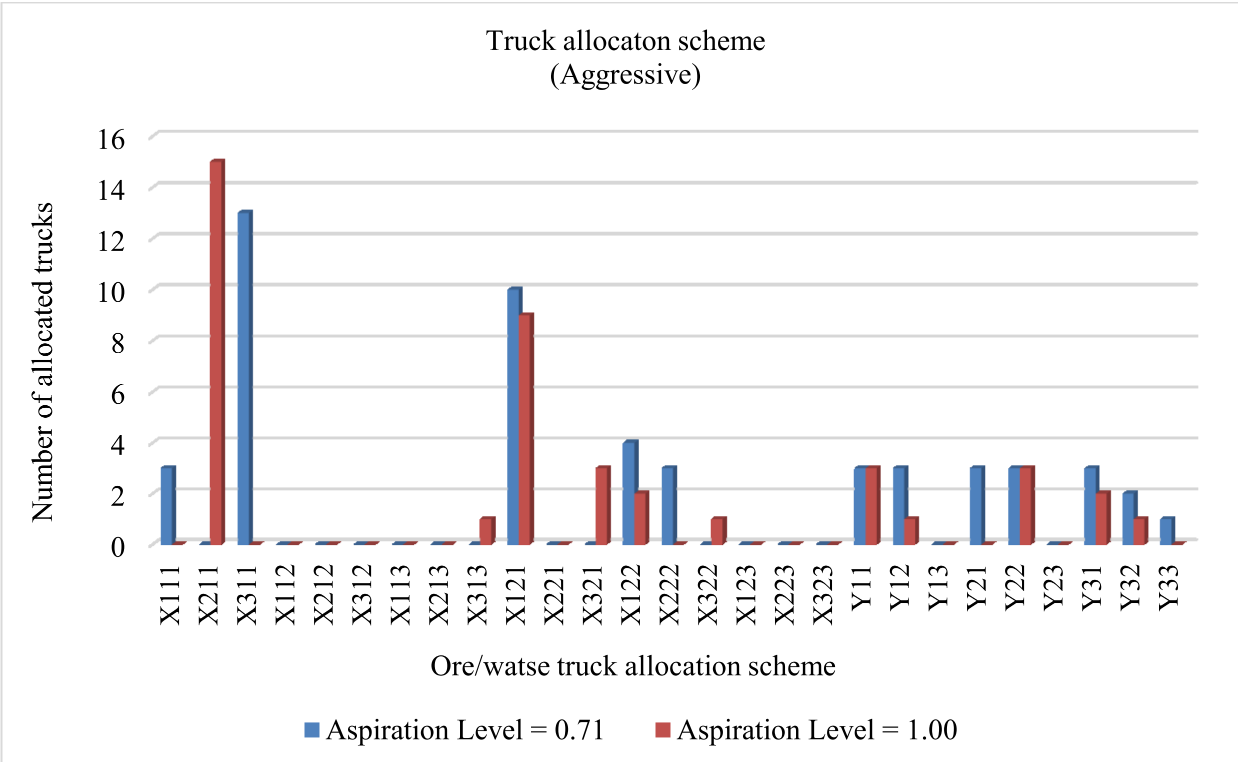


*Figure 7.12 Ore/waste truck allocation scheme under aggressive condition*

In Table 7.16, the ore/waste truck hauling costs are illustrated in terms of 240T, 320T and 360T truck categories. It is shown that the unit value of total system cost is in the range of [50.08, 33.80], while its corresponding interval value of risk function is in the range of [0.645, 0.909]. When aspiration value equals to 1, the least total system cost (unit) at 33.80 can be achieved. However, under this least cost condition, its corresponding value of risk function also achieves the highest one at 0.909. In addition, for ore hauling, the unit value of total hauling cost of 240T truck is in the range of [28.00, 29.00], while the interval unit value of total hauling cost of 320T is in the range of [7.87, 0.00]. However, there is no 360T truck used for ore hauling under aggressive decision making condition.

Furthermore, for waste hauling, the unit value of total hauling cost of 240T truck is in the range of [7.00, 3.00] and the one of 320T truck is in the range of [7.21, 1.80]. However, there is also no 360T truck used for waste hauling under aggressive decision making condition.

Table 7.16 Ore/waste truck hauling costs under aggressive condition

| 0.6-cut | Aggressive Condition | |
|---|---|---|
| | Aspiration Level = 0.71 | Aspiration Level = 1.00 |
| Ore Hauling Cost (unit value) | | |
| 240T Truck | 28.00 | 29.00 |
| 320T Truck | 7.87 | 0.00 |
| 360T Truck | 0.00 | 0.00 |
| Waste Hauling Cost (unit value) | | |
| 240T Truck | 7.00 | 3.00 |
| 320T Truck | 7.21 | 1.80 |
| 360T Truck | 0.00 | 0.00 |
| Total System Cost (unit value) | 50.08 | 33.80 |
| Risk Function | 0.645 | 0.909 |

Figure 7.13 illustrates the ore/waste hauling costs in terms of 240T, 320T and 360T truck categories under aggressive decision making condition. In this figure, the blue bar represents the aggressive condition when aspiration level = 0.71, while the orange bar represents the aggressive condition when aspiration level = 1.00. As illustrated in Figure 7.13, the total unit value of ore hauling cost continues being higher than the waste hauling cost. It is apparent that the unit value of total hauling cost of 240T truck contributes the most on both of ore hauling and waste hauling purposes among three types of trucks. Moreover, when aspiration level = 1.00, the unit value of total system cost achieves its lowest at 33.80. Hence, the decision maker can choose the least cost when the aspiration level = 1.00 under aggressive decision making condition. However, its corresponding value of risk function achieves the highest at 0.909.

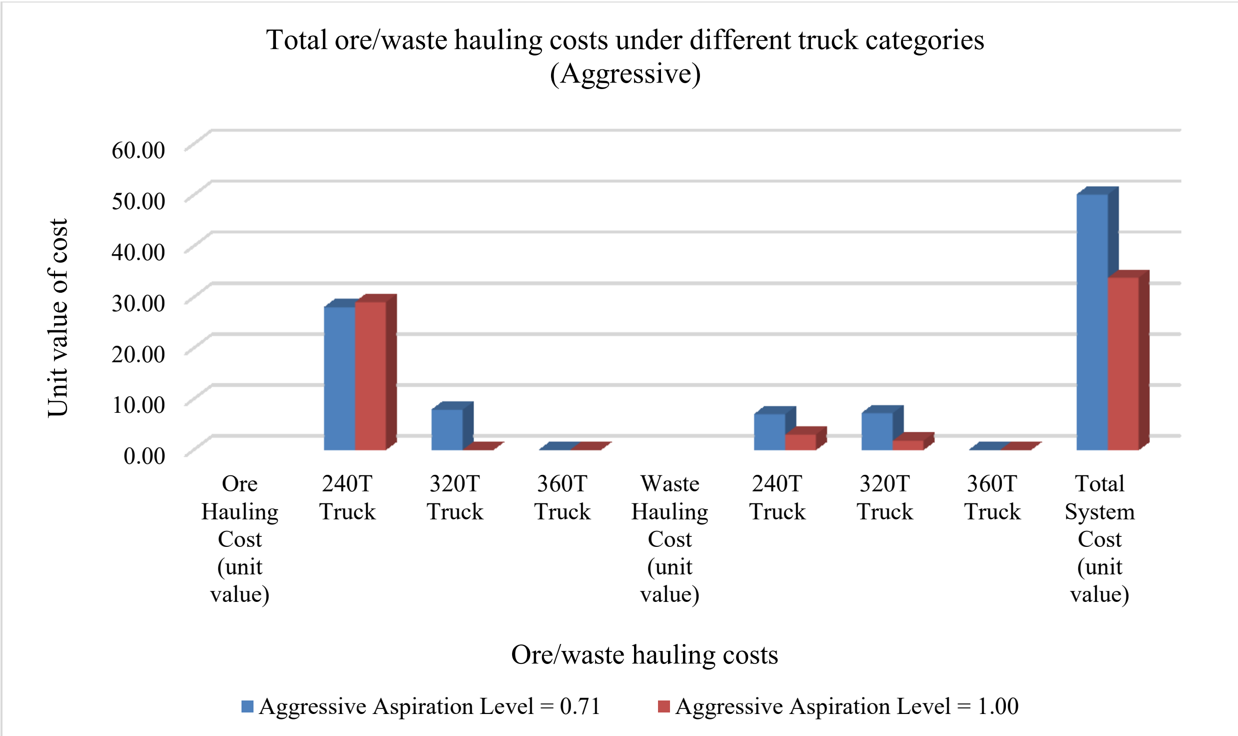


*Figure 7.13 Total ore/waste hauling costs under aggressive condition*

### 7.4.4 Comparison of Three Decision Conditions

Since the solutions are all obtained as interval values, the mean aspiration levels calculated by the lower and upper values are used in this section in order to compare the solutions under conservative, medium and aggressive decision making conditions. For example, from Table 7.7, the interval values of aspiration level are [0.00, 0.29], [0.36, 0.65] and [0.71, 1.00] under conservative, medium and aggressive, respectively. Hence, the mean values of 0.145, 0.505 and 0.855 are used in the following discussion for comparison.

Table 7.17 provides the detailed ore/waste truck allocation schemes under conservative, medium and aggressive conditions. As shown in Table 7.17, the unit value of total system is 83.146 under conservative situation. It is relatively higher than the values of total system costs under medium and aggressive conditions. However, it can achieve the relatively lowest risk value at 0.132 under this conservative condition, while the values of risk function are 0.459 and 0.777 for medium and aggressive conditions, respectively. In the meantime, the unit values of total system cost are 61.388 for medium condition and 43.868 for aggressive condition.

Table 7.17 Comparison of three allocation schemes

| 0.6-cut | Conservative | Medium | Aggressive |
|---|---|---|---|
| Aspiration Level | 0.145 | 0.505 | 0.855 |
| X111 | 0 | 0 | 6 |
| X211 | 18 | 5 | 7 |
| X311 | 1 | 2 | 0 |
| X112 | 0 | 1 | 0 |
| X212 | 0 | 5 | 0 |
| X312 | 1 | 0 | 0 |
| X113 | 3 | 0 | 0 |
| X213 | 0 | 0 | 0 |
| X313 | 2 | 0 | 1 |
| X121 | 13 | 17 | 6 |
| X221 | 0 | 3 | 3 |
| X321 | 0 | 0 | 3 |
| X122 | 0 | 0 | 3 |
| X222 | 0 | 1 | 0 |
| X322 | 9 | 1 | 0 |
| X123 | 0 | 0 | 0 |
| X223 | 0 | 0 | 0 |
| X323 | 0 | 1 | 0 |
| Y11 | 0 | 3 | 5 |
| Y12 | 0 | 3 | 0 |
| Y13 | 6 | 0 | 0 |
| Y21 | 1 | 2 | 0 |
| Y22 | 5 | 1 | 4 |
| Y23 | 0 | 3 | 0 |
| Y31 | 2 | 3 | 0 |
| Y32 | 0 | 1 | 2 |
| Y33 | 4 | 2 | 0 |
| Risk Function | 0.132 | 0.459 | 0.777 |
| Total System Cost (unit value) | 83.146 | 61.388 | 43.868 |

Figure 7.14 illustrates the ore/waste allocation scheme comparison. In this figure, blue bar represents the conservative condition, while orange bar represents the medium condition and the grey bar represents the aggressive condition. It is shown that the allocated number of trucks under conservative condition are more than medium and aggressive conditions. In the meantime, the allocated number of trucks under aggressive condition is the lowest among three decision making conditions. Furthermore, the comparisons of ore/waste truck allocation features and patterns are illustrated in Table 7.18 and Figure 7.15.

In Figure 7.15, it is apparent that the 240T truck takes the dominant place on ore/waste allocation patterns, especially on ore hauling. Meanwhile, 360T trucks are used the least for ore hauling purpose.

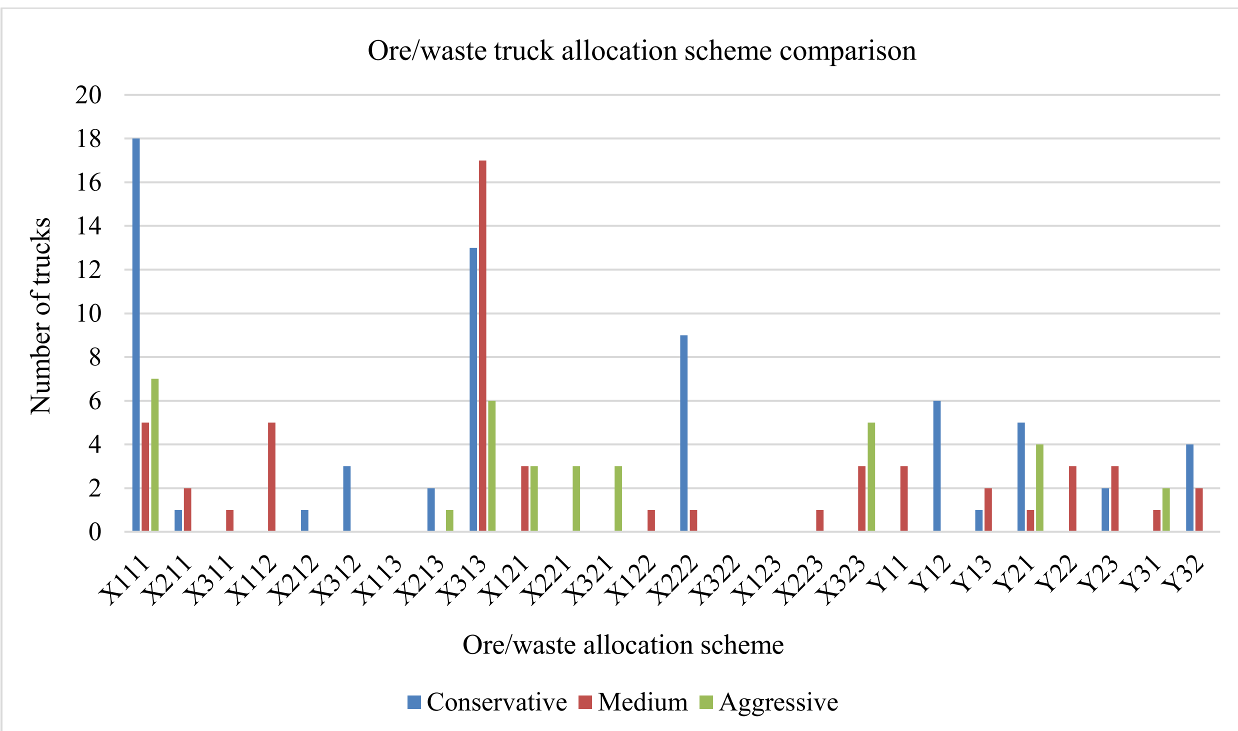


*Figure 7.14 Ore/waste truck allocation scheme comparison*

Table 7.18 Ore/waste truck allocation feature comparison

| 0.6-cut | Conservative | Medium | Aggressive |
|---|---|---|---|
| Ore Hauling | Number of allocated trucks | | |
| 240T Truck | 32 | 27 | 25 |
| 320T Truck | 10 | 8 | 3 |
| 360T Truck | 5 | 1 | 1 |
| Waste Hauling | Number of allocated trucks | | |
| 240T Truck | 3 | 8 | 5 |
| 320T Truck | 5 | 5 | 6 |
| 360T Truck | 10 | 5 | 0 |
| Total System Cost (unit value) | 83.146 | 61.388 | 43.868 |
| Risk Function | 0.132 | 0.459 | 0.777 |

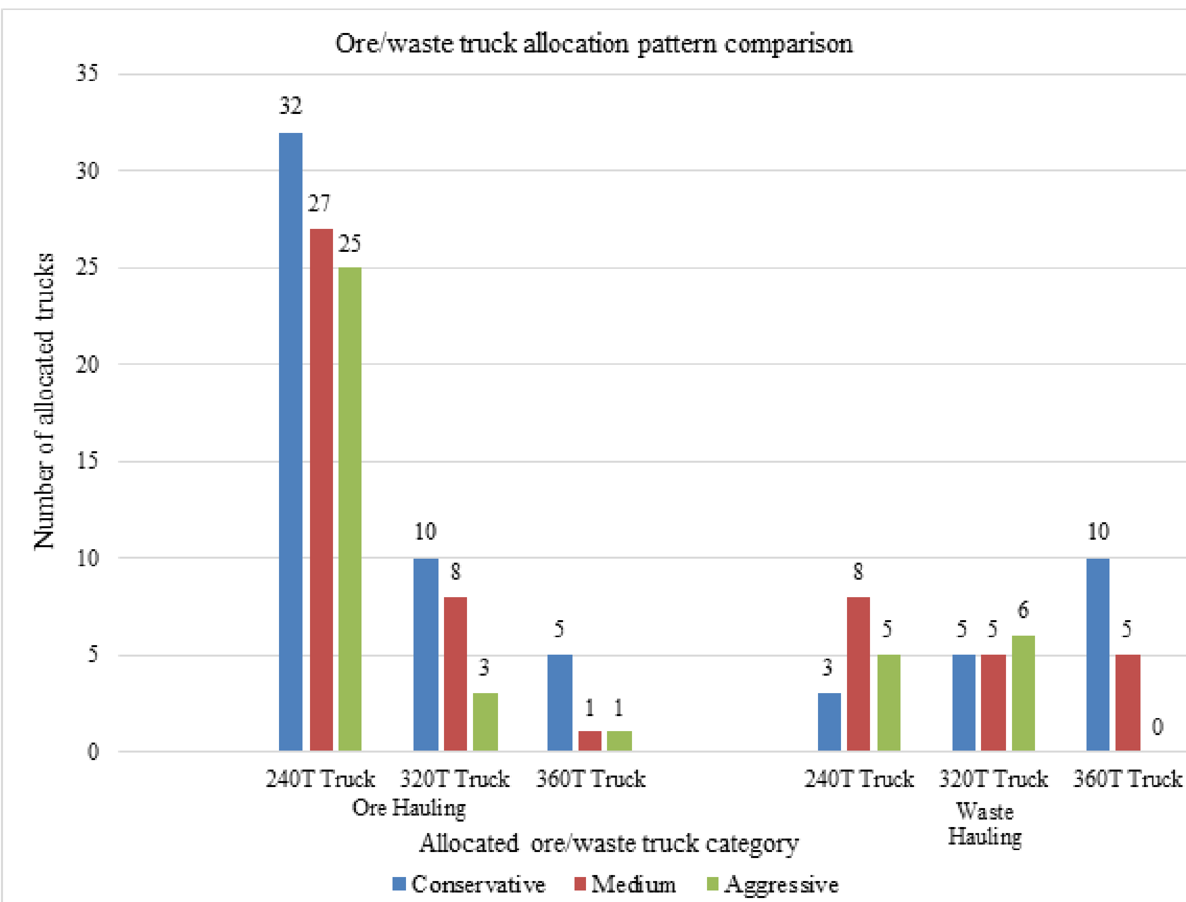


*Figure 7.15 Ore/waste truck allocation pattern comparison*

Table 7.19 presents the comparison of ore/waste hauling costs in terms of 240T, 320T and 360T truck categories.

Table 7.19 Ore/waste hauling costs comparison

| | Conservative | Medium | Aggressive |
|---|---|---|---|
| Ore Hauling Cost (unit value) | | | |
| 240T Truck | 32 | 27 | 25 |
| 320T Truck | 13.33 | 10.64 | 4 |
| 360T Truck | 7.5 | 1.5 | 1.5 |
| Waste Hauling Cost (unit value) | | | |
| 240T Truck | 3 | 8 | 5 |
| 320T Truck | 6.67 | 6.67 | 7.98 |
| 360T Truck | 15 | 7.5 | 0 |
| Total System Cost (unit value) | 83.146 | 61.31 | 43.48 |
| Risk Function | 0.132 | 0.459 | 0.777 |

Figure 7.16 illustrates the comparison of ore/waste hauling costs. As shown in this figure, it is obviously observed that the unit value of total system cost under aggressive condition is the lowest at 43.48 comparing to the unit value of 83.146 under conservative condition and the unit value of 61.31 under medium condition. It is because the aggressive condition assumes the total ore and waste required amount to be at a relative lower level, so that the lower hauling cost can be achieved. The aggressive solution is attractive for the decision maker or the stakeholder from the economical opinion. However, this decision condition comes along with the relatively higher risk of violating the constraints, which means it cannot guarantee the required hauling amount is always under the lower level. Once the required hauling amount happens to be higher than the assumption, then the solution under this aggressive condition will fail to satisfy the real need. Hence, the value of risk function is relatively higher under the aggressive condition than the conservative and medium conditions. On the other hand, the value of risk function under the conservative condition is the lowest at 43.48 among three conditions. However, as presented in Figure 7.16, the conservative condition requires the highest total system cost.

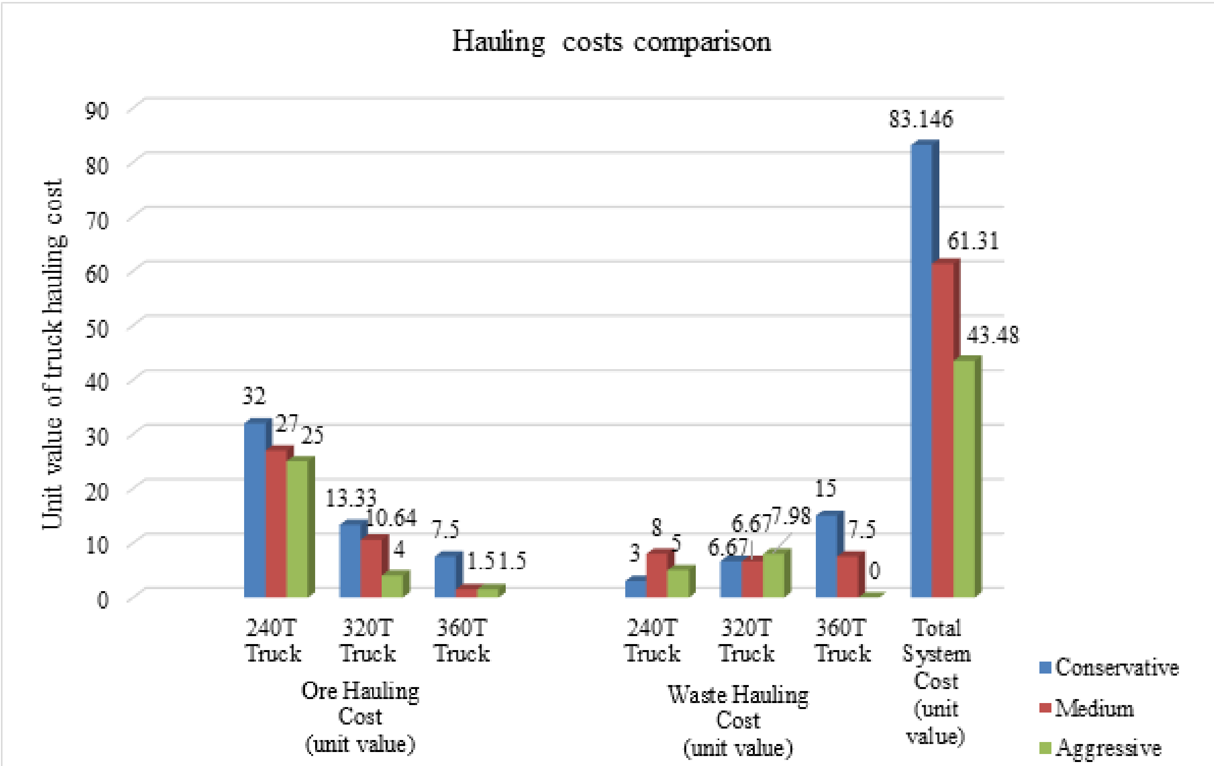


*Figure 7.16 Ore/waste hauling costs comparison*

Figure 7.17 presents the tradeoff between the values of risk function and the unit values of total system cost. In general, the conservative decision support requires the higher unit value of total system cost and assumes the higher required ore/waste hauling amount at a lower risk of violating the constraints. In the meantime, the aggressive decision support results in the lower unit value of total system cost with the assumption of the relatively lower required ore/waste hauling amount, while its corresponding risk level of violating the constraints is higher than the other two conditions. However, the medium decision support is always in between. Each of these conditions is not superior to the others. Therefore, the decision maker or the stakeholder should select the decision making condition based on their preferences or needs for Yalong surface mining project.

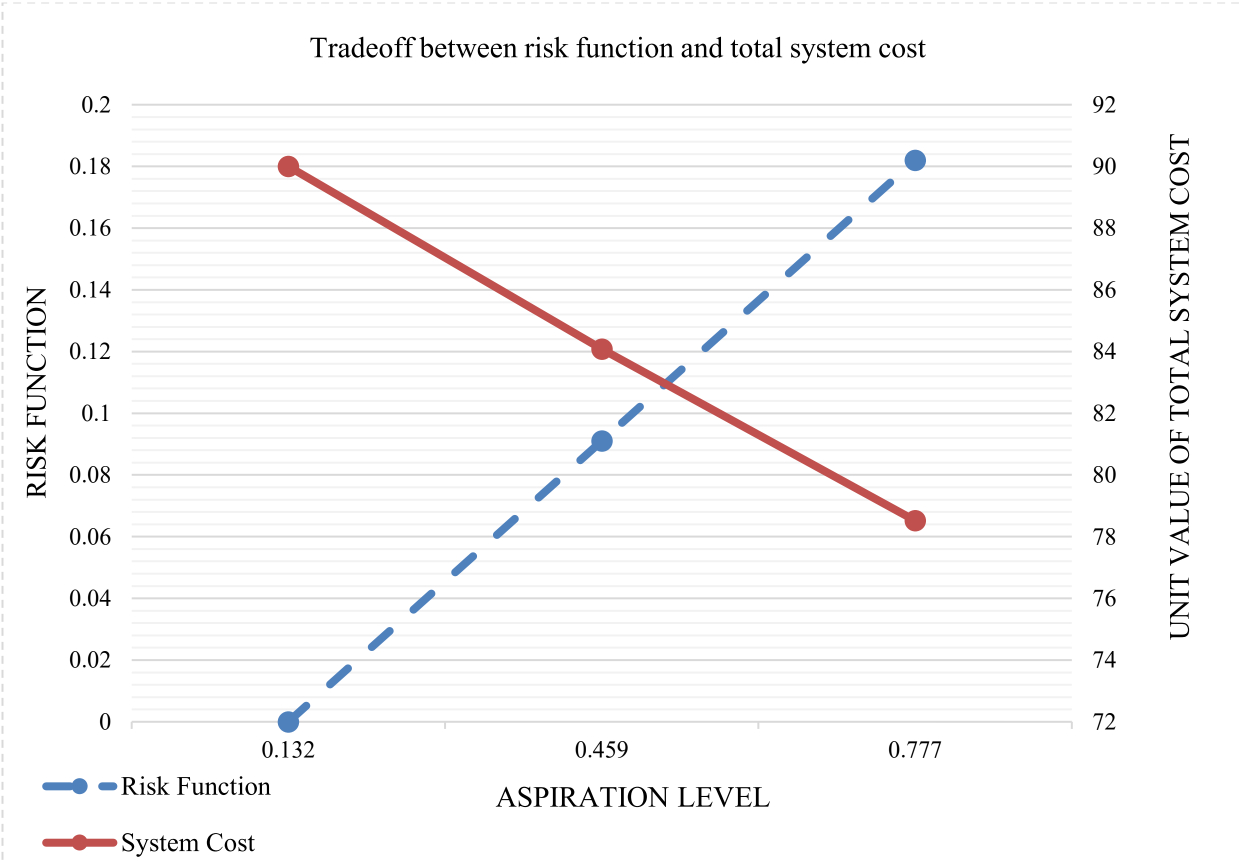


*Figure 7.17 Trade-off between risk functions and total allocation costs*

# CHAPTER 8 CONCLUSIONS

## 8.1 SUMMARY

In this study, a fuzzy risk explicit interval linear programming method is proposed and applied to the optimal management and allocation of hauling truck resources for Yalong mining project in China. The approach can provide a practical decision support through reflecting the tradeoff between system benefits and decision risks. From this study, some conclusions could be summarized as follows:

(1) Monte Carlo simulation algorithm, two-step algorithm and BWC algorithm have been widely used to solve the ILP problems in the past. Among them, Monte Carlo simulation algorithm has the extensive computing requirement and it is not applicable for most practical cases. The other two algorithms have its fundamental flaws in its solution process. In this study, the validity checking for both of BWC and two-step algorithms was conducted by using a numerical example, and the results indicate that both algorithms could produce either infeasible or non-optimal solutions. It is thus recommended that decision risks associated with the model results have to be examined if the algorithms have to be used before their theoretical flaws are fixed.

(2) Problem and examples of REILP approach are examined in this study. Generally, the tradeoff between the total system cost and the decision risk is provided by REILP approach to assist the decision maker to select a crisp decision solution. For this approach, the risk function is defined to find the optimal solutions under the minimum decision risk condition. The minimization or maximization objective function of the original ILP model was converted to a constraint in order to keep the system balanced at its desire. Furthermore, considering the potential hardness of crisp decision making, a FREILP model was developed in this study.

(3) As further development of REILP approach, the FREILP model was developed by restructuring the model formulation and introducing the fuzzy theory to the selected aspiration levels in this study. Comparing to the REILP model, FREILP model is able to offer more practical and reliable decision suggestions. The advantage of this approach is that the optimal solutions associated with their corresponding decision risks can be incorporated in the decision making process. In addition, for those decision makers who cannot define the crisp aspiration value, the FREILP model can provide the decision assistance under conservative, medium and aggressive decision making conditions.

(4) The FREILP model was implemented to the truck allocation planning system of Yalong surface mining project in Guizhou, China. The classic ILP model was firstly applied to minimize the total system cost, including the ore hauling cost and waste hauling cost. It was achieved by setting the allocated ore/waste hauling trucks as variables with the related constraints of ore/waste production amount, required ore/waste hauling amount, truck cycle time and truck resources limitations. Then the original ILP model was transformed to the REILP model. Meanwhile, the original minimization objective function was transformed to risk minimization function, while the aspiration levels playing the dominant roles in the REILP model. 11 solutions are generated from the event model under 11 pre-set aspiration levels in the range of [0, 1] with the step increase of 0.1. The REILP solutions provide specific decision making support for the people who already have a desired aspiration level preference. In the meantime, FREILP provides the decision making support under conservative, medium and aggressive situations, which is benefit for the people who fail to select a crisp aspiration level.

(5) In this study, the FREILP model provides the decision supports under conservative, medium and aggressive conditions. Under the conservative condition, the unit value of total system cost of Yalong truck allocation system is in the range of [90.00, 73.46] with the corresponding risk value range of [0.00, 0.26]. Under the medium condition, the unit value of total system cost is in the range of [69.75, 53.44], while the interval value of risk function is [0.33, 0.59]. In the meantime, for the aggressive condition, the unit value of total system

cost is evaluated as in the range of [50.08, 33.80] and its value of risk function is in the range of [0.65, 0.91].

(6) This study represents that the FREILP model can provide optimal solutions, which can perfectly reflect the tradeoff between the total system cost and the decision risk. Moreover, the decision makers are able to make more reliable decisions based on the tradeoff relationship analysis generated by the FREILP approach.

## 8.2 RESEARCH ACHIEVEMENTS

This study is the first attempt at implementing the REILP and FREILP approach into a practical truck allocation system of a surface mining project. In the meantime, the research experience and knowledge gained from this study become the valuable support for the other research studies or the other practical applications. Moreover, the solutions obtained from this study can also be used by the decision makers or the shareholders for truck allocation system of Yalong surface mining project in Guizhou, China.

## 8.3 RECOMMENDATION FOR FUTURE RESEARCH

In this study, the original objective function of its ILP model is to minimize the total truck hauling costs while satisfying the ore/waste desire. In order to improve the critical parameter accuracy, many other factors can be considered for the model constraints (i.e. local weather characters, different road conditions or even using the truck haul load data accessed from local ore/waste truck hauling database). However, due to failed to proceed with local ore/waste truck hauling road conditions, the yearly ore/waste truck hauling records collected from Yalong sub-project are used in this study. If more updated and comprehensive data can be used, more accurate the model will achieve.

By implementing the REILP and FREILP models for minimizing the allocated truck hauling cost in this study, the generated solutions from either REILP or FREILP are more effective and practical. However, these solutions failed to contain the entire solution space for its original ILP model. It indicates some of the optimal solutions might be missed during this improvement. Hence, a further and more comprehensive method is recommended for researching the ILP and REILP models.

In the meantime, the REILP and FREILP methods could be widely introduced to any other areas regarding the decision making needs. For example, property investing management or stock trading management.

# BIBLIOGRAPHY


Barr, D. (2012). *Stochastic dynamic optimization of cut-off grade in open pit mines*. (Master's thesis, Queen's University).

Bascetin, A., and Kesimal, A. (1999). "The study of a fuzzy set theory for the selection of an optimum coal transportation system from pit to the power plant". *International Journal of Surface Mining, Reclamation and Environment*, 13(3): 97-101.

Beaumont, O. (1998). "Solving interval linear systems with linear programming techniques". *Linear Algebra and its Applications*, 281(1-3): 293-309.

Burt, C. N., and Caccetta, L. (2014). "Equipment selection for surface mining: a review". *Interfaces*, 44(2): 143-162.

Caccetta, L., and Hill, S. P. (2003). "An application of branch and cut to open pit mine scheduling". *Journal of global optimization*, 27(2): 349-365.

Cetin, N. (2004). *Open-pit truck/shovel haulage system simulation*. (Master's thesis, Middle East Technical University).

Chinneck, J. W., and Ramadan, K. (2000). "Linear programming with interval coefficients". *Journal of the Operational Research Society*, 209-220.

China Mining Association. (2015). *2014-2015 annual report*. Retrieved from http://www.gzcoal.gov.cn/article.jsp?id=9883&itemId=73.

Chung, H. T. (2002). *Optimal haul truck allocation in the syncrude mine*. (Doctoral dissertation. University of Alberta).

Chung, H. T., Kresta, J. V., Forbes, J. F., and Marquez, H. J. (2005). "A stochastic optimization approach to mine truck allocation". *International Journal of Surface Mining, Reclamation and Environment*, 19(3): 162-175.

Chung, H. T., Ingolfsson, A., and Doucette, J. (2013). "A linear model for surface mining haul truck allocation incorporating shovel idle probabilities". *European Journal of Operational Research*, 231(3): 770-778.

Emmanuel, T. (2001). *Introduction to the theory and application of data envelopment analysis*. Dordrecht: Kluwer Academic Publishers.

Ercelebi, S. G., and Bascetin, A. (2009). "Optimization of shovel-truck system for surface mining". *Journal of the Southern African Institute of Mining and Metallurgy*, 109(7): 433-439.

Goel, V., and Grossmann, I. E. (2004). “A stochastic programming approach to planning of offshore gas field developments under uncertainty in reserves”. *Computers & Chemical Engineering,* 28(8): 1409-1429.

Guo, P., Huang, G. H., and Li, Y. P. (2010). “An inexact fuzzy-chance-constrained two-stage mixed-integer linear programming approach for flood diversion planning under multiple uncertainties”. *Advances in Water Resources*, 33(1): 81-91.

Hammah, R. E., and Curran, J. H. (1998). “Fuzzy cluster algorithm for the automatic identification of joint sets”. *International Journal of Rock Mechanics and Mining Sciences*, 35(7): 889-905.

Hansen, E., and Walster, G. W. (2003). *Global optimization using interval analysis: revised and expanded* (Vol. 264). CRC Press.

Hartman, H. L. (1992). *SME mining engineering handbook* (Vol. 2). S. G. Britton (Ed.). Denver: Society for Mining, Metallurgy, and Exploration.

Huang, G. H. Baetz, B. W., and Patry, G. G. (1992). “A grey linear programming approach for municipal solid waste management planning under uncertainty”. *Civil Engineering Systems*, 9(4): 319–335.

Huang, G. H., Baetz, B. W., and Patry, G. G. (1995). “Grey fuzzy integer programming: an application to regional waste management planning under uncertainty”. *Socio-Economic Planning Sciences*, 29(1): 17-38.

Huang, G. H., and Cao, M. F. (2011). “Analysis of Solution Methods for Interval Linear Programming”. *Journal of Environmental Informatics*, 17(2): 54-64.

Huang, G. H., Sae-Lim, N., Liu, L., and Chen, Z. (2001). “An interval-parameter fuzzy-stochastic programming approach for municipal solid waste management and planning”. *Environmental Modeling and Assessment*, 6(4): 271-283.

Inuiguchi, M., and Sakawa, M. (1994). “Possible and necessary optimality test in possibilistic linear programming problems”. *Fuzzy Sets and Systems*, 67(1): 29-46.

Inuiguchi, M., and Ramık, J. (2000). “Possibilistic linear programming: a brief review of fuzzy mathematical programming and a comparison with stochastic programming in portfolio selection problem”. *Fuzzy Sets and Systems*, 111(1): 3-28.

Ishibuchi, H., and Tanaka, H. (1990). "Multi-objective programming in optimization of the interval objective function". *European Journal of Operational Research*, 48(2): 219-225.

Javad, S. (2009). *Long-term open-pit planning by ant colony optimization.* (Doctoral dissertation, University Heidelberg).

Kannan, D., Khodaverdi, R., Olfat, L., Jafarian, A., and Diabat, A. (2013). "Integrated fuzzy multi criteria decision making method and multi-objective programming approach for supplier selection and order allocation in a green supply chain". *Journal of Cleaner Production*, 47: 355-367.

Krause, A., and Musingwini, C. (2007). "Modelling open pit shovel-truck systems using the machine repair model". *Journal of the South African Institute of Mining and Metallurgy*, 107(8): 469-476.

Liu, Y., Zou, R., and Guo, H. (2010). "Risk explicit interval linear programming model for uncertainty-based nutrient-reduction optimization for the lake Qionghai watershed". *Journal of Water Resources Planning and Management*, 137(1): 83-91.

Maqsood, I., and Huang, G. H. (2003). "A two-stage interval-stochastic programming model for waste management under uncertainty". *Journal of the Air & Waste Management Association*, 53(5): 540-552.

Mitra, R., and Saydam, S. (2012). "Surface coal mining methods in Australia". *Mining Methods*. InTech.

Moore, R. E. (1979). *Methods and applications of interval analysis*. Society for Industrial and Applied Mathematics.

Pan, X. (2012). "Optimization of mineral processing plant through ROM ore size". *AGH Journal of Mining and Geoengineering*, 36(4): 123-132.

Pei, W. W. (2011). *A FREILP approach for long-term planning of MSW management system in HRM, Canada.* (Master's thesis, Dalhousie University).

Richard, T. (1980). "Toward a positive theory of consumer choice". *Journal of Economic Behavior & Organization*, 1(1): 39-60.

Rommelfanger, H., Hanuscheck, R., and Wolf, J. (1989). "Linear programming with fuzzy objectives". *Fuzzy Sets and Systems*, 29(1): 31-48.

Rubinstein, R. Y., and Marcus, R. (1985). "Efficiency of multivariate control variates in Monte Carlo simulation". *Operations Research*, 33(3): 661-677.

Sahoo, S. (2012). *Truck allocation model using linear programming and queueing theory.* (Doctoral dissertation, National Institute of Technology).

Samanta, B., Sarkar, B., and Mukherjee, S. K. (2002). "Selection of opencast mining equipment by a multi-criteria decision-making process". *Mining Technology*, 111(2): 136-142.

Schultz, R. (2003). "Stochastic programming with integer variables". *Mathematical Programming*, 97(1): 285-309.

Schwarm, A. T., and Nikolaou, M. (1999). "Chance-constrained model predictive control". *AIChE Journal*, 45(8): 1743-1752.

Steuer, R. E. (1981). "Algorithms for linear programming problems with interval objective function coefficients". *Mathematics of Operations Research*, 6(3): 333-348.

Sturgul, J. R., and Li, Z. (1997). "New developments in simulation technology and applications in the minerals industry". *International Journal of Surface Mining, Reclamation and Environment*, 11(4): 159-162.

Tan, Y. F. (2012). *Enhancing simulation models for open pit copper mining using visual basic for applications.* (Master's thesis, Chuo Gakuin University).

Tan, Y., Miwa, K., Chinbat, U., and Takakuwa, S. (2012). "Operations modeling and analysis of open pit copper mining using GPS tracking data". In *Proceedings of the Winter Simulation Conference* (p. 117). Winter Simulation Conference.

Tong, S. C. (1994). "Interval number and fuzzy number linear programmings". *Fuzzy Sets and Systems*, 66(3): 301-306.

Torkamani, E., and Askari-Nasab, H. (2015). "A linkage of truck-and-shovel operations to short-term mine plans using discrete-event simulation". *International Journal of Mining and Mineral Engineering*, 6(2): 97-118.

Usama, F. (1996). "From data mining to knowledge discovery in databases". *AI Magazine*, 17(3): 37.

Zhou, F. Huang, G. H. Chen, G. X., and Guo, H. C. (2009). "Enhanced interval linear programming". *European Journal of Operational Research*, 199(2): 323-333.

Zimmermann, H. J. (1985). "Applications of fuzzy set theory to mathematical programming". *Information Sciences*, 36(1-2): 29-58.

Zou, R., Liu, Y., Liu, L., and Guo, H. (2009). "REILP approach for uncertainty-based decision making in civil engineering". *Journal of Computing in Civil Engineering*, 24(4): 357-364.