

ENHANCING SIGNAL ALIGNMENT IN MIMO Y CHANNELS
WITH SPACE AND TIME SCHEDULING

by

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To My Parents

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Abstract

Spectrum sharing and relaying in wireless networks offer attractive solutions in communication system designs, especially when combined with more advanced signal processing in the terminals and the relays operating in multi-antenna configurations.

A promising technique in this area is signal alignment (SA) that efficiently exploits the available throughput in time and spatial domains.

This thesis examines the design of signal processing algorithms with SA in networks operating in multiple-input multiple-output (MIMO) Y channel configurations, where multiple terminals communicate with each other in a bi-directional manner via the relay. Two main contributions are presented: (i) the integration of distributed space-time block coding (STBC) and SA in MIMO Y channels is presented for enhancing system reliability of MIMO Y channels (ii) a time scheduling scheme is proposed to address the problem of large antenna number requirements on nodes in generalized MIMO Y channels.

List of Abbreviations and Symbols Used

| | |
|---------------------|---|
| 3G | the third generation |
| AF | amplify-and-forward |
| ANC | analog network coding |
| AWGN | additive white Gaussian noise |
| BC | broadcast |
| BER | bit error rate |
| BPSK | binary phase shift keying |
| CDMA | code division multiple access |
| CSI | channel state information |
| DF | decode-and-forward |
| FEC | Forward error control |
| IA | interference alignment |
| IC | interference channel |
| i.i.d | independent identically distributed |
| MAC | medium access control |
| MAI | multiple access interference |
| MATLAB [®] | mathematical laboratory |
| MIMO | multiple-input multiple-output |
| MMSE | minimum mean square error |
| ML | maximum likelihood |
| MUD | multiuser detection |
| MU-MIMO | multi-user multiple-input multiple-output |
| NC | network coding |
| OSTBC | orthogonal space-time block code |
| PNC | physical network coding |
| QAM | quadrature amplitude modulation |
| SA | signal alignment |
| SISO | single-input single-output |

| | |
|-------|--|
| SNR | signal-to-noise ratio |
| STBC | space-time block code |
| TDMA | time division multiple access |
| TS(s) | time slot(s) |
| TSER | ternary symbol error rate |
| TWRC | two-way relay channel |
| WCDMA | wideband code division multiple access |
| WLAN | wireless local area network |
| WN | wireless network |
| WSN | wireless sensor network |
| XOR | exclusive or |
| ZF | zero-forcing |

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Chapter 1

Introduction

With the growing demands for wireless communication services to support higher bit rates, there is an ever-pressing need to increase bandwidth efficiency of existing communication systems. To address this and many other issues, such as extending the coverage area and reducing multiple access interference (MAI), one of the most successful and efficient solutions is the deployment of relays [1]. Relays that receive and re-transmit the signals between the source and the destination terminals have been used in communication networks for a long time, but only recently with the increasing computational and processing capabilities in the terminals and relays, more advanced configurations of relay networks are proposed. These configurations take advantage of more complex signal processing operations in the terminals and relays, where at the relays the transmitted signals are processed versions of the received signals.

In wireless networks, where multiple transmit and receive user pairs communicate using the same radio channel which is a shared medium, simultaneous transmissions introduce MAI and are usually controlled through time or frequency slotting of different transmissions. Allowing better coordination between transmit and receive user pairs enables higher throughput in communication systems and one of the most promising techniques in this area is Interference Alignment (IA). Interference alignment is a linear precoding technique that attempts to align interfering signals in time, frequency, or space.

When a multi-way communication network with three nodes and a relay is considered, it is called the multiple-input multiple-output (MIMO) Y channel. In this system, where three terminals communicate with each other in a bi-directional manner via the relay, there have been proposals deploying IA to increase the throughput measured by the number of symbols transmitted on a single channel access. Specifically, in terminals and the relay deploying MIMO configurations of antennas, IA uses

the spatial dimensions offered by multiple antennas for alignment, and even though the signals interfere at the destination, the required data can be recovered. However, these schemes are demonstrated to offer increased throughput only in channels with high signal-to-noise ratios (SNRs) [2].

In a conventional MIMO system setup where a single terminal communicating with another terminal using multiple antennas, space-time coding involves the use of coding across time and antennas to improve the bit error rate (BER) performance of the communication link. This is achieved at the expense of reduced throughput of the MIMO system operating in the multiplexing gain configurations [3], where information bits are transmitted multiple times in specialized versions from different antennas. In this thesis, to improve the BER performance of the IA schemes in the MIMO Y channel at lower SNRs, we proposed to integrate the space-time block coding (STBC) which is tailored to the unique features of the underlying communication system into the MIMO Y channels. Specifically, we have designed the precoding and time scheduling of the existing STBC to match the antenna configurations and signal processing in the conventional MIMO Y channel with IA alone. We have also designed a transmission scheduling scheme in order to reduce the number of required antennas in generalized MIMO Y channels. As the MIMO Y channel is a good model for communication in ad hoc networks, wireless local area networks (WLAN), and wireless sensor networks (WSN) [4], we expect that the proposed schemes will find practical applications in networks with a relay or where one of the terminals assumes the relay role.

1.1 Relay Networks and Wireless Network Coding

There are two major methods to process received signals at the relay: decode-and-forward (DF) and amplify-and-forward (AF) [5]. AF relays re-transmit the signal without decoding while DF relays decode the received signal, encode the signal again, and transmit. To be specific, the difference between the two schemes is that using DF, the relay detects and potentially processes the received signal at the bit/symbol level and re-transmits it, while with AF, the relay just amplifies the continuous time (modulated) received signal. In both cases, working with multiple antennas (the MIMO configuration) allows to take advantage of either spatial diversity or multiplexing gain.

In the former case, multiple versions of received signals improve the reliability of communication as measured through BER as a function of SNR. In the case of multiplexing gain, through postprocessing at the receiver and/or potentially pre-processing at the transmitter, multiple parallel virtual channels are created, increasing bandwidth efficiency of the single-input single-output (SISO) physical channel. In this thesis, the relay is working in the DF type of a MIMO mode.

The idea of re-transmitting the processed bit level versions of the received data at the relays is attributed to Ahlswede et al. in [6] and is called network coding (NC). It was originally introduced in the context of wire-line networks and has drawn enormous attention due to its efficiency in improving the system capacity [7]. The fundamental idea of NC is to allow immediate node(s)/relay(s) to perform linear operations on received messages from the different sources. This is done through a process such that the receiver(s) can recover the original data from the delivered messages. This way of processing the data at the relays has the objective of reducing the total number of time slots required to transmit data. The wireless extension of NC, physical-layer network coding (PNC) [8], [9] and analog-network coding (ANC) [10], [11] also significantly boost the system throughput, especially in the model of two-way relay channel (TWRC) [12] visualized in Fig. 1.1 [13]. In this case, two

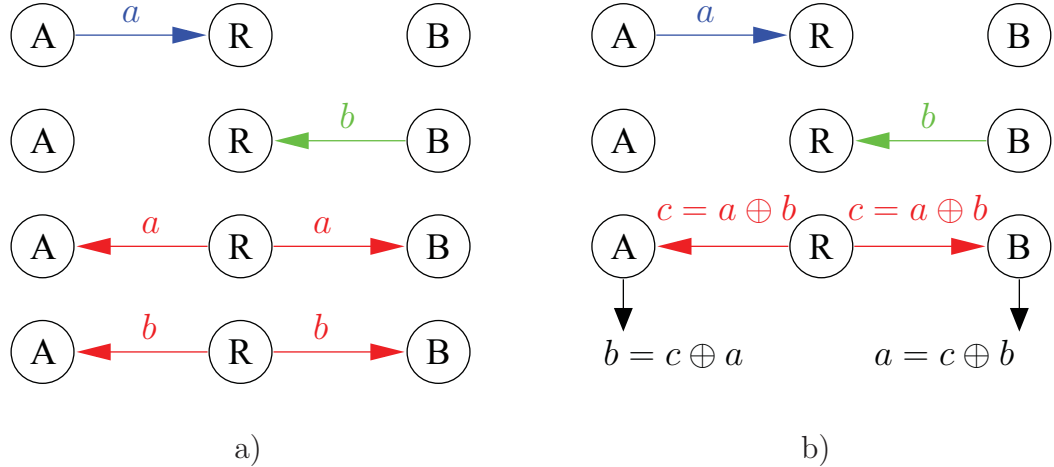


Figure 1.1: Conventional DF vs. Wireless NC of a relay node.

terminals, A and B, intend to exchange data (at the bit level) a and b via the relay R: i.e., A and B do not use overlapping transmission ranges. We assume here, as it is the case in our work, that all devices operate in half-duplex mode: i.e., in any

given time slot (TS) the device either receives or transmits the data. In conventional DF visualized in Fig. 1.1(a), the exchange process proceeds with nodes A and B first broadcasting their data (bits) to node R in two time slots (TSs). Node R then re-broadcasts packet a and subsequently, packet b , which completes the data exchange in four TSs. In wireless NC implementation of the same system in Fig. 1.1(b), instead of using two separate transmissions to send packets a and b , NC allows the relay node R to encode the two received packets and broadcasts a single, XORed packet $c = a \oplus b$. Both nodes A and B can recover desired packets b and a from the c packet by XOR-ing c with their own packets a and b , respectively. The data exchange is completed in the NC mode in three TSs which improves the throughput (bandwidth efficiency) but also improves the energy efficiency as the wireless NC requires less data to be sent. These improvements are possible because we take advantage of the broadcast characteristics of wireless transmissions (both A and B overhear the same signal in the third TS).

1.2 Interference and Signal Alignment

In wireless communication systems where, in order to increase the bandwidth efficiency, multiple users transmit at the same time and in the same frequency band and do not employ orthogonal multiplexing, it is unavoidable to face interference between users because of radio channel broadcast characteristics. A lot of efforts have been made to combat the detrimental effects of this MAI. Some of the most recognized approaches include the development of code division multiple access (CDMA) requiring bandwidth expansion and multiuser detection (MUD) techniques. A new effort in this area is the interference alignment which allows concurrent decoding of transmitter-receiver pairs communications in MIMO networks.

Interference alignment introduced by V. Cadambe and S. Jafar *et al.* [14]–[16] has become an active research area because of its advantage in terms of interference suppression and throughput improvements. The idea of IA is to “combine” all interference signals together and occupy only part of the radio channel resources, such as time, frequency and spatial domains, for “unobstructed” (interference-free) data transmission. An example of IA applied in K -user interference channel (IC) developed in [15] is shown in Fig. 1.2. In this K -user IC, there are K transmitter-

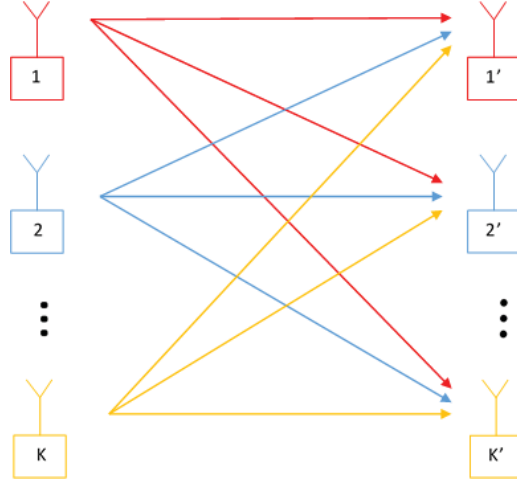


Figure 1.2: K -user interference channel.

receiver pairs sharing one common wireless channel, where interference is coming from other $(K-1)$ users in the common channel. Applying IA, interference signals would be aligned along the same direction in space by carefully designing the precoding vector for each transmitter. Consequently, the capacity of the system is shown as $C(SNR) = \frac{K}{2} \log(SNR) + O(\log(SNR))$ [15]. The result is outstanding because previously the common belief was that $C(SNR) = \log(SNR) + O(\log(SNR))$ would be the best achievable capacity by evenly dividing the resources for the K -users.

Signal alignment (SA) is an extended concept of interference alignment [4, 17]. Instead of treating some of signals as noise, SA views every signal as equally useful and groups those signals based on the requirements from the perspective of individual users. Working along with network coding, SA shows its efficiency particularly in TWRC. For instance, in a $K=6$ -user TWRC model shown in Fig. 1.3, three paired transmitters and receivers intend to convey information through the relay station. In this system, the terminals indexed by j and j' ($j, j' \in \{1, 2, 3\}$) wish to exchange different data color coded with the same color for each of the three communication pairs, which is similar to the generalized case in Fig. 1.1. The transmitted signals are precoded so that paired messages are aligned in one spatial dimension and they are sent in the same time slot. This is assuming the relay is equipped with three antennas, while terminals use just one antenna. Three different dimensions are represented in different colors in Fig. 1.3. Thereafter, the relay station broadcasts this combined signal, and each user is able to obtain the desired message by subtracting its own sent

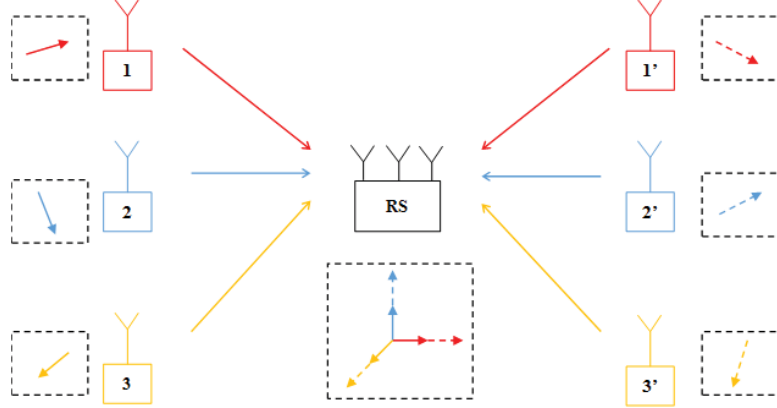


Figure 1.3: 3-paired TWRC with SA.

message – so called self-interference. By doing so, six messages are exchanged within two TSs, taking advantage of ideas from wireless network coding and MIMO channels. It is worth mentioning that this scheme also reduces the antenna configuration complexity. Considering the same system model working in a traditional network coding manner, the relay would have to be equipped with K antennas to receive K separate signals (assuming transmitters to the left and to the right sending in the same time slot, which was not the case in Fig. 1.1). However, the relay only requires $\frac{K}{2}$ antennas for $\frac{K}{2}$ signal dimensions using SA, which is half of the previous requirement.

1.3 MIMO Y Channel Model

The MIMO Y channel is an extended model of a wireless two-way relay channel originally presented by Lee in [4]. The system model of the original three-user MIMO Y channel is illustrated in Fig. 1.4. This is also the model used in the majority of this thesis.

The MIMO Y channel model consists of three user nodes equipped with two antennas and one relay station equipped with three antennas. Every user is intended to exchange one message with other users in the model. The message bit is denoted as b_{ij} , where i is the destination node number and j is the source node number ($j, i \in \{1, 2, 3\}$) and at the bit level $b_{ij} \in \{1, 0\}$. Because the direct links between user nodes are assumed to be unavailable, every user communicates with other user nodes through the relay station. It is assumed that all nodes are working in half-duplex mode and have the perfect global channel state information (CSI) in the system. A

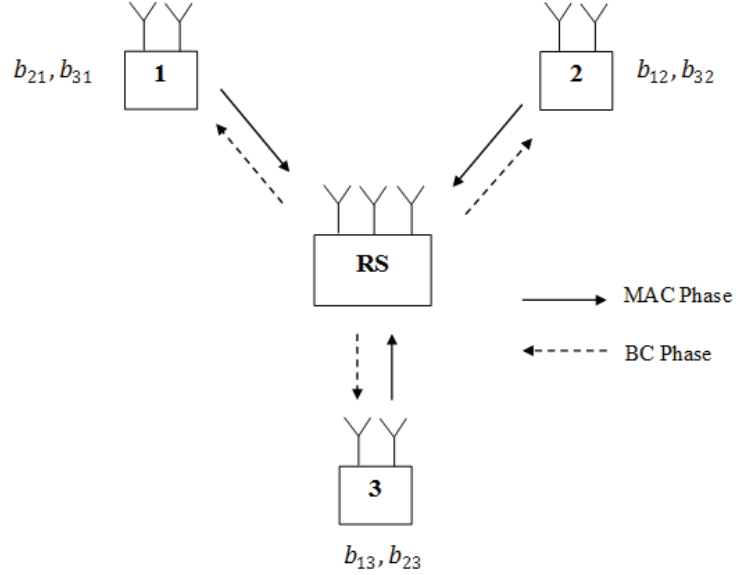


Figure 1.4: 3-user MIMO Y channel model.

typical communication of six messages is completed with two phases: medium access control (MAC) phase and broadcast phase (BC).

1.3.1 MAC Phase

In the MAC phase, all messages are first processed with their precoding vectors/matrices at each user node, and then transmitted to the relay station simultaneously. After receiving, a decode-and-forward scheme is used to obtain the messages at the relay station. The precoding vector for each message is designed to align two independent messages from the same communication pair (referred to as two mutual messages) into one signal dimension. The signal alignment is displayed in Fig. 1.5. Here D_{1-2} , D_{1-3} and D_{2-3} denote three signal dimensions. Colors are used to differentiate between user pairs. The placement of messages in the space is determined by the product of the channel gain matrix H_{Rk} and the precoding vector P_{dk} . The channel gain matrix H_{Rk} is the 3×2 matrix with each element $H_{Rk}(t, f)$ representing the gain from transmit antenna f of the k -th terminal to the receive antenna t of the relay R . While in this section, we differentiate between the channel gain matrix H_{Rk} from the k terminal to the relay in the MAC phase and H_{kR} from the relay R to the k terminal in the BC phase. Later on and in the figures, we refer to these channels just as H_k , which does not necessarily imply the reciprocity of channels but is used

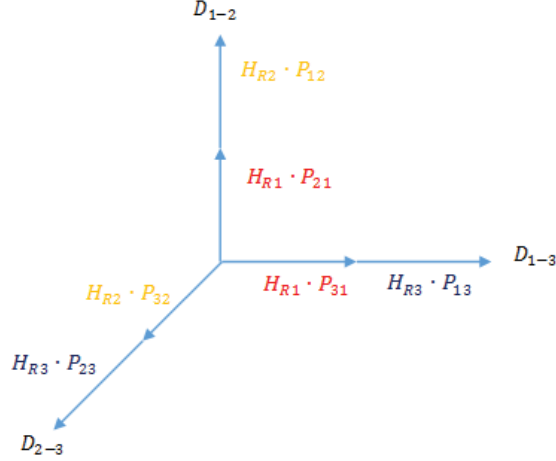


Figure 1.5: 3-user MIMO Y channel SA for MAC phase.

just for the convenience of notation.

The precoding vectors P_{lk} of size 2×1 when concatenated $P_k = [P_{lk} \mid P_{pk}]$ form the precoding matrix P_k of size 2×2 used at the k -th user to beamform (shape) the transmitted data. The transmitted data by terminal k is $P_k \cdot \begin{bmatrix} m_{lk} \\ m_{pk} \end{bmatrix}$ where $l, p \neq k$ and $m_{lk} \in \{1, -1\}$ are the Binary Phase Shift Keyed (BPSK) representations of the bits b_{lk} . The received signal from a single user k at the relay is $H_{Rk} \cdot P_k \cdot \begin{bmatrix} m_{lk} \\ m_{pk} \end{bmatrix}$. Since the channel matrix depends on the channel itself, precoding vectors are the only factors that we can manipulate to tailor messages into the desired spatial (antenna) signal dimensions. As shown in Fig. 1.5, the paired vectors with swiped source and destination in precoding vectors are aligned together in the same signal dimension. This means that mutual messages between two users are also aligned and they are represented by the following system of linear equations:

$$\begin{cases} H_{R1} \cdot P_{21} = H_{R2} \cdot P_{12} \\ H_{R1} \cdot P_{31} = H_{R3} \cdot P_{13} \\ H_{R3} \cdot P_{23} = H_{R2} \cdot P_{32} \end{cases} \quad (1.1)$$

Assuming that precoding vectors meet the conditions imposed in (1.1), after precoding at every node and sending messages to the relay at the same time, the received

signal at relay station is written as:

$$\begin{aligned}
Y_R &= H_{R1} \cdot P_1 \cdot \begin{bmatrix} m_{21} \\ m_{31} \end{bmatrix} + H_{R2} \cdot P_2 \cdot \begin{bmatrix} m_{12} \\ m_{32} \end{bmatrix} + H_{R3} \cdot P_3 \cdot \begin{bmatrix} m_{13} \\ m_{23} \end{bmatrix} \\
&= H_{R1} \cdot P_{21} \cdot m_{21} + H_{R1} \cdot P_{31} \cdot m_{31} + H_{R2} \cdot P_{12} \cdot m_{12} + H_{R2} \cdot P_{32} \cdot m_{32} + \\
&\quad + H_{R3} \cdot P_{13} \cdot m_{13} + H_{R3} \cdot P_{23} \cdot m_{23}
\end{aligned} \tag{1.2}$$

We disregard here initially the effects of the additive white Gaussian noise (AWGN) omnipresent in any communication receiver. Due to the design of the signal alignment presented in this section, the corresponding paired messages (b_{ij} and b_{ji}) can be combined in an analog version of the network-coded form. First, based on (1.1) we defined the effective channel gain matrix in three of the spatial dimensions as follows:

$$G_{i-j} = H_{Ri} \cdot P_{ji} = H_{Rj} \cdot P_{ij} \tag{1.3}$$

With this, (1.2) can be rewritten into:

$$\begin{aligned}
Y_R &= (H_{R1} \cdot P_{21} \cdot m_{21} + H_{R2} \cdot P_{12} \cdot m_{12}) + (H_{R1} \cdot P_{31} \cdot m_{31} + H_{R3} \cdot P_{13} \cdot m_{13}) + \\
&\quad + (H_{R2} \cdot P_{32} \cdot m_{32} + H_{R3} \cdot P_{23} \cdot m_{23}) \\
&= G_{1-2} \cdot (m_{21} + m_{12}) + G_{1-3} \cdot (m_{31} + m_{13}) + G_{2-3} \cdot (m_{23} + m_{32}) \\
&= G \cdot [(m_{21} + m_{12}), (m_{31} + m_{13}), (m_{23} + m_{32})]^T
\end{aligned} \tag{1.4}$$

where G is the 3×3 matrix obtained by concatenating effective channel gain matrices G_{1-2} , G_{1-3} and G_{2-3} each of size 3×1 . With this representation, in the zero-forcing (ZF) approach, the recovery of the signal of interest is obtained using the matrix G inversion.

It is worth observing that in this $K=3$ -user MIMO Y channel configuration, although the three dimensions in Fig. 1.5 seem to be placed as orthogonal to each other, we do not actually impose orthogonality constraints over the directions D_{i-j} of the effective channel gain vector/matrix G_{i-j} . Because the model is in minimum configuration without redundancy antennas and channel is pre-determined, we do not have control over the direction of $\begin{bmatrix} P_{ij} \\ P_{ji} \end{bmatrix}$ in (1.3). Usually we constrain the norm square of the vector $\begin{bmatrix} P_{ij} \\ P_{ji} \end{bmatrix}$ to be one which has the implication on the user powers. The power issue in the system is further discussed in Appendix B.

In most publications on the MIMO Y channel, the relay processes the received signal in DF manner [18]–[21]. By using the ZF, minimum mean square error (MMSE)

or maximum likelihood (ML) decoder, the relay is able to obtain the network-coded version of paired messages. If the transmitted data in the MAC phase are BPSK, i.e., $m_{ij} = \pm 1$, that implies the three levels on the recovered (ternary) data at the relay on the mutual channel, i.e., $m_{lk} + m_{kl} = \pm 2$ or 0. In the transmission phase (called BC phase) from the relay to all the terminals in the same time slot, the recovered ternary symbols $m_{lk} + m_{kl}$ will be re-mapped into the BPSK format $(m_{lk} + m_{kl})_B$ such that ± 2 is mapped into -1 and 0 is mapped into 1 [8]. With this, the transmitted binary signals from the relay $(m_{21} + m_{12})_B$, $(m_{31} + m_{13})_B$, $(m_{23} + m_{32})_B$ represent the messages $(b_{21} \oplus b_{12})$, $(b_{31} \oplus b_{13})$, $(b_{23} \oplus b_{32})$. In the next subsection, when we describe the BC phase, for the brevity of notation, rather than writing $(m_{ij} + m_{ji})_B$ we will just write $(m_{ij} + m_{ji})$ and from the context of the BC phase will infer that this represents the binary representation.

1.3.2 BC Phase

In the broadcast (BC) phase, a combined signal representing three pairs of network-coded signals is generated by applying nulling vectors at the relay, and it is broadcasted to all user nodes in the same time slot as one signal X_R which is of size $[3 \times 1]$. Considering that each user is interested in receiving only two network coded messages/symbols (out of the three available at the relay) and that it operates using two receive antennas, the nulling vector denoted as U_{i-j} of size $[3 \times 1]$ is applied at the relay in order to remove the effects of the undesired signal for each user node. This is with the purpose that each user receives relevant data in the desired signal dimensions and eventually can decode intended messages by conducting XOR operations. The principle here is that the information bearing signal at the relay $[(m_{21} + m_{12}), (m_{31} + m_{13}), (m_{23} + m_{32})]^T$ is precoded with the matrix $U = [U_{1-2} \mid U_{1-3} \mid U_{2-3}]$ of size 3×3 which is the concatenation of nulling vectors and where T stands for transpose. After precoding with the nulling matrix, the re-transmitted signal at the relay is written as:

$$\begin{aligned} X_R &= U \cdot [(m_{21} + m_{12}), (m_{31} + m_{13}), (m_{23} + m_{32})] \\ &= U_{1-2} \cdot (m_{21} + m_{12}) + U_{1-3} \cdot (m_{31} + m_{13}) + U_{2-3} \cdot (m_{23} + m_{32}) \end{aligned} \tag{1.5}$$

To realize the idea described in this section, nulling vectors U' s are designed to satisfy the conditions:

$$\begin{cases} H_{1R} \cdot U_{2-3} = 0 \\ H_{2R} \cdot U_{1-3} = 0 \\ H_{3R} \cdot U_{1-2} = 0 \end{cases} \quad (1.6)$$

where H_{kR} represents the channel gain matrix from the relay R to the user k which is of size 2×3 . To explain the constraints in (1.6), we consider the received signal by the first terminal $Y_{1R} = H_{1R} \cdot X_R$, i.e.:

$$\begin{aligned} Y_{1R} &= H_{1R} \cdot U \cdot [(m_{21} + m_{12}), (m_{31} + m_{13}), (m_{23} + m_{32})]^T \\ &= H_{1R} \cdot U_{1-2} \cdot (m_{21} + m_{12}) + H_{1R} \cdot U_{1-3} \cdot (m_{31} + m_{13}) + H_{1R} \cdot U_{2-3} \cdot (m_{23} + m_{32}) \end{aligned} \quad (1.7)$$

The signal dimension D_{2-3} contains a mutual message between User 2 and 3, but for User 1 this message is irrelevant. Hence U_{2-3} is designed to null this undesired signal for User 1 by letting the product of channel matrix H_{1R} and U_{2-3} to be zero. The other two nulling vectors U_{1-2} and U_{1-3} are constructed in the same manner from the perspective of User 1 and User 2 respectively.

In summary, the common broadcast signal X_R will pass through different channels and reach the users' receiver antennas with only relevant information content for the user. For each user, considering the constraints in (1.6), the received signal of size 2×1 is written as

$$Y_{1R} = H_{1R} \cdot [U_{1-2} \cdot (m_{21} + m_{12}) + U_{1-3} \cdot (m_{31} + m_{13})] \quad (1.8)$$

$$Y_{2R} = H_{2R} \cdot [U_{1-2} \cdot (m_{21} + m_{12}) + U_{2-3} \cdot (m_{32} + m_{23})] \quad (1.9)$$

$$Y_{3R} = H_{3R} \cdot [U_{1-3} \cdot (m_{31} + m_{13}) + U_{2-3} \cdot (m_{32} + m_{23})] \quad (1.10)$$

To recover data of interest at User 1, the received signal is written in a compact form as:

$$\begin{aligned} Y_{1R} &= H_{1R} \cdot [U_{1-2} \mid U_{1-3}] \cdot [(m_{21} + m_{12}), (m_{31} + m_{13})]^T \\ &= H_{1R} \cdot U_1 \cdot [(m_{21} + m_{12}), (m_{31} + m_{13})]^T \end{aligned} \quad (1.11)$$

where U_1 and $H_{1R} \cdot U_1$ are the matrices of size 3×2 and 2×2 , respectively and U_1 is concatenation of nulling vectors U_{1-2} and U_{1-3} . Similarly, as in the MAC phase, the

recovery of signal of interest $[(m_{21} + m_{12}), (m_{31} + m_{13})]$ at User 1 can be performed using the ZF approach by inverting $H_{1R} \cdot U_1$.

What differentiates $(m_{ij} + m_{ji})$ in the BC phase is that these are BPSK representations of $b_{ij} \oplus b_{ji}$, while in the MAC phase $(m_{ij} + m_{ji})$ had the ternary representation. So, for User 1, knowing the network coded messages $b_{21} \oplus b_{12}$ and $b_{31} \oplus b_{13}$ can be used to calculate the desired messages using the XOR operations as:

$$b_{ij} = b_{ji} \oplus (b_{ij} \oplus b_{ji}) \quad (1.12)$$

1.3.3 Implementation Feasibility of SA in MIMO Y Channels

In the original paper [15], the feasibility of implementing IA in MIMO Y channels is derived from the perspective of signal space. Here, we consider it as a problem of finding solutions to a linear system of equations. At the same time, the minimum antenna numbers in the system configuration can also be correspondingly determined.

In the MAC phase, the signals transmitted from each user are precoded to align six messages into three signal dimensions. To achieve this, the constraints given in (1.1) are interpreted here as the linear system of equations.

In general cases, we consider that M is the number of antennas at the user node and N is the number of antennas at the relay node. Each channel matrix H_k in (1.1) would be a $N \times M$ matrix; meanwhile, P_{lk} is a precoding vector which is $M \times 1$. Therefore, there are $2M$ unknowns and N equations in every line of (1.1) defining one dimension. In order to ensure this equation set is solvable and that the solution set is not all zeros as well, the systems of equations in (1.1) have to be an underdetermined system of equations, meaning that there ought to be more unknowns than equations. As a result, the constraint on a number of antennas is written as [4]:

$$N \leq 2M - 1 \quad (1.13)$$

which is met in our example case of $M = 2$ and $N = 3$ when presenting the MIMO Y channel with interference/signal alignment. When in (1.13) we have the equality, we refer to this configuration of antennas as the minimum configuration in a sense that there is no flexibility in the choice of effective channel gain directions.

In the BC phase of three-user case, the relay also precodes the signal before broadcasting to all users. The nulling vectors discussed in Section 1.3.2 ensure that

the received signal at user node is only composed of two signal dimensions of interest. The constraint for each user is presented in (1.6). Using the same reasons as in the MAC phase discussions, (1.6) also has to be underdetermined, which leads to the second constraint in (1.14). Meanwhile, it's worth mentioning that the constraint for BC phase is not applicable to general cases because more than one spatial dimensions need to be nulled for each user.

$$N \geq M + 1 \tag{1.14}$$

The minimum configuration of antenna number for MIMO Y channel and its extensions for higher number of users than three could be discussed accordingly and has been presented in [15]. We discuss the required antenna numbers of $K=4$ user case in Appendix A.

The precoding for signal and interference alignment in the MIMO Y channel in MAC and BC phases is based on the knowledge of the channel gain matrices between different nodes and the relay, and with this knowledge, nodes are permitted to perform advanced (linear) processing of information to increase bandwidth efficiency. Estimates of the channel gains, i.e., CSI, can be obtained using training sequences at the beginning of transmission using known sequences of data. To accommodate time varying channels, the transmission of training sequences is repeated in a periodic fashion which may lead to the reduction of effective throughput. Also, from the implementation point of view, the calculation of the precoding matrices and the distribution of CSI is of concern, as it would have to be implemented either in the centralized fashion or de-centralized fashion. This would require the protocol overhead to distribute the information, which is not in the scope of this thesis.

Noise Enhancement and Error Propagation

In the SA approach presented in this section, we imposed in (1.1) and (1.6) two conditions for aligning the signal of interest in the MAC and nulling the irrelevant signals in the BC phase, respectively. While these conditions have the format of ZF, in this thesis we refer to the ZF approach as the methodology used to recover data of interest involving matrix inversion, which is deployed in this thesis in both MAC and BC phases. Specifically, in the MAC phase, when considering the received signal at the relay as in (1.15), we should account for the AWGN vector $N_R = [n_1, n_2, n_3]^T$

where n_i represents the noise in the i -th receiving antenna at the relay and the components of the N_R vector are independent and identically distributed (i.i.d.), which is the standard assumption in this field. With this, the received signal at the relay is:

$$Y_R = G \cdot [(m_{21} + m_{12}), (m_{31} + m_{13}), (m_{23} + m_{32})]^T + N_R \quad (1.15)$$

Similarly, in the BC phase, the received signal at the User 1 given initially in (1.11), when accounting for the AWGN in the antenna array of the User 1 is:

$$Y_{1R} = H_{1R} \cdot U_1 \cdot [(m_{21} + m_{12}), (m_{31} + m_{13})]^T + N_1 \quad (1.16)$$

With the representations in (1.15) and (1.16), the data of interest and the observable vector, generically referred to henceforth as X and Y , are linked through the square known matrix A as $Y = A \cdot X + N$ where N is the noise in the receiver. In order to recover the data of interest, we use matrix inversion $\hat{x} = A^{-1}y = x + A^{-1} \cdot N$. It is the term $A^{-1} \cdot N$ that may lead the variance of the noise of different sub-channels to become very high causing what is referred to in the literature as noise enhancement. This problem has been known for quite some time and there are some methods like MMSE reception that can partially overcome the negative effects of ZF matrix inversion. However, ZF is an elegant approach and we use it in the thesis so as not to obstruct the real issues addressed here.

Yet another issue in networks deploying NC type processing is what is called error propagation due to hard-decision on the data received at the relay. This is because data received in error on any intermediate links will affect detection on all end-to-end links. It is these problems of noise enhancement and error propagation that make the SA in the MIMO Y channel suitable only in high SNRs.

1.4 Space-Time Block Coding

Space-time coding involves the use of coding across time and transmit antennas to improve the BER performance or throughput of the MIMO communication link as illustrated in Fig. 1.6. The input symbols entering the space-time encoder serially are distributed to parallel sub-streams, which are then emitted at the same time in the same frequency band from the antenna corresponding to that sub-stream. Signals from narrowband modulation schemes, like quadrature amplitude modulation

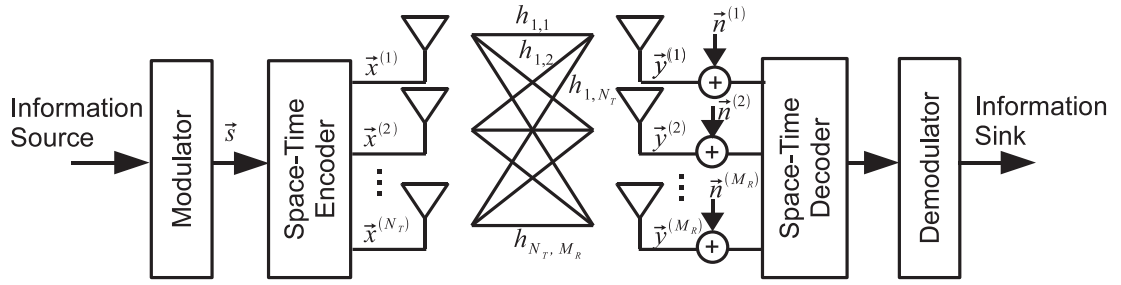


Figure 1.6: Block diagram of a MIMO system with N_T transmit and M_R receive antennas.

(QAM), transmitted simultaneously over each antenna interfere in time and frequency with each other as they propagate through the wireless channel. The flat fading channels represented in Fig. 1.6 with complex random variables h_{kl} also distort the signal waveforms. In the MIMO system, by proper pre- and post-processing of the signals using the space and time dimensions, the distorted and superimposed waveforms detected by each receive antenna are used to estimate the original input symbols at the receiver with improved accuracy as compared with SISO links. STBC is in essence a diversity technique deployed to combat fading effects in a wireless MIMO channel [22].

1.4.1 Alamouti's Space-Time Code

A particularly elegant scheme for MIMO coding was developed by Alamouti [23]. Alamouti's space-time code is a complex valued orthogonal STBC (OSTBC) working with two transmit antennas and can be seen as a specialized version of a repetition code. Assuming the MIMO channel gains are constant over the code duration, this code works with two transmit antennas over two time slots using as the codewords:

$$X_{Alamouti} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \quad (1.17)$$

where the rows represent encoding along the antenna (space) dimension and the columns represent encoding along the time dimension. That is, in the first time slot, two symbols x_1 and x_2 are simultaneously transmitted from antenna 1 and 2, respectively. In the second time slot, symbol $-x_2^*$ is transmitted from antenna 1 and x_1^* is transmitted from antenna 2, where the superscript $*$ denotes complex conjugate.

Because of the orthogonality of codewords, symbols are received in parallel form and can be easily decoded at the receiver through linear processing. The Alamouti scheme can be applied in links with two transmit and an arbitrary number of receive antennas. In addition, Alamouti's code transmits two symbols within two time slots, so the code rate is equal to $R = 2/2 = 1$, which means no data rate is sacrificed comparing to the corresponding SISO model. Due to these two significant features, Alamouti's code has been widely adopted in applications, such as the third generation (3G) cellular standard WCDMA [3]. In Chapter 2 of this thesis, the proposed scheme employs Alamouti's code as the transmission matrix in the BC phase.

1.4.2 Orthogonal STBC in a Single MIMO Link

Orthogonal STBC, studied in [22], is one of the most important classes of linear STBC and generalizes the Alamouti codes. As its name indicates, in OSTBC the space-time codewords are orthogonal. For a model with n transmit antennas and k transmit symbols x_1, x_2, \dots, x_k , the transmitted matrix X of data symbols satisfy the following condition:

$$X^H X = (|x_1|^2 + |x_2|^2 + \dots + |x_k|^2) \cdot I_{n \times n} \quad (1.18)$$

where H stands for Hermitian transpose. This property guarantees the received symbols in a parallel form so that the simple linear decoding can be performed to recover symbols.

Assuming a single receiver, OSTBC can provide the full transmit diversity of n with the code rate $\frac{n}{k}$ [24]. Alamouti's code is the only STBC code of full rate for complex constellations with linear processing. It also has been proven in [25] that for more than two transmit antennas, the data rate can not exceed even $\frac{3}{4}$ for such codes. There are still some extensive investigations to search for high rate OSTBC with a high number of antennas. Table 1.1 summarizes some the research work on the design of OSTBC with high antenna numbers. When integrating STBC into the SA algorithms in the MIMO Y channel as proposed in Chapter 2 of this thesis, we will work with the OSTBC code proposed by O. Tirkkonen in [26] where data are encoded along the three spatial dimensions in a block of four time slots.

Table 1.1: Research progress into the design of OSTBC with high antenna numbers.

| Rate | Author | Number of Spatial Streams |
|------|-----------------------------------|---------------------------|
| 1 | M. Alamouti [23] | 2 |
| 3/4 | V. Tarokh [22], O. Tirkkonen [26] | 3, 4 |
| 2/3 | X. B. Liang [27] | 5 |
| 2/3 | C. Xu [28] | 5, 6 |
| 7/11 | W. Su [29] | 5 |
| 5/8 | C. Xu [28] | 7 |
| 3/5 | W. Su [29] | 6 |
| 1/2 | V. Tarokh [22] | Any |

1.5 Thesis Objectives

The general objective of this work is to study the methods to improve the performance of MIMO Y channels with SA regarding two aspects: (i) system reliability and (ii) antenna array configurations of reduced complexity.

To accomplish the first goal, innovative designs of integrated space-time block codes and SA in two phases of MIMO Y channel are proposed. In particular, the focus is to develop an efficient transmission algorithm that reduces the BER in the MIMO Y channel in low SNR regions as compared to the BER in systems without STBC. The enhancement in the system reliability benefiting from STBC diversity gain is targeted to not lose much of the high data-rate in MIMO Y channel schemes with SA alone. To this end, we devised joint pre-processing and data transmission scheduling in both MAC and BC phases so that that signals are arranged in the proper spatial dimensions and decoding of NC data is possible. Computation complexity issues are also considered in the proposed schemes. Specifically, based on the characteristics of space-time block code structures, our objective is to take advantage of the reuse of precoding vectors and nulling vectors in the proposed scheme.

The second target considers the potential problem of increasing the number of antennas with rising the number of users in the MIMO Y channel. Specifically, in the four-user original MIMO Y channel, each user has to have 4 antennas and the relay is equipped with 7 antennas, [4] while in the five-user MIMO Y channel, each user has to have 6 antennas and the relay is equipped with 13 antennas. The more users in the systems, the higher number of messages exchanged in two time slots - minimum

$\frac{K \cdot (K-1)}{2}$, where K is the number of users. The price to pay is increased complexity of the hardware (number of radio frequency amplifier frontends and antennas) and the processing complexity. To reduce the number of antennas on a device so that the MIMO Y channel configuration could be implemented in a practical environment, this work divides the whole transmission process into multiple parts. The proposed scheme is able to finish the data exchanges with the reduced number of antennas considering the balance between bandwidth efficiency (the increased number of time slots) and practical configuration cost.

In two chapters of the thesis presenting our contributions that follow, although the formal design and verification is carried out for a specific number of users, this thesis formalizes the guidelines for systems with higher number of users and offers opportunities for generalizations. The methodology applied in this thesis is that we first test a typical example, construct a solution and verify its applicability. Then we use the intuition and developed concepts to build general principles and guidelines that can also be applied to general cases.

Our approach in the thesis is consistent with current trends in communication system designs based on cooperative communication and networking. Unlike conventional point-to-point communications, cooperative communication allows users to share resources to create collaboration via distributed transmission and processing of messages. This cooperative diversity concept is similar to the single user (SU) MIMO system but is applied in a networked setting. As a result, it is often called a distributed MIMO or network MIMO. It represents a paradigm shift from a network of conventional point-to-point links to network cooperation. In Chapter 2 of this thesis, we are attempting to work within the framework of collaborative STBC, and in Chapter 3 we are using the divide-and-conquer approach and reuse of antenna resources to reduce the number of antennas in the MIMO Y channels with larger number of users.

1.6 Thesis Organization

The remainder of this thesis is organized as follows:

In Chapter 2, we propose a strategy integrating STBC with MIMO Y channel for enhancing system reliability. The transmission scheme for a three-user scenario

and extension for four-user cases are described in detail and the design guidelines are presented for a generalized case. Specifically, precoding vectors and nulling vectors are carefully designed to accommodate the requirements of forming STBC codeword structures for both MAC and BC phases. Moreover, optimization is consistently considered through the design process to maximize the achievable data rate. Finally, we evaluate the performance of the proposed design in the environment of Rayleigh fading channel in terms of BER. Comparisons with other schemes with the same configuration of MIMO Y channels are carried out using extensive simulations to demonstrate the advantage of the proposed schemes in terms of the coding gain.

In Chapter 3, we balance MIMO Y channel complexity in terms of number of required antenna at the expense of higher number of time slots (lower bandwidth efficiency). We first propose a time scheduling and signal processing scheme for four-user MIMO Y channel as implemented using principles of original three-user MIMO Y channel. Then a generalization is presented through an example of a five-user case as implemented using proposed principles in six time slots (2 cycles) as opposed to two time slots (1 cycle), where the cycle representing MAC and BC phases in the original scheme. In the end, the analysis of the simulation results in terms of antenna numbers on nodes and BER performance is demonstrated.

Chapter 4 provides conclusions and potential future investigations. A discussion of possible research directions in MIMO Y channel area are also offered.

Chapter 2

Integrated STBC and Signal Alignment for MIMO Y Channel

In wireless networks with coordinated transmissions, SA along with the physical-layer network coding (PNC) allows for the increasing number of users with a significantly improved sum data rate in the system. Initial investigations into applications of SA in MIMO Y channels focused on increasing the multiplexing gain. In this chapter, we trade off the multiplexing gain of the SA techniques to achieve the diversity gain. Specifically, we explore the integration of STBC and SA techniques in MIMO Y channels. We provide an innovative approach to use the STBC in a distributed fashion on virtual spatial channels and we design a new matching SA scheme. The introduced transmission schemes for both MAC and BC phases improve the system reliability in term of BER. In addition, the optimization in terms of system complexity and capacity for the proposed scheme is conducted.

This chapter is organized as follows. First, Section 2.1 evaluates related work and introduces the general concepts for the proposed scheme, while Section 2.2 elaborates on the MIMO Y channel model with generalized K ($K \geq 3$) users, where the original three-user MIMO Y channel was covered in Chapter 1. Sections 2.3 and 2.4 illustrate the proposed scheme in detail for MAC and BC phases in MIMO Y channels, respectively. In both sections, the original three-user MIMO Y channel model with STBC and SA is investigated and optimized. In Section 2.5, the proposed scheme is extended to a four-user MIMO Y channel and design guidelines for generalized cases are provided. Finally, Section 2.6 presents the theoretical analysis and Monte Carlo simulation results. The BER performance of the proposed schemes is documented and compared to the system using repetition code in the original MIMO Y channels.

2.1 Related Literature Evaluation

The early work into MIMO Y channels in [30] presents the signaling scheme including precoding and detection for the three-user MIMO Y channel model and realizes great success in terms of achievable capacity. Thereafter in [4], the extensions of the original scheme for higher number of users are discussed, but the research still focus on capacity and the BER performance of the system is not presented. It has been shown in [18] that the BER performance of the original scheme significantly suffers in Rayleigh fading channels and is only applicable in the AWGN channel or in a very high SNR region of the fading channel. However, in practical wireless implementations it is not possible to avoid the fading channels. Without addressing the BER performance in the fading channels, the system remains only a theoretical model. The investigations in [18] and [21] provide a potential solution to this BER performance problem by trading off the configuration and computational complexity. By adding redundant antennas at user nodes, more than one available set of the precoding vectors and nulling vectors are able to be used in the system. Multiple sets of solutions are generated iteratively and eventually the ones with the best BER performance are chosen as the beamforming vectors. However, because of the redundant antennas and iterative calculations, the implementation of this scheme suffers from high system cost and computational complexity which may be difficult to implement in real-world applications. Therefore, in this chapter, we develop a scheme that improves system reliability in fading channels without adding redundant antennas.

2.2 System Model

This section considers a wireless two-way relay network composed of K user nodes ($K \geq 3$) and one relay station denoted as RS . Each user node is equipped with the same number of antennas n_u and the relay station is equipped with n_r antennas. User nodes are assumed to be located at the same distance to the relay station and they communicate with each other only through the relay station to exchange the data between themselves in a bi-directional (two-way) communication. The nodes are synchronized and the relay does not need to know the data being exchanged between the nodes. For the K -user MIMO Y channel model, $K(K - 1)$ messages are

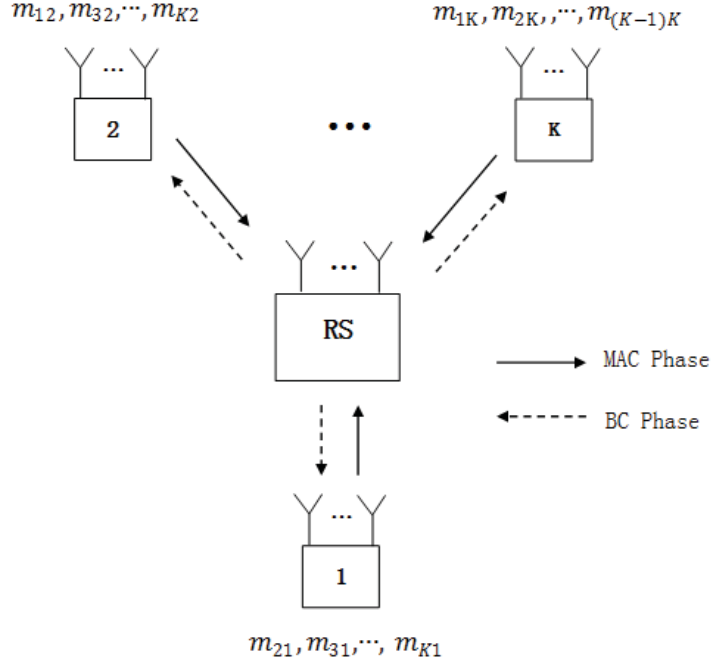


Figure 2.1: K-user MIMO Y channel.

exchanged within two phases: MAC and BC phases using $\frac{K(K-1)}{2}$ mutual messages, which is the same as in the original scheme. Assuming the fading channel is slow and flat, the channel gain matrices from User i to the relay station are defined as $H_{Ri}, i = 1, 2, \dots, K$ and are of size $n_r \times n_u$ with each element in H_{Ri} being $\mathcal{CN}(0, 1)$ (complex normal with variance one and zero mean) and independent of each other.

The conventional application of SA in such a system with K users follows the same reasonings as those developed in Chapter 1 when $K = 3$. Specifically, the conditions for aligning the signal of interest in the MAC phase would cover $\frac{K(K-1)}{2}$ mutual messages (as in (1.2) and (1.1)) and nulling the irrelevant signals in the BC phase would cover $\frac{K(K-1)}{2} - (K - 1)$ mutual messages that one user is not interested in (similar to (1.7) and (1.6)). However, the sizes of precoding and nulling vectors (beamforming and nulling matrices) and postprocessing matrices in ZF algorithms need to be adjusted accordingly, and this is controlled by values n_u and n_r . The number of antennas, n_u and n_r are chosen to meet the feasibility of finding the non-zero solutions in the corresponding systems of equations as in (1.1) and (1.6) for the case of $K = 3$. In the case of $K = 4$, the number of systems of equations for aligning the signal of interest in the MAC phase (equivalent to (1.2)) would be $\frac{K(K-1)}{2} = 6$ and

rather than having three conditions as in (1.1) we would have six conditions (linear systems of equations). Similarly, rather than having three (or rather minimum of $\frac{32}{2}$) nulling vectors in (1.6), we would have minimum of six nulling vectors for $K = 4$. To meet the feasibility conditions of finding the solutions of the corresponding systems of equations (design constraints of beamforming and nulling vectors), when $K = 4$, as originally presented in [4], $n_u = 4$ and $n_r = 7$. The detailed processes are provided in Appendix A. In this thesis, we do not document the precise manipulation of signals with beamforming and nulling vectors/matrices in the general case of arbitrary K , as the lengthy notation would not add much to the discussion that we already had for $K = 4$. Also $K = 4$ is the highest number of users that we work with in this chapter. As in the case of $K = 3$ user MIMO Y channel, we also assume that global CSI is known to all user nodes and the relay station and the corresponding nulling, precoding and ZF matrices are available at the corresponding nodes.

Our signal level model of the MIMO Y channel is similar to the original one with SA alone, but some modifications are introduced to let the STBC fit better into the original framework. In the MAC phase, two of the paired K messages m_{ij}, m_{ji} are aligned into one signal dimension and pass through the corresponding spatial channel. The received information at the relay would be the summation of paired messages $m_{ij} + m_{ji}$, which is defined as mutual symbol x_l with the index l chosen to better match the notation used in STBC. The critical characteristic of our scheme is that each phase (MAC and BC) is going to use multiple TSs instead of one TS as in the conventional (STBC uncoded) MIMO Y channel. During the consecutive time slots in the MAC phase, symbols are transmitted in the STBC form and construct the STBC codeword of mutual symbols visible only at the relay station. In the BC phase, the relay decodes received (mutual) symbols and broadcasts symbols to users. Nulling vectors are properly designed so that user nodes are able to receive symbols in Alamouti's space-time block code form. As a result, three TSs are actually sufficient for all user in the BC phase.

In the following two sections, the proposed scheme will be elaborated based on the minimum configuration: three-user MIMO Y channel.

2.3 MAC Phase

2.3.1 Transmission Matrix Design

First, we introduce a space-time block code as the transmission matrix for the MAC phase as given by:

$$X_3 = \begin{bmatrix} x_1 & -x_2^* & x_3^* & 0 \\ x_2 & x_1^* & 0 & x_3^* \\ x_3 & 0 & -x_1^* & -x_2^* \end{bmatrix} \quad (2.1)$$

The code deployed here is from [26] and it is an orthogonal complex space-time block code of high data rate $3/4$, where rows of the code indicate spatial streams (or rather mapping of data onto parallel data streams in space) and columns indicate time instances (or TSs) when the columns are sent on parallel spatial channels. Originally the code was designed for transmitting 3 messages within 4 TSs by using 3 transmit antennas.

In our proposed scheme, the interpretation of the codeword matrix is slightly modified. Columns of the code still denote the consecutive time slots of the MAC phase. However, we interpret the rows in (2.1) as the three different signal dimensions in MIMO Y channel signal space. After all, we do not have 3 transmit antennas at the user nodes. We actually create 3 parallel data (spatial) streams using 6 transmit antennas (two from each of the 3 users) using SA. Also, entries x_i in the matrix are defined here as mutual symbols, which are the summations of two corresponding (bi-directional) messages in our model. Specifically, three symbols can be written as $x_1 = m_{21} + m_{12}$, $x_2 = m_{31} + m_{13}$, $x_3 = m_{23} + m_{32}$. Consequently, the “transmitted” mutual messages x_1 , x_2 and x_3 are not available at the same location, as is the case in the conventional applications of STBC. The mutual messages x_1 , x_2 and x_3 are created in the wireless channels because of signal additions in a broadcast radio medium. Assuming that the channel conditions (channel gain matrices) are constant over the block duration (four TSs), and with the orthogonal structure of this code, we ensure that the received three mutual symbols at the relay station (over four TSs) allow us to use simple decoding and achieve diversity gain.

2.3.2 Precoding Vector

In order to benefit from the space-time block code at the relay, precoding vectors at each user node must be constructed properly to allow networked or distributed constructions of mutual messages x_i . Applying STBC directly to MIMO Y channel would be difficult. In the original implementations of STBC, messages are transmitted from different antennas of one common node. Therefore, it is possible to interchange data between antennas in different time slots. However, in the model of MIMO Y channel, following the original interpretation of STBC codewords means data is accessible between users, which is impossible because that is the purpose of MIMO Y channel transmissions.

The problem discussed above is solved in this thesis by properly designing and applying the precoding vectors in MIMO Y channel, which is unique to our proposed scheme. As described in Chapter 1, precoding vectors are used to shift the position of signals in the spatial dimension. We further utilize this characteristic, so the goal of precoding vectors is not only to align signals but, more importantly, to push signals into the right signal dimensions as required by the entries in the rows of the codewords.

The operation of our MAC phase is illustrated in Fig. 2.2, where rows of the code denote signal dimensions: D_{1-2} , D_{1-3} and D_{2-3}

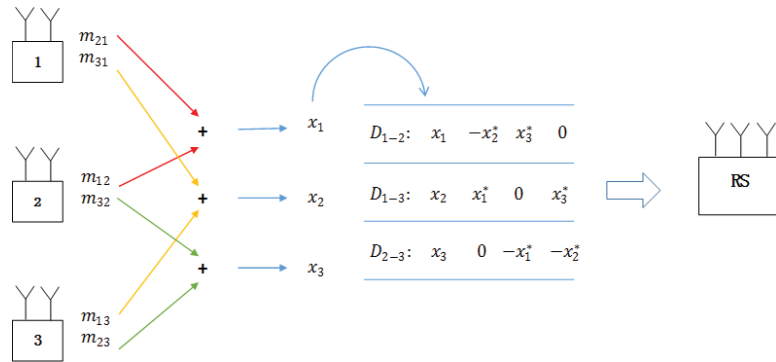


Figure 2.2: 3-user STBC MIMO Y channel scheme.

Firstly, we revisit the effective channel gain matrix G_l as the product of the channel gain matrix H_i and precoding vector P_{ji} from user nodes for each of the three signal dimensions D_{i-j} .

$$G_l = H_i \times P_{ji} \quad (2.2)$$

In Section 1.3.1, when originally defining the effective channel gains in (1.3) we used the notations G_{i-j} to emphasize the mutual signaling channels. For the convenience of the presentation in the context of STBC, we adopt here the single indexing of both the effective channel gains and the signal dimensions.

Symbols from the same row of matrix in Fig. 2.2 should pass through one common channel – that is to say, have the same effective channel gain G . Note that symbols are scalars and the factor that actually shifts the signals in the space is the vector of effective channel gain G . Since H_i is fixed for a block duration and pre-determined, the precoding vector P_{ji} is the only part that we are able to manipulate, and now this has to be obtained for every time slot in the MAC phase for a given STBC codeword. For instance, in dimension $D_{1-2} = D_1$, which is the first row of (2.1), four consecutive symbols $x_1, -x_2^*, x_3^*$ and 0 must have the same effective channel gain G_1 .

Intuitively, we can realize this by changing the precoding vectors every time slots, as given by:

$$G_1 = H^1 \times P^1 = H^2 \times P^2 = H^3 \times P^3 = H^4 \times P^4 \quad (2.3)$$

where the superscripts indicate time slots within the codeword. Taking into consideration both the basic function of precoding vectors - to align signals - and the new requirement presented in (2.3) and Fig. 2.2, the specific precoding vector design of the proposed scheme is summarized in Table 2.1.

Table 2.1: Time slots and dimension allocations in the proposed STBC.

| Dimen- sions | Time Slots | | | | | | | |
|-----------------|------------|---|----------|---|----------|---|----------|---|
| | T_1 | | T_2 | | T_3 | | T_4 | |
| G_1 | x_1 | $H_1 \cdot P_{21}^1 = H_2 \cdot P_{12}^1$ | $-x_2^*$ | $H_1 \cdot P_{31}^2 = H_3 \cdot P_{13}^2$ | x_3^* | $H_2 \cdot P_{32}^3 = H_3 \cdot P_{23}^3$ | 0 | 0 |
| G_2 | x_2 | $H_1 \cdot P_{31}^1 = H_3 \cdot P_{13}^1$ | x_1^* | $H_1 \cdot P_{21}^2 = H_2 \cdot P_{12}^2$ | 0 | 0 | x_3^* | $H_3 \cdot P_{23}^4 = H_2 \cdot P_{32}^4$ |
| G_3 | x_3 | $H_2 \cdot P_{32}^1 = H_3 \cdot P_{23}^1$ | 0 | 0 | $-x_1^*$ | $H_2 \cdot P_{12}^3 = H_1 \cdot P_{21}^3$ | $-x_2^*$ | $H_3 \cdot P_{13}^4 = H_1 \cdot P_{31}^4$ |

In this table, P_{ji}^k means the precoding vector to User j from User i at the k^{th} time slot. As with the STBC matrix, rows of this table define effective channel gain and columns indicate time slots. In each cell, elements in code (2.1) are repeated on the left, followed by the constraint of designing the precoding vector on the right side. Note that every term $H \cdot P$ in the same row defines the same effective channel gain vector G .

To understand the entries of Table 2.1, it is useful to cross reference the entries in the column T_1 , which corresponds to the transmissions of x_1, x_2 and x_3 on 3 spatial dimensions, with the relations that we have developed in (1.4), (1.1) and (1.3), where we were writing the transmitted mutual messages as $(m_{21} + m_{12}), (m_{31} + m_{13}), (m_{23} + m_{32})$. Specifically, for the first time slot T_1 , P_{ji}^1 are designed to align symbols x_1, x_2, x_3 in signal dimensions G_1, G_2 and G_3 , which is identical to the original scheme. In time slot T_1 when “sending” x_1 along D_1 and using the effective channel gain G_1 , the precoding vectors and channel gain matrices representing G_1 are $G_1 = H_1 \cdot P_{21}^1 = H_2 \cdot P_{12}^1$. In time slot T_2 , when “sending” $-x_2^*$ along D_1 and using the same effective channel gain G_1 , this effective channel is now represented with the precoding vectors and channel gain matrices as $G_1 = H_1 \cdot P_{31}^2 = H_3 \cdot P_{13}^2$ which somewhat resembles the indexing next to x_2 in time slot T_1 , at least in terms of subscripts. The reasons for adopting the strategy presented in Table 2.1 are captured through color coding of signal alignment at different users. Specifically, colors indicate precoding vectors from different users, where (i) red is for User 1, (ii) green is for User 2 and (iii) blue is for User 3. By preserving the colors from users transmitting the mutual symbols in T_1 , from one time slot to another when switching the spatial position of symbols, we ensure that this data will be available for precoding in the distributed system of 3 users with 6 message symbols. As compared to time slot T_1 , in T_2 , for example, x_1 and x_2 need to change their spatial positions along dimensions G_2 and G_1 . Thus, P_{21}^1 and P_{13}^1 in User 1 and User 2 in T_1 must be renewed (with superscript 2 in T_1) as P_{21}^2 and P_{13}^2 to push the signals x_1 and x_2 as x_1^* and $-x_2^*$ into the right dimensions. Similar procedures are implemented for other symbols. A cell that has an entry of zero indicates that we do not transmit any message through that channel at the given time. We could simply set the precoding vector as zero at the user node.

In summary, based on Table 2.1, we have derived the new signal alignment scheme that supports the deployment of the distributed STBC in the three-user MIMO Y channel. In this unique alignment, effective channel gain matrices are re-arranged based on the same source node but at different time slots, visualized in Fig. 2.3.

To be specific, there are three signal dimensions in space, corresponding to D_{1-2} , D_{1-3} and D_{2-3} . The black arrows and red color represent the signal alignment scheme in T_1 , which is identical to the original MIMO Y channel. The color coding in

Fig. 2.3 does not match color coding in Table 2.1, as in the table the same color represents the user, while in the figure the same color represents a common time slot (for terms in the form $H \cdot P$). Grey arrows and other colors are added based on the need to develop the proposed scheme. What is important in the understanding of Fig. 2.3 is that subscripts in precoding vectors indicate the user data is being sent from and to. Also, it should be observed that all the vectors in one direction are exactly the same: i.e, we have along D_1 all the vectors from the first row in Table 2.1 and we re-arranged them “slightly”. Based on the first row in Table 2.1, we have $G_1 = H_1 \cdot P_{21}^1 = H_2 \cdot P_{12}^1 = H_1 \cdot P_{31}^2 = H_3 \cdot P_{13}^2 = H_2 \cdot P_{32}^3 = H_3 \cdot P_{23}^3$, and all these terms appear in Fig. 2.3 along D_{1-2} with different colors and different pairing.

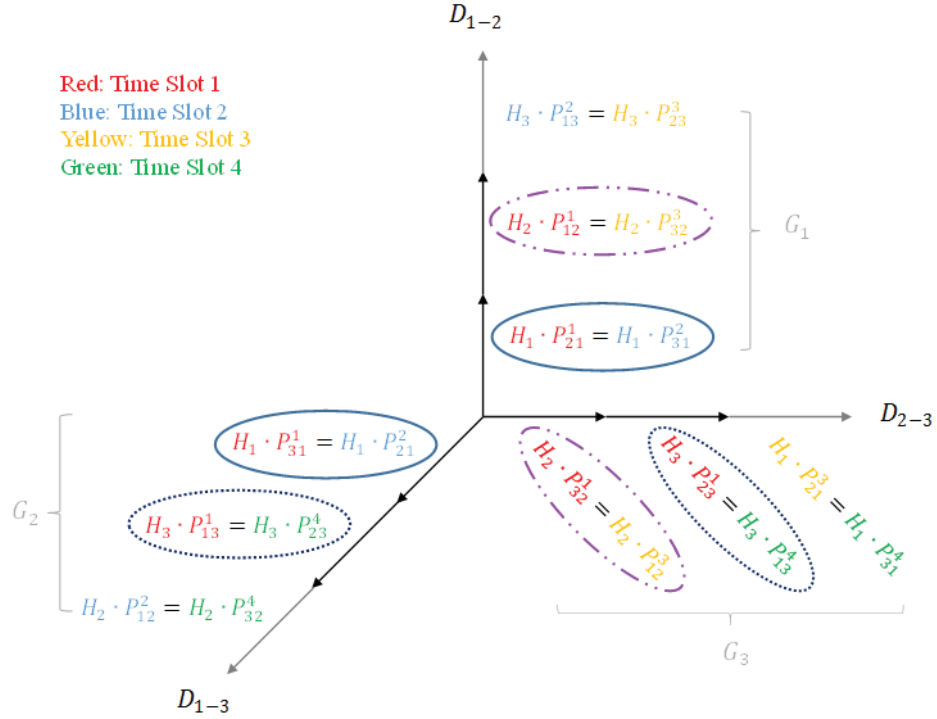


Figure 2.3: New signal alignment.

Based on Fig. 2.3, we deduce next that the precoding vectors for every user do not have to change every time slot. Consider User 1 as an example. The transmitted symbols in T_1 and T_2 are the same with the exchanged order of signal dimensions, circled in Fig. 2.3 with the blue circle (we disregard in these discussions changing of the sign, and conjugation is not critical for our BPSK data). From the figure, the

precoding vectors for User 1 – which in T_1 are P_{21}^1 and P_{31}^1 and in T_2 are P_{21}^2 and P_{31}^2 – they should satisfy

$$\begin{aligned} P_{21}^1 &= P_{31}^2 \\ P_{31}^1 &= P_{21}^2 \end{aligned} \tag{2.4}$$

The latter two relations come from Fig. 2.3 because we could drop the corresponding relations ($H \cdot P$ type) in the common matrix H_1 . The vector relations in (2.4) form the basis for reuse, as they mean that for User 1 (at least), we will need to calculate the precoding vectors (P_{21}^1 and P_{31}^1) only once in the time slot T_1 , and in the time slot T_2 we can reuse these vectors (as P_{21}^2 and P_{31}^2) with the interchanged positions without the need to calculate P_{21}^2 and P_{31}^2 . Based on scheduling transmissions as in Table 2.1, theoretically we need 18 precoding vectors over four time slots. In practice, a lot of these vectors can be reused with interchanged positions in our transmission schemes. For example, similarly to our earlier discussion but working now with magenta circles in Fig. 2.3, the precoding vectors for User 2 – which are P_{12}^1 and P_{32}^1 in T_1 and P_{12}^3 and P_{32}^3 in T_3 – should satisfy $P_{12}^1 = P_{32}^3$ and $P_{32}^1 = P_{12}^3$. This means that for User 2, we will need to calculate the precoding vectors (P_{12}^1 and P_{32}^1) only once in the time slot T_1 , and in the time slot T_3 we can reuse them with the interchanged positions as P_{12}^3 and P_{32}^3 . Yet again, for User 3, working now with dotted circles in Fig. 2.3, the precoding vectors for User 3 – which are P_{13}^1 and P_{23}^1 in T_1 and P_{13}^4 and P_{23}^4 in T_4 – should satisfy $P_{13}^1 = P_{23}^4$ and $P_{23}^1 = P_{13}^4$. This means that for User 3, we will need to calculate the precoding vectors (P_{13}^1 and P_{23}^1) only once in the time slot T_1 , and in the time slot T_4 we can re-use them with the interchanged positions as P_{13}^4 and P_{23}^4 . Working in a similar fashion with other precoding vectors that we have not covered yet, we determined that instead of calculating all 18 precoding vectors in 4 TSs, we actually need to calculate two sets of precoding vectors and reuse them. In the first set we have 6 precoding vectors as already discussed: P_{21}^1 and P_{31}^1 , P_{12}^1 and P_{32}^1 , P_{13}^1 and P_{23}^1 and we reuse them in other time slots as an additional six out of the total 18 in a block. The other set is calculated for precoding vectors P_{12}^2 , P_{13}^2 and P_{21}^3 which are reused as P_{32}^4 , P_{23}^3 and P_{31}^4 , respectively. The explanation for this reuse is much simpler, as it comes from analyzing the remaining relations in Fig. 2.3, where along three dimensions D_{1-3} , D_{1-2} and D_{2-3} , we have $P_{12}^2 = P_{32}^4$ and $P_{13}^2 = P_{23}^3$ and

$P_{21}^3 = P_{31}^4$, respectively. This results in simplified implementations, as we can reuse the precoding vectors from previous time slots. We created Table 2.2 illustrating this reuse of precoding vectors, where superscript * refers to new precoding vector for each user node. For each user node, two kinds of color indicate two sets of precoding vectors. Instead of regenerating all precoding vector sets for every time slot, only two sets would be enough in the proposed scheme with the total of 9 precoding vectors in a block, which reduces the computation complexity by half.

Table 2.2: Optimized precoding vector design.

| From User | To User | T_1 | T_2 | T_3 | T_4 |
|-----------|---------|----------|------------|------------|------------|
| 1 | 2 | P_{21} | P_{31} | P_{21}^* | 0 |
| | 3 | P_{31} | P_{21} | 0 | P_{21}^* |
| 2 | 1 | P_{12} | P_{12}^* | P_{32} | 0 |
| | 3 | P_{32} | 0 | P_{12} | P_{12}^* |
| 3 | 1 | P_{13} | P_{13}^* | 0 | P_{23} |
| | 2 | P_{23} | 0 | P_{13}^* | P_{13} |

2.3.3 Relay Detection Scheme

Assuming that BPSK modulation is applied at the user nodes for transmission, the mutual symbols x_1, x_2, x_3 at the relay station are ternary symbols, i.e., $x_i \in \{+2, 0, -2\}$. After the duration of codeword as in (2.1), baseband symbols in complex STBC form will be received and be ready for relay detection.

In convention, ZF, MMSE and ML decoders are available for decoding STBC. Among those techniques, ML gives the best performance at the cost of highest complexity. Due to the perfect orthogonality of the applied OSTBC, symbols are presented in parallel form at the relay. Therefore, ML simplifies to ZF, which is the simplest technique for decoding.

At the relay, for T_1 , the received signal, after passing through the Rayleigh fading channel in a vector form, is expressed as:

$$Y^1 = \begin{bmatrix} y_1^1 \\ y_2^1 \\ y_3^1 \end{bmatrix} = \begin{bmatrix} G_1 & G_2 & G_3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + n_r^1 \quad (2.5)$$

where y_i^k represents the signal received at i^{th} antenna in k^{th} time slot; G_i is the effective channel gain characterizing a signal dimension. Here n_r^1 is a 3×1 vector corresponding to AWGN added at received antennas.

In the next consecutive three TSs of the proposed MAC phase, the relay station keeps working in receiving mode and receives signals from 3 antennas forming column vectors $Y^i = [y_1^i \ y_2^i \ y_3^i]^T$. We combine four received vectors in four TSs of the proposed MAC phase into one system of equations as:

$$\begin{bmatrix} Y^1 & Y^2 & Y^3 & Y^4 \end{bmatrix} = \begin{bmatrix} G_1 & G_2 & G_3 \end{bmatrix} \cdot \begin{bmatrix} x_1 & -x_2^* & x_3^* & 0 \\ x_2 & x_1^* & 0 & x_3^* \\ x_3 & 0 & -x_1^* & -x_2^* \end{bmatrix} + \begin{bmatrix} n_R^1 \\ n_R^2 \\ n_R^3 \\ n_R^4 \end{bmatrix}^T \quad (2.6)$$

For convenience, we perform complex conjugate operations to the received signals at $T_2 - T_4$ to remove conjugate from mutual symbols. As a result, x_1, x_2, x_3 are condensed into a column vector and (2.6) can be re-written as:

$$\begin{bmatrix} Y^1 \\ Y^{2*} \\ Y^{3*} \\ Y^{4*} \end{bmatrix} = \begin{bmatrix} G_1 & G_2 & G_3 \\ G_2^* & -G_1^* & 0 \\ -G_3^* & 0 & G_1^* \\ 0 & -G_3^* & G_2^* \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} n_R^1 \\ n_R^{2*} \\ n_R^{3*} \\ n_R^{4*} \end{bmatrix} \quad (2.7)$$

We call the channel matrix linking Y^i 's in the column format to $[x_1 \ x_2 \ x_3]^T$ in (2.7), whose entries are specialized versions of G , the equivalent channel matrix and denote it with H .

In order to estimate the transmitted symbols x_1, x_2, x_3 by ZF decoding, the relay station needs to process the received signals by multiplying them with the inverse of H . For a general $m \times n$ matrix, an inverse matrix does not always exist but always can be equivalently represented by the pseudo inverse [31]. Thus, in our model, the pseudo inverse of H is given as $H^{-1} = (H^H \cdot H)^{-1} \cdot H^H$. The term H^H is denoted as the Hermitian matrix and obtained by taking complex conjugate transpose of H , shown as:

$$H^H = \begin{bmatrix} G_1^H & G_2^T & -G_3^T & 0 \\ G_2^H & -G_1^T & 0 & -G_3^T \\ G_3^H & 0 & G_1^T & G_2^T \end{bmatrix} \quad (2.8)$$

Consequently, the product of $H^H \cdot H$ is calculated as in (2.9), where $w = |G_1|^2 + |G_2|^2 + |G_3|^2$ is defined as the weight of channel matrix $H^H H$.

$$H^H \cdot H = \begin{bmatrix} w & 0 & 0 \\ 0 & w & 0 \\ 0 & 0 & w \end{bmatrix} \quad (2.9)$$

Due to the orthogonal characteristic of the transmission matrix, the product $(H^H \cdot H)^{-1}$ is a diagonal matrix, and its pseudo inverse is simply the inverse of elements on the diagonal of the matrix. Therefore, after going through the procedures described above, the estimation of “transmitted” mutual symbols (containing the summation of two mutual messages) are decoded through linear processing at relay station, given as:

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{w} & 0 & 0 \\ 0 & \frac{1}{w} & 0 \\ 0 & 0 & \frac{1}{w} \end{bmatrix} \cdot H^H \cdot \begin{bmatrix} Y^1 \\ Y^{2*} \\ Y^{3*} \\ Y^{4*} \end{bmatrix} \quad (2.10)$$

2.4 BC Phase

2.4.1 STBC Based Transmission Scheme

Conventionally, in the BC phase of the MIMO Y channel, the beamforming vectors are constructed in such a way that only the signal dimensions containing the desired mutual messages are available for each user node, while the irrelevant signals are canceled by the nulling vectors applied at the relay. The scheme has its advantages from the perspective of capacity and hardware cost, but results in poor BER performance. To enhance the system reliability, the proposed scheme in the BC phase designs the transmission matrix at the relay that enables the received symbols at each user node to form the Alamouti’s STBC matrix and as a result gain diversity.

The desired Alamouti’s code for each users is rewritten into

$$X_{Alamouti} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \quad (2.11)$$

where the rows and columns of the matrix refer to the spatial dimensions and time slots, respectively. Obviously, on the first attempt, two TSs are required for the

proposed scheme to form such a code structure for each receiver. As usual, we assume that the channels between the relay and users are Rayleigh fading channels and constant over the code block duration.

Let consider x in (2.11) as the received symbol composed of two messages. For T_1 that is the first column of Alamouti's code, and it can be simply viewed as the original MIMO Y channel BC phase process. Thus, the relay could use the same scheme as the original case. For T_2 , user node would require the same symbols as in T_1 , but in the complex conjugate forms and with the signal dimension swapped. Due to the fact that each user expects two symbols from two different signal dimensions and a symbol could not occupy two signal dimensions at the same time, the transmission matrix could be intuitively constructed as in Table 2.3.

Table 2.3: First attempt transmission scheme for the BC phase.

| Dimen- sions | Time Slots | | | |
|-----------------|------------|----------|----------|----------|
| | T_1 | T_2 | T_3 | T_4 |
| D_1 | x_1 | $-x_2^*$ | $-x_3^*$ | 0 |
| D_2 | x_2 | x_1^* | 0 | $-x_3^*$ |
| D_3 | x_3 | 0 | x_1^* | x_2^* |

This straight-forward scheme completes the transmissions of the BC phase in four TSs. Each of the symbols x_1, x_2 and x_3 represents the summation of two messages in a form of mutual messages, which has the same definition as in Section 1.3.1 in BPSK representations, Here $x_i \in \pm 1$, while in the previous section x_i 's had ternary representation. Color-coding is used in Table 2.3 and tables as follows: red color for User 1, green color for User 2, and blue color for User 3 (except for T_1 when all users receive the relevant data). For the duration of the block transmission, symbols and their revised forms occupy in turns three signal dimensions D_1, D_2 and D_3 . For T_1 , all three symbols are sent as in the original scheme. In the next three TSs, each one will enable one user to form the Alamouti's code. For example, User 1 will be active in T_1 and T_2 , receiving $[x_1 \ x_2]^T$ in T_1 and $[-x_2^* \ x_1^*]^T$ in T_2 to construct the Alamouti's code as arranged through this scheduling.

However, it could be noted that such a structure is cumbersome and non-optimal. First, zeros have to be padded to fill the table, since only two channels and two TSs are required to form the Alamouti's code for each user. In addition, except for T_1 ,

only one user node is served for each time slot, while others are inactive for 50 % of the BC phase duration. More importantly, the total number of required time slots would increase rapidly as the number of users in the system increases.

In order to improve time utilization, an optimized STBC transmission matrix is proposed in Table 2.4. Note that, in this revised transmission matrix, the block duration is reduced to three TSs instead of four TSs as in Table 2.3. This improvement is achieved by replacing zeros in Table 2.3 with symbols and re-organizing the transmission order. For convenience, all negative signs are ignored in order to avoid conflict between two users, requirements and could be added by users after receiving data. The re-organization of data transmissions (time scheduling) in Tables 2.4 and 2.3 are preserving the relevant structure of Alamouti's codewords for different users.

Table 2.4: Proposed transmission scheme for the BC phase.

| Dimen- sions | Time Slots | | |
|-----------------|------------|---------|---------|
| | T_1 | T_2 | T_3 |
| D_1 | x_1 | x_2^* | x_3^* |
| D_2 | x_2 | x_3^* | x_1^* |
| D_3 | x_3 | x_1^* | x_2^* |

In the new scheme as proposed in Table 2.4, the required symbols for each user are distributed over all three TSs. This gives the option to involve all user nodes in each time slot: the users are actively receiving over all 3 TSs, which improves time utilization. Consider User 1 as an example. As usual, in T_1 , the vector $[x_1 \ x_2]^T$ is received and contains all required symbols, not only for User 1, but also for other users. In T_2 , vector $[x_2^* \ x_3^*]^T$ is received through dimension D_1 and D_2 , but only the desired symbol x_2^* is recorded by User 1. In T_3 , the same procedure is conducted and symbol x_1^* passed through D_2 is picked from the vector. We summarize receiving matrices for all three users in Table 2.5, where all user nodes use two spatial dimensions in T_1 and one spatial dimension (out of an available two) at T_2 and T_3 .

After all three TSs, each user is able to form the Alamouti's code. The same decoding procedures of STBC as in last section can be used to obtain the estimated symbols. Messages are extracted from symbols by performing XOR operation on their bit representation as described in Chapter 1. This would complete the bi-directional data exchange in seven ($7 = 4 + 3$) TSs (4 TSs in the MAC and 3 TSs in the BC

Table 2.5: Alamouti’s codeword reception as visible from the perspective of different users.

| Dimen- sions | Time Slots | | |
|-----------------|------------|---------|---------|
| | T_1 | T_2 | T_3 |
| D_1 | x_1 | x_2^* | |
| D_2 | x_2 | | x_1^* |
| D_3 | | | |

(a) User 1

| Dimen- sions | Time Slots | | |
|-----------------|------------|---------|---------|
| | T_1 | T_2 | T_3 |
| D_1 | x_1 | | x_3^* |
| D_2 | | | |
| D_3 | x_3 | x_1^* | |

(b) User 2

| Dimen- sions | Time Slots | | |
|-----------------|------------|---------|---------|
| | T_1 | T_2 | T_3 |
| D_1 | | | |
| D_2 | x_2 | x_3^* | |
| D_3 | x_3 | | x_2^* |

(c) User 3

phases) which represents the loss in the bandwidth efficiency by a factor of $\frac{7}{2}$ as compared to the conventional MIMO Y channel completing the same data exchange in two TSs (one TS for each phase).

2.4.2 Nulling Vector Design

At this point, we have explained the construction of the optimal STBC transmission matrix for the BC phase. However, in order to realize such a signal alignment scheme, nulling vectors at the relay still need to be determined. The major difference between the new and the original nulling vectors comes from the last two TSs of Table 2.4. Specifically, there is one more signal dimension that needs to be nulled in T_2 and T_3 as compared to the original scheme as implemented in T_1 . The impact of this change is directly shown in Table 2.6 as the constraints of generating nulling vectors. The constraints in Table 2.6 are direct outcomes of zero entries in Table 2.5.

Table 2.6: Nulling vector design principles for different users in different TSs.

| User Node | Time Slots | | |
|--------------|--------------------------|--|--|
| | T_1 | T_2 | T_3 |
| User 1 | $H_1 \cdot U_{23}^1 = 0$ | $H_1 \cdot U_{13}^2 = 0$ $H_1 \cdot U_{23}^2 = 0$ | $H_1 \cdot U_{12}^3 = 0$ $H_1 \cdot U_{23}^3 = 0$ |
| User 2 | $H_2 \cdot U_{13}^1 = 0$ | $H_2 \cdot U_{12}^2 = 0$ $H_2 \cdot U_{13}^2 = 0$ | $H_2 \cdot U_{23}^3 = 0$ $H_2 \cdot U_{13}^3 = 0$ |
| User 3 | $H_3 \cdot U_{12}^1 = 0$ | $H_3 \cdot U_{12}^2 = 0$ $H_3 \cdot U_{23}^2 = 0$ | $H_3 \cdot U_{12}^3 = 0$ $H_3 \cdot U_{13}^3 = 0$ |

Specifically, for the first time slot, nulling vectors are identical to the original case because we are applying the original signal alignment scheme. For the next two time

slots, in addition to the same requirement in T_1 , one more constraint is attached for each user. As a consequence, the design principle of nulling vectors U_{12} , U_{13} and U_{23} in each time slot as implemented at the relay is summarized in Table 2.7, where superscript * is used to differentiate from the nulling vectors in T_1 . Notice that the constraint on the nulling vectors in T_2 and T_3 are merely in shifted positions. This indicates that instead of calculating a new set of nulling vectors for each time slot, we can reuse two calculations and apply the same nulling vectors in two different TSs in different positions of the nulling matrix, which is indicated by three colors in Table 2.7. This advantage will show its efficiency as the number of users grows.

Table 2.7: Nulling vectors generating principle at relay.

| Nulling Vector | Time Slots | | |
|----------------|------------------------|---|---|
| | T_1 | T_2 | T_3 |
| U_{12} | $H_3 \cdot U_{12} = 0$ | $\begin{pmatrix} H_2 \\ H_3 \end{pmatrix} \cdot U_{12}^* = 0$ | $\begin{pmatrix} H_1 \\ H_3 \end{pmatrix} \cdot U_{12}^* = 0$ |
| U_{13} | $H_2 \cdot U_{13} = 0$ | $\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \cdot U_{13}^* = 0$ | $\begin{pmatrix} H_2 \\ H_3 \end{pmatrix} \cdot U_{13}^* = 0$ |
| U_{23} | $H_1 \cdot U_{23} = 0$ | $\begin{pmatrix} H_1 \\ H_3 \end{pmatrix} \cdot U_{23}^* = 0$ | $\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \cdot U_{23}^* = 0$ |

2.4.3 Feasibility of Implementation Analysis

In order to realize the proposed design, we should derive the existence proof of such nulling vectors. Since the proof for T_1 is provided in the introduction chapter, we will focus here on the next two TSs. For each nulling vector in T_2 and T_3 , it has to null the effects of two channel gain matrices: e.g., H_2 and H_3 for the first entry in T_2 . Each H denotes the channel gain matrix from the relay station equipped with three antennas, to one of the user nodes. This indicates that each constraint has three unknowns to be calculated. Nevertheless, the equation number, which is determined by user antenna numbers, still needs to be decided. Assuming user nodes decide to keep the same number of receive antennas as in T_1 , which is two, the equations of each constraint are obviously not solvable (4 equations and 3 unknowns). Due to the characteristic that only one signal dimension is available for each user during T_2 - T_3 as visible in Table 2.5, the minimum number of antennas would be one. By conducting the reasoning again, it is easy to find out that one antenna at each user would lead the equations in our nulling constraints to be underdetermined and, as

discussed before, means that at least one non-zero set of solution exists.

2.5 Generalization of the proposed scheme

In this section, we extend the proposed scheme to the four-user ($K = 4$) case in MIMO Y channel and then provide the guidelines for generalizing the scheme in cases $K > 4$.

The extended four-user MIMO Y channel has an analogous setup to the original model, where four users are intended to exchange information with each other via the relay station. The transmission is still divided into two phases: MAC and BC phase. In this case, using conventional MIMO Y channel model with SA, we work with four antennas at the user nodes and seven antennas at the relay node.

2.5.1 MAC Phase

For an extended four-user model, the transmission matrix (STBC codeword) we are looking for should contain at least 6 different symbols since there are 6 network-coded signal dimensions. In addition, the length of the desired code should be as short as possible for a higher data rate.

Based on the criteria above, the transmission matrix (2.12) we adopted here is another rate $3/4$ ($\frac{6}{8}$) complex orthogonal space-time block code that is presented in [28]. Following the same definition of the three-user case, x is denoted as a network-coded symbol in our model and can be extended to 6 symbols $x_1 = m_{21} + m_{12}, x_2 = m_{31} + m_{13}, x_3 = m_{23} + m_{32}, x_4 = m_{41} + m_{14}, x_5 = m_{42} + m_{24}, x_6 = m_{43} + m_{34}$. It is notable that the applied matrix has four rows, which means there are four different signal dimensions in our model. By repeating symbols in certain order with complex conjugates, this OSTBC matrix can be formed at the relay, which ensures the improvement of transmission reliability of the MAC phase.

$$X_6 = \begin{bmatrix} x_1 & -x_2^* & x_3^* & 0 & 0 & x_4^* & x_5^* & -x_6 \\ x_2 & x_1^* & 0 & x_3^* & -x_4^* & 0 & x_6^* & x_5 \\ x_3 & 0 & -x_1^* & -x_2^* & x_5^* & x_6^* & 0 & x_4 \\ 0 & x_4 & x_5 & x_6 & x_1 & x_2 & -x_3 & 0 \end{bmatrix} \quad (2.12)$$

As we described in the previous section, in order to form such a transmission

matrix, proper precoding vectors must be designed to put the symbols into the right signal dimensions. The calculation of these vectors follows a similar procedure as the one used in Section 2.3 for the three-user scheme. According to code (2.12), for each symbol x_i at time slot T , the condition to satisfy is $H_i \times P_{ji}^t = H_j \times P_{ij}^t$, which realizes the alignment of two paired messages. Table 2.8 shows the effective channel gains of each symbol in terms of time slots T_i .

Table 2.8: Time slots and dimension allocations in generalized STBC.

| Dimen- sions | Time Slots | | | | | | | |
|-----------------|------------|---|----------|---|----------|---|----------|---|
| | T_1 | | T_2 | | T_3 | | T_4 | |
| G_1 | x_1 | $H_1 \cdot P_{21}^1 = H_2 \cdot P_{12}^1$ | $-x_2^*$ | $H_1 \cdot P_{31}^2 = H_3 \cdot P_{13}^2$ | x_3^* | $H_2 \cdot P_{32}^3 = H_3 \cdot P_{23}^3$ | 0 | 0 |
| G_2 | x_2 | $H_1 \cdot P_{31}^1 = H_3 \cdot P_{13}^1$ | x_1^* | $H_1 \cdot P_{21}^2 = H_2 \cdot P_{12}^2$ | 0 | 0 | x_3^* | $H_2 \cdot P_{32}^4 = H_3 \cdot P_{23}^4$ |
| G_3 | x_3 | $H_2 \cdot P_{32}^1 = H_3 \cdot P_{23}^1$ | 0 | 0 | $-x_1^*$ | $H_1 \cdot P_{21}^3 = H_2 \cdot P_{12}^3$ | $-x_2^*$ | $H_1 \cdot P_{31}^4 = H_3 \cdot P_{13}^4$ |
| G_4 | 0 | 0 | x_4 | $H_1 \cdot P_{41}^2 = H_4 \cdot P_{14}^2$ | x_5 | $H_4 \cdot P_{24}^3 = H_2 \cdot P_{42}^3$ | x_6 | $H_4 \cdot P_{34}^4 = H_3 \cdot P_{43}^4$ |
| | T_5 | | T_6 | | T_7 | | T_8 | |
| G_1 | 0 | 0 | x_4^* | $H_1 \cdot P_{41}^6 = H_4 \cdot P_{14}^6$ | x_5^* | $H_4 \cdot P_{24}^7 = H_2 \cdot P_{42}^7$ | $-x_6$ | $H_4 \cdot P_{34}^8 = H_3 \cdot P_{43}^8$ |
| G_2 | $-x_4^*$ | $H_1 \cdot P_{41}^5 = H_4 \cdot P_{14}^5$ | 0 | 0 | x_6^* | $H_4 \cdot P_{34}^7 = H_3 \cdot P_{43}^7$ | x_5 | $H_4 \cdot P_{24}^8 = H_2 \cdot P_{42}^8$ |
| G_3 | x_5 | $H_4 \cdot P_{24}^5 = H_2 \cdot P_{42}^5$ | x_6^* | $H_4 \cdot P_{34}^6 = H_3 \cdot P_{43}^6$ | 0 | 0 | x_4 | $H_1 \cdot P_{41}^8 = H_4 \cdot P_{14}^8$ |
| G_4 | x_1 | $H_1 \cdot P_{21}^5 = H_2 \cdot P_{12}^5$ | x_2 | $H_1 \cdot P_{31}^6 = H_3 \cdot P_{13}^6$ | $-x_3$ | $H_2 \cdot P_{32}^7 = H_3 \cdot P_{23}^7$ | 0 | 0 |

Because each row denotes one channel, the terms $H \times P$ in the same row should be equal to each other. Therefore, for each user node, the reusable precoding vectors can be found and redundant denotation will be able to be replaced by a common one. Consequently, each user's precoding vector can be decided.

As discussed in Section 2.3, the design guidelines of the generalized scheme for the MAC phase can be summarized in three parts:

1. Transmission matrix. The suggested transmission matrix for the MAC phase should be a complex OSTBC of the highest possible rate, whose symbol number equals to the dimension number $\frac{K(K-1)}{2}$.

2. Precoding vectors. Precoding vectors in the proposed scheme are designed to meet twofold objectives. The first is to align two paired signals into one signal dimension as in the original scheme. More importantly, from our perspective, they should push symbols into proper signal dimension according to the transmission matrix chosen. By reusing the repeated precoding vector, optimized design reducing computational complexity could be achieved.

3. Relay detection. Once the relay station has all knowledge of the received STBC matrix, simple ZF or ML decoding that is analogous to the three-user case elaborated in Section 2.3 can be applied and estimators of mutual symbols can be obtained at the relay station.

2.5.2 BC Phase

In the BC phase of the four-user model, each user requires three symbols. Therefore, the desired OSTBC code should have three different symbols. Here, an OSTBC of high rate $3/4$ is chosen as:

$$X_3 = \begin{bmatrix} x_1 & -x_2^* & x_3^* & 0 \\ x_2 & x_1^* & 0 & x_3^* \\ x_3 & 0 & -x_1^* & -x_2^* \end{bmatrix} \quad (2.13)$$

Table 2.9 illustrates the corresponding optimal transmission matrix for the four-user case. Each user's desired symbols are distributed evenly over multiple time slots. Table 2.10 shows the corresponding reception matrix for each user. Color is used here to differentiate user nodes: User 1 - red; User 2 - green; User 3 - blue; User 4 - magenta. In T_1 , every user node is able to receive the original version of their desired symbols. In the following four time slots, symbols will pass through certain signal dimensions based on the code chosen. Once the whole transmission matrix has been broadcasted, desired symbols would be picked up by the terminals to form the selected OSTBC matrix (2.13).

Consider User 1 as an example. The desired symbols are x_1 , x_2 and x_4 , which contain information about $m_{21} + m_{12}$, $m_{31} + m_{13}$ and $m_{41} + m_{14}$, respectively. In this case, a specialized version of the codeword in (2.13) requires these three symbols passing through D_1 , D_2 and D_4 in the following time slots. As the color red indicates in Table 2.9 and 2.10, the requirement is met by the proposed scheme. It's worth mentioning that all user nodes would use three antennas at T_1 and two antennas at $T_2 - T_3$ for the reason of dimension constraint discussed in Section 1.3.3.

For the generalized scheme of the BC phase, in order to design the optimal transmission matrix without padding zeros, and using the lowest antenna number, the major design procedures are as follows:

Table 2.9: Suggested transmission scheme for the BC phase.

| Dimen- sions | Time Slots | | | | |
|-----------------|------------|---------|---------|---------|---------|
| | T_1 | T_2 | T_3 | T_4 | T_5 |
| D_1 | x_1 | x_2^* | x_4^* | x_3^* | x_5^* |
| D_2 | x_2 | x_3^* | x_1^* | x_4^* | x_6^* |
| D_3 | x_3 | x_5^* | x_2^* | x_6^* | x_1^* |
| D_4 | x_4 | x_1^* | x_6^* | x_5^* | x_2^* |
| D_5 | x_5 | x_6^* | x_3^* | x_1^* | x_4^* |
| D_6 | x_6 | x_4^* | x_5^* | x_2^* | x_3^* |

Table 2.10: Reception of data by different users.

| Dimen- sions | Time Slots | | | | |
|-----------------|------------|---------|---------|---------|---------|
| | T_1 | T_2 | T_3 | T_4 | T_5 |
| D_1 | x_1 | x_2^* | x_4^* | | |
| D_2 | x_2 | | x_1^* | x_4^* | |
| D_4 | x_4 | x_1^* | | | x_2^* |

(a) User 1

| Dimen- sions | Time Slots | | | | |
|-----------------|------------|---------|---------|---------|---------|
| | T_1 | T_2 | T_3 | T_4 | T_5 |
| D_1 | x_1 | | | x_3^* | x_5^* |
| D_3 | x_3 | x_5^* | | | x_1^* |
| D_5 | x_5 | | x_3^* | x_1^* | |

(b) User 2

| Dimen- sions | Time Slots | | | | |
|-----------------|------------|---------|---------|---------|---------|
| | T_1 | T_2 | T_3 | T_4 | T_5 |
| D_2 | x_2 | x_3^* | | | x_6^* |
| D_3 | x_3 | | x_2^* | x_6^* | |
| D_6 | x_6 | | | x_2^* | x_3^* |

(c) User 3

| Dimen- sions | Time Slots | | | | |
|-----------------|------------|---------|---------|---------|---------|
| | T_1 | T_2 | T_3 | T_4 | T_5 |
| D_4 | x_4 | | x_6^* | x_5^* | |
| D_5 | x_5 | x_6^* | | | x_4^* |
| D_6 | x_6 | x_4^* | x_5^* | | |

(d) User 4

1. Find the qualified reception space-time block code for user nodes. The OSTBC with the highest possible data rate containing the same number of symbols as the users require would be the optimal choice.

2. Construct the transmission matrix according to the reception code chosen. For $\frac{K(K-1)}{2}$ symbols to be transmit at the relay, there will be $\frac{K(K-1)}{2}$ signal dimensions. T_1 will always be the same as the original scheme, which has x_1 to $x_{K(K-1)/2}$.

3. Ensure the required symbols and corresponding signal dimensions are chosen for each user.

4. Distribute symbols as evenly as possible over every time slot (each time slot should contain at least one symbol of each user).

2.6 Performance Evaluation and Comparisons

In this section, classic Monte Carlo simulations in MATLAB[®] are used to demonstrate the BER performance of the proposed integrated STBC and signal alignment scheme. In addition, for comparison purposes, we also provide the BER simulation results of the original MIMO Y channel scheme and the original scheme integrated with the repetition codes in the MAC phase of the rate corresponding to the rate of the STBC used in our design in the MAC phase.

For the simulation model, we use the minimum configuration of MIMO Y channel described in the previous sections, where there are three users with 2 antennas and one relay with 3 antennas. BPSK is used as the modulation scheme. All wireless channels are assumed to be i.i.d. Rayleigh flat fading channels. In MATLAB simulations, we assume the power constraint (average power) on each user is one and the variance of AWGN noise (in one antenna) - at the relay in the MAC phase and at each node in the BC phase is σ_n^2 . So the SNR in the figures with simulation results is given as $1/\sigma_n^2$. Therefore, the simulated BER performance is demonstrated as a function of the SNR per user.

When examining the BER performance, we will be differentiating between (i) the BER performance of the end-to-end bi-directional connections between the nodes sending the data over the two hops in MAC and BC phases and (ii) the single link (one hop) performance in the isolated MAC.

Firstly, we show the end-to-end BER performance of the uncoded original scheme in Fig. 2.4 as the benchmark to our proposed scheme, where two common decoding algorithms ML and ZF are examined in the system. While ZF decoding accomplished through the matrix inversion was described in Section 1.3.3, the ML decoding in the original MIMO Y channel scheme requires a few comments. Using the ML decoding, in the MAC phase with 3-D received signals as in (1.15), we decode three ternary mutual symbols of levels $\pm 2, 0$ using minimum distance decoding from $3^3 = 27$ ideal mutual symbol points. These points are located in 3-D space with positions determined by 3 signaling dimensions given by $(\pm 2, 0) \cdot G_1$, $(\pm 2, 0) \cdot G_2$ and $(\pm 2, 0) \cdot G_3$. In the BC phase, the ML decoding at User 1 uses 2-D received signals as in (1.16), and to decode BPSK encoded representations for $(m_{21} + m_{12})$ and $(m_{31} + m_{13})$, we calculate the minimum distance to 4 ideal points in 2-D with locations determined

by $\pm 1 \cdot H_{1R} \cdot U_{1-2}$ and $\pm 1 \cdot H_{1R} \cdot U_{1-3}$.

In general, when compared to ZF decoding, ML gives better performance but requires higher computational complexity. As we can see from Fig. 2.4, the performance of the original uncoded scheme suffers in the Rayleigh fading environment. To target the BER of 10^{-3} , very high SNR of 25dB and 38dB are required using ML and ZF correspondingly, which proves the necessity of introducing the diversity gain as in the scheme proposed in this thesis. Even though the encoded MIMO Y channel exhibits

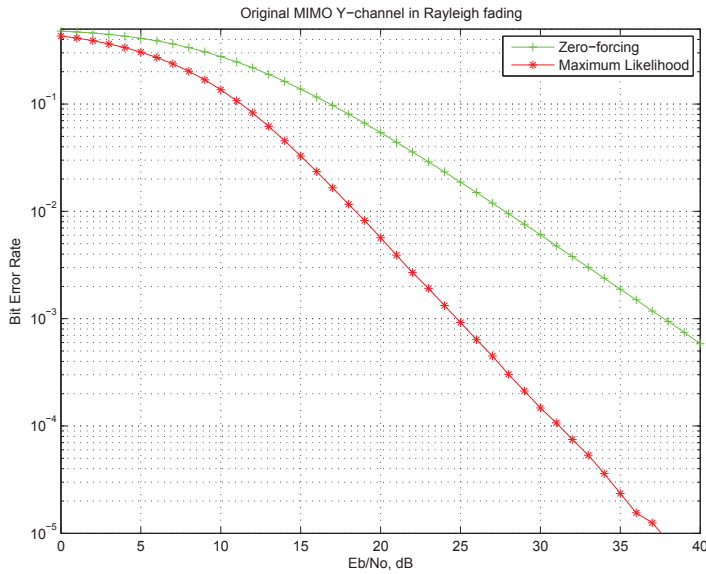


Figure 2.4: End-to-end BER performance of the uncoded 3-user MIMO Y channel in Rayleigh fading environment using ML and ZF decoding.

better BER performance with ML decoding, in all other simulations we will work with the ZF decoding to obtain hard-decisions for STBC processing, as ZF offers the implementation simplicity.

In Fig. 2.5, we present the simulation results and comparisons for the MAC phase alone. The received symbols at the relay are analog-network-coded (ANC) mutual symbols with three levels 0,+2,-2 (ternary symbols), so we use the term ternary symbol error rate (TSER) instead of BER. It's clear to see that the original scheme with ZF decoding only has diversity order of one. To get the targeted TSER 10^{-3} , it requires very high SNR of 38dB. Our focus is on rather low TSER and BER in the range of 10^{-3} as this is where the original scheme suffers the most and the coding gain improvements are the most important (in the “low” SNR region). To improve the

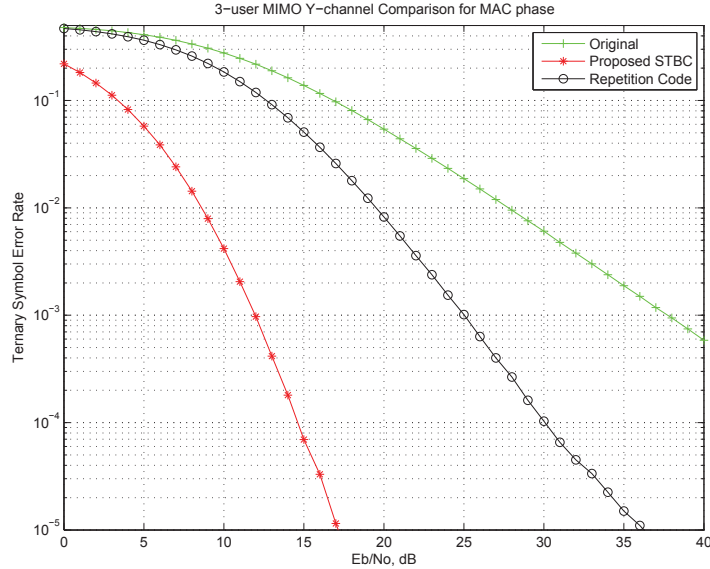


Figure 2.5: Symbol Error Rate performance comparisons for 3-user MIMO Y channel in the MAC phase.

system reliability, data rate could be partly traded off using the conventional Forward Error Control (FEC) techniques. We apply here as a benchmark the repetition code in MAC phase transmission. Specifically, all mutual messages are repeated four times over the four TSs which is the same loss in data rate efficiency as in our proposed scheme. At the relay, after hard decoding with ZF as in the conventional MIMO Y channel, we use the majority criterion. As shown in Fig. 2.5, the repetition code does provide some diversity gain (there is a straight line relation in TSER represented on the log scale vs SNR in dB). The diversity order (the slope of the relation between TSER on the log scale and SNR in dB) is much lower for the repetition code of the same rate as in our scheme when compared with the performance of the proposed STBC-based scheme. Comparing the coding gain to get the targeted TSER of 10^{-3} , the proposed scheme offers 13dB improvement over the repetition code and 26dB improvement over the original scheme.

In Fig. 2.6, we compare the BER improvements in the BC phase alone. For the original MIMO Y channel scheme, the model is equivalent to two parallel SISO transmissions, so the performance is 3dB worse than SISO transmission in a Rayleigh fading channel. This would be true provided that we did not have noise enhancement due to ZF and that the channels were independent and orthogonal, which may not be

the case in the original MIMO Y channel. For our proposed scheme, users are able to form Alamouti's code with two receiving antennas; therefore, a diversity of four and a coding gain of 22dB at BER 10^{-3} are achieved.

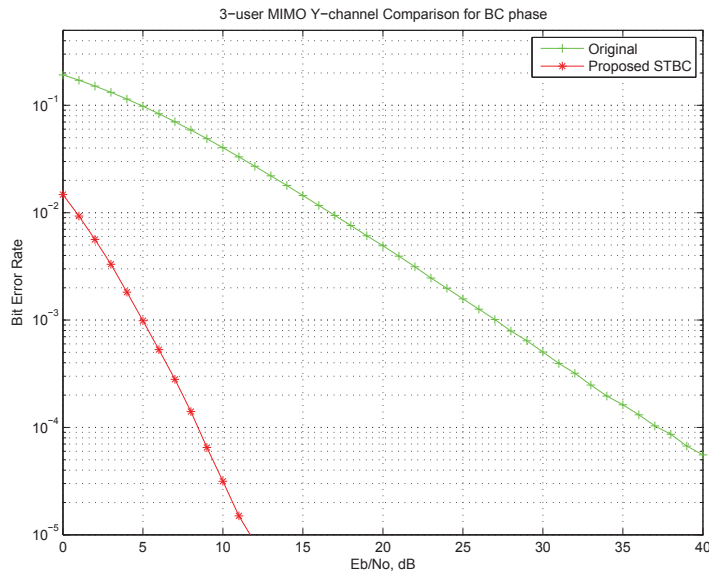


Figure 2.6: BER performance comparisons for 3-user MIMO Y channel in the BC phase.

Finally, we present an overall, end-to-end, BER performance comparison between the original and the proposed scheme in Fig. 2.7. Based on the fact that the system performance is determined by its worst link, the curves for both schemes in Fig. 2.7 are close to the one in the MAC phase. As before, we find that proposed scheme significantly outperforms the original case offering close to 25dB coding at BER 10^{-3} .

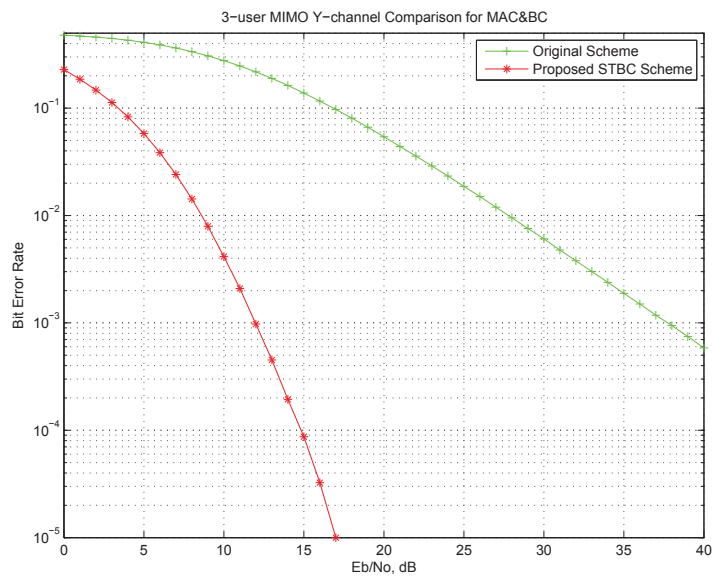


Figure 2.7: End-to-end BER performance comparisons for 3-user MIMO Y channel.

Chapter 3

Time Scheduling Scheme for MIMO Y Channel

In this chapter, we consider the generalized MIMO Y channel in terms of system configuration in order to reduce the required antenna numbers at the user nodes and at the relay. In the original MIMO Y channel scheme, the exchange of all bi-directional messages between users is completed in the MAC phase and the BC phase through the relay, which we define as one cycle of MIMO Y channel transmissions. In the original scheme, the MAC phase and the BC phase each use just one time slot, and the relay station is the bottleneck node of the system model. More importantly, the system complexity at the relay in terms of the antenna numbers rapidly increases as the number of users increases. The requirement of large number of antennas at the relay prevents the scheme from real implementations. In this chapter, we propose a time scheduling scheme to deal with this problem. The transmission process of MIMO Y channel is divided into multiple cycles instead of one and user nodes send messages in two or more MAC and BC phases in a carefully designed order. As a result, the required number of antennas at the relay station is decreased at the expense of reduced effective throughput. The antenna numbers of the relay and most of the users will always remain at the minimum configuration, which makes the whole system more amenable in practical implementations.

3.1 Generalized MIMO Y Channel

In the original MIMO Y channel scheme with three users, users exchange their messages via a relay station in two time slots: one for MAC phase and one for BC phase. The scheme exhibits a huge advantage from the perspective of sum data rate over other traditional transmission schemes, such as in the case of time division multiple access (TDMA) or multi-user multiple-input multiple-output (MU-MIMO). However when it comes to the generalized cases with rising numbers of user nodes, the original scheme still works theoretically, but suffers from the problem of a fast growing

number of antennas. This is true especially for the relay station, as the bottleneck node of the model, where the required antenna number would be at least half of all users' messages ($\frac{K \cdot (K-1)}{2}$ where K is the number of users). In addition, redundant antennas are needed for realization of precoding vectors and nulling vectors, which makes the number of antennas even larger.

Table 3.1 enumerates the required number of antennas at the relay and the users when applying the extended original scheme. It is noticeable from the table that when the number of user nodes keeps increasing, the number of antennas required at both relay and user side are becoming impractical.

Table 3.1: Antenna number requirement at relay and user nodes.

| Num of Users | Num of Relay Antennas | Num of User Antennas |
|--------------|-----------------------|----------------------|
| 3 | 3 | 2 |
| 4 | 7 | 4 |
| 5 | 13 | 6 |
| 6 | 21 | 8 |
| 7 | 31 | 11 |
| 8 | 43 | 15 |
| 9 | 57 | 19 |

Let us consider the system with 6 users as an example. Ideally, the relay station would have 15 antennas, which is the total number of signal dimensions ($5 \cdot 6/2$ is the number of mutual messages). Each user would be equipped with 5 antennas, which is the number of messages from one user. However, as shown in Table 3.1, the redundant requirement of antennas makes the relay and user nodes equip with 21 antennas and 8 antennas respectively to realize the original scheme due to dimension constraints to meet the feasibility conditions of finding precoding and nulling vectors. When the user node number rises to 9, the relay and users would need 57 and 19 antennas, respectively. Obviously, the direct extension of the original scheme is hard to implement in the real world because of the requirement of a large number of antennas.

The major cause of these problems is that all communications have to be finished in a tight schedule of one cycle (two TSs). This time limitation leads to the advantage of a high data rate, but also significantly increases the system configuration cost and brings considerable redundancy. The original three-user configuration is the only

exception. The three-user model in the original paper, which is also the minimum configuration, has the optimal setup in terms of antenna numbers due to the natural characteristic and transmission mechanism of MIMO Y channel. For any model of more users than three, redundant antennas have to be added to ensure that the precoding vectors and nulling vectors can be obtained from the imposed constraints. To address this problem in the generalized MIMO Y channel with a higher number of users, we propose a time scheduling scheme that breaks down the transmission into multiple three-user equivalent cases. As a proof of concept, first a four-user case will be used to elaborate on the proposed scheme and later on we will extend it into the K -user cases.

3.2 Time Scheduling Scheme

3.2.1 Four-User Model

In a four-user MIMO Y channel model, each user node has 3 messages to send, so totally 12 messages are involved in the transmission, which are supposed to be aligned to 6 signal dimensions. The minimum number of required antennas on nodes can be denoted as a set $\{6, 3\}$, indicating antenna numbers at the relay and the users, accordingly. However, due to the constraints of generating precoding vectors and nulling vectors discussed in Chapter 1, redundant antennas must be added, so the applicable configuration for this case would be $\{7, 4\}$. Therefore, the relay station has to be equipped with 7 antennas for only four users and, overall 5 antennas (one at the relay and one at each node) are redundant in the whole model.

In our proposed scheme, we use a multiple-cycle transmission to release the heavy antenna loads on nodes, especially the relay station. In the case of four users, nodes are split into two groups and all messages are exchanged in two cycles instead of one originally. The processes of the scheme are graphically visualized in Fig. 3.1.

The proposed time scheduling scheme decomposes the communication into two cycles and each one contains a MAC phase and a BC phase. The colors blue and red in Fig. 3.1 indicate the messages exchanged in Cycle 1 and Cycle 2, correspondingly. To be specific, Users 1, 2 and 3 are involved in Cycle 1, so the data interchanged among these nodes will be finished in the first cycle. It is notable that the configuration for

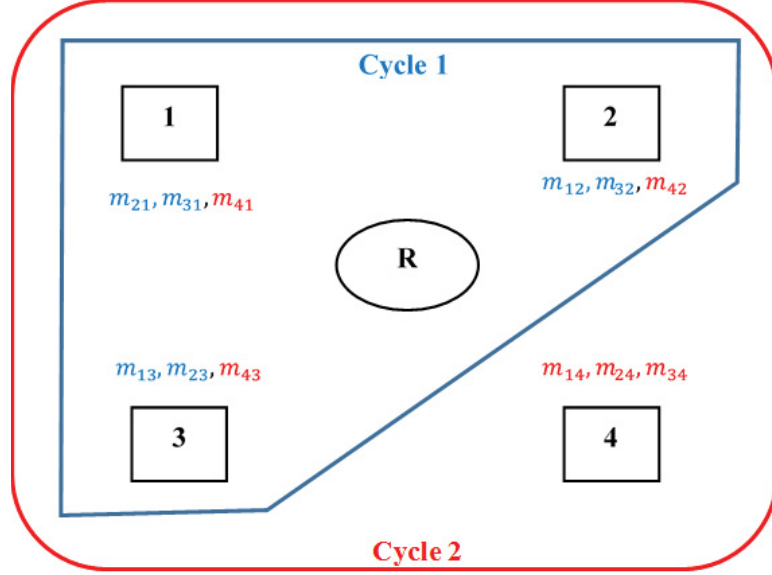


Figure 3.1: Time scheduling for 4-user scenario.

Cycle 1 is identical to the original three-user scenario, hence no redundant antennas are needed. In Cycle 2, all four users are involved. Since the mutual information between Users 1, 2 and 3 have been exchanged in Cycle 1, the messages left for these nodes are only toward User 4. Defined as a special node, User 4 is silent in the first cycle. The data-exchanging among User 4 and other users is the only purpose of Cycle 2. After two cycles of transmission, all data communications among four users would be completed.

Based on the proposed scheme described above, antenna numbers at each node can be analyzed and decided. Due to the fact that the setup in Cycle 1 is the same as the three-user model, the antenna requirement for all three users in Cycle 1 is known from Chapter 1 as $\{3, 2\}$. For Cycle 2, the constraint for the MAC phase will be examined first. In order to align two paired messages into one signal dimension, the constraint in (3.1) must be satisfied in Cycle 2, where $j = 1, 2, 3$ and 4 indicates the “special” User 4.

$$H_j \cdot P_{4j} = H_4 \cdot P_{j4} \quad (3.1)$$

Users 1, 2, 3 and the special User 4 have respectively 1 and 3 messages to send, so the minimum numbers of antennas on the corresponding nodes would be 1 and 3. The relay should have at least 3 antennas because there are 3 pairs of messages converging at the relay. Based on this, we can show the size of each matrix when

rewriting (3.1) as:

$$H_j_{[3 \times 1]} \cdot P_{4j}_{[1 \times 1]} = H_4_{[3 \times 3]} \cdot P_{j4}_{[3 \times 1]} \quad (3.2)$$

The constraint can be viewed as a problem of finding a solution set of 4 unknowns to an equation set of 3 equations, which leads to at least one set of non-zero solution. As a result, the required antenna number set would be $\{3, 2\}$ for Users 1-3, and $\{3, 3\}$ for the special User 4. The overall antenna requirement for MAC phase depends on the highest demanding set and therefore can be written as $\{3, 2, 3\}$, where the entries indicate antenna numbers at the relay, the normal user and the special user, respectively.

For the BC phase, the constraints are formed from the design of nulling vectors. To make irrelevant messages unavailable to each user, the requirements of nulling vectors can be enumerated as:

$$\begin{aligned} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \cdot U_{34} = 0 & \quad \begin{pmatrix} H_1 \\ H_3 \end{pmatrix} \cdot U_{24} = 0 & \quad \begin{pmatrix} H_1 \\ H_4 \end{pmatrix} \cdot U_{23} = 0 \\ \begin{pmatrix} H_2 \\ H_3 \end{pmatrix} \cdot U_{14} = 0 & \quad \begin{pmatrix} H_2 \\ H_4 \end{pmatrix} \cdot U_{13} = 0 & \quad \begin{pmatrix} H_3 \\ H_4 \end{pmatrix} \cdot U_{12} = 0 \end{aligned}$$

The summation of row numbers of two paired channel matrices decides the number of equations. If it is higher than the column number of the channel matrices, which is 3 in this case, additional antennas have to be used to ensure the nulling vector is achievable. Set $\{3, 1, 3\}$ as derived previously can be proved to be not only applicable but also the minimum configuration.

Combining the requirements for both two cycles, the proposed four-user model needs 3 antennas for the relay station, 2 antennas for User 1-3 and 3 antennas for User 4, which is denoted as set $\{3, 2, 3\}$.

As presented above, the proposed time scheduling scheme is able to reduce the required antenna numbers, comparing with the conventional setup of a four-user MIMO Y channel. Specifically, the relay used to have 7 antennas but now it has decreased to 3; all user nodes used to have the same 4 antennas but now three users have 2 antenna and one user has 3 antennas. That is to say, by doubling the required time duration, half of the antennas can be saved. By increasing the time for data exchange or equivalently reducing the throughput by two, the proposed scheme turns the four-user MIMO Y channel to a more acceptable configuration in practical applications from the perspective of antenna numbers.

3.2.2 Five-User Model and Generalization

To extend the proposed scheme to work with more users, a user is added and treated as the special user, which is similar to User 4 in Fig. 3.1. Messages between the special user and other users are dealt within the additional cycle which will require a new MAC and BC phases, each with one TS.

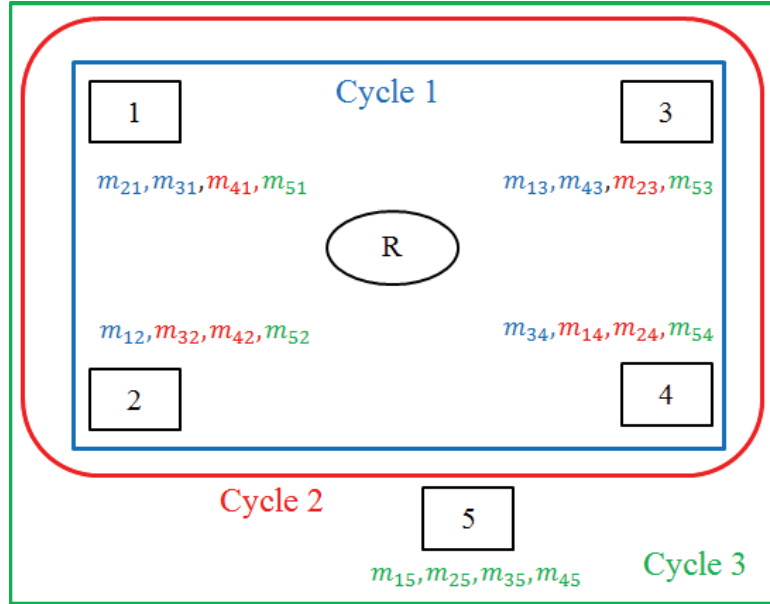


Figure 3.2: Time scheduling for 5-user scenario.

Fig. 3.2 is an example for the processes of the five-user MIMO Y channel with the proposed scheme. As shown in the figure, five users are split into three groups, which indicate three cycles. Note that the first two cycles are similar to the four-user model from Fig. 3.1 and the newly added User 5 is considered as the special user. Comparing with the four-user model, the difference is that one cycle, denoted by the green color, is added to exchange the mutual messages among User 5 and other users. For the normal users, the number of cycles can be decided with the purpose of maintaining the minimum configuration.

The proposed scheme can be recursively extended to general cases with more users by applying the same algorithm. Table 3.2 summarizes the scenarios when there are 4 to 9 user nodes in the system and the general case. Assuming there are K users in the system and User K is the special user, it would require $(K-2)$ cycles to finish the transmission; the relay station and the special user would have the same number of

Table 3.2: General antenna number requirement.

| User Num | Cycle Num | Relay Antenna Num | Normal User Antenna Num | Special User Antenna Num |
|----------|-----------|-------------------|-------------------------|--------------------------|
| 4 | 2 | 3 | 2 | 3 |
| 5 | 3 | 4 | 2 | 4 |
| 6 | 4 | 5 | 2 | 5 |
| 7 | 5 | 6 | 2 | 6 |
| 8 | 6 | 7 | 2 | 7 |
| 9 | 7 | 8 | 2 | 8 |
| K | $K-2$ | $K-1$ | 2 | $K-1$ |

antennas, which is $(K-1)$, and the normal users will always use only 2 antennas.

3.3 Performance Evaluation Comparison

In this section, we provide the comparison results in terms of antenna numbers and BER performance for the original scheme and the proposed time scheduling scheme using MATLAB simulations.

Fig. 3.3 compares the system configuration of two schemes in terms of the required antenna numbers at the relay. Stars and crosses in Fig. 3.3 denote the antenna numbers at a given number of users (K) for the original and the proposed schemes, meanwhile the red and blue lines are their curve fitting results, respectively. As discussed in the previous sections, the proposed scheme can significantly decrease the antenna number requirements. The red line, indicating original scheme antenna numbers, rises rapidly to an unacceptable number 57 when there are 9 users in the system. In the contrast, the blue line for the proposed scheme stays at a low level and increases linearly.

Fig. 3.4 displays the same comparison but from the perspective the number of required antennas at the users. The antenna numbers at the user nodes for the original scheme have a fast increment. For interchanging messages among 9 users, the original scheme would require 19 antennas at each user node, shown as red dotted line in the figure. Though the special user sacrifices its antenna configuration with more antennas than others, the required antenna numbers for most of the users in proposed scheme keep constant at 2, marked by the blue line with triangles.

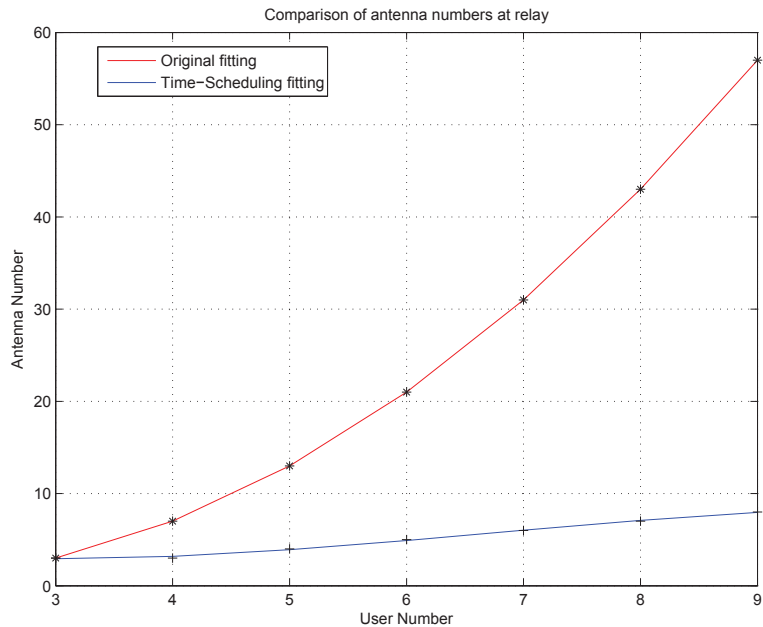


Figure 3.3: Comparison of antenna numbers at the relay.

It is worth mentioning that the huge advantage of the proposed scheme is gained by trading off its data rate and that the comparisons above are focused on the system setup requirements, not based on the same data rate. Results in Fig. 3.3 and Fig. 3.4 clearly imply that the generalized original scheme is not applicable in the reality in terms of system configuration. To solve this, a portion of the bandwidth efficiency advantage of the MIMO Y channel is sacrificed here to make the model practical and this is what we have attempted in this chapter.

Finally, in Fig. 3.5, we compare the four-user model's BER performance for both schemes in Rayleigh fading channels, where the simulation environment is the same as in Section 2.6. As we can see from the Fig. 3.5, the proposed scheme does not provide much reliability improvement. In the low SNR regions, the green line with stars for proposed scheme provides a little improvement but not in the higher SNR environment after 24 dB. The main reasons are the existence of the special user and the multiple cycles algorithm in time scheduling scheme. Both situations lead to a higher probability of bad channel occurrences because of the fact that the system reliability performance depends on the worst link and the worse user. Therefore, the proposed scheme does not show improvement over the original one in terms of BER performance.

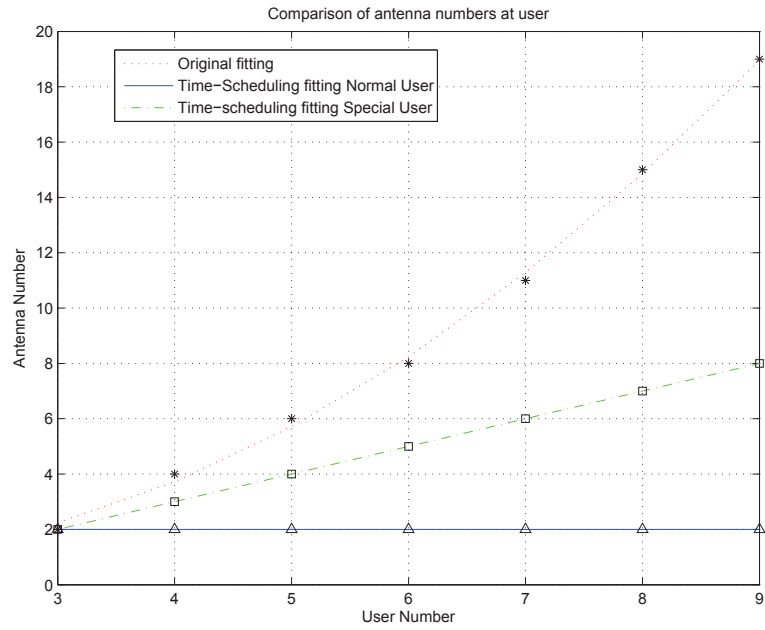


Figure 3.4: Comparison of antenna numbers at the user.

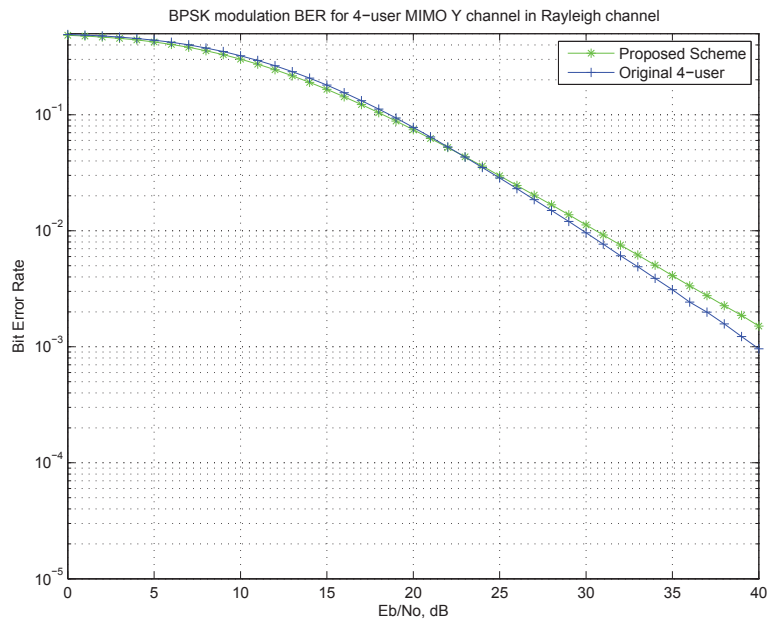


Figure 3.5: BER performance comparison for 4-user scenario.

Chapter 4

Conclusion

This chapter provides an overview of the thesis contributions and suggestions for future work on this topic. Section 4.1 discusses the contributions of this thesis while Section 4.2 suggests potential future work.

4.1 Thesis Contributions

The main contributions of this thesis are two-folded: The first one is the primary contribution of this thesis, where the novel integrated STBC and signal alignment scheme in MIMO Y channel is investigated to enhance the BER performance of the system in low SNR environments. The second contribution of this thesis is the design of a time scheduling scheme in order to reduce the required antenna numbers. In both proposed schemes, we trade off the multiplexing gain of the SA techniques to achieve higher system reliability and lower system configuration, which make the MIMO Y channel more amenable in practical implementations. The main vehicles to evaluate the BER performance of the proposed schemes are simulations in MATLAB, and to examine and optimize the system complexity analytical tools are deployed.

In the area of the first contribution, this thesis innovates by introducing the distributed OSTBC technique into MIMO Y channel to obtain diversity gain. The general goal is to enable the destinations that is the relay node for the MAC phase and the user nodes for the BC phase, to receive and construct the block of messages in a form of the chosen high-rate space-time block code, and therefore improves the general reliability of the system. In both the MAC and BC phases, user nodes generate and apply the proper pre-processing vectors to messages for a block duration. After transmitting, ZF or ML detection of STBC can be used to recover the messages, so that BER enhancement would be accomplished from the diversity gain of STBC.

Specific issues and detailed solutions of the first contribution can be summarized as follows:

1. Precoding vector design and optimization for MAC phase

When designing the precoding vectors in the MAC phase, the goal is to manipulate and push signals into the proper spatial dimensions. This goal has several aspects, including: (i) align messages from two paired users into a common dimension; (ii) ensure that each mutual symbol, which is the combination of two messages, has proper effective channel gain based on the chosen space-time block code; (iii) optimize the efficiency of generating precoding vectors. To accomplish these intentions, we define the concept of the space-time block code entries as symbols containing two paired messages from different physical locations in the wireless network. Each symbol on the same row of selected space-time block code should have the same matrix of effective channel gain, which is directly controlled by precoding vectors. Based on this fact, precoding vectors over time slots for each user can be generated accordingly to form the virtual spatial dimensions. The optimization is achieved by reusing precoding vectors that have been generated on the same user node but at different time slots.

2. Transmission matrix design and optimization for BC phase

The general target of transmission matrix design in the BC phase is to enable each user to receive desired symbols for multiple times and to form space-time block code after the whole block duration. The optimal method to realize this goal is transmitting the primitive version of all desired messages to its destination as original scheme within the first time slot; symbols are designed and transmitted by equally spreading on their designated channel over the rest time slots of the block duration. The transmission matrix is optimal in terms of efficiency because no redundant zeros are padded. Consequently, each user can build the space-time block code with their desired symbols and obtain diversity gain.

In the second part of this research, we deal with the high antenna numbers issue in the generalized MIMO Y channel. The contribution of this work is that we proposed a time scheduling scheme that distributes the heavy loads of antennas on nodes over multiple time slots. This is realized by splitting transmission into multiple cycles, where data-interchanging between the normal users are conducted first for multiple cycles and then the special user is included in the last cycle. Each unit of data-exchanging uses the original MIMO Y channel configuration which is the minimum

setup without redundant antennas.

Although we designed and illustrated the contributions for the minimum number of users, this thesis provides examples on higher number of users and the approach for the generalized case. The methodology applied in this thesis is that we first examine and verify the operations in smaller system configurations and typical cases to test the practicability of the proposed scheme and then expand to generalized cases.

The contributions of this thesis are achieved by exercising the trade-off between bandwidth efficiency and system reliability and setup complexity. The original scheme of MIMO Y channel displays its benefits from the perspective of bandwidth efficiency but without considering other aspects, which brings difficulties in the real world implementations. In this thesis, we sacrifice the minimum amount of bandwidth efficiency for the purpose of making the overall system more practical in terms of two aspects: (i) the system reliability in Chapter 2 (ii) the configuration complexity in Chapter 3.

4.2 Suggested Future Work

In this section, three related topics are suggested for potential future work. Some of these issues are generic in all signal and interference alignment schemes which require overhead to perform functions like synchronization, channel estimation and feedback. There are already some publications dealing with practical verifications of IA schemes using a real-world software defined radios testbed and verifying that the theoretical concept of IA could be transformed into a practical solution. [32]

1. Effects of Imperfect Channel Information

In this thesis, an assumption was made that global CSI, i.e., channel gain matrices, is available at all nodes, so that every node in the decentralized fashion could calculate their precoding or nulling vectors in the pre-processing stages and use the effective channel gain matrices in the ZF detection stages. It also could be considered that a centralized controller, which gathers the global CSI, calculates the required matrices and subsequently distributes the relevant information to different nodes through the control plane of the network. Moreover, we assumed the instantaneous availability of the accurate CSI. However, in practical scenarios, these assumptions about the

availability of CSI either at the centralized or distributed nodes are difficult to fulfil since the channels are usually estimated at the receivers, and then fed back to the transmitters. Therefore, it is of utmost importance to understand the fundamental performance limits of the proposed schemes under the assumption of delayed and only partial knowledge of the CSI.

2. Synchronous Transmissions

A basic assumption made in this thesis is that all message transmissions are perfectly synchronous. In essence, we have a synchronous system that requires globally coordinated operations of nodes. While there are methods and network protocols to achieve synchronous operation of networking nodes, the problem that we have to deal with in this thesis at the relay is that it may not be enough to synchronize the transmissions of mutual symbols at the nodes. Actually, we require the synchronous reception at the relay of the messages from terminals at different locations but, because of the variability in the propagation delays of the signals, this may pose the challenge, especially at high data rates.

3. Power Allocation

This study has concentrated on the bit level transmissions, without considering the factor of various power levels from different nodes. In practical systems, the required power to reach another node is generally determined by the distance; meanwhile power control mechanisms are usually deployed to make all the signals arrive at the same power level at the destination. However, power control is affected by the channel estimation accuracy and the delay in the distribution of the relevant information. Therefore, the effects on the system performance of imperfect power control should also be better understood. Moreover, equal receiving power may not be an optimum power allocation strategy in the signal alignment design of MIMO Y channels and further investigations into the power allocation strategies in this form of cooperative communications could be considered.

Appendix A

Antenna Number Constraints for Four-User MIMO Y Channel

In this appendix, we derive the antenna number requirements for the case of four-user MIMO Y channel. In [4], these requirements have been addressed in terms of available signal space. In this appendix, we revisit this problem for a four-user case as finding solutions to the underdetermined systems of linear equations.

In a four-user MIMO Y channel, each user is intended to exchange one message with other three users within two TSs via the relay. The total number of messages to be exchanged is then $K \cdot (K - 1) = 12$. Following the same assumptions and definitions as in Section 1.3, the antenna numbers at the relay and user nodes are determined by the constraints of generating precoding vectors and nulling vectors in MAC and BC phases, respectively.

A.A MAC Phase

In the MAC phase, each user applies precoding vectors to its messages and then transmits to the relay in the same time slots, as other users do. Therefore, the received signal at the relay can be presented as:

$$\begin{aligned} Y_R &= H_{R1} \cdot P_1 \cdot \begin{bmatrix} m_{21} \\ m_{31} \\ m_{41} \end{bmatrix} + H_{R2} \cdot P_2 \cdot \begin{bmatrix} m_{12} \\ m_{32} \\ m_{42} \end{bmatrix} + H_{R3} \cdot P_3 \cdot \begin{bmatrix} m_{13} \\ m_{23} \\ m_{43} \end{bmatrix} + H_{R4} \cdot P_4 \cdot \begin{bmatrix} m_{14} \\ m_{24} \\ m_{34} \end{bmatrix} \\ &= H_{R1} \cdot P_{21} \cdot m_{21} + H_{R1} \cdot P_{31} \cdot m_{31} + H_{R1} \cdot P_{41} \cdot m_{41} \\ &\quad + H_{R2} \cdot P_{12} \cdot m_{12} + H_{R2} \cdot P_{32} \cdot m_{32} + H_{R2} \cdot P_{42} \cdot m_{42} \\ &\quad + H_{R3} \cdot P_{13} \cdot m_{13} + H_{R3} \cdot P_{23} \cdot m_{23} + H_{R3} \cdot P_{43} \cdot m_{43} \\ &\quad + H_{R4} \cdot P_{14} \cdot m_{14} + H_{R4} \cdot P_{24} \cdot m_{24} + H_{R4} \cdot P_{34} \cdot m_{34} \end{aligned} \tag{A.1}$$

where subscript R refers to the relay and $P_k = [P_{lk} \mid P_{pk} \mid P_{qk}]$, $l, p, q \neq k \in \{1, 2, 3, 4\}$ is the precoding matrix at the k th user which concatenates three precoding vectors from a common k th node.

The precoding vectors in (A.1) are designed in such a manner that, at the relay, they align two mutual messages into a common spatial dimension. Therefore 12 messages from all users, represented at the relay as $H \cdot P \cdot m$, are paired into $\frac{K \cdot (K-1)}{2} = 6$ mutual messages and aligned into 6 dimensions. In this case, the precoding vector constraints are in a form of six systems of linear equations written as:

$$\begin{cases} H_{R1} \cdot P_{21} = H_{R2} \cdot P_{12} \\ H_{R1} \cdot P_{31} = H_{R3} \cdot P_{13} \\ H_{R3} \cdot P_{23} = H_{R2} \cdot P_{32} \\ H_{R1} \cdot P_{41} = H_{R4} \cdot P_{14} \\ H_{R2} \cdot P_{42} = H_{R4} \cdot P_{24} \\ H_{R3} \cdot P_{43} = H_{R4} \cdot P_{34} \end{cases} \quad (\text{A.2})$$

The requirements in (A.2) can be summarized into a general form as:

$$H_{Ri} \cdot P_{ji} = H_{Rj} \cdot P_{ij} \quad (\text{A.3})$$

where $i \neq j \in \{1, 2, 3, 4\}$. The subscripts Ri in the channel gain matrix H_{Ri} indicate the direction from User i to the relay and the size of H_{Ri} for the number of rows and columns is determined by the antenna numbers at the relay and the user nodes, respectively.

Because the total number of spatial dimensions in the system is six and each user has three messages to send, the minimum number of antennas at the relay is six and three at the user nodes, which is denoted here as a set $\{6, 3\}$. However, this configuration set is not able to satisfy the constraint in (A.3), as demonstrated by rewriting (A.3) with sizes of the matrices in the minimal configuration of antennas:

$$[H_{Ri}_{[6 \times 3]} \mid -H_{Rj}_{[6 \times 3]}] \cdot \begin{bmatrix} P_{ji}_{[3 \times 1]} \\ P_{ij}_{[3 \times 1]} \end{bmatrix} = 0 \quad (\text{A.4})$$

As discussed in Section 1.3.3, the precoding vector constraint with 6 equations and 6 unknowns in (A.4) only leads to an all zeros' solution which apparently is not admissible. Therefore, each user has to be equipped with an additional (redundant) antenna. With this, (A.4) has 6 equations and 8 unknowns, and hence there are infinite sets of non-zero solutions that can be obtained from this underdetermined

linear system of equations (for one mutual message). Consequently, the acceptable minimum antenna configuration in the MAC phase is $\{6, 4\}$.

Let us assume now that there is an antenna configuration (in terms of antenna numbers both at the user side and at the relay) that satisfies conditions for obtaining desired precoding vectors as in (A.2). Similarly as in Section 1.3.1, after defining the effective channel gain matrix G_{i-j} as in (1.3), the signals at the relay in (A.1) can be grouped accordingly, and the received signal at the relay is written as:

$$\begin{aligned}
Y_R &= (H_{R1} \cdot P_{21} \cdot m_{21} + H_{R2} \cdot P_{12} \cdot m_{12}) + (H_{R1} \cdot P_{31} \cdot m_{31} + H_{R3} \cdot P_{13} \cdot m_{13}) \\
&+ (H_{R2} \cdot P_{32} \cdot m_{32} + H_{R3} \cdot P_{23} \cdot m_{23}) + (H_{R1} \cdot P_{41} \cdot m_{41} + H_{R4} \cdot P_{14} \cdot m_{14}) \\
&+ (H_{R2} \cdot P_{42} \cdot m_{42} + H_{R4} \cdot P_{24} \cdot m_{24}) + (H_{R3} \cdot P_{43} \cdot m_{43} + H_{R4} \cdot P_{34} \cdot m_{13}) \\
&= G_{1-2} \cdot (m_{21} + m_{12}) + G_{1-3} \cdot (m_{31} + m_{13}) + G_{2-3} \cdot (m_{23} + m_{32}) \\
&+ G_{1-4} \cdot (m_{41} + m_{14}) + G_{2-4} \cdot (m_{42} + m_{24}) + G_{3-4} \cdot (m_{43} + m_{34}) \\
&= G \cdot [(m_{21} + m_{12}), (m_{31} + m_{13}), (m_{23} + m_{32}), (m_{14} + m_{41}), (m_{24} + m_{42}), (m_{34} + m_{43})]^T
\end{aligned} \tag{A.5}$$

where G is the concatenation of 6 effective channel gain matrices G_{i-j} , actually column vectors. Using ZF decoding, the ternary mutual symbols can be obtained at the receiver by multiplying Y_R with the inverse of G , i.e., G^{-1} or its pseudo-inverse, i.e., $G^\dagger = G^H \cdot (G \cdot G^H)^{-1}$. At this stage, we stipulate that the number of receive antennas at the relay is the same as the number of spatial streams (mutual messages) so it may seem that G is a square matrix (6×6). Actually, to find $[(m_{21} + m_{12}), (m_{31} + m_{13}), (m_{23} + m_{32}), (m_{14} + m_{41}), (m_{24} + m_{42}), (m_{34} + m_{43})]$ in (A.5), through pseudo-inverse, it is sufficient that the number of rows in G (the number of receive antennas at the relay) is greater or equal to the number of columns in G , i.e., the number of mutual messages.

A.B BC Phase

In the BC phase, a data vector with six mutual symbols is first multiplied with a nulling matrix U which is the concatenation of $\frac{K \cdot (K-1)}{2} = 6$ nulling (precoding)

vectors. The combined signal is then broadcast to all users, represented as:

$$\begin{aligned}
X_R &= U \cdot [(m_{21} + m_{12}), (m_{31} + m_{13}), (m_{23} + m_{32}), (m_{14} + m_{41}), (m_{24} + m_{42}), (m_{34} + m_{43})]^T \\
&= U_{1-2} \cdot (m_{21} + m_{12}) + U_{1-3} \cdot (m_{31} + m_{13}) + U_{2-3} \cdot (m_{23} + m_{32}) \\
&\quad + U_{1-4} \cdot (m_{41} + m_{14}) + U_{2-4} \cdot (m_{42} + m_{24}) + U_{3-4} \cdot (m_{43} + m_{34})
\end{aligned} \tag{A.6}$$

In the case of a four-user MIMO Y channel, out of the six mutual messages, each user is only interested in $K - 1 = 3$ mutual messages. In order to keep the minimum system configuration of antennas, each user would only receive its desired messages while the mutual messages between other users should be canceled using nulling vectors. For examples, from the perspective of User 1, similarly as when analyzing (1.7), when we consider the received signal by the first terminal $Y_{1R} = H_{1R} \cdot X_R$, we should aim at: $H_{1R} \cdot U_{2-3} = 0$; $H_{1R} \cdot U_{2-4} = 0$ and $H_{1R} \cdot U_{3-4} = 0$. When we analyze the requirements from the perspective of all users, it can be observed that each nulling vector has to null two channel gain matrices at the same time as follows:

$$\left\{ \begin{array}{l}
\begin{bmatrix} H_{3R} \\ H_{4R} \end{bmatrix} \cdot U_{1-2} = 0 \\
\begin{bmatrix} H_{2R} \\ H_{4R} \end{bmatrix} \cdot U_{1-3} = 0 \\
\begin{bmatrix} H_{1R} \\ H_{4R} \end{bmatrix} \cdot U_{2-3} = 0 \\
\begin{bmatrix} H_{2R} \\ H_{3R} \end{bmatrix} \cdot U_{1-4} = 0 \\
\begin{bmatrix} H_{1R} \\ H_{3R} \end{bmatrix} \cdot U_{2-4} = 0 \\
\begin{bmatrix} H_{1R} \\ H_{2R} \end{bmatrix} \cdot U_{3-4} = 0
\end{array} \right. \tag{A.7}$$

The general representation of (A.7) is:

$$\begin{bmatrix} H_{iR} \\ H_{jR} \end{bmatrix} \cdot U_{m-n} = 0 \tag{A.8}$$

where $i \neq j \neq m \neq n \in \{1, 2, 3, 4\}$. Because each user requires 3 messages and the relay has to transmit 6 mutual messages at least along six signalling dimensions, the channel gain matrix H would be at least of size $[3 \times 6]$, so that the stack of two H s in (A.8) would be of size $[6 \times 6]$. However, as discussed in Section A.A, this would result in a solution set of all zeros. To build an underdetermined system of linear equations for the nulling vector constraints in (A.7) and find at least one set of non-zero solutions, one redundant antenna has to be added to the relay node, which leads

the $\{7, 3\}$ antenna configuration with the constraint sizes as:

$$\begin{bmatrix} H_{iR} \\ H_{jR} \end{bmatrix}_{[6 \times 7]} \cdot U_{m-n_{[7 \times 1]}} = 0 \quad (\text{A.9})$$

We consider an example of the reception of data at User 1 in the BC phase. After the broadcast signal passes through channel H_{1R} , only 3 desired mutual symbols are detectable by User 1 as follows:

$$\begin{aligned} Y_{1R} &= H_{1R} \cdot U \cdot [(m_{21} + m_{12}), (m_{31} + m_{13}), (m_{23} + m_{32}), (m_{14} + m_{41}), (m_{24} + m_{42}), (m_{34} + m_{43})]^T \\ &= H_{1R} \cdot U_{1-2} \cdot (m_{21} + m_{12}) + H_{1R} \cdot U_{1-3} \cdot (m_{31} + m_{13}) + H_{1R} \cdot U_{1-4} \cdot (m_{41} + m_{14}) \\ &= H_{1R} \cdot [U_{1-2}|U_{1-3}|U_{1-4}] \cdot [(m_{21} + m_{12}), (m_{31} + m_{13}), (m_{41} + m_{14})]^T \end{aligned} \quad (\text{A.10})$$

As in Section 1.3.2, the recovery of the signal of interest $[(m_{21} + m_{12}), (m_{31} + m_{13}), (m_{41} + m_{14})]^T$ at User 1 based on Y_{1R} can be performed using the ZF approach by inverting $H_{1R} \cdot [U_{1-2}|U_{1-3}|U_{1-4}]$, which is a square matrix.

To sum up, for a four-user MIMO Y channel, user nodes would use 4 antennas in the MAC phase and 3 antennas in the BC phase; the relay can use 6 or 7 antennas in the MAC phase and 7 antennas in the BC phase. Combining the antenna number requirements of two phases, the required antenna number is 7 for the relay and 4 for the users, which can be represented as $\{7, 4\}$.

Appendix B

A Power Constraint on User Transmissions

In the MAC phase of a three-user MIMO Y channel, each user is sending two signals (s_1 and s_2) from two antennas to the relay after applying the precoding matrix P to the BPSK encoded messages, e.g., m_{21} and m_{31} for User 1. The instantaneous transmit power of a user node can thus be represented as $|s_1|^2 + |s_2|^2$ and we apply the expectation operator ($E(\cdot)$) to calculate the average power. Defining a column vector of the transmitted signal as: $s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$, the average power used by one user is:

$$E\{|s_1|^2 + |s_2|^2\} = E\{s^H \cdot s\} \quad (\text{B.11})$$

In this appendix, we derive the transmit power from User 1 as an example. In the MAC phase, User 1 applies the precoding matrix $[P_{21} \ P_{31}]_{2 \times 2}$ to its messages in the column vector form $[m_{21}, m_{31}]^T$, where both P_{21} and P_{31} are column vectors of size 2×1 . Thus considering the properties of Hermitian transpose, the transmitted signal s and its Hermitian transpose s^H can be written as:

$$\begin{aligned} s &= [P_{21} \ P_{31}] \cdot \begin{bmatrix} m_{21} \\ m_{31} \end{bmatrix} \\ s^H &= [m_{21}^* \ m_{31}^*] \cdot \begin{bmatrix} P_{21} \\ P_{31} \end{bmatrix} \end{aligned} \quad (\text{B.12})$$

Combining these representations, the instantaneous transmit power of User 1 is:

$$\begin{aligned} s^H \cdot s &= [m_{21}^* \ m_{31}^*] \cdot \begin{bmatrix} P_{21} \\ P_{31} \end{bmatrix} \cdot [P_{21} \ P_{31}] \cdot \begin{bmatrix} m_{21} \\ m_{31} \end{bmatrix} \\ &= [m_{21}^* \ m_{31}^*] \cdot \begin{bmatrix} |P_{21}|^2 & a \\ b & |P_{31}|^2 \end{bmatrix} \cdot \begin{bmatrix} m_{21} \\ m_{31} \end{bmatrix} \\ &= |P_{21}|^2 \cdot |m_{21}|^2 + a \cdot m_{21} \cdot m_{31}^* + b \cdot m_{31} \cdot m_{21}^* + |P_{31}|^2 \cdot |m_{31}|^2 \end{aligned} \quad (\text{B.13})$$

where $|P_{21}|^2$ and $|P_{31}|^2$ are the norm square of the vector P_{21} and P_{31} respectively, and the parameters a and b represent dot products of the corresponding vectors.

In order to calculate the average transmit power at User 1, expectation should be applied to (B.13). Because m_{21} and m_{31} are BPSK bits = ± 1 , expectations $E\{|m_{21}|^2\}$ and $E\{|m_{31}|^2\}$ would equal to one. In addition, because $E(m_{21}) = E(m_{31}) = 0$ and m_{21} and m_{31} are i.i.d, which means the expectation of their product is zero. Thus, after taking the expectation, two terms out of four in (B.13) will disappear: i.e., $E\{a \cdot m_{21} \cdot m_{31}^*\} = E\{b \cdot m_{31} \cdot m_{21}^*\} = 0$. As a result, the average transmit power at User 1 is:

$$P_{user1} = E\{s^H \cdot s\} = |P_{21}|^2 + |P_{31}|^2 \quad (\text{B.14})$$

Similar derivations for User 2 and User 3 result in the following expressions for their average transmit power, represented as:

$$\begin{cases} P_{user1} = |P_{21}|^2 + |P_{31}|^2 \\ P_{user2} = |P_{12}|^2 + |P_{32}|^2 \\ P_{user3} = |P_{13}|^2 + |P_{23}|^2 \end{cases} \quad (\text{B.15})$$

From (B.15), it is apparent that the transmit power at user nodes is determined by the precoding vectors of the MAC phase. In the conventional systems, the precoding vectors would be designed in such a manner that power of every user is the same $P_{user1} = P_{user2} = P_{user3}$, but in this considered case, this would impose additional limitations in the system that could not be met. Hence we revisit the constraints for generating precoding vectors in 3 user MIMO Y channel as in (1.1), and we rewrite them as follows:

$$[H_{Ri} \quad -H_{Rj}] \cdot \begin{bmatrix} P_{ji} \\ P_{ij} \end{bmatrix} = 0 \quad (\text{B.16})$$

In a original three user MIMO Y channel with minimum antenna configurations, there are three equations and four unknowns as discussed in (B.16), so the non-zero solutions for $\frac{P_{ji}}{P_{ij}}$ are aligned along a line, and we do not exercise the control over the directions of $\frac{P_{ji}}{P_{ij}}$ as it is determined by the channel gain matrices H_{Ri} and H_{Rj} . It is a practice in the field to set the norm square of $\frac{P_{ji}}{P_{ij}}$ to a fixed level W , so that:

$$\begin{cases} |P_{21}|^2 + |P_{12}|^2 = W \\ |P_{13}|^2 + |P_{31}|^2 = W \\ |P_{23}|^2 + |P_{32}|^2 = W \end{cases} \quad (\text{B.17})$$

(In our simulations we worked with norms of $\frac{P_{ji}}{P_{ij}}$ always set to 1). With these settings, some interesting observations about the users' power are as follows. First, the total power for the three users in the system based on (B.15) and (B.17) is constrained as follows:

$$P_{user1} + P_{user2} + P_{user3} = |P_{21}|^2 + |P_{31}|^2 + |P_{12}|^2 + |P_{32}|^2 + |P_{13}|^2 + |P_{23}|^2 = 3W \quad (\text{B.18})$$

where we take the advantage of color coding for the norms square of different vectors by grouping them accordingly. Because we do not really differentiate between the users, even though their short term average power in (B.15) may vary, based on (B.18) the long term average power for every user is W . Also because from (B.17), $|P_{21}|^2 \leq W$ and $|P_{31}|^2 \leq W$, in short term average power for User 1 cannot exceed $2W$, i.e., $P_{user1} = |P_{21}|^2 + |P_{31}|^2 \leq 2 \cdot W$.

Bibliography

- [1] A. H. Mohammed, B. Dai, B. Huang, M. Azhar, G. Xu, P. Qin, and S. Yu, “A survey and tutorial of wireless relay network protocols based on network coding,” *J. Network and Comput. Applicat.*, vol. 36, no. 2, pp. 593 – 610, 2013.
- [2] O. El Ayach, S. W. Peters, and R. W. Heath, Jr, “The Practical Challenges of Interference Alignment,” *ArXiv e-prints*, June 2012.
- [3] E. G. Larsson and P. Stoica, *Space-Time Block Coding for Wireless Communications*. Cambridge University Press, 2003.
- [4] N. Lee, J.-B. Lim, and J. Chun, “Degrees of freedom of the MIMO Y channel: Signal space alignment for network coding,” *IEEE Trans. Inform. Theory*, vol. 56, no. 7, pp. 3332–3342, July 2010.
- [5] A. Nosratinia, T. Hunter, and A. Hedayat, “Cooperative communication in wireless networks,” *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 74–80, Oct 2004.
- [6] R. Ahlswede, N. Cai, S.-Y. Li, and R. Yeung, “Network information flow,” *IEEE Trans. Inform. Theory*, vol. 46, no. 4, pp. 1204–1216, Jul 2000.
- [7] S.-Y. Li, R. Yeung, and N. Cai, “Linear network coding,” *IEEE Trans. Inform. Theory*, vol. 49, no. 2, pp. 371–381, Feb 2003.
- [8] Z. Shengli, S.-C. Liew, and P. P. K. Lam, “Physical Layer Network Coding,” *ArXiv e-prints*, April 2007.
- [9] S. Zhang and S. C. Liew, “Physical layer network coding with multiple antennas,” in *2010 IEEE Wireless Commun. and Networking Conf.*, April 2010, pp. 1–6.
- [10] S. Katti, S. Gollakota, and D. Katabi, “Embracing wireless interference: Analog network coding,” *SIGCOMM Comput. Commun. Rev.*, vol. 37, no. 4, pp. 397–408, Aug. 2007.
- [11] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft, “Xors in the air: Practical wireless network coding,” *IEEE/ACM Trans. Networking*, vol. 16, no. 3, pp. 497–510, June 2008.
- [12] D. Gunduz, A. Goldsmith, and H. Poor, “MIMO two-way relay channel: Diversity-multiplexing tradeoff analysis,” in *2008 Asilomar Conf. Signals, Syst. and Comput.*, Oct 2008, pp. 1474–1478.
- [13] M. Medard and A. Sprintson, *Network Coding Fundamentals and Applications*. Academic Press, 2012.

- [14] M. Maddah-Ali, A. Motahari, and A. Khandani, "Communication over MIMO X channels: Interference alignment, decomposition, and performance analysis," *IEEE Trans. Inform. Theory*, vol. 54, no. 8, pp. 3457–3470, Aug 2008.
- [15] V. Cadambe and S. Jafar, "Interference alignment and degrees of freedom of the K-User interference channel," *IEEE Trans. Inform. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug 2008.
- [16] S. Jafar and S. Shamai, "Degrees of freedom of the MIMO X channel," in *IEEE Global Telecommun. Conf.*, Nov 2007, pp. 1632–1636.
- [17] G. Zheng, I. Krikidis, C. Masouros, S. Timotheou, D.-A. Toumpakaris, and Z. Ding, "Rethinking the role of interference in wireless networks," *IEEE Commun. Mag.*, vol. 52, no. 11, pp. 152–158, Nov 2014.
- [18] N. Wang, Z. Ding, X. Dai, and A. Vasilakos, "On generalized MIMO Y channels: Precoding design, mapping, and diversity gain," *IEEE Trans. Veh. Technol.*, vol. 60, no. 7, pp. 3525–3532, Sept 2011.
- [19] J. She, S. Chen, B. Hu, Y. Wang, and X. Su, "Practical conditions of signal space alignment for generalized MIMO Y channel," *Science China Information Sciences*, vol. 55, no. 10, pp. 2171–2182, 2012.
- [20] Y. Su, Y. Li, and J. Liu, "Amplify-and-Forward MIMO Y channel: Power allocation based signal space alignment," in *2012 IEEE Veh. Technol. Conf.*, Sept 2012, pp. 1–5.
- [21] Z. Zhou and B. Vucetic, "An iterative beamforming optimization algorithm for generalized MIMO Y channels," in *2012 IEEE Int. Conf. Commun.*, June 2012, pp. 4595–4599.
- [22] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456–1467, Jul 1999.
- [23] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct 1998.
- [24] V. Tarokh, N. Seshadri, and A. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, no. 2, pp. 744–765, Mar 1998.
- [25] H. Wang and X.-G. Xia, "Upper bounds of rates of complex orthogonal space-time block codes," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2788–2796, Oct 2003.
- [26] O. Tirkkonen and A. Hottinen, "Square-matrix embeddable space-time block codes for complex signal constellations," *IEEE Trans. Inform. Theory*, vol. 48, no. 2, pp. 384–395, Feb 2002.

- [27] X.-B. Liang, "A high-rate orthogonal space-time block code," *IEEE Commun. Lett.*, vol. 7, no. 5, pp. 222–223, May 2003.
- [28] C. Xu, Y. Gong, and Y.-C. Liang, "High-rate complex orthogonal space-time block codes for high number of transmit antennas," in *2007 IEEE Radio and Wireless Symp.*, Jan 2007, pp. 63–66.
- [29] W. Su and X.-G. Xia, "Two generalized complex orthogonal space-time block codes of rates 7/11 and 3/5 for 5 and 6 transmit antennas," *IEEE Trans. Inform. Theory*, vol. 49, no. 1, pp. 313–316, Jan 2003.
- [30] N. Lee and J.-B. Lim, "A novel signaling for communication on MIMO Y channel: Signal space alignment for network coding," in *2009 IEEE Int. Symp. Inform. Theory*, June 2009, pp. 2892–2896.
- [31] P. D. Lax, *Linear Algebra*. Wiley Press, 1997.
- [32] O. El Ayach, S. Peters, and R. Heath, "The feasibility of interference alignment over measured MIMO-OFDM channels," *IEEE Trans. Vehicular Technol.*, vol. 59, no. 9, pp. 4309–4321, Nov 2010.