THEMATIC DEVELOPMENT OF RECOVERY, REMANUFACTURING, AND SUPPORT MODELS FOR SUSTAINABLE SUPPLY CHAINS

by

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ABSTRACT

The long-term availability of our resources depends on the reduction of their consumption. One method to achieve this goal is to encourage product remanufacturing, as the production of remanufactured products typically costs less than manufacturing new ones. This thesis explores the effects that remanufacturing can have on the collection of end-of-life products, value recovery activities, and after-sale support systems. These three areas constitute the three main themes of this dissertation, which are investigated in the natural sequence of their appearance in the life-cycle of a reverse logistics network.

The first theme addresses the collection of products within the context of closed-loop supply chains by considering design decisions such as the selection of collection centres to serve, the establishment of transportation routes, the scheduling of pick-ups and deliveries, and vehicle assignment. This is important for any organization involved in product recovery to allow them to understand the interactions between these decisions. A case study on the design of a collection network for a Nova Scotian firm recovering recyclables is included.

The second theme investigates production planning within a remanufacturing framework taking uncertainties in product quantity and quality into account. In this context, an existing collection network recovers used products which can then provide components that can be remanufactured and used to repair failed products under warranty. An optimal two-stage control theory model is developed and solved to find the optimal time to start remanufacturing and the quantity of spare parts needed.

The third theme deals with warranty models for reconditioned products. After the current state of the literature in this burgeoning research area is presented, we develop a generalized mathematical model for an unlimited free replacement warranty policy where repairs are carried-out with parts drawn from a lot composed of a mixture of new and reconditioned components. Using the mathematical model, relationships between reconditioning decisions and market responses are analyzed.

This research addresses substantive issues in the design and operation of sustainable manufacturing and logistics. Our models show that remanufactured products can be used to benefit both the economical imperatives of companies and the environmental needs of society.

LIST OF ABBREVIATIONS USED

3PL	Third-Party Logistics Provider
ACO	Ant Colony Optimization
ARP	Assignment Routing Problem
BOM	Bill of Materials
BPP	Bin Packing Problem
cdf	Cumulative Density Function
CFC	Chlorofluorocarbon
CLSC	Closed-Loop Supply Chain
CM	Corrective Maintenance
CPFR	Collaborative Planning and Forecasting Replenishment
CPR	Collective Producer Responsibility
CRM	Customer Relationship Management
CT	Compaction Trailer
CVRP	Capacitated Vehicle Routing Problem
DfR	Design for Recycling
DP	Dynamic Program
DVRP	Dynamic Vehicle Routing Problem
ED	Enviro-Depot
EHF	Environmental Handling Fee
EM	Environmental Management
EoL	End-of-Life
EoU	End-of-Use
EPA	Environmental Protection Agency
EPR	Extended Producer Responsibility
ERP	Enterprise Resource Planning
ES	Evolution Strategy
FRW	Free Replacement Warranty
GA	Genetic Algorithm
GHG	Greenhouse Gas
GHGE	Greenhouse Gas Emission
GIS	Geographic Information Systems
GIS-T	Geographic Information Systems for Transportation
GLS	Guided Local Search
GPS	Global Positioning System
GRASP	Greedy Randomized Adaptive Search Procedure
GSCM	Green Supply Chain Management

HBMO	Honey Bees Mating Optimization
HRM	Halifax Regional Municipality
HVRP	Heterogeneous Vehicle Routing Problem
IPR	Individual Producer Responsibility
IRP	Inventory Routing Problem
ISOP	Integrated Sustainable Operations Planning
IT	Information Technology
JIT	Just-in-Time
LCA	Life-Cycle Analysis
LP	Linear Programming
LSW	Lump-Sum Warranty
MILP	Mixed-Integer Linear Program
MIP	Mixed-Integer Program
MIPS	Material Input per Service Unit
MRP	Material Requirement Planning
MSW	Municipal Solid Waste
NFRW	Non-Renewing Free Replacement Warranty
NLP	Nonlinear Programming
ODRP	Ontario Deposit Return Program
OEM	Original Equipment Manufacturer
OES	Ontario Economic Stewardship
OR	Operations Research
pdf	Probability Density Function
PET	Polyethylene Terephthalate
PLC	Product Life-Cycle
PM	Preventive Maintenance
PoS	Point of Sale
PRM	Product Recovery Management
PRW	Pro-Rata Warranty
PVRP	Periodic Vehicle Routing Problem
QFD	Quality Function Deployment
R&D	Research and Development
RPC	Regional Processing Centre
RRFB	Resource Recovery Fund Board Inc.
RVRP	Rich Vehicle Routing Problem

SA	Simulated Annealing
SCC	Social Cost of Carbon
SCG	Social Cost of GHGEs
SCM	Supply Chain Management
SCOR	Supply-Chain Operations Reference
SDVRP	Site-Dependent Vehicle Routing Problem
SLA	Service-Level Agreement
SPLC	Sustainable Product Life-Cycle
TDP	Truck Dispatching Problem
TMS	Transportation Management Systems
TSP	Travelling Salesman Problem
UFRW	Unlimited Free Replacement Warranty
VNS	Variable Neighbourhood Search
VRP	Vehicle Routing Problem
WMS	Warehouse Management Systems
WEEE	Waste Electrical and Electronic Equipment

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CHAPTER 1

INTRODUCTION

Pollution and greenhouse gas (GHG) emission concerns, scarcity of raw materials, and the pressure on landfills due to increased wastage in society, have led to the adoption of sustainability and sustainable development initiatives across the globe. These initiatives have translated into specific regulations and policies that organizations must integrate in their decision-making.

In this introductory chapter of this dissertation, we begin by discussing sustainability in general terms and then look at what this implies for companies involved in product recovery, manufacturing, or support operations. A sustainable product life-cycle (SPLC) framework with integrated planning activities for sustainable operations is proposed to motivate this research, which deals with the development of mathematical decision support models in three areas of sustainable supply chain operations planning.

1.1 Sustainability

"Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs."
(Brundtland, 1987)

The Brundtland (1987) definition is referred to the most frequently when interpreting the meaning of sustainability. Many countries have adapted and expanded the prior definition based on their national areas of focus. For example, the Environment Canada (2013b) definition of sustainable development is:

> About meeting the needs of today without compromising the needs of future generations. It is about improving the standard of living by protecting human health, conserving the environment, using resources efficiently and advancing long-term economic competitiveness. It requires the integration of environmental, economic and social priorities into policies and programs and requires action at all levels - citizens, industry, and governments.

These two definitions, albeit comparable, are also very broad. Specifically, people have taken different views of what exactly constitutes the needs of the present and future generations. Clift (2003), like many others, believe that limitless consumption is not a *need*, especially when it diminishes the health of the environment:

> It is ignorant, pointless or simply dishonest to state that all inhabitants of the planet should have the opportunity to achieve the level of consumption enjoyed at present in most of the industrialised countries, because to do so would require resources and carrying capacity many times larger than those with which the planet is equipped. Constraints arise from resources: material and energy availability; human and economic capital; or from the capacity of the biosphere to accommodate emissions and wastes from human activities, an obvious example being the effect of emissions on the global climate (which is emerging as the active constraint on the use of fossil fuels, rather than the long-term availability of those fuels).

There has been a lot of research on what constitutes a need. Maslow's hierarchy of needs, which describes the stages of growth in humans from physiological to self-actualization, is considered the formative research in this subject. The annual upgrade of smartphones, for example, may not be considered a basic need from a human survival perspective. In spite of that, some companies intentionally design products with a short life-span, believing that the key to profitability is constantly producing new products. For certain industry groups, the *future needs* requirement is satisfied by establishing the availability of a given resource through the discovery of new sources and/or recycling. However, for some residents in the region where resource extraction or production is taking place, it may not be acceptable if another party generates pollution and contaminates the environment. Firms that are able to perceive the importance of consumer sustainability concerns, can incorporate them into their mission, values, vision, and long-term strategic goals.

There are many dimensions to sustainability, including: the levels of pollutants that can be released; the identification of new depletable resources; the exploitation of renewable resources adhering to the principle of maximum sustainable yield; necessary lifestyle changes; and the influence of the government, technology, and market forces to coerce change (Linton, Klassen, and Jayaraman, 2007). Nonetheless, if we take the view that the ability to survive (as in Maslow), is a basic human need for all generations, then we really have to start considering the earth, in its entirety, as a single system. Here, wasteful production, and resource extraction should not degrade the terrestrial system through the contamination of air, water, soil, and ozone, or induce climate change.

Some examples of these types of degradation are: hydraulic fracturing, provides new sources of natural gas and petroleum, but contaminates adjacent water tables; sourcing cheaper products overseas enables more affordability, but their transportation releases copious amounts of GHGs into the air; chlorofluorocarbons (CFCs) allow a greater standard of living through refrigeration, but contribute to ozone depletion in the upper atmosphere; and processing bitumen in the Athabasca oil sands pollutes the surrounding air, water, and land, but provides a source of crude oil.

Resource conservation reduces the need for extraction and production. Conservation decreases consumption and thereby reduces adverse impacts to the environment, such as air, water, and land pollution, and mitigates against scarcity. In Figure 1.1, Cohen (2007) illustrates the rates of consumption for several resources (indium, hafnium, gallium, etc.), and estimates how long it will take for a particular resource to be fully consumed.

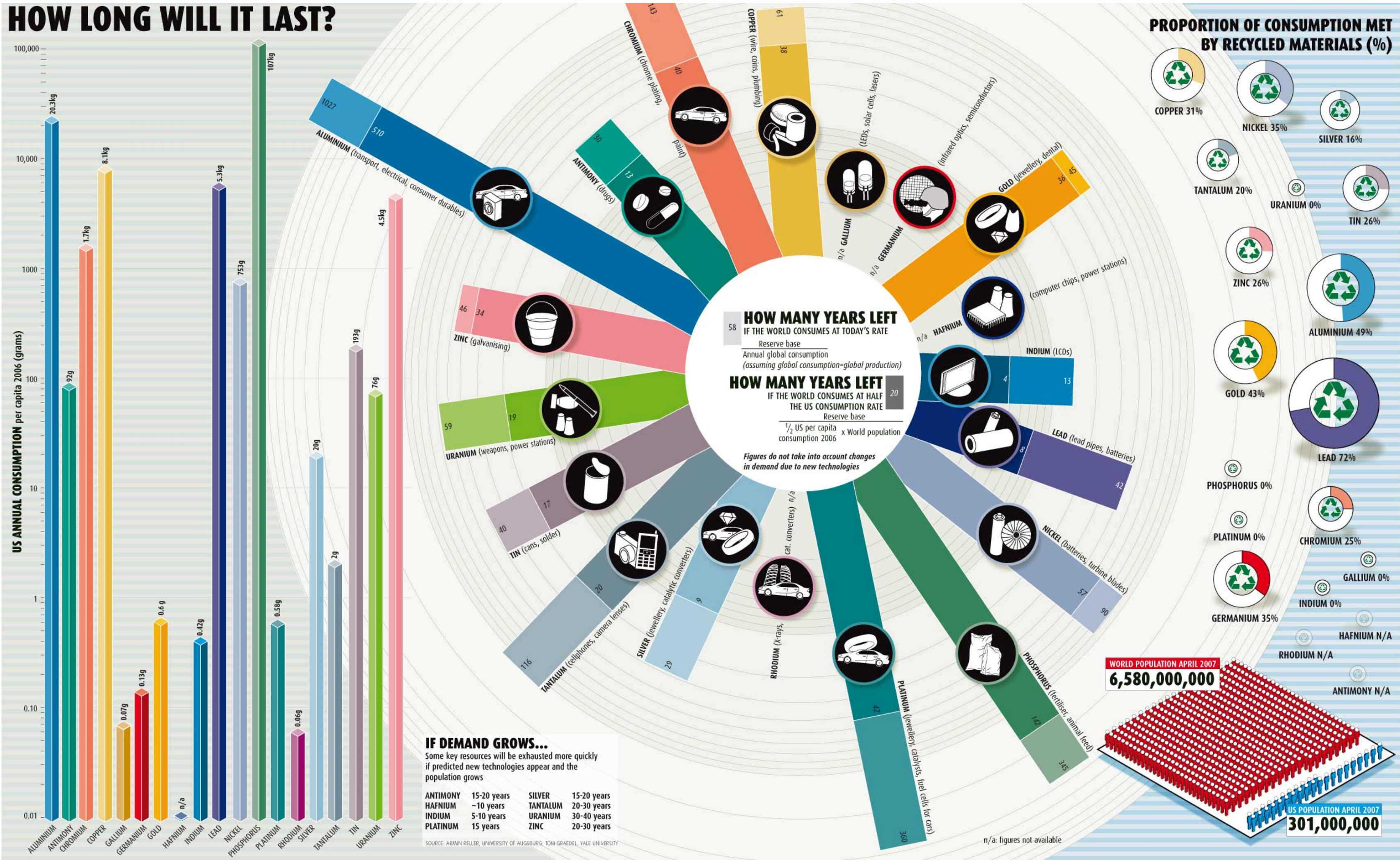


Figure 1.1: Present Rate of Consumption (Cohen, 2007)

These shortages are exacerbated by the unsustainable "current business practice of extracting raw materials from the earth, manufacturing them into products, and then disposing of the products into landfills or incinerators after a short period of use" (Ferguson and Souza, 2010). This mentality of instant disposal creates massive problems in not only conservation, but also waste generation and management, which includes the rapid rising cost of waste disposal (Ayres, Ferrer, and Van Leynseele, 1997). Authors have discussed how consumer electronics, which through obsolescence has a relatively short usage life, contribute greatly towards waste creation. According to Fiksel (2003), electronics "can generate a significant ecological footprint in terms of material, energy, and land use as well as industrial wastes and emissions". For example, Sawyer (2010) estimates the initial service lives of some electronic products including: cell phones (1.5-2.5 years), laptop computers (2-3 years), desktop computers/keyboards (3.3-4 years), printers (3-6 years), and LCD monitors (3-8 years).

A solution to these problems of unsustainability is to design quality products that operate better, are easier to maintain, last longer, are easier to reuse, and have recoverable components. In order for these products to be reused, they have to be recovered from the consumer before entering the waste stream.

> Resource recovery involves diverting used products from the waste stream and seizing their remaining value via reuse, recycling and/or re-manufacturing. This reduces the use of virgin natural resources, mitigates environmental pollution and eases the burden on limited landfill space for the waste stream (Wojanowski, Verter, and Boyaci, 2007).

Product recovery consists of diverting used products from the waste stream, and having them returned to the manufacturer. End-of-life (EoL), end-of-use (EoU), and consumer returns are three types of returns for a used product (Ferguson and Souza, 2010). In the first type, EoL returns are typically no longer functional, and are very old. EoU returns are usually functional, have had significant use, but are no longer technologically current. Finally, with consumer returns, the products are generally not defective, have gone through minimal use, and are technologically current. These returns, if properly collected, can provide a non-negligible source of materials at various grades of quality. In 2007, only 18.4% (by weight) of EoL electronics produced in the US were collected for recycling (U.S. EPA, 2010). Additionally, the cost of reprocessing these returns is always less expensive than starting from the raw material stage to produce a new product (Ferguson, 2010). Similarly, the diminishing amount of raw materials will force manufacturers to implement design methodologies, use materials more efficiently, and recover the products at their EoL (Duelos, Otto, and Konitzer, 2010).

Product recovery management (PRM) refers to the "management of all used and discarded products, components, and materials" by a manufacturer (Thierry, Salomon, van Nunen, and Van Wassenhove, 1995). This may either be voluntary or forced through government intervention. It may also be facilitated by industry groups or government providing financial incentives.

One form of government intervention is in the form of legislation. Mayorga and Subramanian (2009) classify relevant legislation into two main categories: cap and trade, and extended producer responsibility (EPR). Cap and trade establishes a price for the discharge of waste to the producer, while EPR places the financial obligation on the beneficiaries (producers and consumers). Cap and trade, particularly in the form of carbon trading, is a popular policy topic in the political forum; however, its implementation is quite contentious. In the electronic waste (e-waste) domain, EPR legislation is much more prevalent, as it can be implemented via advance recovery fees, recycling subsidies, unit-based pricing, and deposit-refund schemes.

There are two types of EPR models: collective and individual producer. In the collective model, manufacturers jointly share costs and receive waste in terms of market share. In the individual producer model, each producer is responsible for the EoU/EoL waste that is generated by consumers upon disposal. In the latter, Atasu and Van Wassenhove (2010) believe that since the costs of recycling are heavily dependent on the product type, individual producer models are more likely to generate incentives for recyclable product design. Nevertheless, the largest global EPR mandate has been the Waste Electrical and Electronic Equipment (WEEE) Directive, which is a collective model. It enforces producer responsibility for EoL electrical and electronic waste in Europe. With the WEEE Directive, “producers are physically and financially responsible for meeting certain recycling or recovery targets, while the member states must guarantee that 4 kg of such waste is collected per capita per year, at no cost to the end users” (Atasu and Van Wassenhove, 2010).

EPR is meant to encourage recycling and reduce the consumption of virgin material. Producers and manufacturers have to set up their activities to conform to these regulations in the most beneficial way. The intent of EPR is to embolden manufacturers to standardize their products across their industry, design their products for disassembly, and subsequently participate in remanufacturing. This will ultimately enable the cost of remanufacturing to go down and enlarge the scope of what can be remanufactured economically.

The next section discusses different ways in which a manufacturing company can integrate sustainability in its operations.

1.2 Sustainability for Manufacturing Organizations

For a manufacturing organization to engage in sustainable operations, it must first define a strategic vision for its green strategy. This vision can then be translated into real business decisions for implementation at product, operations, and service levels. Products should be designed in a sustainable manner, such as utilizing life-cycle analysis (LCA) and EcoDesign techniques to be constructed for disassembly and exploit the use of reprocessed components. Sustainability should be integrated in the logistics activities of the company, including the engagement of suppliers and purchasers to meet or exceed environmental expectations (Walton, Handfield, and Melnyk, 1998). Finally, the green enterprise must develop support services for consumers using its sustainable products.

1.2.1 Green Business Strategy

Since the mandate of any corporation is to increase shareholder wealth, the green business strategy of the company should be aligned with its mission, values, vision, and long-term goals. Reuse, remanufacturing, and recycling are some aspects of sustainability that may be part of a company's green business strategy. For example, the Volvo Group's corporate values are: quality, safety, and the environment (Bourgeois and Leleux, 2004). According to Guide Jr. and Van Wassenhove (2001), reuse activities must be value-creating and show how the management of product returns influences operations as well as overall profitability. Similarly, some of the business strategy tools that organizations consider before electing to undertake sustainable activities such as remanufacturing are: SWOT, value chain, and market segmentation analyses. Quantifying the benefits and drawbacks of remanufacturing inclusive of marketing, sales, and service, is part of the undertaking while defining a green strategy. In these analyses, customer relationship management (CRM) issues are extremely important, specifically the customer's perception of remanufacturing. The cachet of being environment friendly can be used beneficially in marketing and to cultivate customer loyalty. However, because these products are second-hand, some customers may not be willing to pay as much for them, and even consider them to be less appealing. Furthermore, as remanufactured products have a higher failure rate, it will be more expensive to facilitate their future repairs. Therefore, all of these strategic issues must be considered by the firm while engaging in sustainable product, logistics, and support service design.

SWOT Analysis

The elementary decision to pursue a sustainable business activity such as remanufacturing is whether the particular venture will be beneficial. A typical framework to determine the suitability of a proposition is known as a SWOT analysis (Figure 1.2), which evaluates the strengths, weaknesses, opportunities, and threats of a venture. It is generally presented in the form of a matrix with the top quadrants representing the elements that are internal to the organization, and the bottom that are external to it. The left and right halves comprise factors (helpful/harmful) that have to be taken into account in evaluating the decision.

SWOT has two aspects: matching and conversion. In matching, strengths are matched against weaknesses and opportunities against threats. In conversion, an attempt is made to convert harmful factors into helpful ones (i.e. a weakness into a strength, or a threat into an opportunity). Understanding the impact of SWOT can help structure the sustainable value chain and the market segment(s) to target.

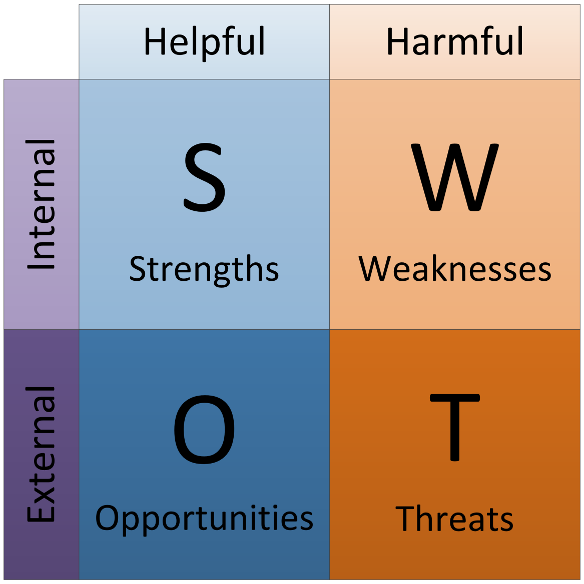


Figure 1.2: SWOT Analysis

In order to illustrate SWOT, consider the case study presented in Bourgeois and Leleux (2004) for the strategic implementation of remanufacturing at Renault Trucks. The internal factors for Renault are its dealers, franchisees, and customers; and the external factors are its competitors. In the SWOT analysis framework, the strengths for Renault are its ability to offer high quality remanufactured products (with warranty assurance) at a higher price-point in comparison to dealers/repair shops. Another strength is Renault's massive international retail network that allows the sale of these remanufactured parts to a large global customer-base. An example of a weakness for Renault is alienating its dealers who have already engaged in profitable remanufacturing. This alienation is an important factor, because the dealers are the direct conduit to the consumer-base and may influence their perception of Renault's remanufactured products.

Opportunities for Renault are: a lower cost of production for remanufactured products, a large retail network and customer base, the ability to offer good warranty programs, and an estimated growth rate of 20% over three years in the European market for remanufacturing parts. A key threat to Renault is not entering the remanufacturing market, because its competitors have already established themselves with the production of remanufactured parts representing 11-21% of their total parts turnover. Another threat is competition by independent repair shops which steeply discount remanufactured parts.

Value Chain Analysis

Achieving corporate sustainability requires the consideration of social, economic, and environmental factors, all having a direct impact on each other (Adams, 2006). This endeavour must also demonstrate that it generates value for the enterprise. The mission statement for the Volvo Group is expressed as:

> By creating value for our customers we create value for our shareholders. We use our expertise to create transport-related hard and soft products of superior quality, safety and environmental care for demanding customers in selected segments. We work with energy, passion and respect for the individual (Bourgeois and Leleux, 2004).

Porter (1985) defines value as the amount that buyers are willing to pay for what a firm provides. In other words, a company is considered successful if the value it generates exceeds the costs involved in delivering its products/services. This leads to the idea of a value chain (Figure 1.3), which encompasses all the actions an enterprise participates in, from the cradle to the grave of a product. A study demonstrating that 47% of consumers are willing to pay 17-19% more for green products is one example of the value of green products amalgamating the three facets of society, economy, and the environment (Mayorga and Subramanian, 2009).

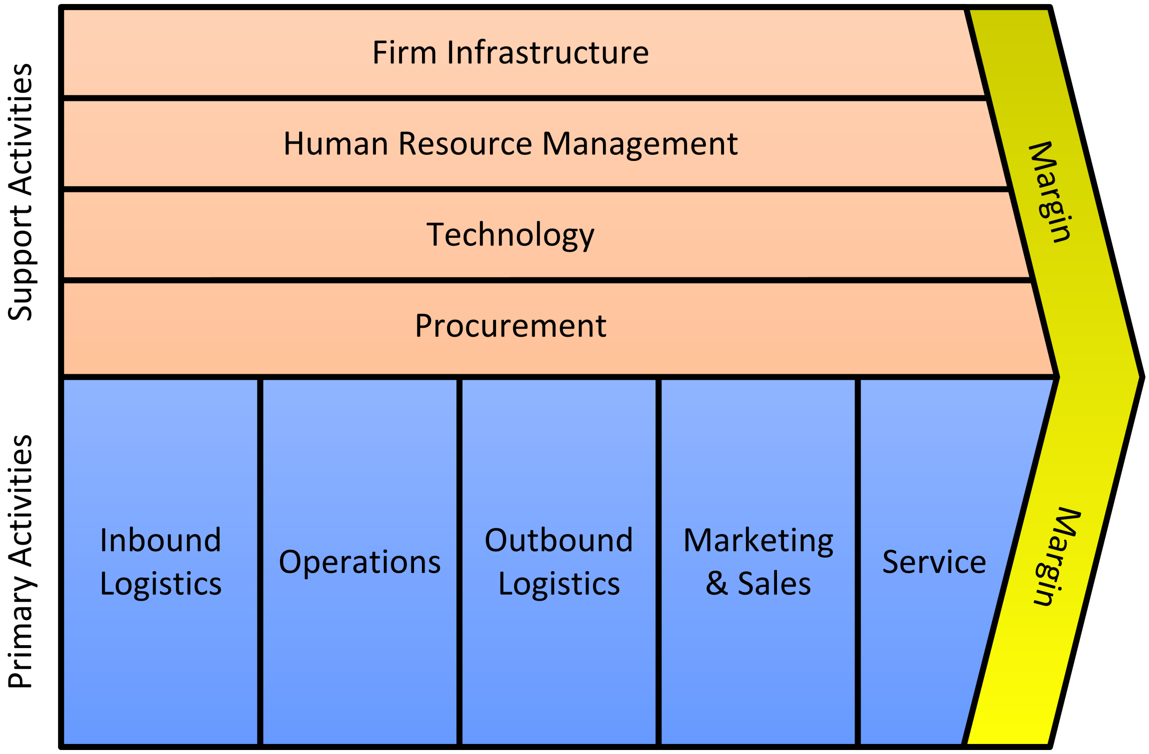


Figure 1.3: Value Chain (Porter, 1985)

The Porter value chain is composed of two sets of activities: primary and support. In the primary category, there are inbound and outbound logistics, operations, marketing, sales, and service; while in the support category we have procurement, technological development, human resource management, and infrastructure (Porter, 1985). We can see in Figure 1.3 that an equal importance is given to operations, logistics, marketing, sales, and service. By understanding the enterprise's value chain, we can determine its core competencies, the type of product(s)/service(s) that should be provided, and the market(s) to target.

Market Segmentation Analysis

The organization must assess what market segment(s) to go after, as this selection dictates several key characteristics in the design and sale of products. The marketing mix (product, price, place, and promotion) is a set of variables based on the selection of the market segment. As Wu (2012a) describes, for green products there is usually a choice to target the green market segment (high-pricing strategy), or the whole market (low-pricing strategy). The green market strategy would entail a high-priced exclusive product that is marketed to be extremely desirable, such as the Tesla Roadster, which is a luxury electric car. Conversely, the opposite would be a refurbished product that is sold at a heavily reduced price, with the primary promotional driver being its low cost. While in either segmentation strategy it is necessary to have a green product, the marketing mix variables would be greatly different.

1.2.2 Green Product Design

Many approaches have been proposed to evaluate the sustainability of products that a company engages in. The most commonly used technique is the LCA and its variants. The LCA methodology can be used to analyze the environmental impact of an existing product/system, or utilized concurrently with the EcoDesign approach to design a new sustainable product/system. In this section, we will present both procedures and discuss how a manufacturing firm can use them to fulfill its sustainability strategy.

Environmental Assessment Tools

Every product, process, or service leaves an environmental impact/footprint. The LCA is a tool used to gauge the environmental costs and benefits associated with the entire life-cycle of a product from the procurement of materials (new/recycled) to disposal or reuse. It examines the resources and materials used in each system, as well as the emissions generated (Craighill and Powell, 1996). This appraisal comprises the wastes and emissions associated with a product, starting with production and including all downstream activities. By using LCA, a company can make an informed decision on suppliers, products, processes, and materials to use in order to balance its economic and sustainable business goals. The LCA also raises awareness within a company on the environmental impacts of its activities.

An LCA study can highlight what types of benefits derive from recycling a certain product, as well as the possible trade-off between environmental and ecological benefits. Sometimes, only the trade-offs are uncovered without a clear quantification thereof, leaving the selection of products or processes to be completed via some other mechanism. For example, Ayres (1995) questions how to "choose between a theoretically biodegradable product manufactured by a dirty process and a non-biodegradable product whose manufacturing process is relatively clean". Chang (2009) provides examples of some of these trade-offs:

> The waste paper recycling scenario has clear energy savings and reduces acid emissions with a similar cost and amount of global warming emissions. The recycling of waste glass and metal packaging leads to clear ecological benefits, whereas recycling of plastic packaging is ecologically advantageous, although it causes much higher collection and treatment costs.

Ayres (1995) proposes an amendment to the LCA framework to classify environmental effects into independent dimensions in order to develop physical measures and derive quantitative ratings for a considerable number of resources, pollutants, and nutrients. Some of these dimensions are: resource depletion, human toxicity, ecotoxicity, acidification, and nutrification. A key criticism of the LCA is that there has been inconsistency with the use of units of mass, energy, and emissions, thereby complicating the comparison between product/process alternatives. Furthermore, there is also a concern that LCA concentrates too much on energy, and pays little attention to other considerations.

One approach that has been proposed to address this issue is material input per service unit (MIPS). This method, developed by Ritthoff, Rohn, and Liedtke (2002), adds up the overall materials required to make a product, or provide a service throughout its life-cycle. MIPS was conceived to encourage sustainability by measuring the eco-efficiency of a product/service. A product/service with a lower MIPS value is considered to be more eco-efficient, and the two ways to reduce the MIPS value are to either decrease the material inputs or increase the amount of service units. Car-pooling for example reduces the MIPS value for a given vehicle by increasing the number of service units (i.e. passengers). Another outcome of this technique has been the growth of purchasing locally-grown food in an effort to decrease transportation expenses.

Emergy accounting is another approach to incorporate sustainability into product design by “including the contribution of natural ecosystems, data relative to human labor and process implementation” (Almeida, Rodrigues, Bonilla, and Giannetti, 2010). In this methodology, the type of available energy (exergy) required to make a product or service is measured in emjoules, which are joules of solar energy (Odum, 1996). By converting different forms of energy and resources into emjoules, an equivalency on the basis of a common energy source can be established.

“The resources necessary to produce the goods that an individual or population consumes offers a simple and intuitive estimate of the production inputs for a given consumption level” (Fiala, 2008). This technique, first proposed by Rees (1992), is known as the ecological footprint. While it can be useful to measure several sustainability metrics, the downside of the ecological footprint is that it is not successful in incorporating land degradation, where the land “can either no longer be used, or it is used at a severely decreased efficiency” (van Kooten and Bulte, 2000). Since, land degradation is one of the most important issues of sustainability, the LCA with all of its faults still remains the preferred procedure to assess environmental impacts.

While the LCA is a tool to quantify environmental impacts, it facilitates EcoDesign, a methodology to incorporate environmental considerations for the concurrent design of sustainable products/systems. The next section presents multiple aspects of EcoDesign and discusses how a manufacturing firm can use it to fulfill its sustainability strategy.

EcoDesign

EcoDesign can be thought of as product design with the following elements: accessibility for recovery of components or modules, easy removal of labels, use of prefabricated breaking points, and using as few joining elements as possible in order to facilitate upgrade, repair, and reprocessing. Luttropp and Lagerstedt (2006) provide a definition and present 10 rules for EcoDesign practice. Other considerations in EcoDesign are: minimizing energy and resource consumption, using recycled and simple materials, investing in high quality materials to promote long life and reduce maintenance, avoiding the use of toxic substances, and utilizing closed-loop supply chains for recovery. This view of EcoDesign moves it into a broader framework. Byggeth and Hochschorner (2006) describe a valuation technique to assess trade-offs in different EcoDesign tools. Design for remanufacturing and EcoDesign include a series of activities such as the design for disassembly, multiple life-cycles, upgrade, and core collection (Charter and Gray, 2008; Hatcher, Ijomah, and Windmill, 2011).

There has been a trend towards sustainable product and process design in the literature and in practice. Without explicitly using the term EcoDesign, other authors discuss sustainable product design and development to minimize energy and material consumption as well as the impacts of waste and emissions, while maximizing the usage of expended resources (Nasr and Thurston, 2006). Misra (2013) describes the design of sustainable products encompassing strategies of waste prevention, dematerialization (using less materials), production, and consumption.

Disassemblability is the degree to which a product can be disassembled without force (Mok, Kim, and Moon, 1997). In their study, Mok et al. (1997) investigate geometrical and material characteristics of parts; these include subassemblies and joining elements to be assembled, the disassembly mechanism between parts and subassemblies, and the weakpoints of the disassembly process. Meacham, Uzsoy, and Venkatadri (1999) consider the optimization problem of meeting demand for recovered components and subassemblies from an inventory of recovered products and propose a column-generation algorithm to solve it. In this setting, products have common components and disassembly capacity is limited. Zwingmann, Aït-Kadi, Coulibaly, and Mutel (2008) develop a framework to identify all the feasible disassembly sequences for a multi-component product, and find an optimal disassembly sequence, according to specific criteria such as cost, duration, and profit.

Disassemblability is also analyzed in a multi-period model by Wu (2012a), which consists of an original equipment manufacturer (OEM) and a remanufacturer. This paper discusses how disassemblability can benefit the remanufacturer because its costs of recovery and production are reduced. The OEM also benefits by having increased production flexibility; however, it is also accompanied by greater fixed costs, such as the investment costs of purchasing new equipment or updating facilities. Furthermore, its future sales may be compromised due to competition with the remanufacturer. In their model, remanufactured products are sold in the second period (where both participants compete directly with each other). The remanufacturer must decide whether to target the green market segment (high-pricing strategy), or the whole market (low-pricing strategy). Similarly, the OEM must decide the level of disassemblability in the design of the product.

Some OEMs attempt to deter remanufacturing by using welding or adhesives, although it has been argued that these low disassemblability techniques are cheaper to utilize in the initial production process. In the situation where the OEM is also the remanufacturer, they are much more inclined to opt for a high level of disassemblability.

While disassemblability dictates the degree to which a product can be taken apart, modularity “permits components to be produced separately and used interchangeably in different product configurations without compromising system integrity” (Mikkola and Gassmann, 2003). Das and Chowdhury (2012) develop a mixed-integer program (MIP) that maximizes profit by using modular product design to fabricate products at different quality levels. One aspect of modularity is standardization, which can drive down the cost of remanufacturing by allowing interchangeable components among many manufacturers.

Design for recycling (DfR) requires the consideration of conceiving future products using some current or legacy components. Such components are chosen if their utility is expected to last over several product generations. Quella and Belli (2013) describe how DfR facilitates easier assembly, disassembly, and lower production costs via modular construction.

In the design of a sustainable product, dependability is also an important factor. The dependability of a system is the ability to deliver a service that can justifiably be trusted by its users (Laprie, 1992). An alternate definition for dependability is the ability to avoid service failures that are more frequent, and more severe than is acceptable (Avižienis, Laprie, Randell, and Landwehr, 2004). Dependability can be perceived in terms of availability, reliability, maintainability, safety, and/or security.

Sherwood, Shu, and Fenton (2000) describe different types of failure and scrap modes in the automotive industry, and then conduct statistical analysis to determine conditions for the design of repair and remanufacturing. A procedure is proposed in Wani and Gandhi (1999) that is based on a digraph and matrix method for the evaluation of a maintainability index of mechanical systems. Their procedure is useful in the design and development of maintainable systems. A safety-based availability assessment method to be used at the design stage is developed in Houssin and Coulibaly (2014). The timely availability of a sufficient quantity of durable, high-quality products is also very important in sustainable design, for which a dependable, well-designed acquisition network is required.

1.2.3 Green Logistics and Supply Chain Management

The integration of environmental aspects into traditional logistics and supply chain management (SCM) activities is known as green supply chain management (GSCM). It covers areas such as supplier selection, material sourcing, manufacturing processes, delivery of finished goods to consumers, EoU/EoL collection, and value recovery operations. In a green/closed-loop supply chain, the product is considered over multiple cycles, each going from its cradle to its grave. In this section, we first review and classify traditional supply chains models, and then present the functions comprising GSCM. Finally, remanufacturing is analyzed through a new SPLC framework.

Traditional Supply Chain Management

Since the early 1980s, SCM has been a dominant topic covering customer relationship management (CRM), marketing, manufacturing, transportation, and distribution. Min and Zhou (2002) describe a supply chain as a system of integrated business processes, with the main objective to enhance operational efficiency, profitability, and the competitive position of a firm and its supply chain partners. The chain encompasses everyone from the end-user to the original suppliers including: the acquisition of raw materials and parts; the transformation of raw materials and parts into finished products; the addition of value to these products; the distribution and promotion of these products to either retailers or customers; and the facilitation of information exchange among various business entities (e.g. suppliers, manufacturers, distributors, third-party logistics providers, and retailers).

In the Porter (1985) definition, inbound logistics, operations, and outbound logistics comprise the first three out of five steps in the primary value chain. This is important, because this delineates the key difference between a supply (logistics oriented), and a value chain (addition of marketing, sales, and service). The Supply Chain Council (2012) describes the major logistical (infrastructural) components of a supply chain in the supply-chain operations reference (SCOR) model as the:

- Inbound: The chain representing inbound material flow from suppliers (including their suppliers in turn) to an organization;
- Organization: An organization within a supply chain;
- Outbound: The chain representing outbound material flow from an organization to its customers (and their customers in turn).

SCOR is based on six distinct management processes: *Plan*, *Source*, *Make*, *Deliver*, *Return*, and *Enable*. *Source*, *Make*, and *Deliver* relate to the inbound, organization, and outbound activities respectively. *Plan* and *Enable* represent higher order processes, and *Return* was recently added to include reverse logistics processes (Stewart, 1997; Supply Chain Council, 2012).

Another way of looking at a supply chain is through its three levels of decision-making, which are: strategic, tactical, and operational. The strategic level refers to the high level of decision-making in the supply chain including pricing and network design. The tactical level entails activities such as product acquisition and planning, inventory planning, and dealing with product returns. Finally, the operational level refers to day-to-day activities such as disassembly planning, lot sizing, priority dispatching, and scheduling.

When considering the supply chain through the infrastructure point of view, the movement of products between facilities is referred to as transportation, while movement within a facility is known as material handling. There has been a lot of work in supply chain literature to minimize the costs and environmental impacts of transportation. Similarly, material handling systems are important, but also very expensive. There is no added value from material movement in the manufacturing process, but it accounts for half of the company's operational costs (Meyers, 1993). It is ideal to limit the amount of handling to minimize cost, waste, and the ecological footprint. The Material Handling Industry of America has developed 10 material handling principles around the themes of planning, standardization, work, ergonomics, unit load, space utilization, system, automation, environment, and life-cycle (Heragu and Ekren, 2009). The environment's inclusion in this list is noteworthy, because it encourages the reduction of natural resource consumption and energy usage.

The access to accurate and timely information by collaboration between all involved parties is another important consideration in SCM. By working together, costs of purchasing, logistics, and manufacturing are all lowered, while also enabling product variety and customization (Chopra and Meindl, 2006). Lambert (1998) is a proponent of more active forms of management in SCM in addition to collaboration that is loosely managed or spontaneous. Each interaction is defined as a business process link, and it varies based on its level of criticality to the firm. These links, classified as: managed, monitored, not managed, and non-member, are illustrated in Figure 1.4:

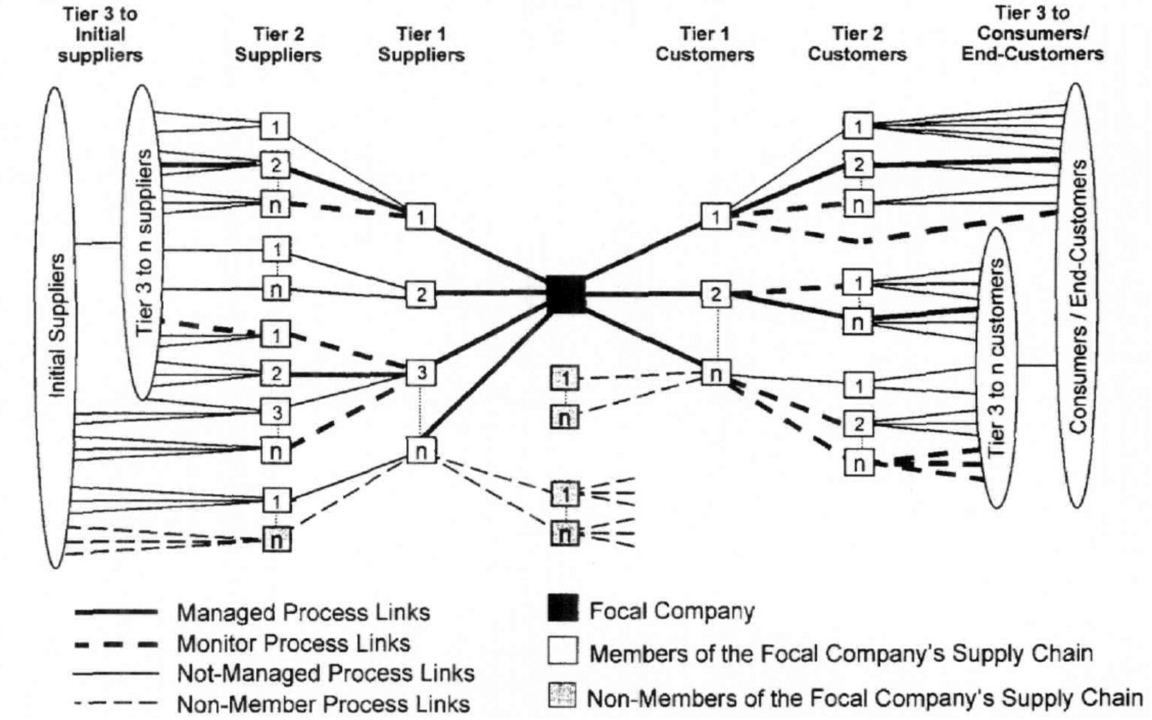


Figure 1.4: Business Process Links (Lambert, 1998)

Managed links connect the firm to one or more supplier; monitored links are not fully controlled by the firm, but monitor the management and integration; non-managed links are not controlled nor monitored by the firm; and non-member links are between partners and those outside the core supply chain (Lambert, 1998). These links play an important strategic role in identifying possible competitive advantages (Porter, 1985); and they also restrict the scope of the model in order to get a more accurate representation.

Classification of Supply Chain Models

Supply chain models are theoretical constructs to represent activities and their relationships within a supply chain. These models are usually normative or prescriptive and can be at the strategic, tactical, or operational level; the general objective to maximize the overall value generated by the supply chain by modelling some aspects of it to fulfill certain explicit visions or goals (Chopra and Meindl, 2006).

Supply chain goals are used to develop performance measures (objectives) which can then be incorporated into the model. Min and Zhou (2002) identify some of these performance measures as: customer service initiatives (product availability and response time); monetary value initiatives (asset utilization, return on investment, and cost behaviour); information/knowledge transactions (real-time communications, technology transfer); and risk elements (risk of quality and information failure).

In addition to goal-setting, the decision variables and the constraints of a model are required for its construction. Typical decision variables are: number and location of facilities, types of equipment in the facilities, number of stages in the supply chain, flow allocation, service sequence, inventory levels, size of workforce, and the extent of outsourcing. Constraints to consider may include capacity, demand, and minimum service levels. The performance measures/objectives are generally expressed as functions of one or more decision variables. Examples of supply chain models are: the location-allocation model, various types of multi-echelon inventory models, and simulation models.

There are numerous taxonomies of supply chain models. Min and Zhou (2002) organize them into four categories: deterministic, stochastic, hybrid, and information technology (IT)-driven. This is depicted in Figure 1.5:

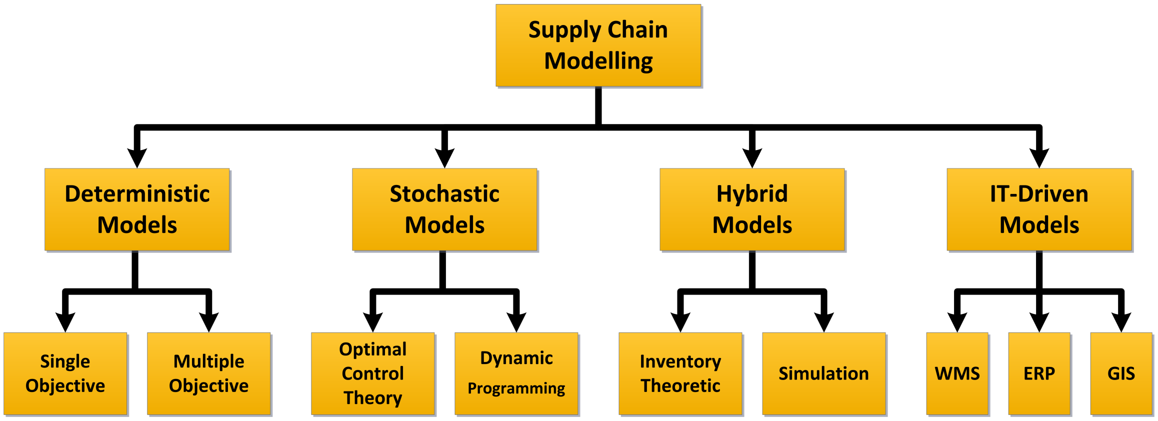


Figure 1.5: Supply Chain Model Taxonomy (Min and Zhou, 2002)

Deterministic models assume that all the model parameters are known and fixed with certainty. Stochastic models take into account uncertain parameters. Hybrid models have elements of both. IT-driven models integrate and coordinate various phases of supply chain planning on a real-time basis using tools such as: transportation management systems (TMS), warehouse management systems (WMS), collaborative planning and forecasting replenishment (CPFR), enterprise resource planning (ERP), and geographic information systems (GIS).

Green Supply Chains

When environmental factors are integrated into SCM, the resulting value chain is called a green supply chain. "Green supply chain strategies refer to efforts to minimize the negative impact of firms and their supply chains on the natural environment" (Mollenkopf, Stolze, Tate, and Ueltschy, 2010). The unification of environmental plans with the firm's operations, particularly procurement and delivery, leads to the idea of GSCM. GSCM covers several areas, including the materials used in product design for the environment, product design processes, supplier evaluation and selection, supplier process improvement, and inbound logistics (Walton et al., 1998).

Mayorga and Subramanian (2009) integrate environmental factors in GSCM by introducing emissions permits and limits, and stochastic net present value of a banked permit in decision variables, constraints, and the objective function. Srivastava (2007) develops a framework to classify GSCM into areas of green manufacturing and remanufacturing, reverse logistics and network design, and waste management. An adaption of this taxonomy, focusing on GSCM is illustrated in Figure 1.6.

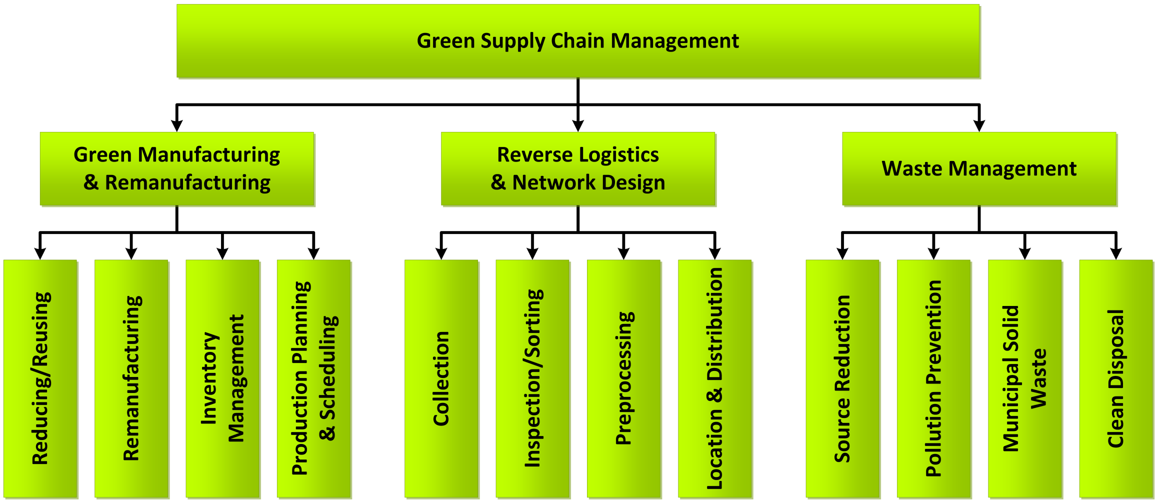


Figure 1.6: GCSM Taxonomy Adapted from Srivastava (2007)

GSCM also deals with improved collaboration among all parties in order to achieve better results for a firm and its partners. Just as collaboration helps to lower costs within a traditional supply chain, Vachon and Klassen (2008) demonstrate that it is also transferable, and beneficial to the sustainable version:

> Environmental collaboration was defined specifically to focus on inter-organizational interactions between supply chain members, including such aspects as joint environmental goal setting, shared environmental planning, and working together to reduce pollution or other environmental impacts.

Research has also been conducted on how supply chain companies use contracts with suppliers to influence environmentally conscious material usage (Reiskin, White, Johnson, and Votta, 1999; Bierma and Waterstraat Jr., 2000; Corbett and DeCroix, 2001). “The most common GSCM practices involve organizations assessing the environmental performance of their suppliers, requiring suppliers to undertake measures that ensure environmental quality of their products, and evaluating the cost of waste in their operating systems” (Darnall, Jolley, and Handfield, 2008).

Remanufacturing

The idea of reducing waste to cut costs is not a new concept; this has been at the heart of lean manufacturing for many decades. In lean manufacturing, anything that does not produce value to the end customer is considered wasteful. By eliminating waste, there is an improvement in productivity, efficiency, quality, and mix complexity (Krafcik, 1988). One aspect of reducing waste is by designing products that are easily reused and reprocessed. This is an important area of focus because there is room for creating value for the company by modifying its supply chain (to a closed-loop) to enable product recovery and remanufacturing.

In the traditional product life-cycle (PLC), a product is only monitored from its cradle to its grave; however there is now a need to also monitor a product after the end of its useful life. “Today, modern environmental management prescribes sustainable manufacturing practices that focus on prevention of waste and responsible care of the earth’s natural resources...described as ‘cradle-to-cradle’ resource management” (Kumar and Putnam, 2008).

A remanufacturer has three options for retrieving products from the marketplace: directly from the customer, via a retailer by providing incentives, or by subcontracting a third party to do the collection (Savaskan, Bhattacharya, and Van Wassenhove, 2004). Upon collection, products are typically aggregated at consolidation centres, before being shipped to treatment centres for recycling, parts harvesting, resale (as-is), internal reuse, remanufacturing, or disposal. Closed-loop supply chain (CLSC) and management models have drawn a lot of attention over the last decade (see (Savaskan et al., 2004; Ferguson and Souza, 2010; Fleischmann, Galbreth, and Tagaras, 2010; Aït-Kadi, Chouinard, Marcotte, and Riopel, 2012) for extensive reviews). An example of a CLSC for leased products is depicted in Figure 1.7.

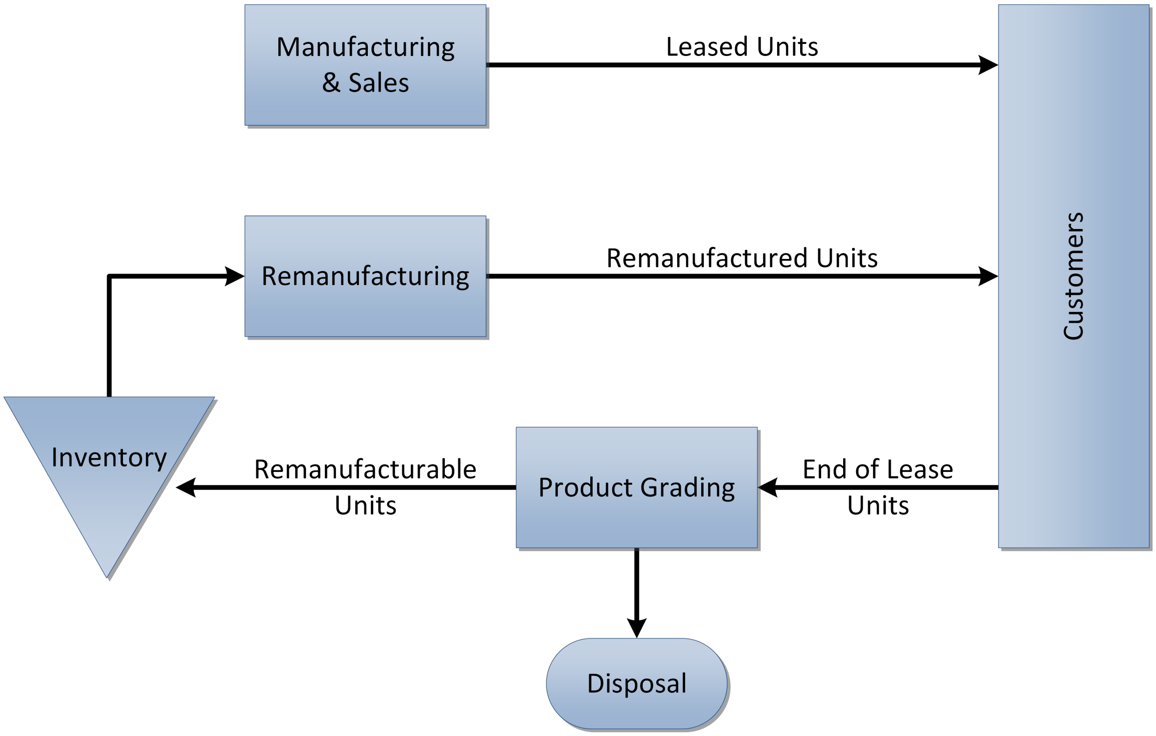


Figure 1.7: CLSC for Leased Products Adapted from Ferguson and Souza (2010)

Lund and Hauser (2003) consider remanufacturing to be a value-added operation that can be of both economic, and environmental benefit to a corporation and consumers, because it has the potential for higher profitability. To understand and place remanufacturing in the context of green logistics and GSCM, the SPLC framework shown in Figure 1.8 is proposed as part of this research. It illustrates *cradle-to-cradle* resource management activities including remanufacturing.

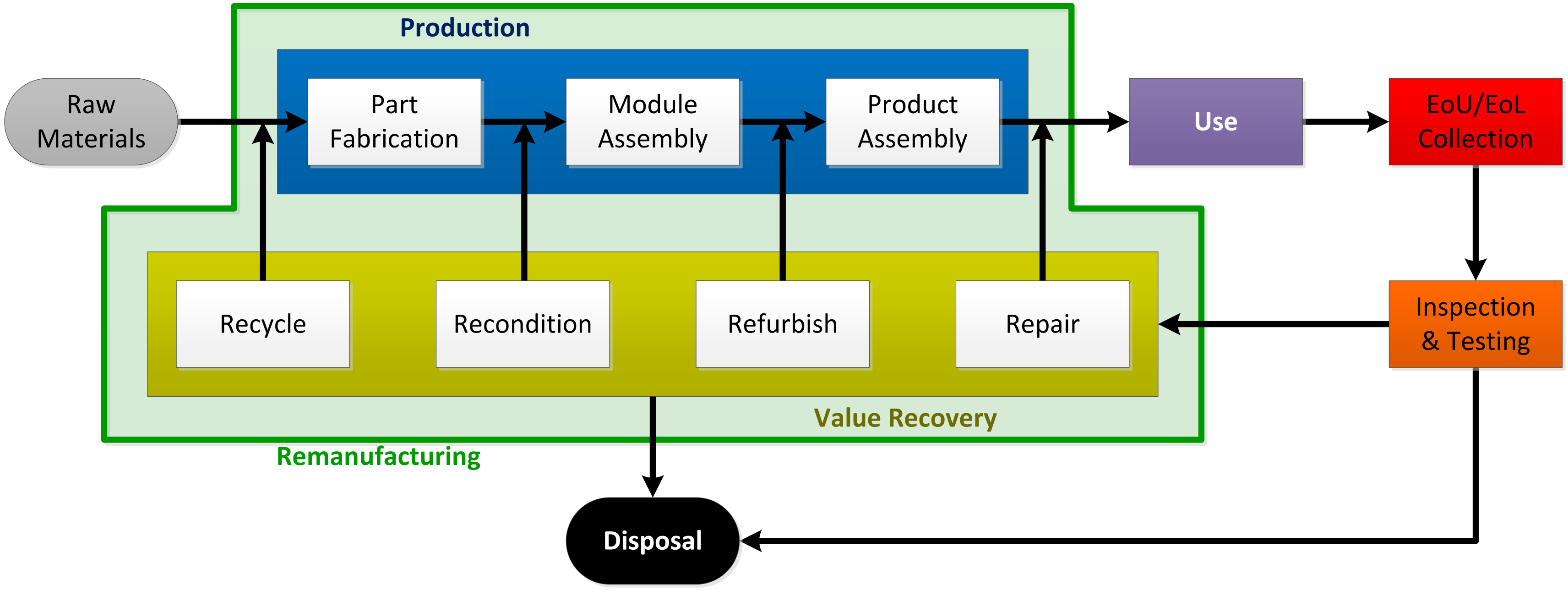


Figure 1.8: Sustainable Product Life-Cycle Framework with Remanufacturing

In Figure 1.8, the major categories are: *EoU/EoL collection* (in red), *inspection & testing* (in orange), *value recovery* (in gold), *production* (in blue), and *use* (in purple). This activity chain begins with the procurement of raw materials which are then manufactured into a finished product through the steps of fabrication, part, module, and product assembly. The complete product is then sold to the consumer who uses it until its EoU or EoL. This part of the diagram is the production chain part of the PLC.

The closed-loop activities begin with EoU/EoL collection. Products are then sent to be sorted, cleaned, tested, triaged, and graded. Based on their state, they undergo one of four value recovery activities (repair, refurbishing, reconditioning, or recycling), or are disposed of (Thierry et al., 1995):

- ⋆ When minimal effort is needed to restore the product to working order, the recovery option usually selected is repair. Repair operations are carried out on a limited number of failed components or modules, which are fixed or replaced by new or reconditioned spare parts.
- ⋆ If the product requires more effort and a higher level of quality, the recovery option selected is refurbishing, where the product is disassembled into modules, which are inspected, repaired, replaced, or upgraded.
- ⋆ If even more effort is required, the reconditioning recovery option can be considered. In this option, products are disassembled, and good quality parts are recovered to be used in other options with or without further quality enhancement to bring them to a like-new condition. The remaining items are either disposed of or recycled.
- ⋆ Finally, recycling is an energy intensive exercise reserved for products or parts that are unfit for the previous three recovery options. Recycling involves sorting, shredding, and melting to generate materials that can be used to fabricate individual parts, and conserve virgin materials.

The value recovery activities yield modules, parts, and/or materials that can be reused in the production process, exclusively or in conjunction with new parts before being sent back into the marketplace for consumers to use. Thus, the amalgamation of both value recovery and production processes as depicted in Figure 1.8 may be viewed as the definition of *remanufacturing* (in green). In this definition, remanufacturing is defined as the superset of the production and value recovery processes. The traditional production and distribution network does not use recovered materials or components. On the other hand, the remanufacturing network, through value recovery, includes the use of recovered materials and components in production.

Remanufacturing procedures correspond to different types of upgrade actions. In reliability theory, repairing a product by replacing one or two broken parts with like-new or new spares is considered minimal repair. Reconditioning a part or module to a *as-good-as-new* condition may be viewed as perfect repair. Refurbishing is an upgrade action that falls somewhere between minimal repair and perfect repair. Refurbishing a product can be modelled as a general imperfect repair process. These reliability considerations of remanufactured products have significant maintenance and service implications, which will be explored later in this dissertation.

1.2.4 Product Support Systems Design

Product support systems are a mechanism to provide assistance to consumers when they have complications with their purchased products that are still under warranty. They represent a vital link between a company and its consumer-base. In the SPLC framework (Figure 1.8), support activities span the *use* and *repair* categories.

Quality function deployment (QFD) is a methodology to incorporate the needs of a customer (both features and quality) into the design of products. QFD is defined as a "method to transform qualitative user demands into quantitative parameters, to deploy the functions forming quality, and to deploy methods for achieving the design quality into subsystems and component parts, and ultimately to specific elements of the manufacturing process" (Mizuno and Akao, 1994). Hauser and Clausing (1988) describe the QFD structure, which "focuses and coordinates skills within an organization, first to design, then to manufacture and market goods that customers want to purchase and will continue to purchase". This sequence of events take into account the voice of the customer through engineering and manufacturing choices. Support systems provide valuable field service data and also customer satisfaction indicators. The ability to support a product, particularly through its warranty, depends on the quality of the product (Hauser and Clausing, 1988; Padmanabhan and Rao, 1993). As a result, support systems, which have an impact on the entire SPLC, have taken a much broader role recently (Markeset and Kumar, 2003):

> Traditionally, support merely constituted maintenance, service and repair. However, as the scope of product support has broadened over the past decade, it has also included such aspects as installation, commissioning, training, maintenance and repair services, documentation, spare parts supply and logistics, product upgrading and modifications, software, and warranty schemes, telephone support, etc.

The first two items in the list described by Markeset and Kumar (2003), installation and commissioning, are generally similar for remanufactured and traditional products. This is because the composition of the product itself is not likely be known to those conducting these activities and there is generally no deviation from standard processes. However, the remaining items in the list are likely to have an effect on the support service procedures employed by the organization. We will explore how the aspects described by Markeset and Kumar (2003) relate to a firm engaged in remanufacturing.

We start by examining technical support, which encompasses employee training and their capability to provide resolution in person or remotely via video conference, telephone, email, etc. It can be assumed that the frequency of technical support that is necessary will increase, because remanufactured products, with lower reliability, will be more prone to failure. This will then translate into an increased training and operational budget as more resources are needed in order to conduct the necessary support services. Furthermore, this training may also be needed to account for newer failure modes (e.g. reconditioned products may be more prone to corrosion).

The remanufacturing facility should make spare parts, documentation, and software readily accessible. For example, a third-party conducting repairs may also need to supply OEM documentation and software to the end consumer. Spare parts may be original from the OEM, generic, or recovered from EoU/EoL products, though the third option poses logistical challenges. In the case of spare parts, the challenge is to maintain inventory levels to fulfill support needs over the production cycle.

The repair kit problem formulation is used to determine the optimum quantity of components of each type in order to minimize total holding and service shortage costs. This is relevant when technicians are sent to different sites to repair products with a kit composed of different types of components. Bijvank, Koole, and Vis (2010) develop a generalized model and an efficient algorithm to determine the contents of the repair kit. In the case of remanufactured products, this problem takes on an added dimension, which is the quality level of kit components – for example, a component in the kit may either be new, or of varying quality grades.

As the condition of a product deteriorates over time, maintenance and repair services are needed. Preventive maintenance is defined as "an activity undertaken regularly at preselected (but not necessarily identical) intervals while the device is satisfactorily operating, to reduce or eliminate the accumulated deterioration"; on the other hand, repair is "an activity where a device is restored to working condition after a failure has rendered it inoperative. If not indicated otherwise, then after repair the device is as good as new" (Sim and Endrenyi, 1988). Since remanufactured products will have generally higher failure rates, the maintenance policy must be redesigned to account for this. Similarly, with remanufactured products, repairs may occur sooner (with an added financial burden to the servicing firm).

Product upgrading and modification improve a consumer's product experience. This could be through enhancements such as a newer version of software for existing hardware, or physical upgrades to the hardware itself. "Remanufactured components, especially engines, often benefited from recent technology upgrades, which sometimes made them more efficient than the original component in terms of performance and emission regulations" (Bourgeois and Leleux, 2004). In other situations, technical modifications are needed in response to safety recalls.

"Remanufacturing...is the lowest level of risk and the most cost effective choice. Of course, it is important to know who did the work and to make certain a warranty is attached" (Birkland, 1996). When designing the warranty policy, particularly with remanufactured products, there are several important aspects to address, including length of the warranty. Consumers have a preference towards products with a longer warranty for the following reasons:

- Increased coverage from the manufacturer is more reassuring.
- The perception that a longer warranty period is synonymous with higher quality and reliability.

For a firm offering a warranty, a longer warranty period implies a larger reserve fund to service failures. However, longer warranty periods can also increase demand and thereby, profit. The use of reconditioned components in warranty servicing adds another level of decision-making complexity for a firm.

To successfully transition into a sustainable business, the organization will need a structured approach to steer and manage the decision-making, the implementation of ideas and solutions, the monitoring and control of the changes, and the continuous identification of opportunities for improvement. This is discussed in the next section.

1.3 Integrated Sustainable Operations Planning

For many organizations, complying with sustainability legislation or voluntarily engaging in sustainable activities will mean a redefinition of their business vision; a readjustment of their policies; and a redesign of their products, operations, logistics, and services. A framework for decision-making and operations planning for a sustainable organization is presented in Figure 1.9. The proposed framework integrates three mutually interacting areas of operations planning: logistics planning, remanufacturing planning, and support systems planning.

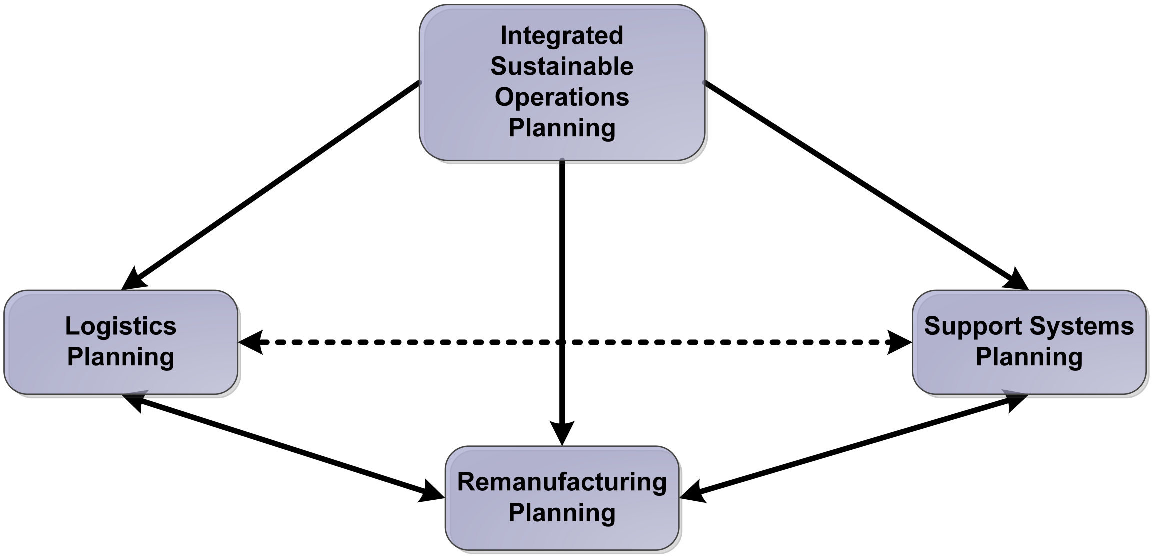


Figure 1.9: Framework for Sustainable Operations Planning

The Integrated Sustainable Operations Planning (ISOP) level is responsible for the implementation of the organization's sustainability vision, monitoring the progress of the sustainability plan, providing feedback to all stakeholders and employees, and promoting and creating a sustainability culture to steer the whole organization toward its goals. The ISOP level also interacts with the three specific planning areas (logistics, remanufacturing, and support systems).

Reverse logistics planning addresses issues related to the handling, shipping, and flow of products/parts within the supply and distribution network of the organization. Examples of some important questions in this planning are:

- ⋆ How will EoU/EoL products be recovered?
- ⋆ What capacity should be built into the reverse logistics?
- ⋆ Where should reprocessing centres be located?
- ⋆ Should the company use a third-party logistics provider (3PL)?
- ⋆ How to design inventory policies for recovered systems?

Remanufacturing planning deals with the decisions, policies, and operations that will make the organization's portfolio of products sustainable. Examples of some important questions covered in this area are:

- ⋆ How modular will the products be?
- ⋆ Which materials will be used or forbidden (see Volvo's White (Nordkil, 1998c), Grey (Nordkil, 1998a), and Black (Nordkil, 1998b) Lists)?
- ⋆ When in the product life-cycle should remanufacturing be introduced?
- ⋆ Where and how will reprocessing of recovered products take place?

Support systems planning refers to the customer interface to provide high quality remanufactured products. The organization must devise a system to provide timely maintenance, repair, technical support, and documentation to consumers who buy second-hand, refreshed, or remanufactured products. Examples of some important questions are:

- ⋆ What type and length of warranty should be offered?
- ⋆ Where should service centres be located?
- ⋆ What should replacement policies be?
- ⋆ What upgrade offers should be made and when should they be presented?
- ⋆ What should the inventory level of replacement parts be?

There are significant interrelations between these three areas of operations planning. For example, in the consumer electronics industry, the decision to refurbish and sell used products to the consumer is dependent on the extent that the products are designed to facilitate the remanufacturing process. It is also easy to see that an efficient collection network must be set up to recover products and parts to make the remanufacturing process economical. Finally, remanufactured products sold to consumers have to be backed by strong warranty and replacement policies to make them attractive.

1.4 Research Objectives & Dissertation Organization

This dissertation thematically explores sustainable supply chain operations planning. The three main chapters of this dissertation, *Chapters 2-4*, correspond to the three themes of Figure 1.9. Each chapter is meant to be self-contained with the following elements: introduction, literature review, problem statement, solution methodology, model formulation, results and discussion, conclusions, and suggestions for future research. The remainder of this dissertation is organized as follows:

Reverse logistics network design is an important element of strategic planning for organizations involved in closed-loop or environmentally sustainable supply chains. In *Chapter 2*, we address the design of reverse logistics networks for the recovery of EoU/EoL products. In this chapter, we present a two-phased mathematical decomposition model to generate collection routes for recyclable materials in a provincial municipal solid waste (MSW) network. Our approach considers a single type of vehicle for the collection of recyclables and allows for depots to be skipped, if they have low volumes or are difficult to incorporate in a routing solution. One of the decisions in this model is the number of vehicles needed to construct a periodic MSW pickup schedule. The results from the case study demonstrate economic savings and a reduction in greenhouse gas emissions (GHGEs). While the illustrative example in this chapter is specific to the recovery of MSW recyclables, this methodology has general applicability for other EoU/EoL products, including electronics.

The research question of developing a cost-optimal production strategy for remanufacturing with reconditioned components is addressed in *Chapter 3*. One of the issues faced by strategic planners in sustainable manufacturing is how to develop long-term production plans in remanufacturing settings with recovered EoU/EoL products. The availability and economic feasibility of product recovery in the remanufacturing case is influenced by the product life-cycle which consists of four phases: introduction, growth, maturity, decline. In this chapter, we extend a two-stage production control model in literature to a setting involving both new and reconditioned components. The key production life-cycle decisions for this problem include the duration of the production cycle, the length of the warranty offered to the customer, and the commencement of remanufacturing in the product life-cycle (with implications for the timing of recovery). The case of an anonymous regional airline using reconditioned spare parts is presented for illustrative purposes. The contribution of this chapter to the literature is the integration of the remanufacturing production plan into the product life-cycle.

As noted, support systems planning is the third key enabler within the ISOP framework, and is the theme for *Chapter 4*. In this chapter, a warranty policy model is developed for the case when replacement parts are sourced from a lot containing both new and reconditioned components. It is expected that this model will be of importance to strategic planners within a sustainable manufacturing organization interested in using remanufactured components. The length of the warranty period is an important decision variable in such settings, since customers need greater assurances for their purchases. At the same time, the organization needs to trade-off cost savings from introducing cheaper reconditioned components with the higher costs of warranty and maintenance ensuing from their lower reliability. This problem is formulated as an unconstrained nonlinear programming (NLP) model to maximize the profitability of the remanufacturer. The key policy parameters are: warranty length, product sale price, average age of reconditioned components, and the proportion of reconditioned components in the input lot. The contribution of this part of the thesis is a quantitative basis to model and optimize profit as a function of warranty policy parameters.

CHAPTER 2

NETWORK CONFIGURATION FOR PRODUCT RECOVERY

Designing the reverse logistics network may be seen as the starting point for a sustainable organization engaging in product value recovery. As illustrated in Figure 1.8, products are recovered from the customer and processed through the value recovery network. The CLSC places emphasis on product return and as mentioned in the previous chapter, an important component within the current version of SCOR.

After EoU/EoL collection, recovered products end up in one of the four value recovery activities in Figure 1.8, namely recycling, reconditioning, refurbishing, or repair. The reverse logistics network is used to recover EoU/EoL products from collection centres and transport them to end-use facilities. Reverse logistics networks are usually complex in nature and flow is from many locations to one facility, unlike forward logistics flow which is one-to-many. The complexity arises from the geographical diversity of customer locations and unpredictability in collection volume, depending on the incentive to return products. Reverse logistics operations are conducted by the companies themselves, industry groups, non-profit organizations, governments (usually municipal), or may be outsourced to a 3PL. Many companies also operate a reverse logistics network for their repair network. In the case of a repair network, products are usually brought in by customers to retail outlets. If the repair cannot be conducted at the retail outlet, the product is sent to a regional or national repair centre. Once repaired, the products are usually channeled back to the customer through the forward logistics network.

Historically, human health was a major reason for government bodies to collect waste and keep cities clean. Landfills were the primary method to process waste, however as populations have grown dramatically, so too has the quantity of waste produced. As this became more expensive, other initiatives to divert waste such as recycling and composting have taken hold. While recycling is at the material level, activities such as reconditioning, refurbishing, or repair are at the product level. The e-waste collection initiatives of governments and private industry groups across the world are designed to enable these higher value recovery activities. Resource recovery offers an alternative to the disposal of unwanted products that are slated to end-up in landfills, or for processes such as incineration or waste-to-energy. Nearly all human activities such as agriculture, industrial production, pharmaceuticals, and power generation create waste. Waste is classified into categories such as: industrial, agricultural, medical, radioactive, and MSW. MSW is composed of everyday items that are discarded by the public, such as recyclables, organic material, and e-waste. The MSW stream in particular has resources that can be valuable for remanufacturing.

In this chapter of the dissertation, we explore recovery network design of recyclable materials using the MSW setting.

2.1 Definitions, Classification, and General Issues

There are three aspects to the product recovery framework: who does the collection, who is responsible, and who funds it. From the collection viewpoint, the product recovery collection network can be either public or private. Public collection entails the acquisition of any product that fulfills the mandate of a government entity through legislation. This also includes the sub-contracting to either for-profit or non-profit organizations. The municipal collection of recyclables, organic material, electronic waste, and garbage are the most recognizable examples of this. Private collection is when an organization recovers products for reuse, value-added processes such as remanufacturing, or to take advantage of available monetary incentives.

From the responsibility viewpoint, government, on behalf of society, attributes responsibility to producers or consumers. If the collection responsibility belongs to the producer, it is known as EPR; if it belongs to the consumer, it is commonly referred to as product stewardship. “In Canada, both EPR and product stewardship programs are used to manage products at their end-of-life” (Environment Canada, 2013a). The goals of these programs from the perspective of governments is to conserve virgin material resources and to reduce waste. Atasu and Van Wassenhove (2010) state:

> From the legislator's perspective, the ultimate goal should be the reduction of the environmental impact by proper recycling and the disposal of e-waste while keeping the social-economic impact at a marginal level. In other words, EPR should maximize social welfare (including the environmental impact). The goal of manufacturers, on the other hand, is usually to comply with the law at the minimum possible cost. Consequently, certain conflicts, such as environmental benefits versus economic impact (increased costs), are inherent in the nature of EPR.

Pertaining to EoU/EoL electronic waste, the WEEE Directive in the EU has been the largest global implementation of EPR. There is also a product stewardship program in Ontario called WEEE.

There are two types of EPR models: individual producer responsibility (IPR) and collective producer responsibility (CPR). IPR initiatives are usually private and designed to fit a firm's strategic policy of sustainability and/or satisfy government legislation. While IPR models are usually exclusively private, CPR models may incorporate public collection. Dell and other producers enable IPR by allowing consumers to return products back to them at no charge by following a convenient process:

> Dell supports legislation consistent with our policy, under which all producers take responsibility for proper end-of-life management of their own electronic products, enabling internalization of the end-of-life costs of a producer's own brand that can in turn enable the continual review of the eco-design of the products...For instance, under the Dell primary direct consumer return process, a consumer can go online, print a prepaid shipping label, package the product and schedule a convenient At-Home pickup for shipment back to Dell. Other manufacturers would have flexibility to implement varying collection systems to allow consumers to return products, as long as those systems are comparable to the convenience of this example (Dell, 2014).

From the funding point of view, producer(s) are mandated to cover the cost of recovery in both IPR and CPR. The primary motivation behind CPR is to exploit economies of scale in reverse logistics operations.

> CPR allows producers to divide up and share in a more arbitrary way the costs of recycling their products. All electronic waste from all producers is collected and recycled and the producers receive one bill, of which they all pay a part. The division of the costs is often done based on the current market share of the producers, and not on the amount of waste collected of each brand, or the recyclability of that waste (Greenpeace, 2008).

In the product stewardship framework, consumers are usually charged at the point of sale (PoS) to fund the cost of accumulating and disposing of products in a sustainable manner. These tariffs are known as environmental handling fees (EHFs) or eco-fees, and are levied on a multitude of products such as tires, paint, and consumer electronics. The EHFs that are charged to the consumer are non-refundable, and because of that, there is a pool of money reserved for companies to collect, consolidate, and recover products. The Ontario Economic Stewardship (OES) is a non-profit organization to oversee the obligations of the Ontario WEEE program. As a result of this, these products can be returned at municipal recycling centres, or large retail stores such as Walmart, Best Buy, Future Shop, etc. There are over 600 collection sites in Ontario, which facilitated the collection of over 75.7 Gg of electronics in 2012 alone (Ontario Electronic Stewardship, 2013).

Thus far, the funding mechanisms that we have discussed are: government taxation, producer responsibility, and consumers paying an upfront fee to cover the cost of recovery. Deposit-refund is another system, where the consumer pays a deposit upon purchase and gets a refund when the product is returned to a drop-off facility. The success of the recovery program depends on the availability and proliferation of these drop-off/collection centres, as well as the educational initiatives employed by the collection agency.

One example of a deposit-return collection network is the Ontario Deposit Return Program (ODRP), which is overseen by the Beer Store. This is a privately held joint venture, headquartered in Mississauga, and owned by three brewers: Molson Coors Brewing Company, Anheuser-Busch InBev, and Sapporo Breweries Limited. Initially, only beer bottles were collected to be cleaned and reused, and the cost-savings from this reuse activity compensated and validated the recovery activity. This was purely a private venture, with no government involvement. However, in 2007, the ODRP was legislated by the Government of Ontario for the collection, reuse, and recycling of all alcoholic beverage containers sold in Ontario. The Beer Store was selected as the primary operator due to its existing recovery network and experience.

Alcoholic containers are assessed a deposit fee at the time of purchase, and a full refund is given when returned to any Beer Store location. According to The Beer Store (2010), 86% of Ontarians are within a five minute drive from a retail beer location in the province, translating to return rates of 95% for beer containers, 82% for aluminum cans, 87% for large glass wine and spirit bottles, and 77% overall for all ODRP containers. In fact, "the majority (>75%) of beverage alcohol consumers return their empty containers while purchasing new product" (The Beer Store, 2010). As the bottles are reused instead of recycled, this translates to lower energy utilization. Furthermore, the high return rate corresponds to a reduction in transportation costs and GHGEs. The consumer's awareness of the collection process is another key component, and with regards to the ODRP we see that:

> Consumer knowledge of the deposit programs was also found to be very high. Almost all consumers surveyed (94%) agreed that they understood how the return process works, found it easy to use (90%), and found TBS [The Beer Store] staff courteous (86%)...one in four respondents identified the bottle return program as one of their top reasons for choosing to buy beer at The Beer Store (The Beer Store, 2010).

Another example of a deposit-return network is found in the province of Nova Scotia. It is governed by Resource Recovery Fund Board Inc. (RRFB), a non-profit corporation responsible for the collection of beverage containers, used tires, paint, and electronics. Here, a hybrid-deposit policy is administered, where the consumer is refunded half the deposit paid on plastic, aluminium, and glass beverage containers. The other half of the initial deposit is set aside to handle the collected products.

2.2 Municipal Solid Waste

Municipalities around the world are under pressure to divert MSW from landfills due to rapidly escalating costs. MSW management seeks to address public health, environmental, and financial concerns by properly diverting household items such as food waste, grass clippings, bottles, newspapers, consumer electronics, and appliances (Chang, 2009). MSW is usually separated into different streams for recyclable materials, organic waste, and the residual, which is sent for disposal. Figure 2.1 displays the composition of MSW generated in the US in 2006:

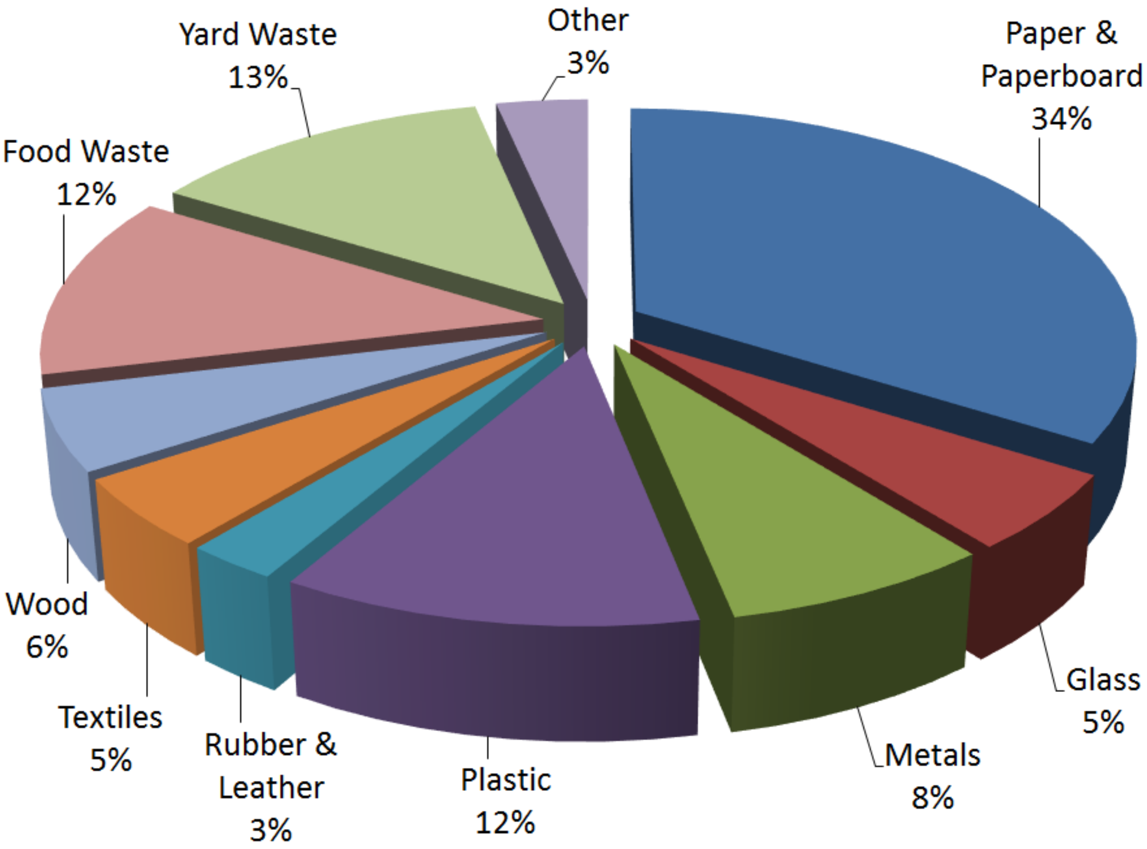


Figure 2.1: Composition of MSW in the US from 1960-2006 (U.S. EPA, 2009)

Table 2.1 presents MSW data generated in the US from 1960 to 2006. During this timeframe, MSW has increased in the US by 285%, and of this, plastics have had the largest growth rate of 7 562%.

Table 2.1: MSW Generated in the US from 1960-2006 (in Gg) (U.S. EPA, 2009)

Component	1960	1970	1980	1990	2000	2004	2006
Paper	29 990	44 310	55 160	72 730	87 740	87 550	85 290
Glass	6 720	12 740	15 130	13 100	12 620	12 650	13 200
Metals	10 820	13 830	15 510	16 550	18 240	18 810	19 130
Plastics	390	2 900	6 830	17 130	25 340	29 210	29 490
Rubber/Leather	1 840	2 970	4 200	5 790	6 530	6 690	6 540
Textiles	1 760	2 040	2 530	5 810	9 440	10 930	11 840
Wood	3 030	3 720	7 010	12 210	13 020	13 730	13 930
Food	12 200	12 800	13 000	20 800	27 110	29 730	31 250
Yard	20 000	23 200	27 500	35 000	30 530	31 770	32 400
Other	1 370	2 550	4 770	6 090	7 690	8 110	8 270
Total:	88 120	121 060	151 640	205 210	238 260	249 180	251 340

In the US, the Environmental Protection Agency (EPA)'s tiered waste management strategy is as follows:

> (1) source reduction (or waste prevention), including reuse of products and on-site (or backyard) composting of yard trimmings; (2) recycling, including off-site (or community) composting; (3) combustion with energy recovery; and (4) disposal through landfilling or combustion without energy recovery (Chang, 2009).

Collection and transportation costs can account for more than 75% of the overall waste management budget (Tavares, Zsigraiova, Semiao, and da Graca Carvalho, 2008). Therefore, it is extremely important to understand the MSW collection network and how to design the collection routing network. In the following section, we present a review of the literature pertaining to collection network design.

2.3 Collection Network Design: A Literature Review

With respect to reverse logistics, Fleischmann, Bloemhof-Ruwaard, Dekker, Van der Laan, van Nunen, and Van Wassenhove (1997) identify reverse distribution planning as a major subject in reverse logistics. "With increasing concern over environmental degradation, waste management and reverse supply-chain management have grown in importance for policymakers and academics, and research on logistics management of all phases of a product's life cycle has expanded" (de Figueiredo and Mayerle, 2008). In the ensuing literature review, we first look at different aspects of the design collection networks and then the mathematical tools that have either been utilized or are potentially useful.

2.3.1 Applications in Product Recovery Collection Networks

Environmental issues and strategies, including the design and operation of product recovery collection networks, are increasingly being analyzed using operations research (OR) methods. A typical application of sustainable product recovery is in recycling.

> A recycling operation must deal with the same transport difficulties and storage problems that challenge any distribution system; a landfill is, in effect, like a warehouse whose inventory never shrinks; garbage barges and toxic trains are the environmental equivalents of production-distribution networks (Kleiner, 1990).

In their survey, Bloemhof-Ruwaard, Van Beek, Hordijk, and Van Wassenhove (1995) discuss the importance of OR in environmental management (EM), specifically focusing on the impacts on the supply and environmental chains. Impacts on the supply chain are defined by Bloemhof-Ruwaard et al. (1995) as:

> Decisions on production planning, logistics, location, allocation and inventory control will change due to legal requirements or consumer pressures to reduce waste and emissions. Therefore, there is a need to adapt OR tools such as production planning algorithms, location models and routing heuristics in order to deal adequately with a new situation requiring 'green supply chain modelling'.

Similarly, impacts on the environmental chain are defined as:

> The amount of waste and the level of emissions caused by the supply chain result in a number of serious environmental effects, such as global warming and acid rain. Frequently, these environmental problems are international and complex. The interaction between OR and EM can result in a clear formulation of these problems and in new insights in the impacts of alternative policy measures (Bloemhof-Ruwaard et al., 1995).

The strength of OR is that the “models can be used as a rational tool in otherwise irrational and emotional debates on environmental issues” (Bloemhof-Ruwaard et al., 1995). Recovery networks in general are sequentially classified into three parts: collection from the disposal site, activities at the recovery facilities, and redistribution to the reuse market; with the collection activities typically being composed of purchasing, transportation, and storage (Fleischmann, Krikke, Dekker, and Flapper, 2000). Two areas of consideration pertaining to the application of OR and EM in collections networks are hazardous waste and MSW.

List, Mirchandani, Turnquist, and Zografos (1991) review papers dealing with hazardous waste disposal optimization covering risk analysis, routing, scheduling, and facility location. For hazardous waste, there are a limited number of treatment and disposal sites, which makes the routing problem very specific. The traditional objective of minimizing total travel distance also generally minimizes risk exposure. However, Batta and Chiu (1988) present single-objective models to incorporate the risk of an accidental release of hazardous waste by assigning accident probabilities to network links and penalties to nodes. List and Turnquist (1998) develop a multi-objective model for the routing and emergency response to the transportation of high-level radioactive waste shipments.

Aside from problems dealing with hazardous waste, many researchers have also studied collection networks for MSW. We will first discuss issues relating to the transportation of residual solid waste, followed by recycling, and e-waste.

Ghose, Dikshit, and Sharma (2006) propose a minimum distance optimization model for the routing system to collect and transport solid waste. Their model considers population density, waste generation capacity, a road network with different types of roads, storage bins, on-site storage, bulk storage, primary collection, and collection vehicles. Their model presents their work “as a decision support tool by municipal authorities for efficient management of the daily operations for transporting solid wastes, load balancing within vehicles, managing fuel consumption and generating work schedules for the workers and vehicles”. In the Ghose et al. (2006) model, the assumptions are that there is a single garage from which vehicles start and end their routes, fixed bin locations, and a single landfill. Extensions to this model could involve multiple destinations, different types of collected materials, multiple vehicles with different origins, a variety of recyclable products with distinct destinations, and the ability to relocate the collection centres based on population density and growth.

de Figueiredo and Mayerle (2008) design a minimum-cost recycling network to meet the required throughput for the collection and transportation of EoL tires. These EoL tires, in the two Southern Brazilian states of Paraná and Santa Catarina, are collected for recycling and are delivered to a bituminous schist industrialization plant after being processed in Piraquara, the state capital of Paraná. This plant produces gas, combustible oil, and sulfur from schist rock. This is a type of facility location problem where the optimal number and location of receiving centers are determined by the recycler. In the minimum-cost model, financial incentives are offered to facilitate product recovery. Using a modified Teitz and Bart procedure coupled with Fibonacci and bisection search methods, the optimal number and configuration of receiving centres, as well as the incentive to be paid to collectors for each recyclable item delivered is determined. A conceptual framework, an analytical model, and a three-stage algorithmic solution is presented and applied to a case study to recycle used tires in Southern Brazil.

Nagurney and Toyasaki (2005) formulate a multi-tiered e-cycling network model to deal with the reverse supply chain network problems of electronic waste management and recycling. The authors derive the governing equilibrium conditions from the variational inequality formulation and optimality conditions for the decision-makers. Thus, the equilibrium model determines material flows, prices, as well as the equilibrium pattern by describing the behaviour of decision makers including recyclers, processors, and consumers. The e-cycling network consists of four tiers: sources of the electronic waste, recyclers of the electronic waste, processors of electronic waste, and the consumers of electronic waste. These consumers could be demand markets for precious metals, refurbished products, or for the extracted components. Each tier also has a landfilling option, where the e-waste can be disposed of. A scenario that consisted of two sources of electronic waste, two recyclers, two processors, and two demand markets was solved numerically with a wide array of parametric values. Nagurney and Toyasaki (2005) state that this model can be applied "to address a variety of recycling issues associated with recent recycling policy instruments, such as the home appliances recycling law in Japan, or the electronic equipment waste directive in the European Union".

List et al. (2003) present a model which can be used to assess the impacts of uncertainty on fleet sizing decisions. The base model, considers postponed shipments, shipments carried, vehicle flows (loaded and empty movements across the network), and vehicle fleet size(s). The costs included in this model are: fleet ownership costs, fleet operating costs, and service quality penalties for delayed and deferred shipments. Delayed shipments are late, but within an acceptable time window, whereas deferred shipments are outside of it. By quantifying these two service quality penalties, a trade-off can be created between costs of fleet ownership and service quality. "Purchasing too small a fleet results in large penalty costs for demand that is served late or not at all, while purchasing too large a fleet results in excessive ownership (and perhaps operating) costs" (List et al., 2003). Moreover, a risk aversion parameter that varies with the fleet size is introduced. In this model, having a large fleet is being very risk averse, while a small fleet is consenting to a high level of risk. Therefore, for different levels of risk acceptance, optimal choices for fleet sizing can be strategically determined. The authors formulate a linear programming (LP) model, a two-stage stochastic optimization, and a robust optimization model. Subsequently, a numerical illustration is provided for the optimization model.

2.3.2 Methodologies for Collection Network Design

There are two components to the design of collection networks. The first deals with the location/placement of the nodes, while the second deals with the routing. Location models consist of "space, customers and facilities. Supplies exist at the facilities, demand occurs at the customer sites, and the goods are 'somehow' transported from the facilities to the customers" (Eiselt and Sandblom, 2010).

GIS is a fundamental tool that brings together data, visualization, and optimization within one framework for organizations to use. GIS are computer-based systems to aid in the collection, maintenance, storage, analysis, output, and distribution of spatial data and information. GIS enables the "ability to communicate, understand, and use information about place and time consistently across jurisdictional, institutional, and technical boundaries" (Fletcher, 2000). GIS data can be very useful to design, solve, and validate problems in MSW collection networks.

GIS can be applied to a variety of problems where geographical data and tracking the movement of objects are needed. This has led to the application of this technology in many different fields. One such advancement is the integration into wildlife biology to greatly improve the ability to study, understand, and manage wildlife populations (Sampson and DelGiudice, 2006). With respect to humans, Jat, Garg, and Khare (2008) focus on the urban sprawl aspect of population density using GIS and remote sensing. Moore, Diez Roux, and Brines (2008) compare the association between the densities of supermarkets in relation to a person's home, and the availability of healthy food. Clarke and Maantay (2005) advocate examining GIS data to determine if barriers to recycling are clustered geographically. It is easy to see why GIS is relevant to MSW, since knowing the geographic distribution of the population is essential to estimating the waste generated and routing the collection.

GIS has proven very useful to supply chain management and network modelling problems: product sourcing and location planning (Camm et al., 1997), facilities management of warehousing and production (Johnston, Taylor, and Visweswaramurthy, 1999), warehouse restructuring strategy (Min and Melachrinoudis, 2001), etc.

GIS for transportation (GIS-T) is defined as "interconnected systems of hardware, software, data, people, organizations, and institutional arrangements for collecting, storing, analyzing, and disseminating information about areas of the earth that are used for, influenced by, or affected by transportation activity" (Fletcher, 2000). Fletcher (2000) estimates that the "public sector GIS-T applications and services will increasingly reflect the shift in national transportation policy, from infrastructure development to asset preservation and transportation operations". Larsen, Madsen, and Solomon (2008) state that due to the recent progress in digital road maps "most industrialized countries now have almost fully detailed road network databases". These features can provide accurate parametric values when solving transportation network models. For example, optimization based on minimal fuel consumption instead of shortest distance yields fuel savings of 52%, and 3D GIS route modelling incorporating terrain relief, and road gradients can be used to minimize fuel consumption for the specific purposes of waste collection/transportation (Tavares et al., 2008).

Guzman, Paningbatan Jr., and Alcantara (2010) develop a system to analyze and simulate the solid waste flow of Tuguegarao City, in the Philippines using GIS. Population, per capita waste generation, average annual growth rates of population, and solid waste composition were used to predict the volume of compostable and recyclable waste generated. The study found via simulation that by 2015, if all the waste generated was composted or recycled, 51 557 m^3/yr of compostable and 31 760 m^3/yr of recyclable waste would be recovered.

Anghinolfi, Paolucci, Robba, and Taramasso (2013) build a dynamic optimization model to collect recyclable material using GIS data. Their model minimizes costs and includes state variables to account for the quantity of waste in each bin every day. Control variables represent the quantity of material that is collected each day and the collection routes for the vehicles. The effectiveness of the model is illustrated in a case study in the Cogoleto municipality in Italy by finding that the total cost of optimized collection is about 2.5 times less than the current policy.

Of all the systems that are available, *Esri's ArcGIS* in particular is a very powerful and versatile platform when working with maps, geographic information, and network analyses. Specifically, the *ArcGIS Network Analyst* extension is a useful tool in modelling and solving many logistics problems such as: route optimization, finding the closest facility, location-allocation, generating service areas, creating origin destination matrices, and solving multi-vehicle routing problems.

Bozkaya, Yanik, and Balcisoy (2010) propose a GIS-based location-routing model to determine the optimum set of locations for a supermarket chain in Istanbul. This model maximizes profit, which is defined as gross profit margin minus logistics costs, and is composed of two parts: location and routing. Initially, a genetic algorithm (GA) is used to decide which locations to open. This is followed by the use of *Network Analyst* and its built-in tabu search algorithm to solve the Vehicle Routing Problem (VRP) component. Results from this case study indicate that re-organizing the configuration of open stores to better respond to the demographic distribution and location of competitors in the region can grow market share over 15%.

Bosona, Nordmark, Gebresenbet, and Ljungberg (2013) evaluate the performance of a Swedish integrated food distribution network. The VRP was built such that:

> (i) a fleet of vehicles serves a set of delivery points; (ii) the demand of each customer per visit must be satisfied by a vehicle assigned on the route; (iii) each vehicle leaves and returns to the depot; (iv) the vehicle capacity should not be exceeded; (v) the delivery stages are single stages (i.e. product collection and distribution activities must be done separately); (vi) the working time constraints for the driver and vehicle on each route should not be violated; and (vii) delivery time window should be satisfied.

Using the *Network Analyst VRP solver*, the optimization lead to a significant reduction in the number of vehicles (routes), stops, transport distance, and transport time.

The Ghose et al. (2006) model, which was referred to earlier, uses *Network Analyst* to determine the minimum cost/distance paths for transporting solid waste to the landfill. Their model considers population density, waste generation capacity, a road network with different types of roads, storage bins, on-site storage, bulk storage, primary collection, and collection vehicles.

Zsigraiova, Semiao, and Beijoco (2013) optimize a system for MSW collection and transportation with respect to collection frequencies and the shortest routes. This study also considers fuel consumption and pollutant emissions, such as CO, CO_2, NO_x, and particulate matter. *Network Analyst* is used to optimize glass-waste collection routes in Barreiro, Portugal.

Tavares, Zsigraiova, Semiao, and da Graca Carvalho (2009) utilize *Network Analyst* as well as the *ArcGIS 3D Analyst* extension to optimize MSW collection routes for minimum fuel consumption using 3D GIS modelling. Their model specifically accounts for the effects of road inclination and vehicle weight. This model is then applied it to two different cases: routing waste collection vehicles in the city of Praia, the capital of Cape Verde, and routing the transport of waste from different municipalities of Santiago Island to an incineration plant.

The formative example of the routing problem is the Travelling Salesman Problem (TSP), which is thought to be one of the most challenging problems in operational research. “The most common interpretation of the travelling salesman is that of a salesman seeking the shortest tour to visit n clients or cities” (Eiselt and Sandblom, 2010). An extension to the TSP, known as the Truck Dispatching Problem (TDP), was derived by Dantzig and Ramser (1959) to optimize the routing of a fleet of delivery trucks between a bulk terminal and a number of service stations. The objective of the TDP is to assign service stations to trucks to satisfy their demands, while minimizing the total distance covered by the fleet. This model is the precursor to what has been collectively referred to as VRPs.

The VRP is a combinatorial optimization model used to determine collection routes to service customers from depots by minimizing costs. The objective of the standard VRP is usually in the form of minimizing either the distance travelled, fuel consumption, or quantity of emissions. However, there are situations where multiple objectives, which usually conflict with each other, need to be considered. In the VRP framework, these could relate to customer satisfaction, environmental impacts, or operational costs. Environmental impacts in particular are usually very difficult to monetize, typically requiring a multi-objective solution (Mavrotas, Skoulaxinou, Gakis, Katsouros, and Georgopoulou, 2013). In their model, Mavrotas et al. (2013) develop a multi-objective model to deal with GHGEs in the context of MSW management. This model has two objective functions: the net present cost of the system and CO_2-equivalent emissions. They develop Pareto solutions for different scenarios which can then be compared. The authors test their model using data from an MSW management system in the Eastern Macedonia-Thrace region of Greece.

The Capacitated Vehicle Routing Problem

The Classical VRP or Capacitated VRP (CVRP) is a generalization of the TSP and is considered one of the most prominent formulations of the VRP. "The traditional CVRP involves designing least cost delivery routes to service a geographically-dispersed customer set, while respecting vehicle capacity constraints" (Vidal, Crainic, Gendreau, and Prins, 2013). It is often described as an amalgamation of a bin packing problem (BPP), which assigns loads to capacitated vehicles, and a TSP that finds the best (minimum cost) route for each vehicle. More specifically,

> In the CVRP all the customers correspond to deliveries, the demands are deterministic, known in advance and may not be split, the vehicles are identical and are based at a single central depot, only the capacity restrictions for the vehicles are imposed, and the objective is to minimize the total cost (i.e., the number of routes and/or their length or travel time) needed to serve all the customers (Toth and Vigo, 2002).

An extensive coverage of the CVRP is provided by Golden, Magnanti, and Nguyen (1977); Laporte, Nobert, and Desrochers (1985); Cordeau, Gendreau, and Laporte (1997); Toth and Vigo (2002); Lim and Wang (2005); Parthanadee and Logendran (2006); Ramos, Gomes-Salema, and Barbosa-povoa (2009).

The CVRP is depicted as a directed graph $G{=}(N, A)$, where $N{=}\{0, 1, ..., n\}$ is the set of $n{+}1$ nodes, and A is the set of arcs. The zeroth node represents the depot, while the remaining nodes characterize the n customers. The set of customers are represented by $N'{=}\{1, 2, ..., n\}$. Each node has a demand q_i that must be transported (either picked-up or delivered) by the vehicle. It is assumed that there is no demand at the depot ($q_0{=}0$). For each arc $(i, j) \in A$, there is a non-negative routing cost of c_{ij} and an integer variable x_{ij}, which indicates the number of times arc ij is traversed in the solution. At the depot, a fixed homogenous fleet of m vehicles, each with a capacity Q, is available. A vehicle route is defined as a sequence of visitations to customers commencing and concluding at the depot. As described by Cordeau, Laporte, Savelsbergh, and Vigo (2006), in the CVRP, a set of m routes are determined to minimize total travel costs such that:

(1) each customer is visited exactly once by one route,
(2) each route starts and ends at the depot,
(3) the total demand of the customers served by a route does not exceed the vehicle capacity Q, and
(4) the length of each route does not exceed a preset limit L.

The CVRP consists of m identical vehicles, each with a capacity Q, and may perform at most one route. S is the set of subtours and it is assumed that m is not smaller than $r(S)$, the minimum number of vehicles needed to serve all the customers. The value of $r(S)$ may be attained by solving the BPP associated with the CVRP (Toth and Vigo, 2002). The Laporte et al. (1985) CVRP formulation is given below:

$$\min \sum_{i,j \in N} c_{ij} x_{ij} \tag{2.1}$$

subject to:

$$\begin{aligned}
\sum_{j \in N'} x_{1j} = &\quad 2m && && (2.2)\\
\sum_{i<k} x_{ik} + \sum_{j>k} x_{kj} = &\quad 2 && \forall k \in N' && (2.3)\\
\sum_{i \in S} \sum_{j \in S} x_{ij} \geq &\quad 2r(S) && S \subseteq N' && (2.4)\\
m \geq &\quad 1 && m \in \mathbb{Z} && (2.5)\\
x_{ij} = &\quad 0,1,2 && i = 1, j \in N' && (2.6)\\
x_{ij} = &\quad 0,1 && i = 1, j \notin N' && (2.7)
\end{aligned}$$

The degree constraints (2.2) and (2.3) ensure that exactly two arcs are incident to each vertex associated with a customer, and $2m$ arcs are incident to the depot vertex, respectively. Constraint (2.4) are the subtour elimination constraints which prohibit the occurrence of illegal subtours. Illegal subtours are ones that are either: disconnected from the depot, connected to the depot and having a capacity exceeding Q, or connected to the depot and having a total length exceeding L. The subtours can also be modified to constrain the total distance and/or time of a given route.

Another topic of consideration is the geometric configuration of items within a vehicle. This is a variant of the BPP and is known as the Loading CVRP. An example of this problem was presented by Gendreau, Iori, Laporte, and Martello (2006), who apply this to the case of an Italian company that produces bedroom furniture. This company pays third-party carriers based on the total distance travelled by their fleet of vehicles. Therefore, it is in the interest of the furniture company to minimize the total distance travelled and pack each vehicle in an optimal manner. The Loading CVRP can either be two-dimensional (2L-CVRP) as in Gendreau, Iori, Laporte, and Martello (2008a); Zachariadis, Tarantilis, and Kiranoudis (2009); Fuellerer, Doerner, Hartl, and Iori (2009); Duhamel, Lacomme, Quilliot, and Toussaint (2011) or three-dimensional (3L-CVRP) as in Gendreau et al. (2006); Tarantilis, Zachariadis, and Kiranoudis (2009); Fuellerer, Doerner, Hartl, and Iori (2010); Tao and Wang (2010).

Rich VRPs (RVRPs) are a more complex set of CVRP variants that were initially thought to be too difficult to solve. They add complexity by considering additional constraints or different objective functions. According to Rieck and Zimmermann (2010), the CVRP is typically modelled as a two-index vehicle-flow formulation, where binary variables indicate whether or not an arc is selected. The RVRP is a three-index vehicle-flow formulation that also considers an arc-vehicle combination binary variable that indicates the particular vehicle traversing the arc. Rieck and Zimmermann (2010) state that "these models seem to be more flexible in incorporating additional constraints, e.g. different capacities of the vehicles". Some common examples of RVRP are: time windows, simultaneous pickup and delivery, heterogeneous vehicle fleets, and a total working time for vehicle operators. Desaulniers et al. (2002) formulate a heterogeneous multi-vehicle pickup and delivery VRP model with capacity constraints. Bräysy, Gendreau, Hasle, and Løkketangen (2002) perform an in-depth survey of RVRP heuristic solution techniques.

The Heterogeneous Vehicle Routing Problem

Another generalization of the VRP is where the fleet encompasses multiple types of vehicles. Known as the Heterogeneous VRP (HVRP), the objective is to minimize the total cost of a heterogeneous fleet of m vehicles of different types k, with capacity Q_k, and fixed vehicle cost F_k to supply n customers with q_i units from the depot.

The HVRP, is also depicted as a directed graph G=(N, A), where N is the set of nodes and A is the set of arcs. There is a non-negative routing cost of c_{ij}^k for each arc $(i, j) \in A$ and for each vehicle type k. This allows for arc specific costs arising from tolls, gradients, etc. in a road network. A route is defined as the pair (R, k), where

R=($i_1, i_2, ..., i_{|R|}$), with i_1=$i_{|R|}$=0, $\{i_2, ..., i_{|R|-1}\} \subseteq N'$, and k is the type of vehicle associated with the route. R is used to refer to the visiting sequence, starting with the depot. A route (R, k) is feasible if the total demand of the customers visited by the route does not exceed the vehicle capacity Q_k. The cost of a route corresponds to the sum of the costs of the arcs forming the route, plus the fixed cost of the vehicle associated with it (since a vehicle can only be assigned one route, the fixed cost is counted only once, if the vehicle is used).

The conventional HVRP is constructed so a set of feasible routes can be attained such that each customer is visited exactly once, and the number of routes does not exceed the number of available vehicles. When developing HVRP models that specifically cater to a particular scenario, constraints with respect to the number of vehicles, their costs, and their capacities can be relaxed. For example, if there are an unlimited number of vehicles for each type, we can set m_k=∞, $\forall k \in M$. If fixed vehicle costs are not considered, then F_k=0, $\forall k \in M$; and if there is only one vehicle type, then k=1. Finally, if the problem is symmetric, $c_{ij}^k = c_{ji}^k$, and if routing costs are independent of the vehicle, then $c_{ij}^{k1} = c_{ij}^{k2} = c_{ij}$, $\forall k_1, k_2 \in M, k_1 \neq k_2$, and $\forall (i, j) \in A$.

There are six main HVRP variants, depending on whether the number of vehicles is limited or not, whether fixed costs are considered or not, and whether the routing costs are vehicle dependent, site dependent, or independent.

Table 2.2: HVRP Variants

Problem Variant	**Fleet Size**	**Fixed Costs**	**Routing Costs**
HVRPFD	Limited	Considered	Vehicle Dependent
HVRPD	Limited	Not Considered	Vehicle Dependent
SDVRP	Limited	Not Considered	Site-Dependent
FSMFD	Unlimited	Considered	Vehicle Dependent
FSMD	Unlimited	Not Considered	Vehicle Dependent
FSMF	Unlimited	Considered	Vehicle Independent

As seen in Table 2.2, the Site-Dependent VRP (SDVRP) is the only HVRP variant where routing costs are dependent on the sites that are visited. Each site (customer) may place restrictions on the types of vehicles that may visit it. For example, some sites may not be able to facilitate vehicles that are too large, or have the ability to handle certain products. In this formulation, for all vehicles types k that are incompatible with node j, the routing cost c_{ij}^k of arcs entering node j is set to infinity.

A formulation for the HVRPFD is given by Baldacci, Battarra, and Vigo (2008) below. The formulation style in Baldacci et al. (2008) is composed for a heterogenous fleet, and as such is different from the Laporte et al. (1985) CVRP formulation.

$$\min \sum_{k \in M} F_k \sum_{j \in N'} x_{0j}^k + \sum_{k \in M} \sum_{\substack{i,j \in N \\ i \neq j}} c_{ij}^k x_{ij}^k \tag{2.8}$$

subject to:

$$\sum_{k \in M} \sum_{i \in N} x_{ij}^k = 1 \qquad \forall j \in N' \tag{2.9}$$

$$\sum_{i \in N} x_{ip}^k - \sum_{j \in N} x_{pj}^k = 0 \qquad \forall p \in N', \forall k \in M \tag{2.10}$$

$$\sum_{j \in N'} x_{0j}^k \leq m_k \qquad \forall k \in M \tag{2.11}$$

$$\sum_{i \in N} y_{ij} - \sum_{i \in N} y_{ji} = q_j \qquad \forall j \in N' \tag{2.12}$$

$$q_j x_{ij}^k \leq y_{ij} \leq (Q_k - q_i) x_{ij}^k \qquad \forall i, j \in N, i \neq j, \forall k \in M \tag{2.13}$$

$$y_{ij} \geq 0 \qquad \forall i, j \in N, i \neq j \tag{2.14}$$

$$x_{ij}^k \in \{0, 1\} \qquad \forall i, j \in N, i \neq j \forall k \in M \tag{2.15}$$

Constraints (2.9) and (2.10) ensure that a customer is visited exactly once and that a vehicle visiting a customer, must depart. The maximum number of vehicles available for each vehicle type is imposed by constraint (2.11). Constraint (2.12) is the commodity flow constraint, specifying that the difference between the quantity of goods a vehicle carries before and after visiting a customer is equal to customer demand. Finally, constraint (2.13) ensures that the vehicle capacity is never exceeded.

The Periodic Vehicle Routing Problem

The Periodic Vehicle Routing Problem (PVRP), first introduced by Beltrami and Bodin (1974), is a formulation where routes are constructed over multiple days with the objective of minimizing the total travel cost. In their study of hoist compacter routing, they examine two techniques: develop routes to allocate to delivery days, and route each day individually and assign customers to delivery days. Russell and Igo (1979) classify the PVRP as the Assignment Routing Problem (ARP), regarding it as an MIP with vehicle capacity and route duration constraints. Francis, Smilowitz, and Tzur (2008) define a schedule as "a collection of days within the planning period in which nodes receive service. Allocating a node to a schedule implies that the node will receive service in every day of that schedule". Christofides and Beasley (1984) describe the PVRP as a set of routes for a specified planning period (typically a day), to satisfy customer requirements including frequency of visitation. Tan and Beasley (1984) derive the PVRP as an extension to the single-day VRP by assigning customers to delivery combinations, and then to the vehicles themselves on each of the chosen delivery days. The following decision variables are introduced:

$$a_{rt} = \begin{cases} 1 & \text{if delivery combination } r \ (r = 1, ..., R) \text{ involves delivery on day } t \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ir} = \begin{cases} 1 & \text{if customer } i \text{ is assigned to delivery combination } r \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ikt} = \begin{cases} 1 & \text{if customer } i \text{ is delivered to by vehicle } k \text{ on day } t \\ 0 & \text{otherwise} \end{cases}$$

Eq. (2.16) is the objective function which minimizes the total distance travelled, subject to constraints (2.17)-(2.21).

$$\min \sum_{i=1}^{n} \sum_{k=1}^{K} \sum_{t=1}^{T} d_{ikt} y_{ikt} \tag{2.16}$$

subject to:

$$\sum_{r \in S_i} x_{ir} = 1 \qquad i = 1, ..., n \tag{2.17}$$

$$\sum_{k=1}^{K} y_{ikt} = \sum_{r \in S_i} a_{rt} x_{ir} \qquad i = 1, ..., n \quad i = t, ..., T \tag{2.18}$$

$$\sum_{i=1}^{n} q_i y_{ikt} \leq Q_k \qquad k = 1, ..., K \quad t = 1, ..., T \tag{2.19}$$

$$x_{ir} \in (0, 1) \quad \forall r \in S_i \quad i = 1, ..., n \tag{2.20}$$

$$y_{ikt} \in (0, 1) \qquad i = 1, ..., n \quad k = 1, ..., K \quad t = 1, ..., T \tag{2.21}$$

where r is each distinct delivery combination, S_i is the set of allowable delivery combinations for customer i, q_i is the demand of customer i (i=1, ..., n) for each delivery, Q_k is the capacity of vehicle k (k=1, ..., K), and d_{ikt} is the distance contribution of customer i to the route followed by vehicle k on day t (t=1, ..., T). Constraint (2.17) dictates than an acceptable delivery combination is chosen for each customer, while constraint (2.18) ensures that a vehicle is used for a delivery to a customer i on day t if the delivery combination for i requires it. Constraints (2.19)-(2.21) are the vehicle capacity and integrality constraints respectively.

Francis et al. (2008) presents a few of the main extensions to the PVRP, including the: Multidepot PVRP (MDPVRP), PVRP with Intermediate Facilities (PVRPIF), PVRP with Time Window constraints (PVRPTW), and PVRP with Service Choice (PVRP-SC).

- MDPVRP: Delivery vehicles are based across a number of depots
- PVRPIF: Delivery vehicles are based in a single depot, but capacity replenishment is possible along the routes
- PVRPTW: Customers may only be visited at certain times
- PVRP-SC: Visitation frequency is an endogenous decision of the problem

The Inventory Routing Problem

Inventory Routing Problems (IRPs) consider the environments of the suppliers, customers, as well as the vehicles (usually capacitated). IRPs are formulations where inventory control and routing decisions are made at the same time to minimize the total cost of inventory holding and transportation, while preventing stock-outs, and having a ceiling on storage capacity (Bertazzi, Savelsbergh, and Speranza, 2008). The VRP is adapted to incorporate supplier/customer inventory capacities and holding costs, production and consumption rates, and time periods. Inventory holding costs may be charged only at the supplier or customer, or at both the suppliers and the customers. Production and consumption may occur continuously or at discrete time instants. These rates can be: constant or vary over time, deterministic or stochastic, and the planning horizon can be finite or infinite (Bertazzi et al., 2008).

Burns, Hall, Blumenfeld, and Daganzo (1985) develop analytic models to solve the IRP by minimizing the distribution cost and finding the optimal trade-off between transportation and inventory costs. Bell, Dalberto, Fisher, Greenfield, Jaikumar, Kedia, Mack, and Prutzman (1983) examine inventory holding capacity and costs in their optimization model when solving their MIP. Oppen, Løkketangen, and Desrosiers (2010) solve a VRP with inventory constraints and loading sequence for the collection of livestock. Archetti, Speranza, and Savelsbergh (2008) utilize a split delivery VRP model to fulfill customer demand at minimum cost and also allow multiple customer visitations.

The Dynamic Vehicle Routing Problem

In an effort to offer a higher level of service to the consumer through superior on-time assistance, Dynamic VRPs (DVRPs) are a VRP extension where real-time information is used in the problem solution. Consumer requests pertaining to the demand for goods or services, travel time, service time, and vehicle availability affect the routing during its operation (Pillac, Gendreau, Gueret, and Medaglia, 2011). Taxi and emergency services, such as police, fire, and ambulance dispatching are some examples of applications of this problem.

> In the VRP all routing and demand information is known with certainty prior to the day of operations, so routes can be planned ahead. In contrast, in the DVRP part or all of the necessary information becomes available only during the day of operation (Larsen et al., 2008).

The lack of known demand data leads to uncertainties in the route planning activity, highlighting the importance on having the most up-to-date information, which is based on communication between the dispatching center, the vehicle drivers, and the customers. The basic method to determine the position of the vehicles is to have the driver report back to the dispatching center every time a customer has been serviced. The availability and quality of information for dynamic route planning can be improved by using more advanced technologies such as a computerized dispatcher, smartphones, and Global Positioning System (GPS) to determine the positions of drivers and customers.

Optimization Metaheuristics

A metaheuristic is a high-level/generic solution scheme or heuristic that can be adapted to solve optimization problems (Glover, 1986). Gendreau, Potvin, Bräysy, Hasle, and Løkketangen (2008b) state that "a good metaheuristic implementation can provide near-optimal solutions in reasonable computation times". The Handbook of Metaheuristics by Glover and Kochenberger (2003) is a comprehensive coverage of the methodologies, algorithms, and applications that have proven to be successful in practice. The most popular types of metaheuristics include: ant colony optimization (ACO), GAs, simulated annealing (SA), tabu search, variable neighbourhood search

(VNS), and greedy randomized adaptive search procedure (GRASP). ACO and GAs are inspired from natural metaphors; and SA, tabu, and VNSs are randomized local search methods.

In the last two decades, a considerable number of methods have been developed based on the efficiency and simplicity of self-optimized processes in nature (Galski, de Sousa, Ramos, and Muraoka, 2004). These global methods are radically different from gradient-based methods, because instead of looking at a single point individually and stepping to a new point for each iteration, a whole population of solutions are iterated towards the optimum at the same time. Using a population lets us explore multiple local optima simultaneously by covering a large design space range. This results in a smaller likelihood than gradient-based methods of getting stuck in local minima solutions and increasing the prospect of finding a global optimum. Other advantages of global methods are: straightforward implementations, the ability to handle noisy objective functions, and the capability to be used for multi-objective optimization. However, these methods are typically computationally expensive in comparison to gradient-based methods, especially for problems with a large number of design variables.

Swarm intelligence is a common natural metaphor whose computational techniques are inspired by the social behaviours of insects and other animals. ACO, for example, attempts to mimic the foraging behaviour of some ant species. Each ant emits a *pheromone* to delineate a favourable path that should be followed by other members of the colony. Additionally, the pheromone trail evaporates over time, which means that the concentration of pheromones is higher on shorter paths in comparison to longer ones. Since the attractiveness of a path is based on the strength and concentration of pheromones, eventually the positive feedback from other ants will lead the entire colony to follow the shortest (optimal) path. When applied to the TSP, "the problem is tackled by simulating a number of artificial ants moving on a graph that encodes the problem itself: each vertex represents a city and each edge represents a connection between two cities" (Dorigo, Birattari, and Stützle, 2006).

Due to the close relationship between the TSP and the VRP, the ACO can be quite easily adapted to solve the VRP. Gambardella, Taillard, and Agazzi (1999) present an ACO-based approach to solve VRPs with time-windows. This model is composed of two ant colonies, each representing an objective function. The first minimizes the number of vehicles, while the second minimizes the distance travelled, however the first takes precedence over the second. Information is exchanged between the two colonies through pheromone updating. Reimann, Doerner, and Hartl (2004) present an algorithm to improve the efficiency of the ACO-VRP solutions. In their approach, the problem is decomposed into several disjoint subproblems, and each subproblem is solved using ACO. The developed model can also be used for VRPs with time windows, backhauls, multiple depots, or DVRPs.

The honey bees mating optimization (HBMO) is another swarm-based approach to optimization by imitating the behaviour of insects. A colony of honey bees contains one queen bee, who is the only fertile female, and thousands of fertile males (drones). The queen goes on mating-flights away from its home colony and mates with multiple drones from other colonies before returning. The sperm from the different drones are accumulated in the spermatheca and form the genetic pool of the colony. The eggs, are then fertilized with a random a mixture of the sperms. Haddad, Afshar, and Mariño (2006) present the following algorithm for the HBMO:

1. The queen (best solution), in its mating flight, probabilistically selects drones to form the spermatheca (list of drones).
2. Drones are randomly selected from the spermatheca and crossed-over with the queen to create new broods (trial solutions).
3. Workers (heuristics) are used to conduct local searches on broods and their fitness is adapted based on the broods' improvement.
4. Queens with weaker fitness are replaced by fitter broods.

Marinakis and Marinaki (2011) create a hybrid model by combining HBMO and neighbourhood search algorithms to solve the VRP. The model first minimizes the number of vehicles required, and then the total distance travelled. The authors found that their algorithm had great performance in quality and computational efficiency when compared to the benchmarks by Christofides, Mingozzi, and Toth (1979).

Another nature-inspired metaheuristic, GAs, attempt to replicate the process of natural selection. This evolution strategy (ES) is based on the twin concepts of reproduction and survival of the fittest, where the objective function of the formulation is the fitness value. The first step in a GA is to initialize a population of randomly generated creatures. Each creature is composed of chromosomes and the goal is to find a creature with the best genetic material (chromosomes) to solve the problem. This population is known as a generation and each subsequent iteration becomes a new generation. In each generation, the fitness of every creature is evaluated, and the creatures with better fitness values are stochastically selected and mated to generate offspring. The mating process, known as the crossover operation, is where a proportion of chromosomes are taken from each parent to produce the offspring. Random mutations are also introduced to increase variability in the solution space. The offspring compose the population of the next generation, and this process repeats until either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population. With regards to VRPs, Gendreau et al. (2008b) describes some alterations to GAs:

> When applied to vehicle routing problems, the classical GA solution scheme is often modified. In particular, the encoding of solutions into chromosomes is either completely ignored (by applying the various operators directly on the solutions) or designed in a very particular way to take advantage of specialized crossover and mutation operators.

He, Wei, Lu, and Huang (2010) use a GA to solve the TDP in the context of an open pit surface mine. In this problem there are a fixed set of transportation routes with the objective to allocate trucks based on the demand of the discharge points, while minimizing freight and maintenance costs. The model was validated and solved using the *MATLAB GA* toolbox, as well as distance, time, and vehicle data. In the primitive generations, the fitness values of the trucks are very high. However, after 70 iterations, the fitness values have decreased drastically, and have essential plateaued for the last 30 iterations. Additionally, in the final iteration, the average fitness value of the trucks is roughly equivalent to the best fitness value for that generation.

Berger and Barkaoui (2003) propose a new hybrid GA to solve the CVRP by minimizing the total distance travelled. This model evolves two populations at the same time, where the best individuals in each population are mutually exchanged through migration over each generation. The authors determined that this algorithm is cost-effective and competitive in comparison to other VRP metaheuristics. Mester and Bräysy (2007) develop a two-phase methodology to solve CVRPs. The first phase generates a set of solutions and then selects the best solution as the starting solution. The second phase improves the starting solution with an iterative two-stage procedure. The first stage uses a guided local search (GLS) metaheuristic, which operates by augmenting the objective function with a penalty term based on particular features not considered to be part of a near-optimal solution. The second stage performs a series of removal and reinsertion operations with an ES metaheuristic. This formulation is tested against the Christofides et al. (1979) benchmark and provided the best-known solutions to 70 of 76 VRPs within reasonable computational times.

Annealing is a heat treatment method used to improve material properties, typically cooling molten metal until it reaches a lower energy state. When cooling is done abruptly, it is known as rapid quenching, whereas when the cooling is conducted gradually it is known as careful annealing. SA is a stochastic neighbourhood search strategy that simulates the process of gradually cooling a metal in stages by successively lowering the temperature in each stage (Laarhoven and Aarts, 1987). Many neighbourhood techniques have a tendency to get stuck in local optima, however SA was designed to circumvent this. In SA, non-improving solutions are accepted with higher probability at higher temperatures (diversification), and this probability of acceptance is reduced at lower temperatures (intensification). In the context of the VRP, stages correspond to each set of routes, and energy is the cost.

Tabu search is also a neighbourhood search strategy, but it incorporates memory in the form of a list of recent solutions (Glover and Laguna, 1997). This list, known as the tabu (forbidden) list, describes the visited solutions or user-provided sets of rules. The algorithm prevents going back to recently visited solution or a solution that violates a user-defined rule. Three important strategies to consider in tabu search are: forbidding, freeing, and short-term. The forbidding strategy manages what goes into the tabu list, the freeing strategy manages what goes out of the tabu list, and the short-term strategy manages the interplay between the forbidding and freeing strategies. Osman (1993) develops SA and tabu search metastrategies and investigates their algorithmic performances for the VRP under capacity and distance constraints. The SA algorithm was able to find a solution, but it displayed large variances with regard to solution quality and computational time. The tabu search outperformed the SA, and its results were also more robust than the SA. These metastrategies algorithms can also be applied to VRPs with different vehicle sizes.

Gendreau, Hertz, and Laporte (1994) present a new tabu search heuristic for VRPs with capacity and route length restrictions. In their algorithm, a sequence of adjacent solutions are obtained by removing and inserting cities into different routes. During the search, infeasible solutions are also allowed through penalty terms in the objective function to reduce the likelihood of local minima. The algorithm has the flexibility to be executed from any starting solution (feasible or not), the number of vehicles can be fixed or bounded, and vehicles can have different characteristics. The performance of this algorithm is tested numerically against the Christofides et al. (1979) benchmark problems.

Gendreau et al. (2008a) solve the 2L-CVRP by using a tabu search algorithm. In this model, transportation costs to deliver goods (certain number of two-dimensional weighted items) to customers by a fleet of vehicles based at a central depot is minimized. In this formulation, a feasibility check of the two-dimensional packing must be executed on each vehicle. The loading component of the problem is solved through heuristics, lower bounds, and a truncated branch-and-bound procedure. In all 360 instances, the tabu search algorithm was able to a find a feasible solution.

The *Esri ArcGIS Network Analyst VRP solver* also uses an algorithm based on the tabu search metaheuristic. Although the exact formulation is proprietary, and thus unknown to the public, there is information on the search procedure in general. The first step is the generation of an origin-destination matrix, which consists of the shortest-path costs between all customers and depots in the network. The second step is to construct an initial solution using the cost matrix by individually inserting each customer into a route. The final step is to iteratively improve the initial solution by moving orders between routes and resequencing them on each route.

VNS is yet another neighbourhood based search algorithm, however this one consists of a series of nested neighbourhoods. When a local optimum is reached, the algorithm tackles another neighbourhood, and keeps repeating until the best solution is attained (Glover and Kochenberger, 2003). Zeng, Ong, and Ng (2005) use VNS to solve the distance-constrained CVRP and the CVRP with multiple trips. This algorithm chooses a certain number of nodes and deletes them from routes, and then tries to insert these nodes into other routes by solving an assignment problem with a corresponding cost matrix. When calculating the cost matrix, only the capacity constraint is considered, and if the demand of the node is greater than the remaining capacity of the corresponding route, the cost value is set to M. The algorithm is tested against the Christofides et al. (1979) benchmark.

Kytöjoki, Nuortio, Bräysy, and Gendreau (2007) apply a two-phase VNS heuristic to very large-scale CVRPs. In the first phase an initial solution is created, and in the second phase the initial solution is improved. This model consists of geographically scattered customers with known demands that are serviced by a fleet of identically capacitated vehicles. The objective is then to design least cost routes for these vehicles. The formulation is tested on scenarios where that are up to 20,000 customers, and the model is able to find solutions with reasonable computational times.

2.4 Illustrated Case Study: RRFB – Nova Scotia

We now present a case study for the problem of designing a routing network for the collection of recyclables in Nova Scotia. Nova Scotia is a province in Atlantic Canada with an area of 52 939.44 km^2, and a population of 921 727 as of 2011 (Statistics Canada, 2013). Its capital region, the Halifax Regional Municipality (HRM), situated at 44.6489°N, 63.5754°W, is the most populous in the province.

RRFB is a non-profit corporation which administers the collection of recyclables such as: beverage containers, used tires, paint, and electronics for the province of Nova Scotia. With the beverage container recycling program, the consumer is refunded half the deposit that they pay on their plastic, aluminium, or glass products. In 2011, Nova Scotia had a beverage container recovery rate of 79% (RRFB, 2011). Having been very successful at educating consumers on the importance of recycling and taking back EoL products, operations have steadily increased. The collection of recyclables through a sparsely populated territory, such as the province of Nova Scotia is even more challenging than in urban centres. RRFB has recognized the need for an engineering approach in designing and improving their logistics network.

There are three steps involved with the handling of recyclable products within an MSW collection network:

1. **Collection:** Products are dropped-off by individuals and accrue at collection centres. They are then sorted and transported to processing depots. Typical recyclables are: aluminium cans, plastic containers, glass bottles, paper, etc.

2. **Consolidation:** At processing depots, products are compacted, baled, and prepared for sale to an end-use recycling facility based on the material type. The primary function of this step is to reduce transportation costs via product compaction.

3. **Product Recovery:** Baled products are transported in bulk to end-use facilities for recycling. Plastics and metals are melted for recasting, while glass can be cleaned and reused or crushed and remelted.

The collection centres, consolidation depots, and end-use facilities are the nodes in the resource recovery network, which are connected by transportation routes. These routes can be traversed by different types of vehicles or even pedestrians depending on the region, its infrastructure, and the costs of transportation. We now describe the RRFB network in detail.

Figure 2.2: RRFB Vehicle Loading Bulk Bags at an Enviro-Depot

The collection centres in the RRFB network are known as Enviro-Depots (EDs). Here, products are sorted by material, type, and colour and stored in bulk bags. The bulk bags have a fixed maximum volume and are used for all products. Figure 2.2 portrays the bulk bag loading process of a pickup at an ED. These are subsequently transported to assigned depots known as regional processing centres (RPCs). The RRFB network shown in Figure 2.3 spans seven regions consisting of 83 privately-owned EDs (black asterisks) and 4 RPCs (red stars).

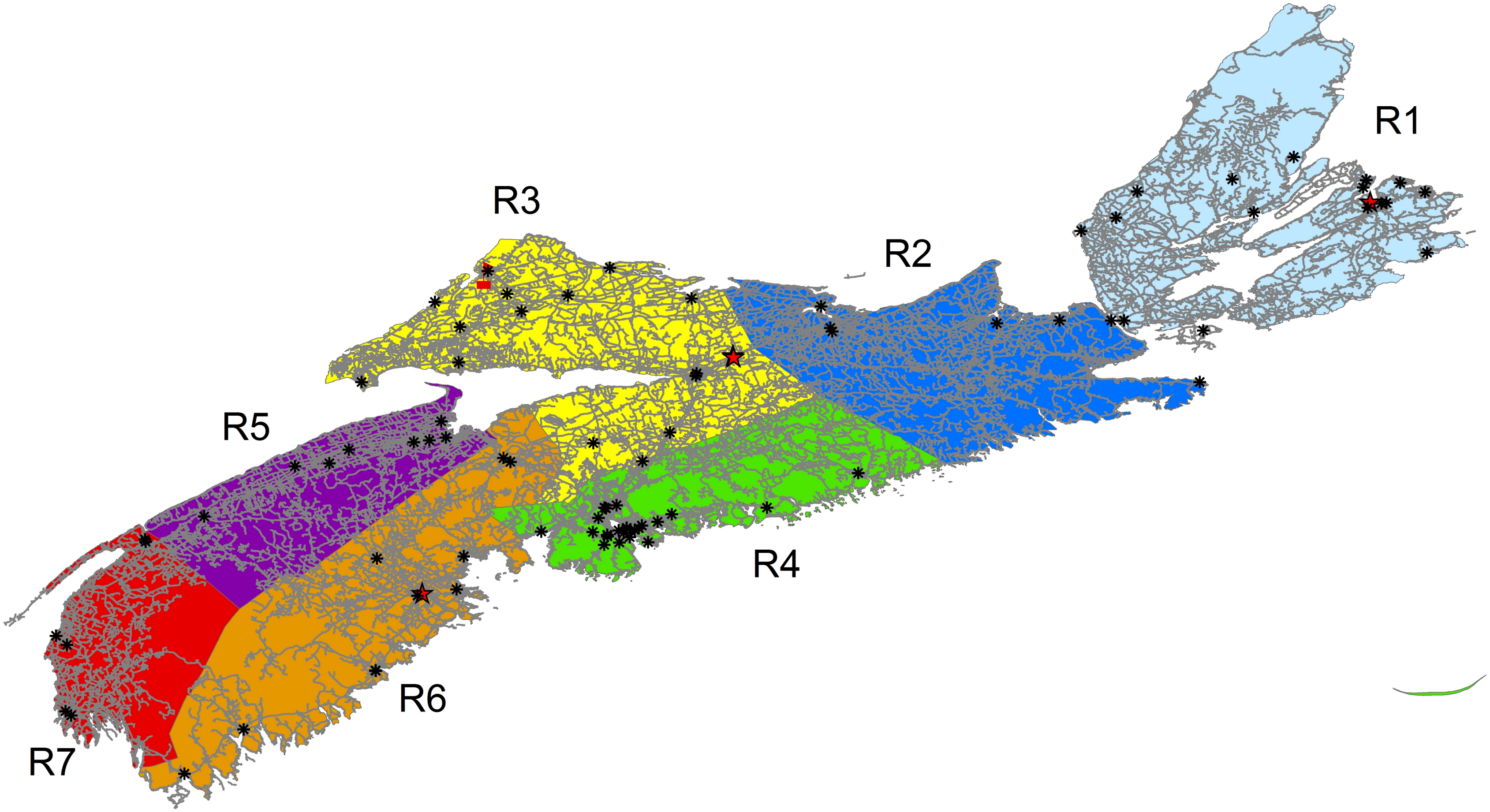


Figure 2.3: RRFB Network

The total annual collection volume from all seven regions is displayed in Figure 2.4:

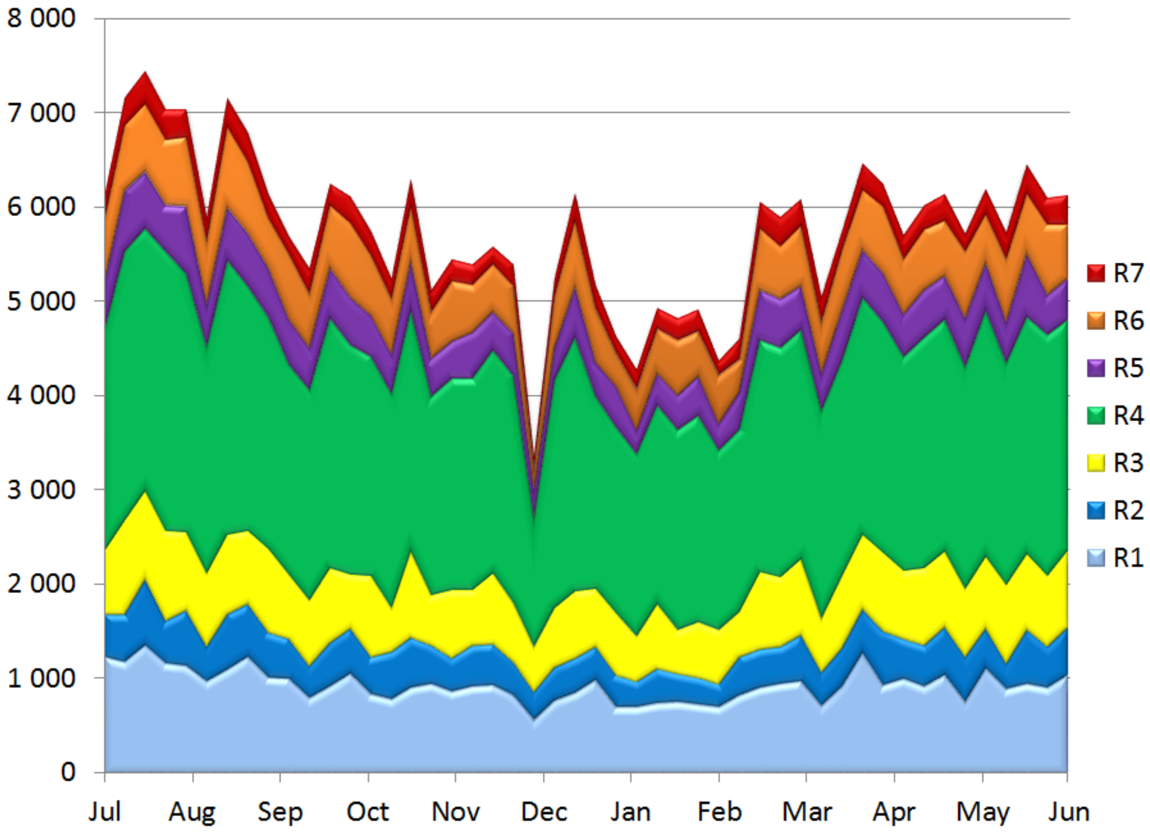


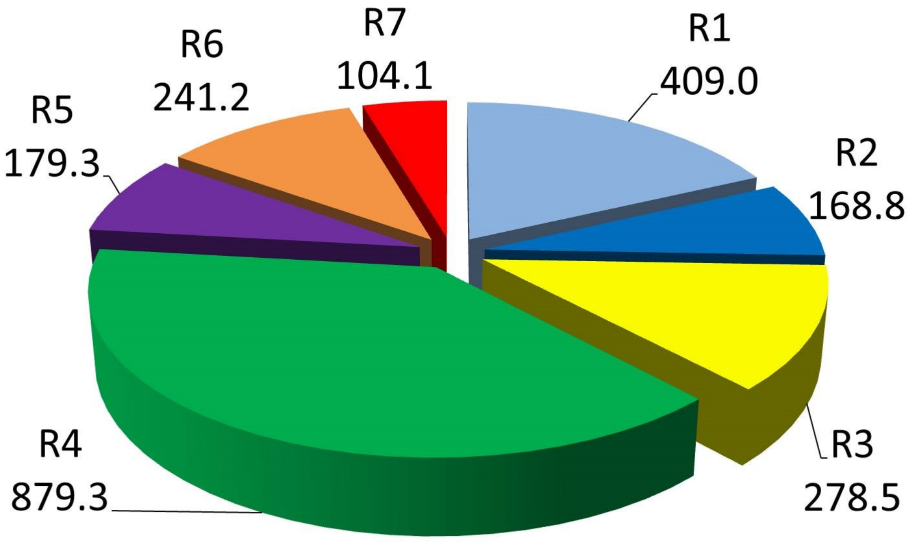
Figure 2.4: Total Annual Waste Across All Regions (in bags)

The material collected from the seven regions flow to one of the four predetermined RPCs, where it is sorted and baled.

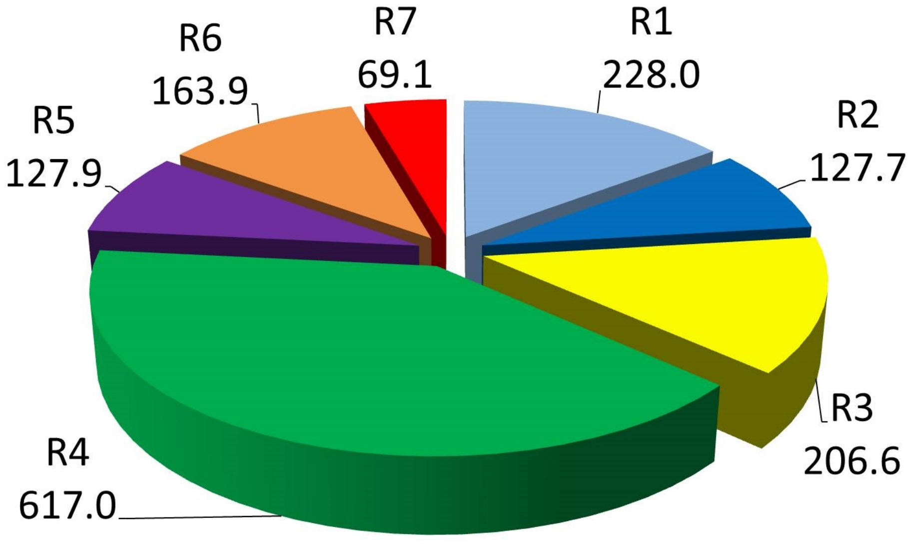
Region 1 → Sydney, Cape Breton (46.1502°N, 60.2293°W)
Regions 2-3 → Colchester (45.4559°N, 63.1048°W)
Region 4 → Kemptown (45.4494°N, 63.1018°W)
Regions 5-7 → Lunenburg (44.3829°N, 64.5069°W)

The material is then consolidated at the central RPC in Kemptown, and is sent to an end-use facility based on the material type.

In this analysis, we consider two material types: clear polyethylene terephthalate (PET) and aluminium. For these materials, the end-use recycling facilities are located in Amherst, Nova Scotia (for clear PET) and Kentucky, USA (for aluminium). Figure 2.5 exhibits the weekly quantity of bags for each region.



(a) Average Weekly Quantity of Bags of Clear PET



(b) Average Weekly Quantity of Bags of Aluminium

Figure 2.5: Average Weekly Quantity of Recyclables

Currently, 3PLs are paid a fixed rate per bag (β=Φ) to recover recyclables and bring them to the assigned RPC. They are responsible for collection routes, schedules, fleet, etc. Collection takes place using traditional trailers (Freightliners) with a maximum capacity of 80 bags. For the two types of recyclables considered, clear PET (p=1) and aluminium (p=2), the in situ baling ratios at the RPCs are 9:1 and 5:1 respectively. After baling, for all RPCs (other than Kemptown), there are transfer costs, including a per km fuel surcharge, to Kemptown. Furthermore, for clear PET there is an additional transfer cost from Kemptown to Recycler 1 in Amherst. These transfer costs are shown in Table 2.3. The transportation costs of aluminium from Kemptown to Kentucky are borne by Recycler 2. To ensure confidentiality, all financial values are manifested as multiples of Φ.

Table 2.3: Transfer Costs for Traditional Trailers

Transfer	Cost
Sydney to Kemptown	123Φ
Lunenburg to Kemptown	91Φ
Colchester to Kemptown	14Φ
Kemptown to Recycler 1	45Φ

In this study, we investigate collection using compaction trailers (CTs), which can be leased/purchased from an OEM. With the CTs, RRFB incurs driver, maintenance, and fuel costs in addition to leasing/purchase costs. The CTs can be set up to collect two types of recyclables: clear PET and aluminium, both with baling ratios of 5:1.

A pilot project was implemented for 14 EDs within the HRM, the region in the with the highest population density. For the CTs, RRFB is responsible for the vehicles and their collection schedules. With compaction, each CT has an effective maximum capacity of approximately 400 bags. Thus, these vehicles make less frequent trips, circumvent the intermediary RPCs, and go directly to Recycler 1 for clear PET, or Kemptown for aluminium. Therefore, the transportation costs in Table 2.3 are saved using CTs. B_{pi} represents the savings from bypassing the intermediary RPCs, and is calculated by pro-rating the transfer cost of a baled product from the regional RPC to Kemptown, and then to Recycler 1 (if applicable for either situation).

For example, a CT transporting a bag of clear PET from ED 4 in Region 1 would bypass both the RPC in Sydney, as well as the central RPC in Kemptown, going directly to the end-use facility in Amherst. Therefore, a proportion of the transfer costs from Sydney to Kemptown, and Kemptown to Recycler 1, shown in Table 2.3 are saved based on the RPC baling ratio (in this case 9:1 for clear PET).
This sample calculation is shown below:

$$B_{14} = \frac{123\Phi}{9} + \frac{45\Phi}{9} = \frac{168\Phi}{9} = 18.67\Phi$$

Therefore, the revenue per bag of product p collected at each collection centre i is equal to: $\rho_{pi} = \beta + B_{pi}$.

2.5 A Two-Phase Approach for the Design of a Collection Network and Activities

The research question for the RRFB network, as addressed in this dissertation, is how to design the collection system using a fleet of CTs, given that a node has to be serviced using either this type of vehicle or subcontracting pickup to a 3PL for a variable cost per bag. The intent is to design a collection network in order to determine the customer drop-off centres to visit, the routes for each EoU/EoL product, the fleet size, and the daily schedules, while maximizing the net revenue.

One of the challenges in routing a high-value asset such as the CT in Nova Scotia, as well as other Canadian provinces, is that the population is concentrated in a few urban areas with a high population density, and there are relatively large rural areas with low population densities. The volume of recyclables is correlated with the population, and as a result, the network will have varying pickup volumes across collection centres.

As we are only considering a homogeneous fleet of CTs, the Laporte et al. (1985) CVRP formulation shown in Eqs. (2.1)-(2.7) provides an excellent baseline model. The Baldacci et al. (2008) HVRPFD formulation shown in Eqs. (2.8)-(2.15) presents a cost minimization model, as well as an explicit formulation for the capacity constraints. Moreover, since each route needs to be realized within a working day (typically 8-10 hours), this requires a total time-constraint on each route. The PVRP model shown in Eqs. (2.16)-(2.21) imparts an example of how to model the time-duration constraint for each route, and also how to develop a schedule. However, the network route design problem we address is more complex than a time-constrained CVRP, due to the following characteristics:

1. Only the collection centres that comprise the most profitable system will be considered based on the number of available routes. It is assumed that when a vehicle is not routed through a collection centre, pickup is conducted by a third-party for a variable cost per bag. For remote depots with low volumes, a 3PL may be the best option.

2. There is a scheduling aspect to the problem. One or more vehicles may be purchased to service collection centres, however designing the routes requires the solution to be feasible over a typical work-week. For example, if all collection in a network requires three vehicles, and each vehicle is available for five days in a week, a maximum of 15 product-routes are available. It is assumed that the product type for pickup is determined at the start of the route.

3. There are inventory considerations in the problem. In regions with low volumes (especially if they are also remote), it may be possible to visit certain collection centres every second or third week. This ensures that a high value asset such as the CT is well utilized. Such a strategy is also practical for low volume collection centres, which may not have a large buildup of collectibles. Conversely, it is possible that some depots need more than one collection per week – this is not the case in the RRFB example, but is possible in a general sense.

The next section will formulate a model to address these additional characteristics.

2.5.1 Model Formulation

The developed model will be formulated with a two-phase approach to jointly determine the routing design and the number of vehicles. It incorporates aspects of a time-constrained CVRP as well as an iterative procedure to perform the sequencing and scheduling. Both phases are modelled as mixed-integer linear programs (MILPs). The overall methodology in this formulation is to solve the first phase repeatedly by specifying the number of routes, generating collection routes for each product type, and then assigning the product-route combinations to vehicles in the second phase. Only variable costs are taken into account in the first phase, while the fixed cost of purchasing the CTs is taken into account in the second phase. The two phases together jointly determine the number of vehicles and the product-routes they service. The notation for this model is shown below:

Parameters

d_{ij}: Distance from node i to node j
t_{ij}: Time required to travel from node i to node j
q_{pj}: Inventory of product p at node j
Q_p: Vehicle volume capacity for product p
λ_p: Loading time per unit volume of product p
t_{pij}: Total time to travel from i to j and load product p at node j
$t_{pij} = t_{ij} + \lambda_p q_{pj}$
ρ_{pi}: Revenue per unit volume of product p collected at node i
δ_p: Unit distance variable cost for product p
τ_p: Unit time variable cost for product p
T_p: Route time limit for product p
m_p: Number of routes to collect product p
M_p: Minimum number of routes allowed for product p
ψ: Number of days in service per week
F: Fixed costs per vehicle amortized over a weekly period
ϵ: Social cost of GHGEs (SCG) in \$/Mg of emissions

Sets

N: Set of all nodes, depot is node 0 $\quad N = \{0, 1, \cdots, |N| - 1\}$
N': Set of collection centre nodes $\quad N' = \{1, 2, \cdots, |N'|\}$
P: Set of recyclable products $\quad P = \{1, 2, \cdots, |P|\}$
S_p: Set of scenarios for product p $\quad S_p = \{0, 1, \cdots, |S_p|\}$
R_{ps}: Set of routes for product p and scenario s $\quad R_{ps} = \{1, 2, \cdots, |R_{ps}|\}$

Decision Variables

z_{pij}: Vehicle load of product p going from i to j

$$y_{pj} = \begin{cases} 1 & \text{if centre } j \text{ is included in the collection routes} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{pij} = \begin{cases} 1 & \text{if vehicle goes from } i \text{ to } j \text{ to pickup product } p \\ 0 & \text{otherwise} \end{cases}$$

v: Number of vehicles, $v \in \mathbb{Z}$

$$w_{psr} = \begin{cases} 1 & \text{if route } r \text{ has been included in scenario } s \text{ for product } p \\ 0 & \text{otherwise} \end{cases}$$

$$u_{ps} = \begin{cases} 1 & \text{if scenario } s \text{ has been selected for product } p \\ 0 & \text{otherwise} \end{cases}$$

Route Generation

The intent of the route generation phase is to create a large database of collection routes, based solely on variable costs, to be the input of the second phase. For each product type, scenarios are grouped based on the specified number of routes. The objective of the route generation formulation is to maximize the revenue of the entire scenario (all routes combined together), however the revenue of individual routes may be negative. Any collection centre included in the phase one model generates revenue through product recovery, while route costs are based on time (driver wages and maintenance requirements), and distance (fuel consumption). The objective function for each route (π_{psr}) only incorporates variable costs of collection and does not pay attention to the number of vehicles needed or their associated fixed costs.

For example, a 10-route scenario will generate the 10 best routes to maximize the net revenue (via product recovery) of the route set. As such, only the optimal collection centres will comprise the scenario's route set. In an 8-route scenario, the routes would be regenerated to determine the eight best routes to maximize the total amount of products collected, and the composition and sequence of all of these routes could be different than the ones generated in the 10-route scenario.

The scenarios are solved separately for each type of product, and these results then provide the input for the second phase. The mathematical model, and the pseudo-code of the iterative search for the first phase are presented next.

Route Generation Model – Formulation P1

Formulation P1 is a modified CVRP model used to generate sets of viable one-day routes. One of the input parameters in the model is the number of routes available to service the customer (m_p). In the traditional CVRP, m_p is equal to the minimum number of routes needed to serve all of the collection centres ($\hat{R}_p$). However, in this modification, m_p is set to be less than $\hat{R}_p$, which allows the exclusion of collection centres. Decreasing this parameter causes more and more collection centres to be dropped, and by repeating this procedure, different sets of collection routes, referred to as scenarios, are generated.

The following MILP is used to generate routes for each product p. The objective function, Eq. (2.22), maximizes the total collection profitability. The first term in the objective function, $\sum_{i \in N} \sum_{\substack{j \in N \\ j \neq i}} [\rho_{pi} q_{pi} y_{pj}]$, represents the revenue obtained by choosing to collect product p through all collection centres j where y_{pj}=1. The second term, $\sum_{i \in N} \sum_{\substack{j \in N \\ j \neq i}} [(\delta_p d_{ij} + \tau_p t_{pij}) x_{pij}]$, represents the distance and time based variable costs incurred by routing the vehicle. As mentioned earlier, fixed costs for the vehicles are not taken into account at this juncture, because the exact number of vehicles required is part of the optimization in the next phase.

$$\max \pi_{psr} = \sum_{i \in N} \sum_{\substack{j \in N \\ j \neq i}} [\rho_{pi} q_{pi} y_{pj} - (\delta_p d_{ij} + \tau_p t_{pij}) x_{pij}] \tag{2.22}$$

subject to:

$$\sum_{i \in N} x_{pij} = y_{pj} \qquad \forall j \in N' \tag{2.23}$$

$$\sum_{i \in N} x_{pih} - \sum_{j \in N} x_{phj} = 0 \qquad \forall h \in N' \tag{2.24}$$

$$\sum_{j \in N'} x_{p0j} = m_p \tag{2.25}$$

$$\sum_{i \in N} z_{pij} - \sum_{i \in N} z_{pji} = -q_{pj} \quad \forall j \in N' \tag{2.26}$$

$$Q_p - q_{pj} x_{pij} \geq z_{pij} \qquad \forall i, j \in N, i \neq j \tag{2.27}$$

$$\sum_{i \in N} \sum_{\substack{j \in N \\ j \neq i}} t_{pij} x_{pij} \leq T_p \tag{2.28}$$

$$z_{pij} \geq 0 \qquad \forall i, j \in N, i \neq j \tag{2.29}$$

$$x_{pij} \in \{0, 1\} \quad \forall i, j \in N \tag{2.30}$$

$$y_{pj} \in \{0, 1\} \quad \forall j \in N' \tag{2.31}$$

Constraint (2.23) assures that if a node is visited for a product, it is included in a route. Constraint (2.24) ensures that if a vehicle visits a node to pick up a product, it must also depart from it. Constraint (2.25) limits the maximum number of routes that can be put in service. Constraint (2.26) requires that if a vehicle leaves node i for node j then it has enough empty space to pick up the inventory at node j. Finally, the vehicle capacity and time constraints are enforced by (2.27) and (2.28), respectively. The algorithm to iteratively solve the first phase is presented next.

```
for $p$=1 to $|P|$ do
    Step 1: Determine $\hat{R}_p$, the minimum number of routes needed to service all customers in the network.
    Step 2: Solve Formulation P1 and generate scenarios
    for $s$=0 to $Max\{\hat{R}_p - 1, M_p\}$ do
        Set $m_p = \hat{R}_p - s$ and solve Formulation P1;
        Store $R_{ps}$, the route information for current scenario
    end
end
```

Algorithm 1: Pseudo-Code for Route Generation

In the first step, the problem is solved to determine $\hat{R}_p$, the minimum number of routes to fulfill the collection requirements in the network. In step 2, Formulation P1 is solved iteratively to yield routes for each scenario. A scenario is the set of routes obtained by solving Formulation P1 for m_p routes. A complete set of scenarios is generated by systematically decreasing the number of routes allowed from its maximum value of $\hat{R}_p$. In this process, collection centres with remote locations and/or low volumes may be dropped from the network. The iterative procedure stops when the last scenario (m_p=1) is run, or the pre-determined minimum number of routes allowed, M_p, is reached. The decision maker can set the value of M_p based on practical considerations. At the end of the iterative procedure, route-scenarios are obtained for each product along with the following outputs:

- Set of routes for each scenario s and product p (R_{ps})
- Time for each route (t_{psr})
- Distance travelled (d_{psr})
- Quantity of recyclables recovered (q_{psr})
- Route revenue (π_{psr})
- Route utilization (η_{psr}=q_{psr}/Q_p)
- Average weekly frequency (f_{psr})
- GHGEs diverted (g_{psr})
- Total GHGEs diverted for a scenario (G_{ps})

Instead of solving Formulation P1 directly, we will be using the *Esri ArcGIS Network Analyst VRP solver* to find solutions to the modified time-constrained CVRP model. The GIS database will be used to manage information pertaining to the locations and inventory of collection centres, depots, and other facilities, as well as road networks. The VRP solver will be used to generate routes to facilitate acquisition from all collection centres, if enough routes are available. However, if there is an insufficient number of routes, then the solver will attempt to maximize the procurement of available products in order to maximize revenue. As previously discussed, $\hat{R}_p$ is the minimum number of routes required to fulfill the collection requirements of the entire network. In each iteration, we supply the value of m_p, which is the number of available routes to the solver. Since $m_p<\hat{R}_p$, the solver will be forced to construct the best set of routes equaling m_p in order to maximize the collection revenue. As a result, certain collection centres will not be included in the set of routes (which we refer to as a scenario), and thus be dropped from the network.

When composing the weekly route schedule, some routes can be serviced less frequently depending on their utilization (η_{psr}). If a route is underutilized, collection can be alternated every other week if $\eta_{psr}\leq 50\%$, or every three weeks if $\eta_{psr}\leq 33\%$. Routes can then be grouped together for a single vehicle as long as the cumulative $\eta_{psr}\leq 100\%$; i.e. two routes where $\eta_{psr}\leq 50\%$ or three routes where $\eta_{psr}\leq 33\%$. Assuming that products can accumulate at collection centres until they are picked-up, the reciprocal of the frequency (f_{psr}) represents the time in weeks between pickups for a given route, allowing the vehicles to service another route during that vacant time.

$$f_{psr} = \begin{cases} 1, & \text{if } \eta_{psr} > 50\% \\ {}^1/_2, & \text{if } 33\% < \eta_{psr} \leq 50\% \\ {}^1/_3, & \text{if } \eta_{psr} \leq 33\% \end{cases} \tag{2.32}$$

Next, in calculating the GHGE diversion factor, γ, the following elements are taken into consideration:

1. Establish the fuel efficiency of the displaced vehicle in km/L
2. Determine the replacement vehicle capacity with respect to the displaced one
3. Find the GHGE factor in kg/L
4. Arrive at γ in kg/km

The mass of g_{psr} in Mg (1000 kg) is calculated using γ, f_{psr}, and d_{psr}. These outputs and indicators are used as parameters in the second phase of the heuristic.

Route Sequencing & Scheduling Model

In phase two, the focus is on using the product-route combinations generated from phase one to create feasible weekly schedules for each vehicle. In this step, the optimal number of vehicles required is determined. Hypothetically, assume that there are three products (plastic, aluminium, and electronics), all with routes where weekly collection is required (η_{psr}>50%). If there are seven plastic, four aluminium, and two electronics routes, then three vehicles would be required to cover the 13 routes assuming that the vehicles are available for 5 days/week.

The MILP developed below assigns routes to vehicles to yield a route schedule with the goal of maximizing total profitability in the network. In this phase, the fixed costs of the vehicles are taken into account, as well as any credit for diverting GHGEs, represented by ϵg_{psr}. The SCG is a metric which quantifies the monetary value of benefits to society for each Mg of GHGE diverted. The SCG is similar to the social cost of carbon (SCC) used in Canada and the United States. However, in addition to $C0_2$, it accounts for emission from other types of GHGs such as CH_4 and N_2O.

Route Sequencing & Scheduling Model – Formulation P2

This formulation determines the collection schedule, the route sequence, and the number of vehicles needed by maximizing the net profitability (Π) of the entire system. The first term, $\sum_{p\in P}\sum_{s\in S_p}\sum_{r\in R_{ps}}(\pi_{psr}+\epsilon g_{psr})\,w_{psr}$, represents the variable revenue from the first phase, and the credits associated with the SCG. The second term, vF, captures the fixed amortized costs of the vehicles over a weekly time horizon.

$$\max \Pi = \sum_{p\in P}\sum_{s\in S_p}\sum_{r\in R_{ps}}(\pi_{psr}+\epsilon g_{psr})\,w_{psr} - vF \tag{2.33}$$

subject to:

$$\sum_{p\in P}\sum_{s\in S_p} u_{ps} = \quad 1 \tag{2.34}$$

$$\sum_{r\in R_{ps}} w_{psr} \leq \quad u_{ps} \qquad \forall p\in P, \forall s\in S_p \tag{2.35}$$

$$\sum_{p\in P}\sum_{s\in S_p}\sum_{r\in R_{ps}} f_{psr} w_{psr} \leq \quad \psi v \tag{2.36}$$

$$v \geq 0 \qquad v\in\mathbb{Z} \tag{2.37}$$

$$u_{ps} \in \quad \{0,1\} \quad \forall p,s,r \tag{2.38}$$

$$w_{psr} \in \quad \{0,1\} \quad \forall p,s,r \tag{2.39}$$

Constraint (2.34) stipulates that exactly one scenario must be chosen for each product. Constraint (2.35) guarantees that only routes within chosen scenarios can be selected. Finally, constraint (2.36) ensures that a sufficient number of vehicles are available for collection.

2.5.2 Results & Discussion

In the RRFB case, all routes have a total time constraint of 600 minutes, commencing and terminating at Kemptown. However, this timeframe is reduced even further when factoring unloading and processing times. The aluminium routes require 30 minutes for unloading and processing at Kemptown, thus T_2=570 minutes. The clear PET routes also require 30 minutes for unloading and processing at Recycler 1, plus an additional 120-minute round-trip to Recycler 1 from the vicinity of Kemptown. Therefore the clear PET routes are limited to T_1=450 minutes. The CT vehicles are limited to one product type per trip, and have a capacity which is the same for both product types of Q_1=Q_2=400 bags. Some other constraints in the RRFB problem are:

- For each product, each ED can only be visited at most once per week, although some EDs may be serviced less frequently.
- The available inventory is collected in full in each visit.

Prospective routes for clear PET and aluminium are generated separately using average weekly collection volumes through the first-phase optimization. The routes are solved using the tabu search heuristic in the *Esri ArcGIS Network Analyst VRP solver*, which allows for constraints on the route duration, vehicle capacity, the number of CT routes available, as well as the revenue function.

The *Full* and *Reduced* networks are two scenario classes that are considered in this analysis. In the *Full* network, all EDs in the province of Nova Scotia are required to be visited. However, in the *Reduced* network, only the EDs with more than 8 bags/week in the HRM (Region 4), and more than 12 bags/week in the other regions are serviced. Both scenario classes are then combined together to be solved in the second phase optimization using a standard open source *GLPK* solver.

For clear PET, when M_1=$\hat{R}_1$ (s=0), there are 15 routes for the *Full* network and 10 for the *Reduced* network. Similarly, for aluminium M_2=$\hat{R}_2$ (s=0), there are 9 routes for the *Full* network and 6 for the *Reduced* network. For both products and scenario classes we allow the iteration to run until M_p=1. This results in 25 scenarios for clear PET and 15 scenarios for aluminium. The total number of routes can be calculated as: $\sum_{i=1}^{15} i + \sum_{i=1}^{10} i + \sum_{i=1}^{9} i + \sum_{i=1}^{6} i$. In total, there are 241 routes in 40 scenarios for both products that must be considered in the second-phase optimization. The two-phase mathematical model is applied to both network types and the following results are obtained. Additionally, the following nomenclature is adopted when naming the routes: $\{product\ class\ m_p\ routeNumber\}$, where *product* is either P (clear PET) or A (aluminium), and *class* is either F (Full) or R (Reduced). Figure 2.6 depicts a CT vehicle in the collection process.

Figure 2.6: CT Vehicle in the Collection Process

Product 1: Clear PET

In the *Full* network, there are 15 routes serving all but two EDs in the entire network (Figure 2.7). These two EDs are excluded as they would violate the total time constraint. Figure 2.8 shows the 10 feasible routes that are generated to cover the *Reduced* network with the exception of one ED, which violates the time constraint.

When $\beta=\Phi$, only 7 of these 15 routes are profitable in the *Full* network after variable costs are integrated in the analysis. Similarly, only 6 of the 10 routes remain profitable in the *Reduced* network after accounting for the variable costs. Applying Algorithm 1 to restrict the number of available CT routes and combining the networks (scenario classes) together yields the results presented in Table 2.4.

Table 2.4: GIS Generated Clear PET Routes, $\beta=\Phi$

	All Routes				**Profitable Routes**			
S_1	r_s	q_s	G_s	π_s	r'_s	q'_s	G'_s	π'_s
PF15	15	2 085	17.3	-13Φ	7	1 544	10.1	677Φ
PF14	14	2 064	16.4	132Φ	7	1 544	10.1	677Φ
PF13	13	2 054	13.9	324Φ	6	1 377	8.2	658Φ
PF12	12	2 054	13.6	367Φ	5	1 370	7.5	730Φ
PF11	11	2 017	13.0	478Φ	6	1 508	8.9	726Φ
PF10	10	2 010	13.1	637Φ	6	1 661	9.7	884Φ
PF09	9	1 900	12.3	662Φ	6	1 646	9.7	875Φ
PF08	8	1 859	10.4	796Φ	7	1 738	9.6	821Φ
PF07	7	1 826	11.7	909Φ	7	1 826	11.7	909Φ
PF06	6	1 779	11.3	959Φ	6	1 779	11.3	959Φ
PF05	5	1 546	8.9	849Φ	5	1 546	8.9	849Φ
PR10	10	1 919	13.1	571Φ	6	1 582	9.4	839Φ
PR09	9	1 900	11.8	714Φ	5	1 473	8.2	874Φ
PR08	8	1 871	12.2	792Φ	6	1 633	10.4	850Φ
PR07	7	1 786	10.0	908Φ	7	1 786	10.0	908Φ
PR06	6	1 688	8.7	931Φ	6	1 688	8.7	931Φ
PR05	5	1 624	9.1	982Φ	5	1 624	9.1	982Φ
PR04	4	1 369	7.1	843Φ	4	1 369	7.1	843Φ

There are two groupings that can be observed in the prior table: All and Profitable Routes. The first group sums together all routes that are included in each scenario, while the latter only aggregates the values of the routes which were profitable in each scenario. Furthermore, r_s, q_s, G_s, and π_s are the total number of routes, quantity recovered, GHGEs diverted, and revenue for each scenario respectively. The Profitable Route equivalents have been appended with the $'$ symbol (r'_s, q'_s, G'_s, and π'_s). We see that the most profitable scenario is *PR05* followed closely by *PF06*.

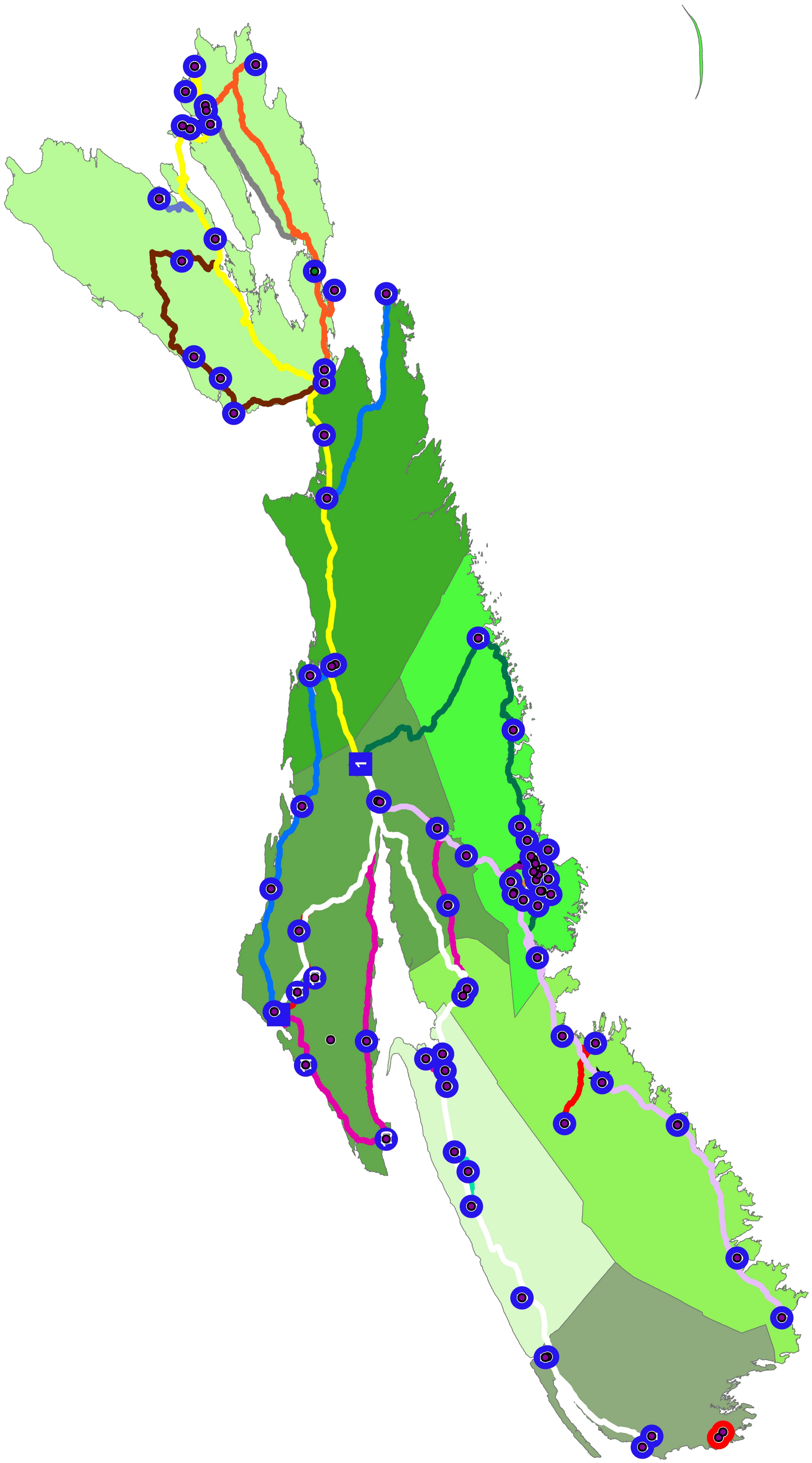


Figure 2.7: Full Network, GIS Generated Clear PET Routes

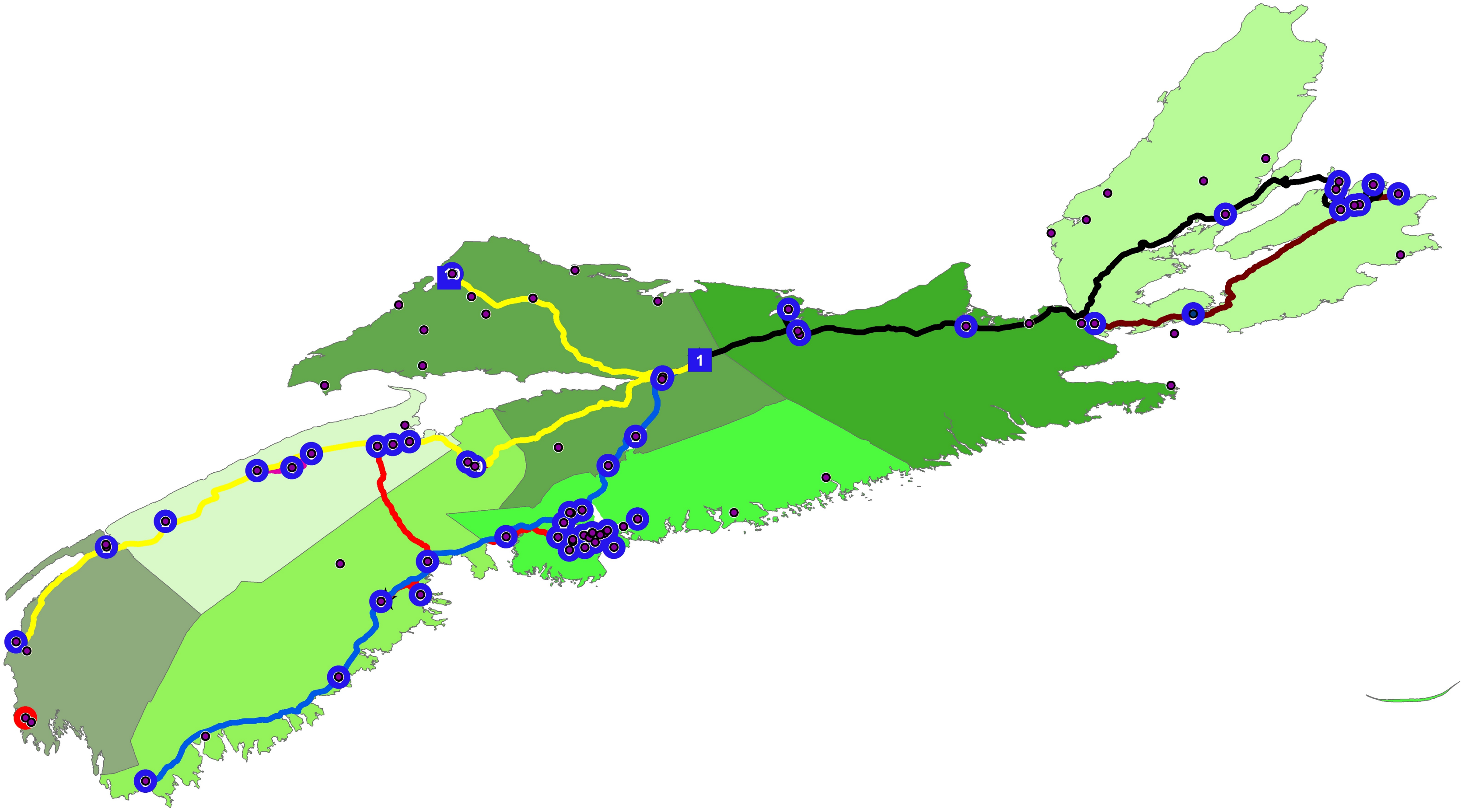


Figure 2.8: Reduced Network, GIS Generated Clear PET Routes

Product 2: Aluminium

The minimum number of routes to satisfy the collection of aluminium using CTs is investigated here. The *Full* network can be satisfied with just nine routes (Figure 2.9), and the *Reduced* in six (Figure 2.10).

When β=Φ, only 5 of these 9 routes are profitable in the *Full* network after variable costs are integrated in the analysis. Similarly, only 5 of the 6 routes remain profitable in the *Reduced* network after accounting for the variable costs. Again, applying Algorithm 1 to restrict the number of available CT routes and combining the networks (scenario classes) together yields the results presented in Table 2.5.

Table 2.5: GIS Generated Aluminium Routes, β=Φ

	All Routes				**Profitable Routes**			
S_2	r_s	q_s	G_s	π_s	r'_s	q'_s	G'_s	π'_s
AF09	9	1 493	8.3	347Φ	5	1 172	5.4	589Φ
AF08	8	1 473	8.0	402Φ	4	1 072	5.0	612Φ
AF07	7	1 452	9.0	530Φ	4	1 178	6.6	678Φ
AF06	6	1 286	7.3	412Φ	3	978	4.6	564Φ
AF05	5	1 371	8.1	679Φ	4	1 216	7.1	686Φ
AF04	4	1 125	5.7	531Φ	3	1 029	4.8	584Φ
AF03	3	879	3.9	417Φ	2	800	3.1	491Φ
AR06	6	1 305	6.3	625Φ	5	1 204	5.5	670Φ
AR05	5	1 291	7.2	670Φ	4	1 182	6.4	705Φ
AR04	4	1 242	7.2	1 065Φ	4	1 242	7.2	1 065Φ
AR03	3	1 050	5.1	629Φ	3	1 050	5.1	629Φ
AR02	2	681	3.3	434Φ	2	681	3.3	434Φ

In Table 2.5 we see that *AR04* has the largest revenue compared to all other scenarios, and also diverts the most GHGEs (G'_s) among all Profitable Routes.

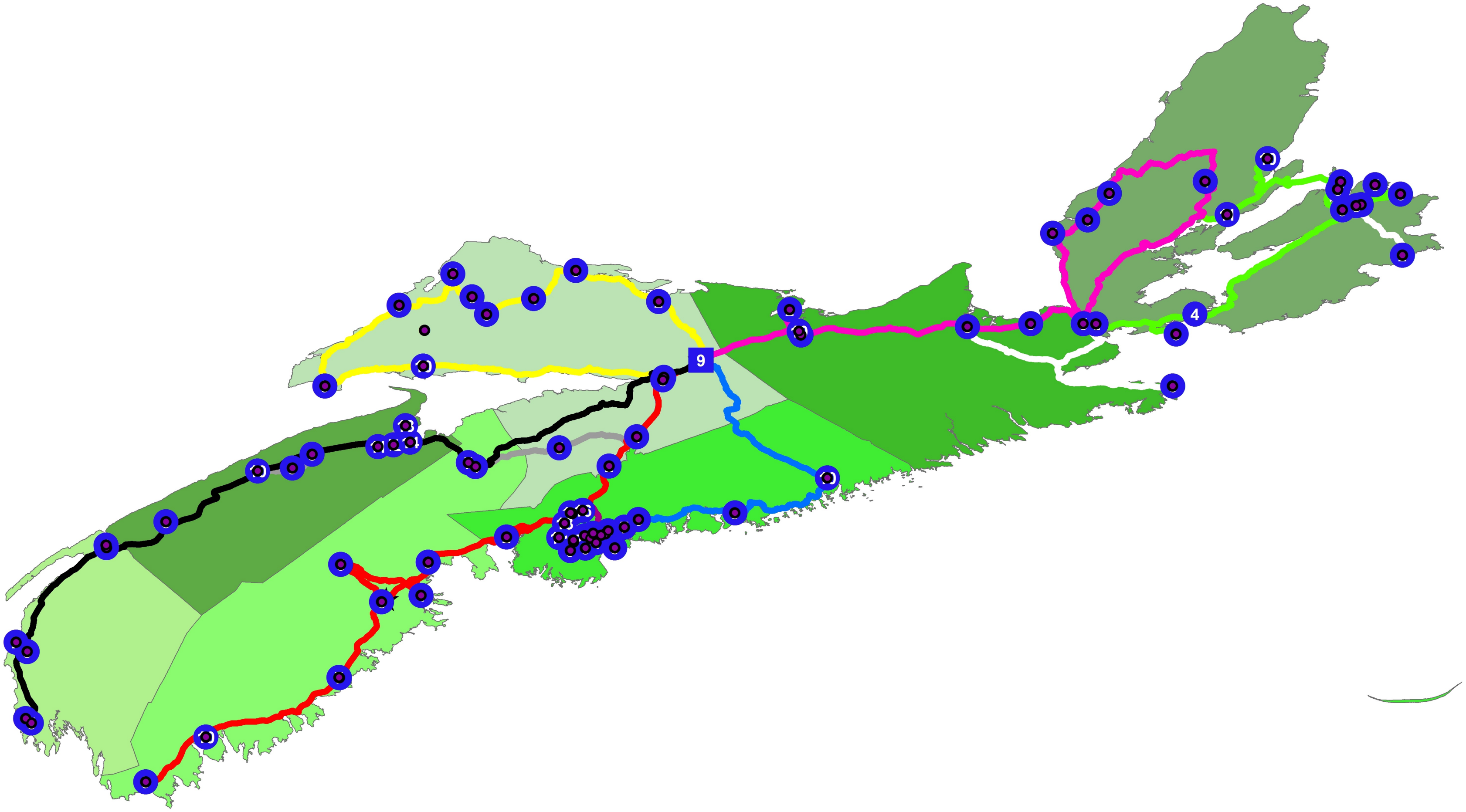


Figure 2.9: Full Network, GIS Generated Aluminium Routes

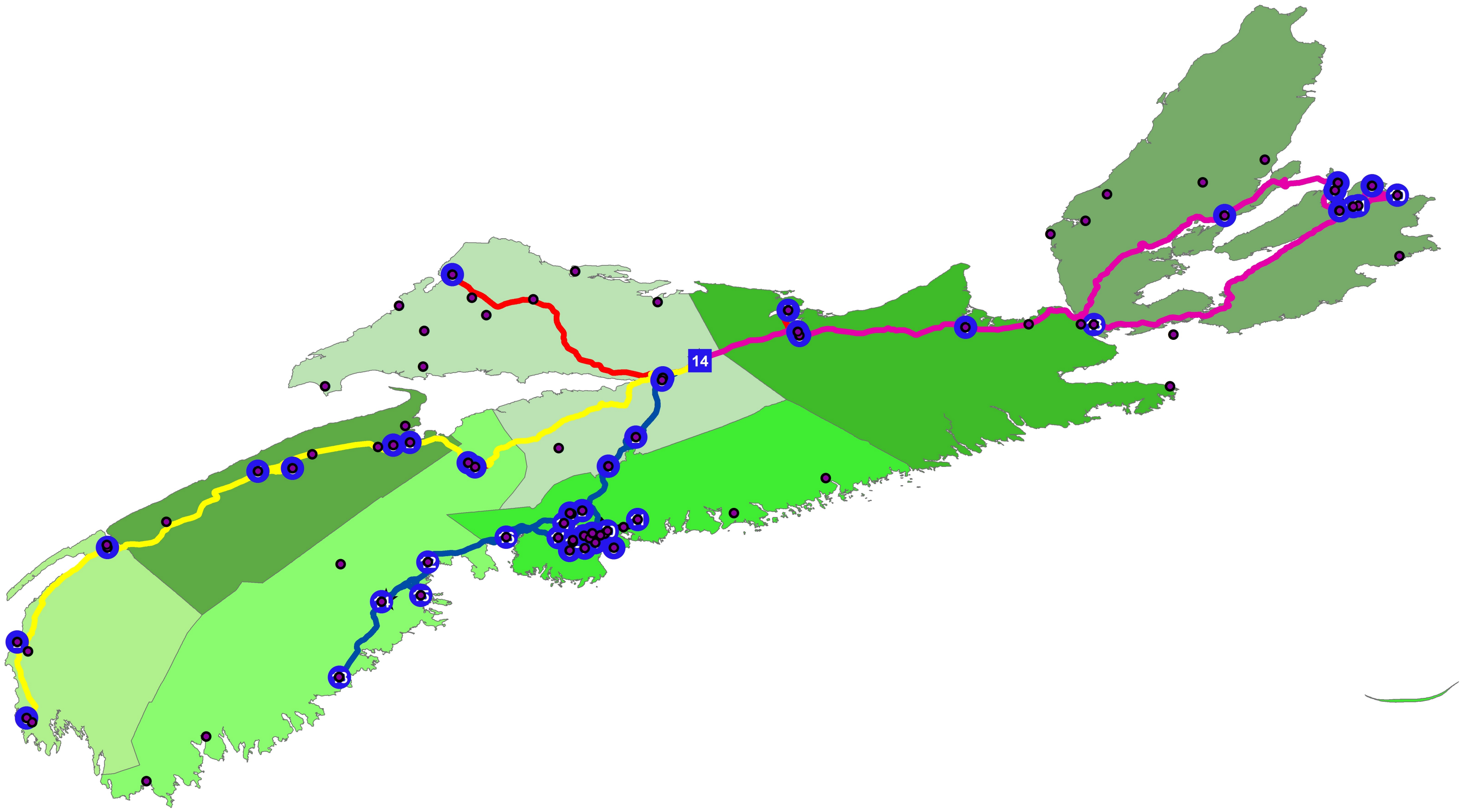


Figure 2.10: Reduced Network, GIS Generated Aluminium Routes

Second Phase Optimization

All routes obtained for each scenario are fed into the second phase of the optimization heuristic. The constraints are that no depot can be visited more than once for a given product type, and each route is dedicated to a single product. The GHGE diversion factor (γ) is calculated below.

The Freightliners have an approximate fuel efficiency of 3.36 km/L (7.9 mpg) (Trucking Info, 2011), and consume diesel, which has a GHGE factor (composed of CO_2, CH_4, and N_2O) equal to 2.6635 kg/L (Environment Canada, 2012). Each CT has a compaction ratio of 5:1, implying that for each route it services, it diverts the equivalent GHGEs of 4 trips on the same route by the 3PL. This results in a GHGE diversion factor (γ) of 3.1722 kg/km for each CT route.

For the ensuing analysis, the CTs can be operational for five days a week (ψ=5), and SCG is not used (ϵ=0). The second phase optimization arrives at a solution where in an average week, 2 CTs are in use $8^1/2$ out of a possible 10 days, net profitability Π^*=1 483Φ, and G^*=16.34 Mg of GHGEs are diverted. The routes comprising the optimal solution are shown in Table 2.6.

Table 2.6: Routes in Optimal Solution

Route	Region	n_{ps}	q_{psr}	η_{psr}	d_{psr}	π_{psr}	g_{psr}
PR05i	3,4,6	11	348	87%	636	215Φ	2.02
PR05ii	4	9	365	91%	502	246Φ	1.59
PR05iii	4-6	10	356	89%	629	235Φ	1.99
PR05iv	2-4	10	369	92%	673	228Φ	2.13
PR05v	1	5	186	47%	874	59Φ	1.39
AR04i	3,4	11	386	97%	490	239Φ	1.56
AR04ii	3,4,6	17	360	90%	542	213Φ	1.72
AR04iii	1,2	12	278	70%	671	164Φ	2.13
AR04iv	3,5-7	9	218	55%	569	448Φ	1.80

The complete schedule for the prior optimal solution is then presented in Table 2.7.

Table 2.7: Optimal Schedule

	Vehicle 1				
	Day 1	Day 2	Day 3	Day 4	Day 5
Week 1	PR05i	PR05ii	PR05iii	PR05iv	PR05v
Week 2	PR05i	PR05ii	PR05iii	PR05iv	–

	Vehicle 2				
	Day 1	Day 2	Day 3	Day 4	Day 5
Week 1	AR04i	AR04ii	AR04iii	AR04iv	–
Week 2	AR04i	AR04ii	AR04iii	AR04iv	–

Finally, Figures 2.11 & 2.12 provide a graphical representation of the routes.

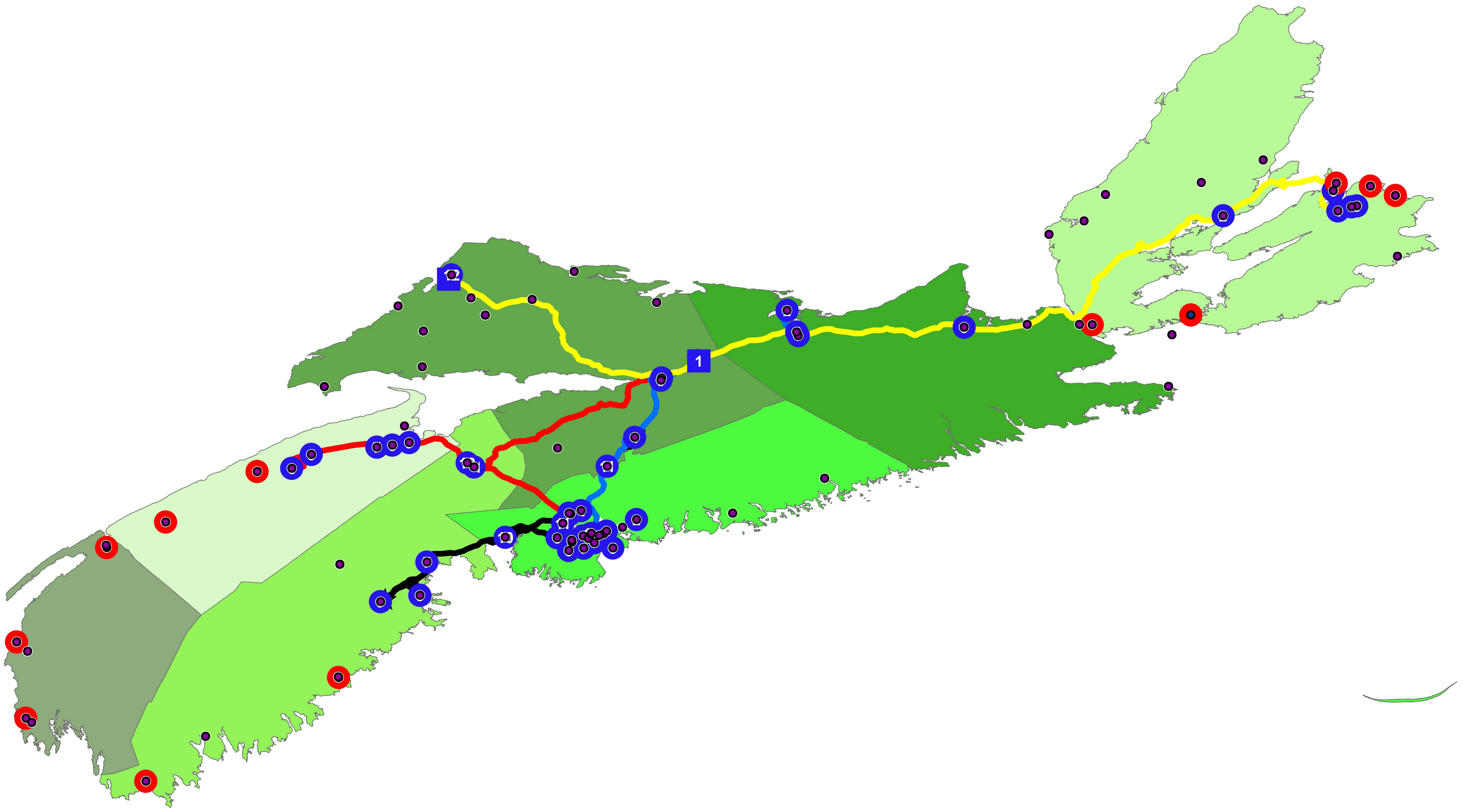


Figure 2.11: Optimal Solution of Clear PET Routes

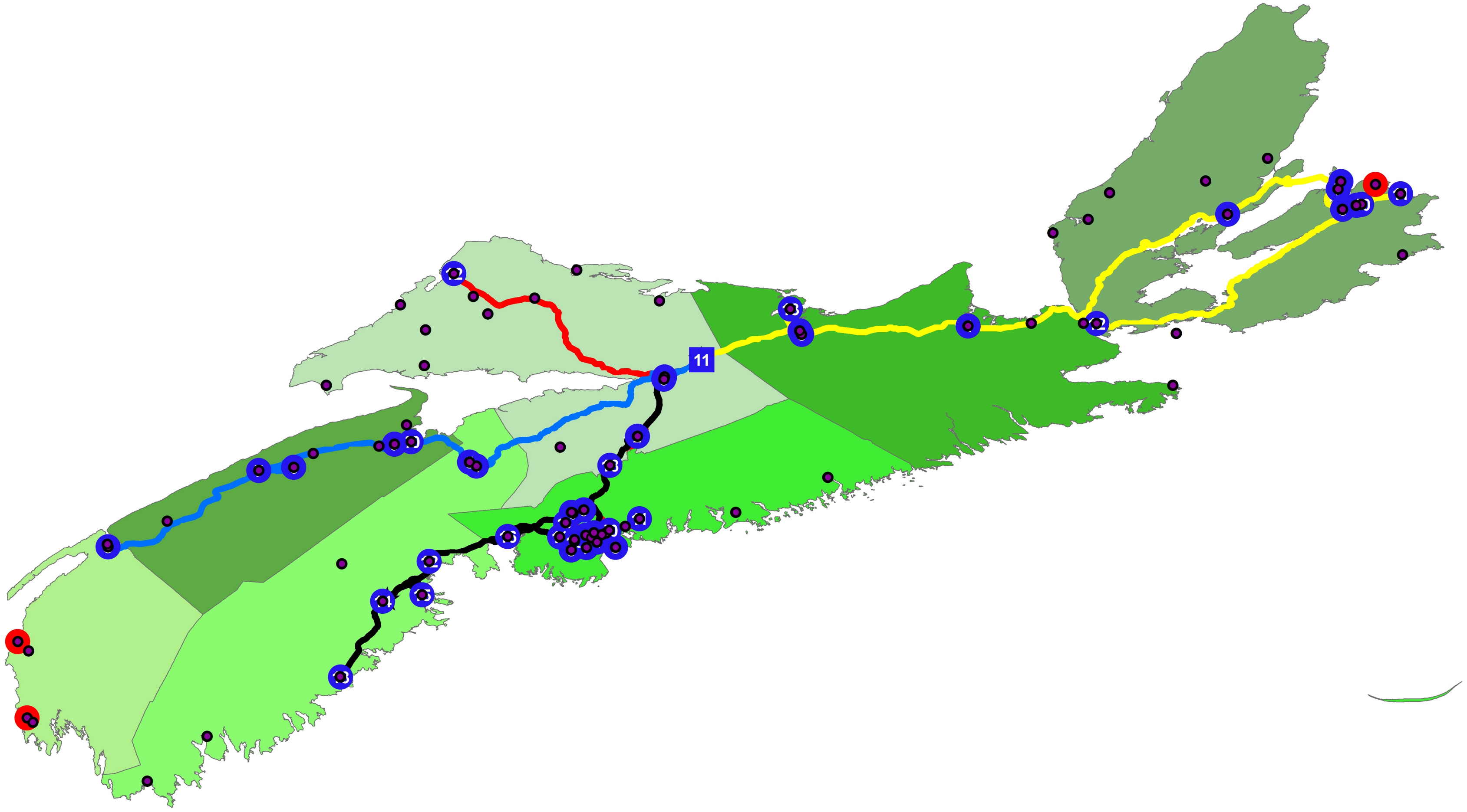


Figure 2.12: Optimal Solution of Aluminium Routes

A review of Table 2.6 presents a notably low utilization rate of 47% for route *PR05v*. This route also has the lowest π_{psr} value, and travels 874 km to visit only five collection centres. Secondly, even though there is capacity for 10 days of collection, only 8½ are being utilized. The next section explores a post-processing modification to further improve the solution.

Route Modification

In this section, *PR05v* will be taken out of the solution set, and the framework for the collection from this region will be slightly altered by defining a new route (*PCB*) over two days. The first day (*PCB1*) will be constrained to pick up from Cape Breton and return to Kemptown within 600 minutes, and the second day (*PCB2*) will be a round-trip to Recycler 1 including handling at that facility.

Table 2.8: Cape Breton Clear PET

Route	Region	n_{ps}	q_{psr}	η_{psr}	d_{psr}	π_{psr}	g_{psr}
PCB1	1,2	12	363	91%	665	276Φ	2.11
PCB2	–	–	–	–	240	-43Φ	0.76
PCB	1,2	12	363	91%	905	233Φ	2.87

This new combined solution has a total weekly profit of 1 658Φ, and a diversion of 17.82 Mg of GHGEs. The optimization method reduces the collection costs by 53.7% and successfully routes CTs to pick up approximately 85.05% of the two recyclable products considered in the case study.

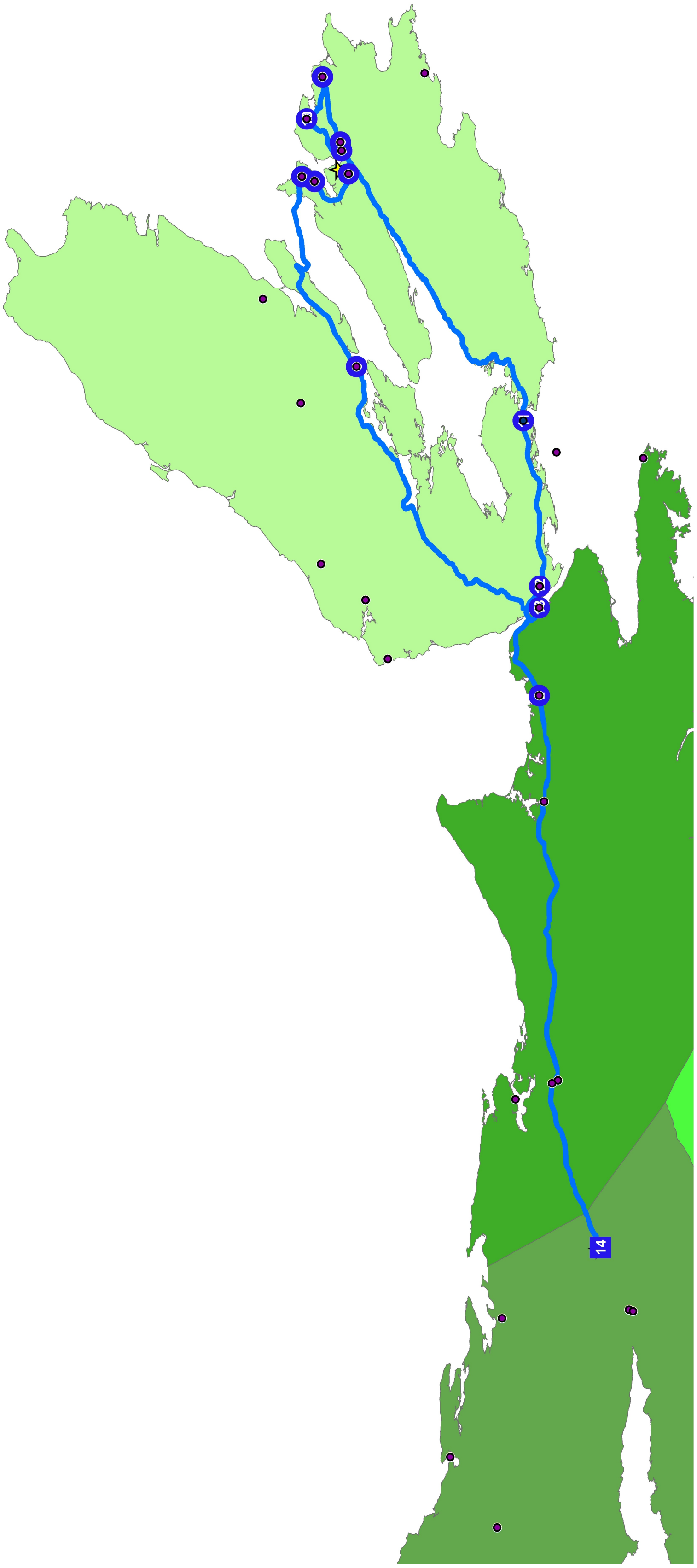


Figure 2.13: Cape Breton Clear PET

Sensitivity Analyses

Here, we perform sensitivity analyses by varying two factors in the pre-modified solution. The first evaluates the effect of changing the value of β, while the second solves the two-phase problem by incorporating SCG credits.

We generate solutions by solving the problem once again with different values of β. The complete solutions are presented in Tables A.1-A.14, which can be found in Appendix A. Figure 2.14 illustrates the total savings Π^* as a function of β, which is expressed as proportions of Φ:

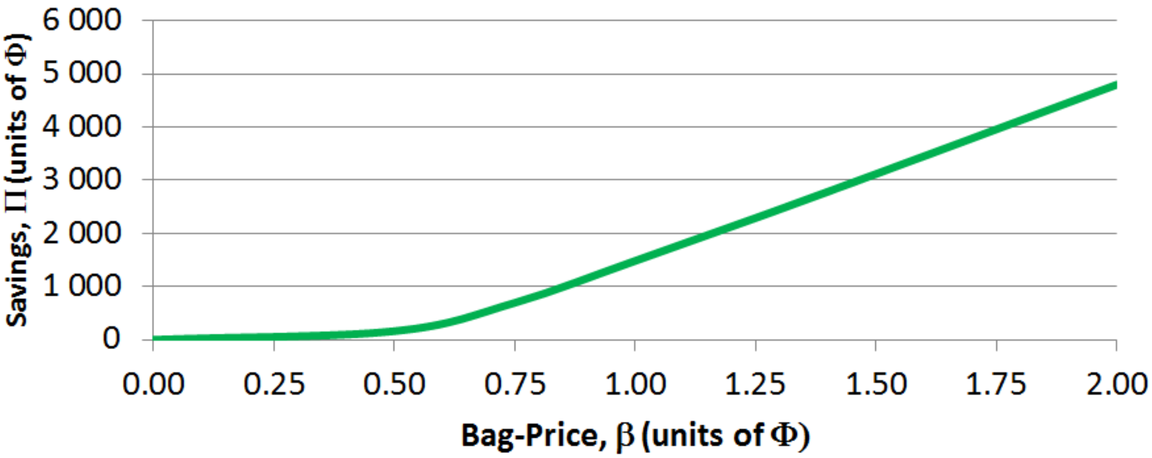


Figure 2.14: Total Savings based on Bag-Price Sensitivity

Table 2.9 summarizes the sensitivity of β, where v is the number of vehicles, S_i is the selected scenario for product i, and r_i is number of routes selected from scenario S_i:

Table 2.9: Bag-Price Sensitivity

β	v	S_1	r_1	S_2	r_2	Π^*
0.50Φ	1	PR06	3	AR04	2	162Φ
0.75Φ	2	PR05	5	AR04	4	694Φ
1.00Φ	2	PR05	5	AR04	4	$1\ 483\Phi$
1.25Φ	2	PF06	6	AR04	4	$2\ 288\Phi$
1.50Φ	2	PF06	6	AR04	4	$3\ 116\Phi$
2.00Φ	2	PF10	8	AR04	4	$4\ 794\Phi$

In Table 2.9 we can observe some key results. The most pronounced is that *AR04* is the preferred scenario in every instance, and its selection does not seem to be influenced by the bag-price. We also notice that the chosen clear PET scenario changes from *PR06* when β=0.50Φ, to *PR05* when β=0.75Φ, to *PF06* when β=1.25Φ, and lastly to *PF10* when β=2.00Φ. Moreover, when β=1.50Φ, this is the first time that the clear PET scenario is more profitable than the aluminium. Finally, we see that even a doubling of the price per bag does not necessitate a third vehicle.

The MILP used in the optimization of the collection network has been set up with the capability to incorporate SCG credits associated with GHGE diversion. In this case study, the value of ϵ is set to zero because of the lack of SCG estimates for this specific application. However, the mathematical model allows for sensitivity analyses to be conducted to study how the routes may be influenced by increasing the SCG value (Table 2.10). As a higher price is placed on GHGEs, the recommended routes will not only become more profitable, but the opportunity to have more CTs may arise as a different set of routes can come into play.

Table 2.10: Incorporating SCG

ϵ	v	Clear PET		Aluminium		Π^*	G
\$0	2	PR05	5	AR04	4	1 483Φ	16.34
\$10	2	PR05	5	AR04	4	1 514Φ	16.34
\$25	2	PR05	5	AR04	4	1 559Φ	16.34
\$100	2	PF06	6	AR04	4	1 806Φ	18.47
\$200	2	PF06	6	AR04	4	2 151Φ	18.47

It is interesting that a large SCG value is needed for there to be substantial change in the optimal solution. Here, we see that the clear PET scenario changes from the *PR05* to *PF06*, but also witness that there is no change in the aluminium scenario.

Summary of Findings

The developed model is successfully applied to a case study pertaining to the collection of clear PET and aluminium in the province of Nova Scotia. Ignoring GHGE savings, the described optimization methodology can reduce current collection costs by 53.7%. The route configuration corresponding to this solution uses two CTs to pick up approximately 85.05% of the two recyclable products investigated in this study.

The sensitivity analyses demonstrated that:

- The price per bag, β, has a substantial impact on the optimal solution, as well as the number of CTs required. When the value is reduced to β=0.50Φ, the schedule contains 5 routes, and needs only one CT. When the price is doubled (β=2.00Φ), the schedule is comprised of 12 routes, but due to the frequencies of these routes, only two CTs are required. It should also be noted that between β=0.75Φ and β=1.50Φ there is very little change in the optimal scheduling, and it would be a very conservative investment to purchase two CTs.
- The SCG parameter, ϵ, has a relatively minor impact on the optimal solution. It appears as though a very large SCG value is needed to significantly effect the schedule.

2.6 Conclusions & Future Research

This research deals with the design of pickup schedules for a collection network with a single vehicle type, multiple products, across many diverse municipalities. A two-phase mathematical model is developed to jointly determine the number of vehicles, the scheduling, as well as the routes constituting the collection network. In the first phase, viable sets of CT routes for each product type are generated from underlying GIS data by repeatedly solving instances of the route-generation problem using the built-in tabu search heuristic in the *Esri ArcGIS Network Analyst VRP solver.* In the second phase, the formulated MILP is solved to determine the optimal combination of product-routes generated in the first phase to maximize net profitability.

Using the data and the methodology developed as a foundation, the following research issues can be considered for future deliberation. This model constrained the number of feasible routes in a given scenario, while still having the possibility to access all the collection centres. An alternate approach is to eliminate collection centres in each scenario while maintaining the same number of routes. The placement of these collection centres based on population density can also be examined. This study did not explicitly route the 3PL vehicles that were used to service the collection centres not visited by CTs. An area of research to investigate is a network designed with multiple vehicle types (CTs and 3PL vehicles) to concurrently service the collection centres.

A greater precision in estimating administrative and loading times will also provide more accurate routes and fiscal savings. A closer inspection into these processes by either altering the manual component, or introducing automation can lead to a decrease in time, make the process more efficient, and allow for a greater range of collection. Factoring facility related costs by circumventing RPCs, exploring the option of consolidating RPCs in an effort to decrease facility processing costs, and establishing the ideal number of EDs and their locations will also increase net savings. Other search heuristics, such as ACO, HBMO, GAs, SA, VNS, and column generation, can be harnessed and have their effectiveness evaluated with respect to tabu. Finally, GPS data from truck movement can be used to track how closely the movement of trucks and recyclables corresponds to those suggested by the model with the view to capture historical data for future analysis and improvement.

In this chapter, we developed a two-phase model to determine the optimal routing sequence in an effort to efficiently collect MSW for an end-use facility to reprocess. In the next chapter, we will investigate how to integrate and use the collected EoU/EoL products in post-consumption processing, also known as remanufacturing.

CHAPTER 3

PLANNING FOR PRODUCTION USING REMANUFACTURED SYSTEMS

The implementation of EPR legislation mandating the recovery of EoU/EoL products increases the volume of recovered products going through post-consumption processing or remanufacturing. Besides the environmental benefits, remanufacturing presents an economic opportunity for manufacturers and consumers. Essentially, it is a set of value recovering and production activities for EoU/EoL products that have been collected, sorted, cleaned, and are then reprocessed through repair, refurbishing, reconditioning, or recycling. Disassembled and reconditioned parts, components, and modules can then be used in the production of new or second-hand products. The goal of an organization engaging in remanufacturing will be to design its production system and activities in order to produce competitive products by recovering maximum value for returns and minimizing its operational costs. Moreover, these decisions will have to account for constraints specific to the context of remanufacturing: uncertainties in the quality, quantity, and timing of the incoming EoU/EoL products, setting up the remanufacturing system, support services, etc.

This chapter is dedicated to the planning and integration of remanufactured systems into the production plan of both new and second-hand products. This chapter is organized as follows. The first section defines, classifies, and discusses value recovery activities and remanufacturing processes. The second section is devoted to the analysis of uncertainties and production planning models for remanufacturing, specifically the solution methodologies of aggregate planning, material requirement planning (MRP), and control theory models. In the third section, a new control theory model is developed to determine optimal remanufacturing strategies for a producer who wishes to recover and reuse returned products that have failed under warranty. Finally, a discussion of results and future areas of research conclude this chapter.

3.1 Definitions, Classification, and General Issues

The *3Rs* (reduce, reuse, and recycle) have historically exemplified consumer environmental practices. The consumer generally perceives recycling as the act of placing an item in a bin, so it can be recovered, reprocessed, and used again. The definition of reprocessing has evolved, primarily due to the proliferation of electronic waste, to incorporate activities other than strictly the recycling of materials, which is very energy intensive. These other reprocessing activities: repair, refurbishing, and reconditioning, are less energy intensive to recover value. All of these reprocessing activities are collectively classified as value recovery, and are the basis of the modern sustainable manufacturing paradigm (Thierry et al., 1995). These *6Rs* create a continuum of sustainable options requiring different amounts of energy.

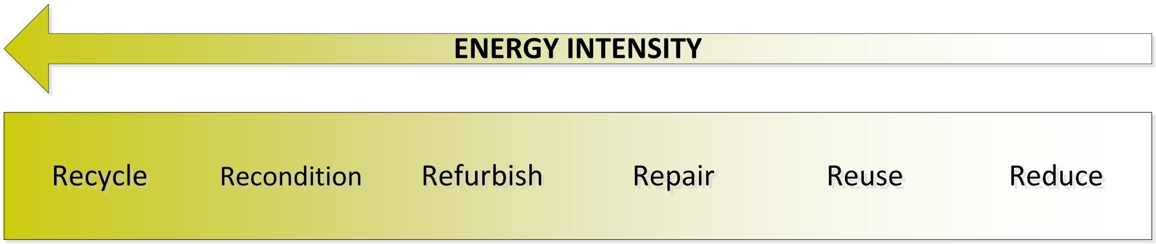


Figure 3.1: Continuum of Sustainable Options

As depicted in Figure 3.1, Reduce consumes the least, followed by Reuse, and then the value recovery activities of: Repair, Refurbish, Recondition, and Recycle. These sustainable options enable manufacturers to consider reverse logistics and value recovery, to lower production costs and project a green corporate image.

Several studies published on reverse logistics and value recovery include: Baetz and Neebe (1994); Thierry et al. (1995); Klausner and Hendrickson (2000); Lee, Lye, and Khoo (2001); Guide Jr. and Van Wassenhove (2001); Nakashima, Arimitsu, Nose, and Kuriyama (2002); Fleischmann, van Nunen, and Grave (2003). The logistics aspect was discussed in the previous chapter, now we will focus on value recovery and other remanufacturing activities.

Remanufacturing is an option available at the end of a product's life to extend its original life (González and Adenso-Díaz, 2005; Zussman, Kruvet, and Salinger, 1994). It is the process of restoring used products to like-new conditions by disassembly, cleaning, repairing and replacing parts, and reassembly (Lund and Hauser, 2003). Disassembly, shredding, and separation are operations in value recovery (Kriwet, Zussman, and Seliger, 1995).

In general, it is difficult and costly to recover used items from assemblies or equipment not designed for remanufacturing (Graedel and Allenby, 1996). Xerox's products however are robust, large, and easy to disassemble, making remanufacturing valuable, and attributing to the company's overall success (Kerr and Ryan, 2001). An overview of the disassembly and recycling of electronic consumer products is presented by de Ron and Penev (1995). Physical relations between all relevant components in a product are represented as a tree graph used to assess which components and/or materials are either valuable or poisonous. "The poisonous components will define the number of compulsory disassembly operations, while the valuable components imply the number of desirable operations" (de Ron and Penev, 1995). Analysis of the tree graph yields the suitability for disassembly. For example, if the joints are fixed or if there are no physical relations between the components, then separation is not possible. An optimal disassembly plan is obtained from a cost/benefit analysis including the value added to the product, the value of the abandoned products, and the revenues from disassembly, dismantling, and separated shredded materials.

Figure 3.2 presents the remanufacturing framework with all the value recovery options, and their integration to the production sequence.

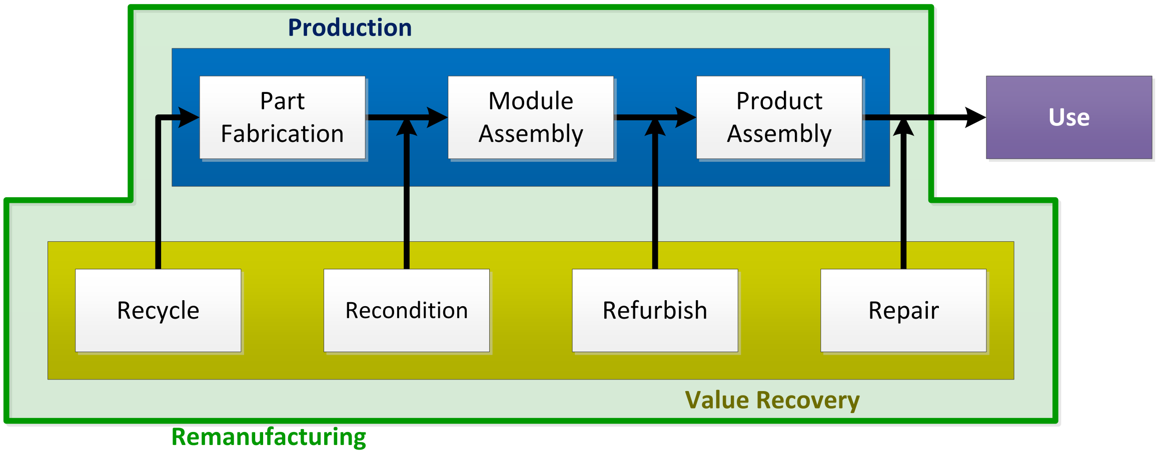


Figure 3.2: Remanufacturing Framework

In Table 3.1, we summarize the different attributes of these value recovery options:

Table 3.1: Value Recovery Options

Option	Disassembly Level	Energy/ Effort	Activities
Repair	Product	Low	Products are inspected and restored to working order. Repaired products are returned back to the consumer.
Refurbish	Product, Module	Medium	Products may have some modules replaced or upgraded. Modules are inspected and then repaired, replaced, or upgraded. Refurbished modules are used for remanufacturing.
Recondition	Part	High	Parts are inspected and then repaired, replaced, or upgraded. Reconditioned parts are used for remanufacturing.
Recycle	Material	Extreme	Materials are sorted, shredded, and melted. Recycled materials are used to fabricate individual parts.

Remanufacturing can be lucrative to both the organization and its consumers due to the potential for higher profitability (Lund and Hauser, 2003). Since parts and components can be reused, the cost to remanufacture a product is generally much less than creating a new one. For example, with certain vehicles, remanufactured parts provide "vehicle owners with a low cost, like-new replacement component with the added environmental benefit of managing their equipment on a 'cradle-to-cradle' versus a 'cradle-to-grave' life cycle" (Bourgeois and Leleux, 2004).

Companies that incorporate sustainable activities as part of their business strategy may also be able to build long-term relationships with customers, which can also become a competitive advantage in supplying future products. This can also counter some of the main challenges such as labour intensity, and uncertainties in supply and quality (The Beer Store, 2010). Economically, the process can be profitable. "In the United States, remanufacturing is at least a \$50 billion industry with direct employment of about 480,000 in 73,000 firms" (Gutowski, Sahni, Boustani, and Graves, 2011). Some of the other socio-economic gains from remanufacturing include providing local skilled jobs and reducing transportation expenses (Gutowski et al., 2011).

Some companies decide not participate in this practice because they favour new products, which will sell for a much higher margin than remanufactured ones. Guide Jr. and Li (2010) state that "the greatest concern to managers is the case when the remanufactured version of a product currently in production is available". Product cannibalization is when consumers develop a preference for remanufactured products over the new version, which also happens to reduce the company's net profit.

In general, consumers perceive remanufactured products as being inferior to new ones, forcing retailers to sell them at a discounted price. Gutowski et al. (2011) report that remanufactured products sell for approximately 50-80% of a new product. Similarly, the Fleischmann et al. (2003) study at IBM found that costs associated with remanufacturing are 80% less than procuring a new part. With these products selling at a discount, the consumer benefits, without having to sacrifice product performance.

Remanufacturing has the potential of being economically profitable and environmentally friendly. Reaping these benefits requires methodological planning, coordination, and execution of remanufacturing activities on recovered products. Production planning and control methodology can be adapted to the context of remanufacturing for the purpose of modelling the recovery of EoU/EoL products and their integration into the manufacturing system.

3.2 Production Planning for Remanufacturing

The goal of manufacturing planning and control is to satisfy customer demand over a specified time horizon by acquiring, utilizing, and allocating resources to production activities in the most effective and efficient manner. The range of problems, formulations, and solution methodologies in production planning cover an extensive scope, however some "typical decisions include work force level, production lot sizes, assignment of overtime and sequencing of production runs" (Graves, 1999).

Production planning in a manufacturing environment is a vast and complex area of research. Hax and Meal (1973) develop a hierarchical planning and scheduling system for multiple plants and products. Bitran and Hax (1977) expand the Hax and Meal (1973) system by analyzing the interactions between the different hierarchical levels. There has been a lot of research in the last half-century dealing with production planning models in general. Some of the prominent review papers in this field have been presented by Moore and Wilson (1967); Graves (1981); Guide Jr. (2000); Silver (2008).

When developing an optimization model for manufacturing planning and control there are many factors that need to be addressed. As mentioned by Graves (1999), the estimation of customer demand, which may or may not be known, is paramount among these considerations. An unknown demand will require a forecast, which in turn will present a degree of uncertainty. Moreover, decisions on production, inventory quantities, resource acquisition, and resource allocation also need to be addressed. Remanufacturing adds additional complexity to the traditional manufacturing paradigm due to the uncertainty in the availability and condition of recovered products.

With the recent development in CLSC management, many production planning models have focused on the integration of remanufacturing with its specific context and challenges. Many of the issues encountered during remanufacturing originate from decisions made at the acquisition and grading stages of EoU/EoL products. Uncertainties in the quality, quantity, and timing of recovered products are major concerns that will be discussed in the next subsection. A review of mathematical models for production planning of remanufactured products will follow. These models are aggregate planning, MRP, and control theory production planning models.

3.2.1 Uncertainties in Remanufacturing Production Planning

Figure 3.3 depicts the key activities in CLSC/reverse logistics networks. EoU/EoL products recovered from the production, distribution, and use activities are graded into different levels, or are disposed of. Subsequently, they are reprocessed through one of the four value recovery activities, and then redistributed to a particular production activity.

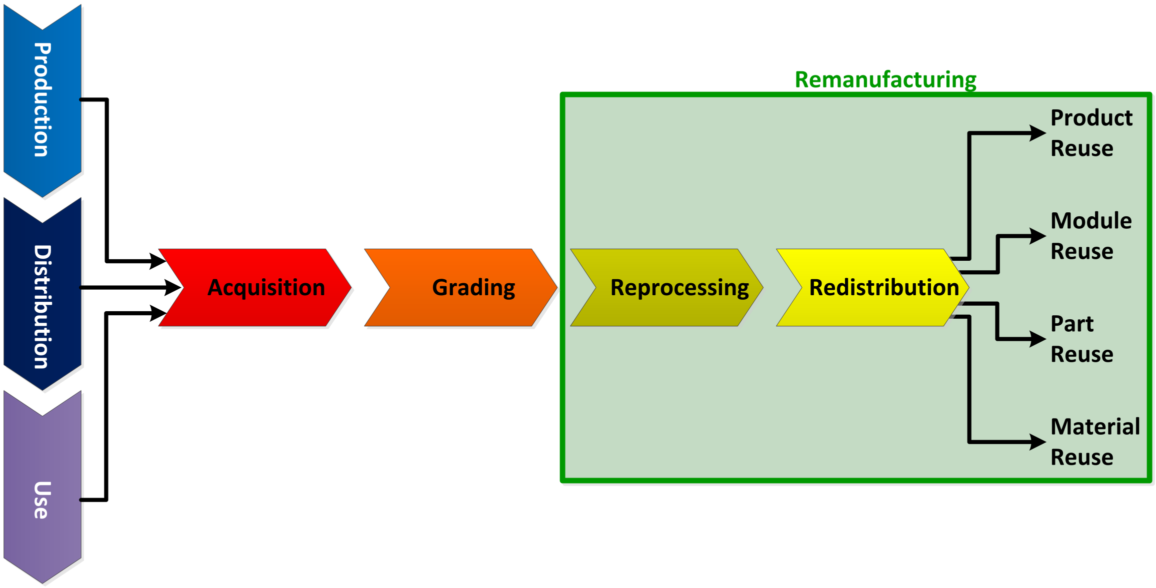


Figure 3.3: Reverse Logistics Chain Adapted from Fleischmann et al. (2005)

Guide Jr. and Van Wassenhove (2001) define two types of CLSCs based on the level of management used in the acquisition of recoverables:

- Waste stream CLSCs, where acquisition management is minimal, all returned items are passively accepted, and then processed at minimum cost.
- Market-driven CLSCs, where acquisition management is very important, only certain items are accepted, and the key objective is to maximize profit.

By increasing the control over the level of acquisition management, the organization can mitigate the consequences from the variability of recovered products.

EoU/EoL products usually lack the homogeneity, quality standards, and availability of traditional raw materials/components. This results in uncertainties of quality, quantity, and timing specific to remanufacturing. Inspection, testing, and grading are important processes to assess and sort incoming recovered products based on their levels of quality. Grading is extremely valuable, because even if a sufficient quantity of products are obtained, they need to be in a state that is economically feasible to reprocess. Ferguson, Guide Jr., Koca, and Souza (2009) justify the need to grade prior to production:

> We need to account for different quality grades in inputs, which may have non-stationary availability (quantity) during the planning horizon. In this situation, we need to track inventories for different quality grades for inputs, and the possibility of salvaging some of this inventory if it becomes excessive, particularly of low-quality inputs.

Costs, processing times, and values are then attached to the recovered products based on their quality grade (Denizel, Ferguson, and Souza, 2010). These parameters are necessary to assess the feasibility and benefits of remanufacturing for each quality grade lot. Furthermore, when the quality grade is below a certain threshold, the option of disposal has to be considered. When the only two options are recoverable and unrecoverable, the term yield (of recoverable products) is used instead of quality grade by many authors in this field.

Ketzenberg, Souza, and Guide Jr. (2003) develop a model with two quality classes (recoverable and unrecoverable), and error-free grading. They study the problem of designing a mixed assembly-disassembly line for remanufacturing, where parts from the disassembly and repair of used products can be used to build new products. They simulate two configurations, one where there are separate lines for assembly and disassembly, and the other where the activities are shared. They find that a two-day production buffer provides the same flow times as having an infinite buffer, and even a relatively small finite buffer offers a significant improvements in flow times. Additionally, as yield losses increase, average flow times and the buffer size necessary to maintain a given flow time level also increase.

Ferrer (2003) examines yield information with respect to supplier responsiveness in remanufacturing operations by providing optimal lot-size policies for several scenarios. The cost function is composed of purchase, disassembly, repair, holding, and shortage costs. Total cost is a function of the number of machines sent to disassembly, the number of parts procured from the external supplier, and the realization of the reclaim yield. The reclaim yield used in this formulation is a random variable between zero and one. They discover that short lead times become increasingly important as shortage costs increase. However, it is more beneficial to have the capability to detect the yield early in the recovery process as yield variance, or the costs of purchase, repair, or holding increases.

Ferrer and Ketzenberg (2004) formulate a multi-period, multi-part extension to the Ferrer (2003) model, by developing four infinite horizon, stochastic dynamic program (Markov decision process) models to evaluate the impact of yield information and supplier lead time on manufacturing costs. Specifically, the ability to identify part yield early and to obtain purchased units from suppliers with a short lead time were examined. It was found that the value of yield information (early detection) is significantly valuable, while supplier responsiveness (short lead time for placing orders) is not. However, the value of short lead times increases as product complexity (large number of target parts) increases. These results were ascertained based on a test-set of 135 cases, representing a wide range of operating conditions.

Pan, Tang, and Liu (2009) also present a problem where returned products are either remanufactured or disposed of. In their capacitated dynamic lot sizing problem, backlogging is not allowed, and there are limited capacities for production, remanufacturing, and disposal. They formulate a dynamic programming model and solve it numerically for several scenarios. They find that the length of the planning horizon plays an important role in the cost savings achieved from expanding the remanufacturing production capacity. A long planning horizon will enable expansion by saving costs. However, with a short planning horizon, any savings from increasing the capacity may be overwhelmed by the cost of expansion.

The quality of the incoming systems can sometimes be decided by the remanufacturer who can procure the items in unsorted lots, and perform the grading processes in-house. Another scenario is to pay a higher price for better quality units, effectively transferring the grading process to the supplier (Fleischmann et al., 2010). A premium is then paid for a reduction of uncertainty in the quality of the products. In the end, it is up to the remanufacturer to determine whether they or the suppliers are responsible for the inspection, testing, and grading stage.

Souza, Ketzenberg, and Guide Jr. (2002) present one of the earliest publications with respect to CLSC grading problems. They examine production planning and control for a remanufacturer who sorts the returned products into three quality levels (superior, average, and inferior) with known proportions. In this model, each quality level requires a different remanufacturing process. Souza et al. (2002) also observe that using a dynamic dispatching rule, which takes into account the workload at each station, can mitigate the effects of grading errors, which vary directly with flow times.

Aras, Boyaci, and Verter (2004) develop a model with two levels of quality, high and low, with both being able to be remanufactured, albeit with different cost structures. The grading is assumed to be error-free, and its costs are considered negligible. The stochastic nature of the quality of returned products has led the authors to develop a continuous-time Markov chain model. This model analyzes the conditions where grading leads to cost savings in hybrid remanufacturing and manufacturing systems. Their numerical analyses find that cost savings due to grading can reach 10%, with the savings being attributed to prioritizing remanufacturing on the basis of the return quality, and disposing of unsuitable returns. The key conclusions from this model is that grading is most cost-effective when: the quality of all returns are relatively low with a high quality-gap between both levels, the demand rate is low, and the return rate relative to the demand rate is high.

Guide Jr., Muyldermans, and Van Wassenhove (2005); Zikopoulos and Tagaras (2008); Tagaras and Zikopoulos (2008); Ferguson et al. (2009) focus on the costs of grading in their models. Ferguson et al. (2009) present a tactical remanufacturing production planning model with returns that have different quality levels. Using data from the mailing systems remanufacturing operations of Pitney-Bowes, they find that as the quality level decreases, so does the salvage value of any unused returns, however the remanufacturing cost increases. They formulate a stochastic dynamic program where the objective (maximum attainable profit) is a function of the number of quality grades. Scenarios with 1, 3, 5, 7, and 10 distinct quality grade levels are studied with the intent to quantify the value of the grading process. An optimal solution is obtained by solving the model using a simple greedy heuristic in the case of deterministic returns and demand. Ferguson et al. (2009) demonstrate numerically using parameter values obtained from multiple industry partners, that by having grading classes, there is an improvement in total profits of about 4% over a system without grading. They find that the savings remain essentially the same for both capacitated and uncapacitated facilities. Although it is beneficial to have more than two categories, as in Aras et al. (2004), negligible benefits are observed beyond five classes.

Fleischmann et al. (2010) explain that when "there is no variation in the quality of the acquired items...the acquisition decision is focused exclusively on quantity". Atasu and Çetinkaya (2006) study the situation where there is no quality variation with respect to inventory optimization for the optimal collection and use of remanufacturable returns. Having a sufficient quantity of recovered systems is also an important consideration, and as observed in the prior chapter, there are many factors that determine the supply quantity. These factors must be adjusted to provide a sufficient supply of recoverables for the remanufacturing process.

The uncertainties that we have discussed above pose significant challenges in the planning of production in the remanufacturing context. In the next section, a review of aggregate planning and MRP models for remanufacturing production planning are provided in addition to the papers already reviewed above, on the grading/quality topics, which also include some aspects of production planning.

3.2.2 Aggregate Planning & MRP Models

The graded products are either disposed of or remanufactured. Remanufacturing, as we have already covered, includes the value recovery activities of repair, refurbishing, reconditioning, and recycling in conjunction with the production of parts, modules, and products. Fleischmann et al. (2003, 2005) conduct a study on IBM's component-dismantling operation, finding that used computer equipment may be recycled for materials, dismantled for spare parts, or refurbished and sold once again.

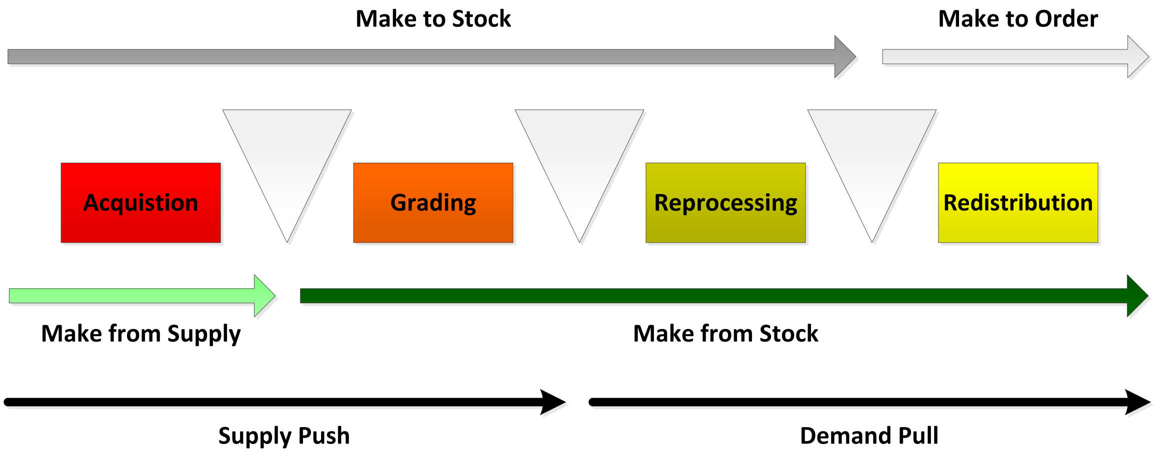


Figure 3.4: Inventory Buffers Adapted from Fleischmann et al. (2005)

As depicted in Figure 3.4, the border between the *make to stock* and *make to order* customer-order decoupling point is important in establishing inventory buffers. Fleischmann et al. (2005) state that in CLSCs, there is also a similar point (the boundary between *make from stock* and *make from supply*), which specifies the distance in the process chain a returned product has to move, thus establishing its inventory buffer quantity. These points may or may not coincide.

Generally, if demand cannot be found, reprocessing should be postponed. However, if demand information is available, postponing reprocessing decisions are based on the trade-off between higher safety stock levels and lower per unit holding costs (Fleischmann et al., 2005). Demand however, is just one uncertainty in remanufacturing. There are also uncertainties in quality, acquisition volumes, structural fluctuations of supply and demand, varying capacity utilization, and interrelation between recovery options (Fleischmann et al., 2010).

A typical production plan model minimizes the variable production costs plus the inventory holding costs for all items over a planning horizon, subject to a variety of constraints such as production, inventory, and capacity requirements.

Hoshino, Yura, and Hitomi (1995) construct a production planning optimization model for a manufacturing system that utilizes recyclable material. Initially, issues pertaining to material flow in an idealized recycling-oriented society are discussed. Then a representation of recycling-oriented manufacturing systems composed of production and disposal divisions is proposed. The production division is the classical forward distribution network, while the disposal division is the reverse logistics network. Recovered products are classified into three levels: remanufacturable (reused in the plant), recyclable (by raw-material suppliers), and unrecoverable. In their model, the firm purchases some parts from raw-material suppliers, assembles products with both new and recyclable parts, and also sells remanufacturable parts to other suppliers. They develop a mathematical model to determine an optimal recycling plan that maximizes total profit and recycling rate. A numerical example is presented.

Golany, Yang, and Yu (2001) present a finite horizon, deterministic demand, multi-period production planning problem with remanufacturing. In each period, the model selects whether collected products are remanufactured, held for future periods, or discarded based on costs.

> Remanufacturing may involve testing, cleaning, disassembly with replacements of some components and then reassembly, etc. Holding the used items in inventory may involve shipping in and out of storage, cost of storage space, monitoring, etc. Disposal of items may incur the costs associated with transportation, disassembly and separation of hazardous materials, etc. Disposal may also be associated with negative costs (i.e. revenues) due to salvage value of the disposed items (Golany et al., 2001).

In addition to these costs, for every period, the model also accounts for production, holding (for both used and new items), remanufacturing, and disposal costs. The remanufacturing costs are linear or concave in the quantity of remanufactured products, and the problem is solved using dynamic programming. The formulation also assumes a single quality grade for all returns, and that remanufactured products are identical to new products.

Souza (2010) describes two approaches for remanufacturing production planning: a forecast-driven aggregate planning optimization and an MRP-based heuristic.

The first approach requires data such as the future forecasts of the number of returns of each quality grade, the demand for remanufactured products over the planning horizon, and all relevant costs. Souza (2010) presents an LP model to minimize the costs of production, salvaging, backlogging, and holding under inventory balance, as well as aggregate capacity constraints for both returns and remanufactured products. This formulation is shown next:

Indices

i: Quality, i=1 (best),...,G (worse)
t: Time period, t=1,...,T

Decision Variables

u_{it}: Inventory of quality i returns at the end of period t
v_{it}: Quantity of quality i returns salvaged in period t
z_{it}: Quantity of quality i returns remanufactured in period t
y_t^+: Inventory of remanufactured products at the end of period t
y_t^-: Backlog of remanufactured products at the end of period t

Parameters

D_t: Demand forecast for remanufactured products at period t
B_{it}: Quantity of returns of quality i at period t
s_i: Unit salvage value for returns of quality i that are not remanufactured
h_i: Unit holding cost for return of quality i
h_r: Unit holding cost for remanufactured product
b: Unit backlogging cost
c_i: Unit remanufacturing cost for return of quality i
a_i: Unit capacity usage by return of quality i
K_t: Total capacity available at time t

Eq. (3.1) is the objective function minimizing the costs of production, salvaging, backlogging, and holding, and is subject to constraints (3.2)-(3.5).

$$\min \sum_{t=1}^{T} \sum_{i=1}^{G} \{h_i u_{it} - s_i v_{it} + c_i z_{it}\} + h_r y_t^+ + b y_t^- \tag{3.1}$$

subject to:

$$\left(y_{t-1}^+ - y_{t-1}^-\right) + \sum_i z_{it} - \left(y_t^+ - y_t^-\right) = D_t \quad t = 1, ..., T \tag{3.2}$$

$$u_{it} - u_{i,t-1} + v_{it} + z_{it} = B_{it} \quad i = 1, ..., G \quad t = 1, ..., T \tag{3.3}$$

$$\sum_i a_i z_{it} \leq K_t \quad t = 1, ..., T \tag{3.4}$$

$$u_{it}, v_{it}, z_{it}, y_t^+, y_t^- \geq 0 \qquad i = 1, ..., G \quad t = 1, ..., T \tag{3.5}$$

The authors imply that salvaging (denoted by v_{it} is a less profitable means of disposition, similar to cannibalization proposed in Thierry et al. (1995). Therefore v_{it} appears in the inventory balance constraint Eq. (3.3) just like z_{it}, which refers to the number of units remanufactured. The Souza (2010) model does not incorporate fixed costs for remanufacturing in a period, and also assumes that there is no uncertainty in demand or returns. These uncertainties can be dealt with by carrying a safety stock of remanufactured products, or solving the problem on a rolling horizon basis.

The second approach incorporates a rolling planning horizon, demand forecast uncertainty, and limited information on return forecasts and their quality levels. The safety stock is represented by a multiplier (S) in Eq. (3.6):

$$S = 1 + k\sigma_{A/F} \tag{3.6}$$

where S is the safety stock multiplier, k is the safety factor, and $\sigma_{A/F}$ is the standard deviation of the ratio of actual demand to forecast (A_t/D_t). The safety factor k is calculated from underage (not meeting demand) and overage (excess supply) costs.

The probability of meeting all the demand for remanufactured products in the same period is presented below in Eq. (3.7):

$$\frac{C_u}{C_u + C_o} \tag{3.7}$$

where C_u and C_o are the underage and overage costs respectively of not meeting demand for remanufactured products in a period. Using this ratio, k can be calculated:

$$k = \mathcal{T}^{-1}\left(\frac{2C_o}{C_u + C_o}, N\right) \tag{3.8}$$

where N is the number of historical forecast data points, and $\mathcal{T}^{-1}(\cdot,\cdot)$ is the inverse function of the t-distribution with two parameters: probability and degrees of freedom. These values of k can then be used in an MRP table, which can be easily set up in a spreadsheet. This MRP model is dynamic and controls the manufacturing system by utilizing the feedback from prior periods. This second model may be more appropriate when there is significant uncertainty in the demand for the remanufactured product, while the first model is more suitable for capacitated problems.

Standard control theory production planning models will be explored as a way to bridge the gap between the two approaches presented above.

3.2.3 Control Theory Production Planning Models

"The objective of optimal control theory is to determine the control signals that will cause a process to satisfy the physical constraints and at the same time minimize (or maximize) some performance criterion" (Kirk, 2012). From a mathematical modelling perspective, optimal control theory has a very deep historical context.

> The development of the mathematical theory of optimal control began in the early to mid 1950's, partially in response to problems in various branches of engineering and economics. Despite its modern origins, optimal control theory, from a mathematical point of view, is a variant of one of the oldest and most important subfields in mathematics – the calculus of variations (Berkovitz, 1976).

Optimal control theory is a particularly interesting approach to use in production planning given the dynamic nature of manufacturing systems and their inputs. Several papers have successfully used optimal control theory to study production planning problems.

Tomiyama (1985) develops a two-stage optimal control problem with a time-delay argument in the objective function to study the optimal investment decision for a firm whose capital goods are subject to a delivery lag. The switching time in this model results in a change in the state equation and the performance index. He provides the model formulation and presents the necessary conditions for a general two-stage problem with an adjustable switching time.

Rossana (1985) develops a dynamic model of a firm holding a stock of unfilled orders for output while facing a fixed delivery lag attached to acquiring new capital goods. The problem is modelled as a two-stage optimal control problem. The first stage corresponds to the immediate sales period where the price is dictated by the demand of the consumers, and the second stage is the period during which the new goods may be delivered. In their study, a correlation between short-run and long-run decisions is demonstrated.

Tomiyama and Rossana (1989) develop a general formulation to find the optimal instrument function and switching time that maximizes the performance index; subject to the state equation, the boundary condition, and the instrument constraint. The authors derive the necessary conditions for the existence of an optimal solution to this optimal control problem. The optimality criteria developed is applied to a problem that analyzes the effect of delivery lags on investment decisions. Previous research only examined delivery lags from the perspective of the suppliers, whereas this extension optimizes the behaviour of both sides of the marketplace. In the example studied, parameters such as the purchase price, delivery, and stock of goods are considered in addition to labour, output price, discount rate, calendar time, delivery lag, and wage rate. The purchase price of the goods is composed of a spot price associated with immediate delivery and a term that captures any discounts or premia imposed by the suppliers if the firm takes forward delivery of new goods. The effect of the optimal delivery lag is examined by performing a sensitivity analysis on parameters typically set by the firm: the spot price, output price, discount rate, and wage rate. It was discovered that a longer delivery lag would be chosen when the spot price or wage rate rises, and shorter delivery lag when the output price increases. The effect of the discount rate was found to be inconclusive.

Makris (2001) extends the model proposed by Tomiyama and Rossana (1989) to have an infinite horizon. He also studies the optimal timing of switching between alternative and consecutive regimes. The necessary optimality conditions for this problem are also provided.

Boucekkine, Saglam, and Vallée (2004) use a two-stage optimal control technique to solve technology adoption problems with a learning curve, fixed costs, and incentives. In their baseline scenario (without learning behaviour), a drop in the consumption level is observed at the time of switching to the new technology. Based on how the obsolescence costs compare to the growth advantage, the system either sticks with the initial technology or immediately switches to the new technology. In their second scenario, learning is introduced by incorporating fixed costs and incentives to delay switching. Here, the system will only make the switch if the growth advantage overwhelms both the obsolescence and learning costs.

Kim and Park (2008) develop a comprehensive production planning control model that incorporates spare parts manufacturing and maintenance considerations. They investigate a warranty policy (known as an EoL warranty), where the coverage period takes effect after the production cycle of the product, and its role on the profitability of a manufacturer. Their model determines the product price, warranty period, and the spare parts production plan that maximizes profit while fulfilling contractual commitments to the customer over a given decision horizon.

Kim and Park (2008) consider a manufacturing company that makes and sells a product over T units of time (first stage/production cycle). The company has to provide the customer with a spare part during a period of length $T{+}w$, where w is the warranty period (second-stage). Figure 3.5 depicts the two stages of the problem.

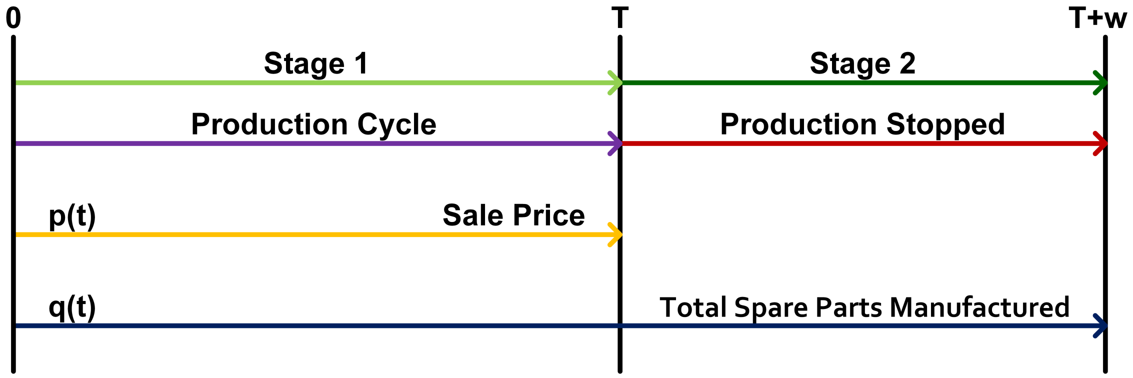


Figure 3.5: Two-Stage Control Variables (Kim and Park, 2008)

During the first stage, the company produces the original products and builds up an inventory of spare parts needed to cover the warranty replacements. It is important to note that in their model the customer is only allowed one spare part during the entire $T+w$ coverage period. If the inventory of spares built up during the first stage is not sufficient to cover the replacements during the second stage, then the company has to manufacture new ones at a higher per unit cost. The spare parts that are produced in the first stage and consumed in the second stage incur a holding cost. On the other hand, spare parts that are produced in the second stage have a higher cost per unit to manufacture. Moreover, in order to maximize profitability, customer demand is a very important factor. The demand varies inversely with the sale price of the product, and directly with the warranty length. Although increasing the warranty length spurs demand, it also amplifies the future support service costs to honour the warranty. Notation for this model is shown below, and further details on these parameters are provided in the following model:

Decision Variables
w: Warranty period
T: Production cycle of the product

Control Variables
$p(t)$: Unit sale price of the product at t
$q(t)$: Spare parts produced at t

State Variables
$D(t)$: Cumulative sales units of the product at t
$F(t)$: Cumulative parts failures at t
$Q(t)$: Cumulative production of spare part at t

Parameters
λ: Failure rate
c_p: Unit cost to produce products
c_{mi}: Unit cost to manufacture spare parts at stage i
c_h: Unit cost to hold spare parts in inventory
c_{ri}: Unit cost to repair/replace spare parts at stage i
d_1: Potential market size when price and warranty are zero
d_2: Price coefficient
d_3: Warranty coefficient
s_1, s_2: Lump sum profit sales parameters

Other Functions

$d(t)$: Demand of the product at t
$f(t)$: Number of failures of the product at t
Π: Net Profit

The objective function in their model consists of three major parts: the profit function of the first stage ($0 \leq t \leq T$), the cost function of the second stage ($T \leq t \leq T+w$), and a lump sum profit based on the cumulative sales at the end of the first stage. It is expressed as:

$$\begin{aligned}\max \Pi = & \int_0^T \left[d(t)\left(p(t)-c_p\right)-\frac{c_{m1}}{2}q(t)^2-c_h\left[Q(t)-F(t)\right]-c_{r1}f(t)\right]dt \\ & + \int_T^{T+w}\left[-\frac{c_{m2}}{2}q(t)^2-c_h\left[Q(t)-F(t)\right]-c_{r2}f(t)\right]dt \\ & + s_1 D(T)-s_2 D(T)^2 \end{aligned} \tag{3.9}$$

subject to:

$$Q(t) \geq F(t) \tag{3.10}$$

$$Q(0)=0, D(0)=0, F(0)=0 \tag{3.11}$$

where:

$$d(t) = \begin{cases} d_1 - d_2 p(t) + d_3 w & \text{if } t \leq T \\ 0 & \text{if } t > T \end{cases}$$

$$f(t) = \lambda\left(D(t)-F(t)\right)$$

$$\dot{D}(t) = d(t)$$

$$\dot{Q}(t) = q(t)$$

They derive the necessary and sufficient conditions for optimality and are able to jointly determine the optimal values of the production cycle (T^*) and the warranty length (w^*). A solution to this problem is illustrated in Figure 3.6:

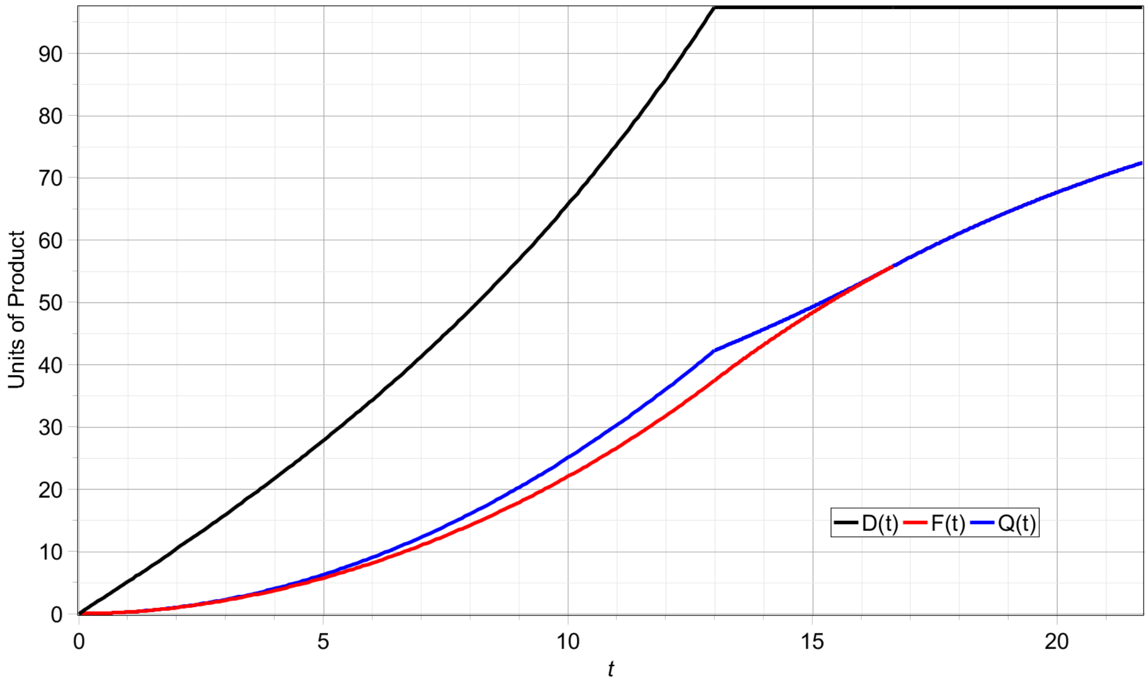


Figure 3.6: Optimal State Variables Dynamics (Kim and Park, 2008)

Kim and Park (2008) use their model to run multiple numerical experiments to gain valuable managerial insights into the behaviour and dynamics of the system. They discover that the relationship between the two decision variables is convex. For very short production cycles, a relatively long warranty length is needed to entice customers in order to sell more products within the limited product life. As the cycle gets longer, the necessity to sell products quickly fades, and the firm can reduce the warranty length. Then, after a certain point, the firm needs to increase its warranty coverage for these relatively longer cycles in order to increase demand that can last long enough for the expected product life. They also determine that the relationship between the warranty period and failure rate is concave.

It should be noted that in the Kim and Park (2008) model, only one warranted replacement is provided upon failure. Because the EoL warranty length w takes effect after the production cycle T, early adopters get a longer warranty coverage than consumers buying their products right before T. By offering warranty coverage of different lengths based on when the product is purchased, their model treats some consumers unfairly. In the marketplace, warranty periods begin at the time of purchase, and are generally not affected by the duration of the production cycle.

In the next section of the dissertation, the Kim and Park (2008) model will be improved by providing a fair warranty policy, and incorporating reconditioned components to remanufacture spare parts.

3.3 Two-Stage Optimal Control Theory Model for Remanufacturing

A new two-stage control theory model is developed to extend the Kim and Park (2008) model to deal with production planning with remanufactured products. With the integration of remanufacturing, the following questions must be considered:

1. What are the economic benefits of these remanufacturing activities?
2. When should these remanufacturing activities be started?
3. How many new and reconditioned spare parts should be produced?

The warranty policy provided during the coverage period will be changed from a single replacement to a free replacement warranty (FRW) policy, specifically the non-renewing FRW (NFRW) policy. Additionally, quality, quantity, and timing uncertainties will be incorporated in the formulation of the model.

3.3.1 Model Formulation

A company produces and sells a product until time T when it is discontinued. The product is sold with an NFRW policy, where "the manufacturer agrees to repair or provide replacements for failed items free of charge up to a time w from the time of the initial purchase" (Blischke and Murthy, 1992). After time w elapses, the warranty expires. According to Blischke and Murthy (1992), "this type of policy is the most common of all consumer warranties and probably of commercial warranties as well".

During the production cycle, spare parts are fabricated and used to repair products that fail during the warranty period w. Initially, all spare parts are new. As more failed products are recovered, components can be harvested and reprocessed into reconditioned spare parts that can also be used to repair failed products. Production of spare parts needed to carry out warranty replacements continues well after the product is discontinued, although it is more expensive after T. Due to economies of scale, it is cheaper to manufacture a spare part in Stage 1 than in Stage 2. However, there are holding costs associated with carrying inventories of spare parts.

Product sales bring revenue to the company. Costs are incurred by manufacturing new products, new spare parts, remanufacturing spare parts from failed products, and holding the inventories of spare parts. The system is also subject to production capacity constraints of both new and remanufactured parts, as well as the availability of remanufacturable products. Our goal is to determine the ideal production cycle length (T), warranty period (w), and time to commence remanufacturing (γT).

The modelling horizon can be split into three distinct periods:

- Stage 1A ($0 \leq t \leq \gamma T$): the company manufactures the product and also manufactures new spare parts using solely new components;
- Stage 1B ($\gamma T \leq t \leq T$): the company manufactures the product and also manufactures spare parts using both new and reconditioned components;
- Stage 2 ($T \leq t \leq T+w$): product production is discontinued, but the manufacture of new and reconditioned spare parts are produced as needed.

The optimal control theory remanufacturing model that has been developed using both new and reconditioned components is illustrated in Figure 3.7:

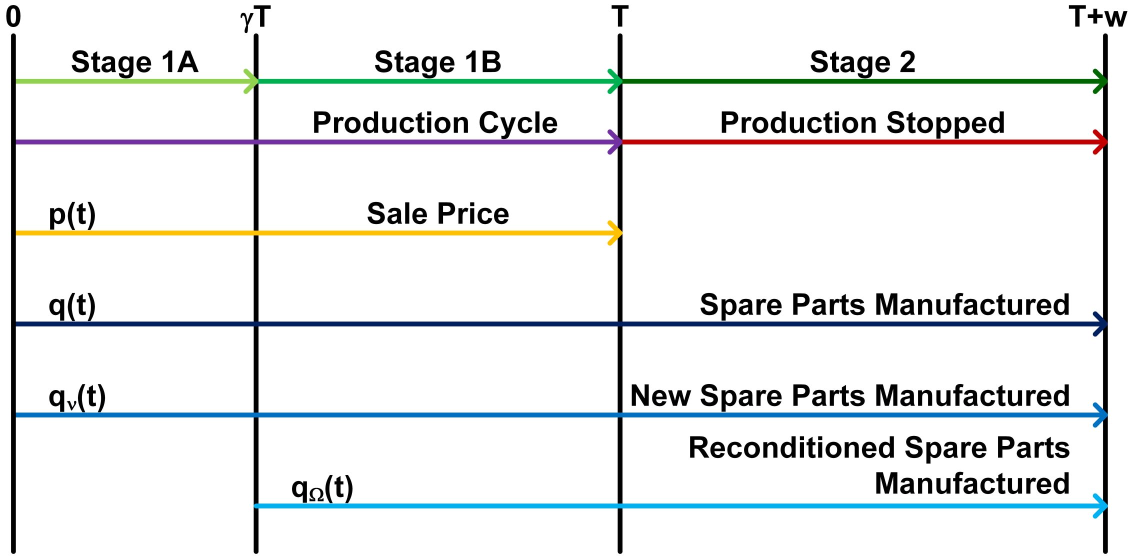


Figure 3.7: Remanufacturing Model Control Variables

The introduction of reconditioned components is a key factor in our study. The time to begin remanufacturing is defined as a fraction (γ) of the production cycle (T) with $0 \leq \gamma \leq 1$. A value of $\gamma = 0$ means that reconditioned components are utilized at time 0, effectively eliminating Stage 1A; whereas $\gamma = 1$ implies that reconditioned components are introduced at the start of the second stage.

The total amount of failed products, $F(t)$, and how much of it is usable, η, are important considerations with regards to availability of reconditioned parts. η represents the proportion of failed products that will be in a state of degradation such that their key components can be tested, removed, and reconditioned economically. Lastly, there is a cost to use reconditioned components, so k_Ω models the production cost of remanufacturing as a function of γ. Furthermore, all pertinent terms relating to spare parts use two indices: ν for new components and Ω for reconditioned components. An extensive notation including decision variables, control variables, state variables, parameters, and other functions for this model is adopted below:

Decision Variables

w: Warranty period
T: Production cycle of the product
γ: Fraction indicating the time at which remanufacturing begins, $0 \leq \gamma \leq 1$

Control Variables

$p(t)$: Unit sale price of the product at t
$q_\nu(t)$: New spare parts produced at t
$q_\Omega(t)$: Reconditioned spare parts produced at t

State Variables

$D(t)$: Cumulative sales units of the product at t
$F(t)$: Cumulative number of failures at t
$Q_\nu(t)$: Cumulative production of spare parts at t using new components
$Q_\Omega(t)$: Cumulative production of spare parts at t using reconditioned components

Parameters

η: Proportion of failed products that can be reconditioned
λ: Failure rate
c_p: Unit cost to produce products
c_{mi}: Unit cost to manufacture spare parts at stage i
c_h: Unit cost to hold spare parts in inventory
c_r: Unit cost to repair/replace spare parts
d_1: Potential market size when price and warranty is zero
d_2: Price coefficient
d_3: Warranty coefficient
k_Ω: Remanufacturing cost parameter
s_1,s_2: Lump sum profit sales parameters

Other Functions

$d(t)$: Demand of the product at t
$f(t)$: Number of failures of the product at t
Π: Net Profit

The objective function is composed of three main parts: the profit function at the first stage (sales revenue minus holding, production, and repair costs), the cost function at the second stage (holding, production, and repair costs), and a constant term of $s_1 D(T) - s_2 D(T)^2$ representing a lump sum profit based on the market share held by the company (installed base - cumulative).

$$\max \Pi = \int_0^T \left[d(t)(p(t) - c_p) - \frac{c_{m1}\left[q_\nu(t)^2 + k_\Omega q_\Omega(t)^2\right]}{2} - c_h\left[Q_\nu(t) + Q_\Omega(t) - F(t)\right] - c_r f(t) \right] dt$$

$$+ \int_T^{T+w} \left[-\frac{c_{m2}\left[q_\nu(t)^2 + k_\Omega q_\Omega(t)^2\right]}{2} - c_h\left[Q_\nu(t) + Q_\Omega(t) - F(t)\right] - c_r f(t) \right] dt$$

$$+ s_1 D(T) - s_2 D(T)^2 \tag{3.12}$$

subject to:

$$Q_\nu(t) + Q_\Omega(t) \geq F(t) \tag{3.13}$$

$$Q_\nu(T+w) + Q_\Omega(T+w) = F(T+w) \tag{3.14}$$

$$Q_\Omega(t) \leq \eta F(t) \tag{3.15}$$

$$D(0) = 0, F(0) = 0, Q_\nu(0) = 0, Q_\Omega(0) = 0 \tag{3.16}$$

$$Q_\Omega(\gamma T) = 0 \tag{3.17}$$

$$t, w, T, \gamma, p(t), q_\nu(t), q_\Omega(t), D(t), F(t), Q_\nu(t), Q_\Omega(t), d(t), f(t) \geq 0 \tag{3.18}$$

where:

$$\dot{D}(t) = d(t), \dot{F}(t) = f(t), \dot{Q}_\nu(t) = q_\nu(t), \dot{Q}_\Omega(t) = q_\Omega(t)$$

The integrand of Stage 1 of the profit function comprises the following terms:

- $d(t)(p(t) - c_p)$ represents the total revenue from the sale of the product. The demand for the product is modelled to grow with the increase of the warranty period and the decrease of the price. The instantaneous demand is:

$$d(t) = \begin{cases} d_1 - d_2 p(t) + d_3 w & \text{if } t \leq T \\ 0 & \text{if } t > T \end{cases} \tag{3.19}$$

- $\frac{c_{m1}}{2}\left[q_\nu(t)^2 + k_\Omega q_\Omega(t)^2\right]$ represents the cost to manufacture new spare parts and the cost to recondition spare parts from the lot of failed products that can be repaired and reused.

- Starting the remanufacturing process at time γT incurs costs that have to be included in the cost of the reconditioned spare parts. The remanufacturing cost parameter is defined as follows:

  $$k_\Omega = \delta_1(1-\gamma) + \delta_2 \tag{3.20}$$

  where δ_1 and δ_2 are positive parameters representing the slope and intercept respectively of the assumed linearly decreasing relationship between the starting time of remanufacturing and the total cost of all the efforts needed to set up the remanufacturing process. To start remanufacturing at time 0 would be very expensive as very few EoU/EoL products would be available. But as time passes, the availability of EoU/EoL products would improve and the costs to recover them would decrease.

- $c_h\left[Q_\nu(t) + Q_\Omega(t) - F(t)\right]$ represents the inventory holding costs for both new and reconditioned spare parts. The difference $Q_\nu(t)+Q_\Omega(t)-F(t)$ represents the instantaneous quantity of products on-hand.

- $c_r f(t)$ represents the cost to replace the key failed component in the product with a spare part.

  - With the NFRW policy as long as a product (key component) fails during the original warranty period, it is replaced with another key component free of charge. The instantaneous number of failures $f(t)$ is assumed to be a proportion (λ) of the total volume of products under warranty coverage. At any given time t,

    $$f(t) = \lambda\left[D(t) - V(t)\right] \tag{3.21}$$

    where $V(t)$ is the volume of products no longer covered by the warranty. The expression of $V(t)$ is given by:

    $$V(t) = \begin{cases} 0 & \text{if } t \leq w \\ D(t-w) & \text{if } t \geq w \end{cases} \tag{3.22}$$

$V(t)$ is the same function as $D(t)$ but only shifted by w because products bought at any given time lose their coverage after exactly w units of time of operation. Therefore:

$$f(t) = \begin{cases} \lambda D(t) & \text{if } 0 \leq t \leq w \\ \lambda \left[D(t) - D(t-w) \right] & \text{if } w \leq t \leq T \\ \lambda \left[D(T) - D(t-w) \right] & \text{if } t \geq T \end{cases} \tag{3.23}$$

The cumulative number of parts failure $F(t)$ is:

$$F(t) = \int_0^t f(x)dx \tag{3.24}$$

Constraint (3.13) is the back-logging constraint, it stipulates that the cumulative production of all spare parts must be greater that the cumulative number of failures at all times. Constraint (3.14) prevents the excess production of spare parts at the end of the planning horizon. Constraint (3.15) limits the production of spare parts using reconditioned components to be a proportion of the number of failed products. Furthermore, we define ϕ to be a point in time where all future failure replacements are carried out using reconditioned components only. Therefore, $Q_\nu(\phi){=}F(T{+}w){-}Q_\Omega(T{+}w)$, or in other words, $q_\nu(t){=}0$, when $t{\geq}\phi$.

Using the solution methodology proposed by Kim and Park (2008), we derive the Hamiltonian, the necessary conditions for optimality, and the optimal solution functions for each stage of the model.

Stage 1 (From $0 \leq t \leq T$)

Using the Pontryagin Maximum Principle, the Hamiltonian is derived in Eq. (3.25), and the necessary conditions are given by Eqs. (3.26)-(3.35):

$$\begin{aligned}\mathcal{H}_1 = {} & (d_1 - d_2 p(t) + d_3 w)(p(t) - c_p) - \frac{c_{m1} q_\nu(t)^2}{2} - \frac{c_{m1} k_\Omega q_\Omega(t)^2}{2} \\ & - c_h \left[Q_\nu(t) + Q_\Omega(t) - F(t)\right] - c_r f(t) \\ & + (d_1 - d_2 p(t) + d_3 w)\varphi + q_\nu(t)\psi + q_\Omega(t)\xi \\ & + \left[Q\nu(t) + Q_\Omega(t) - F(t)\right]\zeta + f(t)\theta \end{aligned} \tag{3.25}$$

The equations of motion for the control variables $p(t)$, $q_\nu(t)$, and $q_\Omega(t)$ can be determined by solving the following relations:

$$\frac{\partial \mathcal{H}_1}{\partial p} = d_1 - 2d_2 p(t) + d_3 w + d_2 c_p - \varphi d_2 = 0 \tag{3.26}$$

$$\frac{\partial \mathcal{H}_1}{\partial q_\nu} = \psi - c_{m1} q_\nu(t) = 0 \tag{3.27}$$

$$\frac{\partial \mathcal{H}_1}{\partial q_\Omega} = \xi - c_{m1} k_\Omega q_\Omega(t) = 0 \tag{3.28}$$

The equations of motion for the costate variables $\psi(t)$, $\xi(t)$, and $\theta(t)$ can be determined by solving the following relations:

$$\dot{\psi}(t) = -\frac{\partial \mathcal{H}_1}{\partial Q_\nu} = c_h - \zeta \tag{3.29}$$

$$\dot{\xi}(t) = -\frac{\partial \mathcal{H}_1}{\partial Q_\Omega} = c_h - \zeta \tag{3.30}$$

$$\dot{\theta}(t) = -\frac{\partial \mathcal{H}_1}{\partial F} = \zeta - c_h \tag{3.31}$$

$$\zeta\left[Q\nu(t) + Q_\Omega(t) - F(t)\right] = 0, \quad \zeta \geq 0, \quad Q\nu(t) + Q_\Omega(t) - F(t) \geq 0 \tag{3.32}$$

It can be assumed that $w<T$ because in practice the warranty period offered is usually a fraction of the production cycle. The first stage is then divided into two sub-stages: sub-stage 1.1 ($0\leq t\leq w$) and sub-stage 1.2 ($w\leq t\leq T$). From Eq. (3.23), we known that $f(t)=\lambda D(t)$ in sub-stage 1.1, and $f(t)=\lambda\left[D(t)-D(t-w)\right]$ in sub-stage 1.2. This provides equations of motion for $\dot{\varphi}(t)$:

$$\dot{\varphi}_{11}(t)=-\frac{\partial\mathcal{H}_{11}}{\partial D}=\lambda c_r-\lambda\theta(t) \tag{3.33}$$

$$\dot{\varphi}_{12}(t)=-\frac{\partial\mathcal{H}_{12}}{\partial D}=0 \tag{3.34}$$

The transversality condition is:

$$\varphi_{12}(T)=0 \tag{3.35}$$

Given that $c_{m1}<c_{m2}$, there is an incentive to make more spare parts than needed during the first stage. Therefore, $Q\nu(t)+Q_\Omega(t)-F(t)>0$, and the only solution to Eq. (3.32) is $\zeta=0$. Thus, we obtain:

$$\psi(t)=\int\dot{\psi}(t)dt=c_ht+X_1 \tag{3.36}$$

$$\xi(t)=\int\dot{\xi}(t)dt=c_ht+X_2 \tag{3.37}$$

$$\theta(t)=\int\dot{\theta}(t)dt=X_3-c_ht \tag{3.38}$$

$$\varphi_{11}(t)=\int\dot{\varphi}_{11}(t)dt=\lambda c_rt+\frac{\lambda c_ht^2}{2}-\lambda X_3t+X_4 \tag{3.39}$$

$$\varphi_{12}(t)=\int\dot{\varphi}_{12}(t)dt=X_5 \tag{3.40}$$

The expressions of $q_\nu(t)$, $q_\Omega(t)$, and $p(t)$ are updated using the above equations:

$$q_\nu(t) = \frac{\psi}{c_{m1}} \tag{3.41}$$

$$q_\Omega(t) = \frac{\xi}{c_{m1}k_\Omega} \tag{3.42}$$

$$p_{11}(t) = \frac{d_1 + d_3w + d_2c_p - \varphi_{11}d_2}{2d_2} \tag{3.43}$$

$$p_{12}(t) = \frac{d_1 + d_3w + d_2c_p - \varphi_{12}d_2}{2d_2} \tag{3.44}$$

Updated expressions for $q_\nu(t)$ and $q_\Omega(t)$ respectively are obtained from Eqs. (3.41) & (3.42):

$$q_\nu(t) = \frac{c_ht + X_1}{c_{m1}} \tag{3.45}$$

$$q_\Omega(t) = \frac{c_ht + X_2}{c_{m1}k_\Omega} \tag{3.46}$$

By setting $q_\nu(0)$=0, from the initial conditions we can determine that X_1=0. Since remanufacturing only commences at time γT, to determine X_2, we set $q_\Omega(\gamma T)$=0:

$$X_2 = -c_h\gamma T \tag{3.47}$$

This leads to:

$$q_\nu(t) = \frac{c_ht}{c_{m1}} \tag{3.48}$$

$$q_\Omega(t) = \frac{c_ht - c_h\gamma T}{c_{m1}k_\Omega} \tag{3.49}$$

Now, taking the integrals of $q_\nu(t)$ and $q_\Omega(t)$, we obtain the expressions for $Q_\nu(t)$ and $Q_\Omega(t)$ respectively for the first stage:

$$Q_\nu(t) = \int q_\nu(t)dt = \frac{c_h t^2}{2c_{m1}} + X_7 \tag{3.50}$$

$$Q_\Omega(t) = \int q_\Omega(t)dt = \frac{c_h t^2}{2c_{m1}k_\Omega} - \frac{c_h \gamma T t}{c_{m1}k_\Omega} + X_8 \tag{3.51}$$

By evaluating $Q_\nu(0)$=0, we can determine that X_7=0. Since remanufacturing only commences at time γT, to determine X_8, we set $Q_\Omega(\gamma T)$=0. Thus,

$$X_8 = \frac{c_h \gamma^2 T^2}{2c_{m1}k_\Omega} \tag{3.52}$$

Therefore the final equations for $Q_\nu(t)$ and $Q_\Omega(t)$ for the first stage are:

$$Q_\nu(t) = \frac{c_h t^2}{2c_{m1}} \tag{3.53}$$

$$Q_\Omega(t) = \frac{c_h t^2}{2c_{m1}k_\Omega} - \frac{c_h \gamma T t}{c_{m1}k_\Omega} + \frac{c_h \gamma^2 T^2}{2c_{m1}k_\Omega} \tag{3.54}$$

We obtain the expressions for $p_{11}(t)$ and $p_{12}(t)$ by taking Eqs. (3.43) & (3.44) and combine them with Eqs. (3.39) & (3.40) respectively:

$$p_{11}(t) = \frac{d_1 + d_3 w}{2d_2} + \frac{c_p - \lambda c_r t + \lambda X_3 t - X_4}{2} - \frac{\lambda c_h t^2}{4} \tag{3.55}$$

$$p_{12}(t) = \frac{d_1 + d_3 w}{2d_2} + \frac{c_p - X_5}{2} \tag{3.56}$$

To determine the integration constants X_4 and X_5, we set $\varphi_{11}(w)$=$\varphi_{12}(w)$ to provide a relationship for X_4, and evaluate Eq. (3.35), $\varphi_{12}(T)$=0, to yield X_5=0:

$$X_4 = \lambda X_3 w - \lambda c_r w - \frac{\lambda c_h w^2}{2} \tag{3.57}$$

Therefore:

$$p_{11}(t) = \frac{d_1 + d_3 w}{2d_2} + \frac{c_p + \lambda c_r(w-t) + \lambda X_3(t-w)}{2} + \frac{\lambda c_h(w^2 - t^2)}{4} \tag{3.58}$$

$$p_{12}(t) = \frac{d_1 + d_3 w}{2d_2} + \frac{c_p}{2} \tag{3.59}$$

Eq. (3.19) is updated with Eqs. (3.58) & (3.59) to get:

$$d(t) = \begin{cases} \frac{1}{2}\left[d_1 + d_3 w - d_2 c_p\right] - \frac{d_2 \lambda}{2}\left[c_r(w-t) + X_3(t-w)\right] - \frac{d_2\lambda}{4} c_h (w^2 - t^2) & \text{if } 0 \leq t \leq w \\ \frac{1}{2}\left[d_1 + d_3 w - d_2 c_p\right] & \text{if } w \leq t \leq T \\ 0 & \text{if } t > T \end{cases} \tag{3.60}$$

$D_{12}(t)$ is then determined by taking the integral of $d_{12}(t)$:

$$D_{12}(t) = \int d_{12}(t)dt = \frac{d_1 t + d_3 w t - d_2 c_p t}{2} + X_9 \tag{3.61}$$

To solve for X_9, we consider the final component of the objective function, the lump sum profit the company garners from the total sales during the first stage $s_1 D(T) - s_2 D(T)^2$. Since this value is a constant, its derivative is equal to zero:

$$s_1 - 2s_2 D(T) = 0 \tag{3.62}$$

Now combining Eq. (3.62) with the fact that $D(T) = D_{12}(T)$, we get:

$$s_1 = 2s_2 \left[\frac{d_1 T + d_3 w T - d_2 c_p T}{2} + X_9\right] \tag{3.63}$$

which yields:

$$X_9 = \frac{s_1}{2s_2} + \frac{d_2 c_p T - d_1 T - d_3 w T}{2} \tag{3.64}$$

Therefore:

$$D_{12}(t) = (d_1 + d_3w)(t - T) + \frac{(d_1 + d_3w + d_2c_p)(T - t)}{2} + \frac{s_1}{2s_2} \tag{3.65}$$

We now integrate $d_{11}(t)$ to get $D_{11}(t)$:

$$D_{11}(t) = \int d_{11}(t)dt$$

$$\begin{aligned} D_{11}(t) =& \frac{d_2\lambda c_h t^3}{12} + \frac{(c_r - X_3)d_2\lambda t^2}{4} + \frac{(X_3 - c_r)d_2\lambda wt}{2} \\ & - \frac{d_2\lambda c_h w^2 t}{4} + \frac{(d_1 - d_2c_p + d_3w)t}{2} + X_{10} \end{aligned} \tag{3.66}$$

Setting $D_{11}(0)$=0 yields X_{10}=0. Since $D_{11}(t)$ and $D_{12}(t)$ are continuous, $D_{11}(w)$ must be equal to $D_{12}(w)$. This condition is used to isolate and solve for X_3:

$$X_3 = c_r + \frac{2wc_h}{3} + \frac{2Tc_p}{\lambda w^2} - \frac{2T(d_1 + d_3)}{d_2\lambda w^2} + \frac{2s_1}{s_2d_2\lambda w^2} \tag{3.67}$$

Stage 2 (From $T \le t \le T + w$)

The Hamiltonian for this second stage is shown in Eq. (3.68) below:

$$\begin{aligned} \mathcal{H}_2 = & -\frac{c_{m2}q_\nu(t)^2}{2} - \frac{c_{m2}k_\Omega q_\Omega(t)^2}{2} - c_h\left[Q_\nu(t) + Q_\Omega(t) - F(t)\right] - c_r f(t) \\ & + q_\nu(t)\psi + q_\Omega(t)\xi + \left[Q_\nu(t) + Q_\Omega(t) - F(t)\right]\zeta + f(t)\theta \end{aligned} \tag{3.68}$$

The equations of motion for $q_\nu(t)$ and $q_\Omega(t)$ are realized from:

$$\frac{\partial \mathcal{H}_2}{\partial q_\nu} = \psi - c_{m2}q_\nu(t) = 0 \tag{3.69}$$

$$\frac{\partial \mathcal{H}_2}{\partial q_\Omega} = \xi - c_{m2}k_\Omega q_\Omega(t) = 0 \tag{3.70}$$

The equations of motion for $\dot{\psi}(t)$ and $\dot{\xi}(t)$ are:

$$\dot{\psi}(t) = -\frac{\partial \mathcal{H}}{\partial Q_\nu} = c_h - \zeta \tag{3.71}$$

$$\dot{\xi}(t) = -\frac{\partial \mathcal{H}}{\partial Q_\Omega} = c_h - \zeta \tag{3.72}$$

For the same reason as in Stage 1 ($c_{m1}<c_{m2}$), ζ is set=0. Then,

$$\psi(t) = \int \dot{\psi}(t)dt = c_h t + X_{11} \tag{3.73}$$

$$\xi(t) = \int \dot{\xi}(t)dt = c_h t + X_{12} \tag{3.74}$$

which combined with Eqs. (3.69) & (3.70) yield:

$$q_\nu(t) = \frac{\psi}{c_{m2}} = \frac{c_h t + X_{11}}{c_{m2}} \tag{3.75}$$

$$q_\Omega(t) = \frac{\xi}{c_{m2}} = \frac{c_h t + X_{12}}{c_{m2} k_\Omega} \tag{3.76}$$

Since $q_\nu(t)$ and $q_\Omega(t)$ must be continuous, $q_\nu(T)$ and $q_\Omega(T)$ from Stage 1 must be equal to $q_\nu(T)$ and $q_\Omega(T)$ from Stage 2. Using this condition, we obtain:

$$q_\nu(T) = \frac{c_h T + X_{11}}{c_{m2}} = \frac{c_h T}{c_{m1}} \tag{3.77}$$

$$q_\Omega(T) = \frac{c_h T + X_{12}}{c_{m2} k_\Omega} = \frac{c_h t - c_h \gamma T}{c_{m1} k_\Omega} \tag{3.78}$$

Thus:

$$X_{11} = c_h T \left(\frac{c_{m2}}{c_{m1}} - 1 \right) \tag{3.79}$$

$$X_{12} = c_h T \left(\frac{c_{m2}}{c_{m1}} - \frac{\gamma c_{m2}}{c_{m1}} - 1 \right) \tag{3.80}$$

Therefore:

$$q_\nu(t) = \frac{c_h t - c_h T}{c_{m2}} + \frac{c_h T}{c_{m1}} \tag{3.81}$$

$$q_\Omega(t) = \frac{c_h t - c_h T}{c_{m2} k_\Omega} + \frac{c_h T - c_h \gamma T}{c_{m1} k_\Omega} \tag{3.82}$$

Now, taking the integrals of $q_\nu(t)$ and $q_\Omega(t)$, we obtain the expressions for $Q_\nu(t)$ and $Q_\Omega(t)$ respectively for the second stage:

$$Q_\nu(t) = \int q_\nu(t) = \frac{c_h t^2}{2c_{m2}} - \frac{c_h T t}{c_{m2}} + \frac{c_h T t}{c_{m1}} + X_{13} \tag{3.83}$$

$$Q_\Omega(t) = \int q_\Omega(t) = \frac{c_h t^2}{2c_{m2} k_\Omega} - \frac{c_h T t}{c_{m2} k_\Omega} + \frac{c_h T t}{c_{m1} k_\Omega} - \frac{c_h \gamma T t}{c_{m1} k_\Omega} + X_{14} \tag{3.84}$$

Since $Q_\nu(t)$ and $Q_\Omega(t)$ must be continuous, $Q_\nu(T)$ and $Q_\Omega(T)$ from Stage 1 must be equal to $Q_\nu(T)$ and $Q_\Omega(T)$ from Stage 2. Using this condition, we obtain:

$$\frac{c_h T^2}{2c_{m2}} - \frac{c_h T^2}{c_{m2}} + \frac{c_h T^2}{c_{m1}} + X_{13} = \frac{c_h T^2}{2c_{m1}} \tag{3.85}$$

$$\frac{c_h T^2}{2c_{m2} k_\Omega} - \frac{c_h T^2}{c_{m2} k_\Omega} + \frac{c_h T^2}{c_{m1} k_\Omega} - \frac{h \gamma T^2}{c_{m1} k_\Omega} + X_{14} = \frac{c_h t^2}{2c_{m1} k_\Omega} - \frac{c_h \gamma T t}{c_{m1} k_\Omega} + \frac{c_h \gamma^2 T^2}{2c_{m1} k_\Omega} \tag{3.86}$$

Thus:

$$X_{13} = \frac{c_h T^2}{2c_{m2}} - \frac{c_h T^2}{2c_{m1}} \tag{3.87}$$

$$X_{14} = \frac{c_h T^2}{2c_{m2} k_\Omega} + \frac{c_h \gamma^2 T^2 - c_h T^2}{2c_{m1} k_\Omega} \tag{3.88}$$

Therefore:

$$Q_\nu(t) = \frac{c_h t^2 - 2c_h T t + c_h T^2}{2c_{m2}} - \frac{2c_h T t - c_h T^2}{2c_{m1}} \tag{3.89}$$

$$Q_\Omega(t) = \frac{c_h t^2 - 2c_h T t + c_h T^2}{2c_{m2} k_\Omega} + \frac{2c_h T t - 2c_h \gamma T t + c_h \gamma^2 T^2 - c_h T^2}{2c_{m1} k_\Omega} \tag{3.90}$$

According to Mangasarian (1966), "Pontryagin's maximum principle furnishes necessary conditions for the optimality of the control of a dynamic system". The theorem below proves the optimality of the derived equations.

Theorem 1. *The solutions that satisfy the necessary conditions are optimal.*

Proof. Concavity of all control variables $p(t)$, $q_\nu(t)$, and $q_\Omega(t)$ is shown through partial twice-differentiation of the objective function Eq. (3.12):

Stage 1:

$$\frac{\partial \Pi_1^2}{\partial^2 p} = -2d_2 < 0 \tag{3.91}$$

$$\frac{\partial \Pi_1^2}{\partial^2 q_\nu} = -c_{m1} < 0 \tag{3.92}$$

$$\frac{\partial \Pi_1^2}{\partial^2 q_\Omega} = -k_\Omega c_{m1} < 0 \tag{3.93}$$

Stage 2:

$$\frac{\partial \Pi_2^2}{\partial^2 p} = 0 \tag{3.94}$$

$$\frac{\partial \Pi_2^2}{\partial^2 q_\nu} = -c_{m2} < 0 \tag{3.95}$$

$$\frac{\partial \Pi_2^2}{\partial^2 q_\Omega} = -k_\Omega c_{m2} < 0 \tag{3.96}$$

Eqs. (3.91)-(3.96) prove that the objective function is concave in $p(t)$, $q_\nu(t)$, and $q_\Omega(t)$. Twice differentiating the constraints results in all terms equaling zero, demonstrating that the constraints are linear in $p(t)$, $q_\nu(t)$, and $q_\Omega(t)$.

$\therefore$ Since the objective is concave and the constraints are linear, the optimum is global and the necessary conditions are also sufficient conditions for optimality. $\square$

3.3.2 Results & Discussion

The model developed above is solved using data from an anonymous regional airline for a rotable spare part. This spare part is the Electronic Horizontal Situation Indicator / Electronic Attitude Directional Indicator (EHSI/EADI) Display. The data obtained from the company has been anonymized, but the proportions have been preserved. This model examines three key factors in the remanufacturing process: the length of the production cycle (T), the warranty period (w), and when to commence remanufacturing (γ). Using the values shown in Table 3.2, the optimal control theory model is solved using *Maplesoft Maple 18*.

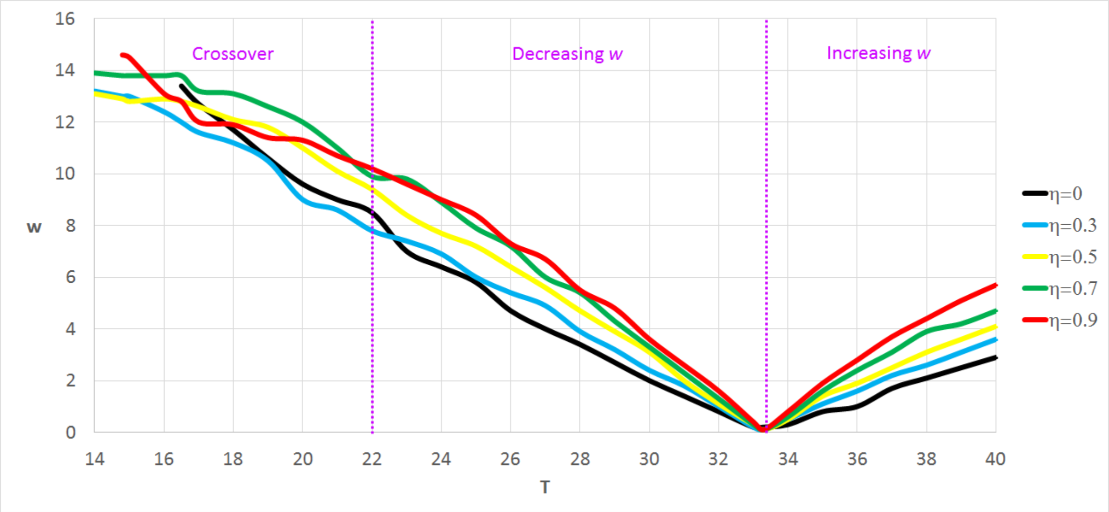
Table 3.2: Parameters Used in Numerical Computations

Parameter	λ	c_p	c_{m1}	c_{m2}	c_h	c_r	d_1	d_2	d_3	δ_1	δ_2	s_1	s_2
Value	0.05	4	1	2	0.5	2	10	1	0.1	0.8	0.4	100	0.5

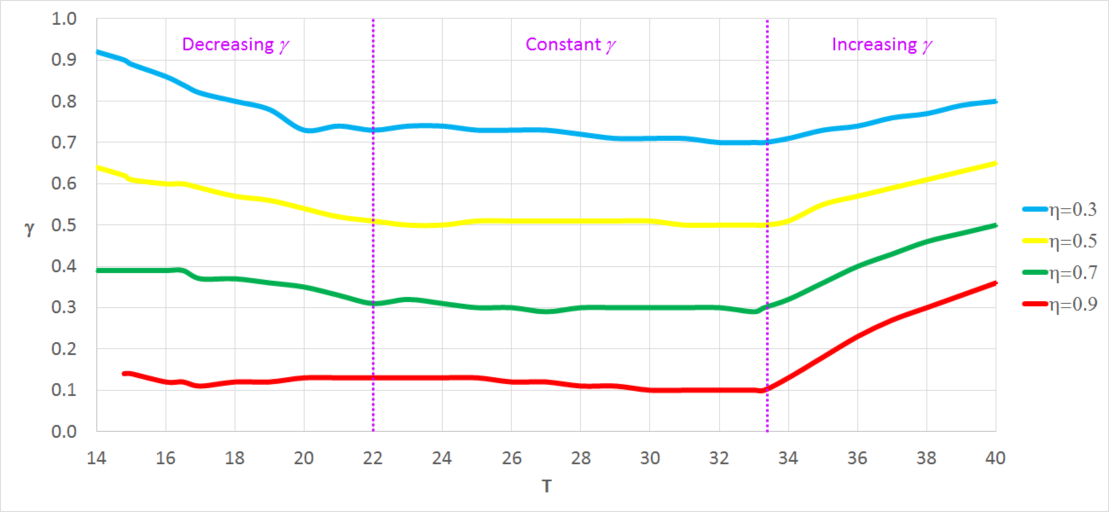
Several experiments are conducted below to study the dynamic relationships between the decision variables and key parameters.

- **The optimal (T,w,γ) for a given η:**

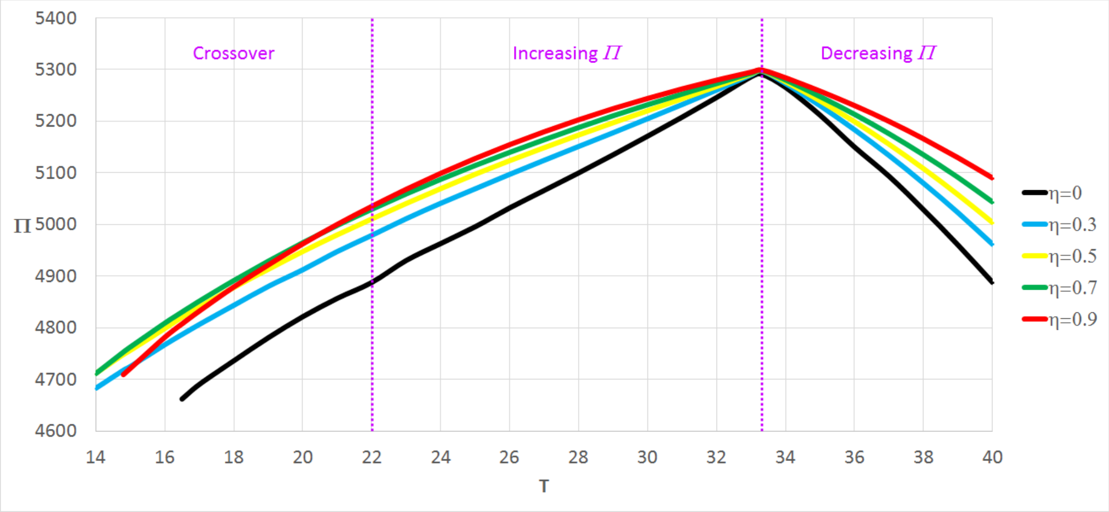
The objective function is maximized for profitability, however it is constrained by Eq. (3.13), which ensures that there are enough spare parts at any given time to repair failed products. Figure 3.8 displays the relationship between the three decision variables: T, w, and γ, and Table 3.3 summarizes the optimal values of T^*, w^*, and γ^* for a given η.



(a) Optimal w-T Pairs



(b) Optimal γ-T Pairs



(c) Optimal Profitability over T

Figure 3.8: Optimal Decision Variables

Figure 3.8a depicts w as the value of T rises, with findings similar to those by Kim and Park (2008). We observe w decreasing to its minima, also where the maximum value of Π is attained, and then starts to increase once again. At $T \approx 18.5$, we notice that $\eta=0$ crosses the curves of $\eta=0.5$, $\eta=0.7$, and $\eta=0.9$. This demonstrates the necessity of having longer warranty periods when no remanufacturable products are available. With no remanufacturing, the firm must charge a higher price for the product, and thus increase the warranty length to sustain demand.

In Figure 3.8b, there appears to be three regions based on the value of γ: decreasing, constant, and increasing. In the decreasing region, the values of γ are higher to defer the start of remanufacturing in order to account for the short production cycle. In the constant region, the value of γ appears to be steady, transitioning to the increasing region after the optimum value of T is reached. Finally, γ increases to compensate for the rising warranty length for very long production cycles. Here, the start of the remanufacturing process is delayed so parts can be produced at a more affordable price at the end of the first stage.

Figure 3.8c illustrates the value of Π as T increases. At $T=15$, the $\eta=0.9$ curve is the least profitable in comparison to all of the other curves where reconditioned components are used. However, by $T \approx 21$, the $\eta=0.9$ curve becomes the most profitable. This informs us that in a short production cycle, an abundance of remanufacturable products may also result in higher costs of production. In this scenario, as the system starts remanufacturing earlier, it produces and stores reconditioned spare parts, which incurs higher inventory holding costs. From this point, Π continues to increase to its optimal value at $T=33.3$, from which it starts to decrease.

Additionally, at time $t=0$ there are not any legacy products in the market. This means that there is also no remanufacturable material (from a prior generation) available. Because products typically tend not to fail so rapidly, it takes some time for products the from current generation to become accessible in order to remanufacture. The availability of failed products contrasts with the model's preference to produce products earlier to exploit economies of scale.

Table 3.3: Optimal Values for a Given η

η	T^*	w^*	γ^*	Π^*
0.0	33.3	0.2	–	5291.28
0.3	33.3	0.1	0.70	5297.41
0.5	33.3	0.1	0.50	5298.14
0.7	33.3	0.1	0.30	5298.69
0.9	33.3	0.1	0.10	5299.27

In Table 3.3, we witness a notably short warranty length, an earlier start to begin remanufacturing (γ^*), and a rise in Π^* as η is increased. While it is quite acceptable to have a long production cycle, competitive forces within industry and/or government regulations may force firms to have a minimum warranty length.

- **The optimal γ for a given w:**

In Table 3.4, we vary the values of w at the value of T=33.3 to mimic warranty lengths offered in practice. For example, many consumer electronics come with a 36-month lifetime and 12-month warranty period, corresponding to T=36 and w=12.

Table 3.4: Optimal Values when T=33.3, η=0.7, and w is varied

w	γ^*	Π^*
3.0	0.36	5260.31
6.0	0.42	5215.14
9.0	0.48	5164.84
12.0	0.54	5111.59
15.0	0.59	5053.78
18.0	0.64	4998.19
21.0	0.67	4935.55
24.0	0.70	4881.32
27.0	0.71	4820.52
30.0	0.72	4770.53

As expected, Π^* decreases as the value of w increases, since larger warranty lengths increase the probability of servicing a failure. An interesting observation is that γ^* also increases with w, meaning that remanufacturing starts later when the warranty period is increased.

- **The optimal (T,γ) for a given w:**

In the previous analysis, we fixed the production cycle value at T=33.3. Now, we will concurrently optimize for both T and γ by varying w:

Table 3.5: Optimal Values when η=0.7, and w is varied

w	T^*	γ^*	$\gamma^* T^*$	Π^*
3.0	32.6	0.35	11.41	5262.67
6.0	31.3	0.39	12.21	5222.76
9.0	30.6	0.44	13.46	5180.42
12.0	28.4	0.46	13.06	5129.87
15.0	27.4	0.50	13.70	5077.96
18.0	26.8	0.54	14.47	5022.39
21.0	26.7	0.59	15.75	4972.13
24.0	26.8	0.63	16.88	4919.36
27.0	27.2	0.66	17.95	4864.85
30.0	30.4	0.70	21.28	4810.77

The results in Table 3.5 show γ^* increasing, the combined value of $\gamma^* T^*$ increasing, and Π^* decreasing as the value of w increases. We also observe T^* decreases to a minimum value at w=21 and starts to increase once again.

- **The optimal (T,γ) for a given λ:**

In Table 3.6, we vary the values of the failure rate λ (evaluated at w=12, η=0.7), to determine its effect on the decision variables.

Table 3.6: Optimal Values when w=12, η=0.7, and λ is varied

λ	T^*	γ^*	Π^*
0.025	30.7	0.53	5256.50
0.035	30.8	0.52	5206.93
0.045	30.3	0.50	5158.42
0.050	30.1	0.49	5134.14
0.060	29.6	0.47	5086.68
0.075	29.1	0.44	5013.94
0.100	29.3	0.41	4889.43
0.150	29.5	0.34	4609.06

In Table 3.6 we witness that as λ is increased, the values of T^*, γ^*, and Π^* decrease. Increasing the failure rate causes more failures, growing the need for a greater number of spare parts, adding more costs, and reducing profit. This requirement for more spare parts forces remanufacturing to be started earlier.

- **The optimal (T,γ) for a given c_h:**

In Table 3.7 we vary the values of the inventory holding cost c_h (evaluated at w=12, η=0.7), to determine its effect on the decision variables.

Table 3.7: Optimal Values when w=12, η=0.7, and c_h is varied

c_h	T^*	γ^*	Π^*
0.25	31.1	0.45	5199.24
0.33	30.6	0.47	5179.10
0.50	30.1	0.49	5134.14
0.75	29.0	0.49	5067.80
1.00	28.5	0.49	5001.36
1.50	27.2	0.48	4872.96
2.00	26.0	0.47	4749.62
2.50	23.4	0.46	4635.83

In Table 3.7, as c_h is increased, the values of T^* and Π^* decrease. The behaviour of γ is interesting as it appears to initially increase, and then subsequently starts to decrease. Increasing the holding cost reduces the length of the production cycle and starts remanufacturing earlier to limit the quantity of spare parts in inventory.

In the figures below, two new terms have been appended: $Q(t)$ and $W(t)$. $Q(t)$ is the combined production of all spare parts, where $Q(t)$=$Q_\nu(t)$+$Q_\Omega(t)$. $W(t)$ represents the quantity of products covered under the warranty policy, and can be calculated as the difference between the cumulative sales and the volume of products no longer covered by the warranty, $W(t)$=$D(t)$−$V(t)$.

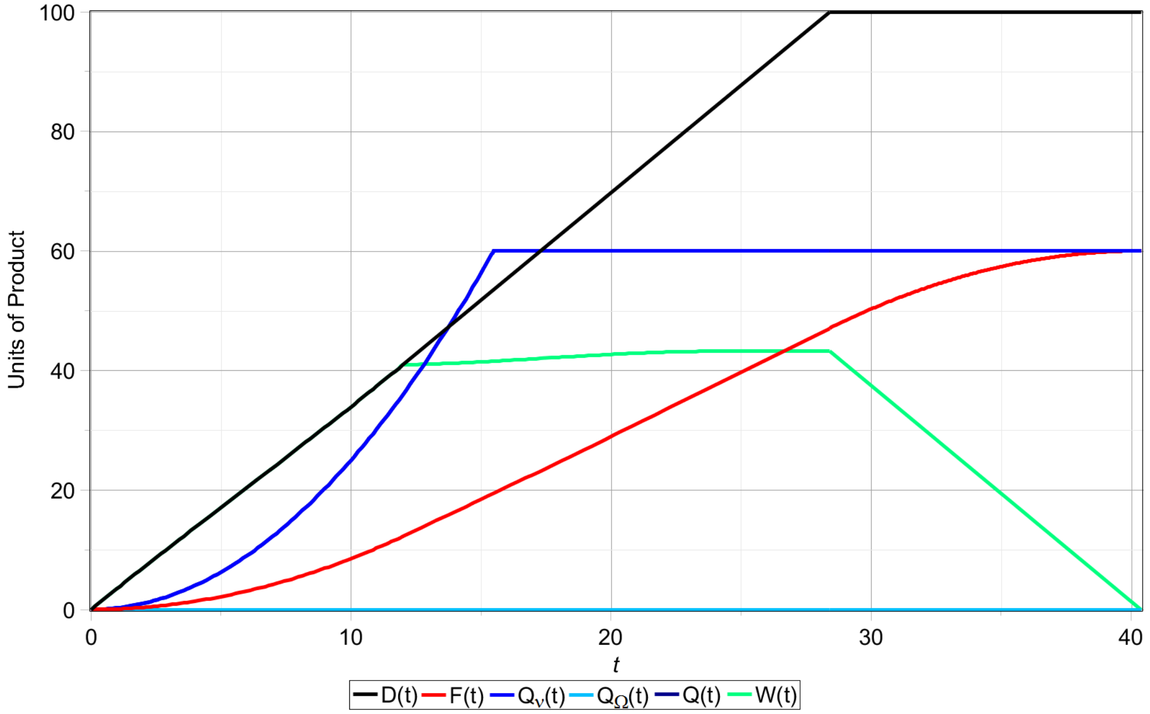


Figure 3.9: Dynamics with No Remanufacturing

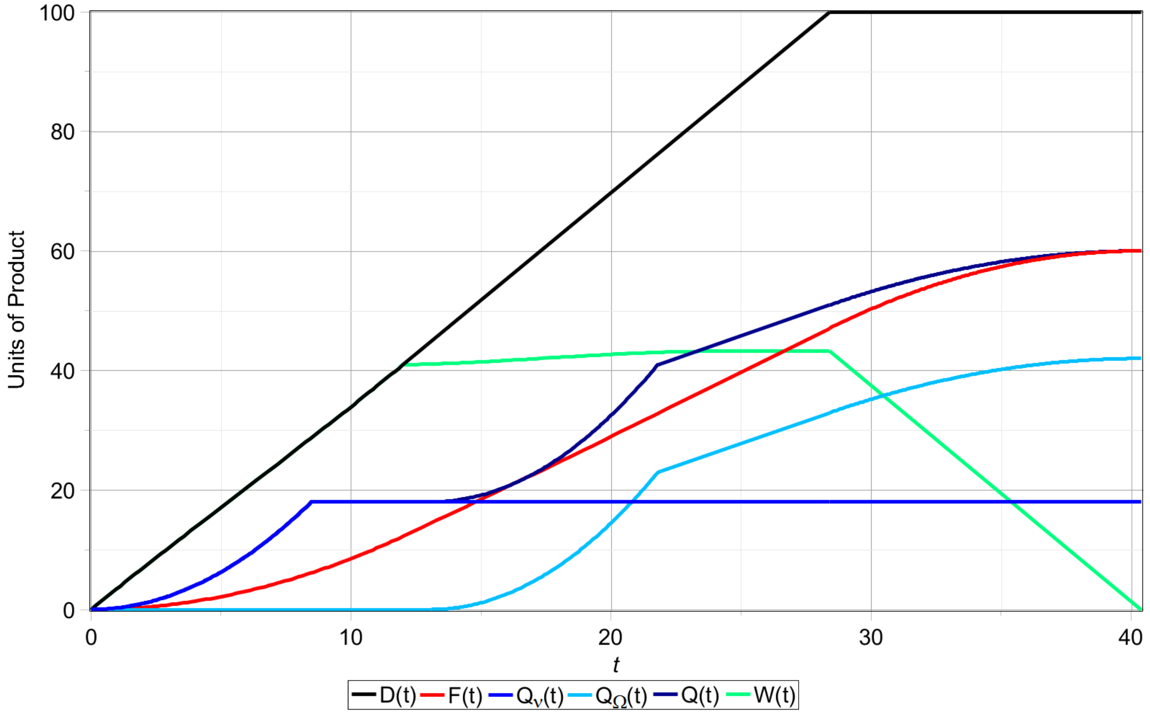


Figure 3.10: Dynamics with Remanufacturing

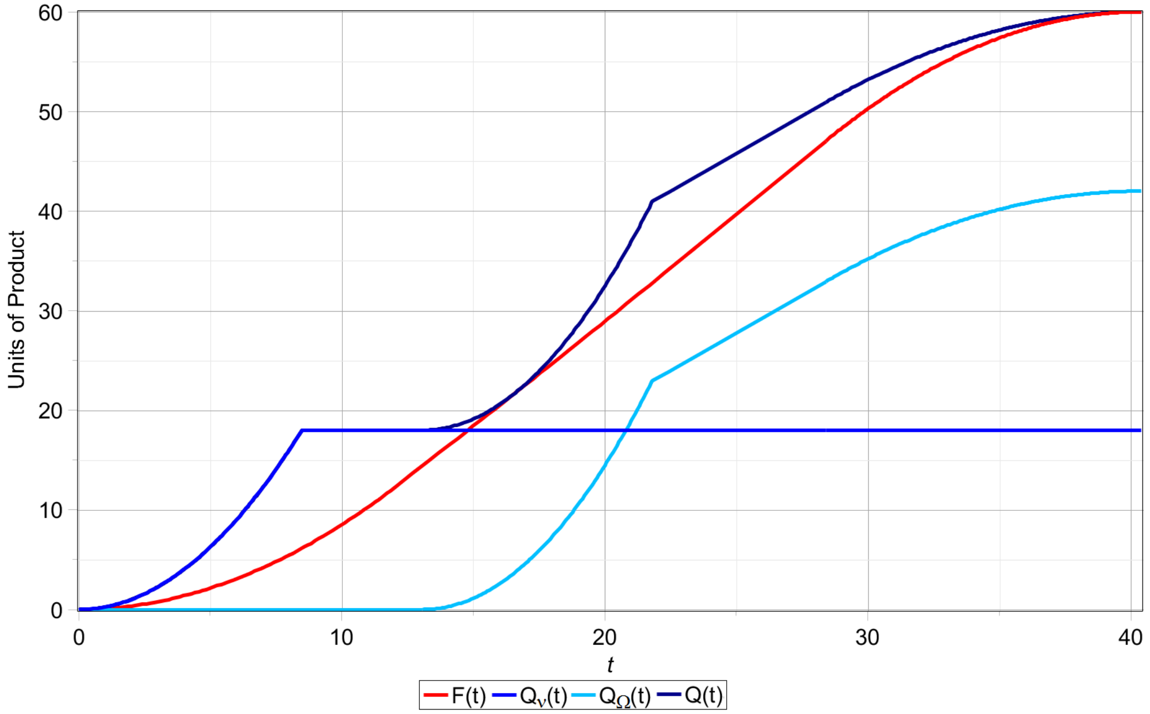


Figure 3.11: Production and Failure Dynamics with Remanufacturing

In Figure 3.9, the dynamics are displayed for the scenario where remanufacturing does not take place, and T=28.4, w=12, and η=0. Figures 3.10 & 3.11 depict the production dynamics of a scenario where there is remanufacturing. In this solution, T=28.4, w=12, γ=0.46, and η=0.7. Figure 3.10 displays all of the control variables, while Figure 3.11 illustrates just the production variables $Q_\nu(t)$, $Q_\Omega(t)$, and $Q(t)$; as well as the cumulative number of failures, $F(t)$.

Summary of Findings

The length of warranty period (w) offered for a product is closely related to the duration of its production cycle (T). Longer warranties amplify the funds that must be kept in reserve to honour the terms of the policy. Figures 3.8a & 3.8b establish that for each value of T, there is a single value of w and γ paired with it respectively. Therefore, triplets of T, w, and γ compose the optimal values of Π shown in Figure 3.8c.

The results from the numerical experiments conducted demonstrate that the model behaves as expected and yields conclusions in line with prior knowledge from optimal control theory. Furthermore, the model shows how an optimal combination of T, w, and γ can yield substantially higher profits in the context of remanufacturing.

The sensitivity analyses showed that:

- When increasing η, we see a decrease in γ^*, and typically an increase in Π^*. With a greater volume of remanufacturable components available, the system starts remanufacturing earlier. In a smaller production cycle, it can actually be less profitable to have a large value of η, as the system will produce and store reconditioned spare parts in inventory.
- When increasing the value of w, we see an increase in γ^*, and a decrease in Π^*. If the value of T is not fixed, we also see a decrease in T^* to a minimum value, before it starts to increase once again. However, an increase in the value of $\gamma^* T^*$ is maintained throughout. With fixed values of w, the model elects to decrease the length of the production cycle, and starts remanufacturing later.
- When increasing the failure rate λ, the values of T^*, γ^*, and Π^* decrease. Increasing the failure rate cause more failures, and therefore the need more for spare parts needed grows. This causes remanufacturing to be started prematurely, adds more costs, and reduces profit.
- When increasing the inventory holding costs c_h, the values of T^* and Π^* decrease. The behaviour of γ is interesting as it appears to initially increase, and then subsequently starts decreasing. Increasing the holding costs reduces the length of the production cycle and starts remanufacturing expeditiously to limit the quantity of spare parts in inventory.

3.4 Conclusions & Future Research

A two-stage optimal control theory model was formulated to determine the best remanufacturing planning strategy for a firm producing new and reconditioned spare parts to support the warranted replacements/repairs of a product. This product is sold with a fixed length NFRW policy. The demand for the product was modelled to be proportional to the length of the warranty period to translate to the perception of better reliability through a longer warranty. The model obtained was solved numerically using *Maplesoft Maple 18*, and yielded valid decision parameters that were discussed and explained. It demonstrated how an appropriate warranty policy and associated production decisions can make reconditioned products attractive from both economic and environmental perspectives.

Areas of future exploration include: using reconditioned components in the primary production in addition to them being spare parts, considering multiple quality grades of reconditioned components, and developing/solving a stochastic version of the problem where the availability of EoU/EoL products and their quality grades can be random. Simulation is also an approach to solving the stochastic version of the problem where the age of available EoU/EoL products is a random variable. The current model assumes a constant failure rate, which is true for electronics as in the case study. For other types of components, the failure rate can be non-constant. Therefore, another extension will be to derive a model for time dependent failure rates.

In this chapter, we addressed three questions: what are the economic benefits of remanufacturing activities, when should remanufacturing activities be started, and how many new and reconditioned spare parts should be produced. The warranty policy was used as a mechanism by the firm to grow the demand for its product. It is assumed that such policies exist. In the next chapter we will investigate how optimal warranty policies can be determined in the context of remanufacturing operations.

CHAPTER 4

WARRANTY POLICIES FOR REMANUFACTURING

A product can continue to have utility after the end of its traditional life-cycle through different remanufacturing processes. This extended use is often referred to as the product's second-life, making it both a sustainable and financial opportunity for the manufacturer. Reconditioning is one remanufacturing operation type that can be used to produce replacement parts for products under warranty. As the integration of reconditioned components in remanufactured/reprocessed/refreshed/second-hand products becomes widespread, manufacturers are increasingly required to provide lower prices and better support to consumers. Getting a satisfactory warranty coverage has proven to be a very important criterion for consumers acquiring new products to protect themselves against poor performance. For consumers pondering the purchase of remanufactured products, the warranty coverage and the discounted sale price are key decision factors, because these products have higher failure rates than new ones. Manufacturers who engage in the sale of remanufactured products or parts must carefully define their warranty offerings to encourage sales and minimize the cost of honouring such warranty policies. These competing objectives can be resolved by the development of mathematical models to determine optimal warranty policies.

This chapter is devoted to the modelling of warranty policies for remanufactured products, and is organized as follows. The first section presents elementary definitions of reliability functions, warranty policies, their classification, and related issues. The second section is devoted to a review of warranty models for remanufactured products. In the third section, we develop a new mathematical model to determine optimal warranty strategies for a manufacturer who uses a mix of new and reconditioned components to fulfill warranty claims. Finally, a discussion of results and future areas of research conclude this chapter.

4.1 Definitions, Classification, and General Issues

As stated by Shafiee, Chukova, Saidi-Mehrabad, and Niaki (2011), “a product warranty is an agreement offered by the manufacturer/dealer to a consumer to repair/replace a faulty item, or to partially or fully reimburse the consumer in the event of product failure during the warranty period”. In effect, this is a contract created between the manufacturer and buyer to establish liability upon failure. As described by Murthy and Djamaludin (2002):

> The use of warranties is widespread and they serve many purposes. These include protection for manufacturer and buyer, signalling of product quality, an important element of marketing strategy, assuring buyers against items which do not perform as promised and play an important role in the dispute resolution between buyer and manufacturer.

From the above definition, many authors focus on the two aspects of protection and promotion (Murthy and Blischke, 2000; Shafiee and Chukova, 2013a). For customers, protection is against product failure due to faulty design, manufacturing defects, etc. Whereas manufacturers are protected against product misuse by consumers, illegal claims, etc. (Shafiee et al., 2011). In addition to manufacturers and customers, others that are impacted by the role of warranty are: legislators, courts, consumer affairs groups, and public policy decision-makers. These stakeholders are collectively referred to as society. All three parties (customers, manufacturers, and society) look to warranty for protection in one form or another. In addition, manufacturers also use warranty as a marketing tool by insinuating that a product with a longer warranty period is more reliable.

A warranty policy is “a statement on the extent of the warranty coverage and the type of compensation provided to the customers in the case of faulty product” (Shafiee et al., 2011). It is of the utmost importance to properly determine the terms of warranty policies, as the cost to comply with them can be very large. According to McGuire (1980), the future costs associated with warranty claims can be anywhere from 2-15% of net sales. In the example provided by Manna, Pal, and Sinha (2007), American manufacturers annually spend over $25 billion on warranty services. The automotive industry in particular, annually spends \$15 billion to service warranty repairs in the United States alone (Yang, 2010).

Due to these high costs, the role of warranty is considered as one of the best ways to convey product quality to consumers (Blischke and Murthy, 1993; Noll, 2004; Anityasari, Kaebernick, and Kara, 2007). Since the costs are dependent on product reliability and warranty terms, for a manufacturer to offer a superior warranty in comparison to a competitor, they need to ensure that the reliability of the product must also be better (Spence, 1977). Therefore, before developing a warranty model, it is crucial to determine the reliability of the concerned product.

4.1.1 Reliability Function

The cost of honouring warranty agreements can be very high. Therefore manufacturers have to carefully evaluate the risk of failure of their products through reliability assessment programs before determining the warranty coverage to offer. The most basic warranty model is determined by the mission length w that ensures that the proportion of failures for a product does not exceed a given pre-determined proportion p. The warranty length w to be offered is then the solution of $R(w) \geq 1-p$, where $R(t)$ is the reliability of the product. According to reliability theory, the reliability is the probability that a system will perform its intended function satisfactorily for a specified period of time under specified operating conditions. Manufacturers typically conduct accelerated life testing experiments to obtain the failure data of their products. This data can be fit to a lifetime probability density function (pdf), $f(.)$, which can used to calculate the reliability function $R(t)$ and other characteristics:

$$R(t) = \int_t^\infty f(x)dx \tag{4.1}$$

The failure or hazard rate function is:

$$h(t) = \frac{f(t)}{R(t)} \tag{4.2}$$

Many probability distributions can be used to represent the lifetime pdf of systems. The Weibull distribution is the most widely used in the literature, and it consists of three important parameters: the scale parameter (θ), the shape parameter (β), and the location parameter (ζ). The pdf, cumulative density function (cdf), reliability, and failure rate functions are given in Eqs. (4.3)-(4.6):

$$f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^{\beta}} \tag{4.3}$$

$$F(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^{\beta}} \tag{4.4}$$

$$R(t) = 1 - F(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}} \tag{4.5}$$

$$h(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \tag{4.6}$$

As witnessed in Eq. 4.6, as the value of β changes, the failure rate function, $h(t)$, takes different shapes:

- For $\beta < 1$ the failure rate is decreasing,
- For $\beta = 1$ the failure rate is constant, and
- For $\beta > 1$ the failure rate is increasing.

These three stages are found in the same sequence in the bathtub curve, which represents the reliability of most products throughout their life. Several researchers, including Xie, Tang, and Goh (2002), have proposed extensions to the Weibull distribution. Hjorth (1980) formulates the IDB distribution which is composed of increasing, decreasing, constant, and bathtub-shaped failure rates.

Once the reliability function of the system under consideration is known, a warranty policy can be formulated and modelled. The optimal warranty length and other design parameters will seek to minimize the proportion of failures, hence reducing the financial burden to the firm. The next section will present a taxonomy of warranty policies.

4.1.2 Taxonomy of Warranty Policies

Many papers have dealt with the modelling of warranty policies and their associated costs for new products (Nguyen and Murthy, 1986; Mamer, 1987; Nguyen and Murthy, 1989; Chukova, Dimitrov, and Rykov, 1993; Blischke and Murthy, 1993; Murthy and Djamaludin, 2002; Karim and Suzuki, 2005). Warranty models are typically mathematical expressions of the total cost to run the warranty policy that is being offered. These functions include the additional costs of addressing the failed item, and can be expressed as warranty costs per unit sale, per unit time, over the lifetime of the item, or over the product life-cycle (Blischke and Murthy, 1993; Chattopadhyay and Murthy, 2000). Most warranty models are developed from the manufacturers' point of view; however, there is a small body of literature dedicated to warranty models from the customers' perspective (Shafiee and Chukova, 2013a). Mamer (1987) investigates a model of a product to evaluate the financial impact of a warranty policy by examining per unit costs and net revenues, and comparing it to the warranty and production costs. A recent review of warranty models can be found in Murthy and Djamaludin (2002).

There are many different types of warranty policies in existence to satiate the needs of consumers, producers, and society. A policy that is based on calendar time alone is considered to be one-dimensional, whereas a two-dimensional warranty is limited by the age and the usage of the product. One-dimensional policies are very common in the marketplace. They last for a fixed timeframe such as one or two years, and are found in most consumer products (cameras, laptops, cell phones, etc.) On the other hand, two-dimensional warranties are much less prevalent, and are usually associated with products that exhibit wear, degradation, or usage. In practice, these are typically found in automotive warranties. For example the Mazda2 is covered for 36 months or 80,000km for factory defects, and 60 months or 100,000km for Powertrain components. Blischke and Murthy (1992); Murthy and Blischke (1992a,b) present a warranty review trilogy (Product Warranty Management – I-III) exploring various topics in this field including a taxonomy shown in Figure 4.1.

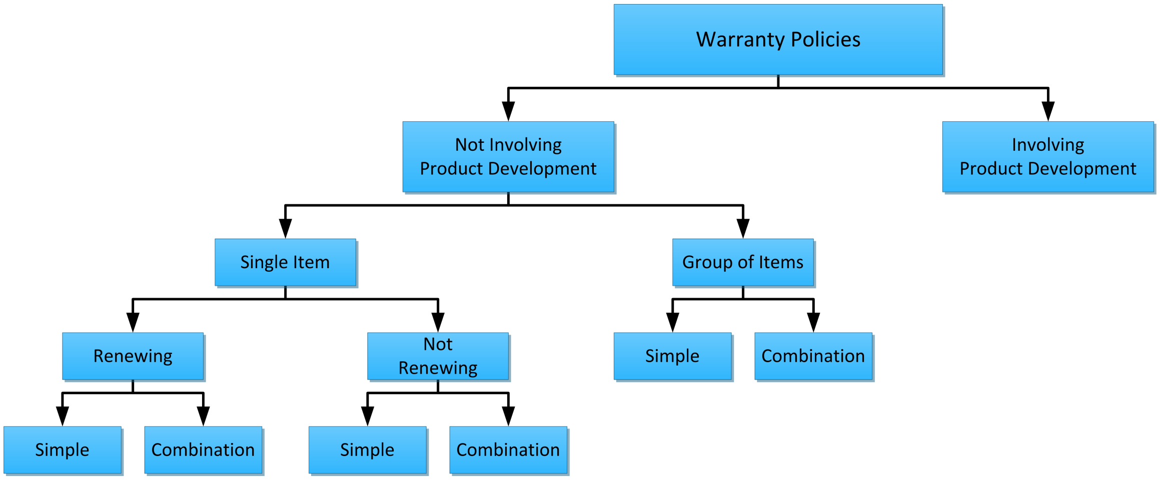


Figure 4.1: Warranty Policy Taxonomy (Blischke and Murthy, 1992)

The first branching point deals with product development after sale. Warranties that include product development after sale “are typically part of a maintenance contract and are used principally in government acquisition of large, complex items - for example, military equipment - or in certain commercial transactions involving large, expensive items such as aircraft” (Blischke and Murthy, 1992). Warranties that exclude product development after sale are normally used for civilian products. These are categorized into whether it is for the sale of a single item or for a group of items. The only difference between the policies for a single item compared to a group of items, is that for the latter, the warranty refers to the lot of items as a single entity (such as a fleet of vehicles).

A policy can either be renewing or non-renewing. Under a renewing policy, if an item fails within the warranty period, it will be replaced by a new item with a new identical warranty, replacing the old one. In other words, a new warranty period begins with each replacement. On the contrary, the non-renewing policy will replace the failed item, but will not add another period of warranty. This means that the product will continue to be covered for the remainder of the initial warranty period.

The FRW, and the pro-rata warranty (PRW) are the two basic sub-policies of the renewing/non-renewing policies (Murthy and Djamaludin, 2002; Mamer, 1987; Huang, Liu, and Murthy, 2007). With the FRW, the seller agrees to repair or replace the failed product free of charge for the entire duration of the warranty period. The unlimited FRW (UFRW) is a renewing FRW policy that is renewed an unlimited number of time as long as the failure occurs during the warranty period. In *Chapter 3*, we introduced the NFRW policy, where the coverage provided to the consumer is not renewed upon repair/replacement. These types of policies are typically found in small consumer electronics. Under a PRW, the seller agrees to refund a fraction of the purchase price, if the product fails during the warranty period. This refund function can be either linear, proportionally linear, or non-linear (Blischke and Murthy, 1995). Lu and Wang (2011) study a PRW policy with multiple periods and differential pricing decisions. Chien (2010) creates a pro-rata rebate warranty with salvage value considerations. Under a rebate policy, manufacturers agree to refund an amount to the maximum of the selling price. Additionally, the FRW and the PRW can be combined to create hybrid policies. Popović and Spasojević-Brkić (2011) create an optimization model comprising these warranty policies. Nguyen and Murthy (1986, 1989); Jack and Murthy (2001) present other optimal repair-replacement strategies for one-dimensional warranties. Chattopadhyay and Murthy (2000); Iskandar and Murthy (2003); Iskandar, Murthy, and Jack (2005) study two-dimensional warranty policies.

Shafiee and Chukova (2013a) develop a unified warranty-maintenance taxonomy characterized by three policy categories: product, warranty, and maintenance. This is shown in Figure 4.2.

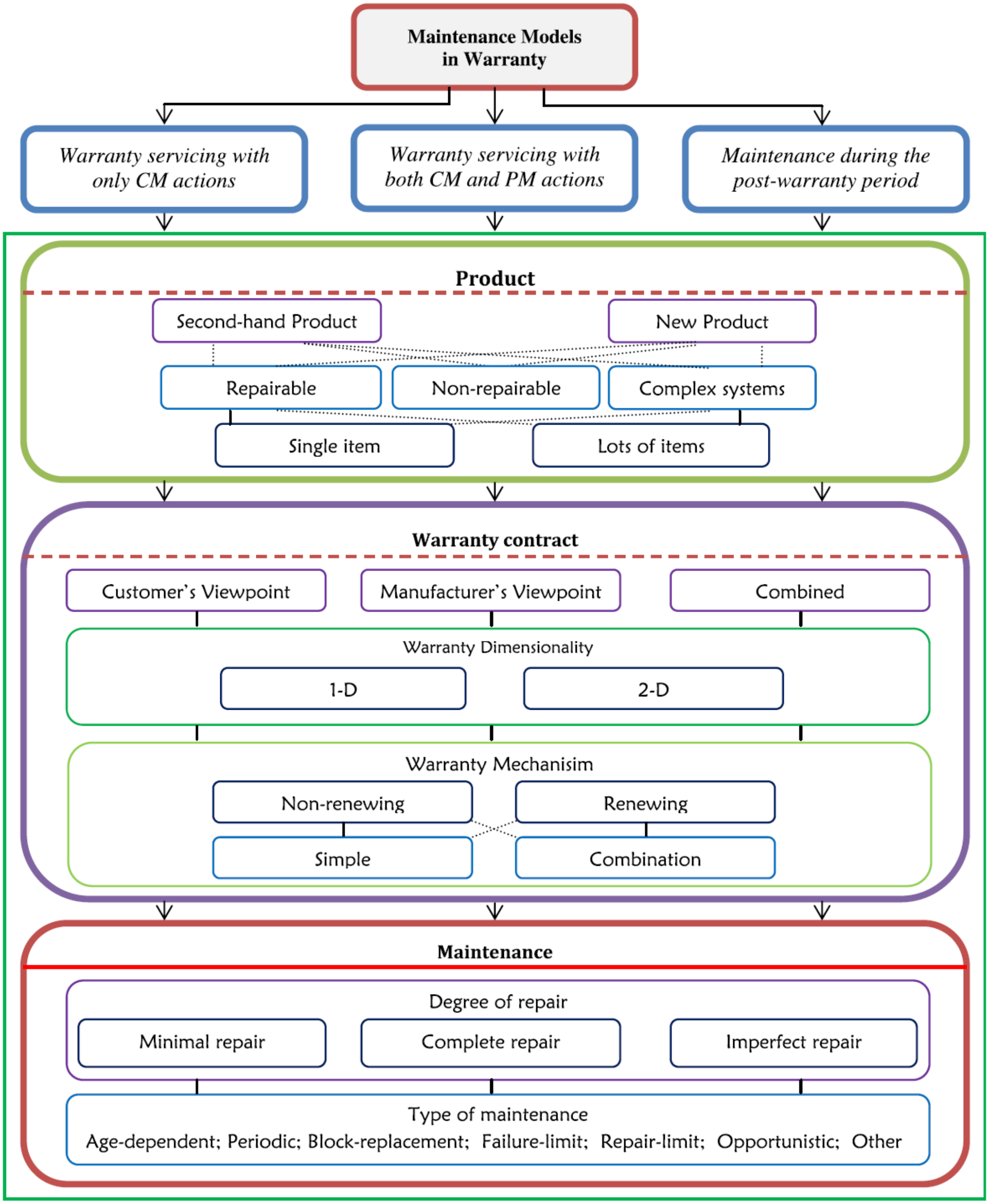


Figure 4.2: Unified Warranty-Maintenance Taxonomy (Shafiee and Chukova, 2013a)

Each category presents multiple options. Each combination of options from these three categories defines a specific warranty-maintenance strategy. All combinations create a spectrum of warranty strategies to suit the specific needs of organizations and customers. The product category focuses on whether the product is new or second-hand, repairable or not, and whether it can be refreshed. The warranty contract group represents the different types of warranty policies that are present, including the viewpoint, dimensionality, and mechanism. Finally, the maintenance group details repair policies including the types of maintenance, and the degrees of repair.

4.1.3 Upgrade, Repair, & Maintenance Policies

Of the four value recovery activities in Figure 1.8, the three restoration activities of repair, refurbish, and recondition can be considered to be upgrade actions. Recycling is not included, because in that process, the product is restored to materials, hence there is no improvement to the product itself. Repairing a product by replacing one or two broken parts with like-new or new spares will amount to a minimal repair from a reliability point of view. Reconditioning a part or module to an *as-good-as-new* condition can be modelled as a perfect repair, while refurbishing a product can be modelled as a general imperfect repair process. Upgrade activities are maintenance procedures that are usually carried out to bring components to a better condition, and thus improving their reliability. The cost of this rejuvenating/refreshing/upgrading action is proportional to the upgrade level carried out, and is an expense that can increase the sale price of the restored system. Paying for the upgrade at the recovery/remanufacturing stage, should improve the reliability of the second-hand products sold, and should reduce the costs of servicing these products during their second warranty coverage. Failures during the warranty period are costly to service, especially when dealing with reconditioned components that are less reliable than new ones.

In the continuum of all upgrade actions the two bounds are: perfect repair, where the component is restored to a condition as if it were new; and minimal repair just restores the component to an operational state, without affecting its effective age. The intermediary lines in between the bounds shown in Figure 4.3 represent some of the infinite repair possibilities of imperfect repair that can occur.

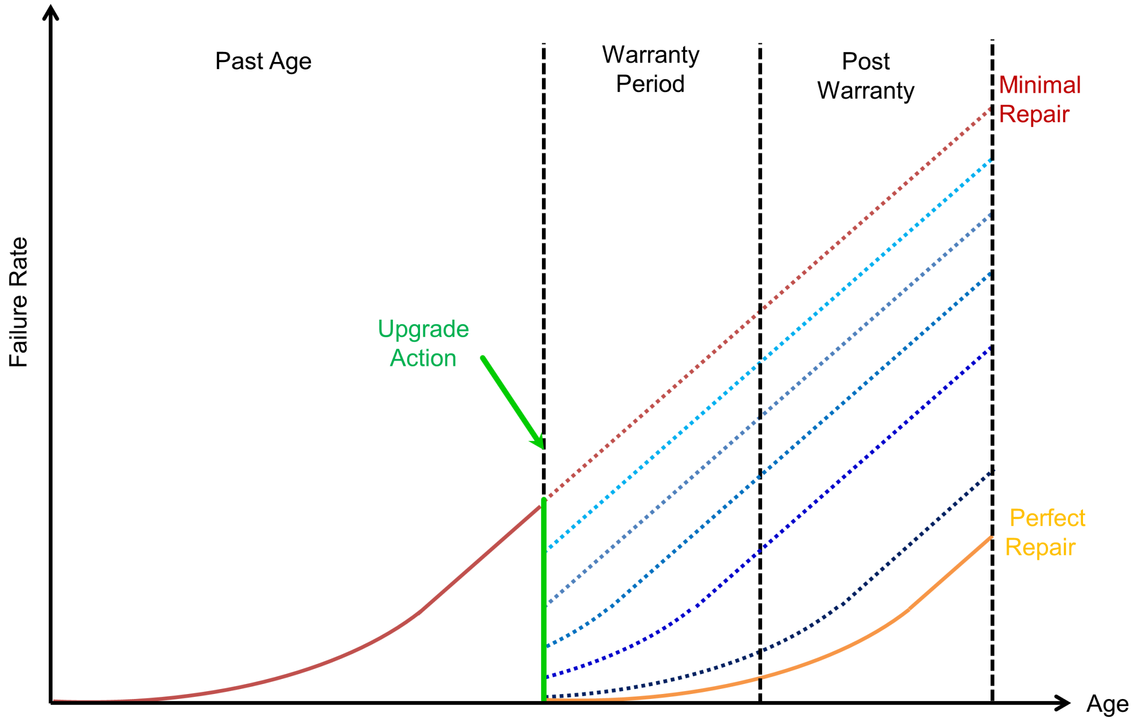


Figure 4.3: Changes in Failure Rate as a Result of Upgrade Action

Naini and Shafiee (2011) state that there are three common approaches to model the upgrade action effect on the reliability: virtual age, improvement factor, and probabilistic. The effective age of the EoL component is diminished in the virtual age approach (Naini and Shafiee, 2011; Kijima, Morimura, and Suzuki, 1988), whereas the failure rate of the item is reduced in the improvement factor approach (Malik, 1979). In the probabilistic approach, the EoL component undergoes either a perfect repair with a given probability, or an imperfect repair as a function of the aforementioned (Brown and Proschan, 1983; Block, Borges, and Savits, 1985). The virtual age model has been used the most frequently in the reviewed literature. By having an accurate estimate of the failure rate, one can better estimate future costs of supporting the item during its warranty, as well as its possible reuse in future products.

Over time, the condition of a product deteriorates, so that maintenance and repair services are needed to keep it in working order. Since remanufactured products will usually have different (higher) failure rates, their maintenance strategies must then be adapted to account for their current age. With these products, repair may occur sooner, more frequently, and therefore add an extra financial burden to the servicing firm. There are two basic types of maintenance procedures: corrective maintenance (CM) and preventive maintenance (PM). With CM, the failed products are restored to an operational state through minimal, perfect, or imperfect repair. PM on the other hand, occurs before the product fails, with the intent to reduce the risk of failure and degradation. For example, the most mundane automotive PM procedures are oil changes, brake inspections, and tire replacement. The type of maintenance procedure(s) implemented depends on the useful life of the product. A product with a relatively short useful life should involve only CM actions; whereas with a long useful life, PM actions should be considered to reduce the future costs to service the warranty (Shafiee and Chukova, 2013a):

> If the useful life of a product is relatively short, then its (basic) warranty is also relatively short, and warranty servicing should involve only CM actions. If a product has a long useful life, then an extended warranty could be also relatively long, and the manufacturer can reduce warranty servicing costs by performing effective PM actions. Optimal maintenance strategies need to be viewed from a life cycle perspective (from buyer's and manufacturer's viewpoints), and hence, there is a close link between warranties (basic warranty and extended warranty) and maintenance.

Yeh, Chen, and Chen (2005) examine non-repairable products under a renewing FRW policy for optimal age replacement, and Chien and Chen (2010) model a combination FRW–PRW policy for non-repairable products. Sim and Endrenyi (1988); Yeh and Lo (2001) develop PM warranty policies for repairable products. Matis, Jayaraman, and Rangan (2008) design a multi-period warranty policy for repairable items, optimizing for the PRW length and product price. In the first period, the product is either replaced or repaired for free, and the second follows a PRW policy. Chien (2008) formulates a replacement model with minimal repair under renewing FRW. Pascual and Ortega (2006); Yun, Murthy, and Jack (2008); Yeo and Yuan (2009) present warranty servicing strategies for different levels of imperfect repair.

Yeh, Lo, and Yu (2011) propose two periodical age reduction PM models for a second-hand product with known age and a pre-specified length of usage. Their objective is to minimize the total expected maintenance cost by obtaining the optimal number (of times) and degree of PM actions. Pongpech, Murthy, and Boondiskulchock (2006) propose a mathematical model to determine the optimal upgrade and PM actions that minimize the total expected maintenance and penalty costs for used equipment under lease. Khatab, Diallo, and Aït-Kadi (2014) investigate the relationship between rejuvenation/upgrade decisions of recovered EoL systems, and the subsequent maintenance costs incurred during their second-life as refreshed products. They develop a mathematical model for the joint determination of the optimal upgrade level and imperfect PM strategy. Khatab et al. (2014) provide a numerical example to illustrate the validity of their model.

4.1.4 General Warranty Issues

It is well known that from a financial perspective, a producer should not offer long warranty coverage for less reliable products, because such a decision will force the company to set aside a larger reserve fund to service these failures. However, these policies can still be found in the marketplace. A study by Agrawal, Richardson, and Grimm (1996) demonstrated "the average correlation between warranty terms and reliability across the five appliance and four electronic products covering a nine-year period shows that the relationship is positive but very weak". The existence of these managerial decisions leads to the importance of briefly discussing the perception of consumers, and other intangible factors.

To the consumer, warranty acts as proxy for reliability since "products become more complex, and less easily evaluated by consumers, warranties are used to indicate the product's performance and reliability" (Murthy and Djamaludin, 2002). Consumers have a preference towards products with a longer warranty for two reasons. The first reason is the assurance from the manufacturer to have the product repaired/replaced over a longer horizon. Secondly, when warranty periods are longer, the consumer assumes that the product is of a higher quality and reliability. Otherwise, the manufacturer would have not taken the risk of incurring expensive repair costs. Boulding and Kirmani (1993) examine whether consumers perceive warranty as an indication of quality using signalling theory. Blair and Innis (1996) perform a study on the effects of warranty length on two groups of consumers (experts and non-experts). This study assessed the perceived product quality for both known and unknown brands (i.e. Schwinn vs. Monarch bicycles), finding that the warranty phenomenon is the most significant for non-experts and unknown brands.

Longer warranty periods can be used as a key promotional driver, where the producer can either charge more for a product and/or induce higher demand. However, a longer warranty on a less reliable product, forces the company to set aside a larger reserve fund to service these failures. Therefore, the profitability of an enterprise can be maximized by accurately setting decision variables pertaining to the servicing of a product, such as its warranty length and price.

Repair logistics planning is another factor that needs to be considered when servicing failed products under warranty. Typically, failed products under warranty are sent back to service facilities for repair/replacement. These facilities benefit from economies of scale by centralizing resources – both parts and expertise. The location of these service facilities and the structure of the service network should be carefully planned. Decentralized organizations with many local service facilities may provide quick service to customers, but cost more to operate. Conversely, a centralized structure will be less expensive to run, but customers may experience longer wait times to get their product fixed. Recently, a growing trend in the decentralization and outsourcing of maintenance operations to third-parties has been observed.

> Some main advantages of outsourcing warranty servicing are (i) access to high expertise and improved maintenance technology, (ii) fixed cost service contracts, (iii) less capital investment, and (iv) frees managers' time for other tasks. However, there could be some disadvantages as well (i) cost of the outsourcing, (ii) dependency on the service agent, and (iii) loss of maintenance knowledge and personnel (Shafiee and Chukova, 2013a).

When offering policies with long warranty periods, we can not only anticipate higher repair costs, but also reverberations based on who facilitates the repair. We can also consider other aspects that influence the consumer such as return ease and repair time, yet it is very difficult to assess how they could be used as promotional tools. However, determining the relationship between return ease, repair time, and product demand can be an interesting analysis.

Warranty terms usually contain exclusion conditions to deny repair or replacement for products that were abused or used in non-standard operating conditions. Wu (2011) introduces new models to account for warranty claims due to misuse, and/or failures caused by various human factors. An up to date review of warranty data analysis can be found in Wu (2012b).

The introduction of reconditioned components, both in the production and service of products will bring forth additional complexity to the issues listed above. The next section will review warranty models, specifically developed for second-hand systems.

4.2 Review of Warranty Models for Second-Hand Systems

There is a booming market for refreshed, remanufactured, second-hand products. “Secondary markets allow access to consumers that cannot afford a new product and provide current owners an outlet to dispose of products that still have market value” (Heese, Cattani, Ferrer, Gilland, and Roth, 2005). Saidi-Mehrabad, Noorossana, and Shafiee (2010) describe how between 1990 and 2005, used car sales have increased from 4.7 to 5.4 million, while new car sales declined from 2.3 to 2.07 million.

> In spite of increasing the market share for second-hand products, often, customers are uncertain about the performance and durability of these products due to the lack of knowledge related to past usage and maintenance history. To reduce this uncertainty and increase sales, dealers are offering warranty to customers. Offering the warranty implies that the dealer incurs additional costs to service any claims made by the customers (Saidi-Mehrabad et al., 2010).

In order to generate demand for reconditioned or second-hand products, manufacturers, dealers, and brokers have had to resort to a combination of initiatives to promote and infer the quality of their products. These initiatives include significant price reductions and generous warranty coverage (same coverage length as new systems, free preventive maintenance in the first year of the refreshed system, etc.) Both the increasing market share and the necessity to offer warranty have brought forth the need to develop new warranty models for second-hand products. However, the number of models in this area is severely lacking (Chukova and Shafiee, 2013):

> A significant amount of academic research has been conducted in modelling warranty policies and costs for new products. In contrast, a brief review of the literature shows that only a few researchers have worked in the area of warranties for second-hand products.

Research on warranty models for reconditioned/upgraded products is still in its nascent stages. Shafiee and Chukova (2013a) provide a current summary of warranty models for second-hand products.

Chattopadhyay and Murthy (2000) is generally accepted as the formative paper in developing a warranty cost analysis for second-hand products. The warranty cost equations developed by Chattopadhyay and Murthy (2000) for second-hand components at the component level for both the FRW (Eq. (4.7)) and PRW (Eq. (4.8)) policies are shown below:

$$C_i(w;\tau_i) = c_i \left[F_{i1}(w) + \int_0^W M_i(t-x)dF_{i1}(x) \right] \tag{4.7}$$

$$C(w) = c_s \left[F_1(w) - \frac{1}{w} \int_0^w x dF_1(x) \right] \tag{4.8}$$

where $F_{i1}(w)$ is the first failure, $M_i(w)$ is the associated renewal function, τ_i is the age of the component, c_i is the cost of each failure replacement, and c_s is the sale cost.

Chattopadhyay and Murthy (2001) also develop statistical models for the analysis of warranty policies for second-hand products. Chattopadhyay and Murthy (2004) propose mathematical models for FRW to improve the reliability of second-hand products. Chattopadhyay and Yun (2006) develop a model to estimate the cost for two-dimensional warranty policies associated with sale of second-hand products.

The lump-sum warranty (LSW) is a common policy for second-hand items, where "the manufacturer refunds a customer some proportion of the sales price if the product fails during the warranty period" (Chien and Chen, 2008). Naini and Shafiee (2011) present a policy, where prior to time w_f ($w_f<w$), faulty items are replaced free of charge, and after w_f it follows an LSW policy. The expected warranty cost per replacement, $E\left[R(t)\right]$, is given by Eq. (4.9):

$$E\left[R(t)\right] = c_r(\tau)\left[(1-k)F_1(w_f) + kF_1(w)\right] \tag{4.9}$$

where $0\leq k\leq 1$ is a refund coefficient and $c_r(\tau)$ is the replacement cost of the failed item with a used one of age τ.

Saidi-Mehrabad et al. (2010) develop reliability improvement strategies for second-hand products sold under multiple warranty policies including: failure-free warranty, rebate warranty, and a combination of free replacement and lump sum policies. Shafiee and Chukova (2013b) develop a comprehensive expected profit optimization problem, that maximizes the per unit profit $\pi_i(\tau, w, k)$ by incorporating the age of the second-hand product (τ), the warranty period (w), and the upgrade level (k).

$$\max\ \pi_i(\tau, w, k) = p(\tau, w, k) - c_\tau - c_k(\tau) - c_{iw}(k) \tag{4.10}$$

The sale price $p(\tau, w, k)$ is modelled as follows:

$$p(\tau, w, k) = p_0\left(1 - \frac{\tau}{L}\right)(w+\eta_w)^a\,(k+\eta_k)^b \tag{4.11}$$

where p_0 is the sale price of the new product (without warranty), L is the expected lifetime, $a>0$ is the warranty length elasticity parameter, $b>0$ is the age reducing elasticity parameter, and η_w and η_k are non-negative constants of warranty/age reducing displacement, which allow for the non-zero price when no warranty and/or no upgrade actions are offered.

The value of a second-hand product of age τ (>0) is given by:

$$c_\tau = \begin{cases} k_0 p_0 \left[1 - \frac{\tau}{L}\right] & \text{if } \tau \in (0, L] \\ 0 & \text{if } \tau > L \end{cases} \tag{4.12}$$

The cost of upgrade action $c_k(\tau)$ is:

$$c_k(\tau) = c_m + c^{\psi} \tau^{\xi} \tag{4.13}$$

where the parameters c, ψ and ξ are greater than zero and c_m is the cost of a minimal repair. Eq. (4.13) implies that the cost of an upgrade action increases as τ and/or k increases. Note that, k=0 implies minimal repair and as a result, $c_k(\tau)$=c_m. The expected warranty cost per product is:

$$c_{iw}(k) = \int_m^M \bar{c} \int_0^w [r_{ik}(t)dt] \; h(\tau) d\tau \tag{4.14}$$

where $\bar{c}$ is the expected cost of each minimal repair over the warranty period and $r_{ik}(t)$ is the failure rate function after the upgrade action model i.

Lo and Yu (2013) develop a mathematical model and provide a practical application to used cars to determine the optimal upgrade level and warranty length to maximize the expected profit for used products. Naini and Shafiee (2011) propose a joint optimal price and upgrade level model for warranted second-hand products. They present an application of their model to solve a problem pertaining to second-hand electric drills. Shafiee and Chukova (2013a) propose a three-parameter optimization model to determine the optimal upgrade strategy, warranty policy, and sale price of a second-hand product to maximize the expected profit of a dealer.

Chari, Diallo, and Venkatadri (2013) develop an optimal mathematical model utilizing reconditioned components for replacements. The product demand is modelled to be proportional to the length of the warranty period to resemble the consumers' perception of better reliability through longer warranty. Additionally, the acquisition cost is modelled to be decreasing with the age of the reconditioned component.

There are a growing number of models for second-hand products. These models have to account for specific issues encountered during the recovery, upgrading, and the operation of remanufactured systems. The next section will present a new warranty model for second-hand products where the replacements are carried out with a mixture of new and reconditioned parts.

4.3 Optimal Warranty Models for a Replacement Strategy using a Mixture of New and Reconditioned Parts

Under a large majority of traditional warranty policies, a failed product is repaired with new components. In the context of EoL/EoU product recovery and remanufacturing, we also have access to reconditioned parts. It is therefore justified to investigate the conditions under which the reconditioned parts can be used to service warranty claims.

Chari et al. (2013) propose an original mathematical model for a one-dimensional UFRW policy of duration w with replacements composed of reconditioned components of age τ. In their study, they assume that the components are always available in sufficient quantity. In practice, it is not always possible to have access to enough reconditioned components to honour the warranty. Due to the fact that the supply of EoL/EoU or returned products is not steady, a manufacturer may consider using a combination of new and reconditioned components to facilitate replacements.

This section is dedicated to the development of a mathematical model for a UFRW policy where the manufacturer uses a mixture of new and reconditioned components to honour warranty claims.

4.3.1 Model Formulation

Under the proposed warranty strategy, a producer manufactures and sells new products containing a key component that is warranted for a coverage period of w. The warranty policy offered is of the UFRW type. If the key component fails during the warranted period, the producer replaces it with one randomly chosen from a pool of new and reconditioned components. The reconditioned parts cost less, and are less reliable than the new ones. The goal then to find the appropriate mixing proportions, age of reconditioned components, warranty length, and sale price of products that maximize the profit of the producer. The following notation is used:

Decision Variables

w: Warranty period
p: Price per unit product sold
τ: Age of reconditioned components
η: Proportion of reconditioned components

Functions

$f(t)$: Lifetime pdf of new components
$F(t)$: Lifetime cdf of new components
$F_\tau(t,\tau)$: Lifetime cdf of reconditioned components
$F_m(t,\tau,\eta)$: Lifetime cdf of mixed components
$R(t)$: Reliability of new components
$R_\tau(t,\tau)$: Reliability of reconditioned components
$R_m(t,\tau,\eta)$: Reliability of the mixture of components
$A(\tau,\eta)$: Acquisition cost for each component
$N(w,\tau,\eta)$: Expected number of failures
$C_r(w,\tau,\eta)$: Reserve warranty cost per unit
$C_u(w,\tau,\eta)$: Cost per unit product
$D(w,p)$: Total demand
$S(w,p,\tau,\eta)$: Scarcity cost
$\pi(w,p,\tau,\eta)$: Profit per unit product sold
$\Pi(w,p,\tau,\eta)$: Total profit

Parameters

β: Weibull distribution shape parameter
θ: Weibull distribution scale parameter
a_i: Age coefficients for acquiring reconditioned components
A_0: Unit cost of product (not including warranty)
d_1: Price coefficient
d_2: Warranty coefficient
D_1: Demand amplitude factor
D_2: Warranty displacement constant
S_0: Fixed cost for component recovery
S_1: Ideal volume of recovered components to minimize fixed scarcity costs

Reliability Function for a Reconditioned Part

A part or component is said to have age τ ($\tau \geq 0$) if:

- It has been operating without failure of τ units of time, or
- It has operated longer and is rejuvenated to age τ.

If $\tau=0$, then the component is new. If $f(t)$ denotes the lifetime pdf function of a new component, then the lifetime pdf function $f_\tau(t)$, cdf $F_\tau(t)$, reliability function $R_\tau(t)$, and failure rate function $h_\tau(t)$ for a reconditioned component of age τ can be realized (Diallo and Aït-Kadi, 2011; Chari et al., 2013):

$$f_\tau(t) = \frac{f(\tau + t)}{R(\tau)} \tag{4.15}$$

$$F_\tau(t) = \frac{F(\tau + t) - F(\tau)}{R(\tau)} \tag{4.16}$$

$$R_\tau(t) = \frac{R(\tau + t)}{R(\tau)} \tag{4.17}$$

$$h_\tau(t) = h(\tau + t) \tag{4.18}$$

Reliability Function for a Mixed Population

The general concept of statistical mixture has been successfully applied to the modelling of heterogeneous populations (Proschan, 1963; Barlow, Proschan, and Hunter, 1967; Wondmagegnehu, Navarro, and Hernandez, 2005). The mixture of two Weibull distributions is used to find the optimal burn-in preventive replacement policy (Jiang and Jardine, 2007), and age-based inspection strategy (Scarf, Cavalcante, Dwight, and Gordon, 2009). Diallo and Aït-Kadi (2011) study the reliability properties of a mixture of two subpopulations of identical components with different ages.

The mixture population of parts used to honour the warranty is made of η% of reconditioned components of age τ, and (1-η)% of new components. The proportion η can also be interpreted as the fraction of time that reconditioned components are available to be used for replacement. The reliability function of such a mixed population of parts is given by Diallo and Aït-Kadi (2011):

$$R_m(t) = (1-\eta)R(t) + \eta R_\tau(t) \tag{4.19}$$

$$R_m(t) = (1-\eta)R(t) + \eta \frac{R(\tau+t)}{R(\tau)} \tag{4.20}$$

$$R_m(t) = \frac{(1-\eta)R(t)R(\tau) + \eta R(\tau+t)}{R(\tau)} \tag{4.21}$$

Similarly, the failure can be characterized as:

$$F_m(t) = (1-\eta)F(t) + \eta F_\tau(t) \tag{4.22}$$

$$F_m(t) = (1-\eta)F(t) + \eta \frac{F(\tau+t)}{F(\tau)} \tag{4.23}$$

$$F_m(t) = \frac{(1-\eta)F(t)F(\tau) + \eta F(\tau+t)}{F(\tau)} \tag{4.24}$$

Components of the Total Profit Function

Figure 4.4 depicts the diagram of influence (conceptual model) of the four decision variables (w,p,τ,η) of the problem. This diagram shows how the net total profit function and the intermediate functions are affected by the decision variables. For example, the warranty length affects both the number of failures and the demand for the products. Starting from the top of the conceptual model, we define successive intermediate functions (decomposition) until reaching the basic decision variables. Then using the defined structure, we proceed backwards and write the expressions of the intermediate functions until the objective function is obtained.

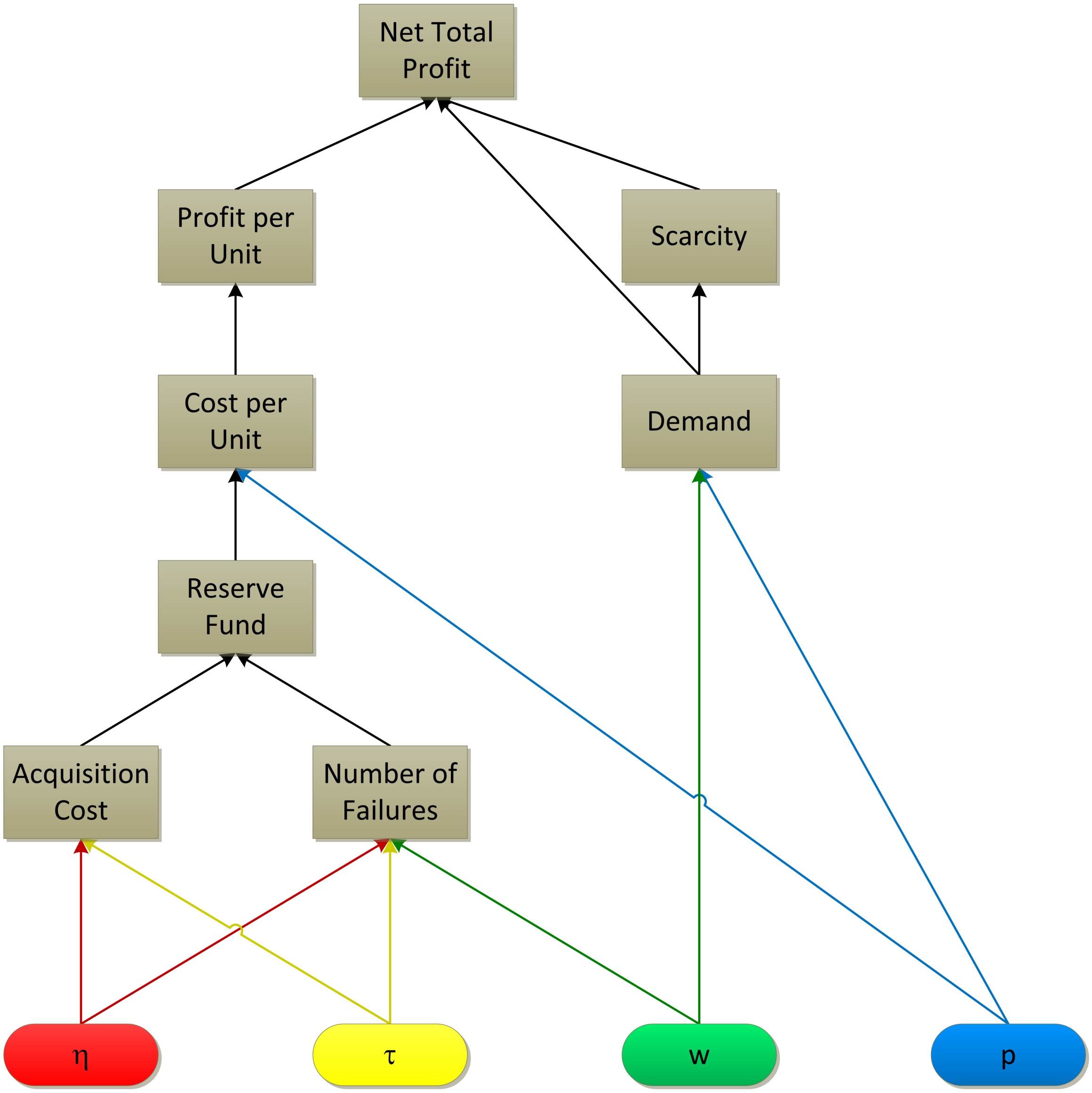


Figure 4.4: Diagram of Influence

Acquisition Cost of Replacement Parts

The pool of components used to carry-out the replacement of failed parts is a perfect mixture of η% reconditioned parts of age τ and (1-η)% of new parts.

It is reasonable to assume that the acquisition cost of a part of age τ is a decreasing function of its age up to a certain age, from where it will start to increase to account for the increased dissassembly and other cleaning efforts (rust, dust, etc.)

The acquisition cost of a part of age τ is defined as:

$$A(\tau) = A_0 \left(\tau + 1\right)^{-a_1} + \tau^{a_2} \tag{4.25}$$

where A_0=$A(\tau$=$0)$ is the cost of a new part, and a_1 and a_2 are positive parameters. Parameter a_1 affects the discount rate offered on the old parts, and parameter a_2 signals an increase in cost due to aging. The profile of $A(\tau)$ is displayed in Figure 4.10:

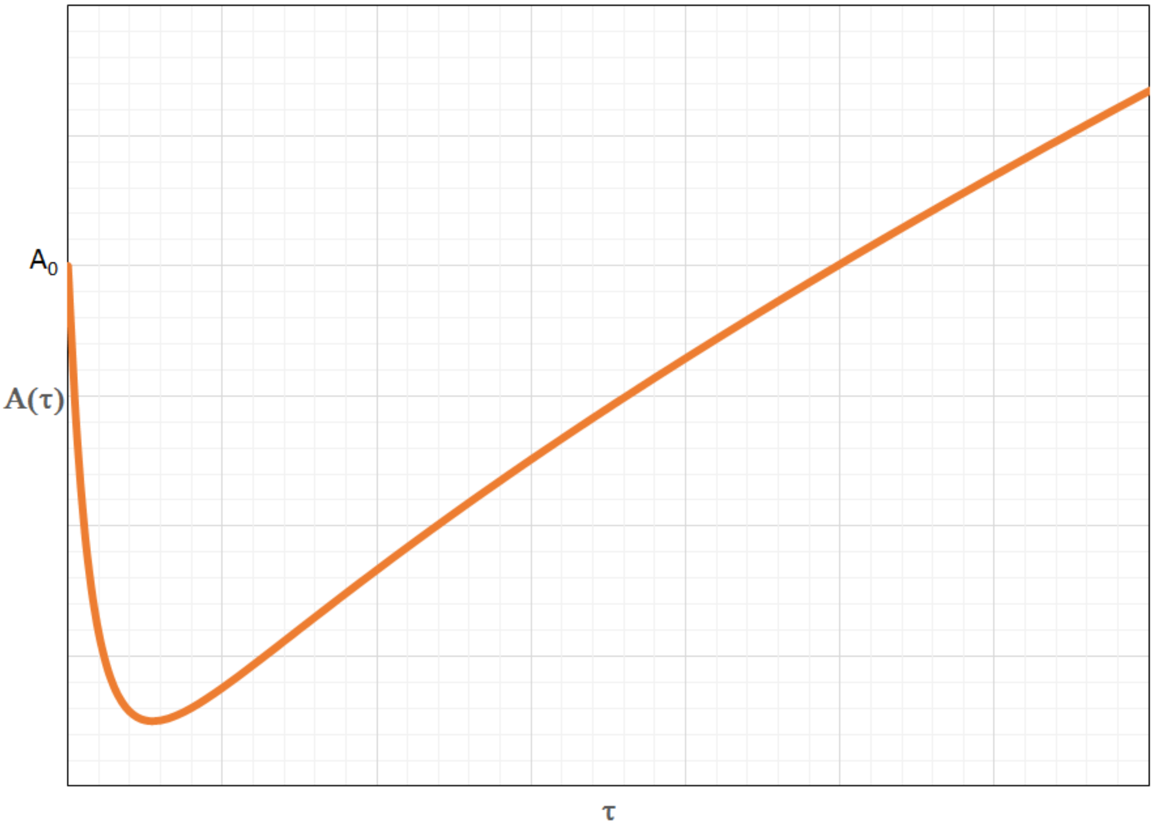


Figure 4.5: Acquisition Cost as a function of τ

The average cost of each part is randomly picked from the pool of replacement parts is then given by Eq. (4.26):

$$A(\tau, \eta) = (1 - \eta)\, A_0 + \eta \left(A_0 \left(\tau + 1\right)^{-a_1} + \tau^{a_2}\right) \tag{4.26}$$

Average Number of Replacements/Failures

The product is sold new and it is assumed that there is only one key component (CPU, motor, etc.) in the product. So each failure generates exactly one replacement. Thus, the average number of replacements is equal to the expected number of failures during the UFRW coverage (Smith and Leadbetter, 1963; Smeitink and Dekker, 1990). For a given product, the key component is first replaced when the original (new) component fails for the first time with probability $F(w)$. The key component will be replaced for the n^{th} time with probability $F(w)F_m(w)^{n-1}$ (i.e. the original replacement failed once and was replaced and failed n-1 times after being replaced by a component picked from the mixed pool). The expected number of failures is:

$$N(w,\tau,\eta) = F(w) + F(w)F_m(w,\tau,\eta) + F(w)F_\mu^2(w,\tau,\eta) + ... \tag{4.27}$$

$$N(w,\tau,\eta) = \sum_{k=0}^{\infty} F(w)F_m^k(w,\tau,\eta) \tag{4.28}$$

$$N(w,\tau,\eta) = \frac{F(w)}{1 - F_m(w,\tau,\eta)} \tag{4.29}$$

$$N(w,\tau,\eta) = \frac{1 - R(w)}{R_m(w,\tau,\eta)} \tag{4.30}$$

$$N(w,\tau,\eta) = \frac{\left[1 - R(w)\right] R(\tau)}{(1-\eta)R(w)R(\tau) + \eta R(\tau + w)} \tag{4.31}$$

Reserve Fund

For each new product sold, the producer holds $C_r(w,\tau,\eta)$ in a reserve fund to cover the cost of replacements for that product. On average there are $N(w,\tau,\eta)$ replacements for each product and each replacement component costs $A(\tau,\eta)$. Therefore, the reserve fund per unit is:

$$C_r(w,\tau,\eta) = N(w,\tau,\eta)A(\tau,\eta) \tag{4.32}$$

$$C_r(w,\tau,\eta) = \frac{R(\tau)\left[1 - R(w)\right]\left[(1-\eta)\, A_0 + \eta\left(A_0\,(\tau+1)^{-a_1} + \tau^{a_2}\right)\right]}{(1-\eta)R(w)R(\tau) + \eta R(\tau + w)} \tag{4.33}$$

It should be noted that Eq. (4.33) gives rise to two special cases: η=0 and η=1. When η=0, Eq. (4.33) is a renewing UFRW using only new parts. This is equivalent to the results obtained by Blischke and Murthy (1993); Anityasari et al. (2007):

$$C_r(w,\tau,0)=\frac{R(\tau)\left[1-R(w)\right]A(0)}{R(w)R(\tau)}=\left[\frac{1}{R(w)}-1\right]A_0 \tag{4.34}$$

When η=1, Eq. (4.33) is identical to the results obtained by Chari et al. (2013):

$$C_r(w,\tau,1)=\frac{A(\tau)R(\tau)\left[1-R(w)\right]}{R(\tau+w)} \tag{4.35}$$

Total Unit Cost

The total unit cost of each new product $C_u(w,\tau,\eta)$ is the sum of the reserve fund $C_r(w,\tau,\eta)$ and the cost to manufacture the initial product, K_0. It is assumed that the cost of the product is mainly the cost of the key component (K_0=A_0). Therefore:

$$C_u(w,\tau,\eta)=C_r(w,\tau,\eta)+A_0 \tag{4.36}$$

Profit per Unit Sold

Each new product costs $C_u(w,\tau,\eta)$ and is sold at a price p. The profit per unit of each product sold is then given by:

$$\pi(w,p,\tau,\eta)=p-C_u(w,\tau,\eta) \tag{4.37}$$

Sales Volume / Total Demand

The expected forecast sales volume/total demand, $D(w,p)$, for the product is modelled as a displaced log-linear function of w and p (Glickman and Berger, 1976; Matis et al., 2008):

$$D(w,p)=D_1p^{-d_1}\left(w+D_2\right)^{d_2} \tag{4.38}$$

where d_1 is the rate of decrease of the sales volume with the increasing price of the product, d_2 is the rate of increase of the sales volume with the increasing warranty length, D_1 is a demand amplitude factor, and D_2 is a warranty displacement constant.

The values of the parameters d_1, d_2, D_1, and D_2 are obtained from customer surveys and market studies. Eq. (4.38) models consumers' preference towards longer warranty periods and aversion to higher prices.

Scarcity

Next, we introduce a term, $S(w, p, \tau, \eta)$, to represent the scarcity of the reconditioned components.

$$S(w, p, \tau, \eta) = S_0 \left[\eta \; N(w, \tau, \eta) \; D(w, p) - S_1\right]^2 \tag{4.39}$$

where S_0 is the fixed cost for product recovery, S_1 is the ideal volume of recovered product to minimize fixed scarcity costs, and the term $\eta \; N(w, \tau, \eta) \; D(w, p)$ represents the expected total volume of reconditioned components required.

Figure 4.6 illustrates how this cost varies based on the required volume of reconditioned components. We see that if no reconditioned components are used in the remanufacturing effort, there is still a fixed cost, $S_0 S_1^2$, which represents the cost of setting-up a recovery network. This cost could be the physical infrastructure of the remanufacturing organization, or a service-level agreement (SLA) with a third-party supplier, where there was a contract in place to purchase a certain volume of reconditioned components (S_1). If the SLA is met, then there is no scarcity cost, however if the quantity deviates from S_1, the cost increases.

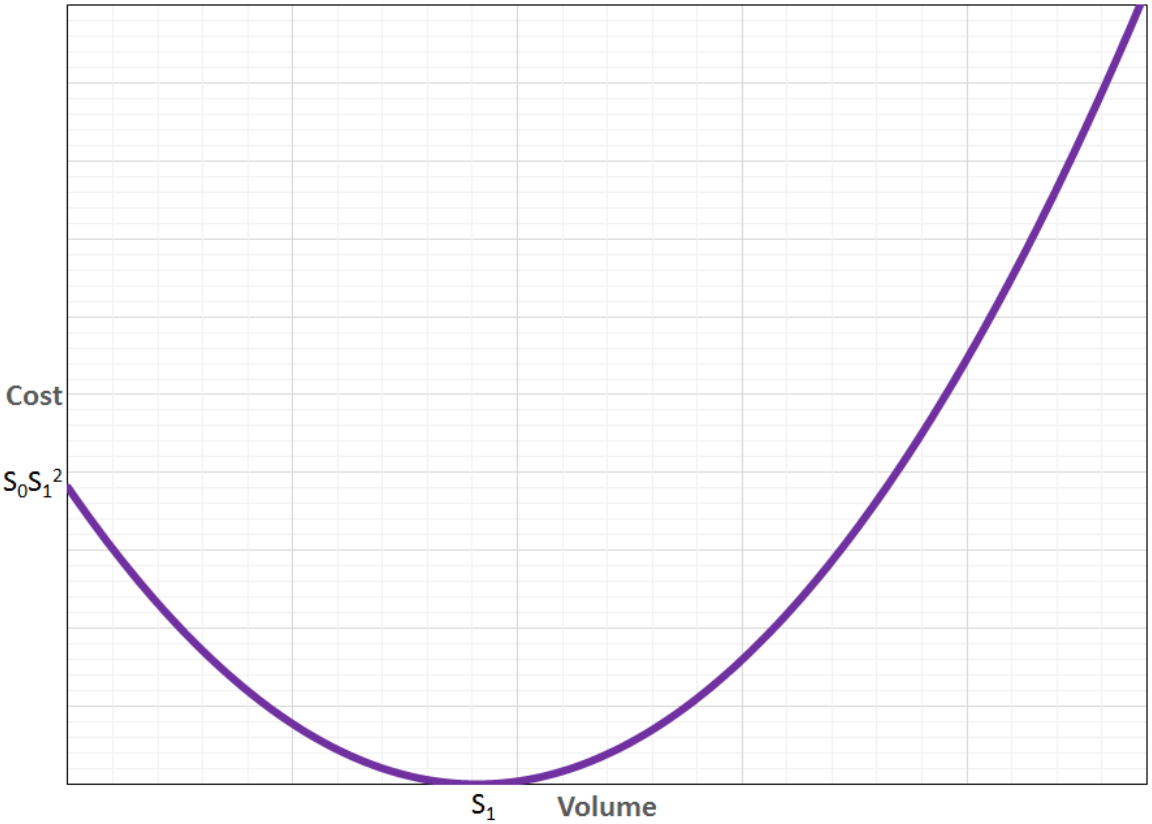


Figure 4.6: Scarcity Cost as a function of the Volume of Reconditioned Components

Total Profit

Finally, the expression of the expected total profit is composed of the profit per unit (Eq. (4.37)), the total demand (Eq. (4.38)), and the scarcity cost (Eq. (4.39)):

$$\Pi(w,p,\tau,\eta) = D(w,p)\ \pi(w,p,\tau,\eta) - S(w,p,\tau,\eta) \tag{4.40}$$

$$\begin{aligned}\Pi(w,p,\tau,\eta) = {} & \left[D_1 p^{-d_1}\left(w+D_2\right)^{d_2}\right]\left[p - C_u(w,\tau,\eta)\right] \\ & - S_0\left[\eta\ N(w,\tau,\eta)\ \left[D_1 p^{-d_1}\left(w+D_2\right)^{d_2}\right] - S_1\right]^2\end{aligned} \tag{4.41}$$

Thus, the optimal strategy is the solution to the following system of equations.

$$\begin{cases} \frac{\partial \Pi}{\partial w} = 0, & at\ w = w^*, p = p^*, \tau = \tau^*, \eta = \eta^* \\ \frac{\partial \Pi}{\partial p} = 0, & at\ w = w^*, p = p^*, \tau = \tau^*, \eta = \eta^* \\ \frac{\partial \Pi}{\partial \tau} = 0, & at\ w = w^*, p = p^*, \tau = \tau^*, \eta = \eta^* \\ \frac{\partial \Pi}{\partial \eta} = 0, & at\ w = w^*, p = p^*, \tau = \tau^*, \eta = \eta^* \end{cases} \tag{4.42}$$

Eq. (4.42) is analytically cumbersome to optimize, therefore numerical optimization techniques are used to obtain the decision variables $(w^*, p^*, \tau^*, \eta^*)$ that maximize the total profit. The ensuing section will present and discuss the results obtained.

4.3.2 Results & Discussion

For the numerical experiments, the products are considered to have a two parameter (ζ=0) Weibull distributed lifetimes. Their pdf, cdf, and reliability functions are given by Eqs. (4.3)-(4.5) respectively. Using the values shown in Table 4.1, an optimal solution $(w, p, \tau, \eta)^*$ that maximizes $\Pi(w, p, \tau, \eta)$ is solved numerically using the NLP solver in *Maplesoft Maple 18.*

Table 4.1: Parameters Used in Total Revenue Solution

Parameter	β	θ	A_0	a_1	a_2	d_1	d_2	D_1	D_2
Value	0.95	2.0	5.0	3.3	0.7	2.6	1.9	100 000	2.0

From this formulation, two special scenarios can be distinguished. The first scenario presents the numerical results when there are no reconditioned components (τ=η=0), as in Blischke and Murthy (1993); Anityasari et al. (2007). The second scenario depicts the situation where only reconditioned components are used (η=1), as in Chari et al. (2013). Finally, the general model is presented, where all four parameters are varied to obtain optimal numerical results.

Scenario 1: No Reconditioned Components

In this special scenario, because there are no reconditioned components, the values of τ and η are both 0. The scarcity parameters (S_0, S_1) are also 0, because in this scenario there is no intention of acquiring EoL/EoU systems for the purposes of remanufacturing. Hence, the optimal solution of $\Pi(w,p)^*$ is solved for. A single solution of w^*=0.26 and p^*=9.36 is obtained, resulting in Π^*=5 036. This provides the baseline to evaluate the numerical results to be obtained in subsequent scenarios and sensitivity analyses. Figure 4.7 displays these parameters:

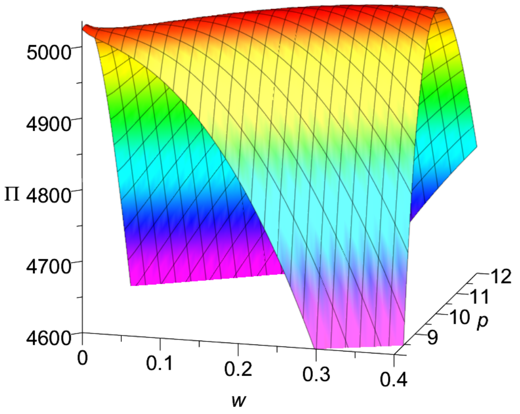


Figure 4.7: Total Profit Solution to Scenario 1

As our solution is not analytically determined, we will calculate the eigenvalues at the optimal solution and if the Hessian matrix is negative definite, we will therefore be able to state that the solution is at least a local maximum. The Hessian ($H(w,p)$), and its eigenvalues (λ_1 and λ_2) at our optimal solution are calculated to be:

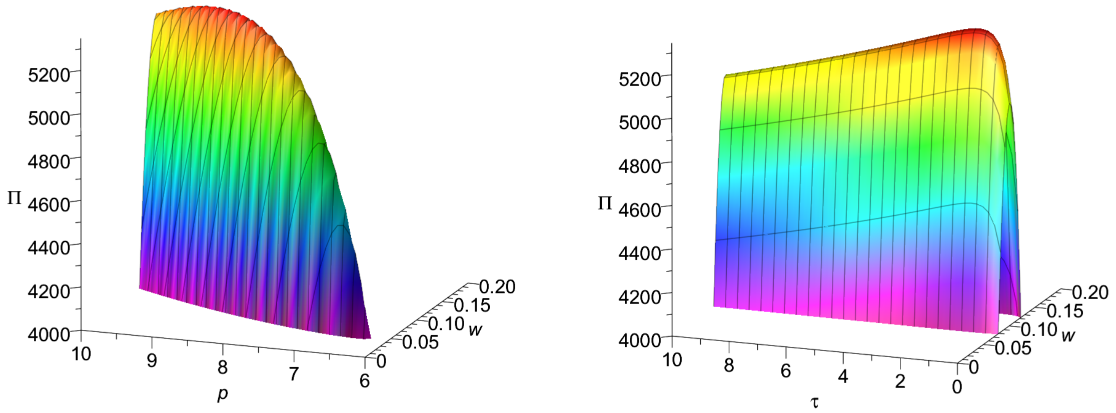
$$H(w,p) = \begin{bmatrix} -6\ 857.26 & 1\ 178.12 \\ 1\ 178.12 & -239.04 \end{bmatrix}$$

$$\lambda_1 = -35.58, \lambda_2 = -7\ 060.72$$

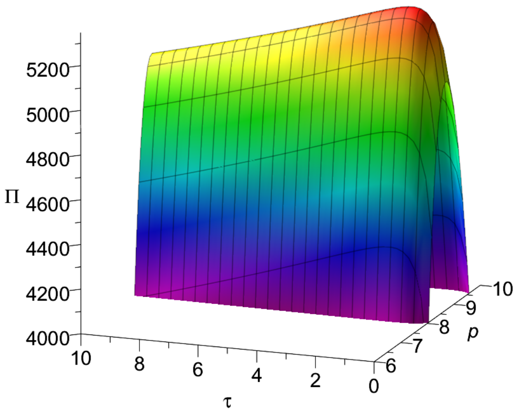
Since both eigenvalues are negative, the Hessian matrix is negative definite and the numerically obtained solution is a local maximum of the profit function ($\Pi(w,p)$).

Scenario 2: Only Reconditioned Components

This scenario is realized by setting the value of η=1 (i.e. all parts are reconditioned), and introducing the scarcity parameters with values of S_0=2.3 and S_1=91. Again, using the values from Table 4.1, we search for the optimal solution $(w, p, \tau)^*$ that maximizes the profit, Π^*. The optimal solution obtained is: w^*=0.10, p^*=8.64, and τ^*=1.11. The introduction of reconditioned parts into the system causes the overall profit (Π^*=5 350) to significantly increase from the baseline value (5 036). Notably, this increased profitability also accounts for the scarcity costs associated with reconditioning. Moreover, the values for both the warranty length and sale price have decreased from the solution in the prior scenario. Figure 4.8 shows three surface plots of the total profit as a function of each 2-out-of-3 combination of the decision variables.



(a) Total Profit as a function of w and p (b) Total Profit as a function of w and τ



(c) Total Profit as a function of p and τ

Figure 4.8: Total Profit Solutions to Scenario 2

As observed graphically, there are distinct optimal values found at the apex of each surface in all three plots. By writing the Hessian ($H(w,p,\tau)$) and determining its eigenvalues (λ_1, λ_2, and λ_3), which are all negative, we can state that the Hessian matrix is negative definite, and thus we have a local maximum.

$$H(w,p,\tau) = \begin{bmatrix} -4\ 160\ 800.44 & 122\ 119.92 & 879.05 \\ 122\ 119.92 & -3788.95 & -25.27 \\ 879.05 & -25.27 & -109.21 \end{bmatrix}$$

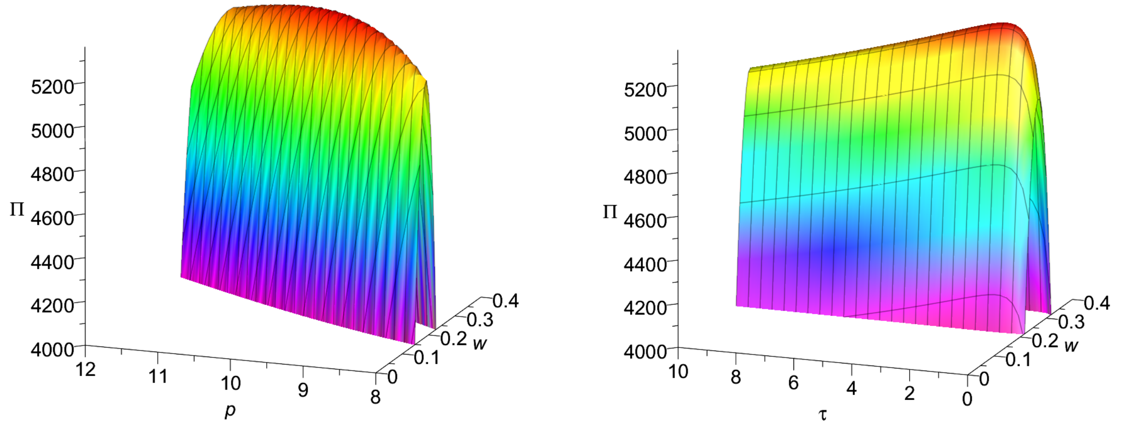
$$\lambda_1 = -109.02, \lambda_2 = -204.55, \lambda_3 = -4\ 164\ 385.04$$

Scenario 2 does not accurately represent what really takes place in practice as it is not always possible to have access to enough reconditioned components to honour the warranty obligations of all failed products. Hence, new components must be used concurrently with reconditioned components

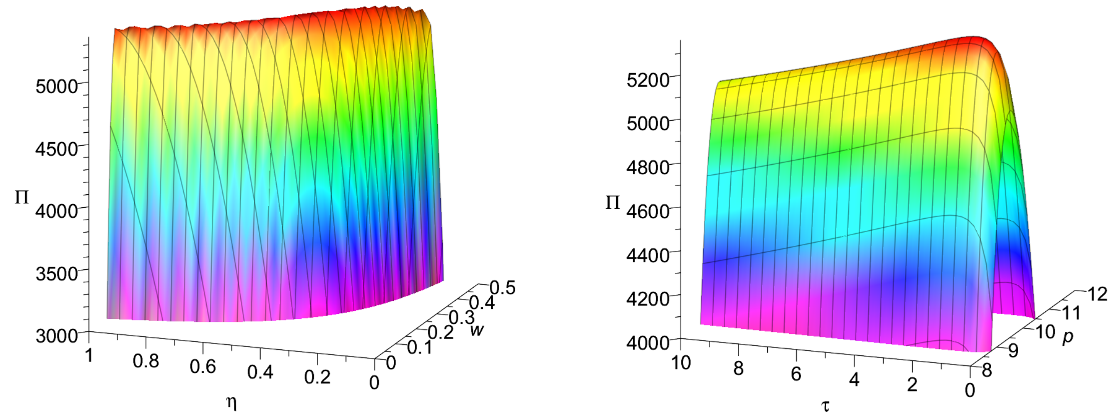
Mixed Population of New and Reconditioned Components

In the general model, η, the proportion of reconditioned components in the lot, is allowed to vary between 0% and 100%. The aim is to find the optimal solution $(w,p,\tau,\eta)^*$ that maximizes the profit, Π^* using the values from Table 4.1. The optimal solution obtained is: w^*=0.27, p^*=9.45, τ^*=1.12, η^*=0.41, and Π^*=5 364. By using a mixture that contains 41% of reconditioned components, the total profit grows from 5 350 (Scenario 2) to 5 364. Moreover, the values of w^* and p^* are slightly higher than both prior cases, and there is also a marginal increase in τ^* from Scenario 2. The access to reconditioned parts at a lower cost, allows for a slightly longer warranty period which combined with a small increase in price and demand results in a higher total profit.

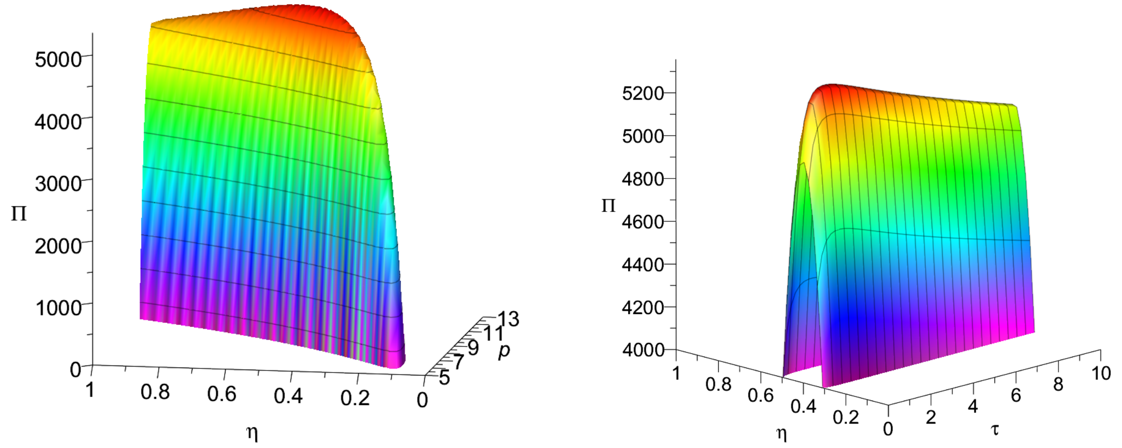
Figure 4.9 shows six surface plots of the total profit as a function of each 2-out-of-4 combination of the decision variables.



(a) Total Profit as a function of w and p (b) Total Profit as a function of w and τ



(c) Total Profit as a function of w and η (d) Total Profit as a function of p and τ



(e) Total Profit as a function of p and η (f) Total Profit as a function of τ and η

Figure 4.9: Total Profit Solutions to Scenario 3

As observed graphically, the optimal values are found at the apex of each surface in all six plots. The Hessian ($H(w, p, \tau, \eta)$) and its eigenvalues (λ_1, λ_2, λ_3, and λ_4) are given by:

$$H(w, p, \tau, \eta) = \begin{bmatrix} -811\ 420.29 & 49\ 696.13 & 395.16 & -428\ 104.79 \\ 49\ 696.13 & -3\ 162.39 & -23.36 & 25\ 831.69 \\ 395.16 & -23.36 & -106.33 & 205.93 \\ -428\ 104.79 & 25\ 831.69 & 205.93 & -227\ 943.78 \end{bmatrix}$$

$$\lambda_1 = -39.04, \lambda_2 = -110.80, \lambda_3 = -1\ 718.76, \lambda_4 = -1\ 040\ 764.18$$

All four eigenvalues are negative. Therefore, the Hessian matrix is negative definite, and we have a local maximum.

Sensitivity Analyses

Several experiments are carried out to glean insight into the behaviour of the model by varying the following parameters:

- β, the shape parameter of the Weibull distribution.
- a_1, the discount rate offered for the acquisition price of reconditioned parts.
- w, the length of the warranty coverage.
- η, the proportion of reconditioned components in the lot.

The selection of the Weibull distribution shape parameter (β) can have a significant impact on the solution as it controls the reliability of the component. Table 4.2 presents the numerical results of key variables and measures by varying β.

Table 4.2: Profit and Optimal solution for Various Values of β

β	w^*	p^*	τ^*	η^*	Π^*	$D(w,p)^*$	$N(w,\tau,\eta)^*$	$R_m(w,\tau,\eta)^*$
0.5	0.01	8.19	1.10	1.000	4 923	1 584	0.057	0.998
0.6	0.02	8.28	1.10	1.000	4 974	1 560	0.058	0.993
0.7	0.04	8.38	1.11	1.000	5 054	1 536	0.059	0.985
0.8	0.06	8.49	1.11	1.000	5 159	1 517	0.060	0.974
0.9	0.09	8.59	1.11	1.000	5 283	1 508	0.061	0.959
1.0	0.38	9.80	1.10	0.325	5 481	**1 369**	0.206	0.829
1.1	0.48	9.99	1.03	0.280	5 722	1 411	0.232	0.804
1.2	0.54	**10.01**	0.95	0.262	5 958	1 474	**0.238**	0.794
1.3	0.59	9.99	0.87	0.252	6 186	1 541	0.236	0.790
1.4	0.64	9.94	0.81	0.245	6 406	1 610	0.232	**0.789**
1.6	0.71	9.84	0.70	0.239	6 824	1 744	0.220	0.791
1.8	0.77	9.73	0.61	**0.236**	7 213	1 872	0.207	0.797
2.0	0.82	9.63	0.55	0.236	7 575	1 993	0.194	0.803
2.5	0.93	9.41	0.46	0.242	8 380	2 265	0.167	0.820
3.0	1.01	9.25	0.42	0.251	9 064	2 500	0.146	0.834
3.5	1.08	9.12	0.39	0.261	9 652	2 704	0.129	0.845
4.0	1.14	9.02	0.37	0.273	10 163	2 883	0.116	0.854
5.0	1.23	8.87	0.33	0.297	11 009	3 183	0.097	0.866
6.0	1.30	8.76	0.31	0.323	11 684	3 425	0.083	0.874
7.0	1.36	8.68	0.29	0.349	12 237	3 624	0.072	0.880
8.0	1.40	8.61	0.27	0.375	12 700	3 792	0.064	0.884
10.0	1.48	8.52	0.24	0.428	13 434	4 059	0.053	0.889

From Table 4.2 we distinguish two distinct cases: $\beta<1$ and $\beta\geq1$

For $\beta<1$, the failure rate is decreasing and therefore reconditioned components are more reliable than new ones. In the classic failure rate bathtub curve, this case corresponds to the decreasing failure rate zone where a new component is worse than a used component (NWU). For a discussion on aging in reliability theory, see Pham (2003). The model then acts by using reconditioned components as much as possible. For this case $\eta=1$ with $\tau\approx1.1$. Very little warranty is offered because there is a need to keep the demand for reconditioned components within the limit of what scarcity permits. As β increases towards a value of 1, the price increases slightly to compensate for the small drop in reliability and the related increase in reserve funds.

For $\beta \geq 1$, the failure rate is non-decreasing and the reliability deteriorates at a higher rate. The model then needs more new parts and younger reconditioned parts to be injected into the mixture (i.e. η and τ decrease), which reduces the replacement costs. As β increases, we see an increase in w^*, $D(w,p)^*$, and Π^*, which is an effort by the model to grow the sales volume and profit. Moreover, p^* reaches its largest value at $\beta \approx 1.2$, at which point it starts to decrease. This parallels the average number of replacements ($N(w,\tau,\eta)^*$), which is the expected number of failures. The reason for such behaviour is because the expected number of failures is an inverse measure of reliability, and thus the need to charge more for a less reliable product.

The choice of the acquisition discount rate offered for the reconditioned parts (a_1) can also have a large impact on the solution as it controls the savings offered on the acquisition of second-hand components. Table 4.3 presents the numerical results of key variables and measures by varying values of a_1.

Table 4.3: Profit and Optimal solution for Various Values of a_1

a_1	w^*	p^*	τ^*	η^*	Π^*	$D(w,p)^*$	$R_m(w,\tau,\eta)^*$
1.0	0.28	9.47	2.01	0.402	5 202	1 382	0.865
1.5	0.28	9.47	1.81	0.403	5 267	1 382	0.865
2.0	0.28	9.46	1.57	0.405	5 307	1 383	0.866
2.5	0.28	9.46	1.36	0.407	5 335	1 384	0.866
3.0	0.27	9.45	1.20	0.408	5 354	1 385	0.866
3.3	0.27	9.45	1.12	0.409	5 364	1 385	0.866
3.5	0.27	9.45	1.07	0.410	5 369	1 386	0.866
5.0	0.27	9.44	0.81	0.413	5 398	1 387	0.867
10.0	0.27	9.42	0.45	0.418	5 436	1 390	0.867
100.0	0.26	9.37	0.06	0.430	5 481	1 397	0.868

As the discount rate for reconditioned components grows, the value of η becomes marginally larger. Since there appears to be an effort by the model to keep the expected reliability value of the mixed population ($R_m(w,\tau,\eta)^*$) fairly constant, we observe a minor decrease in w^* and τ^* to compensate for the increase in η^*. Moreover, the sale price p^* drops slightly in order to moderately increase demand and profitability. It should be noted that the variation of a_1 has the smallest impact on the decision variables as well as the total profitability.

The role of the acquisition cost is central to understanding of the next set of results. Recall that the expression of the average unit acquisition cost is given by:

$$A(\tau, \eta) = (1 - \eta)\, A_0 + \eta\left(A_0\, (\tau + 1)^{-a_1} + \tau^{a_2}\right)$$

This acquisition cost is highest when there are very old reconditioned parts, or a very high proportion of very young reconditioned parts. As illustrated in Figure 4.10, the minimum value of $A(\tau, \eta)$ occurs for mid-range values of τ and η.

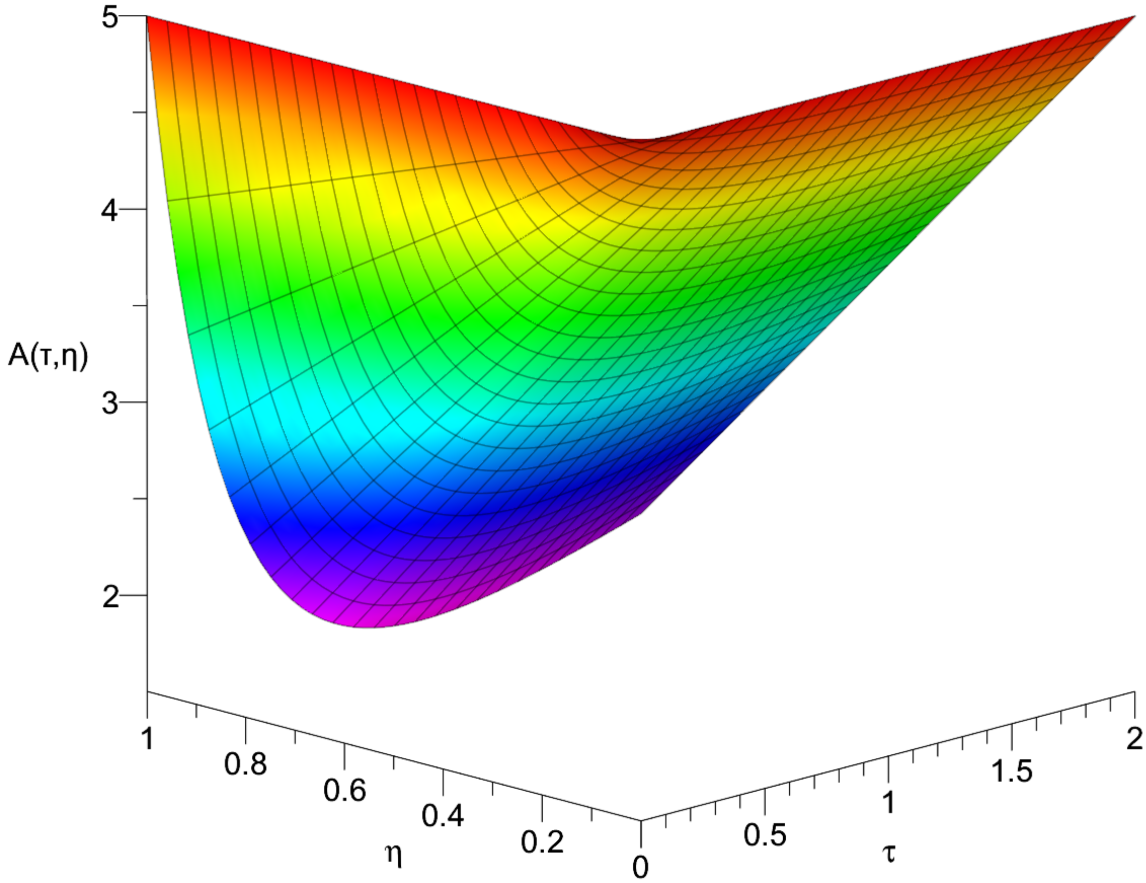


Figure 4.10: Acquisition Cost as a function of τ and η

Although $A(\tau, \eta)$ is independent of both w and p, it plays an important role in the model's sensitivity to the decision variable w. A set of experiments is conducted to study how the model reacts to varying values of w. Table 4.4 summarizes the numerical results obtained.

Table 4.4: Profit Solution of p^*, τ^*, and η^*, by varying w

w	p^*	τ^*	η^*	Π^*	$D(w,p)^*$	$R_m(w,\tau,\eta)^*$	$A(\tau,\eta)^*$
0.1	8.55	1.11	1.0000	5349	1547	0.952	1.5012
0.2	9.09	1.12	0.5411	5361	1440	0.901	3.1069
0.3	9.58	1.12	0.3774	5363	1366	0.854	3.6797
0.4	10.09	1.13	0.2949	5355	1295	0.811	3.9683
0.5	10.62	1.14	0.2455	5334	1225	0.770	4.1412
0.6	11.17	1.15	0.2129	5303	1156	0.732	4.2555
0.7	11.75	1.15	0.1899	5261	1090	0.696	4.3359
0.8	12.35	1.16	0.1730	5210	1026	0.662	4.3949
0.9	12.98	1.16	0.1603	5150	964	0.630	4.4395
1.0	13.63	1.17	0.1505	5081	905	0.599	4.4737
1.1	14.32	1.17	0.1429	5006	848	0.571	4.5005
1.2	15.04	1.18	0.1369	4924	793	0.544	4.5215
1.3	15.79	1.18	0.1322	4837	741	0.518	4.5380
1.4	16.57	1.19	0.1286	4745	691	0.493	4.5508
1.5	17.39	1.19	0.1257	4650	644	0.470	4.5607
1.6	18.25	1.20	0.1236	4550	600	0.448	4.5681
1.7	19.14	1.20	0.1222	4448	558	0.427	4.5733
1.8	20.08	1.21	0.1212	4344	518	0.407	4.5767
1.9	21.06	1.21	0.1207	4239	481	0.388	4.5785
2.0	22.09	1.22	**0.1206**	4132	446	0.370	**4.5788**
2.1	23.16	1.22	0.1209	4024	413	0.353	4.5778
2.2	24.28	1.22	0.1216	3916	382	0.337	4.5756
2.3	25.46	1.23	0.1226	3808	354	0.321	4.5722
2.4	26.68	1.23	0.1239	3700	327	0.307	4.5677
2.5	27.97	1.24	0.1255	3593	302	0.293	4.5621
3.0	35.33	1.25	0.1378	3077	201	0.232	4.5196
4.0	56.08	1.29	0.1847	2195	85	0.147	4.3573
5.0	88.50	1.31	0.2714	1545	35	0.094	4.0567
6.0	139.02	1.34	0.4246	1102	14	0.061	3.5265
7.0	217.55	1.36	0.6982	815	5	0.040	2.5809
8.0	318.24	1.36	1.0000	632	2	0.026	1.5347

The future costs to honour a warranty policy is dependent on its duration. As the warranty length is increased, the model needs a way to improve the reliability in order to reduce costs. As a response to this, η^* is decreased to have more new parts in the mixture. The proportion of reconditioned parts, η^*, reaches its minima (when $w \approx 2.0$), and at the same time the acquisition cost $A(\tau,\eta)^*$ reaches its maxima. Subsequently, η^* starts to increase as $A(\tau,\eta)^*$ decreases. We also observe an increase in the sale price and τ^* to compensate for the longer warranty period.

One last set of experiments is conducted to study how the model reacts to varying values of η. Table 4.5 summarizes the numerical results obtained.

Table 4.5: Profit Solution of w^*, p^*, and τ^*, by varying η

	η	w^*	p^*	τ^*	Π^*
Scenario 1	**0.00**	**0.26**	**9.36**	–	**5 035.95**
	0.10	1.04	12.15	1.15	4 813.87
	0.20	0.58	10.73	1.14	5 298.88
	0.30	0.38	9.93	1.13	5 356.49
	0.40	0.28	9.48	1.12	5 363.83
Optimal	**0.41**	**0.27**	**9.45**	**1.12**	**5 363.85**
	0.42	0.27	9.42	1.12	5 363.82
	0.43	0.26	9.38	1.12	5 363.73
	0.44	0.25	9.35	1.12	5 363.60
	0.45	0.25	9.33	1.12	5 363.43
	0.50	0.22	9.20	1.12	5 362.23
	0.60	0.18	9.01	1.12	5 359.12
	0.70	0.15	8.88	1.11	5 356.12
	0.80	0.13	8.78	1.11	5 353.56
	0.90	0.12	8.70	1.11	5 351.45
Scenario 2	**1.00**	**0.10**	**8.64**	**1.11**	**5 349.72**

In general, we see a decrease in the warranty length, sale price, and age of reconditioned components when η increases. It can also be observed that the solution with 0% reconditioned components (i.e. all new components), can be more profitable than when there is an inappropriately low percentage of reconditioned components.

Summary of Findings

The results from the numerical experiments conducted demonstrate that the model behaves as expected and yields conclusions in line with prior knowledge from reliability theory. Furthermore, the model shows how an optimal mixture of new and reconditioned parts can yield substantially higher profits.

The sensitivity analyses showed that:

- When increasing the value of β, we discern an increase in w^* and Π^*. The value of τ^* also decreases, which is the model's reaction to reduce the costs of replacements.
- When increasing the discount factor a_1, we see an increase in η^*, Π^*, and $D(w,p)^*$. The higher demand and profitability are attributed to the reduction in the sale price p^*. The model keeps the reliability value of the mixed population ($R_m(w,\tau,\eta)^*$) fairly constant by decreasing w^* and τ^*.
- When increasing the warranty length w, we witness an increase in p^* and τ^*, and a decrease in Π^*, $D(w,p)^*$, and $R_m(w,\tau,\eta)^*$. The higher sale price and older reconditioned parts are an attempt to hedge against higher future warranty servicing costs as the warranty length w is increased.
- When increasing η, we generally observe a decrease in the warranty length, sale price, and age of reconditioned components.

4.4 Conclusions & Future Research

This research presented a mathematical model for a one-dimensional UFRW policy, under which replacement components are randomly selected from a lot that is composed of new and reconditioned parts. The product demand is modelled to represent consumer preference towards a longer warranty period, and a lower sale price. This model also incorporates scarcity costs to acquire reconditioned components to account for the initialization of the recovery network, as well as the deviation from the ideal volume of recovered products. This formulation is solved numerically using the NLP solver in *Maplesoft Maple 18* to yield optimal solutions. This model substantiates the use of reconditioned components in the production process by demonstrating the profitability from a warranty cost analysis perspective. Furthermore, the utilization of remanufacturing activities aids in sustainable development through the reduction in pollution, consumption, and the ecological footprint of products.

Future extensions for this research involve several new warranty models suited for remanufactured products. This includes the development of mathematical models for PRW and hybrid PRW policies, integration of reconditioned products of different quality or degradation levels, multi-component systems, and the reuse of non-failed, but reported for claim products. We have also started the development of imperfect maintenance strategies for the second-life of remanufactured or refreshed products.

CHAPTER 5

CONCLUSIONS

This dissertation dealt with three engineering areas to enable sustainable manufacturing. From the perspective of the manufacturing organization, the venture to incorporate sustainable practices must add some sort of value (economical, ecological, promotional, etc.), be part of a comprehensive strategic vision, and the new green activities/processes themselves must be monitored and measured. The proposed SPLC framework illustrates the planning and management issues occurring at the collection, remanufacturing, and reuse stages.

In the framework for sustainable operations planning, the ISOP level covers the implementation of the organization's sustainability vision, monitors the progress of the sustainability plan, provides feedback to all stakeholders and employees, and promotes and creates a culture of sustainability to steer the whole organization toward its goals. The ISOP level in turn motivates and ties together three areas of sustainable supply chain operations planning: logistics, remanufacturing, and support systems.

The importance of these three areas becomes evident when considering the specific problem of remanufacturing. Primarily, in order to enable remanufacturing, there is a requirement to obtain EoU/EoL products (logistics planning). Next, the acquired EoU/EoL products need to have their value recovered and incorporated into a production plan (remanufacturing planning). Finally, the customer needs to be assured that the remanufactured products they have purchased will be supported for a certain duration (support systems planning). These planning activities have implications on customer needs, market factors influencing sales for remanufactured products (quality, price, demand, warranty, etc.), and elements such as facility location, production cycle, and design.

This research conducts a thematic exploration of these three areas/themes, and for each of the three themes a specific problem is further investigated. The problems addressed are: *network configuration for product recovery*, *planning for production using remanufactured systems*, and *warranty policies for remanufacturing.*

5.1 Theme 1: Network Configuration for Product Recovery

In order to enable sustainable manufacturing, it is essential to design an efficient EoU/EoL collection network. This is particularly useful in the recovery of MSW, which includes organic material, recyclables, and EoU/EoL electronics. Collection networks can involve multiple parties, 3PLs, or non-profit collection agencies. In an unplanned system, a multitude of collection centres can come into existence, making collection challenging and expensive. These centres may correlate to different retailers who are either mandated by law to recover products, or by private facilities who view the recovery as an opportunity for profitability. While encouraging returns is better for amassing more product, it places a substantial burden on their subsequent collection and reprocessing. A plethora of collection centres, especially if they are sparsely located, require much more travel, which causes operating costs and emissions to increase. The collection network can be greatly improved by managing the volume of recovered products, and minimizing travel and other operational costs through optimal mathematical models.

In *Chapter 2*, we presented a brief survey of VRP models used in the design of collection and delivery networks. A mathematical model was then developed to help RRFB address the optimization of its recyclables collection network. In this model, only collection centres that have a sufficient supply of products to maximize profitability are considered. This model operates in two phases: route generation and route selection. In the route generation phase, the *Esri ArcGIS Network Analyst VRP solver* is used to generate routes for each product type. The objective is to maximize total collection profitability, subject to vehicle capacity and time constraints. There is also a constraint on the maximum number of routes, which decreases in each iteration. In the route selection phase, the product-route combinations generated in the first phase are used to create weekly schedules for each vehicle. This phase also determines the number of vehicles required to optimally service the province of Nova Scotia given the current collection rates (product value).

In this model, the number of vehicles, routes, and products recovered can be raised by increasing product value. This discovery is important because it also tells the end-use/remanufacturing facility that if they want access to a greater inventory of recovered products, they will have to pay more for it. A higher sale price also mitigates against remanufacturing uncertainties such as quantity, quality, and timing.

There are many aspects with respect to the planning of a reverse logistics network, and thus several significant issues for future research such as: who is responsible for the collection, what products are collected, how are they collected, when are they collected, where are they collected, and why would the consumer return the product?

- In this dissertation we have discussed the legislative responsibilities manufacturers have for the collection of EoU/EoL products, whether this is individual or collective based, and also who provides the funding (consumer, manufacturer, or government). There are also situations that may arise where government would need to intervene to ensure organizations are liable if they fail to meet their obligations. Future research in this area could pertain to designing network models that incorporate penalties for not meeting collection obligations, or subsidies to facilitate the collection of products from isolated regions. A marketplace/exchange for the resale of EoU/EoL products is another economic model that can be developed to represent a system with government oversight.

- Having a marketplace for EoU/EoL products will have some very interesting outcomes. The first will be to assign a value to these products, so there is a financial interest by a third-party to incentivize the collection. Secondly, it will dictate what products are collected, or targeted for collection. Thirdly, this may motivate the segmentation of EoU/EoL products into different grades prior to collection. Fourthly, companies may be willing to have a higher cost structure to collect premium EoU/EoL products. Finally, the outcomes of this research may also decide how the products are collected.

- We examined one collection mechanism (vehicle routing though collection depots), however there are many other methods of acquisition. We would suggest the investigation of whether EoU/EoL products be returned to central collection depots, picked-up curbside, or even returned via the postal service. These in themselves will prove to be interesting mathematical models. If collection centres are utilized, what should their proximity and proliferation to the consumer be, and how does this affect the return volume of the EoU/EoL products. Should the collection be conducted by a 3PL, private non-profit, government, the firm that sells the device, or the one that manufactures it?

- The determination of the collection mechanism will then lead to the location and frequency of collection. A collection system through the postal service for example has drop-off points that may have a greater proliferation and frequency than central depots, but also arrives with higher costs. Furthermore, there is also the aspect of collection reliability that needs to be tackled. Finally, we need to address what motivates the consumer to return the products.

- The consumer may return the product for environmental reasons, or for monetary gain. Since we are considering EoU/EoL products as a commodity within the marketplace, we need to address whether consumers should be compensated for their return, and whether the compensation value should be based on the quality level of the product. This can lead to models which determine the ideal monetary amount to facilitate consumer returns, and how it varies with the quality and quantity of returns. Next, we can consider how different types of compensation (cash payment, rebate on future products, etc.) can influence the volume of returns. Alternatively, instead of an outright sale (with compensation for returns), the effects of applying a leasing model to consumer electronics can also be studied. Finally, merging quantitative motivational models, such as Vroom's Expectancy Theory may help us better understand how to maximize product collection from a motivational perspective.

5.2 Theme 2: Planning for Production using Remanufactured Systems

One of the issues faced by strategic planners in sustainable manufacturing is how to develop a long-term remanufacturing production plan with recovered EoU/EoL products. In the development of a plan, it is pivotal to account for uncertainties such as quantity, quality, and timing. In *Chapter 2*, we observed that a higher value for EoU/EoL products allows the recovery firm to schedule more routes, and thus acquire more products. Therefore, we can ascertain that if a remanufacturing firm is willing to pay more for EoU/EoL products, the uncertainties of quantity and timing can be mitigated by having access to a greater volume of recovered products. Furthermore, if the remanufacturing firm were willing to pay even more, the supplier will sort and test the recovered products, and only provide premium quality EoU/EoL products. Another method to hedge against these uncertainties is for the production plan to dynamically adjust based on the availability of EoU/EoL products in the marketplace.

In *Chapter 3*, we extended a two-stage production control model from literature to incorporate both new and reconditioned components. The key decisions for this problem are the duration of the production cycle, the length of the free replacement warranty offered to the customer, and the remanufacturing starting instant. The state variables of this formulation are the sale price of the product, as well as the production of new and reconditioned spare parts. We observed that for a given set of parameters, there is a clear optimal solution for the three decision variables, and in all examined cases, remanufacturing resulted in a greater profit over the scenario where no reconditioning takes place.

In our model, only the failed products from the current product generation are considered for reconditioning. This in turn requires an additional cost to collect, sort, and return the products back to the remanufacturing firm. Because this is the first generation being reconditioned, there are not any legacy products in the market, and thus none available at time zero. Furthermore, because products take time to fail, it takes even longer for them to become accessible. However, if the remanufacturing firm were to design their second generation of products to utilize components from their first, then there would be a greater quantity of components available in the marketplace. This clearly points to the necessity of EcoDesign principles such as design for disassembly, as well as the standardization of components and modules across industry to essentially treat them as if they were commodities. This would encourage and facilitate sustainable manufacturing.

From the research that has been conducted, one area which could have a substantial impact on remanufacturing production planning is to have more emphasis on the utilization of EcoDesign principles in the manufacture of the initial product. As discussed in the research, DfR, disassemblability, and component/module standardization, would better enable value recovery and remanufacturing. Hence, we feel that there are future research opportunities in investigating ways to reward sustainable design across the industry; and also how the recovered products, modules, and components can be more integrated with the traditional manufacturing production plan. Inevitably, there are also barriers such as market share cannibalization which exist, and research is needed to understand how to mitigate and/or remove them.

5.3 Theme 3: Warranty Policies for Remanufacturing

Support system design is the third key enabler within the ISOP framework. In our review we have found that a negative public perception may exist for remanufactured products. This understanding is attributed to the fact that remanufactured products are perceived as having a lower reliability, and thus more prone to failure. A longer warranty period is sometimes offered to make remanufactured products more appealing. From the customers' point of view, increased coverage from the manufacturer is more reassuring, and a longer warranty period is synonymous with higher quality and reliability. In this chapter, the role of the warranty policy in establishing the overall profitability of a firm is examined, and a model is developed for the case when a product is repaired/replaced with parts coming from a mixture of both new and reconditioned components. It is expected that this model will be of importance to strategic planners within a sustainable manufacturing organization interested in using components from EoU/EoL products to honour various warranty policies.

In *Chapter 4*, the problem is formulated and solved as a nonlinear mathematical model to maximize the profitability of the remanufacturer by optimally choosing the warranty length, product sale price, age of reconditioned components, and the proportion of reconditioned components in the mixture. This analysis showed how optimal warranty policies can be devised to improve the profitability of remanufacturing activities. This research is important as it will aid strategic planners within an organization to determine an appropriate warranty policy that will be beneficial to the firm. This policy can be tweaked to fit the requirements of the firm's desired sale prices, as well as to the reconditioned components that are available to them.

This theme focused on a four-dimensional analysis pertaining to corrective repair of products with a UFRW policy using reconditioned components. In the model, the sale price, warranty length, the age of the reconditioned components, as well as its proportion used was optimized for. A logical extension to this would be to optimize for an age distribution of reconditioned components as it is not likely to receive a specific quantity of a particular component of a given age. Another extension would be to have different proportions of components of different ages.

The other type of warranty policy that is offered instead of an FRW is the PRW. If we were to hypothetically offer a PRW, is it better to offer a monetary incentive for return, a discounted repair, or a discount on replacement? A lease model may be better suited to reclaim EoU/EoL products, we can now also examine whether it is better suited to a FRW or PRW policy.

Moreover, as we dealt solely with corrective repair, models with preventative maintenance may also be explored to reduce total costs over the lifetime of the product. This encompasses: how is the optimal level of repair determined, how does the level of repair vary between different warranty policies, should there be multiple upgrade actions to continuously extend the longevity of the product, and if so, what is the ideal number of upgrade actions? Additionally, we can also probe how to design products to facilitate the implementation of preventative maintenance procedures.

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APPENDIX A

RRFB CASE STUDY SENSITIVITY ANALYSIS

We start by reducing the bag-price (β) to half its value, and display the results in Table A.1 for clear PET and Table A.2 for aluminium.

Table A.1: GIS Generated Clear PET Routes, β=0.50Φ

	All Routes				**Profitable Routes**			
S_1	r_s	q_s	G_s	π_s	r'_s	q'_s	G'_s	π'_s
PF15	15	2 085	17.3	-1 056Φ	2	649	2.9	112Φ
PF14	14	2 064	16.4	-900Φ	2	649	2.9	112Φ
PF13	13	2 054	13.9	-703Φ	2	649	2.9	112Φ
PF12	12	2 054	13.6	-643Φ	2	751	3.0	151Φ
PF11	11	2 017	13.0	-531Φ	2	758	3.0	152Φ
PF10	10	2 010	13.1	-368Φ	3	1 064	5.1	166Φ
PF09	9	1 900	12.3	-288Φ	3	1 067	5.1	175Φ
PF08	8	1 859	10.4	-133Φ	3	1 086	4.9	181Φ
PF07	7	1 826	11.7	-4Φ	3	982	4.9	109Φ
PF06	6	1 779	11.3	70Φ	4	1 366	7.7	135Φ
PF05	5	1 546	8.9	76Φ	4	1 356	7.8	129Φ
PR10	10	1 919	13.1	-389Φ	3	990	4.6	163Φ
PR09	9	1 900	11.8	-236Φ	3	1 028	4.6	180Φ
PR08	8	1 871	12.2	-143Φ	4	1 319	7.7	148Φ
PR07	7	1 786	10.0	15Φ	3	1 101	5.2	183Φ
PR06	6	1 688	8.7	87Φ	3	1 135	5.1	203Φ
PR05	5	1 624	9.1	170Φ	4	1 438	7.7	204Φ
PR04	4	1 369	7.1	158Φ	3	1 183	5.7	193Φ

Table A.2: GIS Generated Aluminium Routes, β=0.50Φ

	All Routes				**Profitable Routes**			
S_2	r_s	q_s	G_s	π_s	r'_s	q'_s	G'_s	π'_s
AF09	9	1 493	8.3	-400Φ	2	642	2.3	99Φ
AF08	8	1 473	8.0	-334Φ	2	690	2.0	104Φ
AF07	7	1 452	9.0	-196Φ	2	686	2.6	93Φ
AF06	6	1 286	7.3	-231Φ	3	978	4.6	75Φ
AF05	5	1 371	8.1	-7Φ	3	1 016	4.9	87Φ
AF04	4	1 125	5.7	-32Φ	2	791	2.9	111Φ
AF03	3	879	3.9	-22Φ	2	800	3.1	91Φ
AR06	6	1 305	6.3	-27Φ	3	959	4.7	123Φ
AR05	5	1 291	7.2	24Φ	3	973	5.2	117Φ
AR04	4	1 242	7.2	299Φ	4	1 242	7.2	299Φ
AR03	3	1 050	5.1	104Φ	3	1 050	5.1	104Φ
AR02	2	681	3.3	94Φ	2	681	3.3	94Φ

For both products we now see that there are only two profitable routes in the zeroth scenario for the *Full* network and three for the *Reduced*; and the most profitable scenarios are all from the *Reduced* network. For clear PET these are *PR05*, closely followed by *PR06*, and for aluminium it is *AR04*. This indicates that the model wants limit the quantity to pick-up as the product has very little value. The optimal solution for this value of β comprises a single CT, and attains a Π^* value of 162Φ. The optimal schedule is shown below in Table A.3.

Table A.3: Optimal Schedule, β=0.50Φ

	Vehicle 1				
	Day 1	Day 2	Day 3	Day 4	Day 5
Week 1	PR06i	PR06ii	PR06iv	AR04i	AR04iv

Now, we determine the results when β is equal to 0.75Φ, and display them in Tables A.4 & A.5 for clear PET and aluminium respectively.

Table A.4: GIS Generated Clear PET Routes, β=0.75Φ

	All Routes				**Profitable Routes**			
S_1	r_s	q_s	G_s	π_s	r'_s	q'_s	G'_s	π'_s
PF15	15	2 085	17.3	-534Φ	5	1 229	7.8	327Φ
PF14	14	2 064	16.4	-384Φ	5	1 229	7.8	327Φ
PF13	13	2 054	13.9	-190Φ	5	1 215	7.1	333Φ
PF12	12	2 054	13.6	-138Φ	4	1 208	6.5	406Φ
PF11	11	2 017	13.0	-27Φ	3	1 029	5.1	397Φ
PF10	10	2 010	13.1	134Φ	5	1 499	8.6	487Φ
PF09	9	1 900	12.3	187Φ	5	1 484	8.6	482Φ
PF08	8	1 859	10.4	331Φ	4	1 272	6.3	464Φ
PF07	7	1 826	11.7	452Φ	6	1 655	10.5	467Φ
PF06	6	1 779	11.3	514Φ	6	1 779	11.3	514Φ
PF05	5	1 546	8.9	463Φ	4	1 356	7.8	468Φ
PR10	10	1 919	13.1	91Φ	5	1 429	8.2	471Φ
PR09	9	1 900	11.8	239Φ	5	1 473	8.2	506Φ
PR08	8	1 871	12.2	325Φ	4	1 319	7.7	478Φ
PR07	7	1 786	10.0	461Φ	5	1 489	7.2	509Φ
PR06	6	1 688	8.7	509Φ	5	1 512	7.6	511Φ
PR05	5	1 624	9.1	576Φ	5	1 624	9.1	576Φ
PR04	4	1 369	7.1	501Φ	4	1 369	7.1	501Φ

Table A.5: GIS Generated Aluminium Routes, β=0.75Φ

	All Routes				**Profitable Routes**			
S_2	r_s	q_s	G_s	π_s	r'_s	q'_s	G'_s	π'_s
AF09	9	1 493	8.3	-26Φ	4	1 034	4.5	325Φ
AF08	8	1 473	8.0	34Φ	4	1 072	5.0	344Φ
AF07	7	1 452	9.0	167Φ	4	1 178	6.6	383Φ
AF06	6	1 286	7.3	90Φ	3	978	4.6	320Φ
AF05	5	1 371	8.1	336Φ	4	1 216	7.1	382Φ
AF04	4	1 125	5.7	250Φ	3	1 029	4.8	327Φ
AF03	3	879	3.9	197Φ	2	800	3.1	291Φ
AR06	6	1 305	6.3	299Φ	4	1 073	4.9	387Φ
AR05	5	1 291	7.2	347Φ	4	1 182	6.4	410Φ
AR04	4	1 242	7.2	682Φ	4	1 242	7.2	682Φ
AR03	3	1 050	5.1	367Φ	3	1 050	5.1	367Φ
AR02	2	681	3.3	264Φ	2	681	3.3	264Φ

For this occurrence, we perceive that there are five profitable routes in the zeroth scenario for both the *Full* and *Reduced* networks for clear PET; and four profitable routes for both networks for aluminium. For clear PET the majority of the profitable scenarios are from the *Reduced* network, with *PR05* being the most profitable. However, here, the second most profitable scenario is *PF06* from the *Full* network. With respect to aluminium, *AR04* still demonstrates its dominance over its counterparts. This time, the optimal solution requires two CTs, and attains a Π^* value of 694Φ. The optimal schedule is shown below in Table A.6.

Table A.6: Optimal Schedule, β=0.75Φ

	Vehicle 1				
	Day 1	Day 2	Day 3	Day 4	Day 5
Week 1	PR05i	PR05ii	PR05iii	PR05iv	PR05v
Week 2	PR05i	PR05ii	PR05iii	PR05iv	–

	Vehicle 2				
	Day 1	Day 2	Day 3	Day 4	Day 5
Week 1	AR04i	AR04ii	AR04iii	AR04iv	–
Week 2	AR04i	AR04ii	AR04iii	AR04iv	–

Now, we raise the value of β by 25% over its original value of Φ. Again, Tables A.7 & A.8 show the results for both products.

Table A.7: GIS Generated Clear PET Routes, β=1.25Φ

	All Routes				**Profitable Routes**			
S_1	r_s	q_s	G_s	π_s	r'_s	q'_s	G'_s	π'_s
PF15	15	2 085	17.3	508Φ	9	1 779	11.9	1 068Φ
PF14	14	2 064	16.4	648Φ	9	1 779	11.9	1 068Φ
PF13	13	2 054	13.9	837Φ	10	1 846	11.3	1 043Φ
PF12	12	2 054	13.6	872Φ	9	1 879	11.0	1 118Φ
PF11	11	2 017	13.0	982Φ	8	1 757	10.6	1 112Φ
PF10	10	2 010	13.1	1 139Φ	8	1 912	11.4	1 326Φ
PF09	9	1 900	12.3	1 137Φ	7	1 767	10.5	1 291Φ
PF08	8	1 859	10.4	1 261Φ	8	1 859	10.4	1 261Φ
PF07	7	1 826	11.7	1 365Φ	7	1 826	11.7	1 365Φ
PF06	6	1 779	11.3	1 404Φ	6	1 779	11.3	1 404Φ
PF05	5	1 546	8.9	1 236Φ	5	1 546	8.9	1 236Φ
PR10	10	1 919	13.1	1 050Φ	8	1 816	11.3	1 243Φ
PR09	9	1 900	11.8	1 189Φ	8	1 828	10.8	1 255Φ
PR08	8	1 871	12.2	1 260Φ	7	1 754	11.2	1 263Φ
PR07	7	1 786	10.0	1 354Φ	7	1 786	10.0	1 354Φ
PR06	6	1 688	8.7	1 353Φ	6	1 688	8.7	1 353Φ
PR05	5	1 624	9.1	1 388Φ	5	1 624	9.1	1 388Φ
PR04	4	1 369	7.1	1 185Φ	4	1 369	7.1	1 185Φ

Table A.8: GIS Generated Aluminium Routes, β=1.25Φ

	All Routes				**Profitable Routes**			
S_2	r_s	q_s	G_s	π_s	r'_s	q'_s	G'_s	π'_s
AF09	9	1 493	8.3	720Φ	6	1 304	6.2	905Φ
AF08	8	1 473	8.0	771Φ	5	1 203	5.7	899Φ
AF07	7	1 452	9.0	893Φ	5	1 314	7.7	989Φ
AF06	6	1 286	7.3	733Φ	4	1 117	5.7	824Φ
AF05	5	1 371	8.1	1 021Φ	5	1 371	8.1	1 021Φ
AF04	4	1 125	5.7	812Φ	3	1 029	4.8	841Φ
AF03	3	879	3.9	637Φ	2	800	3.1	691Φ
AR06	6	1 305	6.3	952Φ	5	1 204	5.5	971Φ
AR05	5	1 291	7.2	992Φ	4	1 182	6.4	1 001Φ
AR04	4	1 242	7.2	1 448Φ	4	1 242	7.2	1 448Φ
AR03	3	1 050	5.1	892Φ	3	1 050	5.1	892Φ
AR02	2	681	3.3	605Φ	2	681	3.3	605Φ

For clear PET, we discover that there are nine profitable routes in the zeroth scenario for the *Full*, and eight for the *Reduced* network. This time we see *PF06* becoming the most profitable scenario over *PR05*. Comparatively, there are six profitable routes in the zeroth scenario for the *Full*, and five for the *Reduced* network. Nevertheless, *AR04* still remains as the most profitable scenario for aluminium. The optimal solution still requires two CTs, and attains a Π^* value of 2 288Φ. The optimal schedule is shown below in Table A.9.

Table A.9: Optimal Schedule, β=1.25Φ

	Vehicle 1				
	Day 1	Day 2	Day 3	Day 4	Day 5
Week 1	PF06i	PF06ii	PF06iii	PF06iv	PF06v
Week 2	PF06i	PF06ii	PF06iii	PF06iv	PF06v

	Vehicle 2				
	Day 1	Day 2	Day 3	Day 4	Day 5
Week 1	AR04i	AR04ii	AR04iii	AR04iv	PF06vi
Week 2	AR04i	AR04ii	AR04iii	AR04iv	–

Now, we raise the value of β by 50% over its original value. Tables A.10 & A.11 present the consequent results.

Table A.10: GIS Generated Clear PET Routes, β=1.50Φ

	All Routes				**Profitable Routes**			
S_1	r_s	q_s	G_s	π_s	r'_s	q'_s	G'_s	π'_s
PF15	15	2 085	17.3	1 029Φ	11	1 979	13.7	1 516Φ
PF14	14	2 064	16.4	1 164Φ	11	1 979	13.7	1 516Φ
PF13	13	2 054	13.9	1 351Φ	11	1 948	12.1	1 507Φ
PF12	12	2 054	13.6	1 378Φ	10	1 981	11.9	1 582Φ
PF11	11	2 017	13.0	1 486Φ	10	1 969	12.1	1 575Φ
PF10	10	2 010	13.1	1 642Φ	8	1 912	11.4	1 804Φ
PF09	9	1 900	12.3	1 612Φ	7	1 767	10.5	1 733Φ
PF08	8	1 859	10.4	1 726Φ	8	1 859	10.4	1 726Φ
PF07	7	1 826	11.7	1 822Φ	7	1 826	11.7	1 822Φ
PF06	6	1 779	11.3	1 849Φ	6	1 779	11.3	1 849Φ
PF05	5	1 546	8.9	1 622Φ	5	1 546	8.9	1 622Φ
PR10	10	1 919	13.1	1 530Φ	8	1 816	11.3	1 697Φ
PR09	9	1 900	11.8	1 664Φ	8	1 828	10.8	1 712Φ
PR08	8	1 871	12.2	1 728Φ	8	1 871	12.2	1 728Φ
PR07	7	1 786	10.0	1 801Φ	7	1 786	10.0	1 801Φ
PR06	6	1 688	8.7	1 775Φ	6	1 688	8.7	1 775Φ
PR05	5	1 624	9.1	1 794Φ	5	1 624	9.1	1 794Φ
PR04	4	1 369	7.1	1 527Φ	4	1 369	7.1	1 527Φ

Table A.11: GIS Generated Aluminium Routes, β=1.50Φ

	All Routes				**Profitable Routes**			
S_2	r_s	q_s	G_s	π_s	r'_s	q'_s	G'_s	π'_s
AF09	9	1 493	8.3	1 093Φ	6	1 304	6.2	1 231Φ
AF08	8	1 473	8.0	1 139Φ	5	1 203	5.7	1 199Φ
AF07	7	1 452	9.0	1 256Φ	5	1 314	7.7	1 317Φ
AF06	6	1 286	7.3	1 055Φ	4	1 117	5.7	1 104Φ
AF05	5	1 371	8.1	1 364Φ	5	1 371	8.1	1 364Φ
AF04	4	1 125	5.7	1 093Φ	3	1 029	4.8	1 099Φ
AF03	3	879	3.9	857Φ	2	800	3.1	891Φ
AR06	6	1 305	6.3	1 278Φ	6	1 305	6.3	1 278Φ
AR05	5	1 291	7.2	1 315Φ	5	1 291	7.2	1 315Φ
AR04	4	1 242	7.2	1 831Φ	4	1 242	7.2	1 831Φ
AR03	3	1 050	5.1	1 154Φ	3	1 050	5.1	1 154Φ
AR02	2	681	3.3	775Φ	2	681	3.3	775Φ

For clear PET, we distinguish that there are now 11 profitable routes in the zeroth scenario for the *Full*, but still eight for the *Reduced* network. *PF06* remains as the most profitable scenario, however *PF07*, and *PF10* closely follow. This is the first time where the majority of the profitable scenario come from the *Full* network. For clear PET, there are now six profitable routes in the zeroth scenario for both the *Full* and *Reduced* networks. This is very interesting, because in *AR06* all routes are profitable, however *AR04* is still the most profitable. Again we see a two-CT optimal solution, with a Π^* value of 3 116Φ. The optimal schedule is shown below in Table A.12.

Table A.12: Optimal Schedule, β=1.50Φ

	Vehicle 1				
	Day 1	Day 2	Day 3	Day 4	Day 5
Week 1	PF06i	PF06ii	PF06iii	PF06iv	PF06v
Week 2	PF06i	PF06ii	PF06iii	PF06iv	PF06v

	Vehicle 2				
	Day 1	Day 2	Day 3	Day 4	Day 5
Week 1	AR04i	AR04ii	AR04iii	AR04iv	PF06vi
Week 2	AR04i	AR04ii	AR04iii	AR04iv	–

Finally, we double the original value of β, and display the results in Tables A.13 & A.14.

Table A.13: GIS Generated Clear PET Routes, β=2.00Φ

	All Routes				**Profitable Routes**			
S_1	r_s	q_s	G_s	π_s	r'_s	q'_s	G'_s	π'_s
PF15	15	2 085	17.3	2 072Φ	11	1 979	13.7	2 506Φ
PF14	14	2 064	16.4	2 196Φ	11	1 979	13.7	2 506Φ
PF13	13	2 054	13.9	2 378Φ	11	1 948	12.1	2 481Φ
PF12	12	2 054	13.6	2 388Φ	10	1 981	11.9	2 556Φ
PF11	11	2 017	13.0	2 495Φ	10	1 969	12.1	2 559Φ
PF10	10	2 010	13.1	2 647Φ	8	1 912	11.4	2 760Φ
PF09	9	1 900	12.3	2 562Φ	7	1 767	10.5	2 617Φ
PF08	8	1 859	10.4	2 655Φ	8	1 859	10.4	2 655Φ
PF07	7	1 826	11.7	2 735Φ	7	1 826	11.7	2 735Φ
PF06	6	1 779	11.3	2 738Φ	6	1 779	11.3	2 738Φ
PF05	5	1 546	8.9	2 395Φ	5	1 546	8.9	2 395Φ
PR10	10	1 919	13.1	2 490Φ	8	1 816	11.3	2 605Φ
PR09	9	1 900	11.8	2 614Φ	8	1 828	10.8	2 626Φ
PR08	8	1 871	12.2	2 663Φ	8	1 871	12.2	2 663Φ
PR07	7	1 786	10.0	2 694Φ	7	1 786	10.0	2 694Φ
PR06	6	1 688	8.7	2 619Φ	6	1 688	8.7	2 619Φ
PR05	5	1 624	9.1	2 606Φ	5	1 624	9.1	2 606Φ
PR04	4	1 369	7.1	2 212Φ	4	1 369	7.1	2 212Φ

Table A.14: GIS Generated Aluminium Routes, β=2.00Φ

	All Routes				**Profitable Routes**			
S_2	r_s	q_s	G_s	π_s	r'_s	q'_s	G'_s	π'_s
AF09	9	1 493	8.3	1 840Φ	8	1 453	7.4	1 920Φ
AF08	8	1 473	8.0	1 875Φ	7	1 393	7.3	1 880Φ
AF07	7	1 452	9.0	1 982Φ	6	1 385	8.2	2 003Φ
AF06	6	1 286	7.3	1 698Φ	6	1 286	7.3	1 698Φ
AF05	5	1 371	8.1	2 050Φ	5	1 371	8.1	2 050Φ
AF04	4	1 125	5.7	1 656Φ	4	1 125	5.7	1 656Φ
AF03	3	879	3.9	1 296Φ	3	826	3.4	1 296Φ
AR06	6	1 305	6.3	1 930Φ	6	1 305	6.3	1 930Φ
AR05	5	1 291	7.2	1 961Φ	5	1 291	7.2	1 961Φ
AR04	4	1 242	7.2	2 597Φ	4	1 242	7.2	2 597Φ
AR03	3	1 050	5.1	1 679Φ	3	1 050	5.1	1 679Φ
AR02	2	681	3.3	1 115Φ	2	681	3.3	1 115Φ

Here we observe that the number of profitable routes in the zeroth scenario remains the same for both clear PET sub-scenarios. The most profitable scenarios are all from the *Full* network with *PF10* followed by *PF06*, and *PF07*. With regards to aluminium, *AF09* now has eight profitable scenarios, and *AR06* remains at its maximum of six. Once again, *AR04* is still the most profitable. We still see a two-CT optimal solution, with a Π^* value of 4 794Φ. The optimal schedule is shown below in Table A.15.

Table A.15: Optimal Schedule, β=2.00Φ

	Vehicle 1				
	Day 1	Day 2	Day 3	Day 4	Day 5
Week 1	PF10i	PF10ii	PF10iii	PF10iv	PF10v
Week 2	PF10vii	PF10ii	PF10iii	PF10iv	PF10v
Week 3	–	PF10ii	PF10iii	PF10iv	PF10v

	Vehicle 2				
	Day 1	Day 2	Day 3	Day 4	Day 5
Week 1	AR04i	AR04ii	AR04iii	AR04iv	PF10vi
Week 2	AR04i	AR04ii	AR04iii	AR04iv	PF10x