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Visible Electronic Emission Spectra of Diatomic Molecules: Analysis of the AlO (B-X) and BI (a-X) Systems, and First Observations of Some ll-valence Electron Cations, $\mathrm{SeO}^{+}$, $\mathrm{PCl}^{+}, \mathrm{PBr}^{+}$, and $\mathrm{AsCl}{ }^{+}$.

## by

## Stavros Naxakis

# Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy 

at

Dalhousie University Halifax, Nova Scotia August, 1990

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#### Abstract

Iwenty-five bands of the $B^{2} \Sigma-X^{2} \Sigma$ system of Alo with $0 \leq v^{\prime} \leq 9$ and $0 \leq v^{\prime \prime} \leq 6$ have been photographed at high resolution. The measured positions of the assigned lines of each band have been fitted by least-squares to obtain estimates of the constants ( $\mathrm{B}^{\prime}, \mathrm{D}^{\prime}, \mathrm{B}^{\prime \prime}, \mathrm{D}^{\prime \prime}$ ) the band origin, and $\Delta \gamma_{u^{\prime} v^{\prime \prime}}$, the difference of the upper and lower state spindoubling constants. The parameters from individual bands have been merged to single-valued estimates, as well as to polynomial representations in ( $v+1 / 2$ ).

New visible systems of red-degraded bands have been observed in emission from reactions of discharged helium with $\mathrm{PCl}_{3}, \mathrm{AsCl}_{3}$ and $\mathrm{SeOCl}_{2}$, and assigned as ${ }^{2} \Pi \rightarrow X^{2} \Pi$ transitions of $\mathrm{PCl}^{+}, \mathrm{AsCl}^{+}$and $\mathrm{SeO}^{+}$. The same excitation technique with $\mathrm{BI}_{3}$ substrate was employed for the observation of the $a^{3} \Pi\left(0^{+}, 1\right) \rightarrow X^{1} \Sigma^{+}$band systems of $B I$; the $a(1) \rightarrow X$ system was observed for the first time. Head positions measured from photoelectrically recorded spectra have been fitted by least-squares to obtain estimates of the vibrational constants of the states involved in the transitions. In addition, eight bands of the $a\left(0^{+}\right) \rightarrow X$ system and two bands of the $a(1) \rightarrow X$ system of ${ }^{11} B I$ have been rotationally analyzed. Three bands ( $0-16,0-17,0-18$ ) of tre ${ }^{2} I I-X^{2}$ II system of $P^{35} \mathrm{Cl}^{+}$have been rotationally analyzed. The determined constants include the first estimated spin-orbit coupling constants for both states. The reliability of these estimates is discussed.

Finally, the results of the reactions of discharged helium with $\mathrm{PBr}_{3}$, and various other substrate molecules are also discussed.


## Definitions of Terms and Symbols

```
H Hamiltonian Operator
\sigma Standard deviation
f Degrees of freedom
h Planck's constant (6.626176 x 10-34 Js)
A Angstrom (1 x 10-10 m)
\mu Reduced mass
A T
A}\mp@subsup{}{}{-1}\quad\mathrm{ Inverse of matrix A
\deltaij Kronecker delta (0, if i * j)
x is a vector
```


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## Chapter 1

## Introduction

### 1.1 General Introduction

The subject matter contained in this thesis deals with the spectroscopic investigation of diatomic molecules and molecular ions. Although, a common factor of this work is that all the diatomic species were observed in emission, the methods of excitation and signal detection broadly divide this thesis into two parts.

In the first part, the $\mathrm{B}^{2} \Sigma^{+} \rightarrow \mathrm{X}^{2} \Sigma^{+}$band system of aluminum monoxide has been recorded and rotationally analyzed using photographic techniques. The excited AlO was produced by flowing trace quantities of oxygen and $\mathrm{AlC}_{\mathrm{i}_{3}}$ through an electrodeless discharge. The analysis of the spectrum has either improved or provided the first reliable vibrational dependences of the various molecular constants of the electronic states involved. Details of this work are given in Chapter 3.

In the second part, a flowing afterglow method has been applied to reactions of helium ions and/or metastable atoms with small molecules and established as a powerful tool for observing new emissions from diatomic molecules. All these emissions have been detected photoelectrically, and are discussed in Chapters 5 through 10. Details of the
excitation method used and of the flowing afterglow system are presented in Chapter 4.

In Chapter 5 , the vibrational analysis of $a^{3} \Pi_{0,1} \rightarrow X^{1} \Sigma^{+}$ system of BI , a molecule with 10 -valence electrons, has been completed with the observation of the $a(1) \rightarrow X^{1} \Sigma^{+}$sub-system for the first time.

In Chapter 7, three isoelectronic ions with ll-valence electrons, $\mathrm{PCl}^{+}, \mathrm{ASCl}^{+}$and $\mathrm{SeO}^{+}$, were identified by analyzing vibrationally their corresponding new visible band emission spectra.

In Chapters 6 and 8, rotational analyses of two of the spectra, namely those of BI and $\mathrm{PCl}^{+}$, were undertaken with the assistance of a digital electronic interface which allows a laboratory mini-computer to assume complete control of a modern, commercial scanning spectrometer. The dual capability of this interface to generate stepping motor drive pulses for control of grating rotation, and to collect photon-counts before passing these data to the computer for storage and eventual processing, has established the above mentioned analyses as comparable in accuracy as the tedious spectrographic techniques.

Furthermore, in Chapters 9 and 10 unassigned bandheads of a new ion, $\mathrm{PBr}^{+}$, are reported and several reactions of discharged helium with various reagents are summarized. Once more, the merit of the flowing afterglow method for generating "clean" spectra is demonstrated.

The correlation of the experimental molecular
parameters to an effective Hamiltonian and some modern fitting procedures, used throughout chis work, are reviewed in Chapter 2.

In conclusion, apart from the obvious spectroscopic and kinetic interest, the discovery of new gaseous molecular ions is important in such systems as electrical discharges and flames.

## Chapter 2

## A Review of Some Spectroscopic Techniques

### 2.1 Introduction

In chis thesis the direct fitting method of Zare et al.
(1) has been employed for the reduction of observed line positions to molecular parameters. This method utilizes the complete data set, in contrast with the traditional combination differences method (2), which uses only part of the data and implicitly assumes that all measurement errors are associated with the state under analysis.

Briefly, in the direct fitting method calculated line positions are iteratively compared to the measured line positions in a nonlinear least-squares fit. The calculated line positions are the differences between the eigenvalues of the upper and lower state Hamiltonians, in which the molecular constants appear as adjustable parameters. Algebraic expressions for the roots of secular determinants are not required since this procedure is numerical. Furthermore, the estimated values of the molecular constants are the minimum-variance, unbiased estimates with statistically meaningful errors.

The development of the model Hamiltonians used to represent the molecular states involved in the observed transitions, will be discussed in the next section.

In the remaining sections of this chapter, the following topics will be presented,
i) the equations and the correlation matriges of the nonlinear and linear band-by-band least-squares fit,
ii) the method of correlated least-squares fitting $(3,4)$ for reducing multiple estimates of the same parameters to the best single valued estimates, and
iii) the method used to construct the potential curves of AlO.

### 2.2 Molecular Hamiltonian

For each of the rotationally analyzed transitions of the present work, namely AlO $\left(B^{2} \Sigma^{+}-\mathrm{X}^{2} \Sigma^{+}\right), \operatorname{PCl}{ }^{+}\left({ }^{2} \Pi I-{ }^{2} \Pi\right.$ ), and $B I\left(a^{3} \Pi_{0,1}-X^{1} \Sigma^{+}\right)$, two effective Hamiltonians are required to describe all the observed line frequencies.

The problem of deriving an effective Hamiltonian for a vibrating rotating molecule has been considered at length by numerous authors. In the present work the methodolog:Zare and co-workers (l) has been adopted. In the absence of external fields, the Hamiltonian can be written as:

$$
\begin{equation*}
\mathrm{H}=\mathrm{H}_{0}+\mathrm{H}_{\mathrm{rot}}+\mathrm{H}_{\mathrm{fs}} \tag{2.1}
\end{equation*}
$$

where $H_{0}$ represents the nonrelativistic Hamiltonian of the nonrotating molecule, $H_{\text {rot }}$ symbolizes the rotational motion of the nuclei, and $H_{f s}$ contains magnetic terms that cause the fine structure. The exact form of $H_{0}$ is of little concern in a rotational analysis, because it depends only on
the electronic and vibrational quantum numbers $n$ and $v$. The rotational part, $\mathrm{H}_{\text {rot }}$ given by

$$
\begin{align*}
& H_{\text {rot }}=B(r) \underline{R}^{2} \\
& =B(r)(\underline{I}-\underline{L}-\underline{S})^{2} \\
& =B(r)\left[\left(\underline{J}^{2}-\underline{J}_{2}^{2}\right)+\left(\underline{\underline{L}}^{2}-\underline{\underline{L}}_{2}^{2}\right)+\left(\underline{S}^{2}-\underline{S}_{2}^{2}\right)+\left(\underline{L_{1}} \underline{S}+\underline{L}_{+}\right)\right. \\
& \left.-\left(\underline{J}_{+} \underline{\underline{L}}_{-}+\underline{J}_{-} \underline{\underline{I}}_{+}\right)-\left(\underline{\mathrm{J}}_{+} \underline{S}_{-}+\underline{\mathrm{J}}_{-} \underline{\underline{S}}_{+}\right)\right] \tag{2.2}
\end{align*}
$$

plays the major role. In Eq. (2.2)

$$
\begin{equation*}
B(r)=h / 8 \pi^{2} c \mu r^{2} \tag{2.3}
\end{equation*}
$$

is the radial part of the rotational operator in wavenumber units, in terms of the internuclear distance $r$ and the reduced mass $\mu$,

$$
\begin{equation*}
\underline{R}=\underline{\mathrm{J}}-\underline{\mathrm{L}}-\underline{\mathrm{S}} \tag{2.4}
\end{equation*}
$$

is the rotational angular momentum operator of the nuclei, in terms of the total angular momentum (J), electronic orbital angular momentum (L), and spin angular momentum $\underline{S}$ operators, and

$$
\begin{equation*}
J_{ \pm}=\underline{I}_{x} \pm i \underline{I}_{y}, S_{ \pm}=\underline{S}_{x} \pm i \underline{S}_{y}, I_{ \pm}=\underline{I}_{x} \pm i \underline{I}_{y} \tag{2.5}
\end{equation*}
$$

The fine structure Hamiltonian represents the major
magnetic interactions by a limited set of parameters; terms that cause shifts rather than splittings of the fine structure components are omitted. The fine structure Hamiltonian is regarded as a sum of three forms,

$$
\begin{equation*}
\mathrm{H}_{\mathrm{fs}}=\mathrm{H}_{\mathrm{SO}}+\mathrm{H}_{\mathrm{SS}}+\mathrm{H}_{\mathrm{SR}} \tag{2.6}
\end{equation*}
$$

where

$$
\begin{equation*}
H_{S O}=A(r) \underline{L} \cdot \underline{S}=A(r)\left[\underline{L}_{z} \underline{S}_{z}+1 / 2\left(\underline{L}_{+} \underline{S}+\underline{L}_{-S_{+}}\right)\right] \tag{2.7}
\end{equation*}
$$

is the spin-crbit interaction,

$$
\begin{equation*}
H_{s s}=\epsilon(r)\left(3 \underline{S}_{z}^{2}-\underline{s}^{2}\right) \tag{2.8}
\end{equation*}
$$

is the spin-spin interaction, and
$H_{S R}=\gamma(r) \underline{N} \cdot \underline{S}=\gamma(r)(\underline{J}-\underline{S}) \cdot \underline{S}=\gamma(r)\left[-\underline{S}^{2}+\underline{J}_{z} \underline{S}_{2}+1 / 2\left(\underline{J}_{+} \underline{S}+\underline{J}_{\underline{S}}\right)\right]$
is the spin rotation interaction. The $z$ axis lies along the internuclear axis, and in Eq. (2.9), $\underline{N}=\underline{\mathrm{I}}$ - $\underline{\mathrm{S}}$ is the operator for total orbital angular momentum apart from spin.

The energy levels of a diatomic molecule are given by the solutions to the time-independent Schrödinger equation

$$
\begin{equation*}
\mathrm{H} \Psi=\mathrm{E} \Psi \tag{2.10}
\end{equation*}
$$

In practice, $\Psi$ can be expanded in terms of a convenient finite basis set $\phi_{i}$. Eq. (2.10) can be solved by finding the roots of the secular determinant

$$
\begin{equation*}
\left|H_{i j}-E \delta_{i j}\right|=0 \tag{2.11}
\end{equation*}
$$

The basis set can be chosen arbitrarily, but it is convenient to partition the determinant into diagonal blocks that weakly interact with one another. The diagonal blocks refer to Born-Oppenheimer states. Hund's case (a) coupling gives a basis set with good quantum numbers $J, S, \Lambda$ and $\Sigma$. $\Omega$ is also good but it is redundant ( $\Omega=\Lambda+\Sigma$ ). For a given electronic state $n, \mid n i s \Lambda \Sigma>$ are eigenfunctions of the operators $\underline{J}^{2}, \underline{S}^{2}, \underline{J}_{z}, \underline{S}_{z}$, and $\underline{L}_{z}$ with eigenvalues $J(J+1)$, $S(S+1), \Omega, \Sigma$, and $\Lambda$ respectively. Matrix elements, off diagonal with respect to $\Omega$ and $\Sigma$ are given according to phase convention (4a) as,

$$
\begin{align*}
& \langle J \Omega \pm 1| J_{\mp} \mid J \Omega>=[J(J+1)-\Omega(\Omega \pm 1)]^{1 / 2}  \tag{2.12}\\
& \langle S \Sigma \pm 1| S_{ \pm}|S \Sigma\rangle=[S(S+1)-\Sigma(\Sigma \pm 1)]^{1 / 2} \tag{2.13}
\end{align*}
$$

An advantage of the case (a) basis is that the absence of intermediate angular momenta ( $\mathrm{N}, \mathrm{J}_{\mathrm{a}}$ ) and the presence of the maximum number of molecule-fixed $z$-components $(\Lambda, \Sigma, \Omega)$ snable $H_{\text {rot }}$ and $H_{\text {so }}$ matrix elements to be evaluated using elementary raising and lowering op rator techniques.

For a given electronic state and vibration level, the energy levels can be found by diagonalizing a $(2 S+1)\left(2-\delta_{\Lambda 0}\right)$ by $(2 S+1)\left(2-\delta_{A 0}\right)$ submatrix which is called the nvJSA block (Born-Oppenheimer state). If the magnitude of the offdiagonal matrix elements that connect different nvJSA blocks are small, their effect can be included simply by perturbation theory. A convenient way to include these perturbations arising from distant states without the knowledge of the exact energy position, is to transform the original Hamiltonian to a new Hamiltonian in which firstorder interactions between the states under consideration and the distant states are eliminated. This transformation is the well-known Van Vleck transformation (5). Therefore, the Van Vleck transformation is an approximate diagonalization procedure which facilitates the factoring of an infinite energy satrix into small submatrices which can be treated individually. The energy matrix can be expanded as:

$$
\begin{equation*}
\mathrm{H}=\mathrm{H}_{0}+\lambda \mathrm{H}_{1}+\lambda^{2} \mathrm{H}_{2}+\ldots \tag{2.14}
\end{equation*}
$$

Elements of $H_{0}$ lie entirely in diagonal blocks, while $H_{1}, H_{2}$ are the perturbation terms that may have elements inside and outside these blocks. The transformation is applied to H to
remove the off-diagonal elements of $\mathrm{H}_{1}$; the most important off-diagonal terms which remain are second-order elements that contribute to the energy in the fourth order, and the energy matrix is now diagonal up through third-order.

A further simplification results if the energy separation between the interacting blocks is much larger than the energy separation between the fine structure of the nvJsi block. Then, the second order correction may be written,

$$
\begin{equation*}
\Sigma_{n}^{\prime} \frac{\langle m| H_{1}|n\rangle\langle n| H_{1}\left|m^{\prime}\right\rangle}{E_{m}-E_{n}}, \tag{2.15}
\end{equation*}
$$

where $E_{m}-E_{n}$ is the energy difference between the unperturbed blocks and $m, m^{\prime}$ are matrix elements of the nvJSA block of interest. Table 2.1 lists case (a) matrix elements within the same nvJSA block (1), that are used in the present work. The derivation of these matrix elements will not be repeated since this has been considered by various authors, (see for example, Hougen (6), Kovacs (7), Freed (8)). The parameters $\mathrm{T}_{\mathrm{v}}, \mathrm{B}_{\mathrm{v}}, \gamma_{\mathrm{v}}, \mathrm{A}_{\mathrm{v}}$ appearing in Table 2.1 represent the expectation values of the radial parts of their respective operators,

Table 2.1: Matrix Elements Within The Same nvJsA Block

$$
\begin{aligned}
\langle\Lambda \Sigma| \mathrm{H}_{\mathrm{o}}|\Lambda \Sigma\rangle & =\mathrm{T}_{\mathrm{v}} \\
\left.<\Lambda \Sigma\left|\mathrm{H}_{\mathrm{rot}}\right| \Lambda \Sigma\right\rangle & =\mathrm{B}_{\mathrm{v}}\left[J(J+1)-\Omega^{2}+\mathrm{S}(\mathrm{~S}+1)-\Sigma^{2}\right] \\
\left.<\Lambda \Sigma\left|\mathrm{H}_{\mathrm{rot}}\right| \Lambda \Sigma \pm 1\right\rangle & =-\mathrm{B}_{\mathrm{v}}\left[J(J+1)-\Omega(\Omega \pm 1]^{1 / 2}[\mathrm{~S}(\mathrm{~S}+1)-\Sigma(\Sigma \pm 1)]^{1 / 2}\right. \\
<\Lambda \Sigma\left|\mathrm{H}_{\mathrm{SR}}\right| \Lambda \Sigma> & =\gamma_{\mathrm{v}}[\Sigma \Omega-\mathrm{S}(\mathrm{~S}+1)] \\
\left.<\Lambda \Sigma\left|\mathrm{H}_{\mathrm{SR}}\right| \Lambda \Sigma \pm 1\right\rangle & =\frac{2}{2} \gamma_{\mathrm{v}}[J(J+1)-\Omega(\Omega \pm 1)]^{1 / 2}[\mathrm{~S}(\mathrm{~S}+1)-\Sigma(\Sigma \pm 1)]^{1 / 2} \\
\left.<\Lambda \Sigma\left|\mathrm{H}_{\mathrm{so}}\right| \Lambda \Sigma\right\rangle & =A_{\mathrm{v}} \Lambda \Sigma
\end{aligned}
$$

$$
\begin{align*}
& \mathrm{T}_{\mathrm{v}}=\langle\mathrm{nv}| \mathrm{H}_{0}+\mathrm{B}(\mathrm{r})\left(\mathrm{I}_{\mathrm{x}}^{2}+\mathrm{I}_{\mathrm{y}}^{2}\right)|\mathrm{nv}\rangle \\
& \mathrm{B}_{\mathrm{v}}=\langle\mathrm{nv}| \mathrm{B}(\mathrm{r})|\mathrm{nv}\rangle \\
& \mathrm{A}_{\mathrm{v}}=\langle\mathrm{nv}| \mathrm{A}(\mathrm{r})|\mathrm{nv}\rangle \\
& \gamma_{\mathrm{v}}=\langle\mathrm{nv}| \gamma(r)|\mathrm{nv}\rangle \tag{2.16}
\end{align*}
$$

The use of the Van Vleck transformation described in the last paragraph becomes necessary because of the radial dependence of the operators of Table 2.1. As the molecule rotates it couples neighbouring vibrational levels within the same electronic state. Probably the most easily observable of the second order terms are the centrifugal distortion effects associated with the off-diagonal matrix elements of $B(r)$, since $B(r)$ has fairly strong dependence on $r$ and also since levels with relatively high values of the quantum number $J$ can be observed. The second order effect of $B(r)$ with itself gives the parameter

$$
\begin{equation*}
D_{v}=-\Sigma_{V^{\prime}} \frac{\langle n v| B(r)\left|n v^{\prime}\right\rangle\left\langle n v^{\prime}\right| B(r)|n v\rangle}{E_{n v}-E_{n v^{\prime}}} \tag{2.17}
\end{equation*}
$$

while the cross-term between $B(x)$ and a function $X(x)$, e.g. spin-orbit function $A(r)$ or the spin-rotation function $\gamma(r)$, gives,
$X_{D v}=\sum_{V^{\prime}}^{\prime} \frac{\left[\langle n v| B(r)\left|n v^{\prime}\right\rangle\left\langle n v^{\prime}\right| X(r)|n v\rangle+\langle n v| X(r)\left|n v^{\prime}\right\rangle\left\langle n v^{\prime}\right| B(r)|n v\rangle\right]}{E_{n v}-E_{n v^{\prime}}}$
Because of the stronger radial dependence of $B(r)$ than other functions, the cross-term needs more extended data in order to be determined experimentally.

To obtain the second-order correction matrix element for $H_{\text {rot }}$ the radial part given in Eq. (2.17) must be multiplied by the angular part of the rotational Hamiltonian, given in Eq. (2.2). It can be seen that centrifugal distortion may be treated as if $H_{\text {rot }}$ were replaced by an effective rotational Hamiltonian of the form

$$
\begin{equation*}
H_{r o t}=B_{v} \underline{R}^{2}-D_{v} \underline{R}^{4}+H_{v} \underline{R}^{6}+\ldots \tag{2.19}
\end{equation*}
$$

The spin-orbit and the spin-rotation interactions may also couple together neighbouring vibrational levels through the weak radial dependence of $A(r)$ and $\gamma(r)$.

So far the nvJSA block separates into two identical diagonal subblocks, one for $+\Lambda$ and one for $-\Lambda$, provided $\Lambda \neq 0$. To remove this degeneracy, perturbations from neighbouring electronic states, must be considered. Different electronic states can be connected by terms of the form $J_{+} L_{-}+J_{-} L_{4}$ arising from $H_{r o t}$ and connecting electronic states differing by one unit in $\Lambda$. This interaction is $\mathcal{J}-$ dependent and affects the $\pm \Lambda$ components differently. Another type of interaction, that is $J$-independent is of the form $L_{+} S_{-}+L_{-} S_{+}$arising from $H_{\text {rot }}$ and $H_{S O}$. It contributes to the $J$-independent shifts in the $\Lambda$ doublets.

A further simplification in the Hamiltonian will result if a transformation is made to a basis set symmetrized with respect to reflection in the plane containing the internuclear axis. These new wavefunctions refer to energy levels wnich can be labeled as either having e or f parity (9), depending on whether a set of levels transforms as
either a ${ }^{1} \Sigma^{+}$or a ${ }^{1} \Sigma^{-}$state. This parity convention, then, designates the electronic parity of a set of levels according to the following scheme (half-integral spin):

$$
\begin{align*}
& \text { e levels } \sigma_{v} \psi=+(-1)^{J-1 / 2} \psi \\
& \text { f levels } \sigma_{v} \psi=-(-1)^{J-1 / 2} \psi \tag{2.20}
\end{align*}
$$

It follows that the resulting wavefunctions necessary to evaluate the Hamiltonian for ${ }^{2} \Sigma^{+}$(AlO) and ${ }^{2} I I$ ( $\mathrm{PCl}^{+}$) states are given by,

$$
\begin{align*}
& { }^{2} \Sigma_{1 / 2}\binom{e}{f}=1 / \sqrt{2} \quad\left\{\left.\right|^{2} \Sigma_{1 / 2}^{+}> \pm\left.\right|^{2} \Sigma_{-1 / 2}^{+}>\right\} \\
& { }^{2} \Pi_{3 / 2}\binom{e}{f}=1 / \sqrt{2} \quad\left\{\left.\right|^{2} \Pi_{3 / 2}> \pm\left.\right|^{2} \Pi_{-3 / 2}>\right\} \\
& { }^{2} \Pi_{1 / 2}\binom{e}{f}=1 / \sqrt{2} \quad\left\{\left.\right|^{2} \Pi_{1 / 2}> \pm\left.\right|^{2} \Pi_{-1 / 2}>\right\} \tag{2.21}
\end{align*}
$$

As a result of these symmetrized wavefunctions the secular determinants of ${ }^{2} \Sigma-^{2} \Sigma$ and ${ }^{2} \Pi-^{2} \Pi$ transitions can be factored into two diagonal blocks of order $1 \times 1$ and $2 \times 2$ respectively, corresponding to levels with definite parity. The energy levels of $a^{2} \Sigma$ electronic state can be represented by analytical expressions. Taking into consideration the matrix elements of Table 2.1 and the rotational centrifugal distortion, the rotational energy levels for $a^{2} \Sigma^{+}$state are given by,

$$
\begin{equation*}
F_{1}(J, e)=B_{v}\left(J-\frac{1}{2}\right)\left(J+\frac{1}{2}\right)-D_{v}\left[\left(J-\frac{1}{2}\right)\left(J+\frac{1}{2}\right)\right]^{2}+\frac{1}{2} \gamma_{v}\left(J-\frac{1}{2}\right) \tag{2.22}
\end{equation*}
$$

and

$$
\begin{equation*}
F_{2}(J, f)=B_{v}\left(J+\frac{1}{2}\right)(J+3 / 2)-D_{v}\left[\left(J+\frac{1}{2}\right)(J+3 / 2)\right]^{2}-\frac{1}{2} \gamma_{v}(J+3 / 2) \tag{2.23}
\end{equation*}
$$

Equations (2.22) and (2.23) have a more familiar form if $J+\frac{1}{2}$ is replaced by the quantum number $N$, so that each rotational level (except the $N=0$ level) occurs as a closely spaced pair. Because of the functional form of the ${ }^{2} \Sigma$
analytical expressions, the molecular constants absorb the effects of any perturbation due to ${ }^{2}$ II electronic states. The result is that there is no simple way to ascertain whether the rotational and spin-rotation coupling constants have the separate mechanical and magnetic meanings usually attributed to them.
${ }^{2}$ II states may be perturbed heterogeneously by ${ }^{2} \Sigma$ and/or ${ }^{2} \Delta$ states. However, the matrix elements $\left.\left.\left\langle{ }^{2} \Pi_{1 / 2}\binom{e}{f}\right| H_{r o t}\right|^{2} \Sigma_{1 / 2}\binom{e}{f}\right\rangle$ are parity dependent because a ${ }^{2} \Sigma_{1 / 2}$ basis function is involved. This parity dependent matrix element is mainly rrsponsible for the splitting of ${ }^{2}$ II rotational levels that is called $\Lambda$-doubling. Therefore, it is reasonable to assume that the $\Lambda$-doubling of $a^{2} I I$ state is usually accounted for only by consideration of ${ }^{2} \Sigma$ states. It should be noted that the observed splitting of a ${ }^{2} \Pi_{1 / 2}$ component of a ${ }^{2} I I$ state is more significant than for ${ }^{2} \Pi_{3 / 2}$, since the ${ }^{2} \Pi_{1 / 2}$ component experiences two types of interactions with a ${ }^{2} \Sigma$ state, namely the spin-orbit, $L_{4} S_{-}+L_{-} S_{+}$, and the L-uncoupling $J_{+} L_{-}$ $+J_{-} L_{4}$; however, the ${ }^{2} \Pi_{3 / 2}$ component experiences only the L-uncoupling.

The weakness of distant ${ }^{2} \Pi-{ }^{2} \Sigma$ interactions permits the use of the Van Vleck transformation to uncouple the interacting states. In Table 2.2 the matrix elements conn-rting a ${ }^{2} I I$ state with perturbing ${ }^{2} \Sigma$ states are given. The derivation of these elements is a direct result of Eq. (2.15) and the matrix elements of Ref. (1) connecting nvJSA blocks with different values of $\Lambda$ or $s$.

Table 2.2: Matrix Elements Connecting $a^{2}$ II State with Perturbing ${ }^{2} \Sigma$ States

$$
\begin{aligned}
H_{3 / 2,3 / 2}\binom{e}{f}= & 1 / 2 q_{v}(J+3 / 2)(J-1 / 2) \\
H_{1 / 2,1 / 2}(e)= & 1 / 2 q_{v}(J-1 / 2)^{2}-1 / 2 p_{v}(J-1 / 2)+o_{v} \\
H_{1 / 2,1 / 2}(f)= & 1 / 2 q_{v}(J+3 / 2)^{2}-1 / 2 p_{v}(J+3 / 2)+o_{v} \\
H_{3 / 2,1 / 2}(e)= & 1 / 2 q_{v}(J+3 / 2)^{1 / 2}(J-1 / 2)^{3 / 2} \\
& -1 / 4 p_{v}(J+3 / 2)^{1 / 2}(J-1 / 2)^{1 / 2} \\
H_{3 / 2,1 / 2}(f)= & -1 / 2 q_{v}(J+3 / 2)^{3 / 2}(J-1 / 2)^{1 / 2} \\
& -1 / 4 p_{v}(J+3 / 2)^{1 / 2}(J-1 / 2)^{1 / 2}
\end{aligned}
$$

The parameters $o_{v}, p_{v}$ and $q_{v}$ introduced in Table (2.2), are defined by,

$$
\begin{align*}
& o_{v}=1 / 4 \underset{n^{\prime}, v^{\prime}}{\Sigma} \frac{\left.\left|\left\langle n^{2} \Pi v\right| A I_{4}\right| n^{\prime} \Sigma v^{\prime}\right\rangle\left.\right|^{2}}{E_{n v}-E_{n^{\prime} v^{\prime}}}  \tag{2.24}\\
& p_{v}=\sum_{n^{\prime}, v^{\prime}} \frac{\mid\left\langle n^{2} \Pi v\right| A I_{H}\left|n^{\prime 2} \Sigma v^{\prime}\right\rangle\left\langle n^{2} \Pi v\right| B L_{\mu}\left|n^{\prime 2} \Sigma v^{\prime}\right\rangle}{E_{n v}-E_{n^{\prime} v^{\prime}}}  \tag{2.25}\\
& q_{v}=\operatorname{m}_{n^{\prime}, v^{\prime}} \Sigma \frac{\left.\left|\left\langle n^{2} \Pi v\right| B L_{H}\right| n^{\prime} \Sigma v^{\prime}\right\rangle\left.\right|^{2}}{E_{n v}-E_{n^{\prime} v^{\prime}}} \tag{2.26}
\end{align*}
$$

The centrifugal distortions of the above parameters $\left(o_{D v}, o_{H v} \ldots, p_{D v}, p_{\text {Hv }} \ldots, q_{D v}, q_{H v} \ldots\right.$ ) are not indicated in Table (2.2).

Comparison of the functional forms of Table 2.2 and analytic expressions (2.22) and (2.23) for ${ }^{2} \Sigma$ states show that ${ }^{2}$ II perturbations cor'iribute a term $o_{v}$ to the effective band origin, a term $q_{v}$ to the effective $B_{v}$ value and a term - $p_{v}$ to the effective spin-rotation constant $\gamma_{v}$. This illustrates how the presence of a ${ }^{2}$ II state impairs the mechanical meaning of the rotational constants of $a^{2} \Sigma$ state, as mentioned in the previous paragraph. In conclusion, Table 2.3 lists matrix elements of the effective ${ }^{2} \Pi$ Hamiltonian used in the analysis of the ${ }^{2} \Pi-{ }^{2} \Pi$ system of $\mathrm{PCl}^{+}$. The matrix elements used in the present nonlinear least-squares fitting routine are essentially identical with those formulated by Brown and co-workers (10). However, the effects of these matrix elements are identical to those obtained by Zare et al (1). The main differences are in the
definitions of $B_{v}$, and $\gamma_{v}$ and of the parameters used to describe centrifugal distortion of $n$-doubling. However, the latter are not required in the $P C l^{+}$analysis.

$$
\begin{aligned}
E(1,1) & 1 \\
(2,2) & 1
\end{aligned}
$$

B $(1,1) x^{2}$
D $(1,1)-x^{4}-x^{2}+1$
$(2,2) \quad n^{2}-2$
$(2,2)-x^{4}+3 x^{2}-3$
$(1,2)-\left(x^{2}-1\right)^{1 / 2}$
$(1,2) \quad 2\left(x^{2}-1\right)^{3 / 2}$
A $(1,1) \quad-\frac{1}{2}$
$\gamma(1,1)-1$
$(2,2) \quad \frac{3}{2}$
$(1,2) \quad \frac{1}{2}\left(x^{2}-1\right)^{1 / 2}$
$q(1,1) \pm x$
$p(1,1) \quad \pm \frac{1}{2} x$
$(1,2) \pm \frac{1}{2} x\left(x^{2}-1\right)^{1 / 2}$

$$
\begin{gathered}
{ }^{\text {a }} 1 \equiv^{2} \Pi_{1 / 2} ; 2 \equiv^{2} \Pi_{3 / 2} ;( \pm)=\binom{e}{f} \\
x=\left(J+\frac{1}{2}\right)
\end{gathered}
$$

### 2.3 Band-by-Band Least-Squares Fits

### 2.3.1 Nonlinear Fitting Procedure ( ${ }^{2} \pi-{ }^{2} \pi$ system)

The nonlinear least-squares fitting routine accepts transitions as input data, together with initial trial values of the spectroscopic constants and their estimated uncertainties. These initial values of the parameters of the band system are used to evaluate the Hamiltonian matrix, which is then diagonalized. The resulting eigenvalues form a set of term energies from which calculated transitions are obtained. These calculated transitions are compared to the experimentally observed frequencies. Corrections to the parameters are computed, and a new set of term energies is constructed using the adjusted parameters.

This procedure is repeated until convergence is achieved. Mention should be made here of the commonly made assumptions that the measured errors of each band are randomly scattered with variance $\hat{\sigma}^{2}$ and zero covariance.

The final molecular parameters which are obtained by the nonlinear least-squares fitting are minimum-variance, linear unbiased estimated (MVLU); this is in accord with the approximately linear nature of the model in the region about the estimated values of the parameters, as follows.

The molecular parameters ( $\beta_{1}, \beta_{2}, \ldots \beta_{p}$ ) can be derived from the measured line wavenumbers $Y\left(Y_{1}, Y_{2}, \ldots Y_{n}\right)$ using the non-linear expression,

$$
\begin{equation*}
\mathbf{Y}=f(X, \beta)+\epsilon \tag{2.27}
\end{equation*}
$$

where $x\left(x_{1}, x_{2}, \ldots x_{m}\right)$ are the independent variables, and $\epsilon\left(\epsilon_{1}, \epsilon_{2}, \ldots \epsilon_{n}\right)$ are the unknown measurement errors. Eq. (2.27) can be expanded by Taylor series with respect to $\left(\Delta \beta_{1}, \Delta \beta_{2} \ldots \Delta \beta_{p}\right)$, the corrections of the molecular parameters. The first term of this expansion for the $i^{\text {th }}$ line measurement will yield,

$$
\begin{align*}
\Delta Y_{i} & \left.=Y_{i} \text { (observed }\right)-Y_{i}(\text { calculated }) \\
& =\left(\partial f / \partial \beta_{1}\right)_{i} \Delta \beta_{1}+\left(\partial f / \partial \beta_{2}\right)_{i} \Delta \beta_{2}+\ldots+\left(\partial f / \partial \beta_{p}\right)_{i} \Delta \beta_{p} \tag{2.28}
\end{align*}
$$

where $\left(\partial f / \partial \beta_{1}\right), \ldots,\left(\partial f / \partial \beta_{p}\right)$ are the partial derivatives of Eq. (2.27), and the initial values of $\left(\beta_{1}, \beta_{2}, \ldots \beta_{p}\right)$ are the given trial values. The linearized equation (2.28) can be expressed in matrix form as,

$$
\begin{equation*}
\Delta \mathbf{Y}=\mathbf{A} \Delta \boldsymbol{\beta} \tag{2.29}
\end{equation*}
$$

where, $\Delta Y$ and $\Delta \beta$ are column vectors of order $n x 1$ and $p x 1$ respectively, and the irst row of $n \times p$ matrix has elements given by,

$$
\left(\partial \mathrm{f} / \partial \beta_{1}\right)_{1},\left(\partial \mathrm{f} / \partial \beta_{2}\right)_{1}, \ldots,\left(\partial \mathrm{f} / \partial \beta_{p}\right)_{1}
$$

The linear least-square fitting of Eq. (2.29) is very well documented in a review article (11), and a summary of the derived least-squares equations will be presented. For every iteration the sum of the squared deviations between the measured and the calculated line positions.

$$
\begin{equation*}
\Delta Y^{\top} Y=\sum_{i=1}^{n}\left(\Delta Y_{i}\right)^{2} \tag{2.30}
\end{equation*}
$$

is minimized, and a set of $\Delta \beta$ is given by,

$$
\begin{equation*}
\Delta \beta=\left(A^{\top} A\right)^{-1} A^{\top} \Delta Y \tag{2.31}
\end{equation*}
$$

This process is repeated until a preset convergence
criterion is satisfied.
The variance, $\hat{\sigma}^{2}$, and standard deviation, $\hat{\sigma}$, of a fit are given by,
and

$$
\begin{equation*}
\hat{\sigma}=\left|\left(\hat{\sigma}^{2}\right)^{1 / 2}\right|, \tag{2.33}
\end{equation*}
$$

where $f$ represents the number of degrees of freedom of the fit,

$$
\begin{equation*}
f=(n-p) \tag{2.34}
\end{equation*}
$$

The variance of the estimated parameters are taken as the corresponding diagonal elements of the square-symmetric variance-covariance matrix, $\hat{\theta}$. Thus, the variance of a parameter $\beta_{i}$ is taken as $\hat{\theta}_{i 1}$, where

$$
\begin{equation*}
\hat{\theta}=\hat{\sigma}^{2} v \tag{2.35}
\end{equation*}
$$

The dispersion matrix, $\nabla$, is given by,

$$
\begin{equation*}
V=\left(A^{\top} A\right)^{-1} \tag{2.36}
\end{equation*}
$$

The covariances are the off-diagonal elements of $\hat{\theta}$, denoted by $\hat{\theta}_{1 j}$. The normalized variance-covariance matrix is called the correlation-coefficient matrix and given by the symbol c,

$$
\begin{equation*}
c_{i j}=\hat{\theta}_{i j} /\left(\hat{\theta}_{i i} \hat{\theta}_{\mathrm{j} j}\right)^{1 / 2} \tag{2.37}
\end{equation*}
$$

This symmetric matrix, having unity valued diagonal elements and off-diagonal elements in the range -1 to +1 , relates the degree of correlation between the estimated parameters of a fit. Thus an off-diagonal element $c_{1 j}$ gives a measure of the interdependence of parameter $\beta_{1}$ upon
parameter $\beta_{j}$. The correlation is large if $\left|c_{i j}\right| \approx 1$ and low ${ }^{2}$ if $\left|c_{i j}\right| \approx 0$.

### 2.3.2 Linear fitting procedure $\left({ }^{2} \Sigma^{+}-{ }^{2} \Sigma^{+}\right.$system)

The expressions (2.22) and (2.23) for the energy levels of a ${ }^{2} \Sigma$ state lead to a linear least-squares solution of the equation,

$$
\begin{equation*}
\mathbf{Y}=\mathbf{A} \boldsymbol{\beta}+\boldsymbol{\epsilon} \tag{2.38}
\end{equation*}
$$

where $Y, \beta$, and $\epsilon$ are the column vectors containing the $n$ known measured line positions of a band, the $p$ molecular parameters to be determined and the $n$ unknown measurement errors respectively. A is the known $n \times p$ coefficient matrix with elements given by Eqs. (2.22) and (2.23).

The resulting least-squares values $\hat{\boldsymbol{\beta}}$ of molecular parameters together with expressions for $\sigma^{2}, \hat{\Theta}$, and $c$ are given by Eqs. (2.31), (2.32), (2.35) and (2.37).

### 2.4 Singie-step merging

The band-by-band reduction of the systems analyzed, presently, leads to multiple estimates of various nolecular parameters. Simple averaging of these overdetermined constants, weighted or unweighted, does not give satisfactory results. The discrepancy appears due to the fact that no account has been taken of the correlations between the various parameters of each fit. Therefore, the problem is to merge the redundant values from the band-by-
band fits into the "best" nonredundant minimum variance, linear, and unbiased, values.

Albritton and co-workers (3) have developed a satisfactory approach for obtaining MVLU nonredundant values. This method is called merged least-squares fitting and it takes the output of the band-by-band fits, $\hat{\beta}$ and $\hat{\theta}$, to be the input of another least-squares fit that "merges" the redundant values.

Since the variance-covariance matrices $\hat{\boldsymbol{\theta}}$ associated with the $\hat{\beta}$ generally do not have equal diagonal elements and zero off-diagonal elements, the so?ution of the leastsquares fit must employ the weighted, correlated leastsquares formalism (ll). Using this, the nonredundant molecular constants that minimize their standard errors subject to the interrelations contained in $\hat{\theta}$ are given by

$$
\begin{equation*}
\hat{\beta}^{M}=\left(X^{T} \hat{\Phi}^{-1} X\right)^{-1} X^{T} \hat{\Phi}^{-1} Y \tag{2.39}
\end{equation*}
$$

where matrix $X$ relates the redundant values of the column vector $y$ to the corresponding nonredundant column vector $\beta$, matrix $\hat{\Phi}$ is a nondiagonal matrix composed of the individual $\hat{\theta}_{i}, i=1,2, \ldots$ number of bands merged,

$$
\hat{\Phi}=\left[\begin{array}{ccc}
\hat{\theta}_{1} & & 0  \tag{2.40}\\
& \hat{\theta}_{2} & \\
0 & & \hat{\theta}_{\mathrm{k}}
\end{array}\right]
$$

where the circumflex denotes that $\hat{\beta}^{M}$ values are MVLU estimates, and the superscript $M$ denotes that they are estimates of the merge nethod.

The precision of the estimates $\beta^{\mathrm{H}}$ is indicated by their standard errors, which are the square roots of the diagonal elements of the variance-covariance matrix associated with $\hat{\beta}^{M}$,

$$
\begin{equation*}
\hat{\boldsymbol{\theta}}^{M}=\hat{\sigma}_{M}^{2} \hat{\mathbf{v}}^{M} \tag{2.41}
\end{equation*}
$$

where the merged dispersion matrix is given by

$$
\begin{equation*}
\mathbf{V}^{M}=\left(\mathbf{X}^{T} \hat{\mathbf{I}}^{-1} \mathbf{X}\right)^{-1} \tag{2.42}
\end{equation*}
$$

The estimated variance of the merged fit $\hat{\sigma}_{M}^{2}$ is given by

$$
\begin{equation*}
\hat{\sigma}_{M}^{2}=\left(\mathbf{Y}-\mathbf{x} \hat{\boldsymbol{\beta}}^{M}\right)^{T \hat{\mathbf{h}}^{-1}}\left(\mathbf{Y}-\mathbf{x} \hat{\boldsymbol{\beta}}^{\mathrm{M}}\right) / \mathbf{f}_{\mathrm{M}}, \tag{2.44}
\end{equation*}
$$

where the degrees 0 . freedom of the merged fit are denoted by $f_{M}$. When the degrees of freedom in the individual band-by-band fits are greater or equal to about 30, the estimated molecular constants and standard errors have their usual statistical meaning. Confidence limits can be constructed by
$\hat{\beta}_{i}-t(f, 1-a / 2) \hat{\theta}_{i i}^{1 / 2} \leq \beta_{i}^{\text {true }} \leq \hat{\beta}_{i}+t(f, 1-a / 2) \theta_{i i}^{1 / 2}$
within which one can be $100(1-a) \%$ confident that the unknown "true" value lies. In other words, if the set of measurements could be repeated many times in an identical fashion except for random measurement errors, then the set of $\left(\hat{\beta}_{i}-\beta_{i}^{\text {true }}\right) / \hat{\theta}_{i i}^{1 / 2}$ value would be distributed symmetrically about zero in a fashion described by the $t(f)$ function, which is a tabulated function of statistics, for the given degrees of freedom f. Similarly if only random errors in the measurements are considered and the degrees of freedom of band-by-band fits are greater than about 40 then the
estimated variance of the merge $\hat{\sigma}_{M}^{2}$ divided by its unknown "true" values has a $\chi^{2} / f$ distribution which again is a commonly tabulated function of statistics for the given degrees of freedom $f$. Unlike the symmetric $t$ function with range $-\infty \leq t \leq+\infty$ and a mean of zero, the asymmetric $\chi^{2} / f$ function has a range of $0 \leq \chi^{2} / f \leq \infty$ and mean of unity. The $100(1-a) \%$ confidence limits for $\hat{\sigma}_{M}^{2}$ are

$$
\begin{equation*}
\hat{\sigma}_{M}^{2} / p(f, 1-a / 2) \leq \hat{\sigma}_{\text {Mtrue }}^{2} \leq \hat{\sigma}_{M} / p(f, a / 2) \tag{2.45}
\end{equation*}
$$

and are asymmetric.
If these limits, whose variation about unity is due to random measurement errors, are exceeded, then one can conclude that it is likely that systematic errors are present in some or all of the data sets or Hamiltonians.

### 2.5 Stepwise Merging

A useful approach for the identification and (if necessary) rejection of data that are likely to contain relative systematic error, is the method of stepwise merging formulated by Coxon (4). In Coxon's work it was established that two or more sets of separately merged parameters can themselves be merged together to a single set of parameters using a least-squares grand merge approach. The output constants and dispersion matrix of such a grand merge are identical with those derived from the equivalent single-step merge. The only difference in the stepwise merge approach is that the weight matrix to be used is derived not from the
variance-covariance matrices of the separately merged input parameters, but from the corresponding dispersion matrices,

$$
\hat{\Phi}_{M}=\left[\begin{array}{lllll}
\left(\hat{v}^{M}\right)_{1} & & & & 0  \tag{2.46}\\
& & \left(\hat{v}^{M}\right)_{3} & & \\
& & \left(\hat{v}^{M}\right)_{3} & & \\
& & & \cdot & \\
& & & & \\
& & & & \\
& & & & \\
& & & \left(\hat{v}^{M}\right)_{k}
\end{array}\right]
$$

It was also demonstrated that in the absence of systematic errors, confidence limits for a set of merged molecular constants can be derived from the dispersion matrix element, $\hat{V}_{i 1}$, rather than estimated variance $\hat{\theta}_{i i}=\hat{\sigma}_{M}^{2} \hat{V}_{i 1}$. Alternatively, if $\hat{\sigma}_{M}^{2}$ lies outside of the confidence limits expected from the $\chi^{2} / f$ distribution the standard errors of $\hat{\beta}^{M}$ derived from $\hat{V}^{M}$ should be multipled by $\hat{c}_{M}(4)$. The estimated variance for a merge performed in a stepwise manner is calculated from:

$$
\begin{equation*}
f_{M 1} \hat{\sigma}_{M 1}^{2}+f_{M 2} \hat{\sigma}_{M 2}^{2}+f_{G 1} \hat{\sigma}_{G M}^{2}=f_{M} \hat{\sigma}_{M 1}^{2} \tag{2.47}
\end{equation*}
$$

with

$$
\begin{equation*}
f_{M 1}+f_{M 2}+f_{G M}=f_{M r} \tag{2.48}
\end{equation*}
$$

where $f_{M 1}$ and $\hat{\sigma}_{M 1}^{2}$ and $f_{M 2}$ and $\hat{\sigma}_{M 2}^{2}$ are the degrees of freedom and variances of two merges, consolidated as a single set of parameters, in the grand merge, with degrees of freedom and variance $f_{G M}$ and $\hat{\sigma}_{G M}^{2}$. The degrees of freedom and variance of the single step merge are denoted by $f_{M}, \hat{\sigma}_{M}^{2}$. Finally, the stepwise approach is simpler and less expensive in computer usage.

### 2.6 RKR Potentials

One of the best methods of obtaining accurate potentials for diatomic molecules is by means of the so-called RKR procedure developed by Rydberg, Klein ani Rees (12).

The RKR method is based on the W.K.B. (Wentzel-KramersBrillouin) approximation (13). In this approximation, the eigenvalues for the one-dimensional motion of a particle in a potential well are given by the phase integral condition,

$$
\begin{equation*}
\oint p(r) d r=h\left(v+\frac{1}{2}\right)=I, \tag{2.49}
\end{equation*}
$$

where $p(r)$ is the total radial momentum of the particle, $I$ is the action variable arising from the quantization of the radial momentum, and $v$ is the vibrational quantum number. $\oint$ denotes the integral taken over a complete cycle of the classical motion.

Klein (12) suggested a procedure to solve the integral (2.49), leading to an explicit relationship between the turning points, $r_{1}$ and $r_{2}$, and the auxiliary terms $f$ and $g$. In the case of a rotationless state of the molecule ( $J=0$ ), $r_{1}$ and $r_{2}$ are given by,

$$
\begin{gather*}
f=\frac{1}{2}\left(r_{2}-r_{1}\right)  \tag{2.50}\\
g=\frac{1}{2}\left(1 / r_{1}-1 / r_{2}\right) \tag{2.51}
\end{gather*}
$$

so that

$$
\begin{equation*}
r_{1,2}(U)=\left(f / g+f^{2}\right)^{1 / 2} \pm f . \tag{2.52}
\end{equation*}
$$

The functions $f$ and $g$ are given by,

$$
\begin{equation*}
f(U)=\left[h / 8 \pi^{2} c \mu\right]^{1 / 2} \int_{-\frac{1}{2}}^{v} \frac{d v^{\prime}}{\sqrt{U} G\left(v^{*}\right)} \tag{2.53}
\end{equation*}
$$

and

$$
\begin{equation*}
g(U)=\left[8 \pi^{2} c \mu / h\right]^{1 / 2} \int_{-\frac{1}{2}}^{v} \frac{B_{v} d v^{\prime}}{\sqrt{U-G\left(v^{\prime}\right)}}, \tag{2.54}
\end{equation*}
$$

where $v^{\prime}$ is regarded as a continuous variable, and $v$ is the vibrational quantum number for $E=U=G(v)$. In order to evaluate these integrals it is necessary that the rotational constants ( $B_{v}$ ) and energy levels ( $G(v)$ ) be known accurately. The integrals in Eqs. (2.53) and (2.54) may be evaluated by numerical integration except at the upper limits, where the denominators of the integrands become infinite.

This difficulty can be overcome (14) by assuming that the intervals between pairs of energy levels can be adequately represented by local Morse parameters $\tilde{\omega}_{\mathrm{e}}$ and $\tilde{\omega}_{\mathrm{e}} \tilde{\mathrm{x}}_{\mathrm{e}}$; the simple Morse polynomial can be integrated to give analytical expressions for $f$ and $g$. This approach seems a good approximation for most molecules, but is not very adequate for light hydride molecules, especially as the molecule approaches dissociation. However, a new program has been developed recently in this laboratory by Coxon and Foster (15) in which the energy levels are fitted to slowly varying function of the type proposed by Gilmore (16). As a consequence of the weak vibrational dependence, accurate interpolation is achieved with a low-order polynomial. Using the aforementioned program, the available rotational29and vibrational constants have been used to determine
potential energy curves for both electronic states of the
Alo molecule.

## Chapter 3

## Rotational Analysis of the B-X

System of AIO

### 3.1 Introduction

The work presented in this chapter is concerned with rotational analyses of twenty-five bands, specifically 0-1, $0-2,1-0,1-2,1-3,2-0,2-1,2-3,2-4,3-1,3-4,3-5,4-1$, $4-2,4-5,5-2,5-3,5-6,6-3,6-4,7-4,7-5,8-5,8-6$, and 9-6 bands, of the $B^{2} \Sigma^{+} \rightarrow X^{2} \Sigma^{+}$system of Al0; this system is also well-known as the blue-green system of AlO. Effective molecular parameters are determined by linear least-squares fitting of the measured line frequencies.

The B $\rightarrow X$ system of A. 10 is readily observed both in emission and absorption. Many different types of sources excite the system in emission. Examples are: [i] an arc running between aluminum electrodes in air or oxygen (1720); [ii] exploding aluminum wires in an oxidizing atmosphere $(21,22)$; [iii] shock excited $\mathrm{Al}_{2} \mathrm{O}_{3}$ or $\mathrm{Al}+\mathrm{O}_{2}$, (23); [iv] in a high current hollow cathode lamp containing pellets of an $\mathrm{Al}-\mathrm{Al}_{2} \mathrm{O}_{3}$ mixture (24); and [v] from a microwave discharge through gaseous $\mathrm{AlCl}_{3}$ and $\mathrm{O}_{2}$ (25). Aluminum monoxide is of interest for a number of reasons. In particular, temperatures and densities in the upper
atmosphere, as well as the differential extinction of solar radiation, have been calculated from observations in AlO $(B \rightarrow X)$ following release of aluminum containing compounds (26-29). It is also known from thermochemical data that Alo is one of the numerous products occurring in the vapour of alumina, the formation and dissociation of which is exceedingly complex (30).

The Alo molecule has already been the subject of a great deal of spectroscopic work. The most complete vibrational analysis published is that of Shimauchi (19) who has extended the observed levels to $v^{\prime \prime}=12$ for the $X^{2} \Sigma^{+}$state and $v^{\prime}=16$ for the $B^{2} \Sigma^{+}$state. In the same paper it was reported that the $v^{\prime \prime}=9$ level of the $X$ state is displaced by a perturbation (due to the low-lying $A^{2} I I$ state).

A rotational analysis of the $B \rightarrow X$ system was first accomplished in 1927 by Pomeroy (20). The 1-0, 0-0, and 0-1 bands were analyzed and the rotational constants, B and D, for both states were reported.

Ten years later sen (31) using a similar experimental arrangement to that described by Pomeroy, photographed in a high dispersion spectrograph, 1-0, $0-0,0-1,2-1,1-1$, and 1-2 bands in emission. The rotational constants obtained by Sen were in good agreement with Pomeroy's values.

Sen initially used a graphical method (2) for determination of the difference of the $B$ and $X$ state spinrotation constants $(\Delta \gamma)$ from the experimentally obtained spectrum. The reported absolute values of the spin-rotation
constants for the $v^{\prime \prime}=0, v^{\prime \prime}=1$ and $v^{\prime}=0$ vibrational levels were derived by solving simultaneously the equations involving $\Delta \gamma$.

In 1957, Lagerqvist et al. (18) rotationally analyzed bands involving the vibrational levels $v^{\prime}=0,1,2$, and 3, $v^{\prime \prime}=0,1,2,3,4$, and 5, and effective rotational parameters $B$ and D were determined graphically. The differences between the splitting constants $\gamma^{\prime}$ and $\gamma^{\prime \prime}$ were determined in a similar way as that of the previously mentioned analysis. The absolute values for $\gamma^{\prime}$ and $\gamma^{\prime \prime}$ were obtained from the $0-0$ band, where the line positions were most accurately known. However, the reported absolute values were poorly determined.

Finally, in 1975, Mahieu et al. (32) attempted to determine the spin-doubling constants in several vibrational levels of the $X^{2} \Sigma^{+}$and $B^{2} \Sigma^{+}$states from high-resolution spectra obtained from low-temperature discharges. In this paper, eight bands of the $\mathrm{B}^{2} \Sigma^{+} \rightarrow \mathrm{X}^{2} \Sigma^{+}$system with $\mathrm{v}^{\prime} \leq 2$ and $v^{\prime \prime} \leq 3$ were analyzed. The advantage of this excitation source was that it gave "sharp lines" and bands with relatively low rotational quantum number. Therefore, the troublesome band overlapping obtained in the previously mentioned high-temperature sources was avoided. The most interesting consequence of the source used was that the analysis could be performed fairly easily and the correct assignment of the $P_{1}, R_{1}$ and $P_{2}, R_{2}$ branches could be deduced. This was accomplished using the Mulliken relation
(33) which indicates that the $P_{1}\left(N+\frac{1}{2}\right)$ line is always a little more intense than the $\mathrm{P}_{2}\left(\mathrm{~N}-\frac{1}{2}\right)$ line, where N is the nuclear rotation quantum number. The photoelectric trace of the 0-0 band obtained from Mahieu's work suggested that the intense component was always that of lowest wavenumbers. However, this result is just opposite to the conclusions of Sen (31) and Lagerqvist et al. (18), derived from their studies of spectra from an arc discharge, a source which is not favorable for obtaining spectral regions with only one branch present. The spin-rotation constants were approximately two orders of magnitude less than those previously reported and the $\gamma^{\prime}$ was negative. However, the lack of smooth vibrational dependence for the $\gamma_{v}^{\prime}$ and $\gamma_{V}^{\prime \prime}$ values and their experimental errors limit the significance of the work by Mahieu et al. (12) for obtaining accurately even the difference of the spin-rotation interactions of the X and B states.

In summary, the high resolution work which has been performed on the $B^{2} \Sigma^{+} \rightarrow X^{2} \Sigma^{+}$system of AlO is of Jess than optimum value given the fact that the evaluated molecular constants from the previous works were determined by analyzing small numbers of bands involving levels $v " \leq 4$ and $v^{\prime} \leq 3$ and by using the combination differences method, a method which does not utilize all the available data and which yields overly optimistic error limits.

In the present work, the rotational analysis has been extended to $v^{\prime \prime}=6$ and $v^{\prime}=9$. Band origins, effective
rotational parameters and spin-rotation coupling constants have been obtained by reducing the line positions of the individuals bands with the method of direct linear leastsquares fitting. Multiple determinations of the parameters are merged to obtain single-valued estimates of the same parameters, as well as Dunham coefficients describing their vibrational dependences. However, despite the improved quality of data in the present work, the accuracy of the measured line positions is still not sufficient to provide a determination of the absolute magnitude of the spin-rotation coupling constants of both states. The spin-rotation interaction parameters have been found, nevertheless, to within a common undetermined constant, This constant would be provided from a future microwave spectrum of Alo in the ground electronic state. Furthermore, no interactions between either or both the $X$ and $B$ states with the $A^{2} I I$ state could be detected for the analyzed vibrational levels. Rotational analysis of bands with higher vibrational numbers of the system under study was not possible with the excitation source used. The present work is concluded with calculations of RKR potentials curves for the two states.

Shortly after this work was published (34), a similar analysis was reported (35) in which ten bands of the B-X system of Alo were photographed with $v^{\prime} \leq 7$ and $v^{\prime \prime} \leq 5$. All the bands analyzed overlapped with the Coxon and Naxakis (34) work. Although rotational constants of the B state ( $B_{v}, D_{v}$ and $\gamma_{v}$ ) and the bands origins were determined by
simultaneous least-squares fit of the line frequencies, the ${ }^{35}$ constants $B_{v}$ and $D_{v}$ of the ground electronic state were constrained from the work of Lagerqvist et al. (18) and the spin-rotation constants were constrained from the work of Mahieu et al. (32). In conclusion, the recent work of Singh et al. (35) does not contribute anything novel to the analysis of the $B-X$ system of AlO.

### 3.2 Nature of the Electronic Transition

The ground state of the AlO molecule can be formed from an oxygen and an aluminum atom in their ground atomic states with term symbols ${ }^{3} \mathrm{P}_{8}$ and ${ }^{2} \mathrm{P}_{\mathrm{u}}$ respectively.

According to Wigner and Witmer (36), $\Sigma^{+}, \Sigma^{+}(2), ~ I I(2)$, and $\Delta$ molecular states can be derivec from the lowest states of the separated atoms. Experimental evidence supported by SCF calculations (37) suggests that the ground state configuration of AlO can be written as $(\mathrm{KKL})(\mathrm{z} \sigma)^{2}(\mathrm{y} \sigma)^{2}(\mathrm{w} \pi)^{4}(\mathrm{x} \sigma)^{12} \Sigma^{+}$. By promotion of one $\mathrm{y} \sigma$ electron to the $x \sigma$ outer orbital, the second excited state (B) can be obtained. Furthermore, because $\Sigma^{+}$states cannot combine with $\Sigma^{-}$states (2), the $B$ state is also $\Sigma^{+}$. Therefore, both the $X$ and $B$ states of AlO have zero component of electronic orbital angular momentum in the direction of the internuclear axis $(\Lambda=0)$. The spin vector $S=\frac{1}{2}$ couples with the nuclear rotation vector ( $R$ ) perpendicular to the internuclear axis, to give the
resultant total angular momentum (J) of the molecuie. This zero coupling of spin to the internuclear axis is the characteristic of Hund's case (b).

In Hund's case (b) the quantum number $N$ of the total angular momentum apart from spin ( $N=R+L$ ) is defined, and if both states belong to case (b), the following selection rule holds for this quantum number (2), $\Delta N=0, \pm 1$ with the added restriction of $\Delta N \neq 0$ for $\Sigma-\Sigma$ transitions.

These rules are similar to the selection rules for $J$ and hold to the extent that the interaction of the electron spin $S$ with $N$ can be neglected. It is well-known that transitions with $\Delta J=1$ (or $\Delta N=1$ ) compose the $R$ branch and with $\Delta \mathrm{J}=-1$ (or $\Delta \mathrm{N}=-1$ ) the P branch. In branches for which $\Delta J \geqslant \Delta N$ the intensity falls off very rapidly with increasing $N$; these branches are called satellite branches, since their lines lie very close to those of the corresponding main branches with the same $\Delta N$ but having $\Delta \mathrm{J}=\Delta \mathrm{N}$. In the AlO molecule, the satellite branches are completely overlapped by the main R and P branches. Finally, the symmetry selection rules for electric dipole radiation hold quite generally; positive parity terms combine only with negative parity and vice versa. The energy level pattern of a ${ }^{2} \Sigma^{+} \rightarrow{ }^{2} \Sigma^{+}$electronic transition is shown schematically in Fig. 3.1. The separation of the two rotational sublevels with $J=N+\frac{1}{2}\left(F_{1}\right)$ and $J=N-\frac{1}{2}\left(F_{2}\right)$ for a given $N$ is very small compared to the separation of successive rotational levels. With the instrumental
resolution of this work, it was normally possible to resolve the doublet $R\left(R_{1}, R_{2}\right)$ and the doublet $P\left(P_{1}, P_{2}\right)$ branches for lines with $\mathrm{N} \geq 15$.

Figure 3.1: Energy level diagram for the first lines of a ${ }^{2} \Sigma^{+}-{ }^{2} \Sigma^{+}$transition. The spin-splitting has been much exaggerated.

Figure 3.1


### 3.3 Experimental Details and Rotational Assignment

Electronically excited Alo ( $\mathrm{B}^{2} \Sigma^{+}$) radicals were produced in the flow system shown in Fig. 3.2. Anhydrous aluminum trichloride (Anachemia) was placed in a side tube connected to the emission cavity and was heated by a nichrome wire. $\mathrm{AlCl}_{3}$ vapour was produced. Trace quantities of oxygen (Linde, commercial, 99.5\%) were flowed through a 2.45 GHz electrodeless discharge (EMS Microtron 200) operated at a power level of $\sim 100 \mathrm{~W}$ and the discharged gases were removed by a $20 \mathrm{~m}^{3} / \mathrm{hr}$ single-stage pump (LeyboldHeraeus S16A).

The pressure in the system was typically less than $\sim 3$ torr. The flow rate of oxygen and the heating temperature of $\mathrm{AlCl}_{3}$ were adjusted to maintain the blue-green Alo discharge and to optimize the emission.

In order to decide which bands would be rotationally analyzed, a low resolution spectrum in the region 4200-5400 $\AA$ was $r \geq$ corded in the first order of a Spex 1704X 1-m spectrometer fitted a 1200 groove/mm (120 x 140 mm ) grating blazed at 5000 A. The reciprocal dispersion in the first order is $\sim 8.0 \AA \mathrm{~mm}^{-1}$. The detection system consisted of an RCA-C31014A GaAs photomultiplier (dark count $\sim 4$ counts/sec at $-20^{\circ} \mathrm{C}$ and cathode voltage -1500 V ) mounted inside an R.F. shielded, water-cooled housing (Products for Research, TE-192-RF) equipped with a heated window assembly and focussing lens. The photomultiplier output was displayed on a

Figure 3.2: The flow system used for the production of Alo ( $\mathrm{B}^{2} \Sigma^{+}$) molecule.


It was decided that high resolution spectra would be recorded photographically in the following regions:
[i] $4400 \AA-4645 \AA(\Delta v=3,2$, sequences),
[ii] $5080 \AA-5490 \AA(\Delta V=-1,-2$ sequences $)$. Spectra in these regions were recorded on Kodak $103 a 0$ and 103aD photographic plates. A 3.5-m R.S.V. Ebert spectrograph was used in the 2nd order of a 1200 line $/ \mathrm{mm}$ grating (reciprocal dispersion of about $1 \AA \mathrm{~mm}^{-1}$ ). The grating was blazed at 10000 A. Exposure times were 4 hours with a slit width of $15 \mu \mathrm{~m}$. A quartz filter was inserted for cutting off the third-order wavelengths. Wavelength calibration was made by reference to emission lines in overlapping orders of a commercial hollow cathode iron/neon discharge lamp (Westinghouse WL22810A) operated at 20 mA with exposure time 30 minutes. Spectral and calibration lines were measured on a horizontal, Abbé type comparator ( $0.5 \mu \mathrm{~m}$ resolution). The dispersion curve of each measured plate was established by fitting the iron and neon standard lines (38) to a second or third-order polynomial in distance; the dispersion curve could be used to generate the wavenumbers of the AlO spectral lines. The standard deviations of these calibration fits were typically 0.004 A .

Figure 3.3: Photoelectric trace of the $\Delta v=-1$ and $\Delta v=-2$ sequences of the $B$ - $X$ system of Alo.


For many of the photographed bands with low $\mathrm{v}^{\prime}$ or $\mathrm{v}^{4}, 4$ combination differences agreed with the constants determined by Lagerqvist et al. (18). Preliminary fits of these bands provided approximate constants for higher levels. Thus, the assignments for other bands were obtained progressively. In the case of blended lines, assignments were made in accord with intensity considerations. Because of the similar magnitude of the rotational constants of the $X$ and $B$ states, the observed vibrational sequences are quite compact; this causes considerable and unavoidable blending of lines, particularly within the latter members of the sequences. A portion of the $\Delta v=2$ sequence is shown in Fig. 3.4. The rotational assignment of the 5-3 band is also indicated. From the spectrum it is obvious that the Alo B-X system is red-degraded with extensive overlapping of R -branch lines of low J near the band heads. As a consequence, it was not possible to make any assignments for the R branch to lines with $\mathrm{N}<27$.

Figure 3.4: A portion of the $\Delta v=2$ sequence and the rotational assignment of the 5-3 band in the $B^{2} \Sigma^{+}-X^{2} \Sigma^{+}$system of AlO.

Figure 3.4


The problem of unresolved doublets at the beginning of each branch was solved by fitting these doublets to the average of the respective calculated branches. Since the components of each doublet have similar intensities, this procedure is quite satisfactory. It also represents a major improvement over the commonly employed alternatives of either inclusion of each blended line twice in the fit to the experimental data, or exclusion of the blended data. The high correlation of the spin-rotation coupling constants did not allow determination of the absolute magnitude of $\gamma$ in both electronic states. Therefore, it was necessary $=0$ constrain one of $\gamma_{v}^{\prime}$ or $\gamma \|$ in order to obtain the magnitude of the other or the difference $\gamma_{V}^{\prime}-\gamma_{V}^{\prime \prime}$ as in the present work. Approximate values for the constrained spin-rotational parameters $\gamma \| \mathrm{V}$ were obtained from the work Mahieu et al. (32). Finally, the present analysis adopted the negative sign of $\gamma^{\prime}$, first because the experimental method used by Mahieu et al. (32) was better for obtaining the absolute intensity of the spectral lines, as explained in the introduction of this chapter, and second, because the variance of the band-by-band reduction of the system was slightly smaller with $\gamma_{v}^{\prime}$ negative.

### 3.4 Results and Discussion

A computer program was written for direct least-squares fitting of the measured line positions of individual bands.

The program employs the well-known expressions for $\mathrm{a}^{2} \Sigma^{+}$ state, as described in Chapter 2,

$$
\begin{align*}
& F_{v}(J, e)=B_{v} x(x-1)-D_{v} x^{2}(x-1)^{2}+1 / 2 \gamma_{v}(x-1)  \tag{3.1}\\
& F_{v}(J, f)=B_{v} x(x+1)-D_{v} x^{2}(x+1)^{2}-1 / 2 \gamma_{v}(x+1), \tag{3.2}
\end{align*}
$$

where $x=(J+1 / 2)$, and $e$ and $f$ denote the parity labellings, corresponding to the $F_{1}$ and $F_{2}$ components respectively. Higher-order parameters ( $H_{v} . . ., \gamma_{D v} . .$. ) can also be fitted, but these were not required for any of the bands. Several parameters associated with the least-squares fits are given in Table 3.1. For each band, the table gives the band origin, the number of lines fitted, the rotational extent and the standard deviation $(\hat{\sigma})$ of the fit. The values of $(\hat{\sigma})$ range from 0.015 to $0.058 \mathrm{~cm}^{-1}$, a spread which is due to the variation of intensity, and hence measurement precision, of the measured bands and/or the degree of accidental blending of branches. Although the sequence with $\Delta v=-1$ is stronger than the other three measured sequences $(\Delta v=3,2,-2)$, the vibrational spacings of the $B$ and $X$ electronic states cause considerable overlapping of the latter members of the $\Delta v=-1$ sequence. This effect is in accord with the standard deviations and the number of lines listed in the Table 3.1. Blending of lines was expected to be least troublesome in the sequence with $\Delta v=3$; accordingly, more bands were analyzed within this sequence, even though it was the weakest one. In order to obtain the parameters given in Table 3.1 , several preliminary leastsquares fits were performed for each '?and with the purpose
of identifying and removing any lines with anomalously large residuals. Such lines were removed from subsequent fits. Systematic error that was not revealed in the individual band fits was identified in preliminary merge fits of the constants. Additional lines were then removed from the data set; the magnitudes of the residuals of such lines were nearly always considerably more than two standard deviations of the final fits.

Table 3.1: Individual Band Fits ${ }^{2}$ in the $B^{2} \Sigma^{+} \rightarrow X^{2} \Sigma^{+}$System of AlO

|  | v" | $\nu_{0}\left(\mathrm{~cm}^{-1}\right)$ | n | $\mathrm{N}_{\text {max }}$ | $\hat{\sigma}\left(\mathrm{cm}^{-1}\right)$ | $\mathrm{rms} / \hat{\sigma}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 19669.792 (6) | 124 | 65 | 0.016 | 1.00 |
| 0 | 2 | 18718.405(8) | 49 | 36 | 0.028 | 1.02 |
| 1 | $0^{\text {b }}$ |  | 26 | 64 | 0.015 | 1.02 |
| 1 | 2 | 19581.512 (6) | 93 | 55 | 0.016 | 0.99 |
| 1 | 3 | 18644.137(7) | 57 | 41 | 0.024 | 1.00 |
| 2 | 0 | 22354.059(12) | 63 | 47 | 0.028 | 1.08 |
| 2 | $1^{\text {b }}$ |  | 38 | 64 | 0.030 | 1.03 |
| 2 | 3 | 19499.921(6) | 76 | 53 | 0.017 | 1.06 |
| 2 | 4 | 18576.598(11) | 47 | 38 | 0.035 | 0.97 |
| 3 | 1 | 22237.069 (9) | 111 | 54 | 0.026 | 0.99 |
| 3 | 4 | 19425.054 (8) | 57 | 42 | 0.023 | 1.10 |
| 3 | 5 | 18515.829(14) | 25 | 30 | 0.026 | 1.83 |
| 4 | 1 | 23078.060(11) | 39 | 33 | 0.031 | 1.10 |
| 4 | 2 | 22126.759 (8) | 74 | 32 | 0.026 | 1.17 |
| 4 | 5 | 19357.057(20) | 41 | 42 | 0.058 | 1.08 |
| 5 | 2 | 22960.453(10) | 52 | 38 | 0.030 | 0.99 |
| 5 | 3 | 22023.177 (9) | 63 | 49 | 0.028 | 1.09 |
| 5 | 6 | 19295.479(17) | 12 | 22 | 0.028 | 2.44 |
| 6 | 3 | 22849.526(12) | 52 | 34 | 0.039 | 1.12 |
| 6 | 4 | 21926.325(9) | 74 | 46 | 0.028 | 1.01 |
| 7 | 4 | 22745.338(13) | 58 | 40 | 0.042 | 1.01 |
| 7 | 5 | 21836.299(10) | 58 | 42 | 0.031 | 1.11 |
| 8 | 5 | 22647.985(20) | 36 | 36 | 0.057 | 1.14 |
| 8 | 6 | 21752.905(15) | 57 | 40 | 0.037 | 1.20 |
| 9 | 6 | 22557.230(20) | 37 | 31 | 0.055 | 1.05 |

${ }^{a}$ For each band fitted, $n$ is the total number of fitted lines, $\hat{\sigma}$ is the standard deviation, and $N_{\text {max }}$ is the maximum value of the quantum number $\mathrm{N}^{\prime \prime}$ of the fitted lines. The band origins ( $\nu_{0}$ ) and estimated standard errors (in parentheses) were obtained from a subsequent merge fit (see text) ; rms/ $\hat{\sigma}$ represents the ratio of the root mean square of the residuals for lines calculated with merged parameters to the estimated standard deviation.
For these bands, the measurements were limited to the doublet separations, $R_{1}(N)-R_{2}(N)$ and $P_{1}(N)-P_{2}(N)$, and the fit led to a single estimated parameter, $\Delta \gamma_{v^{\prime}, v^{\prime}}$.

As indicated in Table 3.1, special considerations were necessary in the fits for four of the bands. For the 3-5 and 5-6 bands, the rotational development did not permit the determination of all four parameters $B^{\prime}, D^{\prime}, B^{\prime \prime}$, and $D^{\prime \prime}$ simultaneously. For the 3-5 band, the fit led to estimates of $B_{3}^{\prime}, B_{5}^{\prime \prime}$, and the difference, $D_{3}^{\prime}$ - $D_{5}^{\prime \prime}$; for the 5-6 band, only two effective rotational parameters, $B!-B_{5}^{\prime \prime}$ and $D!-$ $D_{6}^{\prime \prime}$ were fitted. In the case of the 1-0 and 2-1 bands, the calibration spectrum of the plate was incomplete, and absolute wavenumbers of the lines were obtained necessarily by extrapolation. Despite systematic error in the absolute line positions, the doublet spacings in the $P$ and $R$ branches were obtained reliably, and were fitted as such.

The measured line positions of 25 bands in the $B \rightarrow X$ system of Alo have been employed in a least-squares fitting procedure to give 137 estimated parameters. As mentioned previously, except for the $3-5,5-6,1-0$ and $2-1$ bands, the individual band fits yielded six parameters, $\nu_{0}, B_{v}^{\prime}, D_{v}^{\prime}, B_{v}^{\prime \prime}$, $D_{v}^{\prime \prime}$, and $\left(\gamma_{v}^{\prime}-\gamma_{V}^{\|}\right)$. For the 1-0 and 2-1 bands, only a single estimated parameter, $\left(\gamma_{v}^{\prime}-\gamma_{v}^{\prime}\right)$, is obtained from the spinrotation splittings, $P_{1}(N)-P_{2}(N)$ and $R_{1}(N)-R_{2}(N)$. Several merges of the 137 estimated constants from the individual band fits have been performed in a stepwise fashion (4) to obtain the best single-valued estimates of the constants $\left(X_{v}\right)$ for each level, and of "Dunham coefficients" ( $\mathrm{X}^{(1)}$ ) in polynomial representation of $X_{v}$

$$
\begin{equation*}
X_{v}=\sum_{i=0}^{m} x^{(i)}(v+1 / 2)^{i} \tag{3.3}
\end{equation*}
$$

In the first such merge (A), the reduced set of estimated parameters consists of 23 band origins, 20 parameters for the $B^{2} \Sigma^{+}$state ( $\left.B_{v}, D_{v} ; v=0-9\right), 14$ parameters for the $x^{2} \Sigma^{+}$ state $\left(B_{v}, D_{v} ; V=0,6\right)$ and 25 unmerged ( $\gamma_{v}^{\prime}-\gamma_{v}^{\prime \prime}$ ) constants for individual bands.

The estimated variance of the merge (A) with $f=137$ $82=55$ degrees of freedom is given by $\left(\hat{\sigma}_{M}\right)^{2}=(1.255)^{2}$.

The constraints introduced in Merge A improved the vibrational dependence of the estimated values of ( $\gamma_{v}^{\prime}-\gamma_{v}^{\prime \prime}$ ) over those given by the individual band fits. The estimates of ( $\gamma_{v}^{\prime}-\gamma_{v}^{\prime}$ ) for the 1-0 and 2-1 bands were unchanged as expected. The estimated values of $\left(\gamma_{v}^{\prime}-\gamma_{v}^{\prime}\right)$ output from merge $A$ are listed in the first column of Table 3.2.

It is immediately obvious that $\gamma_{V}^{\prime \prime}$ changes significantly with v', while $\gamma_{v}^{\prime}$ varies slowly with $v^{\prime}$. For example, in the $v^{\prime}=4$ progression, $\left(\gamma_{4}^{\prime}-\gamma_{v}^{\prime \prime}\right)$ changes from -0.01536 to $-0.03151 \mathrm{~cm}^{-1}$ over the range $\mathrm{v}^{\prime \prime}=1$ to $\mathrm{v}^{\prime \prime}=5$; but in the $\mathrm{v}^{\prime \prime}=3$ progression only a small change in ( $\gamma_{V}^{\prime}-\gamma_{3}^{\prime \prime}$ ) occurs from -0.01809 to $-0.01928 \mathrm{~cm}^{-1}$ in the range $v^{\prime}=1$ to $v^{\prime}=6$. Unfortunately, the precision of the measured line positions only permitted the determination of the differences $\left(\gamma_{v}^{\prime}-\gamma_{v}^{\prime}\right)$ from the individual bands. However, it was possible to determine the spin-rotation constants $\gamma$ of the $B$ and X states within an undetermined common constant, as follows. The ground state equilibrium parameter, $\gamma_{\mathrm{e}}^{\mathrm{e}}$, was
arbitrarily selected as the undetermined constant.
Expressing $\gamma \|$ as

$$
\begin{equation*}
\gamma_{v}^{\prime \prime}=\gamma^{\prime}+\gamma^{\prime \prime}(1)\left(v^{\prime \prime}+1 / 2\right)+\ldots \tag{3.4}
\end{equation*}
$$

then

$$
\begin{equation*}
\left(\gamma_{v}^{\prime}-\gamma_{v}^{\|}\right)=\left(\gamma_{v}^{\prime}-\gamma_{\|}^{\|}\right)-\gamma^{\prime(1)}\left(v^{\prime \prime}+1 / 2\right)+\ldots \tag{3.5}
\end{equation*}
$$

Table 3.2: Least-Squares Fitting of ( $\left.\gamma_{v}^{\prime}-\gamma_{v}^{W}\right)\left(\mathrm{cm}^{-1}\right)$, from
Individual Bands of $\mathrm{AlO}\left(\mathrm{B}^{2} \Sigma^{+} \rightarrow \mathrm{X}^{2} \Sigma^{+}\right)$

| $\mathrm{v}^{\prime}-\mathrm{v}^{\prime \prime}$ | Estimated ${ }^{\mathrm{a})}$ | Fitted ${ }^{\mathrm{b})}$ | Residual | Res/ $\sigma$ | Ref. (32) |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 1 | -0.01414 | -0.01399 | -0.00015 | -2.0 | -0.0136 |
| 0 | 2 | -0.01636 | -0.01573 | -0.00063 | -1.8 | -0.0135 |
| 1 | 0 | -0.01276 | -0.01289 | 0.00013 | 2.1 | -0.0119 |
| 1 | 2 | -0.01569 | -0.01587 | 0.00018 | 1.8 | -0.0129 |
| 1 | 3 | -0.01809 | -0.01864 | 0.00055 | 2.1 | -0.0154 |
| 2 | 0 | -0.01400 | -0.01303 | -0.00097 | -3.2 | -0.0124 |
| 2 | 1 | -0.01442 | -0.01428 | -0.00014 | -1.2 | -0.0135 |
| 2 | 3 | -0.01856 | -0.01878 | 0.00022 | 1.9 | -0.0159 |
| 2 | 4 | -0.02379 | -0.02314 | -0.00065 | -1.5 |  |
| 3 | 1 | -0.01449 | -0.01442 | -0.00007 | -0.5 | -0.0137 |
| 3 | 4 | -0.02371 | -0.02328 | -0.00043 | -1.8 |  |
| 3 | 5 | -0.03019 | -0.02979 | -0.00040 | -0.8 |  |
| 4 | 1 | -0.01536 | -0.01456 | -0.00080 | -1.8 | -0.0161 |
| 4 | 2 | -0.01599 | -0.01629 | 0.00030 | 1.6 |  |
| 4 | 5 | -0.03151 | -0.02993 | -0.00158 | -2.5 |  |
| 5 | 2 | -0.01658 | -0.01644 | -0.00014 | -0.4 |  |
| 5 | 3 | -0.01823 | -0.01920 | 0.00097 | 3.4 |  |
| 5 | 6 | -0.03999 | -0.03928 | -0.00071 | -0.8 |  |
| 6 | 3 | -0.01927 | -0.01935 | 0.00008 | 0.2 |  |
| 6 | 4 | -0.02406 | -0.02371 | -0.00035 | -1.5 |  |
| 7 | 4 | -0.02400 | -0.02385 | -0.00015 | -0.3 |  |
| 7 | 5 | -0.03121 | -0.03036 | -0.00085 | -2.7 |  |
| 8 | 5 | -0.03198 | -0.03050 | -0.00148 | -1.9 |  |
| 8 | 6 | -0.03883 | -0.03971 | 0.00088 | 2.3 |  |
| 9 | 6 | -0.03799 | -0.03985 | 0.00206 | 2.1 |  |

[^0]The 25 estimated values of ( $\gamma \dot{v}-\gamma \|)$ ) can be reduced by least-squares to a fitted set of 10 parameters ( $\gamma \mathbf{v}$ - $\gamma!$ ), $v^{\prime}=0-9$, and a small number of coefficients, $\gamma^{\prime \prime}{ }^{(1)}, \gamma^{\prime \prime}(2)$, $\gamma^{\prime \prime}{ }^{(3)}$, representing the vibrational dependence of $\gamma_{V}^{\prime \prime}$. This fit was performed in merge $B$, with the fitted parameters given in Table 3.3. The estimated variance of this merge $B$ with $f=137-70=67$ degrees of freedom is given by $\left(\hat{\sigma}_{M}\right)^{2}=(1.401)^{2}$. The success of the model used in merge $B$. can be demonstrated by the small residuals between the input values and those given by the fitted parameters, as is also indicated in Table 3.2. It is worth mentioning that the residuals are larger on average by a factor of $\sim 1.7$ than the standard errors of the input values. The values of $B_{v}$ and $D_{v}$ for the two states given in Table 3.3 are considered the best single-valued estimates which the data can provide. A further reduction in the number of parameters was obtained in merge $C$ which was performed by adopting Eq. (3.3) for the rotational constants of both states, as well as of $\left(\gamma_{v}^{\prime}-\gamma_{\mathrm{g}}^{\prime}\right)$. The order of the expansion required for each constant was determined by performing fits to increasingly higher-order until the standard deviation of the fitted coefficient ( $\mathrm{X}^{(1)}$ ) was larger than its magnitude. The output of merge $C$ consisted of the 16 Dunham parameters listed in Table 3.4 and 23 new estimates of the band origins, listed in Table 3.1. The variance of the equivalent single-step merge from the constants of the individual bands to the parameters of merge $C$ was $\left(\hat{\sigma}_{M}\right)^{2}=(1.829)^{2}$ with

## $f=137-39=98$ degrees of freedom.

## Table 3.3: Merged Parameters ${ }^{2}\left(\mathrm{~cm}^{-1}\right)$ for the $\mathrm{X}^{2} \mathrm{\Sigma}^{+}$and $\mathrm{B}^{2} \Sigma^{+}$ States of AlO

| $v$ | $\mathrm{B}^{2} \mathrm{E}^{+}$ |  |  | $x^{2} x^{+}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{B}_{\mathrm{v}}$ | $10^{6} \mathrm{D}_{\mathrm{v}}$ | $-10^{2}\left(\gamma_{v}^{\prime}-\gamma_{6}^{\prime \prime}\right)$ | $) \quad \mathrm{B}_{v}$ | $10^{6} \mathrm{D}_{v}$ |
| 9 | 0.56267 (22) | $0.98(20)$ | 1.07(16) |  |  |
| 8 | 0.567008(115) | 1.317(73) | 1.211(95) |  |  |
| 7 | 0.571067 (65) | 1.174(30) | 1.365(52) |  |  |
| 6 | 0.575437 (60) | 1,201(25) | 1.343(38) | 0.603547 (130) | 1.442(85) |
| 5 | 0.579662(55) | 1.145(19) | 1.265(40) | 0.609343 (55) | 1.250 (19) |
| 4 | 0.584132(56) | 1.176(18) | 1.291(36) | 0.615285 (56) | 1.222(20) |
| 3 | $0.588482(53)$ | 1.164(14) | 1.288(31) | $0.621103(56)$ | 1.170(18) |
| 2 | 0.592953(58) | 1.180(20) | 1.269(27) | 0.626928 (53) | 1.136(16) |
| 1 | $0.597389(55)$ | 1.156(17) | 1.224(20) | 0.632775 (54) | 1.140(14) |
| 0 | 0.601936(51) | 1.1834(134) | 1.248 (31) | 0.638576 (67) | 1.133(25) |

a Values in parentheses are estimated standard errors given by $\sigma_{M}\left(V_{i i}\right)^{1 / 2}$, in units of the last significant figure of the corresponding constant. The ground state spin-rotation constant varies with $v^{\prime \prime}$ according to $\gamma_{v}^{\prime \prime}-\gamma_{e}^{\prime \prime}=1.20(38) \times$ $10^{-3}\left(v^{\prime \prime}+1 / 2\right)-2.1(15) \times 10^{-4}\left(v^{\prime \prime}+1 / 2\right)^{2}+1.03(18) \times 10^{-4}$ $\left(v^{\prime \prime}+1 / 2\right)^{3}$.

Table 3.4: Merged Dunham Coefficients ${ }^{a}\left(\mathrm{~cm}^{-1}\right)$ for the $\mathrm{X}^{2} \Sigma^{+} 61$ and $\mathrm{B}^{2} \Sigma^{+}$States of AlO

|  |  | B | D | $\gamma-\gamma_{\bullet}^{\prime \prime}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{B}^{2} \Sigma^{+}$ | X (0) | $6.04160(64) \times 10^{-1}$ | $1.1739(162) \times 10^{-6}$ | -1.206(24) $\times 10^{-2}$ |
|  | X(1) | -4.5338(90) $\times 10^{-3}$ | -4.70(140) $\times 10^{-9}$ | $-1.42(51) \times 10^{-4}$ |
|  | $\mathrm{X}(2)$ | $1.11(24) \times 10^{-5}$ |  |  |
|  | $\mathrm{X}(3)$ | $6.1(22) \times 10^{-7}$ |  |  |
| $\mathrm{X}^{2} \Sigma^{+}$ | $\mathrm{X}(0)$ | 6.41369 (70) $\times 10^{-1}$ | 1.099 (19) $\times 10^{-6}$ |  |
|  | $\mathrm{X}(1)$ | $-5.7302(114) \times 10^{-3}$ | 1.83 (24) $\times 10^{-8}$ | $1.29(34) \times 10^{-3}$ |
|  | $\mathrm{X}(2)$ | $-1.915(160) \mathrm{x}^{-5}$ |  | -1.73(136) $\times 10^{-4}$ |
|  | $\mathrm{X}(3)$ |  |  | $9.23(148) \times 10^{-5}$ |

a Values in parentheses are estimated standard errors given by $\sigma_{M}\left(V_{i i}\right)^{1 / 2}$ in units of the last significant figure of the corresponding constant.

# Table 3.5: Calculated Line Positions and Residuals for the $B \rightarrow X$ System of AlO 

All data are in units of wavenumbers $\left(\mathrm{cm}^{-1}\right)$. Lines flagged with * were excluded from the final least-squares fitting procedure. The columns labelled "Res" are the residuals defined by

$$
\text { Res }=\text { observed position - calculated position }
$$

Unresolved $P_{1}(N)$ and $P_{2}(N)$ or $R_{1}(N)$ and $R_{2}(N)$ blends are listed between the corresponding branches.

CALCULATED LINE POSITIONS AND RESIPUALSFOR THE $O-1$ BAHD OF ALO (B2E- $\times 2$ E)

| H | $\mathrm{P}_{2}$ |  | $\mathrm{P}_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Calc | Res | Cale | Res | Calc | Res | Calc | Res |
| 0 |  |  |  |  | 19671.008 |  | 19670.989 |  |
| 1 |  | 19668.529 |  |  | 19672.157 |  | 19672.124 |  |
| 2 |  | 19667.202 | -0.039* |  | 19673.244 |  | 19673.197 |  |
| 3 |  | 19665.813 | -0.039* |  | 19674.270 |  | 19674.209 |  |
| 4 |  | 19664.363 | -0.029 |  | 19675.234 |  | 19675.159 |  |
| 5 |  | 19662.851 | 0.006 |  | 19676.136 |  | 19676.047 |  |
| 6 |  | 19661.278 | 0.030 |  | 19676.976 |  | 19676.873 |  |
| 7 |  | 19659.643 | 0.001 |  | 19677.754 |  | 19677.637 |  |
| 8 |  | 19657.946 | -0.005 |  | 19678.470 |  | 19678.339 |  |
| 9 |  | 19656.188 | 0.006 |  | 19679.125 |  | 19678.980 |  |
| 10 |  | 19654.369 | 0.001 |  | 19679.717 |  | 19679,558 |  |
| 11 |  | 19652.488 | 0.007 |  | 19680.247 |  | 19680.074 |  |
| 12 |  | 19650.546 | -0.008 |  | 19680.716 |  | 19680.529 |  |
| 13 | 19648.630 | 0.018 | 19648.454 | 0.024 | 19581.112 |  | 19680.921 |  |
| 14 | 19646.572 | -0.020 | 19646.381 | 0.007 | 19681.466 |  | 19681.251 |  |
| 15 | 19644.453 | 0.007 | 19644.248 | -0.026 | 19681.747 |  | 19681.518 |  |
| 16 17 | 19642.272 | 0.011 0.010 | 19642.053 | 0.015 -0.009 | 19681.967 19682.124 |  | 19681.724 19681.867 |  |
| 18 | 19637.726 | 0.013 | 19637.479 | -0.038 | 19682.219 |  | 19681.948 |  |
| 19 | 19635.361 | -0.015 | 19635.100 | -0.018 | 19682.251 |  | 19681.966 |  |
| 20 | 19632.935 | 0.016 | 19632.660 | -0.009 | 19682.221 |  | 19681.922 |  |
| 21 | 19630. 447 | 0.020 | 19630.158 | -0,020 | 19682.129 |  | 19681.816 |  |
| 22 | 19627.898 | 0.009 | 19627.595 | -0.013 | 19681.974 |  | 19681.647 |  |
| 23 | 19625.288 | -0.008 | 19624.971 | -0.003 | 19681.756 |  | 19681.415 |  |
| 24 | 19622.616 | 0.012 | \$9622.286 | -0.015 | 19681.476 |  | 19681.121 |  |
| 25 | 19619.883 | 0.011 | 19619.539 | -0.005 | 19681.133 |  | 19680.764 |  |
| 26 | 19617.089 | 0.002 | 19616.730 | -0.012 | 19680.728 |  | 19680.345 |  |
| 27 | 19614.234 | 0.011 | 19613.861 | -0.004 | 19680.259 |  | $19679.862$ |  |
| 28 | 19611.317 | 0.009 | 19610.930 | -0.000 | 19679.728 |  | 19679.317 |  |
| 29 | 19608.339 | 0.015 | 19607.938 | -0.010 | 19679.134 |  |  |  |
| 30 | 19605.299 | -0.001 | 19604.884 | 0.013 | 19678.476 | -0.042* | 19678.037 | 0.004 |
| 31 | 19602.198 | -0.002 | 19601.770 | 0.003 | 19677.756 | -0.056* | 19677.303 | -0.017 |
| 32 | 19599.036 | 0.019 | 19598.593 | -0.002 | 19676.973 | -0.019* | 19676.506 | 0.012 |
| 33 | 19595.813 | 0.015 | 19595.356 | -0.003 | 19676.126 | -0.030* | 19675.645 | 0.019 |
| 34 | 19592.528 | 0.118* | 19592.057 | -0.036 | 19675.216 | 0.008* | 19674.721 | 0.022 |
| 35 | 19589.182 | 0.111* | 19588.697 | -0.073 | 19674.243 | $0.036 *$ | 19673.734 | 0.007 |
| 36 | 19585.774 |  | 19585.275 |  | 19673.207 | 0.019* | 19672.684 | 0.017 |
| 37 | 19582.305 | -0.006 | 19581:792 | -0.001 | 19672.107 | $0.009 *$ | 19671.570 | 0.013 |
| 38 39 | 19578.775 19575.183 | 0.025 | 19578.248 19574.642 | -0.026 | 19670.944 19669.717 | -0.065* | 19670.393 19669.152 | 0.011 -0.008 |
| 40 | 19571.530 | 0.001 | 19570.975 | 0.028 | 19668.426 | 0.036 | 19667.847 | 0.008 |
| 41 | 19567.815 | 0.071* | 19567.247 | 0.016 | 19667.072 | 0.091* | 19666.479 | 0.006 |
| 42 | 19564.039 | -0.001 | 19563.457 | 0.010 | 19665.654 | 0.120* | 19665.047 | -0.013 |
| 43 | 19560.202 | -0.021 | 19559.605 | 0.010 | 19664.172 | 0.162* | 19663.551 | -0.002 |
| 44 | 19556.303 | 0.083* | 19555.692 | 0.021 | 19662.626 | $0.231 *$ | 19661.991 | -0.004 |
| 45 | 19552.343 | -0.013 | 19551.718 | 0.013 | 19661.017 | 0.051 | 19660.368 | 0.001 |
| 46 | 19548.321 | -0.012 | 19547.682 |  | 19659.343 | -0.026 | 19658.680 | -0.010 |
| 47 | 19544.237 | 0.007 | 19543.585 | -0.012 | 19657.605 | -0.028 | 19656.928 | -0.020 |
| 48 | 19540.092 | 0.007 | 19539.426 | -0.013 | 19655.803 | -0.014 | 19655.112 | 0.001 |
| 49 | 19535.886 | -0.006 | 19535.205 |  | 19653.936 | -0.015 | 19653.231 | 0.010 |
| 50 | 19531.618 | 0.007 | 19530.920 | 0.024 | 19652.005 | 0.003 | 19651.286 | -0.017 |
| 51 | 19527.288 |  | 19526.579 |  | 19650.010 | -0.005 | 19649.277 | -0.039 |
| 52 | 19522.896 | 0.020 | 19522.174 | -0.014 | 19647.950 | -0.011 | 19647.203 | -0.015 |
| 53 | 19518.443 |  | 19517.706 |  | 19545.825 | -0.008 | 19645.064 | -0.033 |
| 54 | 19513.928 | 0.016 | 19513.178 |  | 19643.636 | 0.015 | 19642.861 | 0.026 |
| 55 | 19509.352 |  | 19508.587 |  | 19641.382 | -0.004 | 19640.593 | -0.010 |
| 56 | 19504.713 |  | 19503.935 |  | 19639.063 | 0.010 | 19638.260 | -0.024 |
| 57 | 19500.013 |  | 19499.220 |  | 19636.679 | 0.003 | 19635.862 | 0.010 |
| 58 | 19495.251 |  | 19494.444 |  | 19634.230 | 0.013 | 19633.399 | -0.011 |
| 59 | 19490.427 |  | 19489.606 |  | 19631.716 | 0.013 | 19630.871 | 0.044* |
| 50 | 19485.541 |  | 19484.707 |  | 19629.136 | 0.066* | 19628.277 |  |
|  | 19480.593 |  | 19479.745 |  | 19626.491 | 0.022 | 19625.619 |  |
| E 2 | 19475.584 |  | 19474.721 |  | 19623.781 | 0.066* | 19622.894 |  |
| 63 | 19470.512 |  | 19469.635 |  | 19621.005 | 0.027 | 19620.105 |  |
| 64 | 19465.378 |  | 19464.488 |  | 19618.164 | 0.105* | 19617.249 |  |
| 65 | 19460.182 |  | 19459.278 |  | 19615.257 | -0.010 | 19614.328 |  |
| 66 | 19454.924 |  | 19454.006 |  | 19612.284 |  | 19611.341 |  |
| 67 | 19449.604 |  | 19448.671 |  | 19609.245 |  | 19608.288 |  |


| н | $\mathrm{P}_{2}$ |  | $\mathrm{P}_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Cale | Res | calc | Res | Calc | Re | calc | Res |
|  |  |  |  |  | 18719.622 |  | 18719.502 18720.748 |  |
| $\frac{1}{3}$ |  | 18715.851 | -0.003 |  |  |  |  |  |
| 5 |  | 187711.0.033 | -0.001 |  | -18723.967 ${ }^{18724.928}$ |  | 188723.8885 |  |
| 7 |  | 18711.1335 | -0.014 |  | 118725 5 18726.839 |  | 18825 18725.725 1859 |  |
| 8 |  | 18700:978) | -0.035 |  | 18727.510 |  | 18727.364 |  |
| 10 |  | 187703.662 | -0.007 |  | 18728.979 |  | 18728.802 |  |
| 12 |  | 18700:066 | -0.036 |  | - 18729.638 |  | 16739.445 |  |
| 14 |  | 18699.213 | -0.031 |  | 18830.805 |  | 18730.580 |  |
| 15 |  | 18694.359 | -0.006 |  | 117731.769 |  | 18731.513 |  |
| \% | 18692.481 18680.434 1808 | -0.069* | 186950.174 <br>  <br> 18688 <br> 1065 | -0.026 |  |  | -18732. 244 |  |
| 19 | - 18.8668 .201 | 0.042 | ${ }^{1868565.065}$ | -0.024 | - $187333.89{ }^{187}$ |  | - 187322.738 |  |
| 20 | 10684.008 | 0.038 | 186883.698 | -0.044 | 118733.295 |  | ${ }^{188732.960}$ |  |
| 22 |  | 0.042 | 18677:133 | -0.042 | 18733.550 |  | 18733.184 |  |
| 24 | 1867:742 | -0.015 | 118674, 3 365 | -0, 012 | 11733.602 |  | 18733.204 |  |
|  | 18659.811 | -0.001 | 18669.407 | -0.022 | 18733.450 |  | 18733.021 |  |
|  | ${ }^{186657.271}$ | -0.012 | ${ }^{1866664.851}$ | 0.002 | 18733.297 |  | 18732.852 |  |
| - 30 | - 18665.048 | -0.035 | ${ }^{186551.5892}$ | -0:060 | 118732. ${ }^{1835}$ |  |  |  |
| ${ }^{31}$ |  |  | 118656.134 | - | 18732.175 |  | 18733.668 |  |
| 33 | 18650.993 | 0.048 | -18650.479 | -0.105* | 18731.307 |  | 退18730. ${ }^{1868}$ |  |
| 34 35 | ${ }_{18645.172}^{18649}$ | -0.025 | ${ }_{1}^{1864474.575}$ | 0.004 0.014 | 18730.796 18730.233 |  | -18730.241 |  |
| ${ }^{35}$ | ${ }_{\text {186492.186 }}^{18}$ | 0.032 | 18641.624 | 0.024 | 118729.619 |  | 28729.0.033 |  |
| 38 | ${ }_{18653.068}^{1865}$ |  | 18655.474 |  | 18728.236 |  | 18727. ${ }^{1819}$ |  |
| 40 | 18629.751 |  | 18629.126 |  | 18726.647 |  | 18725.998 |  |
| $\begin{aligned} & 41 \\ & 42 \end{aligned}$ | ${ }_{18623}^{186237}$ |  | ${ }^{186525.8780}$ |  | 18825.775 |  | ${ }_{18724.171}^{1825}$ |  |
| $\begin{aligned} & 43 \\ & 44 \\ & \hline 4 \end{aligned}$ | 186516.524 |  | 18619.233 |  | 188723.875 |  | 187723.1796 |  |
| 45 | 186609.614 |  | 18612.391 |  | - 18721.768 |  |  |  |
| $\begin{aligned} & 47 \\ & 48 \\ & 49 \end{aligned}$ | ${ }^{186606.085}$ |  | 18605.350 18601.756 |  | 18719.453 |  | 18718.6944 |  |
| $$ | ${ }_{185959.208}^{1859}$ |  | 18594.112 |  | 18716.929 |  | 18716:138 ${ }^{1871482}$ |  |
| 51 | ${ }^{185951.474}$ |  | 18590.576 |  | 18714.1966 |  | 18713.374 187121.314 |  |
| 53 54 | ${ }_{18579}^{1858989}$ |  | 185893.042 |  | 18711:254 |  |  |  |


| N | $\mathrm{P}_{2}(\mathrm{~N})-\mathrm{P}_{1}(\mathrm{~N})$ |  | $\mathrm{R}_{2}(\mathrm{~N})-\mathrm{R}_{1}(\mathrm{~N})$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Calc | Rea | calc | Res |
|  <br> 0 <br> 1 <br> 1 <br> 2 <br> 3 <br> 4 <br> 5 <br> 6 <br> 7 <br> 8 <br> 9 <br> 10 <br> 10 <br> 12 <br> 12 <br> 14 <br> 14 <br> 14 <br> 15 <br> 15 <br> 17 <br> 18 <br> 19 <br> 20 <br> 21 <br> 22 <br> 23 <br> 24 <br> 25 <br> 26 <br> 27 <br> 28 <br> 29 <br> 30 <br> 31 <br> 31 <br> 32 <br> 34 <br> 34 <br> 35 <br> 36 <br> 37 <br> 38 <br> 39 <br> 40 <br> 41 <br> 42 <br> 43 <br> 44 <br> 45 <br> 46 <br> 47 <br> 47 <br> 48 <br> 49 <br> 50 <br> 51 |  |  |  |  |

CALCULATED LINE POSITIONS AND EESIDUALS FOR THE $1-2$ 3AND OF A1O (B2E - X2

| $\boldsymbol{N}$ | $\mathrm{P}_{2}$ |  | $\mathrm{p}_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Calc | Rea | Calc | Res | Calc | Rea | Calc | Res |
| 0 |  |  |  |  | 19582.721 |  | 19582.700 |  |
| 1 |  | 19580.262 |  |  | 19583.854 |  | 19583.828 |  |
| 2 |  | 19578.949 |  |  | 19584.949 |  | 19584.897 |  |
| 3 |  | 19577.577 | -0.125* |  | 19585.974 |  | 19585.906 |  |
| 4 |  | 19576.146 | -0.066* |  | 19586.940 |  | 19586.856 |  |
| 5 |  | 19574.656 | -0.010 |  | 19587.847 |  | :9587.747 |  |
| 6 |  | 19573.107 | -0.004 |  | 19588.695 |  | 19588.579 |  |
| 7 |  | 19,771.500 | 0.031 |  | 19589.483 |  | 19589.352 |  |
| 8 |  | 19569.833 | 0.001 |  | $19590.212$ |  | 19590.065 |  |
| 9 |  | 19568.108 | 0.012 |  | $19590.892$ |  | 19590.719 |  |
| 10 |  | 19566.323 | -0.001 |  | 19591.492 |  | 19591.313 |  |
| 11 | 19564.565 | -0.104* | 19554.395 | -0.357* | 19592.043 |  | 19591.848 |  |
| 12 | 19562.671 | -0.013 | 19562.485 | -0.068* | 19592.534 |  | 19592.324 |  |
| 13 | 19560.718 | 0.008 | 19560.516 | 0.020 | 19592.966 |  | 19592.740 |  |
| 14 15 | 1.9558 .707 19556.636 | 0.004 0.006 | 19558.489 19556.403 | -0.006 -0.017 | 19593.339 |  | 19593.096 |  |
| 15 16 | 19556.636 19554.507 | 0.006 -0.003 | 19556.403 19554.258 | -0.017 | 19593.652 19593.805 |  | 19593.393 19593.631 |  |
| 17 | 19552.319 | -0.011 | 19552.054 | -0.002 | 19594.099 |  | 19593.809 |  |
| 18 | 19550.073 | -0.007 | 19549.792 | -0.019 | 19594.233 |  | 19593.927 |  |
| 19 | 19547.768 | 0.016 | 19547.471 | -0.001 | 19594.307 |  | 19593.985 |  |
| 20 | 19545.404 | 0.008 | 19545.091 | 0.007 | 19594.322 |  | 19593.984 |  |
| 21 | 19542.982 | 0.011 | 19542.653 | -0.010 | 19594.276 |  | 19593.923 |  |
| 22 | 19540.501 | -0.017 | 19540.156 | -0.036 | 19594. 171 |  | 19593.802 |  |
| 23 | 19537.961 | -0.014 | 19537.601 | -0.014 | 19594.007 |  | 19593.621 |  |
| 24 | 19535.363 | -0.017 | 19534.987 | -0.012 | 19593.782 |  | 19593.381 |  |
| 25 | 19532.705 | -0.011 | 19532.314 | -0.016 | 19593.497 |  | 19593.080 |  |
| 26 | 19529.991 | -0.004 | 19529.583 | 0.000 | 19593.152 |  | 19592.720 |  |
| 27 | 29527.217 | 0.023 | 19526.793 | 0.021 | 19592.748 |  | 19592.299 |  |
| 28 | 19524.2 r | -0.008 | 29523.945 | -0.015 | 19592.283 |  | 19591.819 |  |
| 29 | 19521.494 | 0.023 | 19521.038 | 0.010 | 19591.758 |  | . 19591.278 |  |
| 30 | 19518.545 | -0.026 | 19518.073 | -0.001 | 19591.173 |  | 19590.677 |  |
| 31 | 19515.537 | 0.015 | 19515.050 | 0.005 | 19590.528 |  | 19590.016 |  |
| 32 | 15312.471 | $0.134 *$ | 19511.968 | -0.015 | 19589.823 |  | 19589.295 |  |
| 33 | 19509.346 | 0.032* | 19508.827 | -0.010 | 19589.057 |  | 19588.513 |  |
| 34 | 19506.163 | -0.012 | 19505.628 | 0.039 | 19588.231 |  | 19587.671 |  |
| 35 | 19502.922 | -0.002 | 19502.371 | 0.006 | -3587.345 |  | 19586.769 |  |
|  |  | -0.016 | 19499.055 | 0.058* | 19586.398 |  | 19585.807 |  |
| 37 | 19496.263 | -0.006 | 19495.680 | 0.020 | 19585.391 |  | 19584.784 |  |
| 38 | 19492.846 | -0.015 | 19492.248 | 0.003 | 19584.323 | 0.003 | 19583.700 | 0.028 |
| 39 | 19489.371 | -0.004 | 19488.757 | 0.007 | 19583.195 | -0.018 | 19582.556 | 0.030 |
| 40 | 19485.838 | 0.010 | 19485.207 | -0.161* | 19582.006 | -0.213* | 19581.351 | -0.021 |
| 41 | 19482.246 | 0.001 | 19481.599 | -0.137* | 19580.757 | 0.001 | 19580.086 | 0.011 |
| 42 | 19478.595 | -0.004 | 19477.933 | 0.008 | 19579.446 | -0.013 | 19578.760 | 0.040 + |
| 43 | 19474.887 | -0.030 | 19474.209 | -0.001 | 19578.075 |  | 19577.373 |  |
| 44 | 19471.119 | -0.060* | 19470.426 | 0.035 | 19576.644 | 0.017 | 19575.925 |  |
| 45 | 19465.294 |  | 19466.584 | -0.029* | 19575.151 | 0.027 | 19574.417 |  |
| 46 | 19463.410 | -0.031 | 19462.685 | 0.014 | 19573.597 | 0.019 | 19572.847 |  |
| 47 | 19459.468 | -0.050* | 19458.727 | -6.014 | 19571.983 | -0.005 | 29571.217 |  |
| 48 | 19455,468 |  | 19454.710 |  | 19570.307 | 0.018 | 19569.526 |  |
| 49 | 19451.409 | -0.006 | 19450.636 | 0.013 | 19568.571 | 0.007 | 19567.773 |  |
| 50 | 19447.292 | -0.004 | 13446.503 | -0.012 | 19566.773 | -0.026 | 19565.960 |  |
| 51 | 19443.126 | 0.013 | 19442.311 | -0.017 | 19564.917 |  | 19564.085 |  |
| 52 | 19438.882 |  | 19438.051 |  | 19562.994 |  | 19562.149 |  |
| 53 | 19434.590 | 0.005 | 19433.753 |  | 19561.013 |  | 19560.152 |  |
| 54 | 19430.240 | 0.001 | 19429.387 | 0.003 | 19558.970 |  | 19558.093 |  |
| 55 | 19425.831 |  | 19424.962 | -0.039* | 19556.866 |  | 19555.973 |  |
| 56 | 19421.364 |  | 19420.479 |  | 19554.701 |  | 19553.792 |  |

CALCULATED LINE POSITYONS AND RESIDUALS FOR THE $1-3$ BAND OF A10 (B2 $-x^{2}$ L)

| N | $\mathrm{P}_{2}$ |  | $\mathrm{P}_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | cale | Res | Cale | Res | Calc | Res | calc | Rea |
| 0 |  |  |  |  | 18645.347 |  | 18645.325 |  |
| 1 |  |  |  |  | 18646.504 |  | 18646.463 |  |
| 2 |  | $18641.610$ |  |  | 28647.613 |  | 18647.554 |  |
| 3 |  | 18640.273 |  |  | 18648.675 |  | 18648.597 |  |
| 4 |  | 18638.889 |  |  | 18649.689 |  | 18649.593 |  |
| 5 |  | 18637.457 | -0.003 |  | 18650.656 |  | 18650.541 |  |
| 6 |  | 18635.979 | 0.011 |  | 18651.575 |  | 18651.441 |  |
| 7 |  | 18634.453 | -0.008 |  | 18652.446 |  | 18652.294 |  |
| 8 |  | 18632.880 | 0.004 |  | 18653.270 |  | 18653.100 |  |
| 9 |  | 18631.259 | 0.002 |  | 18654.047 |  | 18653.857 |  |
| 10 |  | 18629.592 | 0.048 |  | 18654.775 |  | 18654.567 |  |
| 11 |  | 18627.878 | -0.001 |  | 18655.456 |  | 18655,230 |  |
| 12 |  | $18626.116$ | -0.035 -0.030 |  | 18656.090 18656.675 |  | 18655.844 18656.411 |  |
| 13 |  | 18624,307 -0.035 | -0.030 18622.323 |  | 18656.675 18657.213 |  | 18656.411 18656.930 |  |
| 14 | 18620.587 | -0.035 | 18522.323 18620.411 | -0.004 0.029 | 18657.213 |  | 18656.930 18657.401 |  |
| 16 | 18618.747 | 0.002 | 18618.452 | 0.002 | 18658.145 |  | 18657.825 |  |
| 17 | 18616.760 | -0.004 | 18616.446 | -0.023 | 18658.539 |  | 18658.200 |  |
| 18 | 18614.725 | 0.031 | 18614.393 | 0.011 | 18658.885 |  | 18658.528 |  |
| 19 | 18612.644 | 0.042 | 18612.293 | 0.017 | 18659.183 |  | 18658.808 |  |
| 20 | 18610.516 | -0.008 | 18610.147 | -0.024 | 18659.434 |  | 18659.039 |  |
| 22 | 18605.120 | -0.014 | 18605.713 | -0.012 | 18659.791 |  | 18659.359 |  |
| 23 | 18503.852 | -0.033 | 18503.426 | -0.026 | 18659.897 |  | 18659.447 |  |
| 24 | 18601.536 | -0.016 | 18601.092 | 0.013 | 18659.955 |  | 18659.486 |  |
| 25 | 18599.174 | 0.007 | 18598.711 | 0.044 | 18659.965 |  | 18659.478 |  |
| 26 | 18596.766 | 0.023 | 18596.284 | -0.020 | 18659.927 |  | 18659.421 |  |
| 27 | 18597.311. | 0.057* | 18593.810 | -0.012 | 18659.841 |  | 18659.316 |  |
| 29 | 18591.809 | -0.008 | \$9591. 290 | 0.035 | 18659.707 |  | 18659.163 |  |
| 29 | 18589.260 | 0.056* | 18588.722 | -0.046 | 18659.524 |  | 28658.962 |  |
| 30 | 18586.665 | -0.033 | 18586.109 | 0.043 | 18659.293 |  | 18658.712 |  |
| 31 | 18584.023 | -0.038 | 18583.448 | 0.021 | 18659.014 |  | 18658.414 |  |
| 32 33 | 18581.335 | 0.020 -0.002 | 18580.741 | -0.031 | 18658.686 |  | 18658.068 |  |
| 33 34 | 18578.600 18575.818 | -0.002 -0.020 | 18577.988 18575.187 | 0.001 -0.010 | 18658.310 18657.896 |  | 18657.674 |  |
| 35 | 18572.990 | 0.003 | 18572.341 | 0.008 | 18657.414 |  | 18656.740 |  |
| 36 | 18570.116 | -0.024 | 18569.448 | 0.030 | 18656.892 |  | 18656.200 |  |
| 37 | 18567.1195 | -0.029 | 18566.509 | 0.023 | 18656.323 |  | 18655.612 |  |
| 38 | 18564.228 |  | 18563.523 |  | 18655.705 |  | 18654.975 |  |
| 39 | 18561.215 |  | 18560.491 |  | 18655.038 |  | 18654.290 |  |
| 40 | 18558.155 |  | 18557.412 |  | 18654.323 |  | 18653.556 |  |
| 41 | 18555.048 | 0.013 | 18554.287 | 0.036 | 18653. 559 |  | 18652.773 |  |
| 42 | 18551.886 |  | 18551.116 |  | 18652.747 |  | 18651.942 |  |
| 43 | 18548.697 |  | 18547.898 |  | 10651.886 |  | 18651.063 |  |
| 44 | 18545.452 |  | 18544.635 |  | 18650.976 |  | 18650.134 |  |
| 45 | 18542.160 |  | 18541.325 |  | 18650.017 |  | 18649.157 |  |
| 46 | 18538.823 |  | 18537.969 |  | 16649.010 |  | 18648.131 |  |
| 67 | 18535.433 |  | 18534.566 |  | 18647.954 |  | 18647.056 |  |
| 48 | $18532.009$ |  | 18531.118 |  | $18646.849$ |  | $18645,933$ |  |
| 49 | 18528.533 |  | 18527.623 |  | $18645.695$ |  | $18644.761$ |  |
| 50 51 | 18525.011 |  | 18524.032 18520.495 |  | 18644.493 18543.241 |  | 18643.539 18642.269 |  |
| 51 52 | 18521.443 18517.829 |  | 18520.495 18516.862 |  | 18543.241 |  | 18642.269 18640.950 |  |
| 53 | 18514.168 |  | 18513.184 |  | 18640.591 |  | 18639.58? |  |




CALCULATED LINE POSITIONS AND RESIDUALS FOR THE 2-3 BAND OF ALO (B2L- $\mathbf{X}^{2} E$ )

| N | $\mathrm{P}_{2}$ |  | $\mathrm{P}_{1}$ |  | $\mathrm{R}_{2}$ |  | $8_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Cale | Res | Calc | Res | Calc | Res | Calc | Res |
| 0 |  |  |  |  | 19501.123 |  | 19501.101 |  |
| 1 |  | 19498.684 |  |  | 19502.262 |  | 19502.222 |  |
| 2 |  | 19497.386 |  |  | 19503.345 |  | 19503.285 |  |
| 3 |  | 19496.031 |  |  | 19504.370 |  | 19504.292 |  |
| 4 |  | 19494.620 | -0.009 |  | $19505.340$ |  | $19505.243$ |  |
| 5 |  | $19493.153$ | 0.058* |  | $19506.253$ |  | 19506.137 |  |
| 6 |  | 19491.629 | 0.012 |  | 19507.110 |  | 19506.975 |  |
| 7 |  | 19490.050 | 0.004 |  | 19507.909 |  | 19507.756 |  |
| 8 9 |  | 19488.414 | -0.012 -0.025 |  | 19508.653 19509.340 |  | 15508.481 19509.149 |  |
| 10 |  | 19484.974 | -0.072* |  | 19509.970 |  | 19509.760 |  |
| 11 |  | 19483.170 | 0.023 |  | 19510.543 |  | 19510.315 |  |
| 12 |  | 19481.310 | 0.092. |  | 29511.060 |  | 19510.813 |  |
| 13 |  | 19479.393 | -0.009 |  | 19511.520 |  | 19511.254 |  |
| 14 | 19477.551 | -0.046* | 19477.291 | -0.017 | 19511.924 |  | 19511.639 |  |
| 15 | 19475.533 | -0.005 | 19475.254 | -0.051* | 19512.270 |  | 19511.967 |  |
| 16 | 19473.458 | -0.021 | 19473.160 | -0.023 | 19512.560 |  | 19512.238 |  |
| 17 | 19471.327 | -0.021 | 19471.011 | 0.048* | 19512.793 |  | 19512.452 |  |
| 18 | 19469.141 | 0.014 | 19468.806 | -0.049* | 19512.969 |  | 19512.609 |  |
| 19 | 19466.899 | 0.005 | 19466.545 | 0.014 | 19513.088 |  | 19512.710 |  |
| 20 | 19464.601 | 0.009 | 19464.228 | -0.003 | 19513.151 |  | 19512.753 |  |
| 21 | 19462.247 | -0.001 | 19461.855 | 0.017 | 19513.156 |  | 19512.740 |  |
| 22 | 19459.837 | 0.027 | 19459.427 | -0.009 | 19513.105 |  | $19512.670$ |  |
| 23 | 19457.372 | 0.004 -0.042 | 19456.943 | 0.008 | 19512.996 |  | 19512.543 |  |
| 24 25 | 19454.85 | -0.042 -0.005 | 19454.403 19451.807 | 0.014 | 19512.831 |  | 19512.358 |  |
| 26 | 19449.642 | 0.001 | 19449.156 | 0.013 | 19512.329 |  | 19511.819 |  |
| 27 | 13446.954 | -0.007 | 19446.450 | 0.041 | 19511.992 |  | 19511.463. |  |
| 28 | 19444.210 | 0.003 | 19443.687 | 0.002 | 19511.598 |  | 19511.050 |  |
| 29 | 19441.411 | -0.010 | 19440.869 | -0.010 | 19511.147 |  | 25510.581 |  |
| 30 | 19438.556 | 0.228* | 19437.996 | 0.128* | 19510.639 |  | 19510.054 |  |
| 31 | 19435.646 | $0.070 \%$ | 19435.067 | 0.062* | 19510.074 |  | 19509.470 |  |
| 32 | 19432.681 | 0.013 | 19432.083 | -0.056* | 19509.451 |  | 19508.828 |  |
| 33 | 19429.659 | 0.020 | 19429.043 | -0.011 | 19508.771 |  | 19508.130 |  |
| 34 | 19426.583 | -0.027 | 19425.948 | 0.014 | 13508.034 |  | 19507.374 |  |
| 35 | 19423.451 | 0.005 | 19422.797 | -0.028 | 19507.240 |  | 25506.561 |  |
| 36 | 19420.264 | 0.011 | 19419.591 | 0.030 | 19506.388 |  | 19505.690 |  |
| 37 | 19417.021 | -0.104* | 19416.330 | 0.001 | 19505.479 |  | 19504.762 |  |
| 38 | 19413.724 | -0.002 | 19413.013 | 0.013 | 19504.512 |  | 19503.777 |  |
| 39 | 19410.371 | 0.022 | 19409.641 | -0.001 | 19503.489 |  | 19502.734 |  |
| 43 | $19406.962$ | -0.146* | 19406.214 | -0.001 | 19502.407 | $-0.030$ | 19501.634 | $0.022$ |
| 41 | $19403.499$ | 0.017 | 19402.732 | 0.050* | 19501.268 | $-0.0514$ | 19500.477 | $0.031$ |
| 42 43 | 19399.980 19396.406 | -0.012 0.002 | 19399.194 19395.602 | 0.008 -0.019 | 19500.072 | 0.001 | 19499.262 19497.989 | -0.149* |
| 44 | 19392.777 | 0.013 | 19391.954 | -0.004 | 19497.507 | -0.099* | 19496.659 | 0.018 |
| 45 | 19389.093 | 0.028 | 19388.251 | 0.059* | 19496.138 | 0.119* | 19495.271 | -0.023 |
| 46 | 19385.354 | 0.029* | 19384.493 | -0.012 | 19494.711 |  | 19493.826 | -0.022 |
| 47 | 13381.560 | -0.033 | 19380.680 | -0.076* | 19493. 227 | -0.016 | 19492.323 | -0.072* |
| 48 | 19377.711 | -0.010 | 19376.312 | 0.031 | 19491.685 | -0.044* | 19490.762 | -0.017 |
| 49 | 19373.807 |  | 19372.889 |  | 19490.086 | -0.032* | 19489.144 |  |
| 50 | 19369.847 | -0.034 | 19368.911 | -0.008 | 19488. 429 | -0.027* | 19487.468 |  |
| 51 | 19365.833 | -0.018 | 19364.879 | -0.073* | 19486.714 | -0.017* | 19485,734 |  |
| 52 | 19361.765 | 0.061** | 19360.791 | -0.012 | 19484.941 | 0.105* | 19483.943 |  |
| 53 | 19357.641 | -0.027 | 19356.648 |  | 19483.111 |  | 19482.093 |  |
| 54 55 | 19353.452 19349.229 |  | 19352.451 19348.199 |  | 19481.222 |  | 19480.186 |  |
| 55 | 19349.229 |  | 19348.199 |  | 19479.275 |  | 19478.222 |  |

CALCULATED LINE POSITLONS AND RESIDUALS FOR THE 2-4 EAND OF AIO (B2E- $x^{2}$ E)

| N | $\mathrm{P}_{2}$ |  | $\mathrm{P}_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Calc | Res | Calc | Res | Calc | Res | Cale | Res |
| 0 |  |  |  |  | 18577.801 |  | 18577.777 |  |
| 1 |  | 18575.373 |  |  | 18578.954 |  | 18578.907 |  |
| 2 |  | 18574.098 |  |  | 18580.062 |  | 18579.992 |  |
| 3 |  | 18572.779 |  |  | 18581.126 |  | 18581.032 |  |
| 4 |  | 18571.415 |  |  | 18582.144 |  | 18582.028 |  |
| 5 |  | 18570.006 | 0.086* |  | 18583.118 |  | 18582.979 |  |
| 6 |  | 18568.553 | 0.027 |  | 18584.048 |  | $18583.885$ |  |
| 7 |  | 18567.056 | 0.110 |  | 18584.932 |  | 18584.746 |  |
| 8 |  | 18565.514 | 0.005 |  | 18585.772 |  | 18585.563 |  |
| 9 |  | 18563.928 | 0.107* |  | 18585.567 |  | 18586.335 |  |
| 10 |  | 18562.298 | 0.019 |  | 18587.317 |  | 18587.062 |  |
| 11. |  | 18560.624 | -0.040 |  | 18588.62 .2 |  | 18587.744 18588.381 |  |
| 12 |  | 18558.905 | 0.016 0.077 |  | 18588.683 18589.298 |  | 18588.381 18588.973 |  |
| 14 | 18555.496 | 0.013 | 18555.173 | -0.112* | 18589.868 |  | 18589.520 |  |
| 15 | 18553.656 | 0.028 | 18553.310 | -0.004 | 18590.394 |  | 18590.023 |  |
| 16 | 18551.772 | 0.068 | 10551.403 | 0.016 | 18590.875 |  | 18590.480 |  |
| 17 | 18549.844 | 0.024 | 18549.452 | -0.044 | 18591.310 |  | 18590.893 |  |
| 18 | 18547.872 | -0.005 | 18547.457 | -0.011 | 18591.701 |  | 18591.260 |  |
| 19 | 18545.855 | -0.003 | 18545.417 | -0.051 | 18592.046 |  | 18591.583 |  |
| 21 | 18543.796 | -0.033 | 18543.334 | -0.054 | 18592.347 |  | 18.31.860 |  |
| 22 | 18541.692 | -0.105* | 18541.207 | 0.003 -0.036 | 18592.602 |  | 18592.092 18592.279 |  |
| 23 | 18537.353 | 0.008 | 18536.822 | 0.018 | 18592.978 |  | 18592.421 |  |
| 24 | 18535.118 | 0.032 | 18534.563 | -0.044 | 18593.098 |  | 18592.518 |  |
| 25 | 18532.838 |  | 18532.261 |  | 18593.173 |  | 18592.570 |  |
| 26 | 18530.515 | -0.030 | 18529.915 | 0.038 | 18593.202 |  | 18592.577 |  |
| 27 | 18528.149 | -0.028 | 18527.525 | 0.009 | 18593.187 |  |  |  |
| 28 | 18525.738 | -0.023* | 18525.091 | -0.052 | 18593.127 |  | 18592.455 |  |
| 29 | 18523.285 | -0.048* | 18522.614 | 0.009 | 18593.021 |  | 18592.326 |  |
| 30 | 18520.787 | 0.024 | 18520.094 | 0.058 | 18592.870 |  | 18592.152 |  |
| 31 | $18518.246$ | 0.038 | 18517.529 | 0.019 | 18592.674 |  | 18591.932 |  |
| 32 | 18515.662 | -0.024 | 18514.922 | -0.046 | 18592.432 |  | 18591.668 |  |
| 33 | 18513.034 | 0.006 | 18512.271 | -0.042 | 18592.145 |  | 18591.358 |  |
| 34 | 18510.362 | 0.002 | 18509.576 | -0.056 | 18591.813 |  | 18591.003 |  |
| 35 | 18507.648 | -0.014 | 18506;838 | -056 | 18591.436 |  | 18590.602 |  |
| 36 | 18504.890 | 0.019 | 18504.057 | 0.042 | 18591.014 |  | 18590.157 |  |
| 37 | 18502.088 |  | 18501.23 .3 | -0.027 | 18590.546 |  | 18589.666 |  |
| 38 | 18499.244 |  | 18498.365 | -0.018 | 18590.033 |  | 18539.129 |  |
| 39 | 18436.356 |  | 18495.454 |  | 18589.474 |  | 18588.548 |  |
| 40 | 18493.425 |  | 12492.501 |  | 18588.870 |  | 18587.921 |  |
| 41 | 18490.452 |  | 18489.504 |  | 18588.221 |  | 18587.248 |  |
| 42 | 18487.435 |  | 18486.463 |  | 18587.527 |  | 18586.531 |  |
| 43 | 18484.375 |  | 18483.380 |  | 18586.787 |  | 18585.768 |  |
| 44 | 18481.272 |  | 18430.254 |  | 18586.002 |  | 18584.959 |  |
| 45 | 18478.126 |  | 18477.086 |  | 18585.171 |  | 18584.106 |  |
| 45 | 18474.938 |  | 18473.374 |  | 18584.295 |  | 18583.207 |  |

CAICULATED LINE POSITIONS AMD RESIOUALS FOR THE $3-1$ BAND OF A 10 (B2 $2-x 2 E$ )

| N | $\mathrm{P}_{2}$ |  | $P_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Calc | Res | calc | Res | Calc | Res | Cale | Res |
| 0 |  |  |  |  | 22238.261 |  | 22238.241 |  |
| 1 |  | 22235.808 |  |  | 22239.356 |  | 22239.322 |  |
| 2 |  | 22234.454 |  |  | 22240.363 |  | 22240.315 |  |
| 3 |  | 22233.012 |  |  | 22241-281 |  | 22241.218 |  |
| 4 |  | 22231.481 | 0.068* |  | 22242-111 |  | 22242.034 |  |
| 5 |  | 22229.862 | 0.022 |  | 22242.852 |  | 22242.760 |  |
| 6 |  | 22228.154 | 0.010 |  | 22243.504 |  | 22243.398 |  |
| 7 |  | 22226.358 | -0.023 |  | 22244.068 |  | 22243.947 |  |
| 8 |  | 22224.473 | -0.016 |  | 22244.542 |  | 22244.407 |  |
| 9 |  | 22222.500 | -0.014 |  | 22244.928 |  | 22214.778 |  |
| 10 |  | 22220.439 | -0.007 |  | 22245.225 |  | 22245.061 |  |
| 11 |  | 22218.289 | -0.022 |  | 22245.433 |  | 22245.254 |  |
| 12 |  | 22216.051 | -0.002 |  | 22245.552 |  | 22245.359 |  |
| 13 |  | 22213.725 | -0.011 |  | 22245.582 |  | 22245.375 |  |
| 14 |  | 22211.310 | 0.025 |  | 22245.523 |  | 22245.301 |  |
| 15 |  | 22208.808 | 0.032 |  | 22245.375 |  | 22245.139 |  |
| 16 |  | 22206.217 | -0.005 |  | 22245.138 |  | 22244.888 |  |
| 17 |  | 22203.537 | 0.032 |  | 22244.812 |  | 22244.547 |  |
| 18 |  | 22200.770 | 0.022 |  | 22244.397 |  | 22244.117 |  |
| 19 |  | 22197.915 | -0.006 |  | 22243.892 |  | 22243.598 |  |
| 20 |  | 22194,971 | 0.010 |  | 22243.298 |  | 22242.990 |  |
| 21 |  | $<2191.939$ | -0.011 |  | 22242.615 |  | 22242.292 |  |
| 22 |  | 22188.819 | -0.017 |  | 22241.843 |  | 22241.506 |  |
| 23 |  | 22185.611 | 0.002 |  | 22240.981 |  | 22240.629 |  |
| 24 |  | 22182.315 | -0.012 |  | 22240.030 | -0.040 | 22239.664 | -0.030 |
| 25 |  | 22178.930 | -0,000 |  | 22238.989 | -0.046 | 22238.609 | -0.021 |
| 26 27 | 22175.643 22172.089 | $-0.089 \%$ 0.020 | 22175.273 | 0.012 | 22237.859 | 0.028 | 22237.464 | -0.007 |
| 28 | 22168.0848 | 0.020 | 22168.049 | 0.012 -0.001 | 22236.639 | -0.001 0.056 | 22236.230 | 0.003 -0.049 |
| 29 | 22164.718 | -0.003 | 22164.305 | -0.020 | 22233.930 | $0.093 *$ | 22233.492 | 0.008 |
| 30 | 22160.901 | -0.011 | 22160.473 | 0.007 | 22232.442 | -0.082* | 22231. 989 | -0.087\% |
| 31 | 22156.995 | 0.004 | 22156.553 | -0.011 | 22230.863 | 0.018 | 22230.396 | -0.013 |
|  | 22153.001 | -0.031 | 22152.545 | -0.012 | 22229.195 | -0.006 | 22228.714 | 0.022 |
| 33 | 22148.920 | -0.011 | 22140.449 | -0.063 | 22227.437 | -0.023 | 22226.941 | -0.032 |
| 34 | 22144.750 | -0.015 | 22144.265 | 0.001 | 22225.589 | 0.016 | 22225.078 | 0.002 |
| 35 | 22140.492 | 0.022 | 22139.993 | -0.020 | 22223.650 | -0.001 | 22223.126 | -0.058 |
| 36 | 22136.146 | -0.001 | 22135.632 |  | 22221.622 | 0.071* | 22221.083 | 0.009 |
| 37 | 22131.712 | 0.012* | 22131.184 | -0.181* | 22219.504 | 0.002 | 22218.951 | -0.006 |
| 38 | 22127.190 | 0.028* | 22126.648 | 0.042* | 22217.296 | -0.031 | 22216.728 | -0.044 |
| 39 | 22122.580 |  | 22122.023 |  | 22214.997 | 0.009 | 22214.415 | 0.030 |
| 40 | 22117.882 |  | 22117.311 | 0.027 | 22212.609 | 0.029 | 22212.012 | 0.036 |
| 41 | 22113.096 |  | 22112.510 |  | 22210.130 | 0.004 | 22209.519 | 0.020 |
| 42 | 22108.222 | -0.016 | 22107.622 | -0.025 | 22207.560 | 0.011 | 22206.935 | -0.032 |
|  | 22103.260 | -0.047 | 22102.645 | -0.005 | 22204.901 | 0.025 | 22204:261 | -0.013 |
| 44 | 22098.210 | 0.011 | 22097.580 | -0.039 | 22202.150 | -0.026 | 22201.496 | -0.030 |
| 45 | 22093.071 | 0.008 | 22092.428 | 0.025 | 22199.310 | -0.001 | 22198.641 | -0.010 |
| 46 | 22087.845 |  | 22087.187 |  | 22196.378 | 0.037 | 22195.695 | 0.087* |
| 47 | 22082.530 |  | 22081.858 |  | 22193.356 | -0.000 | 22192.659 | 0.034 |
| 48 | $22077.128$ | 0.019 | 22076.441 | 0.036 | 22190.244 | 0.103* | $22189.532$ | 0.069* |
| 49 | 22071.637 | 0.060 | 22070.936 | -0.023 | 22187.040 | -0.010 | 22186.314 | 0.044 |
| 50 | 22066.059 |  | 22065.343 |  | 22183.746 |  | 22183.005 |  |
| 51 | 22060.392 | -0.020 | 22059.562 | -0.043 | 22180.361 | -0.029 | 22179.505 | 0.055 |
| 52 | 22054.637 | 0.029 | 22053.892 | 0.006 | 22176.884 | 0.037 | 22176.115 | -0.002 |
| 53 | 22048.794 |  | 22048.035 |  | 22173.317 | -0.055* | 22172.533 | -0.001 |
| 54 | 22042.863 | -0.042 | 22042.089 | 0.047 | 22169.659 | -0.020 | 22168.860 | -0.068* |
| 55 | 22036.843 |  | 22036.056 |  | 22165.909 |  | 22165.096 |  |
| 56 | 22030.736 |  | 22029.934 |  | 22162.069 |  | 22161.241 |  |

CALCULATED LINE POSITIONS AND RESIDUALS FOR THE $3-4$ BAND OF ALO (B2E- $X^{2}$ L)

| $N$ | $\mathrm{P}_{2}$ |  | $\mathrm{P}_{1}$ |  | $\mathrm{R}_{2}$ |  | $R_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Calc | Res | Calc | Rea | Calc | Res | calc | Res |
| 0 |  |  |  |  | 19426. 250 |  | 19426.225 |  |
| 1 | 19423.841 |  | 19423.819 |  | 19427.385 |  | 19427.337 |  |
| 2 | 19422.569 |  | 19422.523 |  | 19428.466 |  | 19428.395 |  |
| 3 | $19421.243$ |  | 19421.175 |  | 19429.494 |  | 19429.400 |  |
| 4 | 19419.864 |  | 19419.772 |  | 19430.468 |  | 19430.351 |  |
| 5 |  | 19418.374 | 0.036 |  | 19431.389 |  | 19431.248 |  |
| 6 |  | 19416.877 | 0.040 |  | 19432. 256 |  | 19432.092 |  |
| 7 |  | 19415.326 | 0.031 |  | 19433.069 |  | 19432.882 |  |
| 8 |  | 19413.722 | -0.000 |  | 19433.829 |  | 19433.619 |  |
| 9 |  | 19412.065 | 0.033 |  | 19434.535 |  | 19434.301 |  |
| 10 |  | 19410.355 | 0.038 |  | 19435.187 |  | 19434.930 |  |
| 11 |  | 19408.591 | 0.011 |  | 19435.786 |  | 19435.506 |  |
| 12 |  | 19406.774 | 0.042 |  | 19436.331 |  | 19436.027 |  |
| 13 |  | 19404.905 | -0.028 |  | 19436.822 |  | 19436.495 |  |
| 14 |  | 19402.982 | 0.003 |  | 19437.259 |  | 19436.909 |  |
| 15 | 19401.180 | -0.026 | 19400.832 | 0.005 | 19437.642 |  | 19437.269 |  |
| 16 | 19399.163 | 0.039 | 19398.791 | -0.030 | 19437.971 |  | 19437.575 |  |
| 17 | 19397.092 | -0.001 | 19396.697 | -0.011 | 19438.247 |  | 19437.827 |  |
| 18 | 19394.969 | -0.016 | 19394.551 | 0.007 | 19438.469 |  | 19438.025 |  |
| 19 | 19392.793 | -0.003 | 19392.352 | -0.021 | 19438.636 |  | 19438.170 |  |
| 20 | 19390.564 | -0.003 | 19390.099 | -0.029 | 19438.750 |  | 19438.260 |  |
| 21 | 19388.283 | 0.027 | 19387.794 | -0.020 | 19438.810 |  | 19438.297 |  |
| 22 | 19385.948 | 0.014 | 19385.437 | -0.054 | 19438.816 |  | 19438.279 |  |
| 23 | 19383.561 | -0.039 | 19383.026 | -0.037 | 19438.768 |  | 19438.208 |  |
| 24 | 19381.121 | -0.042 | 19380.563 | 0.041 | 19438.666 |  | 19438.083 |  |
| 25 | 19378.628 | -0.001 | 19378.047 | -0.012 | 19438.510 |  | 19437.903 |  |
| 26 | 19376.083 | -0.003 | 19375.479 | 0.005 | 19438.299 |  | 19437.670 |  |
| 27 | 19373.486 | -0.016 | 19372.858 | -0.011 | 19438.~35 |  | 19437.382 |  |
|  | 19370.835 |  | 19370.184 | -0.016* | 19437.:17 |  | 19437.041 |  |
| 29 | 19368.133 |  | 19367.458 |  | 19437.345 |  | 19436.645 |  |
| 30 | 19365.378 | -0.030 | 19364.680 |  | 19436.919 |  | 19436.196 |  |
| 31 | 19362.570 | -0.001 | 19361.849 | -0.023 | 19436.438 |  | 19435.692 |  |
| 32 | 19359.710 | 0.036 | 19358.966 | 0.059* | 19435.904 |  | 19435.135 |  |
| 33 | 19356.798 | -0.008 | 19356.031 | 0.035 | 19435.315 |  | 19434.523 |  |
| 34 | 19353.834 | 0.027 | 19353.043 | -0.009 | 19434.673 |  | 19433.857 |  |
| 35 36 | 19350.817 | 0.006 | 19350.003 | -0.003 | 19433.976 |  | 19433.137 |  |
| 36 37 | 19347.749 19344.628 | 0.010 0.004 | 19346.911 19343.757 | -0.021 0.033 | 19433.225 19432.420 |  | 19432.363 19431.534 |  |
| 37 38 | 19344.628 19341.455 | 0.004 -0.029 | 19343.757 19340.571 | 0.033 -0.043 | 19432.420 19431.561 |  | 19431.534 19430.652 |  |
| 39 | 19338.230 | 0.028 | 19337.323 | 0.057* | 19430.648 |  | 19429.715 |  |
| 40 | 19334.954 | $0.073 *$ | 19334.023 | -0.024 | 19429.680 |  | 19428.725 |  |
| 41 | 19331.625 | 0.033 | 19330.671 | -0.020 | 19428.659 |  | 19427.680 |  |
| 42 | 19328.244 | 0.001 | 19327.267 |  | 19427.583 |  | 19426.581 |  |
| 43 | 19324.812 | 0.074* | 19323.812 |  | 19426.453 |  | 19425.428 |  |
| 44 | 19321:328 |  | 19320.305 19316.746 |  | 19425.269 |  | 19424.220 19422.959 |  |
| 45 | 19317.792 |  | 19316.746 |  | 19424.031 |  | 19422.959 |  |


|  | $\mathrm{P}_{2}$ |  | $\mathrm{P}_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| H | Calc | Res | Calc | Res | Calc | Res | Calc | Res |
| 0 |  |  |  |  | 18517.026 |  | 18516.999 |  |
| 1 | 18514.633 |  | 18514.601 |  | 18518.176 |  | 18518.119 |  |
| 2 | 18513.388 |  | 18513.326 |  | 18519.285 |  | 18519.198 |  |
| 3 | 18512.101 |  | 18512.009 |  | 18520.351 |  | 18520.235 |  |
| 4 | 28510.773 |  | 18510.651 |  | 18521.376 |  | 18521.230 |  |
| 5 | 18509.403 |  | 18509.251 |  | 18522.360 |  | 18522.183 |  |
| 6 | 18507.991 |  | 18507.810 |  | 18523.301 |  | 18523.095 |  |
| 7 8 | 18506.538 18505.044 |  | 18506.328 18504.803 |  | 18524.201 |  | 18523.965 |  |
| 8 9 | 18505.044 18503.508 |  | 18504.803 |  | 18525.058 |  | 18524.793 |  |
| 10 | 18501.931 |  | 18501.631 |  | 18526.648 |  | 18526.323 |  |
| 11 | 18500.313 |  | 18499.983 |  | 18527.380 |  | 18527.025 |  |
| 12 | 18498.654 |  | 18498.294 |  | 18528.071 |  | 18527.686 |  |
| 13 | 18496.953 |  | 18496.563 |  | 18528.719 |  | 18528.304 |  |
| 14 | 18495.211 |  | 18494.791 |  | 18529.325 |  | 18528.881 |  |
| 15 | 18493.428 | -0.019 | 18492.976 | 0.036 | 18529.890 |  | 18529.415 |  |
| 16 | 18491.603 | 0.112* | 18491. 124 | 0.011 | 18530.412 |  | 18529.908 |  |
| 17 | 18489.738 | 0.061 | 18489.229 | -0.019 | 18530.893 |  | 18530.359 |  |
| 18 | 18487.832 | 0.016 | 18487.293 | 0.058 | 18531.331 |  | 18530.767 |  |
| 19 | 18485.885 | 0.067 | 18485.316 | 0.012 | 18531.728 |  | 18531.134 |  |
| 20 | 18483.896 | 0.058 | 18483.298 | 0.044 | 18532.082 |  | 18531.459 |  |
| 21 | 18481.867 | 0.002 | 18481.239 | 0.033 | 18532.395 |  | 18531.742 |  |
| 22 | 18479.797 | 0.036 | 18479.140 |  | 18532.665 |  | 18531.982 |  |
| 23 | 18477.587 | 0.084 | 18476.999 | 0.036 | 18532.894 |  | 18532.181 |  |
| 24 |  | 0.041 | 18474.818 | 0.059 | 18533.080 |  | 18532.338 |  |
| 25 | 18473.343 | 0.051 | 18472.596 | 0.045 | 18533.224 |  | 18532.452 |  |
| 26 | $18471.111$ |  | 18470.334 |  | 18533.327 |  | 18532.525 |  |
| 27 | 18468.837 18466.524 | 0.058 0.077 | 18468.031 18465.687 |  | 18533.387 |  | 18532.555 |  |
| 28 | 18466.524 | 0.077 | 18465.687 | 0.075 0.037 | 18533.405 18533.382 |  | 18532.544 18532.490 |  |
| 30 | 18461.775 | 0.013 | 18460.879 | 0.161* | 18533.316 |  | 18532.395 |  |
| 31 | 18459.340 |  | 18458.414 |  | 18533.208 |  | 18532.257 |  |
| 32 | 18456.865 |  | 18455.909 |  | 18533.058 |  | 18532.077 |  |
| 33 | 18454.350 |  | 18453.364 |  | 18532.866 |  | 18531.856 |  |
| 34 | 18451.794 |  | 18450.779 |  | 18532.633 |  | 18531.592 |  |
| 35 | 18449.198 |  | 18448.153 |  | 18532.357 |  | 18531.287 |  |
| 36 | 18446.563 |  | 18445.488 |  | 18532.039 |  | 18530.939 |  |
| 37 | 18443.887 |  | 18442.782 |  | 18531.679 |  | 18530.549 |  |
| 38 | 18441.172 |  | 18440.037 |  | 18531.277 |  | 18530.118 |  |
| 39 | $18438 \cdot 416$ |  | $18437.252$ |  | $18530.833$ |  | 18529.644 |  |
| 40 | $\begin{aligned} & 18435.621 \\ & 18432.786 \end{aligned}$ |  | $18434.427$ |  | $18530.348$ |  | 18529.129 |  |
| 42 | 18432.786 18429.912 |  | 18431.563 18428.658 |  | 18529.820 18529.250 |  | 18528.571 18527.972 |  |


| N | $\mathrm{P}_{2}$ |  | $P_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Cale | Res | Cale | Res | calc | Res | cale | Res |
| 0 |  |  |  |  | 23079.242 |  | 23079.322 |  |
| 1 |  | 23076.799 23075.436 | 0.006* |  | 23080.320 |  | 233080.285 |  |
| 2 3 |  | 23073.476 | 0.1121* |  | 23081.301 |  | 23001.252 |  |
| 4 |  | 2302.418 | 0.071** |  | 23082.969 |  | 233082.891 |  |
| 6 |  | 23059.012 | $0.054 *$ |  | 23084.248 |  | 23084.140 |  |
| 7 |  | 23067.162 | -0.003 |  | 23084.740 |  | 23084.619 |  |
| 8 |  | 23055.216 | 0.003 |  | 23085.136 |  | 23004.999 |  |
| 10 |  | 23061.031 | -0.009 |  | 2085.633 |  | 23085. 467 |  |
| 11 |  | 23058.794 | 0.007 |  | 23085.735 |  | 23085.555 |  |
| 12 |  | 23055.458 | -0.053 |  | 23085.740 |  | 23085.545 |  |
| 13 |  | 23054.026 | -0.011 |  | 23085.646 |  | 23085.437 |  |
| 15 |  | 23048.871 | -0.060 |  | 23065.166 |  | 23084.927 |  |
| 16 |  | 23046.147 | -0.035 |  | 23084.779 |  | 23084.526 |  |
| 17 |  | 23043.327 | -0.060 |  | 23084.294 |  | 23084.026 |  |
| 18 |  | 23040.410 | -0.003 |  | 23083.711 |  | 23083.429 |  |
| $\frac{19}{20}$ |  | 23037.395 | -0.093* |  | 23083.030 |  | 23082.733 |  |
| 21 |  | 23031.075 | -0.030 |  | 23081.374 |  | 23081:048 |  |
| 22 | 23027.928 | 0.057 | 23027.613 | -0.003 | 23080.395 |  | 23080.058 |  |
| 23 | 23024.533 | -0.088* | 23024.203 | -0.003 | 23079.325 |  | 23078.970 |  |
| 24 | 23021.041 | $\begin{aligned} & -01004 \\ & 23017 \end{aligned}$ | $\begin{aligned} & 23020.697 \\ & 0.0558 \end{aligned}$ | 0.008 | $\begin{aligned} & 23078.153 \\ & 23076.884 \end{aligned}$ |  | $\begin{aligned} & 23077.784 \\ & 23076: 499 \end{aligned}$ |  |
| 26 |  | 23013.580 | 0.102* |  | 23075.515 | 0.037 | 23075.117 | 0.038 |
| 27 | 23009.984 | -0.010 | 23009.596 | -0.056 | 23074.049 | 0.048 | 23073.636 | 0.018 |
| 28 | 23006.128 | -0.042 | 23005.711 | -0.025 | 23072.484 23070.820 | 0.005 | 23072.056 |  |
| 30 | 22998.055 | -0.018 | 22997.624 | -0.053 | 23069.058 | 0.008 | 23068.602 |  |
| 31 | 22993.885 | 0.083* | 22993.439 | -0.001 | 23067.198 | -0.039** | 23066.727 |  |
| 32 33 |  | 0.137* | 22988.158 | $=0.042$ -0.014 | 23065.239 | -0.020** | 23064.753 23062.681 |  |
| 34 | 229850. 794 | 0.058 | 229880. 304 | -0.014 | 23061.025 | -0.015* | 23060.510 |  |
| 35 | 22976.237 |  | 22975.732 |  | 23058.770 | 0.031* | 23058.240 |  |
| 36 | 22971.583 |  | 22971.054 |  | 23055.416 | -0.012* | 23055.872 |  |
| 37 | 22966.832 |  | 22966.298 |  | 23053.963 | $0.052 *$ | 23053.405 |  |
| 39 | 22957.040 |  | 22956.477 |  | 23048.762 | 0.049* | 23048.174 |  |
| 40 | 22951.998 |  | 22951.421 |  | 23045.012 |  | 23045.410 |  |
| 41 | 22946.860 |  | 22946.268 |  | 23043.164 |  | 23042.547 |  |
| 42 | 22941.625 |  | 22941.019 |  | 23040.216 |  | 23039.585 |  |
| 44 | 22930.865 |  | 22930.230 |  | 23034.024 |  | 23033.363 |  |
| 45 | 22925.340 |  | 22524.690 |  | 23030.779 |  | 23030.103 |  |
| 46 | 22919.718 |  | 22919.053 |  | 23027.434 |  | 23026.744 |  |
| 48 | 22908.184 |  | 22907.490 |  | 23020.447 |  | -23023.286 |  |
| 49 | 22902.271 |  | 22901.563 |  | 23016.805 |  | 23016.071 |  |
| 50 | 22896.262 |  | 22895.539 |  | 23013.062 |  | 23012.314 |  |
| 52 | 22883.954 |  | 22883.202 |  | 23005.279 |  | 23004.502 |  |

CALCULATED LIME POSITIONS AND RESIDUALS FOR THE 4-2 BAND OE A10 (B2E- X2E)

| $\cdots$ | $\mathrm{P}_{2}$ |  | $\mathrm{P}_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Calc | Rea | Cale | Rea | Cale | Res | Calc | Res |
| 1 |  |  |  |  | 22127.940 |  | 23127.920 |  |
| 1 |  | 22125.508 | -0.033* |  | 22129.031 |  | 22128.994 |  |
| $\frac{2}{3}$ |  | 2212.184 | -0.105* |  | 22130.954 |  | 22130.884 |  |
| 4 5 |  | 22121.232 | -0.007* |  | 22131.787 22132.534 |  | 22131.701 |  |
| 6 |  | 22117.953 | -0.038* |  | 22133.195 |  | 22133.076 |  |
| 7 |  | 22116.185 | -0.038* |  | 22133.770 |  | 22133.635 |  |
| 8 |  | 22114.332 | $=0.038 *$ -0.010 |  | 22134.259 |  | 22134.108 |  |
| 10 |  | 22110.368 | -0.031 |  | 22134.979 |  | 22134.795 |  |
| 11 |  | 22108.258 | -0.052 |  | 22135. 210 |  | 22135.010 |  |
| 13 |  | 22103.781 | -0.012 |  | 22135.413 |  | 22135.180 |  |
| 14 |  | 22101.415 | -0.069* |  | 22135.385 |  | 22135.136 |  |
| $\stackrel{15}{16}$ |  | 22098.963 | $=0.048$ $=0.064 *$ |  | 22135.272 |  | 22135.006 |  |
| 17 |  | 22093.803 | -0.026 |  | 22134.785 |  | 22134.487 |  |
| 18 |  | 22091.095 | -0.053 |  | 22134.413 |  | $2 \times 134.098$ |  |
| $\frac{19}{20}$ |  | 22085.424 | -0.033 |  | 22133.954 |  | 22133.062 |  |
| 21 |  | 22082.460 | -0.009 |  | 22132.777 |  | 22132.414 |  |
| 22 | 22079.588 | -0.009 | 22079.234 | -0.037 | 22132.059 |  | 22131.679 |  |
| 24 | 22073.250 | -0.015 | 22072.864 | -0.069 | 22130. 363 | -0.062* | 22129.858 |  |
| 25 | 22069.954 | 0.055 | 22069.551 | -0.011 | 22129.385 | -0.043 | 22128.957 | -0.005 |
| 25 | 22066.572 | -0.001 | 22066.153 | -0.058 | 22128.321 | -0.017 | 22127.877 | -0.018 |
| 28 | 22059.554 | 0.065 | 22059.102 | -0.020 | 22125:933 | 0.019 | 22125.456 | -0.020 |
| 29 | 22055.917 | 0.028 | 22055.449 | -0.035 | 22124.609 | 0.033 | 22124.116 | 0.006 |
| 30 | 22052.195 | 0.009 | 22051.711 | O.001 | 22123.199 | 0.071* | 22122.689 | -0.052* |
| 32 | 22044.496 | 0.007 | 22043.979 | 0.027 | 22120.117 | 0.043 | 22119.575 | 0.054 |
| 33 | 22040.519 | 0.023 | 22039.986 | 0.030 | 22118.446 | -0.013 | 22117.888 | c. 027 |
| 34 | 220365.457 | 0.010 | 22035.908 | 0.005 | 22116.688 | 0.028 | 22116.114 | 0.033 |
| 35 35 | 22032.311 | 0.042* | 22031.745 | -0.030* | 22114.844 | 0.043 | 22114.253 | 0.041 |
| 37 | 22023.762 | $0.103 *$ | 22023.164 | 0.081* | 22110.894 | 0.016 | 22110.270 | 0.067* |
| 38 3 | 22019.361 | -0.009 | 22018.746 | 0.013 | 22108.789 | -0.020 | 22108.149 | 0.057* |
| 39 40 | 22014.874 22010 |  | 22014.243 |  | 22106.596 | -0.010 | 22105.940 | 0.068* |
| 41 | 22005.647 | -0.016 | 22004:984 | 0.014 | 22101.951 | -0.037 | 22101.262 | 0.084* |
| 42 | 22000.906 |  | 22000.227 |  | 22099.497 | 0.005 | 22098.792 | 0.123* |
| $\begin{aligned} & 43 \\ & 44 \end{aligned}$ | 21996.081 21991.170 |  | $\begin{aligned} & 21955.385 \\ & 21990.458 \end{aligned}$ |  | $\begin{array}{r} 22096.957 \\ 22094.329 \end{array}$ |  | $\begin{aligned} & 22096.235 \\ & 22093.591 \end{aligned}$ |  |
| 45 | 21986.175 | -0.045 | 21985.447 | -0.013 | 22091.614 |  | 22090.860 |  |
| 4 | 21881.096 | -0.025 | 21980.351 | 0.037 | 22088.812 |  | 22088.041 |  |
| 48 | 21975.931 21970.682 | -0.041 | 21975.170 21959.904 | -0.026 | $\begin{aligned} & 22085.922 \\ & 22082.946 \end{aligned}$ |  | $\begin{aligned} & 22085.136 \\ & 22082.143 \end{aligned}$ |  |
| 49 | 21965.348 | -0.022 | 21964.554 |  | 22079.881 |  | 22079.062 |  |
| 50 | 21959.930 |  | 21959.120 |  | 22076.730 |  | 22075.894 |  |
| 51 52 | 21954.427 | -0.012 | 21953.600 | 0.002 | 22073.491 22070.165 |  | 220772.639 22069.297 |  |
| 53 | 21943.167 |  | 21942.308 |  | 22065.751 |  | 22065.865 |  |
| 54 | 21937.410 |  | 21936.534 |  | 22063.249 |  | 22062.349 |  |

CALCULATED LINE POSITIONS AND RESIDUALS FOR THE 4-5 BAND OF A10 (B2E- $\mathbf{x}^{2} 5$ )

|  | $\mathrm{P}_{2}$ |  | $P_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $N$ | Calc | Res | Calc | Res | Cale | Res | Calc | Res |
| 0 |  |  |  |  | 19358.246 |  | 19358.218 |  |
| 1 | 19355.862 |  | 19355.829 |  | 19359.379 |  | 19359.321 |  |
| 2 | 19354.608 |  | 19354.546 |  | 19360.461 |  | 19360.373 |  |
| 3 | 19353.303 |  | 19353.211 |  | 19361.492 19362.473 |  | 19361.375 |  |
| 4 | 19351.948 |  | 19350.391 |  | 19362.473 |  | 19362.326 19363.226 |  |
| 6 | 19349.088 |  | 19348.906 |  | 19364.283 |  | 19364.076 |  |
| 7 |  | 19347.476 | -0.141* |  | 19365.112 |  | 19364.875 |  |
| 8 | 19346.026 |  | 19345.784 |  | 19365.890 |  | 19365.623 |  |
| 9 | 19344.420 |  | 19344.148 |  | 19366.618 |  | 19366.321 |  |
| 10 | 19342.763 |  | 19342.462 |  | 19367.295 |  | 19366.968 |  |
| 11 | 19341.057 |  | 19340.725 |  | 19367.921 |  | 19367.564 19368.110 |  |
| 12 | 19339.300 19337.494 |  | 19338.939 19337.102 |  | 19368.497 19369.021 |  | 19368.110 19368.605 |  |
| 14 | 19335.637 | -0.050 | 19335.215 | -0.188* | 19369.495 |  | 19369.049 |  |
| 15 | 19333.730 | -0.115 | 19333.279 | -0.090 | 19369.919 |  | 19369.442 |  |
| 16 | 19331.774 | -0.116* | 19331.292 | -0.134 | 19370.291 |  | 19369.785 |  |
| 17 | 19329.767 | -0.038 | 19329.256 | -0.095 | 19370.613 |  | 19370.076 |  |
| 18 | 19327.711 | -0.121 | 19327.170 | -0.005 | 19370.884 |  | 19370.317 |  |
| 19 | 19325.605 | -0.104 | 19325.034 | -0.148* | 19371.104 |  | 19370.508 |  |
| 20 | 19323.449 | -0.064 | 19322.848 | -0.138 | 19371.273 |  | 19370.647 |  |
| 21 | 19321.244 | 0.033 | 19320.613 | -0.201* | 19371.392 |  | 19370.735 |  |
| 22 | 19318.988 | -0.012 | 19318.328 | -0.030 | 19371.459 |  | 19370.773 |  |
| 23 | 19316.684 | 0.013 | 19315.993 | -0.093 | 19371.476 |  | 19370.760 |  |
| 24 | 19314.330 | -0.103 | 19313.609 | -0.033 | 19371.442 |  | 19370.696 |  |
|  | 19311.925 | -0.020 | 19311.175 |  | 19371.357 |  | 19370.581 |  |
| 26 | 19309.473 | -0.008* | 19308.692 | -0.012* | 19371.222 |  | 19370.416 |  |
| 27 | 19306.970 | -0.057* | 19306.160 | 0.006* | 19371.035 |  | 19370.199 |  |
| 28 | 19304.419 | 0.020* | 19303.578 | -0.036* | 19370.798 |  | 19369.932 |  |
| 29 | 19301.818 | -0.003* | 19300.947 | -0.066* | 19370.510 |  | 19369.614 |  |
| 30 | 19299.168 | 0.078* | 19298.267 | -0.003 | 19370.171 |  | 19369.245 |  |
| 31 | 19296.468 | -0.022 | 19295.538 | -0.079 | 19369.781 |  | 19368.826 |  |
| 32 | 19293.720 | -0.020 | 19292.760 | -0.043 | 19369.341 |  | 19368.355 |  |
| 33 | 19290.923 | -0.016 | 19289.933 | -0.082 | 19368.849 |  | 19367.834 |  |
| 34 | 19288.076 | 0.035 | 19287.056 | -0.080 | 19368.307 |  | 19367.262 |  |
| 35 | 19285.181 | 0.006 | 19284.131 | -0.019 | 19367.715 |  | 19366.639 |  |
| 36 | $\frac{19282 \cdot 237}{19290}$ | 0.063 | 19281.158 | 0.019 | 19367.071 |  | 19365.966 |  |
| 37 | 19279.245 | 0.037 | 19278.135 | 0.043* | 19366.377 |  | 19365.241 |  |
| 38 | 19276.204 | 0.044 | 19275.064 | 0.012 | 19365.632 |  | 19364.466 |  |
| 39 | 19273.114 | 0.040 | 19271.944 | -0.046 | 19364.836 |  | 19363.641 |  |
| 80 | 19269.975 | -0.000 | 19268.776 | -0.009 | $19363.989$ |  | 19362.764 |  |
| 41 | 19266.789 |  | 19265.559 |  | 19363.092 |  | 19361.837 |  |
| 42 | 19263.554 | 0.037 | 19262.294 | -0.056 | 19362.144 |  | 19360.860 |  |
| 43 | 19250.270 | 0.066* | 19258.981 |  | 19361.146 |  | 19359.831 |  |
| 44 | 19256.939 | $0.120 \%$ | 19255.619 |  | 19360.097 |  | 19358.752 |  |

CALCULATED LINE POSITIONS AHD RESIDUALS FOR THE $5-2$ BAND OF A1O (B2 $\left.\Sigma-X^{2} E\right)$

| N | $\mathrm{P}_{2}$ |  | $\mathrm{P}_{1}$ |  | ${ }^{8}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Cale | Res | Celc | Res | Calc | Res | Calc | Res |
| 0 |  |  |  |  | 22961.627 |  | 22961. 505 |  |
| 1 | 22959.209 |  | 22959.197 |  | 22962.700 |  | 22962.663 |  |
| 2 | 22957.869 |  | 22957.841 |  | 22963.678 |  | 22963.625 |  |
| 3 |  | 22956.412 | 0.066* |  | 22964.562 |  | 22964.492 |  |
| 4 | 22954.905 |  | 22954.844 |  | 22965.351 |  | 22965.265 |  |
| 5 6 |  | $\begin{aligned} & 22953.243 \\ & 22951.517 \end{aligned}$ | 0.017 $-0.076 \%$ |  | $\begin{aligned} & 22966.046 \\ & 22966.646 \end{aligned}$ |  | $\begin{aligned} & 22965.943 \\ & 22966.526 \end{aligned}$ |  |
| 6 |  | $\begin{aligned} & 22951.517 \\ & 22949.697 \end{aligned}$ | -0.076* |  | 22966.646 |  | 22966.526 |  |
| 8 |  | 22947.782 | -0.007 |  | 22967.561 |  | 22967.408 |  |
| 9 |  | 22945.773 | -0.012 |  | 22967.877 |  | 22967.708 |  |
| 10 |  | 22943.670 | -0.020 |  | 22968.097 |  | 22967.912 |  |
| 11 |  | 22941.472 | 0.009 |  | 22968.223 |  | 22968.021 |  |
| 12 |  | 22939.180 | -0.078* |  | 22968.254 |  | 22968.036 |  |
| 13 |  | 22936.794 | -0.062* |  | 22968.190 |  | 22967.956 |  |
| 14 |  | 22934.314 | 0.028 |  | 22968.032 |  | 22957.780 |  |
| 15 |  | 22931.739 | -0.018 |  | 22967.778 |  | 22967.510 |  |
| 16 |  | 22929.071 | -0.067* |  | 22967.429 |  | 22967.145 |  |
| 17 |  | 22926.308 | 0.041 |  | 22965.985 |  | 22966.685 |  |
| 18 |  | 22923.452 | 0.048 |  | 22966.447 |  | 22966.130 |  |
| 19 |  | 22920.501 | 0.022 |  | 22965.813 |  | 22965.480 |  |
| 20 |  | 22917.456 | 0.039 |  | 22965.084 |  | 22964.734 |  |
| 21 |  | 22914.318 | -0.042 |  | 22964.260 |  | 22963.894 |  |
| 22 |  | 22911.085 | 0.043 |  | 22963.341 |  | 22962.958 |  |
| 23 |  | \%2907.758 | -0.104* |  | 22962.327 |  | 22961.927 |  |
| 24 |  | 22904.338 | 0.004 |  | 22961.217 | 0.102* | 22960.801 | -0.027 |
| 25 |  | 22900.823 | 0.051 |  | 22960.012 | 0.020 | 22959.580 | -0.038 |
| 27 | 22893.732 | ${ }^{22897.215}$ | -0.081** 293 | 0.004 | 22958.712 | -0.039 | 22958.264 | 0.005 |
| 28 | 22889.945 | 0.051 | 22889.489 | -0.012 | 22955.826 | 0.011 | 22955.345 | -0.050 |
| 29 | 22886.063 |  | 22885.591 |  | 22954.240 | -0.031 | 22953.742 | -0.050 |
| 30 | 22882.088 | 0.082* | 22881.600 | 0.004 | 22952.559 | -0.010 | 22952.045 | -0.079* |
| 31 | 22878.019 | 0.027 | 22877.514 | -0.013 | 22950.782 | 0.032 | 22950.251 | 0.029 |
| 32 | 22873.856 | 0.036 | 22873.335 | -0.046 | 22948.910 | -0.004 | 22948.363 |  |
| 33 | 22069.600 | -0.030 | 22869.062 | -0.015 | 22946.942 | 0.020 | 22946.378 | 0.002 |
| 34 | 22865.250 | 0.021 | 22864.696 | -0.070* | 22944.879 | -0.055 | 22944.299 | -0.035 |
| 35 | 22850.806 | \$.110* | 22860.236 | 0.019 | 22942.720 | -0.017 | 22942.124 |  |
| 36 | 22856.269 |  | 22855.682 |  | 22940.465 | -0.018 | 22939.853 | 0.008 |
| 37 | 22851.638 |  | 22851.035 |  | 22938.116 | 0.045* | 22937.486 | -0.036 |
| 38 | 22846.914 |  | 22846.294 |  | 22935.670 | 0.048* | 22935.024 | 0.011 |
| 39 | 22842.096 |  | 22841.460 |  | 22933.129 |  | 22932.467 |  |
| 40 | 22837.184 |  | 22836.532 |  | 22930.492 |  | 22929.813 |  |
| 41 | 22832.190 |  | 22831.510 |  | 22927.759 |  | 22927.064 |  |
| 42 | 22827.081 |  | 22826.395 |  | 22924.931 |  | 22924.220 |  |
| 43 | 22821.889 |  | 22821.187 |  | 22922.007 |  | 22921.279 |  |
| 44 | 22816.604 |  | 22815.886 |  | 22918.987 |  | 2291.8.243 |  |
| 45 | 22811.226 |  | 22810.491 |  | 22915.871 |  | 22915.111 |  |
| 46 | 22805.754 |  | 22805.002 |  | 22912.660 |  | 22911.882 |  |
| 47 | 22800.188 |  | 22799.420 |  | 22909.352 |  | 22908.559 |  |
| 48 | 22794.530 |  | 22793.746 |  | 22905.949 |  | 22905.139 |  |
| 49 | 22788.778 |  | 22787.977 |  | 22902.449 |  | 22901.623 |  |


| N | $\mathrm{p}_{2}$ |  | $\mathrm{P}_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Cale | Res | Calc | Res | Calc | Res | Calc | Ren |
| $\bigcirc$ |  |  |  |  | 22024.353 |  | 22024.331 |  |
| $\frac{1}{2}$ | 22021.948 |  | 22021.932 |  | 22025.439 |  | 22025.397 |  |
|  | $\begin{aligned} & 220190035 \\ & 92019: 235 \end{aligned}$ |  | $22020.598$ |  | 22027.363 |  | 22027.282 |  |
| 4 |  | 22016.144 | 22017.680 |  | 22028.200 22028.954 |  | 22028.101 |  |
| 6 |  | 22014.488 | -0.041 |  | 22029.625 |  | 22023.488 |  |
| 7 |  | 22012.749 | 0.056 |  | 22030.214 |  | 22030.057 |  |
| 9 |  | 22009.024 | 0.037 |  | 22031.141 |  | 22030.946 |  |
| 10 |  | 22007.038 | 0.020 |  | 22031.480 |  | 22031.266 |  |
| 11 12 |  | 22004.969 | 0.029 0.022 |  | 22031.736 |  | 22031.503 |  |
| 13 |  | 22000.584 | -0.007 |  | 22031.999 |  | $22031.72 \%$ |  |
| 14 |  | 21998.267 | -0.039 |  | 22032.005 |  | 22031.714 |  |
| 15 16 |  | 21995.869 21993.387 | -0.069** |  | 22031.929 |  | 22031.518 22031.439 |  |
| 17 |  | 21990.824 | -0.008 |  | 22031.525 |  | 22031:176 |  |
| 18 |  | 21988.178 | -0.038 |  | 22031.199 |  | 22030.831 |  |
| 19 20 |  | 21985.450 | -0.016 |  | $\begin{array}{r} 22030.789 \\ 22030.296 \end{array}$ |  | 22030.402 |  |
| 21 | 21979.947 | 0.013 | 21979.547 | 0.066 | 22029.719 |  | 22029.294 |  |
| 22 | 21975.982 | -0.012 | 21976.563 | 0.000 | 22029.060 |  | 22028.615 |  |
| 23 24 | 21973.935 | -0.033 | 21973.456 21970.348 | 0.028 0.057 | 22028.316 |  | 22027.852 22027.006 |  |
| 25 | 21967.594 | -0.001 | 21967.117 | 0.023 | 22026.580 |  | 22026.077 |  |
| 25 | 21964.301 | 0.033 | 21963.805 | -0.054 | 22025.586 |  | 22025.065 |  |
| 27 28 | 21960.925 | -0.017 | 21960.410 | -0.031 | 22024.510 | -0.104* | 22023.969 22022.789 | 0.026 |
| 29 | 21953:929 | 0.009 | 21953.375 | -0.025 | 22022.105 | -0.009 | 22021.526 | 0.027 |
| 30 | 21950.307 | 0.025 | 21949.734 | 0.002 | 22020.778 | -0.049 | 22020.179 | 0.009 |
| 32 | 21942.820 | ${ }_{-0.049}$ | 21942.208 | 0.024 0.016 | 22017.873 | -0.039 | 22017.236 | 0.015 |
| 33 | 21938.953 | -0.007 | 21938.322 | -0.008 | 22016.295 | -0.056 | 22015.639 | 0.028 |
| 34 | 21935.005 |  | 21934.355 |  | 22014.633 | -0.286* | 22013.958 |  |
| 36 | 21925.863 | -0.003 | 21926.175 | 0.101* | 22011.059 | -0.054 | 22010.346 | -0.022 |
| 37 | 21922.670 | -0.03 | 21921.962 | 0.101* | 22009.147 | -0.086* | 22008.414 | 0.028 |
| -38 | 21918.395 |  | 21917.669 |  | 22007.151 | -0.093* | 22006.399 | 0.009 -0.005 |
| 40 | 21909.601 |  | 21908.836 |  | 22002.909 | -0.074* | 22002.118 | -0.007 |
| 41 | 21905.082 |  | 21904.298 |  | 22000.662 | -0.085* | 21999.852 |  |
| 42 | 21900.481 |  | 21899.578 |  | 21998.331 | -0.103* | 21997.502 | -0.040 |
| 44 | 21891.035 |  | 21990.194 |  | 21993.419 | -0.053* | 21992.551 |  |
| 45 | 21886.192 |  | 21885.331 |  | 21990.837 | -0.021* | 21989.951 | 0.001 |
| 46 | 21881.266 |  | 21880.386 |  | 21988.172 | -0.032* | 21987.266 |  |
| 48 | 21871.171 |  | 21870.252 |  | 21982.590 | 0.000\% | 21881.646 | 0.047 |
| 49 | 21866.002 |  | 21855.064 |  | 21979.673 | -0.060* | 21978.710 | 0.026 |
| S1 | 21860.752 21855.421 |  | 21859.795 |  | 21976.673 21973.583 |  | 21975.690 21972.587 |  |

CALCULATED LINE POSITIONS AND RESIDUALS FOR THE 5-6 BAND OF AIO (B2E-X2E)

| N | $P_{2}$ |  | $\mathrm{P}_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Calc | Res | Calc | Res | Calc | Res | Calc | Rea |
| 0 |  |  |  |  | 19296.664 |  | 19296.631 |  |
| 1 | 19294.305 |  | 19254.259 |  | 19297.796 |  | 19297.724 |  |
| 2 | 19293.070 |  | 19292.985 |  | 19298.880 |  | 19290.769 |  |
| 3 | 19291.789 |  | 19291.664 |  | 19299.917 |  | 19299.766 |  |
| 4 | $19290.460$ |  | 19290.296 |  | 19300.906 |  | 19300.717 |  |
| 5 | 19289.084 19287.651 |  | 19288.881 19287.419 |  | 19301.848 |  | 19301.619 19302.475 |  |
| 6 7 | 19287.651 |  | 19287.419 19285.909 |  | 19302.743 19303.590 |  | 19302.475 19303.282 |  |
| 8 | 19284.674 |  | 19284.353 |  | 19304.350 |  | 19304.043 |  |
| 9 | 19283.116 |  | 19282.749 |  | 19305.142 |  | 19304.756 |  |
| 10 | 19281.498 |  | 19281.099 |  | 19305.846 |  | 19305.421 |  |
| 11 | 19279.840 |  | 19279.401 |  | 19306.504 |  | 19306.039 |  |
| 12 | 19278.135 |  | 19277.657 |  | 19307.113 |  | 19306.609 |  |
| 13 | 19276.383 |  | 19275.866 |  | 19307.675 |  | 19307.132 |  |
| 14 | 19274.585 |  | 19274.028 | 0.003 | 19308.190 |  | 19307.607 |  |
| 15 | $19272.739$ | 0.067 | 19272.143 |  | 19308.657 |  | $19308,035$ |  |
| 16 | 19270.847 19268.908 | 0.069 | 19270.212 19268.234 | 0.007 | 19309.076 19309.448 |  | 19308.415 19308.748 |  |
| 18 | 19266.923 | 0.033 | 19256.209 | 0.065 | 19309.772 |  | 19309.033 |  |
| 19 | 19264.891 | 0.059 | 19264.138 | 0.045 | 19310.049 |  | 19309.270 |  |
| 20 | 19262.813 | 0.068 | 19262.020 | 0.218* | 19310.279 |  | 19309.460 |  |
| 21 | 19260.688 | 0.110 | 19259.856 | 0.108 | 19310.460 |  | 19309.603 |  |
|  | 19258.517 | 0.084 | 19257.646 |  | 19310.594 |  | 19309.698 |  |
| 23 | 19256.300 |  | 19255.389 |  | 19310.681 |  | 19309.745 |  |
| 24 | 19254.036 |  | 19253.086. |  | 19310.720 |  | 19309,745 |  |
| 25 | 19251.726 |  | 19250.737 |  | 19310.712 |  | 19309.698 |  |
| 26 | 19249.371 |  | 19248.342 |  | 19310.656 |  | 19309.602 |  |
| 27 | $19246.969$ |  | $19245.901$ |  | 19310.553 |  | 19309.460 |  |
| - 28 | 19244.521 19242.028 |  | 19243.414 19240.882 |  | 19310.402 |  | 19309.270 |  |
| +29 30 | 19242.028 19239.488 |  | 19240.882 19238.303 |  | 19310.204 |  | 19309.033 19308.748 |  |
| 30 | 19239.488 19236.903 |  | 19238.303 |  | 19309.959 19309.666 |  | 19308.748 19308.416 |  |
| 32 | 19234.273 |  | 19233.009 |  | 19309.326 |  | 19308.036 |  |
| 33 | 19231.597 |  | 19230.293 |  | 19308.938 |  | 19307.609 |  |
| 34 | 19228.875 |  | 19227.532 |  | 19308.504 |  | 19307.135 |  |
| 35 | 19226.108 |  | 19224.726 |  | 19308.021 |  | 19306.614 |  |
| 36 | 19223.296 |  | 19221.875 |  | 19307.1922 |  | 19306.045 |  |
| 3 B | 19217.536 |  | 19216.036 |  | 19306. 292 |  | 19304.767 |  |
| 39 | 19214.588 |  | 19213.049 |  | 19305.621 |  | 19304.056 |  |
| 40 | 19211.596 |  | 19210.017 |  | 19304.903 |  | 19303.299 |  |
| 41 | 19208.558 |  | 19206.941 |  | 19304.138 |  | 19302.495 |  |
| 42 | 19205.476 |  | 19203.820 |  | 19303.326 |  | 19301.644 |  |
| 43 | $19202.350$ |  | $19200.654$ |  | 19302.467 |  | 19300.745 |  |
| 44 | $\begin{aligned} & 19199.179 \\ & 19195.963 \end{aligned}$ |  | 19197.443 19194.188 |  | 19301.561 19300.609 |  | 19299.800 19298.808 |  |
| 46 | 19192.703 |  | 19190.889 |  | 19299.609 |  | 19297.769 |  |



| N | $\mathrm{P}_{2}$ |  | $\mathrm{P}_{1}{ }^{\text {. }}$ |  | $R_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Calc | Res | Calc | Res | Cale | Ren | Calc | Res |
| 0 |  |  |  |  | 21927.495 |  | 21927.470 |  |
| 1 | 21925.113 |  | 21925.090 |  | 21928.578 |  | 21928.529 |  |
| 2 | 21923.815 |  | 21923.768 |  | 21929.581 |  | 21929.509 |  |
| 3 |  | 21922.402 | 0.120* |  | 21930.504 |  | 21930.408 |  |
| 4 |  | 21920.932 | 0.052* |  | 21931.347 |  | 21931.228 |  |
| 5 |  | 21919.383 | 0.005 |  | 21932.111 |  | 21931.967 |  |
| 6 |  | 21917.755 | -0.021 | . | 21932.794 |  | 21932.627 |  |
| 7 |  | 21916.047 | 0.005 |  | 21933.398 |  | 21933.207 |  |
| 8 |  | 21914.259 | -0.044 |  | 21933.921 |  | 21933.707 |  |
| 9 |  | 21912.392 | 0.022 |  | 21934.365 |  | 21934.127 |  |
| 10 |  | 21910.445 | 0.011 |  | 21934.729 |  | 21934.467 |  |
| 11 |  | 21908.419 | -0.048 |  | 21935.013 |  | 21934.727 |  |
| 12 |  | 21906.313 | -0.002 |  | 21935.216 |  | 21934.907 |  |
| 14 |  | 21904-129 | 0.041 |  | 21935.340 |  | 21935.007 |  |
| 15 |  | 21899.521 | -0.009 |  | 21935.347 |  | 21934.967 |  |
| 16 | 21897.288 | -0.009 | 21896.909 | 0.025 | 21935.231 |  | 21934.827 |  |
| 17 | 21894.798 | 0.064 | 21894.396 | 0.002 | 21935.034 |  | 21934.607 |  |
| 18 | 21892.229 | 0.013 | 21891.803 | 0.042 | 21934.758 |  | 21934.306 |  |
| 19 | 21889.581 | 0.014 | 21889.131 | 0.044 | 21934.401 |  | 21933.926 |  |
| 20 | 21886.854 | 0.019 | 21886.380 | 7.045 | 21933.964 |  | 21933.465 |  |
| 21 | 21884.047 | 0.016 | 21883.551 | - 6,003 | 21933.447 |  | 21932.925 |  |
| 22 | 21881.162 21878.198 | 0.044 0.025 | 21880.642 21877.654 | -0.026 -0.702 | 21932.850 21932.173 |  | 21932.304 |  |
| 24 | 21875.155 | $0.084 *$ | 21874.588 | -0.1528 | 21931.415 |  | 21930.822 |  |
| 25 | 21872.034 | -0.014 | 21871.442 | -0.059 | 21930.578 |  | 21929.961 |  |
| 26 | 21868.833 | 0.033 | 21868.218 | -0.007 | 21929.660 |  | 21929.019 |  |
| 27 | 21865.554 | -0.006 | 21864.915 | 0.012 | 21928.663 |  | 21927.998 |  |
| 28 | 21862.196 | 0.032 | 21861.534 | 0.011 | 21927.585 |  | 21926.896 |  |
| 29 | 21858.760 | -0.012 | 21858.074 | -0.027 | 21926.427 |  | 21925.714 | 0.035 |
| 30 | 21855.245 | 0.049 | 21854.535 | -0.046 | 21925.188 | $-0.023$ | 21924.452 | 0.006 |
| 31 | 21851.652 | -0.015 | 21850.918 | 0.025 | 21923.870 | -0.036 | 21923.110 | -0.014 |
| 32 | 21847.980 | -0.013 | 21847.222 | -0.021 | 21922.471 | 0.041 | 21921.688 | $0.164 *$ |
| 33 | 21844.230 |  | 21843.448 |  | 21920.992 | -0.008 | 21920.185 | 0.001 |
| 34 | 21840.401 |  | 21839.596 |  | 21919.433 | -0.044* | 21918.602 | -0.095* |
| 35 | 21836.494 | 0.031 | 21835.656 | 0.000 | 21917.794 | -0.060* | 21916.940 | 0.002 |
| 36 | 21832.509 | -0.010 | $2183 \times .657$ | -0.045 | 21916.075 | -0.023* | 21915.197 | 0.023 |
| 37 | 21928.446 | 0.048 | 2182. 570 | 0.023 | 21914.276 | -0.061* | 21913.374 | 0.002 |
| 38 | 21824.305 |  | 21823.405 | 0.005 | 21912.396 | 0.018* | 21911.470 | 0.005 |
| 39 | 21820.086 | 0.010 | 21819.162 |  | 21910.436 | $0.020 *$ | 21909.487 | 0.050* |
| 40 | 21815.789 | -0.052 | 21814.842 | -0.069 | 21008.397 | -0.026* | 21907.423 | -0.028* |
| 41 | 21811.414 | -0.001 | 21810.443 | -0.129* | 21906.277 | 0.034* | 21905.280 | -0.186* |
| 42 | 21806.961 | -0.033 | 21805.966 | -0.015 | 21904.077 |  | 21903.056 | 0.049* |
| 43 | 21802.430 | -0.003 | 21801.412 | -0.050 | $21901.797$ |  | $21900.752$ | -0.168* |
| 44 | 21797.822 21793.136 |  | 21796.780 21792.070 |  | 21899.437 |  | 21898.369 21895.905 | -0.082* |
| 45 | 21793.136 | -0.008 | 21792.070 |  | 21896.996 |  | 21895.905 | -0.088 |
| 46 | 21788.373 21703.532 | -0.016 | 21787.283 21782.418 | 0.006 | 21894.476 21891.876 |  | 21893.361 21890.737 |  |
| 48 | 21778.613 |  | 21777.476 |  | 21889.196 |  | 21888.033 |  |

CALCULATED LINE POSITIONS AND RESIDUALS FOR THE $7-4$ BAND OF ALO (B2E-X2E)

| N | $\mathrm{P}_{2}$ |  | $\mathrm{P}_{1}$ |  | $\mathrm{R}_{2}$ |  | $R_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Calc | Res | cale | Rea | Cale | Rea | Cale | Res |
| 0 |  |  |  |  | 22746.498 |  | 22746.473 |  |
| $\frac{1}{2}$ | $\begin{aligned} & 22744.125 \\ & 22742.818 \end{aligned}$ |  | 22744.102 2742.772 |  | 22747.564 |  | 22747.515 22748.459 |  |
| 3 3 | 22741.423 2279 |  | 22741.353 |  | 22749.430 |  | 22745.334 |  |
| 5 | 22739.940 | 22738.309 | 0.104*846 |  | 22750.231 22750.943 |  | 22750.110 |  |
| 6 |  | 22736.638 | 0.056 |  | 22751.566 |  | 22751.393 |  |
| 7 |  | 22734.878 | 0.043 |  | 22752.101 |  | 22751.909 |  |
| 8 |  | 22733.030 | 0.023 |  | 22752.548 |  | 22752.332 |  |
| 10 |  | 22729.071 | 0.017 |  | 22753.175 |  | 22752.912 |  |
| 11 |  | 22726.959 | -0.020 |  | 22753.356 |  | 22753.069 |  |
| 12 |  | 22724.759 | -0.000 |  | 22753.449 |  | 22753.137 |  |
| 13 |  | 22722.471 | -0.027 |  | 22753.452 |  | 22753.117 |  |
| 14 15 |  | 22720.096 | $=0.002$ -0.007 |  | 22759.368 22753.194 |  | 22753.009 |  |
| 16 |  | 22715.081 | 0.015 |  | 22752.932 |  | 22752.525 |  |
| 17 |  | 22712.442 | 0.017 |  | 2275.581 |  | 22752.151 |  |
| 18 | 22709.930 | 0.028 | 22709.502 | -0.004 | 22752.142 |  | 22751.687 |  |
| $\frac{19}{20}$ | 22704.237 | -0.028 | 22705.676 | 0.029 0.089 | 22750.614 |  | 22750.495 |  |
| 21 | 22701.260 | -0.032 |  | -0.060 | 22750.292 |  | 22749.766 |  |
| 22 | 22698.195 | 0.045 | 22697.672 | 0.040 | 22749.498 2748.615 |  | 22748.948 |  |
| 24 | 226591.043 | -0.142* | 22691.232 | -0.032 | 22748.615 |  | 2274.041 |  |
| 25 | 22588.476 | -0.065 | 22687.881 | 0.003 | 22746.583 | -0.069* | 22745.962 | 0.016 |
| 26 | 22685.061 | -0.030 | 22684,442 | -0.026 | 22745.434 | -0.007 | 22744.789 |  |
| 27 | 22681.559 | -0.000 | 22680.917 | 0.131* | 22744.197 | -0.057 | 22743.528 | -0.061 |
| 29 | 22674:294 | -0.013 | 22673.604 | -0.010 | 22741.456 | -0.011 | 22740.739 | -0.050 |
| 30 | 22670.531 | 0.059 | 22669.817 | -0.055 | 22739.952 | -0.014 | 22739.212 | 0.075 |
| 31 | 22665.744 | -. 0.038 | 22661.982 | 0.114* | 22738.360 22735.679 | 0.053 | 22737.595 | -0.133 |
| 33 | 22658.720 | 0.068 | 22657.934 | -0.019 | 22734.909 | 0.012* | 22734.097 | -0.079 |
| 34 35 | 22654.609 |  | 22653.799 |  | 22733.051 | 0.002" | 22732.215 | 0.046 |
| 35 | 22646.127 | - | 22645.270 |  | 22732.104 | 0.023* | 22730.244 | -0.049 |
| 37 | 22641.756 |  | 22640.875 |  | 22725.944 | -0.005* | 22726.037 |  |
| -388 | 22537.299 |  | 22636.394 |  | 22724.731 | 0.028* | 22723.800 | -0.026 |
| 40 | 22528.12 ' |  | 22527.171 |  | 22720.040 | $0.054 *$ | 22710.061 |  |
| 41 | $22623.40 \%$ |  | 22622.431 |  | 22717.561 | $0.064 *$ | 22716.558 |  |
| 42 | 22618.604 22613.715 |  | 22617.603 22612.690 |  | 22714.994 | 0.102* | 22713.967 |  |


| $N$ | $\mathrm{P}_{2}$ |  | $\mathrm{P}_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Cale | Rea | cale | Res | calc | Rea | Cale | Res |
| 11234456789910111212131415161618181920212223242525272728293031 |  |  |  |  | 21837.463 |  | 21837.434 |  |
|  | 21835.104 21833.824 |  | 21835.072 21833.761 |  | 21838.543 |  | 21838.485 21839.459 |  |
|  |  | 21832.421 | 21833.761 |  | 21840.475 |  | 21840.355 |  |
|  |  | 21830.974 21829.450 | 0.132* |  | 21841.326 |  | 211841.177 |  |
|  |  | 21827.849 | 0.101* |  | 21842.799 |  | 21842.588 |  |
|  |  | 21826.172 | 0.062 |  | 21843.420 |  | 21843.179 |  |
|  |  | 21824.419 21822.590 | -0.003 |  | 21843.965 |  | 21843.693 |  |
|  |  | 21822.590 21820.685 | 0.064 |  | 21844.433 |  | 21844.131 |  |
|  |  | 21818.703 | -131* |  | 21845.138 |  | 21844,776 |  |
|  | 21814.711 | 21816.646 | O. 031814.314 |  | 21845.376 |  | 21844.984 |  |
|  | 21812.516 | -0.013 | 21812.089 | $0.234 *$ | 21845.622 |  | 21845.114 |  |
|  | 21810.246 | 0.068 | 21809.789 | 0.025 | 21845.629 |  | 21845. 146 |  |
|  | 21807.900 | -0.023 | 21807.412 | -0.063 | 21845.560 |  | 21845.046 |  |
|  | 21802.980 | -0.008 | 21802.431 | -0.004 | 21845.192 |  | 21844.617 |  |
|  | 21800.406 | 0.022 | 21799.827 | -0.002 | 21844.893 |  | 21844,288 |  |
|  | 21797.757 | 0.006 -0.016 | 21797.148 | -0.008 | 21844.517 |  | 21843.881 |  |
|  | 21792.232 | -0.096* | 21791.562 | -0.000 | 21884. 534 |  | 21842.838 |  |
|  | 21789.356 | -0.024 | 21788.656 | -0.024 | 21842.928 |  | 21842.202 |  |
|  | 21786.405 | 0.050 | 21785.574 | -0.007 | 21842.245 |  | 21841.488 |  |
|  | 21780.276 | 0.017 | 21779.465 | 0.030 | 21840.649 |  | 21839.832 |  |
|  | 2177.099 | -0.034 | 21776.277 | -0.004 | 21839.736 |  | 21838.888 |  |
|  | 21770.519 | -0.152* | 21769.636 | $=0.012$ -0.009 | 21838.746 21837.680 |  | 21837.868 |  |
|  | 21767.116 | 0.046 | 21766.203 | -0.013 | 21836.537 | -0.022 | 216315.598 |  |
|  | 21763.638 |  | 21762.695 |  | 21835.317 | 0.020 | 21534.348 | -0.075* |
|  | 21760.086 21756.459 | -0.004 | 21759.112 | -0.034 0.051 | 21834.021 | -0.101* | 21833.021 21831.618 |  |
|  | 21752.757 | 0.036 | 21751.722 | -0.073 | 21831.199 | -0.093* | 21830.138 | -0.003 |
|  | 21748.980 | -0.013 | 21747.915 | -0.039 | 21829.672 |  | 21828.582 |  |
|  | 21741.203 | -0.017* | 21744.034 21740.077 | 0.047 | 21828.070 21826.391 |  | 21826.949 | -0.040 -0.060 |
|  | 21737.203 |  | 21736.047 | 0.019 | 21824.635 |  | 21823.453 | -0.043 |
|  | 21733.128 |  | 21731.942 |  | 21822.803 |  | 21821.591 | -0.066 |
|  | 21728.979 21744.756 | -0.010 0.027 | 21727.763 | -0.035 | 21820.895 |  | 21819.652 |  |
|  | 21720.459 |  | 21719.182 | 0.007 | 21816.849 |  | 21815.545 |  |
|  | 21716.088 |  | 21714.780 |  | 21814.711 |  | 21813.378 |  |


| N | $\mathrm{P}_{2}$ |  | $\mathrm{P}_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Calc | Rea | cale | Res | Calc | Res | Calc | Rea |
|  |  |  |  |  |  |  |  |  |
| $\frac{1}{2}$ | $\begin{aligned} & 22646.790 \\ & 72645.501 \end{aligned}$ |  | 22646.757 32645.438 |  | $\begin{aligned} & 22650.204 \\ & 22651.383 \end{aligned}$ |  | $\begin{aligned} & 22650.145 \\ & 22551.093 \end{aligned}$ |  |
| 3 |  | 22644.082 | 0.027* |  | 22652.076 |  | 22651.956 |  |
| 4 |  | 22642.508 | -0.162* |  | 22652.885 |  | 22652.735 |  |
| 6 |  | 22639.408 | 0.028 |  | 22654.247 |  | 22654.036 |  |
| 7 |  | 22637.6880 | -0.121 |  | 22654.801 |  | 22654.559 |  |
| $\stackrel{8}{9}$ |  | 226353.8681 | -0.102 |  | 22655. 2695 |  | 22654.997 |  |
| 10 |  | 22631.989 | -0.145* |  | 22655.951 |  | 22655.617 |  |
| 11 |  | 22659.923 | -0.068 |  | 225556.164 |  | 22655.800 |  |
| 13 |  | 22625.537 | -0.233* |  | 22656.334 |  | 22655.909 |  |
| 14 15 |  | $\begin{array}{r} 22623.247 \\ 22620.813 \end{array}$ | $\begin{array}{r} 0.100 * \\ -0.285 * \end{array}$ |  | $\begin{aligned} & 22656.291 \\ & 22656.163 \end{aligned}$ |  | $\begin{aligned} & 22655.836 \\ & 22655.677 \end{aligned}$ |  |
| 16 | 22618.569 |  | 22618.079 |  | 22655.950 |  | 22655.434 |  |
| 17 | 22616.012 | 0.064 | 22615.491 22612.819 | -0.133* | 22655.652 22555.269 |  | 22655.105 |  |
| 18 | 22613.370 | 0.261* | 22610.062 | -0.154* | 22654.800 |  | 22654.691 |  |
| 20 | 22607.834 | 0.011 | 22507.222 | -0.038 | 22654.246 |  | 22653.608 |  |
| 21 | 22604.940 | -0.060 | 22604.298 | 0.079 | 22553. 607 |  | 22652.938 |  |
| 23 | 22598.899 | -0.048 | 22598.196 | 0.163* | 22552.074 |  | 22651.344 |  |
| 24 | 22595.753 | -0.024 | 22595.019 | 0.029 | 22651.179 |  | 22650.418 |  |
| 25 | 22592.523 | -0.066 | 22592.759 22588.415 | 0.031 0.057 | 22650.199 | 0.117 | 22649.408 | -0.068 |
| 27 | 22585.812 | -0.014 | $22584 \times 987$ | -0.100 | 22547.985 | 0.125 | 22547.132 |  |
| 28 | 22582.331 | 0.053 | 22581.475 | -0.053 | 22646.749 | 0.079 | 22645.867 |  |
| 29 30 | 22578.767 | $=0.021$ -0.031 | 22577.880 | -0.032 | 22645.429 | -0.101** | 22644.516 |  |
| 31 | 22571.388 | -0.017 | 22570.440 | -0.128** | 22542.534 |  | 22641.560 |  |
| 32 | 22567.573 | -0.031 | 22566.595 | -0.132* | 22640.958 | -0.018 | 22639.954 |  |
| 34 | 22559.694 |  | 22558.655 |  | 22637.552 | 0.007* | 22636.487 | -0.120 |
| 35 | 22555.630 |  | 22554.560 |  | 22635.722 | 0.044* | 22634.626 |  |
| 36 | 22551.483 |  | 22550.383 |  | 22633.807 |  | 22632.680 | -0.039 |
| 38 | 22542.940 |  | 22541.778 |  | 22629.722 | 0.037* | 22628.534 |  |
| 39 | 22533.544 |  | 22537.352 |  | 22627.552 |  | 32626.334 |  |
| 40 | 22534.066 |  | 22532.844 |  | 22625.297 | 0.007* | 22624.049 |  |
| 42 | 22524.861 |  | 22523.578 |  | 22620.533 | -0.005* | 22611.679 |  |
| 43 | 22520.136 |  | 22518.822 |  | 22618.025 |  | 22616.685 |  |

CALCULATED LINE POSITIONS AND RESIDUALS FOR THE B-6 BAMD OF A1O (B2E-X2L)

| N | $\mathrm{p}_{2}$ |  | $\mathrm{P}_{1}$ |  | $R_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Cale | Rea | Calc | Res | calc | Res | Calc | Rea |
|  |  |  |  |  | $\begin{aligned} & 21754.064 \\ & 21755.145 \end{aligned}$ |  | $\begin{aligned} & 21754.031 \\ & 91755.037 \end{aligned}$ |  |
| $\frac{1}{2}$ | 21751.731 21750.471 |  | 21751.684 21750.385 |  | $\begin{aligned} & 21755.145 \\ & 71755 \end{aligned}$ |  | $21755.072$ |  |
| 3 | 21749.138 |  | 21749.013 |  | 21757.085 |  | 21756.934 |  |
| 4 | 21747.733 |  | 21747.567 |  | 21757.947 |  | 21757.755 |  |
| 5 |  | 21746.151 | -0.012 |  | 21758.735 |  | 21758.504 |  |
| 6 | 21744.703 | 21742936 | 21744.458 |  | 21759.450 |  | 21759.179 |  |
| 7 |  | 21742.936 | -0.035 |  | 21760.092 |  | 21759.780 |  |
| 8 |  | 21741.219 |  |  | 21760.660 |  | 21760.309 |  |
| 9 |  |  | -0.078* |  | 21761.155 |  | 21760.765 |  |
| 10 | 21737.769 |  | 21737.365 | -0.030 | 21761.577 |  | 21761.147 |  |
| 11 | 21735.854 |  | 21735.410 |  | 21761.926 |  | 21761.456 |  |
| 12 13 | 21733.866 21731.806 | -0.032 0.023 | 21733.383 21731.283 | -0.156* | 21762.201 21762.404 |  | 21761.692 |  |
| 14 | 21729.673 | -0.148* | 21725.110 | -0.141* | 21762.533 |  | 21761.944 |  |
| 15 | 21727.468 | 0.030 | 21726.865 | -0.1847 | 21762.588 |  | 21761.960 |  |
| 16 | 21725.190 |  | 21724.548 |  | 21752.571 |  | 21761.903 |  |
| 17 | 21722.840 | -0.082 | 21722.158 | 0.026 | 21762.480 |  | 21761.772 |  |
| 18 | 21720.418 | -0.032 | 21719.695 | -0.003 | 21762.317 |  | 21761.569 |  |
| 19 | 21717.923 | -0.050 | 21717.162 | 0.004 | 21762.079 |  | 21761.292 |  |
| 20 | 21715.357 | -0.016 | 21714.556 | 0.079 | 21761.769 |  | 21760.942 |  |
| 21 | 21712.718 | -0.023 | 21711.878 | -0.120\% | 21761.386 |  | 21760.519 |  |
| 22 | 21710.008 | -0.004 | 21709.127 | 0.014 | 21760.929 |  | 21760.022 |  |
| 23 | 21707.225 | -0.119* | 21706.305 | -0.034 | 21760.400 |  | 21759.453 |  |
| 24 | 21704.371 | -0.042 | 21703.411 | -0.050 | 21759.797 |  | 21758.810 |  |
| 25 | 21701.445 | -0.007 | 21700.445 | -0.002 | 21759.121 |  | 21768.055 |  |
| 26 | 21698.447 | -0.009 | 21697.408 | 0.030 | 21758.372 |  | 21757.306 |  |
| 27 | 21695.377 | -0.031 | 21694.299 | 0.053 | 21757.550 |  | 21756.444 |  |
| 23 | 21692.236 | -0.085 | 21691.118 | -0.008 | 21756.654 |  | 21755.509 |  |
| 29 | 21689.024 | -0.031 | 21687.866 | 0.022 | 21755.686 |  | 21754.502 |  |
| 30 | 21685.740 |  | 21684.542 | 0.061* | 21754.645 |  | 21753.421 |  |
| 31 | 21682385 | -0.017* | 21681.148 | -0.069* | 21753.531 | -0.042 | 21752.267 | 0.004 |
| 32 | 2167\%.959 | 0.035 | 21677.682 | 0.071 | 21752.344 | -0.073 | 21751.040 | 0.019 |
| 33 | 21675.462 |  | 21674.145 | -0.026 | 21751.084 | -0.025 | 21749.741 | -0.023 |
| 34 | 21671.893 | 0.038 | 21670.537 | 0.074 | 21749.752 | -0.034 | 21748.369 | -0.005 |
| 35 | $21668.254$ |  | 21665.858 |  | 21748.346 | 0.018 | 21746.923 | 0.026 |
| 36 37 | 21664.544 | 0.079* | 21663.108 21659.287 | 0.065 0.095 | 21746.868 | 0.081 -0.029 | 21745.406 21743.815 |  |
| 37 38 | 21660.763 | 0.109** | 21659.287 21655.396 | 0.095 0.086 | 21745.317 21743.694 | -0.029 | 21743.815 21742.152 | 0.048 -0.053 |
| 38 | 21652.990 | 0.119* | 21651.435 | 0.086 | 21741.998 | 0.101* | 21740.416 | -0.125* |
| 40 | 21648.998 | $0.160 *$ | 21647.403 | 0.144* | 21740.229 | -0.105* | 21738.608 | 0.010 |
| 41 | 21644.935 |  | 21643.301 | $0.074 *$ | 21738.388 |  | 21736.727 | 0.071 |
| 42 | 21640.803 |  | 21639.128 | 0.063* | 21726.475 |  | 21734.774 | -0.037 |
| 43 | 21636.600 |  | 21634.886 | 0.142* | 21734.489 |  | 21732.748 | -0.053* |
| 44 | 21632.327 |  | 21630.573 |  | 21732.431 |  | 21730.650 | 0.041 * |
| 45 | 21627.985 |  | 21625.191 |  | 21730.300 |  | 21728.480 | -0.057 |

CALCULATED LINE POSITIONS AKD RESIDUALS ZOR THE $9-6$ BAHD OF AIO $\left(B 2 \Sigma-X^{2}\right.$ L)

| N | $\mathrm{P}_{2}$ |  | $\mathrm{P}_{1}$ |  | $\mathrm{R}_{2}$ |  | $\mathrm{R}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Cale | Rea | Calc | Rea | Cale | Res | Calc | Rea |
| 1 |  |  |  |  | 22558.382 22559.446 |  | 22558.349 |  |
| $\frac{1}{2}$ | 22554.769 |  | 22554.703 |  | 22560.428 |  | 22560.315 |  |
| 3 | 22553.440 |  | 22553. 313 |  | 22561.329 |  | 22561.176 |  |
| 4 |  | 22551.926 22550.394 | -0.026 |  | 22562.1486 |  | 22561.956 |  |
| 6 |  | 22548.781 | -0.018 |  | 22563.542 |  | 22563.270 |  |
| 7 |  | 22547.086 | 0.027 |  | 22564.117 |  | 22563.805 |  |
| ${ }_{9}^{8}$ |  | 22545.311 22543.454 | -0.095 |  | 22564.610 |  | 22564.258 |  |
| 10 |  | 22541.516 | 0.053 |  | 22565.352 |  | 22564.920 |  |
| 11 |  | 22539.498 | 0.023 |  | 22555.600 |  | 22565.128 |  |
| 12 |  | 22537.398 | $\begin{array}{r} -0.140 * \\ 0.019 \end{array}$ |  | 22565.767 |  | $22565.2{ }^{2}$ |  |
| 14 | 22533.238 | -0.028 | 22532.674 | 0.030 | 22565.856 |  | 22565.264 |  |
| 15 | 22530.916 | -0.059 | 22530.312 | 0.111 | 22565.778 |  | 22565.146 |  |
| 17 | 22526.029 | ${ }^{22508088}$ | 22525.345 | -0.004 | 22565.377 |  | 22564.656 |  |
| 18 | 22523.465 | 0.021 | 22522.741 |  | 22565.054 |  | 22564.303 |  |
| 19 | 22520.820 | -0.057 | 22520.056 | -0.044 | 22564.650 |  | 22563.859 |  |
| 20 | 22518.094 | -0.263* | 22517.291 | 0.076 | 22564.164 22563.597 |  | 22563.334 |  |
| 22 | 22512.402 | -0.084 | 22511,519 | -0.022 | 22562.948 |  | 22562.038 |  |
| 23 | 22509.436 | -0.097 | 22508.513 | 0.076 | 22562.218 |  | 22561.258 |  |
| 29 | 22506.389 | ${ }^{-0.125}$ | 22505.426 | -0.018 | 22561.406 |  | 22560.416 |  |
| 26 | 2250.256 | -0.1005* | 22492.013 | -0.174* | 22559.538 |  | 2255.483 |  |
| 27 | 22496.769 | -0.105 | 22455.687 | -0.009 | 22558.482 |  | 22557.372 |  |
| 28 | 22493.403 |  | 22492.280 | 0.041 | 22557.344 |  | 22556.195 |  |
| 29 | 22489.956 | 0.004 | 22488.794 | -0.019 | 22556.126 |  | 22554.937 |  |
| 30 | 22485.430 | -0.021 | 22485.228 | 0.124* | 22554.826 |  | 22553.597 |  |
| 31 | 22482.825 | -0.026 | 22481.583 |  | 22553.444 |  | 22552.176 |  |
| 33 | 22475.375 |  | 22474:053 |  | 22550.438 |  | 22549.090 |  |

This estimate lies, however, outside the limits $(0.8828)^{2} \leq(\hat{\sigma})^{2} \leq(1.11)^{2}$ given by the $\chi^{2} / 98$ distribution at the 95\% confidence level for random measurement error. Such a result is not unusual; in nearly all applications of the merge method to date, estimated variances of merges have been found to be well outside the limits set by the $\chi^{2} / f_{m}$ distribution. The values of the present work are, in fact, unusually small. It could be concluded, therefore, that the systematic error in the present work is also unusually small, and that the effect on merged parameters ( $\hat{X}_{i}$ ) can be accommodated satisfactorily by calculating standard errors $\left(\hat{\sigma}_{i}\right)$ for each constant from (4)

$$
\begin{equation*}
\hat{\sigma}_{i}=\left(\hat{\sigma}_{M}\right)\left(V_{i i}\right)^{1 / 2} \tag{3.6}
\end{equation*}
$$

In Eq. (3.6), $V$ is the dispersion matrix of the merged constants. All estimated standard errors reported in the present work were obtained from Eq. (3.6). The parameters output in merge $C$, in fact, reproduce the measured line positions very satisfactorily. This is shown in Table 3.5 which lists line positions and residuals calculated from the parameters of Table 3.1 and 3.4.

The ratio of the root mean square of the residuals (rms), $\left\{\Sigma\left(v_{i}^{\text {obs }}-v_{i}^{\text {cal }}\right)^{2} n^{-1}\right\}^{1 / 2}$, for lines calculated with parameters from merge $C$ to the estimated standard deviation, is given in the right-hand column of Table 3.1. The ratios are in general very satisfactory, except the very weak 3-5 and 5-6 bands with few lines fitted.

The presence of significant, but not unusually large,
systematic error was revealed in merge $D$, in which the band origins were reduced to a set of relative term values. In merge $D$, the output consisted of the sixteen term values listed in Table 3,6 which are determined relative to $X^{2} \Sigma^{+}$, $v=0$. The variance of the equivalent single-step merge of the individual band-by-band constants (105 degrees of freedom, $f=137-32$ ) to this output set was $\left(\hat{\sigma}_{M}\right)^{2}=$ $(3.70)^{2}$, significantly larger than those of the merges described earlier; the large value is due to the presence of relative systematic error between different bands.

It is worth noticing, however, that the estimates in merge $D$ of all the Dunham coefficients of Table 3.4 were unchanged to within the estimated one standard errors when these were defined by $\hat{\sigma}_{1}=3.70\left(V_{1 i}\right)^{1 / 2}$; this is in agreement with the general conclusion that contamination of leastsquares parameter estimates due to systematic error is assessed reliably when Eq. (3.6) is used to specify the standard errors. Vibrational Dunham coefficients given by a final fit, merge $E$, are included in Table 3.6. Merge $E$ has variance $\left(\hat{\sigma}_{M}\right)^{2}=(3.84)^{2}$ and $f=137-25=112$ degrees of freedom, for the equivalent single-step merge. It was necessary to determine nine (five for the $B^{2} \Sigma^{+}$state, and four for the $X^{2} \Sigma^{+}$state) vibrational coefficients in order to represent the sixteen relative vibrational term values.

The Dunham coefficients in Tables 3.4 and 3.6 are considered the best available for the $B^{2} \Sigma^{+}$and $X^{2} \Sigma^{+}$states of A10.

Finally, the vibrational dependence of ( $\gamma_{v}^{\prime \prime}-\gamma_{\|}^{\prime \prime}$ ) and ( $\gamma_{v}^{\prime}-\gamma_{\&}^{\prime \prime}$ ) are shown in Fig. 3.5. For ( $\gamma\|=\gamma\|$ ), the curve is calculated from the parameters of Merge B (Table 3.3), and shows that the magnitude of $\gamma_{v}^{\prime \prime}$ changes rapidly with $v^{\prime \prime}$ for $v^{\prime \prime} \leq 6$. The conclusion is valid on the assumption that the magnitude of $\gamma_{0}^{\prime \prime}$ is small. Approximate estimates of $\gamma \|$ of 0.01 and $0.005 \mathrm{~cm}^{-1}$ obtained in references (18) and (32) respectively, would support this assumption. For ( $\gamma_{v}^{\prime}-\gamma_{0}^{\prime \prime}$ ) the points in Fig. 3.5 are the estimates listed in Table 3.3 and the straight line is from merge $C$ (Table 3.4). The different vibrational behavior of ( $\left.\gamma_{v}^{\prime}-\gamma_{v}^{\prime \prime}\right)$ is in accord with the expectation that the major contribution to the spin-rotational constants comes from the low-lying $A^{2} \Pi$ state. The potential minimum of this state lies at $\sim 5400$ $\mathrm{cm}^{-1}$, between the $\mathrm{v}^{\prime \prime}=5$ and 6 levels of the $\mathrm{X}^{2} \Sigma^{+}$state.

It would be of much interest to examine spin-doubling in leve: $s$ above $v^{\prime \prime}=6$ since the levels of the $A$ and $X$ states overlap and large varying spin-doubling would be expected.

The observed $v^{\prime \prime}$-dependence within the individual $\mathrm{v}^{\prime}$ progressions from the present analysis is not reproduced in the work of Mahieu et al. (32), as it can be seen from the comparison made between the first and the last column of Table 3.2 for the $v^{\prime}=0,1$ and 2 progressions. It is clear though that reasonable agreement exists in terms of the absolute ( $\gamma^{\prime}-\gamma^{\prime \prime}$ ) values.

> As a conclusion to this analysis, the effective rotational and vibrational constants were used to calculate the RKR turning points for each state (Table 3.7). These results were obtained using the method described in section 2.6; the data used were the term values of Table 3.6 and the rotational constants ( $B_{v}$ ) of Table 3.3. The reduced mass of 10.041951 amu was based on the atomic masses of ${ }^{16} \mathrm{O}$ and ${ }^{27} \mathrm{Al}$. The RKR curves have been plotted in Fig. 3.6.

| Vierge (D) |  |  |  | Niarge (E) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{T}_{\mathrm{v}}\left(\mathrm{B}^{2} \mathrm{z}^{+}\right)$ | $G_{v}\left(x^{2} \Sigma^{+}\right)$ |  | $B^{2} \Sigma^{+}$ | $x^{2} x^{+}$ |
| 9 | 28133.989(55) |  | $T_{0}$ | 20688.726(49) | 0.0 |
| 8 | 27334.618(45) |  | $\omega_{0}$ | 870 535(32) | 975.256(58) |
| 7 | 26522.831(37) |  | $\omega_{e} x_{e}$ | 3.7206(131) | $6.9182(102)$ |
| 6 | 25703.715(32) | 5581.729(37) | $10^{2} \omega_{0} y_{e}$ | 0.870(207) | -1.93(64) |
| 5 | 24877.260(30) | 4686.570(35) | $10^{3} \omega_{s} z_{e}$ | -0.443(1.09) | 1.10(46) |
| 4 | 24043.508(30) | 3777.418(31) |  |  |  |
| 3 | 23202.452(30) | 2854.120(27) |  |  |  |
| 2 | 22354.034(24) | 1916.762(26) |  |  |  |
| 1 | 21498.278(25) | 965.399(23) |  |  |  |
| 0 | 20635.181(26) | 0.000 |  |  |  | for the $\mathrm{X}^{2} \mathrm{\Sigma}^{+}$and $\mathrm{B}^{2} \mathrm{~S}^{+}$States of Alo.

Kierge (D)
$G_{v}\left(x^{2} \Sigma^{+}\right)$
$\mathrm{B}^{2} \Sigma^{+} \quad \mathrm{x}^{2} \Sigma^{+}$

Figure 3.5: Plots of ( $\gamma_{v}-\gamma_{\theta}^{\prime \prime}$ ) for the $\mathrm{B}^{2} \Sigma^{+}$and $\mathrm{X}^{2} \Sigma^{+}$states of Alo. The points shown for the $B$ state and the smooth curve for X state were obtained from Merge B (Table 3.3). The straight line for the B state is from the fit of Merge C (Table 3.4). Error bars are two standard deviations.


Table 3.7: RKR Turning Points (A) for the $X^{2} \Sigma^{+}$and $B^{2} \Sigma_{1}^{+}$ States of $\mathrm{AlO}^{\mathrm{a}}$

|  | $B^{2} \Sigma^{+}$ |  |  | $\mathrm{X}^{2} \Sigma^{+}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| v | $\mathrm{G}_{\mathrm{v}}$ | $r_{\text {min }}$ | $\mathrm{r}_{\text {max }}$ | $\mathrm{G}_{\mathrm{v}}$ | $r_{\text {min }}$ | $r_{\text {max }}$ |
| 0 | 434.592 | 1.60780 | 1.73227 | 487.998 | 1.56251 | 1.67995 |
| 1 | 1297.689 | 1.56850 | 1.78480 | 1453.397 | 1.52596 | 1.73062 |
| 2 | 2153.445 | 1.54306 | 1.82330 | 2404.760 | 1.50257 | 1.76 70 |
| 3 | 3001.863 | 1.52337 | 1.85614 | 3342.118 | 1.48452 | 1.80102 |
| 4 | 3842.919 | 1.50701 | 1.88570 | 4265.416 | 1.46956 | 1.83071 |
| 5 | 4676.671 | 1.49289 | 1.91307 | 5174.568 | 1.45672 | 1.85855 |
| 6 | 5503.126 | 1.48043 | 1.93889 | 6069.727 | 1.44542 | 1.88507 |
| 7 | 6322.242 | 1.46915 | 1.96344 |  |  |  |
| 8 | 7134.029 | 1.45892 | 1.98709 |  |  |  |
| 9 | 7938.400 | 1.44942 | 2.00992 |  |  |  |

${ }^{a} Y_{o o}(B)=0.2558 \mathrm{~cm}^{-1} ; \quad Y_{o o}(X)=-0.0113 \mathrm{~cm}^{-1}$.

Figure 3.6: RKR potential curves for the $\mathrm{B}^{2} \Sigma^{+}$and $\mathrm{X}^{2} \Sigma^{+}$ states of aluminum monoxide; vibrational energy levels are shown.


## Chapter 4

## Excitation of Spectra by Species of a Helium Discharge and

## Details of the Flowing Afterglow System.

For the second part of this work a fast-flow system was constructed in which a discharge could be excited in helium with the long-lived energetic species then being removed from the active discharge and later mixed downstream with a second gas. Collins and Robertson (39) have experimentally established that in a similar afterglow system the helium active species present were $\mathrm{He}\left(2^{3} \mathrm{~S}\right)$, $\mathrm{He}_{2}\left(\mathrm{a}^{3} \Sigma_{u}^{+}\right)$, the molecular ion ( $\mathrm{He}_{2}^{+}$) and the atomic ion $\mathrm{He}^{+}$.

Interaction of these active helium species with various substrate gas molecules is responsible for the presently observed luminescence.

In the following chapters spectral analysis of these luminescence reactions gives direct information on the electronic states of the products. The mechanism of the reactions leading to the excited products is beyond the purpose of the present thesis. However, energy conservation requirements together with the Franck resonance rule (40), are generally useful in suggesting reactions and the reasons for predominance of certain reactions, established by other considerations, but they are not specific in themselves. The maximum available energy from the active helium species
are given in Ref. (39) as, $\mathrm{He}^{+}(24.58 \mathrm{eV}), \mathrm{He}\left(2^{3} \mathrm{~S} 19.82 \mathrm{eV}\right),{ }^{99}$ $\mathrm{He}_{2}^{+}(18.80-21.80 \mathrm{eV})$ and $\mathrm{He}_{2}\left(\mathrm{a}^{3} \Sigma_{\mathrm{u}}^{+} 14.60-17.40 \mathrm{eV}\right)$.

Three independent investigations $(39,41,42)$ on the intensity of various band systems observed from flowing afterglow systems suggest that the intensity is strongly dependent upon the helium pressure. Three characteristic classes of reactions may be distinguished, namely those that show an intensity peak at $\sim 1$ Torr, $\sim 2$ Torr and $\sim 3$ Torr, with metastable $\mathrm{He}\left(2^{3} \mathrm{~S}\right), \mathrm{He}^{+}$and $\mathrm{He}_{2}^{+}$being dominant active species, respectively. In the following paragraphs the flowing afterglow apparatus used in the present work will be discussed in detail together with the dispersion, the detection and the data acquisjtion systems. More experimental details of the reactions of discharged helium with $\mathrm{BI}_{3}, \mathrm{PCl}_{3}, \mathrm{ASCl}_{3}$ and $\mathrm{SeOCl}_{2}$ will be given in the final part of this chapter.

The experimental arrangement, shown in Figure 4.1, was similar to that used for production of the $A^{2} \Pi_{1}$ state of $\mathrm{SO}^{+}$(43). Prepurified helium (Linde 99.996\%) was purified further by passage through two traps containing molecular sieve (Davison, 4-8 mesh; 0.5 nm pore size), maintained in liquid nitrogen baths. The helium was passed through a dcdischarge maintained between tantalum foil electrodes 4 cm apart. A ballast resistor ( $18 \mathrm{k} \Omega$ ) was included in series with the power supply to reduce power fluctuations. Typical current flows were 15 mA for a supply voltage of $\sim 600 \mathrm{~V}$.

Figure 4.1: The flowing afterglow apparatus.


Small flows of various substrate molecules were admitted from a resistively heated side tube positioned a few centimeters downstream of the discharge. The system was pumped by a $170 \mathrm{~m}^{3} \mathrm{~h}^{-1}$ pump (Leybold Heraeus E 150) at total pressures between 1.5 and 4 Torr. The emissions could be observed visually.

Spectra were recorded in the first-order of $1.26-\mathrm{m}$ scanning monochromator (Spex 1269) fitted with a 2400-groove $\mathrm{mm}^{-1}$ grating (Bausch and Lomb) blazed at $5000 \AA$. The slit width was $200 \mu \mathrm{~m}$, corresponding to a spectral width of 0.6 A. Signals were detected by a GaAs photomultiplier (RCA C31014A) maintained at $-20^{\circ} \mathrm{C}$ in a RF-shielded, water-cooled housing (Products for Research TE-192-RF); the dark current was $\sim 8$ counts $s^{-1}$. The photomultiplier signal was processed with an SSR 1120 Amplifier/Discriminator and a PAR 1105 photon counter, the count rate being displayed on a strip chart recorder (BBC Goerz, Servogor 210).

In the following sections, the automation of the spectrometer and the data acquisition system, which were employed for higher resolution analysis of the spectra of BI and $\mathrm{PCl}^{+}$, will be discussed.

The monochromator was operated under full computer control (Digital MINC MNC11-AA, PDP 11/23). The computer is equipped with two 16-bit parallel digital input modules and a 16-bit parallel digical output module which are used to receive data from and send instructions to an interface (44) which contains logic circuits for operations.

The spectrometer is scanned in a stepwise fashion. Depending on the desired resolution interval, a train of pulses is supplied to the spectrometer stepper motor. Signals from the photomultiplier are counted by the interface for a given time interval. After the signal count has been passed to the computer, the above procedure is repeated enabling a spectrum to be recorded digitally as a set of measured counts at a large number of equally spaced wavelengths. In the present work, the sesolution interval was $0.02 \AA$ in accord with the spectral slit width of $\sim 0.1 \AA$ which corresponds to a mechanical slit width of $20 \mu \mathrm{~m}$. With counting times $\sim 4 \mathrm{~s}$, the total count for the more intense lines was typically a few hundred counts, giving good signal-to-noise ratio for all the bands analyzed presently.

Spectra were calibrated by reference to emission lines of either $\mathrm{Fe} / \mathrm{Ne}$ or U/Ar hollow cathode discharge lamps operated at $\sim 15 \mathrm{~mA}$. The calibration lines were recorded simultareousiy with the recorded spectra using the electronic shutter described in detail in Ref. (44). Several calibration lines were obtained both before and after, as well as within, the band being recorded, and were spaced as evenly as possible. Measured (drum) calibration line positions ( $\lambda_{d}$ ) were fitted by a nonlinear least-squares routine according to

$$
\begin{equation*}
\Delta \lambda(A)=\sum_{i=0}^{2} a_{i} \lambda_{d}^{i}+\alpha \sin \left[2 \pi\left(\lambda_{d}-\delta\right) / 50\right] \tag{4.1}
\end{equation*}
$$

where $\Delta \lambda$ are the residuals $\left(\lambda_{d}-\lambda_{t}\right)$ between the measured ( $\lambda_{d}$ )
and true wavelengths $\left(\lambda_{t}\right)$. Eq. (4.1) takes account of a small periodic ( 50 A ) error in the grating rotation lead screw.

The parameters $\alpha$ and $\delta$ were fitted for each calibration along with the set of $a_{1}$. The spectra were displayed on a graphics terminal (Digital VT105) and measured manually. A computer program was then employed to obtain vacuum wavenumbers from the calibration parameters. The rotationally analyzed bands of BI and $\mathrm{PCl}^{+}$and portions of the low resolution spectra of $\mathrm{PCl}^{+}$and $\mathrm{AsCl}^{+}$were drawn with a digital plotter (Tektronics 4662) through the interface with the MINC computer.

## Boron Iodide

The $a^{3} \Pi_{0,1}$ state of $B I$ was populated by the reaction of discharged helium with boron triiodide (Alpha products 95\%). Small flows of $\mathrm{BI}_{3}$ were admitted from the resistively heated side tube and the emission could be observed as a red diffuse flame. The helium pressure was kept at 1.5 Torr in order to maximize the intensity of the band systems.

The wavelengths of neon atomic emission lines, which were recorded simultaneously, from a light pen source (Oriel) were used to calibrate the low-resolution spectrum. These neon lines were fitted by least-squares to a quadratic in "d", the distance along the trace (38). The standard deviation of this fit was 0.32 A . The band head positions were determined at half-maximum intensity above the baseline, and are accurate to -1 A.

For the higher resolution analysis the spectra were calibrated by reference to emission lines of either $\mathrm{Fe} / \mathrm{Ne}$ or U/Ar hollow cathode lamps. The standord deviations of the calibration lines fits were typically 0.01 A. Phosphorous chloride cation

Electronicaliy excited $\mathrm{PCl}^{+}$was produced by the reaction of $\mathrm{PCl}_{3}$ vapour with discharged helium at total pressures near 4 Torr. The $\mathrm{PCl}_{3}$ flow was introduced into the reaction cell from the liquid (BDH chemicals, 98\%) contained in a side tube 5 cm downstream fitted with a Teflon needle valve to maximize the brightness of the conical bluish flame. Prepurified helium (Linde, 99.996\%) was purified further by passing the gas through the two traps containing molecular sieve cooled by liquid nitrogen.

It was found that the pore size and the physical state of the molecular sieve are more critical factors for obtaining $\mathrm{PCl}^{+}$and $\mathrm{AsCl}^{+}$emission spectrum than any other emission spectra studied. Experimental trials with eitier different pore size (: 4 nm ) or traps only partially free from impurities (i.e. water) did not sive the optimum intensity of the $\mathrm{PCl}^{+}$or $\mathrm{AsCl}^{+}$.

The calibration of the low resolution $\mathrm{PCl}^{+}$spectrum was obtained by emission lines of atomic Cl and He (45) which were fitted by least-squares to a linear function in "d". The standard deviation of this fit was 0.27 A . The $\mathrm{PCl}^{+}$, bandhead positions were determined at half maximum intensity above the baseline and have an estimated
uncertainty of $\sim 0.6 \AA$ (based on a 0.4 mm uncertainty in the distance along the trace).

For the higher resolution work on $\mathrm{PCl}^{+}$, a uranium hollow cathode lamp was used for wavelength calibration. Usually, 25-30 uranium lines were fitted for each sub-band with a standard deviation of about 0.008 A.

## Arsenic chloride cation

Electronically excited AsCl ${ }^{+}$was produced by the reaction of $\mathrm{AsCl}_{3}$ vapour with discharged helium at total pressures approximately 4 torr. $\mathrm{AsCl}_{3}$ ( BDH chemicals, 99\%) was kept in the side tube which was not heated since the room temperature vapour pressure of liquid $\mathrm{AsCl}_{3}$ was sufficient to maintain a constant flow of $\mathrm{AsCl}_{3}$ for the production of excited $\mathrm{AsCl}^{+}$.

The band head positions were measured manually and were determined with an estimated uncertainty of $\sim 0.6 \AA$. Emission lines of atomic $\mathrm{Cl}(45)$, He (45) and As (46) were fitted by least-squares to provide wavelength calibration of the spectrum. The quality of this fit was similar to the corresponding fit of $\mathrm{PCl}^{+}$.

Selenium oxide cation
$\mathrm{SeO}^{+}$was formed in the reaction of selenium dichloride oxide vapour with discharged helium at total pressures of $\sim 3$ torr. Liquid $\mathrm{SeOCl}_{2}$ (Alpha, 97\%) was heated ( $\sim 50^{\circ} \mathrm{C}$ ) in order to maintain a constant flow of $\mathrm{SeOCl}_{2}$. The band head positions were obtained with an internal calibration provided by emission lines of atomic helium and selenium

## $(45,46)$. Estimated uncertainties in the bandhead

## Chapter 5

## Vibrational Analysis of the $\mathbf{a} \rightarrow \mathrm{X}$ System of the BI

### 5.1 Introduction

In the present chapter, an investigation of the reaction of $\mathrm{BI}_{3}$ with discharged helium is described. It was found that the reaction was chemiluminescent, the emission spectrum consisting of systems of violet degraded bands in the 5400-6800 \& region. These bands have been assigned to the $a^{3} \Pi_{0}^{+} \rightarrow X^{1} \Sigma^{+}$and $a^{3} \Pi_{1} \rightarrow X^{1} \Sigma^{+}$systems of $B I$. The $a^{3} \Pi_{0,1}^{+} \rightarrow X^{1} \Sigma^{+}$ Cameron bands of the boron monohalides have been studied principally by Lebreton and coworkers (47-51); the spectra were excited using a Schuler-type discharge. These investigations comprise rotational analyses of the 0-0 band of the $a^{3} I I-X$ system of $B F(47)$, the $0-0$ and $1-1$ bands of the $a^{3} \Pi_{1}-X$ system of BCl (48) and of selected bands of the many bands identified in the spectra of $\operatorname{BBr}(49,50)$ and $B I(51)$. In the case of BBr , rotational constants were derived from an analysis (50) of three bands in the $\Delta v=0$ sequence ( $0-0$, 1-1 and 2-2). The work of Lebreton et al. (51) on BI is the first observation of this molecule and the bands were obtained from a discharge of a mixture of argon and $\mathrm{BI}_{3}$ vapour. Twelve head positions ( $v^{\prime} \leq 9, v^{\prime \prime} \leq 7$ ) of the $a^{3} \Pi_{0}^{*} \rightarrow$ $X^{1} \Sigma^{+}$system were reported in the range $5680-6210 \AA$, together
with a rotational analysis of the $0-0$ and $1-0$ bands.
However, the observation of additional bands was precluded by strong interfering emission due to $I_{2}$. In addition, Lebreton et al. (51) tentatively assigned two bands in the ultraviolet region, obtained by flash photolysis, as due to the $A^{1} \Pi+X^{1} \Sigma^{+}$system of $B I$. Briggs and Piercy (52) observed the same UV bands near 3490 \& but they also observed several other bands in the range 2660-2790 A which were assigned to the $\Delta v=-1,0,1$ sequences of $B I\left(A^{1} \Pi \leftarrow^{1} \Sigma^{+}\right)$. However, the latter bands should be attributed to BCl from $\mathrm{BCl}_{3}$ impurity, as discussed by Lebreton et al. (51) and Huber and Herzberg (53). In the more recent work by Briggs and Simmons (54), there was no discussion of the possibility that $\mathrm{BCl}_{3}$ impurity was responsible for their incorrect interpretation (52) of the flash photolysis experiments. Finally, Bredohl has very recently reported (55) a complete analysis of the $0-0$ band of the $a^{3} \Pi_{r}-{ }^{1} \Sigma^{+}$system of BCl. The spectrum was well-developed and the spin-orbit constant of the $a^{3} I I$ state could be determined. In the present study, on BI, most of the bands observed by Lebreton et al. (51) in the $\Delta V=1$ sequence of the $a^{3} \Pi_{0}-X^{1} \Sigma^{+}$system were also obtained. In addition, the $\Delta v=2$ and 3 sequences were recorded together with several bands of the less abundant ${ }^{10} \mathrm{BI}$ isotope. The vibrational parameters of the $\mathrm{a}^{3} \Pi_{0}-\mathrm{X}^{1} \Sigma^{+}$system were determined with greater reliability and differed significantly from the previous work. Another contribution of the present analysis was the observation of
the $a^{3} \Pi_{1}-X^{1} \Sigma^{+}$system of $B I$ which was completely absent or ${ }^{1}$ overlapped in Lebreton's work (51). Fifteen bands with clearly resolved $P$ and $Q$ heads have been assigned and the molecular parameters of the $a^{3} \Pi_{1}$ state of $B I$ were well determined.

It should be mentioned also that the presently used source of excitation demonstrates an efficient method of obtaining weak emission spectra without interferences from strong extensive systems that can result when a substrate molecule $1 s$ introduced into a microwave discharge cavity.

### 5.2 Results and Discussion

Part of the visible emission spectrum of BI is shown in Figure 5.1. Sequences of violet-degraded bands are observed with the strongest bands near 6000 A. Assignment of bands to the $a^{3} \Pi_{0}^{+}-X^{1} \Sigma^{+}$system was straightforward since some bands have been identified previously (51). Twenty-three bands were assigned to the ${ }^{3} \Pi_{0}^{+}-{ }^{1} \Sigma^{+}$system and are shown in a Deslandres array in Table 5.1. For two sequences ( $\Delta v=1$ and 2), additional heads were observed with lower intensity, and are due to corresponding transitions of the less abundant ( $\sim 20 \%$ ) ${ }^{10}$ BI isotope; Table 5.2 shows the ten heads which were assigned to the $\mathrm{a}^{3} \Pi_{0}^{+}-{ }^{1} \Sigma^{+}$system of ${ }^{10} \mathrm{BI}$. After the assignments of the $a\left(0^{+}\right) \rightarrow X$ system had been made, three sequences remained that did not belong to this system. These bands have been assigned to the $a(1) \rightarrow X$ system of
${ }^{11}$ BI. The validity of this assignment was justified by two principal arguments; i) when the bandheads were arranged in a Deslandres table, Table 5.3, the lower state vibrational intervals were similar to those of the $a\left(0^{+}\right) \rightarrow$ system of ${ }^{11} \mathrm{BI}$; and ii) all the observed bands in the three sequences were double-headed, as expected for a Hund's case (c) $\Delta \Omega=1$ transition for which $\Delta J=0, \pm 1$. (A more detailed description of case (c) coupling case is presented in the following chapter.) The two heads are formed by the $P$ and $Q$ branches, with the $P$ head $\sim 7 \mathrm{~cm}^{-1}$ lower in energy. For $\Delta \Omega=$ 0 , as in $a\left(0^{+}\right) \rightarrow X^{1} \Sigma^{+}, \Delta J=0$ is forbidden, and the corresponding bands of this system show only single heads. It can be noted also that the separations of $\sim 300 \mathrm{~cm}^{-1}$ between the $a\left(0^{+}\right) \rightarrow \mathrm{X}^{1} \Sigma^{+}$and $\mathrm{a}(1) \rightarrow \mathrm{X}^{1} \Sigma^{+}$sub-systems proride an approximate estimate for the spin-orbit coupling constant of the excited state. The corresponding separation of the two sub-systems in $\mathrm{BBr}(50)$ is $\sim 180 \mathrm{~cm}^{-1}$, significantly smaller, as expected.

| Table 5,1: |  | Deslandres table for the $\mathrm{a}^{3} \Pi_{0}^{+}-\mathrm{X}^{1} \mathrm{\Sigma}^{+}$system of ${ }^{11} \mathrm{Br}$ |  |  |  |  |  |  |  |  | 112 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| v' | $v^{\prime}=0$ |  | v"=1 |  | $v^{\prime \prime}=2$ |  | $v^{\prime \prime}=3$ | $v "=4$ |  | $v^{\prime \prime}=5$ |  | V"-5 |
| 0 | 16087.1 | 569.0 | 15518.1 |  |  |  |  |  |  |  |  |  |
|  | 641.0 |  | 641.6 |  |  |  |  |  |  |  |  |  |
| 1 | 15728.1 | 568,4 | 16159.7 | 562.7 | 15597.0 |  |  |  |  |  |  |  |
|  | 629.7 | 629.3 |  |  |  |  |  |  |  |  |  |  |
| 2 | 17357.8 | 568.8 | 16789.0 |  |  |  |  |  |  |  |  |  |
|  | 617.5 |  | 619.8 |  |  |  |  |  |  |  |  |  |
| 3 | 17975.3 | 566.5 | 17408.8 | 563.1 | 16845.7 |  |  |  |  |  |  |  |
|  |  |  | 606.5 |  | 608.8 |  |  |  |  |  |  |  |
| 4 |  |  | 18015.3 | 560.8 | 17454.5 | 558.2 | 16896.3 |  |  |  |  |  |
|  |  |  |  |  | 595.0 |  | 596.4 |  |  |  |  |  |
| 5 |  |  |  |  | 18049.5 | 556.8 | 17492.7549 .0 | 16943.7 |  |  |  |  |
|  |  |  |  |  |  |  | 587.3 | 585.9 |  |  |  |  |
| 6 |  |  |  |  |  |  | $18080.0 \leq 50.4$ | 17529.6 | 544.7 | 16984.9 |  |  |
|  |  |  |  |  |  |  |  | 574.0 |  | 573.0 |  |  |
| 7 |  |  |  |  |  |  |  | 18103.6 | 545.7 | 17557.9 |  |  |
|  |  |  |  |  |  |  |  |  |  | 561.0 |  |  |
| 8 |  |  |  |  |  |  |  |  |  | 18118.9 | 537.3 | 17581.6 |


| $v^{\prime}$ | $v^{\prime \prime}=0$ | $v "=1$ |  | $v "=2$ |  | $v^{\prime \prime}=3$ |  | $v^{\prime \prime}=4$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  | 15495.2 |  |  |  |  |  |  |
| 1 | 16758.5 |  |  | 15575.0 |  |  |  |  |
| 2 |  | 16821.3 |  |  |  |  |  |  |
|  |  | 644.2 |  |  |  |  |  |  |
| 3 |  | $17+65.5$ | 586.4 | 16879.1 |  |  |  |  |
|  |  |  |  | 631.5 |  |  |  |  |
| 4 |  |  |  | 17510.6 | 582.0 | 16928.6 |  |  |
|  |  |  |  |  |  | 625.1 |  |  |
| 5 |  |  |  |  |  | 17553.7 | 575.3 | 16978.4 |

Table 5.3: Deslandres Table for the $a^{3} \Pi_{1}-X^{1} \Sigma^{+}$system of ${ }^{11}{ }_{B I}$

a) For each band, the upper and lower entries denote the positions of the $Q$ and $P$ heads, respectively, heads flagged with an asterisk were overlapped by intense atomic lines.

# Figure 5.1: A portion of the BI emission spectrum from -5890 to ~6325 A. 



Since the present measurements refer to positions of band heads, rather than origins, it is appropriate to determine the reliability of the derived vibrational intervals in Tables 5.1, 5.2, and 5.3. This can be achieved by calculating head-toorigin separations for $P$ or $R$ branches from the equation (2),

$$
\begin{equation*}
\nu_{\text {head }}-\nu_{0}=-\frac{\left(B_{v}^{\prime}+B_{v}^{\prime}\right)^{2}}{4\left(B_{v}^{\prime}-B_{v}^{\prime}\right)} . \tag{5.1}
\end{equation*}
$$

If it is assumed that $\alpha_{0}^{\prime \prime}$ is similar to $\alpha_{0}^{!} \sim 0.004 \mathrm{~cm}^{-1}$ (51), the calculated head-to-origin spacings increase with $\Delta v=$ $v^{\prime}-v^{\prime \prime}$. For example, the spacings are $\sim 4 \mathrm{~cm}^{-1}$ for $\Delta v=-3$, $-5.4 \mathrm{~cm}^{-1}$ for $\Delta \mathrm{V}=0$, and $\sim 10 \mathrm{~cm}^{-1}$ for $\Delta \mathrm{V}=3$. Such separations are of the same order as that of the measurement precision of the present work. It can je expected, therefore, that the head measurements are quite suitable for deriving effective vibrational parameters that differ insignificantly from their true values. For the $a(1) \rightarrow X$ bands, the error introduced with the use of head data is negligible, because the $Q$-heads are available. It is interesting to note that the P - and Q -head separations, Table 5.3, are in accord with the calculated head-to-origin spacings for the $a\left(0^{+}\right) \rightarrow X$ system. The measured $P-$ and $Q$-head positions of the $a\left(0^{+}\right) \rightarrow X$ and $a(1) \rightarrow X$ systems respectively can be represented adequately by the difference between the two power series expansions in vibrational quantum number,
$\nu_{i v^{\prime} v^{\prime \prime}}=T_{0}+\sum_{k=0} Y Y_{k}\left(V^{\prime}+1 / 2\right)^{k}-\sum_{k=0} Y_{i k}^{M}\left(v^{\prime \prime}+1 / 2\right)^{k}$,
where $i$ represents a particular isotope. The $\mathrm{X}_{1 \mathrm{k}}$ coefficients can be related to isotopically invariant parameters, $\mathrm{U}_{\mathrm{k}}$ (56), by,

$$
\begin{equation*}
Y_{i k}=\mu_{i}^{(-k / 2)} U_{k}, \tag{5.3}
\end{equation*}
$$

where $\mu_{i}$ is the reduced mass of the isotopic molecule i. The expression (5.2) is valid strictly only for sub-band origins; however, from the earlier discussion, it was not expected, nor found, that such a model introduces any systematic trend in the residuals between the fitted and observed band positions. Instead, the head-to-origin shifts are accommodated by the determination of effective $T_{e}$ values of the two excited states $\left({ }^{3} \Pi_{0}^{+},{ }^{3} \Pi_{1}\right)$. It was found that the accuracy of the measurements and the vibrational extension of the heads permitted the evaluation of only the first two effective isotopically invariant vibrational parameters ( $k=1,2$ ). The six parameters ( $U_{10}$ and $U_{20}$ for the ${ }^{1} \Sigma^{+},{ }^{3} \Pi_{0}^{+}$, and ${ }^{3} \Pi_{1}$ electronic states) have been derived using a simultaneous least-squares fit to the model given by equations 5.2, 5.3, and are listed in Table 5.4. The standard deviation of the fit was $\hat{\sigma}=1.6 \mathrm{~cm}^{-1}$ with 40 degrees of freedom, and is in accord with estimated uncertainties in the data. In Table 5.5, the equilibrium term values, $T_{0}$, and $\omega_{0}$, $\omega_{\mathrm{e}} \mathrm{X}_{\mathrm{a}},\left(\mathrm{U}_{10} /\left(\mu_{\mathrm{i}}\right)^{1 / 2},-\mathrm{U}_{20} / \mu_{1}\right)$ are reported and compared with the corresponding constants for the $\mathrm{X}, \mathrm{a}\left(\mathrm{O}^{+}\right)$and $\mathrm{a}(1)$ states of BCl and BBr . From Table 5.5, it is immediately obvious that, i) the $a(1)$ states lie at higher energy than the $a\left(0^{+}\right)$states and,
ii) the similar magnitudes of the vibrational parameters of the $\mathrm{a}(1)$ and $\mathrm{a}(0)$ electronic states of BBr and BCl suggest that both states correlate with the same electronic configuration. In accord with Barrow's discussion (57) on the low-lying states of the Group 3B halides, the observed energy ordering of the excited states of BBr and BI can be correlated only with the lowest excited configuration, $(z \sigma)^{2}(y \sigma)^{2}(w \pi)^{4}(x \sigma)(v \pi),{ }^{1} \Pi,{ }^{3} \Pi_{r}$. If a bonding w $m$ electron is promoted to the $x \sigma$ orbital, the resulting state will be ${ }^{3} \Pi_{i}$. Furthermore, Barrow argued that the $a^{3} \Pi_{r}$ excited states of the boron monohalides are correlated with $B\left({ }^{2} P_{3 / 2}\right)$ and $X\left({ }^{2} P_{3 / 2}\right)$ atomic states.

Table 5.4: Isotopically Invariant Dunham Coefficients ${ }^{\text {a }}$ ( $\mathrm{cm}^{-1}$ ) for the $X^{1} \Sigma^{+}$and $a^{3} I I$ States of $B I$

|  | $U_{10}$ | $U_{20}$ |
| :--- | :--- | :--- |
| $\mathrm{X}^{1} \Sigma^{+}$ | $1830.4(21)$ | $-30.1(12)$ |
| $\mathrm{a}^{3} \Pi_{0}+$ | $2076.3(17)$ | $-56.0(7)$ |
| $\mathrm{a}^{3} \mathrm{H}_{1}$ | $2050.5(29)$ | $-55.4(13)$ |

a) Values in parentheses are estimated standard errors in units of the last digit of the corresponding parameters. These errors do not include contributions arising from the use of head data, rather than origin data; see text.

a) Values in parentheses are estimated standard errors in units of the last aigit of the corresponding constant. Reduced masses of BI , calculated from the atomic masses of Ref. (58), are ${ }^{10}{ }_{\text {BI }} 9.2806796$, ${ }^{11}$ BI 10.130461 amu.
b) $\mathrm{T}_{0}$ values
c) $A=49.541(22)$

It is quite clear that the coupling of the angular momenta changes from Hund's case (b) in BF to an intermediate case (a)-case (c) in the $\mathrm{BCl}, \mathrm{BBr}$ and BI molecules. With increasing mass, case (c) coupling becomes predominant; as a consequence, the total electronic angular momentum about the internuclear axis is well-defined, the $\Delta S=0$ selection rule holds less and less rigorously, and larger transition moments for the ${ }^{3} \Pi\left(0^{+}, 1\right) \rightarrow X$ transitions are expected. In addition, the case (c) selection rule $\Delta \Omega=0, \pm 1$ forbids the ${ }^{3} \Pi(2) \rightarrow X$ transition, which is observed in BF (47).

In conclusion, some consideration should be given to the difference in intensity between the two sub-systems of the Cameron bands for the $\mathrm{BCl}, \mathrm{BBr}$ and BI molecules. The coupling of electronic states which are derived from the same configurations, presently $a^{3} \Pi$ and $A^{1} \Pi$, are of greatest importance in understanding the strong spin-orbit mixing toward the case (c) limit. As the selection rule for the spin-orbit operator is always $\Delta \Omega=0$, only the states having $\Omega=1$ can interact. Thus, non-zero spin-orbit matrix elements will occur between the $a(1)$ and $A^{1} \Pi_{1}$ states.

The factors affecting the intensity of forbidden transitions in diatomic molecules are discussed in the work of James (59). He assumed that the $a \rightarrow X$ transition of the Cameron system is allowed by the spin-orbit perturbation which mixes the ${ }^{1} \Pi_{1}$ state with ${ }^{3} \Pi_{1}$. Perturbation theory was followed to derive the first-order correction to the zeroth-
order wavefunctions of the $a(1)$ state. The corrected wavefunction was employed to determine the transition moment matrix element between the $\mathrm{a}(1)$ and $\mathrm{X}^{1} \Sigma^{+}$states.

The approximate final expression for the effective electronic transition moment, the square of which is proportional to the band emission intensity, is given by

$$
\begin{equation*}
\left(R_{e}\right)_{a, X}=A\left(R_{\mathrm{e}}\right)_{A, X} \Delta E^{-1} \tag{5.4}
\end{equation*}
$$

where $\left(R_{e}\right)_{a, X}$ and $\left(R_{e}\right)_{A, X}$ are the electronic transition moments between the forbidden and allowed transitions respectively, $A$ is the spin-orbit constant of the $a^{3} I I$ state and $\Delta E$ is the energy difference between the $A{ }^{1} \Pi_{1}$ and $a(1)$ states. Using Eq. (5.4) with the data of Table 5.5 , the spin-orbit mixing of the $a(1)$ and $A^{1} \Pi$ states becomes more significant with increasing mass, so that the transition probability of the $\mathrm{a}(1) \rightarrow \mathrm{X}$ system relative to the $\mathrm{a}\left(0^{+}\right) \rightarrow \mathrm{X}$ system is expected to be largest in BI. However, the $a\left(0^{+}\right) \rightarrow X$ bands of $B I$ have approximately twice the intensity of the $a(1) \rightarrow X$ bands. It can be concluded, therefore, that the reaction o. discharged helium with $\mathrm{BI}_{3}$ favours population of the lowerlying a( $\left.0^{+}\right)$state.

## Chapter 6

## Rotational Analysis of the $\mathbf{a} \rightarrow \mathbf{X}$

 System of ${ }^{11} \mathrm{BI}$
### 6.1 Introduction

Although the present technique for excitation of emission from BI is relatively clean, with only few atomic lines as significant interference, the intensity of the observed bands is weak compared to a discharge source. However, the present work has established that spectra recorded with a scanning spectrometer and photoelectric detection show sufficient resolution for a useful rotational analysis. For the $a\left(0^{+}\right) \rightarrow X$ system of $B I$, the analysis has been extended considerably to eight bands (2-0, 3-1, 4-2, 1-0, 2-1, 3-2, 0-1, 0-0) with better rotational development than was possible in the work of Lebreton et al. (51).

In addition, two bands $(0-0,0-1)$ of the $a(1) \rightarrow X$ system have been analyzed, and rotational constants for the a(1) state are obtained for the first time.

For the eight bands of the $a(0) \rightarrow X^{1} \Sigma^{+}$system, the measured line positions have been reduced by direct least squares fitting to four parameters ( $\nu_{0}, B_{v}^{\prime}, D_{v}^{\prime}, B_{v}^{\prime \prime}$ ). For the two bands of the $a(1) \rightarrow X^{1} \Sigma^{+}$system, five parameters have been determined $\left(\nu_{0}, B_{v}^{\prime}, D_{v}^{\prime}, B_{v}^{\prime \prime}\right.$ and the $\Omega$-doubling parameter (q) of the excited state). Multiple determinations of the parameters are reduced by the method
of merging to optimum single-valued estimates, as well as Dunham coefficients describing their vibrational dependences. The optical data of the present study are not sufficiently precise and extended to provide reliable estimates of absolute $\mathrm{D}_{\mathrm{v}}^{\prime}, \mathrm{D}_{\mathrm{v}}$ values simultaneously. Accordingly, values of $\mathrm{D}_{\mathrm{v}}$ calculated by Hutson's method (60) from a preliminary RKR potential were used as a constraint in the direct least-squares fitting.

### 6.2 Nature of the Transition, Analysis and Data Fitting

As indicated in Chapter 5, the excited $a\left(0^{+}, 1\right)$ state of BI can be described by a coupling scheme close to Hund's case (c). The interaction between the orbital angular momentum and spin of the electrons ( $L$ and $S$ ) is stronger than the interaction with the internuclear axis of the molecule. In this case, $\Lambda$ and $\Sigma$ (the projections of $L$ and $S$ on the axis) are not defined; rather, $L$ and $s$ first form a resultant $J_{a}$ which is then coupled to the internuclear axis with a component $\Omega$. The electronic angular momentum $\Omega$ and the angular momentum of nuclear rotation ( $R$ ) then form the resultant total angular momentum J. Therefore, electronic states which are represented by case (c) cannot be classified as ${ }^{1} \Sigma,{ }^{1} \Pi{ }^{1}{ }^{1} \Delta \ldots$ states, but instead are simply called $0,1,2, \ldots$. , according to the value of $\Omega$.

Since the excited state of $B I$ is formally $a^{3} I I$ in Hund's case (a), three separate states with $\Omega$ values of 0,1 and 2
are obtained in case (c). However, since the ground state is ${ }^{1} \Sigma^{+}$, the case (c) selection rule $\Delta \Omega=0, \pm 1$ restricts the observed transitions to two independent systems, $a(1) \rightarrow X^{1} \Sigma^{+}$ and $\mathrm{a}\left(0^{+}\right) \rightarrow \mathrm{X}^{1} \Sigma^{+}$. In addition, the $\mathrm{a}(1)-\mathrm{X}$ individual bands show $P(\Delta J=-1), Q(\Delta J=0), R(\Delta J=1)$ structure with $P$ and $Q$ bandheads; while the $a\left(0^{+}\right)+X$ bands only show $P$ and $R$ structure with one $P$ bandhead in accord with the restriction that if both states have $\Omega=0$, no $Q$ branch occurs.

The criterion in the selection of BI bands for analysis was that they be sufficiently intense to yield good signal-to-noise. For this reason, no attempt was made to analyze bands of the less abundant ( $\sim 20 \%$ ) ${ }^{10} \mathrm{BI}$ isotope observed at low resolution.

The line frequencies $\left(\nu_{1}\right)$ of individual bands were fitted by least-squares to determine the parameters in the equation

$$
\begin{align*}
\nu_{i} & =\nu_{0}+\left\{B_{v k}^{\prime} J^{\prime}\left(J^{\prime}+1\right)-D_{v k}^{\prime}\left[J^{\prime}\left(J^{\prime}+1\right)\right]^{2}\right\} \\
& -\left\{B_{v}^{\prime \prime} J^{\prime \prime}\left(J^{\prime \prime}+1\right)-D_{v}^{\prime \prime}\left[J^{\prime \prime}\left(J^{\prime \prime}+1\right)\right]^{2}\right)+\varepsilon_{1}, \tag{6.1}
\end{align*}
$$

where $\nu_{0}$ is the band origin of the $v^{\prime}$ - $v^{\prime \prime}$ band, $B_{v}^{\prime \prime}, D_{v}^{\prime \prime}$ are the ground state rotational constants, $B_{v k}^{\prime}, D_{v k}^{\prime}, k=e$ or $f$ are the effective rotational constants for the two $\Omega$-doubled $e$ and $f$ components of the $a(1)$ state, and $\varepsilon_{i}$ are the unknown measurement errors. For the $a\left(0^{+}\right)$state, the rotational levels are nondegenerate and hence they can be represented by a single set of rotational constants ( $B_{v}^{\prime}, D_{v}^{\prime}$ ), as in the ground state. Furthermore, the sets of $e$ and $f$ levels are
each associated with different branches in the $\mathrm{a}(1) \rightarrow \mathrm{X}$ system; as indicated in the rotational level energy diagram in Figure 6.1, the $P$ and $R$ branches are associated with $e$ levels, and the $Q$ branch with $f$ levels.

In practice, Eq. (6.1) was modified by introduction of the $\Omega$-doubling parameter $q_{v}=B_{v e}-B_{v f}$, so that the constants employed in the computer fits to describe the excited state $a(1)$ were $B_{v f}, D_{v f}$ together with $q_{v}$. Higher-order centrifugal distortion terms were not required in the band-by-band fits. In fact, due to the high correlation of the centrifugal distortion constants, $D_{v}^{\prime}$ and $D_{v}^{\prime \prime}$, and the extension of the available data, $D_{v}^{\prime \prime}$ always constrained at the value $D_{0}^{\|}=$ $5.97 \times 10^{-7} \mathrm{~cm}^{-1}$ calculated using Hutson's method (60) with the $X^{1} \Sigma^{+}$potential energy curve. The $D_{v}^{\prime \prime}$ parameter did not show any significant variation for $v^{\prime \prime}=0,1,2$. In addition, the same parameter at the potential minimum, $D_{e}^{\prime \prime}$ as given by the Dunham relationship (2) $D_{g}^{\prime \prime}=4 \mathrm{~B}_{\mathrm{e}}^{3} / \omega_{\mathrm{e}}^{2}$, had the same value as $D_{0}$.

Firstly, bands belonging to the $a\left(0^{+}\right) \rightarrow X$ system were analyzed. The numbering of the lines in the $P$ and $R$ branches was by no means obvious. In the vicinity of the bandheads and the origins, the lines appear as closely spaced doublets composed of a $P(J)$ line with an $R(J)$ line of low-J or a $\mathrm{P}(\mathrm{J})$ line of high-J. These doublets were not resolved with the present resolution so that the $P(1)$ or $R(0)$ lines could not be identified directly. Although
intensity considerations of the rotational lines in the neighbourhood of the central gap (space between the $P(1)$ and $R(0)$ lines) reduced the possibilities of a correct absolute numbering to two or three different schemes, an unequivocal assignment could not be obtained. However, the problem was resolved as follows. Approximate estimates for $\mathrm{B}_{\mathrm{v}}$ were calculated from $B_{!}^{\prime \prime}$ and $\alpha_{0}^{\prime \prime}$ assuming that $\alpha_{a}^{\prime \prime}$ is similar to $\alpha_{a}^{\prime}$, $\sim 0.004 \mathrm{~cm}^{-1}$ (51).

The $r_{\mathrm{e}}^{\prime \prime}$ value that defines $\mathrm{B}_{\mathrm{e}}^{\prime \prime}$ was estimated by using Badger's rule (2), $\mu \omega_{\mathrm{e}}^{2}\left(\mathrm{r}_{\mathrm{e}}-\delta\right)^{3}=\mathrm{C}$. The parameters $\delta$ and C were calculated from the known $r_{\mathrm{g}}$ and $\omega_{\mathrm{e}}$ for BCl and BBr (53), and gave $r_{8}^{\prime \prime}=2.10 \AA$ for $B I$. This estimate is consistent with the estimate calculated from the covalent radii of $\mathrm{Cl}, \mathrm{Br}$ and I .

For the band-by-band fits of the $a\left(0^{+}\right) \rightarrow X$ system, the rotational constants of the $X$ state were constrained to values less than, close to, and greater than the estimated values. The three absolute trial numberings of the rotational lines led to fits with similar standard deviations. However, for only one case of the absolute numbering were the multiple estimates of the same molecular constant in agreement to within two standard errors. For the other numbering schemes, such multiple estimates differed by many standard errors (typically ~15 standard errors). Furthermoia, the $J$ numbering found by this approach is in agreement with that used by Lebreton et al.
(51) for the two bands common to both studies. It should be noted that the numbering which was given in Ref. (51) was not assumed at the outset of the present work since no discussion about its validity was presented, and it is not obvious from the published spectrum that an unequivocal absolute numbering had been properly considered. It has been mentioned already that the $P(J)$ and $R(J)$ lines near the origins of the analyzed bands of the $a\left(0^{+}\right) \rightarrow X$ system are blended with $P(J)$ lines of high $J$-values in the returning limb of this branch. In order to accommodate these blended lines into the least-squares fit and obtain the best possible results, it was decided to fit the blends of lines of similar intensity as the weighted means of the separate transitions; however, blended lines with an intensity ratio greater than four were fitted as single lines in the corresponding strong branch. For the $a\left(0^{+}\right)-X$ bands, the blends were $(P(J)+P(n-J-1))$ and $(R(J)+P(n+J))$. The range of $n$ is broad, from $n=27$ for $0-1$ band to $n=43$ for the $4-$ 2 band. The weights were taken as the corresponding squares of intensity factors for $a^{1} \Sigma-{ }^{1} \Sigma$ transition given in Ref. (2) by,

$$
\begin{equation*}
I \sim S_{J} \exp \left(-B_{v}^{\prime} J^{\prime}\left(J^{\prime}+1\right) / k T\right) \tag{6.2}
\end{equation*}
$$

where $T$ is the rotational temperature, $k=0.695 \mathrm{~cm}^{-1} \mathrm{~K}^{-1}$ is the Boltzmann constant, and $S_{J}$ is the line strength given by $S_{J}=J^{\prime}+1$ for the $R$ branch, and $S_{J}=J^{\prime}$ for the $P$ branch. From an intensity analysis of the 0-1 band recorded
under stable conditions, and using Eq. (6.2), it was found that the rotational temperature of BI was close to 300 K . In the 3-1 band, the blending of the PP and RP lines was not so close as in the other bands; therefore, the rotational lines in the region of the band origin appear to be broader and with low signal-to-noise ratio. For this reason, only lines which can be assigned as essentially unblended $R(J)$ were included in the fit.

The blending problem in the bands of the $a(1)-x$ system is similar but more complicated. Figure 6.2 shows that many of $R(J)$ lines are blended not only with $P(n+J)$ lines, as in the $0^{+}-\mathrm{X}$ bands, but also with much stronger $Q\left(n^{\prime}+J\right)$ lines. In this case, the weights were obtained from the intensity factors for $a^{1} I I-{ }^{1} \Sigma$ transition; the line strengths are then (2),

$$
\begin{aligned}
& S_{J}=0.25\left(J^{\prime}+1\right) \text { for the } R \text { branch, } \\
& S_{J}=0.25\left(2 J^{\prime}+1\right) \text { for the } Q \text { branch, }
\end{aligned}
$$

and $\quad S_{J}=0.25 \mathrm{~J}^{\prime}$ for the P branch.
The contributions of $P(n+J)$ lines were so much smaller than those of $Q\left(n^{\prime}+J\right)$ that they were neglected entirely. Thus for the $a(1)-X$ bands only blends of $P(J)+P(n, J-1)$ and $R(J)+Q\left(n^{\prime}+J\right)$ were considered.

Figure 6.1: An energy level diagram for the first lines of a ${ }^{1} I I-{ }^{1} \Sigma^{+}$transition. The $\Omega$-doubling of the $\Omega=1$ state has been much exaggerated.

Figure 6.1


Figure 6.2: Rotational structure of the $0-0$ band of the $\mathrm{a}(1) \rightarrow \mathrm{X}^{1} \Sigma^{+}$system of ${ }^{11} \mathrm{BI}$. The trace illustrates the extensive blending of the $R$ and Q branches. The intense emission near the $R(7)$ ine is an unidentified atomic line.


### 6.3 Results and Discussion

A summary of the final individual band least-squares fits is given in Table 6.1. Lines were excluded in all bands when the magnitudes of their residuals exceeded 2.5 standard deviations of the fits. The standard deviations of the fits in Table 6.1 range between 0.018 and $0.042 \mathrm{~cm}^{-1}$, and are consistent with the estimated measurement precision for single lines. The measured line positions of the ten individual bands in the $a \rightarrow X$ system of $B I$ thus provided $a$ set of 42 estimated parameters. Each band yielded four constants, $\nu_{0}, B_{v}^{\prime}, D_{v}^{\prime}, B_{v}^{\prime \prime}$, and for the $0-0$ and $0-1$ bands of the $a(1) \rightarrow X$ system, the $\Omega$-doubling parameter $q$ ! was obtained. The method of merging, discussed in Chapter 2, was employed subsequently to reduce the 42 parameters to a smaller set of parameters which takes account (in the leastsquares sense) of the entire data set simultaneously. Merging was performed in a stepwise fashion. In the first merge (A), account is taken of the fact that many of the 42 parameters are multiple estimates of the same molecular constants. Therefore, the 42 parameters were reduced to a new set of 25 estimated constants (ten origins, ten $B_{v}^{\prime}$ and $D_{v}^{\prime}$ values for $v^{\prime}=0-4$, three $B_{v}^{\prime \prime}$ for $v^{\prime \prime}=0-2$, $B_{0}^{\prime}$ for the a(1) state and the $\Omega$-doubling parameter $q_{0}^{1}$ ). The estimated variance of the merge is $\hat{\sigma}^{2}=(1.140)^{2}$, with $f=17$ degrees of freedom, and the results are given in Table 6.2.

A second step in the fit, merge (B), was performed to

Table 6.1: Individual Band Fits ${ }^{a}$ in the $a^{3} I I\left(0^{+}, 1\right)-X^{1} \Sigma^{+}$ System of ${ }^{11} B I$

|  | v' v' | $\nu_{0}\left(\mathrm{~cm}^{-1}\right)$ | n | $\hat{\sigma}(\mathrm{cm}-1)$ | rms/ $\hat{\sigma}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{+}-\mathrm{X}$ | 0-1 | 15525.316(16) | 38 | 0.029 | 1.00 |
|  | 0-0 | 16095.250(11) | 49 | 0.018 | 1.00 |
|  | 1-0 | 16735.902(13) | 36 | 0.032 | 1.16 |
|  | 2-1 | 16796.967(13) | 35 | 0.032 | 1.09 |
|  | 3-2 | 16853.512(16) | 33 | 0.031 | 1.07 |
|  | 2. - 0 | 17366.931(13) | 30 | 0.026 | 1.31 |
|  | 3-1 | 17418.063(19) | 13 | 0.032 | 0.91 |
|  | 4-2 | 17464.338(19) | 28 | 0.042 | 0.98 |
| $1-\mathrm{x}$ | 0-1 | 15820.780(13) | 52 | 0.029 | 1.00 |
|  | 0-0 | 16390.690(15) | 49 | 0.036 | 1.00 |

${ }^{a}$ For each band, $n$ is the number of fitted lines and $\hat{\sigma}$ is the standard deviation. The band origins ( $\nu_{0}$ ) and estimated standard errors were obtained from Merge (B) (see text). rms/ $\hat{\sigma}$ represents the ratio of the root mean square of the residuals for lines calculated with merged parameters to the estimated standard deviation.
obtain $\mathrm{B}_{*}$ and $\alpha_{\text {. }}$ for the $\mathrm{a}\left(0^{+}\right)$and X states, according to Eq. (6.3),

$$
\begin{equation*}
B_{v}=B_{0}-\alpha_{0}\left(v+\frac{1}{2}\right) \tag{6.3}
\end{equation*}
$$

The results of merge (B) are summarized in Tables 6.1 and 6.3, which list the 10 band origins and the coefficients defined by Eq. (6.3) respectively. The centrifugal distortion constant of the $a\left(0^{+}\right)$state did not show any significant vibrational dependence, so it was kept constant over the range of $v^{\prime}, 0 \leq v^{\prime} \leq 4$. The estimated variance $\left(\hat{\sigma}_{M}^{2}=(1.784)^{2}\right.$ with $\left.f=25\right)$ indicates the presence of only small systematic error in the data, assuming the model is adequate. However, in this case, the results is very acceptable compared with similar merges for other band systems. The output parameters in merge (B) reproduce the measured line positions very satisfactorily. This is shown in detail by the listing of line positions and residuals calculated from the parameters of Tables 6.1 and 6.3, and listed in Tables 6.4 and 6.5 , and in summary by the entries in the last column of Table 6.1.

Table 6.2: Merged Parameters ${ }^{\mathrm{a}}\left(\mathrm{cm}^{-1}\right)$ for the X and a States of ${ }^{11} \mathrm{BI}$

|  | v | $\mathrm{B}_{\mathrm{v}}$ | $10^{7} \mathrm{D}_{\mathrm{v}}$ | $10^{5} \mathrm{q}_{\mathrm{v}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{a}^{3} \Pi\left(0^{+}\right)$ | 4 | $0.37716(10)$ | $5.68(39)$ |  |
|  | 3 | $0.380549(66)$ | $5.72(39)$ |  |
|  | 2 | $0.384286(56)$ | $6.75(23)$ |  |
|  | 1 | $0.387769(72)$ | $6.22(42)$ |  |
|  | 0 | $0.391093(52)$ | $5.59(12)$ |  |
| $\mathrm{a}^{3} \Pi(1)$ | 0 | $0.390581(56)$ | b |  |
| $\mathrm{X}^{1} \Sigma^{+}$ | 2 | $0.359697(71)$ |  |  |
|  | 1 | $0.362350(55)$ |  |  |
|  | 0 | $0.365104(54)$ |  |  |

${ }^{\text {a }}$ Parameters are fitted values output from Merge (A) (see text). Values in parentheses are estimated standard errors given by $\hat{\sigma}_{M}\left(V_{1 i}\right)^{1 / 2}$ in units of the last significant figure of the corresponding constant.
${ }^{b}$ The merge assumes that $D!$ for $v '=0$ of $a^{3} \Pi(1)$ is the same as $D!$ for $v^{\prime}=0$ of $a^{3} I I\left(0^{+}\right)$.


Table 6.4: Line Positions ${ }^{2}\left(\mathrm{~cm}^{-1}\right)$ for the $\mathrm{a}^{3} \mathrm{H}\left(0^{+}\right)-\mathrm{X}^{1} \Sigma^{+}$System of ${ }^{11_{\mathrm{BI}}}$

|  | $0=0$ |  |  | $1 \cdot 0$ |  |  | 2-0 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | p* | Calc | Res | p* | Calc | Res | p* | Calc | Res |
| P(9) | 20 | 16090.558 | 0.036 |  |  |  |  |  |  |
| 8 | 21 | 16090.872 | -0.020 | 25 | 16731.267 | -0.048 | 31 | 17362.130 | -0.028 |
| 7 | 22 | 16091.239 | 0.003 | 26 | 16731.673 | -0.051 | 32 | 17362.591 | -0.021 |
| 6 | 23 | 16091.658 | 0.027 | 27 | 16732.124 | -0.043 | 33 | 17363.090 | -0.064 |
| 5 | 24 | 16092.130 | 0.029 | 28 | 16732.619 | 0.043 | 34 | 17363.627 | -0.064 |
| 4 | 25 | 16092.654 | 0.021 | 29 | 16733.157 | -0.093* | 35 | 17364.202 | -0.110* |
| 3 | 26 | 16093.231 | -0.002 | 30 | 16733.738 | -0.009 | 36 | 17364.813 | 0.000 |
| 2 | 27 | 16093.861 | -0.014 | 31 | 16734.360 | 0.077* | 37 | 17365.461 | 0.023 |
| 1 | 28 | 16094.545 | -0.000 | 32 | 16735.020 | -0.000 | 38 | 17356.144 | -0.123* |
|  | 29 | 16095.284 | -0.007 | 33 | 16735.712 | 0.044 | 39 | 17366.854 | -0.042 |
| R(0) | 30 | 16096.068 | 0.013 | 34 | 16736.479 |  | 40 | 17367.624 |  |
| 1 | 31 | 16096.901 | -0.007 | 35 | 16737.350 | 0.103* | 41 | 17368.463 | -0.031 |
| 2 | 32. | 16097.787 | 0.003 | 36 | 16738.239 | 0.060 | 42 | 17369.321 | -0.002 |
| 3 | 33 | 16098.724 | 0.002 | 37 | 16739.169 | -0.025 | 43 | 17370.212 | -0.078* |
| 4 | 34 | 16099.714 | -0.004 | 38 | 16740.141 | -0.007 | 44 | 17371.139 | -0.044 |
| 5 | 35 | 16100.756 | -0.004 | 39 | 16741.156 | 0.002 | 45 | 17372.102 | -0.049 |
| 6 | 36 | 16101.849 | 0.013 | 40 | 16742.213 | -0.031 |  | 17373.110 | -0.002 |
| 7 | 37 | 16102.995 | 0.011 | 41 | 16743.313 | -0.143* |  | 17374.144 | -0.075* |
| 8 | 38 | 16104.192 | 0.005 | 42 | 16744.457 | 0.039 |  | 17375.217 | 0.005 |
| 9 | 39 | 16105.442 | 0.001 |  | 16745.683 | -0.027 |  | 17376.328 | -0.099* |
| 10 | 40 | 16106.744 | 0.036 |  | 16746.909 | -0.096* |  | 17377.477 | -0.042 |
| 11 | 41 | 16108.097 | -0.014 |  | 16748.180 | -0.055 |  | 17378.664 | 0.026 |
| 12 |  | 16109.479 | 0.066* |  | 16749.495 | -0.031 |  | 17379.889 | 0.048 |
| 13 |  | 16110.937 | 0.013 |  | 16750.856 | -0.068 |  | 17381.151 | 0.009 |
| 14 |  | 16112.448 | 0.022 |  | 16752.261 | -0.025 |  | 17382.452 | 0.004 |
| 15 |  | 16114.010 | 0.012 |  | 16753.712 | -0.004 |  | 17383.791 | 0.034 |
| 16 |  | 16115.624 | -0.025 |  | 16755.207 | -0.030 |  | 17385.167 | 0.027 |
| 17 |  | 16117.290 | 0.009 |  | 16756.747 | -0.042 |  | 17386.581 | 0.005 |
| 18 |  | 16119.007 | -0.021 |  | 16758.332 |  |  | 17388.034 | -0.007 |
| 19 |  | 16120.777 | -0.022 |  | 16759.962 | -0.013 |  | 17389.524 | 0.002 |
| 20 |  | 16122.599 | -0.038 |  | 16761.637 | 0.038 |  | 17391.052 | 0.035 |
| 21 |  | 16124.472 | -0.012 |  | 16763.356 | 0.015 |  | 17392.618 | 0.018 |
| 22 |  | 16126.397 | -0.011 |  | 16765.121 | 0.011 |  | 17394.222 | 0.007 |
| 23 |  | 16128.374 | 0.014 |  | 16766.930 | 0.042 |  | 17395.863 | 0.090* |
| 24 |  | 16130.403 | -0.010 |  | 16768.784 | 0.017 |  | 17397.543 | 0.107* |
| 25 |  | 16132.483 | 0.020 |  | 16770.683 | 0.015 |  | 17399.260 | 0.057 |
| 26 |  | 16134.615 | -0.023 |  | 16772.627 | 0.022 |  | 17401.015 | -0.008 |
| 27 |  | 16136.800 | 0.011 |  | 16774.615 | 0.042 |  | 17402.808 | 0.060 |
| 28 |  | 16139.035 | 0.029 |  | 16776.649 | 0.030 |  | 17404.639 | 0.279* |
| 29 |  | 16141.323 | -0.024 |  | 16778.727 | 0.073 |  | 17406.507 | 0.036 |
| 30 |  | 16143.663 | 0.011 |  | 16780.849 | -0.026 |  | 17408.413 | -0.148* |
| 31 |  | 16146.054 | -0.025 |  | 16783.017 | -0.043 |  |  |  |
| 32 |  | 16148.497 | -0.011 |  | 16785.229 | 0.060 |  |  |  |


| 0-0 |  |  | 1-0 |  |  | 2-0 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| p* | Calc | Res | P* | Calc | Res | P* | Calc | Res |


| 33 | 16150.991 | -0.009 | 16787.486 |  |
| :--- | :--- | :---: | :--- | :--- |
| 34 | 16153.537 | -0.007 | 16789.788 | 0.022 |
| 35 | 16156.935 | -0.028 |  |  |
| 36 | 16158.785 | 0.017 |  |  |
| 37 | 16161.486 | $0.068^{*}$ |  |  |
| 38 | 16164.239 | 0.013 |  |  |
| 39 | 16167.044 | -0.016 |  |  |
| 40 | 16169.901 | 0.023 |  |  |

Table 6.4: Line Positions ${ }^{\left(\mathrm{cm}^{-1}\right)}$ for the $a^{3} \mathrm{n}\left(\mathrm{O}^{+}\right)=X^{1} \Sigma^{+}$System of ${ }^{11} \mathrm{BI}$ (cont ${ }^{\text {(d) }}$ )

| 0-1 |  |  | 2-1 |  |  | 3-1 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P* | Calc | Res | P* | Calc | Res | P* | Calc | Res |


| 9 |  |  |  | 25 | 16791.985 | 0.063 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 |  |  |  | 26 | 16792.357 | -0.021 |  |  |
| 7 | 19 | 15521.421 | -0.157* | 27 | 16792.772 | -0.011 |  |  |
| 6 | 20 | 15521.794 | -0.157* | 28 | 16793.231 | 0.056 |  |  |
| 5 | 21 | 15522.224 | -0.158* | 29 | 16793.733 | 0.001 |  |  |
| 4 | 22 | 15522.711 | -0.110* | 30 | 16794.278 | -0.020 |  |  |
| 3 | 23 | 15523.254 | -0.030 | 31 | 16794.866 | 0.006 |  |  |
| 2 | 24 | 15523.853 | -0.008 | 32 | 16795.495 | 0.047 |  |  |
| 1 | 25 | 15524.508 | -0.023 | 33 | 16796.164 | -0.019 |  |  |
|  | 26 | 15525.217 | 0.056 | 34 | 16796.869 | 0.168* |  |  |
| K(0) | 27 | 15526.000 | 0.085* | 35 | 16797.647 |  |  |  |
| 1 | 28 | 15526.848 | 0.013 | 36 | 16798.476 | 0.112* |  |  |
| 2 | 29 | 15527.753 | 0.017 | 37 | 16799.345 | 0.049 |  |  |
| 3 | 30 | 15528.715 | 0.011 | 38 | 16800.255 |  |  |  |
| 4 | 31 | 15529.735 | -0.007 | 39 | 16801.206 | 0.028 |  |  |
| 5 | 32 | 15530.811 | -0.021 | 40 | 16802.200 | -0.091* |  |  |
| 6 | 33 | 15531.945 | 0.032 | 41 | 16803.235 | -0.065 |  |  |
| 7 | 34 | 15533.135 | 0.048 | 42 | 16804.313 | -0.025 |  |  |
| 8 | 35 | 15534.383 | 0.040 | 43 | 16805.433 | -0.085* |  |  |
| 9 | 36 | 15535.687 | 0.001 |  | 16806.609 | -0.099* | 17427.321 | 0.010 |
| 10 | 37 | 15537.049 | -0.030 |  | 16807.813 | -0.064 | 17428.428 | 0.034 |
| 11 | 38 | 15538.468 | -0.021 |  | 16809.0\%0 | -0.048 | 17429.611 |  |
| 12 | 39 | 15539.943 | 0.037 |  | 16810.350 | -0.064 | 17430.810 |  |
| 13 |  | 15541.499 | 0.033 |  | 16811.683 | -0.023 | 17432.046 | -0.017 |
| 14 |  | 15543.086 | -0.008 |  | 16813.060 | 0.022 | 17433.318 | -0.032 |
| 15 |  | 15544.730 | 0.025 |  | 16814.481 | -0.013 | 17434.626 | -0.035 |
| 16 |  | 15546.431 | 0.012 |  | 16815.945 | 0.045 | 17435.971 | -0.002 |
| 17 |  | 15548.190 | 0.011 |  | 16817.452 | -0.065 | 17437.353 |  |
| 18 |  | 15550.006 | -0.009 |  | 16819.002 | -0.051 | 17438.770 |  |
| 19 |  | 15551.879 | -0.060 |  | 16820.596 | 0.034 | 17440.224 | 0.047 |
| 20 |  | 15553.809 | 0.025 |  | 16822.233 | 0.075* | 17441.715 |  |
| 21 |  | 15555.797 | -0.039 |  | 16823.913 | 0.006 | 17443.241 |  |
| 22 |  | 15557.842 | 0.027 |  | 16825.637 | 0.004 | 17444.804 | -0.016 |
| 23 |  | 15559.945 | -0.017 |  | 16827.404 | -0.004 | 17446.404 | -0.036 |
| 24 |  | 15562.104 | -0.027 |  | 16829.214 | 0.017 | 17448.039 | 0.034 |
| 25 |  | 15564.321 | 0.025 |  | 16831.068 | 0.022 | 17449.711 | -0.021 |
| 26 |  | 15566.595 | -0.025 |  | 16832.965 | 0.019 | 17451.419 |  |
| 27 |  | 15568.927 | -0.037 |  | 16834.905 | 0.027 | 17453.164 | -0.007 |
| 28 |  | 15571.315 | -0.020 |  | 16836.888 | 0.028 | 17454.944 | 0.039 |
| 29 |  | 15573.761 | 0.009 |  | 16838.915 | 0.030 |  |  |
| 30 |  | 15576.264 | -0.038 |  | 16840.985 | 0.010 |  |  |
| 31 |  | 15578.824 | 0.022 |  | 16843.098 | -0.007 |  |  |
| 32 |  | 15581.441 | -0.043 |  | 16845.254 | -0.007 |  |  |
| 33 |  | 15584.116 | 0.038 |  |  |  |  |  |
| 34 |  | 15586.847 | -0.002 |  |  |  |  |  |

Table 6.4: Line Positions ${ }^{a}\left(\mathrm{~cm}^{-1}\right)$ for the $a^{3} \mathrm{II}\left(0^{+}\right)-x^{1} \Sigma^{+}$System of ${ }^{11}$ BI (cont'd.)

|  | 3-2 |  |  | 4-2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | P* | Calc | Res | P* | Calc | Res |
| P(8) | 27 | 16848.914 | -0.026 |  |  |  |
| 7 | 28 | 16849.336 | -0.043 | 35 | 17460.073 | 0.023 |
| 6 | 29 | 16849.801 | -0.013 | 36 | 17460.586 | -0.008 |
| 5 | 30 | 16850. 308 | 0.007 | 37 | 17461.135 | 0.053 |
| 4 | 31 | 16850.856 | -0.000 | 38 | 17461.721 | 0.009 |
| 3 | 32 | 16851.446 | -0.097* | 39 | 17462.345 | 0.059 |
| 2 | 33 | 16852.077 | -0.053 | 40 | 17463.009 | -0.010 |
| 1 | 34 | 16852.748 | -0.029 | 41 | 17463.720 | -0.035 |
|  | 35 | 16853.457 | -0.096* | 42 | 17464.501 |  |
| R(0) | 36 | 16854.226 | 0.081* | 43 |  |  |
| 1 | 37 | 16855.041 | 0.007 | 44 | 17465.976 | -0.044 |
| 2 | 38 | 16855.895 | 0.097* | 45 | 17466.776 | 0.107* |
| 3 | 39 | 16856.789 | -0.053 | 46 | 17467.620 | 0.153* |
| 4 | 40 | 16857.724 | 0.067 | 47 | 17468.504 | 0.013 |
| 5 | 41 | 16858.699 | 0.050 |  | 17469.390 | 0.091* |
| 6 | 42 | 16859.716 | 0.000 |  | 17470.354 | -0.019 |
| 7 | 43 | 16860.774 | 0.081* |  | 17471.354 | -0.010 |
| 8 | 44 | 16861.873 | -0.096* |  | 17472.388 | 0.076 |
| 9 |  | 16863.015 | -0.045 |  | 17473.457 | 0.078 |
| 10 |  | 16864.197 | 0.023 |  | 17474.561 | 0.032 |
| 11 |  | 16865.419 | 0.029 |  | 17475.701 | -0.069 |
| 12 |  | 16866.684 | 0.026 |  | 17476.875 | -0.005 |
| 13 |  | 16867.991 | 0.036 |  | 17478.083 | -0.020 |
| 14 |  | 16869.339 | 0.043 |  | 17479.327 | -0.037 |
| 15 |  | 16870.730 | -0.008 |  | 17480.606 | -0.003 |
| 16 |  | 16872.162 | -0.019 |  | 17481.919 | -0.100* |
| 17 |  | 16873.636 | 0.020 |  | 17483.267 | 0.044 |
| 18 |  | 16875.152 | -0.009 |  | 17484.650 | 0.022 |
| 19 |  | 16876.709 | 0.039 |  | 17486.068 | -0.036 |
| 20 |  | 16878.309 | 0.018 |  | 17487.521 | -0.064 |
| 21 |  | 16879.950 |  |  | 17489.008 | -0.068 |
| 22 |  | 16881.633 | 0.003 |  | 17490.531 | -0.016 |
| 23 |  | 16883.357 | 0.017 |  | 17492.088 | 0.021 |
| 24 |  | 16885.124 | -0.020 |  | 17493.679 | 0.050 |
| 25 |  | 16886.932 | 0.048 |  | 17495.306 | -0.030 |
| 26 |  | 16888.782 | -0.006 |  |  |  |
| 27 |  | 16890.674 | 0.013 |  |  |  |
| 28 |  | 16892.607 | -0.034 |  |  |  |
| 29 |  | 16894.582 | -0.028 |  |  |  |
| 30 |  | 16896.599 | -0.043 |  |  |  |

[^1]Table 6.5: Line Positions ${ }^{a}\left(\mathrm{~cm}^{-1}\right)$ for the $a^{3} \Pi(1)-X^{1} \Sigma^{+}$system of ${ }^{11}{ }_{B 1}$

| 0-0 Band |  | Q-3ranch |  | Blends |  | R-Branch |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Calc | Res | Calc | Res | Cale | Res |
| P(10) | $\mathrm{P}(20)$ |  |  | 16385.746 | 0.046 |  |  |
| 9 | 21 |  |  | 16386.031 | -0.088* |  |  |
| 8 | 22 |  |  | 16386.368 | 0.000* |  |  |
| 7 | 23 |  |  | 16386.758 | 0.057 |  |  |
| 6 | 24 |  |  | 16387.202 | -0.047 |  |  |
| 5 | 25 |  |  | 16387.700 | 0.022 |  |  |
| 4 | 26 |  |  | 16388.254 | -0.001 |  |  |
| 3 | 27 |  |  | 16388.865 | 0.050 |  |  |
| 3 | 28 |  |  | 16389.539 | -0.028 |  |  |
| 1 | 29 |  |  | 16390.283 | -0.017 |  |  |
| $Q(0)$ |  | 16390.690 |  |  |  |  |  |
| 1 |  | 16390.741 |  |  |  |  |  |
| 2 |  | 16390.843 |  |  |  |  |  |
| 3 |  | 16390.997 |  |  |  |  |  |
| 4 |  | 16391.201 |  |  |  |  |  |
| 5 | R(0) | 16391.456 |  |  |  | 16391.471 |  |
| 6 |  | 16391.763 |  |  |  |  |  |
| 7 | 1 | 16392.121 |  |  |  | 16392.304 |  |
| 8 |  | 16392.529 |  |  |  |  |  |
| 9 |  | 16392.989 | -0.017 |  |  |  |  |
| 10 | 2 |  |  | 16393.446 | 0.058 |  |  |
| 11 | 3 |  |  | 16394.074 | -0.007 |  |  |
| 12 |  | 16394.675 | 0.029 |  |  |  |  |
| 13 | 4 |  |  | 16395.289 | 0.108* |  |  |
| 14 | 5 |  |  | 15396.075 | 0.023 |  |  |
| 15 | 6 |  |  | 16396.925 | -0.072 |  |  |
| 16 |  | 16397.639 | 0.023 |  |  |  |  |
| 17 | 7 |  |  | 16398.469 | 0.056 |  |  |
| 18 | 8 | 16399.427 |  |  |  | 16399.556 |  |
| 19 | 9 |  |  | 16400.520 | -0.069 |  |  |
| 20 | 10 | 16401.420 | 0.028 |  |  | 16402.087 | 0.011 |
| 21 |  | 16402.493 | -0.034 |  |  |  |  |
| 22 | 11 |  |  | 16403.551 | 0.046 |  |  |
| 23 | 12 |  |  | 16404.803 | -0.025 |  |  |
| 24 | 13 |  |  | 16406.112 | -0.016 |  |  |
| 25 | 14 | 16407.297 | -0.020 |  |  | 16407.757 | 0.060 |
| 26 | 15 | 16408.626 | 0.044 |  |  | 16409.302 | -0.004 |
| 27 | 16 | 16410.006 | 0.009 |  |  | 16410.898 | -0.042 |
| 28 | 17 | 16411.437 | 0.004 |  |  | 16412.544 | -0.009 |
| 29 |  | 16412.919 | -0.052 |  |  |  |  |
| 30 | 18 |  |  | 16414.353 | 0.067 |  |  |
| 31 | 19 |  |  | 16416.014 | 0.010 |  |  |
| 32 | 20 |  |  | 16417.731 | 0.015 |  |  |
| 33 | 21 |  |  | 16419.503 | 0.020 |  |  |
| 34 | 22 |  |  | 16421.330 | 0.020 |  |  |


|  |  | Q-bran |  |  |  | R-Br | anch |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0. |  | Calc | Res | Calc | Res | Cale | Res |  |
| 35 | 23 | 16422.888 | -0.014 |  |  | 16423.489 | -0.105* |  |
| 36 | 24 | 16424.729 | 0.006 |  |  | 16425.490 | -0.040 |  |
| 37 | 25 | 16426.621 | -0.018 |  |  | 16427.543 | -0.061 |  |
| 38 | 26 | 16428.564 | -0.035 |  |  | 16429.645 | -0.047 |  |
| 39 |  | 16430.558 |  |  |  |  |  |  |
| 40 | 27 | 16432.604 |  |  |  | 16431.799 | 0.014 |  |
| 41 | 28 | 16434.701 |  |  |  | 16434.004 | -0.018 |  |
| 42 | 29 | 16436.849 |  |  |  | 16436.259 |  |  |
| 43 | 30 | 16439.048 |  |  |  | 16438.565 | -0.103* |  |
| 44 | 31 | 16441.299 |  |  |  | 16440.921 | -0.029 |  |
| 45 | 32 | 16443.600 |  |  |  | 16443.328 |  |  |
| 46 | 33 | 16445.954 |  |  |  | 16445.786 | -0.005 |  |
| 47 | 34 | 16448.358 |  |  |  | 16448.294 |  |  |
| 48 | 35 | 16450.814 |  |  |  | 16450.853 | -0.083* |  |
| 49 | 36 | 16453.321 |  |  |  | 16453.463 | -0.053 |  |

Table 6.5: Line Positions ${ }^{a}\left(\mathrm{~cm}^{-1}\right)$ for the $a^{3} \Pi{ }^{3}\left(0^{+}\right)-X^{1} \Sigma^{+}$System of ${ }^{11}$ BI (cont'd,)

| 0-1 | 1 Band | Q-branch |  | Blends |  | R-Branch |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Calc | Res | Cale | Res | Cate | Res |
| P(8) | P(19) |  |  | 15816.633 | 0.019 |  |  |
| 7 | 20 |  |  | 15816.979 | -0.119* |  |  |
| 6 | 21 |  |  | 15817.383 | -0.057 |  |  |
| 5 | 22 |  |  | 15817.847 | 0.025 |  |  |
| 4 | 23 |  |  | 15818.371 | 0.037 |  |  |
| 3 | 24 |  |  | 15818.957 | -0.012 |  |  |
| 2 | 25 |  |  | 15819.608 | -0.006 |  |  |
| 1 | 26 |  |  | 15820.328 | -0.062* |  |  |
| Q(0) |  | 15820.780 |  |  |  |  |  |
| 1 |  | 15820.837 |  |  |  |  |  |
| i |  | 15820.950 |  |  |  |  |  |
| 3 |  | 15821.119 |  |  |  |  |  |
| 4 |  | 15821.345 |  |  |  |  |  |
| 5 | R(0) | 15821.628 |  |  |  | 15821.561 |  |
| 6 |  | 15821.967 |  |  |  |  |  |
| 7 | 1 | 15822.363 |  |  |  | 15822.399 |  |
| 8 |  | 15822.815 | -0.037 |  |  |  |  |
| 9 | 2 | 15823.324 |  |  |  | 15823.293 |  |
| 10 |  | 15823.890 | -0.078* |  |  |  |  |
| 11 | 3 |  |  | 15824.459 | 0.058 |  |  |
| 12 | 4 |  |  | 15825.204 | -0.009 |  |  |
| 13 |  | 15825.925 | -0.095* |  |  |  |  |
| 14 | 5 |  |  | 15826.622 | 0.032 |  |  |
| 15 | 6 |  |  | 15827.532 | 0.006 |  |  |
| 16 | 7 |  |  | 15828.508 | 0.006 |  |  |
| 17 | 8 | 15829.431 | 0.018 |  |  | 15829.843 |  |
| 18 | 9 | 15830.449 | -0.025 |  |  | 15831.131 | -0.019 |
| 19 |  | 15831.524 | -0.006 |  |  |  |  |
| 20 | 10 |  |  | 15832.597 | 0.016 |  |  |
| 21 | 11 |  |  | 15833.854 | -0.008 |  |  |
| 22 | 12 |  |  | 15835.175 | 0.015 |  |  |
| 23 | 13 | 15836.387 | -0.025 |  |  | 15836.850 |  |
| 24 | 14 | 15837.745 | 0.033 |  |  | 15838.420 | 0.024 |
| 25 | 15 | 15839.159 | -0.029 |  |  | 15840.047 | -0.001 |
| 26 | 16 | 15840.630 | 0.026 |  |  | 15841.729 | -0.029 |
| 27 | 17 | 15842.157 | -0.031 |  |  | 15843.468 | -0.037 |
| 28 |  | 15843.741 | -0.049 |  |  |  |  |
| 29 | 18 |  |  | 15845.328 | -0.003 |  |  |
| 30 | 19 |  |  | 15847.095 | 0.013 |  |  |
| 31 | 20 |  |  | 15848.923 | -0.001 |  |  |
| 32 | 21 |  |  | 15850.811 | 0.044 |  |  |
| 33 | 22 | 15852.509 | 0.021 |  |  | 15853.007 | -0.050 |
| 34 | 23 | 15854.433 | 0.013 |  |  | 15855.083 | 0.003 |
| 35 | 24 | 15856.413 |  |  |  | 15857.216 | -0.027 |
| 36 | 25 | 15858.450 | 0.009 |  |  | 15859.404 | -0.027 |
| 37 | 26 | 15860.543 | 0.001 |  |  | 15861.649 | 0.005 |
| 38 | 27 | 15862.694 |  |  |  | 15863.950 | -0.032 |

Table 6.5: Line Positions ${ }^{a}\left(\mathrm{~cm}^{-1}\right)$ for the $a^{3} \Pi(1)-X^{1} \Sigma^{+}$system of ${ }^{11}$ BI (cont'd.)

| 0-1 Band | a-branch |  | Blends |  | R-Branch |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Calc | Res | Calc | Res | Calc | Res |


| 39 | 28 | 15864.901 |  |
| :--- | :--- | :--- | :--- |
| 40 | 29 | 15867.164 | -0.008 |
| 41 | 30 | 15869.485 | 0.018 |
|  | 31 |  |  |
|  | 32 |  |  |
|  | 33 |  |  |
|  | 34 |  |  |
|  | 35 |  |  |
|  | 36 |  |  |
|  | 37 |  |  |


| 15866.307 | $-0.093 *$ |
| :---: | :---: |
| 15868.720 | 0.003 |
| 15871.190 | -0.008 |
| 15875.715 | -0.052 |
| 15876.297 |  |
| 15878.934 | -0.002 |
| 15881.628 |  |
| 15884.378 |  |
| 15887.184 | 0.054 |
| 15890.045 | 0.049 |

${ }^{\text {a }}$ The table lists positions of single lines and intesity-weighted blends calculated from the parameters of Merge (B). The residuals are defined as res $=\nu_{\text {obs }}-\nu_{\text {calc }}$. Data not included in the least-squares fits are indicated by as asterisk.

These entries are the ratios of the root-mean squares (rms) of the residuals to the standard deviations for the individual bands. The final merges, (C) and (D), employed the output of merge (B) as input to obtain estimates of vibrational term values and vibrational constants respectively for the $\mathrm{a}\left(0^{+}\right)$and X .

The ten estimated band origins were reduced in merge (C) to eight term values relative to $G^{\prime \prime}(0)=0 . l$, and in merge (D) to $T!, \omega_{!}^{\prime}, \omega_{e}^{\prime} x_{!}^{!}, \omega_{\mathrm{e}}^{!} y_{!}^{\prime}$ for the $a\left(0^{+}\right)$state, $T_{!}^{\prime}+G^{\prime}(0)$ for the $a(1)$ state, and $\omega_{\theta}^{\prime \prime}, \omega_{e}^{\prime \prime} x_{\mathrm{e}}^{\prime \prime}$ for the X state. The results are given in Table 6.6. The estimated variances of the equivalent single step merges to the parameters of merges (C) and (D) are (2.00) ${ }^{2}$ and (1.97) ${ }^{2}$, indicating the absence of any appreciable systematic error in the absulute wavenumbers.

In conclusion, the following observations can be emphasized:

The estimates in Table 6.2 of $\mathrm{B}!, \mathrm{B} 1, \mathrm{~B} \|$, and these reported by Lebreton et al. (51) $(B!=0.389, ~ B!=0.385$, $\mathrm{B}_{\mathrm{O}}=0.363 \mathrm{~cm}^{-1}$ ), are in very good agreement.

Since the latter constants were derived from conventional spectrographic data, there is much confidence in the reliability of the present approach for gaining useful fundamental data from weak spertra.

The present work demonstrates, then, the advantages of the photoelectric technique. These are: the very high
sensitivity permitting the use of the weak, but specific emission source, without the problems of spectral
interference. The analysis of the $a\left(0^{+}\right) \rightarrow X$ system has thus been extended considerably and the first data on the a(1) state have been oktained.

Table 6.6: Term Values and Vibrational Parameters ${ }^{\text {A }}\left(\mathrm{cm}^{-1}\right)$ for the $X^{1} \Sigma^{+}$and $a^{3} \Pi\left(0^{+}, 1\right)$ States of ${ }^{11} B I$

| $\begin{aligned} & a^{3} \Pi(1) \\ & a^{3} \Pi\left(0^{+}\right) \end{aligned}$ | $\mathrm{E}_{0}=16390.707(16)$ | $\mathrm{T}_{\text {a }}+\mathrm{G}^{\prime}(0)=16677.695(20)$ |
| :---: | :---: | :---: |
|  | $\mathrm{E}_{4}=18598.828$ (34) | $\mathrm{T}_{\mathrm{e}}=16058.399(26)$ |
|  | $\mathrm{E}_{3}=17988.002(26)$ | $\omega_{0}=649.989(44)$ |
|  | $\mathrm{E}_{2}=1.7366 .922(14)$ | $\omega_{\mathrm{g}} \mathrm{x}_{0}=4.585$ (22) |
|  | $\mathrm{E}_{1}=16735.905$ (14) | $10^{2} \omega_{\text {g }} Y_{\mathrm{e}}=-5.14(30)$ |
|  | $\mathrm{E}_{0}=16095.253$ (12) |  |
| $\mathrm{X}^{1} \Sigma^{+}$ | $\mathrm{E}_{2}=1134.489$ (30) | $\omega_{\mathrm{a}}=575.322(26)$ |
|  | $\mathrm{E}_{1}=569.937(13)$ | $\omega_{\text {g }} \mathrm{x}_{\mathrm{E}}=2.693$ (10) |
|  | $\mathrm{E}_{0}=0.0$ |  |

${ }^{\text {a }}$ Values in parentheses are estimated standard errors given by $\hat{\sigma}_{M}\left(V_{1 i}\right)^{1 / 2}$, in units of the last significant figure of the corresponding parameter.

## Chapter 7

## Vibrational Analysis of $\mathrm{PCl}^{+}, \mathrm{AsCl}^{+}$and $\mathrm{SeO}^{+}$. New Visible Emission Spectra

### 7.1 Introduction

A relatively small number of diatomic molecular ions have been observed spectroscopically. The present chapter describes experiments that have been carried out to produce new emission spectra of 11-valence electron diatomic ions. Specifically, two new ionic emissions were analyzed and attributed to Group 5 halides ( $\mathrm{PCl}^{+}, \mathrm{ASCl}{ }^{+}$) and one new emission ( $\mathrm{SeO}^{+}$) extended the knowledge of the relatively well characterized diatomic ions of Group 6. Although the homonuclear $\mathrm{O}_{2}^{+}(61,62), \mathrm{S}_{2}^{+}(63,64,65), \mathrm{Se}_{2}^{+}(66), \mathrm{Te}_{2}^{+}(66)$ and heteronuclear $\mathrm{SO}^{+}(43,67,68)$ and $\mathrm{TeO}^{+}(69)$ ions have been the subjects of varying degrees of analysis, only three observations of the isovalent Group 5 monohalides are known.

Douglas and Frackowiak (70) have rotationally analyzed five red-degraded bands of the $A^{2} \Sigma^{+}-X^{2} \Pi$ band system of the $\mathrm{PF}^{+}$ion, excited in a discharge through helium mixed with a trace of $\mathrm{PF}_{3}$. Dyke and coworkers (71) studied the ground state of $\mathrm{NF}^{+}$via the photoelectron spectrum of $\mathrm{NF}, \mathrm{NF}\left(\mathrm{X}^{3} \Sigma^{-}\right.$, $\left.a^{1} \Delta\right)+\mathrm{NF}^{+}\left(\mathrm{X}^{2} I\right)+\mathrm{e}^{-}$. Finally, in a similar study by the same group (72), the photoelectron spectrum of the products of the $\mathrm{F}+\mathrm{PH}_{3}$ reaction was observed in the ionization energy region 9.5-10.0 eV. This spectrum has been assigned
as the first ionization band of $\mathrm{PF}, \mathrm{PF}\left(\mathrm{X}^{3} \Sigma^{\prime \prime}\right) \rightarrow \mathrm{PF}^{+}\left(\mathrm{X}^{3} \Sigma^{-}\right) \rightarrow$ $\mathrm{PF}^{+}\left(\mathrm{X}^{2} \Pi\right)+\mathrm{e}^{-}$.

## $\mathrm{PCl}^{+}$

Earlier experimental work in this laboratory (73) was concerned with the study of the reaction of phosphorous trichloride with metastable $\operatorname{Ar}\left({ }^{3} \mathrm{P}_{2,0}\right)$ atoms of energy -11.5 eV. Numerous red-degraded emission bands in the 4000-6000 A region were assigned to the $A^{3} \Pi_{r}-X^{3} \Sigma^{-}$system of PCl , and weak system in the $8100-8300 \AA$ region was assigned to the $b^{1} \Sigma^{+}-X^{3} \Sigma^{-}$system of PCI.

Presently, the reaction of $\mathrm{PCl}_{3}$ with the products of discharged helium were examined under the experimental conditions described in Chapter 4. Numerous red-degraded bands have been observed in the $4000-6900 \AA$ region and assigned to $\mathrm{a}^{2} \Pi \rightarrow \mathrm{X}^{2} \Pi$ system of the $\mathrm{PCl}^{+}$ion.

Although this ion has been detected mass spectrometrically from electron-impact ionization of phosphorous trichloride and phosphoryl chloride (74), there have been no previous spectroscopic observations.

## $\mathrm{ASCl}^{+}$

As was the case with most of the ionic species studied in this laboratory, electronic excitation of $\mathrm{AsCl}^{+}$was achieved in the reaction of discharged helium with a suitable substrate, in this case $\mathrm{AsCl}_{3}$. An extensive system of red-
degraded bands has been observed in the visible (4800 \& 7850 A) ; the system is similar in appearance to that of $\mathrm{PCl}^{+}$ and it was assigned as a ${ }^{2} \Pi-X^{2} \Pi$ transition of $A s C l+$.

## $\mathrm{SeO}^{+}$

By analogy with the use of $\mathrm{S}_{2} \mathrm{Cl}_{2}$ as substrate for excitation of $S_{2}^{+}(A \rightarrow X)(63)$, the first observation of an emission spectrum of the selenium monoxide cation has been obtained from the reaction of discharged helium with $\mathrm{SeOCl}_{2}$. The observed spectrum ( $3900 \AA-6700 \AA$ ) is similar in appearance to the $A \rightarrow X$ system of $S 0^{+}$, and is assigned to the same ${ }^{2} \Pi$ ${ }^{2}$ II transition in $\mathrm{SeO}^{+}$. The only previous observation of $\mathrm{SeO}^{+}$ was by mass spectrometric detection in the vapour above $\mathrm{SeO}_{2}$ and $\mathrm{SeO}_{3}$ at high temperature (75).

### 7.2.1 $\mathrm{PCI}^{+}$: Vibrational Assignment

The $\mathrm{PCl}^{+}$emission spectrum consists of an extensive series of double-headed, red-degraded bands in the range 14550-25000 $\mathrm{cm}^{-1}$. A small portion of the spectrum is shown in Figure 7.1, which illustrates the intensity difference of the two spin-orbit components. The shorter wavelength spinorbit sub-bands are approximately three times more intense than the longer wavelength sub-bands. For the less abundant $\mathrm{P}^{37} \mathrm{Cl}^{+}$ion, only the more intense shorter wavelength subbands can be observed.

The absolute vibrational numbering in the ground state
was obtained from the observed isotopic splittings between corresponding bands of $\mathrm{P}^{35} \mathrm{Cl} \mathrm{l}^{+}$and $\mathrm{P}^{37} \mathrm{Cl}{ }^{+}$in the $\mathrm{v}^{\prime}=0$ progression, as follows. Letting $\delta G_{v}$ denote the difference between the vibrational term values for two isotopically related diatomics,

$$
\begin{equation*}
\delta G_{v}=G(v)-G^{1}(v), \tag{7.1}
\end{equation*}
$$

$\delta G_{v}$ is given to a good approximation (73) by

$$
\begin{equation*}
\delta G_{v}=(1-\rho)(v+1 / 2) \Delta G_{v} \tag{7.2}
\end{equation*}
$$

where $\Delta G_{v}$ is the vibrational frequency for the $v^{\text {th }}$ level and $\rho$ is the square root of the ratio of the reduced masses, $\rho=$ $\left(\mu / \mu^{1}\right)^{1 / 2}$. If the frequency of the $v^{\prime}=0 \rightarrow v^{\prime \prime}$ transition is denoted by $\nu_{0, v \prime}$, then the observed isotope shift, $\delta \nu_{0 \mathrm{v}}$ is given by

$$
\begin{align*}
\delta \nu_{\rho v^{\prime \prime}} & =\delta G_{\mathrm{v}^{\prime \prime}}-\delta G_{v^{\prime}}=0 \\
& =(1-\rho)\left[\left(v^{\prime \prime}+1 / 2\right) \Delta G_{v^{\prime \prime}}-1 / 2 \Delta G_{v^{\prime}}=0\right] \tag{7.3}
\end{align*}
$$

or

$$
\begin{equation*}
\delta \nu_{0 v^{\prime}} / \Delta G_{v^{\prime \prime}}=(1-\rho)\left[\left(v^{\prime \prime}+1 / 2\right)-1 / 2 \Delta G_{v^{\prime}}=0 / \Delta G_{v^{\prime \prime}}\right] \tag{7.4}
\end{equation*}
$$

$\Delta G_{v^{\prime \prime}}$ were obtained from the plot of $\Delta G\left(v^{\prime \prime}+1 / 2\right)$ against $\left(v^{\prime \prime}+1 / 2\right)$. The second term of the right-hand side of Eq. (7.4) is small and almost constant. The approximate value used to calculate this term was $\Delta G_{0}=318.5 \mathrm{~cm}^{-1}$. The results are given in Table 7.1 and are plotted according to Eq. (7.4) in Figure 7.2.

It should be noted that only strong sub-band heads with reliable $\mathrm{P}^{37} \mathrm{Cl}^{+}$measurements were used in Table 7.1. The
solid line in Figure 7.2 is drawn with a slope of 0.01277 , which is the value of (1-p) calculated for $P^{35} C l$ and $P^{37} C l$ (58). Ruling out the possibility that the spectrum obtained in the present work was actually due to PCl , there is no doubt that the emitter is $\mathrm{PCl}^{+}$. Further confirmation about the nature of the emitter will be given in the next section. Finally, the ground state numbering is established unequivocally and is presented in the Deslandres Table, Table 7.2.

Figure 7.1: A portion of the $\mathrm{PCl}^{+}$emission spectrum.

Figure 7.1

Wavelength (i)

## Table 7.1: Vibrational Isotope ${ }^{\text {a }}$ Effect in the ${ }^{2} \Pi$ - $\mathrm{X}^{2} \|$ System of $\mathrm{P}^{35} \mathrm{Cl}^{+}$and $\mathrm{P}^{37} \mathrm{Cl}^{+}$

| $\delta \nu_{0 v^{\prime \prime}}$ | $\Delta G_{v^{\prime \prime}}$ | $\mathrm{v}^{\prime \prime}$ | $\left(\mathrm{v}^{\prime \prime}+\frac{1}{2}\right)-\frac{\Delta \mathrm{G}_{0}}{}$ |  |
| :--- | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
|  |  |  |  |  |
| 109.8 | 615.5 | 14 | 14.24 | 0.1784 |
| 118.3 | 610.7 | 15 | 15.24 | 0.1937 |
| 127.2 | 606.0 | 16 | 16.24 | 0.2099 |
| 135.2 | 600.0 | 17 | 17.23 | 0.2253 |
| 141.5 | 593.6 | 18 | 18.23 | 0.2384 |
| 148.5 | 586.2 | 19 | 19.23 | 0.2533 |
|  |  |  |  |  |

${ }^{\text {a }}$ See Eq. (7.4) for definitions of the symbols.

Figure 7.2: Vibrational isotope effect in the $X^{2} I I$ state of $\mathrm{PCl}^{+}$. Plot of sub-bandhead differences of $\mathrm{P}^{35} \mathrm{Cl}^{+}$and $\mathrm{P}^{37} \mathrm{Cl}^{+}$for the $\mathrm{V}^{\prime}=0$ progression according to Eq. (7.4); see text. The solid line corresponds to the theoretical slope, $1-\rho=0.01277$



[^2]7.2.2 $\mathrm{PCl}^{+}$: Results and Discussion

It is clear from Table 7.2 that the spectrum consists of a long progression of bands from $v^{\prime}=0$ and a shorter progression from $v^{\prime}=1$. This intensity distribution is consistent with a considerable difference in the equilibrium internuclear separations and vibrational frequencies of the two states; the single excited state spacing, $\Delta G^{\prime}\left(\frac{1}{2}\right)$ ~ $320 \mathrm{~cm}^{-1}$, is about half the $\Delta G^{\prime \prime}\left(\mathrm{V}+\frac{1}{2}\right)$ intervals, indicating a relatively shallow and weakly bound excited state.

The head positions $\nu$ (inv'v") of both isotopic bands can be represented adequately by Eq. 5.3, involving a difference of two Dunham power series in ( $v^{\prime}+\frac{1}{2}$ ) and ( $v^{\prime \prime}+\frac{1}{2}$ ):

$$
\begin{align*}
\nu\left(i \Omega v^{\prime} v^{\prime \prime}\right) & =T!+\sum_{k=1} U_{k 0}^{\prime}\left(x_{1}^{\prime}\right)^{k}-\sum_{k=1}^{\sum} U_{k 0}^{\prime \prime \prime}\left(x_{1}^{\prime \prime}\right)^{k} \\
& +(\Omega-1)\left(\sum_{k=0} A_{k}^{\prime}\left(x_{1}^{\prime}\right)^{k}-\sum_{k=0}^{\sum} A_{k}^{\prime \prime}\left(x_{1}^{\prime \prime}\right)^{k}\right) . \tag{7.5}
\end{align*}
$$

$\mathrm{U}_{\mathrm{k} 0}$ and $\mathrm{A}_{\mathrm{k} 0}$ are mass-invariant parameters describing the vibrational terms and spin-orbit splittings of the two states as a function of mass reduced quantum number, $x_{i}=\left(v+\frac{1}{2}\right) / \mu_{i}^{1 / 2}$ where $i$ denotes a particular isotope. The isotopically invariant parameters $U_{k 0}$ are related (56) to the usually reported $Y_{k 0}$ Dunham coefficients by Eq, (5.3) given in Chapter 5.

It should be noted that Eq. (5.3) is correct only when higher-order Dunham and/or Born-Oppenheimer breakdown terms are neglected, as in the present case of a low resolution spectrum. For the present data on $\mathrm{PCl}^{+}$, with only two vibrational levels for the excited state being observed, and
with the accuracy of the measurements inadequate for detection of any vibrational dependence of the ground state spin-orbit separation, Eq. (7.5) could be reduced to the 5parameter expression,


When the assignnent of the sub-bands is $\Pi_{\Omega} \leftrightarrow \Pi_{\Omega}$ with, $\Delta \Omega=0$ and the designation $\Omega=1 / 2$ or $3 / 2$ unknown, as in the present case, only $|\Delta A|=|A|-A_{e}^{\prime \prime} \mid$ can be calculateã.

It was found possible to fit simultaneously most of the bandheads of Table 7.2 by weighted least-squares to the model given by Eq. 7.6. In this way, estimates of the vibrational parame'cers of the two states and $\Delta A$ could be obtained. The rejected data from Table 7.2 were mostly weaker heads overlapped by strong atomic lines. The weight of each band was taken as inversely proportional to the square of the estimated wavenumber uncertainty, which corresponds to a constant value of $0.6 \AA$.

This uncertainty varied from $3.6 \mathrm{~cm}^{-1}$ at short wavenumber to $1.2 \mathrm{~cm}^{-1}$ at long wavenumber. The results of this fit according to Eq. (7.6) of 35 heads of $\mathrm{P}^{35} \mathrm{Cl}^{+}$and 10 heads of $\mathrm{P}^{37} \mathrm{Cl}^{+}$are listed in Table 7.3. The estimated standard deviation of the fit was $\hat{\sigma}=1.4$ with $f=40$ degrees of freedom; the difficulty encountered in locating the heads of several of the weaker bands is the principle factor leading to residuals of the fit being somewhat larger than the estimated measurement precision. Confirmation
that the emitting species is $\mathrm{PCl}^{+}$can be established from two observations: i) The relative intensity of the isotope bands is in accord with the ${ }^{35} \mathrm{Cl}:{ }^{37} \mathrm{Cl}$ natural abundance. In addition the presence of spin-orbit doublets indicate a chlorine-containing diatomic with an odd number of electrons. ii) An electron-impact study (74) reported that an energy of $16.5 \pm 0.5 \mathrm{eV}$ is required for the formation of $\mathrm{PCl}^{+}$from dissociative ionization of $\mathrm{PCl}_{3}$. Combining this result with the energy required for formation of the excited $\left(V^{\prime}=0\right)$ state of $P C 1^{+}, \sim 20 \mathrm{eV}$ is needed to form $P C l^{+}$from $\mathrm{PCl}_{3}$. This is larger than the 19.82 eV available from the metastable $\mathrm{He}\left({ }^{3} \mathrm{~s}\right)$ but less than that of $\mathrm{He}_{2}^{+}(\sim 21 \mathrm{eV})$. In accord with this energy consideration the large helium flow rate and pressure (~ 4 Torr) used experimentally to induce the emission, are conditions known to enhance production of $\mathrm{He}_{2}^{+}(39,41,42)$, as mentioned in Chapter 4. Additional evidence for $\mathrm{PCl}^{+}$as the carrier of the emission is provided by a comparison of the ground state vibrational frequency with those of isnvalent species. Values of $T:,|\Delta A|$, $\omega_{\theta}^{\prime}, \omega_{\theta}^{\prime \prime}\left(U_{10} / \mu_{i}^{1 / 2}\right)$, and of $\omega_{\theta}^{\prime \prime} X_{\theta}^{\prime \prime}\left(-U_{20} / \mu_{i}\right)$, calculated for $\mathrm{P}^{35} \mathrm{Cl}^{+}$ from the results in Table 7.3 are listed in Table 7.4, and compared with the corresponding constants of selected isovalent molecules having excited states with $\omega_{\mathrm{e}}^{\prime}<\omega_{\mathrm{g}}^{\prime \prime}$. The ratios of $\omega_{\mathrm{e}}^{\prime \prime}(\mathrm{X})$ of $\mathrm{PCl}^{+}$with those of the isoelectronic species SiCl and PS, 1.26 and 0.93, are similar in magnitude to the ratios of $\omega_{\mathrm{e}}^{\prime \prime}(X)$ of $\mathrm{PF}^{+}$with those of the
corresponding isoelectronic molecules, SiF and PO, 1.23 and 0.85. The ratios $\omega_{!}^{\prime \prime}(\mathrm{X})$ to $\omega!(\mathrm{A})$ for the isovalent $\mathrm{PF}^{+}$and $\mathrm{PCl}^{+}$molecules are 1.70 and 2.15 respectively.

The observed system of $\mathrm{PCl}^{+}$can be attributed to either $a^{2} \Pi \rightarrow X^{2} \Pi$ or to $a^{2} \Sigma \rightarrow X^{2} \Pi$ system. In the latter case, the spin-orbit splitting is directly correlated with the spinorbit coupling constant of the ground state. However, there are several indications that the excited state is a ${ }^{2} \Pi$ state, and not ${ }^{2} \Sigma$, as in the observed $\mathrm{PF}^{+}$system (70). First, the observed spin-orbit coupling constant in $\mathrm{PF}^{+}$ ( $\mathrm{X}^{2} \mathrm{II}$ ) about $324 \mathrm{~cm}^{-1}$, almost twice the sub-band separation in $\mathrm{PCl}^{+}$. However, comparing the spin-orbit constants of the ground state of the isovalent molecules SiF, SiCl and PO, $\mathrm{PF}^{+}$, (Table 7.4), a larger magnitude of A for the heavier molecule is observed; second, work at higher resolution indicates the absence of any $Q$ branches ( $\Delta J=0$ ) which is characteristic for a transition with $\Delta \Lambda=0$. More details of the nature of $a^{2} I I-{ }^{2} I I$ transition will be presented in the next chapter. Finally, a considerable intensity difference between the sub-bands is consistent with a ${ }^{2}$ II excited state where the higher energy spin-orbit component has a lower rate of population than the lower one.

Table 7.3: "Isotopically Invariant Parameters ( $\mathrm{cm}^{-1}$ ) for the $\mathrm{X}^{2} \mathrm{II}$ and $\mathrm{A}^{2} \Pi$ states of $\mathrm{PCl}^{+}$

|  | $A$ | $X^{2} \mathrm{II}$ |
| :--- | :---: | :---: |
| $\mathrm{T}_{\mathbf{a}}$ | $28753.3(64)$ | 0.00 |
| $\mathrm{U}_{10}$ | $1299.2(43)$ | $2795.8(31)$ |
| $\mathrm{U}_{20}$ |  | $-42.69(40)$ |
| $\|\Delta \mathrm{A}\|^{\mathrm{b}}$ |  | $183.1(11)$ |

[^3]| Low-lying Excited States of Selected 11 -valence Electron Diatomics. ${ }^{\text {c }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{T}_{\mathrm{B}}$ | $\omega_{\theta}$ | $\omega_{e} \mathrm{x}_{\mathrm{e}}$ | $A_{B}$ | $\|\triangle A\|$ | Ref. |
| SiF | 22858.3 | 718.5 (8) | 10.167 |  |  |  |
|  | 0.0 | 857.19 | 4.735 | $161.88^{\text {b }}$ |  | (53) |
| PF ${ }^{+}$ | 35434.64 | 619.00 | 4.62 |  |  |  |
|  | 0.0 | 1053.25 | 5.05 | 323.95 |  | (70) |
| PO | 33120.7 | 759.2 | 3.8(5) | $-13.3^{\text {b }}$ |  |  |
|  |  |  |  |  | 237.3 | (53) |
|  | 0.0 | 1233.34 | 6.56 | 224.03 |  |  |
| $s i^{35} \mathrm{cl}$ | 23010.4 | 296.4 | 0.73 |  |  |  |
|  | 0.0 | 535.60 | 2.168 | 207.21 |  | (76) |
| $\mathrm{P}^{35} \mathrm{Cl}^{+}{ }_{\text {A }}$ | 28753.3(64) 320.6(11) |  |  |  |  |  |
|  |  |  |  |  | 183(1) | this |
| $\mathrm{x}^{2} \mathrm{II}$ | 0.0 | 689.8(8) | 2.60(2) |  |  | work |
| PS $\quad B^{2} \Pi$ | 22573.3 | 510.80(2) | $1.79(2)$ | 91.85 |  |  |
|  |  |  |  |  | 230.1 | (77) |
| $\mathrm{x}^{2} \mathrm{II}$ | 0.0 | 739.49(2) | 2.97(2) | $321.93^{\text {b }}$ |  |  |

a Values in parentheses correspond to one standard deviation in units of the last digit of the corresponding parameters.
${ }^{b} v=0$
${ }^{6}$ Reduced masses of $\mathrm{P}^{35} \mathrm{Cl}^{+}$and $\mathrm{P}^{37} \mathrm{Cl}^{+}$, calculated from the atomic masses of ref. [58] are 16.425144 and 16.852792 amu respectively.

The eleven valence electrons of the $\mathrm{PCl}^{+}$ion can be correlated with the following electron configuration, ... $(z \sigma)^{2}(y \sigma)^{2}(\omega \pi)^{4}(x \sigma)^{2}(v \pi)$ leading to a ${ }^{2} \Pi$ regular ground state. A weakly bound excited ${ }^{2}$ II state can be obtained only by promotion of a bonding $\omega \pi$ electron to the $v \pi$ orbital, giving the configuration $(z \sigma)^{2}(y \sigma)^{2}(\omega \pi)^{3}(x \sigma)^{2}(v \pi)^{2}$, with possible states ${ }^{2,4} \Pi,^{2} \Phi$. The corresponding ${ }^{2}$ II states of SiF and SiCl have not been observed. It can be noted, however, that Verma (78) tried to correlate the observed higher lying ${ }^{2} I I$ states of $\operatorname{SiF}$ and SiCl with strong bonding character to a $(\omega \pi)^{3}(\mathrm{x} \sigma)^{2}(\mathrm{v} \pi)^{2}$ configuration. Johns and Barrow (79) suggested that the $(\omega \pi)^{3}(\mathrm{x} \sigma)^{2}(\mathrm{v} \pi)^{2}$ electronic configuration gives rise only to repulsive states in the SiF molecule. A better understanding of the electronic states of the llvalence electrons molecules car be obtained from the ab initio CI calculations on Si.F publishec recently by Bialski and Grein (80). In this work, a minimal basis set was used and states with $\omega_{g}^{\prime}>\omega_{\mathrm{g}}^{\prime \prime}(\mathrm{X})$ and/or $r_{\mathrm{g}}^{\prime}<r_{\mathrm{g}}^{\prime \prime}(\mathrm{X})$ were considered to be Rydberg states. An unobserved stable excited ${ }^{2} \Pi(3)$ state of SiF with calculated $T_{0}=31800 \mathrm{~cm}^{-1}$ was found to have a leading configuration $(\omega \pi)^{3}(\mathrm{x} \sigma)^{2}(\mathrm{~V} \pi)^{2}$ and $r_{e}^{\prime} \sim 2.4 \AA$ compared to $r_{e}^{\prime \prime}=1.6011 \AA$ for the ground ${ }^{2} I I$ state. The spectroscopic constants $\omega_{e}$ and $D_{e}$ for this state were not given due to its shallow potential curve.

From the preceding discussion, it would be reasonable to assume that the excited ${ }^{2}$ II state of $\mathrm{PCl}^{+}$observed
presently, corresponds to the ${ }^{2} I(3)$ state predicted in SiF, and observed in NS as $\mathrm{H}^{2} \Pi$ (81). Another ${ }^{2} \Pi(2)$ state has the same configuration as the ${ }^{2} \Pi(3)$ state but in the $C I$ calculations was found to be repulsive, in accord with the expected correlation with ground state $\operatorname{Si}\left({ }^{3} \mathrm{P}\right)+\boldsymbol{F}\left({ }^{2} \mathrm{P}\right)$ atoms. As in SiF, but not NS, the lower ${ }^{2} \Pi(2)$ state of $P C l^{+}$could be repulsive. In their conclusion, Bialski and Grein reported (80) that the three low-lying ${ }^{2} \Sigma^{+}$valence states of CCl, SiF, NS and Po were all repulsive. The observed $A^{2} \Sigma^{+}$ states in PO and NS should be almost pure Rydberg states corresponding to $\mathrm{V} \pi \rightarrow \mathrm{ns}$ transitions. In the case of $\mathrm{PF}^{+}$ emission observed by Douglas and Frackowiak (70) the $A^{2} \mathbb{F}_{1}^{+}$ state with $T_{0} \sim 35435 \mathrm{~cm}^{-1}$ is probably a higher-lying valence state and not a Rydberg state. Finally, after the present work was complete, Nguyen (82) reported an ab initio investigation, with a split-valence pius diffuse and polarization basis set extended by Moller-Plesset perturbation theory up to fourth-order, of the lower-lying bound electronic states of three diatomic molecules PH , PF . PCl and their cations. In this work the vibrational frequency of the ground state $X^{2} I I$ of $\mathrm{PCl}^{+}$was calculated as $696 \mathrm{~cm}^{-1}$. The value of $690 \mathrm{~cm}^{-1}$ for the same constant in the present experimental work is further confimmation that the emitting molecule is $\mathrm{PCI}^{+}$. It should be noted that within the Moller-Plesset perturbation theory only the lowest state of a given symmetry and multiplicity can be reliably
calculated; hence, the excited ${ }^{2}$ II state with configuration $(\omega \pi)^{3}(\mathrm{x} \sigma)^{2}(\mathrm{v} \pi)^{2}$ could not be obtained in the recent ab initio study. Furthermore, Nguyen reports an excited ${ }^{2} \Sigma$ state derived from..$\left(x_{0}\right)^{1}(v \pi)^{2}$ with an extremely shallow potential and lying $\sim 6696 \mathrm{~cm}^{-1}$ higher than ${ }^{2} I I$ excited state reported in the present work. Therefore, the lack of agreement between the calculated ${ }^{2} \Sigma \rightarrow X^{2} I I$ and the observed ${ }^{2}$ II $\rightarrow X^{2}$ II energies supports the assignment of the latter as the system responsible for the $\mathrm{PCl}^{+}$emission spectrum.

### 7.3.1 $\mathrm{AsCl}^{+}$: Vibrational Assignment

The $A s C l^{+}$emission spectrum consist of a large number of red-degraded sub-bands in the range $12800-21000 \mathrm{~cm}^{-1}$, with the most intense emission around $16000 \mathrm{~cm}^{-1}$.

Figure 7.3 shows a portion of the spectrum. As in the case of $\mathrm{PCl}^{+}$, the higher energy member of the spin-orbit coupled pairs is more intense than the lower energy member; but since the constant separation of the spin-orbit components is $\sim 1390 \mathrm{~cm}^{-1}$, the veaker sub-band of each ( $\mathrm{v}^{\prime}, \mathrm{v}^{\prime \prime}$ ) transition was either very close to or overlapped by the stronger sub-band of the $\left(v^{\prime}, v^{\prime \prime}+3\right)$ transition. For this reason, measurements of the weaker heads were less certain, or impossible. Although complicating the spectrum, the two isotopes of $\mathrm{AsCl}^{+}$provided a means for establishing the correct vibrational numbering for the observed transitions using Eq. (7.4) as for the $\mathrm{PC}^{+}$ion. The approximate value
used for $\Delta G_{0}$, was $240.3 \mathrm{~cm}^{-1}$. The results are given in Table 7.5 and are plotted according to Eq. (7.4) in Fig. 7.4. The solid line in Figure 7.4 is drawn with a slope of 0.01859 , which is the value of ( $1-\rho$ ) calculated for $A s^{35} \mathrm{Cl}$ and $\mathrm{As}^{37} \mathrm{Cl}(58)$. However, since the possibility that the spectrum obtained presently was due to AsCl (83) is ruled out, there is no doubt that the emitter is Ascl ${ }^{+}$. Finally, the head positions of both isotopes with unequivocal numbering are arranged in a Deslandres array in Table 7.6.

Figure 7.3: A portion of the $\mathrm{AsCl}^{+}$emission spectrum, showing bandheads belonging to the ${ }^{2} I I-X^{2} \Pi$ systems of $\mathrm{As}^{35} \mathrm{Cl}^{+}$and $\mathrm{As}^{37} \mathrm{Cl}^{+}$. The long wavelength member of each of spin-orbit coupled pair (----) is weak, and often overlapped (see text)

Intensity (counts $s^{-1}$ )


| $\delta \nu_{0 \mathrm{v}^{\prime \prime}}$ | $\Delta \mathrm{G}_{\mathrm{v}}{ }^{\text {( }}$ | v" | $\left(v^{\prime \prime}+\frac{1}{2}\right)-$ | $\frac{\Delta G_{0}{ }^{\prime}}{2 \Delta G_{\mathrm{v}^{\prime \prime}}}$ | $\delta \nu_{0 v^{\prime \prime}} / \Delta G_{v^{\prime \prime}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 165.1 | 463.0 | 19 | 19.24 |  | 0.3566 |
| 172.8 | 457.3 | 20 | 20.24 |  | 0.3779 |
| 186.9 | 450.0 | 22 | 22.23 |  | 0.4153 |
| 192.6 | 444.7 | 23 | 23.23 |  | 0.4331 |
| 205.4 | 440.0 | 25 | 25.22 |  | 0.4691 |
| 211.6 | 434.3 | 26 | 26.22 |  | 0.4872 |

${ }^{\text {a }}$ See Eq. (7.4) for definitions of the symbols.

Figure 7.4: Vibrational isotope effect in the $\mathrm{X}^{2}$ II state of $\mathrm{AsCl}^{+}$. Plot of sub-bandhead differences of $\mathrm{As}^{35} \mathrm{Cl}^{+}$and $\mathrm{As}^{37} \mathrm{Cl}^{+}$for the $\mathrm{V}^{\prime}=0$ progression according to Eq. (7.4); see text. The solid line corresponds to the theoretical slope $1-\rho=0.01859$.


|  | $\mathrm{As}^{25} \mathrm{Cl}^{+}$ |  |  |  |  |  | $\mathrm{As}^{37} \mathrm{Cl}^{+}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| v |  | $\mathrm{v}^{\prime \prime}=0$ |  |  | $v^{\prime \prime}=1$ |  |  | $v "=0$ |  |  | $\mathrm{v}^{\prime}=1$ |  |
| 9 |  |  | 20004.1 | 243.9 |  |  | 20247.9 |  |  |  |  |  |
|  |  |  |  |  |  |  | 489.0 |  |  |  |  |  |
| 10 | 20892.3 |  |  |  |  |  | 19758.9 |  |  |  |  |  |
|  | 488.9 |  |  |  |  |  | (485) |  |  |  |  |  |
| 11 | 20403.4 | 1375.9 | 19028.3 | 250.6 | 20653.9 | (1380) | 19274.0* | 20510.5 |  |  |  |  |
|  | 483.3 |  | 487.0 |  | 484.0 |  | (491) | (480) |  |  |  |  |
| 12 | 19920.1 | 1378.8 | 18541.3 | 245.7 | 20169.9 | 1386.9 | 18783.0 | $20030.0 *$ |  |  |  |  |
|  | (471) |  |  |  | 483.0 |  |  | (471) |  |  |  |  |
| 13 | 19449.5* |  |  | (237) | 19886.9 |  |  | 19558.8 |  |  |  |  |
|  | (487) |  |  |  | 479.8 |  |  | 469.8 |  |  |  |  |
| 14 | 18962.1 | 1386.1 | 17576.0 | 245.1 | 19207.1 | 1386.0 | 17821.2 | 19089.0 |  |  | 241.1 | 19330.1 |
|  | 474.9 |  |  |  | 477.6 |  |  |  |  |  |  | 470.3 |
| 15 | 18487.2 |  |  | 242.4 | 18729.5 |  |  |  |  |  |  | 18859.8 |
|  | 457.9 |  |  |  | 474.5 |  |  |  |  |  |  | 468.3 |
| 16 | 18019.3 |  |  | 235.7 | 18255.0 |  |  | 18162.2 |  |  | 229.3 | 18391.5 |
|  | 466.1 |  |  |  | 467.7 |  |  | (457) |  |  |  | 457.3 |
| 17 | 17553.2 |  |  | 234.1 | 17787.3 |  |  | 17705.3* |  |  | (229) | 17934.2 |
|  | (465) |  |  |  | 467.5 |  |  | (454) |  |  |  |  |
| 18 | 17087.8* | (1395) | $\$ 5693.2$ | 246.6 | 17319.8 | 1380.0 | 15939.8 | 17251.0* | (1399) | 15851.6 |  |  |
|  | (465) |  | (464) |  | 458.4 |  |  | (463) |  | 455.6 |  |  |
| 19 | 16623.0 | (1394) | 15228.8* | 238.4 | 16861.4 |  |  | 16788.1 | ¢392.0 | 15396.0 |  |  |
|  | 461.0 |  | (455) |  | 458.4 |  |  | 453.3 |  | 449.6 |  |  |
| 20 | 16162.0 | 1387.7 | 14774.3 | 241.0 | 16403.0 |  |  | 16334.8 | 1388.4 | 14946.4 |  |  |
|  | 453.6 |  | 453.2 |  |  |  |  | 448.5 |  | 447.5 |  |  |
| 21 | 15708.4 | 1387.3 | 14324.1 |  |  |  |  | 15888.3 | 1389.4 | 14498.9 |  |  |
|  | 452.7 |  | 448.9 |  |  |  |  | 445.7 |  |  |  |  |
| 22 | 15255.7 | 1383.5 | 13872.2 |  |  |  |  | 15442.6 |  |  |  |  |
|  | 447.2 |  | (445) |  |  |  |  | 441.5 |  |  |  |  |
| 23 | 14808.5 | (1381) | 13427.3* |  |  |  |  | 15001.1 |  |  |  |  |
|  | 442.2 |  | (442) |  |  |  |  | 438.6 |  |  |  |  |
| 24 | 14366.3 | (1381) | 12985.6* |  |  |  |  | 14562.5 |  |  |  |  |
|  | 443.8 |  |  |  |  |  |  | 433.6 |  |  |  |  |
| 25 | 13922.5 |  |  |  |  |  |  | 14128.9 |  |  |  |  |
|  | 436.1 |  |  |  |  |  |  | 430.9 |  |  |  |  |
|  |  |  | *Heads | not in | cluded in | in the |  |  |  |  |  |  |
| 26 | 13486.4 |  | least | -squares | fit, se | ee text. |  | 13698.0 |  |  |  |  |
|  | 432.5 |  |  |  |  |  |  |  |  |  |  |  |
| 27 | 13053.9 |  |  |  |  |  |  |  |  |  |  |  |

### 7.3.2 $\mathrm{AsCl}^{+}$: Results and Discussion

The head positions of the bands of both sub-systems and of both isotores were, fitted simultaneously by weighted least-squares to the same model as that employed for $\mathrm{PCl}^{+}$, as given by Eq. (7.6). The isotopically invariant parameters of Table $7.7, \mathrm{U}_{\mathrm{k} 0}$, are related to the more familiar vibrational Dunham coefficients $Y_{k 0}$ by Eq. (5.3).

Heads not included in the fit are flagged by an asterisk in the Deslandres Table (7.6), and are mostly weak heads overlapped by atomic lines. Weights were chosen as $1 / \sigma^{2}$ where $\sigma$ are the uncertainties in the measurements of band-heads, and varied from $0.9 \mathrm{~cm}^{-1}$ at the red end of the spectrum to $2.7 \mathrm{~cm}^{-1}$ at the blue end.

The 39 heads of $\mathrm{As}^{35} \mathrm{Cl}^{+}$and the 20 heads $\mathrm{As}^{37} \mathrm{Cl}^{+}$were fitted satisfactorily only for the numbering given in Table 7.6. As mentioned earlier, the numbering was established initially from the observed isotope shifts. The standard deviation of the fit was 1.4 with 54 degrees of freedom. This standard deviation increased considerably to ~ 3.0, where the vibrational numbering, was changed up or down by one unit. The spin-orbit coupling separations were represented by a single parameter $|\Delta A|$, which corresponds to the difference between the spin-orbit coupling constants of the upper and lower ${ }^{2} \Pi$ states. Additional parameters representing the v"-dependence could not be obtained from the present experimental data. Although the magnitude of $|\Delta A| \sim 1386 \mathrm{~cm}^{-1}$ is considerably larger than the
corresponding constant of the isovalent phosphorous cation, $\sim 183 \mathrm{~cm}^{-1}$, it is similar to $|\Delta A|$ of the isovalent arsenic species, AsS and AsO, as shown in Table 7.8. The observed transitions of Ascl $^{+}$from $v^{\prime}=0,1$ to the vibrational levels $v^{\prime \prime}=9-27$ of the ground state are consistent with a weakly bound excited state having $r_{\theta}^{\prime}>r_{\theta}^{\prime \prime}(X)$, and can be readily correlated with the similar observed progressions of the isovalent $\mathrm{PCl}^{+}$cation. It is reasonable to assume that the configuration and the nature of the electronic states involved in the observed $\mathrm{AsCl}^{+}$emission are the same as for $\mathrm{PCl}^{+}$. The assignment of the excited state as ${ }^{2} I I$, as opposed to ${ }^{2} \Sigma$, is also in accord with the difference in intensity between the two sub-systems in $\mathrm{AsCl}^{+}$; this observation implies different rates of population of the $F_{1}$ and $F_{2}$ components of the excited ${ }^{2} I I$ state. It should also be noted from Table 7.8 that for the isovalent AsO and Ass molecules, an excited ${ }^{2}$ II state is known at similar energy.

Presently, the problem of whether the Ascl ${ }^{+}$excited state is regular or inverted cannot be resolved unequivocally. However, it is believed that much of the observed spin-orbit separation can be attributed to large spin-orbit coupling in the ground state, as in AsO (84). In addition, an examination of Table 7.8 shows that the ground vibrational frequency of $\mathrm{AsCl}^{+}$is similar to that of the AsS
 significantly lower than those of the ${ }^{2} I I$ states of AsO and

Ass. This anomalously low vibrational spacing in the excited state is also observed in $\mathrm{PCl}^{+}$. Small vibrational intervals in the excited states of other isovalent species have also been found, for example, the $A^{2} \Sigma$ states of the silicon halide molecules. In conclusion, Shimauchi (88) has compared the $\omega_{0}^{\prime \prime}$ values of the phosphorus and arsenic oxide and sulphide ions with those of the neutral molecules; the magnitude of the ratios, $\omega_{\mathrm{a}}^{\prime \prime}(\mathrm{ion}) / \omega_{\mathrm{a}}^{\prime \prime}$ (neutral), is 1.14 in all cases. The ratios of the $\omega$ ! values for $\mathrm{AsCl}^{+}$and Ascl (83), and for $\mathrm{PCl}^{+}$and PCl (73) are 1.24 and 1.25 respectively.

Table 7.7: Isotopically Invariant Parameters ${ }^{n}\left(\mathrm{~cm}^{-1}\right)$ for the $\mathrm{X}^{2} \mathrm{II}$ and Excited ${ }^{2}$ II States of $\mathrm{AsCl}^{+}$


|  |  | T ${ }_{\text {s }}$ | $\omega_{0}$ | $\omega_{\mathrm{a}} \mathrm{x}_{\mathrm{e}}$ | $\mathrm{A}_{\text {e }}$ | $\|\Delta A\|$ | Ref. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Aso | $\mathrm{G}^{2} \mathrm{I}_{1 / 2}$ | 26485.2 | 630.30 | 3.006 | -316.8 | 1342.8 | (84) |
|  |  |  |  |  |  |  |  |
| $\mathrm{X}^{2} \mathrm{I}_{1 / 2}$ |  | $0.0 \quad 967.08$ | 4.850 | 1025.9 |  |  |  |
| SiBr | $\mathrm{A}^{2} \Sigma$ | 2093..6 | 250.3 | 0.5 | 419.2 |  | (85) |
|  | $\mathrm{X}^{2} \mathrm{II}_{1 / 2}$ | 20.0 | 424.5 | 1.5 |  |  |  |
| AsS | $\mathrm{A}^{2} \Pi_{1 / 2}$ | -20475 | 402.26 | 1.21 |  |  |  |
|  |  |  |  |  |  | -1210 | $(86,87)$ |
|  | $\mathrm{X}^{2} \mathrm{I}_{1 / 2}$ | 20.0 | 567.94 | 1.97 |  |  |  |
| $\mathrm{As}^{35} \mathrm{Cl}^{+}$ | ${ }^{2} \Pi$ | 25435(5) | $242.4(9)^{\text {b }}$ |  |  | 1387(1) |  |
|  |  |  |  |  |  |  | ) this |
|  | $\mathrm{X}^{2} \mathrm{II}$ | 0.0 | 527.7(5) | 1.74(1) |  |  | work ${ }^{\text {c }}$ | of Some 11-valence Electron Diatomics

[^4]
### 7.4.1 $\mathrm{SeO}^{+}$: Vibrational Assignment

The numerous red-degraded bands of the $\mathrm{SeO}^{+}$emission are in the range $15000-25500 \mathrm{~cm}^{-1}$. Figure 7.5 , shows a portion of the $\mathrm{SeO}^{+}$spectrum, and its characteristic feature, namely the presence of "double-headed" bands. This eloseness of heads cannot be attributed to any selenium isotope effect, and arises instead from the similarity in magnitude of the spin-orbit splittings and the ground state vibrational spacings. More specifically, one head is the first head of a spin-orbit comp nent of a ( $v^{\prime}, v^{\prime \prime}$ ) transition, while the other is the second head of the neighbouring (v' - v' - 1) band. This accidental overlapping causes considerable difficulty in measuring accurately the longer wavelength member of each pair. The measured head positions are arranged in a Deslandres array given in Table 7.9. In this table, which was obtained taking into consideration the similar intensity of the two spin-orbit coupled pairs of the same (v', v") transition, there is also good agreement between the several estimates of the upper and lower state vibrational intervals, which decrease steadily with increasing $v$.

In addition, the intensity distribution of the bands is consistent with the larger vibrational frequency of the ground state, and is similar to the observed intensity of the $A \rightarrow X$ system of $\mathrm{SO}^{+}$. With this type of distribution, there is little doubt that the absolute vibrational numbering in Table 7.9 is correct.

Figure 7.5: A portion of the $\mathrm{SeO}^{+}$emission spectrum from $\sim 19750$ to $\sim 22050 \mathrm{~cm}^{-1}$

Figure 7.5


Table 7.9: Deslandres Table for the $A^{2} \Pi-X^{2} n$ System of $\mathrm{SeO}^{+}$

| $v^{\prime} v^{\prime \prime}=0$ |  | $v^{\prime \prime}=1$ |  | $v "=2$ |  | $v \prime \prime=3$ |  | $v^{\prime \prime}=4$ |  | $v^{\prime \prime}=5$ |  | $v \prime \prime=6$ |  | $v^{\prime \prime}=7$ | $v^{\prime \prime}=8$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 21913.5 |  | 20943.4 |  | 19981.6 |  | 19031.4 |  | $18104.9{ }^{\text {a }}$ |  | $17179.3^{\text {a }}$ |  | b |  |
| 0 |  |  | 972.0 |  | 962.6 |  | 951.2 |  | 937.2 |  | 928.1 |  | 912.2 |  |  |
|  |  | 21025.0 |  | 20051.0 |  | 19087.7 |  | 18135.6 |  | 17198.4 |  | 16270.3 |  | 15358.1 |  |
|  |  | 659.6 |  | 656.5 |  | 655.8 |  | 658.3 |  |  |  |  |  |  |  |
| 23548.3 |  | 22574.5 |  | 21598.1 |  | 20635.9 |  | $b$ |  |  |  |  |  |  |  |
| 1 | 983.1 |  | 975.2 |  | 936.2 |  | 95\%. 2 |  |  |  |  |  |  |  |  |
| 22675.7 |  | 21686.3 |  | 20709.3 |  | 19745.1 |  | 18793.9 |  |  |  |  |  |  | 15111.7 |
| 649.8 |  | 649.5 |  | 648.1 |  | 649.2 |  |  |  |  |  |  |  |  |  |
| 24194.9 |  | 23212.6 |  | $b$ |  | b |  | 20336.8 ${ }^{\text {a }}$ |  | $b$ |  | $b$ |  |  |  |
| 2 | 987.6 |  | 978.4 |  | 963.1 |  |  |  |  |  |  |  |  |  |  |
| 23328.6 |  | 22335.8 |  | 21357.4 |  | 20394.3 |  | $19470.6^{\text {a }}$ |  | $18509.5^{\text {a }}$ |  | 17576.3 |  |  |  |
| 643.2 |  | 445.0 |  | 641.9 |  |  |  |  |  |  |  |  |  |  |  |
| 24840.9 |  | $23846.6^{\text {a }}$ |  | b |  |  |  |  |  |  |  |  |  |  |  |
| 3 | 988.3 |  | 981.5 |  |  |  |  |  |  |  |  |  |  |  |  |
| 23969.1 |  | 22980.8 |  | 21999.3 |  |  |  |  |  |  |  |  |  |  |  |
| 634.7 |  | 632.1 |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | $24485.6^{\text {a }}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | 990.9 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $-4603.8$ |  | 23612.9 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 628.0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 25231.8 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

a Not included in the least-squares fit
b Band head overlapped by an adjacent pand

### 7.4.2 $\mathrm{SeO}^{+}$: Results and Discussion

The estimates of the vibrational and spin-orbit parameters of the two states were obtained using a leastsquares fitting procedure with the head positions given in Table 7.9. It was found that these data are represented adequately by a simple expression involving a difference of two Dunham series in ( $v^{\prime}+\frac{1}{2}$ ) and ( $v^{\prime \prime}+\frac{1}{2}$ ),

$$
\begin{align*}
V_{v^{\prime}}^{\prime \prime}= & T!+\sum_{k=1} Y_{k}^{\prime}\left(V^{\prime}+\frac{1}{2}\right)^{k}-\sum_{k=1} Y_{X}^{!}\left(V^{\prime \prime}+\frac{1}{2}\right)^{k} \\
+ & (\Omega-1)\left(A_{!}+\alpha_{A}^{\prime}\left(V^{\prime}+\frac{1}{2}\right)-A_{U}^{\prime \prime}-\right. \\
& \left.\alpha_{A}^{\prime \prime}\left(V^{\prime \prime}+\frac{1}{2}\right)\right) \tag{7.7}
\end{align*}
$$

However, $A:$ and $A!$ are totally correlated, so that only the magnitude of the difference of $\Delta A_{0}=A_{a}^{\prime}-A_{a}^{\prime \prime}$ could be determined. The constants $\alpha_{A}^{\prime}$ and $\alpha_{A}^{\prime \prime}$ represent the variation of the unknown individual spin-orbit coupling constants with vibrational quantum number. If $\Delta A_{\mathrm{e}}$ has negative sign, which will be argued in the following sections, $\alpha_{\mathrm{A}}$ and $\alpha_{\mathrm{A}}^{\prime \prime}$ will have positive sign.

In addition, the two vibrational Dunham coefficients obtained for each state are listed in Table 7.10. The least-squares fit had a standard deviation of $2.2 \mathrm{~cm}^{-1}$ and 27 degrees of freedom. The molecular constants given in Table 7.10 follow the expected trends of those of a number of isovalent species, as shown in Table 7.11. It is clear that for all the molecules in this table, the nature of the ground and first excited electronic states is well-known, and that definite overall trends in the values of the constants are clearly established. For example, except for
$\mathrm{O}_{2}^{+}$, the ratios of $\omega_{0}(\mathrm{X}) / \omega_{\mathrm{e}}(\mathrm{A})$ are in the range $1.45-1.62$. For $\mathrm{SeO}^{+}$, this ratio is 1.51 . It should be mentioned also that in all of $T_{0}(A), \omega_{0}(A), \omega_{0} x_{0}(A), \omega_{a}(X)$ and $\omega_{\mathrm{e}} \mathrm{X}_{\mathrm{a}}(\mathrm{X})$, there is a decrease in magnitude with increasing reduced mass.

Although the molecular constants given in Table 7.10 are very similar, as expected, to those of NSe $(90,91)$, the possibility that the spectrum obtained in this work actually belonged to NSe was el minated by comparing the bandhead positions determined in the present work with those of Refs. ( 90,91 ). In a similar way, $\mathrm{Se}_{2}(92)$ and $\operatorname{SeO}(93,94)$ were also eliminated as possible carriers of the bands observed presently.

Finally, the question concerning the sign of $\Delta A_{e}=A!-$ A" can be resolved with reasonable sertainty by examination of the data of Table 7.11. It can be seen from the isoelectronic pairs NS, $\mathrm{SO}^{+}$and $\mathrm{NSe}, \mathrm{SeO}^{+}$that there is a close similarity in the molecular parameters. The spinorbit coupling constant of the ground state, $\mathrm{A}_{\mathrm{e}}^{\prime \prime}$, for $\mathrm{SO}^{+}$is considerably larger than for NS, while the same constants for the excited state, $A!$, are smaller and have opposite signs. Accordingly, $A_{\text {I }}^{\prime \prime}$ for ${S e O^{+}}^{+}$should be not only positive, but probably of larger magnitude than the value for NSe ( $\sim 890 \mathrm{~cm}^{-1}$ ). Taking into consideration the magnitude of $\left|\Delta A_{e}\right|\left(\sim 878 \mathrm{~cm}^{-1}\right)$ that has been obtained in the present work, it is reasonable to expect that the excited $\mathrm{A}^{2}$ il state of $\mathrm{SeO}^{+}$is also regular. There is also little
doubt then that $\Delta A_{0}=A_{0}$ - $A_{0}^{\prime \prime}$ is negative for $S \in O^{+}$, in accord with all the isovalent molecules in Table $7 . i 1$ for which A! and A! are known.

Table 7.10: Molecular Constants ${ }^{\mathrm{a}}\left(\mathrm{cm}^{-1}\right)$ for the $\mathrm{X}^{2} \Pi$ and $A^{2} \Pi$ States of $\mathrm{SeO}^{+}$

|  | $\mathrm{A}^{2} \mathrm{II}$ | $\mathrm{X}^{2} \mathrm{II}$ |
| :---: | :---: | :---: |
| $\mathrm{T}_{\text {e }}$ | 22625.7(17) | 0.00 |
| $\omega_{\text {e }}$ | 663.1 (9) | 999.7(6) |
| $\omega_{\mathrm{e}} \mathrm{X}_{\mathrm{e}}$ | 3.65 (18) | 6.31(8) |
| $\left\|\alpha_{A}\right\|^{b}$ | 3.0 (9) | 5.0(7) |
| $\left\|\Delta A_{e}\right\|^{b, c}$ | 878.4(28) |  |
| ${ }^{\text {a }}$ Values in parentheses correspond to one standard deviation in units of the last digit of the corresponding parameters. |  |  |
| ${ }^{\text {b }}$ Altho param ${ }^{c} \Delta A_{e}=$ | of $\alpha^{\prime}$ and $\alpha^{\prime \prime}$ ign opposite | not known, of $\Delta A_{0}$, $s$ |

rable 7.11: Molecular Constants $\left(\mathrm{cm}^{-1}\right)$ for the $A^{2} \pi$ and $X^{2} \pi$ States of Some Diatomic Species with 11 -valence Electrons.

|  | $\begin{gathered} 0_{2}^{+} \\ (61,89) \end{gathered}$ | $\begin{gathered} \text { NS } \\ (81) \end{gathered}$ | $\begin{gathered} \mathrm{so}^{+} \\ (67) \end{gathered}$ | $\begin{aligned} & \mathrm{NSe}^{\mathrm{d}} \\ & (90) \end{aligned}$ | $\begin{aligned} & \text { Se0 }^{+} \\ & \text {this work } \end{aligned}$ | $\begin{gathered} \mathrm{s}_{2}^{+} \\ (65) \end{gathered}$ | Se ${ }^{+}$ <br> (66) | Tel ${ }^{\circ}$ <br> (85) | $\operatorname{Ta}_{0}^{*}$ <br> (60) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Te ${ }^{\text {' }}$ | 40570.7 | $30296.4{ }^{\text {b }}$ | 31432 | $24205^{6 \theta}$ | 22625.7 | 22344.69 | 19200 | 19700 | 145.0 |
| $\omega_{e}{ }^{\prime}$ | 898.24 | $797.31^{\text {b }}$ | 805.25 | $658.9{ }^{\text {bf }}$ | 663.1 | 552.72 |  |  |  |
| $\omega_{e} x^{\prime}{ }^{\prime}$ | 13.57 | $3.72{ }^{\text {b }}$ | $6.34{ }^{\text {c }}$ |  | 3.65 | 3.14 |  |  |  |
| $A_{e}{ }^{\prime}$ | $-3.496^{\text {a }}$ | 90.4 | -53.91 | $449^{\text {a }}$ |  | 13.5 |  |  |  |
| $\omega_{e}{ }^{\prime \prime}$ | 1904.77 | $1219.14^{\text {b }}$ | 1311.4 | $957.37{ }^{\text {b }}$ | 999.7 | 805.9 |  |  |  |
| $\omega_{e}{ }^{x} e^{\prime \prime}$ | 16.26 | $7.28{ }^{\text {b }}$ | 8.30 | $5.72{ }^{\text {b }}$ | 6.31 | 3.38 |  |  |  |
| $A_{e}$ | $200.289^{\text {a }}$ | 223.15 | 363.8 | 890.84 |  | 469.7 | 1935 | 4840 | 3790 |
| $\left\|\Delta A_{e}\right\|$ | $203.785^{\text {a }}$ | 132.8 | 417.7 | -440 | 878.4 | 456.2 |  |  |  |
| ${ }^{\mathbf{a}} \mathbf{v}=0$ |  |  |  |  |  |  |  |  |  |
| b $2_{\Pi_{1 / 2}}-2_{\Pi_{1 / 2}}$ sub-system |  |  |  |  |  |  |  |  |  |
| ${ }^{\text {c }}$ constrained value |  |  |  |  |  |  |  |  |  |
| d $14 N^{80} \mathrm{Se}$ |  |  |  |  |  |  |  |  |  |
| ${ }^{\text {e }} \mathrm{T}_{0}$ value |  |  |  |  |  |  |  |  |  |
| ${ }^{\mathrm{f}} \Delta \mathrm{G}_{1 / 2}$ value |  |  |  |  |  |  |  |  |  |

## Chapter 8

## Rotational Analysis of the $\mathbf{A} \rightarrow \mathbf{X}$ System of $\mathrm{PCl}^{+}$

### 8.1 Introduction

The only previous work on a rotationally resolved spectrum of a Group 5 monohalide cation is the analysis of five red-degraded bands of $\mathrm{PF}^{+}(70)$. The present chapcer is concerned with the first rotational (spectroscopic) analysis of $\mathrm{A}^{2} I I \rightarrow \mathrm{X}^{2} I$ system of $\mathrm{PCl}^{+}$. In the low-resolution study (Chapter 7) of this system of $\mathrm{PCl}^{+}$, a considerable number of red-degraded bands were observed in the range of $4000-$ 7000 A. In view of the time available for the completion cf the present thesis, it was necessary to limit the present work to the analysis of only three of the most intense bands of $\mathrm{P}^{35} \mathrm{Cl}^{+}$. No attempt was made to investigate any bands of the less abundant isotope molecule $\mathrm{P}^{37} \mathrm{Cl}^{+}$. The recorded spectra were clean and free of overlap from any other species. The bands studied belong to the $v^{\prime}=0$ progression ( $0-16,0-17$ and $0-18$ ), and the molecular constants for the two electronic states were fitted directly to the measured line positions of individual bands. The multiple estimates of the constants for $v^{\prime}=0$ were reduced to single-valued parameters ky the method of merging, and estimates of the spin-or'bit coupling constants for both states are reported for first time. It should be noted here that the three bands analyzed by the author represent a
contribution to the more extensive analysis undertaken subsequently by coxon et al. (95).

### 8.2 Nature of the Transition, and Rotational

## Assignments

Since the observed transition of $\mathrm{PCl}^{+}$has been assigned as $A^{2} \Pi \rightarrow X^{2} \Pi$, the $X$ and $A$ states have a non-zero component of electronic orbital angular momentum in the direction of the internuclear axis ( $\Lambda=1$ ). This vector couples strongly with the corresponding component ( $\Sigma=\frac{1}{2},-\frac{1}{2}$ ) of the spin angular momentum, $s$ to form the resultant electronic angular momentum ( $\Omega=3 / 2,1 / 2$ ). $\Omega$ couples with the nuclear rotation vector ( $R$ ) perpendicular to the internuclear axis, giving the resultant total angular momentum (J) of the molecule. The coupling of $\Lambda$ and $\Sigma$ to the internuclear axis is characteristic of Hund's case (a).

If both states belong to case (a), besides the general selection rules $(\Delta J=0 \pm 1 ; \Delta S=0 ;+*-)$, the following additional restrictions hold: $\Delta \Sigma=0 ; \Delta \Omega=0, \pm 1$. As a result, $a{ }^{2} \Pi$ - ${ }^{2} I I$ band splits into two sub-bands, ${ }^{2} \Pi_{1 / 2}$ ${ }^{2} \Pi_{1 / 2}$ and ${ }^{2} \Pi_{3 / 2}-{ }^{2} \Pi_{3 / 2}$, as presently observed. In general, as a result of the selection rules and the fine splitting of each rotational level into two distinct levels of opposite parity ( $\Lambda$-doubling), each sub-band has six branches which form three close pairs, two $p(\Delta J=-1)$, two $Q(\Delta J=0)$, and two $R(\Delta J=1)$ branches, the two $Q$-branches being very weak.

With the $0.06 \AA$ resolution of the present analysis, there was no evidence for any $\Lambda$-doubling splitting. In addition, there was no indication for the presence of $Q$-branches. The energy level pattern of $a^{2} \Pi \rightarrow{ }^{2} I I$ electronic transition, with the four observed branches $R_{11}, P_{11}, R_{22}, P_{22}$, is shown schematically in Figure 8.1.

As described in Chapter 7, one sub-band is approximately three times more intense than the other. The stronger sub-band always lies at higher energy, and was assigned as the $\Pi_{1 / 2}-\Pi_{1 / 2}$ transition. This conclusion was based on the fact that all the isovalent molecules in Table 7.4 have a regular ground state ( $\Omega=\frac{1}{2}$ component has lower energy), and the spin-orbit splitting of the ground state is larger in magnitude than the corresponding splitting of the excited state. The difference in intensity arises from the different rates of population of the two spin-orbit components of the excited ${ }^{2}$ II state.

The weak spin-orbit components of the $\mathrm{P}^{37} \mathrm{Cl}^{+}$isotope were not observed in the present work, since they were overlapped with the intense sub-band of the $\mathrm{P}^{35} \mathrm{Cl}^{+}$isotope. Both sub-bands of the more abundant isotope $\mathrm{P}^{35} \mathrm{Cl}{ }^{+}$were recorded for the $0-16,0-17$, and $0-18$ bands. These bands are well-separated, free from any overlapping, and showed well-resolved rotational structure even for low J. Therefore, these three bands were selected for preliminary rotational assignment by the method of combination
differences. For two bands with same upper state vibrational number, the combination differences,

$$
\begin{equation*}
R(J)-P(J)=\Delta_{2} F^{\prime}(J) \approx 4 B_{v}^{\prime}\left(J+\frac{1}{2}\right) \tag{8,1}
\end{equation*}
$$

(neglecting centrifugal distortion, D) must agree exactly for each $J$ value. Thus, in order to find the correct relative numbering, a trial numbering of the lines in the two bands is used. The trial numbering is then varied systematicaily until the $R(J)-P(J)$ values agree exactly. This can be achieved sy changing the numbering in one of the branches of each band by $1,2,3 \ldots$ units.

Figures 8.1: Energy level diagram for the first rotational
lines of the $A^{2} \Pi-X^{2} \Pi$ transition of the $\mathrm{PCl}^{+}$ion.


It is obvious that in order to employ this procedure, the rotational lines must be first assigned as $R$ or $P$ branch transitions. In the present work, the red degradation ( $B_{v}^{\prime}<B_{v}^{\|}$) of the bands and the intensity contours of the branches enabled an unequivocal identification of the lines as $P$ or $R$ branches. Once the correct relative numbering has been established, successive $\Delta_{2} F^{\prime}$ values differ by $4 B_{v}^{\prime}$, in accord with Eq. (8.1). Consequently, dividing one of the $\Delta_{2} F^{\prime}$ values by $4 B_{v}^{\prime}$ estimated in this way, the absolute $J$ value can be obtained. Confirmation for the absolute $J$ numbering was provided by the requirement for $D_{0}$ to be close to the theoretical value of $4 \mathrm{~B}_{\mathrm{e}}^{3} / \omega_{\mathrm{e}}^{2}$. For the three bands analyzed, estimates of seven constants ( $\nu_{0}, B^{\prime}, D^{\prime}, A^{\prime}$, $\left.B^{\prime \prime}, D^{\prime \prime}, A^{\prime \prime}\right)$ could be obtained by fitting directly the measured line positions of both sub-bands. Line positions were calculated as the differences of the eigenvalues of a standard ${ }^{2}$ II Hamiltonian containing the molecular constants as variables for the upper and lower states of the transition, as discussed earlier in Chapter 2. Trial values of the constants were then improved iteratively using nonlinear least-squares fitting to yield the final set of estimated molecular parameters.

### 8.3 Results

A portion of the ${ }^{2} \Pi_{1 / 2}-{ }^{2} I_{1 / 2}$ sub-band of the $0-17$ band is shown in Fig. 8.2. This is a characteristic trace which
illustrates the quality of the spectra for all the sub-bands recorded presently. The two branches, $P_{11}$ and $R_{11}$, are easily distinguished, and there is little blending with overiapping weak structure of the other bands. The results of the present investigation are summarized in Table 8.1.

The standard deviations $(\hat{\sigma})$ of the final least-squares fits for individual bands, along with the number and maximum value of $J$ " of lines fitted, can be obtained from this table. The standard deviations, are in accord with the estimated measurement precision. Band origins of the individual bands are also listed in Table 8.1. These estimates were obtained after merging (4) the parameters from the individual band-fits.

The three individual band-fits yielded 21 estimated constants, with three estimates of the same excited state constants ( $B!$, $D$ ! , $A!$ ). Single-valued estimates were obtained by merging the individual band constants to a new set of 15 constants (merge (A)). Since the centrifugal distortion constants of the ground state ( $D_{v}^{\prime \prime}$ ) in the output of merge (A) did not show any significant vibrational dependence, it was decided to reduce these 3 constants to a single vibrationally independent parameter $D^{\prime \prime}$ (merge (B)). The variance of the equivalent single step merge to the 13 output parameters of merge $(B)$ is $\hat{\sigma}^{2}=(1.59)^{2}$ with $f=8$ degrees of freedom. This estimate lies just outside the limits $(0.584)^{2} \leq \hat{\sigma}^{2} \leq(1.40)^{2}$ given by the $\chi^{2} / 8$ distribution
at the 95\% confidence level. This satisfactory merge is considered to indicate the absence of any significant systematic error. The constants from merge (B) are listed in Table 8.2, and the band origins are listed in Table 8.1. Finally, calculated line positions for the bands analyzed have been obtained with the parameters of merge and are listed in Table 8.3, along with the corresponding residuals (observed-calculated). The merged parameters reproduce the analyzed bands very successfully, as indicated by the similar magnitudes of the stan ird deviations and root mean-squares (rms) of the residuals for individual bands. The rms of the residuals for the $0-16,0-17$, and 0-18 bands are also listed in Table 8.1.

Figure 8.2: The $\mathrm{A}^{2} \Pi_{1 / 2}-\mathrm{X}^{2} \Pi_{1 / 2}$ 0-17 sub-band of $\mathrm{P}^{35} \mathrm{Cl}^{+}$. The trace was recorded photoelectrically from the flame of the $\mathrm{He}_{2}^{+}+\mathrm{PCl}_{3}$ reaction. The spectral resolution is about 0.06 A .


Table 8.1: Least squares fits ${ }^{\text {a }}$ for individual bands of the $A^{2} \Pi \rightarrow X^{2} \Pi$ system of $P^{35} \mathrm{Cl}^{+}$

| Band | $\nu_{0}$ | N | $\hat{\sigma}$ | rms | $\mathrm{J}_{\max }$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $0-16$ | $18236.635(13)$ | 115 | 0.0405 | 0.0398 | 52.5 |
| $0-17$ | $17634.819(11)$ | 131 | 0.0383 | 0.0379 | 58.5 |
| $0-18$ | $17038.661(14)$ | 101 | 0.0477 | 0.0507 | 54.5 |

a For each band fitted, $N$ is the total number of fitted lines, $\hat{\sigma}$ is the standard deviation, $\nu_{0}$ is the fitted band origin from merge ( $B$ ), rms is the root mean square of the residuals between the observed and calculated line positions, and $J_{\max }$ is the maximum value of the quantum number $J^{\prime \prime}$ of the fitted lines. All data are in $\mathrm{cm}^{-1}$ units. Values in parentheses are estimated standard errors given by $\hat{\sigma}_{M}\left(V_{1 i}\right)^{1 / 2}$ in units of the last significant figure of the corresponding constant.

Table 8.2: Fitted Constants ${ }^{2}$ for the $A^{2} I I$ and $X^{2} I I$ States of $\mathrm{P}^{35} \mathrm{Cl}^{+}$

$$
\mathrm{A}^{2} \Pi
$$

| v | $\mathrm{B}_{\mathrm{v}}$ | $10^{7} \mathrm{D}_{\mathrm{v}}$ | $\mathrm{A}_{\mathrm{v}}$ |
| :---: | :---: | :---: | :---: |
| 0 | $0.18770(12)$ | $2.09(34)$ | $98.52(320)$ |

## $x^{2} I$

| v | $\mathrm{B}_{\mathrm{v}}$ | $\mathrm{A}_{\mathrm{v}}$ |
| :--- | :--- | :--- |
| 16 | $0.25933(12)$ | $282.56(320)$ |
| 17 | $0.25781(12)$ | $282.19(320)$ |
| 18 | $0.25621(12)$ | $281.43(320)$ |
|  | $\mathrm{D}=1.52(34) \times 10^{-7}$ |  |

a The results, in $\mathrm{cm}^{-1}$ units, are from merge (B) (see text). Values in parentheses are estimated standard errors given by $\hat{\sigma}_{M}\left(V_{i j}\right)^{1 / 2}$ in units of the last significant figure of the corresponding constant.

Table 8.3: Calculated Line Positions for the $0-16$ Band of $p^{35} \mathrm{cl}^{+}(A-X)$.

| J | P1 |  | R1 |  | P2 |  | R2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Calc. | Res. | Calc. | Res. | Calc. | Res. | Calc. | Kes. |
| 0.5 |  |  | 18329.144 |  |  |  | 18145.253 |  |
| 1.5 | 18426.895 |  | 18329.304 |  | 18045.383 |  | 18145.414 |  |
| 2.5 | 18327.072 |  | 18329.320 |  | 18143.176 |  | 18145.433 |  |
| 3.5 | 18326.195 |  | 18329.192 |  | 18142.299 |  | 18145.308 |  |
| 4.5 | 18325.174 |  | 18328.921 |  | 18141.280 |  | 18145.041 |  |
| 5.5 | 18324.011 |  | 18328.506 |  | 18140.117 |  | 18144.630 |  |
| 6.5 | 18322.703 |  | 28327.948 |  | 19138.812 |  | 18144,077 |  |
| 7.5 | 18321.252 |  | 18327.247 |  | 18137.363 |  | 18143.380 |  |
| 8.5 | 18319.658 |  | 18326.401 |  | 18135.771 | -0.106* | 18142.540 |  |
| 9.5 | 18317.920 |  | 18325.412 |  | 18134.037 |  | 18141.557 |  |
| 10.5 | 18316.039 |  | 183?4.280 |  | 16132.159 | -n.127* | 18140.431 |  |
| 11.5 | 18314.014 | -0.018 | 18322.996 |  | 18130.138 | -0.058 | 18139.162 |  |
| 12.5 | 18311.846 | 0.036 | 18321.584 |  | 18127.974 | 0.026 | 18137.750 |  |
| 13.5 | 18309.534 | 0.021 | 18320.021 |  | 18125668 | -0,071 | 18136.194 | -0.073 |
| 14.5 | 18307.079 |  | 18318.314 |  | 18123.218 | -0.138* | 18134.49 E |  |
| 15.5 | 18304.480 | 0.037 | 18316.463 |  | 18120.625 | 0.104* | 18132.654 | -0.021 |
| 16.5 | 18301.737 | -0.016 | 18314.468 | 0.088 | 18117.888 | 0.024 | 18130.668 | -0.067 |
| 17.5 | 18298.851 | 0.041 | 18312.330 | 0.031 | 18115.009 | 0.074 | 18128.540 | -0.059 |
| 18.5 | 18295.821 | -0.133** | 18310.048 | -0.041 | 18111.987 | -0.004 | 18126.266 | $0.149 *$ |
| 19.5 | 18292.648 | 0.001 | 18307.622 | -0.102* | 18108.821 | 0.024 | 18123.853 | -0.010 |
| 20.5 | 18289.330 | -0.051 | 18305.052 | 0.007 | 18105.513 | 0.016 | 18121.294 | -0.084 |
| 21.5 | 18285.870 | -0.026 | 18302.338 | 0.013 | 18102.061 |  | 18118.592 | -0.006 |
| 22.5 | 18282.265 | -0.026 | 18299.481 | 0.021 | 1.8098.465 | 0.050 | 18115,747 | 0.049 |
| 23.5 | 18278.517 | 0.002 | \$8296.479 | 0.048 | 18094.727 | 0.105* | 18112.758 | 0.027 |
| 24.5 | 18274.625 | 0.043 | 18293.334 | -0.049 | 18090.845 | 0.121* | 18109325 | 0.038 |
| 25.5 | 18270.590 | 0.014 | 18290.044 | 0.019 | 18086.820 | 0.066 | 1810.349 | -0.016 |
| 26.5 | 18266.410 | 0.030 | 18286.611 | 0.003 | 18082.652 | 0.000 | 18102.929 | 0.023 |
| 27. 5 | 18262.087 | -0.026 | 18283.035 | -0.011 | 18078.340 | 0.031 | 18099.366 | -0.009 |
| 28.5 | 18257.620 | -0.001 | 18279.311 | 0.000 | 18073.885 | 0.070 | 18095.659 | -0.052 |
| 29.5 | 18253.009 | -0.042 | 18275.445 | 0.095* | 18059.286 | -0.000 | 18091.808 | 0.091* |
| 30.5 | 18248.254 | 0.018 | 18271.435 | -0.013 | 18064.544 | -0.046 | 18087.813 | 0.082* |
| 31.5 | 18243.355 | -0.061 | 18267.281 | 0.056 | 18059.658 | -0.014 | 18083.675 | 0.142 * |
| 32.5 | 18238.312 | 0.010 | 18262.982 | 0.070 | 18054.629 | 0.010 | 18079.393 | 0.089* |
| 33.5 | 18233.125 | -0.007 | 18258.539 | 0.016 | 18049.456 | -0.049 | 18074.966 | 0.021 |
| 34.5 | 18227.793 | -0.071 | 18253.951 | -0.075 | 18044.140 | 0.026 | 18070.396 |  |
| 35.5 | 18222.318 | -0.017 | 28249.219 | -0.022 | 18038.680 | -0.035 | 18065.682 | -0.039 |
| 36.5 | 18216.699 | -0.041 | 18244.342 | -0.034 | 18033.076 | 0.020 | 18060.823 | -0.008 |
| 37.5 | 18210.935 | -0.034 | 18239.321 | -0.025 | 18027.328 | 0.051 | 18055.821 | 0.028 |
| 38.5 | 18205.028 | -0.055 | 18234.155 | -0.011 | 18021.427 | 0.115* | 18050.674 | -0.007 |
| 39.5 | 18198.976 | 0.320*: | 18228.845 | 0.018 | 18015.401 |  | 18045.383 | -0.021 |
| 40.5 | 18192.779 | 0.008 | 18223.390 | -0.027 | 18009.222 | 0.071 | 18039.947 | -0.045 |
| 41.5 | 18186.438 | -0.042 | 18217.790 | -0.001 | 18002.899 |  | 18034.388 | -0.029 |
| 42.5 | 18179.953 | 0.039 | 18212.045 | 0.036 | 17996.431 |  | 18028.643 | 0.034 |
| 43.5 | 18173.323 | 0.076 | 18206.155 | -0.015 | 17989.820 |  | 28022.775 | -0.010 |
| 44.5 | 18166.549 | -0.013 | 18200.121 | 0.002 | 17983.064 |  | 18016.761 | 0.082* |
| 45.5 | 18159.630 | -0.041 | 18193.941 | 0.054 | 17976.164 |  | 18010.603 | -0.103* |
| 46.5 | 18152.567 | -0.043 | 18287.616 |  | 17969.120 |  | 18004.300 |  |
| 47.5 | 18145.359 |  | 18181.146 | -0.016 | 17961.932 |  | 27997.853 |  |
| 48.5 | 18138.006 |  | 18174.531 | -0.073 | 17954.599 |  | 17991.260 |  |
| 49.5 | 18130.508 |  | 18167.771 | 0.010 | 17947.122 |  | 17984.523 |  |
| 50.5 | 18122.865 |  | 18160.865 | 0.031 | 17939.501 |  | 17977.641 |  |
| 51.5 | 18115.078 |  | 18153.813 | 0.071 | 17931.734 |  | 17970.613 |  |
| 52.5 | 18107.145 |  | 18146.617 | 0.012 | 17923.823 |  | 17963.441 |  |

TABLE 8.3 cont'd...

TABLE 8.3 cu.tt'd ${ }^{\text {a) }}$ Calculated Line Positions for the $0-17$ Band of $\mathrm{p}^{35} \mathrm{Cl}^{+}(\mathrm{A}-\mathrm{X})$

| J | P1 |  | R1 |  | P2 |  | R2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | calc. | res. | calc. | Res. | calc. | Res. | calc. | Res. |
| 0.5 |  |  | 17727.146 |  |  |  | 17543.619 |  |
| 1.5 | 17824.901 |  | 17727.310 |  | 17443.754 |  | 17543.785 |  |
| 2. | 17725.085 |  | 17727.333 |  | 17541.554 |  | 17543.811 |  |
| 3.5 | 17724.219 |  | 17727.216 |  | 17540.688 |  | 17543.697 |  |
| 4.5 | 17723.212 |  | 17726.959 |  | 17539.682 |  | 17543.443 |  |
| 5.5 | 17722.065 |  | 17726.561 |  | 17538.536 |  | 17543.049 |  |
| 6.5 | 17720.777 | 0.007 | 17726.022 | 0.071 | 17537.250 |  | 17542.516 |  |
| 7.5 | 17719.349 | -0.009 | 17725.343 | -0.019 | 17535.825 | 0.062 | 17541.842 |  |
| 8.5 | 17717.781 | -0.083 | 17724.524 | 0.007 | 17534.259 |  | 17541.028 |  |
| 9.5 | 17716.072 | -0.013 | 17723.564 | -0.009 | 17532.553 |  | 17540.074 |  |
| 10.5 | 17714.222 | 0.010 | 17722.463 | -0.100 | 17530.707 | 0.008 | 17538.979 | -0.006 |
| 11.5 | 17712.232 | 0.024 | 17721.222 | 0.009 | 17528.721 | -0.102 | 17537.745 |  |
| 12.5 | 17710.101 | -0.009 | 17719.840 | -0.014 | 17526.595 | 0.158* | 17536.371 | -0.033 |
| 13.5 | 17707.830 | 0.025 | 17718.317 | -0.045 | 17524.329 |  | 17534.856 |  |
| 14.5 | 17705.419 | 0.002 | 17716.654 | 0.001 | 17521.923 | 0.032 | 17533.201 | 0.015 |
| 15.5 | 17702.867 | -0.004 | 17714.850 | 0.053 | 17519.377 |  | 17531.406 | -0.045 |
| 16.5 | 17700.174 | -0.075 | 17712.905 | 0.010 | 17516.691 |  | 17529.471 | -0.023 |
| 17.5 | 17697.341 | 0.019 | 17710.1320 | 0.056 | 17513.865 | -0.015 | 17527.396 | -0.068 |
| 18.5 | 17694.367 | -0.010 | 17708.594 | 0.033 | 17510.899 |  | 17525.180 | -0.145* |
| 19.5 | 17691.252 | 0.049 | 17706.227 | 0.038 | 17507.792 | 0.048 | 17522.824 | -0.020 |
| 20.5 | 17687.997 | 0.003 | 17703.719 | -0.031 | 17504.546 | 0.019 | 17520.327 | 0.016 |
| 21.5 | 17684.601 | -0.002 | 17701.070 | -0.036 | 17501.159 | -0,005 | 17517.691 | -0.036 |
| 22.5 | 17681.065 | -0.014 | 17698.281 | -0.052 | 17497.632 | -0.089* | 17514.913 | 0.030 |
| 23.5 | 17677.388 | - 0.034 | 17695.350 | 0.045 | 17493.965 | -0.018 | 17511.996 | 0.015 |
| 24.5 | 17673.570 | -0.010 | 17692.279 | 0.032 | 17490.158 | -0.070 | 17508.937 | 0.094 |
| 25.5 | 17669.611 | -0.041 | 17689.066 | 0.038 | 17486.210 | -0.018 | 17505.739 | -0.051 |
| 26.5 | 17665.512 | 0.019 | 17685.713 | 0.028 | 17482.122 | 0.072 | 17502.399 |  |
| 27.5 | 17661.272 | -0.038 | 17682.218 | -0.003 | 17477.893 | -0.001 | 17498.919 |  |
| 28.5 | 17656.891 | -0.080 | 17678.582 | 0.023 | 17473.525 | -0.049 | 17495.299 | 0.058 |
| 29.5 | 17652.369 | 0.018 | 17674.806 | -0.020 | 17469.016 | 0.045 | 17491.537 | -0.006 |
| 30.5 | 17647.706 | -0.016 | 17670.888 | -0.042 | 17464.366 | -0.079 | 17487.635 | -0.096* |
| 31.5 | 17642.902 | -0.014 | 17666.828 | 0.016 | 17459.576 |  | 17483.592 | 0.006 |
| 32.5 | 17637.958 | 0.043 | 17662.628 | 0.019 | 17454.645 | 0.034 | 17479.408 | -0.012 |
| 33.5 | 17632.872 | 0.025 | 17658.286 | -0.007 | 17449.574 | -0.357* | 17475.084 |  |
| 34.5 | 17627.645 | 0.049 | 17653.803 | 0.042 | 17444.362 |  | 17470.618 |  |
| 35.5 | 17622.277 |  | 17649.178 | 0.022 | 17439.009 |  | 17466.011 |  |
| 36.5 | 17616.768 | 0.018 | 17644.412 | 0.031 | 17433.516 |  | 17461.264 |  |
| 37.5 | 17611.118 | -0.022 | 17639.504 | 0.014 | 17427.882 |  | 17456.375 |  |
| 38.5 | 17605.327 | -0.046 | 17634.454 | 0.055 | 17422.108 |  | 17451.345 |  |
| 39.5 | 17599.394 | -0.013 | 17629.263 | 0.095 | 17416.192 |  | 17446.124 |  |
| 40.5 | 17593.320 | -0.064 | 17623.931 | -0.003 | 17410.135 |  | 17440.861 |  |
| 41.5 | 17587.105 | -0.020 | 17618.456 | 0.062 | 17403.938 |  | 17435.407 |  |
| 42.5 | 17580.748 | 0.156* | 17612.840 | -0.015 | 17397.600 |  | 17429.812 |  |
| 43.5 | 17574.250 | -0.045 | 17607.082 | -0.317* | 17391.120 |  | 17424,075 |  |
| 44.5 | 17567.610 | 0.030 | 17601.182 | -0.011 | 17384.499 |  | 17418.196 |  |
| 45.5 | 17560.829 | -0.031 | 17595.139 | 0.002 | 17377.738 |  | 17412.176 |  |
| 46.5 | 17553.906 | 0.005 | 17588.955 | -0.048 | 17370.835 |  | 17406.015 |  |
| 47.5 | 17546.841 | -0.020 | 17582.629 | -0.023 | 17363.790 |  | 17399.711 |  |
| 48.5 | 17539.635 | -0.016 | 17576.160 | -0.062 | 17356.605 |  | 17393.266 |  |
| 49.5 | 27532.287 |  | 17569.549 | -0.014 | 17349.278 |  | 17386.679 |  |
| 50.5 | 17524.797 | 0.043 | 17562.796 | -0.007 | 17341.809 |  | 17379.949 |  |
| 51.5 | 17517.165 |  | 17555.901 | 0.028 | 17334.199 |  | 17373.078 |  |
| 52.5 | 17509.391 | 0.041 | 17548.862 | -0.021 | 17326.447 |  | 17366.065 |  |
| 53.5 | 17501.475 |  | 17541.682 | 0.000 | 17318.554 |  | 17358.909 |  |
| 54.5 | 17493.417 | -0.027 | 17534.358 | -0.045 | 17310.519 |  | 17351.611 |  |
| 55.5 | 17485.217 | -0.055 | 17526.892 |  | 17302.342 |  | 17344.171 |  |
| 56.5 | 17476.874 | 0.051 | 17519.284 | 0.009 | 17294.023 |  | 17336.588 |  |
| 57.5 | 17468.390 | 0.005 | 17511.532 | -0.067 | 17285.562 |  | 17328.863 |  |
|  |  |  |  |  |  |  | table 8.3 | cont ${ }^{\text {d... }}$ |

TABLE 8.3 cont'd a) Calculated Line Positions for the 0 m 18 Band of $\mathrm{p}^{35} \mathrm{Cl}^{+}(\mathrm{A}-\mathrm{X})$

| J | Pl |  | R1 |  | P2 |  | R2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | calc. | Res. | Calc. | Res. | Calc. | Res. | Calc. | Res. |
| 0.5 |  |  | 17130.608 |  |  |  | 16947.841 |  |
| 1.5 | 17229.367 |  | 17130.776 |  | 16847.980 |  | 16948.012 |  |
| 2.5 | 17128.560 |  | 17130.808 |  | 16945.789 |  | 16948.046 |  |
| 3.5 | 17121.705 |  | 17130.702 |  | 16944.934 |  | 16947.943 |  |
| 4,5 | 17126.712 |  | 17130.459 |  | 16943.943 |  | 16947.704 |  |
| 5.5 | 17125.583 |  | 17130.079 |  | 16942.815 |  | 16947.328 |  |
| 6.5 | 17124.316 |  | 17129.561 |  | 16941.550 |  | 16946.815 |  |
| 7.5 | 17122.912 |  | 17128.906 |  | 16940.148 |  | 16946.165 | 0.046 |
| 8.5 | 17121.371 |  | 17128.114 |  | 16938.609 |  | 16945.378 |  |
| 9.5 | 17119.692 |  | 17127.184 |  | 16936.934 |  | 16944.455 | 0.076 |
| 10.5 | 17117.876 |  | 17126.117 | 0.011 | 16935.122 |  | 16943.394 | 0.065 |
| 11.5 | 17115.923 |  | 17124.913 | -0.027 | 16933.173 | 0.051 | 16942.197 | 0.044 |
| 12.5 | 17113.832 |  | 17123.571 | -0.027 | 16931.087 |  | 16940.863 | -0.121 |
| 13.5 | 17111.605 | -0.214* | 17122.091 | 0.006 | 16928.865 |  | 16939.392 | 0.064 |
| 14.5 | 17109.240 | -0.013 | 17120.475 | -0.034 | 16926.506 |  | 16937.784 | -0.262* |
| 15.5 | 17106.737 |  | 17118.720 | 0.066 | 16924.010 | -0.153* | 16936.039 |  |
| 16.5 | 17104.097 | -0.152* | 17116.829 | 0.000 | 16921.377 | -0.009 | 16934.156 | 0.079 |
| 17.5 | 17101.320 | -0.071 | 17114.799 | 0.050 | 16918,607 | 0.009 | 16932.137 | 0.167* |
| 18.5 | 17098.406 |  | 17112.633 | -0.016 | 16915.700 | 0.030 | 16929.981 | 0.014 |
| 19.5 | 17095.354 | 0.023 | 17110.328 | -0.048 | 16912.656 | -0.039 | 16927.688 | -0.001 |
| 20.5 | 17092.165 | 0.001 | 17107.886 | -0.010 | 16909.476 | 0.143* | 16925.257 | -0.195* |
| 21.5 | 17088.838 | 0.045 | 17105.306 | 0.014 | 16906.158 |  | 16922.690 | -0.049 |
| 22.5 | 17085.374 | -0.003 | 17102.589 | 0.037 | 16902.704 | -0.021 | 16919.985 | -0.026 |
| 23.5 | 17081.772 |  | 17099.734 | 0.047 | 16899.112 | -0.251* | 16917.143 | 0.157* |
| 24.5 | 17078.033 | -0.045 | 17096.741 | -0.010 | 16895.383 | 0.039 | 16914.163 | -0.046 |
| 25.5 | 17074.156 | 0.078 | 17093.611 | 0.000 | 16891.518 |  | 16911.046 | -0.019 |
| 26.5 | 17070.141 |  | 17090.342 | -0.036 | 16887.515 | -0.083 | 16907.792 | 0.046 |
| 27.5 | 17065.989 | 0.002 | 17086.936 | -0.016 | 16883.375 | 0.084 | 16904.401 | -0.072 |
| 28.5 | 17061.700 | -0.017 | 17083.392 | 0.072 | 16879.098 | 0.059 | 16900.872 | -0.108 |
| 29.5 | 17057.273 | 0.059 | 17079.709 | -0.043 | 16874.684 |  | 16897.205 | -0.127 |
| 30.5 | 17052.708 | -0.058 | 17075.889 | -0.071 | 16870.132 |  | 16893.401 | 0.037 |
| 31.5 | 17048.005 | 0.019 | 17071.931 | -0.027 | 16865.443 |  | 16889.460 | -0.136 |
| 32.5 | 17043.165 | 0.079 | 17067.835 | -0.044 | 16860.617 |  | 16885.381 | 0.647 |
| 33.5 | 17038.186 | 0.020 | 17063.600 | 0.011 | 16855.654 |  | 16881.164 | -0.071 |
| 34.5 | 17033.070 | 0.014 | 17059.228 | 0.020 | 16850.553 |  | 16876.809 |  |
| 35.5 | 17027.816 | 0.010 | 17054.717 | -0.026 | 16845.315 |  | 16872.312 |  |
| 36.5 | 17022.424 | 0.044 | 17050.068 | -0.003 | 16839.939 |  | 16867.686 |  |
| 37.5 | 17016.894 | 0.041 | 17045.280 | -0.058 | 16834.426 |  | 16862.918 |  |
| 38.5 | 17011. 227 | -0.017 | 17040.354 |  | 16828.775 |  | 16858.012 |  |
| 39.5 | 17005.421 | -0.052 | 17035.290 | -0.056 | 16822.986 |  | 16852.968 |  |
| 40.5 | 16999.477 | 0.145* | 17030.087 | 0.072 | 16817.060 |  | 16847.786 |  |
| 41.5 | 16993.394 | 0.090 | 17024.746 | 0.090 | 16810.996 |  | 16842.465 |  |
| 42.5 | 16987.174 | -0.004 | 17019.266 | 0.011 | 16804.794 |  | 16837.007 |  |
| 43.5 | 16980.815 | 0.078 | 17013.647 |  | 16798.455 |  | 16831.410 |  |
| 44.5 | 16974.318 | 0.056 | 17007.890 | 0.031 | 16791.977 |  | 16825.674 |  |
| 45.5 | 16967.683 | 0.185* | 17002.994 | 0.009 | 16785.362 |  | 16819.801 |  |
| 46.5 | 16960.909 | 0.073 | 16995.953 | -0.030 | 16778.600 |  | 16813.789 |  |
| 47.5 | 16953.997 | 0.018 | 16989.785 | 0.033 | 16771.717 |  | 16807.638 |  |
| 48.5 | 16946.946 | 0.029 | 16983.472 | 0.021 | 16764.688 |  | 16801.349 |  |
| 49.5 | 16939.757 |  | 16977.020 | 0.044 | 16757.520 |  | 16794.921 |  |
| 50.5 | 16932.429 |  | 16970.429 | 0.034 | 16750.214 |  | 16788.354 |  |
| 51.5 | 16924.962 |  | 16963.698 | 0.060 | 16742.769 |  | 16781.648 |  |
| 52.5 | 16917.357 |  | 16956.829 | -0.002 | 16735.187 |  | 16774.804 |  |
| 53.5 | 16909.613 |  | 16949.820 | $0.123 *$ | 16727.465 |  | 16767.820 |  |
| 54.5 | 16901.730 |  | 16942.671 | -0.021 | 16719.606 |  | 16760.698 |  |

[^5]The estimated molecular constants of the $\mathrm{PCl}^{+}$ion in Table 8.2 merit further discussion. The rotational constants $B$ for both states of the isoelectronic SiCl molecule (76) are similar in magnitude with the corresponding constants of $\mathrm{PCl}^{+}$, as indicated by the following numerical values;

$$
\begin{aligned}
\text { SiCl: } \mathrm{B}_{10}^{\prime \prime} & =0.2397 \mathrm{~cm}^{-1}\left(\mathrm{X}^{2} I I\right) \\
\mathrm{B}_{0}^{\prime} & =0.1983 \mathrm{~cm}^{-1}\left(\mathrm{~A}^{2} \Sigma^{+}\right) \\
\mathrm{PCl}^{+}: \mathrm{B}_{16}^{\prime \prime} & =0.25933(12) \mathrm{cm}^{-1}\left(\mathrm{X}^{2} \Pi\right) \\
\mathrm{B}_{0}^{\prime} & =0.18770(12) \mathrm{cm}^{-1}\left(\mathrm{~A}^{2} \Pi\right)
\end{aligned}
$$

The spin-orbit coupling constant in the ground ${ }^{2}$ II state of $P \mathrm{~F}^{+}$is well-determined (70) from $\mathrm{a}^{2} \Sigma-\mathrm{X}^{2} I I$ transition as $323.95 \mathrm{~cm}^{-1}$. For the isovalent species SiF and Sicl, the ground state A" values are 161.9 (17) and 207.2 (18) $\mathrm{cm}^{-1}$ (53) respectively. If a similar halogen dependence holds for $\mathrm{PF}^{+}$and $\mathrm{PCl}^{+}$, $\mathrm{A}^{\prime \prime}$ for $\mathrm{PCl}^{+}$should be $\sim 370 \mathrm{~cm}^{-1}$, considerably larger than the value in Table 8.2 of - $282 \mathrm{~cm}^{-1}$.

A probable explanation for this anomaly can be obtained by considering the absolute spin-orbit constants obtained recently for a similar ${ }^{2} \Pi-^{2} \Pi$ system, namely SiN $D^{2} \Pi-$ $A^{2} \Pi$. For the 4-3 band of this sys'em Linton (96) estimated $A_{4}=-45.14(97) \mathrm{cm}^{-1}$ and $A_{3}^{\prime \prime}=-72.18(94) \mathrm{cm}^{-1}$. Bredohl et al. (97) analyzed the same band with the $A_{3}^{\prime \prime}$ value already determined from the $K^{2} \Sigma-A^{2} I I$ transition as $A_{3}^{\prime \prime}=-88.65 \mathrm{~cm}^{-1}$. Knowing the spin-orbit constant of the $A^{2} I$ state a
corresponding estimate of $A$ for the $D^{2} I I$ state could be determined from the term values $F_{2}(J)-F_{1}(J)$ using the Hill-Van Vleck expression (2). The calculated value of A! was $-61.80 \mathrm{~cm}^{-1}$. Since the reported spin-orbit constants differed significantly between these two analyses, it was decided to reexamine the siN data from both studies, and if possible to determine the source of disagreement.

Firstly, the program for direct fitting of the measured line positions was used to obtain estimates of the spinorbit coupling constants using first Linton's data then those of Bredohl et al. In these initial fits, trial values of $A$ were taken from Ref. (97) and the spin-rotation constants were constrained to zero. The estimates of $A^{\prime}$ and $A^{\prime \prime}$ obtained from both fits were similar to the values reported by Linton, within the experimental error.

In the second set of fits, the spin-rotational constants of both states were set at approximate trial values calculated using the expression of Brown and Watson (98) relating $\gamma$ and $A_{\mathrm{D}}$, and with Veseth's formula (99) for $A_{D}$ (centrifugal distortion of spin-orbit coupling) and neglecting the first-order effect. The numerical values were calculated to be: $\gamma_{4}^{1} \approx-1.6 \times 10^{-2} \mathrm{~cm}^{-1}$ and $\gamma_{3} \approx 2.9 \times 10^{-3} \mathrm{~cm}^{-1}$. The precision of the measured line positions from both studies was not sufficient to permit the simultaneous determination of $\gamma_{v}^{\prime}$ and $\gamma_{v}^{\|}$. However, when $\gamma_{4}^{\prime}$ was constrained to its trial value, it was of much interest
to find that the estimated spin-orbit parameters were $A_{4}=-57.7(1) \mathrm{cm}^{-1}, A_{3}^{11}=-84.8(1) \mathrm{cm}^{-1}$; very close to the values reported by Bredohl et al. (97). From this discussion, it can be concluded that the absolute values of $A_{V}^{\prime \prime}$ and $A_{0}$ of $\mathrm{PCl}^{+}$determined in Table 8.2 are considerably less reliable than is suggested by the quoted standard errors, and it is of some interest to consider this topic in more detail. The two spin-orbit coupling constants $A^{\prime}$ and $A^{\prime \prime}$ of $a^{2} I I-{ }^{2} I I$ transition are highly correlated. The determination of separate values of $A^{\prime}$ and $A^{\prime \prime}$ with direct fitting program such as that employed presently is made possible only when the $Y=A / B$ values for the two states are significantly different, since the spin-orbit splittings then have different $J$-dependences for the two states, as follows. In Hund's case (a), the analytical expressions for the rotational terin values of the spin-orbit components of a ${ }^{2}$ II state are given quite adequately by expression (2),

$$
\begin{equation*}
F(J)=B_{e f f} J(J+1)-D_{v} J^{2}(J+1)^{2} \tag{8.2}
\end{equation*}
$$

where the effective rotational constant $B_{\text {eff }}$ is slightly different for each spin-orbit component and where terms independent of $J$ have been omitted. Furthermore, Mulliken (33) showed that $B_{\text {eff }}$ can be expanded as a series, with the first two members given by,

$$
\begin{equation*}
B_{\text {eff }}=B(1 \pm B / A) \tag{8.3}
\end{equation*}
$$

where $B$ is the mechanical rotational constant and the "+" and "-" are for the $F_{2}$ and $F_{1}$ components respectively. For
a transition in which both $Y$ values are large, the magnitudes of the J-dependences become small, and can be comparable with higher order effects (centrifugal distortion of spin-orbit coupling and second order spin-rotation, which are strongly correlated (98)). As a result, the absolute magnitude of the fitted spin-orbit coupling constants obtained from a ${ }^{2} \Pi$ - ${ }^{2} \Pi$ transition can depend significantly on the corresponding $\gamma$ (or $A_{D}$ ) values. The precision of the present data on $\mathrm{PCl}^{+}$could not afford determinations of $\gamma_{\mathrm{v}}$ for either state, so that $A^{\prime}$ and $A^{\prime \prime}$ are then strictly effective parameters that absorb unknown spin-rotational/ centrifugal distortion effects. Data of better precision from a future study or from the photoelectron spectrum of PCl might resolve this problem.

## Chapter 9

## Unassigned Bands of $\mathrm{PBr}^{+}$

### 9.1 Introduction

After the observation and analysis of the emission due to the phosphorous monochloride cation, it was of interest to investigate the chemiluminescence reaction of discharged helium with $\mathrm{PBr}_{3}$. The emission observed was not known hitherto, and is tentatively attributed to a new ion, $\mathrm{PBr}^{+}$.

Although it has not been possible to arrange the numerous bands of $\mathrm{PBr}^{+}$in a Deslandres table, and hence to derive the vibrational constants, it is believed that this work will stimulate other spectroscopic investigations on this molecule.

### 9.2 Results and Discussions

The $\mathrm{PBr}^{+}$emission spectrum obtained at low resolution in the present work, under the same conditions as for the other ions, consists of a large number of red-degraded bands in the range $13000-24500 \mathrm{Cl}^{-1}$, with the most intense emission in the range $15300-17350 \mathrm{~cm}^{-1}$. Figure 9.1 shows a portion of the spectrum and illustrates the presence of "quadruple-headed" bands. As discussed later, this characteristic feature is a principal source of difficulty in vibrationally assigning the $\mathrm{PBr}^{+}$bands.

The wavelengths ( $\AA$ ) and vacuum wavenumbers of the
unassigned band-heads are given in Table 9.1. The calibration of the spectrum was obtained by least squares fitting of emission lines of atomic $\mathrm{Br}, \mathrm{P}$ and He (45) to a linear function in $d$, the distance along the trace. The standard deviation of this fit was ~ 0.07 A . The $\mathrm{PBr}^{+}$ single bandhead positions were determined at half maximum intensity above the baseline and have an estimated uncertainty of $\sim 0.6 \AA$.

By comparison with the isovalent ions $\mathrm{FCl}^{+}$and $\mathrm{AsCl}^{+}$, it is reasonable to believe that the two states involved in this emission should have a similar well-bound ${ }^{2}$ II ground state and a weakly-bound excited state, most probably ${ }^{2} \Pi$. However, the two factors that complicate the $\mathrm{PBr}^{+}$spectrum to a greater degree than for the other reported ions are, i) the similarity in magnitude of the spin-orbit splitting and the ground state vibrational spacings, and ii), the existence of two equally abundant ${ }^{79} \mathrm{Br}$ and ${ }^{81} \mathrm{Br}$ isotopes. The corresponding closeness of heads causes difficulty in obtaining accurate measurements of the heads of the overlapped longer wavelength members of each "quadrupleheaded" band, which are very important in determining accurately the isotope shift.

Recently, the vibrational constant $\omega_{\mathrm{O}}^{\prime \prime}(\mathrm{X})$, of the PSe molecule, isoelectronic with $\mathrm{PBr}^{+}$, was reported in Ref. (100) as $556.8 \mathrm{~cm}^{-1}$. Considering that the ratio of $\omega_{0}^{\prime \prime}$ (X) for the isoelectronic species $P C l^{+}+P S$, which was reported
0.93 , is probably very similar to the corresponding ratio for $\mathrm{PBr}^{+}$and PSe , the ground state vibrational constant of $\mathrm{PBr}^{+}$is estimated as $\sim 530 \mathrm{~cm}^{-1}$.

This value is in very good agreement with the observed difference of $\sim 490 \mathrm{~cm}^{-1}$ between the shorter wavelength members of the multiple-headed bands assigned to $\mathrm{PBr}^{+}$. The expected value of $\omega$ ! for the excited state of $\mathrm{PBr}^{+}$, should be similar to the estimate of $\omega$ : for the $A^{2} \Sigma$ state of the SiBr and the ${ }^{2} I I$ excited state of the $\mathrm{AsCl}^{+}$ion. On the other hand, a value for the spin-orbit separation cannot be predicted from the isoelectronic species, since the question of whether the excited state, if ${ }^{2} \Pi$, is regular or inverted, cannot be addressed at the present time. Nevertheless, there is little doubt that much of the spin-orbit separation is due to spin-orbit coupling in the ground state. Therefore, $\Delta A$ is expected to be greater than $\sim 183 \mathrm{~cm}^{-1}$, as for $\mathrm{PCl}^{+}$, assuming the halogen dependence for the ground state spin-orbit coupling constants of $\mathrm{PCl}^{+}$and $\mathrm{PBr}^{+}$is similar to that of their isoelectronic species $\operatorname{sicl}$ ( $A^{\prime \prime} \sim 207.2 \mathrm{~cm}^{-1}$ ) and SiBr ( $A^{\prime \prime} \sim 419.2 \mathrm{~cm}^{-1}$ ), see Tables $7.4,7.8$.

In conclusion, it seems likely that the use of isotopically pure $\mathrm{PBr}_{3}$ as the substrate molecule would be of much help in obtaining a simplified spectrum, from which an unequivocal assignment of the bands could be achieved. It should also be mentioned that photoelectron spectroscopy on PBr will be very helpful in determining the spin-orbit
coupling constants of the ground or of both the electronic states of $\mathrm{PBr}^{+}$.

Table 9.1: Wavelengths and Vacuum Wavenumbers for the Unassigned Band System of $\mathrm{PBr}^{+}$
$\left.\begin{array}{llll}\hline & & & \\ \hline & \nu_{\mathrm{vac}}(\mathrm{Cm}\end{array}{ }^{-1}\right) \quad \lambda(\AA) \quad \nu_{\mathrm{vac}}\left(\mathrm{cm}^{-1}\right)$

Table 9.1: Wavelengths and Vacuum Wavenumbers for the Unassigned Band System of $\mathrm{PBr}^{+}$(cont'd.)

| $\lambda(A)$ | $\nu_{\mathrm{vac}}\left(\mathrm{cm}^{-1}\right)$ | $\lambda(A)$ | $\nu_{\mathrm{vac}}\left(\mathrm{cm}^{-1}\right)$ |
| :---: | :---: | :---: | :---: |
| 5444.7 | 18361.5 | 6475.2 | 15439.2 |
| 5459.7 | 18310.9 | 6489.8 | 15404.6 |
| 5503.8 | 18164.3 | 6497.0 | 15387.4 |
| 5509.7 | 18144.8 | 6511.4 | 15353.5 |
| 5527.7 | 18085.5 |  |  |
| 5531.0 | 18074.7 | 6618.7 | 15401.6 |
|  |  | 6626.7 | 15086.3 |
| 5600.3 | 17851.3 | 6680.7 | 14964.4 |
| 5614.4 | 17806.4 | 6715.1 | 14887.6 |
| 5621.0 | 17785.4 | 6734.9 | 14843.9 |
| 5639.4 | 17727.4 |  |  |
| 5645.0 | 17710.0 | 6836.7 | 14623.0 |
| 5695.2 | 17553.8 | 6844.1 | 14607.1 |
|  |  | 6853.4 | 14587.3 |
| 5753.2 | 17376.9 |  |  |
| 5760.3 | 17355.4 | 6935.5 | 14414.7 |
| 5771.5 | 17321.8 | 6971.3 | 14340.6 |
| 5778.7 | 17300.2 | 6979.1 | 14324.6 |
| 5809.1 | 17209.5 |  |  |
| 5819.4 | 17179.1 | 7083.8 | 14112.9 |
| 5892.7 | 16965.5 | 7094.0 | 14092.6 |
| 5908.7 | 16919.6 | 7121.2 | 14038.8 |
|  |  | 7129.1 | 14023.2 |
| 5920.2 | 16886.7 | 7223.8 | 13839.4 |
| 5928.0 | 16864.4 | 7232.4 | 13822.8 |
| 5939.1 | 16832.9 |  |  |
| 5983.7 | 16707.3 | 7238.6 | 13622.8 |
| 6078.5 | 16447.0 | 7348.0 | 13605.4 |
|  |  | 7349.8 | 13602.1 |
| 6092.5 | 16409.0 | 7384.0 | 13539.0 |
| 6097.3 | 16396.2 | 7393.4 | 13521.8 |
| 6105.0 | 16375.4 |  |  |
|  |  | 7492.6 | 13342.8 |
| 6124.0 | 16324.6 | 7502.6 | 13325.0 |
| 6170.2 | 16202.5 |  |  |
| 6188.0 | 16155.9 | 7610.4 | 13136.4 |
|  |  | 7624.1 | 13112.7 |
| 6280.5 | 15917.9 | 7665.0 | 13042.7 |
| 6285.6 | 15904.9 | 7676.2 | 13023.6 |
| 6290.9 | 15891.6 |  |  |
| 6300.3 | 15867.7 |  |  |
| 6391.9 | 15640.4 |  |  |
| 6412.5 | 155 |  |  |

## Chapter 10

## Additional Reactions of Discharged Helium

The spectra described in Chapters 5-9 from various reactions of discharged helium have been investigated quite thoroughly. In the present chapter, preliminary observations on reactions with additional substrates will be described. Most of the spectra have been observed previously by other means of excitation and have been analyzed with different degrees of completeness by other workers.

The observations are summarized in Table 10.1. Each entry of this table indicates the reagent used, the excited molecule, the nature of the transition, and the spectral range. The following sections provide further detail on the individual reactions:

Table 10.1: Survey of Some Emission Spectra Arising from Discharged Helium Impact Reactions

| Reagent | Observed Emission | Range (A) | Ref. |
| :--- | :--- | :--- | :--- |
| $\mathrm{N}_{3} \mathrm{~S}_{3} \mathrm{Cl}_{3}$ | $\mathrm{NS}\left(\mathrm{B}^{2} \Pi_{r}-\mathrm{X}^{2} \Pi_{r}\right)$ | $3150-5500$ | $(81,102)$ |
| $\mathrm{PSCl}_{3}$ | $\operatorname{PS}\left(\mathrm{~B}^{2} \Pi_{\mathrm{r}}-\mathrm{X}^{2} \Pi_{r}\right)$ | $3400-6300$ | $(77)$ |
|  | $\operatorname{PS}\left(\mathrm{C}^{2} \Sigma-\mathrm{X}^{2} \Pi_{r}\right)$ | $2700-3400$ | $(102,104)$ |
|  | $\mathrm{PS}^{+}\left(\mathrm{A}^{1} \Sigma^{+}-\mathrm{X}^{1} \Sigma^{+}\right)$ | $2480-2750$ | $(105)$ |
| $\mathrm{VOCl}_{3}$ | $\mathrm{VO}\left(\mathrm{C}^{4} \Sigma^{-}-\mathrm{X}^{4} \Sigma^{-}\right)$ | $4300-6200$ | $(106)$ |
| $\mathrm{POCl}_{3}$ | $\mathrm{PO}\left(\mathrm{B}^{2} \Sigma^{+}-\mathrm{X}^{2} \Pi_{\mathrm{r}}\right)$ | $3250-3450$ | $(107)$ |
| $\mathrm{BCl}_{3}$ | $\mathrm{BCl}\left(\mathrm{A}^{1} \Pi-\mathrm{X}^{1} \Sigma^{+}\right)$ | $2700-2850$ | $(107)$ |
| $\mathrm{BBr}_{3}$ | $\operatorname{BBr}\left(\mathrm{a}^{3} \Pi_{\left.1,0^{+}-\mathrm{X}^{1} \Sigma^{+}\right)}\right.$ | $5100-5600$ | $(50)$ |
| $\mathrm{SbCl}_{3}$ | $\operatorname{SbCl}\left(\mathrm{~A}_{1}-\mathrm{X}^{3} \Sigma^{-}\right)$ | $4750-7000$ | $(110,111)$ |
| $\mathrm{NCl}_{3}$ | -- | -- | $(112)$ |

Trithiazyl chloride $\left(\mathrm{N}_{3} \mathrm{~S}_{3} \mathrm{Cl}_{3}\right)$ was prepared following the procedure of Ref. (101). The NS spectrum obtained from the reaction of discharged helium with $\mathrm{N}_{3} \mathrm{~S}_{3} \mathrm{Cl}_{3}$ was free from other interfering species. This can be compared with the method of microwave excitation of NS through a mixture of nitrogen and sulphur $(81,102)$ in which the high v" NS bands are overlapped strongly by the $B^{3} \Sigma_{u}^{-} \rightarrow X^{3} \Sigma_{g}^{-}$system of $S_{2}$. As a consequence, bands of NS with $\mathrm{v}^{\prime \prime} \geq 2$ could not be analyzed. Presently, the $B \rightarrow X$ system of the NS was extended towards longer wavelengths, and bands were observed with $\mathrm{v}^{\prime \prime}=10$. In Figure 10.1 a portion of the NS spectrum is shown. Quite recently, Karna and Grein (103) have performed ab initio CI calculations on $\mathrm{NS}^{+}$, using double-zeta basis sets with diffuse and polarization functions. Nine stable excited states of $\mathrm{NS}^{+}$were found. Among these excited states, only one ${ }^{1}$ II state is stable and lies at $\sim 6 \mathrm{eV}$ above the ${ }^{1} \Sigma^{+}$ground state. It is possible then that the unsuccessful search for a spectrum of $\mathrm{NS}^{+}$emission is due to the lack of response of the present spectrometer/photomultiplier outside the range 2200-7500 A.
(b) $\operatorname{PS}(B, C \rightarrow X)$ and $\operatorname{PS}^{+}(A \rightarrow X)$

The PS and $\mathrm{PS}^{+}$emission spectra observed in the present work are among the best examples of the helium afterglow method for providing very specific excitation. The extensive red-degraded band systems of PS and $\mathrm{PS}^{+}$were free
from overlap by any other bands, in particular emission due to $P_{2}$ or $S_{2}$. In Figures 10.2 and 10.3 portions of the PS and $\mathrm{PS}^{+}$spectra are shown. Jenouvrier and Pascal (77) have studied the $B \rightarrow X$ system of PS and rotationally analyzed 50 sub-bands with vibrational quantum numbers $v^{\prime} \leq 11$ and $v^{\prime \prime} \leq 6$. The $C \rightarrow X$ system has been studied vibrationally by Narasimham and Subramanian (102); the 2-0 and 1-0 bands have been rotationally analyzed by Narasimham and Balasubramanian (104). It was not possible to analyze more bands of this system because of $P_{2}$ and $S_{2}$ interference.

The $\mathrm{A} \rightarrow \mathrm{X}$ emission spectrum of $\mathrm{PS}^{+}$was first observed by Dressler (105). An attempt was made to analyze the present spectrum in order to obtain the first rotational constants of $\mathrm{PS}^{+}$. However, the intensity of the observed bands was not sufficient for high-resolution work. Improved techniques to obtain a more intense $\mathrm{PS}^{+}$emission spectrum would be advantageous.

Figure 10.1: A portion of the NS emission spectrum, showing bandheads belonging to the $B^{2} I I-X^{2} I I$ system.


Figure 10.2: A portion of the $B^{2} I I-X^{2} I I$ emission spectrum of PS.


Figure 10.3: A portion of the PS $^{+}(A \rightarrow X)$ emission spectrum.

(C) $\mathrm{He}_{2}^{*}+\mathrm{BCl}_{3} ; \mathrm{He}_{2}^{*}+\mathrm{BBr}_{3}$

In the pursuit of identification of new ions through their emission spectra, the Group 3 monohalide cations were the prime target since no optical emissions were known hitherto for these molecules. However, the reactions of $\mathrm{BCl}_{3}$ and $\mathrm{BBr}_{3}$ with discharged helium, generated in the fast flow system described previously, only produced emissions assigned to the corresponding neutral diatomics. According to a mass spectrometric study of photoionization of $\mathrm{BCl}_{3}$ by Dibeler and Walker (108), $\Delta E=18.37 \pm 0.02 \mathrm{eV}$ for the formation of $\mathrm{BCl}^{+}$in the ground state. It was not surprising then that even at large helium flow rates and pressure ( $\sim 4$ Torr), conditions which are known to enhance production of $\mathrm{He}_{2}^{+}$, insufficient energy ( $\sim 21 \mathrm{eV}$ ) was available from $\mathrm{He}_{2}^{+}$to form electronically excited $\mathrm{BCl}^{+}$.

However, in recent work by Yamaguchi et al. (109), a new emission spectrum was observed in the $4700-5150 \AA$ region from the flowing afterglow reaction of $\mathrm{BBr}_{3}$ and assigned to the $\mathrm{A}^{2} \Pi_{r}-\mathrm{X}^{2} \Sigma^{+}$transition of $\mathrm{BBr}^{+}$. From the vibrational analysis, the vibrational constants of both states of ${ }^{11} \mathrm{BBr}^{+}$ were determined. Although the afterglow apparatus was similar to the one used during the present study, lower total pressures ( $\sim 0.5$ Torr) of the discharged helium were achieved at high flow rates using a fast $600 \mathrm{~m}^{3} / \mathrm{h}$ mechanical booster pump. This supports the claim of Yamaguchi and coworker (109) that the active helium species responsible for the $\mathrm{BBr}^{+}$emission was $\mathrm{He}^{+}$, and that the minimum energy
required for the production of the $\mathrm{A}^{2} \Pi_{1 / 2}$ state of $\mathrm{BBr}^{+}$was $19.2 \pm 0.5 \mathrm{eV}$.
(d) $\mathrm{He}_{2}^{*}+\mathrm{SbCl}_{3}$.

Most of the red-degraded bands in the range of 17800$20900 \mathrm{~cm}^{-1}$ observed from the reaction of discharged helium with $\mathrm{SbCl}_{3}$ have been assigned to the $\mathrm{A}_{1}-\mathrm{X}$ system of SbCl . This system has been studied vibrationally by two separate groups. Ferguson and Hudes (110) had first seen this emission and assigned the most intense bands to a $v^{\prime}=0$ progression. In the later analysis by Avasthi (111), the spectra were recorded at higher resolution and a larger number of bands were vibrationally assigned. Although the same system seems to have been observed by both groups, the common bands were not assigned to the same quantum numbers. In the present work on $\operatorname{SbCl}$, all the bands of the $A_{1} \rightarrow X$ system reported previously were also observed.

The relative intensity distribution of the bands was more consistent with the original assignment by Ferguson and Hudes (110). Figure 10.4 shows a portion of SbCl spectrum. In addition, however, several new bands were observed in the longer wavelength region. The wavenumbers of the most intense of these bandheads, which were obtained directly from the wavelength drum reading of the spectrometer with an estimated uncertainty of $\sim 4 \mathrm{~cm}^{-1}$, are as follows: 16930, 16576, 16222, 15871, 15519, 15171, $14824 \mathrm{~cm}^{-1}$. It can be
readily seen that the difference between adjacent heads is $\sim 350 \mathrm{~cm}^{-1}$, similar to the corresponding difference of the bands belonging to the $A_{1} \rightarrow X$ system. Therefore, it is suggested that either these bands are members of the $v^{\prime}=0$ progression with high $v^{\prime \prime}$ of the $A_{1} \rightarrow X$ system or that they belong to a new system or sbcl having the same ground state or a state with similar vibrational constant.

The work on this molecule was not completed. Further work was precluded because the substrate molecule ( $\mathrm{SbCl}_{3}$ ), which is a solid compound at room temperature and which boils at $283^{\circ} \mathrm{C}$, requires modifications on the existing apparatus in order to increase the amount of vapour reaching the reaction zone; with more intense spectra it would be possible to identify bands of the $\mathrm{Sb}^{37} \mathrm{cl}$ isotope. In addition, more intense emission would allow the spectrum to be recorded at higher resolution, as is necessary in obtaining more accurate band head measurements for both chlorine isotopes.
(e) $\mathrm{He}_{2}^{*}+\mathrm{NCl}_{3}$.

Recently, Obase and co-workers (112) observed a new emission system in the 4400-5700 $\AA$ region from the reaction of $\mathrm{He}^{+}$with $\mathrm{NF}_{3}$. This emission is assigned to the $N F\left(c^{1} \Pi-b^{1} \Sigma^{+}\right)$transition. Prior to this publication a $12 \%$ mixture of $\mathrm{NCl}_{3}$ in $\mathrm{CCl}_{4}$ was prepared in this laboratory, according to the procedure reported by Noyes (113), and
reacted with discharged helium. The recorded spectrum did not indicate the existence of any emitting diatomic species. It is believed, now, that a more concentrated $\mathrm{NCl}_{3}$ solution might produce a corresponding $c^{1} \Pi-b^{1} \Sigma^{+}$emission of NCl and/or $a^{2} \Pi$ - $X^{2} \Pi$ emission of $\mathrm{NCl}^{+}$.

In conclusion, a significant portion of the research presently undertaken and presented in Chapters 5 through 10 of this thesis might encourage research on the kinetic aspects of the reactions reported.

Figure 10.4: A portion of the $A_{1}-X$ emission spectrum of SbCl .


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[^0]:    a The estimated values for individual bands were given by the output of a fit (Merge A, see text) in which multiple estimates of $B_{v}$ and $D_{v}$ for the two states were reduced to single-valued estimates.
    ${ }^{b}$ The fitted values are those given by the parameters (Table 3.3) of: a fit (Merge B) which separates the spin-rotation coupling in the two states to within an undetermined value of $r$ ".

[^1]:    ${ }^{\text {a }}$ The table lists positions of single lines and intensity-weighted blends calculated from the parameters of Merge (B). Blended positions are those for which $P^{*}(J)$ is listed in addition to $P(J)$ or $R(J)$. The residuals are defined as res $=\nu_{\text {obs }}-\nu_{\text {calc }}$. Data not included in the least-squares fits are indicated by as asterisk.

[^2]:    ${ }^{\text {a }}$ Bands not included in the least squares fits are flagged by an asterisk, see text.

[^3]:    "Values in parentheses correspond to one standard deviation in units of the last digit of the corresponding parameters.
    ${ }^{b} \Delta A=A_{e}{ }^{\prime}-A_{e}{ }^{\prime \prime}$

[^4]:    a Values in parentheses correspond to one standard deviation in units of the last digit of the corresponding parameters.
    b $\Delta G(1 / 2)$
    c Reduced masses of $\mathrm{As}^{35} \mathrm{Cl}^{+}$and $\mathrm{As}^{37} \mathrm{Cl}^{+}$, calculated from the atomic masses of ref. (58) are 23.841219 and 24.752939 amu , respectively.

[^5]:    - Lines flagged by * were excluded from the least squares fits, All calculated lines positions are obtained from the parameters of merge (B) (see text). Residuald are

