Modeling, Simulation and Optimization of Residential and Commercial Energy Systems

by

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DEDICATION

I would like to dedicate this research to the spirit of my beloved father, to my beloved mother, brothers, sisters, to my beloved wife Mbarka and my sweet daughter Sara.

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ABSTRACT

A Residential Energy Management System (REMS) in smart grid can be defined as processes of control systems designed to control, measure, monitor, and modify energy demand and energy consumption profiles. (REMS) provides capability to manage a daily load curve in order to reduce power consumption and energy cost. Consequently, (REMS) offers significant benefits for both the electricity suppliers and consumers in terms of control and schedule time of use of major appliances in the residential and commercial sectors.

In recent years, however, the rate of energy demand has increased rapidly throughout the world while the price of energy has been fluctuating. To find the solution for such problems, (REMS) establishes the optimal daily operation of home appliances. Numerous methods for (REMS) are used; this thesis analyzes many candidate scenarios during peak and off peak load periods comparing to the tariff of the residential sector to reduce the usage and its associated costs. It presents simulated results of proposed (REMS) to provide an automated least cost demand response. The main approach will be to ensure the satisfaction of the requirements with constraints on efficient use of energy. In this thesis, multiphasic system behavior and an individual set of components simulation of smart appliances in residential and commercial energy systems with a realistic manner are proposed.

LIST OF ABBREVIATIONS AND SYMBOLS USED

LIST OF ABBREVIATIONS

REMS	Residential Energy Management System
DSM	Demand-Side Management
EMS	Energy management system
ETP	The equivalent thermal parameters
WHAM	Water Heater Analysis Model
UEC	The average annual unit energy consumption
HVAC	Heating ventilation and air conditioning
DEWH	Domestic Electric Water Heater (A1)
DECW	Domestic Electric Clothes Washer (A2)
DECD	Domestic Electric Clothes Dryer (A3)
DECI	Domestic Electric Clothes Iron (A4)
DEBH	Domestic Electric Baseboard Heater (A5)
DEO	Domestic Electric Oven (A6)
DEDW	Domestic Electric Dishwasher (A7)
F-lamp	Fluorescent-lamp (A8)
CFL	Compact fluorescent lamps
LED	Light emitting diode
LPP	Linear Programming problem
LP	Linear Programming
BILP	Binary Integer Linear Programming
MBLP	Mixed Binary Linear Programming

LIST OF SYMBOLS

$T_H(t)$	Hot water temperature in tank (℃)
T_a	Ambient air temperature outside tank (℃)
T_{in}	Incoming inlet cold water temperature (℃)
T_s	Set-point temperature = (ΔT) (℃)
D_b	Dead band around the Set-point
T_{low}	Low water temperature (℃)
T_{high}	High water temperature (℃)
$W_d(t)$	Average hot water draw per hour (l/sec)
ρ	Density of water (kg/l)
V	Volume of tank (l)
C_p	Specific heat of the water ($J/kg.$℃)
SA	Surface area of tank (m^2)
U	Heat loss coefficient ($W/m^2.$℃)
R	Thermal resistance of tank ($m^2.$℃$/W$)
$Q(t)$	Rate of energy input (W); $Q = Prated$
P_{rated}	Power rated input of the heating resistance element (W)
C	Thermal capacity of water in the tank ($J/$℃)
R'	Thermal resistance $R' = \frac{1}{(G+B)}$ (℃$/W$)
G	Thermal conductance G =A/R ($W/$℃) where, R is the thermal resistance of the tank
$B(t)$	$W_d(t) * \rho * C_p$ ($J/sec.°C$)
τ	Time constant $\tau = R'C$ (sec)
$t_{on\ (startup)}$	Worming-up time
N_{cycles}	Number of cycling time
ΔP	Average power consumption
TOU	time of use
EC	Energy Consumption
$\lvert Z_f \rvert$	The forward impedance

$\lvert Z_b \rvert$	The backward impedance
$\lvert Z_i \rvert$	The input impedance
P_i	The input power
P_g	The air-gap power
P_m	The power mechanical
P_L	Power losses
n_{sync}	Synchronous speed
n_r	Rotor speed
ω_{sync}	The angular synchronous speed
T_m	Output torque
η	The efficiency of the motor
Z_p	The impedance of the parallel brunch of F-lamp circuit
Z_c	The impedance of the series brunch of F-lamp circuit
G_{lamp}	Lamp conductance
$f(u_i)$	Cost function
P_i	The TOU electricity price rate in ($\$/kWh$)
L	Load per appliance

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CHAPTER 1 INTRODUCTION

1.1 MOTIVATION BEHIND THE WORK

The energy demand has been growing over the last decades throughout the world while energy prices have been fluctuating over time. Accordingly, Residential Energy Management System (REMS) plays a significant role since it provides the capability to manage the daily load curve in order to reduce power consumption and energy cost. In addition, REMS offers significant benefits for both the electricity suppliers and consumers in terms of control and schedule time of use (TOU) of major appliances in the residential sector as well as in the commercial sector.

Numerous methods for REMS are used; this study analyzes many candidate scenarios during peak and off peak load periods comparing to the tariff of residential and commercial sectors to reduce the usage and its associated costs.

The investigation of modeling and simulation systems requires collecting some important data in order to design and simulate each model. An individual physical model in place of aggregated models with acceptable level of accuracy is used. It is worth mentioning that, instead of taking the parameter values from the literature, this study examines many mathematical models in order to calculate the thermal capacitance and the thermal conductance in terms of temperature and input power. In addition, to determine the appropriate model that reflects the parameters' impact as well as the system behavior of each individual model; multiphasic system behavior and an individual set of components simulation of appliances with a realistic manner are proposed. On the other hand, the mathematical model [3] with appropriate modification for obtaining expressions to calculate approximate temperature and time duration is determined by using a single-zone lumped-parameter thermal model.

1.2 THESIS OBJECTIVES AND CONTRIBUTION

There are three main objectives of this thesis:

First, all modeling of appliances are designed and implemented by using both the Matlab/Simulink and Matlab/SimScape modeling environments.

The Matlab/Simulink, Matlab/SimScape and MathWorks are used for modeling of heating systems, where Matlab/SimScape libraries contain many blocks that can be used for modeling thermal, hydraulic and mechanical components to model and simulate such systems in order to develop control systems and test system performance [1, 2].

Second, once the model of appliance is designed and simulated, the next step is to compare the simulation results to approximate linear equations and exponential equations; which are derived and used for estimating the amount of temperature, time duration and power/energy consumption [3].

Third, the main contribution of this study is to determine the optimal solution of time of use (TOU) in a case of an individual operation as well as in an aggregating operation, in order to reduce energy cost and to determine the best operation time by using Linear programming (LP, or linear optimization).

1.3 THESIS OUTLINE

This thesis is divided into seven main chapters. The first chapter presents the concept of the thesis and the second chapter which includes the literature reviews, begins with load management principles, basic definitions for power system evaluation and an introduction to the thermal system (heat and temperature).

The third chapter consists of the modeling of domestic electric water heater system in details; mathematical model and the system behavior during operation time. The forth chapter includes the modeling of heating systems such as domestic oven, domestic baseboard heater and domestic clothes iron; all these models are discussed in details of mathematical models and the system behavior during operation time. The fifth chapter contains the modeling of electric machine systems such as clothes washer, clothes dryer and dishwasher.

The sixth chapter contains an optimization method; the objective function of this chapter is to analyze many candidate scenarios during peak and off peak load periods comparing to the tariff of the residential and commercial sectors to reduce the usage and its associated costs. Thus, the optimization function seeks minimization of costs and the optimal daily operation of home appliances in order to find the solution for such problems. The main approach will be to ensure the satisfaction of the requirements with constraints on efficient use of energy [4].

The last chapter provides the conclusion, recommendations, and suggestions for future studies related to this thesis.

CHAPTER 2 LITERATURE REVIEW

2.1 INTRODCTION

An Electrical power system consists of three components:

- Generating Stations.
- Transmission Systems.
- Distribution Systems.

These three components of power system are integrated together to supply electricity to the consumers as shown in Figure 1:

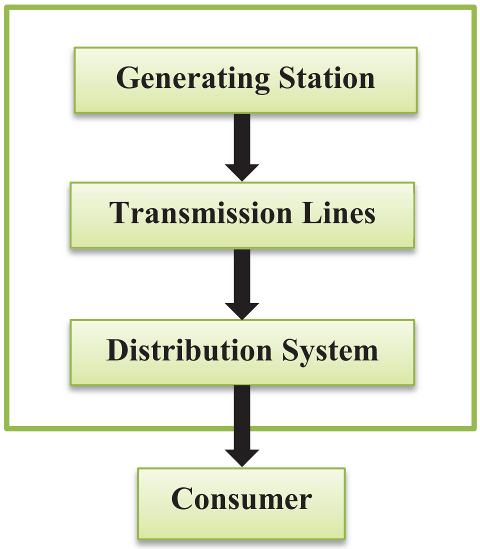

Figure 2.1 Components of electric power system [5]

2.2 LOAD MANAGEMENT

In the literatures, Load Management is defined as sets of objectives designed to control and modify the patterns of demands of various consumers in residential, commercial and industrial sectors of a power utility. This control and modification enables the supply system to meet the demand at all times in economic manner. Load Management can be applied to all the loads experienced by a power utility including cooling loads, heating loads and lighting loads. These loads vary by day, month and season. This means that load on the system is always changing with the time and is never constant. The utility will benefit from load management only to the extent that the actions taken will move energy consumption away from the utility's peak period [6].

2.3 BASIC DEFINITION

In order to understand the concept of Load Management clearly, some basic definitions in electric power systems are discussed [7].

- **Connected Load**: The rating in **(kW)** of the apparatus installed on consumer premises.
- **Maximum Demand**: the maximum load, which a consumer uses at any time.
- **Demand Factor:** The ratio of the maximum demand and connected load is given as:

  Demand factor = Maximum demand / connected load
- **Load Curve**: A curve showing the load demand of a consumer versus the time in hours of a day.
- **Load Factor**: The ratio of average load to the maximum load.

  Load factor = average load / maximum load

- **Load Shifting**: In both residential and commercial sectors, many electric customers cannot change the time they wash clothes, dishes, having shower, decades of results confirm that customers find small ways to manage their time of use in response to dynamic rates. These small changes add up to dramatic impacts. As a result, we can conclude that saving money by Shifting Load [5 and 7].
- **Strategic Conservation:** Strategic conservation is the load shape change that occurs from targeted conservation activities. This strategy is not traditionally considered by the utilities as a load management option since it involves a reduction in sales not necessarily accompanied with peak reduction [5].

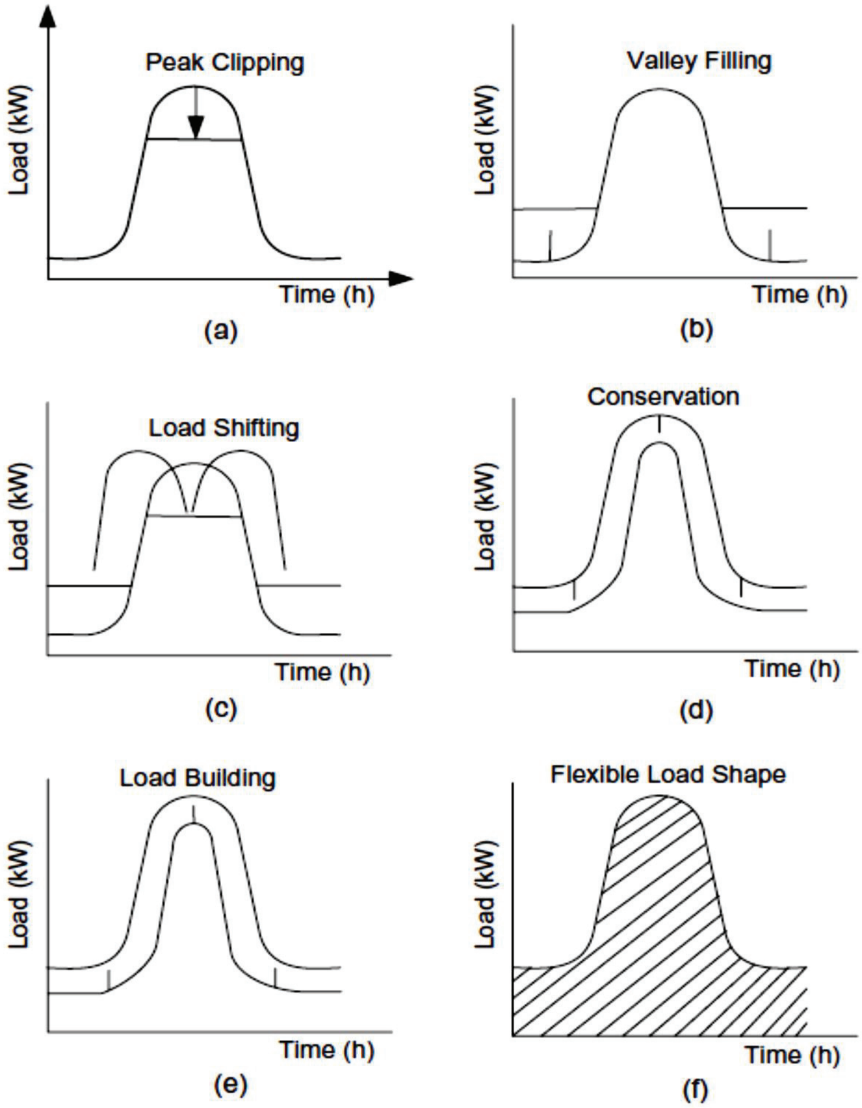

Figure 2.2 The load shape for standard DSM load management [32]

2.4 REVIEW OF SMART GRID

The Smart Grid is characterized by a two-way flow of electricity and information and will be capable of monitoring everything from power plants to customer preferences for individual appliances in residential and commercial sectors. It incorporates into the grid the benefits of distributed computing and communications to deliver real-time information and enable the near-instantaneous balance of supply and demand at the device level [8, 9].

2.5 THE THERMAL SYSTEM (HEAT AND TEMPERATURE)

Thermal systems are those that involve the transfer of heat from one substance to another. There are three different ways that heat can flow through conduction, convection, and radiation. Most thermal processes in process control systems do not involved radiation heat transfer [10].

Principle of Heat Transfer,

Heat transfer (q) is a thermal energy in transit due to a temperature difference $\varDelta\theta$.

Heat transfer (q) is the thermal energy transfer per unit time, can be express as:

$$q = K\,\varDelta\theta \tag{2.1}$$

Where:

q	: rate of heat flow, kcal/sec (W)
$\varDelta\theta$	: temperature difference, ºC
K	: coefficient, kcal/sec ºC

The coefficient (K) is given by

$$K = \frac{kA}{\varDelta X} \qquad \text{(For conduction)}$$

$$K = H\,A \qquad \text{(For convection)}$$

Where

K	: thermal conductivity, kcal/m sec ºC
A	: area normal to heat flow (m²)
ΔX	: thickness of conductor, (m)
H	: convection coefficient, kcal/m² sec ºC

2.6 MATHMATICAL MODEL OF THERMAL SYSTEM USING SIMSCAPE

Differential equations are used to describe physical component in the classical control system analysis. To get the response of the system we use the steps shown in figure (2.3) to derive the mathematical model. Most physical systems include some combination of mechanical, electrical, and hydraulic components. Mathematical models are used to predict system performance with considerable accuracy and adequate quality control of the materials used. Once the control system is described by differential equations, it is desired to solve them for some control variable (output) in response to a desired input function such as a step input, ramp input, or other function. In addition, the control system may have some initial conditions. With the advent of total computer control, it is possible to use iterative techniques for the solution of control systems described by Laplace transforms. The technique of applying these iterative techniques to a control system has described as “simulation”. These techniques are implemented in this thesis as a modeling and simulation of domestic appliances to show the control system response for expected input signals [34 and 35].

Simscape: is a library of basic real world mechanics, hydraulics and electronics components. A system built from these blocks is modeled from the view of energy flow. The model works directly with physical parameters, so it approximates good real systems. Simscape generates differential equations from the modeled system that can communicate with other components from Matlab/Simulink after the simulation has been started [1].

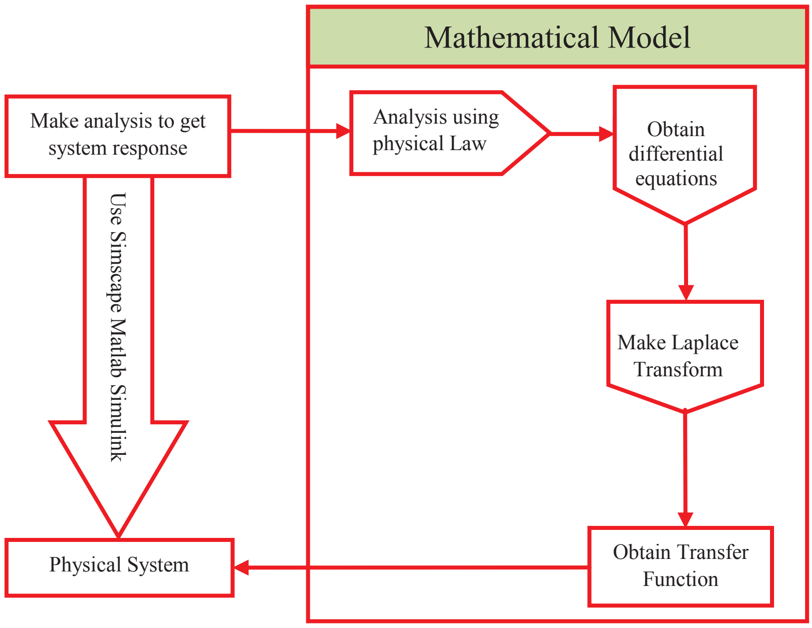

Figure 2.3 Steps of deriving system response [80]

In the literature, the heat transfer rate equation is given [34]

$$Q = K\ A\ \Delta T \tag{2.2}$$

Where,

Q	: heat generation (W)
ΔT	: temperature difference (K or ºC)
A	: heat exchange area (m²)
K	: heat transfer coefficient $W/m.℃$

- **ENERGY EQUATION OF THE THERMAL SYSTEMS:**

In order to investigate thermal behavior of the heater, the energy equation of the thermal systems can be predicted from conservation of energy applied to the heater as follows:

Energy Equation of the system is given [11 and 36]

$$W_{elec} - Q = mc_v \frac{dT}{dt} \tag{2.3}$$

Where

Q	: The rate of heat transfer from the heater
W_{elec}	: The rate of electrical energy input to the heater
mc_v	: Thermal capacity of the heater
m	: Mass of the heater
c_v	: Specific heat
dT/dt	: Temperature variations

By using an energy equation, we can express the rate of temperature change as:

$$\frac{dT}{dt} = \frac{1}{mc_v}[W_{elec} - Q] \tag{2.4}$$

The effect of temperature on electrical resistance of the heater material must be included and so the energy equation can be expressed as following:

$$W_{elec} = \frac{E^2}{R} = \frac{E^2}{R_o[1 + \alpha(T - T_o)]} \tag{2.5}$$

Where

E	: Applied voltage

R : Electrical resistance at operating temperature (T)

R_o : Electrical resistance at standard temperature (ºC)

α : Temperature coefficient of resistance

Modeling and simulation heating systems class is a challenge to predict the system behavior. These kind of systems are thermostatic loads, which means their performances are controlled by a thermostat; such systems include a domestic electric water heater, space heaters, ovens, refrigerators and air conditioning which is called heating ventilation and air conditioning (HVAC). In order to investigate the system behavior and load characteristics, Thermostatic Load Modeling by using thermal systems network in all modeling of systems is considered.

CHAPTER 3 MODELING OF ELECTRIC WATER HEATER

3.1 INTRODUCTION

Numerous models of diverse types of Domestic Electric Water Heaters (DEWHs) under consideration supply hot water for domestic use have been introduced in the literature. The objectives of these models are to analyze, calculate the energy consumption and to control the energy flow. For instance, the Water Heater Analysis Model (WHAM) was designed to calculate the energy consumption per day [13]. Other models were designed to obtain the EWH demand in order to control it by means of Demand Side Management (DSM) [12, 25, and 26].

Network elements for the thermal system as a thermal model are capacitance (C) and Resistance (R). Therefore, the equivalent thermal parameters (ETP) of an electric water heater circuit:

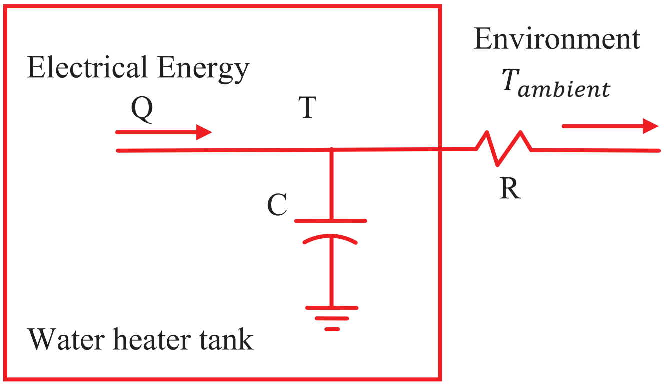

Figure 3.1 ETP heat transfer model of electric water heater

Table 3.1 Thermal to electrical analogies of various parameters

Thermal	Electrical
Heat flow rate (H = Q/ t)	Current (I)
Temperature (T)	Voltage (V)
Thermal Resistance ($R_{thermal}$)	Resistance (R)
Heat Capacity ($C_{thermal}$)	Capacitance (C)

Applying (KCL),

$$Q = (T_{water} - T_{ambient}) / R$$

R = 1/H → Q = $(T_w - T_a)$ H

For capacitor

Q = T/Xc

Q = $\frac{\dot{T}}{1/C} + (T_{water} - T_{ambient})$ H

- **Methodology**

The first-order differential equation (3.1) is used to implement a simple model of a DEWH, which represents the energy flow [3, 12] as follows:

$$C\dot{T}(t) = G\,(T_a - T_H(t)) + HW_d(t)\big(T_{in} - T_H(t)\big) + Q(t) \quad (3.1)$$

C = $\rho C_p V$ $(J/°C)$	**G** = SA.U = SA/R $(W/°C)$	**H** = ρC_p $(J/l.°C)$

Where,

C	: thermal capacity of water in the tank $(J/°C)$
$T_H(t)$	: hot water temperature in tank (°C)
T_a	: ambient air temperature outside tank (°C)
T_{in}	: incoming inlet cold water temperature (°C)
$W_d(t)$	: average hot water draw per hour (l/sec)
ρ	: density of water (kg/l)
V	: volume of tank (l)
C_p	: specific heat of the water $(J/kg.°C)$
SA	: surface area of tank (m^2)
U	: stand-by heat loss coefficient $(W/m^2.°C)$
R	: thermal resistance of tank $(m^2.°C/W)$
$Q(t)$	: rate of energy input (W); $Q = Prated$
P_{rated}	: power rated input of the heating resistance element (W)

3.2 MODELING OF (DEWH)

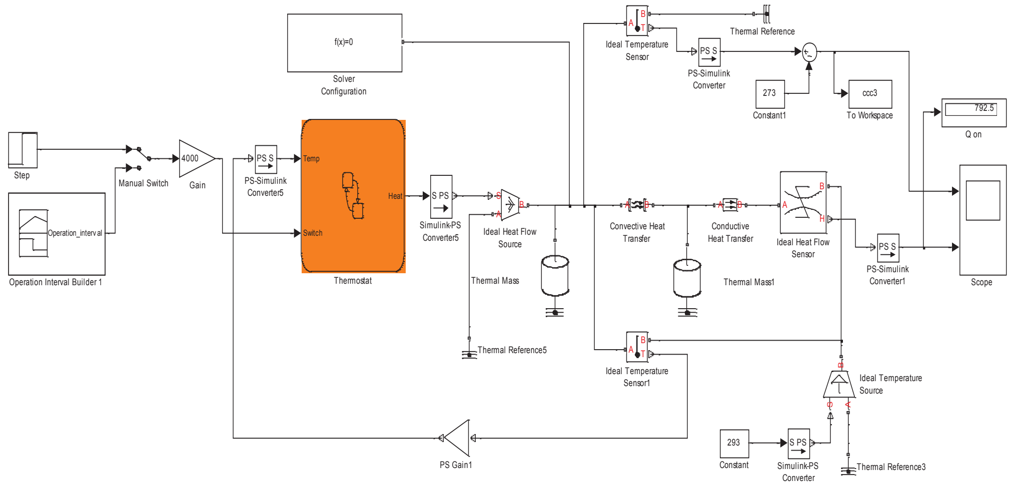

Figure 3.2 Model of Domestic Electric Water Heater

Table 3.2 DEWH parameters

Convection Parameters				
Convective Heat Transfer		Thermal Mass		
Area (m^2)	Heat Transfer coefficient ($W/m^2.$°C)	Mass (kg)	Specific heat Cp ($J/kg.$°C)	Initial Temperature (°C)
2.8	0.4965	189.27	4181	20

Conduction Parameters					
Conductive Heat Transfer			Thermal Mass		
Area (m^2)	Thickness (m)	Thermal Conductivity ($W/m.$°C)	Mass (kg)	Specific heat Cp ($J/kg.$°C)	Initial Temperature (°C)
1.1	0.005	0.8	1010	1.04	20

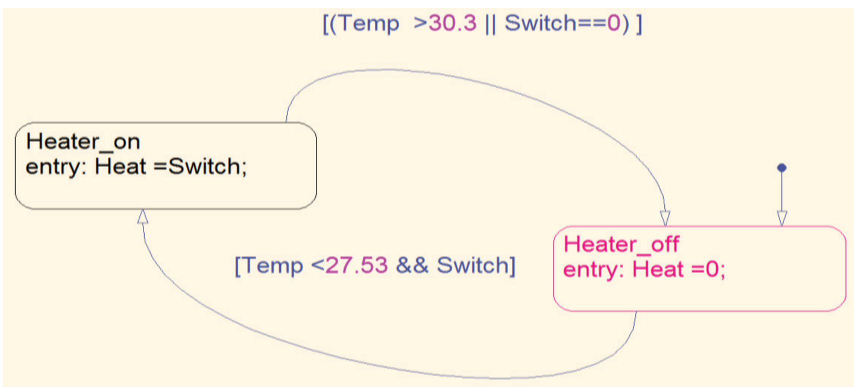

Figure 3.3 State flow model of the thermostat of DEWH

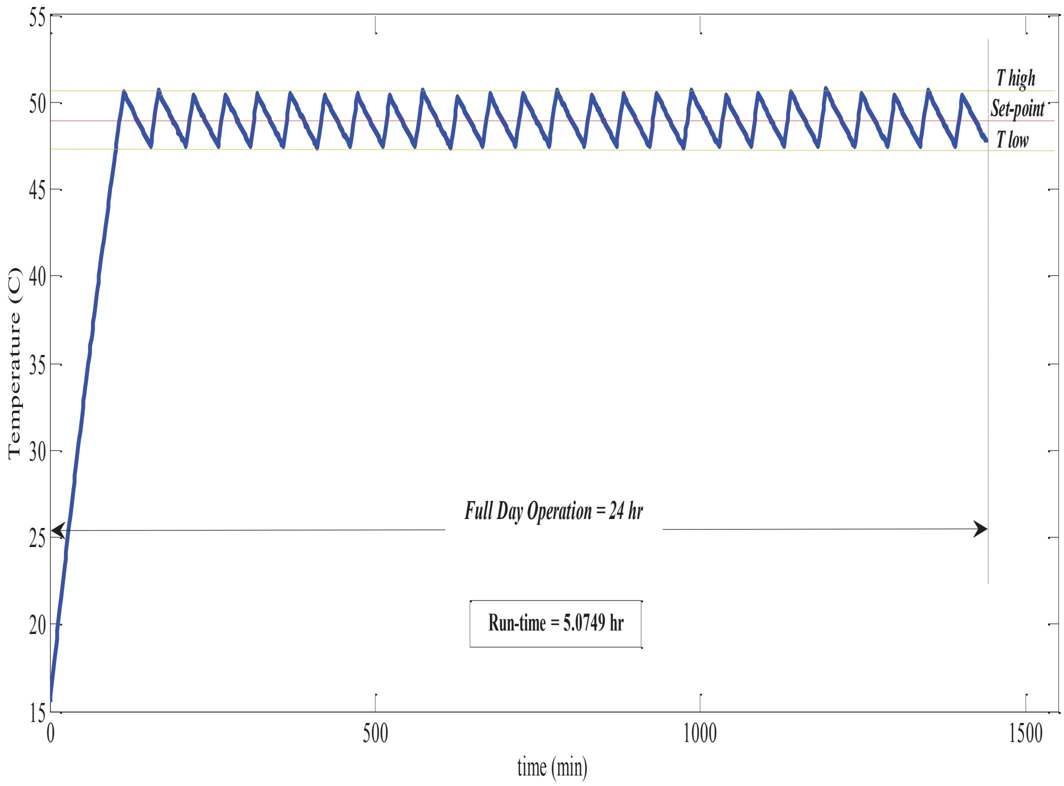

Figure 3.4 DEWH Temperature vs. Time full-day

The Domestic Electric Water Heater (DEWH) with one resistance element is implemented in this study by using a single-zone lumped-parameter thermal model [3].

DEWH is considered as an ideal candidate for Demand-Side Management (DSM) because the hot water in the tank acts as energy storage. Therefore, its operation time usually runs 24/7, which means this appliance should be plugged into electricity 24hrs/day; thus, we can conclude that the approximate run-time is 3hrs/day to keep the desired temperature between (47.64 ºC - 50.4 ºC) where the set-point is 49.02 ºC. A differential equation model of the (DEWH) is presented in the literature [1, 3]. This model is based mainly on energy flow analysis and yields a method to determine the temperature of the water in the tank as a function of time.

An equation that describes how $T_H(t)$ varies in time can be obtained by solving Eq. (3.1) as [12]

$$T(\mathrm{t}) = T_H(t_0)\, e^{-\left(\frac{1}{R'C}\right)(t-t_0)} + (R'G\,T_a + R'B\,T_{in} + R'Q) * \left[1 - e^{-\left(\frac{1}{R'C}\right)(t-t_0)}\right] \qquad (3.2)$$

Where,

τ	: time constant $\tau = R'C \quad (sec)$
T_{in}	: incoming water temperature (℃)
T_a	: ambient air temperature outside tank (℃)
T_{in}	: temperature of water in tank at time t (℃)
$T_H\,(t)$	: temperature of water in tank at time t (℃)
$Q\,(t)$	: energy input rate as a function of time (W)
R'	: thermal resistance $R' = \frac{1}{(G+B)} \quad (℃/W)$
G	: thermal conductance G =A/R $(W/℃)$ where, R is the thermal resistance of the tank $(m^2.℃/W)$
SA	: surface area of tank (m^2)
U	: stand-by heat loss coefficient $(W/m^2.℃)$
B	: B =HW_d
C	: thermal capacitance $(J/℃)$

The (Q) and (B) values are time dependent. Thus, the energy input (Q) is dependent on the element condition (on/off), and (B) is a function of the usage of water. Therefore, the value of (τ) and (T_H(τ)) must be updated every time there is a change in (B) or (Q). All other parameters can be measured, and must be known for accurate prediction of the temperature [12, 13, 14 and 15].

3.2.1 MATHEMATICAL ANALYSIS

The main purpose of this section is to discover and determine the effect of DEWH parameters during its run-time, by developing simple mathematical equations that represent the effect of varying parameters on the system behavior such as power and energy consumption. This can be achieved by deriving equations to obtain temperature of $\{T(t),\ T_{low}$ and $T_{high}\}$ and to determine time duration of $\{t_{on},\ t_{on\,(startup)}$ and $t_{off}\}$, the cycling time, and power consumed by DEWH from the energy flow is also considered.

Consider a DEWH tank that is standing idle. It periodically turns on its heating element in order to maintain the temperature of the water within a certain range. Thus, the approximate water temperature at the time (t) is given by solving Eq. (3.1) and with the appropriate modifications in Eq. (3.2) [3]. The following is an exponential equation that can be used

$$T(\mathrm{t}) = T_H(t_0)e^{-\frac{(t-t_0)}{\tau}} + K\left(1 - e^{-\frac{(t-t_0)}{\tau}}\right) \tag{3.3}$$

Where,

$\tau = \dfrac{C}{(G+B)}$	$K = \dfrac{GT_a + BT_{in} + Q}{(G+B)}$

More recent approaches have developed water draw profiles that are based on user behavior (e.g., showering and hand washing and so on); these events are used for calculation and analysis. These water profiles, measurements of water temperature and heater power consumption with respect to time interval could give results that are more accurate since a single-zone lumped parameter thermal model is considered for DEWH, which has a single resistance element. In this study many scenarios investigation is considered. These kinds of problems are challenging due to the lack of information availability and many factors that are related to the water draw profiles, the thermal resistance, thermal capacity and heat loss [3, 12]. However, this study focuses on time interval and water temperature by using the data which has been collected from system behavior analysis, in order to find suitable calculations that provide determination of the main parameters values that affect such problems. In this study the Simscape library within Matlab program is used, which provides more accuracy in figures data.

Research done by a group of researchers at the University of New Brunswick, 2012 [3] includes calculations considered to determine the thermal parameters, thermal resistance (R_{th}) and the thermal capacity (C_{th}) in terms of time, water temperature and input power. However, instead of taking those parameters values from the literatures, the system behavior in transient condition and steady state condition with material properties is considered for obtaining expressions that can be used to determine the value of all constant parameters by deriving some equations that represent operation cycles of the system performance. The main calculations of the mathematical model is based on the Lumped Parameter Method which is included in that study [3] with a few required modifications to develop simple mathematical equations that reflect this study and represent the effect of varying some parameters in this model of DEWH.

Table 3.3 Known parameters of DEWH

Input power (W)	Thermostat parameters $(°C)$			Ambient and inlet temperature $(°C)$		t_{off} (max) (sec)	$t_{on}(min)$ (sec)
$Q_i = P_{rated}$	T_{high}	T_{low}	$T_{s\,(set-point)}$	T_a	T_{in}	t_{off}^{m}	t_{on}^{m}
4000	50.4	47.64	49.02	20	15.5	54000	550.44

$$\boldsymbol{G} = \frac{Qi\,(\Delta - 1)}{\Delta(T_{high} - T_a) - (T_{low} - T_a)} \quad (3.4)$$

$$\boldsymbol{C} = G\left(\frac{t_{off}^{m}}{\ln(T_{high} - T_a) - \ln(T_{low} - T_a)}\right) \quad (3.5)$$

Where,

$$\Delta = \left(\frac{T_{high} - T_a}{T_{low} - T_a}\right)^{t_{on}^{m}/t_{off}^{m}} \quad (3.6)$$

By applying above formulas the results are:

Δ	G $(W/°C)$	C $(J/°C)$
1.00097	1.3919	789683.5264

In addition, the thermal capacitance can be calculated by $C = \rho C_p V$

ρ (kg/m^3)	C_p $(J/kg.°C)$	V (m^3)	C $(J/°C)$
998	4181	0.18927	789755.1943

The ratio of G/C can be calculated directly or by

$$G/C = \frac{\ln(T_{high} - T_a) - \ln(T_{low} - T_a)}{t_{off}^{m}} \quad (3.7)$$

$$\frac{G}{C} = 1.762566843 \times 10^{-6}\ sec^{-1} = \boldsymbol{\tau}^{-1}$$

G/C (sec^{-1})	$\boldsymbol{\tau} = C/G$ (sec)
1.7626×10^{-6}	567343.6968

Calculations of $T_H(t)$, T_{low} and T_{high} by using an exponential equation (3.3) and two scenarios of (K)

$\boldsymbol{\tau} = \dfrac{C}{(G+B)}$	$\boldsymbol{K} = \dfrac{GT_a + HW_d T_{in} + Q}{(G+B)}$

$B = HW_d\,,\ H = \rho C_p$

$B = HW_d$ $(J/sec.°C)$	Density ρ (kg/l)	Specific heat C_p $(J/kg.°C)$	Water draw W_d (l/sec)
26.3224	0.998	4181	6.30833×10^{-3}

Scenario 1,

$(K = 0)$ and $(t - t_0) = t_{on}^{m} = 550.44\ sec$ in case of natural response, the equation Eq. (3) yield,

$$T(\mathrm{t}) = T_H(t_0)e^{-\frac{(t-t_0)}{\tau}} \quad \rightarrow \quad T(\mathrm{t}) = 20\ °C$$

Scenario 2,

$(K = 2893.7696\ °C)$ In case of $B = 0$ thus, the equation of (K) yield

$$K = \frac{GT_a + Q}{G} = T_a + \frac{Q}{G}$$

By substituting $K = T_a + \frac{Q}{G}$, $\tau = \frac{C}{G}$ and $(t - t_0) = (t_{on}^{m})$ in Eq. (3.3) and rewritten,

$$T(\mathrm{t}) = T_H(t_0)e^{-(t_{on}^{m})\frac{G}{C}} + K\left(1 - e^{-(t_{on}^{m})\frac{G}{C}}\right) \tag{3.8}$$

t_{on}^{m} (sec)	G $(W/°C)$	C $(J/°C)$	K $(°C)$
550.44	1.3919	789683.5264	2893.7696

$T(\mathrm{t}) = 23\ °C$

By applying above equation to obtain the calculation of $T(\mathrm{t})$

	$T_H(t_0)$ $(°C)$	t_{on}^{m} (sec)	τ (sec)	K $(°C)$	$T(\mathrm{t})$ $(°C)$
Scenario 1 (natural response)	20	550.44	567343.6968	0	20
Scenario 2 (forced response)	20	550.44	567343.6968	2893.7696	23

Calculation of (T_{low}) ***and*** (T_{high})

$$T_{low} = T_{high}\ e^{-(t_{off}^{m})G/C} + T_a\left(1 - e^{-(t_{off}^{m})G/C}\right) \tag{3.9}$$

In this case, the heater is (OFF) thus, $Q = 0\,W \quad \rightarrow \quad K = T_a$

T_{high} (°C)	T_a (°C)	t_{off}^{m} (sec)	G (W/°C)	C (J/°C)	K (°C)
50.4	20	54000	1.3919	789683.5264	20

$T_{low} = 47.64\,°C$

$$T_{high} = T_{low}\, e^{-(t_{on}^{m})G/C} + \left(T_a + \frac{Q}{G}\right)\left(1 - e^{-(t_{on}^{m})G/C}\right) \qquad (3.10)$$

In this case the heater is (ON) thus, $Q = 4000\,W \quad \rightarrow \quad K = T_a + \frac{Q}{G}$

T_{low} (°C)	T_a (°C)	t_{on}^{m} (sec)	G (W/°C)	C (J/°C)	K (°C)
47.64	20	550.44	1.3919	789683.5264	202893.7696

$T_{high} = 50.4\,°C$

Set-point (ΔT) $= \frac{T_{high} + T_{low}}{2} = \frac{50.4 + 47.64}{2} = 49.02$ °C

Set-point (ΔT) $= T_s = 49.02$ °C

The dead band (D_b) around the Set-point (ΔT) $= T_s$ is usually varied between ($\pm$ 1.38 to 2.77 °C) [12]. The upper limit T_{high} and the lower limit T_{low} represent the dead-band of the thermostat around the thermostat set-point (ΔT) $= T_s$ which reflect a customer comfort choice.

Thus, in this model of DEWH the Dead band (D_b) value is:

$$\text{Dead band } (D_b) = \frac{T_{high} - T_{low}}{2} = \frac{50.4 - 47.64}{2} = 1.38\text{ °C}$$

Therefore,

$$T_{high} = T_d + D_b = 50.4\text{ °C}$$

$$T_{low} = T_d - D_b = 47.64\text{ °C}$$

Calculation Time: (t_{on}) and (t_{off})

- **Linear Equations of (t_{off}) and (t_{on})**

$$t_{off}^{m} = \frac{C(T_{high} - T_{low})}{G\,(T_s - T_a) + B(T_d - T_{in})} \qquad (3.11)$$

$$t_{on}^{m} = \frac{C(T_{high} - T_{low})}{G\,(T_a - T_s) + B(T_{in} - T_s) + Q_i} \tag{3.12}$$

CASE 1, *in this case, there is no draw of water from **DEWH*** $W_d = 0$

T_{high} (°C)	T_{low} (°C)	T_a (°C)	T_s (°C)	T_{in} (°C)	G (W/°C)	C (J/°C)	W_d (l/hr)	Q_i (W)
50.4	47.64	20	49.02	15.5	1.3919	789683.5264	0	ON/OFF

Linear Equations Results of case 1

t_{off}^{m} (sec)		t_{on}^{m} (sec)	
$Q_i = 0$	$Wd = 0$	$Q_i = 4000$	$Wd = 0$
53958		550.44	

*The results of case 1, indicate that the maximum duration of t_{off}^{m} and the minimum duration of t_{on}^{m}

CASE 2, *in this case, there is draw of water from **DEWH*** $Wd = 22.71\ (l/hr)$

T_{high} (°C)	T_{low} (°C)	T_a (°C)	T_s (°C)	T_{in} (°C)	G (W/°C)	C (J/°C)	W_d (l/hr)	Q_i (W)
50.4	47.64	20	49.02	15.5	1.3919	789683.5264	22.71	ON/OFF

Linear Equations Results of case 2

t_{off}^{m} (sec)		t_{on}^{m} (sec)	
$Q_i = 0$	$Wd = 22.71$	$Q_i = 4000$	$W_d = 22.71$
2362		708	

*The results of case 2, indicate that the minimum duration of t_{off}^{m} and the maximum duration of t_{on}^{m}

$Cycling\ time = t_{total}$ (sec)
$t_{on}^{m} + t_{off}^{m}$ = 708 + 2362 = 3070

- **Exponential Equations of** ***(t_{off}) and (t_{on})***

$$t_{off}^{m} = \frac{C}{G} \ln\left[\frac{(GT_a R'(t)) - T_{high} - R'B(T_s - T_{in})}{(GT_a R'(t)) - T_{low} - R'B(T_s - T_{in})}\right] \tag{3.13}$$

$$t_{on}^{m} = \frac{C}{G} \ln\left[\frac{GT_a R'(t) + Q_i R'(t) - T_{low} - R'B(T_s - T_{in})}{GT_a R'(t) + Q_i R'^{(t)} - T_{high} - R'B(T_s - T_{in})}\right] \tag{3.14}$$

CASE 1, *in this case, there is no draw of water from DEWH* $W_d = 0$

T_{high} $(°C)$	T_{low} $(°C)$	T_a $(°C)$	T_s $(°C)$	T_{in} $(°C)$	G $(W/°C)$	C $(J/°C)$	W_d (l/hr)	Q_i (W)
50.4	47.64	20	49.02	15.5	1.3919	789683.5264	0	ON/OFF

Exponential Equations Results of case 1

t_{off}^{m} (sec)		t_{on}^{m} (sec)	
$Q_i = 0$	$Wd = 0$	$Q_i = 4000$	$Wd = 0$
54000		550.44	

*The results of case 1, indicate that the maximum duration of t_{off}^{m} and the minimum duration of t_{on}^{m}

CASE 2, *in this case, there is draw of water from DEWH* $W_d = 22.71\ (l/hr)$

T_{high} $(°C)$	T_{low} $(°C)$	T_a $(°C)$	T_s $(°C)$	T_{in} $(°C)$	G $(W/°C)$	C $(J/°C)$	W_d (l/hr)	Q_i (W)
50.4	47.64	20	49.02	15.5	1.3919	789683.5264	22.71	ON/OFF

Exponential Equations Results of case 2

t_{off}^{m} (sec)		t_{on}^{m} (sec)	
$Q_i = 0$	$Wd = 22.71$	$Q_i = 4000$	$Wd = 22.71$
2362		708.26	

*The results of case 2, indicate that the minimum duration of t_{off}^{m} and the maximum duration of t_{on}^{m}

$t_{total-one\ period}$ (sec)
$t_{on}^{m} + t_{off}^{m} = 708.26 + 2362 = 3070.26$

Calculation Time: ($t_{on}^{start-up}$) ***<u>Transient Conditions</u>***

T_{high} $(°C)$	T_{low} $(°C)$	T_a $(°C)$	T_s $(°C)$	T_{in} $(°C)$	G $(W/°C)$	C $(J/°C)$	W_d (l/hr)	Q_i (W)
47.64	15.5	20	31.57	15.5	1.3919	789683.5264	22.71	4000

$$t_{on}^{start-up} = \frac{C(T_{high} - T_{low})}{G\,(T_a - T_s) + B(T_{in} - T_s) + Q_i} \tag{3.15}$$

$$t_{on}^{start-up} = \frac{C}{G} \ln\left[\frac{GT_aR'(t) + Q_iR'(t) - T_{low} - R'B(T_s - T_{in})}{GT_aR'(t) + Q_iR'(t) - T_{high} - R'B(T_s - T_{in})}\right] \tag{3.16}$$

Table 3.4 The results of equations and simulation in case 1

Method	Case 1		
	$t_{on}^{start-up}$ (hr)	Q_i (kW)	W_d (l/hr)
Linear Equation	1.7697	4	0
Exponential Equation	1.7697	4	0
Simulation	1.8787	4	0

$$Duty\ Cycle = (t_{on}/Cycling\ time) \tag{3.17}$$

t_{on}^{m} (hr)	$Cycling\ time$ (hr)
0.1967	0.8528

$$Duty\ Cycle = \mathbf{0.2307}$$

Also, in order to calculate the Duty Cycle value when there is hot water draw $W_d > 0$ instead of time calculation; specifically, the change of (W_d) will affect the value of parameter (B) since, $(B = HW_d)$ which means any increasing or decreasing of duty cycle value is regarding to some specific variations of (W_d) and the set-point (T_s) as well . In this case, we can conclude that the set-point (T_s) plays a significant role to adjust water temperature; on other hand, the value of (B) is due to water usage which is governed by consumer behavior. Thus, the value of duty cycle will increase as the value of (T_s) or (B) increases. Therefore, the duty cycle can be calculated by using [12] where, $(G, T_a, T_{in}$ and $Q_i)$ are assumed to be constant.

$$Duty\ Cycle = \frac{G(T_s - T_a) + B(T_s - T_{in})}{Q_i} \tag{3.18}$$

$$Duty\ Cycle = \mathbf{0.2307}$$

The average power consumption by DEWH (P_{DEWH}) from the energy flow can be calculated by

$$\Delta P_{DEWH} = P_{rated} * (t_{on} / t_{total}) \tag{3.19}$$

$$\Delta P_{DEWH} = 4\ kW * 0.2307 = 0.9228\ kW$$

Table 3.5, below shows the results obtained from linear equations, exponential equations, the average power and the simulation of the DEWH model in case of

$\boldsymbol{W_d = 6\ gal/hr = 22.71\ l/hr}$

Table 3.5 The comparison of the simulation results and equations

Method	Q_i (kW)	$t_{on}^{start-up}$ (hr)	t_{on}^{m} (hr)	t_{off}^{m} (hr)	$Cycling_{time}$ (hr)	$Duty\ Cycle$	ΔP_{DEWH} (kW)
Linear Equation	4	1.7697	0.1966	0.6561	0.8527	0.2307	0.9228
Exponential Equation	4	1.7697	0.1967	0.6561	0.8528	0.2307	0.9228
Simulation	4	1.8787	0.1958	0.6417	0.8375	0.2338	0.9352

3.2.2 ANALYSIS OF CYCLING TIME

By calculating the period while the thermostat is in position *ON*, the equations (3.12 and 3.14) show that t_{on} is dependent on the power supply since Q is proportional to the heater rated power P_{rated}. on the other hand, calculation of the period while the thermostat is in position *OFF*, the equation (3.11 and 3.13) show that t_{off} is independent of the power supply since Q = 0

$$\text{Cycling time} = t_{on} + t_{off} \tag{3.20}$$

Cycling time (hr)	t_{on} (hr)	t_{off} (hr)	W_d (l/hr)
0.8528	0.1967	0.6561	22.71

$$N_{cycles} = \frac{full\ operation\ periods - t_{on}(\text{start}-\text{up})}{cycling\ time} \tag{3.21}$$

$\boldsymbol{N_{cycles}}$	Full period Operation (hr)	$\boldsymbol{t_{on}}(\text{start}-\text{up})$ (hr)	Cycling time (hr)	W_d (l/hr)
26	24	1.7697	0.8528	22.71

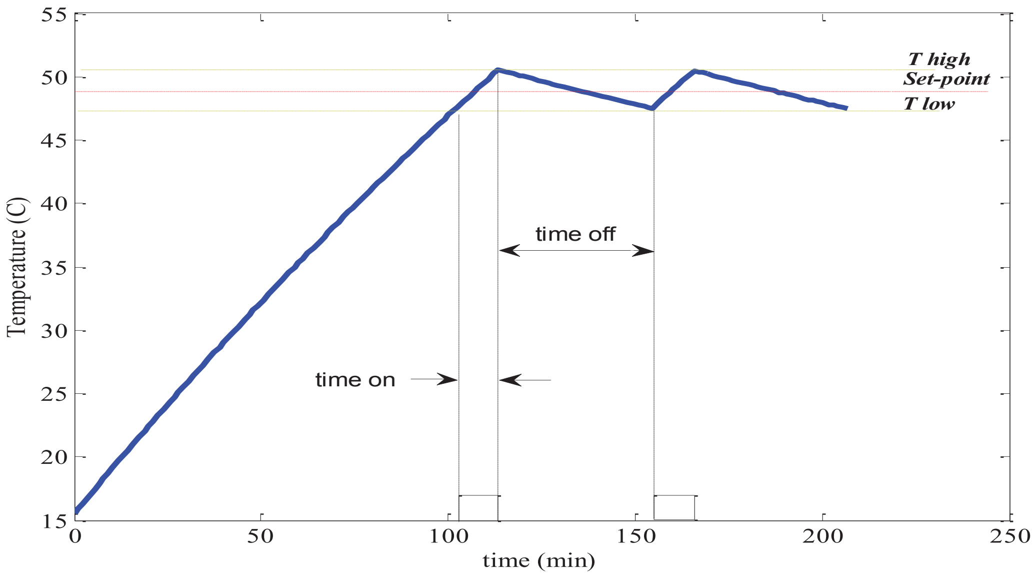

Figure 3.5 Simplified thermal characteristic curve of thermostat (cycling time)

Figure 3.5, of temperature as a function of time indicates that the DEWH has reached a steady state condition where the set-point is 49.02 $°C$.

$$\boldsymbol{t}_{on(total)} = (\boldsymbol{N}_{cycles} * \boldsymbol{t}_{on}) + \boldsymbol{t}_{on(\text{start-up})} \tag{3.22}$$

$t_{on(total)}$ (hr)	N_{cycles}	t_{on} (hr)	$t_{on(\text{start-up})}$ (hr)	W_d (l/hr)
6.8839	26	0.1967	1.7697	22.71

Power / Energy consumption of (DEWH):

$$Q_{on\ (kWh)} = P_{rated} * t_{on} \tag{3.23}$$

Table 3.6 The results of simulation and calculation

method	Q_{on} (kWh)	P_{rated} (kW)	t_{on} (hr)	W_d (l/hr)
Calculation	**0.7868**	4	0.1967	22.71
Simulation	**0.7832**	4	0.1958	22.71

$$Q_{total} = P_{rated} * t_{on(total)} \tag{3.24}$$

$\boldsymbol{Q}_{total}$ (kWh)	P_{rated} (kW)	$t_{on(total)}$ (hr)	W_d (l/hr)
20.4568	4	5.1142	22.71

$$\text{DEWH (kW)} = \frac{Q_{(total)}(kWh)}{t_{on(total)}(hr)} \quad (3.25)$$

Electric Heater (kW)	Q_{total} (kWh)	t_{on} (total) (hr)	W_d (l/hr)
4	20.4568	5.1142	22.71

Comparison of the results of the Exponential equations, Linear equations and the simulation results of the DEWH model.

Table 3.7 The comparison of equations and simulation results

Methods	$t_{on(start-up)}$ (hr)	t_{on} (hr)	t_{off} (hr)	$t_{on(total)}$ (hr)	**P** (kW)
Linear equations	1.7697	0.1966	0.6561	5.1116	4
Exponential equations	1.7697	0.1967	0.6561	5.1142	4
Simulation Result	1.8787	0.1958	0.6417	5.0908	4

Table 3.8 The time response in steady-state & transient conditions

Methods	**P** (kW)	$t_{on}^{start-up}$ (hr)	t_{on} (hr)	t_{off} (hr)	$cycling\ time$ (hr)	$Steady-State\ Condition$	$Transient\ Condition$
						$t_{on(total)}$ (hr)	$t_{on(total)}$ (hr)
Linear Equation	4	1.7697	0.1966	0.6561	0.8527	5.1116	6.8813
Exponential Equation	4	1.7697	0.1967	0.6561	0.8528	5.1142	6.8839
Simulation Result	4	1.8787	0.1958	0.6417	0.8375	5.0908	6.9695

3.2.3 COMPARISON OF MODELS

Comparison of DEWH model with another literature model:

In comparison of DEWH model with another literature model [12] in case of two scenarios by using a selected value of water daily usages ($\mathbf{W_d}$) in steady-state conditions as follows:

W_d		W_d	
(gal/day)	(l/day)	(gal/hr)	(l/hr)
115.97	438.99	6	22.71

	Unit	**DEWH model**	**Compared model**	**% Difference**
W_d	(gal/hr)	6	6	
	(l/hr)	22.71	22.71	
$T_{s\,(set-point)}$	℃	49.02	48.88	
t_{on}	(hr)	0.1967	0.1993	**1.3045**
t_{off}	(hr)	0.6561	0.6533	**0.4285**
$cycling\ time$	(hr)	0.8528	0.8526	**0.0234**
$Duty\ Cycle$		0.2307	0.2338	**1.32591**
ΔP_{DEWH}	(kW)	0.9228	0.9351	**1.3153**
Δ Energy Consumption	(kWh)	0.7868	0.7972	**1.3045**
$Q_i = P_{rated}$	(kW)	4	4	

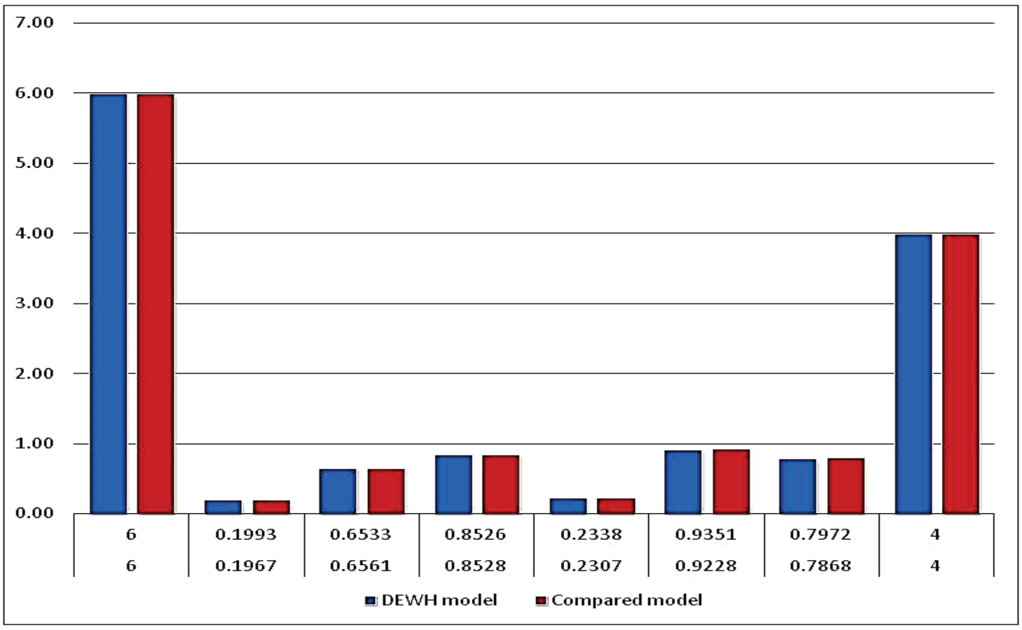

Figure 3.6 Comparison of DEWH model with another literature model

W_d		W_d	
(gal/day)	(l/day)	(gal/hr)	(l/hr)
115.92	438.99	5.3	20.06

	Unit	**DEWH model**	**Compared model**	**% Difference**
W_d	(gal/hr)	5.3	5.3	
	(l/hr)	20.06	20.06	
$T_{s\,(set-point)}$	℃	49.02	48.88	
t_{on}	(hr)	0.1904	0.1929	**1.2960**
t_{off}	(hr)	0.7385	0.7339	**0.6267**
$cycling\ time$	(hr)	0.9289	0.9267	**0.2374**
$Duty\ Cycle$		0.2050	0.2082	**1.5369**
ΔP_{DEWH}	(kW)	0.8198	0.8328	**1.5609**
Δ Energy Consumption	(kWh)	0.7616	0.7716	**1.2960**
Q_i = P	(kW)	4	4	

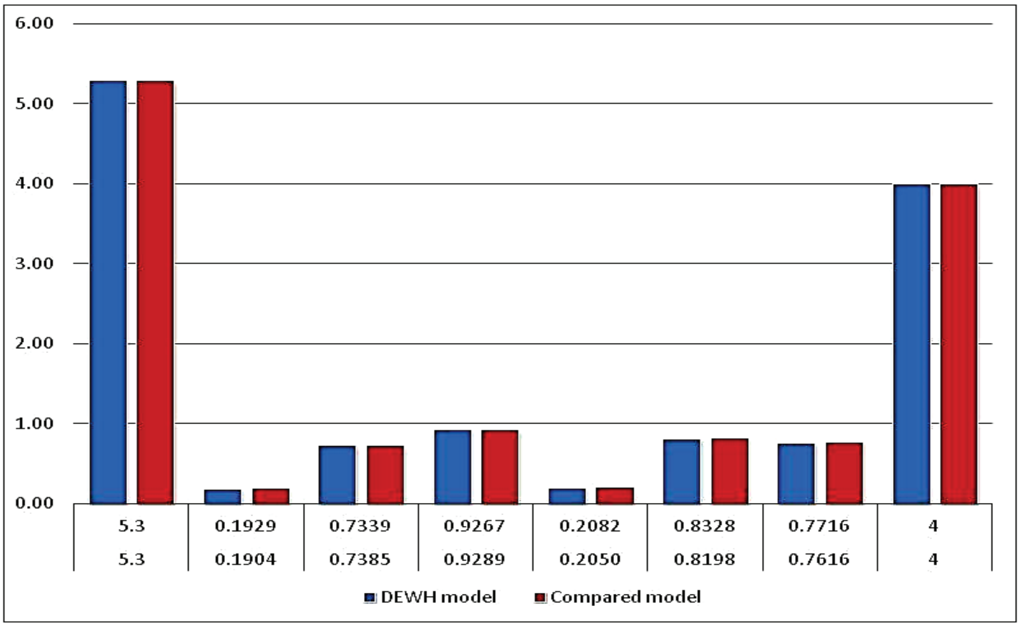

Figure 3.7 Comparison of DEWH model with another literature model

It is worth mentioning that in this study of (DEWH) model:

1. t_{on} is directly proportional with the quantity of hot water usages W_d
2. t_{off} is inversely proportional with the quantity of hot water usages W_d
3. The amount of hot water usages Wd is governed by user behavior, events and family members as well.

4. When there is no water draw $W_d(t) = 0$, in this case, the minimum duration of t_{on} and the maximum duration of t_{off} occurs.

5. The parameter of thermal conductance **(G)** is obtained by calculation in terms of time, temperature and in-put power since in the compared model the value of parameter (G) is taken directly from literature.

3.2.4 LOAD PROFILE AND ENERGY COST

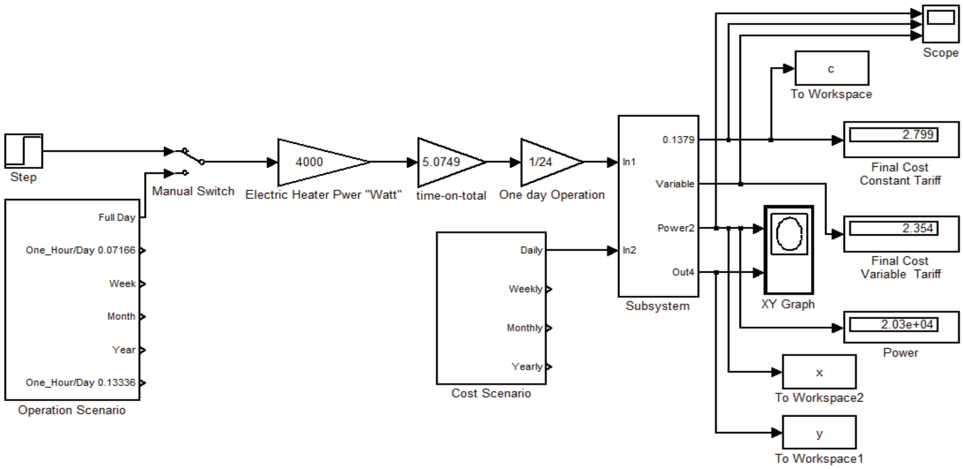

Figure 3.8 Energy Cost Calculator of Electric Water-Heater

Simulation Results of Electric Water-Heater Full Day (TOU)

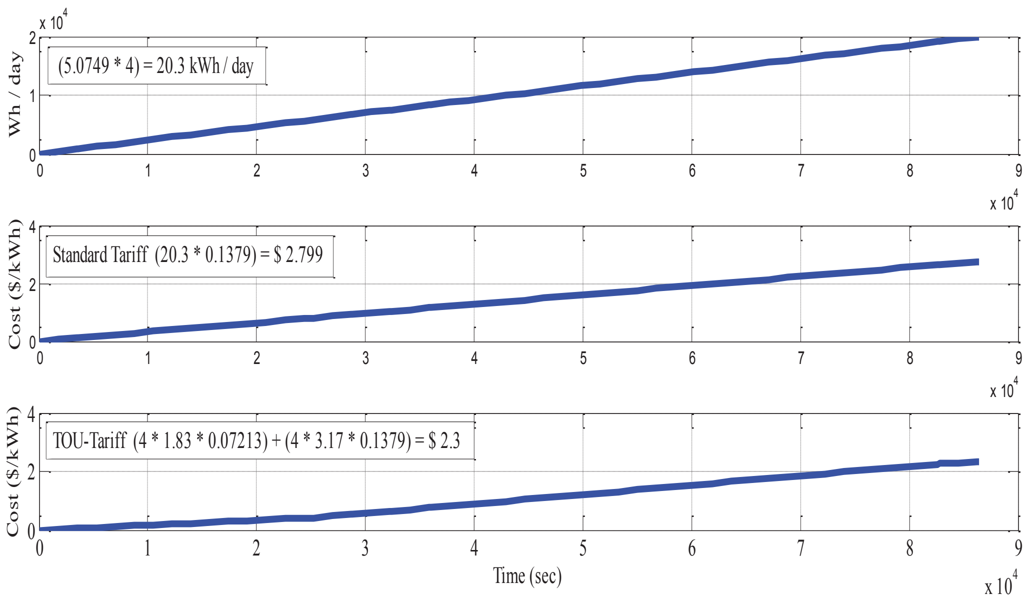

Figure 3.9 Load profile of DEWH Full Day (Load and Cost vs. time)

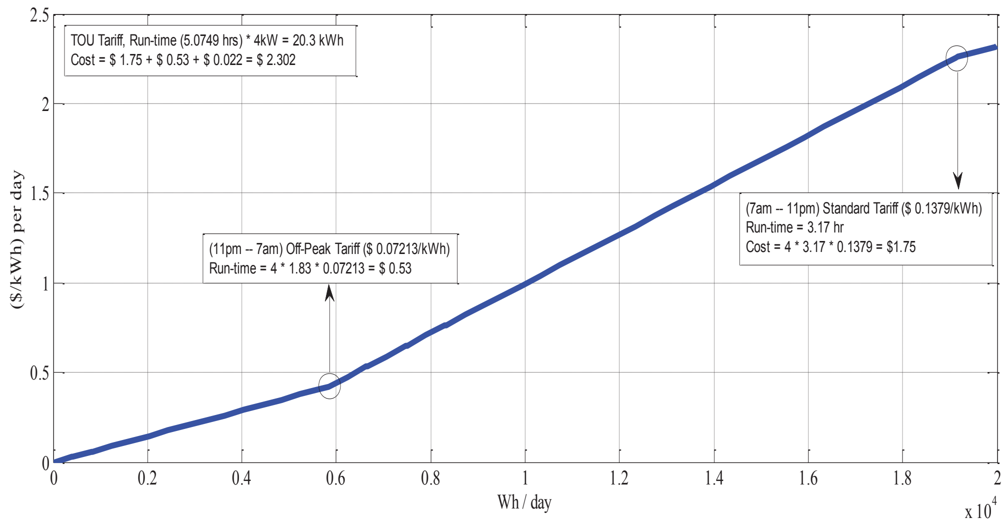

Figure 3.10 Load profile of DEWH Full Day (Load vs. Cost)

For simulations closest to reality, the model of Energy Cost Calculator is designed based on NS power electricity rate [17] as follows:

Table 3.9 The cost-rate of NS power * *Rates effective January 1, 2013* [17]

Time & Hours			Rate/kWh
Weekends & Holidays	Year round (115 Days)	24 Hours / Day	**Off-Peak** 7.213¢/kWh
Weekdays	March to November (182 Days)	7am to 11pm	**Standard Rate** 13.790¢/kWh
		11pm to 7am	**Off-Peak** 7.213 ¢/kWh
Weekdays	December to February (68 Days)	7am to 12pm 4pm to 11pm	**On-Peak** 17.878¢/kWh
		11pm to 7am	**Off-Peak** 7.213 ¢/kWh
		12pm to 4pm	**Standard Rate** 13.790¢/kWh

The scenario of time operating of DEWH assumed to be in March includes power consumption and the energy cost calculations with respect to time and constraints.
As can be seen from figures 3.9 and 3.10, the energy consumption of DEWH is 20.3 kWh per day to keep hot water at comfortable temperature. Moreover, figure 3-9, of load profile of DEWH full day (load and cost vs. time) indicates the following:

- First figure indicates the full day load: 4 kW * 5.0749 hrs. = **20.3** kWh/day
- Second figure illustrates the energy cost under the standard tariff ($ 0.1379/kWh) which is: 20.3 kWh * $ 0.1379 = $ **2.8**/day.

- Third figure illustrates the energy cost in variable tariff (TOU) as follows:
  Standard: 16 hrs. (7 to 23); ($ 0.1379/kWh) where the (Run-time = 3.17 hr.)

  Off-peak: 8 hrs. (23 to 7); ($ 0.07213/kWh) where the (Run-time = 1.83 hr.)

  In this Load profile curves the total variable tariff (TOU) calculation is

  (4 kW * 1.83 hr. * $ 0.07213) + (4 kW * 3.17 hr. * $ 0.1379) = $ 2.3/day

As a result, we can conclude that the energy cost of run-time of DEWH under the standard tariff ($0.1379/kWh) equal $2.799/day and the energy cost of run-time of DEWH under the variable tariff (TOU) equal $2.3/day. Thus, the difference between two costs is ($ 0.499). In other words, if the customer runs the DEWH under these constraints of (TOU) he/she will save up to 17.83% per this appliance. However, calculates an optimal schedule for DEWH based on fluctuating tariffs can suggest more energy efficient; therefore, this idea will discuss later in the optimization chapter.

CHAPTER 4 MODELING OF HEATING SYSTEMS

4.1 INTRODUCTION

The basic operating principles of the heating systems and lighting systems are considered in order to design the models that can be implemented and simulated in the Matlab program; thus, there are four suitable models of these appliances

- Domestic Electric Oven (DEO).
- Domestic Electric Clothes Iron (DECI).
- Domestic Electric Baseboard Heater (DEBH).
- Modeling of Lighting (F-Lamp)

In this chapter, all the above appliances are designed, tested and investigated. This chapter discusses the design of heating system models and a lighting system in order to investigate the impact of its main parameters on its ability to assist with reducing energy consumption. In these kinds of systems, the control strategies developed played a significant role in controlling the major appliances in residential and commercial sectors.

4.2 DOMESTIC ELECTRIC OVEN (DEO)

4.2.1 INTRODUCTION

Numerous models of diverse types of the Domestic Electric Oven (DEO) have been introduced in the literature. The objectives of these models differ from one to another, in order to meet the requirements of the design the purpose of the usage, the size and the material must be considered in the modeling process.

(DEOs) are common appliances used for many purposes and in the literature, are defined as thermally insulated chambers used for the heating, baking, cooking [27-28 and 30]. The heater element itself is very essential in an electric oven system but sometimes can be dangerous if not properly controlled, since the ovens simply run continuously at various temperatures. Thus, in order to control and regulate the amount of heat produced, a thermostat is used [29].

A thermostat device plays a significant role as a controller, which is used for regulating the heat intensity with a certain range of a temperature set point; the thermostat range can be predetermined as required since the user can specify the temperature, toasting time, and start or stop the heating process at any time [31]. In fact, many domestic electric ovens that are more conventional have a simple thermostat. In particular, the task of the thermostat is to turn an electric oven *ON/OFF* automatically and select the temperature when a pre-set temperature has been reached. Thus, we can conclude that the (DEO) is an enclosed compartment where the power and the temperature can be adjusted for heating, baking and drying / cooking food, while the oven temperature is regulated by thermostatic control. It is worth mentioning that the operation time of an electric oven in residential sector is less than in the commercial sector.

In this study the typical DEO is presented and implemented by using an approach for identifying the thermal parameters of physical models of a single- zone lumped-parameter thermal model. This approach has been developed by a group of researchers at the University of New Brunswick, 2012 [3].

4.2.2 MODELING OF (DEO)

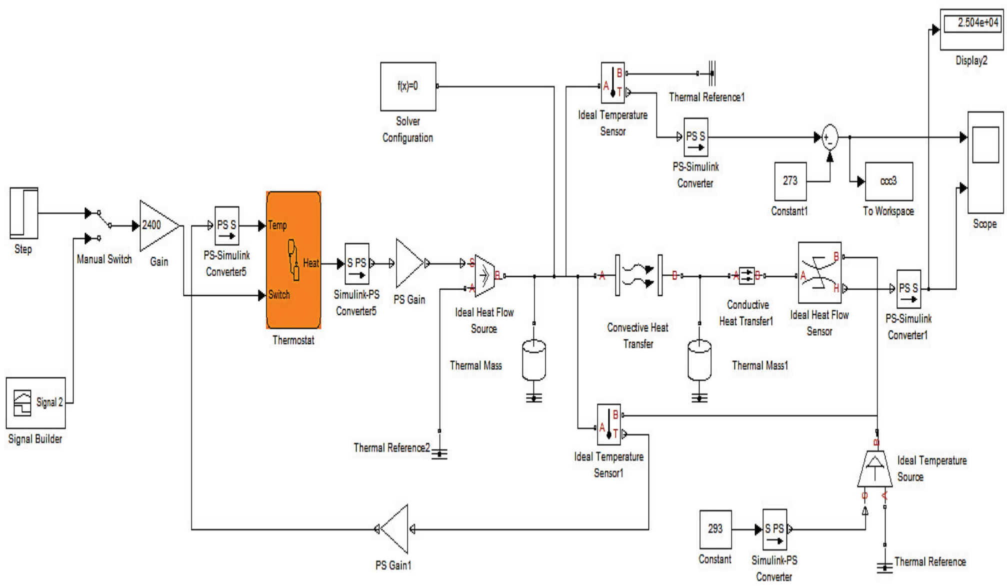

Figure 4.1 Model of Domestic Electric Oven system

Table 4.1 Model of Domestic Electric Oven specifications

Convection Parameters					
Convective Heat Transfer		Thermal Mass			
Area (m^2)	Heat Transfer coefficient ($W/m^2.$°C)	Mass (kg)	Specific heat Cp ($J/kg.$°C)	Initial Temperature (°C)	
0.25	300	22	1005.4	20	
Conduction Parameters					
Conductive Heat Transfer			Thermal Mass		
Area (m^2)	Thickness (m)	Thermal Conductivity ($W/m.$°C)	Mass (kg)	Specific heat Cp ($J/kg.$°C)	Initial Temperature (°C)
1.5	0.04	180	40	502	20

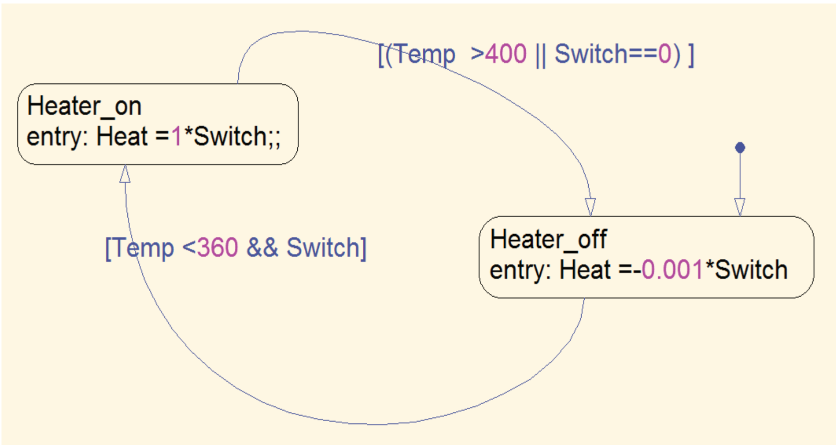

Figure 4.2 State flow model of the thermostat of DEO

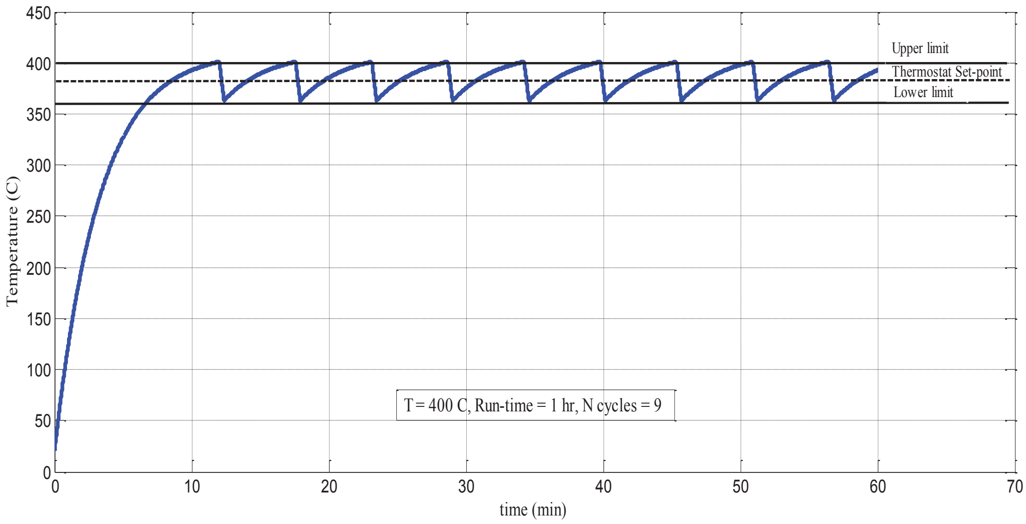

Figure 4.3 DEO Temperature vs. Time

4.2.3 MATHEMATICAL MODEL OF (DEO)

The aim of this section is to discover and determine the effect of (DEO) parameters during its run-time by applying simple mathematical equations that represent the effect of varying parameters on the system behavior such as power and energy consumption. This can be achieved by deriving equations to obtain temperature $T(\mathrm{t})$, T_{low} and T_{high} as in [3]. In this study however, the mathematical model with appropriate modification to meet the requirements; the time duration of $\{t_{on}, t_{on\,(startup)}$ and $t_{off}\}$, the cycling time, and power/energy consumed by DEO from the energy flow during transient condition and steady-state condition response are also considered.

The primary calculations are based on the research that has been done by a group of researchers [3] as mentioned in chapter 3. These include calculations that determine the thermal parameters of DEO such as thermal resistance (R_{th}) and the thermal capacity (C_{th}) in terms of temperature, P_{rated} and time durations. However, instead of taking those parameters values from literature; this study examines and investigates the system behavior with material properties for obtaining expressions that can be used to determine the value of all constant parameters by deriving some equations that represent operation cycles of the system performance. The mathematical model of DEO is based on the Lumped Parameter Method which is included in the studies [3 and 12]. However, in this study a few required modifications to develop simple mathematical equations that reflect the system behavior and show the effect of varying some parameters in this model are represented.

Table 4.2 Known parameters of DEO model

Input power (W)	Thermostat parameters $(°C)$			Ambient temperature $(°C)$	$t_{off}\ (max)$ (sec)	$t_{on}(min)$ (sec)
$Q_i = P_{rated}$	T_{high}	T_{low}	$T_{s\,(set-point)}$	T_a	t_{off}^{m}	t_{on}^{m}
2400	400	365	382.5	20	20.4	312

t_{off}^{max}, indicates that the maximum duration of t_{off}^{m} and t_{on}^{max} the minimum duration of t_{on}^{m}

$$\boldsymbol{G} = \frac{Qi\ (\Delta-1)}{\Delta(T_{high}-T_a)-(T_{low}-T_a)} \tag{4.1}$$

$$\boldsymbol{C} = G\left(\frac{t_{off}^{m}}{\ln(T_{high}-T_a)-\ln(T_{low}-T_a)}\right) \tag{4.2}$$

Where,

$$\Delta = \left(\frac{T_{high}-T_a}{T_{low}-T_a}\right)^{t_{on}^{m}/t_{off}^{m}} \tag{4.3}$$

By applying formulas (4.1, 4.2 and 4.3) the results are

Δ	$G\ (W/°C)$	$C\ \ (J/°C)$
4.3834	6.1484	1298.0619

The ratio of G/C can be calculated directly or by

$$G/C = \frac{\ln(T_{high}-T_a)-\ln(T_{low}-T_a)}{t_{off}^{m}} \tag{4.4}$$

$$\frac{G}{C} = 4.736600003 \times 10^{-3}\ sec^{-1} = \boldsymbol{\tau}^{-1}$$

$G/C\quad (sec^{-1})$	$\boldsymbol{\tau} = C/G\quad (sec)$
$4.736600003 \times 10^{-3}$	211.1219016

Calculations of DEO $\{\boldsymbol{T}(\mathrm{t}), \boldsymbol{T}_{low}, \boldsymbol{T}_{high}, \boldsymbol{t}_{off}\ , \boldsymbol{t}_{on}, t_{on}^{start-up}\}$

In order to calculate the approximate heater element temperature at the time (t) within a certain range; in the literature [12], the equation to obtain $T(t)$ at any time was obtained by solving the step response of the first order differential equation [12], with the appropriate modifications to determine $\{\boldsymbol{T}(t), \boldsymbol{T}_{low}, \boldsymbol{T}_{high}, \boldsymbol{t}_{off}, \boldsymbol{t}_{on}, \boldsymbol{t}_{on}^{start-up}\}$. An equation that describes how $T(t)$ varies in time can be obtained by applying eq. (3.3) in chapter 3 as follows:

$$T(\mathrm{t}) = T_H(t_0)e^{-\frac{(t-t_0)}{\tau}} + K\left(1 - e^{-\frac{(t-t_0)}{\tau}}\right) \tag{4.5}$$

Calculations of $T(t)$, by using an exponential equation (4.5) from **appendix A** of DEO model the results are

$\boldsymbol{\tau} = 211.1219\ sec$	$\boldsymbol{K} = 410.3455\ °C$

By applying equation (4.5) to obtain $T(t)$

	$T_H(t_0)$ (°C)	t_{on}^m (sec)	τ (sec)	K (°C)	$T(t)$ (°C)
Case 1 (natural response)	20	312	211.1219	0	4.5627
Case 2 (forced response)	20	312	211.1219	410.3455	317.6807

Calculation of (T_{low}) and (T_{high}) from **appendix A** of DEO

$$T_{low} = T_{high}\, e^{-\frac{t_{off}^m}{C/G}} + K\left(1 - e^{-\frac{t_{off}^m}{C/G}}\right) \tag{4.6}$$

$$T_{high} = T_{low}\, e^{-\frac{t_{on}^m}{C/G}} + K\left(1 - e^{-\frac{t_{on}^m}{C/G}}\right) \tag{4.7}$$

Case 1, in this case the heater element is (*OFF*) thus, $Q = 0\ W \quad \rightarrow \quad K = T_a$

T_{high} (°C)	T_a (°C)	t_{off}^m (sec)	G (W/°C)	C (J/°C)	K (°C)
400	20	20.4	6.1484	1298.0619	20

$T_{low} = \mathbf{365}$ °C

Case 2, in this case the heater is (*ON*) thus, $Q = 1500\ W \quad \rightarrow \quad K = T_a + \frac{Q}{G}$

T_{low} (°C)	T_a (°C)	t_{on}^m (sec)	G (W/°C)	C (J/°C)	K (°C)
365	20	312	6.1484	1298.0619	410.3455

$T_{high} = \mathbf{400}$ °C

Set-point (ΔT) = T_s = 382.5 °C Thus, the Dead band value (D_b) = 17.5 °C

Calculation of Time duration (t_{off}), (t_{on}) and ($t_{on}^{start-up}$) from appendix A

Constant system parameters

T_{high} (°C)	T_{low} (°C)	T_a (°C)	T_s (°C)	G (W/°C)	C (J/°C)	Q_i (W)
400	365	20	382.5	6.1484	1298.0619	2400

1- Linear Equations

$$t_{off}^{m} = \frac{C(T_{high} - T_{low})}{G(T_s - T_a)} \tag{4.8}$$

$$t_{on}^{m} = \frac{C(T_{high} - T_{low})}{G\,(T_a - T_s) + Q_i} \tag{4.9}$$

$$t_{on}^{start-up} = \frac{C(T_{high} - T_{low})}{G\,(T_a - T_s) + Q_i} \tag{4.10}$$

Linear Equations Results

P_{rated} (kW)	t_{off}^{m} (sec)	t_{on}^{m} (sec)	$t_{on}^{start-up}$ (sec)
0 / 2.4	20.38	265.37	334.35

$\boldsymbol{t_{off}} = 20.38\, sec = 0.34\ min = 0.0057\, hr$

$\boldsymbol{t_{on}} = 265.37\ sec = 4.42\, min = 0.0737\ hr$

$\boldsymbol{t_{on\,(startup)}} = 334.35\ sec = 5.5725\ min = 0.0929\ hr$

2- Exponential Equations

$$t_{off} = RC\ ln\left(\frac{T_{high} - T_a}{T_{low} - T_a}\right) \tag{4.11}$$

$$t_{on} = RC\ ln\left(\frac{T_{low} - T_a - R * P_{rated}}{T_{high} - T_a - R * P_{rated}}\right) \tag{4.12}$$

$$t_{on\,(startup)} = RC\ ln\left(\frac{T_{low} - T_a - R * P_{rated}}{T_{high} - T_a - R * P_{rated}}\right) \tag{4.13}$$

Exponential Equations Results

P_{rated} (kW)	t_{off}^{m} (sec)	t_{on}^{m} (sec)	$t_{on}^{start-up}$ (sec)
0 / 2.4	20.40	312.00	454.49

$\boldsymbol{t_{off}} = 20.40\, sec = 0.34\, min = 0.0057\, hr$

$\boldsymbol{t_{on}} = 312.00\, sec = 5.20\, min = 0.0867\, hr$

$\boldsymbol{t_{on\,(startup)}} = 454.49\, sec = 7.57\, min = 0.1262\, hr$

The results of time duration calculations indicate that the period while the thermostat is in position *ON*, the equations (4.9 and 4.12) show that $\boldsymbol{t_{on}}$ is dependent on the power supply $\boldsymbol{Q_{in}} = 2.4\, kW$ since $\boldsymbol{Q_i}$ is proportional to the heater rated power $\boldsymbol{P_{rated}}$.

Otherwise, during the thermostat is in position *OFF*, the equations (4.8 and 4.11) indicate that $\boldsymbol{t_{off}}$ is independent on the power supply since $\boldsymbol{Q_{in}} = 0$

Table 4.3 Comparison of the equations and the simulation results

Method	$\boldsymbol{t_{on}^{start-up}}$ (hr)	$\boldsymbol{t_{on}}$ (hr)	$\boldsymbol{t_{off}}$ (hr)	$\boldsymbol{t_{on}^{total}}$ (hr)	$\boldsymbol{cycling_{time}}$ (hr)	$\boldsymbol{Run_{time}}$ (hr)
Linear equations	0.0929	0.0737	0.0057	0.7857	0.0794	1
Exponential equations	0.1262	0.0867	0.0057	0.9412	0.0924	1
Simulation Result	0.1253	0.0865	0.0057	0.9384	0.0922	1

From table 4.3, we can conclude that the exponential equation is more accurate than the linear equation compared with the simulation result of (DEO) model.

Power / Energy Consumption of (DEO)

Method	Q_{on} (kWh)	P_{rated} (kW)	t_{on} (hr)
Calculation	**0.2081**	2.4	0.0867
Simulation	**0.2076**	2.4	0.0865

Electric Heater (kW)	$E.C$ (kWh)	$t_{on(total)}$ (hr)	Run-time (hr)
2.4	2.2589	0.9412	1

The average power consumption (ΔP_{DEO}) from the energy flow and the energy consumption per operation cycle (EC) of the (DEO) can be calculated by using Duty Cycle value and input rated power; in **appendix A** the results as follows:

$Duty\ Cycle$	t_{on}^{m} (hr)	$Cycling_{time}$ (hr)
0.9383	0.0867	0.0924

Table 4.4 Comparison of equations and simulation results in steady-state

Method	Q_i (kW)	$\boldsymbol{t_{on}^{m}}$ (hr)	$\boldsymbol{t_{off}^{m}}$ (hr)	$Cycling\ time$ (hr)	$Duty\ Cycle$	ΔP_{DEO} (kW)	EC_{DEO} (kWh)	$\boldsymbol{Run_{time}}$ (hr)
Linear Equation	2.4	0.0737	0.0057	0.0794	0.9282	2.2277	0.1769	1
Exponential Equation	2.4	0.0867	0.0057	0.0924	0.9383	2.2519	0.2081	1
Simulation Result	2.4	0.0865	0.0057	0.0922	0.9382	2.2517	0.2076	1

Table 4.5 Comparison of the results in transient condition

Method	Q_i (kW)	t_{on}^{total} (hr)	$Cycling_{time}$ (hr)	$Duty$ $Cycle$	ΔP_{DEBH} (kW)	EC_{DEBH} (kWh)	$Run\ time$ (hr)
Linear Equation	2.4	0.7857	0.0794	0.9282	2.2277	1.8857	1
Exponential Equation	2.4	0.9412	0.0924	0.9383	2.2519	2.2589	1
Simulation Result	2.4	0.9384	0.0922	0.9382	2.2517	2.2522	1

$$EC_{DEO} = 2.4 * 0.9412 = 2.2589\ kWh$$

Tables [4.4 and 4.5] show the results obtained from linear equations, exponential equations and the simulation results of the DEO model in steady-state condition. Therefore, we can conclude that the exponential equation result is more accurate than the linear equation compared with the simulation result of this model.

Comparison of transient condition and steady-state condition to illustrate the impact of the worming-up time ($t_{on}^{start-up}$).

Table 4.6 The time response of the system

Methods	**P** (kW)	$t_{on}^{start-up}$ (hr)	t_{on} (hr)	t_{off} (hr)	$cycling$ $time$ (hr)	N_{cycles}	$Transient$ $Condition$	$Steady\_State$ $Condition$
							$t_{on(total)}$ (hr)	$t_{on(total)}$ (hr)
Linear Equation	2.4	0.0929	0.0737	0.0057	0.0794	9.4	0.7857	0.6928
Exponential Equation	2.4	0.1262	0.0867	0.0057	0.0924	9.4	0.9412	0.8149
Simulation Result	2.4	0.1253	0.0865	0.0057	0.0922	9.4	0.9384	0.8131

According to thermodynamic principles, the transient condition and steady-state condition can be expressed as follows:

- Transient condition is defined as a time interval that the system is taking during start-up period to reach a desired temperature or taking its time to respond to any disturbances.

- Steady-state condition is defined as a condition where the system behavior is depend on the long run time and will remain stable if there is no disturbance.

Table 4.6, illustrates the time $\boldsymbol{t}_{on(total)}$ in steady-state condition is obviously less than the $\boldsymbol{t}_{on(total)}$ in transient condition; the difference is due to the worming-up time $\boldsymbol{t}_{on}^{start-up}$ to reach the set-point temperature. Thus, this period of time in transient condition should be included in calculation of $\boldsymbol{t}_{on(total)}$ in order to calculate the overall energy consumption of the DEO system. In addition, the thermostat time $\boldsymbol{t}_{off}$ is much less than the time $\boldsymbol{t}_{on}$ which indicates the turn-off temperature T_{high} differs slightly from the turn-on temperature T_{low}. This difference is called hysteresis/dead band of the thermostat around the thermostat set-point. Also, this difference will prevent the oven from switching rapidly and unnecessarily when the temperature is near the set-point since, the setting of set-point set by the consumer choice.

4.2.4 LOAD PROFILE AND ENERGY COST

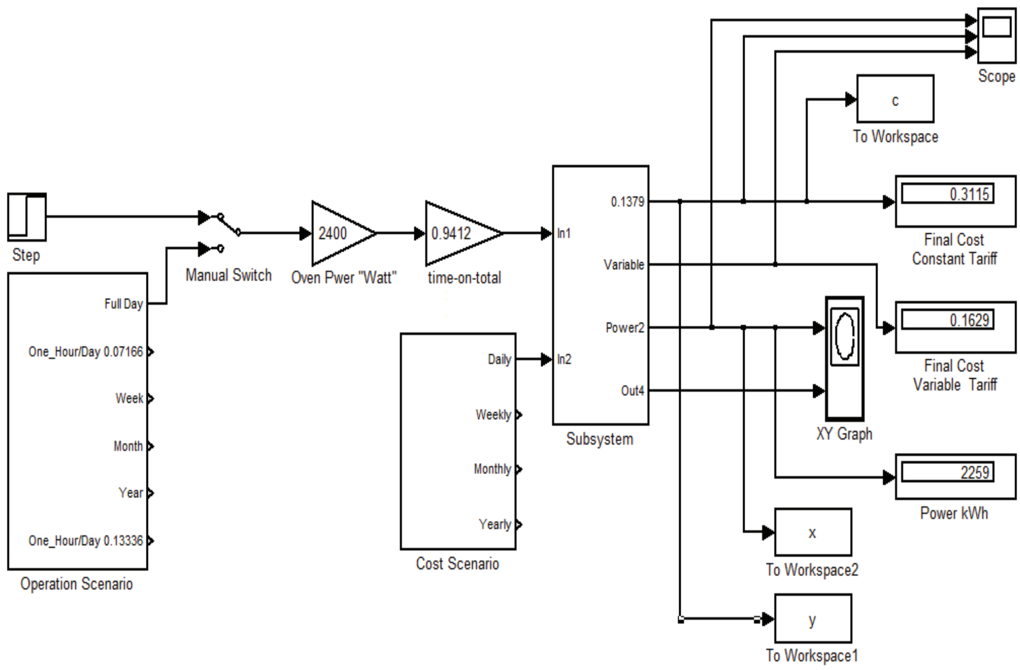

Figure 4.4 Energy Cost Calculator of DEO

Simulation Results of DEO (TOU)

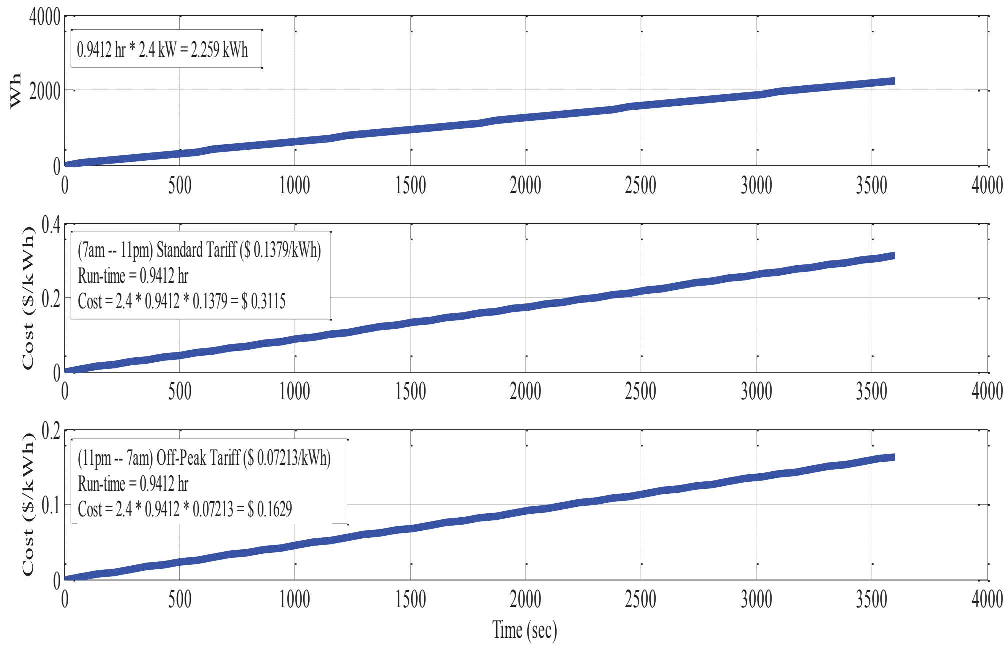

Figure 4.5 Load profile of DEO (Load and Cost vs. time)

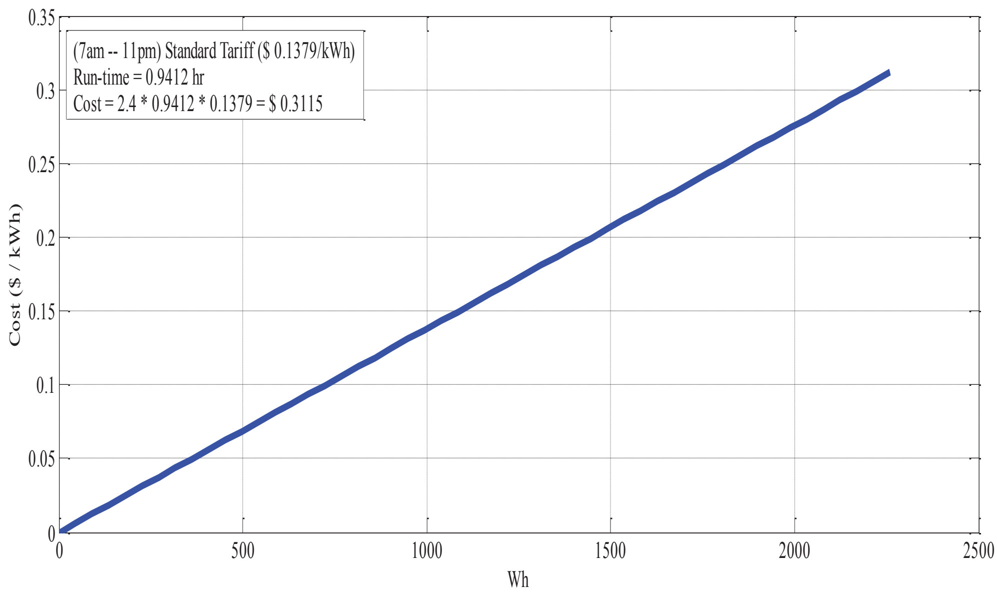

Figure 4.6 Load profile of DEO (Load vs. Cost) Standard tariff

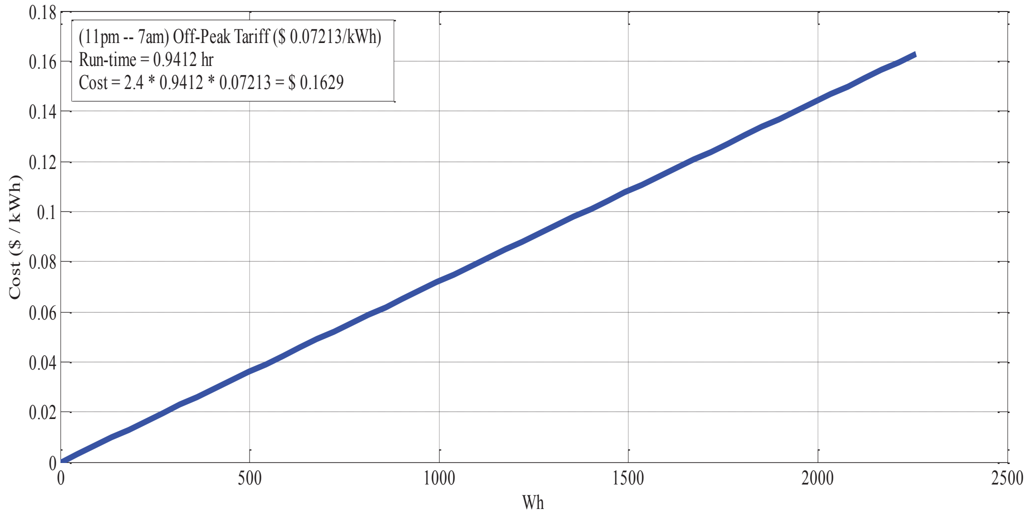

Figure 4.7 Load profile of DEO (Load vs. Cost) Off-Peak tariff

For simulations closest to reality, the model of Energy Cost Calculator is designed based on NS power electricity rate [17] as follows:

Table 4.7 The cost-rate of NS power * *Rates effective January 1, 2013* [17]

Time & Hours			Rate/kWh
Weekends & Holidays	Year round (115 Days)	24 Hours / Day	**Off-Peak** 7.213¢/kWh
Weekdays	March to November (182 Days)	7am to 11pm	**Standard Rate** 13.790¢/kWh
		11pm to 7am	**Off-Peak** 7.213 ¢/kWh

The scenario of time operating of DEO assumed to be in March includes power consumption and the energy cost calculations with respect to time and constraints.
As can be seen from figures (4.5, 4.6 and 4.7), the energy consumption of DEO is 2.259 kWh to keep the oven at the desired temperature. Moreover, figure 4.5, of load profile of DEO (load and cost vs. time) indicates the following:

- First figure indicates the full load: 2.4 kW * 0.9412 hrs. = **2.259** kWh
- Second figure illustrates the energy cost under the standard tariff ($ 0.1379/kWh) which is: 2.259 kWh * $ 0.1379 = $ **0.312**.
- Third figure illustrates the energy cost under off-peak tariff ($ 0.07213/kWh) which is: 2.259 kWh * $ 0.07213 = $ 0.163

In this load profile curves the total variable tariff (TOU) not included where the off-peak tariff calculation is assumed just in case of operating time in period between (11pm to 7am) or in case of weekend.

As a result, we can conclude that the energy cost of run-time of DEO under the standard tariff ($0.138/kWh) equal $ 0.312 and the energy cost of Run-time of DEO under the off-peak tariff equal $ 0.163. Thus, the difference between two costs is ($ 0.15). In other words, if the customer runs the DEO under the off-peak tariff he/she will save up to 47.7% per this appliance. However, calculates an optimal schedule for DEO based on fluctuating tariffs will discuss later in the optimization chapter.

4.3 DOMESTIC ELECTRIC CLOTHES IRON (DECI)

4.3.1 INTRODUCTION

An Electric Clothes Iron for domestic purposes (DECI) is among the most common household appliances in the world. It works as a general household appliance used to press the wrinkles out of the clothes. There are various types of (DECI) such as an electric dry iron and a modern electric steam iron.

An electric clothes iron has a heat-radiating base, which is formed of aluminum or its alloy. The heat-radiating base heated by a heater element inside the iron. A heating element set into the soleplate that heats up in accordance with the heat control mechanism.

In this study the typical DECI is presented and implemented by using an approach for identifying the thermal parameters of physical models of single- zone lumped-parameter thermal model, this approach has been developed by a group of researchers at the University of New Brunswick, 2012 [3].

4.3.2 MODELING OF (DECI)

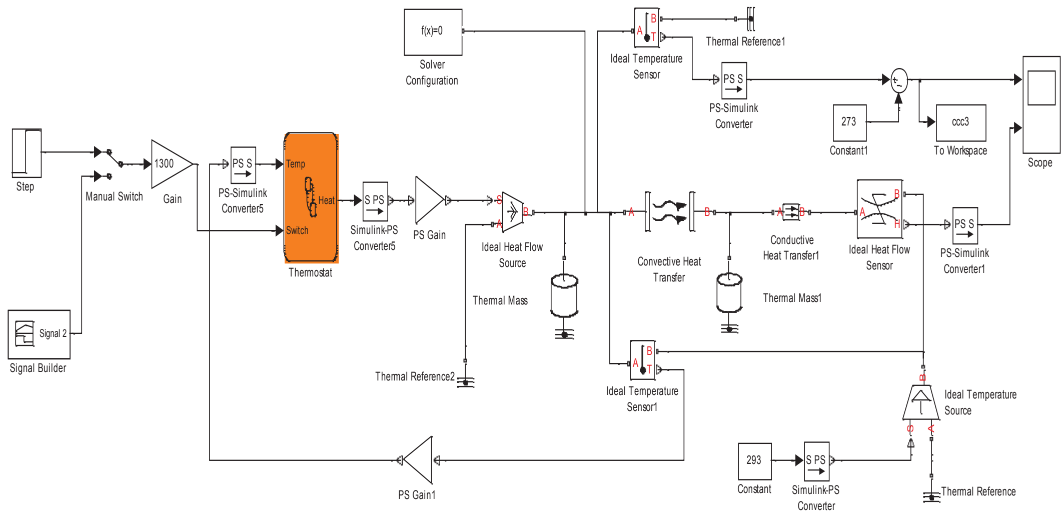

Figure 4.8 Model of Domestic Electric Clothes Iron

Table 4.8 Model of Domestic Electric Clothes Iron specifications

Convection Parameters					
Convective Heat Transfer		Thermal Mass			
Area (m^2)	Heat Transfer coefficient ($W/m^2.°C$)	Mass (kg)	Specific heat Cp ($J/kg.°C$)	Initial Temperature (°C)	
0.041	80	1.134	910	20	
Conduction Parameters					
Conductive Heat Transfer			Thermal Mass		
Area (m^2)	Thickness (m)	Thermal Conductivity ($W/m.°C$)	Mass (kg)	Specific heat Cp ($J/kg.°C$)	Initial Temperature (°C)
0.03	0.003	250	1	1005	20

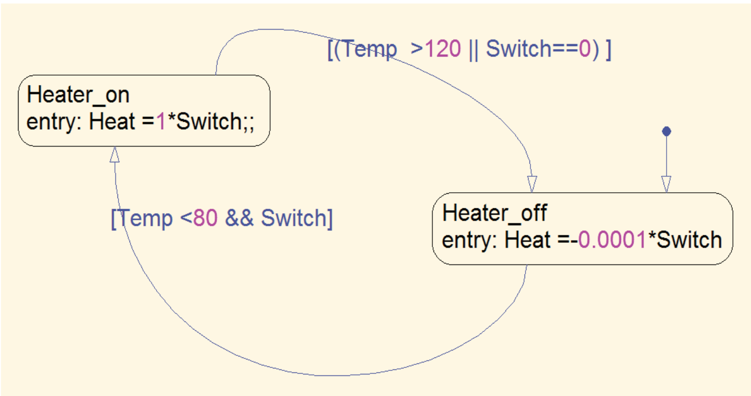

Figure 4.9 State flow of the thermostat of DECI

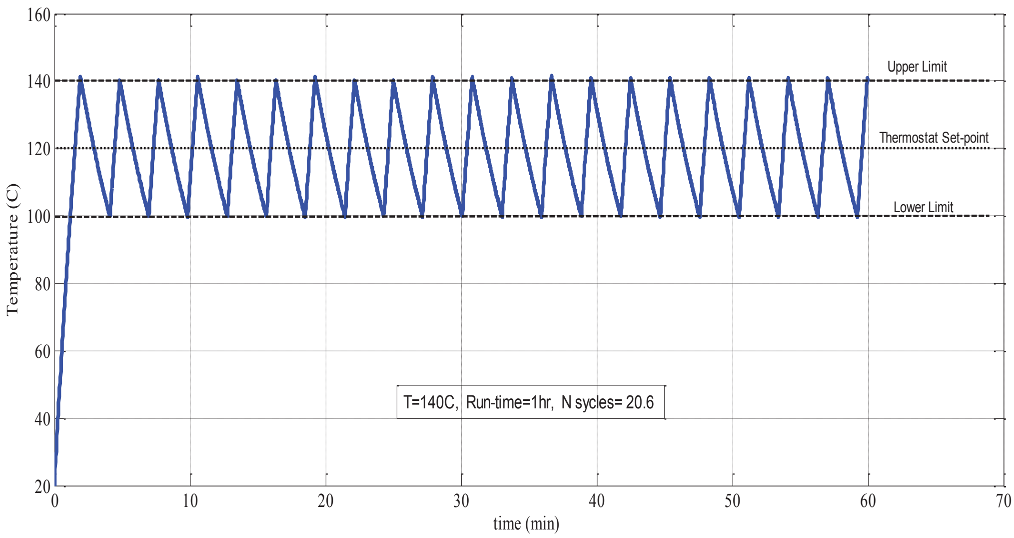

Figure 4.10 DECI Temperature vs. Time (A)

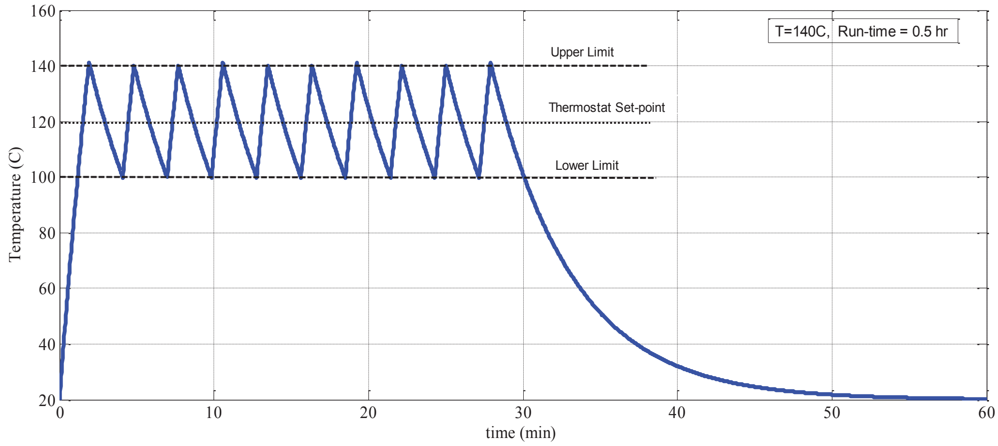

Figure 4.11 DECI Temperature vs. Time (B)

A common clothes iron works in a range of approximately (50 to 230 °C). However, the above figures (4.10 and 4.11) are illustrating just one stage of the user choice in range of (100 to 140 °C). On the other hand, the plot of temperature as a function of time shows that the system has reached a steady state.

4.3.3 MATHEMATICAL MODEL of DECI

The aim of this section is to provide, a sample calculation in order to analyse the DECI system behavior during transient condition and steady-state condition response.

The primary calculations of a mathematical model of DEIC are based on the Lumped Parameter Method which is included in studies [3] and [12] as mentioned in chapter 3. In this section the main parameters of DECI such as thermal resistance (R_{th}) and the thermal capacity (C_{th}) in terms of temperature, P_{rated} and time durations are calculated. However, instead of taking those parameters values from literature; this study examines and investigates the system behavior with material properties for obtaining expressions that can be used to determine the value of all constant parameters by deriving some equations that represent operation cycles of the system performance. The mathematical model of DECI is based on the Lumped Parameter Method which is included in the studies [3 and 12]. However, in this study a few required modifications to develop simple mathematical equations that reflect the system behavior and show the effect of varying some parameters in this model are represented.

Constant parameters of the DECI

$Q = P_{rated}$ (W)	T_{high} $(°C)$	T_{low} $(°C)$	$T_{set-point}$ $(°C)$	T_{amb} $(°C)$	t_{on}^{m} (sec)	t_{off}^{m} (sec)
1300	140	100	120	20	42.42	128.4

t_{off}^{max}, indicates that the maximum duration of t_{off}^{m} and t_{on}^{max} the minimum duration of t_{on}^{m}

By using an approach for identification of physical models of single-zone lumped-parameter thermal model equations (4.1 to 4.7); from **appendix B of DECI** the results are:

Δ	G $(W/°C)$	C $(J/°C)$
1.14334	3.2577	1031.6283

Also, the thermal capacitance can be calculated by $C = \rho C_p V$

ρ (kg/m^3)	C_p $(J/kg.°C)$	V (m^3)	C $(J/°C)$
2700	910	0.00042	1031.94

The ratio of G/C can be calculated directly

G/C (sec^{-1})	$\boldsymbol{\tau} = C/G$ (sec)
3.1578279×10^{-3}	316.67336

Calculations of DECI $\{\boldsymbol{T}(\text{t}), \boldsymbol{T}_{low}, \boldsymbol{T}_{high}, \boldsymbol{t}_{off}, \boldsymbol{t}_{on}, t_{on}^{start-up}\}$

In order to calculate the approximate heater element temperature at the time (t) within a certain range; in the literature [12], the equation to obtain $T(t)$ at any time was obtained by solving the step response of the first order differential equation [12], with the appropriate modifications to determine $\{\boldsymbol{T}(t), \boldsymbol{T}_{low}, \boldsymbol{T}_{high}, \boldsymbol{t}_{off}, \boldsymbol{t}_{on}, \boldsymbol{t}_{on}^{start-up}\}$.
An equation 4.5 that describes how $T(t)$ varies in time can be obtained by applying eq. (3.3) in chapter 3 as follows:

$$T(\text{t}) = T_H(t_0)\, e^{-\frac{(t-t_0)}{\tau}} + K\left(1 - e^{-\frac{(t-t_0)}{\tau}}\right) \quad (4.5)$$

Calculations of $T(t)$, by using an exponential equation (4.5); from **appendix B** of DECI model the results are

$\boldsymbol{\tau} = 316.67434\ sec$	$\boldsymbol{K} = 419.0545\ °C$

By applying equation (4.5) to obtain, the calculation of $T(\mathrm{t})$

	$\boldsymbol{T_H(t_0)}$ $(°C)$	$\boldsymbol{t_{on}^m}$ (sec)	$\boldsymbol{\tau}$ (sec)	$\boldsymbol{K}$ $(°C)$	$\boldsymbol{T}(\mathrm{t})$ $(°C)$
Scenario 1 (natural response)	20	42.42	316.6734	0	17.49
Scenario 2 (forced response)	20	42.42	316.6734	419.0545	70.03

- **Calculation of $(\boldsymbol{T_{low}})$ and $(\boldsymbol{T_{high}})$ from appendix B of DECI**

$$T_{low} = T_{high}\ e^{-\left(t_{off}^m\right)G/C} + K\left(1 - e^{-\left(t_{off}^m\right)G/C}\right) \tag{4.6}$$

$$T_{high} = T_{low}\ e^{-(t_{on}^m)G/C} + K\left(1 - e^{-(t_{on}^m)G/C}\right) \tag{4.7}$$

In this case the heater is (*OFF*) thus, $Q = 0\ W \quad \rightarrow \quad K = T_a$

T_{high} $(°C)$	T_a $(°C)$	t_{off}^m (sec)	G $(W/°C)$	C $(J/°C)$	K $(°C)$
140	20	128.4	3.2577	1031.63	20

$\boldsymbol{T_{low}} = 100.0001\ °C$

In this case the heater is (*ON*) thus, $Q = 1300\ W \quad \rightarrow \quad K = T_a + \frac{Q}{G}$

T_{low} $(°C)$	T_a $(°C)$	t_{on}^m (sec)	G $(W/°C)$	C $(J/°C)$	K $(°C)$
100	20	42.42	3.2577	1031.63	419.0545

$\boldsymbol{T_{high}} = 139.9999\ °C$

Set-point, $(\Delta T) = T_s = 120$ °C. Thus, the Dead band value $(D_b) = 20$ °C

- **Calculation of time durations $(\boldsymbol{t_{off}})$, $(\boldsymbol{t_{on}})$ and $(\boldsymbol{t_{on}^{start-up}})$ from appendix B of DECI**

System parameters

T_{high} $(°C)$	T_{low} $(°C)$	T_a $(°C)$	T_s $(°C)$	G $(W/°C)$	C $(J/°C)$	Q_i (W)
140	100	20	120	3.2577	1031.63	1300

By using the equations (4.8 to 4.22) the results are

1- Linear Equations of (t_{off}), (t_{on}) *and* $(t_{on}^{start-up})$ results

P_{rated} (kW)	t_{off}^{m} (sec)	t_{on}^{m} (sec)	$t_{on}^{start-up}$ (sec)
0 / 1.3	126.7	42.36	70.56

$$\boldsymbol{t_{off}} = 126.7\, sec = 2.11\, min = 0.0352\, hr$$

$$\boldsymbol{t_{on}} = 42.36\, sec = 0.706\, min = 0.0118\, hr$$

$$\boldsymbol{t_{on}^{start-up}} = 70.56\, sec = 1.176\, min = 0.0196\, hr$$

2- Exponential Equations of time $(\boldsymbol{t_{off}})$, $(\boldsymbol{t_{on}})$ and $(\boldsymbol{t_{on}^{start-up}})$ results

P_{rated} (kW)	t_{off}^{m} (sec)	t_{on}^{m} (sec)	$t_{on}^{start-up}$ (sec)
0 / 1.3	128.42	42.42	70.85

$$\boldsymbol{t_{off}} = 128.42\,\text{sec} = 2.14\,\text{min} = 0.0357\, hr$$

$$\boldsymbol{t_{on}} = 42.42\,\text{sec} = 0.707\,\text{min} = 0.0118\, hr$$

$$\boldsymbol{t_{on}^{start-up}} = 70.85\,\text{sec} = 1.18\,\text{min} = 0.0197\, hr$$

Calculation of the period while the thermostat is in position *ON*, the equations (4.9, 4.12) show that $\boldsymbol{t_{on}}$ is dependent on the power supply since $\boldsymbol{Q_{in}}$ is proportional to the heater rated power P_{rated}. Otherwise, during the thermostat is in position *OFF*; the equations (4.8, 4.11) indicate that $\boldsymbol{t_{off}}$ is independent of the power supply since $\boldsymbol{Q_{in}} = 0$

Power / Energy consumption of (DECI)

Method	Q_{on} (kWh)	P_{rated} (kW)	t_{on} (hr)
Calculation	**0.0153**	1.3	0.0118
Simulation	**0.0155**	1.3	0.0119

Q_{total} (kWh)	P_{rated} (kW)	t_{on} (total) (hr)
0.3416	1.3	0.2628

Table 4.9 Comparison of the results of the time duration of the DECI model

Method	$t_{on(start-up)}$ (hr)	t_{on} (hr)	t_{off} (hr)	$t_{on(total)}$ (hr)	**P** (kW)
Linear equations	0.0196	0.0118	0.0352	0.2627	1.3
Exponential equations	0.0197	0.0118	0.0357	0.2628	1.3
Simulation Result	0.0197	0.0119	0.0359	0.2648	1.3

The average power consumption (ΔP_{DECI}) from the energy flow and the energy consumption per operation cycle (EC) of the (DECI) can be calculated by using Duty Cycle value and input rated power; in **appendix B** the results as follows:

Table 4.10 Comparison of the simulation results to equations

Method	Q_i (kW)	$t_{on}^{start-up}$ (hr)	t_{on}^{m} (hr)	t_{off}^{m} (hr)	*Cycling time* (hr)	*Duty Cycle*	ΔP_{DECI} (kW)	$E.C$ (kWh)
Linear Equation	1.3	0.0196	0.0118	0.0352	0.047	0.2511	0.3264	0.3415
Exponential Equation	1.3	0.0197	0.0118	0.0357	0.0475	0.2484	0.3229	0.3416
Simulation	1.3	0.0197	0.0119	0.0359	0.0478	0.2490	0.3237	0.3443

Table 4.10, shows the results obtained from linear equations and exponential equations, the average power, energy consumption and the simulation result of the DECI model. The results obtained from linear equations and exponential equations are almost the same as the results obtained from simulation. Thus the mathematical model has a good agreement with the simulation results.

Table 4.11 Comparison of the time response of the system

Methods	P (kW)	$t_{on}^{start-up}$ (hr)	t_{on} (hr)	t_{off} (hr)	$cycling\ time$ (hr)	$Steady-State\ Condition$	$Transient\ Condition$
						$t_{on(total)}$ (hr)	$t_{on(total)}$ (hr)
Linear Equation	1.3	0.0196	0.0118	0.0352	0.047	0.2431	0.2627
Exponential Equation	1.3	0.0197	0.0118	0.0357	0.0475	0.2431	0.2628
Simulation Result	1.3	0.0197	0.0119	0.0359	0.0478	0.2451	0.2648

From table 4.11, we can conclude that the difference between two results of $t_{on(total)}$ is due to steady-state condition and transient condition therefore, the worming-up time $t_{on}^{start-up}$ will take its (natural response) to reach the set-point temperature. Thus, this period of time in transient condition should be included in calculation of $t_{on(total)}$ in order to calculate the overall energy consumption of the DECI system.

According to thermodynamic principles, the Transient condition and Steady-state condition can be expressed as follows:

- Transient condition is defined as a time interval that the system is taking during start-up period to reach a desired temperature or taking its time to respond to any disturbances.
- Steady-state condition is defined as a condition where the system behavior is depend on the long run time and will remain stable if there is no disturbance.

4.3.4 LOAD PROFILE AND ENERGY COST

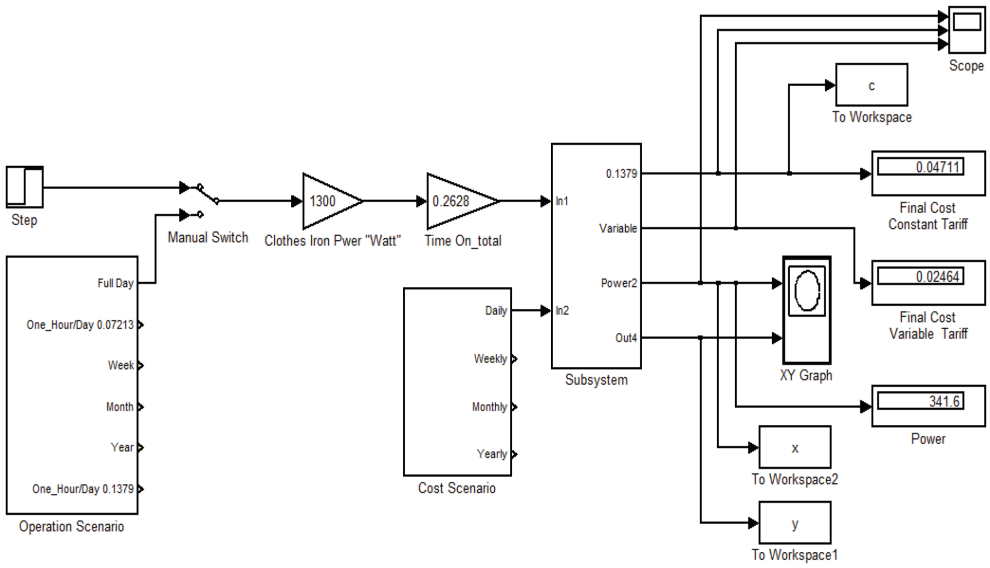

Figure 4.12 Energy Cost Calculator of DECI

Simulation Results of DECI (TOU)

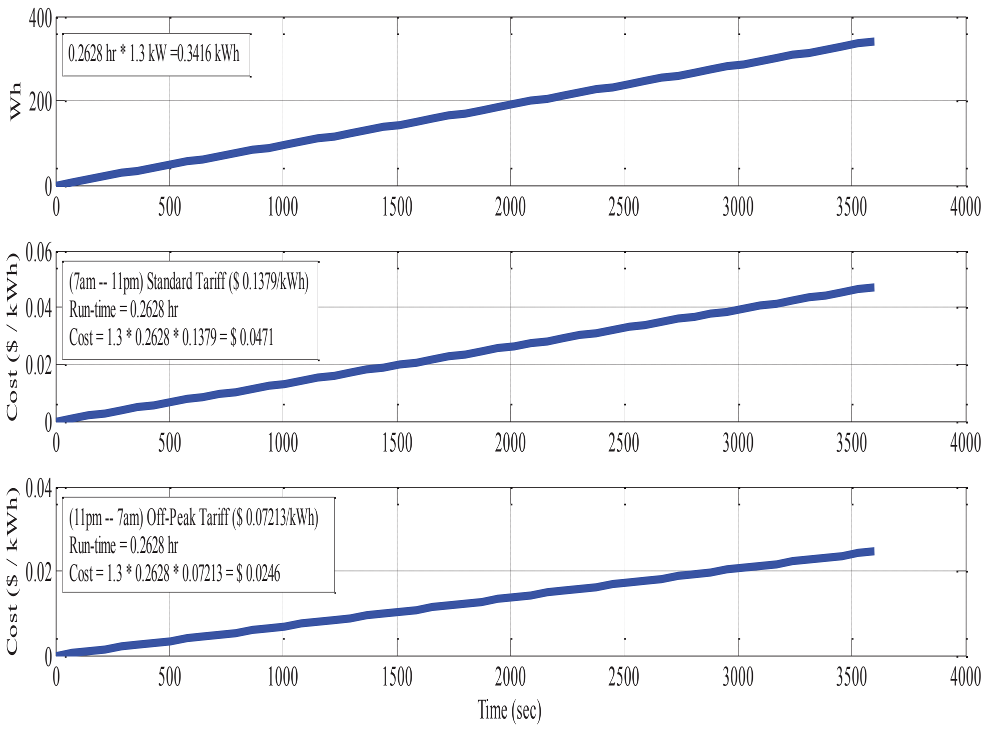

Figure 4.13 Load profile of DECI (Load and Cost vs. time)

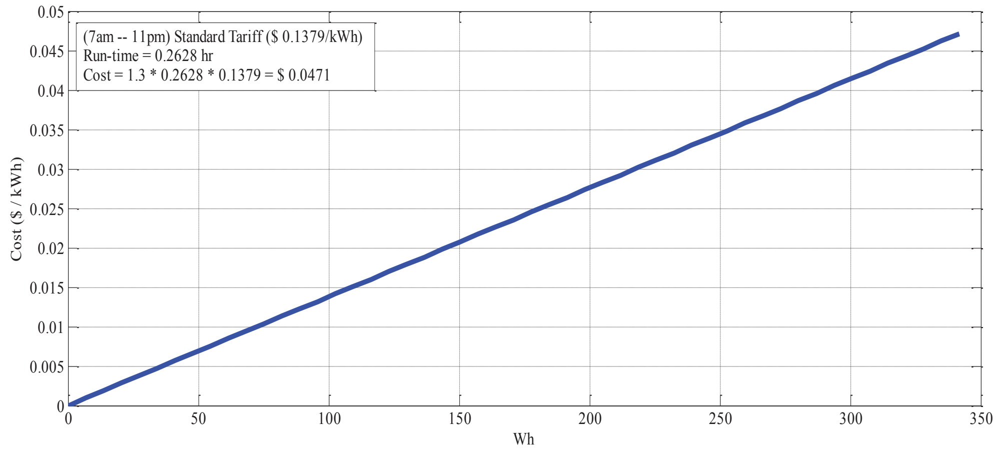

Figure 4.14 Load profile of DECI (Load vs. Cost) Standard tariff

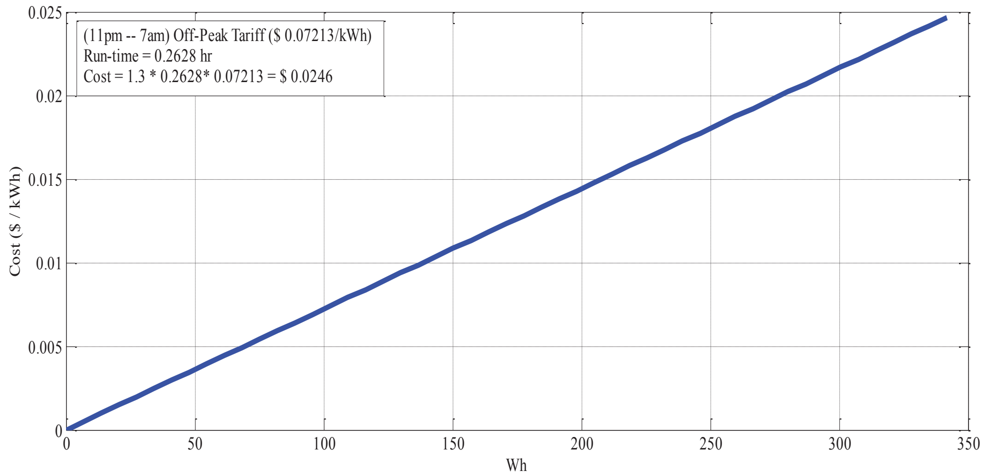

Figure 4.15 Load profile of DECI (Load vs. Cost) Off-Peak tariff

For simulations closest to reality, the model of Energy Cost Calculator is designed based on NS power electricity rate [17] as follows:

Table 4.12 The cost-rate of NS power * *Rates effective January 1, 2013* [17]

Time & Hours			**Rate/kWh**
Weekends & Holidays	Year round (115 Days)	24 Hours / Day	**Off-Peak** 7.213 ¢/kWh
Weekdays	March to November (182 Days)	7am to 11pm	**Standard Rate** 13.790 ¢/kWh
		11pm to 7am	**Off-Peak** 7.213 ¢/kWh

The scenario of time operating of DECI assumed to be in March includes power consumption and the energy cost calculations with respect to time and constraints.

As can be seen from figures (4.13, 4.14 and 4.15), the energy consumption of DECI is 0.3416 kWh to keep the clothes iron at the desired temperature. Moreover, figure 4.13, of load profile of DECI (load and cost vs. time) indicates the following:

- First figure indicates the full load: 1.3 kW * 0.2628 hrs. = 0.**3416** kWh
- Second figure illustrates the energy cost under the standard tariff ($ 0.1379/kWh) which is: 0.3416 kWh * $ 0.1379 = $ **0.047**.
- Third figure illustrates the energy cost under off-peak tariff ($ 0.07213/kWh) which is: 0.3416 kWh * $ 0.07213 = $ 0.025

In this load profile curves the total variable tariff (TOU) not included where the off-peak tariff calculation is assumed just in case of operating time in period between (11pm to 7am) or in case of weekend.

As a result, we can conclude that the energy cost of run-time of DECI under the standard tariff ($0.1379/kWh) equal $ 0.047 and the energy cost of Run-time of DECI under the off-peak tariff equal $ 0.025. Thus, the difference between two costs is ($ 0.023). In other words, if the customer runs the DECI under the off-peak tariff he/she will save up to 47.8% per this appliance. However, calculates an optimal schedule for DECI based on fluctuating tariffs will discuss later in the optimization chapter.

4.4 DOMESTIC ELECTRIC BASEBOARD HEATER (DEBH)

4.4.1 INTRODUCTION

There is a diversity of electric heaters appliances for domestic and commercial use. The main purpose of these heaters is to convert the electrical energy and transfer it in form of heat. There are many types of electric resistance systems that can be supplied by centralized forced-air electric furnaces or by zonal heaters in each room. Electric room heaters can be categorized into various types such as baseboard heaters, electric wall heaters, and electric space heaters [22].

The Domestic Electric Baseboard Heater (DEBH) is used in this study as a model of room heater; where DEBH contains an electric heating element encased in metal pipes; the pipes, surrounded by aluminum fins to aid heat transfer [22]. Typically, the fins materials have a high thermal conductivity to increase the heat transfer from surfaces in order to worm-up a room space to desired temperature where the setting of set-point of DEBH set by the consumer choice. The last main part is the heat control mechanism "thermostat".

4.4.2 MODEL of DEBH

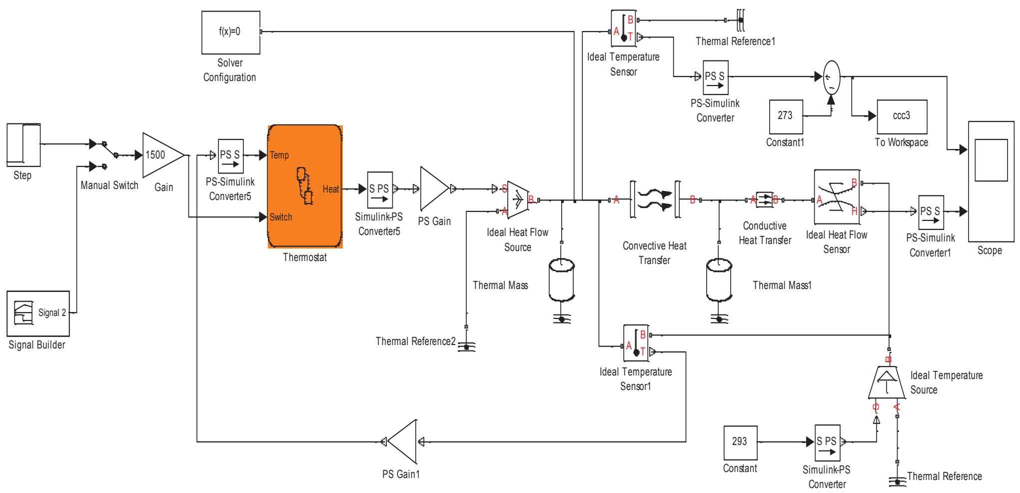

Figure 4.16 Model of Domestic Electric Baseboard Heater (DEBH)

Table 4.13 Model of (DEBH) specifications

<table>
<tr><th colspan="6">Convection Parameters</th></tr>
<tr><td colspan="2">Convective Heat Transfer</td><td colspan="4">Thermal Mass</td></tr>
<tr><td>Area (m^2)</td><td>Heat Transfer Coefficient ($W/m^2.°C$)</td><td>Mass (kg)</td><td>Specific heat Cp ($J/kg.°C$)</td><td colspan="2">Initial Temperature (K)</td></tr>
<tr><td>0.22</td><td>476</td><td>14.58</td><td>910</td><td colspan="2">285</td></tr>
<tr><th colspan="6">Conduction Parameters</th></tr>
<tr><td colspan="3">Conductive Heat Transfer</td><td colspan="3">Thermal Mass</td></tr>
<tr><td>Area (m^2)</td><td>Thickness (m)</td><td>Thermal Conductivity ($W/m.°C$)</td><td>Mass (kg)</td><td>Specific heat Cp ($J/kg.°C$)</td><td>Initial Temperature (K)</td></tr>
<tr><td>0.058</td><td>0.0053</td><td>220</td><td>9.8345</td><td>1005</td><td>293</td></tr>
</table>

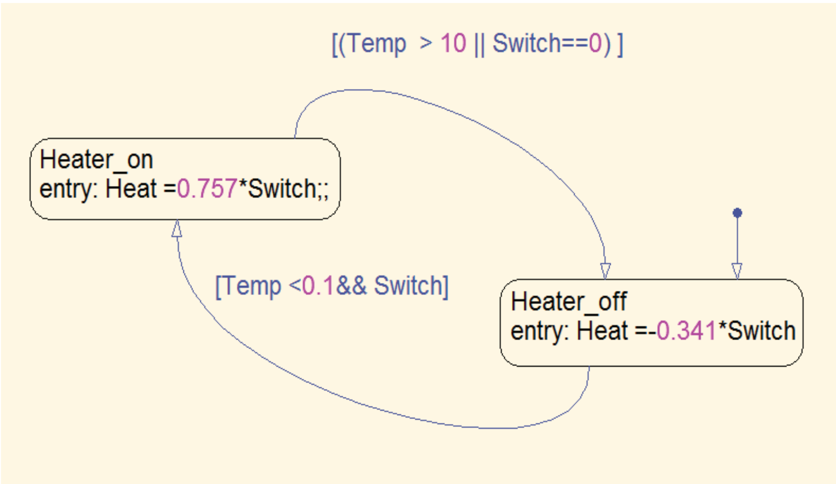

Figure 4.17 State flow model of the thermostat of (DEBH)

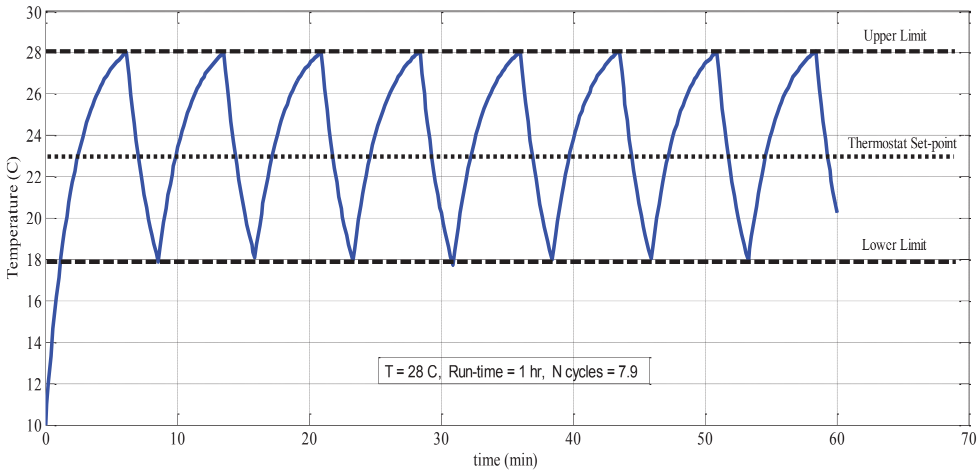

Figure 4.18 DEBH Temperature vs. Time (Run-time = 1hr)

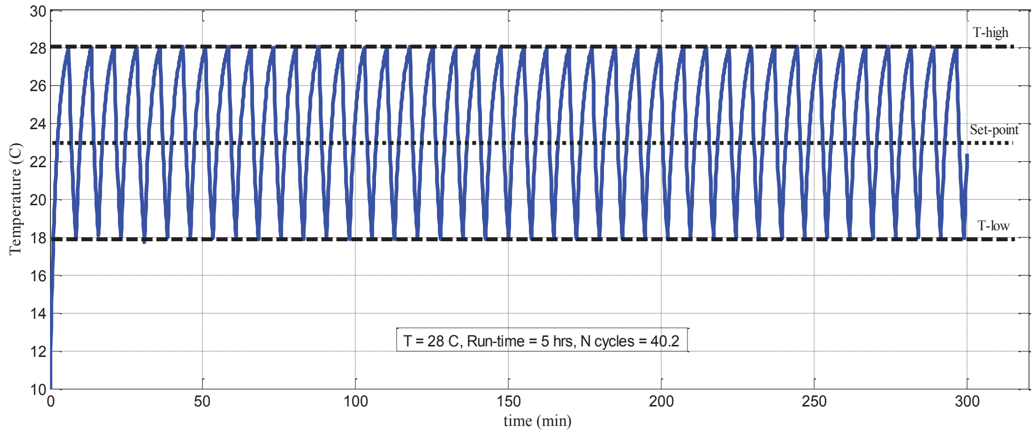

Figure 4.19 DEBH Temperature vs. Time (Run-time = 5 hrs.)

4.4.3 MATHEMATICAL MODEL of DEBH

The aim of this section is to provide, a sample calculation in order to analyse the DEBH system behavior during transient condition and steady-state condition response. The primary calculations of a mathematical model of DEBH are based on the Lumped Parameter Method which is included in studies [3] and [12] as mentioned in chapter 3.

In this section the main parameters of DEBH such as thermal resistance (R_{th}) and the thermal capacity (C_{th}) in terms of temperature, P_{rated} and time durations are calculated. However, instead of taking those parameters values from literature; this study examines and investigates the system behavior with material properties for obtaining expressions that can be used to determine the value of all constant parameters by deriving some equations that represent operation cycles of the system performance. The mathematical model of DEBH is based on the Lumped Parameter Method which is included in the studies [3 and 12]. However, in this study a few required modifications to develop simple mathematical equations that reflect the system behavior and show the effect of varying some parameters in this model are represented.

Constant parameters of the DEBH:

$Q = P_{rated}$ (W)	T_{high} $(°C)$	T_{low} $(°C)$	$T_{set-point}$ $(°C)$	T_{amb} $(°C)$	t_{on}^{m} (sec)	t_{off}^{m} (sec)
1500	28	18	23	10	300	145.8

t_{off}^{max}, indicates that the maximum duration of t_{off}^{m} and t_{on}^{max} the minimum duration of t_{on}^{m}

By using an approach for identification of physical models of single-zone lumped-parameter thermal model equations (4.1 to 4.7); from **appendix C** of DEBH the results are

Δ	G $(W/°C)$	C $(J/°C)$
5.30463	73.8077	13270.1464

Also, the thermal capacitance can be calculated by $C = \rho C_p V$

ρ (kg/m^3)	C_p $(J/kg.°C)$	V (m^3)	C $(J/°C)$
2700	910	0.0054	13267.8

The ratio of G/C can be calculated directly

G/C (sec^{-1})	$\boldsymbol{\tau} = (G/C)^{-1}$ (sec)
5.5619356×10^{-3}	179.7935

Calculations of DEBH $\{\boldsymbol{T}(t), \boldsymbol{T}_{low}, \boldsymbol{T}_{high}, \boldsymbol{t}_{off}, \boldsymbol{t}_{on}, \boldsymbol{t}_{on}^{start-up}\}$

In order to calculate the approximate heater element temperature at the time (t) within a certain range; in the literature [12], the equation to obtain $T(t)$ at any time was obtained by solving the step response of the first order differential equation [12], with the appropriate modifications to determine $\{\boldsymbol{T}(t), \boldsymbol{T}_{low}, \boldsymbol{T}_{high}, \boldsymbol{t}_{off}, \boldsymbol{t}_{on}, \boldsymbol{t}_{on}^{start-up}\}$.
An equation (4.5) that describes how $T(t)$ varies in time can be obtained by applying eq. (3.3) in chapter 3 as follows:

$$T(\mathrm{t}) = T_H(t_0)\, e^{-\frac{(t-t_0)}{\tau}} + K\left(1 - e^{-\frac{(t-t_0)}{\tau}}\right) \tag{4.5}$$

Calculations of $T(t)$, by using an exponential equation (4.5) From **appendix C** of DEBH model the results are

$\boldsymbol{\tau} = 179.7935\ sec$	$\boldsymbol{K} = 30.3231\ °C$

By applying above equation (4.5) to obtain $T(\mathrm{t})$

	$\boldsymbol{T_H(t_0)}$ $(°C)$	$\boldsymbol{t_{on}^{m}}$ (sec)	$\boldsymbol{\tau}$ (sec)	$\boldsymbol{K}$ $(°C)$	$\boldsymbol{T}(\mathrm{t})$ $(°C)$
Scenario 1 (natural response)	10	300	179.7935	0	1.8851
Scenario 2 (forced response)	10	300	179.7935	30.3231	26.49

- ***Calculation of*** $(\boldsymbol{T_{low}})$ ***and*** $(\boldsymbol{T_{high}})$ **from appendix C of DEBH**

$$T_{low} = T_{high}\, \boldsymbol{e}^{-\frac{t_{off}^{m}}{C/G}} + T_a \left(1 - \boldsymbol{e}^{-\frac{t_{off}^{m}}{C/G}}\right) \tag{4.6}$$

$$T_{high} = T_{low}\, \boldsymbol{e}^{-\frac{t_{on}^{m}}{C/G}} + (T_a + \frac{Q}{G}) \left(1 - \boldsymbol{e}^{-\frac{t_{on}^{m}}{C/G}}\right) \tag{4.7}$$

In this case the heater is (*OFF*) thus, $Q = 0\ W \quad \rightarrow \quad K = T_a$

T_{high} $(°C)$	T_a $(°C)$	t_{off}^{m} (sec)	G $(W/°C)$	C $(J/°C)$	K $(°C)$
28	10	145.8	73.8077	13270.1464	10

$\boldsymbol{T_{low}} = 18\ °C$

In this case the heater is (*ON*) thus, $Q = 1500\ W \quad \rightarrow \quad K = T_a + \frac{Q}{G}$

T_{low} $(°C)$	T_a $(°C)$	t_{on}^{m} (sec)	G $(W/°C)$	C $(J/°C)$	K $(°C)$
18	10	300	73.8077	13270.1464	30.3231

$\boldsymbol{T_{high}} = 28\ °C$

Set-point $(\Delta T) = T_s = 23\ °\mathrm{C}$ Thus, the Dead band value $(D_b) = 5\ °\mathrm{C}$

- ***Calculation of Time:*** (t_{off}), (t_{on}) ***and*** $(t_{on}^{start-up})$ **from appendix C of DEBH**

System parameters

$\boldsymbol{T_{high}}$ $(°C)$	$\boldsymbol{T_{low}}$ $(°C)$	$\boldsymbol{T_a}$ $(°C)$	$\boldsymbol{T_s}$ $(°C)$	$\boldsymbol{G}$ $(W/°C)$	$\boldsymbol{C}$ $(J/°C)$	$\boldsymbol{Q_i}$ (W)
28	18	10	23	73.8077	13270.1464	1500

By using the equations (4.8 to 4.22) the results are

1- Linear Equations of (t_{off}) and (t_{on}) results

P_{rated} (kW)	t_{off}^{m} (sec)	t_{on}^{m} (sec)	$t_{on}^{start-up}$ (sec)
0 / 1.5	138.30	245.52	88.12

$\boldsymbol{t_{off}} = 138.30 \text{ sec} = 2.305 \text{ min} = 0.0384\ hr$

$\boldsymbol{t_{on}} = 245.52 \text{ sec} = 4.092 \text{ min} = 0.0682\ hr$

$\boldsymbol{t_{on\,(startup)}} = 88.12 \text{ sec} = 1.47 \text{ min} = 0.0245\ hr$

2- Exponential Equations of (t_{off}), $(t_{on})$and $t_{on\,(startup)}$ results

P_{rated} (kW)	t_{off}^{m} (sec)	t_{on}^{m} (sec)	$t_{on}^{start-up}$ (sec)
0 / 1.5	145.80	300	89.95

$\boldsymbol{t_{off}} = 145.80 \text{ sec} = 2.43 \text{ min} = 0.0405\ hr$

$\boldsymbol{t_{on}} = 300 \text{ sec} = 5 \text{ min} = 0.0833\ hr$

$\boldsymbol{t_{on\,(startup)}} = 89.95 \text{ sec} = 1.50 \text{ min} = 0.025\ hr$

Calculation result of the period while the thermostat is in position *ON*, the equations (4.9, 4.12) show that $\boldsymbol{t_{on}}$ is dependent on the power supply since $\boldsymbol{Q_i}$ is proportional to the heater rated power $\boldsymbol{P_{rated}}$. Otherwise, while the thermostat is in position *OFF*, the equations (4.8, 4.11) indicate that $\boldsymbol{t_{off}}$ is independent of the power supply since $\boldsymbol{Q_{in}} = 0$

Table 4.14 Comparison of equations and the simulation results

Method	$t_{on}^{start-up}$ (hr)	t_{on} (hr)	t_{off} (hr)	t_{on}^{total} (hr)	$cycling_{time}$ (hr)	*Run time* (hr)	**P** (kW)
Linear equations	0.0245	0.0682	0.0384	0.5633	0.1066	1	1.5
Exponential equations	0.0249	0.0833	0.0405	0.6831	0.1238	1	1.5
Simulation Result	0.0196	0.0828	0.0393	0.6791	0.1221	1	1.5

From table 4.14, we can conclude that the exponential equations results are more accurate than the linear equations results comparing with the simulation result.

Power / Energy Consumption of (DEBH):

Method	Q_{on} (kWh)	P_{rated} (kW)	t_{on} (hr)
Calculation	**0.1249**	1.5	0.0833
Simulation	**0.1242**	1.5	0.0828

Q_{total} (kWh)	P_{rated} (kW)	$t_{on(total)}$ (hr)	*Run-time* (hr)
1.0247	1.5	0.6831	1
5.0604	1.5	3.3736	5

Electric Heater (kW)	Q_{total} (kWh)	$t_{on(total)}$ (hr)	Run-time (hr)
1.5	1.0247	0.6831	1
1.5	5.0604	3.3736	5

The average power consumption (ΔP) from the energy flow and the energy consumption per operation cycle (EC) of the (DEBH) can be calculated by using Duty Cycle value and input rated power; in **appendix C** the results as follows:

Table 4.15 The results of equations and simulation in steady-state condition

Method	Q_i (kW)	t_{on}^{m} (hr)	t_{off}^{m} (hr)	$Cycling_{time}$ (hr)	*Duty Cycle*	ΔP_{DEBH} (kW)	EC_{DEBH} (kWh)	*Run time* (hr)
Linear Equation	1.5	0.0682	0.0384	0.1066	0.6398	0.9597	0.1023	1
Exponential Equation	1.5	0.0833	0.0405	0.1238	0.6729	1.0094	0.1249	1
Simulation Result	1.5	0.0828	0.0393	0.1221	0.6781	1.0172	0.1242	1

Table 4.16 The results of equations and simulation in transient condition.

Method	Q_i (kW)	$\boldsymbol{t}_{on}^{total}$ (hr)	$Cycling_{time}$ (hr)	$Duty\ Cycle$	ΔP_{DEBH} (kW)	EC_{DEBH} (kWh)	$Run\ time$ (hr)
Linear Equation	1.5	0.5633	0.1066	0.6398	0.9597	0.8449	1
Exponential Equation	1.5	0.6831	0.1238	0.6729	1.0094	1.0247	1
Simulation Result	1.5	0.6737	0.1221	0.6781	1.0172	1.0106	1

Comparison of transient condition and steady-state condition to illustrate the impact of the worming up time ($t_{on}^{start-up}$).

Table 4.17 Time response of the system

Methods	**P** (kW)	$\boldsymbol{t}_{on}^{start-up}$ (hr)	$\boldsymbol{t_{on}}$ (hr)	$\boldsymbol{t_{off}}$ (hr)	$cycling\ time$ (hr)	$\boldsymbol{N}_{cycles}$	$\boldsymbol{Transient\ Condition}$	$\boldsymbol{Steady-State\ Condition}$
							$\boldsymbol{t_{on(total)}}$ (hr)	$\boldsymbol{t_{on(total)}}$ (hr)
Linear Equation	1.5	0.0245	0.0682	0.0384	0.1066	7.9	0.5633	0.5388
Exponential Equation	1.5	0.0249	0.0833	0.0405	0.1238	7.9	0.6831	0.6581
Simulation Result	1.5	0.0196	0.0828	0.0393	0.1221	7.9	0.6737	0.6541

From table 4.17, we can conclude that the difference between two results of $\boldsymbol{t}_{on(total)}$ of DEBH is due to steady-state condition and transient condition therefore, the worming-up time $\boldsymbol{t}_{on}^{start-up}$ will take its (natural response) to reach the set-point temperature. Thus, this period of time in transient condition should be included in calculation of $\boldsymbol{t}_{on(total)}$ in order to calculate the overall energy consumption of the DEBH system. According to thermodynamic principles, the Transient condition and Steady-state condition can be expressed as follows:

- Transient condition is defined as a time interval that the system is taking during start-up period to reach a desired temperature or taking its time to respond to any disturbances.

- Steady-state condition is defined as a condition where the system behavior is depend on the long run time and will remain stable if there is no disturbance.

4.4.4 LOAD PROFILE AND ENERGY COST

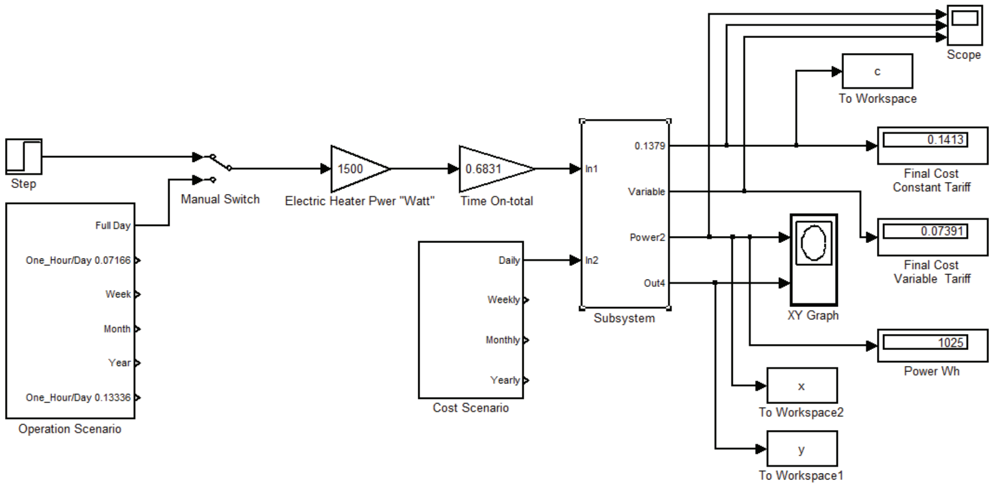

Figure 4.20 Energy Cost Calculator of DEBH

CASE 1, Simulation Results of DEBH (TOU), Run-time = 1 hr.

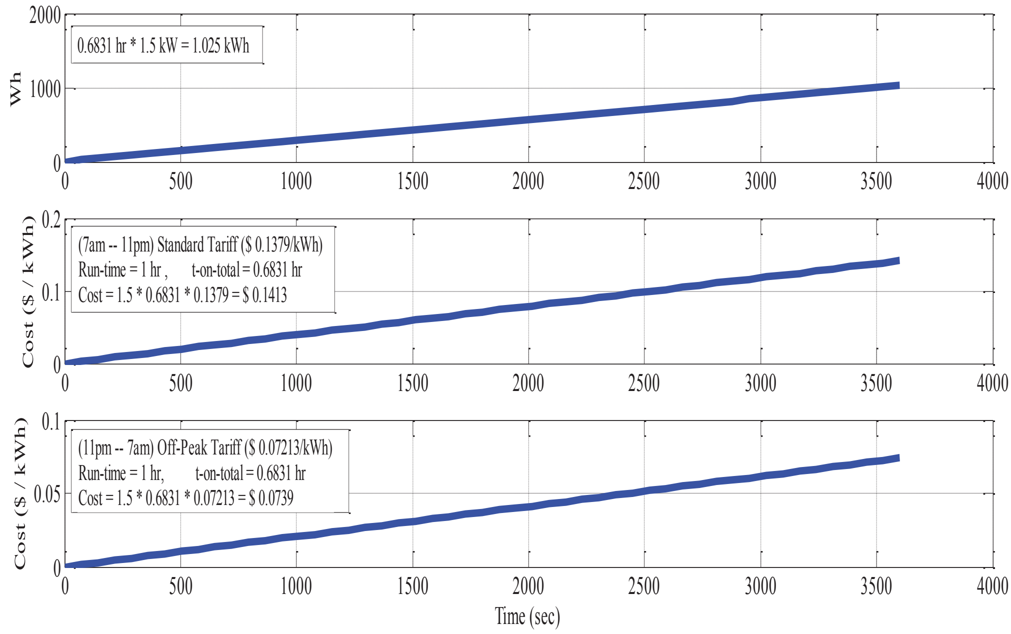

Figure 4.21 DEBH (Load and Cost vs. time) Run-time = 1 hr.

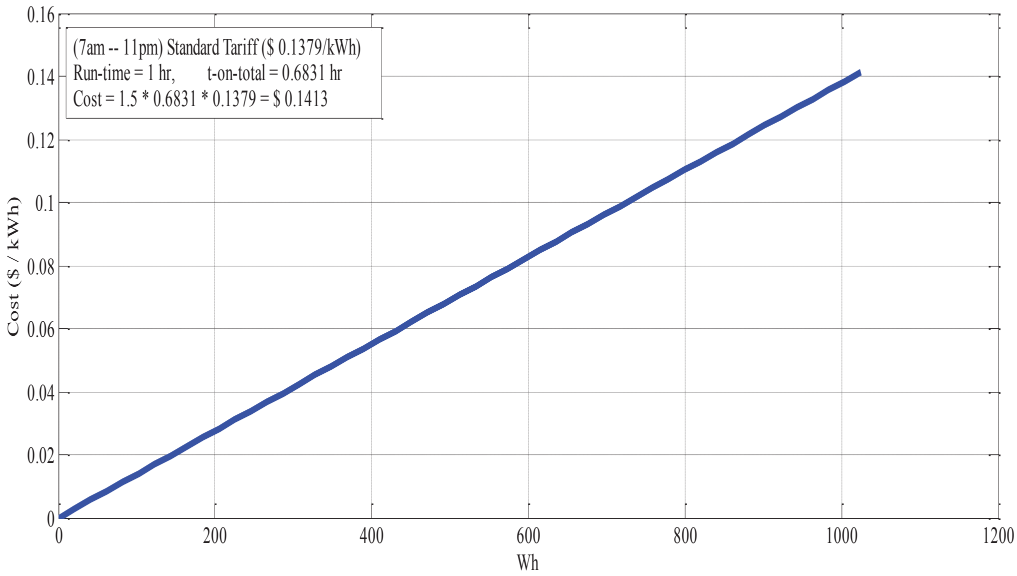

Figure 4.22 DEBH (Load vs. Cost) Standard tariff, Run-time = 1 hr.

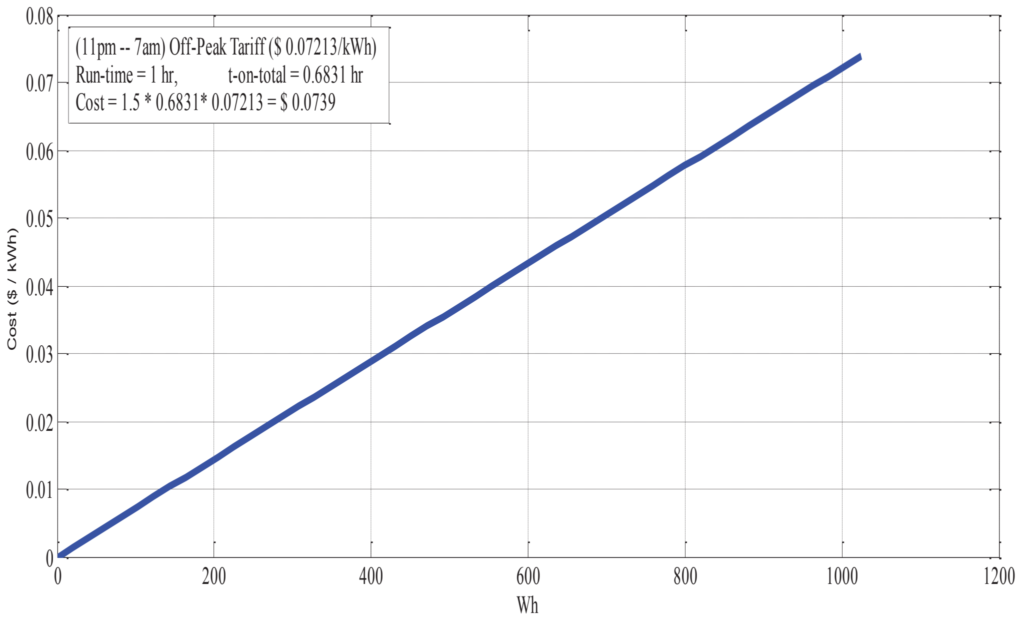

Figure 4.23 DEBH (Load vs. Cost) Off-Peak tariff, Run-time = 1 hr.

CASE 2, Simulation Results of DEBH (TOU), Run-time = 5 hrs.

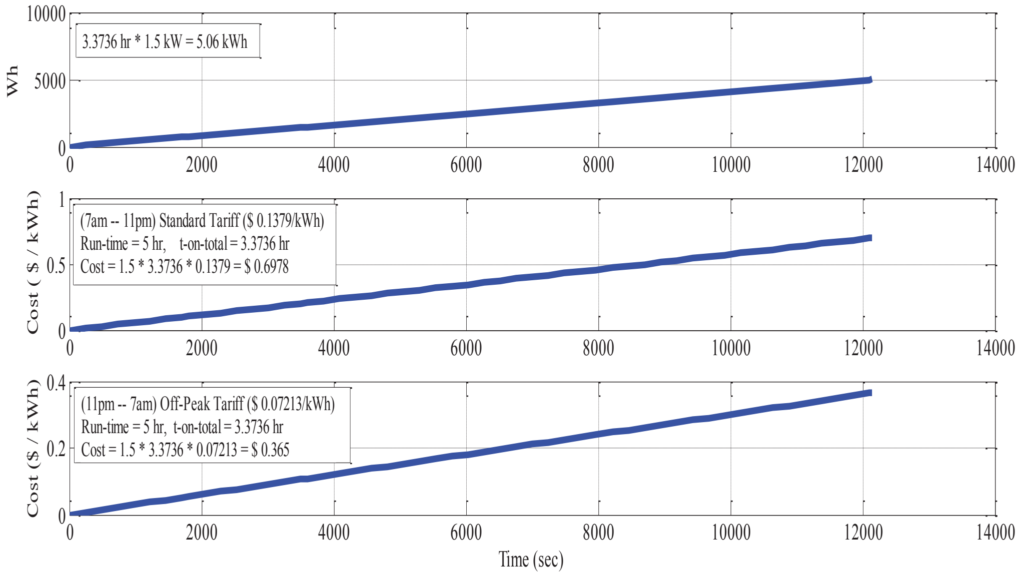

Figure 4.24 DEBH (Load and Cost vs. time) Run-time = 5 hrs.

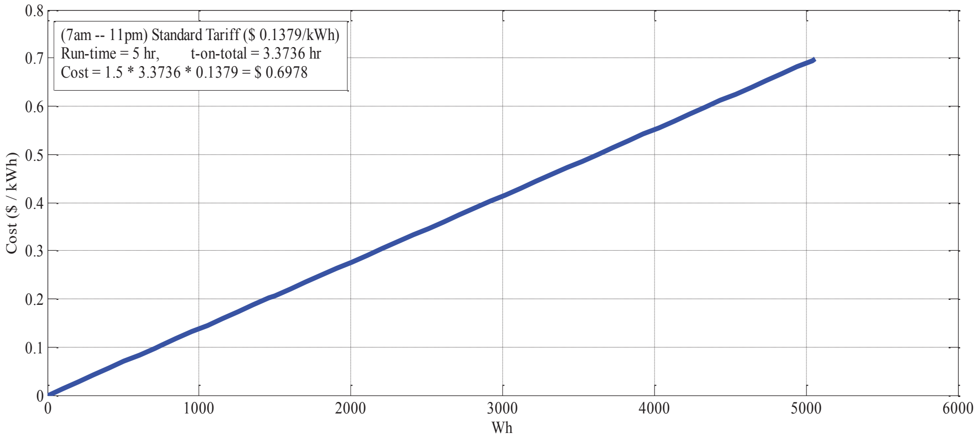

Figure 4.25 DEBH (Load vs. Cost) Standard tariff, Run-time = 5 hrs.

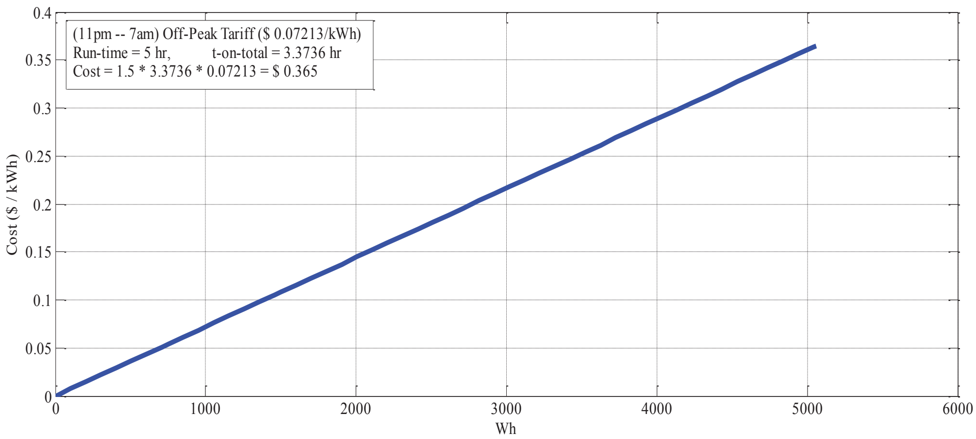

Figure 4.26 DEBH (Load vs. Cost) Off-Peak tariff
Run-time = 5 hrs.

For simulations closest to reality, the model of Energy Cost Calculator is designed based on NS power electricity rate [17] as follows:

Table 4.18 The cost-rate of NS power * *Rates effective January 1, 2013* [17]

Time & Hours			Rate/kWh
Weekends & Holidays	Year round (115 Days)	24 Hours / Day	**Off-Peak** 7.213 ¢/kWh
Weekdays	March to November (182 Days)	7am to 11pm	**Standard Rate** 13.790 ¢/kWh
		11pm to 7am	**Off-Peak** 7.213 ¢/kWh

CASE 1, Run-time = 1 hr. $t_{on(total)}$= 0.6831 hr.

The scenario of time operating of DEBH assumed to be in March includes power consumption and the energy cost calculations with respect to time and constraints.

As can be seen from figures (4.21, 4.22 and 4.23), the energy consumption of DEBH is 1.025 kWh to keep the heater at the desired temperature. Moreover, figure 4.21, of load profile of DEBH (load and cost vs. time) indicates the following:

- First figure indicates the full load: 1.5 kW * 0.6831 hrs. = 1.025 kWh
- Second figure illustrates the energy cost under the standard tariff ($ 0.1379/kWh) which is: 1.025 kWh * $ 0.1379 = $ **0.1413**
- Third figure illustrates the energy cost under off-peak tariff ($ 0.07213/kWh) which is: 1.025 kWh * $ 0.07213 = $ **0.074**

In this load profile curves the total variable tariff (TOU) not included where the off-peak tariff calculation is assumed just in case of operating time in period between (11pm to 7am) or in case of weekend.

As a result, we can conclude that the energy cost of DEBH, (run-time = 1 hr.) under the standard tariff ($0.1379/kWh) equal $ 0.1413 and the energy cost of DEBH, (run-time = 1 hr.) under the off-peak tariff equal $ 0.074. Thus, the difference between two costs is ($ 0.0674). In other words, if the customer runs the DEBH under the off-peak tariff, he/she will save up to 47.7% per this appliance.

CASE 2, Run-time = 5 hrs. $t_{on(total)}$ = 3.3736 hrs.

The scenario of time operating of DEBH assumed to be in March includes power consumption and the energy cost calculations with respect to time and constraints.

As can be seen from figures (4.24, 4.25 and 4.26), the energy consumption of DEBH is 5.0604 kWh to keep the heater at the desired temperature. Moreover, figure 4.24, of load profile of DEBH (load and cost vs. time) indicates the following:

- First figure indicates the full load: 1.5 kW * 3.3736 hrs. = 5.0604 kWh
- Second figure illustrates the energy cost under the standard tariff ($ 0.1379/kWh) which is: 5.0604 kWh * $ 0.1379 = $ **0.698**
- Third figure illustrates the energy cost under off-peak tariff ($ 0.07213/kWh) which is: 5.0604 kWh * $ 0.07213 = $ **0.37**

In this load profile curves the total variable tariff (TOU) not included where the off-peak tariff calculation is assumed just in case of operating time in period between (11pm to 7am) or in case of weekend.

As a result, we can conclude that the energy cost of DEBH, (run-time = 5 hrs.) under the standard tariff ($0.1379/kWh) equal $ 0.698 and the energy cost of DEBH, (run-time = 5 hrs.) under the off-peak tariff equal $ 0.37. Thus, the difference between two costs is ($ 0.3328). In other words, if the customer runs the DEBH under the off-peak tariff, he/she will save up to 47.7% per this appliance.

However, calculates an optimal schedule for DEBH based on fluctuating tariffs will discuss later in the optimization chapter.

4.5 MODEL of LIGHTING

4.5.1 INTRODUCTION

The lighting system plays a significant role in every building, whether it is a natural or artificial source. Numerous light sources are available across the world; the most desirable light source is the natural light from sun referred to as daylight; artificial sources of lighting are incandescent lamps, fluorescent-lamps (F-Lamp), compact fluorescent lamps (CFL) and light emitting diode (LED), etc.

Fluorescent-lamps have become a major method of lighting systems and have become widely accepted in both residential and commercial sectors, particularly in offices, public places, and factories. In addition, fluorescent-lamps commonly are used in a residential sector. It is preferable to conventional tungsten incandescent lamps because of their high luminous efficacy (lumens/watts) [58, 59 and 60]. In this study, the fluorescent-lamp is presented.

Fluorescent-lamp is a device that presents a negative impedance to the power supply and operating at high frequency (20 kHz or above). According to [59], there are two fundamental considerations that are used to evaluate electrical characteristic of any discharge lighting system, the starting and operating scenarios. Thus, the electrical circuit of the fluorescent lamp must contain some form of current-limiting such as an auxiliary device, which is commonly called ballast, to provide a positive resistance or reactance in order to regulate the current flow through the tube [61, 65]. In other words, the ballast provides a suitable starting voltage and positive resistance; a higher voltage is provided by a capacitor, which is placed in parallel to the lamp; in this case the ballast is often called electronic ballasts; the positive resistance limits the ultimate flow of current during lamp operation [61, 62, 65, and 70]. There are two types of ballasts commonly used in residential and commercial lighting, the line-frequency conventional electromagnetic ballast and the high-frequency electronic ballast [65]. Electronic ballasts are more energy-efficient than magnetic ballasts. Fluorescent lamps are available in many shapes in the market such as linear shapes, U shapes, long twin-tube shape, compact fluorescent lamps and circular fluorescent lamps [63, 64, 70 and 71]. The main parts of the fluorescent lamps power circuit are the ballast and the lamp tube. Thus, the overall power consumption of fluorescent lighting system is lamp power rated plus the power that is consumed by ballast during system operation.

4.5.2 MODEL of FLUORESCENT LAMP (F-Lamp)

In recent years however, numerous fluorescent-lamp models and approaches have developed the characteristic and the system behavior of fluorescent lighting systems [56, 57 and 58]. These models are implemented by using the SPICE-based program. Generally, these kinds of models employ a complicated model equation and many parameters; therefore, these models are not suitable for electrical circuit simulation [66]. This kind of problem is challenging due to the lack of information availability. However, the main purpose of this section is to present a simple model that describes main parameters of fluorescent-lamp and the system behavior during run-time in order to calculate the overall energy consumption.

By considering some previously presented models and using the data which has been collected from experimental in [65, 66, 67, 68 and 70]. These models present the fluorescent-lamp by a power dependent resistance. A mathematical model of fluorescent-lamp based on work done in "Electronic Ballast with Wide Dimming Range: Matlab-Simulink Implementation of a Double Exponential Fluorescent-Lamp Model" [68] was chosen for implementation consists in a simple equation capable of describing the electrical characteristics of the lamp at high frequency; in equation (4.23) a regressive method employs an exponential approximation of the equivalent fluorescent lamp resistance as a function power in order to represent a curve fitting to the experimental data of equivalent resistance versus average power also to obtain the lamp current and lamp voltage as well.

Decreasing monotonic double exponential function is

$$\boldsymbol{R_{lamp} = ae^{-bP_{lamp}} + ce^{-dP_{lamp}}} \tag{4.23}$$

Where,

R_{lamp} : fluorescent lamp resistance

P_{lamp} : fluorescent lamp average power

a, b, c, d : the coefficient of the exponential equation

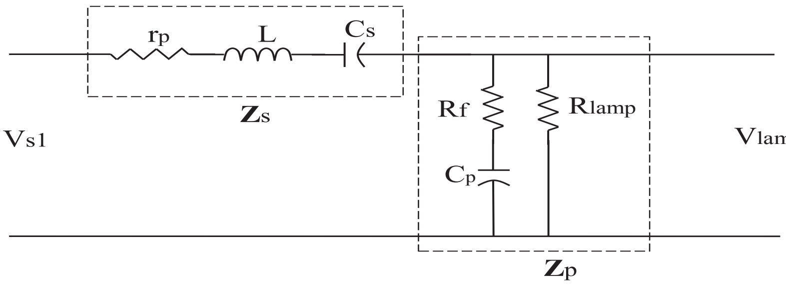

Figure 4.27 Equivalent circuit of the series resonant parallel loaded ballast and f-lamp [61]

From figure 4.27, the lamp voltage can be calculated [61] as follows:

$$\boldsymbol{V}_{lamp} = \frac{V_{s1} * Z_p}{(Z_p + Z_s)} \quad (4.24)$$

Where,

V_{s1} : rms volt of power supply

Z_p : the impedance of the parallel brunch

Z_c : the impedance of the series brunch

$$\mathbf{Z_P} = \frac{1}{\frac{1}{R_{lamp}} + \frac{1}{R_f + \frac{1}{j\omega C_p}}} \quad (4.25)$$

$$\mathbf{Z_S} = j\omega L + \frac{1}{j\omega C_s} + r_p \quad (4.26)$$

Where,

r_p, R_f : noise resistance of inductance and capacitance

$$\mathbf{Z_{total}} = Z_P + Z_S \quad (4.27)$$

The lamp current can be calculated [61] as follows:

$$\boldsymbol{I}_{lamp}(t) = \boldsymbol{G}_{lamp}(t) * \boldsymbol{V}_{lamp}(t) \quad (4.28)$$

Where,

$\boldsymbol{G}_{lamp}(t)$: lamp conductance

In order to calculate $\boldsymbol{R_{lamp}}$ the coefficient of an exponential equation (4.23) and the average power values are obtained from model parameters [68] as follows:

Table 4.19 Simulink model parameters [68]

Double Exponential Model
a = 8147 b = −0.2113 c = 1433 d = −0.05353
Time constant τ = 3E-4 sec
$\boldsymbol{R_f} = 2 \times 9.6\ \Omega$
Electronic Ballast
$\boldsymbol{V_{s,1}} = {}^{800}/_{\pi\sqrt{2}}\ V$
$\boldsymbol{L} =$ 2.2358E − 3 H
$\mathbf{A} =$ 0.1307
$\boldsymbol{C_p} =$ 3.1639E − 9 F
$\boldsymbol{C_s} =$ 2.407E − 8 F

By applying equation 4.23, to calculate $\boldsymbol{R_{lamp}}$ based on the table 4.20

$$\boldsymbol{R_{lamp}} = 8147e^{-\mathbf{0.2113*16.7430}} + 1433e^{-\mathbf{0.05353*16.7430}} = \mathbf{825.21\ \Omega}$$

Table 4.20 of V-I Characteristics of fluorescent lamp 32 W that presented in [61] shows the value of R_{lamp} at high frequency; as can be seen from this table, there are two values of the equivalent resistance of the lamp as follows:

$\boldsymbol{R_{lamp-\mathrm{VI}}}$ is based on the division of V_{rms} and I_{rms} of the lamp circuit

$\boldsymbol{R_{lamp-\mathrm{PI}}}$ is based on the division of the average lamp power P_{avg} by the square root of current I_{rms}

Table 4.20 V-I Characteristics of a 32 W General Electric fluorescent-lamp [61]

Power level	V_{RMS} [V]	I_{RMS} [A]	P_{AVG} [V]	$R_{lamp\text{-}VI}$ [Ω]	$R_{lamp\text{-}PI}$[Ω]	$\frac{\Delta R_{lamp}}{R_{lamp}}$ %	Frequency [KHz]
maximum	99.5393	0.2888	28.2260	344.61	338.3	1.848	51.653
↓	103.1804	0.2576	26.0112	400.49	391.88	2.173	57.339
	104.6403	0.2321	23.8078	450.84	441.93	1.996	61.125
	110.9816	0.1974	21.4655	562.32	551.08	2.019	68.871
	113.2090	0.1821	20.1859	621.71	608.79	2.099	72.046
	114.1565	0.1690	18.8594	675.51	660.38	2.265	75.586
	120.2613	0.1424	16.7430	844.35	825.32	2.279	80.775
	126.6726	0.1134	14.0515	1117.2	1093	2.189	85.470
minimum	143.8952	0.0417	5.8999	3447.4	3386.4	1.785	90.909

4.5.3 MATHEMATICAL MODEL of FLUORESCENT LAMP (FL)

Calculation of equivalent simplified circuit of the series resonant and parallel load ballast:

In order to calculate the current and lamp voltage we have to calculate the series and parallel impedances of the lamp circuit as follows:

Assumptions based on table 4.20, V-I Characteristics a 32 W

V_{s1} (v)	f (kHz)	C_s (F)	C_p (F)	L (H)	ω (rad/sec)	R_f (Ω)
120	80.775	2.2473E-8	2.9372E-9	2.4E-3	507267	2×9.6

- Impedance of parallel brunch (Z_p) can be calculated from equation 4.25

$$\mathbf{Z}_p = R_{lamp} \,\|(R_f + Z_{C_P}) = R_{lamp} \,\|(R_f - \frac{1}{j\omega C_p})$$

$$\overline{Z_P} = \frac{1}{\frac{1}{R_{lamp}} + \frac{1}{R_f + \frac{1}{j\omega C_p}}} = \frac{1}{\frac{1}{825.205} + \frac{1}{19.2 - j\frac{1}{507267 * 2.9372E-9}}}$$

$$\overline{Z_P} = 330.9919 - j392.8198\ \Omega$$

- Impedance of series brunch (Z_s) can be calculated from equation 4.26

$$\mathbf{Z_s} = Z_{L_S} + Z_{C_S} = j\omega L - \frac{1}{j\omega C_s}$$

$$\overline{Z_{L_s}} = j\omega L = j507267 * 2.4E-3 = j1217.4408\ \Omega$$

$$\overline{Z_{C_s}} = -\frac{1}{j\omega C_s} = -j\frac{1}{507267} * 2.2473E-8 = -j87.7208\ \Omega$$

$$\overline{Z_S} = j1129.72\ \Omega$$

$$\overline{Z_T} = \overline{Z_S} + \overline{Z_P} = (j1129.72) + (330.9919 - j392.8198)$$

$$\overline{Z_T} = 330.9919 + j736.9002\ \Omega$$

$$\overline{Z_T} = 807.8227\ \angle 65.8119°$$

Total current of the circuit is given by

$$\mathbf{I}_T = \frac{\overline{V}}{\overline{Z_T}} = \frac{120\angle 0°}{807.8227\ \angle 65.8119°} = \mathbf{0.1485} \angle -65.8119°\ \text{A}$$

Lamp voltage from equation (4.24)

$$\boldsymbol{V}_{lamp} = \frac{V_{s1} * Z_p}{(Z_p + Z_s)} = \frac{(120\angle 0°)(513.676\angle{-49.8823°})}{(513.676\angle{-49.8823°}) + (1129.72\angle 90°)} = 76.3052\angle - 115.69° \text{ v}$$

Lamp Current from equation (4.28)

$$\boldsymbol{I}_{lamp}(t) = \boldsymbol{G}_{lamp}(t) * \boldsymbol{V}_{lamp}(t) = (825.21)^{-1} * (76.3052) = 0.0925 \text{ A}$$

Average lamp power is given by

$$\boldsymbol{P}_{avg} = (I_{rms})^{2*}(Z_T) = (0.1485)^{2*}(807.8227) = 17.8\, W$$

The simulation results of fluorescent-lamp circuit

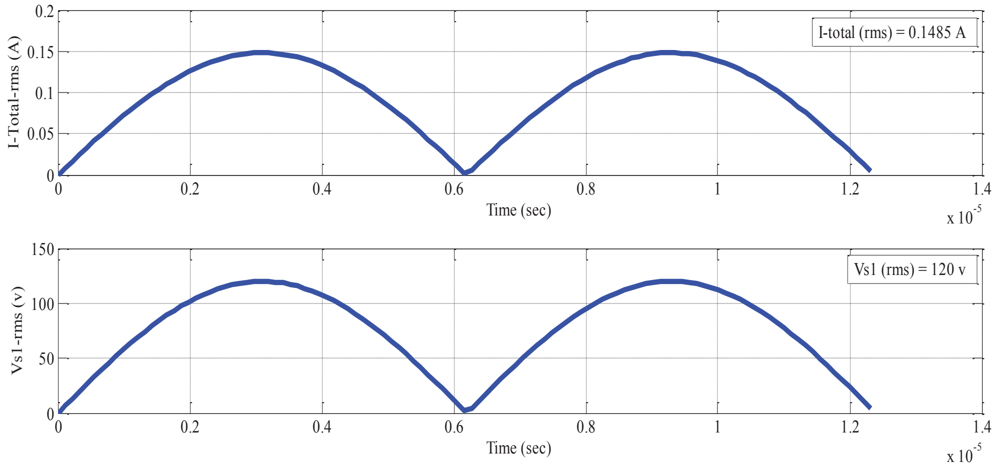

Figure 4.28 Vs1 (rms) and $\mathbf{I_{total}}$ (rms) of F-lamp

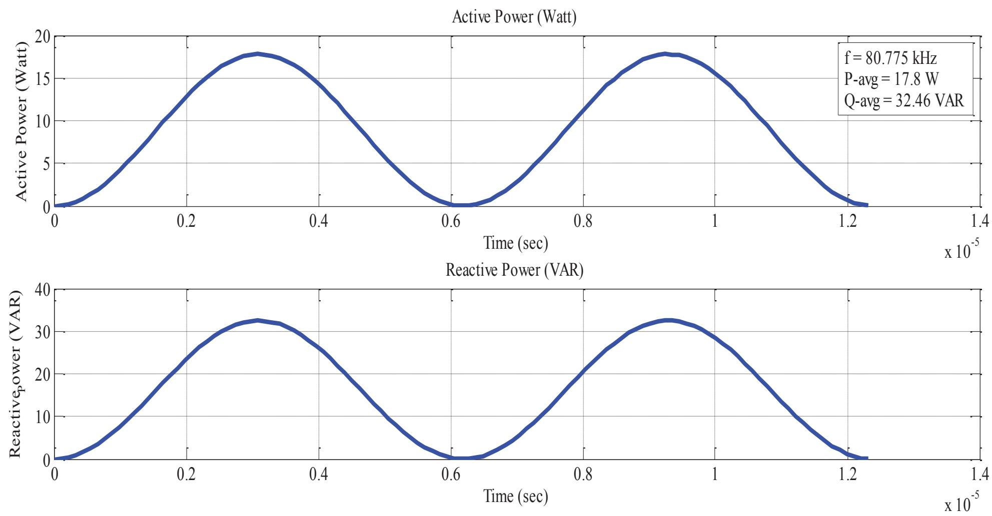

Figure 4.29 Active power and Reactive power of F-lamp for 80.775 kHz

Table 4.21 The average operating time and energy use of fluorescent-lamp

Appliance	Power (kW)	Run-time (hr.)	Energy consumption (kWh)
Fluorescent-lamp	0.032	1	0.032
		3	0.096
		5	0.16
		8	0.256
		12	0.384

CHAPTER 5 MODELING of MACHINES

5.1 INTRODUCTION

In literature , this kind of systems is known as a multi-physical system since it's contain mechanical elements, a hydraulic system, a thermal system and the electronic controls [35]. The basic operating principles of the multi-physical systems is considered in order to design models that can be implemented and simulated in Matlab program; thus, a suitable models of the following three appliances are presented in this chapter:

- Domestic Electrical Clothes Washer (DECW)
- Domestic Electrical Clothes Dryer (DECD)
- Domestic Electrical Dishwasher (DEDW)

In this chapter, all above appliances designed, simulated and investigated. Moreover, in order to gather enough data to be able to model all the machines, this chapter discusses the modeling and parameter identification of all above three systems and investigates the impact of its main parameters on its ability to assist with reducing energy consumption and more efficiency. A modern dishwasher is assumed to be used approximately 200 times/year; in this case, it uses 1.05 kWh per cycle, where 0.924 kWh is electricity for heating. A clothes washer and clothes dryer are assumed to be used approximately 200 times/year; thus, the clothes washer uses 0.95 kWh per cycle, where 0.76 kWh is electricity for heating; while a clothes dryer uses 2.6 kWh per cycle [38, 38 and 39]. Therefore, for three cases most of the energy is required for hot water and hot air rather than for electric motor use [36].

5.2 MODEL of DOMESTIC ELECTRIC CLOTHES WASHER (DECW)

5.2.1 INTRODUCTION

A Clothes Washer is a common appliance used for domestic and commercial purpose. On the other hand, a clothes washer typically contains four parts; mechanical elements that run the system, a hydraulic system that provides the water to the laundry, a thermal system that controls the temperature level and the electronic controls that manage the whole cycle to achieve overall system performance [35, 36]. The motor of the clothes washer has various speeds to operate, a spin speed of 1200 rpm is common and a peak spin speed as high as 1600 rpm. New models have speeds of 1800-2000 rpm [36, 39]. In this study, DECW (Non- Electric Water Heating) is presented; the focus of the simulation section was to study and present in details a performance of a modified version of a single-phase motor [78] of DECW.

5.2.2 MODEL of A SINGLE-PHASE INDUCTION MOTOR of DECW

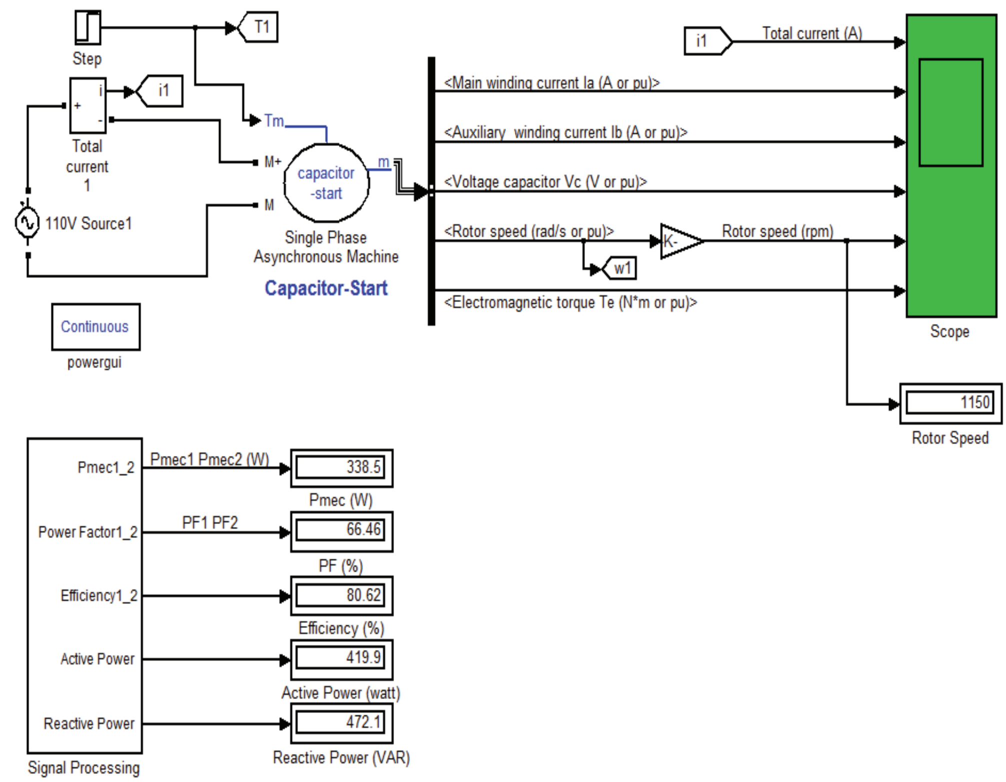

Figure 5.1 Modified version model of a single-phase induction motor of DECW [78]

5.2.3 MATHEMATICAL MODEL of A 1-PHASE INDUCTION MOTOR DECW

The aim of this section is to provide, a sample calculation in order to analyse the DECW system behavior during steady-state condition response. This study presents just in details a single-phase induction motor. In literature, the operation time and energy use of clothes washer as follows:

Table 5.1 The average operating time / energy consumption of DECW [40]

Appliance	Power (W)	Run-time Hours/month	Energy consumption kWh/month
Clothes Washer, Automatic (With Electric Water Heating)	500	7-40	33-196
Clothes Washer, Automatic (Non- Electric Water Heating)	500	7-40	3-16

MATHEMATICAL MODEL of A 1-PHASE INDUCTION MOTOR of DECW

Appendix **D** has all specifications and calculations of equations circuit of a single-phase induction motor of clothes washer DECW. In this model of DECW an induction motor single-phase capacitor start is implemented, the parameters of the motor are

Table 5.2 Induction motor single-phase capacitor start specifications

Volt (v)	Frequency (Hz)	Stator		Rotor		X_m (Ω)	slip	pole
		R_1 (Ω)	X_1 (Ω)	R_2 (Ω)	X_2 (Ω)			
120	60	4.20	1.0305	2.47	0.7962	58.1779	0.05	6

Simulation result of a single-phase induction motor capacitor-start of DECW run-time = 30 min

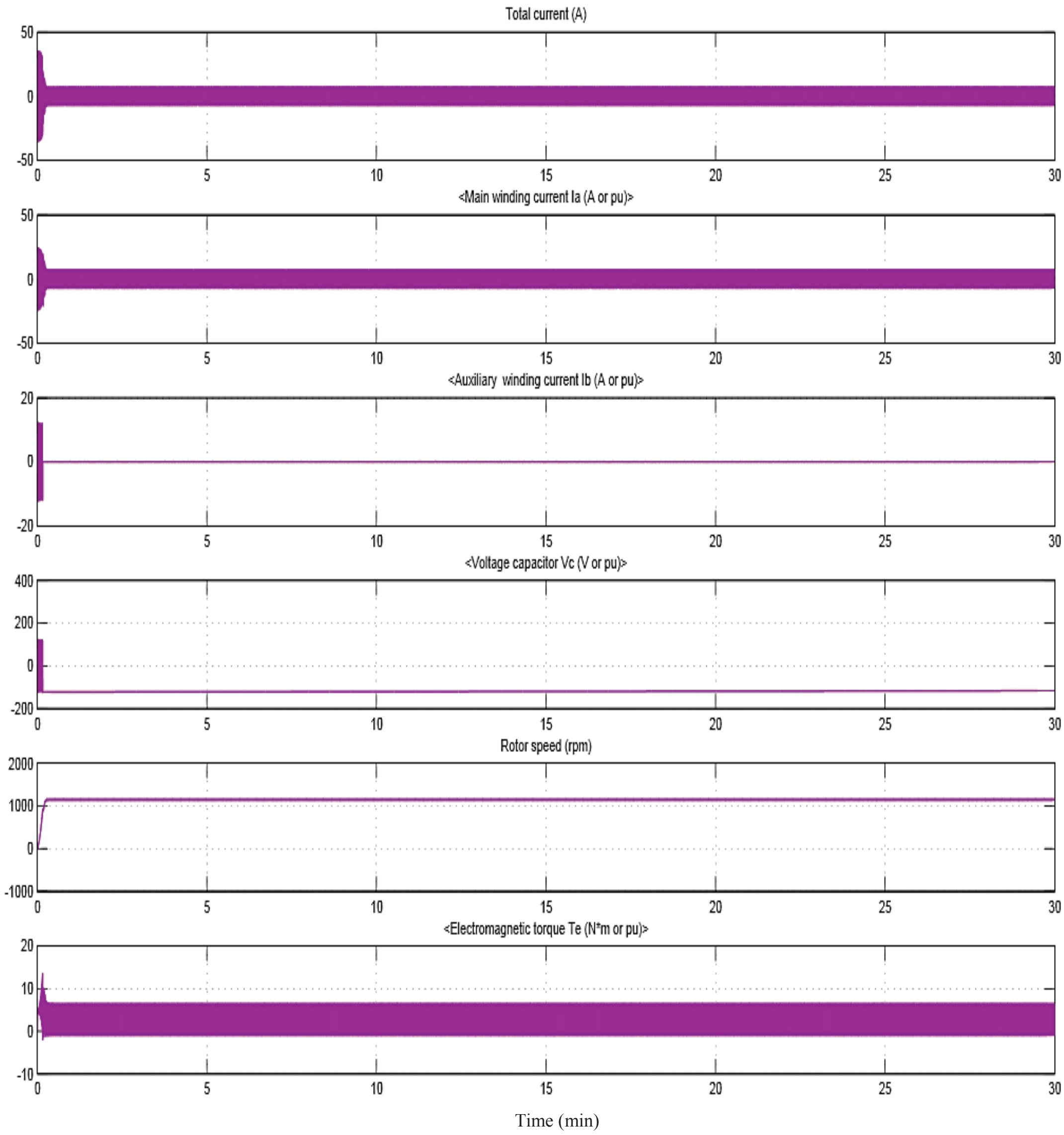

Figure 5.2 System behavior of DECW within run-time 30 min

Based on the calculations of equation circuit of 1-phase induction motor of DECW in **appendix D**, the results as follows:

Table 5.3 Comparison of the calculations and the simulation results

Methods	P_{input} (W)	P_m (W)	**PF**	T_m $(N.m)$	n_s (rpm)	n_r (rpm)	η
Calculation	500.113	338.8424	0.8111	2.8384	1200	1140	0.68
Simulation Result	506.2	336.2	0.8063	2.81	1200	1143	0.67

From table 5.3, we can conclude that the mathematical model of a single-phase induction motor capacitor-start of DECW has a good agreement with the simulation results. It should be pointed out that in the simulation result, the clothes washer machine run-time is assumed to be 30 minutes. However, the run-time depends on the consumer choice and behavior as well as the number of family and their needs which will appear in operation cycle. On the other hand, in commercial sector the run-time in most cases approximately 8 h/day which much greater than domestic sector uses. Virtually, this difference would affect and increase the energy consumption of the clothes washer per day as well as the energy cost.

Energy Consumption of Clothes washer (Non- electric water heating)

Energy consumption of clothes washer depends on many factors; the energy use, clothes load capacity and complete washing process, which consisting of a series of different operations such as wash, rinse, spin, etc. In case of one load per day the energy consumption from **appendix D** as follows:

P_{rated} (kW)	$Run\ time$ (hr)	$E.C$ (kWh)
0.5	0.5	0.25

Case 1, run-time = 0.5 hr. per load

	P_{rated} (kW)	$Run\ time$ (hr)	$E.C$ (kWh)
day	0.5	0.5	0.25
week	0.5	3.5	1.75
month	0.5	15	7.5
year	0.5	180	90

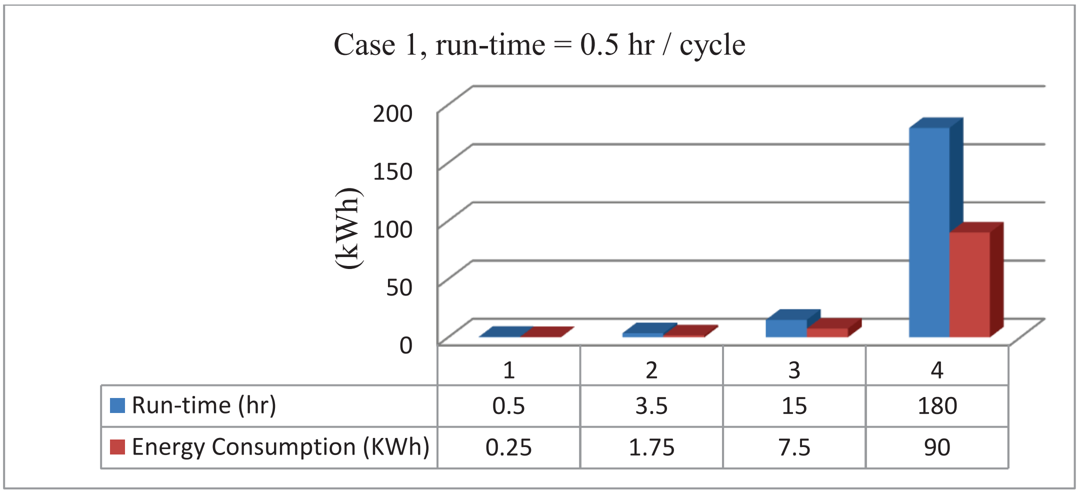

Figure 5.3 Case 1- DECW average energy consumption, run-time every day

Case 2, run-time = 1.5 hr. per load

	P_{rated} (kW)	**Run time** (hr)	$E.C$ (kWh)
day	0.5	1.5	0.75
week	0.5	3	1.5
month	0.5	12	6
year	0.5	144	72

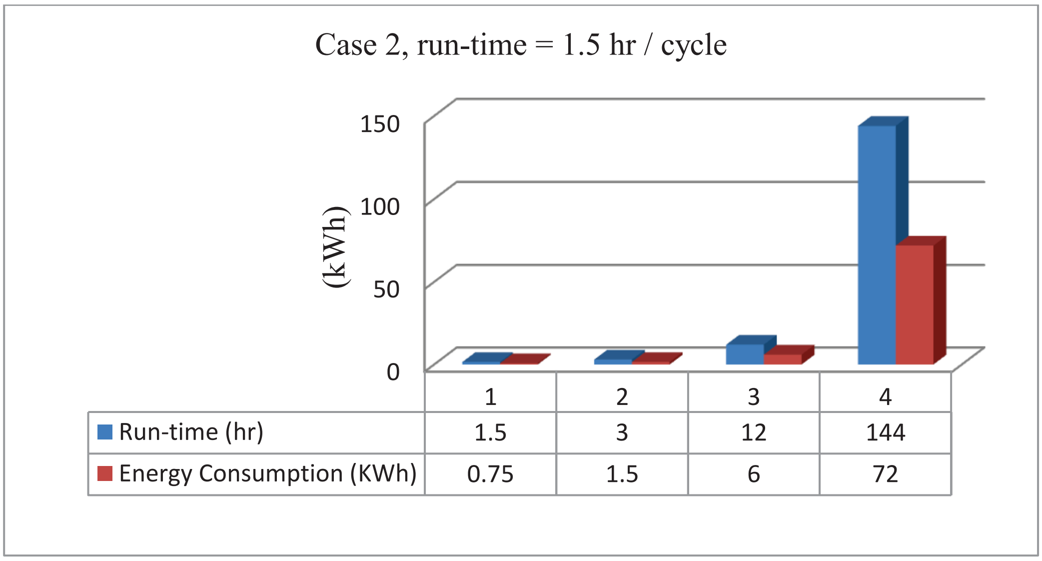

Figure 5.4 Case 2- DECW average energy consumption, run-time twice a week

Although the assumption of using the clothes washer machine every day is realistic of commercial sector but is unrealistic of domestic sector since it's more than the average of household use. According to [46] Energy Consumption of Major Household Appliances Shipped in Canada, Summary Report "The average annual unit energy consumption (UEC) of clothes washer decreased dramatically between 1990 and 2009; In 2009 the average (UEC) was 234 kWh/yr. compared with 1218 kWh/yr. in 1990, mostly due to energy-efficiency improvement and to the increasing popularity of front-loading units which are more energy-efficiency than top-loading units".

In case 2 figure 5.4, we assume that the clothes washer machine un-electrical water heating for a middle-class family is used twice a week since it should be run before the clothes dryer machine with a standard load of dryer 1.5 hr. per cycle. Thus, the difference between two cases is due to the energy consumption which reflects the average of operation time. In comparison, the average annual energy consumption in case-1 is greater than in case-2 as shown in figures (5.3 and 5.4).

5.2.4 LOAD PROFILE AND ENERGY COST

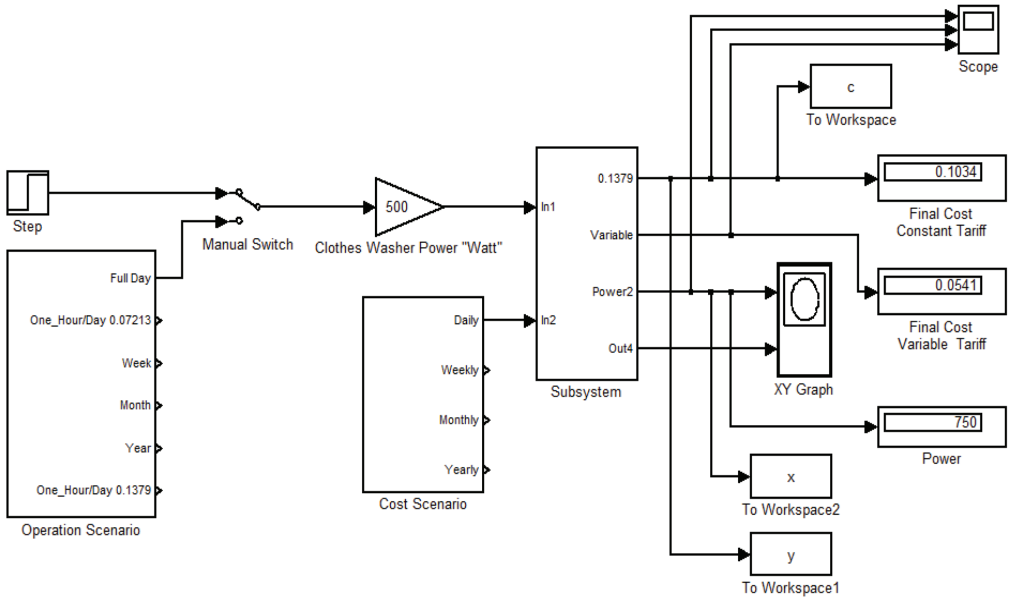

Figure 5.5 Energy Cost Calculator of DECW

Simulation Results of DECW (TOU) case 1, Run-time = 1.5 hr. /cycle

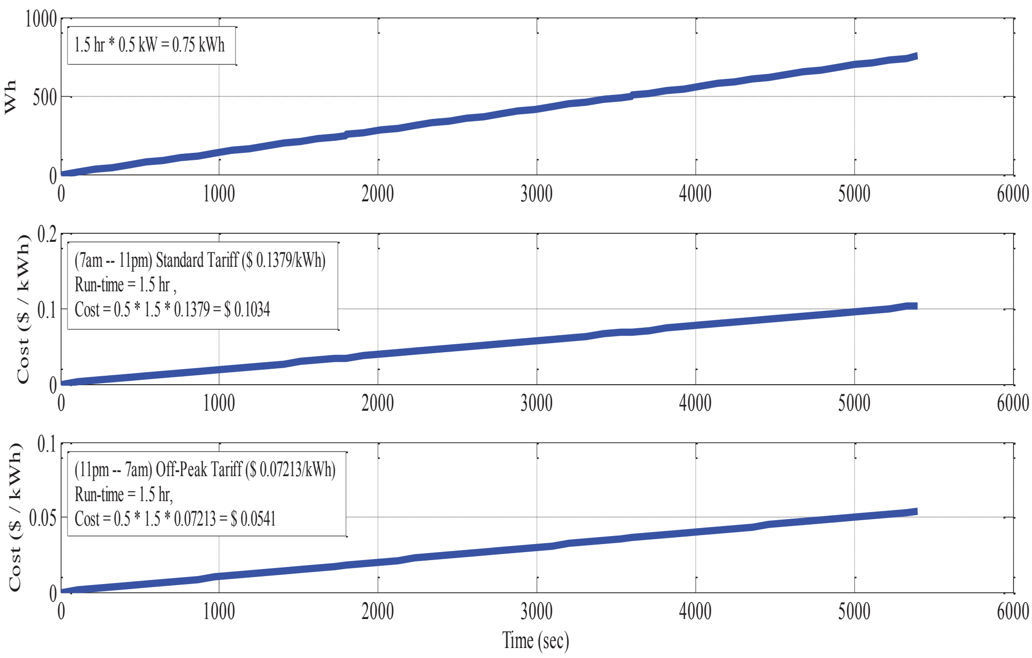

Figure 5.6 Load profile of DECW (Load and Cost vs. time)
Run-time = 1.5. hr. /cycle

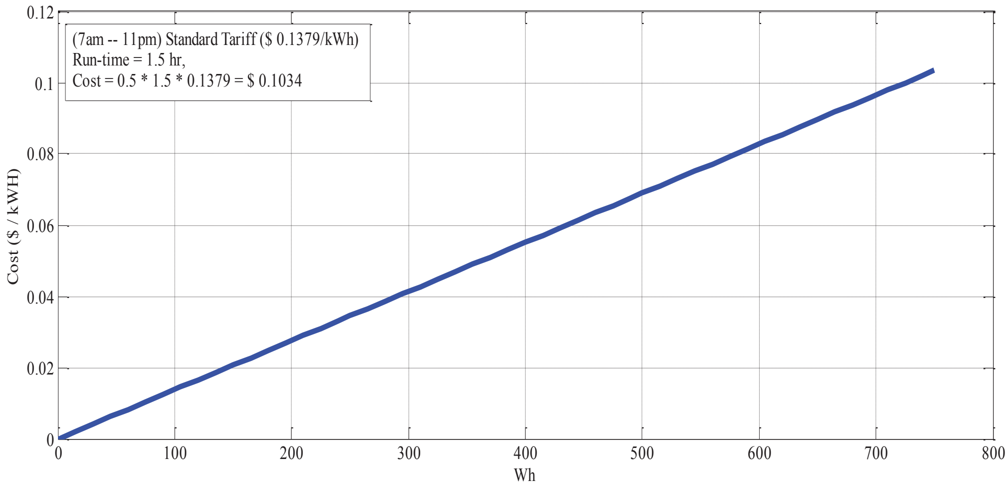

Figure 5.7 Load profile of DECW (Load vs. Cost) Standard tariff
Run-time = 1.5 hr. / cycle

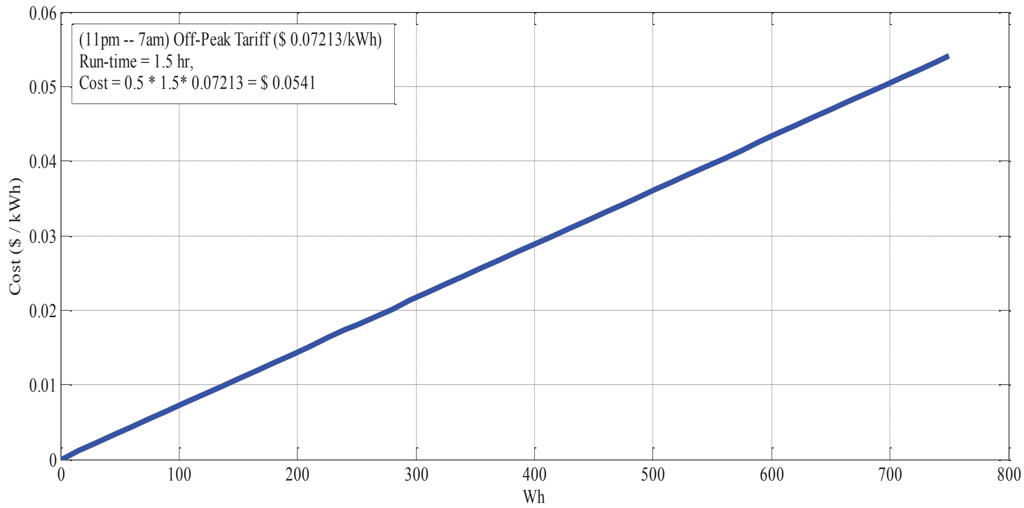

Figure 5.8 Load profile of DECW (Load vs. Cost) Off-Peak tariff
Run-time = 1.5 hr.

For simulations closest to reality, the model of Energy Cost Calculator is designed based on NS power electricity rate [17] as follows:

Table 5.4 The cost-rate of NS power * *Rates effective January 1, 2013* [17]

Time & Hours			**Rate/kWh**
Weekends & Holidays	Year round (115 Days)	24 Hours / Day	**Off-Peak** 7.213 ¢/kWh
Weekdays	March to November (182 Days)	7am to 11pm	**Standard Rate** 13.790 ¢/kWh
		11pm to 7am	**Off-Peak** 7.213 ¢/kWh

The scenario of time operating of DECW assumed to be in March includes power consumption and the energy cost calculations with respect to time and constraints.

As can be seen from figures (5.6, 5.7 and 5.8) the energy consumption of DECW is 1.025 kWh to keep the heater at the desired temperature. Moreover, figure 5.7, of load profile of DECW (load and cost vs. time) indicates the following:

- First figure indicates the full load: 0.5 kW * 1.5 hrs. = 0.75 kWh
- Second figure illustrates the energy cost under the standard tariff ($ 0.1379/kWh) which is: 0.75 kWh * $ 0.1379 = $ **0.1034**
- Third figure illustrates the energy cost under off-peak tariff ($ 0.07213/kWh) which is: 0.75 kWh * $ 0.07213 = $ **0.054**

In this load profile curves the total variable tariff (TOU) not included where the off-peak tariff calculation is assumed just in case of operating time in period between (11pm to 7am) or in case of weekend.

As a result, we can conclude that the energy cost of DECW, (run-time = 1.5 hr.) under the standard tariff ($0.1379/kWh) equal $ 0.1034 and the energy cost of DECW, (run-time = 1.5 hr.) under the off-peak tariff equal $ 0.054. Thus, the difference between two costs is ($ 0.049). In other word, if the customer runs the DECW under the off-peak tariff, he/she will save up to 47.7% per cycle in this appliance.

However, calculates an optimal schedule for DECW based on fluctuating tariffs will discuss later in the optimization chapter.

5.3 MODEL of DOMESTIC ELECTRIC CLOTHES DRYER (DECD)

5.3.1 INTRODUCTION

A clothes dryer, also known as a tumble dryer or drying machine, is a common appliance for domestic and commercial use. In particular, a clothes dryer is perfectly co-ordinated with the clothes washer machine, whose task is to remove moisture from a load of clothes after being cleaned in a clothes washer. A typical clothes dryer consists of four main parts, including a rotating tumbler, an electric-powered heater, an induction motor and a set of controls [41, 42].

- Rotating tumbler which holds clothes and circulates heated air.
- Induction motor is used for rotating the tumbler.
- Electric-heated coils as a heat source to heat up the air.
- A set of controls.

There are several types of clothes dryers available in the market these days for both domestic and commercial use with special design features that can be modified according to the user needs and choice such as drying time, temperature setting, moisture control and load capacity. The main two common types of clothes dryers are the venting tumble dryer and the condensing tumble dryer. One of the most widely used in the residential sector is the condensing tumble dryer; the advantage of this type is that all heat supplied to the dryer eventually ends up in the laundry room [43, 44].However, in this study the domestic electric clothes dryer (DECD) that consists of an induction motor and a heater is presented.

5.3.2 MODEL of DECD

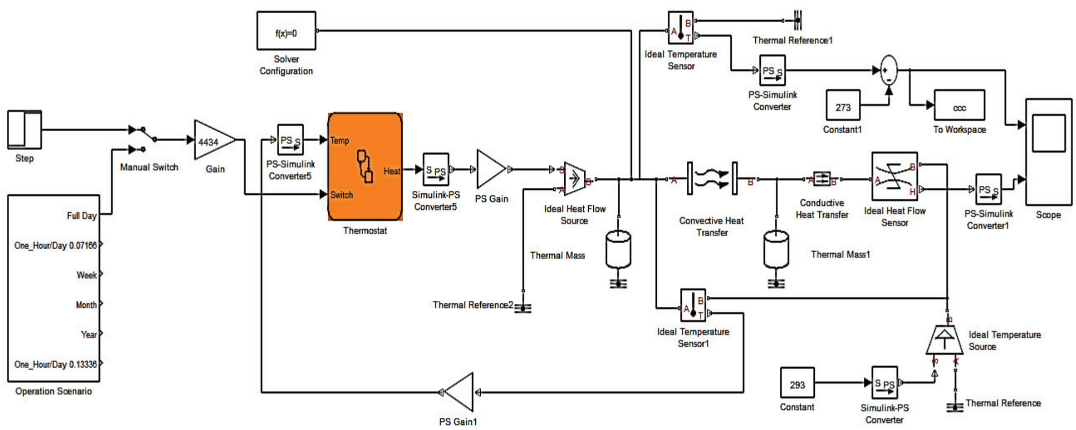

Figure 5.9 Model of Domestic Electric Clothes dryer (heater)

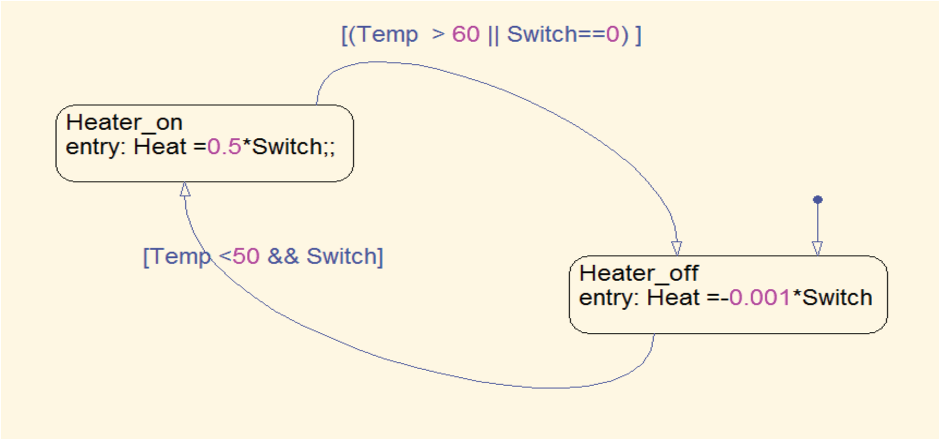

Figure 5.10 State flow model of the thermostat of (DECD-Heater)

Table 5.5 Model of (DECD) heater specifications

Convection Parameters				
Convective Heat Transfer		Thermal Mass		
Area (m^2)	Heat Transfer Coefficient $(W/m^2.°C)$	Mass (kg)	Specific heat Cp $(J/kg.°C)$	Initial Temperature (K)
1.2	70	3.5	640	293

Conduction Parameters					
Conductive Heat Transfer			Thermal Mass		
Area (m^2)	Thickness (m)	Thermal Conductivity $(W/m.°C)$	Mass (kg)	Specific heat Cp $(J/kg.°C)$	Initial Temperature (K)
0.5	0.025	45	15	1005.4	293

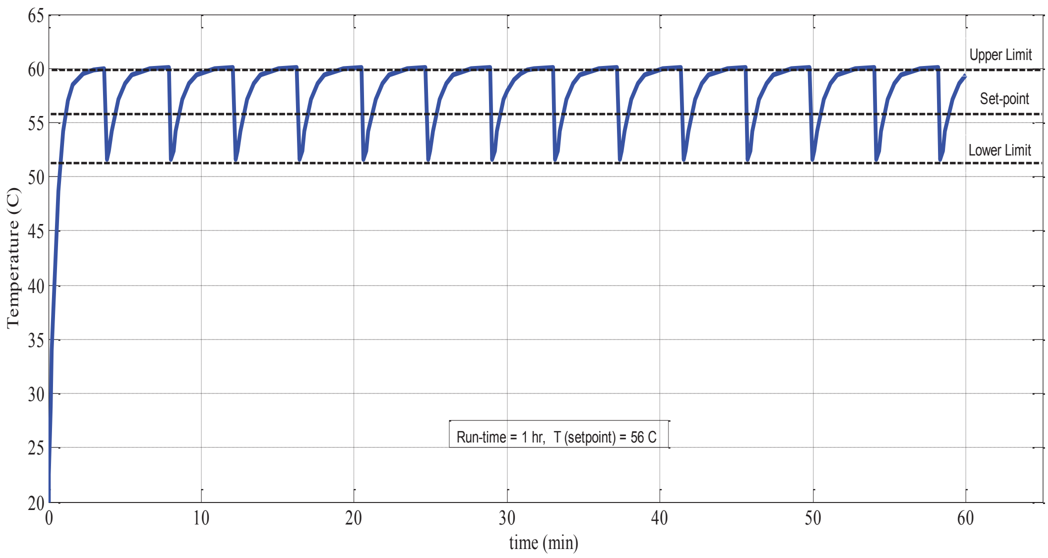

Figure 5.11 Temperature vs. Time of Clothes dryer Run-time = 1 hour

According to [45] the clothes dryer has a certain range of temperature that provide good quality of drying and avoiding any damage might cause in clothes and system itself where the setting of temperature can be adjusted according to the user needs and choice as follows:

Cycle	**Temperature** $(°C)$
Low Heat (Delicate/Gentle)	51
Medium Heat (Permanent/Press)	57
High Heat (Normal/Cottons)	57

On the other hand, figure 5-11, indicates the clothes dryer heater has reached the desired temperature in four minutes where the temperature levels as follows:

T_{high} $(°C)$	T_{low} $(°C)$	$T_{set-point}$ $(°C)$	t_{on} (hr)	t_{off} (hr)
60	51.5	56	0.0674	0.0026

The average power consumption (ΔP) from the energy flow and the energy consumption (EC) per operation cycle of the (DECD) heater can be calculated by using Duty Cycle value and input rated power as follows:

$Duty\ Cycle$	t_{on} (hr)	$Cycling_{time}$ (hr)
0.9629	0.0674	0.07

$$\Delta P_{DECD-heater} = \boldsymbol{P}_{rated} * (Duty\ Cycle)$$
$$\Delta P_{DECD-heater} = 4.434\ kW * 0.9629 = 4.2695\ kW$$

$$Energy\ consumption\ (kWh) = \boldsymbol{P}_{rated} * \boldsymbol{t}_{\boldsymbol{on}\,(total)}$$
$$EC_{DECD} = 4.434 * 0.9773 = 4.3333\ kWh$$

$\Delta P_{DECD-heater}$ (kW)	EC_{DECD} (kWh)
4.2695	4.3333

Calculation of time duration, of the DECD heater

From figure 5-11, the total time of heater $\boldsymbol{t}_{on(total)}$ can be calculated as

$$\boldsymbol{t}_{on(total)} = (\boldsymbol{N}_{cycles} * \boldsymbol{t}_{on}) + \boldsymbol{t}_{on\ (startup)}$$

$\boldsymbol{t}_{on}^{total}$ (hr)	$\boldsymbol{t}_{on}$ (hr)	$\boldsymbol{N}_{cycles}$	$\boldsymbol{t}_{on\ (start-up)}$ (hr)	$Run\ time$ (hr)
0.9773	0.0674	14.3	0.0135	1

Table 5.6 Time duration, power and energy consumption of the DECD heater

$\boldsymbol{P}_{rated}$ (kW)	$\boldsymbol{t}_{on}^{total}$ (hr)	$\boldsymbol{t}_{on}$ (hr)	$\boldsymbol{t}_{off}$ (hr)	$Cycling_{time}$ (hr)	$Duty$ $Cycle$	ΔP_{DECD} (kW)	EC_{DECD} (kWh)	Run $time$ (hr)
4.434	0.0135	0.0674	0.0026	0.07	0.9629	4.2695	4.3333	1

It should be pointed out that in the simulation result, the clothes dryer machine run-time is assumed to be 1 hr. However, the run-time depends on the consumer choice and behaviour as well as the number of family and their needs which will reflect and appear in operation cycle of the clothes dryer. On the other hand, in commercial sector the run-time in most cases approximately 8 h/day which much greater than domestic sector uses. Virtually, this difference would affect and increase the energy consumption of the clothes washer per day as well as the energy cost.

5.3.3 MATHEMATICAL MODEL of A 1-PHASE INDUCTION MOTOR DECD

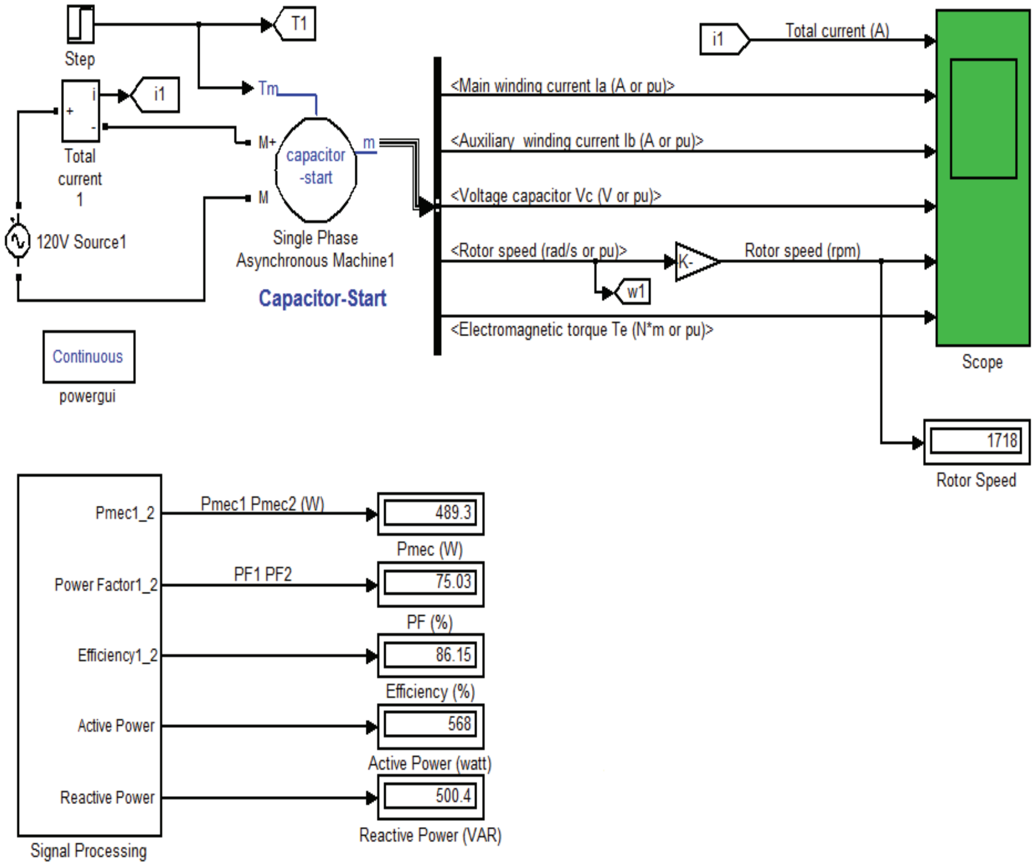

Figure 5.12 Modified version model of a single-phase induction motor of DECD [78]

The aim of this section is to provide, a sample calculation in order to analyse the DECD system behavior during steady-state condition response. This study presents just in details a single-phase induction motor capacitor start [78]. In literature [40], the operation time and energy use of clothes dryer as follows:

Table 5.7 The average operating time and energy consumption of clothes Dryer [40]

Appliance	Power (W)	Run-time Hours/month	Energy consumption kWh/month
Clothes Dryer	5369	6-28	30-140

In this model the single-phase induction motor is used with the following specifications

Table 5.8 Single-phase induction motor capacitor start specifications

Volt (v)	Frequency (Hz)	Stator		Rotor		X_m (Ω)	slip	pole	hp
		R_1 (Ω)	X_1 (Ω)	R_2 (Ω)	X_2 (Ω)				
120	60	0.4	2.4906	1.9	3.2405	73	0.05	4	0.5

Simulation result of a 1-phase induction motor capacitor-start of DECD run-time = 30 min

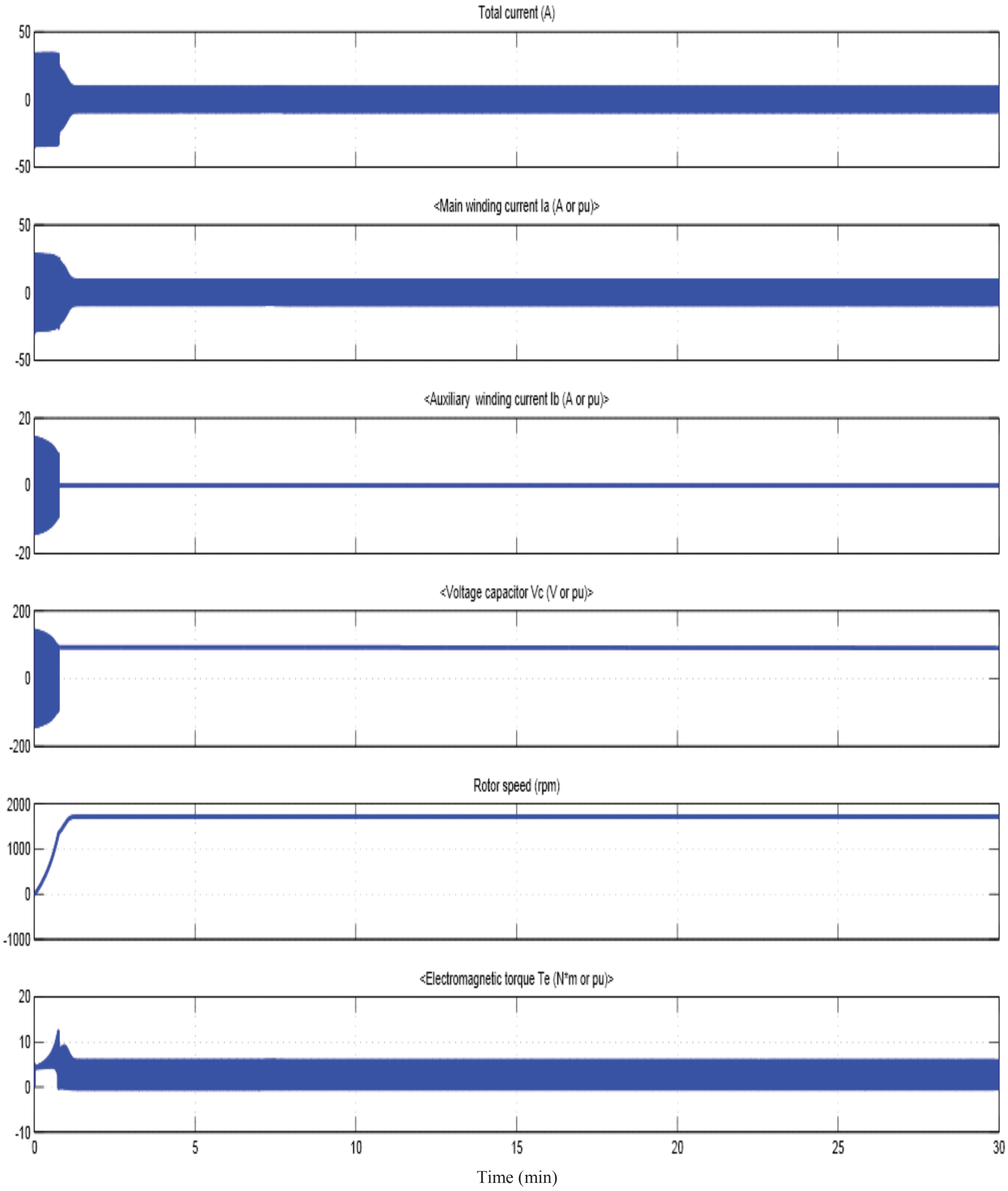

Figure 5.13 System behavior of DECD within run-time 30 min

Based on the calculations of equation circuit of 1-phase induction motor of DECD in **appendix E**, the results as follows:

Table 5.9 Comparison of the calculations and the simulation results

Methods	P_{input} (W)	P_m (W)	**PF**	T_m $(N.m)$	n_{sync} (r/min)	n_r (r/min)	η
Calculation	565.8736	490.6172	0.7626	2.7397	1800	1710	0.87
Simulation Result	567.9	489.3	0.7504	2.72	1800	1718	0.86

From table 5.9, we can conclude that the mathematical model of a single-phase induction motor capacitor-start has a good agreement with the simulation results.

The overall energy consumption of DECD (heater and motor)

Energy consumption of clothes dryer depends on many factors; the energy use by heater and motor, clothes load capacity and complete drying process, which consisting of a series of different operations. In case of a middle-class family, one load per day the average energy consumption based on calculations in **appendix E** of DECD, the results as follows:

$\boldsymbol{Heater}$ (kW)	$\boldsymbol{Motor}$ (kW)	$\boldsymbol{Total\ P_{rated}}$ (kW)
4.434	0.566	5

$$\boldsymbol{E.C = P_{rated}} * Run - time$$

Case 1, run-time = 0.75 hr. per load

	$\boldsymbol{P_{rated}}$ (kW)	$\boldsymbol{Run - time}$ (hr)	$\boldsymbol{E.C}$ (kWh)
Day	5	0.75	3.75
Week	5	5.25	26.25
Month	5	22.5	112.5
Year	5	270	1350

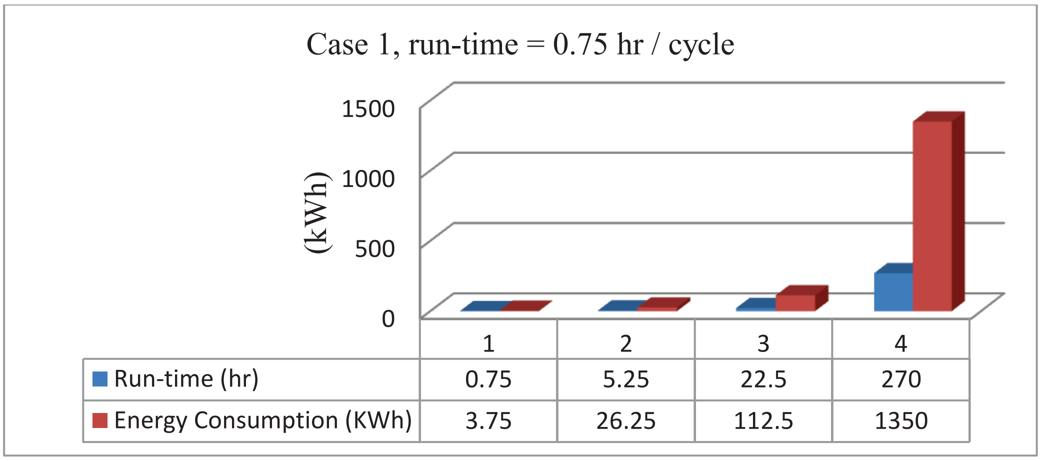

Figure 5.14 Case 1- DECD average energy consumption, run-time every day

Case 2, run-time = 1.5 hr.

	P_{rated} (kW)	$Run - time$ (hr)	$E.C$ (kWh)
Day	5	1.5	7.5
Week	5	3	15
Month	5	12	60
Year	5	144	720

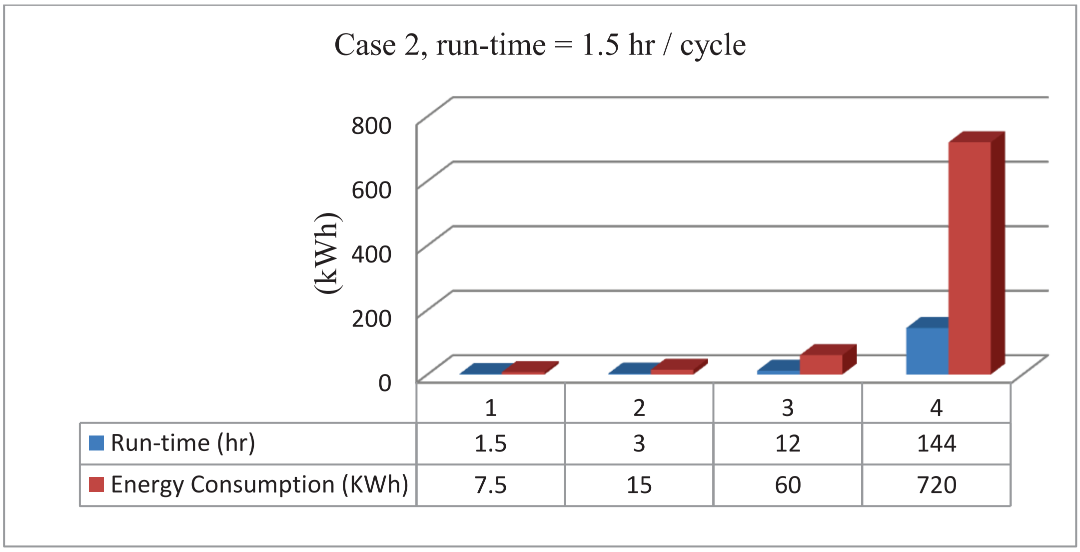

Figure 5.15 Case 2- DECD average energy consumption, run-time twice a week

Although the assumption of using the clothes dryer machine every day is realistic of commercial sector but is unrealistic of domestic sector since it's more than the average of household use. According to [46] Energy Consumption of Major Household Appliances Shipped in Canada, Summary Report "The average annual unit energy consumption (UEC) increased slightly in each year since 2005, reaching 921 kWh/yr. in 2009, mostly due to the use of larger capacity units".

The average annual energy consumption of electric clothes dryer [46]	
1992	2009
983 $kWh/year$	921 $kWh/year$

In case 2 figure 5.15, we assume that the clothes dryer machine for a middle-class family is used twice a week since it should be run after the Clothes washer machine respectively with a standard load of dryer 1.5 hr. per load. Thus, the difference between two cases is due to the energy consumption which reflects the average of operation time. In comparison, the average annual energy consumption in case-1 is greater than case-2 as shown in figures 5.14 and 5.15.

5.3.4 LOAD PROFILE AND ENERGY COST

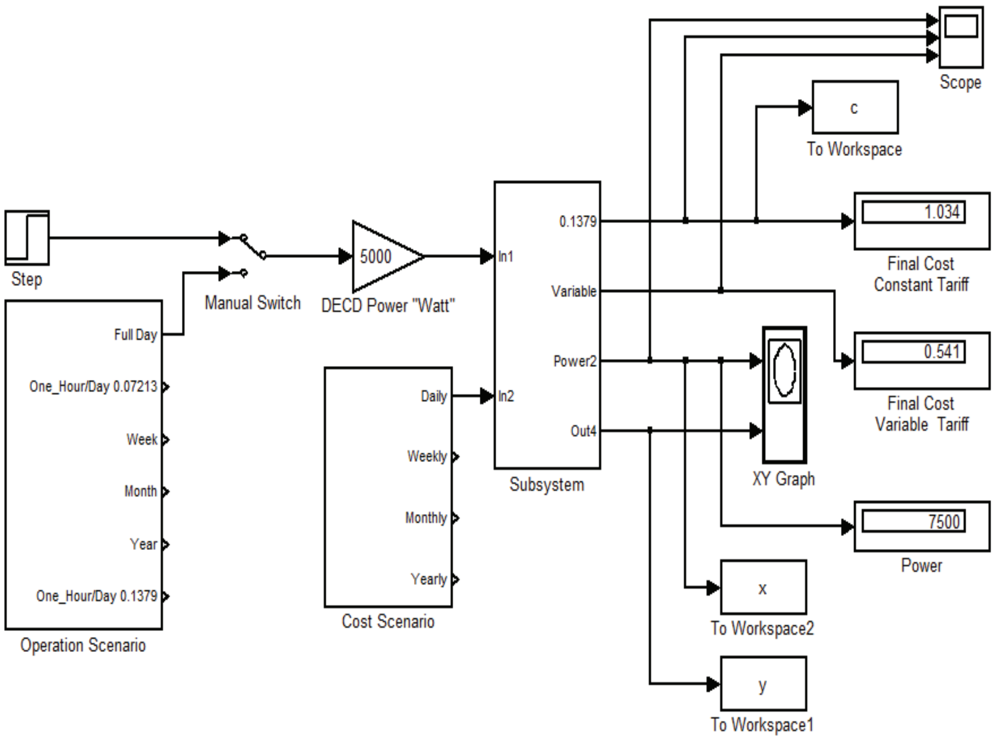

Figure 5.16 Energy Cost Calculator of DECD

Simulation Results of DECD (TOU) case 1, Run-time = 1.5 hr. /cycle

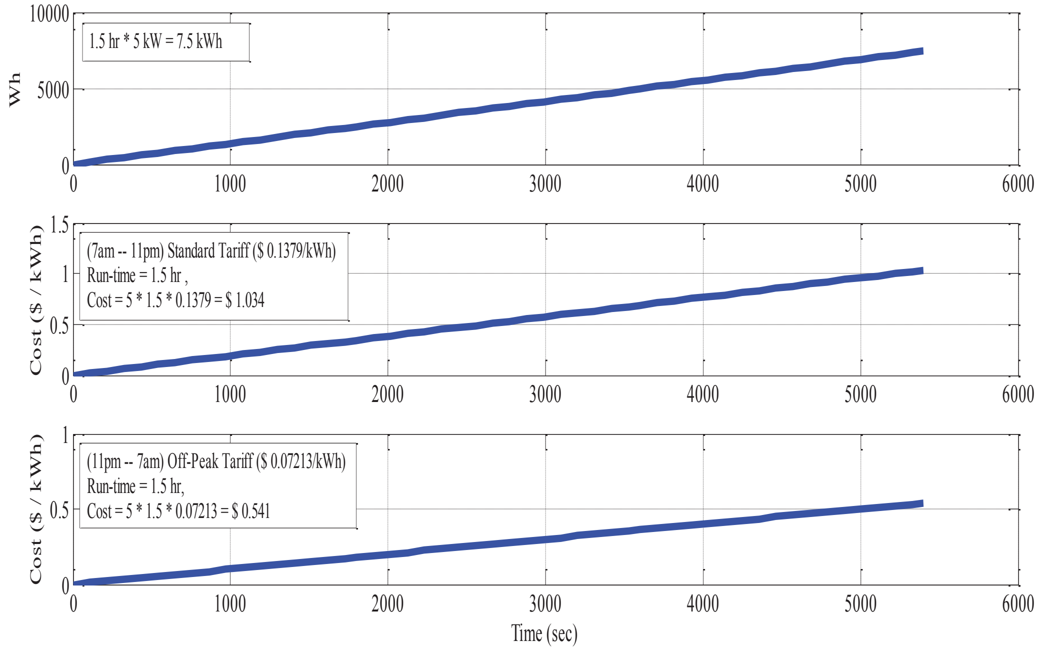

Figure 5.17 Load profile of DECD (Load and Cost vs. time)
Run-time = 1.5. hr. /cycle

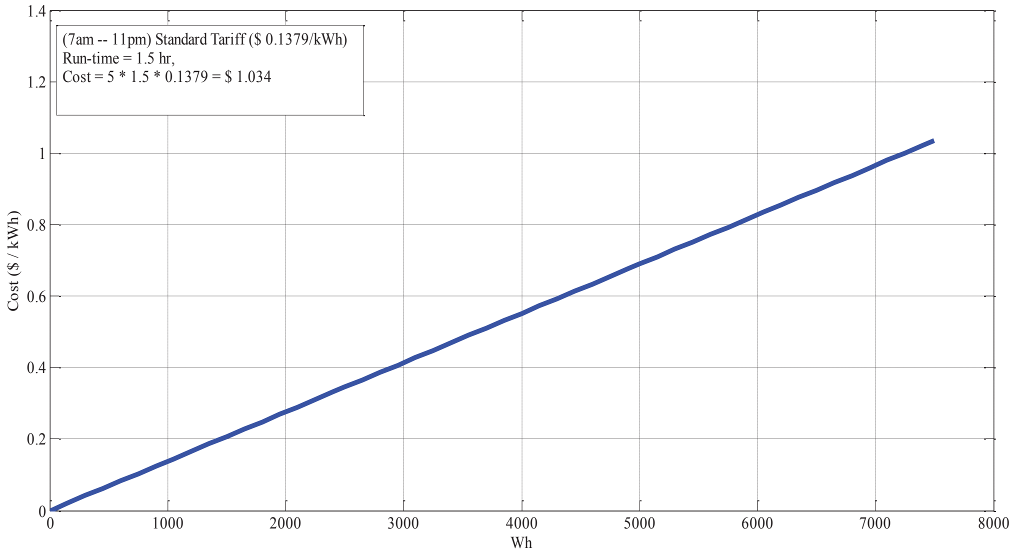

Figure 5.18 Load profile of DECD (Load vs. Cost) Standard tariff
Run-time = 1.5 hr. / cycle

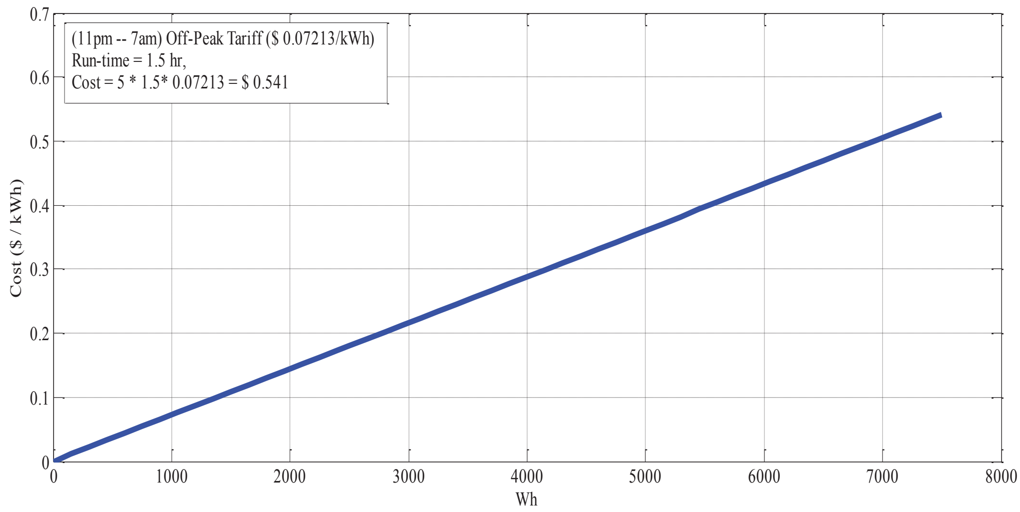

Figure 5.19 Load profile of DECD (Load vs. Cost) Off-Peak tariff
Run-time = 1.5 hr.

For simulations closest to reality, the model of Energy Cost Calculator is designed based on NS power electricity rate [17].

The scenario of time operating of DECD assumed to be in March includes power consumption and the energy cost calculations with respect to time and constraints.

As can be seen from figures (5.17, 5.18 and 5.19) the energy consumption of DECD is 7.5 kWh to keep the heater at the desired temperature. Moreover, figure 5.17, of load profile of DECD (load and cost vs. time) indicates the following:

- First figure indicates the full load: 5 kW * 1.5 hrs. = 7.5 kWh
- Second figure illustrates the energy cost under the standard tariff ($ 0.1379/kWh) which is: 7.5 kWh * $ 0.1379 = $ **1.034**
- Third figure illustrates the energy cost under off-peak tariff ($ 0.07213/kWh) which is: 7.5 kWh * $ 0.07213 = $ **0.54**

In this load profile curves the total variable tariff (TOU) not included where the off-peak tariff calculation is assumed just in case of operating time in period between (11pm to 7am) or in case of weekend.

As a result, we can conclude that the energy cost of DECD, (run-time = 1.5 hr.) under the standard tariff ($0.1379/kWh) equal $ 1.034 and the energy cost of DECD, (run-time = 1.5 hr.) under the off-peak tariff equal $ 0.54. Thus, the difference between two costs is ($ 0.49). In other words, if the customer runs the DECD under the off-peak tariff, he/she will save up to 47.7% per cycle in this appliance.

However, calculates an optimal schedule for DECD based on fluctuating tariffs will discuss later in the optimization chapter.

5.4 MODEL of DOMESTIC ELECTRIC DISHWASHER (DEDW)

5.4.1 INTRODUCTION

Dishwashers are used for cleaning dishes and other eating instruments in the commercial sector such as restaurants, schools, hospitals, nursing homes and in the residential sector. In particular, a dishwasher machine is a complex cleaning system that integrates many components both electrical and mechanical. In modern dishwashers many improvements have been made and implemented by manufacturers to satisfy consumer demands such as high cleaning performance, cleaning time and low energy consumption [47, 48].

A typical dishwasher consists of three main parts, including a thermal energy, an induction motor and a set of controls. The motor of the dishwasher has various speeds to operate; the wash motor operates between (310 – 2800 rpm) [56].

During a wash cycle the heater element 1200 W and the motor will operate at the same time while the thermostat maintains the desired temperature. According to [54] a modern dishwasher consumes 1.05 kWh per cycle, where 0.924 kWh per cycle is electricity for heating. The average run-time of a dishwasher is assumed to be 220 times/year.

There are many types of dishwashers available in the market these days for both domestic and commercial use with special design features that can be modified according to the user needs and choice such as cleaning time, temperature setting, energy consumption and load capacity. The main two common types of dishwashers are with electric water heating and with non-electric water heating [40]. However, in this study the domestic electric dishwasher (DEDW) that consists of a heater and an induction motor is presented.

Table 5.10 The average operating time and energy use of dishwasher [40]

Appliance	Power (W)	Run-time Hours/month	Energy consumption kWh/month
Dishwasher (With Electric Water Heating)	1300	8-40	20-102
Dishwasher (Non-Electric Water Heating)	1300	8-40	3-16

In order to reduce energy consumption of the dishwasher it is common to connect it through a water pipe to the electric water heater to feed the machine by hot water in a certain range of temperature. By assuming the DEDW is connected to the electric water heater DEWH, in chapter 3, we have presented the model of DEWH that has a limit of temperature (47.64 ℃ to 50.4 ℃). However, in the literatures [49, 50, 51, 52 and 53] the temperature required of the water that circulates to wash dishes is varied (76 ℃ to 90 ℃) as a maximum limit where the minimum limit is (48 ℃ to 51 ℃).

In this model, the DEDW with electric water heating under the limit of temperature between (70 ℃ to 80 ℃) is considered since the minimum temperature range is already provided by electric water heater DEWH in chapter 3, thus, the inlet temperature is assumed to be the hot water temperature that provides by DEWH where T_{high} = 51.4 ℃. In other words, the system needs a thermal energy just to reach the desired operating temperature; that can be achieved by adding an electric powered heater element (1200 W) where its task is to maintain the operating temperature as well as to provide further temperature for heating the dishwasher chamber during the drying cycle. Moreover, for the mechanical work of this system, the main task of its parts through an induction motor is very important for spinning, spraying of the water that is needed during the wash cycle as well as for running a fan to improve drying process [57].

In [47] the energy consumption of DEDW has decreased from an average of 3 kWh to below 1kWh per cycle. However, both models (with electric water heating and with non-electric water heating) in this study are considered to compare them in the overall energy consumption calculation of the DEDW where the heater element is assumed to be 1200 W and the power rating of the motor is 1134 W.

5.4.2 MODEL of DEDW Heater

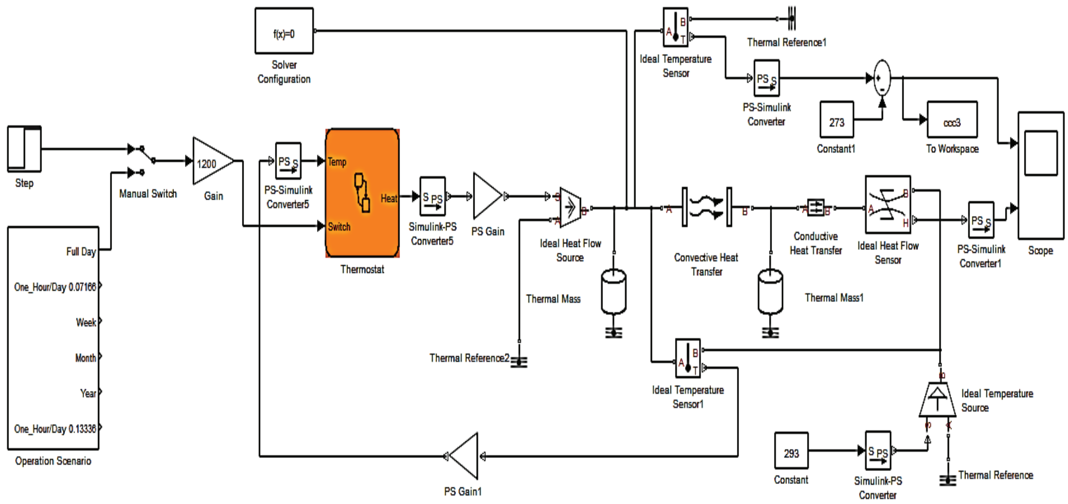

Figure 5.20 Model of Domestic Electric Dishwasher (heater)

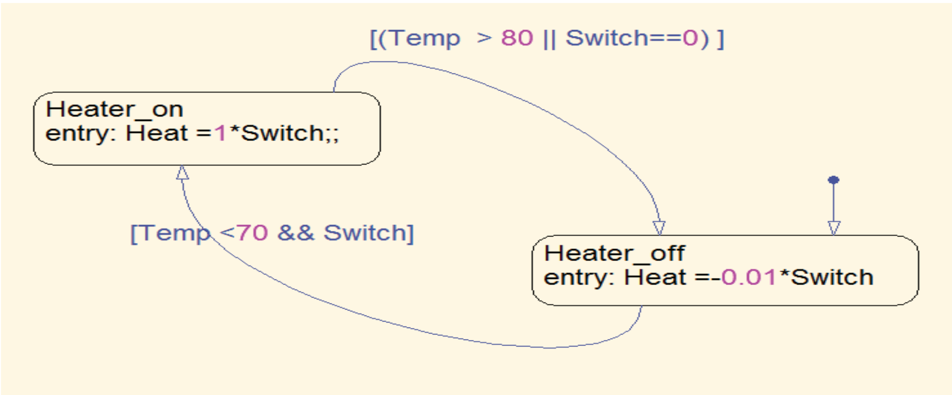

Figure 5.21 State flow model of the thermostat of (DEDW-Heater)

Table 5.11 Model of (DEDW) heater Specifications

Convection Parameters				
Convective Heat Transfer		Thermal Mass		
Area (m^2)	Heat Transfer Coefficient $(W/m^2.°C)$	Mass (kg)	Specific heat Cp $(J/kg.°C)$	Initial Temperature (K)
0.3	125	2.4	440	324.4

Conduction Parameters					
Conductive Heat Transfer			Thermal Mass		
Area (m^2)	Thickness (m)	Thermal Conductivity $(W/m.°C)$	Mass (kg)	Specific heat Cp $(J/kg.°C)$	Initial Temperature (K)
0.19	0.2	45	0.25	1005.4	293

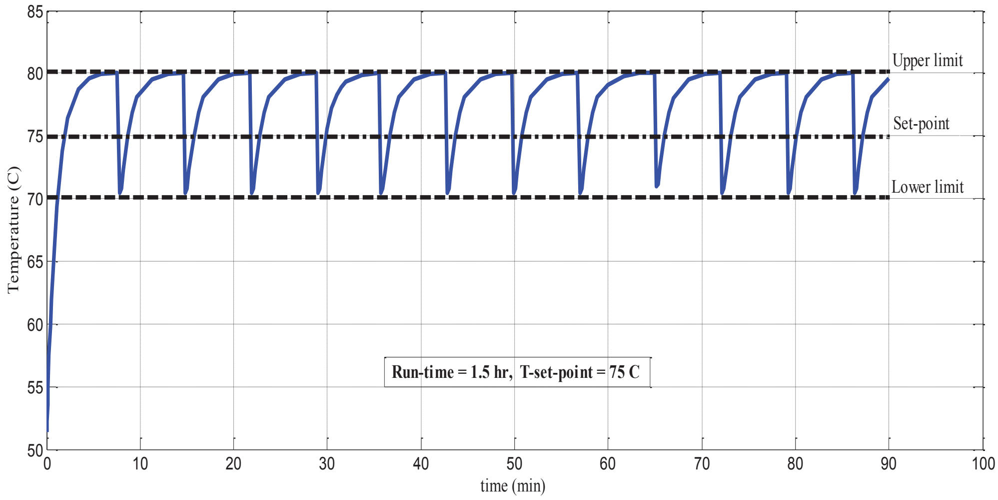

Figure 5.22 Temperature vs. Time of DEDW Run-time = 1.5 hour

According to [49, 50, 51, 52 and 53] the dishwasher has a certain range of temperature that provides good quality of washing and drying; in this model of DEDW figure 5.22, indicates that the DEDW heater started from $(51.4\ °C)$ as T_{inlet} and has reached the desired temperature in 6.8665 minutes where the temperature levels of the system as follows:

T_{high} $(°C)$	T_{low} $(°C)$	$T_{set-point}$ $(°C)$	t_{on} (hr)	t_{off} (hr)
80	70	75	0.1144	0.0034

The average power consumption (ΔP) from the energy flow and the energy consumption (EC) per operation cycle of the (DEDW) heater can be calculated by using Duty Cycle value and the input power rating as follows:

$$Duty\ Cycle = (\boldsymbol{t}_{on}\ /\ Cycling_{time})$$

$Duty\ Cycle$	t_{on} (hr)	$Cycling_{time}$ (hr)
0.9711	0.1144	0.1178

$$\Delta P_{DEDW-heater} = \boldsymbol{P}_{rated} * (Duty\ Cycle)$$

$$\Delta P_{DEDW-heater} = 1.2\ kW * 0.9711 = 1.1653\ kW$$

$$Energy\ consumption\ (kWh) = \boldsymbol{P}_{rated} * \boldsymbol{t}_{\boldsymbol{on}\,(total)}$$

$$EC_{DEDW} = 1.2 * 0.1144 = 0.1373\ kWh$$

Calculation of time duration, of the DEDW heater

From figure 5.22, the total time of heater $\boldsymbol{t}_{on(total)}$ can be calculated as

$$\boldsymbol{t}_{on(total)} = (\boldsymbol{N}_{cycles} * \boldsymbol{t}_{on}) + \boldsymbol{t}_{on\ (startup)}$$

$\boldsymbol{t}_{on}^{total}$ (hr)	$\boldsymbol{t}_{on}$ (hr)	$\boldsymbol{N}_{cycles}$	$\boldsymbol{t}_{on\ (start-up)}$ (hr)	$Run\ time$ (hr)
1.4267	0.1144	12.3	0.0196	1.5

Table 5.12 Time duration, power and energy consumption of the DEDW heater

$\boldsymbol{P}_{rated}$ (kW)	$\boldsymbol{t}_{on}^{total}$ (hr)	$\boldsymbol{t}_{on}$ (hr)	$\boldsymbol{t}_{off}$ (hr)	$Cycling_{time}$ (hr)	$Duty$ $Cycle$	ΔP_{DECD} (kW)	EC_{DECD} (kWh)	Run $time$ (hr)
1.2	1.4267	0.1144	0.0034	0.1178	0.9711	1.1653	0.1373	1.5

It should be pointed out that in the simulation result, the dishwasher machine run-time is assumed to be 1.5 hr. However, the run-time depends on the consumer choice and behaviour as well as the number of family and their needs which will reflect the run-time and appear in operation cycle of the dishwasher. On the other hand, in commercial sector the run-time in most cases approximately (5 to 6 hr/day) which much greater than domestic sector uses. Virtually, this difference would affect and increase the energy consumption of the dishwasher per day as well as the energy cost.

5.4.3 MATHEMATICAL MODEL of A 1-PHASE INDUCTION MOTOR DEDW

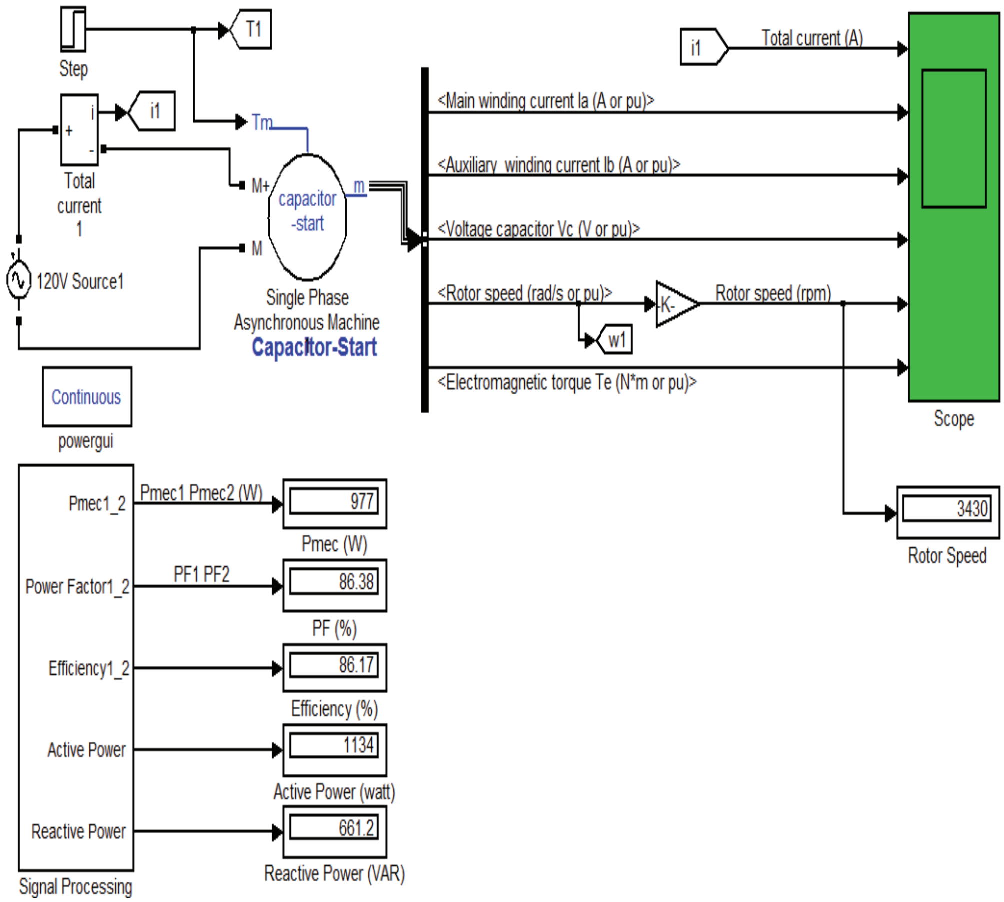

Figure 5.23 Modified version model of a single-phase induction motor of DEDW [78]

The aim of this section is to provide a sample calculation in order to analyse the DEDW system behavior during steady-state condition response. This study presents just in details a single-phase induction motor capacitor start [78] with the following specifications

Table 5.13 Single-phase induction motor capacitor start specifications of DEDW

Volt (v)	Frequency (Hz)	Stator		Rotor		X_m (Ω)	slip	pole	hp
		R_1 (Ω)	X_1 (Ω)	R_2 (Ω)	X_2 (Ω)				
120	60	0.4	1.2246	1.12	1.2246	58	0.05	2	0.3

Simulation result of a single-phase induction motor capacitor-start of DEDW run-time = 30 min

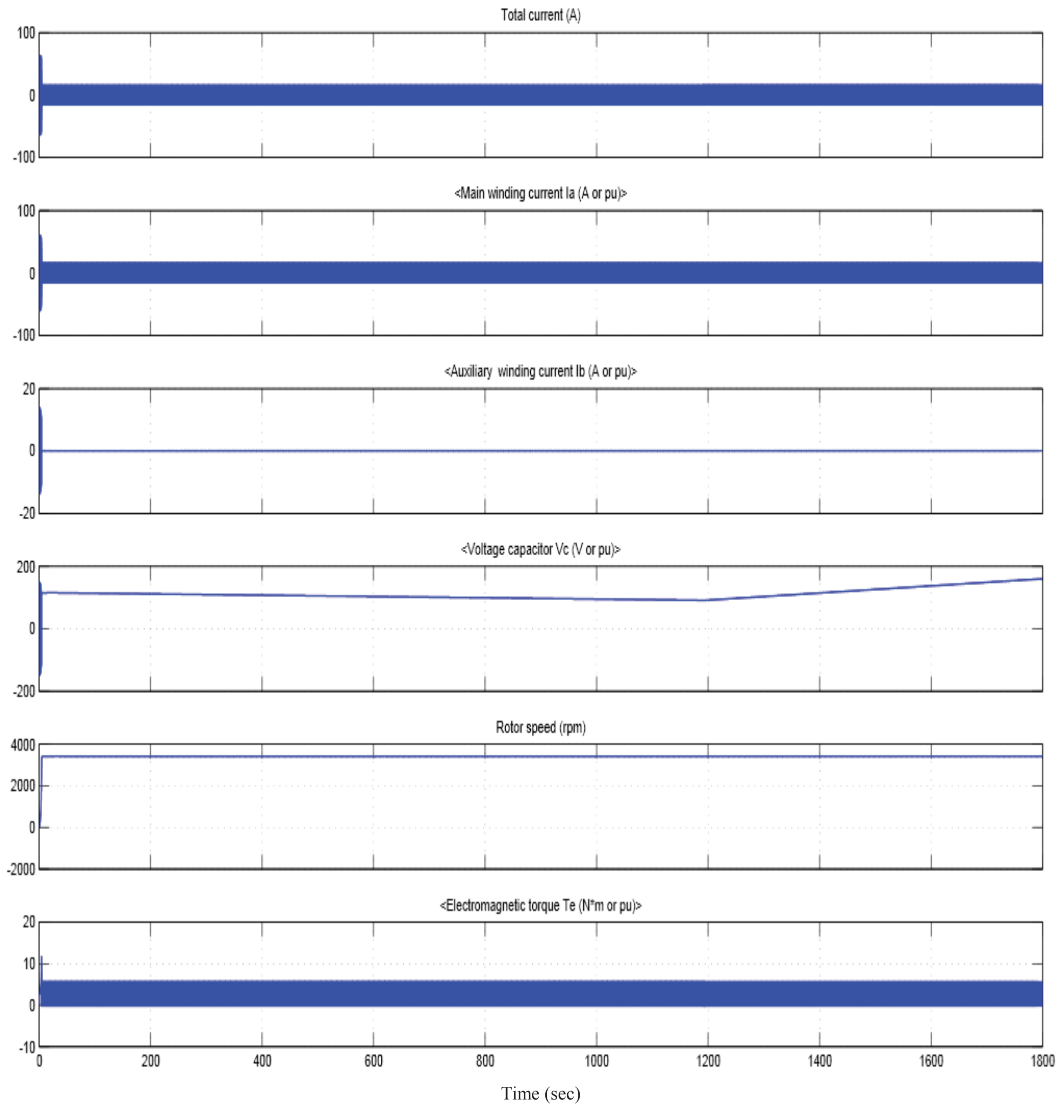

Figure 5.24 System behavior of DEDW within run-time 30 min

Based on the calculations of equation circuit of 1-phase induction motor of DECW model, from **appendix F** the results as follows:

Table 5.14 Comparison of the calculations and the simulation results

Methods	P_{input} (W)	P_m (W)	**PF**	T_m $(N.m)$	n_{sync} $(r.p.m)$	n_r $(r.p.m)$	η
Calculation	1090.3621	947.4327	0.8690	2.6468	3600	3420	0.87
Simulation Result	1134	977	0.8638	2.72	3600	3430	0.86

The overall energy consumption of DEDW "heater and motor"

Energy consumption of the dishwasher depends on many factors; the energy use by heater and motor and the complete washing process, which consisting of a series of different operations. In case of a middle-class family, one load per day the average energy consumption based on calculations in **appendix F** of DEDW, the results as follows:

Heater (kW)	***Motor*** (kW)	***Total*** $\boldsymbol{P_{rated}}$ (kW)
1.2	1.134	2.334

$$\boldsymbol{E.C} = \boldsymbol{P_{rated}}\ (kW) * \textbf{Run-time}\ (hr)$$

Case 1, run-time = 1.5 hr. /cycle, daily, heat wash/dry included

	$\boldsymbol{P_{rated}}$ (kW)	**Run-time** (hr)	$\boldsymbol{E.C}$ (kWh)
Day	2.334	1.5	3.501
Week	2.334	10.5	24.507
Month	2.334	45	105.03
Year	2.334	540	1260.36

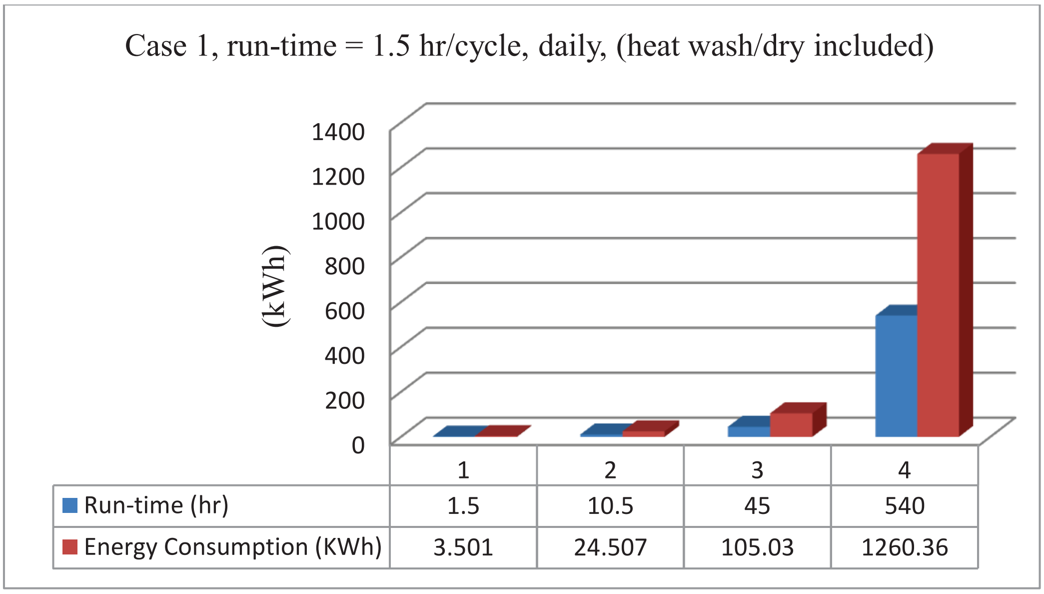

Figure 5.25 Case 1- DEDW average energy consumption, run-time every day (heat wash/dry included)

Case 2, run-time = 1.5 hr. /cycle, daily, heat wash/dry excluded

	P_{rated} (kW)	**Run-time** (hr)	$E.C$ (kWh)
Day	1.134	1.5	1.701
Week	1.134	10.5	11.907
Month	1.134	45	51.03
Year	1.134	540	612.36

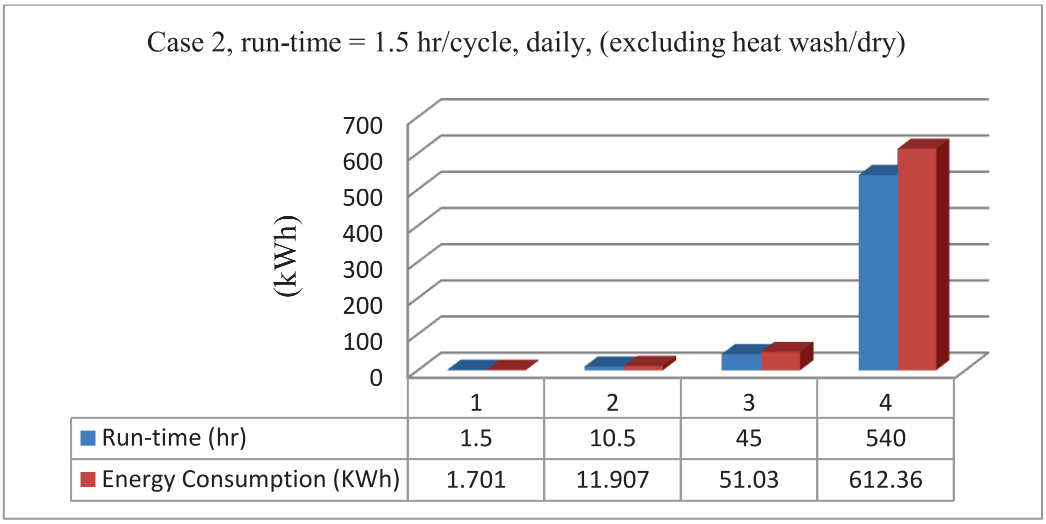

Figure 5.26 Case 2- DEDW average energy consumption, run-time every day (heat wash/dry excluded)

Although the assumption of using the dishwasher machine every day is realistic of commercial sector but is unrealistic of domestic sector since it's more than the average of household use. According to [46] Energy Consumption of Major Household Appliances Shipped in Canada, Summary Report; the average annual unit energy consumption (UEC) of dishwashers improved dramatically between 1990 and 2009 decreased by 68 %, from 1026 kWh/yr. in 1990 to 325 kWh/yr. in 2009.

In case 2, figure (5.26) we assume that the dishwasher machine for a middle-class family is used every day, where the standard load of household dishwasher assumed to be 1.5 hr. per cycle. Thus, the difference between two cases is due to the overall energy consumption by dishwasher system which reflects the use of electric heater during the wash and dry process. On the other hand, the operation time of the system in this study just in the case of long period of operation. In addition, it's controlled by consumer behavior and the load capacity of the dishwasher. In comparison, the average annual energy consumption in case-1 is much greater than case-2 as shown in figures (5.25 and 5.26).

5.4.4 LOAD PROFILE AND ENERGY COST

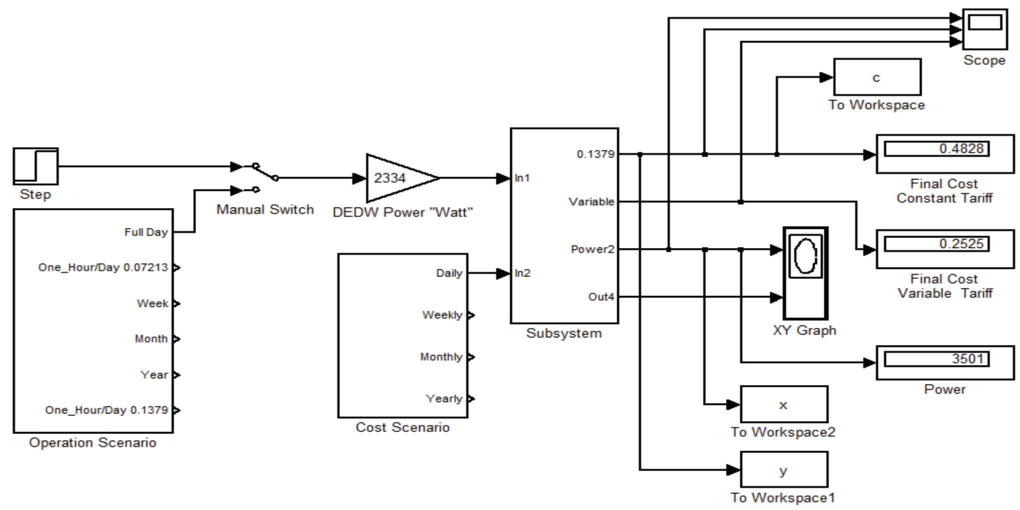

Figure 5.27 Energy Cost Calculator of DEDW

CASE 1, Simulation Results of DEDW (TOU)

Run-time = 1.5 hr. P_{rated} = 2.334 kW

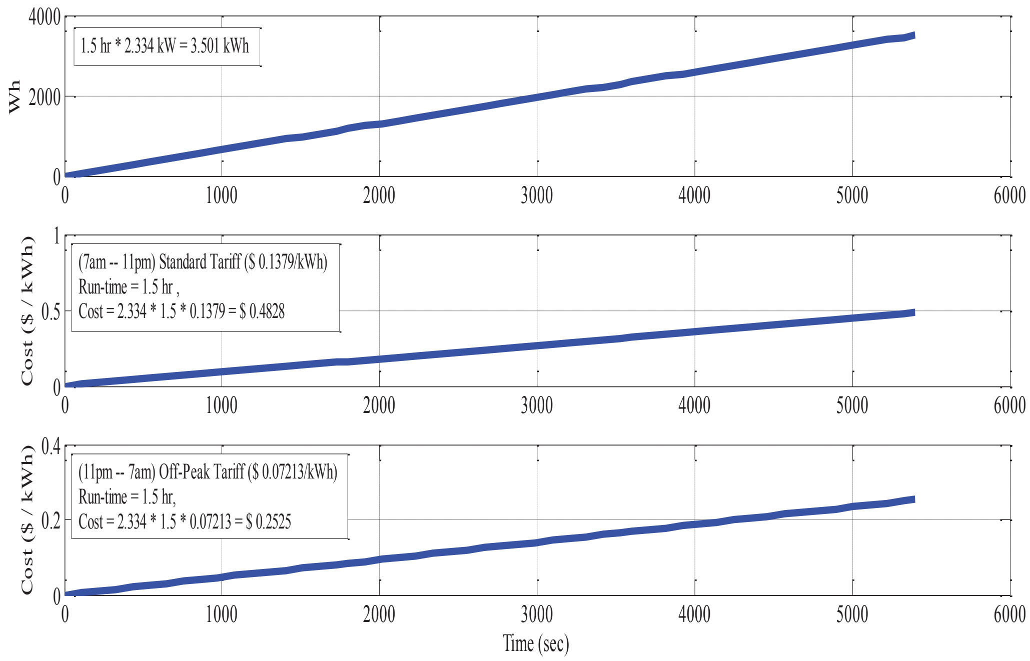

Figure 5.28 Load profile of DEDW (Load and Cost vs. time)
Run-time = 1.5. hr. /cycle (heat wash/dry included)

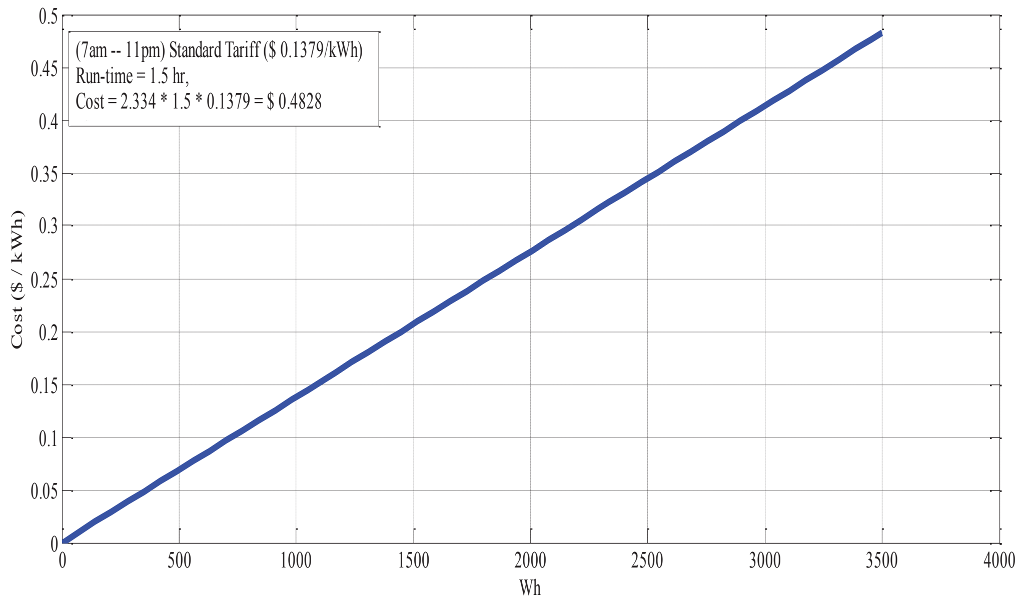

Figure 5.29 Load profile of DEDW (Load vs. Cost) Standard tariff
Run-time = 1.5 hr. / cycle

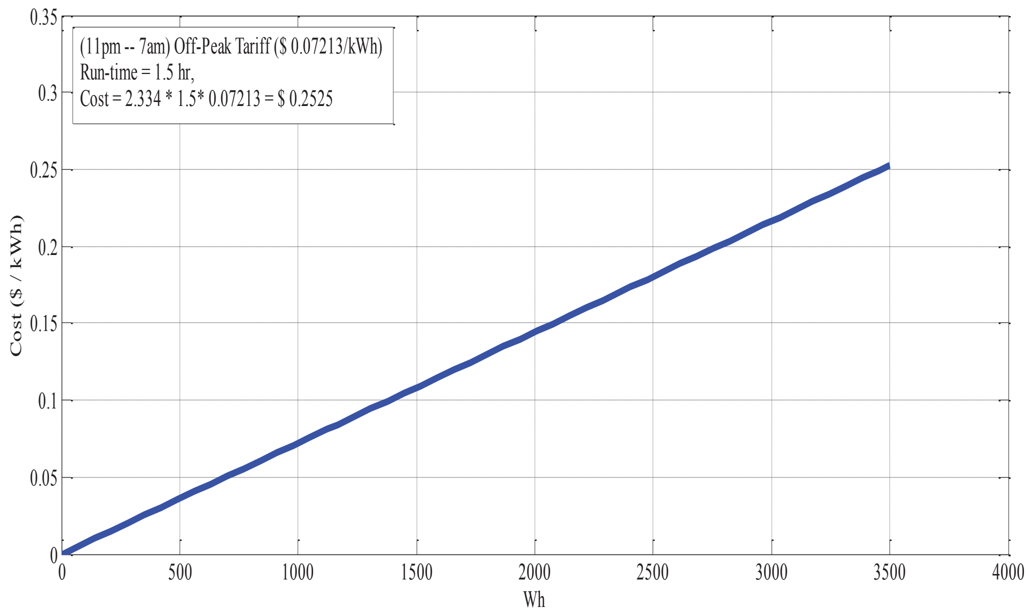

Figure 5.30 Load profile of DEDW (Load vs. Cost) Off-Peak tariff
Run-time = 1.5 hr.

CASE 2, Simulation Results of DEDW (TOU)

Run-time = 1.5 hr. P_{rated} = 1.134 kW

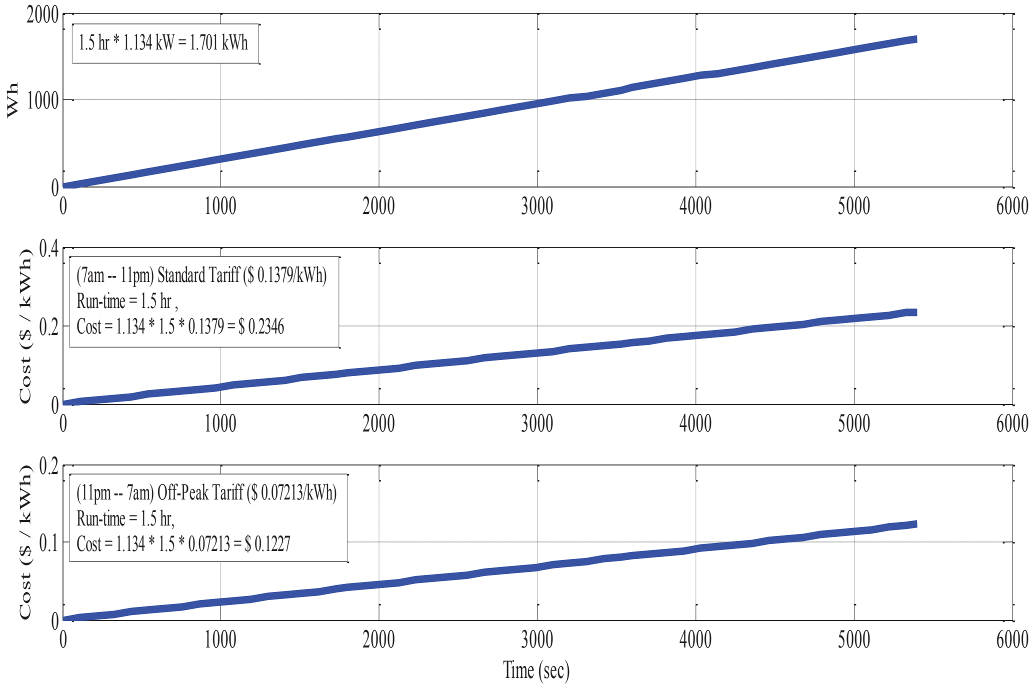

Figure 5.31 Load profile of DEDW (Load and Cost vs. time)
Run-time = 1.5. hr. /cycle (heat wash/dry excluded)

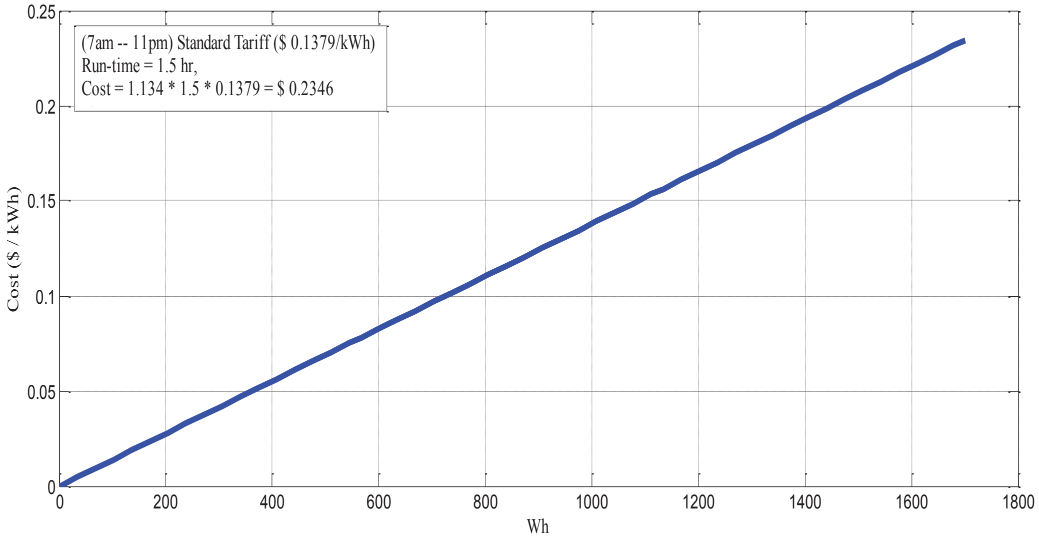

Figure 5.32 Load profile of DEDW (Load vs. Cost) Standard tariff
Run-time = 1.5 hr. / cycle

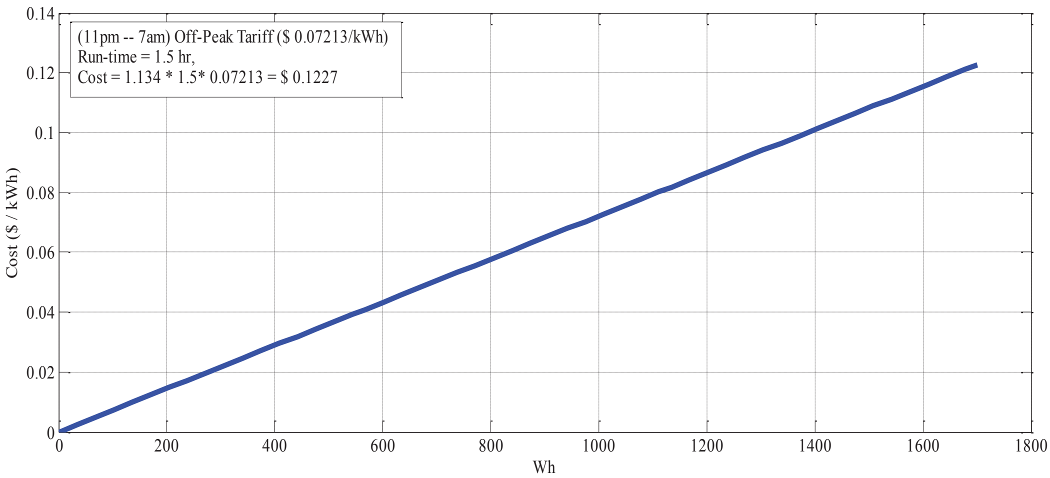

Figure 5.33 Load profile of DEDW (Load vs. Cost) Off-Peak tariff
Run-time = 1.5 hr.

For simulations closest to reality, the model of Energy Cost Calculator is designed based on NS power electricity rate [17].

CASE 1, Run-time = 1.5 hr. P_{rated}= 2.334 kW

The scenario of time operating of DEDW (heat wash/dry included) assumed to be in March includes power consumption and the energy cost calculations with respect to time and constraints. As can be seen from figures (5.28, 5.29 and 5.30), the energy consumption of DEDW in case of (heat wash/dry included) is 3.501 kWh to keep the heater for washing and drying at the desired temperature. Moreover, figure 5.28, of load profile of DEDW (load and cost vs. time) indicates the following:

- First figure indicates the full load: 2.334 kW * 1.5 hrs. = 3.501 kWh
- Second figure illustrates the energy cost under the standard tariff ($ 0.1379/kWh) which is: 3.501 kWh * $ 0.1379 = $ **0.48**
- Third figure illustrates the energy cost under off-peak tariff ($ 0.07213/kWh) which is: 3.501 kWh * $ 0.07213 = $ **0.25**

In this load profile curves the total variable tariff (TOU) not included where the off-peak tariff calculation is assumed just in case of operating time in the period between (11pm to 7am) or in case of weekend. As a result, we can conclude that the energy cost of DEDW, (run-time = 1.5 hr.) under the standard tariff of ($0.1379/kWh) equal $ 0.483 and the energy cost of DEDW, (run-time = 1.5 hr.) under the off-peak tariff equal $ 0.253. Thus, the difference between two costs is ($

0.2303). In other words, if the customer runs the DEDW during the off-peak tariff, he/she will save up to 47.7% per cycle in this appliance.

CASE 2, Run-time = 1.5 hr. P_{rated} = 1.134 kW

The scenario of time operating of DEDW (heat wash/dry excluded) assumed to be in March includes power consumption and the energy cost calculations with respect to time and constraints. As can be seen from figures (5.31, 5.32 and 5.33), the energy consumption of DEDW in case of (heat wash/dry included) is 1.701 kWh to keep the wash heater at the desired temperature. Moreover, figure 5.31, of load profile of DEDW (load and cost vs. time) indicates the following:

- First figure indicates the full load: 1.134 kW * 1.5 hrs. = 1.701 kWh
- Second figure illustrates the energy cost under the standard tariff ($ 0.1379/kWh) which is: 1.701 kWh * $ 0.1379 = $ **0.235**
- Third figure illustrates the energy cost under off-peak tariff ($ 0.07213/kWh) which is: 1.701 kWh * $ 0.07213 = $ **0.123**

In this load profile curves the total variable tariff (TOU) not included where the off-peak tariff calculation is assumed just in case of operating time in the period between (11pm to 7am) or in case of weekend. As a result, we can conclude that the energy cost of DEDW, (run-time = 1.5 hr.) under the standard tariff of ($0.1379/kWh) equal $ 0.235 and the energy cost of DEDW, (run-time = 1.5 hr.) under the off-peak tariff equal $ 0.123. Thus, the difference between two costs is ($ 0.112). In other words, if the customer runs the DEDW during the off-peak tariff, he/she will save up to 47.7% per cycle in this appliance.

Table 5.15 Comparison between Case 1 and Case 2

	$\boldsymbol{P_{rated}}$ (kW)	**Run-time** (hr)	$\boldsymbol{E.C}$ (kWh)
heat wash (**dry included**)	2.334	1.5	3.501
heat wash (**dry excluded**)	1.134	1.5	1.701

However, calculates an optimal schedule for DEDW based on fluctuating tariffs will discuss later in the optimization chapter.

CHAPTER 6 OPTIMIZATION METHOD

6.1 INTRODUCTION

In recent years, however, the rate of energy demand has increased rapidly throughout the world while the price of energy has been fluctuating. In energy management system (EMS) whether residential sector or commercial sector the ultimate goal is to make the best choice by selecting the best least costly decision among the set of permissible decisions under conditions; that can be achieved by finding the optimal time operating of certain appliances under the condition that they satisfy the consumer choice and reduce the overall power consumption and energy cost.

The particular problem of this chapter is time operating and energy cost under conditions. To find the optimal solution for such problems, (REMS) establishes the optimal daily operation of home appliances. Therefore, this study analyzes many candidate scenarios during peak and off peak load periods comparing to the tariff of the residential sector in Nova Scotia based on electricity rate of NS Power [17] to reduce the usage and its associated costs. It presents simulated results of proposed (REMS) to provide an automated least cost demand response. The main approach will be to ensure the satisfaction of the requirements with constraints on efficient use of energy.

6.2 MODEL of LINEAR PROGRAMING PROBLEM (LPP)

For this optimization method that is applicable for the solution of a Linear Programming problem (LPP) where all functions are involved, the objective function and the constraints must be a linear functions of the decision variables [72, 73 and 74].

Linear Programming (LP) model

A linear Programming (LP) model is a technique for optimization of a linear objective function, subject to linear equality and linear inequality constraints, in order to achieve the best optimal solution of the problem such as a maximum profit or a minimum cost [74, 76].

Integer programming (IP) model

An integer programming (IP) model is a linear programming model with the additional requirement that all of the variables are integers [74, 76].

Binary integer programming (BIP) model

A binary integer programming (BIP) model is an integer programming model where each variable has a value of (0 or 1) / (on or off). In other words, this may represent the selection of many situations depends on the problem [74, 75 and 76].

A Linear Programming Problem (LPP) in standard form as a vector and matrix

$$\text{minimize} \quad C^T x \tag{6.1}$$
$$\text{Subject to:} \quad Ax - b \le 0$$
$$x \ge 0$$

Where,

x	: an n-dimensional column vector
C^T	: an n-dimensional row vector
A	: an $m \times n$ matrix
b	: an m-dimensional column vector
$x \ge 0$	: means that each component of x is non-negative

6.2.1 MATHEMATICAL MODEL of (BIP)

The power consumption scheduling problem is formed as a binary-integer linear programming model. The daily power consumption tasks are based on the run-time (starting-time and ending-time). In this problem the objective is to minimize the daily energy cost.

Problem Formulation

In this case study of residential energy system and commercial energy system; the objective of the problem formulation is to determine the optimal time schedule operating of appliances which optimize the total energy consumption in order to reduce its associated cost

$$minimize_U \quad c^T U \qquad (6.2)$$

Subject to: $AU \leq b$

$U \geq 0$

Where, the vectors are (c, U, b) and the matrix is A

The cost rate of electricity is constant every hour thus; the sampling period is 1 hr. therefore, 24hr. /day

Switching/decision variable $u_i \begin{cases} 1; & when\ the\ appliance\ is\ ON \\ 0; & when\ the\ appliance\ is\ OFF \end{cases}$

The cost function is given by

$$f(u_i) = \sum_{i=0}^{23} L\, u_i P_i \qquad (6.3)$$

Where,

L : load per appliance (kW)

P_i : TOU electricity cost rate in (\$/kWh)

TOU electricity price rate is varied in every season; for instance, in summer is given by two periods, Standard Tariff and Off-Peak Tariff as follows:

$$P_i = \begin{cases} 0.07213\ ; & i\ = 23,0,1,2,3,4,5,6 \\ 0.1367\ ; & i\ = 7,8,9,10,\ldots\ldots,22 \end{cases}$$

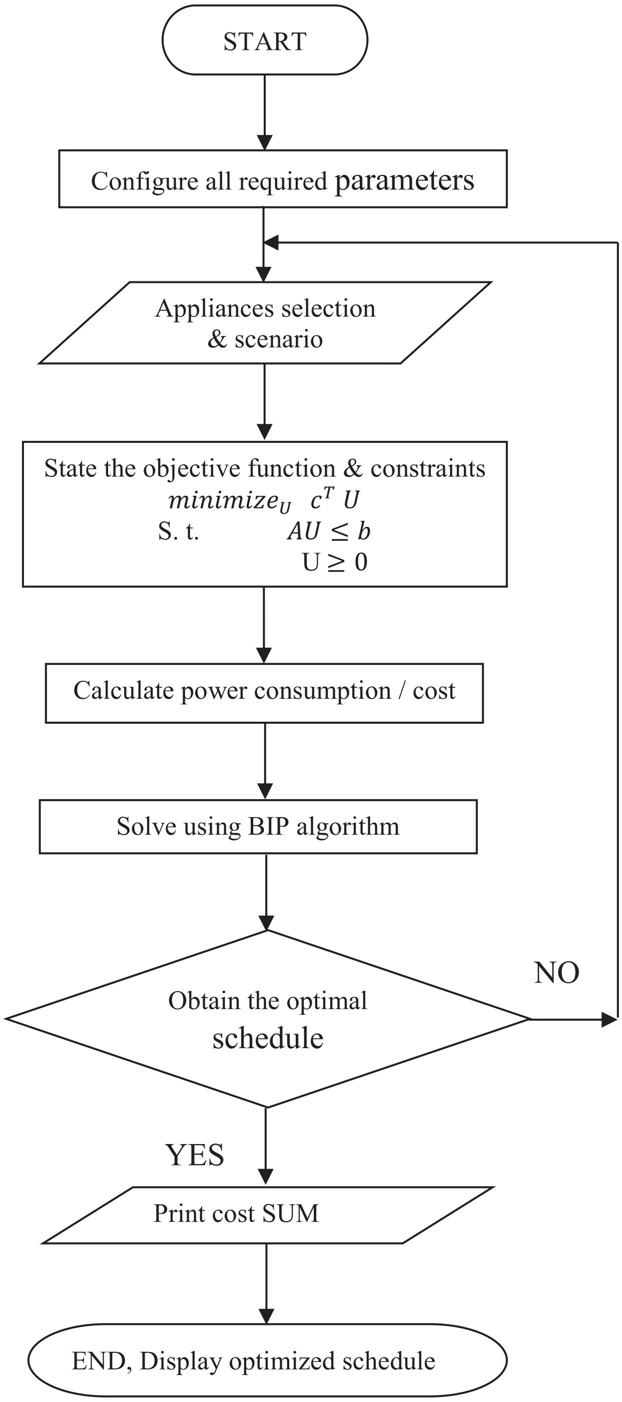

Figure 6.1 Flowchart of the BIP optimization problem

6.2.2 CASE STUDY of RESIDENTIAL ENERGY SYSTEM

In this case study of residential sector, a middle-class family is presented and it's divided to three scenarios as follows:

- Scenario 1 week-days in March (summer season)

➢ Scenario 2 weekends

➢ Scenario 3 week-days in January (winter season)

The domestic service time-of-day tariff is based on N.S power [17] as follows:

Table 6.1 The cost-rate of NS Power * *Rates effective January 1, 2013* [17]

Time & Hours			**Rate/kWh**
Weekends & Holidays	Year round (115 Days)	24 Hours / Day	**Off-Peak** 7.213¢/kWh
Weekdays	March to November (182 Days)	7am to 11pm	**Standard Rate** 13.790¢/kWh
		11pm to 7am	**Off-Peak** 7.213 ¢/kWh
Weekdays	December to February (68 Days)	7am to 12pm 4pm to 11pm	**On-Peak** 17.878¢/kWh
		11pm to 7am	**Off-Peak** 7.213 ¢/kWh
		12pm to 4pm	**Standard Rate** 13.790¢/kWh

Based on previous chapters and its related results the electricity consumption by appliances as follows:

Table 6.2 The average operating time and energy use of residential appliances

Appliance	Power (kW)	Run-time (hr.)	Period-time (hr.)		Appliance number	Energy consumption (kWh/day)
			Start	End		
Water Heater	4	Full day	6am	8am	1	20.2996
			16pm	19pm		
Oven	2.4	2	7am	8am	1	4.5178
			5pm	6pm		
Baseboard Heater	1.5	Full day	12am	9am	1	10.9296
			5pm	12am		
Clothes Washer	0.5	2	9am	11am	1	1
Clothes Dryer	5	2	10am	12am	1	10
Clothes Iron	1.3	1	11am	12am	1	0.3416
Dishwasher	2.334	2	8am	9am	1	4.668
			6pm	7pm		
Fluorescent-lamp	0.032	2	6am	8am	2	0.672
		5	5pm	10pm	3	
		2	10pm	12pm	1	

Since we have 8 appliances the constraints are given by 3 scenarios, for (Summer-weekdays), weekends and (Winter weekdays); we named all appliances as follows:

Water Heater (DEWH)	Clothes Washer (DECW)	Clothes Dryer (DECD)	Clothes Iron (DECI)	Baseboard Heater (DEBH)	Oven (DEO)	Dishwasher (DEDW)	Fluorescent Lamp (F-lamp)
A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**

Scenario 1, weekdays, (March, summer season)

This scenario is assumed to be in March, 2013 where the average temperature is (-5 to 3 °C) [77] and there is a two-part time of use (TOU) tariff as follows:

Off-Peak / weekdays		**Standard Rate (weekdays)**	
Period (hr.)	Rate ($/kWh)	Period (hr.)	Rate ($/kWh)
11pm to 7am	0.07213	7am to 11pm	0.1379

Five appliances are assumed to be run during week-days (A1, A5, A6, A7, and A8); by using the same method of the (BIP) model; the results of nun-optimize run-time load aggregation cost scenario-1 weekdays as follows:

Normal case of non-optimized run-time during week-days Scenario-1

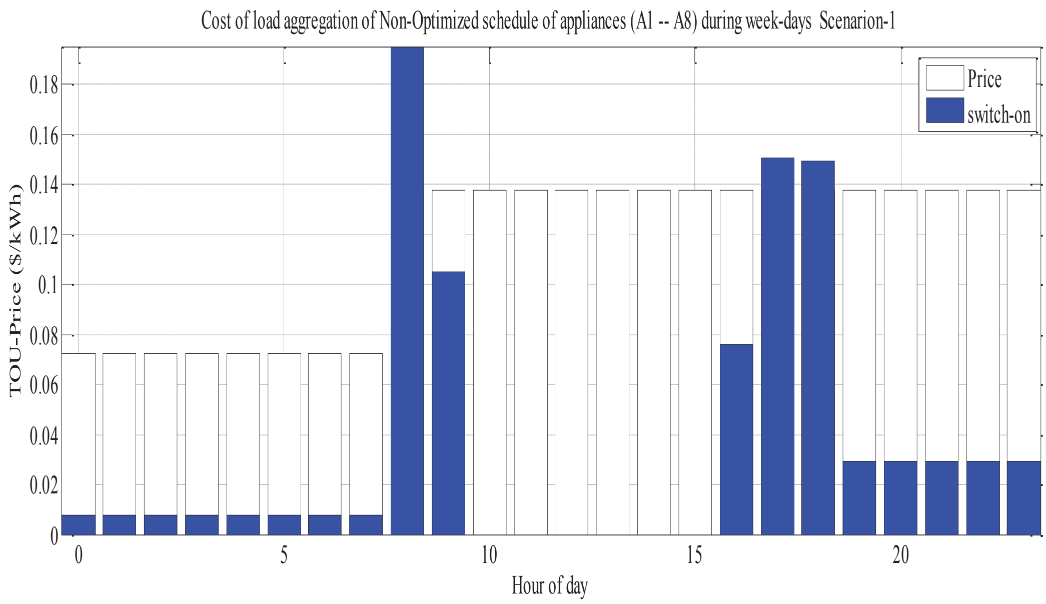

Figure 6.2 Load aggregation cost of non-optimized schedule Scenario-1

Table 6.3 Scenario 1, Load aggregation cost of non-optimized schedule

Time	Appliances	Load (kW)	Tariff ($/kWh)	Energy cost ($)
1	A5	1.5	0.07213	0.1082
2	A5	1.5	0.07213	0.1082
3	A5	1.5	0.07213	0.1082
4	A5	1.5	0.07213	0.1082
5	A5	1.5	0.07213	0.1082
6	A5	1.5	0.07213	0.1082
7	A5	1.5	0.07213	0.1082
8	A5	1.5	0.1379	0.2069
9	A1, A5, A6, A7, A8	10.266	0.1379	1.4157
10	A1, A5, A8	5.532	0.1379	0.7629
17	A1	4	0.1379	0.5516
18	A1, A5, A6, A8	7.932	0.1379	1.0938
19	A1, A5, A7, A8	7.866	0.1379	1.0847
20	A5, A8	1.532	0.1379	0.2113
21	A5, A8	1.532	0.1379	0.2113
22	A5, A8	1.532	0.1379	0.2113
23	A5, A8	1.532	0.1379	0.2113
24	A5, A8	1.532	0.1379	0.2113
Total Energy Cost/day				**6.9292**

Optimized case of run-time schedule during week-days Scenarion-1

In order to demonstrate the (BIP) algorithm; for instance, the mathematical model of DEWH (A1) is presented as follows:

Appliance DEWH (A1) model by using (BIP),

$$minimize_U \quad c^T U$$

$$\text{Subject to:} \quad AU \leq b$$

$$U \geq 0$$

The cost function is given by

$$f(u_i) = \sum_{i=0}^{23} 4\, u_i P_i$$

The TOU electricity cost rate in ($/kWh) is given by

$$P_i = \begin{cases} 0.07213\,; & i = 23,0,1,2,3,4,5,6 \\ 0.1367\,; & i = 7,8,9,10,\ldots\ldots,22 \end{cases}$$

The constraints is given by

$$\sum_{i=0}^{23} u_i \leq 5$$

(At least, 2 hrs. in the morning)

$$\sum_{i=6}^{8} u_i \leq 2$$

(At least, 3 hrs. in the morning)

$$\sum_{i=16}^{19} u_i \leq 3$$

Thus, the vector b = $\begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$

The matrix A is of size (3×24) and its *i*-th row and *j*-th column element is a_{ij} where,

$$a_{ij} = \begin{cases} 1 \quad for \quad a_{1j}, \; j = 0,1,2, \dots \dots ,23 \\ 1 \quad for \quad a_{2j}, \; j = 6,7,8 \quad and \;\; 0 \; elsewhere \\ 1 \quad for \quad a_{3j}, \; j = 16,17,18,19 \quad and \;\; 0 \; elsewhere \end{cases}$$

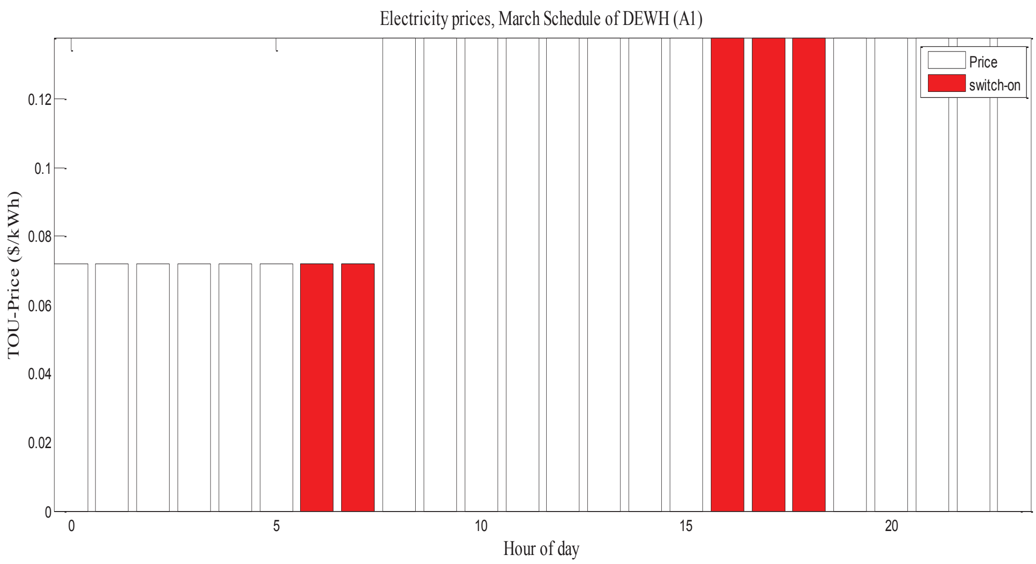

Figure 6.3 DEWH (A1) Optimized run-time schedule Scenarion-1

By applying the same method of (BIP) model; the appliances (A1, A5, A6, A7 and A8) are scheduled in week-days as follows:

Table 6.4 Scenario-1 appliances of residential energy system

Appliance	Time operating interval (hr.)	Run-time (hr.)
DEWH (A1)	(6 to 8) and (16 to 19)	5
DEBH (A5)	(17 to 0) and (0 to 9)	16
DEO (A6)	(7) and (17)	2
DEDW (A7)	(8) and (18)	2
F-lamp (A8)	(6 to8) and (17 to 0)	9

Simulation results of scenario 1, week-days as follows:

- Appliances, DEWH (A1), DEBH (A5) and DEO (A6)

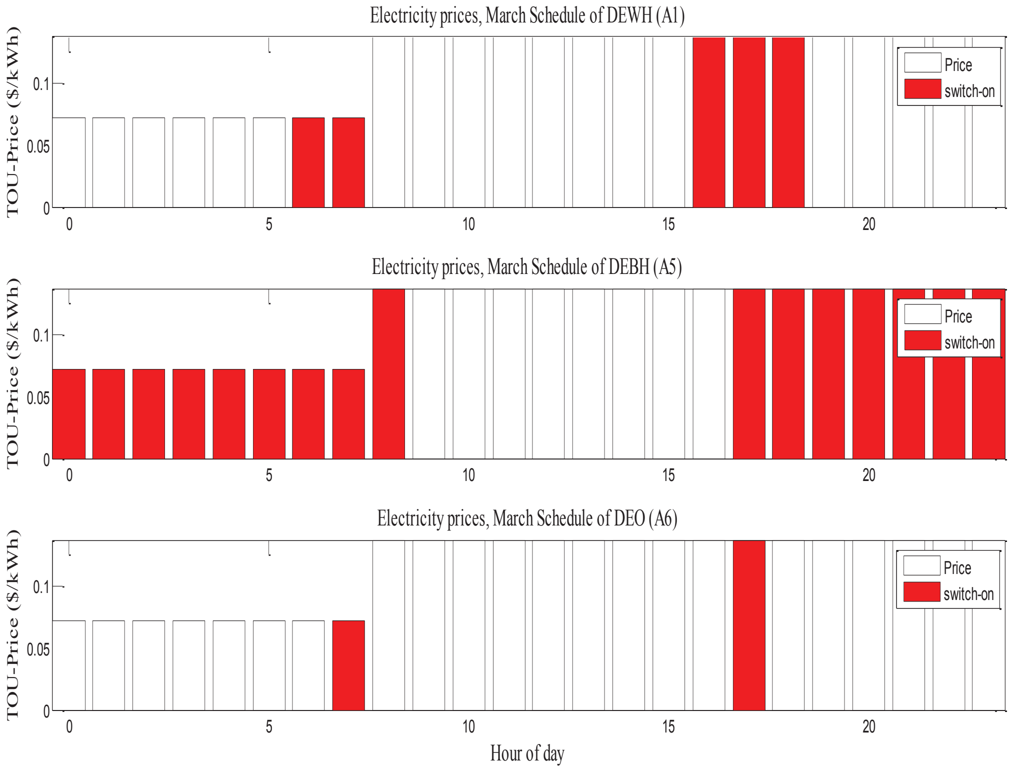

Figure 6.4 TOU tariff of appliances (A1, A5, and A6) Optimized run-time schedule Scenarion-1

- Appliances, DEDW (A7) and F-lamp (A8)

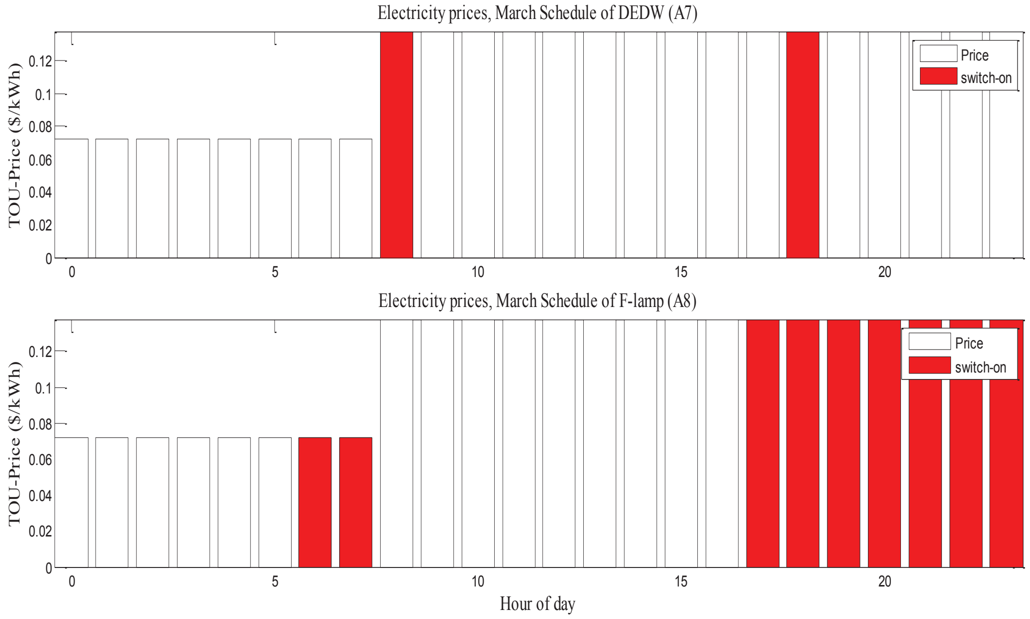

Figure 6.5 TOU tariff of appliances (A7, and A8) Optimized run-time schedule weekdays Scenarion-1

As can be seen in figures (6.4 and 6.5) the simulation results model of (BIP) has a good agreement with mathematical model, where all appliances in scenario 1 week-day (A1, A5, A6, A7 and A8) are operated during schedule time of the requirements with constraints. For instance, figure 6-3, indicates that the appliance A5 has been run two hours as required during off-peak tariff where the constraints is three hours (6 to 8).

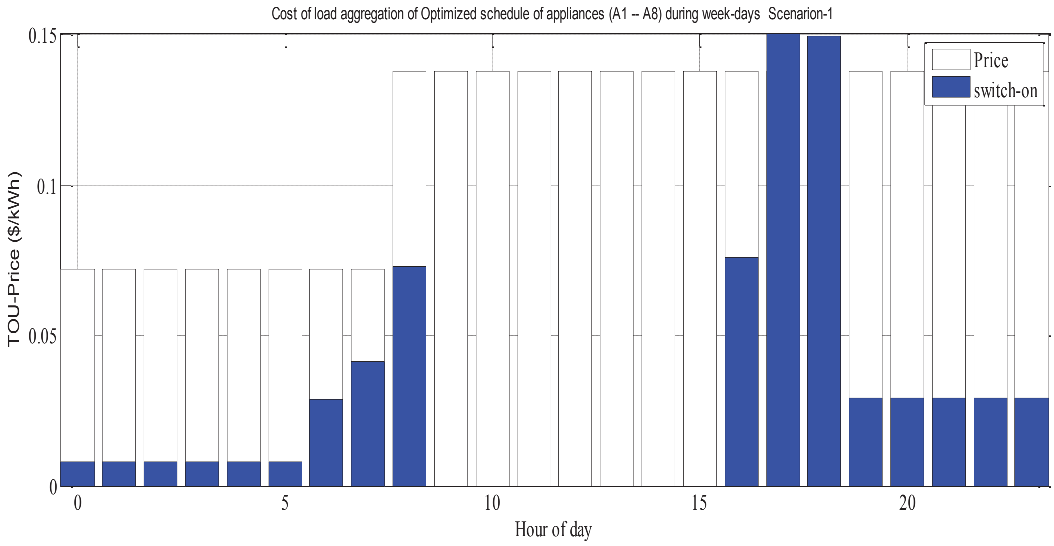

Figure 6.6 Load aggregation cost of optimized schedule Scenarion-1

Table 6.5 Load aggregation cost of optimized schedule Scenarion-1

Time	Appliances	Load (kW)	Tariff ($/kWh)	Energy cost ($)
1	A5	1.5	0.07213	0.1082
2	A5	1.5	0.07213	0.1082
3	A5	1.5	0.07213	0.1082
4	A5	1.5	0.07213	0.1082
5	A5	1.5	0.07213	0.1082
6	A5	1.5	0.07213	0.1082
7	A1, A5, A8	5.532	0.07213	0.3990
8	A1, A5, A6, A8	7.932	0.1379	1.0938
9	A5, A7	3.834	0.1379	0.5287
17	A1	4	0.1379	0.5516
18	A1, A5, A6, A8	7.932	0.1379	1.0938
19	A1, A5, A7, A8	7.866	0.1379	1.0847
20	A5, A8	1.532	0.1379	0.2113
21	A5, A8	1.532	0.1379	0.2113
22	A5, A8	1.532	0.1379	0.2113
23	A5, A8	1.532	0.1379	0.2113
24	A5, A8	1.532	0.1379	0.2113
Total Energy Cost/day				**6.4572**

Comparison of non-optimized and optimized schedule cost Scenario 1

Total Energy Cost/Day of Non-Optimized schedule	Total Energy Cost/Day of Optimized schedule	%
6.9292	6.4572	6.81

Scenario 2, weekends

Three appliances are assumed to be run just during weekends (A2, A3, and A4). In addition we already have five appliances (A1, A5, A6, A7, and A8) that should be run during scenario 2 as well; in this scenario of weekends, just off-peak tariff is applied.

The appliances (A1, A2, A3, A4, A5, A6, A7 and A8) are scheduled in weekends and therefore, by applying the same method of (BIP) algorithm.

Table 6.6 Scenario-2 appliances of residential energy system

Appliance	Time operating interval (hr.)	Run-time (hr.)
DEWH (A1)	(6 to 8) and (16 to 19)	5
DECW (A2)	(9 to 11)	2
DECD (A3)	(10 to 12)	2
DECI (A4)	(10 to 11)	1
DEBH (A5)	(17 to 0) and (0 to 13)	20
DEO (A6)	(7) and (17)	2
DEDW (A7)	(8) and (18)	2
F-lamp (A8)	(6 to8) and (17 to 0)	9

Simulation results of scenario 2 weekends as follows:

- Appliances, DEWH (A1), DECW (A2) and DECD (A3)

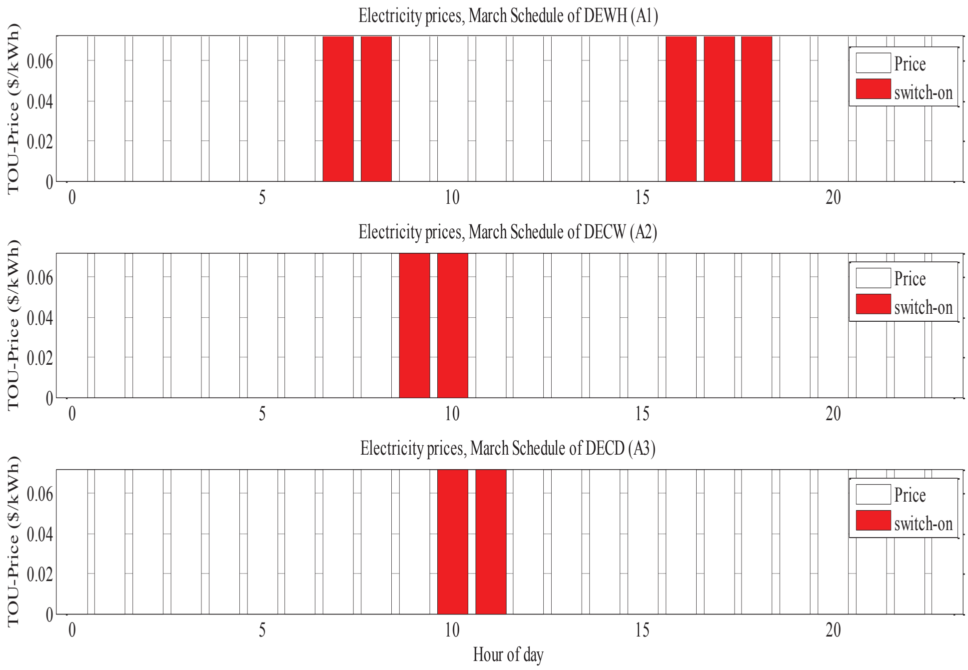

Figure 6.7 Off-Peak tariff of appliances (A1, A2 and A3) Optimized run-time schedule weekends Scenarion-2

- Appliances, DECI (A4), DEBH (A5) and DEO (A6)

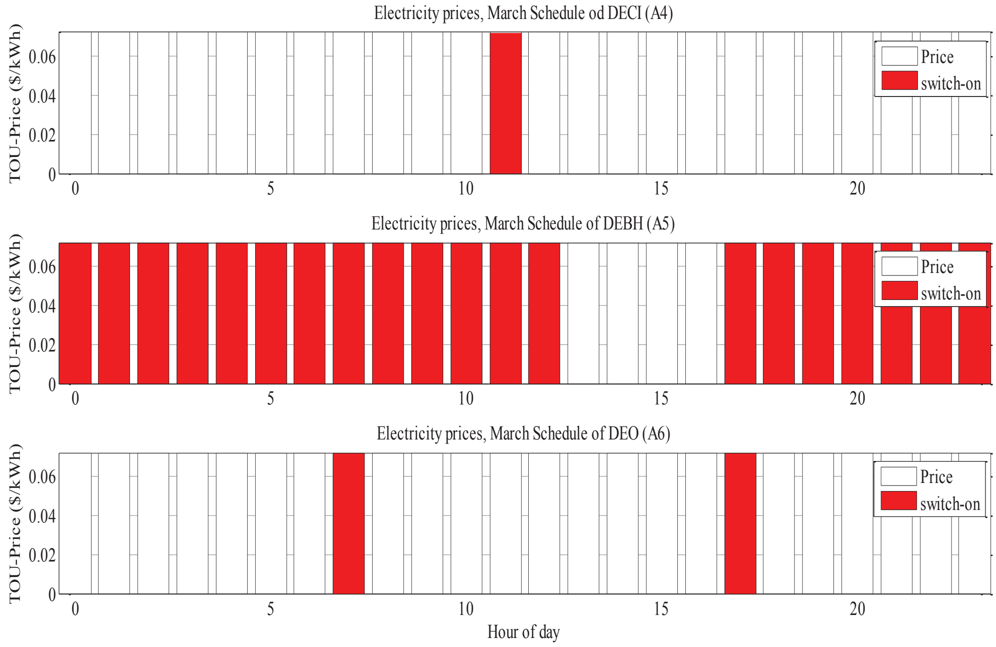

Figure 6.8 Off-Peak tariff of appliances (A4, A5 and A6)
Optimized run-time schedule weekends Scenarion-2

- Appliances, DEDW (A7), F-lamp (A8)

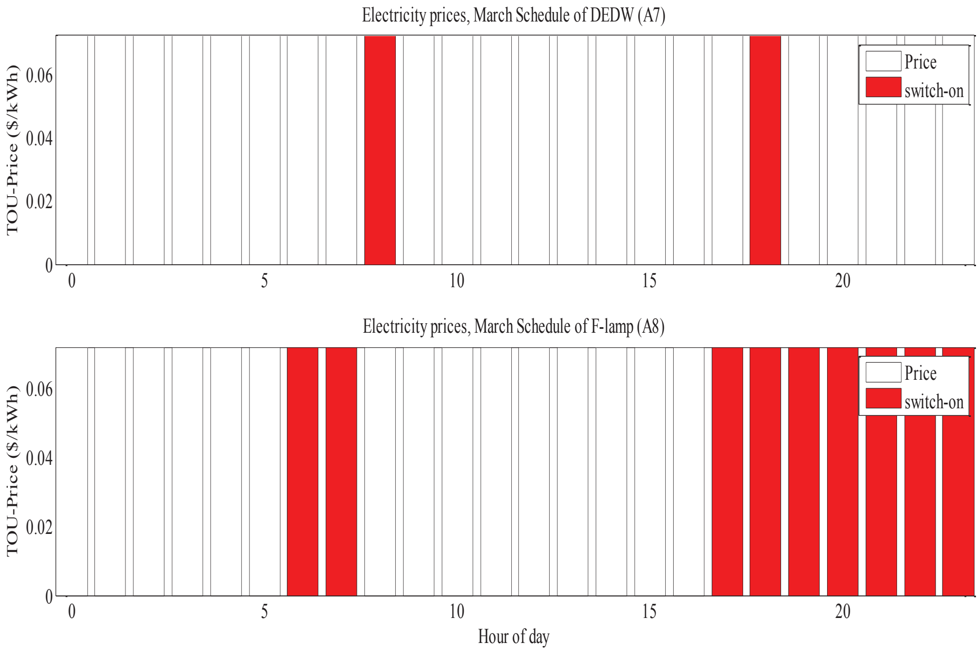

Figure 6.9 Off-Peak tariff of appliances (A7, and A8)
Optimized run-time schedule Scenarion-2

- Load aggregation cost of optimized run-time Scenarion-2

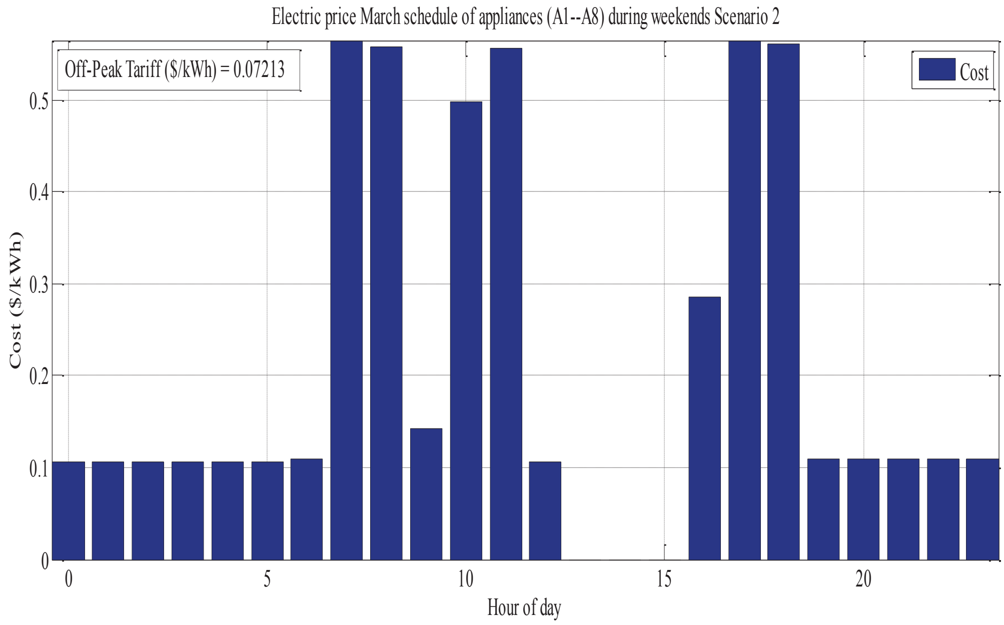

Figure 6.10 Load aggregation cost of run-time schedule Scenarion-2

As can be seen in figures (6.7, 6.8 and 6.9), the simulation results model of (BIP) has a good agreement with mathematical model, where all appliances (A1, A2, A3, A4, A5, A6, A7 and A8) in scenario 2 weekends are operated during schedule time of the requirements with constraints. On the other hand, figure (6.8 and 6.10) indicates that the appliances (A5 and A8) are operated at the same time 6am; where at 7am there are four appliances (A1, A5, A6 and A8); moreover, three appliances (A1, A5 and A7) are operated at 8am; also we can conclude that at 9am the appliance (A2 and A5) are operated for washing clothes and heating the house; therefore, at 10 am there are three appliances (A2 and A3) should be operated respectively followed by (A4, A3) at 11am and appliance (A5) just for heating the house during this time and next hour as well; from (1pm to 4pm) is unoccupied period. In addition, the electric water heater (A1) is operated at 4pm for three hours, where at 5pm there are four appliances (A1, A5, A6 and A8) are operated at same time; appliances (A1, A5, A7, and A8) should be operated at 6pm where dishwasher should be run after cocking time; all appliances should be turned off at 7pm except two appliances (A5 and A8) should be turned on till 11pm; from 12am till 5am just electric baseboard heater should turned on to maintain the temperature at desired level.

Table 6.7 Scenario 2, Load aggregation energy cost of residential sector

Time	Appliances	Energy Cost ($/kWh)
6am	(A5) and (2*A8)	0.113
7am	(A1), (A5), (A6) and (2*A8)	0.574
8am	(A1), (A5) and (A7)	0.565
9am	(A2) and (A5)	0.144
10am	(A2), (A3) and (A5)	0.505
11am	(A3), (A4) and (A5)	0.563
12pm	(A5)	0.108
4pm	(A1)	0.289
5pm	(A1), (A5), (A6) and (3*A8)	0.577
6pm	(A1), (A5), (A7) and (3*A8)	0.572
7pm	(A5) and (3*A8)	0.115
8pm	(A5) and (3*A8)	0.115
9pm	(A5) and (3*A8)	0.115
10pm	(A5) and (A8)	0.111
11pm	(A5) and (A8)	0.111
(0am to 5am	(A5)	0.108
Total cost/day		**4. 685**

Scenario 3, week-days (January, winter season)

This scenario is assumed to be in January 2013 where the average temperature is (-9 ℃) [77]; and there is a three-part time of use (TOU) tariff as follows:

Table 6.8 The cost-rate of NS power * *Rates effective January 1, 2013* [17]

Time & Hours			Rate/kWh
Weekdays	December to February (68 Days)	7am to 12pm 4pm to 11pm	**On-Peak** 17.878¢/kWh
		11pm to 7am	**Off-Peak** 7.213 ¢/kWh
		12pm to 4pm	**Standard Rate** 13.790¢/kWh

Five appliances are assumed to be run during week-days (A1, A5, A6, A7, and A8), by using the same method of the (BIP) model; the results of nun-optimize run-time load aggregation cost scenario-3 weekdays as follows:

Normal case of Non-Optimized run-time during week-days Scenarion-3

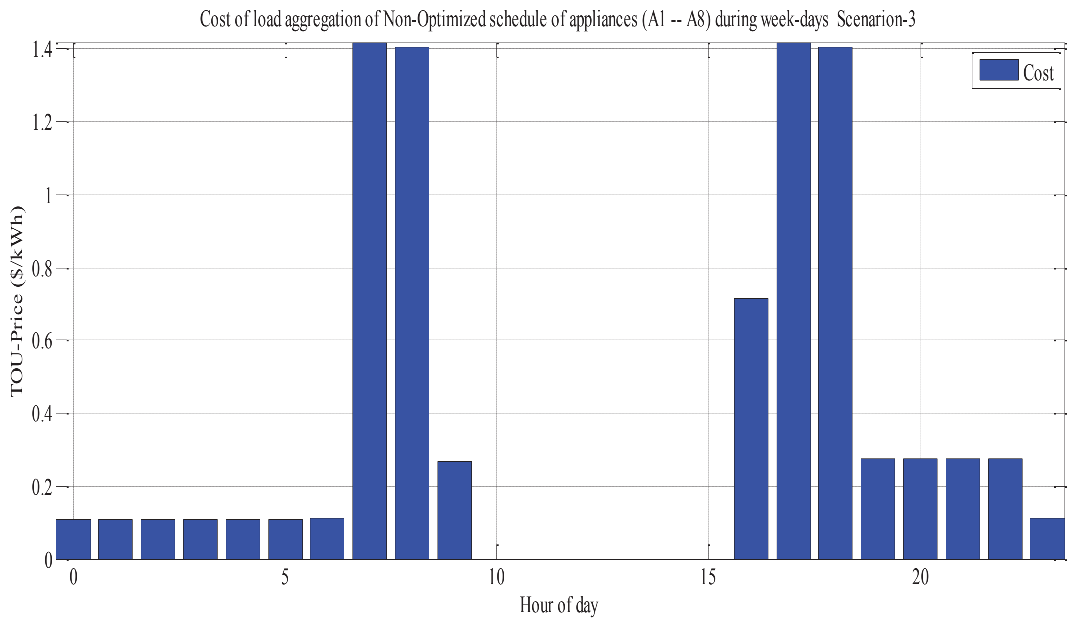

Figure 6.11 Load aggregation cost of non-optimized schedule Scenarion-3

Table 6.9 Scenario 3, Load aggregation cost of Non-Optimized schedule

Time	Appliances	Load (kW)	Tariff ($/kWh)	Energy Cost ($)
1	A5	1.5	0.07213	0.1082
2	A5	1.5	0.07213	0.1082
3	A5	1.5	0.07213	0.1082
4	A5	1.5	0.07213	0.1082
5	A5	1.5	0.07213	0.1082
6	A5	1.5	0.07213	0.1082
7	A5, A8	1.532	0.07213	0.1105
8	A1, A5, A6, A8	7.932	0.17878	1.4181
9	A5, A5, A7, A8	7.866	0.17878	1.4063
10	A5,	1.5	0.17878	0.2682
17	A1	4	0.17878	0.7151
18	A1, A5, A6, A8	7.932	0.17878	1.4181
19	A1, A5, A7, A8	7.866	0.17878	1.4063
20	A5, A8	1.532	0.17878	0.2739
21	A5, A8	1.532	0.17878	0.2739
22	A5, A8	1.532	0.17878	0.2739
23	A5, A8	1.532	0.17878	0.2739
24	A5, A8	1.532	0.07213	0.1105
Total Energy Cost/day				**8.5978**

Optimized case of run-time schedule during week-days Scenarion-3

Table 6.10 Scenario 3, appliances of residential energy system

Appliance	Time operating interval (hr.)	Run-time (hr.)
DEWH (A1)	(6 to 8) and (16 to 19)	5
DEBH (A5)	(17 to 24) and (24 to 9)	16
DEO (A6)	(8) and (18)	2
DEDW (A7)	(9) and (19)	2
F-lamp (A8)	(6 to 8) and (17 to 24)	9

Simulation results of scenario 3, week-days as follows:

- Appliances, DEWH (A1), DEBH (A5) and DEO (A6)

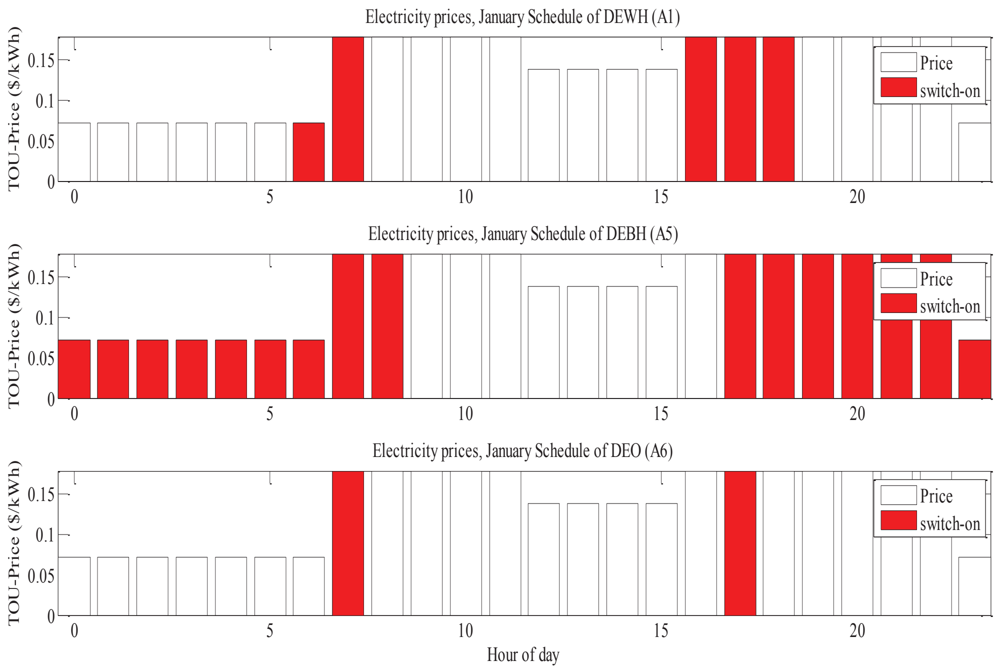

Figure 6.12 TOU tariff of appliances (A1, A5, and A6)
Optimized run-time schedule Scenarion-3

- Appliances, DEDW (A7) and F-lamp (A8)

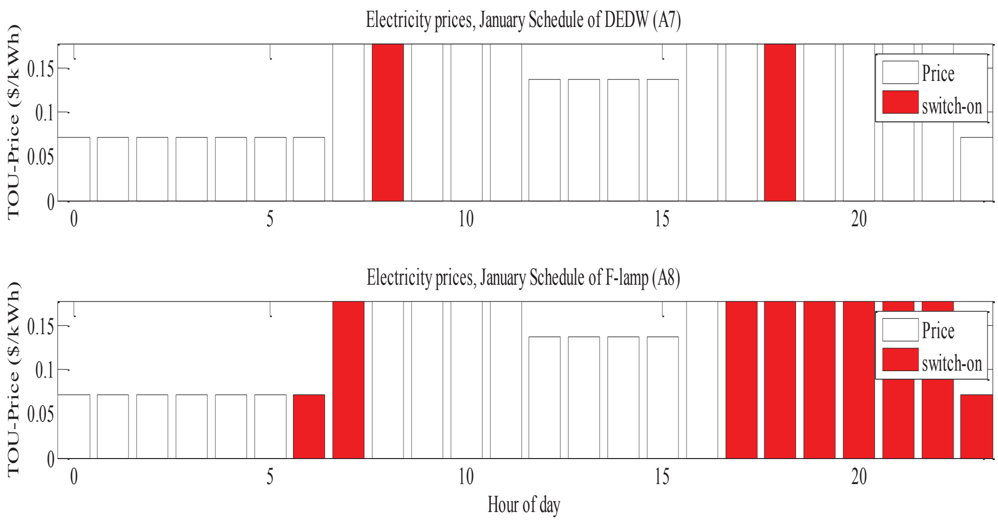

Figure 6.13 TOU tariff of appliances (A7, and A8)
Optimized schedule Scenario-3

- Cost of load aggregation of optimized run-time schedule of residential energy system Scenarion-3

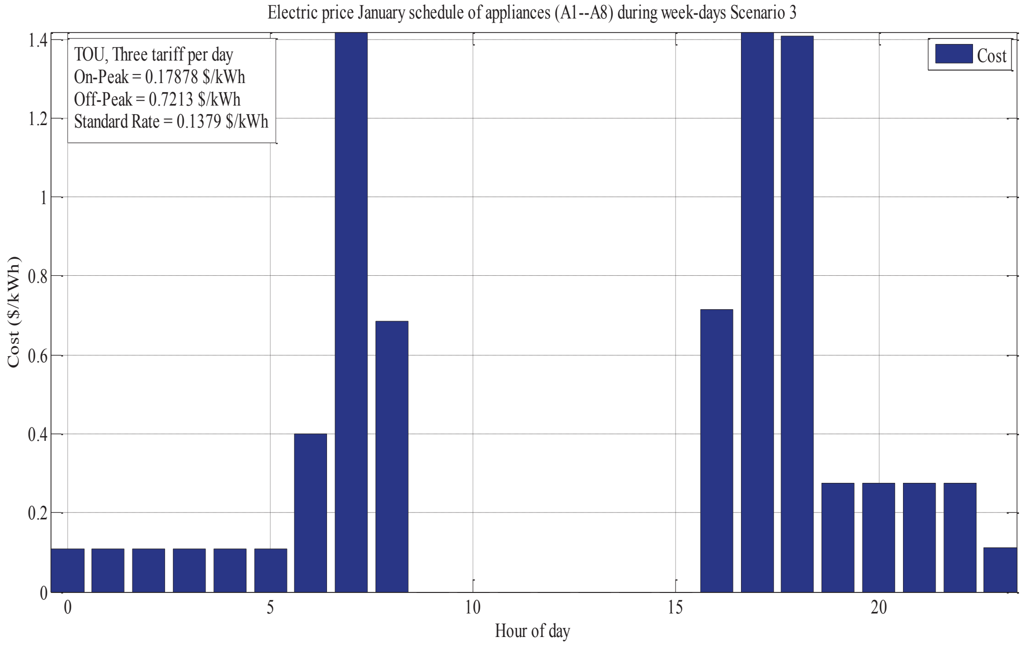

Figure 6.14 Load aggregation cost of optimized schedule Scenarion-3

As can be seen in figures (6.12 and 6.13), the simulation results of (BIP) model has a good agreement with mathematical model, where five appliances (A1, A5, A6, A7 and A8) in scenario-3 week-days are operated during schedule time of the requirements with constraints. On the other hand, figure (6.14) indicates that the period of (23pm to 6am) has two appliances (A5 and A8) both of them are operated at the same time in off-peak tariff; where at 6am there are three appliances runs (A1, A5 and A8); moreover, four appliances (A1, A5, A6 and A8) are operated at 7am during on-peak tariff; at 8am there are two appliances are operated at same time (A5 and A7), where the dishwasher should be run after cocking time; from (9am to 4pm) is unoccupied period; also we can conclude that at 4pm the electric water heater started its cycle to provide a hot water at desired temperature. The period from (5pm to 7pm) where the appliances (A1, A5, A6, A7 and A8) are assumed to be run during On-peak time in order to provide hot water, heating the house, cocking/washing time and lighting as well; this figure also indicates that at 7pm there is just two appliances should be run (A5 and A8) until 11pm where all appliances should be turned off at this time except heating system should be run to maintain the temperature at desired level during sleeping time until 9am.

Table 6.11 Scenario 3, Load aggregation cost of optimized schedule

Time	Appliances	Load (kW)	Tariff ($/kWh)	Energy Cost ($)
1	A5	1.5	0.07213	0.1082
2	A5	1.5	0.07213	0.1082
3	A5	1.5	0.07213	0.1082
4	A5	1.5	0.07213	0.1082
5	A5	1.5	0.07213	0.1082
6	A5	1.5	0.07213	0.1082
7	A1, A5, A8	5.532	0.07213	0.3990
8	A1, A5, A6, A8	7.932	0.17878	1.4181
9	A5, A7	3.834	0.17878	0.6854
17	A1	4	0.17878	0.7151
18	A1, A5, A6, A8	7.932	0.17878	1.4181
19	A1, A5, A7, A8	7.866	0.17878	1.4063
20	A5, A8	1.532	0.17878	0.2739
21	A5, A8	1.532	0.17878	0.2739
22	A5, A8	1.532	0.17878	0.2739
23	A5, A8	1.532	0.17878	0.2739
24	A5, A8	1.532	0.07213	0.1105
Total Energy Cost/Day				**7.8973**

Comparison of non-optimized and optimized schedule cost Scenario-3

Total Energy Cost/Day of Non-Optimized schedule	Total Energy Cost/Day of Optimized schedule	%
8.5978	7.8973	8.15

6.2.3 CASE STUDY of COMMERCIAL ENERGY SYSTEM

In this case study of commercial sector, a small business office is presented and it has just one scenario, based on NS Power [17] a small general tariff as follows:

Table 6.12 The cost-rate of NS Power * *Rates effective January 1, 2013* [17]

First 200 (kWh/month)	For all additional (kWh)
$ 0.14276	$ 0.1256

Table 6.13 Appliances of commercial energy system a case study

Appliance	Time operating interval (hr.)	Run-time (hr.)
EWH (A1)	6am to 6pm	12
EBH (A5)	6am to 6pm	12
EO (A6)	12pm to 1pm	1
F-lamp (A8)	6am to 6pm	12

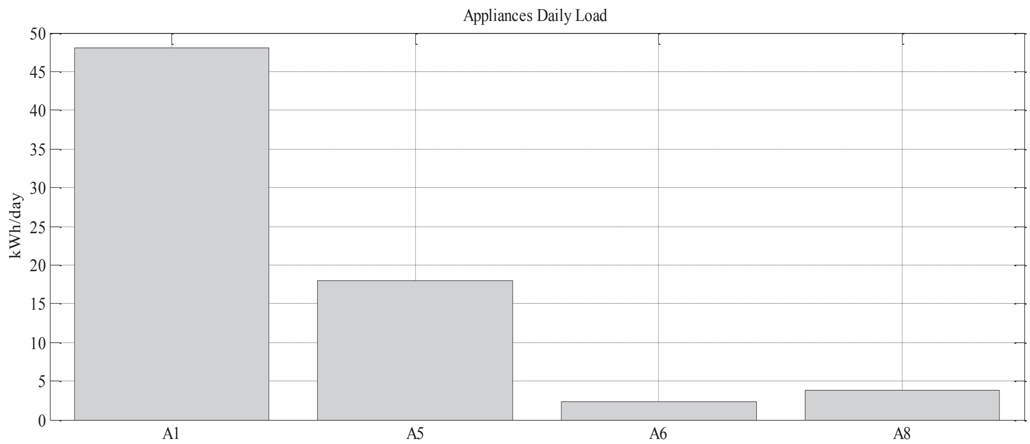

Figure 6.15 Daily load in commercial energy system of a case study

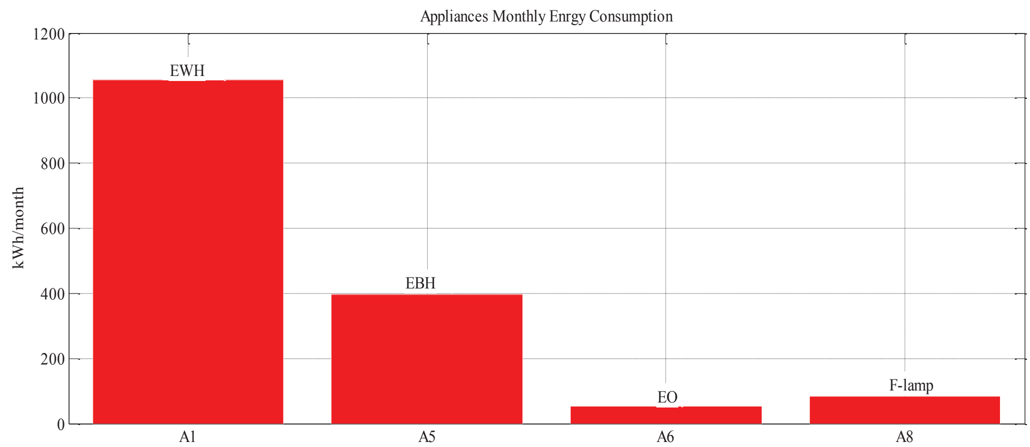

Figure 6.16 Monthly load in commercial energy system of a case study

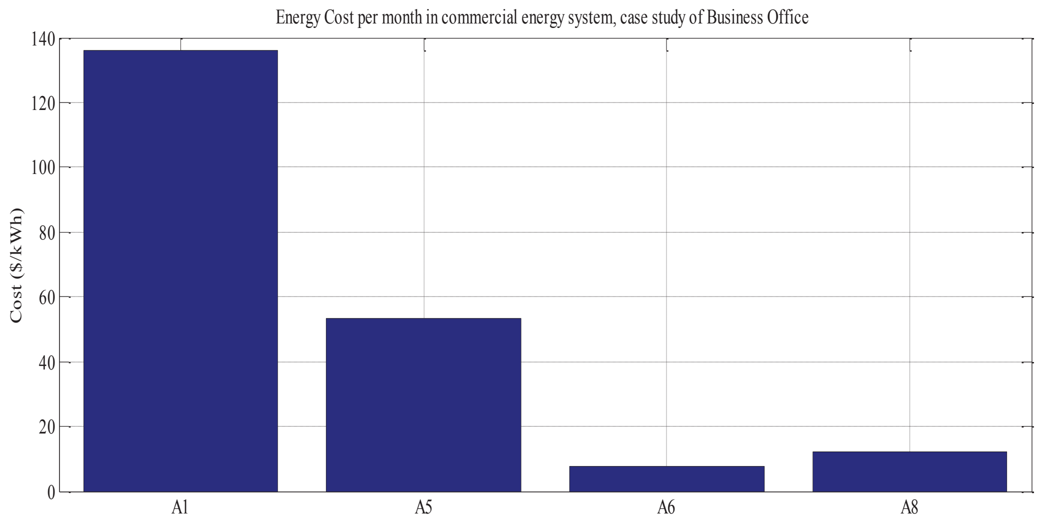

Figure 6.17 Monthly cost in commercial energy system of a case study

As can be seen in figures (6.15, 6.16 and 6.17) the office appliances are assumed to be run as follows:

- An electric water heater that runs 12 hrs. /day and 22 days/month, where its energy consumption is 48 kWh./day and 1065kWh/month, the energy cost is ($136/month).
- An electric baseboard heater that runs 12 hrs./day where the energy consumption is 18 kWh/day and 396 kWh/month, the energy cost is ($53.2/month),
- An electric oven that runs 1 hr./day where the energy consumption is 2.4 kWh/day and 52.8 kWh/month, the energy cost is ($7.54/month),
- A10 F-lamp that run 12 hr./day where the energy consumption are 3.84 kWh/day and 84.5 kWh/month, the energy cost is ($12.1/month).

Monthly energy consumption and cost in commercial energy system

Figure 6.18, indicates overall monthly energy consumption and its associated cost. In this case study of commercial energy system, it is worth mentioning that the optimization technique is not applicable according to the condition of a small general tariff table 6.12. However, tables [6.14 and 6.15] show in details number of appliances in the office and the overall energy consumption and its associated cost based on NS power tariff table 6.12.

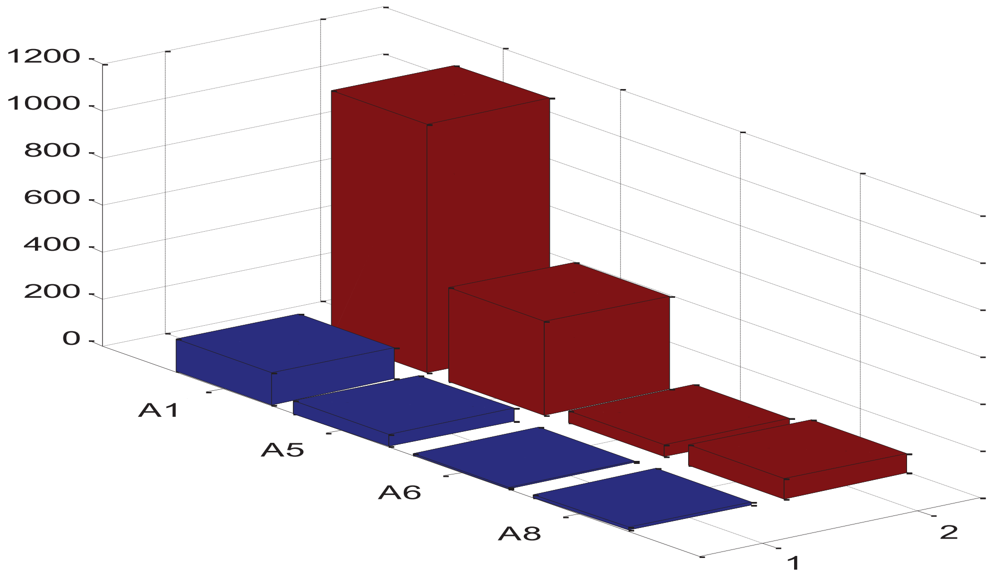

Figure 6.18 Monthly energy consumption and cost in commercial energy system of a case study

Table 6.14 Daily load aggregation of energy consumption

Appliance	kW	Time operating interval (hr.)	Run-time (hr./day)	Appliance number	kWh/day
EWH (A1)	4	6am to 6pm	12	1	48
EBH (A5)	1.5	6am to 6pm	12	3	54
EO (A6)	2.4	12pm to 1pm	1	1	2.4
F-lamp (A8)	0.032	6am to 6pm	12	10	3.84
Total energy consumption					108.2

Table 6.15 Monthly load aggregation of energy (consumption and cost)

Appliance	kW	Time operating interval (hr.)	Run-time (hr./month)	appliance number	kWh/month
EWH (A1)	4	6am to 6pm	264	1	1056
EBH (A5)	1.5	6am to 6pm	264	3	1188
EO (A6)	2.4	12pm to 1pm	22	1	52.8
F-lamp (A8)	0.032	6am to 6pm	264	10	84.48
Total energy consumption (kWh/month)					**2381.3**
Total energy cost ($/month)					**302.52**

6.2.4 COMPARISON OF MODELS

Comparison of optimization method of (BIP) model with another literature model

Numerous researches work has been done conducted linear programing optimization technique in order to minimize the residential energy consumption and its associated cost where the power consumption patterns are classified into power-shiftable or time- shiftable and non- shiftable types.

In this thesis, the optimization problem is solved by binary integer programming (BIP) model which is formulated and numerically simulated by considering various energy consumption patterns of eight home appliances and its associated cost under time of use (TOU) pricing of electricity in Canada based on (NS power) rate [17] with respect to the schedule plan and the consumer choice / comfortable; however, three scenarios are simulated under non-schedule plan and pre-schedule plan of Summer and Winter season divided into week-days and weekends. It is worth mentioning that the optimization technique is not applicable according to the rate of weekends.

In comparison of optimization method of (BIP) model with another literature model [79] in a case study of "Home Appliance Scheduling Optimization with Time-Varying Electricity Price and Peak Load Limitation." In [79] the linear optimization problem solved by using mixed integer linear programming (MBLP) this model is formulated and numerically simulated by considering various energy consumption patterns of five home appliances in a real time-varying electricity price (TOU) with respect to the schedule plan.

Comparison of optimization method in winter season

% optimal Energy Cost/Day of (BIP)	% optimal Energy Cost/Day of Compared model (MBLP)
8.15 %	12%

The difference between two models is due to the electricity rate and scenarios conditions; where in (MBLP) [79], there are three appliances are operated during week-days; (clothes washer, clothes dryer and clothes iron) while in (BIP) model all three appliances clothes washer (DECW), clothes dryer (DECD) and DECI) were operated in weekends only. Moreover, in (BIP) the customer choice/comfortable is considered in constrains of all scenarios.

CHAPTER 7 CONCLUSION AND RECOMMENDATIONS

7.1 CONCLUSION

This study examines mathematical modeling and simulation of some appliances that can be used for domestic and commercial purpose in order to analyze and simulate the behaviour of each system in terms of input power; The domestic electric water heater (DEWH), domestic electric oven (DEO), domestic electric baseboard heater (DEBH) and domestic electric clothes iron (DECI) are modeled and a mathematical model for each system is derived; simulation results are compared with approximate linear equations and exponential equations, which are derived and used for estimating the amount of temperature, time duration and power consumption. The simulation results have a good agreement with a mathematical model; also, we can conclude that the exponential equations are more accurate in some cases compared to the linear equations and the simulated model results. In particular, the domestic electric water heater model in this study has been compared to another literature model.

On the other hand, electric machine systems such as domestic electric clothes washer (DECW), clothes electric clothes dryer (DECD) and domestic electric dishwasher (DEDW) are modeled and simulated in details of mathematical models and the system behaviour during operation time. All modeling of appliances are designed and implemented by using both the Matlab/Simulink and Matlab/SimScape environments in order to develop control systems of heating cycles in these machines and test the system performance of single-phase induction motors model. In addition, fluorescent-lamp (F-lamp) has been presented as a model of lighting system by analysing the equivalent simplified circuit of the series resonant parallel loaded ballast and fluorescent lamp.

Multiphasic system behavior and an individual set of components simulation of appliances is presented in a realistic manner and examined in this thesis in order to determine the appropriate model that reflects the parameters' impact as well as the system behavior of each individual model. Thus, the main parameters and system behaviour of each model in transient condition and steady-state condition were discussed in depth to observe the power consumption of each appliance. Therefore, this thesis presented and implemented an approach for identifying of physical models of a single-zone lumped-parameter thermal model; this approach has been developed by a group of researchers at the University of New Brunswick, 2012.

This thesis focused on energy consumption of residential and commercial sectors, in order to reduce energy cost, and a time of use (TOU) problem solution was optimized to determine the best operation time schedule; in a case of an individual operating as well as in an aggregating operation during Summer and Winter, Linear Binary Integer Programming optimization technique (BIP) was implemented for both an individual physical model and aggregated models to reduce energy consumption and its associated cost during peak period with respect the schedule plan and the customer choice and behavior.

Many candidate scenarios were analyzed during peak and off peak load periods comparing to the tariff of the residential and commercial sectors to reduce the usage and its associated costs. Thus, the optimization function seeks minimization of costs and the optimal daily operation of home appliances in order to find the solution for such problems. The main approach was to ensure the satisfaction of the requirements with constraints on efficient use of energy. The optimal solution of (TOU) problem has been achieved by using (BIP) algorithm based on pricing of electricity in Canada (NS power) rate with respect the schedule plan and the consumer choice/comfortable.

7.2 RECOMMENDATIONS AND FUTURE WORK

Recommendations and suggestions for future studies related to this thesis are as follows:

- Further research is recommended to implement cooling systems in residential and commercial sectors.
- Load flow study of residential sector in large scale such as a small village.
- Load flow study for commercial sector in large business and industrial.
- Adding different power sources such as wind and solar as an alternative source in a smart grid in order to reduce the electricity consumption.

- In a smart grid environment load shifting techniques could be used during peak period.
- Applying other optimization technique on TOU problem in residential and commercial sectors

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APPENDIX A Model of domestic electric oven DEO

The aim of this section is to provide, a sample calculations in order to analyze the DEO system behavior during transient condition and steady-state condition response.

Calculations of DEO $\{\boldsymbol{T}(\mathrm{t}), \boldsymbol{T}_{low}, \boldsymbol{T}_{high}, \boldsymbol{t}_{off}\ , \boldsymbol{t}_{on}, t_{on}^{start-up}\}$

In order to calculate the approximate heater element temperature at the time (t) within a certain range, the first order differential equation [12] can be used as follows:

$$T(\mathrm{t}) = T_H(t_0)e^{-\frac{(t-t_0)}{\tau}} + K\left(1 - e^{-\frac{(t-t_0)}{\tau}}\right) \quad (4.5)$$

$\tau = \frac{C}{G}$	$K = \frac{GT_a + Q}{G}$

Calculations of $T(t)$, by using an exponential equation (4.5)

$\boldsymbol{\tau} = \frac{\mathrm{C}}{\mathrm{G}} = \frac{1298.0619}{6.1484} = 211.1219\ sec$	$\boldsymbol{K} = \frac{\mathrm{G\ T_a + Q}}{\mathrm{G}} = \frac{(6.1484*20) + 2400}{6.1484} = 410.3455\ °C$

By applying above equation to obtain $T(\mathrm{t})$

	$\boldsymbol{T}_H(t_0)$ (°C)	$\boldsymbol{t}_{on}^{m}$ (sec)	$\boldsymbol{\tau}$ (sec)	$\boldsymbol{K}$ (°C)	$\boldsymbol{T}(\mathrm{t})$ (°C)
Case 1 (natural response)	20	312	211.1219	0	4.5627
Case 2 (forced response)	20	312	211.1219	410.3455	317.6807

Calculation of $(\boldsymbol{T}_{low})$ and $(\boldsymbol{T}_{high})$

$$T_{low} = T_{high}\ \boldsymbol{e}^{-\frac{t_{off}^{m}}{C/G}} + T_a\left(1 - \boldsymbol{e}^{-\frac{t_{off}^{m}}{C/G}}\right) \quad (4.6)$$

In this case the heater element is (*OFF*) thus, $Q = 0\ W \quad \rightarrow \quad K = T_a$

T_{high} (°C)	T_a (°C)	t_{off}^{m} (sec)	G (W/°C)	C (J/°C)	K (°C)
400	20	20.4	6.1484	1298.0619	20

$\boldsymbol{T}_{low} = \mathbf{365}\ °C$

$$T_{high} = T_{low}\, \boldsymbol{e}^{-\frac{t_{on}^{m}}{C/G}} + (T_a + \tfrac{Q}{G})\left(1 - \boldsymbol{e}^{-\frac{t_{on}^{m}}{C/G}}\right) \tag{4.7}$$

In this case the heater is (*ON*) thus, $Q = 1500\ W \quad \rightarrow \quad K = T_a + \frac{Q}{G}$

T_{low} (°C)	T_a (°C)	t_{on}^{m} (sec)	G (W/°C)	C (J/°C)	K (°C)
365	20	312	6.1484	1298.0619	410.3455

$\boldsymbol{T_{high}} = \mathbf{400}$ °C

Set-point (ΔT) $= T_s = \frac{T_{high} + T_{low}}{2} = \frac{400 + 365}{2} = 382.5$ °C

Thus, the Dead band value (D_b) is:

$$D_b = \frac{T_{high} - T_{low}}{2} = \frac{400 - 365}{2} = 17.5 \text{ °C}$$

Therefore,

$$T_{high} = T_s + D_b = 400 \text{ °C}$$

$$T_{low} = T_s - D_b = 365 \text{ °C}$$

Calculation of Time duration (t_{off}), (t_{on}) and ($t_{on}^{start-up}$)

1- Linear Equations

$$\boldsymbol{t_{off}^{m}} = \frac{C(T_{high} - T_{low})}{G(T_s - T_a)} \tag{4.8}$$

$$\boldsymbol{t_{on}^{m}} = \frac{C(T_{high} - T_{low})}{G\,(T_a - T_s) + Q_i} \tag{4.9}$$

$$\boldsymbol{t_{on}^{start-up}} = \frac{C(T_{high} - T_{low})}{G\,(T_a - T_s) + Q_i} \tag{4.10}$$

Constant system parameters

T_{high} (°C)	T_{low} (°C)	T_a (°C)	T_s (°C)	G (W/°C)	C (J/°C)	Q_i (W)
400	365	20	382.5	6.1484	1298.0619	2400

Linear Equations Results

P_{rated} (kW)	t_{off}^{m} (sec)	t_{on}^{m} (sec)	$t_{on}^{start-up}$ (sec)
0 / 2.4	20.38	265.37	334.35

$\boldsymbol{t_{off}} = 20.38\ sec = 0.34\ min = 0.0057\ hr$

$\boldsymbol{t_{on}} = 265.37\ sec = 4.42\ min = 0.0737\ hr$

$\boldsymbol{t_{on\,(startup)}} = 334.35\ sec = 5.5725\ min = 0.0929\ hr$

2- Exponential Equations

$$t_{off}^{m} = RC\ ln\left(\frac{T_{high} - T_a}{T_{low} - T_a}\right) \quad (4.11)$$

$$t_{on}^{m} = RC\ ln\left(\frac{T_{low} - T_a - R * P_{rated}}{T_{high} - T_a - R * P_{rated}}\right) \quad (4.12)$$

$$t_{on\ (startup)} = RC\ ln\left(\frac{T_{low} - T_a - R * P_{rated}}{T_{high} - T_a - R * P_{rated}}\right) \quad (4.13)$$

System parameters

T_{high} (°C)	T_{low} (°C)	T_a (°C)	T_s (°C)	R' (°C/W)	C (J/°C)	Q_i (W)
400	365	20	382.5	6.1484	1298.0619	2400

Exponential Equations Results

P_{rated} (kW)	t_{off}^{m} (sec)	t_{on}^{m} (sec)	$t_{on}^{start-up}$ (sec)
0 / 2.4	20.40	312.00	454.49

$t_{off} = 20.40\ sec = 0.34\ min = 0.0057\ hr$

$t_{on} = 312.00\ sec = 5.20\ min = 0.0867\ hr$

$t_{on\ (startup)} = 454.49\ sec = 7.57\ min = 0.1262\ hr$

$$\text{Cycling time} = t_{on} + t_{off} \quad (4.14)$$

$Cycling\_time$ (hr)	t_{on} (hr)	t_{off} (hr)
0.0924	0.0867	0.0057

$$N_{cycles} = \frac{Run_{time} - t_{on\ (start-up)}}{cycling\ time} \quad (4.15)$$

N cycles	Run_{time} (hr)	$t_{on\ (startup)}$ (hr)	$cycling\_time$ (hr)
9.4	1	0.1262	0.0924

$$t_{on(total)} = (N_{cycles} * t_{on}) + t_{on\ (startup)} \quad (4.16)$$

$t_{on(total)}$ (hr)	t_{on} (hr)	N_{cycles}	$t_{on\ (start-up)}$ (hr)	Run_{time} (hr)
0.9412	0.0867	9.4	0.1262	1

Power / Energy Consumption of DEO

$$Q_{on\ (kWh)} = \boldsymbol{P_{rated}} * \boldsymbol{t_{on}} \tag{4.17}$$

Method	Q_{on} (kWh)	P_{rated} (kW)	t_{on} (hr)
Calculation	**0.2081**	2.4	0.0867

$$Q_{total} = P_{rated}\ (kW) * \boldsymbol{t_{on(total)}}\ (hr) \tag{4.18}$$

Q_{total} (kWh)	P_{rated} (kW)	$t_{on(total)}$ (hr)	*Run-time* (hr)
2.2589	2.4	0.9412	1

$$\text{DEO (kW)} = \boldsymbol{Q_{total}} / \boldsymbol{t_{on\ (total)}} \tag{4.19}$$

Electric Heater (kW)	Q_{total} (kWh)	$t_{on(total)}$ (hr)	Run-time (hr)
2.4	2.2589	0.9412	1

$$Duty\ Cycle = (\boldsymbol{t_{on}} / Cycling_{time}) \tag{4.20}$$

$Duty\ Cycle$	t_{on}^{m} (hr)	$Cycling_{time}$ (hr)
0.9383	0.0867	0.0924

$$\Delta P_{DEO} = \boldsymbol{P_{rated}} * (Duty\ Cycle) \tag{4.21}$$

$$\Delta P_{DEO} = 2.4\ kW * 0.9383 = 2.2519\ kW$$

The Energy consumption by DEO ($\boldsymbol{EC_{DEO}}$) can be calculated as follows:

$$Energy\ consumption\ (kWh) = \boldsymbol{P_{rated}} * \boldsymbol{t_{on}} \tag{4.22}$$

$$EC_{DEO} = 2.4 * 0.0867 = 0.2081\ kWh$$

APPENDIX B Model of domestic electric clothes iron DECI

The aim of this section is to provide, a sample calculation in order to analyze the DECI system behavior during transient condition and steady-state condition response.

Calculations of DECI $\{\boldsymbol{T}(\mathrm{t}), \boldsymbol{T}_{low}, \boldsymbol{T}_{high}, \boldsymbol{t}_{off}, \boldsymbol{t}_{on}, t_{on}^{start-up}\}$

Constant parameters of the DECI:

$Q = P_{rated}$ (W)	T_{high} $(°C)$	T_{low} $(°C)$	$T_{set-point}$ $(°C)$	T_a $(°C)$	t_{on}^m (sec)	t_{off}^m (sec)
1300	140	100	120	20	42.42	128.4

By using an approach for identification of physical models of single- zone lumped-parameter thermal model equations (4.1 to 4.7) the results are

$$\boldsymbol{G} = \frac{Qi\,(\Delta - 1)}{\Delta(T_{high} - T_a) - (T_{low} - T_a)} \tag{4.1}$$

$$\boldsymbol{C} = G\left(\frac{t_{off}^m}{\ln(T_{high} - T_a) - \ln(T_{low} - T_a)}\right) \tag{4.2}$$

Where,

$$\Delta = \left(\frac{T_{high} - T_a}{T_{low} - T_a}\right)^{t_{on}^m / t_{off}^m} \tag{4.3}$$

$$G/C = \frac{\ln(T_{high} - T_a) - \ln(T_{low} - T_a)}{t_{off}^m} \tag{4.4}$$

Δ	G $(W/°C)$	C $(J/°C)$
1.14334	3.2577	1031.6283

Also, the thermal capacitance can be calculated by $C = \rho C_p V$

ρ (kg/m^3)	C_p $(J/kg.°C)$	V (m^3)	C $(J/°C)$
2700	910	0.00042	1031.94

The ratio of G/C can be calculated directly

G/C (sec^{-1})	$\boldsymbol{\tau} = C/G$ (sec)
3.1578279×10^{-3}	316.6738

Calculations of DECI $\{\boldsymbol{T}(\mathrm{t}), \boldsymbol{T}_{low}, \boldsymbol{T}_{high}, \boldsymbol{t}_{off}, \boldsymbol{t}_{on}, t_{on}^{start-up}\}$

$$T(\mathrm{t}) = T_H(t_0)\, e^{-\frac{(t-t_0)}{\tau}} + K\left(1 - e^{-\frac{(t-t_0)}{\tau}}\right) \quad (4.5)$$

Calculations of $T(t)$, by using an exponential equation (4.5)

$\boldsymbol{\tau} = \frac{\mathrm{C}}{\mathrm{G}} = \frac{1031.6283}{3.2577} = 316.6738\ sec$	$\boldsymbol{K} = \frac{\mathrm{G\,T_a + Q}}{\mathrm{G}} = \frac{(3.2577 * 20) + 1300}{3.2577} = 419.0545\ °C$

By applying above equation to obtain, the calculation of $T(\mathrm{t})$

	$\boldsymbol{T}_H(t_0)$ $(°C)$	$\boldsymbol{t}_{on}^{m}$ (sec)	$\boldsymbol{\tau}$ (sec)	$\boldsymbol{K}$ $(°C)$	$\boldsymbol{T}(\mathrm{t})$ $(°C)$
Scenario 1 (natural response)	20	42.42	316.6738	0	17.49
Scenario 2 (forced response)	20	42.42	316.6738	419.0545	70.03

- ***Calculation of temperatures*** $(\boldsymbol{T}_{low})$ ***and*** $(\boldsymbol{T}_{high})$

$$T_{low} = T_{high}\, \boldsymbol{e}^{-\frac{t_{off}^{m}}{C/G}} + K\left(1 - \boldsymbol{e}^{-\frac{t_{off}^{m}}{C/G}}\right) \quad (4.6)$$

In this case the heater is (*OFF*) thus, $Q = 0\ W \quad \rightarrow \quad K = T_a$

T_{high} $(°C)$	T_a $(°C)$	t_{off}^{m} (sec)	G $(W/°C)$	C $(J/°C)$	K $(°C)$
140	20	128.4	3.2577	1031.63	20

$\boldsymbol{T}_{low} = 100.0001\ °C$

$$T_{high} = T_{low}\, \boldsymbol{e}^{-\frac{t_{on}^{m}}{C/G}} + K\left(1 - \boldsymbol{e}^{-\frac{t_{on}^{m}}{C/G}}\right) \quad (4.7)$$

In this case the heater is (*ON*) thus, $Q = 1300\ W \quad \rightarrow \quad K = T_a + \frac{Q}{G}$

T_{low} $(°C)$	T_a $(°C)$	t_{on}^{m} (sec)	G $(W/°C)$	C $(J/°C)$	K $(°C)$
100	20	42.42	3.2577	1031.63	419.0545

$\boldsymbol{T}_{high} = 139.9999\ °C$

Set-point $(\Delta T) = T_s = \frac{T_{high} + T_{low}}{2} = \frac{100 + 140}{2} = 120\ °\mathrm{C}$

Thus, the Dead band value (D_b) is

$$D_b = \frac{T_{high} - T_{low}}{2} = \frac{140 - 100}{2} = 20\ °\mathrm{C}$$

Therefore,

$$T_{high} = T_s + D_b = 140\ °C$$

$$T_{low} = T_s - D_b = 100\ °C$$

Calculation of Time duration (t_{off}), (t_{on}) and ($t_{on}^{start-up}$)

1- Linear Equations

$$\boldsymbol{t}_{off}^{m} = \frac{C(T_{high} - T_{low})}{G(T_s - T_a)} \quad (4.8)$$

$$\boldsymbol{t}_{on}^{m} = \frac{C(T_{high} - T_{low})}{G\ (T_a - T_s) + Q_i} \quad (4.9)$$

$$\boldsymbol{t}_{on}^{start-up} = \frac{C(T_{high} - T_{low})}{G\ (T_a - T_s) + Q_i} \quad (4.10)$$

System parameters

$\boldsymbol{T}_{high}$ $(°C)$	$\boldsymbol{T}_{low}$ $(°C)$	$\boldsymbol{T}_a$ $(°C)$	$\boldsymbol{T}_s$ $(°C)$	$\boldsymbol{G}$ $(W/°C)$	$\boldsymbol{C}$ $(J/°C)$	$\boldsymbol{Q}_i$ (W)
140	100	20	120	3.2577	1031.63	1300

Linear Equations Results

P_{rated} (kW)	t_{off}^{m} (sec)	t_{on}^{m} (sec)	$t_{on}^{start-up}$ (sec)
0 / 1.3	126.7	42.36	70.56

$$\boldsymbol{t}_{off} = 126.7\ sec = 2.11\ min = 0.0352\ hr$$

$$\boldsymbol{t}_{on} = 42.36\ sec = 0.706\ min = 0.0118\ hr$$

$$\boldsymbol{t}_{on}^{start-up} = 70.56\ sec = 1.176\ min = 0.0196\ hr$$

2- Exponential Equations

$$\boldsymbol{t}_{off}^{m} = RC\ ln\left(\frac{T_{high} - T_a}{T_{low} - T_a}\right) \quad (4.11)$$

$$\boldsymbol{t}_{on}^{m} = RC\ ln\left(\frac{T_{low} - T_a - R * P_{rated}}{T_{high} - T_a - R * P_{rated}}\right) \quad (4.12)$$

$$\boldsymbol{t}_{on\ (startup)} = RC\ ln\left(\frac{T_{low} - T_a - R * P_{rated}}{T_{high} - T_a - R * P_{rated}}\right) \quad (4.13)$$

Constant System Parameters

$\boldsymbol{T}_{high}$ $(°C)$	$\boldsymbol{T}_{low}$ $(°C)$	$\boldsymbol{T}_a$ $(°C)$	$\boldsymbol{T}_s$ $(°C)$	$\boldsymbol{R}'$ $(°C/W)$	$\boldsymbol{C}$ $(J/°C)$	$\boldsymbol{Q}_i$ (W)
140	100	20	120	0.3070	1031.63	1300

Exponential Equations Results

P_{rated} (kW)	t_{off}^{m} (sec)	t_{on}^{m} (sec)	$t_{on}^{start-up}$ (sec)
0 / 1.3	128.42	42.42	70.85

$\boldsymbol{t_{off}} = 128.42 \text{ sec} = 2.14 \text{ min} = 0.0357\ hr$

$\boldsymbol{t_{on}} = 42.42 \text{ sec} = 0.707 \text{ min} = 0.0118\ hr$

$\boldsymbol{t_{on}^{start-up}} = 70.85 \text{ sec} = 1.18 \text{ min} = 0.0197\ hr$

$$\text{Cycling time} = t_{on} + t_{off} \tag{4.14}$$

$\boldsymbol{Cycling_{time}}$ (hr)	$\boldsymbol{t_{on}}$ (hr)	$\boldsymbol{t_{off}}$ (hr)
0.0475	0.0118	0.0357

$$\boldsymbol{N}_{cycles} = \frac{Run_{time} - \boldsymbol{t}_{on\ (start-up)}}{cycling\ time} \tag{4.15}$$

N cycles	$full\ operation\ period$ (hr)	$\boldsymbol{t}_{on\ (startup)}$ (hr)	$cycling\ time$ (hr)
20.6	1	0.0197	0.0475

$$\boldsymbol{t}_{on(total)} = (\boldsymbol{N}_{cycles} * \boldsymbol{t}_{on}) + \boldsymbol{t}_{on\ (startup)} \tag{4.16}$$

$\boldsymbol{t}_{on(total)}$ (hr)	$\boldsymbol{t}_{on}$ (hr)	$\boldsymbol{N}_{cycles}$	$\boldsymbol{t}_{on\ (start-up)}$ (hr)	$full\ operation\ period$ (hr)
0.2628	0.0118	20.6	0.0197	1
0.1377	0.0118	10.1	0.0197	0.5

Power / Energy consumption of Domestic Clothes Iron (DECI)

$$Q_{on\ (kWh)} = P_{rated} * t_{on}\ (hr) \tag{4.17}$$

Method	Q_{on} (kWh)	P_{rated} (kW)	t_{on} (hr)
Calculation	**0.0153**	1.3	0.0118
Simulation	**0.0155**	1.3	0.0119

$$Q_{total} = P_{rated}\ (kW) * t_{on}\ \text{(total)}\ (hr) \tag{4.18}$$

Q_{total} (kWh)	P_{rated} (kW)	t_{on} (total) (hr)
0.3416	1.3	0.2628

$$\text{DECI (kW)} = \frac{Q_{(total)}(kWh)}{t_{on(total)}(hr)} \tag{4.19}$$

Electric Heater (kW)	Q_{total} (kWh)	t_{on} (total) (hr)
1.3	0.3416	0.2628

$$Duty\ Cycle = (\boldsymbol{t}_{on}\ /\ \boldsymbol{t}_{total}) \tag{4.20}$$

t_{on}^{m} (hr)	$Cycling\ time$ (hr)
0.0118	0.0475

$$Duty\ Cycle = \mathbf{0.2484}$$

The average power consumption by DECI ($\Delta \boldsymbol{P}_{\boldsymbol{DECI}}$) from the energy flow can be calculated as follows:

$$\Delta P_{DECI} = P_{rated} * Duty\ Cycle \tag{4.21}$$

$$\Delta P_{DECI} = 1.3\ kW * 0.2484 = 0.3229\ kW$$

The Energy consumption by DECI ($\boldsymbol{EC}_{\boldsymbol{DECI}}$) can be calculated as follows:

$$Energy\ consumption\ (kWh) = \boldsymbol{P}_{rated} * \boldsymbol{t}_{on(total)} \tag{4.22}$$

$$EC_{DECI} = 1.3 * 0.2628 = 0.3416\ kWh$$

APPENDIX C Model of domestic electric baseboard heater DEBH

The aim of this section is to provide, a sample calculation in order to analyze the DEBH system behavior during transient condition and steady-state condition response.

Calculations of DEBH $\{\boldsymbol{T}(\mathrm{t}), \boldsymbol{T}_{low}, \boldsymbol{T}_{high}, \boldsymbol{t}_{off}, \boldsymbol{t}_{on}, t_{on}^{start-up}\}$

Constant parameters of the DEBH:

$Q = P_{rated}$ (W)	T_{high} $(°C)$	T_{low} $(°C)$	$T_{set-point}$ $(°C)$	T_{amb} $(°C)$	t_{on}^{m} (sec)	t_{off}^{m} (sec)
1500	28	18	23	10	300	145.8

t_{off}^{max}, indicates that the maximum duration of t_{off}^{m} and t_{on}^{max} the minimum duration of t_{on}^{m}

By using an approach for identification of physical models of single- zone lumped-parameter thermal model equations (4.1 to 4.7) the results are

$$\boldsymbol{G} = \frac{Qi\,(\Delta - 1)}{\Delta(T_{high} - T_a) - (T_{low} - T_a)} \quad (4.1)$$

$$\boldsymbol{C} = G\left(\frac{t_{off}^{m}}{\ln(T_{high} - T_a) - \ln(T_{low} - T_a)}\right) \quad (4.2)$$

Where,

$$\Delta = \left(\frac{T_{high} - T_a}{T_{low} - T_a}\right)^{t_{on}^{m}/t_{off}^{m}} \quad (4.3)$$

$$G/C = \frac{\ln(T_{high} - T_a) - \ln(T_{low} - T_a)}{t_{off}^{m}} \quad (4.4)$$

By applying above formulas, the results are

Δ	G $(W/°C)$	C $(J/°C)$
5.30463	73.8077	13270.1464

Also, the thermal capacitance can be calculated by $C = \rho C_p V$

ρ (kg/m^3)	C_p $(J/kg.°C)$	V (m^3)	C $(J/°C)$
2700	910	0.0054	13267.8

The ratio of G/C can be calculated directly

G/C (sec^{-1})	$\tau = (G/C)^{-1}$ (sec)
5.5619356×10^{-3}	179.7935

Calculations of DEBH $\{T(\mathrm{t}), T_{low}, T_{high}, t_{off}, t_{on}, t_{on}^{start-up}\}$

$$T(\mathrm{t}) = T_H(t_0)\, e^{-\frac{(t-t_0)}{\tau}} + K\left(1 - e^{-\frac{(t-t_0)}{\tau}}\right) \tag{4.5}$$

$\mathrm{K} = T_a + Q/G$

Calculations of $T_H(t)$, by using an axponential Equation (4.5) Where,

$\tau = \frac{\mathrm{C}}{\mathrm{G}} = \frac{13270.1464}{73.8077} = 179.7935\ sec$	$K = \frac{\mathrm{G\,T_a + Q}}{\mathrm{G}} = \frac{(73.8077*10)+1500}{73.8077} = 30.3231\ °C$

By applying above equation to obtain $T(\mathrm{t})$

	$T_H(t_0)$ $(°C)$	t_{on}^{m} (sec)	τ (sec)	K $(°C)$	$T(\mathrm{t})$ $(°C)$
Scenario 1 (natural response)	10	300	179.7935	0	1.8851
Scenario 2 (forced response)	10	300	179.7935	30.3231	26.49

Calculation of (T_{low}) ***and*** (T_{high})

$$T_{low} = T_{high}\, e^{-\frac{t_{off}^{m}}{C/G}} + K\left(1 - e^{-\frac{t_{off}^{m}}{C/G}}\right) \tag{4.6}$$

In this case the heater is (*OFF*) thus, $Q = 0\ W \quad \rightarrow \quad K = T_a$

T_{high} $(°C)$	T_a $(°C)$	t_{off}^{m} (sec)	G $(W/°C)$	C $(J/°C)$	K $(°C)$
28	10	145.8	73.8077	13270.1464	10

$T_{low} = 18\ °C$

$$T_{high} = T_{low}\, e^{-\frac{t_{on}^{m}}{C/G}} + K\left(1 - e^{-\frac{t_{on}^{m}}{C/G}}\right) \tag{4.7}$$

In this case the heater is (*ON*) thus, $Q = 1500\ W \quad \rightarrow \quad K = T_a + \frac{Q}{G}$

T_{low} $(°C)$	T_a $(°C)$	t_{on}^{m} (sec)	G $(W/°C)$	C $(J/°C)$	K $(°C)$
18	10	300	73.8077	13270.1464	30.3231

$T_{high} = 28\ °C$

Set-point (ΔT) $= \ T_s = \frac{T_{high} + T_{low}}{2} = \frac{28 + 18}{2} = 23\ ℃$

Thus, the Dead band value (D_b) is:

$$D_b \ = \ \frac{T_{high} - T_{low}}{2} = \frac{28 - 18}{2} = 5\ ℃$$

Therefore,

$$T_{high} = T_s + D_b = 28\ ℃$$

$$T_{low} = T_s - D_b = 18\ ℃$$

Calculation of Time: (t_{off}), (t_{on}) ***and*** $(t_{on}^{start-up})$

1- Linear Equations

$$\boldsymbol{t}_{off}^{m} = \frac{C(T_{high} - T_{low})}{G(T_s - T_a)} \quad (4.8)$$

$$\boldsymbol{t}_{on}^{m} = \frac{C(T_{high} - T_{low})}{G\ (T_a - T_s) + Q_i} \quad (4.9)$$

$$\boldsymbol{t}_{on}^{start-up} = \frac{C(T_{high} - T_{low})}{G\ (T_a - T_s) + Q_i} \quad (4.10)$$

System parameters

T_{high} (°C)	T_{low} (°C)	T_a (°C)	T_s (°C)	G (W/°C)	C (J/°C)	Q_i (W)
28	18	10	23	73.8077	13270.1464	1500

Linear Equations Results

P_{rated} (kW)	t_{off}^{m} (sec)	t_{on}^{m} (sec)	$t_{on}^{start-up}$ (sec)
0 / 1.5	138.30	245.52	88.12

$\boldsymbol{t_{off}} = 138.30\ \text{sec} = 2.305\ \text{min} = 0.0384\ hr$

$\boldsymbol{t_{on}} = 245.52\ \text{sec} = 4.092\ \text{min} = 0.0682\ hr$

$\boldsymbol{t_{on\ (startup)}} = 88.12\ \text{sec} = 1.47\ \text{min} = 0.0245\ hr$

1- Exponential Equations

$$\boldsymbol{t}_{off}^{m} = RC\ ln\left(\frac{T_{high} - T_a}{T_{low} - T_a}\right) \quad (4.11)$$

$$\boldsymbol{t}_{on}^{m} = RC\ ln\left(\frac{T_{low} - T_a - R * P_{rated}}{T_{high} - T_a - R * P_{rated}}\right) \quad (4.12)$$

$$\boldsymbol{t}_{on\ (startup)} = RC\ ln\left(\frac{T_{low} - T_a - R * P_{rated}}{T_{high} - T_a - R * P_{rated}}\right) \quad (4.13)$$

System parameters

T_{high} (°C)	T_{low} (°C)	T_a (°C)	T_s (°C)	R' (°C/W)	C (J/°C)	Q_i (W)
28	18	10	23	0.0135487	13270.1464	1500

Exponential Equations Results

P_{rated} (kW)	t_{off}^{m} (sec)	t_{on}^{m} (sec)	$t_{on}^{start-up}$ (sec)
0 / 1.5	145.80	300	89.95

$\boldsymbol{t_{off}} = 145.80\ \text{sec} = 2.43\ \text{min} = 0.0405\ hr$

$\boldsymbol{t_{on}} = 300\ \text{sec} = 5\ \text{min} = 0.0833\ hr$

$\boldsymbol{t_{on\ (startup)}} = 89.95\ \text{sec} = 1.50\ \text{min} = 0.025\ hr$

$$\text{Cycling time} = t_{on} + t_{off} \quad (4.14)$$

Cycling time (hr)	$\boldsymbol{t_{on}}$ (hr)	$\boldsymbol{t_{off}}$ (hr)
0.1238	0.0833	0.0405

$$\boldsymbol{N}_{cycles} = \frac{Run_{time} - \boldsymbol{t}_{on\ (start-up)}}{cycling\ time} \quad (4.15)$$

N cycles	Run_{time} (hr)	$\boldsymbol{t}_{on\ (startup)}$ (hr)	*cycling time* (hr)
7.9	1	0.025	0.1238
40.2	5	0.025	0.1238

$$\boldsymbol{t}_{on(total)} = (\boldsymbol{N}_{cycles} * \boldsymbol{t}_{on}) + \boldsymbol{t}_{on\ (startup)} \quad (4.16)$$

$\boldsymbol{t}_{on(total)}$ (hr)	$\boldsymbol{t}_{on}$ (hr)	$\boldsymbol{N}_{cycles}$	$\boldsymbol{t}_{on\ (start-up)}$ (hr)	*full operation period* (hr)
0.6831	0.0833	7.9	0.025	1
3.374	0.0833	40.2	0.025	5

Power / Energy Consumption of (DEBH):

$$Q_{on\ (kWh)} = P_{rated} * t_{on}\ (hr) \quad (4.17)$$

Method	Q_{on} (kWh)	P_{rated} (kW)	t_{on} (hr)
Calculation	**0.1249**	1.5	0.0833
Simulation	**0.1242**	1.5	0.0828

$$\boldsymbol{Q}_{total} = P_{rated}\ (kW) * \boldsymbol{t}_{on(total)}\ (hr) \quad (4.18)$$

Q_{total} (kWh)	P_{rated} (kW)	$t_{on(total)}$ (hr)	*Run-time* (hr)
1.0247	1.5	0.6831	1
5.0604	1.5	3.3736	5

$$\text{DEBH (kW)} = \boldsymbol{Q}_{total} \ / \ \boldsymbol{t}_{\boldsymbol{on}\,(total)} \qquad (4.19)$$

Electric Heater (kW)	Q_{total} (kWh)	$t_{on(total)}$ (hr)	Run-time (hr)
1.5	1.0247	0.6831	1
1.5	5.0604	3.3736	5

The average power consumption (ΔP) from the energy flow and the energy consumption per operation cycle (EC) of the (DEBH) can be calculated by using Duty Cycle value and input rated power as follows:

$$Duty\ Cycle = (\boldsymbol{t}_{on} \ / \ Cycling_{time}) \qquad (4.20)$$

$Duty\ Cycle$	t_{on}^{m} (hr)	$Cycling_{time}$ (hr)
0.6729	0.0833	0.1238

$$\Delta P_{DEBH} = P_{rated} * \ Duty\ Cycle \qquad (4.21)$$

$$\Delta P_{DEBH} = 1.5\ kW * \ 0.6729 = 1.0094\ kW$$

The Energy consumption by DEBH (EC_{DEBH}) can be calculated as follows:

$$Energy\ consumption\ (kWh) = \boldsymbol{P}_{rated} * \ \boldsymbol{t}_{on(total)} \qquad (4.22)$$

$$EC_{DEBH} = 1.5 * \ 0.6831 = 1.0247\ kWh$$

APPENDIX D Model of a 1-phase induction motor of DECW

Appendix **D** has all specifications and calculations of equations circuit of a single-phase induction motor of clothes washer DECW. In this model of DECW an induction motor single-phase capacitor start is implemented, the parameters of the motor are

Table 5.16 1-phase induction motor capacitor start specifications of DECW

Volt (v)	Frequency (Hz)	Stator		Rotor		X_m (Ω)	slip	pole
		R_1 (Ω)	X_1 (Ω)	R_2 (Ω)	X_2 (Ω)			
120	60	4.20	1.0305	2.47	0.7962	58.1779	0.05	6

According to [18], the basic concept of the circuit model of the single-phase induction motor and its equivalent circuit as following:

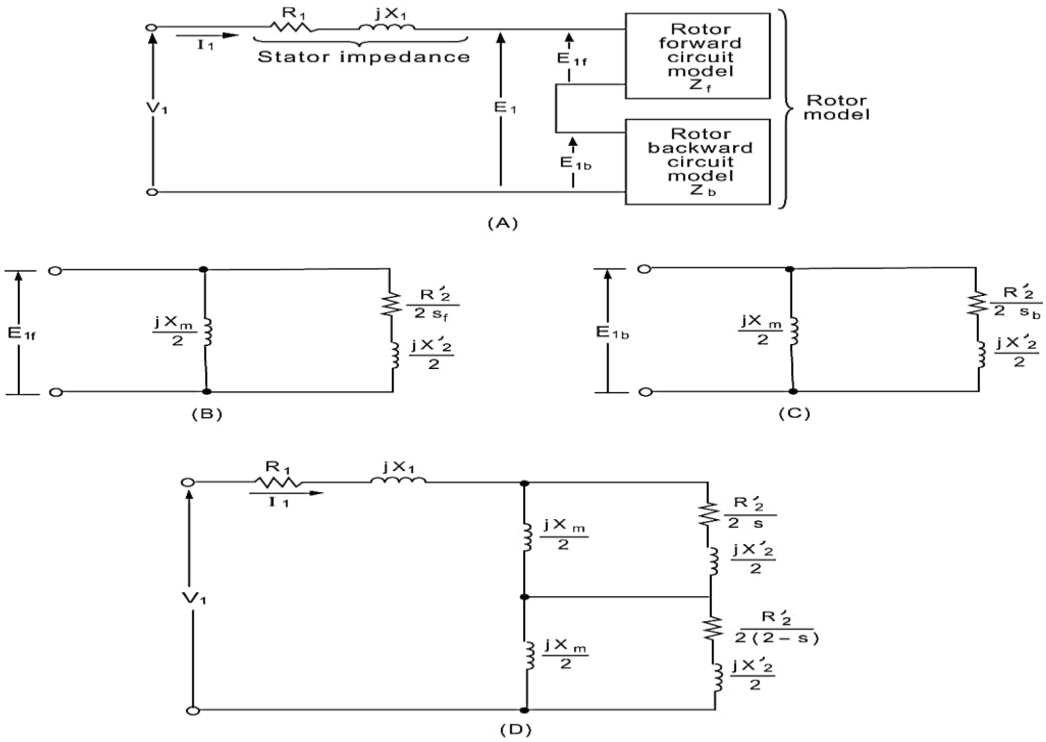

Figure 5.34 Developing an equivalent circuit of for single-phase induction motors [18]

Where,

(A) : basic concept

(B) : forward model

(C) : backward model

(D) : complete equivalent circuit

Calculations of a single-phase induction motor capacitor-start

This calculation is based on [18] "Electric Energy Systems by El-Hawary 2000"

- **The forward impedance (Z_f)**

$$\boldsymbol{Z_f} = (j\,\tfrac{X_m}{2})\ \|\ (\tfrac{R_2'}{2s} + j\,\tfrac{X_2'}{2}) \tag{5.1}$$

$$\boldsymbol{Z_f} = \frac{\boldsymbol{j(X_m/2)\,[(R_2'/2s) + j(X_2'/2)]}}{\boldsymbol{(R_2'/2s) + j[(X_m + X_2')/2]}}$$

$$Z_f = \frac{j29.0889\ (24.7 + j0.3981)}{(24.7 + j29.4871)} = 18.6815\angle 40.8748°$$

$$Z_f = 14.1259 + j12.2253\ \Omega\ = \boldsymbol{R_f} + \ j\boldsymbol{X_f}$$

- **The backward impedance (Z_b)**

$$\boldsymbol{Z_b} = (j\,\tfrac{X_m}{2})\ \|\ (\tfrac{R_2'}{2(2-s)} + j\,\tfrac{X_2'}{2}) \tag{5.2}$$

$$\boldsymbol{Z_b} = \frac{\boldsymbol{j(X_m/2)\,[(R_2'/2(2-s)) + j(X_2'/2)]}}{\boldsymbol{(R_2'/2(2-s)) + j[(X_m + X_2')/2]}}$$

$$Z_b = \frac{j29.0889\ (0.6333 + j0.3981)}{(0.6333 + j29.4871)} = 0.7378\angle 33.3843°$$

$$Z_b = 0.6160 + j0.4059\ \Omega\ = \boldsymbol{R_b} + \ j\boldsymbol{X_b}$$

As a result, $Z_f > Z_b$ the forward impedance $|Z_f|$ is much larger than the Backward impedance $|Z_b|$.

- **The input impedance $|Z_i|$ is the sum of all impedances:**

$$Z_i = Z_1 + Z_f + Z_b = 18.9419 + j13.6617\ \boldsymbol{\Omega}$$

$$Z_i = 23.3546\angle 35.8007°$$

- **The input power to the stator P_i is giving by**

$$P_i = V_1 I_1 \cos \Phi_1 \tag{5.3}$$

$$I_1 = \frac{V_1}{Z_i} = \frac{120\angle 0°}{23.3546\angle 35.8007°} = 5.1382\angle -35.8007°$$

$$\text{Power Factor} = \cos \Phi_1 = \cos\ (35.8007°) = 0.8111\ Lagging$$

$$P_i = (120)(5.1382)(0.8111) = 500.1113\ W$$

- **Stator ohmic losses P_{l_s}**

$$P_{l_s} = |I_1|^2\, R_1 \tag{5.4}$$

$$P_{l_s} = (5.1382)^2(4.20) = 110.8846\, W$$

- **The air-gap power (P_g) is given by:**

$$P_g = P_i - P_{l_s} \tag{5.5}$$

$$P_g = 500.1113 - 110.8846 = 389.2267\, W$$

Also we can calculate P_g by sum of the forward and backward air-gap powers as follows:

$$P_g = P_{gf} + P_{gb} \tag{5.6}$$

$$P_{gf} = |I_1|^2 R_f \tag{5.7}$$

$$P_{gf} = (5.1382)^2(14.1259) = 372.9393\, W$$

$$P_{gb} = |I_1|^2 R_b \tag{5.8}$$

$$P_{gb} = (5.1382)^2(0.6160) = 16.2631\, W$$

$$P_g = P_{gf} + P_{gb} = 372.9393 + 16.2631 = 389.2024\, W$$

- **The static ohmic losses in the rotor circuit:**

. The forward rotor losses

$$P_{l\,rf} = s\,P_{gf} \tag{5.9}$$

$$P_{l\,rf} = (0.05)(372.9393) = 18.6469\, W$$

. The backward rotor losses

$$P_{l\,rb} = (2 - s)P_{gb} \tag{5.10}$$

$$P_{l\,rb} = (1.95)(16.2631) = 31.7130\, W$$

. The sum of the losses

$$P_L = P_{l_s} + P_{lrf} + P_{l\,rb}$$

$$P_L = 110.8846 + 18.6469 + 31.7130 = 161.2445\, W$$

- **The power converted from electrical form to mechanical form is the output power which is given by**

$$P_m = (1 - s)(P_{gf} - P_{gb}) \quad (5.11)$$

$$P_m = (0.95)(372.9393 - 16.2631) = \mathbf{338.8424}\ W$$

Since, we have (P_{gf}) and (P_{gb}) in terms of current (I_1), the resistance of the forward circuit (R_f) and the resistance of backward circuit (R_b) therefore; we can calculate mechanical output power (P_m) as follows:

$$P_m = |I_1|^2\, R_f(1 - s) + |I_1|^2\, R_b[1 - (2 - s)] \quad (5.12)$$

$$P_m = |I_1|^2\ (R_f - R_b)(1 - s)$$

$$P_m = (5.1382)^2(14.1259 - 0.6160)(1 - 0.05) = \mathbf{338.8424}\ W$$

It is worth mentioning that, the sum of output power (P_m) and the losses (P_L) should match the input power (P_i)

$$P_m + P_L = 338.8424 + 161.2445 = \mathbf{500.0869}\ \text{W}$$

- **The speed of the machine, synchronous speed (n_{sync}) and rotor speed (n_r) as following**

  Since the motor has six-poles, 60 Hz

$$(n_{sync}) = \frac{120\, f}{P} = \frac{120 * 60}{6} = 1200\ r/min$$

$$(n_r) = (1 - s)\, n_{sync} = (1 - 0.05)\ (1200) = 1140\ r/min$$

- **The output torque is given by**

$$T_m = \frac{1}{\omega_{sync}}\ (P_{gf} - P_{gb}) \quad (5.13)$$

$$\omega_{sync} = \frac{2\pi\, n_{sync}}{60} = \frac{2\pi\ (1200)}{60} = 125.66\ \text{rad/sec}$$

$$T_m = \frac{1}{125.66}\ (372.9393 - 16.2631) = 2.8384\ \text{N.m}$$

- **The efficiency of the motor is:**

$$\eta = \frac{P_m}{P_i} = \frac{420.3883}{522.6802} = \mathbf{0.6775}$$

APPENDIX E Model of a 1-phase induction motor of DECD

Appendix **E** has all specifications and calculations of equations circuit of a single-phase induction motor of clothes dryer DECD. In this model of DECD an induction motor single-phase capacitor start is implemented, the parameters of the motor are

Table 5.17 1-phase induction motor capacitor start specifications of DECD

Volt	Frequency	Stator		Rotor		X_m	slip	pole	hp
		R_1	X_1	R_2	X_2	(Ω)			
(v)	(Hz)	(Ω)	(Ω)	(Ω)	(Ω)				
120	60	0.4	2.4906	1.9	3.2405	73	0.05	4	0.5

Calculations of a single-phase induction motor capacitor-start

In order to obtain the mathematical model of a single-phase induction motor capacitor-start of DECD with appropriate modification in motor specifications to meet the model requirement, equations of Appendix D (5.1 to 5.5 and 5.9 to 5.13) have been applied as follows:

The impedance for the forward-field circuit (Z_f) and backward-field circuit (Z_b) are given by

- **The forward impedance ($\boldsymbol{Z_f}$)**

$$\boldsymbol{Z_f} = (j\,\frac{\boldsymbol{X_m}}{\boldsymbol{2}}) \parallel (\,\frac{\boldsymbol{R_2'}}{\boldsymbol{2s}} + j\,\frac{\boldsymbol{X_2'}}{\boldsymbol{2}}) \qquad (5.1)$$

$$\boldsymbol{Z_f} = \frac{j36.5\ (19 + j1.6203)}{(19 + j38.1203)} = 16.3411\angle 31.3670°$$

$$\boldsymbol{Z_f} = 13.9529 + j8.5059\ \Omega\ =\ (\boldsymbol{R_f} + \ j\boldsymbol{X_f})$$

- **The backward impedance ($\boldsymbol{Z_b}$)**

$$\boldsymbol{Z_b} = (j\,\frac{\boldsymbol{X_m}}{\boldsymbol{2}}) \parallel (\,\frac{\boldsymbol{R_2'}}{\boldsymbol{2(2-s)}} + j\,\frac{\boldsymbol{X_2'}}{\boldsymbol{2}}) \qquad (5.2)$$

$$\boldsymbol{Z_b} = \frac{j36.5(0.48726 + j1.6203)}{(0.4872 + j38.1203)} = 1.6199\angle 73.9969°$$

$$\boldsymbol{Z_b} = 0.4466 + j1.5571\ \Omega\ =\ (\boldsymbol{R_b} + \ j\boldsymbol{X_b})$$

As a result, $Z_f > Z_b$ the forward impedance $|Z_f|$ is much larger than the backward impedance $|Z_b|$

- **The input impedance $|Z_i|$ is the sum of all impedances**

Total series impedances are

$\boldsymbol{Z_i} = \boldsymbol{Z_1} + \boldsymbol{Z_f} + \boldsymbol{Z_b} = 14.7991 + j12.5536\ \boldsymbol{\Omega}$

$\boldsymbol{Z_i} = 19.4036\angle 40.3069°$

- **The input power to the stator $\boldsymbol{P_i}$ is giving by:**

$$P_i = V_1 I_1 \cos \Phi_1 \tag{5.3}$$

$$I_1 = \frac{V_1}{Z_i} = \frac{120\angle 0°}{19.4063\angle 40.3069°} = 6.1836\angle -40.3069°$$

$\text{Power Factor} = \cos \Phi_1 = \cos\ (40.3069°) = 0.7626\ \ Lagging$

$P_i = (120)(6.1836)(0.7626) = 565.8736\ W$

- **Stator ohmic losses $\boldsymbol{P_{l_s}}$**

$$\boldsymbol{P_{l_s}} = |\boldsymbol{I_1}|^2\ \boldsymbol{R_1} \tag{5.4}$$

$\boldsymbol{P_{l_s}} = (6.1836)^2(0.4) = 15.2948\ W$

- **The air-gap power $\boldsymbol{P_g}$ is given by:**

$$\boldsymbol{P_g} = \boldsymbol{P_i} - \boldsymbol{P_{l_s}} \tag{5.5}$$

$\boldsymbol{P_g} = 565.8736 - 15.2948 = 550.5788\ W$

- **The static ohmic losses in the rotor circuit:**

. The forward rotor losses

$$\boldsymbol{P_{l\,rf}} = \ \boldsymbol{s}\,\boldsymbol{P_{gf}} \tag{5.9}$$

$P_{l\,rf} = (0.05)(533.5158) = 26.6758\ W$

. The backward rotor losses

$$\boldsymbol{P_{l\,rb}} = (\boldsymbol{2} - \boldsymbol{s})\boldsymbol{P_{gb}} \tag{5.10}$$

$P_{l\,rb} = (1.95)(17.0766) = 33.2994\ W$

. The sum of the losses

$\boldsymbol{P_L} = \boldsymbol{P_{l_s}} + \boldsymbol{P_{lrf}} + \boldsymbol{P_{l\,rb}}$

$\boldsymbol{P_L} = 15.2948 + 26.6758\ + 33.2994 = 75.27\ W$

- **The power converted from electrical form to mechanical form is the output power which is given by:**

$$\boldsymbol{P_m} = (\mathbf{1} - \boldsymbol{s})(\boldsymbol{P_{gf}} - \boldsymbol{P_{gb}}) \quad (5.11)$$

$$\boldsymbol{P_m} = (0.95)(533.5158 - 17.0766) = 490.6172\ W$$

Since, we have $(\boldsymbol{P_{gf}})$ and $(\boldsymbol{P_{gb}})$ in terms of current $(\boldsymbol{I_1})$, the resistance of the forward circuit $(\boldsymbol{R_f})$ and the resistance of backward circuit $(\boldsymbol{R_b})$ therefore; we can calculate mechanical output power $(\boldsymbol{P_m})$ as follows:

$$\boldsymbol{P_m} = |\boldsymbol{I_1}|^2\, \boldsymbol{R_f}(\mathbf{1} - \boldsymbol{s}) + |\boldsymbol{I_1}|^2\, \boldsymbol{R_b}[\mathbf{1} - (\mathbf{2} - \boldsymbol{s})] \quad (5.12)$$

$$\boldsymbol{P_m} = |\boldsymbol{I_1}|^2\ (\boldsymbol{R_f} - \boldsymbol{R_b})(\mathbf{1} - \boldsymbol{s})$$

$$\boldsymbol{P_m} = (6.1836)^2(13.9529 - 0.4466)(1 - 0.05) = 490.6390\ W$$

It is worth mentioning that, the sum of output power $(\boldsymbol{P_m})$ and the losses $(\boldsymbol{P_L})$ should match the input power $(\boldsymbol{P_i})$

$\boldsymbol{P_m} + \boldsymbol{P_L}$ = 490.6172 + 75.27 = **565.8872** W

- **The speed of the machine, synchronous speed $(\boldsymbol{n_{sync}})$ and rotor speed $(\boldsymbol{n_r})$ as follows:**

$$(\boldsymbol{n_{sync}}) = \frac{\mathbf{120}\, \boldsymbol{f}}{\boldsymbol{P}} = \frac{120 * 60}{4} = 1800\ r/min$$

$$(\boldsymbol{n_r}) = (\mathbf{1} - \boldsymbol{s})\, \boldsymbol{n_{sync}} = (1 - 0.05)\ (1800) = 1710\ r/min$$

- **The output torque is given by**

$$\boldsymbol{T_m} = \frac{\mathbf{1}}{\boldsymbol{\omega_{sync}}}\ (\boldsymbol{P_{gf}} - \boldsymbol{P_{gb}}) \quad (5.13)$$

$$\boldsymbol{\omega}_{sync} = \frac{2\pi\ n_{sync}}{60} = \frac{2\pi\ (1800)}{60} = 188.5\ \text{rad/sec}$$

$$\boldsymbol{T_m} = \frac{1}{188.5}\ (533.5158 - 17.0766) = 2.7397\ \text{N.m}$$

- **The efficiency of the motor is:**

$$\boldsymbol{\eta} = \frac{P_m}{P_i} = \frac{490.6172}{565.8736} = 0.8670$$

APPENDIX F Model of a 1-phase induction motor of DEDW

Appendix **F** has all specifications and calculations of equations circuit of a single-phase induction motor of dishwasher DEDW. In this model of DEDW the single-phase induction motor is used with the following specifications

Table 5.18 1-phase induction motor capacitor start specifications of DEDW

Volt	Frequency	Stator		Rotor		X_m	slip	pole	hp
		R_1	X_1	R_2	X_2	(Ω)			
(v)	(Hz)	(Ω)	(Ω)	(Ω)	(Ω)				
120	60	0.4	1.2246	1.12	1.2246	58	0.05	2	0.3

Calculations of a single-phase induction motor capacitor-start

In order to obtain the mathematical model of a single-phase induction motor capacitor-start of DEDW with appropriate modification in motor specifications to meet the model requirement, equations of Appendix D (5.1 to 5.5 and 5.9 to 5.13) have been applied as follows:

The impedance for the forward-field circuit (Z_f) and backward-field circuit (Z_b) are given by

- **The forward impedance ($\boldsymbol{Z_f}$)**

$$\boldsymbol{Z_f} = (j\,\frac{\boldsymbol{X_m}}{\boldsymbol{2}})\ \|\ (\frac{\boldsymbol{R_2'}}{\boldsymbol{2s}} + j\,\frac{\boldsymbol{X_2'}}{\boldsymbol{2}}) \tag{5.1}$$

$$\boldsymbol{Z_f} = \frac{j29\ (11.2 + j0.6123)}{(11.2 + j29.6123)} = 10.22745\angle 23.8469°$$

$$\boldsymbol{Z_f} = 9.3973 + j4.1539\ \Omega\ =\ (\boldsymbol{R_f} +\ j\boldsymbol{X_f})$$

- **The backward impedance ($\boldsymbol{Z_b}$)**

$$\boldsymbol{Z_b} = (j\,\frac{\boldsymbol{X_m}}{\boldsymbol{2}})\ \|\ (\frac{\boldsymbol{R_2'}}{\boldsymbol{2(2-s)}} + j\,\frac{\boldsymbol{X_2'}}{\boldsymbol{2}}) \tag{5.2}$$

$$\boldsymbol{Z_b} = \frac{j29(0.2872 + j0.6123)}{(0.2872 + j29.6123)} = 0.6623\angle 65.4267°$$

$$\boldsymbol{Z_b} = 0.2754 + j0.6023\ \Omega\ =\ (\boldsymbol{R_b} +\ j\boldsymbol{X_b})$$

As a result, $Z_f > Z_b$ the forward impedance $|Z_f|$ is much larger than the backward impedance $|Z_b|$

- **The input impedance $|Z_i|$ is the sum of all impedances**

Total series impedances are

$\boldsymbol{Z_i} = \boldsymbol{Z_1} + \boldsymbol{Z_f} + \boldsymbol{Z_b}$

$\boldsymbol{Z_i} = 9.9727 + j5.6793\ \boldsymbol{\Omega}$

$\boldsymbol{Z_i} = 11.4765\angle 29.6609°$

- **The input power to the stator $\boldsymbol{P_i}$ is giving by:**

$$P_i = V_1 I_1 \cos \Phi_1 \tag{5.3}$$

$$I_1 = \frac{V_1}{Z_i} = \frac{120\angle 0°}{11.4765\angle 29.6609°} = 10.4561\angle -29.6609°$$

$$\text{Power Factor} = \cos \Phi_1 = \cos\ (29.6609°) = 0.8690\ \ Lagging$$

$$P_i = (120)(10.4561)(0.8690) = 1090.3621\ W$$

- **Stator ohmic losses $\boldsymbol{P_{l_s}}$**

$$\boldsymbol{P_{l_s}} = |\boldsymbol{I_1}|^2\ \boldsymbol{R_1} \tag{5.4}$$

$$\boldsymbol{P_{l_s}} = (10.4561)^2(0.3) = 32.799\ W$$

- **The air-gap power $\boldsymbol{P_g}$ is given by:**

$$\boldsymbol{P_g} = \boldsymbol{P_i} - \boldsymbol{P_{l_s}} \tag{5.5}$$

$$P_g = 1090.3621 - 32.799 = 1057.5631\ W$$

- **The static ohmic losses in the rotor circuit:**

. The forward rotor losses

$$\boldsymbol{P_{l\,rf}} = \ \boldsymbol{s\,P_{gf}} \tag{5.9}$$

$$P_{l\,rf} = (0.05)(1027.4071) = 51.3704\ W$$

. The backward rotor losses

$$\boldsymbol{P_{l\,rb}} = (\boldsymbol{2} - \boldsymbol{s})\boldsymbol{P_{gb}} \tag{5.10}$$

$$P_{l\,rb} = (2 - 0.05)(30.1095) = 58.7135\ W$$

. The sum of the losses

$$\boldsymbol{P_L} = \boldsymbol{P_{l_s}} + \boldsymbol{P_{lrf}} + \boldsymbol{P_{l\,rb}}$$

$$\boldsymbol{P_L} = 32.799 + 51.3704\ + 58.7135 = 142.8829\ W$$

- **The power converted from electrical form to mechanical form is the output power which is given by**

$$\boldsymbol{P_m} = (\boldsymbol{1} - \boldsymbol{s})(\boldsymbol{P_{gf}} - \boldsymbol{P_{gb}}) \qquad (5.11)$$

$$\boldsymbol{P_m} = (1 - 0.05)(1027.4071 - 30.1095) = 947.4327\, W$$

Since, we have $(\boldsymbol{P_{gf}})$ and $(\boldsymbol{P_{gb}})$ in terms of current $(\boldsymbol{I}_1)$**,** the resistance of the forward circuit $(\boldsymbol{R}_f)$ and the resistance of backward circuit $(\boldsymbol{R}_b)$ therefore; we can calculate mechanical output power $(\boldsymbol{P_m})$ as following

$$\boldsymbol{P_m} = |\boldsymbol{I_1}|^2\, \boldsymbol{R}_f(\boldsymbol{1} - \boldsymbol{s}) + |\boldsymbol{I_1}|^2\, \boldsymbol{R}_b[\boldsymbol{1} - (\boldsymbol{2} - \boldsymbol{s})] \qquad (5.12)$$

$$\boldsymbol{P_m} = |\boldsymbol{I_1}|^2\, (\boldsymbol{R}_f - \boldsymbol{R}_b)(\boldsymbol{1} - \boldsymbol{s})$$

$$\boldsymbol{P_m} = (10.4561)^2\, (9.3973 - 0.2872)(1 - 0.05)$$

$$\boldsymbol{P_m} = 947.4327\, W$$

It is worth mentioning that, the sum of output power $(\boldsymbol{P_m})$ and the losses $(\boldsymbol{P_L})$ should match the input power $(\boldsymbol{P_i})$

$\boldsymbol{P_m} + \boldsymbol{P_L}$ = 947.4327 + 142.8829 = **1090.3156** W

- **The speed of the machine that has (2 poles); the synchronous speed $(\boldsymbol{n_{sync}})$ and rotor speed $(\boldsymbol{n_r})$ as follows:**

$$(\boldsymbol{n_{sync}}) = \frac{\boldsymbol{120\, f}}{\boldsymbol{P}} = \frac{120 * 60}{2} = 3600\, rpm$$

$$(\boldsymbol{n_r}) = (\boldsymbol{1} - \boldsymbol{s})\, \boldsymbol{n_{sync}} = (1 - 0.05)\,(3600) = 3420\, rpm$$

- **The output torque is given by**

$$\boldsymbol{T_m} = \frac{\boldsymbol{1}}{\boldsymbol{\omega_{sync}}}\, (\boldsymbol{P_{gf}} - \boldsymbol{P_{gb}}) \qquad (5.13)$$

$$\boldsymbol{\omega}_{sync} = \frac{2\pi\, n_{sync}}{60} = \frac{2\pi\, (3600)}{60} = 376.8\ \text{rad/sec}$$

$$\boldsymbol{T_m} = \frac{1}{376.8}\, (1027.4071 - 30.1095) = 2.6468\ \text{N.m}$$

- **The efficiency of the machine is:**

$$\boldsymbol{\eta} = \frac{P_m}{P_i} = \frac{947.4327}{1090.3621} = 0.8689$$