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Persistent currents and dissipation in narrow bilayer quantum Hall bars

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Bilayer quantum Hall states support a flow of nearly dissipationless staggered current which can only decay through collective channels. We study the dominant finite-temperature dissipation mechanism which in narrow bars is driven by thermal nucleation of pseudospin solitons. We find the finite-temperature resistivity, predict the resulting staggered current-voltage characteristics, and calculate the associated zero-temperature critical staggered current and gate voltage.

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A 2D electron gas bilayer, subjected to a strong perpendicular magnetic field, can exhibit incompressible quantum Hall (QH) states even for filling fractions corresponding to compressible states of noninteracting layers.[1] The nontrivial, strongly interacting nature of these QH states lies in the fact that they survive the limit of vanishing interlayer tunneling,[2] which to many striking predictions and observations.[3] They are stabilized by the exchange part of the Coulomb interaction, which, in the limit of vanishing single-particle tunneling, sets the scale of the gap and leads to macroscopic interlayer phase coherence.

In addition to exhibiting the QHE for a uniform current, these states support persistent currents that are counterpropagating in the two layers with $J = J_{\text{top}} - J_{\text{bottom}}$. In this Rapid Communication, we study a thermally-driven decay mechanism of J which controls the current-voltage characteristics for staggered currents smaller than the critical current J_c. Because the bilayer system displays a quantum Hall gap Δ in the phase-coherent ground state, dissipation via single-particle mechanisms is strongly suppressed for $k_B T \ll \Delta$. Therefore, as with supercurrents in superconductors,[4] the staggered-current decay rate is dominated, for a range of parameters, by the collective mechanism of soliton nucleation.

A convenient language for describing this strongly correlated quantum-coherent gapped state is in terms of a pseudospin unit vector field $\hat{m}(\vec{r}) = (\vec{m}_\perp, m_z)$,[2] with $m_z = \cos\theta = n_{\text{top}} - n_{\text{bottom}}$ giving the electron charge-density difference between top and bottom layers and $\vec{m}_\perp = \sin\theta(\cos\phi, \sin\phi)$ characterizing the relative phase $\phi = \phi_{\text{top}} - \phi_{\text{bottom}}$ of electrons in two layers. The energy functional describing long length scale (larger than the magnetic length $\ell = \sqrt{\hbar c/Be}$) variations of $\hat{m}(\vec{r})$ is given by[2]

$$H = \int d^2 r \left[\frac{\rho_s^\perp}{2} |\nabla \vec{m}_\perp|^2 + \frac{\rho_s^z}{2} |\nabla m_z|^2 + \beta m_z^2 - \vec{h}\cdot\hat{m} \right], \quad (1)$$

where electron Coulomb interaction is the origin of the effective exchange constants $\rho_s^{z,\perp}$ that drive the transition into the pseudoferromagnetic ground state, corresponding to the interlayer phase-coherent QH state. The electrostatic capacitive energy β introduces a hard-axis anisotropy, which forces the pseudomagnetization to lie in the $\perp$ plane ($m_z = 0$) and thereby reduces the full SU(2) pseudospin symmetry to U(1).[2,5] A combination of the external gate voltage V_g and the single-electron interlayer tunneling t acts as an external pseudomagnetic field $\vec{h} = (t/2\pi\ell^2)\hat{x} + V_g\hat{z}$. Because the tunneling t can be tuned independently of β to be quite small, the low-energy physics of this anisotropic QH pseudoferromagnet, described by the Goldstone mode $\hat{m}$, can be fully explored experimentally.

In the limit of vanishing tunneling t, an essentially exact analytical treatment of narrow (1D limit) QH bars is possible and leads to the following results. The bilayer QH phase exhibits staggered current-carrying states that are metastable and therefore it supports staggered persistent currents for $J < J_c(V_g)$, where the critical current density is given by

$$J_c(V_g) = J_c^0 q_*(v)(1 - v^2/[1 - q_*^2(v)]^2) \quad (2a)$$

$$\xrightarrow{v\to 0} J_c^0[1 - (v/4)^{2/3}], \quad (2b)$$

where $J_c^0 = 2\sqrt{2\beta\rho_s^\perp}/\hbar$ is the critical current at zero gate voltage, which vanishes in the SU(2) invariant $\beta \to 0$ limit,[6] $v = V_g/2\beta$ is a reduced measure of the gate voltage V_g, and $q_*^2(v) = 1 - v^{2/3}[(\sqrt{4+v^2}+2)^{1/3} - (\sqrt{4+v^2}-2)^{1/3}]$. At a finite temperature, $J_c(V_g)$, plotted in Fig. 1, therefore delineates a low resistivity regime, where Ohmic dissipation is dominated by slow thermal soliton nucleation, from a highly resistive state dominated by quasiparticle dissipation.

For $J < J_c(V_g)$, the staggered IV characteristics are plotted in Fig. 2 and given by

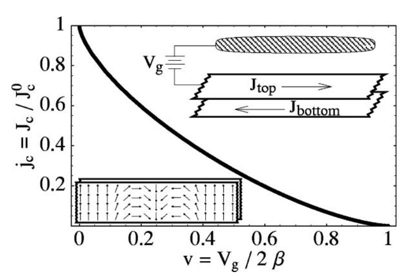

FIG. 1. Main Plot: Critical staggered current j_c as a function of gate voltage v. Upper Inset: Sample geometry considered in this work. Lower Inset: Pseudospin configuration of the 2π-line soliton whose nucleation leads to staggered-current decay.

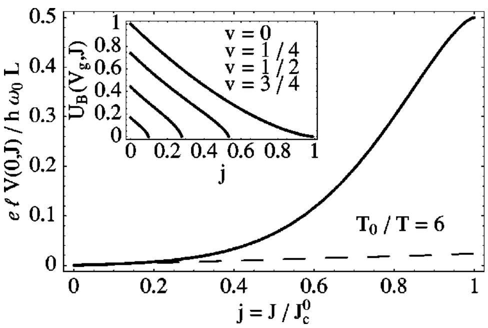

FIG. 2. Main Plot: IV curve for $J<J_c(V_g)$, $\rho_s^z=\rho_s^\perp$, $V_g=0$ and $T_0/T=6$. Inset: Energy barrier for various gate voltages $v=V_g/2\beta$, with the barrier vanishing for $v\to 1$. Qualitatively similar curves are obtained for the full range $0\leqslant\rho_s^z/\rho_s^\perp\leqslant 1$.

$$V(V_g,J)=\frac{h\omega_0 L}{e\ell}\sinh(\pi j T_0/2T)e^{-(U_B+\pi j/2)T_0/T}, \quad (3)$$

where $\omega_0=\sqrt{2\pi\ell^2 t\beta}/\hbar$ is the microscopic attempt frequency, $j=J/J_c^0$ is the reduced (dimensionless) current density, $k_BT_0=\hbar J_c^0 L_y$, with L_y the narrow sample dimension $\ll L_x\equiv L$, and $E_B=k_BT_0U_B$ is the saddle-point energy barrier separating two different current-carrying states. The barrier is plotted in the inset of Fig. 2 and is explicitly given by

$$U_B(V_g,J)=\int_{m_1}^{m_0}dm\left\{\left(1+\frac{\rho_s^z}{\rho_s^\perp}\frac{m^2}{1-m^2}\right)\left[(m_0^2-m^2)\times\left(1-\left(\frac{qm_0}{m}\right)^2\right)+2v(\sqrt{1-m_0^2}-\sqrt{1-m^2})\right]\right\}^{1/2}, \quad (4)$$

where the limits of integration are $m_0^2=j/q$ and $m_1^2=q^2[m_0^2+2\overline{m}^2(1+\sqrt{1+m_0^2/\overline{m}^2})]$, with $\overline{m}^2=(1-m_0^2)(1-q^2)$, and the dimensionless wave vector q is defined implicitly through the current $j=q[1-v^2/(1-q^2)^2]$. The analytic expression for the barrier simplifies considerably when one of its arguments vanishes. We find that $U_B(V_g,J)$ depends weakly on $\varepsilon\equiv 1-(\rho_s^z/\rho_s^\perp)$ and for $\varepsilon\to 1$ is given by

$$U_B(0,J)=\frac{\pi}{4}(1-j)^2, \quad (5a)$$

$$U_B(V_g,0)=\tfrac{1}{2}[\sin^{-1}(\sqrt{1-v^2})-v\sqrt{1-v^2}], \quad (5b)$$

and, for $0\leqslant\epsilon\leqslant 1$,

$$U_B(0,0)=\frac{1}{2}\sqrt{1-\varepsilon}-\frac{1}{4\sqrt{\varepsilon}}\left[\frac{\pi}{2}-\sin^{-1}(1-2\varepsilon)\right]. \quad (5c)$$

For such narrow Hall bars the *staggered* linear resistivity ρ is always finite at finite temperature and is given by

$$\rho(V_g)=\frac{V}{LJe}\overset{J\to 0}{\longrightarrow}\frac{h^2\omega_0 L_y}{2k_BTe^2\ell}e^{-U_B(V_g,0)T_0/T}. \quad (6)$$

From exact diagonalization studies[2] at $d/\ell=1/2$, we have $\beta=5.1\times10^{-3}e^2/\ell^3\epsilon$ and $\rho_s^\perp=1.5\times10^{-2}e^2/\ell\epsilon$. Taking in addition $\ell=20$ nm, $T=1$ K, dielectric constant $\epsilon=13.2$, $t=0.1$ meV, $V_g=0$, and $\varepsilon=0$, we obtain

$$\rho(0)\approx 10^5 N(0.2)^N\Omega, \quad (7)$$

where $N=L_y/\ell$. Setting $N=5$, 10, 15 respectively gives $\rho(0)=245\ \Omega$, 0.2 Ω, and 115 $\mu\Omega$.

For a realistic system, there is a limited range of validity of the above results, with other effects dominating outside of this range. The constraint of quasiequilibrium, which implies low decay rate, together with the requirement that the thermal collective dissipation mechanism dominates over single-particle current decay requires $k_BT<E_B<\Delta$. At the same time, however, T must be sufficiently high so that thermal nucleation dominates over quantum tunneling of phase slips.[7] Furthermore, in order for the bulk nucleation rate to be experimentally observable, it is necessary that it dominates over phase slips nucleated at surfaces, contacts, and sample inhomogeneities. Since the bulk nucleation rate scales with the Hall bar length L, we expect that the bulk mechanism dominates over surface nucleation for $L\gg\ell$. Also, for the staggered current decay rate to be dominated by the 1D *line* solitons studied here, the saddle-point energy barrier given in Eq. (4) must be lower than barriers for the competing mechanism of $\pm$ vortex pair nucleation. For a sufficiently wide Hall bar, the latter scenario will dominate, with the crossover occurring for $L_y\approx\xi=\sqrt{\rho_s^\perp/\beta}$.

We now present the highlights of calculations that lead to these results. Although quite distinct in detail, the spirit of our analysis follows the classic work of Langer and Ambegaokar.[4]

The Euler–Lagrange (EL) equations for Eq. (1) admit the uniform current-carrying solutions:

$$\phi(k)=kx, \quad (8a)$$

$$m_\perp^2(v,q)=1-v^2/(1-q^2)^2, \quad (8b)$$

where $q^2=k^2\rho_s^\perp/2\beta$ is the dimensionless wave vector, and we have taken our sample to lie in the x–y plane with dimensions $L_x\equiv L\gg L_y$. Equation (8b) is valid in the region $q^2\leqslant 1-v$ and $v=V_g/2\beta\leqslant 1$. (We consider V_g to be non-negative.) The staggered current for this solution is $J=2\rho_s^\perp m_\perp^2 k/\hbar$, or, equivalently, $j=m_\perp^2 q$.

For nonuniform solutions, the EL equations can be combined into a single equation which, after some manipulation, can be written as

$$\tfrac{1}{2}M(\partial_x m_\perp)^2+V_{\rm eff}=E_{\rm eff}, \quad (9)$$

for some constant $E_{\rm eff}$, with

$$M(m_\perp)=\rho_s^\perp+\rho_s^z m_\perp^2/(1-m_\perp^2), \quad (10a)$$

$$V_{\rm eff}(m_\perp)=\beta\left[\frac{j^2}{m_\perp^2}-(1-m_\perp^2)+2v\sqrt{1-m_\perp^2}\right]. \quad (10b)$$

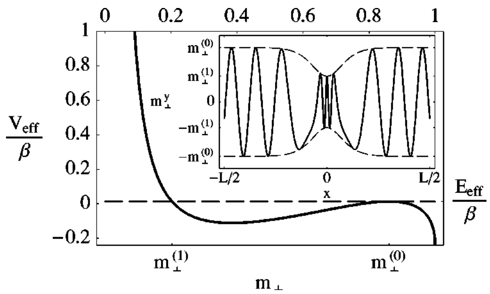

FIG. 3. Main Plot: Effective potential, Eq. (10b), for the mechanical analogy of the EL equations plotted for $v=0.5$ and $j=0.1$. The physically stable configuration is the mechanically unstable point $m_\perp^{(0)}$. Inset: Schematic drawing of the saddle-point (bounce) solution—the dominant contribution to the transition probability.

In the usual mechanical analogy, Eq. (9) represents the energy $E_{\rm eff}$ of a particle at ''position'' $m_\perp$ and ''time'' x, moving in a potential $V_{\rm eff}(m_\perp)$ and with a space-dependent mass $M(m_\perp)$. This potential is plotted in Fig. 3 for $v=0.5$ ($j_c \approx 0.28$) and $j=0.1$.

The ''conservation of energy,'' Eq. (9), immediately implies the existence of two extended solutions.[8] A uniform current-carrying solution is given by Eq. (8), corresponding to the particle forever remaining on top of the hill at $m_\perp^{(0)}$ with angular velocity k. For this solution, $\vec{m}_\perp$ winds azimuthally at a constant rate as a function of x with a constant amplitude $m_\perp^{(0)}$ deviating from the equatorial plane ($m_\perp=1$) with increasing V_g. For fixed winding k, this uniform current-carrying solution is a local minimum of H and is hence metastable. Since the energy is lowered with decreasing k, we expect that at finite temperature the system will thermally activate down to the $k=0$ zero-current ground state via successive thermal transitions $k \rightarrow k-2\pi/L$. To calculate the rate of such transitions we need to compute the saddle-point barrier separating two ''neighboring'' current-carrying metastable states. Brief reflection on the mechanical analogy shows that the second solution is in fact a saddle-point solution corresponding to the particle starting at ''position'' $m_\perp^{(0)}$ and ''time'' $x=-\infty$, spiraling down hill to $m_\perp^{(1)}$ at $x=0$ while conserving angular momentum but increasing its angular velocity $k(m_\perp) \propto 1/m_\perp^2$, and finally bouncing back out to $m_\perp^{(0)}$ for $x=+\infty$. The resulting ''energy conservation'' equation allows an exact determination of the saddle-point solution, written in terms of a 1D integral,

$$x=\frac{1}{\sqrt{2}}\int_{m_\perp^{(1)}}^{m_\perp(x)} dm_\perp \sqrt{M(m_\perp)/[E_{\rm eff}-V_{\rm eff}(m_\perp)]}, \qquad (11)$$

for an appropriate value of $E_{\rm eff}$, and is shown in the inset of Fig. 3.

The saddle-point solution is the nucleation site for the eventual singularity where $m_\perp \rightarrow 0$ ($m^z \rightarrow \pm 1$), and whereby the system can slip a loop reducing the total phase winding $\Delta\phi$ by 2π, and therefore leading to staggered-current decay. Contrary to what is often tacitly assumed, the barrier for defect nucleation is *not* determined by the energy of the defect—which may be either singular, as it is in superconductors, or nonsingular, as it is in the present case of the full SU(2) space—although we expect them to be close in energy. Instead, the barrier is controlled by a *non*-singular saddle-point field configuration (inset of Fig. 3), which is in the same topological sector as the current-carrying metastable minimum. It is also important to note that for sufficiently small β, the energy for such phase slips is much smaller than the quantum Hall gap Δ and that therefore the entire system remains in the fully gapped quantum Hall state throughout the phase-slip process. This is to be contrasted with phase slips inside a superconductor, where the order parameter, and therefore the bulk gap, are suppressed within the vortex core and one in principle has to take into account the core-confined low energy (normal) quasiparticle degrees of freedom.

The energy barrier is defined as the difference in energy between the saddle-point solution, Eq. (11), and the uniform current-carrying solution, Eq. (8). By exploiting the mechanical analogy, and in particular the conservation law, Eq. (9), we obtain the expression quoted in Eq. (4).

In the steady-state regime, the staggered voltage $V(V_g,J)$ is proportional to the net rate of phase slips, $\partial_t \Delta\phi/2\pi$, which itself is the difference between the rate of current-decreasing transitions ($=\omega_0 \exp[-E_B/k_B T]$) and the rate of current-increasing transitions ($=\omega_0 \exp[-(E_B+\pi\hbar J L_y)/k_B T]$). In a sample of length L, there are approximately L/ℓ possible nucleation sites. These considerations lead directly to the expression in Eq. (3).

In contrast to the energy barrier E_B, which is a static quantity, we must consider pseudospin dynamics in order to compute the attempt frequency ω_0 appearing in Eq. (3). Within the microscopic dynamical model valid at low temperatures, $T \ll t$, for finite interlayer tunneling, ω_0^2 is given by the ratio of the curvature of the metastable well ($\sim t/2\pi\ell^2$) and the dynamical mass term ($\sim \hbar^2/4\pi^2\ell^4\beta$), leading to $\hbar\omega_0=\sqrt{2\pi\ell^2 t\beta}$. In contrast, for $T \gg t$, we expect classical Langevin dynamics,[3] characterized by a kinetic ''drag'' coefficient γ, and which in the simplest estimate gives $\omega_0=\beta/\gamma$.

When the transverse (narrow) dimension L_y of the sample becomes sufficiently large, the energy barrier $E_B \approx (\rho_s^\perp \beta)^{1/2} L_y$ for nucleating a line-soliton defect becomes comparable to the energy of nucleating a $\pm$ vortex pair—a competing mechanism for inducing phase slips. Up to weak logarithmic corrections, the energy of such a vortex pair is $E_v \approx \rho_s^\perp = \beta\xi^2$, where $\xi=(\rho_s^\perp/\beta)^{1/2}$ is the core size. Vortex nucleation should therefore dominate the 1D soliton nucleation considered here for $L_y > \xi$, which can be tuned independently of the QH gap Δ. This is in contrast with superconductors, where the corresponding 1D to 2D crossover scale is the Ginzburg–Landau correlation length, controlled by the superconducting gap.

Although it is difficult to extend our exact 1D analysis to the 2D limit, we can estimate the staggered current decay rate using simple scaling arguments. In 2D, the phase slip rate is controlled by $\pm$ vortex pair nucleation, analogously to superfluids and superconductors. However, in contrast to those more familiar systems, here vortices (half-skyrmion, i.e., merons) carry $\pm 1/2$ electromagnetic charge in addition to their $U(1)$ topological charge, and there are therefore four elementary vortex defects $(+1/2,L)$, $(-1/2,L)$, $(+1/2,R)$, and $(-1/2,R)$, with L,R respectively corresponding to $\pm 2\pi$ circulation of ϕ. Correspondingly, within the interlayer phase (ϕ) coherent state, the L and R vortices are bound into two types of topologically neutral pairs: (i) electromagnetically neutral pairs $[(+1/2,L)–(-1/2,R)]$ and (ii) electromagnetically charged pairs $[(+1/2,L)–(+1/2,R)]$.[9] Nevertheless, we do not expect the Coulomb interaction, which is subdominant to the topological-charge confining potential, to play a role in staggered-current-induced vortex ionization processes. Hence, in the limit of vanishing t, standard nucleation analysis for dissipation in superconducting films[10] can be easily extended to our system. It predicts a highly *nonlinear* power-law staggered IV, $E \sim J^{\alpha}$, with $\alpha(T) \geqslant 3$ in the interlayer coherent state, a result that contrasts strikingly with the *linear* staggered IV found in the 1D limit.

Nonvanishing interlayer tunneling, t explicitly breaks $U(1)$ symmetry and leads to nonuniform staggered current-carrying states composed of a lattice of solitons of width $\delta = \ell(2\pi\rho_{\perp}/t)^{1/2}$ and density n akin to a periodic array of Rayleigh–Benard current rolls.[7]

In the dense soliton limit, $n\delta \gg 1$, (relevant for small t and large J) the current $J(x)$ is nearly uniform and our 1D results directly apply. In the dilute limit, $n\delta \ll 1$ (which is always reached for sufficiently low $J > J_{c1} \sim \sqrt{t}$, [see Refs. 6 and 11]), for $\xi \ll \delta$, phase slips are confined to a single soliton, inside which $\partial_x \phi \approx 2\pi/\delta$ is uniform and the tunneling energy is on average zero. Hence, our 1D, $t=0$ analysis again applies with the J-independent wave vector $k_{\mathrm{eff}} = 2\pi/\delta$. Furthermore, scaling analysis[7] suggests that in the opposite limit $\xi > \delta$, this solution is still valid, but with the nucleation width set by δ, rather than ξ.

In the 2D limit, the staggered current decays by ionization of $L–R$ vortex pairs, whose energy for $t>0$ grows *linearly* with the separation R as $(\rho_s t)^{1/2} R/\ell$. This therefore suggests the existence of a true staggered *critical* current $J_c = (e/\hbar)\sqrt{\rho_s t/\ell^2}$, with $E(J<J_c)=0$ even at finite T (up to corrections that vanish in the thermodynamic limit), and for $J > J_c$, $E(J) \approx |J - J_c|^{\alpha(T)}$.

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[8] Although an infinite number of solutions exist, only two satisfy the physically-motivated requirement that a saddle-point solution should be ''close'' in form to the uniform current carrying state, only deviating from it *locally*.

[9] Interestingly, such an electromagnetically charged but topologically neutral vortex pair corresponds to a charged fermionic excitation which is gapped in the QH state. Although disorder will undoubtedly induce such charged quasiparticles, the QH state should survive as long as they remain localized. The delocalization of such topologically neutral vortex dipoles will destroy the QHE, but will preserve the interlayer phase-coherence, therefore suggesting the possibility of an exotic *gapless* interlayer phase-coherent state in a narrow sliver around $\nu = 1/2$. L.R. thanks Steve Girvin for discussion on this point.

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