ART. V.—ON THE MEASUREMENT OF TEMPERATURE AND TIME—BY PROF. J. G. MACGREGOR, D. SC.

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(Abstract.)

THE object of this paper was to point out the analogy between the so-called measurement of time and of temperature.

The time of the occurrence of any event may be described by the aid of any series of recurring events. The daily passage across the meridian of the first point of Aries may be chosen, for example. In that case the time of the occurrence of an event is described as between the nth and $(n+1)$th transits of this point. To make the description more definite we may use a rapidly oscillating pendulum and describe the event as occurring between the mth and the $(m+1)$th oscillations of the pendulum after the nth transit of the first point of Aries. By thus selecting a series of events occurring with sufficient frequency it is possible to give our descriptions of instants of time as great precision as may be desirable.

It is consequently possible to record the magnitudes of variable quantities (e. g. distances, angles, etc.,) at definite instants, and therefore to compare the changes which the positions of bodies may have undergone in any required interval of time.

To facilitate the comparison of the contemporaneous changes of position or motions of bodies among one another, the motion of some one body is chosen as a standard, and all other motions are compared with it. It is obviously desirable that the moving body chosen as a standard of reference should so move that as many as possible of the laws of the motions of other bodies, when expressed in terms of its motion, should be (1) simple and (2) permanent, i. e., independent of the date of their determination. The selection of such a moving body is rendered possible by the records of astronomers, which extend over more than 2000 years. Their observations shew that if the motions of other bodies are compared with the contemporaneous rotation of the earth relative to the fixed stars, the laws of their motions take forms which are simpler and more permanent than if any other motion be taken as standard. Hence, by common consent, the motion of the earth about its axis is taken as a standard with which other motions are compared.

It is obvious that if the interval of time in which the earth makes a complete rotation were always the same, the laws of the motions of bodies, expressed by reference to the contemporaneous rotation of the earth, would be identical with the laws of their motions, expressed in terms of time. Usually in stating the laws of the motions of bodies it is assumed that the rotation of the earth is uniform, and these laws are expressed in terms of time. But though the terminology of time is employed the laws of their motions are always really expressed in terms of the standard motion.

Recent discussion of astronomical observations * seems to shew that the laws of the motions of heavenly bodies would take simpler forms, and would be more permanent, if the standard motion were that of an ideal earth, rotating so that its rate of rotation would slowly gain on the rate of rotation of the actual earth. If the time of the rotation of this ideal earth be assumed to be uniform, the time of the earth's rotation, i. e., the sidereal day, must be regarded as increasing at a slow rate; and when the sidereal day is said to be increasing, nothing more is meant than that, as time goes on, a greater and greater number of rotations of this ideal earth occur during one rotation of the actual earth. We have no means of knowing whether the time of the rotation of the ideal earth is more or less variable than that of the real earth. But as the laws of the motions of bodies generally are simpler and more permanent when expressed in terms of the rotation of the ideal earth than when expressed in terms of the rotation of the real earth, it is convenient to assume the time of the former uniform and that of the latter variable.

While therefore it is possible to describe instants of time with any degree of precision, it is not possible to measure the interval

* See Thomson & Tait's "Treatise on Natural Philosophy," Part II., p. 830.

of time between two instants, i. e., to compare one interval with another as to magnitude; and when we express the laws of the motions of bodies by reference to time, and thus seem to claim to be able to measure time, we are in reality only expressing the laws of the motions of bodies in terms of the contemporaneous motion of some one body.

The temperatures of bodies may be described by reference to any quantity which varies with temperature, as the volume of a body under constant pressure, or its pressure under constant volume. Thus the temperature of a body is usually described as being the same as that of the mercury in a thermometer when the apparent volume of the mercury has a specified value. Except for the difficulty of making thermometers which are exactly comparable, temperatures may be described in this way with as great precision as may be desired.

It is therefore possible, as it is also important, to compare the changes of volume, pressure, &c., of different bodies, involved in given changes of temperature. Changes of volume, pressure, &c., consequent upon the same change of temperature, may be called co-thermal changes, the term *co-thermal* having the same signification with respect to temperature as *contemporaneous* has with respect to time.

To facilitate the comparison of co-thermal changes, some one such change is chosen, and all others are expressed in terms of it. Usually the change chosen as standard is the change in the apparent volume of the mercury in the ordinary thermometer. And when the laws of the variation of the volume, pressure, etc., of bodies with temperature are expressed in terms of the co-thermal change in the apparent volume of the mercury of the thermometer, the laws of the variation of volume, pressure, etc., thus expressed, are said to be expressed in terms of temperature. They are no more really laws of variation in terms of temperature however than laws of the motions of bodies expressed in terms of the contemporaneous rotations of the earth are laws of their motions with respect to time. When we speak of them as laws of variation with respect to temperature, we assume, for the sake of a convenient terminology, that increments of the apparent volume of the mercury in the thermometer, which are the same fraction of its apparent volume at the temperature, say, of melting ice, are due to equal changes of temperature.

It is found that if we employ thermometers containing different liquids, and compare the changes of volume, pressure, &c., of bodies due to change of temperature, with the co-thermal changes in the apparent volumes of these liquids, the laws of the variation of volume, pressure, &c., thus obtained have different forms, but that if gases, far removed from their temperatures of condensation, be employed instead of liquids the laws obtained have the same form. Hence it is manifestly advantageous that laws of the variation of quantities with temperature should be expressed in terms of the co-thermal changes in the apparent volume of a gas enclosed in a glass vessel.

Sir William Thomson has shown that if the variations of the volume, pressure, &c., of bodies, due to a change from one temperature to another, be compared, not with the co-thermal change in the apparent volume of a liquid or a gas enclosed in a glass vessel, but with the work done by a reversible heat engine, working with its source at the one temperature and its refrigerator at the other, and taking in at the source an amount of heat sufficient to raise the entropy of the working substance by a fixed amount, the laws of the variation of the volume, pressure, &c., of bodies, expressed in terms of the work thus done, will be the same, whatever the working substance of the heat engine may be. Hence it is manifestly still more advantageous that laws of variation with temperature should be expressed in this way. But laws of the variation of volume, pressure, &c., expressed in this way, are no more truly laws of the variation of these quantities with respect to temperature, than those expressed by the aid of the Mercury Thermometer.

In fine, neither time nor temperature can be measured, And when we seem to claim to measure them by expressing laws of the variation of quantities with time or temperature, in terms of time or temperature respectively, we are simply expressing these laws of variation in terms of the contemporaneous or co-thermal changes respectively, of some body chosen as a standard.