EKMAN'S THEORY APPLIED TO WATER REPLACEMENTS ON THE SCOTIAN SHELF.*

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ABSTRACT.

A rough inverse correlation exists between excess S.W. wind mileage (S.W.—N.E.) at Sable Is. and September mean surface water temperatures in Halifax Harbour. In the absence of steep atmospheric pressure gradients over the neighboring ocean, to the south and west, in September 1934, a high correlation exists between the direction of the component winds (N.E. or S.W.) at Sable Is. and the daily mean surface water temperatures in Halifax Harbour. This indicates that wind, over suitable water areas, is an important factor in the phenomenon of replacement of coastal waters. Analyzed on the basis of the Ekman theory, N.E. winds over the Scotian gulf are favourable to the shoreward movement of offshore surface waters, while S.W. winds are favourable to offshoreward movement of surface waters with consequent upwelling. A modification of McEwen's application of Ekman's theory to upwelling indicates that waters of the "intermediate layer" are dissipated through coastal upwelling resulting from wind action. A quantitative estimate is made of the necessary enhancement of the "intermediate layer" to offset the loss due to upwelling. A southwesterly movement of approximately 0.6 nautical mi./day (1.0 Km./day) through a section (50 m. in depth and 60 Km. in width) normal to the coast is necessary to offset the loss through upwelling caused by an average monthly S.W. wind component of 7.0 mi./hr. (11.3 Km./hr.). The importance of the bottom configuration of the Scotian shelf to the circulation in a vertical section normal to the coast is indicated by the fixed condition (d > 2D) necessary for the establishment of the "surface", "mid water", and "bottom" currents. Generally speaking, depths greater than 150 m. (found chiefly within the margins of the Scotian gulf) are necessary for the establishment of such a system of currents on the Scotian shelf. Within the limited depths, the current system requires a period of approximately 3 days in order to attain 0.7 of its maximum efficiency.

Introduction.

Illustrations of water replacements on the Scotian shelf have been furnished in earlier papers[1]. In attempting to deal with the factors associated with the phenomenon, difficulty has always been encountered in attempting to distinguish between the effect of wind and that of a steep atmospheric pressure gradient. These replacements are more easily

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[1]Hachey, H. B.   *J. Biol. Bd. Can.*, **1**, (4) 227-237. (1935).   *Prog. Rep. (Atl.) Biol. Bd. Canad.*, **11**, 6-9.   (1934).

Leim, A. H. and H. B. Hachey.   *Trans. Amer. Fish. Soc.*, **65**, 279-283. (1935).

detected during the months of late summer as the waters are then more highly stratified. During late August and early September, it is usual for large scale replacements (in part associated with steep atmospheric pressure gradients over the neighboring ocean) to occur, and these are readily followed in an area such as Halifax Harbour.

During the month of September 1934, the western North Atlantic, to the south and west, was comparatively free from storms[2]. A North Atlantic tropical cyclone was reported on the 5th of the month near the Bahama Islands. It ceased to be of importance after reaching Cape Hatteras on September 8th. About the middle of the month, a depression was noted to be moving northwestward, passing close to the Virgin Islands, but it seems never to have reached marked strength, and by the 21st, between the Bahamas and Bermuda, it ceased to be identifiable. Steep atmospheric pressure gradients over the western North Atlantic, to the south and west, were therefore relatively absent in September 1934. A large scale replacement of waters occurred[3] associated at first with the North Atlantic tropical cyclone of September 5th. The replacements continued for the greater part of the month. We may therefore look to an analysis of wind data, on the basis of Ekman's theory of wind driven currents[4], in an attempt to arrive at the possible relation of wind to this phenomenon of replacement.

The Effect of a Steady and Uniform Wind.

In Ekman's study of the influence of the earth's rotation on ocean currents, a most successful theory of wind driven currents was developed from a number of type problems which he solved by ordinary hydrodynamical methods. A particular problem was concerned with a steady and uniform wind blowing in a constant direction everywhere outside a straight and infinitely long coast, where the sea is considered to be of uniform depth. In such a case, a slope of the surface waters

[2]Hunter, H. C. *Mon. Weather Rev.*, **62**, (9), p. 350. (1934).

[3]Leim, A. H. and H. B. Hachey. loc. cit.

[4]Ekman, V. W. *Arkiv. f. Matematik, Astron. och Fysik.*, **2**, 1-53. (1905-06).

will arise, perpendicular to the coast, and gradually increase until the total flow, perpendicular to the coast, due to wind and pressure gradient, is zero.

The equation for a steady current under such conditions (density differences neglected) has been derived by Ekman, and he has shown that the character of the motion will depend upon the depth of the sea "d" compared to the depth of the wind current "D", and on the angle between the wind direction and the coast line. If the depth is sufficiently great (d > 2D) three currents will be present as follows:

(a) a surface current which will be a pure drift current deflected to the right of the wind's direction,

(b) a mid-water current parallel to the coast, and

(c) a bottom current moving more or less in a direction at right angles to the coast.

The bottom current is the bottom part of the gradient current and, in order to satisfy the condition of continuity,
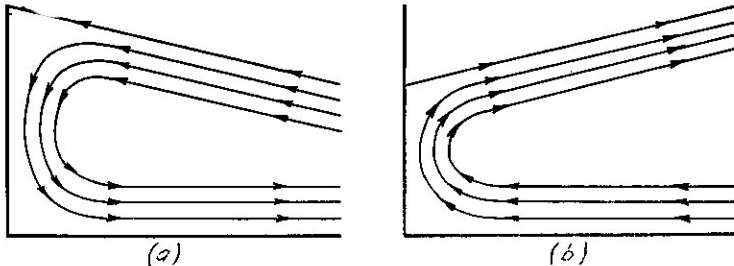

Figure I.   Circulation in a section caused by a steady uniform wind, blowing in a constant direction everywhere, outside a straight and infinitely long coast (d>2D): (a) wind normal to the paper, towards the reader, (b) wind normal to the paper, away from the reader.

the flow towards or away from the coast in this layer must equal the flow away from or towards the coast in the surface layer. Thus in a cross section normal to the coast, the components of flow normal to the coast in the surface and bottom layers are productive of circulation as illustrated in Figures I (a) and I (b). In Figure I (a), a steady uniform wind (nor-

mal to the paper, towards the reader) blowing in a constant direction everywhere, outside a straight and infinitely long coast is causing a sinking of surface waters in the neighborhood of the coast, and thus a replacement of bottom waters by surface waters. In Figure I (b), the wind is represented as blowing in the opposite direction (normal to the paper and away from the reader). In such a case, the resultant vertical circulation is responsible for offshoreward movement of surface waters with consequent upwelling.

Theoretically, too, a wind of given strength would have its greatest effect when directed a little more than thirteen degrees to the left of the coast line, and perpendicular to this direction would have its smallest effect.

Application to the Scotian Shelf.

On the basis of the above exposition of the theoretical results of Ekman, winds most effective in setting up the illustrated system of currents are from the northeast where shoreward movements of surface waters are concerned, and from the southwest where offshoreward movements of surface waters are concerned. Our concern is not with local winds, but with those representative of conditions over the Scotian shelf. Hence, wind observations at Sable Is. furnish the most representative data available.

The Scotian shelf, with the exception of the Scotian gulf[5], consists of depths generally less than 100 m. The system of currents, illustrated in the diagram (Figure I), can be set up by the winds only where d > 2D. For a given latitude, D varies with the wind velocity according to the formula[6] $D = \dfrac{7.6}{\sqrt{\sin \varphi}} V_w$, where V_w is the wind velocity and φ is the latitude. For wind velocities between 5 and 10 mi./hr. (8 and 10 Km./hr.), in latitude 44° N, D has a value between 50 and 100 m. With such winds, the illustrated system of

[5]Hachey, H. B. *Trans. Amer. Fish. Soc.* (in press), 66, (1936).

[6]McEwen, G. F. *Internat. Rev. d. ges. Hydrobiol. u. Hydrograph.* 243-285. (1912).

currents is possible where the depths range between 100 and 200 m. Such depths are found chiefly within the Scotian gulf.

The time required for such a system of currents to attain 0.7 of its final value is given by the formula[6]

$$t = \frac{d}{D \sin \varphi} + .0036 \, \frac{x^2 \sin \varphi}{D} \text{ days,}$$

where x is the distance from the coast in kilometres across the stream, and d and D are depths in metres. Giving x a value of 60 mi. (96 Km.), d a value of 150 m., D a value of 75 m., and solving we get t equal to 3 days. With these points in mind, our next interest is concerned with a method of detecting the reaction of the waters of the Scotian shelf to the wind.

The heat energy directly received from the sun is, of course, the most direct and important factor in the determination of surface water temperatures. An interchange of heat is continually taking place, and it is when this interchange is in favour of the surface waters that increasing surface water temperatures are recorded. During the period of increasing surface water temperatures, the process is modified considerably on the open coast through the upwelling of deeper and colder waters. As autumn approaches, the amount of heat absorbed by the surface waters is offset by the amount of heat lost, and the mean daily temperatures tend to vary but little from day to day. It is during this period that other factors, which tend to control the surface water temperatures, are most apparent. In particular, continued shoreward movement of warmer offshore waters on the Scotian shelf is readily detected by following the changing hydrographic conditions in Halifax Harbour. It can be readily shown that a rough inverse relationship (correlation coefficient more than twice the probable error) exists between excess southwest wind mileage (S.W.—N.E.) at Sable Is. and the September mean surface water temperature in Halifax Harbour. Wind records for Sable Is. (as furnished by the Meteorological Service of Canada) for the month of September have been analyzed for the years 1928 to 1936 inclusive (with the exception of 1932,

when no records for the month were available). The wind mileages for the month were converted to N.E. and S.W. components, and differences (S.W.—N.E.) were plotted against the corresponding September mean surface water temperatures in Halifax Harbour. The result is indicated in Figure II.
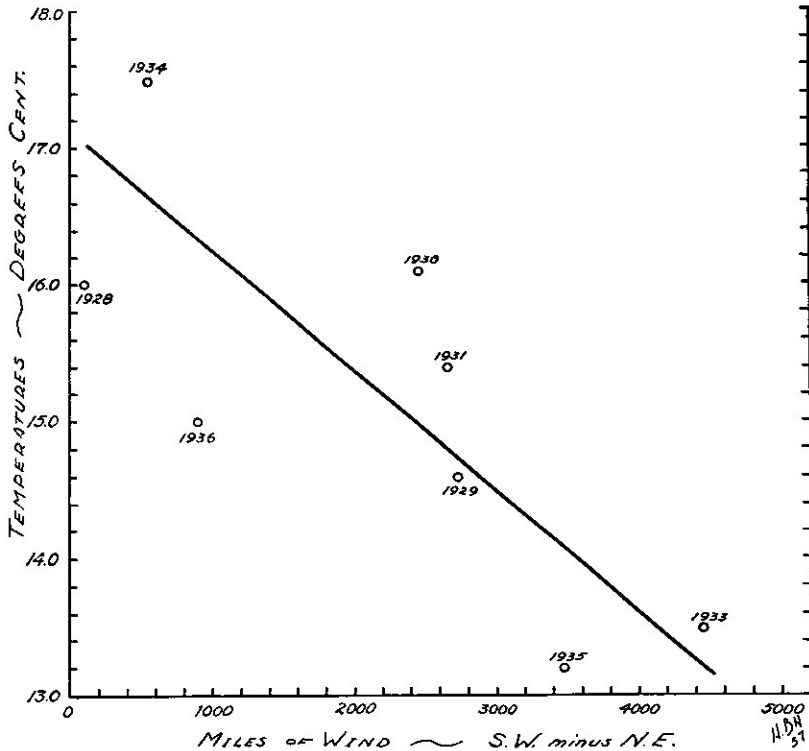

Figure II. S.W. minus N.E. component wind mileage for September at Sable Is. plotted against the corresponding September mean surface water temperatures in Halifax Harbour.

Further than this, the month of September 1934, with almost complete freedom from storms, offers an exceptional opportunity for a detailed examination of the relation between daily winds at Sable Is. and daily mean surface water tem-

peratures in Halifax Harbour. The N.E. and S.W. components of the wind mileage at Sable Is. for September and part of October 1934 are plotted synoptically with the corresponding daily mean surface water temperature in Halifax Harbour in Figure III. According to the Ekman theory, N.E. winds would be favourable to onshore movements of surface waters within the Scotian gulf and this shoreward movement would
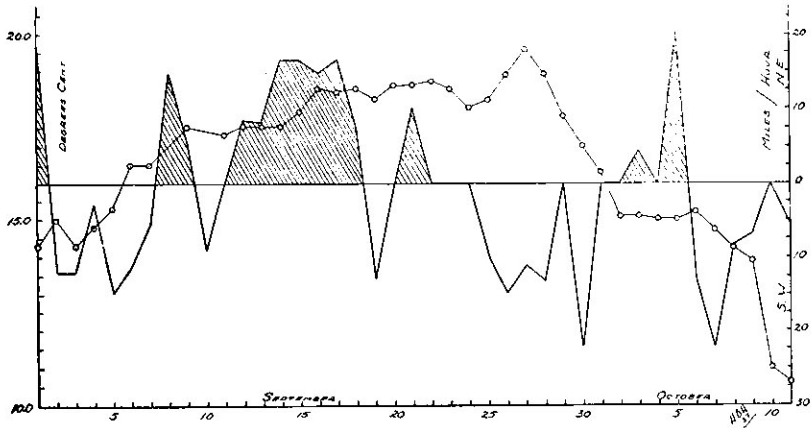

Figure III. Synoptic representation of N.E. and S.W. components of the wind mileage at Sable Is. for September and October 1934, and the corresponding daily mean surface water temperatures in Halifax Harbour.

be reflected in higher coastal surface water temperatures. Again, S.W. winds would be favourable to offshore movement of surface waters within the Scotian gulf resulting in upwelling which would favour lower coastal surface water temperatures. Keeping in mind the existence of a North Atlantic tropical cyclone during the first eight days of the month of September, the lag in the development of the circulation with respect to the wind, and the possibilities with respect to a North Atlantic tropical cyclone of short duration previous to September 21st, a very high correlation between wind direction and inshore surface water temperatures is indicated. The N.E. component winds of September 8th and 9th might be considered responsible for shoreward movements of surface

waters bringing about an increase in the temperature of inshore surface waters. This increase in inshore surface water temperatures is followed by a slight drop as the winds moved into the S.W. Continued winds, with strong N.E. components from the 12th to the 18th inclusive, are probably responsible for a steady and considerable increase in temperatures. A change to the S.W. on the 19th is recorded by a slight drop in water temperature. S.W. components of the winds on the 21st and zero components on the 22nd, 23rd, and 24th may be correlated with the very slight temperature changes recorded. The rise in temperature from the 24th to the 27th does not seemingly follow from wind action and may possibly be related to the storm conditions previous to September 21st. Continued S.W. component winds are followed by a sharp and continued drop in temperature until October 2nd, when N.E. winds tend to hold the temperature at a steady level. After October 6th, a further drop in temperature is experienced until the normal temperature for the middle of October is reached (October monthly normal as determined for a ten year period is 12.0°C.).

Wind Action and Upwelling.

McEwen[7] successfully applied the theoretical considerations of Ekman to the problem of upwelling on the west coast of North America. The successful method of McEwen is considerably modified in formulating a temperature problem for the coastal waters of southern Nova Scotia. Some interesting theoretical results are obtained.

Hunter[8] states that, for August 1934, previous to the 24th of the month, no storm of any consequence was encountered in the waters to the south and west of Nova Scotia. A survey of these waters was made within the period August 8th to 12th inclusive. In particular, the western half of the survey was made on August 11th and 12th. The average S.W. component of the wind, as recorded on Sable Is. for the 8th,

[7]McEwen, G. F. loc. cit.
[8]Hunter, H. C. *Mon. Weather Rev.*, **62**, (8), p. 302. (1934).

9th and 10th of August, was 11 mi./hr. (17.7 Km./hr.). The distribution of surface water temperatures is shown in Figure IV. Offshore surface water temperatures were about 17.0°C. while inshore temperatures ranged between 12.5°C. and 17.0°C. Over the western part of the area, the isotherm of 17.0°C. is, on the average, a distance of 40 nautical mi. (74.1 Km.) from the coast. Hence the horizontal temperature gradient
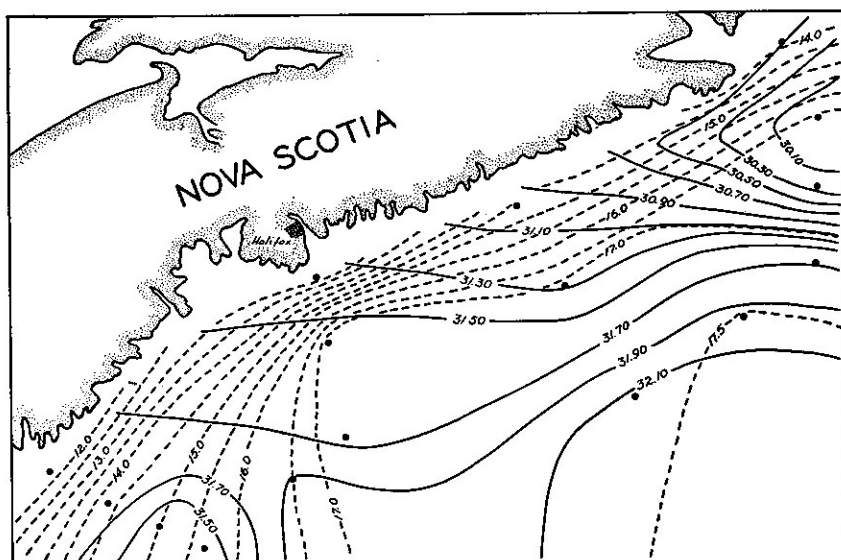

Figure IV. Distribution of surface temperatures and salinities on the Scotian shelf in August 1934. (Courtesy of the American Fisheries Society).

of 4.5° occurs in a band of coastal water of an average width of 40 nautical mi. (74.1 Km.).

Let us assume that, if it were not for upwelling, the temperature of inshore surface waters for August 11th and 12th would be the same as that offshore (approx. 17.0°C.). Let us assume also that the lowered temperatures, following upwelling, are caused by the circulation set up by wind action, and that this causative wind is indicated by the average S.W. component as recorded at Sable Is. for the 8th, 9th and 10th

of August. Let t be the mean temperature of offshore surface waters for the three day period, and let that mean be represented by the temperature observed on August 11th and 12th (approx. 17.0°C.). Let T be the corresponding mean temperature for inshore surface waters (12.5°C.). Let x_1 be the width of the band of coastal water in which the average horizontal temperature gradient of t-T exists, and let x_2 be the width of the band of offshore waters of a temperature t. Let the temperature be uniform to a depth y.

As compared to a similar volume of water at 0°C., the volume represented by unit length of shore line, width x_1+x_2 and thickness y, has excess heat to the amount of

$$Ay \left\{ \left(\frac{T+t}{2} \right) x_1 + tx_2 \right\} \quad \text{units.}$$

Assume that this amount of heat is the same as if the total volume $y(x_1+x_2)$ was, to begin with, at a temperature t, and some volume xy at temperature t was removed from the region in the three day period (referred to above), and replaced by an equal volume xy upwelling from below at temperature t_1, and we have

$$Ay \left\{ \left(\frac{T+t}{2} \right) x_1 + tx_2 \right\} = Ay (x_1 + x_2) t - Ayxt + Ayxt_1 \quad (1)$$

and symplifying we get

$$t - T = \frac{2x}{x_1} (t - t_1) \quad (2)$$

Where the temperature is uniform down to a depth of 5 m. and the depth of the wind current is taken as 75 m., McEwen[9] has shown that the amount of upwelling per unit length of coast line per unit time is

$$\frac{.02}{\sqrt{\sin \varphi}} V_w \quad (3)$$

where V_w is the wind component parallel to the coast in miles per hour, and φ is the latitude. Hence xy, the amount of

[9]McEwen, G. F. loc. cit.

upwelling in a three day period, is equal to the amount removed from the coast in the surface flow

$$= \frac{.02}{\sqrt{\sin \varphi}} V_w \times 3 \times 24 \times 3600$$

and therefore $x = \frac{.02}{5 \sqrt{\sin \varphi}} V_w \times 3 \times 24 \times 3600$ (4)

where V_w is the average S.W. component of the wind for the three day period concerned (11 mi./hr. = 17.7 Km./hr.). Substituting for x in (2) and solving we get

$$t - T = \frac{2074 \ V_w}{x_1 \sqrt{\sin \varphi}} \ (t - t_1)$$ (5)

In the particular distribution of surface temperatures as obtained in August 1934, we have

t = 17.0° C.,   T = 12.5° C., φ = 44° N., x_1 = 73600 m.

Solving (5) we get

t_1 (the temperature of the upwelling water) = 4.9°C.

For this region, such a result is a reasonable one, and indicates, on the basis of our assumptions, that the waters of the "intermediate layer[10]" are entering into this upwelling process. In August 1934 such temperatures were found inshore at depths greater than 25 m.

Discussion.

The preceding application of Ekman's theory to the waters of the Scotian shelf is possible only during those periods where wind action is the predominating cause of circulation in a vertical section normal to the coast. The typical circulation is brought about under certain stated ideal conditions which are only approximated to in the actual case. Particular attention must be given to the relationship between d (the depth of the water) and D (the depth of the wind current). It is formulated that the depth of the water must be, at least, twice that of the wind current. With the depth of the wind

[10]Hachey, H. B. *J. Biol. Bd. Can.*, **1**, (4), 227-237. (1935).

current taken as 75 m., it is postulated that the depth of the water necessary for the existence of circulation as illustrated in Figures I (a) and I (b) must be at least 150 m. Such depths are encountered on the Scotian shelf chiefly within the boundaries of the Scotian gulf[11]. Consequently, it might be expected that the most efficient system of circulation in a vertical section normal to the coast would be experienced, as a result of favourable wind action, in a section through the Scotian gulf and normal to the coast. When invasions of foreign waters[12] occur on the Scotian shelf and organisms typical of marginal waters are found within and about the margins of the Scotian gulf, the question is raised as to the reason for the limitation of the distribution within the area of the Scotian gulf. The above considerations of the circulation set up through wind action seemingly offer a reasonable answer to the question.

From equation (3) we may estimate the amount of upwelling U along the western half of the south coast of Nova Scotia (approximately 200 Km.), assuming that the circulation in a vertical section normal to the coast is efficiently produced by favourable winds. From (3) $U = \dfrac{.02}{\sqrt{\sin \varphi}} V_w$ per unit length of coast line per unit time. In September 1935, the average S.W. component of the wind was 7.0 mi./hr. (11.3 Km./hr.). Hence solving for U, we get the amount of upwelling along the western half of the south coast during the month of September 1935 (due to wind action only) as 9×10^{10} cu. m. It has been shown that the waters of the "intermediate layer" of the Scotian shelf enter into this upwelling. If the "intermediate layer" is to show no signs of diminishing as the season progresses, there must be sufficient enhancement to offset the amount of upwelling. Hence in a section normal to the coast, 50 m. in depth and 60 Km. in width, the enhancement necessary to offset the calculated upwelling for the month of September 1935 involves a movement through this vertical

[11]Hachey, H. B. *Trans. Amer. Fish. Soc.* (in press), **66**, (1936).
[12]Leim, A. H. and H. B. Hachey. loc. cit.

section of 3 x 10⁴ metres per month or approximately 0.6
nautical mi./day (1.0 Km./day).

It is readily seen from theoretical considerations of the
system of currents set up by winds that upwelling is favoured
by winds from the southwest, and further that the "mid water"
current tends to offset water movement from the northeast,
which movement is necessary for enhancement of the "inter-
mediate layer". Northeast winds tend to prevent upwelling
by bringing about shoreward movements of surface waters.
The "mid water" current would aid the enhancement of the
"intermediate layer", and it is therefore probable that annual
differences in the nature of the "intermediate layer" are
related, in part, to the nature of the winds of the region.

The predominating wind during the summer months is
from a general southwest direction. Hence, in summer,
offshore movements of surface waters and onshore movements
of bottom waters are favoured. The reverse is true in the
winter months, when the predominating wind is from a norther-
ly direction, favouring onshore movements of surface waters
and offshore movements of bottom waters. Such movements
of the bottom waters are attested to by the higher bottom
salinities in the summer months (greater than 34.00% and
as high as 34.90%) as compared to the bottom salinities in
the winter months (lower than 34.00%).