RISK ASSESSMENT OF SOIL LINERS: INFLUENCE OF CORRELATION LENGTH

by

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To my family

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Abstract

Estimates of flow through soil liners are often performed deterministically. Recent advances in the state of practice is suggesting estimates of reliability (i.e., risk) associated with this flow will be required. In this study, probabilistic simulations are performed to examine the influence of correlation length on the level of risk associated with flow through soil liner systems. Influence of correlation length on the statistics of effective hydraulic conductivity is examined for different geometries of liner. Predictions are made for the statistics of effective hydraulic conductivity. The predicted results are used to investigate the influence of correlation length, hydraulic conductivity mean and coefficient of variation and aspect ratio of liner on the probability of exceeding regulatory hydraulic conductivity. It is shown that probability of exceedance increases with increasing correlation length and mean and decreasing aspect ratio of liner and coefficient of variation. Numerical examples are presented to illustrate the proposed methodology.

List of Abbreviations and Symbols Used

A	Plan area of the liner
D	Effective dimension of the liner
i	Hydraulic gradient across the liner
k	Hydraulic conductivity of the liner
k_A	Arithmetic average of point-scale hydraulic conductivity
k'_A	Normalized arithmetic average of point-scale hydraulic conductivity
k_{crit}	Regulatory hydraulic conductivity
k_{eff}	Effective hydraulic conductivity
k'_{eff}	Normalized effective hydraulic conductivity
k_G	Geometric average of point-scale hydraulic conductivity
k'_G	Normalized geometric average of point-scale hydraulic conductivity
k_H	Harmonic average of point-scale hydraulic conductivity
k_i	Hydraulic conductivity of the i^{th} sample
$\ln k$	Log-hydraulic conductivity
LSD	Limit states design
n	Number of samples
$P(E)$	Probability of exceedance
Q	Total flow through soil liner
RBD	Reliability based design
T_i	Dimension of the random field in the ith direction
WSD	Working stress design
X	Thickness of the liner
Y	Width of the liner

X/Y	Aspect ratio of liner
θ_i	Correlation length in the ith direction of the random field
θ_k	Correlation length of hydraulic conductivity
θ'_k	Normalized correlation length of hydraulic conductivity
$\frac{\theta_k}{D}$	Normalized correlation length of hydraulic conductivity
μ_k	Mean of hydraulic conductivity
μ'_k	Normalized mean of hydraulic conductivity
μ_{k_A}	Mean of arithmetic average of point-scale hydraulic conductivity
$\mu_{k_{eff}}$	Mean of effective hydraulic conductivity
μ_{k_G}	Mean of geometric average of point-scale hydraulic conductivity
$\mu_{\ln k}$	Mean of log-hydraulic conductivity
$\mu_{\ln k_{eff}}$	Mean of log-effective hydraulic conductivity
$\mu_{\ln k_G}$	Mean of log-geometric average of point-scale hydraulic conductivity
σ_{k_A}	Standard deviation of arithmetic average of point-scale hydraulic conductivity
$\sigma_{k_{eff}}$	Standard deviation of effective hydraulic conductivity
σ_{k_G}	Standard deviation of geometric average of point-scale hydraulic conductivity
$\sigma_{\ln k}$	Standard deviation of log-hydraulic conductivity
$\sigma_{\ln k_G}$	Standard deviation of log-geometric average of point-scale hydraulic conductivity
σ_k	Standard deviation of hydraulic conductivity
$\sigma_{\ln k_{eff}}$	Standard deviation of log-effective hydraulic conductivity
ν_k	Coefficient of variation of hydraulic conductivity
ρ	Correlation coefficient
γ	Variance function

τ_i	Distance between points in the ith direction of the random field
Φ	Cumulative density function of the standard normal variate

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Chapter 1

Introduction

Waste containment facilities rely on liner systems placed between the facility and the underlying aquifer to minimize the migration of contaminant and thereby to limit the contamination of the surrounding soil and the groundwater. These liner systems may be naturally placed or engineered and may be comprised of different materials of different areal extents as well as thickness. The equation governing the total advective flow through a saturated soil liner is given by

$$Q = kiA \tag{1.1}$$

where k is the hydraulic conductivity of the liner, i is the hydraulic gradient across the liner and A is the plan area of the liner. Hydraulic conductivity is a spatially variable property both for natural soil (Byers & Stephens, 1983; Freeze & Cherry, 1979) and compacted soil liners (Rogowski, Weinrich & Simmons, 1985; Benson, 1993). Due to this spatial variability in hydraulic conductivity, there is always a risk that the flow through a soil liner will exceed some desired value. This may compromise the operation of a reservoir or, if contaminants are present, it may impart an unacceptable risk to the environment. In this thesis, a probabilistic simulation technique is utilized to determine the proper average for the hydraulic conductivity of a soil liner system which characterizes the total flow rate through a variable soil liner. This averaging technique is further used to evaluate the risk associated with the flow through a soil liner. The following sections will review soil liners and their reliability based design.

1.1 Soil Liners

Three types of soil liners are commonly used in the base of waste containment facilities; natural undisturbed clayey deposits, compacted soil liners and geosynthetic clay liners. Natural clayey liner systems and compacted soil liners are the subject of this thesis.

Natural clayey deposits consist of thick (up to 30 to 40m) undisturbed clay soil having low hydraulic conductivity. The hydraulic conductivity of these natural clay deposits will depend on the clay mineralogy, the manner of deposition and the stress history of the deposits (Rowe et al., 2004). In-situ hydraulic conductivity of these deposits can be assessed by triaxial or fixed wall hydraulic conductivity apparatus in the laboratory; variable head or constant head tests conducted in piezometers in the field as well as interpretation of pumping tests on underlying aquifer in the field.

Compacted clay liners, usually 0.6 to 3m thick, consist of natural clay which is recompacted in the field to obtain the desired hydraulic strength properties. Good engineering practice and quality assurance programs can result in good quality, low hydraulic conductivity soil liners (Rowe et al., 2004). The hydraulic conductivity of compacted clay liners depends on the clay mineralogy and the manner of placement of the liner. Tests assessing the hydraulic conductivity of compacted clay liners can be conducted either in the field or in the laboratory. Laboratory tests include triaxial and fixed wall hydraulic conductivity tests and field tests include large ring infiltrometers, lysimeters and falling head tests in short boreholes into liners.

1.2 Reliability Based Design

One of the major issues considered in engineering design is safety. Different design approaches will evaluate safety differently. Three design approaches used in geotechnical engineering are working stress design (WSD), limit states design (LSD) and reliability based design (RBD). In WSD and LSD, global factors of safety and partial factors of safety are often used respectively to consider uncertainty in design and construction. These two approaches do not quantify the level of risk associated with the system due to uncertainty. Reliability and probability based design is an advancement of WSD and LSD approaches, where a quantitative assessment of the level of risk associated with a system is made. Reliability based design offers the advantages of being more realistic, rational, consistent and widely applicable (Becker, 1996) where design parameters are treated as random variables. As will be discussed in this thesis, probabilities of exceedance of flow through a liner can be calculated if the distribution of hydraulic conductivity are, at least approximately, known (Menzies, 2008). Current design practice for soil liners considers deterministic hydraulic conductivity of soil for contaminant flow modeling through liners and hence the risk or probability of exceedance associated with the liner due to the uncertainty in hydraulic conductivity is not currently incorporated into design. Reliability based design in practice would allow for the uncertainty in the hydraulic conductivity of soil liners to be accounted for and the risk associated with this uncertainty to be assessed.

1.3 Objectives

This thesis has three distinct objectives:

1. to examine several different averaging techniques for soil liner systems and assess them for several geometries of liners;

2. to develop an approximation for the mean and standard deviation of the effective hydraulic conductivity to allow for use in calculating risk of excess flow through various liner systems; and,

3. to examine the influence of the correlation length on the risk or probability of

exceedance related to the flow through soil liners using the approximation for the mean and standard deviation of the effective hydraulic conductivity.

1.4 Thesis Organization

This thesis is organized as follows:

- Chapter 2 is a literature review on the effective hydraulic conductivity and the risk or probability of exceedance related to the flow through soil liners.
- Chapter 3 presents the methodology utilized in this research work. Steps followed in the simulations, performed using a random finite element model, mrflow3d, are presented.
- Chapter 4 contains results used to develop an approximation to the mean and standard deviation of the effective hydraulic conductivity of liner systems of different geometries. Using this approximation, chapter 4 also presents the risks associated with the spatial variability of hydraulic conductivity.
- Chapter 5 summarizes the results obtained in chapter 4 and draws conclusions from these results. This chapter also presents recommendations for further study.

Chapter 2

Literature Review

2.1 Background

Research into reliability based design of soil liners has been conducted over two decades, however, there are few publications related to this topic. The intent of this chapter is to review the literature on this subject. The literature review consists of two distinct sections. The first section contains a review of the effective hydraulic conductivity, k_{eff}, and the second contains a review of the risk or probability of exceedance, P(E), associated with flow through soil liners.

Before proceeding, it is useful to define several terms such as the random field, the arithmetic, geometric and harmonic averages of point-scale hydraulic conductivity to facilitate discussion of the literature review.

Random field theory is used to model variable engineering properties (Fenton & Griffiths, 2008). In a random field, the property of interest is considered as a random variable. Local Average Subdivision (LAS) is a method to generate the random field. This method generates realizations of local averages over selected volumes. In generating the random field, the LAS algorithm preserves the spatial correlation between local averages of the property. Correlation between local averages can be represented by a distance called a correlation length, θ. The correlation length is a distance over which the property of interest is significantly correlated and beyond which is largely uncorrelated. The concept of LAS algorithm arises from the fact that instead of point-to-point measurements, engineering properties are measured over some selected volume, thus representing the average property over that volume. Local averaging reduces the variance of the average of the random field. The amount of variance reduction depends on the volume selected for local averaging. The amount of variance reduction increases as the volume of local averaging increases. The variance function, $\gamma(T)$, is used to express the amount of variance reduction when averaged over some length T (Fenton & Griffiths, 2008).

The averaging can be performed using either the arithmetic average, k_A, the geometric average, k_G and the harmonic average, k_H, defined as:

$$k_A = \frac{1}{n}\sum_{i=1}^{n} k_i = \frac{1}{D}\int_D k(\underset{\sim}{x})\, d\underset{\sim}{x} \tag{2.1}$$

$$k_G = \exp\frac{1}{n}\sum_{i=1}^{n} \ln k_i = exp\left[\frac{1}{D}\int_D \ln k(\underset{\sim}{x})\, d\underset{\sim}{x}\right] \tag{2.2}$$

$$k_H = \left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{k_i}\right]^{-1} = \left[\frac{1}{D}\int_D \frac{1}{k(\underset{\sim}{x})}\, d\underset{\sim}{x}\right]^{-1} \tag{2.3}$$

where k_i is the point-scale hydraulic conductivity, n is the sample size and D is the averaging domain. The arithmetic, k_A, geometric, k_G and harmonic averages, k_H, have some physical meanings. The arithmetic, k_A and harmonic averages, k_H, are representative of the two extreme flow fields, namely parallel and series flow respectively as shown in Figure 2.1. Parallel flow is the case where the flow is parallel to the layers of soil, whereas, series flow is the case where the flow is perpendicular to the layers of soil. Bouwer (1969) showed that for parallel flow, a heterogeneous medium of soil can be replaced by a homogeneous medium having single hydraulic conductivity value equal to the arithmetic average, k_A, provided that the layers have equal thickness and hydraulic gradient constant along each layer. For series flow, he showed that layered medium can be replaced by a single hydraulic conductivity value equal to the harmonic average, k_H. However, in

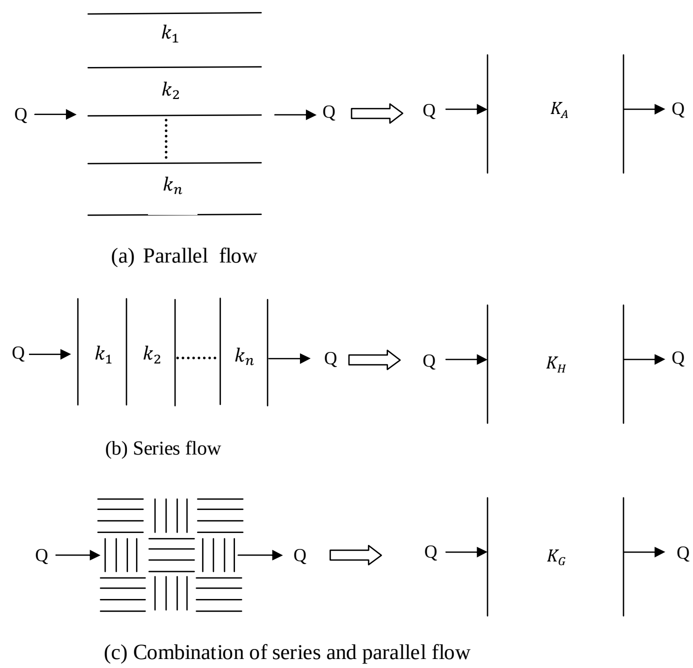

Figure 2.1: Porous media flow scenarios (from Donald, 1990)

reality, the flow will rarely be either perfectly parallel or series, but somewhere between the two, as shown in Figure 2.1. The geometric average is another term which is used to represent the actual flow field. The geometric average, k_G, is a value that lies between k_A and k_H.

2.2 Effective Hydraulic Conductivity

The effective hydraulic conductivity, k_{eff}, is defined as a single value of hydraulic conductivity which is equivalent to a heterogeneous medium of hydraulic conductivity in terms of the total flow through the medium (Bogardi et al., 1990). Although effective hydraulic conductivity is an average hydraulic conductivity of a soil liner, simple averages of point-scale hydraulic conductivity such as the arithmetic, geometric, and harmonic averages do not always adequately represent the total flow. Effective hydraulic conductivity is representative of interconnected zones of different hydraulic conductivity in the flow field (Benson et al., 1994).

Several researchers have published information related to defining the effective hydraulic conductivity. The earliest work relates to when Warren and Price (1961) studied the influence of three-dimensional heterogeneities of hydraulic conductivity on flow across a cube and radial flow from an injection well and found the effective hydraulic conductivity to be the geometric average of individual hydraulic conductivities. In their investigation, they used a simulation technique. Unlike Warren and Price (1961), Bouwer (1969) considered a uniform and binomial distribution instead of lognormal distribution of hydraulic conductivity. He used analog simulations to determine effective hydraulic conductivity for a two-dimensional flow field and found that the geometric average of hydraulic conductivity was the best approximation for effective conductivity. Smith and Freeze (1979) used Monte Carlo techniques to find effective conductivity for two-dimensional steady state groundwater flow. For an unbounded domain under uniform gradient field, they found the effective conductivity was described well by the geometric average and for a bounded domain under non-uniform gradient field, they found the effective hydraulic conductivity greater than the geometric average of point-scale hydraulic conductivity. One of the earliest attempts to define effective hydraulic conductivity analytically is that presented by Gutjahr et al. (1979). They used a spectral perturbation method to determine the effective conductivity for an unbounded domain under uniform gradient and no external stresses. They proposed the following expressions:

One-dimensional flow:

$$k_{eff} = e^{\mu_{\ln k}} \left(1 - \frac{\sigma^2_{\ln k}}{2}\right) \tag{2.4}$$

Two-dimensional flow:

$$k_{eff} = e^{\mu_{\ln k}} \tag{2.5}$$

Three-dimensional flow:

$$k_{eff} = e^{\mu_{\ln k}} \left(1 + \frac{\sigma^2_{\ln k}}{6}\right) \tag{2.6}$$

where $\mu_{\ln k}$ and $\sigma^2_{\ln k}$ are the arithmetic mean and variance of log-hydraulic conductivity distribution. According to Gutjahr, these expressions will be valid for $\sigma^2_{\ln k} \leq 0.5$.

Using a self consistent model, Dagan (1982) also attempted to define the effective hydraulic conductivity for an infinite domain of flow under a uniform gradient and having no external stresses. His results were the same as Gutjahr et al. (1978). Fenton and Griffiths (1993) utilized Monte Carlo simulation technique to examine the influence of the correlation length and aspect ratio of site on the effective hydraulic conductivity distribution. For two-dimensional finite domain of flow, they found the effective conductivity to be the geometric average of point-scale hydraulic conductivity for a square domain. For small aspect ratios (ratio between the dimension of the site parallel to the flow to that perpendicular to the flow) of the site, they found the effective hydraulic conductivity to be the arithmetic average and for large aspect ratio, they found it as the harmonic average of hydraulic conductivities. Expressions of the effective hydraulic conductivity for an unbounded domain of flow will not be valid for the liner system where the flow domain is finite in dimension. In order to define the effective conductivity for a soil liner, Benson et al. (1994) used a Monte Carlo simulation approach and examined the influence of the mean and variance of point-scale hydraulic conductivity and liner thickness on the effective hydraulic conductivity. By regression, they found following expression:

$$k_{eff} = k_G \exp\left[\sigma^2_{\ln k}\left(\frac{1.671}{N} - 0.452\right)\right] \tag{2.7}$$

where k_G is the geometric average of point-scale hydraulic conductivity, $\sigma^2_{\ln k}$ is the variance of log-hydraulic conductivity and N is the number of lifts.

2.3 Probability of Exceedance

Limited amounts of research has been conducted on the reliability of soil liners since late 80's. The earliest attempt to estimate post-construction reliability of soil liners is that presented by Bogardi et al.(1990). In the study six systems of reliability measures are defined to estimate post-construction reliability from a given set of direct and indirect hydraulic conductivity measurements from a test fill. Benson and Charbeneau (1991) presented a method to estimate the reliability of compacted soil liners based on first-passage time (time when leakage first starts from the base of the liner) which is a function of the variability of hydraulic conductivity and liner thickness. They showed that the reliability (probability that total flow through the liner is below the flow through a regulatory liner having the same area, i.e., $P[k_{eff} < k_{crit}]$) of a liner increased with increase in hydraulic conductivity variance and liner thickness. Benson and Daniel (1994a, 1994b) also investigated the influence of liner thickness and the mean and variance of hydraulic conductivity on the performance of compacted soil liners using one-dimensional and three-dimensional stochastic models. The performance criteria considered were first passage time and flux. It was found that the effective hydraulic conductivity decreased with a decrease in the mean and an increase in the coefficient of variation of point-scale hydraulic conductivity and the liner thickness of a multi-lift liner. Based on their results, they recommended a minimum thickness of 60 to 90 cm in order to obtain lower probability of exceedance. Benson et al. (1994) utilized a Monte Carlo simulation approach to examine the influence of the coefficient of variation of point-scale hydraulic conductivity and the liner thickness on the effective hydraulic conductivity. They showed that the effective hydraulic conductivity decreased with increased coefficient of variation and increased liner thickness. In the analysis, no-correlation between point-scale hydraulic conductivity was assumed. Menzies (2008) was the first who examined the influence of the correlation length on probability of exceedance (probability that total flow through the liner exceeds the flow through a regulatory liner having the same area, i.e., $P[k_{eff} > k_{crit}]$) associated with the flow through compacted soil liners. The influences of the hydraulic conductivity mean and variance on probability of exceedance were also examined. In the study, a two-dimensional stochastic model was used to perform simulations and the probability of exceedance was calculated using the arithmetic average. It was found that probability of exceedance of the liner increases with increase in the mean and decrease in the variance of point-scale hydraulic conductivity. Probability of exceedance was found to reach a maximum at a correlation length of 10-20% of liner size in any direction. In his study, lognormal distribution of hydraulic conductivity and isotropic correlation length within the random field were assumed.

2.4 Summary

Results from Monte Carlo simulations and analytical approaches to the solution of the steady state ground water flow equation indicate that for an unbounded domain under uniform gradient fields and under no external stresses, the effective hydraulic conductivity is best estimated by the geometric average of point-scale hydraulic conductivity for two-dimensional domain of flow. For the three dimensional case, the effective hydraulic conductivity is larger than the geometric average and is a function of the variance of log-hydraulic conductivity. Monte Carlo simulations for bounded domains show that for a two-dimensional case, the effective hydraulic conductivity is best approximated by the geometric average for square domains and is larger than the geometric average for small aspect ratios and smaller than the geometric average for large aspect ratios of the site. Monte Carlo simulations for three-dimensional domain of soil liner reveal that the effective hydraulic conductivity is larger than the geometric average and is a function of variance of log-hydraulic conductivity.

The risk or probability of exceedance associated with a soil liner system increases with increase in the mean and decrease in the variance of point-scale hydraulic conductivity and liner thickness. The correlation length has maximum impact on the risk when its value is 10-20% of the liner size in any direction.

In evaluating the risk, previous studies considered arithmetic averaging. Hence, proper averaging techniques should be used for the hydraulic conductivity characterizing the total flow rate through the liner as well as for evaluating the risk associated with the flow through a soil liner system.

Chapter 3

Methodology

This chapter discusses the methodology utilized to obtain the distribution (mean and standard deviation) of effective hydraulic conductivity as well as the methods to quantify risk associated with flow through a soil liner system. The chapter is sub-divided into following two sections:
(i) simulations and
(ii) parameters utilized in simulations.

3.1 Simulations

In this study, Monte Carlo simulations are performed using the three-dimensional random finite element model, mrflow3d, designed by Fenton and Griffiths to analyze stochastic fluid flow problems. The model is described by Griffiths and Fenton (1997). The mesh discretization used in the simulations is shown in Figure 3.1.

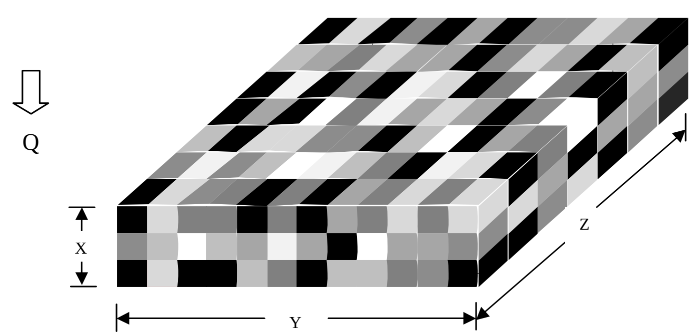

Figure 3.1: Schematic of mesh discretization used in simulation

For the modeling performed, it was assumed that an impervious boundary exists on vertical edges and a uniform unit pressure head was applied on top which directs the flow vertically downward. The inputs to the model are the mean and standard deviation of point-scale hydraulic conductivity, correlation lengths (either isotropic or anisotropic), number of cells in each direction and size of cells. For specified inputs, the model generates a random field of log-normally distributed hydraulic conductivity, which is subsequently analyzed for flow using the finite element method. The mrflow3d software is freely available (www.engmath.dal.ca /rfem/rfem_pubs.html) and the outputs are scalable. There is also provisions for users to modify the model, if desired. Limited simulations were also performed using the two-dimensional random finite element model, mrflow2d, in order to compare with three-dimensional results.

Steps followed in the simulations were as follows:

1. For each set of input parameters of mean, standard deviation and correlation length of point-scale hydraulic conductivity, a normally distributed random field of $y = \ln k$ was first generated using Local Average Subdivision (LAS) algorithm (Fenton & Vanmarcke, 1991).

2. The hydraulic conductivity of each element, which represents the average over the element, was specified using $k = e^y$.

3. The field was analyzed for flow using the finite element method.

4. The total flow, Q, through the field was obtained by summing the flow from all elements.

5. The effective hydraulic conductivity, k_{eff}, the arithmetic average, k_A, and geometric average, k_G, of point-scale hydraulic conductivity were calculated using the following expressions.

$$k_{eff} = \mu_k \left(\frac{Q}{Q_{\mu_k}} \right) \tag{3.1}$$

$$k_A = \frac{1}{n}\sum_{i=1}^{n} k_i \tag{3.2}$$

$$k_G = \exp\left\{\frac{1}{n}\sum_{i=1}^{n} \ln k_i\right\} \tag{3.3}$$

where

Q=total flow through the random field considering random hydraulic conductivity throughout,

Q_{μ_k}=total flow through the field having constant hydraulic conductivity, μ_k, throughout,

k_i=local average of hydraulic conductivity over the ith element, and

n=number of elements.

6. The mean and standard deviation of each output quantity over 1,000 realizations (single generation of random field and subsequent analysis for flow is termed a realization) were then computed.

If k_{eff} is lognormally distributed, the exceedance probability is,

$$P(E) = 1 - \Phi\left[\frac{\ln k_{crit} - \mu_{\ln k_{eff}}}{\sigma_{\ln k_{eff}}}\right] \tag{3.4}$$

where Φ=cumulative density function of standard normal variate, $\mu_{\ln k_{eff}}$ and $\sigma_{lnk_{eff}}$ are the mean and standard deviation of log-effective hydraulic conductivity respectively.

3.2 Parameters Used in Simulations

In simulations, the mean of point-scale hydraulic conductivity of the input distribution, μ_k, the arithmetic average, k_A, the geometric average, k_G, of point-scale hydraulic conductivity and the effective hydraulic conductivity, k_{eff}, are normalized with respect to the regulatory hydraulic conductivity, k_{crit}.

$$\mu_k' = \frac{\mu_k}{k_{crit}} \tag{3.5}$$

$$k_A' = \frac{k_A}{k_{crit}} \tag{3.6}$$

$$k_G' = \frac{k_G}{k_{crit}} \tag{3.7}$$

$$k_{eff}' = \frac{k_{eff}}{k_{crit}} = \frac{\mu_k \left(\frac{Q}{Q_{\mu_k}}\right)}{k_{crit}} \tag{3.8}$$

The correlation length, θ_k, is normalized by the effective dimension of the liner, D, where $D = \sqrt[3]{XYZ}$ and X, Y and Z are the dimensions of the liner.

$$\theta_k' = \frac{\theta_k}{D} \tag{3.9}$$

Normalization of hydraulic conductivity by regulatory hydraulic conductivity and correlation length by the effective dimension of the liner, enable the results to be scaled to any desired regulatory hydraulic conductivity and any liner size with same aspect ratio respectively.

Parametric variations considered in the simulations were:

- Normalized mean hydraulic conductivity, μ_k'=0.5, 1.0, 1.5 and 2.0,
- Coefficient of variation, ν_k =0.5, 1.0 and 2.0,
- Normalized correlation length, $\frac{\theta_k}{D}$ =0.01, 0.02, 0.04, 0.08, 0.1, 0.2, 0.4, 0.8, 1.0, 10.0 and 100.0, and,
- Aspect ratio of liner, X/Y=0.1, 0.3 and 1.0.

In the simulations performed, the parametric variations considered for the mean and standard deviation were intended to simulate a wide range of field conditions. Smaller values of aspect ratio (i.e., 0.1) simulate liners used in landfill having large areal extent compared to the thickness and larger aspect ratios (i.e., 0.3 and 1) simulate liners used in small leachate lagoons.

In this study, the correlation length was assumed to be equal in all three directions. In the simulations, the correlation function (ρ) is assumed to be Markovian with exponentially decaying correlation between points in the field (Vanmarcke, 1983):

$$\rho\left(\tau_1, \tau_2, \tau_3\right) = \exp\left(-2\sqrt{\frac{\tau_1^2}{\theta_1^2} + \frac{\tau_2^2}{\theta_2^2} + \frac{\tau_3^2}{\theta_3^2}}\right) \tag{3.10}$$

where

θ_i=correlation length in the ith direction.

τ_i=distance in the ith direction between points where correlation coefficient is desired.

The corresponding variance function is

$$\begin{aligned} \gamma\left(T_1, T_2, T_3\right) = & \frac{8}{T_1^2 T_2^2 T_3^2} \int_0^{T_1} \int_0^{T_2} \int_0^{T_3} \left(T_1 - \tau_1\right)\left(T_2 - \tau_2\right)\left(T_3 - \tau_3\right) \\ & \rho\left(\tau_1, \tau_2, \tau_3\right) d\tau_3 d\tau_2 d\tau_1 \end{aligned} \tag{3.11}$$

where T_i is the dimension of the averaging domain in the ith direction.

A sensitivity analysis was performed in order to examine the influence of the element mesh size on the distribution of output quantities (i.e., the effective hydraulic conductivity, k_{eff}, the arithmetic average, k_A, and the geometric average, k_G of point-scale hydraulic conductivity). For the same domain size, element sizes considered were 0.05, 0.0417, 0.0357 and 0.03125 corresponding to an element mesh of $20 \times 20 \times 20$, $24 \times 24 \times 24$, $28 \times 28 \times 28$ and $32 \times 32 \times 32$ respectively. As shown by the results included in Appendix A, all sizes give similar results. Based on the least computing time, a $0.05 \times 0.05 \times 0.05$ element was selected for all simulations.

Twenty elements of $0.05 \times 0.05 \times 0.05$ size were specified in length and breadth directions in order to obtain a plan liner area of 1 and the number of elements was varied in thickness direction (i.e., "X") according to the aspect ratio. In the simulations consideration of liner of plan area of 1×1 enables the results to be scaled to any liner size. For each set of input parameters, 1,000 realizations (single generation of random field and subsequent analysis for flow is called a realization) were performed.

Chapter 4

Results

4.1 Influence of Correlation Length on Mean of Effective Hydraulic Conductivity

Figure 4.1 shows the influence of normalized correlation length on the arithmetic and geometric averages of point-scale hydraulic conductivity as well as on the effective hydraulic conductivity, for a coefficient of variation of point-scale hydraulic conductivity of 1.0 and an aspect ratio of liner (ratio between the thickness to the width of the liner) of 0.1. Each point on the plot is the corresponding average of 1,000 realizations. Figure 4.1 indicates that as the correlation length increases, the mean of the arithmetic average and geometric average of point-scale hydraulic conductivity and the effective hydraulic conductivity increases. These results are as expected, because as the correlation length increases, fewer low hydraulic conductivity zones are developed in each realization in the random field. Deviations in this increasing trend with correlation lengths are due to the sampling error which could be avoided somewhat by using more than 1,000 realizations at those correlation lengths. It is also shown that for a particular correlation length, the mean of the effective hydraulic conductivity approaches the mean of the arithmetic average for an aspect ratio, X/Y, of 0.1.
For example, for the case of μ'_k=1, ν_k=1 and $\dfrac{\theta_k}{D}$=0.8, when X/Y=0.1,
$\hat{\mu}_{k_A}$=0.969, $\hat{\mu}_{k_G}$=0.748, and $\hat{\mu}_{k_{eff}}$=0.937.

Results for a coefficient of variation of 1.0 and an aspect ratio of liner of 0.3

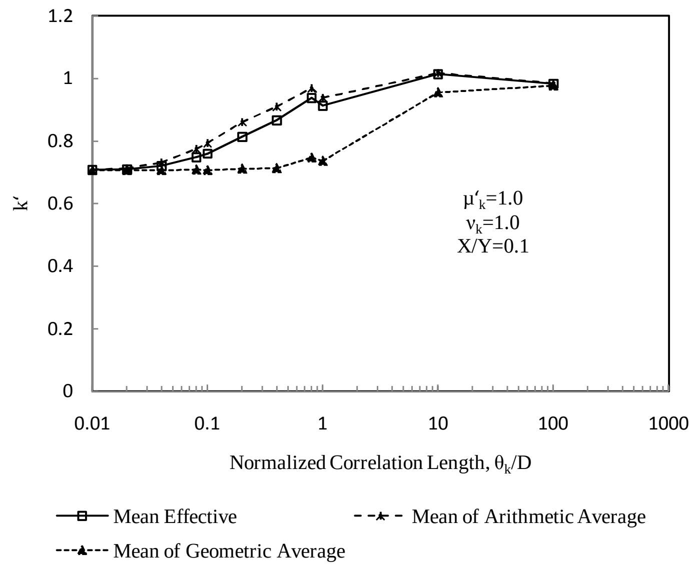

Figure 4.1: Influence of correlation length on the mean of effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 0.1

are presented in Figure 4.2. Similar to the results in Figure 4.1 for an aspect ratio of 0.1, increasing trends with correlation lengths are observed for the mean of the arithmetic and geometric average of point-scale hydraulic conductivity as well as the effective hydraulic conductivity. It is shown that compared to the case of aspect ratio of liner of 0.1, the mean of the effective hydraulic conductivity deviates away from the mean of the arithmetic average and becomes closer to the mean of the geometric average.

For example, for the case of μ'_k=1, ν_k=1 and $\dfrac{\theta_k}{D}$=0.8, when X/Y=0.3,

$\hat{\mu}_{k_A}$=0.954, $\hat{\mu}_{k_G}$=0.751, and $\hat{\mu}_{k_{eff}}$=0.886.

Figure 4.3 shows the influence of the correlation length on the mean of the arithmetic and geometric average of point-scale hydraulic conductivity and the effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 1.0. As with the other aspect ratios examined, increasing trends in the

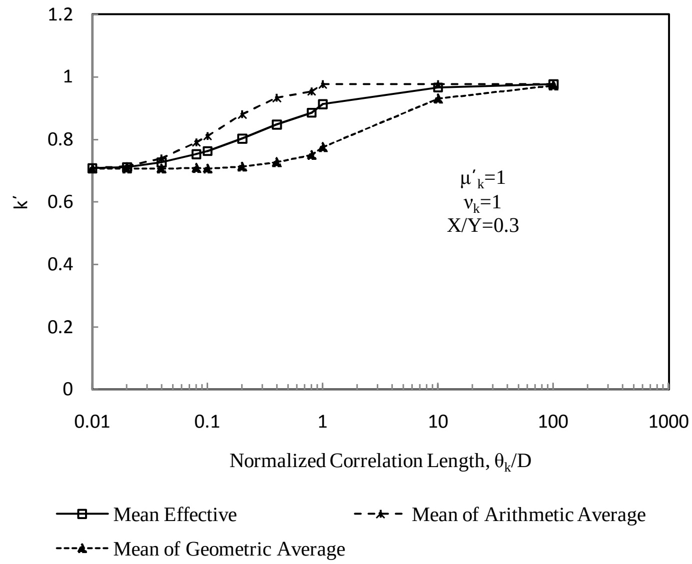

Figure 4.2: Influence of correlation length on the mean of effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 0.3

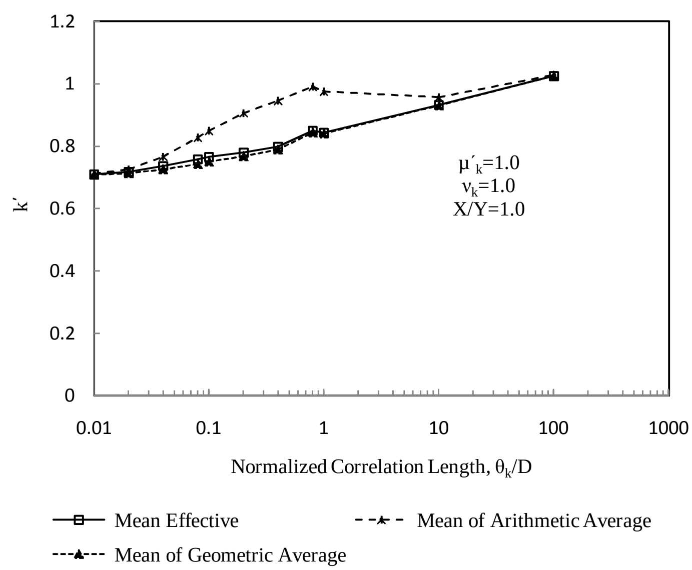

Figure 4.3: Influence of correlation length on the mean of effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 1.0

mean of the arithmetic and geometric average of point-scale hydraulic conductivity and the effective hydraulic conductivity with increasing correlation lengths are observed. Figure 4.3 also shows that for an aspect ratio, X/Y, of 1.0, the mean of

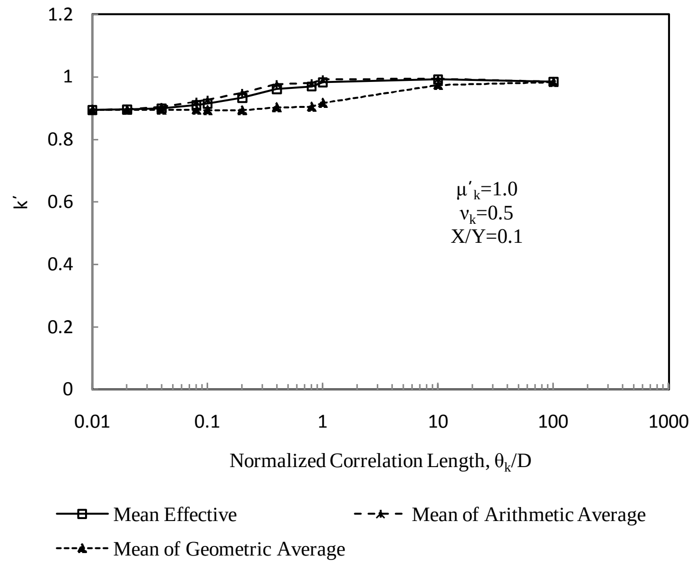

Figure 4.4: Influence of correlation length on the mean of effective Hydraulic conductivity for a coefficient of variation of 0.5 and an aspect ratio of liner of 0.1

the effective hydraulic conductivity approaches the mean of the geometric average, moving away from the arithmetic average.

For example, for the case of μ'_k=1, ν_k=1 and $\dfrac{\theta_k}{D}$=0.8, when X/Y=1.0, $\hat{\mu}_{k_A}$=0.991, $\hat{\mu}_{k_G}$=0.843, and $\hat{\mu}_{k_{eff}}$=0.843.

The influence of correlation length on the mean of the effective hydraulic conductivity, for a coefficient of variation of 0.5 and an aspect ratio of liner of 0.1, shown in Figure 4.4, indicates that the mean of the effective hydraulic conductivity also approaches the mean of the arithmetic average of point-scale hydraulic conductivity. Compared to a coefficient of variation of 1.0, it is also shown that the differences between the effective hydraulic conductivity and the arithmetic average at smaller correlation lengths are lower.

Figure 4.5 presents the results of the influence of correlation length on the mean of the arithmetic and the geometric average of point-scale hydraulic conductivity and the effective hydraulic conductivity for a coefficient of variation of 0.5 and an aspect ratio of liner of 1.0. It is observed that the mean of the effective hydraulic conductivity approaches the mean of the geometric average, similar to a coefficient of variation of 1.0, but differences between them is comparatively not so large.

When the coefficient of variation is 2.0 and the aspect ratio of liner is 0.1, the mean of the effective hydraulic conductivity continues to approach the mean of the arithmetic average which is shown in Figure 4.6. It is also noted that at lower correlation lengths, the means are lower than those obtained for coefficients of variation of 1.0 and 0.5.

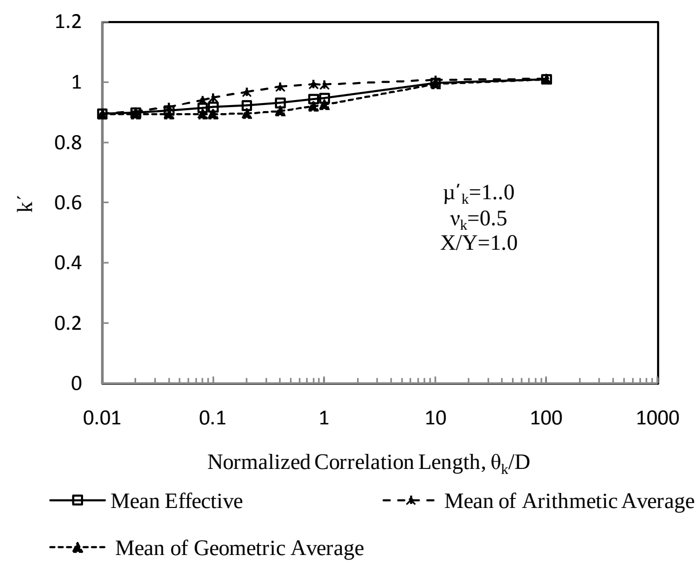

Figure 4.5: Influence of correlation length on the mean of effective hydraulic conductivity for a coefficient of variation of 0.5 and an aspect ratio of liner of 1.0

Results for the influence of correlation length on the mean of the effective hydraulic conductivity, for a coefficient of variation of 2.0 and an aspect ratio of liner of 1.0, presented in Figure 4.7, indicate that the mean of the effective hydraulic conductivity approaches the mean of the geometric average.

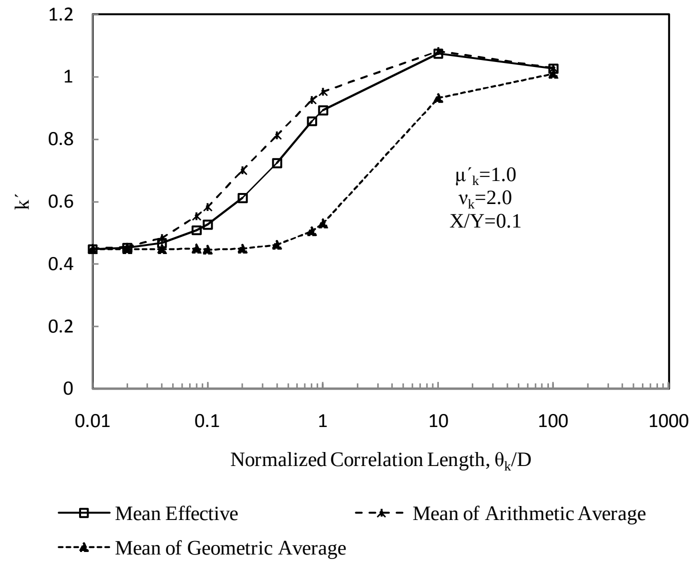

Figure 4.6: Influence of correlation length on the mean of effective hydraulic conductivity for a coefficient of variation of 2.0 and an aspect ratio of liner of 0.1

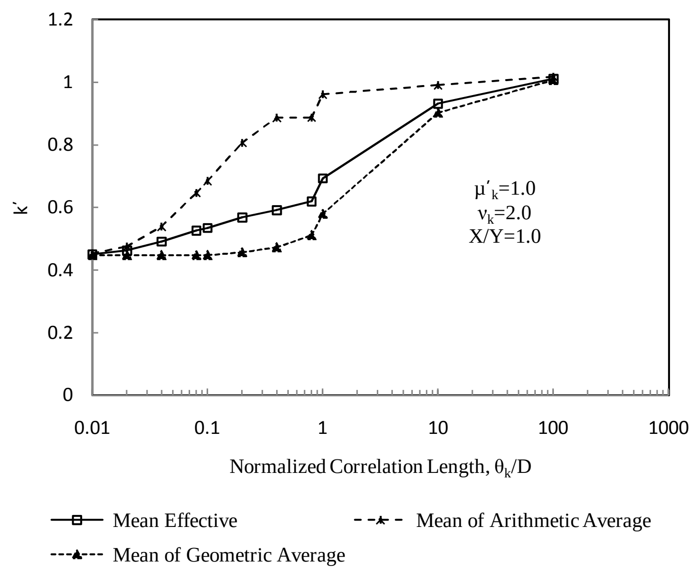

Figure 4.7: Influence of correlation length on the mean of effective hydraulic conductivity for a coefficient of variation of 2.0 and an aspect ratio of liner of 1.0

For all cases in which the coefficient of variations are 0.5 and 2.0, as well as for all aspect ratios of liner, the mean of the arithmetic and geometric average of point-scale hydraulic conductivity and the effective hydraulic conductivity increase with increasing correlation lengths and the deviations from this increasing trend are found for some correlation lengths due to sampling error. In order to reduce sampling error, more than 1,000 realizations is suggested.

The decreasing trend in the effective hydraulic conductivity with increasing aspect ratio of liner is as expected. As the aspect ratio of liner increases, the freedom of the flow to avoid low hydraulic conductivity zones increases (Griffiths & Fenton, 1997).

4.2 Prediction of Mean of Effective Hydraulic Conductivity

Results showing the influence of correlation length on the mean of the effective hydraulic conductivity indicates that for small aspect ratios of liner (i.e., X/Y=0.1), the arithmetic average of point-scale hydraulic conductivity is the best approximation for the effective hydraulic conductivity and for larger aspect ratio of liner (i.e., X/Y=1.0), the geometric average is the best approximation for the effective hydraulic conductivity. However, many liners will have different aspect ratios. Proper averaging for the effective hydraulic conductivity needs to be determined for other aspect ratios in order to quantify the risk of flow exceeding that regulated.

Based on the results presented in the previous section, a prediction of the mean of effective hydraulic conductivity can be made

$$\hat{\mu}_{k_{eff}} = e^{-1.21(X/Y)} \hat{\mu}_{k_A} + \left[1 - e^{-1.21(X/Y)}\right] \hat{\mu}_{k_G} \tag{4.1}$$

where

$\hat{\mu}_{k_{eff}}$=estimated mean of the effective hydraulic conductivity from the simulation,

$\hat{\mu}_{k_A}$=estimated mean of the arithmetic average of point-scale hydraulic conductivity from the simulation, and

$\hat{\mu}_{k_G}$=estimated mean of the geometric average of point-scale hydraulic conductivity from the simulation.

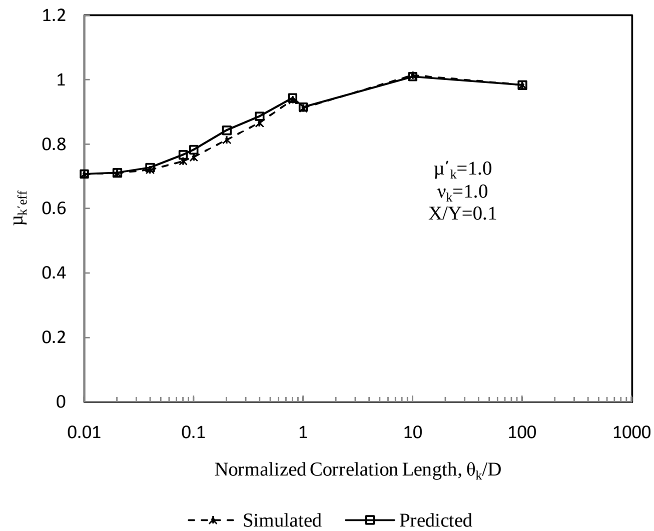

Figure 4.8: Comparison between simulated and predicted mean of effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 0.1

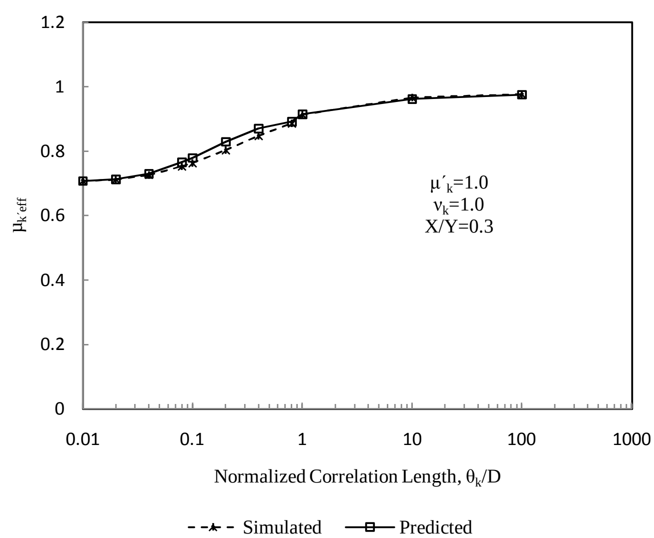

Figure 4.9: Comparison between simulated and predicted mean of effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 0.3

Figures 4.8 to 4.14 compare the simulated results of the mean of effective hydraulic conductivity to those predicted with equation 4.1 for different coefficients of variation and aspect ratios. Comparison between simulated and predicted results show excellent agreement for all cases.

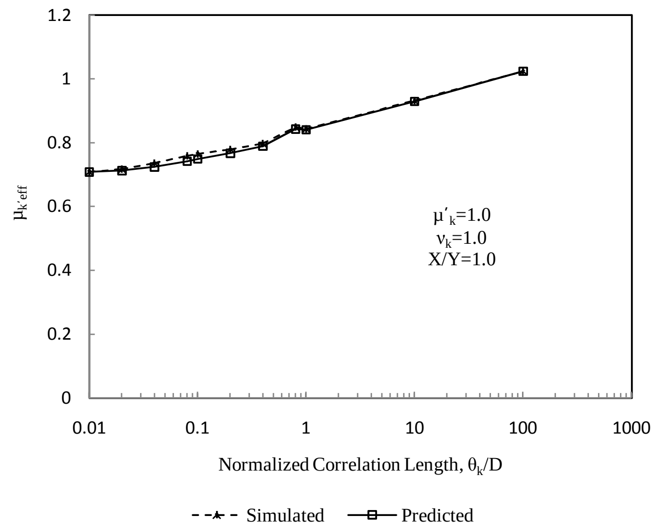

Figure 4.10: Comparison between simulated and predicted mean of effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 1.0

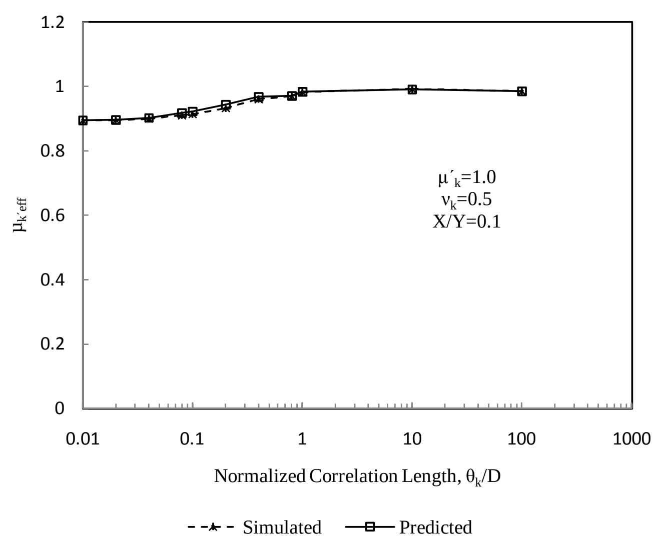

Figure 4.11: Comparison between simulated and predicted mean of effective hydraulic conductivity for a coefficient of variation of 0.5 and an aspect ratio of liner of 0.1

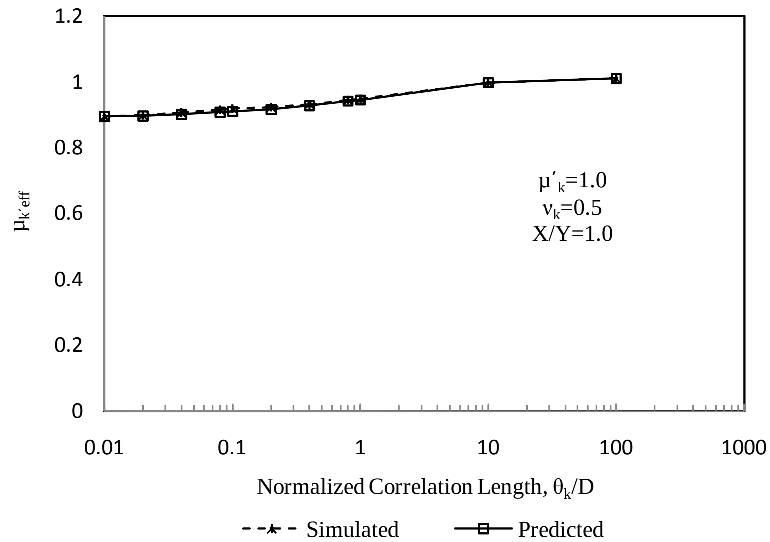

Figure 4.12: Comparison between simulated and predicted mean of effective hydraulic conductivity for a coefficient of variation of 0.5 and an aspect ratio of liner of 1.0

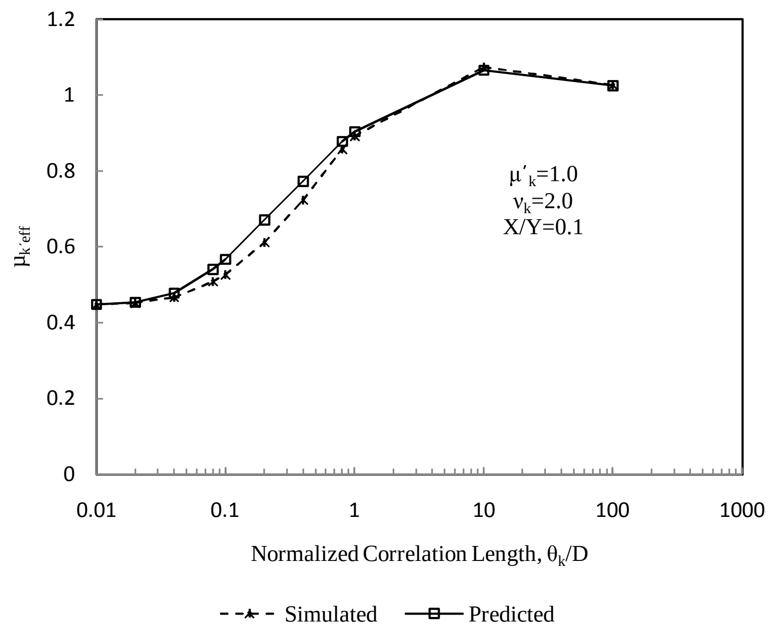

Figure 4.13: Comparison between simulated and predicted mean of effective hydraulic conductivity for a coefficient of variation of 2.0 and an aspect ratio of liner of 0.1

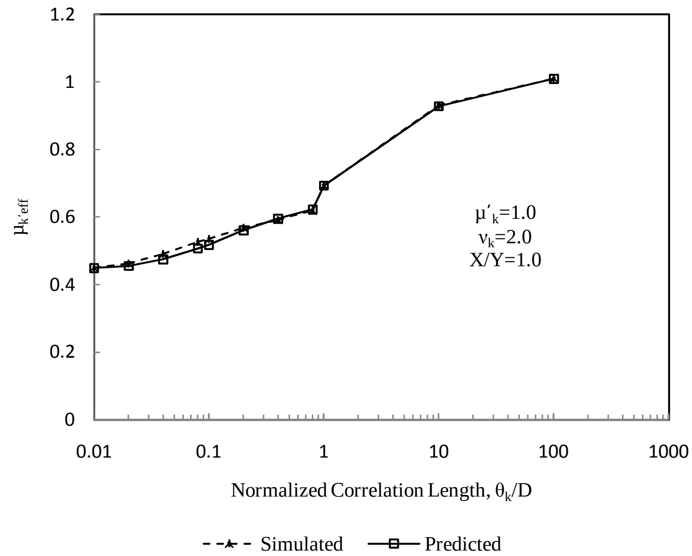

Figure 4.14: Comparison between simulated and predicted mean of effective hydraulic conductivity for a coefficient of variation of 2.0 and an aspect ratio of liner of 1.0

4.3 Influence of Correlation Length on Standard Deviation of Effective Hydraulic Conductivity

Simulations are performed in order to have an approximation for the standard deviation of the effective hydraulic conductivity which is used to evaluate the risk associated with the flow through a soil liner system. To achieve this target, the influence of the correlation length on the standard deviation of the effective hydraulic conductivity is examined.

Figure 4.15 shows the influence of the correlation length on the standard deviation of the arithmetic and the geometric average of point-scale hydraulic conductivity as well as the effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 0.1. Each point on the plot is obtained by taking the corresponding standard deviation of 1,000 realizations. Results show that as the correlation length increases, the standard deviation of the effective hydraulic conductivity also increases. The reason for this increase is because in statistics, it is well known that as the number of independent samples in an average decreases, the variance of the average increases. In the random field context, the increase in variance of the flow rate is to be expected due to the decrease in the number of independent samples that results with increasing correlation length (Griffiths & Fenton, 1997). It is also found that the standard deviation of the effective hydraulic conductivity approaches the standard deviation of the arithmetic average for this case.

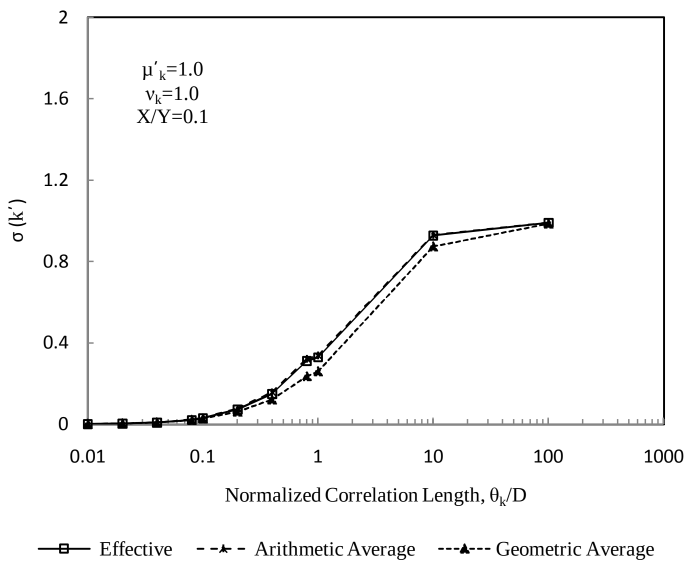

Figure 4.15: Influence of correlation length on the standard deviation of effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 0.1

For example, for μ'_k=1, ν_k=1 and $\dfrac{\theta_k}{D}$=0.8, when X/Y=0.1, $\hat{\sigma}_{k_A}$=0.321, $\hat{\sigma}_{k_G}$=0.236, and $\hat{\sigma}_{k_{eff}}$=0.321.

The influence of correlation length on the standard deviation of the effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 0.3, presented in Figure 4.16, indicates that the standard deviation of the effective hydraulic conductivity trends closer to the standard deviation of the geometric average and away from the standard deviation of the arithmetic average

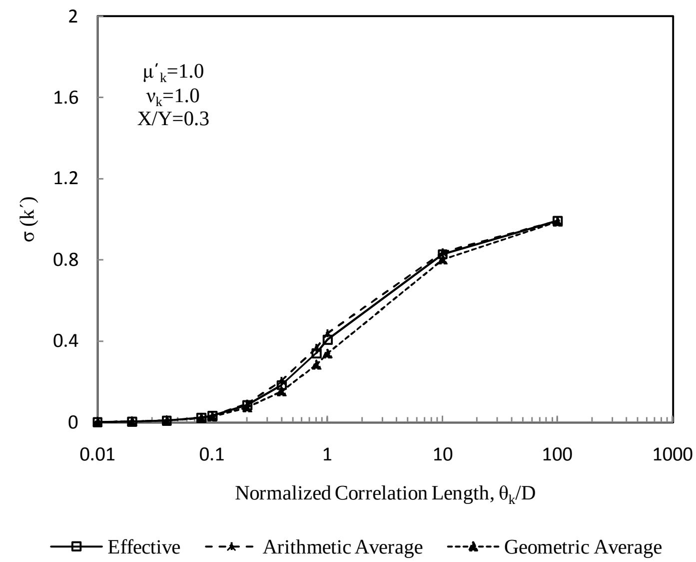

Figure 4.16: Influence of correlation length on the standard deviation of effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 0.3

(i.e., for higher X/Y).
For example, for μ'_k=1, ν_k=1 and $\dfrac{\theta_k}{D}$=0.8, when X/Y=0.3,
$\hat{\sigma}_{k_A}$=0.366, $\hat{\sigma}_{k_G}$=0.284, and $\hat{\sigma}_{k_{eff}}$=0.339.

Similarly, Figure 4.17 presents the results on the influence of correlation length on the standard deviation of the effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 1.0. For this case, the standard deviation of the effective hydraulic conductivity trends even more towards the standard deviation of the geometric average.
For example, for μ'_k=1, ν_k=1 and $\dfrac{\theta_k}{D}$=0.8, when X/Y=1.0,
$\hat{\sigma}_{k_A}$=0.429, $\hat{\sigma}_{k_G}$=0.332, and $\hat{\sigma}_{k_{eff}}$=0.365.

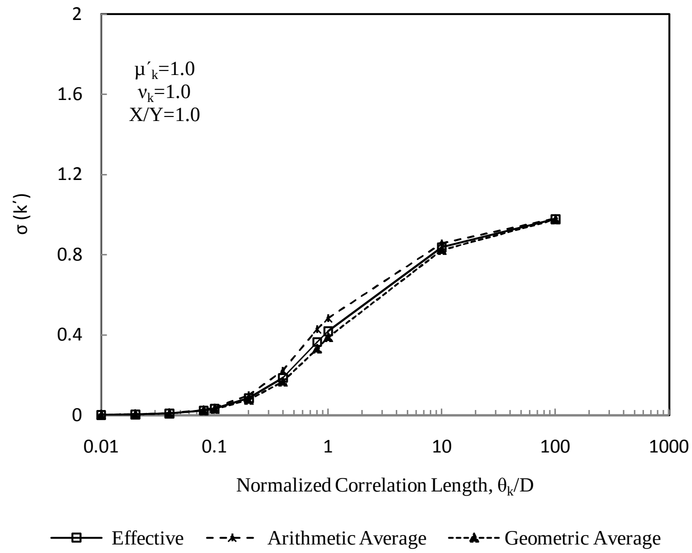

Figure 4.17: Influence of correlation length on the standard deviation of effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 1.0

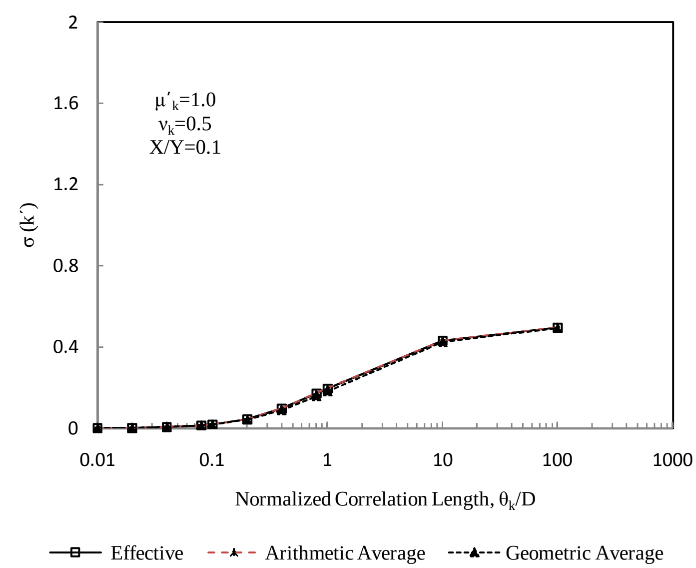

Figure 4.18: Influence of correlation length on the standard deviation of effective hydraulic conductivity for a coefficient of variation of 0.5 and an aspect ratio of liner of 0.1

Figures 4.18 to 4.21 show the influence of correlation length on the standard deviation of the effective hydraulic conductivity for different coefficients of variation

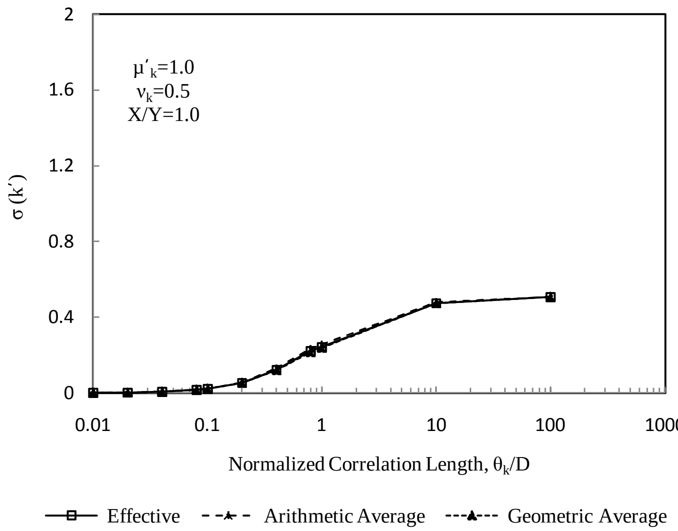

Figure 4.19: Influence of correlation length on the standard deviation of effective hydraulic conductivity for a coefficient of variation of 0.5 and an aspect ratio of liner of 1.0

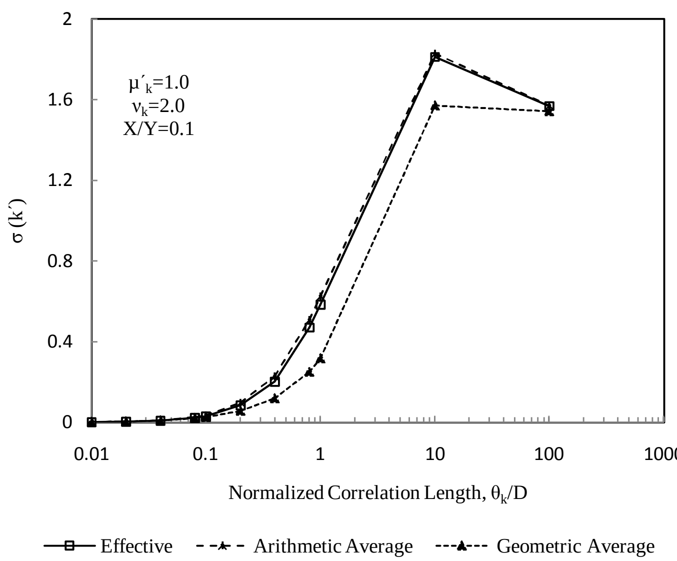

Figure 4.20: Influence of correlation length on the standard deviation of effective hydraulic conductivity for a coefficient of variation of 2.0 and an aspect ratio of liner of 0.1

and aspect ratios of liner. Generally speaking, similar trends as that discussed above for the mean of the effective hydraulic conductivity are obtained.

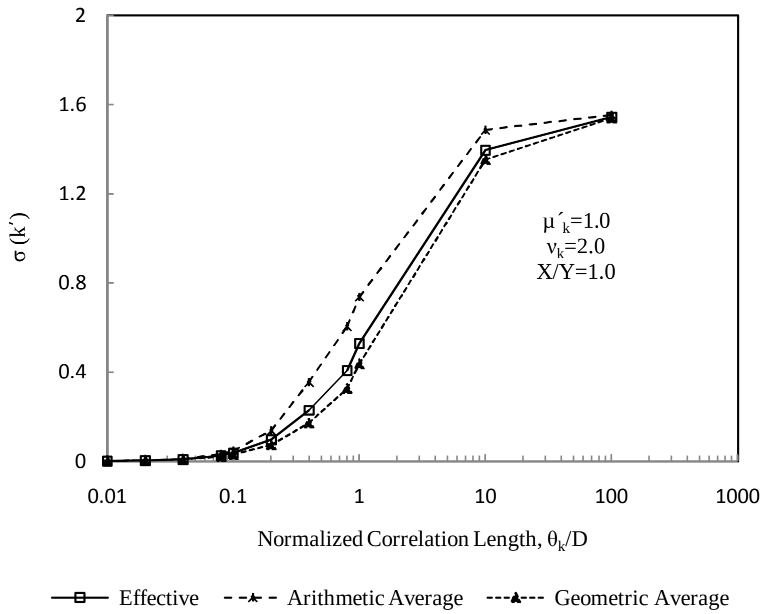

Figure 4.21: Influence of correlation length on the standard deviation of effective hydraulic conductivity for a coefficient of variation of 2.0 and an aspect ratio of liner of 1.0

4.4 Prediction of Standard Deviation of Effective Hydraulic Conductivity

In order to evaluate risk that associates with the flow through a soil liner system, using equation 3.4, one needs to know the standard deviation of the effective hydraulic conductivity. For this reason, prediction of the standard deviation of the effective hydraulic conductivity is made as follows, based on the previous results that as the aspect ratio of liner decreases, the standard deviation of the effective hydraulic conductivity approaches the standard deviation of the arithmetic average and as it increases, the standard deviation of the effective hydraulic conductivity approaches the standard deviation of the geometric average.

$$\hat{\sigma}_{k_{eff}} = e^{-1.21(X/Y)}\hat{\sigma}_{k_A} + \left[1 - e^{-1.21(X/Y)}\right]\hat{\sigma}_{k_G} \tag{4.2}$$

where

$\hat{\sigma}_{k_{eff}}$=estimated standard deviation of the effective hydraulic conductivity from the simulation,

$\hat{\sigma}_{k_A}$=estimated standard deviation of the arithmetic average from the simulation, and

$\hat{\sigma}_{k_G}$=estimated standard deviation of the geometric average from the simulation.

The good agreement obtained between simulated and results predicted by equation 4.2, for different coefficients of variation and aspect ratios of liner, is illustrated in Figures 4.23-4.28.

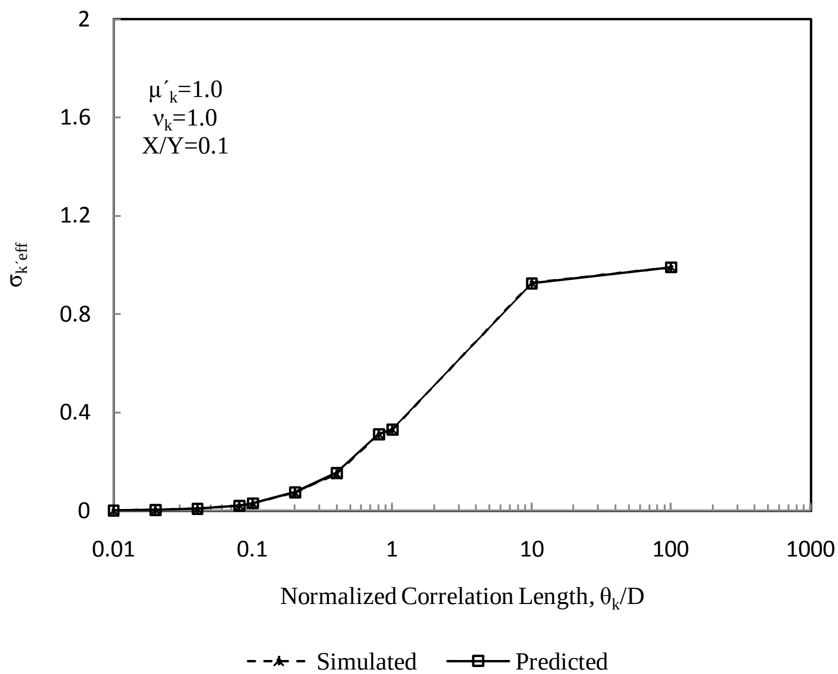

Figure 4.22: Comparison between simulated and predicted standard deviation of effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 0.1

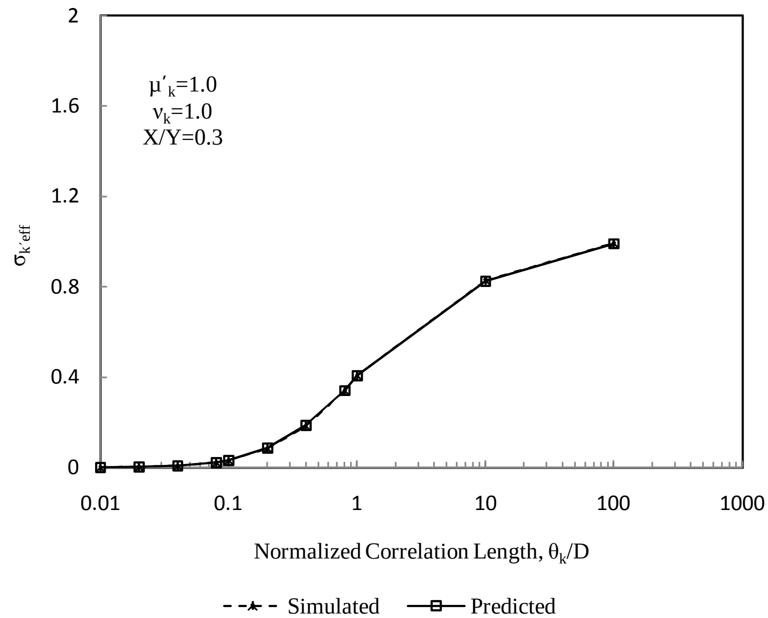

Figure 4.23: Comparison between simulated and predicted standard deviation of effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 0.3

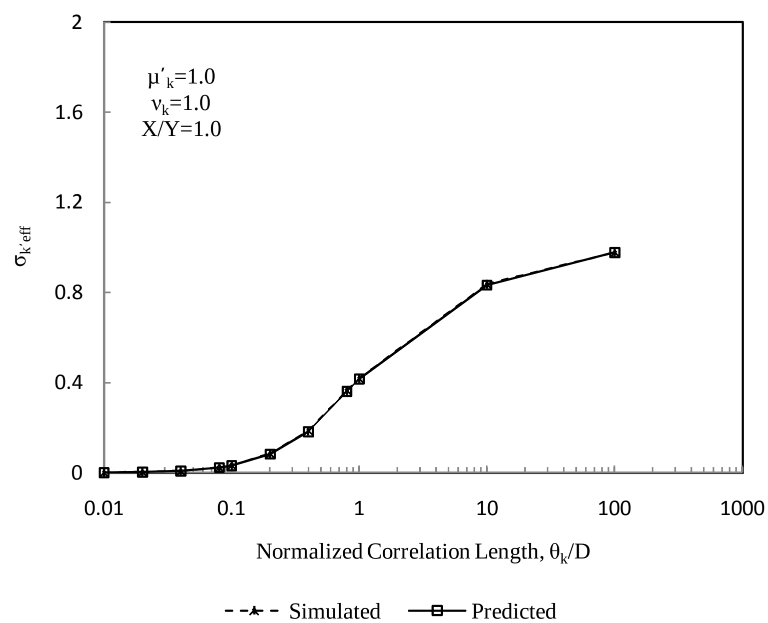

Figure 4.24: Comparison between simulated and predicted standard deviation of effective hydraulic conductivity for a coefficient of variation of 1.0 and an aspect ratio of liner of 1.0

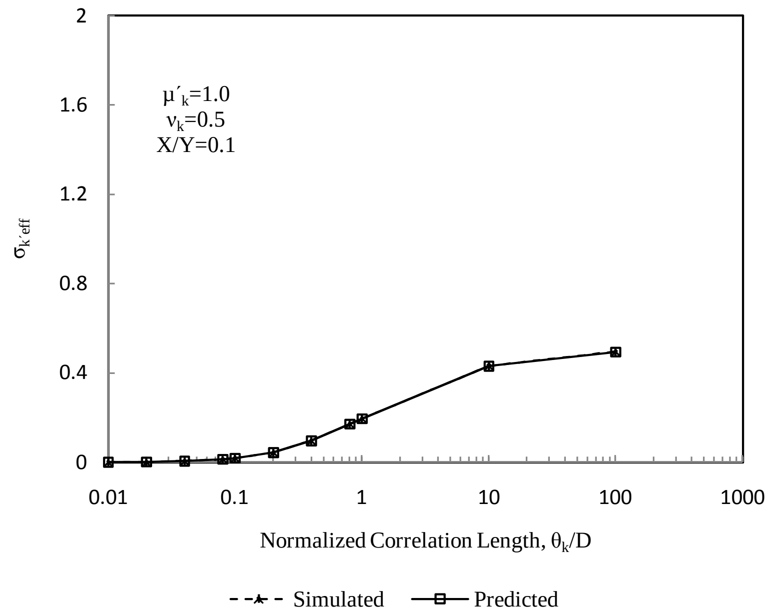

Figure 4.25: Comparison between simulated and predicted standard deviation of effective hydraulic conductivity for a coefficient of variation of 0.5 and aspect ratio of liner of 0.1

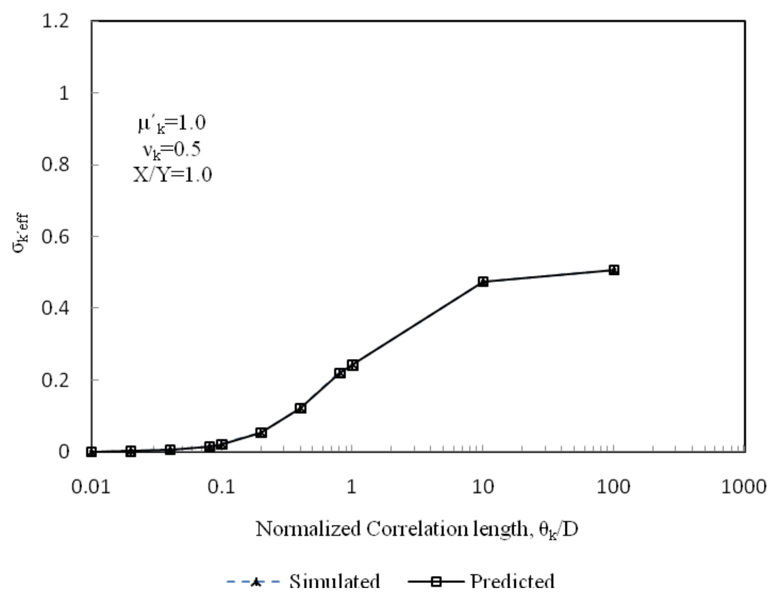

Figure 4.26: Comparison between simulated and predicted standard deviation of effective hydraulic conductivity for a coefficient of variation of 0.5 and an aspect ratio of liner of 1.0

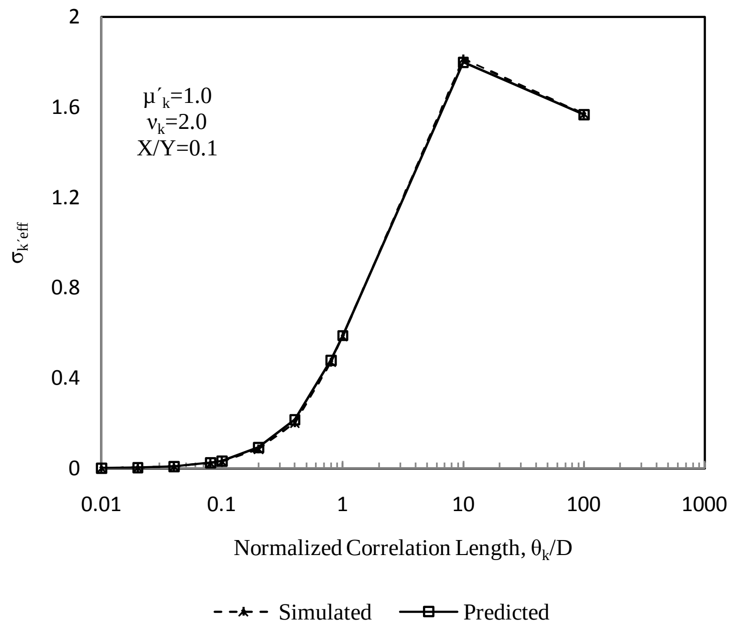

Figure 4.27: Comparison between simulated and predicted standard deviation of effective hydraulic conductivity for a coefficient of variation of 2.0 and an aspect ratio of liner of 0.1

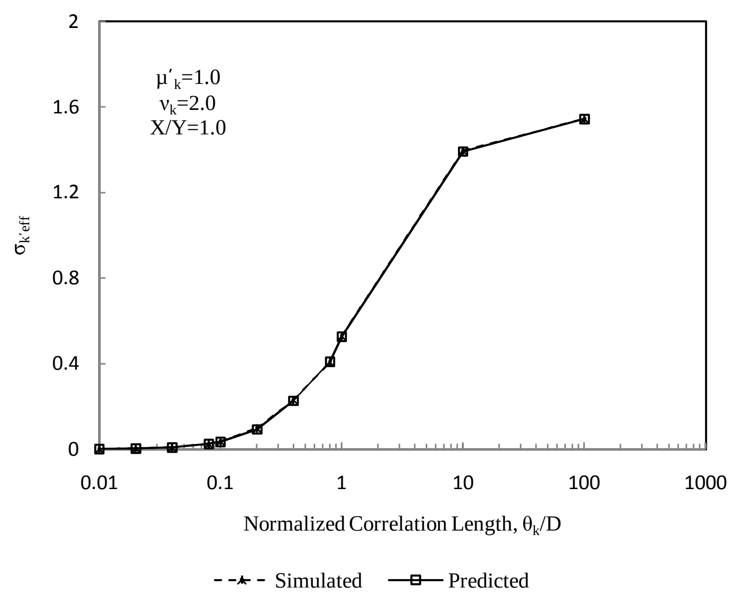

Figure 4.28: Comparison between simulated and predicted standard deviation of effective hydraulic conductivity for a coefficient of variation of 2.0 and an aspect ratio of liner of 1.0

4.5 Comparison between Simulated and Predicted Probability of Exceedance

Simulations were performed to examine the influence of the correlation length on the mean and the standard deviation of the effective hydraulic conductivity. Based on the findings, predictions were made for the mean and the standard deviation of the effective hydraulic conductivity in order to obtain the risk associated with the flow through soil liner systems (i.e., equation 3.4).

Figures 4.29 to 4.35 show comparison between simulated and predicted probability of exceedance for different coefficients of variation and aspect ratios of liner. Each of the figures indicates good agreement between simulated and predicted results.

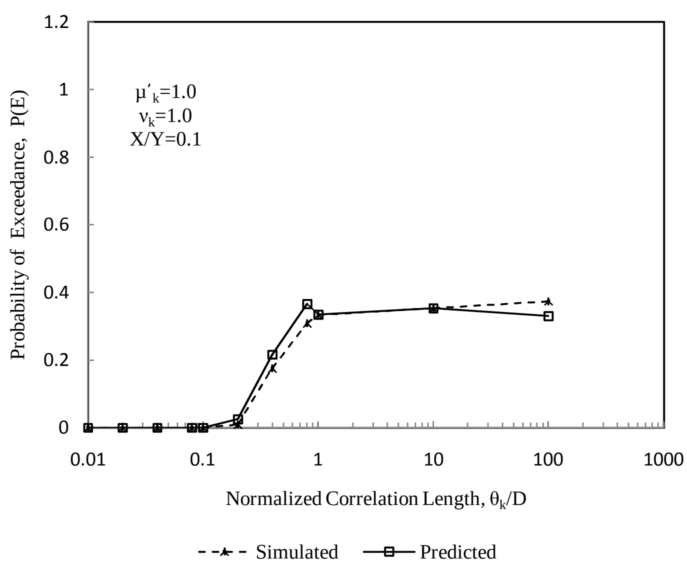

Figure 4.29: Comparison between simulated and predicted probability of exceedance for a coefficient of variation of 1.0 and an aspect ratio of liner of 0.1

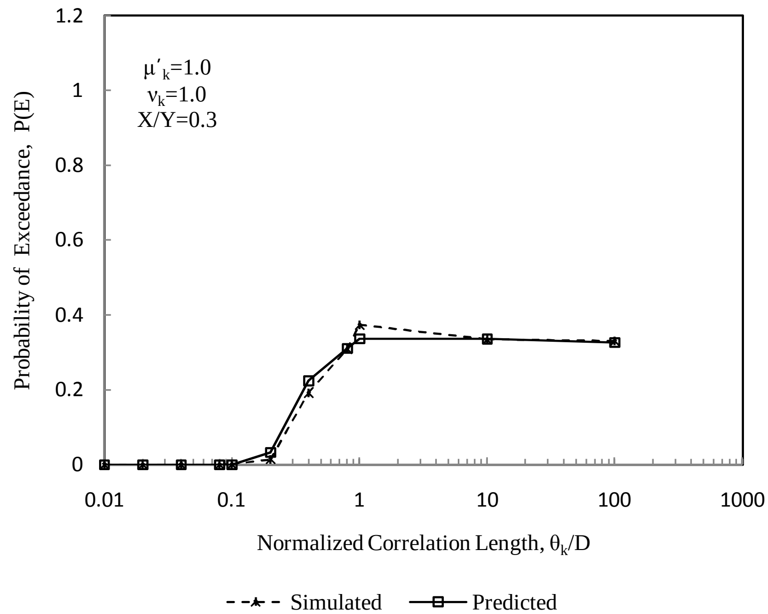

Figure 4.30: Comparison between simulated and predicted probability of exceedance for a coefficient of variation of 1.0 and an aspect ratio of liner of 0.3

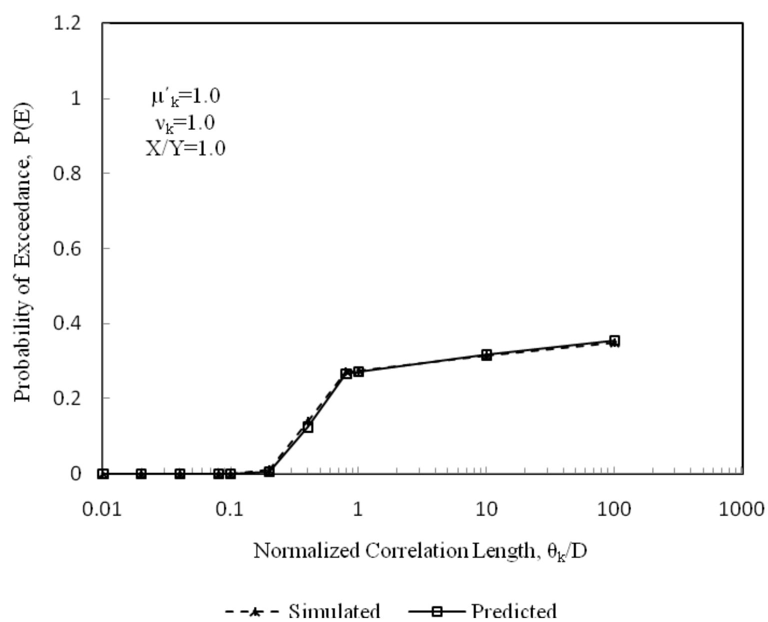

Figure 4.31: Comparison between simulated and predicted probability of exceedance for a coefficient of variation of 1.0 and aspect ratio of liner of 1.0

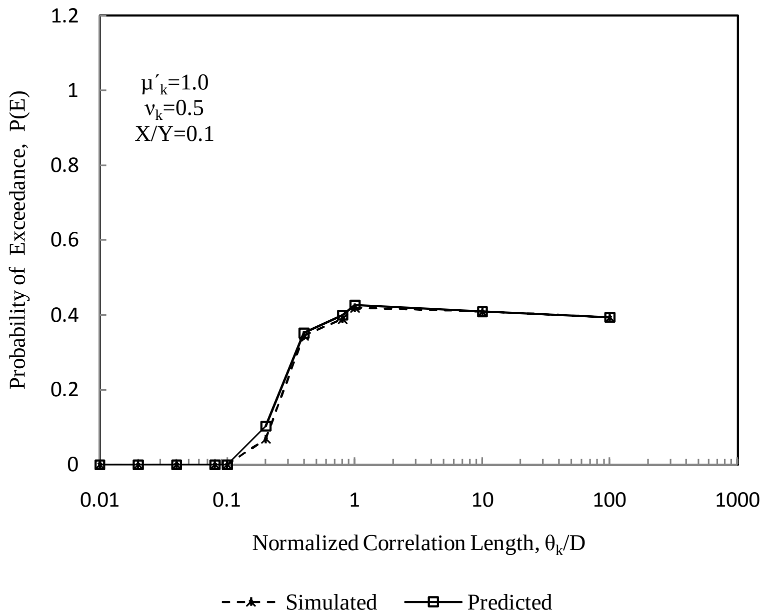

Figure 4.32: Comparison between simulated and predicted probability of exceedance for a coefficient of variation of 0.5 and an aspect ratio of liner of 0.1

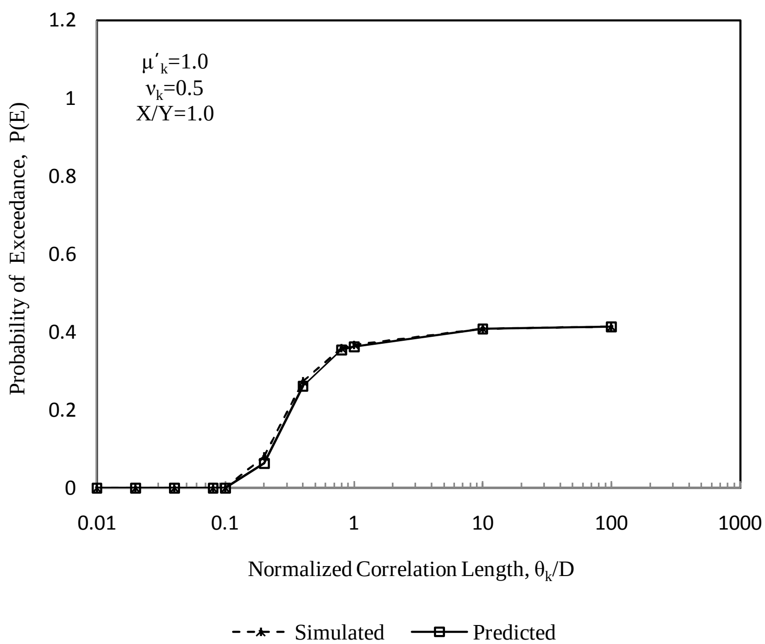

Figure 4.33: Comparison between simulated and predicted probability of exceedance for a coefficient of variation of 0.5 and aspect ratio of liner of 1.0

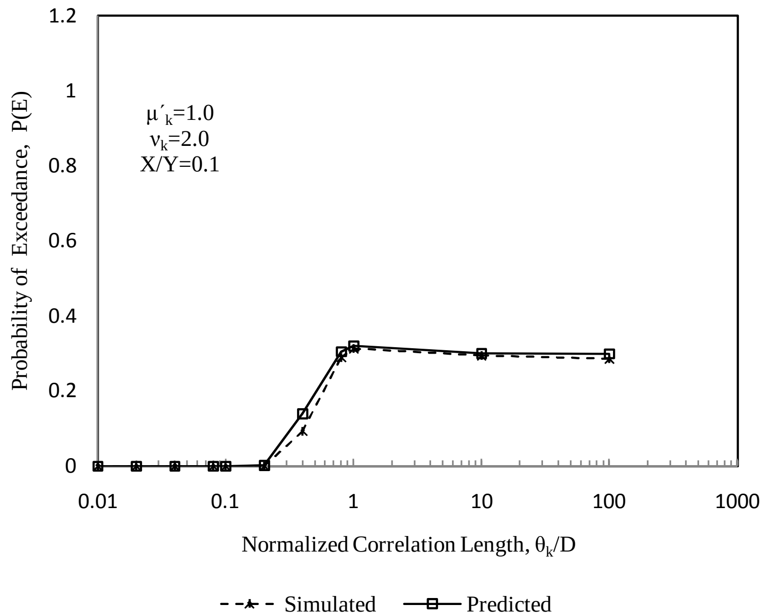

Figure 4.34: Comparison between simulated and predicted probability of exceedance for a coefficient of variation of 2.0 and aspect ratio of liner of 0.1

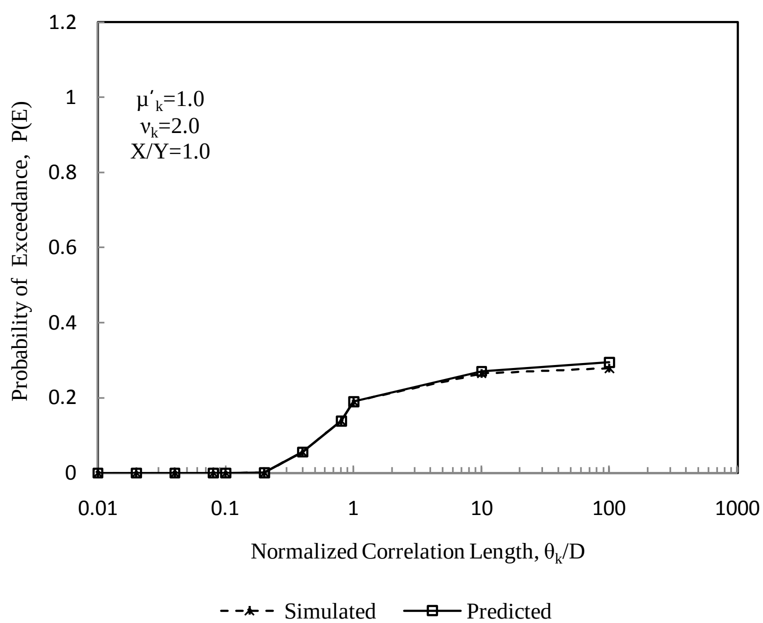

Figure 4.35: Comparison between simulated and predicted probability of exceedance for a coefficient of variation of 2.0 and aspect ratio of liner of 1.0

4.6 Influence of Correlation Length on Probability of Exceedance

The two limiting values for the random field correlation length for hydraulic conductivity are when the correlation length is equal to zero or when it is equal to infinity. When θ_k is equal to 0, points within the field have no correlation with each other. In other words, they are independent (Fenton and Griffiths, 2008). In this case, each local average sample will consist of an infinite number of independent values whose average is a constant. For this limiting case, probability of exceedance would be either 1 or 0, depending on whether the specified mean hydraulic conductivity is above or below regulatory hydraulic conductivity, k_{crit}. For the other extreme of when θ_k is equal to infinity, points within the random field are perfectly correlated with each other. In other words, for a particular realization, the field can be represented by a single hydraulic conductivity value. For this case, the mean and standard deviation of the effective hydraulic conductivity will be the specified mean and standard deviation of point-scale hydraulic conductivity.

In this section, the results related to the influence of the correlation length on the probability of exceedance for different specified means and coefficients of variation of point-scale hydraulic conductivity and aspect ratios of liners are presented. All results are based on the predictions made in the previous sections for the mean and standard deviation of the effective hydraulic conductivity. Figure 4.36 shows the influence of the correlation length on the probability of exceedance for different aspect ratios of liner. The specified mean and coefficient of variation of point-scale hydraulic conductivity for all cases are 1.0. For μ_k' and ν_k of 1.0 and a particular X/Y, each point on the plot is the calculation for a particular value of $\frac{\theta_k}{D}$.

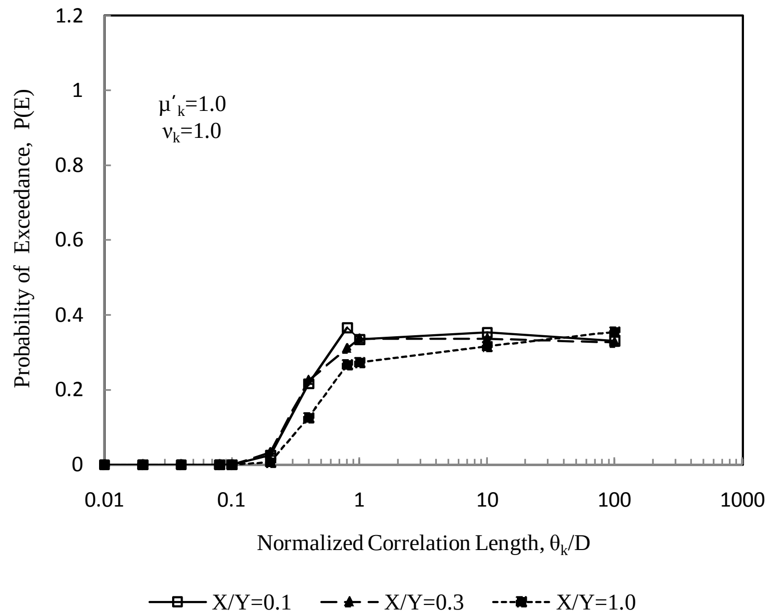

Figure 4.36: Influence of correlation length on probability of exceedance for different aspect ratios of liner

Figure 4.36 indicates that as the correlation length increases, probability of exceedance of soil liner increases due to increasing mean and standard deviation of effective hydraulic conductivity. The deviations in the increasing trend due to the sampling error can be avoided by taking more than 1,000 realizations. The increase in probability of exceedance with the correlation length is as expected. Because increase in the correlation length of hydraulic conductivity increases the uniformity of the random field and the flow finds less low-k zones along the flow path. The consequence of this is a higher probability of exceedance. For μ_k' of 1.0, the results for all aspect ratios show expected probability of exceedance of zero at correlation length of 0.01. For an aspect ratio of 0.1, worst case correlation length (correlation length at which the probability of exceedance is maximum) is obtained at a value of 0.8. For all other aspect ratios, at higher correlation length, probability of exceedance is tending towards the limiting value of 0.339 which results from the mean and standard deviation of effective hydraulic conductivity of 1.0 when θ_k is equal to infinity. Results also indicate that the probability of exceedance decreases with increasing aspect ratio of liner.

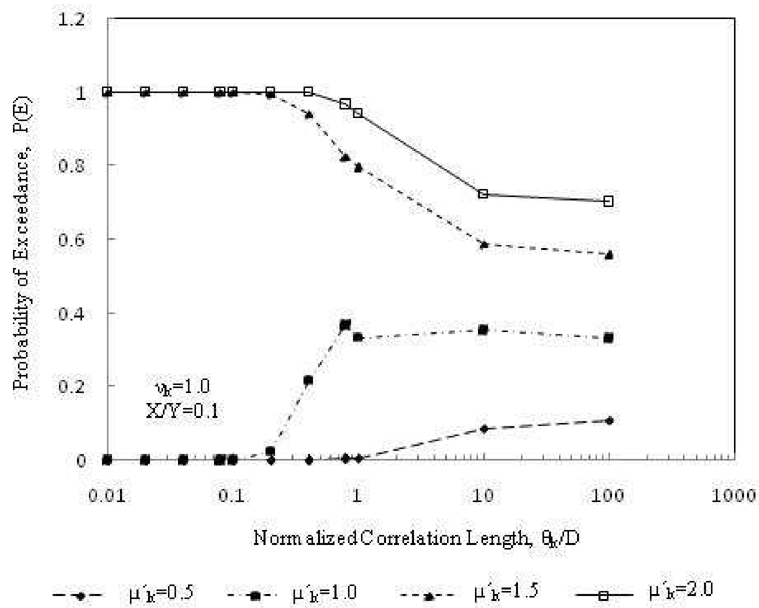

Figure 4.37: Influence of correlation length on probability of exceedance for different means

Figure 4.37 presents the results for the influence of the correlation length on the probability of exceedance for different means, μ'_k. Each point on the plots is the predicted result for a coefficient of variation of 1.0, aspect ratio of liner of 0.1 and a particular mean. When μ'_k=0.5 or 1.0, an increasing trend in the probability of exceedance with the correlation length is obtained. For a mean of 1.5 and 2.0 times of the regulatory hydraulic conductivity, the probability of exceedance decreases from 1 (as expected for the limiting case at $\theta'_k = 0$) as the correlation length increases. Figure 4.37 also indicates that for a particular correlation length, the probability of exceedance increases as the mean increases. This is as expected because increasing mean increases the probability of high-k zones along the flow-paths.

The influence of the correlation length on the probability of exceedance for different coefficients of variation is shown in Fig.4.38. The mean and aspect ratio of liner for this case are 1.0 and 0.1 respectively. For this case, the probability of exceedance increases with increasing correlation length. For a particular

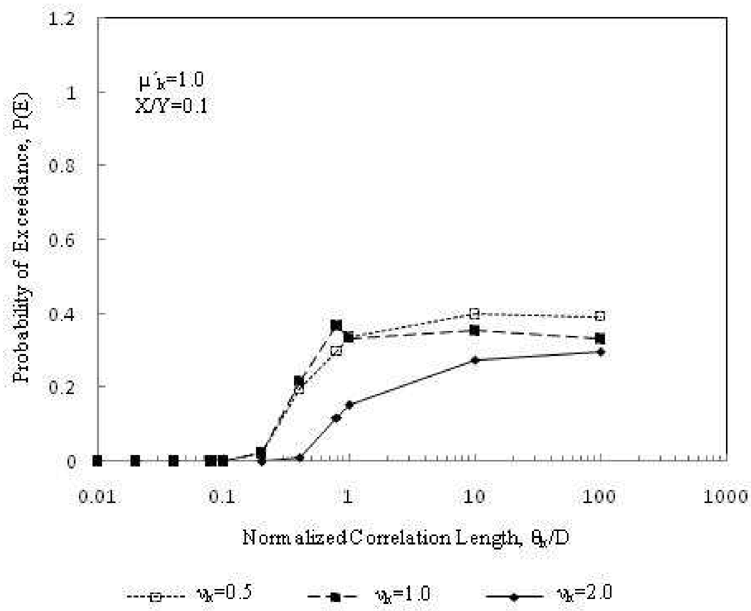

Figure 4.38: Influence of correlation length on probability of exceedance for different coefficients of variations

correlation length, probability of exceedance decreases with increasing coefficient of variation. This is as expected, because for a particular mean, an increase in the coefficient of variation increases the positive skew of a lognormal hydraulic conductivity distribution, which results in an increase in the probability of low hydraulic conductivities more compared to the increase in the probability of high hydraulic conductivities. Thus, a small change in the probability of high k_{eff} and a large change in the probability of low k_{eff} results in an increase in the coefficient of variation of hydraulic conductivity (Benson & Daniel, 1994a). This consequences decrease in probability of exceedance with increasing coefficient of variation.

4.7 Predicted Probability of Exceedance Based on the Simulated and Analytical Parameters

In this study, predictions have made for the mean and standard deviation of the effective hydraulic conductivity using simulation derived sample means and standard deviations according to equations 4.1 and 4.2. The mean and standard deviation of arithmetic and geometric averages can be estimated analytically, which allows the mean and standard deviation of effective hydraulic conductivity to be obtained without the need of simulation. To this end, equations 4.1 and 4.2 can be modified to become

$$\mu_{k_{eff}} = e^{-1.21(X/Y)}\mu_{k_A} + \left[1 - e^{-1.21(X/Y)}\right]\mu_{k_G} \tag{4.3}$$

$$\sigma_{k_{eff}} = e^{-1.21(X/Y)}\sigma_{k_A} + \left[1 - e^{-1.21(X/Y)}\right]\sigma_{k_G} \tag{4.4}$$

According to Fenton and Griffiths (2008), predictions of μ_{k_A}, μ_{k_G} , σ_{k_A}, and σ_{k_G} can be made analytically as follows:

$$\mu_{k_A} = \mu_k \tag{4.5}$$

$$\mu_{k_G} = e^{\left(\mu_{\ln k_G} + \frac{1}{2}\sigma^2_{\ln k_G}\right)} \tag{4.6}$$

$$\sigma_{k_A} = \sqrt{\gamma\left(T_1, T_2, T_3\right)}\sigma_k \tag{4.7}$$

$$\sigma_{k_G} = \sqrt{\left[e^{\left(2\mu_{\ln k_G} + \sigma^2_{\ln k_G}\right)}\right]\left[e^{\sigma^2_{\ln k_G}} - 1\right]} \tag{4.8}$$

where

$$\mu_{\ln k_G} = \ln \mu_k - \frac{1}{2}\ln(1 + \nu_k^2)$$

$$\sigma_{\ln k_G} = \sqrt{\gamma(T_1, T_2, T_3)}\sigma_{\ln k}$$

$\gamma\left(T_1, T_2, T_3\right)$ is the three-dimensional Markovian variance function. It is calculated here considering correlation structure to be separable so that the variance function is also separable.

$$\gamma\left(T_1, T_2, T_3\right) = \gamma\left(T_1\right)\gamma\left(T_2\right)\gamma\left(T_3\right) \tag{4.9}$$

where

$$\gamma\left(T_i\right) = \frac{\theta_i^2}{2T_i^2}\left[\frac{2|T_i|}{\theta_i} + \exp\left\{-\frac{2|T_i|}{\theta_i}\right\} - 1\right]$$

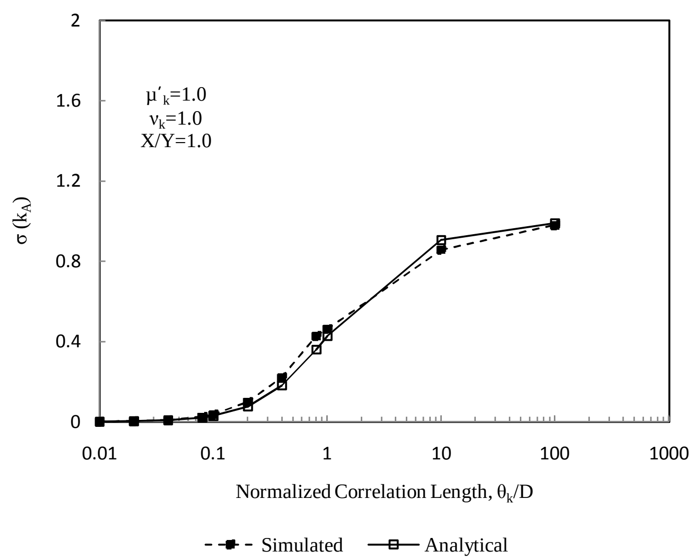

Figure 4.39: Comparison between simulated and analytical standard deviation of the arithmetic average

An example can be given for the agreement between the simulated, $\hat{\sigma}_{k_A}$, and the analytical, σ_{k_A}, for aspect ratio of liner of 1.0, which is shown in Figure 4.39. Nice agreement between the simulated and analytical results is obtained for this case.

Figure 4.40 shows nice agreement between the simulated, $\hat{\sigma}_{k_G}$, and the analytical, σ_{k_G}.

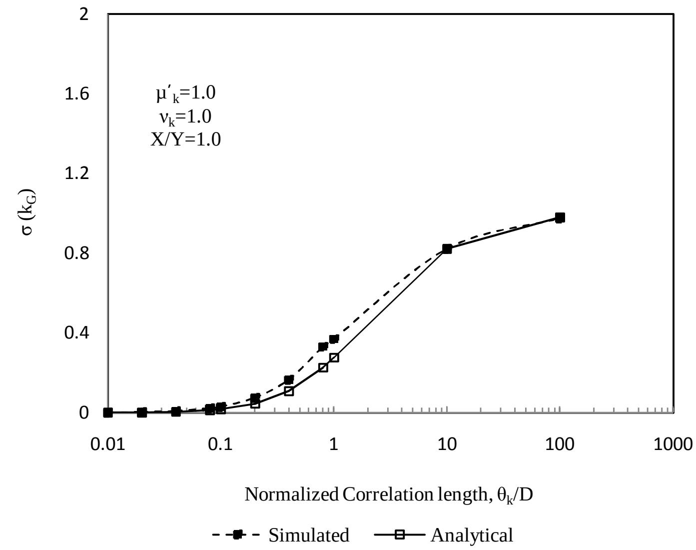

Figure 4.40: Comparison between simulated and analytical standard deviation of the geometric average

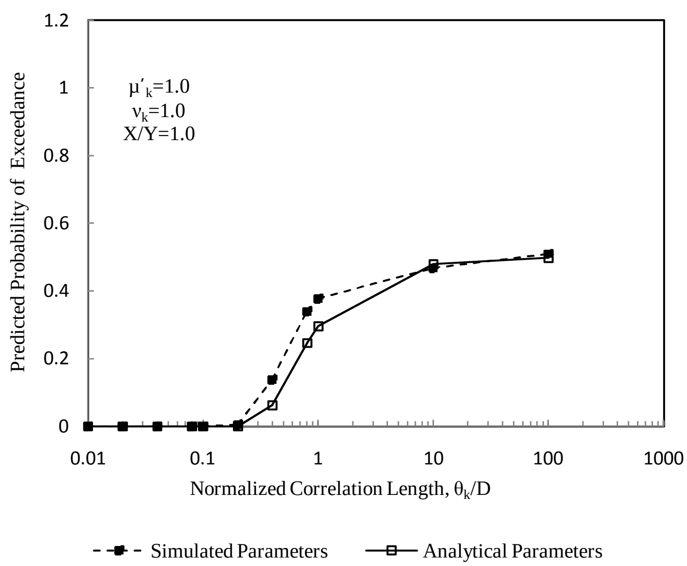

Figure 4.41: Predicted probability of exceedance for simulated and analytical parameters

Figure 4.41 shows good agreement between the predicted probability of exceedance calculated using equation 3.4, based on the simulated, $\hat{\mu}_{k_A}$, $\hat{\mu}_{k_G}$, $\hat{\sigma}_{k_A}$, and $\hat{\sigma}_{k_G}$

and the analytical μ_{k_A}, μ_{k_G} , σ_{k_A}, and σ_{k_G}.

4.8 Comparison between 3-D and 2-D Probability of Exceedance

Simulations are performed using the two-dimensional model, mrflow2d, to investigate if two-dimensions are sufficient for modeling purposes. The advantage of two-dimensional modeling is that it requires less computing time relative to three-dimensional modeling. Figure 4.42 shows comparison between 3-D and 2-D

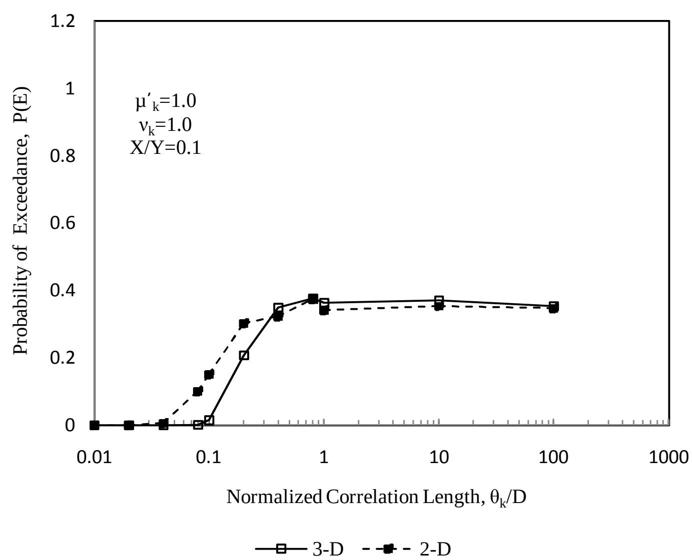

Figure 4.42: Comparison between 3-D and 2-D probability of exceedance for aspect ratio of liner of 0.1

probability of exceedance vs. correlation length for an aspect ratio of liner of 0.1, Figures 4.43 and 4.44 show the same comparison for aspect ratios of 0.3 and 1.0 respectively.

Results show that for a particular correlation length, 3-D case gives similar results of probability of exceedance as that for 2-D case for all cases of aspect ratios of liner. The exception to this statement may be at smaller correlation lengths for an aspect ratio of 0.1. A comparison between 3-D and 2-D results for the mean and standard deviation of the effective hydraulic conductivity for all aspect ratios

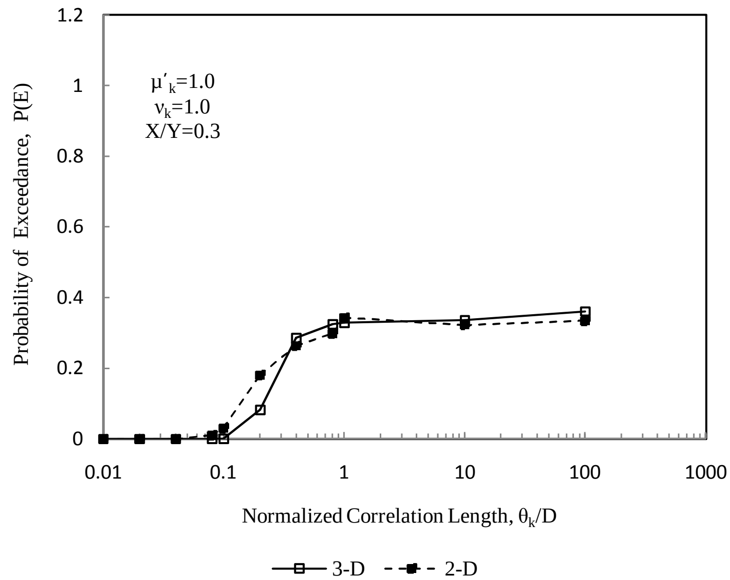

Figure 4.43: Comparison between 3-D and 2-D probability of exceedance for aspect ratio of liner of 0.3

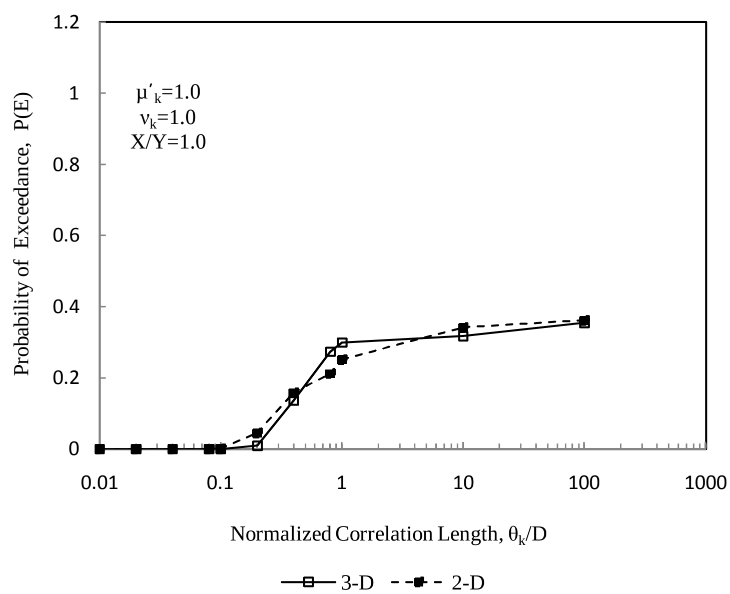

Figure 4.44: Comparison between 3-D and 2-D probability of exceedance for aspect ratio of liner of 1.0

are included in Appendix B.

4.9 Using The Proposed Method for Calculating Probability of Exceedance: Two Examples

Two examples can be given to clarify the proposed method in this study to evaluate the risk (i.e., probability of exceedance) associated with a soil liner.

Example 1

Consider a liner to be constructed that has a plan area of 100 m$\times$100 m. Testing on readily available clay soil revealed mean hydraulic conductivity to be 2×10^{-9} m/s with a coefficient of variation of 2.0. The correlation length is assumed to be $\theta_k = 3$ m in all three directions. The regulatory requirement for the liner construction is X=1 m thick clayey liner with hydraulic conductivity of 1×10^{-9} m/s. It is necessary to quantify the risk associated with the proposed liner.

Given the mean and coefficient of variation of point-scale hydraulic conductivity, the standard deviation and mean of log-k of the clayey liner are as follows:

$$
\begin{aligned}
\sigma_{\ln k} &= \sqrt{\ln(1+\nu_k^2)} \\
&= \sqrt{\ln(1+2^2)} \\
&= 1.269
\end{aligned}
$$

$$
\begin{aligned}
\mu_{\ln k} &= \ln \mu_k - \frac{1}{2}\sigma_{\ln k}^2 \\
&= \ln(2 \times 10^{-9}) - \frac{1}{2}(1.269)^2 \\
&= -20.835
\end{aligned}
$$

Using $\gamma(T_1 \times T_2 \times T_2) = \gamma(T_1) \times \gamma(T_2) \times \gamma(T_3)$ where $T_1 = T_2$=100 and T_3=1 and $\gamma\left(T_i\right) = \frac{\theta_i^2}{2T_i^2}\left[\frac{2|T_i|}{\theta_i} + \exp\left\{-\frac{2|T_i|}{\theta_i}\right\} - 1\right]$, the variance function is calculated as 7.08×10^{-4}.

The geometric average of point-scale hydraulic conductivity can be calculated to be,

$$\begin{aligned} k_G &= e^{\mu_{\ln k}} \\ &= e^{-20.835} \\ &= 8.944 \times 10^{-10} m/s \end{aligned}$$

The mean and standard deviation of the arithmetic and geometric averages of point-scale hydraulic conductivity can be calculated as follows:

$$\begin{aligned} \mu_{k_A} &= \mu_k \\ &= 2 \times 10^{-9} m/s \end{aligned}$$

$$\begin{aligned} \mu_{k_G} &= \exp\left\{ \mu_{\ln k_G} + \frac{1}{2}\sigma^2_{\ln k_G} \right\} \\ &= 8.95 \times 10^{-10} m/s \end{aligned}$$

where

$$\begin{aligned} \mu_{\ln k_G} &= \ln \mu_k - \frac{1}{2}\ln(1+\nu_k^2) \\ &= \ln(2 \times 10^{-9}) - \frac{1}{2}\ln(1+2^2) \\ &= -20.83 \end{aligned}$$

$$\begin{aligned} \sigma_{\ln k_G} &= \sqrt{\gamma(T_1, T_2, T_3)}\sigma_{\ln k} \\ &= \sqrt{(7.08 \times 10^{-4})}(1.269) \\ &= 0.0337 \end{aligned}$$

$$\begin{aligned}\sigma_{k_A} &= \sqrt{\gamma(T_1,T_2,T_3)}\sigma_k \\ &= \sqrt{\gamma(T_1,T_2,T_3)}(\nu_k \times \mu_k) \\ &= \sqrt{(7.08\times10^{-4})}(2\times 2\times 10^{-9}) \\ &= 1.06\times10^{-10}\end{aligned}$$

$$\begin{aligned}\sigma_{k_G} &= \sqrt{\left[e^{\left(2\mu_{\ln k_G}+\sigma^2_{\ln k_G}\right)}\right]\left[e^{\sigma^2_{\ln k_G}}-1\right]} \\ &= \sqrt{[e^{2(-20.83)+(0.0337)^2}]\,[e^{(0.0337)^2}-1]} \\ &= 3.02\times10^{-11}\end{aligned}$$

The mean and standard deviation of the effective hydraulic conductivity can be calculated as follows:

$$\begin{aligned}\mu_{k_{eff}} &= e^{-1.21(X/Y)}\mu_{k_A} + \left[1-e^{-1.21(X/Y)}\right]\mu_{k_G} \\ &= e^{-1.21(1/100)}(2\times10^{-9}) + \left[1-e^{-1.21(1/100)}\right](8.95\times10^{-10}) \\ &= 1.986\times10^{-9} m/s\end{aligned}$$

$$\begin{aligned}\sigma_{k_{eff}} &= e^{-1.21(X/Y)}\sigma_{k_A} + \left[1-e^{-1.21(X/Y)}\right]\sigma_{k_G} \\ &= e^{-1.21(1/100)}(1.06\times10^{-10}) + \left[1-e^{-1.21(1/100)}\right](3.02\times10^{-11}) \\ &= 1.055\times10^{-10}\end{aligned}$$

The standard deviation and mean of log-effective hydraulic conductivity can be calculated as,

$$\begin{aligned}
\sigma_{\ln k_{eff}} &= \sqrt{\ln(1+\nu_{k_{eff}}^2)} \\
&= \ln\left[1+\left(\frac{1.055\times10^{-10}}{1.986\times10^{-9}}\right)^2\right] \\
&= 0.05306
\end{aligned}$$

$$\begin{aligned}
\mu_{\ln k_{eff}} &= \ln \mu_{k_{eff}} - \frac{1}{2}\sigma_{\ln k_{eff}}^2 \\
&= \ln(1.986\times10^{-9}) - \frac{1}{2}(0.05306)^2 \\
&= -20.04
\end{aligned}$$

The probability of exceedance can be calculated as:

$$\begin{aligned}
P(E) &= 1-\Phi\left[\frac{\ln(k_{crit})-\mu_{\ln k_{eff}}}{\sigma_{\ln k_{eff}}}\right] \\
&= 1-\Phi\left[\frac{\ln(1)-(-20.04)}{0.05306}\right] \\
&= 1.0
\end{aligned}$$

Example 2

It is desired to quantify the risk associated with the flow though a soil liner having a size of 10 m×10 m×10 m, to be used under a 10 m×10 m leachate lagoon. All other data related to the clay will remain same as in example 1.

Given the mean and coefficient of variation of point-scale hydraulic conductivity, the mean and standard deviation of log-k of the clayey liner are as in the previous example,

$$\mu_{\ln k} = -20.835$$

$$\sigma_{\ln k} = 1.269$$

Using $\gamma(T_1 \times T_2 \times T_2) = \gamma(T_1) \times \gamma(T_2) \times \gamma(T_3)$ where $T_1 = T_2 = T_3$=10 and $\gamma\left(T_i\right) = \dfrac{\theta_i^2}{2T_i^2}\left[\dfrac{2|T_i|}{\theta_i} + \exp\left\{-\dfrac{2|T_i|}{\theta_i}\right\} - 1\right]$, the variance function is calculated as 0.0527.

The geometric mean of point-scale hydraulic conductivity can be calculated as,

$$k_G = 8.94 \times 10^{-10} m/s$$

The mean and standard deviation of the arithmetic and geometric averages of point-scale hydraulic conductivity can be calculated as follows:

$$\mu_{k_A} = 2 \times 10^{-9} m/s$$

$$\begin{aligned} \mu_{k_G} &= \exp\left\{\mu_{\ln k_G} + \frac{1}{2}\sigma^2_{\ln k_G}\right\} \\ &= 9.33 \times 10^{-10} m/s \end{aligned}$$

$$\begin{aligned}
\sigma_{k_A} &= \sqrt{\gamma(T_1, T_2, T_3)}\sigma_k \\
&= (0.0527)(2 \times 3 \times 10^{-9}) \\
&= 9.184 \times 10^{-10}
\end{aligned}$$

$$\begin{aligned}
\sigma_{k_G} &= \sqrt{\left[e^{\left(2\mu_{\ln k_G} + \sigma^2_{\ln k_G}\right)}\right]\left[e^{\sigma^2_{\ln k_G}} - 1\right]} \\
&= 2.777 \times 10^{-10}
\end{aligned}$$

The mean and standard deviation of the effective hydraulic conductivity can be calculated as follows,

$$\begin{aligned}
\mu_{k_{eff}} &= e^{-1.21(X/Y)}\mu_{k_A} + \left[1 - e^{-1.21(X/Y)}\right]\mu_{k_G} \\
&= e^{-1.21(10/10)}(2 \times 10^{-9}) + \left[1 - e^{-1.21(10/10)}\right](9.33 \times 10^{-10}) \\
&= 1.25 \times 10^{-9} m/s
\end{aligned}$$

$$\begin{aligned}
\sigma_{k_{eff}} &= e^{-1.21(X/Y)}\sigma_{k_A} + \left[1 - e^{-1.21(X/Y)}\right]\sigma_{k_G} \\
&= e^{-1.21(10/10)}(9.184 \times 10^{-10}) + \left[1 - e^{-1.21(10/10)}\right](2.777 \times 10^{-10}) \\
&= 9.11 \times 10^{-10}
\end{aligned}$$

The mean and standard deviation of log-effective hydraulic conductivity can be calculated as,

$$\begin{aligned}
\mu_{\ln k_{eff}} &= \ln \mu_{k_{eff}} - \frac{1}{2}\sigma^2_{\ln k_{eff}} \\
&= \ln(1.25 \times 10^{-9}) - \frac{1}{2}(9.11 \times 10^{-10})^2 \\
&= -20.71
\end{aligned}$$

$$\begin{aligned}\sigma_{\ln k_{eff}} &= \sqrt{\ln(1+\nu_{k_{eff}}^2)} \\ &= 0.507\end{aligned}$$

Probability of exceedance can be calculated as:

$$\begin{aligned}P(E) &= 1 - \Phi\left[\frac{\ln(k_{crit}) - \mu_{\ln k_{eff}}}{\sigma_{\ln k_{eff}}}\right] \\ &= 0.507\end{aligned}$$

Chapter 5

Conclusion

5.1 Summary and Conclusions

In this study, Monte Carlo simulations were performed using a random field finite element model, mrflow3d, to evaluate the risk associated with the flow through soil liner systems. In this regard, an attempt was first made to determine the proper average to use to characterize the effective hydraulic conductivity, which in turn, characterizes the total flow rate through a saturated soil liner. A similar prediction was also made for the standard deviation of the effective hydraulic conductivity. The influence of the correlation length was considered in obtaining these predictions. The risk that soil liner system fails to perform adequately was evaluated using the predicted distribution of the effective hydraulic conductivity. Using this proposed method, designers will be able to evaluate the risk associated with the flow through a soil liner at the design stage on set limits on the variability expected for the construction.

Based on the results obtained in this study, the following conclusions can be drawn:

- The mean of the effective hydraulic conductivity increases with a increase in the correlation length and a decrease in the aspect ratio of the liner.

- A prediction for the mean effective hydraulic conductivity is,

$$\hat{\mu}_{k_{eff}} = e^{-1.21(X/Y)}\hat{\mu}_{k_A} + \left[1 - e^{-1.21(X/Y)}\right]\hat{\mu}_{k_G}$$

- The standard deviation of the effective hydraulic conductivity increases with an increase in the correlation length and a decrease in the aspect ratio of liner.

- A prediction for the standard deviation of the effective hydraulic conductivity is,

$$\hat{\sigma}_{k_{eff}} = e^{-1.21(X/Y)}\hat{\sigma}_{k_A} + \left[1 - e^{-1.21(X/Y)}\right]\hat{\sigma}_{k_G}$$

- The probability of exceedance increases with increasing correlation length of hydraulic conductivity.

- The probability of exceedance increases with increasing hydraulic conductivity mean.

- The probability of exceedance increases with decreasing liner aspect ratio.

- The probability of exceedance increases with decreasing hydraulic conductivity variance.

- A comparison between 3-D and 2-D case for the probability of exceedance indicates that the 2-D model is a reasonable approximation.

5.2 Recommendations for further study

Although reliability of soil liner is an issue of research for over two decades (Bogardi et al., 1989, 1990; Benson and Charbeneau, 1991; Benson et al., 1994; Benson and Daniel, 1994a, 1994b; Benson et al. 1999) there are still some issues which should be included into future research, such as,

- This study assumes equal correlation length in all three directions. But for a layered soil mass, the horizontal correlation length are generally larger than the vertical correlation length due to the natural stratification of many soil deposits. Consideration of this anisotropy in the correlation length in future research will be more rational.

- Future study should examine the influence of sampling on the probability of exceedance.

- In further study, uncertainty in hydraulic gradient across the liner could be considered to evaluate the probability of exceedance associated with the flow through soil liner systems.

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Appendix A

Influence of Element Size on Results

Table A.1: Sensitivity Analysis

Element Size	Effective Hydraulic Conductivity		Arithmetic Average		Geometric Average		Time (hr)
	μ	σ	μ	σ	μ	σ	
0.05 × 0.05 × 0.05	0.818	0.236	0.968	0.280	0.744	0.211	0.62
0.0417 × 0.0417 × 0.0417	0.812	0.229	0.968	0.276	0.739	0.205	3.83
0.0357 × 0.0357 × 0.0357	0.815	0.232	0.974	0.283	0.739	0.207	10.16
0.03125 × 0.03125 × 0.03125	0.812	0.235	0.971	0.283	0.736	0.208	15.26

Appendix B

Statistics of Effective Hydraulic Conductivity: 3D vs. 2D

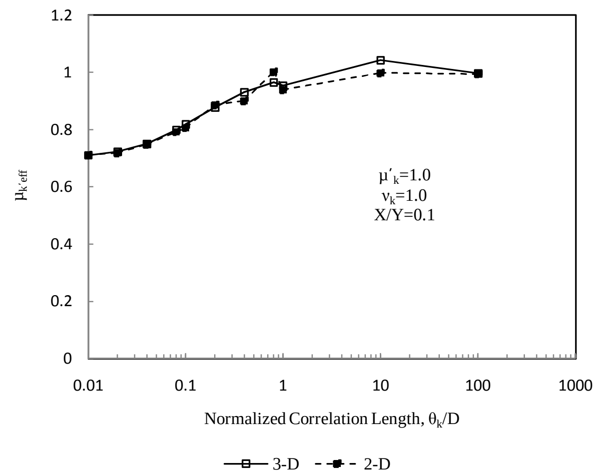

Figure B.1: Comparison between 3-D and 2-D mean of effective hydraulic conductivity for aspect ratio of liner of 0.1

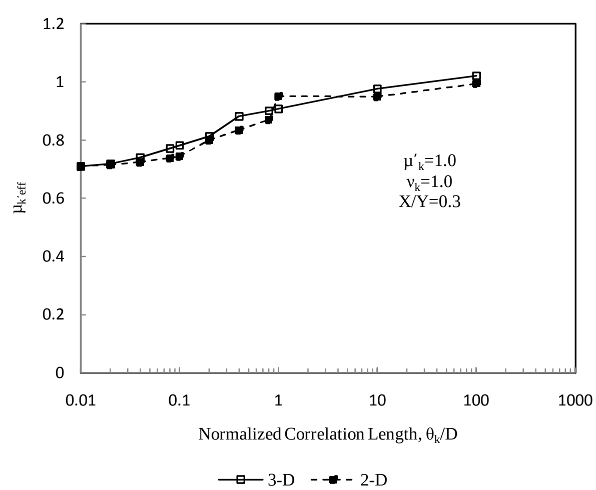

Figure B.2: Comparison between 3-D and 2-D mean of effective hydraulic conductivity for aspect ratio of liner of 0.3

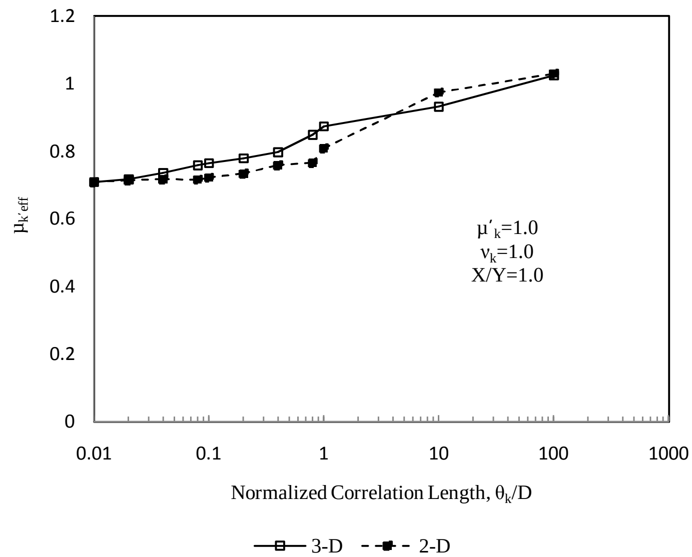

Figure B.3: Comparison between 3-D and 2-D mean of effective hydraulic conductivity for aspect ratio of liner of 1.0

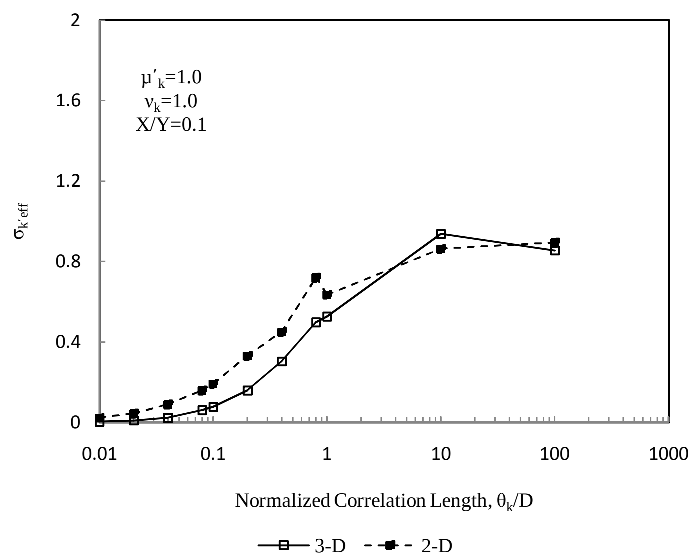

Figure B.4: Comparison between 3-D and 2-D standard deviation of effective hydraulic conductivity for aspect ratio of liner of 0.1

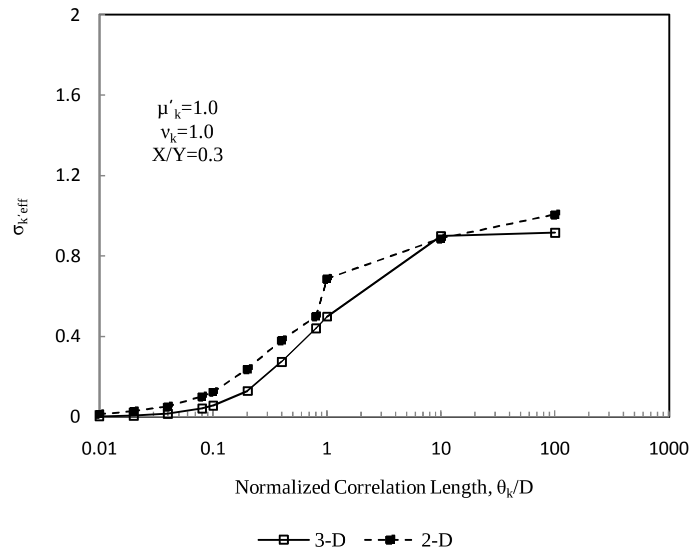

Figure B.5: Comparison between 3-D and 2-D standard deviation of effective hydraulic conductivity for aspect ratio of liner of 0.3

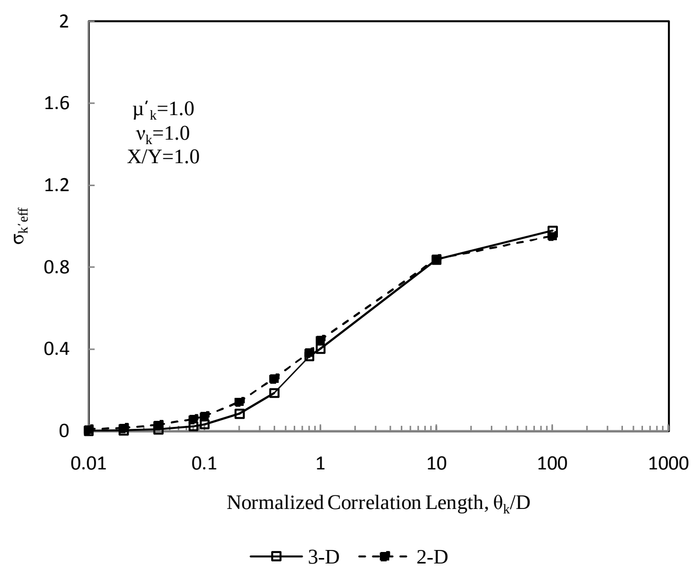

Figure B.6: Comparison between 3-D and 2-D standard deviation of effective hydraulic conductivity for aspect ratio of liner of 1.0