XI.—On the Variation of the Rigidity of Vulcanized India-Rubber, with Tension.—By Thomas C. Hebb, B. A., *Dalhousie College, Halifax, N. S.*

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Mr. W. A. Macdonald* found in the course of experiments conducted in Dalhousie College last year, that the rigidity (kinetically determined) of a fresh or partially fatigued vulcanized india-rubber cord, when subjected to increasing tension, at first diminished, then reached a minimum, and finally increased; while in the case of a sufficiently fatigued cord, the minimum point seemed to disappear. But owing to a doubtful mode of gripping the ends of the cord, his experiments were not conclusive.

At Prof. MacGregor's suggestion, I have made the experiments described below with the object of settling this question and finding out what I could about the phenomenon.

For this purpose I have (1) used the method of gripping the cord which Mr. Macdonald employed in his last series of experiments in order to exclude the source of error affecting his earlier observations; (2) applied the static as well as the kinetic method of determining the rigidity; (3) made experiments both on the cord which Mr. Macdonald used and on fresh cords, and (4) adopted modes of procedure, suggested by the results of my earlier experiments, with respect to the time between the loading of the cord and the determination of the rigidity, and to the magnitude of the angle of torsion.

The cords used were cylindrical in section, about forty inches in length, and one-third of an inch in diameter. The grips consisted of pieces of brass tubing of the same diameter (inside) as the cord, in one end of each of which three longitudinal cuts had been made. The ends of the rubber cord were drawn into

* Proc. N. S. Inst. Sci., 10, 28, 1898-99.

the cut ends of these brass tubes and firmly fastened there by wire twisted around the tubes outside. Cords of considerable length were used, in order that the effect of the gripping at the ends might be inappreciable.

The cords were suspended from an iron bracket, moveable on vertical guide-posts which were attached to the wall of the laboratory, and capable of being firmly clamped to these posts at any desired elevation.

The upper brass tube passed through a wooden socket firmly fixed in the bracket. It was held in this socket by friction, and while it could be rotated by hand, there was no danger of its shifting its position otherwise. The brass tube projected above the socket, and carried a wooden disk, on which was a divided circle. A pointer fixed over the disk indicated the number of degrees through which the cord was twisted. Thus any desired torsion could be given to the cord at the upper end.

The brass tube at the lower end of the cord carried, in a plane perpendicular to it, a light wooden arm for the application of the twisting force in the static experiments, and which served as a platform for the stretching weights.

These weights were square leaden plates of about four inches' edge. They had holes of the size of the brass tube cut in the centres, and slits leading to them from the edge, so that they could be easily put on and taken off.

In applying the kinetic method, the cord was kept fixed at the top, while the lower end, with the plates attached, was twisted through some angle and then let go. The time of oscillation was then determined by means of a stop-watch. This datum, together with other data easily obtained, viz., length and diameter of cord, and moment of inertia of plates, gave the means of finding the rigidity. In determining the time of oscillation, it was soon noticed that it varied with the angle through which the cord was twisted. Hence the cord was always twisted through known angles. In the static, as in the kinetic method, the rigidity varied with the angle, and here, also, definite angles of twist were always used.

In using the static method, the twisting force was applied at the end of the arm carried by the lower brass tube. In the earlier experiments it was applied by means of a thin silk string, horizontal and perpendicular to the arm, which passed over the pulley of a set of frictionless wheels taken from an Attwood's machine, and carried a small plummet of known weight. The plummet was so light that the cord was not appreciably deflected from the vertical. In order to make the friction as nearly as possible the same in all experiments with the same plummet, I observed the position of the end of the arm before the plummet was attached or the cord twisted, and then having attached the plummet, I determined the amount of twist to be applied in order that the arm might make small oscillations about this position.

Even with this procedure, however, successive observations showed a lack of agreement which was traceable to friction. Hence, in the later experiments, I used Mallock's* method of applying the force, which I found not only to give more consistent results, but to occupy less time. A small plummet of known weight which was suspended from the end of the arm by a fine silk string was drawn aside by a second silk string, which was kept horizontal, the two strings being in a plane perpendicular to the arm. The distance to which it was drawn aside was determined by the aid of a second plummet hanging freely from the end of the arm. The horizontal force at the end of the arm was then equal to the weight of the first plummet multiplied by the ratio of the distance to which it was drawn aside to the distance below the end of the arm of the point of junction of the two strings. These distances, together with the length of the arm, could be measured with considerable accuracy, and thus the torque to which the cord was subjected determined.

* Proc. R. S. L., 46, 233, 1889.

In most of the measurements made, the quantity under consideration was determined as a mean of several observations.

Lengths were measured by means of a beam compass reading to .01 inch. The limit of error of a mean value was found by comparing a number of such mean values with their mean, the greatest divergence being taken to be the possible error of a determination. It was found to be different according as it was the length of the cord or of a side of the Mallock triangle or of the arm, that was determined. In the two former cases the greatest divergence from the mean was .01 in., in the latter .005 in.

The diameters of the cords, which were approximately cylindrical, were found by means of a screw-gauge reading to .001 inch. The possible error was found by the above method to be .0005 in. Owing to the difference in diameter at different parts of the cord, it was found necessary to have marks on the cord, at which the measurements were always made.

The error that might be made in determining the angle of twist in the static method was estimated to be about a quarter of a degree.

The weight of the small plummet used in twisting the cord was found by means of a balance weighing to .001 grm. The method of weighing was that of substitution, and the limit of error was estimated to be .0005 grm.

The time of oscillation was found by means of a stop-watch divided into fifth-seconds, but capable of estimation to .1 sec. The limit of error was determined in the same way as in the case of length, and found to be about .04 sec.

In the static method the formula used for the calculation of the rigidity was the following: $n = 2\,T\,l/\pi\,r^4\theta$ in which T is the torque in lb.-inch units, applied at lower end of cord, l is length of cord in inches, r is radius of cord in inches, and θ is angle twisted through measured in radians. For the kinetic method the formula: $n = 8\,\pi\,l\,I/t^2r^4g$ was used, in which l and r were expressed as in the previous formula, I is moment of inertia expressed in lbs. and inches, and t is the time of a complete oscillation expressed in seconds. The moment of inertia of the plates used in stretching the cords was found by means of the following formula: $I = M(a^2 + b^2)/12$, in which M is mass of plates in lbs. and a and b are the lengths of sides of plates in inches. The moment of inertia of the brass tube at the end of the cord was found to be negligible.

The values of the rigidity determined as above would thus be expressed in inch-lb-second gravitational units.

The effects on the calculated values of the rigidity, of the above possible errors of the component observations, were calculated in a few cases, and were found in the static observations to be between 1.5% and 2.5%, and in the case of the kinetic observations to be between 2.5% and 3%.

My first observations were made on the cord which Mr. Macdonald had previously used. He had subjected it to repeated and prolonged extension, and found that the minimum point which in the earlier series of observations seemed to characterize the variation of its rigidity with tension finally disappeared. Since the time of his experiments it had lain for six months unstretched.

The static method was the only one employed, the torque being applied by means of the frictionless wheels. The procedure was as follows: First a weight was hung on the cord, and left there for at least a day. Then its rigidity was determined, after which the load was increased and the cord left for another day. On the third day the same process was repeated, and so on. It was not until later on in the experiments that the importance of allowing the loads to act for corresponding intervals of time was realized. The following table gives the results obtained:

TABLE I.

Date.	Temp. (°C.)	Length. (inches)	Diam. (inches).	Angle of Torsion, (degrees.)	Torque (lb.-in.)	Rigidity.	Load (lbs.)
Oct. 20..	.....	....	....	....	....	....	1.611
" 23 .	17.5	46.82	.322	917	.0209	58	"
" 24 .	17.4	49.68	.313	1013	.0209	62	2.150
" 26..	21.4	52.98	.303	625	.0117	69	2.678
" 26..	20.5	53.07	.302	1093	.0209	71	"
" 27..	19.5	56.96	.291	1235	.0209	78	3.216
" 30..	18.5	57.28	.291	713	.0117	77	"
" 30..	18.8	57.28	.291	1256	.0209	78	"
" 31.	17.8	61.82	.278	1373	.0209	92	3.758
Nov. 1..	19.5	66.88	.270	1545	.0209	99	4.289

Although the above table shows considerable disagreement between successive observations, due probably in some measure to friction, to difference of temperature slightly, and to inequality in the times of application of the loads, the values obtained clearly increase with the tension, and give no indication of a minimum point. This result is in agreement with Mr. Macdonald's last series of observations, and seems to show that the rigidity increases steadily with tension in a cord which has been subjected to sufficiently prolonged extension, provided at least a day is allowed to intervene between increasing the tension and determining the rigidity.

The next observations were made on a fresh cord of a different rubber from Mr. Macdonald's. It was harder, and contained 60 per cent of pure rubber, according to the statement of the manufacturer. A freshly-cut surface showed a dark gray colour.

All the series of observations given below were made on rubber cords of this kind.

Both static and kinetic methods were applied, very little time being allowed to intervene between the two determinations, in order that the cord might be in the same state, as

TABLE II.

Date.	Diam.	Kinetic Observations.						Static Observations.				Kinetic Rigidity.			Static Rigidity.		Load, (lbs.)
		Length.	Temp.	Mo. of Inertia.	Time of Oscillation.			Length.	Temp.	Torque.							
					360°.	180°.	90°.			90°.	31 5°.	360°.	180.°	90°	90°.	31°5.	
Jan. 11	......	......	....	....	......	......	....	......	....	.....	......	.....	...	....	.....	......	.561
" 16	.397	35.06	18.2	1.44	3.65	3.42	3.13	35.06	17.9	.01536	.00574	159	181	217	141	150	"
" 17	.394	35.58	18.9	2.88	5.22	5.19	4.85	35.64	17.5	.01444	.00531	163	165	188	139	146	1.089
" 18	.389	36.23	14.7	4.42	6.60	6.37	6.23	36.24	15.5	.01444	.00555	167	179	183	148	163	1.644
" 19	.385	36.85	16.6	5.90	7.97	7.65	7.34	36.88	18.2	.01304	.00483	162	176	191	142	150	2.183
" 19	.381	37.54	17.9	7.37	9.16	8.96	8.76	37.57	19.8	.01201	.00433	163	170	178	139	143	2.711
" 20	.381	37.61	17.0	7.37	9.17	8.99	8.69	37.62	17.4	.01191	.00422	163	169	178	138	140	"
" 22	.380	37.64	18.4	7.37	9.34	9.18	9.03	37.69	18.4	.01226	.00449	159	165	169	144	150	"
" 22	.376	38.32	18.4	8.83	10.63	10.34	10.02	33.35	18.3	.01116	.00411	156	165	176	139	146	3.249
" 23	.376	38.43	18.3	8.83	10.47	9.93	9.41	38.46	18.3	.01120	.00397	161	180	199	140	142	"
" 24	.372	39.24	18.5	10.30	11.59	11.11	10.60	39.26	18.2	.01055	.00381	164	178	196	140	145	3.791
" 25	.366	40.18	15.0	11.74	12.67	12.41	11.93	40.18	15.5	.01026	.00368	170	177	192	149	153	4.322
" 26	.362	41.05	17.4	13.24	13.97	13.71	13.41	41.05	17.7	.00945	.....	169	175	183	147	......	4.869
" 27	.356	42.11	14.8	14.75	15.10	14.76	14.53	42.14	14.5	.00891	.....	176	185	191	152	......	5.422

nearly as possible, in both. In the static determinations of this, and of all subsequent series, Mallock's mode of applying the twisting force was used. The kinetic observations were made with different amplitudes of angle of oscillation, and the static observations with different angles of torsion. The general procedure was as in the former case. Table II contains the results.

In none of the columns of rigidity values of this table do the values found indicate any simple law of variation with tension. They do not even increase or decrease continuously as tension increases, but appear to oscillate between increment and decrement, and by amounts which are not accounted for merely by errors of observation. The variations cannot be accounted for even by errors of method, because in general both methods give similar variations. They may, perhaps, be partially at least, accounted for by defective procedure. Nevertheless, two conclusions may be drawn :—(1) The smaller the angle of torsion in the static determinations and the angle of oscillation in the kinetic determinations, the greater is the value of the rigidity obtained. Mallock drew the same conclusion as to kinetic rigidities from his observations. (2) The kinetic determinations show a point of minimum rigidity as tension increases; but the static determinations are not sufficiently exact to be decisive as to whether or not the existence of this point is independent of the method. Thus the kinetic observations bear out Mr. Macdonald's result that the kinetic rigidity exhibits the minimum point in the case of a cord previously unstretched.

The cord used in the last experiment being now in a state of tension, was experimented on in a reverse manner. It was left a day under the full load, when its rigidity was determined. Then one of the weights was taken off, and it was again left for a day under the diminished load, and its rigidity determined; and so on.

TABLE III.

Date		Diam.	Kinetic Observations.						Static Observations.			Kinetic Rigidity.			Static Rigidity.	Load, (lbs.)
			Length.	Temp.	Mo. of Inertia.	Time of Oscillation.			Length.	Temp.	Torque.	360°.	180°.	90°.	90°.	
						360°.	180°.	90°.			90°.					
Jan.	27	.356	42.11	14.8	14.75	15.10	14.76	14.53	42.14	14.5	.00891	176	185	191	152	5.422
“	30	.357	42.20	18.5	14.75	15.20	14.91	14.63	42.21	18.5	.00845	172	179	186	142	“
“	31	.360	41.41	15.4	13.21	14.29	14.03	13.79	41.43	15.4	.00917	166	173	179	147	4.869
Feb.	1	.366	40.50	18.5	11.74	13.16	12.93	12.68	40.51	18.5	.00929	159	165	171	136	4.322
“	2	.370	39.69	19.0	10.30	12.09	11.92	11.51	39.71	19.0	.00911	155	160	171	129	3.791
“	3	.373	38.95	14.0	8.83	10.93	10.67	10.30	38.97	14.0	.00979	155	163	175	123	3.249
“	5	.377	38.20	13.0	7.37	9.80	9.35	8.90	38.20	13.0	.00998	151	165	183	122	2.711
“	6	.382	37.44	18.6	5.90	8.52	8.32	7.74	37.44	18.6	.01090	148	156	180	124	2.183
“	7	.385	36.76	19.0	4.42	7.10	6.82	6.33	36.76	19.0	.01166	153	165	191	127	1.644
“	8	.390	36.09	15.4	2.88	5.65	5.19	4.65	36.09	15.4	.01323	146	174	217	134	1.089
“	9	.395	35.34	17.0	1.44	3.90	3.57	3.16	35.34	17.0	.01418	143	172	218	134	.561

Table III gives the results of the observations. There is a greater uniformity in the way in which the values of the rigidity vary with change of tension than there was when the tension was increasing, which may be ascribed in part to the somewhat greater uniformity of the time intervals between changing the load and determining the rigidity, and in part to a greater permanence of internal structure produced by the previous prolonged extension. It will be noticed (1) that the values of the rigidity run through pretty much the same course as they did in Table II, when the tension was being increased, though the final values of Table III, in the case of the static rigidity for the greater angle of torsion and in the case of the kinetic rigidity for the greater angles of oscillation, are less than the initial values of Table II, and (2) that the minimum point is given not only by the kinetic results for the amplitudes 180° and 90°, but also by the static results, which shows that the occurrence of the minimum point is not due to a defect peculiar to the kinetic method.

To see what effect the time interval between the putting on of the load and the finding of the rigidity, had on the rigidity, a new cord was experimented with in the following manner:—A weight having been put on the cord, the rigidity was determined both immediately afterwards and after the lapse of certain intervals of time. Then another weight was added and the previous process repeated. Owing to lack of time, only the kinetic method was used. Table IV gives the results.

TABLE IV.

Date.	Temp.	Diam.	Mo of Inertia.	Length.	Time of Oscillation.			Kinetic Rigidity.			Load. (lbs)
					90°.	180°.	360°.	90°.	180°.	360°.	
February 1....	19.0	.382	3.08	36.81	5.31	5.66	5.80	197	173	165	1.133
" 2....	19.0	.381	3 08	36 92	4.85	5 32	5 75	239	199	170	"
" 2....	19.5	.374	5.99	38.12	7.80	8.17	8.42	200	182	171	2.206
" 3....	14.0	.374	5.99	38.25	7.16	7.70	8.27	238	206	178	"
" 5....	13.0	.373	5.99	38.33	7.28	7.92	8.36	233	197	177	"
" 6....	18.6	.374	5.99	38.38	7.45	8 08	8.57	220	187	167	"
" 7....	19.0	.374	5.99	38.40	7.20	8.00	8.50	236	191	169	"
" 7....	19.0	.368	8.92	39.72	10 45	10.73	11.03	184	175	165	3.272
" 8....	16.7	.366	8.92	39.90	10.01	10.58	10.93	206	184	173	"
" 9 ...	17.0	.366	8.92	39.95	10.03	10.48	10.93	205	188	173	"
" 10....	14.4	.366	8.92	40.01	9.65	10 33	10.98	222	194	172	"

It appears from these results that increase of tension in a cord which has been under tension for some time immediately decreases the rigidity, but that if the cord is left under the tension the rigidity increases again. According to the longest series of observations made, the rigidity of a cord thus left under a constant load seems to pass through a maximum point, but that is perhaps doubtful.

Since the immediate effect of increase of tension is to decrease the rigidity, it might be expected that the twisting of the cord in the determination of its rigidity would increase the rigidity, and that consequently the greater the angle the cord is twisted through the smaller will be the value of the rigidity found. This expectation is borne out by the results as given in the tables. It was also noticed when taking several observations of the time of oscillation in order to get a mean value, that the first values were always the smallest, a fact which seems to indicate that the rigidity decreases with strain.

From the preceding it is easily seen that the procedure followed above was not such as could give a simple relation between the observed rigidity and the tension. For since the rigidity of a cord under tension varies with time, the experiments must always be made, if they are to give a definite result, when the rigidity is at a minimum or a maximum. Now the minimum value of the rigidity of a cord under tension appears from the last table to be immediately after the tension is applied. If, however, we decide to determine the rigidity when at its minimum, not only must the rigidity be found immediately after the tension is applied, but the rigidity of the cord due to its previous tension must not have had time to change from the minimum value. Hence the procedure should be as follows: Load the cord and find the rigidity immediately, then increase the load and find the rigidity immediately, and so on, the whole series of experiments being carried out in the shortest time possible.

With a new cord of the same kind as before, this procedure was followed, and the results of Table V obtained.

TABLE V.

Date.	Temp.	Diam.	Length.	Mo. of Inertia.	Time of Oscillation.			Torque.	Kinetic Rigidity.			Static Rig.	Load, (lbs.)
					90°.	180°.	360°.	90°.	90°.	180°.	360°.	90°.	
Feb. 10	14.6	.372	34.58	1.44	4.25	4.41	4.50	.00851	150	139	135	100	.561
"	14.6	.368	35.39	2.88	6.18	6.32	6.47	.00827	151	144	139	103	1.089
"	14.4	.363	36.34	4.42	7.91	8.15	8.32	.00745	154	145	139	101	1.644
"	14.7	.357	37.45	5.90	9.52	9.75	10.03	.00715	156	149	140	107	2.183
"	15.0	.351	38.72	7.37	11.06	11.26	11.55	.00687	160	155	147	114	2.711
"	16.0	.345	40.03	8.83	12.53	12.78	13.18	.00636	166	159	150	117	3.249
"	16.7	.339	41.44	10.30	14.13	14.40	14.60	....	169	162	158	....	3.791

There is some lack of regularity in the results of this table so far as the determinations with smaller loads are concerned, but the divergence from regularity is probably within the limit of error of the observations. The results obtained with the larger loads exhibit much greater regularity than the results of the previous series of observations. The table shows that if the tension be increased as rapidly as is consistent with the determination of the rigidity at successive stages, the rigidity increases with the tension continuously, at first comparatively slowly, and finally with greater rapidity.

The comparatively slow increment of the rigidity under the smaller loads would suggest the possibility that the rigidity may not appreciably vary with the tension at all under the circumstances aimed at in the experiments. For in the light of the results of Table IV the larger values of the rigidity under the greater loads may be due entirely to the time effect of the previous increments of load.