Plane Trigonometry

Definitions.

1. Every circle is supposed to be divided into 360 equal parts called degrees, every degree into 60 minutes, every minute into 60 seconds &c.

2. An angle is measured by an arc of a circle contained between the lines which form the angle, the angular point being the centre, and estimated by the number of degrees contained in the arc

3. The complement of an arc is what it wants of a quadrant, and of an angle what it is less than a right angle

4. The Supplement of an arc is ~~the~~ what it wants of a semicircle.

5. The Chord of an arc is a right line joining the extremities of the arc —

6. The Sine of an arc is a right line drawn from one end of the arc perpendicular on the radius passing through the other end of the arc

7. The Tangent of an arc is a right line at right angles from the extremity of the Diameter passing thro' one end of the arc, and produced without the circle till it meet with the Secant which is - a line drawn from the center thro the other end of the arc. —

8. The versed Sine of an arc is that part of the Diameter intercepted between the arc and its Sine

9. The Co.sine, Co.Tangent and Co. Secant of an arc are ~~is~~ the sine tangent and secant of the complement of that arc. —

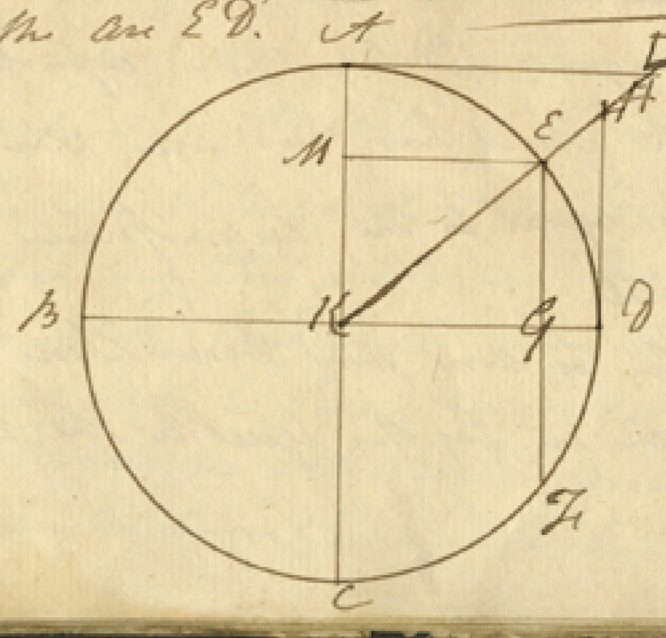
2 AC complement of ED _ 3 EAB the suppt. of ED

1. Arc ED is the measure of the ∠ EKD

EF the Chord of EDF

4. EG the sine. DH the tangent ~~and~~ KH and the secant ~~of the arc ED~~. GD the versed sine of the arc ED.

EM the Co. sine. AL the Co. tangent, KL the Co Secant and AM the Co. Versed sine of the arc ED.



Corollaries

1. The sine of a quadrant or 90°. is equal to the Radius of the circle, and the greatest possible

2. An arc and its supplement have the same Sine Tangent and Secant, the two latter are called Negative when the Arc is greater than a quadrant –

3. When the Arc is nothing (0) the sine and Tangent are 0, but the secant is then the radius. When the arc is a quadrant the sine is the radius and the Tangent and Secant are infinite.

4. The Chord has two arcs which together are equal to the circumference of the circle.

5. The Co-sine and Versed Sine of an arc are together equal to the radius

6. The Triangles KCH, KGE, KAL & KME are all right angled and similar – ~~these~~ therefore

7. ~~R : T :: Ct : S : T~~
~~R : S :: Cs : S : R~~
The Radius is a mean proportional between the tangent and Cotangent, and between the Co-sine and Secant of any angle –

8. The Sine, Tangent, Secant of an arch which measures any angle, is to the Sin T. Sec, or V. sine of any other arch which measures the same angle as the radius of the first arch is to the radius of the second.

9. In every triangle there are six parts, three sides and three angles, any three of which, except the three angles, being given the rest may be found by Trigonometry. Triangles are divided into Right angled triangles, and Oblique L Triangles

Of Right angled Triangles

Theorem

In every right angled plane triangle, if the Hypothenuse be made the radius of a circle, the sides are the sines of their opposite angles. But if one the sides be made radius, the other side will be the Tangent of the opposite angle, and the Hypothenuse the Secant of the same angle — By this theorem all the cases of right angled triangle may be solved

1 — In stating the proportion, the 1st 2d & 3 terms must be given

2 State first for one of those parts of which there are fewest given

3. In stating for a side begin with the name of a Given side — as Radius, sine, Tangent or Secant.

4 In stating for an angle begin with a side

Plane Trigonometry

1 The Circumference of every circle is divided or supposed to be divided into 360 equal parts called Degrees, and every degree into 60 minutes, every minute into 60 seconds &c —

2. An angle is said to be of such a number of degrees as is contained in the arc which measures the angle —

3. The complement of an arc or angle is what it wants of a quadrant or 90°: and the supplement ~~what~~ it wants of a semicircle or 180°. —

4. The Chord of an arc is the straight line which joins the extremities of the arc

5. The Sine of an arc is a straight line drawn from one end of the arc perpendicular to the radius passing thro' the other end of the arc.

6. The tangent of an arc is a straight line drawn perpendicular from the end of the diameter passing thro' one end of the arc, and produced without the circle 'till it meet with the ~~Secant~~ Diameter produced passing thro' the other end of the arc —

7. The Secant is a right line, drawn from the center thro' one end of the arc, and produced without the circle 'till it meet the Tangent from the other end of the arc. —

8. The versed Sine of an arc is that part of the Diameter between ~~the~~ the Sine of the arc, and the circumference of the circle —

9: The Co. Sine, Co. Tangent or Co. Secant of an arc is the Sine, Tangent ~~&~~ or Secant of the complement of that arc. —

~~Figure~~ 10th

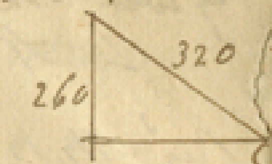
10th The Sine Tangent Line of an Arc. is ~~the~~ also ~~Figure with~~ the Sine, Tangent & Secant ~~or~~ of the angle which that arc measures.

3760
5
25800

Case 1st

In the right angled triangle ABC at A

Given AC & AB = 320 & 260



Required the ∠s A, C

and side BC

AC : R :: AB : SC = ~~[illegible]~~ = 35°, 39'

R : AC :: S.A : BC =

or

S.C : AB :: S.A : BC

R : AB :: T.A : BC

~~R : AB :: T.A~~

T.C : AB :: R : BC

Sec. A : AC :: T.A : BC

S.A C : AC :: R : BC

= 106.5

Case 2^{d}

Given AC = 400
∠A = 40° 30′
Req^{d} AB, BC & ∠C

The angle C = Comp^{t} A =

R : AC :: S, A : BC

R : AC :: S, C : AB
other proport^{s} for AB
S, A : BC :: S, C : AB
R : BC T, C AB
T, A : BC :: R : AB
Sec^{t} C : AC :: T, C : AB
Sec^{t} A : AC :: R : AB.

=

Case 3

Given BC =
∠C =

Req^{d} AC,
AB
∠A

S, A : BC :: S, C : AB

S, A : BC :: R : AC
S, C : AB :: R : AC
T, A : BC : Sec^{t} A : AC
R : BC :: Sec^{t} C : AC
R : AB :: Sec^{t} A : AC
T, C : AB : Sec^{t} C : AC

=

Case 4th

Given AB =
BC =

Req^{d} ∠s A, C
and side AC

AB : R

or

BC : R :: AB : T, C

S, A : BC :: R : AC =
and S, C : AB :: R : AC
T, A : BC :: Sec^{t} A : AC
T, C : AB :: Sec^{t} C : AC
R : AB :: Sec^{t} A : AC
R : BC :: Sec^{t} C : AC.

Case 2d Triangles —

Given AC = 400 … 1st

In every triangle the sides are to one another as the sines of their opposite ∠s

Theorem 2

In every plane triangle the sum of any two sides containing a given is to their difference as the tang.t of half the sum of the other two angles at the base is to the tangent of half their difference

Theorem 3

In a plane triangle if the perpendicular drawn to the base from the opposite angle fall within the ~~base~~ triangle, the Base is to the sum of the sides as the difference of the sides is to difference of the segments of the base; but if the perp.r fall without the triangle, the Base is to the sum of the sides as the Diff.t sides to the sum of the segments of the base.

Lemma

Half the difference of two unequal magnitudes added to half their sum ~~[illegible]~~ is equal to the greater ~~and~~ and ½ Diff sub: from ½ the sum is equal to the less.

Case 1st

Given AB =
~~B~~C =
∠ C =

Req.d ∠s A, B and
Side AB —

AB : S.C :: AC : S.B

S.C : AB :: S.A : BC

or S.B : AC :: S.A : B

Case 2

Given { ∠ A 110° 30′
∠ B
and side BC

Req.d { side AB
AC
∠ C

S.A : BC :: S.C : AB

S.A BC : S.B : AC

Because the sine of an angle and its supplement —

Sub: 110° 30′ from 180 and take the sine of 61° 30′ the remain.r

Case 2^d

Given AC = 400 ft

AB = 300

$\angle$ A = 50° 12′

Req.^{d} $\angle$s B, C

and side BC

Sum sides : Diff :: T. ½ sum angles : Tang^t of half Diff

$AB + AC : AB - AC :: T. \frac{B+C}{2} : T. \frac{B-C}{2}$ ——— = 10° 59′

S, B : AC :: S. A : BC = 475.39 —

Half added —

Sub: from

— Case 4 —

Given the three sides AB = BC = AC

Req.^{d} the angles —

Base : Sum sides :: Diff sides : Diff Seg.^{s} base

or BC : AB + AC :: AB − AC : BD − DC