

Instrained motions.

A body in motion may meet with obstacles & will totally destroy, or it may diminish, or otherwise change its motion. Obstacles & destroy may either be quite invincible, so as to destroy any motion whatever, or they may be just able to destroy it if motion under consideration. An invincible obstacle too may be of such a nature as not to permit any motion to begin, as when a body pulls by an inflexible line fastened to a fixed point. or it may be such as only to allow a motion in its own direction.

1. If a body in motion meets an immoveable and impene- trable plane at right angles, it will rest. For it must either move to one side or return back. There is no cause to determine it to either side, therefore it must either return back or rest. It cannot return back, for it must previously rest, and once it is at rest, it cannot move, unless some moving force act on it, & we do not suppose to take place in the present case.

2. Since then an immoveable plane destroys motion, it must be considered as exerting a power, measured by its change of motion it produces; and its direction in it this power is exerted is to be esteemed in its direction of its change.

is it produces, that is in a direction tending
 proportional to it. This exertion is resistance. We
 conclude if the resistance of resisting planes is perpen-
 dicular to their surface.

3. If a body A (fig. 19) moving in the direction AB meets an
 immovable plane BD , inclined to its direction in \angle CBD , it will move in \angle direction BD , with a velocity w is
 to its former velocity as \cosine CBD to Radius, and \angle loss of
 velocity w it sustains will be proportional to \angle versed Sine
 of that angle.

At \angle point B if body is acted on by two forces - its inherent
 force in \angle direction BC , & \angle resistance of \angle plane in \angle direction
 BE perpendicular to BD . Draw CD parallel to
 BE , and DE parallel to BC . Then it is clear that BD is
 the the motion w must result from \angle joint action of \angle
 forces BC, BE . This motion is uniform, for \angle action of \angle
 resistance is instantaneous. BD therefore is to BC as \angle velocity
 after \angle stroke to \angle former velocity. But if BC is Radius
 then BD is \angle cosine and DE \angle versed sine of \angle angle CBD .

Lemma (fig. 19 No 2)
 The versed sine of \angle angle of contact is an infinitesimal of the

3. About \angle point of contact A in the chord AB
 describe \angle arc of a circle BC cutting \angle tangent
 in C ; draw CD perpendicular to AB .
 Then we have $AB:BC = BC:BD$.

Let B continually approach A & at last unite to it. Then BC
 is infinitely small or vanishes w respect to AB and BD
 is infinitely small in respect of BC .

4. If a body A , moving in \angle direction AB , (fig. 20) meets a
 curve channel BCD , in whose plane it lies, & \angle AB touches
 in B , it will move in \angle path BCD to an uniform motion,
 and will lose none of its velocity.

For its loss of velocity in each nascent chord of the
 curve is as \angle versed sine of \angle angle of contact that is an
 infinitesimal of the second order. an infinity therefore
 of such diminutions will make but an infinitely small
 part of its velocity. It therefore moves on to an undeter-
 mined velocity, and its motion is therefore uniform.

N. B. This may be rigorously demonstrated as follows (fig. 21)

Lemma

The subtense of \angle angle of contact in any circle is directly
 as \angle square of \angle nascent arch & inversely as \angle Radius.

This is demonstrated in the last propo-
sition (Fig. 17) when it was shewn AD / or

making AE the diameter then if radius is equal
AE & BC = $\frac{AB^2}{R}$

5. Let now a body move by its inherent force along a
curved channel ABC (Fig. 24) in every point B it en-
deavours to proceed in its direction of the tangent BD, &
if AB, BC are two equal nascent arches, & BD = BC, then
DC will be its effect of its resistance canal to its rectilinear
motion along BD. But its resistance was exerted at its point
B, & therefore in a direction BE perpendicular to its tangent
BD. Now if its curve ABC be conceived as produced by its
evolution of some other curve, BE will be its position of its
momentary radius while it was describing its nascent arch
AB. Let E be its momentary centre, or the centre of its equi-
curve circle. Then BD is its subtense of its angle of contact and
therefore = $\frac{AB^2}{R}$. But CD = BF its resistance of its curve, now
its resistance is exerted against an equal & contrary pressure
by which its body tends to recede from the centre E of its equicurve
circle, and it may therefore be called its centrifugal force of the
body, arising from its curvilinear motion.

The versed sine of its angle of

Centrifugal force therefore is always proportional to
it is always equal to $\frac{AB^2}{2R}$. But AB will represent
velocity, being $\frac{S}{t}$. Therefore if body in its motion along its
curve canal always presses it to a force = $\frac{V^2}{R}$, or its centrifugal
force always is proportional to $\frac{V^2}{R}$

N. B. It is easy to see that its law of its centrifugal force
is precisely its same to its formerly demonstrated of centripetal
forces, it were formerly demonstrated to be directly as its
square of its velocity, and inversely as its chord of the equi-
curve circle, passing thro its centre of forces. In its pre-
sent case this chord is always the diameter because
the resistance of its canal is always perpendicular to the
tangent, & therefore its direction always passes thro its
centre.

It appears therefore its resistance of its solid
canal performs its power of a centripetal force, its sole
difference that it is not directed to a fixed point within
the curve, but always to its momentary centre of its nas-
cent arch, i. e. it is always directed to a point in its evolving
curve by its evolution it is produced.

All its analogies therefore it can be demonstrated of

centripetal forces of bodies revolving in circles
demonstrated of these centrifugal forces in their motions.

This was first done by Mr. Huggens, & shall be
mentioned in my present course, because it gives us a
method of mustering by experiments & doctrine of central forces in
free motions, and at the same time gives us our only argu-
ment against a system of philosophy, which was published
by Descartes, & an explanation of the planetary motions.

$$1. F \div D \div \frac{D}{P}$$

$$2. \text{ If } P \div p \text{ then } F:Q \div D:d$$

$$3. \text{ If } P^2:p^2 \div D:d \text{ if } \frac{D}{P^2}:\frac{d}{p^2} \div \frac{D}{D}:\frac{d}{d} = 1:1$$

and therefore $F \div Q$

$$4. \text{ If } P:p \div d:D \text{ then } P^2:p^2 \div d^2:D^2 \text{ and } \frac{D}{P^2}:\frac{d}{p^2} \div \frac{D}{d^2}:\frac{d}{D^2} \div d:D \text{ therefore } F:Q \div d:D$$

$$5. \text{ If } P^3:p^3 \div D^3:d^3 \text{ then } F:Q \div \frac{D}{P^3}:\frac{d}{p^3} \div \frac{D}{D^3}:\frac{d}{d^3} \div \frac{1}{D^2}:\frac{1}{d^2}$$

6. The centrifugal force is proportional to the square of
the angular velocity multiplied by the distance. For the angular
velocity, or the angle described in any given time, is inversely
as the time of describing the whole circumference, that is, calling
the angular velocity $A/A \div \frac{1}{P}$. Therefore $\frac{D}{P^2} \div D \times A^2$, and

It is much used

The centrifugal force is to the weight of the body as the height
thro' which the body must fall by the uniform action of its weight,
in order to acquire the velocity of the revolution, is to half the
radius of the circle in which it revolves.

For it has been shewn that if a body be retained in a
circular orbit by a centripetal force, the uniform action
of the force would communicate the velocity of the revolution
by a fall thro' $\frac{1}{2}$ of the radius.

The uniform action of the gravity is supposed to
communicate the same velocity by a fall thro' a certain
height H . It was also formerly shewn that if two powers
 F & G , acting uniformly, communicate equal velocities
by different falls, the falls are inversely proportional
to the forces therefore $F:G \div H:\frac{1}{2}R$. But what was
formerly demonstrated of centripetal forces is also true of
centrifugal forces of bodies revolving in circles. Therefore
the corollary is true.

Hence we acquire the method of obtaining the absolute
quantity of the centrifugal force of a body in any point of
any curve. For it may be considered as revolving for a

moment in \dot{r} nascent arch of \dot{r} equicurve
universally, \dot{r} centrifugal force of a body in any
motion in any curve is to the weight of \dot{r} body as H is to
radius of the equicurve circle.

Nature exhibits many instances of centrifugal
forces. A stone swinging round in a sling is a well known
instance, & \dot{r} strain upon \dot{r} string is \dot{r} effect and measure
of \dot{r} centrifugal force. The fanners of a boring mill, the
fanners for winnowing grain, & \dot{r} circular ventilators,
are machines acting on this principle. The experiment
is formerly mentioned for ascertaining \dot{r} difference between
absolute & relative motion is another instance. In con-
sequence of \dot{r} centrifugal force arising from \dot{r} Earth's mo-
tion round her axis, she has deviated from \dot{r} spherical
figure, & acquired a sort of turnip shape. From \dot{r} same
cause gravity is diminished as we approach the
Equator, and did she revolve about her axis about 17 times

she does bodies at \dot{r} Equator would have no
all. Jupiter is flattened for \dot{r} same cause.
Of the Equilibrium of central Forces.
If two bodies are connected together & made to re-
volve round a point, from \dot{r} their distances are inver-
sely as their quantities of matter, their centrifugal forces
will ballance each other, and \dot{r} point will rest.

Before quitting this subject we must not neglect a
curious observation, it has appeared a great difficulty in
 \dot{r} science of motion, & has led one of \dot{r} first Geometers of
Europe into mistakes concerning \dot{r} revolution of bodies
in constrained curvilinear paths.

It has been demonstrated that \dot{r} revolving body suffers no
loss of velocity. It has also been demonstrated that it exerts
a determined pressure on \dot{r} canal, it must be ballanced
by a contrary resistance.

Have considered only those changes in were
caused by a constrained path on the motion of a body
its own inertia. But if ~~the~~ important phenomena
occur in those motions which are of production
of external forces.

All of important questions on this subject
may be reduced to two.

1. to find the velocity in any part of the path. 2. to find
the time of arriving to any given point.

1. Let a body A Fig. 25 be impelled by a force directed
to S along a line ATR , all lying in a plane passing
thru the line AS . About the centre S describe the circle TD .

Then knowing the distance AS from which the body begins
to move, & the law of the centripetal, we can find the velocity
acquired by the rectilinear motion at the point D ; and it
may be demonstrated in the same manner as in the case
of free motions, that the velocity at T is equal to the velocity
at D . 2. Divide AS into innumerable equal parts, of which
 $D\delta$ be one - draw $A\lambda$ perpendicular to AS describe the arch

δT . Now $D\delta$ being given $T\delta$ is also given, for $D\delta : T\delta = \text{Rad} : \text{Tan}$
Secant of $\angle ST\delta$. Since the velocities at D & T are equal the time

of describing AT is given, being equal to $\frac{AT}{v}$,
 constant & therefore equal to 1 the time of descri-
ing AT . Make DN perpendicular to AS and pro-
 portional to this quantity, and let PNO be a curve which
 passes thro' the extremities of all δ lines such as DN . Then
 the times of describing AT will always be proportional to
 the area $2PNO$. For it is plain that $D\delta \times DN$ or the area
 $D\delta nN$ is the fluxion of the time, & the area $2PNDA$ will
 be the whole time.

In motions of this kind it is evident that
 the path sustains a twofold pressure one arising from
 the centrifugal force, is was proportional to $\frac{v^2}{r}$, and
 another arising from the resolution of its central force
 OT into TK is promotes the motion in its curve, and
 KO , is increases its pressure on the path.

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In these disquisitions concerning the constrained motions of bodies along determined paths, if they cannot quit, we have conceived them confined to the paths by its solidity & immobility.

But there are other, and more common ways in which these constrained motions along curvilinear paths are produced. Suppose, for instance, that the body *A* [Fig. 26] to be fastened to the thread *AE*, is applied to the curve *GKF*, by its evolution of which the curve *GAH* is produced. Suppose that the body is acted on by an accelerating force directed to a point *S*, so situated that a line drawn from it to any point of the curve lies wholly without it. It is plain that the body will be made in this curve *GAH*. For at any point *A*, the accelerating force, if may be represented by any line *AB* may be resolved in

AC directed along the tangent, & *AD* directed towards the momentary radius *FA*. By means of the force *AC* it is impelled along the curve, and by the force *AD* it stretches the thread, and is opposed by an equal & contrary force *AE*, exerted by the cohesion of the thread, and by its means propagated to the momentary centre *F*. The evolute, therefore, performs in the present case the same effect as was performed by the solidity of the path in the former cases. Now however various these curves may be, the motions in them are precisely the same as in the nascent arches of the circles, having lines such as *AF* for their momentary Radii, & points such as *F* for their momentary centres. If the thread is fastened to a fixed point, & does not unfold from any curve, the curve described by the body *A* is a circle, as in the case of a common pendulum. There is yet another method which deserves our particular considerations, as it is the foundation of all mechanics. Suppose a body *A* [Fig. 27] connected with a fixed point *P* by means of an inflexible line *AP*, which is at liberty to move round *P*. The body is by this means constrained to move in a circular path, having in every point the direction of the tangent

The motion of such a body is to be determined
 there. Let the accelerating force act in the direction
 represented by AF . Drawing FB perpendicular to
 the direction of motion and FC perpendicular to AF ,
 I claim that the part FC acts only in pressing the line down
 on the point P , & is opposed by the resistance of its path in the
 direction AE . The force AB impells it along the curve, and if AF
 represents the space thro' which the accelerating force would have
 carried the body if moving freely, AB will represent the space along
 which it would impell the body in the direction of its constrained motion.
 Mean while the point P must be sustained by a force equal
 to AE or AC . Draw PD perpendicular to AF ; then on account
 of the similarity of the triangles FAB, APD , we have this
 analogy $FA:AB = PA:PD$. Therefore if PA represents the abso-
 lute force of F , PD will represent its effort on the body A , by which
 it impells it along the arch of the circle, or produces its rotatory
 motion. Therefore its absolute quantity is to this effort as the
 distance of the point of application from the fixed point to the
 perpendicular distance of its direction from the same point.
 Let the same force act in another direction Af , and drawing as be-
 fore b, c, Pd , its effort will be represented by Pd .
 Hence the efforts of a force applied to the body are as the per-
 pendiculars let fall from the point P on these directions.

P is called the centre of rotation or motion.

Let any other force Q

... you be ascribed to the other qualities of
matter, which therefore must be considered by us
as powers. Our only notion of these powers is that
they produce changes of motion, this is the general
point of view in which we are to consider them &
by which they are all brought into one class, however
they may be distinguished by circumstances peculiar
to each. Thus the actions of animals, the force of gravity,
the impulse of wind, water, or solid bodies all agree in
producing motion, and are therefore considered by us as
powers. But their nature is different in each. But what
notion have we of this difference, but the difference in
motion which they produce. We must, therefore, have
some principles previously established with respect
to motion before we can assert the existence or influence
of any moving power. And we must perceive the circum-
stances in which all ^{powers} agree in producing or modifying
motion, before we can ascertain those in which they differ,
and seek to characterize each particular class. This first
object of our attention, therefore, must be the general
laws of motion and moving powers. These I shall consider
under the general expression Dynamics —
In this part of our subject I shall consider a particle of

matter as agitated by moving powers every general case in which it can be found, and thus establish a set of general laws, by which we are afterwards to proceed in investigating the nature of powers whose exertions we observe in the different classes of natural Phenomena. I, E, in investigating the nature of the subordinate properties which distinguish ⁱⁿ both the different classes or situations of natural bodies. This disquisition must be looked on as the abstract and difficult part of the course. For as I shall dwell entirely on general ideas, & abstract from every distinction among bodies, I cannot give instances where these differences unavoidably take place. But it will be attended with this advantage that the subsequent parts of the course acquire a connection which you will hardly find in system of Natural Philosophy, and the investigation will be, upon the whole more perspicuous and shorter. For in the usual manner of treating this science each branch of it had a set of first principles belonging to itself which it was necessary to investigate, because not dependant on more

general principles. I flatter myself, therefore that you will find the close attention which you will be obliged to give to the first part of our course amply repaid by the facility you will afterwards acquire in following out my explanation of the Phenomena of nature. Having thoroughly established these general principles of Dynamics, I shall in the next place proceed to the philosophical history of the principle classes of natural Bodies. In different classing natural bodies different arrangements suit different purposes. The nomenclator or as he is commonly called the natural historian chooses for his principles of arrangement the resemblance in external forms. The Chymist chooses the resemblance in their relations to heat and moisture. The Physician their resemblances in their mode of affecting the animal constitutions. I take for the principle of my arrangement the resemblance in their properties with respect to motion. Thus as all bodies on which we can make experiments are heavy, I, E, when unsupported fall or begin the application of our general ^{doctrines} of Dynamics with the Phenomena from which the truth or reality of these

of gravitating bodies, shewing the Phenomena from which the truth or reality of general property is inferred, and then explaining all the subordinate Phenomena. This will include Physical Astronomy, the theory of gunnery, the motion of pendulums, and vulgar Mechanics.

The next mechanical property of matter, in point of generality, is that by which the parts of matter remain attached to each other and resist the efforts to separate them. This is not so general as the other, for we shall find that it is not competent to all matter, nor to the same matter in all situations, being susceptible of two principle modifications called solidity and fluidity, and each of these being susceptible of other modifications called compressibility and incompressibility, the compressible bodies of each class showing a remarkable property called Elasticity, which produces very great variety in their properties with respect to motion.

The facts which occur when considering the Phenomena of heavy bodies will point out to us an analogy by which we are enabled to make some probable conjectures with respect to the nature of this power of Cohesion, and its

different modifications. After discussing these I shall proceed to explain the mechanical Phenomena consequent on the combination of the general principles of Dynamics with the new properties of gravity and cohesion.

This will lead to the consideration of the strength of bodies, the communication of motion by impulse, or collision, the laws of Hydrostatics, Hydraulics, and Pneumatics, Acoustics Resistance of media, and Friction. Having thus exhausted the Phenomena which depend on the most general properties of matter, I shall in the next place proceed to consider such as arise from its more particular affections. Here the most remarkable is that which certain bodies acquire in various ways of attracting and repelling other bodies. I mean Electricity, of which I shall attempt a mechanical theory, from analogy. The ~~the~~ ~~not~~ mechanical Phenomena of Electricity are generally accompanied by some others very wonderful and important. These the not mechanical, shall be considered, in hopes that in future time their connection with the mechanical properties may be investigated. Analogous to this are the Phenomena of magnetism, of which I shall also deliver a mechanical

ical theory from analogy, which perfectly coincides with every Phenomena.

I shall in this manner have assisted all the Phenomena of the ^{sensible} motions and actions of bodies upon each other. In the prosecution of the study we shall have seen such variety of moving powers, that we shall find great analogues between these Phenomena and those which take place in the sensible motions & actions of matter. I shall thus have an opportunity of informing you of the attempts made by ingenious men to explain the Phenomena of heat, mixture, and the growth of plants and animals, and give you an account of the best supported opinion concerning electrical attractions.

This remains a very numerous and important collection of Phenomena, which I dare affirm ^{to be} causes of moving ^{bodies}, and therefore properly comprehended in our plan. I mean the Phenomena of light. But it has also been considered in a course of this kind, and I hope that I shall suggest reasons which will be sufficient to induce you to believe that it may be reduced under the Phenomena of moving bodies.

Such is the arrangement that I propose to follow in the course of Lectures on which we are now about to follow enter. During our progress there will occur many applications to the arts of life which would take up too much of our time to consider fully, but which are of too great importance to be passed altogether. I shall make a collection of them and connecting them in the best manner I can make them the subject of a particular course.

Introduction

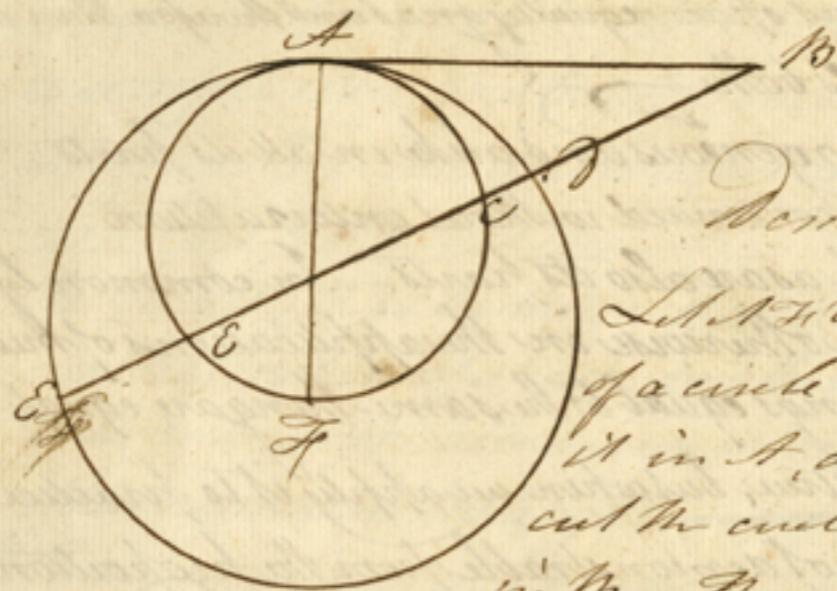
Of Space, Time, and Motion

1. While a body moves time flows, and the body passes thro' the contiguous parts of space during the successive parts of time. In order therefore to have a distinct notion of the motion of bodies, it is necessary to have clear conceptions of space and time.
2. Space in general is an abstract term expressing a simple idea, and therefore incapable of any definition; and all we can do is to reflect on the manner in which we form this idea.
3. Two points give us the notion of distance. — This is the simplest notion of space and the distance we call a line. Extension of one dimension or measure, because it has it has no measure but length.
4. Three points may give us ~~another~~ the idea of another kind of space if one of them does not lie between the other two, or in the straight line joining them. This kind of space is a surface and has two dimensions, for it may be measured both in length and breadth.
5. Four points may give us the idea of another kind of space, if none of them lies in the line or surface joining the others — a solid, having three dimensions, for it may be measured

in length, breadth, and thickness.

6. A point separates the parts of lines but makes no ~~part~~ part of either; it is only the boundary where one ends and the other begins. — in like manner is the separation of two contiguous surfaces, but is not a part of either, and a surface separates two contiguous solids but makes no part of either.
7. Space is unbounded or infinite. What is beyond any assigned bounds? Space — it is infinite, for no space is so great but we can conceive a contiguous space equally great and therefore there is a space which holds both.
8. It is perfectly homogeneous and alike in all its parts.
9. It is contiguous or continued without interruption.
10. It is immovable as are also its parts. In common life we act as if it were otherwise in the applications of measure, saying that two things equal to the same thing are equal. The proportion is true, but when we apply it to practice, our practice is not demonstrable from the proportion. when I measure two lines by means of a third, which I successively apply to each, I do not apply the same numerical measure or part of space, but a different.
11. Space is infinitely divisible. For as space is continued and uninterrupted, and as ^{no} point in any part of a line, it follows that between every two points a line is interposed a 3^d point may be placed between them without touching either.

for it may be placed either equally or unqually between them
 if it is equally it cant touch one without touching both,
 and thus there will be but one point instead of 3 or 2,
 if it is placed unqually and be supposed to touch one,
 it may be brought farther from it without touching
 the other.



Demonstration

Let AB be the Diameter
 of a circle and AB touching
 it in A , and any line PCE
 cut the circle in C, E , and AD
 in D . — Then $AP^2 = EP \times PC$

Describe now another circle cutting PE in E
 and D . — Then $AP^2 = EP \times PD$, or $PD = \frac{AP^2}{EP}$, the
 larger that EP is the smaller will PD be,
 but there is no bounds to the enlargement
 of EP , therefore none to the diminution of
 PD .

12. Space is the place of bodies.

13. What is now said is properly to space in the abstract, but
 our real perceptions of space are very different & imper-
 fect, the space of itself could not have been conceived
 by us, and it is only by means of bodies that we perceive
 it, and perceive it so far as it is the place of bodies, and
 while bodies do not ^{only} simply change their distances from
 each other and from us, we imagine the part of space
 to be the same. When I return to this room tomorrow,
 I think I come to the same place, even altho reflection
 tells me that the place I am in now will be at an im-
 mense distance from that part of space in which I
 shall be tomorrow when in this Room.

14. Should the whole universe be moved from its place,
 we could not perceive any change of place, and should
 imagine ourselves, in the same part of space. Nay
 should it all contract or swell in any proportion
 the change could not be ~~more~~ sensible provided only
 that the laws of attraction and repulsion were changed
 accordingly.

15. On the contrary should a change happen in the
 positions of objects, this would result a change in
 our Ideas, but this change will be the same, whether

The change of position happens to ourselves or to external objects. The idea is perfectly the same whether the sun turns round the Earth or Earth round the sun; so different is sensible place from real and relative place, from absolute.

16. It is thus sensible space which we call place. This also refers to contiguous bodies and therefore were there but one body in the universe which could assign it no place. The true place of a body is that part of absolute space which it fills. Its sensible place is its situation not place which it has among other bodies. Its sensible place therefore may vary while its real place remains the same, namely by the change of the situations of surrounding bodies. The sun nearly retains his real place, but his sensible place continually varies, and is not the same to any two spectators. We can never, ^{therefore} assign the real place of any thing unless there is some body which never changes its real place; and to which we may refer all the others. These sensible measures of space may well enough answer the purposes of common life, but they are unfit for the purposes of Philosophy, in

we must abstract from appearances & consider realities.

Of Time

17. Much of what has been said of space may be affirmed of time.

It is incapable of definition. We got our notions of it by the successions of our thoughts or perceptions. A beat of my watch gives me no idea of time; but two beats convey the idea of duration interposed between them. It is evident that a 3^d beat will give me the idea of another duration. We can feel an equality in the intervals, but have no distinct idea of their proportion. We follow the same method as in space we take what we think a common measure. The intervals between the beats of a watch or the like. But whether ^{we can} or not ascertain the perfect equality of two portions of duration, we know that two portions may be equal, and therefore a double of one of them, that there may be a portion which is triple another &c. Thus we know that time is a proper quantity altho we may not be able to measure it distinctly.

18. The beats of my watch are not parts of time, but only the

boundaries of these portions of time which lie between them, and are called instants; while the very small parts of duration which flow between them are called moments.

19. In all these considerations there is such an analogy between Space and time that we cannot talk of time without borrowing language from space. Thus the terms of space, Interval, long, short, near, before, after belong properly to space but are applied, in our language at least, to time. The Russian language has a set of words appropriated to each *пространство*.

20. Hence points are mentioned in time if it is represented by a line. one remarkable difference. The quantity observable in space may be measured in three different ways that observable in time only in one way.

21. As no two points of space can be coincident, so no two instants of Time can be simultaneous or coexistent. The identity or numerical difference of two points of space is that they are at distance, that of two instants of time that they are successive.

22. Hence it follows that Time may be divided without end. no interval so short but an event may happen after its beginning but before its ending.

23. Time like space must be unbounded, that is eternal, uninterrupted, homogeneous, immovable or unchangeable in the order of its parts.

24. Time is the Area of every event. All things are placed in space in the order of situation, and are placed in time in the order of succession.

25. All that is here delivered is true only of Time in the most strict and abstract sense. But we cannot perceive this time. As all its parts are perfectly alike we cannot distinguish them each other or tell in which of them any event happened. Had the whole world existed many millions of years sooner or later we could not know it provided only that the same intervals were interposed between the events which we perceive. Had the duration of the universe been lengthened or shortened in any

proportion it does not appear that we could know it; for we have no information by which we can learn the change or immutability of the succession of our ideas, and therefore we are always disposed to refer the change to changes in the interval. But reason, that is observation makes us correct this hasty judgement. A man intent on study thinks the time short but trusts his Watch

26. Thus then the real time is unknown - but there is a sensible time like a sensible space, bearing a relation to some trains of events. Thus we say that the fire of London happened in 1666 altho this by no means points out the real Era of that event -

And time is measured by successions of events - beats of a clock mark seconds, successive sunnings mark days, successive changes of the moon mark months, and successive changes of the seasons mark years. These are deemed equal altho their differences amount to a considerable quantity which Astronomers have discovered and call'd the equation, and by which they endeavour to point out the difference between true time and the sensible measure of it.

which we commonly use in its stead.

27. Our sensible time may therefore be very different from the real. A Person who sails round the Earth Eastward and returns to the same place will have reckoned two days more time than the man who made a similar voyage westward.

phenomena which they occasion among bodies in relative motion are sufficient authorities for judging of their nature. May we may go farther. There are many circumstances in nature from w^{ch} we can with certainty and accuracy discover absolute motion, and assign its quantity. These are the causes which distinguish them in nature, namely the powers which produce motion. True motion can neither be produced nor changed with force being impressed on the same body. But a relative motion in a body may be produced by only changing the situation of the neighbouring bodies. Again true motion is always changed by any force impressed but a relative motion is not so changed for if the same forces are impressed on the bodies to which the reference is made their situations remain if same, and no change happens in their relative motions. Thus if two ships are sailing from or towards each other on any point of the compass, in still water, and both of them come at one time to be carried away by the same current their relative motions will not be changed.

There are also effects by which we can at all times distinguish real from relative motions. The chief of these is that tendency which bodies, moving in a circle, have of receding from the centre. If the circular motions in or by apparent this tendency is nothing, but in absolute motions is greater or less according to the quantity of absolute motion.

Sir I N illustrates this by a beautiful and simple experiment. Let a bucket of water be hanged by a string and twisted about till the string is much twisted. When every thing is still let it now be suddenly turned round the other way. The water will not immediately acquire the motion of the bucket, and therefore it has a relative motion contrary to that of the bucket. But by degrees friction of the sides will communicate motion to the water, and therefore its motion relative to the bucket will continually decrease, and if the motion is long continued the water will at last move as fast as the bucket, and therefore will be relatively at rest in it.

But it will be found that if the surface of the water will be level at first, and will gradually heap up at the sides, the water receding from the centre as the real motion increases and the relative diminishes. The true circular motion can by no means be ascertained by the relative motion but it may be ascertained by means of this centrifugal force.

The relative quantities therefore are not the quantities whose names they bear but only sensible measures, true or false, which the vulgar mistake for the quantities themselves. But we see that we can distinguish the absolute from the relative, partly by the apparent motions,

which are the differences of the real ones, and partly by the forces which are the causes or effects of the real ones.

Thus if two spheres connected by a thread revolve round that point where they are in equilibrium, the tension of the thread would show their endeavours to recede from the centre, and therefore show us the quantity of their circular motion.

If now we impress on both bodies in opposite directions equal forces. if these conspire with their circular motion it will increase the tension of the thread, but if it counteracts their circular motion it will diminish the tension. Thus by trying all sides of the bodies, we can find what side is that which upon being struck produces the greatest increment of the tension. This will doubtless be the posterior side. Thus the direction of the circular motion is found out.

From such an observation as this we can discover all the real motions which obtain in the observed relative motions. For we can compare the discovered motion of our spheres with the apparent motion of the constellations and thus discover whether they have any motion and what it is. Knowing this it is easy to determine all the other motions in the universe.

This is to constitute our future business
Having thus established some general notions of space
Time, and motion, we come in the next place to consider the
more general properties of motion, deduced from a consi-
deration of the circumstances which concur in forming
the Idea. All that we can observe in motion is the
space pass'd over, and the time employ'd. Motion has a
beginning and end in space; it has also a beginning
and an end in time.

The portion of time elapsed is the duration, or time of
the motion.

The portion of space pass'd over is the path of the
motion. It is susceptible of three varieties, in position,
form and length.

The position of the path along w^{ch} motion is performed
is called the ^{Direction} of that motion.

If the direction of the motion never changes, the path
is a straight line, and the motion is called rectilinear.
but a change in the direction makes a change in the
path, which therefore ceases to be a straight line, and
may have any other assignable form, for any figure
may be describ'd by motion. In every point of the
path the motion has a certain direction, according to
which it is describ'g the part of the path pass'g

that point. When the path is a curve, in which case
the motion is termed curvilinear, the direction is the posi-
tion of the tangent to the curve in that point. These
varieties in the path of motion determine its kind.

The other variety in the length of the path determines
the quantity of motion, for it determines the quantity
of translation of the moving point.

A motion is similar to another when its duration,
path, and direction are similar. It is dissimilar when
any of these are dissimilar. If the time is longer, it is
called slower, if shorter, faster. If the path is longer,
it is called swifter, if shorter, it is called slower.

The terms swift and slow, therefore, do not depend on the
space or time alone, but on both jointly. The quality
by which motions differ in this combination of Ideas
of space & time is called velocity in the language of Na-
tural Philosophy. It is there an improper term be-
cause it alludes to one form of this combination only,
namely when the Idea of a great portion of space is com-
bin'd with that of a small portion of time.

We have in the English language a very convenient
term expressing such a combination or expressing how
much of one thing corresponds to a certain quantity of
another. We say that the money was lent at the rate
of 5 p^{er} cent. that a field produced at the rate of 40

bushels an Acre, and we say that a ship sails
of nine miles an hour.

The term rate in all these cases has the same meaning.
It does not express a proportion, for this can obtain only
among homogeneous quantities. But rate always means
quantities of different kinds. Even when we say five p^{er}
Cent. we mean different kinds of quantity, namely in-
terest and principal, and in general, the term rate ex-
presses how much of one thing corresponds to a determined
unity of another.

It is in this sense that we are to use the term velocity, not
as expressing any proportion subsisting between space
and time, but only the proportions which subsist a-
mong the different spaces corresponding to some deter-
mined unity of space. The first however, is the most useful.
We seldom say that a man walks at the rate of two hours
to the league, but we say that he walks at the rate of half a league
an hour.

When the Natural Philosopher speaks absolutely of ve-
locities he speaks of the space corresponding to some unity
of time, usually a second. But when he speaks of them
comparatively, he seldom assigns them absolute mea-
sures. Then when he says that two velocities are as 10
to 11 he says nothing from which we can infer the real

quantity of either, and the expression only ascertains the
proportion of the spaces corresponding to any indetermi-
ned unit of time, and the numbers themselves express the
proportions which each space bears to some one space con-
sidered as unit or that each velocity bears to some one velo-
city considered as unit, so that the proportion of these pro-
portions - 10 to 11 is the proportion of these proportions.

It must be here observed that this Idea of the velocity is
only an Idea of the measure of that quality of motion
by which a certain quantity of space comes to correspond
to a certain quantity of time, and that the term velocity
is only an expression for the measure of this quality.

The quality of itself is an abstract Idea which we can-
not define, but only describe by its measure. When a
body falls, it moves faster and faster every moment,
and we say that its velocity is continually augmen-
ting. Now in any one instant of time it does not move
over any space but is only moving over it, and it is im-
possible from the fact to assign any rate of the space
and time; for let us take any time however small, its mo-
tion at the end of that time is swifter than at the begin-
ning, yet we pretend to assign its different velocities at every
point of its fall. The measures which we assign in these

cases are the measures of the velocities when they
-posed to remain constant for some length of time or
space. Here then the velocity means a quality of motion
which we consider as a quantity, not directly, but by
means of some measure with which it is invariably con-
-nected; and in order to obtain such a measure we are ob-
-liged in imagination to suppose that the quality or ve-
-locity remains for some time unchanged. But it is
evident that in cases where it is continually changing
we shall always err from the true proportions. But
the smaller the time, the nearer will our measures
approach to the true proportions, and therefore when
the time vanishes the proportions of our measures will
be the same with those of the quality.

This is the foundation of the maxim which we shall
be continually employing, that during an infi-
-nity small part of time, or along an infinity small
part of space, the velocity is constant.

Uniform motion is that when the velocity is constant.
Accelerated motion is that in which the velocity increa-
-ses.

Retarded motion is that where the velocity is dimin-
-ished.

uniformly accelerated, or retarded, motion is that in which
the velocity receives equal increments, or decrements, in
equal times.

I may just observe that it is in the respect only of the time
that the motion of Natural Philosopher differs from that
of the Geometer. In elementary Geometry figures are con-
-sidered as produced by motion, but without any regard to
the time, and it was reserved -- for Sir J. N. to create a new
and sublime space of Geometry by introducing this Idea
into the motions considered.

Since in all varied motions the circumstances are contin-
-ually changing, in order to have accurate measures of
them we are obliged to suppose them uniform for a small
moment of time, and from the properties of uniform
motion we determine the momentary properties of any
motion however varied.

It is therefore necessary to consider the chief properties
of uniform motions. They may be reduced to three propo-
-sitions.

Propositions

In uniform motions the spaces are proportional to the
velocities, the times being the same.

This follows from our conception of velocities, which
mean nothing but the measures of a certain quality.

in one time measured by CG , the spaces will be proportional to CB , BC , or to the rectangles BF , CG .

Now the space described in the first motion is to that described in the third motion as the rectangle AC , to the rectangle BF , and the space described in the third is to that described in the second as the rectangle BF to the rectangle CG , Therefore the space in the first motion is to the space in the second motion as AC , CG or as the rectangles of the lines which measure the times and velocity of the first motion to the rectangle of the lines which measure the times and velocities of the second motion.

Thus then there arises from the nature of our measures of time and velocity a natural measure of the space, a space may therefore always be represented by a rectangle of an Area proportional to the space, but its sides undetermined, and now having determined any one of them by a knowledge either of the time or velocity, the other can be determined by that problem of elementary Geometry where we are taught to apply to a straight line a rectangle equal to a given rectangle. The other side of the rectangle formed by this Problem will be the measure of the velocity or the time, according as the given side was the measure

the time or the velocity.

This is the meaning of the Latin expressions *spatium ut proportionale velocitati ductae in tempus*, and *velocitas ut proportionale spatio applicato ad tempus*, and *tempus ut proportionale spatio applicato ad velocitatem*. As a product of numbers is analogous to a rectangle of lines, it follows that the spaces are in the proportion of the products of numbers which measure the times & velocities. This is a much more convenient measure than the other, and though some writers choose to abide by the Geometrical measure, from an affectation of accuracy in quantities which are incommensurable, and therefore can't have their proportions represented by numbers, yet this is an unjustifiable necessity. For our measures of velocity and time refer to some quantity of each reckoned as unit in every case where we give them absolute measures, and therefore we can represent their proportions by numbers without relinquishing the Geometrical expressions when it is more convenient.

Since Division in numbers corresponds to applying a rectangle to a line representing the Divisor, and since the Quotient will in this case be represented by the other side of the rectangle it follows that when a space is given and we

want to find the velocity, we need only divide the number
representing the space by that representing the time and
the Quotient will be the number representing the velocity.
Hence the Arithmetical expression, the spaces are propor-
tional to t times multiplied by the velocities, and the times
are proportional to the spaces divided by the velocities,
and the velocities are proportional to the spaces divided by
the times.

When two rectangles are equal to t bases are reciprocally
proportional to the heights, when two dividends are equal
the Quotients are reciprocally proportional to the Divisors.
The greater the height the shorter will be t base the
greater the divisor the smaller will be the Quotient.

When a side of two rectangles is equal, the other side is direct-
ly proportional to the rectangle - when two Divisors are e-
qual the Quotients are directly proportional to the dividends.
Hence the truth of these proportions.

These spaces are directly proportional to times and velocities.

The spaces are directly as the rectangles or products of t times
and velocities.

The spaces are in the ratio compounded of the ratios of the

velocities.

The velocities are in the direct ratios of spaces and inverse ratio
of the times.

The Times are directly as the spaces and inversely as the velocities.
All these propositions are more conveniently represented by
general symbols or formulas.

$$1. S \div V T,$$

$$2. V \div \frac{S}{T}$$

$$3. T \div \frac{S}{V}$$

$$4. S \div T$$

$$5. S \div V$$

$$6. T \div \frac{S}{V}$$

$$7. V \div \frac{S}{T}$$

I may here make one observation w will be of frequent use.
If any effect changes at the same rate with each of
many different causes, the effect will be directly as the
product of all the causes which increase it while they
increase, and inversely as the product of all those which
diminish it while they increase.

It is demonstrated in the very same way in every instance
as in the present, and therefore I shall without hesita-
-tion apply it as I go on - I shall use it at present in es-
-tablishing another expression which is to be of frequent use.
As the change of place is measured by the path of the motion,

and this depends on the velocity, the quantity of matter of any point will increase with its velocity. But the absolute quantity of motion which may be the subject of our consideration will always with the number of moving points. When therefore a body is in motion, and of consequence each particle in motion, the absolute quantity of motion must be made up of the sum of all the particular quantities of motion. It may therefore be represented by a number. If all the particles have one velocity, this number will be the product of the number of particles multiplied by the number representing the velocities; or it will be by a rectangle one of whose sides is analogous the velocity and the other to the number of particles.

Hence if Q represents the quantity of motion, M the quantity of matter, & V the velocity of the body we have the following expression—

$$Q = M \cdot V$$

ed on by a smooth current: while another ship in sight is fixed in still water, each appears to the other to move—their distance or situation, or both, are continually changing, but the appearance is the same in both ships, and it does not appear whether this effect is produced by a motion in the one or in the other, and by spectators in each the motion ascribed to the other.

In like manner that motion which is the cause of raising and setting of the heavenly bodies is ascribed by a spectator on the Earth to a motion in them from East to west; but a motion of the Earth from west to east would produce the same appearance, viz. a continual change of the situation of the heavenly bodies with respect to the Earth—Which of these is the cause has been the subject of long and difficult investigation.

Thus tho' these necessary effects of real motions are continually approaching to us, being the difference of real motions, the real motions themselves are very difficult to be discovered. For when we are carried along by a motion of which we are insensible, we imagine that we are at rest, and assuming this false principle in our hasty & imperceptible reasonings about the apparent effects of real motion, we mistake our false conclusions for real perceptions, and we call these errors of judgment deceptions of sense.

It would appear therefore that we must remain ignorant of the absolute and real motions of bodies, and consequently

by ignorant of the powers of nature which
regulate motion. For since we have no notion of
powers but by their effects, and since their effects are of
changes of motion, it would appear that the real causes
must be as different from the apparent causes as the real
motions are from the apparent, and that we must there-
-fore remain ignorant of them. But this is not the case.
Whatever are the absolute motions of two Bodies, their rel-
-ative motions are the differences of their absolute motions.
Suppose a man should walk from the stern to the stern
of a ship while she sails a mile, his relative motion is a ship
length, this is also the difference between the absolute motion
of the ship and man. This relative motion would have
been the same and the difference of the absolute motions
equal to it altho' the absolute motion had been any how
different. Now it will be found that the action of bodies on each
other in y way of impulse does not depend on their absolute motions
but the differences of these motions. The sensible actions there-
-fore of bodies on each other in respect of impulse are sufficient
for giving us a just notion of the powers of impelling bodies. It
will also be shown that the powers of gravity, electricity, magne-
-tism and the like, are not affected by any motions, and therefore

if at midsummer noon, I discharge a 24 pounder,
from some part of the earth which has no latitude, due East, &
suppose if it were possible to make if bullet fly with about
150 times if velocity if our greatest charge has ever been able
to give. The consequence will be if it will set out to an in-
conceivable swiftness, it will gradually diminish, & at last
if bullet, after having flown a few miles to if East, will touch
if ground, where it will rest. Here is a passage from a most
evident motion to rest. But if fact is just if inverse. The
bullet if moment it is out of the piece is nearly at rest. But
if air is rushing to if westward, to a velocity of near 100,000
feet in a second, & by its producing resistance gradually commu-
nicates its motion to if bullet, till at last, if bullet is moved by if
Earth, & acquires if whole of its motion in its orbit round if sun.
A person who remains for twelve hours in if place where if gun
was discharged has acquired an addition to his motion of about
2500 feet in a second, three times swifter than that of 24 pounder
when employed to batter in breach, yet if if change he is quite
insensible.

What then remains to be done in order to settle if
important point, the foundation of every enquiry of Nat. philosophy.
It is not so difficult as we would imagine altho we cannot observe
if absolute motions of bodies, yet their relative motion will enable
us to judge of if laws of nature as obtain in if absolute motion.

Now daily & uncontradicted experiment shews us if
if same force at mid summer noon to move a body Eastward
westward. The one is a diminution & the other an equal augmentation
of absolute motion. A body therefore shews no greater tendency
to diminish than to increase motion.

Further: if relative motions of bodies are if differences
of their absolute motions. changes of relative motions are there-
fore equal changes of if differences of absolute motions. If two bo-
dies persevere in their absolute motions they persevere in if
differences of these absolute motions, & therefore in their relative
motions, & if differences of their relative motions; on if contrary
if two bodies persevere in their difference of relative motions, they
persevere in their difference of absolute motions, and in
their absolute motions themselves. If therefore, in
all instances, bodies shew equal tendencies to retain their
differences of relative motions, they ipso facto shew
equal tendencies to retain their differences of absolute
motions, & to retain their absolute motions themselves.

How is if tendency to be observed? by putting two
bodies in such a state if they cannot on account of their
impenetrability, or if mutual actions of any powers both
preserve their present state, or present differences of absolute
motions.

change produced in if relative motion of if one is the effect,
& if only measure of if tendency of the other to retain its
present state. This change must be measured by the quantity
of motion; for if perseverance is considered as a property
or not of every particle of m^r in each body. The
proportion therefore of these changes will indicate if
proportion of if tendencies in question.

Let if following experiment be made at mid-summer
noon. let **A** moving apparently Westward come up to **B**
apparently at rest to if velocity of three feet in a second.

Suppose it really happens in if experiment, if **A**
meeting **B** at rest carries it forward to undiminished velocity.
It proves if **B** resists no augmentation of motion but if **A**
resists a diminution of motion, **B** therefore will shew no ten-
dency to remain at rest, but **A** will shew a tendency to
continue in motion.

On if other hand suppose that when **A** has come
up to **B** it stops. That is to say **B** receives no increment of
motion from **A**, but **A** loses all its excess of motion. Here
then **B** shews a tendency to remain at rest, but **A** no tendency
to continue in motion.

Again suppose if, after meeting, both move on to if velocity of
one foot in a second. Then **B** has received an increment of motion
it may be called 1, & if is if effect & measure of **A**'s tendency to

continue in motion. A has suffered a diminution
it may be called 2 , it is its effect & its measure of B 's tendency
remain at rest. The genuine conclusion from this experi-
ment would be its tendency of B to remain at rest was double
its tendency of A to continue in motion.

Again suppose its both move on to the velocity of $\frac{1}{2}$ feet
in a second. Then its change in B is equal to its in A , &
therefore its tendency of B to continue at rest is equal to the
tendency of A to continue in motion.

Lastly suppose its $A=2B$, & its both move on
to its velocity of 2 feet in a second. Then B has received an increment
of 2 seconds. This is the effect & measure of its tendency of the whole
mass in A to continue in motion & half of it is its effect & measure of
its tendency of half its mass in A to continue in motion. A has
suffered a diminution of motion it may be called 2 , & for each half
has suffered a diminution 1 , it is its effect & measure of its tendency
of its mass in B to continue at rest. But 1 was also its measure of
its tendency of A to continue in motion. Therefore A shows its
same tendency to continue in motion as B does to remain
at rest. The just conclusion therefore from such results will
be that mass has an equal tendency to continue at rest & in motion.
I chuse mid-summer noon, & a westerly motion of A , its its
relative motions might be its simple difference of absolute motions.