THE BIVARIATE LUMINOSITY-COLOR DISTRIBUTION OF IRAS GALAXIES AND IMPLICATIONS FOR THE HIGH-REDSHIFT UNIVERSE

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ABSTRACT

We present a characterization of the local luminosity-color bivariate distribution of IRAS galaxies from the 1.2 Jy sample, selected at 60 µm. The $R(60,100)$ infrared (IR) color is used as the best single-parameter description of the IR spectral energy distribution of galaxies. We derive an analytical form of the distribution and use it to constrain the effect of the IR color distribution on evolution models for high-redshift, far-infrared (FIR)-luminous galaxies. Our adopted evolution retains the locally observed correlation between luminosity and color, such that the larger characteristic luminosities at higher redshift have a warmer characteristic color. The width of the color distribution at a given luminosity remains constant for all redshifts. We demonstrate that there is the potential for both hotter and colder sources to be missed in cosmological surveys. An evolving bivariate luminosity function coupled with the cold-source bias of submillimeter-selected surveys suggests the existence of a large population of cold sources appearing in such surveys. Likewise, a hot-source bias for most SIRTF wave bands together with a bivariate model suggests an excess of hot sources being selected. We test the evolutionary form against available data for higher redshift, FIR galaxies. The data do not reveal evidence for any strong evolution in the characteristic luminosity-color distribution as a function of redshift over $0 < z < 1$. However, there is marginal evidence for a broadening of the color distribution at higher redshifts, consistent with our locally characterized trend of a broadening in the IR color distribution at the highest luminosities.

Subject headings: galaxies: evolution — galaxies: formation — infrared: galaxies — radio continuum: galaxies — submillimeter

1. INTRODUCTION

The local infrared-luminous galaxies detected by the IRAS satellite exhibit a vast array of source properties. Infrared (IR) color has typically been used to attempt to parametrize the IRAS population (Soifer & Neugebauer 1991). Dale et al. (2001) demonstrated that the $S_{60}/S_{100}$ flux ratio or color [hereafter referred to as $R(60,100)$] provides the best single-parameter characterization of the IRAS galaxy population, in addition to luminosity. While a complicated array of dust properties contributes to the spectral energy distribution (SED) of each galaxy, studies of IRAS galaxies have typically reduced the description to a best-fit single dust temperature, $T_d$, with a one-to-one mapping to $R(60,100)$. Indeed, changing the dust temperature has been demonstrated to have a significantly larger effect on the galaxy SED than on dust emissivity, mid-IR spectral index, or cosmology (Blain et al. 2002). The inferred luminosity of an infrared galaxy for a fixed observed mid-IR flux density increases by a factor of 10 if the dust temperature is doubled.

It has been demonstrated that low-redshift IRAS galaxies exhibit slowly varying correlations of $R(60,100)$ with luminosity (Dale et al. 2001; Dunne et al. 2000; Andreani & Franceschini 1996). However, over a large spread of luminosities, the $R(60,100)$ ratio of infrared galaxies does change systematically. Fitting single dust temperature ($T_d$) models to $R(60,100)$, we find ~20 K for low-redshift spirals (Reach et al. 1995; Alton et al. 2000; Dunne & Eales 2001) and 30–60 K for the high-luminosity objects typically detected by IRAS (Soifer & Neugebauer 1991; Stanford et al. 2000). High-redshift, hyperluminous galaxies can show dust temperatures of up to 110 K (e.g., Lewis et al. 1998), implying a continuation in the luminosity-$T_d$ relation out to the higher luminosities characteristic of the distant universe.

However, while a statistical relation exists between the $R(60,100)$ and IR luminosity, the distribution is broad. We find in substantial numbers both extremely luminous, yet cold, galaxies and low-luminosity, hot galaxies. One surprisingly cold and luminous galaxy, Arp 302 (or UGC 9618/NGC 5051), has been identified from the IRAS bright galaxy sample (BGS) with $S_{60} = 6.8$ Jy, $S_{100} = 15.3$ Jy, and $L_{FIR} = 3.89 \times 10^{11} L_\odot$. This is the system with the largest deviation from the median $R(60,100)$ for its luminosity. Lo, Gao, & Gruendl (1997) suggest that this galaxy is the most massive known (in terms of CO gas mass). By contrast, galaxies with very hot IR color, and without obvious active galactic nucleus (AGN) contributions, have been identified from the faintest $L_{FIR}$ sources detected by IRAS. NGC 1377, NGC 4491, and IRAS 1953 with $R(60,100) \sim 1$ and $L_{FIR} \sim 10^9 L_\odot$ are as hot as or hotter than the ultraluminous infrared galaxy (ULIRG) Arp 220, but with $10^{-3}$ times the far-infrared (FIR) luminosity (Roussel et al. 2003).

The possible importance of cold, luminous galaxies to FIR and submillimeter (submm) surveys has been pointed out by Eales et al. (1999, 2000). This has recently been highlighted observationally by Chapman et al. (2002c), who demonstrated that cold and luminous sources exist at higher redshift, identifying two Infrared Space Observatory/FIRBACK sources (FB1-40 and FB1-64) with $L_{FIR} > 10^{12} L_\odot$ with $z \lesssim 1$ galaxies and finding best-fit single-temperature graybodies of 26 and 31 K, respectively (dust...
emissivity $\beta = 1.6$). The local cold source, Arp 302, approximately matches the color and luminosity of FB1-40. However, FB1-40 and FB1-64 were discovered from a random sampling of the submm-luminous FIRBACK sources, and they cannot be considered as an insignificant portion of the high-z ULIRG population.

It is therefore a concern that studies of the evolving galaxy populations typically assume a small range of template galaxy SEDs, sampling only the monotonic relation of dust temperature to luminosity in choosing the SED templates (Blain et al. 1999a, 1999b; Malkan & Stecker 2001; Rowan-Robinson 2001; Chary & Elbaz 2001; Chapman et al. 2002b; Franceschini et al. 2002). Modeling efforts have in effect tied the dust temperature directly to the FIR luminosity, ignoring the $T_d$ distribution for each luminosity class. This in part may be a result of the fact that the $R(60, 100)$ distribution for local IRAS galaxies has never been carefully studied, and no analytical form for the distribution has been presented.

Importantly, an evolving distribution bivariate in luminosity and color, $\Phi(L, C)$, consistent with the broad distribution observed locally should subsume a nonnegligible fraction of both cold, luminous galaxies and hot, faint galaxies. Surveys that select objects at either the cold Rayleigh-Jeans tail of the dust SED or the hot Wien tail will preferentially detect appropriately cold or hot objects for a given luminosity class, if they exist in nonnegligible numbers. For instance, the cold-source bias of FIR- (e.g., 170 $\mu$m) and submm-selected surveys will result in any existing population of cold sources being overrepresented.

At the highest redshifts, the poorly understood submm population is thought to dominate the most luminous IR galaxies. Since the discovery of high-redshift submm sources (Smail, Ivison, & Blain 1997), there has been an ongoing debate about their nature and their dust properties. In the absence of redshifts, it is difficult to understand whether they represent hot sources at very high redshifts or colder sources at more modest redshifts (e.g., Eales et al. 1999, 2000), similar to FB1-40 and FB1-64 (Chapman et al. 2002c). Chapman et al. (2003a) have measured spectroscopic redshifts for radio-identified submm galaxies, claiming a typical dust temperature of $\sim 40$ K by assuming the empirical relation between the FIR and radio observed locally (e.g., Helou et al. 1985). However, without additional SED measurements, it is unclear what distribution in dust properties for the submm galaxies remains consistent with their currently measured properties.

In this paper, we characterize the local luminosity-color bivariate distribution, $\Phi(L, C)$, of IRAS galaxies from the 1.2 Jy sample, selected at 60 $\mu$m. We represent the total infrared luminosity with $L_{\text{TIR}}$, and the $R(60, 100)$ IR color is used as the best single-parameter description of the IR SED of galaxies; $L_{\text{TIR}}$ is defined as in Dale et al. (2001), integrating over the SED from 3 to 1100 $\mu$m. These authors define a bolometric conversion between the more typical FIR luminosity (e.g., Helou et al. 1985) as

$$\log(\text{TIR}/\text{FIR}) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4,$$

where $x = \log f_{60}(60 \mu m)/f_{100}(100 \mu m)$ and $[a(z = 0)] = [0.2738, -0.0282, 0.7281, 0.6208, 0.9118]$. The first part of the paper is directed at the derivation of an analytical form for the local IRAS color distribution. In order to constrain the effect of the IR color distribution on our understanding of high-redshift FIR-luminous galaxies, we evolve the bivariate distribution according to luminosity evolution prescriptions used in the literature. For convenience, we use the terms “color” (meaning 60/100 $\mu$m flux ratio) and “temperature” interchangeably, even though we do not collapse the SED description to a single-temperature blackbody. The color is taken to be a diagnostic of the typical heating conditions in the interstellar medium of a galaxy and, therefore, would be indicative of a characteristic dust temperature. Section 2 describes our sample used to construct $\Phi(L, C)$. Section 3 provides a statistical analysis of the 1.2 Jy sample and provides a best-fit analytical form to $\Phi(L, C)$. Section 4 explores the evolutionary behavior of $\Phi(L, C)$, while §5 compares the evolving $\Phi(L, C)$ to existing data sets at higher redshift.

2. SAMPLE SELECTION

Our starting point for this study is the $S_{60 \mu m} > 1.2$ Jy sample of galaxies (Fisher et al. 1995). We plot the $R(60, 100)$ distribution of these galaxies as a function of both $L_{\text{FIR}}$ and $L_{\text{TIR}}$ in Figure 1. The parameter $L_{\text{FIR}}$ is calculated directly from the 60 and 100 $\mu$m flux densities and the redshift as described in Helou et al. (1985). The $L_{\text{TIR}}$ parameter represents a bolometric correction for the flux from 3 to 1100 $\mu$m and represents a larger correction for sources with cooler $R(60, 100)$ colors, as seen in the comparison of the two panels of Figure 1. The analytical expression mapping the average FIR luminosity to TIR luminosity from Dale et al. (2001) was reproduced in §1.

While studying this sample, we noticed a significant tail of very cold and luminous galaxies, departing bimodally from the main distribution (see Fig. 1). On closer inspection of the 60 and 100 $\mu$m IRAS fluxes using the XSCANPI application provided by the Infrared Processing and Analysis
Center (IPAC), we found that almost 100% of these sources have spurious measurements. First, the cirrus contamination can be large, affecting the 100 μm more than 60 μm and leading to an apparently small 60 μm/100 μm ratio. Second, the 60 μm flux measurements found for some of the sources from XSCANPI were less than 1.2 Jy and should not be included in our catalog.

We flagged all such sources in our sample and removed them from subsequent consideration. To ensure that this effect did not significantly contaminate the main distribution, we randomly selected 200 of the sources with \( L_{\text{FIR}} > 10^{11} L_\odot \) for XSCANPI analysis, finding them to have correct flux estimates to within 5% of the Fisher et al. (1995) values. We then derive the TIR luminosity function with the revised catalog of 1.2 Jy sources. We adopt an accessible volume technique as described in Avni & Bahcall (1980). Our general luminosity function is represented as

\[
\Phi(L) dL = \frac{1}{V_i},
\]

with \( \Phi(L) dL \) as the number density of sources (Mpc\(^{-3}\)) in the luminosity range \( L-L+\Delta L \). The accessible volume, \( V_i \), represents the \( i \)th source in the sample, the maximum volume in which the object could be located and still be detected in the IRAS 1.2 Jy 60 μm catalog. The sum is then over all sources within the luminosity range. We then map sources to their TIR luminosity using the above definition. Our constructed TIR luminosity function is therefore directly related to the 60 μm luminosity function for this sample.

Figure 1 also shows the median \( R(60, 100) \) values and the interquartile range, revealing the dust temperature to luminosity relation. In §§ 3–6, we study and characterize this relation.

3. RESULTS

3.1. Statistics

In choosing a luminosity variable in \( \Phi(L, C) \) to describe the distribution of galaxies, it is desirable to minimize the dependencies between \( L \) and \( C \). The parameter \( L_{\text{TIR}} \) has been shown to provide less dependency than \( L_{\text{FIR}} \) on IRAS colors (Dale et al. 2001). Similarly, \( R(60, 100) \) has been demonstrated to parameterize the variation in mid-IR to FIR properties better than any other combination of IRAS bands (Dale et al. 2001).

Having chosen our variable representation of \( \Phi(L, C) \), we can proceed to quantify the relation for the \( S_{60} > 1.2 \) Jy IRAS sample and explore the implications. We begin by calculating the median and first/last quartile statistics for the 1.2 Jy sample, corrected for spurious cold, luminous sources as described in § 2. The analytic relation in the distribution median is well fitted by a dual power law, as shown in Figure 2:

\[
\log R \left( \frac{60}{100} \right) = 0.162 \times \log \left( \frac{L_{\text{TIR}}}{L_\odot} \right) - 2.080, \quad \text{if} \quad L_{\text{TIR}} > 5 \times 10^{10} L_\odot ,
\]

\[
\log R \left( \frac{60}{100} \right) = 0.022 \times \log \left( \frac{L_{\text{TIR}}}{L_\odot} \right) - 0.593, \quad \text{if} \quad L_{\text{TIR}} < 5 \times 10^{10} L_\odot .
\]

We express this composite function with a smooth transition as

\[
R \left( \frac{60}{100} \right) = C_s \times \left( 1 + \frac{L_s}{L_{\text{TIR}}} \right)^{-\delta} \times \left( 1 + \frac{L_{\text{TIR}}}{L_s} \right)^{\gamma},
\]

with \( \gamma = 0.16, \delta = 0.02, C_s = 0.45, \) and \( L_s = 5.0 \times 10^{10} L_\odot \).

The statistics and the fitted relation are shown in Figure 2a. We also fitted the width of the interquartile statistics after removing the median (Fig. 2b). The width of the distribution described in terms of this statistic is essentially
constant in \( \log[R(60, 100)] \) as a function of \( L_{\text{TIR}} \), therefore showing a slight broadening in \( R(60, 100) \) as shown in Figure 2b. At the highest luminosities there is some evidence for significant broadening of even the \( \log[R(60, 100)] \), although the statistics are poor and there remains a concern that all spurious sources have not been removed from the sample (§ 2).

These equations provide a simple, first-order description of the \( \Phi(L, C) \) distribution of IRAS galaxies from which to gauge a more comprehensive analysis. This expression should be suitable for many applications and evolutionary extrapolations.

### 3.2. Fitting the \( \Phi(L, C) \) Distribution

We now consider the detailed distributions in \( R(60, 100) \) for the 1.2 Jy sample. Histograms of the distribution from each luminosity bin are plotted in Figure 3. For clarity, a fixed constant is introduced to offset each class of \( L_{\text{TIR}} \). With no a priori assumptions about the \( R(60, 100) \) distribution of the IRAS sources, we begin by testing whether the population might be well represented by a Gaussian in the \( R(60, 100) \) color. A Gaussian in both linear \( R(60, 100) \) and \( \log R(60, 100) \) was first fitted to each bin of 380 sources. The functional form

\[
C + Bx + D \exp \left[-\frac{1}{2} \left( \frac{x - A}{E} \right)^2 \right],
\]

where \( x \) represents \( R(60, 100) \), was fitted to each histogram of 380 sources. An equal weighting of the points in the histogram was assumed in the fitting. The residuals were compared, with the \( \chi^2 \) being a factor of \( \sim 2 \) smaller for the Gaussian in \( \log \) (or lognormal distribution), suggesting the lognormal as the more appropriate representation.

The results of our fitting are shown in Figure 4a, with each luminosity bin of the color histogram offset for clarity. The median has been removed for comparison of luminosities. Bottom: Subtracted residuals are shown for both lognormal (heavy line) and linear Gaussian fits (dashed light line). Note that an improved fit, in the sense of reduced \( \chi^2 \), cannot be obtained by a first-order term of the form \( xG \). A higher order term is required to obtain a better fit, which is not justified for the present data set.

![Figure 3](image1)

**Fig. 3.** \( \Phi(L, C) \) distribution in log IR color, shown as a smoothed histogram. The effective histogram bin size is \( dR(60, 100) = 0.05 \). Histograms of \( L_{\text{TIR}} \) are shown offset by a fixed constant for clarity and labeled on the right in \( L_{\odot} \).

![Figure 4](image2)

**Fig. 4.** Top: \( R(60, 100) \) distribution (heavy solid lines) with lognormal fits (light solid lines) and linear Gaussian fits (light dashed lines). Offsets for each luminosity class are inserted as in Fig. 3. The median \( R(60, 100) \) has been removed for comparison of luminosities. Bottom: Subtracted residuals are shown for both lognormal (heavy line) and linear Gaussian fits (dashed light line). Note that an improved fit, in the sense of reduced \( \chi^2 \), cannot be obtained by a first-order term of the form \( xG \). A higher order term is required to obtain a better fit, which is not justified for the present data set.
fit of the lognormal, as well as the skewed nature of the distribution. Heavy lines show the residuals from a log Gaussian fit, while the light dashed lines show the residuals from a linear Gaussian fit. While the skew is an apparently large and systematic effect (it appears as a double S, integral shape), the most significant deviations occur in the wings of the distribution, where our error bars are larger than the skew due to the rapidly diminishing number of sources in each histogram bin. For completeness we attempted to fit the skewed component with a multiplicative factor \( \lambda \exp(-z^2/2) \), but that did not improve the \( \chi^2 \). The systematic skew to the distribution therefore cannot be removed without a term that is too high an order to justify from the statistics of the sample. We therefore use the lognormal distribution fit as the best representation of the 1.2 Jy IRAS galaxy sample. Note also that the residual skew is negligible near the intermediate luminosity bins of the population (the best sampled region) and becomes more pronounced toward the luminosity extremes. The skew may therefore partially be a result of including broader luminosity ranges in the distribution to retain equal-number bins.

The distribution over \( R(60, 100) \) color can then be expressed as

\[
G(C) = \exp \left[ -\frac{1}{2} \left( \frac{C - C_0}{\sigma_C} \right)^2 \right],
\]

where \( \sigma_C = 0.065 \) and \( C = \log R(60/100) \);

\[
C_0 = C_\star \times \left( 1 + \frac{L_\star}{L_{\text{TIR}}} \right)^{-\delta} \left( 1 + \frac{L_{\text{TIR}}}{L_\star} \right)^{\gamma},
\]

with \( \gamma = 0.16, \delta = 0.02, C_\star = 0.45, \) and \( L_\star = 5.0 \times 10^{10} L_\odot \).

As the residual skew is close to symmetrical about the Gaussian fit peak, we find that our fit to the distribution describes nearly the same function as the median and first/last quartile expression (a lognormal) presented in \( \S \) 3.1. The formal \( \Phi(L, C) \) is then expressed as follows, where we have derived the TIR luminosity function directly from the 1.2 Jy catalog as described in \( \S \) 3.1.

\[
\Phi(L, C) dL dC = \Phi_1(L) \times \Phi_2(C) dL dC
\]

\[
\quad = \rho_\star \times \left( \frac{L}{L_\star} \right)^{(1-\alpha)} \times \left( 1 + \frac{L}{L_\star} \right)^{-\beta} \times \exp \left[ -\frac{1}{2} \left( \frac{C - C_0}{\sigma_C} \right)^2 \right] dL dC, \quad (1 - \alpha),
\]

with \( L = L_{\text{TIR}}, \rho_\star = 4.34 \text{ Mpc}^{-3} L_\odot^{-1}, L_\star = 10^{10.7} L_\odot, \alpha = 1.5, \beta = 2.2, \quad C = \log(S_{60 \mu m}/S_{100 \mu m}), \quad C_0 \) as above, and \( \sigma_C = 0.065 \).

Our analysis has suggested that the \( L-C \) distribution is best described with a dual power law in \( L \) with a break luminosity \( 5.0 \times 10^{10} L_\odot \), a faint-end slope of \( \delta = 0.02 \), and a bright-end slope of \( \gamma = 0.16 \). We offer two possible physical interpretations of this result.

First, this may be the point of transition from cirrus-dominated luminosity to active star formation (i.e., high-density photodissociation regions, etc.) dominating the luminosity. The different power laws then arise because in the former case the luminosity increases mostly by making the emitting dust mass larger, whereas for active star formation, the heating drives the luminosity.

Second, this may be the point of transition in the 60 \( \mu m \) band from where fluctuating grain emission still contributes to where large dust grains dominate. Once large dust grains dominate the 60 \( \mu m \) flux, the \( R(60, 100) \) ratio begins to look like a blackbody ratio, leading, naturally, to the broken power-law relation. However in this case, the high-luminosity, steep portion of the relation should scale with dust temperature as \( L \propto T^3 \). Since this is not observed, this second explanation cannot dominate the observed relation.

4. CONSEQUENCES OF THE \( L-C \) RELATION FOR EVOLUTION MODELS

Several authors have recently modeled the evolution of dusty galaxies using pure, or nearly pure, luminosity evolution, reproducing the observed counts and backgrounds at IR through submm wavelengths (e.g., Blain et al. 1999a, 1999b; Malkan & Stecker 2001; Rowan-Robinson 2001; Chary & Elbaz 2001; Chapman et al. 2002b; Franceschini et al. 2002). None of these models include a bivariate luminosity function (LF) and, at best, map the dust temperature or spectral shape monotonically to the luminosity.

The spectral shape of the FIR background (FIRB) detected by DIRBE at 140 and 240 \( \mu m \) (Puget et al. 1996; Fixsen et al. 1998) indicates a peak at ~200 \( \mu m \). However, the width of the peak suggests that galaxies over a large range in redshifts and/or dust temperature contribute to the FIRB. There is a degeneracy of dust temperature with redshift (Blain 1999), since both translating a source to higher redshift and decreasing the dust temperature will shift the SED to lower frequency. This suggests an urgency to explore the effect of the bivariate LF on evolutionary models.

4.1. A Color-added Evolution Model

Our goal is to understand the key differences in \( \Phi(L, C) \) from the single-variable \( \Phi(L) \). We turn to Monte Carlo simulations of a pure luminosity evolution paradigm, similar to the models of the above authors. In companion papers (Lewis et al. 2003; S. Chapman 2003, in preparation), we demonstrate that our model is able to simultaneously fit the FIR background and the counts at various wavelength bands.

We evolve the local FIR LF using \( \Phi(L, \nu) = \Phi_0[L/g(z), \nu_0(1 + z)] \). Our evolution function follows a power law in redshift, \( g(z) = (1 + z)^\delta \), out to \( z = 2.6 \). Beyond \( z = 2.6 \) the function drops again as \( g(z) = (1 + z)^{-4} \) to avoid overpredicting the FIR background. This power-law index is chosen on the basis of evolutionary models fitted to both optical- and submm-wavelength data (Blain et al. 1999a, 1999b). The evolution to \( z = 2.6 \) was chosen as this provided the best fit to the joint radio/submm sample of galaxies (Chapman, Lewis, & Helou 2002a; Lewis et al. 2003)—currently the best constraint on high-z FIR galaxy evolution (Blain et al. 1999a). The redshift \( z = 2.6 \) is also close to the consensus of the median redshift for SCUBA sources suggested, for example, by photometric redshifts in Ivison et al. (2002). In order to match the pure luminosity evolution, we truncate the evolved functions at low luminosity and at both extremes of color, so that they integrate to identical numbers of sources per comoving volume.

One additional assumption in this model is that the bivariate distribution, \( \Phi(L, C) \), evolves in \( L \) as a function of
redshift, while the relation as observed locally between $L$ and $C$ holds at all redshifts. The width of the distribution in $C$ follows our measured relation from $\S$ 3, which remains close to constant in $\log[R(60, 100)]$. This implies that the characteristic dust temperature of higher redshift sources will be greater because of the rise in $L$.

We note, however, that while the no-evolution hypothesis for $C$ is the simplest, it is not necessarily the obvious choice physically. Higher redshift sources may have distinctly different dust properties than local analogs. For instance, metallicities will likely be lower, there may be less dust to heat, and dust may be less centrally concentrated. This form of evolution might lead to the $R(60, 100)$ distribution being biased to cooler IR colors with increasing redshift. In the absence of any strong constraint from the counts or background (Lewis et al. 2003; S. Chapman 2003, in preparation), this assumption needs to be tested by future observational data sets. However, we shall examine in $\S$ 4.2 the preliminary evidence in support of this simple hypothesis.

We then incorporate this hypothesis into our Monte Carlo models, drawing $R(60, 100)$ values from the evolving $\Phi(L, C)$ to fill our desired survey volume. This model does not include observational error but should accurately represent the chance of observing a source with a given IR color in a particular survey. The simulation is cut off at $10^8 L_\odot$ in order to avoid truncation effects from the fixed comoving number density in the luminosity functions.

In order to tie our evolving $L(T_{\text{IR}})$ luminosity function to observable quantities, we map each galaxy with a given $L_{\text{TIR}}$ and $R(60, 100)$ value to an SED shape. Normalization of the SEDs is accomplished by integrating the SED over the 3–1100 $\mu$m range and scaling to the adopted $L_{\text{TIR}}$. Spectral templates are taken from the Dale et al. (2001; Dale & Helou 2002) catalog, divided into 64 classes from $R(60, 100) = 0.29$ to 1.64, corresponding roughly to single-component dust temperature models of 19–56 K. Representative SEDs from our catalog are shown in Figure 5. The global galaxy SEDs adopted here use a power-law distribution of dust over heating intensity $U$ in order to reproduce the range of photometric and spectroscopic properties observed by IRAS and ISO for galaxies in the local universe. Dale & Helou (2002) specifically investigate the dust emissivity of the local IRAS galaxies, using the submm data presented in Dunne et al. (2000). They find that the dust emissivity should vary with environment with $\beta = 2.5–0.4 \log U$, where $U$ is the local radiation field.

**4.2. Model Results and Implications**

To underscore the effect of not including the $\Phi(L, C)$ distribution in the redshift evolution, we show in Figures 6, 7, and 8 the output from our luminosity evolution models, with and without the broad local color distribution in the LF.

In Figure 6, we plot the $L-T_d$ plane showing points drawn from our Monte Carlo sampling of the evolving bivariate LF. We compare directly the bivariate $\Phi(L, C)$ to an equivalent $\Phi(L)$, with a very narrow range of colors for each luminosity (bold, offset points). The ordinate has been mapped from $R(60, 100)$ to dust temperature ($T_d$) using the single-temperature graybody that provides the best fit to the SED template. The quantized appearance of the points in $T_d$ results from the individual 64 template SED classes considered. The LF with narrow range of IR color closely
approximates a single-variable LF whereby luminosity maps monotonically to $T_d$ or IR color. Figure 6 provides an overview of $R(60, 100)$ as a function of $L$, simply extrapolating the local $L$-$C$ relation to the required luminosities—our baseline assumption in this model. The distribution is symmetric in log($T_d$) about each luminosity. As the $\Phi(L, C)$ distribution remains identical to our locally characterized form (§3) for all redshifts, any deviations from the median local relation reflect only the scatter in the relatively small numbers of luminous galaxies in this simulated survey volume.

In Figure 7, we show the $L_{\text{TIR}}$ distribution as a function of redshift. The left panels show the $\Phi(L, C)$ model, while the right panels show the single-variable LF model. Until we apply a flux limit to the figure (darker points), there is no difference between the visualizations since they have the same luminosity evolution formalism. Figure 7 is our Monte Carlo representation of the evolving LF; vertical slices reveal the dual power law $\Phi(L)$ at each redshift. Note that with comoving number density, in the absence of luminosity evolution, a local physical density of $N$ objects per Mpc$^3$...
corresponds to a physical density of $N \times (1 + z)^3$ at redshift $z$. In this type of model the evolution is not necessarily meant to imply that ULIRGs fade into less luminous LIRGs with time. Instead, it provides a picture where, for instance, the local counterparts with volume densities similar to ULIRGs at $z \sim 1$ have $10^{11.5} L_{\odot}$ at the present day. This form of evolution is then not describing the internal physics of the IR-luminous population; high-$z$ ULIRGs do not evolve into similar local ULIRGs, but instead likely form ellipticals (e.g., Sanders & Mirabel 1996; Tacconi et al. 2003), fading away from the LF.

When flux-limited surveys are considered in the context of Figure 7, differences in the two models become manifest. In the case of the single-variable distribution, each $L_{\text{TIR}}$ point maps uniquely to a flux for a given wavelength. However in the bivariate LF, each $L_{\text{TIR}}$ point corresponds to a probability distribution of fluxes corresponding to the lognormal distribution in $R(60, 100)$ and the associated range of SED templates that can be tied to the $L_{\text{TIR}}$ value. The sensitivity limits in the 850 and 24 $\mu$m bands are shown for characteristic survey depths with SCUBA and SIRTF (2 and 0.1 mJy rms, respectively). The structure in the 24 $\mu$m survey is a result of polycyclic aromatic hydrocarbon bands (rest $\sim 10$ $\mu$m) being redshifted through the SIRTF 24 $\mu$m filter. At 850 $\mu$m, the $K$-correction from the rising graybody dust SED results in a flat luminosity-flux relation for redshifts $z \gtrsim 1$.

The effect is subtle in Figure 7, as both the luminosity function and the color distribution are scattering the observed fluxes, largely canceling dramatic differences in the effective luminosity limits probed with redshift. However, the most important difference between the bivariate and luminosity-only models is apparent in Figure 7: in the simpler $\Phi(L)$ model, the flux limit for a given wavelength translates at each redshift into a transition range of luminosities, within which galaxies are or are not detected depending on their color. We draw lines on the single-variable model for $L$ corresponding to $T_D$ of 35, 38, and 50 K, using the mapping illustrated in Figure 6.

The implications of this model difference are therefore much more apparent in a direct analysis of $T_D$, as demonstrated in Figure 8. Histograms of sources versus redshift are plotted for the 850 and 24 $\mu$m bands, comparing the detectability of hotter and cooler IR colors (again mapped to the equivalent single component dust temperature). Heavy dashed lines represent the evolving $\Phi(L, C)$ distribution, while lighter lines represent our approximation to a single-variable LF.

We first consider the model differences for a SCUBA 850 $\mu$m survey in Figure 8. A cut for detectable sources colder than $T_D < 35$ K (shown in Fig. 7) lies along the sensitivity limit of SCUBA. This results in the most dramatic difference between the models, since in the bivariate model, the steep luminosity function scatters many more cold, underluminous sources into the sample than the corresponding loss of hotter, higher luminosity sources. The selection along the cold side of the graybody peak for SCUBA therefore produces a substantial population of cold sources that are
strikingly absent when only the single-variable LF is included in the evolution. This difference results in an important prediction of the existence of cold and luminous galaxies at moderate to high redshifts—a prediction that has been verified through the detection of ULIRGs with \( T_d < 30 \) K (Chapman et al. 2002c). The opposite effect is shown in the middle panel of Figure 8 for the range \( 35 \) K < \( T_d < 38 \) K, producing a band in luminosity lying just above the sensitivity limit of our SCUBA survey in Figure 7. The bivariate model now scatters the warmer sources below our detection limit, resulting in a dearth of sources relative to the single-variable LF. For the hottest sources (\( T_d > 50 \) K), the luminosities are large enough (Fig. 7) that the sensitivity limit of the survey has no bearing on the observed sources, and the comparison of models simply reflects the steep luminosity function that asymmetrically scatters more low-luminosity hot sources into the temperature cut than luminous cold sources that fall beneath the cut. The bivariate \( \Phi(L, C) \) model thus predicts almost twice as many warm galaxy detections, even though the underlying luminosity distribution is the same as in the \( \Phi(L) \) model.

For surveys selecting sources along the hot dust side of the graybody peak, that being the case for all the accessible wavelengths of SIRTF except 170 \( \mu \)m, colder luminous sources will be missed and hotter low-luminosity sources will be preferentially detected. This is exactly in the opposite sense to the situation for submm-selected surveys (such as with SCUBA), preferentially detecting colder sources at a given luminosity (Eales et al. 1999; Blain et al. 2002).

The sensitivity limit of SIRTF is adversely affected by the steep Wien slope of the far- to mid-IR SED, making more distant sources difficult to detect and resulting in the steep sensitivity curve in Figure 7. The result in Figure 8 is that the histograms near the sensitivity limit of SIRTF will be dominated by the large numbers of sources lying both above and below any fixed temperature cut. The histograms in the cold and warm 24 \( \mu \)m panels therefore continue to show the asymmetric shifting of sources due to the steep luminosity function, but only as a small perturbation on the large number of sources that are unaffected by the sensitivity limit. However, for hotter dust temperatures, an excess of a factor greater than 2 of sources is predicted by the \( \Phi(L, C) \) model. This excess occurs for the SIRTF 24 \( \mu \)m band for the same reason discussed above for SCUBA. All sources are luminous enough to be detected regardless of their temperature, and there are far more lower luminosity sources that are boosted by the bivariate LF into the \( T_d > 50 \) K bin than vice versa.

Consideration of the model differences in Figure 8 also addresses the question of the SCUBA/SIRTF galaxy overlap. As the SCUBA and SIRTF observing windows lie on opposite sides of the dust spectral energy peak, the direct overlap of the populations is a strong function of the dominant dust temperature. In particular, a cold (\( T_d < 30 \) K) SCUBA population may be difficult to detect even in the deepest SIRTF exposures, as revealed in the high-redshift excess of cold SCUBA sources compared with the SIRTF population. Scritiny of Figure 7 along the survey flux limits reveals that at \( z \leq 3 \), the individual SCUBA galaxies detected in either model are mostly detected by SIRTF at 24 \( \mu \)m at this survey depth. However, deeper submm surveys by instruments such as the Large Millimeter Telescope (Hughes 2001) may uncover fainter cold galaxies at lower redshifts that would require very deep SIRTF exposures to detect. By contrast, most or all of the SCUBA sources should be detected by SIRTF over all redshifts when the characteristic dust temperatures are higher, as suggested by the similar \( N(z) \) in the lower panels of Figure 8.

Figures 7 and 8 are also suggestive of the effect of the evolutionary form on the detectability of galaxies at different wavelengths. The properties of the sources detectable at high redshift are a sensitive function of the evolutionary form adopted—in our scenario, lower redshift peaks in the evolution function would result in an appreciably smaller fraction of high-redshift galaxies detected (effectively generating differences in the redshift distributions). For example, a direct application of our \( \Phi(L, C) \) model to the SCUBA population (Chapman et al. 2002a; Lewis et al. 2003) has suggested a best-fit peak at a lower redshift of \( z \sim 2.5 \). However, this is extremely sensitive to any evolution in the IR color distribution (but see § 5 to follow). We defer detailed fitting of our model to the available high-redshift counts and FIRB to a companion paper (Lewis et al. 2003). Ultimately, redshift surveys of SIRTF and SCUBA galaxies will be required to test the detailed form of the evolution function (see Chapman et al. 2003a for preliminary results on the SCUBA galaxy redshift distribution).

5. COMPARISON WITH DATA OUT TO HIGHER REDSHIFTS

Our comparison of evolving a single-variable LF versus a bivariate \( \Phi(L, C) \) (Figs. 6–8) clearly demonstrates that an oversimplified representation of the galaxy population can miss large classes of detectable sources at high redshifts. It is then crucial to understand whether the IR color distribution itself has evolved with redshift.

While much attention has been paid to the luminosity evolution of ULIRGs (e.g., Blain et al. 1999a, 1999b; Chary & Elbaz 2001; Malkan & Stecker 2001; Rowan-Robinson 2001; Chapman et al. 2002b; Ivison et al. 2002), there has been no study of the \( R(60, 100) \) evolution. The simplest scenario, which we considered above, is that the \( R(60, 100) \) correlation with \( L_{\text{IR}} \) continues to higher luminosities, extrapolating the expressions derived in this work for the local \( R(60, 100) \) distribution.

Existing data sets are largely inadequate for characterization of the high-\( z \) IR color distribution. ISOPHOT and SCUBA galaxies do not generally have accurate redshifts or even detections at sufficient wavelengths to measure the rest frame \( R(60, 100) \). However, a survey of moderate-redshift, luminous IRAS galaxies (Stanford et al. 2000) provides an initial foray into the higher redshift \( R(60, 100) \) distribution (for a complementary analysis of this population, see also Blain, Barnard, & Chapman 2003). This section will focus on the Stanford et al. (2000) IRAS galaxies, as well as the microwave radio sources (Chapman et al. 2003b), to assess our \( \Phi(L, C) \) characterization.

5.1. 60 \( \mu \)m–selected LIRGs/ULIRGs at 0.1 < \( z < 0.9 \)

A recent sample of distant LIRGs and ULIRGs from Stanford et al. (2000) has an effective sensitivity only 2–3 times higher than that of the deepest ISOPHOT 170 \( \mu \)m surveys, and we can use this sample to study the higher redshift \( \Phi(L, C) \). The Stanford et al. (2000) sample was selected from a positional cross-correlation of the IRAS Faint Source Catalog with the FIRST database. Objects from this set
were selected for spectroscopy by virtue of following the well-known star-forming galaxy correlation between 1.4 GHz and 60 μm flux and by being optically faint on the Palomar Observatory Sky Survey. Optical identification and spectroscopy were obtained for 108 targets at the Lick Observatory 3 m telescope. Most objects show spectra typical of starburst galaxies and do not show the high-ionization lines of active galactic nuclei. The redshift distribution covers 0.1 < z < 0.9, with 13 objects at z > 0.5 and an average redshift of z = 0.31.

5.1.1. The Dependence of R(60, 100) on L

We first address the issue of the dependence of width in C (= R(60, 100)) on the luminosity. Figure 9a shows the R(60, 100) versus L distribution of the Stanford et al. (2000) sample relative to our fit C relation from § 3, overlaying the local BGS IRAS galaxies observed with SCUBA by Dunne et al. (2000) for reference. The rest frame R(60, 100) has been calculated by fitting the Dale et al. (2001, 2002) template SEDs to the observed frame 60 μm, 100 μm, and radio points, and extracting the rest frame 60 and 100 μm fluxes. The moderate-z sources from Stanford et al. (2000) are not, however, all detected at 100 μm. We have run the IPAC application XSCANPI over all 103 Stanford sources to derive improved estimates of the 60 and 100 μm fluxes, finding 46 with spurious 100 μm measurements. The sources without 100 μm detections are presented as lower limits to R(60, 100), while those with only marginal 100 μm detections are differentiated with open symbols. The fits to the distribution are then derived, using only sources with secure 100 μm detections. The fit result is shown as the lighter solid line, with the ±1 σ lines shown as lighter dashed lines in the top panel of Figure 9. The sources with 100 μm limits appear to be uniformly distributed amongst the 100 μm detected sources, and a large systematic skew is unlikely.

The median relation between L_{FIR} and R(60, 100) is similar to that observed locally, although it falls to the −1 σ deviation of the local relation for the most luminous Stanford sources. However, these most luminous sources

![Graph showing the dependence of rest frame R(60, 100) on L_{FIR}](image)
are not very numerous and contribute negligibly to the fit lines. Moreover, the fit did not take into account the lower limits to \( R(60, 100) \). The median of this luminous tail of Stanford sources on their own is in fact more consistent with the local relation.

The width of the Stanford distribution matches the extrapolation from the 1.2 Jy envelope at \( L_{\text{FIR}} = 3.1 \times 10^{12} L_\odot \) (Fig. 9a). However, the width of the Stanford sources exceed the local distribution by a factor of \( \sim 50\% \) at \( L_{\text{FIR}} = 4.6 \times 10^{11} L_\odot \), and there is an apparent excess of LIRG/ULIRG class sources as cold as the two FIRBACK galaxies discussed in § 1, FB1-40 and FB1-64 at their respective FIR luminosities (see § 5.1.3). As the Stanford et al. (2000) sources were selected in the 60 \( \mu \)m band, a direct comparison with our 1.2 Jy catalog is justified, with the assumption that the fainter flux limits of the Stanford et al. (2000) sample simply allow a probe of the higher luminosity distribution out to moderate redshifts, where evolution is still not likely to be a large effect. However, even for sources with detected 100 \( \mu \)m fluxes, the signal-to-noise ratio is not high, and photometric scatter may affect the width of the diagram. A further complication is that while the rest frame \( R(60, 100) \) has been demonstrated to correlate well with cold dust temperature using 850 \( \mu \)m measurements (Dunne et al. 2000), the observed \( R(60, 100) \) becomes less effective at constraining the cold-temperature properties as redshifts increase beyond \( z \sim 0.5 \). The majority of the Stanford sources are at \( z \lesssim 0.4 \), and this should not result in a large effect on the average sample properties. Observations at 850 \( \mu \)m will be required to properly assess the contribution of the Stanford sources at higher redshifts to the width of the distribution.

We therefore conclude that the direct extrapolation from the local IRAS galaxies is consistent with the width of the Stanford et al. (2000) sources, with a possibility of an increasing width at lower luminosities in the Stanford sample. We note, however, that the width of even the local distribution at the highest luminosities is highly uncertain, and a change in the slope of the broadening function could be in agreement to within the errors.

5.1.2. The Dependence of \( R(60, 100) \) on Redshift

We now address the possible evolution in the \((L, C)\) relation with redshift. In Figure 9b, the \( R(60, 100) \) ratio is shown as a function of redshift for the same sources. The overlapping luminosity regions can be compared directly. Triangles have \( \log L_{\text{FIR}}/L_\odot \sim 11.5 \pm 0.5 \), in both IRAS-BGS and Stanford et al. (2000) galaxies. A fit to the overlapping luminosity populations (shown by oversized symbols) is shown with a solid line, along with 1 and 2 \( \sigma \) envelopes.

The salient point of Figure 9b is that the Stanford sources do not appear significantly different in the median than the local comparison sample. The large fraction of Stanford sources with lower limits and uncertain 100 \( \mu \)m fluxes may be responsible for the median point lying somewhat lower than local galaxies. The current data therefore do not support a scenario in which the median \( R(60, 100) \) in \( C \) varies significantly with redshift out to \( z \sim 1 \). The width in \( C \) of both luminosity bins is similar in Figure 9b and wider than the local distribution. This is consistent with the broadening of \( C \) with luminosity and suggests that the redshift evolution affects the width in \( C \) only insofar as higher redshift sources have characteristically higher \( L \). However, the statistics for sources with \( z > 0.5 \) are poor. With identified redshifts for increased numbers of submm galaxies (see Chapman et al. 2003a), this issue will be directly addressable.

5.1.3. ISOPHOT 170 \( \mu \)m Galaxies at Intermediate Redshift

Recently, Keck spectra and UKIRT high spatial resolution near-IR imagery for two of the proposed highest redshift sources from the FIRBACK-N1 170 \( \mu \)m survey were obtained (Chapman et al. 2002c). These authors found that the redshifts of counterparts to the 170 \( \mu \)m sources confirm that both sources are ULIRGs, but that their redshifts are significantly lower than implied by fitting a typical ULIRG SED to their FIR/submm/radio SEDs. This indicates that they have cooler dust temperatures, \( T_d \sim 30 \) K, than the canonical ULIRG values (\( T_d \sim 45 \) K). As previous models have failed to predict the existence of such cold and luminous sources, these two galaxies therefore confirm the importance of considering the full \( \Phi(L, C) \) in evolutionary models.

In Figure 9, we overplot these two FIRBACK ULIRGs on the local IRAS BGS and the Stanford LIRG/ULIRG sample; FN1-40 and FN1-64 are cold objects, lying near the \( \sim 2 \) \( \sigma \) level below the local median relation. While FB1-40 and FB1-64 appear as cold objects relative to the local distribution in \( \Phi(L, C) \), there is not yet enough follow-up to ISOPHOT sources to know whether \( C \) has substantially evolved out to \( z \sim 1 \). FN1-40 and FB1-64 remain at the periphery of the distribution, but they are certainly not unusual objects when compared to similar-redshift ULIRGs from the Stanford et al. (2000) sample.

5.2. The L-C Relation for Microjansky Radio Sources

In this subsection, we test the lower luminosity \( L-C \) relation out to \( z \sim 0.6 \) using a sample of radio galaxies with 20 cm fluxes ranging from 30 to 500 \( \mu \)Jy from Chapman et al. (2003b). These radio sources were observed in the submm, and all have spectroscopic redshifts in the range \( z = 0.1-0.6 \). From this information we can estimate \( T_d \) and \( L_{\text{FIR}} \) by assuming the FIR-radio correlation for star-forming galaxies (Helou et al. 1985) and a synchrotron spectrum with an index of \( \alpha = -0.8 \) (Richards 1999). The temperature \( T_d \) is estimated by taking the SED template from Dale & Helou (2002), which has the appropriate temperature to fit the radio and submm points at the fixed redshift of the source. Note that we are only indirectly testing the \( L-C \) relation with these galaxies (we are estimating both \( L \) and \( C \) through local empirical correlations), and we cannot deconvolve variations in the \( L-C \) relation from the FIR-radio correlation.

We then restrict the sample to those sources with \( L_{\text{FIR}} < 10^{11} L_\odot \) as the spectroscopic incompleteness introduces a severe bias for more luminous (and typically optically fainter) galaxies (as discussed in Chapman et al. 2003b). This also reduces the radio-bright AGN contribution to the sample, which will have much larger apparent \( L_{\text{FIR}} \) calculated in this manner because of the AGN-generated radio excess.

In Figure 10 we plot \( T_d \) versus FIR luminosity, along with our derived local IRAS galaxy relation from § 3. The average points in Figure 10 do not include sources individually detected in the submm, and the variance from measurement errors is expected to be comparable to that from scatter in the dust properties. Thus our points can be used only as a characterization of the average \( L-C \) relation.
Figure 10.—Dust temperature ($T_d$) vs. the log of FIR luminosity for radio sources with $L_{\text{FIR}} < 10^{11} L_\odot$ and spectroscopic redshifts. Sources span $z = 0.1–0.6$ and have been divided into three equal-number bins. The $T_d$ and FIR values have been calculated from the redshift and the submillimeter/radio measurements as described in the text. Our derivation of the median and interquartile range of local $IRAS$ galaxies from the 1.2 Jy catalog are shown. The agreement of the higher redshift sources with the local relation is remarkable.

Figure 10 demonstrates that the $L_{\text{FIR}} < 10^{11} L_\odot$ radio sources, spanning the redshift range $z = 0.1–0.6$, appear to follow the local $IRAS$ color distribution very well. This suggests jointly that neither the $L-C$ relation nor the FIR-radio correlation can deviate significantly from local values for these sources. The analysis provides a direct consistency check of the lower luminosity tail of local $IRAS$ galaxies with higher redshift star-forming galaxies, of which the microjansky radio population should represent higher redshift specimens (e.g., Richards 1999).

6. CONCLUSIONS

We have analyzed the $R(60, 100)$ distribution of local IR-luminous galaxies, finding a best-fit, low-order analytical expression for the bivariate luminosity function, $\Phi(L, C)$. A lognormal distribution about a dual power law in the median $R(60, 100)$ versus $L_{\text{TIR}}$ is demonstrated to provide the best fit of the $IRAS$ population over 4 orders of magnitude in $L_{\text{TIR}}$.

We then studied the redshift evolution of $\Phi(L, C)$, using a luminosity evolution paradigm. We demonstrated that while similar luminosity regimes are detectable at 24 and 850 μm, the dust temperatures represented by the detectable sources can differ by factors of greater than 2. For a flux-limited survey, the bivariate $\Phi(L, C)$ model predicts about twice as many warm galaxy detections as the equivalent single-variable model, even though the underlying luminosity distributions are the same in the two models. In a similar manner, recently discovered populations of cold, luminous galaxies can be predicted naturally within our model. Consideration of the model differences also addresses the question of the SCUBA/$SIRTF$ galaxy overlap, a strong function of the dominant dust temperature. A cold ($T_d < 30$ K) SCUBA population may be difficult to detect even in the deepest $SIRTF$ exposures, as revealed in the high-redshift excess of cold SCUBA sources compared with the $SIRTF$ population.

We compare our derived color relation with existing data for higher redshift IR galaxies. General agreement with the local color relation is found among all higher redshift galaxy populations considered. We therefore find no significant evidence for a variation in the median $R(60, 100)$ ratio as a function of redshift.

We do find tentative evidence for a broadening of the IR color distribution at higher $L$ from both our local 1.2 Jy sample and the moderate-redshift high-luminosity sample of Stanford et al. (2000). This suggests that redshift evolution does not in itself affect the width in $C$. The baseline assumption that the locally measured $L-C$ relation holds over all redshifts is therefore a reasonable working hypothesis, to be verified with $SIRTF$ data.

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