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# MODELLING THE OBSERVED VELOCITY OF OUR GALAXY RELATIVE TO THE COSMIC MICROWAVE BACKGROUND

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The aim of this work is to model the observed velocity of our galaxy relative to the cosmic microwave background that was recently discovered by Smoot et al. A homogenous and isotropic, flat, relativistic two fluid cosmological model is considered in which two separate fluids act as the source of the gravitational field. In this model one fluid is a comoving radiative perfect fluid modelling the cosmic microwave background and the second a non-comoving imperfect fluid modelling the observed material content of the universe. The model that is obtained is represented as a solution of Einstein's equations and the laws of thermodynamics in which all the physical quantities occurring in the solution are suitably well behaved. In addition, the model is in good agreement with current observations.

Le but de ce travail est de modeler la vitesse observée de notre galaxie par rapport aux microondes cosmiques de fond découvertes récemment par Smoot et al. On considère un modèle cosmologique qui est homogène, isotropique, plat et relativistique composé de deux fluides distincts qui sont la source du champ gravitationnel. Dans ce modèle, un des fluides est un fluide parfait, radiatif et codéplacent qui représente les microondes cosmiques de fond et l'autre fluide représente le contenu matériel observé de l'univers. Le modèle ainsi obtenu est représenté sous forme d'une solution aux équations d'Einstein et les lois de la thermodynamique dans laquelle toutes les quantités physiques qui s'y trouvent se comportent normalement. En plus, le modèle s'accorde bien avec les observations actuelles.

## Introduction

It is currently believed that except for very early times the evolution of the universe is adequately described by "an FRW model". The conventional wisdom is that the universe evolved initially from a radiation-like state to a matter-like universe at later times.

The original Friedmann-Robertson-Walker (FRW) models were solutions of Einstein's equations in which the source of the gravitational field was either a comoving radiative perfect fluid or a comoving matter perfect fluid. Radiation models were to govern the evolution of the universe for early times while models such as the Einstein de Sitter model were supposed to be applicable at more advanced times. Later, in order to obtain a single model governing the evolution of the universe, known radiation models and known matter models were matched continuously at the time when the energy densities of the radiation and matter fields were equivalent (see, for example, Gamow [1956] or Jacobs [1967]).

Many authors (McIntosh [1968a] or [1968b]) were stimulated to investigate FRW models which include both radiation matter fields for all times by the discovery in 1965 of the 2.7°K isotropic cosmic microwave background (Penzias and Wilson [1965] and Roll and Wilkinson [1966]), which was presumed to be a remnant of the "primeval fireball". In these models two (separate) comoving perfect fluids are assumed to act as the source of the gravitational field. (A review of the standard theory of two fluid cosmological models is given in Coley and Tupper [1986]).

Recently it was discovered (Smoot et al. [1977]) that there is an observed motion of our galaxy relative to the cosmic microwave background, which has stimulated our present interest in attempting to model this observed feature of the universe. That is, we shall seek a model in which there are two cosmological fluids, the first a comoving fluid representing the background radiation field and the second a matter fluid constituting the observed material content of the universe which is non-comoving so that there is a relative motion between the two fluids.

It is observed that both the cosmic microwave background and the observed matter are approximately isotropic and homogeneous. Consequently, we wish to study models in which isotropy and homogeneity is preserved, namely FRW models. In particular we shall consider zero curvature FRW models here, for which the metric is given by

$$ds^2 = -c^2dt^2 + R^2(t)\,(dx^2 + dy^2 + dz^2)\ , \tag{1}$$

in "axial" coordinates, where t is the cosmic time and R the expansion factor. Our assumed model is therefore an FRW model in which there are two fluids present which act as the source for the gravitational field (as represented by (1) ), a comoving radiation field and a tilting matter field. We note that this is only possible if the matter field is represented by an imperfect fluid (Coley and Tupper [1983], [1984], and [1986] ).

It has been established that FRW models can be interpreted as solutions of Einstein's equations for a variety of different sources (Coley and Tupper [1983] and [1984] ). From the comments above we can see how the study of FRW models due to increasingly more sophisticated matter distributions is associated with progressive discoveries concerning the nature of the universe.

## Preliminaries

Henceforward we shall be seeking a two fluid FRW model of the following description. We shall assume that the first fluid is a comoving, perfect fluid with the (black body) radiative equations of state

$$\rho_r = aT_r^4,\quad \frac{p_r}{c^2} = \frac{1}{3}\rho_r \tag{2}$$

where $\rho_r$, $p_r$ and $T_r$ are the density, pressure and temperature of the radiation and $a = 7.57 \times 10^{-15}$ erg cm$^{-3}$deg$^{-4}$ is Stefan's constant. The comoving 4-velocity of the radiation is represented by $v_i = (-c,0,0,0)$. This fluid will model the observed cosmic microwave background. The second fluid will be taken to be a non-comoving imperfect fluid with density $\rho_m$, pressure $p_m$, temperature T, 4-velocity $u^i$, shear tensor $\sigma^{ij}$, shear viscosity coefficient $\eta$ and heat conduction vector $q^i$. This second fluid will model the observed matter in the universe.

In this configuration Einstein's equations take on the form

$$\frac{c^4}{8\pi G}G_{ij} = \frac{\rho_r}{3}(4v_iv_j + c^2g_{ij}) + (\rho_m + \frac{1}{c^2}p_m)\,u_iu_j + p_mg_{ij} - 2\eta\sigma_{ij} + q_iu_j + q_ju_i. \tag{3}$$

The matter is assumed to be moving axially relative to the comoving radiation, thus modelling the observed relative velocity between the centre of our galaxy and the cosmic microwave background. Consequently, we shall take $u_i$ to be of the form

$$u_i = (-\alpha, 0, 0, \beta R) \tag{4}$$

where $\alpha^2 - \beta^2 = c^2$, and $\alpha$ and $\beta$ are assumed to be functions of t alone. We also assume that

$$q_i = \frac{Q}{c}(\beta, 0, 0, -\alpha R) \tag{5}$$

so that $q_i u^i = 0$ and $Q^2 \equiv q_i q^i$.

Using Eqns. (1), (3) and (4), Eqns. (2) become

$$\rho_m = \frac{1}{c^2}\left\{\frac{1}{8\pi G}\left[\frac{\dot{R}^2}{R^2}(3\alpha^2 - \beta^2) - \frac{2\beta^2 \ddot{R}}{R}\right] - \frac{\rho_r}{3}(3\alpha^2 + \beta^2)\right\} \tag{6}$$

$$3p_m = \frac{1}{8\pi G}\left\{\frac{\dot{R}^2}{R^2}(5\beta^2 - 3\alpha^2) - \frac{2\ddot{R}}{R}(3\alpha^2 - 2\beta^2)\right\} - \frac{\rho_r}{3}(3\alpha^2 + \beta^2) \tag{7}$$

$$\frac{\eta\dot{\alpha}}{c^3} = \frac{-\beta^2}{2c^2}\left\{\frac{1}{8\pi G}\left[\frac{2\dot{R}^2}{R^2} - \frac{2\ddot{R}}{R}\right] - \frac{4}{3}\rho_r\right\} \tag{8}$$

$$Q = \frac{1}{c}\left\{\frac{1}{8\pi G}\left[\frac{2\dot{R}^2}{R^2} - \frac{2\ddot{R}}{R}\right] - \frac{4}{3}\rho_r\right\}\alpha\beta \tag{9}$$

where a dot denotes differentiation with respect to t. The right-hand sides of Eqns. (6) and (7) are positive which always ensures that the terms in braces on the right-hand sides of Eqns. (8) and (9) are positive, so that Q is the same sign as $\beta$ and $\eta$ is non-negative if and only if

$$\dot{\alpha} \leq 0. \tag{10}$$

We shall not take $T_r$ and T to be equal in the model. This means that the two fluids will not be in thermodynamical equilibrium throughout the history of the universe. This is precisely what is to be expected if an imperfect fluid with non-zero heat conduction vector is present. We note that it is presently believed that the current temperature of the "matter" in the universe is about four times higher than that of the cosmic microwave background.

It will be assumed that the physical quantities associated with the imperfect fluid satisfy the following set of thermodynamical laws.

(i) Baryon conservation law: $(nu^\mu)_{;\mu} = 0$. This equation can be integrated so that $n = n_0 R^{-3}\alpha^{-1}$. n is the particle density and $n_0$ a constant.

(ii) Gibbs' relation: $T\,d(Sn^{-1}) = d(\rho_m n^{-1}) + p_m d(n^{-1})$. We note that if the "integrating factor" T is a function of t alone, a solution of the Gibbs' relation, which can now be thought of as an equation to determine the entropy density S, is guaranteed.

(iii) Temperature gradient law: $q^\mu = -\kappa c^{-6}(g^{\mu\nu} + u^\mu u^\nu)(c^2 T_{,\nu} + T u_{\nu;\pi} u^\pi)$. We note that the condition that the thermal conductivity $\kappa$ is non-negative ensures that entropy production is also non-negative. In the situation under investigation we find that the temperature gradient law reduces to a single equation, which can be used to determine $\kappa$. It can be shown that the condition for $\kappa \geq 0$ then reduces to

$$\frac{\dot{T}}{T} + \frac{\dot{\beta}}{\beta} + \frac{\dot{R}}{R} \geq 0. \tag{11}$$

We note that although the two fluids in the model are not in thermal equilibrium, deviations from thermodynamical equilibrium are not so large that the applicability of the assumed laws of thermodynamics is brought into question.

In addition, we demand that the energy conditions $\rho_r > 0, \rho_m > 0, \rho_m - c^{-2} p_m \geq 0$ and $p_m \geq 0$ are all satisfied in the model. We also expect T to be a decreasing function of t. Finally, we impose the following "boundary conditions" on the model:

$$\begin{aligned} \alpha &\to c \quad \text{as} \quad t \to \infty \\ \alpha &\to \infty \quad \text{as} \quad t \to 0 \end{aligned} \tag{12}$$

## The Model

In our model we shall take R(t) to be of the form

$$R(t) = t^{1/2} (1 + \ell t^{3/5})^{5/18}, \tag{13}$$

where $\ell$ is a positive constant such that $\ell t^{3/5}$ is dimensionless. With this choice of R(t), which is a monotonically increasing function of time, we have that $R(t) \to t^{1/2}$ as $t \to 0$, so that initially the universe was in a pure radiation state (initially even the matter field was in a radiation-like state), and $R(t) \to t^{2/3}$ as $t \to \infty$, so that the model evolves towards the Einstein de Sitter universe (in which the matter field will completely dominate). We remark that the specification of R(t) above (and $\alpha$ and T below) is effectively equivalent to specifying the equations of state for the fluid; for example a relationship results between $\rho_m$ and $p_m$ (which we note is allowed to vary with time). The question of whether such a "choice" of equations of state is physically acceptable is answered when the solution is obtained and all the physical quantities appearing in the resulting model are found to be suitably well behaved. Finally, from $8\pi G R^2 \rho_r \equiv -3(2\ddot{R}R + \dot{R}^2)$ we find that

$$\rho_r = \frac{3}{32\pi G} (t^{-2}) (1 + \ell t^{3/5})^{-2} (1 + \frac{8}{15} \ell t^{3/5}). \tag{14}$$

We assume that $\alpha$ and $\beta$ are of the form

$$\frac{\alpha}{c} = \frac{1 + ht^{-q}}{(1 + 2ht^{-q})^{1/2}}, \quad \frac{\beta}{c} = \frac{ht^{-q}}{(1 + 2ht^{-q})^{1/2}}, \tag{15}$$

where h and q are positive constants. With this choice of $\alpha$ we note that conditions (12) are satisfied, and a brief calculation yields $\dot{\alpha} < 0$ guaranteeing $\eta > 0$ for all t (from Eqn. (10)). We also assume T is of the form

$$T = T_0 t^{-b} R^{-p} \alpha^s, \tag{16}$$

where b, p and s are positive constants.

Condition (11), ensuring $\kappa$ is non-negative, takes on the form of an algebraic inequality involving t and the arbitrary parameters h and $\ell$ and b, p, q and s. It can be shown that this inequality will be satisfied for all t, and all h and $\ell$, if either of the following two sets of inequalities are satisfied:

$$s \leq 1 \quad \text{and} \quad \frac{1}{2} - \frac{p}{2} - b - q \geq 0 \tag{17a}$$

$$s > 1 \quad \text{and} \quad 1 - p - 2b - q - sq \geq 0. \tag{17b}$$

There will be various solutions depending on the desired behaviour of physical quantities (such as T) as $t \to 0$ or $t \to \infty$. Here, we shall make the following assumptions. First, since $R(t) \to t^{1/2}$ as $t \to 0$, we shall assume that $\rho_m^{-1} T^4 \to$ constant as $t \to 0$, which implies that $7/5 + q - 4b - 2p - 2qs = 0$. Second, we shall assume that as $t \to \infty$, $\rho_m^{-1} T^a \to$ constant where $a \leq 4$, which implies that $b/2 + p/3 \geq 1/4$. Finally, for simplicity we shall assume that $b = 0$ and $s \leq 1$ so that the conditions to be satisfied become:

$$p \geq \frac{3}{4}, \quad \frac{7}{5} + q - 2p - 2qs = 0, \quad 1 - p - 2q \geq 0. \tag{18}$$

From these conditions we find that $p \leq 19/25$. This suggests two straightforward solutions in which $p = 3/4$ or $p = 19/25$. Let us concentrate on the model in which $p = 19/25$; a complete solution of (18) is then

$$p = \frac{19}{25}, q = \frac{3}{25}, \quad s = 0, \quad b = 0. \tag{19}$$

In this case $T = T_0 R^{-19/25}$ and $\rho_m \sim T^{75/19}$ for large t. The end conditions for this model have been described above. Using Eqns. (6), (7), (8), (9) and the aforementioned laws of thermodynamics the full solution (representing the model) is given by

$$\begin{aligned}
\rho_m &= \frac{\ell}{5\pi G}(1+\ell t^{3/5})^{-2}\left(1+\frac{5}{6}\ell t^{3/5}\right)(1+2ht^{-3/25})^{-1}(1+ht^{-3/25})^2 t^{-7/5} \\
\frac{3p_m}{c^2} &= \frac{\ell h^2}{5\pi G}(1+\ell t^{3/5})^{-2}\left(1+\frac{5}{6}\ell t^{3/5}\right)(1+2ht^{-3/25})^{-1} t^{-41/25} \\
\eta &= \frac{5c^2\ell}{6\pi G}(1+\ell t^{3/5})^{-2}\left(1+\frac{5}{6}\ell t^{3/5}\right)(1+2ht^{-3/25})^{1/2} t^{-2/5} \\
\frac{Q}{c} &= \frac{\ell h}{5\pi G}(1+\ell t^{3/5})^{-2}\left(1+\frac{5}{6}\ell t^{3/5}\right)(1+2ht^{-3/25})^{-1}(1+ht^{-3/25}) t^{-38/25} \\
n &= \frac{n_0}{c}(1+\ell t^{3/5})^{-5/6}(1+2ht^{-3/25})^{1/2}(1+ht^{-3/25}) t^{-3/2} \\
\kappa &= \frac{5c^2}{T_0\pi G}(1+\ell t^{3/5})^{-71/90}\left(1+\frac{5}{6}\ell t^{3/5}\right)(1+2ht^{-3/25})^{1/2}(1+ht^{-3/25}) t^{-31/50} \\
&\quad \times \left[1+\frac{3h}{\ell}t^{-18/25}+5ht^{-3/25}\right]^{-1}.
\end{aligned} \tag{20}$$

We note that in this model $\eta$ and $\kappa$ are always positive. In addition, $\rho_m$ and $p_m$ are always positive and monotonically decreasing, and $3p_m/c^2\rho_m \to 1$ as $t \to 0$ and $3p_m/c^2\rho_m \to 0$ as $t \to \infty$. The model is physically acceptable from a theoretical point of view since Einstein's equations and the laws of thermodynamics are satisfied, and all physical quantities appearing in the model are suitably well-behaved. The observational predictions of this model will be discussed below.

## Observations

We let the subscript zero denote the present time. All numerical values will be calculated to three significant places only. We shall assume that the value of the arbitrary positive constant $\ell$ is given by $\ell = 1.06 \times 10^{-7}$ (see Coley [1985] ). Based upon a Hubble parameter $H_0 = 55$ km sec$^{-1}$ Mpc$^{-1}$ we find that $t_0^* \equiv H_0^{-1} = 5.67 \times 10^{17}$ secs, so that from the definition of $H_0$ and Eqn. (13) we find that $t_0 = 3.78 \times 10^{17}$ secs (the age of the universe). We note that $\ell t_0^{3/5} = 3.73 \times 10^3$.

We shall assume that the present velocity of our galaxy relative to the cosmic microwave background is three hundred kilometers per second (Smoot et al. [1977]), so that from Eqns. (15) we find that $ht_0^{-3/25} = 1.00 \times 10^{-3}$, which fixes h as $h = 1.29 \times 10^{-1}$.

From Eqn. (20) we find that $\rho_{m,0} = 5.57 \times 10^{-30}$ g cm$^{-3}$. From Eqn. (14) we find that $\rho_{r,0} = 4.47 \times 10^{-34}$ g cm$^{-3}$ and hence from $\rho_r = aT_r^4$ we find that $T_{r,0} = 2.70°$K. In addition, Eqn. (20) yields $c^{-2}\, p_{m,0} = 1.86 \times 10^{-36}$ g cm$^{-3}$, and finally we obtain $p_{m,0}/c^2\rho_{m,0} = 3.33 \times 10^{-7}$.

Let $t_e$ be the time when $\rho_r = \rho_m$. From Eqns. (14) and (20) we then obtain a quadratic equation in $\ell t_e^{3/5}$. Taking the positive root, and using the established valued of $\ell$, we find that $t_e = 1.01 \times 10^{11}$ secs. The rate of energy transfer per unit volume from radiation to matter $E_m$ (as defined by Davidson [1962]) is found to be

$$E_m = \frac{c^2\ell}{50\pi G}\,(t^{-12/5})\,(1 + \ell t^{3/5})^{-3}\,(1 - \frac{1}{6}\,\ell t^{3/5}). \tag{21}$$

Since $E_m \neq 0$ in this model, there will be interaction (that is, energy transfer) between the two fluids. For small t, $E_m$ is positive (as desired). For larger values of t, such as at present, it is believed that there is a conversion or net rate of gain of energy per unit volume from radiation to matter (ie., $E_{m,0} < 0$) due to the nuclear burning of stars in galaxies. In our model, from Eqn. (21) we find that $E_{m,0} = -7.16 \times 10^{-32}$ erg cm$^{-3}$ sec$^{-1}$.

Comparing the above with current numerical estimates in the literature (Coley and Tupper [1986]) we see that the predictions of the model are in excellent agreement with actual observations. The model is consequently a physically acceptable cosmological model. Moreover, our model is also able to predict a relative velocity of the galaxy with respect to the cosmic microwave background of 300 km sec$^{-1}$. We conclude by remarking that through the model described above we have achieved our goal of modelling the observed motion of our galaxy relative to the cosmic microwave background.

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